

YEAR 9

CambridgeMATHS NSW

STAGE 5.1 / 5.2 / 5.3

SECOND EDITION

STUART PALMER, DAVID GREENWOOD
SARA WOOLLEY, JENNY GOODMAN,
JENNIFER VAUGHAN

INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

CAMBRIDGE UNIVERSITY PRESS

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

© Stuart Palmer, David Greenwood, Sara Woolley, Jenny Goodman,
Jennifer Vaughan 2014, 2019

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2014
Second Edition 2019
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6

Cover designed by Sardine Designs
Text design by Loupe Studios
Typeset by diacriTech
Printed in China by C & C Offset Printing Co., Ltd.

A catalogue record is available for this book from the National Library of Australia at www.nla.gov.au

ISBN 978-1-108-46816-9 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

Reproduction and Communication for educational purposes
The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this publication, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact:

Copyright Agency Limited
Level 12, 66 Goulburn Street
Sydney NSW 2000
Telephone: (02) 9394 7600
Facsimile: (02) 9394 7601
Email: memberservices@copyright.com.au

Reproduction and Communication for other purposes
Except as permitted under the Act (for example a fair dealing for the purposes of study, research, criticism or review) no part of this publication may be reproduced, stored in a retrieval system, communicated or transmitted in any form or by any means without prior written permission. All inquiries should be made to the publisher at the address above.

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate. Information regarding prices, travel timetables and other factual information given in this work is correct at the time of first printing but Cambridge University Press does not guarantee the accuracy of such information thereafter.

Cambridge University Press acknowledges the Australian Aboriginal and Torres Strait Islander peoples of this nation. We acknowledge the traditional custodians of the lands on which our company is located and where we conduct our business. We pay our respects to ancestors and Elders, past and present. Cambridge University Press is committed to honouring Australian Aboriginal and Torres Strait Islander peoples' unique cultural and spiritual relationships to the land, waters and seas and their rich contribution to society.

Table of Contents

About the authors viii
Introduction and guide to this book ix
Overview of the digital resources xiv
Acknowledgements xviii

1 Computation and financial mathematics 2

Number and Algebra
Computation with integers (S4)
Fractions, decimals and percentages (S4)
Financial mathematics (S4, 5.1, 5.2◊)
MA4–4NA, MA4–5NA, MA5.1–4NA, MA5.2–4NA

Pre-test 4
1A Computation with integers REVISION 5
1B Decimal places and significant figures 10
1C Rational numbers REVISION 14
1D Computation with fractions REVISION 19
1E Ratios, rates and best buys REVISION 24
1F Computation with percentages and money REVISION 29
1G Percentage increase and decrease REVISION 34
1H Profits and discounts REVISION 38
1I Income 43
1J The PAYG income tax system 50
1K Simple interest 56
1L Compound interest and depreciation 62
1M Using a formula for compound interest and depreciation 67
Investigation 73
Puzzles and challenges 75
Review: Chapter summary 76
Multiple-choice questions 77
Short-answer questions 78
Extended-response questions 79

2 Expressions, equations and inequalities 80

Number and Algebra
Algebraic techniques (S4)
Equations (S4, 5.2)
MA4–10NA, MA5.2–8NA

Pre-test 82
2A Algebraic expressions REVISION 83
2B Simplifying algebraic expressions REVISION 88
2C Expanding algebraic expressions 92
2D Linear equations with pronumerals on one side 96
2E Linear equations with brackets and pronumerals on both sides 100
2F Using linear equations to solve problems 104
2G Linear inequalities 108
2H Using formulas 112
2I Linear simultaneous equations: substitution 117
2J Linear simultaneous equations: elimination 121
2K Using linear simultaneous equations to solve problems 126
2L Quadratic equations of the form $ax^2 = c$ 130

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

	Investigation	135
	Puzzles and challenges	137
	Review: Chapter summary	138
	Multiple-choice questions	139
	Short-answer questions	140
	Extended-response questions	141

3 Right-angled triangles 142

Measurement and Geometry

Right-angled triangles (Pythagoras) (S4)
Right-angled triangles (trigonometry) (S5.1/5.2◊)
MA4–16MG, MA5.1–10MG, MA5.2–13MG

	Pre-test	144
3A	Pythagoras' theorem REVISION	145
3B	Finding the shorter sides REVISION	152
3C	Using Pythagoras' theorem to solve two-dimensional problems REVISION	157
3D	Using Pythagoras' theorem to solve three-dimensional problems	162
3E	Introducing the trigonometric ratios	167
3F	Finding unknown sides	173
3G	Solving for the denominator	178
3H	Finding unknown angles	182
3I	Using trigonometry to solve problems	186
3J	Bearings	192
	Investigation	198
	Puzzles and challenges	200
	Review: Chapter summary	201
	Multiple-choice questions	202
	Short-answer questions	203
	Extended-response questions	205

4 Linear relationships 206

Number and Algebra

Linear relationships (S5.1, 5.2, 5.3§)
MA5.1–6NA, MA5.2–9NA, MA5.3–8NA

	Pre-test	208
4A	Introducing linear relationships	209
4B	Graphing straight lines using intercepts	215
4C	Lines with only one intercept	219
4D	Gradient	224
4E	Gradient and direct proportion	231
4F	Gradient–intercept form	236
4G	Finding the equation of a line using $y = mx + b$	242
4H	Midpoint and length of a line segment from diagrams	247
4I	Perpendicular lines and parallel lines	251
4J	Linear modelling FRINGE	255
4K	Graphical solutions to linear simultaneous equations	259
	Investigation	264
	Puzzles and challenges	266

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Review: Chapter summary 267
Multiple-choice questions 268
Short-answer questions 269
Extended-response questions 271

5 Length, area, surface area and volume 272

Measurement and Geometry

Area and surface area (S4, 5.1, 5.2, 5.3)
Volume (S4, 5.2)
MA5.1–8MG, MA5.2–11MG, MA5.3–13MG, MA5.2–12MG, MA5.3–14MG

Pre-test 274
5A Length and perimeter REVISION 275
5B Circumference and perimeter of sectors REVISION 280
5C Area REVISION 286
5D Perimeter and area of composite shapes 294
5E Surface area of prisms and pyramids 300
5F Surface area of cylinders 305
5G Volume of prisms 309
5H Volume of cylinders 315
Investigation 319
Puzzles and challenges 320
Review: Chapter summary 321
Multiple-choice questions 322
Short-answer questions 323
Extended-response questions 325

Semester review 1 326

6 Indices and surds 334

Number and Algebra

Indices (S4, 5.1, 5.2)
Surds and indices (S5.3§)
MA4–9NA, MA5.1–5NA, MA5.2–7NA, MA5.3–6NA

Measurement and Geometry

Numbers of any magnitude (S5.1)
MA5.1–9MG

Pre-test 336
6A Index notation 337
6B Index laws for multiplying and dividing 343
6C The zero index and power of a power 348
6D Index laws extended 352
6E Negative indices 356
6F Scientific notation 361
6G Scientific notation using significant figures 367
6H Fractional indices and surds 372
6I Simple operations with surds 377
Investigation 381
Puzzles and challenges 383
Review: Chapter summary 384
Multiple-choice questions 385
Short-answer questions 386
Extended-response questions 387

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Properties of geometrical figures 388

Measurement and Geometry

Properties of geometrical figures (S4, 5.1, 5.2, 5.3§)
MA4–17MG, MA4–18MG, MA5.1–11MG, MA5.2–14MG, MA5.3–16MG

	Pre-test	390
7A	Angles and triangles REVISION	391
7B	Parallel lines REVISION	399
7C	Quadrilaterals and other polygons	406
7D	Congruent triangles REVISION	412
7E	Using congruence in proof	419
7F	Enlargement and similar figures	424
7G	Similar triangles	431
7H	Proving and applying similar triangles	438
	Investigation	443
	Puzzles and challenges	445
	Review: Chapter summary	446
	Multiple-choice questions	447
	Short-answer questions	448
	Extended-response questions	451

8 Quadratic expressions and algebraic fractions 452

Number and Algebra

Algebraic techniques (S5.2, 5.3§)
Equations (S5.2, 5.3§)
MA5.2–6NA, MA5.2–8NA, MA5.3–5NA, MA5.3–7NA

	Pre-test	454
8A	Expanding binomial products	455
8B	Perfect squares and the difference of two squares	459
8C	Factorising algebraic expressions	463
8D	Factorising the difference of two squares	467
8E	Factorising by grouping in pairs	471
8F	Factorising monic quadratic trinomials	474
8G	Factorising non-monic quadratic trinomials	479
8H	Simplifying algebraic fractions: multiplication and division	483
8I	Simplifying algebraic fractions: addition and subtraction	488
8J	Further addition and subtraction of algebraic fractions	492
8K	Equations involving algebraic fractions	496
	Investigation	501
	Puzzles and challenges	503
	Review: Chapter summary	504
	Multiple-choice questions	505
	Short-answer questions	506
	Extended-response questions	507

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 Probability and single variable data analysis 508

Statistics and Probability

Probability (S5.1, 5.2)
Single variable data analysis (S5.1, 5.2◊)
MA5.1–13SP, MA5.1–12SP, MA5.2–17SP, MA5.2–15SP

	Pre-test	510
9A	Probability review REVISION	511
9B	Venn diagrams and two-way tables	516
9C	Using set notation FRINGE	522
9D	Using arrays for two-step experiments	527
9E	Using tree diagrams	532
9F	Using relative frequencies to estimate probabilities	538
9G	Mean, median and mode REVISION	543
9H	Stem-and-leaf plots	548
9I	Grouping data into classes FRINGE	554
9J	Measures of spread: range and interquartile range	560
9K	Box plots	565
	Investigation	571
	Puzzles and challenges	573
	Review: Chapter summary	574
	Multiple-choice questions	575
	Short-answer questions	576
	Extended-response questions	578

10 Quadratic equations and graphs of parabolas 580

Number and Algebra

Equations (S5.2, 5.3§)
Non-linear relationships (S5.1, 5.2◊, 5.3§)
MA5.2–8NA, MA5.1–7NA, MA5.3–7NA, MA5.2–10NA, MA5.3–9NA

	Pre-test	582
10A	Quadratic equations	583
10B	Solving $ax^2 + bx = 0$ and $x^2 - d^2 = 0$ by factorising	588
10C	Solving $x^2 + bx + c = 0$ by factorising	592
10D	Using quadratic equations to solve problems	596
10E	The parabola	600
10F	Sketching $y = ax^2$ with dilations and reflections	608
10G	Translations of $y = x^2$	615
10H	Sketching parabolas using intercept form	623
	Investigation	630
	Puzzles and challenges	632
	Review: Chapter summary	633
	Multiple-choice questions	634
	Short-answer questions	635
	Extended-response questions	637

Semester review 2 638

Answers *646*
Index *723*

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

About the authors



Stuart Palmer was born and educated in NSW. He is a high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at The University of Sydney.



David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 20 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.



Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, and is currently teaching both junior and senior mathematics in Victoria. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.



Justin Robinson is Head of Positive Education and a mathematics teacher at Geelong Grammar School. Prior to this, he spent 20 years teaching mathematics and was a key writer of in-house maths material. He has a keen interest in engaging all students through a wide variety of effective teaching methods and materials.



Jenny Goodman has worked for over 20 years in comprehensive State and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.



Jennifer Vaughan has taught secondary mathematics for over 30 years in NSW, WA, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Introduction and guide to this book

The **second edition** of this popular resource features a new interactive digital platform powered by Cambridge HOTmaths, together with improvements and updates to the textbook, and additional online resources such as video demonstrations of all the worked examples, Desmos-based interactives, carefully chosen HOTmaths resources including widgets and walkthroughs, and worked solutions for all exercises, with access controlled by the teacher. The Interactive Textbook also includes the ability for students to complete textbook work, including full working-out online, where they can self-assess their own work and alert teachers to particularly difficult questions. Teachers can see all student work, the questions that students have 'red-flagged', as well as a range of reports. As with the first edition, the complete resource is structured on detailed teaching programs for teaching the NSW Syllabus, now found in the Online Teaching Suite.

The chapter and section structure has been retained, and remains based on a logical teaching and learning sequence for the syllabus topic concerned, so that chapter sections can be used as ready-prepared lessons. Exercises have questions graded by level of difficulty and are grouped according to the **Working Mathematically components** of the NSW Syllabus, with enrichment questions at the end. Working programs for three ability levels (Building Progressing and Mastering) have been subtly embedded inside the exercises to facilitate the management of differentiated learning and reporting on students' achievement (see page x for more information on the Working Programs). In the second edition, the *Understanding* and *Fluency* components have been combined, as have *Problem-Solving* and *Reasoning*. This has allowed us to better order questions according to difficulty and better reflect the interrelated nature of the Working Mathematically components, as described in the NSW Syllabus.

Topics are aligned exactly to the NSW Syllabus, as indicated at the start of each chapter and in the teaching program, except for topics marked as:

- REVISION — prerequisite knowledge
- EXTENSION — goes beyond the Syllabus
- FRINGE — topics treated in a way that lies at the edge of the Syllabus requirements, but which provide variety and stimulus.

See the Stage 5 books for their additional curriculum linkage.

The parallel ***CambridgeMATHS Gold*** series for Years 7–10 provides resources for students working at Stages 3, 4, and 5.1. The two series have a content structure designed to make the teaching of mixed ability classes smoother.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

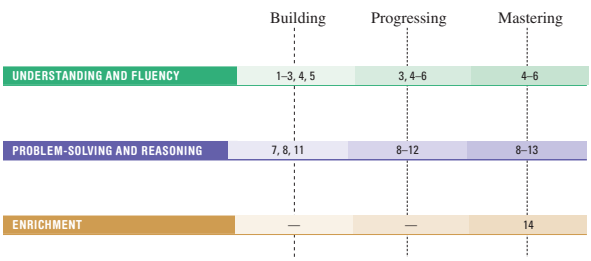
Guide to the working programs

It is not expected that any student would do every question in an exercise. The print and online versions contain working programs that are subtly embedded in every exercise. The suggested working programs provide three pathways through each book to allow differentiation for Building, Progressing and Mastering students.

Each exercise is structured in subsections that match the Working Mathematically strands, as well as Enrichment (Challenge).

The questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest-shaded colour) is the Building pathway
- The middle column (medium-shaded colour) is the Progressing pathway
- The right column (darkest-shaded colour) is the Mastering pathway.



Gradients within exercises and question subgroups

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise, where there is an increasing level of mathematical sophistication required in the Problem-solving and Reasoning group of questions than in the Understanding and Fluency group, and within each group the first few questions are easier than the last.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway will likely need more practice at Understanding and Fluency but should also attempt the easier Problem-Solving and Reasoning questions.

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them if the chapter pre-tests can be used as a diagnostic tool.

For classes grouped according to ability, teachers may wish to set one of the Building, Progressing or Mastering pathways as the default setting for their entire class and then make individual alterations, depending on student need. For mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e … or b, d, f, …)
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- —: do not complete any of the questions in this section.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

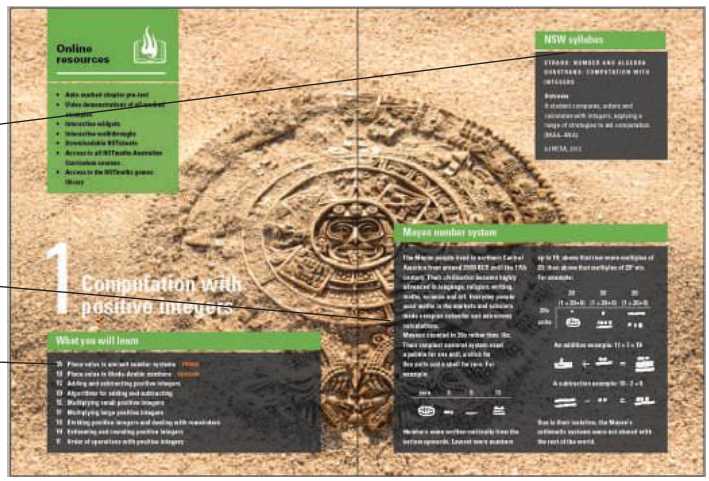
Guide to this book

Features:

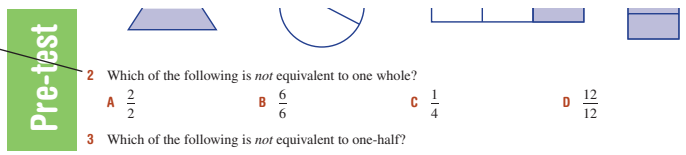
NSW Syllabus: strands, substrands and content outcomes for chapter (see teaching program for more detail)

Chapter introduction: use to set a context for students

What you will learn: an overview of chapter contents

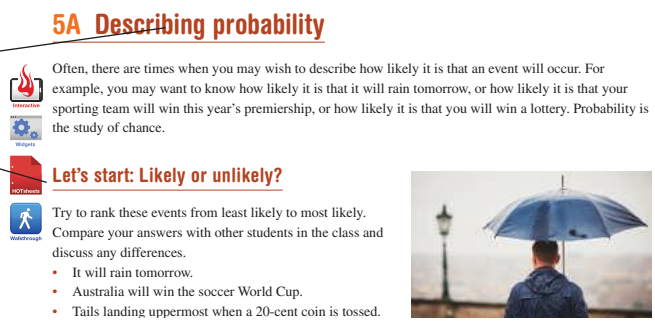


Pre-test: establishes prior knowledge (also available as an auto-marked quiz in the Interactive Textbook as well as a printable worksheet)

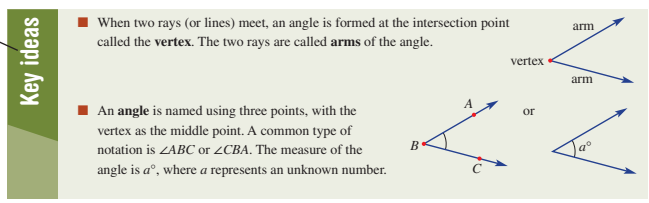


Topic introduction: use to relate the topic to mathematics in the wider world

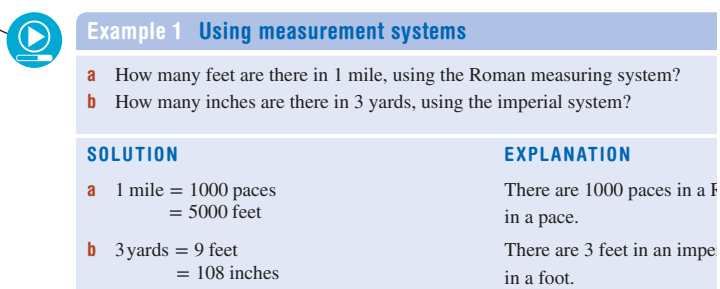
Let's start: an activity (which can often be done in groups) to start the lesson



Key ideas: summarises the knowledge and skills for the lesson



Examples: solutions with explanations and descriptive titles to aid searches. Video demonstrations of every example are included in the Interactive Textbook.



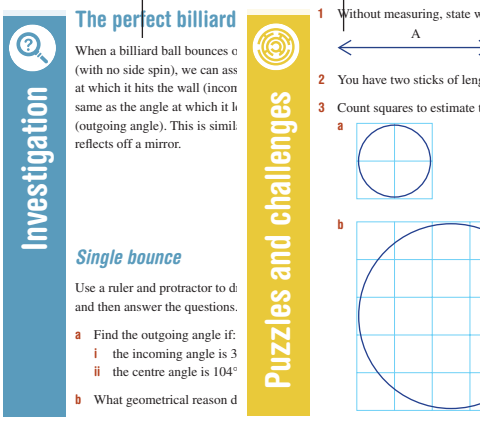
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

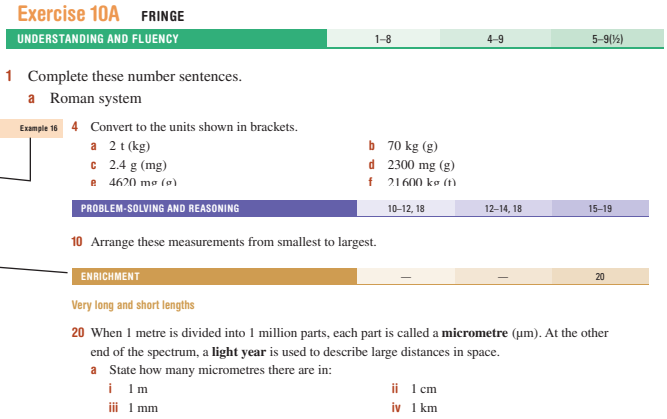
Exercise questions categorised by the **working mathematically components** and **enrichment**

Example references link exercise questions to worked examples.

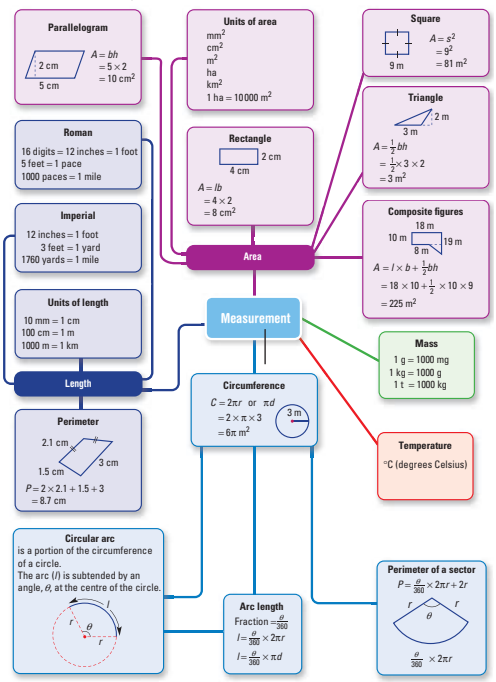
Investigations: inquiry-based activities

Puzzles and challenges

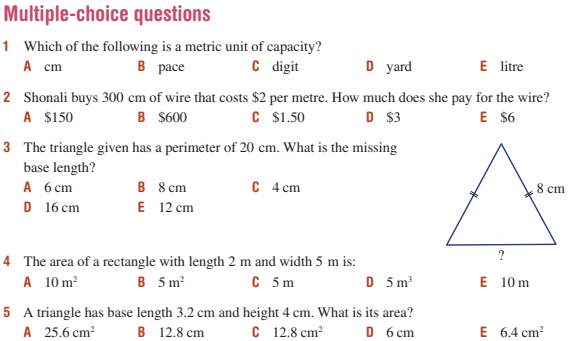




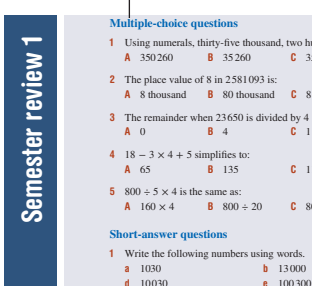
Chapter summary: mind map of key concepts & interconnections



Chapter reviews with **multiple-choice, short-answer** and **extended-response questions**



Two **Semester reviews** per book



Textbooks also include:

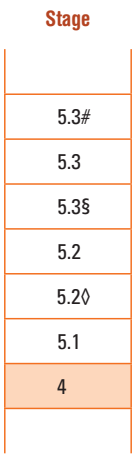
- Complete **answers**
- **Index**

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Stage Ladder icons

Shading on the ladder icons at the start of each section indicate the Stage covered by most of that section.

This key explains what each rung on the ladder icon means in practical terms. For more information see the teaching program and teacher resource package:



Stage	Past and present experience in Stages 4 and 5	Future direction for Stage 6 and beyond
5.3#	These are optional topics which contain challenging material for students who will complete all of Stage 5.3 during Years 9 and 10.	These topics are intended for students who are aiming to study Mathematics at the very highest level in Stage 6 and beyond.
5.3	Capable students who rapidly grasp new concepts should go beyond 5.2 and study at a more advanced level with these additional topics.	Students who have completed 5.1, 5.2 and 5.2 and 5.3 are generally well prepared for a calculus-based Stage 6 Mathematics course.
5.3§	These topics are recommended for students who will complete all the 5.1 and 5.2 content and have time to cover some additional material.	These topics are intended for students aiming to complete a calculus-based Mathematics course in Stage 6.
5.2	A typical student should be able to complete all the 5.1 and 5.2 material by the end of Year 10. If possible, students should also cover some 5.3 topics.	Students who have completed 5.1 and 5.2 without any 5.3 material typically find it difficult to complete a calculus-based Stage 6 Mathematics course.
5.2◊	These topics are recommended for students who will complete all the 5.1 content and have time to cover some additional material.	These topics are intended for students aiming to complete a non-calculus course in Stage 6, such as Mathematics Standard.
5.1	Stage 5.1 contains compulsory material for all students in Years 9 and 10. Some students will be able to complete these topics very quickly. Others may need additional time to master the basics.	Students who have completed 5.1 without any 5.2 or 5.3 material have very limited options in Stage 6 Mathematics.
4	Some students require revision and consolidation of Stage 4 material prior to tackling Stage 5 topics.	

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Overview of the digital resources

Interactive Textbook: general features

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available separately. (A **downloadable PDF textbook** is also included for offline use). These are its features, including those enabled when the students' ITB accounts are linked to the teacher's **Online Teaching Suite (OTS)** account.

The features described below are illustrated in the screenshot below.

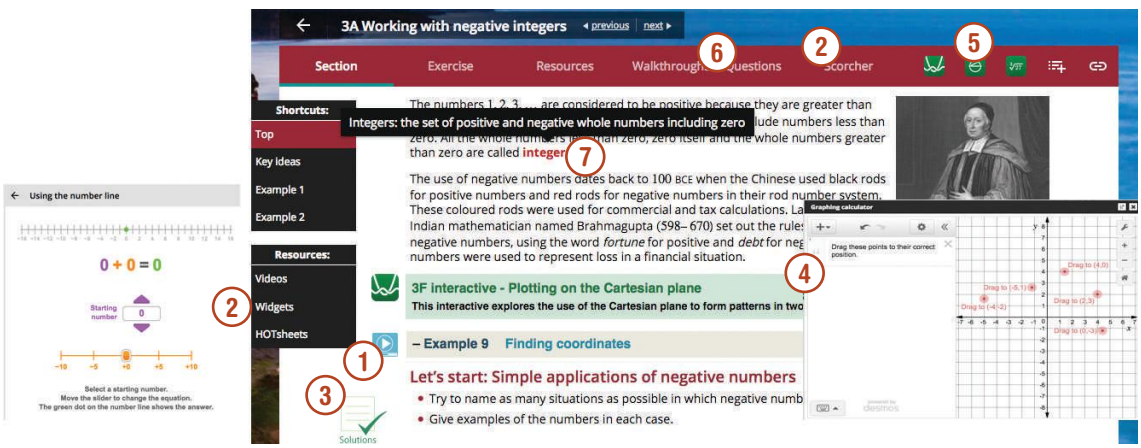
1. Every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the 'flipped classroom'
2. Seamlessly blend with Cambridge HOTmaths, including hundreds of interactive widgets, walkthroughs and games and access to Scorcher
3. **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher
4. **Desmos interactives** based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics
5. The **Desmos scientific calculator** is also available for students to use (as well as the graphics calculator and geometry tools)
6. Auto-marked practice quizzes in each section with saved scores
7. **Definitions** pop up for key terms in the text, and access to the Hotmaths **dictionary**

Not shown but also included:

- Access to alternative HOTmaths lessons is included, including content from previous year levels.
- Auto-marked pre-tests and multiple-choice review questions in each chapter.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Interactive Textbook: Workspaces and self-assessment tools

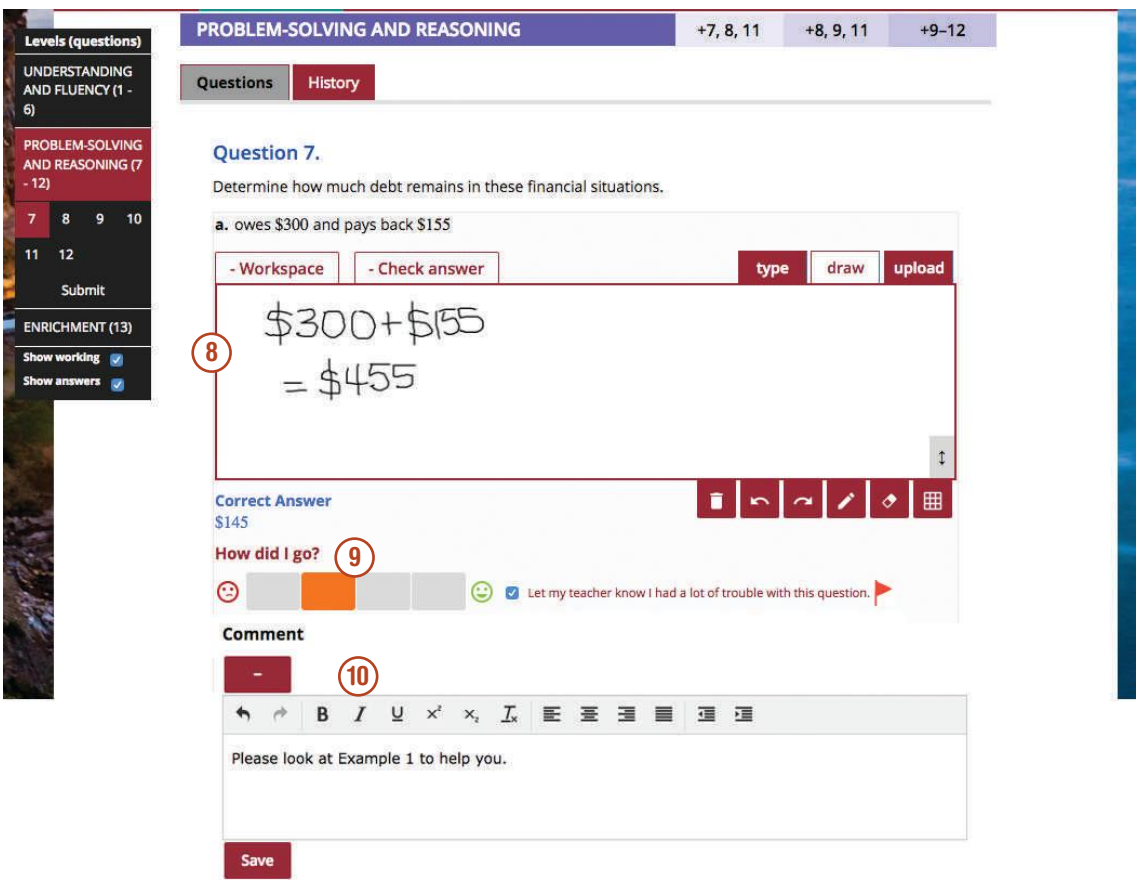
Almost every question in ***CambridgeMATHS NSW Second Edition*** can be completed and saved by students, including showing full working-out and students critically assessing their own work. This is done via the workspaces and self-assessment tools that are found below every question in the Interactive Textbook.

8 The new **workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploding images of writing or drawing.

9 The new **self-assessment tools** enable students to check answers including questions that have been red-flagged, and can rate their confidence level in their work, and alert teachers to questions the student has had particular trouble with. This self-assessment helps develop responsibility for learning and communicates progress and performance to the teacher.

10 Teachers can view the students' self-assessment individually or provide feedback. They can also view results by class.

WORKSPACES AND SELF-ASSESSMENT



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Downloadable PDF Textbook

The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

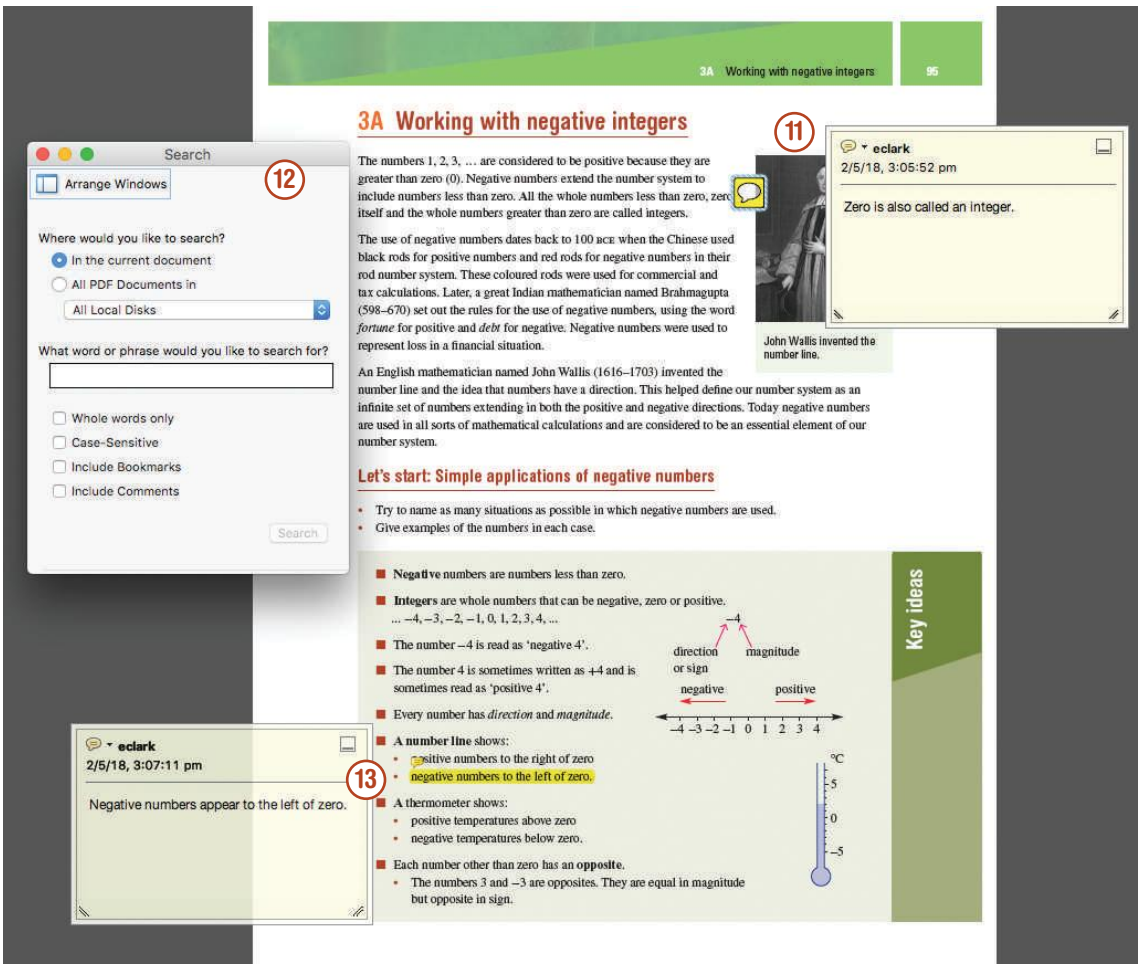
The features include:

11 PDF note-taking

12 PDF search features are enabled

13 highlighting functionality.

PDF TEXTBOOK



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online Teaching Suite

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the assets and resources are in one place for easy access. The features include:

14 The HOTmaths learning management system with class and student analytics and reports, and communication tools

15 Teacher's view of a student's working and self-assessment, including multiple progress and completion reports viewable at both student and class level, as well as seeing the questions that a class has flagged as being difficult

16 A HOTmaths-style test generator

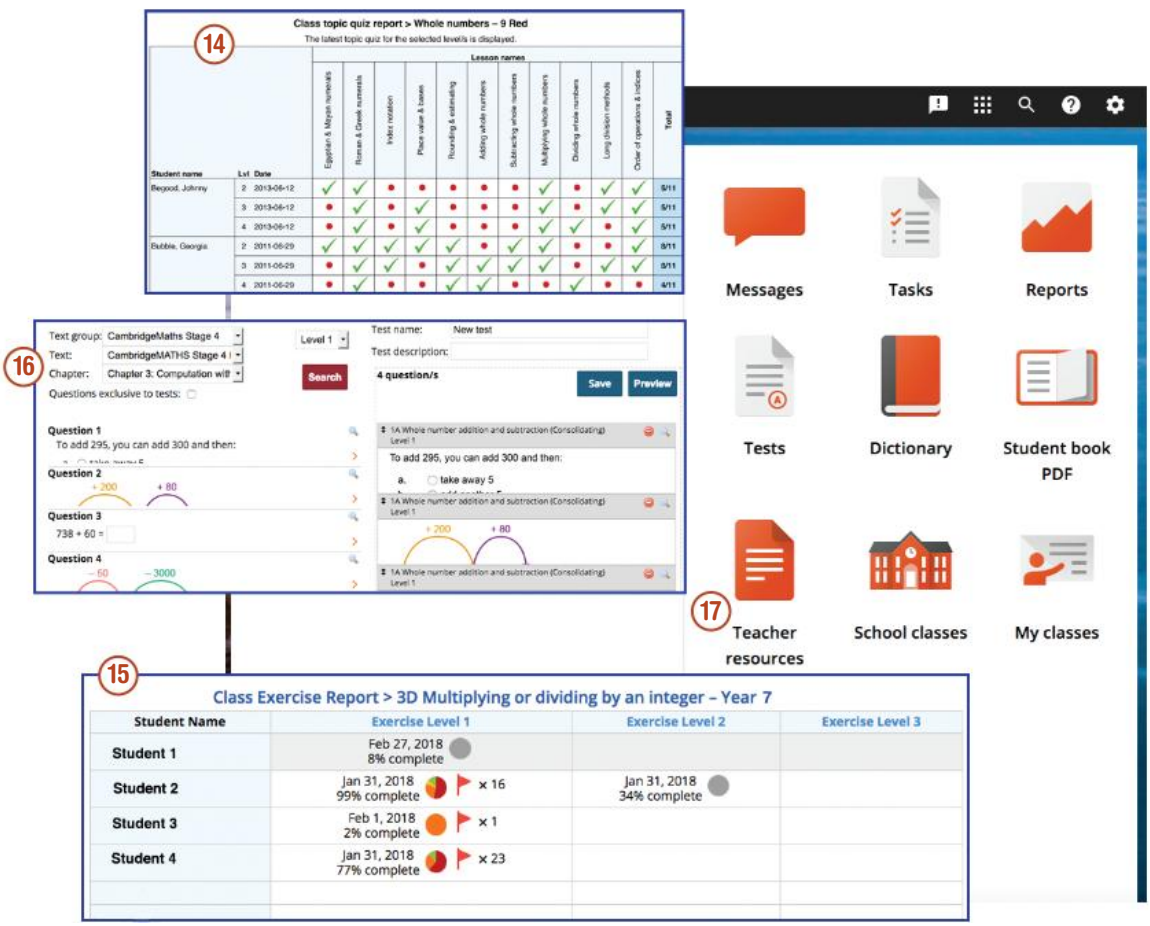
17 Chapter tests and worksheets

Not shown but also available:

- Editable teaching programs and curriculum grids.

ONLINE TEACHING SUITE POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Acknowledgements

The author and publisher wish to thank the following sources for permission to reproduce material:

Cover: © Getty Images /

Images: © Getty Images / Cuhrig, Chapter 1 Opener / Lebedinski Vladislav, 1A (1) / Sven Hansche, 1A (2) / Mark Colbe, 1A (3) / Becky Stares, 1B (1) / AJ James, 1C (1) / Design56, 1D (1) / BValentyn Semenov, 1D (2) / Wavebreakmedia, 1F (2) / brown54486, 1F (1) / Macnan, 1H (1) / Mascot, 1I (1) / Stuart Miller, 1L (2) / UYonghao Wu, 1L (3) / Imagevixen, 1L (4) / Jose Luis Pelaex, 1M (1) / Theerapan Bhumirat, 1M (2) / bruev, 1M (3) / nikom khotjan, 1M (4) / andrsr, 1M (5) / Lottie Davies, 1M (6) / Fairfax Media, 2E (1) / Sigrid Olsson, 2E (1) / M Vaisanen, 2F (1) / filadendron, 2F (2) / Hero Images, 2F (3) / fatcamera, 2H (1) / Monkey Business Images, 2I (1) / Yaorucheng, 2J (1) / marcduf, 2K (2) / John Lund / Heather Hryciw, 2K (3) / Ktsdesign, 2K (4) / Klaus Vedfelt, 2K (5) / German – skydiver, 2L (1) / Photos.com, 3A (1) / Henglein and Steets, 3A (1) / Busa Photography, 3C (2) / imagevavi, 3D (2) / dolomo84, 3D (2) / Sebastian Arning, 3D (3) / Spike Mafford, 3I (1) / Martin Deja, 3I (3) / MarioGuti, 3I (4) / Maskot, 3J (1) / bauhaus1000, 3J (3) / Nawaj Panichphol, 4E (1) / Luca Pellagrini, 4E (3) / Amaia Arozena & Gotzon Iraola, 4G (1) / Luboslava Racekova, 4K (1) / Ivan Ferdian, 4K (2) / Hero Images, 4K (3) / knapjames, 5A (1) / timmewman, 5B (1) / Matti Keshi-Kohtamaki, 5B (1) / David Person, 5C (2) / Ellen van Bodegom, 6A (1) / WitR, 6A (1) / Digital Vision, 6F (1) / Shingo Tamura, 6F (1) / Callista Images, 6G (1) / Hero Images, 8A (1) / Elleen Groome, Chapter 5, Ch Review / zentilia, Chapter 9 Opener / Robert Daly, 9B (1) / rozikassim, 9E (1) / Dan Brownsword, 9E (1) / Fotosearch, 9E (2) / Monty Rakusen, 9F (1) / Jovanmandic, 9F (3) / Dave And Les Jacobs / Lloyd Dobbie, 9F (1) / Hero Images, 9I (1) / Torwai Suebsri, 9J (1) / Mint Images, 9J (1) / Bruno Guerreiro, 9J (3) / Paul Bradbury, 9K (1) / Westend61, 9K (3) / John Lamb, 9K (4) / M_y_y_a, 9K (2) / Erik Isadson, 10A (1) / Erik Isakson, 10A (1) / Tatiana Popova, 10B (1) / Pasieda, 10C (1) / Universal Images Group, 10C (2) / Dmitry Rogulin, 10D (1) / Lazio Podor, 10G (1); © Shutterstock / Dirk Ott, 1E (1) / cozyta, 1F (1) / Robynmac, 1K (1) / Wouter Tolenaars, 2A (1) / Shebeko, 2B (1) / Xphantom, 2C (1) / marekuliasz, Chapter 3 Opener / Bjorn Heller, 3B (1) / Doyn and Margaret Loyd, 3B (1) / tungtopgun, 3D (1) / Aitornnfoto, 3I (2) / italianestro, 4E (1) / Chubykin Arkady, Chapter 6 Opener / Minorov, 6B (1) / Apollofoto, 6C (1) / videowokart, 6F (1) / Michael Taylor, 6I (1) / pistolseven, 7C (2) / Ian 2020, 7F (1) / Dainis Derics, 8B (1) / Bigstock, 8C (1) / evantravels, 9A (1) / Epicstockmedia, Chapter 10 Opener / Kristina Postnikova, 10E (1); © Alamy / Juice Images, Chapter 5 Opener / aerial-photos.com, Chapter 7 Opener / Haydn Denman, Chapter 8 Opener.

Every effort has been made to trace and acknowledge copyright. The publisher apologises for any accidental infringement and welcomes information that would redress this situation.

NSW NESA – Mathematics K-10 Syllabus Stage 4 © NSW Education Standards Authority for and on behalf of the Crown in right of the State of New South Wales, 2012.

The NSW Education Standards Authority (NESA) does not endorse model answers prepared by or for the publisher and takes no responsibility for errors in the reproduction of the material supplied by the NESA. The publisher wishes to acknowledge that this study guide has been independently published by Cambridge University Press for use by teachers and students, and that although material contained has been reproduced with permission, this publication is in no way endorsed by the NESA.

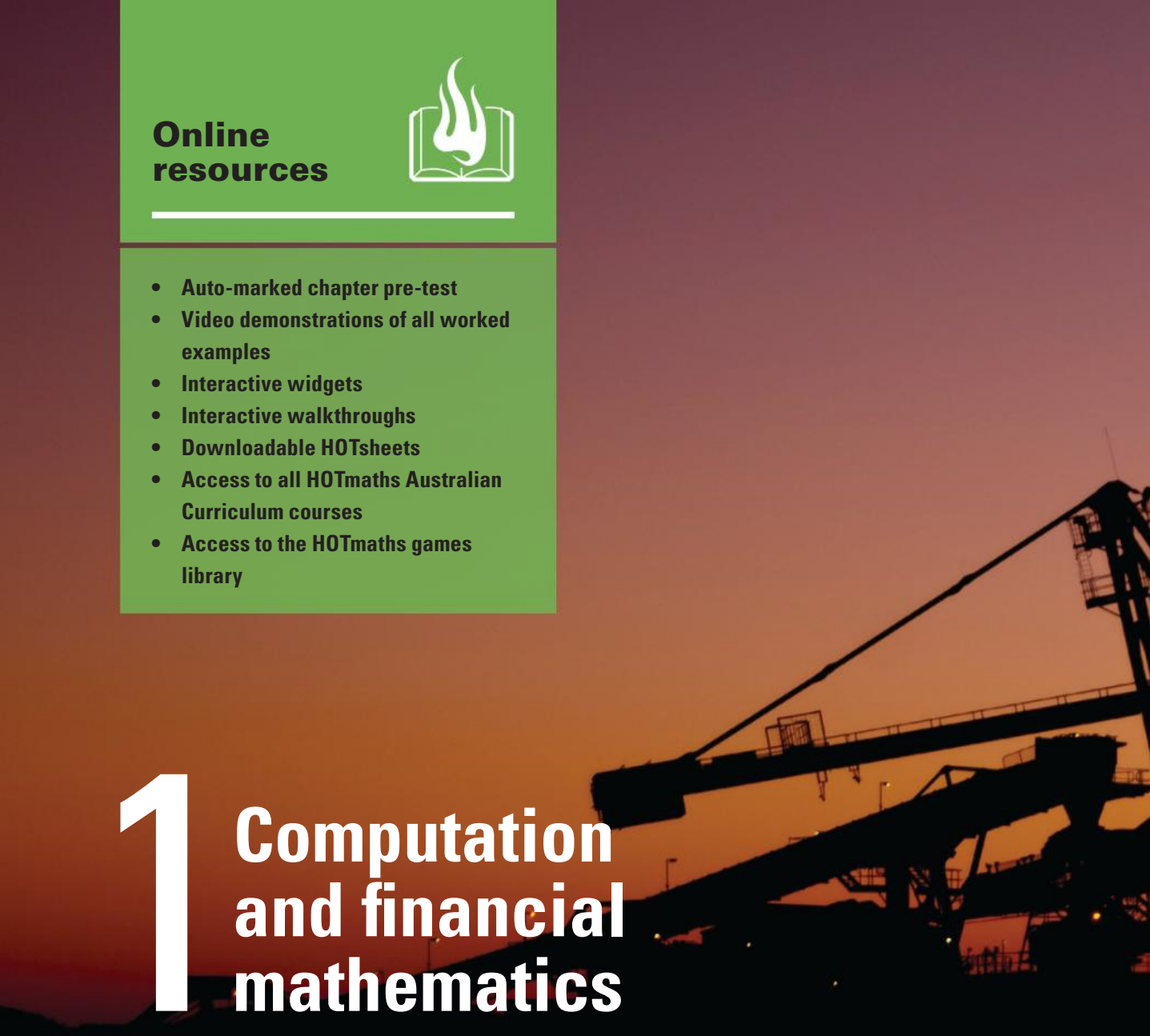
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library



1 Computation and financial mathematics

What you will learn

1A Computation with integers REVISION
1B Decimal places and significant figures
1C Rational numbers REVISION
1D Computation with fractions REVISION
1E Ratios, rates and best buys REVISION
1F Computation with percentages and money REVISION
1G Percentage increase and decrease REVISION
1H Profits and discounts REVISION
1I Income
1J The PAYG income tax system
1K Simple interest
1L Compound interest and depreciation
1M Using a formula for compound interest and depreciation

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: COMPUTATION WITH INTEGERS FRACTIONS, DECIMALS AND PERCENTAGES (S4) FINANCIAL MATHEMATICS (S4, 5.1, 5.2◊)

Outcomes

A student compares, orders and calculates with integers, applying a range of strategies to aid computation. (MA4–4NA)

A student operates with fractions, decimals and percentages. (MA4–5NA)

A student solves financial problems involving earning, spending and investing money. (MA5.1–4NA)

A student solves financial problems involving compound interest. (MA5.2–4NA)

© NESA, 2012

Global financial crisis

In 2007 the world's economy was rocked by what is considered the worst financial crisis since the Great Depression of the 1930s. Stock prices dropped and some of the world's most influential economies were in danger of collapsing.

Like the Great Depression, the Global Financial Crisis (GFC) originated in the United States, yet the effects were felt around the world.

The GFC of 2007, and the five years following, is linked to the real estate market of the US. Low interest rates enticed people to borrow more, and banks issued more and more loans to potential home owners, causing house prices to increase as demand for property increased. As with most trends, it cannot continue indefinitely, so when people were not able to repay their loans as their interest rates increased, the real estate bubble burst, banks foreclosed and this put pressure on the economy. This impacted the global money markets and ordinary citizens around the world, as many other financial institutions had invested money in US real estate.

Australia survived this period due to our strong mining industry and trade with Asia. Interest rates were lowered to stimulate growth in our economy and the housing market. It remains to be seen what the future brings as interest rates fluctuate over time.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Evaluate each of the following.

a $5 + 6 \times 2$

b $12 \div 4 \times 3 + 2$

c $12 \div (4 \times 3) + 2$

d $3 + (18 - 2 \times (3 + 4) + 1)$

e $8 - 12$

f $-4 + 3$

g -2×3

h $-18 \div (-9)$

2 Write $\frac{11}{5}$ as a:

a mixed numeral

b decimal

3 Evaluate each of the following.

a 3^2 **b** $\sqrt{25}$ **c** $(-4)^2$ **d** 2^3

4 Determine which is larger, $\frac{3}{4}$ or $\frac{7}{9}$ by:

a rewriting with the lowest common denominator

b converting to decimals (to 3 decimal places where necessary)

5 Arrange the numbers in each of the following sets in descending order.

a 2.645, 2.654, 2.465 and 2.564

b 0.456, 0.564, 0.0456 and 0.654

6 Evaluate each of the following.

a $4.26 + 3.73$ **b** $3.12 + 6.99$ **c** $10.89 - 3.78$

7 Evaluate each of the following.

a 7×0.2 **b** 0.3×0.2 **c** 2.3×1.6

d 4.2×3.9 **e** $14.8 \div 4$ **f** $12.6 \div 0.07$

8 Evaluate each of the following.

a 0.345×100

b $3.74 \times 100\,000$

c $37.54 \div 1000$

d $3.754 \div 100\,000$

9 Find the lowest common denominator for these pairs of fractions.

a $\frac{1}{3}$ and $\frac{1}{5}$ **b** $\frac{1}{6}$ and $\frac{1}{4}$ **c** $\frac{1}{5}$ and $\frac{1}{10}$

10 Evaluate each of the following.

a $\frac{2}{7} + \frac{3}{7}$ **b** $2\frac{1}{2} - \frac{3}{2}$ **c** $\frac{2}{3} \times \frac{3}{4}$ **d** $\frac{1}{2} \div 2$

11 Find:

a 50% of 26

b 10% of 600

c 9% of 90

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1A Computation with integers REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Throughout history, mathematicians have developed number systems to investigate and explain the world in which they live. The Egyptians used hieroglyphics to record whole numbers as well as fractions, the Babylonians used a place-value system based on the number 60 and the ancient Chinese and Indians developed systems using negative numbers. Our current base-10 decimal system (the Hindu-Arabic system) has expanded to include positive and negative numbers, fractions (rational numbers) and also numbers that cannot be written as fractions (irrational numbers), for example, π and $\sqrt{2}$. All the numbers in our number system, not including imaginary numbers, are called real numbers.



Markets used number systems in ancient times to enable trade through setting prices, counting stock and measuring produce.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Special sets of numbers

Here are some special groups of numbers. Can you describe what special property each group has? Try to use the correct vocabulary, for example, factors of 12.

- 7, 14, 21, 28, ...
- 1, 4, 9, 16, 25, ...
- 1, 2, 3, 4, 6, 9, 12, 18, 36.
- 1, 8, 27, 64, 125, ...
- 0, 1, 1, 2, 3, 5, 8, 13, ...
- 2, 3, 5, 7, 11, 13, 17, 19, ...

Key ideas

- The **integers** include …, −3, −2, −1, 0, 1, 2, 3, ...
- If a and b are positive integers the following rules apply
  - $a + (-b) = a - b$ For example: $5 + (-2) = 5 - 2 = 3$
  - $a - (-b) = a + b$ For example: $5 - (-2) = 5 + 2 = 7$
  - $a \times (-b) = -ab$ For example: $3 \times (-2) = -6$
  - $-a \times (-b) = ab$ For example: $-4 \times (-3) = 12$
  - $a \div (-b) = -\frac{a}{b}$ For example: $8 \div (-4) = -2$
  - $-a \div (-b) = \frac{a}{b}$ For example: $-8 \div (-4) = 2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- **Squares** and **cubes**
  - $a^2 = a \times a$ and $\sqrt{a^2} = a$ (if $a \geq 0$) — For example: $6^2 = 36$ and $\sqrt{36} = 6$
  - $a^3 = a \times a \times a$ and $\sqrt[3]{a^3} = a$ — For example: $4^3 = 64$ and $\sqrt[3]{64} = 4$
- **LCM**, **HCF** and **primes**
  - The **lowest common multiple** (LCM) of two numbers is the smallest multiple shared by both numbers.
    For example: the LCM of 6 and 9 is 18.
  - The **highest common factor** (HCF) of two numbers is the largest factor shared by both numbers.
    For example: the HCF of 24 and 30 is 6.
  - **Prime numbers** have only two factors, 1 and the number itself. The number 1 is not a prime number.
  - **Composite numbers** have more than two factors.
- **Order of operations**
  - Deal with brackets first.
  - Then, evaluate powers.
  - Next, do multiplication and division, from left to right.
  - Then, do addition and subtraction, from left to right.

Example 1 Operating with integers

Evaluate the following.

a $-2 - (-3 \times 13) + (-10)$

b $(-20 \div (-4) + (-3)) \times 2$

c $\sqrt[3]{8} - (-1)^2 + 3^3$

SOLUTION

a $-2 - (-3 \times 13) + (-10) = -2 - (-39) + (-10)$
$= -2 + 39 + (-10)$
$= 37 - 10$
$= 27$

b $(-20 \div (-4) + (-3)) \times 2 = (5 + (-3)) \times 2$
$= 2 \times 2$
$= 4$

c $\sqrt[3]{8} - (-1)^2 + 3^3 = 2 - 1 + 27$
$= 28$

EXPLANATION

Deal with the operations in brackets first.

$-a - (-b) = -a + b$
$a + (-b) = a - b$

$-a \div (-b) = \frac{a}{b}$

Deal with the operations inside brackets before doing the multiplication.
$5 + (-3) = 5 - 3$.

Evaluate powers first.
$\sqrt[3]{8} = 2$ since $2^3 = 8$
$(-1)^2 = -1 \times (-1) = 1$
$3^3 = 3 \times 3 \times 3 = 27$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 1A REVISION

UNDERSTANDING AND FLUENCY	1, 2, 3–4(½), 5–7	3–4(½), 5, 6, 7–8(½)	4–8(½)

1 Write down these sets of numbers.

a The factors of 16
b The factors of 56
c The HCF (highest common factor) of 16 and 56
d The first 7 multiples of 3
e The first 6 multiples of 5
f The LCM (lowest common multiple) of 3 and 5
g The first 10 prime numbers starting from 2
h All the prime numbers between 80 and 110

2 Evaluate the following.

a 11^2 **b** 15^2 **c** $\sqrt{144}$ **d** $\sqrt{400}$
e 3^3 **f** 5^3 **g** $\sqrt[3]{8}$ **h** $\sqrt[3]{64}$

3 Evaluate the following.

a $5 - 10$ **b** $-6 - 2$ **c** $-3 + 2$ **d** $-9 + 18$
e $2 + (-3)$ **f** $-6 + (-10)$ **g** $11 - (-4)$ **h** $-21 - (-30)$
i $2 \times (-3)$ **j** -21×4 **k** $-11 \times (-2)$ **l** $-3 \times (-14)$
m $18 \div (-2)$ **n** $-36 \div 6$ **o** $-100 \div (-10)$ **p** $-950 \div (-50)$

Example 1

4 Evaluate the following, showing your steps.

a $-4 - 3 \times (-2)$ **b** $-3 \times (-2) + (-4)$
c $-2 \times (3 - 8)$ **d** $2 - 7 \times (-2)$
e $2 - 3 \times 2 + (-5)$ **f** $4 + 8 \div (-2) - 3$
g $(-24 \div (-8) + (-5)) \times 2$ **h** $-7 - (-4 \times 8) - 15$
i $-3 - 12 \div (-6) \times (-4)$ **j** $4 \times (-3) \div (-2 \times 3)$
k $(-6 - 9 \times (-2)) \div (-4)$ **l** $10 \times (-2) \div (-7 - (-2))$
m $6 \times (-5) - 14 \div (-2)$ **n** $(-3 + 7) - 2 \times (-3)$
o $-2 + (-4) \div (-3 + 1)$ **p** $-18 \div ((-2 - (-4)) \times (-3))$
q $-2 \times 6 \div (-4) \times (-3)$ **r** $(7 - 14 \div (-2)) \div 2$
s $2 - (1 - 2 \times (-1))$ **t** $20 \div (6 \times (-4 \times 2) \div (-12) - (-1))$

5 Find the LCM of these pairs of numbers.

a 4, 7
b 8, 12
c 11, 17
d 15, 10

6 Find the HCF of these pairs of numbers.

a 20, 8
b 100, 65
c 37, 17
d 23, 46

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Evaluate the following.

a $2^3 - \sqrt{16}$ **b** $5^2 - \sqrt[3]{8}$ **c** $(-1)^2 \times (-3)$

d $(-2)^3 \div (-4)$ **e** $\sqrt{9} - \sqrt[3]{125}$ **f** $1^3 + 2^3 - 3^3$

g $\sqrt[3]{27} - \sqrt{81}$ **h** $\sqrt[3]{27} - \sqrt{9} - \sqrt[3]{1}$ **i** $(-1)^{101} \times (-1)^{1000} \times \sqrt[3]{-1}$

8 Evaluate these expressions by substituting $a = -2$, $b = 6$ and $c = -3$.

a $a^2 - b$ **b** $a - b^2$ **c** $2c + a$ **d** $b^2 - c^2$

e $a^3 + c^2$ **f** $3b + ac$ **g** $c - 2ab$ **h** $abc - (ac)^2$

PROBLEM-SOLVING AND REASONING 9, 10, 14 | 9–11, 14 | 11–15

9 Insert brackets into these statements to make them true.

a $-2 \times 11 + (-2) = -18$ **b** $-6 + (-4) \div 2 = -5$ **c** $2 - 5 \times (-2) = 6$

d $-10 \div 3 + (-5) = 5$ **e** $3 - (-2) + 4 \times 3 = -3$ **f** $(-2)^2 + 4 \div (-2) = -2^2$

10 How many different answers are possible if any number of pairs of brackets is allowed to be inserted into this expression?

$-6 \times 4 - (-7) + (-1)$

11 Margaret and Mildred meet on a Eurostar train travelling from London to Paris. Margaret visits her daughter in Paris every 28 days. Mildred visits her son in Paris every 36 days. When will Margaret and Mildred have a chance to meet again on the train?



12 **a** The sum of two numbers is 5 and their difference is 9. What are the two numbers?

b The sum of two numbers is -3 and their product is -10. What are the two numbers?

13 Two opposing football teams have squad sizes of 24 and 32. For a training exercise, each squad is to divide into smaller groups of equal size. What is the largest number of players in a group if the group size for both squads is the same?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

14 **a** Evaluate:

i 4^2

ii $(-4)^2$

b If $a^2 = 16$, write down the possible values of a.

c If $a^3 = 27$, write down the value of a.

d Explain why there are two values of a for which $a^2 = 16$ but only one value of a for which $a^3 = 27$.

e Find $\sqrt[3]{-27}$.

f Explain why $\sqrt{-16}$ cannot exist (using real numbers).

g -2^2 is the same as -1×2^2. Now evaluate:

i -2^2 **ii** -5^3

iii $-(-3)^2$ **iv** $-(-4)^2$

h Decide if $(-2)^2$ and -2^2 are equal.

i Decide if $(-2)^3$ and -2^3 are equal.

j Explain why the HCF of two distinct prime numbers is 1.

k Explain why the LCM of two distinct prime numbers a and b is $a \times b$.

15 If a and b are both positive numbers and $a > b$, decide if the following are true or false.

a $a - b < 0$ **b** $-a \times b > 0$ **c** $-a \div (-b) > 0$

d $(-a)^2 - a^2 = 0$ **e** $-b + a < 0$ **f** $2a - 2b > 0$

ENRICHMENT — — 16

Special numbers

16 **a** Perfect numbers are positive integers that are equal to the sum of all their factors, excluding the number itself.

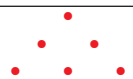
i Show that 6 is a perfect number.

ii There is one perfect number between 20 and 30. Find the number.

iii The next perfect number is 496. Show that 496 is a perfect number.

b Triangular numbers are the number of dots required to form triangles as shown in this table.

i Complete this table.

Number of rows	1	2	3	4	5	6
Diagram						
Number of dots (triangular number)	1	3				

ii Find the 7th and 8th triangular numbers.

c Fibonacci numbers are a sequence of numbers where each number is the sum of the two preceding numbers. The first two numbers in the sequence are 0 and 1.

i Write down the first 10 Fibonacci numbers.

ii If the Fibonacci numbers were to be extended in the negative direction, what would the first four negative Fibonacci numbers be?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1B Decimal places and significant figures

Interactive
Widgets
HOTsheets
Walkthrough

Numbers with and without decimal places can be rounded depending on the level of accuracy required. When using numbers with decimal places, it is common to round off the number to leave only a certain number of decimal places. The time for a 100-metre sprint race, for example, might be 9.94 seconds.



For road construction purposes, the volume of sand in these piles would only need to be known to two or three significant figures.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Due to the experimental nature of science and engineering, not all the digits in all numbers are considered important or 'significant'. In such cases we are able to round numbers to within a certain number of significant figures (sometimes abbreviated to sig. fig.). The number of cubic metres of gravel required for a road, for example, might be calculated as 3485 but is rounded to 3500. In this example, two siginificant figures have been used.

Let's start: Plausible but incorrect

Johnny says that the number 2.748 when rounded to one decimal place is 2.8 because:

- the 8 rounds the 4 to a 5
- then the new 5 rounds the 7 to an 8.

What is wrong with Johnny's theory?

Key ideas

- To round a number to a required number of **decimal places**:
  - Locate the digit in the required decimal place.
  - Round down (leave as is) if the next digit (**critical digit**) is 4 or less.
  - Round up (increase by 1) if the next digit is 5 or more.

  For example:
  - To 2 decimal places, 1.543 rounds to 1.54 and 32.9283 rounds to 32.93.
  - To 1 decimal place, 0.248 rounds to 0.2 and 0.253 rounds to 0.3.

- To round a number to a required number of **significant figures**:
  - Locate the first non-zero digit counting from left to right.
  - From this first significant digit, count out the number of significant digits including zeros.
  - Stop at the required number of significant digits and round this last digit.
  - Replace any non-significant digit to the left of a decimal point with a zero.

  For example, these numbers are all rounded to 3 significant figures:
  $2.5391 \approx 2.54$, $0.002713 \approx 0.00271$, $568\,810 \approx 569\,000$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 2 Rounding to a number of decimal places

Round each of these to 2 decimal places.

a 256.1793 **b** 0.04459 **c** 4.8972

SOLUTION	EXPLANATION
a $256.1793 \approx 256.18$	The number after the second decimal place is 9, so round up (increase the 7 by 1).
b $0.04459 \approx 0.04$	The number after the second decimal place is 4, so round down. 4459 is closer to 4000 than 5000.
c $4.8972 \approx 4.90$	The number after the second decimal place is 7, so round up. Increasing by 1 means 0.89 becomes 0.90.

Example 3 Rounding to a number of significant figures

Round each of these numbers to 2 significant figures.

a 2567 **b** 23067.453 **c** 0.04059

SOLUTION	EXPLANATION
a $2567 \approx 2600$	The first two digits are the first two significant figures. The third digit is 6, so round up. Replace the last two non-significant digits with zeros.
b $23067.453 \approx 23000$	The first two digits are the first two significant figures. The third digit is 0, so round down.
c $0.04059 \approx 0.041$	Locate the first non-zero digit, i.e. 4. So 4 and 0 are the first two significant figures. The next digit is 5, so round up.

Example 4 Estimating using significant figures

Estimate the answer to the following by rounding each number in the expression to 1 significant figure and use your calculator to check how reasonable your answer is.

$27 + 1329.5 \times 0.0064$

SOLUTION	EXPLANATION
$27 + 1329.5 \times 0.0064$	
$\approx 30 + 1000 \times 0.006$ $= 30 + 6$ $= 36$	Round each number to one significant figure and evaluate. Recall multiplication occurs before the addition.
The estimated answer is reasonable.	By calculator (to 1 decimal place): $27 + 1329.5 \times 0.0064 = 35.5$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 1B

UNDERSTANDING AND FLUENCY	1–6, 9(½)	3(½), 4, 5, 6(½), 8, 9(½)	3(½), 5, 7(½), 8, 9(½)

1 Choose the number to answer each question.

a Is 44 closer to 40 or 50?

b Is 266 closer to 260 or 270?

c Is 7.89 closer to 7.8 or 7.9?

d Is 0.043 closer to 0.04 or 0.05?

2 Choose the correct answer if the first given number is rounded to 3 significant figures.

a 32 124 is rounded to 321, 3210 or 32 100

b 431.92 is rounded to 431, 432 or 430

c 5.8871 is rounded to 5.887, 5.88 or 5.89

d 0.44322 is rounded to 0.44, 0.443 or 0.44302

e 0.0019671 is rounded to 0.002, 0.00197 or 0.00196

Example 2

3 Round each of the following numbers to 2 decimal places.

a 17.962	**b** 11.082	**c** 72.986
d 47.859	**e** 63.925	**f** 23.807
g 804.5272	**h** 500.5749	**i** 821.2749
j 5810.2539	**k** 1004.9981	**l** 2649.9974

4 Round these numbers to the nearest whole number.

a 6.814

b 73.148

c 129.94

d 36 200.49

5 Use division to write these fractions as decimals rounded to 3 decimal places.

a $\frac{1}{3}$ **b** $\frac{2}{7}$

c $\frac{13}{11}$ **d** $\frac{400}{29}$

Example 3

6 Round the numbers in Question **3** to 2 significant figures.

7 Round the numbers in Question **3** to 1 significant figure.



8 Using one significant figure rounding, 324 rounds to 300, 1.7 rounds to 2 and 9.6 rounds to 10.

a Calculate $300 \times 2 \div 10$.

b Use a calculator to calculate $324 \times 1.7 \div 9.6$.

c What is the difference between the answer in part **a** and the exact answer in part **b**?

Example 4

9 Estimate the answers to the following by rounding each number in the problem to 1 significant figure. Check how reasonable your answer is with a calculator.



a $567 + 3126$	**b** $795 - 35.6$	**c** 97.8×42.2
d $965.98 + 5321 - 2763.2$	**e** $4.23 - 1.92 \times 1.827$	**f** $17.43 - 2.047 \times 8.165$
g $0.0704 + 0.0482$	**h** 0.023×0.98	**i** $0.027 \div 0.0032$
j 41.034^2	**k** 0.078×0.9803^2	**l** $1.8494^2 + 0.972 \times 7.032$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING | 10, 11, 14 | 10–12, 14, 15 | 11–13, 15, 16

10 An electronic timer records the time for a running relay between two teams A and B. Team A's time is 54.283 seconds and team B's time is 53.791 seconds. Find the difference in the times for teams A and B if the times were written down using:

a 1 decimal place
b 4 significant figures
c 2 significant figures
d 1 significant figure

11 $28.4 \times 2.94 \times 11.31$ is calculated by first rounding each of the three numbers. Describe the type of rounding that has taken place if the answer is:

a 900 **b** 893.2 **c** 924

12 150 m of fencing and 18 posts are used to create an area in the shape of an equilateral triangle. Posts are used in the corners and are evenly spaced along the sides.
Find the distance between each post. Write your answer in metres rounded to the nearest centimetre.

