MARK BARNES | STEVEN MORRIS

JACARANDA MATHS QUEST
GENERAL MATHEMATICS 11
FOR QUEENSLAND

UNITS 1&2

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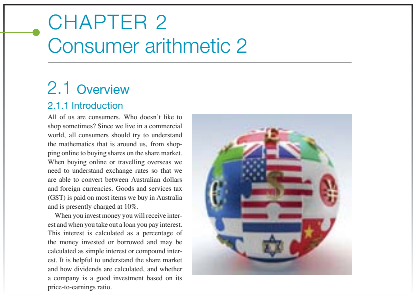
ABOUT THIS RESOURCE

Jacaranda Maths Quest 11 General Mathematics Units 1 & 2 for Queensland is expertly tailored to address comprehensively the intent and structure of the new syllabus. The *Jacaranda Maths Quest for Queensland* series provides easy-to-follow text and is supported by a bank of resources for both teachers and students. At Jacaranda we believe that every student should experience success and build confidence, while those who want to be challenged are supported as they progress to more difficult concepts and questions.

Preparing students for exam success

Chapter openers place mathematics in real-world contexts to drive engagement.

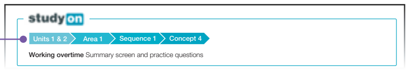


CHAPTER 2
Consumer arithmetic 2

2.1 Overview

2.1.1 Introduction

FREE access to studyON — our study, revision and exam practice tool — is included with every title. studyON allows you to revise at the concept, chapter, curriculum topic or unit level.

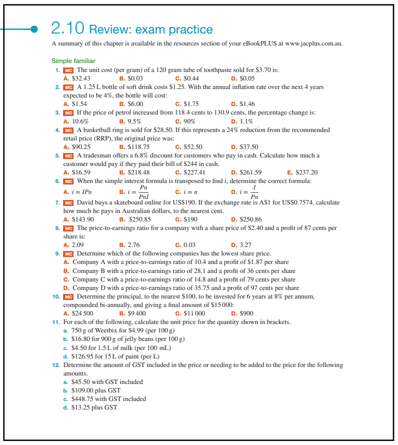


studyON

Units 1 & 2 › Area 1 › Sequence 1 › Concept 4

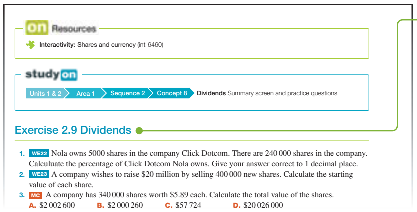
Working overtime Summary screen and practice questions

Every chapter concludes with exam practice questions classified as Simple familiar, Complex familiar and Complex unfamiliar.



2.10 Review: exam practice

Each subtopic concludes with carefully graded questions.

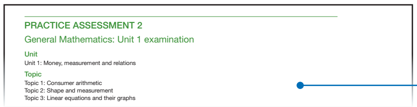


Resources

studyON

Exercise 2.9 Dividends

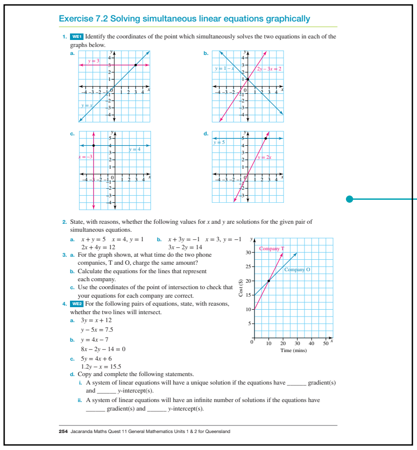
Two complete sets of practice assessments modelled on QCAA guidelines — a set for student revision and a quarantined set for teachers — are included. Exemplary responses and worked solutions are provided for teachers.



PRACTICE ASSESSMENT 2

General Mathematics: Unit 1 examination

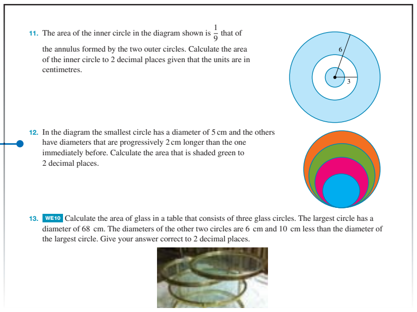
Chapter questions and activities are aligned with Marzano and Kenall's taxonomy of cognitive process — retrieval, comprehension, analysis and knowledge utilisation.



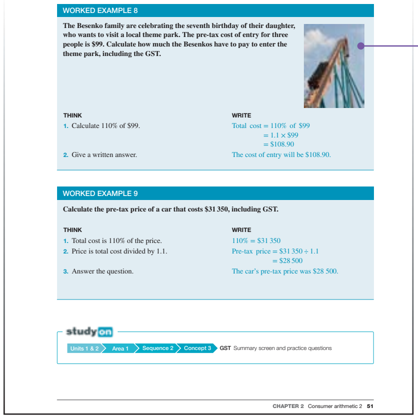
Exercise 7.2 Solving simultaneous linear equations graphically

Features of the *Maths Quest* series

Questions and topics are sequenced from lower to higher levels of complexity; ideas and concepts are logically developed and questions are carefully graded, allowing every student to achieve success.



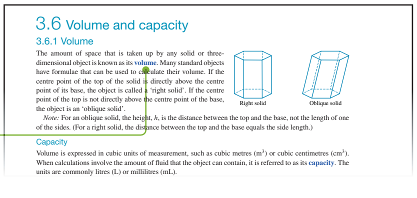
Fully worked examples in the Think/Write format provide guidance and are linked to questions.



WORKED EXAMPLE 8

WORKED EXAMPLE 9

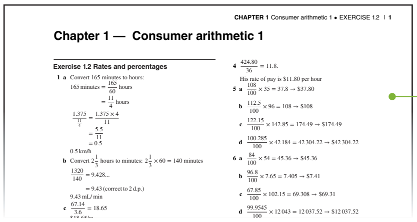
An extensive glossary of mathematical terms is provided in print and as a hover-over feature in the eBookPLUS.



3.6 Volume and capacity

3.6.1 Volume

Free fully worked solutions are provided, enabling students to get help where they need it, whether at home or in the classroom — help at the point of learning is critical. Answers are provided at the end of each chapter in the print and offline PDF.



Chapter 1 — Consumer arithmetic 1

eBookPLUS features

Fully worked solutions for every question

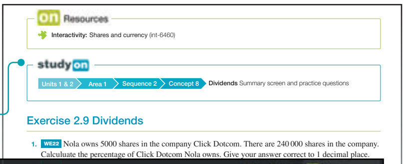
Digital documents: downloadable SkillSHEETS to support skill development and SpreadSHEETS to explore mathematical relationships and concepts

Chapter summaries in downloadable format to assist in study and exam preparation

A downloadable PDF of the entire chapter of the print text

Interactivities and video eLessons placed at the point of learning to enhance understanding and correct common misconceptions

Concept summary links to studyON for study, revision and exam practice

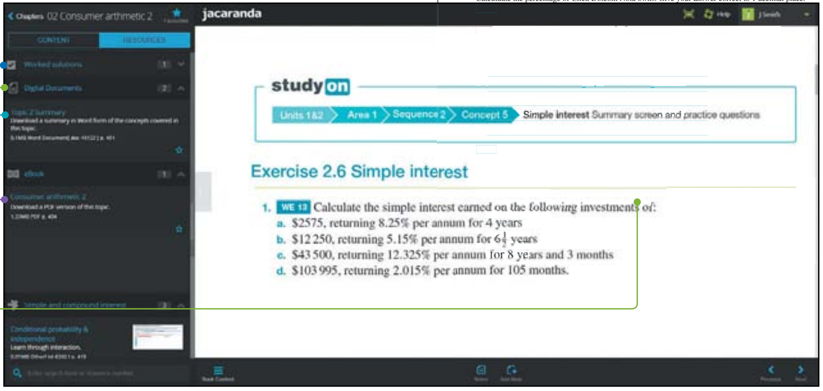


Interactivity: Shares and currency (int-6460)

Dividends Summary screen and practice questions

Exercise 2.9 Dividends

1. WE22 Nola owns 5000 shares in the company Click Dotcom. There are 240 000 shares in the company. Calculuate the percentage of Click Dotcom Nola owns. Give your answer correct to 1 decimal place.



Simple interest Summary screen and practice questions

Exercise 2.6 Simple interest

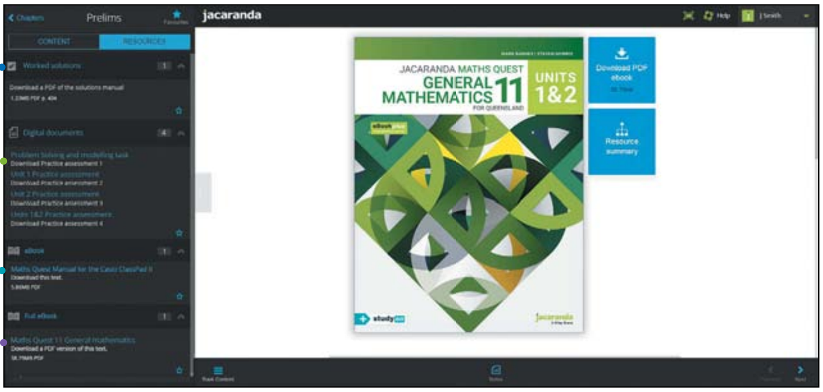
1. WE 13 Calculate the simple interest earned on the following investments of:
   a. \$2575, returning 8.25% per annum for 4 years
   b. \$12 250, returning 5.15% per annum for $6\frac{1}{2}$ years
   c. \$43 500, returning 12.325% per annum for 8 years and 3 months
   d. \$103 995, returning 2.015% per annum for 105 months.

In the Prelims section of your eBooKPLUS

A downloadable PDF of the entire solutions manual, containing worked solutions for every question in the text

A set of four practice assessments: a problem solving and modelling task and three examination-style assessments

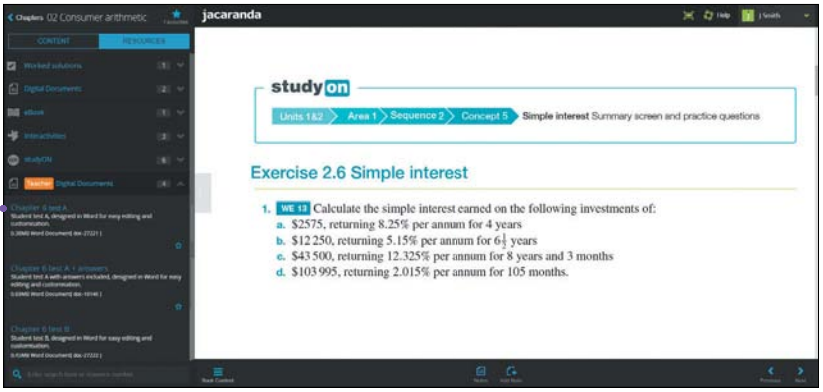
FREE copies of the *Maths Quest Manual for the TI-Nspire CAS calculator* and the *Maths Quest Manual for the Casio Classpad II calculator*



A downloadable PDF of the entire print text

Additional resources for teachers available in the eGuidePLUS

In the Resources tab of every chapter there are two chapter tests in downloadable, customisable Word format with worked solutions.



Simple interest Summary screen and practice questions

Exercise 2.6 Simple interest

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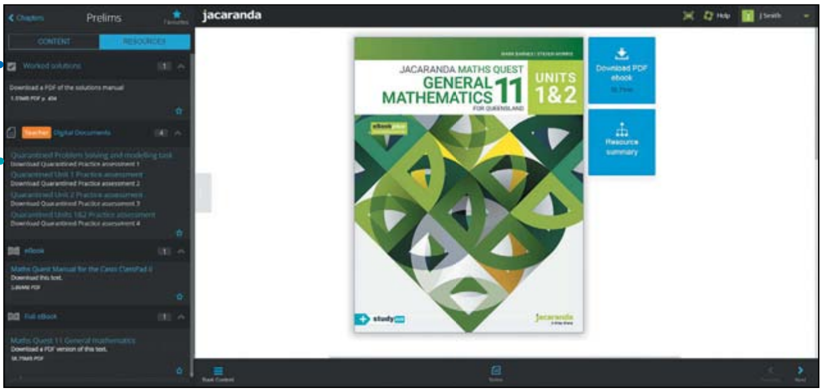
   c. \$43 500, returning 12.325% per annum for 8 years and 3 months
   d. \$103 995, returning 2.015% per annum for 105 months.

In the Prelims section of the eGuidePLUS

Work programs are provided to assist with classroom planning.

Practice assessments: in addition to the four provided in the eBookPLUS, teachers have access to a further four quarantined assessments. Modelled on QCAA guidelines, the problem solving and modelling tasks are provided with exemplary responses while the examination-style assessments include annotated worked solutions. They are downloadable in Word format to allow teachers to customise as they need.



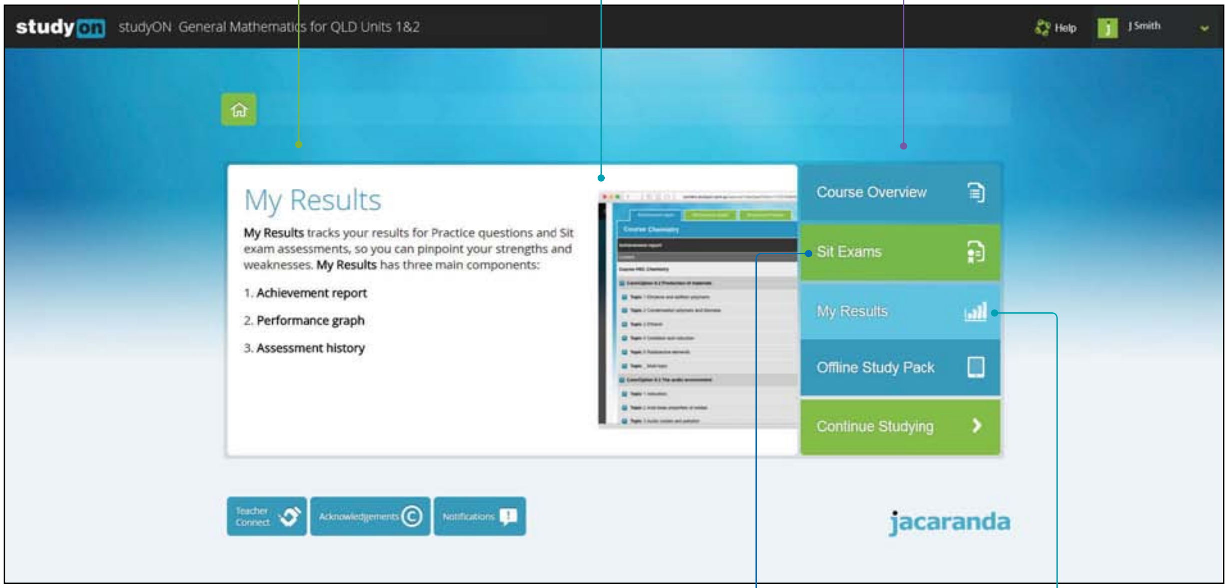
studyON — an invaluable exam preparation tool

studyON provides a complete study solution. An interactive and highly visual online study, revision and exam practice tool, it is designed to help students and teachers maximise exam results.

Concept summary screens and interactivities summarise key concepts and help prevent misconceptions.

Direct links from the eBookPLUS help scaffold students' understanding and study practices.

The studyON question hierarchy allows students in the *Continue Studying* feature to revise across the entire course, or to drill down to concept level for a more granular set of questions.



My Results

My Results tracks your results for Practice questions and Sit exam assessments, so you can pinpoint your strengths and weaknesses. **My Results** has three main components:

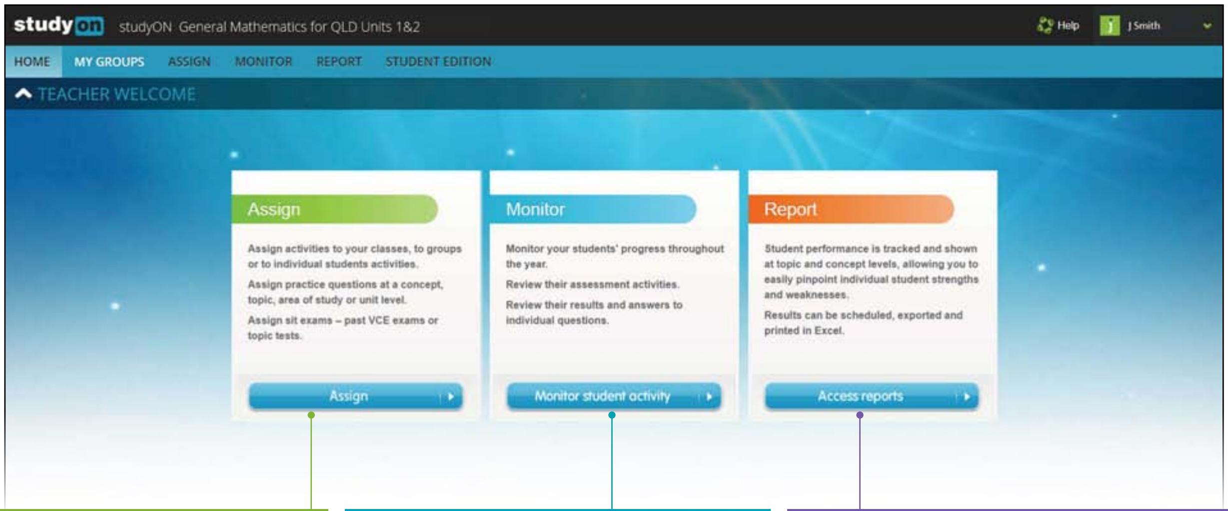
1. Achievement report
2. Performance graph
3. Assessment history

studyON prepares students for actual exams:

- The *Sit Exams* feature allows students to sit timed practice exams.
- Exam-style questions have been authored by our team of highly qualified teachers.
- From 2020, official past QCAA exam questions will be available for Units 3 & 4 with exemplary worked solutions to provide feedback for every question.

studyON's built-in progress tracker enables self-diagnosis of strengths and weaknesses at a topic and concept level, so students know exactly what needs extra revision and can sit their exams with confidence.

studyON Teacher edition is a powerful diagnostic tool



Assign activities to your classes, to groups or to individual students activities.

Assign practice questions at a concept, topic, area of study or unit level.

Assign sit exams – past VCE exams or topic tests.

Monitor your students' progress throughout the year.

Review their assessment activities.

Review their results and answers to individual questions.

Student performance is tracked and shown at topic and concept levels, allowing you to easily pinpoint individual student strengths and weaknesses.

Results can be scheduled, exported and printed in Excel.

Enables teachers to assign activities for extra revision and practice, and track progress at an individual, group and classroom level

Allows teachers to monitor students' activities and results to pinpoint strengths and weaknesses. Armed with evidence-based insights, teachers can intervene at the right time.

Alignment with the Jacaranda text helps with planning, and instant feedback saves marking time. Built-in reporting functionality lets teachers easily schedule, export and print reports in Excel.

About eBookPLUS and studyON

Access your online Jacaranda resources anywhere, anytime, from any device in three easy steps:

STEP 1 Go to **www.jacplus.com.au** and create a user account.
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eBookPLUS is an electronic version of the textbook, together with a targeted range of supporting multimedia resources.

eBookPLUS features:

- **eBook** — the entire textbook in electronic format
- **Digital documents** designed for easy customisation and editing
- **Interactivities** to reinforce and enhance students' learning
- **eLessons** — engaging video clips and supporting material
- **Weblinks** to relevant support material on the internet

eGuidePLUS features assessment and curriculum material to support teachers.

studyON is an interactive and highly visual online study, revision and exam practice tool designed to help students and teachers maximise exam results.

studyON features:

- **Concept summary screens** provide concise explanations of key concepts, with relevant examples.
- **Access exam questions** that have been written by experienced examiners for practice at a concept, area or entire course level, and receive immediate feedback. **From 2020, QCAA questions** will be included with exemplary worked solutions.
- **Sit past QCAA exams** (Units 3 & 4) or **topic tests** (Units 1 & 2) in exam-like situations.
- **Video animations and interactivities** demonstrate concepts to provide a deep understanding (Units 3 & 4 only).
- **All results and performance in practice and sit questions** are tracked to a concept level to pinpoint strengths and weaknesses.

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- **Contact** John Wiley & Sons Australia, Ltd.
  Email: support@jacplus.com.au
  Phone: 1800 JAC PLUS (1800 522 7587)

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JacarandaPLUS requires you to use a supported internet browser and version, otherwise you will not be able to access your resources or view all features and upgrades. Please view the complete list of JacPLUS minimum system requirements at **http://jacplus.desk.com**.

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CHAPTER 1
Consumer arithmetic 1

1.1 Overview

1.1.1 Introduction

Mathematics is an essential part of our daily lives. Everyone needs to have an understanding of consumer mathematics and its use in real-life applications. Accountants, financial planners, book keepers and managers all use consumer arithmetic in their jobs. We all need to know how to check wages and salaries, and how to calculate overtime payments, based on time-and-a-half or double time. When shopping we use percentages to calculate the prices of sale items and percentage increase and decrease. Many employees, such as salespeople, factory workers and fruit pickers, are paid a commission or by piecework for doing their jobs. Piecework means that workers are paid for the amount of work that they have completed rather than an hourly or weekly rate. Government allowances and pensions are also an important form of income for many people.

LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

1.2 Rates and percentages

1.2.1 Definition of rates

Rates and percentages are used to compare how different quantities change. Rates compare quantities of different units of measurement and percentages represent a portion out of 100.

A **rate** is a measure of change between two variables of different units. A rate is calculated per unit or per item. Common examples of rates include speed in kilometres per hour (km/h) or metres per second (m/s), costs and charges in dollars per hour ($/h), and electricity usage in kilowatts per hour (kW/h).

WORKED EXAMPLE 1

Aditi jogs 4.3 kilometres in 24 minutes. What rate is she running in kilometres per hour (km/h)?

THINK	WRITE
1. The units are kilometres and minutes.	4.3 kilometres, 24 minutes
2. Convert minutes to hours.	$\frac{24\ \text{minutes}}{60\ \text{minutes}} = 0.4\ \text{hours}$
3. Write the rate as a fraction in terms of kilometres and hours.	$\frac{4.3\ \text{km}}{0.4\ \text{hrs}} = 10.75$
4. Answer the question including the units.	Aditi is running at 10.75 km/h

WORKED EXAMPLE 2

At what rate (in km/h) are you moving if you are on a bus that travels 11.5 km in 12 minutes?

THINK	WRITE
1. Identify the two measurements: distance and time. As speed is commonly expressed in km/h, convert the time quantity units from minutes to hours.	The quantities are 11.5 km and 12 minutes. $\frac{12}{60} = \frac{1}{5}$ or 0.2 hours
2. Write the rate as a fraction and express in terms of kilometres and hours.	$\frac{11.5\ \text{km}}{0.2\ \text{hrs}} = 57.5$
3. State the final answer including the units.	You are travelling at 57.5 km/h

WORKED EXAMPLE 3

James works as a barista at the local café and is paid $99 for 6 hours. Calculate his rate of pay per hour.

THINK	WRITE
1. James receives $99 for 6 hours.	6 hours = $99
2. Calculate the amount for 1 hour.	$1 \text{ hour} = \frac{99}{6}$ 1 hour = 16.5
3. Answer the question.	James is paid $16.50 per hour

1.2.2 Percentages

Per cent means *per hundred*. A **percentage** means ‘out of 100’ and can be written as a fraction or a decimal. Percentages are used in many everyday applications including sales, commission, profit and loss and many other situations.

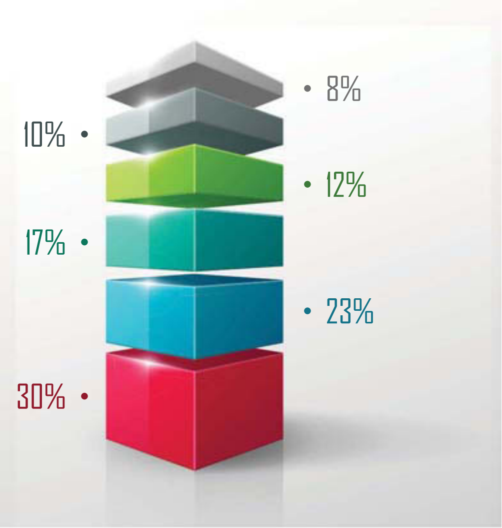
Consider the situation of reducing the price of an item by 18% when it would normally sell for $500. The reduced selling price can be found by evaluating the amount of the reduction and then subtracting it from the original value as shown in the following calculations:

Reduction of 18%: $\frac{18}{100} \times 500 = \$90$

Reduced selling price: $500 − $90 = $410

The selling price can also be obtained with a one-step calculation of $\frac{82}{100} \times 500 = \$410$.

In other words, reducing the price by 18% is the same as multiplying by 82% or (100 − 18) %.



To reduce something by x%, multiply by $(100 - x)$%.
To increase something by x%, multiply by $(100 + x)$%.

WORKED EXAMPLE 4

Calculate an increase to $76 by 15%

THINK	WRITE
1. The original percentage is always 100% so an increase of 15% means a total of 115%.	(100 + 15) % = 115%
2. Write 115% as a decimal.	$115\% = \frac{115}{100}$ 115% = 1.15
3. Multiply the amount $76 by 1.15.	76 × 1.15 = 87.40
4. Answer the question.	$87.40

WORKED EXAMPLE 5

Sarah bought a car for \$7500 and sold it 4 years later at a discount of 30%. Calculate how much she sold the car for.

THINK	WRITE
1. The original percentage is always 100% so a discount of 30% means a total of 70%.	$(100 - 30)\% = 70\%$
2. Write 70% as a decimal.	$70\% = \frac{70}{100}$ $70\% = 0.70$ or 0.7
3. Multiply the amount \$7500 by 0.7.	$7500 \times 0.7 = 5250$
4. Answer the question.	Sarah sold it for \$5250

1.2.3 Percentage increase and decrease

Percentage increase and decrease can be used to calculate sale prices, discounts, profits and many other quantities. It is calculated as a percentage of the original amount.

The formula for calculating the percentage increase/decrease is:

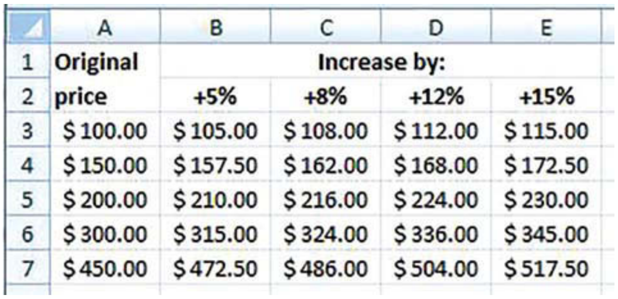
$$\textbf{Percentage increase/decrease} = \frac{\textbf{amount of increase/decrease}}{\textbf{original amount}} \times \mathbf{100}$$

WORKED EXAMPLE 6

Ramon bought a laptop in a sale for \$774.40. If the original price was \$968, calculate the percentage discount.

THINK	WRITE
1. Calculate the amount of the discount by subtracting \$774.40 from \$968.	Discount $= \$968 - \$774.40$ $= \$193.60$
2. Calculate the percentage discount by using the formula: Percentage discount $= \frac{\text{Amount of discount}}{\text{Original amount}} \times 100$.	Percentage discount $= \frac{193.60}{968} \times 100$ $= 20$
3. Write the answer with a percentage symbol.	20%
4. Answer the question.	The percentage discount is 20%.

When a large number of values are being considered in a problem involving percentages, spreadsheets or other technologies can be useful to help carry out most of the associated calculations. For example, a spreadsheet can be set up so that entering the original price of an item will automatically calculate several different percentage increases for comparison.



	A	B	C	D	E
1	Original		Increase by:		
2	price	+5%	+8%	+12%	+15%
3	\$ 100.00	\$ 105.00	\$ 108.00	\$ 112.00	\$ 115.00
4	\$ 150.00	\$ 157.50	\$ 162.00	\$ 168.00	\$ 172.50
5	\$ 200.00	\$ 210.00	\$ 216.00	\$ 224.00	\$ 230.00
6	\$ 300.00	\$ 315.00	\$ 324.00	\$ 336.00	\$ 345.00
7	\$ 450.00	\$ 472.50	\$ 486.00	\$ 504.00	\$ 517.50

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Units 1 & 2 > Area 1 > Sequence 1 > Concepts 1 & 2

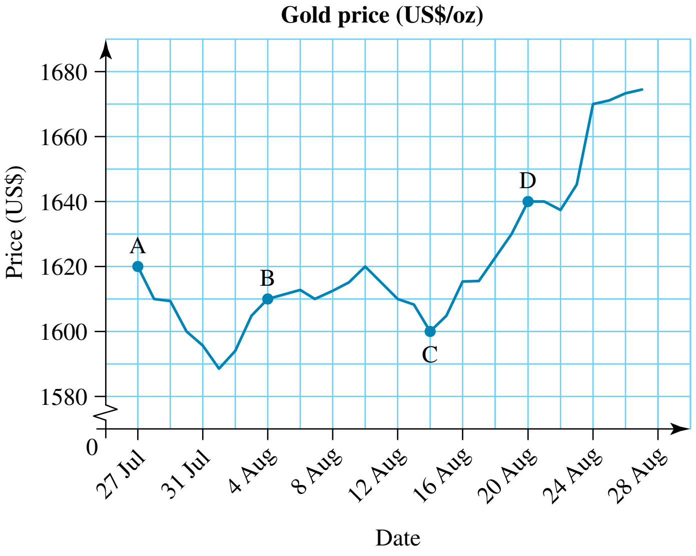
Rates and percentages Summary screen and practice questions
Percentage increase and decrease Summary screen and practice questions

Exercise 1.2 Rates and percentages

1. **WE1** Using the units stated, calculate the rates for:
   - a. a yacht that travels 1.375 km in 165 minutes expressed in km/h
   - b. a tank that loses 1320 mL of water in $2\frac{1}{3}$ hours expressed in mL/ min
   - c. a 3.6-metre-long carpet that costs \$67.14 expressed in \$/m
   - d. a basketball player who has scored a total of 833 points in 68 games expressed in points/game.
2. Calculate the following rates when the units are changed as indicated. Where necessary, give answers correct to 2 decimal places.
   - a. 1.5 m/s to km/h
   - b. 60 km/h to m/s
   - c. 65 cents per gram to \$/kg
   - d. \$5.65 per kilogram to cents per gram
3. **WE2** Calculate the rate (in km/h) that you are moving if you are in a passenger aircraft that travels 1770 km in 100 minutes.

4. **WE3** Michael works as an apprentice chef and is paid \$424.80 for 36 hours. Determine his rate of pay per hour.
5. **WE4** Calculate the following increase:
   - a. \$35 by 8%
   - b. \$96 by 12.5%
   - c. \$142.85 by 22.15%
   - d. \$42184 by 0.285%.
6. Calculate the following decrease:
   - a. \$54 by 16%
   - b. \$7.65 by 3.2%
   - c. \$102.15 by 32.15%
   - d. \$12043 by 0.0455%.
7. **WE5** A clothing shop is closing down and offers a discount of 25% on all items. Calculate how much Jade would pay for a pair of jeans that were originally \$80.
8. The price of a bottle of wine was originally \$19.95. After it received an award for wine of the year, the price was increased by 12.25%. Twelve months later the price was reduced by 15.5%.
   - a. Calculate the final price of a bottle of this wine.
   - b. Calculate the percentage change of the final price from the original price.
9. **WE6** Determine the percentage discount if a piece of silverware has a price tag of \$168 at a market, but the seller is bartered down and sells it for \$147.
10. a. An advertisement for bedroom furniture states that you save \$55 off the recommended retail price when you buy it for \$385. Calculate by what percentage the price has been reduced.
    - b. If another store was advertising the same furniture for 5% less than the sale price of the first store, calculate the percentage the price has been reduced from the recommended retail price.

11. The following graph shows the change in the price of gold (in US dollars per ounce) from 27 July to 27 August.



a. Calculate the percentage change from:

i. the point marked A to the point marked B

ii. the point marked C to the point marked D.

b. Calculate the percentage change from the point marked A to the point marked D.

12. A student's test results in Mathematics are shown in the table.

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8
Mark	$\frac{16}{20}$	$\frac{14}{21}$	$\frac{26}{34}$	$\frac{36}{45}$	$\frac{14.5}{20}$	$\frac{13}{39}$	$\frac{42}{60}$	$\frac{26}{35}$
Percentage								

a. Complete the table by calculating the percentage for each test, giving values correct to 2 decimal places where necessary.

b. Determine the student's overall result from all eight tests as a percentage correct to 2 decimal places.

13. A house originally purchased for \$320 000 is sold to a new buyer at a later date for \$377 600.

a. Calculate the percentage change in the value of the house over this time period.

b. The new buyer pays a deposit of 15% and borrows the rest from a bank. They are required to pay the bank 5% of the total amount borrowed each year. If they purchased the house as an investment, calculate how much they should charge in rent per month in order to fully cover their bank payments.

14. The ladder for the top four teams in the A-League is shown in the following table:

Team	Win	Loss	Draw	Goals for	Goals against
1. Western Sydney Wanderers	18	6	3	41	21
2. Central Coast Mariners	16	5	6	48	22
3. Melbourne Victory	13	9	5	48	45
4. Adelaide United	12	10	5	38	37

Use a spreadsheet to:

a. express the win, loss and draw columns as a percentage of the total games played, correct to 2 decimal places.

b. express the goals for as a percentage of the goals against, correct to 2 decimal places.

1.3 Wages

1.3.1 Wages and salaries

Employees earn an income for doing a job for their employer. This income may be a wage or a salary.

A **wage** is an amount paid to an employee according to an hourly rate. The weekly wage is the hourly rate multiplied by the hours worked. Hours worked outside the normal work period are paid at a higher rate.

A **salary** is a fixed amount paid to an employee to do a job. It is usually based on an annual amount divided into weekly, fortnightly or monthly payments. There is no extra pay for hours worked outside the normal work period. To make calculations about salaries, the following are important:

$$1 \text{ year} = 52 \text{ weeks} = 26 \text{ fortnights} = 12 \text{ months}$$

WORKED EXAMPLE 7

Dimitri works as an accountant and receives an annual salary of \$63 700. Calculate the pay that Dimitri is paid each fortnight.

THINK	WRITE
1. There are 26 fortnights in a year, so we divide \$63 700 by 26.	$\text{Fortnightly pay} = \$63\,700 \div 26$
2. Evaluate.	$= \$2450$

WORKED EXAMPLE 8

Grace is a solicitor who is paid \$6500 per month. Calculate Grace's annual salary.

THINK	WRITE
1. There are 12 months in a year, so multiply \$6500 (monthly pay) by 12.	$\text{Annual salary} = \$6500 \times 12$
2. Evaluate.	$= \$78\,000$

WORKED EXAMPLE 9

Charlotte works as a laboratory technician and is paid an annual salary of \$41 560. If Charlotte works an average of 42 hours per week, calculate her hourly rate of pay.

THINK	WRITE
1. Calculate the weekly pay by dividing the salary by 52.	Weekly pay = \$41 560 ÷ 52 = \$799.23
2. Calculate the hourly rate by dividing the weekly pay by 42.	Hourly rate = \$799.23 ÷ 42 = \$19.03

1.3.2 Wages and Inflation

Inflation is a measure of how an economy is performing over a period of time. It is an increase in the price of goods and services and a decrease in the value of our money. Inflation is a rate that is expressed as a percentage and in Australia is called the consumer price index, (CPI).

As inflation increases the spending power of a set amount of money will decrease. For example, if the cost of a loaf of bread was \$4.00 and rose with inflation then in five years it might cost \$4.50. As inflation gradually decreases the spending power of the dollar, peoples' salaries often increase in line with inflation. This increase counterbalances the decreasing spending power of money.

WORKED EXAMPLE 10

George received a weekly salary of \$1050 in 2017. The rate of inflation was 3.4% in 2018. Calculate his weekly salary at the end of 2018 if it increased with the CPI.

THINK	WRITE
1. An increase in inflation of 3.4% means a total of 103.4%.	$(100 + 3.4)\% = 103.4\%$
2. Write 103.4% as a decimal.	$103.4\% = \frac{103.4}{100}$ $103.4\% = 1.034$
3. Multiply the weekly salary \$1050 by 1.034.	$1050 \times 1.034 = 1085.7$
4. Answer the question.	His salary will be \$1085.70

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Units 1 & 2 > Area 1 > Sequence 1 > Concept 3

Wages and inflation Summary screen and practice questions

on Resources

Digital document: SkillSHEET Converting units of time (doc-10849)
Digital document: SkillSHEET Multiplying and dividing a quantity (money) by a whole number (doc-10850)
Digital document: SpreadSHEET Payroll calculations (doc-1439)

Exercise 1.3 Wages

1. **WE7** Nga is paid a salary of $44 200 per annum. Calculate Nga's fortnightly pay.
2. Roger is paid a salary of $49 920 per annum. Calculate Roger's weekly pay.
3. Frieda is paid a salary of $54 000 per annum. Calculate Frieda's monthly pay.
4. Wendy works as an office secretary and is paid a salary of $38 740 per annum. Calculate Wendy's pay if she is paid:
   a. weekly
   b. fortnightly
5. **WE8** Maxine is paid a salary. She receives $860 per week. Calculate Maxine's annual salary.
6. Thao receives $1250 per fortnight. Calculate Thao's annual salary.
7. **MC** Determine which of the following people receives the greatest salary.
   A. Goran, who receives $530 per week
   B. Bryan, who receives $1075 per fortnight
   C. Wayne, who receives $2330 per month
   D. Chris, who receives $1100 per fortnight
8. Use a spreadsheet to complete the table below for food production employees.

Annual salary	Weekly pay	Fortnightly pay	Monthly pay
$30 000			
$39 500			
$42 250			
$54 350			
$86 475			

9. **WE9** Fiona receives a salary of \$29 700 per annum. If Fiona works an average of 40 hours per week, calculate the equivalent hourly rate of pay.

10. Jade receives a salary of \$33 000 per annum.
    a. Calculate Jade's weekly pay, correct to the nearest cent.
    b. Jade works an average of 36 hours each week. Calculate the hourly rate to which Jade's salary is equivalent. Give your answer correct to the nearest cent.

11. **WE10** Tina receives a weekly salary of \$890. Tina receives a salary increase equal to the rate of inflation. If the rate of inflation is 2.7%, what will her new salary be?

12. Lisa is on an annual salary of \$35 776. Letitia is on a wage and is paid \$16.00 per hour.
    a. Calculate Lisa's weekly pay.
    b. If Lisa works an average of 42 hours per week, calculate whether Lisa or Letitia receives the better rate of pay.

13. Garry earns \$42 500 per year while his friend Henry earns \$18.50 per hour. Calculate the number of hours that Henry will need to work each week to earn more money than Garry does.

14. Use a spreadsheet to calculate the wages for each of the following eight employees.

Employee	Hourly pay rate	Hours worked per week	Wages per week	Wages per annum
A. Richardson	\$35.50	20		
S. Provis	\$21.75	35		
N. Liu	\$17.45	8		
W. Naidoo	\$29.55	28		
G. Riddell	\$15.75	16		
C. Ve	\$23.60	38		
O. Chang	\$15.50	26		
D. Evans	\$22.00	13		

1.4 Earning wages

A **wage** is paid at an hourly rate. The wage for each week is calculated by multiplying the normal rate of pay by the number of hours worked during that week.

WORKED EXAMPLE 11

Sadiq works as a mechanic and is paid \$23.65 per hour.
Calculate Sadiq's wage in a week during which he works 38 hours.

THINK	WRITE
1. Multiply \$23.65 (the hourly rate) by 38 (the number of hours worked).	Wage = \$23.65 × 38 = \$898.70

To compare two people's wages, we need to consider the number of hours each has worked. Wages are compared by looking at the hourly rate. To calculate the hourly rate of an employee, we need to divide the wage by the number of hours worked.

WORKED EXAMPLE 12

Georgina works 42 hours as a computer technician. Her wage for the week totalled $945. Calculate Georgina's hourly rate of pay.

THINK	WRITE
1. Divide $945 (the wage) by 42 (number of hours worked).	Hourly rate = $945 ÷ 42 = $22.50

Sometimes wages are increased because an **allowance** is paid for working in unfavourable conditions. An allowance is an additional payment made when the working conditions are difficult or unpleasant. For example, a road worker may be paid an allowance for working in the rain or in severe weather conditions.

WORKED EXAMPLE 13

Ryan is a road worker and is paid $22.50 per hour for a 35-hour week. For working on wet days he is paid a wet weather allowance of $2.46 per hour. Calculate Ryan's pay if for 12 hours of the week he works in the rain.

THINK	WRITE
1. Calculate Ryan's normal pay by multiplying $22.50 (hourly rate) by 35 (number of hours worked).	Normal pay = $22.50 × 35 = $787.50
2. Calculate the wet weather allowance by multiplying $2.46 (the wet weather allowance) by 12 (number of hours worked in the wet).	Allowance = $2.46 × 12 = $29.52
3. Add the normal pay to the wet weather allowance to calculate the total pay.	Total pay = $787.50 + $29.52 = $817.02

Exercise 1.4 Earning wages

1. **WE11** Allan works for a cleaning company and is paid $12.95 per hour. Calculate Allan's wage in a week during which he works 40 hours.
2. Alicia is an apprentice chef. In the first year of her apprenticeship she earns $11.80 per hour. Calculate Alicia's wage in a week during which she works:
   - a. 36 hours
   - b. 48 hours
   - c. 42.5 hours.
3. Domonic is a fully qualified chef. He earns $23.50 per hour. Calculate Domonic's wage in a week during which he works:
   - a. 32 hours
   - b. 37 hours
   - c. 44.5 hours.

4. Katherine works as a casual waitress. Casual workers earn 20% more per hour than full-time workers to compensate for their lack of holidays and sick leave.
   - a. A full-time waitress earns $14.45 per hour. Calculate the casual rate earned by casual waitresses.
   - b. Calculate Katherine's wage in a week during which she works 6 hours on Saturday and 7 hours on Sunday.
5. **MC** Which of the following workers earns the highest wage for the week?
   - A. Dylan, who works 35 hours at $13.50 per hour
   - B. Lachlan, who works 37 hours at $12.93 per hour
   - C. Connor, who works 38 hours at $12.67 per hour
   - D. Cameron, who works 40 hours at $12.19 per hour
6. **WE12** Calculate the hourly rate of a person who works 40 hours for a wage of $387.20.
7. Julie earns $11.42 per hour. Calculate the number of hours worked by Julie during a week in which she is paid $445.38.
8. Copy and complete the table below for these casual workers and their wages.

Name	Wage	Hours worked	Hourly rate
Brent	$416.16	36	
Sandra	$538.80	40	
Ann	$369.63	37	
Chris	$813.96		$19.38
Anna	$231.30		$15.42
Toni	$776.72		$20.44

9. **MC** Which of the following workers is paid at the highest hourly rate?
   - A. Melissa, who works 35 hours for $366.45
   - B. Belinda, who works 36 hours for $376.20
   - C. April, who works 38 hours for $399.76
   - D. Nicole, who works 40 hours for $419.60
10. **MC** Which of the following people worked the greatest number of hours?
    - A. Su-Li, who earned $439.66 at $11.57 per hour
    - B. Denise, who earned $576.00 at $14.40 per hour
    - C. Vera, who earned $333.20 at $9.52 per hour
    - D. Camille, who earned $707.25 at $17.25 per hour

11. **WE13** Richard works as an electrician and is paid \$20.94 per hour for a 38-hour week. When he has to work at heights, he is paid a \$3.50 per hour 'height allowance'. Calculate Richard's pay during a week in which he spends 15 hours working at heights.
12. Ingrid works as an industrial cleaner and is paid \$14.60 per hour for a 35-hour working week. When Ingrid is working with toxic substances, she is paid an allowance of \$1.08 per hour. Calculate Ingrid's pay if she works with toxic substances all week.
13. Rema works as a tailor and earns \$19.45 per hour.
    a. Calculate Rema's wage for a week in which she works 37 hours.
    b. Zhong is Rema's assistant and earns \$18.20 per hour. Determine the least time Zhong must work if he is to earn more money than Rema does.
14. Tamarin works 38 hours per week at \$12.40 per hour. Zoe earns the same amount each week as Tamarin does, but Zoe works a 40-hour week. Calculate Zoe's hourly rate of pay.

1.5 Working overtime

1.5.1 Overtime and penalities

Overtime is paid when an employee works more than the regular hours each week. When an employee works overtime, a higher rate is paid. This higher rate of pay is called a **penalty rate**. The rate is normally calculated at either:

1. **time and a half**, which means that the person is paid $1\frac{1}{2}$ times the usual rate of pay, or
2. **double time**, which means that the person is paid twice the normal rate of pay.

A person may also be paid these overtime rates for working at unfavourable times such as at night or during weekends.

To calculate the hourly rate earned when working overtime, we multiply the normal hourly rate by the overtime factor, which is $1\frac{1}{2}$ for time and a half and 2 for double time.

WORKED EXAMPLE 14

Gustavo is paid \$15.78 per hour in his job as a childcare worker. Calculate Gustavo's hourly rate when he is being paid for overtime at time and a half.

THINK	WRITE
1. Multiply \$15.78 (the normal hourly rate) by $1\frac{1}{2}$ (the overtime factor for time and a half).	$\text{Time and a half rate} = \$15.78 \times 1\frac{1}{2}$ $= \$23.67$

WORKED EXAMPLE 15

Adrian works as a shop assistant and his normal rate of pay is $12.84 per hour. Calculate the amount that Adrian earns for 6 hours work on Saturday, when he is paid time and a half.

THINK	WRITE
1. Multiply $12.84 (the normal pay rate) by $1\frac{1}{2}$ (the overtime factor) and by 6 (hours worked at time and a half).	$\text{Pay} = \$12.84 \times 1\frac{1}{2} \times 6$ $= \$115.56$

When we need to calculate the total pay for a week that involves overtime, we need to calculate the normal pay and then add the amount earned for any overtime.

WORKED EXAMPLE 16

Natasha works as a waitress and is paid $14.80 per hour for a 38-hour week. Calculate Natasha's pay in a week during which she works 5 hours at time and a half in addition to her regular hours.

THINK	WRITE
1. Calculate Natasha's normal pay.	$\text{Normal pay} = \$14.80 \times 38$ $= \$562.40$
2. Calculate Natasha's pay for 5 hours at time and a half.	$\text{Pay at time and a half} = \$14.80 \times 1\frac{1}{2} \times 5$ $= \$111.00$
3. Add the normal pay and the time and a half pay together.	$\text{Total pay} = \$562.40 + \$111.00$ $= \$673.40$

Some examples of jobs will have more than one overtime rate to consider and some will require you to calculate how many hours have been worked at each rate.

WORKED EXAMPLE 17

Graeme is employed as a car assembly worker and is paid $20.40 per hour for a 36-hour week. If Graeme works overtime, the first 6 hours are paid at time and a half and the remainder at double time. Calculate Graeme's pay in a week during which he works 45 hours.

THINK	WRITE
1. Calculate the number of hours overtime Graeme worked.	$\text{Overtime} = 45 - 36$ $= 9 \text{ hours}$
2. Of these nine hours, calculate how much was at time and a half and how much was at double time.	$\text{Time and a half} = 6 \text{ hours}$ $\text{Double time} = 3 \text{ hours}$
3. Calculate Graeme's normal pay.	$\text{Normal pay} = \$20.40 \times 36$ $= \$734.40$
4. Calculate what Graeme is paid for 6 hours at time and a half.	$\text{Time and a half} = \$20.40 \times 1\frac{1}{2} \times 6$ $= \$183.60$
5. Calculate what Graeme is paid for 3 hours at double time.	$\text{Double time} = \$20.40 \times 2 \times 3$ $= \$122.40$
6. Calculate Graeme's total pay by adding the time and a half and double time payments to his normal pay.	$\text{Total pay} = \$734.40 + \$183.60 + \$122.40$ $= \$1040.40$

on Resources

Digital document: SkillSHEET Multiplying and dividing a quantity (money) by a fraction (doc-10851)

studyon

Units 1 & 2 > Area 1 > Sequence 1 > Concept 4

Working overtime Summary screen and practice questions

Exercise 1.5 Working overtime

1. **WE14** Reece works in a restaurant and is paid a normal hourly rate of \$11.30. Calculate the amount Reece earns each hour when he is being paid time and a half.
2. Gareth works in a warehouse and is normally paid \$11.48 per hour. For working on a Sunday, he is paid time and a half and on a public holiday at double time. Calculate Gareth's hourly rate of pay on:
   a. a Sunday **b.** a public holiday.
3. **WE15** Ben works in a hotel and is paid \$11.88 per hour. Calculate the total amount Ben will earn for an 8-hour shift on Saturday when he is paid at time and a half.
4. **MC** Ernie works as a chef and is paid \$19.90 per hour. What will Ernie's hourly rate be when he is paid time and a half for overtime?
   A. \$18.40
   B. \$22.90
   C. \$29.85
   D. \$35.82

5. MC Stephanie works in a florists shop and is paid $9.40 per hour. Calculate how much more Stephanie will earn for 8 hours work at time and a half than she would at normal pay rates.
A. $37.60
B. $75.20
C. $112.80
D. $150.40

6. MC Eric works on the wharves unloading containers. Eric is paid $14.20 per hour. Calculate the number of hours at time and a half that Eric will have to work to earn the same amount of money that he will earn in 9 hours at normal pay rates.
A. 4.5 hours **B.** 6 hours **C.** 8 hours **D.** 10.5 hours

7. WE16 Rick works 37 hours at normal time each week and receives $12.64 per hour. Calculate Rick's pay in a week when, in addition to his normal hours, he works 4 hours overtime at time and a half.

8. Grant works as a courier and is paid $13.25 per hour for a 35-hour working week. Calculate Grant's pay for a week during which he works 4 hours at time and a half and 2 hours at double time in addition to his regular hours.

9. Copy and complete the table below.

Name	Ordinary rate	Normal hours	Time and a half hours	Double time hours	Total pay
W. Clark	$8.60	38	4	—	
A. Hurst	$9.85	37	—	6	
S. Gannon	$14.50	38	5	2	
G. Dymock	$16.23	37.5	4	1.5	
D. Colley	$24.90	36	6	8	

10. MC Jenny is a casual worker at a motel. The normal rate of pay is $10.40 per hour. Jenny works 8 hours on Saturday for which she is paid time and a half. On Sunday, she works 6 hours for which she is paid double time. Jenny's pay for Saturday and Sunday is equivalent to how many hours work at the normal rate of pay?
A. 14 **B.** 21 **C.** 24 **D.** 28

11. **MC** Patricia works a 35-hour week and is paid \$14.15 per hour. Any overtime that Patricia does is paid at time and a half. Patricia wants to work enough overtime so that she earns more than \$600 each week. What is the minimum number of hours that Patricia will need to work to earn this amount of money?
A. 40 **B.** 41 **C.** 42 **D.** 43

12. **WE17** Steven works on a car assembly line and is paid \$12.40 per hour for a 36-hour working week. The first 4 hours overtime he works each week is paid at time and a half with the rest paid at double time. Calculate Steven's earnings for a week in which he works 43 hours.

13. Zac works in a supermarket. He is paid at an ordinary rate of \$8.85 per hour. If Zac works more than 8 hours on any one day, the first two hours are paid at time and a half and the rest at double time. Calculate Zac's pay
if the hours worked each day are:
Monday — 8 hours
Tuesday — 9 hours
Wednesday — 12 hours
Thursday — 7 hours
Friday — 10.5 hours.

14. Megan works a 38-hour week and for any extra time she is paid at time and a half. When she worked a 45-hour week, she received \$582. What would she earn for a week in which she worked 40 hours?

1.6 Earnings — commission and piecework

1.6.1 Commission

Commission is paid to a salesperson to motivate them to sell more products. It is calculated as a percentage of the total value of the goods sold. In addition to commission, most salespeople receive a fixed amount per week, called a **retainer**. A retainer does not depend on sales.

WORKED EXAMPLE 18

Jack is a computer salesman who is paid a commission of 12% of all sales. Calculate the commission that Jack earns in a week if he makes sales to the value of \$15 000.

THINK	WRITE
1. Calculate 12% of \$15 000.	Commission = 12% of \$15 000 $= \frac{12}{100} = 0.12$ $0.12 \times \$15\,000 = \$1800$
2. Answer the question.	Jack earns \$1800 commission.

WORKED EXAMPLE 19

Peter works in a menswear store. He earns 7.5% commission on all sales on top of his retainer of $450 per week. Calculate Peter's wage in a week when his sales are $7400.

THINK	WRITE
1. Calculate the commission by finding 7.5% of $7400.	$\text{Commission} = 7.5\% \text{ of } \$7400$ $= \dfrac{7.5}{100} = 0.075$ $= 0.075 \times 7400$ $= 555$
2. Find the total wage by adding the retainer to the commission.	$\text{Peter's wage} = \text{Retainer} + \text{commission}$ $= 450 + 555$ $= 1005$
3. Answer the question.	Peter's wage is $1005

WORKED EXAMPLE 20

A real estate agent is paid commission on her sales at the following rate:

- **5% on the first $75 000**
- **2.5% on the balance of the sale price.**

Calculate the commission earned on the sale of a property for $235 000.

THINK	WRITE
1. Calculate 5% of $75 000.	$\text{Commission 1} = 5\% \text{ of } \$75\,000 = \$3750$
2. Calculate the balance of the sale.	$\text{Balance} = \$235\,000 - \$75\,000$ $= \$160\,000$
3. Calculate 2.5% of $160 000.	$\text{Commission 2} = 2.5\% \text{ of } \$160\,000 = \$4000$
4. Add up each portion to calculate the commission.	$\text{Total commission} = \$3750 + \$4000$ $= \$7750$

1.6.2 Piecework

Piecework means that workers are paid for the amount of work that they have completed rather than an hourly or weekly pay rate. Permanent employees are not paid by piecework as it is usually factory workers, fruit pickers or packers who are paid piece rates.

WORKED EXAMPLE 21

Sian delivers flyers to letterboxes and is paid \$25 for 1000 brochures. How much does she earn for delivering 3500 brochures?

THINK	WRITE
1. Calculate the number of thousands of brochures delivered.	$\frac{3500}{1000} = 3.5$
2. Multiply by \$25 to find out what Sian is paid.	$3.5 \times 25 = 87.5$
3. Answer the question.	Sian earns \$87.50

WORKED EXAMPLE 22

Michelle works as a machine worker in a factory. She is paid \$2.75 each for the first 150 items that she sews and then \$3.35 per item thereafter. How much does she get paid if she sews 295 items?

THINK	WRITE
1. Find how much she is paid for 150 items.	Payment 1 $= 150 \times 2.75 = 412.50$
2. Calculate how many items more than 150 she sews.	Number of items more than 150 $= 295 - 150 = 145$
3. Find how much she is paid for 145 items.	Payment 2 $= 145 \times 3.35$ $= 485.75$
4. Calculate how much she is paid for 295 items.	Total payment $= 412.50 + 485.75$ $= 898.25$
5. Answer the question.	Michelle is paid \$898.25

WORKED EXAMPLE 23

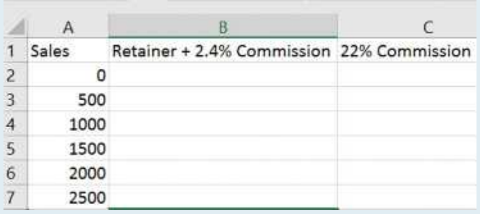
Bob and Sanjeev are employees of a mobile phone company. Bob is paid \$600 plus 2.4% of sales per week and Sanjeev is on a commission-only contract, paid 22% of sales per week.

a. **Use a spreadsheet to compare their salaries for sales up to \$2500. Who earns more money for sales up to \$2500?**

b. **For what value of sales is it better to work on a commission-only contract?**

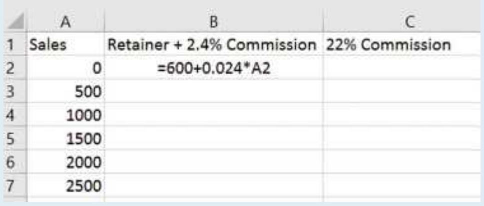
THINK | **WRITE**

1. Set up headings and value amount of sales in $500 increments to $2500.

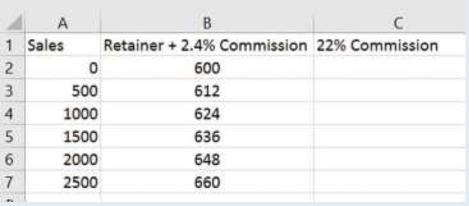


	A	B	C
1	Sales	Retainer + 2.4% Commission	22% Commission
2	0		
3	500		
4	1000		
5	1500		
6	2000		
7	2500		

2. Enter formula in cell B2 to calculate a salary of $600 plus 2.4% commission on sales and fill down.

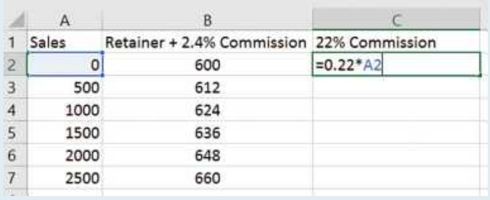


	A	B	C
1	Sales	Retainer + 2.4% Commission	22% Commission
2	0	=600+0.024*A2	
3	500		
4	1000		
5	1500		
6	2000		
7	2500		



	A	B	C
1	Sales	Retainer + 2.4% Commission	22% Commission
2	0	600	
3	500	612	
4	1000	624	
5	1500	636	
6	2000	648	
7	2500	660	

3. Enter formula in cell C2 to calculate a salary of 22% commission on sales and fill down.



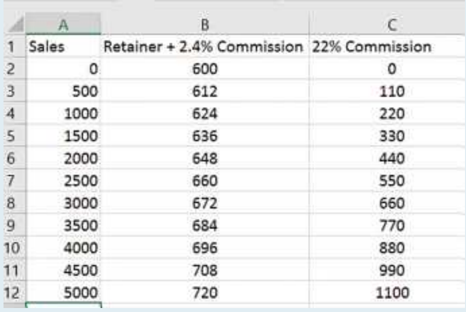
	A	B	C
1	Sales	Retainer + 2.4% Commission	22% Commission
2	0	600	=0.22*A2
3	500	612	
4	1000	624	
5	1500	636	
6	2000	648	
7	2500	660	

	A	B	C
1	Sales	Retainer + 2.4% Commission	22% Commission
2	0	600	0
3	500	612	110
4	1000	624	220
5	1500	636	330
6	2000	648	440
7	2500	660	550

4. Answer part **a.**

A retainer of $600 plus 2.4% commission on sales is better for sales up to $2500, so Bob earns more money.

5. Complete sales column for amounts up to $5000.



	A	B	C
1	Sales	Retainer + 2.4% Commission	22% Commission
2	0	600	0
3	500	612	110
4	1000	624	220
5	1500	636	330
6	2000	648	440
7	2500	660	550
8	3000	672	660
9	3500	684	770
10	4000	696	880
11	4500	708	990
12	5000	720	1100

6. Answer part **b.**

Sales of $3500 or more earn a better wage when 22% commission is paid.

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Units 1 & 2 > Area 1 > Sequence 1 > Concept 5

Commission and piecework Summary screen practice questions

Exercise 1.6 Earnings — commission and piecework

1. **WE18** Kylie is an insurance salesperson and she is paid 8% of the value of any insurance that she sells. Calculate the amount that Kylie is paid for selling insurance to the value of $25 000.

2. **MC** Ursula is a computer software salesperson. Ursula's sales total $105 000 and she is paid a commission of 0.8%. Calculate how much Ursula receives in commission.
   - **A.** $105
   - **B.** $840
   - **C.** $1050
   - **D.** $8400

3. **MC** Asif is a sales representative for a hardware firm. Asif earns $870 commission on sales of $17 400. Calculate the rate of commission Asif receives.
   - **A.** 0.05%
   - **B.** 0.5%
   - **C.** 5%
   - **D.** 10%

4. **WE19** Stanisa is a car salesman who is paid a retainer of $250 per week plus a commission of 2% of any sales he makes. Calculate Stanisa's pay in a week where his sales total $35 000.

5. **MC** A group of sales representatives each has $10 000 in sales for a week. Identify who earns the most money.
   - **A.** Averil, who is paid a commission of 8%
   - **B.** Bernard, who is paid $250 plus 6% commission
   - **C.** Cathy, who is paid $350 plus 4% commission
   - **D.** Darrell, who is paid $540 plus 2.5% commission

6. **WE20** A real estate agent charges commission at the following rate:
   - 5% on the first $75 000
   - 2.5% on the balance of the sale price.

   Calculate the commission charged on the sale of a property for $250 000.

7. Jade sells cosmetics, and the company pays her a fixed weekly wage plus 2.5% commission on all sales she makes. Last week she sold $600 worth of cosmetics and her total pay was $490. How much would she expect to earn in a week where she sold $1400 worth of cosmetics?

8. **WE21** Matthew delivers pamphlets to local letterboxes. He is paid \$21.80 per thousand pamphlets delivered. Calculate what Matthew will be paid for delivering 15 000 pamphlets
9. Julia works after school at a car yard detailing cars. If Julia is paid \$10.85 per car, calculate what she will earn in an afternoon when she details 7 cars.
10. Keith is an uber taxi driver. He is paid \$3.00 plus \$1.60 per kilometre. Calculate the amount Keith will earn for a journey of:
    a. 5 km b. 15.5 km
11. **WE22** Hamish makes leather belts for a local farmers' market. He is paid \$4.50 for each belt for the first 50 belts and \$5.10 thereafter. Determine what his income is for a day in which he produces 68 belts.

12. A production line worker is paid \$3.45 for each of the first 75 toasters assembled then \$4.90 per toaster thereafter. Calculate how much she earns if she produces 120 toasters.
13. Charlie works in a caryard as a detailer. Charlie is paid \$11.60 per car.
    a. What will Charlie earn in an afternoon during which he details 15 cars?
    b. If it takes Charlie 8 hours to detail the cars, calculate his hourly rate of pay.
    c. If Charlie could finish in 6 hours, calculate the hourly rate of pay he would earn.
14. **WE23** Tasha and Neesha work at a bridal store. Tasha is paid \$700 plus 3.8% of sales per week. Neesha is on a commission-only contract and is paid 27% of sales per week.

    a. Use a spreadsheet to compare their salaries for sales up to \$8000. Identify who earns more money for sales up to \$8000.
    b. For what value of sales is it better to work on a commission-only contract?

1.7 Payments — government allowances and pensions

1.7.1 Youth allowance

Some people rely on government allowances for income. These allowances include youth allowance, age pension, unemployment benefit, disability payment and other welfare benefits. The amount of the benefit is affected by age, income and assets.

The government pays youth allowance to people aged 16 to 24 who are studying fulltime, based on the following circumstances:

Your circumstances	Your maximum fortnightly payment
Single, no children, younger than 18 years, and live at your parent's home	$244.10
Single, no children, younger than 18 years, and need to live away from your parent's home to study, train or look for work	$445.80
Single, no children, 18 years or older and live at parent's home	$293.60
Single, no children, 18 years or older and need to live away from parent's home	$445.80
Single, with children	$584.20
Member of a couple, with no children	$445.80
Member of a couple, with children	$489.60

Parents' annual income affects how much youth allowance you can claim. If your parents' income is less than a certain amount, around $50 000, then you may be able to claim the full allowance. If your parents' income is more than the threshold amount the allowance is reduced by 20 cents in each dollar over (this is also affected by the number of children in your family group).

1.7.2 Austudy

Austudy is financial assistance offered by the government, if you are:

- at least 25 years old
- a fulltime student in an approved course or a fulltime Australian apprentice or trainee
- under the income and assets test limits.

If you're	The highest payment per fortnight is
single, no children	$445.80
single, with children	$584.20
in a couple, no children	$445.80
in a couple, with children	$489.60

WORKED EXAMPLE 24

Calculate the youth allowance payment or Austudy payment for each of the following per fortnight.

a. Ben is 30 years old, a single dad and a fulltime student.

b. Chloe lives at home and is 17 years old. She is completing a carpentry course at TAFE.

THINK	WRITE
a. Ben would receive Austudy because he is over 18 years old and studying fulltime.	**a.** Because Ben is a single parent, he will be paid at the 'single with children' rate $584.20.
b. Chloe would receive youth allowance because she is under 18 years old.	**b.** Chloe will be paid at the rate for the category 'single, no children, younger than 18 years, and live at your parent's homes' $244.10.

1.7.3 Age pension

People who are aged over 65 years and 6 months may be entitled to receive the age pension. However, it depends on their income and assets.

Payment per fortnight is shown in the following table:

Per fortnight	Single	Couple each	Couple combined	Couple apart due to ill health
Maximum basic rate	$826.20	$622.80	$1245.60	$826.20
Maximum pension supplement	$67.30	$50.70	$101.40	$67.30
Energy Supplement	$14.10	$10.60	$21.20	$14.10
Total	$907.60	$684.10	$1368.20	$907.60

Income per fortnight is shown below:

Single person

If your income per fortnight is	your pension will reduce by
up to $168	$0
over $168	50 cents for each dollar over $168

Couple living together or apart due to ill health

If your combined income per fortnight is	Your combined pension will reduce by
up to $300	$0
over $300	50 cents each dollar over $300

WORKED EXAMPLE 25

Rhonda is 78 years old and lives alone. She receives an income of $280 per fortnight from her investments. Use the table above and at the end of the previous page to calculate how much Rhonda would receive from the age pension per fortnight if she is entitled to the maximum basic rate.

THINK	WRITE
1. Read the table to find the maximum basic rate for a single person.	Single age pension = $826.20
2. Find the income above $168 that Rhonda receives.	Amount above $168 = $280 – $168 = $112
3. Calculate the amount by which her pension is reduced by finding 50 cents for each dollar over $168.	Reduction = $112 × 0.50 = $56
4. Calculate how much pension Rhonda receives.	$826.20 – $56 = $770.20
5. Answer the question.	Rhonda would receive $770.20.

on Resources

🔗 **Weblink:** Youth Allowance
🔗 **Weblink:** Austudy
🔗 **Weblink:** Age pension

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Units 1 & 2 > Area 1 > Sequence 1 > Concept 6

Government allowances and pensions Summary screen and practice questions

Exercise 1.7 Payments — government allowances and pensions

1. **WE24** Sean is 16 years old and a fulltime student. He lives at home with his parents, who are both unemployed. Calculate how much youth allowance he is entitled to per fortnight.
2. Grace is 17 years old, completing a nursing degree and living with her father, who does not work. Calculate how much youth allowance Grace receives per year.
3. Rebecca is a fulltime student living at home. Identify the factors that need to be considered to determine if she is entitled to receive youth allowance.

4. **MC** Matt is 16 years old, studies fulltime at high school and lives with his grandparents. Calculate how much he receives in youth allowance per week.
   A. \$244.10 **B.** \$122.05 **C.** \$222.90 **D.** \$240.25
5. **WE25** Emily is 70 years old and lives alone. She receives an income of \$425 per fortnight from her investments. How much would Emily expect to receive from the age pension, per fortnight, if she is entitled to the maximum basic rate?

6. George is 62 years old, lives alone and earns \$150 per fortnight. How much age pension does he receive per fortnight?
7. Mitch is 74 years old and lives with his wife. How much age pension does he receive per year?
8. Ayesha qualifies for youth allowance and is entitled to \$489.60 per fortnight. It is reduced by 50 cents in the dollar for any income earned over \$143 per fortnight. Ayesha gets a part-time job as a waitress and earns \$200 per fortnight.
   a. What will be the reduction in her youth allowance?
   b. Calculate how much youth allowance payment Ayesha receives per fortnight.
   c. Calculate how much youth allowance payment Ayesha receives per year.
9. Jarrod studies fulltime and lives at home with his parents. He is entitled to receive the maximum youth allowance per fortnight for a single person over 18 years old. Jarrod has a part-time job and earns \$500 per fortnight. His payment is reduced by 60 cents for every dollar he earns over \$250. Calculate how much youth allowance payment Jarrod receives per fortnight.

10. Angus is married with children and receives the maximum youth allowance per fortnight. His payments are reduced by 50 cents in the dollar for any income earned over \$143 per fortnight.
   a. He started a part-time job earning \$18.50 per hour. Determine how many hours per fortnight he can work without affecting his youth allowance payment.
   b. If Angus worked 23 hours one fortnight, calculate by how much his youth allowance payment was reduced.

1.8 Personal budgets

1.8.1 Expenses

A budget is an estimate of income and expenditure.

A personal budget helps you to make various financial decisions. The expenses in a personal budget can be divided into two major categories: fixed expenses and variable or discretionary expenses. Fixed expenses may include rent or mortgage, medical insurance, car registration and other regular payments that must be paid and can't be varied. Expenses that are non-essential and/or can be controlled or varied are called discretionary expenses and include items such as food, entertainment and clothing. To reduce your expenses in order to save more money, you would look at reducing your discretionary expenses.

A weekly, monthly or yearly budget can be prepared. Expenses may be calculated weekly (such as food), monthly (health insurance), quarterly (electricity bills) or yearly (car registration). Depending on the budget duration, all expenses should be converted to weekly, monthly or yearly amounts. The following table of conversion will help you in preparing a budget.

Purpose	Convert from	Convert to	Operation
Weekly budget	Monthly cost	Weekly cost	× 12, then ÷ 52
	Yearly cost	Weekly cost	÷ 52
Monthly budget	Weekly cost	Monthly cost	× 52, then ÷ 12
	Yearly cost	Monthly cost	÷ 12
Yearly budget	Weekly cost	Yearly cost	× 52
	Monthly cost	Yearly cost	× 12

WORKED EXAMPLE 26

Karla's fortnightly budget is as follows:

INCOME

Salary after tax	$2050
Dividends from shares	$23
Total:	$2073

EXPENSES

Rent of 1-bedroom flat	$520
Electricity	$70
Phone and internet	$40
Health insurance	$30
House contents insurance	$10
Car registration	$27
Car insurance	$42
Petrol	$35
Food	$110
Clothing	$150
Entertainment	$150
Gym membership	$100
Miscellaneous	$40
Total:	$1324

Use the table to

a. **Calculate the total of the fixed expenses. Note that Karla views health insurance, house contents insurance and car insurance as important expenses, and their regular payments are fixed.**

b. **Calculate the total of the discretionary expenses.**

c. **Calculate the amount available for saving per fortnight.**

d. **If Karla wishes to take a vacation and travel to New Zealand (estimated cost $3500), for how many weeks does she have to save?**

THINK	WRITE
a. Identify and find the total of the fixed expenses and add them.	**a.** Fixed expenses: Rent $520 Health insurance $30 House contents insurance $10 Car registration $27 Car insurance $42 Total expenses = 520 + 30 + 10 + 27 + 42 = $629
b. Calculate the total of the discretionary expenses by subtracting fixed expenses from the total.	**b.** Total of discretionary expenses = total expenses − total of fixed expenses = $1324 − $629 = $695
c. Calculate fortnightly savings by subtracting expenses from the income.	**c.** Fortnightly savings = fortnightly income − fortnightly expenses = $2073 − $1324 = $749
d. Calculate the number of weeks required to save for the holiday.	**d.** Karla saves $749 per fortnight which is $374.50 per week. To save $3500 at the rate of $374.50 per week: 3500 ÷ 374.50 = 9.3 or 10 weeks (rounded up). Karla needs to save for 10 weeks.

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Units 1 & 2 > Area 1 > Sequence 1 > Concept 7

Personal budgets Summary screen and practice questions

Exercise 1.8 Personal budgets

1. Explain the difference between fixed and discretionary expenses.
2. MC Which of the following can be considered as a fixed cost?
   - **A.** Speeding ticket
   - **B.** Soccer club membership
   - **C.** Holiday to Thailand
   - **D.** Visit to the dentist

3. Identify as many discretionary expenses as you can that an average 17 year old would have in a month.
4. **WE26** The table below shows the fortnightly budget for a couple with one school-aged child.

Income		**Expenses**	
Combined fortnightly salary after tax	$3800	Mortgage repayments	$1100
		Rates	$75
		Building insurance	$20
		Contents insurance	$15
		Electricity	$120
		Gas	$25
		Telephones and internet	$125
		Car registration	$35
		Car insurance	$50
		Health insurance	$60
		School fees	$110
		Food	$300
		Clothing	$60
		Entertainment	$70
		Miscellaneous	$40
		Maintanance and repairs	$20
		Petrol	$125
Total:	$3800	Total:	$2350

Use the table to calculate the following:
- a. the total of fixed expenses
- b. the total of discretionary expenses
- c. the total weekly savings
- d. the number of weeks needed for the family to save enough for a trip to Bali (estimated cost $3000).

5. Complete the following table of income and expenses to determine the monthly savings for the Saheed family:

Annual salary	Monthly bank interest	Monthly fixed expenses	Monthly discretionary expenses	Monthly savings
$73 700	$784	$3623	$2563	

6. A university student who lives with her parents has the following expenses: she pays her parents $70 per week for board and food; a monthly ticket for public transport costs her $80; she spends on average $45 a month on books and stationery; her single health insurance premium is $68.55 a month; entertainment costs her about $80 a month; the university enrolment fee is $900 a year; clothes cost $40 per month and her mobile phone costs approximately $50 per month.
   - a. Use a spreadsheet to prepare a monthly budget if the student's income consists of Austudy (which is $445.80 per fortnight) plus birthday and Christmas presents ($250 per year).
   - b. Calculate the amount of money that she can save per month.

7. Determine the total monthly cost from the following list of fixed and variable costs. The variable costs represent the amounts spent in a month.

Fixed costs			Variable costs	
Item	**Frequency**	**Amount**	**Item**	**Amount**
Rent	Weekly	$205	Food	$494
Health insurance	Monthly	$89	Clothing	$205
Vehicle registration	Yearly	$540	Entertainment	$123
Vehicle insurance	Yearly	$499	Car repairs	$72

8. Use a spreadsheet to determine the total yearly expenses from the following list of fixed and variable expenses. The variable expenses are monthly expenses.

Fixed costs			Variable costs	
Item	**Frequency**	**Amount**	**Item**	**Amount**
Rent	Weekly	$230	Food	$423
Health insurance	Monthly	$78	Clothing	$107
Vehicle registration	Yearly	$620	Entertainment	$85
Vehicle insurance	Yearly	$389	Car repairs	$325

9. Use the figures in the table below to calculate the total of the weekly expenses.

Item	Cost and period
Rent	$600 per month
Food	$90 per week
Electricity	$420 per 3 months
Gas	$40 per 2 months
Phone	$360 per 3 months
Car registration	$430 per year
Car insurance	$500 per year
Petrol	$50 per week
Health insurance	$175 per 3 months
Contents insurance	$125 per year
Clothes	$100 per month
Entertainment	$80 per month

10. Parents on a kindergarten committee are discussing the budget for the next year. Their income will come from three sources: a government subsidy of $4200; annual enrolment fees of $1250 per child and profits from the various events. They estimate that the cake stall will bring in $400, and profits from the two sausage sizzles will yield about $800 each. They also estimate that they will have 22 children enrolled. The money will be spent as follows: rent of the premises at $500 a month; publishing *the newsletter* $240 per quarter; new equipment will be $7200 per year electricity and phone bills at $220 a month; public liability and contents insurance, $1860 per year. Advertising will cost $30 per month and stationery $250 per year. A new computer will cost $3500 and $2000 is allowed for unexpected expenses.

a. Use a spread sheet to prepare a yearly budget of income and expenses for the kindergarten commitee.
b. Calculate the amount of money that can be saved for future renovations of the kindergarten.
c. Express your answer to **b** as a percentage of the annual income.

1.9 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. Identify which of the following is the same as 1.06.

A. 10.6%
B. 106%
C. $1\frac{6}{10}$
D. 160%

2. Australia needs to make 280 runs in 50 overs to beat Pakistan. Calculate how many runs per over this is.

A. 6.5
B. 5
C. 6
D. 5.6

3. Sasha used to weigh 96 kg. After joining a gym he lost 12 kg. The percentage of the original weight lost was:

A. 8%
B. 10%
C. 12%
D. 12.5%

4. A jewellery salesperson earns 12% commission on all sales. If she sold two rings, one valued at \$400 and the other at \$280, which of the following *could not* be used to evaluate her commission?
   A. $\frac{12}{100} \times 680$
   B. 680×0.12
   C. $\frac{12}{100} \times 280 + \frac{12}{100} \times 400$
   D. $(400 + 280) \times 12$

5. Natasha is picking apples on a farm. She is paid \$6.50 per basket plus a bonus of \$1.80 for every basket after the first ten. Calculate how much she will be paid on the day when she picks 16 baskets of apples.
   A. \$105.80
   B. \$75.80
   C. \$114.80
   D. \$128.80
6. The ring that will cost \$78 after a discount of $33\frac{1}{3}\%$ is:
   A. a friendship ring, normally \$104
   B. a mother of pearl ring, normally \$130
   C. a sapphire ring, normally \$260
   D. a Russian band ring, normally \$117
7. Michael earns a retainer of \$220 per week plus 7.5% commission on all sales. The total of Michael's sales during the week when he earned \$448.50 was:
   A. \$6513.33 **B.** \$8913.33 **C.** \$3420 **D.** \$3046.67
8. Identify which of the following is not a fixed expense.
   A. Car insurance **B.** Mortgage repayments
   C. Council rates **D.** Sport expenses
9. The calculation that *could not* be used to find the sale price of a \$64 item after a discount of 12.5% is:
   A. $\frac{12.5 \times 64}{100}$ **B.** $64 - \frac{12.5 \times 64}{100}$ **C.** $\frac{87.5 \times 64}{100}$ **D.** $\frac{7}{8}$ of 64
10. Stan is paid \$13.20 per hour for the first 40 hours, time-and-a-half for a further 3 hours and double time thereafter. If last week Stan worked 10 hours overtime, his total earning for the week were:
   A. \$770 **B.** \$677.60 **C.** \$772.20 **D.** \$707.20
11. A mobile phone is sold for \$1275. If this represents a 15% reduction from the RRP, calculate the original price.

12. A person has to pay the following bills out of their weekly income of $1100.

Food	$280
Electricity	$105
Telephone	$50
Petrol	$85
Rent	$320

Giving answers correct to 2 decimal places where necessary:

a. express each bill as a percentage of the total bills

b. express each bill as a percentage of the weekly income.

Complex familiar

13. Kay is working in a large department store. She earns a retainer of $300 per week for 5 working days plus 10% commission on all sales. During a 'facials' promotion week she also earns $20 for each facial that she conducts. In one day she did 7 facials and sold her clients different products to the total value of $1089. Calculate Kay's earnings for that day.

14. To make a T-shirt, parts are traced onto the material using a pattern and then cut. A cutter is paid $20 for cutting every 100 standard T-shirts. If a T-shirt contains any extra parts, the cutter is paid an extra $4.50 for each 100 parts. Calculate the amount of money that a cutter will earn cutting an order of 360 T-shirts with two extra parts.

15. A power company claims that if you install solar panels for $1800, you will make this money back in savings on your electricity bill in 2 years. If you usually pay $250 per month, calculate what percentage your bill will be reduced by if their claims are correct.

16. The table below shows the share prices of different companies on the first day of two consecutive months. For each company, identify whether there was an increase or decrease in price, calculate the amount of increase/decrease in dollars and express it as a percentage of the original price (to 2 decimal places).

Company Name	Price on 1.9.98	Price on 1.10.98	Increase (I) or Decrease (D)?	Change of price ($)	Percentage change
a BHT	$12.82	$8.19			
b Super Cole	$5.14	$7.25			
c SuNatCo	$21.35	$19.00			
d AMB	$4.70	$5.76			
e ANX Bank	$9.52	$9.80			
f Pronto Co	$0.45	$0.61			
g EIK Gold	$25.40	$29.12			
h Motors International	$7.80	$7.06			
i Optocom au	$5.70	$4.98			
j National Metro	$18.28	$19.15			

Complex unfamiliar

17. Jim sells women's clothes, and receives $200 retainer and 12% commission on all sales. Jill sells men's clothes, and receives $250 retainer and 10% commission.
    a. If in a certain week each of them sold clothes to the total value of $2000, identify who is better off and by how much?
    b. Calculate the value of total sales Jim and Jill have to make so that they both receive $600.

18. Use a spreadsheet to complete the following time sheet and calculate the wages of an employee who earns $10.50 per hour. Normal pay is paid for an 8 hour day Monday to Friday. Any overtime on weekdays is paid time-and-a-half for the first 3 hours and double time thereafter. Any hours worked on the weekend are paid at double time.

Name			Time Sheet				
Day	On	Off	On	Off	Normal Hours	Time-and-a-half-hours	Double time hours
Monday	7.30	11.30	12.00	4.30			
Tuesday	7.30	11.30	12.00	4.00			
Wednesday	7.30	12.00	12.30	5.30			
Thursday	7.30	11.30	12.30	4.30			
Friday	7.30	11.30	1.00	5.30			
Saturday	8.00	12.00					
Sunday	8.00	11.00					
				Total:			

19. Copy and complete the following table.

Item	Cost price ($)	Percentage discount	Discount ($)	Selling price ($)
a	200	12%		
b	150			142.50
c	98		9.80	
d			16.25	113.75
e		20%		332.80
f		$33\frac{1}{3}\%$	76	

20. The following table shows a list of Rose's expenses.

Item	Cost	Period
Rent	$434	Monthly
Electricity	$130	Quarterly
Gas	$60	Every 2 months
Phone	$300	Quarterly
Car registration	$420	Yearly
Car insurance	$450	Yearly
Contents insurance	$155	Yearly
Health insurance	$40	Monthly
Food	$100	Weekly
Sport	$30	Weekly
Entertainment	$20	Weekly
Clothes	$120	Monthly
Holidays	$1200	Yearly

a. Use a spreadsheet to prepare a weekly budget for Rose.

b. Rose has a part-time job as a receptionist and earns $470 per week. Calculate the amount that she can save per year.

studyon

Units 1 & 2 Sit chapter test

Answers

Chapter 1 Consumer arithmetic 1

Exercise 1.2 Rates and percentages

1. **a.** 0.5 km/h **b.** 9.43 mL/ min **c.** $18.65 per metre **d.** 12.25 points/game

2. **a.** 5.4 km/h **b.** 16.67 m/s **c.** $650 per kilo **d.** 0.57 cents per gram

3. 1062 km/h

4. $11.80 per hour

5. **a.** $37.80 **b.** 108 **c.** $174.49 **d.** $42 304.22

6. **a.** $45.36 **b.** $7.41 **c.** $69.31 **d.** $12 037.52

7. $60

8. **a.** $18.92 **b.** Decrease of 5.16%

9. 12.5%

10. **a.** 12.5% **b.** 16.875%

11. **a.** **i.** reduction of 0.62% **ii.** increase of 2.5%

b. increase of 1.23%

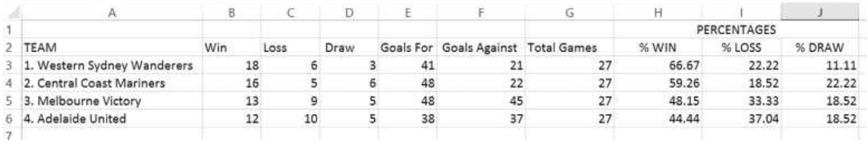
12. **a.**

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8
Mark	$\frac{16}{20}$	$\frac{14}{21}$	$\frac{26}{34}$	$\frac{36}{45}$	$\frac{14.5}{20}$	$\frac{13}{39}$	$\frac{42}{60}$	$\frac{26}{35}$
Percentage	80%	66.67%	76.47%	80%	72.5%	33.33%	70%	74.29%

b. 68.43%

13. **a.** 18% increase **b.** $1337.33

14. **a.**



	A	B	C	D	E	F	G	H	I	J
1									PERCENTAGES	
2	TEAM	Win	Loss	Draw	Goals For	Goals Against	Total Games	% WIN	% LOSS	% DRAW
3	1. Western Sydney Wanderers	18	6	3	41	21	27	66.67	22.22	11.11
4	2. Central Coast Mariners	16	5	6	48	22	27	59.26	18.52	22.22
5	3. Melbourne Victory	13	9	5	48	45	27	48.15	33.33	18.52
6	4. Adelaide United	12	10	5	38	37	27	44.44	37.04	18.52
7										

b.

	A	B	C	D	E	F	G	H
1								PERCENTAGES
2	TEAM	Win	Loss	Draw	Goals For	Goals Against	Total Games	% Goals FOR
3	1. Western Sydney Wanderers	18	6	3	41	21	27	195.24
4	2. Central Coast Mariners	16	5	6	48	22	27	218.18
5	3. Melbourne Victory	13	9	5	48	45	27	106.67
6	4. Adelaide United	12	10	5	38	37	27	102.70
7								

Exercise 1.3 Wages

1. $1700

2. $960

3. $4500

4. **a.** $745 **b.** $1490

5. $44 720

6. $32 500

7. D

8.

Annual salary	Weekly pay	Fortnightly pay	Monthly pay
$30 000	$576.92	$1153.85	$2500.00
$39 500	$759.62	$1519.23	$3291.67
$42 250	$812.50	$1625.00	$3520.83
$54 350	$1045.19	$2090.38	$4529.17
$86 475	$1662.98	$3325.96	$7206.25

9. $14.28

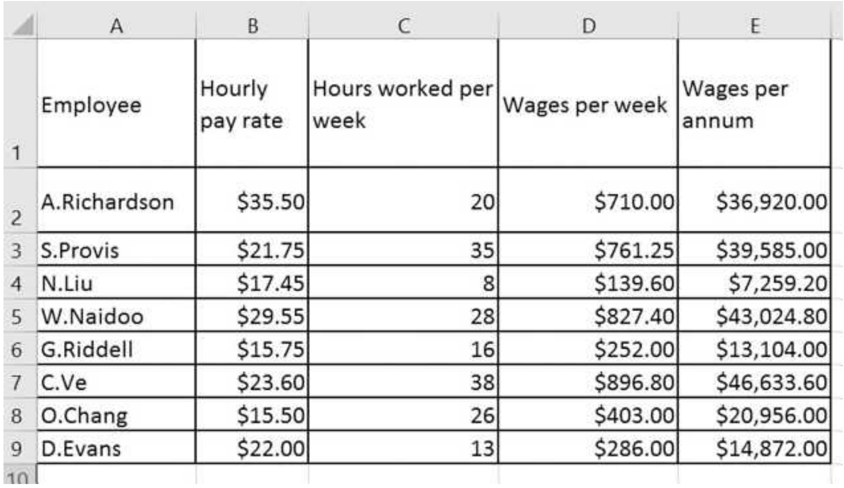
10. **a.** $634.62 **b.** $17.63

11. $914.05

12. a. $688 b. Lisa ($16.38 per hour)

13. 45 hours

14.



	A	B	C	D	E
1	Employee	Hourly pay rate	Hours worked per week	Wages per week	Wages per annum
2	A.Richardson	$35.50	20	$710.00	$36,920.00
3	S.Provis	$21.75	35	$761.25	$39,585.00
4	N.Liu	$17.45	8	$139.60	$7,259.20
5	W.Naidoo	$29.55	28	$827.40	$43,024.80
6	G.Riddell	$15.75	16	$252.00	$13,104.00
7	C.Ve	$23.60	38	$896.80	$46,633.60
8	O.Chang	$15.50	26	$403.00	$20,956.00
9	D.Evans	$22.00	13	$286.00	$14,872.00
10					

Exercise 1.4 Earning wages

1. $518

2. a. $424.80 b. $566.40 c. $501.50

3. a. $752.00 b. $869.50 c. $1045.75

4. a. $17.34/hour b. $225.42

5. D

6. $9.68

7. 39

8.

Name	Wage	Hours worked	Hourly rate
Brent	$416.16	36	$11.56
Sandra	$538.80	40	$13.47
Ann	$369.63	37	$9.99
Chris	$813.96	42	$19.38
Anna	$231.30	15	$15.42
Toni	$776.72	38	$20.44

9. C

10. D

11. $848.22

12. $548.80

13. a. $719.65 b. 40 hours

14. $11.78/hour

Exercise 1.5 Working overtime

1. $16.95

2. a. $17.22/hour b. $22.96/hour

3. $142.56

4. C

5. A

6. B

7. $543.52

8. $596.25

9.

Name	Ord. rate	Normal hours	Time and a half hours	Double time hours	Total pay
W. Clark	$8.60	38	4		$378.40
A. Hurst	$9.85	37		6	$482.65
S. Gannon	$14.50	38	5	2	$717.75
G. Dymock	$16.23	37.5	4	1.5	$754.70
D. Colley	$24.90	36	6	8	$1518.90

10. C

11. A

12. $595.20

13. $455.78

14. $492

Exercise 1.6 Earnings — commission and piecework

1. $2000
2. B
3. C
4. $950
5. B
6. $8125
7. $510
8. $327
9. $75.95

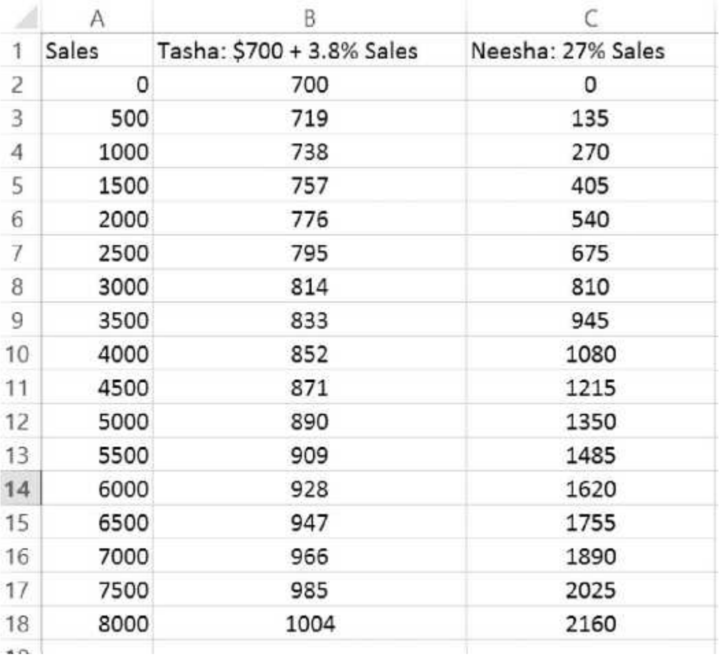
10. a. $11 **b.** 27.80

11. $316.80

12. $479.25

13. a. $174 **b.** $21.75/h **c.** $29.00/h

14. a.



	A	B	C
1	Sales	Tasha: $700 + 3.8% Sales	Neesha: 27% Sales
2	0	700	0
3	500	719	135
4	1000	738	270
5	1500	757	405
6	2000	776	540
7	2500	795	675
8	3000	814	810
9	3500	833	945
10	4000	852	1080
11	4500	871	1215
12	5000	890	1350
13	5500	909	1485
14	6000	928	1620
15	6500	947	1755
16	7000	966	1890
17	7500	985	2025
18	8000	1004	2160

b. Commission only contract is better for sales above $3500 (or equal to $3500)

Exercise 1.7 Payments — government allowances and pensions

1. $244.10

2. $6346.60

3. Factors to consider are: Is she in the 16–24 year bracket and what her parents’ income is.

4. C

5. $697.70

6. George doesn’t receive the Age pension as he is under 65 years and 6 months.

7. $32 385.60

8. **a.** $28.50 **b.** $461.10 **c.** $11 988.60

9. $143.60

10. **a.** 7 hours **b.** $141.25

Exercise 1.8 Personal budgets

1. Fixed expenses are essential expenses like rent, electricity and gas. Discretionary expenses are non-essential expenses and/or can be controlled or varied like entertainment and clothes.

2. D

3. Sample answer: Expenses may include sporting events, entertainment, food, drinks, driving lessons, gym membership and clothing etc.

4. **a.** $1735 **b.** $615 **c.** $725 **d.** 5 weeks

5.

Annual salary	Monthly bank interest	Monthly fixed expenses	Monthly discretionary expenses	Monthly savings
$73700	$784	$3623	$2563	$739.65

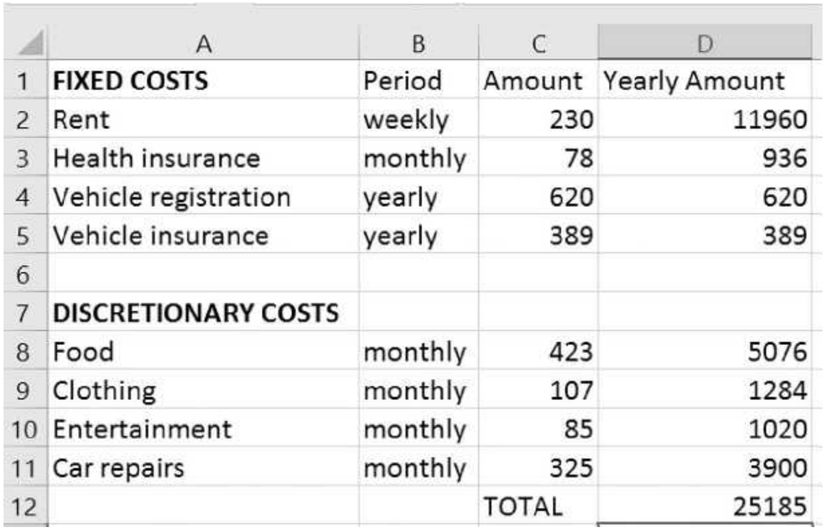
6. **a.** Set up a spreadsheet with the following headings and formulas to calculate the monthly income and expenses.

	A	B	C	D	E
1	**INCOME**	Amount	Period	Monthly Cost	
2	Austudy	445.8	fortnightly	965.90	
3	Presents	250	yearly	20.83	
4				**Total Income**	986.73
5	**EXPENSES**				
6	Board & Food	70	weekly	303.33	
7	Public Transport	80	monthly	80	
8	Books & stationary	45	monthly	45	
9	Health insurance	68.55	monthly	68.55	
10	Entertainment	80	monthly	80	
11	Uni Enrolment Fee	900	yearly	75	
12	Clothes	40	monthly	40	
13	Mobile Phone	50	monthly	50	
14				**Total Expenses**	741.88

b. $244.85

7. $1957.91

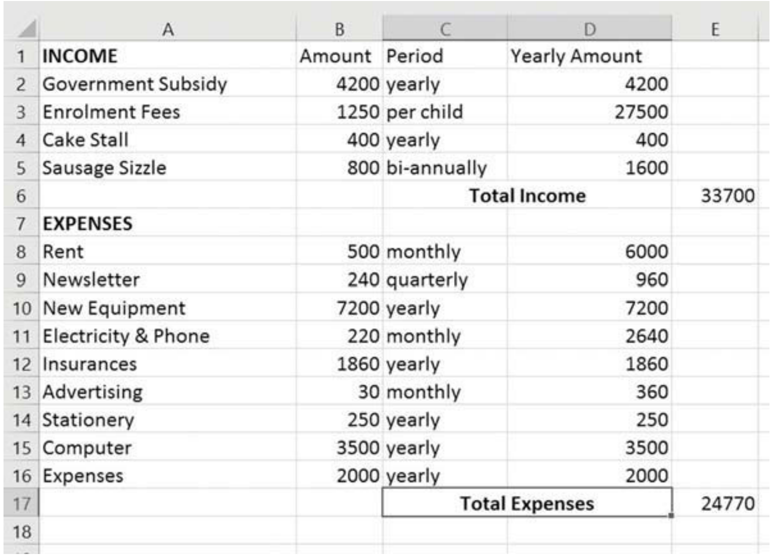
8.



	A	B	C	D
1	**FIXED COSTS**	Period	Amount	Yearly Amount
2	Rent	weekly	230	11960
3	Health insurance	monthly	78	936
4	Vehicle registration	yearly	620	620
5	Vehicle insurance	yearly	389	389
6				
7	**DISCRETIONARY COSTS**			
8	Food	monthly	423	5076
9	Clothing	monthly	107	1284
10	Entertainment	monthly	85	1020
11	Car repairs	monthly	325	3900
12			TOTAL	25185

9. $418.37

10. a.



	A	B	C	D	E
1	INCOME	Amount	Period	Yearly Amount	
2	Government Subsidy	4200	yearly	4200	
3	Enrolment Fees	1250	per child	27500	
4	Cake Stall	400	yearly	400	
5	Sausage Sizzle	800	bi-annually	1600	
6			Total Income		33700
7	EXPENSES				
8	Rent	500	monthly	6000	
9	Newsletter	240	quarterly	960	
10	New Equipment	7200	yearly	7200	
11	Electricity & Phone	220	monthly	2640	
12	Insurances	1860	yearly	1860	
13	Advertising	30	monthly	360	
14	Stationery	250	yearly	250	
15	Computer	3500	yearly	3500	
16	Expenses	2000	yearly	2000	
17			Total Expenses		24770
18					

b. $8930

c. 26.5%

1.9 Review: exam practice

1. B
2. D
3. D
4. D
5. C
6. D
7. D
8. D
9. A
10. C

11. $1500

12. a. Food: 33.33%; electricity: 12.5%; telephone: 5.95%; petrol: 10.12%; rent: 38.10%
    b. Food: 25.45%; electricity: 9.55%; telephone: 4.55%; petrol: 7.73%; rent: 29.09%

13. $308.90

14. $104.40

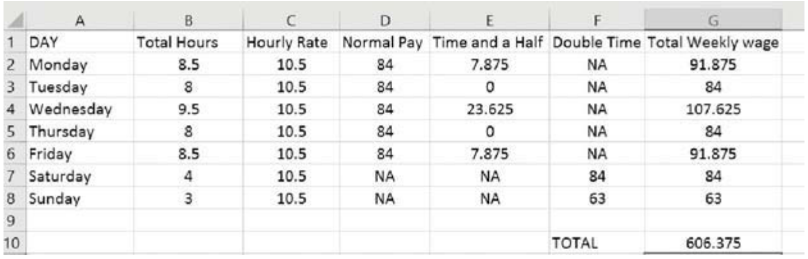
15. Bills will be reduced by 30%

16.

Company name	Price on 1/9/98	Price on 1/10/98	Increase (I) or decrease (D)?	Change of price ($)	% Change
a BHT	$12.82	$8.19	D	$4.63	36.12%
b Super Cole	$5.14	$7.25	I	$2.11	41.05%
c SuNatCo	$21.35	$19.00	D	$2.35	11.01%
d AMB	$4.70	$5.76	I	$1.06	22.55%
e ANX Bank	$9.52	$9.80	I	$0.28	2.94%
f Pronto Co	$0.45	$0.61	I	$0.16	35.56%
g EIK Gold	$25.40	$29.12	I	$3.72	14.65%
h Motors International	$7.80	$7.06	D	$0.74	9.49%
i Optocom au	$5.70	$4.98	D	$0.72	12.63%
j National Metro	$18.28	$19.15	I	$0.87	4.76%

17. a. Jill by $10
    b. Jim must have sales of $3333.33. Jill must have sales of $3500

18.



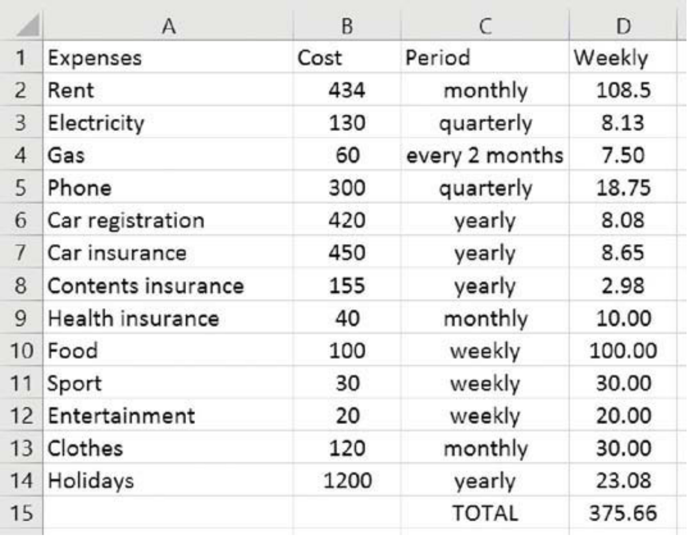
	A	B	C	D	E	F	G
1	DAY	Total Hours	Hourly Rate	Normal Pay	Time and a Half	Double Time	Total Weekly wage
2	Monday	8.5	10.5	84	7.875	NA	91.875
3	Tuesday	8	10.5	84	0	NA	84
4	Wednesday	9.5	10.5	84	23.625	NA	107.625
5	Thursday	8	10.5	84	0	NA	84
6	Friday	8.5	10.5	84	7.875	NA	91.875
7	Saturday	4	10.5	NA	NA	84	84
8	Sunday	3	10.5	NA	NA	63	63
9							
10						TOTAL	606.375

His wages were \$606.40

19.

Item	Cost price (\$)	Percentage discount	Discount (\$)	Selling price (\$)
a	200	12%	24	176
b	150	5%	7.50	142.50
c	98	10%	9.80	88.20
d	130	12.5%	16.25	113.75
e	416	20%	83.20	332.80
f	228	$33\frac{1}{3}\%$	76	152

20. a. Set up a spreadsheet with the following headings and enter formulas to calculate the weekly expenditure.



	A	B	C	D
1	Expenses	Cost	Period	Weekly
2	Rent	434	monthly	108.5
3	Electricity	130	quarterly	8.13
4	Gas	60	every 2 months	7.50
5	Phone	300	quarterly	18.75
6	Car registration	420	yearly	8.08
7	Car insurance	450	yearly	8.65
8	Contents insurance	155	yearly	2.98
9	Health insurance	40	monthly	10.00
10	Food	100	weekly	100.00
11	Sport	30	weekly	30.00
12	Entertainment	20	weekly	20.00
13	Clothes	120	monthly	30.00
14	Holidays	1200	yearly	23.08
15			TOTAL	375.66

b. Rose can save \$4903.60 per year

CHAPTER 2
Consumer arithmetic 2

2.1 Overview

2.1.1 Introduction

All of us are consumers. Who doesn't like to shop sometimes? Since we live in a commercial world, all consumers should try to understand the mathematics that is around us, from shopping online to buying shares on the share market. When buying online or travelling overseas we need to understand exchange rates so that we are able to convert between Australian dollars and foreign currencies. Goods and services tax (GST) is paid on most items we buy in Australia and is presently charged at 10%.

When you invest money you will receive interest and when you take out a loan you pay interest. This interest is calculated as a percentage of the money invested or borrowed and may be calculated as simple interest or compound interest. It is helpful to understand the share market and how dividends are calculated, and whether a company is a good investment based on its price-to-earnings ratio.

LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

2.2 Unit cost

2.2.1 Unit cost method

Unit cost means the cost per unit of an item. It can be the price per litre, per 100mL or per 100g. It is usually used for items sold by weight or volume. It is useful to compare the prices of different sized items and to find which item is the best buy. Supermarkets in Australia are required to show the unit cost on most products.

It is similar to the process used when simplifying rates. If x items cost $\$y$, divide the cost by x to find the price of one item.

$$x \text{ items} = \$y$$

$$1 \text{ item} = \$\frac{y}{x}$$

WORKED EXAMPLE 1

Calculate the cost per 100 grams of pet food if a 1.25 kg box costs $7.50.

THINK	WRITE
1. Identify the cost and the weight. As the final answer is to be referenced in grams, convert the weight from kilograms to grams.	Cost: \$7.50 Weight: 1.250 kg = 1250 g
2. Find the unit cost for 1 gram by dividing the cost by the weight.	$\text{Unit cost} = \frac{7.50}{1250}$ $= 0.006$
3. Find the cost for 100 g by multiplying the unit cost by 100.	$\text{Cost for } 100\text{ g} = 0.006 \times 100$ $= 0.60$ Therefore the cost per 100 grams is \$0.60.

The unit cost method is used to compare the price per unit of items of different sizes. Items must be in the same units when making comparisons.

WORKED EXAMPLE 2

Three shampoos are sold in the following quantities.
Brand A: 200 mL for $5.38
Brand B: 300 mL for $5.98
Brand C: 400 mL for $8.04
Which shampoo is the best buy?

THINK	WRITE
1. Determine the number of 100 mL units for each shampoo.	$\text{Number of 100 mL units of Brand A} = \frac{200}{100} = 2 \text{ units}$ $\text{Number of 100 mL units of Brand B} = \frac{300}{100} = 3 \text{ units}$ $\text{Number of 100 mL units of Brand C} = \frac{400}{100} = 4 \text{ units}$
2. Determine the price per unit for each shampoo. $\text{Price per unit} = \frac{\text{Price}}{100 \text{ mL units}}$	$\text{Brand A} = \frac{5.38}{2} = \$2.69 \text{ per } 100 \text{ mL}$ $\text{Brand B} = \frac{5.98}{3} = \$1.99 \text{ per } 100 \text{ mL}$ $\text{Brand C} = \frac{8.04}{4} = \$2.01 \text{ per } 100 \text{ mL}$ So, \$1.99 per 100 mL is the lowest unit cost.
3. Answer the question for the best buy.	Brand B: 300 mL of shampoo for \$5.98 is the best buy.

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Units 1 & 2 > Area 1 > Sequence 2 > Concept 1 **Unit cost** Summary screen and practice questions

Exercise 2.2 Unit cost

1. Products (and services) other than supermarket items can be sold with a unit pricing scheme. Identify what unit pricing might be used in the following cases.
   - a. Petrol
   - b. A lawyer's fee
   - c. Hotel accommodation
   - d. Lounge room carpet
   - e. Floor tiling
   - f. Wages at a fast-food restaurant
2. Some products are easy to convert to unit prices. Without a calculator, determine the price per 100 grams.
   - a. 1 kg apples at \$3.65
   - b. 500 g laundry powder at \$4.50
3. Without a calculator, calculate the following prices by determining the price per 100 grams.
   - a. 400 g tin of canned peaches at \$2.12
   - b. 5 kg potatoes at \$6.50
4. **WE1** Calculate the cost in dollars per 100 grams for:
   - a. a 650 g box of cereal costing \$6.25
   - b. a 350 g packet of biscuits costing \$3.25

5. Calculate the cost in dollars per 100 grams for:
   a. a 425 g jar of hazelnut spread costing \$3.98
   b. a 550 g container of yoghurt costing \$3.69.
6. Calculate the cost:
   a. per litre if a box of 24 cans that each contains 375 mL costs \$18.00
   b. per 100 mL if a 4 litre bottle of cooking oil costs \$16.75
7. Calculate the cost:
   a. per kilogram if a 250 g pack of cheese slices costs \$5.66.
   b. per kilogram if a 400 g frozen chicken dinner costs \$7.38
8. If 6 avocadoes cost \$13.50, determine how much 9 avocadoes cost.
9. **WE2** Brand H baked beans sell for \$1.86 for 400 grams, while Brand E are \$1.67 for 350 grams. Identify which product has the lower unit price.

10. **MC** Liquids are unit priced according to the price per 100 mL (0.1 litre) instead of per 100 grams. Calculate the unit price of a 1.25 L bottle of cola that sells for \$1.87.
    A. \$0.187
    B. \$0.1496
    C. \$0.1558
    D. \$0.2338
11. Complete the following table by calculating the missing entries.

	Item	Size	Selling price	Unit price (per 100 g or 100 mL)
a	Cheese	450 grams	\$4.78	
b	Onions	2.5 kg		\$0.23
c	BBQ sauce	750 g	\$3.12	
d	Milk	3 litres	\$3.12	

12. A particular car part is shipped in containers that hold 2054 items. Give answers to the following questions correct to the nearest cent.
    a. If each container costs the receiver \$8000, calculate the cost of each item.
    b. If the car parts are sold for a profit of 15%, determine how much is charged for each item.
    c. The shipping company also has smaller containers that cost the receiver \$7000, but only hold 1770 items. If the smaller containers are the only ones available, calculate how much the car part seller must charge to make the same percentage profit.
13. A butcher has the following pre-packed meat specials.

BBQ lamb chops in packs of 12 for \$15.50
Porterhouse steaks in packs of 5 for \$13.80
Chicken drumsticks in packs of 11 for \$11.33

Calculate the price per single unit of each pre-packed meat, correct to the nearest cent.

14. Another nearby butcher matched the price of the butcher in question 13 but decided to bundle the three different meats into packages. Each package contains 2 packs of each type of meat from question 13. Weights for the packages are shown in the table.

Meat	Package 1	Package 2
BBQ lamb chops	2535 grams	2602 grams
Porterhouse steak	1045 grams	1068 grams
Chicken drumsticks	1441 grams	1453 grams

Calculate the price per kilogram for each package correct to the nearest cent.

2.3 Mark-ups and discounts

2.3.1 Mark-ups

A **mark-up** is an amount or a percentage by which goods or services are increased. For example, petrol is frequently marked-up.

To calculate the amount after a mark-up:

- sale price (%) = 100% + mark-up (%)
- convert sale price percentage to a decimal
- multiply the original price by this decimal to calculate the sale price.

WORKED EXAMPLE 3

The price of petrol was marked up by 8.8% from the wholesaler to the retailer. If the wholesaler bought it at 129.5 cents per litre, what is the sale price per litre after the mark-up? Give answer correct to 2 decimal places.

THINK	WRITE
1. Find the total percentage of the sale price including the mark-up.	$\text{Sale price (\%)} = 100\% + 8.8\%$ $= 108.8\%$
2. Convert 108.8% to a decimal.	$\text{Sale price (decimal)} = \frac{108.8}{100}$ $= 1.088$
3. Find the sale price by multiplying the original price by 1.088.	$\text{Sale price per litre} = 129.5 \times 1.088$ $= 140.896$
4. Answer the question, correct to 2 decimal places.	The sale price is 140.90 cents per litre.

When we know the original amount and the sale price and need to find the mark-up as a percentage, we use the following formula:

$$\text{Mark-up \%} = \frac{\text{amount of mark-up}}{\text{original amount}} \times 100$$

WORKED EXAMPLE 4

Cooper bought Bluetooth speakers for \$345 and sold them for \$500. What was the percentage mark-up, correct to the nearest whole number?

THINK	WRITE
1. Find the amount of the mark-up.	$\text{Mark-up} = \$500 - \$345$ $= \$155$
2. Write the amount of the mark-up as a fraction of the original amount.	$\frac{155}{345}$
3. Find the percentage mark-up by multiplying by 100.	$\text{Percentage mark-up} = \frac{155}{345} \times 100$ $= 44.9\%$
4. Answer the question correct to the nearest whole number.	Percentage mark-up was 45%

2.3.2 Discounts

A **discount** is an amount of money by which the price of an item is reduced. If expressed as a percentage of the original price, it is called a **percentage discount**.

$$\textbf{Discount} = \textbf{original price} - \textbf{sale price}$$

$$\textbf{Percentage discount} = \frac{\textbf{discount}}{\textbf{original price}} \times 100\%$$

WORKED EXAMPLE 5

A Playstation game is discounted from \$180 to \$126. Calculate the percentage discount.

THINK	WRITE
1. Find the discount in dollars.	$\text{Discount} = \text{Original price} - \text{sale price}$ $= \$180 - \$126$ $= \$54$

2. Write the formula for the percentage discount.	$\%\ \text{Discount} = \dfrac{\text{discount}}{\text{original price}} \times 100\%$
3. Substitute the values of the discount and the original price into the formula and evaluate.	$\%\ \text{Discount} = \dfrac{54}{180} \times 100\%$ $= 30\%$
4. Answer the question.	The game was discounted by 30%.

Sometimes we are required to find the original price and are given the sale price and the discount, as shown in Worked example 6.

WORKED EXAMPLE 6

Aamir bought a baseball cap, in a 20% off everything sale, for $16. Determine the original price of the cap.

THINK	WRITE
1. The original percentage is 100%, so a discount of 20% means a sale price of 80%.	$(100 - 20)\,\% = 80\%$ $80\%\ \text{of original price} = \$16$
2. Write 80% as a fraction.	$\dfrac{80}{100}\ \text{of original price} = \$16$ $\dfrac{80}{100} = 0.80\ \text{or}\ 0.8$
3. Divide the sale price of $16 by 0.8.	$0.8\ \text{of original price} = \$16$ $\text{Original price} = \dfrac{\$16}{0.8}$ $= \$20$
4. Answer the question.	The original price was $20.

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Units 1 & 2 > Area 1 > Sequence 2 > Concept 2

Mark-ups and discounts Summary screen and practice questions

Exercise 2.3 Mark-ups and discounts

1. **WE3** The wholesale price of petrol was marked up by 5.5% from 108 cents per litre. Calculate the retail price per litre.
2. A transport company adjusts its charges as the price of petrol changes. Calculate what percentage, correct to two decimal places, do their fuel costs change if the price per litre of petrol increases from $1.36 to $1.42.
3. The price of jeans was marked up by 20%. If the original price was $99, calculate the marked up price.
4. **WE4** Smoothies were increased from $4.80 to $5.45 at the local cafe. Calculate the percentage mark-up, correct to the nearest whole number.

5. Calculate the percentage mark-up on an item that was originally $108.90 and sold for $185.50. Give answer correct to 2 decimal places.

6. **WE5** Determine the percentage discount for each of the following items.
   a. A dress, discounted from $80 to $60
   b. A watch, discounted from $365 to $185

7. Compare the following. Which is the greatest percentage discount?
   a. A clock, discounted from $47 to $34
   b. A lamp, discounted from $59 to $42

8. Healthway is promoting health and beauty products with a discount. For the item price tag shown, calculate:
   a. the amount of the discount
   b. the percentage discount.

9. **WE6** Elijah bought some secondhand school books for $280. He got them 60% cheaper than if he purchased them new. Calculate what he would have paid for the books if purchased new.

10. The items shown are from an online electrical store. Next to each item is the retail price and the online price. For each item, calculate:
   i. the discount amount in dollars when the goods are purchased direct
   ii. the percentage discount.

a

MAGIC BLENDER
Blends drinks, sauces, grinds coffee, chops nuts. 12 Month Warranty.
Retail $69.95
ONLINE PRICE $59.90

b

SANDWICH TOASTER
Toasted sandwiches to go, Easy clean. 12 Month Warranty.
Retail $59.95
ONLINE PRICE $47.00

c

TEFLON–BASED IRON
Light weight, easy glide iron. 12 Month Warranty.
Retail $99.95
ONLINE PRICE $78.95

d

RETRO–TOASTER
Your choice of colours, automatic variable control. 12 Month Warranty.
Retail $49.95
ONLINE PRICE $42.50

11. A department store announced a 15% discount on every purchase for one day only. Elena decided to use the opportunity to buy new clothes for her daughter. She bought a dress normally priced at $29, a 3-piece shorts set (normally $30), pants (normally $16), an embroidered top (normally $18) and sandals (normally $26). Calculate:
   a. the total cost of the clothes
   b. the amount she paid after the 15% discount was applied
   c. the amount of money Elena was able to save on these purchases by shopping on that day.

12. Before the beginning of a winter sale, a shop assistant was asked to reduce the prices of all items in the store by 12.5%. She calculated the new prices and attached new tags to the goods. At the end of the sale she was asked to put the old prices back. Unfortunately, the shop assistant had thrown the old tags away as she did not think she would need them again. She decided to add 12.5% to the sale prices. If the shop assistant proceeds in this manner, will she get back to the original prices? Explain your answer with calculations and mathematical reasoning.

13. A carpet company offers a trade discount of 12.5% to a builder for supplying the floor coverings on a new housing estate.
  a. If the builder spends $32 250, calculate how much the carpet was before the discount was applied. Round your answer to the nearest 5 cents.
  b. If the builder charges his customers a total of $35 000, calculate the percentage discount they have received compared to buying direct from the carpet company.

14. The following table shows the mark-ups and discounts applied by a clothing store.

Item	**Cost price**	**Normal retail price (255% mark-up)**	**Standard discount (12.5% mark-down of normal retail price)**	**January sale (32.25% mark-down of normal retail price)**	**Stocktake sale (55% mark-down of normal retail price)**
Socks	$1.85				
Shirts	$12.35				
Trousers	$22.25				
Skirts	$24.45				
Jackets	$32.05				
Ties	$5.65				
Jumpers	$19.95				

Use a spreadsheet to answer these questions.
  a. What calculation is required in order to determine the stocktake sale price?
  b. Enter the information in your spreadsheet and use it to evaluate the normal retail prices and discount prices for each column as indicated.
  c. Calculate the percentage change between the standard discount price and the stocktake sale price of a jacket.

2.4 Goods and services tax (GST)

The **goods and services tax (GST)** is a tax added onto most purchases and services. In Australia it is 10% of the purchase price of an item or service. There are some items and services that are exempt from the GST. These include fresh food, some educational costs and some medical costs. The tax is collected, as part of the total price, at the point of sale.

To calculate the amount of GST payable on an item, we simply calculate 10% of the purchase price.

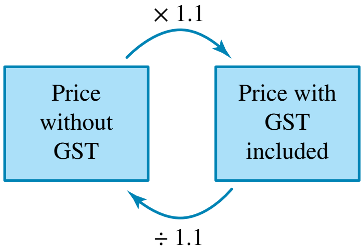
WORKED EXAMPLE 7

A cricket bat has a pre-GST price of $127.50. Calculate the GST payable on the purchase of the bat.

THINK	**WRITE**
Calculate 10% of $127.50.	$\text{GST payable} = 10\%\text{ of }\$127.50$ $= 0.1 \times \$127.50$ $= \$12.75$

2.4.1 Calculating GST

When finding the price of an item with GST included we multiply by 1.1 (110%) and when finding the price excluding GST we divide by 1.1 (110%) as follows:



WORKED EXAMPLE 8

The Besenko family are celebrating the seventh birthday of their daughter, who wants to visit a local theme park. The pre-tax cost of entry for three people is $99. Calculate how much the Besenkos have to pay to enter the theme park, including the GST.

THINK	WRITE
1. Calculate 110% of \$99.	$\text{Total cost} = 110\% \text{ of } \$99$ $= 1.1 \times \$99$ $= \$108.90$
2. Give a written answer.	The cost of entry will be \$108.90.

WORKED EXAMPLE 9

Calculate the pre-tax price of a car that costs $31 350, including GST.

THINK	WRITE
1. Total cost is 110% of the price.	$110\% = \$31\,350$
2. Price is total cost divided by 1.1.	$\text{Pre-tax price} = \$31\,350 \div 1.1$ $= \$28\,500$
3. Answer the question.	The car's pre-tax price was \$28 500.

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Units 1 & 2 › Area 1 › Sequence 2 › Concept 3 › **GST** Summary screen and practice questions

on Resources

Digital document: SkillSHEET Increasing a quantity by a percentage (doc-6902)

Digital document: SkillSHEET Decreasing a quantity by a percentage (doc-5349)

Exercise 2.4 Goods and services tax (GST)

1. **WE7** Calculate the GST payable on a book which has a pre-tax price of \$35.60.
2. Calculate the GST payable on each of the following items (prices given are pre-tax):
   a. a bottle of dishwashing liquid at \$2.30
   b. a basketball at \$68.90
   c. a pair of cargo pants at \$98.50
   d. a bus fare at \$1.30.
3. Calculate the GST payable on each of the following items (correct to the nearest cent):
   a. a barbecued chicken with a pre-tax price of \$7.99
   b. a tin of shoe polish with a pre-tax price of \$4.81
   c. a tin of dog food with a pre-tax price of 93c
   d. a pack of toilet rolls with a pre-tax price of \$6.25.
4. **WE8** A pair of sports shoes that cost \$112.50 has the 10% GST added to its cost. Calculate the total cost of the sports shoes.
5. Calculate the total cost of each of the following items after the 10% GST has been added (prices given are pre-tax):
   a. a football jersey priced at \$114.90
   b. a CD priced at \$29.90
   c. a bunch of flowers priced at \$14.70
   d. a birthday card priced at \$4.95.

6. **WE9** A restaurant bill totals \$108.35, including the 10% GST. Calculate the actual price of the meal before the GST was added.
7. A bus fare was \$2.09, including the 10% GST. Calculate:
   a. the bus fare without the GST
   b. how much GST was paid.
8. Austin travels to the USA. In the state of Utah the VAT is levied at 11%. Calculate what Austin will pay for four nights accommodation in a hotel which charges \$78.40 per night.
9. Calculate the amount of GST included in an item purchased for a total of:
   a. \$34.98
   b. \$586.85
   c. \$56 367.85
   d. \$2.31.
10. Two companies are competing for the same job. Company A quotes a total of \$5575 inclusive of GST. Company B quotes \$5800 plus GST, but offers a 10% reduction on the total price for payment in cash. Determine which is the cheaper offer, and by how much.
11. Sachin decides to purchase a new car. The pre-tax cost for the basic model of the car is \$30 500. It is an extra \$1200 for an automatic car, an extra \$1600 for airconditioning, \$1000 for power steering, \$600 for metallic paint and \$450 for alloy wheels. Calculate the cost of each of the following cars, after the 10% GST has been added:
    a. the basic model car
    b. an automatic car with airconditioning
    c. a car with metallic paint and alloy wheels
    d. a car with all of the above added extras.

12. A plumber quotes his clients the cost of any parts required plus $74.50 per hour for his labour, and then adds on the required GST.
   a. Calculate how much he should quote for a job that requires $250 in parts (excluding GST) and should take 4 hours to complete.
   b. If the job ends up being faster than he first thought, and he ends up charging the client for only 3 hours labour, calculate the percentage discount on the original quote this represents.

13. Jules is shopping for groceries and buys the following items.
   Bread —$3.30*
   Fruit juice —$5.50*
   Meat pies — $5.80
   Ice-cream — $6.90
   Breakfast cereal — $5.00*
   Biscuits — $2.90
   All prices are listed before GST has been added.
   a. The items marked with an asterisk (*) are exempt from GST. Calculate the total amount of GST Jules has to pay for his shopping.
   b. Calculate the additional amount Jules would have to pay if all of the items were eligible for GST.
   c. Jules has a voucher that gives him a 10% discount from this shop. Use your answer from part **a** to calculate how much Jules pays for his groceries.
14. The Australian government is considering raising the GST tax from 10% to 12.5% in order to raise funds and cut the budget deficit.
   The following shopping bill lists all items exclusive of GST. Calculate the amount by which this shopping bill would increase if the rise in GST did go through.
   Note: GST must be paid on all of the items in this bill.
   1 litre of soft drink — $2.80
   Large bag of pretzels — $5.30
   Frozen lasagne — $6.15
   Bottle of shampoo — $7.60
   Box of chocolate — $8.35
   2 tins of dog food —$1.75 each

2.5 Profit and loss

2.5.1 Calculating profit and loss

When an item is sold for more than it costs the difference is said to be a **profit**. It is customary to express profit as a percentage of the cost price:

$$\textbf{Profit} = \textbf{selling price} - \textbf{cost price}$$

$$\textbf{Percentage profit} = \frac{\textbf{profit}}{\textbf{cost price}} \times 100\%$$

$$\textbf{Loss} = \textbf{cost price} - \textbf{selling price}$$

$$\textbf{Percentage loss} = \frac{\textbf{loss}}{\textbf{cost price}} \times 100\%$$

WORKED EXAMPLE 10

Calculate the percentage profit on an item that was bought for \$30 and later sold for \$38.

THINK	WRITE
1. Identify the cost price (CP) and the selling price (SP).	$\text{CP} = \$30;\ \text{SP} = \$38$
2. Write the formula for the profit. (SP > CP)	$\text{Profit} = \text{SP} - \text{CP}$
3. Substitute the values of CP and SP into the formula and evaluate.	$\text{Profit} = \$38 - \$30$ $= \$8$
4. Write the formula for the percentage profit.	$\text{Percentage profit} = \dfrac{\text{profit}}{\text{CP}} \times 100\%$
5. Substitute the values of profit and CP into the formula and evaluate.	$\text{Percentage profit} = \dfrac{8}{30} \times 100\%$ $\approx 26.67\%$

The selling price of an item can be determined if the cost price and profit or loss (*x*, given as a percentage) are known. The profit (or loss) can considered as a percentage change of the cost price.

For profit: selling price = cost price × (100 + *x*)%

For loss: selling price = cost price × (100 − *x*)%

where *x* is the profit or loss expressed as a percentage.

WORKED EXAMPLE 11

Amira buys a watch online for \$485 and sells it to her friend at a profit of 35%. Calculate how much she sold the watch for.

THINK	WRITE
1. The selling price % is the sum of the cost price 100% and the profit (x) 35%	$\text{Selling price }\% = (100 + x)\%\text{ of }\$485$ $= (100 + 35)\%\text{ of }\$485$ $= 135\%$
2. Write 135% as a decimal.	$135\% = \dfrac{135}{100}$ $= 1.35$
3. Using the formula $\text{selling price} = \text{cost price} \times (100 + x)\%$ Multiply the cost price \$485 by 1.35	$\text{Selling price} = \text{Cost price} \times 135\%$ $= 485 \times 1.35$ $= \$654.75$
4. Answer the question	Amira sold the watch for \$654.75.

To find the cost price of an item, when the selling price and profit or loss (x, given as a percentage) are known, we use the following:

$$\textbf{For profit: cost price} = \frac{\textbf{selling price}}{(100 + x)\%}$$

$$\textbf{For loss: cost price} = \frac{\textbf{selling price}}{(100 - x)\%}$$

where x is the profit or loss, expressed as a percentage.

WORKED EXAMPLE 12

A retailer sells a smartphone for \$732, making a profit of 22%. Calculate the wholesale price of the smartphone.

THINK	WRITE
1. The selling price % is the sum of the cost price 100% and the profit (x) 22%.	$\text{Selling price}\,\% = (100 + x)\,\%$ $= (100 + 22)\,\%$ $= 122\%$
2. Write 122% as a decimal.	$122\% = \frac{122}{100}$ $122\% = 1.22$
3. Using the formula $\text{Cost price} = \frac{\text{Selling price}}{(100 + x)\%}$ divide selling price \$732 by 1.22.	$\text{Cost price} = \frac{\text{Selling price}}{122\%}$ $\text{Cost price} = \frac{732}{1.22}$ $= 600$
4. Answer the question	The wholesale price was \$600

on Resources

Interactivity: Calculating percentage change (int-6459)

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Units 1 & 2 > Area 1 > Sequence 2 > Concept 4 > **Profit and loss** Summary screen and practice questions

Exercise 2.5 Profit and loss

1. **WE10** Calculate the percentage profit (to 2 decimal places) for each of the following items.

	Item	Cost price (\$)	Selling price (\$)
a	Tracksuit	80	139.95
b	T-shirt	16	22.50
c	Tennis shoes	49.95	89.95
d	Tank top	6	9

2. Calculate the percentage profit (to 2 decimal places) for each of the following items.

	Item	Cost price ($)	Selling price ($)
a	Swimsuit	38	59
b	Short socks	2	5.95
c	Training pants	20	29
d	Tennis skirt	22	36

3. The following goods were sold at a garage sale. Calculate the percentage loss for each of the items, correct to 2 decimal places.

	Item	Cost price ($)	Selling price ($)
a	Cutlery	40	8
b	Two bedside lamps	100	22
c	Vase	35	5
d	Toaster	19.95	1.50

4. Calculate the percentage loss for each item (to 2 decimal places).

	Item	Cost price ($)	Selling price ($)
a	Electric kettle	42	6
b	Set of golf clubs	150	45
c	Set of building blocks	16	4
d	Five paperback books by Sydney Sheldon	60	2.50

5. A shopkeeper buys 20 kg of cooking chocolate for $50 and sells it in 500 g packets at $3 each. Determine the profit made and express it as a percentage of the cost price.
6. Alex had a collection of 5 Betallica CDs, which he purchased over a period of time at $29.95 each. A friend offered to pay $70 for the whole set. Calculate the loss in dollars and express it as a percentage of the cost price.
7. **WE11** A pair of running shoes is advertised at $140. What does a customer pay if a 25% discount is applied?
8. A shopkeeper at the Southbank Markets buys sheepskin boots from the wholesaler at the following prices: children's sizes — $120 per pair; adults' sizes — $170 per pair, and extra-large sizes — $190 per pair. If the shopkeeper wants to make a 20% profit, determine what the sale price should be for each type.

9. Michael buys a car for $12 000. It depreciates at a rate of $900 per year. If Michael wants his losses to be no more than 30% of the cost price, determine how many years from the purchase he has to sell the car.
10. **WE12** Hans sells a restaurant for $198 000. He calculates that he has made a 10% profit on the buying price. Calculate what he paid for the restaurant originally.

11. By selling a collection of coins for \$177, Igor makes a profit of 18%. Calculate the original cost of the collection.

12. A retailer has purchased a particular style of jumper which is proving to be unpopular. After attempting to sell them for two consecutive seasons, the retailer decides to put them on sale at \$15 each to recover part of the cost. Calculate the wholesale price of each jumper if the retailer suffers a 40% loss.
13. John paid \$50 for a dozen trophies. If he sells them for \$14 each, calculate the percentage profit.
14. P-Mart make 30% on all their sales. If they pay \$600 for a dining suite, calculate what price they should sell it for to make the desired profit.

2.6 Simple interest

2.6.1 Calculating simple interest

When you invest money you receive interest and when you borrow money you pay interest. Simple interest is calculated as a percentage of the amount of money invested or borrowed. It remains constant for the term of the investment or loan.

To calculate simple interest, we use the following formula:

$$\textbf{Simple interest} = \textbf{principal} \times \frac{\textbf{rate}}{\textbf{100}} \times \textbf{time}$$

$$I = Pin$$

- **Principal: money to be invested or borrowed, *P***
- **rate: interest rate as a percentage per time period, *i***
- **time: length of time of investment or loan, *n*.**

WORKED EXAMPLE 13

Calculate the amount of simple interest earned on an investment of \$4450 that returns 6.5% per annum for 3 years.

THINK	WRITE
1. Identify the components of the simple interest formula.	$P = \$4450$ $i = 6.5\%$ $n = 3$
2. Substitute the values into the formula and evaluate the amount of interest.	$I = Pin$ $= 4450 \times \frac{6.5}{100} \times 3$ $= 867.75$
3. State the answer	The amount of simple interest earned is \$867.75.

The amount of an investment is the sum of the initial investment, P, and the simple interest, I.

Amount = principal + interest

$$A = P + I$$

WORKED EXAMPLE 14

$20 000 is invested for 6 years at 7.5% per annum simple interest. Calculate the amount of the investment after 6 years.

THINK	WRITE
1. Amount is the sum of initial investment, principal, and the interest, simple interest.	$A = P + I$
2. Calculate simple interest using the formula $I = Pin$	$I = Pin,\ P = 20\,000, i = \dfrac{7.5}{100}, n = 6$ $I = 20\,000 \times \dfrac{7.5}{100} \times 6$ $I = 9000$
3. Find the amount by adding the interest to the principal.	$A = P + I$ $A = 20\,000 + 9000$ $A = 29\,000$
4. Answer the question	The amount after 6 years is $29 000.

To calculate the principal, rate or time, we transpose the simple interest formula to derive the following formulas:

To find the time: $n = \dfrac{I}{Pi}$

To find the interest rate: $i = \dfrac{I}{Pn}$

To find the principal: $P = \dfrac{I}{in}$

WORKED EXAMPLE 15

Determine how long it will take an investment of $2500 to earn $1100 with a simple interest rate of 5.5% p.a.

THINK	WRITE
1. Identify the components of the simple interest formula.	$P = 2500$ $I = 1100$ $i = 5.5$
2. Substitute the values into the formula and evaluate for n.	$n = \dfrac{I}{Pi}$ $= \dfrac{1100}{2500 \times \frac{5.5}{100}}$ $= \dfrac{1100}{137.5}$
3. State the answer.	It will take 8 years for the investment to earn $1100.

Interest rates, i, are usually stated in terms of a yearly rate, per annum. However, it is very common for interest to be calculated quarterly, monthly, weekly or even daily.

For example, a bank may have a yearly interest rate of 6% per annum, but if interest is calculated weekly, the rate is $\frac{6}{52} = 0.11538\%$ per week.

If i is the rate per annum, we can convert to different time periods as follows:

- Quarterly: $\dfrac{i}{4}$
- Monthly: $\dfrac{i}{12}$
- Weekly: $\dfrac{i}{52}$

The interest rate, i, and the time period, n, must be calculated in the same time periods.

For example, 6% per annum for 2 years, with interest calculated monthly, becomes 0.5% per month $\left(\frac{6}{12} = 0.5\right)$ for 24 months.

WORKED EXAMPLE 16

Calculate the simple interest calculated on a loan of \$665 for 3 years at 7% p.a. calculated quarterly.

THINK	WRITE
1. Identify the components of the simple interest formula, $I = Pin$	$P = \$665$ $i = 7\%$per annum $n = 3$ years
2. Interest is calculated quarterly, so find the interest rate per quarter.	$7\% = \dfrac{7}{4} = 1.75\%$ Interest rate per quarter = 1.75%
3. Calculate the number of quarters in 3 years.	$3 \times 4 = 12$ quarters
4. Use simple interest formula to calculate interest for 3 years	$I = Pin$ $= 665 \times \dfrac{1.75}{100} \times 12$ $= 139.65$
5. Answer the question *Note:* the simple interest after 3 years will be the same if calculated yearly, quarterly or daily.	The simple interest is \$139.65

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Units 1 & 2 Area 1 Sequence 2 Concept 5 **Simple interest** Summary screen and practice questions

Exercise 2.6 Simple interest

1. **WE13** Calculate the simple interest earned on the following investments of:
   - **a.** \$2575, returning 8.25% per annum for 4 years
   - **b.** \$12 250, returning 5.15% per annum for $6\frac{1}{2}$ years
   - **c.** \$43 500, returning 12.325% per annum for 8 years and 3 months
   - **d.** \$103 995, returning 2.015% per annum for 105 months.

2. **WE14** Calculate the amount of the investment, at simple interest, on:
   a. \$500, after returning 3.55% per annum for 3 years
   b. \$2054, after returning 4.22% per annum for $7\frac{3}{4}$ years
   c. \$3500, after returning 11.025% per annum for 9 years and 3 months
   d. \$10 201, after returning 1.008% per annum for 63 months.
3. **WE15** Determine how long it will take an investment of:
   a. \$675 to earn \$216 with a simple interest rate of 3.2% p.a.
   b. \$1000 to earn \$850 with a simple interest rate of 4.25% p.a.
   c. \$5000 to earn \$2100 with a simple interest rate of 5.25% p.a.
   d. \$2500 to earn \$775 with a simple interest rate of 7.75% p.a.
4. a. If \$2000 earns \$590 in 5 years, calculate the simple interest rate.
   b. If \$1800 earns \$648 in 3 years, calculate the simple interest rate.
   c. If \$408 is earned in 6 years with a simple interest rate of 4.25%, calculate how much was invested.
   d. If \$3750 is earned in 12 years with a simple interest rate of 3.125%, calculate how much was invested.
5. **WE16** Calculate the simple interest, for each of the following, if interest is calculated monthly:
   a. a \$8000 loan that is charged simple interest at a rate of 12.25% p.a. for 3 years
   b. a \$23 000 loan that is charged simple interest at a rate of 15.35% p.a. for 6 years
   c. a \$21 050 loan that is charged simple interest at a rate of 11.734% p.a. for 6.25 years
   d. a \$33 224 loan that is charged simple interest at a rate of 23.105% p.a. for 54 months.

6. Calculate how much simple interest is paid on each of the following if invested for 4 years:
   a. \$1224 at 3.6% p.a. calculated yearly
   b. \$955 at 6.024% p.a. calculated monthly
   c. \$2445.50 at 4.8% p.a. calculated yearly
   d. \$13 728.34 at 9.612% p.a. calculated yearly.
7. A savings account with a minimum monthly balance of \$800 earns \$3.60 interest in a month. Calculate the annual rate of simple interest.
8. \$25 000 is invested for 5 years in an account that pays 6.36% p.a. simple interest.
   a. Determine how much interest is earned each year.
   b. Determine what will be the value of the investment after 5 years.
   c. If the money was reinvested for a further 2 years, calculate what simple interest rate would result in the investment amounting to \$35 000 by the end of that time.
9. Rohan invests \$1000 at a simple interest rate of 4.5% p.a. and Ria invests \$800 at a simple interest rate of 8.8%. Use a spreadsheet to calculate when Ria's investment will be greater than Rohan's investment. Give your answer correct to the nearest year.
10. A borrower has to pay 7.8% p.a. simple interest on a 6-year loan. If the total interest paid is \$3744:
    a. calculate how much was borrowed
    b. calculate what the repayments are if they have to be made fortnightly.
11. \$19 245 is invested in a fund that pays a simple interest rate of 7.8% p.a. for 42 months.
    a. Calculate how much simple interest is earned on this investment.
    b. The investor considers an alternative investment with a bank that offers a simple interest rate of 0.625% per month for the first 2.5 years and 0.665% per month after that. Determine which is the best investment.

12. A bank offers a simple interest loan of \$35 000 with monthly repayments of \$545.

- **a.** Calculate what the rate of simple interest is if the loan is paid in full in 15 years.
- **b.** After 5 years of payments the bank offers to reduce the total time of the loan to 12 years if the monthly payments are increased to \$650. Calculate how much interest would be paid over the life of the loan under this arrangement.
- **c.** Calculate the average rate of simple interest over the 12 years under the new arrangement.

13. \$100 is invested in an account that earns \$28 of simple interest in 8 months.

- **a.** Evaluate the annual rate of simple interest.
- **b.** Calculate the amount of interest that would have been earned in the 8 months if the annual interest rate was increased by 0.75%.

14. A bank account pays simple interest at a rate of 7.2% p.a. calculated daily.

- **a.** Determine the daily rate of interest.
- **b.** The account was opened with a deposit of \$250 on 1 July, and regular deposits of \$350 were made every month for the following 6 months. Use a spreadsheet to calculate the interest payable after 6 months.

2.7 Compound interest and inflation on costs

2.7.1 Compound interest

Simple interest is calculated on the original investment or loan and is a constant amount for the duration of the investment or loan. However, compound interest is calculated on the amount plus interest from the previous time period. Sometimes this is expressed as earning interest on interest. Compound interest is not constant as it changes for every period of the investment or loan.

For example, consider an investment of \$5000 that earns 5% p.a. compounding annually. At the end of the first year, the interest amounts to $\frac{5}{100} \times 5000 = \$250$, so the total investment will become \$5250. At the end of the second year, the interest now amounts to $\frac{5}{100} \times 5250 = \$262.50$. As time progresses, the amount of interest becomes larger at each calculation. In contrast, a simple interest rate calculation on this balance would be a constant, unchanging amount of \$250 each year.

2.7.2 The compound interest formula

To find the final amount, *A*, of a compound interest investment or loan, we use the following formula:

$$\boldsymbol{A = P(1 + i)^n}$$

where *A* is the final amount, *P* is the principal, *i* is the rate of interest per period and *n* is the number of compounding periods.

To find the compound interest, *I*, we subtract the initial investment or loan, *P*, from the final amount, *A*, as follows:

$$\text{Interest} = \text{Amount} - \text{Principal}$$

$$I = A - P$$

WORKED EXAMPLE 17

Use the compound interest formula to calculate the amount of interest on an investment of $2500 at 3.5% p.a. compounded annually for 4 years, correct to the nearest cent.

THINK	WRITE
1. Identify the components of the compound interest formula, $A = P\ (1 + i)^n$	$P = 2500$ $i = 3.5\%$ $n = 4$
2. Substitute the values into the formula and evaluate the amount of the investment.	$A = P\,(1 + i)^n$ $= 2500\left(1 + \frac{3.5}{100}\right)^4$ $= 2868.81$ (to 2 decimal places)
3. Subtract the principal from the final amount of the investment to calculate the interest.	$I = A - P$ $= 2868.81 - 2500$ $= 368.81$
4. State the answer.	The amount of compound interest is $368.81.

WORKED EXAMPLE 18

Use the compound interest formula to calculate the amount of interest accumulated on $1735 at 7.2% p.a. for 4 years if the compounding occurs monthly. Give your answer correct to the nearest 5 cents.

THINK	WRITE
1. Identify the components of the compound interest formula, $A = P\,(1 + i)^n$.	$P = \$1735$ $i = 7.2\%$ per annum $n = 4$ years
2. Interest is calculated monthly so find the interest rate per month.	$7.2\% = \frac{7.2}{12} = 0.6\%$
3. Determine the number of months in 4 years.	$4 \times 12 = 48$ months
4. Use the compound interest formula to calculate total amount after 4 years, using $r = 0.6\%$ and $n = 48$ months.	$A = P\,(1 + i)^n$ $A = 1735\left(1 + \frac{0.6}{100}\right)^{48}$ $A = 2312.08$
5. Find the compound interest by subtracting the principal from the amount.	$I = A - P$ $I = 2312.08 - 1735$ $I = 577.08$
6. Answer the question.	The compound interest is $577.10

2.7.3 Inflation on costs

Inflation is a measure of how an economy is performing over a period of time. It is an increase in the price of goods and services and a decrease in the value of our money. Inflation is a rate that is expressed as a percentage and in Australia is called the consumer price index (CPI).

The compound interest formula is used to calculate the future cost of an item due to inflation.

WORKED EXAMPLE 19

An investment property is purchased for \$300 000 and is sold 3 years later for \$320 000. If the average annual inflation is 2.5% p.a., determine if this has been a profitable investment.

THINK	WRITE
1. Recall that inflation is an application of compound interest and identify the components of the formula.	$P = 300\,000$ $i = 2.5\%$ $n = 3$
2. Substitute the values into the formula and evaluate the amount.	$A = P(1+i)^n$ $= 300\,000\left(1+\frac{2.5}{100}\right)^3$ $= 323\,067.19$ (to 2 decimal places)
3. Compare the inflated amount to the selling price.	Inflated amount: \$323 067.19 Selling price: \$320 000
4. State the answer.	This has not been a profitable investment, as the selling price is less than the inflated purchase price.

on Resources

Interactivity: Simple and compound interest (int-6265)

studyon

Units 1 & 2 › Area 1 › Sequence 2 › Concept 6

Compound interest and inflation Summary screen and practice questions

Exercise 2.7 Compound interest and inflation on costs

Unless otherwise directed, where appropriate give all answers to the following questions correct to 2 decimal places or the nearest cent.

1. **WE17** Use the compound interest formula to calculate the amount of compound interest on an investment of:
   a. \$4655 at 4.55% p.a. for 3 years
   b. \$12 344 at 6.35% p.a. for 6 years
   c. \$3465 at 2.015% p.a. for 8 years
   d. \$365 000 at 7.65% p.a. for 20 years.

2. **WE18** Use the compound interest formula to calculate the final amount for:
   - a. $675 at 2.42% p.a. for 2 years compounding weekly
   - b. $4235 at 6.43% p.a. for 3 years compounding quarterly
   - c. $85 276 at 8.14% p.a. for 4 years compounding fortnightly
   - d. $53 412 at 4.329% p.a. for 1 year compounding daily.
3. Use the compound interest formula to calculate the principal required to yield a final amount of:
   - a. $15 000 after compounding at a rate of 5.25% p.a. for 8 years
   - b. $22 500 after compounding at a rate of 7.15% p.a. for 10 years
   - c. $1000 after compounding at a rate of 1.25% p.a. for 2 years
   - d. $80 000 after compounding at a rate of 6.18% p.a. for 15 years.

4. **WE19** An investment property was purchased for $325 000 and sold 5 years later for $370 000. If the average annual inflation is 2.73% p.a., decide if this has been a profitable investment.
5. A business is purchased for $180 000 and is sold 2 years later for $200 000. If the annual average inflation is 1.8% p.a., determine if a real profit has been made.
6. An $8000 investment earns 7.8% p.a. compound interest over 3 years. Calculate how much interest is earned if the amount is compounded:
   - a. annually
   - b. monthly
   - c. weekly
   - d. daily.
7. Shivani is given $5000 by her grandparents on the condition that she invests it for at least 3 years. Her parents help her to find the best investment options and come up with the following choices.
   - i. A local business promising a return of 3.5% compounded annually, with an additional 2% bonus on the total sum paid at the end of the 3-year period
   - ii. A building society paying a fixed interest rate of 4.3% compounded monthly
   - iii. A venture capitalist company guaranteeing a return of 3.9% compounded daily
     - a. Calculate the expected return after 3 years for each of the options.
     - b. Assuming each option is equally secure, decide where Shivani should invest her money.

8. The costs of manufacturing a smart watch decrease by 10% each year.
   - a. If the watch initially retails at $200 and the makers decrease the price in line with the manufacturing costs, calculate how much it will cost at the end of the first 3 years.
   - b. Inflation is at a steady rate of 3% over each of these years, and the price of the watch also rises with the rate of inflation. Recalculate the cost of the watch for each of the 3 years according to inflation. (*Note:* Apply the inflation price increase before the manufacturing cost decrease.)
9. a. Use a spreadsheet to compare $1000 compounding annually with compounding quarterly at a rate of 12% p.a. for 5 years.
   - b. Determine the effect of compounding at more regular intervals during the year.

10. Francisco is a purchaser of fine art, and his two favourite pieces are a sculpture he purchased 17 years ago for \$12 000 and a series of prints he purchased 9 years ago for \$17 000.

- **a.** If inflation averaged 3.3% for the period between when he bought the sculpture and when he bought the prints, which item cost more in real terms?
- **b.** The value of the sculpture has appreciated at a rate of 7.5% since it was purchased, and the value of the prints has appreciated at a rate of 6.8% since they were purchased. Calculate how much they are both worth. Round your answers correct to the nearest dollar.

2.8 Exchange rates

2.8.1 Foreign currency

When travelling overseas or buying online from overseas companies, we need to exchange or convert Australian dollars (AUD or A\$) to the foreign currency. Exchange rates are the amount at which one currency can be exchanged for another currency. These rates change daily, and are published in the financial section of newspapers and reported in the news. For example, one Australian dollar is typically equivalent to:

Currency	Symbol	Exchange rate
Canadian dollar, CAD	\$	0.9643
European euro, EUR	€	0.6416
Hong Kong dollar, HKD	\$	5.9146
Japanese yen, JPY	¥	85.28
Malaysian ringgit, MYR	RM	3.1527
New Zealand dollar, NZD	\$	1.1067
Thai baht, THB		24.89
UK pound sterling, GBP	£	0.5725
US dollar, USD	\$	0.7574

To convert from Australian dollars to a foreign currency, we use:

$$\textbf{Foreign currency} = \textbf{AUD} \times \textbf{exchange rate}$$

WORKED EXAMPLE 20

Convert A$500 to Malaysian ringgit.

THINK	WRITE
1. Use the table to find the exchange rate for $1 converted to RM.	A$1 = RM3.1527
2. Multiply by 500 to determine the value of A$500.	$3.1527 \times 500 = 1576.35$
3. Write the answer	1576.35 ringgit or RM1576.35

To convert to Australian dollars from a foreign currency, we use:

$$\text{AUD} = \frac{\text{foreign currency}}{\text{exchange rate}}$$

WORKED EXAMPLE 21

Convert €415 to Australian dollars.

THINK	WRITE
1. Use the table to find the exchange rate for €1 converted into AUD.	A$1 = €0.6416
2. Divide by 0.6416.	$\frac{415}{0.6416} = 646.82$
3. Write the answer.	A$646.80

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Units 1 & 2 > Area 1 > Sequence 2 > Concept 7 **Exchange rates** Summary screen and practice questions

Exercise 2.8 Exchange rates

1. **WE20** Convert A$35 to Thai baht.
2. Convert each of the following, given in Australian dollars:
   a. $390 to US dollars
   b. $5000 to British pounds sterling
   c. $950 to Thai baht
   d. $34 000 to Hong Kong dollars.
3. **WE21** Convert RM3896 to Australian dollars

4. Calculate how much you would receive in Australian dollars, if you exchange:

a. US$4500 **b.** 880 baht **c.** £3500 **d.** ¥700.

5. Shae is going on an overseas trip and has budgeted to spend A$800 in each country she visits. How much foreign currency can she spend in each of the following countries?

a. Italy **b.** Japan **c.** Scotland **d.** USA

6. Emily buys a pair of boots online from Italy for €200, with free delivery. Calculate how much this is in Australian dollars.

7. Hiromi is on exchange with a family in Sydney and has ¥45 800 to spend. Calculate how much this is in Australian dollars.

8. Michael is going hiking in New Zealand and gets a travelcard loaded with NZ$1090. Calculate how much it costs him in Australian dollars if he has to pay A$20 for the card.

9. Determine the amount in Australian dollars, to the nearest cent, to buy:

a. a bottle of Coke in San Francisco for $8
b. a bowl of noodles in Tokyo for ¥600
c. a watch in Madrid for €540
d. a ticket to the theatre in London for £30.

10. Harry plans to visit Tokyo on business. He changes A$800 into Japanese yen.

a. Calculate how much he receives in yen.
b. If he spends ¥50 000 in Tokyo, calculate how many yen he has left.
c. On return to Australia, Harry changes his leftover yen to Australian dollars. Calculate how much he will have in dollars.

11. Holly travels to Germany. She changes A$660 into European euro.

a. Calculate how many euros she has.
b. When Holly is in Germany she spends 220 euros. Calculate how many euros she has left.
c. If she changes these back to Australian dollars, calculate how much she will she have.

12. During an economic crisis in 1998, Indonesia experienced severe inflation. In one week, on Monday, A$1 would have bought 9500 rupiah whereas on Thursday A$1 would have bought 10 900 rupiah. On holidays in Indonesia at this time, Joel exchanged A$120 and paid for a camera on Monday. Calculate how much he would have saved if he had waited to make the transaction on Thursday (assuming the marked price did not change). Give your answer in Australian dollars.

2.9 Dividends

2.9.1 Shareholders and investing

A dividend is a payment that a company makes to its shareholders for investing in their company. This means that shareholders own part of the company. When the company makes a profit, the shareholders are paid a percentage of this profit, which is called a dividend. Shares can be bought and sold on the share market or the stock exchange.

WORKED EXAMPLE 22

Shane owns 350 shares in Waves Surf Company. There is a total of 7000 shares in the company. Calculate the percentage of the company Shane owns.

THINK	WRITE
1. Find the fraction of the company that Shane owns.	$\% \text{ ownership} = \dfrac{350}{7000} \times 100$
2. Multiply by 100 to convert to a percentage.	$= 5\%$
3. Answer the question.	Shane owns 5% of the company.

WORKED EXAMPLE 23

A company wishes to raise 1 million dollars by selling 25 000 new shares. Calculate the starting value of each share.

THINK	WRITE
1. Since the total money to be raised is \$1 000 000 and there are 25 000 shares, divide to find the starting value of each share.	$\text{Starting value} = \dfrac{1\,000\,000}{25\,000}$ $= \$40$
2. Answer the question.	The starting price of each share is \$40.

A dividend can be calculated in one of the following ways:

1. the number of dollars paid per share
2. a percentage of the price of the shares, called a dividend yield.

2.9.2 Dividend paid per share

To calculate a dividend, the profit shared is divided by the total number of shares in the company.

$$\textbf{Dividend} = \frac{\textbf{profit}}{\textbf{total number of shares}}$$

WORKED EXAMPLE 24

Calculate the dividend payable for a company with 2 500 000 shares when \$525 000 of its annual profit is distributed to the shareholders.

THINK	WRITE
1. Divide the profit by the number of shares.	$\text{Dividend} = \dfrac{525\,000}{2\,500\,000} = \$0.21$
2. State the final answer.	The dividend payable will be 21 cents per share.

2.9.3 Dividend yield

Shares in different companies can vary drastically in price, from cents up to hundreds of dollars for a single share. As a company becomes more successful the share price will rise, and as a company becomes less successful the share price will fall.

An important factor that investors look at when deciding where to invest is the **percentage dividend** (also known as **dividend yield**) of a company. The dividend yield is calculated by dividing the dividend per share by the share price per share.

$$\textbf{Dividend yield} = \frac{\textbf{dividend}}{\textbf{share price}} \times 100$$

WORKED EXAMPLE 25

Calculate the dividend yield of a share that costs \$13.45 with a dividend per share of \$0.45. Give your answer correct to 2 decimal places.

THINK	WRITE
1. To determine the dividend yield, divide the dividend per share by the price of a share.	$\text{Dividend yield} = \frac{0.45}{13.45}$
2. Express the result as a percentage.	$= 3.3457\%, 0.03457 \ldots \times 100$
3. Round your answer to 2 decimal places and state the answer.	$= 3.35\%$ (to 2 decimal places) The dividend yield per share is 3.35%.

2.9.4 The price-to-earnings ratio

The price-to-earnings ratio (P/E ratio) refers to the market price of a company's share price divided by its earnings or profit per share. It gives an indication of how much shares cost per dollar of profit earned. The higher the price-to-earnings ratio, the more you have to invest for each dollar of profit.

The formula to calculate price-to-earnings ratio is:

$$\textbf{Price-to-earnings ratio} = \frac{\textbf{market price per share}}{\textbf{annual earnings per share}}$$

WORKED EXAMPLE 26

Calculate the price-to-earnings ratio for a company whose shares are \$12.75 per share and give an annual earning per share of \$1.50.

THINK	WRITE
1. Substitute share price per share of \$12.75 and annual earnings per share of \$1.50 into the formula: $\frac{\text{market price per share}}{\text{annual earnings per share}}$	$\text{Price-to-earnings ratio} = \frac{12.75}{1.5}$ $= 8.5$
2. Answer the question.	Price-to-earnings ratio is 8.5

on Resources

Interactivity: Shares and currency (int-6460)

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Units 1 & 2 > Area 1 > Sequence 2 > Concept 8 > **Dividends** Summary screen and practice questions

Exercise 2.9 Dividends

1. **WE22** Nola owns 5000 shares in the company Click Dotcom. There are 240 000 shares in the company. Calculate the percentage of Click Dotcom Nola owns. Give your answer correct to 1 decimal place.
2. **WE23** A company wishes to raise $20 million by selling 400 000 new shares. Calculate the starting value of each share.
3. **MC** A company has 340 000 shares worth $5.89 each. Calculate the total value of the shares.
   A. $2 002 600 **B.** $2 000 260 **C.** $57 724 **D.** $20 026 000
4. Complete the following table:

	Number of shares	Single share price	Total value of shares
a		$36.16	$4 520 000
b	35 500	$27.56	
c	789 500		$3 503 562
d	205 500	$2.05	

5. **WE24** Calculate the dividend payable per share for a company with:
   a. 32 220 600 shares, when $1 995 000 of its annual profit is distributed to the shareholders
   b. 44 676 211 shares, when $5 966 000 of its annual profit is distributed to the shareholders
6. Determine how many shares are in a company that declares a dividend of:
   a. 28.6 cents per share when $1 045 600 of its annual profit is distributed.
   b. $2.34 per share when $3 265 340 of its annual profit is distributed.
   c. $16.62 per share when $9 853 000 of its annual profit is distributed.
   d. $34.95 per share when $15 020 960 of its annual profit is distributed.