13 A tonne (1000 kg) of soil is to be equally divided between 7 garden beds. How much soil is placed in each garden bed? Write your answer in tonnes rounded to the nearest kilogram.

14 Should 2.14999 be rounded down or up if it is to be rounded to 1 decimal place? Give reasons.

15 A scientific experiment uses very small amounts of magnesium (0.0025 g) and potassium (0.0062 g). Why does it make sense to use 2 significant figures instead of 2 decimal places when recording numbers in a situation like this?

16 Consider the two numbers 24 and 26.

a Calculate:
  i $24 + 26$
  ii 24×26
b Find the sum (+) of the numbers after rounding each number using 1 significant figure.
c Find the product (×) of the numbers after rounding each number using 1 significant figure.
d What do you notice about the answers for parts **b** and **c** as compared to part **a**? Give an explanation.

ENRICHMENT | — | — | 17

*n*th decimal place

17 **a** Express $\frac{2}{11}$ as a decimal correct to 8 decimal places.
b Using the decimal pattern from part **a**, find the digit in the:
  i 20th decimal place
  ii 45th decimal place
  iii 1000th decimal place
c Express $\frac{1}{7}$ as a decimal correct to 13 decimal places.
d Using the decimal pattern from part **c**, find the digit in the:
  i 20th decimal place
  ii 45th decimal place
  iii 1000th decimal place
e Can you find any fraction whose decimal representation is non-terminating and has no pattern? Use a calculator to help.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1C Rational numbers REVISION

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

Under the guidance of Pythagoras around 500 BCE, it was discovered that some numbers could not be expressed as a fraction. These special numbers, called irrational numbers, when written as a decimal, continue forever and do not show any pattern. So to write these numbers exactly, you need to use special symbols such as $\sqrt{2}$ and π. If, however, the decimal places in a number terminate or if a pattern exists, the number can be expressed as a fraction. These numbers are called rational numbers.

This is $\sqrt{2}$ to 100 decimal places:

1. 41421356237309504880168872420969807856967187537694807317667973799073247846210703885038753432764157273501384623091229702492483605585073721264412149709993583141322266592750559275579995050115278206057147

Let's start: Approximating π

To simplify calculations, the ancient and modern civilisations used fractions to approximate π.

- Write down the decimal value of π that appears on your calculator.
- Search the internet and write down the value of π correct to 12 decimal places.
- What is the current world record for memorising and reciting pi?

Key ideas

- A **non-terminating** decimal is one in which the decimal places continue indefinitely.

Real numbers

Rational numbers (fractions)
(e.g. $\frac{48}{6}, \frac{1}{3}, 7$)

Terminating decimals
(e.g. 8, 1.2, 6.123)

Recurring decimals
(e.g. $0.\dot{5}$, $1.\dot{4}$, $2.\overline{345}$)

Irrational numbers
(e.g. $\sqrt{2}$, pi (π), phi (ϕ) – the golden ratio)

Non-terminating non-recurring decimals
(e.g. $\pi = 3.14159\ldots$, $\phi = 1.618033\ldots$, $\sqrt{2} = 1.414213\ldots$)

- If $\frac{a}{b}$ is a **proper fraction**, then $a < b$. For example: $\frac{2}{7}$
- If $\frac{a}{b}$ is an **improper fraction**, then $a > b$. For example: $\frac{10}{3}$
- A **mixed numeral** is written as a whole number plus a proper fraction. For example: $2\frac{3}{5}$
- **Equivalent fractions** have the same value. For example: $\frac{2}{3} = \frac{6}{9}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- A fraction is simplified by dividing the **numerator** and **denominator** by their highest common factor.
- Fractions can be compared using a **common denominator**. This should be the lowest common multiple of both denominators.
- A dot or bar is used to show a pattern in a recurring decimal number.

  For example: $\frac{1}{6} = 0.16666\ldots = 0.1\dot{6}$ or $\frac{3}{11} = 0.272727\ldots = 0.\overline{27}$ or $0.\dot{2}\dot{7}$

Example 5 Writing fractions as decimals

Write these fractions as decimals.

a $3\frac{3}{8}$ **b** $\frac{5}{13}$

SOLUTION

a
$$8\overline{)3.^{3}0^{6}0^{4}0}\ \ 0.375$$
$$3\frac{3}{8} = 3.375$$

b
$$13\overline{)5.^{5}0^{11}0^{6}0^{8}0^{2}0^{7}0^{5}0}\ \ 0.3\ 8\ 4\ 6\ 1\ 5\ 3$$
$$\frac{5}{13} = 0.\overline{384615}$$

EXPLANATION

Find a decimal for $\frac{3}{8}$ by dividing 8 into 3 using the short division algorithm.

Divide 13 into 5 and continue until the pattern repeats. Add a bar over the repeating pattern.

Writing $0.\dot{3}84615\dot{}$ is an alternative.

Example 6 Writing decimals as fractions

Write these decimals as fractions.

a 0.24 **b** 2.385

SOLUTION

a
$$0.24 = \frac{24}{100} = \frac{6}{25}$$

b
$$2.385 = \frac{2385}{1000} = \frac{477}{200} = 2\frac{77}{200}$$

OR

$$2\frac{385}{1000} = 2\frac{77}{200}$$

EXPLANATION

Write as a fraction using the smallest place value (hundredths) then simplify using the HCF of 4.

The smallest place value is thousandths.

Simplify to an improper fraction or a mixed numeral.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 7 Comparing fractions

Decide which fraction is larger.

$\frac{7}{12}$ or $\frac{8}{15}$

SOLUTION

LCM of 12 and 15 is 60.

$\frac{7}{12} = \frac{35}{60}$ and $\frac{8}{15} = \frac{32}{60}$

$\therefore \frac{7}{12} > \frac{8}{15}$

EXPLANATION

Find the lowest common multiple of the two denominators (lowest common denominator). Write each fraction as an equivalent fraction using the common denominator. Then compare numerators (i.e.35 > 32) to determine the larger fraction.

Exercise 1C REVISION

UNDERSTANDING AND FLUENCY	1–4, 5–8(½), 10	3, 4–8(½), 9, 10	5–8(½), 9, 10

1 Write these numbers as mixed numerals.

a $\frac{7}{5}$ **b** $\frac{13}{3}$ **c** $\frac{48}{11}$ **d** $\frac{326}{53}$

2 Write these numbers as improper fractions.

a $1\frac{4}{7}$ **b** $5\frac{1}{3}$ **c** $9\frac{1}{2}$ **d** $18\frac{4}{13}$

3 Simplify these numbers by cancelling.

a $\frac{4}{10}$ **b** $\frac{8}{58}$ **c** $4\frac{20}{120}$ **d** $-72\frac{125}{1000}$

4 Write down the missing number.

a $\frac{3}{5} = \frac{\square}{15}$ **b** $\frac{5}{6} = \frac{20}{\square}$ **c** $\frac{3}{7} = \frac{9}{\square}$ **d** $\frac{5}{11} = \frac{\square}{77}$

e $\frac{\square}{4} = \frac{21}{28}$ **f** $\frac{\square}{9} = \frac{42}{54}$ **g** $\frac{3}{\square} = \frac{15}{50}$ **h** $\frac{11}{\square} = \frac{121}{66}$

Example 5a

5 Write these fractions as decimals.

a $\frac{11}{4}$ **b** $\frac{7}{20}$ **c** $3\frac{2}{5}$ **d** $\frac{15}{8}$

e $2\frac{5}{8}$ **f** $3\frac{4}{5}$ **g** $\frac{37}{16}$ **h** $\frac{7}{32}$

Example 5b

6 Write these fractions as recurring decimals.

a $\frac{3}{11}$ **b** $\frac{7}{9}$ **c** $\frac{9}{7}$ **d** $\frac{5}{12}$

e $\frac{10}{9}$ **f** $3\frac{5}{6}$ **g** $7\frac{4}{15}$ **h** $\frac{29}{11}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 6

7 Write these decimals as fractions.

a 0.35	**b** 0.06	**c** 3.7	**d** 0.56
e 1.07	**f** 0.075	**g** 3.32	**h** 7.375
i 2.005	**j** 10.044	**k** 6.45	**l** 2.101

Example 7

8 Decide which is the larger fraction in the following pairs.

a $\frac{3}{4}, \frac{5}{6}$	**b** $\frac{13}{20}, \frac{3}{5}$	**c** $\frac{7}{10}, \frac{8}{15}$	**d** $\frac{5}{12}, \frac{7}{18}$
e $\frac{7}{16}, \frac{5}{12}$	**f** $\frac{26}{35}, \frac{11}{14}$	**g** $\frac{7}{12}, \frac{19}{30}$	**h** $\frac{7}{18}, \frac{11}{27}$

9 Place these fractions in descending order.

a $\frac{3}{8}, \frac{5}{12}, \frac{7}{18}$

b $\frac{1}{6}, \frac{5}{24}, \frac{3}{16}$

c $\frac{8}{15}, \frac{23}{40}, \frac{7}{12}$

10 Express the following quantities as simplified fractions.

a \$45 out of \$100

b 12 kg out of 80 kg

c 64 baskets out of 90 shots in basketball

d 115 mL out of 375 mL

PROBLEM-SOLVING AND REASONING 11, 12, 15 | 11(½), 12, 13, 15 | 12–16

11 These sets of fractions form a pattern. Find the next two fractions in the pattern.

a $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

b $\frac{6}{5}, \frac{14}{15}, \frac{2}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

c $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{\square}{\square}, \frac{\square}{\square}$

d $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \frac{\square}{\square}, \frac{\square}{\square}$

12 The 'Weather Forecast' website says there is a 0.45 chance that it will rain tomorrow. The 'Climate Control' website says that the chance of rain is $\frac{14}{30}$. Which website gives the least chance that it will rain?



13 A jug has 400 mL of $\frac{1}{2}$ strength orange juice. The following amounts of full-strength juice are added to the mix. Find a fraction to describe the strength of the orange drink after the full-strength juice is added.

a 100 mL

b 50 mL

c 120 mL

d 375 mL

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

14 If x is an integer, determine what values x can take in the following.

a The fraction $\frac{x}{3}$ is a number between (and not including) 10 and 11.

b The fraction $\frac{x}{7}$ is a number between (and not including) 5 and 8.

c The fraction $\frac{34}{x}$ is a number between 6 and 10.

d The fraction $\frac{23}{x}$ is a number between 7 and 12.

e The fraction $\frac{x}{14}$ is a number between (and not including) 3 and 4.

f The fraction $\frac{58}{x}$ is a number between 9 and 15.

15 $a\frac{b}{c}$ is a mixed numeral with unknown digits a, b and c. Write it as an improper fraction.

16 If $\frac{a}{b}$ is a fraction, answer the given questions with reasons.

a Is it possible to find a fraction that can be simplified by cancelling if one of a or b is prime?

b Is it possible to find a fraction that can be simplified by cancelling if both a and b are prime? Assume $a \neq b$.

c If $\frac{a}{b}$ is a fraction in simplest form, can a and b both be even?

d If $\frac{a}{b}$ is a fraction in simplest form, can a and b both be odd?

ENRICHMENT — — 17

Converting recurring decimals to fractions

17 Here are two examples of conversion from recurring decimals to fractions.

$0.\dot{6} = 0.6666\ldots$
Let $x = 0.6666\ldots$ (1)
$10x = 6.6666\ldots$ (2)
(2) − (1) $9x = 6$
$x = \frac{6}{9} = \frac{2}{3}$
$\therefore\ 0.\dot{6} = \frac{2}{3}$

$1.\overline{27} = 1.272727\ldots$
Let $x = 1.27272\ldots$ (1)
$100x = 127.2727\ldots$(2)
(2) − (1) $99x = 126$
$x = \frac{126}{99}$
$\therefore\ 1.\overline{27} = \frac{126}{99} = 1\frac{27}{99} = 1\frac{3}{11}$

Convert these recurring decimals to fractions using the above method.

a $0.\dot{8}$ **b** $1.\dot{2}$ **c** $0.\overline{81}$ **d** $3.\overline{43}$

e $9.\overline{75}$ **f** $0.\overline{132}$ **g** $2.\overline{917}$ **h** $13.\overline{8125}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1D Computation with fractions REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Computation with integers can be extended to include rational and irrational numbers. The operations include addition, subtraction, multiplication and division. Addition and subtraction of fractions is generally more complex than multiplication and division because there is the added step of finding common denominators.



Fractions are all around you and part of everyday life.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The common errors

Here are incorrect solutions to four computations involving fractions.

- $\frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$
- $\frac{7}{6} \div \frac{7}{3} = \frac{7}{6} \div \frac{14}{6} = \frac{\frac{1}{2}}{6} = \frac{1}{12}$
- $\frac{2}{3} + \frac{1}{2} = \frac{2+1}{3+2} = \frac{3}{5}$
- $1\frac{1}{2} - \frac{2}{3} = 1\frac{3}{6} - \frac{4}{6} = -1\frac{1}{6}$

In each case describe what is incorrect and give the correct solution.

Key ideas

- To add or subtract fractions, first convert each fraction to **equivalent fractions** that have the same **denominator**.
  - Choose the lowest common denominator.
  - Add or subtract the numerators and retain the denominator.

  For example: $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$
- To multiply fractions:
  - Cancel the highest common factor between any numerator and any denominator before multiplication.
  - Convert mixed numerals to improper fractions before multiplying.
  - Multiply the numerators and multiply the denominators.
- The **reciprocal** of a number multiplied by the number itself is equal to 1.

  For example: the reciprocal of 2 is $\frac{1}{2}$ since $2 \times \frac{1}{2} = 1$.

  the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$ since $\frac{3}{5} \times \frac{5}{3} = 1$.
- To divide a number by a fraction, multiply by its reciprocal.

  For example: $\frac{2}{3} \div \frac{5}{6}$ becomes $\frac{2}{3} \times \frac{6}{5}$
  - Whole numbers can be written using a denominator of 1.

    For example: 6 can be written as $\frac{6}{1}$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8 Adding and subtracting fractions

Evaluate the following.

a $\frac{1}{2}+\frac{3}{5}$ **b** $1\frac{2}{3}+4\frac{5}{6}$ **c** $3\frac{2}{5}-2\frac{3}{4}$

SOLUTION

a $\frac{1}{2}+\frac{3}{5}=\frac{5}{10}+\frac{6}{10}$

$=\frac{11}{10}$ or $1\frac{1}{10}$

b $1\frac{2}{3}+4\frac{5}{6}=\frac{5}{3}+\frac{29}{6}$

$=\frac{10}{6}+\frac{29}{6}$

$=\frac{39}{6}$

$=\frac{13}{2}$ or $6\frac{1}{2}$

Alternative method: $1\frac{2}{3}+4\frac{5}{6}=1\frac{4}{6}+4\frac{5}{6}$

$=5\frac{9}{6}$

$=6\frac{3}{6}$

$=6\frac{1}{2}$

c $3\frac{2}{5}-2\frac{3}{4}=\frac{17}{5}-\frac{11}{4}$

$=\frac{68}{20}-\frac{55}{20}$

$=\frac{13}{20}$

EXPLANATION

The lowest common denominator of 2 and 5 is 10. Rewrite as equivalent fractions using a denominator of 10.
Add the numerators.

Change each mixed numeral to an improper fraction.
Remember the lowest common denominator of 3 and 6 is 6. Change $\frac{5}{3}$ to an equivalent fraction with denominator 6, then add the numerators.
Simplify using the HCF (3).

Add whole numbers and fractions separately, obtaining a common denominator for the fractions $\left(\frac{2}{3}=\frac{4}{6}\right)$.
$\frac{9}{6}=1\frac{3}{6}$

Convert to improper fractions, then rewrite as equivalent fractions with the same denominator.

Subtract the numerators.

Example 9 Multiplying fractions

Evaluate the following.

a $\frac{2}{3}\times\frac{5}{7}$ **b** $1\frac{2}{3}\times 2\frac{1}{10}$

SOLUTION

a $\frac{2}{3}\times\frac{5}{7}=\frac{2\times 5}{3\times 7}$

$=\frac{10}{21}$

b $1\frac{2}{3}\times 2\frac{1}{10}=\frac{\cancel{5}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{21}^{7}}{\cancel{10}_{2}}$

$=\frac{7}{2}$ or $3\frac{1}{2}$

EXPLANATION

No cancelling is possible as there are no common factors between numerators and denominators.
Multiply the numerators and denominators.

Rewrite as improper fractions.
Cancel common factors between numerators and denominators and then multiply numerators and denominators.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 10 Dividing fractions

Evaluate the following.

a $\frac{4}{15} \div \frac{12}{25}$ **b** $1\frac{17}{18} \div 1\frac{1}{27}$

SOLUTION

a $\frac{4}{15} \div \frac{12}{25} = \frac{4}{15} \times \frac{25}{12}$

$= \frac{{}^{1}\cancel{4}}{{}_{3}\cancel{15}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$

$= \frac{5}{9}$

b $1\frac{17}{18} \div 1\frac{1}{27} = \frac{35}{18} \div \frac{28}{27}$

$= \frac{{}^{5}\cancel{35}}{{}_{2}\cancel{18}} \times \frac{\cancel{27}^{3}}{\cancel{28}_{4}}$

$= \frac{15}{8}$ or $1\frac{7}{8}$

EXPLANATION

To divide by $\frac{12}{25}$ we multiply by its reciprocal $\frac{25}{12}$.

Cancel common factors between numerators and denominators then multiply fractions.

Rewrite mixed numerals as improper fractions.

Multiply by the reciprocal of the second fraction.

Exercise 1D REVISION

UNDERSTANDING AND FLUENCY	1(½), 2, 3, 4–8(½), 9, 10(½)	3, 4–8(½), 9, 10(½)	5–10(½)

1 Find the lowest common denominator for these pairs of fractions.

a $\frac{1}{2}, \frac{1}{3}$ **b** $\frac{3}{7}, \frac{5}{9}$ **c** $\frac{2}{5}, \frac{1}{13}$ **d** $\frac{3}{10}, \frac{8}{15}$

e $\frac{1}{2}, \frac{1}{4}$ **f** $\frac{9}{11}, \frac{4}{33}$ **g** $\frac{5}{12}, \frac{7}{30}$ **h** $\frac{13}{29}, \frac{2}{3}$

2 Convert these mixed numerals to improper fractions.

a $2\frac{1}{3}$ **b** $7\frac{4}{5}$ **c** $10\frac{1}{4}$ **d** $22\frac{5}{6}$

3 Copy and complete the given working.

a $\frac{3}{2} + \frac{4}{3} = \frac{\square}{6} + \frac{\square}{6}$

$= \frac{\square}{6}$

b $1\frac{1}{3} - \frac{2}{5} = \frac{\square}{3} - \frac{2}{5}$

$= \frac{\square}{15} - \frac{\square}{15}$

$= \frac{14}{\square}$

c $\frac{5}{3} \div \frac{2}{7} = \frac{5}{3} \times \frac{\square}{\square}$

$= \frac{\square}{6}$

Example 8a

4 Evaluate the following.

a $\frac{2}{5} + \frac{1}{5}$ **b** $\frac{3}{9} + \frac{1}{9}$ **c** $\frac{5}{7} + \frac{4}{7}$ **d** $\frac{3}{4} + \frac{1}{5}$

e $\frac{1}{3} + \frac{4}{7}$ **f** $\frac{3}{8} + \frac{4}{5}$ **g** $\frac{2}{5} + \frac{3}{10}$ **h** $\frac{4}{9} + \frac{5}{27}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8b

5 Evaluate the following.

a $3\frac{1}{4}+1\frac{3}{4}$ **b** $2\frac{3}{5}+\frac{4}{5}$ **c** $1\frac{3}{7}+3\frac{5}{7}$

d $2\frac{1}{3}+4\frac{2}{5}$ **e** $2\frac{5}{7}+4\frac{5}{9}$ **f** $10\frac{5}{8}+7\frac{3}{16}$

6 Evaluate the following.

a $\frac{4}{5}-\frac{2}{5}$ **b** $\frac{4}{5}-\frac{7}{9}$ **c** $\frac{3}{4}-\frac{1}{5}$ **d** $\frac{2}{5}-\frac{3}{10}$

e $\frac{8}{9}-\frac{5}{6}$ **f** $\frac{3}{8}-\frac{1}{4}$ **g** $\frac{5}{9}-\frac{3}{8}$ **h** $\frac{5}{12}-\frac{5}{16}$

Example 8c

7 Evaluate the following.

a $2\frac{3}{4}-1\frac{1}{4}$ **b** $3\frac{5}{8}-2\frac{7}{8}$ **c** $3\frac{1}{4}-2\frac{3}{5}$

d $3\frac{5}{8}-2\frac{9}{10}$ **e** $2\frac{2}{3}-1\frac{5}{6}$ **f** $3\frac{7}{11}-2\frac{3}{7}$

Example 9

8 Evaluate the following.

a $\frac{2}{5}\times\frac{3}{7}$ **b** $\frac{3}{5}\times\frac{5}{6}$ **c** $\frac{2}{15}\times\frac{5}{8}$ **d** $\frac{6}{21}\times1\frac{5}{9}$

e $6\times\frac{3}{4}$ **f** $8\times\frac{2}{3}$ **g** $\frac{5}{6}\times9$ **h** $1\frac{1}{4}\times4$

i $2\frac{1}{2}\times6$ **j** $1\frac{5}{8}\times16$ **k** $\frac{10}{21}\times1\frac{2}{5}$ **l** $\frac{25}{44}\times1\frac{7}{15}$

m $1\frac{1}{2}\times1\frac{1}{2}$ **n** $1\frac{1}{2}\times2\frac{1}{3}$ **o** $2\frac{2}{3}\times2\frac{1}{4}$ **p** $1\frac{1}{5}\times1\frac{1}{9}$

9 Write down the reciprocal of these numbers.

a 3 **b** $\frac{5}{7}$ **c** $\frac{1}{8}$ **d** $\frac{13}{9}$

Example 10

10 Evaluate the following.

a $\frac{4}{7}\div\frac{3}{5}$ **b** $\frac{3}{4}\div\frac{2}{3}$ **c** $\frac{5}{8}\div\frac{7}{9}$ **d** $\frac{3}{7}\div\frac{4}{9}$ **e** $\frac{3}{4}\div\frac{9}{16}$

f $\frac{4}{5}\div\frac{8}{15}$ **g** $\frac{8}{9}\div\frac{4}{27}$ **h** $\frac{15}{42}\div\frac{20}{49}$ **i** $15\div\frac{5}{6}$ **j** $6\div\frac{2}{3}$

k $12\div\frac{3}{4}$ **l** $24\div\frac{3}{8}$ **m** $\frac{4}{5}\div8$ **n** $\frac{3}{4}\div9$ **o** $\frac{8}{9}\div6$

p $14\div4\frac{1}{5}$ **q** $6\div1\frac{1}{2}$ **r** $1\frac{1}{3}\div8$ **s** $2\frac{1}{4}\div1\frac{1}{2}$ **t** $4\frac{2}{3}\div5\frac{1}{3}$

PROBLEM-SOLVING AND REASONING 12–14, 17 | 11(½), 12, 14, 15, 17, 18 | 11(½), 14–16, 18, 19

11 Evaluate these mixed-operation computations.

a $\frac{2}{3}\times\frac{1}{3}\div\frac{7}{9}$ **b** $\frac{4}{5}\times\frac{3}{5}\div\frac{9}{10}$ **c** $\frac{4}{9}\times\frac{6}{25}\div\frac{1}{150}$

d $2\frac{1}{5}\times\frac{3}{7}\div1\frac{3}{14}$ **e** $5\frac{1}{3}\times\frac{13}{24}\div1\frac{1}{6}$ **f** $2\frac{4}{13}\times\frac{3}{8}\div3\frac{3}{4}$

12 To remove impurities a mining company filters $3\frac{1}{2}$ tonnes of raw material. If $2\frac{5}{8}$ tonnes are removed, what quantity of material remains?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 When a certain raw material is processed it produces $3\frac{1}{7}$ tonnes of mineral and $2\frac{3}{8}$ tonnes of waste. How many tonnes of raw material were processed?

14 In a $2\frac{1}{2}$ hour maths exam, $\frac{1}{6}$ of that time is allocated as reading time. How long is the reading time?

15 A road is to be constructed with $15\frac{1}{2}$ m^3 of crushed rock. If a small truck can carry $2\frac{1}{3}$ m^3 of crushed rock, how many truckloads will be needed?



16 Regan worked for $7\frac{1}{2}$ hours in a sandwich shop. Three-fifths of her time was spent cleaning up and the rest was spent serving customers. How much time did she spend serving customers?

17 Here is an example involving the subtraction of fractions where improper fractions are not used.

$$2\frac{1}{2} - 1\frac{1}{3} = 2\frac{3}{6} - 1\frac{2}{6}$$
$$= 1\frac{1}{6}$$

Try this technique on the following problem and explain the difficulty that you encounter.

$$2\frac{1}{3} - 1\frac{1}{2}$$

18 **a** A fraction is given by $\frac{a}{b}$. Write down its reciprocal.

b A mixed numeral is given by $a\frac{b}{c}$. Write an expression for its reciprocal.

19 If a, b and c are integers, simplify the following.

a $\frac{b}{a} \times \frac{a}{b}$ **b** $\frac{a}{b} \div \frac{b}{a}$ **c** $\frac{a}{b} \div \frac{a}{b}$

d $\frac{a}{b} \times \frac{c}{a} \div \frac{a}{b}$ **e** $\frac{abc}{a} \div \frac{bc}{a}$ **f** $\frac{a}{b} \div \frac{b}{c} \times \frac{b}{a}$

ENRICHMENT — — 20

Fraction operation challenge

20 Evaluate the following, leaving your answers as improper fractions where necessary.

a $2\frac{1}{3} - 1\frac{2}{5} \times 2\frac{1}{7}$ **b** $1\frac{1}{4} \times 1\frac{1}{5} - 2\frac{1}{2} \div 10$ **c** $1\frac{4}{5} \times 4\frac{1}{6} + \frac{2}{3} \times 1\frac{1}{5}$

d $\left(1\frac{2}{3} + 1\frac{3}{4}\right) \div 3\frac{5}{12}$ **e** $4\frac{1}{6} \div \left(1\frac{1}{3} + 1\frac{1}{4}\right)$ **f** $\left(1\frac{1}{5} - \frac{3}{4}\right) \times \left(1\frac{1}{5} - \frac{3}{4}\right)$

g $\left(2\frac{1}{4} - 1\frac{2}{3}\right) \times \left(2\frac{1}{4} + 1\frac{2}{3}\right)$ **h** $\left(3\frac{1}{2} + 1\frac{3}{5}\right) \times \left(3\frac{1}{2} - 1\frac{3}{5}\right)$ **i** $\left(2\frac{2}{3} - 1\frac{3}{4}\right) \times \left(2\frac{2}{3} + 1\frac{3}{4}\right)$

j $\left(4\frac{1}{2} - 3\frac{2}{3}\right) \div \left(1\frac{1}{3} + \frac{1}{2}\right)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1E Ratios, rates and best buys REVISION

Interactive

Widgets

HOTsheets

Walkthrough

Fractions, ratios and rates are used to compare quantities. A lawnmower, for example, might require $\frac{1}{6}$ of a litre of oil to make a petrol mix of 2 parts oil to 25 parts petrol, which is an oil to petrol ratio of 2 to 25 or 2 : 25. The mower's blades might then spin at a rate of 1000 revolutions per minute (1000 rev/min).

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



Two-stroke lawnmowers run on petrol and oil mixed in a certain ratio.

Let's start: The lottery win

$100 000 is to be divided up for three lucky people into a ratio of 2 to 3 to 5 (2 : 3 : 5). Work out how the money is to be divided.

- Clearly write down your method and answer. There may be many different ways to solve this problem.
- Write down and discuss the alternative methods suggested by other students in the class.

Key ideas

- **Ratios** are used to compare quantities with the same units.
  - The ratio of a to b is written $a : b$.
  - Ratios in simplest form use whole numbers that have no common factor.
- The **unitary method** involves finding the value of one part of a total.
  - Once the value of one part is found the value of several parts can easily be determined.
- A **rate** compares related quantities with different units.
  - The rate is usually written with one quantity compared to a single unit of the other quantity. For example: 50 km per 1 hour or 50 km/h.
- Ratios and rates can be used to determine **best buys** when purchasing products.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11 Simplifying ratios

Simplify these ratios.

a $38 : 24$ **b** $2\frac{1}{2} : 1\frac{1}{3}$ **c** $0.2 : 0.14$

SOLUTION	EXPLANATION
a $38 : 24 = 19 : 12$	The HCF of 38 and 24 is 2 so divide both sides by 2.
b $2\frac{1}{2} : 1\frac{1}{3} = \frac{5}{2} : \frac{4}{3}$	Write as improper fractions using the same denominator.
$= \frac{15}{6} : \frac{8}{6}$	Then multiply both sides by 6 to write as whole numbers.
$= 15 : 8$	
c $0.2 : 0.14 = 20 : 14$	Multiply by 100 to convert both numbers into whole numbers.
$= 10 : 7$	Divide by the HCF to simplify the ratio.

Example 12 Dividing into a given ratio

$300 is to be divided into the ratio 2 : 3.
Find the value of the larger portion using the unitary method.

SOLUTION	EXPLANATION
Total number of parts is $2 + 3 = 5$	Use the ratio 2 : 3 to get the total number of parts.
5 parts = $300	
1 part $= \frac{1}{5}$ of $300	Calculate the value of each part.
$= $60	
Larger portion $= 3 \times 60	Calculate the value of 3 parts.
$= $180	

Example 13 Simplifying rates

Write these rates in simplest form.

a 120 km every 3 hours

b 5000 revolutions in $2\frac{1}{2}$ minutes

SOLUTION	EXPLANATION
a 120 km per 3 hours $= \frac{120}{3}$ km/h	Divide by 3 to write the rate compared to 1 hour.
$= 40$ km/h	
b 5000 revolutions per $2\frac{1}{2}$ minutes	
$= 10\,000$ revolutions per 5 minutes	First multiply by 2 to remove the fraction.
$= \frac{10\,000}{5}$ rev/min	Then divide by 5 to write the rate using 1 minute.
$= 2000$ rev/min	

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14 Finding best buys

a Which is better value?
5 kg of potatoes for $3.80 or 3 kg for $2.20

b Shampoo A: 400 mL for $3.20
Shampoo B: 320 mL for $2.85
Compare the cost of 100 mL of each product
Assuming they are of equal quality, which is the best buy?

SOLUTION	EXPLANATION
a **Method A**: Price per kg.	
5 kg bag	
1 kg costs $3.80 ÷ 5 = $0.76	Divide each price by the number of kilograms to find the price per kilogram.
3 kg bag	
1 kg costs $2.20 ÷ 3 = $0.73	
∴ The 3 kg bag is better value.	Then compare.
Method B: Amount per $1.	
5 kg bag	
$1 buys 5 ÷ 3.8 = 1.32 kg	Divide each amount in kilograms by the cost to find the weight per $1 spent.
3 kg bag	
$1 buys 3 ÷ 2.2 = 1.36 kg	
∴ The 3 kg bag is better value.	Then compare.
b Shampoo A	
100 mL costs $3.20 ÷ 4 = $0.80	Alternatively, divide by 400 to find the cost of 1 mL then multiply by100.
Shampoo B	
100 mL costs $2.85 ÷ 3.2 = $0.89	Alternatively, divide by 320 to find the cost of 1 mL then multiply by 100.
∴ Shampoo A is the best buy.	

Exercise 1E REVISION

UNDERSTANDING AND FLUENCY 1–4, 5–6(½), 7, 8, 10–12 | 3, 5–6(½), 7, 9, 11, 12 | 9–11, 13

1 Write down the missing number.

a 2 : 5 = ☐ : 10 **b** 3 : 7 = ☐ : 28 **c** 5 : 8 = 15 : ☐ **d** 7 : 12 = 42 : ☐

e ☐ : 12 = 1 : 4 **f** 4 : ☐ = 16 : 36 **g** 8 : ☐ = 640 : 880 **h** ☐ : 4 = 7.5 : 10

2 Consider the ratio of boys to girls of 4 : 5.

a What is the total number of parts? **b** What fraction of the total are boys?

c What fraction of the total are girls?

d If there are 18 students in total, how many of them are boys?

e If there are 18 students in total, how many of them are girls?

3 A car is travelling at a rate (speed) of 80 km/h.

a How far would it travel in:

i 3 hours? **ii** $\frac{1}{2}$ hour? **iii** $6\frac{1}{2}$ hours?

b How long would it take to travel:

i 400 km? **ii** 360 km? **iii** 20 km?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party. Updated March 2022

4 Find the cost of 1 kg if:

a 2 kg costs $8 **b** 5 kg costs $15 **c** 4 kg costs $10

Example 11

5 Simplify these ratios.

a 6 : 30 **b** 8 : 20 **c** 42 : 28 **d** 52 : 39 **e** $1\frac{1}{2} : 3\frac{1}{3}$ **f** $2\frac{1}{4} : 1\frac{2}{5}$

g $\frac{3}{8} : 1\frac{3}{4}$ **h** $1\frac{5}{6} : 3\frac{1}{4}$ **i** 0.3 : 0.9 **j** 0.7 : 3.5 **k** 1.6 : 0.56 **l** 0.4 : 0.12

6 Write each of the following as a ratio in simplest form. *Hint:* convert to the same units first.

a 80c : $8 **b** 90c : $4.50 **c** 80 cm : 1.2 m

d 0.7 kg : 800 g **e** 2.5 kg : 400 g **f** 30 min : 2 hours

g 45 min : 3 hours **h** 4 hours : 50 min **i** 40 cm : 2 m : 50 cm

j 80 cm : 600 mm : 2 m **k** 2.5 hours : 1.5 days **l** 0.09 km : 300 m : 1.2 km

Example 12

7 Divide $500 into these given ratios.

a 2 : 3 **b** 3 : 7 **c** 1 : 1 **d** 7 : 13

8 420 g of flour is to be divided into a ratio of 7 : 3 for two different recipes. Find the smaller amount.

9 Divide $70 into these ratios.

a 1 : 2 : 4 **b** 2 : 7 : 1 **c** 8 : 5 : 1

Example 13

10 Write these rates in simplest form.

a 150 km in 10 hours **b** 3000 revolutions in $1\frac{1}{2}$ minutes

c 15 swimming strokes in $\frac{1}{3}$ of a minute **d** 56 metres in 4 seconds

e 180 mL in 22.5 hours **f** 207 heartbeats in $2\frac{1}{4}$ minutes

11 Hamish rides his bike at an average speed of 22 km/h. How far does he ride in:

a $2\frac{1}{2}$ hours? **b** $\frac{3}{4}$ hours? **c** 15 minutes?

Example 14a

12 Determine the best buy in each of the following.

a 2 kg of washing powder for $11.70 or 3 kg for $16.20

b 1.5 kg of red delicious apples for $4.80 or 2.2 kg of royal gala apples for $7.92

c 2.4 litres of orange juice for $4.20 or 3 litres of orange juice for $5.40

d 0.7 GB of internet usage for $14 or 1.5 GB for $30.90 with different service providers

Example 14b

13 Find the cost of 100 g of each product below then decide which is the best buy.

a 300 g of coffee A at $10.80 or 220 g of coffee B at $8.58

b 600 g of pasta A for $7.50 or 250 g of pasta B for $2.35

c 1.2 kg of cereal A for $4.44 or 825 g of cereal B for $3.30

PROBLEM-SOLVING AND REASONING 14, 15, 22 | 15–18, 22, 23 | 17–21, 23, 24

14 Kirsty manages a restaurant. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

15 If a prize of $6000 was divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much did each girl get?

16 When a crate of twenty 375 mL soft-drink cans is purchased, the cost works out to be $1.68 per litre. A crate of 30 of the same cans is advertised as being a saving of 10 cents per can compared with the 20-can crate. Calculate how much the 30-can crate costs.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

17 The dilution ratio for a particular chemical with water is 2 : 3 (chemical to water). If you have 72 litres of chemical, how much water is needed to dilute the chemical?

18 Amy, Belinda, Candice and Diane invested money in the ratio of 2 : 3 : 1 : 4 in a publishing company. If the profit was shared according to their investment, and Amy's profit was $2400, find the profit each investor made.

19 Julie is looking through the supermarket catalogue for her favourite cookies-and-cream ice-cream. She can buy 2 L of triple-chocolate ice-cream for $6.30 while the cookies-and-cream ice-cream is usually $5.40 for 1.2 L. What saving does there need to be on the price of the 1.2 L container of cookies-and-cream ice-cream for it to be of equal value to the 2 L triple-chocolate container?

20 The ratio of the side lengths of one square to another is 1 : 2. Find the ratio of the areas of the two squares.

21 A quadrilateral (with angle sum 360°) has interior angles in the ratio 1 : 2 : 3 : 4. Find the size of each angle.

22 2.5 kg of cereal A costs $4.80 and 1.5 kg of cereal B costs $2.95. Write down at least two different methods to find which cereal is a better buy.

23 If $a : b$ is in simplest form, state whether the following are true or false.

a a and b must both be odd.
b a and b must both be prime.
c At least one of a or b is odd.
d The HCF of a and b is 1

24 A ratio is $a : b$ with $a < b$ and a and b are positive integers. Write an expression for the:

a total number of parts
b fraction of the smaller quantity out of the total
c fraction of the larger quantity out of the total

ENRICHMENT — — 25

Mixing drinks

25 Four jugs of cordial have a cordial-to-water ratio as shown and a given total volume.

Jug	Cordial-to-water ratio	Total volume
1	1 : 5	600 mL
2	2 : 7	900 mL
3	3 : 5	400 mL
4	2 : 9	330 mL

a How much cordial is in:
i jug 1? **ii** jug 2?

b How much water is in:
i jug 3? **ii** jug 4?

c If jugs 1 and 2 were mixed together to give 1500 mL of drink:
i how much cordial is in the drink?
ii what is the ratio of cordial to water in the drink?

d Find the ratio of cordial to water if the following jugs are mixed.
i Jugs 1 and 3 **ii** Jugs 2 and 3 **iii** Jugs 2 and 4 **iv** Jugs 3 and 4

e Which combination of two jugs gives the strongest cordial-to-water ratio?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1F Computation with percentages and money REVISION

Interactive
Widgets
HOTsheets
Walkthrough

We use percentages for many different things in our daily lives. Some examples are loan rates, the interest given on term deposits and discounts on goods.

We know from our previous studies that a percentage is a fraction that has a denominator of 100. 'Per cent' comes from the Latin word *per centum* and means 'out of 100'.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Which is the largest piece?

Four people receive the following portions of a cake:

Milly	25.5%
Tom	$\frac{1}{4}$
Adam	0.26
Mai	The left over

- Which person gets the most cake and why?
- How much cake does Mai get? What is her portion written as a percentage, decimal and fraction?

Key ideas

- Conversions:

Decimal 0.3 Fraction $\frac{3}{10}$	$\times 100$ → ← $\div 100$	Percentage 30%

- A percentage of a number can be found using multiplication.

  For example: $25\% \text{ of } \$26 = 0.25 \times \$26$
  $= \$6.50$

- To find an original amount, use the **unitary method** or use division.

  For example: if 3% of an amount is \$36:
  - Using the unitary method: 1% of the amount is $\$36 \div 3 = \$12$
    $\therefore 100\%$ of the amount is $\$12 \times 100 = \$1200$
  - Using division: 3% of the amount is \$36
    $0.03 \times \text{amount} = \$36$
    $\text{amount} = \$36 \div 0.03$
    $= \$1200$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 15 Converting between percentages, decimals and fractions

a Express 0.45 as a percentage.
b Express 25% as a decimal.
c Express $3\frac{1}{4}\%$ as a fraction.

SOLUTION

a $0.45 \times 100 = 45$
$\therefore 0.45 = 45\%$

b $25\% = 25 \div 100 = 0.25$

c $3\frac{1}{4}\% = 3\frac{1}{4} \div 100$
$= \frac{13}{4} \times \frac{1}{100}$
$= \frac{13}{400}$

EXPLANATION

Multiply by 100. This moves the decimal point 2 places to the right.

Divide by 100. This moves the decimal point 2 places to the left.

Divide by 100.
Write the mixed numeral as an improper fraction and multiply by the reciprocal of 100 $\left(\text{i.e. } \frac{1}{100}\right)$.

Example 16 Writing a quantity as a percentage

Write 50c out of \$2.50 as a percentage.

SOLUTION

50c out of \$2.50 $= \frac{{}^{1}\cancel{50}}{{}_{5}\cancel{250}} \times 100\%$
$= 20\%$

EXPLANATION

Convert to the same units (\$2.50 = 250c) and write as a fraction. Multiply by 100%, cancelling first.

Example 17 Finding a percentage of a quantity

Find 15% of \$35.

SOLUTION

15% of \$35 $= \frac{{}^{3}\cancel{15}}{{}_{20}\cancel{100}} \times \$35$
$= \$5.25$

EXPLANATION

Write the percentage as a fraction out of 100 and multiply by \$35.
Note: 'of' means to 'multiply'.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18 Finding the original amount

Determine the original amount if 5% of the amount is $45.

SOLUTION

Method 1: Unitary

5% of the amount = $45

1% of the amount = $9

100% of the amount = $900

So the original amount is $900

EXPLANATION

To use the unitary method, find the value of 1 part or 1% then multiply by 100 to find 100%.

Method 2: Division

5% of amount = $45

$0.05 \times$ amount = $45

amount = $45 $\div$ 0.05

= $900

Write 5% as a decimal then divide both sides by this number to find the original amount.

Exercise 1F REVISION

UNDERSTANDING AND FLUENCY | 1–3, 4–6(½), 7, 8–10(½) | 3, 4–6(½), 7–10 | 4–6(½), 8–11(½)

1 Convert these percentages to a simplified fraction.

a 3%

b 11%

c 35%

d 8%

2 Convert these percentages to decimals.

a 4%

b 23%

c 86%

d 46.3%

3 Convert these decimals and fractions to percentages.

a 0.5 **b** 0.6 **c** 0.25 **d** 0.9

e $\frac{3}{4}$ **f** $\frac{1}{2}$ **g** $\frac{1}{5}$ **h** $\frac{1}{8}$

Example 15a

4 Write each of the following as a percentage.

a 0.34 **b** 0.4 **c** 0.06 **d** 0.7

e 1 **f** 1.32 **g** 1.09 **h** 3.1

Example 15b

5 Write each of the following as a decimal.

a 67% **b** 30%

c 250% **d** 8%

e $4\frac{3}{4}\%$ **f** $10\frac{5}{8}\%$

g $30\frac{2}{5}\%$ **h** $44\frac{1}{4}\%$

Example 15c

6 Write each part of Question **5** as a simplified fraction.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Copy and complete this table. Use the simplest form for fractions.

Percentage	Fraction	Decimal
10%		
	$\frac{1}{2}$	
5%		
		0.25
		0.2
	$\frac{1}{8}$	
1%		
	$\frac{1}{9}$	
		$0.\dot{2}$

Percentage	Fraction	Decimal
	$\frac{3}{4}$	
15%		
		0.9
37.5%		
$33\frac{1}{3}\%$	$\frac{1}{3}$	
$66\frac{2}{3}\%$	1	
		0.625
	$\frac{1}{6}$	

Example 16

8 Convert each of the following to a percentage.

a \$3 out of \$12 **b** \$6 out of \$18 **c** \$0.40 out of \$2.50
d \$44 out of \$22 **e** \$140 out of \$5 **f** 45c out of \$1.80

Example 17

9 Find:

a 10% of \$360 **b** 50% of \$420 **c** 75% of 64 kg
d 12.5% of 240 km **e** 37.5% of 40 apples **f** 87.5% of 400 m
g $33\frac{1}{3}\%$ of 750 people **h** $66\frac{2}{3}\%$ of 300 cars **i** $8\frac{3}{4}\%$ of \$560

Example 18

10 Determine the original amount if:

a 10% of the amount is \$12 **b** 6% of the amount is \$42
c 3% of the amount is \$9 **d** 40% of the amount is \$2.80
e 90% of the amount is \$0.18 **f** 35% of the amount is \$140

11 Determine the value of x in the following if:

a 10% of x is \$54 **b** 15% of x is \$90 **c** 25% of x is \$127
d 18% of x is \$225 **e** 105% of x is \$126 **f** 110% of x is \$44

PROBLEM-SOLVING AND REASONING 12, 13, 18 | 13–15, 18 | 15–19

12 Bad weather stopped a cricket game for 35 minutes of a scheduled $3\frac{1}{2}$ hour match. What percentage of the scheduled time was lost?



13 Joe lost 4 kg and now weighs 60 kg. What percentage of his original weight did he lose?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

14 About 80% of the mass of the human body is water. If Clare is 60 kg, how many kilograms of water make up her body weight?

15 In a class of 25 students, 40% have been to England. How many students have not been to England?

16 20% of the cross-country runners in a school team weigh between 60 kg and 70 kg. If 4% of the school of 1125 students are in the cross-country team, how many students in the team weigh between 60 kg and 70 kg?



17 One week Grace spent 16% of her weekly wage on a new bookshelf that cost $184. What is her weekly wage?

18 Consider the equation $P\%$ of $a = b$ (like 20% of 40 = 8 or 150% of 22 = 33).

- **a** For what value of P is $P\%$ of $a = a$?
- **b** For what values of P is $P\%$ of $a > a$?
- **c** For what values of P is $P\%$ of $a < a$?

19 What can be said about the numbers x and y if:

- **a** 10% of x = 20% of y?
- **b** 10% of x = 50% of y?
- **c** 5% of x = 3% of y?
- **d** 14% of x = 5% of y?

ENRICHMENT — — 20

More than 100%

20 **a** Find 120% of 60.

- **b** Determine the value of x if 165% of x = 1.5.
- **c** Write 2.80 as a percentage.
- **d** Write 325% as a fraction.
- **e** $2000 in a bank account increases to $5000 over a period of time. By how much has the amount increased as a percentage?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1G Percentage increase and decrease REVISION

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4









Percentages are often used to describe the proportion by which a quantity has increased or decreased. The price of a car in the new year might be increased by 5%. On a $70 000 car, this equates to a $3500 increase. The price of a shirt might be marked down by 30% and if the shirt originally cost $60, this provides an $18 discount. It is important to note that the increase or decrease is calculated on the original amount.

Let's start: The quicker method

Two students, Nicky and Mila, consider the question: $250 is increased by 15%. What is the final amount?
Nicky puts his solution on the board with two steps.

Step 1. 15% of $250 = 0.15 × $250
= $37.50
Step 2. Final amount = $250 + $37.50
= $287.50

Mila says that the same problem can be solved with only one step using the number 1.15.

- Can you explain Mila's method? Write it down.
- What if the question was altered so that $250 is decreased by 15%. How would Nicky's and Mila's methods work in this case?
- Which of the two methods do you prefer and why?

Key ideas

- To increase an amount by a given percentage, multiply by the sum of 100% and the given percentage.

  For example: to increase by 30%, multiply by (100% + 30%) = 130% = 1.3

- To decrease an amount by a given percentage, multiply by 100% minus the given percentage.

  For example: to decrease by 25%, multiply by (100% − 25%) = 75% = 0.75

- To find a **percentage change** use:

$$\text{percentage change} = \frac{\text{change}}{\text{original amount}} \times 100\%$$

Example 19 Increasing by a percentage

Increase $70 by 15%.

SOLUTION	EXPLANATION
100% + 15% = 115% = 1.15	First add 15% from 100%.
$70 × 1.15 = $80.50	Multiply by 1.15 to give $70 plus the increase in one step.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 20 Decreasing by a percentage

Decrease \$5.20 by 40%.

SOLUTION

$100\% - 40\% = 60\%$
$= 0.6$
$\$5.20 \times 0.6 = \$3.12$

EXPLANATION

First subtract 40% from 100% to find the percentage remaining.
Multiply by $60\% = 0.6$ to get the result.

Example 21 Finding a percentage change

a The price of a mobile phone increases from \$250 to \$280. Find the percentage increase.
b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to 1 decimal place.

SOLUTION

a Increase $= \$280 - \$250$
$= \$30$

Percentage increase $= \frac{30}{250} \times \frac{100}{1}\%$
$= 12\%$

b Decrease $= 3220 - 2985$
$= 235$

Percentage decrease $= \frac{235}{3220} \times \frac{100}{1}\%$
$= 7.3\%$ (to 1 d.p)

EXPLANATION

First find the actual increase.

Divide the increase by the original amount and multiply by 100%.

First find the actual decrease.

Divide the decrease by the original population. Round as indicated.

Example 22 Finding the original amount

After rain, the volume of water in a tank increased by 24% to 2200 L. How much water was in the tank before it rained? Round to the nearest litre.

SOLUTION

$100\% + 24\% = 124\%$
$= 1.24$
Original volume $\times 1.24 = 2200$
$\therefore$ original volume $= 2200 \div 1.24$
$= 1774$ L

EXPLANATION

Write the total percentage as a decimal.

The original volume is increased by 24% to give 2200 litres. Divide to find the original volume. Round off to nearest litre.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 1G REVISION

UNDERSTANDING AND FLUENCY	1–3, 4–5(½), 6–8	3, 4–5(½), 6, 8, 9	4–5(½), 7, 9, 10

1 Write the missing number.

a To increase a number by 40%, multiply by _____.
b To increase a number by 26%, multiply by _____.
c To increase a number by _____, multiply by 1.6.
d To increase a number by _____, multiply by 1.21.
e To decrease a number by 20%, multiply by _____.
f To decrease a number by 73%, multiply by _____.
g To decrease a number by _____, multiply by 0.94.
h To decrease a number by _____, multiply by 0.31.

2 The price of a watch increases from $120 to $150.

a What is the price increase?
b Write this increase as a percentage of the original price.

3 A person's weight decreases from 108 kg to 96 kg.

a What is the weight decrease?
b Write this decrease as a percentage of the original weight. Round to 1 decimal place.

Example 19

4 Increase the given amounts by the percentage given in the brackets.

a $50 (5%) **b** 35 min (8%) **c** 250 mL (50%) **d** 1.6 m (15%)
e 24.5 kg (12%) **f** 25 watts (44%) **g** $13 000 (4.5%) **h** $1200 (10.2%)

Example 20

5 Decrease the given amounts by the percentage given in the brackets.

a 24 cm (20%) **b** 35 cm (30%) **c** 42 kg (7%) **d** 55 min (12%)
e $90 (12.8%) **f** 220 mL (8%) **g** 25°C (28%) **h** $420 (4.2%)

Example 21a

6 The length of a bike sprint race is increased from 800 m to 1200 m. Find the percentage increase.

7 From the age of 10 to the age of 17, Nick's height increased from 125 cm to 180 cm. Find the percentage increase.

Example 21b

8 The temperature at night decreased from 25°C to 18°C. Find the percentage decrease.

9 Brett, a rising sprint star, lowered his 100 m time from 13 seconds flat to 12.48 seconds. Find the percentage decrease.

10 Find the percentage change in these situations, rounding to 1 decimal place in each case.

a 22 g increases to 27 g **b** 86°C increases to 109°C
c 136 km decreases to 94 km **d** $85.90 decreases to $52.90

PROBLEM-SOLVING AND REASONING	11–13, 17	11–14, 17, 18	13–16, 18–20

Example 22

11 After a price increase of 20% the cost of entry to a museum rose to $25.80. Find the original price.

12 Average attendance at a sporting match rose by 8% in the past year to 32 473. Find the average in the previous year to the nearest whole number.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 A car decreased in value by 38% to $9235 when it was resold. What was the original price of the car to the nearest dollar?

14 After the redrawing of boundaries, land for a council electorate had decreased by 14.5% to 165 420 hectares. What was the original size of the electorate to the nearest hectare?

15 The total price of an item including GST (at 10%) is $120. How much GST is paid to the nearest cent?

16 A consultant charges a school a fee of $300 per hour including GST (at 10%). The school hires the consultant for 2 hours but can claim back the GST from the government. Find the net cost of the consultant for the school to the nearest cent.

17 An investor starts with $1000.

- **a** After a bad day, the initial investment is reduced by 10%. Find the balance at the end of the day.
- **b** The next day is better and the balance is increased by 10%. Find the balance at the end of the second day.
- **c** The initial amount decreased by 10% on the first day and increased by 10% on the second day. Explain why the balance on the second day did not return to $1000.

18 During a sale in a bookstore all travel guides are reduced from $30 by 20%. What percentage increase is required to take the price back to $30?

19 The cost of an item is reduced by 50%. What percentage increase is required to return to its original price?

20 The cost of an item is increased by 75%. What percentage decrease is required to return to its original price? Round to 2 decimal places.

ENRICHMENT	—	—	21, 22

Repeated increase and decrease

21 If the cost of a pair of shoes was increased three times by 10%, 15% and 8% from an original price of $80, then the final price would be

$80 × 1.10 × 1.15 × 1.08 = $109.30

Use a similar technique to find the final price of these items. Round to the nearest cent.

- **a** Skis starting at $450 and increasing by 20%, 10% and 7%
- **b** A computer starting at $2750 and increasing by 6%, 11% and 4%
- **c** A DVD player starting at $280 and decreasing by 10%, 25% and 20%
- **d** A circular saw starting at $119 and increasing by 18%, 37% and 11%

22 If an amount is increased by the same percentage each time, powers can be used.

For example: 50 kg increased by 12% three times would increase to:

$50 \text{ kg} \times 1.12 \times 1.12 \times 1.12$

$= 50 \text{ kg} \times (1.12)^3$

$= 70.25 \text{ kg}$ (to 2 d.p.)

Use a similar technique to find the final value in these situations. Round to 2 decimal places.

- **a** The mass of a rat initially at 60 grams grows at a rate of 10% every month for 3 months.
- **b** The cost of a new lawnmower initially at $80 000 increases by 5% every year for 4 years.
- **c** The value of a house initially at $380 000 decreases by 4% per year for 3 years.
- **d** The length of a pencil initially at 16 cm decreases through being sharpened by 15% every week for 5 weeks.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1H Profits and discounts REVISION

Interactive

Widgets

HOTsheets

Walkthrough

Percentages are widely used in the world of finance. Profits, losses, commissions, discounts and taxation are often expressed and calculated using percentages.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The best discount

Two adjacent book shops are selling the same book at a discounted price. The recommended retail price for the book is the same for both shops. Each shop has a sign near the book with the given details:

- Shop A. Discounted by 25%
- Shop B. Reduced by 20% then take a further 10% off that.

Which shop offers the bigger discount? Is the difference equal to 5% of the retail price?



Book shops use percentages to advertise discounts during a sale.

Key ideas

- **Profit** is the amount of money made on a sale. If the profit is negative, we say that a loss has been made.
  - Profit = selling price − cost price
- **Mark-up** is the amount added to the cost price to produce the selling price.
  - Selling price = cost price + mark-up
- The **percentage profit** or loss can be found by dividing the profit or loss by the cost price and multiplying by 100.
  - $\%\ \text{Profit/Loss} = \dfrac{\text{profit or loss}}{\text{cost price}} \times 100\%$
- **Discount** is the amount by which an item is marked down.
  - New price = original price − discount
  - Discount = % discount × original price

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 23 Determining profit

A manufacturer produces an item for $400 and sells it for $540.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

SOLUTION

a Profit $= \$540 - \$400$
$= \$140$

b $\%\text{ profit} = \dfrac{140}{400} \times 100\%$
$= 35\%$

EXPLANATION

Profit = selling price − cost price

$\%\text{ profit} = \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

Example 24 Calculating selling price from mark-up

An electrical store marks up all entertainment systems by 30%.
If the cost price of one entertainment system is $8000, what will be its selling price?

SOLUTION

Selling price = 130% of cost price
$= 1.3 \times 8000$
$= \$10\,400$

Alternative method

Mark-up = 30% of $8000
$= 0.3 \times 8000$
$= \$2400$
$\therefore$ selling price $= 8000 + 2400$
$= \$10\,400$

EXPLANATION

Since there is a 30% mark-up added to the cost price (100%), it follows that the selling price is 130% of the cost price.

Change percentage to a decimal and evaluate.

Selling price = cost price + mark-up

Example 25 Finding the discount amount

Harvey Norman advertises a 15% discount on all equipment as a Christmas special. Find the sale price on a projection system that has a marked price of $18 000.

SOLUTION

New price = 85% of $18 000
$= 0.85 \times 18\,000$
$= \$15\,300$

Alternative method

Discount = 15% of $18 000
$= 0.15 \times 18\,000$
$= \$2700$
$\therefore$ new price $= 18\,000 - 2700$
$= \$15\,300$

EXPLANATION

Discounting by 15% means the new price is 85%, i.e. (100 − 15)% of the original price.

Change the percentage to a decimal and evaluate.

New price is original price minus discount.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 26 Calculating sale saving

A toy shop discounts a toy by 10%, due to a sale. If the sale price was $10.80, what was the original price?

SOLUTION

Let $\$x$ be the original price.

$0.9 \times x = 10.8$

$x = 10.8 \div 0.9$

$x = 12$

The original price was $12.

EXPLANATION

The discount factor $= 100\% - 10\% = 90\% = 0.9$. Thus $10.80 is 90% of the original price. Write an equation representing this and solve.

Write the answer in words.

Exercise 1H REVISION

UNDERSTANDING AND FLUENCY 1–7, 11, 13 | 2–5, 7, 8, 11–13 | 4, 6, 7, 10, 11, 13, 14

1 Write the missing numbers in these tables. Use 1 decimal place for percentages.

a

Cost price ($)	7	18	24.80		460.95	3250
Selling price ($)	10	15.50	26.20	11.80	395	
Profit/Loss ($)				4.50 profit		1180 loss
Profit/Loss (%)						

b

Cost price ($)		99.95	199.95			18000
Mark-up ($)	10			700	16700	
Selling price ($)	40.95	179.95	595.90	1499.95	35499	26995
Mark-up (%)						

c

Original price ($)	100	49.95	29.95			2215
New price ($)	72	40.90		176	299.95	
Discount ($)			7.25	23	45.55	178
Discount (%)						

2 The following percentage discounts are given on the price of various products. State the percentage of the original price that each product is selling for.

a 10% **b** 20% **c** 15% **d** 8%

Example 23

3 A manufacturer produces and sells items for the prices shown.

i Determine the profit made.

ii Express this profit as a percentage of the cost price.

a Cost price $10, selling price $12

b Cost price $20, selling price $25

c Cost price $120, selling price $136.80

d Cost price $1400, selling price $3850

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 Dom runs a pizza business. Last year he took in $88 000 and it cost him $33 000 to run. What is his percentage profit for the year? Round to 2 decimal places.

5 It used to take 20 hours to fly to Los Angeles. It now takes 12 hours. What is the percentage decrease in travel time?

6 Rob goes to the races with $600 in his pocket. He leaves at the end of the day with $45. What is his percentage loss?

Example 24

7 Helen owns a handicrafts store that has a policy of marking up all of its items by 25%. If the cost price of one article is $30, what will be its selling price?

8 Lenny marks up all computers in his store by 12.5%. If a computer cost him $890, what will be the selling price of the computer, to the nearest dollar?

9 A dining room table sells for $448. If its cost price was $350, determine the percentage mark-up on the table.

10 A used-car dealer purchases a vehicle for $13 000 and sells it for $18 500. Determine the percentage mark-up on the vehicle to 1 decimal place.

Example 25

11 A store is offering a 15% discount for customers who pay with cash. Rada wants a microwave oven marked at $175. How much will she pay if she is paying with cash?

12 A camera store displays a camera marked at $595 and a lens marked at $380. Sam is offered a discount of 22% if he buys both items. How much will he pay for the camera and lens?

Example 26

13 A refrigerator is discounted by 25%. If Paula pays $460 for it, what was the original price? Round to the nearest cent.

14 Maria put a $50 000 deposit on a house. What is the cost of the house if the deposit is 15% of the total price? Round to the nearest dollar.

PROBLEM-SOLVING AND REASONING 15, 18 | 15, 16, 18, 19 | 16, 17, 19, 20

15 A store marks up a $550 widescreen television by 30%. During a sale it is discounted by 20%. What is the percentage change in the original price of the television?

16 An armchair was purchased for a cost price of $380 and marked up to a retail price. It was then discounted by 10% to a sale price of $427.50. What is the percentage mark-up from the cost price to the retail price?

17 Pairs of shoes are manufactured for $24. They are sold to a warehouse with a mark-up of 15%. The warehouse sells the shoes to a distributor after charging a holding fee of $10 per pair. The distributor sells them to Fine Shoes for a percentage profit of 12%. The store then marks them up by 30%.

a Determine the price of a pair of shoes if you buy it from one of the Fine Shoes stores (round to the nearest 5 cents).

b What is the overall percentage mark-up of a pair of shoes to the nearest whole per cent?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

18 An item before being sold includes a percentage mark-up as well as a sale discount. Does it make a difference in which order the mark-up and discount occur? Explain your answer.

19 Depreciation relates to a reduction in value. A computer depreciates in value by 30% in its first year. If its original value is $3000, find its value after one year.

20 John buys a car for $75 000. The value of the car depreciates at 15% per year. After 1 year the car is worth 85% of its original value, i.e. 85% of $75 000 = $0.85 \times 75\,000 = \$63\,750$.

a What is the value of the car, to the nearest cent, after:

i 2 years?

ii 5 years?

b After how many years will the car first be worth less than $15 000?



ENRICHMENT — — 21

Deposits and discounts

21 A car company offers a special discount deal. After the cash deposit is paid, the amount that remains to be paid is discounted by a percentage that is one tenth of the deposit percentage.

For example, a deposit of $8000 on a $40 000 car represents 20% of the cost. The remaining $32 000 will be discounted by 2%.

Find the amount paid for each car given the following car price and deposit. Round to 2 decimal places where necessary.

a Price = $35 000, deposit = $7000

b Price = $45 000, deposit = $15 000

c Price = $28 000, deposit = $3500

d Price = $33 400, deposit = $5344

e Price = $62 500, deposit = $5000

f Price = $72 500, deposit = $10 150

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Income

Interactive

Widgets

HOTsheets

Walkthrough

Stage

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

For many people, income is made up largely of the money they receive from their paid work – their job. Depending on the job, this payment can be made by a number of different methods. Many professional workers will receive an annual fixed *salary*, which may be paid monthly or fortnightly. Casual workers, including those working in the retail area or restaurants, may receive a *wage* where they are paid a rate per hour worked – this rate may be higher out of regular working hours such as weekends or public holidays. Many sales people, including some real estate agents, may receive a weekly fee (a *retainer*) but may also receive a set percentage of the amount of sales they make (a *commission*). From their income, people have to pay living costs such as electricity, rent, groceries and other items. In addition, they have to pay tax to the government, which funds many of the nation's infrastructure projects and welfare. The method in which this tax is paid from their income may also vary.



Let's start: Which job pays better?

Ben and Nick are both looking for part-time jobs and they spot the following advertisements.

Kitchen hand	Office assistant
$9.40 per hour, $14.10 per hour on weekends	Receive $516 per month for 12 hours work per week

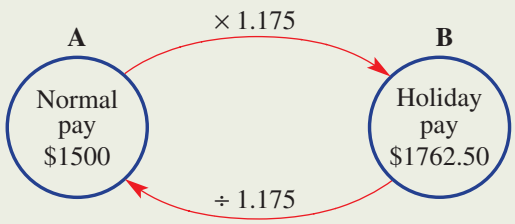
- Nick chooses to work as the kitchen hand. If in his first week he works 4 hours during the week and 8 hours at the weekend, how much will he earn?
- Ben works as the office assistant. How much does he earn per week if he works 4 weeks in a month? What does his hourly rate turn out to be?
- If Nick continues to work 12 hours in a week, does he earn more than Ben if he only works on week days? How many weekday hours must Nick work to match Ben's pay?
- Out of the 12 hours, what is the minimum number of hours Nick must work at the weekend to earn at least as much as Ben?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Workers who earn a **wage** (for example, a casual waiter) are paid a fixed rate per hour. Hours outside the normal working hours (public holidays etc.) are paid at a higher rate called **overtime**. This can occur in a couple of common ways:
  - **Time and a half**: pay is 1.5 times the usual hourly rate
  - **Double time**: pay is twice the usual hourly rate
- Workers who earn a **salary** (for example, an engineer) are paid a fixed amount per year, say $95 000. This is often paid monthly or fortnightly.
  - 12 months in a year and approximately 52 weeks in a year = 26 fortnights
- Some wage and salary earners are paid **leave loading**. When they are on holidays, they earn their normal pay plus a bonus called leave loading. This is usually 17.5% of their normal pay.

$$100\% + 17.5\% = 117.5\%$$
$$= 1.175$$



Leave loading = 17.5% of A = $262.50

- Some people are paid according to the number of items they produce. This is called **piecework**.
- **Commission** is a proportion of the overall sales amount. Salespeople may receive a commission of their sales as well as a set weekly or monthly fee called a **retainer**.
  - Commission = % commission × total sales
- A person's **gross income** is the total income that they earn. The **net income** is what is left after deductions, such as tax and union fees, are subtracted.
  - Net income = gross income − deductions
- **Taxation** is paid to the government once a person's taxable income passes a set amount. The amount paid depends on the person's taxable income.

Example 27 Comparing wages and salaries

Ken earns an annual salary of $59 735 and works a 38-hour week. His wife Brooke works part time in retail and earns $21.80 per hour.

a Calculate how much Ken earns per week.

b Determine who has the higher hourly rate of pay.

c If Brooke works on average 18 hours per week, what is her yearly income?

d Ken takes a 4-week holiday on normal pay plus leave loading. How much will he be paid during his holiday?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
a Weekly rate = \$59 735 ÷ 52 = \$1148.75 ∴ Ken earns \$1148.75 per week	\$59 735 pay in a year. There are approximately 52 weeks in a year.
b Brooke: \$21.80/h Ken: \$1148.75 ÷ 38 = \$30.23/h ∴ Ken is paid more per hour.	Ken works 38 hours in week. Hourly rate = weekly rate ÷ number of hours. Round to the nearest cent. Compare hourly rates.
c Weekly income = \$21.80 × 18 = \$392.40 Yearly income = \$392.4 × 52 = \$20 404.80	Weekly income = hourly rate × number of hours. Multiply by 52 weeks to get yearly income.
d Ken earns \$1148.75 per week. 1148.75 × 4 = \$4595 normal pay \$4595 × 1.175 = \$5399.13 (to 2 d.p.)	Ken receives his normal pay for 4 weeks. To add on leave loading, multiply normal pay by 1.175.

Example 28 Calculating overtime

Georgie works some weekends and late nights in addition to normal working hours and has overtime pay arrangements with her employer.

a Calculate how much Georgie earns if she works 16 hours during the week at the normal hourly rate of \$18.50 and 6 hours on the weekend at time and a half.

b Georgie's hourly rate is changed. In a week she works 9 hours at the normal rate, 4 hours at time and a half and 5 hours at double time. If she earns \$507.50, what is her hourly rate?

SOLUTION	EXPLANATION
a Earnings at normal rate = 16 × \$18.50 = \$296	16 hours at standard rate
Earnings at time and half = 6 × 1.5 × \$18.50 = \$166.50	Time and a half is 1.5 times the standard rate.
Total earnings = \$296 + \$166.50 = \$462.50	Combine earnings.
Alternative method: 6 hours of time and half = 6 × 1.5 = 9 hours of normal time 9 + 16 = 25 25 × \$18.50 = \$462.50	Georgie's pay is equivalent to working 25 hours of normal time.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
b Equivalent hours worked in week $= 9 + (4 \times 1.5) + (5 \times 2)$ $= 9 + 6 + 10$ $= 25$ hours	Calculate the number of equivalent hours worked. 4 hours at time and half is the same pay as for working 6 hours (4×1.5), 5 hours at double time is the same as working 10 hours (5×2).
Hourly rate $= \$507.50 \div 25$ $= \$20.30$	Divide weekly earnings by the 25 equivalent hours worked.
$\therefore$ Georgie earns \$20.30 per hour	

Example 29 Calculating commission and piecework

a A saleswoman is paid a retainer of \$1500 per month. She also receives a commission of 1.25% on the value of goods she sells. If she sells goods worth \$5600 during the month, calculate her earnings for that month.

b David is paid for picking fruit. He is paid \$35 for every bin he fills. Last week he worked 8 hours per day for 6 days and filled 20 bins. Calculate his hourly rate of pay.

SOLUTION	EXPLANATION
a Commission $=$ 1.25% of \$5600 $= 0.0125 \times \$5600$ $= \$70$	Calculate the commission on sales. Change the percentage to a decimal and evaluate.
Earnings $= \$1500 + \$70$ $= \$1570$	Earnings = retainer + commission
b $\$35 \times 20 = \$700$	This is David's pay for the week: 20 bins at \$35 each.
$8 \times 6 = 48$ hours	
$\$700 \div 48 = \$14.58$ (to 2 d.p.)	This is David's hourly rate.

Exercise 1I

UNDERSTANDING AND FLUENCY | 1–6, 8–11 | 3, 5, 6, 8–11 | 5, 7, 10–13

1 An employee has an annual salary of \$47 424 and works 38-hour weeks. Find the average earnings:
a per month **b** per week **c** per hour

2 A shop assistant is paid \$11.40 per hour. Calculate the assistant's earnings from the following hours worked.
a 7 hours
b 5.5 hours
c 4 hours at twice the hourly rate (double time)
d 6 hours at 1.5 times the hourly rate (time and a half)

3 Find the commission earned on the given sales figures if the percentage commission is 20%.
a \$1000 **b** \$280 **c** \$4500 **d** \$725.50

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 Find the net weekly income given the following payslip.

Earnings	$2037.25
Leave loading	$356.52
Tax	$562.00
Union	$15.77
Social club	$5.00
Health insurance	$148.00

Example 27a–c

5 **a** Calculate the hourly rate of pay for a 38-hour working week with an annual salary of:
i $38 532 **ii** $53 352 **iii** $83 980

b Calculate the yearly income for someone who earns $24.20 per hour and in a week works, on average:
i 24 hours **ii** 35 hours **iii** 16 hours

Example 27d

6 Mary earns $800 per week. Calculate her holiday pay for 4 weeks, including leave loading at 17.5%.

Example 28a

7 A job has a normal working hours pay rate of $9.20 per hour. Calculate the pay including overtime from the following hours worked.
a 3 hours and 4 hours at time and a half
b 4 hours and 6 hours at time and a half
c 14 hours and 3 hours at double time
d 20 hours and 5 hours at double time
e 10 hours and 8 hours at time and a half and 3 hours at double time
f 34 hours and 4 hours at time and a half and 2 hours at double time

8 Calculate how many hours at the standard hourly rate the following working hours are equivalent to.
a 3 hours and 2 hours at double time
b 6 hours and 8 hours at time and a half
c 15 hours and 12 hours at time and a half
d 10 hours time and a half and 5 hours at double time
e 20 hours and 6 hours at time and a half and 4 hours at double time
f 32 hours and 4 hours at time and a half and 1 hour at double time

Example 28b

9 Jim, a part-time gardener, earned $261 in a week. If he worked 12 hours during normal working hours and 4 hours overtime at time and a half, what was his hourly rate of pay?

10 Sally earned $329.40 in a week. If she worked 10 hours during the week and 6 hours on Saturday at time and a half and 4 hours on Sunday at double time, what was her hourly rate of pay?

Example 29a

11 Amy works at Best Bookshop. During one week she sells books valued at $800. If she earns $450 per week plus 5% commission, how much does she earn in this week?

12 Jason works for a caravan company. If he sells $84 000 worth of caravans in a month, and he earns $650 per month plus 4% commission on sales, how much does he earn that month?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 For each of the following, find the:
- i annual net income
- ii percentage of gross income paid as tax. Round to 1 decimal place where necessary.

a Gross annual income = $48 241, tax withdrawn = $8206
b Gross annual income = $67 487, tax withdrawn = $13 581.20
c Gross monthly income = $4041, tax withdrawn = $606.15
d Gross monthly income = $3219, tax withdrawn = $714.62

PROBLEM-SOLVING AND REASONING 14, 15, 19 | 15–17, 19, 20 | 16–18, 20–22

14 Arrange the following workers in order of most to least earned in a week of work.
- Adam has an annual salary of $33 384.
- Bill works for 26 hours at a rate of $22.70 per hour.
- Cate earns $2964 per month. (Assume 52 weeks in a year.)
- Diana does shift work 4 days of the week between 10 p.m. and 4 a.m. She earns $19.90 per hour before midnight and $27.50 per hour after midnight.
- Ed works 18 hours at the normal rate of $18.20 per hour, 6 hours at time and a half and 4 hours at double time in a week.

15 Stephen earns an hourly rate of $17.30 for the first 38 hours, time and a half for the next 3 hours and double time for each extra hour above that. Calculate his earnings if he works 44 hours in a week.

16 Jessica works for Woods Real Estate and earns $800 per week plus 0.05% commission. If this week she sold three houses valued at $334 000, $210 000 and $335 500, respectively, how much will she have earned?

17 A door-to-door salesman sells 10 security systems in one week at $1200 each. For the week, he earns a total of $850, including a retainer of $300 and a commission. Find his percentage commission correct to 2 decimal places.

18 Mel is taking her holidays. She receives $2937.50. This includes her normal pay and 17.5% leave loading. How much is the leave loading?

Example 29b

19 Mike delivers the local newspaper. He is paid $30 to deliver 500 newspapers. How much is he paid for each one?

20 Karl is saving and wants to earn $90 from a casual job paying $7.50 per hour.
a How many hours must he work to earn $90?
b Karl can also work some hours for which he is paid time and a half. He decides to work x hours at the normal rate and y hours at time and a half to earn $90. If x and y are both positive whole numbers, find the possible combinations for x and y.

21 A car salesman earns 2% commission on sales up to $60 000 and 2.5% on sales above that.
a Determine the amount earned on sales worth:
- i $46 000
- ii $72 000

b Write a rule for the amount (A dollars) earned on sales of $\$x$ if:
- i $x \leq 60000$
- ii $x > 60000$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

22 Kim has a job selling jewellery. She is about to enter into one of two new payment plans:

- Plan A: $220 per week plus 5% of sales
- Plan B: 9% of sales and no set weekly retainer

a What value of sales gives the same return from each plan?

b Explain how you would choose between plan A and plan B.

ENRICHMENT — — 23

Reading and interpreting an example of a payslip including superannuation

23 The following payslip is given at the end of every fortnight to a part-time teacher who is employed for 3 days per week (6 days per fortnight).

PAYSLIP

EMPLOYER: ABC School
EMPLOYEE: J. Bloggs
Fortnight ending: 17/06/2012 AWARD: Teachers

EARNINGS	QUANTITY	RATE	AMOUNT
Ordinary pay	6.00	339.542307	$2037.25

DEDUCTIONS	AMOUNT	DETAILS
Tax	$562.00	

PROVISIONS	EARNED	TAKEN	BALANCE
Superannuation	$183.35	0.00	$2756.02

SUMMARY

PAY DATE	17/06/2012	Year to date
GROSS	$2037.25	$30766.42
TAX	$562.00	$8312.00
DEDUCTIONS	$0.00	$0.00
NET	$1475.25	$22454.42

a Use the internet to research and answer parts **i–iv**.

- **i** Find the meaning of the word 'superannuation'.
- **ii** Why is superannuation compulsory for all workers in Australia?
- **iii** What percentage of gross income is compulsory superannuation?
- **iv** What happens to the superannuation that is paid to employees?

b This employee earned $2037.25 (before tax) plus $183.35 superannuation. Was this the correct amount of superannuation?

c According to the 'year to date' figures, how much tax has been paid?

d Using the 'year to date' figures, express the tax paid as a percentage of the gross income.

e Complete this sentence: For this worker, approximately _____ cents in every dollar goes to the Australian Taxation Office.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1J The PAYG income tax system

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

It has been said that there are only two sure things in life: death and taxes! The Australian Taxation Office (ATO) collects taxes on behalf of the government to pay for education, hospitals, roads, railways, airports and services such as the police and fire brigades.

In Australia, the financial year runs from 1 July to 30 June the following year. People engaged in paid employment are normally paid weekly or fortnightly. Most of them pay some income tax every time they are paid for their work. This is known as the Pay-As-You-Go (PAYG) system.

At the end of the financial year (30 June), workers complete an income tax return to determine if they have paid the correct amount of income tax during the year. If they have paid too much, they will receive a tax refund. If they have not paid enough, they will be required to pay more tax.

The tax system is very complex. Tax laws change frequently. This section covers the main aspects only.

Let's start: The ATO website

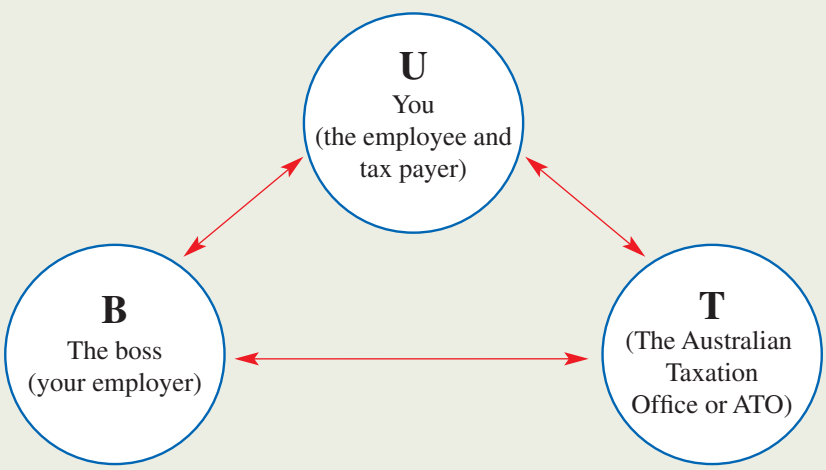
The ATO website has some income tax calculators. Use one to find out how much income tax you would need to pay if your taxable income is:

- $5200 per annum ($100 per week)
- $10 400 per annum ($200 per week)
- $15 600 per annum ($300 per week)
- $20 800 per annum ($400 per week)
- $26 000 per annum ($500 per week)

Does a person earning $1000 per week pay twice as much tax as a person earning $500 per week?
Does a person earning $2000 per week pay twice as much tax as a person earning $1000 per week?

Key ideas

The PAYG tax system can be illustrated as follows:



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- U works for and gets paid by B every week, fortnight or month.
- B calculates the tax that U should pay for the amount earned by U.
- B sends that tax to T every time U gets paid.
- T passes the income tax to the Federal Government.
- On 30 June, B gives U a **payment summary** to confirm the amount of tax that has been paid to T on behalf of U.
- Between 1 July and 31 October, U completes a **tax return** and sends it to T. Some people pay a registered tax agent to do this return for them.
- On this tax return, U lists the following:
  - *all* forms of income, including interest from investments.
  - legitimate **tax deductions** shown on receipts and invoices, such as work-related expenses and donations.
- **Taxable income** is calculated using the formula:
  Taxable income = gross income − tax deductions
- The ATO website includes tables and calculators, such as the following:

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3 572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

  This table can be used to calculate the amount of tax U *should have* paid (the **tax payable**), as opposed to the tax U *did* pay during the year (the **tax withheld**). Each row in the table is called a tax bracket.
- U may also need to pay the Medicare levy. This is a scheme in which all Australian taxpayers share in the cost of running the medical system. In the 2017–2018 financial year, it was 2% of taxable income.
- It is possible that U may have paid too much tax during the year and will receive a **tax refund**.
- It is also possible that U may have paid too little tax and will receive a letter from T asking for the **tax liability** to be paid.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 30 Calculating tax

During the 2017–18 financial year, Richard earned $1050 per week ($54 600 per annum) from his employer and other sources such as interest on investments. He has receipts for $375 for work-related expenses and donations.

a Calculate Richard's taxable income.

b Use this tax table to calculate Richard's tax payable.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

Source: ATO website

c Richard also needs to pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy.

e Express the total tax in part **d** as a percentage of Richard's taxable income.

f During the financial year, Richard's employer sent a total of $7797 in tax to the ATO. Has Richard paid too much tax, or not enough? Calculate his refund or liability.

SOLUTION

a Gross income = $54 600
Deductions = $375
Taxable income = $54 225

EXPLANATION

Taxable income = gross income − deductions

b Tax payable:
$3572 + 0.325 × ($54 225 − $37 000) = $9170.13

Richard is in the middle tax bracket in the table, in which it says:
$3572 plus 32.5c for each $1 over $37 000
Note: 32.5 cents is $0.325.

c $\frac{2}{100} \times \$54\,225 = \$1084.50$

Medicare levy is 2% of the taxable income

d $9170.13 + $1084.50 = $10 254.63

This is the total amount of tax that Richard should have paid.

e $\frac{10\,254.63}{54\,225} \times 100\% = 18.9\%$ (to 1 d.p.)

This implies that Richard paid approximately 18.9% tax on every dollar. This is sometimes read as '18.9 cents in the dollar'.

f Richard paid $7797 in tax during the year.
He should have paid $10 254.63.
Richard has not paid enough tax.
He needs to pay another $2457.63 in tax.

This is known as a shortfall or a liability. He will receive a letter from the ATO requesting payment.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 1J

UNDERSTANDING AND FLUENCY	1–6, 8, 9, 11, 12	4–8, 10–12	5, 8, 9, 11–13

The questions in this exercise relate to the tax table given in Example 30, unless stated otherwise.

1 Complete this statement:
Taxable income = _____ income minus _____.

2 Is the following statement true or false?
The highest income earners in Australia pay 45 cents tax for every dollar they earn.

3 In the 2017–18 financial year, Greg paid no income tax. What could his taxable income have been?

4 Ann's taxable income was $87 000, which puts her at the very top of the middle tax bracket in the tax table. Ben's taxable income was $87 001, which puts him in a higher tax bracket.
Ignoring the Medicare levy, how much more tax does Ben pay than Ann?

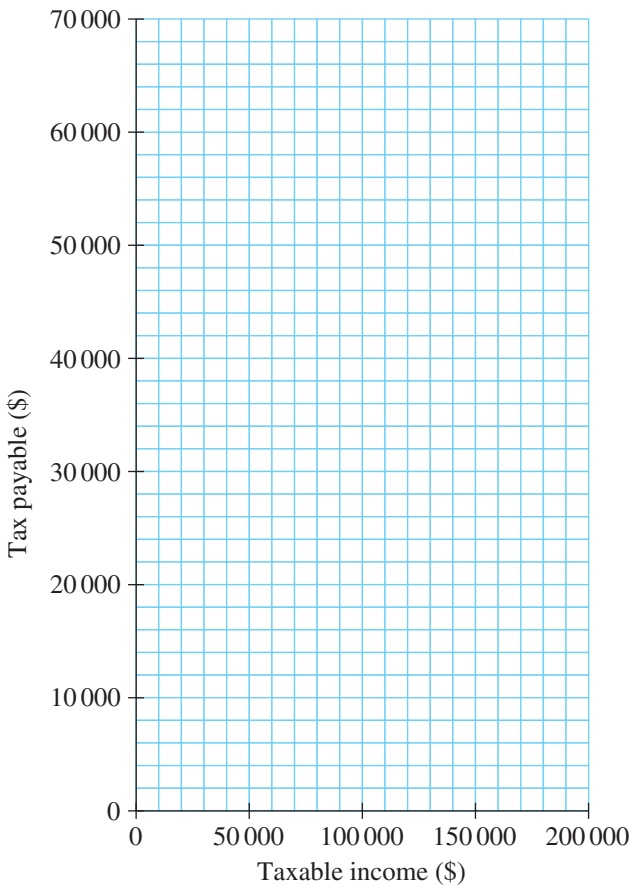
Example 30b

5 Use the tax table to calculate the income tax payable on these taxable incomes.
a $30 000 **b** $60 000 **c** $150 000 **d** $200 000

6 **a** Use the tax table to complete this table (ignoring the Medicare levy).

Taxable income	$0	$18 200	$37 000	$87 000	$180 000	$200 000
Tax payable						

b Copy this set of axes, plot the points, then join the dots with straight line segments.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Jim worked for three different employers. They each paid him $15 000. How much income tax should he have paid?

Example 30

8 Lee has come to the end of her first financial year as a teacher. On 30 June she made the notes in the table to the right about the financial year.

Gross income from employer	$58 725
Gross income from casual job	$7500
Interest on investments	$75
Donations	$250
Work-related expenses	$425
Tax paid during the financial year	$13 070

a Calculate Lee's taxable income.

b Use the tax table to calculate Lee's tax payable.

c Lee also needs to pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy.

e Express the total tax in part **d** as a percentage of Lee's taxable income.

f Has Lee paid too much tax, or not enough? Calculate her refund or liability.

9 Brad's Medicare levy was $1750. This was 2% of his taxable income. What was his taxable income?

10 Tara is saving for an overseas trip. Her taxable income is usually about $20 000. She estimates that she will need $5000 for the trip, so she is going to do some extra work to raise the money. How much extra will she need to earn in order to have $5000 after tax?

11 When Sue used the tax table to calculate her income tax payable, it turned out to be $22 722. What was her taxable income?

12 Explain the difference between gross income and taxable income.

13 Explain the difference between a tax refund and a tax liability.

PROBLEM-SOLVING AND REASONING 14, 15 | 14, 15 | 15, 16

14 Josh looked at the last row of the tax table and said 'It is unfair that people in that tax bracket pay 45 cents in every dollar in tax'. Explain why Josh is incorrect.

15 Consider the tax tables for the two consecutive financial years. Note that in the first table, the value of $6000 is often called the tax-free threshold.

Financial year 2011–12

Taxable income	Tax on this income
0–$6000	Nil
$6001–$37 000	15c for each $1 over $6000
$37 001–$80 000	$4650 plus 30c for each $1 over $37 000
$80 001–$180 000	$17 550 plus 37c for each $1 over $80 000
$180 001 and over	$54 550 plus 45c for each $1 over $180 000

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Financial year 2012–13

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3 572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

a There are some significant changes from the first year to the second. Describe three of them.

b The following people had the same taxable income in both years. Were they advantaged or disadvantaged by the changes, or not affected at all?

- **i** Ali: taxable income = $5000
- **ii** Bill: taxable income = $15 000
- **iii** Col: taxable income = $30 000
- **iv** Di: taxable income = $50 000

16 This was the 2012–13 tax table for people who were not residents of Australia but were working in Australia.

Taxable income	Tax on this income
0–$80 000	32.5c for each $1
$80 001–$180 000	$26 000 plus 37c for each $1 over $80 000
$180 001 and over	$63 000 plus 45c for each $1 over $180 000

Compare this table to the one Question **15** for 2012–13 for Australian residents.
What difference would it make to the tax paid by these people in 2012–13 if they were non-residents rather than residents?

a Ali: taxable income = $5000

b Bill: taxable income = $15 000

c Col: taxable income = $30 000

d Di: taxable income = $50 000

ENRICHMENT — — 17

What are legitimate tax deductions?

17 a Choose an occupation or career in which you are interested. Imagine that you are working in that job. During the year you will need to keep receipts for items you have bought that are legitimate work-related expenses. Do some research on the internet and write down some of the things that you will be able to claim as work-related expenses in your chosen occupation.

b i Imagine your taxable income is $80 000. What is your tax payable?

ii You just found a receipt for a $100 donation to a registered charity. This decreases your taxable income by $100. By how much does it decrease your tax payable?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1K Simple interest

Interactive
Widgets
HOTsheets
Walkthrough

When paying back the amount borrowed from a bank or other financial institution, the borrower pays interest to the lender. It's like rent paid on the money borrowed. A financial institution can be a lender, giving you a loan, or it can be a borrower, when you invest your savings with them (you effectively lend them your money). In either case, interest is calculated as a percentage of the amount borrowed. With simple interest, the percentage is calculated on the amount originally borrowed or invested and is paid at agreed times, such as once a year.



Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Developing the rule

$5000 is invested in a bank and 5% simple interest is paid every year. In the table on the right, the amount of interest paid is shown for Year 1, the amount of accumulated total interest is shown for Years 1 and 2.

- Complete the table, writing an expression in the last cell for the accumulated total interest after t years.
- Now write a rule using $\$P$ for the initial amount, t for the number of years and r for the interest rate to find the total interest earned, $\$I$.

Year	Interest paid that year	Accumulated total interest
0	\$0	\$0
1	$\frac{5}{100} \times \$5000 = \$250$	$1 \times \$250 = \$250$
2		$2 \times \$250 = \$500$
3		
4		
t		

Key ideas

- The **simple interest** formula is:

  $I = PRN$, where

  I is the amount of simple interest (in \$)

  P is the **principal** amount; the money invested, borrowed or loaned (in \$)

  R is the interest rate per time period (expressed as a fraction or decimal)

  N is the number of time periods

- When using simple interest, the principal amount is constant and remains unchanged from one period to the next.
- The total amount ($\$A$) equals the principal plus interest:

  $A = P + I$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 31 Using the simple interest formula

Calculate the simple interest earned if the principal is \$1000, the rate is 5% p.a. and the time is 3 years.

SOLUTION	EXPLANATION
$P = 1000$ $R = 5\% = 5 \div 100 = 0.05$ $N = 3$	List the information given. Convert the interest rate to a decimal.
$I = P \times R \times N$ $= 1000 \times 0.05 \times 3$ $= 150$	Write the formula and substitute the given values.
Interest = \$150	Answer the question.

Example 32 Calculating the final balance

Allan and Rachel plan to invest some money for their child Kaylan. They invest \$4000 for 30 months in a bank that pays 4.5% p.a. Calculate the simple interest and the amount available at the end of the 30 months.

SOLUTION	EXPLANATION
$P = 4000$ $R = 4.5 \div 100 = 0.045$ $N = 30 \div 12 = 2.5$	N is written in years since interest rate is per annum.
$I = P \times R \times N$ $= 4000 \times 0.045 \times 2.5$ $= 450$	Write the formula, substitute and evaluate.
Interest = \$450	
Total amount = \$4000 + \$450 = \$4450	Total amount = principal + interest

Example 33 Determining the investment period

Remy invests \$2500 at 8% p.a. simple interest, for a period of time, to produce \$50 interest. For how long did she invest the money?

SOLUTION	EXPLANATION
$I = 50$ $P = 2500$ $R = 8 \div 100 = 0.08$	List the information. Convert the interest rate to a decimal.
$I = P \times R \times N$ $50 = 2500 \times 0.08 \times N$ $50 = 200N$	Write the formula, substitute the known information and simplify.
$N = \frac{50}{200}$ $= 0.25$	Solve the remaining equation for N.
Time = 0.25 years $= 0.25 \times 12$ months = 3 months	Convert decimal time to months where appropriate.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 34 Purchasing on terms with simple interest

Lucy wants to buy a fridge advertised for $1300, but she does not have enough money at the moment. She signs an 'on terms' or 'hire purchase' agreement to pay a deposit of $300 now, then $60 per month for 2 years.

a How much will Lucy pay for this fridge?
b Why does it cost more than the advertised price to purchase the fridge in this manner?
c How much would she have saved by paying the advertised price rather than purchasing on terms?
d What was the annual simple interest rate, expressed as a percentage of the original amount borrowed?

SOLUTION	EXPLANATION
a $\$300 + \$60 \times 24 = \$1740$	She agreed to pay $300 deposit then $60 per month for 2 years (24 months)
b She only paid $300 deposit so she is effectively borrowing $1000. She is expected to re-pay the $1000 and also pay some interest on that loan.	The 'loan' amount can be calculated using: 'advertised price' minus 'deposit paid'
c $\$1740 - \$1300 = \$440$	This is the interest that she will pay on the loan of $1000.
d $\$440 \div 2 = \$220$ interest per year $\frac{\$220}{\$1000} \times 100\% = 22\%$ p.a.	Let us assume that the interest was spread equally over the 2 years. The interest paid for one year can be compared with the original amount borrowed and converted to a percentage to show what is known as a 'flat' interest rate.

Interest is usually *not* calculated in this way for most of the loans currently on offer. Every fortnight or month, the interest is calculated as a percentage of the amount you *owe*, which will be less than the amount you *borrowed*. See Example 35 for a more realistic and up-to-date view of loan repayments.

Example 35 Repaying a reducible interest loan

Rachel bought a new car for $35 000. She used $5000 that she had saved and borrowed the remaining $30 000. She signed a loan agreement and agreed to pay interest at 12% p.a., which is 1% per month. She decided she could afford to repay $800 per month.

a Complete the next three rows of this table, in which the interest is 1% of the opening balance for each month. Round off to 2 decimal places when necessary.

Month	Opening balance (A)	Interest (B)	Re-payment (C)	Closing balance ($A + B - C$)
1	$30 000	$300	$800	$29 500
2	$29 500	$295	$800	$28 995
3	$28 995	$289.95	$800	$28 484.95
4	$28 484.95	$284.85 (to 2 d.p.)	$800	$27 969.80
5				
6				
7				

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b How much has Rachel paid in instalments in these seven months?
c How much interest has she paid in these seven months?
d By how much has Rachel reduced the amount she owes on this loan?
e Describe what might happen if Rachel defaults on the loan by failing to make the agreed regular repayments on her loan.
f Use your calculator and the instructions in the Explanation column below to estimate the number of months it will take Rachel to repay this loan, if she continues to pay $800 every month.

SOLUTION	EXPLANATION
a The value in the last cell in month 7 is $26 393.24	This is the amount Rachel still owes at the end of the seventh month.
b Instalments paid $= 7 \times \$800 = \$5600$	This is easy to calculate because she is paying the same amount every month.
c Interest paid = $1993.24	This is the sum of the amounts in the interest column. Every month is different because the balance is getting smaller.
d $\$30\,000 - \$26\,393.24 = \$3606.76$	The amount owed has fallen from $30 000 to $26 393.24. She has reduced the amount owing by $3606.76.
e This depends on the wording of the contract that Rachel signed. If Rachel does not make the agreed minimum repayments every month, the lender may have the right to repossess the car and sell it for whatever they can get, which may be less than the amount she owes. Then they may force her to pay the remaining amount owing. Repossession does not remove the financial debt.	In loans such as this, there is usually a minimum amount that must be repaid every month. It is wise to pay more than the minimum, if possible, because: • the loan will be repaid faster and therefore the amount of interest paid will be reduced • there may be some months later in the loan period in which she does not have enough money to make the minimum repayment, so she may be able to re-draw the extra payments she has made above the minimum, if her loan contract allows this.
f It will take 48 months (4 years). By the end of the 47th month she owes a small amount of approximately $190, so she will pay less than $800 in the last month of the loan.	100% + 1% = 101% 101% is 1.01 as a decimal. For a scientific calculator, press 30000 then = Then press × 1.01 − 800 = = = = = = etc. and watch the balance owing at the end of each month fall slowly at first then more and more rapidly. The amount owing decreases at an increasing rate. Count the number of months it will take for the loan to be repaid (when the balance owing is less than zero).

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 1K

UNDERSTANDING AND FLUENCY 1–6 2–7 3–8

1 Complete the table:

	a	b	c	d	e	f
Principal	$1000	$1000	$1000	$1000	$1000	$1000
Interest rate	6% pa	6% pa	3% pa	3% pa	5% pa	10% pa
Years	3	6	6	12	8	16
Interest						
Total amount						

2 $12 000 is invested at 6% p.a. for 42 months.

- **a** What is the principal amount?
- **b** What is the interest rate?
- **c** What is the time period in years?
- **d** How much interest is earned each year?
- **e** How much interest is earned after 2 years?
- **f** How much interest is earned after 42 months?

Example 31

3 Use the formula $I = PRN$ to find the simple interest earned in these financial situations.

- **a** Principal $10 000, rate 10% p.a., time 3 years
- **b** Principal $6000, rate 12% p.a., time 5 years
- **c** Principal $5200, rate 4% p.a., time 24 months
- **d** Principal $3500, rate 6% p.a., time 18 months

Example 32

4 Annie invests $22 000 at a rate of 4% p.a. for 27 months. Calculate the simple interest and the amount available at the end of 27 months.

5 A finance company charges 14% p.a. simple interest. If Lyn borrows $2000 to be repaid over 2 years, calculate her total repayment.

Example 33

6 Zac invests $3500 at 8% p.a. simple interest, for a period of time, to produce $210 interest. For how long did he invest the money?

7 If $4500 earns $120 simple interest at a flat rate of 2% p.a., calculate the duration of the investment.

8 Calculate the principal amount that earns $500 simple interest over 3 years at a rate of 8% p.a. Round to the nearest cent.

PROBLEM-SOLVING AND REASONING 9–11, 13 10–14 12–15

9 Wendy wins $5000 during a chess tournament. She wishes to invest her winnings, and has the two choices given below. Which one gives her the greater total at the end of the time?
Choice 1: 8.5% p.a. simple interest for 4 years
Choice 2: 8% p.a. simple interest for 54 months

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Charlotte borrows $9000 to buy a second-hand car. The loan must be repaid over 5 years at 12% p.a. simple interest. Calculate the:
- **a** total amount to be repaid
- **b** monthly repayment amount if the repayments are spread equally over the 5 years

11 If $5000 earns $6000 simple interest in 12 years, find the interest rate.

12 An investor invests $\$P$ and wants to double this amount of money.
- **a** How much interest must be earned to double this initial amount?
- **b** What simple interest rate is required to double the initial amount in 8 years?
- **c** If the simple interest rate is 5% p.a.:
  - **i** how many years will it take to double the investment?
  - **ii** how many years will it take to triple the investment amount?
  - **iii** how do the investment periods in parts **i** and **ii** compare?

Example 34

13 **a** Sophie borrowed $2000 and took 3 years to repay the loan and $900 interest. What was the per annum simple interest rate?
- **b** If Sophie's interest was calculated at the same rate on the balance owing, how much would she have owed after 6 months if she repaid $40 per month? Give your answer to the nearest dollar.

Example 35

14 To find the total amount $\$T$ including simple interest, the rule is $T = P(1 + RN)$.
- **a** Use this rule to find the total amount after 10 years if $30 000 is invested at 7% p.a.
- **b** Use the rule to find the time that it takes for an investment to grow from $18 000 to $22 320 invested at 6% p.a. simple interest.

15 Rearrange the formula $I = PRN$ to find a rule for:
- **a** P in terms of I, R and N
- **b** N in terms of I, P and R
- **c** R in terms of I, $P$and N

ENRICHMENT – – 16

Property investing with 'interest only' loans

16 Many investors use 'interest only' loans to buy shares or property. This is where the principal stays constant and only the interest is paid back each month.

Sasha buys an investment property for $300 000 and borrows the full amount at 7% p.a. simple interest. She rents out the property at $1500 per month and it costs $3000 per year in rates and other costs to keep the property.
- **a** Find the amount of interest Sasha needs to pay back every month.
- **b** Find Sasha's yearly income from rent.
- **c** By considering the other costs of keeping the property, calculate Sasha's overall loss in a year.
- **d** Sasha hopes that the property's value will increase enough to cover any loss she is making. By what percentage of the original price will the property need to increase in value per year?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1L Compound interest and depreciation

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

When interest is added onto an investment total before the next amount of interest is calculated, we say that the interest is compounded. Interest on a $1000 investment at 8% p.a. gives $80 in the first year and if compounded, the interest calculated in the second year is 8% of $1080. This is repeated until the end of the investment period. Other forms of growth and decay work in a similar manner.



In compound interest, the balance grows faster as time passes.

Let's start: Power play

$10000 is invested at 5% compounded annually. Complete this table showing the interest paid and the balance (original investment plus interest) at the end of each year.

- What patterns can you see developing in the table?
- How can you use the *power* button on your calculator to help find the balance at the end of each year?
- How would you find the balance at the end of 10 years without creating a large table of values?
- Do you know how to build this table in a computerised spreadsheet?

Year	Opening balance	Interest paid this year	Closing balance
1	$10000	$0.05 \times \$10000 = \$$_____	$\$$ _____
2	$\$$ _____	$0.05 \times \$$_____ $= \$$_____	$\$$ _____
3	$\$$ _____	$0.05 \times \$$_____ $= \$$_____	$\$$ _____

Key ideas

- A repeated product can be written and calculated using a power.
  For example: $1.06 \times 1.06 \times 1.06 \times 1.06 = (1.06)^4$
  $0.85 \times 0.85 \times 0.85 = (0.85)^3$
- **Compound interest** is interest that is added to the investment amount before the next amount of interest is calculated.
  For example: $5000 invested at 6% compounded annually for 3 years gives
  $\$5000 \times 1.06 \times 1.06 \times 1.06 = 5000 \times (1.06)^3$
  - 6% compounded annually can be written as 6% p.a.
  - p.a. means 'per annum' or 'per year'.
  - The initial investment or loan is called the **principal**.
  - Total interest earned = final amount − principal
- **Depreciation** is the reverse situation in which the value of an object decreases by a percentage year after year.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 36 Calculating a balance using compound interest

Find the total value of the investment if $8000 is invested at 5% compounded annually for 4 years.

SOLUTION

$100\% + 5\% = 105\%$
$= 1.05$

Investment total $= \$8000 \times (1.05)^4$
$= \$9724.05$

EXPLANATION

Add 5% to 100% to find the multiplying factor.

Multiplying by $(1.05)^4$ is the same as multiplying by $1.05 \times 1.05 \times 1.05 \times 1.05$.
Note: on a scientific calculator this can also be done by pressing 8000 = then × 1.05 = = = =
This will show the balance at the end of every year.

Example 37 Finding the initial amount

After 6 years a loan grows to $62 150. If the interest was compounded annually at a rate of 9%, find the size of the initial loan to the nearest dollar.

SOLUTION

$100\% + 9\% = 109\%$
$= 1.09$

Initial amount $\times (1.09)^6 = \$62\,150$
Initial amount $= \$62\,150 \div (1.09)^6$
$= \$37\,058$

EXPLANATION

Add 9% to 100% to find the multiplying factor.

Write the equation including the final total. Divide by $(1.09)^6$ to find the initial amount and round as required.

Example 38 Calculating the depreciating value of a car

A car worth $30 000 will depreciate by 15% p.a.
What will the car be worth at the end of the fifth year?



SOLUTION

$100\% - 15\% = 85\%$
$= 0.85$

Depreciated value $= \$30\,000 \times (0.85)^5$
$= \$13\,311.16$ (to 2 d.p.)

EXPLANATION

Subtract 15% from 100%.

Multiply by $(0.85)^5$ for 5 years.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 1L

UNDERSTANDING AND FLUENCY	1–4, 7–9	3–5, 7–9	4–6, 8, 9

1 $2000 is invested at 10% compounded annually for 3 years.
Copy and complete the table.

Year	Opening balance	Interest	Closing balance
1		a	b
2		c	d
3		e	f

2 Find the value of the following correct to 2 decimal places.

a $2500 \times 1.03 \times 1.03 \times 1.03$

b $420 \times 1.22 \times 1.22 \times 1.22 \times 1.22$

c $2500 \times (1.03)^3$

d $420 \times (1.22)^4$

3 Fill in the missing numbers to describe each situation.

a $4000 is invested at 20% compounded annually for 3 years.

$\$4000 \times (_____)^3$

b $15000 is invested at 7% compounded annually for 6 years.

$\$_____ \times (1.07)^{\square}$

c $825 is invested at 11% compounded annually for 4 years.

$\$825 \times (_____)^{\square}$

d $825 is depreciating at 11% annually for 4 years.

$\$825 \times (_____)^{\square}$

Example 36

4 Find the total balance of these investments with the given interest rates and time period. Assume interest is compounded annually in each case. Round to the nearest cent.

a $4000, 5%, 10 years

b $6500, 8%, 6 years

c $25000, 11%, 36 months

d $4000, 7%, 60 months

5 Barry borrows $200000 from a bank for 5 years and does not pay any money back until the end of the period. The compound interest rate is 8% p.a. How much does he need to pay back at the end of the 5 years? Round to the nearest cent.

6 Find the total percentage increase in the value of these amounts, compounded annually at the given rates. Round to 1 decimal place.

a $1000, 4% p.a., 5 years

b $20000, 6% p.a., 3 years

c $125000, 9% p.a., 10 years

d $500000, 7.5% p.a., 4 years

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 37

7 After 5 years a loan grows to $45 200. If the interest was compounded annually at a rate of 6% p.a., find the size of the initial loan to the nearest dollar.

8 Find the initial investment amount to the nearest dollar given these final balances, annual interest rates and time periods. Assume interest is compounded annually in each case.

- **a** $26 500, 4%, 3 years
- **b** $42 000, 6%, 4 years
- **c** $35 500, 3.5%, 6 years
- **d** $28 200, 4.7%, 2 years

Example 38

9 Find the depreciated value of these assets to the nearest dollar.

- **a** $2000 computer at 30% p.a. for 4 years
- **b** $50 000 car at 15% p.a. for 5 years
- **c** $50 000 car at 15% p.a. for 10 years

PROBLEM-SOLVING AND REASONING 10, 11, 15 | 11–13, 15 | 13–16

10 Average house prices in Hobart are expected to grow by 8% per year for the next 5 years. What is the expected average value of a house in Hobart in 5 years time, to the nearest dollar, if it is currently valued at $370 000?



11 The population of a country town is expected to fall by 15% per year for the next 8 years due to the downsizing of the iron ore mine. If the population is currently 22 540 people, what is the expected population in 8 years time? Round to the nearest whole number.

12 It is proposed that the mass of a piece of limestone exposed to the wind and rain has decreased by 4.5% per year for the last 15 years. Its current mass is 3.28 kg. Find its approximate mass 15 years ago. Round to 2 decimal places.



13 Charlene wants to invest $10 000 long enough for it to grow to at least $20 000. The compound interest rate is 6% p.a. How many whole number of years does she need to invest the money for so that it grows to her $20 000 target?

14 A forgetful person lets a personal loan balance grow from $800 to $1440 with a compound interest rate of 12.5% p.a. Approximately how many years did the person forget about the loan?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 $400 is invested for 5 years under the following conditions.

- **i** Simple interest at 7% p.a.
- **ii** Compound interest at 7% p.a.

a Find the percentage increase in the total value of the investment using condition **i**.

b Find the percentage increase in the total value of the investment using condition **ii**. Round to 2 decimal places.

c Explain why the total for condition **i** is less than the total for condition **ii**.

16 Find the total percentage increase in these compound interest situations to 2 decimal places.

a 5% p.a. for 3 years

b 12% p.a. for 2 years

c 4.4% p.a. for 5 years

d 7.2% p.a. for 9 years

e r% for t years

ENRICHMENT – – 17–19

Comparing simple and compound interest

17 $16 000 is invested for 5 years at 8% compounded annually.

a Find the total interest earned over the 5 years to the nearest cent.

b Find the simple interest rate that would deliver the same overall interest at the end of the 5 years. Round to 2 decimal places.

18 $100 000 is invested for 10 years at 5.5% compounded annually.

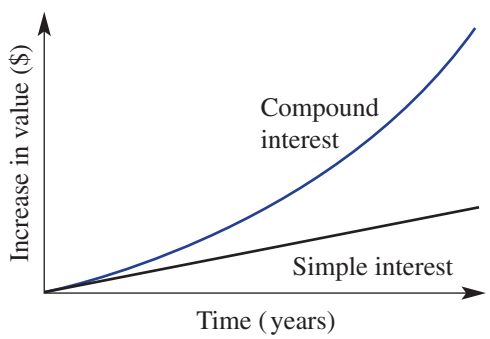
a Find the total percentage increase in the investment to 2 decimal places.

b Find the simple interest rate that would deliver the same overall interest at the end of the 10 years. Round to 2 decimal places.

19 Find the simple interest rate that is equivalent to these annual compound interest rates for the given periods. Round to 2 decimal places.

a 5% p.a. for 4 years

b 10.5% p.a. for 12 years



This graph shows the effect of the same rate of compound and simple interest on the increase in value of an investment or loan, compared over time. Simple interest is calculated only on the principal, so yields a balance that increases by the same amount every year, forming a straight-line graph. After the first year, compound interest is calculated on the principal and what has been added by the previous years' interest, so the balance increases by a greater amount each year. This forms an exponential graph.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1M Using a formula for compound interest and depreciation

Interactive
Widgets
HOTsheets
Walkthrough

We saw in section 1L that interest can be calculated, not on the principal investment, but on the actual amount present in the account. Interest can be calculated using updated applications of the simple interest formula, as done in section 1L, or by developing a new formula known as the compound interest formula.

If the original amount loses value over time, it is said to have depreciated. The compound interest formula can be adapted to cover this scenario as well. Cars, computers, boats and mobile phones are examples of consumer goods that depreciate in value over time.



Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Repeated increases and decreases

If $4000 increased by 5% in the first year, and then this amount was increased by 5% in the second year:

- what was the final amount at the end of the second year?
- what was the overall increase in value?
- what could you do to calculate the value at the end of 100 years, if the interest was added to the amount each year before the new interest was calculated?

If $4000 decreased in value by 5% each year, how would you find the value at the end of:

- 1 year?
- 2 years?
- 100 years?

Key ideas

- The **compound interest** formula is:
  $A = P(1 + R)^n$, where
  A is the total amount (i.e. the principal and interest)
  P is the principal investment (i.e. the original amount invested)
  R is the interest rate per compounding period, expressed as a decimal
  n is the number of compounding periods

  The formula is sometimes written as:
  $FV = PV(1 + R)^n$, where
  FV is the future value
  PV is the present value

- Compound interest = amount (A) − principal (P)

- **Depreciation** occurs when the principal decreases in value over time. In this case the value of R becomes a negative value. The compound interest formula can be manipulated to cover this case by writing it as:
  $A = P(1 + (-R))^n$ or $A = P(1 - R)^n$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 39 Using the compound interest formula

Callum invested $10000 with the Mental Bank. The bank pays interest at the rate of 7% p.a., compounded annually.

a What is the value of Callum's investment at the end of 5 years?

b How much interest was earned over this time?

SOLUTION	EXPLANATION
a $A = P(1 + R)^n$	Write down the compound interest formula.
$P = 10000$	Write down the values of the terms you know.
$R = 0.07$	Remember to write R as a decimal $\left(\frac{7}{100} = 0.07\right)$
$n = 5$	$n = 5$ (once a year for 5 years)
$A = 10000(1 + 0.07)^5$ $= 14025.517 \ldots$	Substitute and evaluate.
Final value is $14025.52.	
b Interest = $4025.52	The interest is found by subtracting the principal from the final amount.

Example 40 Finding compounding amounts using months

Victoria's investment of $6000 is invested at 8.4% p.a. for 4 years. Calculate the final value of her investment if interest is compounded monthly.



SOLUTION	EXPLANATION
$A = P(1 + R)^n$	Write down the compound interest formula.
$P = 6000$	Write down the values of the terms you know.
$R = 0.084 \div 12 = 0.007$	The interest rate of 8.4% is *divided by* 12 to find the *monthly* rate.
$n = 4 \times 12 = 48$	The number of compounding periods (n) is 12 times a year for 4 years or $n = 48$.
$A = 6000(1 + 0.007)^{48}$ $= 8386.2119 \ldots$	Substitute and evaluate.
Final amount is $8386.21.	

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 41 Calculating the depreciated value

Naomi bought a computer three years ago for $5680. She assumed the value of the computer would depreciate at a rate of 25% p.a.
What is the value of the computer at the end of the three years (assuming her assumption was correct)?

SOLUTION

$A = P(1 + R)^n$ or
$A = P(1 - R)^n$

$A = 5680(1 - 0.25)^3$
$= 2396.25$

The value at the end of the three years is $2396.25.

EXPLANATION

We write the compound interest formula with a negative R, as depreciation is a decrease in value over time: $((1 + (-0.25)) = (1 - 0.25)$.
Substitute $P = 5680$, $R = 0.25$ and $n = 3$.

Exercise 1M

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 7	3, 4, 5–6(½), 7	5–7(½), 8

1 Which of the following can be used to calculate the final value of a $1000 investment at 10% p.a. for 3 years compounded annually?

A $1000 \times 0.1 \times 0.1 \times 0.1$ **B** $1000 \times 1.1 \times 3$
C $1000(1.1)^3$ **D** $1000(0.9)^3$

2 Complete the table of values showing how a car worth $20000 depreciates at a rate of 10% p.a. over 4 years.

Year	Value at start of year	10% depreciation	Value at end of year
1	$20000	$2000	$18000
2	$18000	$1800	
3			
4			

3 Which of the following gives the value of a $40000 car once it has depreciated for 4 years at a rate of 30% p.a.?

A 40000×1.3^4 **B** $40000 \times 0.7 \times 4$
C $40000 \times ((1 + (-0.3))^4$ **D** $40000 - 0.7^4$

4 **a** Write down the value of R and the value of n for each of the following situations.
- **i** $100 invested at 12% p.a. compounded annually for 1 year
- **ii** $100 invested at 12% p.a. compounded monthly for 1 year
- **iii** $100 invested at 12% p.a. compounded quarterly for 1 year
- **iv** $100 depreciating at 10% p.a. for 1 year

b Which of the situations in part **a** do you expect to yield the highest interest over the year?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 39

5 Calculate the final investment amount if the interest is compounded annually.

- **a** $4000 at 5% p.a. for 8 years
- **b** $2000 at 5.4% p.a. for 3 years
- **c** $20000 at 8% p.a. for 6 years
- **d** $10000 at 16% p.a. for 6 years
- **e** $500 at 3.8% p.a. for 25 years
- **f** $350 at 6.78% p.a. for 2 years
- **g** $68000 at 2.5% p.a. for 10 years
- **h** $1000000 at 1% p.a. for 100 years

Example 40

6 Calculate the final investment amount if the interest is compounded monthly.

- **a** $4000 at 12% p.a. for 4 years
- **b** $5000 at 6% p.a. for 2 years
- **c** $15000 at 9% p.a. for 3 years
- **d** $950 at 6.6% p.a. for 30 months
- **e** $90000 at 7.2% p.a. for $1\frac{1}{2}$ years
- **f** $34500 at 4% p.a. for 1 year

Example 41

7 It can be assumed that most cars depreciate at a rate of about 15% p.a. Calculate the projected value of each of the following cars at the end of 3 years, by using the rate of 15% p.a. Answer correct to the nearest dollar.

- **a** Bentley Continental GT $189900
- **b** Audi A3 $30850
- **c** Hyundai Accent $16095
- **d** Lexus RX $44000
- **e** Toyota Camry $29500
- **f** Mazda 6 $29800



8 **a** Find the compound interest when $10000 is invested at 6% p.a. over 2 years compounding:

- **i** annually
- **ii** every 6 months (twice a year)
- **iii** quarterly
- **iv** monthly
- **v** daily

b What conclusions can be made about the interest earned versus the number of compounding periods?

PROBLEM-SOLVING AND REASONING	9–11, 15	10–12, 15, 16	14–17

9 How much interest is earned on an investment of $400 if it is invested at:

- **a** 6% p.a. simple interest for 2 years?
- **b** 6% p.a. compounded annually over 2 years?
- **c** 6% p.a. compounded monthly over 2 years?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 **a** Tom invests \$1500 in an account earning $6\frac{1}{2}\%$ p.a. compounded annually. He plans to keep this invested for 5 years. How much interest will he earn on this investment?

b Susan plans to invest \$1500 in an account earning a simple interest rate of $6\frac{1}{2}\%$ p.a. How long does Susan need to invest her money to ensure that her return is equal to Tom's?

c If Susan wishes to invest her \$1500 for 5 years as Tom did, yet still insists on a simple interest rate, what annual rate should she aim to receive so that her interest remains the same as Tom's?

11 What annual simple interest rate corresponds to an investment of \$100 at 8% p.a. compounded monthly for 3 years?

12 Inflation is another application of the compound interest formula, as the cost of goods and services increases by a percentage over time. Find the cost of these household items, in 3 years, if the annual rate of inflation is 3.5%.

a Milk \$3.45 for 2 L

b Loaf of bread \$3.70

c Can of cola \$1.80

d Monthly electricity bill of \$120

e Dishwashing powder \$7.99

13 The compound interest formula consists of the pronumerals P, R, A and n. Given any three of these values, the fourth can be found.
Using your calculator, complete the following table, given the interest is compounded annually. (Trial and error is used when looking for the value of n.)

P(\$)	R	n	A(\$)
	0.1	5	402.63
	0.08	6	952.12
15000		10	24433.42
1		2	1.44
100	0.3		219.70
8500	0.075		15159.56

14 Talia wishes to invest a lump sum today so that she can save \$25000 in 5 years for a European holiday. She chooses an account earning 3.6% p.a. compounded monthly. How much does Talia need to invest to ensure that she reaches her goal?

15 **a** Calculate the interest earned on an investment of \$10000 at 5% p.a. compounded annually for 4 years.

b Calculate the interest earned on an investment of \$20000 at 5% p.a. compounded annually for 4 years. Is it double the interest in part **a**?

c Calculate the interest earned on an investment of \$10000 at 10% p.a. compounded annually for 2 years. Is this the same amount of interest as in part **a**?

d Comment on the effects of doubling the principal invested.

e Comment on the effect of doubling the interest rate and halving the time period.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

16 **a** Calculate the interest earned when $1000 is invested at 6% p.a. over 1 year compounded:

i annually
ii six-monthly
iii quarterly
iv monthly
v fortnightly
vi daily

b Explain why increasing the number of compounding periods increases the interest earned on the investment.

17 Write down a scenario that could explain each of the following lines of working for compound interest and depreciation.

a $300(1.07)^{12}$
b $300(1.02)^{36}$
c $1000(1.005)^{48}$
d $1000(0.95)^{5}$
e $9000(0.8)^{3}$

ENRICHMENT — — 18

Doubling your money

18 How long does it take to double your money?

Tony wishes to find out how long he needs to invest his money at 9% p.a., compounding annually, for it to double in value.

He sets up the following situation, letting P be his initial investment and $2P$ his desired final amount.

He obtains the formula $2P = P(1.09)^n$, which becomes $2 = (1.09)^n$ when he divides both sides by P.

a Complete the table below to find out the number of years needed for Tony to double his money.

n	1	2	3	4	5	6	7	8	9
$(1.09)^n$									

b How long does it take if the interest rate is:

i 8% p.a.?
ii 12% p.a.?
iii 18% p.a.?

c What interest rate is needed to double Tony's money in only 3 years?

d Use your findings above to complete the following.

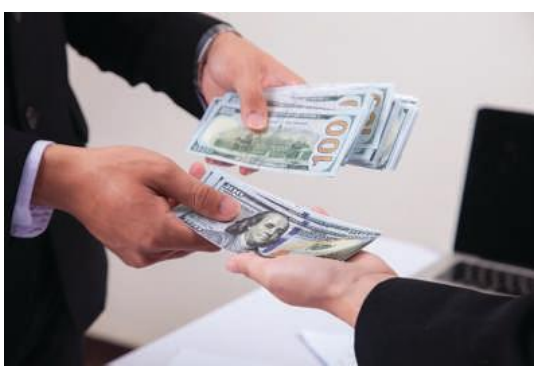
Years to double × interest rate = _____

e Use your rule to find out the interest rate needed to double your money in:

i 2 years
ii $4\frac{1}{2}$ years
iii 10 years

f Investigate the rule needed to triple your money.

g Investigate a rule for depreciation. Is there a rule for the time and rate of depreciation if your investment is halved over time?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Compounding investments

Banks offer many types of investments paying compound interest. Recall that for compound interest you gain interest on the money you have invested over a given time period. This increases your investment amount and therefore the amount of interest you gain in the next period.

Calculating yearly interest

Mary invests \$1000 at 6% per annum. This means Mary earns 6% of the principal every year in interest. That is, after 1 year the interest earned is:

$$6\% \text{ of } \$1000 = \frac{6}{100} \times 1000 = \$60$$

Mary has \$1060 after one year.

a The interest earned is added to the principal at the end of the year, and the total becomes the principal for the second year.
- **i** Assuming the same rate of interest, how much interest will Mary earn at the end of the second year?
- **ii** Calculate the interest earned for the third year.
- **iii** What total amount will Mary have at the end of the third year?
- **iv** How much interest will her money earn altogether over the 3 years?

b Write down a rule that calculates the total value of Mary's investment after t years. Use an initial investment amount of \$1000 and an annual interest rate of 6% p.a.

c Use your rule from part **b** to calculate the:
- **i** value of Mary's investment after 5 years
- **ii** value of Mary's investment after 10 years
- **iii** time it takes for Mary's investment to grow to \$1500
- **iv** time it takes for Mary's investment to grow to \$2000

Using a spreadsheet

This spreadsheet will calculate the compound interest for you if you place the principal in cell B3 and the rate in cell D3. In Mary's case, put 1000 in B3 and $\frac{6}{100}$ in D3.

	A	B	C	D
1	COMPOUND	INTEREST	SIMULATOR	
2				
3	PRINCIPAL	. . .	RATE OF PERIOD	. . .
4				
5	PERIOD	OPENING BALANCE	INTEREST EARNED	NEW BALANCE
6	1	=B3	=B6*D3	=B6 + C6
7	2	=D6	=B7*D3	=B7 + C7
8	3	=D7	=B8*D3	=B8 + C8
9	4	=D8	=B9*D3	=B9 + C9

a Copy the spreadsheet shown using 'fill down' at cells B7, C6 and D6.

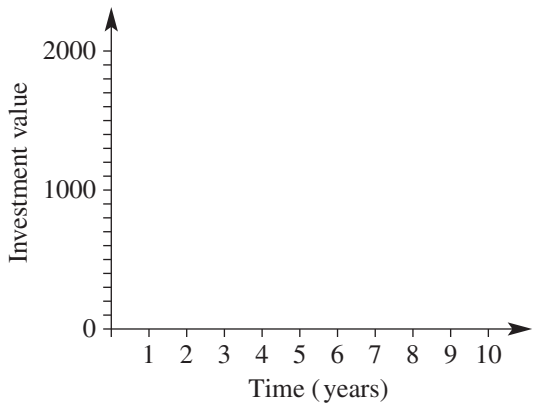
b Determine how much money Mary would have after 4 years.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Investigating compound interest

a What will Mary's balance be after 10 years? Extend your spreadsheet to find out.

b Draw a graph of Investment value versus time as shown. Plot points using the results from your spreadsheet and join them with a smooth curve. Discuss the shape of the curve.



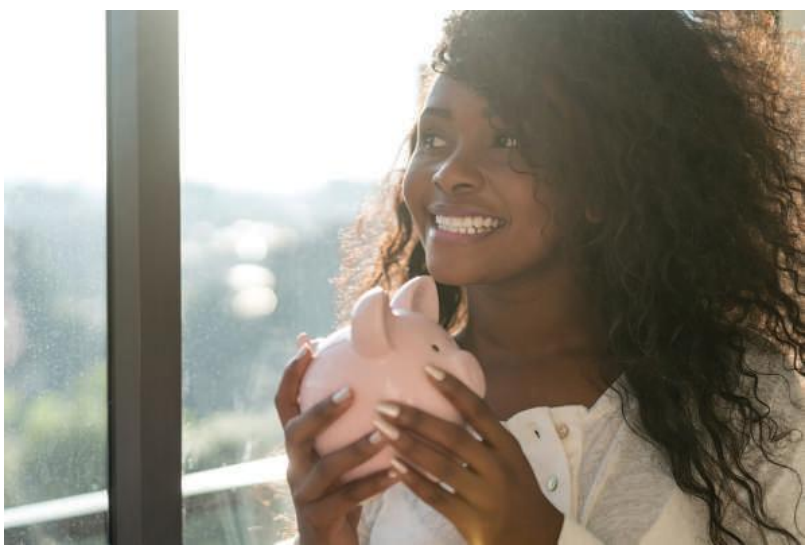
c How long does it take for Mary's investment to grow to $2000? Show this on your spreadsheet.

d Now try altering the interest rate.

- **i** What would Mary's investment grow to in 10 years if the interest rate was 10%?
- **ii** What would Mary's investment grow to in 10 years if the interest rate was 12%?

e What interest rate makes Mary's investment grow to $2000 in 8 years? Use trial and error to get an answer correct to 2 decimal places. Record your investigation results using a table.

f Investigate how changing the principal changes the overall investment amount. Record your investigations in a table, showing the principal amounts and investment balance for a given interest rate and period.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1 By only using the four operations $+$, $-$, $\times$ and $\div$ as well as brackets and square root ($\sqrt{\ }$), the number 4 can be used exactly 4 times to give the answer 9 in the following way $4 \times \sqrt{4} + 4 \div 4$. Use the number 4 exactly 4 times (and no other numbers) and any of the operations (as many times as you like) to give the answer 0 or 1 or 2 or 3 or … or 10.

2 What is the value of n if n is the largest of 5 consecutive numbers that multiply to 95 040?

3 Evaluate the following.

a $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{3}}}$

b $\dfrac{2}{1+\dfrac{2}{1+\dfrac{2}{5}}}$

4 A jug has 1 litre of 10% strength cordial. How much pure cordial needs to be added to make the strength 20%? (The answer is not 100 mL.)



5 An old table in a furniture store is marked down by 10% from its previous price on five separate occasions. What is the total percentage reduction in price correct to 2 decimal places?

6 What simple interest rate is equivalent to a compound interest rate of 6% p.a. over 10 years correct to 2 decimal places?

7 Brendon has a rectangular paved area in his yard.

a If he increases both the length and width by 20%, what is the percentage increase in area?

b If the length and width were increased by the same percentage and the area increases by 21%, what is the percentage increase of the length and width?

8 A rectangular sandpit is shown on a map, which has a scale of 1 : 100. On the map the sandpit has an area of 20 cm^2. What is its actual area?

9 Arrange the numbers 1 to 15 in a row so that each adjacent pair of numbers sum to a square number.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Computation and financial mathematics

Review

$2 + (-3) = -1$
$-4 - (-2) = -2$
$15 \div (-5) = -3$
$-7 \times (-3) = 21$
$(-2)^2 = 4$
$4^3 = 64$

For 8 and 12
HCF is 4
LCM is 24

$\sqrt{36} = 6$
$\sqrt[3]{27} = 3$

Operations with fractions

$$1\frac{5}{6} - \frac{7}{12} = \frac{11}{6} - \frac{7}{12}$$
$$= \frac{22}{12} - \frac{7}{12}$$
$$= \frac{15}{12}$$
$$= 1\frac{1}{4}$$

$$1\frac{1}{2} \div \frac{7}{2} = \frac{3}{\cancel{2}_1} \times \frac{\cancel{2}^1}{7}$$
$$= \frac{3}{7}$$

Rounding

1.284 to 1 d.p. is 1.3
2 sig. fig. is 1.3
472.543 to 1 d.p. is 472.5
1 sig. fig. is 500
0.0624 to 2 d.p. is 0.06
2 sig. fig. is 0.062

Rational numbers (fractions)

$1\frac{3}{8} = \frac{11}{8} = 1.375$
mixed numeral — improper fraction — terminating decimal

$\frac{1}{6} = 0.166\ldots = 1.\dot{1}\dot{6}$
proper fraction — recurring decimal

$\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions

Rates, ratios and best buy

180 km per 3 hrs = 60 km/h
$\frac{3}{7} : \frac{1}{2} = \frac{6}{14} : \frac{7}{14} = 6 : 7$
\$200 divided into 7 : 3
10 parts is \$200
1 part is \$20
3 parts is \$60
7 parts is \$140
Best buy: 3 kg carrots for \$6.45
5 kg for \$10.20
3 kg bag: $6.45 \div 3 = \$2.15$/kg
5 kg bag : $10.20 \div 5 = \$2.04$/kg
$\therefore$ 5 kg is best buy

Irrational numbers

Cannot be expressed as fractions
$\pi, \sqrt{2}$

Compound interest

$A = P(1 + R)^n$
\$3000 at 5% for 3 years
$A = \$3000 \times (1.05)^3$
$= \$3472.88$

Percentages

$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$
$25\frac{1}{2}\% = 25.5 \div 100\% = 0.255$
20% of $60 = 0.2 \times 60 = 12$
5% of amount = 32
$\therefore$ amount $= 32 \div 0.05$
$= 640$

Simple interest

$I = PRN = \$2000 \times \frac{5}{100} \times 4$
$= \$400$
Amount $= P + I$

Income and tax

Employees can be paid:
- wage: hourly rate with overtime at time and a half = 1.5
- salary: annual amount
- commission: % of sales
- net income = gross income – deductions
- 17.5% leave loading for holiday pay

Applications of percentages

Profit or loss
Percentage profit or loss
$= \frac{\text{profit or loss}}{\text{cost price}} \times 100\%$
Mark-up and discount
Commission/tax

Percentage increase and decrease

Increase	Decrease
20 by 6%	20 by 5%
$20 \times 1.06 = 21.2$	$20 \times 0.95 = 19$

Percentage change $= \frac{\text{change}}{\text{original}} \times 100\%$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Multiple-choice questions

1 $\frac{2}{7}$ written as a decimal is:

A 0.29 **B** 0.286 **C** $0.2\dot{8}5$ **D** $0.\overline{285714}$ **E** $0.285714\dot{}$

2 3.0456 written to 3 significant figures is:

A 3.04 **B** 3.05 **C** 3.045 **D** 3.046 **E** 3.45

3 2.25 written as a fraction in simplest form is:

A $2\frac{1}{2}$ **B** $\frac{5}{4}$ **C** $\frac{9}{4}$ **D** $9\frac{1}{4}$ **E** $\frac{225}{100}$

4 $1\frac{1}{2} - \frac{5}{6}$ is equal to:

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $-\frac{1}{2}$ **D** $\frac{2}{6}$ **E** $\frac{1}{2}$

5 $\frac{2}{7} \times \frac{3}{4}$ is equivalent to:

A $\frac{8}{11}$ **B** $\frac{3}{7}$ **C** $\frac{5}{11}$ **D** $\frac{8}{12}$ **E** $\frac{3}{14}$

6 $\frac{3}{4} \div \frac{5}{6}$ is equivalent to:

A $\frac{5}{8}$ **B** 1 **C** 21 **D** $\frac{4}{5}$ **E** $\frac{9}{10}$

7 Simplifying the ratio 50 cm : 4 m gives:

A 50 : 4 **B** 8 : 1 **C** 25 : 2 **D** 1 : 8 **E** 5 : 40

8 28% as a fraction in its simplest form is:

A 0.28 **B** $\frac{28}{100}$ **C** $\frac{0.28}{100}$ **D** $\frac{2.8}{100}$ **E** $\frac{7}{25}$

9 15% of $1600 is equal to:

A 24 **B** 150 **C** $240 **D** $24 **E** 240

10 Jane is paid a wage of $7.80 per hour. If she works 12 hours at this rate during a week plus 4 hours on a public holiday in the week when she gets paid at time and half, her earnings in the week are:

A $140.40 **B** $124.80 **C** $109.20 **D** $156 **E** $62.40

11 Simon earns a weekly retainer of $370 and 12% commission of any sales he makes. If he makes $2700 worth of sales in a particular week, he will earn:

A $595 **B** $652 **C** $694 **D** $738.40 **E** $649.60

12 $1200 is increased by 10% for two years with compound interest. The total balance at the end of the two years is:

A $252 **B** $1452 **C** $1450 **D** $240 **E** $1440

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Short-answer questions

1 Evaluate the following.

a $-4 \times (2 - (-3)) + 4$ **b** $-3 - 4 \times (-2) + (-3)$ **c** $(-8 \div 8 - (-1)) \times (-2)$

d $\sqrt{25} \times \sqrt[3]{8}$ **e** $(-2)^2 - 3^3$ **f** $\sqrt[3]{1000} - (-3)^2$

2 Round these numbers to 3 significant figures.

a 21.483 **b** 29130 **c** 0.15271 **d** 0.002414

3 Estimate the answer by firstly rounding each number to 1 significant figure.

a $294 - 112$ **b** 21.48×2.94 **c** $1.032 \div 0.493$

4 Write these fractions as decimals.

a $2\frac{1}{8}$ **b** $\frac{5}{6}$ **c** $\frac{13}{7}$

5 Write these decimals as fractions.

a 0.75 **b** 1.6 **c** 2.55

6 Simplify the following.

a $\frac{5}{6} - \frac{1}{3}$ **b** $1\frac{1}{2} + \frac{2}{3}$ **c** $\frac{13}{8} - \frac{4}{3}$ **d** $3\frac{1}{2} \times \frac{4}{7}$ **e** $5 \div \frac{4}{3}$ **f** $3\frac{3}{4} \div 1\frac{2}{5}$

7 Simplify these ratios.

a 30 : 12 **b** 1.6 : 0.9 **c** $7\frac{1}{2} : 1\frac{2}{5}$

8 Divide 80 into the given ratio.

a 5 : 3 **b** 5 : 11 **c** 1 : 2 : 5

9 Dry dog food can be bought from store A for $18 for 8 kg or from store B for $14.19 for 5.5 kg.

a Determine the cost per kilogram at each store and state which is the best buy.

b Determine to the nearest whole number how many grams of each brand you get per dollar.

10 Copy and complete the table (right).

Decimal	Fraction	Percentage
0.6		
	$\frac{1}{3}$	
		$3\frac{1}{4}\%$
	$\frac{3}{4}$	
1.2		
		200%

11 Find:

a 25% of $310

b 110% of 1.5

12 Determine the original amount if:

a 20% of the amount is 30

b 72% of the amount is 18

13 **a** Increase 45 by 60%.

b Decrease 1.8 by 35%.

c Find the percentage change if $150 is reduced by $30.

14 The mass of a cat increased by 12% to 14 kg over a 12-month period. What was its previous mass?

15 Determine the discount given on a $15000 car if it is discounted by 12%.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

16 The cost price of an article is $150. If it is sold for $175:
- **a** determine the profit made
- **b** express the profit as a percentage of the cost price

17 Determine the hourly rate of pay for each of the following cases.
- **a** A person with an annual salary of $36 062 working a 38-hour week
- **b** A person who earns $429 working 18 hours at the hourly rate and 8 hours at time and a half

18 Jo's monthly income is $5270. However, 20% of this is paid straight to the government in taxes. What is Jo's net yearly income?

19 Find the simple interest earned on $1500 at 7% p.a. for 5 years.

20 Rob invests $10 000 at 8% p.a. simple interest to produce $3600. How long was the money invested for?

21 Find the total value of an investment if $50 000 is invested at 4% compounded annually for 6 years. Round to the nearest cent.

22 After 8 years a loan grows to $75 210. If the interest was compounded annually at a rate of 8.5%, find the size of the initial loan to the nearest dollar.

Extended-response questions

1 Pauline buys a debutante dress at cost price from her friend Tila. Pauline paid $420 for the dress, which is normally marked up by 55%.
- **a** How much did she save?
- **b** What is the normal selling price of the dress?
- **c** If Tila gets a commission of 15%:
  - **i** how much commission did she get?
  - **ii** how much commission did she lose by selling the dress at cost price rather than the normal selling price?

2 Matilda has two bank accounts with the given details.
Account A: Investment. Principal $25 000, interest rate 6.5% compounded annually
Account B: Loan. 11.5% compounded annually
- **a** Find Matilda's investment account balance after:
  - **i** 1 year
  - **ii** 10 years (to the nearest cent)
- **b** Find the total percentage increase in Matilda's investment account after 10 years, correct to 2 decimal places.
- **c** After 3 years Matilda's loan account has increased to $114 250. Find the initial loan amount to the nearest dollar.
- **d** Matilda reduces her $114 250 loan by $30 000. What is this reduction as a percentage to 2 decimal places?
- **e** For Matilda's investment loan, what simple interest rate is equivalent to 5 years of the compounded interest rate of 6.5%? Round to 1 decimal place.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

2 Expressions, equations and inequalities

What you will learn

2A Algebraic expressions REVISION
2B Simplifying algebraic expressions REVISION
2C Expanding algebraic expressions
2D Linear equations with pronumerals on one side
2E Linear equations with brackets and pronumerals on both sides
2F Using linear equations to solve problems
2G Linear inequalities
2H Using formulas
2I Linear simultaneous equations: substitution
2J Linear simultaneous equations: elimination
2K Using linear simultaneous equations to solve problems
2L Quadratic equations of the form $ax^2 = c$

(Equations involving algebraic fractions are located in section 2D and also in Chapter 8 – section 8K.)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: ALGEBRAIC TECHNIQUES (S4) EQUATIONS (S4, 5.2)

Outcomes

A student uses algebraic techniques to solve simple linear and quadratic equations.
(MA4–10NA)

A student solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques.
(MA5.2–8NA)

© NESA, 2012

Arcade games

Nearly every occupation uses the concept of solving equations in their daily job. This can be as easy as *change equals money given subtract money owed*, to more complicated equations related to the physics involved in a satellite launch.