7. **WE25** Calculate the dividend yields of the following shares.
   a. A share price of $14.60 with a dividend of 93 cents
   b. A share price of $22.34 with a dividend of 87 cents
   c. A share price of $45.50 with a dividend of $2.34
   d. A share price of $33.41 with a dividend of $2.88

8. **MC** Alexandra is having trouble deciding which of the following companies to invest in. She wants to choose the company with the highest dividend yield. Calculate the dividend yield for each company to find out which Alexandra should choose.
   - **A.** A clothing company with a share price of \$9.45 and a dividend of 45 cents
   - **B.** A mining company with a share price of \$53.20 and a dividend of \$1.55
   - **C.** A financial company with a share price of \$33.47 and a dividend of \$1.22
   - **D.** A technology company with a share price of \$7.22 and a dividend of 41 cents
9. **WE26** Calculate the price-to-earnings ratio for a company with:
   - **a.** a current share price of \$12.50 per share and an annual earning of 25 cents per share
   - **b.** a current share price of \$43.25 per share and an annual earning of \$1.24 per share
   - **c.** a current share price of \$79.92 per share and an annual earning of \$3.32 per share
   - **d.** a current share price of \$116.46 per share and an annual earning of \$7.64 per share.
10. Calculate the earnings per share for a company with:
    - **a.** a price-to-earnings ratio of 25.5 and a current share price of \$8.75
    - **b.** a price-to-earnings ratio of 20.3 and a current share price of \$24.35
    - **c.** a price-to-earnings ratio of 12.2 and a current share price of \$10.10
    - **d.** a price-to-earnings ratio of 26 and a current share price of \$102.
11. The details of two companies are shown in the following table.

Company	Share price	Net profit	Total shares
Company A	\$34.50	\$8 600 000	6 500 000
Company B	\$1.48	\$1 224 000	5 550 000

    - **a.** Calculate the dividend per share payable for shareholders in each company if each of the companies re-invests 12.5% of the net profit.
    - **b.** Calculate the price-to-earnings ratio for each company. Remember earnings are also referred to as profit.
    - **c.** If a shareholder has 500 shares in Company A and 1000 shares in Company B, calculate how much they will receive from their dividends.
    - **d.** Which company represents the best investment?
12. The share price of a mining company over several years is shown in the following table.

Year	2012	2013	2014	2015	2016
Share price	\$44.50	\$39.80	\$41.20	\$31.80	\$29.60
Dividend per share	\$1.73	\$3.25	\$2.74	\$3.15	\$3.42

    - **a.** If there are a total of 10 000 000 shares in the company, and 35% of the net profit was reinvested each year, use a spreadsheet or other technology to calculate the net profit for each of the years listed.
    - **b.** Determine the price-to-earnings ratios for each of the years listed.
    - **c.** Determine which was the best year to purchase shares in the company.

2.10 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. **MC** The unit cost (per gram) of a 120 gram tube of toothpaste sold for \$3.70 is:

A. \$32.43 **B.** \$0.03 **C.** \$0.44 **D.** \$0.05

2. **MC** A 1.25 L bottle of soft drink costs \$1.25. With the annual inflation rate over the next 4 years expected to be 4%, the bottle will cost:

A. \$1.54 **B.** \$6.00 **C.** \$1.75 **D.** \$1.46

3. **MC** If the price of petrol increased from 118.4 cents to 130.9 cents, the percentage change is:

A. 10.6% **B.** 9.5% **C.** 90% **D.** 1.1%

4. **MC** A basketball ring is sold for \$28.50. If this represents a 24% reduction from the recommended retail price (RRP), the original price was:

A. \$90.25 **B.** \$118.75 **C.** \$52.50 **D.** \$37.50

5. **MC** A tradesman offers a 6.8% discount for customers who pay in cash. Calculate how much a customer would pay if they paid their bill of \$244 in cash.

A. \$16.59 **B.** \$218.48 **C.** \$227.41 **D.** \$261.59 **E.** \$237.20

6. **MC** When the simple interest formula is transposed to find i, determine the correct formula:

A. $i = IPn$ **B.** $i = \dfrac{Pn}{PnI}$ **C.** $i = n$ **D.** $i = \dfrac{I}{Pn}$

7. **MC** David buys a skateboard online for US\$190. If the exchange rate is A\$1 for US\$0.7574, calculate how much he pays in Australian dollars, to the nearest cent.

A. \$143.90 **B.** \$250.85 **C.** \$190 **D.** \$250.86

8. **MC** The price-to-earnings ratio for a company with a share price of \$2.40 and a profit of 87 cents per share is:

A. 2.09 **B.** 2.76 **C.** 0.03 **D.** 3.27

9. **MC** Determine which of the following companies has the lowest share price.

A. Company A with a price-to-earnings ratio of 10.4 and a profit of \$1.87 per share
B. Company B with a price-to-earnings ratio of 28.1 and a profit of 36 cents per share
C. Company C with a price-to-earnings ratio of 14.8 and a profit of 79 cents per share
D. Company D with a price-to-earnings ratio of 35.75 and a profit of 97 cents per share

10. **MC** Determine the principal, to the nearest \$100, to be invested for 6 years at 8% per annum, compounded bi-annually, and giving a final amount of \$15 000:

A. \$24 500 **B.** \$9 400 **C.** \$11 000 **D.** \$900

11. For each of the following, calculate the unit price for the quantity shown in brackets.

a. 750 g of Weetbix for \$4.99 (per 100 g)
b. \$16.80 for 900 g of jelly beans (per 100 g)
c. \$4.50 for 1.5 L of milk (per 100 mL)
d. \$126.95 for 15 L of paint (per L)

12. Determine the amount of GST included in the price or needing to be added to the price for the following amounts.

a. \$45.50 with GST included
b. \$109.00 plus GST
c. \$448.75 with GST included
d. \$13.25 plus GST

Complex familiar

13. Determine the unknown variable for each of the following.

- **a.** Calculate the amount of simple interest earned on an investment of \$4500 that returns 6.87% per annum for 5.5 years.
- **b.** Determine how long it will take an investment of \$1260 to earn \$350 with a simple interest rate of 4.08%.
- **c.** Calculate the simple interest rate on an investment that earns \$645 in 3 years when the initial principal was \$5300.
- **d.** Calculate the monthly repayments for a \$6250 loan that is charged simple interest at a rate of 9.32% per annum for 7.25 years.

14. Use the compound interest formula to calculate:

- **a.** The interest on an investment of \$3655 at 6.54% per annum for 2.5 years
- **b.** How much \$478 will amount to after 10 years if invested at 2.27% per annum
- **c.** The interest on an investment of \$3550 at 5% per annum in 3 years compounded quarterly
- **d.** The principal required to yield a final amount of \$22 000 after compounding at a rate of 11.2% per annum for 15 years

15. Sophie purchased an investment property for \$250 000, and 4 years later she sold the property for \$275 000. Given the average annual inflation was 2.82% per annum, determine if this has been a profitable investment for Sophie.

16. Use a spreadsheet to calculate the price-to-earnings for each of the following companies shown in this table.

Company	Company A	Company B	Company C	Company D	Company E
Currency	Australian dollars	US dollars	European euros	Chinese yuan	Indian rupees
Share price	\$23.35	\$26.80	€16.20	¥133.5	₹1288
Profit per share	\$1.46	\$1.69	€0.94	¥8.7	₹65.5

Complex unfamiliar

17. John is comparing different brands of lollies at the local supermarket. A packet of Brand A lollies costs \$7.25 and weighs 250 g. A packet of Brand B lollies weighs 1.2 kg and costs \$22.50.

- **a.** Decide which brand is the best value for money. Provide mathematical evidence to support your answer.
- **b.** If the more expensive brand was to reconsider their price, calculate the price for their lollies that would match the unit price of the cheaper brand.

18. Four years ago a business was for sale at \$130 000. Amanda and Callan had the money to purchase the business but missed out at the auction. Four years later the business is again for sale, but now at \$185 000.

- **a.** Determine the percentage increase in the price over the 4 years.
- **b.** Amanda and Callan will now need to borrow the increase in the price amount. Calculate how much interest they will have to pay on a loan compounded annually over 5 years with a rate of 12.75%.

19. Adam purchased 1600 shares in a company that recently announced it had achieved an annual profit of \$890 500. The company has 200 000 shareholders.

- **a.** Determine the dividend payable to Adam.
- **b.** He originally purchased the shares for \$2.17 each. Calculate the dividend yield of the value of the shares.
- **c.** Adam decides to reinvest the dividend payment in a compound interest account at 14.4% p.a. compounded weekly. Calculate how much interest he will earn if he invests for 1.5 years.

20. Tayla is planning to backpack around Australia in a year and estimates that she needs \$1400 for the trip. She has already saved \$1095 and is considering the following investments to help her reach her target:

Option A:
Invest at a rate of 9.85% p.a. simple interest.

a. Determine how much interest Tayla will earn on this investment in one year.

Option B:
Open a Super-Saver investment account that pays 9.55% p.a. compounded daily.

b. Determine how much interest Tayla will earn on this investment in one year.

c. Tayla chooses Option B. After one year of this investment she realises that she doesn't have enough money so she borrows some money from her parents. She re-invests the original savings, interest earned in part **b** and the money she borrowed for another 12 months at the same rate.
Calculate how much she must borrow from her parents.

studyon

Units 1 & 2 Sit chapter test

Answers

Chapter 2 Consumer arithmetic 2

Exercise 2.2 Unit cost

1. a. per litre **b.** per hour **c.** per day
d. per square metre **e.** per square metre **f.** per hour

2. a. \$0.37 **b.** \$0.90

3. a. \$0.53 **b.** \$0.13

4. a. \$0.96 **b.** \$0.93

5. a. \$0.94 **b.** \$0.67

6. a. \$2 per litre **b.** \$0.42 per 100 mL

7. a. \$22.64 per kg **b.** \$18.45 per kg

8. \$20.25

9. Brand H

10. B

11.

Item	Size	Selling price	Unit price (per 100 g or 100 mL)
Cheese	450 g	\$4.78	\$1.06
Onions	2.5kg	\$5.75	\$0.23
BBQ sauce	750 g	\$3.12	\$0.42
Milk	3 litres	\$3.12	\$0.10

12. a. \$3.89 **b.** \$4.47 **c.** \$4.54

13. BBQ lamb chops: \$1.29
Porterhouse steak: \$2.76
Chicken drumsticks: \$1.03

14. Package 1: \$16.18/kg
Package 2: \$15.86/kg

Exercise 2.3 Mark-ups and discounts

1. 113.94 cents

2. 4.41%

3. \$118.80

4. 14%

5. 70.34%

6. a. 25% **b.** 49.3%

7. a. 27.7%
b. 28.8%, so b is the greatest discount

8. a. \$50 **b.** 50.25%

9. \$700

10. i. a. \$10.05 **ii. a.** 14.37%
b. \$12.95 **b.** 21.60%
c. \$21.00 **c.** 21.01%
d. \$7.45 **d.** 14.91%

11. a. \$119 **b.** \$101.15 **c.** \$17.85

12. No, as the 12.5% is calculated from different amounts. For example, a \$60.00 item reduced by 12.5% (\$7.50) is \$52.50. A \$52.50 item increased by 12.5% (\$6.56) is \$59.06.

13. a. \$36 857.15 **b.** 5.04%

14. **a.** Normal retail price × 0.45

b.

Item	Cost price	Normal retail price (255% mark-up)	Standard discount (12.5% mark-down of normal retail price)	January sale (32.25% mark-down of normal retail price)	Stocktake sale (55% mark-down of normal retail price)
Socks	$1.85	$6.57	$5.75	$4.45	$2.96
Shirts	$12.35	$43.84	$38.36	$29.70	$19.73
Trousers	$22.25	$78.99	$69.12	$53.52	$35.55
Skirts	$24.45	$86.80	$75.95	$58.81	$39.06
Jackets	$32.05	$113.78	$99.56	$77.09	$51.20
Ties	$5.65	$20.06	$17.55	$13.59	$9.03
Jumpers	$19.95	$70.82	$61.97	$47.98	$31.87

c. 48.57%

Exercise 2.4 Goods and services tax (GST)

1. $3.56

2. **a.** $0.23 **b.** $6.89 **c.** $9.85 **d.** $0.13

3. **a.** 80c **b.** 48c **c.** 9c **d.** 63c

4. $123.75

5. **a.** $126.39 **b.** $32.89 **c.** $16.17 **d.** $5.45

6. $98.70

7. **a.** $1.90 **b.** $0.19

8. $348.10

9. **a.** $3.18 **b.** $53.35 **c.** $5124.35 **d.** $0.21

10. Company A by $167

11. **a.** $33 550 **b.** $36 630 **c.** $34 705 **d.** $38 885

12. **a.** $602.80 **b.** 13.59%

13. **a.** $1.56 **b.** $1.38 **c.** $27.86

14. $0.84

Exercise 2.5 Profit and loss

1. **a.** 74.94% **b.** 40.63% **c.** 80.08% **d.** 50%

2. **a.** 55.26% **b.** 197.5% **c.** 45% **d.** 63.64%

3. **a.** 80% **b.** 78% **c.** 85.71% **d.** 92.48%

4. **a.** 85.71% **b.** 70% **c.** 75% **d.** 95.83%

5. $70, 140%

6. $79.75; 53.26%

7. $105

8. Children's price: $144, adult's price: $204, extra-large: $228

9. 4 years

10. $180 000

11. $150

12. $25

13. 236%

14. $780

Exercise 2.6 Simple interest

1. **a.** $849.75 **b.** $4100.69 **c.** $44 231.34 **d.** $18 335.62

2. **a.** $553.25 **b.** $2725.76 **c.** $7069.34 **d.** $10 740.84

3. **a.** 10 years **b.** 20 years **c.** 8 years **d.** 4 years

4. **a.** 5.9% **b.** 12% **c.** $1600 **d.** $10 000

5. **a.** $2939.99 **b.** $21 183.10 **c.** $15 437.54 **d.** $34 543.88

6. **a.** $176.26 **b.** $230.12 **c.** $469.54 **d.** $5278.27

7. 5.4%

8. **a.** $1590 **b.** $32 950 **c.** 3.11%

9. In the 8th year

10. **a.** $8000 **b.** $75.28

11. **a.** $5253.89
b. The first investment at 7.8%

12. **a.** 12.02% **b.** $52 300 **c.** 12.45%

13. **a.** 42% **b.** $28.50

14. **a.** 0.02% **b.** $41.33

Exercise 2.7 Compound interest and inflation on costs

1. **a.** $664.76 **b.** $5515.98 **c.** $599.58 **d.** $1 229 312.85

2. **a.** $708.47 **b.** $5128.17 **c.** $118 035.38 **d.** $55 774.84

3. **a.** $9961.26 **b.** $11 278.74 **c.** $975.46 **d.** $32 542.37

4. No, a loss of $1851.73 occurred.

5. Yes, a profit of $13461.68 has been made

6. **a.** $2021.81 **b.** $2101.50 **c.** $2107.38 **d.** $2108.90

7. **a.** **i.** $5654.46 **ii.** $5690.54 **iii.** $5620.26
b. The building society

8. **a.** $145.80
b. Year 1: $185.40, Year 2: $171.86, Year 3: $159.32

9. **a.**

Compounding annually	
Year	**Amount**
1	$1000.00
2	$1120.00
3	$1254.40
4	$1404.93
5	$1573.52

Compounding quarterly	
Quarter	**Amount**
1	$1000.00
2	$1030.00
3	$1060.90
4	$1092.73
5	$1125.51
6	$1159.27
7	$1194.05
8	$1229.87
9	$1266.77
10	$1304.77
11	$1343.92
12	$1384.23
13	$1425.76
14	$1468.53
15	$1512.59
16	$1557.97
17	$1604.71
18	$1652.85
19	$1702.43
20	$1753.51

b. Compounding at more regular intervals pays more interest.

10. **a.** The prints ($17 000) cost more than the sculpture ($15 559.08) in real terms.
b. Sculpture was worth $41 032. Prints were worth $30 732.

Exercise 2.8 Exchange rates

1. ฿871.15

2. **a.** US$295.39 **b.** £2862.50 **c.** ฿23 646 **d.** HK$201 096

3. A$1235.77

4. **a.** A$5941.38 **b.** A$35.36 **c.** A$6113.54 **d.** A$8.21

5. **a.** €513.28 **b.** ¥68 224 **c.** £458 **d.** US$605.92

6. $311.72

7. $537.05

8. A$1004.91

9. **a.** $10.56 **b.** $7.04 **c.** $841.65 **d.** $52.40

10. **a.** ¥68 224 **b.** ¥18 224 **c.** A$213.70

11. **a.** €423.46 **b.** €203.46 **c.** $317.11

12. A$15.41

Exercise 2.9 Dividends

1. 2.08%

2. $50 per share

3. A

4. **a.** 125 000 **b.** $978 380 **c.** $4.44 **d.** $421 275

5. **a.** $0.06 **b.** $0.13

6. **a.** 3 655 944 shares **b.** 1 395 444 shares **c.** 592 840 shares **d.** 429 784 shares

7. **a.** 6.37% **b.** 3.89% **c.** 5.14% **d.** 8.62%

8. D

9. **a.** 50 **b.** 34.9 **c.** 24.1 **d.** 15.2

10. **a.** 34 cents/share **b.** $1.20/share **c.** 83 cents/share **d.** $3.92/share

11. **a.** Company A: $1.16; Company B: $0.19 **b.** Company A: 29.74; Company B: 7.79
c. $770 **d.** Company B

12. **a, b**

Year	Net profit	Price- to-earnings ratio
2012	$26 615 384.62	25.72
2013	$50 000 000.00	12.25
2014	$42 153 846.15	15.04
2015	$48 461 538.46	10.10
2016	$52 615 384.62	8.65

c. 2016

2.10 Review: exam practice

1. B

2. D

3. A

4. D

5. C

6. D

7. D

8. B

9. B

10. B

11. **a.** \$0.67 **b.** \$1.88 **c.** \$0.30 **d.** \$8.46

12. **a.** \$4.14 **b.** \$10.90 **c.** \$40.79 **d.** \$1.33

13. **a.** \$1700.33 **b.** 7 years **c.** 4.06% **d.** \$120.38

14. **a.** \$627.22 **b.** \$598.29 **c.** \$570.68 **d.** \$4475.60

15. She made a loss of \$4415.44 when inflation is taken into account.

16.

Company	Currency	Share price	Profit per share	P/E Ratio
Company A	AU Dollars	\$23.35	\$1.46	15.99
Company B	US Dollars	\$26.80	\$1.69	15.86
Company C	European Euros	€16.20	€0.94	17.23
Company D	Chinese Yuan	¥133.50	¥8.70	15.34
Company E	Indian Rupees	₹1, 288.00	₹65.50	19.66

17. **a.** Brand B (\$1.88 compared to \$2.90 per 100 g) **b.** \$4.70

18. **a.** 42.3% **b.** \$45 218

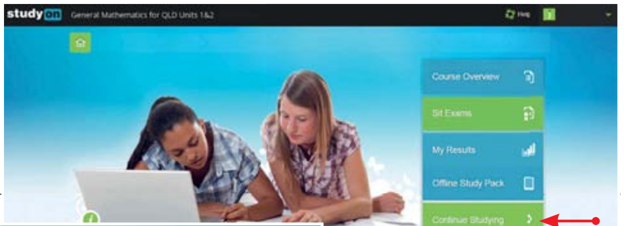
19. **a.** \$7120 **b.** 205.07% **c.** \$1714.01

20. **a.** \$107.86 **b.** \$109.71 **c.** \$67.79

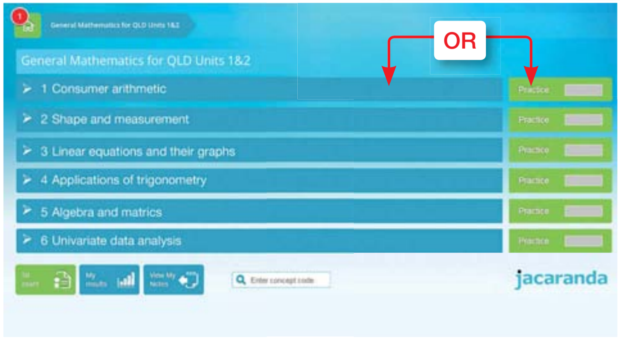
REVISION UNIT 1 Money, measurement and relations

TOPIC 1 Consumer arithmetic

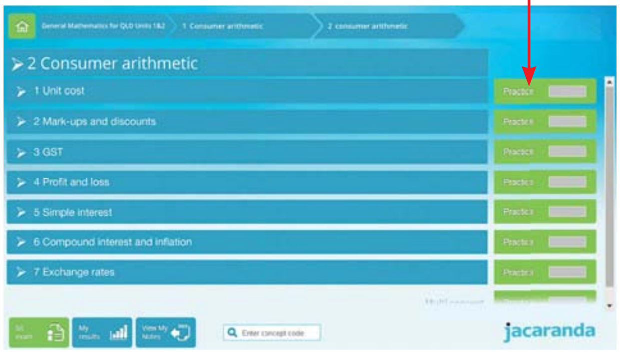
- For revision of this entire topic, go to your **studyON** title in your bookshelf at **www.jacplus.com.au**.
- Select **Continue Studying** to access hundreds of revision questions across your entire course.



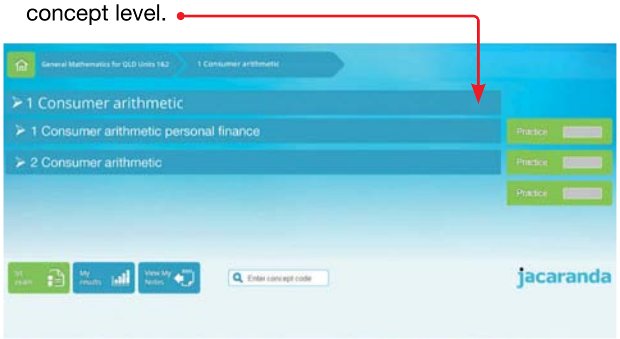
- Select your **course** *General Mathematics for Queensland Units 1&2* to see the entire course divided into syllabus topics.
- Select the **area** you are studying to navigate into the sequence level *OR* select **Practice** to answer all practice questions available for each area.



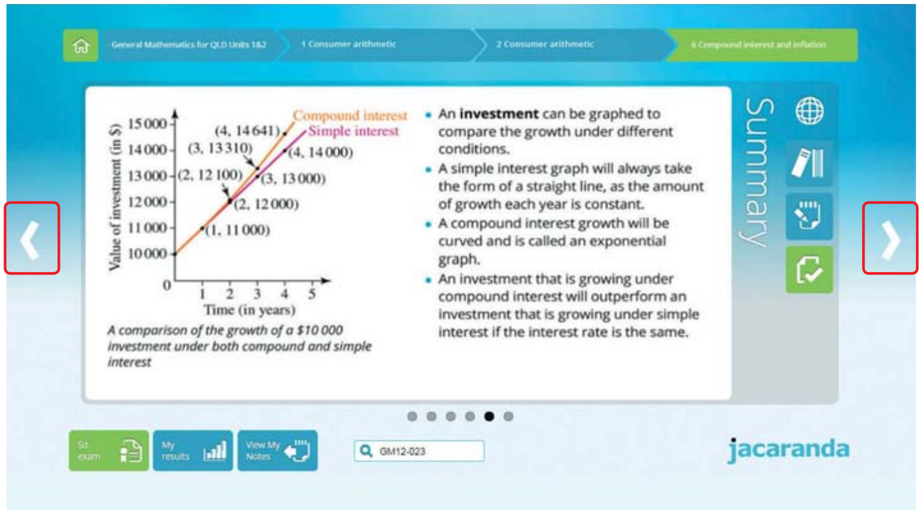
- Select **Practice** at the sequence level to access all questions in the sequence.



- At **sequence level**, drill down to concept level.



- **Summary screens** provide revision and consolidation of key concepts. Select the **next arrow** to revise all concepts in the sequence and practice questions at the concept level for a more granular set of questions.



A comparison of the growth of a $10 000 investment under both compound and simple interest

- An **investment** can be graphed to compare the growth under different conditions.
- A simple interest graph will always take the form of a straight line, as the amount of growth each year is constant.
- A compound interest growth will be curved and is called an exponential graph.
- An investment that is growing under compound interest will outperform an investment that is growing under simple interest if the interest rate is the same.

PRACTICE ASSESSMENT 1

General Mathematics: Problem solving and modelling task

Unit

Unit 1: Money, measurement and relations

Topic

Topic 1: Consumer arithmetic

Conditions

Duration	**Mode**	**Individual/group**
4 weeks	Written report, up to 10 pages (maximum 2000 words) excluding appendix	Individual

Resources permitted

The use of technology is required, for example:
- computer
- internet
- spreadsheet program
- calculator
- other software/technology

Milestones

Week 4		
Week 5		
Week 6		
Week 7 (assessment submission)		

Criterion*	**Marks allocated**	**Result**
Formulate • Assessment objectives 1, 2, 5	4	
Solve • Assessment objectives 1, 6	7	
Evaluate and verify • Assessment objectives 4, 5	5	
Communicate • Assessment objectives 3	4	
Total	20	

Scaffolding

Please refer to the flow chart on the following page describing an appropriate approach to problem solving and modelling.

*Queensland Curriculum & Assessment Authority, *Specialist Mathematics General Senior Syllabus 2019 v1.1*, Brisbane, 2018.
For the most up to date assessment information, please see www.qcaa.qld.edu.au/senior.

Context

A family holiday to Disneyworld, Orlando is a holiday that many families would love to go on. They would be able to visits the four worlds: Magic Kingdom, Epcot, Universal Studios and Animal Kingdom.

For a family to go on this holiday, they will need to understand the financial requirements of such a trip, including fixed and discretionary spending, in order to develop a family budget based on their income.

Task

Investigate the financial requirements of a family of four (2 adults and 2 children) taking a family holiday from Australia to Orlando (USA).

Produce a personal budget based on the results of your mathematical research, then refine the model to ensure they can afford to take the trip. You are to assume:

- the father has a full-time job and the mother works part-time
- the family pays rent on their house
- the family will go to all four worlds and will also look at the possibility of visiting other major attractions in Orlando.

You must use:

- the approach to problem-solving and mathematical modelling provided
- different data and assumptions to other students in your class and school.

You will have four weeks to complete the assessment, including three hours of class time.

To complete this task you must:

- Use the problem-solving and mathematical modelling approach outlined in the General Mathematics syllabus and on the flow chart on the follow page to develop your response.
- Respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.
- Provide a unique response that highlights the real-life application of mathematics.
- Develop a written report that can be read and interpreted independently of the instrument task sheet.
- Use both analytic procedures and technology.

Approach to problem-solving and modelling

Formulate

In this task you will investigate the financial requirements of a family aiming to travel to Orlando (USA) for a family holiday. Your report should consider:

- the family income from the father's full-time work and the mother's part-time work
- the total costs of the trip (including potential increases due to inflation)
- exchange rates for foreign currency while on the holiday.

Design a detailed plan, identifying the mathematical procedures required to solve this problem. Remember to state the necessary assumptions, variables and observations. You must also explain how you will make use of technology.

Solve

Develop a budget to explain how the family could afford the trip. Consider any necessary assumptions, variables and observations in your calculations. You will make further refinements as necessary. The budget should include:

- income from the parent's jobs
- weekly/fortnightly living expenses
- weekly/fortnightly savings.

How will the family finance all the expenses related to their trip? You will need to show the cost of air fares, accommodation, food and entertainment, including potential increases due to inflation, and the use of exchange rates to buy and sell foreign currency while on the holiday and on their return.

You must use technology efficiently and show detailed calculations demonstrating the procedures used to plan and budget for the holiday.

Is it solved?

Evaluate and verify

Evaluate the reasonableness of your original solution.

Based on your budget, consider whether the family can afford the holiday. Look at the strengths and limitations of your plan and make any necessary changes, e.g. extra costs associated with travelling overseas that were not initially considered, or changes to spending in the initial budget to assist in saving more money.

Justify and explain all procedures you have used and decisions you have made. Considering the original task, how valid is your solution?

Is the solution verified?

Commuicate

Once you have completed all necessary calculations, you should consider how you have communicated all aspects of your report. Communicate using appropriate language that refers to the calculations and tables included in previous sections. Your response should be coherently and concisely organised.

Ensure you have:

- used mathematical, statistical and everyday language
- considered the strengths and limitations of your solution
- drawn conclusions by discussing your results
- included recommendations.

CHAPTER 3
Pythagoras' theorem and mensuration

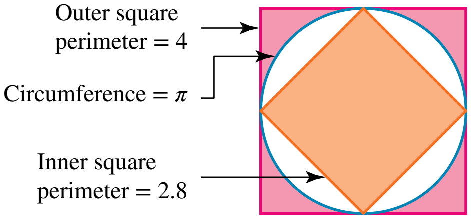
3.1 Overview

When using formulae related to circles we continually use pi. We either use 3.14 or just push the π button on our calculator, but what really is pi and where did it come from?

Pi goes back to work done by Archimedes more than 2000 years ago, when calculators and computers did not exist. He was able to find pi to 99.9% accuracy and this was without decimal places. The techniques he used to do this helped build the foundations of calculus.

Pi is linked to circles because it is the circumference of a circle with a diameter of 1 unit. Archimedes did not know the circumference of a circle, so how did he work out its value? In the diagram below, using a square of side length 1 and hence a circle of diameter 1, trigonometry shows that the value of pi is between the outer square perimeter and the inner square perimeter; thus $2.8 < \pi < 4$. If you take the centre of these two values, you calculate pi to be equal to 3.4.

Archimedes didn't actually use squares; he started with hexagons (six-sided polygons) to find the range of values pi is between. He then continued to 12- , 24- , 48- and 96-sided figures and stopped when he found $3\frac{10}{71} < \pi < 3\frac{1}{7}$. Decimals were not invented until 250 BC, let alone any spreadsheet that could easily perform these repeat calculations, so Archimedes had to spend a lot of time working through his formulae using fractions. The midpoint between $3\frac{10}{71} < \pi < 3\frac{1}{7}$ is 3.14185, which is over 99.9% accurate. This is an amazing achievement considering it was calculated more than 2000 years ago.



LEARNING PATHWAY

Fully worked solutions for this chapter are available in the Resources section of your eBookplus at www.jacplus.com.au.

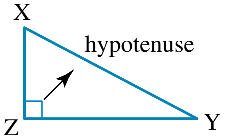
3.2 Pythagoras' theorem in two dimensions

3.2.1 Pythagoras' theorem

The properties of right-angled triangles enable us to calculate lengths and angles which are sometimes not able to be measured. The first property we will investigate is Pythagoras' theorem.

Even though the theorem that describes the relationship between the side lengths of right-angled triangles bears the name of the famous Greek mathematician Pythagoras, who is thought to have lived around 550 BC, evidence exists in some of humanity's earliest relics showing that it was known and used much earlier than that.

Pythagoras' theorem allows us to calculate the length of a side of a right-angled triangle, if we know the lengths of the other two sides. Consider Δ XYZ shown.

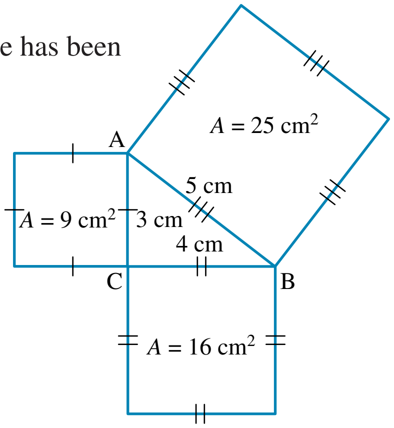


XY is the **hypotenuse** (the longest side). It is opposite the right angle.

Note that the sides of a triangle can be named in either of two ways.

1. A side can be named by the two capital letters given to the vertices at each end. This is what has been done in the figure at right to name the hypotenuse XY.
2. We can also name a side by using the lower-case letter of the opposite vertex. In the figure at right, we could have named the hypotenuse '*z*'.

Consider the right-angled triangle ABC with sides 3 cm, 4 cm and 5 cm. Squares have been constructed on each of the sides. The area of each square has been calculated ($A = S^2$) and indicated.



Note that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

$$25\text{ cm}^2 = 16\text{ cm}^2 + 9\text{ cm}^2$$

Alternatively: $(5\text{ cm})^2 = (4\text{ cm})^2 + (3\text{ cm})^2$

Which means: $\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$

This result is known as **Pythagoras' theorem**. Pythagoras' theorem states:

In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. That is,

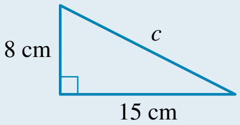
$$\textbf{hypotenuse}^2 = \textbf{base}^2 + \textbf{height}^2$$

It is more commonly expressed as: $c^2 = a^2 + b^2$

This is the formula used to find the length of the hypotenuse in a right-angled triangle when we are given the lengths of the two shorter sides.

WORKED EXAMPLE 1

Calculate the length of the hypotenuse in the triangle given.



THINK	WRITE
1. Write the formula.	$\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$
2. Substitute the lengths of the shorter sides.	$c^2 = 15^2 + 8^2$
3. Evaluate the expression for c^2.	$= 225 + 64$ $= 289$
4. Find the value of c by taking the positive square root.	$c = \sqrt{289}$ $= 17\text{ cm}$

In this example, the answer is a whole number because we are able to find $\sqrt{289}$ exactly. In most examples this will not be possible. In such cases, we are asked to write the answer rounded to a given number of decimal places.

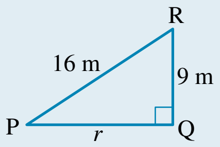
By rearranging Pythagoras' theorem, we can write the formula to find the length of a shorter side of a triangle.

$$\begin{aligned} &\text{Since} & \text{hypotenuse}^2 &= \text{base}^2 + \text{height}^2 \\ &\text{it follows that} & \text{base}^2 &= \text{hypotenuse}^2 - \text{height}^2 \\ &\text{and} & \text{height}^2 &= \text{hypotenuse}^2 - \text{base}^2 \end{aligned}$$

The method of solving this type of question is the same as in the previous example, except that here we use subtraction instead of addition. For this reason, it is important to look at each question carefully to determine whether you are finding the length of the hypotenuse or one of the shorter sides.

WORKED EXAMPLE 2

Calculate the length of side PQ in triangle PQR, rounded to 1 decimal place.



THINK	WRITE
1. Write the formula.	$\text{base}^2 = \text{hypotenuse}^2 - \text{height}^2$
2. Substitute the lengths of the known sides.	$r^2 = 16^2 - 9^2$
3. Evaluate the expression.	$= 256 - 81$ $= 175$
4. Find the answer by finding the square root.	$r = \sqrt{175}$ $= 13.2\text{ m}$

Pythagorean triads (or **Pythagorean triples**) are sets of 3 numbers which satisfy Pythagoras' theorem. The first right-angled triangle we dealt with in this section had side lengths of 3 cm, 4 cm and 5 cm. This satisfied Pythagoras' theorem, so the numbers 3, 4 and 5 form a Pythagorean triad or triple. In fact, any multiple of these numbers, for example 6, 8 and 10; and 1.5, 2 and 2.5 would also form a Pythagorean triad or triple. Some other triads are:

5, 12, 13
8, 15, 17
9, 40, 41.

WORKED EXAMPLE 3

Is the set of numbers 4, 6, 7 a Pythagorean triad?

THINK	WRITE
1. Find the sum of the squares of the two smaller numbers	$4^2 + 6^2 = 16 + 36$ $= 52$
2. Find the square of the largest number.	$7^2 = 49$
3. Compare the two results. The numbers form a Pythagorean triad if the results are the same.	$7^2 \neq 4^2 + 6^2$
4. Write your answer.	4, 6, 7 is *not* a Pythagorean triad.

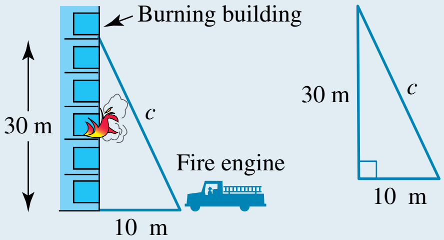
Resources

- **Digital document:** SkillSHEET Pythagoras' theorem (doc-29488)
- **Digital document:** SpreadSHEET Pythagoras (doc-29489)
- **Digital document:** SpreadSHEET Triads (doc-29490)
- **Digital document:** SpreadSHEET Pythagoras' theorem (doc-29491)

Pythagoras' theorem can be used to solve more practical problems. In these cases, it is necessary to draw a diagram that will help you decide the appropriate method for finding a solution. The diagram simply needs to represent the triangle; it does not need to show details of the situation described.

WORKED EXAMPLE 4

The fire brigade attends a blaze in a tall building. They need to rescue a person from the 6th floor of the building, which is 30 m above ground level. Their ladder is 32 m long and must be at least 10 m from the foot of the building. Assess whether the ladder can be used to reach the people needing rescue.

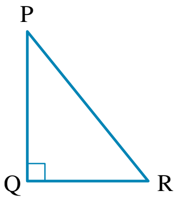
THINK	WRITE
1. Draw a diagram and show all given information.	
2. Write the formula after deciding if you are finding the hypotenuse or a shorter side.	$\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$

3. Substitute the lengths of the known sides.	$c^2 = 10^2 + 30^2$
4. Evaluate the expression.	$= 100 + 900$ $= 1000$
5. Find the answer by taking the square root.	$c = \sqrt{1000}$ $= 31.62$ m
6. Give a written answer.	The ladder will be long enough to make the rescue, since it is 32 m long.

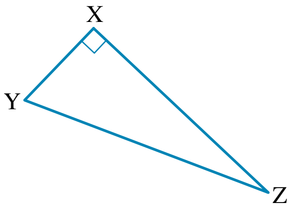
Exercise 3.2 Pythagoras' theorem in two dimensions

1. Identify the hypotenuse in each of the following triangles.

a.

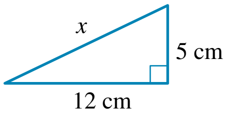


b.

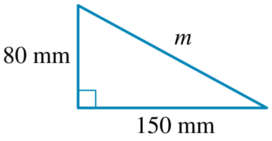


2. **WE1** Calculate the length of the hypotenuse in each of the following triangles.

a.

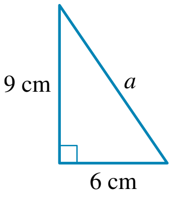


b.

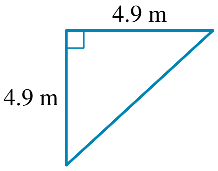


3. In each of the following, calculate the length of the hypotenuse, correct to 2 decimal places.

a.

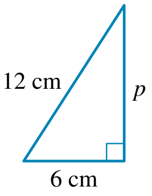


b.

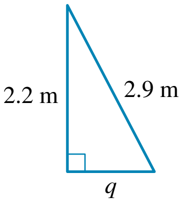


4. **WE2** Calculate the length of each unknown shorter side in the right-angled triangles given. Round your answer to 1 decimal place.

a.

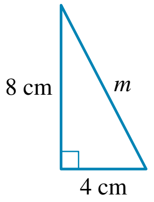


b.

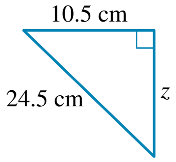


5. In each of the following right-angled triangles, calculate the length of the side marked with a pronumeral, rounded to 1 decimal place.

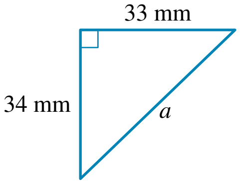
a.



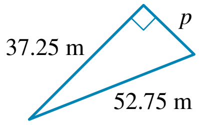
b.



c.

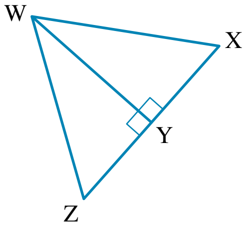


d.



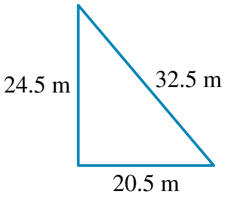
6. **MC** The hypotenuse in ΔWXY at right is:

A. WX
B. XY
C. YZ
D. ZW

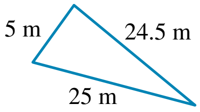


7. **MC** Decide which of the following triangles is definitely right-angled.

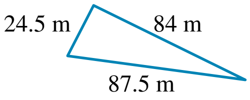
A.



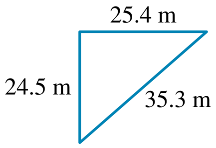
B.



C.



D.



8. **WE3** Are the following sets of numbers Pythagorean triads?

a. 9, 12, 15
b. 4, 5, 6
c. 30, 40, 50
d. 3, 6, 9
e. 0.6, 0.8, 1.0
f. 7, 24, 25

9. Complete the following Pythagorean triads.

a. 9, ____, 15
b. __, 24, 25
c. 1.5, 2.0, __
d. 3, __, 5
e. 11, 60, __
f. 10, __, 26
g. __, 40, 41
h. 0.7, 2.4, __

10. **MC** Decide which of the following is a Pythagorean triad.

A. 7, 14, 21 **B.** 1.2, 1.5, 3.6 **C.** 3, 6, 9 **D.** 15, 20, 25

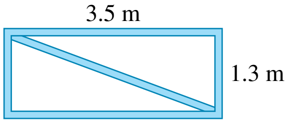
11. **MC** Determine which of the following is *not* a Pythagorean triad?

A. 5, 4, 3 **B.** 6, 9, 11 **C.** 13, 84, 85 **D.** 0.9, 4.0, 4.1

12. **WE4** A television antenna is 12 m high. To support it, wires are attached to the ground 5 m from the foot of the antenna. Calculate the length of each wire.

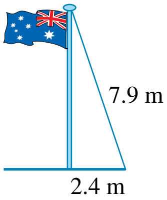
13. Susie needs to clean the guttering on her roof. She places her ladder 1.2 m back from the edge of the guttering that is 3 m above the ground. How long will Susie's ladder need to be (rounded to 2 decimal places)?

14. A rectangular gate is 3.5 m long and 1.3 m wide. The gate is to be strengthened by a diagonal brace as shown at right. How long should the brace be (correct to 2 decimal places)?



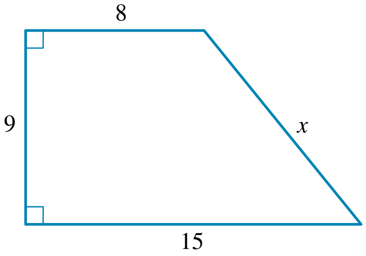
15. A 2.5 m ladder leans against a brick wall. The foot of the ladder is 1.2 m from the foot of the wall. How high up the wall will the ladder reach (correct to 1 decimal place)?

16. Use the measurements in the diagram to determine the height of the flagpole, correct to 1 decimal place.



17. An isosceles, right-angled triangle has a hypotenuse of 10 cm. Calculate the length of the shorter sides. (*Hint:* Call both shorter sides x.)

18. Calculate the length of the unknown side in the following diagram, giving your answer correct to 2 decimal places.



3.3 Pythagoras' theorem in three dimensions

3.3.1 Using Pythagoras' theorem in three dimensions

Many three-dimensional objects contain right-angled triangles that can be modelled with two-dimensional drawings. Using this method we can calculate missing side lengths of three-dimensional objects.

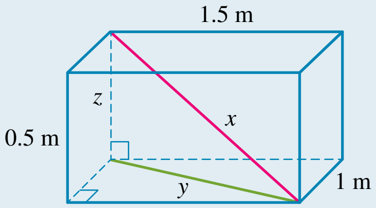
WORKED EXAMPLE 5

Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions 1 m × 1.5 m × 0.5 m.

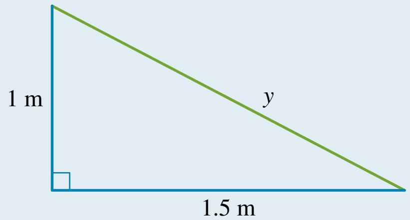
THINK

1. Draw a diagram of a rectangular box with a rod in it, labelling the dimensions.
2. Draw in a right-angled triangle that has the metal rod as one of the sides, as shown in pink and labelled x. The length of y in this right-angled triangle is not known. Draw in another right-angled triangle to calculate the length of y, as shown in green.

WRITE/DRAW

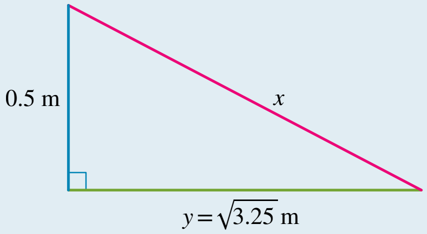


3. Calculate the length of y using Pythagoras' theorem as an exact value.



$$c^2 = a^2 + b^2$$
$$y^2 = 1.5^2 + 1^2$$
$$= 3.25$$
$$y = \sqrt{3.25}$$

4. Draw the right-angled triangle containing the rod and use Pythagoras' theorem to calculate the length of the rod (x).



$$c^2 = a^2 + b^2$$
$$x^2 = \left(\sqrt{3.25}\right)^2 + 0.5^2$$
$$= 3.25 + 0.25$$
$$= 3.5$$
$$x = \sqrt{3.5}$$
$$\approx 1.87$$

5. Answer the question.

The maximum length of the metal rod is 1.87 m (correct to 2 decimal places).

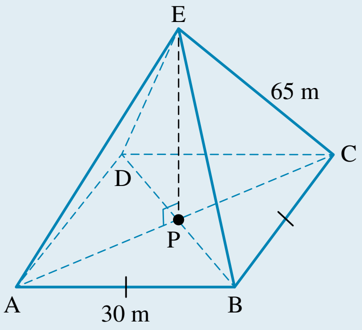
WORKED EXAMPLE 6

A square pyramid has a base length of 30 m and a slant edge of 65 m. Determine the height of the pyramid, rounding your answer to 1 decimal place.

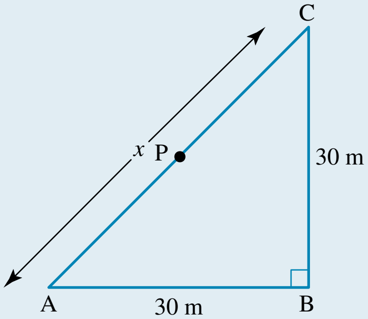
THINK

WRITE/DRAW

1. Draw a diagram to represent the situation. Add a point in the centre of the diagram immediately below the apex of the pyramid.



2. Determine the diagonal distance across the base of the pyramid by using Pythagoras' theorem, leaving your answer in exact form.

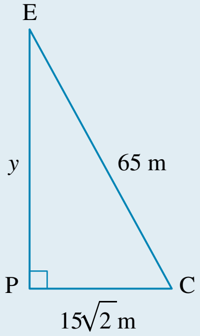


$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
x^2 &= 30^2 + 30^2 \\
&= 900 + 900 \\
&= 1800 \\
x &= \sqrt{1800} \\
&= \sqrt{900 \times 2} \\
&= 30\sqrt{2}
\end{aligned}
$$

3. Calculate the distance from one of the corners on the base of the pyramid to the centre of the base of the pyramid.

$$
\begin{aligned}
AP &= \frac{1}{2}AC \\
&= \frac{1}{2} \times 30\sqrt{2} \\
&= 15\sqrt{2}
\end{aligned}
$$

4. Draw the triangle that contains the height of the pyramid and the distance from one of the corners on the base of the pyramid to the centre of the base of the pyramid.



5. Use Pythagoras' theorem to calculate the height of the pyramid, rounding your answer to 1 decimal place.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
65^2 &= y^2 + (15\sqrt{2})^2 \\
4225 &= y^2 + 450 \\
y^2 &= 4225 - 450 \\
y &= 3775 \\
&= \sqrt{3775} \\
&\approx 61.4
\end{aligned}
$$

6. State the answer.

The height of the pyramid is 61.4 m (correct to 1 decimal place).

Resources

Interactivity: Pythagoras' theorem (int-6473)

studyon

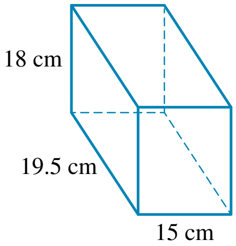
Units 1 & 2 > Area 2 > Sequence 1 > Concept 1

Pythagoras' theorem in two and three dimensions Summary screen and practice questions

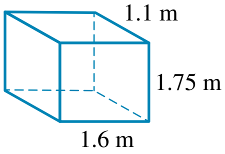
Exercise 3.3 Pythagroras' theorem in three dimensions

1. **WE5** Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions 1.2 m × 83 cm × 55 cm.
2. Determine whether a metal rod of length 2.8 m would be able to fit into a rectangular crate with dimensions 2.3 m × 1.2 m × 0.8 m.
3. **WE6** A square pyramid has a base length of 25 m and a edge height of 45 m. Determine the height of the pyramid, rounding your answer to 1 decimal place.
4. Determine which of the following square pyramids has the greater height.
   Pyramid 1: base length of 18 m and slant height of 30 m
   Pyramid 2: base length of 22 m and slant height of 28 m
5. Calculate the length of the longest metal rod that can fit diagonally into the boxes shown below.

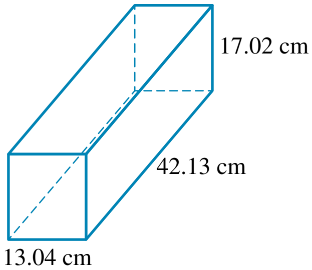
a.



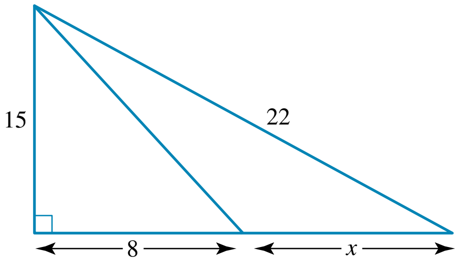
b.



c.

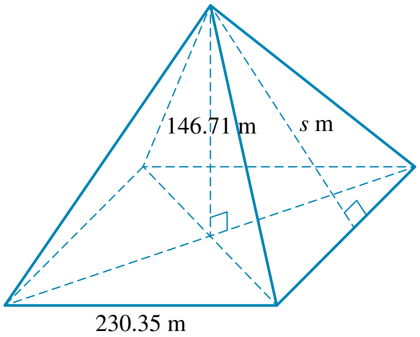


6. Calculate, correct to 2 decimal places, the height of a square pyramid with base width twice the height if the slant edge is:
   a. 20 cm **b.** 48 cm **c.** 5.5 cm **d.** 166 cm.
7. A friend wants to pack an umbrella into her suitcase.
   a. If the suitcase measures 89 cm × 21 cm × 44 cm, will her 1 m long umbrella fit in?
   b. Give the length of the longest object that will fit into the suitcase.
8. Calculate the value of x, correct to 2 decimal places, in the following diagram.

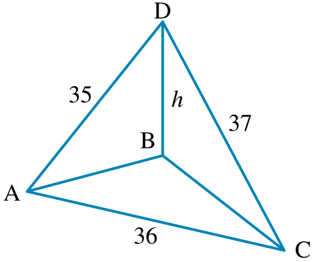


9. Stephano is renovating his apartment, which he accesses through two corridors. The corridors of the apartment building are 2 m wide with 2 m high ceilings, and the first corridor is at right angles to the second. Show that he can carry lengths of timber up to 6 m long to his apartment.

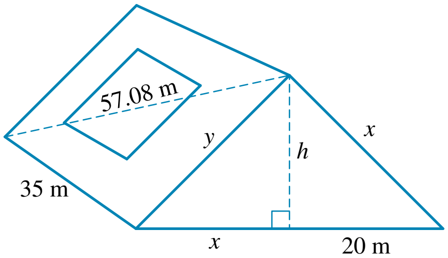
10. The Great Pyramid in Egypt is a square-based pyramid. The square base has a side length of 230.35 m and the perpendicular height is 146.71 m. Find the slant height, *s* m, of the great pyramid. Round your answer correct to 1 decimal place.



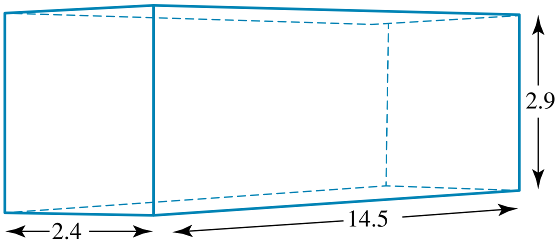
11. Angles ABD, CBD and ABC are right angles. Calculate the value of h, correct to 3 decimal places.



12. The roof of a squash centre is constructed to allow for maximum use of sunlight. Calculate the value of *h*, giving your answer correct to 1 decimal place.



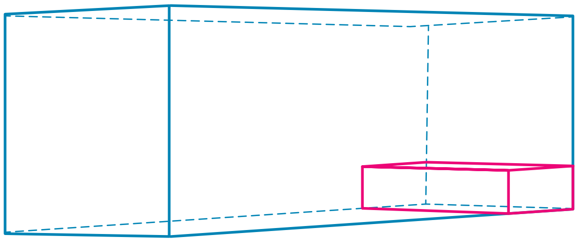
13. A semi-trailer carries a container that has the following internal dimensions: length 14.5 m, width 2.4 m and height 2.9 m. Give your answers to the following questions correct to 2 decimal places.



a. Calculate the length of the longest object that can be placed on the floor of the container.

b. Calculate the length of the longest object that can be placed in the container if only one end is placed on the floor.

c. If a rectangular box with length 2.4 m, width 1.2 m and height 0.8 m is placed on the floor at one end so that it fits across the width of the container, calculate the length of the longest object that can now be placed inside if it touches the floor adjacent to the box.



14. An ultralight aircraft is flying at an altitude of 1000 m and a horizontal distance of 10 km from its landing point.

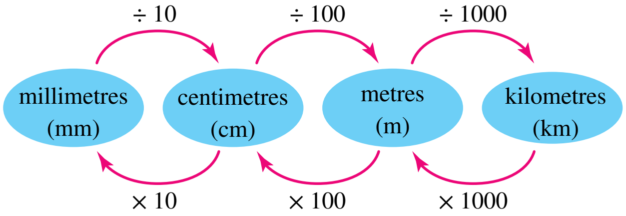
- **a.** If the aircraft travels in a straight line from its current position to its landing point, how far does it travel correct to the nearest metre? (Assume the ground is level.)
- **b.** If the aircraft maintained the same altitude for a further 4 km, what would be the straight-line distance from the new position to the same landing point, correct to the nearest metre?
- **c.** From the original starting point the pilot mistakenly follows a direct line to a point on the ground that is 2.5 km short of the correct landing point. He realises his mistake when he is at an altitude of 400 m and a horizontal distance of 5.5 km from the correct landing point. He then follows a straight-line path to the correct landing point. Calculate the total distance travelled by the aircraft from its starting point to the correct landing point, correct to the nearest metre.

3.4 Perimeter and area I

3.4.1 Units of length and area

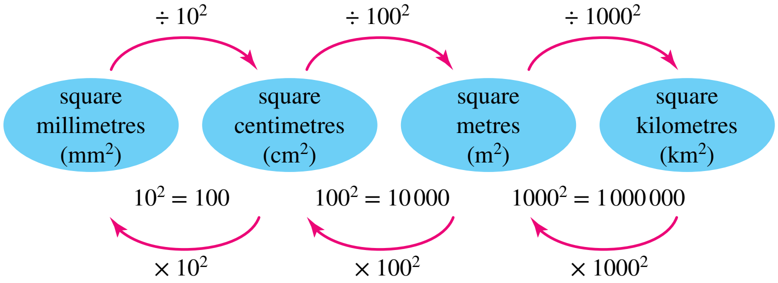
Units of length are used to describe the distance between any two points.

The standard unit of length in the metric system is the metre. The most commonly used units of length are the millimetre (mm), centimetre (cm), metre (m) and kilometre (km). These are related as shown in the following diagram.



Units of **area** are named by the side length of the square that encloses that amount of space. For example, a square metre is the amount of space enclosed by a square with a side length of 1 m.

The most common units of area are related as shown in the following diagram:



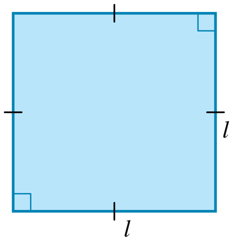
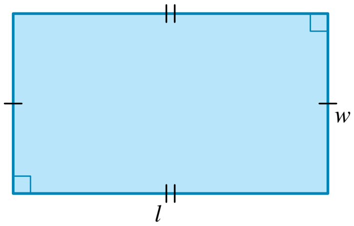
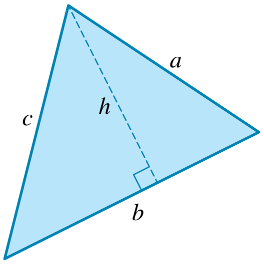
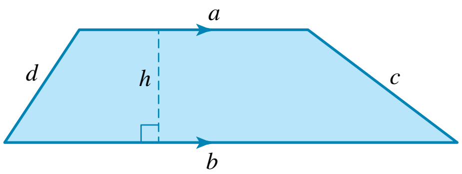
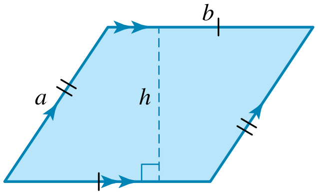
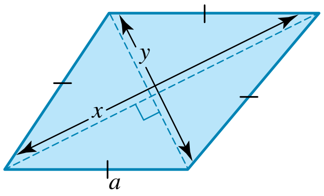
on Resources

Interactivity: Conversion of units of area (int-6269)

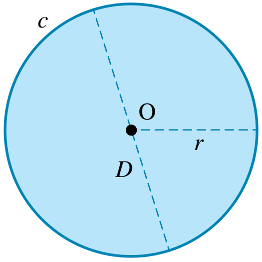
SkillSHEET Conversion of length units (doc-29492)

3.4.2 Perimeter and area of standard shapes

You should now be familiar with the methods and units of measurement used for calculating the **perimeter** (distance around an object) and area (two-dimensional space taken up by an object) of standard **polygons** and other shapes. These are summarised in the following table.

Shape	Perimeter and area
Square 	Perimeter: $P = 4l$ Area: $A = l^2$
Rectangle 	Perimeter: $P = 2l + 2w$ Area: $A = lw$
Triangle 	Perimeter: $P = a + b + c$ Area: $A = \frac{1}{2}bh$
Trapezium 	Perimeter: $P = a + b + c + d$ Area: $A = \frac{1}{2}(a + b)h$
Parallelogram 	Perimeter: $P = 2a + 2b$ Area: $A = bh$
Rhombus 	Perimeter: $P = 4a$ Area: $A = \frac{1}{2}xy$

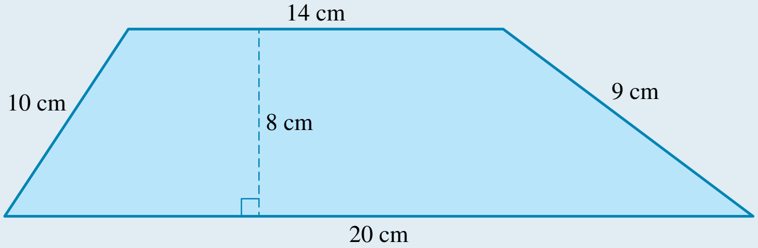
Continued

Shape	Perimeter and area
Circle 	Circumference (perimeter): $C = 2\pi r = \pi D$ Area: $A = \pi r^2$

Note: The approximate value of π is 3.14. However, when calculating **circumference** and area, always use the π button on your calculator and make rounding off to the required number of decimal places your final step.

WORKED EXAMPLE 7

Calculate the perimeter and area of the shape shown in the diagram.



THINK	WRITE
1. Identify the shape.	Trapezium
2. Identify the components for the perimeter formula and evaluate.	$P = 10 + 20 + 14 + 9$ $= 53$
3. State the perimeter including the units.	$P = 53\text{ cm}$
4. Identify the components for the area formula and evaluate.	$A = \frac{1}{2}(a + b)h$ $= \frac{1}{2}(20 + 14)8$ $= \frac{1}{2} \times 34 \times 8$ $= 136$
5. State the area and give the units.	$A = 136\text{ cm}^2$

on Resources

- **Interactivity:** Area and perimeter (int-6474)
- **Digital document:** SpreadSHEET Rounding (doc-29493)
- **Digital document:** SpreadSHEET Area of a square (doc-29494)
- **Digital document:** SpreadSHEET Area of a rectangle (doc-29495)
- **Digital document:** SpreadSHEET Area of a triangle (doc-29496)
- **Digital document:** SpreadSHEET Area of a circle (doc-29497)

studyon

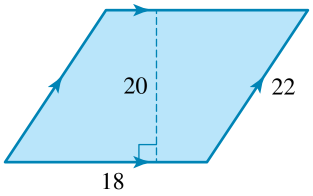
Units 1 & 2 > Area 2 > Sequence 1 > Concepts 2 & 3

Perimeter Summary screen and practice questions
Area Summary screen and practice questions

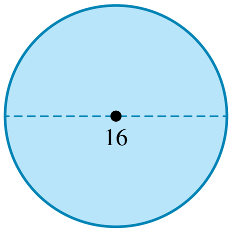
Exercise 3.4 Perimeter and area I

In the following questions, assume all measurements are in centimetres unless otherwise indicated.

1. **WE7** Calculate the perimeter and area of the shape shown in the diagram.

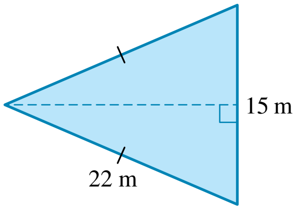


2. Calculate the circumference and area of the shape shown in the diagram, round your final answers to 2 decimal places.

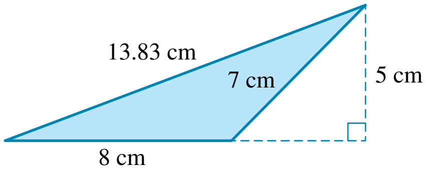


3. Calculate the perimeter and area of each of the following shapes, round answers to 2 decimal places where appropriate.

a.

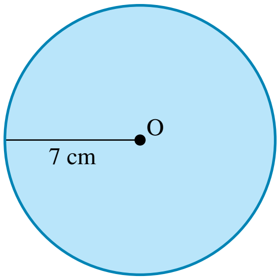


b.

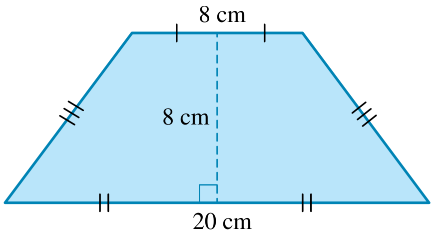


4. Calculate the perimeter and area of each of the following shapes, giving answers rounded to 2 decimal places where appropriate.

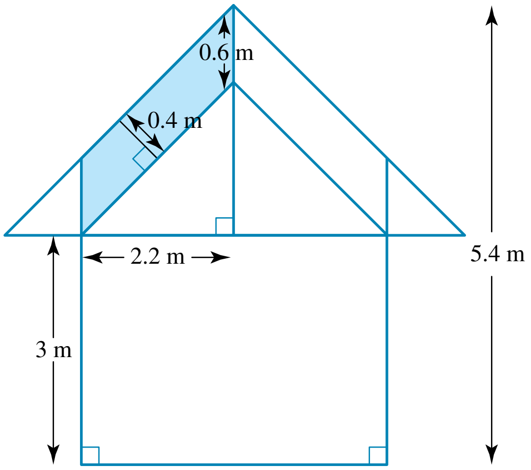
a.



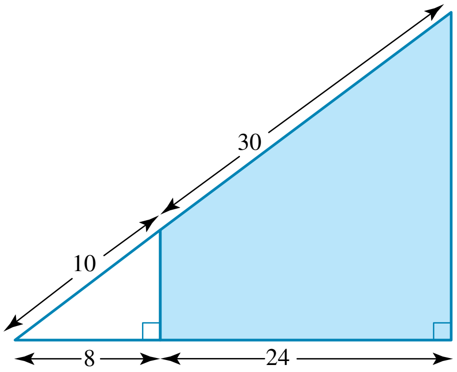
b.



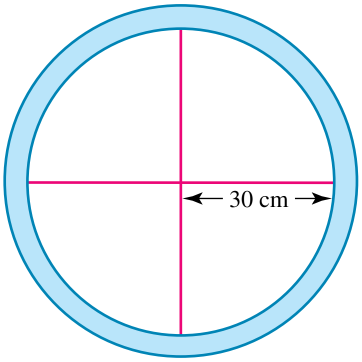
5. Correct to 2 decimal places, calculate the circumference and area of:
   a. a circle of radius 5 cm
   b. a circle of diameter 18 cm.
6. Calculate the perimeter and area of a parallelogram with side lengths of 12 cm and 22 cm, and a perpendicular distance of 16 cm between the short sides.
7. Calculate the area of a rhombus with diagonals of 11.63 cm and 5.81 cm.
8. Calculate the area of the shaded region shown in the diagram, giving your answer correct to 2 decimal places.



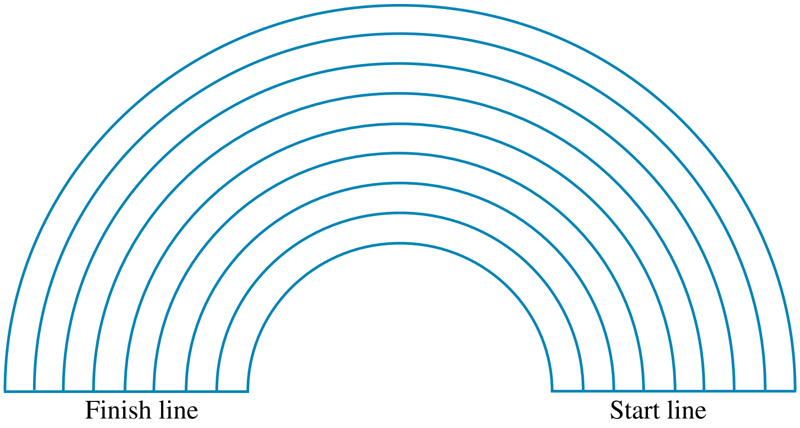
9. Calculate the perimeter of the large triangle and hence find the shaded area.



10. A circle has an area of 3140 cm^2. Determine its radius correct to 2 decimal places.
11. A rectangle has a side length that is twice as long as its width. If it has an area of 968 cm^2, determine the length of its diagonal correct to 2 decimal places.
12. A window consists of a circular metal frame 2 cm wide and two straight pieces of metal that divide the inner region into four equal segments, as shown in the diagram.
    a. If the window has an inner radius of 30 cm, calculate, correct to 2 decimal places:
       i. the outer circumference of the window
       ii. the total area of the circular metal frame.
    b. If the area of the metal frame is increased by 10% by reducing the size of the inner radius, calculate the circumference of the new inner circle.



13. A semicircular section of a running track consists of 8 lanes that are 1.2 m wide.

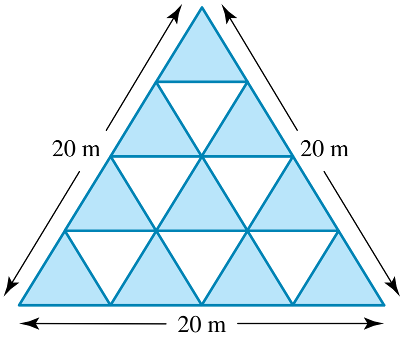


The innermost line of the first lane has a total length of 100 m.

a. How much further will someone in lane 8 run around the curve from the start line to the finish line?

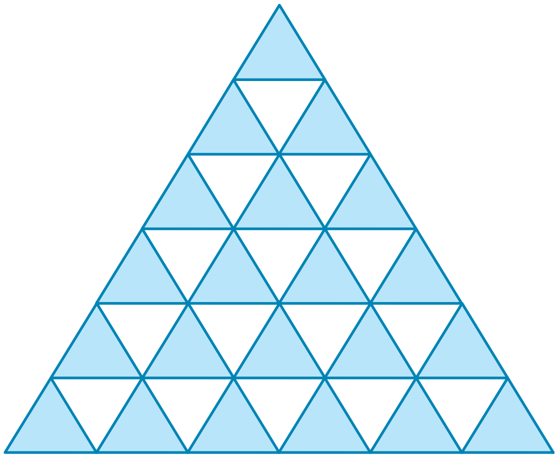
b. What is the total area of the curved section of the track?

14. A paved area of a garden courtyard forms an equilateral triangle with a side length of 20 m. It is paved using a series of identically sized blue and white triangular pavers as shown in the diagram.

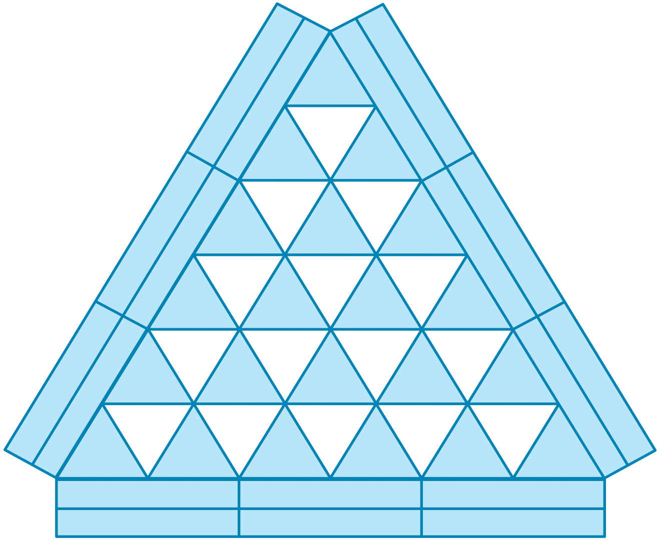


a. Calculate the total area of the paving correct to 2 decimal places.

b. If the pattern is continued by adding two more rows of pavers, calculate the new perimeter and area of the paving correct to 2 decimal places.



c. After the additional two rows are added, the architects decide to add two rows of rectangular pavers to each side. Each rectangular paver has a length that is twice the side length of a triangular paver, and a width that is half the side length of a triangular paver. If this was done on each side of the triangular paved area, calculate the perimeter and area of the paving.



3.5 Perimeter and area II

3.5.1 Composite shapes

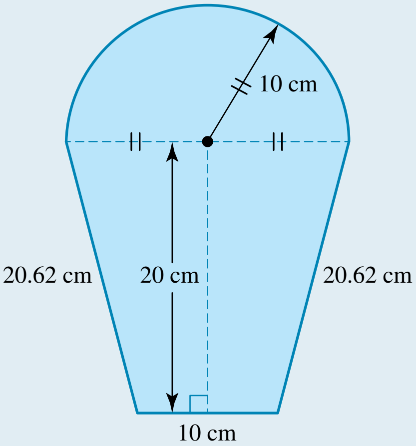
Many objects are not standard shapes but are combinations of them. For example:

To find the areas of composite shapes, split them up into standard shapes, calculate the individual areas of these standard shapes and sum the answers together.

To find the perimeters of composite shapes, it is often easiest to calculate each individual side length and to then calculate the total, rather than applying any specific formula.

WORKED EXAMPLE 8

Calculate the area of the object shown correct to 2 decimal places.

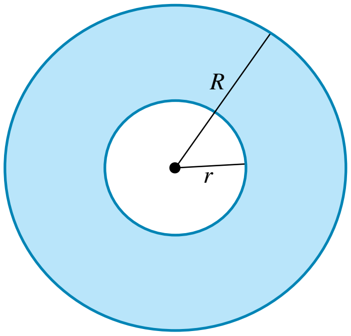


THINK	WRITE
1. Identify the given information.	The shape is a combination of a trapezium and a semicircle.
2. Find the area of each component of the shape.	Area of trapezium: $A = \frac{1}{2}(a + b)h$ $= \frac{1}{2}(10 + 20)\,20$ $= 300\text{ cm}^2$ Area of semicircle: $A = \frac{1}{2}\pi r^2$ $= \frac{1}{2}\pi(10)^2$ $\approx 157.08\text{ cm}^2$
3. Sum the areas of the components.	Total area: $300 + 157.08 = 457.08$
4. State the answer.	The area of the shape is 457.08 cm^2.

Some composite shapes do have specific formulae.

3.5.2 Annulus

The area between two circles with the same centre is known as an **annulus**. It is calculated by subtracting the area of the inner circle from the area of the outer circle.



Area of annulus = area of outer circle − area of inner circle

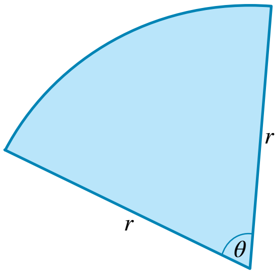
$$\begin{aligned} A &= \pi R^2 - \pi r^2 \\ &= \pi\left(R^2 - r^2\right) \end{aligned}$$

3.5.3 Sectors

Sectors are fractions of a circle. Because there are 360 degrees in a whole circle, the area of the sector can be found using, $A = \frac{\theta}{360} \times \pi r^2$, where θ is the angle between the two radii that form the sector.

The perimeter of a sector is a fraction of the circumference of the related circle plus two radii:

$$\begin{aligned} P &= \left(\frac{\theta}{360} \times 2\pi r\right) + 2r \\ &= 2r\left(\frac{\theta}{360}\pi + 1\right) \end{aligned}$$

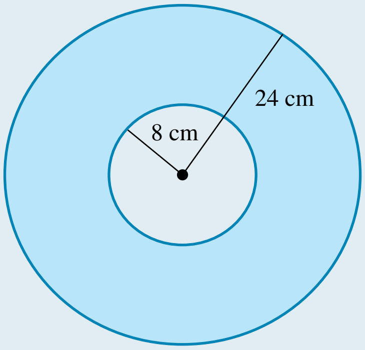


$$\textbf{Area of a sector} = \frac{\theta}{360} \times \pi r^2$$

$$\textbf{Perimeter of a sector} = 2r\left(\frac{\theta}{360}\pi + 1\right)$$

WORKED EXAMPLE 9

Calculate the area of the annulus shown in the diagram correct to 1 decimal place.



THINK	WRITE
1. Identify the given information.	The area shown is an annulus. The radius of the outer circle is 24 cm. The radius of the inner circle is 8 cm.
2. Substitute the information into the formula and simplify.	$A = \pi R^2 - \pi r^2$ $= \pi(R^2 - r^2)$ $= \pi(24^2 - 8^2)$ $= 512\pi$ ≈ 1608.5
3. State the answer.	The shaded area is 1608.5 cm^2.

on Resources

Digital document: SkillSHEET Perimeter of composite shapes (doc-29498)
Digital document: SkillSHEET Finding the size of a sector (doc-29499)
Digital document: SkillSHEET Area of composite shapes (doc-29500)

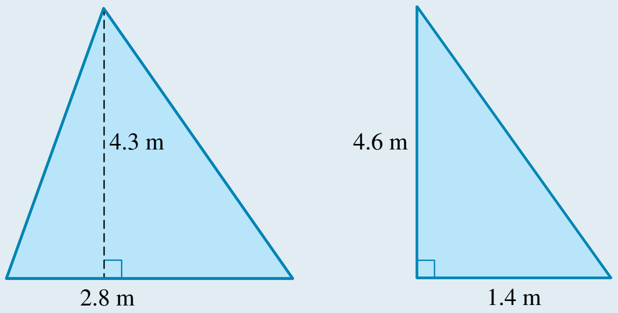
3.5.4 Applications

Calculations for perimeter and area have many and varied applications, including building and construction, painting and decorating, real estate, surveying and engineering.

When dealing with these problems it is often useful to draw diagrams to represent the given information.

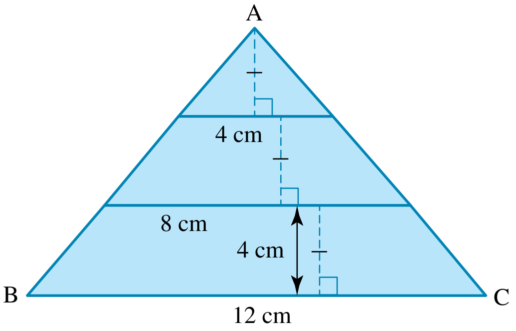
WORKED EXAMPLE 10

Calculate the total area of the sails on a yacht correct to 2 decimal places, if the apex of one sail is 4.3 m above its base length of 2.8 m, and the apex of the other sail is 4.6 m above its base of length of 1.4 m.

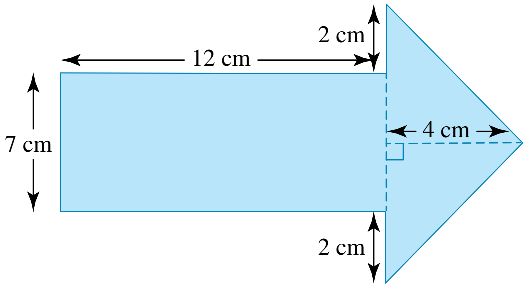
THINK	**WRITE/DRAW**
1. Draw a diagram of the given information.	
2. Identify the formulae required from the given information.	For each sail, use the formula for area of a triangle: $a = \frac{1}{2}bh$
3. Substitute the information into the required formulae for each area and simplify.	$\begin{aligned}\text{Sail 1: } A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 2.8 \times 4.3 \\ &= 6.02\end{aligned}$ $\begin{aligned}\text{Sail 2: } A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 1.4 \times 4.6 \\ &= 3.22\end{aligned}$
4. Add the areas of each of the required parts.	$\begin{aligned}\text{Area of sail} &= 6.02 + 3.22 \\ &= 9.24\end{aligned}$
5. State the answer.	The total area of the sails is 9.24 m^2.

Exercise 3.5 Perimeter and area II

1. **WE8** Calculate the area of the object shown.

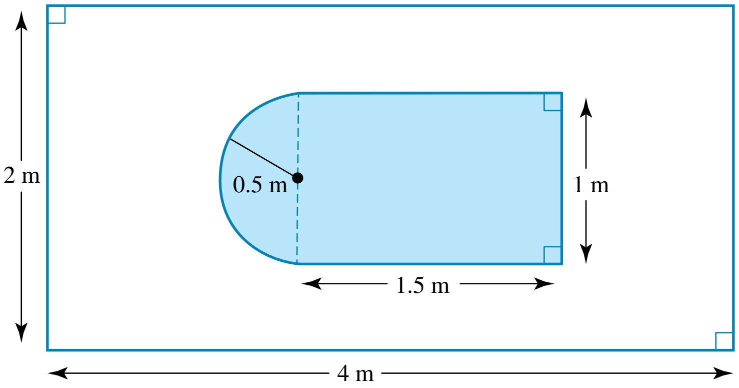


2. Calculate the perimeter and area of the object shown.



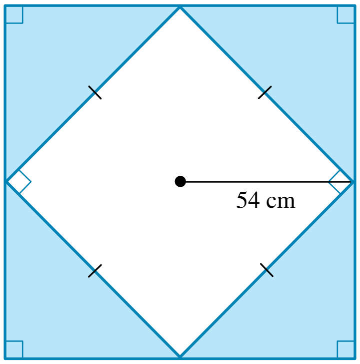
3. A circle of radius 8 cm is cut out from a square of side length 20 cm. How much of the area of the square remains? Round your answer to 2 decimal places.

4. a. Calculate the perimeter of the shaded area inside the larger rectangle shown in the diagram correct to 2 decimal places.

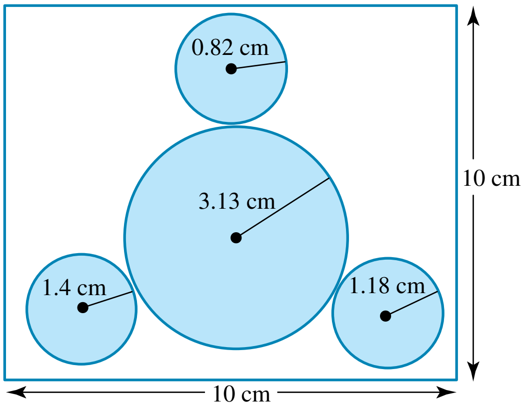


b. If the shaded area inside the rectangle is removed, what area remains?

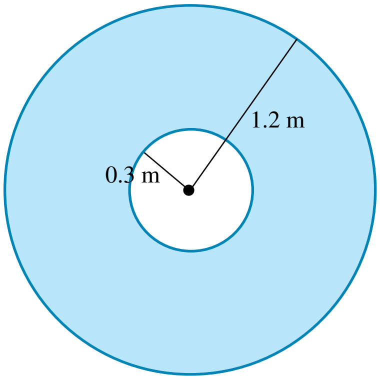
5. a. Calculate the shaded area in the diagram.



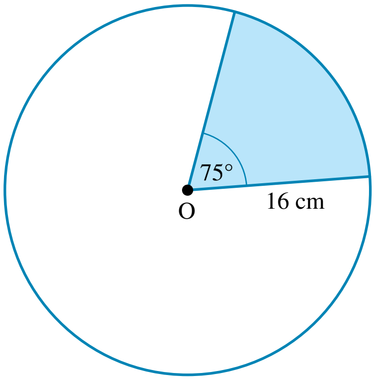
b. Calculate the unshaded area inside the square shown in the diagram, giving your answer correct to 2 decimal places.



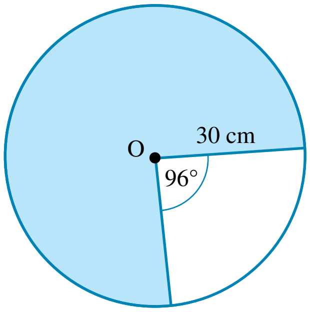
6. **WE9** Calculate the shaded area shown in the diagram to 2 decimal places.



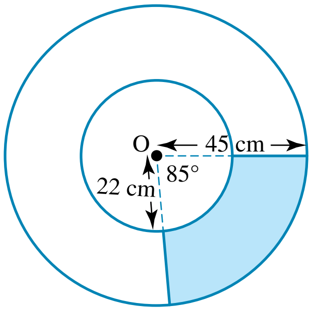
7. Calculate the area and perimeter of the shaded region shown in the diagram to 2 decimal places.



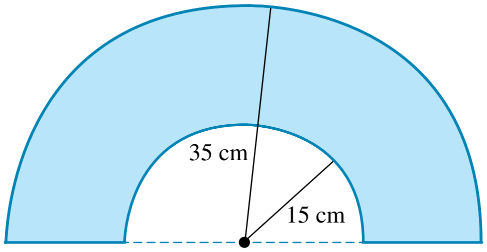
8. Calculate the area and perimeter of the shaded region shown in the diagram to 2 decimal places.



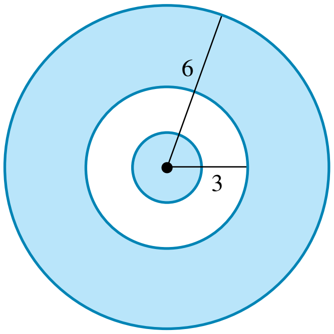
9. Calculate the area and perimeter of the shaded region shown in the diagram to 2 decimal places.



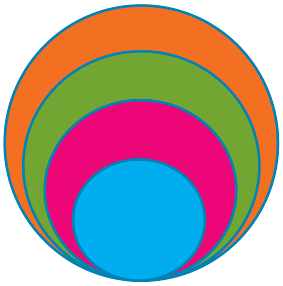
10. Calculate the perimeter and area of the shaded region in the half-annulus formed by 2 semicircles shown in the diagram. Give your answers correct to the nearest whole number.



11. The area of the inner circle in the diagram shown is $\frac{1}{9}$ that of the annulus formed by the two outer circles. Calculate the area of the inner circle to 2 decimal places given that the units are in centimetres.

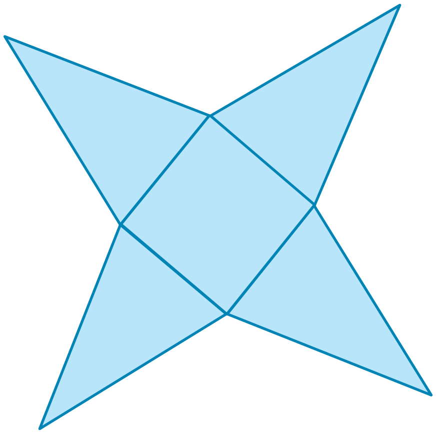


12. In the diagram the smallest circle has a diameter of 5 cm and the others have diameters that are progressively 2 cm longer than the one immediately before. Calculate the area that is shaded green to 2 decimal places.

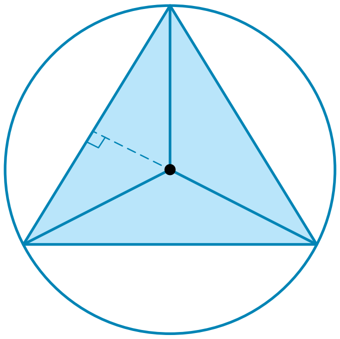


13. **WE10** Calculate the area of glass in a table that consists of three glass circles. The largest circle has a diameter of 68 cm. The diameters of the other two circles are 6 cm and 10 cm less than the diameter of the largest circle. Give your answer correct to 2 decimal places.

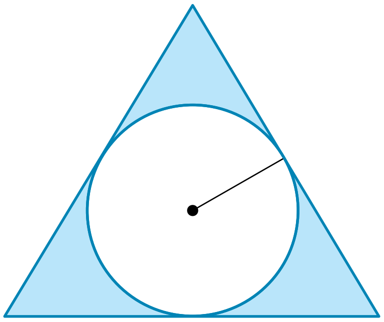
14. Part of the floor of an ancient Roman building was tiled in a pattern in which four identical triangles form a square with their bases. If the triangles have a base length of 12 cm and a height of 18 cm, calculate the perimeter and area they enclose, correct to 2 decimal places. (That is, calculate the perimeter and area of the shaded region shown on the right.)



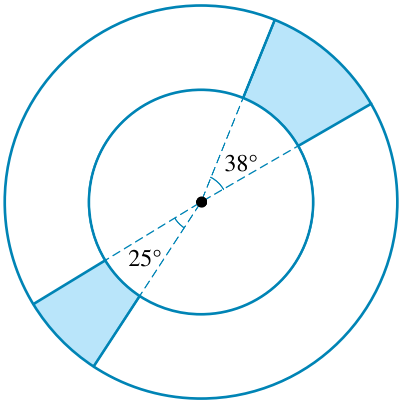
15. The vertices of an equilateral triangle of side length 2 m touch the edge of a circle with radius 1.16 m, as shown in the diagram. Calculate the area of the unshaded region correct to 2 decimal places.



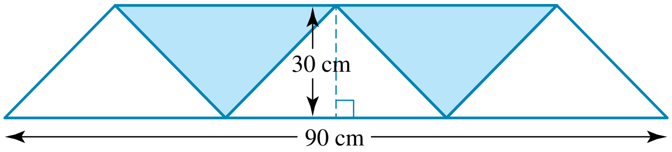
16. A circle of radius 0.58 m sits inside an equilateral triangle of side length 2 m so that it touches the edges of the triangle at three points. If the circle represents an area of the triangle to be removed, how much area would remain once this was done?



17. An annulus has an inner radius of 20 cm and an outer radius of 35 cm. Two sectors are to be removed. If one sector has an angle at the centre of 38° and the other has an angle of 25°, what area remains? Round your answer to 2 decimal places.

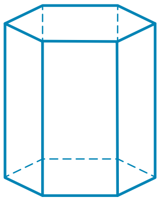


18. A trapezium is divided into five identical triangles of equal size with dimensions as shown in the diagram. Find the area and perimeter of the shaded region.

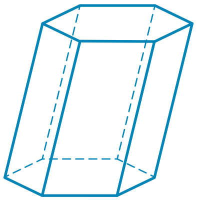


3.6 Volume and capacity

3.6.1 Volume



Right solid



Oblique solid

The amount of space that is taken up by any solid or three-dimensional object is known as its **volume**. Many standard objects have formulae that can be used to calculate their volume. If the centre point of the top of the solid is directly above the centre point of its base, the object is called a 'right solid'. If the centre point of the top is not directly above the centre point of the base, the object is an 'oblique solid'.

Note: For an oblique solid, the height, *h*, is the distance between the top and the base, not the length of one of the sides. (For a right solid, the distance between the top and the base equals the side length.)

Capacity

Volume is expressed in cubic units of measurement, such as cubic metres (m^3) or cubic centimetres (cm^3). When calculations involve the amount of fluid that the object can contain, it is referred to as its **capacity**. The units are commonly litres (L) or millilitres (mL).

To convert cubic centimetres to millilitres, use $1\,\text{cm}^3 = 1\,\text{mL}$

$$\begin{aligned}\therefore 1000\,\text{cm}^3 &= 1000\,\text{mL}\\ &= 1\,\text{L}\\ 1\text{m}^3 &= 1000\,\text{L}\\ &= 1\,\text{kL}\end{aligned}$$

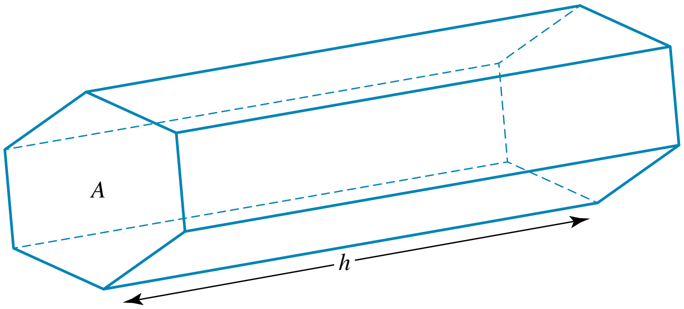
on Resources

Digital document: SpreadSHEET Capacity (doc-29501)

Digital document: SpreadSHEET Coversion of volume units (doc-29502)

3.6.2 Volume of prisms

If a solid object has identical ends which are polygons that are joined by flat surfaces and the object's cross-section is the same along its length, the object is a **prism**. The volume of a prism is calculated by taking the product of the base area and its height (or length).

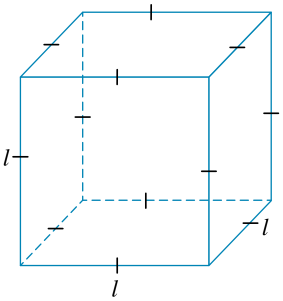
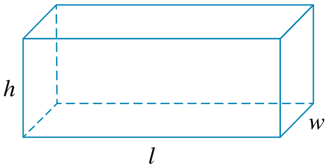
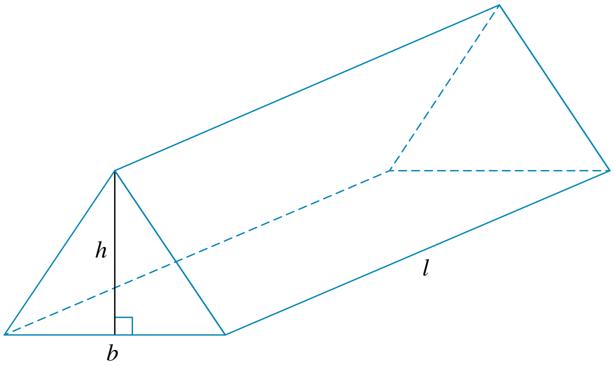


$$V = A \times h$$

Where A = base area

Common prisms

The formulae for calculating the volume of some of the most common prisms are summarised in the following table.

Prism	**Volume**
Cube  	$V = A \times h$ $= (l \times l) \times l$ $= l^3$
Rectangular prism  	$V = A \times h$ $= (l \times w) \times h$ $= l \times w \times h$
Triangular prism  	$V = A \times h$ $= \left(\frac{1}{2}bh\right) \times l$ $= \frac{1}{2}bhl$

Note: These formulae apply to both right prisms and oblique prisms, as long as you remember that the height of an oblique prism is its perpendicular height (the distance between the top and the base).

WORKED EXAMPLE 11

Calculate the volume of a triangular prism with length $l = 12$ cm, triangle base length $b = 6$ cm and triangle height $h = 4$ cm. Hence calculate its capacity in mL.

THINK	**WRITE**
1. Identify the given information.	Triangular prism, $l = 12$ cm, $b = 6$ cm, $h = 4$ cm

2. Substitute the information into the appropriate formula for the solid object and evaluate.

$$V = \frac{1}{2}bhl$$
$$= \frac{1}{2} \times 6 \times 4 \times 12$$
$$= 144$$

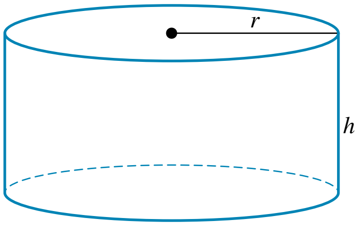
3. State the answer.

The volume is 144 cm^3.

4. Since $1\text{cm}^3 = 1\text{mL}$.

Capacity is 144 mL.

3.6.3 Cylinders



A **cylinder** is a solid object with ends that are identical circles and a cross-section that is the same along its length (like a prism). As a result it has a curved surface along its length.

As for prisms, the volume of a cylinder is calculated by taking the product of the base area and the height:

$$\boldsymbol{V = \text{Base area} \times \text{height}}$$
$$\boldsymbol{V = \pi r^2 h}$$

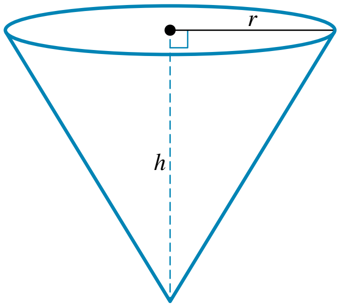
WORKED EXAMPLE 12

Calculate the volume of a cylinder of radius 10 cm and height 15 cm correct to 2 decimal places. Hence calculate its capacity in L to 2 decimal places.

THINK	WRITE
1. Identify the given information.	Cylinder, $r = 10$ cm, $h = 15$ cm
2. Substitute the information into the appropriate formula for the solid object and evaluate.	$V = \pi r^2 h$ $= \pi \times 10 \times 10 \times 15$ ≈ 4712.39
3. State the answer.	The volume is 4712.39 cm^3 .
4. Since $1000\,\text{cm}^3 = 1\text{L}$.	Capacity $= \dfrac{4712.39}{1000}$ $= 4.71\,\text{L}$

3.6.4 Cones and pyramids

Cones



A **cone** is a solid object that is similar to a cylinder in that it has one end that is circular, but different in that at the other end it has a single vertex.

It can be shown that if you have a cone and a cylinder with identical circular bases and heights, the volume of the cylinder will be three times the volume of the cone. (The proof of this is beyond the scope of this course.) The volume of a cone can therefore be calculated by using the formula for a comparable cylinder and dividing by three.

$$\boldsymbol{V = \frac{1}{3} \times \text{ base area} \times \text{height}}$$
$$\boldsymbol{V = \frac{1}{3}\pi r^2 h}$$

WORKED EXAMPLE 13

Calculate the volume of a cone of radius 20 cm and a height of 36 cm correct to 1 decimal place.

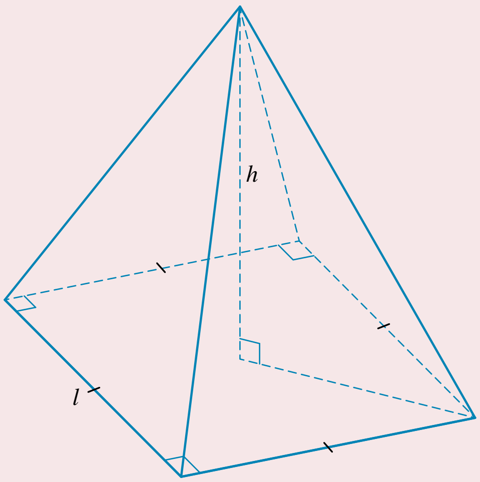
THINK	WRITE
1. Identify the given information.	Cone with $r = 20\text{ cm}$ and $h = 36\text{ cm}$
2. Substitute the information into the appropriate formula for the solid object and evaluate.	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3} \times \pi \times 20 \times 20 \times 36$ $\approx 15\ 079.6$
3. State the answer.	The volume is $15\,079.6\ \text{cm}^3$.

Pyramids

A **pyramid** is a solid object whose base is a polygon and whose sides are triangles that meet at a single point. The most famous examples are the pyramids of Ancient Egypt, which were built as tombs for the pharaohs.

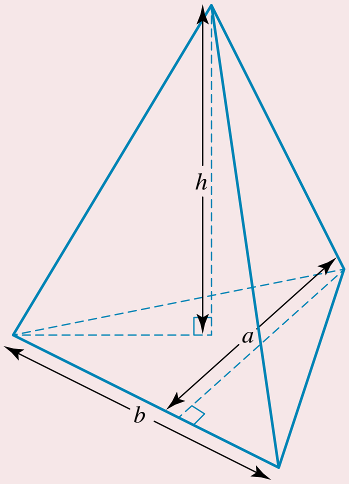
A pyramid is named after the shape of its base. For example, a hexagonal pyramid has a hexagon as its base polygon. The most common pyramids are square pyramids and triangular pyramids. As with cones, the volume of a pyramid can be calculated by using the formula of a comparable prism and dividing by three.

Square pyramid



$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$
$$= \frac{1}{3}l^2 h$$

Triangular pyramid



$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$
$$= \frac{1}{3}\left(\frac{1}{2}ab\right)h$$
$$= \frac{1}{6}abh$$

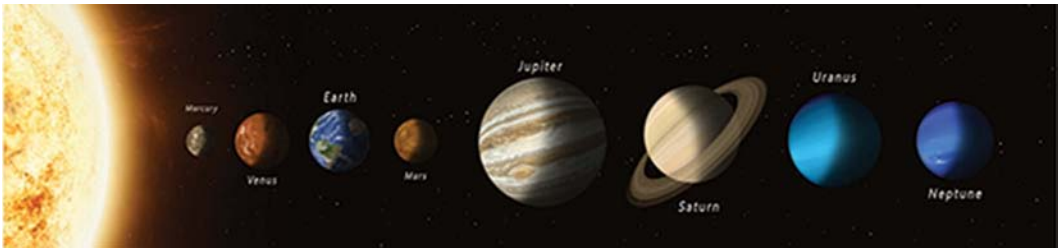
WORKED EXAMPLE 14

Calculate the volume of a pyramid that is 75 cm tall and has a rectangular base with dimensions 45 cm by 38 cm.

THINK	WRITE
1. Identify the given information.	Pyramid with a rectangular base of $l = 45,\ w = 38$ and $h = 75$ cm
2. Substitute the information into the appropriate formula for the solid object and evaluate.	$V = \frac{1}{3}lwh$ $= \frac{1}{3} \times 45 \times 38 \times 75$ $= 42\,750$
3. State the answer.	The volume is 42 750 cm^3.

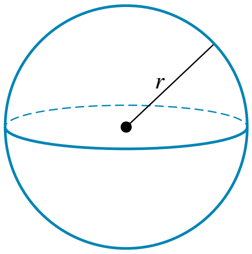
3.6.5 Spheres

A **sphere** is a solid object that has a curved surface such that every point on the surface is the same distance (the radius of the sphere) from a central point.



The formula for calculating the volume of a sphere has been attributed to the ancient Greek mathematician Archimedes.

$$V = \frac{4}{3}\pi r^3$$



WORKED EXAMPLE 15

Calculate the volume of a sphere of radius 63 cm correct to 1 decimal place.

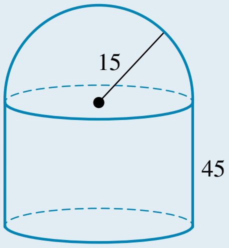
THINK	WRITE
1. Identify the given information.	Sphere of $r = 63$ cm
2. Substitute the information into the appropriate formula for the solid object and evaluate.	$V = \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times \pi \times 63 \times 63 \times 63$ $= 1\,047\,394.4$
3. State the answer.	The volume is 1 047 394.4 cm^3 .

3.6.6 Volumes of composite solids

As with calculations for perimeter and area, when a solid object is composed of two or more standard shapes, we need to identify each part and add their volumes to evaluate the overall volume.

WORKED EXAMPLE 16

Calculate the volume of an object that is composed of a hemisphere (half a sphere) of radius 15 cm that sits on top of a cylinder of height 45 cm, correct to 1 decimal place.



THINK	WRITE
1. Identify the given information.	Hemisphere with $r = 15$ cm and a cylinder of $r = 15$ cm and $h = 45$ cm
2. Substitute the information into the appropriate formula for each component of the solid object and evaluate. A hemisphere is half a sphere, hence the volume of a hemisphere is half the volume of a sphere.	Hemisphere: $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ $= \frac{1}{2} \times \left(\frac{4}{3} \times \pi \times 15 \times 15 \times 15\right)$ ≈ 7068.6 Cylinder: $V = \pi r^2 h$ $= \pi \times 15 \times 15 \times 45$ ≈ 31808.6
3. Add the volume of each component.	Composite object: $V = 7068.6 + 31\,808.6$ $= 38\,877.2$
4. State the answer.	The volume is $38\,877.2\ \text{cm}^3$.

on Resources

Interactivity: Volume (int-6476)

Digital document: SkillSHEET Volume (doc-29503)

studyon

Units 1 & 2 > Area 2 > Sequence 1 > Concept 4

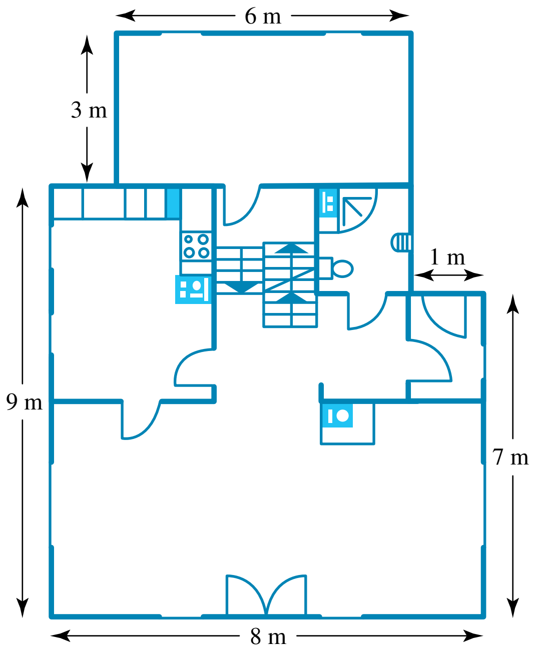
Volume and capacity Summary screen and practice questions

Exercise 3.6 Volume and capacity

1. **WE11** Calculate the volume of a triangular prism with length $l = 2.5$ m, triangle base length $b = 0.6$ m and triangle height $h = 0.8$ m.
2. Giving answers correct to the nearest cubic centimetre, calculate the volume of a prism that has:
   - a. a base area of 200 cm^2 and a height of 1.025 m
   - b. a rectangular base 25.25 cm by 12.65 cm and a length of 0.42 m
   - c. a right-angled triangular base with one side length of 48 cm, a hypotenuse of 73 cm and a length of 96 cm
   - d. a height of 1.05 m and a trapezium-shaped base with parallel sides that are 25 cm and 40 cm long and 15 cm apart.
3. The Gold Medal Pool Company sells three types of above-ground swimming pools, with base shapes that are square, rectangular or circular. Use the information in the table to list the volumes of each type in order from largest to smallest, giving your answers in litres.

Type	Depth	Base dimensions
Square pool	1.2 m	Length: 3 m
Rectangular pool	1.2 m	Length: 4.1 m Width: 2.25 m
Circular pool	1.2 m	Diameter: 3.3 m

4. A builder uses the floor plan of the house he is building to calculate the amount of concrete he needs to order for the foundations supporting the brick walls.



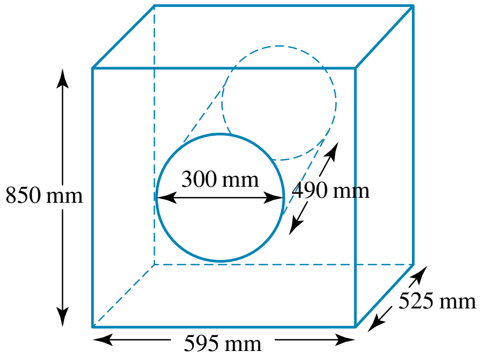
   - a. The foundation needs to go around the perimeter of the house with a width of 600 mm and a depth of 1050 mm. How many cubic metres of concrete are required?
   - b. The builder also wants to order the concrete required to pour a rectangular slab 3 m by 4 m to a depth of 600 mm. How many cubic metres of extra concrete should he order?
5. **WE12** Giving answers correct to the nearest cubic centimetre, calculate the volume of a cylinder of radius 22.5 cm and a height of 35.4 cm. Hence calculate its capacity rounding to the nearest mL.

6. Calculate the volume of a cylinder that has:

- **a.** a base circumference of 314 cm and a height of 0.625 m, giving your answer correct to the nearest cubic centimetre
- **b.** a height of 425 cm and a radius that is three-quarters of its height, giving your answer correct to the nearest cubic metre.

7. A company manufactures skylights in the shape of a cylinder with a hemispherical lid. When they are fitted onto a house, three-quarters of the length of the cylinder is below the roof. If the cylinder is 1.5 m long and has a radius of 30 cm, calculate the volume of the skylight that is above the roof and the volume that is below it. Give your answers correct to the nearest cubic centimetre.

8. The outer shape of a washing machine is a rectangular prism with a height of 850 mm, a width of 595 mm and a depth of 525 mm. Inside the machine, clothes are washed in a cylindrical stainless steel drum that has a diameter of 300 mm and a length of 490 mm.



- **a.** Determine the maximum volume of water, in litres, that the stainless steel drum can hold.
- **b.** Calculate the volume of the washing machine, in cubic metres, after subtracting the volume of the stainless steel drum.

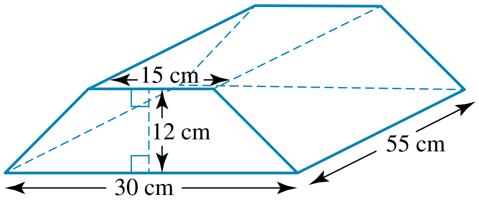
9. **WE13** Calculate the volume of a cone of radius 30 cm and a height of 42 cm correct to 1 decimal place. Hence calculate its capacity in litres to 2 decimal place.

10. Calculate the volume of a cone that has:

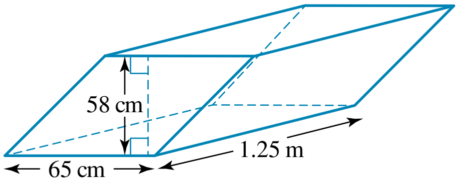
- **a.** a base circumference of 628 cm and a height of 0.72 m, correct to the nearest whole number
- **b.** a height of 0.36 cm and a radius that is two-thirds of its height, correct to 3 decimal places.

11. Calculate the volumes of the solid objects shown in the following diagrams.

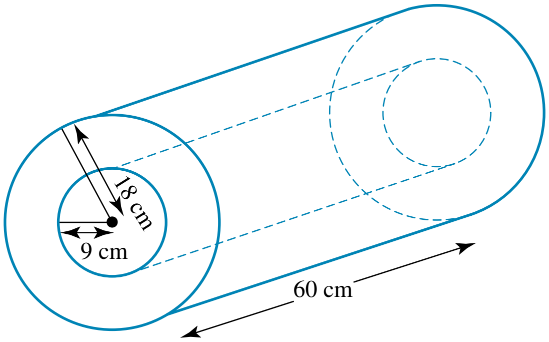
a.



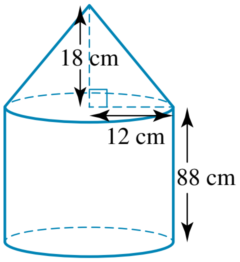
b.



c.



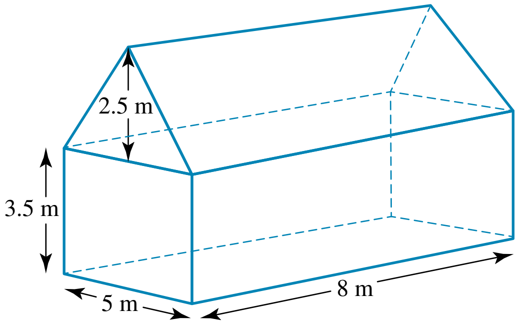
d.



12. **WE14** Calculate the volume of a pyramid that is 2.025 m tall and has a rectangular base with dimensions 1.05 m by 0.0745 m, correct to 4 decimal places. Hence calculate its capacity in litres to 1 decimal place.

13. Calculate the volume of a pyramid that has:
    a. a base area of 366 cm^2 and a height of 1.875 m
    b. a rectangular base 18.45 cm by 26.55 cm and a length of 0.96 m
    c. a height of 3.6 m and a triangular base with one side length of 1.2 m and a perpendicular height of 0.6 m.

14. The diagram shows the dimensions for a proposed house extension. Calculate the volume of insulation required in the roof if it takes up an eighth of the overall roof space.

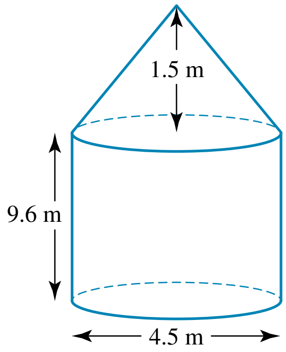


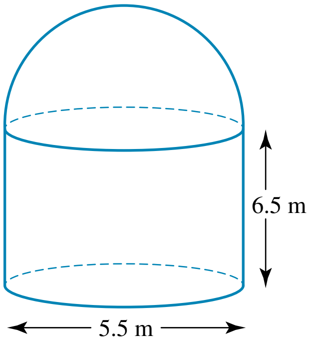
15. **WE15** Calculate the volume of a sphere of radius 0.27 m correct to 4 decimal places. Hence calculate its capacity in litres to 1 decimal place.

16. Calculate the radius, correct to the nearest whole number, of a sphere that has:
    a. a volume of 248 398.88 cm^3
    b. a volume of 4.187 m^3.

17. **WE16** Calculate the volume of an object that is composed of a hemisphere (half a sphere) of radius 1.5 m that sits on top of a cylinder of height 2.1 m. Give your answer correct to 2 decimal places.

18. A wheat farmer needs to purchase a new grain silo and has the choice of two sizes.
One is cylindrical with a conical top, and the other is cylindrical with a hemispherical top. Use the dimensions shown in the diagrams to determine which silo holds the greatest volume of wheat and by how much.





19. The glass pyramid in the courtyard of the Louvre Museum in Paris has a height of 22 m and a square base with side lengths of 35 m.

- **a.** Determine the volume of the glass pyramid in cubic metres.
- **b.** A second glass pyramid at the Louvre Museum is called the Inverted Pyramid as it hangs upside down from the ceiling. If its dimensions are one-third of those of the larger glass pyramid, what is its volume in cubic centimetres?

20. Tennis balls are spherical with a diameter of 6.7 cm. They are sold in packs of four in cylindrical canisters whose internal dimensions are 26.95 cm long with a diameter that is 5 mm greater than that of a ball. The canisters are packed vertically in rectangular boxes; each box is 27 cm high and will fit exactly eight canisters along its length and exactly four along its width.

- **a.** Calculate the volume of free space that is in a canister containing four tennis balls.
- **b.** Calculate the volume of free space that is in a rectangular box packed full of canisters.

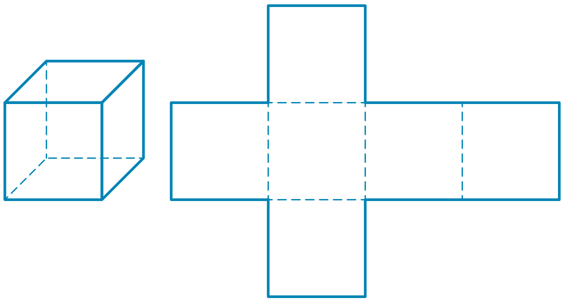
3.7 Surface area of three-dimensional objects

3.7.1 Surface area

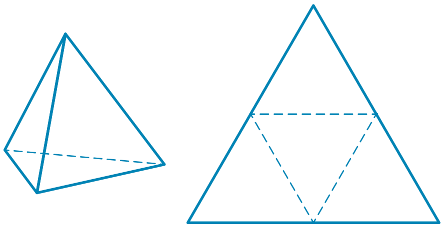
The **surface area** of a solid object is equal to the combined total of the areas of each individual surface that forms it. Some objects have specific formulae for the calculation of the total surface area, whereas others require the calculation of each individual surface in turn. Surface area is particularly important in design and construction when considering how much material is required to make a solid object. In manufacturing it could be important to make an object with the smallest amount of material that is capable of holding a particular volume. Surface area is also important in aerodynamics, as the greater the surface area, the greater the potential air resistance or drag.

Nets

The **net** of a solid object is like a pattern or plan for its construction. Each surface of the object is included in its net. Therefore, the net can be used to calculate the total surface area of the object. For example, the net of a cube will have six squares, whereas the net of a triangular pyramid (or tetrahedron) will have four triangles.



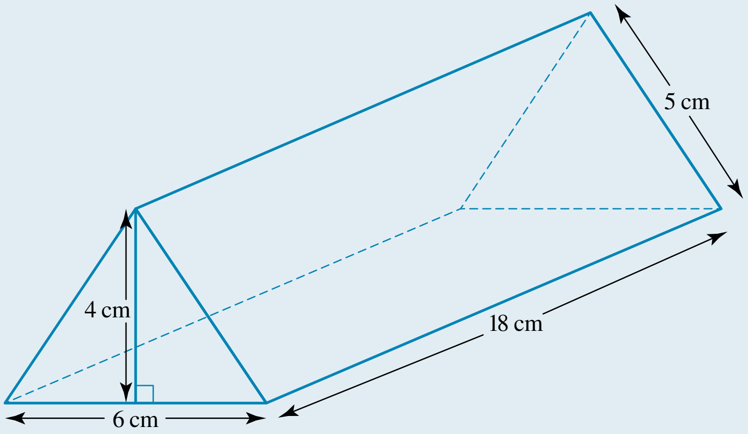
The net of a cube

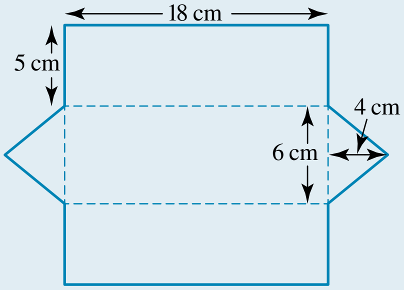


The net of a tetrahedron

WORKED EXAMPLE 17

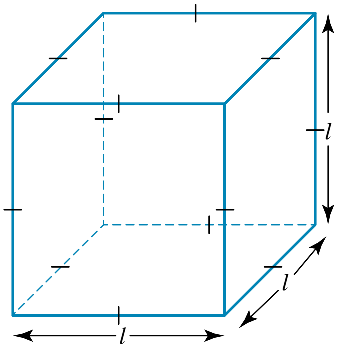
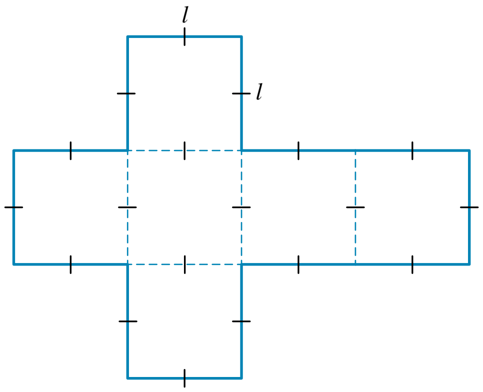
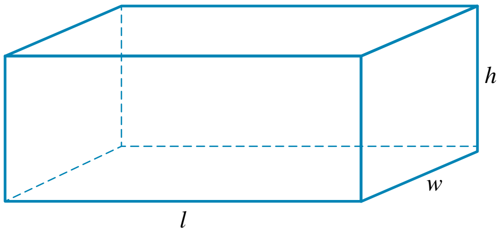
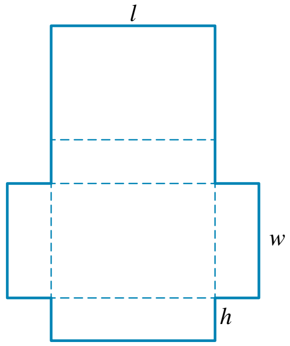
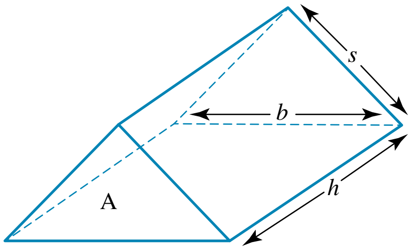
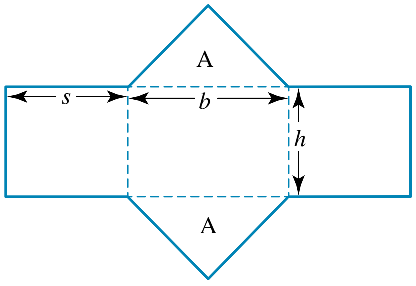
Calculate the surface area of the prism shown by first drawing its net.

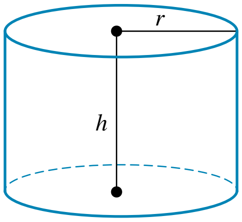
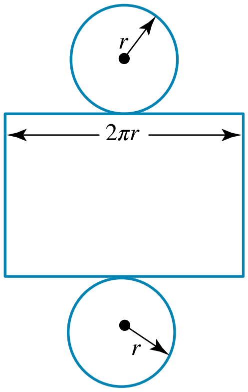
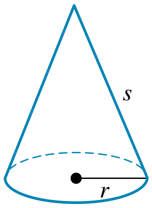
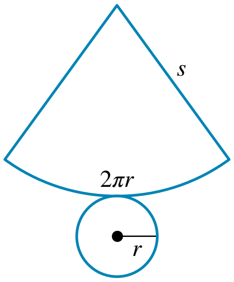
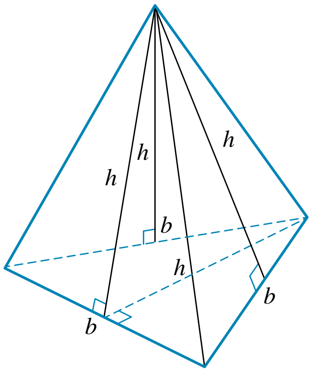
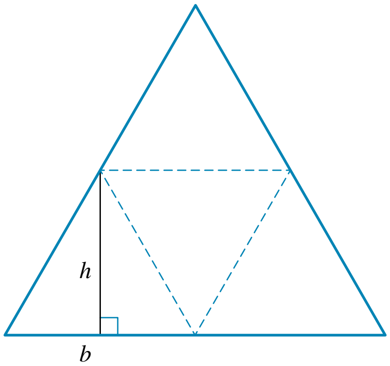


THINK	WRITE/DRAW
1. Identify the prism and each surface in it.	The triangular prism consists of two identical triangular ends, two identical rectangular sides and one rectangular base.
2. Redraw the given diagram as a net, making sure to check that each surface is present.	
3. Calculate the area of each surface identified in the net.	Triangular ends: $A = 2 \times \left(\frac{1}{2}bh\right)$ $= 2 \times \left(\frac{1}{2} \times 6 \times 4\right)$ $= 24$ Rectangular sides: $A = 2 \times (lw)$ $= 2 \times (18 \times 5)$ $= 180$ Rectangular base: $A = lw$ $= 18 \times 6$ $= 108$
4. Add the component areas and state the answer.	Total surface area: $= 24 + 180 + 108$ $= 312\,\text{cm}^2$

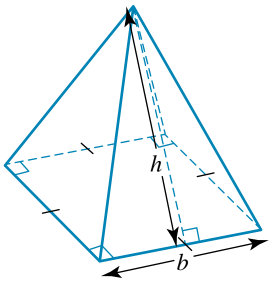
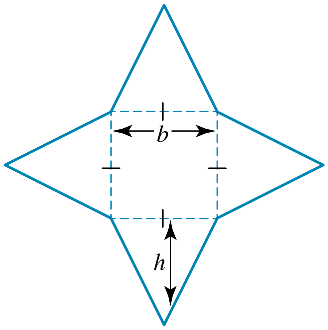
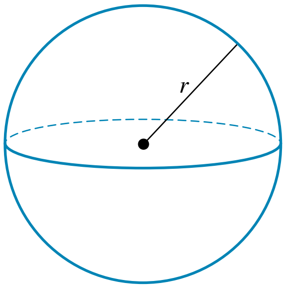
3.7.2 Surface area formulae

The surface area formulae for common solid objects are summarised in the following table.

Object	**Surface area**
Cube  	 SA = $6l^2$
Rectangular prism  	 SA $= 2lw + 2lh + 2wh$
Triangular prism  	 SA $= 2\text{A} + 2hs + bh$ (where A is the area of the triangular end)

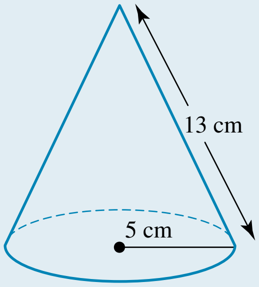
<table>
<tr><th>Object</th><th>Surface area</th></tr>
<tr><td>Cylinder
 </td><td>
$\text{SA} = 2\pi r^2 + 2\pi rh$
$= 2\pi r(r + h)$</td></tr>
<tr><td>Cone
 </td><td>
$\text{SA} = \pi rs + \pi r^2$
$= \pi r(s + r)$
(including the circular base)</td></tr>
<tr><td>Tetrahedron
 </td><td>
$\text{SA} = 4 \times \left(\frac{1}{2}bh\right)$</td></tr>
</table>

Continued

Object	**Surface area**
Square right pyramid 	 $\text{SA} = 4 \times \left(\frac{1}{2}bh\right) + b^2$
Sphere 	$\text{SA} = 4\pi r^2$

WORKED EXAMPLE 18

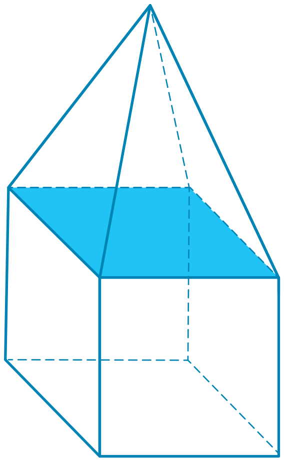
Calculate the surface area of the object shown by selecting an appropriate formula. Round your answer to 1 decimal place.



THINK	**WRITE**
1. Identify the object and the appropriate formula.	Given the object is a cone, the formula is $\text{SA} = \pi rs + \pi r^2$.
2. Substitute the given values into the formula and evaluate.	$\text{SA} = \pi rs + \pi r^2$ $= \pi r(s + r)$ $= \pi \times 5(5 + 13)$ $= \pi \times 90$ ≈ 282.7
3. State the final answer.	The surface area of the cone is $282.7\ \text{cm}^2$.

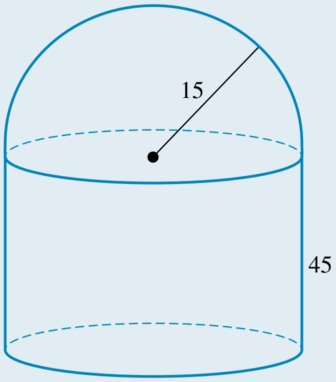
3.7.3 Surface areas of composite solids

For composite solids, be careful to include only those surfaces that form the outer part of the object. For example, if a solid consisted of a pyramid on top of a cube, the internal surface highlighted in blue would not be included.



WORKED EXAMPLE 19

Calculate the surface area of the object shown correct to 2 decimal places.



THINK	WRITE
1. Identify the components of the composite solid.	The object consists of a hemisphere that sits on top of a cylinder.
2. Substitute the given values into the formula for each surface of the object and evaluate.	Hemisphere: $SA = \frac{1}{2}(4\pi r^2)$ $= \frac{1}{2}(4 \times \pi \times 15^2)$ ≈ 1413.72 Cylinder (no top): $SA = \pi r^2 + 2\pi rh$ $= \pi \times 15^2 + 2 \times \pi \times 15 \times 45$ ≈ 4948.01
3. Add the area of each surface to obtain the total surface area.	Total surface area: $SA = 1413.72 + 4948.01$ $= 6361.73$
4. State the answer.	The total surface area of the object is 6361.73 cm^2.

on Resources

Interactivity: Surface area (int-6477)

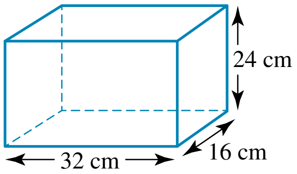
studyon

Units 1 & 2 › Area 2 › Sequence 1 › Concept 5

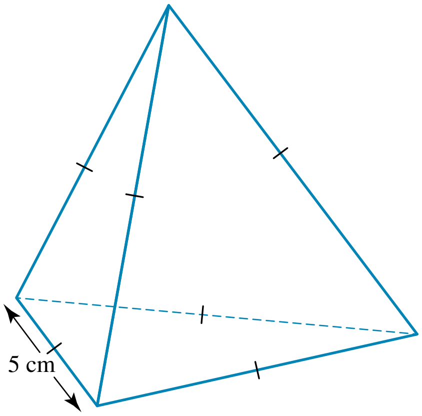
Surface area of three-dimensional objects Summary screen and practice questions

Exercise 3.7 Surface area of three-dimensional objects

1. **WE17** Calculate the surface area of the prism shown by first drawing its net.

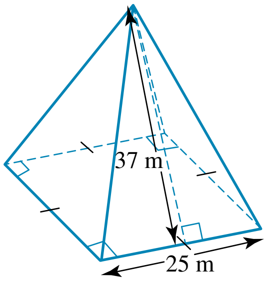


2. Calculate the surface area of the tetrahedron shown by first drawing its net.

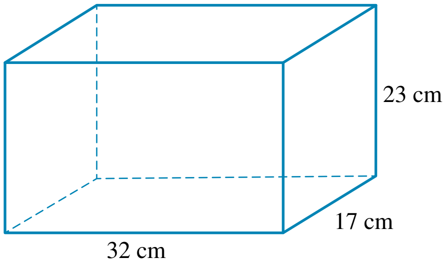


3. **WE18** Calculate the surface areas of the objects shown by selecting appropriate formulae.

a.



b.

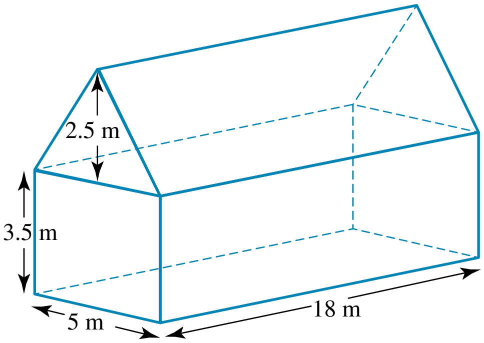


4. Calculate (correct to 2 decimal places where appropriate) the surface area of:
- **a.** a pyramid formed by four equilateral triangles with a side length of 12 cm
- **b.** a sphere with a radius of 98 cm
- **c.** a cylinder with a radius of 15 cm and a height of 22 cm
- **d.** a cone with a radius of 12.5 cm and a slant height of 27.2 cm.

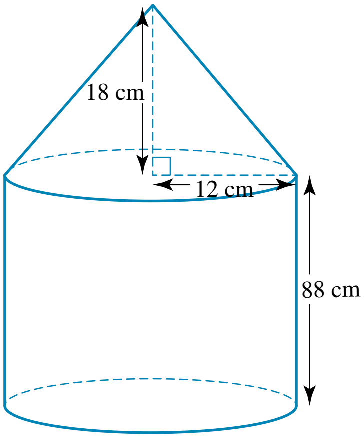
5. Calculate (correct to 2 decimal places where appropriate) the total surface area of:
- **a.** a rectangular prism with dimensions 8 cm by 12 cm by 5 cm
- **b.** a cylinder with a base diameter of 18 cm and a height of 20 cm
- **c.** a square pyramid with a base length of 15 cm and a vertical height of 18 cm
- **d.** a sphere of radius 10 cm.

6. A prism is 25 cm high and has a trapezoidal base whose parallel sides are 8 cm and 12 cm long respectively, and are 10 cm apart.
- **a.** Construct the net of the prism.
- **b.** Calculate the total surface area of the prism.

7. **WE19** Calculate the surface area of the object shown.



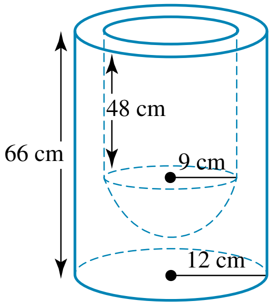
8. Calculate the surface area of the object shown. Round your answer to 2 decimal places.



9. A hemispherical glass ornament sits on a circular base that has a radius of 5 cm.
   a. Calculate its total surface area to the nearest square centimetre.
   b. If an artist attaches it to an 8-cm-tall cylindrical stand with the same circumference, what is the new total surface area of the combined object that is created? Give your answer to the nearest square centimetre.

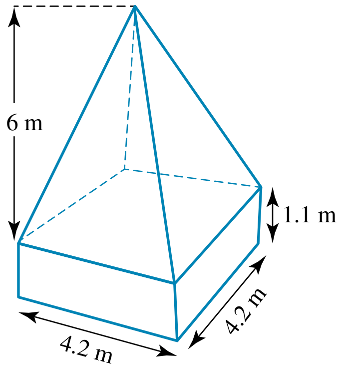
10. An ice-cream shop sells two types of cones. One is 6.5 cm tall with a radius of 2.2 cm. The other is 7.5 cm tall with a radius of 1.7 cm. By first calculating the slant height of each cone correct to 2 decimal places, determine which cone (not including any ice-cream) has the greater surface area and by how much.

11. A cylindrical plastic vase is 66 cm high and has a radius of 12 cm. The centre has been hollowed out so that there is a cylindrical space with a radius of 9 cm that goes to a depth of 48 cm and ends in a hemisphere, as shown in the diagram.
    Giving your answers to the nearest square centimetre:
    a. calculate the area of the external surfaces of the vase
    b. calculate the area of the internal surface of the vase.

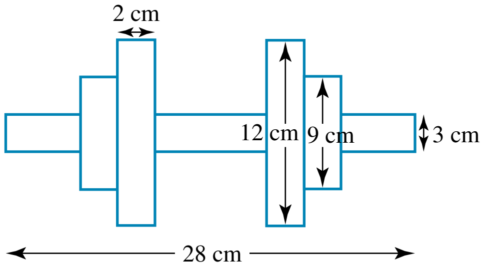


12. The top of a church tower is in the shape of a square pyramid that sits on top of a rectangular prism base that is 1.1 m high. The pyramid is 6 m high with a base length of 4.2 m.

    Calculate the total external surface area of the top of the church tower if the base of the prism forms the ceiling of a balcony. Give your answer correct to 2 decimal places.



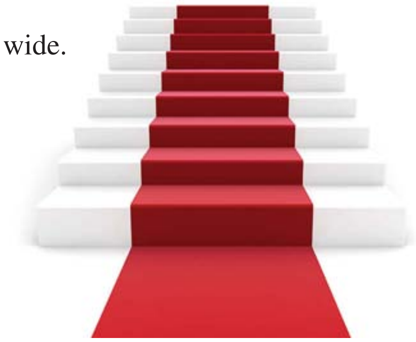
13. A dumbbell consists of a cylindrical tube that is 28 cm long with a diameter of 3 cm, and two pairs of cylindrical discs that are held in place by two locks. The larger discs have a diameter of 12 cm and a width of 2 cm, and the smaller discs are the same thickness with a diameter of 9 cm.
Calculate the total area of the exposed surfaces of the discs when they are held in position as shown in the diagram. Give your answer to the nearest square centimetre.



14. A staircase has a section of red carpet down its centre strip. Each of the nine steps is 16 cm high, 25 cm deep and 120 cm wide. The red carpet is 80 cm wide and extends from the back of the uppermost step to a point 65 cm beyond the base of the lower step.

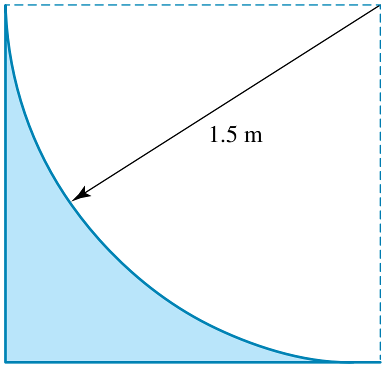
a. Determine the area of the red carpet.

b. If all areas of the front and top of the stairs that are not covered by the carpet are to be painted white, what is the area to be painted?



15. A rectangular swimming pool is 12.5 m long, 4.3 m wide and 1.5 m deep. If all internal surfaces are to be tiled, calculate the total area of tiles required.

16. A quarter-pipe skateboard ramp has a curved surface that is one-quarter of a cylinder with a radius of 1.5 m. If the surface of the ramp is 2.4 m wide, calculate the total surface area of the front, back and sides.

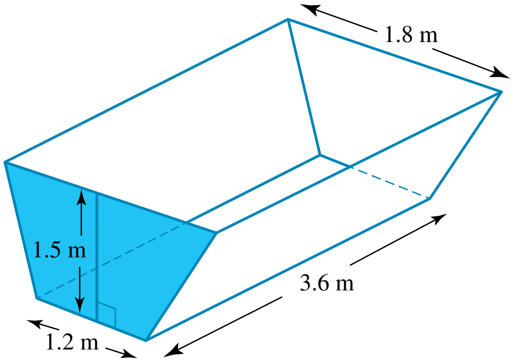


3.8 Review: exam practice

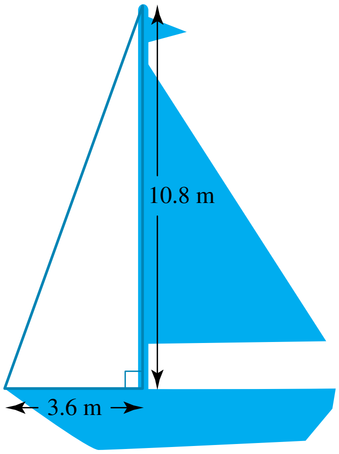
A summary of this chapter is available in the Resources section of your eBookplus at www.jacplus.com.au.

Simple familiar

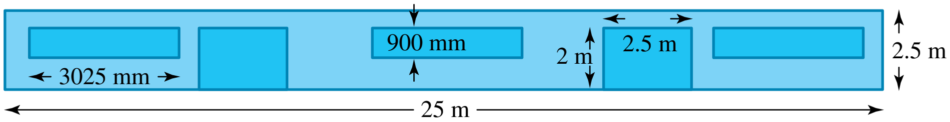
1. **MC** Which group of three numbers would be the side lengths of a right-angled triangle?
   A. 6, 24, 25 **B.** 13, 14, 15 **C.** 7, 24, 25 **D.** 9, 12, 16
2. **MC** If a right-angled isosceles triangle has a hypotenuse of length 32 units, the other sides will be closest to:
   A. 21.54 **B.** 21.55 **C.** 16 **D.** 22.63
3. **MC** An equilateral triangle with a side length of 4 units will have an altitude (height) closest to:
   A. 3.46 **B.** 4.47 **C.** 4 **D.** 3.47
4. **MC** A trapezium has a height of 8 cm and an area of 148 cm^2. Its parallel sides could be:
   A. 11 cm and 27 cm
   B. 12 cm and 24 cm
   C. 12 cm and 26 cm
   D. 12 cm and 25 cm
5. **MC** A circle has a circumference of 75.4 cm. Its area is closest to:
   A. 440 cm^2
   B. 475 cm^2
   C. 461 cm^2
   D. 452 cm^2
6. **MC** A cylinder with a volume of 1570 cm^3 and a height of 20 cm will have a diameter that is closest to:
   A. 5 cm
   B. 12 cm
   C. 15 cm
   D. 10 cm
7. **MC** A cone with a surface area of 2713 cm^2 and a diameter of 24 cm will have a slant-height that is closest to:
   A. 58 cm **B.** 48 cm **C.** 60 cm **D.** 46 cm
8. **MC** A hemisphere with a radius of 22.5 cm will have a volume and total surface area respectively that are closest to:
   A. 47 689 cm^3 and 4764 cm^2
   B. 47 689 cm^3 and 6358 cm^2
   C. 23 845 cm^3 and 6359 cm^2
   D. 23 856 cm^3 and 4771 cm^2
9. **MC** A square pyramid with a volume of 500 cm^3 and a vertical height of 15 cm will have a surface area that is closest to:
   A. 416 cm^2 **B.** 492 cm^2 **C.** 359 cm^2 **D.** 316 cm^2
10. **MC** An open rubbish skip-container is in the shape of a trapezoidal prism with the dimensions indicated in the diagram.
    The surface area (m^2) and volume (m^3) respectively are:
    A. 8.1 and 19.8
    B. 19.8 and 8.1
    C. 17.75 and 8.1
    D. 8.1 and 26.3



11. Calculate the length of wire required to support the mast of a yacht if the mast is 10.8 m long and the support wire is attached to the horizontal deck at a point 3.6 m from the base of the mast.



12. The side pieces of train carriages are made from rectangular sheets of pressed metal of length 25 m and height 2.5 m. Rectangular sections for the doors and windows are cut out. The dimensions of the spaces for the doors are 2 m high by 2.5 m wide. The window spaces are 3025 mm wide by 900 mm high. Each sheet must have spaces cut for two doors and three windows.

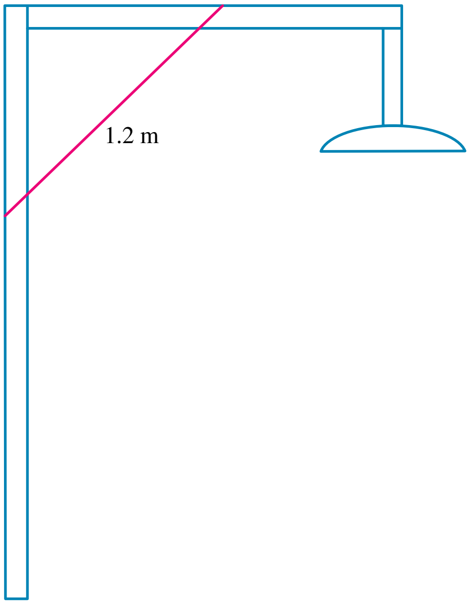


a. Calculate the total area of pressed metal that remains once the sections for the doors and windows have been removed.

b. A thin edging strip is placed around each window and around the top and sides of the door opening. What is the total length of edging required?

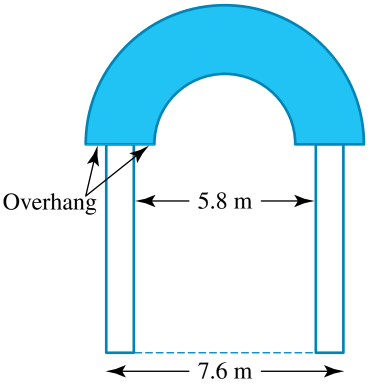
Complex familiar

13. The supporting strut of a streetlight must be attached so that its ends are an equal distance from the top of the pole. If the strut is 1.2 m long, how far are the ends from the top of the pole?

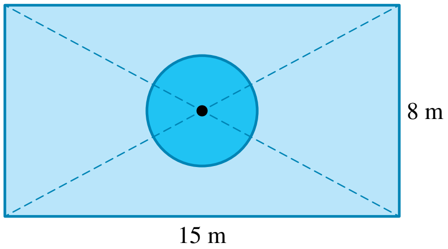


14. A surveyor is measuring a building site and wants to check that the guidelines for the foundations are square (i.e. at right angles). He places a marker 3600 mm from a corner along one line, and another marker 4800 mm from the corner along the other line. How far apart must the markers be for the lines to be square?

15. A semicircular arch sits on two columns as shown in the diagram at right. The outer edges of the columns are 7.6 m apart and the inner edges are 5.8 m apart. The width of each column is three-quarters the width of the arch, and the arch overhangs the columns by one-eighth of its width on each edge. The face of the arch (the shaded area) is to be tiled. What area will the tiles cover?



16. A circular pond is placed in the middle of a rectangular garden that is 15 m by 8 m .



If the radius of the pond is a quarter of the distance from the centre to the corner of the garden, calculate:

a. the circumference of the pond

b. the area of the garden, not including the pond

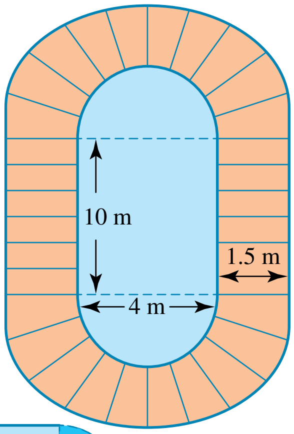
c. the volume of water in the pond in litres if it is filled to a depth of 850 mm.

Complex unfamiliar

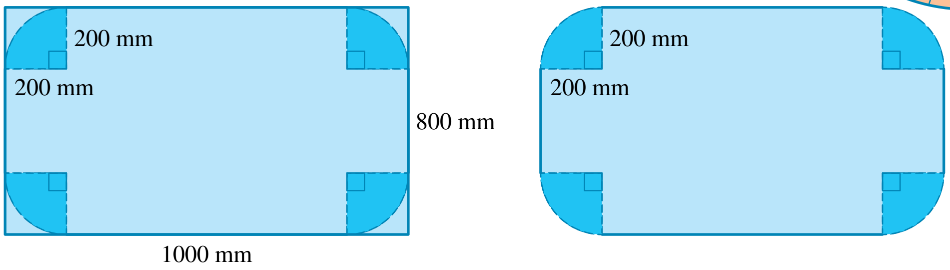
17. When viewed from above, a swimming pool can be seen as a rectangle with a semicircle at each end, as shown in the diagram at right. The area around the outside of the pool extending 1.5 m from the edge is to be paved.

a. Calculate the paved area around the pool.

b. If the pool is to be filled to a depth of 900 mm in the semicircular sections and 1500 mm in the rectangular section, what is the total volume of water in the pool to the nearest litre?



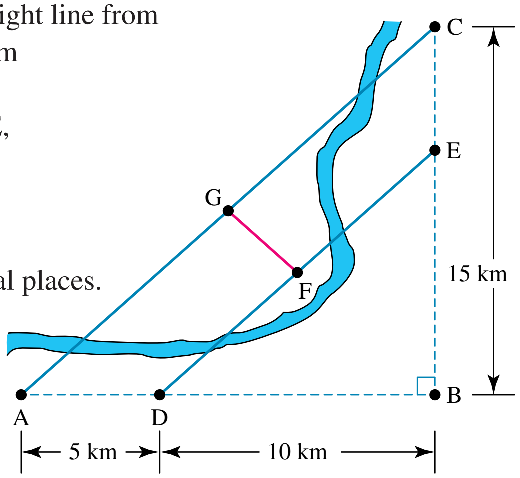
18. A rectangular piece of glass with side lengths 1000 mm and 800 mm has its corners removed for safety, as shown in the diagrams below.



a. Calculate the surface area of the glass after the corners have been removed.

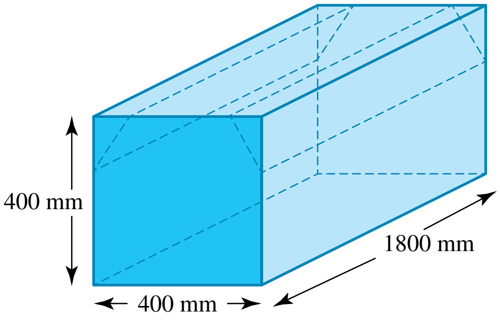
b. Calculate the perimeter of the glass after the corners have been removed.

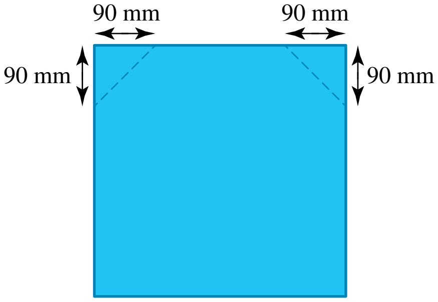
19. Two tunnels run under a bend in a river. One runs in a straight line from point A to point C, and the other runs in a straight line from point D to point E. Points A and D are 5 km apart, as are points C and E. Point B is 10 km from both D and E, and BD is perpendicular to BE. An access tunnel GF is to be constructed between the midpoints of AC and DE.



- **a.** Calculate the lengths of AC and DE correct to 2 decimal places.
- **b.** Calculate the length of GF correct to 2 decimal places.
- **c.** The tunnels running from A to C and from D to E are cylindrical in shape with an outer diameter of 40 m. Calculate the volume of material that was removed to create the two tunnels.
- **d.** The trucks used to remove the excavated material during the construction of the tunnels can carry a maximum of 85 m^3. How many truck loads were required to make the two tunnels? Round your answer correct to the nearest whole number.
- **e.** The inner walls of the tunnels are formed of concrete that is 3.5 m thick. Calculate the total volume of concrete used for the tunnels, correct to 1 decimal place.
- **f.** The inner surface of the concrete in the tunnels is sprayed with a sealant to prevent water seeping through. Calculate the total area that is sprayed with the sealant, correct to 1 decimal place.

20. A piece of timber has the dimensions 400 mm by 400 mm by 1800 mm. The top corners of the piece of timber are removed along its length. The cuts are made at an angle a distance of 900 mm from the corners, so the timber that is removed forms two triangular prisms.





- **a.** Calculate the volume of the piece of timber in cubic metres after the corners are removed. Give your answer correct to 3 decimal places.
- **b.** Calculate the surface area of the piece of timber after the corners are removed.
- **c.** What is the total volume and surface area of the two smaller pieces of timber that are cut from the corners, assuming they each remain as one piece?

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Units 1 & 2 Sit chapter test

Answers

Chapter 3 Pythagoras’ theorem and mensuration

Exercise 3.2 Pythagoras’ theorem in two dimensions

1. a. PR b. YZ
2. a. 13 cm b. 170 mm
3. a. 10.82 cm b. 6.93 m
4. a. 10.4 cm b. 1.9 m
5. a. 8.9 cm b. 22.1 cm c. 47.4 mm d. 37.3 m
6. A
7. C
8. a. Yes b. No c. Yes
   d. No e. Yes f. Yes
9. a. 9, 12, 15 b. 7, 24, 25 c. 1.5, 2.0, 2.5
   d. 3, 4, 5 e. 11, 60, 61 f. 10, 24, 26
   g. 9, 40, 41 h. 0.7, 2.4, 2.5
10. D
11. B
12. 13 m
13. 3.23 m
14. 3.73 m
15. 2.2 m
16. 7.5 m
17. 7.07 cm
18. 11.40

Exercise 3.3 Pythagoras’ theorem in three dimensions

1. 1.56 m
2. No, the maximum length rod that could fit would be 2.71 m long.
3. 41.4 m
4. Pyramid 1 has the greatest height.
5. a. 30.48 cm b. 2.61 cm c. 47.27 cm
6. a. 11.55 cm b. 27.71 cm
   c. 3.18 cm d. 95.84 cm
7. a. Yes b. 1.015 m
8. 8.09
9. 6 m; sample response can be found in the worked solutions in your online resources.
10. $s = 186.5$ m
11. $h = 25.475$
12. $h = 28.5$ m
13. a. 14.70 m b. 14.98 m c. 13.82 m
14. a. 10 050 m b. 6083 m c. 10 054 m

Exercise 3.4 Perimeter and area I

1. Perimeter = 80 cm, area = 360 cm^2
2. Circumference ≈ 50.27 cm, area ≈ 201.06 cm^2
3. a. Perimeter = 59 m, area = 155.12 m^2
   b. Perimeter = 28.83 cm, area = 20 cm^2
4. a. Perimeter = 43.98 cm, area = 153.94 cm^2
   b. Perimeter = 48 cm, area = 112 cm^2
5. a. Circumference = 31.42 cm, area = 78.54 cm^2
   b. Circumference = 56.55 cm, area = 254.47 cm^2
6. Perimeter = 68 cm, area = 192 cm^2
7. 33.79 cm^2
8. Area = 1.14 m^2
9. Perimeter = 96 cm, area = 360 cm^2
10. 31.61 cm
11. 49.19 cm
12. a. i. 201.06 cm ii. 389.56 cm^2
    b. 187.19 cm
13. a. 26.39 m b. 1104.73 m^2
14. a. 173.21 m^2
    b. Perimeter = 90 m, area = 389.71 m^2
    c. Perimeter = 120 m, area = 839.7 m^2

Exercise 3.5 Perimeter and area II

1. 72 cm^2
2. Perimeter = 48.6 cm, area = 106 cm^2
3. 198.94 cm^2
4. a. 5.57 m b. 6.11 m^2
5. a. 5831.62 cm^2 b. 56.58 cm^2
6. 4.24 cm^2
7. Perimeter = 52.94 cm, area = 167.55 cm^2
8. Perimeter = 198.23 cm, area = 2073.45 cm^2
9. Perimeter = 145.40 cm, area = 1143.06 cm^2
10. Perimeter = 197 cm, area = 1571 cm^2
11. 9.42 cm^2
12. 25.13 cm^2
13. 9292.83 cm^3
14. Perimeter = 151.76 cm, area = 576 cm^2
15. 2.50 m^2
16. 0.67 m^2
17. 2138.25 cm^2
18. Area = 900 cm^2, perimeter = 194.16 cm

Exercise 3.6 Volume and capacity

1. 0.6 m^3
2. a. 20 500 cm^3 b. 13 415 cm^3
   c. 126 720 cm^3 d. 51 188 cm^3
3. Rectangular pool: 11 070 L
   Square pool: 10 800 L
   Circular pool: 10 263.58 L
4. a. 25.2 m^3 b. 7.2 m^3
5. 56 301 cm^3, 56301 mL
6. a. 490 376 cm^3 b. 136 m^3
7. Volume above = 162 578 cm^3, volume below = 318 086 cm^3
8. a. 34.64 L b. 0.231 m^3
9. 39 584.1 cm^3, 39.58 L
10. a. 753 218 cm^3 b. 0.022 cm^3
11. a. 14 850 cm^3 b. 471 250 cm^3
    c. 45 804.4 cm^3 d. 42 524.6 cm^3
12. 0.0528m^3, 52.8 L
13. a. 22 875 cm^3 b. 15 675.12 cm^3 c. 0.432 m^3
14. 6.25 m^3
15. 0.0824 m^3, 82.4 L

16. a. 39 cm **b.** 1 m

17. 21.91 m^3

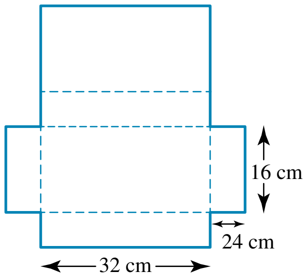
18. The hemispherical-topped silo holds 37.35 m^3 more.

19. a. 8983.3 m^3 **b.** 332 716 049.4 cm^3

20. a. 467.35 cm^3 **b.** 9677.11 cm^3

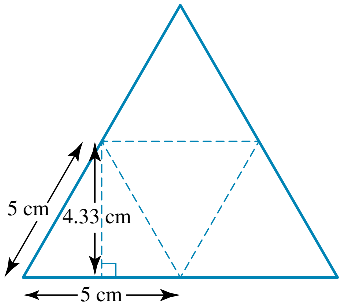
Exercise 3.7 Surface area of three-dimensional objects

1.



3328 cm^2

2.



43.3 cm^2

3. a. 2475 m^2 **b.** 3342 cm^2

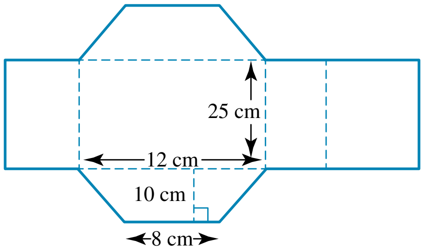
4. a. 249.42 cm^2 **b.** 120 687.42 cm^2

c. 3487.17 cm^2 **d.** 1559.02 cm^2

5. a. 392 cm^2 **b.** 1639.91 cm^2

c. 810 cm^2 **d.** 1256.64 cm^2

6. a.



b. 1210 cm^2

7. 390.94 m^2

8. 7902.86 cm^2

9. a. 236 cm^2 **b.** 487 cm^2

10. The cone with height 6.5 cm and radius 2.2 cm has the greater surface area by 6.34 cm^2.

11. a. 5627 cm^2 **b.** 3223 cm^2

12. 71.90 m^2

13. 688 cm^2

14. a. 34 720 cm^2 **b.** 14 760 cm^2

15. 104.15 m^2

16. 10.22 m^2

3.8 Review: exam practice

1. C **2.** D **3.** A **4.** D **5.** D
6. D **7.** C **8.** D **9.** A **10.** B

11. 11.38 m

12. a. 44.33 m^2 **b.** 36.55 m

13. 0.849 m **14.** 6000 mm **15.** 12.63 m^2

16. a. 13.352 m **b.** 105.8 m^2 **c.** 12 058.32 L

17. a. 55.92 m^2 **b.** 71 310 L

18. a. 765 663.7 mm^2 **b.** 3256.6 mm

19. a. 21.21 km and 14.14 km
b. 3.54 km
c. 44 422 120 m^3
d. 522 613
e. 14 187 314.6 m^3
f. 3 664 824.9 m^2

20. a. 0.273 m^3
b. 2 994 005 mm^2
c. Volume = 14 580 000 mm^3
Surface area = 1 122 405 mm^2

CHAPTER 4
Similar figures and scale factors

4.1 Overview

The construction industry continues to grow. You only need to look at any major city in the world to see cranes on buildings and new buildings being built. Buildings are used as dwellings and shelters. They are also places for companies to conduct business, as well as being hospitals, schools or even places of leisure. But the most familiar building is our home. To produce all these buildings and make sure they are functional, designers must apply the principals of mathematics. These principals involve scale drawings in the form of plans, where the actual sizes are reduced proportionally to a size where they can be viewed on paper. Architects will often use a different set of scales to that of engineers, surveyors or even furniture designers. This all depends on the size of what is being designed, as well as the complexity of the design.