The coders of computer games require an understanding not just of basic algebra, linear, quadratics and, at times, simultaneous equations, but of the coding programs as well. The more complicated the game the more detailed the equations and coding needs to be.

Pac-Man is a famous arcade game from the 1980s available now as a computer game and app. It even has its own cartoon spin-off. The equations used in the coding of such a game can be as simple as defining the 'heat' of Pac-Man (heat is a nickname for the property that makes the Ghosts want to chase Pac-Man): 'heat = a positive value', or 'heat = a negative value'. Other more complicated equations are also incorporated into the code. Equations of the type 'heat = $k \times$ {heat (up) + heat (down) + …}' are employed throughout the code for the game to activate the features that make it unique.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Write algebraic expressions for:

a 3 more than x

b the product of a and b

c 2 lots of y less 3

d the sum of x and 2 divided by 3

2 Evaluate the following if $a = 3$ and $b = -2$.

a $2a - 5$ **b** $ab + 4$

c $\frac{9}{a} - b$ **d** $2a(b + 1)$

3 Simplify by collecting like terms.

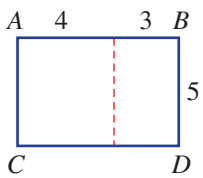
a $2x + 5x - 4x$ **b** $7y + xy - y$ **c** $-4x - (-4x)$

d $3x + 12y - 3x + 5y$ **e** $3a - 4a^2 + 7a + 5a^2$ **f** $3xy - 4y + 2yx - 3y$

4 Simplify the following.

a $3 \times 2a$ **b** $7x \times (-3y)$ **c** $\frac{8b}{2}$ **d** $\frac{9mn}{6n}$

5 Describe and calculate in two different ways how the area of rectangle $ABCD$ can be found.



6 Expand the following.

a $2(x + 3)$ **b** $3(a - 5)$

c $4x(3 - 2y)$ **d** $-3(2b - 1)$

7 Which of the following equations is $x = 4$ a solution to?

A $2x + 3 = 9$ **B** $\frac{x}{2} + 3 = 5$ **C** $\frac{2x + 1}{3} = 3$ **D** $5 - 2x = -1$

8 Find the value of a that makes the following true.

a $a + 4 = 13$ **b** $a - 3 = 7$ **c** $2a + 1 = 9$ **d** $\frac{a - 1}{4} = 5$

9 State if each of the following are true or false.

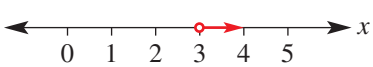
a $5 > 3$ **b** $6 < 2$ **c** $4 \leq 4$

d $-1 < 5$ **e** $-7 > -2$ **f** $-3 < -1$

10 Find the value of y in each of the following for the given value of x.

a $y = 2x - 3, x = 5$ **b** $y = -3x + 7, x = 2$ **c** $2x + y = 7, x = -1$ **d** $x - 2y = 3, x = 15$

11 Which inequality describes this number line?

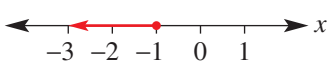


A $x > 3$ **B** $x \geq 3$

C $x < 3$ **D** $x \leq 3$

E $3 \leq x < 4$

12 Which inequality describes this number line?



A $x \leq -2$ **B** $x > -1$

C $x \geq -1$ **D** $x < -1$

E $x \leq -1$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2A Algebraic expressions REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Algebra is central to the study of mathematics and is commonly used to solve problems in a vast array of theoretical and practical problems. Algebra involves the represention and manipulation of unknown or varying quantities in a mathematical context. Pronumerals are used to represent these unknown quantities.



Pronumerals representing unknown quantities are used in a wide range of jobs and occupations.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Remembering the vocabulary

State the parts of the expression $5x - 2xy + (4a)^2 - 2$ that match these words:

- pronumeral
- term
- coefficient
- constant term
- squared term.

Key ideas

- In algebra, letters are used to represent numbers. These letters are called **pronumerals** or **variables**.
- An **expression** is a combination of numbers and pronumerals connected by the four operations $+$, $-$, $\times$ and $\div$. Brackets can also be used.
  For example: $5x^2 + 4y - 1$ and $3(x + 2) - \frac{y}{5}$
- A **term** is a combination of numbers and pronumerals connected with only multiplication and division. Terms are separated with the operations $+$ and $-$.
  For example: $5x + 7y$ is a two-term expression
- **Coefficients** are the numbers being multiplied by pronumerals.
  For example: the 3 in $3x$ and $\frac{1}{2}$ in $\frac{x^2}{2}$ are coefficients
- **Constant terms** consist of a number only.
  For example: -2 in $x^2 + 4x - 2$ (The sign must be included.)
- Expressions can be **evaluated** by substituting a number for a pronumeral.
  For example: if $a = -2$ then $a + 6 = -2 + 6 = 4$
- **Order of operations** should be followed when evaluating expressions:
  1. Operations in brackets, then
  2. Powers, then
  3. Multiplication and division (from left to right), then
  4. Addition and subtraction (from left to right).

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 1 Writing algebraic expressions for worded statements

Write an algebraic expression for the:

a number of tickets needed for 3 boys and r girls

b cost of P pies at \$3 each

c number of grams of peanuts for one child if 300 g of peanuts is shared equally among C children

SOLUTION	EXPLANATION
a $3 + r$	3 tickets plus the number of girls
b $3P$	3 multiplied by the number of pies
c $\frac{300}{C}$	300 g divided into C parts

Example 2 Converting words to expressions

Write an algebraic expression for:

a five less than x

b three more than twice x

c the sum of a and b is divided by 4

d the square of the sum of x and y

SOLUTION	EXPLANATION
a $x - 5$	5 subtracted from x
b $2x + 3$	Twice x plus 3
c $\frac{a + b}{4}$	The sum of a and b is done first $(a + b)$ and the result is divided by 4.
d $(x + y)^2$	The sum of x and y is done first and then the result is squared.

Example 3 Substituting values into expressions

Evaluate these expressions if $a = 5$, $b = -2$ and $c = 3$.

a $7a - 2(a - c)$

b $b^2 - ac$

SOLUTION	EXPLANATION
a $7a - 2(a - c) = 7 \times 5 - 2(5 - 3)$ $= 35 - 2 \times 2$ $= 35 - 4$ $= 31$	Substitute the values for a and c. When using order of operations, evaluate brackets before moving to multiplication and division then addition and subtraction.
b $b^2 - ac = (-2)^2 - 5 \times 3$ $= 4 - 15$ $= -11$	Evaluate powers before the other operations. $(-2)^2 = -2 \times (-2) = 4$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 2A REVISION

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	3, 4, 5–6(½), 7	4, 5–6(½), 7

1 Consider the expression $3x^2 + 5d + \frac{w}{4} + 9$. Complete the following.

- **a** The expression contains _____ terms
- **b** The coefficient of d is _____
- **c** The coefficient of w is _____
- **d** The constant term is _____

2 Match an item in the left column with an item in the right column.

A	Product	**a**	Division
B	Sum	**b**	Subtraction
C	Difference	**c**	Multiplication
D	Quotient	**d**	Addition
E	x^2	**e**	The reciprocal of a
F	$\frac{1}{a}$	**f**	The square of x

3 State the coefficient in these terms.

a $5xy$ **b** $-2a^2$ **c** $\frac{x}{3}$ **d** $-\frac{2a}{5}$

Example 1

4 Write an algebraic expression for the following.

- **a** The number of tickets required for:
  - **i** 4 boys and r girls
  - **ii** t boys and 2 girls
  - **iii** b boys and g girls
  - **iv** x boys, y girls and z adults
- **b** The cost of:
  - **i** P pies at \$6 each
  - **ii** 10 pies at \$$n$ each
  - **iii** D drinks at \$2 each
  - **iv** P pies at \$5 and D drinks at \$2
- **c** The number of grams of lollies for one child if 500 g of lollies is shared equally among C children.

Example 2

5 Write an algebraic expression for each of the following.

a	The sum of 2 and x	**b**	The sum of ab and y
c	5 less than x	**d**	The product of x and 3
e	The difference between $3x$ and $2y$	**f**	Three times the value of p
g	Four more than twice x	**h**	The sum of x and y is divided by 5
i	10 less than the product of 4 and x	**j**	The square of the sum of m and n
k	The sum of the squares of m and n	**l**	The square root of the sum of x and y
m	The sum of a and its reciprocal	**n**	The cube of the square root of x

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 3

6 Evaluate these expressions if $a = 4$, $b = -3$ and $c = 7$.

a $b - ac$ **b** $bc - a$ **c** $a^2 - c^2$ **d** $b^2 - ac$

e $\frac{a+b}{2}$ **f** $\frac{b^2+c}{a}$ **g** $\frac{1}{c} \times (a - b)$ **h** $a^3 - bc$

7 Evaluate these expressions if $x = -2$, $y = -\frac{1}{2}$ and $z = \frac{1}{6}$.

a $xy + z$ **b** $y^2 + x^2$ **c** xyz **d** $\frac{xz+1}{y}$

PROBLEM-SOLVING AND REASONING 8, 9, 11 | 8, 9, 11, 12 | 9, 10, 12, 13

8 A rectangular garden bed is 12 m long and 5 m wide.

a Find the area of the garden bed.

b The length is increased by x cm and the width is decreased by y cm. Find the new length and width of the garden.

c Write an expression for the area of the new garden bed.

9 The expression for the area of a trapezium is $\frac{1}{2}h(a + b)$ where a and b are the lengths of the two parallel sides and h is the distance between the two parallel sides.

a Find the area of the trapezium with $a = 5$, $b = 7$ and $h = 3$.

b A trapezium has $h = 4$ and area 12. If a and b are whole numbers, what possible values can the pronumeral a have?

10 The cost of 10 identical puzzles is $\$P$.

a Write an expression for the cost of one puzzle.

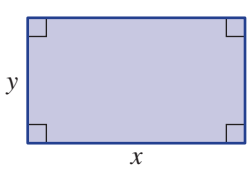
b Write an expression for the cost of n puzzles.

11 For each of these shapes, write an expression for the:

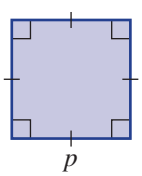
i perimeter

ii area

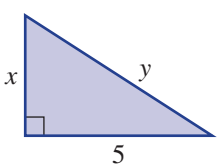
a



b



c



12 Decide if the following statements refer to the same or different expressions. If they are different, write an expression for each statement.

a **A** Twice the sum of x and y

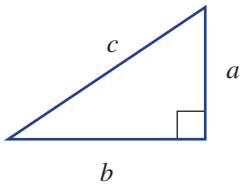
B The sum of $2x$ and y

b **A** The difference between half of x and half of y

B Half of the difference between x and y

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 For a right-angled triangle with hypotenuse c and shorter sides a and b, Pythagoras' theorem states that $c^2 = a^2 + b^2$.



a Which of these two descriptions also describes Pythagoras' theorem?

A The square of the hypotenuse is equal to the square of the sum of the two shorter sides.

B The square of the hypotenuse is equal to the sum of the squares of the two shorter sides.

b For the incorrect description, write an equation to match.

ENRICHMENT — — 14

The sum of the first n positive integers

14 The rule for the sum of the first n positive integers is given by:

The product of n and one more than n all divided by 2**.**

a Write an expression for the above description.

b Test the expression to find these sums.

i $1 + 2 + 3 + 4$ $(n = 4)$

ii $1 + 2 + 3 + \ \ldots + 10$ $(n = 10)$

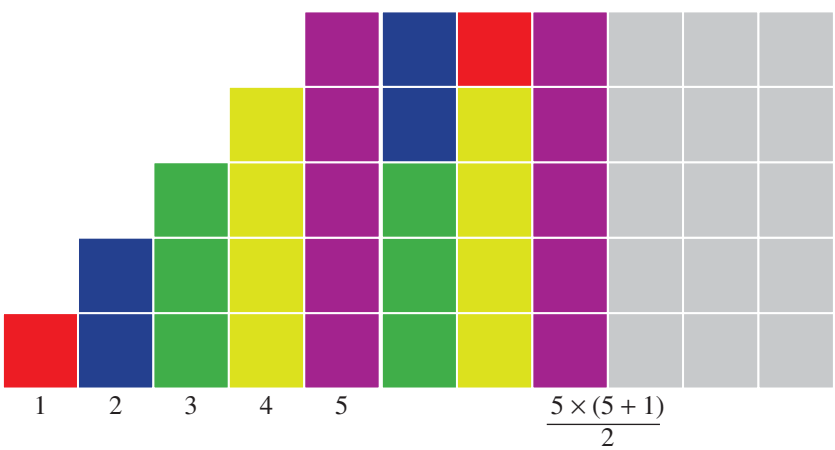
c Another way to describe the same expression is:

The sum of half of the square of n and half of n.

Write the expression for this description.

d Check that your expressions in parts **a** and **c** are equivalent (the same) by testing $n = 4$ and $n = 10$.

e $\frac{1}{2}(n^2 + n)$ is also equivalent to the above two expressions. Write this expression in words.



This diagram represents the sum of the first five positive integers arranged according to the expression in question 14.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2B Simplifying algebraic expressions REVISION

Interactive

Widgets

HOTsheets

Walkthrough

Just as $2 + 2 + 2 + 2 = 4 \times 2$, so $x + x + x + x = 4 \times x$ or $4x$.
We say that the expression $x + x + x + x$ is simplified to $4x$.
Similarly, $3x + 5x = 8x$ and $8x - 3x = 5x$.

All these expressions have like terms and can be simplified to an expression with a smaller number of terms.

A single term such as $2 \times 5 \times x \div 10$ can also be simplified using multiplication and division, so: $2 \times 5 \times x \div 10 = \frac{10x}{10} = x$.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Are they equivalent?

All these expressions can be separated into two groups. Group them so that the expressions in each group are equivalent.

$2x$ $\quad 2x - y$ $\quad 4x - x - x$ $\quad 10x - y - 8x$

$\frac{24x}{12}$ $\quad y + x - y + x$ $\quad 2 \times x - 1 \times y$ $\quad -y + 2x$

$0 + \frac{1}{2} \times 4x$ $\quad \frac{x}{\left(\frac{1}{2}\right)} + 0y$ $\quad \frac{6x^2}{3x}$ $\quad -1 \times y + \frac{x^2}{\frac{1}{2}x}$

Key ideas

- The symbols for multiplication ($\times$) and division ($\div$) are usually not shown in simplified algebraic terms.
  For example: $5 \times a \times b = 5ab$ and $-7 \times x \div y^2 = -\frac{7x}{y^2}$
- When dividing algebraic expressions common factors can be cancelled.
  For example: $\frac{7x}{14} = \frac{x}{2}$ $\quad \frac{a^2b}{a} = \frac{{}^{1}\cancel{a} \times a \times b}{{}^{1}\cancel{a}} = ab$
  $\frac{7xy}{14y} = \frac{x}{2}$ $\quad \frac{15a^2b}{10a} = \frac{3 \times \cancel{5} \times a \times \cancel{a} \times b}{2 \times \cancel{5} \times \cancel{a}} = \frac{3ab}{2}$
- **Like terms** have the same pronumeral factors.
  For example: $5x$ and $7x$ are like terms and $3a^2b$ and $-2a^2b$ are like terms.
  - Since $a \times b = b \times a$, then ab and ba are also like terms
- The letters in a term are usually written in alphabetical order.
- Like terms can be collected (added and subtracted) to form a single term.
  For example: $5ab + 8ab = 13ab$
  $4x^2y - 2yx^2 = 2x^2y$
- **Unlike terms** do not have the same pronumeral factors.
  For example: $5x$, x^2, xy and $\frac{4xyx}{5}$ are all unlike terms.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 4 Multiplying algebraic terms

Simplify the following.

a $3 \times 2b$

b $-2a \times 3ab$

SOLUTION

a $3 \times 2b = 3 \times 2 \times b = 6b$

b $-2a \times 3ab = -2 \times 3 \times a \times a \times b = -6a^2b$

EXPLANATION

Multiply the coefficients.

Multiply the coefficients and simplify.

Example 5 Dividing algebraic terms

Simplify the following.

a $\dfrac{6ab}{18b}$

b $12a^2b \div (3ab)$

SOLUTION

a $\dfrac{{}^{1}\cancel{6}ab^{1}}{{}_{3}\cancel{18}b_{1}} = \dfrac{a}{3}$

b $12a^2b \div (3ab) = \dfrac{{}^{4}\cancel{12}a^{\cancel{2}1}b^{1}}{{}_{1}\cancel{3}a_{\cancel{1}}b_{1}}$
$= 4a$

EXPLANATION

Deal with numerals and promumerals separately, cancelling where possible.

Write as a fraction first. Cancel where possible, recall $a^2 = a \times a$.

Example 6 Collecting like terms

Simplify the following by collecting like terms.

a $3x + 4 - 2x$

b $3x + 2y + 4x + 7y$

c $8ab^2 - 9ab - ab^2 + 3ba$

SOLUTION

a $3x + 4 - 2x = 3x - 2x + 4$
$= x + 4$

b $3x + 2y + 4x + 7y = 3x + 4x + 2y + 7y$
$= 7x + 9y$

c $8ab^2 - 9ab - ab^2 + 3ba$
$= 8ab^2 - ab^2 - 9ab + 3ab$
$= 7ab^2 - 6ab$

EXPLANATION

Collect like terms ($3x$ and $-2x$). The sign belongs to the term that follows. Combine their coefficients $3 - 2 = 1$.

Collect like terms and combine their coefficients.

Collect like terms. Remember $ab = ba$ and $ab^2 = 1ab^2$.
$8 - 1 = 7$ and $-9 + 3 = -6$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 2B REVISION

UNDERSTANDING AND FLUENCY	1, 2–7(½), 9–11(½)	3–11(½)	5–11(½)

1 True or False?

a $1a$ can be written as a **b** $0b$ is equal to 0

c $a \times b = b \times a = ab$ **d** x^2 and $2x$ are like terms

2 Simplify these products.

a $a \times b$ **b** $6 \times a$ **c** $a \times 8$

d $4 \times b \times a$ **e** $5 \times n \times m \times 2$ **f** $9 \times x \times x$

3 Are these like terms or unlike terms?

a $4ab$ and $3ab$ **b** $2x$ and $7xy$ **c** 5 and $4m$

d $3t$ and $-6tw$ **e** $7yz$ and yz **f** $2mn$ and $9nm$

g $5a$ and a **h** $3x^2y$ and $7xy^2$ **i** $4ab^2$ and $-3b^2a$

Example 4a **4** Simplify the following.

a $5 \times 2m$ **b** $2 \times 6b$ **c** $3 \times 5p$ **d** $3x \times 2y$

e $3p \times 6r$ **f** $4m \times 4n$ **g** $-2x \times 7y$ **h** $5m \times (-3n)$

i $-4c \times 3d$ **j** $2a \times 3b \times 5$ **k** $-4r \times 3 \times 2s$ **l** $5j \times (-4) \times 2k$

Example 4b **5** Simplify the following.

a $4n \times 6n$ **b** $-3q \times q$ **c** $5s \times 2s$

d $7a \times 3ab$ **e** $5mn \times (-3n)$ **f** $-3gh \times (-6h)$

g $3xy \times 4xy$ **h** $-4ab \times (-2ab)$ **i** $-2mn \times 3mn$

Example 5a **6** Simplify the following by cancelling.

a $\frac{8b}{2}$ **b** $-\frac{2a}{6}$ **c** $\frac{4ab}{6}$ **d** $\frac{3mn}{6n}$ **e** $-\frac{5xy}{20y}$

f $\frac{10st}{6t}$ **g** $\frac{u^2v}{u}$ **h** $\frac{5r^2s^2}{8rs}$ **i** $\frac{5ab^2}{9b}$ **j** $\frac{7y}{y^2}$

Example 5b **7** Simplify the following by first writing in fraction form.

a $2x \div 5$ **b** $-4 \div (-3a)$ **c** $11mn \div 3$ **d** $12ab \div 2$

e $-10 \div (2gh)$ **f** $8x \div x$ **g** $-3xy \div (yx)$ **h** $7mn \div (3m)$

i $-27pq \div (6p)$ **j** $24ab^2 \div (8ab)$ **k** $25x^2y \div (-5xy)$ **l** $9m^2n \div (18mn)$

8 Simplify the following.

a $x \times 4 \div y$ **b** $5 \times p \div 2$ **c** $6 \times (-a) \times b$ **d** $a \times (-3) \div (2b)$

e $-7 \div (5m) \times n$ **f** $5s \div (2t) \times 4$ **g** $6 \times 4mn \div (3m)$ **h** $8x \times 3y \div (8x)$

i $3ab \times 12bc \div (9abc)$ **j** $4x \times 3xy \div (2x)$ **k** $10m \times 4mn \div (8mn)$ **l** $3pq \times pq \div p$

Example 6a **9** Simplify the following by collecting like terms.

a $6x + x$ **b** $6x - x$ **c** $x - 6x$

d $x + x$ **e** $x - x$ **f** $x - x - x$

g $3x + 4x + x$ **h** $3 + 4x + x$ **i** $3 + 4 + x$

j $3a + 7a$ **k** $3a - 7a$ **l** $-3a + 7a$

m $5x + 2x + 4x$ **n** $6ab - 2ab - ba$ **o** $7mn + 2mn - 2mn$

p $4y - 3y + 8$ **q** $7x + 5 - 4x$ **r** $6xy + xy + 4y$

s $5ab + 3 + 7ba$ **t** $2 - 5m - m$ **u** $4 - 2x + x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 6b

10 Simplify the following by collecting like terms.

a $2a + 4b + 3a + 5b$ **b** $4x + 3y + 2x + 2y$ **c** $6t + 5 - 2t + 1$ **d** $5x + 1 + 6x + 3$

e $xy + 8x + 4xy - 4x$ **f** $3mn - 4 + 4nm - 5$ **g** $4ab + 2a + ab - 3a$ **h** $3st - 8ts + 2st + 3ts$

Example 6c

11 Simplify the following by collecting like terms.

a $5xy^2 - 4xy^2$ **b** $3a^2b + 4ba^2$ **c** $8m^2n - 6nm^2 + m^2n$

d $7p^2q^2 - 2p^2q^2 - 4p^2q^2$ **e** $2x^2y - 4xy^2 + 5yx^2$ **f** $10rs^2 + 3rs^2 - 6r^2s$

g $x^2 - 7x - 3x^2$ **h** $a^2b - 4ab^2 + 3a^2b + b^2a$ **i** $10pq^2 - 2qp - 3pq^2 - 6pq$

j $12m^2n^2 - 2mn^2 - 4m^2n^2 + mn^2$

PROBLEM-SOLVING AND REASONING	12, 13, 16	12–14, 16	13–17

12 A farmer has x pigs and y chickens.

a Write an expression for the total number of heads.

b Write an expression for the total number of legs.

13 A rectangle's length is three times its breadth x metres. Write an expression for the rectangle's:

a perimeter **b** area

14 A right-angled triangle has side lengths $5x$ cm, $12x$ cm and $13x$ cm. Write an expression for the triangle's:

a perimeter **b** area

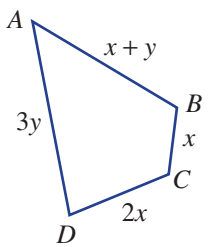
15 The average (mean) mark on a test for 20 students is x. Another student who scores 75 in the test is added to the list. Write an expression for the new average (mean).

16 Decide if the following are always true for all real numbers.

a $a \times b = b \times a$ **b** $a \div b = b \div a$ **c** $a + b = b + a$

d $a - b = b - a$ **e** $a^2b = b^2a$ **f** $1 \div \frac{1}{a} = a$

17 The diagram shows the route taken by a salesperson who travels from A to D via B and C.



a If the salesperson then returns directly to A, write an expression (in simplest form) for the total distance travelled.

b If $y = x + 1$, write an expression for the total distance the salesperson travels in terms of x only. Simplify your expression.

c When $y = x + 1$, how much would the distance have been reduced by (in terms of x) if the salesperson had travelled directly from A to D and straight back to A?

ENRICHMENT	—	—	18

Higher powers

18 For this question, note this example: $\frac{2a^3}{4a} = \frac{2_1 \times a \times a \times \cancel{a}_1}{\cancel{2}^2 \times \cancel{a}_1} = \frac{a^2}{2}$

Simplify these expressions with higher powers.

a $\frac{a^4}{a}$ **b** $\frac{3b^3}{9b}$ **c** $\frac{4ab^3}{12ab}$ **d** $\frac{6a^4b^2}{16a^3b}$

e $-\frac{2a^5}{3a^2}$ **f** $-\frac{8a^5}{20a^7}$ **g** $\frac{4a^3}{10a^8}$ **h** $\frac{3a^3b}{12ab^4}$

i $\frac{15a^4b^2}{5ab}$ **j** $\frac{28a^3b^5}{7a^4b^2}$ **k** $\frac{2a^5b^2}{6a^2b^3}$ **l** $-\frac{5a^3b^7}{10a^2b^{10}}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2C Expanding algebraic expressions

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

A mental technique to find the product of 5 and 23 might be to find 5×20 and add 5×3 to give 115. This technique uses the distributive law over addition.

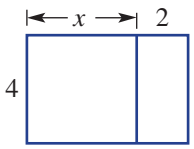
So: $5 \times 23 = 5 \times (20 + 3)$
$= 5 \times 20 + 5 \times 3$

Since pronumerals represent numbers, the same law applies for algebraic expressions.

Let's start: Rectangular distributions

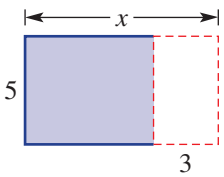
This diagram shows two joined rectangles with the given dimensions.

- Find two different ways to write expressions for the combined area of the two rectangles.
- Compare your two expressions. Are they equivalent?



This diagram shows a rectangle of length x reduced by a length of 3.

- Find two different ways to write expressions for the remaining area (shaded).
- Compare your two expressions. Are they equivalent?





Examples of the distributive law can often be found in everyday tasks.

Key ideas

- The **distributive law** is used to expand and remove brackets.
  - A term on the outside of the brackets is multiplied by each term inside the brackets.
    $a(b + c) = ab + ac$ or $a(b - c) = ab - ac$
    $-a(b + c) = -ab - ac$ or $-a(b - c) = -ab + ac$
  - If the number in front of the bracket is negative, the sign of each of the terms inside the brackets will change when expanded.
    For example: $-2(x - 3) = -2x + 6$, since $-2 \times x = -2x$ and $-2 \times (-3) = 6$.

Example 7 Expanding simple expressions with brackets

Expand the following.

a $3(x + 4)$ **b** $5(x - 11)$ **c** $-2(x - 5)$

SOLUTION	EXPLANATION
a $3(x + 4) = 3x + 12$	$3 \times x = 3x$ and $3 \times 4 = 12$
b $5(x - 11) = 5x - 55$	$5 \times x = 5x$ and $5 \times (-11) = -55$
c $-2(x - 5) = -2x + 10$	$-2 \times x = -2x$ and $-2 \times (-5) = +10$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8 Expanding brackets and simplifying

Expand the following.

a $4(x + 3y)$ **b** $-2x(4x - 3)$

SOLUTION

a $4(x + 3y) = 4 \times x + 4 \times 3y$
$= 4x + 12y$

b $-2x(4x - 3) = -2x \times 4x + (-2x) \times (-3)$
$= -8x^2 + 6x$

EXPLANATION

Multiply each term inside the brackets by 4.
$4 \times x = 4x$ and $4 \times 3 \times y = 12y$.

Each term inside the brackets is multiplied by $-2x$.
$-2 \times 4 = -8$ and $x \times x = x^2$ and $-2 \times (-3) = 6$

Example 9 Simplifying by removing brackets

Expand the following and collect like terms.

a $2 - 3(x - 4)$ **b** $3(x + 2y) - (3x + y)$

SOLUTION

a $2 - 3(x - 4) = 2 - (3x - 12)$
$= 2 - 3x + 12$
$= 14 - 3x$

b $3(x + 2y) - (3x + y)$
$= 3x + 6y - 3x - y$
$= 3x - 3x + 6y - y$
$= 5y$

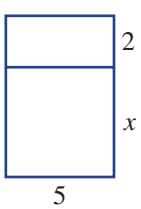
EXPLANATION

$3(x - 4) = 3x - 12$.
$-(3x - 12) = -1(3x - 12)$ so multiplying by negative 1 changes the sign of each term inside the brackets.
$-(3x + y) = -1(3x + y) = -3x - y$.
Collect like terms and simplify.

Exercise 2C

UNDERSTANDING AND FLUENCY	1–2, 3–6(½)	2–7(½)	3–7(½)

1 This diagram shows two joined rectangles with the given dimensions.



a Write an expression for the area of the:
i larger rectangle (x by 5) **ii** smaller rectangle (2 by 5)
b Use your answers from part **a** to find the combined area of both rectangles.
c Write an expression for the total side length of the side involving x.
d Use your answer from part **c** to find the combined area of both rectangles.
e Complete this statement: $5(x + 2) =$ _____ + _____

2 **a** Substitute the value $x = 5$ into these expressions.
i $-2(x + 3)$ **ii** $-2x + 6$
b Do your answers from part **a** suggest that $-2(x + 3) = -2x + 6$? When expanded what should $-2(x + 3)$ equal?
c Substitute the value $x = 10$ into these expressions.
i $-3(x - 1)$ **ii** $-3x - 3$
d Do your answers from part **c** suggest that $-3(x - 1) = -3x - 3$? When expanded what should $-3(x - 1)$ equal?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 7a, b

3 Expand the following.

a $2(x + 3)$	**b** $5(x + 12)$	**c** $2(x - 7)$	**d** $7(x - 9)$
e $3(2 + x)$	**f** $7(3 - x)$	**g** $4(7 - x)$	**h** $2(x - 6)$
i $6(2x + 1)$	**j** $4(2 - 3w)$	**k** $9(2w + 3h)$	**l** $4(3d - 2w - 1)$

Example 7c

4 Expand the following.

a $-3(x + 2)$	**b** $-2(x + 11)$	**c** $-5(x - 3)$	**d** $-6(x - 6)$
e $-4(2 - x)$	**f** $-13(5 + x)$	**g** $-20(9 + x)$	**h** $-300(1 - x)$
i $-5(2w - 3)$	**j** $-3(w + 8)$	**k** $-2(3x - 7)$	**l** $-(w - x + 9)$

Example 8

5 Expand the following.

a $2(a + b)$	**b** $5(a - 2)$	**c** $3(m - 4)$	**d** $-8(2x + 5)$
e $-3(4x + 5)$	**f** $-4x(x - 2y)$	**g** $-9t(2y - 3)$	**h** $a(3a + 4)$
i $d(2d - 5)$	**j** $-2b(3b - 5)$	**k** $2x(4x + 1)$	**l** $5y(1 - 3y)$

Example 9a

6 Expand the following and collect like terms.

a $3 + 2(x + 4)$	**b** $4 + 6(x - 3)$	**c** $2 + 5(3x - 1)$	**d** $5 + (3x - 4)$
e $3 + 4(x - 2)$	**f** $7 + 2(x - 3)$	**g** $2 - 3(x + 2)$	**h** $1 - 5(x + 4)$
i $5 - (x - 6)$	**j** $9 - (x - 3)$	**k** $5 - (3 + 2x)$	**l** $4 - (3x - 2)$

Example 9b

7 Expand the following and collect like terms.

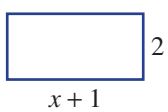
a $2(x + 3) + 3(x + 2)$	**b** $2(x - 3) + 2(x - 1)$	**c** $3(2x + 1) + 5(x - 1)$
d $4(3x + 2) + 5(x - 3)$	**e** $-3(2x + 1) + (2x - 3)$	**f** $-2(x + 2) + 3(x - 1)$
g $2(4x - 3) - 2(3x - 1)$	**h** $-3(4x + 3) - 5(3x - 1)$	**i** $-(x + 3) - 3(x + 5)$
j $-2(2x - 4) - 3(3x + 5)$	**k** $3(3x - 1) - 2(2 - x)$	**l** $-4(5 - x) - (2x - 5)$

PROBLEM-SOLVING AND REASONING 8, 9, 12 | 8–10, 12, 13(½) | 9–13

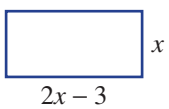
8 A rectangle's length is 4 more than its width, x. Find an expanded expression for its area.

9 Find the area of these basic shapes in expanded form.

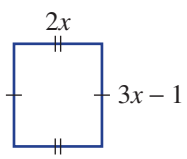
a



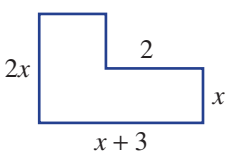
b



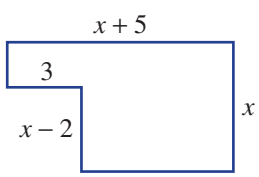
c



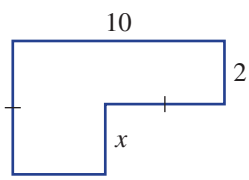
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Gary gets a bonus \$20 for every computer he sells over and above his quota of 10 per week. If he sells n computers in a week and $n > 10$, write an expression for Gary's bonus in that week (in expanded form).

11 Jill pays tax at 20c in the dollar for every dollar earned over \$10000. If Jill earns \$$x$ and $x > 10000$, write an expression for Jill's tax in expanded form.

12 Identify the errors in these expressions, then write out the correct expansion.

a $2(x + 6) = 2x + 6$

b $x(x - 4) = 2x - 4x$

c $-3(x + 4) = -3x + 12$

d $-7(x - 7) = -7x - 49$

e $5 - 2(x - 7) = 5 - 2x - 14$
$= -9 - 2x$

f $4(x - 2) - 3(x + 2) = 4x - 8 - 3x + 6$
$= x - 2$

13 In Years 7 and 8 we explored how to use the distributive law to find products mentally.

For example: $7 \times 104 = 7 \times (100 + 4)$
$= 7 \times 100 + 7 \times 4$
$= 728$

and $4 \times 298 = 4 \times (300 - 2)$
$= 4 \times 300 - 4 \times 2$
$= 1192$

Use the distributive law to evaluate these products mentally.

a 6×52 **b** 9×102 **c** 5×91 **d** 4×326

e 3×99 **f** 7×395 **g** 9×990 **h** 6×879

ENRICHMENT — — 14

Pronumerals and taxes

14 A progressive income tax system increases the tax rate for higher incomes. Here is an example.

Income	Tax
0 – \$20000	\$0
\$20001 – \$50000	\$0 + 20% of income above \$20000
\$50001 – \$100000	a + 30% of income above \$50000
\$100000 –	b + 50% of income above \$100000

a Find the values of a and b in the above table.

b Find the tax payable for these incomes.

i \$35000 **ii** \$72000 **iii** \$160000

c Find an expression for the tax payable for an income of \$$x$ if:

i $0 \leq x \leq 20000$

ii $20000 < x \leq 50000$

iii $50000 < x \leq 100000$

iv $x > 100000$

d Check that you have fully expanded and simplified your expressions for part **c**. Add steps if required.

e Use your expressions from parts **c** and **d** to check your answers to part **b** by choosing a particular income amount and checking against the table above.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2D Linear equations with pronumerals on one side

Interactive

Widgets

HOTsheets

Walkthrough

A mathematical statement containing an equals sign, a left-hand side and a right-hand side is called an equation. For example, $5 = 10 \div 2$, $3x = 9$, $x^2 + 1 = 10$ and $\frac{1}{x} = \frac{x}{5}$ are equations. Linear equations can be written in the form $ax + b = c$ where the power of x is 1. For example, $4x - 1 = 6$, $3 = 2(x + 1)$ and $\frac{5x}{3} = \frac{2x + 1}{4}$ are all linear equations. Equations are solved by finding the value of the pronumeral that makes the equation true. This can be done by inspection for very simple linear equations (for example, if $3x = 15$, then $x = 5$ since $3 \times 5 = 15$). More complex linear equations can be solved through a series of steps where each step produces an equivalent equation.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Why are they equivalent?

The following list of equations can be categorised into two groups. The equations in each group should be equivalent.

$5x = 20$ $\quad 2x - 1 = -3$ $\quad x = 4$ $\quad 1 - x = -3$

$7x = -7$ $\quad 3 - 5x = -17$ $\quad \frac{8x}{5} - \frac{3x}{5} = -1$ $\quad x = -1$

- Discuss how you divided the equations into the two groups.
- How can you check to see if the equations in each group are equivalent?

Key ideas

- **Equivalent equations** are created by:
  - adding a number to or subtracting a number from both sides of the equation
  - multiplying or dividing both sides of the equation by the same number (not including 0)
  - swapping the left-hand side (LHS) with the right-hand side (RHS).
- Solve a linear equation by creating simpler equivalent equations.
- The solution to an equation can be checked by substituting the solution into the original equation and checking that both sides are equal.
- Equations involving algebraic fractions are introduced in this section and extended in Chapter 8.

Example 10 Solving simple linear equations

Solve each of the following equations.

a $2x + 3 = 4$ **b** $\frac{x}{4} - 3 = 7$ **c** $5 - 2x = 12$

d $\frac{2x}{3} + 5 = 7$ **e** $\frac{2x + 4}{9} = 2$ **f** $10 = 5 + \frac{2x}{3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
a $2x + 3 = 4$	
$2x = 1$	Subtract 3 from both sides.
$x = \frac{1}{2}$	Divide both sides by 2.
Check: LHS $= 2 \times \left(\frac{1}{2}\right) + 3 = 4 =$ RHS	Check the answer by substituting $x = \frac{1}{2}$ into the original equation.
b $\frac{x}{4} - 3 = 7$	
$\frac{x}{4} = 10$	Add 3 to both sides.
$x = 40$	Multiply both sides by 4.
Check: LHS $= \frac{(40)}{4} - 3 = 10 - 3 = 7 =$ RHS	Check the solution by substituting $x = 40$ into $\frac{x}{4} - 3$. Since this equals 7, $x = 40$ is the solution.
c $5 - 2x = 12$	
$-2x = 7$	Subtract 5 from both sides.
$x = \frac{-7}{2}$ or $-3\frac{1}{2}$	Divide both sides by -2.
Check: LHS $= 5 - 2 \times \left(\frac{-7}{2}\right) = 5 + 7 = 12 =$ RHS	Check the solution.
d $\frac{2x}{3} + 5 = 7$	
$\frac{2x}{3} = 2$	Subtract 5 from both sides first then multiply both sides by 3.
$2x = 6$	Divide both sides by 2.
$x = 3$	Check the solution.
Check: LHS $= \frac{2 \times (3)}{3} + 5 = 2 + 5 = 7 =$ RHS	
e $\frac{2x + 4}{9} = 2$	Multiply both sides by 9 first to eliminate the fraction. Solve the remaining equation by subtracting 4 from both sides and then dividing both sides by 2.
$2x + 4 = 18$	
$2x = 14$	
$x = 7$	Check the solution.
Check: LHS $= \frac{2 \times (7) + 4}{9} = \frac{18}{9} = 2 =$ RHS	
f $10 = 5 + \frac{2x}{3}$	
$5 + \frac{2x}{3} = 10$	Swap LHS and RHS.
$\frac{2x}{3} = 5$	Subtract 5 from both sides.
$2x = 15$	Multiply both sides by 3.
$x = 7\frac{1}{2}$	Divide both sides by 2.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 2D

UNDERSTANDING AND FLUENCY	1–2(½), 3, 4–8(½)	3, 4–8(½)	4–8(½)

1 Solve these equations by inspection.

a $x + 2 = 6$ **b** $x - 2 = 6$ **c** $6 - x = 2$ **d** $2 - x = 6$

e $2x = 6$ **f** $6x = 2$ **g** $\frac{x}{6} = 2$ **h** $\frac{6}{x} = 2$

2 Solve the following equations.

a $2x + 1 = 7$ **b** $11x - 1 = 21$ **c** $4 - x = 2$ **d** $\frac{x}{3} + 1 = 5$

e $2 + \frac{x}{3} = 6$ **f** $3x - 1 = -16$ **g** $1 = \frac{x + 1}{7}$ **h** $1 = \frac{3x + 2}{5}$

3 Which of the following equations are equivalent to $3x = 12$?

A $3x + 1 = 13$ **B** $12 = 3x - 1$ **C** $12x = 12$ **D** $-3x = -12$

E $3 = \frac{3x}{4}$ **F** $x = 4$ **G** $\frac{3x}{5} = 10$ **H** $3x - x = 12 - x$

Example 10a

4 Solve each of the following equations. Check your answers.

a $2x + 5 = 9$ **b** $5a + 6 = 11$ **c** $8 = 3m - 4$

d $2x - 4 = -6$ **e** $2n + 13 = 7$ **f** $2x + 5 = -7$

g $2b + 15 = 7$ **h** $3y - 2 = -13$ **i** $7 = 3a + 2$

j $4b + 7 = 25$ **k** $24x - 2 = 10$ **l** $6x - 5 = 3$

m $7y - 3 = -8$ **n** $2a + \frac{1}{2} = \frac{1}{4}$ **o** $-1 = 5n - \frac{1}{4}$

Example 10b

5 Solve each of the following equations.

a $\frac{x}{4} + 3 = 5$ **b** $\frac{x}{2} + 4 = 5$ **c** $\frac{b}{3} + 5 = 9$ **d** $\frac{t}{2} + 5 = 2$

e $\frac{a}{3} + 4 = 2$ **f** $\frac{y}{5} - 4 = 2$ **g** $\frac{x}{3} - 7 = -12$ **h** $\frac{s}{2} - 3 = -7$

i $\frac{x}{4} - 5 = -2$ **j** $\frac{m}{4} - 2 = 3$ **k** $1 - \frac{y}{5} = 2$ **l** $2 - \frac{x}{5} = 4$

Example 10c

6 Solve each of the following equations.

a $12 - 2x = 18$ **b** $9 = 2 - 7x$ **c** $15 - 5x = 5$ **d** $-13 = 3 - 2x$

e $2 - 5x = 9$ **f** $23 = 4 - 7x$ **g** $5 - 8x = 2$ **h** $-10 = -3 - 4x$

Example 10d, f

7 Solve these equations.

a $\frac{2b}{3} = 6$ **b** $\frac{3x}{2} = 9$ **c** $\frac{4x}{3} = -9$ **d** $\frac{2x}{5} = -3$

e $\frac{3x}{4} = \frac{1}{2}$ **f** $\frac{5n}{4} = -\frac{1}{5}$ **g** $\frac{2x}{3} - 1 = 7$ **h** $\frac{3x}{4} - 2 = 7$

i $3 + \frac{2x}{3} = -3$ **j** $5 + \frac{3d}{2} = -7$ **k** $11 - \frac{3f}{2} = 2$ **l** $3 - \frac{4z}{3} = 5$

m $3 + \frac{x}{2} = 0$ **n** $\frac{3x}{2} + 1 = 1$ **o** $4 = \frac{2p}{5}$ **p** $4 = \frac{2p}{5} + 4$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 10e

8 Solve each of the following equations. Check your answers.

a $\frac{x+1}{3} = 4$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{4+y}{3} = -2$ **d** $\frac{6+b}{2} = -3$

e $\frac{1-a}{2} = 3$ **f** $\frac{5-x}{3} = 2$ **g** $\frac{3m-1}{5} = 4$ **h** $\frac{2x+2}{3} = 4$

i $\frac{7x-3}{3} = 9$ **j** $5 = \frac{3b-6}{2}$ **k** $\frac{4-2y}{6} = 3$ **l** $-2 = \frac{9-5t}{3}$

m $\frac{3+x}{2} = 0$ **n** $\frac{4-w}{4} = 4$ **o** $\frac{4-w}{4} = 0$ **p** $\frac{4-w}{4} = 1$

PROBLEM-SOLVING AND REASONING	9, 10	9, 10	9(½), 10, 11

9 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

a If 8 is added to a certain number, the result is 34.

b Seven less than a certain number is 21.

c I think of a number, double it and add 4. The result is 10.

d I think of a number, halve it and subtract 4. The result is 10.

e Four less than three times a number is 20.

f A number is multiplied by 7 and the product is divided by 3. The final result is 8.

g Five Easter eggs are added to my initial collection of Easter eggs. I share them between myself and two friends and each person gets exactly four. How many were there initially?

h My weekly pay is increased by $200 per week. Half of my pay now goes to pay the rent and $100 to buy groceries. If this leaves me with $450, what is my original weekly pay?

10 Describe the error made in each of these incorrect solutions.

a
$$\begin{aligned} 2x - 1 &= 4 \\ x - 1 &= 2 \\ x &= 3 \end{aligned}$$

b
$$\begin{aligned} \frac{5x+2}{3} &= 7 \\ \frac{5x}{3} &= 5 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

c
$$\begin{aligned} 5 - x &= 12 \\ x &= 7 \end{aligned}$$

d
$$\begin{aligned} \frac{x}{3} - 4 &= 2 \\ x - 4 &= 6 \\ x &= 10 \end{aligned}$$

11 An equation like $2(x + 3) = 8$ can be solved without expanding the brackets. The first step is to divide both sides by 2.

a Use this approach to solve these equations.

i $3(x - 1) = 12$ **ii** $4(x + 2) = -4$ **iii** $7(5x + 1) = 14$

iv $5(1 - x) = -10$ **v** $-2(3x + 1) = 3$ **vi** $-5(1 - 4x) = 1$

b By considering your solutions to the equations in part **a**, when do you think this method is most appropriate?

ENRICHMENT	—	—	12

Changing the subject

12 Make a the subject of each of the following equations.

a $a - b = c$ **b** $2a + b = c$ **c** $c - ab = 2d$ **d** $b = \frac{a}{c} - d$

e $\frac{ab}{c} = -d$ **f** $\frac{2a}{b} = \frac{1}{c}$ **g** $\frac{2ab}{c} - 3 = -d$ **h** $b - \frac{ac}{2d} = 3$

i $\frac{a+b}{c} = d$ **j** $\frac{b-a}{c} = -d$ **k** $\frac{ad-6c}{2b} = e$ **l** $\frac{d-4ac}{e} = 3f$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2E Linear equations with brackets and pronumerals on both sides

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

More complex linear equations may have pronumerals on both sides of the equation and/or brackets. Examples are $3x = 5x - 1$ and $4(x + 2) = 5x$. Brackets can be removed by expanding, and equations with pronumerals on both sides can be solved by collecting like terms using addition and subtraction.

Let's start: Steps in the wrong order

The steps to solve $3(2 - x) = -2(2x - 1)$ are listed here in the incorrect order.

$3(2 - x) = -2(2x - 1)$
$x = -4$
$6 + x = 2$
$6 - 3x = -4x + 2$

- Arrange them in the correct order working from top to bottom.
- By considering all the steps in the correct order, explain what has happened in each step.

Key ideas

- Equations with brackets can be solved by first expanding the brackets.
  For example: $3(x + 1) = 2$ becomes $3x + 3 = 2$.
- If an equation has pronumerals on both sides, collect to one side by adding or subtracting one of the terms.
  For example: $3x + 4 = 2x - 3$ becomes $x + 4 = -3$ by subtracting $2x$ from both sides.

Example 11 Solving equations with brackets and pronumerals on both sides

Solve each of the following equations.

a $2(3x - 4) = 11$
b $2(x + 3) - 4x = 8$
c $5x - 2 = 3x - 4$
d $3(2x + 4) = 8(x + 1)$

SOLUTION

a $2(3x - 4) = 11$
$6x - 8 = 11$
$6x = 19$
$x = \frac{19}{6}$ or $3\frac{1}{6}$

EXPLANATION

Expand the brackets,
$2(3x - 4) = 2 \times 3x + 2 \times (-4)$.

Add 8 to both sides, then divide both sides by 6, leaving your answer in fraction form.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
b $2(x + 3) - 4x = 8$	
$2x + 6 - 4x = 8$	Expand the brackets and collect any like terms,
$-2x + 6 = 8$	i.e. $2x - 4x = -2x$.
$-2x = 2$	Subtract 6 from both sides.
$x = -1$	Divide by -2.
c $5x - 2 = 3x - 4$	
$2x - 2 = -4$	Collect x terms on one side by subtracting $3x$ from both sides.
$2x = -2$	
$x = -1$	Add 2 to both sides and then divide both sides by 2.
d $3(2x + 4) = 8(x + 1)$	Expand the brackets on each side.
$6x + 12 = 8x + 8$	Subtract $6x$ from both sides. Alternatively subtract $8x$ to end up with $-2x + 12 = 8$. (Subtracting $6x$ keeps the x-coefficient positive.)
$12 = 2x + 8$	
$4 = 2x$	
$2 = x$	
$\therefore x = 2$	Solve the equation and make x the subject.

Exercise 2E

UNDERSTANDING AND FLUENCY	1, 2, 3–5(½)	2, 3–6(½)	3–6(½)

1 Expand these expressions and simplify.

a $3(x - 4) + x$
b $2(1 - x) + 2x$
c $3(x - 1) + 2(x - 3)$
d $5(1 - 2x) - 2 + x$
e $2 - 3(3 - x)$
f $7(2 - x) - 5(x - 2)$

2 Show the next step only for the given equations and instructions.

a $2(x + 3) = 5$ (expand the brackets)
b $5 + 2(x - 1) = 7$ (expand the brackets)
c $3x + 1 = x - 6$ (subtract x from both sides)
d $4x - 3 = 2x + 1$ (subtract $2x$ from both sides)

Example 11a

3 Solve each of the following equations by first expanding the brackets.

a $2(x + 3) = 11$
b $5(a + 3) = 8$
c $3(m + 4) = 31$
d $5(y - 7) = -12$
e $4(p - 5) = -35$
f $2(k - 5) = 9$
g $4(5 - b) = 18$
h $2(1 - m) = 13$
i $5(3 - x) = 19$
j $7(2a + 1) = 8$
k $4(3x - 2) = 30$
l $3(3n - 2) = 0$
m $5(3 - 2x) = 6$
n $6(1 - 2y) = -8$
o $4(3 - 2a) = 13$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11b

4 Expand and simplify, then solve each of the following equations.

a $(x + 4) + 2x = 7$
b $2(x - 3) - 3x = 4$
c $4(x - 1) + x - 1 = 0$
d $3(2x + 3) - 1 - 4x = 4$
e $3(x - 4) + 2(x + 1) = 15$
f $2(x + 1) - 3(x - 2) = 8$
g $6(x + 3) + 2x = 26$
h $3(x + 2) + 5x = 46$
i $3(2x - 3) + x = 12$
j $4(3x + 1) + 3x = 19$

Example 11c

5 Solve each of the following equations.

a $5b = 4b + 1$
b $8a = 7a - 4$
c $4t = 10 - t$
d $3m - 8 = 2m$
e $5x - 3 = 4x + 5$
f $9a + 3 = 8a + 6$
g $12x - 3 = 10x + 5$
h $3y + 6 = 2 - y$
i $5m - 4 = 1 - 6m$

Example 11d

6 Solve each of the following equations.

a $5(x - 2) = 2x - 13$
b $3(a + 1) = a + 10$
c $3(y + 4) = y - 6$
d $2(x + 5) = x - 4$
e $5b - 4 = 6(b + 2)$
f $2(4m - 5) = 4m + 2$
g $3(2a - 3) = 5(a + 2)$
h $4(x - 3) = 3(3x + 1)$
i $3(x - 2) = 5(x + 4)$
j $3(n - 2) = 4(n + 5)$
k $2(a + 5) = -2(2a + 3)$
l $-4(x + 2) = 3(2x + 1)$

PROBLEM-SOLVING AND REASONING	7, 8, 10, 11	7, 8, 10, 11	8, 9, 10–12

7 Using x for the unknown number, write down an equation and then solve it to find the number.

a The product of 2 and 3 more than a number is 7.
b The product of 3 and 4 less than a number is -4.
c When 2 less than 3 lots of a number is doubled the result is 5.
d When 5 more than 2 lots of a number is tripled the result is 10.
e 2 more than 3 lots of a number is equivalent to 8 lots of the number.
f 2 more than 3 times the number is equivalent to 1 less than 5 times the number.
g 1 less than a doubled number is equivalent to 5 more than 3 lots of the number.

8 Since Tara started work her original hourly wage has been tripled, then decreased by $6. It is now to be doubled so that she gets $18 an hour. What was her original hourly wage?

9 At the start of lunch, Jimmy and Jake each brought out a new bag of x marbles to play with their friends. By the end of lunch they were surprised to see they had an equal number of marbles, even though overall Jimmy had gained 5 marbles and Jake had ended up with double the result of 3 less than his original amount. How many marbles were originally in each bag?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Consider the equation $3(x - 2) = 9$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

11 Consider the equation $3(x - 2) = 7$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

12 Consider the equation $3x + 1 = 5x - 7$.

- **a** Solve the equation by first subtracting $3x$ from both sides.
- **b** Solve the equation by first subtracting $5x$ from both sides.
- **c** Which method above do you prefer and why? Describe the differences.

ENRICHMENT — — 13

Literal solutions with factorisation

13 Literal equations contain a pronumeral (such as x) and other pronumerals such as a, b and c. To solve such an equation for x, factorisation can be used as shown here.

$$ax = bx + c$$
$$ax - bx = c \quad \text{Subtract } bx \text{ from both sides}$$
$$x(a - b) = c \quad \text{Factorise by taking out } x$$
$$x = \frac{c}{a - b} \quad \text{Divide both sides by } (a - b)$$

Solve each of the following for x in terms of the other pronumerals by using factorisation.

- **a** $ax = bx + d$
- **b** $ax + 1 = bx + 3$
- **c** $5ax = bx + c$
- **d** $3ax + 1 = 4bx - 5$
- **e** $ax - bc = xb - ac$
- **f** $a(x - b) = x - b$
- **g** $ax - bx - c = d + bd$
- **h** $a(x + b) = b(x - c) - x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2F Using linear equations to solve problems

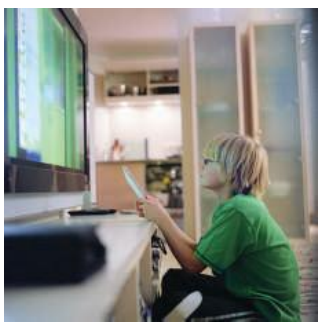
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Many types of problems can be solved by writing and solving linear equations. Often problems are expressed only in words. Reading and understanding the problem, defining a pronumeral and writing an equation become important steps in solving the problem.

Interactive
Widgets
HOTsheets
Walkthrough

Let's start: Too much television?

Three friends, Rick, Kate and Sue, compare how much television they watch in a week at home. Kate watches 3 times the amount of television of Rick, and Sue watches 4 hours less television than Kate. In total they watch 45 hours of television. Find the number of hours of television watched by Rick.



- Let x hours be the number of hours of television watched by Rick.
- Write expressions for the number of hours of television watched by Kate and by Sue.
- Write an equation to represent the information above.
- Solve the equation.
- Answer the question in the original problem.

Key ideas

- To solve a **word problem** using algebra:
  - **Read** the problem and find out what the question is asking for.
  - **Define** a pronumeral and write a statement such as: 'Let x be the number of …' The pronumeral is often what you have been asked to find.
  - **Write** an equation using your defined pronumeral.
  - **Solve** the equation.
  - **Answer** the question in words.
  - **Check** that the solution makes sense.

Example 12 Turning a word problem into an equation

Five less than a certain number is 9 less than three times the number. Write an equation and solve it to find the number.

SOLUTION

Let x be the number.
$x - 5 = 3x - 9$
$-5 = 2x - 9$
$4 = 2x$
$x = 2$

The number is 2.

EXPLANATION

Define the unknown as a pronumeral.
5 less than x is $x - 5$ and this equals 9 less than three times x, i.e. $3x - 9$.
Subtract x from both sides and solve the equation.

Write the answer in words.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 13 Solving word problems

David and Mitch made 254 runs between them in a cricket match. If Mitch made 68 more runs than David, how many runs did each of them make?

SOLUTION	EXPLANATION
Let the number of runs for David be r.	Define the unknown value as a pronumeral.
Number of runs Mitch made is $r + 68$.	Write all other unknown values in terms of r.
$r + (r + 68) = 254$ $2r + 68 = 254$ $2r = 186$ $r = 93$	Write an equation: number of runs for David + number of runs for Mitch = 254 Subtract 68 from both sides and then divide both sides by 2.
David made 93 runs and Mitch made $93 + 68 = 161$ runs.	Express the answer in words.

Exercise 2F

UNDERSTANDING AND FLUENCY	1–6	1(½), 2–7	2, 4, 5, 7–9

Example 12

1 For each of the following examples, make x the unknown number and write an equation.
- **a** Three less than a certain number is 9 less than four times the number.
- **b** Seven is added to a number and the result is then multiplied by 3. The result is 9.
- **c** I think of a number, take away 9, then multiply the result by 4. This gives an answer of 12.
- **d** A number when doubled results in a number that is 5 more than the number itself.
- **e** Eight less than a certain number is 2 more than three times the number.

Example 13

2 Leonie and Emma scored 28 goals between them in a netball match. Leonie scored 8 more goals than Emma.
- **a** Define a pronumeral for the number of goals scored by Emma.
- **b** Write the number of goals scored by Leonie in terms of the pronumeral in part **a**.
- **c** Write an equation in terms of your pronumeral to represent the problem.
- **d** Solve the equation in part **c** to find the unknown value.
- **e** How many goals did each of them score?

3 A rectangle is four times as long as it is wide and its perimeter is 560 cm.
- **a** Define a pronumeral for the unknown width.
- **b** Write an expression for the length in terms of your pronumeral in part **a**.
- **c** Write an equation involving your pronumeral to represent the problem.
- **d** Solve the equation in part **c**.
- **e** What is the length and width of the rectangle?

4 Toby rented a car for a total cost of $290. If the rental company charged $40 per day, plus a hiring fee of $50, for how many days did Toby rent the car?

5 Andrew walked a certain distance, and then ran twice as far as he walked. He then caught a bus for the last 2 km. If he travelled a total of 32 km, find how far Andrew walked and ran.

6 A prize of $1000 is divided between Adele and Benita so that Adele receives $280 more than Benita. How much did each person receive?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Kate is three times as old as her son. If Kate is 30 years older than her son, what are their ages?

8 A train station is between the towns Antville and Bugville. The station is four times as far from Bugville as it is from Antville. If the distance from Antville to Bugville is 95 km, how far is it from Antville to the station?

9 Andrew, Brenda and Cammi all work part time at a supermarket. Cammi earns $20 more than Andrew and Brenda earns $30 less than twice Andrew's wage. If their total combined wage is $400, find how much each of these workers earns.

PROBLEM-SOLVING AND REASONING

10, 11, 13, 16 | 10, 12, 13, 16, 17 | 10, 12–15, 17, 18

10 Macy bought a total of 12 fiction and non-fiction books. The fiction books cost $12 each and the non-fiction books cost $25 each. If she paid $248 altogether, how many of each kind of book did she purchase?

11 If I multiply my age in six years' time by three, the resulting age is my mother's age now. If my mother is currently 48 years old, how old am I?

12 Twelve years ago Eric's father was seven times as old as Eric was. If Eric's father is now 54 years old, how old is Eric now?

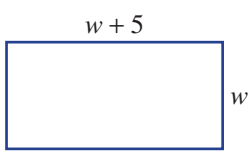
13 In a yacht race the second leg was half the length of the first leg, the third leg was two thirds of the length of the second leg, and the last leg was twice the length of the second leg. If the total distance was 153 km, find the length of each leg.

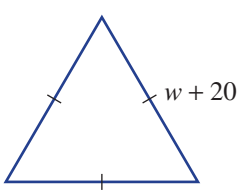
14 The Ace Bicycle Shop charges a flat fee of $4, plus $1 per hour, for the hire of a bicycle. The Best Bicycle Shop charges a flat fee of $8, plus 50 cents per hour. Connie and her friends hire three bicycles from Ace, and David and his brother hire two bicycles from Best. After how many hours will their hire costs be the same?



15 Car A left Melbourne for Adelaide at 11 a.m. and travelled at an average speed of 70 km per hour.
Car B left Melbourne for Adelaide at 1 p.m. on the same day and travelled at an average speed of 90 km per hour. At what time will car B catch up to car A?

16 Two paddocks in the shapes shown below are to be fenced with wire. If the same total amount of wire is used for each paddock, what are the side lengths of each paddock in metres?





ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

17 Consecutive integers can be represented algebraically as x, $x + 1$, $x + 2$ etc.

- **a** Find three consecutive numbers that add to 84.
- **b** **i** Write three consecutive even numbers starting with x.
  - **ii** Find three consecutive even numbers that add to 18.
- **c** **i** Write three consecutive odd numbers starting with x.
  - **ii** Find three consecutive odd numbers that add to 51.
- **d** **i** Write three consecutive multiples of 3 starting with x.
  - **ii** Find three consecutive multiples of 3 that add to 81.

18 Tedco produces a teddy bear that sells for $24. Each teddy bear costs the company $8 to manufacture and there is an initial start-up cost of $7200.

- **a** Write a rule for the total cost, $\$T$, of producing x teddy bears.
- **b** If the cost of a particular production run was $9600, how many teddy bears were manufactured in that run?
- **c** If x teddy bears are sold, write a rule for the revenue, $\$R$, received by the company.
- **d** How many teddy bears were sold if the revenue was $8400?
- **e** If they want to make an annual profit of $54 000, how many teddy bears do they need to sell?

ENRICHMENT — — 19–21

Worded challenges

19 An art curator was investigating the price trends of two art works that had the same initial value. The first painting, 'Green poles', doubled in value in the first year and then lost $8000 in the second year. In the third year its value was three quarters of the previous year.
The second painting, 'Orchids', added $10 000 to its value in the first year then the second year its value was only a third of the previous year. In the third year its value improved to double that of the previous year.
If the value of the paintings was the same in the third year, write an equation and solve it to find the initial value of each painting.

20 Julia drives to her holiday destination over a period of five days. On the first day she travels a certain distance, on the second day she travels half that distance, on the third day a third of that distance, on the fourth day one quarter of the distance and on the fifth day one fifth of the distance. If her destination is 1000 km away, write an equation and solve it to find how far she travels on the first day to the nearest kilometre.



21 Anna King is x years old. Her brother Henry is two thirds of her age and her sister Chloe is three times Henry's age. The twins who live next door are 5 years older than Anna. If the sum of the ages of the King children is equal to the sum of the ages of the twins, find the ages of all the children.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2G Linear inequalities

Stage: 5.3# 5.3 5.3§ 5.2 5.2◊ 5.1 4

Interactive

Widgets

HOTsheets

Walkthrough

An inequality (or inequation) is a mathematical statement that uses a $<$, $\leq$, $>$ or $\geq$ sign. Some examples of inequalities include:

$2 < 6$, $5 \geq -1$, $3x + 1 \leq 7$ and $2x + \frac{1}{3} > \frac{x}{4}$.

Inequalities can represent an infinite set of numbers. For example, the inequality $2x < 6$ means that $x < 3$ and this is the infinite set of all real numbers less than 3.

Let's start: Infinite solutions

Greg, Kevin and Greta think that they all have a correct solution for:

$4x - 1 \geq x + 6$

Greg says $x = 4$ is a solution.

Kevin says $x = 10$ is a solution.

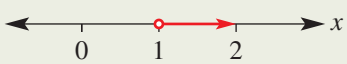
Greta says $x = 100$ is a solution.

- Use substitution to show that they are all correct.
- Can you find the smallest whole number that is a solution to the inequality?
- Can you find the smallest number (including fractions) that satisfies the inequality? What method leads you to your answer?

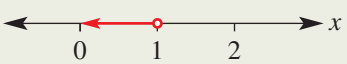
Key ideas

- **Inequalities** can be illustrated using a number line because a line represents an infinite number of points.
  - Use an **open circle** when showing $>$ (greater than) or $<$ (less than).

    For example: $x > 1$

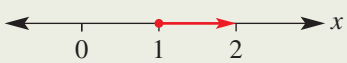
    

    $x < 1$

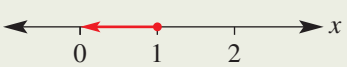
    

  - Use a **closed circle** when showing $\geq$ (greater than or equal to) or $\leq$ (less than or equal to).

    For example: $x \geq 1$

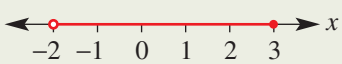
    

    $x \leq 1$

- A **set** may have both an upper and lower bound.

  For example: $-2 < x \leq 3$

- Linear inequalities can be solved in a similar way to linear equations.
  - If, we multiply or divide both sides of an inequality by a negative number, the inequality sign is reversed.

    For example: $5 < 8$ but $-5 > -8$ so if $-x > 1$ then $x < -1$.
  - If we swap the sides of an inequality, then the inequality sign is reversed.

    For example: $3 < 7$ but $7 > 3$ so if $2 > x$ then $x < 2$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14 Representing inequalities on a number line

Show each of the following examples on a number line.

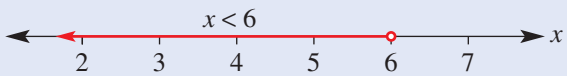
a x is less than 6 ($x < 6$).

b x is greater than or equal to 2 ($x \geq 2$).

c x is greater than -2 but less than or equal to 3 ($-2 < x \leq 3$).

SOLUTION

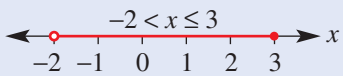
a



b



c



EXPLANATION

An open circle is used to indicate that 6 is not included.

A closed circle is used to indicate that 2 is included.

An open circle is used to indicate that -2 is not included and a closed circle is used to indicate that 3 is included.

Example 15 Solving inequalities

Find the solution set for each of the following inequalities.

a $x - 3 < 7$ **b** $5 - 2x > 3$ **c** $\frac{d}{4} - 3 \geq -11$ **d** $2a + 7 \leq 6a + 3$ **e** $6 \leq 2x + 9$

SOLUTION	EXPLANATION
a $x - 3 < 7$	
$x < 10$	Add 3 to both sides
b $5 - 2x > 3$	Subtract 5 from both sides.
$-2x > -2$	Divide both sides by -2, and reverse the inequality sign.
$x < 1$	
c $\frac{d}{4} - 3 \geq -11$	Add 3 to both sides.
$\frac{d}{4} \geq -8$	Multiply both sides by 4; the inequality sign does not change.
$d \geq -32$	
d $2a + 7 \leq 6a + 3$	Gather pronumerals on one side by subtracting $2a$ from both sides.
$7 \leq 4a + 3$	
$4 \leq 4a$	Subtract 3 from both sides and then divide both sides by 4.
$1 \leq a$	Place the pronumeral a on the left and reverse the inequality.
$a \geq 1$	
e $6 \leq 2x + 9$	Swap LHS and RHS and reverse the inequality sign.
$2x + 9 \geq 6$	Subtract 9 from both sides and divide both sides by 2.
$2x \geq -3$	
$x \geq -\frac{3}{2}$	

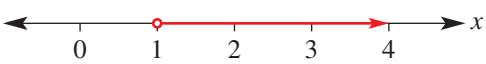
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 2G

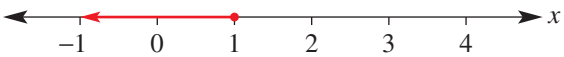
UNDERSTANDING AND FLUENCY 1–6 2–7(½) 3–7(½)

1 Match the graph with the statement.

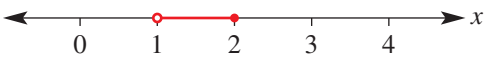
a



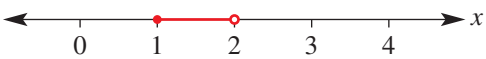
b



c



d



A $x \leq 1$

B $1 \leq x < 2$

C $1 < x \leq 2$

D $x > 1$

Example 14

2 Show each of the following inequalities on a number line.

a $x > 2$ **b** $x \geq 1$ **c** $x \leq 4$ **d** $x < -2$

e $x \geq -1$ **f** $x < -8$ **g** $-1 < x < 1$ **h** $-2 \leq x \leq 2$

i $0 \leq x \leq 3$ **j** $2 \leq x \leq 5$ **k** $-3 < x \leq 4$ **l** $-6 \leq x < -2$

Example 15a

3 Find the solution set for each of the following inequalities.

a $x + 5 < 8$ **b** $b - 2 > 3$ **c** $y - 8 > -2$

d $-12 + m < -7$ **e** $5x \geq 15$ **f** $4t > -20$

g $\frac{x}{3} \geq 4$ **h** $y + 10 \geq 0$ **i** $3m - 7 < 11$

j $12 \leq 6 + 4a$ **k** $2 > 7x - 5$ **l** $9 < 2x - 7$

Example 15b

4 Find the solution set for each of the following inequalities.

a $4 - 3x > -8$ **b** $2 - 4n \geq 6$ **c** $4 - 5x \leq 1$

d $7 - a \leq 3$ **e** $5 - x \leq 11$ **f** $7 - x \leq -3$

g $-2x - 3 > 9$ **h** $-4t + 2 \geq 10$ **i** $-6m - 14 < 15$

Example 15c

5 Find the solution set for each of the following inequalities.

a $\frac{x}{2} - 5 \leq 3$ **b** $3 - \frac{x}{9} \geq 4$ **c** $8 \geq \frac{2x}{5}$

d $\frac{2x + 6}{7} < 4$ **e** $\frac{3x - 4}{2} > -6$ **f** $3 \geq \frac{1 - 7x}{5}$

6 Solve each of the following inequalities.

a $4(x + 2) < 12$ **b** $-3(a + 5) > 9$ **c** $25 \leq 5(3 - x)$

d $2(3 - x) > 1$ **e** $5(y + 2) < -6$ **f** $-11 > -7(1 - x)$

Example 15d

7 Find the solution set for each of the following inequalities.

a $2x + 9 \leq 6x - 1$ **b** $6t + 2 > t - 1$ **c** $7y + 4 \leq 7 - y$

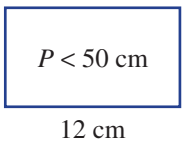
d $3a - 2 < 4 - 2a$ **e** $1 - 3m \geq 7 - 4m$ **f** $7 - 5b > -4 - 3b$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING 8–10, 13 | 9–11, 13, 14 | 10–12, 14, 15

8 Wendy is x years old and Jay is 6 years younger. The sum of their ages is less than 30. Write an inequality involving x and solve it. What can you say about Wendy's age?

9 The perimeter of a particular rectangle needs to be less than 50 cm. If the length of the rectangle is 12 cm and the breadth is b cm, write an inequality involving b and solve it. What breadth does the rectangle need to be?



10 How many integers satisfy both of the given equations?

a $2x + 1 \leq 5$ and $5 - 2x \leq 5$

b $7 - 3x > 10$ and $5x + 13 > -5$

c $\frac{x+1}{3} \geq -2$ and $2 - \frac{x}{3} > 3$

d $\frac{5x+1}{6} < 2$ and $\frac{x}{3} < 2x - 7$

11 The breadth of a rectangle is 10 m and the height is $(2x - 4)$ m. If the area is less than 80 m^2, what are the possible whole number values for x?

12 Two car rental companies have the following payment plans:
Carz: \$90 per week and 15c per kilometre
Renta: \$110 per week and 10c per kilometre
What is the maximum whole number of kilometres that can be travelled in one week with Carz if it is to cost less than it would with Renta?

13 **a** Consider the inequality $2 > x$.

i List five values of x between -1 and 2 which make the inequality true.

ii What must be true about all the values of x if the inequality is true?

b Consider the inequality $-x < 5$.

i List five values of x which make the inequality true.

ii What must be true about all the values of x if the inequality is true?

c Complete these statements.

i If $a > x$, then x __ __.

ii If $-x < a$, then x __ __.

14 Consider the equation $9 - 2x > 3$.

a Solve the equation by first adding $2x$ to both sides then solve for x.

b Solve the equation by first subtracting 9 from both sides.

c What did you have to remember to do in part **b** to ensure that the answer is the same as in part **a**?

15 Combine all your knowledge from this chapter so far to solve these inequalities.

a $\frac{2(x+1)}{3} > x + 5$

b $2x + 3 \geq \frac{x-6}{3}$

c $\frac{2-3x}{2} < 2x - 1$

d $\frac{4(2x-1)}{3} \leq x + 3$

e $1 - x > \frac{7(2-3x)}{4}$

f $2(3 - 2x) \leq 4x$

ENRICHMENT — | — | 16

Literal inequalities

16 Given a, b, c and d are positive numbers (such that $1 < a < b$), solve each of the following for x.

a $ax - b > -c$

b $b - x \leq a$

c $\frac{x}{a} - b \leq c$

d $a \geq \frac{bx}{c}$

e $\frac{ax+b}{c} < d$

f $\frac{b-2x}{c} \leq d$

g $a(x + b) < c$

h $\frac{ax-b}{c} > -d$

i $a(b - x) > c$

j $ax + b \leq x - c$

k $ax + b > bx - 1$

l $b - ax \leq c - bx$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2H Using formulas

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

A formula (or rule) is an equation that relates two or more pronumerals. You can find the value of one of the pronumerals if you are given the value of all other unknowns. Some common formulas contain squares, square roots, cubes and cube roots. The following are some examples of formulas.

- $A = \pi r^2$ is the formula for finding the area, A, of a circle given its radius, r.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.
- $d = vt$ is the formula for finding the distance, d, given the velocity, v, and time, t.

A, F and d are said to be the subjects of the formulas given above.

Let's start: Common formulas

As a class group, try to list at least 10 formulas that you know.

- Write down the formulas and describe what each pronumeral represents.
- Which pronumeral is the subject of each formula?

Key ideas

- The **subject** of a **formula** is a pronumeral that usually sits on its own on the left-hand side. For example: the C in $C = 2\pi r$ is the subject of the formula.
- A pronumeral in a formula can be evaluated by substituting numbers for all other pronumerals.
- A formula can be **transposed** (rearranged) to make another pronumeral the subject.

  $C = 2\pi r$ can be transposed to give $r = \frac{C}{2\pi}$.

Example 16 Substituting values into formulas

Substitute the given values into the formula to evaluate the subject.

a $S = \frac{a}{1 - r}$, when $a = 3$ and $r = 0.4$

b $E = \frac{1}{2}mv^2$, when $m = 4$ and $v = 5$

SOLUTION

a $S = \frac{a}{1 - r}$

$S = \frac{3}{1 - 0.4}$

$= \frac{3}{0.6}$

$= 5$

EXPLANATION

Substitute $a = 3$ and $r = 0.4$ and evaluate.

b $E = \frac{1}{2}mv^2$

$E = \frac{1}{2} \times 4 \times 5^2$

$= \frac{1}{2} \times 4 \times 25$

$= 50$

Substitute $m = 4$ and $v = 5$ and evaluate.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 17 Finding the unknown value in a formula

The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$. Substitute $A = 12$, $a = 5$ and $h = 4$, then find the value of b.

SOLUTION

$A = \frac{1}{2}h(a + b)$
$12 = \frac{1}{2} \times 4 \times (5 + b)$
$12 = 2(5 + b)$
$6 = 5 + b$
$b = 1$

EXPLANATION

Write the formula and substitute the given values of A, a and h. Then solve for b.

Example 18 Transposing formulas

Transpose each of the following to make b the subject.

a $c = a(x + b)$ **b** $c = \sqrt{a^2 + b^2}$ $(b > 0)$

SOLUTION

a

$c = a(x + b)$ — Treat pronumerals as you would numbers.
$\frac{c}{a} = x + b$ — Divide both sides by a.
$\frac{c}{a} - x = b$ — Subtract x from both sides.
$b = \frac{c}{a} - x \left(\text{or } b = \frac{c - ax}{a}\right)$ — Make b the subject on the left-hand side.

b

$c = \sqrt{a^2 + b^2}$
$c^2 = a^2 + b^2$ — Square both sides to remove the square root.
$c^2 - a^2 = b^2$ — Subtract a^2 from both sides.
$b^2 = c^2 - a^2$ — Make b^2 the subject.
$b = \sqrt{c^2 - a^2}$ — Take the square root of both sides, $b = \sqrt{c^2 - a^2}$ if b is positive.

Exercise 2H

UNDERSTANDING AND FLUENCY	1, 2–4(½)	2–4(½)	3–4(½)

1 State the letter that is the subject of these formulas.

a $A = \frac{1}{2}bh$

b $D = b^2 - 4ac$

c $M = \frac{a + b}{2}$

d $A = \pi r^2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 16

2 Substitute the given values into each of the following formulas to evaluate the subject. Round to 2 decimal places where appropriate.

a $A = bh$, when $b = 3$ and $h = 7$

b $F = ma$, when $m = 4$ and $a = 6$

c $m = \dfrac{a + b}{4}$, when $a = 14$ and $b = -6$

d $t = \dfrac{d}{v}$, when $d = 18$ and $v = 3$

e $A = \pi r^2$, when $\pi = 3.14$ and $r = 12$

f $V = \dfrac{4}{3}\pi r^3$, when $\pi = 3.14$ and $r = 2$

g $c = \sqrt{a^2 + b^2}$, when $a = 12$ and $b = 22$

h $Q = \sqrt{2gh}$, when $g = 9.8$ and $h = 11.4$

i $I = \dfrac{MR^2}{2}$, when $M = 12.2$ and $R = 6.4$

j $x = ut + \dfrac{1}{2}at^2$, when $u = 0$, $t = 4$ and $a = 10$

Example 17

3 Substitute the given values into each of the following formulas then solve the equations to determine the value of the unknown pronumeral each time. Round to 2 decimal places where appropriate.

a $m = \dfrac{F}{a}$, when $m = 12$ and $a = 3$

b $A = lb$, when $A = 30$ and $l = 6$

c $A = \dfrac{1}{2}h(a + b)$, when $A = 64$, $b = 12$ and $h = 4$

d $C = 2\pi r$, when $C = 26$ and $\pi = 3.14$

e $S = 2\pi r^2$, when $S = 72$ and $\pi = 3.14$

f $v^2 = u^2 + 2as$, when $v = 22$, $u = 6$ and $a = 12$

g $m = \sqrt{\dfrac{x}{y}}$, when $m = 8$ and $x = 4$

Example 18

4 Transpose each of the following formulas to make the pronumeral shown in brackets the subject.

a $A = 2\pi rh$ (r)

b $I = \dfrac{Prt}{100}$ (r)

c $p = m(x + n)$ (n)

d $d = \dfrac{a + bx}{c}$ (x)

e $V = \pi r^2 h \ (r > 0)$ (r)

f $P = \dfrac{v^2}{R} \ (v > 0)$ (v)

g $S = 2\pi rh + 2\pi r^2$ (h)

h $A = (p + q)^2$ (p)

i $T = 2\pi\sqrt{\dfrac{l}{g}}$ (g)

j $\sqrt{A} + B = 4C$ (A)

PROBLEM-SOLVING AND REASONING	5, 6, 9	5–7, 9, 10	6–8, 9(½), 10

5 The formula $s = \dfrac{d}{t}$ gives the speed s km/h of a car that has travelled a distance of d km in t hours.

a Find the speed of a car that has travelled 400 km in 4.5 hours. Round to 2 decimal places.

b **i** Transpose the formula $s = \dfrac{d}{t}$ to make d the subject.

ii Find the distance covered if a car travels at 75 km/h for 3.8 hours.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 The formula $F = \frac{9}{5}C + 32$ converts degrees Celsius, C, to degrees Fahrenheit, F.

a Find what each of the following temperatures is in degrees Fahrenheit.

i 100°C

ii 38°C

b Transpose the formula to make C the subject.

c Calculate what each of the following temperatures is in degrees Celsius. Round to 1 decimal place where necessary.

i 14°F

ii 98°F

7 The velocity, v m/s, of an object is described by the rule $v = u + at$, where u is the initial velocity in m/s, a is the acceleration in m/s^2 and t is the time in seconds.

a Find the velocity after 3 seconds if the initial velocity is 5 m/s and the acceleration is 10 m/s^2.

b Find the time taken for a body to reach a velocity of 20 m/s if its acceleration is 4 m/s^2 and its initial velocity is 12 m/s.

8 The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$, where t is the time in seconds after a tap is turned on.

a Over time, does the water volume increase or decrease according to the formula?

b Find the volume after 2 minutes.

c Find the time it takes for the volume to reach 1500 litres. Round to the nearest minute.

d How long, to the nearest minute, does it take to completely empty the tank?

9 Write a formula for the following situations. Make the first listed pronumeral the subject.

a $\$D$ given c cents

b d cm given e metres

c The discounted price $\$D$ that is 30% off the marked price $\$M$

d The value of an investment $\$V$ that is 15% more than the initial amount $\$P$

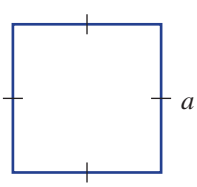
e The cost $\$C$ of hiring a car at \$50 upfront plus \$18 per hour for t hours

f The distance d km remaining in a 42 km marathon after t hours if the running speed is 14 km/h

g The cost $\$C$ of a bottle of soft drink if b bottles cost $\$c$

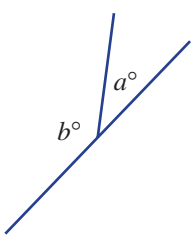
10 Write a formula for the value of a in these diagrams.

a

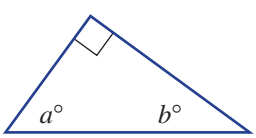


Perimeter = P

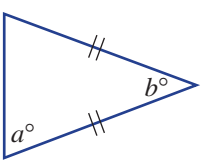
b



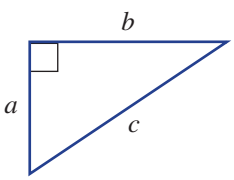
c



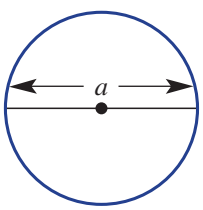
d



e



f



Area = A

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

ENRICHMENT — — 11

Basketball formulas

11 The formula $T = 3x + 2y + f$ can be used to calculate the total number of points made in a basketball game where:

x = number of three-point goals y = number of two-point goals

f = number of free throws made T = total number of points

a Find the total number of points for a game where 12 three-point goals, 15 two-point goals and 7 free throws were made.

b Find the number of three-point goals made if the total number of points was 36 with 5 two-point goals made and 5 free throws made.

c The formula $V = \left(p + \frac{3r}{2} + 2a + \frac{3s}{2} + 2b\right) - \frac{1.5t + 2f + m - o}{g}$ can be used to calculate the value, V, of a basketball player where:

p = points earned r = number of rebounds

a = number of assists s = number of steals

b = number of blocks t = number of turnovers

f = number of personal fouls m = number of missed shots

o = number of offensive rebounds g = number of games played

Calculate the value of a player with 350 points earned, 2 rebounds, 14 assists, 25 steals, 32 blocks, 28 turnovers, 14 personal fouls, 24 missed shots, 32 offensive rebounds and 10 games.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2I Linear simultaneous equations: substitution

Interactive
Widgets
HOTsheets
Walkthrough

A linear equation with one unknown usually has one unique solution. For example, $x = 2$ is the only value of x that makes the equation $2x + 3 = 7$ true.

The linear equation $2x + 3y = 12$ has two unknowns and it has an infinite number of solutions. Each solution is a pair of x and y values that makes the equation true, for example $x = 0$ and $y = 4$ or $x = 3$ and $y = 2$ or $x = 4\frac{1}{2}$ and $y = 1$.

However, if we are told that $2x + 3y = 12$ and also that $y = 2x - 1$, we can find a single solution that satisfies both equations. Equations like this are called simultaneous linear equations, because we can find a pair of x and y values that satisfy both equations at the same time (simultaneously).

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



A share trader examining computer models of financial data, which can involve finding values that satisfy two equations simultaneously.

Let's start: Multiple solutions

There is more than one pair of numbers x and y that satisfies the equation $x - 2y = 5$.

- Write down at least 5 pairs (x, y) that make the equation true.

A second equation is $y = x - 8$.

- Do any of your pairs that make the first equation true also make the second equation true? If not, can you find the special pair of numbers that satisfies both equations simultaneously?

Key ideas

- An algebraic method called **substitution** can be used to solve **simultaneous equations**. It is used when at least one of the equations has a single pronumeral as the subject. For example, y is the subject in the equation $y = 3x + 1$.
- To solve simultaneous equations using substitution:
  1. Substitute one equation into the other.
  2. Solve for the remaining pronumeral.
  3. Substitute to find the value of the second pronumeral.

$2x + 3y = 8$ and $y = x + 1$

$2x + 3(x + 1) = 8$

$2x + 3x + 3 = 8$

$5x + 3 = 8$

$5x = 5$

$x = 1$

$\therefore y = 1 + 1 = 2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 19 Solving using substitution

Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 10$
$y = 4x$

b $4x - y = 6$
$y = 2x - 4$

c $3x + 2y = 19$
$y = 2x - 8$

SOLUTION

a

$$x + y = 10 \quad (1)$$
$$y = 4x \quad (2)$$
$$x + (4x) = 10$$
$$5x = 10$$
$$x = 2$$

From (2) $y = 4x$
$= 4 \times 2$
$= 8$

Solution: $x = 2, y = 8$

Check: $2 + 8 = 10$ and $8 = 4 \times 2$

EXPLANATION

Number the equations for reference.

Substitute $y = 4x$ into (1).
Combine like terms and solve for x.

Substitute $x = 2$ into (2) to find the value of y.

Check your answer by substituting $x = 2$ and $y = 8$ into (1) and (2).

b

$$4x - y = 6 \quad (1)$$
$$y = 2x - 4 \quad (2)$$
$$4x - (2x - 4) = 6$$
$$4x - 2x + 4 = 6$$
$$2x + 4 = 6$$
$$2x = 2$$
$$x = 1$$

From (2) $y = 2x - 4$
$= 2 \times 1 - 4$
$= -2$

Solution: $x = 1, y = -2$

Check: $4 + 2 = 6$ and $-2 = 2 \times 1 - 4$

Substitute $y = 2x - 4$ into (1) using brackets.
Use the distributive law and solve for x.

Substitute $x = 1$ into (2) to find the value of y.

Check: substitute $x = 1$ and $y = -2$ into (1) and (2).

c

$$3x + 2y = 19 \quad (1)$$
$$y = 2x - 8 \quad (2)$$
$$3x + 2(2x - 8) = 19$$
$$3x + 4x - 16 = 19$$
$$7x - 16 = 19$$
$$7x = 35$$
$$x = 5$$

From (2) $y = 2x - 8$
$= 2 \times 5 - 8$
$= 2$

Solution: $x = 5, y = 2$

Check: $3 \times 5 + 2 \times 2 = 19$ and $2 = 2 \times 5 - 8$

Substitute $y = 2x - 8$ into (1).
Use the distributive law and solve for x.

Substitute $x = 5$ into (2) to find the value of y.

Check: substitute $x = 5$ and $y = 2$ into (1) and (2).

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 2I

UNDERSTANDING AND FLUENCY	1–5	2–4, 5(½), 6	4–6(½)

1 Find the value of x or y by substituting the known value.

a $y = 2x - 3 \quad (x = 4)$

b $x = 5 - 2y \quad (y = 4)$

c $2x + 4y = 8 \quad (y = -3)$

2 Choose the correct option.

a When substituting $y = 2x - 1$ into $3x + 2y = 5$ the second equation becomes:

A $3x + 2(2x - 1) = 5$

B $3(2x - 1) + 2y = 5$

C $3x + 2y = 2x - 1$

b When substituting $x = 1 - 3y$ into $5x - y = 6$ the second equation becomes:

A $1 - 3x - y = 6$

B $5(1 - 3y) = 6$

C $5(1 - 3y) - y = 6$

3 Check whether $x = -2$ and $y = 2$ is a solution to each of the following pairs of simultaneous equations.

a $x + y = 0$ and $x - y = -4$

b $x - 2y = -6$ and $2x + y = 0$

c $3x + 4y = -2$ and $x = -3y - 4$

d $2x + y = -2$ and $x = 4y - 10$

Example 19a

4 Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 3$, $y = 2x$

b $x + y = 6$, $x = 5y$

c $x + 5y = 8$, $y = 3x$

d $x - 5y = 3$, $x = 2y$

e $3x + 2y = 18$, $y = 3x$

f $x + 2y = 15$, $y = -3x$

Example 19b

5 Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 12$, $y = x + 6$

b $2x + y = 1$, $y = x + 4$

c $5x + y = 5$, $y = 1 - x$

d $3x - y = 7$, $y = x + 5$

e $3x - y = 9$, $y = x - 1$

f $x + 2y = 6$, $x = 9 - y$

g $y - x = 14$, $x = 4y - 2$

h $3x + y = 4$, $y = 2 - 4x$

i $4x - y = 12$, $y = 8 - 6x$

Example 19c

6 Solve each of the following pairs of simultaneous equations by using substitution.

a $3x + 2y = 8$, $y = 4x - 7$

b $2x + 3y = 11$, $y = 2x + 1$

c $4x + y = 4$, $x = 2y - 8$

d $2x + 5y = -4$, $y = x - 5$

e $2x - 3y = 5$, $x = 5 - y$

f $3x + 2y = 5$, $y = 3 - x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING	7, 8, 10	7, 8, 10	8, 9, 10, 11

7 The sum of two numbers is 48 and the larger number is 14 more than the smaller number. Write two equations and solve them to find the two numbers.

8 The combined mass of two trucks is 29 tonnes. The heavier truck is 1 tonne less than twice the mass of the smaller truck. Write two equations and solve them to find the mass of each truck.

9 The perimeter of a rectangle is 11 cm and the length is 3 cm more than half the width. Find the dimensions of the rectangle.

10 One of the common errors when applying the method of substitution is made in this working. Find the error and describe how to avoid it.

Solve $y = 3x - 1$ and $x - y = 7$.

$x - 3x - 1 = 7$ substituting $y = 3x - 1$ into $x - y = 7$

$-2x - 1 = 7$

$-2x = 8$

$x = -4$

11 If both equations have the same pronumeral as the subject, substitution is still possible. For example, solve $y = 3x - 1 \ldots (1)$ and $y = 2 - x \ldots (2)$

Substitute (1) into (2)

$3x - 1 = 2 - x$

$4x = 3$

$x = \frac{3}{4}$ and $y = \frac{5}{4}$

Use this method to solve these simultaneous equations.

a $y = 4x + 1$
$y = 3 - 2x$

b $y = 3 - 4x$
$y = 2x + 8$

c $y = \frac{1}{2}x + 4$
$y = \frac{x + 1}{3}$

ENRICHMENT	—	—	12

Literally challenging

12 Use substitution to solve each of the following pairs of simultaneous equations for x and y in terms of a and b.

a $ax + y = b$
$y = bx$

b $ax + by = b$
$x = by$

c $x + y = a$
$x = y - b$

d $ax - by = a$
$y = x - a$

e $ax - y = a$
$y = bx + a$

f $ax - by = 2a$
$x = y - b$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2J Linear simultaneous equations: elimination

Interactive
Widgets
HOTsheets
Walkthrough

Another method used to solve simultaneous linear equations is called elimination. This involves the addition or subtraction of the two equations to eliminate one of the pronumerals. We can then solve for the remaining pronumeral and substitute to find the value of the second pronumeral.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: To add or subtract?

To use the method of elimination you need to decide if using addition or using subtraction will eliminate one of the pronumerals.

Decide if the terms in these pairs should be added or subtracted to give the result of 0.

- $3x$ and $3x$
- $2y$ and $-2y$
- $-x$ and x
- $-7y$ and $-7y$

Describe under what circumstances addition or subtraction should be used to eliminate a pair of terms.

Key ideas

- **Elimination** involves the addition or subtraction of two equations to remove one pronumeral.
- Elimination is often used when both equations are of the form $ax + by = d$ or $ax + by + c = 0$.
- Add equations to eliminate terms of opposite sign:

$$\begin{array}{rl} & 3x - y = 4 \\ + & 5x + y = 4 \\ \hline & 8x \quad\; = 8 \end{array}$$

- Subtract equations to eliminate terms of the same sign:

$$\begin{array}{rl} & 2x + 3y = 6 \\ - & 2x - 5y = 7 \\ \hline & 8y = -1 \end{array}$$

- If terms cannot be eliminated just by using addition or subtraction, first multiply one or both equations to form a matching pair.
  For example:

matching pair

1 $3x - 2y = 1$ → $3x - 2y = 1$
$2x + y = 3$ → $4x + 2y = 6$ (Multiply both sides by 2)

matching pair

2 $7x - 2y = 3$ → $28x - 8y = 12$ (Multiply both sides by 4)
$4x - 5y = -6$ → $28x - 35y = -42$ (Multiply both sides by 7)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 20 Solving simultaneous equations using elimination

Solve the following pairs of simultaneous equations by using elimination.

a $x - 2y = 1$
$-x + 5y = 2$

b $3x - 2y = 5$
$5x - 2y = 11$

c $5x + 2y = -7$
$x + 7y = 25$

d $4x + 3y = 18$
$3x - 2y = 5$

SOLUTION	EXPLANATION
a $x - 2y = 1 \quad (1)$ $-x + 5y = 2 \quad (2)$	
(1) + (2) $3y = 3$ $y = 1$	Add the two equations to eliminate x since $x + (-x) = 0$. Then solve for y.
From (1) $x - 2y = 1$ $x - 2 \times (1) = 1$ $x - 2 = 1$ $x = 3$	Substitute $y = 1$ into equation (1) to find x.
$\therefore x = 3, y = 1$	
Check $3 - 2(1) = 1$ $-3 + 5(1) = 2$	Substitute $x = 3$ and $y = 1$ into the original equations to check.
b $3x - 2y = 5 \quad (1)$ $5x - 2y = 11 \quad (2)$	Subtract the two equations to eliminate y since they are the same sign, i.e. $-2y - (-2y) = -2y + 2y = 0$. Alternatively, you could do (1) − (2) but (2) − (1) avoids negative coefficients.
(2) − (1) $2x = 6$ $x = 3$	Solve for x.
From (1) $3x - 2y = 5$ $3 \times (3) - 2y = 5$ $-2y = -4$ $y = 2$	Substitute $x = 3$ into equation (1) to find y.
$\therefore x = 3, y = 2$	
Check $3(3) - 2(2) = 5$ $(5)3 - 2(2) = 11$	Substitute $x = 3$ and $y = 2$ into the original equations to check.
c $5x + 2y = -7 \quad (1)$ $x + 7y = 25 \quad (2)$	There are different numbers of x and y in each equation so multiply equation (2) by 5 to make the coefficient of x equal in size to (1).
$5 \times (2) \quad 5x + 35y = 125 \quad (3)$ $5x + 2y = -7 \quad (1)$	
(3) − (1) $33y = 132$ $y = 4$	Subtract the equations to eliminate x.
From (2) $x + 7y = 25$ $x + 7 \times (4) = 25$ $x + 28 = 25$ $x = -3$	Substitute $y = 4$ in equation (2) to find x.
$\therefore x = -3, y = 4$	Substitute $x = -3$ and $y = 4$ into the original equations to check.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

d

$4x + 3y = 18$ (1)
$3x - 2y = 5$ (2)
$2 \times (1)$ $8x + 6y = 36$ (3)
$3 \times (2)$ $9x - 6y = 15$ (4)
$(3) + (4)$ $17x = 51$
$x = 3$

Multiply equation (1) by 2 and equation (2) by 3 to make the coefficients of y equal in size but opposite in sign.
Add the equations to eliminate y.

From (1) $4x + 3y = 18$
$4 \times (3) + 3y = 18$
$12 + 3y = 18$
$3y = 6$
$y = 2$

Substitute $x = 3$ into equation (1) to find y.

$\therefore x = 3, y = 2$

Substitute $x = 3$ and $y = 2$ into the original equations to check.

Exercise 2J

UNDERSTANDING AND FLUENCY 1, 2, 3(½), 4–5(a, b), 6(½) | 2, 3(½), 4–5(a, b), 6–7(½) | 3(½), 4–5(c), 6–7(½)

1 a Add these equations to eliminate x or y.

i $2x + y = 11$
$3x - y = 5$

ii $3x - 2y = 0$
$4x + 2y = -3$

iii $4x + y = 7$
$-4x + 3y = 11$

b Subtract these equations to eliminate x or y.

i $2x + 5y = 11$
$2x + y = 7$

ii $2x + 5y = 11$
$x + 5y = 7$

iii $3x + 2y = 3$
$x + 2y = -2$

2 a Decide if addition or subtraction should be chosen to eliminate the pronumeral x in these simultaneous equations.

i $x + 2y = 3$
$x - 5y = -4$

ii $-2x - y = -9$
$2x + 3y = 11$

iii $y - x = 0$
$3y - x = 8$

b Decide if addition or subtraction will eliminate the pronumeral y in these simultaneous equations.

i $4x - y = 6$
$x + y = 4$

ii $7x - 2y = 5$
$-3x - 2y = -5$

iii $10y + x = 14$
$-10y - 3x = -24$

Example 20a

3 Solve these simultaneous equations by first adding the equations.

a $x + 2y = 3$
$-x + 3y = 2$

b $x - 4y = 2$
$-x + 6y = 2$

c $-2x + y = 1$
$2x - 3y = -7$

d $3x - y = 2$
$2x + y = 3$

e $2x - 3y = -2$
$-5x + 3y = -4$

f $4x + 3y = 5$
$-4x - 5y = -3$

4 Solve these simultaneous equations by first subtracting the equations.

a $3x + y = 10$
$x + y = 6$

b $2x + 7y = 9$
$2x + 5y = 11$

c $2x + 3y = 14$
$2x - y = -10$

Example 20b

5 Solve these simultaneous equations by first subtracting the equations.

a $5x - y = -2$
$3x - y = 4$

b $-5x + 3y = -1$
$-5x + 4y = 2$

c $9x - 2y = 3$
$-3x - 2y = -9$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 20c

6 Solve the following pairs of simultaneous linear equations by using elimination.

a $4x + y = -8$
$3x - 2y = -17$

b $2x - y = 3$
$5x + 2y = 12$

c $-x + 4y = 2$
$3x - 8y = -2$

d $3x + 2y = 0$
$4x + y = -5$

e $4x + 3y = 13$
$x + 2y = -3$

f $3x - 4y = -1$
$6x - 5y = 10$

g $-4x - 3y = -5$
$7x - y = 40$

h $3x - 4y = -1$
$-5x - 2y = 19$

i $5x - 4y = 7$
$-3x - 2y = 9$

Example 20d

7 Solve the following pairs of simultaneous linear equations by using elimination.

a $3x + 2y = -1$
$4x + 3y = -3$

b $7x + 2y = 8$
$3x - 5y = 21$

c $6x - 5y = -8$
$-5x + 2y = -2$

d $2x - 3y = 3$
$3x - 2y = 7$

e $7x + 2y = 1$
$4x + 3y = 8$

f $5x + 7y = 1$
$3x + 5y = -1$

g $5x + 3y = 16$
$4x + 5y = 5$

h $3x - 7y = 8$
$4x - 3y = -2$

i $2x - 3y = 1$
$3x + 2y = 8$

j $2x - 7y = 11$
$5x + 4y = -37$

k $3x + 5y = 36$
$7x + 2y = -3$

l $2x - 4y = 6$
$5x + 3y = -11$

PROBLEM-SOLVING AND REASONING 8, 9, 12 | 8–10, 12, 13 | 9–11, 13, 14

8 The sum of two numbers is 30 and their difference is 12.
Write two equations and find the numbers.

9 Two supplementary angles differ by 24°. Write two equations and find the two angles.

10 The perimeter of a rectangular city block is 800 metres and the difference between the length and width is 123 metres. What are the dimensions of the city block?



11 A teacher collects a total of 17 mobile phones and iPods before a group of students heads off on a bushwalk. From a second group of students, 40 phones and iPods are collected. The second group had twice the number of phones and 3 times the number of iPods than the first group. How many phones and iPods did the first group have?

12 Consider the pair of simultaneous equations
$2x + y = 5 \ldots (1)$
$5x + y = 11 \ldots (2)$

a Solve the equations by first subtracting equation (2) from equation (1), i.e. (1) − (2).

b Now solve the equations by first subtracting equation (1) from equation (2), i.e. (2) − (1).

c Which method **a** or **b** is preferable and why?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 To solve any of the pairs of simultaneous equations in this section using the method of substitution, what would need to be done before the substitution is made?

Try these using substitution.

a $x + y = 5$
$2x - y = 7$

b $3x - y = -2$
$x + 4y = -5$

14 Find the solution to these pairs of simultaneous equations. What do you notice?

a $2x + 3y = 3$ and $2x + 3y = 1$

b $7x - 14y = 2$ and $y = \frac{1}{2}x + 1$

ENRICHMENT — — 15

Literal elimination

15 Use elimination to solve the following pairs of simultaneous equations to find the value of x and y in terms of the other pronumerals.

a $x + y = a$
$x - y = b$

b $ax + y = 0$
$ax - y = b$

c $x - by = a$
$-x - by = 2a$

d $2ax + y = b$
$x + y = b$

e $bx + 5ay = 2b$
$bx + 2ay = b$

f $ax + 3y = 14$
$ax - y = -10$

g $2ax + y = b$
$3ax - 2y = b$

h $2ax - y = b$
$3ax + 2y = b$

i $-x + ay = b$
$3x - ay = -b$

j $ax + 2y = c$
$2ax + y = -c$

k $ax - 4y = 1$
$x - by = 1$

l $ax + by = a$
$x + y = 1$

m $ax + by = c$
$-ax + y = d$

n $ax - by = a$
$-x + y = 2$

o $ax + by = b$
$3x - y = 2$

p $ax - by = b$
$cx - y = 2$

q $ax + by = c$
$dx - by = f$

r $ax + by = c$
$dx + by = f$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2K Using linear simultaneous equations to solve problems

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

Many problems can be described mathematically using a pair of simultaneous linear equations from which a solution can be obtained algebraically.

Let's start: The tyre store

In one particular week a total of 83 cars and motorcycles are checked into a garage to have their tyres changed. Each motorcycle has 2 tyres changed and each car has 4 tyres changed. The total number of tyres sold in the week is 284.

If you have to find the number of motorcycles and the number of cars that have their tyres changed in the week:

- what two variables should you define?
- what two equations can you write?
- which method (substitution or elimination) would you use to solve the equations?
- what is the solution to the simultaneous equations?
- how would you answer the question in words?



Key ideas

- To solve worded problems with simultaneous equations:
  - Define two pronumerals by writing down what they represent.
    For example: Let $\$C$ be the cost of …
    Let x be the number of …
  - Write a pair of simultaneous equations from the given information using your two pronumerals.
  - Solve the equations simultaneously using substitution or elimination.
  - Check the solution by substituting into the original equations.
  - Express the answer in words.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 21 Solving word problems with simultaneous equations

Andrea bought two containers of ice-cream and three bottles of maple syrup for a total of $22. At the same shop, Bettina bought one container of ice-cream and two bottles of maple syrup for $13. How much does each container of ice-cream and each bottle of maple syrup cost?

SOLUTION	EXPLANATION
Let: $\$x$ be the cost of a container of ice-cream $\$y$ be the cost of a bottle of maple syrup	Define the unknowns. Ask yourself what you are being asked to find.
$2x + 3y = 22$ (1) $x + 2y = 13$ (2)	2 containers of ice-cream and 3 bottles of maple syrup for a total of $22 1 container of ice-cream and 2 bottles of maple syrup for $13. Choose the method of elimination to solve.
$2 \times (2)$ $2x + 4y = 26$ (3) $2x + 3y = 22$ (1)	Multiply (2) by 2 to obtain a matching pair. Subtract equation (1) from (3).
(3) − (1) $y = 4$	
From (2) $x + 2y = 13$	
$x + 2 \times (4) = 13$	Substitute $y = 4$ into (2).
$x + 8 = 13$	Solve for x.
$x = 5$	Substitute $y = 4$ and $x = 5$ into original equations to check.
The cost of one container of ice-cream is $5 and the cost of one bottle of maple syrup is $4.	Answer the question in a sentence.

Exercise 2K

UNDERSTANDING AND FLUENCY	1, 3–7	2, 4–8	4, 6, 8, 9

1 The sum of two numbers is 42 and their difference is 6. Find the two numbers x and y by completing the following steps.
- **a** Write a pair of simultaneous equations relating x and y.
- **b** Solve the pair of equations using substitution or elimination.
- **c** Write your answer in words.

2 The length l cm of a rectangle is 5 cm longer than its breadth b cm. If the perimeter is 84 cm, find the dimensions of the rectangle by completing the following steps.
- **a** Write a pair of simultaneous equations relating l and b.
- **b** Solve the pair of equations using substitution or elimination.
- **c** Write your answer in words.

3 A rectangular block of land has a perimeter of 120 metres and the length l m of the block is three times the breadth b m. Find the dimensions of the block of land by completing the following steps.
- **a** Write a pair of simultaneous equations relating l and b.
- **b** Solve the pair of equations using substitution or elimination.
- **c** Write your answer in words.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 21

4 Mal bought 3 bottles of milk and 4 bags of chips for a total of $17. At the same shop, Barbara bought 1 bottle of milk and 5 bags of chips for $13. Find how much each bottle of milk and each bag of chips cost by:
- **a** defining two pronumerals to represent the problem
- **b** writing a pair of simultaneous equations relating the two pronumerals
- **c** solving the pair of equations using substitution or elimination
- **d** writing your answer in words

5 Leonie bought seven lip glosses and two eye shadows for a total of $69 and Chrissie bought four lip glosses and three eye shadows for a total of $45. Find how much each lip gloss and each eye shadow costs by completing the following steps.
- **a** Define two pronumerals to represent the problem.
- **b** Write a pair of simultaneous equations relating the two pronumerals.
- **c** Solve the pair of equations using substitution or elimination.
- **d** Write your answer in words.

6 Steve bought five cricket balls and fourteen tennis balls for $130. Ben bought eight cricket balls and nine tennis balls for $141. Find the cost of a cricket ball and the cost of a tennis ball.

7 At a birthday party for 20 people each person could order a hot dog or chips. If there were four times as many hot dogs as chips ordered, calculate how many hot dogs and how many chips were bought.



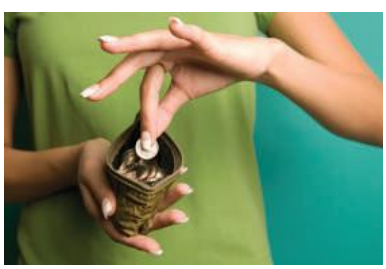
8 The entry fee for a fun run is $10 for adults and $3 for children. A total of $3360 was collected from the 420 competitors. Find the number of adults running and the number of children running.

9 Mila plants 820 hectares of potatoes and corn. To maximise his profit he plants 140 hectares more of potatoes than of corn. How many hectares of each does he plant?

PROBLEM-SOLVING AND REASONING

10, 11, 14 | 10–12, 14, 15 | 11–13, 15, 16

10 Carrie has 27 coins in her purse. All the coins are 5-cent or 20-cent coins. If the total value of the coins is $3.75, how many of each type does she have?



11 Michael is 30 years older than his daughter. In five years' time Michael will be 4 times as old as his daughter. How old is Michael now?

12 Jenny has twice as much money as Kristy. If I give Kristy $250, she will have three times as much as Jenny. How much did each of them have originally?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 At a particular cinema the cost of an adult movie ticket is $15 and the cost of a child's ticket is $10. The seating capacity of the cinema is 240. For one movie session all seats are sold and $3200 is collected from the sale of tickets. How many adult and how many children's tickets were sold?



14 Wilfred and Wendy have a long distance bike race. Wilfred rides at 20 km/h and has a 2 hour head start. Wendy travels at 28 km/h. How long does it take for Wendy to catch up to Wilfred?
Use distance = speed × time.



15 Andrew travelled a distance of 39 km by jogging for 4 hours and cycling for 3 hours. He could have travelled the same distance by jogging for 7 hours and cycling for 2 hours. Find the speed at which he was jogging and the speed at which he was cycling.

16 Malcolm's mother is 27 years older than he is and their ages are both two-digit numbers. If Malcolm swaps the digits in his age, he gets his mother's age.

a How old is Malcolm if the sum of the digits in his age is 5?

b What is the relationship between the digits in Malcolm's age if the sum of the digits is unknown.

c If the sum of the digits in Malcolm's two-digit age is unknown, how many possible ages could he be? What are these ages?

ENRICHMENT	—	—	17, 18

Digit swap

17 The digits of a two-digit number sum to 10. If the digits swap places, the number is 36 more than the original number. What is the original number? Can you show an algebraic solution?

18 The difference between the two digits of a two-digit number is 2. If the digits swap places, the number is 18 less than the original number. What is the original number? Can you show an algebraic solution?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2L Quadratic equations of the form $ax^2 = c$

Interactive
Widgets
HOTsheets
Walkthrough

In a linear equation, if the highest power of the variable is 2, the equation is called a quadratic equation. Since $4 \times 4 = 16$ and $(-4) \times (-4) = 16$, we would say that the equation $x^2 = 16$ has two possible solutions $x = 4$ and $x = -4$. In this section we will solve some simple quadratic equations. Quadratic equations may arise in areas such as the path of a projectile, a falling object or a measurement calculation.



Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: More than one solution?

Is it possible to have more than one solution to an equation?

Consider the following.

What do you notice when you evaluate:

- 3^2?
- $(-3)^2$?
- $(-5)^2$?
- 5^2?

From the above results, what possible value(s) of x make the following equations true?

- $x^2 = 9$
- $x^2 = 25$

Can you find a value of x to make the equation $x^2 = -9$ true? Why or why not?

Key ideas

- A **quadratic equation** is one in which the highest power of the pronumeral is 2. In this section you will solve quadratic equations of the form $ax^2 = c$, such as these:
  $x^2 = 0$ $x^2 = 9$ $2x^2 = 18$ $2x^2 - 6 = 7$
- To solve equations of the form $ax^2 = c$, make x^2 the subject, then take the square root of both sides of the equation. This process may produce two solutions, one solution or no real solutions.
  - If $2x^2 = 18$, then $x^2 = 9$. This equation has **two solutions**. These are found by taking the square root of both sides of the equation. We know that $3^2 = 9$ and also $(-3)^2 = 9$, so there are two solutions: $x = 3$ and $x = -3$. This can be abbreviated to $x = \pm 3$.
  - If $2x^2 = 0$, then $x^2 = 0$. This equation only has **one solution**, $x = 0$, because the square root of zero is zero.
  - If $2x^2 = -18$, then $x^2 = -9$. This equation is said to have '**no real solutions**' because the square root of -9 has no value in the set of real numbers. In future years you may study a topic called *Complex Numbers*, in which equations like $x^2 = -9$ can be solved.
- Some quadratic equations have non-integer solutions. These can be expressed using **surds**. For example, the solution of $x^2 = 10$ is $x = \pm\sqrt{10}$. These are the exact solutions, but they could be expressed as $x = \pm 3.2$ (to 1 d.p.), for example.
- In practical problems such as those involving measurement, the negative solution is usually rejected. For example, if the area of a square is 10 square metres, the exact side length is $\sqrt{10}$ metres, not $-\sqrt{10}$ metres or $\pm\sqrt{10}$ metres.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 22 Solving equations of the form $ax^2 = c$

Find all possible solutions to the following equations.

a $x^2 = 25$

b $2x^2 = 18$

c $\frac{1}{5}x^2 = 20$

SOLUTION	EXPLANATION
a $x^2 = 25$	Take the square root of both sides to solve for x. If $x^2 = c$, $c > 0$, then $x = \pm\sqrt{c}$.
$\therefore x = \pm\sqrt{25}$	Since 25 is a square number, $\sqrt{25}$ is a whole number.
$x = \pm 5$	$\pm$ represents two solutions: $+5$ since $(+5)^2 = 25$ and -5 since $(-5)^2 = 25$.
b $2x^2 = 18$	
$x^2 = 9$	First make x^2 the subject by dividing both sides by 2.
$\therefore x = \pm\sqrt{9}$	Take the square root of both sides to solve for x.
$x = \pm 3$	This gives two possible values for x, $+3$ and -3.
c $\frac{1}{5}x^2 = 20$	$\frac{1}{5}$ of x^2 is the same as $\frac{x^2}{5}$, so rewrite equation in this form.
$\frac{x^2}{5} = 20$	Multiply both sides by 5 (or divide both sides by $\frac{1}{5}$ after the first line, but multiplying by 5 is easier.)
$x^2 = 100$	
$\therefore x = \pm\sqrt{100}$	Take the square root of both sides to give the two possible solutions for x.
$x = \pm 10$	

Example 23 Solving equations with solutions that are surds

Find all possible solutions to the following equations.

a $x^2 = 17$ (answer in exact surd form)

b $3x^2 = 21$ (give answers to 1 decimal place)

SOLUTION	EXPLANATION
a $x^2 = 17$	Take the square root of both sides.
$\therefore x = \pm\sqrt{17}$	Since 17 is not a square number, leave answer in exact surd form $\pm\sqrt{17}$ as required.
b $3x^2 = 21$	Divide both sides by 3.
$x^2 = 7$	Take the square root of both sides.
$\therefore x = \pm\sqrt{7}$	Use a calculator to evaluate $\sqrt{7} = 2.64575\ldots$ and round to 1 decimal place as required.
$x = \pm 2.6$ (to 1 d.p.)	

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 24 Rearranging and solving equations of the form $ax^2 + b = c$

Find exact solution(s), where possible, to the following equations.

a $x^2 - 64 = 0$ **b** $x^2 + 7 = 0$ **c** $3x^2 - 5 = 7, x > 0$ **d** $13 - x^2 = 3, x < 0$

SOLUTION	EXPLANATION
a $x^2 - 64 = 0$ $x^2 = 64$ $\therefore x = \pm\sqrt{64}$ $x = \pm 8$	Make x^2 the subject by adding 64 to both sides. Solve for x by taking the square root of both sides.
b $x^2 + 7 = 0$ $x^2 = -7$ $\therefore$ No possible solution	Make x^2 the subject by subtracting 7 from both sides. Since x^2 is positive for all values of x, $x^2 = c$, where c is negative, has no real solution.
c $3x^2 - 5 = 7$ $3x^2 = 12$ $x^2 = 4$ $\therefore x = \pm\sqrt{4}$ $x = \pm 2$ $x = 2$ since $x > 0$	Make x^2 the subject and then take the square root of both sides to solve for x. Note the restriction that x is a positive value so reject the solution $x = -2$.
d $13 - x^2 = 3$ $-x^2 = -10$ $x^2 = 10$ $\therefore x = \pm\sqrt{10}$ $x = -\sqrt{10}$ since $x < 0$	Subtract 13 from both sides and then divide both sides by -1. Alternatively, add x^2 to both sides to give $13 = x^2 + 3$ and then solve. Leave answer in exact surd form, eliminating $x = \sqrt{10}$ since $x < 0$.

Exercise 2L

UNDERSTANDING AND FLUENCY | 1–3, 4(½), 5 | 3, 4(½), 5, 6(½) | 4(½), 5, 6(½)

1 **a** If $x^2 = 16$, then $x = \pm\square$

b If $x^2 = 0$, then $x = \square$

c If $x^2 = -16$, how many real solutions can be found for x?

2 Copy and complete:

a $2x^2 - 16 = 0$
$2x^2 = \square$
$x^2 = \square$
$x = \pm\square$

b $2x^2 + 16 = 16$
$2x^2 = \square$
$x^2 = \square$
$x = \square$

c $2x^2 + 16 = 0$
$2x^2 = \square$
$x^2 = \square$
No solutions

3 How many solutions will each equation have (two, one or none)?

a $x^2 = 81$ **b** $81x^2 = 0$

c $x^2 + 81 = 0$ **d** $2x^2 - 32 = 0$

e $2x^2 + 32 = 0$ **f** $2x^2 + 32 = 32$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 22

4 Find the possible solutions for x in the following equations.

a $x^2 = 9$ **b** $x^2 = 121$ **c** $x^2 = 1$ **d** $x^2 = 400$

e $5x^2 = 20$ **f** $6x^2 = 600$ **g** $8x^2 = 200$ **h** $4x^2 = 196$

i $-3x^2 = -108$ **j** $-4x^2 = -64$ **k** $\frac{x^2}{2} = 32$ **l** $\frac{x^2}{3} = 27$

m $\frac{x^2}{10} = 12.1$ **n** $0.1x^2 = 1.6$ **o** $\frac{1}{2x^2} = 72$ **p** $\frac{1}{5x^2} = 20$

Example 23

5 **a** Solve the following equations, leaving your answer in exact surd form.

i $x^2 = 15$ **ii** $x^2 = 10$ **iii** $x^2 = 42$ **iv** $x^2 = 3$

v $3x^2 = 21$ **vi** $8x^2 = 88$ **vii** $\frac{x^2}{2} = 17$ **viii** $\frac{x^2}{3} = 13$

b Solve these equations, giving your answers to 1 decimal place.

i $x^2 = 5$ **ii** $x^2 = 17$ **iii** $3x^2 = 120$ **iv** $\frac{x^2}{7} = 20$

Example 24

6 Find the possible solutions to the following equations. Leave your answers in exact form.

a $x^2 - 49 = 0$ **b** $x^2 - 36 = 0$

c $x^2 - 13 = 0$ **d** $x^2 + 4 = 0$

e $x^2 + 14 = 0$ **f** $2x^2 - 44 = 0$

g $3x^2 - 75 = 0, x > 0$ **h** $2x^2 + 32 = 36, x > 0$

i $5x^2 - 32 = 43$ **j** $4 - x^2 = 0, x < 0$

k $10 - x^2 = -12, x < 0$ **l** $8 - 4x^2 = 15$

PROBLEM-SOLVING AND REASONING 7(½), 8, 9, 13, 14 | 7(½), 9–12, 14, 15 | 7(½), 11, 12, 14–17

7 Solve the following equations by first writing in the form $ax^2 = c$. Answer in exact form.

a $3x^2 = 2x^2 + 9$

b $2x^2 = 4x^2 - 32$

c $x^2 + 3x = 3x + 4$

d $x^2 - 4x = 64 - 4x$

e $5x^2 + 3 = 3x^2 + 15, x > 0$

f $x^2 + 2x + 7 = 2x - 5, x > 0$

g $7 - x^2 = 2x^2 + 4, x < 0$

h $-x^2 - 2x + 4 = 2 - 2x, x < 0$

8 15 is subtracted from the square of a number and the result is 66. What are the possible numbers?

9 A square vegetable garden in a small backyard has side length x metres and area 14 m^2.

a Write an equation to represent this information and find the possible values for x to 1 decimal place.

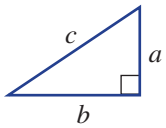
b Comment on the validity of your result in part **a**, given x represents a side length.

10 Determine the radius of a circle with the following areas, A. Answer to 1 decimal place where necessary. Recall the area of a circle is given by πr^2, where r is the radius.

a $A = 24$ m^2 **b** $A = 45$ cm^2 **c** $A = 4\pi$ m^2

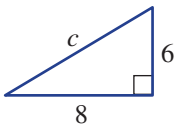
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Recall Pythagoras' theorem: in a right-angled triangle the sum of the squares of the two shorter sides equals the square of the hypotenuse; $a^2 + b^2 = c^2$

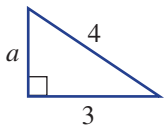


a Solve for the unknown side length in these right-angled triangles. Answer in exact form.

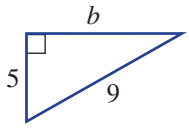
i



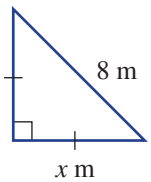
ii



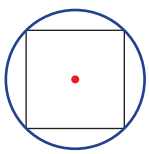
iii



b In the isosceles triangle shown, with hypotenuse 8 m, what is the value of x in exact form?



12 Determine the side length of the largest square that fits inside a circle of diameter 12 m. Round to 1 decimal place.



13 A ball is dropped from the top of a tall building. The height of the object above ground after t seconds is given by $h = 60 - 5t^2$.

a What is the height of the ball above ground after 3 seconds?

b After how many seconds will the ball reach ground level? Answer to 2 decimal places.

c After how many seconds was the ball at a height of 40 m?

14 Explain why $x^2 = c$, where $c < 0$, has no solutions.

15 Determine for which values of c the equation $x^2 + c = 0$ has:

a 0 solutions **b** 1 solution **c** 2 solutions

16 Billy says to Maddy: 'When I add 10 to the square of a number, the result is 6.' Maddy replies that this is impossible. Explain why Maddy is correct.

17 **a** Write down the possible value(s) for x in each of the following.

i $x^2 = 16$ **ii** $x = \sqrt{16}$

b Are the answers in part **a** the same or different? If they are different, explain how.

ENRICHMENT — — 18, 19

Kinematics

Kinematics is the study of motion. If the acceleration of an object is constant, there are a number of formulas that can be used to calculate velocity, displacement (distance from starting point), acceleration and time. One such equation is of the form $s = ut + \frac{1}{2}at^2$, where u is the initial velocity of the object, t is time, a is acceleration and s is displacement. Use this formula to answer the following questions.

18 A remote-control car starts from rest and accelerates constantly at 2 m s^{-2}. How long will the car take to travel 100 m?

19 A skydiver is dropped from a plane with an initial velocity of 0 m s^{-1}. If the acceleration (due to gravity) is 9.8 m s^{-2}, how long does the skydiver take to fall 200 m? Answer to 1 decimal place.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Fire danger

In many countries fire indices have been developed to help predict the likelihood of fire occurring. One of the simplest fire-danger rating systems devised is the Swedish Angstrom Index. This index only considers the relationship between relative humidity, temperature and the likelihood of fire danger.

The index, I, is given by: $$I = \frac{H}{20} + \frac{27 - T}{10}$$

where H is the percentage of relative humidity and T is the temperature in degrees Celsius. The table below shows the likelihood of a fire occurring for different index values.

Index	Likelihood of fire occurring
$I > 4.0$	Unlikely
$2.5 < I < 4.0$	Medium
$2.0 < I < 2.5$	High
$I < 2.0$	Very likely

Constant humidity

a If the humidity is 35% ($H = 35$), how hot would it have to be for the occurrence of fire to be:
- **i** very likely?
- **ii** unlikely?

Discuss your findings with regard to the range of summer temperatures for your capital city or nearest town.

b Repeat part **a** for a humidity of 40%.

c Describe how the 5% change in humidity affects the temperature at which fires become:
- **i** very likely
- **ii** unlikely

Constant temperature

a If the temperature was 30°C, investigate what humidity would make fire occurrence:
- **i** very likely
- **ii** unlikely

b Repeat part **a** for a temperature of 40°C.

c Determine how the ten-degree change in temperature affects the relative humidity at which fire occurrence becomes:
- **i** very likely
- **ii** unlikely

Reflection

Is this fire index more sensitive to temperature or to humidity? Explain your answer.

Investigate

Use the internet to investigate fire indices used in Australia. You can type in key words, such as *Australia, fire danger* and *fire index*.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Families of equations

If a set of equations has something in common, then it may be possible to solve all the equations in the family at once using literal equations. Literal equations use pronumerals in place of numbers.

Family of linear equations

a Solve these linear equations for x.

i $2x + 3 = 5$

ii $2x + 1 = 5$

iii $2x - 1 = 5$

b Now solve the literal equation $2x + a = 5$ for x. Your answer will be in terms of a.

c For part **a i**, the value of a is 3. Substitute $a = 3$ into your rule for x in part **b** to check the result.

Literal equations

a Solve these literal equations for x in terms of the other pronumerals.

i $ax + b = 10$ **ii** $\frac{x - a}{b} = c$ **iii** $\frac{ax}{b} + c = d$

b Use your results from part **a** to solve these equations.

i $-3x + 2 = 10$ **ii** $\frac{x - 5}{7} = -4$ **iii** $\frac{-3x}{4} + 1 = 2$

Factorising to solve for x

To solve for x in an equation like $ax + 1 = bx + 2$, you can use factorisation as shown here.

$$ax + 1 = bx + 2$$
$$ax - bx = 2 - 1$$
$$x(a - b) = 1$$
$$x = \frac{1}{a - b}$$

a Use the above idea to solve these literal equations.

i $ax + 5 = 1 - bx$

ii $a(x + 2) = b(x - 1)$

iii $\frac{ax}{2} + bx = 1$

iv $\frac{b(x - 1)}{a} = x - 2$

b Solve these literal simultaneous equations for x and y.

i $ax + y = 1$, $bx - y = -11$ **ii** $y = ax + b$, $2x + y = 2b$ **iii** $ax + by = c$, $x - y = 0$ **iv** $ax + by = 1$, $bx + ay = 1$

c Check your solutions to parts **a** and **b** above by choosing an equation or a pair of simultaneous equations and solving in the normal way. Choose your equations so that they belong to the family described by the literal equation.

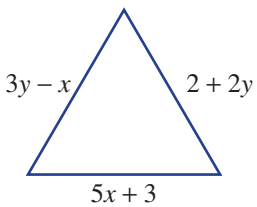
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1 In a magic square, all the rows, columns and main diagonals add to the same total. The total for this magic square is 15. Find the value of x then complete the magic square.

$2x - 2$	$3x$	$\frac{x+1}{2}$
x		

2 A group of office workers had some prize money to distribute among themselves. When all but one took \$9 each, the last person only received \$5. When they all took \$8 each, there was \$12 left over. How much had they won?

3 The sides of an equilateral triangle have lengths $3y - x$, $5x + 3$ and $2 + 2y$. Find the length of the sides.



4 **a** If $a > b > 0$ and $c < 0$, insert an inequality sign to make a true statement.

i $a + c ___ b + c$

ii $ac ___ bc$

iii $a - b ___ 0$

iv $\frac{1}{a} ___ \frac{1}{b}$

b Place a, b, c and d in order from smallest to largest given:

$a > b$

$a + b = c + d$

$b - a > c - d$

5 Find the values of x, y and z if:

$$x - 3y + 2z = 17$$
$$x - y + z = 8$$
$$y + z = 3$$

6 Solve these equations for x.

a $\frac{1}{x} + \frac{1}{a} = \frac{1}{b}$

b $\frac{1}{2x} + \frac{1}{3x} = \frac{1}{4}$

c $\frac{x-1}{3} - \frac{x+1}{4} = x$

d $\frac{2x-3}{4} - \frac{1-x}{5} = \frac{x+1}{2}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Expressions, equations and inequalities

Substitution
The process of replacing a pronumeral with a given value
e.g. if $x = 3$, $2x + 4 = 2 \times 3 + 4$
$= 6 + 4$
$= 10$
e.g. if $x = 2$, $y = -4$, $xy = 2 \times (-4)$
$= -8$

Expanding brackets
e.g. $2(x + 3) = 2 \times x + 2 \times 3$
$= 2x + 6$
$-3(4 - 2x) = -12 + 6x$

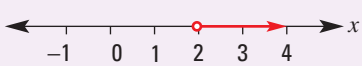
Expressions
e.g. $2x - 3y + 5$
term, coefficient, constant
e.g. 2 more than 3 lots of m is $3m + 2$

Addition/subtraction
Only like terms can be combined under addition or subtraction.
Like terms, e.g. $3a$ and $7a$
$2ab$ and $5ab$
not $4a$ and $7ab$
e.g. $3x + 2y - x + 4y + 7$
$= 3x - x + 2y + 4y + 7$
$= 2x + 6y + 7$

Solving linear equations
Finding the value that makes an equation true
e.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)
e.g. $5(x - 1) = 2x + 7$
$5x - 5 = 2x + 7$ (expand)
$3x - 5 = 7$ (subtract $2x$)
$3x = 12$ (add 5)
$x = 4$ (divide by 3)

Solving word problems
1 Define pronumeral(s).
2 Set up equation(s).
3 Solve equation(s).
4 Check each answer and write in words.

Inequalities
These can be represented using $>, <, \geq, \leq$ rather than $=$.
e.g. $x > 2$
2 not included



Solving inequalities uses the same steps as solving equations except when multiplying or dividing by a negative number. In this case, the inequality sign must be reversed.
e.g. $4 - 2x < 10$ (-4)
$-2x < 6$ $(\div -2)$
$x > -3$ reverse sign

Simultaneous equations
Use substitution or elimination to find the solution that satisfies 2 equations.

Substitution
e.g. $2x + y = 12$ (1)
$y = x + 3$ (2)
In (1) replace y with (2)
$2x + (x + 3) = 12$
$3x + 3 = 12$
$x = 3$
Sub. $x = 3$ to find y
In (2) $y = 3 + 3 = 6$

Elimination
Ensure both equations have a matching pair.
Add 2 equations if matching pair has different sign; subtract if same sign.

e.g. $x + 2y = 2$ (1)
$2x + 3y = 5$ (2)
(1) $\times 2$ $2x + 4y = 4$ (3)
(3) − (2) $y = -1$
In (1) $x + 2(-1) = 2$
$x - 2 = 2$
$\therefore x = 4$

Formulas
Some common formulas
e.g. $A = \pi r^2$, $C = 2\pi r$
An unknown value can be found by substituting values for the other pronumerals.
A formula can be transposed to make a different pronumeral the subject.
e.g. make r the subject in $A = \pi r^2$
$A = \pi r^2$
$\frac{A}{\pi} = r^2$
$\pm\sqrt{\frac{A}{\pi}} = r$
$\therefore r = \sqrt{\frac{A}{\pi}}$ where $r > 0$

Simple quadratic equations
$x^2 = 9$
$x = \pm\sqrt{9}$
$x = \pm 3$
$x^2 = -7$ has no solutions

$16x^2 = 9$
$x^2 = \frac{9}{16}$
$x = \pm\sqrt{\frac{9}{16}}$
$x = \pm\frac{3}{4}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Multiple-choice questions

1 The algebraic expression that represents 2 less than 3 lots of n is:

A $3(n - 2)$ **B** $2 - 3n$ **C** $3n - 2$ **D** $3 + n - 2$ **E** n

2 The simplified form of $6ab + 14a \div 2 - 2ab$ is:

A $8ab$ **B** $8ab + 7a$ **C** $ab + 7a$ **D** $4ab + 7a$ **E** $4ab + \frac{7a}{2}$

3 The solution to $\frac{x}{3} - 1 = 4$ is:

A $x = 13$ **B** $x = 7$ **C** $x = 9$ **D** $x = 15$ **E** $x = \frac{5}{3}$

4 The result when a number is tripled and increased by 21 is 96. The original number is:

A 22 **B** 32 **C** 25 **D** 30 **E** 27

5 The solution to the equation $3(x - 1) = 5x + 7$ is:

A $x = -4$ **B** $x = -5$ **C** $x = 5$ **D** $x = 3$ **E** $x = 1$

6 $\$x$ is raised from a sausage sizzle. Once the \$50 running cost is taken out, the money is shared equally among three charities so that they each get \$120. An equation to represent this is:

A $\frac{x - 50}{3} = 120$ **B** $\frac{x}{3} - 50 = 120$ **C** $\frac{x}{50} = 360$

D $\frac{x}{3} = 310$ **E** $3x + 50 = 120$

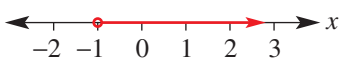
7 If $A = 2\pi rh$ with $A = 310$ and $r = 4$, then the value of h is closest to:

A 12.3 **B** 121.7 **C** 24.7 **D** 38.8 **E** 10.4

8 The formula $d = \sqrt{\frac{a}{b}}$ transposed to make a the subject is:

A $a = \sqrt{bd}$ **B** $a = d\sqrt{b}$ **C** $a = \frac{d^2}{b^2}$ **D** $a = \sqrt{\frac{d}{b}}$ **E** $a = bd^2$

9 The inequality representing the x values on the number line below is:



A $x < -1$ **B** $x > -1$ **C** $x \leq -1$ **D** $x \geq -1$ **E** $-1 < x < 3$

10 The solution to the inequality $1 - 2x > 9$ is:

A $x < -4$ **B** $x < 4$ **C** $x < -5$ **D** $x > -4$ **E** $x > 5$

11 The solution to the simultaneous equations $x + 2y = 16$ and $y = x - 4$ is:

A $x = 4, y = 0$ **B** $x = 8, y = 4$ **C** $x = 6, y = 2$

D $x = 12, y = 8$ **E** $x = 5, y = 1$

12 The solution to the simultaneous equations $2x + y = 2$ and $2x + 3y = 10$ is:

A $x = 0, y = 2$ **B** $x = 2, y = 2$ **C** $x = 2, y = -2$

D $x = -1, y = 4$ **E** $x = -2, y = 6$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Short-answer questions

1 Write algebraic expressions to represent the following.

a The product of m and 7

b Twice the sum of x and y

c The cost of 3 movie tickets at m dollars each

d n divided by 4 less 3

2 Evaluate the following if $x = 2$, $y = -1$ and $z = 5$.

a $yz - x$ **b** $\dfrac{2z - 4y}{x}$ **c** $\dfrac{2z^2}{x} + y$ **d** $x(y + z)$

3 Simplify the following.

a $2m \times 4n$ **b** $\dfrac{4x^2y}{12x}$ **c** $3ab \times 4b \div (2a)$

d $4 - 5b + 2b$ **e** $3mn + 2m - 1 - nm$ **f** $4p + 3q - 2p + q$

4 Expand and simplify the following.

a $2(x + 7)$ **b** $-3(2x + 5)$ **c** $2x(3x - 4)$

d $-2a(5 - 4a)$ **e** $5 - 4(x - 2)$ **f** $4(3x - 1) - 3(2 - 5x)$

5 Solve the following linear equations for x.

a $5x + 6 = 51$ **b** $\dfrac{x + 2}{4} = 7$ **c** $\dfrac{2x}{5} - 3 = 3$ **d** $\dfrac{2x - 5}{3} = -1$

e $7x - 4 = 10$ **f** $3 - 2x = 21$ **g** $1 - \dfrac{4x}{5} = 9$ **h** $2 - 7x = -3$

6 Write an equation to represent each of the following and then solve it for the pronumeral.

a A number n is doubled and increased by 3 to give 21.

b A number of lollies l is decreased by 5 and then shared equally among 3 friends so that they each get 7.

c 5 less than the result of Toni's age x divided by 4 is 0.

7 Solve the following linear equations.

a $2(x + 4) = 18$ **b** $3(2x - 3) = 2$ **c** $8x = 2x + 24$

d $5(2x + 4) = 7x + 5$ **e** $3 - 4x = 7x - 8$ **f** $1 - 2(2 - x) = 5(x - 3)$

8 Nick makes an initial bid of $\$x$ in an auction for a signed cricket bat. By the end of the auction he has paid \$550, \$30 more than twice his initial bid. Set up and solve an equation to determine Nick's initial bid.

9 Represent each of the following on a number line.

a $x > 1$ **b** $x \leq 7$ **c** $x \geq -4$

d $x < -2$ **e** $2 < x \leq 8$ **f** $-1 < x < 3$

10 Solve the following inequalities.

a $x + 8 < 20$ **b** $2m - 4 > -6$ **c** $3 - 2y \leq 15$

d $\dfrac{7 - 2x}{3} > -9$ **e** $3a + 9 < 7(a - 1)$ **f** $-4x + 2 \leq 5x - 16$

11 A car salesman earns \$800 per month plus a 10% commission on the amount of sales for the month. If he is aiming to earn a minimum of \$3200 a month, what is the possible sales amount that will enable this?