Scale is not just used for plans: the plans can be taken a step further to create a scale model of the design. A scale model is generally a physical representation of an object that maintains accurate relationships between all important aspects of the object's design. The scale model demonstrates some of the behaviour of the original object without investigating the original object itself. Scale models are used in many fields including engineering, film making, military, salesmanship and hobby model building. To be considered a true scale model, all important aspects must be accurately modelled, not just the scale of the object but also the material properties. An example of a scale model could be an aerospace company wanting to test a new wing design. They could construct a scaled down model and test it in a wind tunnel under simulated conditions.

The photo above shows a famous model that was made by Dr Maxime Faget, from NASA. It is a model of the Space Shuttle. He needed a model because he had to demonstrate to his NASA co-workers a very different concept — that the Space Shuttle would be able to glide back to land on Earth as an unpowered glider.

LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookplus at www.jacplus.com.au.

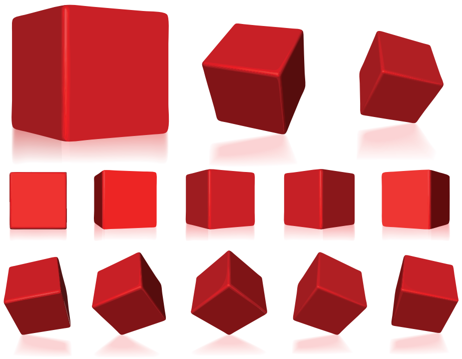
4.2 Similarity of two-dimensional figures

4.2.1 Conditions for similarity

Objects are called **similar** when they are exactly the same shape but have different sizes. Objects that are exactly the same size and shape are called **congruent**.

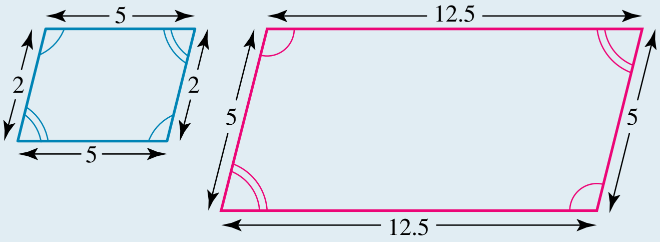
Similarity is an important mathematical concept that is often used for planning purposes in areas such as engineering, architecture and design. Scaled-down versions of much larger objects allow designs to be trialled and tested before their construction.

Two-dimensional objects are similar when their corresponding internal angles are the same and their corresponding side lengths are **proportional**. This means that the ratios of corresponding side lengths are always equal for similar objects. We use the symbols ~ or ||| to indicate that objects are similar.



WORKED EXAMPLE 1

Show that these two objects are similar.



THINK	WRITE
1. Confirm that the internal angles for the objects are the same.	The diagrams indicate that all corresponding angles in both objects are equal.
2. Calculate the ratio of the corresponding side lengths and simplify.	Ratios of corresponding sides: $\frac{12.5}{5} = 2.5$ and $\frac{5}{2} = 2.5$
3. State the answer.	The two objects are similar as their angles are equal and the ratios of the corresponding side lengths are equal.

4.2.2 Similar triangles

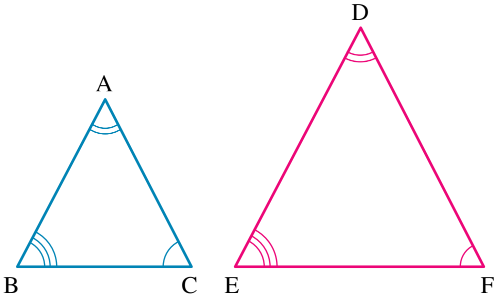
The conditions for similarity apply to all objects, but not all of them need to be known in order to demonstrate similarity in triangles. If pairs of triangles have any of the following conditions in common, they are similar.

Note: In this chapter we will put the image first when calculating ratios of corresponding lengths. The original is blue and the image is red.

1. Angle–angle–angle (AAA)

AAA: If two different-sized triangles have all three angles identified as being equal, they will be similar.

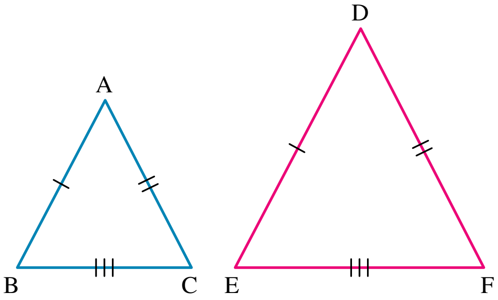
$\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$



2. Side–side–side (SSS)

SSS: If two different-sized triangles have all three sides identified as being in proportion, they will be similar.

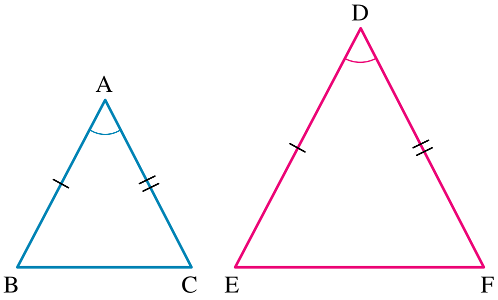
$$\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$$



3. Side–angle–side (SAS)

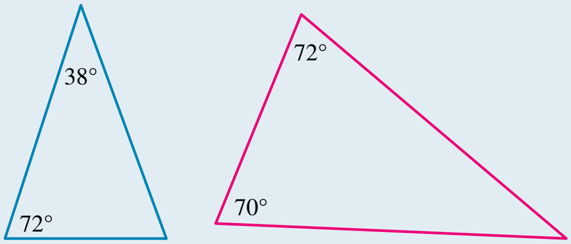
SAS: If two different-sized triangles have two pairs of sides identified as being in proportion and their included angles are equal, they will be similar.

$$\frac{DE}{AB} = \frac{DF}{AC} \text{ and } \angle A = \angle D$$



WORKED EXAMPLE 2

Show that these two triangles are similar.



THINK	WRITE
1. Identify all possible angles and side lengths.	The angles in the blue triangle are: 38°, 72° and 180 $-(38+72)=70°$. The angles in the red triangle are: 70°, 72° and $180-(70+72)=38°$
2. Use one of AAA, SSS or SAS to check for similarity.	All three angles in the two triangles are equal.
3. State the answer.	The two triangles are similar as they satisfy the condition AAA.

on Resources

Interactivity: Similar triangles (int-6273)

studyon

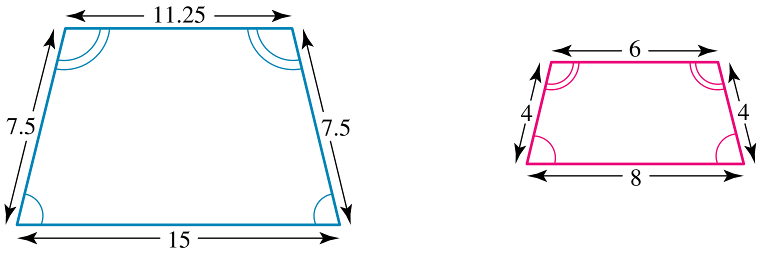
Units 1 & 2 > Area 2 > Sequence 2 > Concept 1

Similarity of two-dimensional figures Summary screen and practice questions

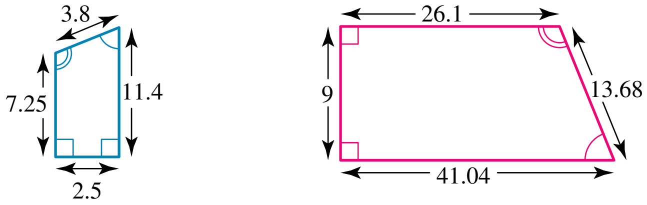
Exercise 4.2 Similarity of two-dimensional figures

1. **WE1** Show that the two objects in each of the following pairs are similar.

a.



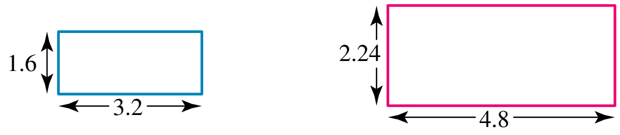
b.



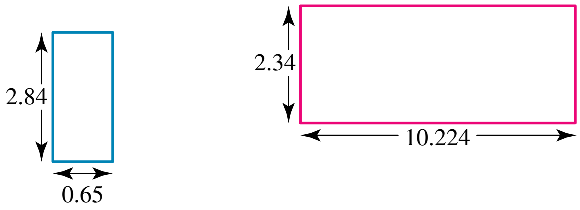
2. Show that a rectangle with side lengths of 4.25 cm and 18.35 cm will be similar to one with side lengths of 106.43 cm and 24.65 cm.

3. Determine which of the following pairs of rectangles are similar.

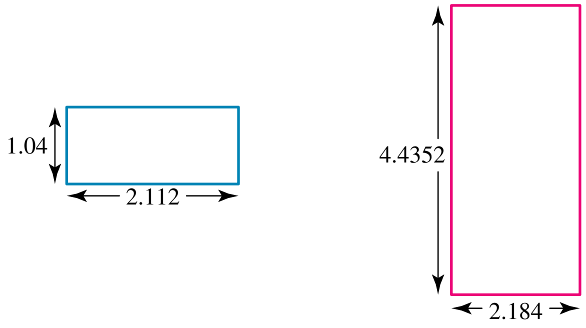
a.



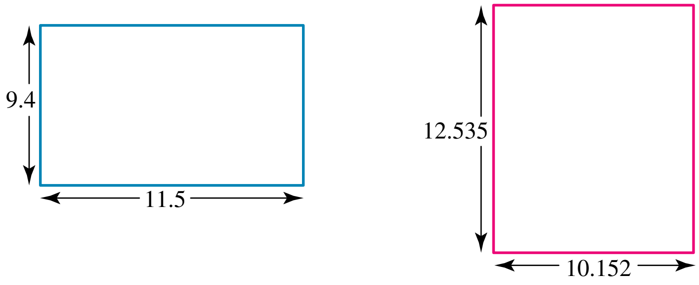
b.



c.

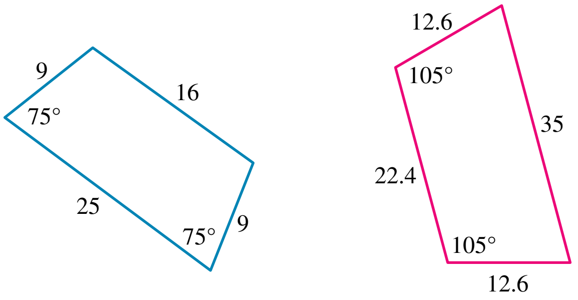


d.

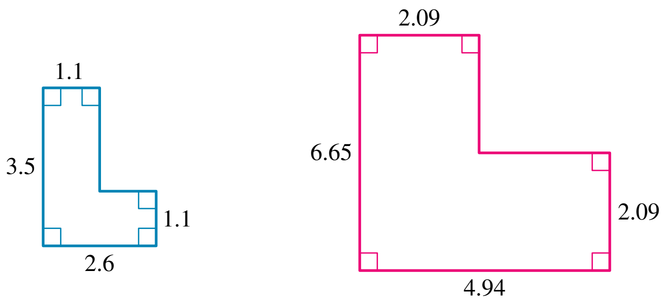


4. Determine which of the following pairs of polygons are similar?

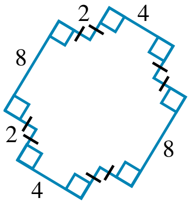
a.

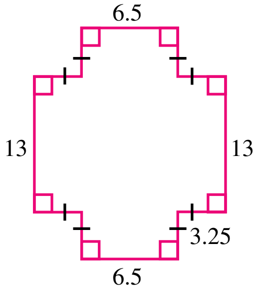


b.

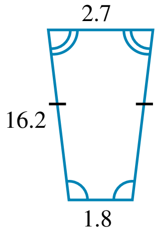


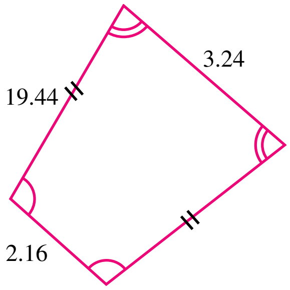
c.





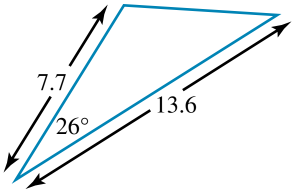
d.

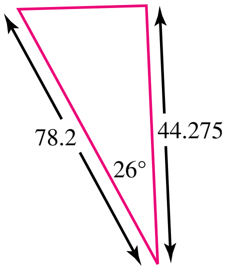




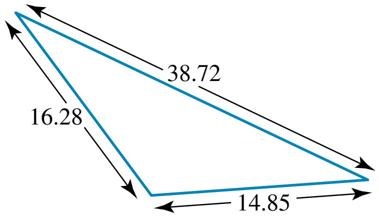
5. **WE2** Show that the two triangles in each of the following pairs are similar.

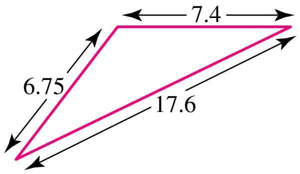
a.





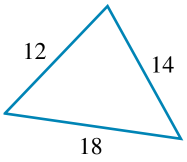
b.

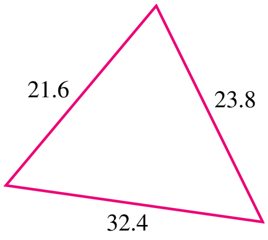




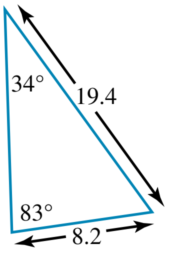
6. Which of the following pairs of triangles are similar?

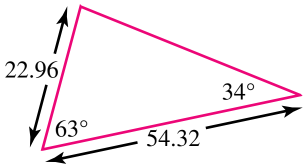
a.





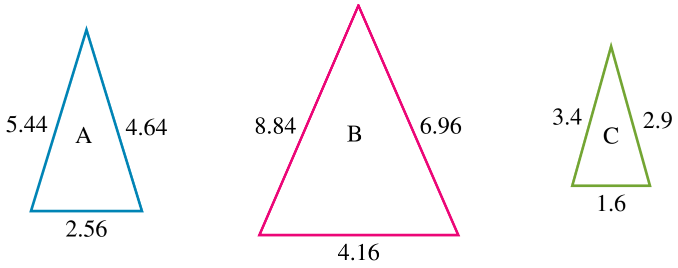
b.



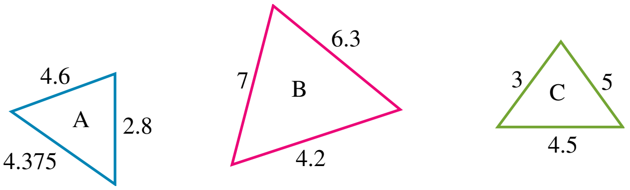


7. In each of the following groups, which two triangles are similar?

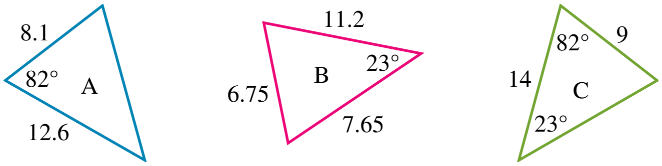
a.



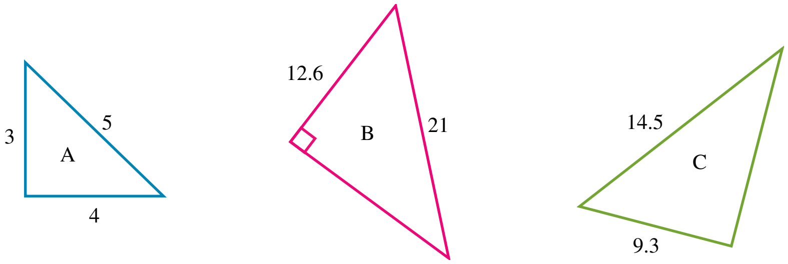
b.



c.

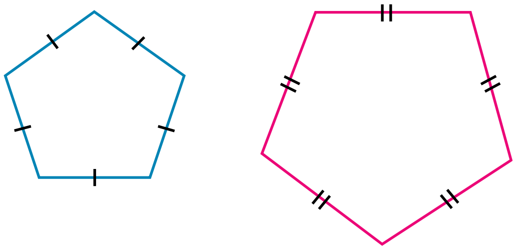


d.

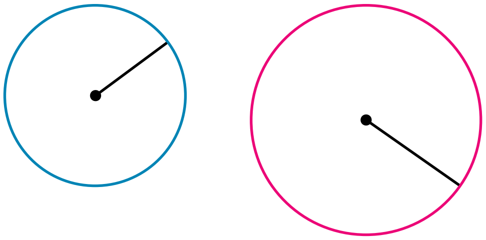


8. Explain why each of the following pairs of objects must be similar.

a.

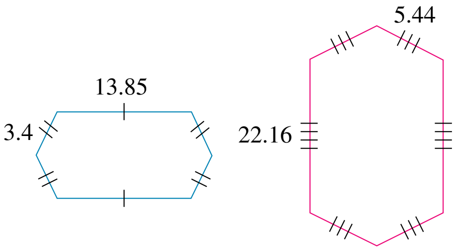


b.

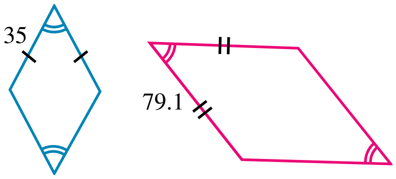


9. Calculate the ratios of the corresponding sides for the following pairs of objects.

a.

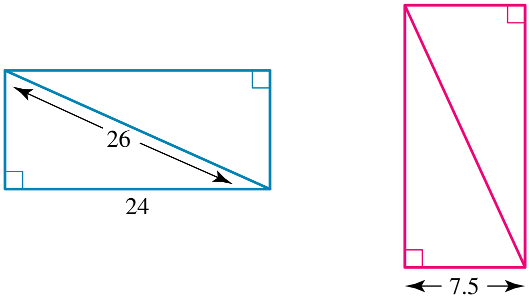


b.

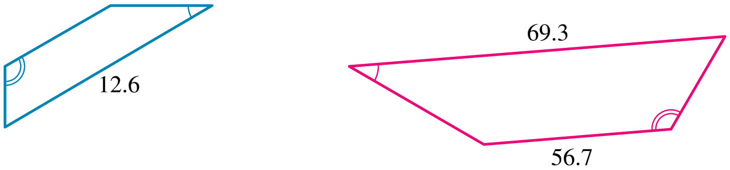


10. Evaluate the ratios of the corresponding side lengths in the following pairs of similar objects.

a.

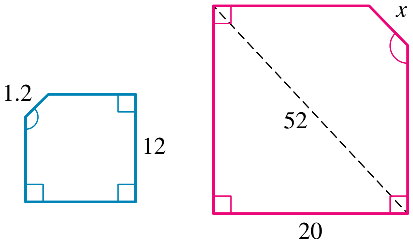


b.

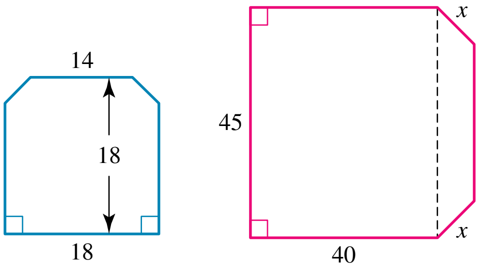


11. Evaluate the unknown side lengths in the following pairs of similar objects.

a.



b.



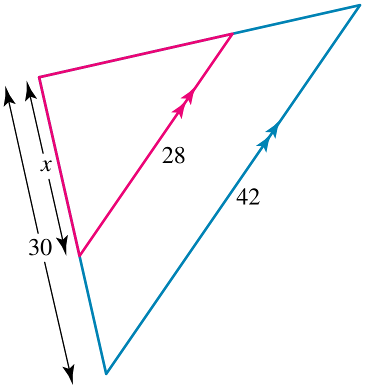
12. Verify that the following are similar.

a. A square of side length 8.2 cm and a square of side length 50.84 cm

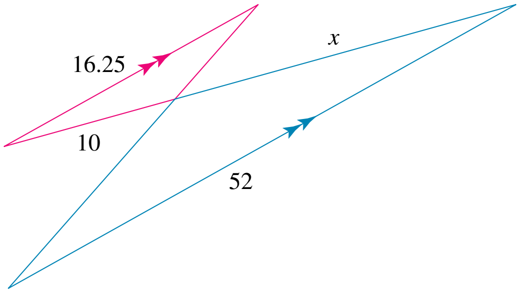
b. An equilateral triangle of side length 12.6 cm and an equilateral triangle of side length 14.34 cm

13. Calculate the value of x required to make the pairs of objects similar in each of the following diagrams.

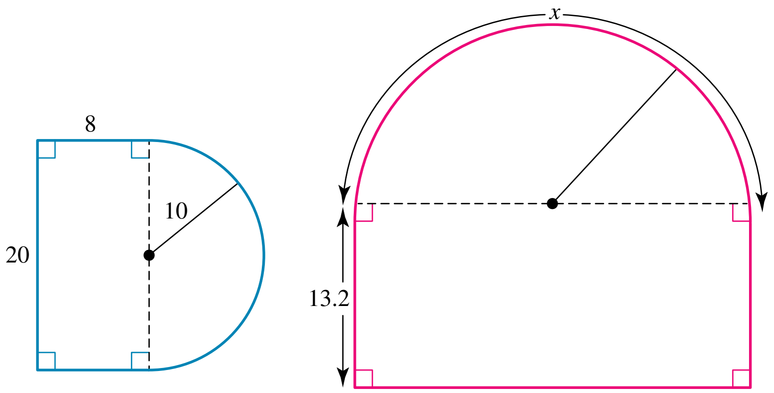
a.



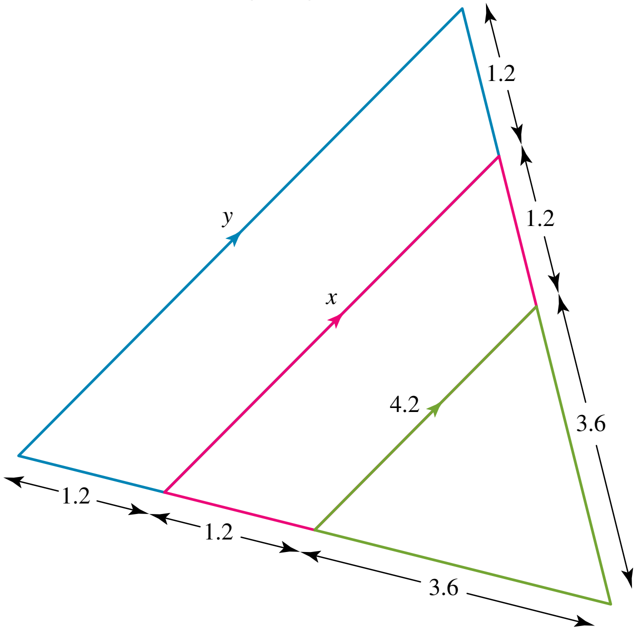
b.



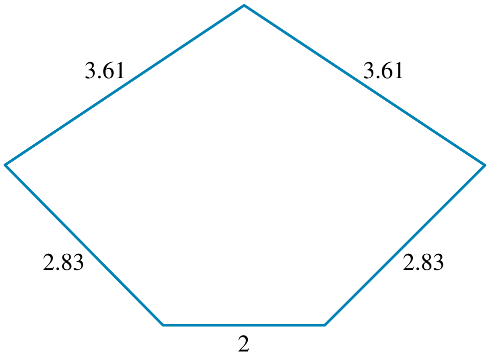
14. Calculate the value of x for the following similar shapes. Give your answer correct to 1 decimal place.



15. Calculate the values of x and y in the following diagram.



16. For the polygon shown, draw and label a similar polygon where:



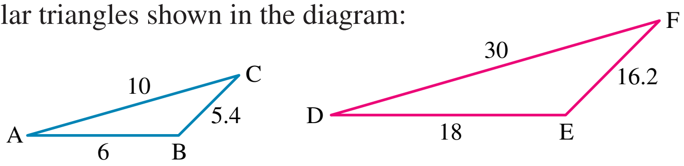
a. the corresponding sides are $\frac{4}{3}$ the size of those shown

b. the corresponding sides are $\frac{4}{5}$ the size of those shown.

4.3 Linear scale factors

4.3.1 Similar triangles

Consider the pair of similar triangles shown in the diagram:



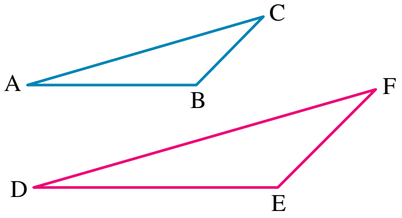
The ratios of the corresponding side lengths are:

$$\begin{aligned} DE : AB &= 18 : 6 \\ &= 3 : 1 \\ EF : BC &= 16.2 : 5.4 \\ &= 3 : 1 \\ DF : AC &= 30 : 10 \\ &= 3 : 1 \end{aligned}$$

Note: In this topic we will put the image first when calculating ratios of corresponding lengths. The original is blue and the image is red.

In fact, the side lengths of triangle DEF are all three times the lengths of triangle ABC. In this case we would say that the **linear scale factor** is 3. The linear scale factor for similar objects can be evaluated using the ratio of the corresponding side lengths.

$\Delta ABC \sim \Delta DEF$



Linear scale factor:

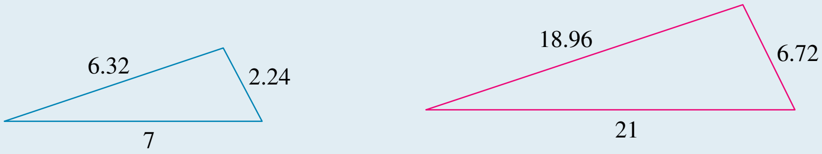
$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = k$$

$$\textbf{Linear scale factor} = \frac{\textbf{length of image}}{\textbf{length of object}}$$

A linear scale factor greater than 1 indicates enlargement, and a linear scale factor less than 1 indicates reduction.

WORKED EXAMPLE 3

Calculate the linear scale factor for the pair of triangles shown in the diagram.

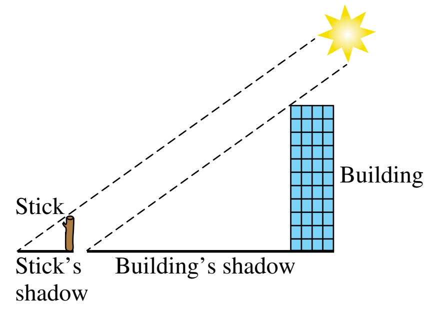


THINK	WRITE
1. Calculate the ratio of the corresponding side lengths and simplify.	$\frac{21}{7} = \frac{18.96}{6.32} = \frac{6.72}{2.24} = 3$
2. State the answer.	The linear scale factor is 3.

4.3.2 Similar triangles and shadow sticks

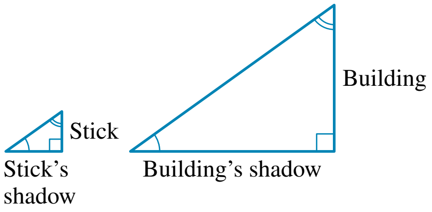
Sometimes it is not possible to measure heights of objects such as trees or tall buildings. It is, however, often possible to measure the length of a shadow cast by these objects. If we compare the length of these shadows with the length of a shadow cast by a stick of known height under the same conditions, it is possible to calculate heights which are difficult to measure.

Consider the situation on the right.



We need to determine the height of the building. Take a stick whose height we know (or can measure) and place it, vertically, in the sunshine, near the building. The rays from the sun are parallel, so we have two **similar** right-angled triangles. (The building and the stick are at right angles to the ground.) We can measure both shadows

The larger right-angled triangle formed by the building and its shadow is some **scale factor** of the smaller right-angled triangle formed by the stick and the stick's shadow.

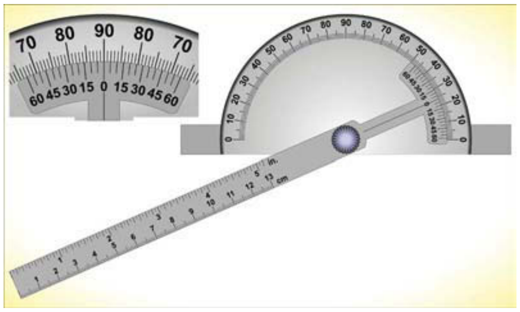


$$\text{Scale factor} = \frac{\text{length of building shadow}}{\text{length of stick shadow}}$$

We can then apply the same scale factor to the height of the stick to determine the height of the building. So,

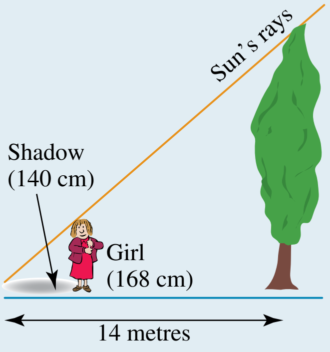
$$\text{Height of building} = \text{height of stick} \times \text{scale factor}$$

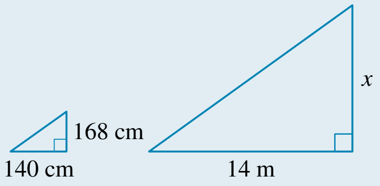
You can use a clinometer like the one pictured at right to make sure the angle of the two objects is the same.



WORKED EXAMPLE 4

At the same time as a tree cast a shadow of 14 m, a 168 cm-tall girl cast a shadow of 140 cm. Calculate the height of the tree. Give the answer to 1 decimal place.



THINK	WRITE
1. Identify the two similar triangles and draw them separately. (We assume that both the girl and the tree are perpendicular to the ground.)	

2. Identify the side of the triangle whose length is required.	
3. Calculate the scale factor. *Note*: Measurements must be in the same units.	$\text{Scale factor} = \dfrac{\text{length of tree shadow}}{\text{length of girl's shadow}}$ $= \dfrac{14\text{ m}}{1.4\text{ m}}$ $= 10$
4. Apply the scale factor to the girl's height.	$\text{Height of tree} = \text{girl's height} \times \text{scale factor}$ $= 1.68\text{ m} \times 10$ $= 16.8\text{ m}$
5. Write the final answer, specifying units.	Height of tree is 16.8 m

on Resources

Digital document: SkillSHEET Similar triangles (doc-29504)

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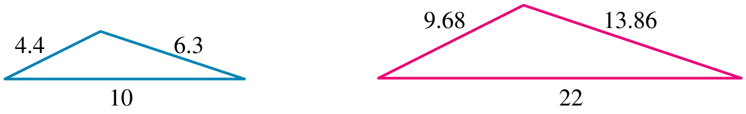
Units 1 & 2 > Area 2 > Sequence 2 > Concept 2

Linear scale factors Summary screen and practice questions

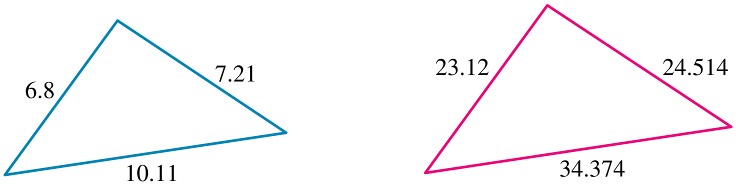
Exercise 4.3 Linear scale factors

1. **WE3** Calculate the linear scale factor for the pairs of triangles shown.

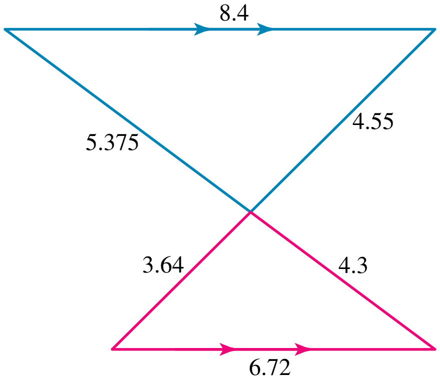
a.



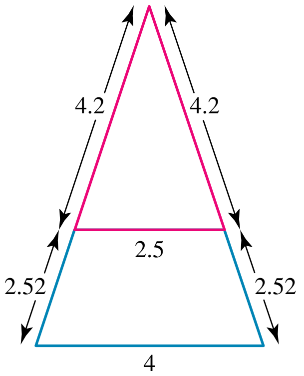
b.



c.

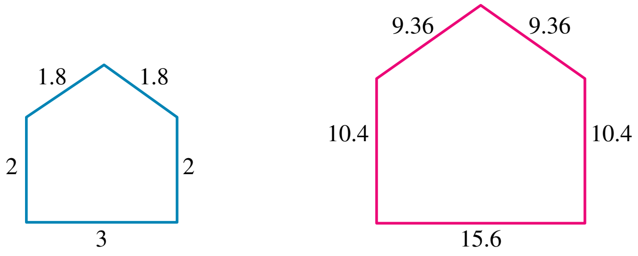


d.

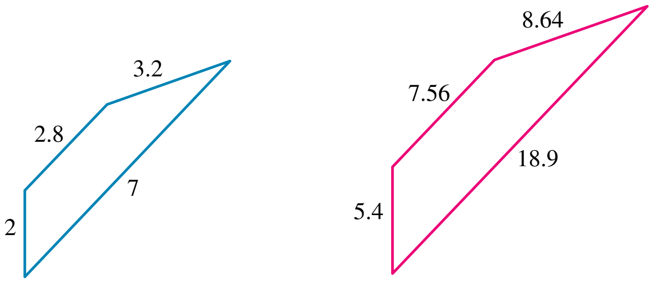


2. Calculate the linear scale factors for the pairs of similar objects shown.

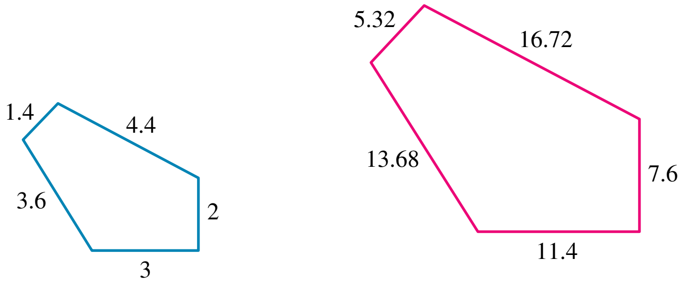
a.



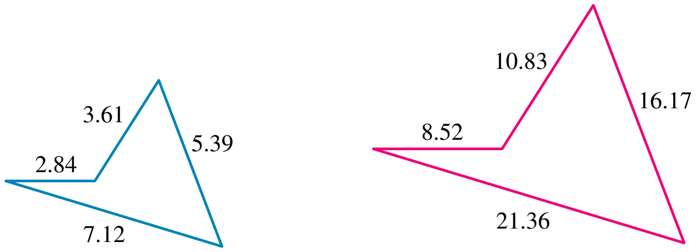
b.



c.



d.



3. Calculate the linear scale factors for the following ratios of corresponding side lengths.

a. $3:2$ **b.** $12:5$ **c.** $3:4$ **d.** $85:68$

4. Calculate the missing values for the following.

a. $\frac{3}{\square} = \frac{\square}{12} = 16$

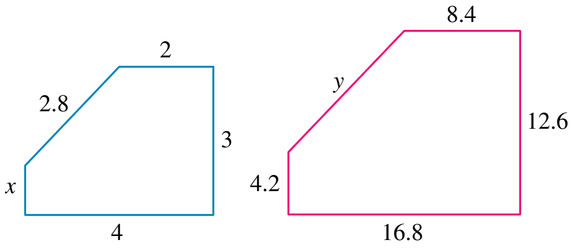
b. $\frac{5}{\square} = \frac{44}{11} = \square$

c. $\frac{\square}{7} = \frac{81}{9} = \square$

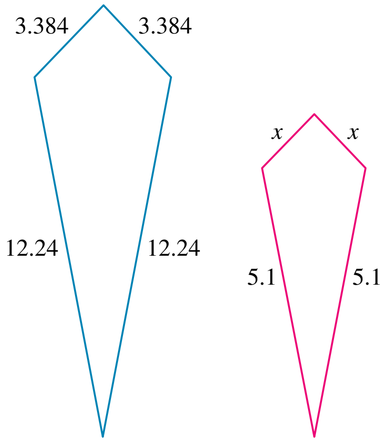
d. $\frac{16.5}{\square} = \frac{\square}{34} = 5.5$

5. Calculate the unknown side lengths in the pairs of similar shapes shown

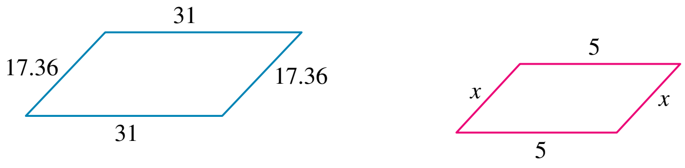
a.



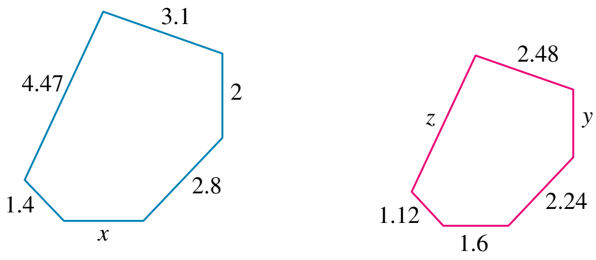
b.



c.

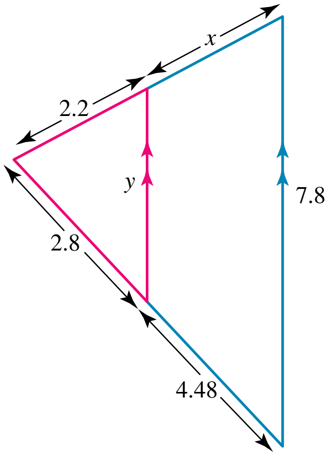


d.

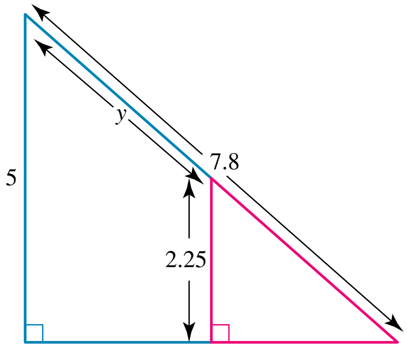


6. Calculate the unknown side lengths in the diagrams shown.

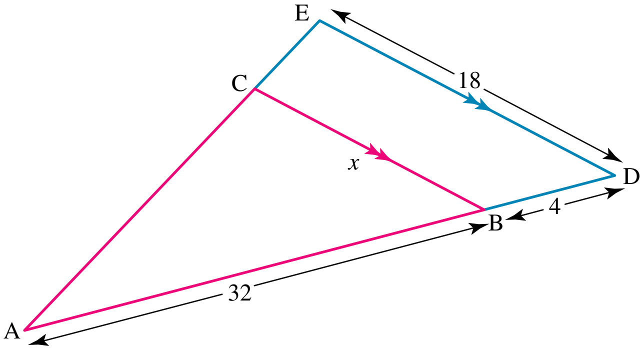
a.



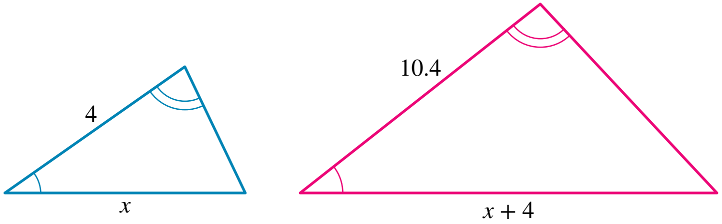
b.



7. Calculate the length of BC.

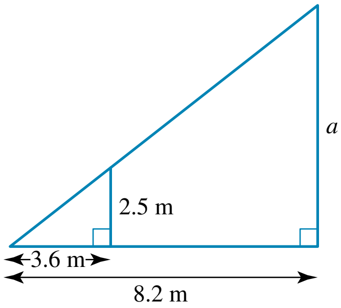


8. Calculate the value of *x*.

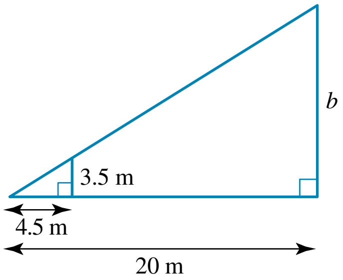


9. The following diagrams represent the measurements taken from shadows of sticks and objects. Use the figures to determine the heights of the objects.

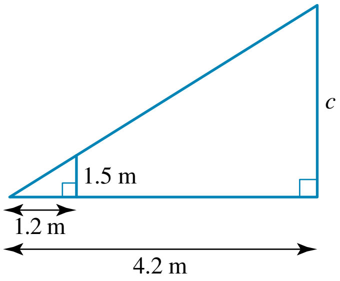
a.



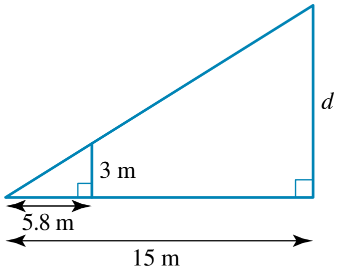
b.



c.



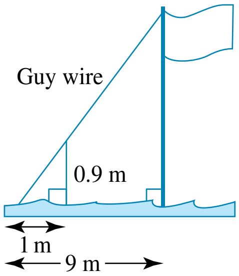
d.



10. At the same time as a building cast a shadow of 14.3 metres, a 2-metre stick cast a shadow of 5.3 metres. What is the height of the building?

11. A boulder on the shoreline cast a shadow of 15.8 m on the beach at the same time as a 1.5-metre stick cast a shadow of 3.7 m. Determine the height of the boulder.

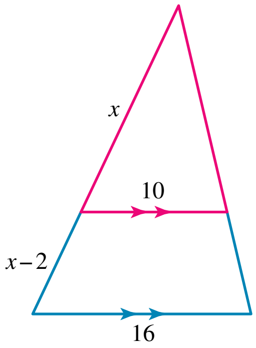
12. Find the height of the flagpole shown in the diagram given.



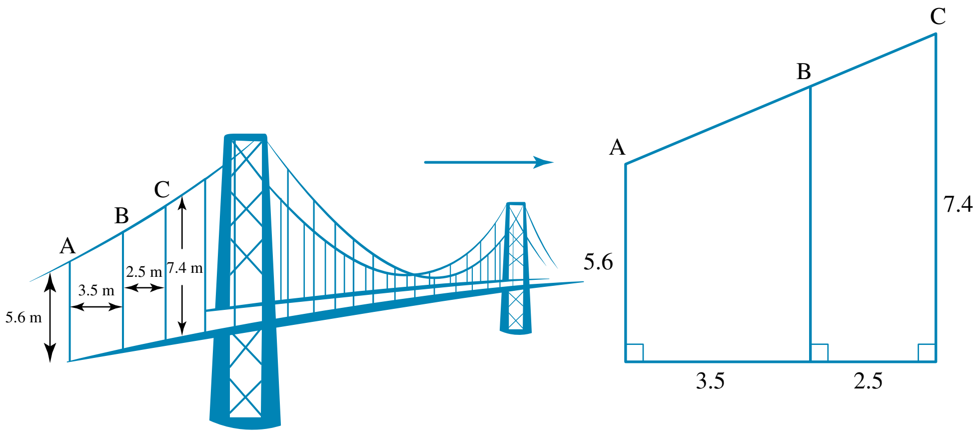
13. The side of a house casts a shadow that is 8.4 m long on horizontal ground.

- **a.** At the same time, an 800-mm vertical garden stake has a shadow that is 1.4 m long. What is the height of the house?
- **b.** When the house has a shadow that is 10 m long, how long is the garden stake's shadow?

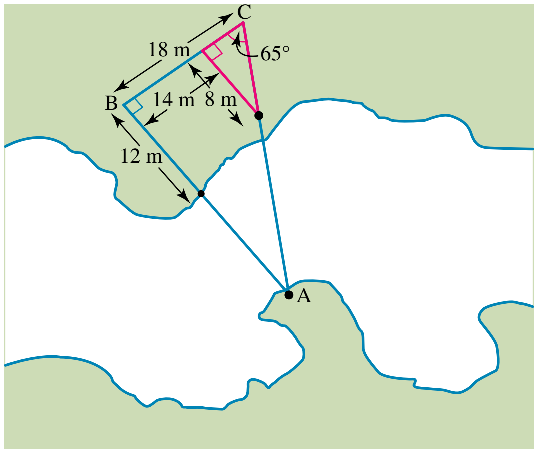
14. Calculate the value of x in the diagram.



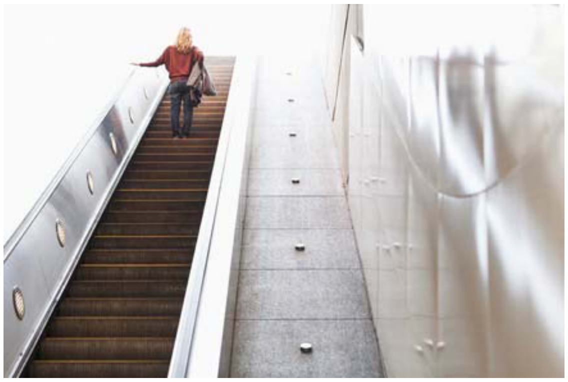
15. A section of a bridge is shown in the diagram. How high is point B above the roadway of the bridge?



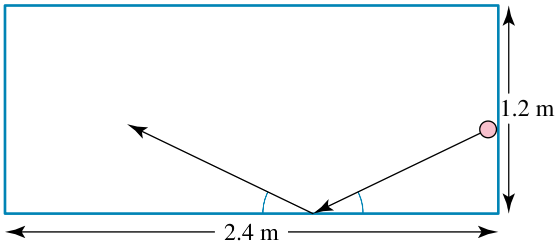
16. To calculate the distance across a ravine, a surveyor took a direct line of sight from the point B to a fixed point A on the other side and then measured out a perpendicular distance of 18 m. From that point the surveyor measured out a smaller similar triangle as shown in the diagram. Calculate the distance across the ravine along the line AB.



17. Over a horizontal distance of 6.5 m, an escalator rises 12.75 m. If you travel on the escalator for a horizontal distance of 4.25 m, what vertical distance have you risen?



18. In a game of billiards, a ball travels in a straight line from a point one-third of the distance from the bottom of the right side and rebounds from a point three-eighths of the distance along the bottom side. The angles between the bottom side and the ball's path before and after it rebounds are equal.



a. Calculate the perpendicular distance, correct to 2 decimal places, from the bottom side after the ball has travelled a distance of 0.8 m parallel with the bottom side after rebounding.

b. If the ball has been struck with sufficient force, at what point on an edge of the table will it next touch? Give your answer correct to 2 decimal places.

4.4 Scale drawings — maps and plans

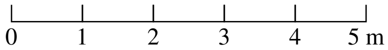
4.4.1 Scale drawings

Some families choose to either design and build a new house or renovate their existing house. The first stage is to select a design, which involves choosing a floor plan that caters for the needs of the family. In order to be able to read plans correctly, you must first understand scale drawings.

Plans can be represented in different ways. The following statements

$$1:100$$

$$1\text{ cm} \Leftrightarrow 1\text{ m}$$



all represent the same scale — that is, 1 cm on the plan represents a distance of 1 metre in the field. The scale factor is expressed as a ratio.

$$\text{Scale factor} = \frac{\text{plan length}}{\text{field length}} = \text{plan length} : \text{field length}$$

In the above examples, the scale factor is $\frac{1}{100}$, meaning that the plan measurements are one hundredth of the field measurements, or the field measurements are 100 times the size of the plan measurements. Notice that in the first representation, no units are mentioned. This means that the user can supply the appropriate units for the particular situation (depending on whether the measurements are small or large). That is,

At a scale of 1:100:
1 mm on the plan or drawing represents 100 mm in the field or original
or
1 cm on the plan or drawing represents 100 cm (1 m) in the field or original

Where the scale factor is a number greater than 1, the plan or drawing represents an **enlargement** of the original; a scale factor smaller than 1 represents a **reduction** of the original.

WORKED EXAMPLE 5

A house plan is drawn to a scale of 1 : 100.

a. Determine the size of a bedroom measuring 3 cm square on the plan.

b. The patio is 3500 mm wide. What would be its measurement on the plan?

THINK	WRITE
a. 1. Use cm as the unit in the scale because the plan unit is cm.	**a.** Scale is $1\text{ cm} : 100\text{ cm}$
2. The actual measurements are 100 times the plan measurements, so multiply by 100.	$3\text{ cm} : 3 \times 100\text{ cm}$
3. Calculate the actual length and convert to the appropriate unit.	$3\text{ cm} : 300\text{ cm}$
4. Write a concluding statement.	3 cm on the plan represents 3 m in the house. ∴ The bedroom is 3 m square.
b. 1. The plan is $\frac{1}{100}$ the size of the house, so divide by 100, remembering the answer is in the same units.	**b.** Patio width $= 3500 \div 100\text{ mm}$ $= 35\text{ mm}$

2. Calculate the plan length, converting to the appropriate unit if necessary.
3. Write a concluding statement.

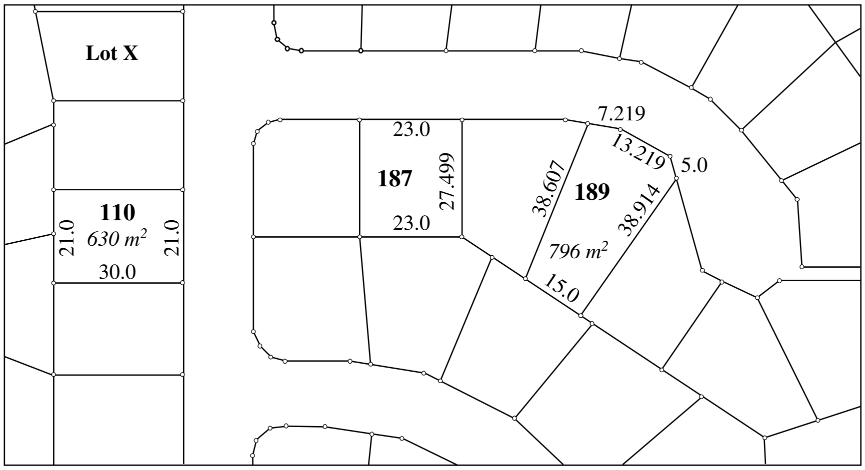
The width of the patio on the plan is 35 mm or 3.5 cm.

4.4.2 Building plans

Detailed building plans are necessary so that builders and other tradespersons know exactly what is required to complete a project. By the time it is completed, any building will have incorporated information drawn from many facets of construction. Drawings for a domestic structure will include a survey plan, a site plan, floor plans and elevations.

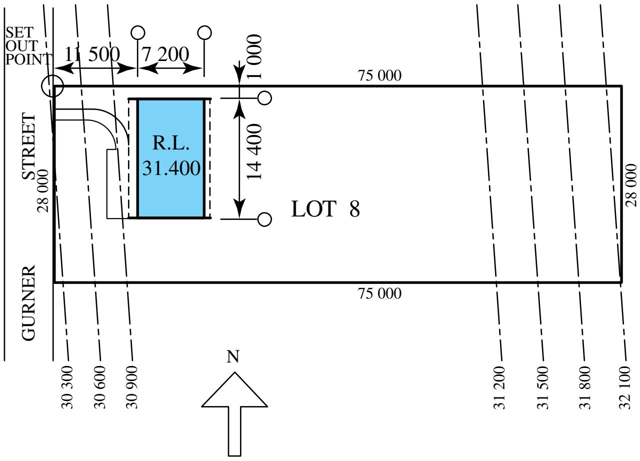
Survey plan

A **survey plan** shows all boundaries of the block of land and includes the position of roadways and nearby lots, as shown in the figure.



Site plan

A **site plan** shows the boundaries of the lot that is to be built on, and where the structure is to be situated on this lot. It may also show contour lines which demonstrate the slope of the land. Contour lines are typically shown as height of the land above sea level.

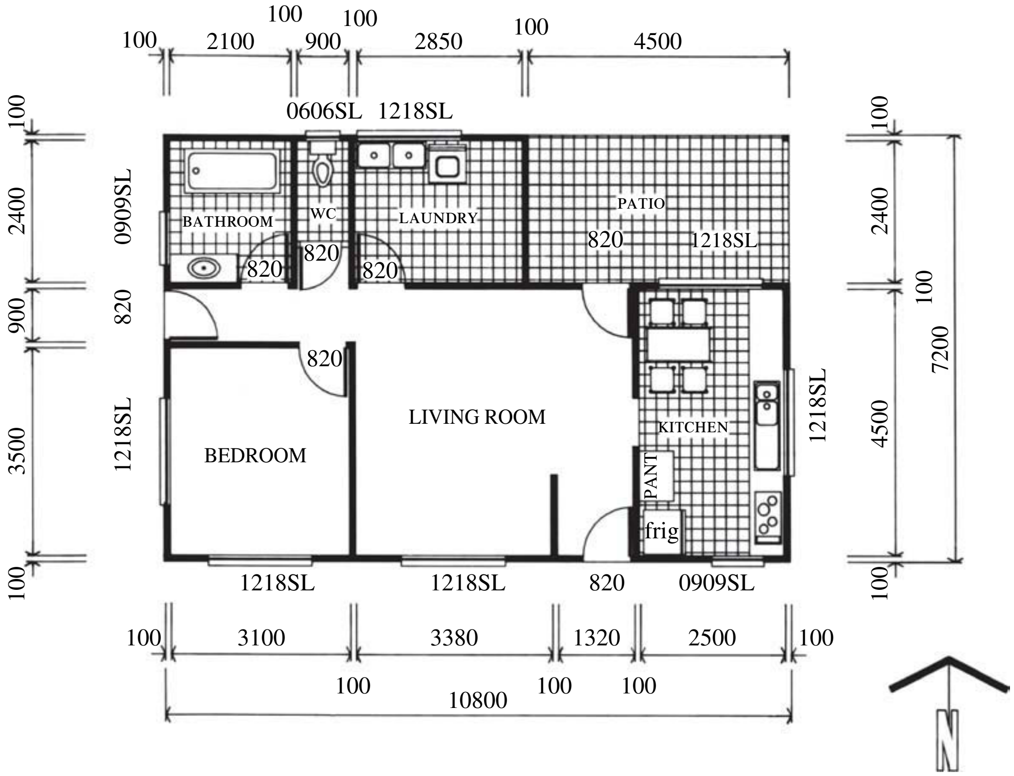


Reproduced by courtesy of Standards Australia

Floor plan

A **floor plan** shows the exact dimensions of the building, including dimensions and names of all rooms, the size and position of doors and windows, the direction in which doors open, the thickness of walls and the location of stairs.

In simple constructions, the roof plan and the locations of all electrical and plumbing fittings are superimposed on the floor plan. For more complex constructions, plans for these services would be made separately. A floor plan without electrical or plumbing fittings is shown in the following figure. If units are not indicated on a plan, it is assumed that dimensions are in millimetres.



WORKED EXAMPLE 6

This question refers to Lot X on the previous survey plan in 4.4.2.

a. What shape is Lot X?

b. The two parallel sides measure 30 m and 33 m, while the front and back boundaries measure 21 m and 23 m respectively. Calculate the area of Lot X.

c. Lot 110 is for sale for $119 700. Suggest a reasonable sale price for Lot X by comparing its area with the area of Lot 110.

THINK	WRITE
a. Lot X is 4-sided with one pair of parallel sides.	a. Lot X has the shape of a trapezium.
b. 1. Recall the formula for the area of a trapezium.	b. Area $= \frac{1}{2}(a + b)h$
2. Substitute for the variables.	$= \frac{1}{2}(30 + 33) \times 21 \text{ m}^2$
3. Calculate the area.	$= 661.5 \text{ m}^2$

c. 1. Calculate the price per m^2 based on Lot 110 information.	**c.** $\text{Price per m}^2 = \frac{\$119\,700}{630\text{ m}^2}$ $= \$190\text{ per m}^2$
2. Multiply by the number of square metres in Lot X.	$\text{Price} = \$190 \times 661.5$ $= \$125\ 685$ A reasonable price for Lot X would be \$125 700.

REMEMBER

1. Scales can be represented in different ways.
2. If no units are indicated on a scale representation, any unit can be inserted, so long as the same unit is used for both the plan length and the field length.
3. The scale factor compares the plan length with the field length; that is

   scale factor = plan length: field length
4. A scale factor greater than 1 represents an enlargement and a scale factor smaller than 1 represents a reduction in size.
5. In converting from a plan length to a field length, multiply by the scale factor. Divide by the scale factor when progressing from the field to the plan.

on Resources

Digital document: SkillSHEET Conversion of length units (doc-29492)
Digital document: SkillSHEET Reading scales (How much is each interval worth?) (doc-29506)
Digital document: SpreadSHEET Map scales 1 (doc-29507)
Digital document: SpreadSHEET Map scales 2 (doc-29508)

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Units 1 & 2 > Area 2 > Sequence 2 > Concept 3

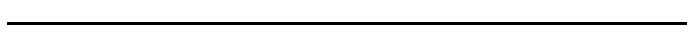
Scale drawing Summary screen and practice questions

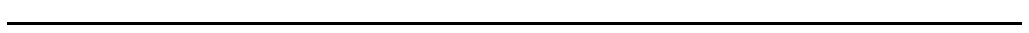
Exercise 4.4 Scale drawings — maps and plans

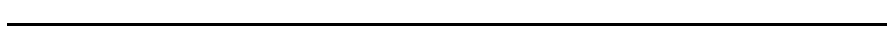
1. Convert to metres.

a. 8215 mm **b.** 350 cm **c.** 89 km **d.** 26 mm **e.** 4 cm **f.** 6.4 km

2. Measure these lengths to the nearest millimetre.

a.


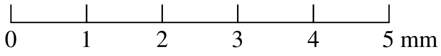
b.


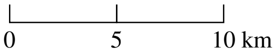
c.


3. Classify the following as enlargements or reductions.

a. 1 : 200

b. 1 mm ⇔ 1m

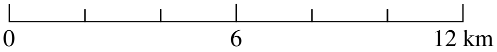
c.


d.


e. 10 : 1

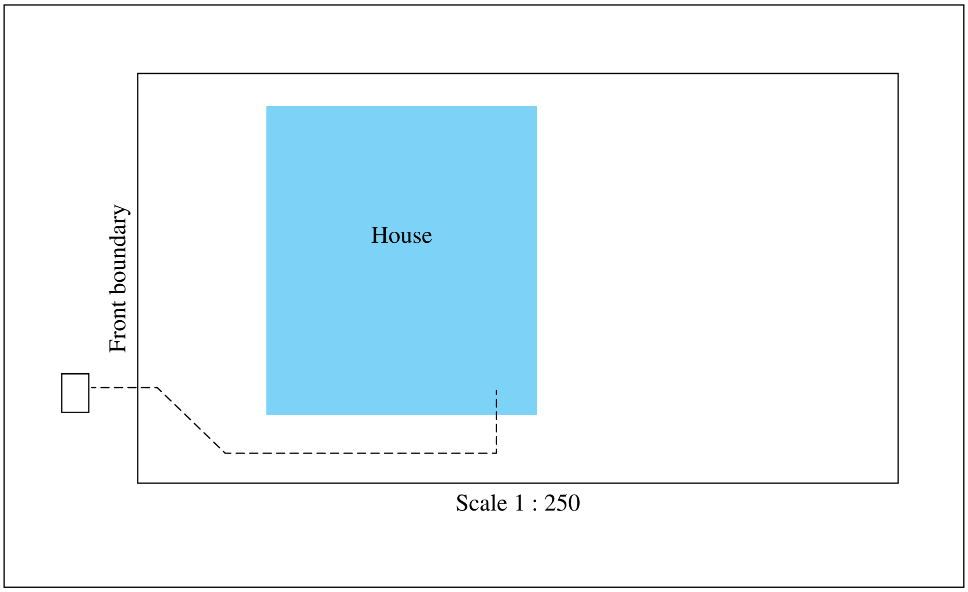
f. $\frac{1}{10\,000}$

4. Express the following in the form of a simplified ratio.
   a. 2 cm ⇔ 100 m
   b.



5. **WE5a** A plan of a building site uses a scale of 1 : 150. What field lengths would be represented by the following plan lengths? (Answer in metres.)
   a. 12 mm
   b. 4.5 cm
   c. 150 mm
   d. 2.25 cm
6. **WE5b** A plan of a house is drawn using a scale of 1 : 125. What would be the measurements of the following on the plan?
   a. Patio 3 m long and 1.75 m wide
   b. Bedroom 3.75 m × 2.25 m
7. **WE6** In the survey plan on page 151
   a. what is the area in square metres of Lot 110?
   b. what is the length and breadth of Lot 110?
   c. what scale has been used to draw the survey plan?
   d. redraw Lot 110 using a 1 : 500 scale.
   e. the area of Lot 187 is not shown. Find this area.
8. Before the metric system was introduced, the area of house blocks was measured in perches (1 perch = 25.3 m^2).
   a. A block of 42 perches is advertised for sale at $160 000. Convert the area to square metres and find the price per square metre.
   b. One lot is 850 m^2 and another is 28 perches. Which is the larger?
9. Lot 110 is for sale at $119 700, and Lot 189 is for sale at $159 500.
   a. Which represents the better value per square metre?
   b. What features of a block of land might attract a purchaser even though its dollar value per square metre may be higher than surrounding blocks? (Comparing the positions of Lots 110 and 189 can assist in your answer, but include as many other features as possible.)
10. The site plan on page 151 shows Lot 8 on Gurner St. All dimensions given are in millimetres.
    a. Find the area of Lot 8 in square metres and perches.
    b. The shaded sketch shows the area of the dwelling proposed to be erected on this lot. What is the area of the proposed dwelling?
    c. What scale has been used to produce this diagram? (*Note:* This scale may not be a simple ratio.)
    d. The dotted lines are contour lines (lines of height). All points along the 31 800 line are 31 800 mm above sea level.
       i. Is the block rising or falling as I walk from the Gurner St entrance to the rear of the block?
       ii. Calculate the angle of rise or fall from the front to the rear.
11. The floor plan on page 152 shows a plan of a one-bedroom dwelling. All dimensions have been given in mm. (*Note*: This scale may not be a simple ratio.)
    a. Calculate the area of this dwelling in square metres. (Include the patio.)
    b. The patio is to be tiled using tiles costing $35 per square metre. Find the cost. Include an extra 5% for cutting.
    c. Find the area of the bedroom, kitchen, laundry and bathroom.
12. A building block measuring 48.4 m by 41.25 m is drawn on a plan with measurements of 8.8 cm by 7.5 cm. What scale was used to draw the plan?

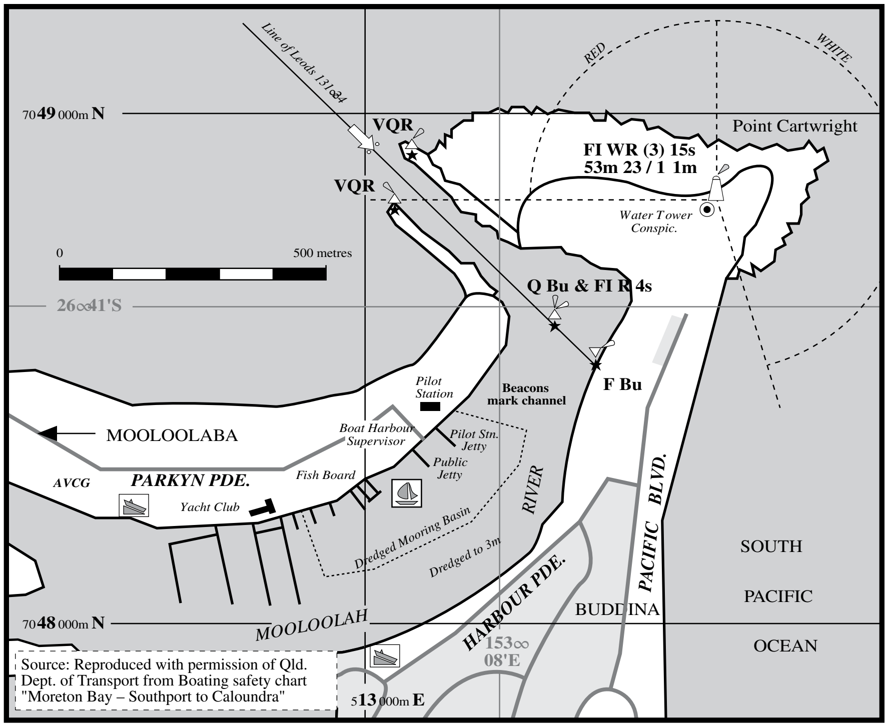
13. The figure below shows a house on a block, drawn using a 1 : 250 scale.



a. How far is the house from the front boundary?
b. The owner intends to fence this property at a cost of $32.00 per metre, plus $250 for gates. What would it cost to fence and install gates on this property?
c. What is the length of the sewer line (dotted)?

14. The map below shows Mooloolaba Boat Harbour.
a. How wide is the entrance to the harbour.
b. How far is it, as the seagull flies, from the yacht club to the water tower at Point Cartwright?
c. What area (in km^2) does this map cover?

Mooloolaba Boat Harbour

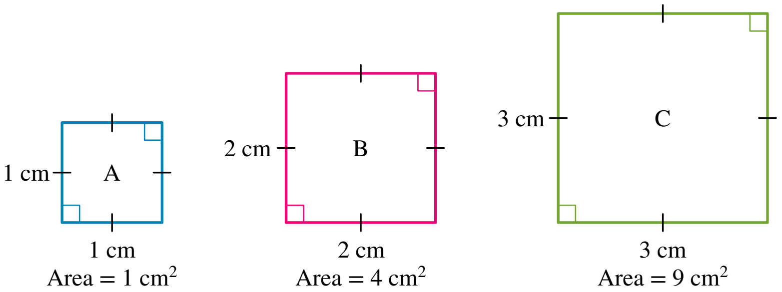


Source: Reproduced with permission of Qld. Dept. of Transport from Boating safety chart "Moreton Bay – Southport to Caloundra"

4.5 Area and volume scale factors

4.5.1 Area scale factor

Consider three squares with side lengths of 1, 2 and 3 cm. Their areas will be 1 cm^2, 4 cm^2 and 9 cm^2 respectively.



The linear scale factor between square A and square B will be 2, and the linear scale factor between square A and square C is 3. When we look at the ratio of the areas of the squares, we get 1 : 4 for squares A and B and 1 : 9 for squares A and C. In both cases, the **area scale factor** is equal to the linear scale factor raised to the power of two.

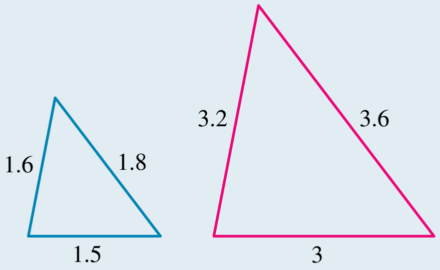
	Linear scale factor	**Area scale factor**
B : A	2	$2^2 = 4$
C : A	3	$3^2 = 9$

Comparing squares B and C, the ratio of the side lengths is 2 : 3, resulting in a linear scale factor of $\frac{3}{2}$ or 1.5. From the ratio of the areas we get 4 : 9, which once again indicates an area scale factor $\frac{9}{4} = 2.25$; that is, the linear scale factor to the power of two.

In general, if the linear scale factor for two similar objects is x, the area scale factor will be x^2.

WORKED EXAMPLE 7

Calculate the area scale factor for the pair of triangles shown in the diagram.

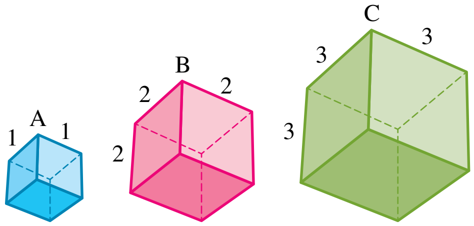


THINK	WRITE
1. Calculate the ratio of the corresponding side lengths.	Linear scale factor $= \frac{3}{1.5} = \frac{3.6}{1.8} = \frac{3.2}{1.6} = 2$
2. Square the linear scale factor to obtain the area scale factor.	$2^2 = 4$
3. State the answer.	The area scale factor is 4.

4.5.2 Volume scale factor

Three-dimensional objects of the same shape are similar when the ratios of their corresponding dimensions are equal. When we compare the volumes of three similar cubes, we can see that if the linear scale factor is x, the **volume scale factor** will be x^3.

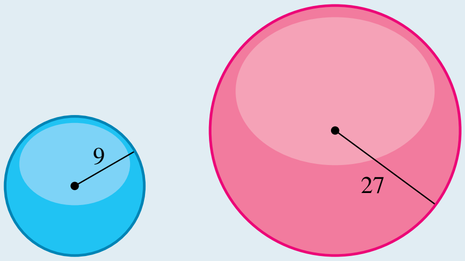
	Cube B : Cube A	Scale factor
Linear	2 : 1	2
Area	4 : 1	$2^2 = 4$
Volume	8 : 1	$2^3 = 8$



If the linear scale factor for two similar objects is x, the volume scale factor will be x^3.

WORKED EXAMPLE 8

Calculate the volume scale factor for the pair of spheres shown in the diagram.



THINK	WRITE
1. Calculate the ratio of the corresponding dimensions.	$\frac{27}{9} = 3$
2. Cube the result to obtain the volume scale factor.	$3^3 = 27$
3. State the answer.	The volume scale factor is 27.

on Resources

Interactivity: Area scale factor (int-6478)
Interactivity: Volume scale factor (int-6479)

studyon

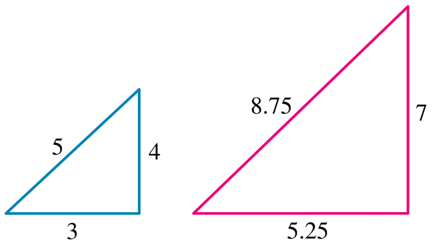
Units 1 & 2 > Area 2 > Sequence 2 > Concept 4

Area and volume scale factors Summary screen and practice questions

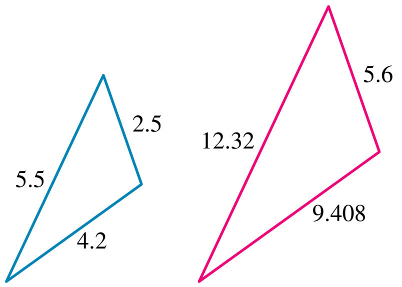
Exercise 4.5 Area and volume scale factors

1. **WE7** Calculate the area scale factor for each of the pairs of triangles shown.

   a.

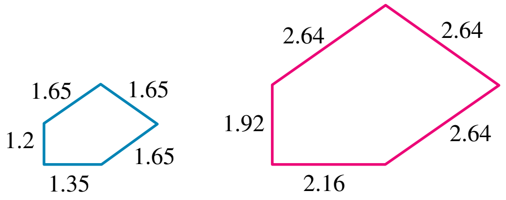


   b.

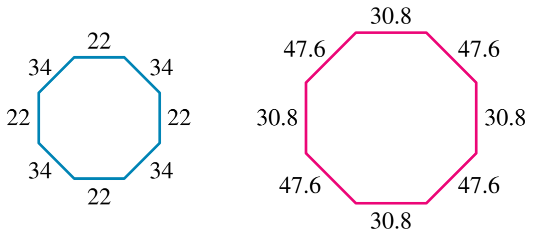


2. Calculate the area scale factor for each of the pairs of similar objects shown.

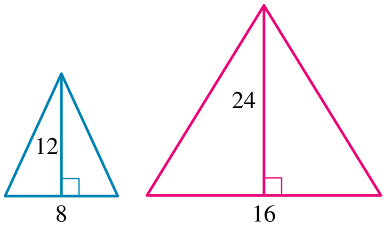
   a.



   b.

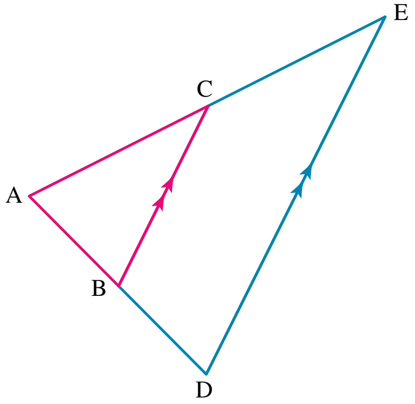


3. a. Calculate the areas of the two similar triangles shown in the diagram.
   b. How many times larger in area is the biggest triangle?
   c. Calculate the linear scale factor.
   d. Calculate the area scale factor.



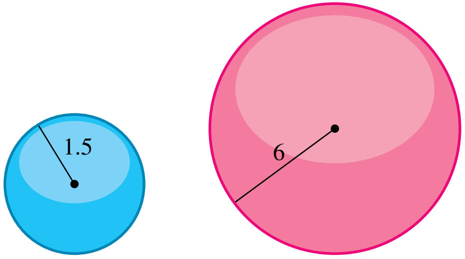
4. A hexagon is made up of six equilateral triangles of side length 2 cm. If a similar hexagon has an area of $24\sqrt{3}\ \text{cm}^2$, calculate the linear scale factor.
5. A rectangular swimming pool is shown on the plans for a building development with a length of 6 cm and a width of 2.5 cm. If the scale on the plans is shown as 1 : 250:
   a. calculate the area scale factor
   b. calculate the surface area of the swimming pool.

6. The area of the triangle ADE in the diagram is 100 cm^2, and the ratio of DE : BC is 2 : 1. Calculate the area of triangle ABC.



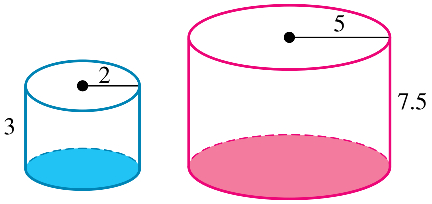
7. The floor of a square room has an area of 12 m^2. Calculate the area that the room takes up in a diagram with a scale of 1 : 250.

8. **WE8** Calculate the volume scale factor for the pair of spheres shown.

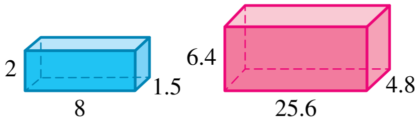


9. Calculate the volume scale factor for each of the pairs of similar objects shown.

a.



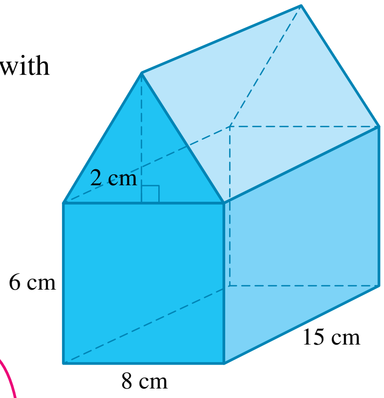
b.



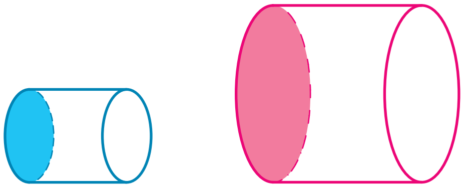
10. An architect makes a small scale model of a house out of balsa wood with the dimensions shown in the diagram.

   a. If the actual length of the building is 26.25 m, what is the scale of the model?

   b. What is the ratio of the volume of the building to the volume of the model?



11. Two similar cylinders have volumes of 400 cm^3 and 50 cm^3 respectively.



   a. What is the linear scale factor?

   b. If the length of the larger cylinder is 8 cm, what is the length of the smaller one?

12. If a cube has a volume of 25 cm^3 and is then enlarged by a linear scale factor of 2.5, what will the new volume be?

13. Calculate the linear scale factor between two similar drink bottles if one has a volume of 600 mL and the other has a volume of 1.25 L.

14. If an area of 712 m^2 is represented on a scale drawing by an area of 44.5 cm^2, what is the actual length that a distance of 5.3 cm on the drawing represents?

15. A model car is an exact replica of the real thing reduced by a factor of 12.

- **a.** If the actual surface area of the car that is spraypainted is 4.32 m^2, what is the equivalent painted area on the model car?
- **b.** If the actual storage capacity of the car is 1.78 m^3, what is the equivalent volume for the model car?

16. A company sells canned fruit in two sizes of similar cylindrical cans. For each size, the height is four-fifths of the diameter.

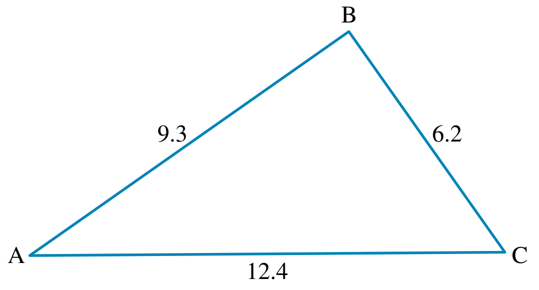
- **a.** Write an expression for calculating the volume of a can of fruit in terms of its diameter.
- **b.** If the linear dimensions of the larger cans are 1.5 times those of the smaller cans, write an expression for calculating the volume of the larger cans of fruit in terms of the diameter of the smaller cans.

4.6 Review: exam practice

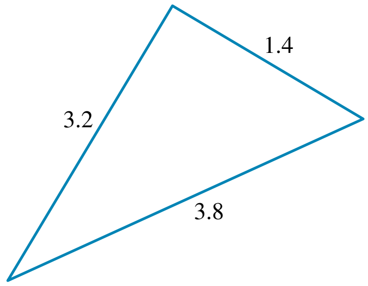
A summary of this chapter is available in the Resources section of your eBookplus at www.jacplus.com.au.

Simple familiar

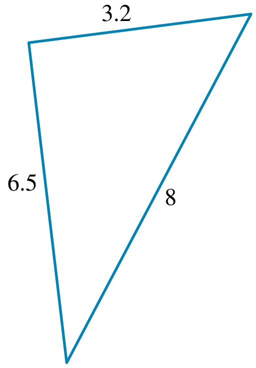
1. **MC** The triangle that is similar to ΔABC is:



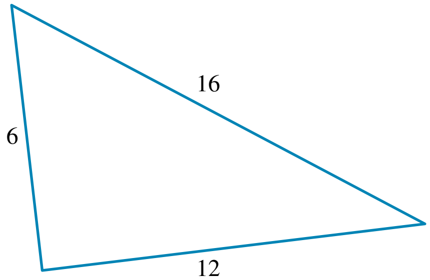
A.



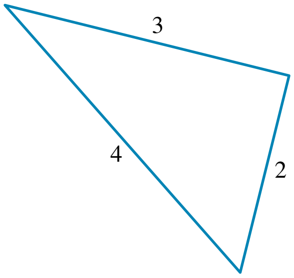
B.



C.



D.



2. **MC** If a map has a scale factor of 1 : 50 000, an actual distance of 11 km would have a length on the map of:

A. 21 cm **B.** 22 cm **C.** 21.5 cm **D.** 11 cm

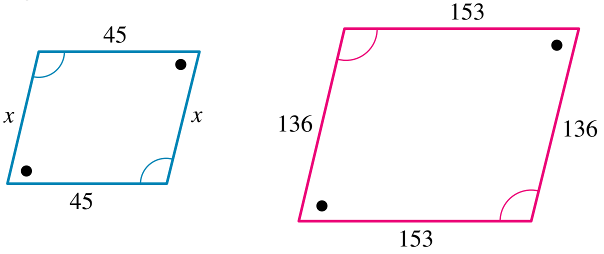
3. **MC** If the volumes of two similar solids are 16 cm^3 and 128 cm^3 respectively, the area scale factor will be:

A. 2 **B.** 3 **C.** 4 **D.** 8

4. **MC** A tree casts a shadow that is 5.6 m long. At the same time a 5–m light pole casts a shadow that is 3.5 m long. The height of the tree is:

A. 4.4 m **B.** 7.5 m **C.** 8.0 m **D.** 6.0 m

5. **MC** The value of x in the diagram is:



A. 38 **B.** 41 **C.** 43 **D.** 40

6. **MC** If the scale factor of the volumes of two similar cuboids is 64 and the volume of the larger one is 1728 cm^3, the surface area of the smaller cuboid will be:

A. 64 cm^2 **B.** 27 cm^2 **C.** 54 cm^2 **D.** 9 cm^2

7. **MC** The plans of a house show the side of a building as 12.5 cm long. If the actual building is 15 m long, the scale of the plan will be:

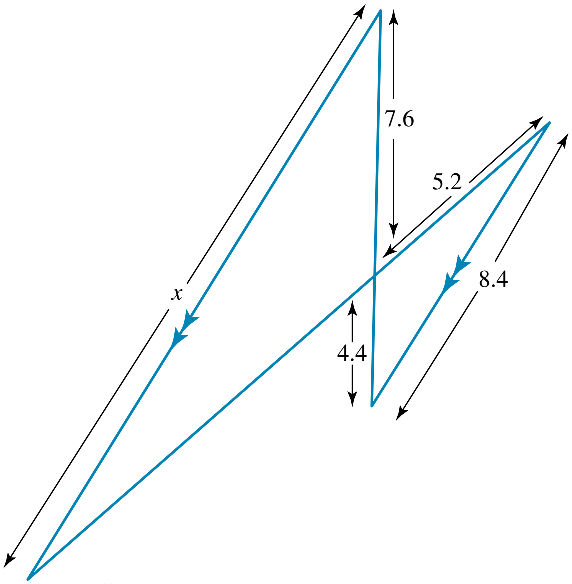
A. 1 : 250 **B.** 1 : 150 **C.** 1 : 220 **D.** 1 : 120

8. **MC** The plans for a building show a concrete slab covering an area of 12.5 cm × 8.4 cm to a depth of 0.25 cm. If the plans are drawn to a scale of 1 : 225, the actual volume of concrete will be closest to:

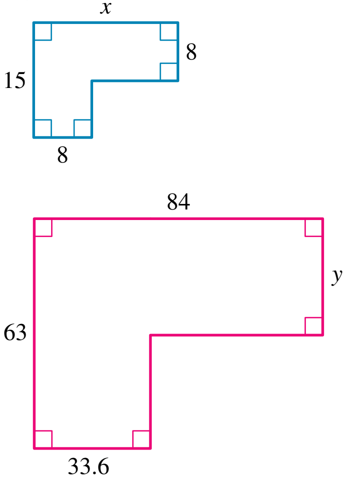
A. 26.25 m^3 **B.** 262.5 m^3 **C.** 5906.25 m^3 **D.** 299 m^3

9. **MC** Calculate the values of the pronumerals in the following diagrams.

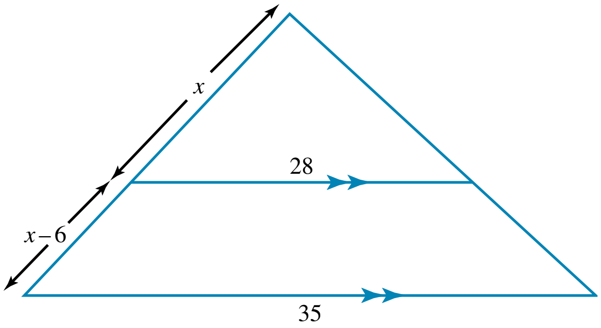
a.



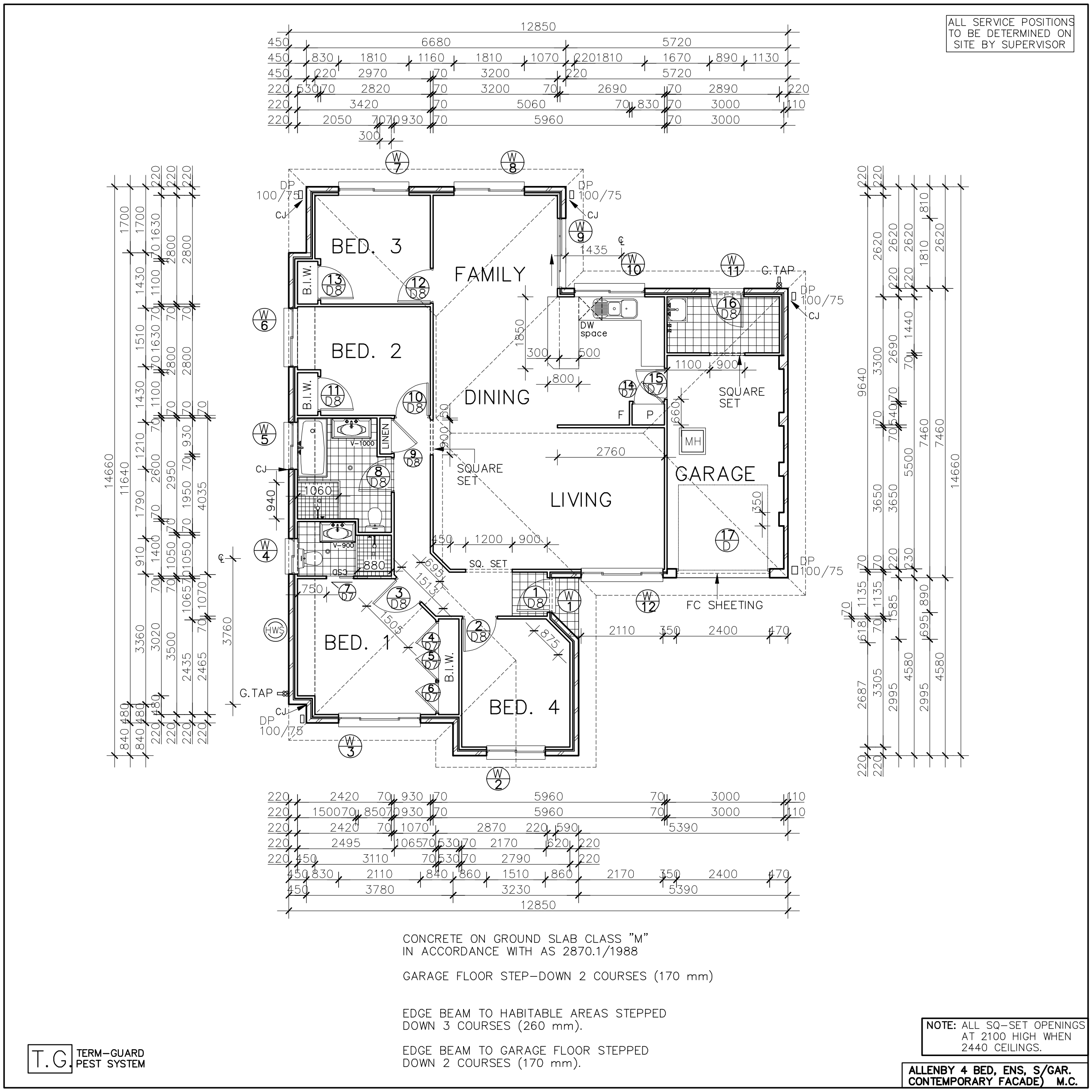
b.



c.



Questions **10–12** draw together the techniques practised in this chapter and focus on applying mathematics to the plans and elevations of a typical residence. Consult the plans or diagrams of the Allenby home from Masterton Homes where necessary to answer the following questions.



CONCRETE ON GROUND SLAB CLASS "M"
IN ACCORDANCE WITH AS 2870.1/1988

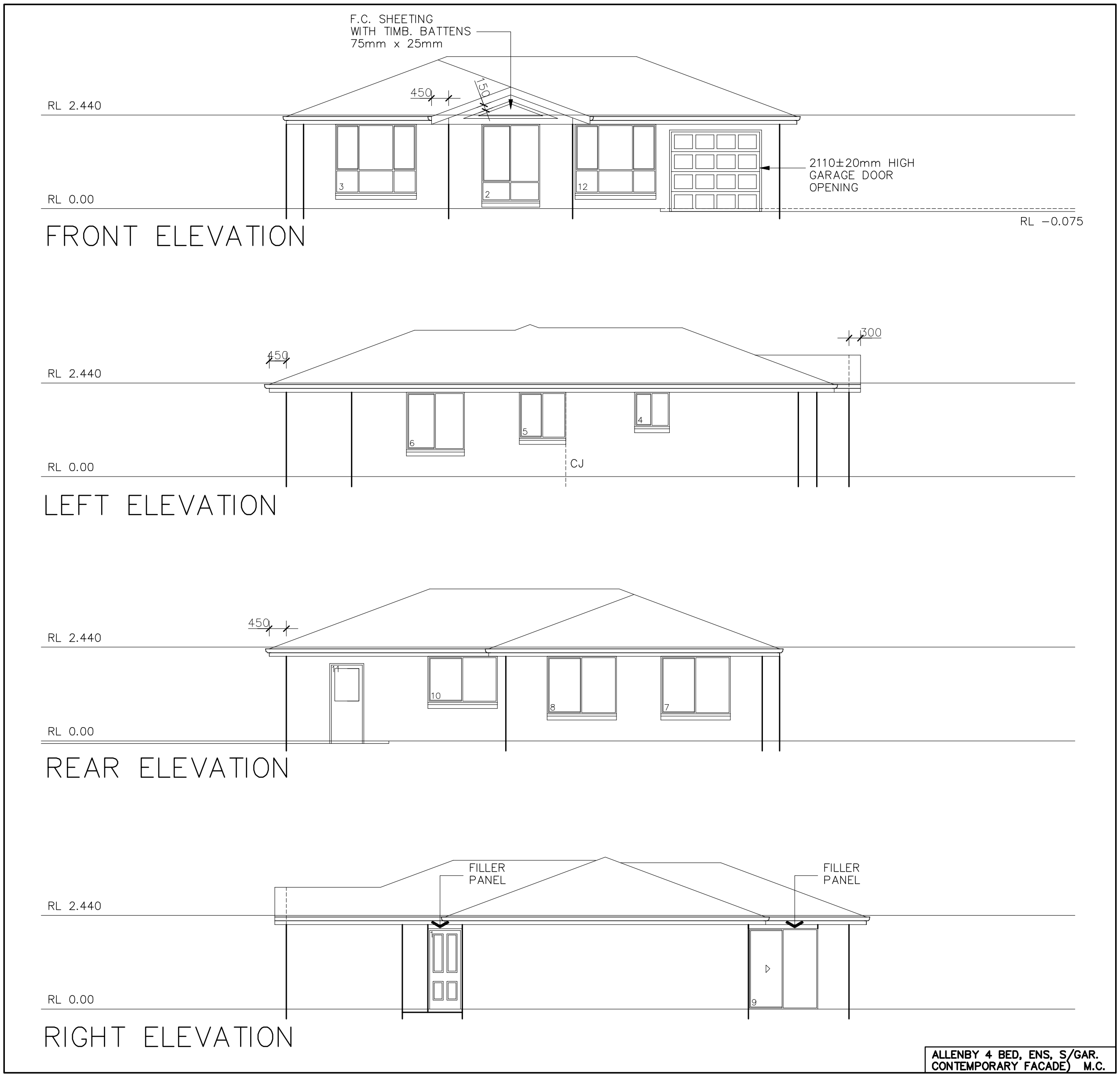
GARAGE FLOOR STEP–DOWN 2 COURSES (170 mm)

EDGE BEAM TO HABITABLE AREAS STEPPED
DOWN 3 COURSES (260 mm).

EDGE BEAM TO GARAGE FLOOR STEPPED
DOWN 2 COURSES (170 mm).

NOTE: ALL SQ–SET OPENINGS AT 2100 HIGH WHEN 2440 CEILINGS.

ALLENBY 4 BED, ENS, S/GAR.
CONTEMPORARY FACADE) M.C.


F.C. SHEETING
WITH TIMB. BATTENS
75mm x 25mm
450
150
RL 2.440
RL 0.00
3
2
12
2110±20mm HIGH
GARAGE DOOR
OPENING
RL −0.075
FRONT ELEVATION
450
300
RL 2.440
RL 0.00
6
5
4
CJ
LEFT ELEVATION
450
RL 2.440
RL 0.00
11
10
8
7
REAR ELEVATION
FILLER
PANEL
FILLER
PANEL
RL 2.440
RL 0.00
1
9
RIGHT ELEVATION
ALLENBY 4 BED, ENS, S/GAR.
CONTEMPORARY FACADE) M.C.

WINDOW SCHEDULE

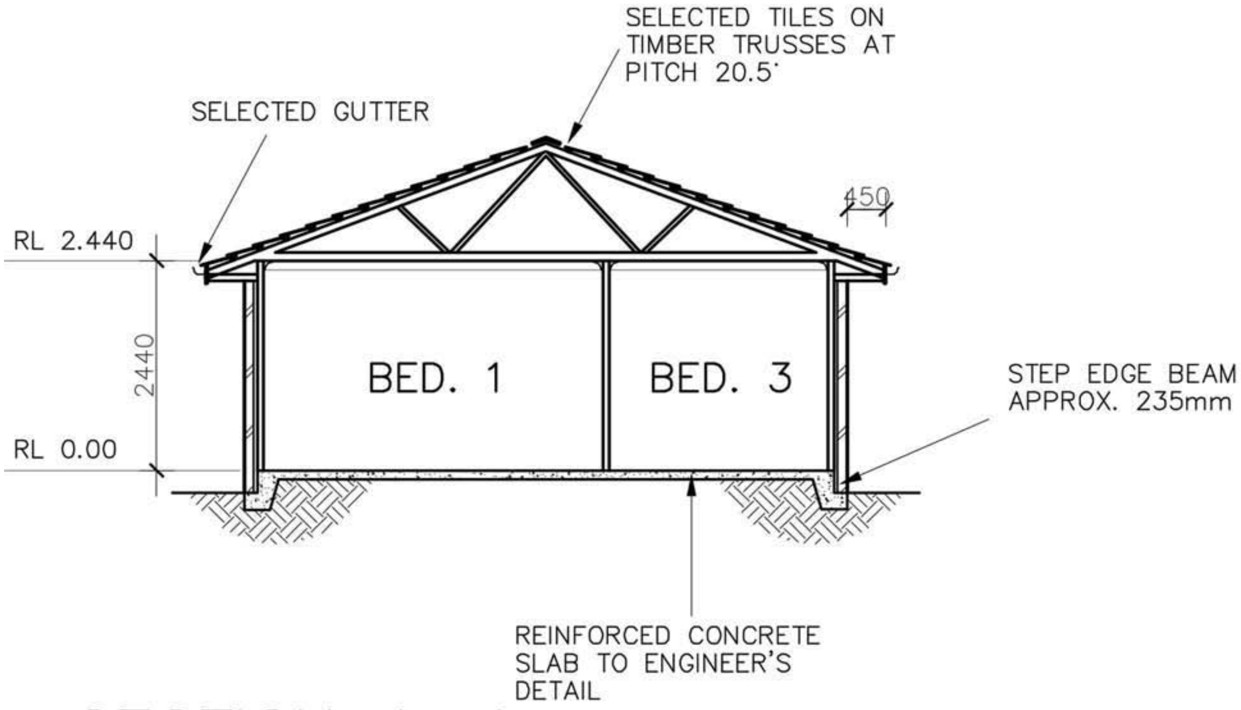
No	TYPE	HEIGHT	WIDTH	GLAZING/REMARK
1	DF1	2090	890	T. E. F.
2	SF2015	2000	1510	
3	SFS1821	1800	2110	
4	SF0909	900	910	
5	SF1212	1200	1210	TOUGHENED GLASS
6	SF1515	1500	1510	
7	SF1518	1500	1810	
8	SF1518	1500	1810	
9	XF2118	2100	1810	AL. SL. DOOR
10	SF1218	1200	1810	
11	DF1	2090	890	T. E. F.
12	SFS1821	1800	2110	

F.W.G. WINDOW CODES

TYPE	HEIGHT	WIDTH	REMARK	QTY
DF1	2090	890	T. E. F.	2
SF0909	900	910		1
SF1212	1200	1210	TOUGHENED GLASS	1
SF1218	1200	1810		1
SF1515	1500	1510		1
SF1518	1500	1810		2
SF2015	2000	1510		1
SFS1821	1800	2110		2
XF2118	2100	1810	AL. SL. DOOR	1

WINDOW COLOUR:T.B.A.

ALLENBY 4 BED, ENS, S/GAR.
CONTEMPORARY FACADE) M.C.



SECTION A—A

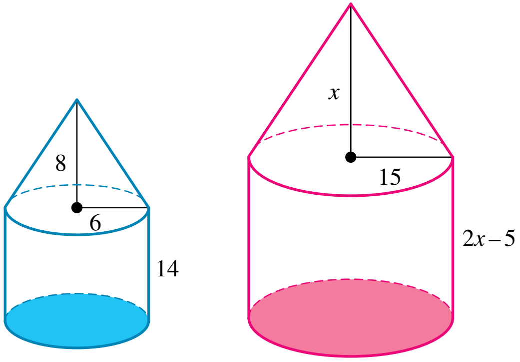
10. a. What is the scale of the floor plan?
    b. How many bedrooms are in the residence?
    c. What are the features of the main bedroom?
    d. Give the dimensions of the garage.
    e. The doors are numbered. How many doors are in the residence?
    f. The windows are also numbered.
        i. How many windows are in the residence?
        ii. How can you tell from the floor plan that the window marked as w9 is actually a sliding door?
    g. How many down-pipes are there around the perimeter of the residence?
    h. Is the garage at the same level as the house? Explain.
    i. How long and wide is the built-in wardrobe in Bedroom 1?

11. a. Give the overall width and depth of the house.
    b. If a minimum of 3 metres is allowed from a dwelling to a boundary line, what is the smallest block of land on which this house could be built?
    c. The house has an area of 143.4 m^2. What percentage of the land would it occupy in this case?

12. If the garage were positioned on the site so that its outside wall faced the west:
    a. in which direction would the back of the house face?
    b. in which direction would the front door face?

Complex familiar

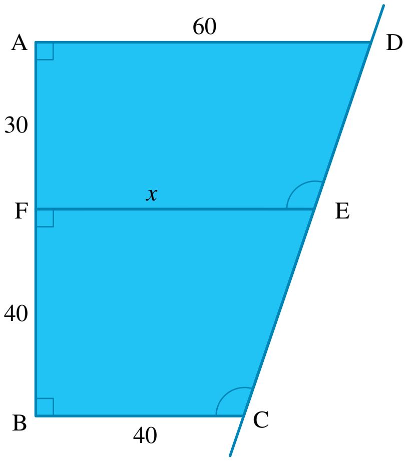
13. Calculate the value of x in the diagram of two similar objects shown.



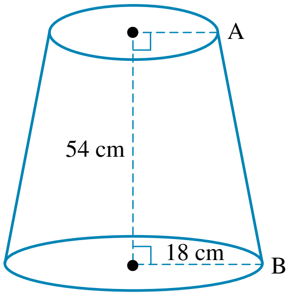
14. A triangle has side lengths of 32 mm, 45 mm and 58 mm.
    a. Calculate the side lengths of a larger similar triangle using a corresponding side ratio of 2 : 3.
    b. What would be the side lengths of the larger triangle in **a** in a drawing with a scale of 5 : 2?

15. The volume of a solid is 1600 cm^3. If the ratio of the corresponding dimensions between this solid and a similar solid is 4 : 5, calculate the volume of the similar solid.

16. A farmer divides paddock ABCD into two separate paddocks along the line FE as shown in the diagram. All distances are shown in metres.
    a. Express the ratio of the corresponding sides of the original paddock, ABCD, with paddock BCEF in its simplest form.
    b. Calculate the length of fencing required to separate the two paddocks along the line FE. Give your answer correct to the nearest centimetre.



Complex unfamiliar

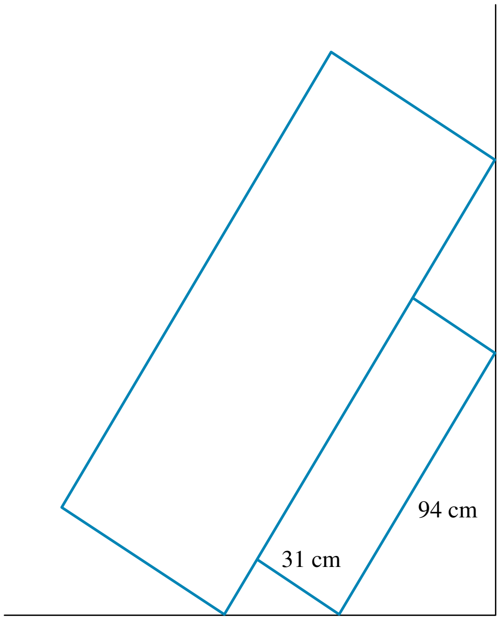


17. The top third of an inverted right cone is removed as shown in the diagram.

- **a.** Calculate the height of the cone that has been removed.
- **b.** Calculate the distance along the edge of the remaining part of the cone from A to B.

18. On a map that is drawn to a scale of 1 : 225 000, the distance between two points is 88 cm.

- **a.** Determine the actual distance between the two points.
  A boat sets out to travel from one point to the other but the navigator makes an error. After travelling 100 km, the crew realise they are directly south of a point that they should have reached after travelling 90 km in a direct line to their destination.
- **b.** If they continue in their current direction, how much further do they have to travel to be directly south of the intended destination?
- **c.** How far away from the intended destination will the boat be when it reaches the point on their course that is directly to the south?
- **d.** When the boat is at the point directly to the south of their intended destination what will be the distance on the map?



19. A rectangular box with dimensions 94 cm × 31 cm leans against a wall as shown in the diagram. A larger box leans against the first box.

- **a.** If the ratio of the corresponding side lengths between the two boxes is 4 : 7, determine the dimensions of the larger box.
- **b.** If the smaller box touches the floor at a point that is 52 cm from the base of the wall, calculate how far up the wall it reaches.
- **c.** Calculate how far up the wall the larger box reaches.

20. A caterer sells takeaway coffee in three different-sized cups whose dimensions are in proportion.

- **a.** If the small cup has a capacity of 200 mL and the medium cup has a capacity of 300 mL, determine the linear scale factor correct to 2 decimal places.
- **b.** If the ratio of corresponding dimensions between the smaller and larger cups is 4 : 5, what is the capacity of the larger cup?
- **c.** If the caterer charges $3.00 for a small cup, $3.50 for a medium cup and $4.00 for a large cup, which is the better value for the customer?

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Units 1 & 2 Sit chapter test

Answers

Chapter 4 Similar figures and scale factors

Exercise 4.2 Similarity of two-dimensional figures

1. a. $\frac{15}{8} = \frac{11.25}{6} = \frac{7.5}{4} = 1.875$, and all angles are equal.

b. $\frac{41.04}{11.4} = \frac{26.1}{7.25} = \frac{9}{2.5} = \frac{13.68}{3.8} = 3.6$, and all angles are equal.

2. $\frac{106.43}{18.35} = \frac{24.65}{4.25} = 5.8$, and all angles are equal.

3. a. $\frac{4.8}{3.2} = 1.5$, $\frac{2.24}{1.6} = 1.4$; not similar

b. $\frac{10.224}{2.84} = 3.6$, $\frac{2.34}{0.65} = 3.6$; similar

c. $\frac{4.4352}{2.112} = 2.1$, $\frac{2.184}{1.04} = 2.1$; similar

d. $\frac{12.535}{11.5} = 1.09$, $\frac{10.152}{9.4} = 1.08$; not similar

4. a. $\frac{35}{25} = 1.4$, $\frac{22.4}{16} = 1.4$, $\frac{12.6}{9} = 1.4$ and all angles are equal; similar

b. $\frac{4.94}{2.6} = 1.9$, $\frac{6.65}{3.5} = 1.9$, $\frac{2.09}{1.1} = 1.9$ and all angles are equal; similar

c. $\frac{13}{8} = 1.625$, $\frac{6.5}{4} = 1.625$, $\frac{3.25}{2} = 1.625$ and all angles are equal; similar

d. $\frac{3.24}{2.7} = 1.2$, $\frac{19.44}{16.2} = 1.2$, $\frac{2.16}{1.8} = 1.2$ and all angles are equal; similar

5. a. $\frac{44.275}{7.7} = \frac{78.2}{13.6} = 5.75$, SAS

b. $\frac{38.72}{17.6} = \frac{16.28}{7.4} = \frac{14.85}{6.75} = 2.2$, SSS

6. a. $\frac{32.4}{18} = \frac{21.6}{12} = 1.8$, $\frac{23.8}{14} = 1.7$; not similar

b. $\frac{54.32}{19.4} = \frac{22.96}{8.2} = 2.8$ and all angles are equal; similar

7. a. A and C **b.** B and C

c. A and C **d.** A and B

8. a. All angles are equal and side lengths are in proportion.

b. All measurements (radius and circumference) are in proportion.

9. a. 1.6 : 1 **b.** 2.26 : 1

10. a. 4 : 3 **b.** 5.5 : 1

11. a. 4.8 **b.** 7.07

12. a. $\frac{50.84}{8.2} = 6.2$, all sides are in proportion and all angles equal.

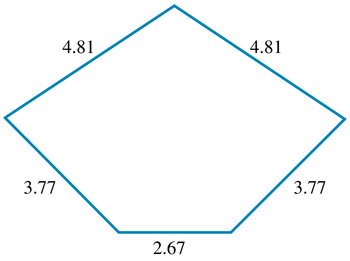
b. $\frac{14.34}{12.6} = 1.138$, all sides are in proportion and all angles are equal.

13. a. 20 **b.** 32

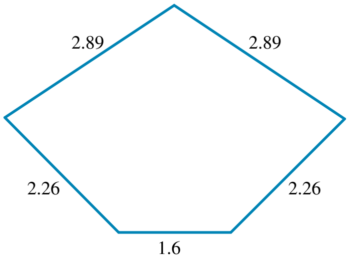
14. 51.8

15. $x = 5.6$, $y = 7$

16. a.



b.



Exercise 4.3 Linear scale factors

1. a. 2.2 **b.** 3.4 **c.** 1.25 **d.** 1.6

2. a. 5.2 **b.** 2.7 **c.** 3.8 **d.** 3

3. a. 1.5 **b.** 2.4 **c.** 0.75 **d.** 1.25

4. a. $\frac{3}{\boxed{0.5}} = \frac{\boxed{72}}{12} = 6$

b. $\frac{5}{\boxed{1.25}} = \frac{44}{11} = \boxed{4}$

c. $\frac{\boxed{63}}{7} = \frac{81}{9} = \boxed{9}$

d. $\frac{16.5}{\boxed{3}} = \frac{\boxed{187}}{34} = 5.5$

5. a. $x = 1$, $y = 11.76$

b. 1.41

c. 2.8

d. $x = 2$, $y = 1.6$, $z = 3.576$

6. a. $x = 3.52$, $y = 3$ **b.** $y = 4.29$

7. 16

8. 2.5

9. a. 5.7 m **b.** 15.6 m

c. 5.3 m **d.** 7.8 m

10. 5.4 m

11. 6.4 m

12. 8.1 m

13. a. 4.8 m **b.** 1.67 m

14. 5

15. 6.65 m

16. 24 m

17. 8.34 m

18. a. 0.36 m

b. 0.67 m from the bottom of the left side

Exercise 4.4 Scale drawings—maps and plans

1. a. 8.215 m
b. 3.5 m
c. 89 000 m
d. 0.026 m
e. 0.04 m
f. 6400 m

2. a. 45 mm
b. 67 mm
c. 58 mm

3. a. Reduction
b. Reduction
c. Enlargement
d. Reduction
e. Enlargement
f. Reduction

4. a. 1 : 5000
b. 1 : 200 000

5. a. 1.8 m
b. 6.75 m
c. 22.5 m
d. 3.375 m

6. a. 2.4 cm long × 1.4 cm wide
b. 3 cm × 1.8 cm

7. a. 630 m^2
b. 30 m, 21 m
c. Approx. 1 : 1500
d. Rectangle 6 cm by 4.2 cm
e. 632 m^2

8. a. 1063 m^2, \$151/m^2
b. 850 m^2 is larger

9. a. Lot 110
b. Does it front a main road? Is it low lying? Slope of land, views, aspect.

10. a. 2100 m^2, 83 perches
b. 104 m^2
c. Approx. 1 : 800
d. **i.** Rising **ii.** 1.4°

11. a. 77.8 m^2
b. Approx. \$400
c. In order 10.85 m^2, 11.25 m^2, 6.84 m^2, 5.04 m^2

12. 1 : 550

13. a. 4.75 m **b.** \$3034 **c.** 18.75 m

14. a. Approx. 75 m
b. Approx. 1000 m
c. Approx. 2 km^2

Exercise 4.5 Area and volume of scale factors

1. a. $\frac{49}{16} = 3.0625$
b. $\frac{3136}{625} = 5.0176$

2. a. $\frac{64}{25} = 2.56$
b. $\frac{49}{25} = 1.96$

3. a. 48 and 192 square units
b. 4
c. 2
d. 4

4. 2

5. a. 62 500
b. 93.75 m^2

6. 25 cm^2

7. 1.92 cm^2

8. 64

9. a. 15.625
b. 32.768

10. a. 1 : 175
b. 5 359 375 : 1

11. a. 2
b. 4 cm

12. 390.625 cm^3

13. 1.28

14. 21.2 m

15. a. 300 cm^2
b. 1030 cm^3

16. a. $V = \frac{\pi D^3}{5}$
b. $V_2 = \frac{27\pi D_1^3}{40}$

4.6 Review: exam practice

1. D

2. B

3. C

4. C

5. D

6. C

7. D

8. D

9. a. 14.51
b. $x = 20$, $y = 33.6$
c. 8

10. a. Approx.1 : 195
b. 4
c. Built-in wardrobes and ensuite
d. 5.5 m × 3 m
e. 17
f. **i.** 12
ii. The arrow on the floor
g. 5
h. The garage is 170 mm lower than the floor of the house.
i. 2465 mm long and 530 mm wide

11. a. 12.85 m wide and 14.66 m deep
b. 18.85 m × 20.66 m or 389.4 m^2
c. 37%

12. a. south **b.** west

13. 20

14. a. 48 mm, 67.5 mm, 87 mm
b. 120 mm, 168.75 mm, 217.5 mm

15. 3125 cm^3

16. a. 7 : 4
b. 5143 cm

17. **a.** 27 cm
b. 55.32 cm

18. **a.** 198 km
b. 120 km
c. 95.90 km
d. 42.62 cm

19. **a.** 164.5 cm × 54.25 cm
b. 78.31 cm
c. 137.04 cm

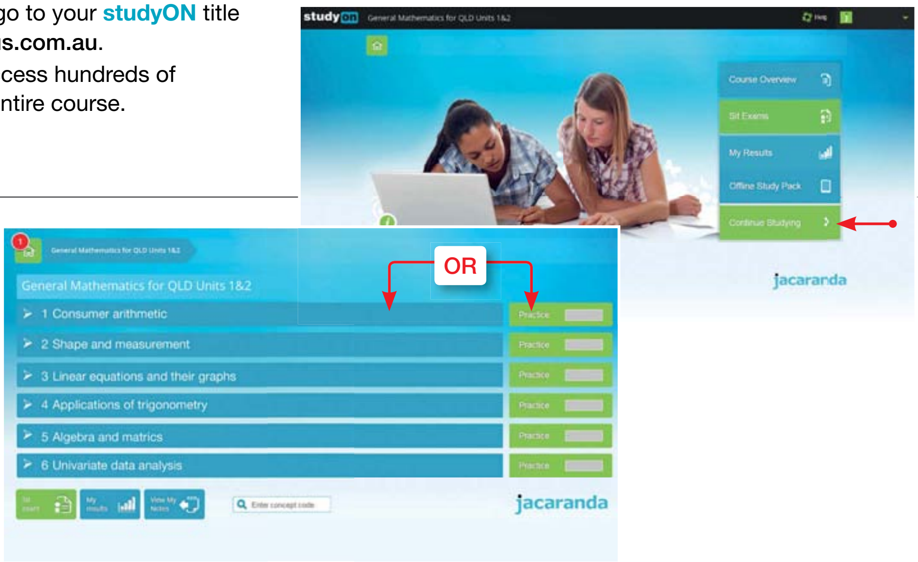
20. **a.** 1.14
b. 390.6 mL
c. Large cup

REVISION UNIT 1 Money, measurement and relations

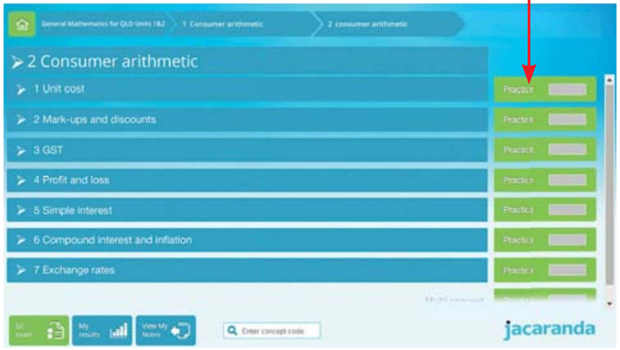
TOPIC 2 Shape and measurement

- For revision of this entire topic, go to your **studyON** title in your bookshelf at **www.jacplus.com.au**.
- Select **Continue Studying** to access hundreds of revision questions across your entire course.

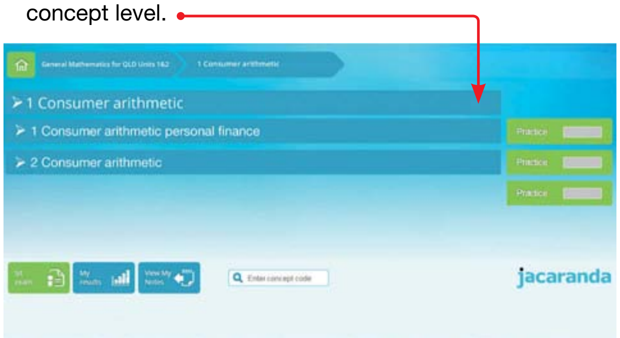
- Select your **course** *General Mathematics for Queensland Units 1&2* to see the entire course divided into syllabus topics.
- Select the **area** you are studying to navigate into the sequence level ***OR*** select **Practice** to answer all practice questions available for each area.



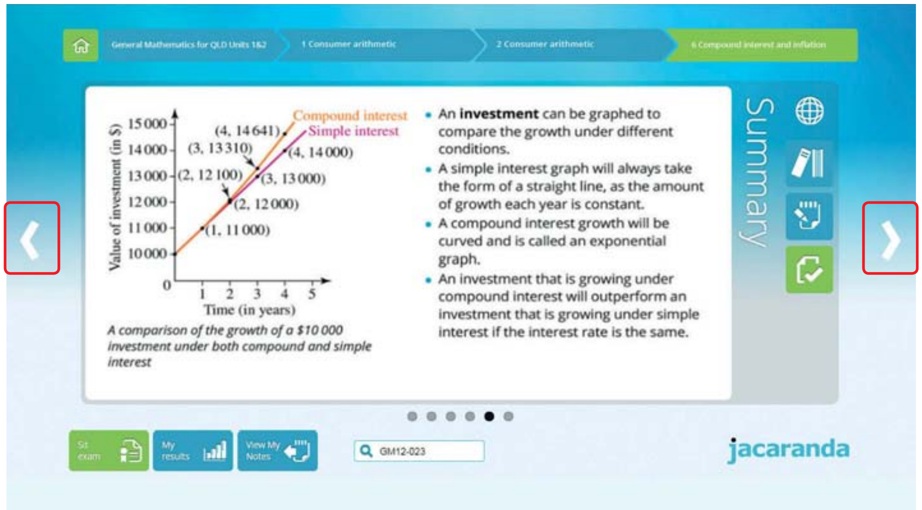
- Select **Practice** at the sequence level to access all questions in the sequence.



- At **sequence level**, drill down to concept level.



- **Summary screens** provide revision and consolidation of key concepts. Select the **next arrow** to revise all concepts in the sequence and practice questions at the concept level for a more granular set of questions.



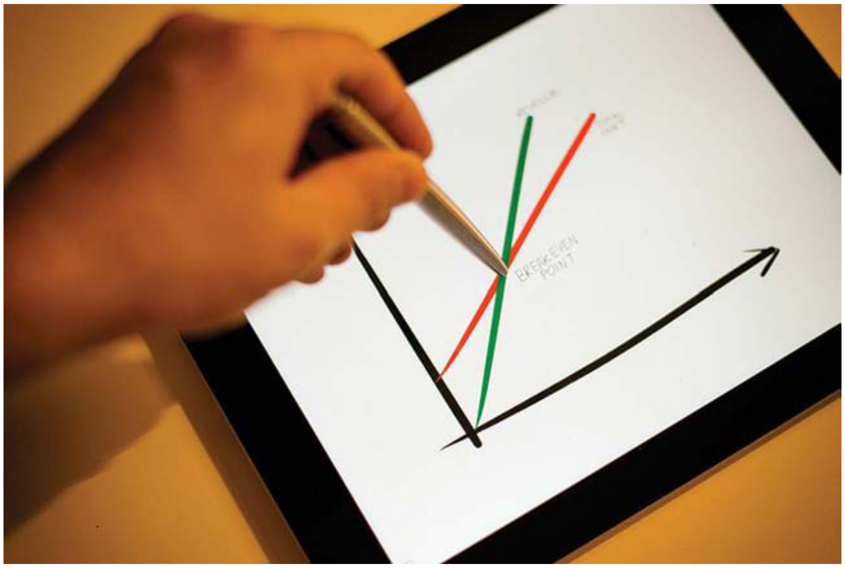
A comparison of the growth of a $10 000 investment under both compound and simple interest

- An **investment** can be graphed to compare the growth under different conditions.
- A simple interest graph will always take the form of a straight line, as the amount of growth each year is constant.
- A compound interest growth will be curved and is called an exponential graph.
- An investment that is growing under compound interest will outperform an investment that is growing under simple interest if the interest rate is the same.

CHAPTER 5
Linear equations

5.1 Overview

5.1.1 Introduction



Linear equations have been around for more than 4000 years. A simple 2 by 2 linear equation system with two unknowns was solved by the people of Babylon. Around 200 BC the Chinese demonstrated the ability to solve a 3 by 3 system of equations. However, it wasn't until the seventeenth century that progress was made in linear algebra by the founder of calculus, Leibnitz. This was followed by work by Cramer and was adapted further by Gauss. Linear equations themselves were invented in 1843 by Irish mathematician Sir William Rowan Hamilton. He made important contributions to mathematics and his work was also used with quantum mechanics. Sir William was seen as a genius since at the age of 13 he reportedly spoke 13 languages, and at 22 he was a professor at the University of Dublin. This work has been used in many areas because there are many situations when there is a direct relationship between two variables. Examples of this could be water being added to a tank at a constant rate, or a taxi trip being charged at a constant rate per kilometre. Using a linear equation to model the cost of a taxi trip allows you to compare one taxi company to another. The break-even point refers to the point where the costs are the same for each taxi company. This is the point where the two linear graphs intersect and this point can be found using a graphical technique, or using substitution or elimination techniques of simultaneous equations.

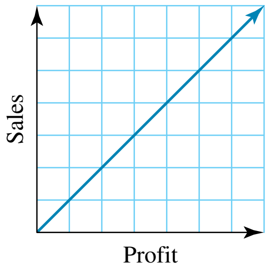
LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookplus at www.jacplus.com.au

5.2 Linear patterns

5.2.1 Identifying linear relations

A **linear relation** is a relationship between two **variables** that when plotted gives a straight line. Many real-life situations can be described by linear relations, such as water being added to a tank at a constant rate, or money being saved when the same amount of money is deposited into a bank at regular time intervals.



When a linear relation is expressed as an equation, the highest power of both variables in the equation is 1. Remember x can be written as x^1.

WORKED EXAMPLE 1

Identify which of the following equations are linear.

a. $y = 4x + 1$

b. $b = c^2 - 5c + 6$

c. $y = \sqrt{x}$

d. $m^2 = 6(n - 10)$

e. $d = \dfrac{3t + 8}{7}$

f. $y = 5^x$

THINK	WRITE
a. 1. Identify the variables.	a. y and x
2. Write the power of each variable.	y has a power of 1. x has a power of 1.
3. Check if the equation is linear.	Since both variables have a power of 1, this is a linear equation.
b. 1. Identify the two variables.	b. b and c
2. Write the power of each variable.	b has a power of 1. c has a power of 2.
3. Check if the equation is linear.	c has a power of 2, so this is not a linear equation.
c. 1. Identify the two variables.	c. y and x
2. Write the power of each variable. Note: A square root is a power of $\frac{1}{2}$.	y has a power of 1. x has a power of $\frac{1}{2}$.
3 Check if the equation is linear.	x has a power of $\frac{1}{2}$, so this is not a linear equation.

d. 1. Identify the two variables.	**d.** m and n
2. Write the power of each variable.	m has a power of 2. n has a power of 1.
3. Check if the equation is linear.	m has a power of 2, so this is not a linear equation.
e. 1. Identify the two variables.	**e.** d and t
2. Write the power of each variable.	d has a power of 1. t has a power of 1.
3. Check if the equation is linear.	Since both variables have a power of 1, this is a linear equation.
f. 1. Identify the two variables.	**f.** y and x
2. Write the power of each variable.	y has a power of 1. x is the power.
3. Check if the equation is linear.	Since x is the power, this is not a linear equation.

5.2.2 Rules for linear relations

Rules define or describe relationships between two or more variables. Rules for linear relations can be found by determining the **common difference** between consecutive terms of the pattern formed by the rule.

Consider the number pattern 4, 7, 10 and 13. This pattern is formed by adding 3s (the common difference is 3). If each number in the pattern is assigned a term number as shown in the table, then the expression to represent the common difference is $3n$ (i.e. $3 \times n$).

Term number, n	1	2	3	4
$3n$	3	6	9	12
$3n + 1$	4	7	10	13

Each term in the number pattern is 1 greater than $3n$, so the rule for this number pattern is $3n + 1$.

If a rule has an equals sign, it is described as an **equation**. For example, $3n+1$ is referred to as an **expression**, but if we define the term as t, then $t = 3n + 1$ is an equation.

WORKED EXAMPLE 2

Determine the equations for the linear relations formed by the following number patterns, where *n* is the term number and *t* is the term.

a. **3, 7, 11, 15**

b. **8, 5, 2, −1**

THINK	WRITE
a. 1. Determine the common difference.	**a.** $7 - 3 = 4$ $15 - 11 = 4$
2. Write the common difference as an expression using the term number n.	$4n$
3. Substitute any term number into $4n$ and evaluate.	$n = 3$ $4 \times 3 = 12$

4. Check the actual term number against the one found.	The actual 3rd term is 11.
5. Add or subtract a number that would result in the actual term number.	$12-1=11$
6. Write the equation for the linear relation.	$t=4n-1$
b. 1. Determine the common difference.	b. $5-8=-3$ $2-5=-3$
2. Write the common difference as an expression using the term number n.	$-3n$
3. Substitute any term number into $-3n$ and evaluate.	$n=2$ $-3\times 2=-6$
4. Check the actual term number against the one found.	The actual 2nd term is 5.
5. Add or subtract a number that would result in the actual term number.	$-6+11=5$
6. Write the equation for the linear relation.	$t=-3n+11$

Note: It is good practice to substitute a second term number into your equation to check that your answer is correct.

5.2.3 Transposing linear equations

If we are given a **linear equation** between two variables, we are able to **transpose** this relationship. That is, we can change the equation so that the variable on the right-hand side of the equation becomes the stand-alone variable on the left-hand side of the equation.

WORKED EXAMPLE 3

Transpose the linear equation $y = 4x + 7$ to make x the subject of the equation.

THINK	WRITE
1. Isolate the variable on the right-hand side of the equation (by subtracting 7 from both sides).	$y-7=4x+7-7$ $y-7=4x$
2. Divide both sides of the equation by the coefficient of the variable, x (in this case 4).	$\frac{y-7}{4}=\frac{4x}{4}$ $\frac{y-7}{4}=x$
3. Transpose the relation by interchanging the left-hand side and the right-hand side.	$x=\frac{y-7}{4}$

on Resources

Interactivity: Transposing linear equations (int-6449)

studyon

Units 1 & 2 > Area 3 > Sequence 1 > Concepts 1 & 2

Linear relations and rules Summary screen and practice questions

Transposing linear relations Summary screen and practice questions

Exercise 5.2 Linear patterns

1. **WE1** Identify which of the following equations are linear.

   a. $y^2 = 7x + 1$
   b. $t = 7x^3 - 6x$
   c. $y = 3(x + 2)$
   d. $m = 2^{x+1}$
   e. $4x + 5y - 9 = 0$
   f. $x = \dfrac{6 - y}{4}$

2. Bethany was asked to identify which equations from a list were linear. The following table shows her responses.

Equation	Bethany's response
$y = 4x + 1$	Yes
$y^2 = 5x - 2$	Yes
$y + 6x = 7$	Yes
$y = x^2 - 5x$	No
$t = 6d^2 - 9$	No
$m^3 = n + 8$	Yes

   a. Insert another column into the table and add your responses with justification, identifying which of the equations are linear.
   b. Provide advice to Bethany to help her to correctly identify linear equations.

3. Identify which of the following are linear equations.

   a. $y = 2t + 5$
   b. $x^2 = 2y + 5$
   c. $m = 3(n + 5)$
   d. $d = 80t + 25$
   e. $y^2 = 7x + 12$
   f. $\sqrt{y} = x + 5$

4. Samson was asked to identify which of the following were linear equations. His responses are shown in the table.

   a. Based on Samson's responses, would he state that $6y^2 + 7x = 9$ is linear? Justify your answer.
   b. What advice would you give to Samson to ensure that he can correctly identify linear equations?

Equation	Samson's response
$y = 5x + 6$	Yes, linear
$y^2 = 6x - 1$	Yes, linear
$y = x^2 + 4$	Not linear
$y^3 = 7(x + 3)$	Yes, linear
$y = \frac{1}{2}x + 6$	Yes, linear
$\sqrt{y} = 4x + 2$	Yes, linear
$y^2 + 5x^3 + 9 = 0$	Not linear
$10y - 11x = 12$	Yes, linear

5. **WE2** Find the equations for the linear relations formed by the following number patterns, where n is the term number and t is the term.

   a. 2, 6, 10, 14, 18, …
   b. 4, 4.5, 5, 5.5, 6, …

6. Jars of vegetables are stacked in ten rows. There are 8 jars in the third row and 5 jars in the sixth row. The number of jars in any row can be represented by a linear relation.

   a. Calculate the common difference.
   b. Determine an equation that will express the number of jars in any of the ten rows.
   c. Determine the total number of jars of vegetables in rows 1 to 6 inclusive.
   d. Determine the number of jars in the 10th row.

7. A number pattern is formed by multiplying the previous term by 1.5. The first term is 2.

   a. Determine the next four terms in the number pattern.
   b. Could this number pattern be represented by a linear equation? Justify your answer.

8. Determine equations for the linear relations formed by the following number patterns.

   a. 3, 7, 11, 15, 19, …
   b. 7, 10, 13, 16, 19, ...
   c. 12, 9, 6, 3, 0, −3, …
   d. 13, 7, 1, −5, −11, …

9. Consider the following number pattern: 1.2, 2.0, 2.8, 3.6, 4.4, …

   a. Calculate the first common difference.
   b. Could this number pattern be represented by a linear equation? Justify your answer.

10. **WE3** Transpose the linear equation $y = 6x - 3$ to make x the subject of the equation.

11. Transpose the linear equation $6y = 3x + 1$ to make x the subject of the equation.

12. Transpose the following linear equations to make x the subject.

   a. $y = 2x + 5$
   b. $3y = 6x + 8$

13. Water is leaking from a water tank at a linear rate. The amount of water, in litres, is measured at the start of each day. At the end of the first day there are 950 litres in the tank, and at the end of the third day there are 850 litres in the tank.

   a. Complete the following table.

Day	1	2	3	4	5
Amount of water (L)	950		850		

   b. Determine the amount of water that was initially in the tank (i.e. at day 0).
   c. Determine an equation that finds the amount of water, w, in litres, at the end of any day, d.

14. At the start of the year Yolanda has \$1500 in her bank account. At the end of each month she deposits an additional \$250.

   a. How much, in dollars, does Yolanda have in her bank account at the end of March?
   b. Develop an equation that determines the amount of money, A, Yolanda has in her bank account at the end of each month, m.
   c. At the start of the following year, Yolanda deposits an additional \$100 each month. How does this change the equation found in part b?

15. On the first day of Sal's hiking trip, she walks halfway into a forest. On each day after the first, she walks exactly half the distance she walked the previous day. Could the distance travelled by Sal each day be described by a linear equation? Justify your answer.

16. Antonia is a runner who has a goal to run a total of 350 km over 5 weeks to raise money for charity.
- **a.** If each week she runs 10 km more than she did on the previous week, how far does she run in week 3?
- **b.** Develop an equation that determines the distance (d) Antonia runs each week.

5.3 Solving simple linear equations

Equations are mathematical statements that show two equal expressions. This means that the left-hand side and the right-hand side of an equation are equal.

A linear equation has pronumerals whose highest power is 1; for example $y = 2x + 5$.

Linear equations can be solved using inverse operations. When solving equations, the last operation performed on the pronumeral when building the equation is the first operation undone by applying inverse operations to both sides of the equation.

WORKED EXAMPLE 4

Solve $6(a - 3) = 72$.

THINK	WRITE
1. To create the equation, a had 3 subtracted from it and the result was multiplied by 6 to get 72. The last operation performed was multiplying by 6. To undo the × 6 operation, divide both sides 6, as shown in blue, and simplify.	$\frac{{}^{1}\cancel{6}(a-3)}{{}^{1}\cancel{6}} = \frac{\cancel{72}^{12}}{{}^{1}\cancel{6}}$
2. Looking at the equivalent equation, ($a - 3 = 12$), a has had 3 subtracted from it. To undo the -3 operation, add 3 to both sides, as shown in red, and simplify.	$a - 3 = 12$ $a - 3 + 3 = 12 + 3$ $a = 15$

5.3.1 Equations with pronumerals on both sides

When a pronumeral appears on both sides of an equation, inverse operations are used to collect the pronumerals into a single term. To solve linear equations containing fractions, multiply both sides of the equation by the lowest common denominator first, to remove the fraction.

WORKED EXAMPLE 5

Solve $3x - 12 = 5x + 4$.

THINK	WRITE
1. Move all the x values to one side by subtracting $3x$ from both sides.	$3x - 12 = 5x + 4$ $3x - 3x - 12 = 5x - 3x + 4$ $-12 = 2x + 4$
2. To get the x values by themselves, subtract 4 from both sides.	$-12 = 2x + 4$ $-12 - 4 = 2x + 4 - 4$ $-16 = 2x$
3. Divide both sides by 2.	$-16 = 2x$ $\frac{-16}{2} = \frac{2x}{2}$ $-8 = x$ $x = -8$

WORKED EXAMPLE 6

Solve $\frac{3a - 2}{2} = \frac{a + 11}{3}$.

THINK	WRITE
1. The lowest common multiple of 2 and 3 is 6. Rewrite each fraction so that the numerator and denominator are converted into equivalent fractions, as shown in blue.	$\frac{3a - 2}{2} \times \frac{3}{3} = \frac{a + 11}{3} \times \frac{2}{2}$ $\frac{3(3a - 2)}{6} = \frac{2(a + 11)}{6}$
2. To remove the fractions, multiply each side by 6, as shown in red, and simplify.	$\frac{3(3a - 2)}{{}^{1}\cancel{6}} \times \frac{\cancel{6}}{1} = \frac{2(a + 11)}{{}^{1}\cancel{6}} \times \frac{\cancel{6}}{1}$ $3(3a - 2) = 2(a + 11)$
3. Expand the brackets then collect the pronumeral terms by subtracting $2a$ from both sides, as shown in green, and simplify.	$9a - 6 = 2a + 22$ $9a - 6 - 2a = 2a + 22 - 2a$ $7a - 6 = 22$
4. Add 6 to both sides, as shown in pink, and simplify. Divide both sides by 7, as shown in pink, and simplify.	$7a - 6 + 6 = 22 + 6$ $7a = 28$ $\frac{{}^{1}\cancel{7}a}{{}^{1}\cancel{7}} = \frac{{}^{4}\cancel{28}}{{}^{1}\cancel{7}}$ $a = 4$

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Solving numerical linear equations Summary screen and practice questions

Exercise 5.3 Solving simple linear equations

1. Solve the following equations.

a. $x - 7 = 12$ **b.** $m + 7 = 15$ **c.** $p + 12 = 25$

d. $q - 23 = 27$ **e.** $y + 12 = 8$ **f.** $k + 21 = 10$

2. Solve the following equations.

a. $5x = 30$ **b.** $4y = 24$ **c.** $7z = 56$

d. $9b = 36$ **e.** $3g = -39$ **f.** $-8p = 88$

3. Solve the following equations.

a. $2m - 5 = 7$ **b.** $3d - 12 = 24$ **c.** $5x - 16 = 14$ **d.** $9x - 29 = 25$

4. Solve the following equations.

a. $3x - 5 = 2$ **b.** $4x - 12 = 13$ **c.** $4x + 9 = 27$ **d.** $3x + 4 = -13$

5. Solve the following equations.

a. $5b + 3 = 38$ **b.** $4d - 7 = 9$ **c.** $3f - 2 = -14$ **d.** $0.9 + 3y = 15$

6. Solve the following equations

a. $2.8 + 0.2b = 18$ **b.** $4 - 2x = 7$ **c.** $2 - 4g = -6$ **d.** $5 = 2x + 13$

7. Solve the following equations.

a. $3(x - 5) = 24$ **b.** $5(2g + 3) = 7$ **c.** $8 = 3(x - 2)$ **d.** $7 + 2(2x - 9) = 0$

8. Solve the following equations.

a. $3m - 4 = m + 8$ **b.** $6g - 18 = g + 52$ **c.** $4t + 14 = 10t + 80$ **d.** $3w - 27 = 7w - 75$

9. Solve the following equations.

a. $-7(-2 + 5x) - 6x = 13$ **b.** $8(2 - x) + 1 = 9$

c. $-(3 - 4x) + 3 = 4$ **d.** $2(x + 1) + 3(x + 2) = 6$

10. Solve the following equations.

a. $\frac{x}{2} + 3 = 5$ **b.** $\frac{2s}{3} - 7 = 2$ **c.** $3 + \frac{3x}{2} = 12$

d. $-11 - \frac{x}{5} = -5$ **e.** $\frac{a - 5}{3} = -2$ **f.** $8 = \frac{a + 4}{11}$

11. Solve the following equations.

a. $3j - 22 = -4 - 3j$ **b.** $26x - 125 = x$

c. $2(3x + 1) = 5(2x + 3)$ **d.** $-2(3a + 1) = 5(1 - a)$

12. Solve the following equations.

a. $\frac{b + 4}{3} = \frac{b - 8}{5}$ **b.** $\frac{3y - 2}{7} = \frac{y + 6}{4}$ **c.** $\frac{4 - 3y}{2} = \frac{1 - 5y}{3}$ **d.** $4 + t = \frac{4t + 1}{3}$

5.4 Solving further linear equations

5.4.1 Solving linear equations with one variable

To solve linear equations with one variable, all operations performed on the variable need to be identified in order, and then the opposite operations need to be performed in reverse order.

In practical problems, solving linear equations can answer everyday questions such as the time required to have a certain amount in the bank, the time taken to travel a certain distance, or the number of participants needed to raise a certain amount of money for charity.

WORKED EXAMPLE 7

Solve the following linear equations to find the unknowns.

a. $5x = 12$ **b.** $8t + 11 = 20$ **c.** $12 = 4(n - 3)$ **d.** $\frac{4x - 2}{3} = 5$

THINK	WRITE
a. 1. Identify the operations performed on the unknown.	**a.** $5x = 5 \times x$ So the operation is $\times 5$.
2. Write the opposite operation.	The opposite operation is $\div 5$.
3. Perform the opposite operation on both sides of the equation.	Step 1 ($\div 5$): $5x = 12$ $\frac{5x}{5} = \frac{12}{5}$ $x = \frac{12}{5}$
4. Write the answer in its simplest form.	$x = \frac{12}{5}$
b. 1. Identify the operations performed in order on the unknown.	**b.** $8t + 11$ The operations are $\times 8$, $+11$.
2. Write the opposite operations.	$\div 8, - 11$
3. Perform the opposite operations in reverse order on both sides of the equation, one operation at a time.	Step 1 (-11): $8t + 11 = 20$ $8t + 11 - 11 = 20 - 11$ $8t = 9$ Step 2 ($\div 8$): $8t = 9$ $\frac{8t}{8} = \frac{9}{8}$ $t = \frac{9}{8}$
4. Write the answer in its simplest form.	$t = \frac{9}{8}$
c. 1. Identify the operations performed in order on the unknown. (Remember operations in brackets are performed first.)	**c.** $4(n - 3)$ The operations are $-3, \times 4$.
2. Write the opposite operations.	$+3, \div 4$
3. Perform the opposite operations on both sides of the equation in reverse order, one operation at a time.	Step 1 ($\div 4$): $12 = 4\,(n - 3)$ $\frac{12}{4} = \frac{4\,(n-3)}{4}$ $3 = n - 3$ Step 2 ($+3$): $3 = n - 3$ $3 + 3 = n - 3 + 3$ $6 = n$

4. Write the answer in its simplest form.	$n = 6$
d. 1. Identify the operations performed in order on the unknown.	**d.** $\frac{4x-2}{3}$ The operations are $\times 4, -2, \div 3$.
2. Write the opposite operations.	$\div 4, +2, \times 3$
3. Perform the opposite operations on both sides of the equation in reverse order, one operation at a time.	Step 1 $(\times 3)$: $\frac{4x-2}{3} = 5$ $3 \times \frac{4x-2}{3} = 5 \times 3$ $4x - 2 = 15$ Step 2 $(+2)$: $4x - 2 = 15$ $4x - 2 + 2 = 15 + 2$ $4x = 17$ Step 3 $(\div 4)$: $4x = 17$ $\frac{4x}{4} = \frac{17}{4}$ $x = \frac{17}{4}$
4. Write the answer in its simplest form.	$x = \frac{17}{4}$

5.4.2 Substituting into linear equations

If we are given a linear equation between two variables and we are given the value of one of the variables, we can **substitute** this into the equation to determine the other value.

WORKED EXAMPLE 8

Substitute $x = 3$ into the linear equation $y = 2x + 5$ to determine the value of y.

THINK	WRITE
1. Substitute the variable (x) with the given value.	$y = 2(3) + 5$
2. Equate the right-hand side of the equation.	$y = 6 + 5$ $y = 11$

5.4.3 Literal linear equations

A **literal equation** is an equation that includes several pronumerals or variables. Literal equations often represent real-life situations.

The equation $y = mx + c$ is an example of a literal linear equation that represents the general form of a straight line.

To solve literal linear equations, you need to isolate the variable for which you are trying to solve.

WORKED EXAMPLE 9

Solve the linear literal equation $y = mx + c$ for x.

THINK	WRITE
1. Isolate the terms containing the variable you want to solve on one side of the equation.	$y - c = mx$
2. Divide by the coefficient of the variable you want to solve for.	$\frac{y-c}{m} = x$
3. Transpose the equation.	$x = \frac{y-c}{m}$

Resources

Interactivity: Solving linear equations (int-6450)

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Solving literal linear equations Summary screen and practice questions

Exercise 5.4 Solving further linear equations

1. **WE7** Solve the following linear equations to find the unknowns.
   a. $2(x + 1) = 8$
   b. $n - 12 = -2$
   c. $4d - 7 = 11$
   d. $\frac{x+1}{2} = 9$
2. a. Write the operations in order that have been performed on the unknowns in the following linear equations.
      i. $10 = 4a + 3$
      ii. $3(x + 2) = 12$
      iii. $\frac{s+1}{2} = 7$
      iv. $16 = 2(3c - 9)$
   b. Find the exact values of the unknowns in part **a** by solving the equations. Show all of the steps involved.
3. Find the exact values of the unknowns in the following linear equations.
   a. $14 = 5 - x$
   b. $\frac{2(3-x)}{3} = 5$
4. Solve the following literal linear equations for the pronumerals given in brackets.
   a. $v = u + at \quad (a)$
   b. $\frac{x}{p} - r = s \quad (x)$
5. **WE8** Substitute $x = 5$ into the equation $y = 5 - 6x$ to determine the value of y.

6. Substitute $x = -3$ into the equation $y = 3x + 3$ to determine the value of y.
7. The equation $w = 10t + 120$ represents the amount of water in a tank, w (in litres), at any time, t (in minutes). Find the time, in minutes, that it takes for the tank to have the following amounts of water.
   - a. 450 litres
   - b. 1200 litres
8. **WE9** Solve the literal linear equation $px - q = r$ for x.
9. Solve the literal linear equation $C = \pi d$ for d.
10. Yorx was asked to solve the linear equation $5w - 13 = 12$. His solution is shown.

Step 1: $\times 5, -13$

Step 2: Opposite operations $\div 5, +13$

Step 3: $5w - 13 = 12$

$$\frac{5w-13}{5} = \frac{12}{5}$$

$$w - 13 = 2.4$$

Step 4: $w - 13 + 13 = 2.4 + 13$

$$w = 15.4$$

   - a. Show that Yorx's answer is incorrect by finding the value of w.
   - b. What advice would you give to Yorx so that he can solve linear equations correctly?
11. The literal linear equation $F = 1.8(K - 273) + 32$ converts the temperature in Kelvin (K) to Fahrenheit (F). Solve the equation for K to give the formula for converting the temperature in Fahrenheit to Kelvin.
12. Consider the linear equation $y = \dfrac{3x+1}{4}$. Find the value of x for the following y-values.
   - a. 2
   - b. -3
   - c. $\dfrac{1}{2}$
   - d. 10

13. The distance travelled, d (in kilometres), at any time t (in hours) can be found using the equation $d = 95t$. Find the time in hours that it takes to travel the following distances.
    Give your answers correct to the nearest minute.
   - a. 190 km
   - b. 250 km
   - c. 65 km
   - d. 356.5 km
14. The amount, A, in dollars in a bank account at the end of any month, m, can found using the equation $A = 150m + 400$.
   - a. How many months would it take to have the following amounts of money in the bank account?
     - i. \$1750
     - ii. \$3200
   - b. How many years would it take to have \$10 000 in the bank account? Give your answer correct to the nearest month.

15. The temperature, C, in degrees Celsius can be found using the equation $C = \dfrac{5(F-32)}{9}$, where F is the temperature in degrees Fahrenheit. Nora needs to set her oven at 190°C, but her oven's temperature is measured in Fahrenheit.
   - a. i. Write the operations performed on the variable F.
     - ii. Write the order in which the operations need to be performed to find the value of F.
   - b. Determine the temperature in Fahrenheit that Nora should set her oven to.

16. The equation that determines the surface area of a cylinder with a radius of 3.5 cm is $A = 2 \times 3.5\pi(3.5 + h)$.
Determine the height in cm of cylinders with radii of 3.5 cm and the following surface areas. Round your answers to 2 decimal places.

a. $200\ \text{cm}^2$ **b.** $240\ \text{cm}^2$

17. Solve the following equations to find the unknowns. Express your answer in exact form.

a. $\dfrac{2-5x}{8} = \dfrac{3}{5}$ **b.** $\dfrac{6(3y-2)}{11} = \dfrac{5}{9}$

c. $\left(\dfrac{4x}{5} - \dfrac{3}{7}\right) + 8 = 2$ **d.** $\dfrac{7x+6}{9} + \dfrac{3x}{10} = \dfrac{4}{5}$

18. The height of a plant can be found using the equation $h = \dfrac{2(3t+15)}{3}$, where h is the height in cm and t is time in weeks.

a. Determine the time the plant takes to grow to the following heights. Give your answers correct to the nearest week.

i. 20 cm   **ii.** 30 cm
iii. 35 cm   **iv.** 50 cm

b. How high is the plant initially?
When the plant reaches 60 cm it is given additional plant food. The plant's growth each week for the next 4 weeks is found using the equation $g = t + 2$, where g is the growth each week in cm and t is the time in weeks since additional plant food was given.

c. Determine the height of the plant in cm for the next 4 weeks.

5.5 Developing linear equations

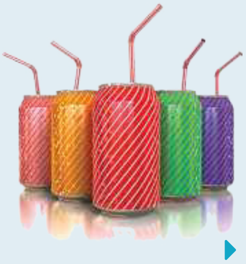
5.5.1 Developing linear equations from word descriptions

To write a worded statement as a linear equation, we must first identify the unknown and choose a **pronumeral** to represent it. We can then use the information given in the statement to write a linear equation in terms of the pronumeral.

The linear equation can then be solved as before, and we can use the result to answer the original question.

WORKED EXAMPLE 10

Cans of soft drinks are sold at SupaSave in packs of 12 costing $5.40. Form and solve a linear equation to determine the price of 1 can of soft drink.



THINK	WRITE
1. Identify the unknown and choose a pronumeral to represent it.	S = price of a can of soft drink
2. Use the given information to write an equation in terms of the pronumeral.	$12S = 5.4$
3. Solve the equation.	$\frac{12S}{12} = \frac{5.4}{12}$ $S = 0.45$
4. Interpret the solution in terms of the original problem.	The price of 1 can of soft drink is \$0.45 or 45 cents.

5.5.2 Word problems with more than one unknown

In some instances a word problem might contain more than one unknown. If we are able to express both unknowns in terms of the same pronumeral, we can create a linear equation as before and solve it to determine the value of both unknowns.

WORKED EXAMPLE 11

Georgina is counting the number of insects and spiders she can find in her back garden. All insects have 6 legs and all spiders have 8 legs. In total, Georgina finds 43 bugs with a total of 290 legs. Form a linear equation to determine exactly how many insects and spiders Georgina found.

THINK	WRITE
1. Identify one of the unknowns and choose a pronumeral to represent it.	Let s = the number of spiders.
2. Define the other unknown in terms of this pronumeral.	Let $43 - s$ = the number of insects.
3. Write expressions for the total numbers of spiders' legs and insects' legs.	Total number of spiders' legs $= 8s$ Total number of insects' legs $= 6(43 - s)$ $= 258 - 6s$
4. Create an equation for the total number of legs of both types of creature.	$8s + (258 - 6s) = 290$
5. Solve the equation.	$8s + 258 - 6s = 290$ $8s - 6s = 290 - 258$ $2s = 32$ $s = 16$
6. Substitute this value back into the second equation to determine the other unknown.	The number of insects $= 43 - 16$ $= 27$
7. Answer the question using words.	Georgina found 27 insects and 16 spiders.

WORKED EXAMPLE 12

The amount of water that is filling a tank is found by the rule $W = 100t + 20$, where W is the amount of water in the tank in litres and t is the time in hours.

a. **Construct a table of values that shows the amount of water, W, in the tank every hour for the first 8 hours (i.e. $t = 0, 1, 2, 3, \ldots, 8$).**

b. **Using your table, how long in hours will it take for there to be over 700 litres in the tank?**

THINK

WRITE

a. 1. Enter the required values of t into the formula to calculate the values of W.

a.

$t = 0$:
$$W = 100\,(0) + 20 \\ = 20$$

$t = 1$:
$$W = 100\,(1) + 20 \\ = 120$$

$t = 2$:
$$W = 100\,(2) + 20 \\ = 220$$

$t = 3$:
$$W = 100\,(3) + 20 \\ = 320$$

$t = 4$:
$$W = 100\,(4) + 20 \\ = 420$$

$t = 5$:
$$W = 100\,(5) + 20 \\ = 520$$

$t = 6$:
$$W = 100\,(6) + 20 \\ = 620$$

$t = 7$:
$$W = 100\,(7) + 20 \\ = 720$$

$t = 8$:
$$W = 100\,(8) + 20 \\ = 820$$

2. Enter the calculated values into a table of values.

t	0	1	2	3	4	5	6	7	8
W	20	120	220	320	420	520	620	720	820

b. 1. Using your table of values, locate the required column.

b.

t	0	1	2	3	4	5	6	7	8
W	20	120	220	320	420	520	620	720	820

2. Read the corresponding values from your table and answer the question.

$t = 7$

It will take 7 hours for there to be over 700 litres of water in the tank.

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Developing linear equations from worked problems Summary screen and practical questions

Exercise 5.5 Developing linear equations

1. **WE10** Artists' pencils at the local art supply store sell in packets of 8 for $17.92. Form and solve a linear equation to determine the price of 1 artists' pencil.
2. Natasha is trying to determine which type of cupcake is the best value for money. The three options Natasha is considering are:
   - 4 red velvet cupcakes for $9.36
   - 3 chocolate delight cupcakes for $7.41
   - 5 caramel surprise cupcakes for $11.80.

   Form and solve linear equations for each type of cupcake to determine which has the cheapest price per cupcake.

3. Three is added to a number and the result is then divided by four, giving an answer of nine. Determine the number.
4. The sides in one pair of sides of a parallelogram are each 3 times the length of a side in the other pair. Determine the side lengths if the perimeter of the parallelogram is 84 cm.
5. **WE11** Fredo is buying a large bunch of flowers for his mother in advance of Mother's Day. He picks out a bunch of roses and lilies, with each rose costing $6.20 and each lily costing $4.70. In total he picks out 19 flowers and pays $98.30. Form a linear equation to determine exactly how many roses and lilies Fredo bought.

6. Miriam has a sweet tooth, and her favourite sweets are strawberry twists and chocolate ripples. The local sweet shop sells both as part of their pick and mix selection, so Miriam fills a bag with them. Each strawberry twist weighs 5 g and each chocolate ripple weighs 9 g. In Miriam's bag there are 28 sweets, weighing a total of 188 g. Determine the number of each type of sweet that Miriam bought by forming and solving a linear equation.
7. One week Jordan bought a bag of his favourite fruit and nut mix at the local market. The next week he saw that the bag was on sale for 20% off the previously marked price. Jordan purchased two more bags at the reduced price. Jordan spent $20.54 in total for the three bags. Calculate the original price of a bag of fruit and nut mix.
8. Six times the sum of four plus a number is equal to one hundred and twenty-six. Calculate the number.
9. **WE12** Libby enjoys riding along Beach Road on a Sunday morning. She rides at a constant speed of 0.4 kilometres per minute.
   a. Construct a table of values that shows how far Libby has travelled for each of the first 10 minutes of her journey.
   b. One Sunday Libby stops and meets a friend 3 kilometres into her journey. Between which minutes does Libby stop?

10. Tommy is saving for a remote-controlled car that is priced at \$49. He has \$20 in his piggy bank. Tommy saves \$3 of his pocket money every week and puts it in his piggy bank. The amount of money in dollars, M, in his piggy bank after w weeks can be found using the rule $M = 3w + 20$.

- **a.** Construct a table of values that shows the amount of money, M, in Tommy's piggy bank every week for the 12 weeks (i.e. $w = 0, 1, 2, 3, \ldots, 12$).
- **b.** Using your table, how many weeks will it take for Tommy to have saved enough money to purchase the remote-controlled car?

11. Fred is saving for a holiday and decides to deposit \$40 in his bank account each week. At the start of his saving scheme he has \$150 in his account.

- **a.** Calculate how much money Fred will have in his account at the end of the fourth week.
- **b.** Construct a table of values detailing how much Fred will have in his account at the end of each of the first 8 weeks.
- **c.** The holiday Fred wants to go on will cost \$720 dollars. Determine a rule to describe Fred's savings. Hence, calculate how many weeks it will take Fred to save up enough money to pay for his holiday.

12. Sabrina is a landscape gardener and has been commissioned to work on a rectangular piece of garden. The length of the garden is 6 metres longer than the width, and the perimeter of the garden is 64 m. Find the parameters of the garden.

13. Yuri is doing his weekly grocery shop and is buying both carrots and potatoes. He calculates that the average weight of a carrot is 60 g and the average weight of a potato is 125 g. Furthermore, he calculates that the average weight of the carrots and potatoes that he purchases is 86 g. If Yuri's shopping weighed 1.29 kg in total, how many of each did he purchase?

14. Ho has a water tank in his back garden that can hold up to 750 L in water. At the start of a rainy day (at 0:00) there is 165 L in the tank, and after a heavy day's rain (at 24:00) there is 201 L in the tank.

- **a.** Assuming that the rain fell consistently during the 24-hour period, set up a linear equation to represent the amount of rain in the tank at any point during the day.
- **b.** Generate a table of values that shows how much water is in the tank after every 2 hours of the 24-hour period.
- **c.** At what time of day did the amount of water in the tank reach 192 L?

15. A large fish tank is being filled with water. After 1 minute the height of the water is 2 cm and after 4 minutes the height of the water is 6 cm. The height of the water, h, in cm after t minutes can be modelled by a linear equation.

- **a.** Was the fish tank empty of water before being filled? Justify your answer by using calculations.
- **b.** Construct an equation between the height of water in the fish tank and the time.
- **c.** Determine the height of the water in the fish tank after five minutes.

16. Michelle and Lydia live 325 km apart. On a Sunday they decide to drive to each other's respective towns. They pass each other after 2.5 hours. If Michelle drives an average of 10 km/h faster than Lydia, calculate the speed at which they are both travelling.

5.6 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. **MC** Which one of the following number patterns can't be represented by a linear expression?

A. 5, 8, 12, 17, … **B.** $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$ **C.** 13, 10.5, 8, 5.5, … **D.** 21.5, 23, 24.5, 26, …

2. **MC** Which of the following is the correctly transposed version of $y = 3x - 6$ that makes x the subject of the equation?

A. $x = \frac{y-6}{3}$ **B.** $x = \frac{y+6}{3}$ **C.** $x = 3y - 6$ **D.** $x = 3y + 6$

3. **MC** The value of x in the linear equation $3(2x + 5) = 12$ is:

A. −1.5 **B.** −1 **C.** −0.5 **D.** 2

4. **MC** The literal linear equation $v = u + at$ is an equation for motion, given an initial velocity, a rate of acceleration and a period of time. The correct solution to this equation for a is:

A. $a = \frac{v-u}{t}$ **B.** $a = \frac{u-v}{t}$ **C.** $a = v - tu$ **D.** $a = u - tv$

The following information relates to **5** and **6** .

Juliana and Alyssa live in two towns 232 km apart. One weekday they both have to drive to each other's town for a business meeting, leaving at the same time in the morning. Alyssa drives an average of 12 km/h faster than Juliana, and they pass each other after 2 hours.

5. **MC** If Juliana drives at an average speed of j km/h, what is Alyssa's average driving speed?

A. $(232 - j)$ km/h **B.** $(232 + j)$ km/h **C.** $(j - 12)$ km/h **D.** $(j + 12)$ km/h

6. **MC** What is Juliana's average driving speed?

A. 50 km/h **B.** 52 km/h **C.** 60 km/h **D.** 64 km/h

7. **MC** The value of x in the linear equation $4(x - 1) = 12$ is:

A. 3 **B.** 4 **C.** 2 **D.** 5

8. **MC** The solution to the equation $5 + \frac{x}{2} = 12$ is:

A. $x = 1$ **B.** $x = 19$ **C.** $x = 16$ **D.** $x = 14$

9. **MC** Which of the following is not a linear equation?

A. $3 - \frac{x}{5} = y$ **B.** $13 + \frac{x}{2} = -y$ **C.** $y = \frac{2}{x} + 1$ **D.** $p = 2q - \frac{1}{2}$

10. **MC** Five is added to 3 times a number and the result is 17. The number is:

A. 5 **B.** 12 **C.** 3 **D.** 4

11. a. i. Complete the following table of values for the linear equation $p = 5(2n + 4)$.

n	1	2	3	4	5	6
p	30	40				

ii. Write down the common difference.

b. What is the value of p when $n = 12$?

12. Solve each of the following equations for the unknown.

a. $3x + 15 = 14$ **b.** $\frac{5-2m}{3} = -2$

Complex familiar

13. Find the equations for the linear relations formed by the following number patterns, where n is the term number and t is the term.

a. 5, 9, 13, 17, … **b.** 6, 2, −2, −6, … **c.** 1.2, 1.7, 2.2, 2.7, … **d.** 103, 106, 109, 112, …

14. Petra is doing a survey of how humans and pets are in her extended family. She gives the following information to her friend Juliana:

- There are 33 humans and pets (combined).
- The combined number of legs (pets and owners) is 94.
- Each human has 2 legs and each pet has 4 legs.

a. If h is the number of humans, express the number of pets in terms of h.

b. Write expressions for the total number of human legs and the total number of pet legs in terms of h.

15. Solve each of the following equations for the unknown.

a. $\frac{3s}{4} + 6 = 10$ **b.** $\frac{-2(3t + 1)}{5} + 3 = 9$

16. Using technology or otherwise, determine an equation that describes the number pattern shown in the table below.

Term number, *n*	1	2	3	4	5
Value, *V*	−4	−2	0	2	4

Complex unfamiliar

17. The terms in a number sequence are found by multiplying the term number, n, by 4 and then subtracting 1. The first term of the sequence is 3.

a. Find an equation that determines the terms in the sequence.

b. Using technology or otherwise, find the first 10 terms of the sequence.

c. Show that the common difference is 4.

18. Jett is starting up a small business selling handmade surfboard covers online. The start-up cost is $250. He calculates that each cover will cost $14.50 to make. The rule that finds the cost, C, to make n covers is $C = 14.50n + 250$.

a. Using technology or otherwise, generate a table of values to determine the cost of producing 10 to 20 surfboard covers.

b. If Jett sells the covers for $32.95, construct a table of values to determine the revenue for selling 10 to 20 surfboard covers.

c. The profit Jett makes is the difference between his selling price and the cost price. Explain how the profit Jett makes can be calculated using the tables of values constructed in parts **a** and **b**.

d. Using your explanation in part **c** and your table of values, determine the profits made by Jett if he sells 10 to 20 surfboard covers.

19. A study of the homework habits of a group of students showed that the amount of weekly homework, in hours, completed by the students had an effect on their performance on the weekly assessment tasks. The table shown represents the number of weekly homework hours spent by the group of students and the average percentage mark they achieved on their weekly assessment tasks.

Hours of homework, *h*	1	2	3	4	5	6	7
Average percentage mark, *m*	22	31	40	49	58	67	76

a. On average, how many marks are gained for each additional hour spent doing homework?
b. Using any appropriate method, determine the rule that finds the average percentage mark, m, for each hour spent doing homework, h.
c. Using the rule you found in part b, determine the average percentage mark for the following numbers of hours spent doing homework:
   i. 9 hours
   ii. 5.5 hours
d. Seth scored 51.25% on the assessment task. Determine how many hours he spent doing homework during the week according to the rule.
e. Nerada did not do any homework during the week. What will be her expected percentage mark on the assessment task according to the rule?
f. Freda argues that the rule does not apply for students who spend more than 10 hours each week doing their homework. Find the average percentage mark for a student who spends 10 hours doing homework. Does Freda have a valid argument?

20. Hank is cooking a Sunday dinner of roast lamb and roast beef for 20 guests. He has a 2.5 kg leg of lamb and a 4.2 kg cut of beef. The recommended cooking time for the lamb is 62.5 minutes; the recommended cooking time for the beef is 105 minutes. Hank's cookbook recommends that the meat be left to rest for 15 minutes before carving.

The cooking time is the same per kilogram for both cuts of meats and increases at a constant rate per kilogram.

a. Find the cooking time in minutes, t, per kilogram of meat, k.
b. Construct an equation that finds the cooking time, in minutes, including the resting time per kilogram of meat (lamb and beef).
c. Using a spreadsheet and your equation from part b, complete the following table for different-sized cuts of meat. Write your answers correct to the nearest whole number.

Weight (g)	Cooking time (minutes)	Weight (g)	Cooking time (minutes)
500		2250	
750		2500	
1000		2750	
1250		3000	
1500		3250	
1750		3500	
2000		3750	

Marcia uses the equation from part b to help her with the cooking time of her Christmas turkey, which weighs 5.5 kg.

d. Using the equation you found in part **b**, determine the cooking time in hours and minutes for the turkey.

e. Marcia finds that the cooking time is incorrect. Explain why the equation did not help her to accurately determine the cooking time.

Marcia finds a cookbook which suggests that the cooking time for a turkey is $\frac{3}{4}$ of an hour per kilogram.

f. **i.** If the resting time for roast turkey is 30 minutes, construct an equation that finds the cooking time per kilogram of turkey.

ii. Using the equation you found in part **i**, determine the recommended cooking time, in hours and minutes, for Marcia's turkey.

studyon

Units 1 & 2 Sit Chapter test

Answers

Chapter 5 Linear equations

Exercise 5.2 Linear patterns

1. **a.** Non-linear **b.** Non-linear **c.** Linear
d. Non-linear **e.** Linear **f.** Linear

2. **a.**

Equation	Bethany's response	Correct response
$y = 4x + 1$	Yes	Yes
$y^2 = 5x - 2$	Yes	No
$y + 6x = 7$	Yes	Yes
$y = x^2 - 5x$	No	No
$t = 6d^2 - 9$	No	No
$m^3 = n + 8$	Yes	No

b. Bethany should look at both variables (pronumerals or letters). Both variables need to have a highest power of 1.

3. **a.** Linear **b.** Non-linear **c.** Linear
d. Linear **e.** Non-linear **f.** Non-linear

4. **a.** Yes, as the power of x is 1.
b. The power of both variables in a linear relation must be 1.

5. **a.** $t = 4n - 2$ **b.** $t = 0.5n + 3.5$

6. **a.** -1 **b.** $t = -n + 11$
c. 45 jars **d.** 1 jar

7. **a.** 3, 4.5, 6.75, 10.125
b. No, as there is no common difference.

8. **a.** $t = 4n - 1$ **b.** $t = 3n + 4$
c. $t = -3n + 15$ **d.** $t = -6n + 19$

9. **a.** 0.8
b. Yes, as it has a common difference.

10. $x = \dfrac{y+3}{6}$

11. $x = 2y - \dfrac{1}{3}$

12. **a.** $x = \dfrac{y-5}{2}$ **b.** $x = \dfrac{3y-8}{6}$

13. **a.**

Day	1	2	3	4	5
Amount of water (L)	950	900	850	800	750

b. 1000 L
c. $w = -50d + 1000$

14. **a.** \$2250
b. $A = 1500 + 250m$
c. This changes the equation to $A = 4500 + 350m$.

15. No, because her distance each day is half the previous distance, so there is no common difference.

16. **a.** 70 km
b. $d = 10w + 40$, where
$w = \text{number of weeks}, \; d = \text{distance}$

Exercise 5.3 Solving simple linear equations

1. **a.** $x = 19$ **b.** $m = 8$ **c.** $p = 13$
d. $q = 50$ **e.** $y = -4$ **f.** $k = -11$

2. **a.** $x = 6$ **b.** $y = 6$ **c.** $z = 8$
d. $b = 4$ **e.** $g = -13$ **f.** $p = -11$

3. **a.** $m = 6$ **b.** $d = 12$
c. $x = 6$ **d.** $x = 6$

4. **a.** $x = \dfrac{7}{3}$ **b.** $x = \dfrac{25}{4}$
c. $x = \dfrac{9}{2}$ **d.** $x = \dfrac{-17}{3}$

5. **a.** $b = 7$ **b.** $d = 4$
c. $f = -4$ **d.** $y = 4.7$

6. **a.** $b = 76$ **b.** $x = \dfrac{-3}{2}$
c. $g = 2$ **d.** $x = -4$

7. **a.** $x = 13$ **b.** $g = \dfrac{-4}{5}$
c. $x = \dfrac{14}{3}$ **d.** $x = \dfrac{11}{4}$

8. **a.** $m = 6$ **b.** $g = 14$
c. $t = -11$ **d.** $w = 12$

9. **a.** $x = \dfrac{1}{41}$ **b.** $x = 1$
c. $x = 1$ **d.** $x = \dfrac{-2}{5}$

10. **a.** $x = 4$ **b.** $s = \dfrac{27}{2}$
c. $x = 6$ **d.** $x = -30$
e. $a = -1$ **f.** $a = 84$

11. **a.** $j = 3$ **b.** $x = 5$
c. $x = \dfrac{-13}{4}$ **d.** $a = -7$

12. **a.** $b = -22$ **b.** $y = 10$
c. $y = -10$ **d.** $t = 11$

Exercise 5.4 Solving further linear equations

1. **a.** $x = 3$ **b.** $n = 10$
c. $d = 4.5$ **d.** $x = 17$

2. **a.** **i.** $\times 4, +3$ **ii.** $+2, \times 3$
iii. $+1, \div 2$ **iv.** $\times 3, -9, \times 2$
b. **i.** $a = \dfrac{7}{4}$ **ii.** $x = 2$
iii. $s = 13$ **iv.** $c = \dfrac{17}{3}$

3. **a.** $x = -9$ **b.** $x = -4.5$

4. **a.** $a = \dfrac{v-u}{t}$ **b.** $x = p(r+s)$

5. $y = -25$

6. $y = -6$

7. **a.** 33 minutes **b.** 108 minutes

8. $x = \dfrac{r+q}{p}$

9. $d = \dfrac{C}{\pi}$

10. **a.** $w = 5$ **b.** Operations need to be performed in reverse order.

11. $K = \dfrac{F - 32}{1.8} + 273$

12. **a.** $x = \dfrac{7}{3}$ **b.** $x = \dfrac{-13}{3}$ **c.** $x = \dfrac{1}{3}$ **d.** $x = 13$

13. **a.** 2 hours **b.** 2 hours 38 minutes **c.** 41 minutes **d.** 3 hours 45 minutes

14. **a.** **i.** 9 months **ii.** 19 months (= 18.67 months)
b. 5 years, 4 months

15. **a.** **i.** $-32, \times 5, \div 9$ **ii.** $\times 9, \div 5, +32$
b. 374°F

16. **a.** 5.59 cm **b.** 7.41 cm

17. **a.** $x = \dfrac{-14}{25}$ **b.** $y = \dfrac{163}{162}$ **c.** $x = \dfrac{-195}{28}$ **d.** $x = \dfrac{12}{97}$

18. **a.** **i.** 5 weeks **ii.** 10 weeks **iii.** 13 weeks **iv.** 20 weeks
b. 10 cm
c. 63 cm, 67 cm, 72 cm, 78 cm

Exercise 5.5 Developing linear equations

1. $2.24

2. The red velvet cupcakes are the cheapest per cupcake — $2.34, compared to $2.47 for chocolate delight and $2.36 for caramel surprise.

3. 33

4. 10.5 cm and 31.5 cm

5. 6 roses and 13 lilies

6. 16 strawberry twists and 12 chocolate ripples

7. $7.90

8. 17

9. **a.**

Minute	1	2	3	4	5	6	7	8	9	10
Distance (km)	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4

b. Between the 7th and 8th minute

10. **a.**

Week	0	1	2	3	4	5	6	7	8	9	10	11	12
Money ($)	20	23	26	29	32	35	38	41	44	47	50	53	56

b. 10 weeks

11. **a.** $310

b.

Week	0	1	2	3	4	5	6	7	8
Amount ($)	150	190	230	270	310	350	390	430	470

c. $A(\$) = 150 + 40W$; 15 weeks

12. 13 m by 19 m

13. 9 carrots and 6 potatoes

14. **a.** $w = 165 + 1.5h$

b.

Hour	2	4	6	8	10	12	14	16	18	20	22	24
Water in Tank (L)	168	171	174	177	180	183	186	189	192	195	198	201

c. 6:00 pm

15. a. At $t = 0$, $h = \frac{2}{3}$cm. Hence, the fish tank was not empty of water before being filled.

b. $h = \frac{4}{3}t + \frac{2}{3}$

c. At $t = 5$, $h = 7\frac{1}{3}$cm

16. Michelle: 70 km/h, Lydia: 60 km/h

5.6 Review: exam practice

1. A
2. B
3. C
4. A
5. D
6. B
7. B
8. D
9. C
10. D

11. a. i.

n	1	2	3	4	5	6
p	30	40	50	60	70	80

ii. 10

b. 140

12. a. $x = 3$ **b.** $m = 5.5$

13. a. $t = 4n + 1$ **b.** $t = -4n + 10$ **c.** $t = 0.5n + 0.7$ **d.** $t = 3n + 100$

14. a. $33 - h$

b. Total number of human legs $= 2h$; total number of pet legs $4(33 - h)$

15. a. $s = 5\frac{1}{3}$ **b.** $t = -5\frac{1}{3}$

16. $v = 2n - 6$

17. a. $t = 4n - 1$

b. 3, 7, 11, 15, 19, 23, 27, 31, 35, 39

c.
$$\begin{aligned} 7 - 3 &= 4 \\ 11 - 7 &= 4 \\ 15 - 11 &= 4 \end{aligned}$$
and so on ...

18. a.

Number of boards	10	11	12	13	14	15	16	17	18	19	20
Cost ($)	395	409.50	424	438.50	453	467.50	482	496.50	511	525.50	540

b.

Number of boards	10	11	12	13	14	15	16	17	18	19	20
Revenue ($)	329.50	362.45	395.40	428.35	461.30	494.25	527.20	560.15	593.10	626.05	659

c. Subtract the values in the first table from the values in the second table.

d.

Number of boards	10	11	12	13	14	15	16	17	18	19	20
Profit ($)	−65.50	−47.05	−28.60	−10.15	8.30	26.75	45.20	63.65	82.10	100.55	119

19. a. 9 marks

b. $m = 9h + 13$

c. i. 94%

ii. 62.5%

d. 4.25 hours

e. 13%

f. According to the equation, the average percentage mark for a student spending 10 hours on their homework would be 103%, so Freda has a valid argument.

20. a. 25 minutes per kilogram

b. $t = 25k + 15$

c.

Weight (g)	**Cooking time (minutes)**	**Weight (g)**	**Cooking time (minutes)**
500	27.5	2250	71.25
750	33.75	2500	77.5
1000	40	2750	83.75
1250	46.25	3000	90
1500	52.5	3250	96.25
1750	58.75	3500	102.5
2000	65	3750	108.75

d. 2 hours, 32.5 minutes

e. This equation will probably vary for different types of meat.

f. **i.** $t = 45k + 30$

ii. 4 hours, 37.5 minutes

CHAPTER 6

Straight-line graphs and their applications

6.1 Overview

A useful application of linear equations is to make predictions about what will happen in the future. For example, if a linear profit equation is modelled then this equation could be used to predict future profits. One way of modelling a linear equation is to plot the relevant data and then draw a linear line of best fit that is modelled. The graph that the data is plotted on is known as a scatter plot. This enables people to determine the relationship between the two variables.

It is interesting to note that a lot of world records follow a linear trend over time. One event that challenges this is the men's long jump world record. At the 1968 Summer Olympics Bob Beamon (pictured above) smashed the record by an amazing 55 cm with a jump of 8.90 m. This jump certainly went against the linear trend. This record stood until 1991 when Mike Powell jumped 8.95 m at the World Championships. If you plot the previous world records and draw a line of best fit, it clearly shows this went against the linear trend over the previous 60 or so years.

LEARNING PATHWAY

6.1 Overview
6.2 Constructing straight line graphs
6.3 Determine and interpret the slope and intercepts of straight line graphs
6.4 Modelling practical situations with straight line graphs
6.5 Review: exam practice

Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

6.2 Constructing straight line graphs

6.2.1 Linear functions

A function is a relationship between a set of inputs and outputs, such that each input is related to exactly one output. Each input and output of a function can be expressed as an ordered pair, with the first element of the pair being the input and the second element of the pair being the output.

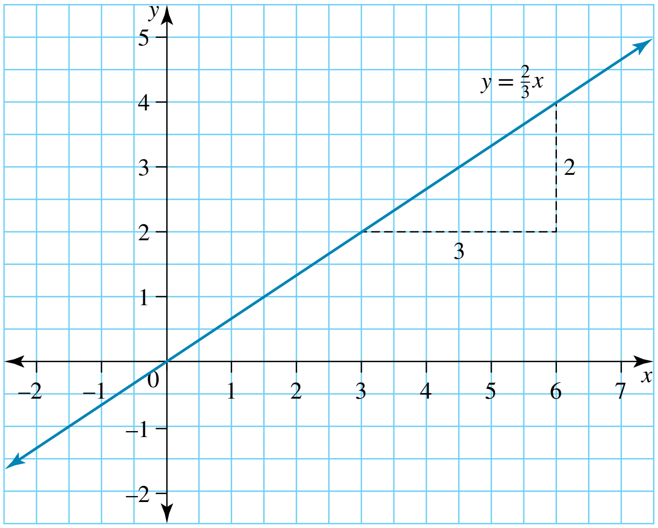
A function of x is denoted as $f(x)$. For example, if we have the function $f(x) = x + 3$, then each output will be exactly 3 greater than each input.

A linear function is a set of ordered pairs that form a straight line when graphed.

The gradient of a linear function

The **gradient** of a straight-line function, also known as the slope, determines the change in the y-value for each change in x-value. The gradient can be found by analysing the equation, by examining the graph or by finding the change in values if two points are given. The gradient is typically represented with the pronumeral m.

A positive gradient means that the y-value is increasing as the x-value increases, and a negative gradient means that the y-value is decreasing as the x-value increases.



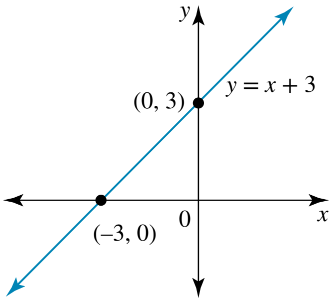
A gradient of $\frac{a}{b}$ means that for every increase of b in the x-value, there is an increase of a in the y-value. For example, a gradient of $\frac{2}{3}$ means that for every increase of 3 in the x-value, the y-value increases by 2.

x- and y-intercepts

The ***x*-intercept** of a linear function is the point where the graph of the equation crosses the x-axis. This occurs when $y = 0$.

The ***y*-intercept** of a linear function is the point where the graph of the equation crosses the y-axis. This occurs when $x = 0$.

In the graph of $y = x + 3$, we can see that the x-intercept is at $(-3, 0)$ and the y-intercept is at $(0, 3)$. These points can also be determined algebraically by putting $y = 0$ and $x = 0$ into the equation.



Gradient–intercept form

All linear equations relating the variables x and y can be rearranged into the form $y = mx + c$, where m is the gradient. This is known as the gradient–intercept form of the equation.

If a linear equation is in **gradient–intercept form**, the number and sign in front of the x-value gives the value of the gradient of the equation. For example, in $y = 4x + 5$, the gradient is 4.

The value of c in linear equations written in gradient–intercept form **is the y-intercept** of the equation. This is because the y-intercept occurs when $x = 0$, and when $x = 0$ the equation simplifies to $y = c$. The value of c in $y = 4x + 5$ is 5.

The gradient–intercept form can also be written as $y = a + bx$ where a is the y-intercept and b is the gradient. This form is commonly used when solving linear problems using technology.

WORKED EXAMPLE 1

State the gradients and y-intercepts of the following linear equations.

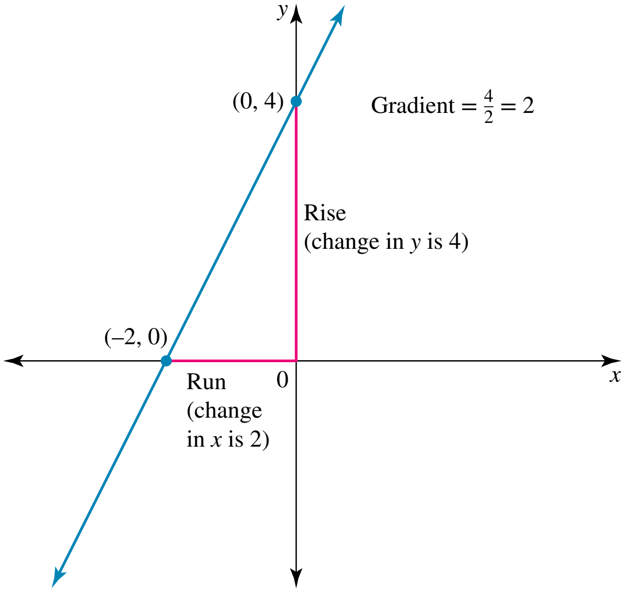
a. $y = 5x + 2$
b. $y = \frac{x}{2} - 3$
c. $y = -2x + 4$
d. $2y = 4x + 3$
e. $3y - 4x = 12$

THINK	WRITE
a. 1. Write the equation. It is in the form $y = mx + c$.	**a.** $y = 5x + 2$
2. Identify the coefficient of x.	The coefficient of x is 5.
3. Identify the value of c.	The value of c is 2.
4. Answer the question.	The gradient is 5 and the y-intercept is 2.
b. 1. Write the equation. It is in the form $y = mx + c$.	**b.** $y = \frac{x}{2} - 3$
2. Identify the coefficient of x.	x has been multiplied by $\frac{1}{2}$, so the coefficient is $\frac{1}{2}$.
3. Identify the value of c.	The value of c is -3.
4. Answer the question.	The gradient is $\frac{1}{2}$ and the y-intercept is -3.
c. 1. Write the equation. It is in the form $y = mx + c$.	**c.** $y = -2x + 4$
2. Identify the coefficient of x.	The coefficient of x is -2 (the coefficient includes the sign).
3. Identify the value of c.	The value of c is 4.
4. Answer the question.	The gradient is -2 and the y-intercept is 4.
d. 1. Write the equation. Rearrange the equation so that it is in the form $y = mx + c$.	**d.** $2y = 4x + 3$ $\frac{2y}{2} = \frac{4x}{2} + \frac{3}{2}$ $y = 2x + \frac{3}{2}$
2. Identify the coefficient of x.	The coefficient of x is 2.
3. Identify the value of c.	The value of c is $\frac{3}{2}$.
4. Answer the question.	The gradient is 2 and the y-intercept is $\frac{3}{2}$.

e. 1. Write the equation. Rearrange the equation so that it is in the form $y = mx + c$.	**e.** $3y - 4x = 12$ $3y - 4x + 4x = 12 + 4x$ $3y = 4x + 12$ $\frac{3y}{3} = \frac{4}{3}x + \frac{12}{3}$ $y = \frac{4}{3}x + 4$
2. Identify the coefficient of x.	The coefficient of x is $\frac{4}{3}$.
3. Identify the value of c.	The value of c is 4.
4. Answer the question.	The gradient is $\frac{4}{3}$ and the y-intercept is 4.

6.2.2 Determining the gradient from a graph

The value of the gradient can be found from a graph of a linear function. The gradient can be found by selecting two points on the line, then finding the change in the y-values and dividing by the change in the x-values.

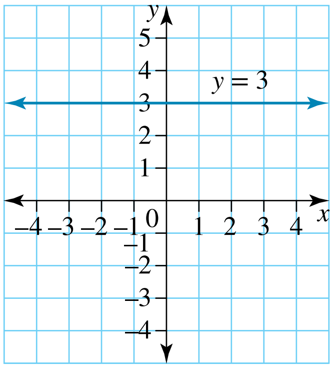


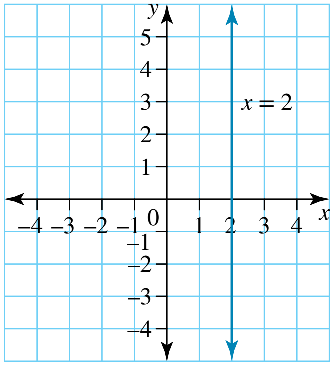
In other words, the general rule to find the value of a gradient that passes through the points (x_1, y_1) and (x_2, y_2) is:

$$\textbf{Gradient, } \boldsymbol{m} = \frac{\textbf{rise}}{\textbf{run}} = \frac{\boldsymbol{y_2 - y_1}}{\boldsymbol{x_2 - x_1}}$$

For all horizontal lines the y-values will be equal to each other, so the numerator of $\frac{y_2 - y_1}{x_2 - x_1}$ will be 0. Therefore, the gradient of horizontal lines is 0.

For all vertical lines the x-values will be equal to each other, so the denominator of $\frac{y_2 - y_1}{x_2 - x_1}$ will be 0. Dividing a value by 0 is undefined; therefore, the gradient of vertical lines is undefined.

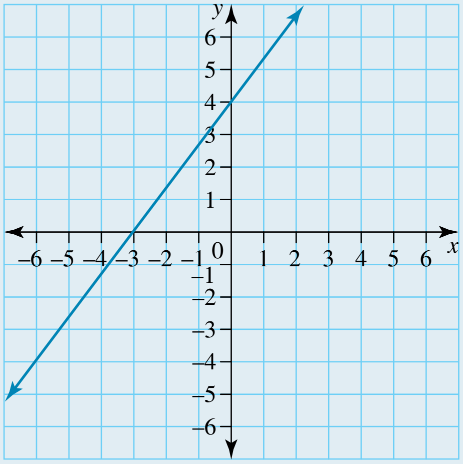




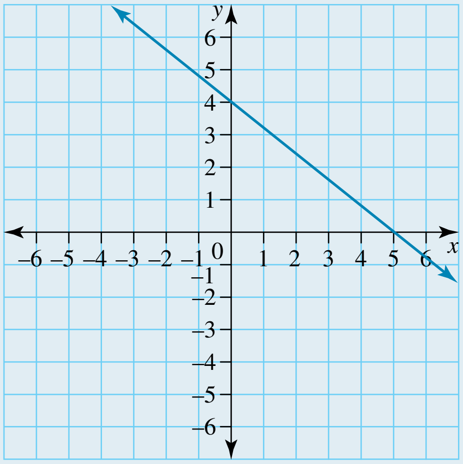
WORKED EXAMPLE 2

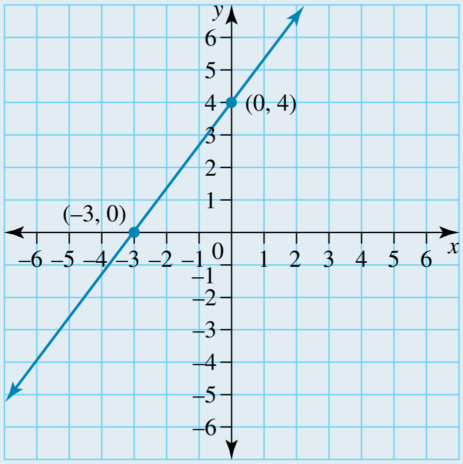
Determine the values of the gradients of the following graphs.

a.



b.



THINK	WRITE
a. 1. Find two points on the graph. (Select the x- and y-intercepts.)	**a.**  (−3, 0) and (0, 4)

2. Determine the rise in the graph (change in y-values).

$4 - 0 = 4$

3. Determine the run in the graph (change in x-values).

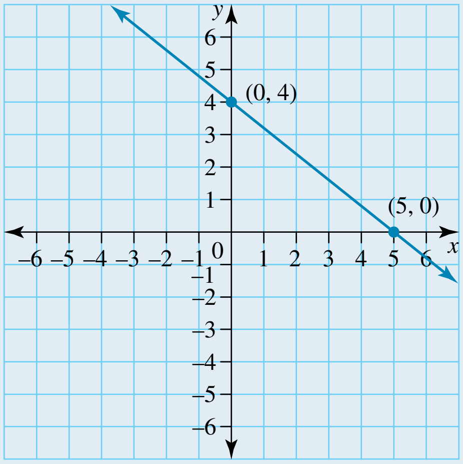
$0 - -3 = 3$

4. Substitute the values into the formula for the gradient.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{3}$$

b. 1. Find two points on the graph. (Select the x- and y-intercepts.)

b.



(0, 4) and (5, 0)

2. Determine the rise in the graph (change in y-values).

$0 - 4 = -4$

3. Determine the change in the x-values.

$5 - 0 = 5$

4. Substitute the values into the formula for the gradient.

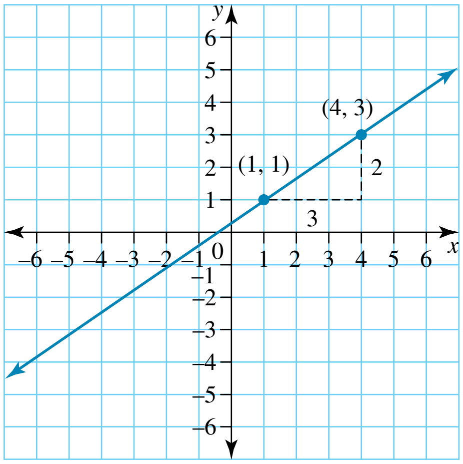
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= -\frac{4}{5}$$

6.2.3 Finding the gradient given two points

If a graph is not provided, we can still find the gradient if we are given two points that the line passes through. The same formula is used to find the gradient by finding the difference in the two y-coordinates and the difference in the two x-coordinates:

For example, the gradient of the line that passes through the points (1, 1) and (4, 3) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - 1} = \frac{2}{3}$$



WORKED EXAMPLE 3

Determine the value of the gradients of the linear graphs that pass through the following points.

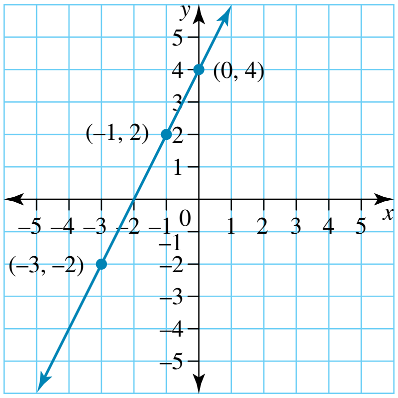
a. **(4, 6) and (5, 9)**
b. **(2, −1) and (0, 5)**
c. **(0.5, 1.5) and (−0.2, 1.8)**

THINK	WRITE
a. 1. Number the points.	**a.** Let $(4, 6) = (x_1, y_1)$ and $(5, 9) = (x_2, y_2)$.
2. Write the formula for the gradient and substitute the values.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9 - 6}{5 - 4}$ $= \frac{3}{1}$
3. Simplify the fraction and answer the question.	The gradient is 3 or $m = 3$.
b. 1. Number the points.	**b.** Let $(2, -1) = (x_1, y_1)$ and $(0, 5) = (x_2, y_2)$.
2. Write the formula for the gradient and substitute the values.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - -1}{0 - 2}$ $= \frac{6}{-2}$
3. Simplify the fraction and answer the question.	The gradient is −3 or $m = -3$.
c. 1. Number the points.	**c.** Let $(0.5, 1.5) = (x_1, y_1)$ and $(-0.2, 1.8) = (x_2, y_2)$.
2. Write the formula for the gradient and substitute the values.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1.8 - 1.5}{-0.2 - 0.5}$ $= \frac{0.3}{-0.7}$
3. Simplify the fraction and answer the question.	The gradient is $-\frac{3}{7}$ or $m = -\frac{3}{7}$.

Resources

Interactivity: Linear graphs (int-6484)

6.2.4 Plotting linear graphs



Linear graphs can be constructed by plotting the points and then ruling a line between the points as shown in the diagram.

If the points or a table of values are not given, then the points can be found by substituting x-values into the rule and finding the corresponding y-values. If a table of values is provided, then the graph can be constructed by plotting the points given and joining them.

WORKED EXAMPLE 4

Construct a linear graph that passes through the points $(-1, 2)$, $(0, 4)$, $(1, 6)$ and $(3, 10)$:

a. without technology

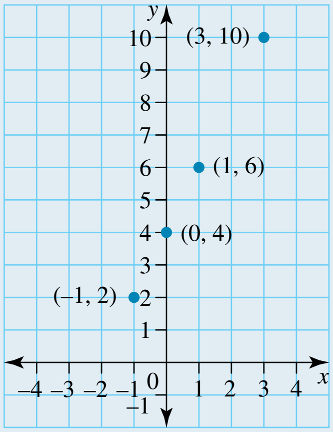
b. using technology.

THINK

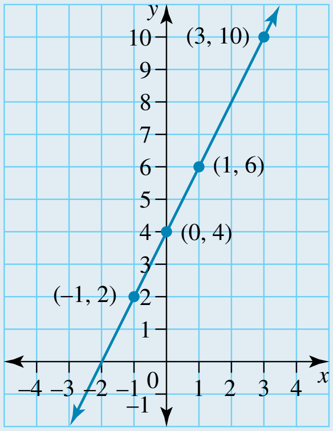
DRAW/DISPLAY

a. 1. Using grid paper, rule up the Cartesian plane (set of axes) and plot the points.

a.

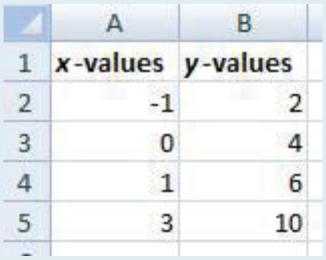


2. Using a ruler, rule a line through the points.



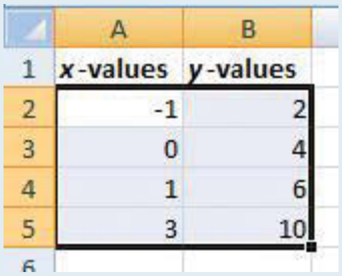
b. 1. Enter the points into your calculator or spreadsheet (the first number corresponds to the *x*-values and the second to the *y*-values).

b.



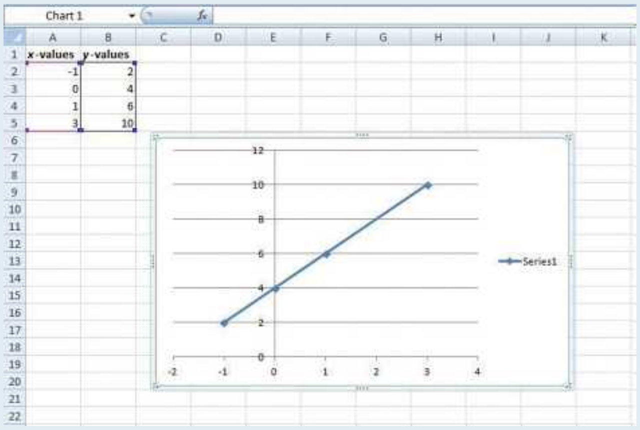
	A	B
1	*x*-values	*y*-values
2	-1	2
3	0	4
4	1	6
5	3	10

2. Highlight the cells.



	A	B
1	*x*-values	*y*-values
2	-1	2
3	0	4
4	1	6
5	3	10
6		

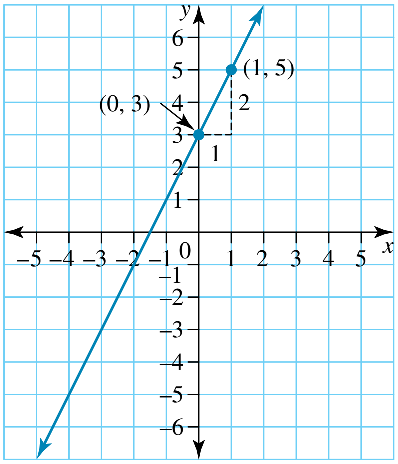
3. Use the scatterplot function of your calculator or spreadsheet to display the plot and the trend line.



	A	B
1	x-values	y-values
2	-1	2
3	0	4
4	1	6
5	3	10

Sketching graphs using the gradient and *y*-intercept method

A linear graph can be constructed by using the gradient and *y*-intercept. The *y*-intercept is marked on the *y*-axis, and then another point is found by using the gradient. For example, a gradient of 2 means that for an increase of 1 in the *x*-value, the *y*-value increases by 2. If the *y*-intercept is (0, 3), then add 1 to the *x*-value (0 + 1) and 2 to the *y*-value (3 + 2) to find another point that the line passes through, (1, 5).



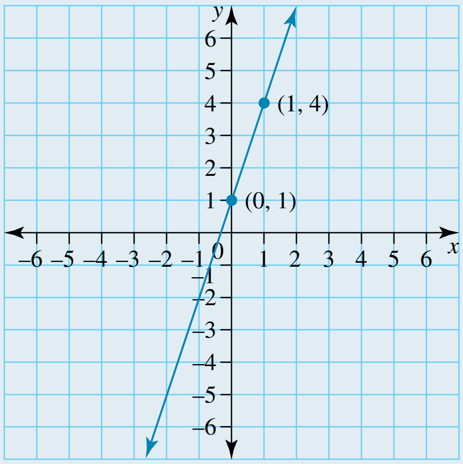
WORKED EXAMPLE 5

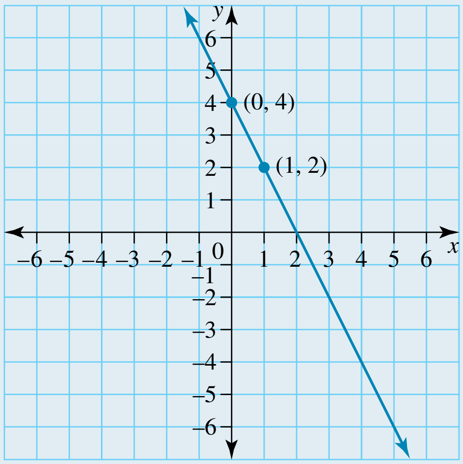
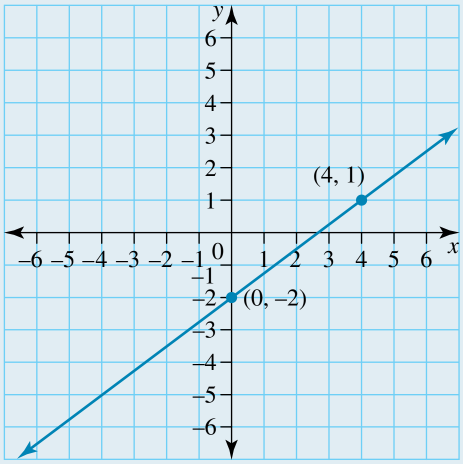
Using the gradient and the y-intercept, sketch the graph of each of the following.

a. **A linear graph with a gradient of 3 and a y-intercept of 1**

b. $y = -2x + 4$

c. $y = \frac{3}{4}x - 2$

THINK	WRITE
a. 1. Interpret the gradient.	**a.** A gradient of 3 means that for an increase of 1 in the x-value, there is an increase of 3 in the y-value.
2. Write the coordinates of the y-intercept.	y-intercept: (0, 1)
3. Find the x- and y-values of another point using the gradient.	New x-value $= 0 + 1 = 1$ New y-value $= 1 + 3 = 4$ Another point on the graph is (1, 4).
4. Construct a set of axes and plot the two points. Using a ruler, rule a line through the points.	
b. 1. Identify the value of the gradient and y-intercept.	**b.** $y = -2x + 4$ has a gradient of -2 and a y-intercept of 4.
2. Interpret the gradient.	A gradient of -2 means that for an increase of 1 in the x-value, there is a decrease of 2 in the y-value
3. Write the coordinates of the y-intercept.	y-intercept: (0, 4)
4. Find the x- and y-values of another point using the gradient.	New x-value $= 0 + 1 = 1$ New y-value $= 4 - 2 = 2$ Another point on the graph is (1, 2).

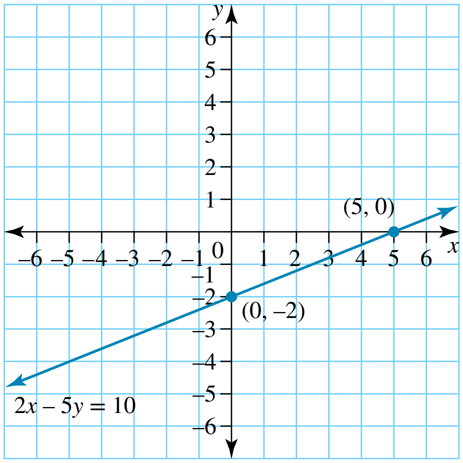
5. Construct a set of axes and plot the two points. Using a ruler, rule a line through the points.	
c. 1. Identify the value of the gradient and y-intercept.	**c.** $y = \frac{3}{4}x - 2$ has a gradient of $\frac{3}{4}$ and a y-intercept of -2.
2. Interpret the gradient.	A gradient of $\frac{3}{4}$ means that for an increase of 4 in the x-value, there is an increase of 3 in the y-value
3. Write the coordinates of the y-intercept.	y-intercept: $(0, -2)$
4. Find the x- and y-values of another point using the gradient.	New x-value $= 0 + 4 = 4$ New y-value $= -2 + 3 = 1$ Another point on the graph is $(4, 1)$.
5. Construct a set of axes and plot the two points. Using a ruler, rule a line through the points.	

Sketching graphs using the x- and y-intercepts

If the points of a linear graph where the line crosses the x- and y-axes (the x- and y-intercepts) are known, then the graph can be constructed by marking these points and ruling a line through them.

To find the x-intercept, substitute $y = 0$ into the equation and then solve the equation for x.

To find the y-intercept, substitute $x = 0$ into the equation and then solve the equation for y.



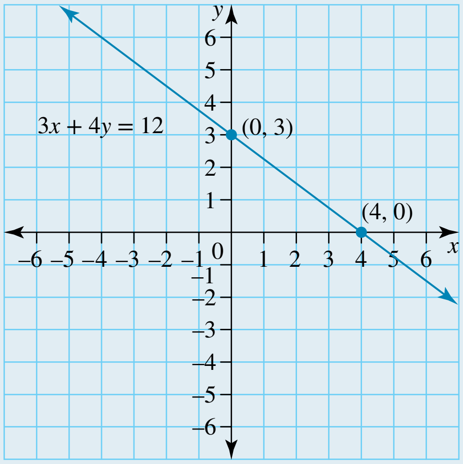
WORKED EXAMPLE 6

Calculate the value of the *x*- and *y*-intercepts for the following linear equations, and hence sketch their graphs.

a. $3x + 4y = 12$

b. $y = 5x$

c. $3y = 2x + 1$

THINK	WRITE
a. 1. To find the *x*-intercept, substitute $y = 0$ and solve for x.	**a.** x-intercept: $y = 0$ $3x + 4y = 12$ $3x + 4 \times 0 = 12$ $3x = 12$ $\frac{3x}{3} = \frac{12}{3}$ $x = 4$ x-intercept: (4, 0)
2. To find the *y*-intercept, substitute $x = 0$ into the equation and solve for y.	y-intercept: $x = 0$ $3x + 4y = 12$ $3 \times 0 + 4y = 12$ $\frac{4y}{4} = \frac{12}{4}$ $y = 3$ y-intercept: (0, 3)
3. Draw a set of axes and plot the *x*- and *y*-intercepts. Draw a line through the two points.	

b. 1. To find the x-intercept, substitute $y = 0$ into the equation and solve for x.

b. x-intercept: $y = 0$

$$y = 5x$$
$$0 = 5x$$
$$x = 0$$

x-intercept:$(0,\ 0)$

2. To find the y-intercept, substitute $x = 0$ into the equation and solve for y.

y-intercept: $x = 0$

$$\begin{aligned} y &= 5x \\ &= 5 \times 0 \\ &= 0 \end{aligned}$$

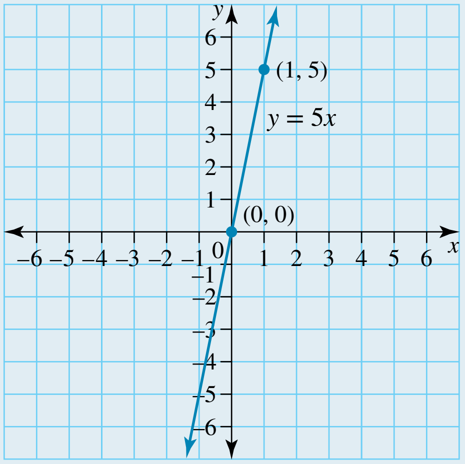
y-intercept: $(0,\ 0)$

3. As the x- and y-intercepts are the same, we need to find another point on the graph. Substitute $x = 1$ into the equation.

$$\begin{aligned} y &= 5x \\ &= 5 \times 1 \\ &= 5 \end{aligned}$$

Another point on the graph is $(1,\ 5)$.

4. Draw a set of axes. Plot the intercept and the second point. Draw a line through the intercepts.



c. 1. To find the x-intercept, substitute $y = 0$ into the equation.

c. x-intercept: $y = 0$

$$3y = 2x + 1$$
$$3 \times 0 = 2x + 1$$
$$0 = 2x + 1$$

2. Solve the equation for x.

$$0 - 1 = 2x + 1 - 1$$
$$-1 = 2x$$
$$\frac{-1}{2} = \frac{2x}{2}$$
$$x = \frac{-1}{2}$$

x-intercept: $\left(-\frac{1}{2},\ 0\right)$

3. To find the y-intercept, substitute $x = 0$ into the equation and solve for y.

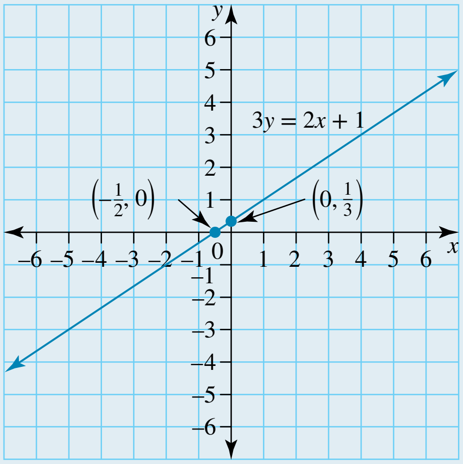
y-intercept: $x = 0$

$$3y = 2 \times 0 + 1$$
$$3y = 1$$
$$\frac{3y}{3} = \frac{1}{3}$$

$$y = \frac{1}{3}$$

$$y\text{-intercept: } \left(0, \frac{1}{3}\right)$$

4. Draw a set of axes and mark the *x*- and *y*-intercepts. Draw a line through the intercepts.



Resources

Interactivity: Equations of straight lines (int-6485)

studyon

Units 1 & 2 › Area 3 › Sequence 2 › Concepts 1 & 2

Linear functions Summary screen and practice questions
Plotting linear graphs Summary screen and practice questions

Exercise 6.2 Constructing straight line graphs

1. **WE1** State the gradients and *y*-intercepts of the following linear equations.

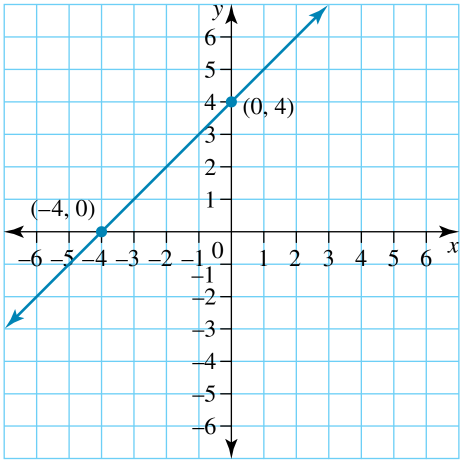
- **a.** $y = 2x + 1$
- **b.** $y = -x + 3$
- **c.** $y = \frac{1}{2}x + 4$
- **d.** $4y = 4x + 1$

2. Determine the gradients and *y*-intercepts of the following linear equations.

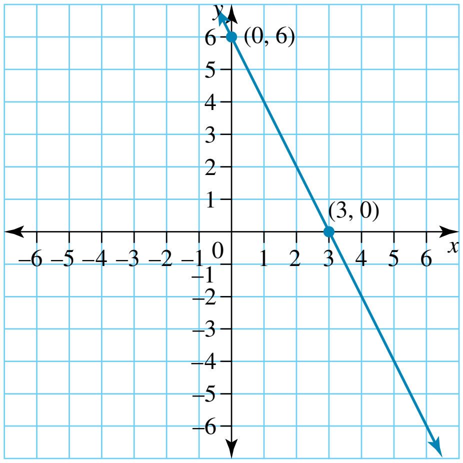
- **a.** $y = \frac{3x - 1}{5}$
- **b.** $y = 5\,(2x - 1)$
- **c.** $y = \frac{3 - x}{2}$
- **d.** $2y + 3x = 6$

3. **WE2** Determine the value of the gradient of each of the following graphs.

a.

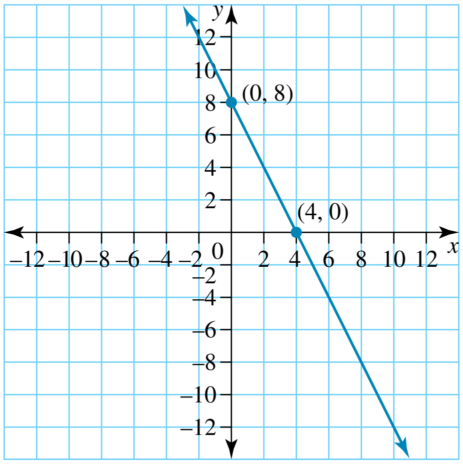


b.

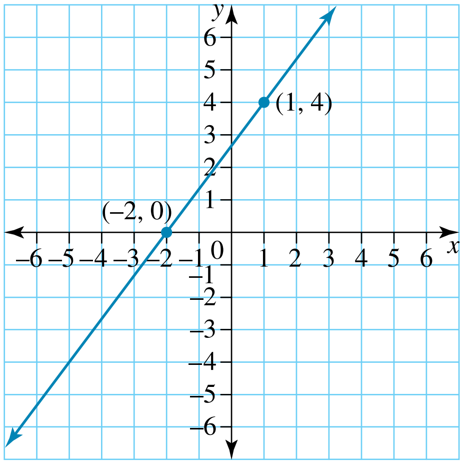


4. **MC** Which of the following graphs has a gradient of $-\frac{1}{4}$?

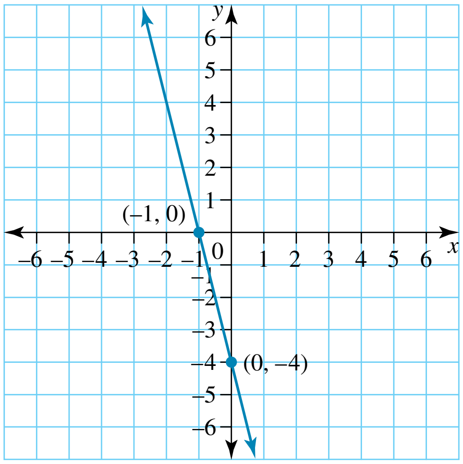
A.



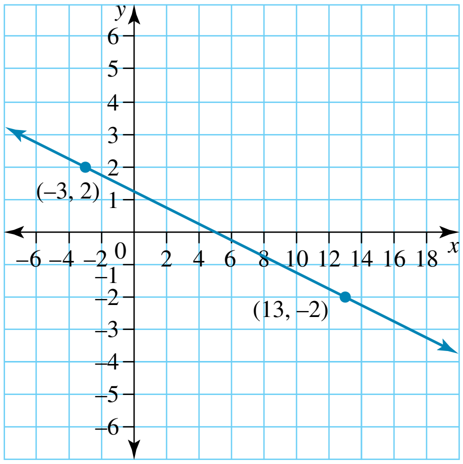
B.



C.



D.



5. Using the gradient, find another point in addition to the y-intercept that lies on each the following straight lines. Hence, sketch the graph of each straight line.

a. Gradient $= 4$, y-intercept $= 3$

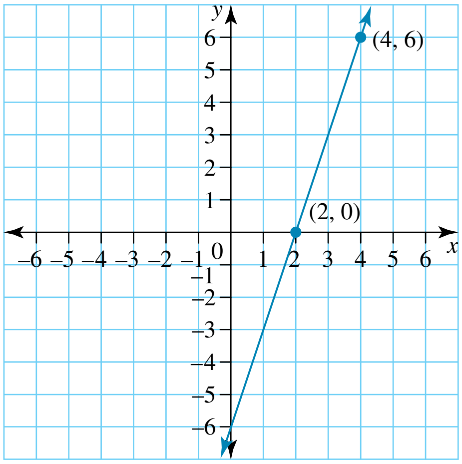
b. Gradient $= -3$, y-intercept $= 1$

c. Gradient $= \frac{1}{4}$, y-intercept $= 4$

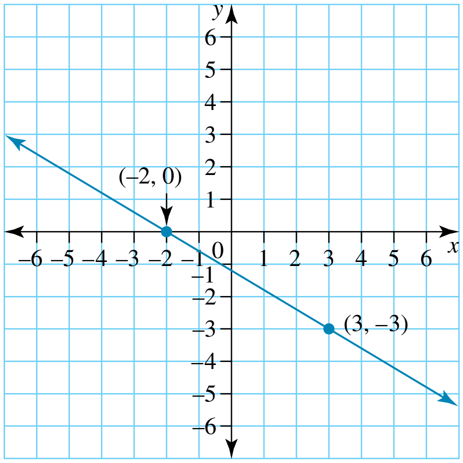
d. Gradient $= -\frac{2}{5}$, y-intercept $= -2$

6. Determine the value of the gradient and y-intercept of each of the following graphs.

a.



b.



7. **WE3** Determine the value of the gradients of the straight-line graphs that pass through the following points.

a. $(2, 3)$ and $(5, 12)$

b. $(-1, 3)$ and $(2, 7)$

c. $(-0.2, 0.7)$ and $(0.5, 0.9)$

d. $(-2, 0)$ and $(3, 0)$

8. A line has a gradient of -2 and passes through the points $(1, 4)$ and $(a, 8)$. Find the value of a.

9. Calculate the values of the gradients of the straight-line graphs that pass through the following points.

a. $(3, 6)$ and $(2, 9)$

b. $(-4, 5)$ and $(1, 8)$

c. $(-0.9, 0.5)$ and $(0.2, -0.7)$

d. $(1.4, 7.8)$ and $(3.2, 9.5)$

e. $\left(\frac{4}{5}, \frac{2}{5}\right)$ and $\left(\frac{1}{5}, -\frac{6}{5}\right)$

f. $\left(\frac{2}{3}, \frac{1}{4}\right)$ and $\left(\frac{3}{4}, -\frac{2}{3}\right)$

10. **WE4** Construct a straight-line graph that passes through the points $(2, 5)$, $(4, 9)$ and $(0, 1)$:

a. without technology

b. using a spreadsheet or otherwise.

11. A straight line passes through the following points: $(3, 7)$, $(0, a)$, $(2, 5)$ and $(-1, -1)$. Construct a graph and hence find the value of the unknown, a.

12. A straight line passes through the points $(2, 5)$, $(0, 9)$, $(-1, 11)$ and $(4, a)$. Construct a graph of the straight line and hence find the value of the unknown, a.

13. A line has a gradient of 5. If it passes through the points $(-2, b)$ and $(-1, 7)$, find the value of b.

14. **WE5** Using the gradient and the y-intercept, sketch the following linear graphs.

a. Gradient $= 2$, y-intercept $= 5$

b. Gradient $= -3$, y-intercept $= 0$

c. Gradient $= \frac{1}{2}$, y-intercept $= 3$

d. Gradient $= 2$, y-intercept $= 4$

15. Using an appropriate method, find the gradients of the lines that pass through the following points.

a. $(0, 5)$ and $(1, 8)$

b. $(0, 2)$ and $(1, -2)$

c. $(0, -3)$ and $(1, -5)$

d. $(0, -1)$ and $(2, -3)$

16. **WE6** Calculate the values of the x- and y-intercepts for the following linear equations, and hence sketch their graphs.

a. $2x + 5y = 20$

b. $4y = 3x + 5$

17. Calculate the values of the x- and y-intercepts for the following linear equations, and hence sketch their graphs.

a. $2x + y = 6$

b. $y = 3x + 9$

c. $2y = 3x + 4$

d. $3y - 4 = 5x$

18. Otis was asked to find the gradient of the line that passes through the points (3, 5) and (4, 2). His response was $\frac{5-2}{4-3} = 3$.

a. Explain the error in Otis's working out. Hence find the correct gradient.

b. What advice would you give Otis so that he can accurately find the gradients of straight lines given two points?

19. A straight-line graph passes through the points $(2, 0)$, $(0, a)$ and $(1, 3)$.

a. Explain why the value of a must be greater than 3.

b. **i.** Which two points can be used to determine the gradient of the line? Justify your answer by finding the value of the gradient.

ii. Using your answer from part **b**, find the value of a.

20. The table shows the value of x- and y-intercepts for the linear equations shown.

Equation	x-intercept	y-intercept
$y = 2x + 7$	$-\frac{7}{2}$	7
$y = 3x + 5$	$-\frac{5}{3}$	5
$y = 4x - 1$	$\frac{1}{4}$	−1
$y = 2x - 4$	$\frac{4}{2} = 2$	−4
$y = x + 2$	−2	2
$y = \frac{1}{2}x + 1$	$\frac{-1}{\frac{1}{2}} = -2$	1
$y = \frac{x}{3} + 2$	$\frac{-2}{\frac{1}{3}} = -6$	2

a. Explain how you can find the x- and y-intercepts for equations of the form shown. Does this method work for all linear equations?

b. Using your explanation from part **a**, write the x- and y-intercept for the equation $y = mx + c$.

c. A straight line has x-intercept $= -\frac{4}{5}$ and y-intercept $= 4$. Write its rule.

6.3 Determine and interpret the slope and intercepts of straight line graphs

6.3.1 Finding the equation of straight lines

Given the gradient and *y*-intercept

When we are given the gradient and y-intercept of a straight line, we can enter these values into the equation $y = mx + c$ to determine the equation of the straight line. Remember that m is equal to the value of the gradient and c is equal to the value of the y-intercept.

For example, if we are given a gradient of 3 and a y-intercept of 6, then the equation of the straight line would be $y = 3x + 6$.

Given the gradient and one point

When we are given the gradient and one point of a straight line, we need to establish the value of the y-intercept to find the equation of the straight line. This can be done by substituting the coordinates of the given point into the equation $y = mx + c$ and then solving for c. Remember that m is equal to the value of the gradient, so this can also be substituted into the equation.

Given two points

When we are given two points of a straight line, we can find the value of the gradient of a straight line between these points as discussed in section 6.2 (by using $m = \frac{y_2 - y_1}{x_2 - x_1}$). Once the gradient has been found, we can find the y-intercept by substituting one of the points into the equation $y = mx + c$ and then solving for c.

WORKED EXAMPLE 7

Determine the equations of the following straight lines.

a. A straight line with a gradient of 2 passing through the point (3, 7)

b. A straight line passing through the points (1, 6) and (3, 0)

c. A straight line passing through the points (2, 5) and (5, 5)

THINK	WRITE
a. 1. Write the gradient–intercept form of a straight line.	**a.** $y = mx + c$
2. Substitute the value of the gradient into the equation (in place of m).	Gradient $= m = 2$ $y = 2x + c$
3. Substitute the values of the given point into the equation and solve for c.	$(3, 7)$ $7 = 2(3) + c$ $7 = 6 + c$ $c = 1$
4. Substitute the value of c back into the equation and write the answer.	The equation of the straight line is $y = 2x + 1$.
b. 1. Write the formula to find the gradient given two points.	**b.** $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Let one of the given points be (x_1, y_1) and let the other point be (x_2, y_2).	Let $(1, 6) = (x_1, y_1)$. Let $(3, 0) = (x_2, y_2)$.

3. Substitute the values into the equation to determine the value of m.	$m = \frac{0-6}{3-1}$ $= \frac{-6}{2}$ $= -3$
4. Substitute the value of m into the equation $y = mx + c$.	$y = mx + c$ $y = -3x + c$
5. Substitute the values of one of the points into the equation and solve for c. *Note:* The point (1, 6) could also be used.	(3, 0) $0 = -3\,(3) + c$ $0 = -9 + c$ $c = 9$
6. Substitute the value of c back into the equation and write the answer.	The equation of the straight line is $y = -3x + 9$.
c. 1. Write the equation to find the gradient given two points.	**c.** $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Let one of the given points be (x_1, y_1) and let the other point be (x_2, y_2).	Let $(2, 5) = (x_1, y_1)$. Let $(5, 5) = (x_2, y_2)$.
3. Substitute the values into the equation to determine the value of m.	$m = \frac{5-5}{5-2}$ $= \frac{0}{3}$ $= 0$
4. A gradient of 0 indicates that the straight line is horizontal and the equation of the line is of the form $y = c$.	$y = c$
5. Substitute the values of one of the points into the equation and solve for c.	(2, 5) $5 = c$ $c = 5$
6. Substitute the value of c back into the equation and write the answer.	The equation of the straight line is $y = 5$.

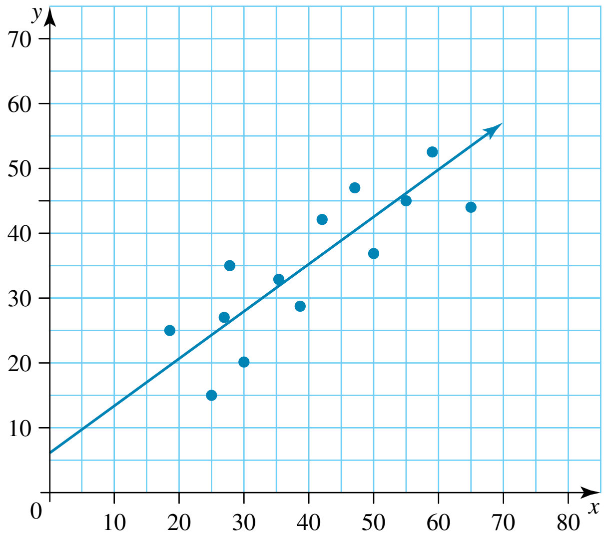
For part **b** of Worked example 7, try substituting the other point into the equation at step **5**. You will find that the calculated value of c is the same, giving you the same equation as an end result.

6.3.2 Lines of best fit by eye

Sometimes the data for a practical problem may not be in the form of a perfect linear relationship, but the data can still be modelled by an approximate linear relationship.

When we are given a scatterplot representing data that appears to be approximately represented by a linear relationship, we can draw a line of best fit by eye so that approximately half of the data points are on either side of the line of best fit.

After drawing a line of best fit, the equation of the line can be determined by picking two points on the line and determining the equation, as demonstrated in the previous section.



Creating a line of best fit when given only two points

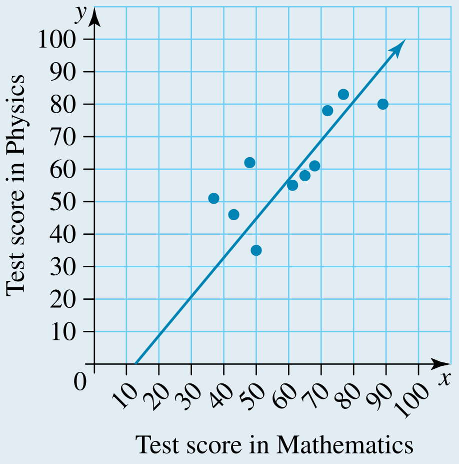
In some instances we may be given only two points of data in a data set. For example, we may know how far someone had travelled after 3 and 5 hours of their journey without being given other details about their journey. In these instances we can make a line of best fit using these two values to estimate other possible values that might fit into the data set.

Although this method can be useful, it is much less reliable than drawing a line of best fit by eye, as we do not know how typical these two points are of the data set. Also, when we draw a line of best fit through two points of data that are close together in value, we are much more likely to have an inaccurate line for the rest of the data set.

WORKED EXAMPLE 8

The following table and scatterplot represent the relationship between the test scores in Mathematics and Physics for ten Year 11 students. A line of best fit by eye has been drawn on the scatterplot.

Test score in Mathematics	65	43	72	77	50	37	68	89	61	48
Test score in Physics	58	46	78	83	35	51	61	80	55	62



Choose two appropriate points that lie on the line of best fit and determine the equation for the line.

THINK	WRITE
1. Look at the scattergraph and pick two points that lie on the line of best fit.	Two points that lie on the line of best fit are (40, 33) and (80, 81).
2. Calculate the value of the gradient between the two points.	Let $(40, 33) = (x_1, y_1)$. Let $(80, 81) = (x_2, y_2)$. $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{81 - 33}{80 - 40}$ $= 1.2$
3. Substitute the value of m into the equation $y = mx + c$.	$y = 1.2x + c$
4. Substitute the values of one of the points into the equation and solve for c.	(40, 33) $33 = 1.2 \times 40 + c$ $33 = 48 + c$ $33 - 48 = c$ $c = -15$
5. Substitute the value of c back into the equation and write the answer.	The line of best fit for the data is $y = 1.2x - 15$.

Note: If you use CAS, there are shortcuts you can take to find the equation of a straight line given two points. Refer to the CAS instructions available in the eBookPLUS.

6.3.3 Making predictions

Interpolation

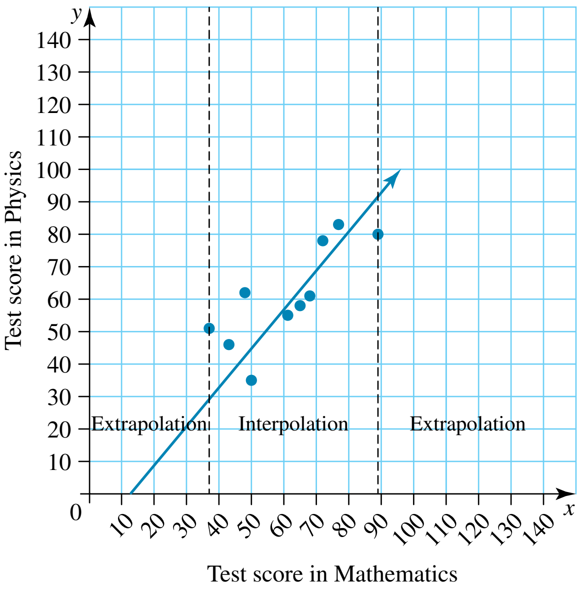
When we use interpolation, we are making a prediction from a line of best fit that appears within the parameters of the original data set.

If we plot our line of best fit on the scatterplot of the given data, then interpolation will occur between the first and last points of the scatterplot.

Extrapolation

When we use extrapolation, we are making a prediction from a line of best fit that appears outside the parameters of the original data set.

If we plot our line of best fit on the scatterplot of the given data, then extrapolation will occur before the first point or after the last point of the scatterplot.



Reliability of predictions

The more pieces of data there are in a set, the better the line of best fit you will be able to draw. More data points allow more reliable predictions.

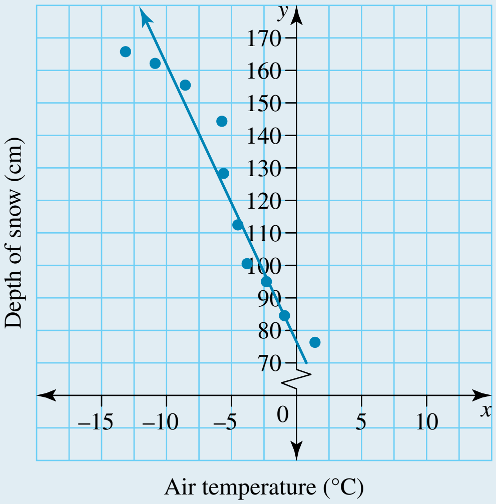
In general, interpolation is a far more reliable method of making predictions than extrapolation. However, there are other factors that should also be considered. Interpolation closer to the centre of the data set will be more reliable than interpolation closer to the edge of the data set. Extrapolation that appears closer to the data set will be much more reliable than extrapolation that appears further away from the data set.

A strong correlation between the points of data will give a more reliable line of best fit to be used. This is shown when all of the points appear close to the line of best fit. The more points there are that appear further away from the line of best fit, the less reliable other predictions will be.

When making predictions, always be careful to think about the data that you are making predictions about. Be sure to think about whether the prediction you are making is realistic or even possible!

WORKED EXAMPLE 9

The following data represent the air temperature (°C) and depth of snow (cm) at a popular ski resort.



Air temperature (°C)	−4.5	−2.3	−8.9	−11.0	−13.3	−6.2	−0.4	1.5	−3.7	−5.4
Depth of snow (cm)	111.3	95.8	155.6	162.3	166.0	144.7	84.0	77.2	100.5	129.3

The line of best fit for this data set has been calculated as $y = -7.2x + 84$.

a. Use the line of best fit to estimate the depth of snow if the air temperature is −6.5°C.

b. Use the line of best fit to estimate the depth of snow if the air temperature is 25.2°C.

c. Comment on the reliability of your estimations in parts a and b.

THINK	**WRITE**
a. 1. Enter the value of x into the equation for the line of best fit.	**a.** $x = -6.5$ $y = -7.2x + 84$ $= -7.2 \times -6.5 + 84$
2. Evaluate the value of x.	$= 130.8$
3. Write the answer.	The depth of snow if the air temperature is −6.5°C will be approximately 130.8 cm.

▶

b. 1. Enter the value of x into the equation for the line of best fit.	**b.** $x = 25.2$ $y = -7.2x + 84$ $= -7.2 \times 25.2 + 84$
2. Evaluate the value of x.	$= -97.4$ (1 d.p)
3. Write the answer.	The depth of snow if the air temperature is 25.2°C will be approximately -97.4 cm.
c. Relate the answers back to the original data to check their reliability.	**c.** The estimate in part **a** was made using interpolation, with the point being comfortably located within the parameters of the original data. The estimate appears to be consistent with the given data and as such is reliable. The estimate in part **b** was made using extrapolation, with the point being located well outside the parameters of the original data. This estimate is clearly unreliable, as we cannot have a negative depth of snow.

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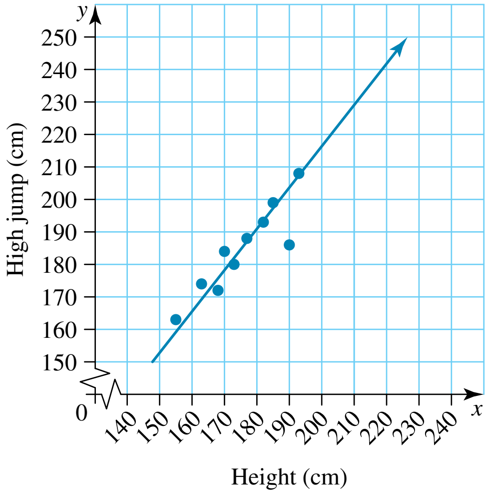
Units 1 & 2 > Area 3 > Sequence 2 > Concepts 3, 4 & 5

Finding the equation of straight lines Summary screen and practice questions
Lines of best fit Summary screen and practice questions
Interpolations and extrapolations Summary screen and practice questions

Exercise 6.3 Determine and interpret the slope and intercepts of straight line graphs

1. **WE7** Determine the equations of the following straight lines.
   - **a.** A straight line with a gradient of 5 passing through the point $(-2, -5)$
   - **b.** A straight line passing through the points $(-3, 4)$ and $(1, 6)$
   - **c.** A straight line passing through the points $(-3, 7)$ and $(0, 7)$
2. **MC** Which of the following equations represents the line that passes through the points $(3, 8)$ and $(12, 35)$?
   - **A.** $y = 3x + 1$
   - **B.** $y = -3x + 1$
   - **C.** $y = 3x - 1$
   - **D.** $y = \frac{1}{3}x + 1$
3. Steve is looking at data comparing the size of different music venues across the country and the average ticket price at these venues. After plotting his data in a scatterplot, he calculates a line of best fit for his data as $y = 0.04x + 15$, where y is the average ticket price in dollars and x is the capacity of the venue.
   - **a.** What does the value of the gradient (m) represent in Steve's equation?
   - **b.** What does the value of the y-intercept represent in Steve's equation?
   - **c.** Is the y-intercept a realistic value for this data?

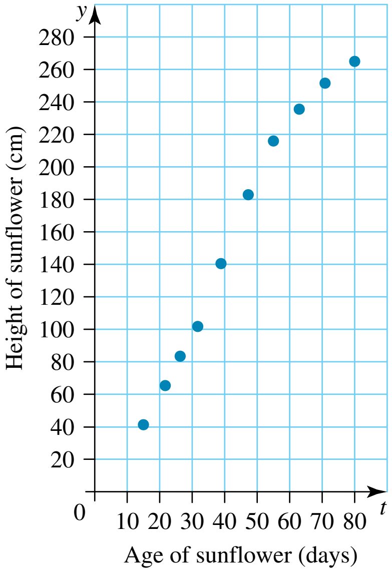
4. **WE8** A sports scientist is looking at data comparing the heights of athletes and their performance in the high jump. The following table and scatterplot represent the data they have collected. A line of best fit by eye has been drawn on the scatterplot. Choose two appropriate points that lie on the line of best fit and determine the equation for the line.



Height (cm)	168	173	155	182	170	193	177	185	163	190
High jump (cm)	172	180	163	193	184	208	188	199	174	186

5. Nidya is analysing the data from question 4, but a clerical error means that she only has access to two points of data: (170, 184) and (177, 188).
   a. Determine Nidya's equation for the line of best fit, rounding all decimal numbers to 2 places.
   b. Add Nidya's line of best fit to the scatterplot of the data.
   c. Comment on the similarities and differences between the two lines of best fit.
6. The following table and scatterplot shows the age and height of a field of sunflowers planted at different times throughout summer.

Age of sunflower (days)	63	71	15	33	80	22	55	47	26	39
Height of sunflower (cm)	237	253	41	101	264	65	218	182	82	140

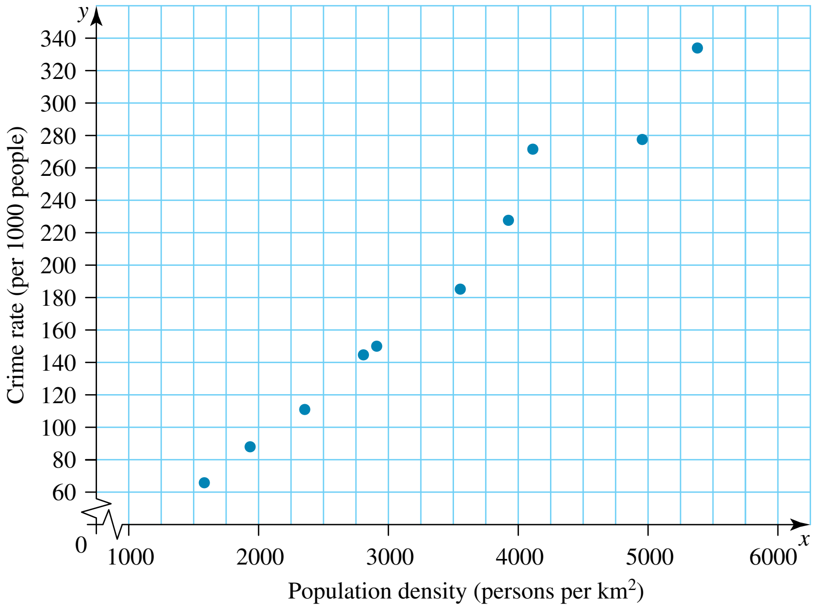


a. Xavier draws a line of best fit by eye that goes through the points (10, 16) and (70, 280). Draw his line of best fit on the scatterplot and comment on his choice of line.
b. Calculate the equation of the line of best fit using the two points that Xavier selected.
c. Patricia draws a line of best fit by eye that goes through the points (10, 18) and (70, 258). Draw her line of best fit on the scatterplot and comment on her choice of line.
d. Calculate the equation of the line of best fit using the two points that Patricia selected.
e. Why is the value of the y-intercept not 0 in either equation?

7. Olivia is analysing historical figures for the prices of silver and gold. The price of silver (per ounce) at any given time (x) is compared with the price of gold (per gram) at that time (y). She asks her assistant to note down the points she gives him and to create a line of best fit from the data.
On reviewing her assistant's notes, she has trouble reading his handwriting. The only complete pieces of information she can make out are one of the points of data (16, 41.5) and the gradient of the line of best fit (2.5).
a. Use the gradient and data point to determine an equation of the line of best fit.
b. Use the equation from part **a** to answer the following questions.
  i. What is the price of a gram of gold if the price of silver is \$25 per ounce?
  ii. What is the price of an ounce of silver if the price of gold is \$65 per gram?
  iii. What is the price of a gram of gold if the price of silver is \$11 per ounce?
  iv. What is the price of an ounce of silver if the price of gold is \$28 per gram?

8. A government department is analysing the population density and crime rate of different suburbs to see if there is a connection. The following table and scatterplot display the data that has been collected so far.
a. Draw a line of best fit on the scatterplot of the data.
b. Choose two points from the line of best fit and find the equation of the line.
c. What does the value of the x-intercept mean in terms of this problem?
d. Is the x-intercept value realistic? Explain your answer.

Population density (persons per km^2)	3525	2767	4931	3910	1572	2330	2894	4146	1968	5337
Crime rate (per 1000 people)	185	144	279	227	65	112	150	273	87	335



9. Kari is calculating the equation of a straight line passing through the points (−2, 5) and (3, 1). Her working is shown below.

$$\begin{aligned}\frac{y_2-y_1}{x_2-x_1} &= \frac{1-5}{-2-3}\\ &= \frac{-4}{-5}\\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}y &= \frac{4}{5}x+c\\ 1 &= \frac{4}{5}\times 3+c\\ c &= 1-\frac{12}{5}\\ &= \frac{-7}{5}\\ y &= \frac{4}{5}x-\frac{7}{5}\end{aligned}$$

a. Identify the error in Kari's working.

b. Calculate the correct equation of the straight line passing through these two points.

10. Horace is a marine biologist studying the lives of sea turtles. He collects the following data comparing the number of sea turtle egg nests and the number of survivors from those nests. The following table and scatterplot display the data he has collected.

Number of sea turtle nests	45	62	12	91	27	5	53	33	41	105
Number of surviving turtles	55	78	13	120	25	5	61	39	46	133

a. Horace draws a line of best fit for the data that goes through the points (0, −5) and (100, 127). Determine the equation for Horace's line of best fit.

b. What does the gradient of the line represent in terms of the problem?

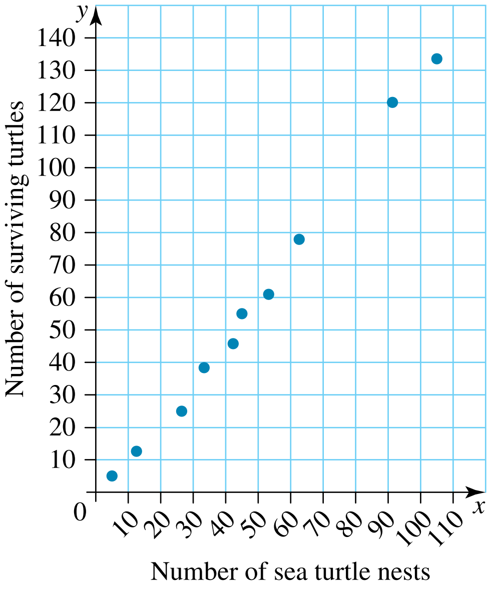
c. What does the y-intercept represent in terms of the problem? Is this value realistic?

d. Use the equation to answer the following questions.

  i. Estimate how many turtles you would expect to survive from 135 nests.

  ii. Estimate how many eggs would need to be laid to have 12 surviving turtles.

e. Comment on the reliability of your answers to part **d**.



11. A straight line passes through the points (−2, 2) and (−2, 6).

a. What is the gradient of the line?

b. Determine the equation of this line.

12. **WE9** An owner of an ice-cream parlour has collected data relating the outside temperature to ice-cream sales.

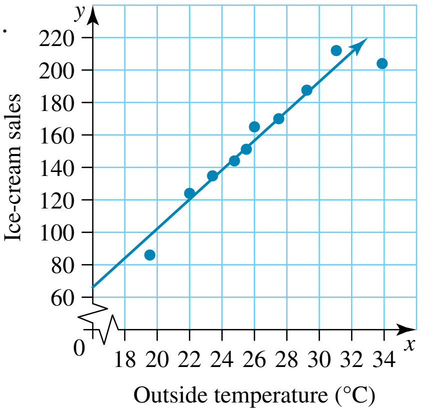
Outside temperature (°C)	23.4	27.5	26.0	31.1	33.8	22.0	19.7	24.6	25.5	29.3
Ice-cream sales	135	170	165	212	204	124	86	144	151	188

A line of best fit for this data has been calculated as $y = 9x - 77$.

a. Use the line of best fit to estimate ice-cream sales if the outside temperature is 27.9 °C.

b. Use the line of best fit to estimate ice-cream sales if the air temperature is 15.2 °C.

c. Comment on the reliability of your answers to parts **a** and **b**.



13. Georgio is comparing the cost and distance of various long-distance flights, and after drawing a scatterplot he creates an equation for a line of best fit to represent his data. Georgio's line of best fit is $y = 0.08x + 55$, where y is the cost of the flight and x is the distance of the flight in kilometres.

a. Estimate the cost of a flight between Melbourne and Sydney (713 km) using Georgio's equation.

b. Estimate the cost of a flight between Melbourne and Broome (3121 km) using Georgio's equation.

c. All of Georgio's data came from flights of distances between 400 km and 2000 km. Comment on the suitability of using Georgio's equation for shorter and longer flights than those he analysed. What other factors might affect the cost of these flights?

14. Mariana is a scientist and is collecting data measuring lung capacity (in L) and time taken to swim 25 metres (in seconds). Unfortunately a spillage in her lab causes all of her data to be erased apart from the records of a person with a lung capacity of 3.5 L completing the 25 metres in 55.8 seconds and a person with a lung capacity of 4.8 L completing the 25 metres in 33.3 seconds.

a. Use the remaining data to construct an equation for the line of best fit relating lung capacity (x) to the time taken to swim 25 metres (y). Give any numerical values correct to 2 decimal places.

b. What does the value of the gradient (m) represent in the equation?

c. Use the equation to estimate the time it takes people with the following lung capacities to swim 25 metres.

   i. 3.2 litres

   ii. 4.4 litres

   iii. 5.3 litres

d. Comment on the reliability of creating the equation from Mariana's two remaining data points.

15. Mitch is analysing data comparing the kicking efficiency (x) with the handball efficiency (y) of different AFL players. His data is shown in the first table.

a. A line of best fit for the data goes through the points (66, 79) and (84, 90). Determine the equation for the line of best fit for this data. Give any numerical values correct to 2 decimal places.

b. Use the equation from part **a** and the figures for kicking efficiency to create a table for the predicted handball efficiency of the same group of players.

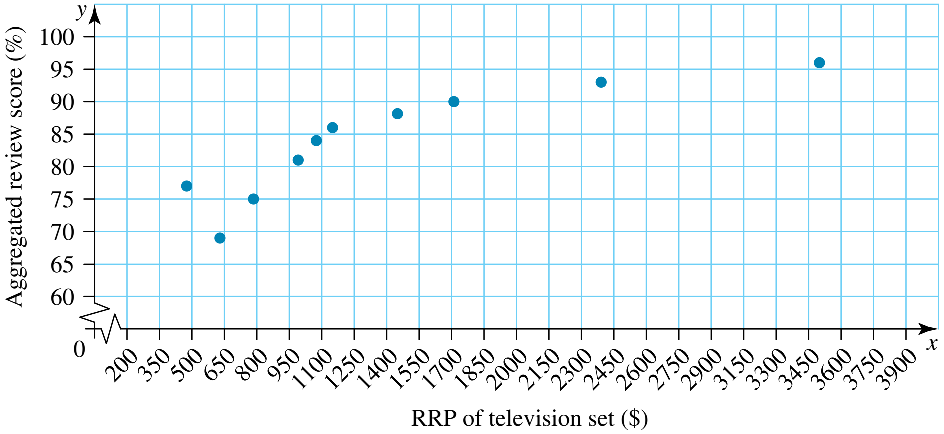
Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Handball efficiency (%)	84.6	79.8	88.5	85.2	87.1	86.7	78.0	81.3	82.4	90.3

Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Predicted handball efficiency (%)										

c. Comment on the differences between the predicted kicking efficiency and the actual kicking efficiency.

16. Chenille is comparing the price of new television sets versus their aggregated review scores (out of 100). The following table and scatterplot display the data she has collected.

RRP of television set ($)	799	1150	2399	480	640	999	1450	1710	3500	1075
Aggregated review score (%)	75	86	93	77	69	81	88	90	96	84



a. Using a spreadsheet or otherwise, calculate the equations of the straight lines that pass through each of the following pairs of points.

i. (400, 80) and (3400, 97)

ii. (300, 78) and (3200, 95)

iii. (400, 75) and (2400, 95)

iv. (430, 67) and (1850, 95)

b. Add these lines on the scatterplot of the data.

c. Which line do you think is the most appropriate line of best fit for the data? Give reasons for your answer.

17. Karyn is investigating whether the salary of the leading actors/actresses in movies has any impact on the box office receipts of the movie. The following table and scatterplot display the data that Karyn has collected.

Actor/actress salary ($m)	0.2	3.5	8.0	2.5	10.0	6.5	1.6	14.0	4.7	7.5
Box office receipts	135	72	259	36	383	154	25	330	98	232

a. Explain why a line of best fit for the data would never go through the point of data (0.2, 135).

b. Construct a line of best fit on the scatterplot.

c. Use two points from your line of best fit to determine the equation for this line.

d. Explain what the value of the gradient (m) means in the context of this problem.

e. Calculate the expected box office receipts for films where the leading actor/actress is paid the following amounts.

   i. $1.2 million
   ii. $11 million
   iii. $50 000
   iv. $20 million

f. Comment on the reliability of your answers to part **e**.

18. Giant sequoias are the world's largest trees, growing up to 100 or more metres in height and 10 or more metres in diameter. Throughout their lifetime they continue to grow in size, with the largest of them among the fastest growing organisms that we know of. Sheila is examining the estimated age and diameter of giant sequoias. The following table and scatterplot show the data she has collected.

Estimated age (years)	450	1120	330	1750	200	1400	630	800	980	2050
Diameter (cm)	345	485	305	560	240	525	390	430	465	590

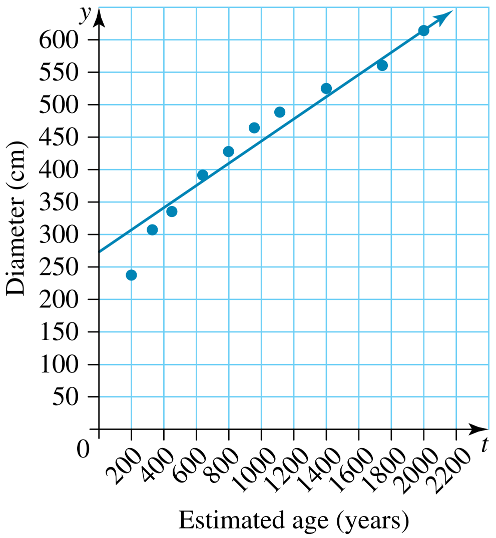
a. The line of best fit shown on the scatterplot passes through the points (0, 267) and (2000, 627). Determine the equation for the line of best fit.

b. Using a spreadsheet, CAS or otherwise, calculate the average (mean) age of the trees in Sheila's data set.

c. Using a spreadsheet, CAS or otherwise, calculate the average (mean) diameter of the trees in Sheila's data set.

d. Subtract the y-intercept from your equation in part **a** from the average diameter calculated in part **c**, and divide this total by the average age calculated in part **b**.

e. How does the answer in part **d** compare to the gradient of the equation calculated in part **a**?



6.4 Modelling practical situations with straight line graphs

6.4.1 Recognising linear models

Practical problems in which there is a constant change over time can be modelled by linear equations. The constant change, such as the rate at which water is leaking or the hourly rate charged by a tradesperson, can be represented by the gradient of the equation. Usually the y-value is the changing quantity and the x-value is time.

The starting point or initial point of the problem is represented by the y-intercept, when the x-value is 0. This represents the initial or starting value. In situations where there is a negative gradient the x-intercept represents when there is nothing left, such as the time taken for a leaking water tank to empty.

Identifying the constant change and the starting point can help to construct a linear equation to represent a practical problem. Once this equation has been established we can use it to calculate specific values or to make predictions as required.

WORKED EXAMPLE 10

Elle is an occupational therapist who charges an hourly rate of \$35 on top of an initial charge of \$50. Construct a linear equation to represent Elle's charge, C, for a period of t hours.

THINK	WRITE
1. Find the constant change and the starting point.	Constant change = 35 Starting point = 50
2. Construct the equation in terms of C by writing the value of the constant change as the coefficient of the pronumeral (t) that affects the change, and writing the starting point as the y-intercept.	$C = 35t + 50$

6.4.2 Solving practical problems

Once an equation is found to represent the practical problem, solutions to the problem can be found by sketching the graph and reading off important information such as the value of the x- and y-intercepts and the gradient. Knowing the equation can also help to find other values related to the problem.

Interpreting the parameters of linear models

When we have determined important values in practical problems, such as the value of the intercepts and gradient, it is important to be able to relate these back to the problem and to interpret their meaning.

For example, if we are given the equation $d = -60t + 300$ to represent the distance a car is in kilometres from a major city after t hours, the value of the gradient (-60) would represent the speed of the car in km/h (60 km/h), the y-intercept (300) would represent the distance the car is from the city at the start of the problem (300 km), and the x-intercept ($t = 5$) would represent the time it takes for the car to reach the city (5 hours).

Note that in the above example, the value of the gradient is negative because the car is heading towards the city, as opposed to away from the city, and we are measuring the distance the car is from the city.

WORKED EXAMPLE 11

A bike tyre has 500 cm^3 of air in it before being punctured by a nail. After the puncture, the air in the tyre is leaking at a rate of 5 cm^3/minute.

- **a.** **Construct an equation to represent the amount of air, A, in the tyre t minutes after the puncture occurred.**
- **b.** **Interpret what the value of the gradient in the equation means.**
- **c.** **Determine the amount of air in the tyre after 12 minutes.**
- **d.** **By solving your equation from part a, determine how long, in minutes, it will take before the tyre is completely flat (i.e. there is no air left).**

THINK	WRITE
a. 1. Find the constant change and the starting point.	**a.** Constant change = -5 Starting point = 500
2. Construct the equation in terms of A by writing the value of the constant change as the coefficient of the pronumeral that affects the change, and writing the starting point as the y-intercept.	$A = -5t + 500$
b. 1. Identify the value of the gradient in the equation.	**b.** $A = -5t + 500$ The value of the gradient is -5.
2. Identify what this value means in terms of the problem.	The value of the gradient represents the rate at which the air is leaking from the tyre. In this case it means that for every minute, the tyre loses 5 cm^3 of air.

c. 1. Using the equation found in part **a**, substitute $t = 12$ and evaluate.	**c.** $A = -5t + 500$ $= -5 \times 12 + 500$ $= 440$
2. Answer the question using words.	There are 440 cm^3 of air in the tyre after 12 minutes.
d. 1. When the tyre is completely flat, $A = 0$.	**d.** $0 = -5t + 500$
2. Solve the equation for t.	$0 - 500 = -5t + 500 - 500$ $-500 = -5t$ $\frac{-500}{-5} = \frac{-5t}{-5}$ $100 = t$
3. Answer the question using words.	After 100 minutes the tyre will be flat.

6.4.3 The domain of a linear model

When creating a linear model, it is important to interpret the given information to determine the domain of the model, that is, the values for which the model is applicable. The domain of a linear model relates to the values of the independent variable in the model (x in the equation $y = mx + c$). For example, in the previous example about air leaking from a tyre at a constant rate, the model will stop being valid after there is no air left in the tyre, so the domain only includes x-values for when there is air in the tyre.

Expressing the domain

The domains of linear models are usually expressed using the less than or equal to sign ($\leq$) and the greater than or equal to sign ($\geq$). If we are modelling a car that is travelling at a constant rate for 50 minutes before it arrives at its destination, the domain would be $0 \leq t \leq 50$, with t representing the time in minutes.

WORKED EXAMPLE 12

Express the following situations as linear models and give the domain of each of the models.

a. A truck drives across the country for 6 hours at a constant speed of 80 km/h before reaching its destination.

b. The temperature in an ice storage room starts at −20°C and falls at a constant rate of 0.8°C per minute for the next 22 minutes.

THINK	WRITE
a. 1. Use pronumerals to represent the information given in the question.	**a.** Let d = the distance travelled by the truck in km. Let t = the time of the journey in hours.
2. Represent the given information as a linear model.	$d = 80t$
3. Determine the domain for which this model is valid.	The model is valid from 0 to 6 hours.
4. Express the domain with the model in algebraic form.	$d = 80t,\ 0 \leq t \leq 6$

b. 1. Use pronumerals to represent the information given in the question.	**b.** Let i = the temperature of the ice room. Let t = the time in minutes.
2. Represent the given information as a linear model.	$i = -20 - 0.8t$
3. Determine the domain for which this model is valid.	The model is valid from 0 to 22 minutes.
4. Express the domain with the model in algebraic form.	$i = -20 - 0.8t,\ 0 \le t \le 22$

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Units 1 & 2 > Area 3 > Sequence 2 > Concept 6

Linear models Summary screen and practice questions

Exercise 6.4 Modelling practical situations with straight line graphs

1. **WE10** An electrician charges a call out fee of \$90 plus an hourly rate of \$65 per hour.
   Construct an equation that determines the electrician's charge, C, for a period of t hours.
2. An oil tanker is leaking oil at a rate of 250 litres per hour. Initially there was 125 000 litres of oil in the tanker. Construct an equation that represents the amount of oil, A, in litres in the oil tanker t hours after the oil started leaking.
3. **WE11** A children's swimming pool is being filled with water. The amount of water in the pool at any time can be found using the equation $A = 20t + 5$, where A is the amount of water in litres and t is the time in minutes.

   a. Explain why this equation can be represented by a straight line.
   b. State the value of the y-intercept and what it represents.
   c. Construct the graph of $A = 20t + 5$ on a set of axes.
   d. The pool holds 500 litres. By solving an equation, determine how long it will take to fill the swimming pool. Write your answer correct to the nearest minute.

4. A yoga ball is being pumped full of air at a rate of 40 cm^3/second. Initially there is 100 cm^3 of air in the ball.
   - a. Construct an equation that represents the amount of air, A, in the ball after t seconds.
   - b. Interpret what the value of the y-intercept in the equation means.
   - c. How much air, in cm^3, is in the ball after 2 minutes?
   - d. When fully inflated the ball holds 100 000 cm^3 of air. Determine how long, in minutes, it takes to fully inflate the ball. Write your answer to the nearest minute.

5. Kirsten is a long-distance runner who can run at a rate of 12 km/h. The distance, d, in km she travels from the starting point of a race can be represented by the equation $d = at - 0.5$.
   - a. Write the value of a.
   - b. Write the y-intercept. In the context of this problem, explain what this value means.
   - c. How far is Kirsten from the starting point after 30 minutes?
   - d. The finish point of this race is 21 km from the starting point. Determine how long, in hours and minutes, it takes Kirsten to run the 21 km. Give your answer correct to the nearest minute.

6. Petrol is being pumped into an empty tank at a rate of 15 litres per minute.
   - a. Construct an equation to represent the amount of petrol in litres, P, in the tank after t minutes.
   - b. What does the value of the gradient in the equation represent?
   - c. If the tank holds 75 litres of petrol, determine the time taken, in minutes, to fill the tank.
   - d. The tank had 15 litres of petrol in it before being filled. Write another equation to represent the amount of petrol, P, in the tank after t minutes.
   - e. State the domain of the equation formulated in part **d**.

7. Gert rides to and from work on his bike. The distance and time taken for him to ride home can be modelled using the equation $d = 37 - 22t$, where d is the distance from home in km and t is the time in hours.
   - a. Determine the distance, in km, between Gert's work and home.
   - b. Explain why the gradient of the line in the graph of the equation is negative.
   - c. By solving an equation determine the time, in hours and minutes, taken for Gert to ride home. Write your answer correct to the nearest minute.
   - d. State the domain of the equation.
   - e. Sketch the graph of the equation.

8. A large fish tank is being filled with water. After 1 minute the height of the water is 2 cm and after 4 minutes the height of the water is 6 cm. The height of the water in cm, h, after t minutes can be modelled by a linear equation.
   a. Determine the gradient of the graph of this equation.
   b. In the context of this problem, what does the gradient represent?
   c. Using the gradient found in part **a**, determine the value of the y-intercept. Round your answer to 2 decimal places.
   d. Was the fish tank empty of water before being filled? Justify your answer using calculations.
9. Fred deposits $40 in his bank account each week. At the start of the year he had $120 in his account. The amount in dollars, A, that Fred has in his account after t weeks can be found using the equation $A = at + b$.
   a. State the values of a and b.
   b. In the context of this problem, what does the y-intercept represent?
   c. How many weeks will it take Fred to save $3000?

10. Michaela is a real estate agent. She receives a commission of 1.5% on house sales, plus a payment of $800 each month. Michaela's monthly wage can be modelled by the equation $W = ax + b$, where W represents Michaela's total monthly wage and x represents her house sales in dollars.
    a. State the values of a and b.
    b. Is there an upper limit to the domain of the model? Explain your answer.
    c. In March Michaela's total house sales were $452 000. Determine her monthly wage for March.
    d. In September Michaela earned $10 582.10. Determine the amount of house sales she made in September.
11. An electrician charges a call-out fee of $175 on top of an hourly rate of $60.
    a. Construct an equation to represent the electrician's fee in terms of his total charge, C, and hourly rate, h.
    b. Claire is a customer and is charged $385 to install a hot water system. Determine how many hours she was charged for.
    The electrician changes his fee structure. The new fee structure is summarised in the following table.

Time	Call-out fee	Quarter-hourly rate
Up to two hours	$100	$20
2–4 hours	$110	$25
Over 4 hours	$115	$50

    c. Using the new fee structure, how much, in dollars, would Claire be charged for the same job?
    d. Construct an equation that models the new fee structure for between 2 and 4 hours.

12. **WE12** Express the following situations as linear models and give the domains of the models.
   a. Julie works at a department store and is paid \$19.20 per hour. She has to work for a minimum of 10 hours per week, but due to her study commitments she can work for no more than 20 hours per week.
   b. The results in a driving test are marked out of 100, with 4 marks taken off for every error made on the course. The lowest possible result is 40 marks.

13. Monique is setting up a new business selling T-shirts through an online auction site. Her supplier in China agrees to a deal whereby they will supply each T-shirt for \$3.50 providing she buys a minimum of 100 T-shirts. The deal is valid for up to 1000 T-shirts.
   a. Set up a linear model (including the domain) to represent this situation.
   b. Explain what the domain represents in this model.
   c. Why is there an upper limit to the domain?

14. The Dunn family departs from home for a caravan trip. They travel at a rate of 80 km/h. The distance they travel from home, in km, can be modelled by a linear equation.

   a. Write the value of the gradient of the graph of the linear equation.
   b. Write the value of the y-intercept. Explain what this value means in the context of this problem.
   c. Using your values from parts **a** and **b**, write an equation to represent the distance the Dunn family are from home at any given point in time.
   d. How long, in hours, have the family travelled when they are 175 km from home? Give your answer to the nearest minute.
   e. The Dunns travel for 2.5 hours before stopping. Determine the distance they are from home.

15. The table shows the amount of money in Kim's savings account at different dates. Kim withdraws the same amount of money every five days.

Date	26/11	1/12	6/12
Amount	\$1250	\$1150	\$1050

   a. The amount of money at any time, t days, in Kim's account can be modelled by a linear equation. Explain why.
   b. Using a calculator, spreadsheet or otherwise, construct a straight line graph to represent the amount of money Kim has in her account from 26 November.
   c. Determine the gradient of the line of the graph, and explain the meaning of the gradient in the context of this problem.
   d. Determine the linear equation that models this situation.
   e. In the context of this problem, explain the meaning of the x-intercept.
   f. Is there a limit to the domain of this problem? If so, do we know the limit?
   g. Kim will need at least \$800 to go on a beach holiday over the Christmas break (starting on 21 December). Show that Kim will not have enough money for her holiday.

16. There are two advertising packages for Get2Msg.com. Package A charges per cm^2 and package B charges per letter. The costs for both packages increase at a constant rate. The table shows the costs for package A for areas from 4 to 10 cm^2.

B'N'B BEDDING

UP TO 50%

SALE

Open 9–5 pm every day
Cnr High Road and Main Street
CityTown
Ph: 5556789
www.bnbbedd.com.au

Area, in cm^2	**Cost (excluding administration charge of $25)**
4	30
6	45
8	60
10	75

a. Determine the cost per cm^2 for package A. Write your answer correct to 2 decimal places.

b. Construct an equation that determines the cost per cm^2.

c. Using your equation from part **b**, determine the cost for an advertisement of 7.5 cm^2.

d. Package B costs 58 cents per standard letter plus an administration cost of $55. Construct an equation to represent the costs for package B.

e. Betty and Boris of B'n'B Bedding want to place this advertisement on Get2Msg.com. Which package would be the better option for them? Justify your answer by finding the costs they would pay for both packages.

17. A basic mobile phone plan designed for school students charges a flat fee of $15 plus 13 cents per minute of a call. Text messaging is free.

a. Construct an equation that determines the cost, in dollars, for any time spent on the phone, in minutes.

b. In the context of this problem what do the gradient and y-intercept of the graph of the equation represent?

c. Using a spreadsheet or technology of your choice, complete the following table to determine the cost at any time, in minutes.

Time (min)	**Cost ($)**	**Time (min)**	**Cost ($)**
5		35	
10		40	
15		45	
20		50	
25		55	
30		60	

d. Bill received a monthly phone bill for $66.50. Determine the number of minutes he spent using his mobile phone. Give your answer to the nearest minute.

18. Carly determines that the number of minutes she spends studying for History tests affects her performances on these tests. She finds that if she does not study, then her test performance in History is 15%. She records the number of minutes she spends studying and her test scores, with her results shown in the following table.

Time in minutes studying, t	15	55	35
Test scores, y (%)	25	a	38

Carly decides to construct an equation that determines her test scores based on the time she spends studying. She uses a linear equation because she finds that there is a constant increase between her number of minutes of study and her test results.

a. Determine the value of a.
b. Using a spreadsheet or otherwise, represent Carly's results from the table on a graph.
c. Explain why the y-intercept is 15.
d. Construct an equation that determines Carly's test score, in %, given her studying time in minutes, t. State the domain of the equation.
e. Carly scored 65% on her final test. Using your equation from part d, determine the number of minutes she spent studying. Give your answer correct to the nearest second.

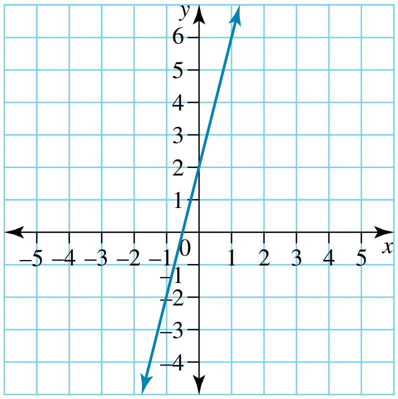
6.5 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

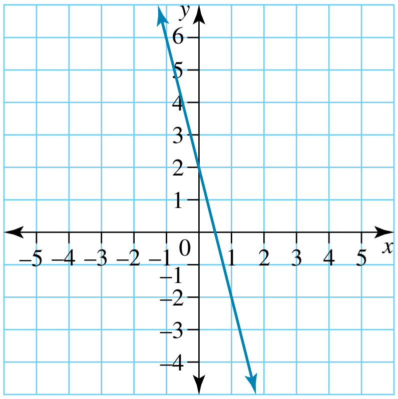
Simple familiar

1. **MC** The gradient of the line passing through the points (4, 6) and (−2, −6) is:
   A. −2 **B.** −0.5 **C.** 0 **D.** 2
2. **MC** The x- and y-intercepts of the linear graph with equation $3x - y = 6$ are:
   A. (2, 6) **B.** (0, 2) and (−6, 0) **C.** (0, 2) and (6, 0) **D.** (2, 0) and (0, −6)
3. **MC** Which of the following is a sketch of the graph with equation $y = 4x + 2$?

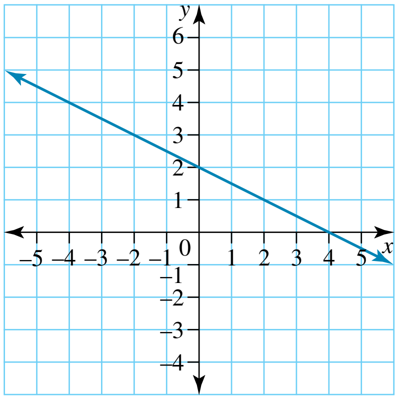
A.



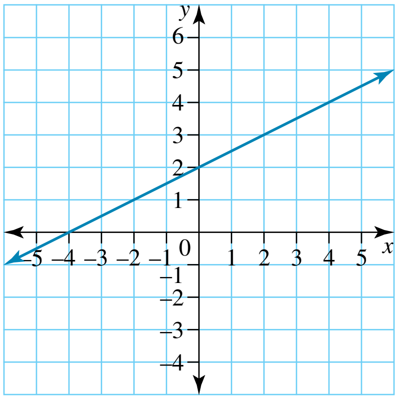
B.



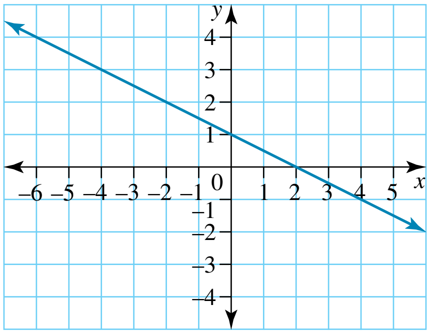
C.



D.



4. **MC** The gradient of the graph shown in the following diagram is:



A. -2 **B.** -1 **C.** $-\frac{1}{2}$ **D.** 1

5. **MC** Bertha knits teddy bears and sells them at the local farmers' market. Bertha spends $120 in wool, and it costs her an additional $4.50 to make each teddy bear. She sells the bears for $14.50 each. How many teddy bears does Bertha need to sell to cover her costs?

A. 8 **B.** 9 **C.** 10 **D.** 12

6. **MC** The line that passes through the point (2, –1) is:

A. $y = -2x + 5$ **B.** $y = 2x - 1$ **C.** $y = -2x + 1$ **D.** $x + y = 1$

The following information relates to questions **7** and **8**.

An inflated party balloon has a small hole and is slowly deflating. The initial volume of the balloon is 1000 cm^3 and the balloon loses 5 cm^3 of air every minute.

7. **MC** If V represents the volume of the balloon in cm^3 and t represents the time in minutes, an equation to represent the volume of the balloon after t minutes is:

A. $V = \dfrac{1000}{5t}$ **B.** $V = \dfrac{1000–5t}{60}$

C. $V = 1000 + 5t$ **D.** $V = 1000–5t$

8. **MC** The time taken for the balloon to have lost 650 cm^3 of air is:

A. 70 minutes **B.** 130 minutes

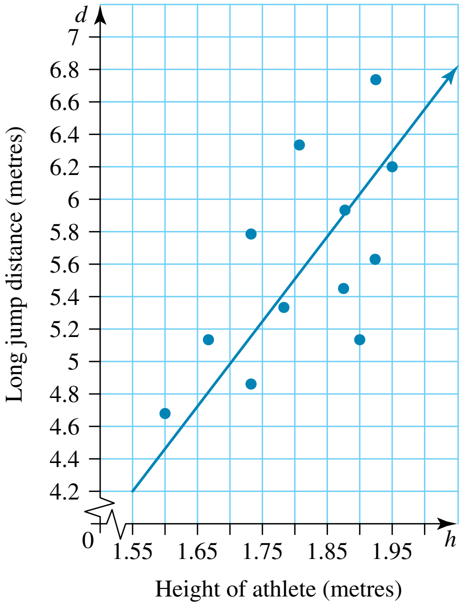
C. 270 minutes **D.** 330 minutes

9. **MC** The equation of a straight line passing through the points (–2, 3) and (5, 1) is:

A. $y = -\frac{2}{7}x + 2\frac{3}{7}$ **B.** $y = 2\frac{3}{7}x - \frac{2}{7}$

C. $y = \frac{2}{7}x + 2\frac{3}{7}$ **D.** $y = \frac{2}{7}x - 2\frac{3}{7}$

10. **MC** The following scatterplot represents the relationship between the height of an athlete and the distance that they can long jump in metres.



Using the line of best fit, the estimated distance that a 1.8-metre-tall athlete could long jump would be:

A. 5 m **B.** 6.02 m **C.** 5.46 m **D.** 5.52 m

11. Sketch the following graphs by finding the x- and y-intercepts. Hence, state the gradient of each graph.

a. $2x + y = 5$ **b.** $y - 4x = 8$ **c.** $4(x + 3y) = 16$ **d.** $3x + 4y - 10 = 0$

12. Find the gradients of the lines passing through the following pairs of points.

a. $(3, -2)$ and $(0, 4)$ **b.** $(5, 11)$ and $(-2, 18)$

c. $(0.3, 4.1)$ and $(1.2, 5.3)$ **d.** $\left(\frac{2}{5}, \frac{1}{4}\right)$ and $\left(-\frac{1}{4}, \frac{3}{5}\right)$

Complex familiar

13. A line has a gradient of $-\frac{3}{4}$. If the line passes through the points $(-a, 3)$ and $(-2, 6)$, find the value of a.

14. Complete the following table.

	Equation	Gradient	y-intercept	x-intercept
a	$y = 5x - 3$	5		
b	$y = 3x + 1$			
c	$3y = 6x - 9$			
d	$2y + 4x = 8$			
e			5	−5
f		2		2

15. Miriam has a sweet tooth, and her favourite sweets are strawberry twists and chocolate ripples. The local sweet shop sells both as part of their pick and mix selection, so Miriam fills a bag with them. Each strawberry twist weighs 5 g and each chocolate ripple weighs 9 g. In Miriam's bag there are 28 sweets, weighing a total of 188 g. Determine the number of each type of sweet that Miriam bought by forming and solving a linear equation.

16. Tommy is saving for a remote-controlled car that is priced at \$49. He has \$20 in his piggy bank. Tommy saves \$3 of his pocket money every week and puts it in his piggy bank. The amount of money in dollars, M, in his piggy bank after w weeks can be found using the rule $M = 3w + 20$.

- **a.** Generate a table of values that shows the amount of money, M, in Tommy's piggy bank every week for the 12 weeks (i.e. $w = 0, 1, 2, 3, \dots, 12$).
- **b.** Using your table, how many weeks will it take for Tommy to have saved enough money to purchase the remote-controlled car?

Complex unfamiliar

17. The recommended maximum heart rate during exercise is given by the equation

$$H = 0.85\,(220 - A)$$

where H is the person's heart rate in beats per minute and A is their age in years.

- **a.** Explain why the maximum heart rate is given by a linear equation.
- **b.** Determine the recommended maximum heart rate for a 25-year-old person. Write your answer correct to the nearest whole number.
- **c.** Determine the gradient and y-intercept of the linear equation.
- **d.** Using your answers from part **c**, sketch the graph that shows the recommended maximum heart rate for persons aged 20 to 70 years.
- **e.** Charlie is working out at the recommended maximum heart rate. His measured heart rate is 162 beats per minute. By solving a linear equation, find Charlie's age.
- **f.** In the context of this problem, explain why finding the x-intercept would be meaningless.

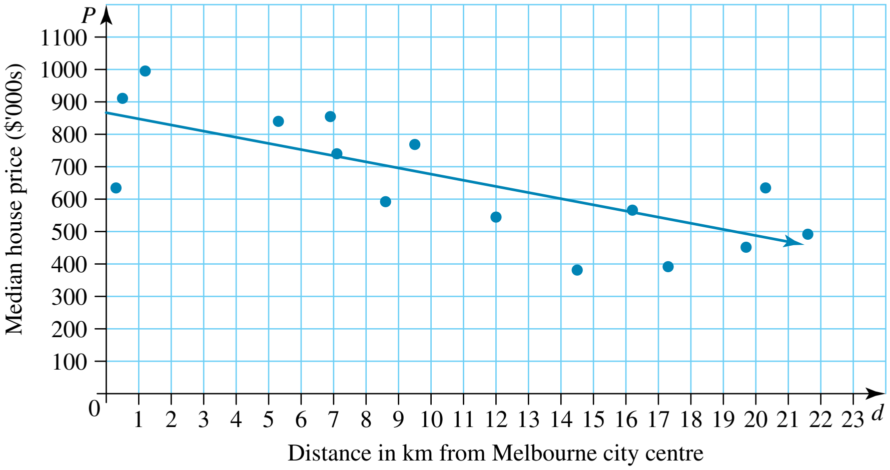
18. A currency converter is showing one Australian dollar would buy 50.48 Indian rupees. Ben is a cricket fan and plans to fly to India for a test match.

- **a.** Find a relationship between Australian dollars and Indian rupees.
- **b.** How much would Ben have if he changed \$1200 to Indian rupees?
- **c.** Tickets for a match range from 350 to 2500 rupees. Find this range in Australian dollars.
- **d.** Is this an example of direct variation? Explain.
- **e.** Sketch the linear model relating Australian dollars and Indian rupees.

19. Trudy is unaware that there is a small hole in the petrol tank of her car. Petrol is leaking out of the tank at a constant rate of 5 mL/ min. Trudy has parked her car in a long-term carpark at the airport and gone on a holiday. Initially there is 45 litres of petrol in the tank.

- **a.** How many litres of petrol leak out of the tank each hour? What is the assumption that is being made about the rate of petrol leaking each hour?
- **b.** How many litres of petrol are lost after four hours?
- **c.** After how many hours will there be 39.75 litres of petrol in the tank?
- **d.** An equation is used to represent the amount of petrol left in the tank, l, after t hours.
  - **i.** Explain why the amount of petrol in the tank would be best modelled by a linear equation.
  - **ii.** Explain why the linear equation will have a negative gradient
  - **iii.** Write an equation to determine the amount of petrol left in the tank, l, after t hours.
- **e.** Using a spreadsheet or other technology of your choice, sketch the graph for the equation found in part **d iii**. Clearly label the x- and y-intercepts.
- **f.** Determine how many hours it will take for the petrol tank to become empty.

20. The median house prices $(\$'000)$ from fifteen western suburbs and the distance in kilometres from the centre of Melbourne were collected. The graph shows these results.



To determine an equation that could be used to determine the median house price, p, at a distance, d km west from the city centre, the following points were used: $(3,\ 808)$ and $(20,\ 468)$.

a. Using these two points, determine the equation of the line of best fit relating the median house price, p, to the distance, d, from the city centre. Write your answers correct to the nearest whole number.

b. In the context of this problem, explain the meaning of the gradient and y-intercept.

c. Using your equation from part **a**, determine the median house price in dollars for the suburbs at the following distances from the city centre.

i. 15.5 km **ii.** 4.8 km **iii.** 18.7 km

d. Xena and Hertz purchase a house for $650 800 in the western suburbs. Using your equation from part **a**, determine how far in kilometres their house is likely to be located from the city centre. Round your answer to 1 decimal place.

e. Determine the value of the x-intercept and explain the meaning of this in the context of the problem.

f. Freda and Benny are planning on purchasing a holiday house in the Grampians located 229 km west of the city centre. Explain why the equation would not be reliable in determining their expected house price.

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Answers

Chapter 6 Straight-line graphs and their applications

Exercise 6.2 Constructing straight line graphs

1. a. Gradient = 2, y-intercept = 1

b. Gradient = −1, y-intercept = 3

c. Gradient = $\frac{1}{2}$, y-intercept = 4

d. Gradient = 1, y-intercept = $\frac{1}{4}$

2. a. Gradient = $\frac{3}{5}$, y-intercept = $-\frac{1}{5}$

b. Gradient = 10, y-intercept = −5

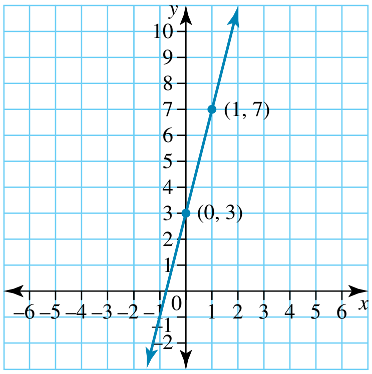
c. Gradient = $-\frac{1}{2}$, y-intercept = $\frac{3}{2}$

d. Gradient = $-\frac{3}{2}$, y-intercept = 3

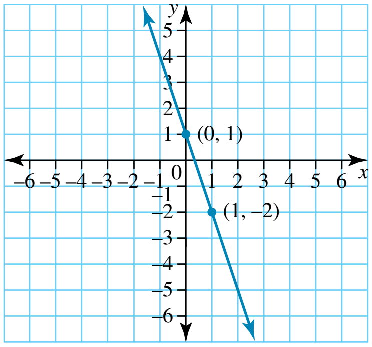
3. a. 1 **b.** −2

4. D

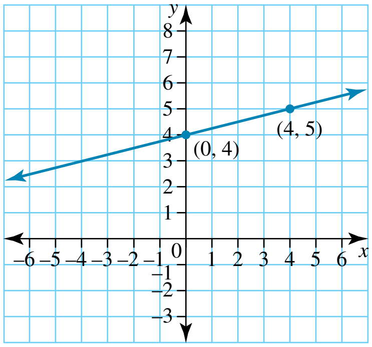
5. a. (1, 7)



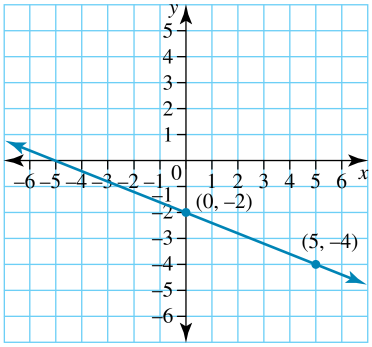
b. (1, −2)



c. (4, 5)



d. (5, −4)



6. a. Gradient = 3, y-intercept = −6

b. Gradient = $-\frac{3}{5}$, y-intercept = $-\frac{6}{5}$

7. a. 3 **b.** $\frac{4}{3}$

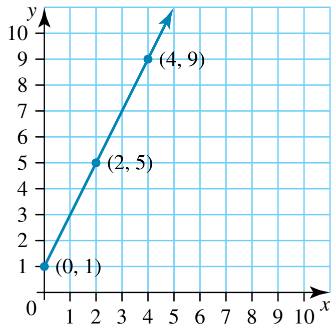
c. $\frac{2}{7}$ **d.** 0

8. −1

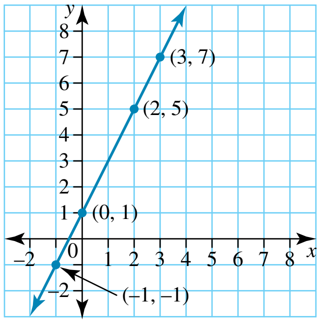
9. a. −3 **b.** $\frac{3}{5}$ **c.** $-\frac{12}{11}$

d. $\frac{17}{18}$ **e.** $\frac{8}{3}$ **f.** −11

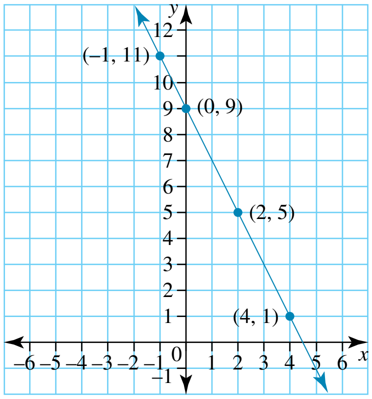
10. a, b



11. $a = 1$



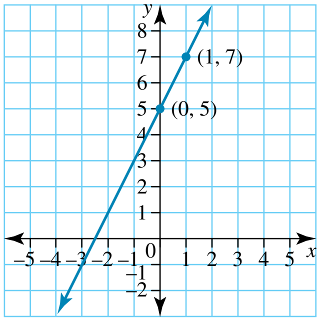
12.



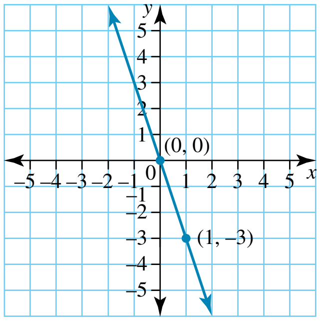
$a = 1$

13. $b = 2$

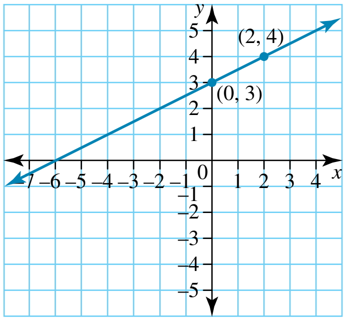
14. a. (1, 7)



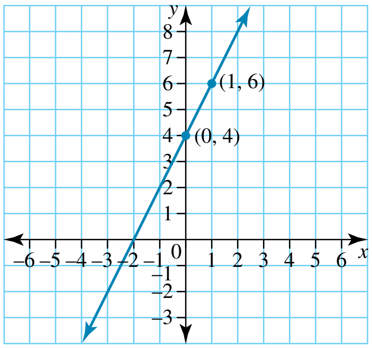
b. (1, −3)



c. (2, 4)

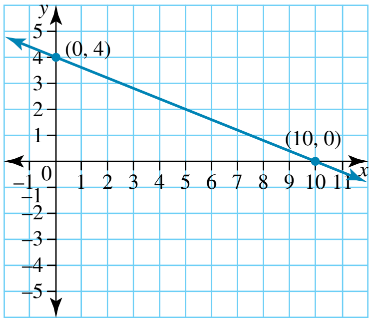


d. (0, 4) and (1, 6)

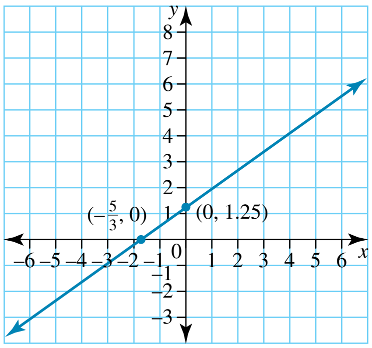


15. a. 3 **b.** −4 **c.** −2 **d.** −1

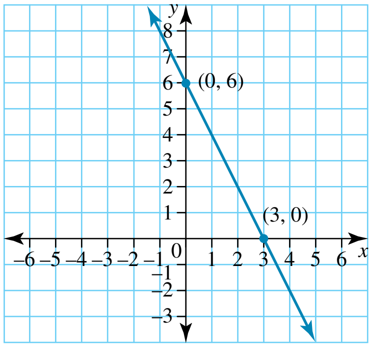
16. a. (10, 0) and (0, 4)



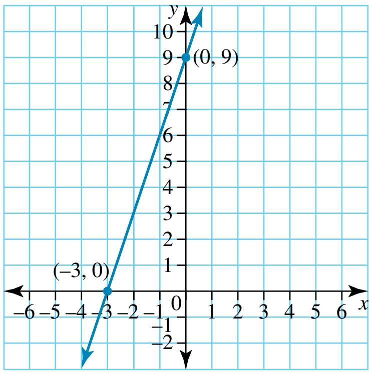
b. $\left(-\frac{5}{3}, 0\right)$ and $\left(0, \frac{5}{4}\right)$



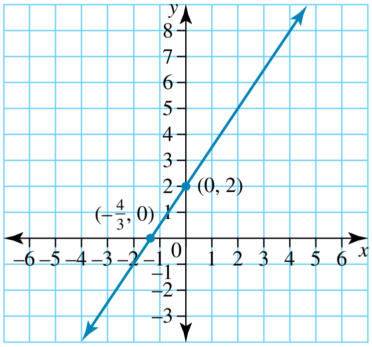
17. a. (3, 0) and (0, 6)



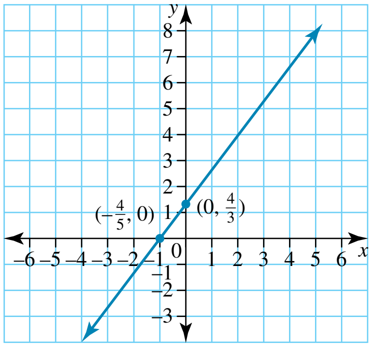
b. (−3, 0) and (0, 9)



c. $\left(-\frac{4}{3}, 0\right)$ and (0, 2)



d. $\left(-\frac{4}{5}, 0\right)$ and $\left(0, \frac{4}{3}\right)$



18. a. Otis swapped the x- and y-values, calculating $\frac{y_2 - y_1}{x_1 - x_2}$.

The correct gradient is −3.

b. Label each x and y pair before substituting them into the formula.

19. a. The points (1, 3) and (2, 0) tell us that the graph has a negative gradient, so the y-intercept must have a greater value than 3.

b. **i.** (2, 0) and (1, 3); gradient = −3 **ii.** $a = 6$

20. a. The y-intercept is the number separate from the x (the constant), and the x-intercept is equal to $\frac{-y\text{-intercept}}{\text{gradient}}$.

This method only works when the equation is in the form $y = mx + c$.

b. y-intercept = c, x-intercept = $\frac{-c}{m}$

c. $y = 5x + 4$

Exercise 6.3 Determine and interpret the slope and interprets of straight line graphs

1. a. $y = 5x + 5$

b. $y = 0.5x + 5.5$

c. $y = 7$

2. C

3. a. The increase in price for every additional person the venue holds

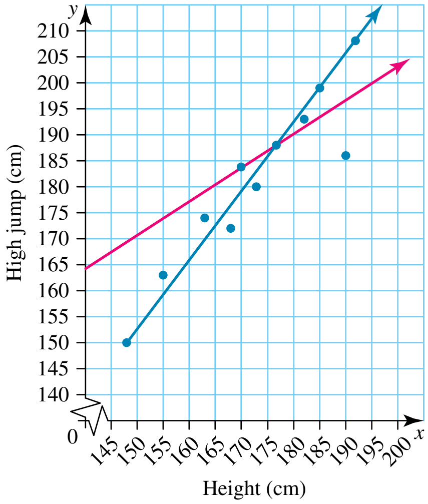
b. The price of a ticket if a venue has no capacity

c. No, as the smallest venues would still have some capacity

4. $y = 1.25x - 33.75$

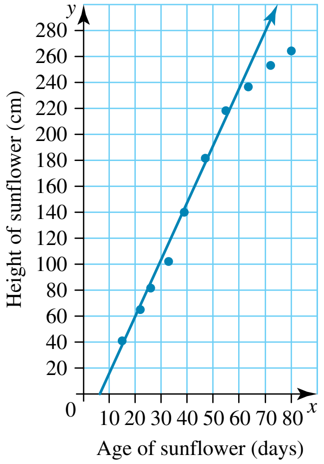
5. a. $y = 0.57x + 86.86$

b.



c. Nidya's line of best fit is not a good representation of the data. In this instance having only two points of data to create the line of best fit was not sufficient.

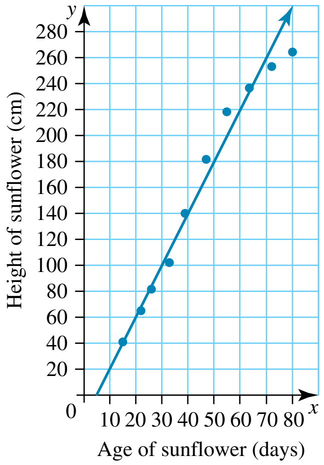
6. a.



Xavier's line is closer to the values above the line than those below it, and there are more values below the line than above it, so this is not a great line of best fit.

b. $y = 4.4x - 28$

c.



Patricia's line is more appropriate as the data points lie on either side of the line and the total distance of the points from the line appears to be minimal.

d. $y = 4x - 22$

e. The line of best fit does not approximate the height for values that appear outside the parameters of the data set, and the y-intercept lies well outside these parameters.

7. a. $y = 2.5x + 1.5$

b. **i.** $64.00 **ii.** $25.40 **iii.** $29.00 **iv.** $10.60

8. a. Lines of best fit will vary but should split the data points on either side of the line and minimise the total distance from the points to the line.

b. Answers will vary.

c. The amount of crime in a suburb with 0 people

d. No; if there are 0 people in a suburb there should be no crime.

9. a. Kari did not assign the x- and y-values for each point before calculating the gradient, and she mixed up the values.

b. $y = -\frac{4}{5}x + \frac{17}{5}$

10. a. $y = 1.32x - 5$

b. The number of surviving turtles from each nest

c. The y-intercept represents the number of surviving turtles from 0 nests. This value is not realistic as you cannot have a negative amount of turtles.

d. **i.** 173 **ii.** 13

e. The answer to **di** was made using extrapolation, so it is not as reliable as the answer to part **dii**, which was made using interpolation. However, due to the nature of the data in question, we would expect this relationship to continue and for both answers to be quite reliable.

11. a. The gradient is undefined. **b.** $x = -2$

12. a. 174 ice-creams **b.** 60 ice-creams

c. The estimate in part **a** is reliable as it was made using interpolation, it is located within the parameters of the original data set, and it appears consistent with the given data.

The estimate in part **b** is unreliable as it was made using extrapolation and is located well outside the parameters of the original data set.

13. a. $112

b. $305

c. All estimates outside the parameters of Georgio's original data set (400 km to 2000 km) will be unreliable, with estimates further away from the data set being more unreliable than those closer to the data set.

Other factors that might affect the cost of flights include air taxes, fluctuating exchange rates and the choice of airlines for various flight paths.

14. a. $y = -17.31x + 116.38$

b. For each increase of 1 L of lung capacity, the swimmer will take less time to swim 25 metres.

c. **i.** 61.0 seconds **ii.** 40.2 seconds

iii. 24.6 seconds

d. As Mariana has only two data points and we have no idea of how typical these are of the data set, the equation for the line of best fit and the estimates established from it are all very unreliable.

15. a. $y = 0.61x + 38.67$

b. *

c. The predicted and actual kicking efficiencies are very similar in values. A couple of the results are identical, and only a couple of the results are significantly different.

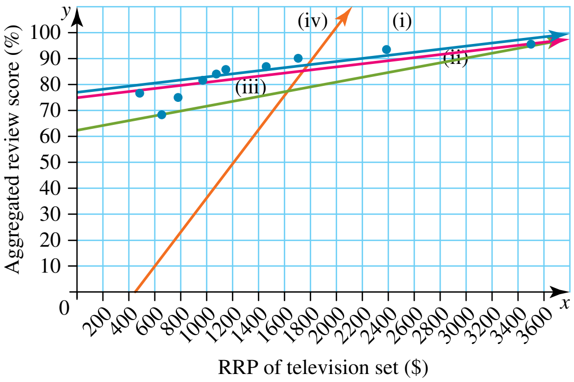
16. a. **i.** $y = 0.0057x + 77.7333$

ii. $y = 0.0059x + 76.2414$

iii. $y = 0.01x + 71$

iv. $y = 0.0197x + 58.5211$

b.



c. Line **iii** is the most appropriate line of best fit for this data

17. a. This point of data is clearly an outlier in terms of the data set.

b. Lines of best fit will vary but should split the data points on either side of the line and minimise the total distance from the points to the line.

c. Answers will vary.

d. The increase in box office taking per $1m increase in the leading actor/actress salary.

e. Answers will vary.

f. The answers to parts **iii** and **iv** are considerably less reliable than the answers to parts **i** and **ii**, as they are created using extrapolation instead of interpolation.

15. b. *

Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Predicted handball efficiency (%)	84.6	78.7	89.4	83.7	86.9	90.3	78.0	82.8	80.6	87.6

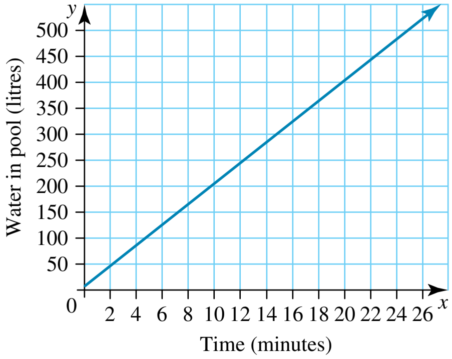
18. a. $y = 0.18x + 267$
b. 971 years
c. 433.5 cm
d. 0.171
e. Differ slightly (by approximately 0.01). They are very similar.

Exercise 6.4 Modelling practical situations with straight line graphs

1. $C = 65t + 90$

2. $a = -250t + 125\,000$

3. a. Both variables in the equation have a power of 1.
b. y-intercept = 5. This represents the amount of water initially in the pool.
c.



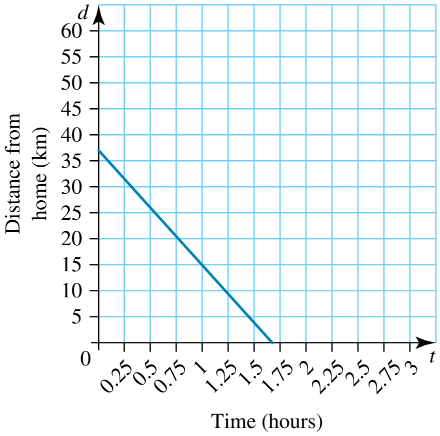
d. 25 minutes

4. a. $A = 40t + 100$
b. How much air was initially in the ball
c. 4900 cm^3
d. 41 minutes 38 seconds

5. a. 12
b. y-intercept = −0.5. This means that Kirsten starts 0.5 km before the starting point of the race.
c. 5.5 km
d. 1 hour, 48 minutes

6. a. $P = 15t$
b. The additional amount of petrol in the tank each minute
c. 5 minutes
d. $P = 15t + 15$
e. $0 \le t \le 4$

7. a. 37 km
b. The distance to Gert's home is reducing as time passes.
c. 1 hour, 41 minutes
d. $0 \le t \le 1.682$ h
e.



8. a. $\frac{4}{3}$
b. The increase in the height of the water each minute
c. 0.67 or $\frac{2}{3}$
d. No, the y-intercept calculated in part **c** is not 0, so there was water in the tank to start with.

9. a. $a = 40$, $b = 120$
b. The amount of money in Fred's account at the start of the year
c. 72 weeks

10. a. $a = 0.015$, $b = 800$
b. No, there is no limit to how much Michaela can earn in a month.
c. \$7580
d. \$652 140

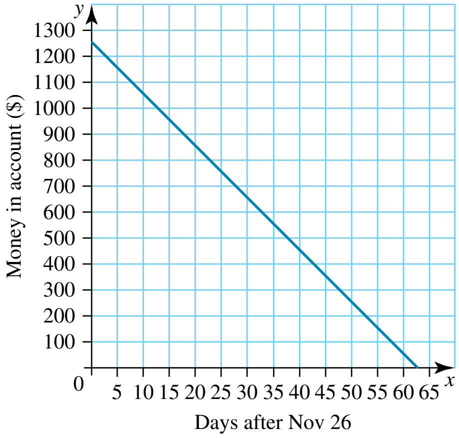
11. a. $C = 60h + 175$
b. 3.5 hours
c. \$460
d. $C = 110 + 100h,\ 2 \le h \le 4$

12. a. $P = 19.2t,\ 10 \le t \le 20$
b. $R = 100–4e,\ 0 \le e \le 15$

13. a. $C = 3.5t,\ 100 \le t \le 1000$
b. The domain represents the number of T-shirts Monique can buy.
c. There is an upper limit as the deal is valid only up to 1000 T-shirts.

14. a. 80
b. y-intercept = 0. This means that they start from home.
c. $d = 80t$
d. 2 hours, 11 minutes
e. 200 km

15. a. Kim withdraws the same amount each 5 days, so there is a constant decrease.
b.



c. Gradient = −20. This means that Kim withdraws an average of \$20 each day.
d. $M = 1250 - 20t$
e. The x-intercept represents when there will be no money left in Kim's account.
f. There will be a limit to the domain, possibly $0 \le t \le 62.5$ days, but we do not know this limit as it depends on how much Kim's account can be overdrawn.
g. After 25 days (on December 21) Kim will have \$750 in her account, so she will not have enough for her holiday.

16. **a.** \$7.50/cm^2

b. $C = 7.5A + 25$

c. \$81.25

d. $C = 0.58l + 55$

e. Package A = \$126.25; Package B = \$113.00. Package B is the better option.

17. **a.** $C = 0.13t + 15$

b. The gradient represents the call cost per minute and the y-intercept represens the flat fee.

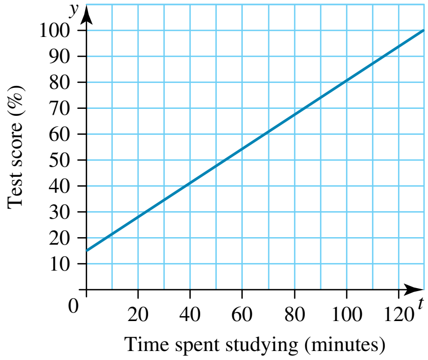
c.

Time	Cost (\$)
5	15.65
10	16.30
15	16.95
20	17.60
25	18.25
30	18.90
35	19.55
40	20.20
45	20.85
50	21.50
55	22.15
60	22.80

d. 396 minutes

18. **a.** $a = 51$

b.



c. If Carly doesn't study, she will score 15%.

d. $y = 0.65t + 15,\ 0 \le t \le 130.77$

e. 76 minutes, 55 seconds

6.5 Review: exam practice

1. D

2. D

3. A

4. C

5. D

6. D

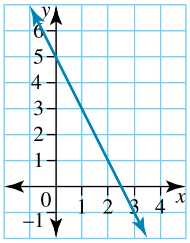
7. D

8. B

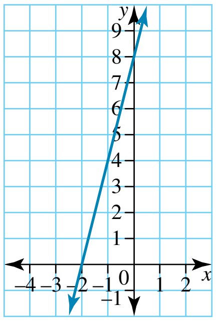
9. A

10. D

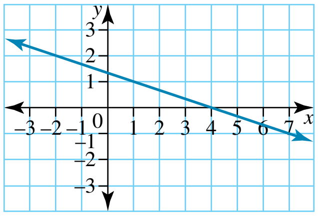
11. **a.** x-intercept: (2.5, 0), y-intercept: (0, 5)



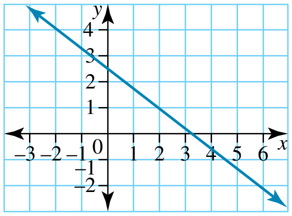
b. x-intercept: (−2, 0), y-intercept: (0, 8)



c. x-intercept: (4, 0), y-intercept: $\left(0,\ \frac{4}{3}\right)$



d. x-intercept: $\left(\frac{10}{3},\ 0\right)$, y-intercept: (0, 2.5)



12. **a.** −2 **b.** −1 **c.** $\frac{4}{3}$ **d.** $-\frac{7}{13}$

13. −2

14.

	Equation	Gradient	y-intercept	x-intercept
a	$y = 5x - 3$	5	−3	0.6
b	$y = 3x + 1$	3	1	$-\frac{1}{3}$
c	$3y = 6x - 9$	2	−3	1.5
d	$2y + 4x = 8$	−2	4	2
e	$y = x + 5$	1	5	−5
f	$y = 2x - 4$	2	−4	2

15. 16 strawberry twists and 12 chocolate ripples.

16. a.

Week	0	1	2	3	4	5	6
Money ($)	20	23	26	29	32	35	38
Week	7	8	9	10	11	12	
Money ($)	41	44	47	50	53	56	

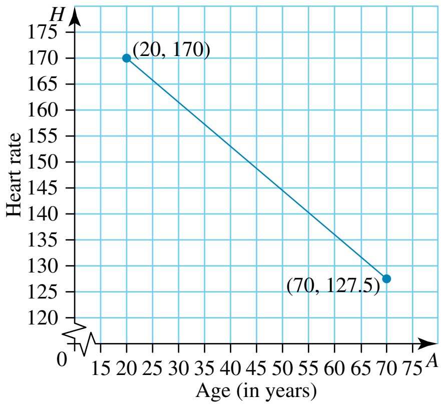
b. 10 weeks

17. a. The power of both variables in the equation (H and A) is 1.

b. 166

c. Gradient = −0.85, y-intercept = 187

d.



e. 29

f. At the x-intercept, heart rate = 0; therefore, the person would no longer be alive.

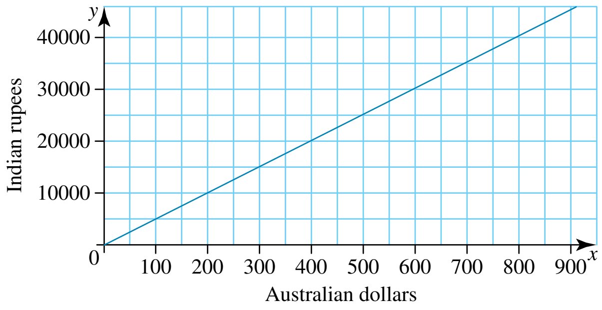
18. a. $y = 50.48x$

b. 60 576 Indian rupees

c. $6.93 to $49.52

d. Yes, this is an example of direct variation. It is of the form $y = mx$. The y-value is always 50.48 times the x-value, so it is increasing at a constant rate and starts at (0, 0).

e.



19. a. 0.3 L/h. It is assumed that the petrol is leaking at a constant rate.

b. 1.2 L

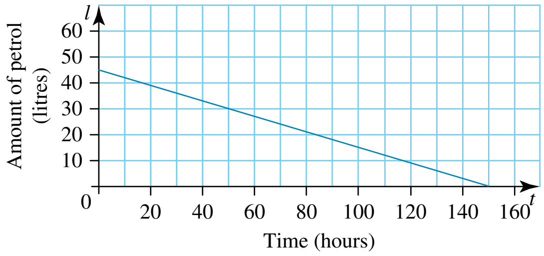
c. 17.5 hours

d. i. Petrol is leaking at a constant rate (the gradient).

ii. Petrol is leaking from the tank; therefore, the amount of petrol is decreasing.

iii. $l = 45–0.3t$

e.



f. 150 hours

20. a. $p = -20d + 868$

b. Gradient = −20 means that for every 1 km from the city centre, the median house price decreases by $20 000. y-intercept = 868 means that the median house price in the city centre is $868 000.

c. i. $558 000 **ii.** $772 000 **iii.** $494 000

d. 10.9 km

e. x-intercept = 43.4. The x-intercept is where the value of properties would equal 0, which is not possible in the context of this problem.

f. A distance of 229 km from the city centre is well outside the data set. Therefore, the equation would not be reliable.

CHAPTER 7
Simultaneous equations and their applications

7.1 Overview

7.1.1 Introduction

Ways of finding the solutions to simultaneous linear equations have important roles in engineering, physics, chemistry, computer science and economics, but also have many simple applications. For example, imagine you decide to make and sell handmade bracelets at a local market. Depending on how much it will cost you to hire and set up your stall and the cost of the materials, you will want to know how many bracelets you need to sell to make a profit. The cost of the number of bracelets and the profit you make from selling them can be set up as linear equations and solved using a graph or algebraically.

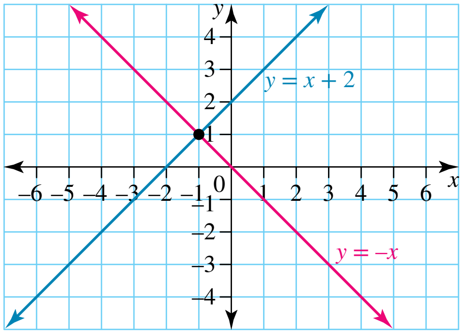
LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookplus at www.jacplus.com.au.

7.2 Solving simultaneous linear equations graphically

7.2.1 Solutions to simultaneous equations

Simultaneous equations are a set of equations that can be solved together. They often represent practical problems that have two or more unknowns. For example, you can use simultaneous equations to find the cost of individual apples and oranges when different amounts of each are bought.

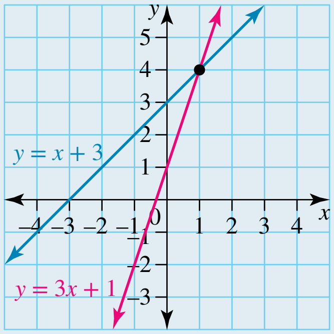


By graphing a pair of simultaneous equations, a point of intersection can be found. The coordinates of the point of intersection are the *x*- and *y*- values that satisfy both of the equations.

WORKED EXAMPLE 1

a. **Identify the coordinates of the point which simultaneously solves the two equations graphed below.**

b. **Check, by substitution, that the solution to part a is correct.**

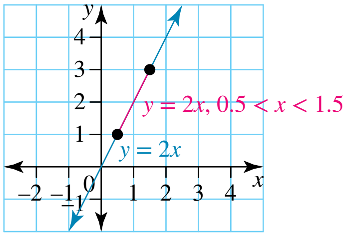


THINK	WRITE
a. Identify the coordinates of the point of intersection (the point where the two lines meet).	(1, 4)

b. The point of intersection (1, 4) represents an x-value of 1 and a y-value of 4. Check that the point of intersection satisfies both equations by substituting $x = 1$ and $y = 4$ into each equation to see if the LHS of the equation is equal to the RHS of the equation.

$y = x + 3$
Let $x = 1$ and $y = 4$
$\text{LHS} = y$
$= 4$
$\text{RHS} = x + 3$
$= 1 + 3$
$= 4$
$\text{LHS} = \text{RHS}$
The solution is correct.

$y = 3x + 1$
Let $x = 1$ and $y = 4$
$\text{LHS} = y$
$= 4$
$\text{RHS} = 3x + 1$
$= 3(1) + 1$
$= 4$
$\text{LHS} = \text{RHS}$
The solution is correct.

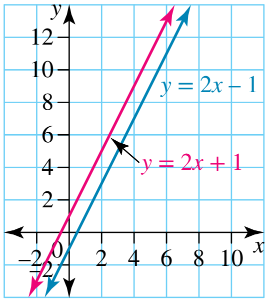


Two lines are **coincident** if they lie one on top of the other. For example, the line and line segment shown are coincident.

For coincident lines, every point where the lines coincide satisfies both equations and hence is a solution to the simultaneous equations. So there are an infinite number of solutions to simultaneous equations of coincident lines.

Coincident lines have essentially the same equation, although the equations may have been altered or multiplied by a constant so they appear different. For example, $y = 2x + 3$ and $2y - 4x = 6$ are coincident lines; $y = 2x + 3$ and $2y - 4x = 6$ have the same gradient and y-intercept.

7.2.2 Equations with no solutions



If two linear lines do not intersect, there is no solution to the equations.

For linear equations, which are represented by straight lines, the only situation in which the lines do not intersect is if the lines are parallel. Parallel lines have the same gradient but different y-intercepts. For example, $y = 2x - 1$ and $y = 2x + 1$ are equations of parallel lines.

WORKED EXAMPLE 2

For the following pairs of equations, state, with reasons, whether the two lines will intersect.

a. $y + x = 2$
$3x - 5 = 2y$

b. $y = 2x + 5$
$3y - 6x = 15$

c. $5y = 4x + 6$
$10y - 8x = 15$

THINK

WRITE

a. 1. Both equations are for straight lines since the highest power of x is 1.

$y + x = 2$ $\quad$ $3x - 5 = 2y$

2. Transpose both formulas into the form $y = mx + c$, where m is the gradient and c is the y-intercept.

$$\begin{aligned} y + x &= 2 \\ y + x - x &= 2 - x \\ y &= 2 - x \\ y &= 2 + -x \\ y &= -x + 2 \end{aligned}$$

$$\begin{aligned} 2y &= 3x - 5 \\ \frac{2y}{2} &= \frac{3x - 5}{2} \\ y &= \frac{3x}{2} - \frac{5}{2} \end{aligned}$$

3. The gradients are not the same so the lines are not parallel.

$m = -1,\ c = 2$ $\qquad$ $m = \frac{3}{2},\ c = -\frac{5}{2}$

The two lines will intersect as their gradients are not the same.

b. 1. Both equations are for straight lines. Transpose both formulas into the form $y = mx + c$.

$$\begin{aligned} y &= 2x + 5 \\ m &= 2,\ c = 5 \end{aligned}$$

$$\begin{aligned} 3y - 6x &= 15 \\ 3y - 6x + 6x &= 15 + 6x \\ 3y &= 15 + 6x \\ \frac{3y}{3} &= \frac{15 + 6x}{3} \\ y &= \frac{15}{3} + \frac{6x}{3} \\ y &= 5 + 2x \\ m &= 2,\ c = 5 \end{aligned}$$

2. The gradients are the same so the lines are parallel. In fact the lines also have the same y-intercept ($c = 5$), so the lines are coincident.

The two lines are coincident (they lie one on top of the other), since they have the same value for their gradient ($m = 2$) and the same value for their y-intercept ($c = 5$). The lines intersect along their whole length.

c. 1. Both equations are equations for straight lines. Transpose both formulas into the form $y = mx + c$.

$$\begin{aligned} 5y &= 4x + 6 \\ \frac{5y}{5} &= \frac{4x + 6}{5} \\ y &= \frac{4x}{5} + \frac{6}{5} \\ m &= \frac{4}{5},\ c = \frac{6}{5} \end{aligned}$$

$$\begin{aligned} 10y - 8x &= 15 \\ 10y - 8x + 8x &= 15 + 8x \\ 10y &= 15 + 8x \\ \frac{10y}{10} &= \frac{15 + 8x}{10} \\ y &= \frac{15}{10} + \frac{8x}{10} \\ y &= \frac{3}{2} + \frac{4x}{5} \\ m &= \frac{4}{5},\ c = \frac{3}{2} \end{aligned}$$

2. The gradients are the same so the lines are parallel. The y-intercepts are different so the lines do not lie one on top of the other. Parallel lines do not intersect.

The two lines will not intersect. Their gradients are the same but their y-intercepts are different and hence they are parallel lines.

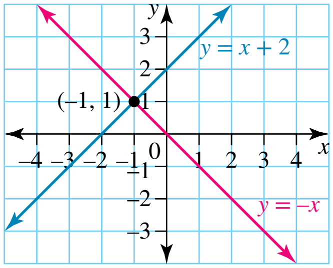
7.2.3 Graphical solutions

The solution to a pair of simultaneous equations can be found by graphing the two equations and identifying the coordinates of the point of intersection.

To sketch a straight line, you need to find two points. There are a variety of ways to sketch a straight line:

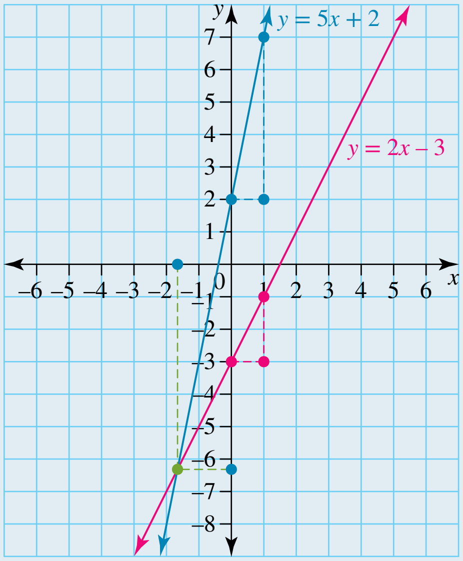
- Calculate the x- and y-intercepts.
- Use the gradient and y-intercept method.
- Calculate the value of the two points by substituting values into the equations.
- Use technology.

The coordinate of the point of intersection $(-1, 1)$ is shown, and this is found by graphing the straight lines $y = x + 2$ and $y = -x$.



WORKED EXAMPLE 3

Solve the simultaneous equations $y = 5x + 2$ and $y = 2x - 3$ graphically.

THINK	WRITE
a. 1. To solve the simultaneous equations, sketch each of the equations on the same axes. Both equations are in the form $y = mx + c$, so identify the gradient and y-intercept in each equation.	$y = 5x + 2$ $\quad$ $y = 2x - 3$ $m = 5$ $\quad$ $m = 2$ $c = 2$ $\quad$ $c = -3$
2. For the gradient, identify the rise and the run.	$m = \frac{\text{rise}}{\text{run}} = \frac{5}{1}$ $\quad$ $m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$ rise = 5, run = 1 $\quad$ rise = 2, run = 1
3. Draw a set of axes and sketch both lines. For the line $y = 5x + 2$: • place a point on the y-axis at the y-intercept ($c = 2$), as shown in blue • from the y-intercept, run 1 and rise 5 and place a point, as shown in blue • draw a line between the two points and label the line, as shown in blue. For the line $y = 2x - 3$: • place a point on the y-axis at the y-intercept ($c = -3$), as shown in pink • from the y-intercept, run 1 and rise 2 and place a point, as shown in pink • draw a line between the two points and label the line, as shown in pink.	
4. Carefully read the coordinates of the point of intersection. The point of intersection appears to be $\left(-1\frac{2}{3}, -6\frac{1}{3}\right)$, as shown in green.	

▶

5. Write the solution.	The solution is $x = -1\frac{2}{3}$ and $y = -6\frac{1}{3}$.
6. Check that the solution satisfies both equations.	$y = 5x + 2$ LHS $= y$ $= -6\frac{1}{3}$ RHS $= 5x + 2$ $= 5\left(-1\frac{2}{3}\right) + 2$ $= \frac{5}{1} \times \frac{-5}{3} + 2$ $= \frac{-25}{3} + 2$ $= \frac{-25}{3} + \frac{6}{3}$ $= \frac{-19}{3}$ $= -6\frac{1}{3}$ = LHS LHS = RHS The solution is correct.
	$y = 2x - 3$ LHS $= y$ $= -6\frac{1}{3}$ RHS $= 2x - 3$ $= 2\left(-1\frac{2}{3}\right) - 3$ $= \frac{2}{1} \times \frac{-5}{3} - 3$ $= \frac{-10}{3} - 3$ $= \frac{-10}{3} - \frac{9}{3}$ $= \frac{-19}{3}$ $= -6\frac{1}{3}$ = LHS LHS = RHS The solution is correct.