12 Find the value of the unknown in each of the following formulas.

a $E = \sqrt{PR}$ when $P = 90$ and $R = 40$ **b** $v = u + at$ when $v = 20$, $u = 10$, $t = 2$

c $V = \dfrac{1}{3}Ah$ when $V = 20$, $A = 6$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 Rearrange the following formulas to make the pronumeral in brackets the subject.

a $v^2 = u^2 + 2ax$ (x)

b $A = \frac{1}{2}r^2\theta$ (θ)

c $P = RI^2, I > 0$ (I)

d $S = \frac{n}{2}(a + l)$ (a)

14 Solve the following simultaneous equations using an appropriate method.

a $x + 2y = 12$
$x = 4y$

b $2x + 3y = -6$
$y = x - 1$

c $7x - 2y = 6$
$y = 2x + 3$

d $x + y = 15$
$x - y = 7$

e $3x + 2y = -19$
$4x - y = -7$

f $3x - 5y = 7$
$5x + 2y = 22$

15 Billy went to the country show and spent \$78 on a combined total of 9 items including rides and showbags. If each showbag cost \$12 and each ride cost \$7, how many of each did Billy buy?

16 Solve the following quadratic equations.

a $2x^2 = 8$

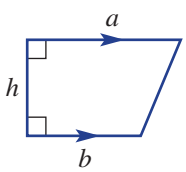
b $5x^2 = 35$ (answer in surd form)

c $-3x^2 = -30$ (answer in surd form)

d $6x^2 + 2 = 8, x < 0$

Extended-response questions

1 The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$. A new backyard deck in the shape of the trapezium shown is being designed.



Currently the dimensions are set such that $a = 12$ m and $h = 10$ m.

a What range of b values is required for an area of at most 110 m^2?

b Rearrange the area formula to make b the subject.

c Use your answer to part **b** to find the length b m required to give an area of 100 m^2.

d Rearrange the area formula to make h the subject.

e If b is set as 8 m, what does the width of the deck (h m) need to be reduced to for an area of 80 m^2?

2 Members of the Hayes and Thompson families attend the local Regatta by the Bay.

a The entry fee for an adult is \$18 and the entry fee for a student is \$8. The father and son from the Thompson family notice that after paying the entry fees and after 5 rides for the son and 3 for the adult, they have each spent the same amount. If the cost of a ride is the same for an adult and a student, write an equation and solve it to determine the cost of a ride.

b For lunch, each family purchases some buckets of hot chips and some drinks.
The Hayes family buys 2 drinks and 1 bucket of chips for \$11 and the Thompson family buys 3 drinks and 2 buckets of chips for \$19. To determine how much each bucket of chips and each drink costs, complete the following:

i Define two pronumerals to represent the problem.

ii Set up two equations relating the pronumerals.

iii Solve your equations in part **b ii** simultaneously.

iv What is the cost of a bucket of chips and the cost of a drink?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library



3 Right-angled triangles

What you will learn

3A Pythagoras' theorem REVISION
3B Finding the shorter sides REVISION
3C Using Pythagoras' theorem to solve two-dimensional problems REVISION
3D Using Pythagoras' theorem to solve three-dimensional problems
3E Introducing the trigonometric ratios
3F Finding unknown sides
3G Solving for the denominator
3H Finding unknown angles
3I Using trigonometry to solve problems
3J Bearings

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY
SUBSTRANDS: RIGHT-ANGLED TRIANGLES (PYTHAGORAS) (S4)
RIGHT-ANGLED TRIANGLES (TRIGONOMETRY) (S5.1/5.2◊)

Outcomes

A student applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems.
(MA4–16MG)

A student applies trigonometry, given diagrams, including problems involving angles of elevation and depression.
(MA5.1–10MG)

A student applies trigonometry to solve problems, including problems involving bearings.
(MA5.2–13MG)

© NESA, 2012

UAVs: Unmanned aerial vehicles

UAVs, such as quadcopters, octocopters and drones, all use the same principles to navigate.

Before UAVs could be developed, the complex dynamics of their flight had to be represented mathematically. These equations were then coded to allow computer control. When in action, the flight controlling program uses data input from several miniature, electronic sensors including a GPS (for location), an accelerometer (for distance travelled) and a magnetometer (for direction of travel).

The GPS device receives time signals and position data from 4 of the 24 GPS satellites and calculates the UAV's distance from each satellite. Pythagoras' theorem and trigonometry is applied in three dimensions to compute the UAV's exact position on the earth giving its latitude, longitude and altitude.

Using data input from the GPS, magnetometer and accelerometer the UAV's computer program applies trigonometry to calculate the direction and distance to fly to a given destination. Using these same principles, UAVs can hold a position in windy conditions or automatically return to the start if its battery is running low or the control signal is broken.

UAVs can fly autonomously following a list of distances and bearings or GPS coordinates and also follow a variety of instructions such as where to photograph, turn, hover, orbit or land.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Calculate each of the following.

a 3^2 **b** 20^2 **c** $2^2 + 4^2$ **d** $50^2 - 40^2$

2 Use a calculator to calculate each of the following correct to 2 decimal places.

a $\sqrt{35}$ **b** $\sqrt{236}$ **c** $\sqrt{23.6}$ **d** $\sqrt{65.4}$

3 Given that x is a positive number, find its value in each of the following equations.

a $x^2 = 4$ **b** $x^2 = 16$ **c** $x^2 = 3^2 + 4^2$ **d** $12^2 + 5^2 = x^2$

4 Given that x is a positive number, solve each equation to find its value.

a $x^2 + 16 = 25$ **b** $x^2 + 9 = 25$ **c** $49 = x^2 + 24$ **d** $100 = x^2 + 36$

5 Round off each number correct to 4 decimal places.

a 0.45678 **b** 0.34569 **c** 0.04562 **d** 0.27997

6 Round off each number correct to 2 decimal places.

a 4.234 **b** 5.678 **c** 76.895 **d** 23.899

7 Solve each of the following equations to find the value of x.

a $3x = 6$ **b** $4x = 12$ **c** $5x = 60$ **d** $8x = 48$

e $\frac{x}{3} = 4$ **f** $\frac{x}{5} = 6$ **g** $\frac{x}{7} = 4$ **h** $\frac{x}{13} = 14$

i $\frac{4x}{3} = 12$ **j** $\frac{6x}{5} = 3$ **k** $\frac{2x}{3} = 8$ **l** $\frac{3x}{4} = 6$

8 Solve each of the following equations to find the value of x correct to 2 decimal places.

a $\frac{x}{3.2} = 4.7$ **b** $\frac{x}{2.1} = 7.43$ **c** $\frac{x}{0.3456} = 4$ **d** $\frac{x}{1.235} = 5.1$

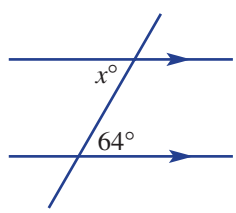
9 Solve each of the following equations to find the value of x correct to 1 decimal place.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 7$ **c** $\frac{32}{x} = 15$ **d** $\frac{14}{x} = 27$

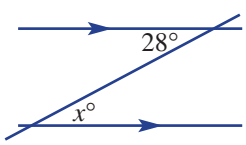
e $\frac{3.8}{x} = 6.9$ **f** $\frac{17}{x} = 8.4$ **g** $\frac{29.34}{x} = 3.24$ **h** $\frac{2.456}{x} = 0.345$

10 Find the value of x in these diagrams.

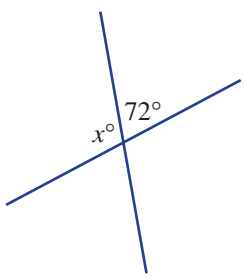
a



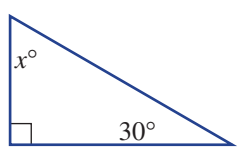
b



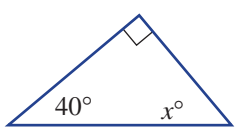
c



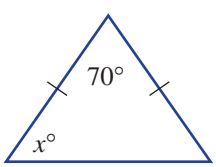
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3A Pythagoras' theorem REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Pythagoras was born on the Greek island of Samos in the 6th century BCE. He received a privileged education and travelled to Egypt and Persia where he developed his ideas in mathematics and philosophy. He settled in Crotone, Italy, where he founded a school. His many students and followers were called the Pythagoreans and under the guidance of Pythagoras, lived a very structured life with strict rules. They aimed to be pure, self-sufficient and wise, where men and women were treated equally and all property was considered communal. They strove to perfect their physical and mental form and made many advances in their understanding of the world through mathematics.

The Pythagoreans discovered the famous theorem, which is named after Pythagoras, and the existence of irrational numbers such as $\sqrt{2}$, which cannot be written down as a fraction or terminating decimal. Such numbers cannot be measured exactly with a ruler with fractional parts and were thought to be unnatural. The Pythagoreans called these numbers 'unutterable' numbers and it is believed that any member of the brotherhood who mentioned these numbers in public would be put to death.



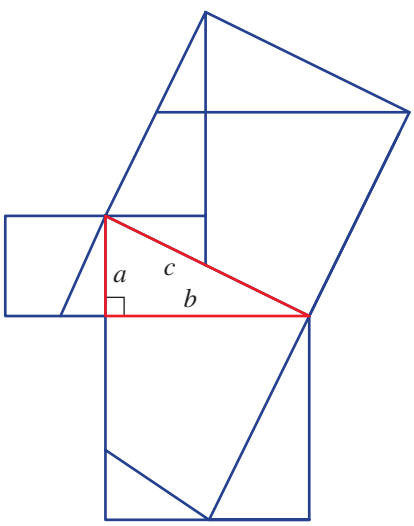
The Pythagorean brotherhood in Ancient Greece.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Matching the areas of squares

Look at this right-angled triangle and the squares drawn on each side. Each square is divided into smaller sections.

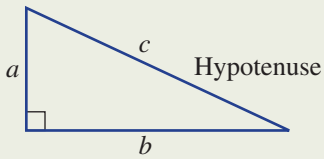
- Can you see how the parts of the two smaller squares would fit into the larger square?
- What is the area of each square if the side lengths of the right-angled triangle are a, b and c as marked?
- What do the answers to the above two questions suggest about the relationship between a, b and c?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- The side opposite the right angle in a right-angled triangle is called the **hypotenuse**. It is the longest side.



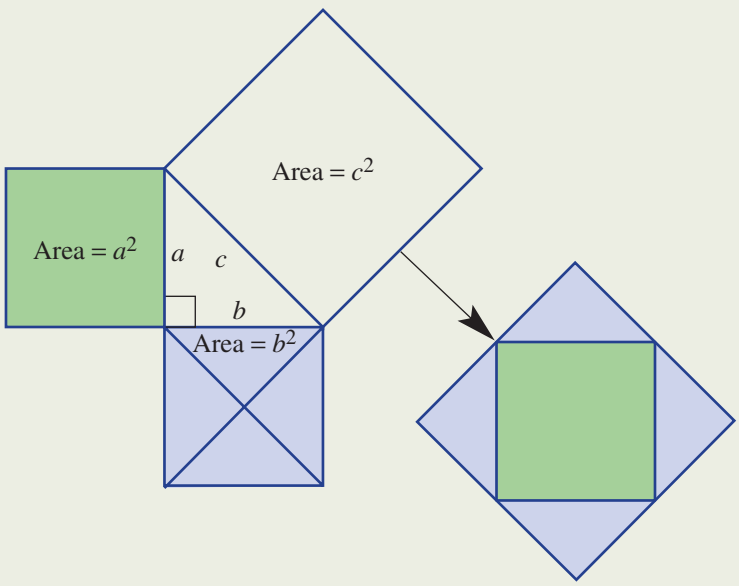
- **Pythagoras' theorem** says that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
  For the triangle shown, it is:

$$c^2 = a^2 + b^2$$

square of the hypotenuse

square of the two shorter sides

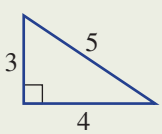
  - The theorem can be illustrated in a diagram like the one shown. The sum of the areas of the

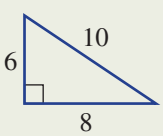


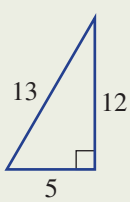
    two smaller squares ($a^2 + b^2$) is the same as the area of the largest square (c^2).

- Lengths can be expressed with **exact values** using **surds**. $\sqrt{2}$, $\sqrt{28}$ and $2\sqrt{3}$ are examples of surds.
  - When expressed as a decimal, a surd is a non-terminating, non-recurring decimal with no pattern, for example, $\sqrt{2} = 1.4142135623\ldots$

- A Pythagorean triad is a set of three whole numbers that can be used to form a right-angled triangle. The best-known examples are:





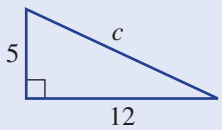


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

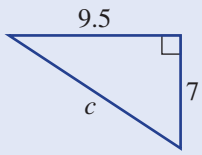
Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse in these right-angled triangles. Round to 2 decimal places in part **b**.

a



b



SOLUTION

a $c^2 = a^2 + b^2$
$= 5^2 + 12^2$
$= 169$
$\therefore\ c = \sqrt{169}$
$= 13$

Alternative method:
$c = 13$ because 5, 12, 13 is a Pythagorean triad.

b $c^2 = a^2 + b^2$
$= 7^2 + 9.5^2$
$= 139.25$
$\therefore\ c = \sqrt{139.25}$
$= 11.80$ (to 2 d.p.)

EXPLANATION

Write the rule and substitute the lengths of the two shorter sides.

If $c^2 = 169$ then $c = \sqrt{169} = 13$.

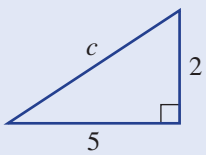
If you know some of the triangles that form a Pythagorean triad, you can use them to write down the length of missing sides.

The order for a and b does not matter since $7^2 + 9.5^2 = 9.5^2 + 7^2$.

Round as required.

Example 2 Finding the length of the hypotenuse using exact values

Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



SOLUTION

$c^2 = a^2 + b^2$
$= 5^2 + 2^2$
$= 29$
$\therefore\ c = \sqrt{29}$

EXPLANATION

Apply Pythagoras' theorem to find the value of c.

Express the answer exactly using a surd.

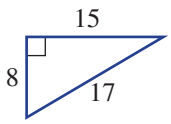
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 3A REVISION

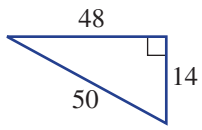
UNDERSTANDING AND FLUENCY | 1–7 | 3–8(½) | 4–8(½)

1 State the length of the hypotenuse in these right-angled triangles.

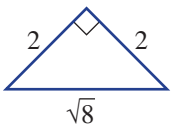
a



b

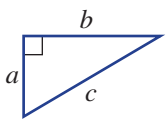


c

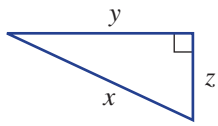


2 Write down Pythagoras' theorem using the given pronumerals for these right-angled triangles. For example: $z^2 = x^2 + y^2$.

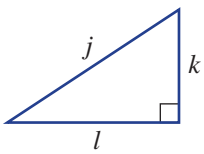
a



b



c



3 Evaluate the following, rounding to 2 decimal places in parts **g** and **h**.

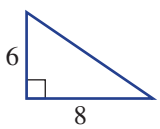
a 9^2 **b** 3.2^2 **c** $3^2 + 2^2$ **d** $9^2 + 5^2$

e $\sqrt{36}$ **f** $\sqrt{64 + 36}$ **g** $\sqrt{24}$ **h** $\sqrt{3^2 + 2^2}$

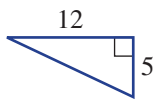
Example 1a

4 Find the length of the hypotenuse in each of the following right-angled triangles.

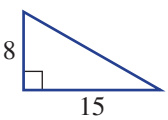
a



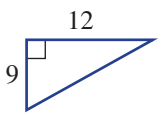
b



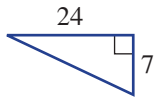
c



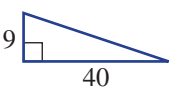
d



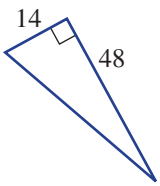
e



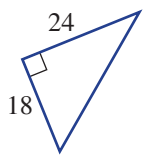
f



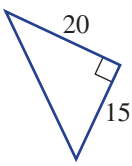
g



h



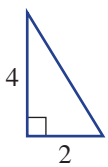
i



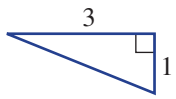
Example 1b

5 Find the length of the hypotenuse in each of these right-angled triangles, correct to 2 decimal places.

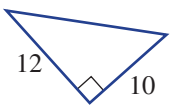
a



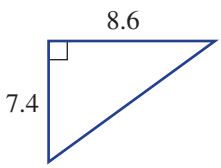
b



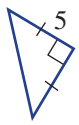
c



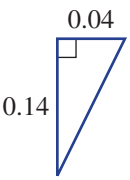
d



e



f

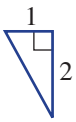


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

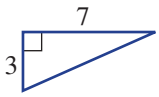
Example 2

6 Find the length of the hypotenuse in these triangles, leaving your answer as an exact value.

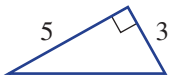
a



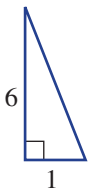
b



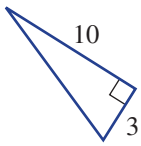
c



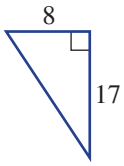
d



e

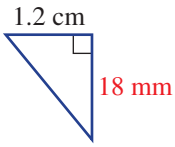


f

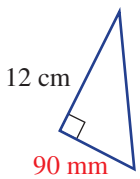


7 Find the length of the hypotenuse in each of these right-angled triangles, rounding to 2 decimal places where necessary. Convert to the units indicated in red.

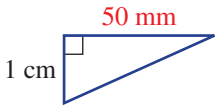
a



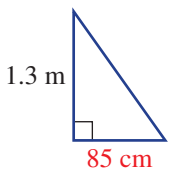
b



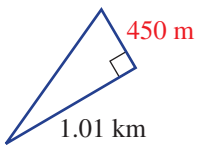
c



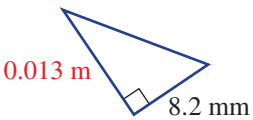
d



e

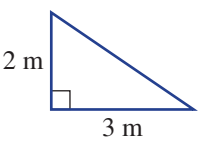


f

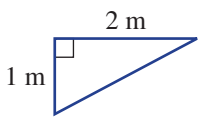


8 For each of these triangles, first calculate the length of the hypotenuse then find the perimeter, correct to 2 decimal places.

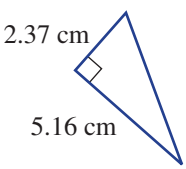
a



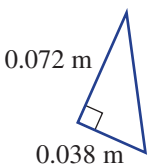
b



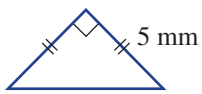
c



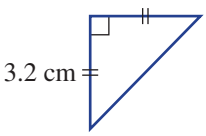
d



e

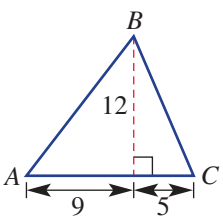


f



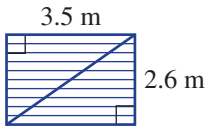
PROBLEM-SOLVING AND REASONING 9–11, 15 | 9–11, 13, 15, 16(½) | 11–14, 16(½), 17

9 Find the perimeter of this triangle. *Hint*: you will need to find *AB* and *BC* first.



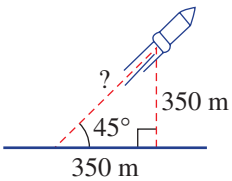
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Find the length of the diagonal steel brace required to support a wall of length 3.5 m and height 2.6 m. Give your answer correct to 1 decimal place.

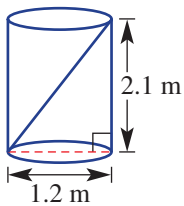


11 A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 200 m from a beacon on the ground. Find the direct distance of the helicopter from the beacon.

12 A miniature rocket blasts off at an angle of 45° and, after a few seconds, reaches a height of 350 m above the ground. At this point it has also covered a horizontal distance of 350 m. How far has the rocket travelled to the nearest metre?



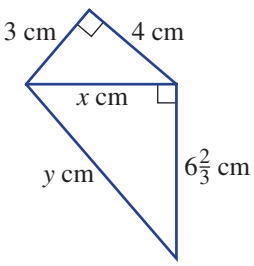
13 Find the length of the longest rod that will fit inside a cylinder of height 2.1 m and with circular end surface of 1.2 m diameter. Give your answer correct to 1 decimal place.



14 For the shape on the right, find the value of:

a x

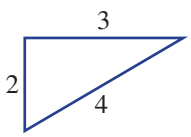
b y (as a fraction)



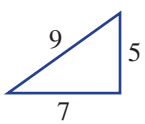
15 One way to check whether a four-sided figure is a rectangle is to ensure that both its diagonals are the same length. What should the length of the diagonals be if a rectangle has side lengths 3 m and 5 m? Answer to 2 decimal places.

16 We know that if the triangle has a right angle, then $c^2 = a^2 + b^2$. The converse of this is that if $c^2 = a^2 + b^2$, then the triangle must have a right angle. Test if $c^2 = a^2 + b^2$ to see if these triangles must have a right angle. They may not be drawn to scale.

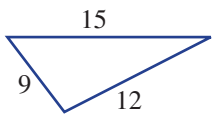
a



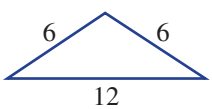
b



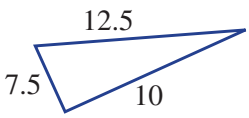
c



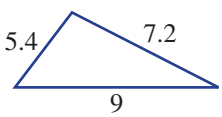
d



e



f

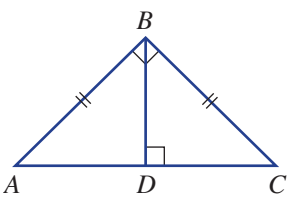


17 Triangle ABC is a right-angled isosceles triangle, and BD is perpendicular to AC. If $DC = 4$ cm and $BD = 4$ cm:

a find the length of BC correct to 2 decimal places

b state the length of AB and hence AC correct to 2 decimal places

c use Pythagoras' theorem and ΔABC to check that the length of AC is twice the length of DC

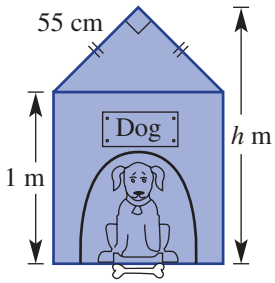


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

ENRICHMENT — — 18, 19

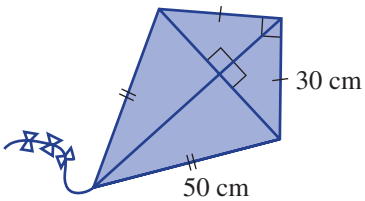
Kennels and kites

18 A dog kennel has the dimensions shown in the diagram on the right. Give your answers to each of the following correct to 2 decimal places.



- **a** What is the width of the kennel?
- **b** What is the total height, h m, of the kennel?
- **c** If the sloping height of the roof was to be reduced from 55 cm to 50 cm, what difference would this make to the total height of the kennel? (Assume that the width is the same as in part **a**.)
- **d** What is the length of the sloping height of the roof of a new kennel if it is to have a total height of 1.2 m? (The height of the kennel without the roof is still 1 m and its width is unchanged.)

19 The frame of a kite is constructed with six pieces of timber dowel. The four pieces around the outer edge are two 30 cm pieces and two 50 cm pieces. The top end of the kite is to form a right angle. Find the length of each of the diagonal pieces required to complete the construction. Answer to 2 decimal places.





ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3B Finding the shorter sides REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Throughout history, mathematicians have utilised known theorems to explore new ideas, discover new theorems and solve a wider range of problems. Similarly, Pythagoras knew that his right-angled triangle theorem could be manipulated so that the length of one of the shorter sides of a triangle can be found if the length of the other two sides are known.

We know that the sum $7 = 3 + 4$ can be written as a difference $3 = 7 - 4$. Likewise, if $c^2 = a^2 + b^2$, then $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.

Applying this to a right-angled triangle means that we can now find the length of one of the shorter sides if the other two sides are known.

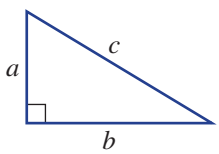


If we know the length of the crane jib and the horizontal distance it extends, Pythagoras' theorem enables us to calculate its vertical height.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: True or false

Below are some mathematical statements relating to a right-angled triangle with hypotenuse c and the two shorter sides a and b.



Some of these mathematical statements are true and some are false. Can you sort them into true and false groups?

$a^2 + b^2 = c^2$ $\quad a = \sqrt{c^2 - b^2}$ $\quad c^2 - a^2 = b^2$ $\quad a^2 - c^2 = b^2$

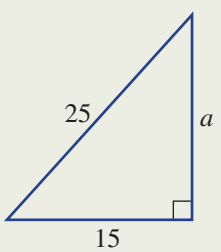
$c = \sqrt{a^2 + b^2}$ $\quad b = \sqrt{a^2 - c^2}$ $\quad c = \sqrt{a^2 - b^2}$ $\quad c^2 - b^2 = a^2$

Key ideas

- When finding the length of a side:
  - substitute known values into Pythagoras' theorem.
  - solve this equation to find the unknown value.

  For example:
  - If $a^2 + 16 = 30$, then subtract 16 from both sides.
  - If $a^2 = 14$, then take the square root of both sides.
  - $a = \sqrt{14}$ is an **exact** answer (a surd).
  - $a = 3.74$ is a rounded decimal answer.

$c^2 = a^2 + b^2$
$25^2 = a^2 + 15^2$
$625 = a^2 + 225$
$400 = a^2$
$a = 20$

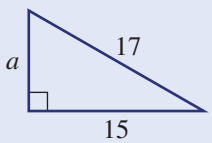


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

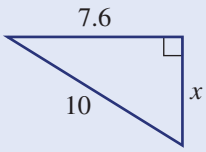
Example 3 Finding a shorter side

In each of the following, find the value of the pronumeral. Round your answer in part **b** to 2 decimal places and give an exact answer to part **c**.

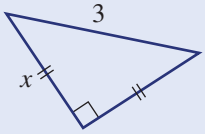
a



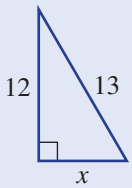
b



c



d



SOLUTION

a $a^2 + 15^2 = 17^2$
$a^2 + 225 = 289$
$a^2 = 64$
$\therefore a = \sqrt{64}$
$a = 8$

b $x^2 + 7.6^2 = 10^2$
$x^2 + 57.76 = 100$
$x^2 = 42.24$
$x = \sqrt{42.24}$
$x = 6.50$ (to 2 d.p.)

c $x^2 + x^2 = 3^2$
$2x^2 = 9$
$x^2 = \frac{9}{2}$
$\therefore x = \sqrt{\frac{9}{2}}$

d $x = 5$ because 5, 12, 13 is a Pythagorean triad.

EXPLANATION

Write the rule and substitute the known sides.
Square 15 and 17.
Subtract 225 from both sides.
Take the square root of both sides.

Write the rule.
Subtract 57.76 from both sides.
Take the square root of both sides.
Round to 2 decimal places.

Two sides are of length x.
Add like terms.
Divide both sides by 2.
Take the square root of both sides. To express as an exact answer, do not round.

If you know some triads, you can use them to write down the length of the unknown side. Alternatively, use the rule.

Exercise 3B REVISION

UNDERSTANDING AND FLUENCY	1–5	2, 3–5(½), 6	3–5(½), 6

1 Find the value of a or b in these equations. (Both a and b are positive numbers.)

a $a = \sqrt{196}$ **b** $a = \sqrt{121}$ **c** $a^2 = 144$ **d** $a^2 = 400$
e $b^2 + 9 = 25$ **f** $b^2 + 49 = 625$ **g** $36 + b^2 = 100$ **h** $15^2 + b^2 = 17^2$

2 If $a^2 + 64 = 100$, decide if the following are true or false.

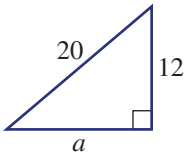
a $a^2 = 100 - 64$ **b** $64 = 100 + a^2$ **c** $100 = \sqrt{a^2 + 64}$
d $a = \sqrt{100 - 64}$ **e** $a = 6$ **f** $a = 10$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

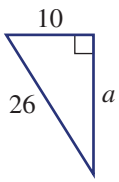
Example 3a, d

3 In each of the following find the value of the pronumeral.

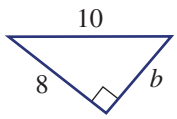
a



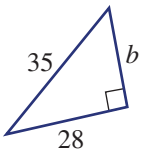
b



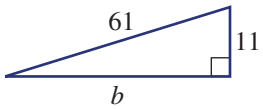
c



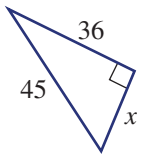
d



e



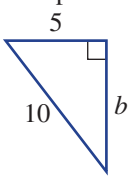
f



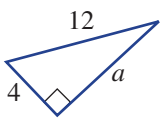
Example 3b

4 In each of the following, find the value of the pronumeral. Express your answers correct to 2 decimal places.

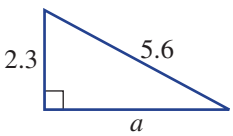
a



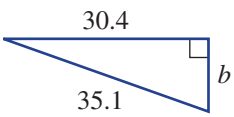
b



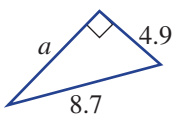
c



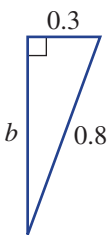
d



e

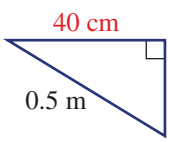


f

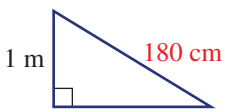


5 Find the length of the unknown side of each of these triangles, correct to 2 decimal places where necessary. Convert to the units shown in red.

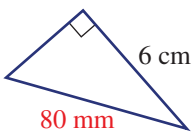
a



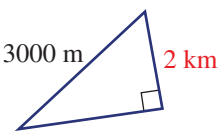
b



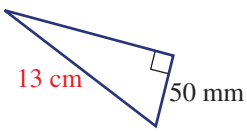
c



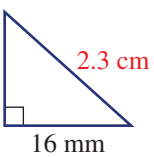
d



e



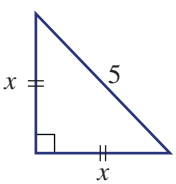
f



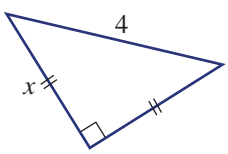
Example 3c

6 In each of the following, find the value of x as an exact answer.

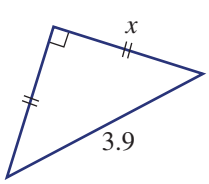
a



b



c

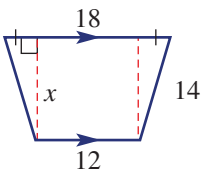


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

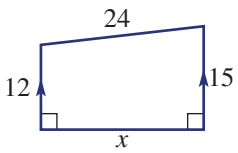
PROBLEM-SOLVING AND REASONING 7–9, 12 | 7, 8, 10, 12 | 9–11, 12, 13

7 For each of the following diagrams, find the value of x. Give an exact answer each time.

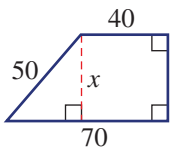
a



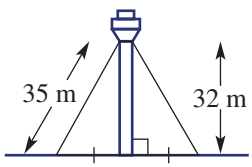
b



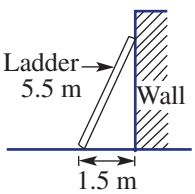
c



8 A 32 m communication tower is supported by 35 m cables stretching from the top of the tower to a position at ground level. Find the distance from the base of the tower to the point where the cable reaches the ground, correct to 1 decimal place.

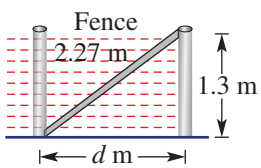


9 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, find how high the top of the ladder is above the ground, correct to 1 decimal place.



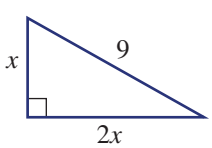
10 If a television has a screen size of 63 cm it means that the diagonal length of the screen is 63 cm. If the vertical height of a 63 cm screen is 39 cm, find how wide the screen is to the nearest centimetre.

11 A 1.3 m vertical fence post is supported by a 2.27 m bar, as shown in the diagram on the right. Find the distance (d metres) from the base of the post to where the support enters the ground. Give your answer correct to 2 decimal places.

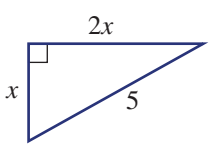


12 For these questions note that $(2x)^2 = 4x^2$ and $(3x)^2 = 9x^2$.
In each of the following find the value of x as an exact answer.

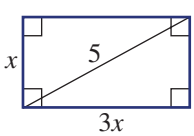
a



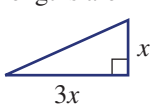
b



c



13 A right-angled triangle has a hypotenuse measuring 5 m. Find the lengths of the other sides if their lengths are in the given ratio. Give an exact answer. *Hint*: you can draw a triangle like this for part **a**.



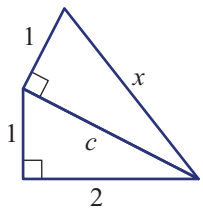
a 1 to 3 **b** 2 to 3 **c** 5 to 7

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

ENRICHMENT — — 14, 15

The power of exact values

14 Consider this diagram and the unknown length x.



a Explain what needs to be found before x can be calculated.

b Now try calculating the value x as an exact value.

c Was it necessary to calculate the value of c or was c^2 enough?

d What problems might be encountered if the value of c was calculated and rounded off before the value of x is found?

15 In the diagram, $OD = 3$ and $AB = BC = CD = 1$.

a Using exact values find the length of:

i OC

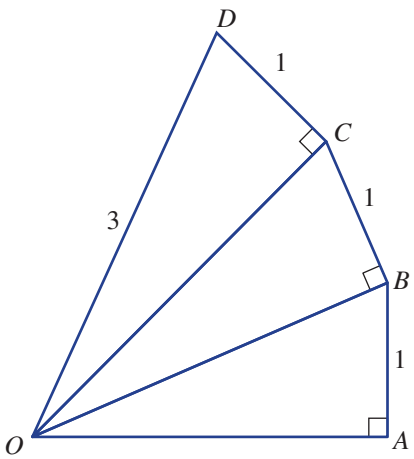
ii OB

iii OA

b Round your answer in part **a iii** to 1 decimal place and use that length to recalculate the lengths of OB, OC and OD (correct to 2 decimal places) starting with ΔOAB.

c Explain the difference between the given length $OD = 3$ and your answer for OD in part **b**.

d Investigate how changing the side length AB affects your answers to parts **a** to **c**.





Pythagoras' theorem can be used to ensure that the corners of land plots and building foundations are right angles.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3C Using Pythagoras' theorem to solve two-dimensional problems REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Initially it may not be obvious that Pythagoras' theorem can be used to help solve a particular problem. With further investigation, however, it may be possible to identify and draw in a right-angled triangle that can help solve the problem. As long as two sides of the right-angled triangle are known, the length of the third side can be found.

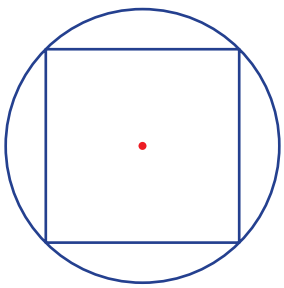


The length of each cable on the Anzac Bridge, Sydney, can be calculated using Pythagoras' theorem.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The biggest square

Imagine trying to cut the largest square from a circle of a certain size and calculating the side length of the square. Drawing a simple diagram as shown does not initially reveal a right-angled triangle.



- If the circle has a diameter of 2 cm, can you find a good position to draw the diameter that also helps to form a right-angled triangle?
- Can you determine the side length of the largest square?
- What percentage of the area of a circle does the largest square occupy?

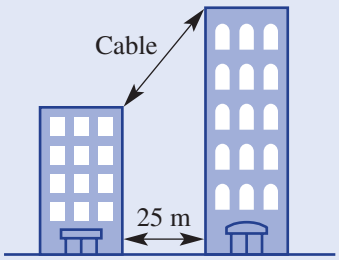
Key ideas

- When applying Pythagoras' theorem:
  - Identify and draw right-angled triangles that may help to solve the problem.
  - Label the sides with their lengths or with a letter (pronumeral) if the length is unknown.
  - Use Pythagoras' theorem to solve for the unknown.
  - Solve the problem by making any further calculations and answering in words.
  - Check that your answer makes sense.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

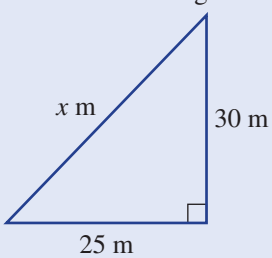
Example 4 Applying Pythagoras' theorem

Two skyscrapers are located 25 m apart and a cable links the tops of the two buildings. Find the length of the cable if the buildings are 50 m and 80 m in height. Give your answer correct to 2 decimal places.



SOLUTION

Let x m be the length of the cable.

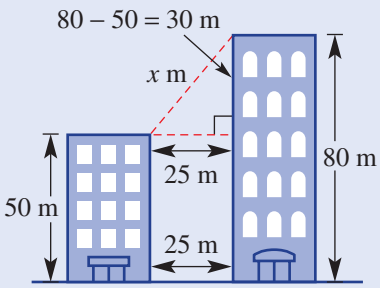


$c^2 = a^2 + b^2$
$x^2 = 25^2 + 30^2$
$x^2 = 625 + 900$
$= 1525$
$\therefore x = \sqrt{1525}$
$= 39.05$ (to 2 d.p.)

$\therefore$ The cable is 39.05 m long.

EXPLANATION

Draw a right-angled triangle and label the measurements and pronumerals.



Set up an equation using Pythagoras' theorem and solve for x.

Answer the question in words.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 3C REVISION

UNDERSTANDING AND FLUENCY	1–5	1, 2, 4, 6	2, 4, 6(½)

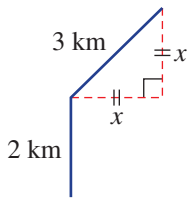
1 Match each problem (**a**, **b** or **c**) with both a diagram (**A**, **B** or **C**) and its solution (**I**, **II**, **III**).

a Two trees stand 20 m apart and they are 32 m and 47 m tall. What is the distance between the tops of the two trees?

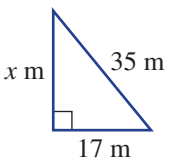
b A man walks due north for 2 km then north-east for 3 km. How far north is he from his starting point?

c A kite is flying with a kite string of length 35 m. Its horizontal distance from its anchor point is 17 m. How high is the kite flying?

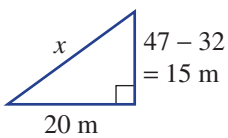
A



B



C

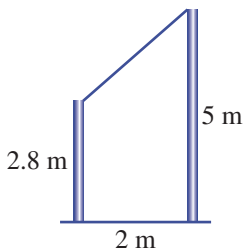


I The kite is flying at a height of 30.59 m.

II The distance between the top of the two trees is 25 m.

III The man has walked a total of $2 + 2.12 = 4.12$ km north from his starting point.

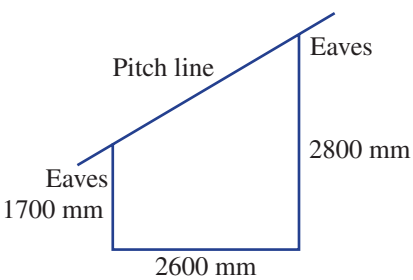
2 Two poles are located 2 m apart. A wire links the tops of the two poles. Find the length of the wire if the poles are 2.8 m and 5 m in height. Give your answer correct to 1 decimal place.



Example 4

3 Two skyscrapers are located 25 m apart and a cable of length 62.3 m links the tops of the two buildings. If the taller building is 200 metres tall, what is the height of the shorter building? Give your answer correct to 1 decimal place.

4 A garage is to be built with a skillion roof (a roof with a single slope). The measurements are given in the diagram. Calculate the pitch line length, to the nearest millimetre. Allow 500 mm for each of the eaves.

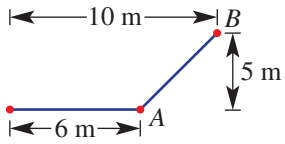


5 Two bushwalkers are standing on different mountain sides. According to their maps, one of them is at a height of 2120 m and the other is at a height of 1650 m. If the horizontal distance between them is 950 m, find the direct distance between the two bushwalkers. Give your answer correct to the nearest metre.

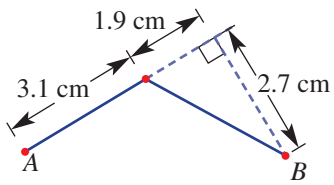
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Find the direct distance between the points A and B in each of the following, correct to 1 decimal place.

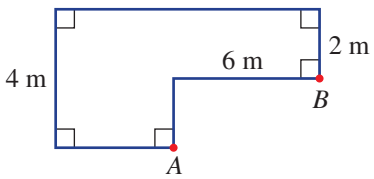
a



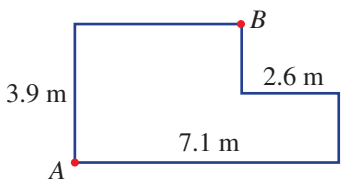
b



c



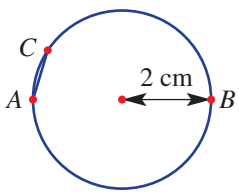
d



PROBLEM-SOLVING AND REASONING	7, 8, 11	8, 9, 11	8–12

7 A 100 m radio mast is supported by six cables in two sets of three cables. They are anchored to the ground at an equal distance from the mast. The top set of three cables is attached at a point 20 m below the top of the mast. Each cable in the lower set of three cables is 60 m long and is attached at a height of 30 m above the ground. If all the cables have to be replaced, find the total length of cable required. Give your answer correct to 2 decimal places.

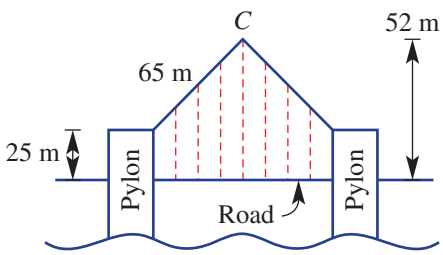
8 In a particular circle of radius 2 cm, AB is a diameter and C is a point on the circumference. Angle ACB is a right angle. The chord AC is 1 cm in length.



a Draw the triangle ABC as described, and mark in all the important information.

b Find the length of BC correct to 1 decimal place.

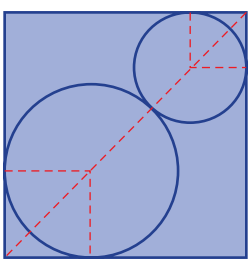
9 A suspension bridge is built with two vertical pylons and two straight beams of equal length that are positioned to extend from the top of the pylons to meet at a point C above the centre of the bridge, as shown in the diagram on the right.



a Calculate the vertical height of the point C above the tops of the pylons.

b Calculate the distance between the pylons, that is, the length of the span of the bridge correct to 1 decimal place.

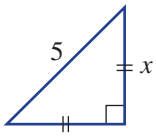
10 Two circles of radius 10 cm and 15 cm respectively are placed inside a square. Find the perimeter of the square to the nearest centimetre. *Hint*: first find the diagonal length of the square using the diagram below.



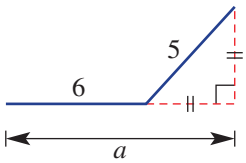
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 It is possible to find the length of the shorter sides of a right isosceles triangle if only the hypotenuse length is known.

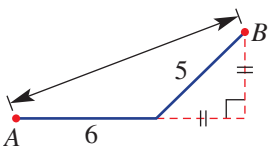
a Find the exact value of x in this right isosceles triangle.



b Now find the exact value of a in this diagram.



c Finally, use your results from above to find the length of AB in this diagram correct to 1 decimal place.



12 Use the method outlined in Question **11** for this problem.

In an army navigation exercise, a group of soldiers hiked due south from base camp for 2.5 km to a water hole. From there, they turned 45° to the left, to head south-east for 1.6 km to a resting point. When the soldiers were at the resting point, how far (correct to 1 decimal place):

a east were they from the water hole?

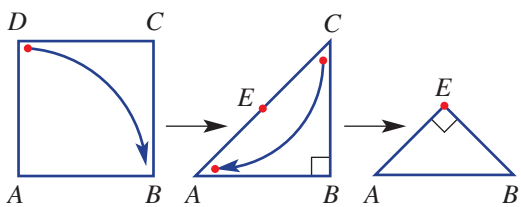
b south were they from the water hole?

c were they in a straight line from base camp?

ENRICHMENT — — 13

Folding paper

13 A square piece of paper, $ABCD$, of side length 20 cm is folded to form a right-angled triangle ABC. The paper is folded a second time to form a right-angled triangle ABE as shown in the diagram below.



a Find the length of AC correct to 2 decimal places.

b Find the perimeter of each of the following, correct to 1 decimal place where necessary:

i square $ABCD$ **ii** triangle ABC **iii** triangle ABE

c Use Pythagoras' theorem and your answer for part **a** to confirm that $AE = BE$ in triangle ABE.

d Investigate how changing the initial side length changes the answers to the above.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3D Using Pythagoras' theorem to solve three-dimensional problems

Interactive

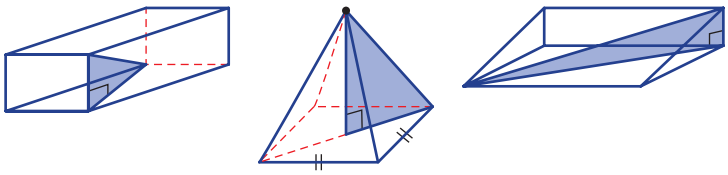
Widgets

HOTsheets

Walkthrough

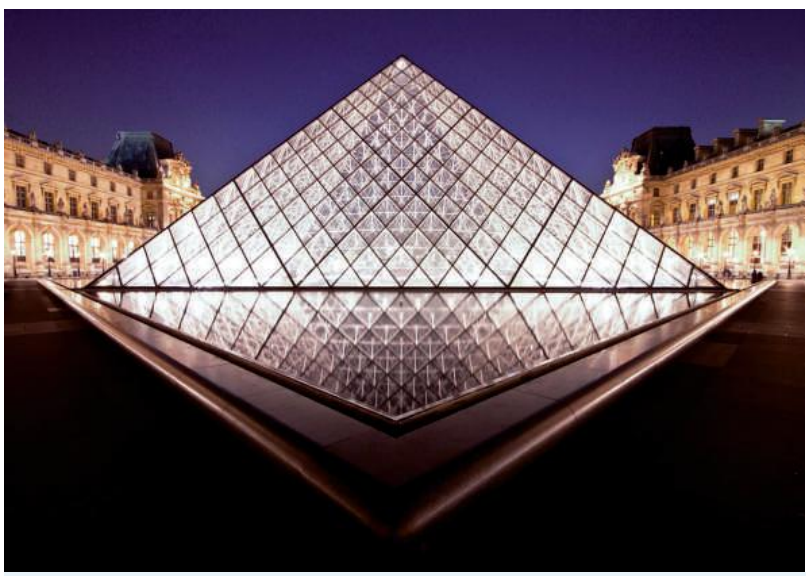
If you cut a solid to form a cross-section a two-dimensional shape is revealed. From that cross-section it may be possible to identify a right-angled triangle that can be used to find unknown lengths. These lengths can then tell us information about the three-dimensional solid.

You can visualise right-angled triangles in all sorts of different solids.



Stage

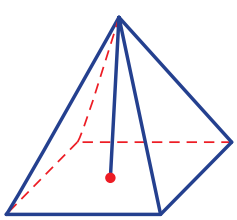
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



The glass pyramid at the Palais du Louvre, Paris, is made up of a total of 70 triangular and 603 rhombus-shaped glass segments together forming many right-angled triangles.

Let's start: How many triangles in a pyramid?

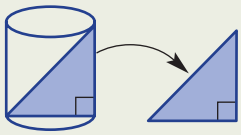
Here is a drawing of a square-based pyramid. By drawing lines from any vertex to the centre of the base and another point, how many different right-angled triangles can you visualise and draw? The triangles could be inside or on the outside surface of the pyramid.

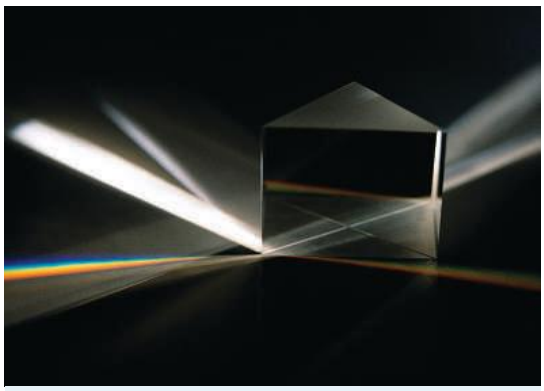


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Right-angled triangles can be identified in many three-dimensional solids.
- It is important to try to draw any identified right-angled triangle using a separate diagram.

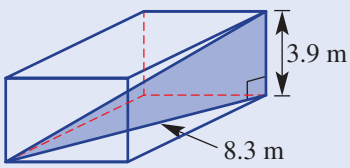




The cross-sections of this glass prism will be either rectangles or triangles depending on how it is cut.

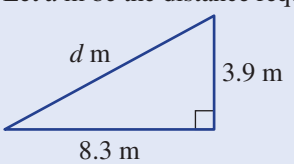
Example 5 Using Pythagoras in 3D

The length of the diagonal on the base of a rectangular prism is 8.3 m and the rectangular prism's height is 3.9 m. Find the distance from one corner of the rectangular prism to the opposite corner. Give your answer correct to 2 decimal places.



SOLUTION

Let d m be the distance required.



$d^2 = 3.9^2 + 8.3^2$
$= 84.1$
$\therefore\ d = 9.17$ (to 2 d.p.)

The distance from one corner of the rectangular prism to the opposite corner is approximately 9.17 m.

EXPLANATION

Draw a right-angled triangle and label all the measurements and pronumerals.

Use Pythagoras' theorem.
Round $\sqrt{84.1}$ to 2 decimal places.

Write your answer in words.

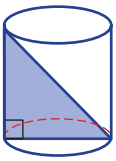
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 3D

UNDERSTANDING AND FLUENCY	1–5	1, 2–3(a, b), 5, 6	2–4(c), 5, 6

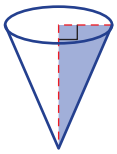
1 Decide if the following shaded regions form right-angled triangles.

a



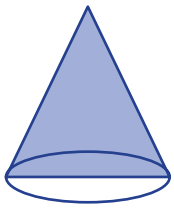
Cylinder

b



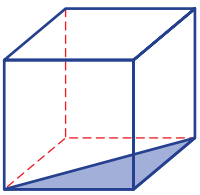
Cone

c



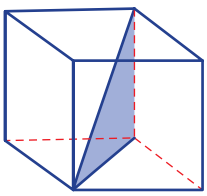
Cone

d



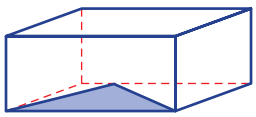
Cube

e



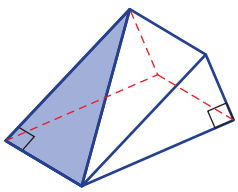
Cube

f



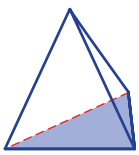
Rectangular prism

g



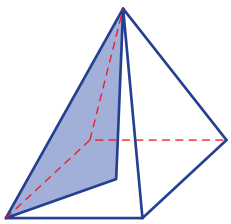
Triangular prism

h



Tetrahedron
regular triangular-based pyramid

i

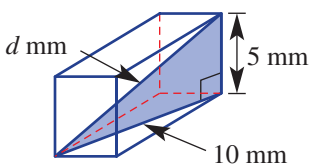


Right square-based pyramid (apex above centre of base)

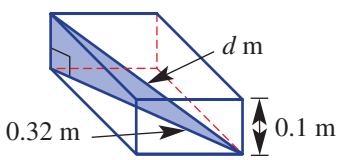
Example 5

2 Find the distance, d units, from one corner to the opposite corner in each of the following rectangular prisms. Give your answers correct to 2 decimal places.

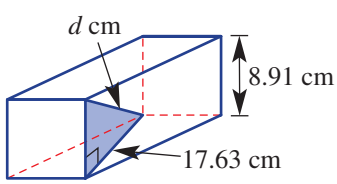
a



b



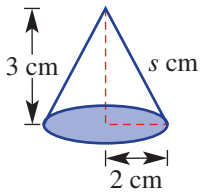
c



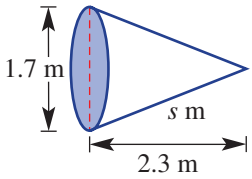
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 Find the slant height, s units, of each of the following cones. Give your answers correct to 1 decimal place.

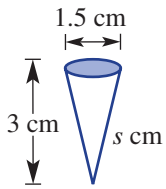
a



b



c

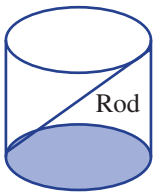


4 Find the length to the nearest millimetre of the longest rod that will fit inside a cylinder of the following dimensions.

a Diameter 10 cm and height 15 cm

b Radius 2.8 mm and height 4.2 mm

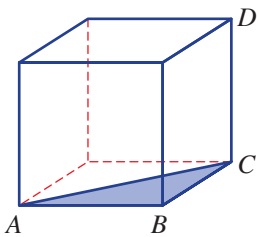
c Diameter 0.034 m and height 0.015 m



5 The cube in the diagram on the right has 1 cm sides.

a Find the length of AC as an exact value.

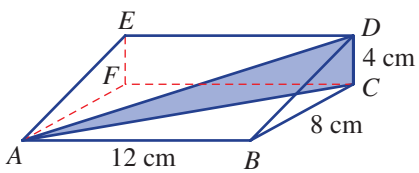
b Hence, find the length of AD correct to 1 decimal place.



6 For the shape shown:

a Find the length of AC as an exact value.

b Hence, find the length of AD correct to 1 decimal place.



PROBLEM-SOLVING AND REASONING 7, 10 | 7, 8, 10 | 8–11

7 A miner makes claim to a circular piece of land with a radius of 40 m from a given point, and is entitled to dig to a depth of 25 m. If the miner can dig tunnels at any angle, find the length of the longest straight tunnel that he can dig, to the nearest metre.



8 A bowl is in the shape of a hemisphere (half sphere) with radius 10 cm. The surface of the water in the bowl has a radius of 7 cm. How deep is the water? Give your answer to 1 decimal place.

9 A cube of side length s sits inside a sphere of radius r so that the vertices of the cube sit on the sphere. Find the ratio $r : s$.

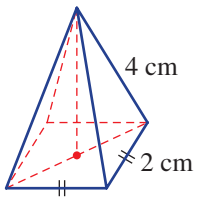
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 There are different ways to approach finding the height of a pyramid depending on what information is given. For each of the following square-based pyramids, find the:

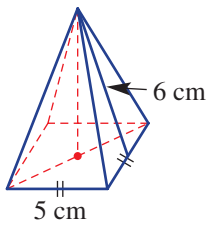
i exact length (using a surd) of the diagonal on the base

ii height of the pyramid correct to 2 decimal places

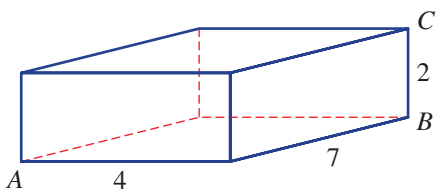
a



b



11 For this rectangular prism answer these questions.



a Find the exact length AB.

b Find AB correct to 2 decimal places.

c Find the length AC using your result from part **a** and then round to 2 decimal places.

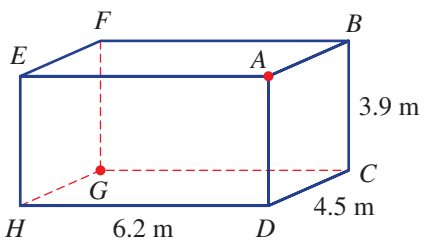
d Find the length AC using your result from part **b** and then round to 2 decimal places.

e How can you explain the difference between your results from parts **c** and **d** above?

ENRICHMENT — — 12

Spider crawl

12 A spider crawls from one corner, A, of the ceiling of a room to the opposite corner, G, on the floor. The room is a rectangular prism with dimensions as given in the diagram below.



a Find how far (correct to 2 decimal places) the spider crawled if it crawls from A to G via:

i B

ii C

iii D

iv F

b Investigate other paths to determine the shortest distance that the spider could crawl in order to travel from point A to point G.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3E Introducing the trigonometric ratios

Interactive
Widgets
HOTsheets
Walkthrough

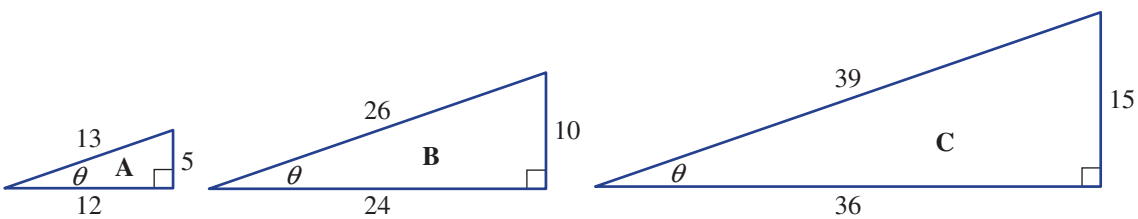
The branch of mathematics called trigonometry deals with the relationship between the side lengths and angles in triangles. Trigonometry dates back to the ancient Egyptian and Babylonian civilisations where a basic form of trigonometry was used in the building of pyramids and in the study of astronomy. The first table of values including chord and arc lengths on a circle for a given angle was created by Hipparchus in the 2nd century BCE in Greece. These tables of values helped to calculate the position of the planets. About three centuries later, Claudius Ptolemy advanced the study of trigonometry, writing 13 books called the *Almagest*. Ptolemy also developed tables of values linking the sides and angles of a triangle and produced many theorems which use the sine, cosine and tangent functions.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

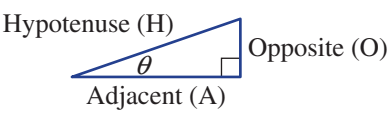
Let's start: What is the sine ratio?

The triangles below are similar:

- The sides of A have been multiplied by 2 to give B.
- The sides of A have been multiplied by 3 to give C.
- The three angles in A are the same as in B and C.
- The Greek letter θ (theta) has been used to mark an angle.



In this topic, the three sides are labelled as follows:



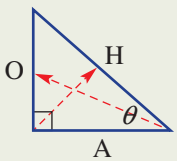
- In triangle A, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square}$ (in simplest form)
- In triangle B, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)
- In triangle C, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)

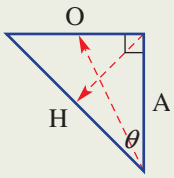
When a right-angled triangle is enlarged, the angles stay the same and the sine ratio remains constant.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- In this topic, the Greek letter θ (theta) is often used to label an unknown angle.
- If a right-angled triangle has another angle, θ, then the:
  - longest side opposite the right angle is called the **hypotenuse** (H)
  - side opposite θ is called the **opposite** (O)
  - remaining side is called the **adjacent** (A).

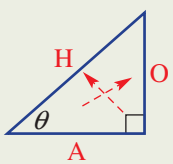




- For a right-angled triangle with a given angle θ, the three ratios **sine** (sin), **cosine** (cos) and **tangent** (tan) are given by the:
  - sine ratio: $\sin\theta = \dfrac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
  - cosine ratio: $\cos\theta = \dfrac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
  - tangent ratio: $\tan\theta = \dfrac{\text{length of the opposite side}}{\text{length of the adjacent side}}$
- The word **SOHCAHTOA** is useful when trying to memorise and recall the three ratios.

SOH CAH TOA

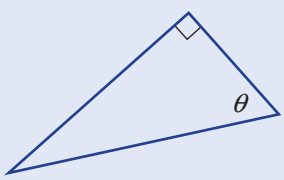
$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$



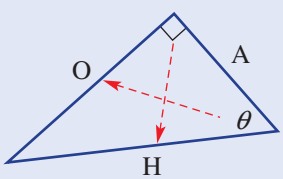
- When a right-angled triangle is enlarged using a scale factor, the three ratios remain constant.

Example 6 Labelling the sides of triangles

Copy this triangle and label the sides as opposite to θ (O), adjacent to θ (A) or hypotenuse (H).



SOLUTION



EXPLANATION

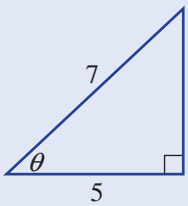
Draw the triangle and label the side opposite the right angle as hypotenuse (H), the side opposite the angle θ as opposite (O) and the remaining side next to the angle θ as adjacent (A).

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

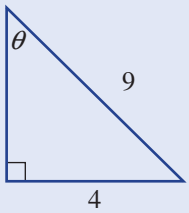
Example 7 Writing trigonometric ratios

Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles.

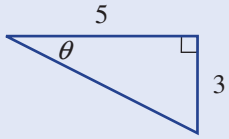
a



b



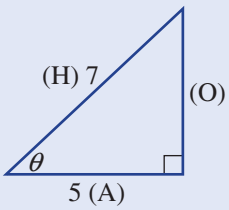
c



SOLUTION

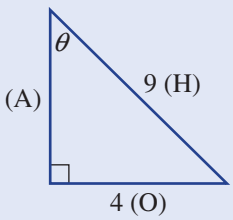
a $\cos\theta = \frac{A}{H}$

$= \frac{5}{7}$



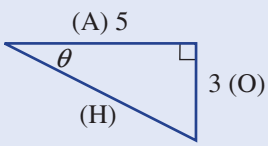
b $\sin\theta = \frac{O}{H}$

$= \frac{4}{9}$



c $\tan\theta = \frac{O}{A}$

$= \frac{3}{5}$



EXPLANATION

Side length 7 is opposite the right angle so it is the hypotenuse (H). Side length 5 is adjacent to angle θ so it is the adjacent (A).

Side length 9 is opposite the right angle so it is the hypotenuse (H). Side length 4 is opposite angle θ so it is the opposite (O).

Side length 5 is the adjacent side to angle θ so it is the adjacent (A). Side length 3 is opposite angle θ so it is the opposite (O).

Exercise 3E

UNDERSTANDING AND FLUENCY	1, 2(½), 3, 4(½), 5, 6	3, 4(½), 5–7	4(½), 5–7

1 Write the missing word in these sentences.

a H stands for the word _____.

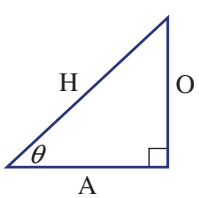
b O stands for the word _____.

c A stands for the word _____.

d $\sin\theta$ = _____ ÷ hypotenuse.

e $\cos\theta$ = adjacent ÷ _____.

f $\tan\theta$ = opposite ÷ _____.

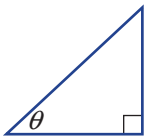


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

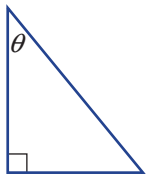
Example 6

2 Copy each of these triangles and label the sides as opposite to θ (O), adjacent to θ (A) or hypotenuse (H).

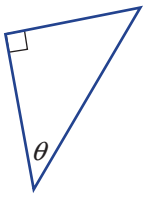
a



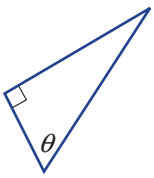
b



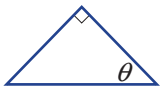
c



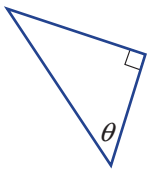
d



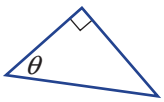
e



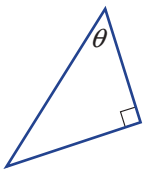
f



g

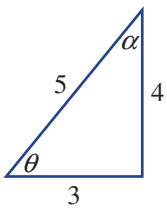


h



3 For the triangle shown, state the length of the side which corresponds to the:

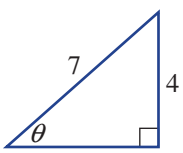
a hypotenuse
b side opposite angle θ
c side opposite angle α
d side adjacent to angle θ
e side adjacent to angle α



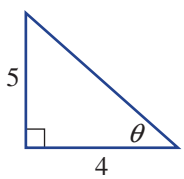
Example 7

4 Use the given sides to write a trigonometric ratio (in fraction form) for each of the following triangles and simplify where possible.

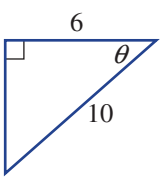
a



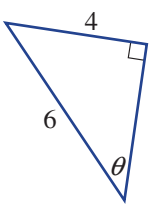
b



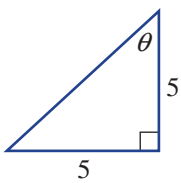
c



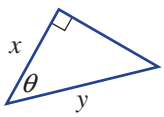
d



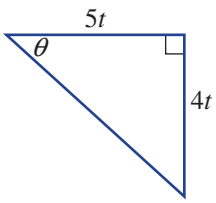
e



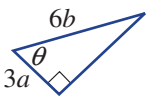
f



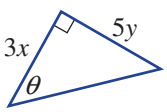
g



h

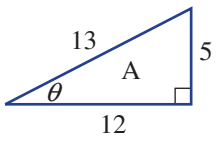


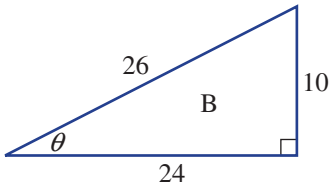
i



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 Here are two similar triangles A and B.





a i Write the ratio $\sin\theta$ (as a fraction) for triangle A.

ii Write the ratio $\sin\theta$ (as a fraction) for triangle B.

iii What do you notice about your two answers from parts **a i** and **a ii**?

b i Write the ratio $\cos\theta$ (as a fraction) for triangle A.

ii Write the ratio $\cos\theta$ (as a fraction) for triangle B.

iii What do you notice about your two answers from parts **b i** and **b ii**?

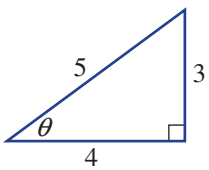
c i Write the ratio $\tan\theta$ (as a fraction) for triangle A.

ii Write the ratio $\tan\theta$ (as a fraction) for triangle B.

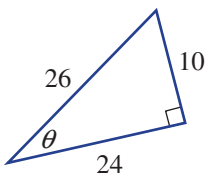
iii What do you notice about your two answers from parts **c i** and **c ii**?