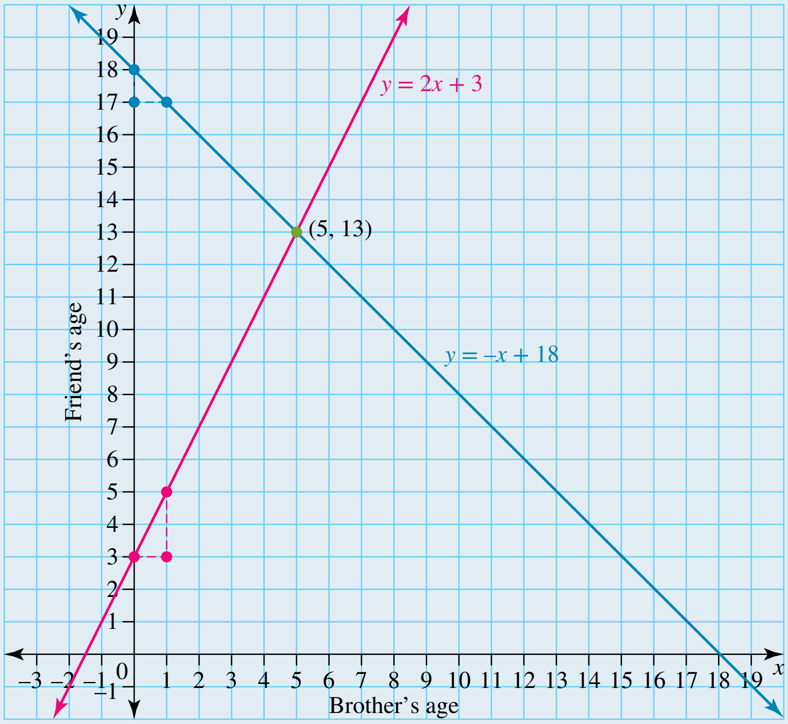
WORKED EXAMPLE 4

Your friend is three years older than twice the age of her brother. The sum of their ages is 18. Calculate the age of both your friend and her brother using two simultaneous equations.

THINK	WRITE
1. To create two equations, allocate a variable for your friend's age and another variable for her brother's age.	Let y = your friend's age. Let x = her brother's age.
2. Use the information in the question to form two equations.	
• Your friend (y) is three years older than twice the age of her brother (x).	$y = 2x + 3$
• The sum of their ages is 18.	$y + x = 18$

3. Graph the two equations.

4. Identify the coordinates of the point of intersection (the point where the two lines meet).



5. Check that the point of intersection satisfies both equations by substituting $x = 5$ and $y = 13$ into each equation to see if the LHS of the equation is equal to the RHS of the equation.

Point of intersection: (5, 13)

$y = 2x + 3$	$y + x = 18$
Let $x = 5$ and $y = 13$	Let $x = 5$ and $y = 13$
LHS = 13	LHS = 13 + 5
RHS = 2(5) + 3	= 18
= 13	RHS = 18
= LHS	= LHS
LHS = RHS	LHS = RHS
The solution is correct	The solution is correct.

6. Answer the question.

The friend is 13 years old and her brother is 5 years old.

on Resources

Interactivity: Solving simultaneous equations graphically (int-6452)

studyon

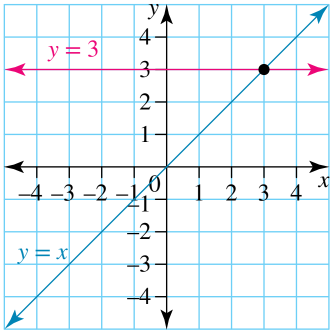
Units 1 & 2 > Area 3 > Sequence 3 > Concept 1

Solving simultaneous linear equations graphically Summary screen and practice questions

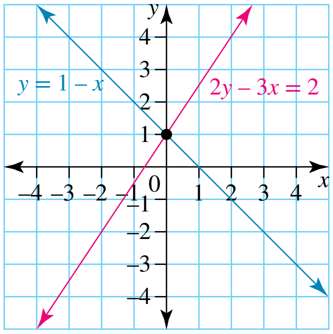
Exercise 7.2 Solving simultaneous linear equations graphically

1. **WE1** Identify the coordinates of the point which simultaneously solves the two equations in each of the graphs below.

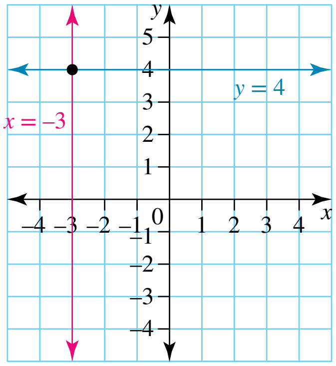
a.



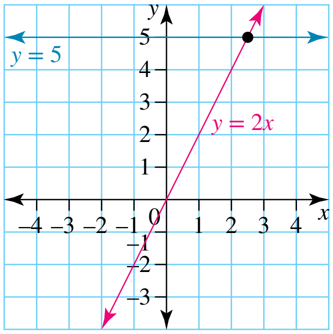
b.



c.



d.



2. State, with reasons, whether the following values for x and y are solutions for the given pair of simultaneous equations.

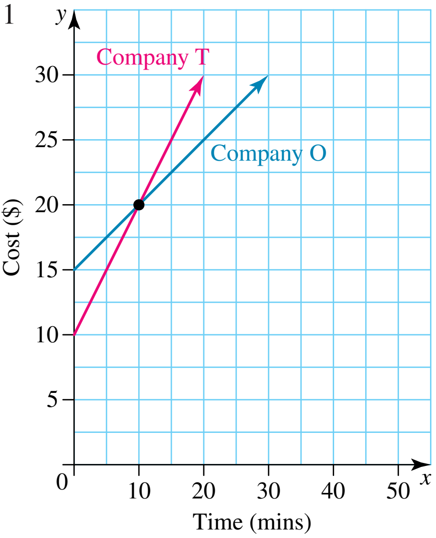
a. $x + y = 5 \quad x = 4,\ y = 1$
$2x + 4y = 12$

b. $x + 3y = -1 \quad x = 3,\ y = -1$
$3x - 2y = 14$

3. a. For the graph shown, at what time do the two phone companies, T and O, charge the same amount?

b. Calculate the equations for the lines that represent each company.

c. Use the coordinates of the point of intersection to check that your equations for each company are correct.



4. **WE2** For the following pairs of equations, state, with reasons, whether the two lines will intersect.

a. $3y = x + 12$
$y - 5x = 7.5$

b. $y = 4x - 7$
$8x - 2y - 14 = 0$

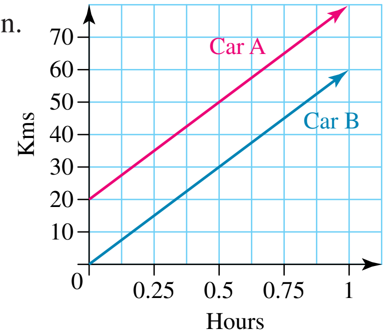
c. $5y = 4x + 6$
$1.2y - x = 15.5$

d. Copy and complete the following statements.

i. A system of linear equations will have a unique solution if the equations have _______ gradient(s) and _______ y-intercept(s).

ii. A system of linear equations will have an infinite number of solutions if the equations have _______ gradient(s) and _______ y-intercept(s).

5. The graph below shows the distance of two cars from a particular town.



a. How far does each car travel in one hour?

b. Calculate the gradient of each line.

c. Are the cars ever at the same spot at the same time? Explain.

d. What do the y-intercepts indicate about the two cars?

6. **WE3** Solve the following pairs of simultaneous equations graphically

a.	$y = -2$ $y = 2x - 4$	**b.**	$x = 2$ $y = 4$
c.	$y = x - 3$ $y = 1 - x$	**d.**	$y = 3x + 6$ $y = 1 - 2x$
e.	$x - y = 0$ $2y + x = 0$	**f.**	$x - y = 3$ $3x + y = 7$

7. Determine the coordinates of the point of intersection of the lines. $9x + 3y = 27$ and $y = 3x - 3$.

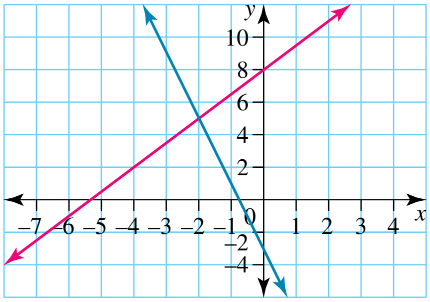
8. **WE4** Your friend is nine years younger than three times the age of her brother. The sum of their ages is 11. Calculate the age of both your friend and her brother using two simultaneous equations.

9. In a fruit shop, one shopper has 3 apples and 5 oranges in their basket and another shopper has 4 apples and 2 oranges. The total cost of those pieces of fruit is shown below.

a. How much do the apples cost?

b. What is the difference in cost between an apple and an orange?

10. A pair of simultaneous equations is solved graphically as shown in the diagram. From the diagram, determine the solution for this pair of simultaneous equations.



11. The following equations represent a pair of simultaneous equations.

$$y = 5x + 1 \text{ and } y = 2x - 5$$

Using technology or otherwise, sketch both graphs on the same set of axes and solve the equations.

12. Using technology or otherwise, sketch and solve the following three simultaneous equations.

$$y = 3x + 7,\ y = 2x + 8 \text{ and } y = -2x + 12$$

13. Using technology or otherwise, solve the following groups of simultaneous equations graphically.

a.	$y = 4x + 1$ and $y = 3x - 1$	**b.**	$y = x - 5$ and $y = -3x + 3$
c.	$y = 3(x - 1)$ and $y = 2(2x + 1)$	**d.**	$y = \frac{x}{2} - 1$ and $y = \frac{x}{2} + 4$

14. Consider the following groups of graphs.
   - i. $y_1 = 5x - 4$ and $y_2 = 6x + 8$
   - ii. $y_1 = -3x - 5$ and $y_2 = 3x + 1$
   - iii. $y_1 = 2x + 6$ and $y_2 = 2x - 4$
   - iv. $y_1 = -x + 3$, $y_2 = x + 5$ and $y_3 = 2x + 6$
      - a. Where possible, determine the point of intersection for each group of graphs using any method.
      - b. Are there solutions for all of these groups of graphs? If not, for which group of graphs is there no solution, and why is this?

7.3 Solving simultaneous equations algebraically

7.3.1 Solving simultaneous equations using substitution

Simultaneous equations can also be solved algebraically. One algebraic method is known as substitution. This method requires one of the equations to be substituted into the other by replacing one of the variables. The second equation is then solved and the value of one of the variables is found. The substitution method is often used when one or both of the equations are written with variables on either side of the equals sign; for example, $c = 12b - 15$ and $2c + 3b = -3$, or $y = 4x + 6$ and $y = 6x + 2$.

WORKED EXAMPLE 5

Solve the following pairs of simultaneous equations using substitution;

a. $\boldsymbol{c = 12b - 15}$ **and** $\boldsymbol{2c + 3b = -3}$
b. $\boldsymbol{y = 4x + 6}$ **and** $\boldsymbol{y = 6x + 2}$
c. $\boldsymbol{3x + 2y = -1}$ **and** $\boldsymbol{y = x - 8}$

THINK	WRITE
a. 1. Identify which variable will be substituted into other equation.	a. $c = 12b - 15$
2. Substitute the variable $c = 12b - 15$ into equation.	$2c + 3b = -3$ $2(12b - 15) + 3b = -3$
3. Expand and simplify the left-hand side, and solve the equation for unknown variable.	$24b - 30 + 3b = -3$ $27b - 30 = -3$ $27b = -3 + 30$ $27b = 27$ $b = 1$
4. Substitute the value for the unknown back into one of the equations.	$c = 12b - 15$ $= 12(1) - 15$ $= -3$
5. Answer the question.	The solution is $b = 1$ and $c = -3$.
b. 1. Both equations are in the form $y =$, so let them equal each other.	b. $4x + 6 = 6x + 2$
2. Move all of the variables to one side.	$4x - 4x + 6 = 6x - 4x + 2$ $6 = 6x - 4x + 2$ $6 = 2x + 2$

3. Solve for the unknown.

$$\begin{aligned} 6 - 2 &= 2x + 2 - 2 \\ 4 &= 2x \\ \frac{4}{2} &= \frac{2x}{2} \\ 2 &= x \end{aligned}$$

4. Substitute the value found, $x = 2$, into either of the original equations.

$$\begin{aligned} y &= 4x + 6 \\ &= 4 \times 2 + 6 \\ &= 8 + 6 \\ &= 14 \end{aligned}$$

5. Answer the question.

The solution is $x = 2$ and $y = 14$.

c. 1. One equation is in the form $y =$, so substitute $y = x - 8$ into the other equation.

c. $$\begin{aligned} 3x + 2y &= -1 \\ 3x + 2(x - 8) &= -1 \end{aligned}$$

2. Expand and simplify the equation.

$$\begin{aligned} 3x + 2x - 16 &= -1 \\ 5x - 16 &= -1 \end{aligned}$$

3. Solve for the unknown.

$$\begin{aligned} 5x - 16 + 16 &= -1 + 16 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

4. Substitute the value found, $x = 3$, into either of the original equations.

$$\begin{aligned} y &= x - 8 \\ &= 3 - 8 \\ &= -5 \end{aligned}$$

5. Answer the question.

The solution is $x = 3$ and $y = -5$.

on Resources

Interactivity: Solving simultaneous equations using substitution (int-6453)

7.3.2 Solving simultaneous equations using elimination

Solving simultaneous equations using **elimination** requires the equations to be added or subtracted so that one of the pronumerals is eliminated or removed. Simultaneous equations that have both pronumerals on the same side are often solved using elimination. For example, $3x + y = 5$ and $4x - y = 2$ both have x and y on the same side of the equation, so they can be solved with this method.

WORKED EXAMPLE 6

Solve the following pairs of simultaneous equations using elimination:

a. $3x + y = 5$ **and** $4x - y = 2$

b. $2a + b = 7$ **and** $a + b = 5$

c. $3c + 4d = 5$ **and** $2c + 3d = 4$

THINK	WRITE
a. 1. Write the simultaneous equations with one on top of the other.	**a.** $3x + y = 5$ [1] $4x - y = 2$ [2]

2. Select one pronumeral to be eliminated.	Select y.
3. Check the coefficients of the pronumeral being eliminated.	The coefficients of y are 1 and -1.
4. If the coefficients are the same number but with different signs, add the equations together.	[1] + [2]: $3x+4x+y-y=5+2$ $7x=7$
5. Solve the equation for the unknown pronumeral.	$7x=7$ $\frac{7x}{7}=\frac{7}{7}$ $x=1$
6. Substitute the pronumeral back into one of the equations.	$3x+y=5$ $3(1)+y=5$
7. Solve the equation to find the value of the other pronumeral.	$3+y=5$ $3-3+y=5-3$ $y=2$
8. Answer the question.	The solution is $x=1$ and $y=2$.
b. 1. Write the simultaneous equations with one on top of the other.	**b.** $2a+b=7$ [1] $a+b=5$ [2]
2. Select one pronumeral to be eliminated.	Select b.
3. Check the coefficients of the pronumeral being eliminated.	The coefficients of b are both 1.
4 If the coefficients are the same number with the same sign, subtract one equation from the other.	[1] − [2]: $2a-a+b-b=7-5$ $a=2$
5. Solve the equation for the unknown pronumeral.	$a=2$
6. Substitute the pronumeral back into one of the equations.	$a+b=5$ $2+b=5$
7. Solve the equation to find the value of the other pronumeral.	$b=5-2$ $b=3$
8. Answer the question.	The solution is $a=2$ and $b=3$.
c. 1. Write the simultaneous equations with one on top of the other.	**c.** $3c+4d=5$ [1] $2c+3d=4$ [2]
2. Select one pronumeral to be eliminated.	Select c.
3. Check the coefficients of the pronumeral being eliminated.	The coefficients of c are 3 and 2.
4. If the coefficients are different numbers, then multiply them both by another number, so they both have the same coefficient value.	$3\times2=6$ $2\times3=6$
5. Multiply the equations (all terms in each equation) by the numbers selected in step **4**.	[1] × 2: $6c+8d=10$ [2] × 3: $6c+9d=12$

6. Check the sign of each coefficient for the selected pronumeral.	$6c + 8d = 10 \quad [3]$ $6c + 9d = 12 \quad [4]$ Both coefficients of c are positive 6.
7. If the signs are the same, subtract one equation from the other and simplify.	[3] − [4]: $6c - 6c + 8d - 9d = 10 - 12$ $-d = -2$
8. Solve the equation for the unknown.	$d = 2$
9. Substitute the pronumeral back into one of the equations.	$2c + 3d = 4$ $2c + 3(2) = 4$
10. Solve the equation to find the value of the other pronumeral.	$2c + 6 = 4$ $2c + 6 - 6 = 4 - 6$ $2c = -2$ $\frac{2c}{2} = \frac{-2}{2}$ $c = -1$
11. Answer the question.	The solution is $c = -1$ and $d = 2$.

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Units 1 & 2 > Area 3 > Sequence 3 > Concept 2

Solving simultaneous linear equations algebraically Summary screen and practice questions

Exercise 7.3 Solving simultaneous equations algebraically

1. **WE5** Solve the following pairs of simultaneous equations using substitution.

a. $y = 2x + 1$ and $2y - x = -1$

b. $m = 2n + 5$ and $m = 4n - 1$

2. Calculate the solutions to the following pairs of simultaneous equations using substitution.

a. $2(x + 1) + y = 5$ and $y = x - 6$

b. $\frac{x + 5}{2} + 2y = 11$ and $y = 6x - 2$

3. **MC** Which one of the following pairs of simultaneous equations would best be solved using the substitution method?

A. $4y - 5x = 7$ and $3x + 2y = 1$

B. $3c + 8d = 19$ and $2c - d = 6$

C. $12x + 6y = 15$ and $9x - y = 13$

D. $n = 9m + 12$ and $3m + 2n = 7$

4. Using the substitution method, solve the following pairs of simultaneous equations

a. $y = 2x + 5$ and $y = 3x - 2$

b. $y = 5x - 2$ and $y = 7x + 2$

c. $y = 2(3x + 1)$ and $y = 4(2x - 3)$

d. $y = 5x - 9$ and $3x - 5y = 1$

e. $3(2x + 1) + y = -19$ and $y = x - 1$

f. $\frac{3x + 5}{2} + 2y = 2$ and $y = x - 2$

5. Solve the following pair of simultaneous equations using the substitution method.

$$3x + y = 8 \text{ and } 2x - y = 7$$

6. **WE6** Solve the following pairs of simultaneous equations using elimination:
   - a. $4x + y = 6$ and $x - y = 4$
   - b. $x + y = 7$ and $x - 2y = -5$
   - c. $2x - y = -5$ and $x - 3y = -10$
   - d. $4x + 3y = 29$ and $2x + y = 13$
   - e. $5x - 7y = -33$ and $4x + 3y = 8$
   - f. $\frac{x}{2} + y = 7$ and $3x + \frac{y}{2} = 20$

7. Consider the following pair of simultaneous equations:

$$ax - 3y = -16 \textit{ and } 3x + y = -2.$$

If $y = 4$, calculate the values of a and x.

8. **MC** The first step when solving the following pair of simultaneous equations using the elimination method is:

$$\begin{aligned} 2x + y &= 3 \qquad [1] \\ 3x - y &= 2 \qquad [2] \end{aligned}$$

   - A. equations [1] and [2] should be added together.
   - B. both equations should be multiplied by 2.
   - C. equation [1] should be subtracted from equation [2].
   - D. equation [1] should be multiplied by 2 and equation [2] should be multiplied by 3.

9. Brendon and Marcia were each asked to solve the following pair of simultaneous equations.

$$\begin{aligned} 3x + 4y &= 17 \qquad [1] \\ 4x - 2y &= 19 \qquad [2] \end{aligned}$$

Marcia decided to use the elimination method. Her solution steps were:

Step 1: [1] × 4:

$12x + 16y = 68 \qquad [3]$

[2] × 3:

$12x - 6y = 57 \qquad [4]$

Step 2: [3] + [4]:

$10y = 125$

Step 3: $y = 12.5$

Step 4: Substitute $y = 12.5$ into [1]:

$3x + 4(12.5) = 17$

Step 5: Solve for x:

$$\begin{aligned} 3x &= 17 - 50 \\ 3x &= -33 \\ x &= -11 \end{aligned}$$

Step 6: The solution is $x = -11$ and $y = 12.5$.

   - a. Marcia has made an error in step 2. Explain where she has made her error, and hence correct her mistake.
   - b. Using the correction you made in part **a**, find the correct solution to this pair of simultaneous equations.

Brendon decided to eliminate y instead of x.

   - c. Using Brendon's method of eliminating y first, show all the appropriate steps involved to reach a solution.

10. In a ball game, a player can kick the ball into the net to score a goal or place the ball over the line to score a behind. The scores in a game between the Rockets and the Comets were:

Rockets: 6 goals 12 behinds, total score 54

Comets: 7 goals 5 behinds, total score 45

The two simultaneous equations that can represent this information are shown.

Rockets: $6x + 12y = 54$

Comets: $7x + 5y = 45$

- **a.** By solving the two simultaneous equations, determine the number of points that are awarded for a goal and a behind.
- **b.** Using the results from part **a**, determine the scores for the game between the Jetts, who scored 4 goals and 10 behinds, and the Meteorites, who scored 6 goals and 9 behinds.

11. Mick and Minnie both work part time at an ice-cream shop. The simultaneous equations shown represent the number of hours Mick (x) and Minnie (y) work each week.

Equation 1: Total number of hours worked by Minnie and Mick: $x + y = 15$

Equation 2: Number of hours worked by Minnie in terms of Mick's hours: $y = 2x$

- **a.** Explain why substitution would be the best method to use to solve these equations.
- **b.** Using substitution, determine the number of hours worked by Mick and Minnie each week.

  To ensure that he has time to do his Mathematics homework, Mick changes the number of hours he works each week. He now works $\frac{1}{3}$ of the number of hours worked by Minnie. An equation that can be used to represent this information is $x = \dfrac{y}{3}$.
- **c.** Calculate the number of hours worked by Mick, given that the total number of hours that Mick and Minnie work does not change.

12. Using technology or otherwise, solve the following groups of simultaneous equations. Write your answers correct to 2 decimal places.

- **a.** $4(x + 6) = y - 6$ and $2(y + 3) = x - 9$
- **b.** $6x + 5y = 8.95$, $y = 3x - 1.36$ and $2x + 3y = 4.17$

7.4 Solving practical problems using simultaneous equations

7.4.1 Setting up simultaneous equations

The solutions to a set of simultaneous equations satisfy all equations that were used. Simultaneous equations can be used to solve problems involving two or more variables or unknowns, such as the cost of 1 kg of apples and bananas, or the number of adults and children attending a show.

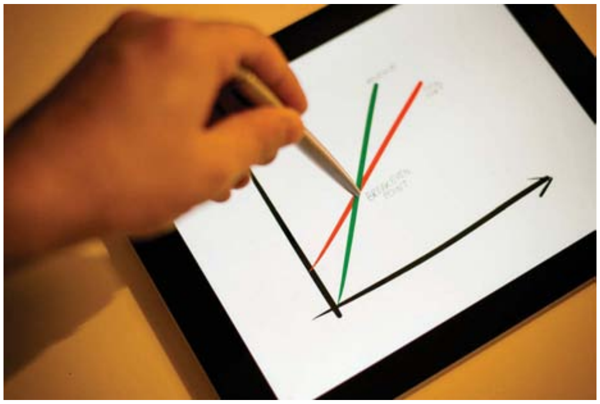
WORKED EXAMPLE 7

At a fruit shop, 2 kg of apples and 3 kg of bananas cost $13.16, and 3 kg of apples and 2 kg of bananas cost $13.74. Represent this information in the form of a pair of simultaneous equations.

THINK	WRITE
1 Identify the two variables.	The cost of 1 kg of apples and the cost of 1 kg of bananas
2 Select two pronumerals to represent these variables. Define the variables.	a = cost of 1 kg of apples b = cost of 1 kg of bananas
3 Identify the key information and rewrite it using the pronumerals selected.	2 kg of apples can be written as $2a$. 3 kg of bananas can be written as $3b$.
4 Construct two equations using the information.	$2a + 3b = 13.16$ $3a + 2b = 13.74$

7.4.2 Break-even points

A **break-even point** is a point where the costs equal the selling price. It is also the point where there is zero profit. For example, if the equation $C = 45 + 3t$ represents the production cost to sell t shirts, and the equation $R = 14t$ represents the revenue from selling the shirts for $14 each, then the break-even point is the number of shirts that need to be sold to cover all costs.



To find the break-even point, the equations for cost and revenue are solved simultaneously.

WORKED EXAMPLE 8

Santo sells shirts for $25. The revenue, R, for selling n shirts is represented by the equation $R = 25n$. The cost to make n shirts is represented by the equation $C = 2200 + 3n$.

a. **Solve the equations simultaneously to determine the break-even point.**

b. **Determine the profit or loss, in dollars, for the following shirt orders.**
   - i. **75 shirts**
   - ii. **220 shirts**

THINK	**WRITE**
a.. **1** Write the two equations.	**a.** $C = 2200 + 3n$ $R = 25n$
2. Equate the equations ($R = C$).	$2200 + 3n = 25n$
3. Solve for the unknown.	$2200 + 3n = 25n$ $2200 + 3n - 3n = 25n - 3n$ $2200 = 22n$ $\frac{2200}{22} = n$ $n = 100$
4. Substitute back into either equation to determine the values of C and R.	$R = 25n$ $= 25 \times 100$ $= 2500$
5. Answer the question in the context of the problem.	The break-even point is (100, 2500). Therefore 100 shirts need to be sold to cover the production cost, which is \$2500.
b. i. **1.** Write the two equations.	**b. i.** $C = 2200 + 3n$ $R = 25n$
2. Substitute the given value into both equations.	$n = 75$ $C = 2200 + 3 \times 75$ $= 2425$ $R = 25 \times 75$ $= 1875$
3. Determine the profit/loss by subtracting the cost, C, from the revenue, R.	Profit/loss $= R - C$ $= 1875 - 2425$ $= -550$ Since the answer is negative, it means that Santo lost \$550 (i.e. selling 75 shirts did not cover the cost to produce the shirts).
ii. **1.** Write the two equations.	**ii.** $C = 2200 + 3n$ $R = 25n$
2. Substitute the given value into both equations.	$n = 220$ $C = 2200 + 3 \times 220$ $= 2860$ $R = 25 \times 220$ $= 5500$
3. Determine the profit/loss by subtracting the cost, C, from the revenue, R.	Profit/loss $= R - C$ $= 5500 - 2860$ $= 2640$ Since the answer is positive, it means that Santo made \$2640 profit from selling 220 shirts.

Resources

Interactivity: Break-even points (int-6454)

studyon

Units 1 & 2 > Area 3 > Sequence 3 > Concept 3

Solving simultaneous linear equations from word problems Summary screen and practice questions

Exercise 7.4 Solving practical problems using simultaneous equations

1. **WE7** Mary bought 4 donuts and 3 cupcakes for \$10.55, and Sharon bought 2 donuts and 4 cupcakes for \$9.90. Letting d represent the cost of a donut and c represent the cost of a cupcake, set up a pair of simultaneous equations to represent this information.

2. A pair of simultaneous equations representing the number of adults and children attending the zoo is shown below.
   Equation 1: $a + c = 350$
   Equation 2: $25a + 15c = 6650$
   a. By solving the pair of simultaneous equations, determine the total number of adults and children attending the zoo.
   b. In the context of this problem, what does equation 2 represent?

3. **WE8** Yolanda sells handmade bracelets at a market for \$12.50.
   The revenue, R, for selling n bracelets is represented by the equation $R = 12.50n$.
   The cost to make n bracelets is represented by the equation $C = 80 + 4.50n$.
   a. i. By solving the equations simultaneously, determine the break-even point.
      ii. In the context of this problem, what does the break-even point mean?
   b. Determine the profit or loss, in dollars, if Yolanda sells:
      i. 8 bracelets
      ii. 13 bracelets.

4. The entry fee for a charity fun run event is \$18. It costs event organisers \$2550 for the hire of the tent and \$3 per entry for administration. Any profit will be donated to local charities.

   An equation to represent the revenue for the entry fee is $R = an$, where R is the total amount collected in entry fees, in dollars, and n is the number of entries.
   a. Write an equation for the value of a.
      The equation that represents the cost for the event is $C = 2550 + bn$.
   b. Write an equation for the value of b.

c. By solving the equations simultaneously, determine the number of entries needed to break even.

d. A total of 310 entries are received for this charity event. Show that the organisers will be able to donate \$2100 to local charities.

e. Determine the number of entries needed to donate \$5010 to local charities.

5. A school group travelled to the city by bus and returned by train. The two equations show the adult, a, and student, s, ticket prices to travel on the bus and train.

Bus: $3.5a + 1.5s = 42.50$

Train: $4.75a + 2.25s = 61.75$

a. Write the cost of a student bus ticket, s, and an adult bus ticket, a.

b. Solve the simultaneous equations and hence determine the number of adults and the number of students in the school group.

6. The following pair of simultaneous equations represents the number of adult and concession tickets sold and the respective ticket prices for the premier screening of the blockbuster *Aliens attack*.

Equation 1: $a + c = 544$

Equation 2: $19.50a + 14.50c = 9013$

a. What are the costs, in dollars, of an adult ticket, a, and a concession ticket, c?

b. In the context of this problem, what does equation 1 represent?

c. By solving the simultaneous equations, determine how many adult and concession tickets were sold for the premier.

7. Charlotte has a babysitting service and charges \$12.50 per hour. After Charlotte calculated her set-up and travel costs, she constructed the cost equation $C = 45 + 2.50h$, where C represents the cost in dollars per job and h represents the hours Charlotte babysits for.

a. Write an equation that represents the revenue, R, earned by Charlotte in terms of number of hours, h.

b. By solving the equations simultaneously, determine the number of hours Charlotte needs to babysit to cover her costs (that is, the break-even point).

c. In one week, Charlotte had four babysitting jobs as shown in the table.

Babysitting job	1	2	3	4
Number of hours (h)	5	3.5	4	7

i. Determine whether Charlotte made a profit or loss for each individual babysitting job.

ii. Did Charlotte make a profit this week? Justify your answer using calculations.

d. Charlotte made a \$50 profit on one job. Determine the total number of hours she babysat for.

8. Trudi and Mia work part time at the local supermarket after school. The following table shows the number of hours worked for both Trudi and Mia and the total wages, in dollars, paid over two weeks.

Week	**Trudi's hours worked**	**Mia's hours worked**	**Total wages**
Week 1	15	12	\$400.50
Week 2	9	13	\$328.75

a. Construct two equations to represent the number of hours worked by Trudi and Mia and the total wages paid for each week. Write your equations using the pronumerals t for Trudi and m for Mia.

b. In the context of this problem, what do t and m represent?

c. By solving the pair of simultaneous equations, determine the values of t and m.

9. Brendan uses carrots and apples to make his special homemade fruit juice. One week he buys 5 kg of carrots and 4 kg of apples for \$31.55. The next week he buys 4 kg of carrots and 3 kg of apples for \$24.65.

a. Set up two simultaneous equations to represent the cost of carrots, x, in dollars per kg, and the cost of apples, y, in dollars per kg.

b. By solving the simultaneous equations, determine how much Brendan spends on 1 kg each of carrots and apples.

c. Determine the amount Brendan spends the following week when he buys 2 kg of carrots and 1.5 kg of apples. Give your answer correct to the nearest 5 cents.

10. The table shows the number of 100 g serves of strawberries and grapes and the total kilojoule intake.

Fruit	**100 g serves**	
Strawberry, s	3	4
Grapes, g	2	3
Total kilojoules	**1000**	**1430**

a. Construct two equations to represent the number of serves of strawberries, s, and grapes, g, and the total kilojoules using the pronumerals shown.

b. By solving the pair of simultaneous equations constructed in part **a**, determine the number of kilojoules (kJ) for a 100-g serve of strawberries

11. Two budget car hire companies offer the following deals for hiring a medium size family car.

Car company	**Deal**
FreeWheels	\$75 plus \$1.10 per km travelled
GetThere	\$90 plus \$0.90 per km travelled

a. Construct two equations to represent the deals for each car hire company. Write your equations in terms of cost, C, and km travelled, k.

b. By solving the two equations simultaneously, determine the value of k at which the cost of hiring a car will be the same.

c. Rex and Jan hire a car for the weekend. They expect to travel a distance of 250 km over the weekend. Which car hire company should they use and why? Justify your answer using calculations.

12. The following table shows the number of boxes of three types of cereal bought each week for a school camp, as well as the total cost for each week.

Cereal	**Week 1**	**Week 2**	**Week 3**
Corn Pops, c	2	1	3
Rice Crunch, r	3	2	4
Muesli, m	1	2	1
Total cost, $	**27.45**	**24.25**	**36.35**

Wen is the cook at the camp. She decides to work out the cost of each box of cereal using simultaneous equations. She incorrectly sets up the following equations:

$$\begin{aligned} 2c + c + 3c &= 27.45 \\ 3r + 2r + 4r &= 24.25 \\ m + 2m + m &= 36.35 \end{aligned}$$

a. Explain why these simultaneous equations will not determine the cost of each box of cereal.

b. Write the correct simultaneous equations.

c. Using technology or otherwise, solve the three simultaneous equations, and hence write the total cost for cereal for week 4's order of 3 boxes of Corn Pops, 2 boxes of Rice Crunch and 2 boxes of muesli.

13. Sally and Nem decide to sell cups of lemonade from their front yard to the neighbourhood children. The cost to make the lemonade using their own lemons can be represented using the equation $C = 0.25n + 2$, where C is the cost in dollars and n is the number of cups of lemonade sold.

a. If they sell cups of lemonade for 50 cents, write an equation to represent the selling price, S, for n number of cups of lemonade.

b. By solving two simultaneous equations, determine the number of cups of lemonade Sally and Nem need to sell in order to break even (i.e. cover their costs).

c. Sally and Nem increase their selling price. If they make a $7 profit for selling 20 cups of lemonade, what is the new selling price?

14. The CotX T-Shirt Company produces T-shirts at a cost of $7.50 each after an initial set-up cost of $810.

a. Determine the cost to produce 100 T-shirts.

b. Using technology or otherwise, complete the following table that shows the cost of producing T-shirts.

n	0	20	30	40	50	60	80	100	120	140
C										

c. Write an equation that represents the cost, C, to produce n T-shirts.

d. CotX sells each T-shirt for $25.50. Write an equation that represents the amount of sales, S, in dollars for selling n T-shirts.

e. By solving two simultaneous equations, determine the number of T-shirts that must be sold for CotX to break even.

f. If CotX needs to make a profit of at least $5000, determine the minimum number of T-shirts they will need to sell to achieve this outcome.

15. There are three types of fruit for sale at the market: starfruit, s, mango, m, and papaya, p. The following table shows the amount of fruit bought and the total cost in dollars.

Starfruit, s	**Mango, m**	**Papaya, p**	**Total cost, \$**
5	3	4	19.40
4	2	5	17.50
3	5	6	24.60

a. Using the pronumerals s, m and p, represent this information with three equations.

b. Using technology or otherwise, calculate the cost of one starfruit, one mango and one papaya.

c. Using your answer from part **b**, determine the cost of 2 starfruit, 4 mangoes and 4 papayas.

16. The Comet Cinema offers four types of tickets to the movies: adult, concession, senior and member. The table below shows the number and types of tickets bought to see four different movies and the total amount of tickets sales in dollars.

Movie	**Adult, a**	**Concession, c**	**Seniors, s**	**Members, m**	**Total sales, \$**
Wizard boy	24	52	12	15	1071.00
Champions	35	8	45	27	1105.50
Pixies on ice	20	55	9	6	961.50
Horror nite	35	15	7	13	777.00

a. Represent this information in four simultaneous equations, using the pronumerals given in the table.

b. Using technology or otherwise, determine the cost, in dollars, for each of the four different movie tickets.

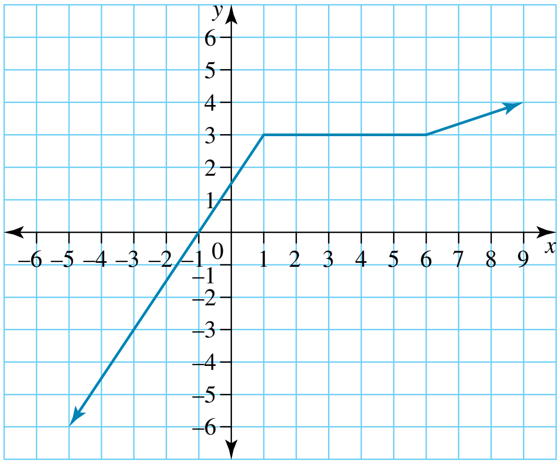
c. The blockbuster movie *Love hurts* took the following tickets sales: 77 adults, 30 concessions, 15 seniors and 45 members. Using your values from part **b**:

- **i.** write the expression that represents this information
- **ii.** determine the total ticket sales in dollars and cents.

7.5 Piecewise linear graphs and step graphs

7.5.1 Open and closed end points

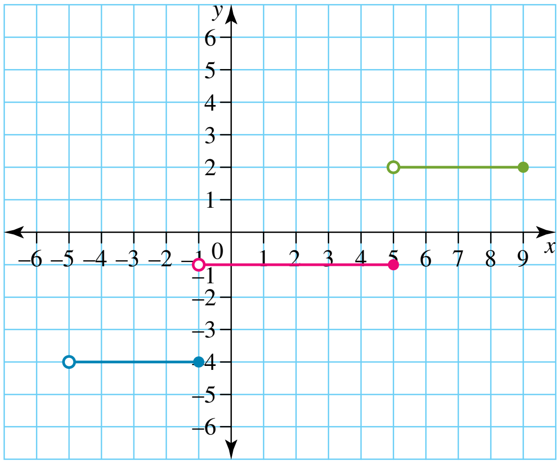
Piecewise graphs are formed by two or more linear graphs that are joined at points of intersection. A piecewise graph is continuous, which means there are no breaks or gaps in the graph, as shown in the diagram.



Step graphs are formed by two or more linear graphs that have zero gradients. Step graphs have breaks, as shown in the second diagram.

The end points of each line depend on whether the point is included in the interval. For example, the interval $-1 < x \le 5$ will have an open end point at $x = -1$, because x does not equal -1 in this case. The same interval will have a closed end point at $x = 5$, because x is less than or equal to 5.

A closed end point means that the x-value is also 'equal to' the value. An open end point means that the x-value is not equal to the value; that is, it is less than or greater than only.



WORKED EXAMPLE 9

A piecewise linear graph is constructed from the following linear graphs.

$$y = 2x + 1,\ x \le a$$
$$y = 4x - 1,\ x > a$$

a. **By solving the equations simultaneously, find the point of intersection and hence state the value of a.**

b. **Sketch the piecewise linear graph.**

THINK	WRITE/DRAW
a. 1. Find the intersection point of the two graphs by solving the equations simultaneously.	a. $y = 2x + 1$ $y = 4x - 1$

Solve by substitution:

$$\begin{aligned} 2x + 1 &= 4x - 1 \\ 2x - 2x + 1 &= 4x - 2x - 1 \\ 1 &= 2x - 1 \\ 1 + 1 &= 2x - 1 + 1 \\ 2 &= 2x \\ x &= 1 \end{aligned}$$

Substitute $x = 1$ to find y:

$$\begin{aligned} y &= 2(1) + 1 \\ &= 3 \end{aligned}$$

The point of intersection is $(1, 3)$.

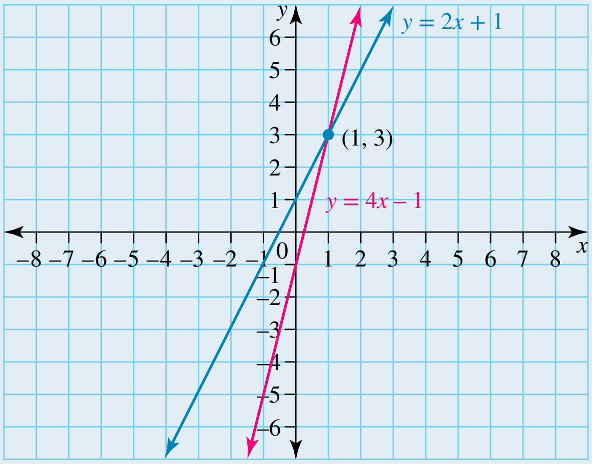
$x = 1$ and $y = 3$

2. The x-value of the point of intersection determines the x-intervals for where the linear graphs meet.

$x = 1$, therefore $a = 1$.

b. 1. Using a spreadsheet or otherwise, sketch the two graphs without taking into account the intervals.

b.

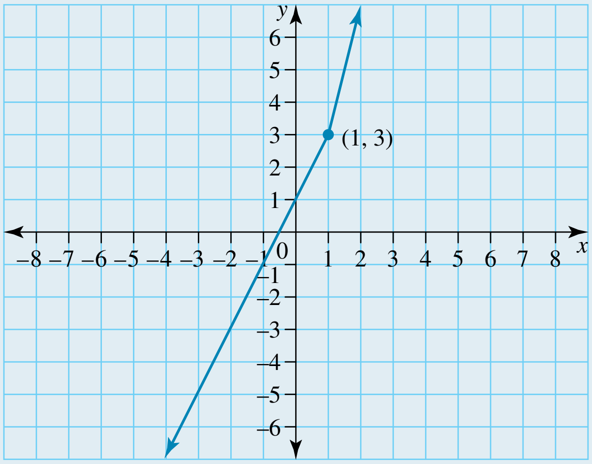


2. Identify which graph exists within the stated x-intervals to sketch the piecewise linear graph.

$y = 2x + 1$ exists for $x \leq 1$.

$y = 4x - 1$ exists for $x > 1$.

Remove the sections of each graph that do not exist for these values of x.



on Resources

Interactivity: Piecewise linear graphs (int-6486)

WORKED EXAMPLE 10

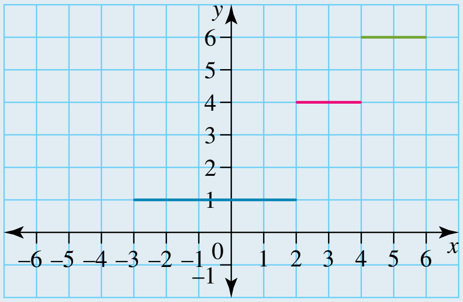
Construct a step graph from the following equations, making sure to take note of the relevant end points.

$$\mathbf{y = 1, -3 < x \leq 2}$$
$$\mathbf{y = 4, 2 < x \leq 4}$$
$$\mathbf{y = 6, 4 < x \leq 6}$$

THINK

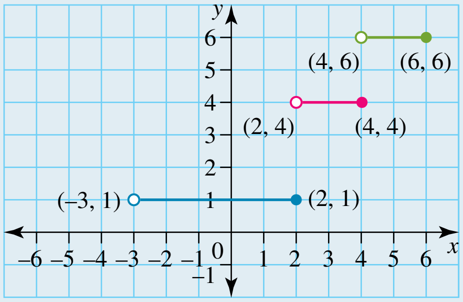
1. Construct a set of axes and draw each line within the stated x-intervals.

WRITE/DRAW



2. Draw in the end points.

For the line $y = 1$:
$-3 < x \leq 2$
$x > -3$ is an open circle.
$x \leq 2$ is a closed circle.
For the line $y = 4$:
$2 < x \leq 4$
$x > 2$ is an open circle.
$x \leq 4$ is a closed circle.
For the line $y = 6$:
$4 < x \leq 6$
$x > 4$ is an open circle.
$x \leq 6$ is a closed circle.



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Interactivity: Step functions (int-6281)

7.5.2 Modelling with piecewise linear and step graphs

Consider the real-life situation of a leaking water tank. For the first 3 hours it leaks at a constant rate of 12 litres per minute; after 3 hours the rate of leakage slows down (decreases) to 9 litres per minute. The water leaks at a constant rate in both situations and can therefore be represented as a linear graph. However, after 3 hours the slope of the line changes because the rate at which the water is leaking changes.

WORKED EXAMPLE 11

The following two equations represent the distance travelled by a group of students over 5 hours. Equation 1 represents the first section of the hike, when the students are walking at a pace of 4 km/h. Equation 2 represents the second section of the hike, when the students change their walking pace.

Equation 1: $d = 4t,\ 0 \leq t \leq 2$

Equation 2: $d = 2t + 4,\ 2 \leq t \leq 5$

The variable d is the distance in km from the campsite, and t is the time in hours.

a. **Determine the time, in hours, for which the group travelled in the first section of the hike.**

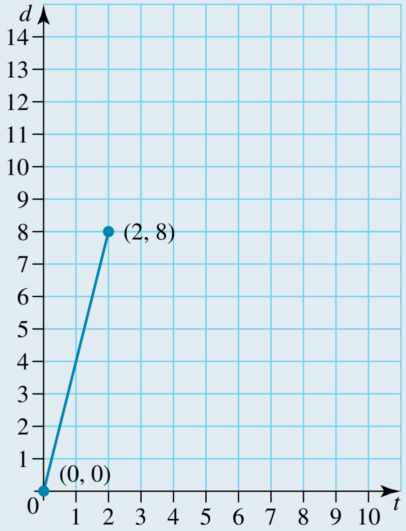
b. **i.** **What was their walking pace in the second section of their hike?**

  ii. **For how long, in hours, did they walk at this pace?**

c. **Sketch a piecewise linear graph to represent the distance travelled by the group of students over the five hour hike.**

THINK	WRITE/DRAW
a. **1.** Determine which equation the question applies to.	**a.** This question applies to Equation 1.
2. Look at the time interval for this equation.	$0 \leq t \leq 2$
3. Interpret the information.	The group travelled for 2 hours.
b. i. **1.** Determine which equation the question applies to.	**b. i.** This question applies to Equation 2.
2. Interpret the equation. The walking pace is found by the coefficient of t, as this represents the gradient.	$d = 2t + 4,\ 2 \leq t \leq 5$ The coefficient of t is 2
3. Answer the question.	The walking pace is 2 km/h.
ii. **1.** Look at the time interval shown.	**ii.** $2 \leq t \leq 5$
2. Interpret the information and answer the question.	They walked at this pace for 3 hours.
c. **1.** Find the distance travelled before the change of pace.	**c.** Change after $t = 2$ hours: $d = 4t$ $d = 4 \times 2$ $d = 8$ km

2. Using a calculator, spreadsheet or otherwise sketch the graph $d = 4t$ between $t = 0$ and $t = 2$.



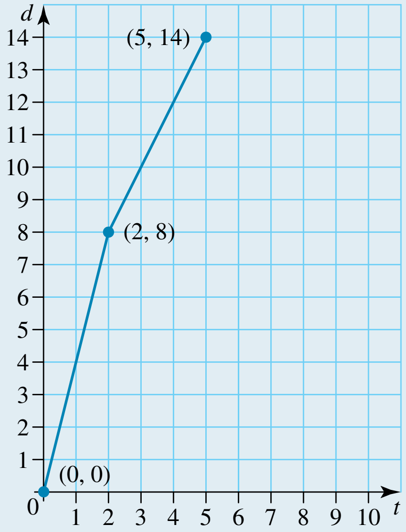
3. Solve the simultaneous equations by substitution to find the point of intersection; $d = 4t$ and $d = 2t + 4$ $\therefore 4t = 2t + 4$.

$$
\begin{aligned}
4t &= 2t + 4 \\
4t - 2t &= 2t - 2t + 4 \\
2t &= 4 \\
t &= 2
\end{aligned}
$$

Substitute $t = 2$ into $d = 4t$:

$d = 4 \times 2 = 8$

4. Using a spreadsheet or otherwise, sketch the graph of $d = 2t + 4$ between $t = 2$ and $t = 5$.



WORKED EXAMPLE 12

The following sign shows the car parking fees in a shopping carpark.

CARPARK FEES

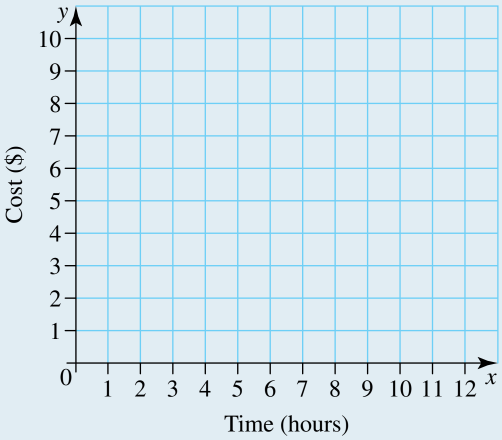
0 < 2 hours	**\$1.00**
2 < 4 hours	**\$2.50**
4 < 6 hours	**\$5.00**
6+ hours	**\$6.00**

Construct a step graph to represent this information.

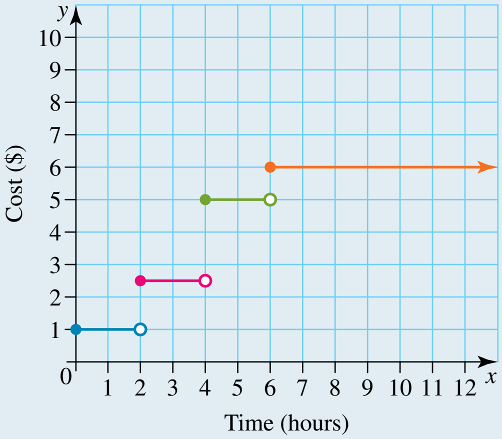
THINK

WRITE/DRAW

1. Draw up a set of axes, labelling the axes in terms of the context of the problem; that is, the time and cost. There is no change in cost during the time intervals, so there is no rate (i.e. the gradient is zero). This means we draw horizontal line segments during the corresponding time intervals.



2. Draw segments to represent the different time intervals. As the cost changes at the start of each time interval, this is represented by a closed circle. Hence, the end of a time period must be represented by an open circle.



studyon

Units 1 & 2 > Area 3 > Sequence 3 > Concept 4

Piecewise and step graphs Summary screen and practice questions

Exercise 7.5 Piecewise linear graphs and step graphs

1. **WE9** A piecewise linear graph is constructed from the following linear graphs.

$$y = -3x - 3,\ x \le a$$
$$y = x + 1,\ x \ge a$$

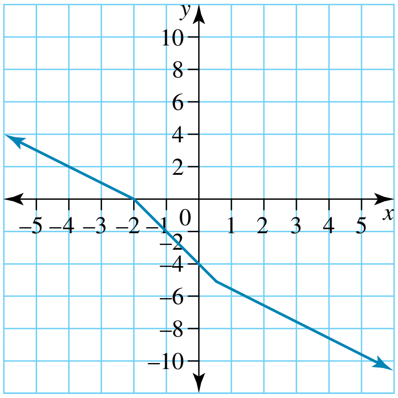
   a. By solving the equations simultaneously, find the point of intersection and hence state the value of a.
   b. Sketch the piecewise linear graph.

2. Consider the following linear graphs that make up a piecewise linear graph.

$$y = 2x - 3, x \le a$$
$$y = 3x - 4, a \le x \le b$$
$$y = 5x - 12, x \ge b$$

a. Using a spreadsheet or otherwise, sketch the three linear graphs.
b. Determine the two points of intersection.
c. Using the points of intersection, find the values of a and b.
d. Sketch the piecewise linear graph.

3. **MC** The diagram shows a piecewise linear graph. Which one of the following options represents the linear graphs that make up the piecewise graph?



A. $y = -2x - 4, x \le -2$
$y = -x - 2, \; -2 \le x \le 0.5$
$y = -x - 4.5, x \ge 0.5$

B. $y = -x - 2, \; x \le -2$
$y = -2x - 4, \; -2 \le x \le 0.5$
$y = -x - 4.5, \; x \ge 0.5$

C. $y = -2x - 4, x \le 0$
$y = -x - 2, 0 \le x \le -5$
$y = -x - 4.5, x \ge -5$

D. $y = -x - 2, \; x \le 0$
$y = -2x - 4, \; 0 \le x \le -5$
$y = -x - 4.5, \; x \ge -5$

4. The growth of a small tree was recorded over 6 months. It was found that the tree's growth could be represented by three linear equations, where h is the height in centimetres and t is the time in months.

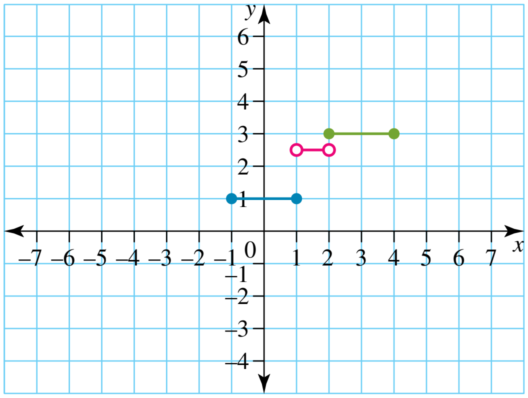
Equation 1: $h = 2t + 20, \; 0 \le t \le a$
Equation 2: $h = t + 22, \; a \le t \le b$
Equation 3: $h = 3t + 12, \; b \le t \le c$

a. i. By solving equations 1 and 2 simultaneously, determine the value of a.
   ii. By solving equations 2 and 3 simultaneously, determine the value of b.
b. Explain why $c = 6$.
c. During which time interval did the tree grow the most?
d. Sketch the piecewise linear graph that shows the height of the tree over the 6-month period.

5. **WE10** Construct a step graph from the following equations, making sure to take note of the relevant end points.

$$y = 3, \; 1 < x \le 4$$
$$y = 1.5, \; 4 < x \le 6$$
$$y = -2, \; 6 < x \le 8$$

6. A step graph is shown below. Write the equations that make up the graph.

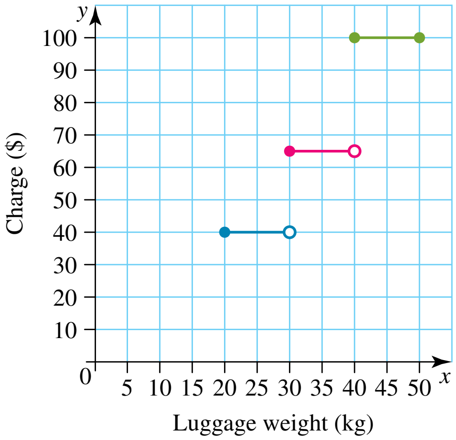


7. The following table shows the costs to hire a plumber.

Time (minutes)	Cost ($)
0 < 15	45
15 < 30	60
30 < 45	80
45 < 60	110

a. Represent this information on a step graph.
b. Anton hired the plumber for a job that took 23 minutes. How much will Anton be expected to be charged for this job?

8. Airline passengers are charged an excess for any luggage that weighs 20 kg or over. The following graph shows these charges for luggage weighing over 20 kg.



a. How much excess would a passenger be charged for luggage that weighs 31 kg?
b. Nerada checks in her luggage and is charged $40. What is the maximum excess luggage she could have without having to pay any more?
c. Hilda and Hanz have two pieces of luggage between them. One piece weighs 32 kg and the other piece weighs 25 kg. Explain how they could minimise their excess luggage charges.

9. **WE11** The following two equations represent water being added to a water tank over 15 hours, where w is the water in litres and t is the time in hours.
Equation 1: $w = 25t,\ 0 \le t \le 5$
Equation 2: $w = 30t - 25,\ 5 \le t \le 15$
a. Determine how many litres of water are in the tank after 5 hours.
b. i. At what rate is the water being added to the tank after 5 hours?
   ii. For how long is the water added to the tank at this rate?
c. Sketch a piecewise graph to represent the water in the tank at any time, t, over the 15-hour period.

10. A car hire company charges a flat rate of $50 plus 75 cents per kilometre up to and including 150 kilometres. An equation to represent this cost, C, in dollars is given as $C = 50 + ak,\ 0 \le k \le b$, where k is the distance travelled in kilometres.
a. Write the values of a and b.
b. Using a spreadsheet or otherwise, sketch this equation on a set of axes, using appropriate values.
   The cost charged for distances over 150 kilometres is given by the equation $C = 87.50 + 0.5k$.

c. Determine the charge in cents per kilometre for distances over 150 kilometres.

d. By solving the two equations simultaneously, find the point of intersection and hence show that the graph will be continuous.

e. Sketch the equation $C = 87.50 + 0.5k$ for $150 \leq k \leq 300$ on the same set of axes as part **b**.

11. The temperature of a wood-fired oven, T°C, steadily increases until it reaches 200°C. Initially the oven has a temperature of 18°C and it reaches the temperature of 200°C in 10 minutes.

a. Construct an equation that finds the temperature of the oven during the first 10 minutes. Include the time interval, t, in your answer.

Once the oven has heated up for 10 minutes, a loaf of bread is placed in the oven to cook for 20 minutes. An equation that represents the temperature of the oven during the cooking of the bread is $T = 200,\ a \leq t \leq b$.

b. i. Write the values of a and b.

ii. In the context of this problem, what do a and b represent?

After the 20 minutes of cooking, the oven's temperature is lowered. The temperature decreases steadily, and after 30 minutes the oven's temperature reaches 60°C. An equation that determines the temperature of the oven during the last 30 minutes is $T = mt + 340,\ d \leq t \leq e$.

c. Determine the values of m, d and e.

d. What does m represent in this equation?

e. Using your values from the previous parts, sketch the graph that shows the changing temperature of the wood fired oven during the 60-minute interval.

12. **WE12** The costs to hire a paddle boat are listed in the following table. Construct a step graph to represent the cost of hiring a paddle boat for up to 40 minutes.

Time (minutes)	Hire cost ($)
$0 < 20$	15
$20 < 30$	20
$30 < 40$	25

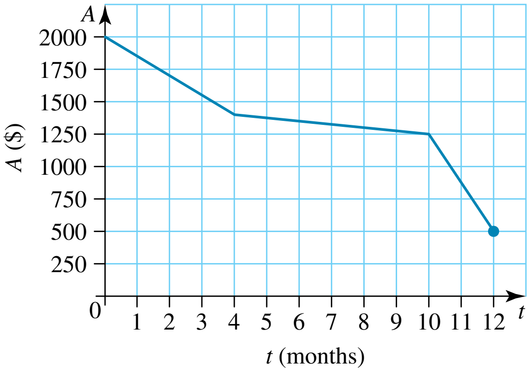
13. The postage costs to send parcels from the Northern Territory to Sydney are shown in the following table:

Weight of parcel (kg)	Cost ($)
$0 < 0.5$	6.60
$0.5 < 1$	16.15
$1 < 2$	21.35
$2 < 3$	26.55
$3 < 4$	31.75
$4 < 5$	36.95

a. Represent this information in a step graph.

b. Pammie has two parcels to post to Sydney from the Northern Territory. One parcel weighs 450 g and the other weighs 525 g. Is it cheaper to send the parcels individually or together? Justify your answer using calculations.

14. The amount of money in a savings account over 12 months is shown in the following piecewise graph, where A is the amount of money in dollars and t is the time in months.



One of the linear graphs that make up the piecewise linear graph is $A = 2000 - 150t$, $0 \leq t \leq a$.

a. Determine the value of a.

b. The equation that intersects with $A = 2000 - 150t$ is given by $A = b - 50t$. If the two equations intersect at the point (4, 1400), show that $b = 1600$.

c. The third equation is given by the rule $A = 4100 - 300t$. By solving a pair of simultaneous equations, find the time interval for this equation.

d. Using an appropriate equation, determine the amount of money in the account at the end of the 12 months.

15. The following linear equations represent the distance sailed by a yacht from the yacht club during a race, where d is the distance in kilometres from the yacht club and t is the time in hours from the start of the race.

Equation 1: $d = 20t$, $0 \leq t \leq 0.75$

Equation 2: $d = 15t + 3.75$, $0.75 \leq t \leq 1.25$

Equation 3: $d = -12t + 37.5$, $1.25 \leq t \leq b$

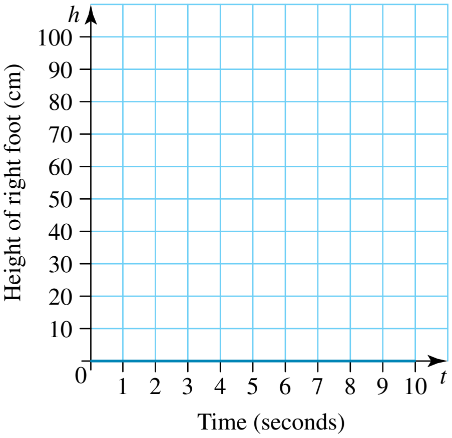
a. Using a spreadsheet or otherwise, determine the points of intersection.

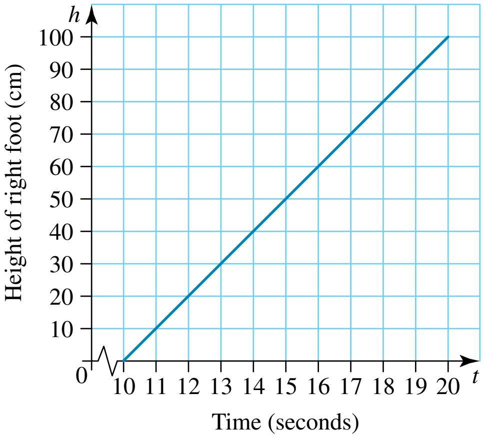
b. In the context of this problem, explain why equation 3 has a negative gradient.

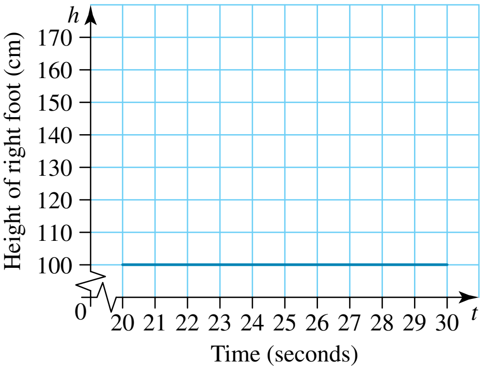
c. Calculate how far the yacht is from the starting point before it turns and heads back to the yacht club.

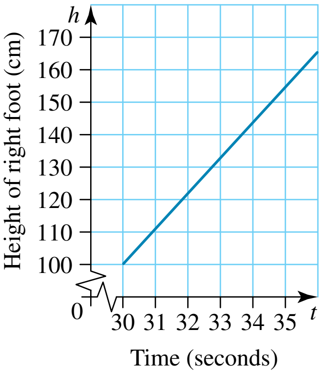
d. Determine the duration, to the nearest minute, of the yacht's sailing time for this race. Hence, find the value for b. Write your answer correct to 2 decimal places.

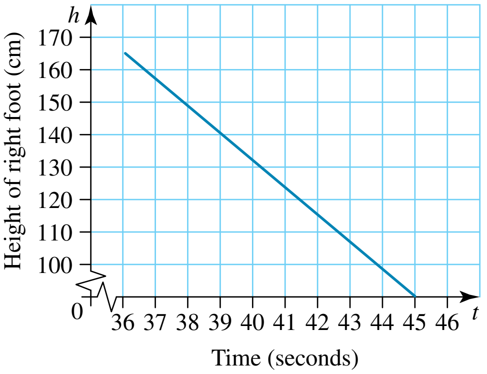
16. The distance of a dancer's right foot from the floor during a dance recital can be found using the following linear graphs, where h is the height in centimetres from the floor and t is the duration of the recital in seconds.

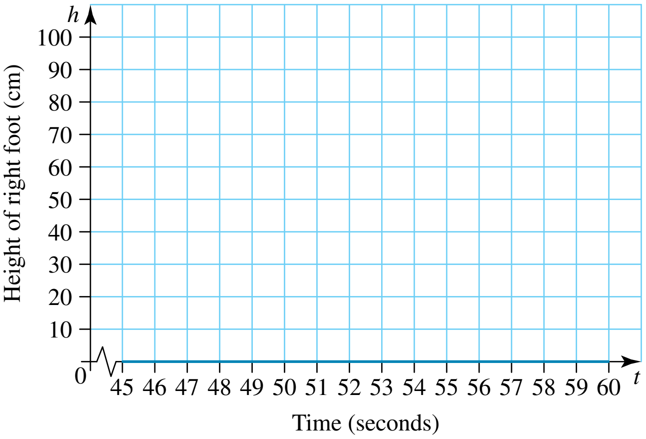












a. During which time interval(s) was the dancer's right foot on the floor? Explain your answer.

b. Calculate the maximum height the dancer's right foot was from the floor.

c. How long, in seconds, was the recital?

d. Construct the graph that shows the distance of the dancer's right foot from the floor at any time during the recital.
Clearly label all key features.

17. Stamp duty is a government charge on the purchase of items such as cars and houses. The table shows the range of stamp duty charges for purchasing a car in South Australia.

Car price ($$P$)	Stamp duty (S)
0–1000	1%
1000–2000	$10 + 2%($P$ – 1000)
2000–3000	$30 + 3%($P$ – 2000)
3000+	$60 + 4%($P$ – 3000)

a. Explain why the stamp duty costs for cars can be modelled by a piecewise linear graph.
The stamp duty charge for a car purchased for \$1000 or less can be expressed by the equation $S = 0.01P$, where S is the stamp duty charge and P is the purchase price of the car for $0 \leq P \leq 1000$. Similar equations can be used to express the charges for cars with higher prices.

Equation 1: $S = 0.01P,\ 0 \leq P \leq 1000$
Equation 2: $S = 0.02P - 10,\ a < P \leq b$
Equation 3: $S = 0.03P - c,\ 2000 < P \leq d$
Equation 4: $S = fP - e,\ P > 3000$

b. For equations 2, 3 and 4, determine the values of a, b, c, d, e and f.

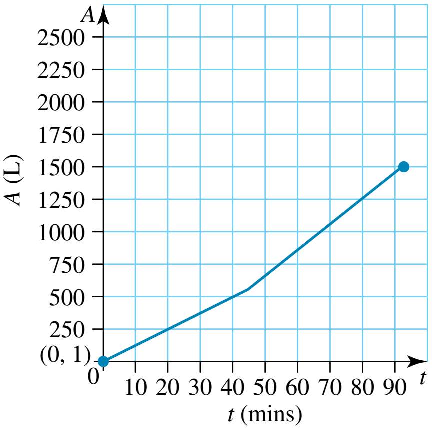
c. Using a spreadsheet or otherwise, find the points of intersections for the equations in part **b**.

d. Suki and Boris purchase a car and pay \$45 in stamp duty. What price did they pay for their car?

18. A small inflatable swimming pool that holds 1500 litres of water is being filled using a hose. The amount of water, A, in litres in the pool after t minutes is shown in the following graph.

a. Estimate the amount of water, in litres, in the pool after 45 minutes.

b. Determine the amount of water being added to the pool each minute during the first 45 minutes.
After 45 minutes the children become impatient and turn the hose up.



The equation $A = 20t - 359$ determines the amount of water, A, in the pool t minutes after 45 minutes.

c. Using this equation, determine the time taken, in minutes, to fill the pool. Give your answer to the nearest whole minute.

19. a. Using a spreadsheet or otherwise, find the points of intersection for the following four linear graphs.

i. $y = x + 4,\ x \leq a$
ii. $y = 2x + 3,\ a \leq x \leq b$
iii. $y = x + 6,\ b \leq x \leq c$
iv. $y = 3x + 1,\ x \geq c$

b. Using your values from part **a**, complete the x-intervals for the linear graphs by finding the values of a, b and c.

c. What problem do you encounter when trying to sketch a piecewise linear graph formed by these four linear graphs?

20. The Slippery Slide ride is a new addition to a famous theme park. The slide has a horizontal distance of 20 metres and is comprised of four sections. The first section is described by the equation $h = -3x + 12,\ 0 \leq x \leq a$, where h is the height in metres from the ground and x is the horizontal distance in metres from the start. In the first section, the slide drops 3 metres over a horizontal distance of 1 metre before meeting the second section.

a. What is the maximum height of the slide above ground?

b. State the value of a.

The remaining sections of the slides are modelled by the following equations.

Section 2: $h = -\frac{2x}{3} + \frac{29}{3},\ a \le x \le b$

Section 3: $h = -2x + 13,\ b \le x \le c$

Section 4: $h = -\frac{5x}{16} + \frac{25}{4}, c \le x \le d$

c. Using a spreadsheet or otherwise, find the points of intersection between each section of the slide and hence find the values of b and c.

d. Explain why $d = 20$.

e. Sketch the graph that shows the height at any horizontal distance from the start of the slide.

7.6 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

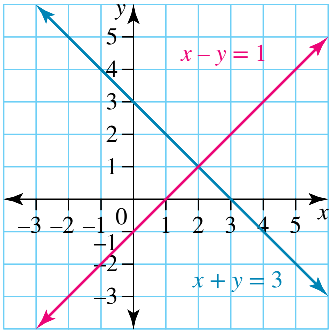
Simple familiar

The following information relates to Questions 1 and 2.

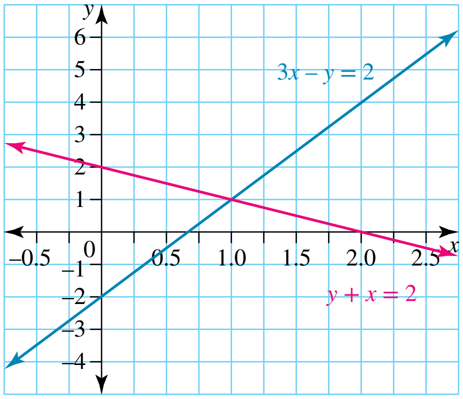
Juliana and Alyssa live in two towns 232 km apart. One weekday they both have to drive to each other's town for a business meeting, leaving at the same time in the morning. Alyssa drives an average of 12 km/h faster than Juliana, and they pass each other after 2 hours.

1. For each of the following graphs, determine the coordinates of the point of intersection. Justify your answers.

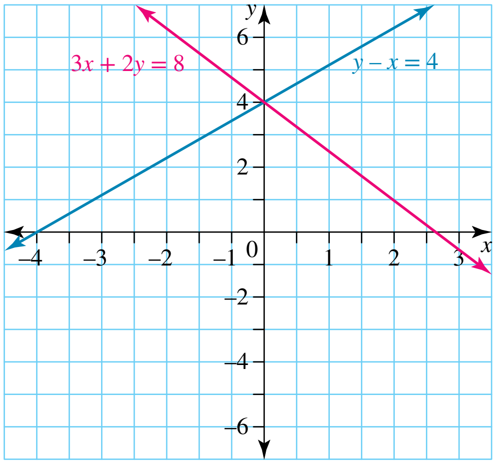
a.



b.



c.



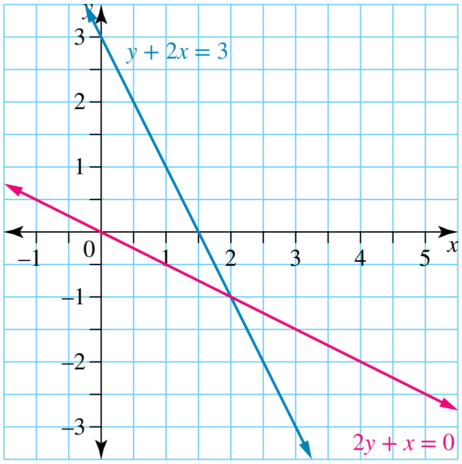
2. **MC** The solution to the following pair of simultaneous equations is:

$$x + 5y = 7$$
$$y = 5 - 2x$$

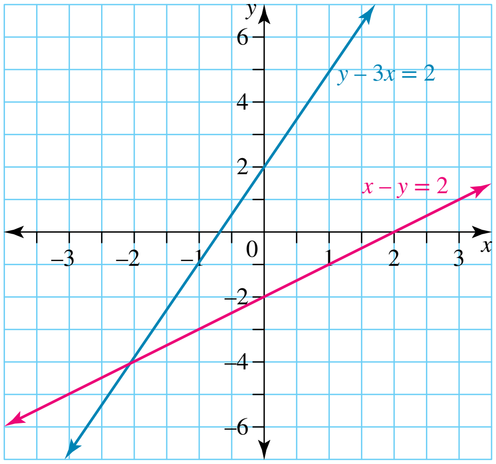
A. $x = -8,\ y = 3$
B. $x = 2.25,\ y = 0.5$
C. $x = 2,\ y = 1$
D. $x = 1,\ y = 2$

3. For each of the following graphs, determine the coordinates of the point of intersection. Justify your answers.

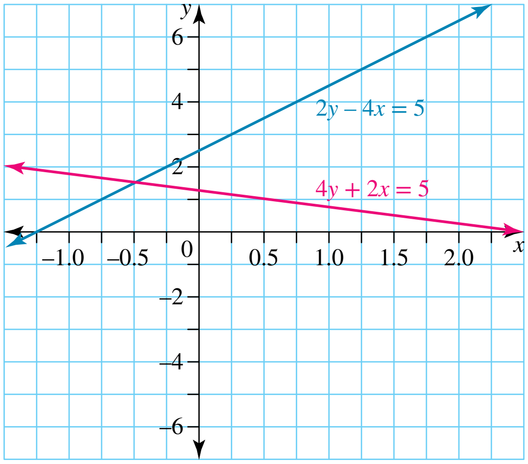
a.



b.



c.



4. Solve the following pairs of simultaneous equations by graphing the equations and identifying the point of intersection.

a. $y = 2x + 3$
$y = 8 - 3x$

b. $y + 2x = -8$
$y = 2x + 4$

c. $2y = 12x + 16$
$3y = -6x - 24$

d. $2x - y = -1$
$3x + y = 11$

5. For each of the pairs of equations below, calculate the gradients and indicate whether the lines are parallel or perpendicular or neither.

a. $y = 2x - 4$
$6y - 12x = 20$

b. $2y = 5 - 6x$
$3y = -9x + 18$

c. $3y + 2x = 9$
$6y + 4x = 18$

d. $y = -2x + 3$
$y = \frac{1}{2}x - 5$

e. $y = 4x + 5$
$2y - 15x = 10$

f. $4y = 6 - x$
$y = 4x + 6$

6. Solve the system of equations $y = 5x$ and $y = -3x + 8$ by substitution.

7. Solve the system of equations $x + 3y = 1$ and $y = 2x + 5$ by elimination by first rearranging the equations.

8. Solve the following pairs of simultaneous equations by substitution and identifying the point of intersection.

a. $31 = y - 2x$
$2x + 2y = 14$

b. $x + 2y = 4$
$y = 2x - 3$

c. $x + y = 6$
$2x + 4y = 20$

d. $y = \frac{2x}{3} + 2$
$y = 2x - 2$

9. Solve the following simultaneous equations by using elimination.

a. $y = 3x - 5$ and $3x + 2y = 17$

b. $2x + y = 7$ and $3x - y = 3$

c. $3x - 2y = -16$ and $y = 2(x + 9)$

d. $4x + 3y = 17$ and $3x + 2y = 13$

10. Determine the break-even points for the following cost and revenue equations. Where appropriate, give your answers correct to 2 decimal places.

a. $C = 150 + 2x$ and $R = 7.5x$

b. $C = 13.5x + 25$ and $R = 19.7x$

Complex familiar

11. Use substitution to solve each of the following pairs of simultaneous equations.

a. $5x + 2y = 17$
$y = \frac{3x - 7}{2}$

b. $2x + 7y = 17$
$x = \frac{1 - 3y}{4}$

c. $2x + 3y = 13$
$y = \frac{4x - 15}{5}$

d. $-2x - 3y = -14$
$x = \frac{2 + 5y}{3}$

12. Solve the simultaneous equations $y + x = 10$ and $-2y + x = -5$ using technology.

13. **MC** Two adults and four children went to the circus. They paid a total of $55.00 for their tickets. One adult and three children paid $35.00 to enter the same circus. Which one of the following sets of simultaneous equations represents this situation, where a is the cost for an adult and c is the cost of a child's entry?

A. $a + c = 55$
$a + c = 35$

B. $2a + c = 55$
$a + 3c = 35$

C. $2a + 3c = 55$
$a + 4c = 35$

D. $2a + 4c = 55$
$a + 3c = 35$

14. **MC** Bertha knits teddy bears and sells them at the local farmers' market. Bertha spends $120 in wool, and it costs her an additional $4.50 to make each teddy bear. She sells the bears for $14.50 each. How many teddy bears does Bertha need to sell to break even?

A. 6

B. 8

C. 9

D. 12

15. A step graph is formed by the following equations:

$$y = -3,\ 0 < x \le 3$$
$$y = 2,\ 3 < x < 5$$
$$y = 5,\ x \ge 5$$

Construct a step graph to represent this information.

16. The following two linear equations make a piecewise linear graph.

$$y = -2x + 1,\ x \le a$$
$$y = -3x + 2,\ x \ge a$$

a. Solve the equations simultaneously, and hence find the value of a.

b. Sketch the piecewise linear graph.

Complex unfamiliar

17. Use substitution to solve each of the following pairs of simultaneous equations for x and y in terms of m and n.

a. $mx - y = n$
$y = nx$

b. $mx - ny = n$
$y = x$

c. $mx - ny = -m$
$x = y - n$

d. $mx - y = m$
$x = \frac{y + m}{n}$

18. Determine the values of a and b so that the pair of equations $ax + by = 17$ and $2ax - by = -11$ has only one solution of $(-2, 3)$.

19. Suzanne is starting a business selling homemade cupcakes. It will cost her \$250 to buy all of the equipment, and each cupcake will cost \$2.25 to make. Suzanne models her costs, C, in dollars, to make n cupcakes using the following equation:

$$C = 2.25n + \$250$$

a. How much will it cost Suzanne to make 50 cupcakes?

b. Suzanne receives an order to supply cupcakes for an afternoon tea. If it costs Suzanne \$373.75 to make the order, how many cupcakes did she make?

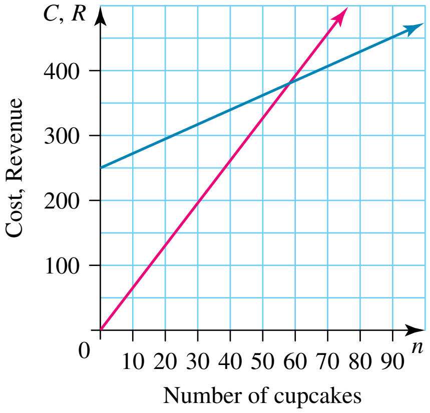
c. Suzanne sells each cupcake for \$6.50. Write an equation to represent the revenue Suzanne earns, R, from selling n cupcakes.

d. One week, Suzanne sells 150 cupcakes. Determine the total profit, in dollars, that Suzanne makes for that week.

e. By solving a pair of simultaneous equations, calculate the total number of cupcakes Suzanne needs to sell to break even. Give your answer correct to the nearest whole number.

f. The graph shows Suzanne's cost, C, and revenue, R, for making and selling n cupcakes.
On the graph, add labels for:

- **i.** the line that represents the cost, C
- **ii.** the line that represents the revenue, R
- **iii.** the break-even point.



20. Jerri and Samantha have both entered a 10-km fun run for charity.
The distance travelled by Jerri can be modelled by the linear equation

$$d = 6t - 0.1$$

where d is the distance in km from the starting point and t is time in hours.

a. Determine the time taken for Jerri to run the 10 kilometres. Give your answer correct to the nearest minute.

b. In the context of this problem, explain the meaning of the d-intercept (y-intercept).

The distance Samantha is from the starting point at any time, t hours, can be modelled by the piecewise linear graph

$$d = 4t,\ 0 \leq t \leq \frac{1}{2}$$

$$d = 8t - 2,\ \frac{1}{2} \leq t \leq b$$

c. How far, in kilometres, did Samantha travel in the first 30 minutes?

d. Determine the speed at which Samantha was travelling in the first 30 minutes.

e. Explain how Samantha's run changed after 30 minutes.

f. i. Determine the value of b.
   ii. Hence, show that Samantha crossed the finishing line ahead of Jerri by 11 minutes.

studyon

Units 1 & 2 Sit chapter test

Answers

Chapter 7 Simultaneous equations and their applications

Exercise 7.2 Solving simultaneous linear equations graphically

1. a. (3, 3) **b.** (0, 1)
c. (−3, 4) **d.** $\left(\frac{5}{2}, 5\right)$

2. a. Yes, because the coordinates of the point satisfy both equations.
b. No, because the point doesn't satisfy both equations.

3. a. 10 minutes
b. Company T: $y = x + 10$
Company O: $y = \frac{1}{2}x + 15$
c. By substitution $y = 20$ when $x = 10$.

4. a. Since the gradients are different the lines will intersect.
b. The graphs intersect along the whole length since the lines have the same gradient and y-intercept.
c. Since the gradients are different the lines will intersect.
d. i. A system of linear equations will have a unique solution if the equations have **different** gradients and **different** y-intercepts.
ii. A system of linear equations will have an infinite number of solutions if the equations have **the same** gradient and **the same** y-intercept.

5. a. Car A travels 60 km, Car B travels 60 km.
b. Car A: $m = \frac{60}{1} = 60$
Car B: $m = \frac{60}{1} = 60$
c. The cars are never in the same spot at the same time because Car A is always 20 km ahead of Car B.
d. The y-intercepts indicate that Car A started at 20 km from the town, while Car B started from the town.

6. a. $x = 1, y = -2$ **b.** $x = 2, y = 4$
c. $x = 2, y = -1$ **d.** $x = -1, y = 3$
e. $x = 0, y = 0$ **f.** $x = \frac{5}{2}, y = -\frac{1}{2}$

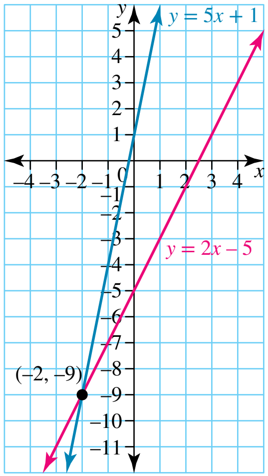
7. $x = 2, y = 3$

8. Your friend is 6 years old and her brother is 5 years old.

9. a. Apples cost $0.25.
b. Oranges cost $0.50, so the difference is $0.25.

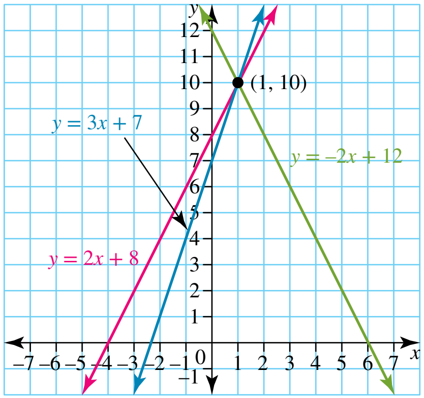
10. $x = -2, y = 5$

11.



$x = -2, y = -9$

12.



$x = 1, y = 10$

13. a. $x = -2, y = -7$
b. $x = 2, y = -3$
c. $x = -5, y = -18$
d. No solution, lines are parallel

14. a. i. (−12, −54) **ii.** (−1, −2)
iii. No solution **iv.** (−1, 4)
b. No, the graphs in part **iii** are parallel (they have the same gradient).

Exercise 7.3 Solving simultaneous equations algebraically

1. a. $x = -1, y = -1$ **b.** $m = 11, n = 3$

2. a. $x = 3, y = -3$ **b.** $x = 1, y = 4$

3. D

4. a. $x = 7, y = 19$ **b.** $x = -2, y = -12$
c. $x = 7, y = 44$ **d.** $x = 2, y = 1$
e. $x = -3, y = -4$ **f.** $x = 1, y = -1$

5. Add the two equations and solve for x, then substitute x into one of the equations to solve for y
$x = 3, \ y = -1$

6. a. $x = 2, y = -2$ **b.** $x = 3, y = 4$
c. $x = -1, y = 3$ **d.** $x = 5, y = 3$
e. $x = -1, y = 4$ **f.** $x = 6, y = 4$

7. $a = 2$ and $x = -2$

8. A

9. a. Marcia added the equations instead of subtracting. The correct result for step 2 is $22y = 11$
b. $x = 5, \ y = \frac{1}{2}$
c. $x = 5 \ y = \frac{1}{2}$

10. a. Goal = 5 points
Behind = 2 points
b. Jetts = 40 points
Meteorites = 48 points

11. a. Equation 2 has unknowns on each side of the equal sign and can be substituted into equation 1.
b. Mick works 5 hours and Minnie works 10 hours.
c. 3 hours 45 minutes (3.75 hours)

12. a. $x = -10.71, y = -12.86$
b. $x = 0.75, y = 0.89$

Exercise 7.4 Solving practical problems using simultaneous equations

1. $4d + 3c = 10.55$ and $2d + 4c = 9.90$

2. a. 140 adults and 210 children

b. The cost of an adult's ticket is \$25, the cost of a children's ticket is \$15, and the total ticket sales is \$6650.

3. a. i. 10

ii. Yolanda needs to sell 10 bracelets to cover her costs.

b. i. \$16 loss

ii. \$24 profit

4. a. $a = 18$

b. $b = 3$

c. 170 entries

d. $R = \$5580, C = \$3480, P = \$2100$

e. 504 entries

5. a. The price per adult ticket is \$3.50 and the price per student ticket is \$1.50.

b. 4 adults and 19 students

6. a. $a = \$19.50, c = \$14.50$

b. The total number of tickets sold (both adult and concession)

c. 225 adult tickets and 319 concession tickets

7. a. $R = 12.50h$

b. 4.5 hours

c. i. Charlotte made a profit for jobs 1 and 4, and a loss for jobs 2 and 3.

ii. Yes, she made \$15 profit.

$(25 + 5 - (10 + 5)) = 30 - 15 = \$15$

d. 9.5 hours

8. a. $15t + 12m = 400.50$ and $9t + 13m = 328.75$

b. t represents the hourly rate earned by Trudi and m represents the hourly rate earned by Mia.

c. $t = \$14.50, m = \$15.25$

9. a. $5x + 4y = 31.55$ and $4x + 3y = 24.65$

b. $x = \$3.95, y = \$2.95$

c. \$12.35

10. a. $3s + 2g = 1000$ and $4s + 3g = 1430$

b. 140 kJ

11. a. $C = 75 + 1.10k$ and $C = 90 + 0.90k$

b. $k = 75$ km

c. $C_{\text{Freewheels}} = \$350$, $C_{\text{GetThere}} = \$315$. They should use GetThere.

12. a. The cost is for the three different types of cereal, but the equations only include one type of cereal.

b. $2c + 3r + m = 27.45$

$c + 2r + 2m = 24.25$

$3c + 4r + m = 36.35$

c. \$34.15

13. a. $S = 0.5n$

b. 8 cups of lemonade

c. 70 cents

14. a. \$1560

b. *

c. $C = 810 + 7.5n$

d. $S = 25.50n$

e. 45 T-shirts

f. 323 T-shirts

15. a. $5s + 3m + 4p = 19.4$

$4s + 2m + 5p = 17.5$

$3s + 5m + 6p = 24.6$

b. $s = \$1.25, m = \$2.25, p = \$1.60$

c. \$17.90

16. a. $24a + 52c + 12s + 15m = 1071$

$35a + 8c + 45s + 27m = 1105.5$

$20a + 55c + 9s + 6m = 961.5$

$35a + 15c + 7s + 13m = 777$

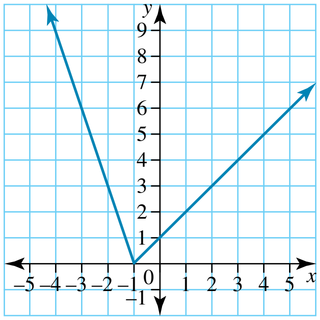
b. Adult ticket = \$13.50, Concession = \$10.50, Seniors = \$8.00, Members = \$7.00

c. i. $77 \times 13.50 + 30 \times 10.50 + 15 \times 8.00 + 45 \times 7.00$

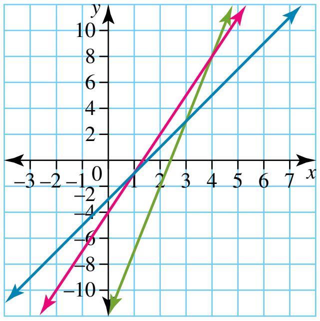
ii. \$1789.50

Exercise 7.5 Piecewise linear graphs and step graphs

1. a. Point of intersection = $(-1, 0)$, $a = -1$



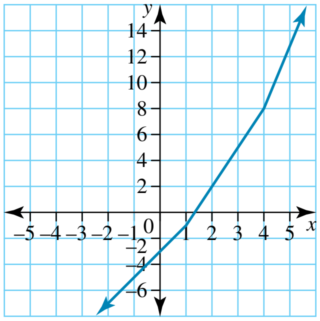
2. a.



b. $(1, -1)$ and $(4, 8)$

c. $a = 1$ and $b = 4$

d.



14. b. *

n	0	20	30	40	50	60	80	100	120	140
C	810	960	1035	1110	1185	1260	1410	1560	1710	1860

3. B

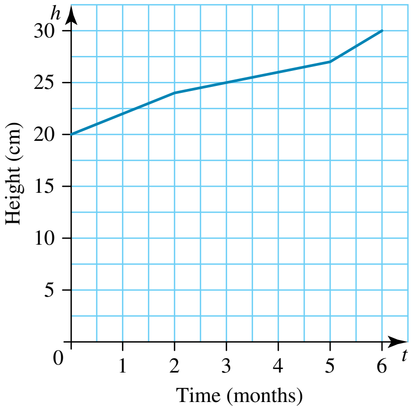
4. a. i. $a = 2$

ii. $b = 5$

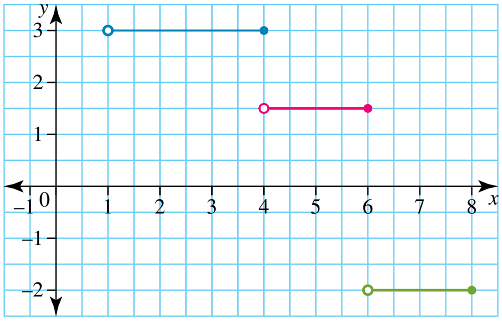
b. The data is only recorded over 6 months.

c. $5 \leq t \leq 6$ (between 5 and 6 months)

d.

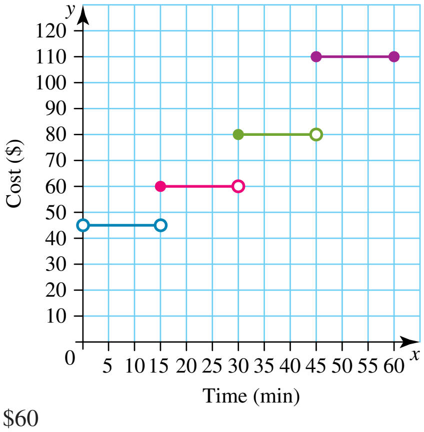


5.



6. $y = 1, -1 \leq x \leq 1; y = 2.5,\ 1 < x < 2; y = 3,\ 2 \leq x \leq 4$

7. a.



b. \$60

8. a. \$65

b. 10 kg

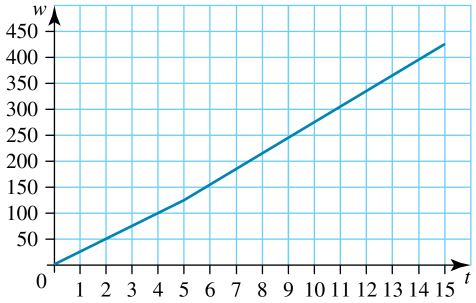
c. Place 2–3kg from the 32kg bag into the 25kg bag and pay \$80 rather than \$105

9. a. 125 L

b. i. 30 L/h

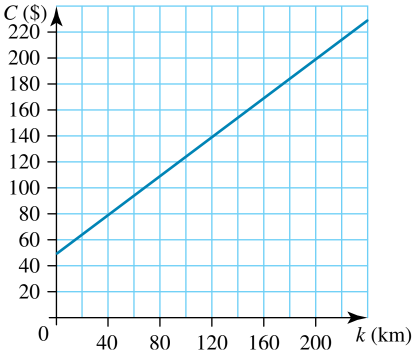
ii. 10 h

c.



10. a. $a = 0.75, b = 150$

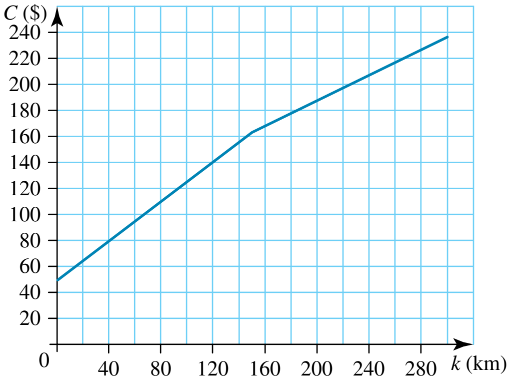
b.



c. 50 cents/km

d. $k = 150, C = 162.50$. This means that the point of intersection (150, 162.5) is the point where the charges change. At this point both equations will have the same value, so the graph will be continuous.

e.



11. a. $T = 18 + 18.2t,\ 0 \leq t \leq 10$

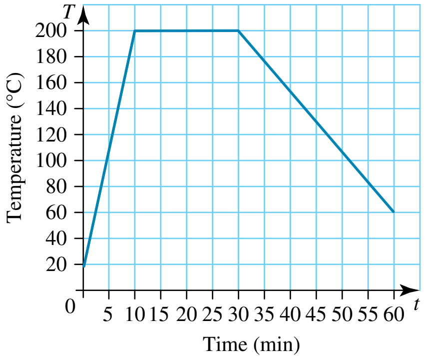
b. i. $a = 10,\ b = 30$

ii. a is the time the oven first reaches 200°C and b is the time at which the bread stops being cooked.

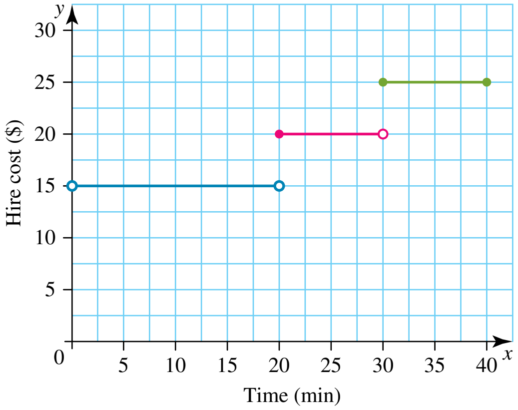
c. $m = \dfrac{-14}{3},\ d = 30,\ e = 60$

d. The change in temperature for each minute in the oven

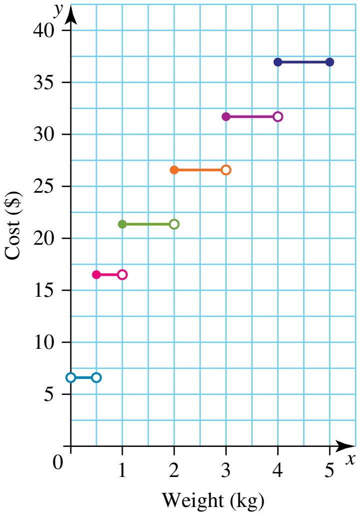
e.



12.



13. a.



b. It is cheaper to post them together ($16.15 together versus $22.75 individually).

14. a. $a = 4$

b. Sample responses can be found in the worked solutions in the online resources.

c. $10 \le t \le 12$

d. $500

15. a. $(0.75,\ 15)$ and $(1.25,\ 22.5)$

b. The yacht is returning to the yacht club during this time period.

c. 22.5 km

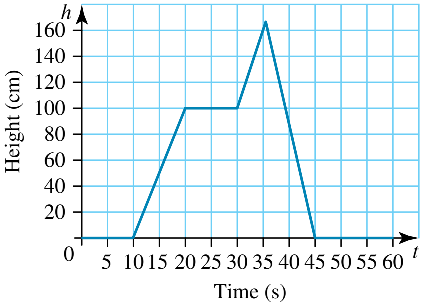
d. 3 hours, 8 minutes; $b = 3.13$

16. a. $0 \le t \le 10$ and $45 \le t \le 60$; these are the intervals when $y = 0$.

b. 165 cm

c. 60 seconds

d.



17. a. There is a change in the rate for different x-values (i.e. different car prices).

b. $a = 1000, b = 2000, c = 30, d = 3000, e = 60, f = 0.04$

c. $(1000,\ 10)$, $(2000,\ 30)$ and $(3000,\ 60)$

d. $2500

18. a. 540 L **b.** 12 L/ min **c.** 93 min

19. a. $(1,\ 5)$, $(3,\ 9)$ and $(2.5,\ 8.5)$

b. $a = 1, b = 3, c = 2.5$

i. $x \le 1$ **ii.** $1 \le x \le 3$ **iii.** $3 \le x \le 2.5$ **iv.** $x \le 2.5$

c. $b > c$, which means that graph **iii** is not valid and the piecewise linear graph cannot be sketched.

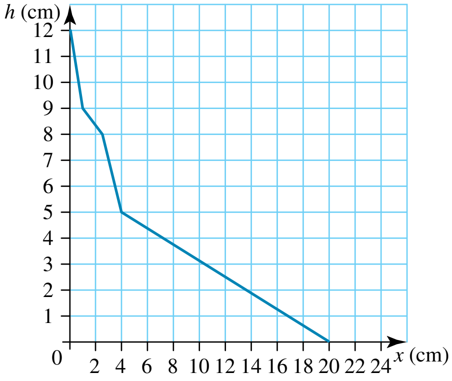
20. a. 12 m

b. $a = 1$

c. $(1,\ 9)$, $(2.5,\ 8)$ and $(4,\ 5)$; $b = 2.5, c = 4$

d. The horizontal distance of the slide is 20 m.

e.



7.6 Review: exam practice

1. a. $(2,\ 1)$ **b.** $(1, 1)$ **c.** $(0, 4)$

2. C

3. a. $(2, -1)$ **b.** $(-2, -4)$ **c.** $(-0.5, 1.5)$

4. a. $(1, 5)$ **b.** $(-3, -2)$

c. $(-2, -4)$ **d.** $(2, 5)$

5. a. $m_1 = 2 \quad m_2 = 2$ Parallel

b. $m_1 = -3 \quad m_2 = -3$ Parallel

c. $m_1 = -\frac{2}{3} \quad m_2 = -\frac{2}{3}$ Parallel

d. $m_1 = -2 \quad m_2 = \frac{1}{2}$ Perpendicular

e. $m_1 = 4 \quad m_2 = 7.5$ Neither

f. $m_1 = -\frac{1}{4} \quad m_2 = 4$ Perpendicular

6. $(1, 5)$

7. $(-2,\ 1)$

8. a. $(-8, 15)$ **b.** $(2, 1)$

c. $(2, 4)$ **d.** $(3, 4)$

9. **a.** $x = 3,\ y = 4$

b. $x = 2,\ y = 3$

c. $x = -20,\ y = -22$

d. $x = 5,\ y = -1$

10. **a.** $x = 27.27$ **b.** $x = 4.03$

11. **a.** $(3, 1)$ **b.** $(-2, 3)$

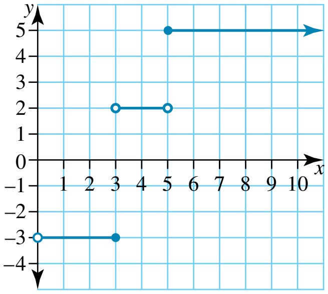
c. $(5, 1)$ **d.** $(4, 2)$

12. $(5, 5)$

13. D

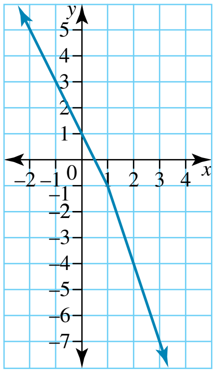
14. D

15.



16. **a.** 1

b.



17. **a.** $\left(\dfrac{n}{m-n}, \dfrac{n^2}{m-n}\right)$ **b.** $\left(\dfrac{n}{m-n}, \dfrac{n}{m-n}\right)$

c. $\left(\dfrac{n^2-m}{m-n}, \dfrac{mn-m}{m-n}\right)$ **d.** $(0, -m)$

18. $a = -1,\ b = 5.$

19. **a.** \$362.50

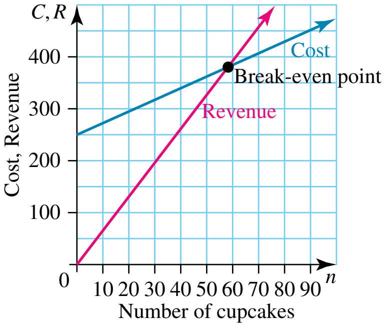
b. 55

c. $R = 6.5n$

d. \$387.50

e. 59

f.



20. **a.** 1 hour, 41 minutes

b. Jerri started 0.1 km (100 metres) behind the starting line.

c. 2 km

d. The gradient of the equation equals the speed; therefore, Samantha was travelling at 4 km/h.

e. After 30 minutes, Samantha increased her speed from 4 km/h to 8 km/h.

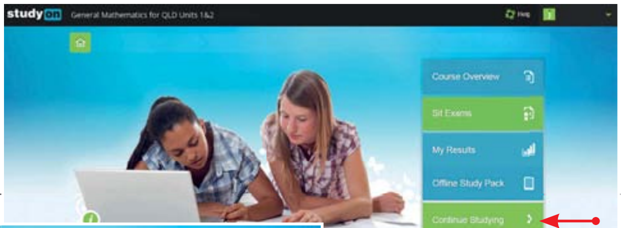
f. **i.** 1.5

ii. Samantha took 1 hour, 30 minutes hours to run 10 km; Jerri took 1 hour 41 minutes. Difference: 41–30 minutes = 11 minutes

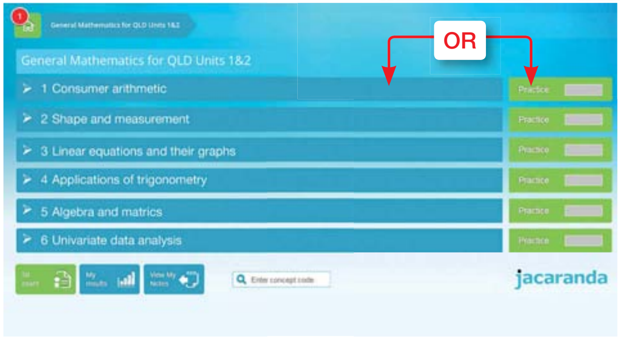
REVISION UNIT 1 Money, measurement and relations

TOPIC 3 Linear equations and their graphs

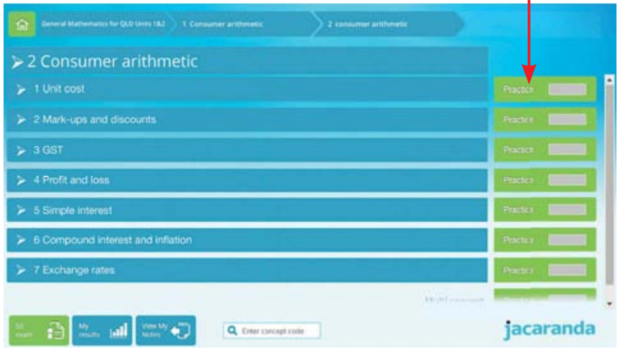
- For revision of this entire topic, go to your **studyON** title in your bookshelf at **www.jacplus.com.au**.
- Select **Continue Studying** to access hundreds of revision questions across your entire course.



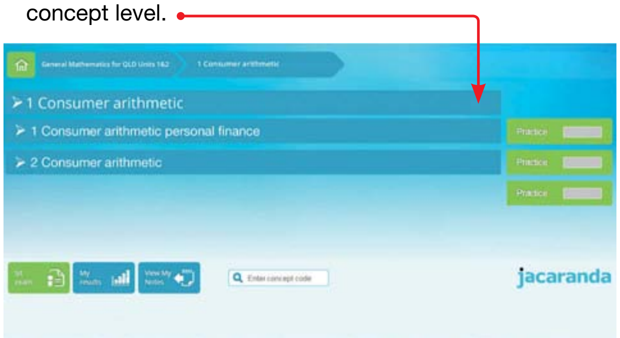
- Select your **course** *General Mathematics for Queensland Units 1&2* to see the entire course divided into syllabus topics.
- Select the **area** you are studying to navigate into the sequence level *OR* select **Practice** to answer all practice questions available for each area.



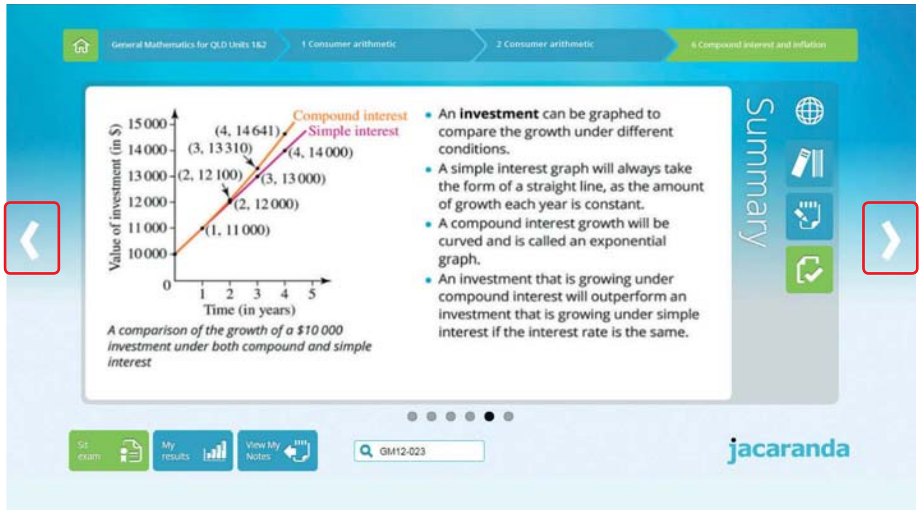
- Select **Practice** at the sequence level to access all questions in the sequence.



- At **sequence level**, drill down to concept level.



- **Summary screens** provide revision and consolidation of key concepts. Select the **next arrow** to revise all concepts in the sequence and practice questions at the concept level for a more granular set of questions.



A comparison of the growth of a $10 000 investment under both compound and simple interest

- An **investment** can be graphed to compare the growth under different conditions.
- A simple interest graph will always take the form of a straight line, as the amount of growth each year is constant.
- A compound interest growth will be curved and is called an exponential graph.
- An investment that is growing under compound interest will outperform an investment that is growing under simple interest if the interest rate is the same.

PRACTICE ASSESSMENT 2

General Mathematics: Unit 1 examination

Unit

Unit 1: Money, measurement and relations

Topic

Topic 1: Consumer arithmetic
Topic 2: Shape and measurement
Topic 3: Linear equations and their graphs

Conditions

Response Type	**Duration**	**Reading**
Short response	120 minutes	5 minutes
Resources	**Instructions**	
• QCAA formula sheet • Notes not permitted • Scientific calculator permitted	• Show all working. • Write responses using a black or blue pen. • Unless otherwise instructed, give answers to **two decimal places**.	

Criterion	**Marks allocated**	**Result**
Foundational knowledge and problem solving *Assessment objectives 1, 2, 3, 4, 5 and 6	65	

* Queensland Curriculum & Assessment Authority, *Specialist Mathematics General Senior Syllabus 2019 v1.1*, Brisbane, 2018.
For the most up to date assessment information, please see www.qcaa.qld.edu.au/senior.

A detailed breakdown of the examination marks summary can be found in the PDF version of this assessment instrument in your eBookPLUS.

Part A: Simple familiar — total marks: 39

Question 1 (3 marks)

The formula for simple interest is $I = \frac{PRT}{100}$, where I = the interest earned, P = the principal invested, R = the annual interest rate and T = the number of years the money is invested.

a. Rearrange the formula to make T the subject of the formula.

b. Determine the number of years that \$2500 should be invested at 5% p.a. to earn \$500 interest.

Question 2 (3 marks)

Calculate the original price of a TV that sold for \$845.75 after it was discounted by 15%.

Question 3 (2 marks)

A cube has a volume of 12.5 cm^3. Determine its volume in mm^3.

Question 4 (3 marks)

What is the cost of a taxi trip over 32 km if the flag fall is \$3.50 and the cost per km is \$2.25?

Question 5 (3 marks)

A TV and a surround sound system package deal is priced at $2880. On a sale it is discounted by 25%.

a. Determine the discounted amount to the nearest cent.

b. Calculate the new price of the package deal.

Question 6 (3 marks)

Matthew sells plants from growing seedlings. He can source the seedlings from two different suppliers. Supplier A charges $650 for 500 seedlings, whereas Supplier B charges $1000 for 800 seedlings.

a. Use the unit cost method to identify which supplier gives the best deal.

b. If Supplier A was on sale with a 7.5% discount, how would this affect the choice of preferred supplier?

Question 7 (5 marks)

Construct a step graph to represent the hire cost of a surfboard in Hawaii.

Surfboard hire	
Hours	Rate ***($)***
0–1	$20
> 1–2	$30
> 2–4	$60
Daily charge:	$75

Question 8 (2 marks)

Kyle is going to the USA for a holiday and wants to take some $US with him. The selling rate of $US is 0.78, and he has $\$750$ in Australian dollars to convert. Determine the amount he receives in $US to the nearest cent.

Question 9 (4 marks)

Joanne wants to re-carpet her bedroom. From her plans with a scale of $1:100$, the room is 3.5 cm by 4.2 cm.

a. Determine the actual length and width of the bedroom.

b. Calculate the area of the bedroom in m^2.

Question 10 (2 marks)

Complete the table to state the gradients and y-intercepts of the following equations.

Equation	Gradient	y-intercept
$y = -3x + 2$		
$2y - 6x = 10$		

Question 11 (5 marks)

Nathaniel has a casual job that earns him $\$17.50$ per hour. He can also earn time-and-a-half on a Saturday and double time on a Sunday. When he works four or more hours he takes a 20-minute unpaid break. Calculate his weekly pay given he works the following hours.

Day	Times worked
Friday	5 pm to 9 pm
Saturday	9 am to 2 pm
Sunday	11 am to 4 pm

Question 12 (4 marks)

Solve for x and y:

$y = 4x - 6$

$x = 2y + 5$

Part B: Complex familiar — total marks: 13

Question 13 (6 marks)

Jane and Jack went to the supermarket to buy some fruit. Jane bought 6 apples and 4 oranges for $\$2.90$, while Jack bought 9 apples and 5 oranges for $\$4.00$. Set up simultaneous equations to calculate the individual price of an apple and an orange.

Question 14 (3 marks)

Shane purchased 500 shares in a company at $\$12.30$ per share. Two years later he sold the shares for $\$15.50$ per share. He was charged a brokerage fee of 3% for buying and selling the shares.

Determine the percentage gain over the two-year period.

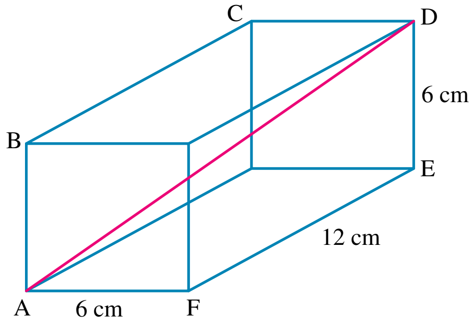
Question 15 (4 marks)

Calculate the surface area of a cube with a volume of $64\,\text{cm}^3$.

Part C: Complex unfamiliar — total marks: 13

Question 16 (5 marks)

Determine the length from A to D for the following rectangular prism.



Question 17 (5 marks)

A tennis ball manufacturer makes tennis balls with a diameter of 7 cm. They are sold in sets of 4 which fit exactly into a cylindrical can.

a. The manufacturer wants to create a new label which will completely cover the side of the can. Determine the surface area of the new label.

b. The manufacturer decides to redesign the packaging of the 4 tennis balls. They decide to encase each ball in a plastic shell which is 3% larger than each individual tennis ball. Assuming the new plastic shells are joined at a single point, determine the volume of the new plastic packaging.

Question 18 (3 marks)

Electricity is charged to households at 18.924 cents per kilowatt hour (kWh) along with a fixed supply charge. Kasey used 664 kWh over 61 days. Her bill for this usage was $\$438.31$. Determine the fixed supply charge.

CHAPTER 8
Applications of trigonometry

8.1 Overview

Trigonometry is a branch of mathematics that describes the relationship between angles and side lengths of triangles. Trigonometry is widely used in many areas:

- **Architecture and engineering:** much of architecture and engineering relies on the formation of triangles for support structures. When an architect wants to correctly lay out a curved wall, work out the slope of a roof, or its correct height, trigonometry is used.
- **Music theory:** music theory involves sound waves, and sound waves travel in a repeating wave pattern. This repeating pattern can be represented graphically by sine and cosine functions. A single note can be modelled on a sine curve and a chord can be modelled with multiple sine curves.
- **Electrical engineers:** the electricity sent to our house requires an understanding of trigonometry. Power companies use what is called alternating current (AC) to send electricity over long distances. This is due to the use of transformers, which require the use of alternating current to function. The alternating current signal has a sinusoidal behaviour.
- **Video games:** when you see a character smoothly glide over road blocks, they don't jump vertically straight up the y-axis, but follow a slightly curved path or a parabolic path. Trigonometric calculations help animators ensure their characters jump over these obstacles in a realistic manner.

Trigonometry is not limited to the areas listed above. It is used in many others, such as flight engineering, physics, archaeology, criminology and marine biology. This shows how important an understanding of trigonometry is since it is used in so many different fields.

LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

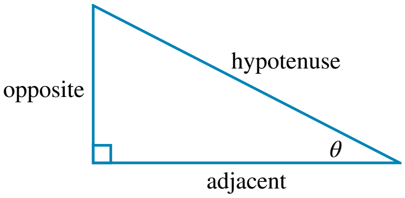
8.2 Review of trigonometric ratios

8.2.1 sin, cos and tan

We have already looked at Pythagoras' theorem, which enabled us to find the length of one side of a right-angled triangle given the lengths of the other two. However, to deal with other relationships in right-angled triangles, we need to turn to **trigonometry**.

Trigonometry allows us to work with the angles also; that is, deal with relationships between angles and sides of right-angled triangles. For example, trigonometry enables us to find the length of a side, given the length of another side and the magnitude of an angle.

So that we are clear about which lines and angles we are describing, we need to identify the given angle, and name the shorter sides with reference to it. For this reason, we label the sides **opposite** and **adjacent** — that is, the sides opposite and adjacent to the given angle. The diagram shows this relationship between the sides and the angle, θ.



The trigonometric ratios are constant for a particular angle, and this is the reason the shadow-stick method worked, as demonstrated in Chapter 4.

Trigonometry uses the ratio of side lengths to calculate the lengths of sides and the size of angles. The ratio of the opposite side to the adjacent side is called the **tangent ratio**. This ratio is fixed for any particular angle.

The tangent ratio for any angle, θ, can be found using the result:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Calculators require a particular sequence of button presses in order to perform this calculation. Investigate the sequence required for your particular calculator.

For all calculations in trigonometry you will need to make sure that your calculator is in **DEGREE MODE**. Check the set-up on your calculator to ensure that this is the case.

WORKED EXAMPLE 1

Using your calculator, calculate the following, correct to 3 decimal places.

a. $\tan 60°$ **b.** $15\tan 75°$ **c.** $\dfrac{8}{\tan 69°}$ **d.** $\tan 49° 32'$

THINK	WRITE/DISPLAY
a. With a scientific calculator, press tan and enter 60, then press =.	**a.** $\tan 60° = 1.732$
b. Enter 15, press x and tan, enter 75, then press =.	**b.** $15\tan 75° = 55.981$
c. Enter 8, press ÷ and tan, enter 69, then press =.	**c.** $\dfrac{8}{\tan 69°} = 3.071$
d. Press tan, enter 49, press DMS, enter 32, press DMS, then press =.	**d.** $\tan 49°32' = 1.172$

Note: Some calculators require that the angle size be entered before the trigonometric functions.

The tangent ratio is used to solve problems involving the opposite side and the adjacent side of a right-angled triangle. The tangent ratio does not allow us to solve problems that involve the **hypotenuse**.

The **sine ratio** (abbreviated to sin; pronounced *sine*) is the name given to the ratio of the opposite side and the hypotenuse.

Regardless of the size of the triangle in any right-angled triangle with equal angles, the ratio of the length of the opposite side to the length of the hypotenuse will remain the same. The formula for the sine ratio is:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

The value of the sine ratio for any angle is found using the sin function on the calculator.

$$\sin 30^\circ = 0.5$$

Check this on your calculator.

WORKED EXAMPLE 2

Calculate, correct to 3 decimal places:

a. $\sin 57^\circ$ **b.** $9 \sin 45^\circ$ **c.** $\frac{18}{\sin 44^\circ}$ **d.** $9.6 \sin 26^\circ 12'$.

THINK	WRITE/DISPLAY
a. With a scientific calculator, press sin and enter 57, then press =.	**a.** $\sin 57^\circ = 0.839$
b. Enter 9, press x and sin, enter 45, then press =.	**b.** $9 \sin 45^\circ = 6.364$
c. Enter 18, press ÷ and sin, enter 44, then press =.	**c.** $\frac{18}{\sin 44^\circ} = 25.912$
d. Enter 9.6, press x and sin, enter 26, press DMS, enter 12, press DMS, then press =.	**d.** $9.6 \sin 26^\circ 12' = 4.238$

Note: Check the sequence of button presses required by your calculator.

Cosine stands for the sine of the complementary angle. The **cosine ratio** is found using the formula:

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

To calculate the cosine ratio for a given angle on your calculator, use the cos function. On your calculator check the calculation:

$$\cos 30^\circ = 0.866$$

WORKED EXAMPLE 3

Calculate, correct to 3 decimal places:

a. $\cos 27^\circ$ **b.** $6\cos 55^\circ$ **c.** $\dfrac{21.3}{\cos 74^\circ}$ **d.** $\dfrac{4.5}{\cos 82^\circ 46'}$.

THINK	WRITE/DISPLAY
a. With a scientific calculator, press **COS** and enter 27, then press **=**.	**a.** $\cos 27^\circ = 0.891$
b. Enter 6, press **X** and **COS**, enter 55, then press **=**.	**b.** $6\cos 55^\circ = 3.441$
c. Enter 21.3, press **÷** and **COS**, enter 74, then press **=**.	**c.** $\dfrac{21.3}{\cos 74^\circ} = 77.275$
d. Enter 4.5, press **÷** and **COS**, enter 82, press **DMS**, enter 46, press **DMS**, then press **=**.	**d.** $\dfrac{4.5}{\cos 82^\circ 46'} = 35.740$

Note: Check the sequence requirements for your calculator.

Similarly, if we are given the sine, cosine or tangent of an angle, we are able to calculate the size of that angle using the calculator. We do this using the inverse functions. On most calculators these are the second function of the sin, cos and tan functions and are denoted $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.

WORKED EXAMPLE 4

Calculate θ, correct to the nearest degree, given that $\sin\theta = 0.738$.

THINK	WRITE/DISPLAY
1. With a scientific calculator, press **2nd F** (or **SHIFT**) **[sin−1]** and enter .738, then press **=**.	
2. Round your answer to the nearest degree.	$\theta = 48^\circ$

Note: Check the sequence requirements for your calculator.

Problems sometimes supply angles in degrees, minutes and seconds, or require answers to be written in the form of degrees, minutes and seconds. On scientific calculators, you will use the **DMS** (Degrees, Minutes, Seconds) function or the **° ' "** function.

WORKED EXAMPLE 5

Given that $\tan\theta = 1.647$, calculate θ to the nearest minute.

THINK	WRITE/DISPLAY
1. With a scientific calculator, press **2nd F** (or **SHIFT**) **[tan−1]** and enter 1.647, then press **=**.	
2. Convert your answer to degrees and minutes by pressing **DMS**.	$\theta = 58^\circ 44'$

studyon

Units 1 & 2 > Area 4 > Sequence 1 > Concept 1

Trigonometric ratios Summary screen and practice questions

Exercise 8.2 Review of trigonometric ratios

1. **WE1** Calculate the value of each of the following, correct to 3 decimal places.
   - a. $\tan 57^\circ$
   - b. $9 \tan 63^\circ$
   - c. $\dfrac{8.6}{\tan 12^\circ}$
   - d. $\tan 33^\circ 19'$
2. **WE2** Calculate the value of each of the following, correct to 3 decimal places.
   - a. $\sin 37^\circ$
   - b. $9.3 \sin 13^\circ$
   - c. $\dfrac{14.5}{\sin 72^\circ}$
   - d. $\dfrac{48}{\sin 67^\circ 40'}$
3. **WE3** Calculate the value of each of the following, correct to 3 decimal places.
   - a. $\cos 45^\circ$
   - b. $0.25 \cos 9^\circ$
   - c. $\dfrac{6}{\cos 24^\circ}$
   - d. $5.9 \cos 2^\circ 3'$
4. Calculate the value of each of the following, correct to 3 decimal places, if necessary.
   - a. $\sin 30^\circ$
   - b. $\cos 15^\circ$
   - c. $\tan 45^\circ$
   - d. $48 \tan 85^\circ$
5. Calculate the value of each of the following, correct to 3 decimal places, if necessary.
   - a. $128 \cos 60^\circ$
   - b. $9.35 \sin 8^\circ$
   - c. $\dfrac{4.5}{\cos 32^\circ}$
   - d. $\dfrac{0.5}{\tan 20^\circ}$
6. Calculate the value of each of the following, correct to 2 decimal places.
   - a. $\sin 24^\circ 38'$
   - b. $\tan 57^\circ 21'$
   - c. $\cos 84^\circ 40'$
   - d. $\dfrac{15}{\sin 72^\circ}$
7. Calculate the value of each of the following, correct to 2 decimal places.
   - a. $9 \cos 55^\circ 30'$
   - b. $4.9 \sin 35^\circ 50'$
   - c. $2.39 \tan 8^\circ 59'$
   - d. $\dfrac{19}{\tan 67^\circ 45'}$
8. Calculate the following to 2 decimal places.
   - a. $\dfrac{49.6}{\cos 47^\circ 25'}$
   - b. $\dfrac{0.84}{\sin 75^\circ 5'}$
9. **WE4** Calculate θ, correct to the nearest degree, given that:
   - a. $\sin \theta = 0.167$
   - b. $\sin \theta = 0.277$.
10. Calculate θ, correct to the nearest degree, given that:
    - a. $\sin \theta = 0.698$
    - b. $\cos \theta = 0.173$
11. **WE5** Calculate θ, correct to the nearest minute, given that:
    - a. $\cos \theta = 0.058$
    - b. $\tan \theta = 1.517$.
12. Calculate θ, correct to the nearest minute, given that:
    - a. $\tan \theta = 0.931$
    - b. $\cos \theta = 0.854$

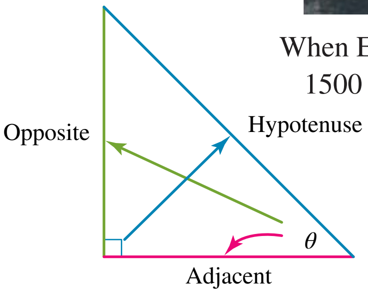
8.3 Applications of trigonometric ratios

8.3.1 Trigonometric ratios

It is important to identify and label the features given in a right-angled triangle. The labelling convention of a right-angled triangle is as follows:

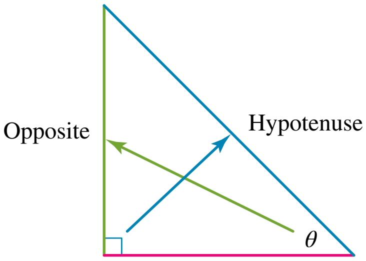
The longest side of a right-angled triangle is always called the hypotenuse and is opposite the right angle. The other two sides are named in relation to the reference angle, θ. The opposite side is opposite the reference angle, and the adjacent side is next to the reference angle.

When Egyptians first used a sundial around 1500 BC they were using trigonometry.



8.3.2 The sine ratio

The sine ratio is used when we want to find an unknown value given two out of the three following values: opposite, hypotenuse and reference angle.



The sine ratio of θ is written as $\sin(\theta)$ and is defined as follows:

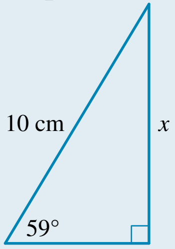
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \text{ or } \sin(\theta) = \frac{O}{H}$$

The inverse sine function is used to find the value of the unknown reference angle given the lengths of the hypotenuse and opposite side.

$$\theta = \sin^{-1}\left(\frac{O}{H}\right)$$

WORKED EXAMPLE 6

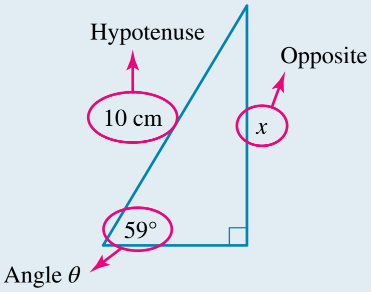
Calculate the length of x correct to 2 decimal places.



THINK

WRITE/DRAW

1. Label all the given information on the triangle.



2. Since we have been given the combination of opposite, hypotenuse and the reference angle θ, we need to use the sine ratio. Substitute the given values into the ratio equation.

$$\sin(\theta) = \frac{O}{H}$$

$$\sin(59°) = \frac{x}{10}$$

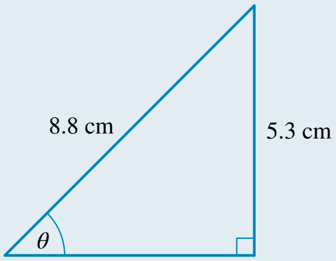
3. Rearrange the equation to make the unknown the subject and solve.
Make sure your calculator is in degree mode.

$$x = 10\sin(59°)$$
$$= 8.57$$

The opposite side length is 8.57 cm.

WORKED EXAMPLE 7

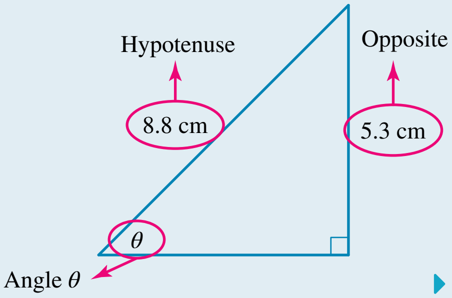
Calculate the value of the unknown angle, θ, correct to 2 decimal places.



THINK

WRITE/DRAW

1. Label all the given information on the triangle.



2. Since we have been given the combination of opposite, hypotenuse and the reference angle θ, we need to use the sine ratio. Substitute the given values into the ratio equation.

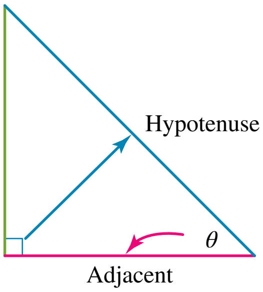
$$\sin(\theta) = \frac{O}{H} = \frac{5.3}{8.8}$$

3. To find the angle θ, we need to use the inverse sine function.
   Make sure your calculator is in degree mode.

$$\theta = \sin^{-1}\left(\frac{5.3}{8.8}\right) = 37.03°$$

8.3.3 The cosine ratio

The cosine ratio is used when we want to find an unknown value given two out of the three following values: adjacent, hypotenuse and reference angle.



The cosine ratio of θ is written as $\cos(\theta)$ and is defined as follows:

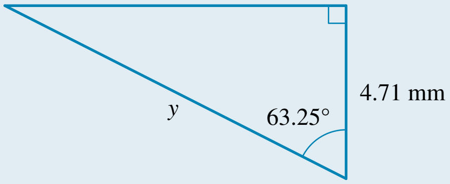
$$\cos(\theta) = \frac{\textbf{adjacent}}{\textbf{hypotenuse}} \quad \textbf{or} \quad \cos(\theta) = \frac{A}{H}$$

The inverse cosine function is used to find the value of the unknown reference angle when given lengths of the hypotenuse and adjacent side.

$$\theta = \cos^{-1}\left(\frac{A}{H}\right)$$

WORKED EXAMPLE 8

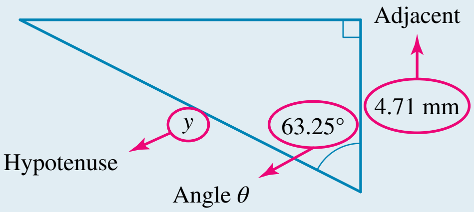
Calculate the length of y correct to 2 decimal places.



THINK

WRITE/DRAW

1. Label all the given information on the triangle.



2. Since we have been given the combination of adjacent, hypotenuse and the reference angle θ, we need to use the cosine ratio. Substitute the given values into the ratio equation.

$$\cos(\theta) = \frac{A}{H}$$
$$\cos(63.25°) = \frac{4.71}{y}$$

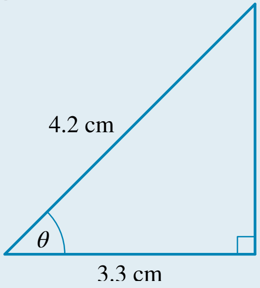
3. Rearrange the equation to make the unknown the subject and solve.
   Make sure your calculator is in degree mode.

$$y = \frac{4.71}{\cos(63.25°)}$$
$$= 10.46$$

The length of the hypotenuse is 10.46 mm.

WORKED EXAMPLE 9

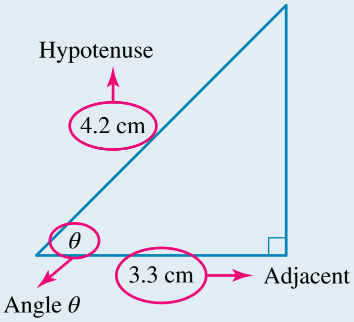
Calculate the value of the unknown angle, θ, correct to 2 decimal places.



THINK

WRITE/DRAW

1. Label all the given information on the triangle.



2. Since we have been given the combination of adjacent, hypotenuse and the reference angle θ, we need to use the cosine ratio. Substitute the given values into the ratio equation.

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{A}{H}$$

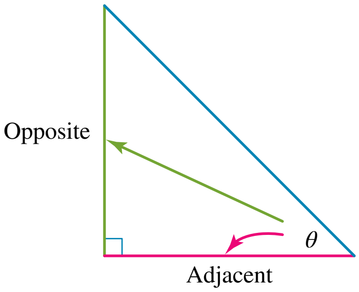
$$\cos(\theta) = \frac{3.3}{4.2}$$

3. To find angle θ, we need to use the inverse cosine function.
Make sure your calculator is in degree mode.

$$\theta = \cos^{-1}\left(\frac{3.3}{4.2}\right)$$
$$= 38.21°$$

8.3.4 The tangent ratio

The tangent ratio is used when we want to find an unknown value given two out of the three following values: opposite, adjacent and reference angle.



The tangent ratio of θ is written as $\tan(\theta)$ and is defined as follows:

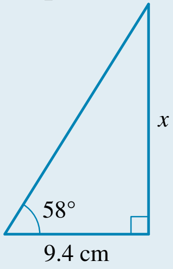
$$\tan(\theta) = \frac{\textbf{opposite}}{\textbf{adjacent}} \quad \textbf{or} \quad \tan(\theta) = \frac{O}{A}$$

The inverse tangent function is used to find the value of the unknown reference angle given the lengths of the adjacent and opposite sides.

$$\theta = \tan^{-1}\left(\frac{O}{A}\right)$$

WORKED EXAMPLE 10

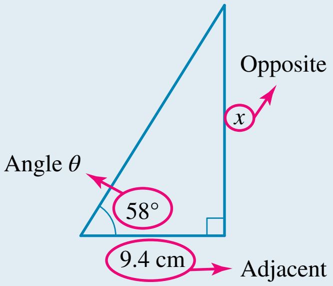
Calculate the length of x correct to 2 decimal places.



THINK

WRITE/DRAW

1. Label all the given information on the triangle.



2. Since we have been given the combination of opposite, adjacent and the reference angle θ, we need to use the tangent ratio. Substitute the given values into the ratio equation.

$$\tan(\theta) = \frac{O}{A}$$

$$\tan(58°) = \frac{x}{9.4}$$

3. Rearrange the equation to make the unknown the subject and solve.
Make sure your calculator is in degree mode.

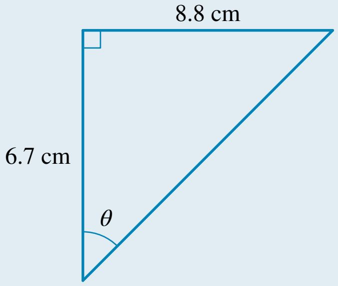
$$x = 9.4\tan(58°)$$

$$x = 15.04$$

The opposite side length is 15.04 cm.

WORKED EXAMPLE 11

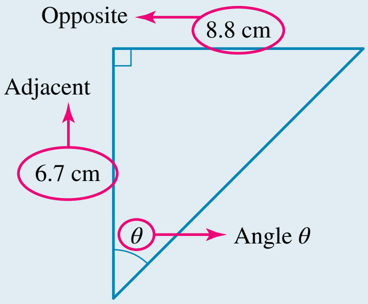
Calculate the value of the unknown angle, θ, correct to 2 decimal places.



THINK

WRITE/DRAW

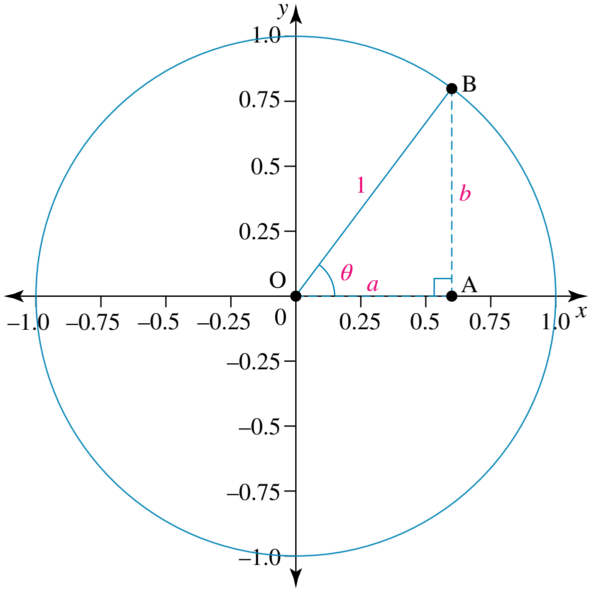
1. Label all the given information on the triangle.



2. Since we have been given the combination of opposite, adjacent and the reference angle θ, we need to use the tangent ratio. Substitute the given values into the ratio equation.	$\tan(\theta) = \frac{O}{A}$ $= \frac{8.8}{6.7}$
3. To find the angle θ, we need to use the inverse tangent function. Make sure your calculator is in degree mode.	$\theta = \tan^{-1}\left(\frac{8.8}{6.7}\right)$ $= 52.72°$

8.3.5 The unit circle

If we draw a circle of radius 1 in the Cartesian plane with its centre located at the origin, then we can locate the coordinates of any point on the circumference of the circle by using right-angled triangles.

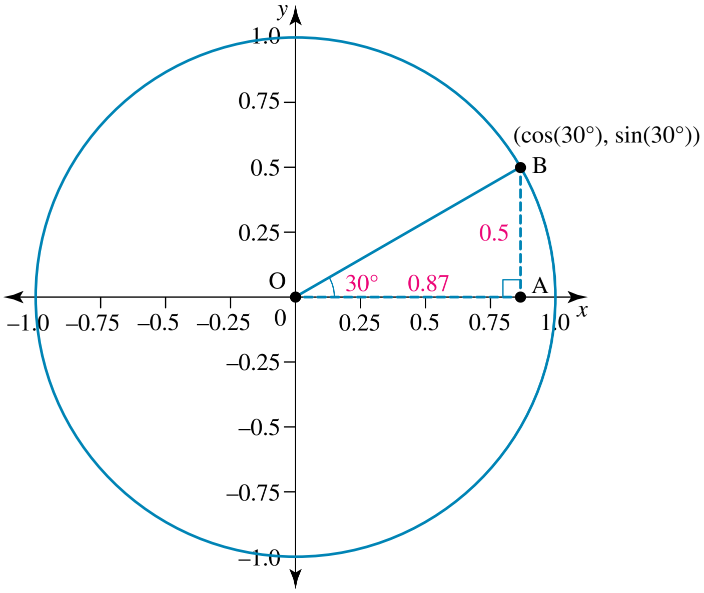


In this diagram, the length of the hypotenuse is 1 and the coordinates of B can be found using the trigonometric ratios.

$$\cos(\theta) = \frac{A}{H} = \frac{a}{1} = a \quad \text{and} \quad \sin(\theta) = \frac{O}{H} = \frac{b}{1} = b$$

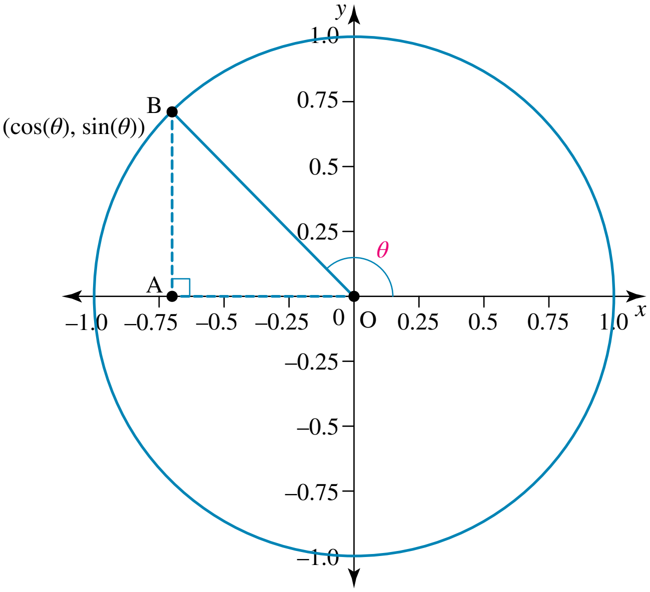
Therefore the base length of the triangle, a, is equal to $\cos(\theta)$, and the height of the triangle, b, is equal to $\sin(\theta)$. This gives the coordinates of B as $(\cos(\theta), \sin(\theta))$.

For example, if we have a right-angled triangle with a reference angle of 30° and a hypotenuse of length 1, then the base length of the triangle will be 0.87 and the height of the triangle will be 0.5, as shown in the following triangle.



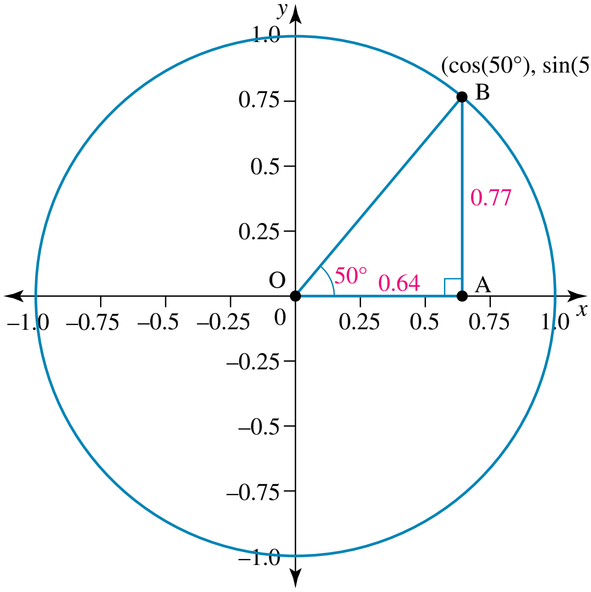
Similarly, if we calculate the value of cos(30°) and sin(30°), we get 0.87 and 0.5 respectively.

We can actually extend this definition to any point B on the unit circle as having the coordinates (cos(θ), sin(θ)), where θ is the angle measured in an anticlockwise direction.



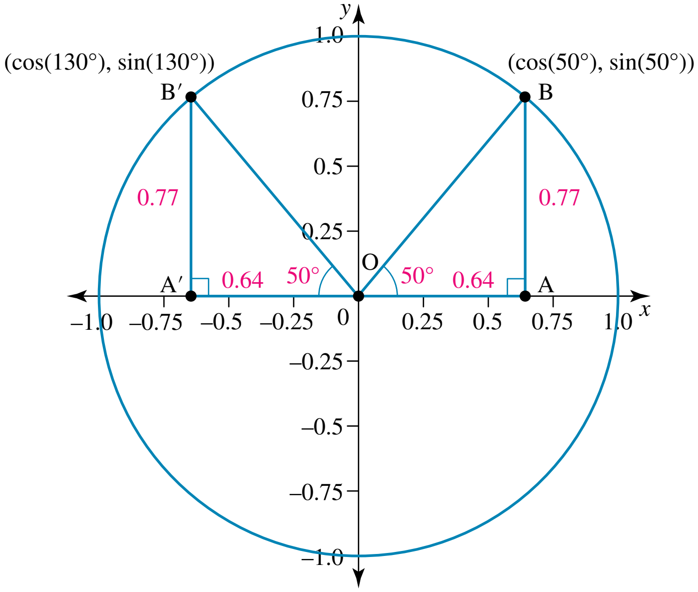
Extending sine and cosine to 180°

We can place any right-angled triangle with a hypotenuse of 1 in the unit circle so that one side of the triangle lies on the positive *x*-axis. The following diagram shows a triangle with base length 0.64, height 0.77 and reference angle 50°.



The coordinates of point B in this triangle are (0.64, 0.77) or (cos(50°), sin(50°)).

Now reflect the triangle in the *y*-axis as shown in the following diagram.

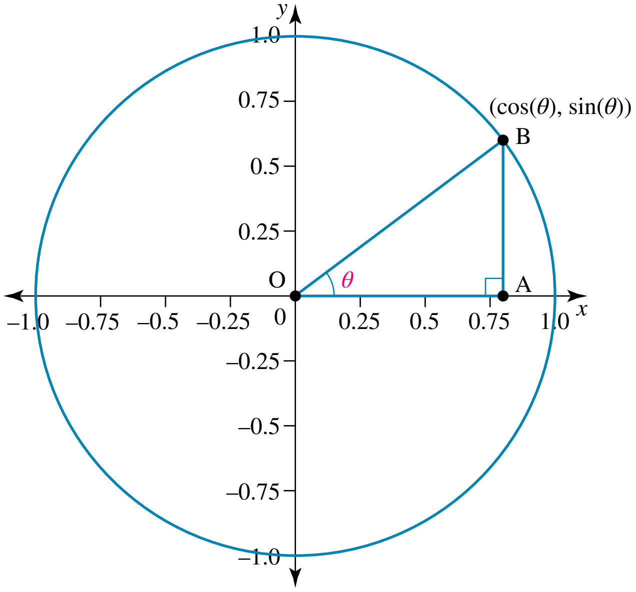


We can see that the coordinates of point B' are (–0.64, 0.77) or (– cos(50°), sin(50°)).

We have previously determined the coordinates of any point B on the circumference of the unit circle as (cos(θ), sin(θ)), where θ is the angle measured in an anticlockwise direction. In this instance the value of $\theta = 180° - 50 = 130°$.

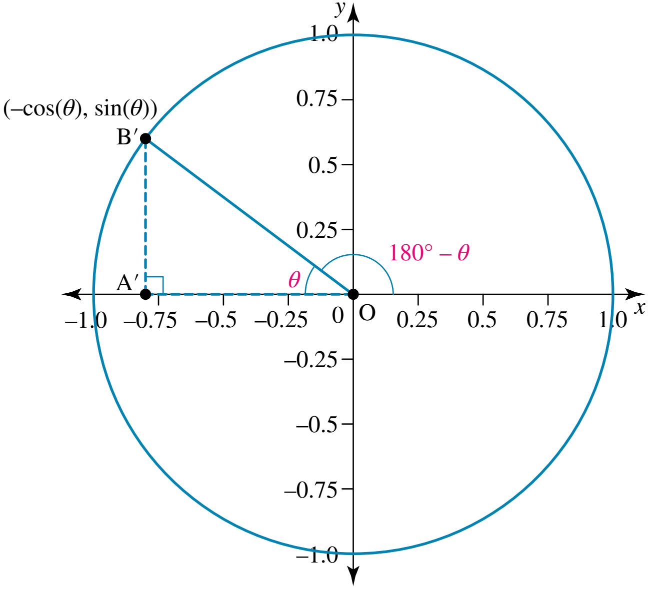
Therefore, the coordinates of point B' are (cos(130°), sin(130°)).

This discovery can be extended when we place any right-angled triangle with a hypotenuse of length 1 inside the unit circle.



As previously determined, the point B has coordinates (cos(θ), sin(θ)).

Reflecting this triangle in the y-axis gives:



So the coordinates of point $B^{'}$ are $(-\cos(\theta), \sin(\theta))$. We also know that the coordinates of $B^{'}$ are $(\cos(180-\theta), \sin(180-\theta))$ from the general rule about the coordinates of any point on the unit circle.

Equating the two coordinates for $B^{'}$ gives us the following equations:

$$-\cos(\theta) = \cos(180-\theta)$$

$$\sin(\theta) = \sin(180-\theta)$$

So, to calculate the values of the sine and cosine ratios for angles up to 180°, we can use:

$$\boldsymbol{\cos(\theta) = -\cos(180-\theta)}$$

$$\boldsymbol{\sin(\theta) = \sin(180-\theta)}$$

Remember that if two angles sum to 180°, then they are supplements of each other. So if we are calculating the sine or cosine of an angle between 90° and 180°, then start by finding the supplement of the given angle.

WORKED EXAMPLE 12

Calculate the values of:

a. $\sin(140^{\circ})$ **b.** $\cos(160^{\circ})$

giving your answers to 2 decimal places.

THINK	WRITE
a. 1. Calculate the supplement of the given angle.	**a.** $180^{\circ} - 140^{\circ} = 40^{\circ}$
2. Calculate the sine of the supplement angle correct to 2 decimal places.	$\sin(40^{\circ}) = 0.642787\ldots$ $= 0.64$ (to 2 decimal places)
3. The sine of an obtuse angle is equal to the sine of its supplement.	$\sin(140^{\circ}) = \sin(40^{\circ})$ $= 0.64$ (to 2 decimal places)
b. 1. Calculate the supplement of the given angle.	**b.** $180^{\circ} - 160^{\circ} = 20^{\circ}$
2. Calculate the cosine of the supplement angle correct to 2 decimal places.	$\cos(20^{\circ}) = 0.939\,692\ldots$ $= 0.94$ (to 2 decimal places)
3. The cosine of an obtuse angle is equal to the negative cosine of its supplement.	$\cos(160^{\circ}) = -\cos(20^{\circ})$ $= -0.94$ (to 2 decimal places)

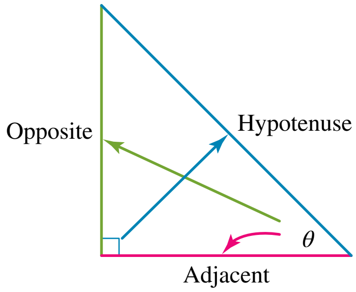
8.3.6 SOH–CAH–TOA

Trigonometric ratios are relationships between the sides and angles of a right-angled triangle.

In solving trigonometric ratio problems for sine, cosine and tangent, we need to:

1. determine which ratio to use
2. write the relevant equation
3. substitute values from given information
4. make sure the calculator is in degree mode
5. solve the equation for the unknown lengths, or use the inverse trigonometric functions to find unknown angles.

To assist in remembering the trigonometric ratios, the mnemonic **SOH – CAH – TOA** has been developed.



SOH – CAH – TOA stands for:
- **S**ine is **O**pposite over **H**ypotenuse
- **C**osine is **A**djacent over **H**ypotenuse
- **T**angent is **O**pposite over **A**djacent.

on Resources

Interactivity: Finding the angle when two sides are known (int-6046)
Interactivity: Trigonometric ratios (int-2577)

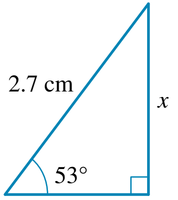
studyon

Units 1 & 2 > Area 4 > Sequence 1 > Concept 2

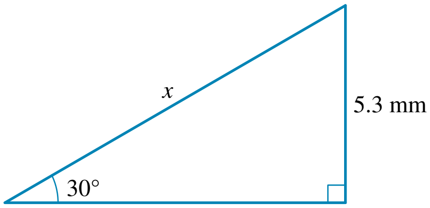
The unit circle Summary screen and practice questions

Exercise 8.3 Applications of trigonometric ratios

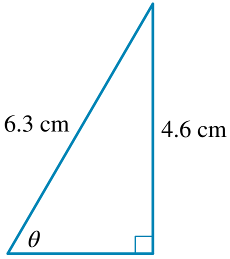
1. **WE6** Calculate the value of x correct to 2 decimal places.



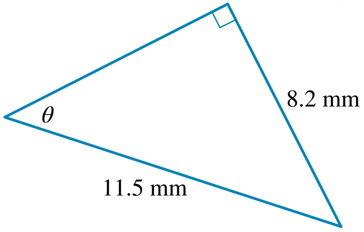
2. Calculate the value of x.



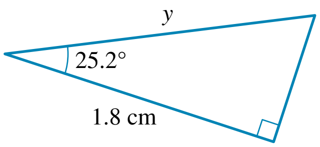
3. **WE7** Calculate the value of the unknown angle, θ, correct to 2 decimal places.



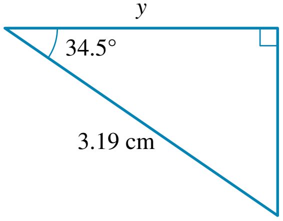
4. Calculate the value of the unknown angle, θ, correct to 2 decimal places.



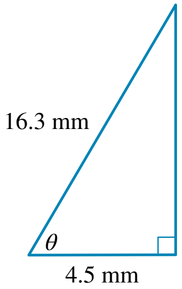
5. **WE8** Calculate the value of y correct to 2 decimal places.



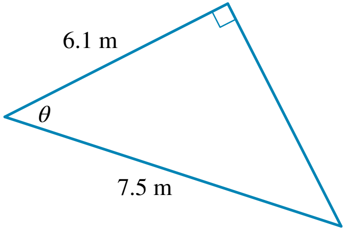
6. Calculate the value of y correct to 2 decimal places.



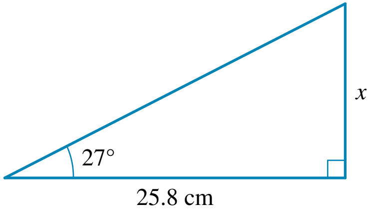
7. **WE9** Calculate the value of the unknown angle, θ, correct to 2 decimal places.



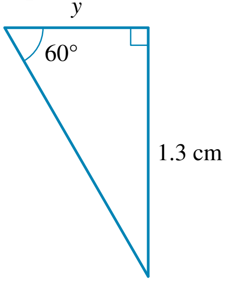
8. Calculate the value of the unknown angle, θ, correct to 2 decimal places.



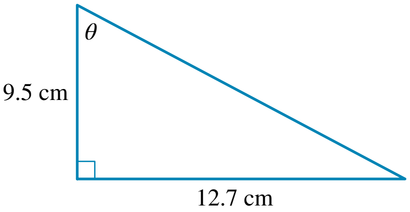
9. **WE10** Calculate the value of x correct to 2 decimal places.



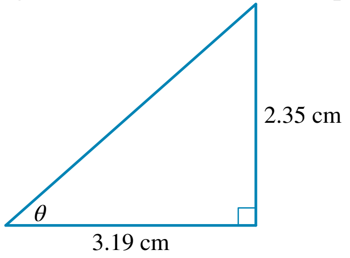
10. Calculate the value of y correct to 2 decimal places.



11. **WE11** Calculate the value of the unknown angle, θ, correct to 2 decimal places.



12. Calculate the value of the unknown angle, θ, correct to 2 decimal places.



13. **WE12** Calculate the values of:

   a. $\sin(125°)$ b. $\cos(152°)$

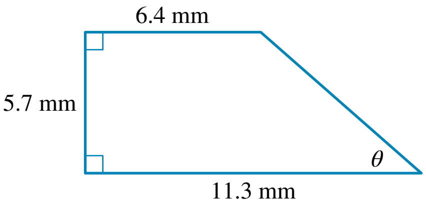
   giving your answers correct to 2 decimal places.

14. Calculate the values of:

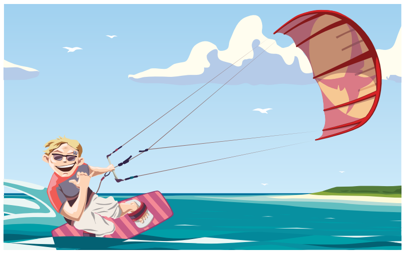
   a. $\sin(99.2°)$ b. $\cos(146.7°)$

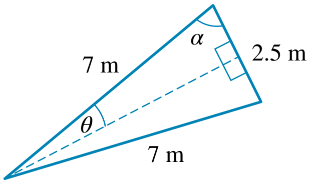
   giving your answers correct to 2 decimal places.

15. Calculate the value of the unknown angle, θ, correct to 2 decimal places.



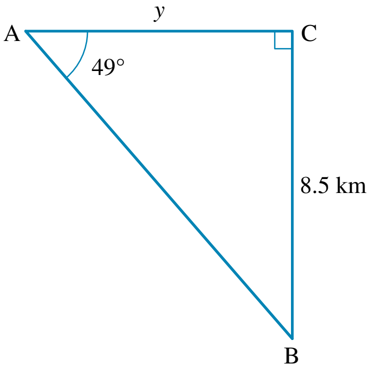
16. A kitesurfer has a kite of length 2.5 m and strings of length 7 m as shown.





Calculate the values of the angles θ and α, correct to 2 decimal places.

17. A yacht race follows a triangular course as shown below. Calculate, correct to 1 decimal place:



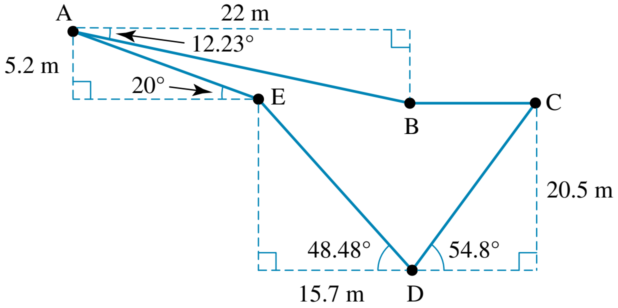
   a. the distance of the final leg, y b. the total distance of the course.

18. A truss is used to build a section of a roof. If the vertical height of the truss is 1.5 metres and the span (horizontal distance between the walls) is 8 metres wide, calculate the pitch of the roof (its angle with the horizontal) correct to 1 decimal place.

19. A 2.5 m ladder is placed against a wall. The base of the ladder is 1.7 m from the wall.

a. Calculate the angle, correct to 2 decimal places, that the ladder makes with the ground.
b. Calculate how far the ladder reaches up the wall, correct to 2 decimal places.

20. A dog training obstacle course ABCDEA is shown in the diagram below with point B vertically above point D.



Calculate the total length of the obstacle course in metres, giving your answer correct to 2 decimal places.

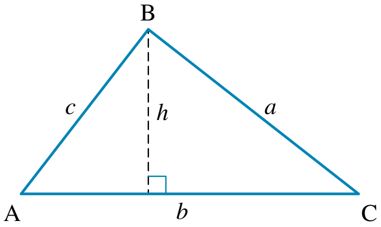
8.4 Area of triangles

8.4.1 Area of triangles

You should be familiar with calculating the area of a triangle using the rule: area $= \frac{1}{2} bh$ where b is the base length and h is the perpendicular height of the triangle. However, for many triangles we are not given the perpendicular height, so this rule cannot be directly used.

Take the triangle ABC as shown in right.

If h is the perpendicular height of this triangle, then we can calculate the value of h by using the sine ratio:



$$\sin(A) = \frac{h}{c}$$

Transposing this equation gives $h = c \sin(A)$, which we can substitute into the rule for the area of the triangle to give:

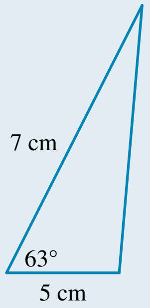
$$\textbf{Area} = \frac{1}{2}bc\sin(A)$$

Note: We can label any sides of the triangle a, b and c, and this formula can be used as long as we have the length of two sides of a triangle and know the value of the included angle.

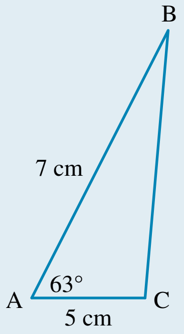
WORKED EXAMPLE 13

Calculate the area of the following triangles. Give both answers correct to 2 decimal places.

a.

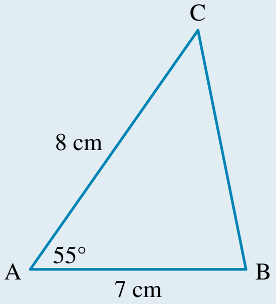


b. **A triangle with sides of length 8 cm and 7 cm, and an included angle of 55°.**

THINK	WRITE/DRAW
a. 1. Label the vertices of the triangle.	**a.** 
2. Write down the known information.	$b = 5\text{ cm}$ $c = 7\text{ cm}$ $A = 63°$
3. Substitute the known values into the formula to calculate the area of the triangle.	$\text{Area} = \frac{1}{2}bc\sin(A)$ $= \frac{1}{2} \times 5 \times 7 \times \sin(63°)$ $= 15.592\ldots$ $= 15.59$ (to 2 decimal places)
4. Write the answer, remembering to include the units.	The area of the triangle is 15.59 cm^2 correct to 2 decimal places.

b. 1. Draw a diagram to represent the triangle.

b.



2. Write down the known information.

$b = 8$ cm

$c = 7$ cm

$A = 55°$

3. Substitute the known values into the formula to calculate the area of the triangle.

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 8\times 7\times \sin(55°)\\ &= 22.936\ldots\\ &= 22.94\ \text{(to 2 decimal places)}\end{aligned}$$

4. Write the answer, remembering to include the units.

The area of the triangle is 22.94 cm^2 correct to 2 decimal places.

8.4.2 Heron's formula

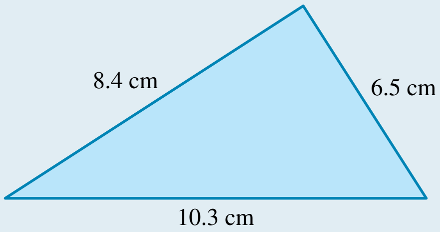
Heron's formula is a way of calculating the area of the triangle if you are given all three side lengths. It is named after Hero of Alexandria, who was a Greek engineer and mathematician.

Step 1: Calculate s, the value of half of the perimeter of the triangle: $s = \dfrac{a+b+c}{2}$

Step 2: Use the following formula to calculate the area of the triangle: $A=\sqrt{s(s-a)(s-b)(s-c)}$

WORKED EXAMPLE 14

Use Heron's formula to calculate the area of the following triangle. Give your answer correct to 1 decimal place.



THINK

WRITE

1. Calculate the value of s.

$$\begin{aligned}s &= \frac{a+b+c}{2}\\ &= \frac{6.5+8.4+10.3}{2}\\ &= \frac{25.2}{2}\\ &= 12.6\end{aligned}$$

2. Use Heron's formula to calculate the area of the triangle correct to 1 decimal place.	$A = \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{12.6(12.6-6.5)(12.6-8.4)(12.6-10.3)}$ $= \sqrt{12.6 \times 6.1 \times 4.2 \times 2.3}$ $= \sqrt{742.4676}$ ≈ 27.2
3. State the area and give the units.	$A = 27.2\,\text{cm}^2$

WORKED EXAMPLE 15

Calculate the area of a triangle with sides of 4 cm, 7 cm and 9 cm, giving your answer correct to 2 decimal places.

THINK	**WRITE**
1 Write down the known information.	$a = 4\,\text{cm}$ $b = 7\,\text{cm}$ $c = 9\,\text{cm}$
2 Calculate the value of *s* (the semi-perimeter).	$s = \frac{a+b+c}{2}$ $= \frac{4+7+9}{2}$ $= \frac{20}{2}$ $= 10$
3 Substitute the values into Heron's formula to calculate the area.	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{10(10-4)(10-7)(10-9)}$ $= \sqrt{10 \times 6 \times 3 \times 1}$ $= \sqrt{180}$ $= 13.416\ldots$ ≈ 13.42
4 Write the answer, remembering to include the units.	The area of the triangle is $13.42\,\text{cm}^2$ correct to 2 decimal places.

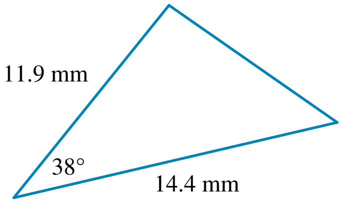
on Resources

- **Interactivity:** Using Heron's formula to find the area of a triangle (int-6475)
- **Interactivity:** Area of triangles (int-6483)

studyon
Units 1 & 2 > Area 4 > Sequence 1 > Concept 3 > **Area of triangles** Summary screen and practice questions

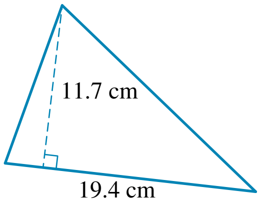
Exercise 8.4 Area of triangles

1. **WE13** Calculate the area of the following triangle correct to 2 decimal places.

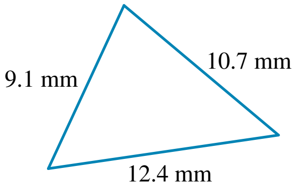


2. Calculate the area of a triangle with sides of length 14.3 mm and 6.5 mm, and an included angle of 32°. Give your answer correct to 2 decimal places.
3. A triangle has one side length of 8 cm and an adjacent angle of 45.5°. If the area of the triangle is 18.54 cm^2, calculate the length of the other side that encloses the 45.5° angle, correct to 2 decimal places.
4. The smallest two sides of a triangle are 10.2 cm and 16.2 cm respectively, and the largest angle of the same triangle is 104.5°. Calculate the area of the triangle correct to 2 decimal places.
5. **WE15** Calculate the area of a triangle with sides of 11 cm, 12 cm and 13 cm, giving your answer correct to 2 decimal places.
6. Calculate the area of a triangle with sides of 22.2 mm, 13.5 mm and 10.1 mm, giving your answer correct to 2 decimal places.
7. Calculate the area of the following triangles, correct to 2 decimal places where appropriate.

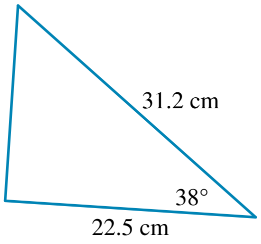
a.



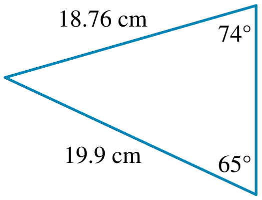
b.



c.

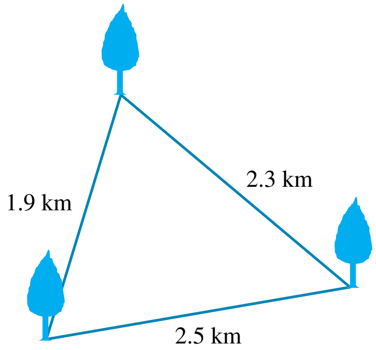


d.



8. Calculate the area of the following triangles, correct to 2 decimal places where appropriate.
   - a. Triangle ABC, given $a = 12\,\text{cm}$, $b = 15\,\text{cm}$, $c = 20\,\text{cm}$
   - b. Triangle ABC, given $a = 10.5\,\text{mm}$, $b = 11.2\,\text{mm}$ and $C = 40°$

9. A triangular field is defined by three trees, each of which sits in one of the corners of the field, as shown in the following diagram.

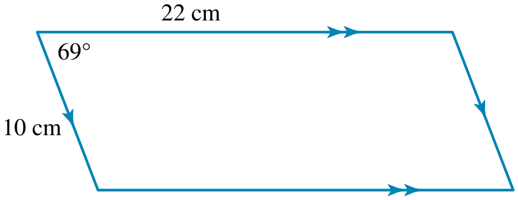


Calculate the area of the field in km^2 correct to 3 decimal places.

10. A triangle has side lengths of $3x$, $4x$ and $5x$. If the area of the triangle is 121.5 cm^2, use any appropriate method to determine the value of x.

11. A triangular-shaped piece of jewellery has two side lengths of 8 cm and an area of 31.98 cm^2. Use trial and error to calculate the length of the third side correct to 1 decimal place.

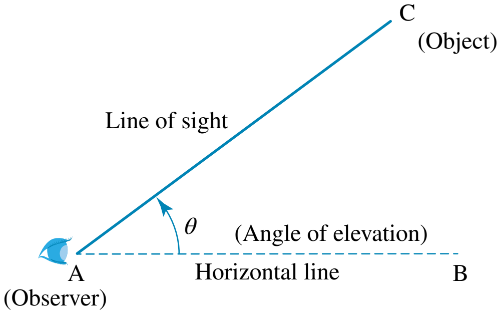
12. Calculate the area of the following shape correct to 2 decimal places.



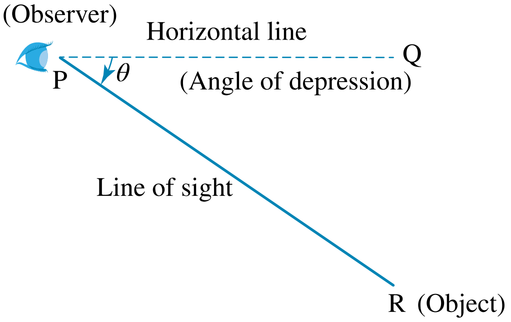
8.5 Angles of elevation and depression

8.5.1 Angles of elevation and depression

An **angle of elevation** is the angle between a horizontal line from the observer to an object that is above the horizontal line.

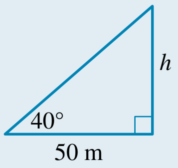


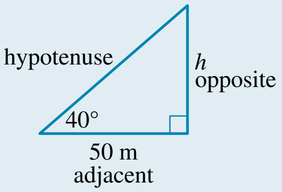
An **angle of depression** is the angle between a horizontal line from the observer to an object that is below the horizontal line.



WORKED EXAMPLE 16

From a point 50 m from the foot of a building, the angle of elevation to the top of the building is measured as 40°.
Calculate the height, *h*, of the building, correct to the nearest metre.

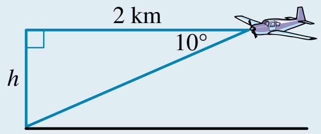


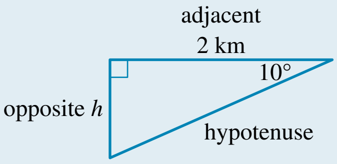
THINK	WRITE
1. Label the sides of the triangle opposite, adjacent and hypotenuse.	
2. Choose the tangent ratio because we are finding the opposite side and have been given the adjacent side.	
3. Write the formula.	$\tan\,\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
4. Substitute for θ and the adjacent side.	$\tan 40° = \dfrac{h}{50}$
5. Make h the subject of the equation.	$h = 50\,\tan\,40°$
6. Calculate.	$= 42\,\text{m}$
7. Give a written answer.	The height of the building is approximately 42 m.

A similar method for finding the solution is used for problems that involve an angle of depression.

WORKED EXAMPLE 17

When an aeroplane in flight is 2 km from a runway, the angle of depression to the runway is 10° .
Calculate the altitude of the aeroplane, correct to the nearest metre.



THINK	WRITE
1. Label the sides of the triangle opposite, adjacent and hypotenuse.	
2. Choose the tangent ratio because we are finding the opposite side given the adjacent side.	
3. Write the formula.	$\tan\,\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
4. Substitute for θ and the adjacent side, converting 2 km to metres.	$\tan\,10° = \dfrac{h}{2000}$

5. Make h the subject of the equation.

$h = 2000 \tan 10°$

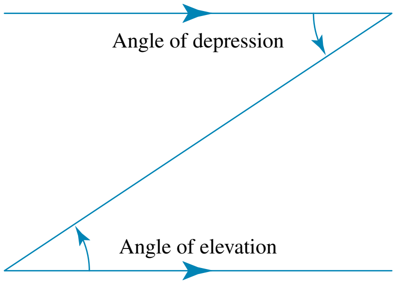
6. Calculate.

$= 353 \text{ m}$

7. Give a written answer.

The altitude of the aeroplane is approximately 353 m.

We use angles of elevation and depression to locate the positions of objects above or below the horizontal (reference) line. Angles of elevation and angles of depression are equal as they are alternate angles.



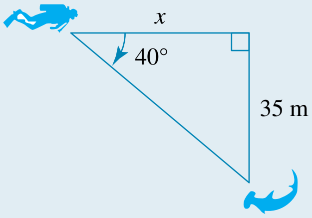
WORKED EXAMPLE 18

The angle of depression from a scuba diver at the water's surface to a hammerhead shark on the sea floor of the Great Barrier Reef is 40°. The depth of the water is 35 m. Calculate the horizontal distance from the scuba diver to the shark, correct to 2 decimal places.

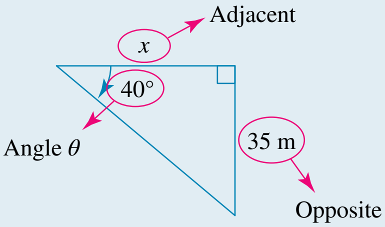
THINK

WRITE/DRAW

1. Draw a diagram to represent the information.



2. Label all the given information on the triangle.



3. Since we have been given the combination of opposite, adjacent and the reference angle θ, we need to use the tangent ratio. Substitute the given values into the ratio equation.

$$\tan(\theta) = \frac{O}{A}$$

$$\tan(40°) = \frac{35}{x}$$

4. Rearrange the equation to make the unknown the subject and solve.
Make sure your calculator is in degree mode.

$$x = \frac{35}{\tan(40°)}$$
$$= 41.71$$

The horizontal distance from the scuba diver to the shark is 41.71 m.

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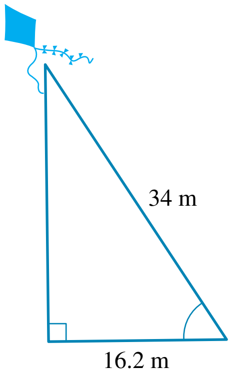
Units 1 & 2 > Area 4 > Sequence 1 > Concept 4

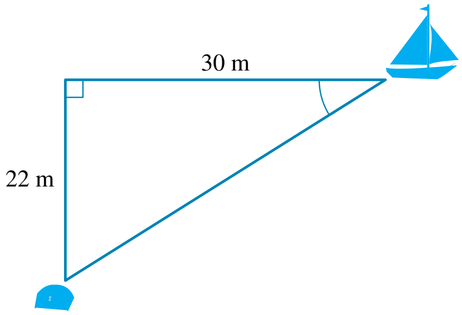
Angles of elevation and depression Summary screen and practice questions

Exercise 8.5 Angles of elevation and depression

1. **WE17** A plane is 36 km from an airstrip, measured in a line from the cockpit to start of the airstrip. The angle of depression is 65°. How high is the plane correct to 2 decimal places?
2. The angle of elevation from a hammerhead shark on the sea floor of the Great Barrier Reef to a scuba diver at the water's surface is 35°. The depth of the water is 33 m. Calculate the horizontal distance from the shark to the scuba diver.
3. **WE18** The angle of depression from a scuba diver at the water's surface to a hammerhead shark on the sea floor of the Great Barrier Reef is 41°. The depth of the water is 32 m. Calculate the horizontal distance from the scuba diver to the shark.
4. Calculate the angle of elevation of the kite from the ground, correct to 2 decimal places.

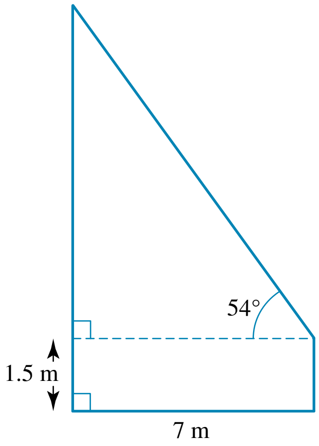


5. Calculate the angle of depression from the boat to the treasure at the bottom of the sea, correct to 2 decimal places.



6. A crocodile is fed on a 'jumping crocodile tour' on the Adelaide River. The tour guide dangles a piece of meat on a stick at an angle of elevation of 60° from the boat, horizontal to the water. If the stick is 2 m long and held 1 m above the water, determine the vertical distance the crocodile has to jump out of the water to get the meat, correct to 2 decimal places.
7. A ski chair lift operates from the Mt Buller village and has an angle of elevation of 45° to the top of the Federation ski run. If the vertical height is 707 m, calculate the ski chair lift length, correct to 2 decimal places.
8. A student uses a clinometer to measure an angle of elevation of 50° from the ground to the top of Uluru. If the student is standing 724 m from the base of Uluru, determine the height of Uluru correct to 2 decimal places.

9. A tourist in Melbourne looks down from the glass floor of Eureka Tower's Skydeck to see people below on the footpath. If the angle of depression is 88° and the people are 11 m from the base of the tower, calculate how high up the tourist is standing in the glass cube.
10. A student uses a clinometer to measure the height of his house. The angle of elevation is 54°. He is 1.5 m tall and stands 7 m from the base of the house. Calculate the height of the house correct to 2 decimal places.



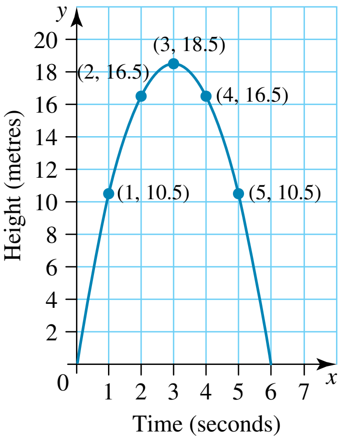
11. A tourist 1.72 m tall is standing 50 m away from the base of the Sydney Opera House. The Opera House is 65 m tall. Calculate the angle of elevation, to the nearest degree, from the tourist to the top of the Opera House.

12. A parachutist falls from a height of 5000 m to the ground while travelling over a horizontal distance of 150 m. Determine the angle of depression of the descent, correct to 2 decimal places.

13. Air traffic controllers in two control towers, which are both 87 m high, spot a plane at an altitude of 500 metres. The angle of elevation from tower A to the plane is 5° and from tower B to the plane is 7°. Calculate the distance between the two control towers correct to the nearest metre.

14. A footballer takes a set shot at goal, with the graph showing the path that the ball took as it travelled towards the goal.
If the footballer's eye level is at 1.6 metres, calculate the angle of elevation from his eyesight to the ball, correct to 2 decimal places, after:

a. 1 second
b. 2 seconds
c. 3 seconds
d. 4 seconds
e. 5 seconds



8.6 The sine rule

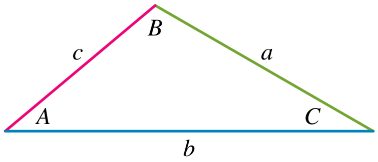
8.6.1 The sine rule

The **sine rule** can be used to find the side length or angle in non-right-angled triangles.

To help us solve non-right-angled triangle problems, the labelling convention of a non-right-angled triangle, ABC, is as follows:

Angle *A* is opposite side length *a*.
Angle *B* is opposite side length *b*.
Angle *C* is opposite side length *c*.

The largest angle will always be opposite the longest side length, and the smallest angle will always be opposite to the smallest side length.



Formulating the sine rule

We can divide an acute non-right-angled triangle into two right-angled triangles as shown in the following diagrams.

If we apply trigonometric ratios to the two right-angled triangles we get:

$$\frac{h}{c} = \sin(A) \quad \text{and} \quad \frac{h}{a} = \sin(C)$$

$$h = c\sin(A) \qquad h = a\sin(C)$$

Equating the two expressions for h gives:

$$c\sin(A) = a\sin(C)$$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

In a similar way, we can split the triangle into two using side a as the base, giving us:

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

This gives us the sine rule:

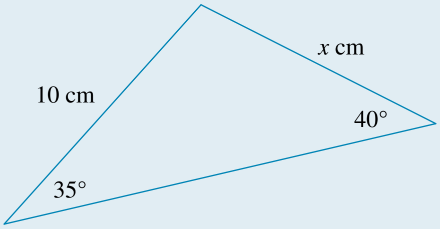
$$\boldsymbol{\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}}$$

We can apply the sine rule to determine all of the angles and side lengths of a triangle if we are given either:

- 2 side lengths and 1 corresponding angle
- 1 side length and 2 angles.

WORKED EXAMPLE 19

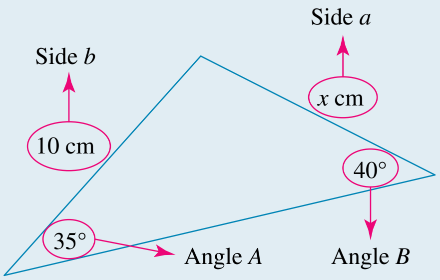
Find the value of the unknown length x, correct to 2 decimal places.



THINK

1. Label the triangle with the given information, using the conventions for labelling.
Angle A is opposite to side a.
Angle B is opposite to side b.

WRITE/DRAW



2. Substitute the known values into the sine rule.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

$$\frac{x}{\sin(35°)} = \frac{10}{\sin(40°)}$$

3. Rearrange the equation to make x the subject and solve.
   Make sure your calculator is in degree mode.

$$\frac{x}{\sin(35°)} = \frac{10}{\sin(40°)}$$

$$x = \frac{10\sin(35°)}{\sin(40°)}$$

$$x = 8.92$$

4. Write the answer.

The unknown side length x is 8.92 cm.

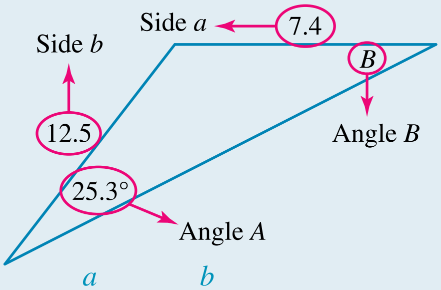
WORKED EXAMPLE 20

A non-right-angled triangle has values of side b = 12.5, angle A = 25.3° and side a = 7.4. Calculate the value of angle B, correct to 2 decimal places.

THINK

WRITE/DRAW

1. Draw a non-right-angled triangle, labelling with the given information.
   Angle A is opposite to side a.
   Angle B is opposite to side b.



2. Substitute the known values into the sine rule.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

$$\frac{7.4}{\sin(25.3°)} = \frac{12.5}{\sin(B°)}$$

3. Rearrange the equation to make sin(B) the subject and solve.
   Make sure your calculator is in degree mode.

$$\frac{7.4}{\sin(25.3°)} = \frac{12.5}{\sin(B)}$$

$$7.4\sin(B) = 12.5\sin(25.3°)$$

$$\sin(B) = \frac{12.5\sin(25.3°)}{7.4}$$

$$B = \sin^{-1}\left(\frac{12.5\sin(25.3°)}{7.4}\right)$$

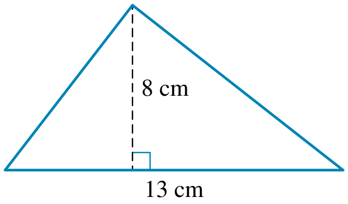
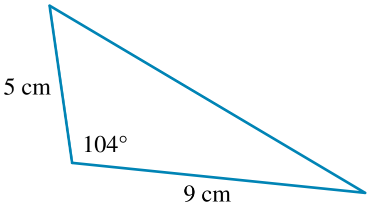
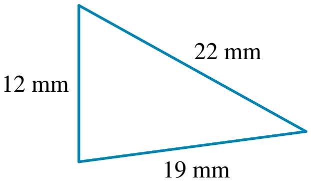
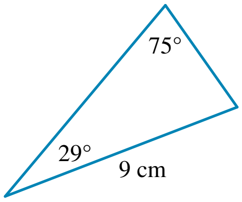
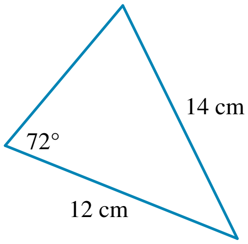
$$= 46.21°$$

4. Write the answer.

Angle B is 46.21°.

8.6.2 Determining which area formula to use

In some situations you may have to perform some calculations to determine either a side length or angle size before calculating the area. This may involve using the sine rule. The following table should help if you are unsure what to do.

Given	**What to do**	**Example**
The base length and perpendicular height	Use Area $= \frac{1}{2}bh$.	
Two side lengths and the included angle	Use Area $= \frac{1}{2}bc\ \sin(A)$.	
Three side lengths	Use Heron's formula: Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$	
Two angles and one side length	Use the sine rule to determine a second side length, and then use Area $= \frac{1}{2}bc\ \sin(A)$. *Note:* The third angle may have to be calculated.	
Two side lengths and an angle opposite one of these lengths	Use the sine rule to calculate the other angle opposite one of these lengths, then determine the final angle before using Area $= \frac{1}{2}bc\ \sin(A)$. *Note:* Check if the ambiguous case is applicable.	

on Resources

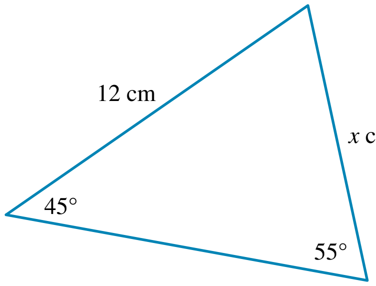
Interactivity: The sine rule (int-6275)

studyon

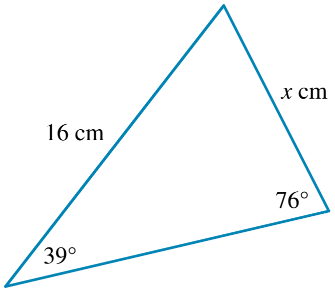
Units 1 & 2 > Area 4 > Sequence 1 > Concept 5 > **The sine rule** Summary screen and practice questions

Exercise 8.6 The sine rule

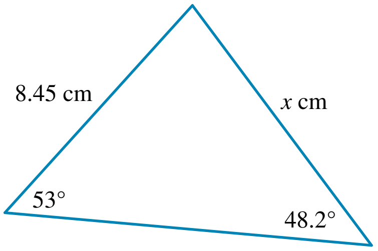
1. **WE19** Calculate the value of the unknown length x correct to 2 decimal places.



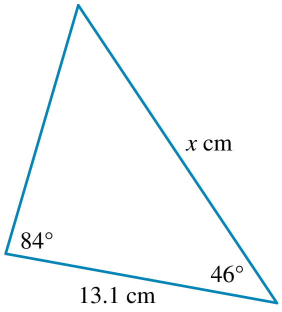
2. Calculate the value of the unknown length x correct to 2 decimal places.



3. Calculate the value of the unknown length x correct to 2 decimal places.



4. Calculate the value of the unknown length x correct to the nearest cm.

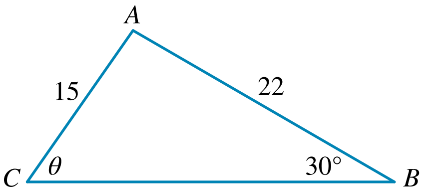


5. Calculate the area of the following triangles, correct to 2 decimal places where appropriate.
   a. Triangle DEF, given $d = 19.8$ cm, $e = 25.6$ cm and $D = 33°$
   b. Triangle PQR, given $p = 45.9$ cm, $Q = 45.5°$ and $R = 67.2°$

6. **WE20** A non-right-angled triangle has values of side $b = 10.5$, angle $A = 22.3°$ and side $a = 8.4$. Calculate the value of angle B correct to 1 decimal place.

7. A non-right-angled triangle has values of side $b = 7.63$, angle $A = 15.8°$ and side $a = 4.56$. Calculate the value of angle B correct to 1 decimal place.

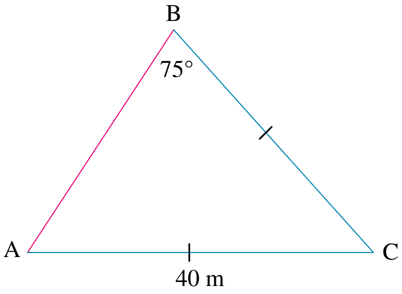
8. For triangle ABC shown, find the acute value of θ correct to 1 decimal place.



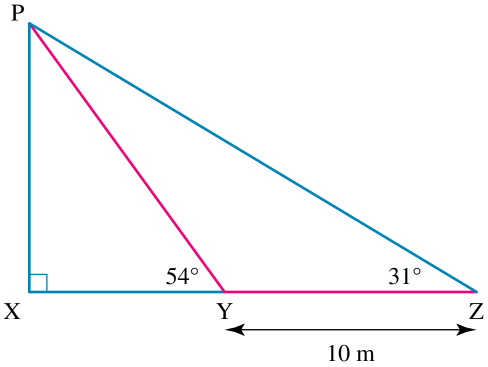
9. If triangle ABC has values $b = 19.5$, $A = 25.3°$ and $a = 11.4$, find both possible angle values of B correct to 2 decimal places.

10. Find all the side lengths, correct to 2 decimal places, for the triangle ABC , given $a = 10.5$, $B = 60°$ and $C = 72°$.

11. Part of a roller-coaster track is in the shape of an isosceles triangle, ABC, as shown in the following triangle. Calculate the track length AB correct to 2 decimal places.

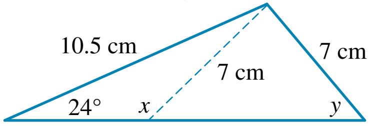


12. The shape and length of a water slide follows the path of PY and YZ in the following diagram.



Calculate, correct to 2 decimal places:
   a. the total length of the water slide
   b. the height of the water slide, PX.

13. Calculate the two unknown angles shown in the diagram below, correct to 1 decimal place.



14. At a theme park, the pirate ship swings back and forth on a pendulum. The centre of the pirate ship is secured by a large metal rod that is 5.6 metres in length. If one of the swings covers an angle of 122°, determine the distance between the point where the rod meets the ship at both extremes of the swing. Give your answers correct to 2 decimal places.

15. Andariel went for a ride on her dune buggy in the desert. She rode east for 6 km, then turned 125° to the left for the second stage of her ride. After 5 minutes riding in the same direction, she turned to the left again, and from there travelled the 5.5 km straight back to her starting position.

How far did Andariel travel in the second section of her ride, correct to 2 decimal places?

16. A triangle ABC has values $a = 11$ cm, $b = 14$ cm and $A = 31.3°$. Answer the following correct to 2 decimal places.

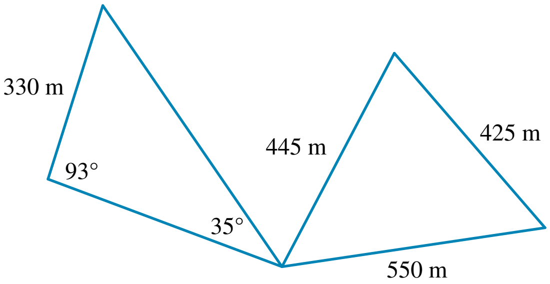
a. Calculate the size of the other two angles of the triangle.

b. Calculate the other side length of the triangle.

c. Calculate the area of the triangle.

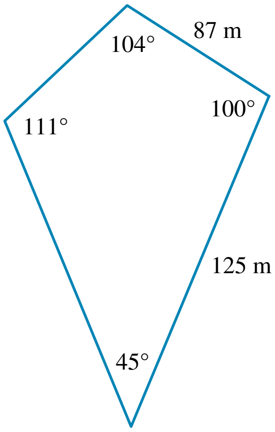
17. A triangle has two sides of length 9.5 cm and 13.5 cm, and one angle of 40.2°. Calculate all three possible areas of the triangle correct to 2 decimal places.

18. A BMX racing track encloses two triangular sections, as shown in the following diagram.



Calculate the total area that the race track encloses to the nearest m^2.

19. A dry field is in the shape of a quadrilateral, as shown in the following diagram. The longest diagonal is 164.228 m.



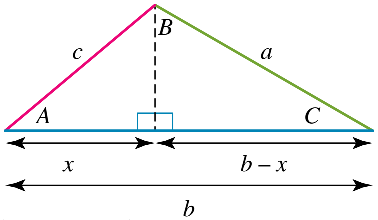
Determine how much grass seed is needed to cover the field in 1 mm of grass seed.
Give your answer correct to 2 decimal places.

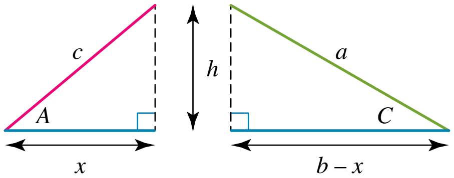
8.7 The cosine rule

8.7.1 Formulating the cosine rule

The cosine rule, like the sine rule, is used to find the length or angle in a non-right-angled triangle. We use the same labelling conventions for non-right-angled triangles as when using the sine rule.

As with the sine rule, the cosine rule is derived from a non-right-angled triangle being divided into two right-angled triangles, where the base side lengths are equal to $(b - x)$ and x.





Using Pythagoras' Theorem we get:

$$\begin{aligned} c^2 &= x^2 + h^2 \\ h^2 &= c^2 - x^2 \end{aligned} \quad \text{and} \quad \begin{aligned} a^2 &= (b - x)^2 + h^2 \\ h^2 &= a^2 - (b - x)^2 \end{aligned}$$

Equating the two expressions for h^2 gives:

$$\begin{aligned} c^2 - x^2 &= a^2 - (b - x)^2 \\ a^2 &= (b - x)^2 + c^2 - x^2 \\ a^2 &= b^2 - 2bx + c^2 \end{aligned}$$

Substituting the trigonometric ratio $x = c\cos(A)$ from the right-angled triangle into the expression, we get:

$$\begin{aligned} a^2 &= b^2 - 2b(c\cos(A)) + c^2 \\ &= b^2 + c^2 - 2bc\cos(A) \end{aligned}$$

This is known as the cosine rule, and we can interchange the pronumerals to get:

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\ \cos A}$$

$$\boldsymbol{b^2 = a^2 + c^2 - 2ac\ \cos B}$$

$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\ \cos C}$$

We can apply the cosine rule to determine all of the angles and side lengths of a triangle if we are given either:

- 3 side lengths or
- 2 side lengths and the included angle.

The cosine rule can also be transposed to give:

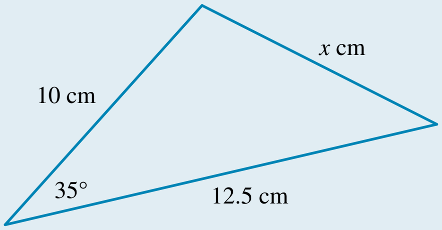
$$\boldsymbol{\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}}$$

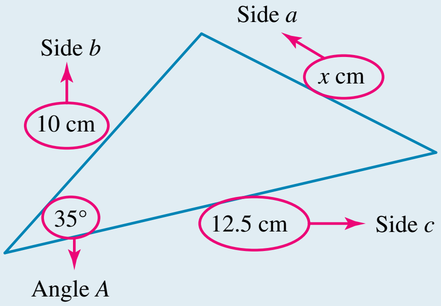
$$\boldsymbol{\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}}$$

$$\boldsymbol{\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}}$$

WORKED EXAMPLE 21

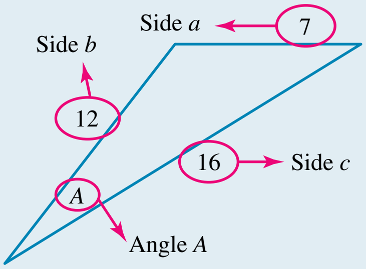
Calculate the value of the unknown length x correct to 2 decimal places.



THINK	**WRITE/DRAW**
1. Draw the non-right-angled triangle, labelling with the given information. Angle A is opposite to side a. If three side lengths and one angle are given, always label the angle as A and the opposite side as a.	
2. Substitute the known values into the cosine rule.	$a^2 = b^2 + c^2 - 2bc\cos(A)$ $x^2 = 10^2 + 12.5^2 - 2 \times 10 \times 12.5\cos(35°)$
3. Solve for x. Make sure your calculator is in degree mode.	$x^2 = 51.462$ $x = \sqrt{51.462}$ ≈ 7.17
4. Write the answer.	The unknown length x is 7.17 cm.

WORKED EXAMPLE 22

A non-right-angled triangle ABC has values $a = 7$, $b = 12$ and $c = 16$. Calculate the magnitude of angle A correct to 2 decimal places.

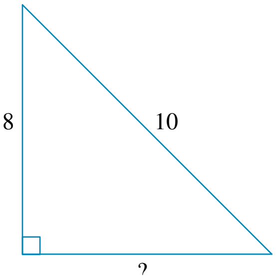
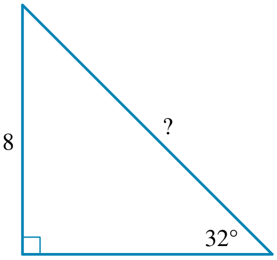
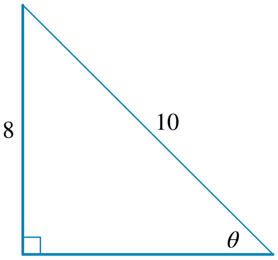
THINK	**WRITE/DRAW**
1. Draw the non-right-angled triangle, labelling with the given information.	
2. Substitute the known values into the cosine rule.	$a^2 = b^2 + c^2 - 2bc\cos(A)$ $7^2 = 12^2 + 16^2 - 2 \times 12 \times 16\cos(A)$

3. Rearrange the equation to make cos (*A*) the subject and solve. Make sure your calculator is in degree mode.	$\cos(A) = \dfrac{12^2 + 16^2 - 7^2}{2 \times 12 \times 16}$ $A = \cos^{-1}\left(\dfrac{12^2 + 16^2 - 7^2}{2 \times 12 \times 16}\right)$ $\approx 23.93^\circ$
4. Write the answer.	The magnitude of angle *A* is 23.93°.

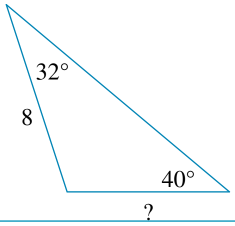
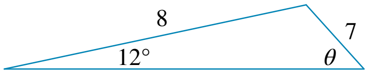
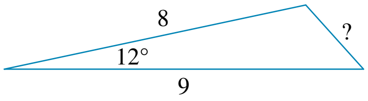
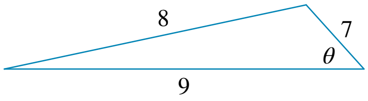
Note: In the example above, it would have been quicker to substitute the known values directly into the transposed cosine rule for cos (*A*).

8.7.2 Determine unknown sides or angles with given information

Knowing which rule to use for different problems will save time and help to reduce the chance for errors to appear in your working. The following table should help you determine which rule to use.

Type of triangle	What you want	What you know	What to use	Rule	Example
Right-angled	Side length	Two other sides	Pythagoras' theorem	$a^2 + b^2 = c^2$	
	Side length	A side length and an angle	Trigonometric ratios	$\sin(\theta) = \dfrac{O}{H}$ $\cos(\theta) = \dfrac{A}{H}$ $\tan(\theta) = \dfrac{O}{A}$	
	Angle	Two side lengths	Trigonometric ratios	$\sin(\theta) = \dfrac{O}{H}$ $\cos(\theta) = \dfrac{A}{H}$ $\tan(\theta) = \dfrac{O}{A}$	

Continued

Type of triangle	What you want	What you know	What to use	Rule	Example
Non-right-angled	Side length	Angle opposite unknown side and another side/angle pair	Sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$	
	Angle	Side length opposite unknown side and another side/angle pair	Sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$	
	Side length	Two sides and the angle between them	Cosine rule	$a^2 = b^2+c^2-2bc\cos(A)$	
	Angle	Three sides	Cosine rule	$\cos(A) = \frac{b^2+c^2-a^2}{2bc}$	

on Resources

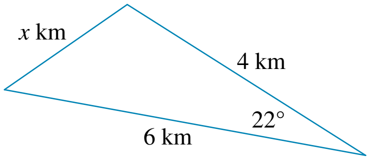
- **Interactivity:** The cosine rule (int-6276)
- **Interactivity:** Solving non-right angled triangles (int-6482)

studyon

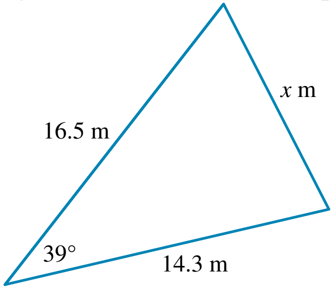
Units 1 & 2 › Area 4 › Sequence 1 › Concept 6 › **The cosine rule** Summary screen and practice questions

Exercise 8.7 The cosine rule

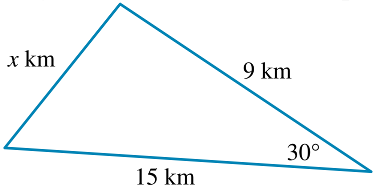
1. **WE21** Calculate the value of the unknown length *x* correct to 2 decimal places.



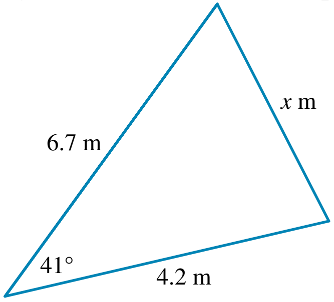
2. Calculate the value of the unknown length x correct to 2 decimal places.



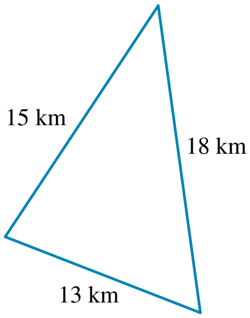
3. Calculate the value of the unknown length x correct to 1 decimal place.



4. Calculate the value of the unknown length x correct to 2 decimal places.

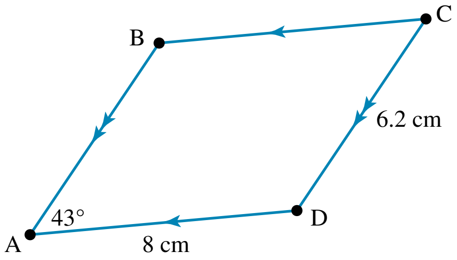


5. **WE22** A non-right-angled triangle ABC has values $a = 8$, $b = 13$ and $c = 17$. Calculate the magnitude of angle A correct to 2 decimal places.
6. A non-right-angled triangle ABC has values $a = 11$, $b = 9$ and $c = 5$. Calculate the magnitude of angle A correct to 2 decimal places.
7. For triangle ABC, calculate the magnitude of angle A correct to 2 decimal places, given $a = 5$, $b = 7$ and $c = 4$.
8. For triangle ABC with $a = 12$, $B = 57°$ and $c = 8$, calculate the side length b correct to 2 decimal places.
9. Calculate the largest angle, correct to 2 decimal places, between any two legs of the following sailing course.

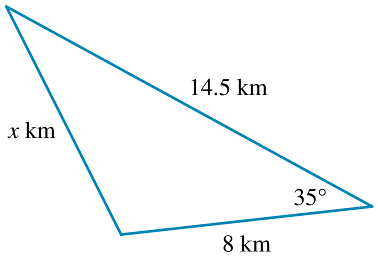


10. A triangular paddock has sides of length 40 m, 50 m and 60 m. Calculate the magnitude of the largest angle between the sides, correct to 2 decimal places.
11. A triangle has side lengths of 5 cm, 7 cm and 9 cm. Calculate the size of the smallest angle correct to 2 decimal places.

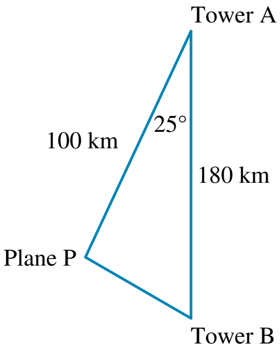
12. ABCD is a parallelogram. Calculate the length of the diagonal AC correct to 2 decimal places.



13. An orienteering course is shown in the following diagram. Calculate the total distance of the course correct to 2 decimal places.



14. Two air traffic control towers are 180 km apart. At the same time, they both detect a plane, P. The plane is at a distance 100 km from Tower A at the bearing shown in the diagram below. Calculate the distance of the plane from Tower B correct to 2 decimal places.



15. Britney is mapping out a new running path around her local park. She is going to run west for 2.1 km, before turning 105° to the right and running another 3.3 km. From there, she will run in a straight line back to her starting position.

How far will Britney run in total? Give your answer correct to the nearest metre.

16. A cruise boat is travelling to two destinations. To get to the first destination it travels for 4.5 hours at a speed of 48 km/h. From there, it takes a 98° turn to the left and travels for 6 hours at a speed of 54 km/h to reach the second destination.

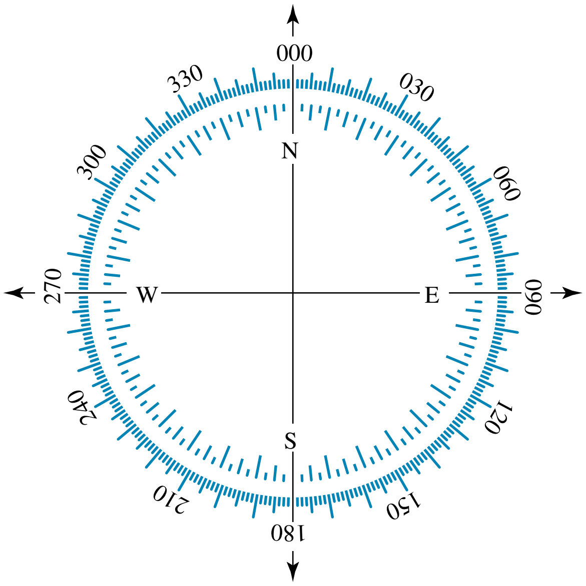
The boat then travels directly back to the start of its journey. Determine how long this leg of the journey will take if the boat is travelling at 50 km/h. Give your answer correct to the nearest minute.

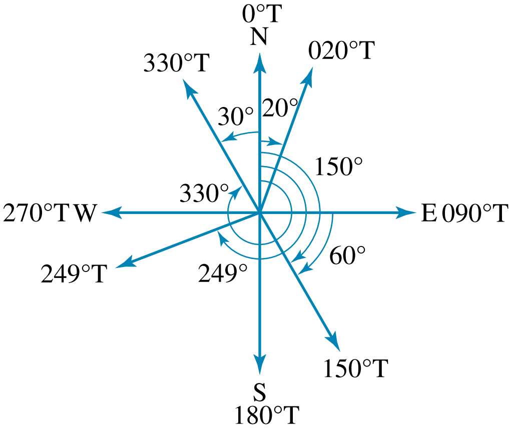
8.8 True bearings

8.8.1 True bearings

Bearings are used to locate the positions of objects or the direction of a journey on a two-dimensional plane.

The four main directions or standard bearings of a directional compass are known as cardinal points. They are North (N), South (S), East (E) and West (W).





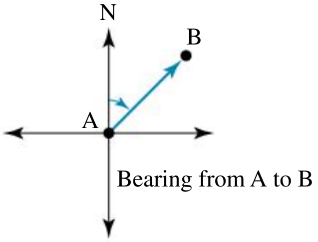
True bearings

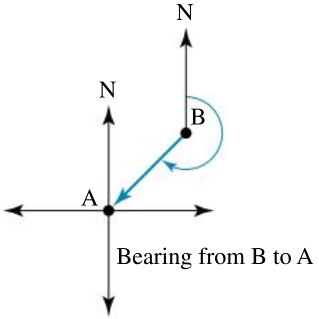
True bearings are measured in a clockwise direction from the north–south line. They are written with all three digits of the angle stated.

If the angle measured is less than 100°, place a zero in front of the angle. For example, if the angle measured is 20°clockwise from the north–south line the bearing is 020°T

8.8.2 Bearings from A to B

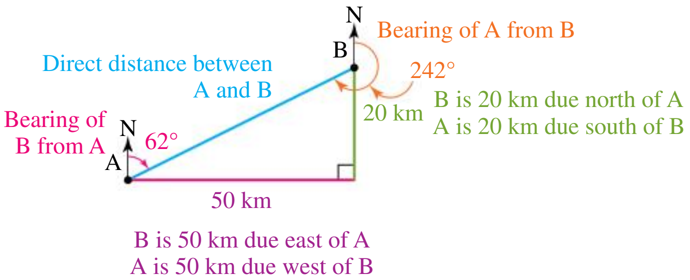
The bearing from A to B is **not** the same as the bearing from B to A.





When determining a bearing from a point to another point, it is important to follow the instructions and draw a diagram. Always draw the centre of the compass at the starting point of the direction requested.

When a problem asks to find the bearing of B from A, mark in north and join a directional line to B to work out the bearing. To return to where you came from is a change in bearing of 180°.

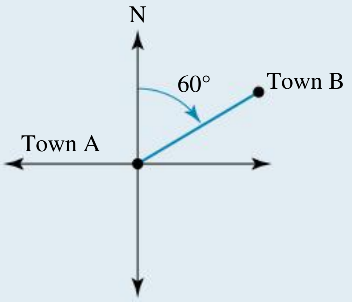


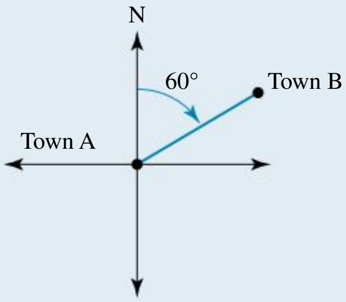
WORKED EXAMPLE 23

Determine the true bearing from:

a. Town A to Town B

b. Town B to Town A.



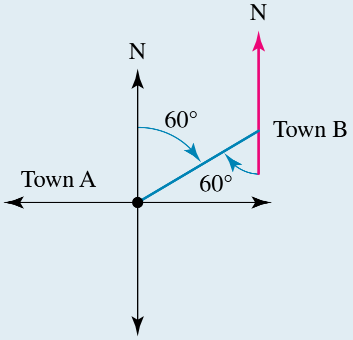
THINK	WRITE/ DRAW
a. 1. To determine the bearing from Town A to Town B, make sure the centre of the compass is marked at town A. The angle is measured clockwise from north to the bearing line at Town B.	a.  The angle measure from north is 60°.

2. A true bearing is written with all three digits of the angle followed by the letter T.

The true bearing from Town A to Town B is 060°T.

b. 1. To find the bearing from Town B to Town A, make sure the centre of the compass is marked at Town B. The angle is measured clockwise from north to the bearing line at Town A.

b.



The angle measure from north is 60° + 180° = 240°.

2. A true bearing is written with all three digits of the angle followed by the letter T.

The true bearing from Town B to Town A is 240°T.

8.8.3 Using trigonometry in bearings problems

As the four cardinal points (N, E, S, W) are at right angles to each other, we can use trigonometry to solve problems involving bearings.

When solving a bearings problem with trigonometry, always start by drawing a diagram to represent the problem. This will help you to identify what information you already have, and determine which trigonometric ratio to use.

WORKED EXAMPLE 24

A boat travels for 25 km in a direction of 310°T.

a. Determine how far north the boat travels, correct to 2 decimal places.

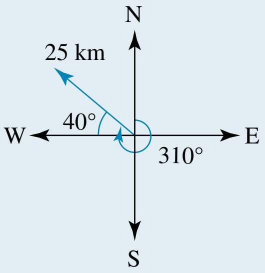
b. Determine how far west the boat travels, correct to 2 decimal places.

THINK

WRITE/ DRAW

a. 1. Draw a diagram of the situation, remembering to label the compass points as well as all of the given information.

a.



2. Identify the information you have in respect to the reference angle, as well as the information you need.

Reference angle = 40°
Hypotenuse = 25
Opposite = ?

3. Determine which of the trigonometric ratios to use.

$$\sin(\theta) = \frac{O}{H}$$

4. Substitute the given values into the trigonometric ratio and solve for the unknown.

$$\sin(40°) = \frac{O}{25}$$

$$25\sin(40°) = O$$

$$O = 16.069\ldots$$

$$= 16.07 \text{ (to 2 decimal places)}$$

5. Write the answer.

The ship travels 16.07 km north.

b. 1. Use your diagram from part **a** and identify the information you have in respect to the reference angle, as well as the information you need.

b. Reference angle = 40°
Hypotenuse = 25
Adjacent = ?

2. Determine which of the trigonometric ratios to use.

$$\cos(\theta) = \frac{A}{H}$$

3. Substitute the given values into the trigonometric ratio and solve for the unknown.

$$\cos(40°) = \frac{A}{25}$$

$$25\cos(40°) = A$$

$$A = 19.151\ldots$$

$$= 19.15 \text{ (to 2 decimal places)}$$

4. Write the answer.

The boat travels 19.15 km west.

studyon

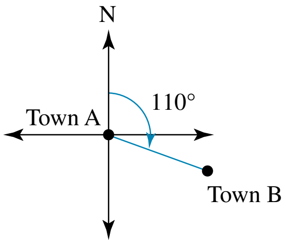
Units 1 & 2 > Area 4 > Sequence 1 > Concept 7 > **True bearings** Summary screen and practice questions

on Resources

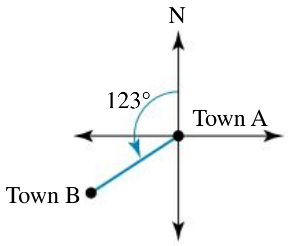
Interactivity: Bearings (int-6481)

Exercise 8.8 True bearings

1. **WE23** In the figure, determine the true bearing from:
   - a. Town A to Town B
   - b. Town B to Town A.

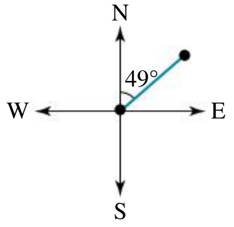


2. In the figure, determine the true bearing from:
   a. Town A to Town B
   b. Town B to Town A.

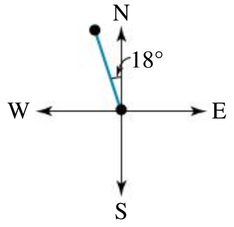


3. **WE24** A boat travels for 36 km in a direction of 155°T.
   a. Determine how far south the boat travels, correct to 2 decimal places.
   b. Determine how far east the boat travels, correct to 2 decimal places.
4. A boat travels north for 6 km, west for 3 km, then south for 2 km. Determine the boat's true bearing from its starting point. Give your answer in decimal form to 1 decimal place.
5. State each of the following as a true bearing.

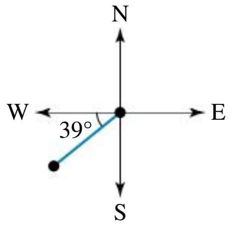
a.



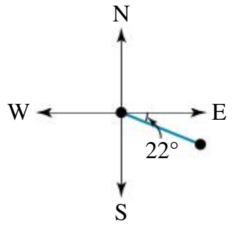
b.



c.

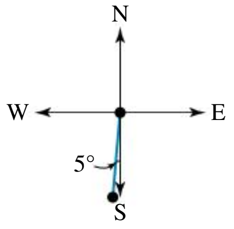


d.

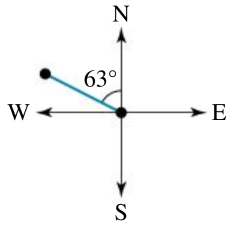


6. State the following as a true bearing.

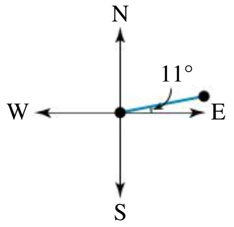
a.



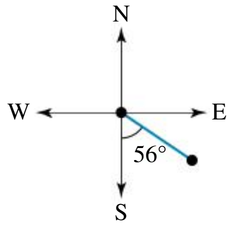
b.



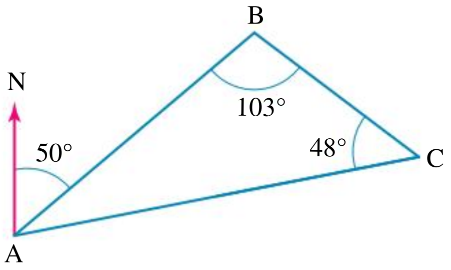
c.



d.

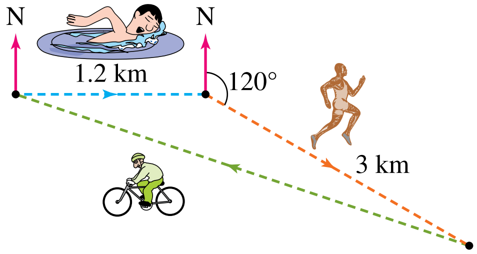


7. From the figure, determine the true bearing of



a. B from A
b. C from B
c. A from C
d. C from A.

8. A yacht race travels a triangular course. The first leg of the race, from the start to buoy 1, is 13 km due south. The second leg, from buoy 1 to buoy 2, is due west. The last leg, from buoy 2 back to the start, is 18 km.
   a. Represent the above information in a diagram.
   b. Calculate the length of the second leg of the race.
   c. Calculate the total length of the course.
   d. What is the bearing of the starting point from buoy 2 to the nearest degree?
9. A car travelled 5.6 km due east, then turned and travelled 800 m due south?
   a. Construct a diagram to show the path of the car.
   b. What is the compass bearing of the finishing point to the nearest degree?
   c. If the car could travel directly from its starting point to its finishing point, what would be the difference in the distance travelled?
10. An athlete practising for a triathlon competition completes a swim, run, cycle circuit as shown in the diagram. Use the information given to calculate:
   a. the distance that the athlete has to ride to get back to the start
   b. the bearing on which the athlete rides back to the start to the nearest degree.



8.9 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

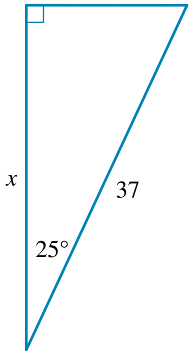
1. **MC** The length x in the triangle shown can be calculated by using:

   A. $37\cos(25°)$

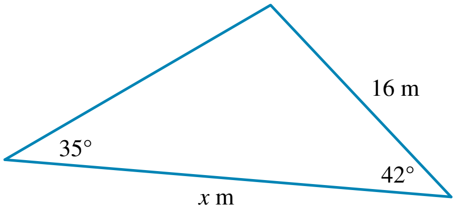
   B. $37\sin(25°)$

   C. $\dfrac{37}{\cos(25°)}$

   D. $\dfrac{37}{\sin(25°)}$

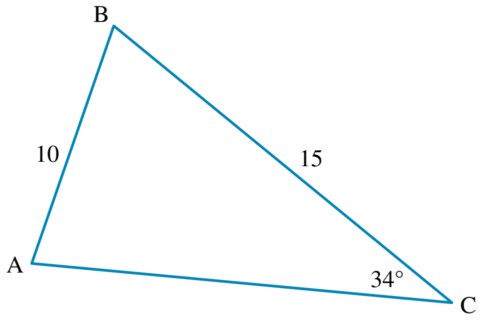


2. **MC** The length x in the triangle shown, correct to the nearest metre, is:



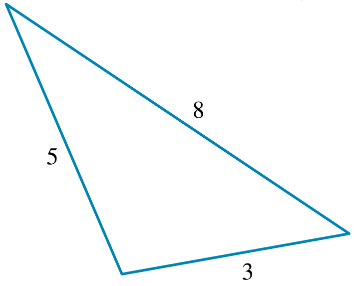
A. 9 **B.** 14 **C.** 19 **D.** 27

3. **MC** The magnitude of angle *A* in the triangle shown, correct to the nearest degree, is:



A. 22° **B.** 30° **C.** 34° **D.** 57°

4. **MC** The largest angle in the triangle shown can be calculated by using:



A. $\cos^{-1}\left(\frac{8^2+3^2-5^2}{2\times3\times8}\right)$

B. $\cos^{-1}\left(\frac{8^2+5^2-3^2}{2\times5\times8}\right)$

C. $\cos^{-1}\left(\frac{5^2+3^2-8^2}{2\times3\times5}\right)$

D. $\cos^{-1}\left(\frac{5^2+3^2-8^2}{2\times3\times8}\right)$

5. **MC** The acute and obtuse angles that have a sine of approximately 0.52992, correct to the nearest degree, are respectively:

A. 31° and 149°

B. 32° and 58°

C. 31° and 59°

D. 32° and 148°

6. **MC** Using Heron's formula, the area of a triangle with sides 4.2 cm, 5.1 cm and 9 cm is:

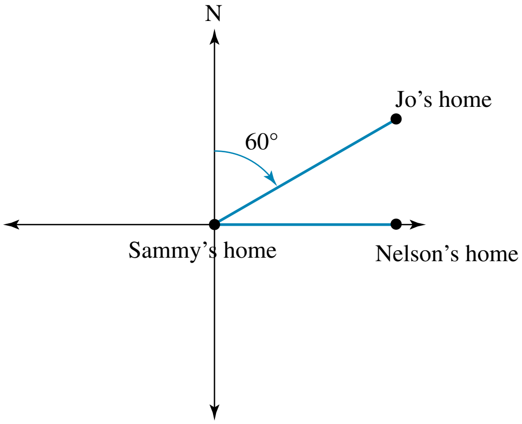
A. 5.3 cm^2

B. 9.2 cm^2

C. 13.7 cm^2

D. 18.3 cm^2

7. **MC** The locations of Jo's, Nelson's and Sammy's homes are shown in the diagram. Jo's home is due north of Nelson's home. The bearings of Jo's and Nelson's homes from Sammy's home are respectively:



A. 030°T and 090°T

B. 060°T and 090°T

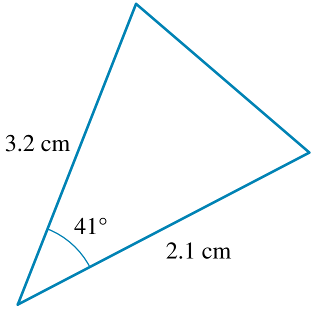
C. 030°T and 180°T

D. 060°T and 180°T

8. **MC** A boy is standing 150 m away from the base of a building. His eye level is 1.65 m above the ground. He observes a hot-air balloon hovering above the building at an angle of elevation of 30°. If the building is 20 m high, the distance the hot air balloon is above the top of the building is closest to:

A. 64 m **B.** 65 m **C.** 66 m **D.** 68 m

9. **MC** The area of the triangle shown, correct to 2 decimal places, is:

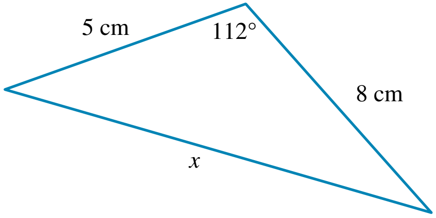


A. 2.20 cm^2 **B.** 2.54 cm^2 **C.** 3.36 cm^2 **D.** 4.41 cm^2

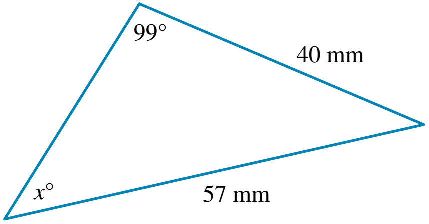
10. **MC** A unit of cadets walked from their camp for 7.5 km on a bearing of 064°T. They then travelled on a bearing of 148°T until they came to a signpost that indicated they were 14 km in a straight line from their camp. The bearing from the signpost back to their camp is closest to:

A. 032°T **B.** 064°T **C.** 096°T **D.** 296°T

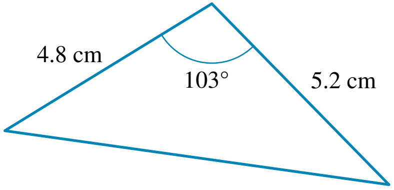
11. **a.** Calculate the value of the unknown length x correct to 2 decimal places.



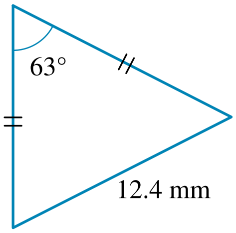
b. Calculate the value of x correct to the nearest degree.



12. **a.** Calculate the area of the triangle shown, correct to 2 decimal places.



b. Calculate the area of the triangle shown, correct to 2 decimal places.

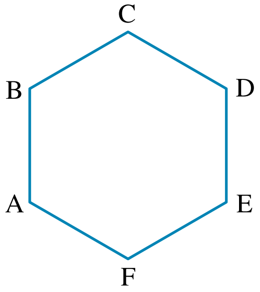


Complex familiar

13. In the triangle ABC, $a = 18.5$ m, $c = 12.6$ m and $C = 31.35°$.

- **a.** Draw a diagram with this information.
- **b.** Determine the value of A and hence the value of B.

14. ABCDEF is a regular hexagon with the point B being due north of A. Calculate the true bearing of the point:

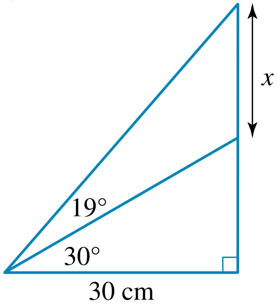


a. C from B **b.** D from C **c.** F from E **d.** E from B.

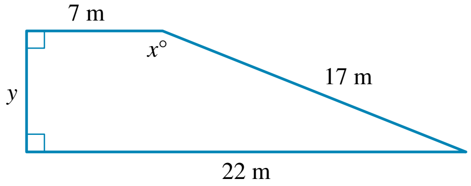
15. Three immunity idols are hidden on an island for a TV reality game. Idol B is 450 m on a bearing of 072°T from Idol A. Immunity idol C is 885 m on a bearing of S75°E from Idol A.

- **a.** Calculate the distance between immunity idols B and C to 2 decimal places.
- **b.** Calculate the triangular area to the nearest metre between immunity idols A, B and C that contestants need to search to find all three idols.

16. **a.** Calculate the value of the unknown length x correct to 2 decimal places.



b. Calculate the values of x and y, correct to 2 decimal places where appropriate.



Complex unfamiliar

17. Three treasure chests are buried on an island. Treasure chest B is 412 m on a bearing of 073°T from treasure chest A. Treasure chest C is 805 m on a bearing of 108°T from treasure chest A.

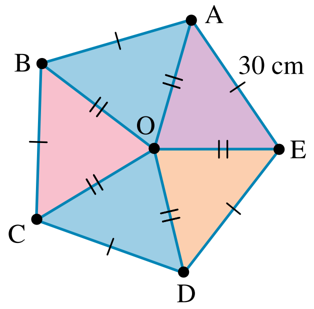
- **a.** Draw a diagram to represent the information.
- **b.** Show that angle A is 35°.
- **c.** Calculate the distance between treasure chests B and C correct to 1 decimal place.
- **d.** Calculate the triangular area between treasure chests A, B and C correct to 1 decimal place.
- **e.** A treasure hunter misreads the information as ‘Treasure chest B is 412 m on a bearing of 078°T from treasure chest A’ rather than ‘Treasure chest B is 412 m on a bearing of 073°T from treasure chest A.’
  - **i.** Construct a diagram to represent the misread information.
  - **ii.** Find how far the treasure hunter has travelled from the actual position of treasure chest B to his incorrect location of treasure chest B, correct to 1 decimal place.

- **iii.** Calculate the true bearing from his incorrect location of treasure chest B to the actual location of treasure chest B.

18. A dog kennel is placed in the corner of a triangular garden at point C. The dog kennel is positioned 30.5 m at an angle of 32.8° from one corner of the backyard fence (A) and 20.8 m from the other corner of the backyard fence (B).

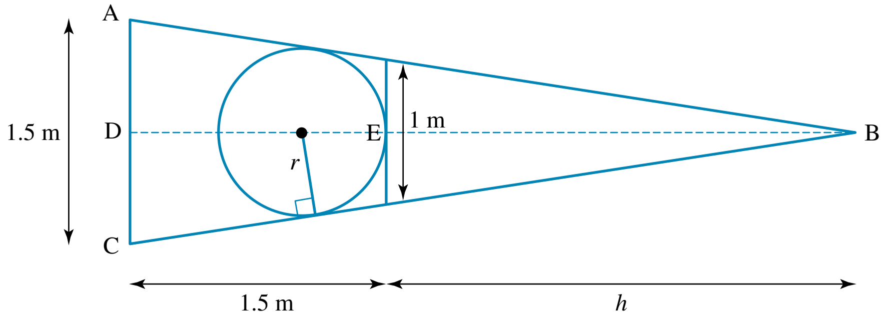
- **a.** Draw a diagram to represent the information.
- **b.** Calculate, correct to 1 decimal place:
  - **i.** the shortest distance between the dog kennel and the backyard fence
  - **ii.** the length of the backyard fence between points A and B.
- **c.** Using Heron's formula, calculate the triangular area between the dog kennel and the two corners of the backyard fence, correct to 1 decimal place.

19. A stained glass window frame consisting of five triangular sections is to be made in the shape of a regular pentagon with a side length of 30 cm.



- **a.** Show that $\angle AOB = 72°$.
- **b.** Calculate $\angle ABO$.
- **c.** Use the sine rule to find the length of OB to 1 decimal place.
- **d.** Calculate the total length of frame required to construct the window, correct to 1 decimal place.
- **e.** Three of the triangular panels must have coloured glass. Calculate the total area of coloured glass required correct to the nearest cm^2.

20. A triangular flag ABC has a printed design with a circle touching the sides of a flag and a 1-metre vertical line as shown in the diagram.



- **a.** Calculate h, the horizontal distance from the vertex point of the flag B to the vertical line at point E.
- **b.** Calculate $\angle ACB$ correct to 2 decimal places.
- **c.** Calculate r, the radius of the circle, correct to 2 decimal places.
- **d.** The circle printed design in the flag is to be coloured yellow. Calculate the area of the circle correct to 2 decimal places.
- **e.** Calculate the total area of the flag correct to 2 decimal places.

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Units 1 & 2 Sit chapter test

Answers

Chapter 8 Applications of trigonometry

Exercise 8.2 Review of trigonometric ratios

1. a. 1.540 b. 17.663
   c. 40.460 d. 0.657
2. a. 0.602 b. 2.092
   c. 15.246 d. 51.893
3. a. 0.707 b. 0.247
   c. 6.568 d. 5.896
4. a. 0.5 b. 0.966
   c. 1 d. 548.643
5. a. 64 b. 1.301
   c. 5.306 d. 1.374
6. a. 0.42 b. 1.56
   c. 0.09 d. 15.77
7. a. 5.10 b. 2.87
   c. 0.38 d. 7.77
8. a. 73.30 b. 0.87
9. a. 10° b. 16°
10. a. 44° b. 80°
11. a. 86°40′ b. 56°36′
12. a. 42°57′ b. 31°21′

Exercise 8.3 Applications of trigonometric ratios

1. $x = 2.16$ cm
2. $x = 10.6$ mm
3. $\theta = 46.90°$
4. $\theta = 45.48°$
5. $y = 1.99$ cm
6. $y = 2.63$ cm
7. $\theta = 73.97°$
8. $\theta = 35.58°$
9. $x = 13.15$ cm
10. $y = 0.75$ cm
11. $\theta = 53.20°$
12. $\theta = 36.38°$
13. a. 0.82
    b. −0.88
14. a. 0.99
    b. −0.84
15. $\theta = 49.32°$
16. $\theta = 10.29°$, $a = 79.71°$
17. a. 7.4 km
    b. 27.2 km
18. 20.56°
19. a. 47.16°
    b. 1.83 m
20. 100.94 m

Exercise 8.4 Area of triangles

1. 52.75 mm^2
2. 24.63 mm^2
3. 6.50 cm
4. 79.99 cm^2
5. 61.48 cm^2
6. 43.92 mm^2
7. a. 113.49 cm^2 b. 47.45 mm^2
   c. 216.10 cm^2 d. 122.46 cm^2
8. a. 89.67 cm^2 b. 37.80 mm^2
9. 2.082 km^2
10. $x = 4.5$ cm
11. 11.1 cm
12. 205.39 cm^2

Exercise 8.5 Angles of elevation and depression

1. 32.63 km
2. 47.13 m
3. 36.81 m
4. 61.54°
5. 36.25°
6. 2.73 m
7. 999.85 m
8. 862.83 m
9. 315 m
10. 11.13 m
11. 52°
12. 88.28°
13. 1357 m
14. a. 83.59° b. 82.35° c. 79.93°
    d. 74.97° e. 60.67°

Exercise 8.6 The sine rule

1. $x = 10.36$ cm
2. $x = 10.38$ cm
3. $x = 9.05$ cm
4. $x = 17$ cm
5. a. 247.68 cm^2
   b. 750.79 cm^2
6. $B = 28.3°$
7. $B = 27.1°$
8. $\theta = 47.2°$
9. $B = 46.97°$ or $133.03°$
10. $b = 12.24$, $c = 13.44$
11. 20.71 m
12. a. 23.18 m
    b. 10.66 m
13. $x = 142.4°$, $y = 37.6°$
14. 9.80 m
15. 5.91 km

16. **a.** $B = 41.39°$, $C = 107.31°$
b. $c = 20.21$ cm
c. 73.51 cm^2

17. 41.39 cm^2, 61.41 cm^2 and 59.12 cm^2

18. 167 330 m^2

19. 8.14 m^3

Exercise 8.7 The cosine rule

1. $x = 2.74$ km

2. $x = 10.49$ m

3. $x = 8.5$ km

4. $x = 4.48$ m

5. $A = 26.95°$

6. $A = 99.59°$

7. $A = 44.42°$

8. $b = 10.17$

9. 79.66°

10. 82.82°

11. 33.56°

12. 13.23 cm

13. 31.68 km

14. 98.86 km

15. 8822 m

16. 7 hours, 16 minutes

Exercise 8.8 True bearings

1. **a.** 110°T **b.** 290°T

2. **a.** 237°T **b.** 057°T

3. **a.** 32.63 km **b.** 15.21 km

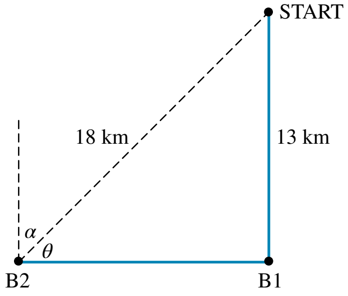
4. 323.1°T

5. **a.** 049°T **b.** 342°T
c. 231°T **d.** 112°T

6. **a.** 185°T **b.** 297°T
c. 079°T **d.** 124°T

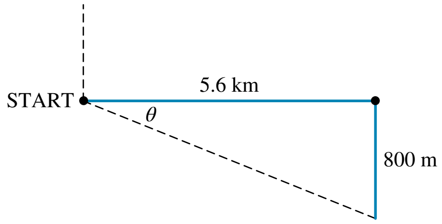
7. **a.** 050°T **b.** 127°T
c. 259°T **d.** 079°T

8. **a.**



b. 12.45 km
c. 43.45 km
d. 044°T

9. **a.**



b. 098°T
c. 0.74 km

10. **a.** 4.08 km **b.** 292°T

8.9 Review: exam practice

1. A

2. D

3. D

4. C

5. D

6. A

7. B

8. D

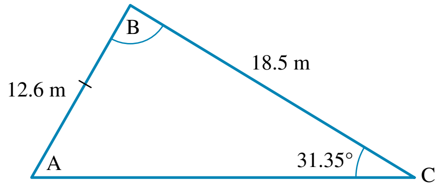
9. A

10. D

11. **a.** 10.91 cm **b.** 44°

12. **a.** 12.16 cm^2 **b.** 62.73 mm^2

13. **a.**



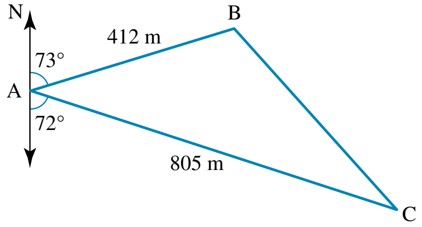
b. $A = 49.81°$, $B = 98.84°$

14. **a.** 060°T
b. 120°T
c. 240°T
d. 120°T

15. **a.** 563.67 m
b. 108 451 m^2

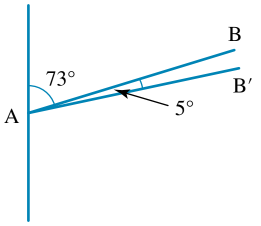
16. **a.** 17.19 cm
b. $x = 151.93°$, $y = 8$ m

17. **a.**



b. $A = 180 - (73 + 72) = 35°$
c. 523.8 m
d. 95 116.2 m^2

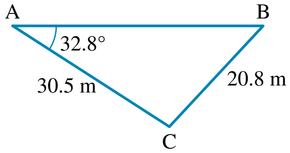
e. **i.**



ii. 35.9 m

iii. 345.6°T

18. **a.**



b. **i.** 16.5 m **ii.** 38.3 m

c. 316.1 m^2

19. **a.** 72° **b.** 54°

c. 25.5 cm **d.** 277.5 cm

e. 928 cm^2

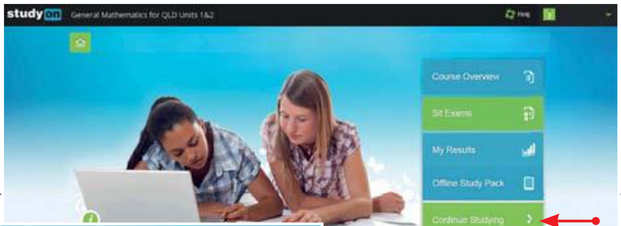
20. **a.** 3 m **b.** 80.54° **c.** 0.59 m

d. 1.09 m^2 **e.** 3.38 m^2

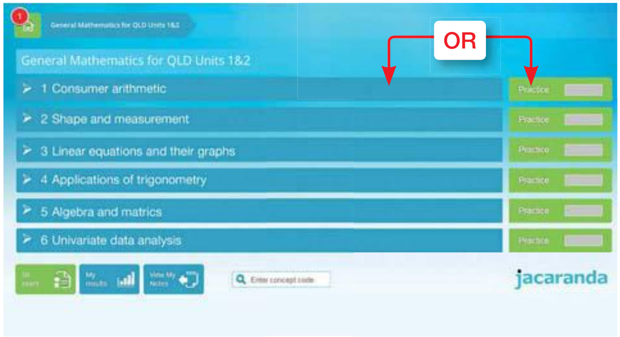
REVISION UNIT 2 Applied trigonometry, algebra, matrices and univariate data

TOPIC 1 Applications of trigonometry

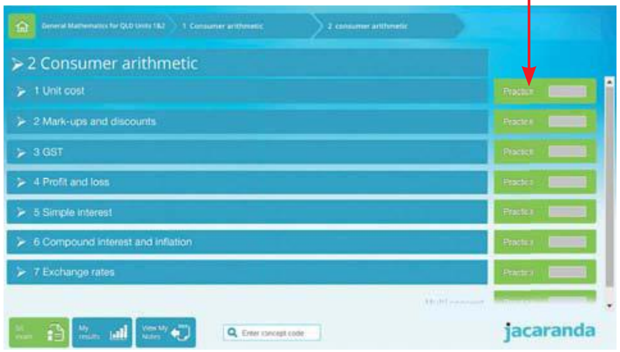
- For revision of this entire topic, go to your **studyON** title in your bookshelf at **www.jacplus.com.au**.
- Select **Continue Studying** to access hundreds of revision questions across your entire course.



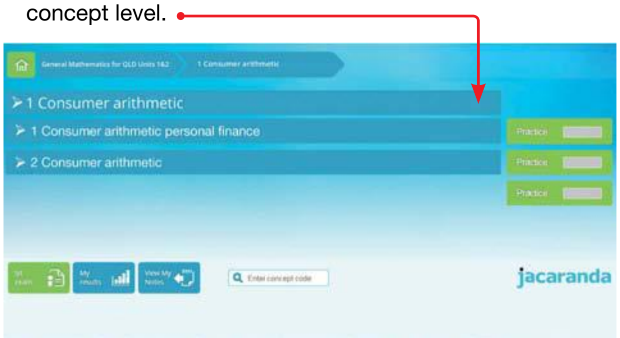
- Select your **course** *General Mathematics for Queensland Units 1&2* to see the entire course divided into syllabus topics.
- Select the **area** you are studying to navigate into the sequence level *OR* select **Practice** to answer all practice questions available for each area.



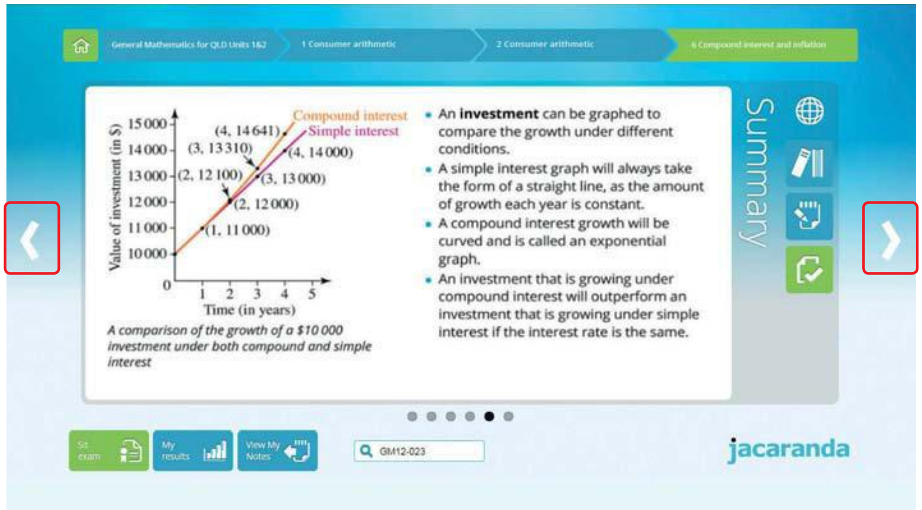
- Select **Practice** at the sequence level to access all questions in the sequence.



- At **sequence level**, drill down to concept level.



- **Summary screens** provide revision and consolidation of key concepts. Select the **next arrow** to revise all concepts in the sequence and practice questions at the concept level for a more granular set of questions.



A comparison of the growth of a $10 000 investment under both compound and simple interest

- An **investment** can be graphed to compare the growth under different conditions.
- A simple interest graph will always take the form of a straight line, as the amount of growth each year is constant.
- A compound interest growth will be curved and is called an exponential graph.
- An investment that is growing under compound interest will outperform an investment that is growing under simple interest if the interest rate is the same.

CHAPTER 9
Linear and non-linear relationships

9.1 Overview

9.1.1 Introduction

Formulas are used every day, such as to calculate areas, volumes or lengths. These are the types of formulas that you may be aware of from previous studies. There are many very famous formulas that you may not have heard of directly, but which have had a huge impact on true development of our understanding of the universe.

Here are three of the most famous formulas in history:

1. Isaac Newton's Law of Universal Gravitation:

   $F = G\frac{m_1 m_2}{r^2}$

   This formula explains why the planets move the way they do and how gravity works, both on Earth and in the wider universe. This formula was first published in 1687.

2. Albert Einstein's Theory of Relativity:

   $E = mc^2$

   Einstein's most famous theory, that looks at the relationship between space and time. This theory was first proposed in 1905. The Theory of Relativity rocked the world of physics, and deepened our knowledge of the universe.

3. The Pythagorean Theorem:

   $c^2 = a^2 + b^2$

This ancient theorem was first recorded around 500 BC. It is a fundamental principle of Euclidean geometry, and the basis for the distance between two points. You would be more familiar with it being used with right angled triangles.

All formulas express a relationship between different variables and constants. This chapter demonstrates how formulas can be used and manipulated to solve problems.

LEARNING SEQUENCE

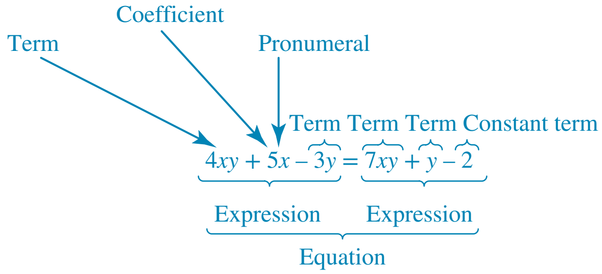
Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

9.2 Substitution

9.2.1 Understanding algebra

Algebra is a type of language used in mathematics that includes pronumerals (letters or groups of letters) that are used to represent unknown numbers. Pronumerals can also be used to describe **variables** (varying values).

The terminology used in algebra is shown in the equation below:



Term: a group of letters and numbers. These form an expression and are separated by an addition or subtraction sign, i.e. $4xy$

Pronumeral: the letter part of the term, i.e. x in $4xy$

Coefficient: the number multiplying (out the front) the pronumeral, i.e. 4 in $4xy$

Constant term: the term that does not have a pronumeral, i.e. -2 in $7xy + y - 2$

Expression: a mathematical statement made up of terms, operation symbols and/or brackets. It does not contain an equality sign, i.e. $7xy + y - 2$

Equation: a mathematical statement containing a left- and right-hand side separated by an equality sign

Like terms: like terms have identical pronumerals, i.e. $-2y$ and y

Sum: only like terms can be added together, i.e. $2x + x = 3x$

Difference: only like terms can be subtracted, i.e. $5y - 2y = 3y$

Product: To find the product of algebraic terms, the terms are multiplied, i.e. $5y \times 2x = 10xy$

9.2.2 Substitution

A **formula** or **rule** is an expression or equation that expresses the relationship between certain quantities.

e.g. OUT = IN + 3 or $y = x + 3$.

When a variable in a formula is replaced by a number, we say that the number is substituted into the formula.

WORKED EXAMPLE 1

If $y = 5x - 4$, substitute the given value of x into the formula to calculate the value of y in each case.

a. $x = 2$ **b.** $x = 6$

THINK	WRITE
a. 1. Write the formula.	$y = 5x - 4$
2. Substitute 2 for x.	$y = 5 \times 2 - 4$
3. Equate for y and write the answer.	$y = 10 - 4$ $y = 6$
b. 1. Write the formula.	$y = 5x - 4$
2. Substitute 6 for x.	$y = 5 \times 6 - 4$
3. Equate for y and write the answer.	$y = 30 - 4$ $y = 26$

This Worked example could also be set up as a table like this: Complete the following table given the rule $y = 5x - 4$.

x	$y = 5x - 4$
2	6
6	26

Resources

- **Interactivity:** Input and output tables (int-4001)
- **Interactivity:** Finding a formula (int-4002)
- **Interactivity:** Substitution (int-4003)

9.2.3 Single substitution

An algebraic expression contains a letter that represents a number and this is known as a pronumeral (or variable). In algebraic expressions the pronumerals are referred to as unknowns since they are specific numbers that are not yet known.

If the value of the pronumerals (or variables) are known, it is possible to **evaluate** (work out the value of) an algebraic expression by using **substitution**. This is done by replacing the pronumeral with its corresponding number.

When evaluating algebraic expressions the rules for **BIDMAS (Brackets, Indices, Division, Multiplication and Subraction)** must still apply and be remembered; for example, $3x$ means $3 \times x$ and ab means $a \times b$.

WORKED EXAMPLE 2

Determine the value of the following expressions if $a = 4$ and $b = 3$.

a. $5b$ **b.** $\frac{8a}{4}$ **c.** $4(b - 1)$

THINK	WRITE
a. 1. Substitute $b = 3$ into the expression.	$5b = 5 \times 3$
2. Evaluate and write the answer.	$= 15$
b. 1. Substitute $a = 4$ into the expression.	$\frac{8a}{4} = \frac{8 \times 4}{4}$
2. Evaluate and write the answer.	$= \frac{32}{4}$ $= 8$
c. 1. Substitute $b = 3$ into the expression.	$4(b - 1) = 4(3 - 1)$
2. Evaluate and write the answer.	$= 4(2)$ $= 8$

Both positive and negative numbers can be substituted into an equation. When substituting negative numbers into an equation be careful of the signs.

WORKED EXAMPLE 3

Determine the value of the following expressions if $p = -2$ and $q = 5$.

a. $-2p + 2$ **b.** $\frac{3}{4q}$ **c.** $7(5 - 3p^2)$

THINK	WRITE
a. 1. Substitute $p = -2$ into the expression.	$-2p + 2 = -2 \times -2 + 2$
2. Evaluate and write the answer.	$= 4 + 2$ $= 6$
b. 1. Substitute $q = 5$ into the expression.	$\frac{3}{4q} = \frac{3}{4 \times 5}$
2. Evaluate and write the answer.	$= \frac{3}{20}$
c. 1. Substitute $p = -2$ into the expression.	$7(5 - 3p^2) = 7(5 - 3 \times (-2)^2)$
2. Evaluate and write the answer.	$= 7(5 - 3 \times 4)$ $= 7(5 - 12)$ $= 7 \times -7$ $= -49$

studyon

Units 1 & 2 › Area 5 › Sequence 1 › Concept 1 **Substitution** Summary screen and practice questions

Exercise 9.2 Substitution

1. **WE1** Evaluate the following equations to find the value of y in each case, if $x = 3$ and $z = 8$:
   - **a.** $y = 5 + x$ **b.** $y = x + 14$ **c.** $y = z + 2$ **d.** $y = 17 + z$
   - **e.** $y = x - 2$ **f.** $y = 21 - x$ **g.** $y = z - 3$ **h.** $y = 21 - z$
2. **WE2** Determine the value of the following expressions, if $a = 3$ and $b = 6$:
   - **a.** $2a$ **b.** $-5b$ **c.** $7a$ **d.** $15b$
   - **e.** $12a$ **f.** $-3 \times 2b$ **g.** $5a \times 2$ **h.** $-8a$
3. Evaluate the following expressions, if $m = 8$ and $n = 3$:
   - **a.** $\frac{m}{2}$ **b.** $\frac{n}{3}$ **c.** $\frac{24}{m}$ **d.** $\frac{15}{n}$
   - **e.** $-\frac{88}{m}$ **f.** $-\frac{36}{n}$ **g.** $\frac{4m}{5}$ **h.** $\frac{20n}{15}$
4. Evaluate the following expressions, if $j = 1$ and $k = 6$:
   - **a.** $3(j + 5)$ **b.** $2(k + 5)$ **c.** $7(11 - j)$ **d.** $12(17 - k)$
   - **e.** $5j(j + 4)$ **f.** $2k(k - 2)$ **g.** $6j(j - 3)$ **h.** $5k(k - 2)$
5. **WE3** Determine the value of the following expressions, if $p = -3$ and $q = 12$:
   - **a.** $5q$ **b.** $-3p$ **c.** $5 + q$ **d.** $13 - p$
   - **e.** $-\frac{39}{p}$ **f.** $\frac{2q}{8}$ **g.** $5(2 - p)$ **h.** $q(q - 8)$

6. Determine the value of the following expressions, if $u = -2$ and $v = -10$:

a. $-7u$ **b.** $-3v$ **c.** $15 + v$ **d.** $27 - u$

e. $-\frac{56}{u}$ **f.** $\frac{6v}{4}$ **g.** $9(2 - u)$ **h.** $v(v - 8)$

7. Determine the value of the following expressions, if $m = -6$ and $n = 0$:

a. $13 - n$ **b.** $-2(3 + m)$ **c.** m^2

d. $7m + 50$ **e.** $9n(10 - n)$ **f.** $\frac{7n}{(2 - n)}$

8. **WE1** Complete the following tables, given the rule:

a. $y = 5x - 4$

x	y
0	
1	
2	
3	

b. $y = -2x + 16$

x	y
3	
6	
9	
12	

9. Complete the following tables, given the rule:

a. $y = 4x + 10$

x	y
−4	
−2	
0	
2	

b. $y = -15x + 35$

x	y
−8	
−4	
0	
4	

10. The area of a circle is calculated by using the formula $A = \pi r^2$. Calculate the area of the following circles to two decimal places:

a. A circular garden bed of radius 5 m.

b. A circular dinner plate of radius 8.75 cm.

c. A dartboard of diameter 45.1 cm.

11. If a phone company charged $19 per month and 9 cents per SMS, what would the monthly phone bill be if the following number of SMSs were made in the month (assuming the phone was only used for SMSs)?

a. 48 SMSs **b.** 30 SMSs

c. 70 SMSs **d.** Derive the rule for the SMS charges.

12. Calculate the value of the algebraic expression $\frac{x^2(5x - 1)}{2x}$ when $x = 4$.

13. Determine the value of x that makes the algebraic expression $6x(3x - 1)$ equal 60.

14. **MC** The value of x that makes the algebraic expression $\frac{3x^2(7x + 3)}{2x}$ equal 33 is:

A. $x = 1$ **B.** $x = 2$ **C.** $x = -1$ **D.** $x = -2$

9.3 Formulas

9.3.1 Evaluating a formula

A formula is a special equation or rule that describes the relationship between different quantities.

To be able to use a formula, you need to know:

- what the pronumerals represent
- the correct units for the pronumerals
- what the formula is used for and the relationship between the variables. For example, in directly proportional expressions, as one variable increases so does the other; in inversely proportional expressions, as one variable increases the other decreases; and in polynomial expressions, as one variable increases the other increases at a much greater rate.

Two examples are shown in the table.

Formula	**What it is used for**	**What the pronumerals represent**	**Unit requirements**
$A = lw$ (directly proportional)	Calculating the area of a rectangle	A represents the area of the rectangle. l represents the length of the rectangle. w represents the width of the rectangle.	l and w must be in the same units.
$s = \frac{d}{t}$ (inversely proportional)	Calculating speed given distance and time	s represents speed d represents distance t represents time	s must reflect the units for d and t e.g. km/h
$E = mc^2$ (a polynomial)	Calculating the energy contained in a given mass	E represents the amount of energy. m represents the mass. c represents the speed of light.	m must be in kg. c must be in m/s.

Often equations are related to mathematical formulas and require substitution to calculate the required answer.

WORKED EXAMPLE 4

The surface area of a cylinder is calculated by using the formula, $SA = 2\pi r^2 + 2\pi rh$, where r is the radius of the circular base and h is the height of the cylinder. (*Note:* This formula is both directly proportional and polynomial.)

a. If a small paint can is a cylinder with a height of 8 cm and a radius of 5 cm, calculate its surface area to two decimal places.

b. If the height of the cylinder is doubled to 16 cm, calculate the effect on the surface area to two decimal points. Comment on your results using mathematical reasoning.

THINK	WRITE
a. 1. Identify the formula required to answer the question.	$SA = 2\pi r^2 + 2\pi rh$
2. Identify the variables given and make sure the units are consistent.	$h = 8$ cm and $r = 5$ cm

3. Substitute the variable value into the formula.	$SA = 2\pi r^2 + 2\pi rh$ $SA = 2\pi(5)^2 + 2\pi(5)(8)$ $SA = 408.41$
4. Write the answer with correct units.	Surface area = 408.41 cm^2
b. 1. Substitute the variable value into the formula.	$SA = 2\pi(5)^2 + 2\pi(5)(16)$ $SA = 659.73$ Surface area = 659.73 cm^2. The height doubling has directly and proportionally increased the surface area.

on Resources

- **Interactivity:** Equations (int-4005)
- **Interactivity:** Writing equations (int-4041)

9.3.2 Using technology to evaluate formulas

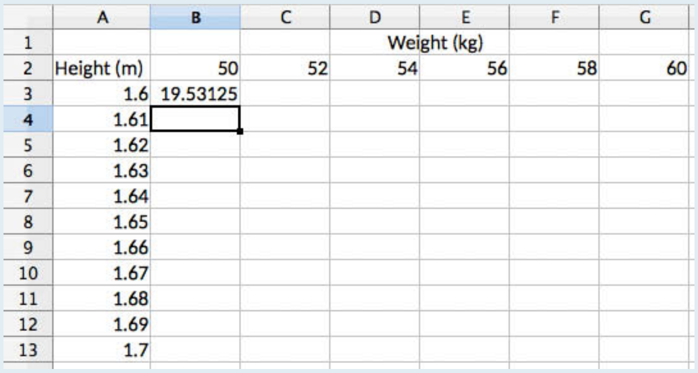
Often multiple calculations are to be done using the same formula. This can be done quickly with a spreadsheet. The following example looks at calculating the body mass index (BMI) of people with different weights and heights. BMI is an indicator of whether a person is underweight, overweight, or a healthy weight. BMI is calculated using a person's weight and height. Overweight is defined as a BMI of 25–29.9; obesity is defined as a BMI equal to or greater than 30. Your BMI is calculated by dividing your weight (in kilograms) by your height squared (in metres)

WORKED EXAMPLE 5

Use a spreadsheet to calculate the BMI of people with weights of 50 kg to 60 kg with increments of 2 kg and with heights of 1.60 m to 1.70 m with increments of 0.01 m.

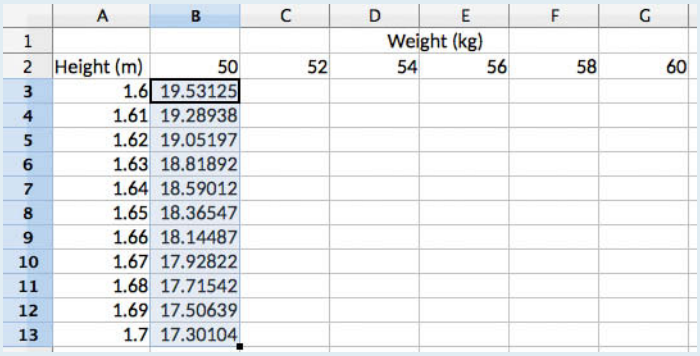
THINK	WRITE
1. Identify the formula required to calculate the BMI.	$\text{BMI} = \dfrac{\text{Weight}}{(\text{Height})^2}$
2. Identify the variables given and make sure the units used are correct.	Height (m) and Weight (kg)

3. Set up the spreadsheet with the specified increments. In cell B3 input the formula (=$B$2/(A3)^2). This calculates the BMI of a 50 kg and 1.60 m tall person. It also uses the $ to lock the cell for copying the formula.



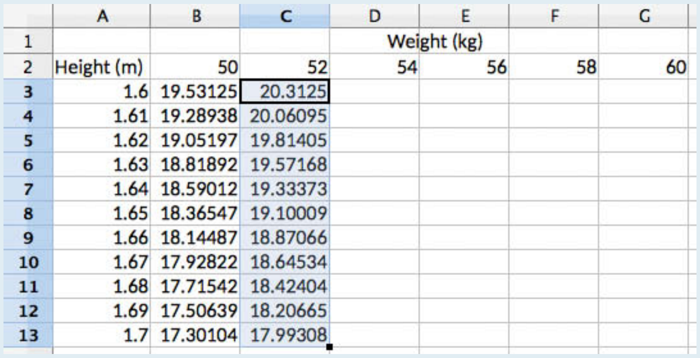
	A	B	C	D	E	F	G
1				Weight (kg)			
2	Height (m)	50	52	54	56	58	60
3	1.6	19.53125					
4	1.61						
5	1.62						
6	1.63						
7	1.64						
8	1.65						
9	1.66						
10	1.67						
11	1.68						
12	1.69						
13	1.7						

4. Copy this cell down vertically to calculate the BMI for all the heights for a 50 kg person. Do this by clicking on the B3 cell and moving the cursor to the bottom right-hand corner; the cross will go black. Then drag this down to cell B13.



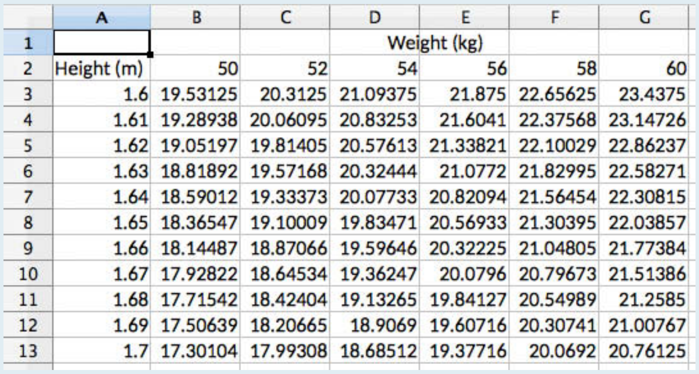
	A	B	C	D	E	F	G
1				Weight (kg)			
2	Height (m)	50	52	54	56	58	60
3	1.6	19.53125					
4	1.61	19.28938					
5	1.62	19.05197					
6	1.63	18.81892					
7	1.64	18.59012					
8	1.65	18.36547					
9	1.66	18.14487					
10	1.67	17.92822					
11	1.68	17.71542					
12	1.69	17.50639					
13	1.7	17.30104					

5. Do the same thing for cell C3, but adjust to use the 52 kg weight. In cell C3 type (=$C$2/(A3)^2). Then copy this vertically the same way as in the previous section.



	A	B	C	D	E	F	G
1				Weight (kg)			
2	Height (m)	50	52	54	56	58	60
3	1.6	19.53125	20.3125				
4	1.61	19.28938	20.06095				
5	1.62	19.05197	19.81405				
6	1.63	18.81892	19.57168				
7	1.64	18.59012	19.33373				
8	1.65	18.36547	19.10009				
9	1.66	18.14487	18.87066				
10	1.67	17.92822	18.64534				
11	1.68	17.71542	18.42404				
12	1.69	17.50639	18.20665				
13	1.7	17.30104	17.99308				

6. Repeat this for each of the columns to complete the table of BMI values.



	A	B	C	D	E	F	G
1				Weight (kg)			
2	Height (m)	50	52	54	56	58	60
3	1.6	19.53125	20.3125	21.09375	21.875	22.65625	23.4375
4	1.61	19.28938	20.06095	20.83253	21.6041	22.37568	23.14726
5	1.62	19.05197	19.81405	20.57613	21.33821	22.10029	22.86237
6	1.63	18.81892	19.57168	20.32444	21.0772	21.82995	22.58271
7	1.64	18.59012	19.33373	20.07733	20.82094	21.56454	22.30815
8	1.65	18.36547	19.10009	19.83471	20.56933	21.30395	22.03857
9	1.66	18.14487	18.87066	19.59646	20.32225	21.04805	21.77384
10	1.67	17.92822	18.64534	19.36247	20.0796	20.79673	21.51386
11	1.68	17.71542	18.42404	19.13265	19.84127	20.54989	21.2585
12	1.69	17.50639	18.20665	18.9069	19.60716	20.30741	21.00767
13	1.7	17.30104	17.99308	18.68512	19.37716	20.0692	20.76125

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Units 1 & 2 › Area 5 › Sequence 1 › Concept 2 **Formulas** Summary screen and practice questions

Exercise 9.3 Formulas

1. Evaluate the following formulas for C, if $a = -5$ and $b = 0$:

a. $C = a + b$ **b.** $C = 2a$ **c.** $C = b - a$ **d.** $C = -2a(3 + b)$

e. $C = 12b$ **f.** $C = 2ab$ **g.** $C = 4b(b - a)$ **h.** $C = \dfrac{7b}{(2 - a)}$

2. Evaluate the following formulas for R, if $p = -2$ and $q = -5$:

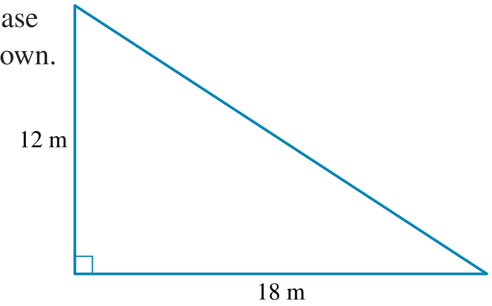
a. $R = \dfrac{(p - q)}{-3p}$ **b.** $R = \dfrac{4q^2}{-10p}$ **c.** $R = p^2(3p - 2q)$ **d.** $R = p^2 - q^2$

3. **WE4** The surface area of a cylinder is calculated by using the formula $SA = 2\pi r^2 + 2\pi rh$, where r is the radius of the circular base and h is the height of the cylinder.

a. Given a cylinder has a height of 13 cm and a radius of 4 cm, calculate its surface area to two decimal places.

b. Calculate its surface area to two decimal places if the height is doubled to 26 cm.

4. The area of a triangle is $A = \frac{1}{2}bh$, where b is the length of the base and h is the vertical height. Calculate the area of the triangle shown.



5. The surface area of a cylinder and a sphere are found by using the following formulas, where r stands for the radius and h represents the height.

$SA_{Cylinder} = 2\pi r^2 + 2\pi rh$ $SA_{Sqhere} = 4\pi r^2$

Calculate the surface area to two decimal places of:

a. Cylinder with a radius of 3 cm and height of 8 cm

b. Cylinder with a radius of 15 cm and height of 9 cm

c. Cylinder with a radius of 12.5 m and height of 18 m

d. Cylinder with a radius of 2 300 cm and height of 35.8 m

e. Sphere with a radius of 5 cm

f. Sphere with a radius of 12 m

g. Sphere with a radius of 7.25 cm

h. Sphere with a diameter of 25 m

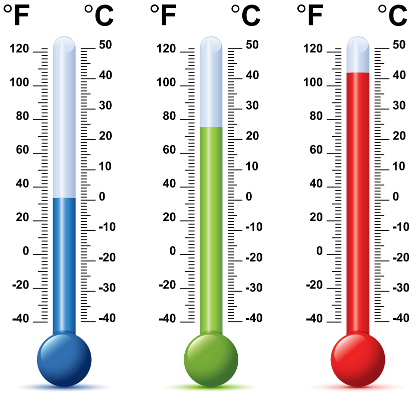
6. **WE5** Use a spreadsheet to calculate the BMI of people with weights of 60 kg to 70 kg with increments of 2 kg and with heights of 1.70 m to 1.80 m with increments of 0.01 m.

7. Use a spreadsheet to calculate the net force (F) for masses (m) of 100 kg to 200 kg, with increments of 10 kg and with accelerations (a) of 0.5 m/s^2 to 1.5 m/s^2, with increments of 0.1 m/s^2, given Newton's Second Law is $F = m \times a$.

8. Use a spreadsheet to calculate the kinetic energy (KE) of an object with masses (m), of 100 kg to 200 kg with increments of 10 kg and with velocities (v) 15 m/s to 25 m/s with increments of 1 m/s given $KE = \frac{1}{2}mv^2$.

9. In Australia we measure our daily temperature in degrees Celsius, °C, whereas in the United States for example, they measure temperature in degrees Fahrenheit, °F. To convert between the two, the following formulas can be used and rounded to 2 decimal places:

$C = \frac{5}{9}(F - 32)$ $F = \left(C \times \frac{9}{5}\right) + 32$



a. Convert the following Fahrenheit temperatures to Celsius:

i. $F = 100°$ **ii.** $F = 50°$

iii. $F = 78°$ **iv.** $F = 25°$

b. Convert the following Celsius temperatures to Fahrenheit:

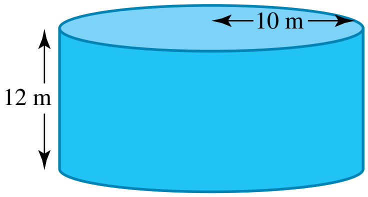
i. $C = 45°$ **ii.** $C = 0°$

iii. $C = 25°$ **iv.** $C = 33°$

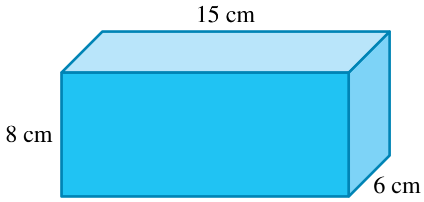
c. Use a spreadsheet to calculate the Celsius (C) values for Fahrenheit (F) values of 80 °F to 90 °F with increments of 1 °F. Then use a spreadsheet to calculate the Fahrenheit (F) values for Celsius (C) values of 25 °C to 35 °C with increments of 1 °C.

10. Determine the volume of the following shapes after identifying the appropriate formula to use.

a.

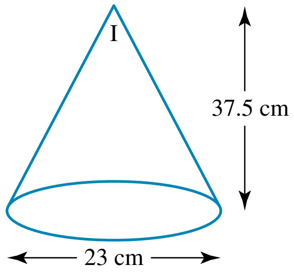


b.

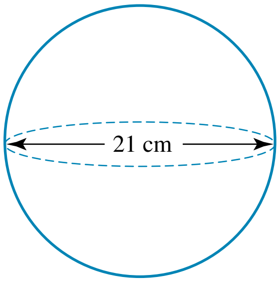


11. Determine the volume of the following shapes after investigating the appropriate formula to use.

a.



b.



12. A mobile phone company uses the formula $C = 0.025n + 29$, where C represents the monthly cost and n represents the number of SMSs made during the month. Calculate the following bills:

a. The monthly bill when 66 SMSs were made.

b. The monthly bill when 116 SMSs were made.

c. The total half yearly bill when 34, 48, 21, 103, 87, 77 were made for each of the months.

d. The yearly bill when on average 58 SMSs were made each month.

13. The volume of a rectangular prism is given by $V = L \times W \times H$. Identify the dimensions of two possible rectangular prisms that have a volume of $120\,m^3$.

14. Two electricity bills are calculated by using the formulas $C_A = 0.16K + 75$ and $C_B = 0.12K + 90$, where C is the cost of the bill and K is the amount of kilowatt-hours (kWh) of electricity used.

a. Decide which bill is cheaper when 200 kWh are used.

b. Calculate the amount of kilowatt-hours used when the two bills are the same.

9.4 Transposition

9.4.1 Rearranging formulas

A formula is an equation showing the relationship between two or more quantities. For example, as was mentioned earlier in the chapter, body mass index (B) is calculated using a person's mass (M) in kilograms and their height (H) in metres.

$$B = \frac{M}{H^2}$$

Sometimes formulas are rearranged so that a relationship is more obvious, or to make the formula easier to use. This is often called making a different variable the subject. This is when the new variable is to the left of the equal sign. For example, making the mass the subject of the body mass index formula is:

$$M = B \times H^2$$

To make a different variable the subject requires you to rearrange or transpose the formula. This is done by using inverse operations performed to both sides of the equation.

$+$ and $-$ are inverse operations.
$\times$ and $\div$ are inverse operations.
Squared $(^2)$ and Square root $\left(\sqrt{\ }\right)$ are inverse operations

WORKED EXAMPLE 6

Transpose the formula $B = \frac{M}{H^2}$ to make M the subject of the formula.

THINK	WRITE
1. Write down the formula.	$B = \frac{M}{H^2}$
2. To make M the subject, use inverse operations to leave M on the right-hand side by itself. Undo the division by H^2 by multiplying both sides by H^2, as shown in blue.	$H^2 \times B = \frac{M}{\cancel{H}^2} \times \cancel{H}^2$ $H^2 \times B = M$
3. Rewrite the equation with the subject first and the variables alphabetically.	$M = H^2 \times B$ $= BH^2$

WORKED EXAMPLE 7

Transpose the formula $a = \frac{v - u}{t}$ to make v the subject.

THINK	WRITE
1. Write the formula.	$a = \frac{v - u}{t}$
2. Undo the steps used to build the formula around v, starting with the last step first. Multiply both sides by t, as shown in blue.	$a \times t = \frac{v - u}{{}^1\cancel{t}} \times {}^1\cancel{t}$

▶

3. Then add u to both sides, as shown in red.	$a \times t + u = v - u + u$
4. Rewrite the formula with the subject on the left-hand side.	$v = at + u$

WORKED EXAMPLE 8

Transpose the formula $B = \frac{M}{H^2}$ to make H the subject.

THINK	WRITE
1. Write the formula.	$B = \frac{M}{H^2}$
2. Undo the steps used to build the formula around H, starting with multiplying both sides by H^2.	$B \times H^2 = \frac{M}{\cancel{H^2}} \times \cancel{H^2}$ $B \times H^2 = M$
3. Then divide both sides by B.	$\frac{B \times H^2}{B} = \frac{M}{B}$ $H^2 = \frac{M}{B}$
4. Then take the square root of both sides.	$H^2 = \frac{M}{B}$ $\sqrt{H^2} = \sqrt{\frac{M}{B}}$ (positive square root as height is positive) $H = \sqrt{\frac{M}{B}}$
5. Rewrite the formula.	$H = \sqrt{\frac{M}{B}}$

on Resources

- **Interactivity:** Backtracking (int-4045)
- **Interactivity:** Inverse operations (int-4043)
- **Interactivity:** Rearranging formulas (int-6040)

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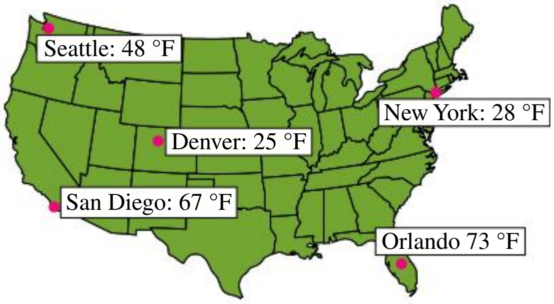
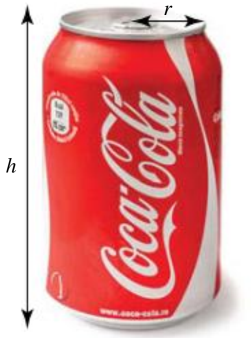
Units 1 & 2 > Area 5 > Sequence 1 > Concept 3 **Transposition** Summary screen and practice questions

Exercise 9.4 Transposition

1. **WE6** Transpose each of the following formulas to make the variable shown in the brackets the subject of the formula.
   - **a.** $C = AB + D \quad (A)$ **b.** $v = u + at \quad (t)$ **c.** $V = \dfrac{Ah}{3} \quad (h)$ **d.** $A = \dfrac{h(a+b)}{2} \quad (b)$
2. **WE7** Transpose each of the following formulas to make the variable shown in the brackets the subject of the formula.
   - **a.** $A = 2\pi r(r + h) \quad (h)$ **b.** $S = \dfrac{a}{1 - r} \quad (r)$
   - **c.** $I = \dfrac{PRT}{100} \quad (R)$ **d.** $A = PR^n \quad (P)$
3. **WE8** Transpose each of the following formulas to make the variable shown in the brackets the subject of the formula.
   - **a.** $A = x^2 \quad (x)$ **b.** $V = \dfrac{\pi r^2 h}{3} \quad (r)$ **c.** $v^2 = u^2 + 2as \quad (u)$ **d.** $c = \sqrt{a^2 + b^2} \quad (a)$
4. The formula $F = \dfrac{9C}{5} + 32$ converts degrees Celsius, °C, to degrees Fahrenheit, °F. While travelling in the USA you discover that the weather reports give the weather forecast in degrees Fahrenheit.
   - **a.** Calculate 40 °C in degrees Fahrenheit.
   - **b.** Transpose the formula to make C the subject.
   - **c.** Use your transposed formula from part **b** to convert the temperatures shown to degrees Celsius.

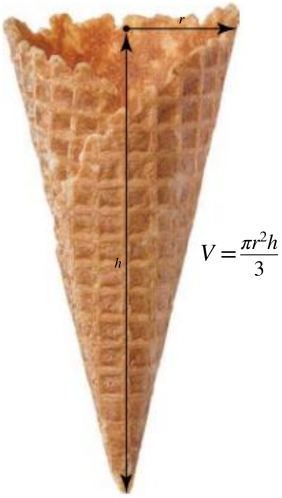


5. A local courier company uses the formula $C = 3.5h + 5$, where h is the number of kilometres and C is the cost of the delivery in dollars, to calculate the total delivery cost.
   - **a.** Calculate the cost of a delivery for a journey of 18 km.
   - **b.** Transpose the formula to make h the subject of the formula.
   - **c.** If the cost of a delivery is \$43.50, calculate how many kilometres the delivery was from the courier company base.
6. **a.** Calculate the approximate volume of the cylinder shown if $r = 5$ cm and $h = 10$ cm.
   - **b.** Transpose the formula to make h the subject.
   - **c.** Calculate the height of the cylinder if $V = 210\text{ cm}^3$ and $r = 3$ cm.



$V = \pi r^2 h$

7. A student was discussing the formula for a hire car company, $C = 0.95D + 30$, where C is the total cost of the car hire and D is the distance travelled in kilometres. The student says that to hire a car from this rental company costs 95 cents a kilometre. Is the student correct? Explain.
8. For the waffle cone shown:
   - **a.** calculate the value of V when $r = 1.5$ cm and $h = 8$ cm by substituting into the formula shown
   - **b.** transpose the formula to make r the subject
   - **c.** use the transposed formula to calculate the value of r when $V = 105\text{ cm}^3$ and $h = 8$ cm.



9. Transpose the equation $T = \dfrac{x - 3}{x - 1}$ to make x the subject.
10. Transpose the following equations to make the variable in brackets the subject.
    - **a.** $z = \dfrac{a(x - w)^2 + 3}{e} - p \quad (w)$ **b.** $w = \dfrac{8w - x}{3(q - r)} + 10 \quad (r)$
    - **c.** $a = b\sqrt{cd + e} \quad (d)$ **d.** $m = \dfrac{e + 3}{z} - tr \quad (z)$

9.5 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. **MC** The value of the expression $3(6x - 8)$ when $x = 9$ is:

   A. 138 **B.** 186 **C.** 124 **D.** 46

2. **MC** Kinetic energy is calculated using the formula $KE = \frac{1}{2}mv^2$, where m is the mass of the object in kg, v is its velocity in m/s and KE is in joules (J). If an object of mass 12 kg is moving at a velocity of 10 m/s, then it has a kinetic energy of:

   A. 1200 J **B.** 120 J **C.** 1000 J **D.** 600 J

3. **MC** The algebraic expression $3p(5q - 7)$, when $p = -2$ and $q = -3$, is:

   A. 144 **B.** −144 **C.** 132 **D.** −132

4. **MC** If Qua is getting paid $12.50 per hour at his part-time job and over the weekend he works 6 hours of normal time and 4 hours of double time, the amount he earns over the weekend is:

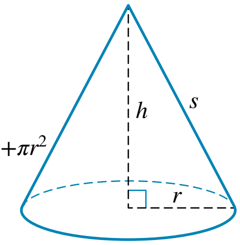
   A. $75 **B.** $100 **C.** $125 **D.** $175

5. **MC** There are 1.0936133 yards for every metre. The number of yards Usain Bolt ran to win the 100 m sprint at the Olympics, to two decimal places, was:

   A. 109.63 **B.** 91.44 **C.** 190.36 **D.** 109.36

6. **MC** The surface area of a cone is shown. If the cone has a sloping height of 12 m and a base radius of 6 m, then the surface area of the cone is closest to:

   A. 339.29 m^2 **B.** 120 m^2 **C.** 303.29 m^2 **D.** 297.87 m^2



$SA = \pi rs + \pi r^2$

7. **MC** A formula for calculating velocity is $v = u + at$, where v is the final velocity, u the initial velocity, a is the acceleration and t represents time. If an object initially starts from rest and accelerates at 2 m/s^2 for 12 seconds, it reaches a velocity of:

   A. 12 m/s **B.** 24 m/s **C.** 36 m/s **D.** 10 m/s

8. Transpose the circular motion formula to make the velocity, v, the subject.

$$a = \frac{v^2}{r}$$

9. Transpose the circular motion formula to make the radius, r, the subject.

$$a = \frac{v^2}{r}$$

10. To calculate the displacement of an object the formula used is $x = \frac{1}{2}(u + v)t$. Transpose the formula to make the velocity, v, the subject of the formula.
11. To calculate the displacement of an object the formula used is $x = \frac{1}{2}(u + v)t$. Transpose the formula to make the time, t, the subject of the formula.
12. The volume of a cone is calculated using the formula $V = \frac{\pi r^2 h}{3}$. Calculate the radius, r, when the cone has a volume of 100 cm^3 and $h = 12$ cm to two decimal places.

Complex familiar

13. Given the values $a = 3$ and $b = -3$, determine the value of the following expressions:

    a. $7a(a + 1)$ **b.** $\frac{a}{12}$ **c.** $13 - b$ **d.** $3a - 5$.

14. The circumference of a circle is found using the formula $C = 2\pi r$. Calculate the circumference of the following circles to 2 decimal places:

- **a.** with radius of 14.5 m
- **b.** with radius of 15.9 cm
- **c.** with diameter of 18 m
- **d.** with diameter 8.8 cm.

15. Given the values $h = -4$ and $k = 9$, calculate the values of the following expressions:

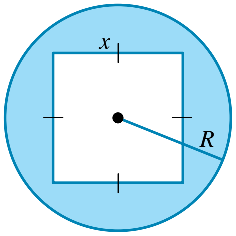
a. $2k - h$ **b.** $\frac{8k}{h}$ **c.** $h^2(12 - k)$ **d.** $-\frac{2hk}{h^2}$.

16. Transpose each of the following formulas.

- **a.** If $E = 4F - G$, make G the subject.
- **b.** If $E = \frac{FG - D}{H}$ make G the subject.
- **c.** If $E = \frac{FG - D}{H}$ make D the subject.

Complex unfamiliar

17. A new game has been created by students for the school fair. To win the game you need to hit the target with 5 darts in the shaded region.



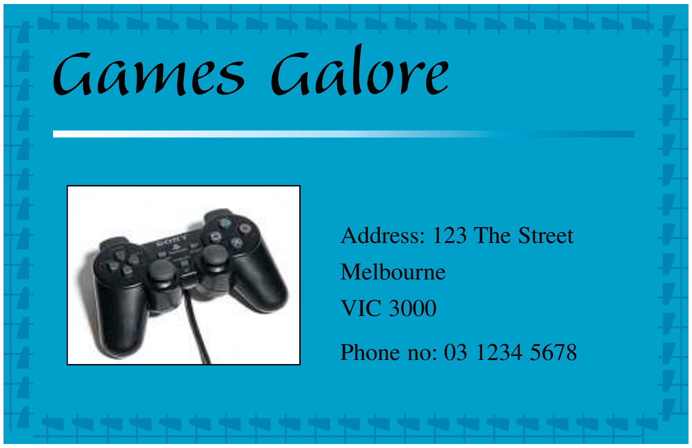
- **a.** Write an expression for the area of the shaded region.
- **b.** If $R = 7.5$ cm and $x = 4$ cm, find the area of the shaded region.
- **c.** Show that $R = \sqrt{\frac{A + x^2}{\pi}}$ by transposing the formula found in part a.
- **d.** If $A = 80\text{ cm}^2$ and $x = 3$ cm, calculate the value of R.
- **e.** The students found that the best size for the game board is when $R = 10$ and $x = 5$. What is the percentage of the total board that is shaded, to the nearest whole number?

18. For the expressions given explain the following using mathematical reasoning:

- **a.** Why the expression $15 - 5x$ will always be positive if x is a negative number?
- **b.** Why the expression $-4 - x^2$ will always be negative?
- **c.** What are the two x values that makes the expression $36 - x^2$ equal to zero?

19. You are investigating getting your business card printed for your new game store. A local printing company charges $250 for the cardboard used and an hourly rate for labour of $40.

- **a.** If h is the number of hours of labour required to print the cards, construct an equation for the cost of the cards, C.
- **b.** You have budgeted $1000 for the printing job. Calculate how many hours of labour you can afford. Give your answer to the nearest minute.



Games Galore

Address: 123 The Street
Melbourne
VIC 3000
Phone no: 03 1234 5678

c. The company estimates that it can print 1000 cards per hour of labour. Calculate how many cards you will get printed with your current budget.

d. An alternative to printing is photocopying. The company charges 15 cents per side for the first 10 000 cards and then 10 cents per side for the remaining cards. Decide which is the cheaper option for 18750 single-sided cards and by how much.

20. You have inherited \$2000 and you decided to invest this money into an interest-bearing account paying $r\,\%$ interest compounded annually. The amount $\$\,A$ to which your investment will grow is given by the formula $A = P\left(1 + \frac{r}{100}\right)^n$, where P is the principal amount and n is the number of years.

a. Calculate A if you invest your inheritance at 8.5% p.a. for 5 years.

b. By rearranging the above formula, what principal amount do you need to invest at 6% p.a. to produce an amount of \$10 000 over 12 years?

c. Calculate r when your \$2000 grows to \$2500 over 2 years.

d. If the interest rate is 7%, use trial and error to find the time it takes (to the nearest year) for your initial inheritance of \$2000 to double in value.

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Units 1 & 2 Sit chapter test

Answers

Chapter 9 Linear and non-linear relationships

Exercise 9.2 Substitution

1. a. 8 **b.** 17 **c.** 10 **d.** 25
e. 1 **f.** 18 **g.** 5 **h.** 13

2. a. 6 **b.** −30 **c.** 21 **d.** 90
e. 36 **f.** −36 **g.** 30 **h.** −24

3. a. 4 **b.** 1 **c.** 3 **d.** 5
e. −11 **f.** −12 **g.** $\frac{32}{5}$ **h.** 4

4. a. 18 **b.** 22 **c.** 70 **d.** 132
e. 25 **f.** 48 **g.** −12 **h.** 120

5. a. 60 **b.** 9 **c.** 17 **d.** 16
e. 13 **f.** 3 **g.** 25 **h.** 48

6. a. 14 **b.** 30 **c.** 5 **d.** 29
e. 28 **f.** −15 **g.** 36 **h.** 180

7. a. 13 **b.** 6 **c.** 36 **d.** 8
e. 0 **f.** 0

8. a.

x	y
0	−4
1	1
2	6
3	11

b.

x	y
3	10
6	4
9	−2
12	−8

9. a.

x	y
−4	−6
−2	2
0	10
2	18

b.

x	y
−8	155
−4	95
0	35
4	−25

10. a. 78.54 m^2 **b.** 240.53 cm^2 **c.** 1597.51 cm^2

11. a. \$23.32 **b.** \$21.70
c. \$25.30 **d.** $C = 0{\cdot}09n + 19$

12. 38

13. $x = 2$; using technology $x = 2, x = \frac{-5}{3}$.

14. D

Exercise 9.3 Formulas

1. a. −5 **b.** −10 **c.** 5 **d.** 30
e. 0 **f.** 0 **g.** 0 **h.** 0

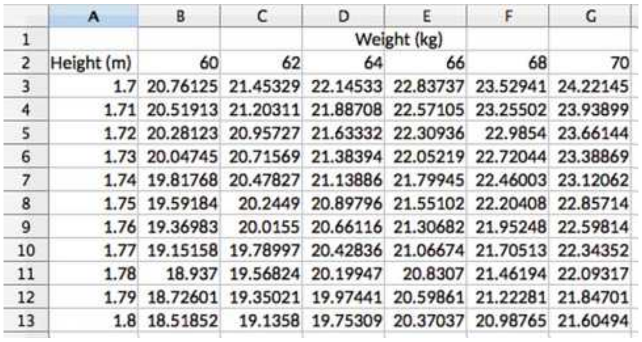
2. a. $\frac{1}{2}$ **b.** 5 **c.** 16 **d.** −21

3. a. 427.26 cm^2 **b.** 753.98 cm^2

4. 108 m^2

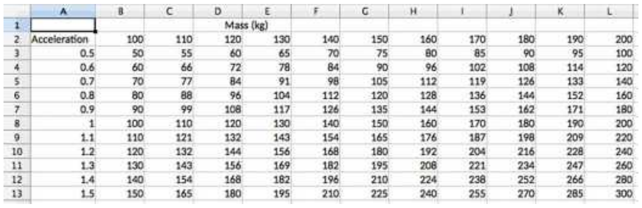
5. a. 207.35 cm^2 **b.** 2 261.95 cm^2 **c.** 2 395.46 m^2 **d.** 8 497.38 m^2
e. 314.16 cm^2 **f.** 1 809.56 m^2 **g.** 660.52 cm^2 **h.** 1 963.50 m^2

6.



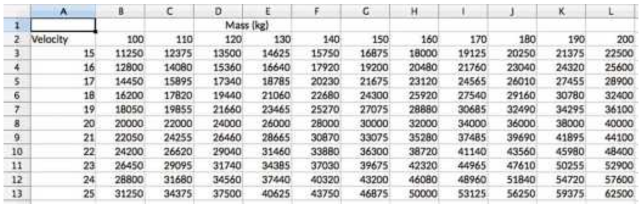
	A	B	C	D	E	F	G
1				Weight (kg)			
2	Height (m)	60	62	64	66	68	70
3	1.7	20.76125	21.45329	22.14533	22.83737	23.52941	24.22145
4	1.71	20.51913	21.20311	21.88708	22.57105	23.25502	23.93899
5	1.72	20.28123	20.95727	21.63332	22.30936	22.9854	23.66144
6	1.73	20.04745	20.71569	21.38394	22.05219	22.72044	23.38869
7	1.74	19.81768	20.47827	21.13886	21.79945	22.46003	23.12062
8	1.75	19.59184	20.2449	20.89796	21.55102	22.20408	22.85714
9	1.76	19.36983	20.0155	20.66116	21.30682	21.95248	22.59814
10	1.77	19.15158	19.78997	20.42836	21.06674	21.70513	22.34352
11	1.78	18.937	19.56824	20.19947	20.8307	21.46194	22.09317
12	1.79	18.72601	19.35021	19.97441	20.59861	21.22281	21.84701
13	1.8	18.51852	19.1358	19.75309	20.37037	20.98765	21.60494

7.



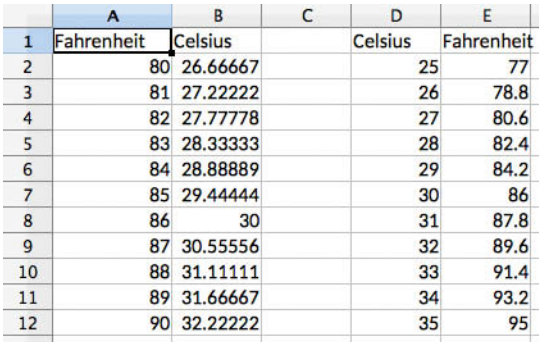
	A	B	C	D	E	F	G	H	I	J	K	L
1				Mass (kg)								
2	Acceleration	100	110	120	130	140	150	160	170	180	190	200
3	0.5	50	55	60	65	70	75	80	85	90	95	100
4	0.6	60	66	72	78	84	90	96	102	108	114	120
5	0.7	70	77	84	91	98	105	112	119	126	133	140
6	0.8	80	88	96	104	112	120	128	136	144	152	160
7	0.9	90	99	108	117	126	135	144	153	162	171	180
8	1	100	110	120	130	140	150	160	170	180	190	200
9	1.1	110	121	132	143	154	165	176	187	198	209	220
10	1.2	120	132	144	156	168	180	192	204	216	228	240
11	1.3	130	143	156	169	182	195	208	221	234	247	260
12	1.4	140	154	168	182	196	210	224	238	252	266	280
13	1.5	150	165	180	195	210	225	240	255	270	285	300

8.



	A	B	C	D	E	F	G	H	I	J	K	L
1				Mass (kg)								
2	Velocity	100	110	120	130	140	150	160	170	180	190	200
3	15	11250	12375	13500	14625	15750	16875	18000	19125	20250	21375	22500
4	16	12800	14080	15360	16640	17920	19200	20480	21760	23040	24320	25600
5	17	14450	15895	17340	18785	20230	21675	23120	24565	26010	27455	28900
6	18	16200	17820	19440	21060	22680	24300	25920	27540	29160	30780	32400
7	19	18050	19855	21660	23465	25270	27075	28880	30685	32490	34295	36100
8	20	20000	22000	24000	26000	28000	30000	32000	34000	36000	38000	40000
9	21	22050	24255	26460	28665	30870	33075	35280	37485	39690	41895	44100
10	22	24200	26620	29040	31460	33880	36300	38720	41140	43560	45980	48400
11	23	26450	29095	31740	34385	37030	39675	42320	44965	47610	50255	52900
12	24	28800	31680	34560	37440	40320	43200	46080	48960	51840	54720	57600
13	25	31250	34375	37500	40625	43750	46875	50000	53125	56250	59375	62500

9. a. **i.** 37.78 °C **ii.** 10.00 °C **iii.** 25.56 °C **iv.** −3.89 °C
b. **i.** 113.00 °F **ii.** 32.00 °F **iii.** 77.00 °F **iv.** 91.40 °F
c.



	A	B	C	D	E
1	Fahrenheit	Celsius		Celsius	Fahrenheit
2	80	26.66667		25	77
3	81	27.22222		26	78.8
4	82	27.77778		27	80.6
5	83	28.33333		28	82.4
6	84	28.88889		29	84.2
7	85	29.44444		30	86
8	86	30		31	87.8
9	87	30.55556		32	89.6
10	88	31.11111		33	91.4
11	89	31.66667		34	93.2
12	90	32.22222		35	95

10. a. 3 769.91 m^3 **b.** 720 cm^3

11. a. 5 193.45 cm^3 **b.** 4 849.05 cm^3

12. a. \$30.65 **b.** \$31.90 **c.** \$183.25 **d.** \$365.40

13. There could be a number of different dimension combinations. A few examples are:

- **i.** $L = 4\,\text{m},\ W = 3\,\text{m}$ and $H = 10\,\text{m}$
- **ii.** $L = 6\,\text{m},\ W = 2\,\text{m}$ and $H = 10\,\text{m}$
- **iii.** $L = 6\,\text{m},\ W = 4\,\text{m}$ and $H = 5\,\text{m}$

14. a. $C_A = \$107,\ C_B = \$114$
Hence, C_A is the cheaper bill.
b. 375 kWh

Exercise 9.4 Transposition

1. a. $A = \dfrac{C-D}{B}$
b. $t = \dfrac{v-u}{a}$
c. $h = \dfrac{3V}{A}$
d. $b = \dfrac{2A}{h} - a$

2. a. $h = \dfrac{A}{2\pi r} - r$
b. $r = 1 - \dfrac{a}{S}$
c. $R = \dfrac{100I}{PT}$
d. $P = \dfrac{A}{R^n}$

3. a. $x = \sqrt{A}$
b. $r = \sqrt{\dfrac{3V}{\pi h}}$
c. $u = \sqrt{v^2 - 2as}$
d. $a = \sqrt{c^2 - b^2}$

4. a. 104° F
b. $C = \frac{5}{9}(F - 32)$
c. Seattle: 8.9 °C
San Diego: 19.4 °C
Denver: −3.9 °C
New York: −2.2 °C
Orlando: 22.8 °C

5. a. \$68
b. $h = \dfrac{C-5}{3.5}$
c. 11 km

6. a. $785.40\,\text{cm}^3$
b. $h = \dfrac{V}{\pi r^2}$
c. 7.43 cm

7. The student is partially correct. It does cost 95 cents per kilometre; however, there is also an initial charge of \$30 for the car hire.

8. a. $18.85\,\text{cm}^3$
b. $r = \sqrt{\dfrac{3V}{\pi h}}$
c. 3.54 cm

9. $x = \dfrac{T-3}{T-1}$

10. a. $w = x - \sqrt{\dfrac{e(z+p)-3}{a}}$
b. $r = q - \dfrac{8w-x}{3(w-10)}$
c. $d = \dfrac{\left(\frac{a}{b}\right)^2 - e}{c}$
d. $z = \dfrac{e+3}{m+tr}$

9.5 Review: exam practice

1. A
2. D
3. C
4. D
5. D
6. A
7. B
8. $v = \sqrt{ar}$
9. $r = \dfrac{v^2}{a}$
10. $v = \dfrac{2x}{t} - u$
11. $t = \dfrac{2x}{u+v}$
12. 2.82 cm

13. a. 84
b. $\frac{1}{4}$
c. 16
d. 4

14. a. 91.11 m
b. 99.90 cm
c. 56.55 m
d. 27.65 cm

15. a. 22
b. −18
c. 48
d. $\frac{9}{2}$

16. a. $G = 4F - E$
b. $G = \dfrac{EH + D}{F}$
c. $D = FG - EH$

17. a. $A = \pi R^2 - x^2$
b. $160.71\,\text{cm}^2$
c. $A = \pi R^2 - x^2$ transposed becomes $R = \sqrt{\dfrac{A + x^2}{\pi}}$
d. 5.32 cm
e. 92%

18. a. Two multiplied negatives means we are always adding a positive number to 15.
b. A number squared is always positive, so when we subtract that from −4 it will always be negative.
c. Need to subtract 36 from 36 to get zero. So $x = 6$ or $x = -6$, because they both equal 36 when squared.

19. a. $C = 40h + 250$
b. 18 hours, 45 minutes
c. 18 750
d. The printing is cheaper by \$1375.

20. a. \$3007.31
b. \$4969.69
c. 11.8%
d. 10 years

CHAPTER 10
Matrices and matrix arithmetic

10.1 Overview

10.1.1 Introduction

Matrices are aligned with the study of solutions of linear simultaneous systems. For example, if you have two equations with two unknowns you can use matrices to solve the two unknowns. The initial use of matrices dates back to the second century BC; however, it was not until the end of the seventeenth century that the ideas were developed further.

Between 200 BC and 100 BC, during the Han Dynasty, the Chinese used matrix-type methods recorded in the text *Nine chapters on the mathematical art*. There was no further development until 1683, when the Japanese mathematician Seki wrote *Method of solving the dissimulated problems*. This work used matrix methods in tables in the same way as the earlier work of the Chinese. It wasn't until 1850 when the term 'matrix' was first used by Sylvester. He defined a matrix to be an oblong arrangement of terms and saw it as something which led to various determinants. Sylvester worked with Cayley, whose *Memoir on the theory of matrices* was published in 1858. Since this time there has been continual development in the field of matrices and they are now used in a vast array of important fields. Olga Taussky-Todd, who described herself as a torchbearer for matrix theory, used matrices to analyse vibrations of airplanes (like that pictured above) during World War II at the National Physical Laboratory in the United Kingdom. More recently, matrices are used to describe the quantum mechanics of atomic structure, designing the graphics for computer games and even in plotting complicated dance steps.

LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

10.2 Types of matrices

A **matrix** is a rectangular array of rows and columns that is used to store and display information. Matrices can be used to represent many different types of information, such as the models of cars sold in different car dealerships, the migration of people to different countries and the shopping habits of customers at different department stores. Matrices also play an important role in encryption. Before sending important information, programmers encrypt or code messages using matrices; the people receiving the information will then use inverse matrices as the key to decode the message. Engineers, scientists and project managers also use matrices to help them to perform various everyday tasks.

10.2.1 Describing matrices

A matrix is usually displayed in square brackets with no borders between the rows and columns.

The table below left shows the number of participants attending three different dance classes (rumba, waltz and chacha) over the two days of a weekend. The matrix below right displays the information presented in the table.

Number of participants attending the dance classes

	Saturday	Sunday
Rumba	9	13
Tango	12	8
Chacha	16	14

Matrix displaying the number of participants attending the dance classes

$$\begin{bmatrix} 9 & 13 \\ 12 & 8 \\ 16 & 14 \end{bmatrix}$$

WORKED EXAMPLE 1

The table below shows the number of adults and children who attended three different events over the school holidays. Construct a matrix to represent this information.

	Circus	Zoo	Show
Adults	140	58	85
Children	200	125	150

THINK	WRITE
1. A matrix is like a table that stores information. What information needs to be displayed?	The information to be displayed is the number of adults and children attending the three events: circus, zoo and show.

2. Write down how many adults and children attend each of the three events

	Circus	Zoo	Show
Adults	140	58	85
Children	200	125	150

3. Write this information in a matrix. Remember to use square brackets.

$$\begin{bmatrix} 140 & 58 & 85 \\ 200 & 125 & 150 \end{bmatrix}$$

10.2.2 Networks

Matrices can also be used to display information about various types of **networks**, including road systems and social networks. The following matrix shows the links between a group of schoolmates on Facebook, with a 1 indicating that the two people are friends on Facebook and a 0 indicating that the two people aren't friends on Facebook.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

From this matrix you can see that the following people are friends with each other on Facebook:

- person A and person B
- person A and person C
- person B and person D
- person C and person D.

WORKED EXAMPLE 2

The distances, in kilometres, along three major roads between the Tasmanian towns Launceston (L), Hobart (H) and Devonport (D) are displayed in the matrix below.

$$\begin{array}{c} \\ \mathbf{H} \\ \mathbf{D} \\ \mathbf{L} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{H} & \mathbf{D} & \mathbf{L} \end{array} \\ \begin{bmatrix} \mathbf{0} & \mathbf{207} & \mathbf{160} \\ \mathbf{207} & \mathbf{0} & \mathbf{75} \\ \mathbf{160} & \mathbf{75} & \mathbf{0} \end{bmatrix} \end{array}$$

a. **What is the distance, in kilometres, between Devonport and Hobart?**

b. **Victor drove 75 km directly between two of the Tasmanian towns. Which two towns did he drive between?**

c. **The Goldstein family would like to drive from Hobart to Launceston, and then to Devonport. Determine the total distance in kilometres that they will travel.**

THINK	WRITE
a. 1. Reading the matrix, locate the first city or town, i.e. Devonport (D), on the top of the matrix	**a** $\begin{array}{cc} & \begin{array}{ccc} & \downarrow & \\ H & D & L\end{array} \\ \begin{array}{c}H\\D\\L\end{array} & \begin{bmatrix}0 & 207 & 160\\ 207 & 0 & 75\\ 160 & 75 & 0\end{bmatrix}\end{array}$
2. Locate the second city or town, i.e. Hobart (H), on the side of the matrix.	$\begin{array}{cc} & \begin{array}{ccc} & \downarrow & \\ H & D & L\end{array} \\ \begin{array}{c}\rightarrow H\\D\\L\end{array} & \begin{bmatrix}0 & \enclose{circle}{207} & 160\\ 207 & 0 & 75\\ 160 & 75 & 0\end{bmatrix}\end{array}$
3. The point where both arrows meet gives you the distance between the two towns.	207 km
b. 1. Locate the entry '75' in the matrix.	**b** $\begin{array}{cc} & \begin{array}{ccc}H & D & L\end{array} \\ \begin{array}{c}H\\D\\L\end{array} & \begin{bmatrix}0 & 207 & 160\\ 207 & 0 & \enclose{circle}{75}\\ 160 & 75 & 0\end{bmatrix}\end{array}$
2. Locate the column and row 'titles' (L and D) for that entry.	$\begin{array}{cc} & \begin{array}{ccc}H & D & L\\ & & \uparrow\end{array} \\ \begin{array}{c}H\\D \leftarrow\\L\end{array} & \begin{bmatrix}0 & 207 & 160\\ \cancel{207} & \cancel{0} & \enclose{circle}{75}\\ 160 & 75 & 0\end{bmatrix}\end{array}$
3. Refer to the title headings in the question.	Victor drove between Launceston and Devonport.
c. 1. Locate the first city or town, i.e. Hobart (H), on the top of the matrix and the second city or town, i.e. Launceston (L), on the side of the matrix.	**c** $\begin{array}{cc} & \begin{array}{ccc}\downarrow & & \\ H & D & L\end{array} \\ \begin{array}{c}H\\D\\\rightarrow L\end{array} & \begin{bmatrix}0 & 207 & 160\\ 207 & 0 & 75\\ \enclose{circle}{160} & 75 & 0\end{bmatrix}\end{array}$
2. Where the row and column meet gives the distance between the two towns.	160 km
3. Determine the distance between the second city or town, i.e. Launceston, and the third city or town, i.e. Devonport.	$\begin{array}{cc} & \begin{array}{ccc} & & \downarrow\\ H & D & L\end{array} \\ \begin{array}{c}H\\\rightarrow D\\L\end{array} & \begin{bmatrix}0 & 207 & 160\\ 207 & 0 & \enclose{circle}{75}\\ 160 & 75 & 0\end{bmatrix}\end{array}$

4. Where the row and column meet gives the distance between the two towns.	75 km
5. Add the two distances together.	$160 + 75 = 235$ km

10.2.3 Defining matrices

The order of a matrix is defined by the number of rows, m, and number of columns, n, in the matrix.

Consider the following matrix, A.

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & -6 & 5 \end{bmatrix}$$

Matrix A has two rows and three columns, and its order is 2×3 (read as a 'two by three' matrix).

A matrix that has the same number of rows and columns is called a **square matrix**.

$$B = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$$

Matrix B has two rows and two columns and is a 2×2 square matrix.

A **row matrix** has only one row.

$$C = \begin{bmatrix} 3 & 7 & -4 \end{bmatrix}$$

Matrix C has only one row and is called a row matrix.

A **column matrix** has only one column.

$$D = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$$

Matrix D has only one column and is called a column matrix.

WORKED EXAMPLE 3

At High Vale College, 150 students are studying General Mathematics and 85 students are studying Mathematical Methods. Construct a column matrix to represent the number of students studying General Mathematics and Mathematical Methods, and state the order of the matrix.

THINK	WRITE
1. Read the question and highlight the key information	150 students study General Mathematics. 85 students study Mathematical Methods.
2. Display this information in a column matrix.	$\begin{bmatrix} 150 \\ 85 \end{bmatrix}$
3. How many rows and columns are there in this matrix?	The order of the matrix is 2×1.

10.2.4 Elements of matrices

The entries in a matrix are called **elements**. The position of an element is described by the corresponding row and column. For example, a_{21} means the entry in the 2nd row and 1st column of matrix A, as shown below.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \enclose{circle}{a_{21}} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

WORKED EXAMPLE 4

Write the element a_{23} for the matrix $A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & -6 & 5 \end{bmatrix}$.

THINK	WRITE
1. The element a_{23} means the element in the 2nd row and 3rd column. Draw lines through the 2nd row and 3rd column to help you identify this element.	$A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & -6 & 5 \end{bmatrix}$
2. Identify the number that is where the lines cross over.	$a_{23} = 5$

10.2.5 Special matrices

Identity matrix

An **identity matrix**, I, is a square matrix in which all of the elements on the diagonal line from the top left to bottom right are 1s and all of the other elements are 0s.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

As you will see later in this topic, identity matrices are used to find inverse matrices, which help solve matrix equations.

Zero matrix

A **zero matrix**, 0, is a square matrix that consists entirely of '0' elements.

The matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is an example of a zero matrix.

Units 1 & 2 > Area 5 > Sequence 2 > Concepts 1 & 2

Definition of a matrix Summary screen and practice questions

Defining a matrix Summary screen and practice questions

Exercise 10.2 Types of matrices

1. **WE1** Cheap Auto sells three types of vehicles: cars, vans and motorbikes. They have two outlets at Valley Heights and Hill Vale. The number of vehicles in stock at each of the two outlets is shown in the table.

	Cars	**Vans**	**Motorbikes**
Valley Heights	18	12	8
Hill Vale	13	10	11

Construct a matrix to represent this information.

2. Newton and Isaacs played a match of tennis. Newton won the match in five sets with a final score of 6–2, 4–6, 7–6, 3–6, 6–4. Construct a matrix to represent this information.

3. **WE2** The distance in kilometres between the towns Port Augusta (P), Coober Pedy (C) and Alice Springs (A) are displayed in the following matrix.

$$\begin{array}{c} \\ P \\ C \\ A \end{array} \begin{array}{c} \begin{array}{ccc} P & C & A \end{array} \\ \begin{bmatrix} 0 & 545 & 1225 \\ 545 & 0 & 688 \\ 1225 & 688 & 0 \end{bmatrix} \end{array}$$

a. Determine the distance in kilometres between Port Augusta and Coober Pedy.
b. Greg drove 688 km between two towns. Which two towns did he travel between?
c. A truck driver travels from Port Augusta to Coober Pedy, then onto Alice Springs. He then drives from Alice Springs directly to Port Augusta. Determine the total distance in kilometres that the truck driver travelled.

4. A one-way economy train fare between Melbourne Southern Cross Station and Canberra Kingston Station is $91.13. A one-way economy train fare between Sydney Central Station and Melbourne Southern Cross Station is $110.72, and a one-way economy train fare between Sydney Central Station and Canberra Kingston Station is $48.02.

a. Represent this information in a matrix.
b. Drew travelled from Sydney Central to Canberra Kingston Station, and then onto Melbourne Southern Cross. Determine how much, in dollars, he paid for the train fare.

5. **WE3** An energy-saving store stocks shower water savers and energy-saving light globes. In one month they sold 45 shower water savers and 30 energy-saving light globes. Construct a column matrix to represent the number of shower water savers and energy-saving light globes sold during this month, and state the order of the matrix.

6. Happy Greens Golf Club held a three-day competition from Friday to Sunday. Participants were grouped into three different categories: experienced, beginner and club member. The table shows the total entries for each type of participant on each of the days of the competition.

Category	Friday	Saturday	Sunday
Experienced	19	23	30
Beginner	12	17	18
Club member	25	33	36

   a. How many entries were received for the competition on Friday?
   b. Calculate the total number of entries for the three day competition.
   c. Construct a row matrix to represent the number of beginners participating in the competition for each of the three days.

7. Write the order of matrices A, B and C.

$$A = [3], \ B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}, \ C = \begin{bmatrix} 4 & -2 \end{bmatrix}$$

8. Which of the following represent matrices? Justify your answers.

   a. $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$
   b. $\begin{bmatrix} 4 & 0 \\ & 3 \end{bmatrix}$
   c. $\begin{bmatrix} & 5 \\ 4 & \\ & 7 \end{bmatrix}$
   d. $\begin{bmatrix} a & c & e & g \\ b & d & f & h \end{bmatrix}$

9. **WE4** Write down the value of the following elements for matrix D.

$$D = \begin{bmatrix} 4 & 5 & 0 \\ 2 & -1 & -3 \\ 1 & -2 & 6 \\ 0 & 3 & 7 \end{bmatrix}$$

   a. d_{12}
   b. d_{33}
   c. d_{43}

10. Consider the matrix $E = \begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{4} \\ -1 & -\frac{1}{2} & -3 \end{bmatrix}$.

   a. Explain why the element e_{24} does not exist.
   b. Which element has a value of -3?
   c. Nadia was asked to write down the value of element e_{12} and wrote -1. Explain Nadia's mistake and state the correct value of element e_{12}.

11. Matrices D and E are shown. Determine the value of the following elements.

$$D = \begin{bmatrix} 5 & 0 & 2 & -1 \\ 8 & 1 & 3 & 6 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.5 & 0.3 \\ 1.2 & 1.1 \\ 0.4 & 0.9 \end{bmatrix}$$

a. d_{23} **b.** d_{14} **c.** d_{22} **d.** e_{11} **e.** e_{32}

12. a. The following matrix represents an incomplete 3×3 identity matrix. Complete the matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & & 0 \\ & 0 & \end{bmatrix}$$

b. Construct a 2×2 zero matrix.

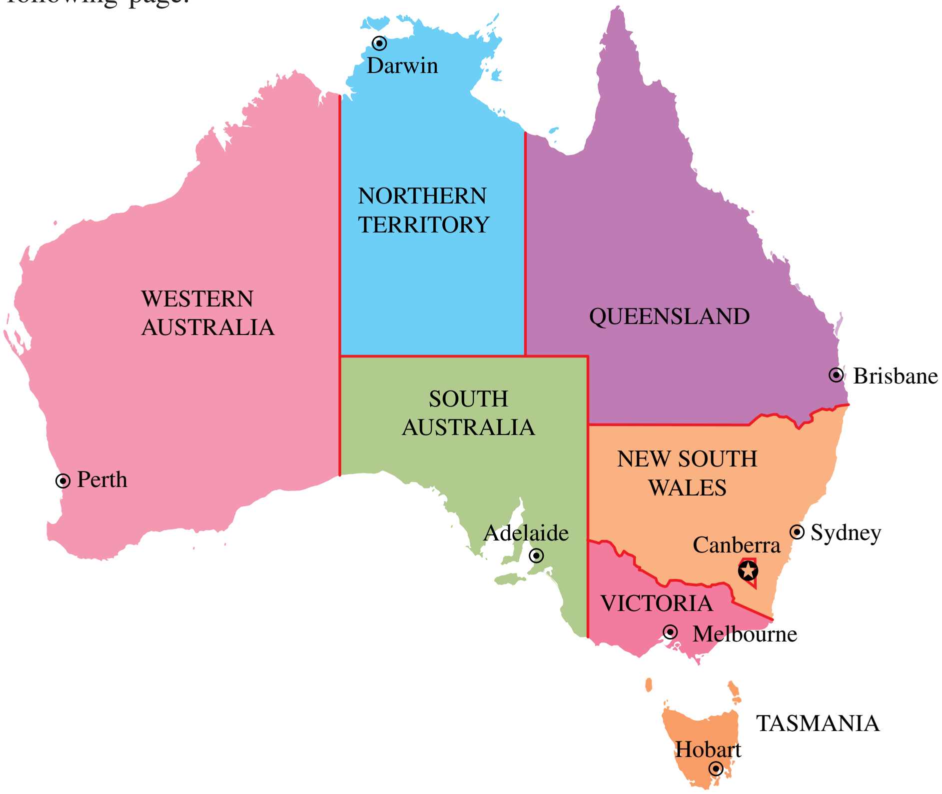
13. The elements in matrix H are shown below.

$h_{12} = 3$
$h_{11} = 4$
$h_{21} = -1$
$h_{31} = -4$
$h_{32} = 6$
$h_{22} = 7$

a. Determine the order of matrix H.

b. Construct matrix H.

14. The land area and population of each Australian state and territory were recorded and summarised in the table on the following page.