6 For each of these triangles, write a ratio (in simplified fraction form) for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

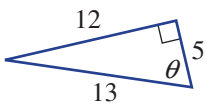
a



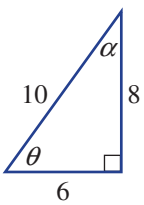
b



c



7 For the triangle shown on the right, write a ratio (in fraction form) for:

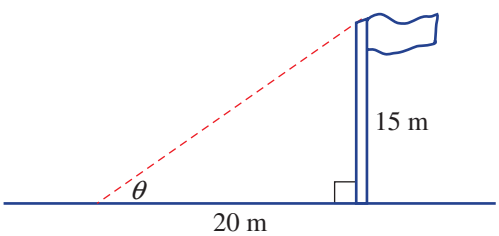


a $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$

d $\tan\alpha$ **e** $\cos\alpha$ **f** $\tan\theta$

PROBLEM-SOLVING AND REASONING 8, 9, 12 | 9, 10, 12, 13 | 10–14

8 A vertical flag pole casts a shadow 20 m long. If the pole is 15 m high, find the ratio for $\tan\theta$.

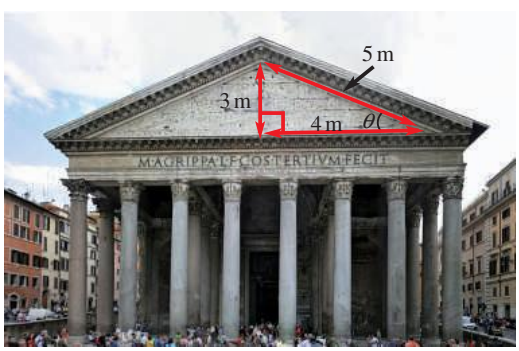


9 The facade of a Roman temple has the measurements shown in the picture on the right. Write down the ratio for:

a $\sin\theta$

b $\cos\theta$

c $\tan\theta$



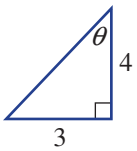
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 For each of the following:

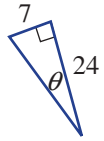
i use Pythagoras' theorem to find the unknown side

ii find the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$

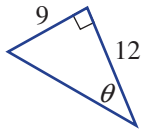
a



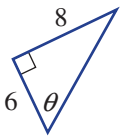
b



c



d

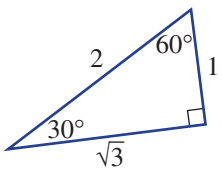


11 **a** Draw a right-angled triangle and mark one of the angles as θ. Mark in the length of the opposite side as 15 units and the length of the hypotenuse as 17 units.

b Using Pythagoras' theorem, find the length of the adjacent side.

c Determine the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

12 This triangle has angles 90°, 60° and 30° and side lengths 1, 2 and $\sqrt{3}$.



a Write a ratio for:

i $\sin 30°$ **ii** $\cos 30°$

iii $\tan 30°$ **iv** $\sin 60°$

v $\cos 60°$ **vi** $\tan 60°$

b What do you notice about the following pairs of ratios?

i $\cos 30°$ and $\sin 60°$ **ii** $\sin 30°$ and $\cos 60°$

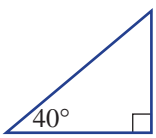
13 **a** Measure all the side lengths of this triangle to the nearest millimetre.

b Use your measurements from part **a** to find an approximate ratio for:

i $\cos 40°$ **ii** $\sin 40°$ **iii** $\tan 40°$

iv $\sin 50°$ **v** $\tan 50°$ **vi** $\cos 50°$

c Do you notice anything about the trigonometric ratios for 40° and 50°?



14 Decide if it is possible to draw a right-angled triangle with the given properties. Explain.

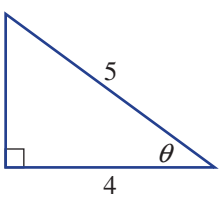
a $\tan\theta = 1$ **b** $\sin\theta = 1$

c $\cos\theta = 0$ **d** $\sin\theta > 1$ or $\cos\theta > 1$

ENRICHMENT — — 15

Pythagorean extensions

15 **a** Given that θ is acute and $\cos\theta = \frac{4}{5}$, find $\sin\theta$ and $\tan\theta$.
Hint: use Pythagoras' theorem.



b For each of the following, draw a right-angled triangle, then use it to find the other two trigonometric ratios.

i $\sin\theta = \frac{1}{2}$

ii $\cos\theta = \frac{1}{2}$

iii $\tan\theta = 1$

c Use your results from part **a** to calculate $(\cos\theta)^2 + (\sin\theta)^2$. What do you notice?

d Evaluate $(\cos\theta)^2 + (\sin\theta)^2$ for other combinations of $\cos\theta$ and $\sin\theta$. Research and describe what you have found.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3F Finding unknown sides

Interactive
Widgets
HOTsheets
Walkthrough

For similar triangles we know that the ratio of corresponding sides is always the same. This implies that the three trigonometric ratios for similar right-angled triangles are also constant if the internal angles are equal. Since ancient times, mathematicians have attempted to tabulate these ratios for varying angles. Here are the ratios for some angles in a right-angled triangle, expressed as decimals.

Angle (θ)	sin θ	cos θ	tan θ
0°	0	1	0
15°	0.259	0.966	0.268
30°	0.5	0.866	0.577
45°	0.707	0.707	1
60°	0.866	0.5	1.732
75°	0.966	0.259	3.732
90°	1	0	undefined

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Note that some of these have been rounded to three decimal places but some are exactly 0, 1 or 0.5.

In modern times these values can be evaluated using calculators to a high degree of accuracy and can be used to help solve problems involving triangles with unknown side lengths.

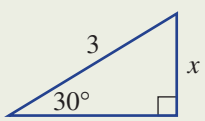
Let's start: Calculator start up

All scientific or CAS calculators can produce accurate values of sin θ, cos θ and tan θ.

- Ensure that your calculator is in degree mode.
- Check the values in the above table to ensure that you are using the calculator correctly.
- Use trial and error to find (to the nearest degree) an angle θ which satisfies these conditions:
  - sin $\theta = 0.454$
  - cos $\theta = 0.588$
  - tan $\theta = 9.514$

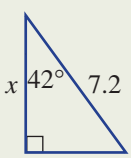
Key ideas

- If θ is in degrees, the ratios for sin θ, cos θ and tan θ can accurately be found using a calculator in **degree mode**.
- If the angles and one side length of a right-angled triangle are known then the other side lengths can be found using the sine ratio, cosine ratio or tangent ratio.



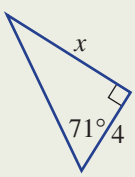
$\sin 30° = \frac{x}{3}$

$\therefore x = 3 \times \sin 30°$



$\cos 42° = \frac{x}{7.2}$

$\therefore x = 7.2 \times \cos 42°$



$\tan 71° = \frac{x}{4}$

$\therefore x = 4 \times \tan 71°$

- Angles that are not whole numbers are sometimes expressed in **degrees**, **minutes** and **seconds**. For example: $22.5° = 20°30'$ and $50.175° = 50°10'30''$.
- There are 60 minutes in 1 degree and 60 seconds in 1 minute. Your calculator can be used to make these conversions. Learn how it is done.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8 Using a calculator

Use a calculator in degree mode to evaluate the following, correct to 2 decimal places.

a sin 50° **b** cos 16° **c** tan 77° **d** tan 15.6° **e** sin 22°30′

SOLUTION

a sin 50° = 0.77 (to 2 d.p.)

b cos 16° = 0.96 (to 2 d.p.)

c tan 77° = 4.33 (to 2 d.p.)

d tan 15.6° = 0.28 (to 2 d.p.)

e sin 22°30′ = 0.38 (to 2 d.p.)

EXPLANATION

sin 50° = 0.766044 … the 3rd decimal place is greater than 4 so round up.

cos 16° = 0.961261 … the 3rd decimal place is less than 5 so round down.

tan 77° = 4.331475 … the 3rd decimal place is less than 5 so round down.

tan 15.6° = 0.2792 … the 3rd decimal place is 9 so round up.

22°30′ = 22.5°(30′ = 30 minutes = 0.5°)

Example 9 Solving for x in the numerator of a trigonometric ratio

Find the value of x in the equation $\cos 20° = \frac{x}{3}$, correct to 2 decimal places.

SOLUTION

$$\cos 20° = \frac{x}{3}$$
$$x = 3 \times \cos 20°$$
$$= 2.82 \text{ (to 2 d.p.)}$$

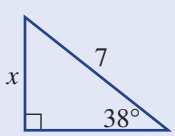
EXPLANATION

Ensure your calculator is in degree mode. Multiply both sides of the equation by 3 and round as required.

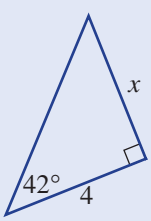
Example 10 Finding side lengths

For each triangle, find the value of x correct to 2 decimal places.

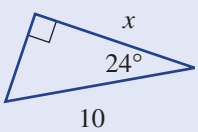
a



b



c



SOLUTION

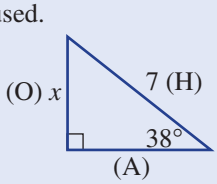
a $\sin 38° = \frac{\text{O}}{\text{H}}$

$$\sin 38° = \frac{x}{7}$$
$$x = 7 \sin 38°$$
$$= 4.31 \text{ (to 2 d.p.)}$$

EXPLANATION

Since the opposite side (O) and the hypotenuse (H) are involved, the sin θ ratio must be used.

Multiply both sides by 7 and evaluate using a calculator.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b $\tan 42° = \frac{O}{A}$

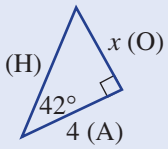
$\tan 42° = \frac{x}{4}$

$x = 4 \tan 42°$

$= 3.60$ (to 2 d.p.)

Since the opposite side (O) and the adjacent side (A) are involved, the tan θ ratio must be used.

Multiply both side by 4 and evaluate.



c $\cos 24° = \frac{A}{H}$

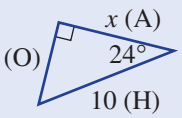
$\cos 24° = \frac{x}{10}$

$x = 10 \cos 24°$

$= 9.14$ (to 2 d.p)

Since the adjacent side (A) and the hypotenuse (H) are involved, the cos θ ratio must be used.

Multiply both sides by 10.

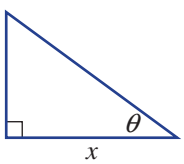


Exercise 3F

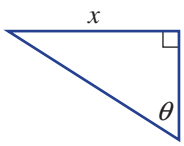
UNDERSTANDING AND FLUENCY | 1–3, 4–5(½) | 3–5(½) | 4–5(½)

1 For the marked angle θ, decide if x represents the length of the opposite (O), adjacent (A) or hypotenuse (H) side.

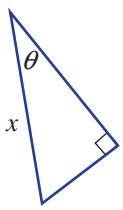
a



b



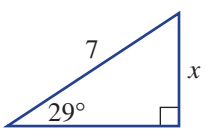
c



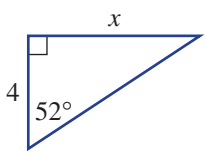
2 Decide if you would use $\sin \theta = \frac{O}{H}$, $\cos \theta = \frac{A}{H}$ or $\tan \theta = \frac{O}{A}$ to help find the value of x in these triangles.

Do not find the value of x, just state which ratio would be used.

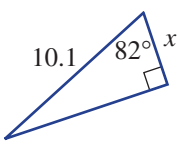
a



b



c



Example 8

3 Use a calculator to evaluate the following correct to 2 decimal places.

a sin 20° **b** cos 37° **c** tan 64° **d** sin 47°

e cos 84° **f** tan 14.1° **g** sin 27°24′ **h** cos 76°12′

Example 9

4 In each of the following, find the value of x correct to 2 decimal places.

a $\sin 50° = \frac{x}{4}$ **b** $\tan 81° = \frac{x}{3}$ **c** $\cos 33° = \frac{x}{6}$

d $\cos 75° = \frac{x}{3.5}$ **e** $\sin 24° = \frac{x}{4.2}$ **f** $\tan 42° = \frac{x}{10}$

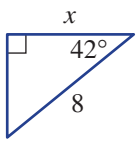
g $\frac{x}{7.1} = \tan 18.4°$ **h** $\frac{x}{5.3} = \sin 64°42′$ **i** $\frac{x}{12.6} = \cos 52°54′$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

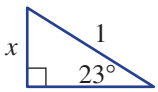
Example 10

5 For the triangles given below, find the value of x correct to 2 decimal places.

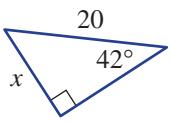
a



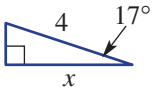
b



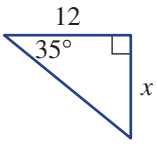
c



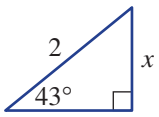
d



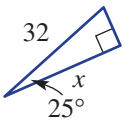
e



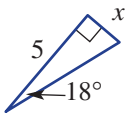
f



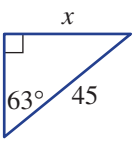
g



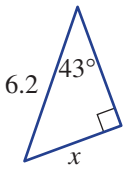
h



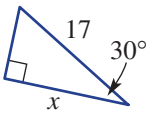
i



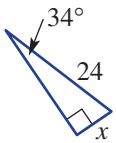
j



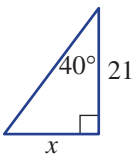
k



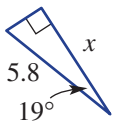
l



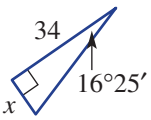
m



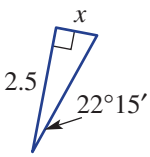
n



o

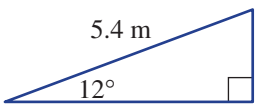


p

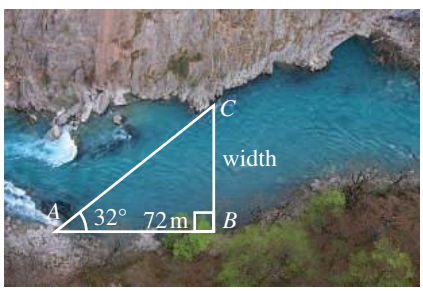


PROBLEM-SOLVING AND REASONING 6, 7, 10 | 6–8, 10 | 7–11

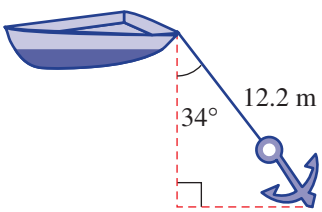
6 Amy walks 5.4 m up a ramp that is inclined at 12° to the horizontal. How high (correct to 2 decimal places) is she above her starting point?



7 Kane wanted to measure the width of a river. He placed two markers, A and B, 72 m apart along the bank. C is a point directly opposite marker B. Kane measured angle CAB to be 32°. Find the width of the river correct to 2 decimal places.



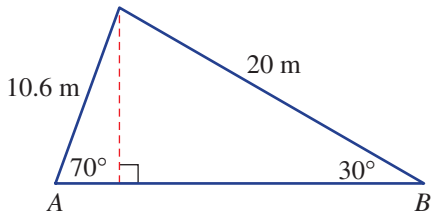
8 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° with the vertical, how deep is the water? Give your answer correct to 2 decimal places.



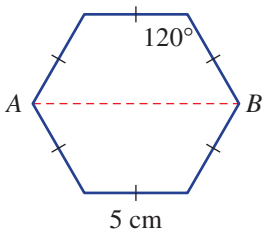
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 Find the length AB in these diagrams. Round to 2 decimal places where necessary.

a



b

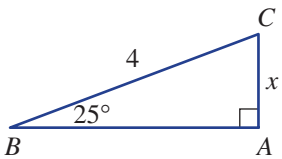


10 Consider the right-angled triangle on the right.

a Find the angle $\angle C$.

b Calculate the value of x correct to 3 decimal places using the sine ratio.

c Calculate the value of x correct to 3 decimal places but instead use the cosine ratio.



11 Complementary angles sum to 90°.

a Find the complementary angles to these angles.

i 10° **ii** 28° **iii** 54° **iv** 81°

b Evaluate:

i sin 10° and cos 80°

ii sin 28° and cos 62°

iii cos 54° and sin 36°

iv cos 81° and sin 9°

c What do you notice in part **b**?

d Complete the following.

i sin 20° = cos _____

ii sin 59° = cos _____

iii cos 36° = sin _____

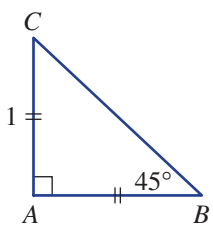
iv cos 73° = sin _____

ENRICHMENT — — 12

Exact values

12 $\sqrt{2}$, $\sqrt{3}$ and $\frac{1}{\sqrt{2}}$ are examples of exact values.

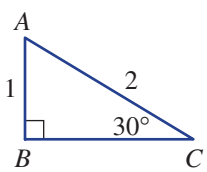
a For the triangle shown (right), use Pythagoras' theorem to find the exact length BC.



b Use your result from part **a** to write down the exact values of:

i sin 45° **ii** cos 45° **iii** tan 45°

c For this triangle (right) use Pythagoras' theorem to find the exact length BC.



d Use your result from part **c** to write down the exact values of:

i sin 30° **ii** cos 30° **iii** tan 30°

iv sin 60° **v** cos 60° **vi** tan 60°

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3G Solving for the denominator

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

So far we have constructed trigonometric ratios using a pronumeral that has always appeared in the numerator.

For example: $\frac{x}{5} = \sin 40°$

This makes it easy to solve for x where both sides of the equation can be multiplied by 5.

If, however, the pronumeral appears in the denominator, there are a number of algebraic steps that can be taken to find the solution.

Let's start: Solution steps

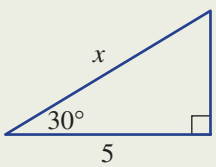
Three students attempt to solve $\sin 40° = \frac{5}{x}$ for x. Nick says $x = 5 \times \sin 40°$. Sharee says $x = \frac{5}{\sin 40°}$. Dori says $x = \frac{1}{5} \times \sin 40°$.

- Which student has the correct solution?
- Can you show the algebraic steps that support the correct answer?

Key ideas

- If the unknown value of a trigonometric ratio is in the **denominator**, you need to rearrange the equation to make the pronumeral the subject.

For example, in the triangle shown:



$\cos 30° = \frac{5}{x}$

Multiply both sides by x. $x \times \cos 30° = 5$

Divide both sides by $\cos 30°$. $x = \frac{5}{\cos 30°}$

Example 11 Solving for x in the denominator

Solve for x in the following equations. Round to 2 decimal places in part **b**.

a $7 = \frac{15}{x}$.

b $\cos 35° = \frac{2}{x}$.

SOLUTION

a
$7 = \frac{15}{x}$
$7x = 15$
$x = \frac{15}{7}$

EXPLANATION

Multiply both sides of the equation by x.
Divide both sides of the equation by 7.

b
$\cos 35° = \frac{2}{x}$
$x \cos 35° = 2$
$x = \frac{2}{\cos 35°}$
$= 2.44$ (to 2 d.p.)

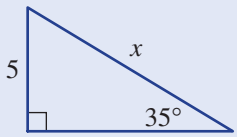
Multiply both sides of the equation by x.
Divide both sides of the equation by $\cos 35°$.
Evaluate and round to 2 decimal places.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

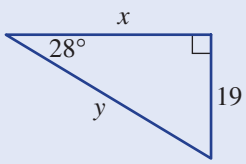
Example 12 Finding side lengths

Find the values of the pronumerals correct to 2 decimal places.

a



b



SOLUTION

a

$$\sin 35° = \frac{O}{H}$$

$$\sin 35° = \frac{5}{x}$$

$$x \sin 35° = 5$$

$$x = \frac{5}{\sin 35°}$$

$$= 8.72 \text{ (to 2 d.p.)}$$

b

$$\tan 28° = \frac{O}{A}$$

$$\tan 28° = \frac{19}{x}$$

$$x \tan 28° = 19$$

$$x = \frac{19}{\tan 28°}$$

$$= 35.73 \text{ (to 2 d.p.)}$$

$$y^2 = x^2 + 19^2$$

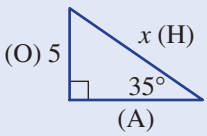
$$= 1637.63$$

$$y = \sqrt{1637.63}$$

$$= 40.47 \text{ (to 2 d.p.)}$$

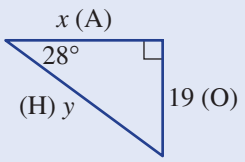
EXPLANATION

Since the opposite side (O) is given and we require the hypotenuse (H), use sin θ.



Multiply both sides of the equation by x, then divide both sides of the equation by sin 35°. Evaluate on a calculator and round off to 2 decimal places.

Since the opposite side (O) is given and the adjacent (A) is required, use tan θ.

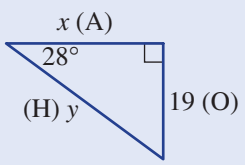


Multiply both sides of the equation by x.

Divide both sides of the equation by tan 28° and round answer to two decimal places.

Find y by using Pythagoras' theorem and substitute the exact value of x, i.e. $\frac{19}{\tan 28°}$.

Alternatively, y can be found by using sin θ.



Exercise 3G

UNDERSTANDING AND FLUENCY	1–4(½)	2–4(½)	3–4(½)

Example 11a

1 Solve these simple equations for x.

a $\frac{4}{x} = 2$

b $\frac{20}{x} = 4$

c $\frac{15}{x} = 5$

d $25 = \frac{100}{x}$

e $5 = \frac{35}{x}$

f $\frac{10}{x} = 2.5$

g $\frac{2.5}{x} = 5$

h $12 = \frac{2.4}{x}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11b

2 For each of the following equations, find the value of x correct to 2 decimal places.

a $\cos 43° = \frac{3}{x}$ **b** $\sin 36° = \frac{4}{x}$ **c** $\tan 9° = \frac{6}{x}$

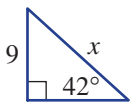
d $\tan 64° = \frac{2}{x}$ **e** $\cos 67° = \frac{5}{x}$ **f** $\sin 12° = \frac{3}{x}$

g $\sin 38.3° = \frac{5.9}{x}$ **h** $\frac{45}{x} = \tan 21°24'$ **i** $\frac{18.7}{x} = \cos 32°$

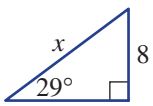
Example 12a

3 Find the value of x correct to 2 decimal places using the sine, cosine or tangent ratios.

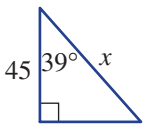
a



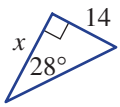
b



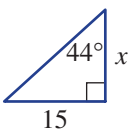
c



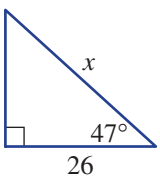
d



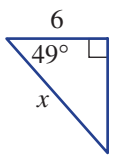
e



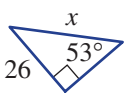
f



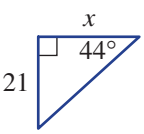
g



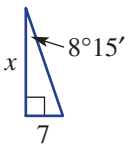
h



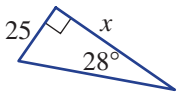
i



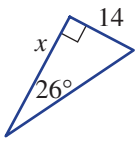
j



k



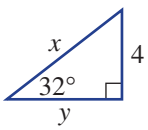
l



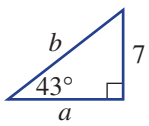
Example 12b

4 Find the value of each pronumeral correct to 1 decimal place.

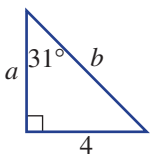
a



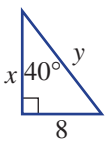
b



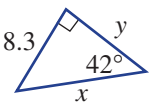
c



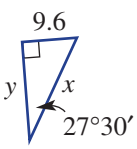
d



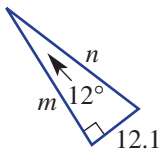
e



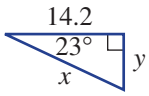
f



g

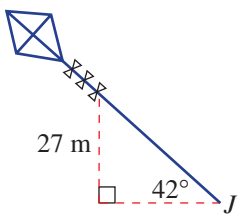


h

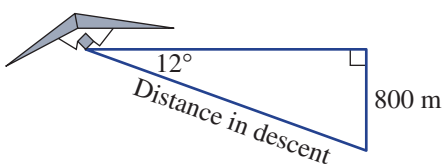


PROBLEM-SOLVING AND REASONING 5–8 6–8 7–9

5 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at 42° to the horizontal, find the length of the string correct to the nearest metre.

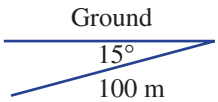


6 A paraglider flying at a height of 800 m descends at an angle of 12° to the horizontal. How far (to the nearest metre) has it travelled in descending to the ground?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 A 100 m mine shaft is dug at an angle of 15° to the horizontal.
How far (to the nearest metre):

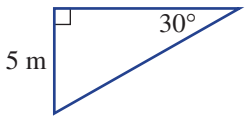


a below ground level is the end of the shaft?

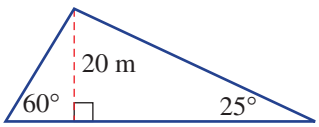
b is the end of the shaft horizontally from the opening?

8 Find the perimeter of these triangles, correct to 1 decimal place.

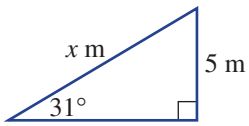
a



b



9 In calculating the value of x for this triangle, accurate to 2 decimal places, two students provide these answers.



A $x = \dfrac{5}{\sin 31°} = \dfrac{5}{0.52} = 9.62$

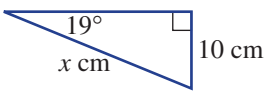
B $x = \dfrac{5}{\sin 31°} = 9.71$

a Which of the two answers is more accurate and why?

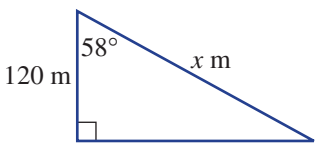
b What advice would you give to the student whose answer is not accurate?

c Find the difference in the answers if the different methods (A and B) are used to calculate the value of x correct to 2 decimal places in these triangles.

i



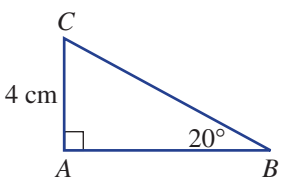
ii



ENRICHMENT — — 10

Linking tan θ to sin θ and cos θ

10 **a** For this triangle find, correct to 3 decimal places:



i AB

ii BC

b Calculate these ratios to 2 decimal places.

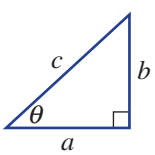
i sin 20°

ii cos 20°

iii tan 20°

c Evaluate $\dfrac{\sin 20°}{\cos 20°}$ using your results from part **b**. What do you notice?

d For this triangle with side lengths a, b and c, find an expression for:



i $\sin\theta$ **ii** $\cos\theta$

iii $\tan\theta$ **iv** $\dfrac{\sin\theta}{\cos\theta}$

e Simplify your expression for part **d iv**. What do you notice?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3H Finding unknown angles

Interactive

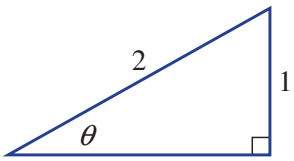
Widgets

HOTsheets

Walkthrough

Logically, if you can use trigonometry to find a side length of a right-angled triangle given one angle and one side, you should be able to find an angle if you are given two sides.

For the triangle below, we know that $\sin 30° = \frac{1}{2}$ so if we were to determine θ if $\sin \theta = \frac{1}{3}$, the answer would be $\theta = 30°$.

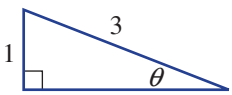


Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Trial and error can be slow

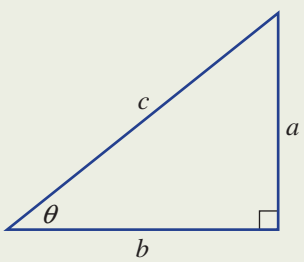
We know that for this triangle, $\sin \theta = \frac{1}{3}$.



- Guess the angle θ.
- For your guess use a calculator in degree mode to see if $\sin \theta = \frac{1}{3} = 0.333\ldots$
- Update your guess and use your calculator to check once again.
- Repeat this guess-and-check process until you think you have the angle θ correct to 3 decimal places.
- Your calculator has a button labelled $\boxed{\sin^{-1}}$ that is useful. Learn how to use it so you don't need to guess. Use it to solve $\sin \theta = \frac{1}{3}$ and check your guess.

Key ideas

- Buttons on your calculator labelled **inverse sine** ($\sin^{-1}$), **inverse cosine** ($\cos^{-1}$) and **inverse tangent** ($\tan^{-1}$) can be used to find unknown angles in right-angled triangles. Ensure your calculator is in degree mode.
  - If $\sin \theta = \frac{a}{c}$, press $\sin^{-1}\left(\frac{a}{c}\right)$.
  - If $\cos \theta = \frac{b}{c}$, press $\cos^{-1}\left(\frac{b}{c}\right)$.
  - If $\tan \theta = \frac{a}{b}$, press $\tan^{-1}\left(\frac{a}{b}\right)$.



- Note that $\sin^{-1} x$ does *not* mean $\frac{1}{\sin x}$.
- You can use your calculator to convert angles into degrees, minutes and seconds. For example: $28.75° = 28°45'$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 13 Finding an angle in degrees

Find the value of θ to the level of accuracy indicated.

a $\sin\theta = 0.3907$ (nearest degree)

b $\tan\theta = \frac{1}{2}$ (1 decimal place)

SOLUTION

a $\sin\theta = 0.3907$

$\theta = 22.998\ldots°$

$\theta = 23°$ (nearest degree)

b $\tan\theta = \frac{1}{2}$

$\theta = 26.565\ldots°$

$\theta = 26.6°$ (to 1 d.p.)

EXPLANATION

Ensure your calculator is in degree mode.

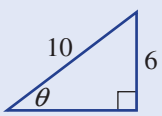
Use the $\sin^{-1}$ key on your calculator, i.e. $\sin^{-1}(0.3907)$.

Round off to the nearest whole number.

Use the $\tan^{-1}$ key on your calculator and round the answer to 1 decimal place.

Example 14 Finding an angle correct to the nearest minute

Find the value of θ to the nearest minute.



SOLUTION

$\sin\theta = \frac{O}{H}$

$= \frac{6}{10}$

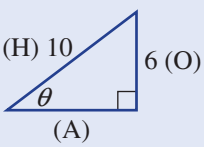
$\theta = 36.869\ldots°$

$\theta = 36°52'11.63''$

$\theta = 36°52'$ (nearest minute)

EXPLANATION

Since the opposite side (O) and the hypotenuse (H) are given, use $\sin\theta$.



Use the $\sin^{-1}$ key on your calculator. Use your calculator to convert the angle into degrees, minutes and seconds.

Round off to the nearest minute. Since there are 60 seconds in a minute, round up for 30 seconds or more and round down for less than 30 seconds.

Exercise 3H

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4, 5–6(½), 7	4, 5–7(½)	6–7(½)

1 Complete the following by converting to degrees, minutes and seconds.

a $25.1° = 25°___'$ **b** $25.2° = 25°___'$ **c** $25.25° = 25°___'$

d $25.5° = 25°___'$ **e** $25.625° = 25°___'___''$ **f** $25\frac{2°}{3} = 25°___'$

2 Convert the following angles into degrees and minutes and round off to the nearest minute.

a 25.2875° **b** 45.16666° **c** 67.4916°

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 Calculate each of the following to the nearest degree.

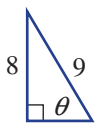
a $\sin^{-1}(0.7324)$ **b** $\cos^{-1}(0.9763)$ **c** $\tan^{-1}(0.3321)$

d $\tan^{-1}(1.235)$ **e** $\sin^{-1}(0.4126)$ **f** $\cos^{-1}(0.7462)$

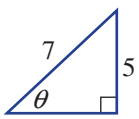
g $\cos^{-1}(0.1971)$ **h** $\sin^{-1}(0.2247)$ **i** $\tan^{-1}(0.0541)$

4 Which trigonometric ratio should be used to solve for θ?

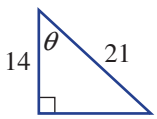
a



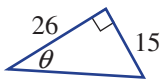
b



c



d



Example 13a

5 Find the value of θ to the nearest degree.

a $\sin\theta = 0.5$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 1$

d $\cos\theta = 0.8660$ **e** $\sin\theta = 0.7071$ **f** $\tan\theta = 0.5774$

g $\sin\theta = 1$ **h** $\tan\theta = 1.192$ **i** $\cos\theta = 0$

j $\cos\theta = 0.5736$ **k** $\cos\theta = 1$ **l** $\sin\theta = 0.9397$

Example 13b

6 Find the angle θ correct to 2 decimal places.

a $\sin\theta = \frac{4}{7}$ **b** $\sin\theta = \frac{1}{3}$ **c** $\sin\theta = \frac{9}{10}$

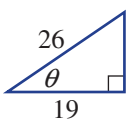
d $\cos\theta = \frac{1}{4}$ **e** $\cos\theta = \frac{4}{5}$ **f** $\cos\theta = \frac{7}{9}$

g $\tan\theta = \frac{3}{5}$ **h** $\tan\theta = \frac{8}{5}$ **i** $\tan\theta = 12$

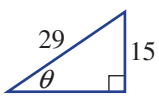
Example 14

7 Find the value of θ to the nearest minute.

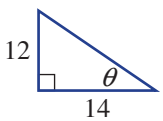
a



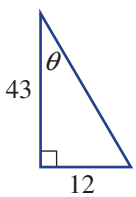
b



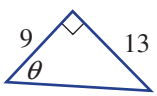
c



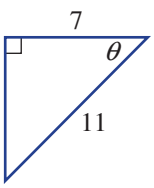
d



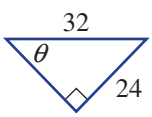
e



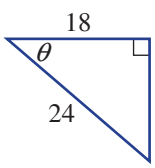
f



g

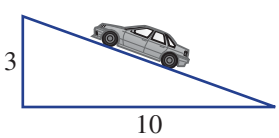


h



PROBLEM-SOLVING AND REASONING 8–10, 13 | 9–11, 13, 14 | 10–12, 14, 15

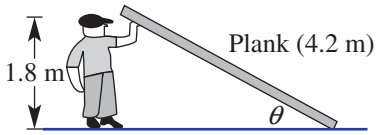
8 A road rises at a grade of 3 in 10. Find the angle (to the nearest degree) the road makes with the horizontal.



9 When a 2.8 m long seesaw is at its maximum height it is 1.1 m off the ground. What angle (correct to 2 decimal places) does the seesaw make with the ground?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Adam, who is 1.8 m tall, holds up a plank of wood 4.2 m long. Find the angle that the plank makes with the ground, correct to 1 decimal place.

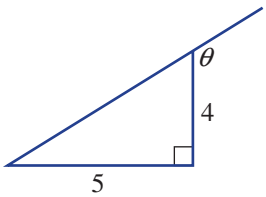


11 A children's slide has a length of 5.8 m. The vertical ladder is 2.6 m above the ground. Find the angle the slide makes with the ground, correct to the nearest minute.

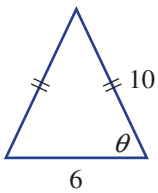


12 Find the value of θ in these diagrams, correct to 1 decimal place.

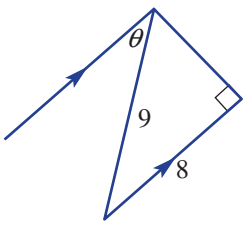
a



b



c

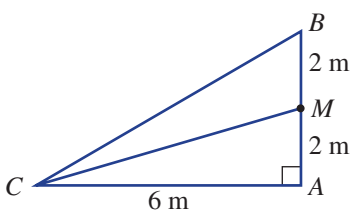


13 Find all the angles to the nearest degree in right-angled triangles with these side lengths.

a 3, 4, 5 **b** 5, 12, 13 **c** 7, 24, 25

14 For what value of θ is $\sin\theta = \cos\theta$?

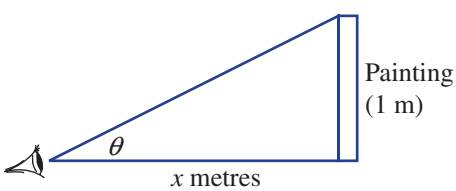
15 If M is the midpoint of AB, decide if $\angle ACM$ is exactly half of angle $\angle ACB$. Investigate and explain.



ENRICHMENT — — 16

Viewing angle

16 Old Joe has trouble with his eyesight but every Sunday goes to view his favourite painting at the gallery. His eye level is at the same level as the base of the painting and the painting is 1 metre tall.



Answer the following to the nearest degree for angles and to 2 decimal places for lengths.

a If $x = 3$, find the viewing angle θ. **b** If $x = 2$, find the viewing angle θ.

c If Joe can stand no closer than 1 metre to the painting, what is Joe's largest viewing angle?

d When the viewing angle is 10°, Joe has trouble seeing the painting. How far is he from the painting at this viewing angle?

e What would be the largest viewing angle if Joe could go as close as he would like to the painting?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3I Using trigonometry to solve problems

Interactive
Widgets
HOTsheets
Walkthrough

In many situations, angles are measured up or down from the horizontal. These are called angles of elevation and depression. Combined with the mathematics of trigonometry, these angles can be used to solve problems, provided right-angled triangles can be identified. The line of sight to a helicopter 100 m above the ground, for example, creates an angle of elevation inside a right-angled triangle.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Illustrate the situation

For the situation below, draw a detailed diagram showing these features:

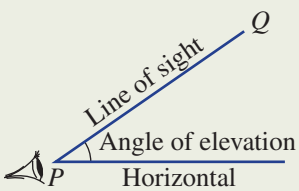
- an angle of elevation
- an angle of depression
- any given lengths
- a right-angled triangle that will help to solve the problem.

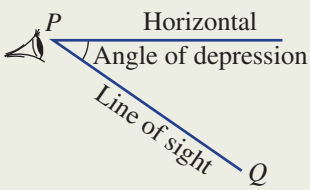
A cat and a bird eye each other from their respective positions. The bird is 20 m up a tree and the cat is on the ground 30 m from the base of the tree. Find the angle their line of sight makes with the horizontal.

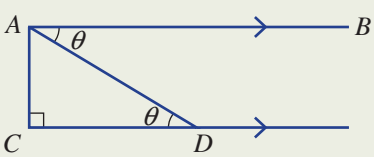
Compare your diagram with others in your class. Is there more than one triangle that could be drawn and used to solve the problem?

Key ideas

- To solve application problems involving trigonometry:
  - Draw a diagram and label the key information.
  - Identify and draw the appropriate right-angled triangles separately.
  - Solve using trigonometry to find the missing measurements.
  - Express your answer in words.
- The **angle of elevation** or **depression** of a point, Q, from another point, P, is given by the angle the line PQ makes with the horizontal.





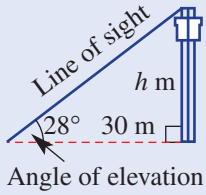
- Angles of elevation or depression are always measured from the horizontal.
- 

  In the diagram AB is parallel to CD so $\angle BAD = \angle ADC$ because they are alternate angles in parallel lines.
  $\therefore$ Angle of elevation from D to A = angle of depression from A to D

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 15 Using angles of elevation

The angle of elevation of the top of a tower from a point on level ground 30 m away from the base of the tower is 28°. Find the height of the tower to the nearest metre.



SOLUTION

Let the height of the tower be h m.

$$\tan 28° = \frac{\text{O}}{\text{A}}$$

$$\tan 28° = \frac{h}{30}$$

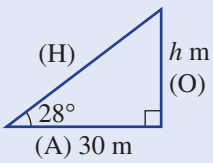
$$h = 30 \tan 28°$$

$$= 15.951\ldots$$

The height is to the nearest metre 16 m.

EXPLANATION

Since the opposite side (O) is required and the adjacent (A) is given, use tan θ.



Multiply both sides by 30 and evaluate.

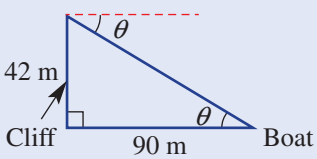
Round to the nearest metre.

Write the answer in words.

Example 16 Finding an angle of depression

From the top of a vertical cliff Andrea spots a boat out at sea. If the top of the cliff is 42 m above sea level and the boat is 90 m away from the base of the cliff, find Andrea's angle of depression to the boat to the nearest degree.

SOLUTION



$$\tan\theta = \frac{\text{O}}{\text{A}}$$

$$\tan\theta = \frac{42}{90}$$

$$\theta = 25.016\ldots°$$

$$\theta = 25°\text{(nearest degree)}$$

The angle of depression is to the nearest degree 25°.

EXPLANATION

Draw a diagram and label all the given measurements.
Use alternate angles in parallel lines to mark θ inside the triangle.

Since the opposite (O) and adjacent sides (A) are given, use tan θ.

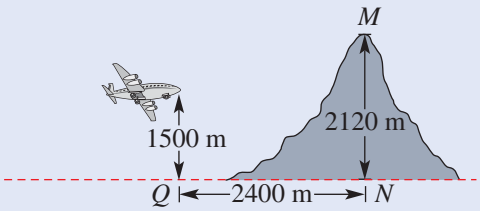
Use the $\tan^{-1}$ key on your calculator.
Round off to the nearest degree.

Express the answer in words.

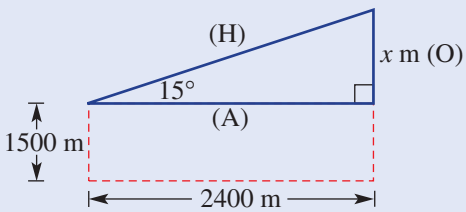
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 17 Applying trigonometry

A plane flying at an altitude of 1500 m starts to climb at an angle of 15° to the horizontal when the pilot sees a mountain peak 2120 m high, 2400 m away from him horizontally. Will the pilot clear the mountain?



SOLUTION



$\tan 15° = \frac{x}{2400}$

$x = 2400 \tan 15°$

$= 643.078 \ldots$

$2120 - 1500 = 620$

Since $x > 620$ m the plane will clear the mountain peak.

EXPLANATION

Draw a diagram, identifying and labelling the right-angled triangle to help solve the problem.

The plane will clear the mountain if the opposite (O) is greater than

$(2120 - 1500)$ m $= 620$ m

Set up the trigonometric ratio using tan.

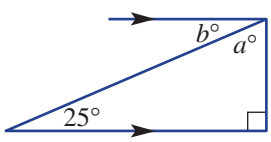
Multiply by 2400 and evaluate.

Answer the question in words.

Exercise 3I

UNDERSTANDING AND FLUENCY	1–9	3–6, 8–10	4, 6, 8–11

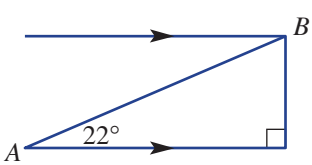
1 Find the values of the pronumerals in this diagram.



2 For this diagram, what is the:

a angle of elevation of B from A?

b angle of depression of A from B?



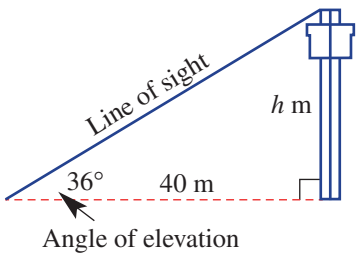
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 Draw a labelled diagram to represent each of the following.

- **a** A building is standing on flat ground. A point is 100 m from the base of the building. From that point the angle of elevation of the top of the building is 36 degrees.
- **b** The angle of depression from the top of a 60 m cliff to a boat is 25 degrees.
- **c** The foot of a ladder is on horizontal ground. The ladder, which is 2 m long, is leaning up against a vertical wall. The angle between the ladder and the wall is 52 degrees.

Example 15

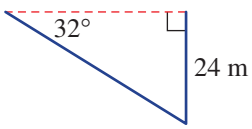
4 The angle of elevation of the top of a tower from a point on the ground 40 m from the base of the tower is 36°. Find the height of the tower to the nearest metre.



5 The angle of elevation of the top of a castle wall from a point on the ground 25 m from the base of the castle wall is 32°. Find the height of the castle wall to the nearest metre.

6 From a point on the ground, Emma measures the angle of elevation of an 80 m tower to be 27°. Find how far Emma is from the base of the tower, correct to the nearest metre.

7 From a pedestrian overpass, Chris spots a landmark at an angle of depression of 32°. How far away (to the nearest metre) is the landmark from the base of the 24 m high overpass?



8 From a lookout tower, David spots a bushfire at an angle of depression of 25°. If the lookout tower is 42 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?

Example 16

9 From the top of a vertical cliff, Josh spots a swimmer out at sea. If the top of the cliff is 38 m above sea level and the swimmer is 50 m away from the base of the cliff, find the angle of depression from Josh to the swimmer, to the nearest degree.



10 From a ship, a person is spotted floating in the sea 200 m away. If the viewing position on the ship is 20 m above sea level, find the angle of depression from the ship to person in the sea. Give your answer to the nearest degree.

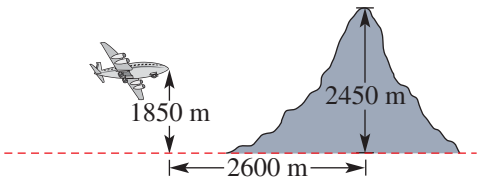
11 A power line is stretched from a pole to the top of a house. The house is 4.1 m high and the power pole is 6.2 m high. The horizontal distance between the house and the power pole is 12 m. Find the angle of elevation of the top of the power pole from the top of the house, to the nearest degree.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING 12, 13, 17(a, b) | 12–14, 17 | 12, 15–18

Example 17

12 A plane flying at 1850 m starts to climb at an angle of 18° to the horizontal when the pilot sees a mountain peak 2450 m high, 2600 m away from him in a horizontal direction. Will the pilot clear the mountain?

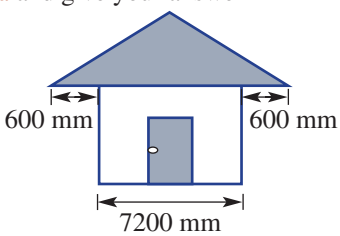


13 A road has a steady gradient of 1 in 10.

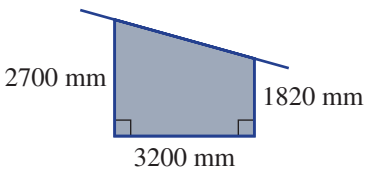
a What angle does the road make with the horizontal? Give your answer to the nearest degree.

b A car starts from the bottom of the inclined road and drives 2 km along the road. How high vertically has the car climbed? Use your rounded answer from part **a** and give your answer correct to the nearest metre.

14 A house is to be built using the design shown on the right. The eaves are 600 mm and the house is 7200 mm wide, excluding the eaves. Calculate the length (to the nearest mm) of a sloping edge of the roof, which is pitched at 25° to the horizontal.

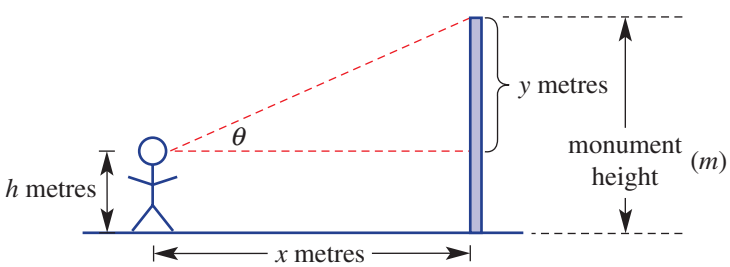


15 A garage is to be built with measurements as shown in the diagram on the right. Calculate the sloping length and pitch (angle) of the roof if the eaves extend 500 mm on each side. Give your answers correct to the nearest unit.



16 The chains on a swing are 3.2 m long and the seat is 0.5 m off the ground when it is in the vertical position. When the swing is pulled as far back as possible, the chains make an angle of 40° with the vertical. How high off the ground, to the nearest cm, is the seat when it is at this extreme position?

17 A person views a vertical monument x metres away as shown.



a If $h = 1.5$, $x = 20$ and $\theta = 15°$ find the height of the monument to 2 decimal places.

b If $h = 1.5$, $x = 20$ and $y = 10$ find θ correct to 1 decimal place.

c Let the height of the monument be m. Write expressions for:

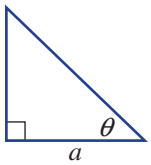
i m using (in terms of) y and h

ii y using x and θ

iii m using (in terms of) x, θ and h

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

18 Find an expression for the area of this triangle using a and θ.



ENRICHMENT	—	—	19–21

Plane trigonometry

19 An aeroplane takes off and climbs at an angle of 20° to the horizontal, at 190 km/h along its flight path for 15 minutes.

a Find the:

i distance the aeroplane travels in 15 minutes

ii height the aeroplane reaches after 15 minutes correct to 2 decimal places

b If the angle at which the plane climbs is twice the original angle but its speed is halved, will it reach a greater height after 15 minutes? Explain.

c If the plane's speed is doubled and its climbing angle is halved, will the plane reach a greater height after 15 minutes? Explain.



20 The residents of Skeville live 12 km from an airport. They maintain that any plane flying lower than 4 km disturbs their peace. Each Sunday they have an outdoor concert from noon till 2 p.m.

a Will a plane taking off from the airport at an angle of 15° over Skeville disturb the residents?

b When the plane in part **a** is directly above Skeville, how far (to the nearest m) has it flown?

c If the plane leaves the airport at 11:50 a.m. on Sunday and travels at an average speed of 180 km/h, will it disturb the start of the concert?

d Investigate what average speed the plane can travel at so that it does not disturb the concert. Assume it leaves at 11:50 a.m.



21 Peter observes a plane flying directly overhead at a height of 820 m. Twenty seconds later, the angle of elevation of the plane from Peter is 32°. Assume the plane flies horizontally.

a How far (to the nearest metre) did the plane fly in 20 seconds?

b What is the plane's speed in km/h correct to 2 decimal places?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3J Bearings

Interactive
Widgets
HOTsheets
Walkthrough

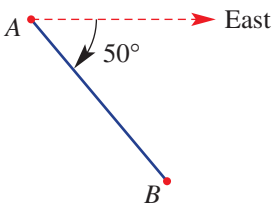
Bearings are used to indicate direction and therefore are commonly used to navigate the sea or air in ships or planes. Bushwalkers use bearings with a compass to help follow a map and navigate a forest. The most common type of bearing is the true bearing measured clockwise from north.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Opposite directions

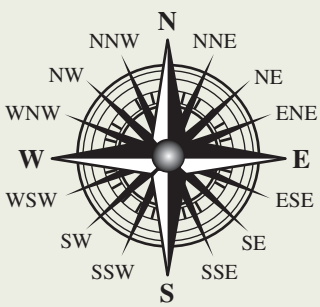
Marg at point A and Jim at point B start walking toward each other. Marg knows that she has to face 50° south of due east.

- Measured clockwise from north, can you help Marg determine her true compass bearing that she should walk on?
- Can you find what bearing Jim should walk on?
- Draw a detailed diagram which supports your answers above.

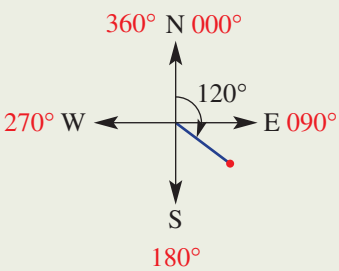


Key ideas

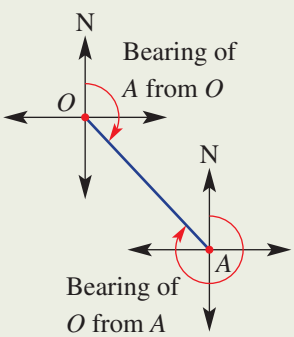
- This mariner's compass shows 16 different directions.



- A **bearing** is an angle measured clockwise from north.
  - It is written using three digits.
    For example: 008°, 032° and 144°.
  - Sometimes the letter T is used to stand for true north.
    For example: NE is 045°T.



- To describe the bearing of an object positioned at A from an object positioned at O, we need to start at O, face north then turn clockwise through the required angle to face the object at A.
- When solving problems with bearings, draw a diagram including four point compass directions (N, E, S, W) at each point.



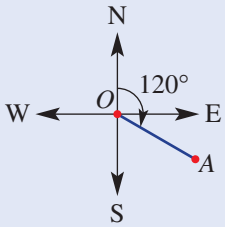
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18 Stating bearings

For the diagram shown, give the bearing of:

a A from O

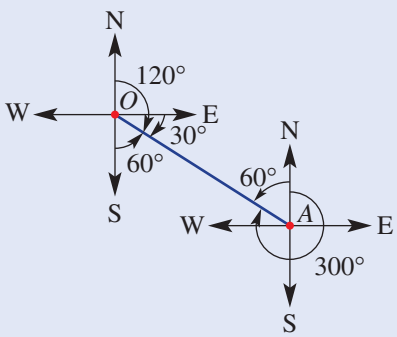
b O from A



SOLUTION

a The bearing of A from O is 120°.

b



The bearing of O from A is: $(360 - 60)° = 300°$

EXPLANATION

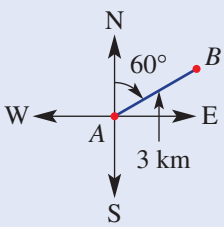
Start at O, face north and turn clockwise until you are facing A.

Start at A, face north and turn clockwise until you are facing O. Mark in a compass at A and use alternate angles in parallel lines to mark 60° angle.

Bearing is then 60° short of 360°.

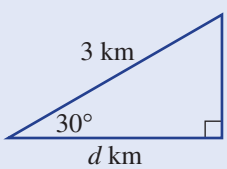
Example 19 Using bearings with trigonometry

A bushwalker walks 3 km on a bearing of 060° from point A to point B. Find how far east (correct to 1 decimal place) point B is from point A.



SOLUTION

Let the distance travelled towards the east be d km.



$$\cos 30° = \frac{d}{3}$$

$$d = 3 \times \cos 30°$$

$$= 2.598 \ldots$$

$\therefore$ The distance east is 2.6 km (to 1 d.p.).

EXPLANATION

Define the distance required and draw and label the right-angled triangle.

Since the adjacent (A) is required and the hypotenuse (H) is given, use $\cos \theta$.
Multiply both sides of the equation by 3 and evaluate rounding off to 1 decimal place.

Express the answer in words.

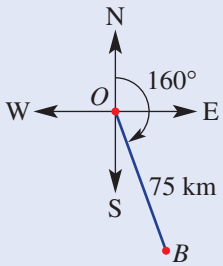
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 20 Calculating a bearing

A fishing boat starts from point O and sails 75 km on a bearing of 160° to point B.

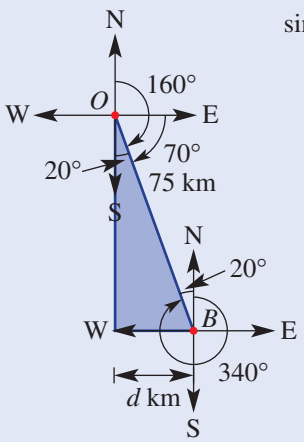
a How far east (to the nearest kilometre) of its starting point is the boat?

b What is the bearing of O from B?



SOLUTION

a Let the distance travelled towards the east be d km.



$$\sin 20° = \frac{d}{75}$$
$$d = 75 \sin 20°$$
$$= 25.651\ldots$$

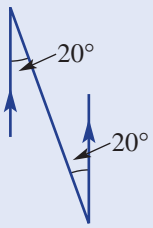
The boat has travelled 26 km to the east of its starting point, to the nearest km.

b The bearing of O from B is $(360° - 20°) = 340°$

EXPLANATION

Draw a diagram and label all the given measurements. Mark in a compass at B and use alternate angles to label extra angles.
Set up a trigonometric ratio using sine and solve for d.

Alternate angle = 20°



Round to the nearest kilometre and write the answer in words.

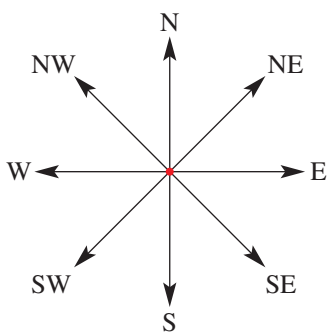
Start at B, face north then turn clockwise to face O.

Exercise 3J

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4–6	2, 3(½), 4–7	3(½), 4, 6, 7

1 Give the bearings for these directions. Use three digits, for example, 045°.

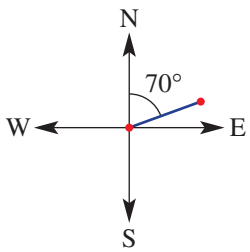
a North (N)
b North-east (NE)
c East (E)
d South-east (SE)
e South (S)
f South-west (SW)
g West (W)
h North-west (NW)



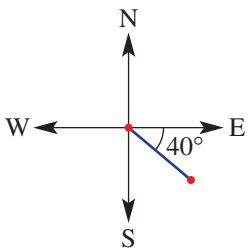
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Write down the bearings shown in these diagrams. Use three digits, for example, 045°.

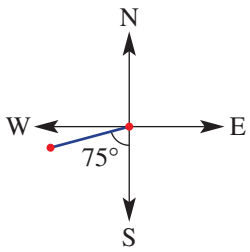
a



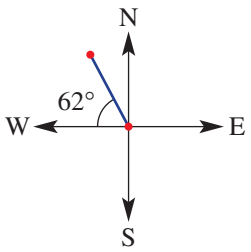
b



c



d

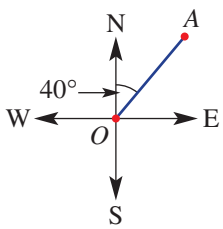


Example 18

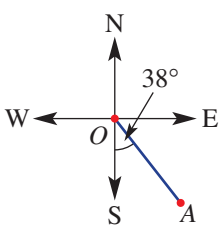
3 For each diagram shown, write the bearing of:

i A from O **ii** O from A

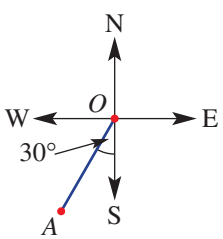
a



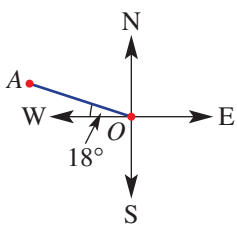
b



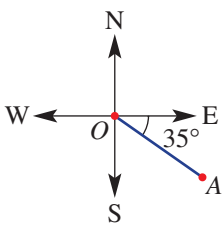
c



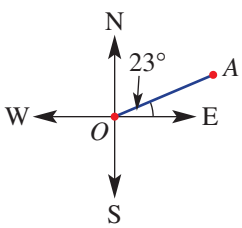
d



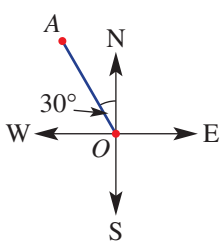
e



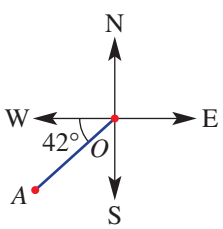
f



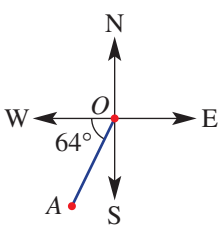
g



h



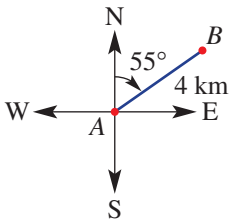
i



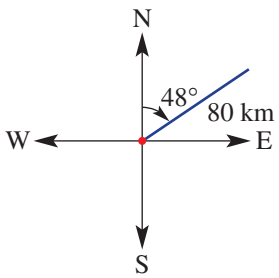
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 19

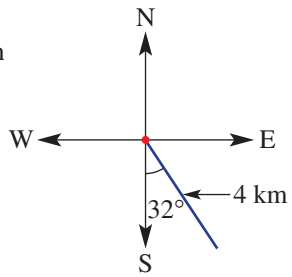
4 A bushwalker walks 4 km on a bearing of 055° from point A to point B. Find how far east point B is from point A, correct to 2 decimal places.



5 A speed boat travels 80 km on a bearing of 048°. Find how far east of its starting point the speed boat is, correct to 2 decimal places.



6 After walking due east, then turning and walking due south, a hiker is 4 km and on a bearing of 148° from her starting point. Find how far she walked in a southerly direction, correct to 1 decimal place.



7 A four-wheel drive vehicle travels for 32 km on a bearing of 200°. How far west (to the nearest km) of its starting point is it?

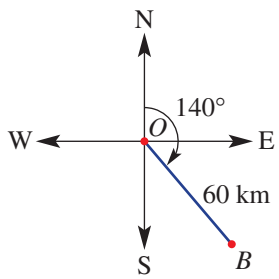
PROBLEM-SOLVING AND REASONING 8–10, 13 8–11, 13 9–14

Example 20

8 A fishing boat starts from point O and sails 60 km on a bearing of 140° to point B.

a How far east of its starting point is the boat, to the nearest kilometre?

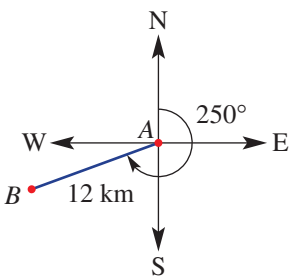
b What is the bearing of O from B?



9 Two towns, A and B, are 12 km apart. The bearing of B from A is 250°.

a How far west of A is B, correct to 1 decimal place?

b Find the bearing of A from B.



10 A helicopter flies on a bearing of 140° for 210 km then flies due east for 175 km. How far east (to the nearest kilometre) has the helicopter travelled from its starting point?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Christopher walks 5 km south then walks on a bearing of 036° until he is due east of his starting point. How far is he from his starting point, to 1 decimal place?

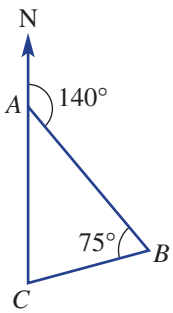
12 Two cyclists leave from the same starting point. One cyclist travels due west while the other travels on a bearing of 202°. After travelling for 18 km, the second cyclist is due south of the first cyclist. How far (to the nearest metre) has the first cyclist travelled?



13 A bearing is $a°$. Write an expression for the bearing of the opposite direction of $a°$ if:

- **a** a is between 0 and 180
- **b** a is between 180 and 360

14 A hiker walks on a triangular pathway starting at point A, walking to point B then C, then A again as shown.



- **a** Find the bearing from B to A.
- **b** Find the bearing from B to C.
- **c** Find the bearing from C to B.
- **d** If the initial bearing was instead 133° and $\angle ABC$ is still 75°, find the bearing from B to C.
- **e** If $\angle ABC$ was 42°, with the initial bearing of 140°, find the bearing from B to C.

ENRICHMENT	—	—	15, 16

Speed trigonometry

15 A plane flies on a bearing of 168° for two hours at an average speed of 310 km/h. How far (to the nearest kilometre):

- **a** has the plane travelled?
- **b** south of its starting point is the plane?
- **c** east of its starting point is the plane?

16 A pilot intends to fly directly to Anderly, which is 240 km due north of his starting point. The trip usually takes 50 minutes. Due to a storm, the pilot changes course and flies to Boxleigh on a bearing of 320° for 150 km, at an average speed of 180 km/h.

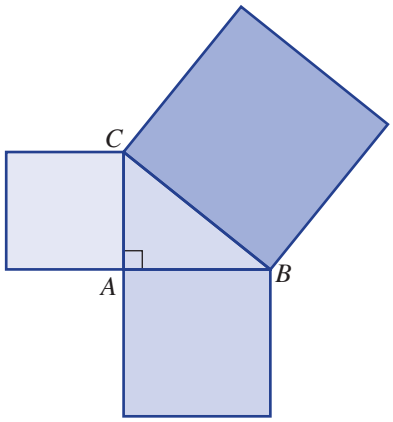
- **a** Find (to the nearest kilometre) how far:
  - **i** north the plane has travelled from its starting point
  - **ii** west the plane has travelled from its starting point
- **b** How many kilometres is the plane off-course?
- **c** From Boxleigh the pilot flies directly to Anderly at 240 km/h.
  - **i** Compared to the usual route, how many extra kilometres has the pilot travelled in reaching Anderly?
  - **ii** Compared to the usual trip, how many extra minutes did the trip to Anderly take?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Investigation

Illustrating Pythagoras

It is possible to use a computer geometry package ('Cabri Geometry' or 'Geometers Sketchpad') to build this construction, which will illustrate Pythagoras' theorem.



Construct

a Start by constructing the line segment AB.
b Construct the right-angled triangle ABC by using the 'Perpendicular Line' tool.
c Construct a square on each side of the triangle. Circles may help to ensure your construction is exact.

Calculate

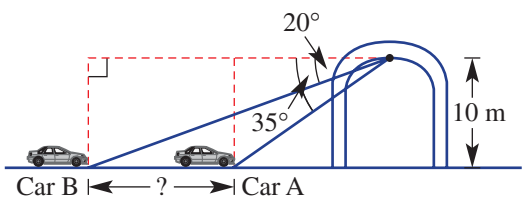
a Measure the areas of the squares representing AB^2, AC^2 and BC^2.
b Calculate the sum of the areas of the two smaller squares by using the 'Calculate' tool.
c i Drag point A or point B and observe the changes in the areas of the squares.
ii Investigate how the areas of the squares change as you drag point A or point B. Explain how this illustrates Pythagoras' theorem.

Constructing triangles to solve problems

Illustrations for some problems may not initially look as if they include right-angled triangles. A common mathematical problem-solving technique is to construct right-angled triangles so that trigonometry can be used.

Car gap

Two cars are observed in the same lane from an overpass bridge 10 m above the road. The angles of depression to the cars are 20° and 35°.

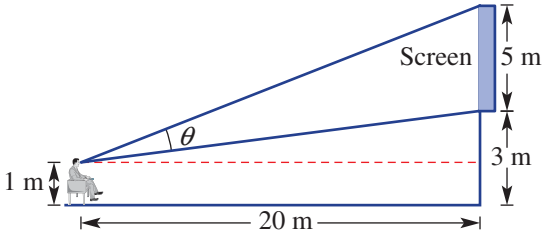


a Find the horizontal distance from the front of car A to the overpass. Show your diagrams and working.
b Find the horizontal distance from the front of car B to the overpass.
c Find the distance between the fronts of the two cars.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Cinema screen

A 5 m vertical cinema screen sits 3 m above the floor of the hall and Wally sits 20 m back from the screen. His eye level is 1 m above the floor.



a Find the angle of elevation from Wally's eye level to the base of the screen. Illustrate your method using a diagram.

b Find the angle of elevation as in part **a** but from his eye level to the top of the screen.

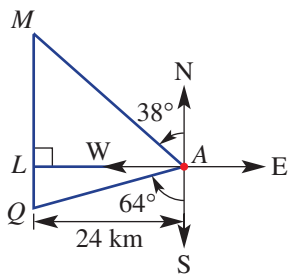
c Use your results from parts **a** and **b** to find Wally's viewing angle θ.

Problem-solving without all the help

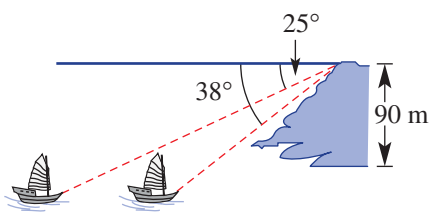
Solve these similar types of problems. You will need to draw detailed diagrams and split the problem into parts. Refer to the previous two problems if you need help.

a An observer is 50 m horizontally from a hot air balloon. The angle of elevation to the top of the balloon is 60° and to the bottom of the balloon's basket is 40°. Find the total height of the balloon (to the nearest metre) from the base of the basket to the top of the balloon.

b A ship (at A) is 24 km due east of a lighthouse (L). The captain takes bearings from two landmarks, M and Q, which are due north and due south of the lighthouse respectively. The bearings of M and Q from the ship are 322° and 244° respectively. How far apart are the two landmarks?



c From the top of a 90 m cliff the angles of depression of two boats in the water, both directly east of the lighthouse, are 25° and 38° respectively. What is the distance between the two boats to the nearest metre?



d A person on a boat 200 m out to sea views a 40 m high castle wall on top of a 32 m high cliff. Find the viewing angle between the base and top of the castle wall from the person on the boat.

Design your own problem

Design a problem similar to the ones above that involve a combination of triangles.

a Clearly write the problem.

b See if a friend can understand and solve your problem.

c Show a complete solution including all diagrams.