State/territory	Land area (km^2)	Population (millions)
Australian Capital Territory	2 358	0.4
Queensland	1 727 200	4.2
New South Wales	801 428	6.8
Northern Territory	1 346 200	0.2
South Australia	984 000	1.6
Western Australia	2 529 875	2.1
Tasmanian	68 330	0.5
Victoria	227 600	5.2

- **a.** Construct an 8 × 1 matrix that displays the population, in millions, of each state and territory in the order shown in the table.
- **b.** Construct a row matrix that represents the land area of each of the states in ascending order.
- **c.** Town planners place the information on land area, in km^2, and population, in millions, for the states New South Wales, Victoria and Queensland respectively in a matrix.
  - **i.** State the order of this matrix.
  - **ii.** Construct this matrix.

15. The estimated number of Indigenous Australians living in each state and territory in Australia in 2006 is shown in the following table.

State and territory	Number of Indigenous persons	% of population that is Indigenous
New South Wales	148 178	2.2
Victoria	30 839	0.6
Queensland	146 429	3.6
South Australia	26 044	1.7
Western Australia	77 928	3.8
Tasmania	16 900	3.4
Northern Territory	66 582	31.6
Australian Capital Territory	4 043	1.2

- **a.** Construct an 8 × 2 matrix to represent this information
- **b.** Determine the total number of Indigenous persons living in the following states and territories in 2006:
  - **i.** Northern Territory
  - **ii.** Tasmania
  - **iii.** Queensland, New South Wales and Victoria (combined).
- **c.** Determine the total number of Indigenous persons who were estimated to be living in Australia in 2006.

16. AeroWings is a budget airline specialising in flights between four mining towns: Olympic Dam (O), Broken Hill (B), Dampier (D) and Mount Isa (M).

The cost of airfares (in dollars) to fly from the towns in the top row to the towns in the first column is shown in the matrix below.

$$\begin{array}{ccc} & & \text{From} \\ & & \begin{array}{cccc} \text{O} & \text{B} & \text{D} & \text{M} \end{array} \\ \text{To} & \begin{array}{c} \text{O} \\ \text{B} \\ \text{D} \\ \text{M} \end{array} & \begin{bmatrix} 0 & 70 & 150 & 190 \\ 89 & 0 & 85 & 75 \\ 175 & 205 & 0 & 285 \\ 307 & 90 & 101 & 0 \end{bmatrix} \end{array}$$

a. In the context of this problem, explain the meaning of the zero entries.

b. Calculate the cost, in dollars, to fly from Olympic Dam to Dampier.

c. Yen paid \$101 for his airfare with AeroWings. At which town did he arrive?

d. AeroWings offers a 25% discount for passengers flying between Dampier and Mount Isa, and a 15% discount for passengers flying from Broken Hill to Olympic Dam. Construct another matrix that includes the discounted airfares (in dollars) between the four mining towns.

17. The matrix below displays the number of roads connecting five towns: Ross (R), Stanley (S), Thomastown (T), Edenhope (E) and Fairhaven (F).

$$\begin{array}{ccc} & \begin{array}{ccccc} \text{R} & \text{S} & \text{T} & \text{E} & \text{F} \end{array} & \\ N = & \begin{bmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} \text{R} \\ \text{S} \\ \text{T} \\ \text{E} \\ \text{F} \end{array} \end{array}$$

a. Construct a road map using the information shown.

b. Determine whether the following statements are true or false.

- **i.** There is a road loop at Stanley.
- **ii.** You can travel directly between Edenhope and Stanley.
- **iii.** There are two roads connecting Thomastown and Edenhope.
- **iv.** There are only three different ways to travel between Ross and Fairhaven.

c. A major flood washes away part of the road connecting Ross and Thomastown. Which elements in matrix N will need to be changed to reflect the new road conditions between the towns?

18. Mackenzie is sitting a Mathematics multiple choice test with ten questions. There are five possible responses for each question: A, B, C, D and E. She selects A for the first question and then determines the answers to the remaining questions using the following matrix.

$$\begin{array}{ccc} & & \text{Answers} \\ & & \text{This question} \\ & & \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{array}{c} \text{Answers} \\ \text{Next question} \end{array} & \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

a. Using the matrix on the previous page, what is Mackenzie's answer to question 2 on the test?

b. Write Mackenzie's responses to the remaining eight questions.

c. Explain why it is impossible for Mackenzie to have more than one answer with response A.

Mackenzie used another matrix to help her answer the multiple choice test. Her responses using this matrix are shown in this grid.

Question	1	2	3	4	5	6	7	8	9	10
Response	A	D	C	B	E	A	D	C	B	E

d. Complete the matrix that Mackenzie used for the test by finding the values of the missing elements.

$$\begin{array}{ccc} & & \text{This question} \\ & & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \text{Next question} & \begin{array}{c} A\\B\\C\\D\\E \end{array} & \begin{bmatrix} 0&0&0&0& \\ 0& & &0&0 \\ 0& & & &0 \\ 1&0&0& &0 \\ 0& &0&0& \end{bmatrix} \end{array}$$

10.3 Operations with matrices

10.3.1 Matrix addition and subtraction

Matrices can be added and subtracted using the same rules as in regular arithmetic. However, matrices can only be added and subtracted if they are the same order (that is, if they have the same number of rows and columns).

Adding matrices

To add matrices, you need to add the corresponding elements of each matrix together (that is, the numbers in the same position).

WORKED EXAMPLE 5

If $A = \begin{bmatrix} 4 & 2 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$, find the value of $A + B$.

THINK	WRITE
1. Write down the two matrices in a sum.	$\begin{bmatrix} 4 & 2 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$
2. Identify the elements in the same position. For example, 4 and 1 are both in the first row and first column. Add the elements in the same positions together.	$\begin{bmatrix} 4+1 & 2+0 \\ 3+5 & -2+3 \end{bmatrix}$
3. Work out the sums and write the answer.	$\begin{bmatrix} 5 & 2 \\ 8 & 1 \end{bmatrix}$

Subtracting matrices

To subtract matrices, you need to subtract the corresponding elements in the same order as presented in the question.

WORKED EXAMPLE 6

If $A = \begin{bmatrix} 6 & 0 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$, calculate the value of $A - B$.

THINK	WRITE
1. Write the two matrices.	$\begin{bmatrix} 6 & 0 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$
2. Subtract the elements in the same position together.	$\begin{bmatrix} 6-4 & 0-2 \\ 2-1 & -2-3 \end{bmatrix}$
3. Work out the subtractions and write the answer.	$\begin{bmatrix} 2 & -2 \\ 1 & -5 \end{bmatrix}$

on Resources

Interactivity: Adding and subtracting matrices (int-6463)

studyon

Units 1 & 2 › Area 5 › Sequence 2 › Concept 3

Matrix addition and subtraction Summary screen and practice questions

Exercise 10.3 Operations with matrices

1. a. **WE5** If $A = \begin{bmatrix} 2 & -3 \\ -1 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 9 \\ 0 & 11 \end{bmatrix}$, determine the value of $A + B$.

   b. If $A = \begin{bmatrix} 0.5 \\ 0.1 \\ 1.2 \end{bmatrix}$, $B = \begin{bmatrix} -0.5 \\ 2.2 \\ 0.9 \end{bmatrix}$ and $C = \begin{bmatrix} -0.1 \\ -0.8 \\ 2.1 \end{bmatrix}$, determine the matrix sum $A + B + C$.

2. Consider the matrices $C = \begin{bmatrix} 1 & -3 \\ 7 & 5 \\ b & 8 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & a \\ -5 & -4 \\ 2 & -9 \end{bmatrix}$.

   If $C + D = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -4 & -1 \end{bmatrix}$, determine the values of a and b.

3. If $A = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}$ and $C = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$, calculate the following.

a. $A + C$ **b.** $B + C$ **c.** $A - B$ **d.** $A + B - C$

4. **WE6** If $A = \begin{bmatrix} 1 & 1 \\ -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -2 & 4 \end{bmatrix}$, calculate the value of $A - B$.

5. Consider the following.

$$B - A = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \quad A + B = \begin{bmatrix} 4 \\ 2 \\ 8 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

a. Explain why matrix B must have an order of 3×1.

b. Determine matrix B.

6. Evaluate the following.

a. $[0.5 \quad 0.25 \quad 1.2] - [0.75 \quad 1.2 \quad 0.9]$

b. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 6 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 12 & 17 & 10 \\ 35 & 20 & 25 \\ 28 & 32 & 29 \end{bmatrix} - \begin{bmatrix} 13 & 12 & 9 \\ 31 & 22 & 22 \\ 25 & 35 & 31 \end{bmatrix}$

d. $\begin{bmatrix} 11 & 6 & 9 \\ 7 & 12 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 8 & 8 \\ 6 & 7 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -1 & 10 \\ 4 & 9 & -3 \end{bmatrix}$

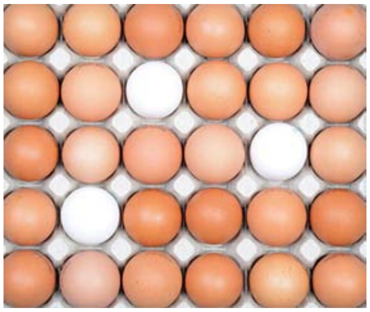
7. If $\begin{bmatrix} 3 & 0 \\ 5 & a \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ -b & 1 \end{bmatrix} = \begin{bmatrix} c & 2 \\ 3 & -4 \end{bmatrix}$, calculate the values of a, b and c.

8. If $\begin{bmatrix} 12 & 10 \\ 25 & 13 \\ 20 & a \end{bmatrix} - \begin{bmatrix} 9 & 11 \\ 26 & c \\ b & 9 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 8 \\ 21 & -3 \end{bmatrix}$, calculate the values of a, b and c.

9. By calculating the order of each of the following matrices, identify which of the matrices can be added and/or subtracted to each other and explain why.

$$A = \begin{bmatrix} 1 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ -8 \end{bmatrix} \qquad C = \begin{bmatrix} -1 \\ -9 \end{bmatrix} \qquad D = [-4] \qquad E = \begin{bmatrix} -3 & 6 \end{bmatrix}$$

10. Hard Eggs sells both free-range and barn-laid eggs in three different egg sizes (small, medium and large) to two shops, Appleton and Barntown. The number of cartons ordered for the Appleton shop is shown in the table below.



Eggs	Small	Medium	Large
Free range	2	3	5
Barn laid	4	6	3

a. Construct a 2 × 3 matrix to represent the egg order for the Appleton shop.
The total orders for both shops are shown in the table below.

Eggs	**Small**	**Medium**	**Large**
Free range	3	4	8
Barn laid	6	8	5

b. **i.** Set up a matrix difference that would determine the order for the Barntown shop.
  ii. Use the matrix difference from part **bi** to determine the order for the Barntown shop. Show the order in a table.

11. Marco was asked to complete the matrix sum $\begin{bmatrix} 8 & 126 & 59 \\ 17 & 102 & -13 \end{bmatrix} + \begin{bmatrix} 22 & 18 & 38 \\ 16 & 27 & 45 \end{bmatrix}$.

He gave $\begin{bmatrix} 271 \\ 194 \end{bmatrix}$ as his answer.

a. By referring to the order of matrices, explain why Marco's answer must be incorrect.

b. By explaining how to add matrices, write simple steps for Marco to follow so that he is able to add and subtract any matrices. Use the terms 'order of matrices' and 'elements' in your explanation.

12. Frederick, Harold, Mia and Petra are machinists who work for Stitch in Time. The table below shows the hours worked by each of the four employees and the number of garments completed each week for the last three weeks.

Employee	**Week 1**		**Week 2**		**Week 3**	
	Hours worked	**Number of garments**	**Hours worked**	**Number of garments**	**Hours worked**	**Number of garments**
Frederick	35	150	32	145	38	166
Harold	41	165	36	152	35	155
Mia	38	155	35	135	35	156
Petra	25	80	30	95	32	110

a. Construct a 4 × 1 matrix to represent the number of garments each employee made in week 1.

b. **i.** Create a matrix sum that would determine the total number of garments each employee made over the three weeks.
  ii. Using your matrix sum from part **bi**, determine the total number of garments each employee made over the three weeks.

c. Nula is the manager of Stitch in Time. She uses the following matrix sum to determine the total number of hours worked by each of the four employees over the three weeks.

$$\begin{bmatrix} 35 \\ \\ 38 \\ 25 \end{bmatrix} + \begin{bmatrix} \\ 36 \\ \\ 30 \end{bmatrix} + \begin{bmatrix} 38 \\ 35 \\ 35 \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

Complete the matrix sum by filling in the missing values.

13. There are three types of fish in a pond: speckles, googly eyes and fantails. At the beginning of the month there were 12 speckles, 9 googly eyes and 8 fantails in the pond. By the end of the month there were 9 speckles, 6 googly eyes and 8 fantails in the pond.

a. Construct a matrix sum to represent this information.

b. After six months, there were 12 speckles, 4 googly eyes and 10 fantails in the pond. Starting from the end of the first month, construct another matrix sum to represent this information.

14. Consider the following matrix sum: $A - C + B = D$. Matrix D has an order of 3×2.

a. State the order of matrices A, B and C. Justify your answer.

A has elements $a_{11} = x$, $a_{21} = 20$, $a_{31} = 3c_{31}$, $a_{12} = 7$, $a_{22} = y$ and $a_{32} = -8$.

B has elements $b_{11} = x$, $b_{21} = 2x$, $b_{31} = 3x, b_{12} = y$, $b_{22} = 5$ and $b_{32} = 6$

C has elements $c_{11} = 12$, $c_{21} = \frac{1}{2}a_{21}$, $c_{31} = 5, c_{12} = 9$, $c_{22} = 2y$ and $c_{32} = 3x$.

b. Define the elements of D in terms of x and y.

c. If $D = \begin{bmatrix} -8 & 1 \\ 14 & 2 \\ 16 & -8 \end{bmatrix}$, show that $x = 2$ and $y = 3$.

15. Using technology or otherwise, evaluate the matrix sum

$$\begin{bmatrix} \frac{1}{2} & \frac{3}{4} & \frac{5}{6} \\ \frac{3}{5} & \frac{2}{7} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{4} & \frac{2}{9} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{1}{3} \\ \frac{1}{10} & \frac{3}{14} & \frac{4}{9} \\ \frac{1}{6} & \frac{1}{2} & \frac{2}{3} \end{bmatrix} - \begin{bmatrix} \frac{1}{8} & \frac{1}{2} & \frac{7}{6} \\ \frac{2}{15} & \frac{8}{21} & \frac{4}{3} \\ \frac{2}{9} & \frac{5}{8} & \frac{10}{9} \end{bmatrix}.$$

16. Consider the matrices A and B.

$$A = \begin{bmatrix} 21 & 10 & 9 \\ 18 & 7 & 12 \end{bmatrix} \qquad B = \begin{bmatrix} -10 & 19 & 11 \\ 36 & -2 & 15 \end{bmatrix}$$

The matrix sum $A + B$ was performed using a spreadsheet. The elements for A were entered into a spreadsheet in the following cells: a_{11} was entered in cell A1, a_{21} into cell A2, a_{12} in cell B1, a_{22} in cell B2, a_{13} in cell C2 and a_{23} in cell C3.

a. If the respective elements for B were entered into cells D1, D2, E1, E2, F1 and F2, write the formulas required to find the matrix sum $A + B$.

b. Hence, using a spreadsheet, state the elements of $A + B$.

10.4 Matrix multiplication

10.4.1 Scalar multiplication

If $A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 0 & 7 \end{bmatrix}$, then $A + A$ can be found by multiplying each element in matrix A by the scalar number 2, because $A + A = 2A$.

$$A + A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 10 & 2 \\ 0 & 14 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 \times 3 & 2 \times 2 \\ 2 \times 5 & 2 \times 1 \\ 2 \times 0 & 2 \times 7 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 10 & 2 \\ 0 & 14 \end{bmatrix}$$

The number 2 is known as a scalar quantity, and the matrix $2A$ represents a **scalar multiplication**. Any matrix can be multiplied by any scalar quantity and the order of the matrix will remain the same. A scalar quantity can be any real number, such as negative or positive numbers, fractions or decimal numbers.

WORKED EXAMPLE 7

Consider the matrix $A = \begin{bmatrix} \mathbf{120} & \mathbf{90} \\ \mathbf{80} & \mathbf{60} \end{bmatrix}$.

Evaluate the following.

a. $\frac{1}{4}A$ **b.** $0.1A$

THINK	WRITE
a. 1. Identify the scalar for the matrix. In this case it is $\frac{1}{4}$, which means that each element in A is multiplied by $\frac{1}{4}$ (or divided by 4).	**a.** $\frac{1}{4}\begin{bmatrix} 120 & 90 \\ 80 & 60 \end{bmatrix}$
2. Multiply each element in A by the scalar.	$\begin{bmatrix} \frac{1}{4} \times 120 & \frac{1}{4} \times 90 \\ \frac{1}{4} \times 80 & \frac{1}{4} \times 60 \end{bmatrix}$
3. Simplify each multiplication by finding common factors and write the answer.	$\begin{bmatrix} \cancel{120}^{30} \times \frac{1}{\cancel{4}^{1}} & \cancel{90}^{45} \times \frac{1}{\cancel{4}^{2}} \\ \cancel{80}^{20} \times \frac{1}{\cancel{4}^{1}} & \cancel{60}^{15} \times \frac{1}{\cancel{4}^{1}} \end{bmatrix}$ $= \begin{bmatrix} 30 & \frac{45}{2} \\ 20 & 15 \end{bmatrix}$
b. 1. Identify the scalar. In this case it is 0.1, which means that each element in A is multiplied by 0.1 (or divided by 10).	**b.** $0.1\begin{bmatrix} 120 & 90 \\ 80 & 60 \end{bmatrix}$

▶

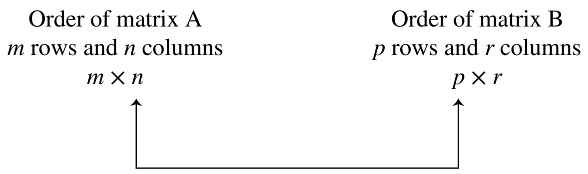
2. Multiply each element in A by the scalar.	$\begin{bmatrix} 0.1 \times 120 & 0.1 \times 90 \\ 0.1 \times 80 & 0.1 \times 60 \end{bmatrix}$
3. Calculate the values for each element and write the answer.	$\begin{bmatrix} 12 & 9 \\ 8 & 6 \end{bmatrix}$

10.4.2 The product matrix and its order

Not all matrices can be multiplied together. However, unlike with addition and subtraction, matrices do not need to have the same order to be multiplied together.

For matrices to be able to be multiplied together (have a product), the *number of columns* in the first matrix must equal the *number of rows* in the second matrix.

For example, consider matrices A and B, with matrix A having an order of $m \times n$ (m rows and n columns) and matrix B having an order of $p \times r$ (p rows and r columns).



Columns in matrix A must equal number of rows in matrix B.

For A and B to be multiplied together, the number of columns in A must equal the number of rows in B; that is, n must equal p. If n does equal p, then the **product matrix** AB is said to exist, and the order of the product matrix AB will be $m \times r$.

Given matrix A with an order of $m \times n$ and matrix B with an order of $p \times r$, matrix AB will have an order of $m \times r$ if $n = p$.

WORKED EXAMPLE 8

If $A = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$, show that the product matrix AB exists and hence write down the order of AB.

THINK	WRITE
1. Write the order of each matrix.	A: 2×1 B: 1×2
2. Write the orders next to each other.	$2 \times 1 \quad 1 \times 2$
3. Circle the two middle numbers.	$2 \times \boxed{1 \quad 1} \times 2$

4. If the two numbers are the same, then the product matrix exists.	Number of columns in A = number of rows in B, therefore the product matrix AB exists.
5. The order of the resultant matrix (the product) will be the first and last number.	$\textcircled{2} \times 1 \quad 1 \times \textcircled{2}$ The order of AB is 2×2.

10.4.3 Multiplying matrices

To multiply matrices together, use the following steps.

Step 1: Confirm that the product matrix exists (that is, the number of columns in the first matrix equals the number of rows in the second matrix).

Step 2: Multiply the elements of each row of the first matrix by the elements of each column of the second matrix.

Step 3: Add the products in each element of the product matrix.

Consider matrices A and B.

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

As previously stated, the order of the product matrix AB will be 2×2.

$$AB = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \end{bmatrix}$$

1st row × 1st column: $3 \times 1 = 3$
1st row × 2nd column: $3 \times 2 = 6$
2nd row × 1st column: $2 \times 1 = 2$
2nd row × 2nd column: $2 \times 2 = 4$

$$AB = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

Unlike when multiplying with real numbers, when multiplying matrices together the order of the multiplication is important. This means that in most cases $AB \neq BA$.

Using matrices A and B as previously defined, the order of product matrix BA is 1×1.

$$BA = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

As when calculating AB, to multiply the elements in these matrices you need to multiply the rows by the columns. Each element in the first row must be multiplied by the corresponding element in the first column, and the total sum of these will make up the element in the first row and first column of the product matrix.

For example, the element in the first row and first column of the product matrix BA is found by the sum $1 \times 3 + 2 \times 2 = 7$.

So the product matrix BA is $[7]$.

WORKED EXAMPLE 9

If $A = \begin{bmatrix} 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, determine the product matrix AB.

THINK	**WRITE**
1. Set up the product matrix.	$\begin{bmatrix} 3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

2. Determine the order of product matrix AB by writing the order of each matrix A and B	$A \times B$ $\enclose{circle}{1} \times 2 \quad \times \quad 2 \times \enclose{circle}{1}$ AB has an order of 1×1.
3. Multiply each element in the first row by the corresponding element in the first column; then calculate the sum of the results.	$3 \times 2 + 5 \times 6 = 36$
4. Write the answer as a matrix.	$[36]$

WORKED EXAMPLE 10

Determine the product matrix MN if $M = \begin{bmatrix} 3 & 6 \\ 5 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 8 \\ 5 & 4 \end{bmatrix}$.

THINK	WRITE
1. Set up the product matrix	$\begin{bmatrix} 3 & 6 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 8 \\ 5 & 4 \end{bmatrix}$
2. Determine the order of product matrix MN by writing the order of each matrix M and N	$M \times N$ $\enclose{circle}{2} \times 2 \quad \times \quad 2 \times \enclose{circle}{2}$ MN has an order of 2×2.
3. To find the element MN_{11}, multiply the corresponding elements in the first row and first column and calculate the sum of the results.	$\begin{bmatrix} \enclose{circle}{3 \quad 6} \\ 5 \quad 2 \end{bmatrix} \times \begin{bmatrix} \enclose{circle}{\begin{matrix} 1 \\ 5 \end{matrix}} & \begin{matrix} 8 \\ 4 \end{matrix} \end{bmatrix}$ $3 \times 1 + 6 \times 5 = 33$
4. To find the element MN_{12}, multiply the corresponding elements in the first row and second column and calculate the sum of the results	$\begin{bmatrix} \enclose{circle}{3 \quad 6} \\ 5 \quad 2 \end{bmatrix} \times \begin{bmatrix} \begin{matrix} 1 \\ 5 \end{matrix} & \enclose{circle}{\begin{matrix} 8 \\ 4 \end{matrix}} \end{bmatrix}$ $3 \times 8 + 6 \times 4 = 48$
5. To find the element MN_{21}, multiply the corresponding elements in the second row and first column and calculate the sum of the results.	$\begin{bmatrix} 3 \quad 6 \\ \enclose{circle}{5 \quad 2} \end{bmatrix} \times \begin{bmatrix} \enclose{circle}{\begin{matrix} 1 \\ 5 \end{matrix}} & \begin{matrix} 8 \\ 4 \end{matrix} \end{bmatrix}$ $5 \times 1 + 2 \times 5 = 15$
6. To find the element MN_{22}, multiply the corresponding elements in the second row and second column and calculate the sum of the results.	$\begin{bmatrix} 3 \quad 6 \\ \enclose{circle}{5 \quad 2} \end{bmatrix} \times \begin{bmatrix} \begin{matrix} 1 \\ 5 \end{matrix} & \enclose{circle}{\begin{matrix} 8 \\ 4 \end{matrix}} \end{bmatrix}$ $5 \times 8 + 2 \times 4 = 48$
7. Construct the matrix MN by writing in each of the elements.	$\begin{bmatrix} 33 & 48 \\ 15 & 48 \end{bmatrix}$

WORKED EXAMPLE 11

Determine the product matrix PQ if $P = \begin{bmatrix} 2 & -1 \\ 3 & 1 \\ 4 & -2 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & 3 & 5 \\ 1 & 2 & -3 \end{bmatrix}$

THINK	WRITE
1. Set up the product matrix	$\begin{bmatrix} 2 & -1 \\ 3 & 1 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} -1 & 3 & 5 \\ 1 & 2 & -3 \end{bmatrix}$
2. Determine the order of product matrix PQ by writing the order of each matrix P and Q.	$P \times Q$ $3 \times 2 \ \times \ 2 \times 3$ PQ has an order of 3×3.
3. To find PQ_{11}, multiply the corresponding elements in the first row and first column and calculate the sum of the results.	$PQ_{11} = (2 \times -1) + (-1 \times 1) = -3$
4. To find PQ_{12}, multiply the corresponding elements in the first row and second column and calculate the sum of the results.	$PQ_{12} = (2 \times 3) + (-1 \times 2) = 4$
5. To find PQ_{13}, multiply the corresponding elements in the first row and third column and calculate the sum of the results.	$PQ_{13} = (2 \times 5) + (-1 \times -3) = 13$
6. To find PQ_{21}, multiply the corresponding elements in the second row and first column and calculate the sum of the results.	$PQ_{21} = (3 \times -1) + (1 \times 1) = -2$
7. To find PQ_{22}, multiply the corresponding elements in the second row and second column and calculate the sum of the results.	$PQ_{22} = (3 \times 3) + (1 \times 2) = 11$
8. To find PQ_{23}, multiply the corresponding elements in the second row and third column and calculate the sum of the results.	$PQ_{23} = (3 \times 5) + (1 \times -3) = 12$
9. To find PQ_{31}, multiply the corresponding elements in the third row and first column and calculate the sum of the results.	$PQ_{31} = (4 \times -1) + (-2 \times 1) = -6$
10. To find PQ_{32}, multiply the corresponding elements in the third row and second column and calculate the sum of the results.	$PQ_{32} = (4 \times 3) + (-2 \times 2) = 8$
11. To find PQ_{33}, multiply the corresponding elements in the third row and third column and calculate the sum of the results.	$PQ_{33} = (4 \times 5) + (-2 \times -3) = 26$
12. Construct the matrix PQ by writing in each of the elements.	$\begin{bmatrix} -3 & 4 & 13 \\ -2 & 11 & 12 \\ -6 & 8 & 26 \end{bmatrix}$

10.4.4 Multiplying by the identity matrix

As previously stated, an identity matrix is a square matrix with 1s in the top left to bottom right diagonal and 0s for all other elements, for example $[1]$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Just like multiplying by the number 1 in the real number system, multiplying by the identity matrix will not change a matrix.

If the matrix $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$ is multiplied by the identity matrix on the left, that is IA, it will be multiplied by a 2×2 identity matrix (because A has 2 rows). If A is multiplied by the identity matrix on the right, that is AI, then it will be multiplied by a 3×3 identity matrix (because A has 3 columns).

$$\begin{aligned} IA &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 2 + 0 \times 3 & 1 \times 4 + 0 \times 5 & 1 \times 6 + 0 \times 7 \\ 0 \times 2 + 1 \times 3 & 0 \times 4 + 1 \times 5 & 0 \times 6 + 1 \times 7 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \\ &= A \end{aligned}$$

$$\begin{aligned} AI &= \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 1 + 4 \times 0 + 6 \times 0 & 2 \times 0 + 4 \times 1 + 6 \times 0 & 2 \times 0 + 4 \times 0 + 6 \times 1 \\ 3 \times 1 + 5 \times 0 + 7 \times 0 & 3 \times 0 + 5 \times 1 + 7 \times 0 & 3 \times 0 + 5 \times 0 + 7 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \\ &= A \end{aligned}$$

Therefore, $AI = IA = A$.

Powers of square matrices

When a square matrix is multiplied by itself, the order of the resultant matrix is equal to the order of the original square matrix. Because of this fact, whole number powers of square matrices always exist.

WORKED EXAMPLE 12

If A = $\begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$, calculate the value of A^3.

THINK	WRITE
1. Write the matrix multiplication in full.	$A^3 = AAA$ $= \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$

2. Calculate the first matrix multiplication (AA).

$$AA = \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$$

$$AA_{11} = 3 \times 3 + 5 \times 5 = 34$$
$$AA_{21} = 5 \times 3 + 1 \times 5 = 20$$
$$AA_{12} = 3 \times 5 + 5 \times 1 = 20$$
$$AA_{22} = 5 \times 5 + 1 \times 1 = 26$$

$$AA = \begin{bmatrix} 34 & 20 \\ 20 & 26 \end{bmatrix}$$

3. Rewrite the full matrix multiplication, substituting the answer found in the previous part.

$$A^3 = AAA$$
$$= \begin{bmatrix} 34 & 20 \\ 20 & 26 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$$

4. Calculate the second matrix multiplication (AAA).

$$AAA_{11} = 34 \times 3 + 20 \times 5 = 202$$
$$AAA_{12} = 34 \times 5 + 20 \times 1 = 190$$
$$AAA_{21} = 20 \times 3 + 26 \times 5 = 190$$
$$AAA_{22} = 20 \times 5 + 26 \times 1 = 126$$

$$AAA = \begin{bmatrix} 202 & 190 \\ 190 & 126 \end{bmatrix}$$

5. Write the answer.

$$A^3 = \begin{bmatrix} 202 & 190 \\ 190 & 126 \end{bmatrix}$$

on Resources

Interactivity: Matrix multiplication (int-6464)

studyon

Units 1 & 2 > Area 5 > Sequence 2 > Concepts 4, 5 & 6

Scalar multiplication Summary screen and practice questions

Matrix multiplication Summary screen and practice questions

Matrix multiplication and powers Summary screen and practice questions

Exercise 10.4 Matrix multiplication

1. **WE7** Consider the matrix $C = \begin{bmatrix} 2 & 3 & 7 \\ 1 & 4 & 6 \end{bmatrix}$. Evaluate the following.

a. $4C$ **b.** $\frac{1}{5}C$ **c.** $0.3C$ **d.** $-C$

2. Matrix D was multiplied by the scalar quantity x. If $3D = \begin{bmatrix} 15 & 0 \\ 21 & 12 \\ 33 & 9 \end{bmatrix}$ and $xD = \begin{bmatrix} 12.5 & 0 \\ 17.5 & 10 \\ 27.5 & 7.5 \end{bmatrix}$, calculate the value of x.

3. **MC** Consider the matrix $M = \begin{bmatrix} 12 & 9 & 15 \\ 36 & 6 & 21 \end{bmatrix}$. Which of the following is equal to the matrix M?

A. $0.1\begin{bmatrix} 1.2 & 0.9 & 1.5 \\ 3.6 & 0.6 & 2.1 \end{bmatrix}$ **B.** $3\begin{bmatrix} 3 & 3 & 5 \\ 9 & 2 & 7 \end{bmatrix}$ **C.** $3\begin{bmatrix} 4 & 3 & 5 \\ 12 & 2 & 7 \end{bmatrix}$ **D.** $3\begin{bmatrix} 36 & 27 & 45 \\ 108 & 18 & 63 \end{bmatrix}$

4. **a.** **WE8** If $X = \begin{bmatrix} 3 & 5 \end{bmatrix}$ and $Y = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, show that the product matrix XY exists and state the order of XY.

   b. Determine which of the following matrices can be multiplied together and state the order of any product matrices that exist.

$$D = \begin{bmatrix} 7 & 4 \\ 3 & 5 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 5 & 7 \\ 8 & 9 \end{bmatrix} \text{ and } E = \begin{bmatrix} 4 & 1 & 2 \\ 6 & 2 & 6 \end{bmatrix}$$

5. The product matrix ST has an order of 3×4. If matrix S has 2 columns, write down the order of matrices S and T.

6. Which of the following matrices can be multiplied together? Justify your answers by determining the order of the product matrices.

$$D = \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix}, E = \begin{bmatrix} 5 & 8 \\ 7 & 1 \\ 9 & 3 \end{bmatrix}, F = \begin{bmatrix} 12 & 7 & 3 \\ 15 & 8 & 4 \end{bmatrix}, G = \begin{bmatrix} 13 & 15 \end{bmatrix}$$

7. **a.** **WE9** If $M = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $N = \begin{bmatrix} 7 & 12 \end{bmatrix}$, determine the product matrix MN.

   b. Does the product matrix NM exist? Justify your answer by calculating the product matrix NM and stating its order.

8. Matrix $S = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix}$, matrix $T = \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix}$ and the product matrix $ST = [5]$. Calculate the value of t.

9. **WE10** Determine the product matrix PQ if $P = \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$.

10. For a concert, three different types of tickets can be purchased; adult, senior and child. The cost of each type of ticket is \$12.50, \$8.50 and \$6.00 respectively. The number of people attending the concert is shown in the following table.

Ticket type	Number of people
Adult	65
Senior	40
Child	85

   a. Construct a column matrix to represent the cost of the three different tickets in the order adult, senior and child.

If the number of people attending the concert is written as a row matrix, a matrix multiplication can be performed to determine the total amount in ticket sales for the concert.

b. By determining the orders of each matrix and then the product matrix, explain why this is the case.

c. By completing the matrix multiplication from part **b**, determine the total amount (in dollars) in ticket sales for the concert.

11. Determine the product matrices when the following pairs of matrices are multiplied together.

a. $\begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 10 & 15 \end{bmatrix}$

b. $\begin{bmatrix} 6 & 5 \\ 8 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 9 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 6 \\ 2 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 & 4 \\ 1 & 2 & 5 \end{bmatrix}$

d. $\begin{bmatrix} 5 & 7 & 1 \\ 6 & 5 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & 7 \\ 3 & 1 \end{bmatrix}$

12. Evaluate the following matrix multiplications.

a. $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

c. Using your results from parts **a** and **b**, when will AB be equal to BA?

d. If A and B are not of the same order, is it possible for AB to be equal to BA?

13. **WE12** If $P = \begin{bmatrix} 8 & 2 \\ 4 & 7 \end{bmatrix}$, calculate the value of P^2.

14. If $T = \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix}$, calculate the value of T^3.

15. The 3×3 identity matrix, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

a. Calculate the value of I^2.

b. Calculate the value of I^3.

c. Calculate the value of I^4.

d. Comment on your answers to parts **a**–**c**.

16. The table below shows the percentage of students who are expected to be awarded grades A–E on their final examinations for Mathematics and Physics.

Grade	A	B	C	D	E
Percentage of students	5	18	45	25	7

The number of students studying Mathematics and Physics is 250 and 185 respectively.

a. Construct a column matrix, S, to represent the number of students studying Mathematics and Physics.

b. Construct a 1×5 matrix, A, to represent the percentage of students expected to receive each grade, expressing each element in decimal form.

c. **i.** In the context of this problem, what does product matrix SA represent?

ii. Determine the product matrix SA. Write your answers correct to the nearest whole numbers.

d. In the context of this problem, what does element SA_{12} represent?

17. A product matrix, $N = MPR$, has order 3×4. Matrix M has m rows and n columns, matrix P has order $1 \times q$, and matrix R has order $2 \times s$. Determine the values of m, n, s and q.

18. Dodgy Bros sell vans, utes and sedans. The average selling price for each type of vehicle is shown in the first table.

Type of vehicle	Selling price ($)
Vans	$4 000
Utes	$12 500
Sedans	$8 500

The second table shows the total number of vans, utes and sedans sold at Dodgy Bros in one month.

Type of vehicle	Number of sales
Vans	5
Utes	8
Sedans	4

Stan is the owner of Dodgy Bros and wants to determine the total amount of monthly sales.

a. Explain how matrices could be used to help Stan determine the total amount, in dollars, of monthly sales.

b. Perform a matrix multiplication that finds the total amount of monthly sales.

c. Brian is Stan's brother and the accountant for Dodgy Bros. In finding the total amount of monthly sales, he performs the following matrix multiplication.

$$\begin{bmatrix} 5 \\ 8 \\ 4 \end{bmatrix} \begin{bmatrix} 4000 & 12\,500 & 8500 \end{bmatrix}$$

Explain why this matrix multiplication is not valid for this problem.

19. In an AFL game of football, 6 points are awarded for a goal and 1 point is awarded for a behind. St Kilda and Collingwood played two games, with the two results given by the following matrix multiplication.

$$\begin{bmatrix} 9 & 14 \\ 10 & 8 \\ 16 & 12 \\ 7 & 10 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} C_1 \\ S_1 \\ C_2 \\ S_2 \end{bmatrix}$$

Complete the matrix multiplication to determine the scores in the two games.

20. By using technology or otherwise, calculate the following powers of square matrices.

a. $\begin{bmatrix} 4 & 8 \\ 7 & 2 \end{bmatrix}^4$

b. $\begin{bmatrix} 3 & 1 & 7 \\ 4 & 2 & 8 \\ 5 & 6 & 9 \end{bmatrix}^3$

10.5 Applications of matrices

10.5.1 Solving problems with matrices

In the previous section, we discussed for matrices to be multiplied together, the number of columns in the first matrix must equal the number of rows in the second matrix. This understanding can be applied to solve application type questions.

WORKED EXAMPLE 13

The Sounds-Good store has three types of televisions priced at \$550, \$970 and \$1200 and three types of sound bars priced at \$99, \$150 and \$320. The manager decides to mark up the televisions by 12% and mark down the sound bars by 10%.

a. Show the prices of the televisions and sound bars in a matrix.

b. Show the matrix obtained by the mark-up of the televisions and the mark-down of the sound bars.

c. Use matrix multiplication to calculate the new prices, to the nearest dollar.

	THINK	WRITE
a.	Place the televisions and sound bars in columns and the prices in the rows. This produces a 3 × 2 matrix.	$\begin{bmatrix} 550 & 99 \\ 970 & 150 \\ 1200 & 320 \end{bmatrix}$
b. 1.	A 12% mark-up is the same as 112% or 1.12. A mark-down of 10% is the same as 90% or 0.90.	$100\% + 12\% = 112\%$ $100\% - 10\% = 90\%$

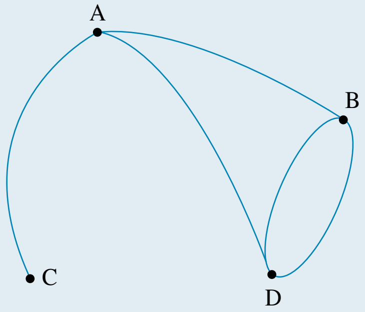
2. We need to multiply the first column of the price matrix from part **a**. by 1.12 and multiply the second column by 0.90. This is a diagonal matrix.	$\begin{bmatrix} 1.12 & 0 \\ 0 & 0.90 \end{bmatrix}$
c. 1. Multiplying the two matrices together we calculate the new prices. We are multiplying a 3×2 by a 2×2 matrix, thus resulting in a 3×2 matrix.	New price $= \begin{bmatrix} 550 & 99 \\ 970 & 150 \\ 1200 & 320 \end{bmatrix} \times \begin{bmatrix} 1.12 & 0 \\ 0 & 0.90 \end{bmatrix}$ $= \begin{bmatrix} (550 \times 1.12) + (99 \times 0) & (550 \times 0) + (99 \times 0.90) \\ (970 \times 1.12) + (150 \times 0) & (970 \times 0) + (150 \times 0.90) \\ (1200 \times 1.12) + (320 \times 0) & (1200 \times 0) + (320 \times 0.90) \end{bmatrix}$ $= \begin{bmatrix} 616 & 89.1 \\ 1086.40 & 135 \\ 1344 & 288 \end{bmatrix}$
2. Round the answers to the nearest dollar.	The marked-up price for the televisions will be \$616, \$1086 and \$1344. The marked-down prices for the sound bars will be \$89, \$135 and \$288.

10.5.2 Adjacency matrices

Matrices can be used to determine the number of different connections between objects, such as towns or people. They can also be used to represent tournament outcomes and determine overall winners. To determine the number of connections between objects, a matrix known as an **adjacency matrix** is set up to represent these connections.

WORKED EXAMPLE 14

The diagram at right shows the number of roads connecting between four towns, A, B, C and D. Construct an adjacency matrix to represent this information.

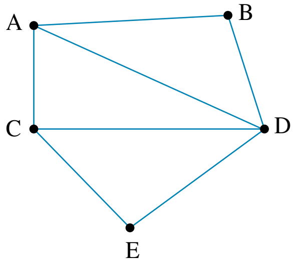


THINK	WRITE
1. Since there are four connecting towns, a 4 × 4 adjacency matrix needs to be constructed. Label the row and columns with the relevant towns A, B, C and D.	$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \end{array}$
2. There is one road connecting town A to town B, so enter 1 in the cell from A to B.	$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} - & - & - & - \\ \textcircled{1} & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \end{array}$
3. There is also only one road between town A and towns C and D; therefore, enter 1 in the appropriate matrix positions. There are no loops at town A (i.e. a road connecting A to A); therefore, enter 0 in this position.	$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & - & - & - \\ 1 & - & - & - \\ 1 & - & - & - \\ 1 & - & - & - \end{bmatrix} \end{array}$
4. Repeat this process for towns B, C and D. Note that there are two roads connecting towns B and D, and that town C only connects to town A.	$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \end{array}$

Determining the number of connections between objects

An adjacency matrix allows us to determine the number of connections either directly between or via objects. If a direct connection between two objects is denoted as one ‘step’, ‘two steps’ means a connection between two objects via a third object, for example the number of ways a person can travel between towns A and D via another town.

You can determine the number of connections of differing ‘steps’ by raising the adjacency matrix to the power that reflects the number of steps in the connection.



For example, the following diagram shows the number of roads connecting five towns, A,B, C, D and E. There are a number of ways to travel between towns A and D. There is one direct path between the towns; this is a one-step path.

A one-step path matrix for the roads connecting the five towns can be shown as:

$$\begin{array}{ccc} & & \text{TO} \\ & & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

FROM

However, you can also travel between towns A and D via town C or B. These are considered two-step paths as there are two links (or roads) in these paths. This is referred to as the link length. The power on the adjacency matrix would therefore be 2 in this case.

WORKED EXAMPLE 15

The following adjacency matrix shows the number of pathways between four attractions at the zoo: lions (L), seals (S), monkeys (M) and elephants (E).

$$\begin{array}{c|cccc|} & \mathbf{L} & \mathbf{S} & \mathbf{M} & \mathbf{E} \\ \mathbf{L} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{S} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{M} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{E} & \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{0} \end{array}$$

Determine how many ways a family can travel from the lions to the monkeys via one of the other two attractions.

THINK	WRITE
1. Determine the link length.	The required path is between two attractions via a third attraction, so the link length is 2.
2. As the link length is 2, this will raise the matrix to a power of 2. Evaluate the matrix using technology such as graphing or a CAS calculator.	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}^2 = \begin{bmatrix} 3 & 2 & 3 & 3 \\ 2 & 3 & 3 & 3 \\ 3 & 3 & 6 & 2 \\ 3 & 3 & 2 & 6 \end{bmatrix}$
3. Interpret the information in the matrix and answer the question by locating the required value.	$\begin{bmatrix} 3 & 2 & \textcircled{3} & 3 \\ 2 & 3 & 3 & 3 \\ 3 & 3 & 6 & 2 \\ 3 & 3 & 2 & 6 \end{bmatrix}$ There are 3 ways in which a family can travel from the lions to the monkeys via one of the other two attractions.

Resources

Interactivity: The adjacency matrix (int-6466)

Exercise 10.5 Applications of matrices

1. **WE13** A store has three types of dishwashers, priced at \$650, \$900 and \$1200, and three types of fridges, priced at \$750, \$1000 and \$1350. The prices of the dishwashers are to be marked down by 8% and the prices of the fridges are to be marked up by 5%.
   a. Construct a suitable matrix to show the prices of the dishwashers and fridges.
   b. Determine the diagonal matrix that would mark down the dishwashers by 8% and mark up the fridges by 5%.
   c. Calculate the new prices (correct to the nearest dollar) using the matrices from part **a** and **b**.

2. The number of Google Home and Google Home Minis sold by four stores is given in the table shown.

	Google Home	**Google Home Mini**
Store A	25	8
Store B	12	9
Store C	20	12
Store D	15	10

If the Google Homes were priced at \$155 each and the Google Home Minis were priced at \$60 each, use matrix operations to determine:

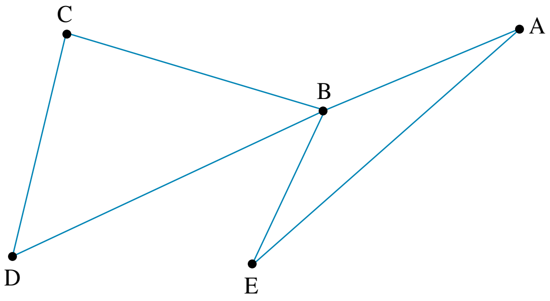
a. the total sales figures of Google Homes at each store.

b. the total sales figures for each store.

c. the store that had the highest sales figures for:

  i. Google Homes ii. Google Home Minis.

3. **WE14** The diagram below shows the network cable between five main computers (A, B, C D and E) in an office building.



Construct an adjacency matrix to represent this information.

4. There are five friends on a social media site: Peta, Seth, Tran, Ned and Wen. The number of communications made between these friends in the last 24 hours is shown in the adjacency matrix below.

$$\begin{array}{c c} & \begin{array}{ccccc} P & S & T & N & W \end{array} \\ \begin{array}{c} P \\ S \\ T \\ N \\ W \end{array} & \begin{bmatrix} 0 & 1 & 3 & 1 & 0 \\ 1 & 0 & 0 & 0 & 4 \\ 3 & 0 & 0 & 2 & 1 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

a. How many times did Peta and Tran communicate over the last 24 hours?

b. Did Seth communicate with Ned at any time during the last 24 hours?

c. In the context of this problem, explain the existence of the zeros along the diagonal.

d. Using the adjacency matrix, construct a diagram that shows the number of communications between the five friends.

5. **WE15** The adjacency matrix below shows the number of roads between three country towns, Gladstone (G), Rockhampton (R) and Bundaberg (B).

$$\begin{array}{c c} & \begin{array}{ccc} G & R & B \end{array} \\ \begin{array}{c} G \\ R \\ B \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$

Using a technology of your choice, determine the number of ways a person can travel from Gladstone to Rockhampton via Bundaberg.

6. The direct Cape Air flights between five cities, Boston (B), Hyannis (H), Martha's Vineyard (M), Nantucket (N) and Providence (P), are shown in the adjacency matrix.

$$\begin{array}{c c}
 & \begin{array}{ccccc} B & H & M & N & P \end{array} \\
\begin{array}{c} B \\ H \\ M \\ N \\ P \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}
\end{array}$$

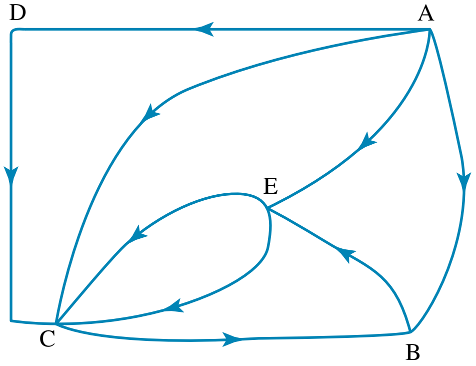
a. Construct a diagram to represent the direct flights between the five cities.

b. Construct a matrix that determines the number of ways a person can fly between two cities via another city.

c. Explain how you would determine the number of ways a person can fly between two cities via two other cities.

d. Is it possible to fly from Boston and stop at every other city? Explain how you would answer this question.

7. For the directed network shown determine the number and name of the:



a. **i.** one-stage paths to get to C. **ii.** two-stage paths to get to C.

b. Represent the one-stage and two-stage pathways of the directed network in matrix form.

8. Given the following communication matrix, answer the following questions

$$\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}
\end{array}$$

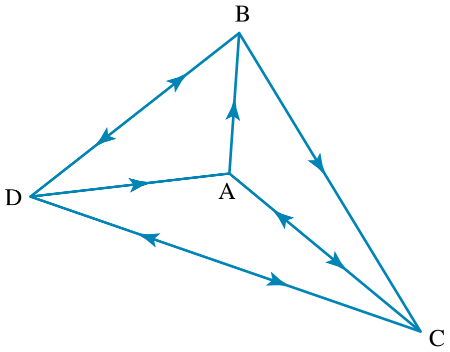
a. To whom can C talk?

b. Who can B receive calls from?

c. Why is the main diagonal all zeros?

d. Who can B not call?

9. Construct the communication matrix from the following communication network.



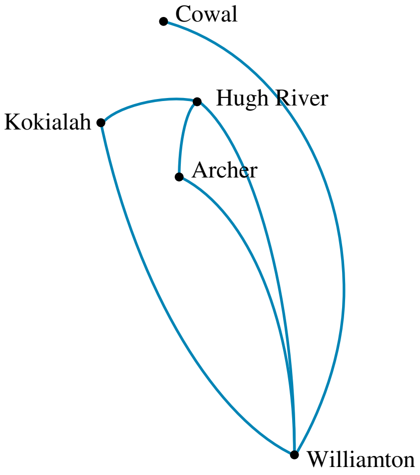
10. The adjacency matrix below shows the number of text messages sent between three friends, Stacey (S), Ruth (R) and Toiya (T), immediately after school one day

$$\begin{array}{c} \\ S \\ R \\ T \end{array}\begin{array}{c} \begin{array}{ccc} S & R & T \end{array} \\ \begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$

a. State the number of text messages sent between Stacey and Ruth.

b. Determine the total number of text messages sent between all three friends.

11. Airlink flies charter flights in the Cape Lancaster region. The direct flights between Williamton, Cowal, Hugh River, Kokialah and Archer are shown in the diagram.



a. Using the diagram, construct an adjacency matrix that shows the number of direct flights between the five towns

b. How many ways can a person travel between Williamton and Kokialah via another town?

c. Is it possible to fly between Cowal and Archer and stop over at two other towns? Justify your answer.

12. The senior school manager developed a matrix formula to determine the number of school jackets to order for Years 11 and 12 students. The column matrix, J_0, shows the number of jackets ordered last year.

$$J_0 = \begin{bmatrix} 250 \\ 295 \end{bmatrix}$$

J_1 is the column matrix that lists the number of Year 11 and 12 jackets to be ordered this year. J_1 is given by the matrix formula

$$J_1 = AJ_0 + B, \text{ where } A = \begin{bmatrix} 0.65 & 0 \\ 0 & 0.82 \end{bmatrix} \text{ and } B = \begin{bmatrix} 13 \\ 19 \end{bmatrix}.$$

a. Determine J_1.

b. Using your value from part a and the same matrix formula, determine the jacket order for the next year. Write your answer to the nearest whole number.

10.6 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. **MC** Consider the matrix equation $\begin{bmatrix} 6 & a \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$.

The values of a and b respectively are:

A. $a = 1, b = 1$ **B.** $a = 1, b = -1$

C. $a = 2, b = 1$ **D.** $a = 2, b = -1$

2. **MC** The order of the matrix $\begin{bmatrix} 6 & 1 & 2 \\ 7 & 3 & 5 \\ 9 & 5 & 0 \\ 0 & 7 & 2 \end{bmatrix}$ is:

A. 3 × 4 **B.** 4 × 2 **C.** 4 × 3 **D.** 8

The following information relates to questions **3** and **4**.

The following matrix shows the airline ticket price between four cities: Perth, Melbourne, Hobart and Sydney.

$$\begin{array}{cc} & \text{From} \\ & \begin{array}{cccc} \text{P} & \text{M} & \text{H} & \text{S} \end{array} \\ \text{TO} \begin{array}{c} \text{P} \\ \text{M} \\ \text{H} \\ \text{S} \end{array} & \begin{bmatrix} 0 & 140 & 450 & 190 \\ 180 & 0 & 90 & 50 \\ 350 & 80 & 0 & 80 \\ 240 & 60 & 110 & 0 \end{bmatrix} \end{array}$$

3. **MC** The cost to fly from Sydney to Hobart is:

A. \$50 **B.** \$80 **C.** \$90 **D.** \$110

4. **MC** A passenger flies from one city and then on to another, with the total cost of the flight being \$320. The order in which the plane flew between the three cities was:

A. Sydney to Melbourne to Perth **B.** Hobart to Melbourne to Perth

C. Perth to Melbourne to Sydney **D.** Perth to Sydney to Hobart

5. **MC** Sarah purchased 2 apples and 3 bananas from her local market for \$3.80. Later that week she went back to the market and purchased 4 more apples and 4 more bananas for \$6.20. A matrix equation that could be set up to determine the price of a single apple and banana is:

A. $\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.8 \\ 6.2 \end{bmatrix}$ **B.** $\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 3.8 \\ 6.2 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.8 \\ 6.2 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 3.8 \\ 6.2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

6. **MC** The matrix that represents the product $\begin{bmatrix} 5 & 1 & 2 \\ 3 & 7 & 8 \end{bmatrix}\begin{bmatrix} 12 \\ 10 \\ 9 \end{bmatrix}$ is:

A. $\begin{bmatrix} 88 \\ 178 \end{bmatrix}$ **B.** $\begin{bmatrix} 60 & 10 & 18 \\ 36 & 63 & 72 \end{bmatrix}$ **C.** $\begin{bmatrix} 88 & 178 \end{bmatrix}$ **D.** $\begin{bmatrix} 48 \\ 49 \end{bmatrix}$

7. **MC** The population age structure (in percentages) in 2010 for selected countries is shown in the following table.

Country	**Percentage of population between the age groups**		
	0–14 years	**15–64 years**	**Over 65 years**
Australia	18.9	67.6	13.5
China	16.4	69.5	14.1
Indonesia	27.0	67.4	5.6

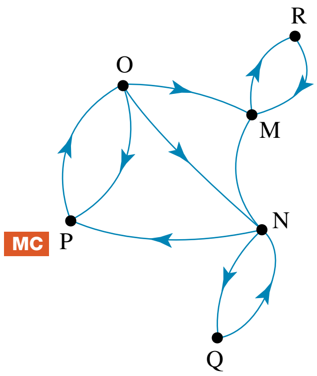
A 3 × 1 matrix that could be used to represent the percentage of population across the three age groups for Indonesia is:

A. $\begin{bmatrix} 18.9 \\ 16.4 \\ 27.0 \end{bmatrix}$ **B.** $\begin{bmatrix} 27.0 & 67.4 & 5.6 \end{bmatrix}$ **C.** $\begin{bmatrix} 18.9 & 67.6 & 13.5 \\ 16.4 & 69.5 & 14.1 \\ 27.0 & 67.4 & 5.6 \end{bmatrix}$ **D.** $\begin{bmatrix} 27.0 \\ 67.4 \\ 5.6 \end{bmatrix}$

8. **MC** Matrix A has an order of 3×2. Matrix B has an order of 1×3. Matrix C has an order of 2×1. Which one of the following matrix multiplications is not possible?

A. AC **B.** BA **C.** BC **D.** CB

9. **MC** Running paths through a park are shown in the diagram. The only way to get to O is from?



A. M **B.** N **C.** P **D.** Q

10. Given matrices $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, evaluate the following.

a. $C + B$ **b.** $B - 2C$ **c.** $AB + C$ **d.** $1.5A$

11. Matrix D has an order of 3×2, matrix E has an order of $1 \times p$ and matrix F has an order of 2×2.

a. For what value of p would the product matrix ED exist?

b. If the product matrix H exists and $H = EDF$, state the order of H.

12. **a.** For each of the following pairs of matrices, state the order of the matrices, and hence state the order of the product matrix.

i. $\begin{bmatrix} 3 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ **ii.** $\begin{bmatrix} 4 & 6 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$

iii. $\begin{bmatrix} -1 & 9 \\ 10 & 5 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 5 & 11 \end{bmatrix}$ **iv.** $\begin{bmatrix} 2 \\ 7 \\ 8 \end{bmatrix}\begin{bmatrix} 5 & 3 & 4 \end{bmatrix}$

b. Determine the product matrices of the matrix multiplications given in part **a**.

Complex familiar

13. **MC** To help him to answer his ten multiple choice questions, Trei is using the following matrix.

$$\begin{array}{c} \text{Answers} \\ \text{This question} \end{array}$$

$$\text{Answers Next question} \begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Trei answered B to question 1 and then used the matrix to answer the remaining nine questions. What was Trei's answer to question 6?

A. C **B.** D **C.** E **D.** A

14. The table below shows the three different ticket prices, in dollars, and the number of tickets sold for a school concert.

Ticket type	Ticket price	Number of tickets sold
Adult	$12.50	140
Child/student	$6.00	225
Teacher	$10.00	90

a. Construct a column matrix to represent the ticket prices for adults, children/students and teachers respectively.

b. Perform a matrix multiplication to determine the total amount of ticket sales in dollars.

15. Rhonda was asked to perform the following matrix multiplication to determine the product matrix GH.

$$GH = \begin{bmatrix} 6 & 5 \\ 3 & 8 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 10 \\ 13 \end{bmatrix}$$

Rhonda's answer was $\begin{bmatrix} 60 & 65 \\ 30 & 104 \\ 50 & 117 \end{bmatrix}$.

a. By stating the order of product matrix GH, explain why Rhonda's answer is obviously incorrect.

b. Determine the product matrix GH.

c. Explain Rhonda's method of multiplying matrices and why this is the incorrect method.

d. Provide simple steps to help Rhonda multiply matrices.

16. The number of adults, children and seniors attending the zoo over Friday, Saturday and Sunday is shown in the table.

Day	Adults	Children	Seniors
Friday	125	245	89
Saturday	350	456	128
Sunday	421	523	102

Entry prices for adults, children and seniors are \$35, \$25, \$20 respectively.

a. Using technology or otherwise, perform a matrix multiplication that will determine the entry fee collected for each of the three days.

b. Write the calculation that finds the entry fee collected for Saturday.

c. Is it possible to perform a matrix multiplication that would find the total for each type of entry fee (adults, children and seniors) over the three days? Explain your answer.

Complex unfamiliar

17. The energy content and amounts of fat and protein contained in each slice of bread and cheese and one teaspoon of margarine is shown in the table below.

Food	Energy content (kilojoules)	Fat (grams)	Protein (grams)
Bread	410	0.95	3.7
Cheese	292	5.5	1.6
Margarine	120	3.3	0.5

Pedro made toasted cheese sandwiches for himself and his friends for lunch. The total amount of fat and protein (in grams) for each of the three foods — bread, cheese and margarine — in the prepared lunch were recorded in the following matrix.

$$\begin{bmatrix} 7.6 & 29.6 \\ 44.0 & x \\ 13.2 & 2 \end{bmatrix}$$

a. How many bread slices did Pedro use?

b. If each sandwich used two pieces of bread, how many cheese sandwiches did Pedro make?

c. **i.** Show that each sandwich had two slices of cheese.

ii. Hence, calculate the exact value of x.

d. Construct a 1×3 matrix to represent the number of slices of bread and cheese and servings of margarine for each sandwich.

18. Tootin' Travel Agents sell three different types of train travel packages on the Midnight Express: Platinum, Gold and Red class. The price for each travel package is shown in the table.

Class	Price
Platinum	\$3890
Gold	\$2178
Red	\$868

a. Construct a column matrix, C, to represent the price of each of the three travel packages: Platinum, Gold and Red. State the order of C.

b. In the last month, Tootin' Travel Agents sold the following number of train travel packages.

- Platinum: 62
- Gold: 125
- Red: 270

Construct a row matrix, P, to represent the number of train travel packages sold over the last month

c. To determine the total amount in dollars for train travel packages in the month, a product matrix, PC is found. State the order of product matrix PC.

Travellers who book in a year in advance receive a 5% discount. To calculate the discounted price, matrix C is multiplied by a scalar product, d.

d. **i.** Write down the value of d.

ii. Using your value for d, construct a new matrix, E, that represents the discounted travel prices. Write your answer correct to the nearest cent.

19. TruSport owns two stores at LeisureLand and SportLand shopping centres. The number of tennis racquets, baseball bats and soccer balls sold in the last week at the two stores is shown in the table below.

Store	Tennis racquets	Baseball bats	Soccer balls
LeisureLand	10	8	9
SportLand	9	12	11

The selling price of each item is shown in the table below.

	Tennis racquet	Baseball bat	Soccer ball
Selling price	\$45.95	\$25.50	\$18.60

a. Construct a 3×2 matrix to represent the number of tennis racquets, baseball bats and soccer balls sold at each of the two stores.

b. Construct a row matrix to represent the selling prices of each of the items.

c. Set up a matrix multiplication that finds the total amount, in dollars, that each store made in the last week.

d. Hence find the total amount, in dollars, that each store made in the last week.

20. There are 472 students studying History and 424 studying Economics at a university. At the end of the academic year, 25% of students will be awarded a Pass grade, 38% will be awarded a Credit grade, 19% will be awarded Distinction grade, 8% will be awarded a High Distinction grade and the remaining students will not pass.

The column matrix $N = \begin{bmatrix} 472 \\ 424 \end{bmatrix}$ represents the number of students studying History and Economics.

a. Write down the order of matrix N.

b. Construct a row matrix, G, to represent the percentages, in decimal form, of students who will be awarded one of the five grades

c. Evaluate the product matrix $A = NG$. Write your answer to the nearest whole number.

d. In the context of this problem, explain what the element A_{24} means.

The cost for textbooks for a student studying History is \$125; for Economics, the textbooks costs \$235.

e. Construct a matrix calculation that will give the total cost for textbooks, C, paid in dollars by the students studying History and Economics.

f. Calculate the total cost for the textbooks.

studyon

Units 1 & 2 **Sit chapter test**

Answers

Chapter 10 Matrices and matrix arithmetic

Exercise 10.2 Types of matrices

1. $\begin{bmatrix} 18 & 12 & 8 \\ 13 & 10 & 11 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 4 & 7 & 3 & 6 \\ 2 & 6 & 6 & 6 & 4 \end{bmatrix}$

3. a. 545 km **b.** Coober Pedy and Alice Springs **c.** 2458 km

4.

a. $\begin{array}{c} \\ M \\ C \\ S \end{array}\begin{array}{c} \begin{array}{ccc} M & C & S \end{array} \\ \begin{bmatrix} 0 & 91.13 & 110.72 \\ 91.13 & 0 & 48.02 \\ 110.72 & 48.02 & 0 \end{bmatrix} \end{array}$

b. \$139.15

5. $\begin{bmatrix} 45 \\ 30 \end{bmatrix}$, order 2×1

6. a. 56 **b.** 213 **c.** $\begin{bmatrix} 12 & 17 & 18 \end{bmatrix}$

7. A: 1×1, B: 3×1, C: 1×2

8. **a** and **d** are matrices with orders of 2×1 and 2×4 respectively. The matrix shown in **b** is incomplete, and the matrix shown in **c** has a different number of rows in each column.

9. a. 5 **b.** 6 **c.** 7

10. a. There is no 4th column.

b. e_{23}

c. Nadia thought that e_{12} was read as 1st column, 2nd row. The correct value is 0.

11. a. 3 **b.** -1 **c.** 1 **d.** 0.5 **e.** 0.9

12. a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

13. a. 3×2 **b.** $\begin{bmatrix} 4 & 3 \\ -1 & 7 \\ -4 & 6 \end{bmatrix}$

14. a. $\begin{bmatrix} 0.4 \\ 4.2 \\ 6.8 \\ 0.2 \\ 1.6 \\ 2.1 \\ 0.5 \\ 5.2 \end{bmatrix}$

b. $\begin{bmatrix} 2358 & 68330 & 227600 & 801428 & 984000 & 1346200 & 1727200 & 2529875 \end{bmatrix}$

c. i. 3×2 **ii.** $\begin{bmatrix} 801428 & 6.8 \\ 227600 & 5.2 \\ 1727200 & 4.2 \end{bmatrix}$

15. a. $\begin{bmatrix} 148178 & 2.2 \\ 30839 & 0.6 \\ 146429 & 3.6 \\ 26044 & 1.7 \\ 77928 & 3.8 \\ 16900 & 3.4 \\ 66582 & 31.6 \\ 4\,043 & 1.2 \end{bmatrix}$

b. i. 66 582 **ii.** 16 900 **iii.** 325 446

c. 516 943

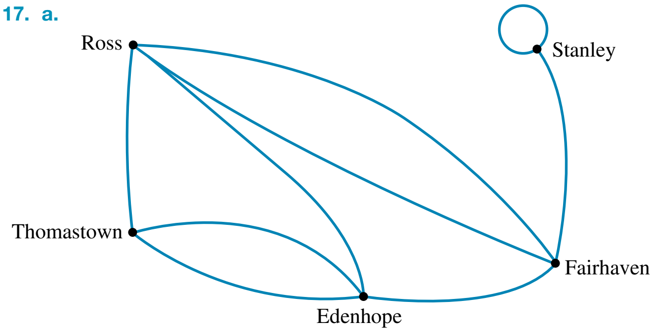
16. a. The zeros mean they don't fly from one place back to the same place.

b. \$175

c. Mount Isa

d.
$$\begin{array}{c@{}c} & \begin{array}{cccc} O & B & D & M \end{array} \\ \begin{array}{c} O \\ B \\ D \\ M \end{array} & \begin{bmatrix} 0 & 59.50 & 150 & 190 \\ 89 & 0 & 85 & 75 \\ 175 & 205 & 0 & 213.75 \\ 307 & 90 & 75.75 & 0 \end{bmatrix} \end{array}$$

17. a.



b. **i.** True **ii.** False **iii.** True **iv.** False

c. N_{31} and N_{13}

18. a. D

b.

Question	1	2	3	4	5	6	7	8	9	10
Response	A	D	B	E	D	B	E	D	B	E

c. There are no 1s in row A, just 0s.

d.
$$\begin{array}{c@{}c@{}c} & & \text{This question} \\ & & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \text{Next question} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Exercise 10.3 Operations with matrices

1. a. $\begin{bmatrix} 1 & 6 \\ -1 & 3 \end{bmatrix}$ **b.** $\begin{bmatrix} -0.1 \\ 1.5 \\ 4.2 \end{bmatrix}$

2. $a = 4,\ b = -6$

3. a. $\begin{bmatrix} 1 \\ 7 \\ 0 \end{bmatrix}$ **b.** $\begin{bmatrix} -5 \\ 3 \\ 6 \end{bmatrix}$ **c.** $\begin{bmatrix} 6 \\ 4 \\ -6 \end{bmatrix}$ **d.** $\begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix}$

4. $\begin{bmatrix} -1 & -2 \\ -1 & -5 \end{bmatrix}$

5. a. Both matrices must be of the same order for it to be possible to add and subtract them.

b. $B = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$

6. a. $\begin{bmatrix} -0.25 & -0.95 & 0.3 \end{bmatrix}$ **b.** $\begin{bmatrix} 3 & -1 \\ 9 & 1 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 5 & 1 \\ 4 & -2 & 3 \\ 3 & -3 & -2 \end{bmatrix}$ **d.** $\begin{bmatrix} 15 & 15 & 7 \\ 9 & 10 & 8 \end{bmatrix}$

7. $a = -5,\ b = 2,\ c = 5$

8. $a = 6,\ b = -1,\ c = 5$

9. A and E have the same order, 1×2.
B and C have the same order, 2×1.

10. a. $\begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 3 \end{bmatrix}$

b. **i.** $\begin{bmatrix} 3 & 4 & 8 \\ 6 & 8 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 3 \end{bmatrix}$

ii. $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 2 & 2 \end{bmatrix}$

Eggs	**Small**	**Medium**	**Large**
Free range	1	1	3
Barn laid	2	2	2

11. a. Both matrices are of the order 2×3; therefore, the answer matrix must also be of the order 2×3. Marco's answer matrix is of the order 2×1, which is incorrect.

b. Sample responses can be found in the worked solutions in the online resources. A possible response is:
Step 1: Check that all matrices are the same order.
Step 2: Add or subtract the corresponding elements.

12. a. $\begin{bmatrix} 150 \\ 165 \\ 155 \\ 80 \end{bmatrix}$

b. **i.** $\begin{bmatrix} 150 \\ 165 \\ 155 \\ 80 \end{bmatrix} + \begin{bmatrix} 145 \\ 152 \\ 135 \\ 95 \end{bmatrix} + \begin{bmatrix} 166 \\ 155 \\ 156 \\ 110 \end{bmatrix}$

ii. $\begin{bmatrix} 461 \\ 472 \\ 446 \\ 285 \end{bmatrix}$

c. $\begin{bmatrix} 35 \\ 41 \\ 38 \\ 25 \end{bmatrix} + \begin{bmatrix} 32 \\ 36 \\ 35 \\ 30 \end{bmatrix} + \begin{bmatrix} 38 \\ 35 \\ 35 \\ 32 \end{bmatrix} = \begin{bmatrix} 105 \\ 112 \\ 108 \\ 87 \end{bmatrix}$

13. a. $\begin{bmatrix} 12 \\ 9 \\ 8 \end{bmatrix} + \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$ or $\begin{bmatrix} 12 \\ 9 \\ 8 \end{bmatrix} - \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$

b. $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ 10 \end{bmatrix}$ or $\begin{bmatrix} 12 \\ 4 \\ 10 \end{bmatrix} - \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$

14. a. To add or subtract matrices, all matrices must be of the same order. Since the resultant matrix D is of the order 3×2, all other matrices must also be of the order 3×2.

b. $\begin{bmatrix} 2x - 12 & y - 2 \\ 2x + 10 & 5 - y \\ 3x + 10 & -2 - 3x \end{bmatrix}$

c. Sample responses can be found in the worked solutions in the online resources.

15. $\begin{bmatrix} \frac{5}{8} & \frac{5}{8} & 0 \\ \frac{17}{30} & \frac{5}{42} & \frac{-5}{9} \\ \frac{11}{18} & \frac{1}{8} & \frac{-2}{9} \end{bmatrix}$

16. a. Sample responses can be found in the worked solutions in the online resources. Possible answer:
= sum (A1 + D1), = sum (B1 + E1), = sum (C1 + F1)
= sum (A2 + D2), = sum (B2 + E2), = sum (C2 + F2)

b. $\begin{bmatrix} 11 & 29 & 20 \\ 54 & 5 & 27 \end{bmatrix}$

Exercise 10.4 Matrix multiplication

1. a. $\begin{bmatrix} 8 & 12 & 28 \\ 4 & 16 & 24 \end{bmatrix}$ **b.** $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & \frac{7}{5} \\ \frac{1}{5} & \frac{4}{5} & \frac{6}{5} \end{bmatrix}$

c. $\begin{bmatrix} 0.6 & 0.9 & 2.1 \\ 0.3 & 1.2 & 1.8 \end{bmatrix}$ **d.** $\begin{bmatrix} -2 & -3 & -7 \\ -1 & -4 & -6 \end{bmatrix}$

2. $x = 2.5$

3. C

4. a. $1 \times \boxed{2\ 2} \times 1$
Number of columns = number of rows, therefore XY exists and is of order 1×1.

b. DE: 3×3, DC: 3×2, ED: 2×2, CE: 2×3

5. S: 3×2, T: 2×4

6. DG: 3×2, FD: 2×1, FE: 2×2, EF: 3×3, GF: 1×3

7. a. $MN = \begin{bmatrix} 28 & 48 \\ 21 & 36 \end{bmatrix}$ **b.** Yes, [64] is of the order 1×1.

8. $t = -3$

9. $PQ = \begin{bmatrix} 41 & 45 \\ 36 & 32 \end{bmatrix}$

10. a. $\begin{bmatrix} 12.50 \\ 8.50 \\ 6.00 \end{bmatrix}$

b. Total tickets requires an order of 1×1, and the order of the ticket price is 3×1. The number of people must be of order 1×3 to result in a product matrix of order 1×1. Therefore, the answer must be a row matrix.

c. \$1662.50

11. a. $\begin{bmatrix} 70 & 105 \\ 20 & 30 \\ 90 & 135 \end{bmatrix}$ **b.** $\begin{bmatrix} 57 \\ 43 \end{bmatrix}$

c. $\begin{bmatrix} 18 & 32 & 46 \\ 9 & 16 & 23 \\ 10 & 17 & 17 \end{bmatrix}$ **d.** $\begin{bmatrix} 30 & 60 \\ 35 & 49 \end{bmatrix}$

12. a. $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

c. When either A or B is the identity matrix

d. No. Consider matrix A with order of $m \times n$ and matrix B with order of $p \times q$, where $m \neq p$ and $n \neq q$. If AB exists, then it has order $m \times q$ and $n = p$. If BA exists, then it has order $p \times n$ and $q = m$. Therefore $AB \neq BA$, unless $m = p$ and $n = q$, which is not possible since they are of different orders.

13. $\begin{bmatrix} 72 & 30 \\ 60 & 57 \end{bmatrix}$

14. $\begin{bmatrix} 27 & 315 \\ 0 & 216 \end{bmatrix}$

15. a. $I^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **b.** $I^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $I^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **d.** Whatever power you raise I to, the matrix stays the same.

16. a. $\begin{bmatrix} 250 \\ 185 \end{bmatrix}$

b. $\begin{bmatrix} 0.05 & 0.18 & 0.45 & 0.25 & 0.07 \end{bmatrix}$

c. i. The number of expected grades (A–E) for students studying Mathematics and Physics.

ii. $\begin{bmatrix} 13 & 45 & 113 & 63 & 18 \\ 9 & 33 & 83 & 46 & 13 \end{bmatrix}$

d. 45 students studying maths are expected to be awarded a B grade.

17. $n = 1$, $m = 3$, $s = 4$, $q = 2$

18. a. Possible answer:
Represent the number of vehicles in a row matrix and the cost for each vehicle in a column matrix, then multiply the two matrices together. The product matrix will have an order of 1×1.

b. $\begin{bmatrix} 154\,000 \end{bmatrix}$ or \$ 154 000

c. Possible answer:
In this multiplication each vehicle is multiplied by the price of each type of vehicle, which is incorrect. For example, the ute is valued at \$12 500, but in this multiplication the eight utes sold are multiplied by \$4000, \$12 500 and \$8500 respectively.

19. $C_1 = 68$, $S_1 = 68$, $C_2 = 108$, $S_2 = 52$. The two results were 68 – 68 and 108 – 52.

20. a. $\begin{bmatrix} 4 & 8 \\ 7 & 2 \end{bmatrix}^4 = \begin{bmatrix} 7200 & 6336 \\ 5544 & 5616 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 1 & 7 \\ 4 & 2 & 8 \\ 5 & 6 & 9 \end{bmatrix}^3 = \begin{bmatrix} 792 & 694 & 1540 \\ 984 & 868 & 1912 \\ 1356 & 1210 & 2632 \end{bmatrix}$

Exercise 10.5 Applications of matrices

1. a. $\begin{bmatrix} 650 & 750 \\ 900 & 1000 \\ 1200 & 1350 \end{bmatrix}$

b. $\begin{bmatrix} 0.92 & 0 \\ 0 & 1.05 \end{bmatrix}$

c. The marked-down price for the dishwashers will be \$598, \$828 and \$1104. The marked-up prices for the fridges will be \$788, \$1050 and \$1418.

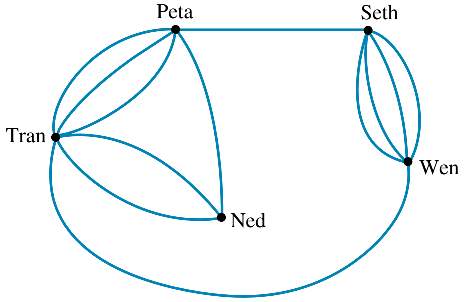
2. a. Store A: \$3875
Store B: \$1860
Store C: \$3100
Store D: \$2325

b. Sales figures:
Store A: \$4355
Store B: \$2400
Store C: \$3820
Store D: \$2925

c. i. Store A: \$3875
ii. Store C: \$720

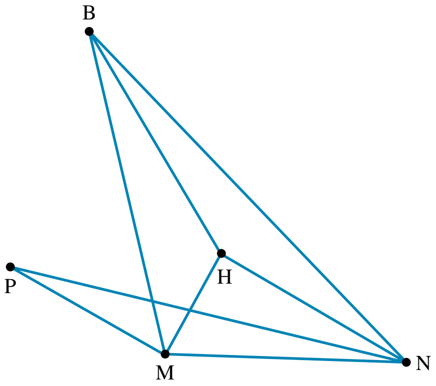
3. $\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$

4. a. 3
b. No
c. They did not communicate with themselves.
d.



5. 2

6. a.



b. $\begin{array}{c} \\ B \\ H \\ M \\ N \\ P \end{array} \begin{array}{c} \begin{array}{ccccc} B & H & M & N & P \end{array} \\ \begin{bmatrix} 3 & 2 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 & 2 \\ 2 & 2 & 4 & 3 & 1 \\ 2 & 2 & 3 & 4 & 1 \\ 2 & 2 & 1 & 1 & 2 \end{bmatrix} \end{array}$

c. Raise the matrix to a power of 2.

d. Yes. Raise the matrix to a power of 4, as there are five cities in total.

7. a. i. A to C
D to C
E to C

ii. A to D to C
A to E to C

b. $A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix}$

8. a. C can talk to A, B and D.
b. B can receive calls from A, C and D.
c. Since they don't communicate with themselves.
d. B cannot call C.

9. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

10. a. 3 **b.** 6

11. a. $\begin{array}{c} \\ W \\ C \\ H \\ K \\ A \end{array} \begin{array}{c} \begin{array}{ccccc} W & C & H & K & A \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$

b. 1
c. Yes. The matrix raised to the power of 3 will provide the number of ways possible.

12. a. $J_1 = \begin{bmatrix} 175.5 \\ 260.9 \end{bmatrix}$
b. 127 Year 11 jackets and 233 Year 12 jackets

10.6 Review: exam practice

1. A **2.** C **3.** B **4.** D **5.** A
6. A **7.** D **8.** C **9.** C

10.
a. $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ **b.** $\begin{bmatrix} -5 \\ -4 \end{bmatrix}$ **c.** $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ **d.** $\begin{bmatrix} 3 & 4.5 \\ 1.5 & 6 \end{bmatrix}$

11. a. 3 **b.** 1×2

12. a. i. 1×2 and 2×1; product matrix: 1×1
ii. 1×2 and 2×2; product matrix: 1×2
iii. 2×2 and 2×2; product matrix: 2×2
iv. 3×1 and 1×3; product matrix: 3×3

b. i. $[36]$ **ii.** $\begin{bmatrix} 32 & 54 \end{bmatrix}$

iii. $\begin{bmatrix} 42 & 101 \\ 55 & 35 \end{bmatrix}$ **iv.** $\begin{bmatrix} 10 & 6 & 8 \\ 35 & 21 & 28 \\ 40 & 24 & 32 \end{bmatrix}$

13. C

14. a. $\begin{bmatrix} 12.5 \\ 6 \\ 10 \end{bmatrix}$ **b.** $4000

15. a. Matrix G is of order 3×2 and matrix H is of order 2×1; therefore, GH is of order 3×1. Rhonda's matrix has an order of 3×2.

b. $\begin{bmatrix} 125 \\ 134 \\ 167 \end{bmatrix}$

c. Possible answer:
Rhonda multiplied the first column with the first row, and then the second column with the second row.

d. Possible answer:
Step 1: Find the order of the product matrix.
Step 2: Multiply the elements in the first row by the elements in the first column.

16. a. $\begin{bmatrix} 12\,280 \\ 26\,210 \\ 29\,850 \end{bmatrix}$; Friday $12 280, Saturday $26 210, Sunday $29 850

b. $350 \times 35 + 456 \times 25 + 128 \times 20$

c. No, because you cannot multiply the entry price (3×1) by the matrix number of people (3×3).

17. a. 8 slices of bread

b. 4 sandwiches

c. i. There are 4 sandwiches, a total of 44.0 g of fat means:
$5.5 \times x = 44.0$
$\therefore x = 8$
So 2 slices per sandwich

ii. 12.8

d. $\begin{bmatrix} 2 & 2 & 1 \end{bmatrix}$

18. a. $\begin{bmatrix} 3890 \\ 2178 \\ 868 \end{bmatrix}$, 3×1

b. $\begin{bmatrix} 62 & 125 & 270 \end{bmatrix}$

c. 1×1

d. i. 0.95 **ii.** $\begin{bmatrix} 3695.50 \\ 2069.10 \\ 824.60 \end{bmatrix}$

19. a. $\begin{bmatrix} 10 & 9 \\ 8 & 12 \\ 9 & 11 \end{bmatrix}$

b. $\begin{bmatrix} 45.95 & 25.50 & 18.60 \end{bmatrix}$

c. $\begin{bmatrix} 45.95 & 25.50 & 18.60 \end{bmatrix}\begin{bmatrix} 10 & 9 \\ 8 & 12 \\ 9 & 11 \end{bmatrix}$

d. LeisureLand: $830.90, SportLand: $924.15

20. a. 2×1

b. $\begin{bmatrix} 0.1 & 0.25 & 0.38 & 0.19 & 0.08 \end{bmatrix}$

c. $\begin{bmatrix} 47 & 118 & 176 & 90 & 38 \\ 42 & 106 & 161 & 81 & 34 \end{bmatrix}$

d. 81 students studying Economics will receive Distinction grades.

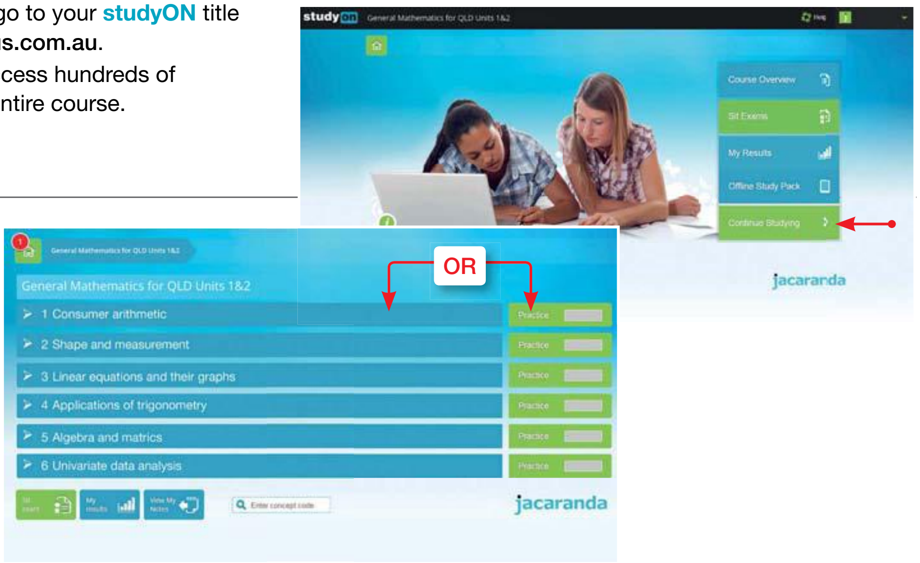
e. $C = PN$, where $N = \begin{bmatrix} 472 \\ 424 \end{bmatrix}$ and $P = \begin{bmatrix} 125 & 235 \end{bmatrix}$

f. $158 640

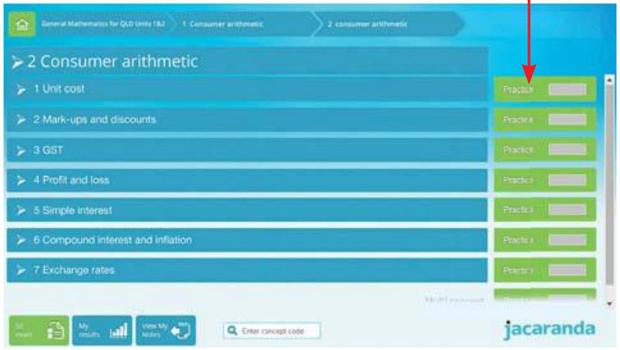
TOPIC 2 Algebra and matrices

- For revision of this entire topic, go to your **studyON** title in your bookshelf at **www.jacplus.com.au**.
- Select **Continue Studying** to access hundreds of revision questions across your entire course.

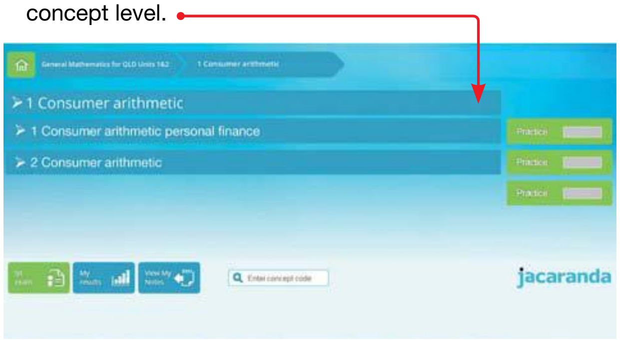
- Select your **course** *General Mathematics for Queensland Units 1&2* to see the entire course divided into syllabus topics.
- Select the **area** you are studying to navigate into the sequence level *OR* select **Practice** to answer all practice questions available for each area.



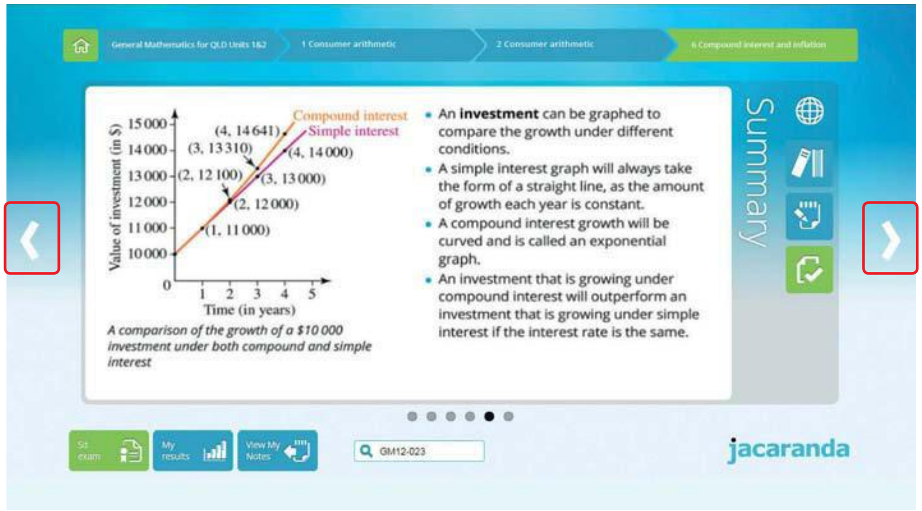
- Select **Practice** at the sequence level to access all questions in the sequence.



- At **sequence level**, drill down to concept level.



- **Summary screens** provide revision and consolidation of key concepts. Select the **next arrow** to revise all concepts in the sequence and practice questions at the concept level for a more granular set of questions.



A comparison of the growth of a $10 000 investment under both compound and simple interest

- An **investment** can be graphed to compare the growth under different conditions.
- A simple interest graph will always take the form of a straight line, as the amount of growth each year is constant.
- A compound interest growth will be curved and is called an exponential graph.
- An investment that is growing under compound interest will outperform an investment that is growing under simple interest if the interest rate is the same.

CHAPTER 11
Univariate data analysis

11.1 Overview

11.1.1 Introduction

A boxplot is a method of graphically representing groups of numerical data through their quartiles, where the spacings between the different sections of the box indicate the degree of spread and skewness in the data. Boxplots are also used to determine and show outliers. The boxplot gives a snapshot of a number of values, such as interquartile range, maximum value, minimum value, range and median of the data.

The boxplot was introduced by the mathematician John Tukey, who is regarded as one of the most influential statisticians of the past 50 years. Some of his work in modern statistics led to concepts that played a central role in the creation of today's telecommunication technology. He is credited with the invention of the computer term 'bit'.

John Tukey was born in New Bedford, Massachusetts in 1915. He obtained a BA and a MSc in chemistry from Brown University in 1937, before moving to Princeton University where he completed his PhD in mathematics. He became a professor at 35 and founding chairman of the Princeton statistics department in 1965. He was awarded the IEEE Medal of Honor in 1982 for his contributions to the spectral analysis of random processes and the fast Fourier transform (FFT) algorithm. He introduced the boxplot in his book *Exploratory data analysis* in 1977.

LEARNING SEQUENCE

11.1 Overview
11.2 Classifying and displaying data
11.3 Construct, describe and interpret dot plots and stem-and-leaf plots
11.4 Construct, describe and interpret column graphs and histograms
11.5 Measures of centre
11.6 Measures of spread
11.7 Review: exam practice

Fully worked solutions for this chapter are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

11.2 Classifying and displaying data

11.2.1 Data types

When analysing data it is important to know what type of data you are dealing with. This can help to determine the best way to both display and analyse the data.

Data can be split into two major groups: **categorical data** and **numerical data**. Both of these can be further divided into two subgroups.

Categorical data

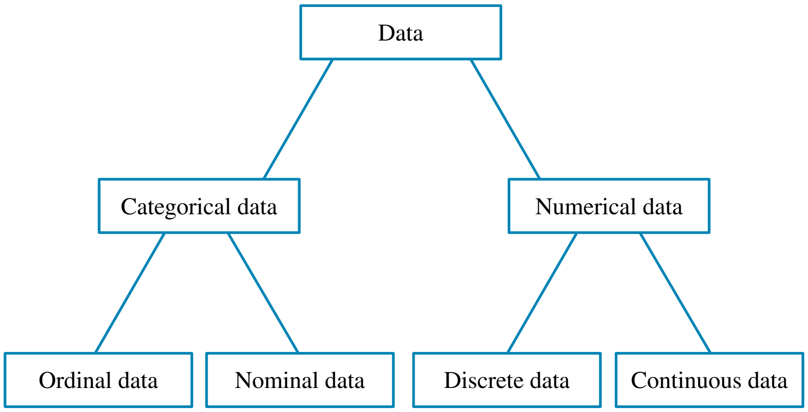
Data that can be organised into groups or categories is known as categorical data. Categorical data is often an ‘object’, ‘thing’ or ‘idea’, with examples including brand names, colours, general sizes and opinions. Categorical data can be classified as either ordinal or nominal. **Ordinal data** is placed into a natural order or ranking, whereas **nominal data** is split into subgroups with no particular order or ranking.

For example, if you were collecting data on income in terms of whether it was ‘High’, ‘Medium’ or ‘Low’, the assumed order would be to place the ‘Medium’ category between the other two, so this is ordinal data. On the other hand, if you were investigating preferred car colours the order doesn’t really matter, so this is nominal data.

Numerical data

Data that can be counted or measured is known as numerical data. Numerical data can be classified as either discrete or continuous. **Discrete data** is counted in exact values, with the values often being whole numbers, whereas **continuous data** can have an infinite number of values, with an additional value always possible between any two given values.

For example, the housing industry might consider the number of bedrooms in residences offered for sale. In this case, the data can only be a restricted group of numbers (1, 2, 3, etc.), so this is discrete data. Now consider meteorological data, such as the maximum daily temperatures over a particular time period. Temperature data could have an infinite number of decimal places (23°C, 25.6°C, 18.21°C, etc.), so this is continuous data.



WORKED EXAMPLE 1

Data on the different types of cars on display in a car yard is collected.

Verify that the collected data is categorical, and determine whether it is ordinal or nominal.

THINK	WRITE
1. Identify the type of data.	The data collected is the brand or model of cars, so this is categorical data.
2. Identify whether the order of the data is relevant.	When assessing the types of different cars, the order is not relevant, so this is nominal data.
3. State the answer.	The data collected is nominal data.

WORKED EXAMPLE 2

Data on the number of people attending matches at sporting venues is collected.

Verify that the collected data is numerical, and determine whether it is discrete or continuous.

THINK	WRITE
1. Identify the type of data.	The data collected is the number of people in sporting venues, so this is numerical data.
2. Does the data have a restricted or infinite set of possible values?	The data involves counting people, so only whole number values are possible.
3. State the answer.	The data collected is discrete data.

11.2.2 Displaying categorical data

Once raw data has been collected, it is helpful to summarise the information into a table or display. Categorical data is usually displayed in either **frequency tables** or **bar charts**. Both of these display the frequency (number of times) that a piece of data occurs in the collected data.

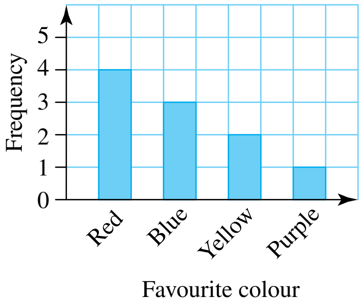
Frequency tables

Frequency tables split the collected data into defined categories and register the frequency of each category in a separate column. A tally column is often included to help count the frequency.

For example, if we have collected the following data about people's favourite colours, we could display it in a frequency table.

Red, Blue, Yellow, Red, Purple, Blue, Red, Yellow, Blue, Red

Favourite colour	Tally	Frequency
Red	||||	4
Blue	|||	3
Yellow	||	2
Purple	|	1



Bar charts

Bar charts display the categories of data on the horizontal axis and the frequency of the data on the vertical axis. As the categories are distinct, there should be a space between all of the bars in the chart.

The bar chart above displays the previous data about favourite colours.

WORKED EXAMPLE 3

The number of students from a particular school who participate in organised sport on weekends is shown in the frequency table.

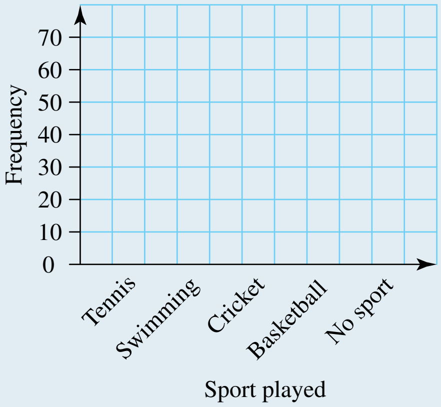
Display the data in a bar chart.

Sport	Frequency
Tennis	40
Swimming	30
Cricket	60
Basketball	50
No sport	70

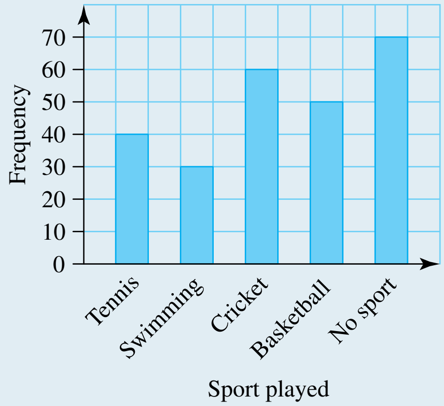
THINK

1. Choose an appropriate scale for the bar chart. As the frequencies go up to 70 and all of the values are multiples of 10, we will mark our intervals in 10s. Display the different categories along the horizontal axis.

WRITE



2. Draw bars to represent the frequency of each category, making sure there are spaces between the bars.



The mode

For categorical data, the **mode** is the category that has the highest frequency. When displaying categorical data in a bar chart, the modal category is the highest bar.

Identifying the mode allows us to know which category is the most common or most popular, which can be particularly useful when analysing data.

In some instances there may be either no modal category or more than one modal category. If the data has no modal category then there is no mode, if it has 2 modal categories then it is bimodal, and if it has 3 modal categories it is trimodal.

WORKED EXAMPLE 4

Thirty students were asked to pick their favourite time of the day between the following categories: Morning (M), Early afternoon (A), Late afternoon (L), Evening (E)

The following data was collected:

A, E, L, E, M, L, E, A, E, M, E, L, E, A, L, M, E, E, L, M, E, A, E, M, L, L, E, E, A, E.

a. **Represent the data in a frequency table.**

b. **Construct a bar chart to represent the data.**

c. **Determine which time of day is the most popular.**

THINK

a. 1. Create a frequency table to capture the data.

WRITE

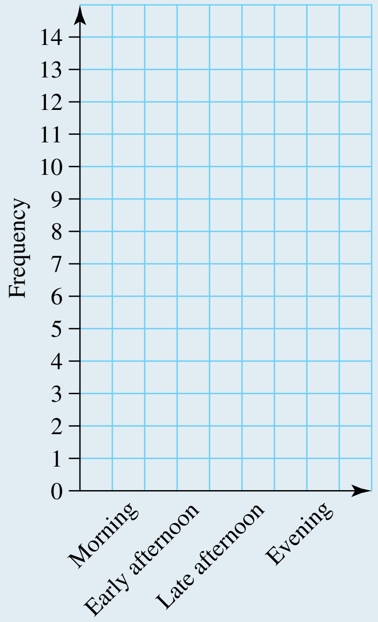
a.

Time of day	Tally	Frequency
Morning		
Early afternoon		
Late afternoon		
Evening		

2. Go through the data, filling in the tally column as you progress. Sum the tally columns to complete the frequency column.

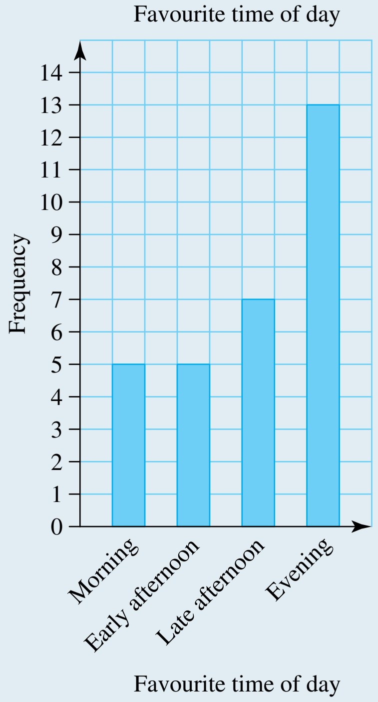
Time of day	Tally	Frequency
Morning	𝍸	5
Early afternoon	𝍸	5
Late afternoon	𝍸 ||	7
Evening	𝍸 𝍸 |||	13

b. 1. Choose an appropriate scale for the bar chart. As the frequencies only go up to 13, we will mark our intervals in single digits. Display the different categories along the horizontal axis.



2. Draw bars to represent the frequency of each category, making sure there are spaces between the bars.

b.



c. 1. The highest bar is the modal category. This is the most popular category. Write the answer.

c. Evening is the most popular time of day among the students.

on Resources

Interactivity: Create a bar chart (int-6493)

studyon

Units 1 & 2 Area 6 Sequence 1 Concepts 1 & 2

Classifying data types Summary screen and practice questions

Categorial data Summary screen and practice questions

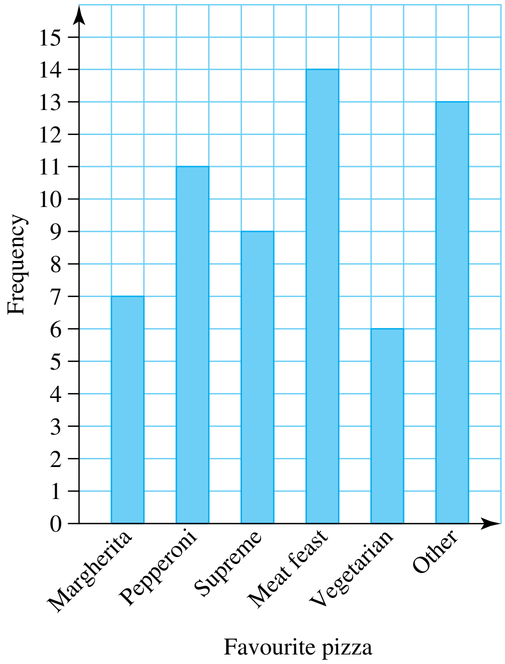
Exercise 11.2 Classifying and displaying data

1. **WE1** Data on the different types of cereal on supermarket shelves is collected.
   Verify that the collected data is categorical, and determine whether it is ordinal or nominal.
2. Data on the rating of hotels from ‘one star’ to ‘five star’ is collected.
   Verify that the collected data is categorical, and determine whether it is ordinal or nominal.
3. **WE2** Identify whether the following is categorical or numerical data and whether any numerical data is discrete or continuous.
   a. The amount of daily rainfall in Geelong
   b. The heights of players in the National Basketball League
   c. The number of children in families
   d. The type of pet owned by families
4. Identify whether the following categorical or numerical data is nominal, ordinal, discrete or continuous.
   a. The times taken for the place getters in the Olympic 100 m sprinting final
   b. The number of gold medals won by countries competing at the Olympic Games
   c. The type of medals won by a country at the Olympic Games
   d. The countries that won at least one gold medal in any Olympics Games
5. **WE3** The preferred movie genre of 100 students is shown in the following frequency table.

Favourite movie genre	Frequency
Action	32
Comedy	19
Romance	13
Drama	15
Horror	7
Musical	4
Animation	10

Construct a bar chart to represent the data.

6. The favourite pizza type of 60 students is shown in the following bar chart.



Construct a frequency table to represent the data.

7. A group of students at a university were surveyed about their usual method of travel, with the results shown in the following table.

Student	Transport method	Student	Transport method
A	Bus	N	Car
B	Walk	O	Bus
C	Train	P	Car
D	Bus	Q	Bus
E	Car	R	Bicycle
F	Bus	S	Car
G	Walk	T	Train
H	Bicycle	U	Bus
I	Bus	V	Walk
J	Car	W	Car
K	Car	X	Train
L	Train	Y	Bus
M	Bicycle	Z	Bus

a. What type of data is being collected?
b. Organise the data into a frequency table.
c. Construct a bar chart to represent the data.

8. In a telephone survey people were asked the question, ‘Do you agree that convicted criminals should be required to serve their full sentence and not receive early parole?’ They were required to respond with either ‘Yes’, ‘No’ or ‘Don’t care’ and the results are as follows.

Person	Opinion	Person	Opinion
A	Yes	N	Yes
B	Yes	O	No
C	Yes	P	No
D	Yes	Q	Yes
E	Don’t care	R	Yes
F	No	S	Yes
G	Don’t care	T	Yes
H	Yes	U	No
I	No	V	Yes
J	No	W	Yes
K	Yes	X	Don’t care
L	No	Y	Yes
M	Yes	Z	Yes

a. Organise the data into an appropriate table.
b. Contruct a bar graph to represent the data.
c. Identify the data as either nominal or ordinal. Explain your answer.

9. Complete the following table by indicating the type of data.

Data	Type	
Example: The types of meat displayed in a butcher shop.	Categorical	Nominal
a. Wines rated as high, medium or low quality		
b. The number of downloads from a website		
c. Electricity usage over a three-month period		
d. The volume of petrol sold by a petrol station per day		

10. **WE4** Twenty-five students were asked to pick their favourite type of animal to keep as a pet. The following data was collected.
Dog, Cat, Cat, Rabbit, Dog, Guinea pig, Dog, Cat, Cat, Rat, Rabbit, Ferret, Dog, Guinea pig, Cat, Rabbit, Rat, Dog, Dog, Rabbit, Cat, Cat, Guinea pig, Cat, Dog

a. Construct a frequency table to represent the data.
b. Construct a bar chart to represent the data.
c. Which animal is the most popular?

11. Thirty students were asked to pick their favourite type of music from the following categories: Pop (P), Rock (R), Classical (C), Folk (F), Electronic (E).

The following data was collected:

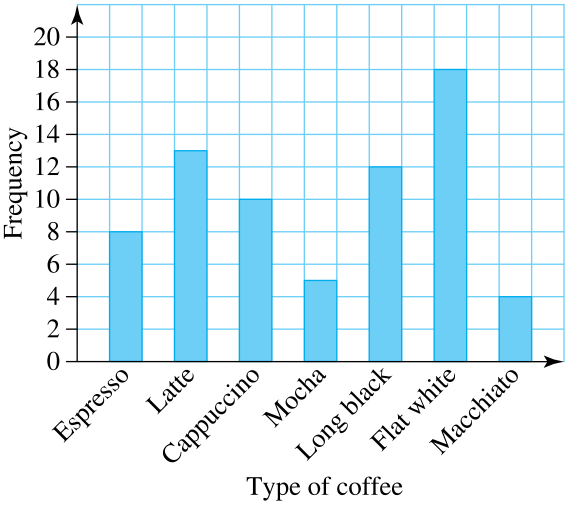
E, R, R, P, P, E, F, E, E, P, R, C, E, P, E, P, C, R, P, F, E, P, P, E, R, R, E, F, P, R

a. Construct a frequency table to represent the data.

b. Construct a bar chart to represent the data.

c. Which type of music is the most popular?

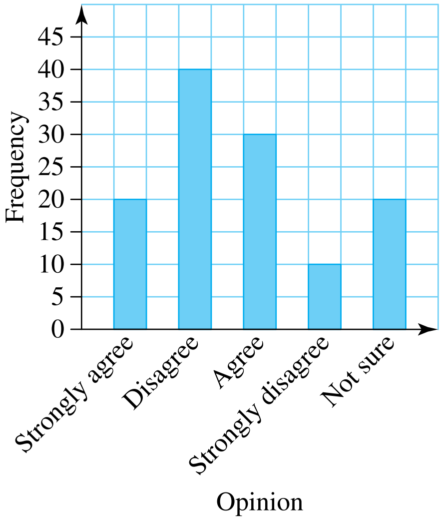
12. The different types of coffee sold at a café in one hour are displayed in the following bar chart.



a. Determine the modal category of the coffees sold.

b. How many coffees were sold in that hour?

13. The results of an opinion survey are displayed in the following bar chart.



a. What type of data is being displayed?

b. Explain what is wrong with the current data display.

c. Redraw the bar chart displaying the data correctly.

14. Exam results for a group of students are shown in the following table.

Student	Result	Student	Result	Student	Result	Student	Result
1	A	6	C	11	B	16	C
2	B	7	C	12	C	17	A
3	D	8	C	13	C	18	C
4	E	9	E	14	C	19	D
5	A	10	D	15	D	20	E

a. Construct a frequency table to represent the exam result data.
b. Construct a bar chart to represent the data.
c. What is the type of data collected?

15. The number of properties sold in the capital cities of Australia for a particular time period is shown in the following table.

	Number of bedrooms			
City	**2**	**3**	**4**	**5**
Adelaide	8	12	5	4
Brisbane	15	11	8	6
Canberra	8	12	9	2
Hobart	3	9	5	1
Melbourne	16	18	12	11
Sydney	23	19	15	9
Perth	7	9	12	3

Use the given information to construct a bar graph that represents the number of bedrooms of properties sold in the capital cities during this time period.

16. The maximum daily temperatures (°C) in Adelaide during a 15-day period in February are listed in the following table.

Day	1	2	3	4	5	6	7	8
Temp(°C)	31	32	40	42	32	34	41	29

Day	9	10	11	12	13	14	15
Temp(°C)	25	33	34	24	22	24	30

Temperatures greater than or equal to 39°C are considered above average and those less than 25°C are considered below average.

a. Organise the data into three categories and display the results in a frequency table.
b. Construct a bar graph to display the data.
c. What is the type of data displayed in your bar graph?

17. The following frequency table displays the different categories of purchases in a shopping basket.

Category	Frequency
Fruit	6
Vegetables	8
Frozen goods	5
Packaged goods	11
Toiletries	3
Other	7

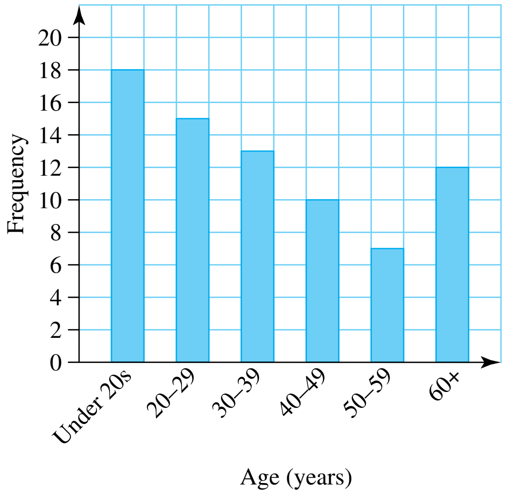
a. Determine how many items were purchased in total.
b. Calculate what percentage of the total purchases were fruit.

18. The birthplaces of 200 Australian citizens were recorded and are shown in the following frequency table.

Birthplace	Frequency
Australia	128
United Kingdom	14
India	10
China	9
Ireland	6
Other	33

a. What type of data is being collected?
b. Construct a bar chart to represent this information.
c. Determine what percentage of the respondents were born in Australia.

19. The following bar chart represents the ages of attendees at a local sporting event.



a. Construct a frequency table to represent the data.
b. Determine the modal category.
c. The age groups are changed to Under '20', '20–39', '40–59' and '60+'. Redraw the bar chart with these new categories.
d. Does this change the modal category?

20. Data for the main area of education and study for a selected group of people aged 15 to 64 during a particular year in Australia is shown in the following table.

Number of people (thousands)					
Main area of education and study	**15–19**	**20–24**	**25–34**	**35–44**	**45–64**
Agriculture	10	9	14	5	5
Creative arts	36	51	20	10	9
Engineering	59	75	50	13	6
Health	44	76	64	32	32
Management and commerce	71	155	135	86	65

a. Construct separate bar charts for each area of education and study to represent the data.
b. Construct separate bar charts for each age group to represent the data.

11.3 Construct, describe and interpret dot plots and stem-and-leaf plots

11.3.1 Stem plots

Stem plots (or stem-and-leaf plots) can be used to display both discrete and continuous numerical data. The data is grouped according to its numerical place value (the 'stem') and then displayed horizontally as a single digit (the 'leaf'). In an unordered stem plot, the data values have been placed into categories but do not appear in order. In an ordered stem plot, the values are placed in numerical order with the smallest values closest to the stem. When answering questions relating to stem plots, present your final answer as an ordered stem plot.

If there are 4 or fewer different place values in your data, it may be preferable to make the stems of the plot represent a class set of 5 instead of a class set of 10. This can be done by inserting an asterisk (∗) after the second of the stems with the same number, as shown in the following example.

Key: 1 |4 = 14
1*|7 = 17

Stem	Leaf
1	4
1*	7 7 9
2	2 3 4 4
2*	6 8
3	0 1 3
3*	5 6 9
4	0

Note that the data has been presented in neat vertical columns, making it easy to read.

Always remember to include a key with your stem plot to indicate what the stem and the leaf represents when put together.

Back-to-back stem plots

As we will see later in this chapter back-to-back stem plots can be used to compare two different sets of data. Back-to-back stem plots share the same stem, with one data set appearing on the left of the stem and the other data set appearing on the right.

Key: 1 |4 = 14
1*|7 = 17

Leaf	Stem	Leaf
3 2	1	4
9 8 6 6	1*	7 7 9
4 2 1 0	2	2 3 4 4
7 6	2*	6 8
4 2 1	3	0 1 3
6 5	3*	5 6 9
	4	0

WORKED EXAMPLE 5

The following data set (of 31 values) shows the maximum daily temperature during the month of January in a particular area.
26, 22, 24, 26, 28, 28, 27, 42, 25, 25, 29, 31, 23, 33, 34, 27, 39, 44, 35, 34, 27, 30, 36, 30, 30, 28, 33, 23, 24, 34, 37
Construct a stem plot to represent the data.

THINK

WRITE

1. Identify the place values for the data. If there are 4 or less different place values, split each into two.

The temperature data has values in the 20s, 30s and 40s.

Stem	Leaf
2	
2*	
3	
3*	
4	

2. Write the units for each stem place value in numerical order, with the smallest values closest to the stem. Make sure to keep consecutive numbers level as they move away from the stem. Remember to add a key to your plot.

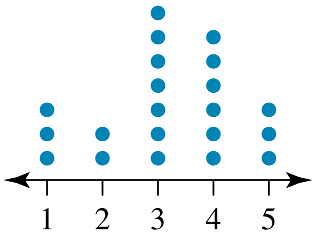
Key: 2 |2 = 22°C

Stem	Leaf
2	2 3 3 4 4
2*	5 5 6 6 7 7 7 8 8 8 9
3	0 0 0 1 3 3 4 4 4
3*	5 6 7 9
4	2 4

11.3.2 Dot plots

Discrete numerical data can also be displayed as a **dot plot**. In these plots every data value is represented by a dot. The most common values can then be clearly identified. You can also use dot plots to represent categorical data.

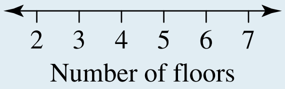
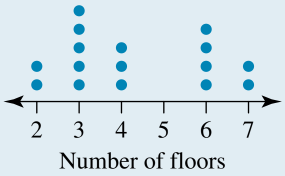
When drawing a dot plot, be careful to make sure that the dots are evenly and consistently spaced.



WORKED EXAMPLE 6

The frequency table shows the number of floors in apartment buildings in a particular area. Construct a dot plot to represent the data.

Number of floors	Frequency
2	2
3	5
4	3
5	0
6	4
7	2

THINK	WRITE
1. Draw a horizontal scale using the discrete data values shown.	The discrete data values are given by the number of floors. 
2. Place one dot directly above the number on the scale for each discrete data value present, making sure to keep corresponding dots at the same level.	

on Resources

- **Interactivity:** Stem plots (int-6242)
- **Interactivity:** Create stem plots (int-6495)
- **Interactivity:** Dot plots, frequency tables and histograms, and bar charts (int-6243)

studyon

Units 1 & 2 > Area 6 > Sequence 1 > Concept 3

Stem-and-leaf plots and dot plots Summary screen and practice questions

Exercise 11.3 Construct, describe and interpret dot plots and stem-and-leaf plots

1. **WE5** Construct a stem plot for the following data set.
The dollars spent per day on lunch by a group of 15 people:
22, 21, 22, 24, 19, 22, 24, 21, 22, 23, 25, 26, 22, 23, 22

2. Construct a stem plot for the following data set. The number of hours spent per week playing computer games by a group of 20 students at a particular school:
14, 21, 25, 7, 25, 20, 21, 14, 21, 20, 6, 23, 26, 23, 17, 13, 9, 24, 17, 24

3. Construct a stem plot for the following data set, the number of passengers per day transported by a taxi driver (40 values):
33, 27, 44, 47, 23, 24, 22, 35, 42, 36, 17, 25, 34,
13, 15, 27, 28, 23, 37, 34, 22, 27, 23, 20, 12, 15,
43, 30, 27, 15, 27, 36, 20, 23, 35, 36, 28, 17, 14, 15

4. Construct a stem plot for the number of patients per day treated by a doctor (40 values):
44, 38, 55, 56, 23, 34, 31, 43, 51, 45, 26, 25, 45, 23,
24, 38, 37, 32, 46, 41, 21, 28, 20, 34, 30, 22, 25, 51, 60, 17, 23, 24,
26, 30, 33, 41, 26, 35, 17, 24, 25

5. **WE6** Construct a dot plot to represent each of the following collections of data.
   a. The number of wickets per game taken by a bowler in a cricket season.

Number of wickets	Frequency
0	4
1	6
2	4
3	2
4	1
5	1

   b. The number of hours per week spent checking emails by a group of workers at a particular company.

Hours checking emails	Frequency
1	1
2	1
3	2
4	4
5	8
6	4

6. Construct dot plots to represent the following collections of data.

a. The scores per round of a golfer over a particular time period (40 values):

73, 77, 74, 77, 73, 74, 72, 75, 72, 76, 77, 75, 74, 73, 75, 77, 78, 73, 77, 74,
72, 77, 73, 70, 72, 75, 73, 70, 77, 75, 77, 76, 70, 73, 75, 76, 78, 77, 74, 75

b. The scores out of 10 in a multiple choice test for a group of students (30 values):

6, 7, 4, 7, 3, 7, 7, 5, 7, 6, 7, 5, 1, 3, 5, 7, 8, 3, 7, 4, 9, 5, 4, 6, 7, 9, 10, 5, 7, 4

7. The data below give the time taken (in minutes) for each of 40 runners on a 10 km fun run. Prepare a stem-and-leaf diagram for the data using a class size of 10 minutes.

36 42 52 38 47 59 72 68 57 82
66 75 45 42 55 38 42 46 48 39
42 58 40 41 47 53 68 43 39 48
71 42 50 46 40 52 37 54 48 52

8. The typing speed (in words per minute) of 30 word processors is recorded below. Prepare a stem-and-leaf diagram of the data using a class size of 5.

96 102 92 96 95 102 95 115 110 108
88 86 107 111 107 108 103 121 107 96
124 95 98 102 108 112 120 99 121 130

9. Twenty transistors are tested by applying increasing voltage until they are destroyed. The maximum voltage that each could withstand is recorded below. Prepare a stem-and-leaf plot of the data using a class size of 0.5.

14.8 15.2 13.8 14.0 14.8 15.7 15.5 15.6 14.7 14.3
14.6 15.2 15.9 15.1 14.3 14.6 13.9 14.7 14.5 14.2

Questions **10** and **11** refer to the following. Each student in a class has been assigned a newly planted tree to look after, and must provide a weekly report on its growth and condition. From the latest reports, the teacher recorded the height of each tree (in mm), and entered these in the stem-and-leaf plot shown below.

Key: 12|1 = 1210 mm
12*|5 = 1250 mm

Stem	Leaf
12	1 2 4
12*	5 7 7 9 9
13	0 1 1 2 3 4 4
13*	5 6 6 7 9 9
14	0 2 3 4
14*	6 7

10. **MC** The class size used in the stem-and-leaf plot is:

A. 1 **B.** 10 **C.** 33 **D.** 50

11. **MC** The number of scores that have been recorded is:

A. 21 **B.** 27 **C.** 33 **D.** 1210

12. The following set of data indicates the number of people who attend early morning fitness classes run by a business for its workers:

14, 17, 13, 8, 16, 21, 25, 16, 19, 17, 21, 8, 13

Display the data as a stem plot.

13. The total number of games played by the players from two basketball squads is shown in the following stem plots.

Key: 0 |1 = 1 game played

Stem	Leaf
0	1
1	4 7
2	4 4 8
3	3 3 5 6
4	1 2 3
5	1 1
6	5
7	
8	
9	1

Key: 2 |4 = 24 games played

Stem	Leaf
2	4
3	1 2 6
4	3 4 5
5	2
6	
7	
8	2 5 7
9	3

a. Describe the shape of each distribution.

b. Construct a stem plot that combines the data for the two teams.

14. Consider the set of data in the stem plot shown.

a. Instead of grouping the data in 10s, the stems could be split in half to use groups of 5. For example, a split stem plot for the same data set could place the data values from 10 to 14 in a row labelled ′1′, while data values from 15 to 19 are put in a row labelled ′1∗′. Use the data from the original stem plot to complete the split stem plot.

b. Comment on the effect of splitting the stem for the data in this question.

Key: 0|1 = 1

Stem	Leaf
0	1
1	1 1 1 4 4 6 6 7 8
2	3 3 4 4 7 7 9

11.4 Construct, describe and interpret column graphs and histograms

11.4.1 Grouped data

Numerical data may be represented as either grouped data or ungrouped data. When assessing ungrouped data, the analysis we do is exact; however, if we have a large data set, the data can be difficult to work with. Grouping data allows us to gain a clearer picture of the data's distribution, and the resultant data is usually easier to work with.

When grouping data, we try to pick class sizes so that between 5 and 10 classes are formed. Ensure that all of the classes are distinct and that there are no overlaps between classes.

When creating a frequency table to represent grouped continuous data, we will represent our class intervals in the form 12 –< 14. This interval covers all values from 12 up to 14 but does not include 14.

WORKED EXAMPLE 7

The following data represents the time (in seconds) it takes for each individual in a group of 20 students to run 100 m.

18.2, 20.1, 15.6, 13.5, 16.7, 15.9, 19.3, 22.5, 18.4, 15.9, 12.4, 14.1, 17.7, 19.4, 21.0, 20.4, 18.2, 15.8, 16.1, 14.6

Group and display the data in a frequency table.

THINK

1. Identify the smallest and largest values in the data set. This will help you to choose your class size and decide what the first class should be.

WRITE

Smallest value = 12.4
Largest value = 22.5
We will have class intervals of 2, starting with 12–<14.

THINK

2. Draw a frequency table to represent the data. Complete the tally column in your table, and use this to fill in the frequency column.

WRITE

Time (seconds)	Tally	Frequency
12–<14	||	2
14–<16	𝍸 |	6
16–<18	|||	3
18–<20	𝍸	5
20–<22	|||	3
22–<24	|	1

11.4.2 Displaying numerical distributions

The types of display we chose to represent numerical data depend on whether that data is discrete or continuous. Representations of discrete data should imply that irrelevant values are impossible, so we usually insert gaps between the data values. On the other hand, continuous data displays often have no gaps between whole numbers, as all possible values between the listed values are possible.

A **column graph** (or **bar graph**) is used when we wish to show a quantity. Categories are written on the horizontal axis and frequencies on the vertical axis.

WORKED EXAMPLE 8

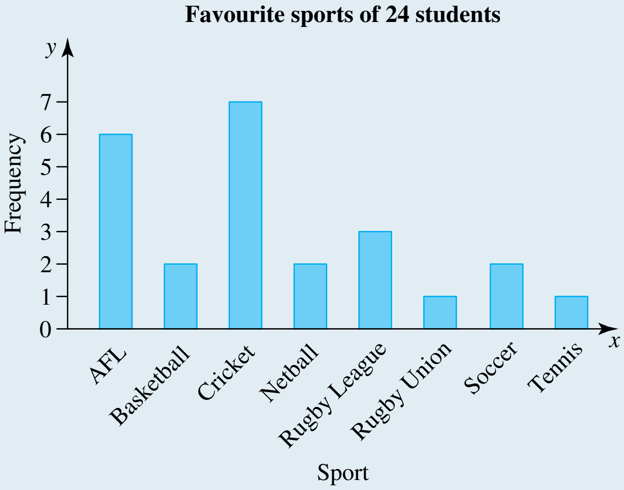
The table below shows the results of the survey on favourite sports.
Construct a column graph to represent this information.

Sport	Frequency
AFL	6
Basketball	2
Cricket	7
Netball	2
Rugby League	3
Rugby Union	1
Soccer	2
Tennis	1

THINK

1. Draw the horizontal axis showing each sport.
2. Draw a vertical axis to show frequencies up to 7.
3. Draw the columns all the same width with gaps between.
4. Use a ruler.
5. Label the axes.
6. Give the graph a title.

WRITE



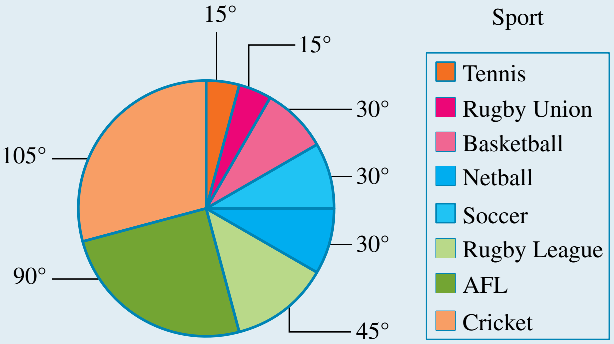
Sector graphs

A **sector graph** (circle graph, or pie graph) is used when we want the graph to display a comparison of quantities. An angle is drawn at the centre of the circle that is the same fraction of 360° as the fraction of people making each response.

WORKED EXAMPLE 9

The table below shows the results of a survey on favourite sports. Draw a sector graph of the results.

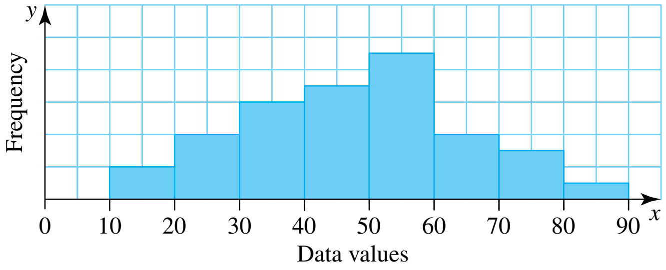
Sport	Frequency
AFL	6
Basketball	2
Cricket	7
Netball	2
Rugby League	3
Rugby Union	1
Soccer	2
Tennis	1

THINK	WRITE
1. Calculate each angle as a fraction of 360° by dividing the frequency of each sport by the total frequency and multiplying by 360°.	$\text{AFL} = \frac{6}{24} \times 360° = 90°$ $\text{Basketball} = \frac{2}{24} \times 360° = 30°$ $\text{Cricket} = \frac{7}{24} \times 360° = 105°$ $\text{Netball} = \frac{2}{24} \times 360° = 30°$ $\text{Rugby League} = \frac{3}{24} \times 360° = 45°$ $\text{Rugby Union} = \frac{1}{24} \times 360° = 15°$ $\text{Soccer} = \frac{2}{24} \times 360° = 30°$ $\text{Tennis} = \frac{1}{24} \times 360° = 15°$
2. Construct the graph, labelling each sector or providing a legend.	

Histogram

We can represent continuous numerical data using a **histogram**, which is very similar to a bar chart with a few essential differences.

In a histogram, the width of each column represents a range of data values, while the height represents their frequencies. For example, in the following histogram the first column represents the frequency of data values that are greater than or equal to 10 but less than 20 ($10 \le x < 20$).



WORKED EXAMPLE 10

The following frequency table represents the heights of players in a basketball squad.

Height (cm)	175–<180	180–<185	185–<190	190–<195	195–<200	200–<205
Frequency	1	3	6	3	1	1

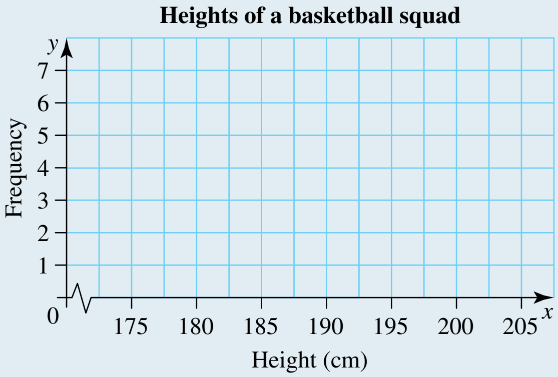
Construct a histogram to represent this data.

THINK

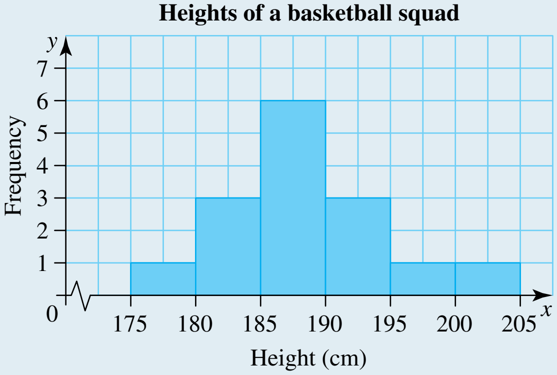
1. Look at the data range and use the leading values from each interval in the table for the scale of the horizontal axis.

WRITE

The height data in the table has intervals starting from 175 cm and increasing by 5 cm.



2. Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



Choosing which plot to use

Grouped data should be represented by a histogram, boxplot or dot plot. On the other hand, we should usually represent ungrouped data by using stem plots.

We should not use a stem plot to represent our data if the range of values in the data set is large, or if the data values have a high number of units in them (ignoring decimal places), as these stem plots can become unwieldy and difficult to use.

on Resources

Interactivity: Create a histogram (int-6494)

studyon

Units 1 & 2 › Area 6 › Sequence 1 › Concept 4

Column graphs and histograms Summary screen and practice questions

Exercise 11.4 Construct, describe and interpret column graphs and histograms

1. **WE7** The following data represents the time (in seconds) it takes for each individual in a group of 20 students to swim 50 m.

48.5, 54.1, 63.0, 39.7, 51.3, 57.7, 68.4, 59.4, 37.5, 41.8,

72.3, 56.3, 45.4, 39.2, 60.3, 56.6, 48.1, 42.9, 53.3, 64.1

Group and display the data in a frequency table.

2. The following data set indicates the time, in seconds, it takes for a tram to travel between two stops on 20 weekday mornings.

95, 112, 99, 91, 105, 110, 97, 122, 108, 101, 95, 89, 100, 115, 124, 98, 87, 111, 115, 106

a. Group and display the data in a frequency table with intervals of width 10 seconds.

b. Group and display the data in a frequency table with intervals of width 5 seconds.

3. The data below show the number of customers that entered a shop each day in a certain month.

114, 195, 175, 163, 180, 120, 204, 199, 178, 216, 200, 147, 168, 173, 102, 150,
169, 185, 173, 164, 130, 199, 158, 163, 141, 155, 132, 143, 190, 179, 200

Display the data in a frequency table of 5 groups.

4. Construct a column graph to display the data from question 3.

5. The marks scored on a Maths exam, out of 100, by 25 Year 11 students are shown below.

87, 44, 95, 66, 78, 69, 66, 92, 78, 54, 60, 66, 69, 66, 77, 79, 66, 71, 71, 83,
74, 81, 69, 70, 57

Copy and complete the table.

Mark	Tally	Frequency
40–49		
50–59		
60–69		
70–79		
80–89		
90–99		

6. Construct a column graph to display the data from question 5.

7. Construct a sector graph to compare the number of people in each category from question 5.

8. A class of students was asked to identify the make of car their family owned. Their responses are shown below.

Holden	Ford	Nissan	Mazda	Toyota	Holden
Ford	Holden	Ford	Mitsubishi	Toyota	Toyota
Nissan	Holden	Holden	Ford	Toyota	Mazda
Mazda	Toyota	Ford	Holden	Holden	Ford
Mitsubishi	Toyota	Holden	Ford	Ford	Toyota

Construct a frequency table to display these results.

9. **WE8** Construct a column graph to display the data from question 8.

10. **WE9** Construct a sector graph to display the data from question 8.

11. **WE10** The following frequency table represents the cholesterol levels measured for a group of people. Construct a histogram to represent this data.

Cholesterol level (*mmol* / *L*)	1−<2.5	2.5−<4.0	4.0−<5.5	5.5−<7.0	7.0−<8.5
Frequency	2	8	12	14	10

12. The following frequency table represents the distances travelled to school by a group of students. Construct a histogram to represent this data.

Distance travelled (*km*)	0−<2	2−<4	4−<6	6−<8	8−<10
Frequency	18	26	14	8	2

13. Organise each of the following data sets into a frequency table using intervals of five, commencing from the lowest value. Then draw a histogram to represent the data.

5, 7, 14, 17, 13, 24, 22, 15, 12, 26, 17, 15, 14, 13, 15, 7, 8, 13, 17, 24,
22, 7, 13, 20, 12, 15, 23, 20, 17, 15, 17, 16, 20, 23, 15, 16, 8, 17, 14, 15

14. Organise each of the following data sets into a frequency table using intervals of five, commencing from the lowest value. Then draw a histogram to represent the data.

34, 28, 45, 46, 13, 24, 11, 33, 41, 35, 16, 15, 35, 13, 14, 28, 27, 22, 36, 31,
11, 18, 24, 20, 12, 15, 41, 50, 27, 13, 14, 16, 20, 23, 31, 26, 25, 27, 34, 35

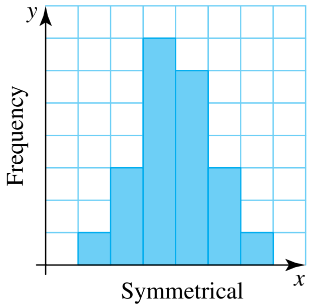
11.5 Measures of centre

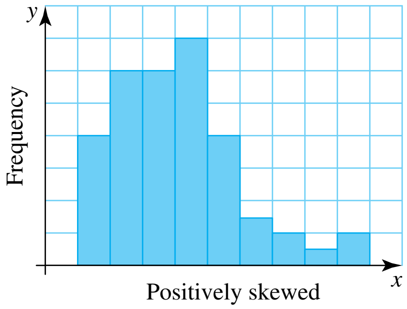
11.5.1 Describing distributions

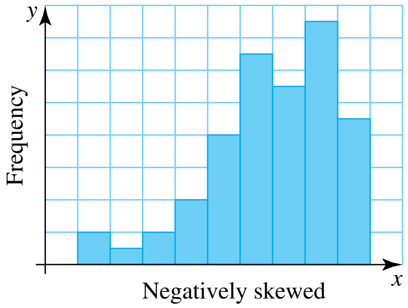
The distribution of a set of data can be described in terms of a number of key features, including shape, modality, spread and outliers.

Shape

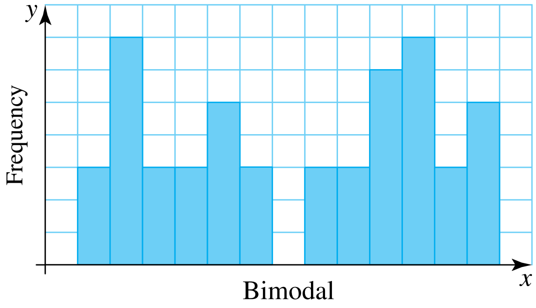
The shape of a numerical distribution is an important indicator of some of the key measures for further analysis and is one of the most important reasons for displaying the data in a graphical form. Shape will generally be described in terms of symmetry or skew. Symmetrical data distributions have higher frequencies around their centres with a relatively evenly balanced spread to either side, while skewed distributions have the majority of their values towards one end. Distributions with higher frequencies on the left side of the graph are positively skewed, while those with higher frequencies on the right side are negatively skewed.







Modality

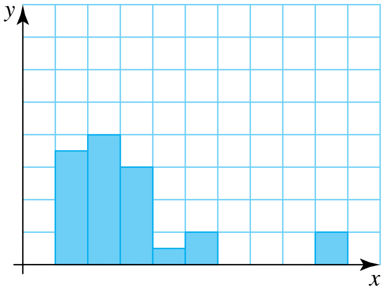


The mode of a distribution is the data value or class interval that has the highest frequency. This will be the column or row on the display that is the longest. When there is more than one mode, the data distribution is multimodal. This can indicate that there may be subgroups within the distribution that may require further investigation. Bimodal distributions can occur when there are two distinct groups present, such as in data values that typically have clear differences between male and female measurements.

Spread

An awareness of how widely spread the data is can be an important consideration when conducting any further analysis. Common indicators of spread include the measures of **range** and the **standard deviation**. The graph will again point to which measures might be most appropriate to use.

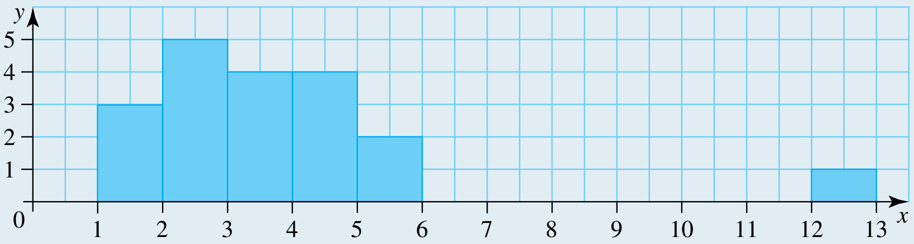
Outliers



An **outlier** is a data value that is an anomaly when compared to the majority of the sample. Sometimes outliers are just unusual readings or measurements, but they can also be the result of errors when recording the data. Outliers can have a significant effect on some of the measures that are used for further data analysis, and they are sometimes removed from the sample for those calculations. The graphical display of the data can alert us to the presence of potential outliers.

WORKED EXAMPLE 11

Describe the distribution of the data shown in the following histogram.



THINK	WRITE
1. Look for the mode and comment on its value.	The distribution has one mode with data values most frequently in the $2 \le x < 3$ interval.
2. Identify the presence of any potential outliers.	There is one potential outlier in the interval between 12 and 13.
3. Describe the shape in terms of symmetry or skewness.	If we include the outlier, the data set can be described as positively skewed as it is clustered to the left. If we don't include the outlier, the distribution can be considered to be approximately symmetrical.

11.5.2 Measures of centre

In many practical settings it is common to use a single measurement to represent an entire set of data. For example, discussions about fuel costs will often focus on the average price of petrol, while in real estate the median house price is considered an important measurement. These representative values are known as measures of centre as they are located in the central region of the data. The **mean**, **median** and mode are all measures of centre, and the most appropriate one to use depends on various characteristics of the data set.

11.5.3 The mean (or arithmetic mean)

The mean of a data set is what we commonly refer to as the average. It is calculated by dividing the sum of the data values by the number of data values. If the data set is a sample of the population, the symbol used for the mean is $\bar{x}$ (pronounced 'x-bar'), whereas if the data set is the whole population, we use the Greek letter μ (pronounced 'mu').

$$\begin{aligned}\bar{x} &= \frac{\text{sum of data values}}{\text{number of data values}}\\ &= \frac{\Sigma x_i}{n}\\ (\text{or } \mu &= \frac{\Sigma x_i}{n})\end{aligned}$$

The Greek letter Σ(sigma) indicates calculating the sum of these values.

WORKED EXAMPLE 12

Calculate the mean of the following data set, correct to 2 decimal places.
6, 3, 4, 5, 7, 7, 4, 8, 5, 10, 6, 10, 9, 8, 3, 6, 5, 4

THINK	WRITE
1. Calculate the sum of the data values.	$6+3+4+5+7+7+4+8+5+10+6+10+9+8+3+6+5+4 = 110$
2. Divide the sum by the number of data values.	$\bar{x} = \frac{110}{18}$ $= 6.111...$
3. State the answer.	The mean of the data set is 6.11.

Calculating the mean for grouped data

To calculate the mean from a table of data that has been organised into groups, we first need to calculate the midpoints of the intervals. We then multiply the values of the midpoints by the corresponding frequencies, and find the sum of these values. Finally, we divide this sum by the total of the frequencies.

If f = the values of the frequencies and x = the values of the midpoints, then $\bar{x} = \dfrac{\Sigma xf}{\Sigma f}$.

WORKED EXAMPLE 13

Calculate the mean of the data set displayed in the following frequency table.

Intervals	Frequency
0−<5	3
5−<10	12
10−<15	3
15−<20	2

THINK

1. Add a column to the table and enter the midpoints for the corresponding intervals.

WRITE

Intervals	Frequency	Midpoint (x)
0−<5	3	2.5
5−<10	12	7.5
10−<15	3	12.5
15−<20	2	17.5

2. Add a column to the table and enter the product of the frequencies and midpoints (xf) for the corresponding intervals.

Intervals	Frequency	Midpoint (x)	xf
0−<5	3	2.5	7.5
5−<10	12	7.5	90
10−<15	3	12.5	37.5
15−<20	2	17.5	35

3. Calculate the totals of the f and xf columns.

Intervals	Frequency	Midpoint (x)	xf
0– < 5	3	2.5	7.5
5– < 10	12	7.5	90
10– < 15	3	12.5	37.5
15– < 20	2	17.5	35
	$f = 20$		$fx = 170$

4. Calculate the mean using the formula $\bar{x} = \dfrac{\sum xf}{\sum f}$.

$$\bar{x} = \frac{\sum xf}{\sum f}$$
$$= \frac{170}{20}$$
$$= 8.5$$

5. State the answer.

The mean of the distribution is 8.5.

11.5.4 The median

When considering a value that truly indicates the centre of a distribution, it would make sense to look at the number that is actually in the middle of the data set. The median of a distribution is the middle value of the ordered data set if there are an odd number of values. If there are an even number of values, the median is halfway between the two middle values. It can be found using the rule:

$$\textbf{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \textbf{ data value}$$

WORKED EXAMPLE 14

Calculate the median of the following data sets.

a. 5, 3, 4, 5, 7, 7, 4, 8, 5, 10, 6, 10, 9, 8, 3, 6, 5, 4

b. 16, 3, 4, 5, 17, 27, 14, 18, 15, 10, 6, 10, 9, 8, 23, 26, 35

THINK	WRITE
a. 1. Put the data set in order from lowest to highest.	**a.** 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 7, 7, 8, 8, 9, 10, 10
2. Identify the data value in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.	There are 18 data values, so the median will be in position $\left(\frac{18+1}{2}\right) = 9.5$, or halfway between position 9 and position 10.

$$3, 3, 4, 4, 4, 5, 5, 5, \boxed{5, 6,}\ 6, 7, 7, 8, 8, 9, 10, 10$$
$$\downarrow$$
$$\text{median} = 5.5$$

3. State the answer.	The median of the data set is 5.5.
b. 1. Put the data set in order from lowest to highest.	**b.** 3, 4, 5, 6, 8, 9, 10, 10, 14, 15, 16, 17, 18, 23, 26, 27, 35
2. Identify the data value in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.	There are 17 data values, so the median will be in position $\left(\frac{17+1}{2}\right) = 9$. $\downarrow$ 3, 4, 5, 6, 8, 9, 10, 10, $\boxed{14,}$ 15, 16, 17, 18, 23, 26, 27, 35 $\downarrow$ median = 14
3. State the answer.	The median of the data set is 14.

11.5.5 Limitations of measures of centre

The mean

An important property of the mean is that it includes all the data in its calculation. As such, it has genuine credibility as a representative value for the distribution. On the other hand, this property also makes it susceptible to being adversely affected by the presence of extreme values when compared to the majority of the distribution.

Consider the data set: 3, 4, 5, 6, 7, 8, 9, which would have a mean of

$$\frac{3+4+5+6+7+8+9}{7} = 6.$$

Compare this to a data set with the same values with the exception of the largest one: 3, 4, 5, 6, 7, 8, 90, which has a mean of

$$\frac{3+4+5+6+7+8+90}{7} = 17.6.$$

As we can see, the mean has been significantly influenced by the one extreme value.

When the data is skewed or contains extreme values, the mean becomes less reliable as a measure of centre.

The median

In the previous example we saw that the mean is affected by an extreme value in the data set; however, the same cannot be said for the median. In both data sets the median will be the value in the fourth position.

3, 4, 5, $\boxed{6,}$ 7, 8, 9

3, 4, 5, $\boxed{6,}$ 7, 8, 90

median

The median is therefore more reliable than the mean when the data is skewed or contains extreme values. Another potential advantage of median is that it is often one of the data values, whereas the mean often isn't. However, the median can be considered unrepresentative as it is not calculated by taking into account the actual values in the data set.

Choosing between measures of centre

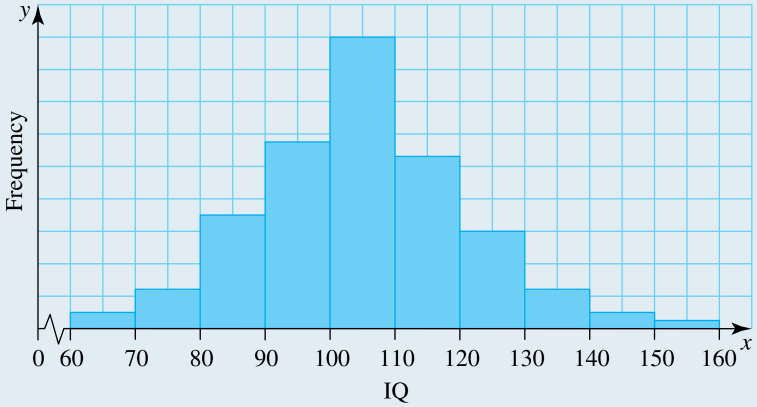
In most situations it is preferable to give both the median and the mean as a measure of centre, as between them they portray a more accurate picture of the data set. However, sometimes it is only possible to give one of these values to represent our data set.

When choosing which measure of centre to use to represent a data set, take into account the distribution of the data. If the data has no outliers and is approximately symmetrical, then the mean is probably the better measure of centre to represent the data. If there are outliers, the median will be significantly less affected by these and would be a better choice to represent the data. The median is also a good choice to represent skewed data.

Also consider what each measure of centre tells you about the data. The values of the mean and median can vary significantly, so choosing which one to represent the data set can be important, and you will need to justify your choice.

WORKED EXAMPLE 15

The following histogram represents the IQ test results for a group of people.



Determine which measure of centre is best to represent the data set.

THINK	WRITE
1. Look at the distribution of the data set.	The data set is approximately symmetrical and has no outliers.
2. If the data set is approximately symmetrical with no outliers, the mean is probably the better measure of centre to represent the data set. If there are outliers or the data is skewed, the median is probably the best measure of centre to use. State the answer.	The mean is the better measure of centre to represent this data set.

on Resources

Interactivity: Mean, median, mode and quartiles (Int-6496)

study on

Units 1 & 2 › Area 6 › Sequence 1 › Concepts 5 & 6

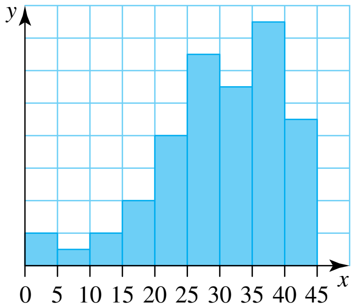
Describing distributions Summary screen and practice question

Measures of centre Summary screen and practice question

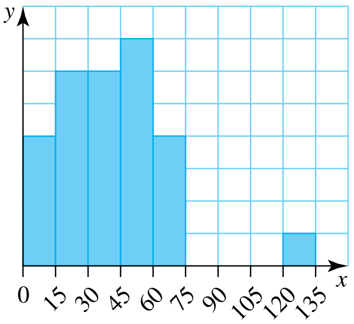
Exercise 11.5 Measures of centre

1. **WE11** Describe the numerical distributions shown by the following histograms.

a.



b.



2. Describe the distribution of the following data sets after drawing histograms with intervals of 10 commencing with the smallest values.
   - **a.** 105, 70, 140, 127, 132, 124, 122, 125, 123, 126, 107, 105, 104, 113, 125, 70, 88, 103, 107, 124, 122, 76, 103, 120, 112, 115, 123, 120, 117, 115, 107, 106, 120, 123, 115, 74, 128, 119
   - **b.** 4, 18, 35, 26, 12, 25, 21, 34, 43, 37, 6, 25, 25, 23, 34, 38, 37, 22, 36, 31, 21, 28, 34, 30, 32, 25, 31, 40, 37, 33, 24, 26, 10, 13, 21, 36, 35, 37, 24, 25

3. A group of 26 students received the following marks on a test:
   6, 4, 3, 8, 6, 9, 5, 6, 9, 7, 7, 8, 5, 7, 4, 3, 8, 6, 5, 7, 9, 5, 6, 6, 7, 8
   - **a.** Construct a dot plot to display the data.
   - **b.** Describe the distribution.

4. **WE12** Calculate the mean of the following data set.

   108, 135, 120, 132, 113, 138, 125, 138, 107, 131, 113, 136, 119, 152, 134, 158, 136, 132, 113, 128

5. **a.** Calculate the mean of the following data set correct to 2 decimal places.
   25, 23, 24, 25, 27, 26, 23, 28, 24, 20, 25, 20, 29, 28, 23, 27, 24
   - **b.** Replace the highest value in the data set from part **a** with the number 79, and then calculate the mean again, correct to 2 decimal places.
   - **c.** How did changing the highest value in the data set affect the mean?

6. **WE13** Calculate the means of the data sets displayed in the following tables, giving your answer correct to 2 decimal places.

a.

Intervals	Frequency
0– < 5	12
5– < 10	10
10– < 15	1
15– < 20	4

b.

Intervals	Frequency
20– < 35	12
35– < 50	6
50– < 65	13
65– < 80	4

7. For each of the following sets of data, estimate the mean by creating a table using intervals that commence with the lowest data value and increase by an amount that is equal to the difference between the highest and lowest data value divided by 5. Give your answers correct to 2 decimal places.

a. 205, 203, 204, 205, 207, 216, 213, 218, 214, 220, 225, 220, 229, 228, 233, 238, 234

b. 5, 13, 24, 5, 27, 16, 13, 18, 24, 10, 5, 20, 30, 18, 13, 7, 14

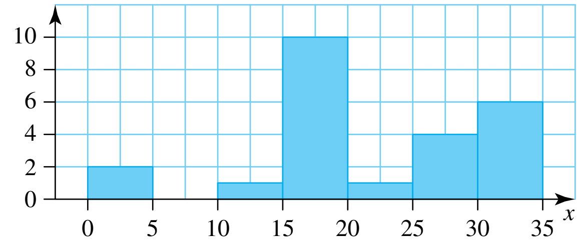
8. Calculate the means of the following data sets correct to 2 decimal places.

a.

Key: 1|2 = 12

Stem	Leaf
0	1 1 5 7
1	2 6
2	3 4 4 5
3	1 3
4	0 0 3
5	5
6	5

b.



9. **WE14** Calculate the medians of the following data sets.

a. 15, 3, 54, 53, 27, 72, 41, 85, 15, 11, 62, 16, 49, 81, 53, 56, 75, 42

b. 126, 301, 422, 567, 179, 267, 149, 198, 165, 170, 602, 180, 109, 85, 223, 206, 335

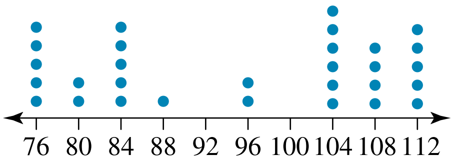
10. **a.** Calculate the median of the following data set.
21, 22, 23, 24, 27, 26, 22, 27, 23, 21, 24, 20, 31, 25, 24, 28, 23

b. Replace the highest value in the data set from part **a** with the number 96 and then calculate the median again.

c. How does changing the highest value in the data set affect the median?

11. Calculate the median of the following data sets.

a.



b. 1.02, 2.01, 3.21, 4.63, 1.49, 3.45, 1.17, 1.38, 1.47, 1.70, 5.02, 1.38, 1.91, 8.54

12. **WE15** The following stem plot represents the lifespan of different animals at an animal sanctuary. Determine which measure of centre is better to represent the data set.

Key: 1|2 = 12

Stem	Leaf
0	3 5 9
1	2 4 6 8
2	0 1 4 5 5 7 9
3	0 2 6
4	
5	
6	0 3

13. The following data set represents the salaries (in \$ 000s) of workers at a small business.
45, 50, 55, 55, 55, 60, 65, 65, 70, 70, 75, 80, 220

a. Calculate the mean of the salaries correct to 2 decimal places.

b. Calculate the median of the salaries.

c. When it comes to negotiating salaries, the workers want to use the mean to represent the data and the management want to use the median. Explain why this might be the case.

14. a. Calculate the mean (correct to 2 decimal places) and median for the following data set.

Average annual rainfall in selected Australian cities	
City	**Rainfall (*mm*)**
Sydney	1276
Melbourne	654
Brisbane	1194
Adelaide	563
Perth	745
Hobart	576
Darwin	1847
Canberra	630
Alice Springs	326

b. Which would be the most appropriate measure of centre to represent this data?

15. On a particular weekend, properties are sold at auction for the following 30 prices:
\$4 700 000, \$3 160 000, \$2 725 000, \$2 616 000, \$2 560 000, \$241 000, \$265 000, \$266 000, \$310 000, \$320 000, \$3 010 000, \$2 580 000, \$2 450 000, \$2 300 000, \$2 275 000, \$286 000, \$325 000, \$330 000, \$435 500, \$456 000, \$1 350 000, \$1 020 000, \$900 000, \$735 000, \$733 000, \$305 000, \$330 000, \$347 000, \$357 000, \$408 000

a. Calculate the mean and median for the data.

b. Construct a histogram of the data using intervals commencing at the lowest value and increasing by amounts of \$250 000.

c. Mark in the location of the mean and median on the histogram.

d. Which would be the more appropriate measure of centre to represent this data?

16. The heights in metres of fruit trees in an orchard were measured with the following results:
1.83, 1.94, 1.98, 1.91, 1.88, 1.76, 2.12, 2.05, 2.11, 2.01, 2.04, 2.08, 2.07, 2.06, 2.05, 2.03, 1.94, 1.96, 2.12, 2.14, 2.04, 2.01, 2.03, 2.06, 2.02, 1.94, 1.98, 2.25, 2.04, 2.06

a. Use intervals of 0.05 m starting with 1.75–<1.80 to display the data in a frequency table.

b. Construct a histogram to display the data and use it to comment on the appropriateness of using either the mean or the median to represent the data.

c. Use the frequency table to calculate the mean.

17. The winning margins in the NRL over a particular period of time were as follows.

Winning margin	Frequency
2	4
4	12
6	8
8	5
10	4
12	4
16	1
20	1
34	1

a. Calculate the mean and the median.
b. Which is the more appropriate measure of centre for this data set and why?

18. The value of the Australian dollar in US cents over a particular period of time was as follows:

93, 91, 88, 94, 86, 90, 93, 95, 84, 81, 91, 96, 99, 101, 106, 104, 104, 99, 99, 96, 94, 95, 91, 90, 89, 88, 89, 86, 88, 87, 83, 88, 84, 85, 86, 86, 87, 88, 87, 84

a. Calculate the mean and median of the raw data.
b. Construct a histogram to display the data, using intervals commencing at 80 – < 85.
c. Mark in the positions of the mean and median on the histogram.
d. Comment on the positions of the mean and median.

19. The annual earnings of a group of professional tennis players are as follows:

\$5 700 000, \$1 125 000, \$620 000, \$4 950 000, \$275 000, \$220 000, \$242 000, \$350 000, \$375 000, \$300 000, \$422 000, \$2 150 000, \$270 000, \$420 000, \$300 000, \$245 000, \$385 000, \$284 000, \$320 000, \$444 000, \$185 000, \$200 500, \$264 000, \$290 000

a. Calculate the mean and median of the raw data. Give your answers correct to the nearest dollar.
b. Construct a histogram to display the data, using intervals commencing with \$180 000– < \$380 000.
c. Mark in the positions of the mean and median on the histogram.
d. Which is the more appropriate measure of centre for this data. Justify your response.

20. The body mass index (BMI) is an accepted measure of obesity with a value of 30 or more being the obese category. The BMI results for a group of people are shown in the table.

22.5	31.4	28.4	18.5	33.2	26.3
27.1	28.6	31.2	21.2	19.8	20.4
20.7	26.4	29.4	27.1	31.6	21.4
34.1	32.1	26.3	21.4	27.3	23.2
28.3	21.4	26.1	26.3	28.4	29.1
22.8	23.7	20.4	28.1	30.4	22.4
18.1	22.5	24.3	25.2	24.7	30.2

a. Display the data in two frequency tables and draw the corresponding histograms.
  i. In the first frequency table, use intervals commencing at the lowest value and increasing by an amount that is calculated by dividing the difference between the lowest and highest data value by 5.
  ii. In the second frequency table, use intervals commencing at the lowest value and increasing by an amount that is calculated by dividing the difference between the lowest and highest data value by 10.

b. Describe the two histograms.

c. Calculate the mean for each frequency table and compare them to the mean of the raw data. Give your answers correct to 2 decimal places.

d. Which measure of centre is the better representation of this data? Justify your response.

11.6 Measures of spread

11.6.1 Measures of centre and measures of spread

While measures of centre such as the mean or median give valuable information about a set of data, taken in isolation they can be quite misleading. Take for example the data sets $\{36, 43, 44, 59, 68\}$ and $\{1, 2, 44, 80, 123\}$. Both groups have a mean of 50 and a median of 44, but the values in the second set are much further apart from each other. Measures of centre tell us nothing about how variable the data values in a set might be; for this we need to consider the measures of spread of the data.

11.6.2 Range and quartiles

Range

In simplest terms the spread of a data set can be determined by looking at the difference between the smallest and largest values. This is called the **range** of the distribution. While the range is a useful calculation, it can also be limited. Any extreme values (outliers) will result in the range giving a false indication of the spread of the data.

$$\textbf{Range} = \textbf{largest value} - \textbf{smallest value}$$

Quartiles

A clearer picture of the spread of data can be obtained by looking at smaller sections. A common way to do this is to divide the data into quarters, known as quartiles.

The **lower quartile** (Q_1) is the value that indicates the median of the lower half of the data.

The second quartile (Q_2) is the median of the distribution of data.

The **upper quartile** (Q_3) is the value that indicates the median of the upper half of the data.

When calculating the values of the lower and upper quartiles, the median should not be included. If the median is between values, then these values should be considered in your calculations.

The interquartile range

The **interquartile range** is found by calculating the difference between the third and first quartiles ($Q_3 - Q_1$), which gives an indication of the spread of the middle 50% of the data.

WORKED EXAMPLE 16

Calculate the interquartile range of the following set of data.

23, 34, 67, 17, 34, 56, 19, 22, 24, 56, 56, 34, 23, 78, 22, 16, 15, 35, 45

THINK	WRITE
1. Put the data in order.	15, 16, 17, 19, 22, 22, 23, 23, 24, 34, 34, 34, 35, 45, 56, 56, 56, 67, 78
2. Identify the median.	There are 19 data values, so the median will be in position $\left(\frac{19+1}{2}\right) = 10.$ $15, 16, 17, 19, 22, 22, 23, 23, 24, \overset{\substack{\text{median}\\ \downarrow}}{\boxed{34}}$ The median is 34.
3. Identify Q_1 by finding the median of the lower half of the data.	There are 9 values in the lower half of the data, so Q_1 will be the 5th of these values. $15, 16, 17, 19, \overset{\substack{Q_1\\ \downarrow}}{\boxed{22,}} 22, 23, 23, 24$ $Q_1 = 22$
4. Identify Q_3 by finding the median of the upper half of the data.	There are 9 values in the upper half of the data, so Q_3 will be the 5th of these values. $34, 34, 35, 45, \overset{\substack{Q_3\\ \downarrow}}{\boxed{56,}} 56, 56, 67, 78$ $Q_3 = 56$
5. Calculate the interquartile range using IQR = $(Q_3 - Q_1)$.	IQR = $(Q_3 - Q_1)$ = 56–22 = 34
6. State the answer.	The interquartile range is 34.

11.6.3 Spread around the mean

When the mean is used as a representative value for data, it makes sense to take note of how much the data varies in comparison to the mean. Two indicators of the spread of data around the mean are the **variance** and the **standard deviation**. These measures generally only apply to continuous numerical data. The larger the variance and standard deviation are, the more spread out the data is away from the mean.

Variance

The variance is calculated by finding the difference between each data value and the mean. To adjust for the fact that values below the mean will result in a negative number, the results are then squared. These values are then averaged to give a single number. The variance is calculated using the following formula:

$$\textbf{Sample variance: } s^2 = \frac{\sum f(x-\bar{x})^2}{(\sum f)-1}$$

Standard deviation

The standard deviation is calculated by taking the square root of the variance.

$$\textbf{Sample standard deviation: } s = \sqrt{\frac{\sum f(x-\bar{x})^2}{(\sum f)-1}}$$
$$= \sqrt{\frac{\sum (x_i-\bar{x})^2}{n-1}}$$

This reverses the previous mathematical process of squaring the differences between the data values and the mean, so that the standard deviation reverts to a comparative unit of measurement for the original data.

The following example shows that the variance and standard deviation can become very messy to calculate once you have large groups of data. Spreadsheets, calculators and similar technologies are a more practical and reliable option for these computations.

The table shows a grouped distribution of a sample of data with a mean of 6.5.

Intervals	**Frequency (f)**	**Midpoint (x)**	xf
0– < 5	2	2.5	5
5– < 10	8	7.5	60
			$\sum xf = 65$

The second last column in the lower table shows the square of the difference between the midpoint and the mean, and the last column shows this value multiplied by the frequency for the interval.

Intervals	**Frequency**	**Midpoint (x)**	xf	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0– < 5	2	2.5	5	$(2.5-6.5)^2 = 16$	32
5– < 10	8	7.5	60	$(7.5-6.5)^2 = 1$	8
	$\sum f = 10$		$\sum xf = 65$		$\sum f(x-\bar{x})^2 = 40$

The sum of the final column can then be used with the sum of the frequency column in the formulas to calculate the variance and standard deviation of the sample.

$$\text{Sample variance: } s^2 = \frac{\sum f(x-\bar{x})^2}{(\sum f)-1}$$
$$= \frac{40}{9}$$
$$\approx 4.44$$

$$\text{Sample standard deviation: } s = \sqrt{4.44}$$
$$\approx 2.11$$

WORKED EXAMPLE 17

Calculate the variance and standard deviation for the sample from the information shown in the table. Give your answers correct to 2 decimal places.

Intervals	Frequency (f)	Midpoint (x)	xf	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
10– < 15	8	12.5	100	$(12.5–15.5)^2 = 9$	72
15– < 20	12	17.5	210	$(17.5–15.5)^2 = 4$	48
	$\sum f = 20$		$\sum xf = 310$		

THINK	WRITE
1. Sum the $f\left(x-\bar{x}\right)^2$ column.	$\sum f\left(x-\bar{x}\right)^2 = 72 + 48$ $= 120$
2. Substitute the values into the formulas for variance and standard deviation.	$s^2 = \frac{120}{19}$ ≈ 6.32 $s = \sqrt{6.32}$ ≈ 2.51
3. State the answer.	The variance of the sample is 6.32 and the standard deviation of the sample is 2.51.

11.6.4 Preferred measures of spread

The standard deviation is generally considered the preferred measure of the spread of a distribution when there are no outliers and no skew, as all of the data contributes to its calculation. When there are outliers or the data is skewed, the interquartile range is a better option as it is not adversely influenced by extreme values.

As the interquartile range is calculated on the basis of just two numbers that may or may not be actual values from the data set, it could be considered to be unrepresentative of the data set.

on Resources

Interactivity: The median, the interquartile range, the range and the mode (int-6244)

Interactivity: The mean and the standard deviation (int-6246)

studyon

Units 1 & 2 > Area 6 > Sequence 1 > Concept 7

Measures of spread Summary screen and practice question

Exercise 11.6 Measures of spread

1. **WE16** Calculate the interquartile range of the following set of data.
421, 331, 127, 105, 309, 512, 129, 232, 124, 154, 246, 124, 313, 218, 112, 136, 155, 305, 415
2. Calculate the interquartile range of the following set of data.
3.11, 3.16, 1.13, 1.56, 3.19, 4.43, 1.98, 4.89, 2.12, 4.78, 3.21, 8.88, 1.21, 5.67, 2.22, 3.34

3. The results for a multiple choice test for 20 students in two different classes are as follows.
Class A: 7, 13, 14, 13, 14, 14, 12, 8, 18, 13, 14, 12, 16, 14, 12, 11, 13, 14, 13, 15
Class B: 18, 19, 12, 12, 11, 17, 9, 18, 17, 14, 13, 11, 17, 13, 17, 14, 14, 15, 13, 12
   a. Compare the spread of the marks for each class by using the range.
   b. Compare the spread of the marks for each class by using the interquartile range.
4. The competition ladder of the Australian and New Zealand netball championship is as follows.

Position	Team	Win	Loss	Goals for	Goals against
1	Adelaide Thunderbirds	12	1	688	620
2	Melbourne Vixens	9	4	692	589
3	Waikato BOP Magic	9	4	749	650
4	Queensland Firebirds	9	4	793	691
5	Central Pulse	8	5	736	706
6	Southern Steel	6	7	812	790
7	West Coast Fever	5	8	715	757
8	NSW Swifts	4	9	652	672
9	Canterbury Tactix	2	11	700	882
10	Northern Mystics	1	12	699	879

   a. Calculate the spread for the 'Goals for' column by using the range.
   b. Calculate the spread for the 'Goals for' column by using the interquartile range.
   c. Compare the spread of the 'Goals for' column with the spread of the 'Goals against' column.
5. **WE17** Calculate the variance and standard deviation for the sample from the information shown in the table. Give your answers correct to 2 decimal places.

Intervals	Frequency	Midpoint (x)	xf	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0– < 10	14	5	70	$(5-8.3)^2 = 10.89$	152.46
10– < 20	7	15	105	$(15-8.3)^2 = 44.89$	314.23
	$\sum f = 21$		$\sum xf = 175$		

6. Complete the table and calculate the variance and standard deviation for the following sample correct to 3 decimal places.

Intervals	Frequency	Midpoint (x)	xf	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0– < 50	35				
50– < 100	125				
	$\sum f =$		$\sum xf =$		$\sum f(x-\bar{x})^2 =$

7. Complete the details of the following table, which shows the results of a survey of the ages of a sample of workers in the hospitality industry.

Age group (years)	Frequency (f)	Midpoint (x)	xf
15– < 20	14		
20– < 25	18		
25– < 30	11		
30– < 35	7		
35– < 40	5		
	$\sum f =$		$\sum xf =$

8. A survey of the number of motor vehicles that pass a school between 8.30 am and 9.30 am on 10 days during a term are as follows.
72, 89, 94, 78, 83, 84, 88, 97, 82, 88
   a. Calculate the standard deviation of the sample correct to 2 decimal places.
   b. Calculate the interquartile range of the sample.
   c. The lowest number is reduced by 10 and the highest value increased by 10. Recalculate the values of the standard deviation and interquartile range.
   d. How is each of the measures affected by the change in the values?

9. A survey of a large sample of people from particular areas of employment found the following average Australian salary ranges.

Employment area	Average minimum	Average maximum
Mining	\$65 795	\$262 733
Management	\$66 701	\$240 000
Engineering	\$56 572	\$233 451
Legal	\$53 794	\$193 235
Building and construction	\$46 795	\$186 412
Telecommunications	\$47 354	\$193 735
Science	\$47 978	\$211 823
Medical	\$42 868	\$228 806
Sales	\$42 917	\$176 783

   a. Calculate the interquartile range for the average minimum salaries.
   b. Calculate the interquartile range for the average maximum salaries.
   c. Comment on the two interquartile ranges.

10. A sample of crime statistics over a two-year period are shown in the following table.

Crime	Year 1	Year 2
Theft from motor vehicle	46 700	42 900
Theft (steal from shop)	19 800	20 600
Theft of motor vehicle	15 650	14 670
Theft of bicycle	4 200	4 660
Theft (other)	50 965	50 650

a. Calculate the interquartile range and standard deviation (correct to 1 decimal place) for both years.
b. Recalculate the interquartile range and standard deviation for both years after removing the smallest category.
c. Comment on the effect of removing the smallest category on the interquartile ranges and standard deviations.

11. The table shows the number of registered passenger vehicles in two particular years for the states and territories of Australia.

Number of passenger vehicles		
	Year 1	Year 2
New South Wales	3 395 905	3 877 515
Victoria	2 997 856	3 446 548
Queensland	2 138 364	2 556 581
South Australia	915 059	1 016 590
Western Australia	1 205 266	1 476 743
Tasmania	271 365	305 913
Northern Territory	73 302	91 071
Australian Capital Territory	191 763	229 060

a. Calculate the interquartile range and standard deviation (correct to 1 decimal place) for both years.
b. Recalculate the interquartile range and standard deviation for both years after removing the three smallest values.
c. Comment on the effect of the removal of the three smallest values on the interquartile ranges and standard deviations.

12. Data collected on the number of daylight hours in Alice Springs is as shown.

10.3, 9.8, 9.6, 9.5, 8.5, 8.4, 9.1, 9.8, 10.0, 10.0, 10.1, 10.0, 10.1, 10.1, 10.6, 8.7, 8.8, 9.0, 8.0, 8.5, 10.6, 10.8, 10.5, 10.9, 8.5, 9.5, 9.3, 9.0, 9.4, 10.6, 8.3, 9.3, 9.0, 10.3, 8.4, 8.9

a. Calculate the range of the data.
b. Calculate the interquartile range of the data.
c. Comment on the difference between the two measures and what this indicates.

13. The volume of wine (′000 litres) available for consumption in Australia for a random selection of months over a 10-year time period is shown in the following table.

38 595	41 301	44 212	39 362	38 914	38 273	39 456	38 823
41 123	42 981	44 567	41 675	41 365	42 845	43 987	41 583
39 347	42 673	44 835	39 773	38 586	38 833	39 756	39 095
42 946	46 382	44 892	41 038	41 402	42 587	43 689	41 209

a. Calculate the mean and standard deviation of the data correct to 2 decimal places.

b. Calculate the median and interquartile range of the data.

c. What percentage, correct to 2 decimal places, of the actual data values from the sample are within one standard deviation of the mean (i.e. between the number obtained by subtracting the standard deviation from the mean and the number obtained by adding the standard deviation to the mean)?

d. What percentage of the actual data values from the sample are between the first and third quartiles?

e. Comment on the differences between your answers for parts c and d.

14. A random sample of the monthly consumer price indices in various cities of Australia is shown in the following table. Answer the following questions, giving answers correct to 2 decimal places where appropriate.

Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
0.4	0.8	0.9	0.7	0.6	0.3	1.2	0.8
0.9	1.0	1.1	1.0	0.8	0.8	0.3	1.0
1.4	1.3	1.3	1.5	1.4	1.3	0.9	1.4
1.5	1.2	1.7	1.3	1.6	1.0	1.5	1.2
1.1	1.2	1.4	1.3	1.0	1.1	1.8	1.5
0.1	0.3	0.2	0.2	0.1	0.2	0.1	0.3
0.4	0.3	0.5	0.5	0.9	0.5	1.1	0.6
1.1	0.5	1.4	1.1	0.8	1.2	1.9	0.9
0.5	0.6	0.3	0.4	0.5	0.6	0.1	0.4
0.8	1.3	0.7	0.5	1.2	0.7	0.5	0.6

a. Calculate the standard deviation and interquartile range of the entire data set.

b. Calculate the standard deviation and interquartile range for each city.

c. Which city bears the closest similarity to the entire data set?

d. Which city bears the least similarity to the entire data set?

15. Answer the questions on the data in the following table. Where appropriate, give answers correct to 2 decimal places.

Carbon dioxide emissions (million metric tons of carbon dioxide)						
Country	2001	2002	2003	2004	2005	2006
Australia	374.05	382.65	380.68	391.03	416.89	417.06
Canada	553.55	573.25	602.46	614.69	632.01	614.33
China	3107.99	3440.60	4061.64	4847.33	5429.30	6017.69
Germany	877.71	857.35	874.04	871.88	852.57	857.60
India	1035.42	1033.52	1048.11	1151.33	1194.01	1293.17
Indonesia	300.18	314.88	305.44	323.29	323.51	280.36
Japan	1197.15	1203.33	1253.29	1257.89	1249.62	1246.76
Russia	1571.14	1571.77	1626.86	1663.44	1698.56	1704.36
United Kingdom	575.19	563.89	575.17	582.29	584.65	585.71
United States	5762.33	5823.80	5877.73	5969.28	5994.29	5902.75

a. Calculate the interquartile range and standard deviation for the Australian data.

b. Compare the measures of spread for the Australian data with those for India, China, the United Kingdom and the United States.

c. For this data, which measure of spread is more appropriate?

16. Answer the questions on the data in the following table. Where appropriate, give your answers correct to 2 decimal places.

Alcohol consumption per adult (litres)	
Country	**Consumption per adult (litres)**
Australia	10.21
Canada	10.01
France	12.48
Germany	12.14
Greece	11.01
Indonesia	0.56
Ireland	14.92
New Zealand	9.99
Russia	16.23
South Africa	10.16
Spain	11.83
Sri Lanka	0.81
United Kingdom	13.24
United States	9.7
Yemen	0.2

a. Calculate the interquartile range and variance for the data set correct to 2 decimal places.
b. Calculate the interquartile range and variance after removing the three lowest values, correct to 2 decimal places.
c. Compare the results from parts a and b.

11.7 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. **MC** The interquartile range of the data distribution shown in the stem plot is:

Key: 2 | 6 = 26

Stem	Leaf
0	2
1	1 5
2	6 6 7 8
3	8 8 9
4	3 4
5	2

A. 41 **B.** 50 **C.** 28 **D.** 20.5

2. **MC** The mean of the data distribution shown in the table is:

Interval	Frequency (*f*)
0– < 15	5
15– < 30	7
30– < 45	6
45– < 60	2

A. 22.4 **B.** 26.25 **C.** 24.35 **D.** 25.65

3. **MC** Data gathered on the number of home runs in a baseball season would be classified as:

A. discrete **B.** nominal
C. continuous **D.** ordinal

4. **MC** For the sample data set 2, 3, 5, 2, 3, 6, 3, 8, 9, 2, 8, 9, 2, 6, 7, the mean and standard deviation respectively would be closest to:

A. 5 and 6 **B.** 5 and 2.6
C. 2.6 and 5 **D.** 5 and 2.7

5. **MC** For the following stem plot, the median and range respectively are:

Key: 5 | 1 = 51

Stem	Leaf
5	1 2
6	2 3 4
7	3 4 4 5
8	6 6
9	2

A. 73 and 41 **B.** 73.5 and 41 **C.** 71 and 39 **D.** 71 and 41

6. State whether each of the following data types is categorical or numerical.
   a. The television program that people watch at 7: 00 pm
   b. The number of pets in each household
   c. The amount of water consumed by athletes in a marathon run
   d. The average distance that students live from school
7. For each of the numerical data types below, determine if the data are discrete or continuous.
   a. The dress sizes of Year 11 girls
   b. The volume of backyard swimming pools
   c. The amount of water used in households
   d. The number of viewers of a particular television program
8. A group of Year 11 students was asked to state the number of movies that they had purchased in the last year. The results are shown below.

12, 1, 13, 20, 5, 22, 35, 12, 17, 20,
9, 5, 11, 0, 14, 25, 3, 8, 10, 9,
12, 6, 18, 7, 10, 9, 6, 23, 14, 19

   a. Put the results into a table using the categories 0– 4, 5–9, 10–14 etc.
   b. Draw a column graph to represent the results.
9. The data below give the number of errors made each week by 20 machine operators. Prepare a stem-and-leaf diagram of the data using a stem of 0, 1, 2, etc.

6, 15, 20, 25, 28, 18, 32, 43, 52, 27, 17, 26, 38, 31, 26, 29, 32, 46, 13, 20

10. The time taken (in seconds) for a test vehicle to accelerate from 0 to 100 km/h is recorded during a test of 24 trials. The results are represented by the stem-and-leaf plot below.
Calculate the median of the data.

Key: 7|2 = 7.2 s
7*|6 = 7.6 s

Stem	Leaf
7	2 4 4
7*	5 5 7 9
8	0 0 1 2 4 4 4
8*	5 5 6 8 9
9	2 2 3
9*	5 7

11. The stem-and-leaf plot below gives the exact mass of 24 packets of biscuits. Find the mean and range of the data.

Key: 248|4 = 248.4 g

Stem	Leaf
248	4 7 8
249	2 3 6 6
250	0 0 1 1 6 9 9
251	1 5 5 5 6 7
252	1 5 8
253	0

12. The frequency table below shows the crowds at football matches for a team over a season.

Class	Classcentre	Frequency
5000–9999		1
10 000–14 999		5
15 000–19 999		9
20 000–24 999		3
25 000–29 999		2
30 000–34 999		2

a. Copy the frequency table and complete the class centre column.

b. Show the information in a frequency histogram.

Complex familar

13. The price of a barrel of oil in US dollars over a particular 18-month time period is shown in the following table.

Month	Price $(US\$)$
Jan	102.96
Feb	97.63
Mar	108.76
Apr	105.25
May	106.17
Jun	83.17
Jul	83.72
Aug	88.99
Sep	95.34
Oct	92.44
Nov	87.05
Dec	88.69
Jan	93.14
Feb	97.46
Mar	90.71
Apr	97.1
May	90.74
Jun	93.41

a. Calculate the mean and median for this data set. Give your answers correct to 1 decimal place.

b. Calculate the standard deviation for this data set. Give your answer correct to 2 decimal places.

14. The table below shows the number of sales made each day over a month in a car yard.

Number of sales	Frequency
0	2
1	5
2	12
3	6
4	2
5	0
6	1

Show this information in a frequency histogram.

15. Display the following scores in a stem-and-leaf plot.

45, 21, 38, 46, 42, 41, 42, 49, 35, 29, 24, 28,
36, 21, 38, 45, 44, 40, 29, 28, 35, 35, 33, 38,
40, 41, 48, 39, 34, 38, 45, 28, 23, 29, 30, 40

16. Use the stem-and-leaf plot drawn in the previous question to find:

a. the range **b.** the median

Complex unfamiliar

17. Use the data on the incidence of communicable diseases in Australia to answer the following questions.

Incidence of communicable diseases in Australia over two consecutive years

Disease	Year 1	Year 2
Hepatitis C	11 089	7 286
Typhoid Fever	116	96
Legionellosis	302	298
Meningococcal disease	259	230
Tuberculosis	1 324	1 327
Influenza (laboratory confirmed)	59 090	13 419
Measles	104	70
Mumps	165	95
Chickenpox	1 753	1 743
Shingles	2 716	2 978
Dengue virus infection	1 406	1 201
Malaria	508	399
Ross River virus infection	4 796	5 147

a. Calculate the mean (correct to 1 decimal place) and median number of cases of communicable diseases of the sample for each year.

b. Comment on the differences between the mean and median values calculated in part **a**.

18. The number of passengers arriving from overseas during a particular time period at various airports in Australia is shown in the following table.

Airport	Number of passengers
Adelaide	5 743
Brisbane	480 625
Cairns	5 110
Coolangatta	7 655
Darwin	5 318
Melbourne	594 286
Perth	318 493

Calculate the mean and standard deviation for the sample. Give your answers correct to 1 decimal place.

19. Use the data shown to answer the following questions.

Women who gave birth and Indigenous status by states and territories, 2009

Status	NSW	Vic.	Qld	WA	SA	Tas	ACT	NT	Aust
Indigenous	2 904	838	3 332	1 738	607	284	107	1 474	11 284
Non-Indigenous	91 958	70 328	57 665	29 022	18 994	5 996	5 601	2 369	281 933

a. Display the data in an appropriate display.
b. Calculate the mean births per state/territory of Australia in 2009 for both Indigenous and Non-Indigenous groups. Give your answers correct to 1 decimal place.
c. Calculate the median births per state/territory of Australia in 2009 for both Indigenous and Non-Indigenous groups.
d. Calculate the standard deviation (correct to 1 decimal place) for the data on births per state/territory of Australia in 2009 for both Indigenous and non-Indigenous groups.

20. Use the data on Tokyo's average maximum temperatures to answer the questions.

Tokyo average maximum temperature, 1980–89 and 2003–12

Year	Temp. (°C)	Year	Temp. (°C)	Year	Temp. (°C)	Year	Temp. (°C)
1980	19.3	1985	19.4	2003	19.6	2008	20.1
1981	19.0	1986	18.8	2004	21.2	2009	20.3
1982	19.6	1987	20.0	2005	20.4	2010	20.6
1983	19.6	1988	19.0	2006	19.9	2011	20.2
1984	18.8	1989	19.9	2007	20.6	2012	20.0

a. Calculate the mean and standard deviation of the temperature data for the two 10-year periods of 1980–89 and 2003–12. Give your answers correct to 2 decimal places.
b. What do the means and standard deviations calculated indicate about the two 10-year periods?
c. Calculate the mean and standard deviation of the total 20 years of the sample data. Give your answers correct to 2 decimal places.
d. How do the measurements in part c compare to the calculations you made in part a?

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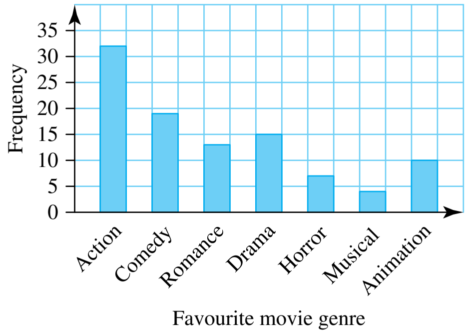
Answers

Chapter 11 Univariate data analysis

Exercise 11.2 Classifying and displaying data

1. Nominal
2. Categorical, ordinal
3. a. Numerical and continuous
   b. Numerical and continuous
   c. Numerical and discrete
   d. Categorical
4. a. Continuous
   b. Discrete
   c. Ordinal
   d. Nominal
5.



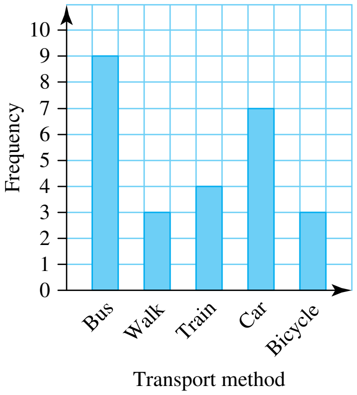
6.

Favourite pizza	Frequency
Margherita	7
Pepperoni	11
Supreme	9
Meat feast	14
Vegetarian	6
Other	13

7. a. Nominal categorical
   b.

Transport method	Frequency
Bus	9
Walk	3
Train	4
Car	7
Bicycle	3

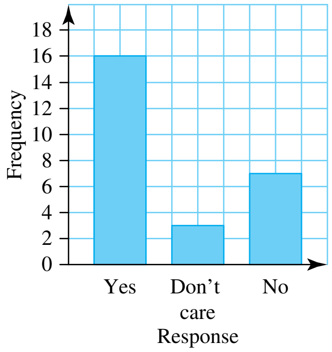
   c.



8. a.

Response	Frequency
Yes	16
Don't care	3
No	7

   b.



   c. Ordinal, as it makes sense to arrange the data in order from 'Yes' to 'No', with 'Don't care' between them.

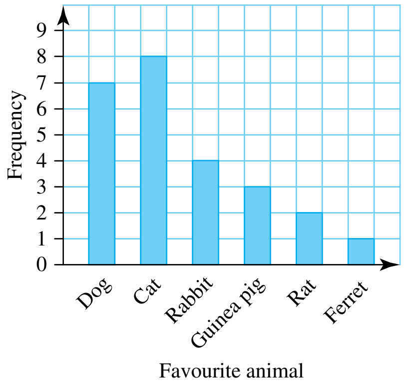
9.

	Data	Type	
a	Wines rated as high, medium or low quality	Categorical	Ordinal
b	The number of downloads from a website	Numerical	Discrete
c	The electricity usage over a three-month period	Numerical	Continuous
d	A volume of petrol sold by a petrol station per day	Numerical	Continuous

10. a.

Favourite animal	Frequency
Dog	7
Cat	8
Rabbit	4
Guinea pig	3
Rat	2
Ferret	1

   b.

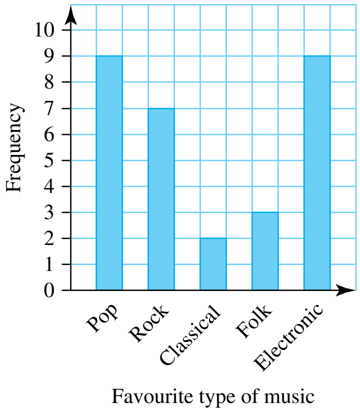


   c. Cat

11. a.

Favourite type of music	Frequency
Pop	9
Rock	7
Classical	2
Folk	3
Electronic	9

b.



c. Pop and electronic

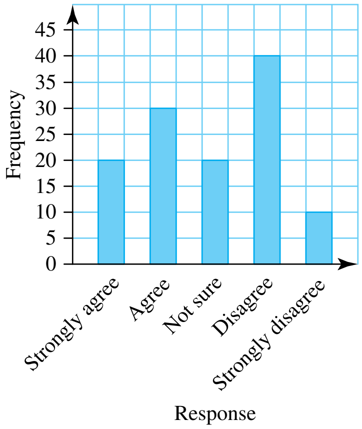
12. a. Flat white

b. 70

13. a. Ordinal categorical

b. The data should be in order from ‘Strongly agree’ through to ‘Strongly disagree’.

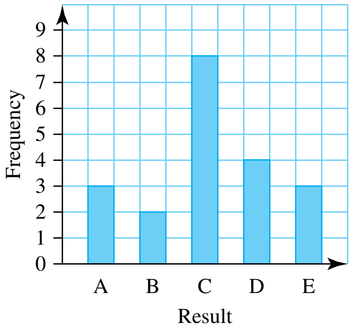
c.



14. a.

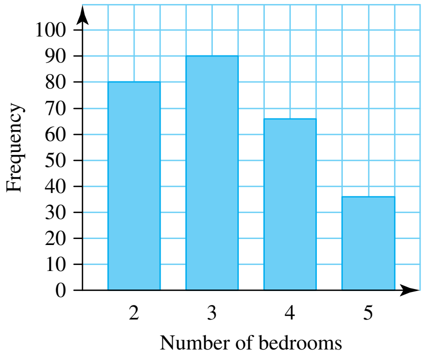
Result	Frequency
A	3
B	2
C	8
D	4
E	3

b.



c. Ordinal categorical

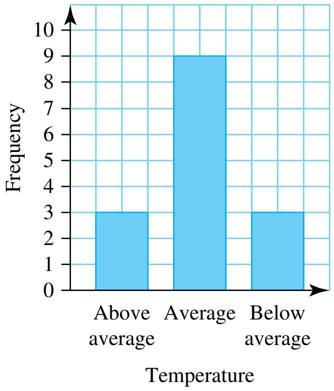
15.



16. a.

Temperature	Frequency
Above average	3
Average	9
Below average	3

b.



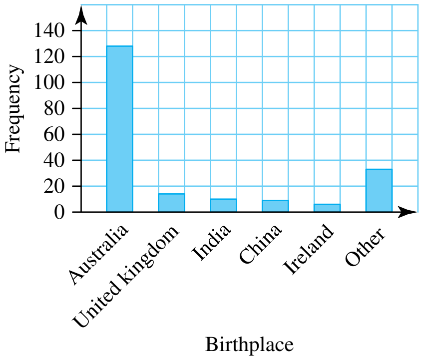
c. Ordinal categorical

17. a. 40

b. 15%

18. a. Nominal categorical

b.



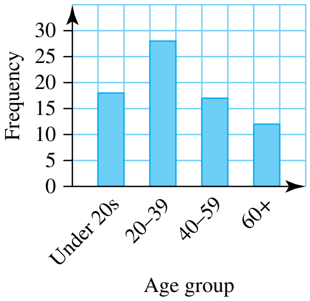
c. 64%

19. a.

Age group	Frequency
Under 20s	18
20 – 29	15
30 – 39	13
40 – 49	10
50 – 59	7
60+	12

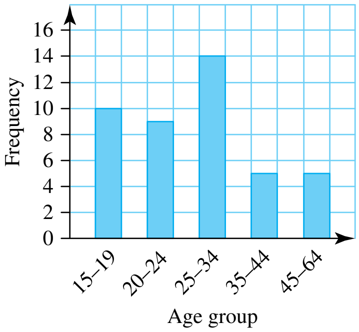
b. Under 20s

c.

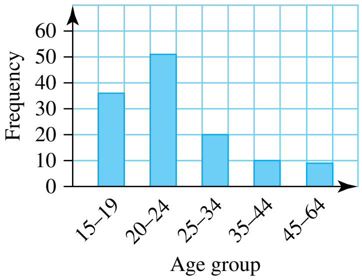


d. Yes, the modal category is now 20–39.

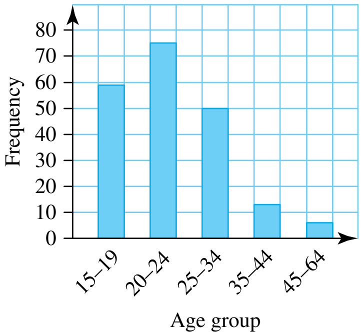
20. a. Agriculture



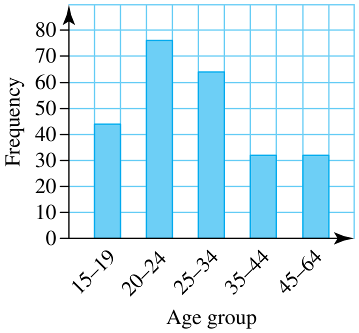
Creative arts



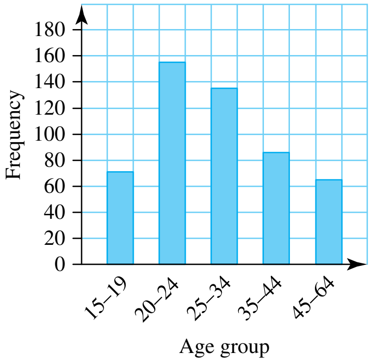
Engineering



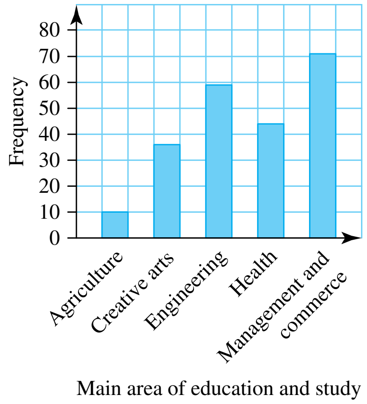
Health



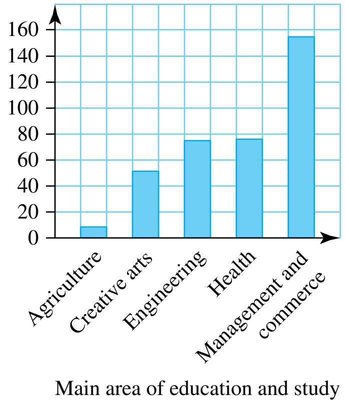
Management and commerce



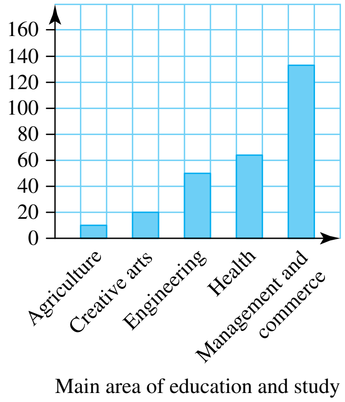
15–19



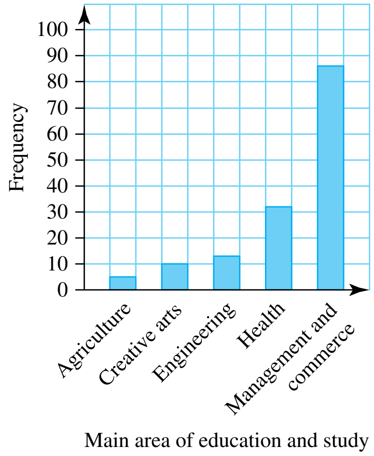
20–24



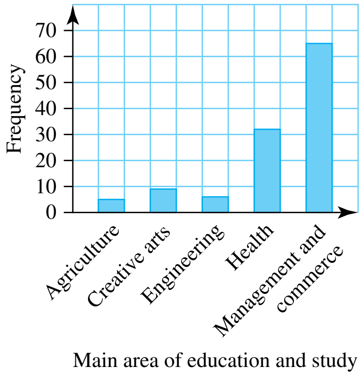
25–34



35–44



45–64



Exercise 11.3 Construct, describe and interpret dot plots and stem-and-leaf plots

1. Key: 1*|9 = $19

Stem	Leaf
1*	9
2	1 1 2 2 2 2 2 2 3 3 4 4
2*	5 6

2. Key: 0*|6 = 6 hours

Stem	Leaf
0*	6 7 9
1	3 4 4
1*	7 7
2	0 0 1 1 1 3 3 4 4
2*	5 5 6

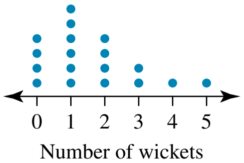
3. Key: 1|2 = 12 passengers

Stem	Leaf
1	2 3 4
1*	5 5 5 5 7 7
2	0 0 2 2 3 3 3 3 4
2*	5 7 7 7 7 7 8 8
3	0 3 4 4
3*	5 5 6 6 6 7
4	2 3 4
4*	7

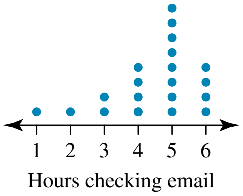
4. Key: 1|7 = 17 patients

Stem	Leaf
1	7 7
2	1 2 3 3 3 4 4 4 5 5 5 6 6 6 8
3	0 0 1 2 3 4 4 5 7 8 8
4	1 1 3 4 5 5 6
5	1 1 5 6
6	0

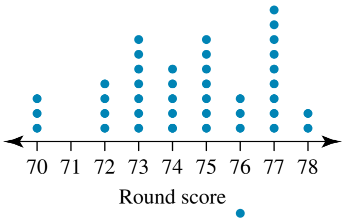
5. a.



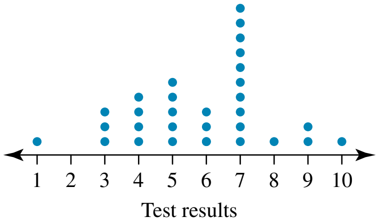
b.



6. a.



b.



7. Key: 3|6 = 36 min

Stem	Leaf
3	6 7 8 8 9 9
4	0 0 1 2 2 2 2 2 3 5 6 6 7 7 8 8 8
5	0 2 2 2 3 4 5 7 8 9
6	6 8 8
7	1 2 5
8	2

8. Key: 10|1 = 101 wpm 10*|6 = 106 wpm

Stem	Leaf
8*	6 8
9	2
9*	5 5 5 6 6 6 8 9
10	2 2 2 3
10*	7 7 7 8 8 8
11	0 1 2
11*	5
12	0 1 1 4
12*	
13	0

9. Key: 14|3 = 14.3 V 14*|8 = 14.8 V

Stem	Leaf
13*	8 9
14	0 2 3 3
14*	5 6 6 7 7 8 8
15	1 2 2
15*	5 6 7 9

10. D

11. B

12. Key: 0*|8 = 8 people

Stem	Leaf
0*	8 8
1	3 3 4
1*	6 6 7 7 9
2	1 1
2*	5

13. a. The first stem plot has one mode with data values that are most frequent in the 30 –< 40 interval. There is a possible outlier at 91, and the distribution appears to be symmetrical.

The second stem plot has 3 modes and two distinct groups of data. There are no obvious outliers, and there is a slight positive skew to the distribution.

b. Key: 0|1 = 1 game played

Stem	Leaf
0	1
1	4 7
2	4 4 4 8
3	1 2 3 3 5 6 6
4	1 2 3 3 4 5
5	1 1 2
6	5
7	
8	2 5 7
9	1 3

14. a. Key: 0|1 = 1

Stem	Leaf
0	1
0*	
1	1 1 1 4 4
1*	6 6 7 8
2	3 3 4 4
2*	7 7 9

b. Splitting the stem for this data gives a clearer picture of the spread and shape of the distribution of the data set.

Exercise 11.4 Construct, describe and interpret column graphs and histograms

1.

Time (seconds)	Frequency
30– < 40	3
40– < 50	5
50– < 60	7
60– < 70	4
70– < 80	1

2. a.

Time (seconds)	Frequency
80– < 90	2
90– < 100	6
100– < 110	5
110– < 120	5
120– < 130	2

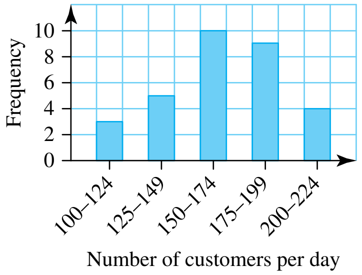
b.

Time(Seconds)	Frequency
85– < 90	2
90– < 95	1
95– < 100	5
100– < 105	2
105– < 110	3
110– < 115	3
115– < 120	2
120– < 125	2

3.

Class interval	Frequency
100–124	3
125–149	5
150–174	10
175–199	9
200–224	4

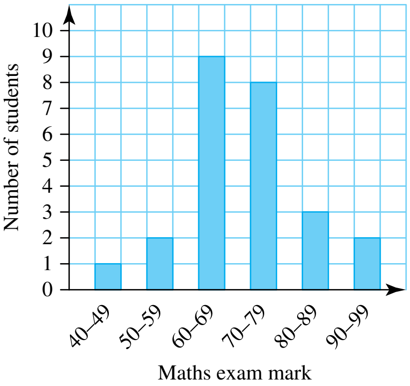
4.



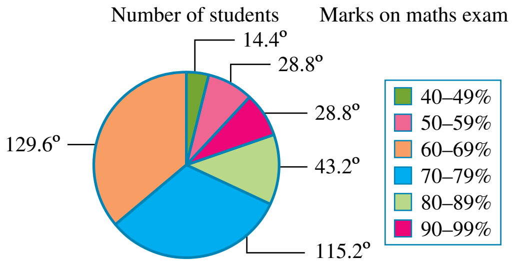
5.

Mark	Tally	Frequency
40–49	|	1
50–59	||	2
60–69	𝍸 ||||	9
70–79	𝍸 |||	8
80–89	|||	3
90–99	||	2

6.



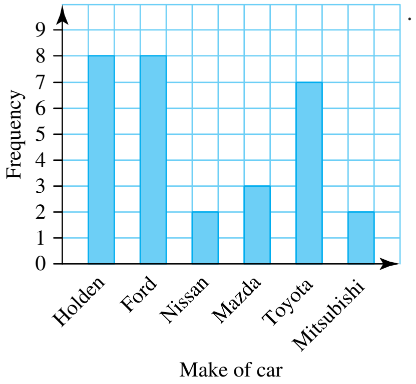
7.



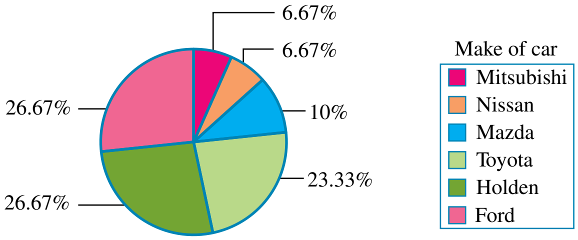
8.

Make	Tally	Frequency
Holden	𝍸 |||	8
Ford	𝍸 |||	8
Nissan	||	2
Mazda	|||	3
Toyota	𝍸 ||	7
Mitsubishi	||	2

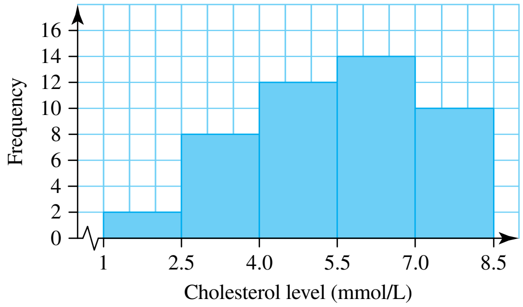
9.



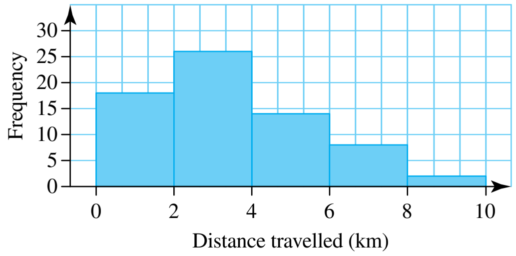
10.



11.

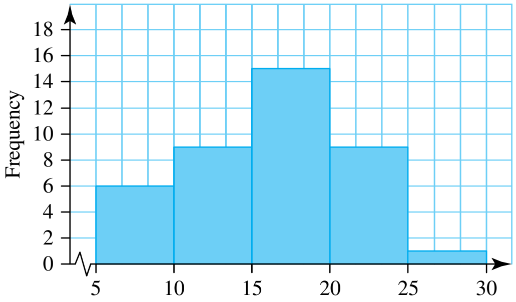


12.



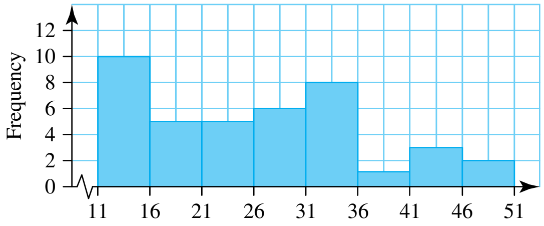
13.

Class interval	Frequency
5– < 10	6
10– < 15	9
15– < 20	15
20– < 25	9
25– < 30	1



14.

Class interval	Frequency
11– < 16	10
16– < 21	5
21– < 26	5
26– < 31	6
31– < 36	8
36– < 41	1
41– < 46	3
46– < 51	2

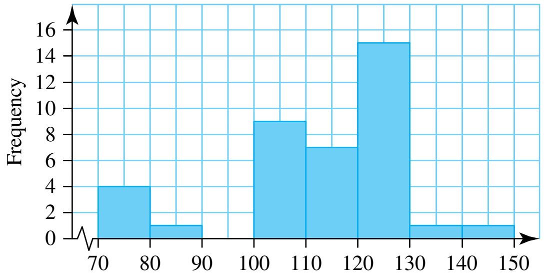


Exercise 11.5 Measures of centre

1. a. The distribution has one mode with data values that are most frequent in the 35– < 40 interval. There are no obvious outliers, and there is a negative skew to the distribution.

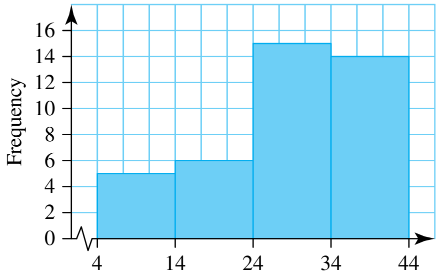
b. The distribution has one mode with data values that are most frequent in the 45– < 60 interval. There are potential outliers in the 120– < 135 interval, and the distribution is either symmetrical (excluding the outliers) or has a slight positive skew (including the outliers).

2. a.



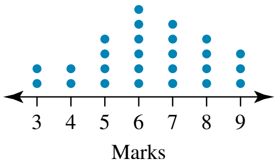
The distribution has one mode with data values that are most frequent in the 120– < 130 interval. There are potential outliers in the 70– < 80 interval, and there is a negative skew to the distribution.

b.



The distribution has one mode with data values that are most frequent in the 24– < 34 interval. There are no obvious outliers, and there is a negative skew to the distribution.

3. a.



b. The distribution has one mode with a value of 6. There are no obvious outliers and there is a slight negative skew to the distribution.

4. 128.4

5. a. 24.76

b. 27.71

c. As the highest value increased, the mean increased significantly.

6. a. 6.94 **b.** 46.36

7. a.

Interval	Frequency (f)	Midpoint (x)	xf
203– < 210	5	206.5	1032.5
210– < 217	3	213.5	640.5
217– < 224	3	220.5	661.5
224– < 231	3	227.5	682.5
231– < 238	2	234.5	469
238– < 245	1	241.5	241.5
	$\sum f = 17$		$\sum fx = 3727.5$

$\bar{x} = 219.26$

b.

Interval	Frequency (f)	Midpoint (x)	xf
5– < 10	4	7.5	30
10– < 15	5	12.5	62.5
15– < 20	3	17.5	52.5
20– < 25	3	22.5	67.5
25– < 30	1	27.5	27.5
30– < 35	1	32.5	32.5
	$\sum f = 17$		$\sum fx = 272.5$

$\bar{x} = 16.03$

8. a. 26.18 **b.** 21.67

9. a. 51 **b.** 198

10. a. 24
b. 24
c. The median is unchanged.

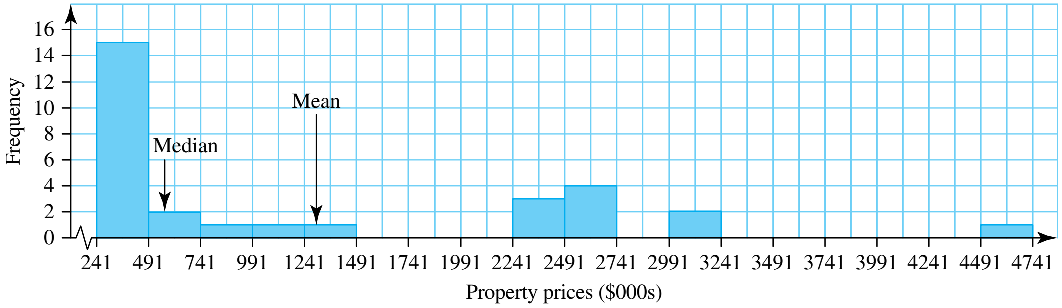
11. a. 100 **b.** 1.805

12. The median, as the data set has two clear outliers

13. a. $74 230.77
b. $65 000
c. It would be in the workers' interest to use a higher figure when negotiating salaries, whereas it would be in the management's interest to use a lower figure.

14. a. Mean = 867.89 mm, Median = 654 mm
b. The median, as it is not affected by the extreme values present in the data set.

15. a. Mean = $1 269 850, Median = $594 500
b., c

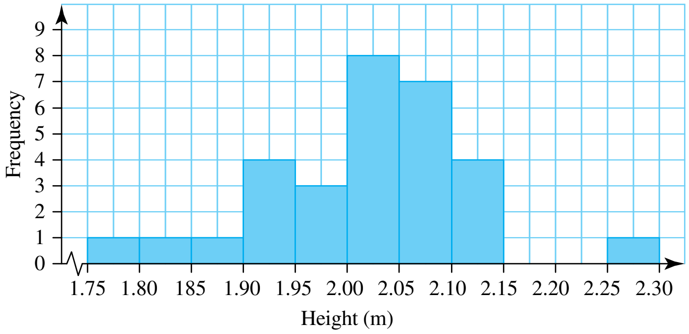


d. The median, as the mean is affected by a few very high values.

16. a.

Interval	Frequency
1.75– < 1.80	1
1.80– < 1.85	1
1.85– < 1.90	1
1.90– < 1.95	4
1.95– < 2.00	3
2.00– < 2.05	8
2.05– < 2.10	7
2.10– < 2.15	4
2.15– < 2.20	0
2.20– < 2.25	0
2.25– < 2.30	1

b.



The median would be the preferred choice due to the extreme values in the data set.

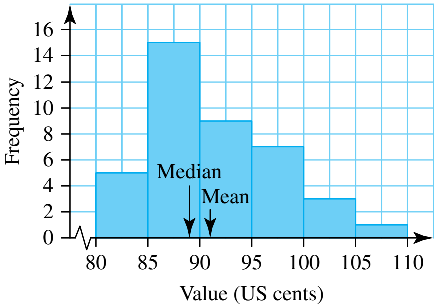
c. 2.02 m

17. a. Mean= 7.55, median = 6

b. The median would be the preferred choice due to the extreme value of 34.

18. a. Mean = 91.125, median = 89.5

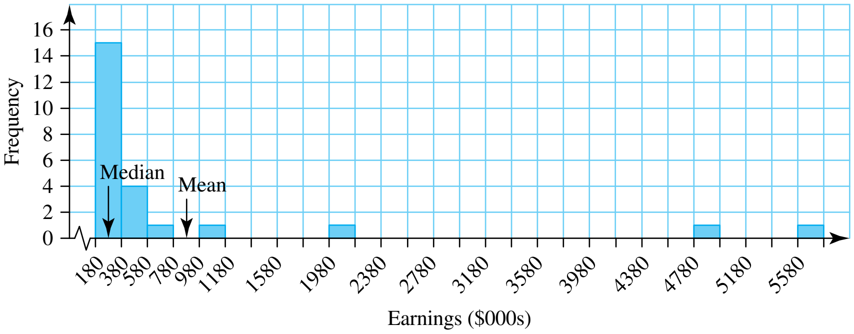
b., c



c. The mean is higher than the median as it has been more influenced by the values at the higher end of the distribution.

19. a. Mean = $847 354, median = $310 000

b., c

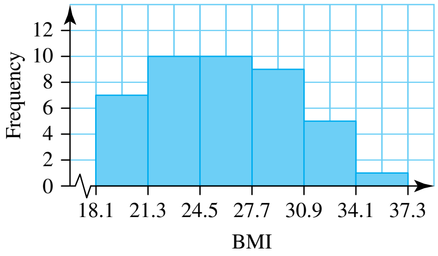


*

d. The median is the best measure as the mean is affected by the extreme values.

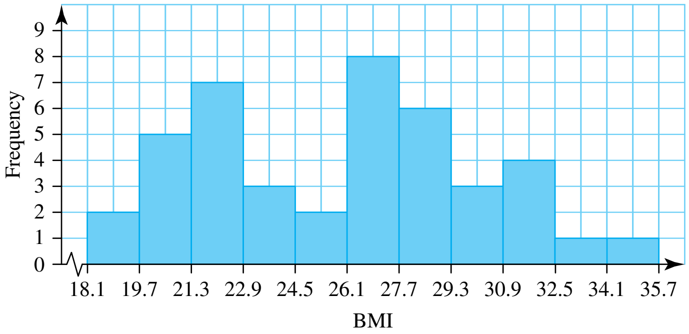
20. a. i.

Interval	Frequency
18.1– < 21.3	7
21.3– < 24.5	10
24.5– < 27.7	10
27.7– < 30.9	9
30.9– < 34.1	5
34.1– < 37.3	1



ii.

Interval	Frequency
18.1– < 19.7	2
19.7– < 21.3	5
21.3– < 22.9	7
22.9– < 24.5	3
24.5– < 26.1	2
26.1– < 27.7	8
27.7– < 29.3	6
29.3– < 30.9	3
30.9– < 32.5	4
32.5– < 34.1	1
34.1– < 35.7	1



b. The first histogram has two modes and is near symmetrical, with a slight positive skew.
The second histogram shows two distinct groups, with a symmetrical lower group and a positively skewed upper group.

c. Table 1: 25.95, Table 2: 25.91, Raw data: 25.76
Both of the tables give a higher value for the mean than the raw data, although the differences are small.

d. The total data set is generally symmetrical with no obvious outliers, so the mean is the best measure of centre.

Exercise 11.6 Measures of spread

1. 186

2. 2.555

3. **a.** Class A = 11, Class B = 10
b. Class A = 2, Class B = 5

4. **a.** 160
b. 57
c. Goals against: range = 293, interquartile range = 140
The 'Goals against' column is significantly more spread out than the 'Goals for' column.

5. Variance = 23.33, standard deviation = 4.83

6.

Interval	Frequency (f)	Midpoint (x)	xf	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0– < 50	35	25	875	1525.88	53405.8
50– < 100	125	75	9375	119.63	14953.6
	$\sum f = 160$		$\sum xf = 10\,250$		$\sum f(x-\bar{x})^2 = 68\,359.4$

Variance = 429,933, standard deviation = 20,735

7.

Age group (years)	Frequency (f)	Midpoint (x)	xf
15– < 20	14	17.5	245
20– < 25	18	22.5	405
25– < 30	11	27.5	302.5
30– < 35	7	32.5	227.5
35– < 40	5	37.5	187.5
	$\sum f = 55$		$\sum fx = 1367.5$

8. **a.** 7.37
b. 7
c. Standard deviation = 11.49, interquartile range = 7
d. The standard deviation increased by 4.12, while the interquartile range was unchanged.

9. **a.** \$16 327.50
b. \$46 902
c. There is a much larger spread in the maximum salaries than the minimum salaries.

10. **a.** Year 1: standard deviation = 20 382.8, interquartile range = 38 907.5
Year 2: standard deviation = 19 389.0, interquartile range = 37 110.0
b. Year 1: standard deviation = 18 123.5, interquartile range = 31 107.5
Year 2: standard deviation = 17 289.2, interquartile range = 29 140
c. Both values are reduced by a similar amount, but there is a larger impact on the standard deviation than the interquartile range.

11. **a.** Year 1: standard deviation = 1 301 033.5, interquartile range = 2 336 546
Year 2: standard deviation = 1 497 303.5, interquartile range = 2 734 078
b. Year 1: standard deviation = 1 082 470.9, interquartile range = 2 136 718
Year 2: standard deviation = 1 228 931.0, interquartile range = 2 415 365
c. Both values are reduced, but there is a bigger impact on the interquartile range than the standard deviation.

12. **a.** 2.9
b. 1.25
c. The range is slightly more $(2.9 > 2.1.25)$ than double the value of the interquartile range. This indicates that the data is bunched with no outliers.

13. **a.** Mean = 41 440.78, standard deviation = 2248.92
b. Median = 41 333, interquartile range = 3609
c. 59.38%
d. 50%

e. There is a greater percentage of the sample within one standard deviation of the mean than between the first and third quartiles.

14. a. Standard deviation = 0.46, interquartile range = 0.7.

b.

	Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
Std dev.	0.46	0.40	0.51	0.45	0.44	0.38	0.67	0.41
IQR	0.7	0.7	0.9	0.8	0.6	0.6	1.2	0.6

c. Sydney

d. Darwin

15. a. Standard deviation = 18.81, interquartile range = 36.21

b. India: standard deviation = 105.86, interquartile range = 158.59
China: standard deviation = 1143.53, interquartile range = 1988.7
United Kingdom: standard deviation = 8.21, interquartile range = 9.48
USA: standard deviation = 87.34, interquartile range = 145.48

c. The standard deviation is appropriate, as there appear to be no obvious outliers in the data for any country.

16. a. Interquartile range = 2.78, variance = 25.40

b. Interquartile range = 2.78, variance = 4.43

c. The interquartile range has stayed the same value, while the variance has reduced significantly.

11.7 Review: exam practice

1. D

2. B

3. A

4. D

5. B

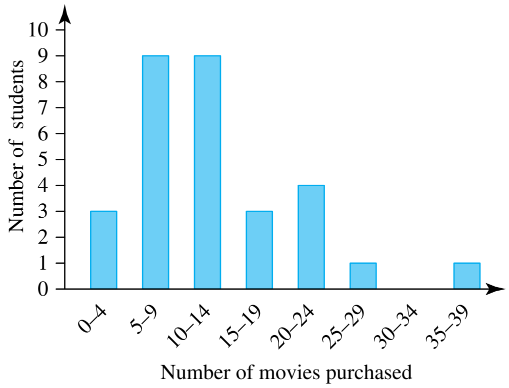
6. a. Categorical **b.** Numerical **c.** Numerical **d.** Numerical

7. a. Discrete **b.** Continuous **c.** Continuous **d.** Discrete

8. a.

Number of DVDs	Tally	Number of students
0–4	|||	3
5–9	𝍸 |||	9
10–14	𝍸 |||	9
15–19	|||	3
20–24	||||	4
25–29	|	1
30–34		0
35–39	|	1

b.



9. Key: 0 | 6 = 6 errors

Stem	Leaf
0	6
1	3 5 7 8
2	0 0 5 6 6 7 8 9
3	1 2 2 8
4	3 6
5	2

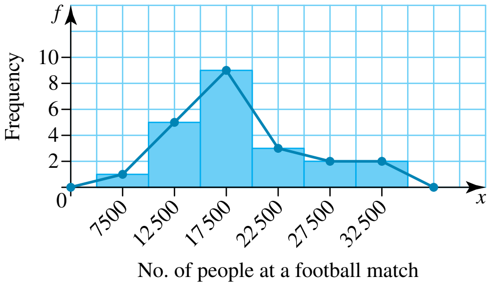
10. 8.4s

11. Mean = 250.65 g, Range = 4.6 g

12. a.

Class	Class centre	Frequency
5000–9999	7 500	1
10 000–14 999	12 500	5
15 000–19 999	17 500	9
20 000–24 999	22 500	3
25 000–29 999	27 500	2
30 000–34 999	32 500	2

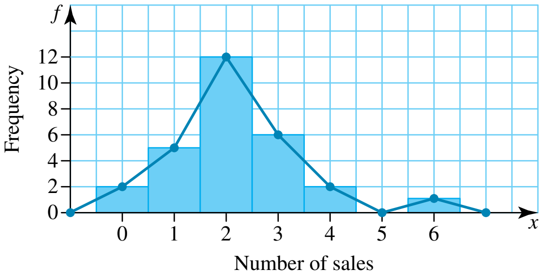
b.



13. a. Mean = 94.6, median = 93.3

b. Standard deviation = 7.49

14.



15. Key 2|1 = 21

Stem	Leaf
2	1 1 3 4 8 8 8 9 9 9
3	0 3 4 5 5 5 6 8 8 8 8 9
4	0 0 0 1 1 2 2 4 5 5 5 6 8 9

16. a. 28

b. 38

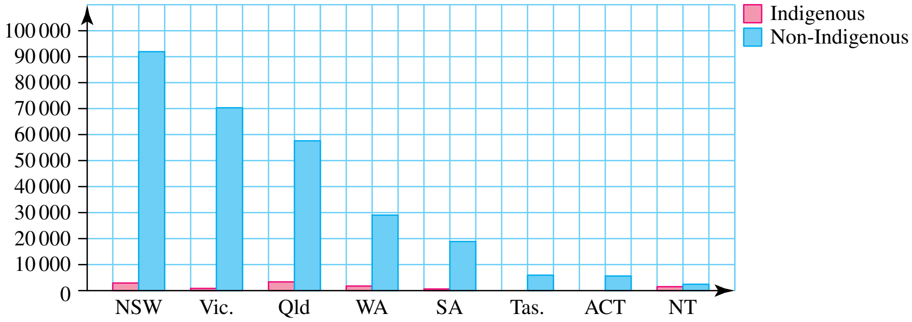
17. a. Year 1: mean = 6432.9, median = 1324

Year 2: mean = 2637.6, median = 1201

b. The mean values are significantly different but the medians are very similar. This would seem to indicate the presence of extreme values in the data.

18. Mean = 202 461.4, standard deviation = 257 819.6

19. a.



b. Indigenous mean = 1410.5, Non-Indigenous mean = 35 241.6

c. Indigenous median = 1156, Non-Indigenous median = 24 008

d. Indigenous: standard deviation = 1193.8, Non-Indigenous: standard deviation = 33 949.0

20. a. 1980 – 89: mean = 19.34, standard deviation = 0.44
2003 – 12: mean = 20.29, standard deviation = 0.45

b. The mean temperature is about one degree higher in the period 2003–12, but the standard deviations indicate that the data have similar spreads.

c. Total data:mean = 19.82, standard deviation = 0.65

d. The mean of the total data is halfway between the two separate time periods. The standard deviation indicates a much greater variation from the mean for the total data.

CHAPTER 12
Univariate data comparisons

12.1 Overview

12.1.1 Introduction

Imagine that a new drug for the relief of cold symptoms has been developed. To test the drug, 40 people were exposed to a cold virus. Twenty patients were then given a dose of the drug while another 20 patients were given a placebo. (In medical tests a control group is often given a placebo drug. The subjects in this group believe that they have been given the real drug, but their dose contains no drug at all.) All participants were then asked to indicate the time when they first felt relief of symptoms. The number of hours from the time the dose was administered to the time when the patients first felt relief of symptoms was recorded. How would these two sets of data be compared? In this chapter we will look at one method of data comparison called parallel boxplots, which are based on the work completed in Chapter 11.

LEARNING SEQUENCE

12.1 Overview
12.2 Constructing boxplots
12.3 Outliers and fences
12.4 Parallel boxplots
12.5 Review: exam practice

Fully worked solutions for this chapter are available in the Resources section of your eBookplus at www.jacplus.com.au.

12.2 Constructing boxplots

12.2.1 The five-number summary

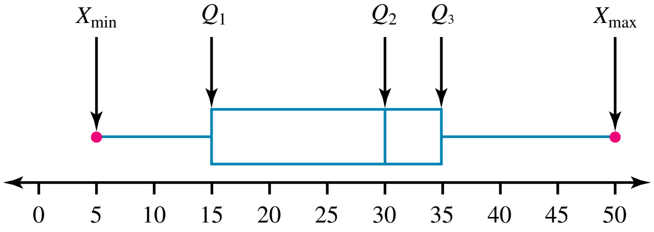
The five-number summary gives five key values that provide information about the spread of a data set. These values are:

1. the lowest score ($X_{\min}$)
2. the lower quartile (Q_1)
3. the median (Q_2)
4. the upper quartile (Q_3)
5. the highest score ($X_{\max}$).

12.2.2 Boxplots

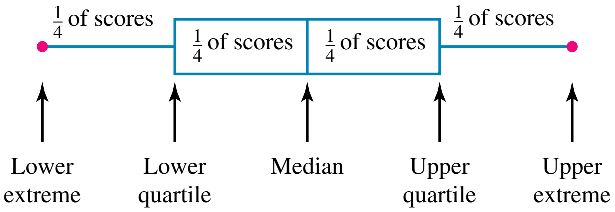
We use a **boxplot** to represent the five-number summary in a graphical form. The boxplot is often displayed either above or below a number line, which allows easy identification of the key values.

Boxplots usually consist of both a central box and 'whiskers' on either side of the box. The box represents the interquartile range (IQR) of the data set, with the distance between the start of the first whisker and the end of the second whisker representing the range of the data. If either the lowest score is equal to the lower quartile or the highest score is equal to the upper quartile, then there will be no whisker on that side of the boxplot.



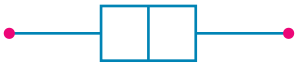
The shape of boxplots

The shape of a boxplot will mirror the distribution of the data set. For example, a boxplot with a small central box and large whiskers will indicate that the majority of the data is clustered around the median, whereas a boxplot with a large central box and small whiskers will indicate that the data is spread more evenly across the range.

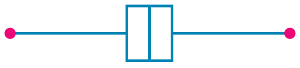


Positively skewed data will have the central box on the left-hand side of the boxplot with a large whisker to the right, while negatively skewed data will have the central box on the right-hand side of the boxplot with a large whisker to the left.

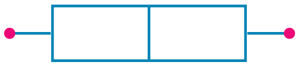
Learning to interpret the shape of boxplots will help you to better understand the data that the boxplot represents.



Normally distributed data



Clustered data



Spread data

Positively skewed data

Negatively skewed data

WORKED EXAMPLE 1

For the set of scores below, develop a five-number summary.
12 15 46 9 36 85 73 29 64 50

THINK	**WRITE**
1. Re-write the list in ascending order.	9 12 15 29 36 46 50 64 73 85
2. Write the lowest score.	Lower extreme = 9
3. Calculate the lower quartile.	Lower quartile = 15
4. Calculate the median.	$\text{Median} = \frac{36+46}{2}$ $= 41$
5. Calculate the upper quartile.	Upper quartile = 64
6. Write the upper extreme.	Upper extreme = 85
7. Combine the data into a five-number summary.	Five-number summary = 9, 15, 41, 64, 85

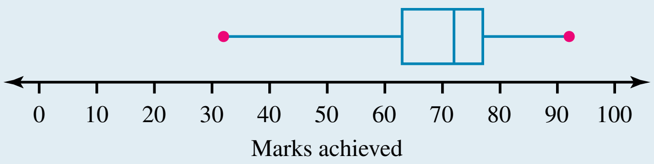
WORKED EXAMPLE 2

The box-and-whisker plot shows the marks achieved by students on their end of year exam.

a. **State the median.**

b. **Find the interquartile range.**

c. **What was the highest mark in the class?**



THINK	**WRITE**
a. The mark in the box shows the median (72).	**a.** Median = 72 marks
b. 1. The lower end of the box shows the lower quartile (63).	**b.** Lower quartile = 63 marks
2. The upper end of the box shows the upper quartile (77)	Upper quartile = 77 marks
3. Subtract the lower quartile from the upper quartile.	Interquartile range = 77 − 63 = 14 marks
c. The top end of the whisker gives the top mark (92).	**c.** Top mark = 92 marks.

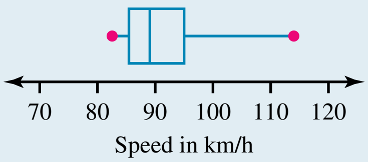
WORKED EXAMPLE 3

After analysing the speed (in km/h) of motorists through a particular intersection, the following five-number summary was developed.
The lowest score is 82 km/h.
The lower quartile is 84 km/h.
The median is 89 km/h.
The upper quartile is 95 km/h.
The highest score is 114 km/h.
Show this information in a box-and-whisker plot.

THINK

1. Draw a scale from 70 to 120 using 1 cm = 10 km/h.
2. Draw the box from 84 to 95, representing Q_1 to Q_3.
3. Mark the median at 89 (Q_2).
4. Draw the whiskers to 82 (X_{min}) and 114 (X_{max}).

WRITE



WORKED EXAMPLE 4

Construct a boxplot for the data contained in the following stem plot, which shows the number of coffees sold by a café each day over a 21-day period.

Key: 6|3 = 63 coffees

Stem	**Leaf**
6	**3 5 8**
7	**0 2 4 5 7 9**
8	**1 1 3 6 8**
9	**0 1 5 6 7**
10	**1 4**

THINK

1. Determine the median of the data, recalling the median formula $\left(\frac{n+1}{2}\right)^{\text{th}}$ data value.

WRITE

There are 21 values, so the median is in the $\left(\frac{21+1}{2}\right) = 11\text{th}$ position.

63, 65, 68, 70, 72, 74, 75, 77, 79, 81, $\overset{\substack{\text{median}\\ \downarrow}}{\textcircled{81}}$

$Q_2 = 81$

THINK

2. Determine the value of the lower quartile, by calculating the median of the lower half of the data set.

WRITE

$\left(\frac{10+1}{2}\right) = 5.5$

There are 10 values in the lower half of the data, so Q_1 will be between the 5th and 6th values.

63, 65, 68, 70, 72, $\overset{\substack{Q_1\\ \downarrow}}{}$ 74, 75, 77, 79, 81

$$\begin{aligned} Q_1 &= \frac{72+74}{2} \\ &= 73 \end{aligned}$$

3. Determine the value of the upper quartile, by calculating the median of the upper half of the data set.

$$\left(\frac{10+1}{2}\right) = 5.5$$

There are 10 values in the upper half of the data, so Q_3 will between the 5th and 6th values.

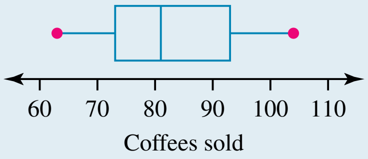
$$83, 86, 88, 90, 91, \overset{Q_3}{\downarrow} 95, 96, 97, 101, 104$$

$$\begin{aligned} Q_3 &= \frac{91+95}{2} \\ &= 93 \end{aligned}$$

4. Write the five-number summary.

$$\begin{aligned} X_{\text{min}} &= 63 \\ Q_1 &= 73 \\ Q_2 &= 81 \\ Q_3 &= 93 \\ X_{\text{max}} &= 104 \end{aligned}$$

5. Rule a suitable scale for your boxplot which covers the full range of values. Draw the central box first (from Q_1 to Q_3, with a line at Q_2) and then draw in the whiskers from the edge of the box to the minimum and maximum values.



on Resources

Interactivity: Boxplots (int-6245)
Digital document: SpreadSHEET Interquartile range (doc-29509)
Digital document: SpreadSHEET Boxplots (doc-29510)

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Units 1 & 2 › Area 6 › Sequence 2 › Concept 1 › **Boxplots** Summary screen and practice questions

Exercise 12.2 Constructing boxplots

1. Copy and complete the following sentences.
   - a. When you want to calculate the _____ of a data set, the first thing you must do is put the data in order from _____ to _____.
   - b. When the median falls between two values, you need to calculate the _____ of those two values.
   - c. _____, the _____ and _____ divides the data into _____.
2. Put these words in order from the one with the smallest value to the one with the largest value.
   Upper quartile; Minimum; Median; Maximum; Lower quartile.

A calculator can be used for many of the following questions.

3. **WE 1** For the data set below, develop a five-number summary.

   15 17 16 8 25 18 20 15 17 14

4. For each of the data sets below, develop a five-number summary.
   - a. 23 45 92 80 84 83 43 83
   - b. 2 6 4 2 5 7 1
   - c. 60 75 29 38 69 63 45 20 29 93 8 29 93

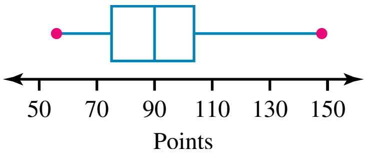
5. **WE 2** From the five-number summary 6, 11, 13, 16, 32 identify:
   - a. the median
   - b. the interquartile range
   - c. the range.

6. From the five-number summary 101, 119, 122, 125, 128 identify:
   - a. the median
   - b. the interquartile range
   - c. the range.

7. **WE 3** A five-number summary is given below.

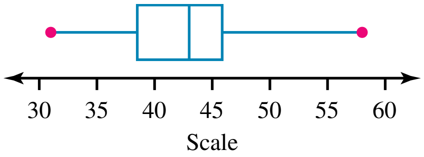
   Lower extreme= 39.2 Upper quartile = 52.3
   Lower quartile= 46.5 Upper extreme = 57.8
   Median = 49.0
   Construct a box-and-whisker plot of the data.

8. The box-and-whisker plot shows the distribution of final points scored by a football team over a season's roster.



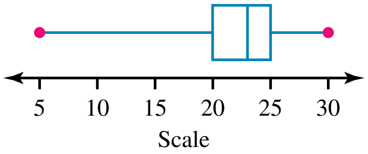
   - a. Identify the team's greatest points score.
   - b. Identify the team's smallest points score.
   - c. Identify the team's median points score.
   - d. Calculate the range of points scored.
   - e. Calculate the interquartile range of points scored.

9. The box-and-whisker plot below shows the distribution of data formed by counting the number of honey bears in each of a large sample of packs.



   In any pack, what was:
   - a. the largest number of honey bears?
   - b. the smallest number of honey bears?
   - c. the median number of honey bears?
   - d. the range of numbers of honey bears?
   - e. the interquartile range of honey bears?

Questions 10, 11 and 12 refer to the box-and-whisker plot shown.



10. **MC** The median of the data is:

A. 5 B. 20 C. 23 D. 25

11. **MC** The interquartile range of the data is:

A. 5 B. 20 C. 25 D. 20 to 25

12. **MC** Which of the following is *not* true of the data represented by the box-and-whisker plot?

A. One-quarter of the scores is between 5 and 20.
B. One-half of the scores is between 20 and 25.
C. The lowest quarter of the data is spread over a wide range.
D. Most of the data are contained between the scores of 5 and 20.

13. The number of sales made each day by a salesperson is recorded over a fortnight:
25, 31, 28, 43, 37, 43, 22, 45, 48, 33

a. Write a five-number summary of the data.
b. Construct a box-and-whisker plot of the data.

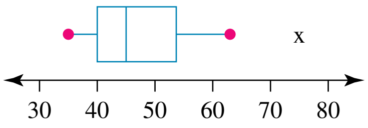
14. The data below show monthly rainfall in millimetres.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
10	12	21	23	39	22	15	11	22	37	45	30

a. Provide a five-number summary of the data.
b. Construct a box-and-whisker plot of the data.

12.3 Outliers and fences

12.3.1 Identifying possible outliers



If there is an outlier (an extreme value) in the data set, then rather than extending the whiskers to reach this value, we extend the whiskers to the next smallest or largest value and indicate the outlier value with an 'x', as shown in the diagram.

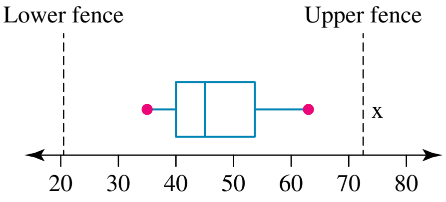
We can identify whether a value in our data set is an outlier or not by calculating the lower and upper fences of our data set.

Lower and upper fences

To calculate the **lower fence** and **upper fence** of the data set, we first need to calculate the interquartile range (IQR). Once this has been calculated, the lower and upper fences are given by the following rules:

$$\textbf{Lower fence} = Q_1 - 1.5 \times \textbf{IQR}$$
$$\textbf{Upper fence} = Q_3 + 1.5 \times \textbf{IQR}$$

If a data value lies outside the lower or upper fence then it can be considered an outlier.



WORKED EXAMPLE 5

The following stem plot represents the time taken (in minutes) for 25 students to finish a maths puzzle.

Key: 1|5 = 1.5 minutes

Stem	**Leaf**
1	**5**
2	**8**
3	**4 6 6 9**
4	**0 0 2 3 5 5 7 8 8**
5	**0 2 4 4 5 8 9**
6	**0 4**
7	
8	**5**

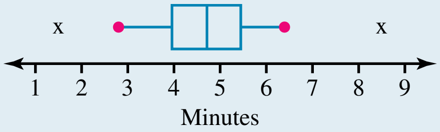
Q_1 ↓

1.5, 2.8, 3.4, 3.6, 3.6, 3.9, 4.0, 4.0, 4.2, 4.3, 4.5, 4.5

a. **Calculate the values of the lower and upper fences.**
b. **Identify any outliers in the data set.**
c. **Construct a boxplot to represent the data.**

THINK	**WRITE**
a. 1. To calculate the upper and lower fences we must first determine the median (Q_2) by recalling the formula $\left(\frac{n+1}{2}\right)^{\text{th}}$ data value.	**a.** There are 25 values, so the median is in the $\left(\frac{25+1}{2}\right) = 13\text{th position.}$ median ↓ 1.5, 2.8, 3.4, 3.6, 3.6, 3.9, 4.0, 4.0, 4.2, 4.3, 4.5, 4.5, 4.7 $Q_2 = 4.7$
2. The lower quartile (Q_1) can then be calculated.	$\left(\frac{12+1}{2}\right) = 6.5$ There are 12 values in the lower half of the data so Q_1 will be between the 6th and 7th values. Q_1 ↓ 1.5, 2.8, 3.4, 3.6, 3.6, 3.9, 4.0, 4.0, 4.2, 4.3, 4.5, 4.5 $Q_1 = \frac{3.9+4.0}{2}$ $= 3.95$
3. The upper quartile (Q_3) can then be calculated.	$\left(\frac{12+1}{2}\right) = 6.5$ There are 12 values in the upper half of the data, so Q_3 will be between the 6th and 7th values. Q_3 ↓ 4.8, 4.8, 5.0, 5.2, 5.4, 5.4, 5.5, 5.8, 5.9, 6.0, 6.4, 8.5 $Q_3 = \frac{5.4+5.5}{2}$ $= 5.45$

▶

4. Calculate the IQR.	$\text{IQR} = Q_3 - Q_1$ $= 5.45 - 3.95$ $= 1.5$
5. Calculate the values of the lower and upper fences by recalling the formulas: Lower fence $= Q_1 - 1.5 \times \text{IQR}$ Upper fence $= Q_3 + 1.5 \times \text{IQR}$	Lower fence $= Q_1 - 1.5 \times \text{IQR}$ $= 3.95 - 1.5 \times 1.5$ $= 3.95 - 2.25$ $= 1.7$ Upper fence $= Q_3 + 1.5 \times \text{IQR}$ $= 5.45 + 1.5 \times 1.5$ $= 5.45 + 2.25$ $= 7.7$
b. 1. Identify whether any values lie below the lower fence or above the upper fence.	**b.** Values below the lower fence (1.7) : 1.5 Values above the upper fence (7.7) : 8.5
2. State the answer.	There are two outliers: 1.5 and 8.5.
c. 1. Write the five-number summary, giving the minimum and maximum values as those that lie within the lower and upper fences.	**c.** $X_{\min} = 2.8$ $Q_1 = 3.95$ $Q_2 = 4.7$ $Q_3 = 5.45$ $X_{\max} = 6.4$
2. Rule a suitable scale for your boxplot to cover the full range of values. Draw the central box first (from Q_1 to Q_3, with a line at Q_2) and then draw in the whiskers from the edge of the box to the minimum and maximum values. Mark the outliers with an 'x'.	

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Units 1 & 2 > Area 6 > Sequence 2 > Concept 2

Outliers and fences Summary screen and practice questions

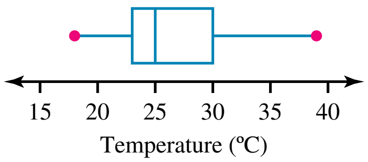
Exercise 12.3 Outliers and fences

1. a. Construct a boxplot for the data contained in the following stem plot, which shows the number of sandwiches sold by a café per day over a 21-day period.
   b. Calculate the upper and lower fences.

Key: $2|9 = 29$ sandwiches

Stem	Leaf
2	9
3	1 3 6 8 9
4	2 4 5 5 6 7 7 8
5	0 0 3 5 8
6	1 2

2. The boxplot shows the temperatures in Brisbane over a 23-day period.
   a. What is the median temperature?
   b. What is the range of the temperatures?
   c. What is the interquartile range of temperatures?
   d. Are there any outliers?

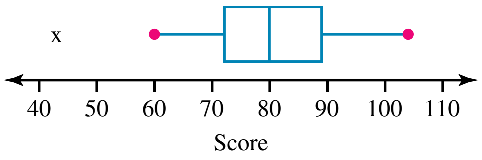


3. **WE 5** The following stem plot represents the time taken (in minutes) for 25 students to finish a logic problem.
   a. Calculate the values of the lower and upper fences.
   b. Identify any outliers in the data set.
   c. Construct a boxplot to represent the data.

Key: 4|4 = 4.4 minutes

Stem	Leaf
4	4
5	
6	2 6 9
7	0 4 7 7 8
8	0 3 3 5 6 8 9
9	1 2 4 6 7
10	2 4 4
11	5
12	

4. The boxplot represents the scores made by an Australian football team over a season.



   a. What was the highest amount of points the team scored in the season?
   b. What was the lowest amount of points the team scored in the season?
   c. What was the range of points scored?
   d. What was the interquartile range of points scored?

5. The following stem plot shows the ages of 25 people when they had their first child.

Key: 1*|7 = 17 years old

Stem	Leaf
1*	7 8 8
2	0 2 3 3 4
2*	5 6 6 7 8 9
3	0 0 1 2 2 4
3*	6 8 9
4	1 3

   a. Prepare a five-number summary of the data.
   b. Construct a boxplot of the data.
   c. Comment on the distribution of the data.

6. **MC** The five-number summary for a data set is 45, 56, 70, 83, 92.
Which of the following statements is definitely *not* true?
A. There are no outliers in the data set. **B.** Half of the scores are between 56 and 70.
C. The range is 47. **D.** The value of the lower fence is 15.5.

7. **MC** The formula for the lower fence of a set of data is:
A. $Q_1 - 1.5 \times \text{IQR}$ **B.** $Q_1 + 1.5 \times \text{IQR}$ **C.** $Q_2 - 1.5 \times \text{IQR}$ **D.** $Q_1 - 2.5 \times \text{IQR}$

8. **MC** Determine whether the following statements are true or false.
a. You can always determine the median from a boxplot.
b. A stem plot contains every piece of data from a data set.
c. Boxplots show the complete distribution of scores within a data set.

9. **MC** The formula for the upper fence of a set of data is:
A. $Q_1 - 1.5 \times \text{IQR}$ **B.** $Q_1 + 1.5 \times \text{IQR}$ **C.** $Q_3 - 1.5 \times \text{IQR}$ **D.** $Q_3 + 1.5 \times \text{IQR}$

10. **MC** The formula for the range of values which excludes outliers is:
A. $Q_3 - 1.5 \times \text{IQR} \le x \le Q_1 + 1.5 \times \text{IQR}$ **B.** $Q_1 - 1.5 \times \text{IQR} \le x \le Q_1 + 1.5 \times \text{IQR}$
C. $Q_3 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$ **D.** $Q_1 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$

11. **MC** From the five-number summary 15, 24, 33, 42, 51, which of the following is true?
A. The range is 35. **B.** The value of the upper fence is 70.
C. The data is symmetrical. **D.** The IQR is 22.

12. The five-number summary of a data set is 15, 29, 43, 57, 96.
a. Calculate the value of the upper and lower fence.
b. Is there an outlier in the data set?

13. From the data set: 12, 18, 21, 16, 9, 15, 21, 32, 15, 18, 27, 24, 19, 24, 30.
a. Calculate the five-number summary of the data.
b. Calculate the IQR.
c. Calculate the upper and lower fences.
d. Identify the outliers, if there are any.

14. Explain why outliers are considered an obstacle to making estimates of data and how this might be overcome.

15. If an outlier is added to the top range of a data set and included in the calculation of the five-number summary, describe the effect on each of the five numbers.

12.4 Parallel boxplots

12.4.1 Comparing data sets

In some situations you will be required to compare two or more data sets. We can use two different graphical representations to easily compare and contrast data sets.

Back-to-back stem plots

As mentioned in Chapter 11, back-to-back stem plots are plotted with the same stem, with one of the plots displayed to the left of the stem and the other plot to the right.

Remember to start the numbering on both sides with the smallest values closest to the stem and increasing in value as you move away from the stem. Also include a key with your back-to-back stem plot.

Key: 7|5 = 7.5

Leaf	Stem	Leaf
7 3	6	
9 6 2	7	5 7
5 4 2	8	0 1 4 8
8 6 6 0 0	9	2 6 6 9 9
9 5 3	10	3 5 7
	11	1 4
	12	2

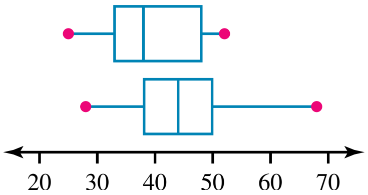
After drawing back-to-back stem plots you can easily identify key points such as:

- which data set has the lowest and/or highest values
- which data set has the largest range
- the spread of both data sets

Parallel boxplots

Parallel boxplots are plotted with one of the boxplots above the other. Both boxplots share the same scale.

Parallel boxplots allow us to easily make comparisons between data sets, as we can see the key features of the boxplots in the same picture. The position and size of the interquartile ranges of the data sets can be seen, as well as their range. However, while parallel boxplots do display information about the general distribution of the data sets they cover, they lack the detail about this distribution that a histogram or stem plot gives.



WORKED EXAMPLE 6

The following back-to-back stem plot shows the size (in kg) of two different breeds of dog.

a. Construct parallel boxplots of the two sets of data.

b. Compare and contrast the two sets of data.

Key: 2|6 = 26 kg

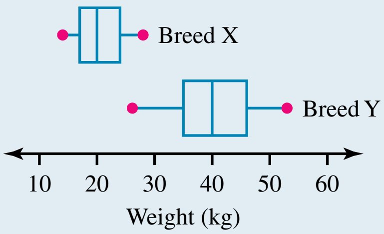
Breed X Leaf	**Stem**	**Breed Y Leaf**
9 8 7 7 6 4 4	1	
8 7 5 4 3 3 1 0	2	6 9
	3	3 5 5 7 8
	4	0 2 4 5 6 9
	5	1 3

THINK	WRITE
a. 1. Calculate the five-number summary for the first data set (Breed X).	a. $X_{\text{min}} = 14$ $X_{\text{max}} = 28$ There are 15 pieces of data, so the median is the $\left(\frac{15+1}{2}\right)$ = 8th piece of data. $Q_2 = 20$ There are 7 pieces of data in the lower half, so Q_1 is the 4th value. $Q_1 = 17$ There are 7 pieces of data in the upper half, so Q_3 is the 4th value. $Q_3 = 24$ Five-number summary: 14, 17, 20, 24, 28

2. Calculate the five-number summary for the second data set (Breed Y).

$X_{\min} = 26$
$X_{\max} = 53$
There are 15 pieces of data, so the median is the $\left(\frac{15+1}{2}\right) = 8$th piece of data.
$Q_2 = 40$
There are 7 pieces of data in the lower half, so Q_1 is the 4th value.
$Q_1 = 35$
There are 7 pieces of data in the upper half, so Q_3 is the 4th value.
$Q_3 = 46$
Five-number summary: 26, 35, 40, 46, 53

3. Use the five-number summaries to plot the parallel boxplots. Use a suitable scale that will cover the full range of values for both data sets.



b. Compare and contrast the data sets, looking at where the key points of each data set lie. Comment on any noticeable differences in the centre and spread of the scores, as well as the shape of the distributions.

b. On the whole, Breed X is considerably lighter than Breed Y, with only a small overlap in the data sets. Breed X has a smaller interquartile range than Breed Y, although both spreads are balanced with no noticeable skew.

on Resources

Interactivity: Back-to-back stem plots (int-6252)
Interactivity: Parallel boxplots (int-6248)

studyon

Units 1 & 2 › Area 6 › Sequence 2 › Concept 3

Comparing datasets and parallel boxplots Summary screen and practice questions

Exercise 12.4 Parallel boxplots

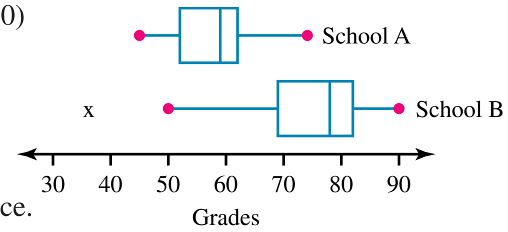
1. **WE6** The following back-to-back stem plot shows the amount of sales (in \$000s) for two different high street stores.

Key: 3|4 = \$3400

Store 1 Leaf	Stem	Store 2 Leaf
	3	4 7 9
7 4	4	2 4 6 8
6 2 1	5	1 2 5 5 9
8 8 5 5	6	3 5 7
5 3 2	7	
6 1	8	
0	9	

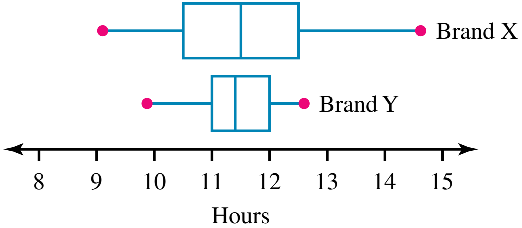
   a. Construct a parallel boxplot of the two sets of data.
   b. Compare and contrast the two sets of data.

2. The parallel boxplot shows the difference in grades (out of 100) between students at two schools.



   a. Which school had the highest overall grade?
   b. Which school had the lowest overall grade?
   c. Calculate the difference between the interquartile ranges of the grades of the two schools, then interpret the difference.

3. The parallel boxplot shows the performance of two leading brands of battery in a test of longevity.



   a. Which brand had the better median performance? Justify your response in a concise way.
   b. Which brand gave the most consistent performance? Justify your response in a concise way.
   c. Which brand had the worst performing battery? Justify your response in a concise way.
   d. Which brand had the best performing battery? Justify your response in a concise way.

4. The prices of main meals at two restaurants which appear in the *Good Food Guide* are shown in the following back-to-back stem plot.

Key: 1|8 = \$18

Restaurant A Leaf	Stem	Restaurant B Leaf
	1	8 9
9 9 8 5 5 4	2	2 5 5 7
8 6 5 5 2	3	0 0 2 5 5 8
2 0 0	4	0 3 6
	5	
9	6	

a. Identify any outliers in either set of data.
b. Prepare five-number summaries for the price of the meals at each restaurant.
c. Construct a parallel boxplot to compare the two data sets.
d. Compare and contrast the cost of the main meals at each restaurant.

5. The following table displays the number of votes that two political parties received in 15 different constituencies in the local elections.

Party A	425	630	813	370	515	662	838	769
Party B	632	924	514	335	748	290	801	956

Party A	541	484	745	833	497	746	651
Party B	677	255	430	789	545	971	318

a. Prepare five-number summaries for both parties’ votes.
b. Display the data sets on a parallel boxplot.
c. Comment on the distributions of both data sets.

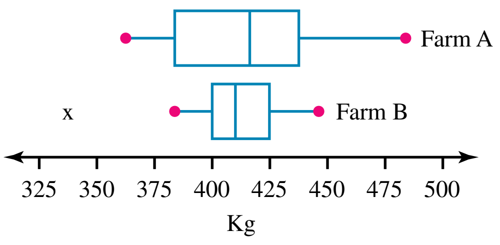
6. The following back-to-back stem plot shows the share prices (in $) of two companies from 18 random months out of a 10-year period.

Key: 1*|7 = $17

Company A Leaf	Stem	Company B Leaf
4 2	1	
9 7 5	1*	7 9
4 4 1	2	0 3 4
9 8 6 6	2*	7 8 8
3 3 2 0	3	1 2 4
8 6	3*	6 9
	4	0 1 2
	4*	5 6

a. Display the data in two frequency tables with intervals of 5.
b. Display the data on a parallel boxplot.
c. Comment on the distributions of both data sets.

7. The following parallel boxplot details the amount of strawberries harvested in kg at two different farms for the month of March over a 15 year period.

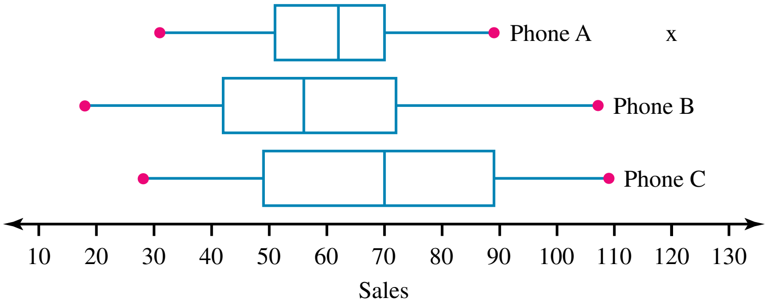


Decide whether the following statements are true or false.
a. Farm A produced a larger harvest of strawberries in March than Farm B more often than not.
b. The strawberry harvest at Farm B in March is much more reliable than the strawberry harvest at farm A.

c. Farm A had the highest producing month for strawberries on record.

d. Farm A had the lowest producing month for strawberries on record.

8. The following parallel boxplot shows the weekly sales figures of three different mobile phones across a period of six months.



a. Which phone had the highest weekly sales overall?

b. Which phone had the most consistent sales?

c. Which phone had the largest range in sales?

d. Which phone had the largest interquartile range in sales?

e. Which phone had the highest median sales figure?

9. The five-number summaries for the amount collected by three different charities in collection tins over a series of weeks are as follows.

Charity 1: 225, 310, 394, 465, 580
Charity 2: 168, 259, 420, 493, 667
Charity 3: 262, 312, 349, 388, 445

a. Construct a parallel boxplot to compare the collections for the three charities.

b. Compare and contrast the amount collected by the three charities, referring to the boxplots to support your observations.

10. The following data sets show the daily sales figures for three new drinks across a 21-day period.

Drink 1: 35, 51, 47, 56, 53, 64, 44, 39, 50, 47, 62, 66, 58, 41, 39, 55, 52, 59, 47, 42, 60
Drink 2: 48, 53, 66, 51, 37, 44, 70, 59, 41, 68, 73, 62, 56, 40, 65, 77, 74, 63, 54, 49, 61
Drink 3: 57, 49, 51, 49, 52, 60, 46, 48, 53, 56, 52, 49, 47, 54, 61, 50, 33, 48, 54, 57, 50

a. Prepare a five-number summary for each drink, excluding any outliers.

b. Plot a parallel boxplot to compare the sales of the three drinks.

c. Compare and contrast the sales of the three drinks.

Questions **11** and **12** refer to the following stem-and-leaf plots.

Key: 12|2 = 122

Group B		**Group A**
Leaf	Stem	Leaf
	12	2
	13	3 8
6	14	0 4 4 6
8 5 4 2 2	15	2 3 5 7 8
8 5 5 3 0 0	16	2 4 4 5
7 4 4 1 0	17	2 6
1 1	18	1

11. **MC** The lower quartile of Group B is:

A. 156.5.
B. 144.
C. 155.
D. 152.

12. **MC** Which of the following statments is false?

A. Data from Group A show less consistency than the data from Group B.
B. Data from Group B have a lower interquartile range.
C. Group B has a greater median.
D. None of the above. (All of the statements are true.)

This data set is for questions **13** and **14**. The following back-to-back stem plot displays the rental price (in $) of one-bedroom apartments in two different suburbs.

Key: 25|0 = $250

Suburb A		Suburb B
Leaf	Stem	Leaf
	25	0
	26	5 9
5 5	27	0 0 5
9 9 5 0	28	5 9 9
5 5 0	29	0
5 5 0 0 0 0	30	0 0 0
5 5 0 0	31	0 5 5
	32	9 9
	33	
	34	0 0
0	35	

13. **a.** Prepare a five-number summary for each suburb, excluding any outliers.
b. Construct a parallel boxplot to compare the data sets.
c. Compare and contrast the rental price in the two suburbs.

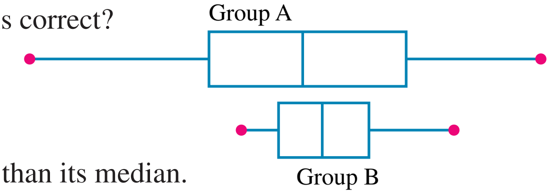
14. **a.** The rental prices in a third suburb, Suburb C, were also analysed, with the data having a five-number summary of 280, 310, 325, 340, 375. Add the third data set to your parallel boxplot.
b. Compare the rent in the third suburb with the other two suburbs.

12.5 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. **MC** For the following parallel boxplots, which statement is correct?



A. Group A has a smaller IQR than Group B.
B. Group B has a greater range than Group A.
C. Group A has a higher median than Group B.
D. 25% of Group B is greater than Group A's Q_1 and less than its median.

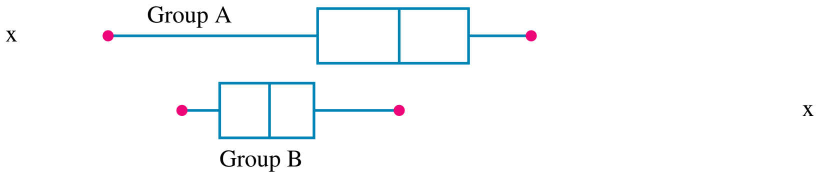
2. **MC** For the data set 789, 211, 167, 321, 432, 222, 234, 456, 456, 234, the five-figure summary in order from the smallest value to the largest is:

A. 167, 222, 321, 456, 789.
B. 167, 222, 277.5, 456, 789.
C. 167, 234, 432, 456, 789.
D. 167, 234, 432, 456, 789.

3. **MC** The following boxplot would best be described as:

A. positively skewed.
B. symmetrical with an outlier.
C. positively skewed with an outlier.
D. negatively skewed with an outlier.

4. **MC** For the following parallel boxplots, which statement is **not** correct?



A. 75% of Group A is larger than 75% of Group B.
B. 25% of Group A has a larger spread than 75% of Group B.
C. The median of Group A is equal to the highest value of Group B
D. Group A is negatively skewed and Group B is positively skewed.

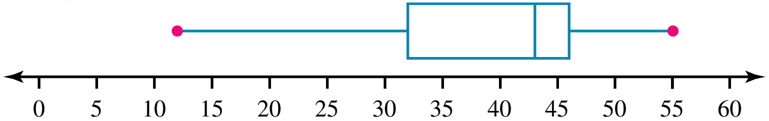
5. **MC** For the data set 21, 56, 110, 15, 111, 45, 250, 124, 78, 24, the number of outliers and the value of 1.5 × IQR will respectively be:

A. 0 and 87. **B.** 1 and 87. **C.** 1 and 111. **D.** 1 and 130.5.

6. For the data set below, give a five-number summary.

24 53 91 57 29 69 29 15 84 6

7. For the box-and-whisker plot shown:



a. state the median
b. calculate the range
c. calculate the interquartile range.

8. The number of babies born each day at a hospital over a year is tabulated and the five-number summary is given below.

Lower extreme = 1
Upper quartile = 16
Lower quartile = 8
Upper extreme = 18
Median = 14

Show this information in a box-and-whisker plot.

9. Construct side-by-side boxplots for the following pair of five-number summaries.
Group X: 14, 18.5, 21.5, 27.5, 33
Group Y: 11, 17.5, 21, 26.5, 35

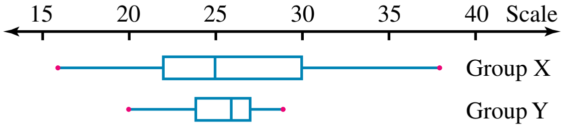
10. The following stem-and-leaf plots give the age at marriage of a group of 10 women and a group of 10 men.

Key: 1|8 = 18 years old

Men Leaf	Stem	**Women** Leaf
8 7	1	8 8
9 8 7 5 1	2	0 2 3 4 4 5
6 3	3	0 1
0	4	

a. Construct side-by-side boxplots of the data.
b. Make comparisons about the distribution of the sets of data.

Questions **11** and **12** refer to the following boxplots.



11. **MC** Which of the following statements is a correct comparison of the data?
A. Group X has a higher median and shows more variability than Group Y.
B. Group X has a lower median and shows more variability than Group Y.
C. Group X has a higher median and shows less variability than Group Y.
D. It is impossible to make comparisons like this without seeing the data displayed on a stem-and-leaf plot.

12. **MC** Which of the following statements is untrue of the boxplots?
A. One-quarter of all Group X data is greater than any of Group Y data.
B. The median of Group X is 25.
C. The interquartile range of Group X is 25.
D. The range of Group Y is 9.

Complex familiar

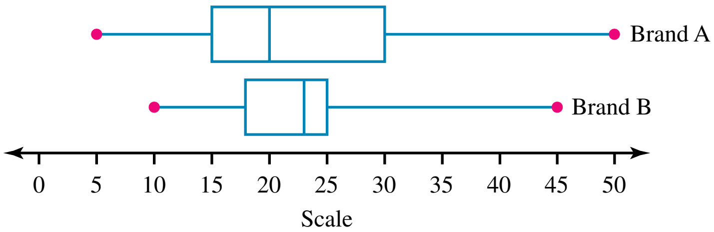
13. The stem-and-leaf plot below represents the number of typing errors recorded by a class of students in 1 page of typing.

Key: 1 | 2 = 12   1* | 5 = 15

Stem	Leaf
0*	0 1 4
0*	6 7 8 9
1*	0 0 1 1 2 3 3 4
1*	5 6 8 9
2*	3

a. Determine how many students are in the class.
b. What is the median number of errors?
c. State the value of the lower quartile.
d. Determine the interquartile range.
e. Construct a boxplot of the data.

14. The box-and-whisker plots show the sales of two different brands of washing powder at a supermarket each day.



a. Calculate the range for Brand A.
b. Calculate the interquartile range for Brand A.
c. Calculate the range for Brand B.
d. Calculate the interquartile range for Brand B.
e. Describe the spread of the sales for each brand of washing powder.

15. The number of words in each of the first 12 sentences is counted in each of 3 different types of book: a children's book, a Year 12 geography text, and a major daily newspaper. The results are as follows:

Children's book	6	8	12	15	6	8	10	8	5	11	10	8
Geography text	16	18	25	13	10	25	29	18	7	22	28	22
Newspaper	12	6	8	14	18	7	12	10	21	17	16	8

a. Construct side-by-side boxplots of the data.
b. Make comparisons about the sentence length of each type of publication. Use statistics in your answer.

16. The stem-and-leaf plot below gives the batting scores of two cricket players — Smith and Jones — who share the responsibility of 'opening the batting' for their side.

Key: 1|2 = 12

Jones Leaf	Stem	**Smith** Leaf
3	0	0 0 1
	1	2 6 9
8 7 4 2	2	
9 9 8 7 7 5	3	
8 4 4 2 0	4	6 6 8
5 2 0	5	7 8 8 9 9
1	6	0 4 6
	7	2 4
	8	5

a. Derive a five-number summary for each player.
b. Construct side-by-side boxplots of the data.
c. Make comparisons between the two sets of data. Use statistics in your answer.
d. Which player do you consider to be the best 'opening bat' and why?

Complex unfamiliar

17. The following table shows data on the Top 10 tourist destinations in Europe in 2009.

Country	Nights in country (× 1000)
Spain	200 552
Italy	158 527
France	98 700
United Kingdom	80 454
Austria	72 225
Germany	54 097
Greece	46 677
Portugal	25 025
Netherlands	25 014
Czech Republic	17 747

a. Display the data as a boxplot.
b. Describe the distribution of the data using the five-figure summary and identify any outliers.

18. The biggest winning margins in AFL Grand Finals up to the year 2013 are shown in the following table.

Winning margin	Year	Winning team	Winning score	Losing team	Losing score
119	2007	Geelong	163	Port Adelaide	44
96	1988	Hawthorn	152	Melbourne	56
83	1983	Hawthorn	140	Essendon	57
81	1980	Richmond	159	Collingwood	78
80	1994	West Coast	143	Geelong	63
78	1985	Essendon	170	Hawthorn	92
73	1949	Essendon	125	Carlton	52
73	1956	Melbourne	121	Collingwood	48
63	1946	Essendon	150	Melbourne	87
61	1995	Carlton	141	Geelong	80
61	1957	Melbourne	116	Essendon	55
60	2000	Essendon	135	Melbourne	75

a. i. Display the winning margin data as a boxplot.
   ii. Display the winning score data as a boxplot.
   iii. Display the losing score data as a boxplot.
b. Describe each boxplot from part a.

19. The following table shows the AFL Grand Final statistics for a sample of players who have kicked a total of 5 or more goals from the clubs Carlton and Collingwood.

Player	Team	Kicks	Marks	Handballs	Disposals	Goals	Behinds
Alex Jesaulenko	Carlton	23	11	9	32	11	0
John Nicholls	Carlton	29	3	1	30	13	1
Wayne Johnston	Carlton	78	19	17	95	5	7
Robert Walls	Carlton	19	9	5	24	11	1
Craig Bradley	Carlton	61	11	37	98	6	2
Mark MacLure	Carlton	34	16	14	48	5	4
Stephen Kernahan	Carlton	44	26	8	52	17	5
Ken Sheldon	Carlton	36	5	12	48	5	2
Syd Jackson	Carlton	13	3	1	14	5	1
Rodney Ashman	Carlton	25	4	10	35	5	2
Greg Williams	Carlton	30	6	29	59	6	4
Alan Didak	Collingwood	46	17	24	70	6	2
Peter Moore	Collingwood	42	22	13	55	11	7
Ricky Barham	Collingwood	42	15	16	58	5	5
Travis Cloke	Collingwood	26	16	9	35	5	4
Ross Dunne	Collingwood	17	6	6	23	5	2
Craig Davis	Collingwood	27	8	8	35	6	3

a. Use the data for goals to compare the two clubs using parallel boxplots.
b. Comment on what the parallel boxplots indicate about the data for goals.
c. Compare the data for kicks and handballs using parallel boxplots.
d. Comment on what the parallel boxplots indicate about the data for kicks and handballs.

20. The following table shows some key nutritional information about a sample of fruits and vegetables.

Food	Calcium (mg)	Serve weight (g)	Water (%)	Energy (kcal)	Protein (g)	Carbohydrate (g)
Avocado	19	173	73	305	4.0	12.0
Blackberries	46	144	86	74	1.0	18.4
Broccoli	205	180	90	53	5.3	10
Cantaloupe	29	267	90	94	2.4	22.3
Carrots	19	72	88	31	0.7	7.3
Cauliflower	17	62	92	15	1.2	2.9
Celery	14	40	95	6	0.3	1.4
Corn	2	77	70	83	2.6	19.4
Cucumber	4	28	96	4	0.2	0.8
Eggplant	10	160	92	45	1.3	10.6
Lettuce	52	163	96	21	2.1	3.8
Mango	21	207	82	135	1.1	35.2
Mushrooms	2	35	92	9	0.7	1.6
Nectarines	6	136	86	67	1.3	16.0
Peaches	4	87	88	37	0.6	9.6
Pears	19	166	84	98	0.7	25.1
Pineapple	11	155	86	76	0.6	19.2
Plums	10	95	84	55	0.5	14.4
Spinach	55	56	92	12	1.6	2.0
Strawberries	28	255	73	245	1.4	66.1

a. Use a spreadsheet or otherwise to convert the water data into its equivalent weight in grams.

b. Compare the data for serve weight with your data for the weight of the water content using parallel boxplots.

c. Comment on the parallel boxplots from part **b**.

d. Use a spreadsheet or otherwise to compare the data for protein and carbohydrate using parallel boxplots.

e. Comment on the parallel boxplots from part **d**.

studyon

Units 1 & 2 **Sit chapter test**

Answers

Chapter 12 Univariate data comparisons

Exercise 12.2 Constructing boxplots

1. a. median, smallest, largest
   b. mean
   c. Q_1, median, Q_3, four

2. Minimum, Lower quartile, Median, Upper quartile, Maximum

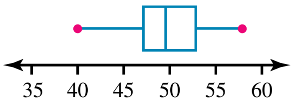
3. 8, 15, 16.5, 18, 25

4. a. 23, 44, 81.5, 83.5, 92
   b. 1, 2, 4, 6, 7
   c. 8, 29, 45, 72, 93

5. a. 13   **b.** 5   **c.** 26

6. a. 122   **b.** 6   **c.** 27

7.



8. a. 147   **b.** 56   **c.** 90   **d.** 91   **e.** 28

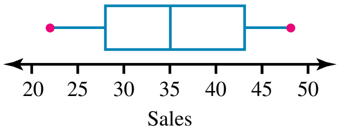
9. a. 58   **b.** 31   **c.** 43   **d.** 27   **e.** 8

10. C

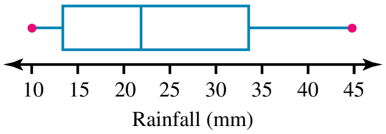
11. A

12. D

13. a. 22, 28, 35, 43, 48
   b.

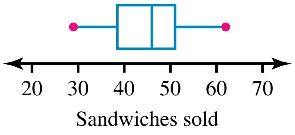


14. a. 10 mm, 13.5 mm, 22 mm, 33.5 mm, 45 mm.
   b.



Exercise 12.3 Outliers and fences

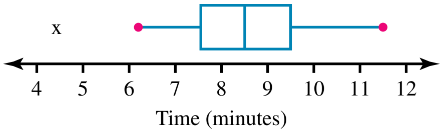
1. a.



   b. Lower fence = 19
   Upper fence = 71

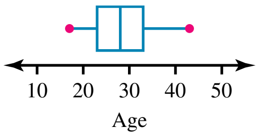
2. a. 25°C   **b.** 21°C   **c.** 7°C   **d.** No

3. a. Lower fence = 4.625, upper fence = 12.425
   b. 4.4 is an outlier.
   c.



4. a. 104   **b.** 43   **c.** 61   **d.** 17

5. a. 17, 23, 28, 33, 43
   b.



   c. The data is fairly symmetrical with no obvious outliers.

6. B

7. A

8. a. True   **b.** True   **c.** False

9. D

10. D

11. C

12. a. Lower fence = −13
   Upper fence = 99
   b. Since the lowest and highest values of 15 and 96 fit within the upper and lower fences of −13 and 99, there are not any outliers in the data.

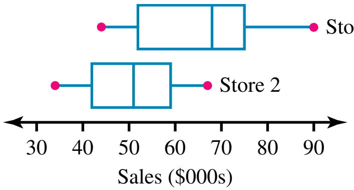
13. a. Five-number summary: 9, 15, 19, 24, 32.
   b. IQR = 9
   c. Lower fence = 1.5
   Upper fence = 37.5
   d. There are no outliers since the 9 and 32 lie within the lower and upper fences.

14. Outliers can unfairly skew data, so can dramatically alter the five-number summary. Identify and remove any outliers from the data before determining the five-number summary.

15. The minimum number will not be increased as it will remain as the minimum.

Exercise 12.4 Parallel boxplots

1. a.

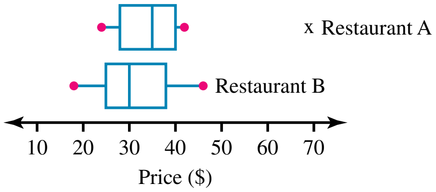


   b. On the whole, store 2 has fewer sales than store 1; however, the sales of store 2 are much more consistent than store 1's sales.
   The sales of store 1 have a negative skew, while the sales of store 2 are symmetrical. There are no obvious outliers in either data set.

2. a. School B
   b. School B
   c. 3 (School B has a bigger interquartile range.)

3. a. Brand X   **b.** Brand Y
   c. Brand X   **d.** Brand X
   Sample responses can be found in the worked solutions in the online resources.

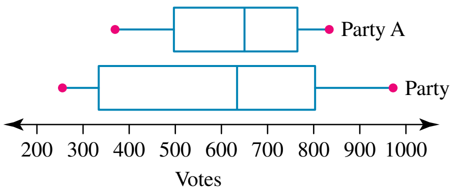
4. a. $69 in Restaurant A is an outlier
   b. Restaurant A: 24, 28, 35, 40, 42
   Restaurant B: 18, 25, 30, 38, 46
   c.



d. The meals in Restaurant A are more consistently priced, but are also in general higher priced. The distribution of prices at Restaurant A has a positive skew, while the distribution of prices at Restaurant B is nearly symmetrical.

5. a. Party A: 370, 497, 651, 769, 838
Party B: 255, 335, 632, 801, 971

b.



c. The spread of votes for Party B is far larger than it is for Party A. Party A polled more consistently and had a higher median number of votes. Party A had a nearly symmetrical distribution of votes, while Party B's votes had a slight negative skew.

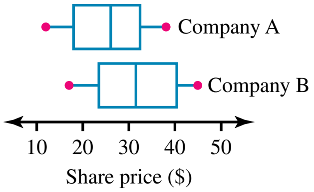
6. a. Company A

Share price ($)	Frequency
10 – < 15	2
15 – < 20	3
20 – < 25	3
25 – < 30	4
30 – < 35	4
35 – < 40	2

Company B

Share price ($)	Frequency
15 – < 20	2
20 – < 25	3
25 – < 30	3
30 – < 35	3
35 – < 40	2
40 – < 45	3
45 – < 50	2

b.

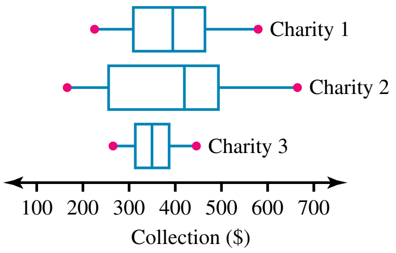


c. On the whole, the share price of Company B is greater than the share price of Company A. However, the share price of Company A is more consistent than the share price of Company B. The share price of Company A has a negative skew, while the share price of Company B has a nearly symmetrical distribution.

7. a. True **b.** True **c.** True **d.** False

8. a. Phone A **b.** Phone A
c. Phone B **d.** Phone C
e. Phone C

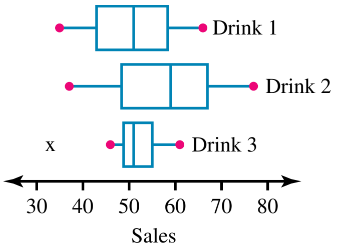
9. a.



b. The collections for Charity 3 were the most consistent of the three charities. Charity 2 collected more money on average than the other charities, but also had the poorest performing week in total. There are no outliers in any of the data sets.

10. a. Drink 1: 35, 43, 51, 58.5, 66
Drink 2: 37, 48.5, 59, 67, 77
Drink 3: 46, 48.5, 51, 55, 61

b.



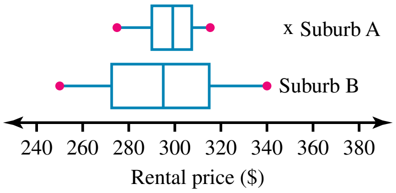
c. The sales of Drink 3 are by far the most consistent, although overall Drink 2 has the highest sales. Drink 2′s sales are also the most inconsistent of all the drinks. There is one outlier in the data sets (33 in Drink 3).

11. C

12. D

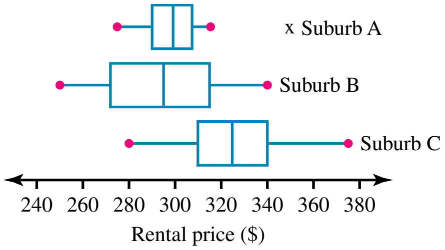
13. a. Suburb A: 275, 289, 300, 307.5, 315
Suburb B: 250, 272.5, 295, 315, 340

b.



c. The rental prices in Suburb A are far more consistent than the rental prices in Suburb B. There is one outlier in the data sets ($350 in Suburb A). Although Suburb A has a higher median rental price, you could not say that it was definitely more expensive than Suburb B.

14. a.



b. Suburb C has a higher average rental price than either Suburb A or B. The spread of the prices in Suburb C is more similar to those in Suburb B than Suburb A.

12.5 Review: exam practice

1. D

2. B

3. B

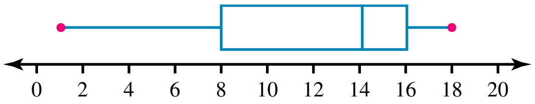
4. C

5. D

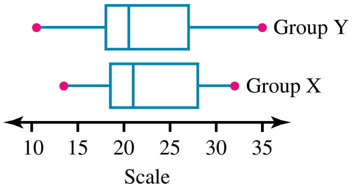
6. 6, 24, 41, 69, 91

7. a. 43 **b.** 43 **c.** 14

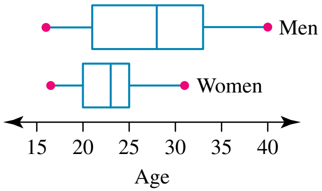
8.



9.



10. a.



b. Women had a smaller range of 13 compared to men with the range of 23. The interquartile range for women was 5 compared to 12 for men. The median age was smaller for women (23.5) compared to men (27.5). Hence, there is less variability in the age at marriage for women.

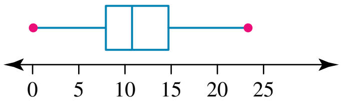
11. B

12. C

13. a. 20 **b.** 11

c. 7.5 **d.** 7

e.



14. a. 45

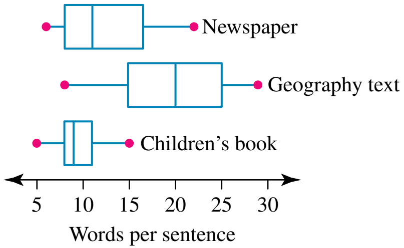
b. 15

c. 35

d. 7

e. The number of daily sales for Brand A varies more than those for Brand B. The lower extreme for Brand A is lower than that for Brand B, and the upper extreme for Brand A is higher than that for Brand B.

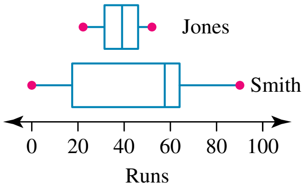
15. a.



b. The longest sentence was in the Geography text book with 29 words. The newspaper’s longest sentence was 21 words, while the children’s book’s longest sentence was 15 words. The shortest sentence went in the order: children (5), newspaper (6) and the geography text (7). The variability was greatest in the geography text (range = 22, IQR = 10.5). Overall the geography text also had a larger central value (median of 20) than the others, children (8) and newspaper (12)

16. a. Smith: 0, 17.5, 57.5, 62, 85; Jones: 3, 31.5, 39, 46, 61

b.



c. **Jones**

Range = 61 − 3 = 58

Interquartile range = 46 − 31.5
= 14.5

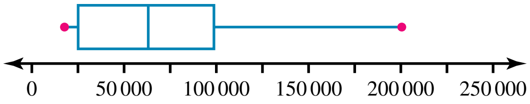
Smith

Range = 85 − 0 = 85

Interquartile range = 62 − 17.5
= 44.5

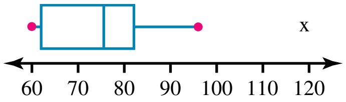
d. Jones’ results are more clustered than Smith’s results indicating less variability in scores obtained. Both players have obtained large ranges overall. Jones’ consistency may indicate that he is better as an opening batsman, while a big hitter like Smith might be better at the end.

17. a.

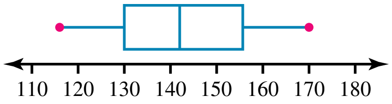


b. Five-figure summary: 17 747, 25 025, 63 161, 98 700, 200 552
The data shows a positive skew with the upper 25% having a much greater spread than the lower 25%. The middle 50% is approximately symmetrical. There are no outliers.

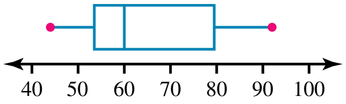
18. a. i.



ii.

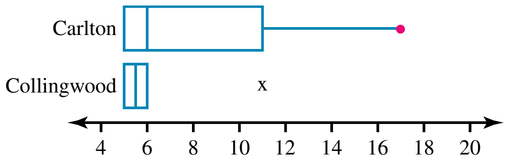


iii.



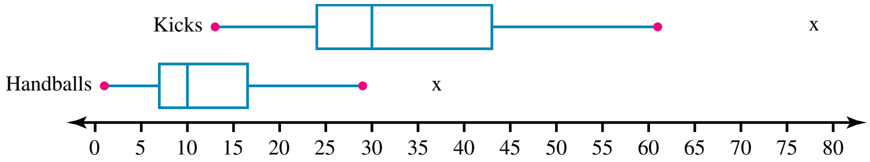
b. i. Negative skew with an upper outlier
ii. Symmetrical with no outliers
iii. Positive skew with no outliers

19. a.



b. Goals scored in Grand Finals for this sample of players is greater but more variable among the Carlton players, as indicated by the larger range and IQR. Collingwood players are concentrated at 5 or 6 with the exception of the one upper outlier of 11.

c.

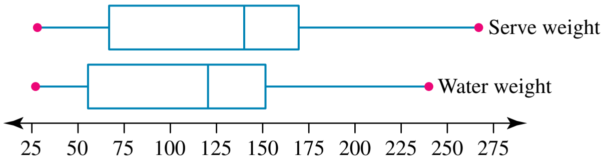


d. Kicks for this sample of players are greater but more variable than the handballs, as indicated by the larger range and IQR. Both are positively skewed with one upper outlier.

20. a.

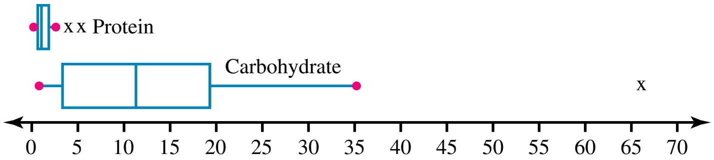
Food	Serve weight (g)	Water (%)	Water weight (g)
Avocado	173	73	126
Blackberries	144	86	124
Broccoli	180	90	162
Cantaloupe	267	90	240
Carrots	72	88	63
Cauliflower	62	92	57
Celery	40	95	38
Corn	77	70	54
Cucumber	28	96	27
Eggplant	160	92	147
Lettuce	163	96	156
Mango	207	82	170
Mushrooms	35	92	32
Nectarines	136	86	117
Peaches	87	88	77
Pears	166	84	139
Pineapple	155	86	133
Plums	95	84	80
Spinach	56	92	52
Strawberries	255	73	186

b.



c. The boxplots appear to indicate that there are only slight differences between the serve weights and water weights of the samples. The distributions are very similar in shape, with the water weights being slightly less overall.

d.

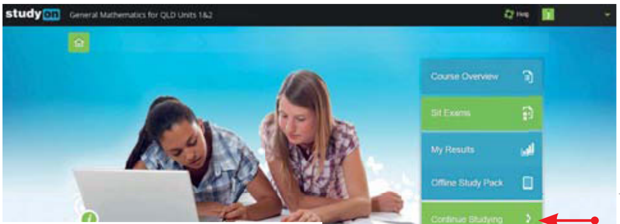


e. Carbohydrate for this sample of foods is much greater but more variable than protein, as indicated by the larger range and IQR. The protein amounts are all less than the Q_1 for the carbohydrate amounts, with the exception of two upper outliers for protein that lie between the Q_1 and median for carbohydrate. Carbohydrate is positively skewed with one upper outlier.

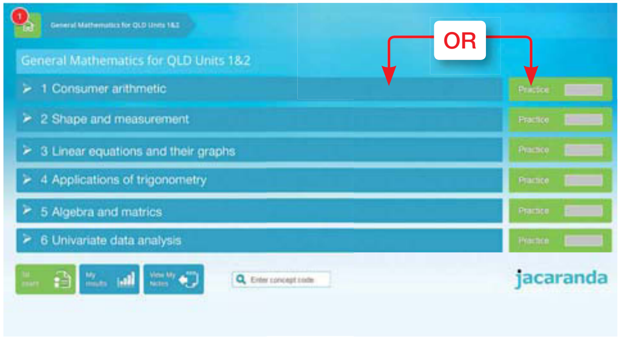
REVISION UNIT 2 Applied trigonometry, algebra, matrices and univariate data

TOPIC 3 Univariate data analysis

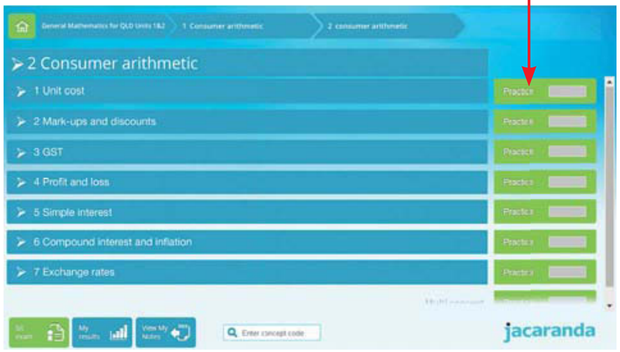
- For revision of this entire topic, go to your **studyON** title in your bookshelf at **www.jacplus.com.au**.
- Select **Continue Studying** to access hundreds of revision questions across your entire course.



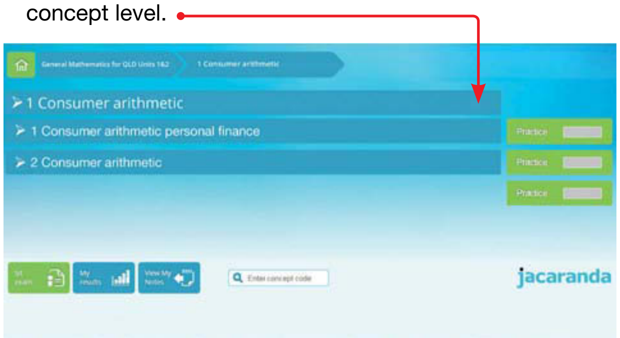
- Select your **course** *General Mathematics for Queensland Units 1&2* to see the entire course divided into syllabus topics.
- Select the **area** you are studying to navigate into the sequence level *OR* select **Practice** to answer all practice questions available for each area.



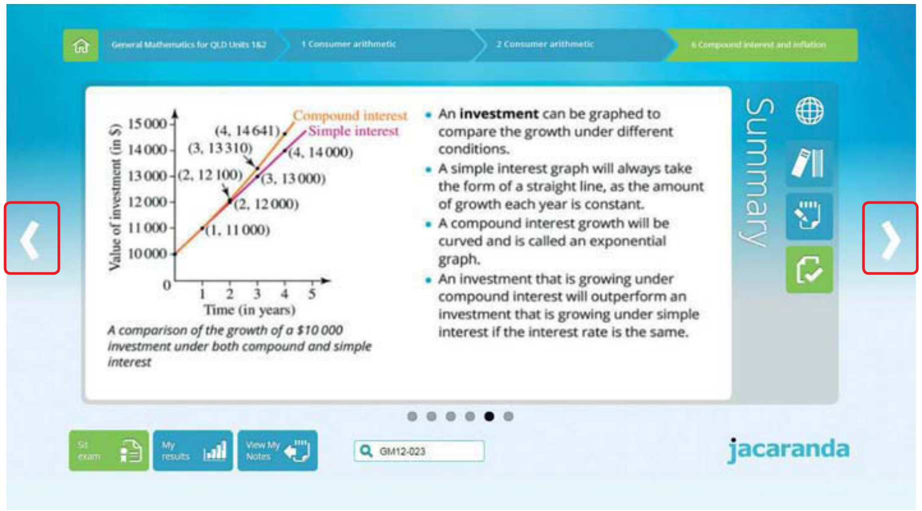
- Select **Practice** at the sequence level to access all questions in the sequence.



- At **sequence level**, drill down to concept level.



- **Summary screens** provide revision and consolidation of key concepts. Select the **next arrow** to revise all concepts in the sequence and practice questions at the concept level for a more granular set of questions.



A comparison of the growth of a $10 000 investment under both compound and simple interest

- An **investment** can be graphed to compare the growth under different conditions.
- A simple interest graph will always take the form of a straight line, as the amount of growth each year is constant.
- A compound interest growth will be curved and is called an exponential graph.
- An investment that is growing under compound interest will outperform an investment that is growing under simple interest if the interest rate is the same.

PRACTICE ASSESSMENT 3

General Mathematics: Unit 2 examination

Unit

Unit 2: Applied trigonometry, algebra, matrices and univariate data

Topic

Topic 1: Applications of trigonometry
Topic 2: Algebra and matrices
Topic 3: Univariate data analysis

Conditions

Response type	Duration	Reading
Short response	120 minutes	5 minutes
Resources	**Instructions**	
• QCAA formula sheet: • Notes not permitted • Scientific calculator permitted	• Show all working. • Write responses using a black or blue pen. • Unless otherwise instructed, give answers to **two decimal places**.	

Criterion	Marks allocated	Result
Foundational knowledge and problem solving *Assessment objectives 1, 2, 3, 4, 5, and 6	60	

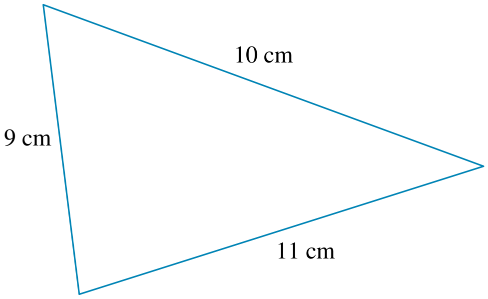
* Queensland Curriculum & Assessment Authority, *Specialist Mathematics General Senior Syllabus 2019 v1.1*, Brisbane, 2018.
For the most up to date assessment information, please see www.qcaa.qld.edu.au/senior.

A detailed breakdown of the examination marks summary can be found in the PDF version of this assessment instrument in your eBookPLUS.

Part A: Simple familiar — total marks: 36

Question 1 (3 marks)

Determine the largest angle in the triangle shown, to 2 decimal places.

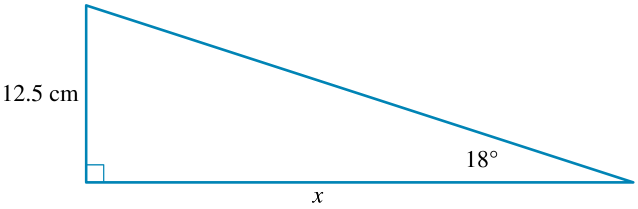


Question 2 (3 marks)

Use Heron's formula to determine the area of a triangle with side lengths of 6.8 cm, 4.3 cm and 8.4 cm.

Question 3 (2 marks)

Determine the value of x in the following triangle, to 2 decimal places.



Question 4 (3 marks)

Evaluate the expression $-7p\,(3q - p)$ when $p = -2$ and $q = 3$.

Question 5 (3 marks)

If Paddy is getting paid $\$13.25$ per hour at his part-time job and over the weekend he works 7 hours of normal time and 5 hours of double time, determine the amount he earns over the weekend.

Question 6 (3 marks)

To calculate the displacement of an object the formula used is $x = \frac{1}{2}(u + v)t$. Transpose the formula to make the initial velocity, u, the subject.

Question 7 (3 marks)

Consider the matrix equation

$$\begin{bmatrix} 8 & a \\ 5 & -3 \end{bmatrix} - \begin{bmatrix} -3 & -3 \\ 0 & 2b \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 5 & 7 \end{bmatrix}.$$

Determine the values of a and b.

Question 8 (2 marks)

If matrices A, B and C are of order 2×2, 3×1 and 2×3 respectively, the product of which two matrices will give a 2×3 matrix? The order of the matrices also needs to be stated.

Question 9 (4 marks)

If $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 0 \\ -2 & 2 \end{bmatrix}$, determine the matrix A.

Question 10 (2 marks)

Data gathered on the number of tackles made by a football team each week is classified as which type of data?

Question 11 (4 marks)

Determine the mean of the data distribution shown in the table.

Interval	**Frequency *(f)***
0– < 15	3
15– < 30	5
30 < 45	8
45– < 60	4

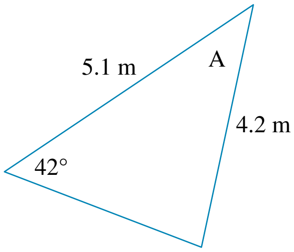
Question 12 (4 marks)

Determine the five-figure summary of the data set shown.
12 34 78 49 18 21 33 81 66 53

Part B: Complex familiar — total marks: 12

Question 13 (4 marks)

Consider the triangle shown.



a. Calculate the angle A.

b. Determine the area of the triangle.

Question 14 (4 marks)

Given matrices $A = \begin{bmatrix} 4 & 3 \\ 7 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -5 & -7 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -2 \\ 12 \end{bmatrix}$, evaluate the following.

a. $2A - B$

b. $2AC - 2C$

Question 15 (4 marks)

You are looking at investing $\$5000$ into an interest-bearing account paying $r\%$ interest compounding annually. The amount, $\$A$, to which your investment will grow is given by the formula

$A = P\left(1 + \frac{r}{100}\right)^n$, where P is the principal amount and n is the number of years.

a. Find A if you invest at 6.25% p.a. for 3 years.

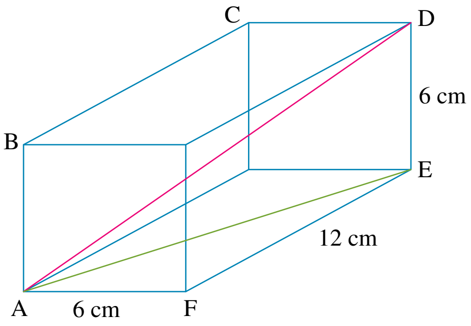
b. By rearranging the formula, find the principal amount you need to invest at 6.5% p.a. to produce an amount of $\$12\,201.33$ over 10 years.

c. Determine r when your $\$5000$ grows to $\$6200$ over 2 years.

Part C: Complex unfamiliar — total marks: 12

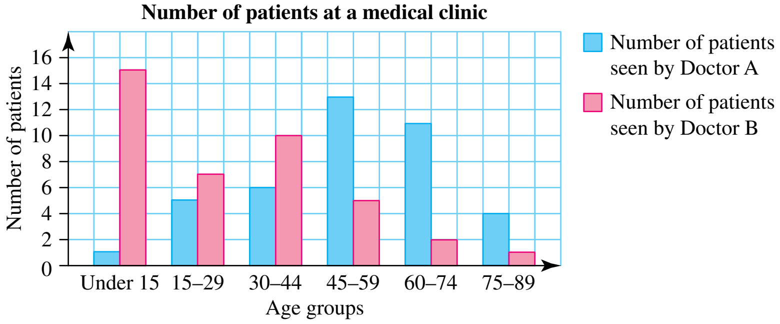
Question 16 (3 marks)

Determine the angle ∠DAE from the following rectangular prism.



Question 17 (2 marks)

The following graphical display summarises the ages at the last birthday of patients seen by two doctors in a medical surgery during one particular day.



a. How many more patients aged under 15 did Doctor B consult compared to Doctor A?

b. Doctor A tends to consult patients aged over 45, whereas Doctor B tends to consult patients aged under 45. True or False?

Question 18 (7 marks)

The stem-and-leaf plot gives the batting scores of two cricketers, Rowe and Harper.

Key 1|3 = 13

Rowe		Harper
Leaf	Stem	Leaf
3		008
9	1	2
73	2	1389
653	3	07
9	4	
971	5	26
	6	6
	7	
	8	7
3	9	
379	10	0

a. Derive a five-number summary for each player.

b. Draw a side-by-side boxplot of the data.

c. Make comparisons between the two data sets, using statistics in your answer.

d. Which player do you consider to be the best at batting and why?

PRACTICE ASSESSMENT 4

General Mathematics : Units 1 & 2 examination

Topic

Unit 1 Topic 1: Consumer arithmetic
Topic 2: Shape and measurement
Topic 3: Linear equations and their graphs
Unit 2 Topic 1: Applications of trigonometry
Topic 2: Algebra and matrices
Topic 3: Univariate data analysis

Conditions

Technique	Response Type	Duration	Reading
Paper 1: Simple familiar Paper 2: Simple familiar, Complex familiar, Complex unfamiliar	Short response	Paper 1: 60 minutes Paper 2: 60 minutes	5 minutes

Resources	Instructions
• QCAA formula sheet: • Notes not permitted • Scientific calculator permitted	• Show all working. • Write responses using a black or blue pen. • Unless otherwise instructed, give answers to **two decimal places**.

Criterion	Marks allocated Paper 1	Marks allocated Paper 2	Result
Foundational knowledge and problem solving *Assessment objectives 1, 2, 3, 4, 5, and 6	40	40	

* Queensland Curriculum & Assessment Authority, *Specialist Mathematics General Senior Syllabus 2019 v1.1*, Brisbane, 2018. For the most up to date assessment information, please see www.qcaa.qld.edu.au/senior.

A detailed breakdown of the examination marks summary can be found in the PDF version of this assessment instrument in your eBookPLUS.

Paper 1 – Simple familiar

Simple familiar — total marks: 40

Question 1 (5 marks)

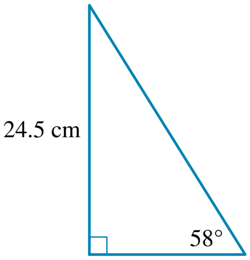
A car salesperson is paid a set weekly wage of $\$325$ per week and a commission of 1.25% on their weekly sales. Calculate their fortnightly earnings to the nearest cent, given their sales over two weeks.

Week 1: $\$55\,000$

Week 2: $\$72\,500$

Question 2 (3 marks)

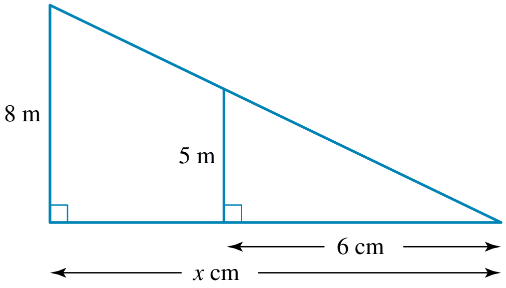
Consider the following right-angled triangle.



Calculate the length of the side adjacent to 58°.

Question 3 (3 marks)

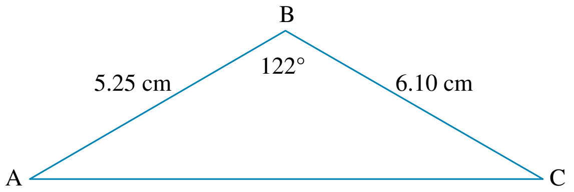
Calculate the value of x in the figure.



Question 4 (5 marks)

a. Calculate the length of AC.

b. Calculate the area of the triangle.



Question 5 (4 marks)

A person is looking at the top of a building in New York at an angle of elevation of $62°$. If their eye level is 1.80 m and they are standing 85 m from the base of the building, determine is the height of the building.

A diagram is required as a part of your answer.

Question 6 (4 marks)

A plumber charges a callout fee plus an hourly rate on top of that.

Fees	Cost
Callout fee	\$125
$0 <$ time ≤ 1 hours	\$95 per hour
$1 <$ time ≤ 3 hours	\$80 per hour
$3 <$ time ≤ 6 hours (max.)	\$60 per hour

Construct a step graph to display this information.

Question 7 (2 marks)

Transpose the formula $s = ut + \frac{1}{2}at^2$ to make a the subject.

Question 8 (2 marks)

John is comparing prices of washing machines and dryer at three different stores, stores A, B and C. He finds the prices for the washing machine to be \$598, \$625 and \$565 respectively, whereas the prices for the dryers are \$425, \$380 and \$420 respectively.

Construct a matrix to show these prices..

Question 9 (2 marks)

Determine which size bag of dogfood is the best value for money. Use mathematics to justify your response.

a. 18 kg for \$49

b. 7 kg for \$16

c. 3 kg for \$7.50

Question 10 (4 marks)

The marks for a General Mathematics examination are shown for a class of 20 students.

90 87 71 52 78 83 93 55 67 70
42 60 70 84 88 39 76 99 50 78

Construct an ordered stem-and-leaf plot to display the data collected.

Question 11 (2 marks)

The label on a packet of chips states it has a mass of 170 grams. A sample of 12 packets were weighed and were found to have the following masses.

165 178 172 169 170 166
170 181 161 173 176 173

Use a calculator to determine the mean and standard deviation for this sample.

Question 12 (4 marks)

A sample of Year 11 students' average number of texts per day is listed below.

9 18 23 26 29 33 41 50 55 59 62 66 99

Identify any outlier/s that may be present in this data. Provide statistical calculations to justify your answer.

Paper 2 – Simple familiar, Complex familiar and Complex unfamiliar

Part A: Simple familiar — total marks: 8

Question 1 (4 marks)

If $A = \begin{bmatrix} 2 & 1 \\ -3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$, calculate the value of:

a. $2A + 2C$

b. AB

Question 2 (4 marks)

Choose from the following words to fill in the blanks in the following sentences.
Categorical, numerical, nominal, ordinal, discrete, continuous

a. People at a local supermarket were surveyed on their favourite soft drink. The data collected is classified as __________________.

b. Data is collected on the number of people attending a local rugby game. This data is classified as __________________ and the variable is identified as __________________.

c. A farmer measures the daily rainfall on his property in his rain gauge. The data collected is classified as __________________

Part B: Complex familiar — total marks: 16

Question 3 (5 marks)

Calculate the value of x and y if:

$$2\begin{bmatrix} 3y & 7 \\ 3 - x & -1 \end{bmatrix} + \begin{bmatrix} 2x - 1 & -1 \\ 2y & 5 \end{bmatrix} = \begin{bmatrix} -1 & 13 \\ -2 & 3 \end{bmatrix}$$

Question 4 (6 marks)

A new board game is a composite shape. It has two sides which are at right angles to each other, which are 30 cm and 40 cm long. The hypotenuse to these two sides forms the diameter of a semicircle.

Determine the area of the board.

A diagram is required as a part of your response.

Question 5 (5 marks)

Sharyn was on a trip to the United States and purchased some clothing when the exchange rate for Australian dollars was $\$0.78$ for $\$\text{US}\,1$. She paid $\$\text{US}\,95$ for a pair of shoes, $\$\text{US}89.50$ for a dress and got 20% off a handbag that was priced at $\$\text{US}\,120$. What did Sharyn pay in total for her purchases in Australian Dollars?

Part C: Complex unfamiliar — total marks: 16

Question 6 (6 marks)

Two presents are to be wrapped. One is the shape of a cube and has a volume of $20\,\text{cm}^3$, and the other is a sphere with a volume of $16\,\text{cm}^3$. Which present requires the least wrapping paper?

Show mathematical calculations to justify your response.

Question 7 (5 marks)

The following two linear equations make a piecewise linear graph.

$y = -2x + 3, x < 2$

$y = x - 2, x \geq 2$

a. Calculate the value of y when $x = 4$.

b. Sketch the piecewise graph.

c. Change the equation of the line $y = x - 2$ so that the graphs intersect at $x = 2$.

Question 8 (5 marks)

Narelle is starting up a gift card business. It will cost her $\$441$ to buy all the equipment, and each card will cost $\$2.75$ to make. Narelle models her costs, C, in dollars, to make n cards using the following equation:

$C = 2.75n + D$

a. Identify the value of D.
b. How much will it cost Narelle to make 150 cards.
c. Narelle has an order from one of her customers. If it cost Narelle $\$922.25$ to make the order, calculate how many cards the order was for.
d. Narelle sells each card for $\$7.25$. Write an equation to represent the revenue, R, for selling n cards.
e. One week, Narelle sells 275 cards. Determine the profit Narelle makes in that week.
f. By solving a pair of simultaneous equations, find the number of cards Narelle needs to sell to break even. Give your answer to the nearest whole number.

GLOSSARY

adjacency matrix a matrix that represents the number of connections between objects in a network or the number of edges that connect the vertices of a graph.

adjacent the side next to the angle used for reference in a right-angled triangle.

allowance an extra payment made to a worker for working in unfavourable conditions.

angle of depression the angle measured down from the horizontal line (through the observation point) to the line of vision.

angle of elevation the angle measured up from the horizontal line (through the observation point) to the line of vision.

annulus the area between two circles with the same centre. The formula is $A = \pi(R^2 - r^2)$, where R is the radius of the outer circle and r is the radius of the inner circle.

area the two-dimensional space taken up by an object.

area scale factor the ratio of the corresponding areas of similar objects. It is equal to the linear scale factor raised to the power of two.

bar charts displays with the categories of data on one axis (usually the horizontal axis) and the frequency of the data on the other (usually the vertical axis).

BIDMAS the order of operations to evaluate an expression: brackets, indices, division, multiplication, addition and subtraction.

boxplot a graphical representation of the five-number summary.

break-even point the point at which revenue begins to exceed the cost of production.

capacity the amount of fluid an object can contain

categorical data data that can be organised into groups or categories and is often an ‘object’, ‘thing’ or ‘idea’. Examples include brand names, colours, general sizes and opinions.

circumference the perimeter of a curved figure, the circumference of a circle is $\text{C} = 2\pi r = \pi \text{D}$

coefficient the number multiplying (out the front) the pronumeral, e.g. 4 in $4xy$.

coincident two lines are coincident if they lie one on top of the other.

column graph graph where the data are displayed in vertical columns.

column matrix a matrix that has only one column.

commission payment made to a salesperson. A commission is usually paid as a percentage of sales.

common difference the difference between each term in an arithmetic sequence: $d = t_{n+1} - t_n$.

cone a solid object in which one end is circular and the other end is a single vertex. Its cross-section is a series of circles that gradually get smaller as they approach the vertex.

congruent objects that are the same size and shape.

constant term the term that does not have a pronumeral, e.g. -2 in $7xy + y - 2$.

continuous data numerical data that can take any value that lies within an interval. Continuous data values are subject to the accuracy of the measuring device being used.

cosine ratio the ratio of the adjacent side and hypotenuse in a right-angled triangle.

cylinder a solid object with ends that are identical circles and a cross-section that is the same along its length.

difference only like terms can be subtracted; e.g. $5y - 2y = 3y$.

discount the amount of money which the price of an item is reduced by.

discrete data numerical data that is counted in exact values, with the values often being whole numbers.

dividend yield (percentage dividend) the dividend paid per share as a percentage of the share price,

$$\text{dividend yield} = \frac{\text{dividend}}{\text{shareprice}} \times 100$$

DMS degrees, minutes, seconds.

dot plot a plot in which every data value is represented by a dot, used to identify the most common values.

double time a penalty rate that pays the employee twice the normal hourly rate.

elements entries in a matrix.

elimination the process of simplifying a mathematical expression by removing a variable; common when solving simultaneous equations

enlargement a figure is drawn similar to, but larger than the original. The corresponding sides will be in equal ratio and all corresponding angles will be equal.

equation a mathematical statement containing a left and right-hand side separated by an equality sign.

evaluate find a numerical answer for an equation.

expression a mathematical statement made up of terms, operation symbols and/or brackets. It does not contain an equality sign, e.g. $7xy + y - 2$.

floor plan a plan showing the floor dimensions of a structure and detailed dimensions of features such as doors, windows, wall thicknesses and stairs.

formula or **rule** an equation showing the relationship between two or more quantities.

frequency tables displays that tabulate data according to the frequencies of predetermined groupings.

gradient also known as the slope; determines the change in the y-value for each change in x-value. This measures the steepness of a line as the ratio $m = \dfrac{rise}{run}$. If (x_1, y_1) and (x_2, y_2) are two points on the line, $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$.

gradient–intercept form the form of a linear equation expressed as $y = mx + c$, where m is the gradient and c is the y-intercept.

goods and services tax (GST) a consumption tax charged on most goods and services.

Heron's formula a way of calculating the area of the triangle if you are given all three side lengths, $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

histogram a display of continuous numerical data similar to a bar chart, in which the width of each column represents a range of data values and the height of the column represents that range's frequency.

hypotenuse the longest side of a right-angled triangle. The hypotenuse is opposite the right angle.

identity matrix a square matrix in which all the elements on the diagonal line from the top left to bottom right are 1s and all of the other elements are 0s.

inflation the rate at which the price of goods and services increase

interquartile range the difference between the upper and lower quartiles of a data set.

like terms like terms have identical pronumerals, e.g. $-2y$ and y.

linear equation an equation in which the highest power of any variable is one.

linear relation a relation between up to two variables of degree 1 that produces a straight line.

linear scale factor the ratio of the corresponding side lengths of similar objects.

literal equation an equation that contains pronumerals rather than numerals as terms or coefficients.

lower fence the lower boundary beyond which a data value is considered to be an outlier: $Q_1 - 1.5 \times \text{IQR}$.

lower quartile (Q_1) the median of the lower half of an ordered data set.

matrix a rectangular array of rows and columns that is used to store and display information.

mark-up the amount or percentage increase added to the costs of goods or services.

mean commonly referred to as the average; a measure of the centre of a set of data. The mean is calculated by dividing the sum of the data values by the number of data values.

median the middle value of the ordered data set if there are an odd number of values, or halfway between the two middle values if there are an even number of values, $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ data value

mode the category or data value(s) with the highest frequency. It is the most frequently occurring value in a data set.

net a 2-dimensional plan of the surfaces that make up a 3-dimensional object

networks an arrangement of interconnecting lines which shows the pathways between points.

nominal data categorical data that has no natural order or ranking.

numerical data data that can be counted or measured.

opposite the side opposite to the angle used for reference in a right-angled triangle.

order the indice or power of a number expressed as a base number and an indice.

ordinal data categorical data that can be placed into a natural order or ranking.

outlier an extreme value or unusual reading in the data set, generally considered to be any value beyond the lower or upper fences.

overtime when a person earns more than the regular hours each week.

parallel boxplots displays in which two or more boxplots share the same scale to enable comparisons between data sets.

penalty rate a higher rate of pay made to a person who is working overtime.

percentage a rate per hundred.

percentage discount the discount of the price of an item expressed as a percentage of the original cost.

perimeter the distance around an object.

piecewise graphs continuous graphs formed by two or more linear graphs that are joined at points of intersection.

piecework payment for the amount of work completed.

polygons 2-dimensional shapes consisting of at least three straight sides.

prism a solid object that has identical ends that are joined by flat surfaces, and a cross-section that is the same along its length.

product to find the product of algebraic terms, the terms are multiplied; e.g. $5y \times 2x = 10xy$.

product matrix the result of multiplying two matrices where the number of columns in the first matrix equals the number of rows in the second matrix. For matrices of order $m \times n$ and $n \times r$, the product matrix will have order $m \times r$.

profit the positive difference between what a product is sold for less than what it cost.

pronumeral a letter or symbol representing a number (usually a variable) in a mathematical expression or equation (twice).

proportional when two quantities have the same ratio; therefore, they always have the same size in relation to each other.

pyramid a solid object whose base is a polygon and whose sides are triangles that meet at a single point.

Pythagoras' theorem in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides, $c^2 = a^2 + b^2$

Pythagorean triads (or **Pythagorean triples**) sets of three numbers which satisfy Pythagoras' theorem.

range a measure of the spread of a numerical data set determined by calculating the difference between the smallest and largest values.

rate a measure of how one quantity is changing compared to another.

reduction a similar figure, drawn smaller in size than the original.

retainer a fixed payment usually paid to someone receiving commission. They receive the retainer regardless of the number of sales made.

row matrix a matrix that has only one row.

salary a form of payment where a person is paid a fixed amount to do their job. A salary is usually based on an annual amount divided into weekly or fortnightly instalments.

scalar multiplication each element of the matrix is multiplied by the same number, called a 'scalar'. A scalar quantity can be any real number, such as negative or positive numbers, fractions or decimal numbers.

scale factor a number by which the side lengths on the first of two similar figures is multiplied by to obtain the measurements on the second of the figures.

sectors fractions of a circle. The area of a sector can be calculated using $A = \frac{\theta}{360}\pi r^2$.

sector graph a graph where a circle is cut into sectors. Each sector then represents a section of the data set. Each sector is the same proportion of the circle as the part of the data set it represents.

similar objects that are the same shape but have different sizes.

simultaneous equations equations belonging to a system of equations in which the solutions for the values of the unknowns must satisfy each equation.

sine ratio the ratio of the opposite side and the hypotenuse in a right-angled triangle.

sine rule the ratios of a side length with the sine of the angle opposite it are equal throughout a triangle: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$. This can be used to find the unknown side length or angle in non-right-angled triangles.

site plan a plan showing the boundaries of a block of land and the position of the structure on the lot.

sphere a solid object that has a curved surface such that every point on the surface is the same distance (the radius of the sphere) from a central point.

square matrix a matrix that has the same number of rows and columns.

standard deviation the most common measure of the spread of data around the mean; found by taking the square root of the variance, $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

stem plots arrangements used for numerical data in which data points are grouped according to their numerical place values (the 'stem') and then displayed horizontally as single digits (the 'leaf').

step graphs discontinuous graphs formed by two or more linear graphs that have zero gradients.

substitute replace with an equivalent value or expression.

sum only like terms can be added together; e.g. $2x + x = 3x$.

surface area the combined total of the areas of each individual surface that forms a solid object.

survey plan a plan showing all boundaries of blocks of land and the position of roadways.

tangent ratio the ratio of the opposite side and the adjacent side in a right-angled triangle.

term a group of letters and numbers. These form an expression and are separated by an addition or subtraction sign; i.e. $4xy$

time and a half a penalty rate where the employee is paid 1.5 times the normal hourly rate.

transpose to rearrange an expression or formula.

trigonometry a branch of mathematics in which sides and angles of triangles are calculated.

unit cost for cost of a single unit of an item, this may be a standard weight or volume, or individual item.

upper fence the upper boundary beyond which a data value is considered to be an outlier: $Q_3 + 1.5 \times \text{IQR}$.

upper quartile (Q_3) the median of the upper half of an ordered data set.

variables a quantity that can take on a range of values depending on its relationship to other values; typically represented by pronumerals.

variance a measure of the spread of a data set from the mean: $s^2 = \frac{\sum f(x-\bar{x})^2}{(\sum f) - 1}$.

volume the amount of space that is taken up by any solid or 3-dimensional object.

volume scale factor the ratio of the corresponding volumes of similar objects. This is equal to the linear scale factor raised to the power of 3.

wage a form of payment that is based on an hourly rate.

***x*-intercept** the point where the graph of an equation crosses the x-axis. This occurs when $y = 0$.

***y*-intercept** the point where the graph of an equation crosses the y-axis. This occurs when $x = 0$.

zero matrix a square matrix that consists entirely of '0' elements.

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