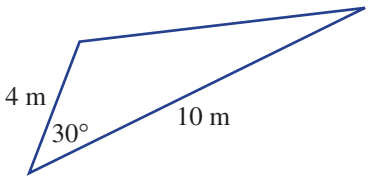
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Puzzles and challenges

1 A right-angled isosceles triangle has area of 4 square units. Determine the exact perimeter of the triangle.

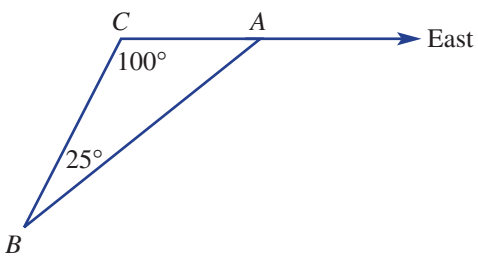
2 Find the area of this triangle using trigonometry.

Hint: insert a line showing the height of the triangle.



3 A rectangle $ABCD$ has sides $AB = CD = 34$ cm. E is a point on CD such that $CE = 9$ cm and $ED = 25$ cm. AE is perpendicular to EB. What is the length of BC?

4 Find the bearing from B to C in this diagram.



5 Which is a better fit? A square peg in a round hole or a round peg in a square hole. Use area calculations and percentages to investigate.

6 Boat A is 20 km from port on a bearing of 025° and boat B is 25 km from port on a bearing of 070°. Boat B is in distress. What bearing should boat A travel on to reach boat B?

7 For positive integers m and n such that $n < m$, the Pythagorean triads (like 3, 4, 5) can be generated using $a = m^2 - n^2$ and $b = 2mn$, where a and b are the two shorter sides of the right-angled triangle.

a Using the above formulas and Pythagoras' theorem to calculate the third side, generate the Pythagorean triads for:

i $m = 2, n = 1$

ii $m = 3, n = 2$

b Using the expressions for a and b and Pythagoras' theorem, find a rule for c (the hypotenuse) in terms of n and m.

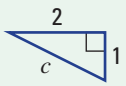
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Right-angled triangles

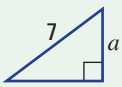
Finding the hypotenuse

$c^2 = 2^2 + 1^2$
$= 5$
$c = \sqrt{5}$
$= 2.24$ (to 2 d. p.)

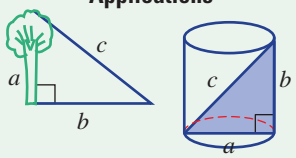


Shorter sides

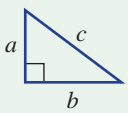
$a^2 + 5^2 = 7^2$
$a^2 = 7^2 - 5^2$
$= 24$
$\therefore a = \sqrt{24}$



Applications



Pythagoras' theorem



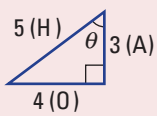
$c^2 = a^2 + b^2$

SOHCAHTOA

$\sin\theta = \frac{O}{H} = \frac{4}{5}$

$\cos\theta = \frac{A}{H} = \frac{3}{5}$

$\tan\theta = \frac{O}{A} = \frac{4}{3}$

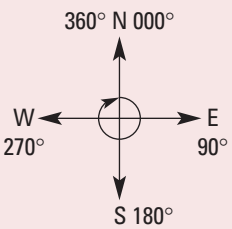


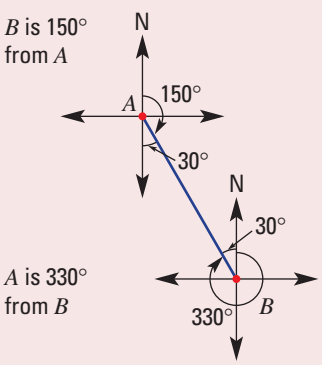
Useful triads

3, 4, 5
6, 8, 10
5, 12, 13

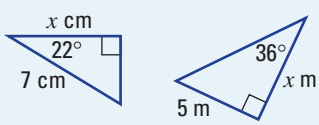
Bearings

Bearings are measured clockwise from North.





Finding side lengths



$\cos 22° = \frac{x}{7}$

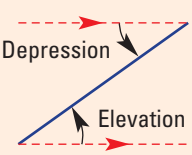
$x = 7 \cos 22°$
$= 6.49$ (to 2 d.p)

$\tan 36° = \frac{5}{x}$

$x \times \tan 36° = 5$

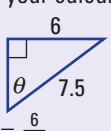
$x = \frac{5}{\tan 36°}$
$= 6.88$ (to 2 d.p)

Elevation and depression



Finding angles

Use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ key on your calculator.



$\sin\theta = \frac{6}{7.5}$

$\theta = 53.13°$ (to 2 d.p)

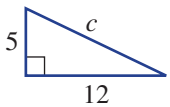
or $\theta = 53° 8'$ (nearest minute)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter review

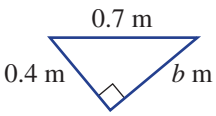
Multiple-choice questions

1 For the right-angled triangle shown, the length of the hypotenuse is given by:



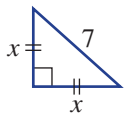
A $c^2 = 5^2 + 12^2$ **B** $c^2 = 5^2 - 12^2$
C $c^2 = 12^2 - 5^2$ **D** $c^2 = 5^2 \times 12^2$
E $(5 + 12)^2$

2 For the right-angled triangle shown, the value of b is given by:



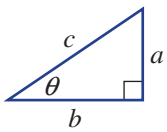
A $\sqrt{0.7^2 + 0.4^2}$ **B** $\sqrt{0.7^2 - 0.4^2}$
C $\sqrt{0.4^2 - 0.7^2}$ **D** $\sqrt{0.7^2 \times 0.4^2}$
E $\sqrt{(0.7 - 0.4^2)^2}$

3 For the right-angled triangle shown:



A $x^2 = \frac{49}{2}$ **B** $7x^2 = 2$
C $x^2 = \frac{7}{2}$ **D** $x^2 + 7^2 = x^2$
E $x^2 = \frac{2}{7}$

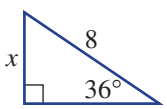
4 For the triangle shown:



A $\sin\theta = \frac{a}{b}$ **B** $\sin\theta = \frac{c}{a}$
C $\sin\theta = \frac{a}{c}$ **D** $\sin\theta = \frac{b}{c}$
E $\sin\theta = \frac{c}{b}$

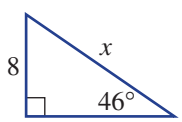
5 The value of cos 46° correct to 4 decimal places is:
A 0.7193 **B** 0.6947
C 0.594 **D** 0.6532
E 1.0355

6 In the diagram, the value of x, correct to 2 decimal places, is:



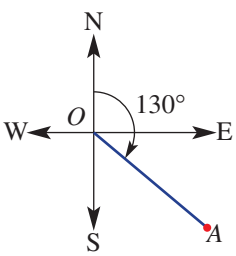
A 40 **B** 13.61
C 4.70 **D** 9.89
E 6.47

7 The length of x in the triangle is given by:



A $8\sin 46°$ **B** $8\cos 46°$
C $\frac{8}{\cos 46°}$ **D** $\frac{8}{\sin 46°}$
E $\frac{\cos 46°}{8}$

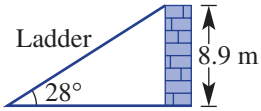
8 The bearing of A from O is 130°. The bearing of O from A is:



A 050° **B** 220°
C 310° **D** 280°
E 170°

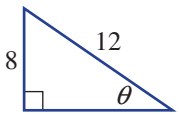
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 A ladder is inclined at an angle of 28° to the horizontal. If the ladder reaches 8.9 m up the wall, the length of the ladder correct to the nearest metre is:



A 19 m **B** 4 m
C 2 m **D** 10 m
E 24 m

10 The value of θ in the diagram, correct to 2 decimal places, is:

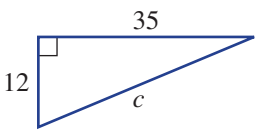


A 0.73° **B** 41.81°
C 48.19° **D** 33.69°
E 4.181°

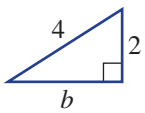
Short-answer questions

1 Find the unknown length in these triangles. Give an exact answer.

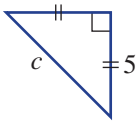
a



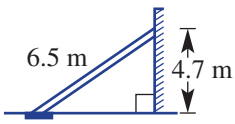
b



c



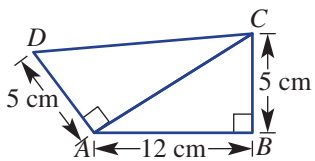
2 A steel support beam of length 6.5 m is connected to a wall at a height of 4.7 m from the ground. Find the distance (to the nearest cm) between the base of the building and the point where the beam is joined to the ground.



3 For this double triangle find:

a AC

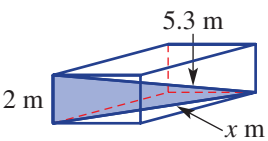
b CD (correct to 2 decimal places)



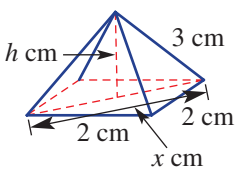
4 Two different cafés on opposite sides of an atrium in a shopping centre are respectively 10 m and 15 m above the ground floor. If the cafés are linked by a 20 m escalator, find the horizontal distance (to the nearest m) across the atrium, between the two cafés.

5 Find the values of the pronumerals in the three-dimensional objects shown below, correct to 2 decimal places.

a



b



6 Find the value of each of the following, correct to 2 decimal places.

a sin 40°

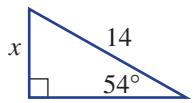
b tan 66°

c cos 44°

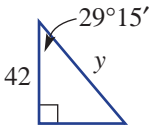
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Find the value of each pronumeral, correct to 2 decimal places.

a



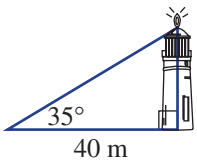
b



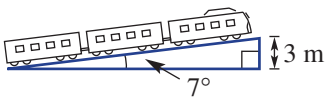
c



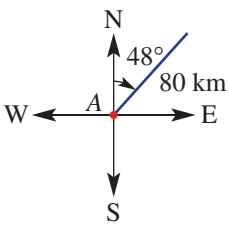
8 The angle of elevation of the top of a lighthouse from a point on the ground 40 m from its base is 35°. Find the height of the lighthouse to 2 decimal places.



9 A train travels up a slope, making an angle of 7° with the horizontal. When the train is at a height of 3 m above its starting point, find the distance it has travelled up the slope, to the nearest metre.



10 A yacht sails 80 km on a bearing of 048°.



a How far east of its starting point is the yacht, correct to 2 decimal places?

b How far north of its starting point is the yacht, correct to 2 decimal places?

11 From a point on the ground, Geoff measures the angle of elevation of a 120 m tower to be 34°. How far from the base of the tower is Geoff, correct to 2 decimal places?

12 A ship leaves Coffs Harbour and sails 320 km east. It then changes direction and sails 240 km due north to its destination. What will the ship's bearing be from Coffs Harbour when it reaches its destination, correct to 2 decimal places?

13 From the roof of a skyscraper, Aisha spots a car at an angle of depression of 51° from the roof of the skyscraper. If the skyscraper is 78 m high, how far away is the car from the base of the skyscraper, correct to the nearest minute?

14 Penny wants to measure the width of a river. She places two markers, A and B, 10 m apart along one bank. C is a point directly opposite marker B. Penny measures angle BAC to be 28°. Find the width of the river to 1 decimal place.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 An aeroplane takes off and climbs at an angle of 15° to the horizontal, at 210 km/h along its flight path for 15 minutes. Find the:

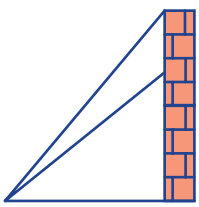
- **a** distance the aeroplane travels
- **b** height the aeroplane reaches, correct to 2 decimal places



Extended-response questions

1 An extension ladder is initially placed so that it reaches 2 metres up a wall. The foot of the ladder is 80 centimetres from the base of the wall.

- **a** Find the length of the ladder, to 2 decimal places, in its original position.
- **b** Without moving the foot of the ladder it is extended so that it reaches one metre further up the wall. How far (to 2 decimal places) has the ladder been extended?
- **c** The ladder is placed so that its foot is now 20 cm closer to the base of the wall.
  - **i** How far up the wall can the ladder length found in part **b** reach? Round to 2 decimal places.
  - **ii** Is this further than the distance in part **a**?



2 From the top of a 100 m cliff Skevi sees a boat out at sea at an angle of depression of 12°.

- **a** Draw a diagram for this situation.
- **b** Find how far out to sea the boat is to the nearest metre.
- **c** A swimmer is 2 km away from the base of the cliff and in line with the boat. What is the angle of depression to the swimmer to the nearest degree?
- **d** How far away is the boat from the swimmer, to the nearest metre?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



Online resources

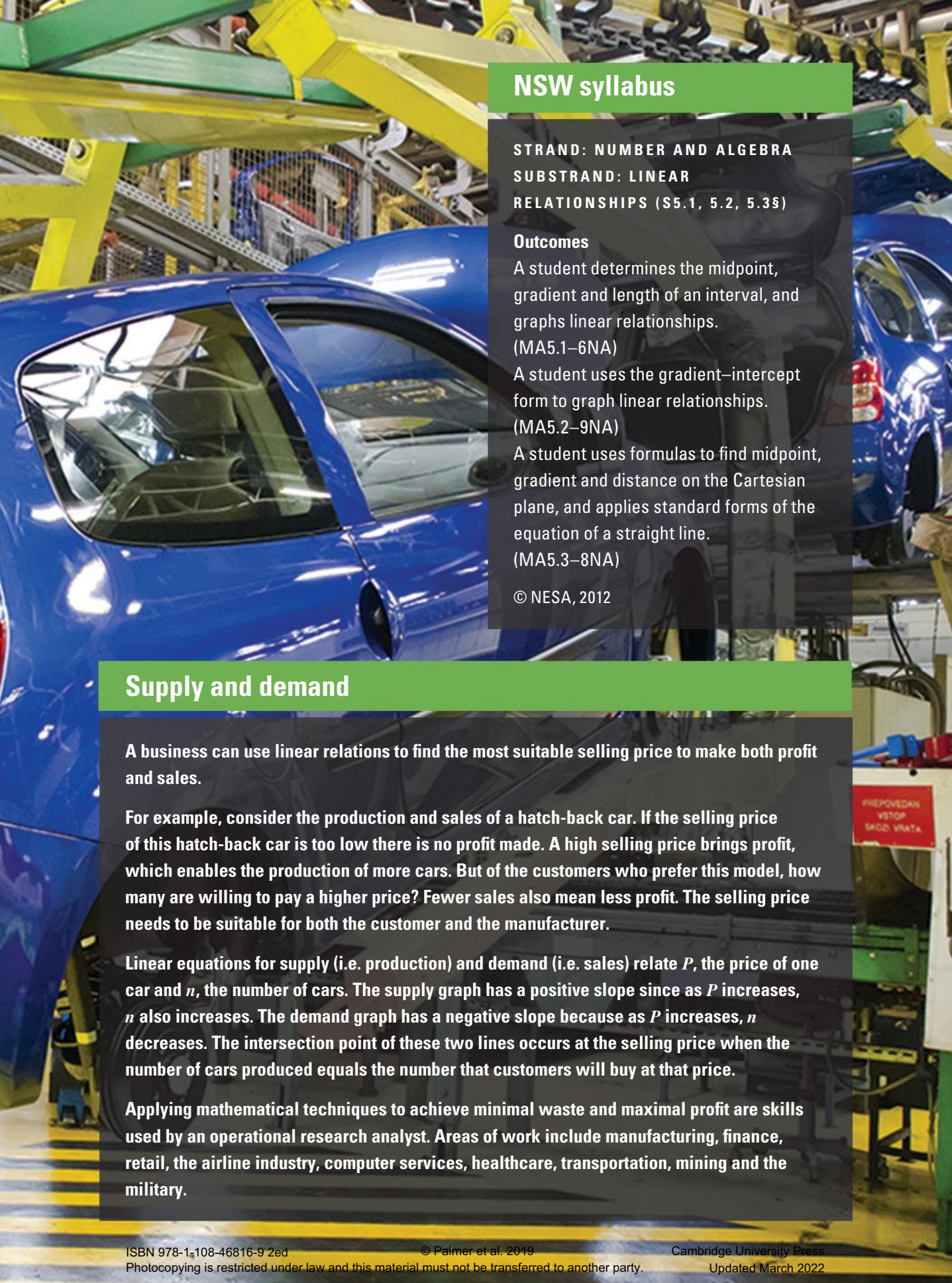
- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

4 Linear relationships

What you will learn

4A Introducing linear relationships
4B Graphing straight lines using intercepts
4C Lines with only one intercept
4D Gradient
4E Gradient and direct proportion
4F Gradient–intercept form
4G Finding the equation of a line using $y = mx + b$
4H Midpoint and length of a line segment from diagrams
4I Perpendicular lines and parallel lines
4J Linear modelling FRINGE
4K Graphical solutions to linear simultaneous equations

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: LINEAR RELATIONSHIPS (S5.1, 5.2, 5.3§)

Outcomes

A student determines the midpoint, gradient and length of an interval, and graphs linear relationships.
(MA5.1–6NA)

A student uses the gradient–intercept form to graph linear relationships.
(MA5.2–9NA)

A student uses formulas to find midpoint, gradient and distance on the Cartesian plane, and applies standard forms of the equation of a straight line.
(MA5.3–8NA)

© NESA, 2012

Supply and demand

A business can use linear relations to find the most suitable selling price to make both profit and sales.

For example, consider the production and sales of a hatch-back car. If the selling price of this hatch-back car is too low there is no profit made. A high selling price brings profit, which enables the production of more cars. But of the customers who prefer this model, how many are willing to pay a higher price? Fewer sales also mean less profit. The selling price needs to be suitable for both the customer and the manufacturer.

Linear equations for supply (i.e. production) and demand (i.e. sales) relate P, the price of one car and n, the number of cars. The supply graph has a positive slope since as P increases, n also increases. The demand graph has a negative slope because as P increases, n decreases. The intersection point of these two lines occurs at the selling price when the number of cars produced equals the number that customers will buy at that price.

Applying mathematical techniques to achieve minimal waste and maximal profit are skills used by an operational research analyst. Areas of work include manufacturing, finance, retail, the airline industry, computer services, healthcare, transportation, mining and the military.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Find the value of y in each of the following when $x = 0$.

a $y = 2x + 3$ **b** $y = 3x - 4$ **c** $x + 2y = 8$ **d** $3x - 4y = 12$

2 Given $y = 3x - 2$, find the value of y when:

a $x = 1$ **b** $x = 3$ **c** $x = -1$ **d** $x = -2$

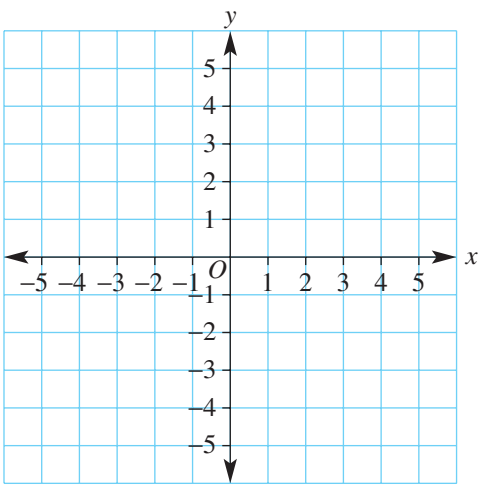
3 Solve the following equations for x.

a $0 = x - 4$ **b** $0 = 3x + 6$ **c** $0 = 2x - 8$

d $0 = \frac{1}{2}x + 2$ **e** $3x = 15$ **f** $\frac{1}{3}x = 2$

4 Plot and label the following points on a Cartesian plane (x–y axes).

a (3, 1) **b** (−2, 4) **c** (0, 0) **d** (5, −3)



5 Determine the average (mean) of the following pairs of numbers.

a 4, 10 **b** 2, 11 **c** −1, 7 **d** −3, −7

6 Find the vertical distance between the following pairs of points.

a (3, 2) and (3, 7) **b** (−1, 1) and (−1, −3)

c (2, 3) and (2, −2) **d** (1, −4) and (1, −1)

7 Find the horizontal distance between the following pairs of points.

a (1, 3) and (5, 3) **b** (−1, 2) and (4, 2)

c (−2, −3) and (5, −3) **d** (−4, 1) and (−1, 1)

8 Rewrite each of the following in the form $y = mx + b$ by making y the subject.

a $y - 2x = 5$ **b** $3x + y = 2$ **c** $-4x + 2y = 6$ **d** $3x - \frac{1}{2}y = 4$

9 If $x = 2$ and $y = 1$, decide if the following equations are true or false.

a $y = 2x + 1$ **b** $y = 3x - 5$ **c** $y = -2x + 5$ **d** $5x - 2y = 6$

10 Determine the value of b in each of the following given:

a $y = 3x + b$ is true when $x = 2$ and $y = 8$

b $y = -2x + b$ is true when $x = -1$ and $y = -3$

c $y = 5x + b$ is true when $x = 1$ and $y = 2$

d $y = \frac{1}{2}x + b$ is true when $x = -4$ and $y = 7$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4A Introducing linear relationships

Interactive

Widgets

HOTsheets

Walkthrough

If two variables are related in some way, we can use mathematical rules to more precisely describe this relationship. The most simple kind of mathematical relationship is one that can be illustrated with a straight line graph. These are called linear relations. The volume of petrol in your car at a service bowser, for example, might initially be 10 L, then be increasing by 1.2 L per second after that. This is an example of a linear relationship between *volume* and *time* because the volume is increasing at a constant rate of 1.2 L/s.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Is it linear?

Here are three rules linking x and y.

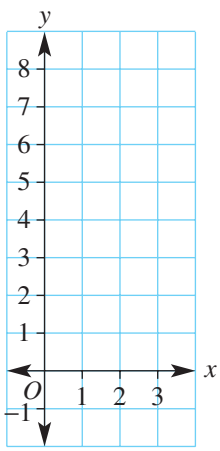
1 $y_1 = \frac{2}{x} + 1$

2 $y_2 = x^2 - 1$

3 $y_3 = 3x - 4$

First complete this simple table and graph.

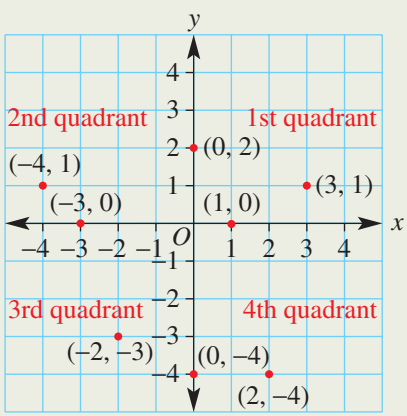
x	1	2	3
y_1			
y_2			
y_3			



- Which of the three rules do you think is linear?
- How do you think the table and graph help you decide it is linear?

Key ideas

- **Coordinate geometry** provides a link between geometry and algebra.
- The **Cartesian plane** (or number plane) consists of two axes which divide the number plane into four **quadrants**.
  - The horizontal x-axis and vertical y-axis intersect at right angles at the **origin** O $(0, 0)$.
  - A point is precisely positioned on a Cartesian plane using the **coordinate pair** (x, y) where x describes the horizontal position and y describes the vertical position of the point from the origin.



- A **linear relationship** is a set of ordered pairs (x, y) that when graphed give a straight line.
- Linear relationships have rules that may be of the form:
  - $y = mx + b$ For example: $y = 2x + 1$
  - $ax + by = d$ or $ax + by + c = 0$ For example: $2x - 3y = 4$ or $2x - 3y - 4 = 0$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

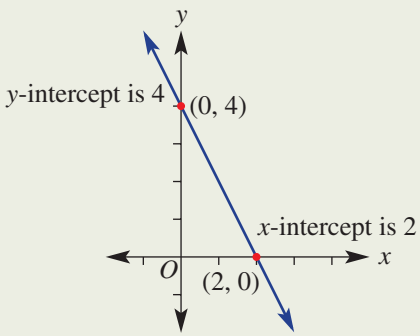
Key ideas

- The y-intercept is the y coordinate of the point at which the line meets the y-axis.
- The x-intercept is the x coordinate of the point at which the line meets the x-axis.

x-intercept is 2

x	−2	−1	0	1	2	3
y	8	6	4	2	0	−2

y-intercept is 4



Example 1 Plotting points to graph straight lines

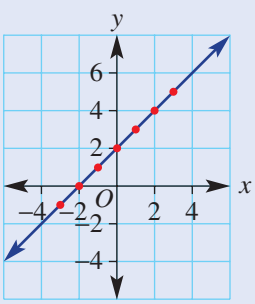
Using $-3 \leq x \leq 3$, construct a table of values and plot a graph for these linear relationships.

a $y = x + 2$ **b** $y = -2x + 2$

SOLUTION

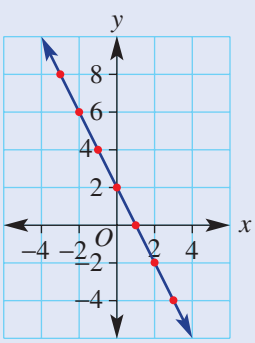
a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



b

x	−3	−2	−1	0	1	2	3
y	8	6	4	2	0	−2	−4



EXPLANATION

Use $-3 \leq x \leq 3$ as instructed and substitute each value of x into the rule $y = x + 2$.

The coordinates of the points are read from the table, i.e. $(-3, -1)$, $(-2, 0)$, etc.

Plot each point and join to form a straight line. Extend the line to show it continues in either direction.

Use $-3 \leq x \leq 3$ as instructed and substitute each value of x into the rule $y = -2x + 2$.

For example:
When $x = -3$
$y = -2 \times (-3) + 2$
$= 6 + 2$
$= 8$

Plot each point and join to form a straight line. Extend the line beyond the plotted points.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

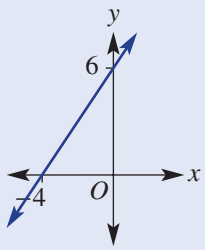
Example 2 Reading off the x-intercept and y-intercept

Write down the x-intercepts and y-intercepts from this table and graph.

a

x	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4

b



SOLUTION

a The x-intercept is 1.

The y-intercept is 2.

b The x-intercept is -4.

The y-intercept is 6.

EXPLANATION

The x-intercept is at the point where $y = 0$ (on the x-axis).

The y-intercept is at the point where $x = 0$ (on the y-axis).

The x-intercept is at the point where $y = 0$ (on the x-axis).

The y-intercept is at the point where $x = 0$ (on the y-axis).

Example 3 Deciding if a point is on a line

Decide if the point $(-2, 4)$ is on the line with the given rules.

a $y = 2x + 10$

b $y = -x + 2$

SOLUTION

a $y = 2x + 10$

Substitute $x = -2$

$y = 2(-2) + 10$

$= 6$

$\therefore$ the point $(-2, 4)$ is not on the line.

b $y = -x + 2$

Substitute $x = -2$

$y = -(-2) + 2$

$= 4$

$\therefore$ the point $(-2, 4)$ is on the line.

EXPLANATION

Find the value of y on the graph of the rule for $x = -2$.

The y value is not 4 so $(-2, 4)$ is not on the line.

By substituting $x = -2$ into the rule for the line, y is 4.

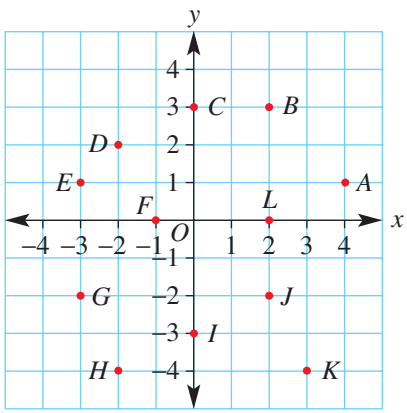
So $(-2, 4)$ is on the line.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4A

UNDERSTANDING AND FLUENCY 1–6 3, 4–7(½) 4–7(½)

1 Refer to the Cartesian plane shown.



- **a** Give the coordinates of all the points A–L.
- **b** Which points lie on the:
  - **i** x-axis?
  - **ii** y-axis?
- **c** Which points lie inside the:
  - **i** 2nd quadrant?
  - **ii** 4th quadrant?
- **d** Which point is closest to the origin?
- **e** Which point is furthest from the origin?

2 For the rule $y = 3x - 4$, find the value of y for these x values.

a $x = 2$ **b** $x = 1$ **c** $x = 0$ **d** $x = -1$

3 For the rule $y = -2x + 1$, find the value of y for these x values.

a $x = 0$ **b** $x = 3$ **c** $x = -1$ **d** $x = -10$

Example 1

4 Using $-3 \leq x \leq 3$, construct a table of values and plot a graph for these linear relationships.

a $y = x - 1$ **b** $y = x + 3$ **c** $y = -2x - 1$

d $y = 2x - 3$ **e** $y = -x + 4$ **f** $y = -3x$

Example 2

5 Write down the x-intercept and y-intercept for each table or graph.

a

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	1	2

b

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

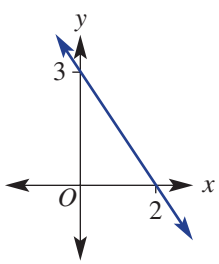
c

x	−1	0	1	2	3	4
y	10	8	6	4	2	0

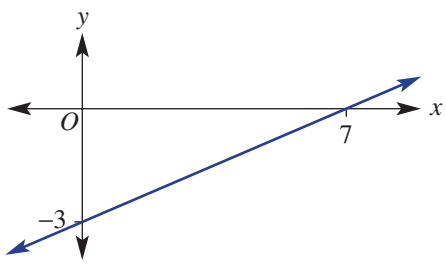
d

x	−5	−4	−3	−2	−1	0
y	0	2	4	6	8	10

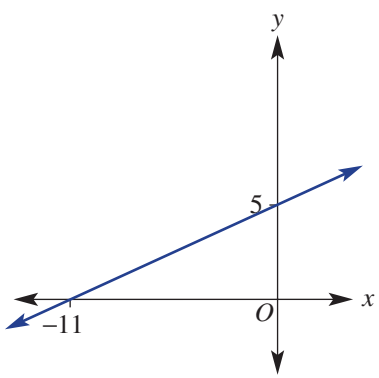
e



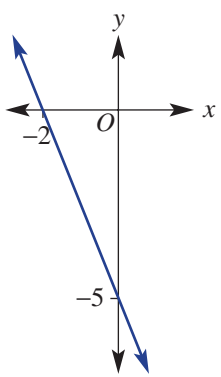
f



g



h



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 3

6 Decide if the point (1, 2) is on the line with the given equation.

a $y = x + 1$ **b** $y = 2x - 1$ **c** $y = -x + 3$

d $y = -2x + 4$ **e** $y = -x + 5$ **f** $y = -\frac{1}{2}x + \frac{1}{2}$

7 Decide if the point (−3, 4) is on the line with the given equation.

a $y = 2x + 8$ **b** $y = x + 7$ **c** $y = -x - 1$

d $y = \frac{1}{3}x + 6$ **e** $y = -\frac{1}{3}x + 3$ **f** $y = \frac{5}{6}x + \frac{11}{2}$

PROBLEM-SOLVING AND REASONING 8, 9, 11 | 8–11 | 8, 10–12

8 Find a rule in the form $y = mx + b$ (e.g. $y = 2x - 1$), which matches these tables of values.

a

x	0	1	2	3	4
y	2	3	4	5	6

b

x	−1	0	1	2	3
y	−2	0	2	4	6

c

x	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7

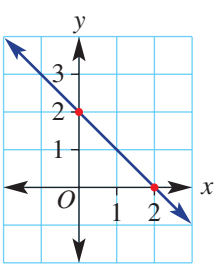
d

x	−3	−2	−1	0	1	2
y	5	4	3	2	1	0

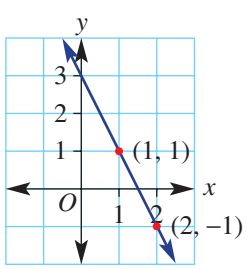
9 Match the rules **A–D** to the graphs **a–d**.

A $y = 2x + 1$ **B** $y = -x - 1$ **C** $y = -2x + 3$ **D** $y = -x + 2$

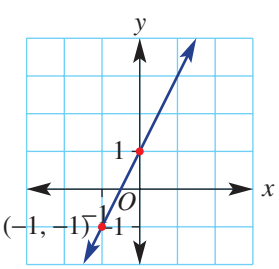
a



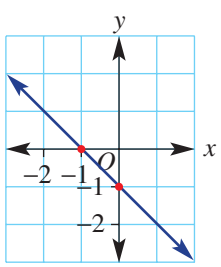
b



c



d



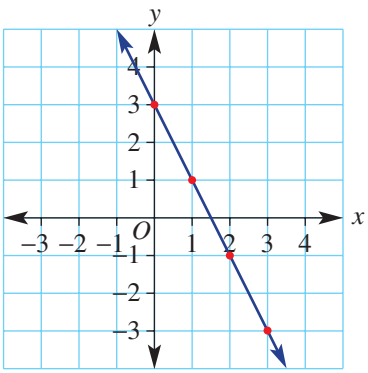
10 Decide if the following equations are true or false.

a $\frac{2x + 4}{2} = x + 4$ **b** $\frac{3x - 6}{3} = x - 2$ **c** $\frac{1}{2}(x - 1) = \frac{1}{2}x - \frac{1}{2}$ **d** $\frac{2}{3}(x - 6) = \frac{2}{3}x - 12$

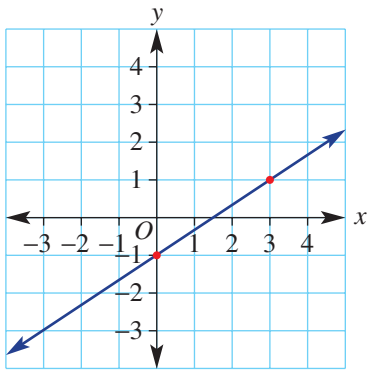
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Give reasons why the x-intercept of these lines is $\frac{3}{2}$.

a



b



12 Decide if the following rules are equivalent.

a $y = 1 - x$ and $y = -x + 1$

b $y = 1 - 3x$ and $y = 3x - 1$

c $y = -2x + 1$ and $y = -1 - 2x$

d $y = -3x + 1$ and $y = 1 - 3x$

ENRICHMENT — — 13

Tough rule finding

13 Find the linear rule linking x and y in these tables.

a

x	−1	0	1	2	3
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	22	21	20	19	18

c

x	0	2	4	6	8
y	−10	−16	−22	−28	−34

d

x	−5	−4	−3	−2	−1
y	29	24	19	14	9

e

x	1	3	5	7	9
y	1	2	3	4	5

f

x	−14	−13	−12	−11	−10
y	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	$3\frac{1}{2}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4B Graphing straight lines using intercepts

Interactive
Widgets
HOTsheets
Walkthrough

When a linear rule is graphed, all the points lie in a straight line. It is possible to graph a straight line using only two points. Two critical points that help draw these graphs are the x-intercept and y-intercept introduced in the previous section.

Stage

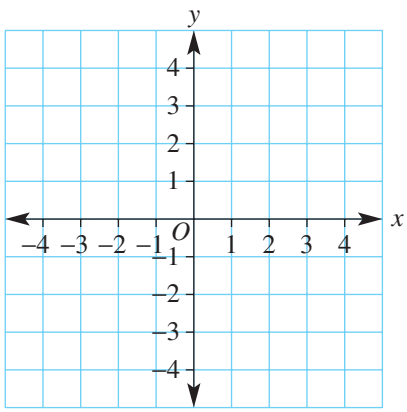
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Two key points

Consider the relation $y = \frac{1}{2}x + 1$ and complete this table and graph.

x	−4	−3	−2	−1	0	1	2
y							

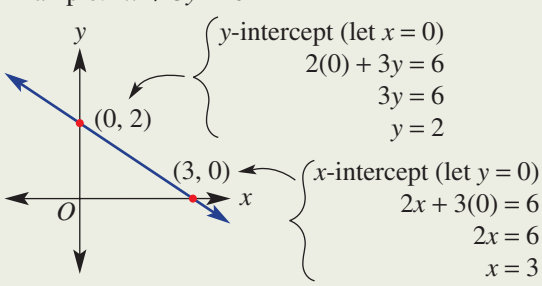
- What are the coordinates of the point where the line crosses the y-axis? That is, state the coordinates of the y-intercept.
- What are the coordinates of the point where the line crosses the x-axis? That is, state the coordinates of the x-intercept.
- Discuss how you might find the coordinates of the x- and y-intercepts without drawing a table and plotting points. Explain your method.



Key ideas

- The y-intercept is the y coordinate of the point at which the line meets the y-axis.
  - Substitute $x = 0$ to find the y-intercept.
- The x-intercept is the x coordinate of the point at which the line meets the x-axis.
  - Substitute $y = 0$ to find the x-intercept.

Example: $2x + 3y = 6$



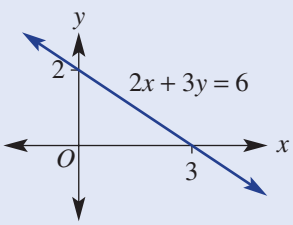
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 4 Graphing lines using intercepts

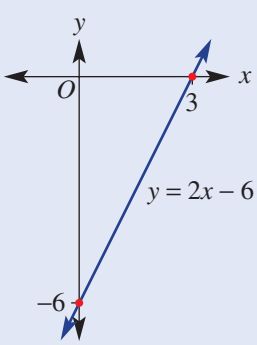
Sketch the graph of the following, showing the x- and y-intercepts.

a $2x + 3y = 6$ **b** $y = 2x - 6$

SOLUTION	EXPLANATION
a $2x + 3y = 6$	Only two points are required to generate a straight line. For the y-intercept, substitute $x = 0$ into the rule and solve for y by dividing each side by 3.
y-intercept (let $x = 0$): $2(0) + 3y = 6$ $3y = 6$ $y = 2$	
$\therefore$ The y-intercept is 2.	State the y-intercept. Plot the point (0, 2).
x-intercept (let $y = 0$): $2x + 3(0) = 6$ $2x = 6$ $x = 3$	Similarly to find the x-intercept, substitute $y = 0$ into the rule and solve for x.
$\therefore$ The x-intercept is 3.	State the x-intercept. Plot the point (3, 0). Mark and label the intercepts on the axes and sketch the graph by joining the two intercepts.



b $y = 2x - 6$	Substitute $x = 0$ for the y-intercept. Simplify to find the y coordinate.
y-intercept (let $x = 0$): $y = 2(0) - 6$ $y = -6$	
$\therefore$ The y-intercept is -6.	Plot the point $(0, -6)$.
x-intercept (let $y = 0$): $0 = 2x - 6$ $6 = 2x$ $x = 3$	Substitute $y = 0$ for the x-intercept. Solve the remaining equation for x by adding 6 to both sides and then dividing both sides by 2.
$\therefore$ The x-intercept is 3.	Plot the point (3, 0). Mark in the two intercepts and join to sketch the graph.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4B

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	3–7(½)	5–7(½)

1 For the line $y = x + 3$.
- **a** Find the value of y when $x = 0$.
- **b** Find the value of x when $y = 0$.
- **c** At what point does this line meets the y-axis?
- **d** At what point does this line meets the x-axis?
- **e** Sketch the line.

2 Repeat Question **1** for the line $x + y = 3$.

3 For these equations find the y-intercept by letting $x = 0$.
- **a** $x + y = 4$
- **b** $x - y = 5$
- **c** $2x + 3y = 9$
- **d** $y = 2x - 4$

4 For these equations find the x-intercept by letting $y = 0$.
- **a** $x + 2y = 5$
- **b** $2x - y = -4$
- **c** $4x - 3y = 12$
- **d** $y = 3x - 6$

Example 4a

5 Sketch the graph of the following relations, by finding the x- and y-intercepts.

a $x + y = 2$	**b** $x + y = 5$	**c** $x - y = 3$	**d** $x - y = -2$
e $2x + y = 4$	**f** $3x - y = 9$	**g** $4x - 2y = 8$	**h** $3x + 2y = 6$
i $3x - 2y = 6$	**j** $y - 3x = 12$	**k** $-5y + 2x = -10$	**l** $-x + 7y = 21$

Example 4b

6 Sketch the graph of the following relations, showing the x- and y-intercepts.

a $y = 3x + 3$	**b** $y = 2x + 2$	**c** $y = x - 5$
d $y = -x - 6$	**e** $y = -2x - 2$	**f** $y = -3x - 6$
g $y = -2x + 4$	**h** $y = 2x - 3$	**i** $y = -x + 1$

7 Sketch the graph of each of the following mixed linear relations.

a $x + 2y = 8$	**b** $3x - 5y = 15$	**c** $3x + 4y = 12$
d $y = 3x - 6$	**e** $3y - 4x = 12$	**f** $5y - x = 10$
g $2x - y - 4 = 0$	**h** $2x - y + 5 = 0$	**i** $x - 4y + 2 = 0$

PROBLEM-SOLVING AND REASONING	8, 9, 11	8, 9, 11, 12	9, 10, 12, 13

8 The distance d metres of a vehicle from an observation point after t seconds is given by the rule $d = 8 - 2t$.
- **a** Find the distance from the observation point initially (at $t = 0$).
- **b** Find after what time t the distance d is equal to 0 (substitute $d = 0$).
- **c** Sketch a graph of d against t with t on the horizontal axis.

9 The height h, in metres, of a lift above ground after t seconds is given by $h = 100 - 8t$.
- **a** How high is the lift initially (at $t = 0$)?
- **b** How long does it take for the lift to reach the ground ($h = 0$)?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Find the x- and y-axis intercepts of the graphs with the given rules. Write answers using fractions.

a $3x - 2y = 5$

b $x + 5y = -7$

c $y - 2x = -13$

d $y = -2x - 1$

e $2y = x - 3$

f $-7y = 1 - 3x$

11 Use your algebra and fraction skills to help sketch graphs for these relations by finding x- and y-intercepts.

a $\frac{x}{2} + \frac{y}{3} = 1$

b $y = \frac{8 - x}{4}$

c $\frac{y}{2} = \frac{2 - 4x}{8}$

12 Explain why the graph of the equation $ax + by = 0$ must pass through the origin for any values of the constants a and b.

13 Write down the rule for the graph with these axes intercepts. Write the rule in the form $ax + by = d$.

a (0, 4) and (4, 0)

b (0, 2) and (2, 0)

c (0, −3) and (3, 0)

d (0, 1) and (−1, 0)

e (0, k) and (k, 0)

f (0, $-k$) and ($-k$, 0)

ENRICHMENT — — 14

Intercept families

14 Find the x- and y-intercepts in terms of the constants a, b and c for these relations.

a $ax + by = c$

b $y = \frac{a}{b}x + c$

c $\frac{ax - by}{c} = 1$

d $ay - bx = c$

e $ay = bx + c$

f $a(x + y) = bc$



Using a linear graph we can model the time it takes for a cyclist to reach the top of a hill.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4C Lines with only one intercept

Interactive

Widgets

HOTsheets

Walkthrough

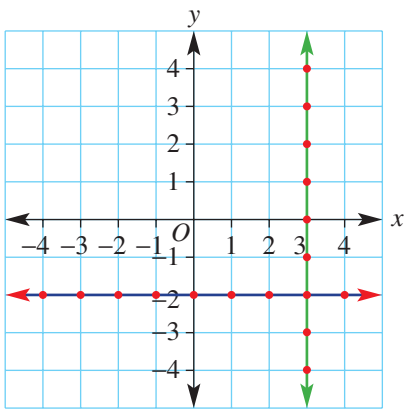
Lines with only one intercept include vertical lines, horizontal lines and lines that pass through the origin.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: What rule satisfies all points?

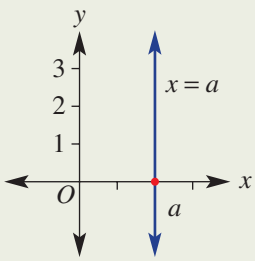
Here is one vertical and one horizontal line.

- For the vertical line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- What simple equation describes every point on the line?
- For the horizontal line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- What simple equation describes every point on the line?
- Where do the two lines intersect?

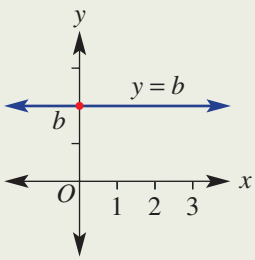


Key ideas

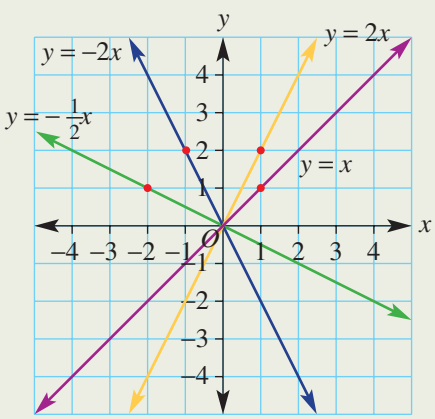
- Vertical line: $x = a$
  - Parallel to the y-axis
  - Equation of the form $x = a$, where a is a constant
  - x-intercept is a



- Horizontal line: $y = b$
  - Parallel to the x-axis
  - Equation of the form $y = b$, where b is a constant
  - y-intercept is b



- Lines through the origin $(0, 0)$: $y = mx$
  - y-intercept is 0
  - x-intercept is 0
  - Substitute $x = 1$ or some value of x other than 0 to find a second point



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 5 Graphing vertical and horizontal lines

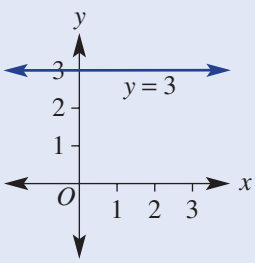
Sketch the graph of the following vertical and horizontal lines.

a $y = 3$

b $x = -4$

SOLUTION

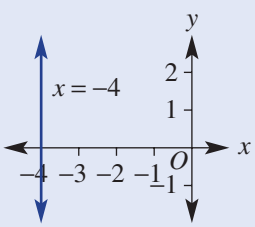
a



EXPLANATION

y-intercept is 3.
Sketch a horizontal line through all points where $y = 3$.

b



x-intercept is -4.
Sketch a vertical line through all points where $x = -4$.

Example 6 Sketching lines which pass through the origin

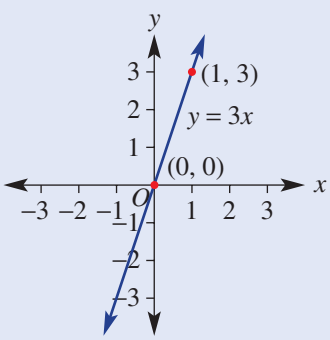
Sketch the graph of $y = 3x$.

SOLUTION

The x- and y-intercepts are 0.
Another point (let $x = 1$):

$$y = 3 \times 1$$
$$y = 3$$

Another point is at (1, 3).



EXPLANATION

The equation is of the form $y = mx$.
Since two points are required to generate the straight line, find another point by substituting $x = 1$.
Other x values could also be shown.

Plot and label both points and sketch the graph by joining the points in a straight line.

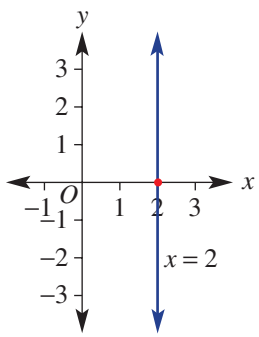
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4C

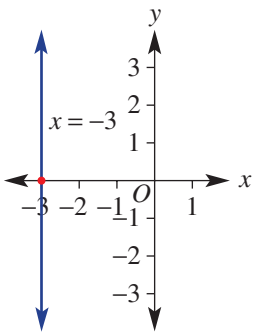
UNDERSTANDING AND FLUENCY	1–3, 4–6(½), 7	3, 4–7(½)	4–7(½)

1 State the x-intercepts for these graphs.

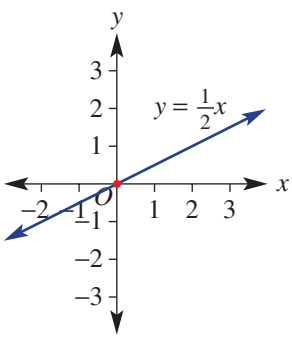
a



b

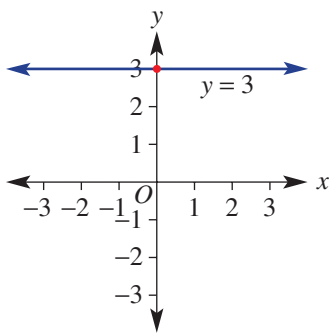


c

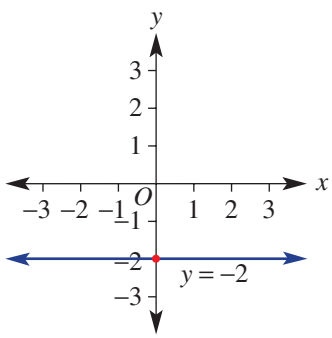


2 State the y-intercepts for these graphs.

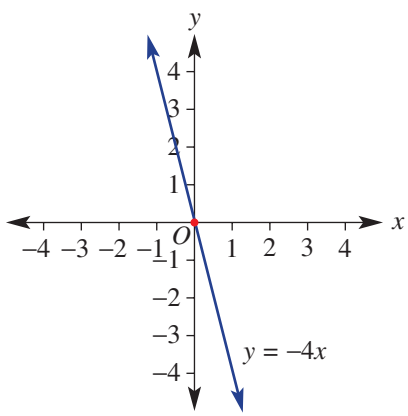
a



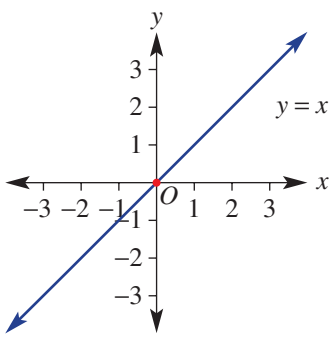
b



c



d



3 Find the value of y if $x = 1$, using these rules.

a $y = 5x$ **b** $y = \frac{1}{3}x$ **c** $y = -4x$ **d** $y = -0.1x$

Example 5

4 Sketch the graphs of the following vertical and horizontal lines.

a $x = 2$ **b** $x = 5$ **c** $y = 4$ **d** $y = 1$
e $x = -3$ **f** $x = -2$ **g** $y = -1$ **h** $y = -3$

Example 6

5 Sketch the graphs of the following linear relations which pass through the origin.

a $y = 2x$ **b** $y = 5x$ **c** $y = 4x$ **d** $y = x$
e $y = -4x$ **f** $y = -3x$ **g** $y = -2x$ **h** $y = -x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Sketch the graphs of these special lines all on the same set of axes and label them with their equations.

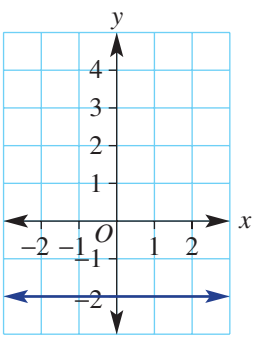
a $x = -2$ **b** $y = -3$ **c** $y = 2$ **d** $x = 4$

e $y = 3x$ **f** $y = -\frac{1}{2}x$ **g** $y = -1.5x$ **h** $x = 0.5$

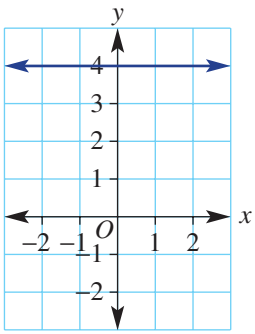
i $x = 0$ **j** $y = 0$ **k** $y = 2x$ **l** $y = 1.5x$

7 What is the equation of each of the following graphs?

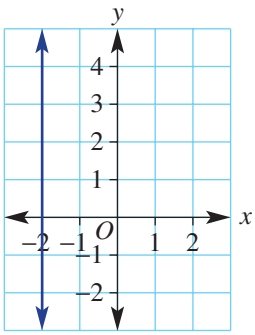
a



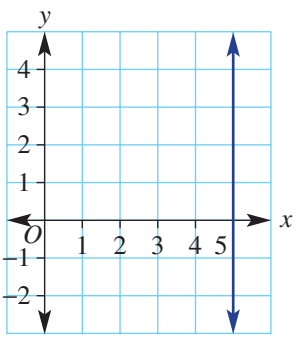
b



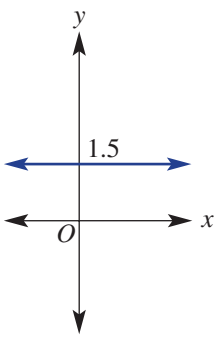
c



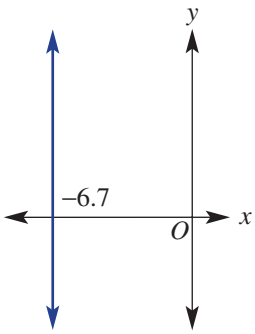
d



e



f



PROBLEM-SOLVING AND REASONING 8, 9, 13 | 9, 10(½), 11, 13–14(½) | 10(½), 11, 12, 13–14(½)

8 Find the equation of the straight line that is:

a parallel to the x-axis and passes through the point (1, 3)

b parallel to the y-axis and passes through the point (5, 4)

c parallel to the y-axis and passes through the point (−2, 4)

d parallel to the x-axis and passes through the point (0, 0)

9 If in a picture, the surface of the sea is represented by the x-axis, state the equation of the following paths.

a A plane flies horizontally at 250 m above sea level. One unit is 1 metre.

b A submarine travels horizontally 45 m below sea level. One unit is 1 metre.

10 Each pair of lines intersects at a point. Find the coordinates of the point.

a $x = 1, y = 2$ **b** $x = -3, y = 5$ **c** $x = 0, y = -4$

d $x = 4, y = 0$ **e** $y = -6x, x = 0$ **f** $y = 3x, x = 1$

g $y = -9x, x = 3$ **h** $y = 8x, y = 40$ **i** $y = 5x, y = 15$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Find the area of the rectangle contained within the following four lines.

a $x = 1$, $x = -2$, $y = -3$, $y = 2$ **b** $x = 0$, $x = 17$, $y = -5$, $y = -1$

12 The lines $x = -1$, $x = 3$ and $y = -2$ form three sides of a rectangle. Find the possible equation of the fourth line if the:

a area of the rectangle is:

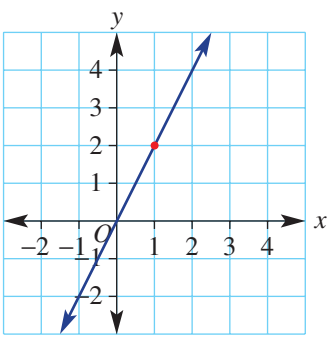
i 12 square units **ii** 8 square units **iii** 22 square units

b perimeter of the rectangle is:

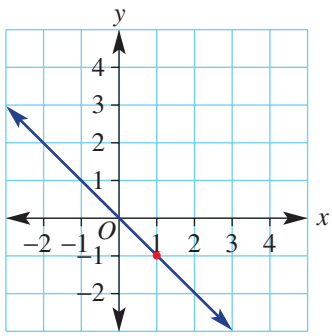
i 14 units **ii** 26 units **iii** 31 units

13 The rules of the following graphs are of the form $y = mx$. Use the points marked with a dot to find m and hence state the equation.

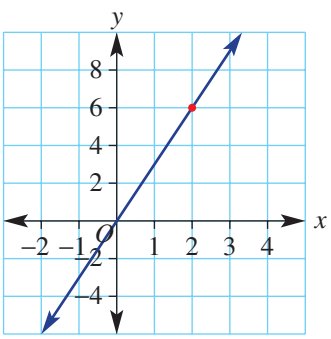
a



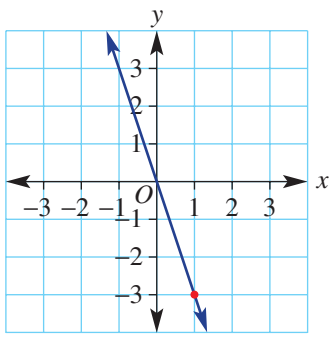
b



c



d



14 Find the equation of the line which passes through the origin and the given point.

a (1, 3) **b** (1, 4) **c** (1, −5) **d** (1, −2)

ENRICHMENT — — 15–17

Trisection

15 A vertical line, horizontal line and another line that passes through the origin all intersect at $(-1, -5)$. What are the equations of the three lines?

16 If $y = b$, $x = a$ and $y = mx$ all intersect at one point:

a state the coordinates of the intersection point **b** find m in terms of a and b

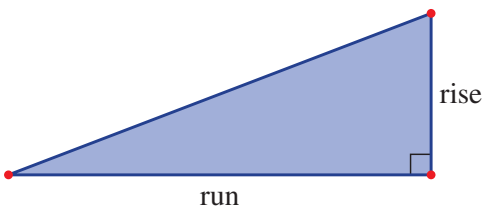
17 The area of a triangle formed by $x = 4$, $y = -2$ and $y = mx$ is 16 square units. Find the value of m, given $m > 0$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4D Gradient

Interactive
Widgets
HOTsheets
Walkthrough

The gradient of a line is a measure of its slope. It is a number which describes the steepness of a line and is calculated by considering how far a line rises or falls between two points within a given horizontal distance. The horizontal distance between two points is called the *run* and the vertical distance is called the *rise*.





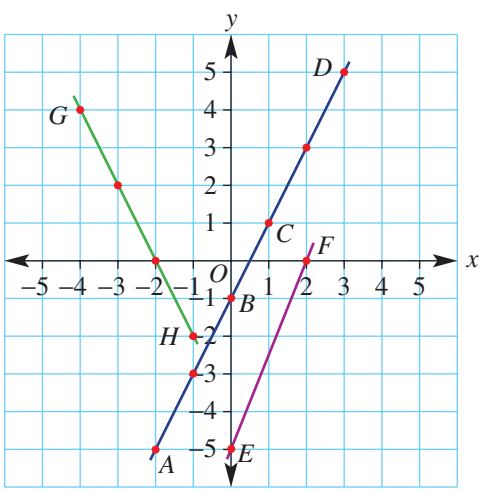
Gradients can be used to describe the steepness of this rollercoaster track.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Which line is the steepest?

The three lines here connect the points A, B, C, D, E, F, G and H.

- Calculate the rise and run (working from left to right) and also the fraction $\frac{\text{rise}}{\text{run}}$ for these segments.
  - **i** AB
  - **ii** BC
  - **iii** BD
  - **iv** EF
  - **v** GH
- What do you notice about the fractions $\left(\frac{\text{rise}}{\text{run}}\right)$ for parts **i**, **ii** and **iii**?
- How does the $\frac{\text{rise}}{\text{run}}$ for EF compare with the $\frac{\text{rise}}{\text{run}}$ for parts **i**, **ii** and **iii**? Which of the two lines is the steepest?
- Your $\frac{\text{rise}}{\text{run}}$ for GH should be negative. Why is this the case?
- Discuss whether or not GH is steeper than AD.



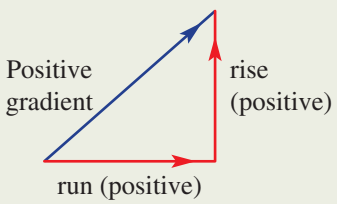
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

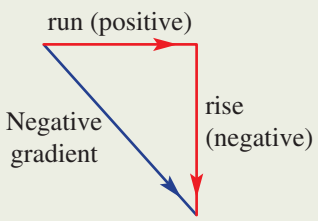
Use computer software (dynamic geometry) to produce a set of axes and grid.

- Construct a line segment with end points on the grid. Show the coordinates of the end points.
- Calculate the rise (vertical distance between the end points) and the run (horizontal distance between the end points).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the end points and explore the effect on the gradient.
- Can you drag the end points but retain the same gradient value? Explain why this is possible.
- Can you drag the end points so that the gradient is zero or undefined? Describe how this can be achieved.

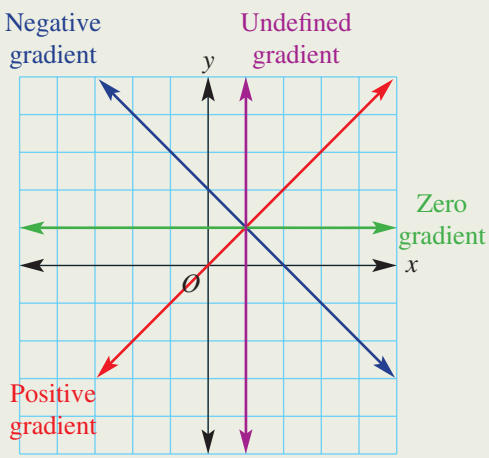
Key ideas

- **Gradient**
- $m = \dfrac{\text{rise}}{\text{run}}$





- We work from left to right, so the run is always positive.
- The gradient can be positive, negative, zero or undefined.
- A vertical line has an undefined gradient.
- Gradient is usually expressed as a simplified proper or improper fraction (not a mixed numeral).

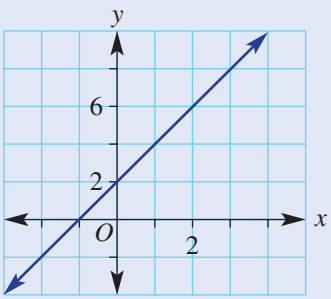


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

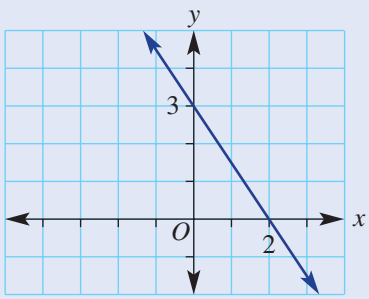
Example 7 Finding the gradient of a line

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

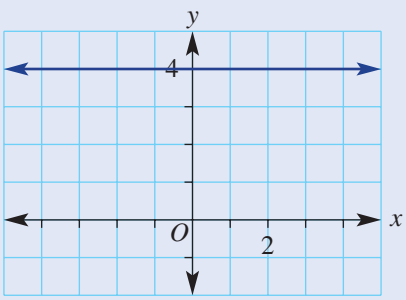
a



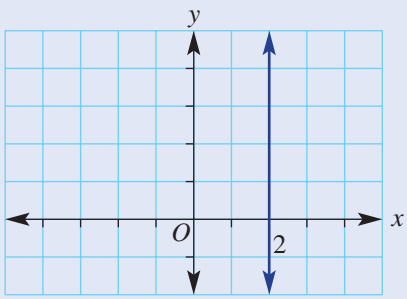
b



c

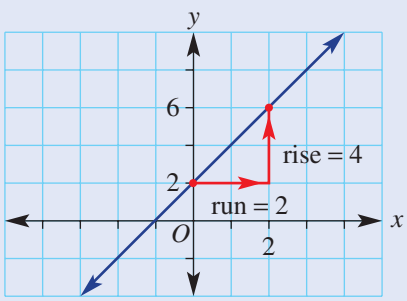


d



SOLUTION

a The gradient is positive.



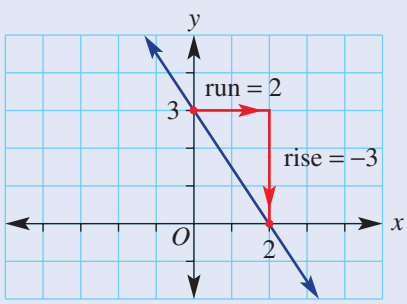
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{2}$$
$$m = 2$$

EXPLANATION

By inspection, the gradient will be positive since the graph rises from left to right. Select any two points and create a right-angled triangle to determine the rise and run.

Substitute rise = 4 and run = 2.

b The gradient is negative.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-3}{2}$$
$$m = -\frac{3}{2}$$

By inspection, the gradient will be negative since y values decrease from left to right.

Rise = −3 and run = 2.

c The gradient is 0.

The line is horizontal.

d The gradient is undefined.

The line is vertical.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8 Finding the gradient between two points

Find the gradient (m) of the line joining the given points.

a $A(3, 4)$ and $B(5, 6)$ **b** $A(-3, 6)$ and $B(1, -3)$

SOLUTION

a $m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{6 - 4}{5 - 3}$

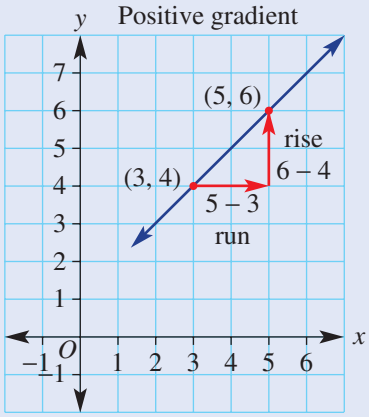
$= \dfrac{2}{2}$

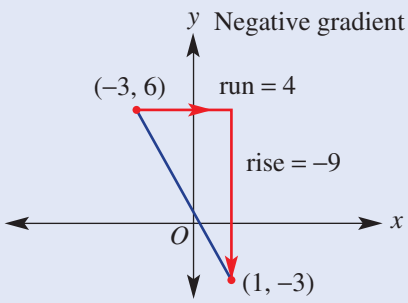
$= 1$

b $m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-9}{4}$ or $-\dfrac{9}{4}$ or -2.25

EXPLANATION



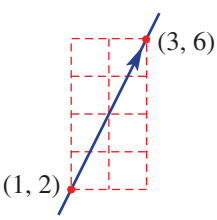


Exercise 4D

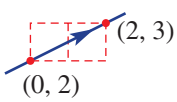
UNDERSTANDING AND FLUENCY	1, 2, 3–4(½)	2, 3–4(½), 5	3–4(½), 5

1 Calculate the gradient using $\dfrac{\text{rise}}{\text{run}}$ for these lines. Remember to give a negative answer if the line is sloping downward from left to right.

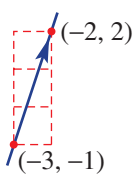
a



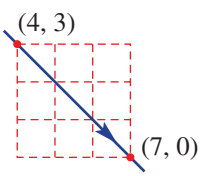
b



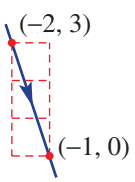
c



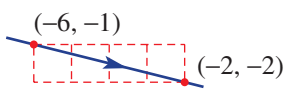
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Use the words: positive, negative, zero or undefined to complete each sentence.

a The gradient of a horizontal line is _____.

b The gradient of the line joining (0, 3) with (5, 0) is _____.

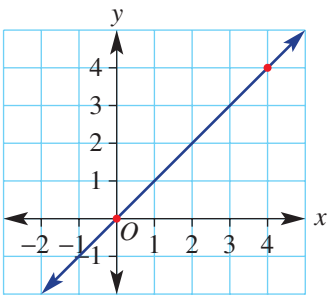
c The gradient of the line joining (−6, 0) with (1, 1) is _____.

d The gradient of a vertical line is _____.

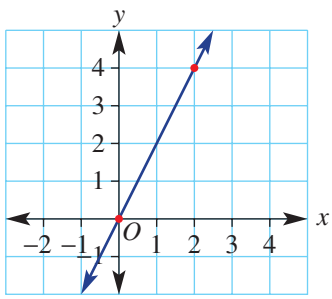
Example 7

3 For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

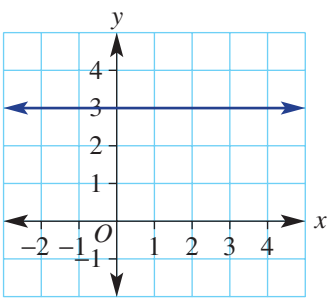
a



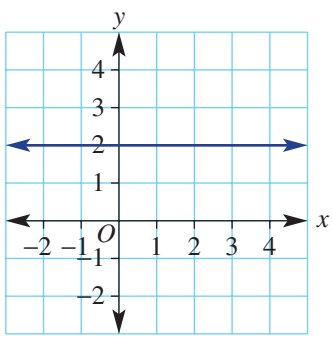
b



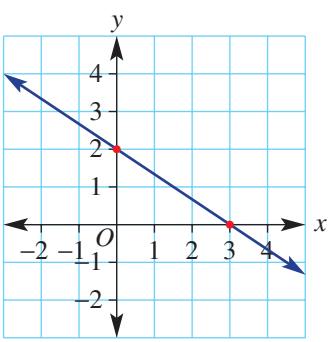
c



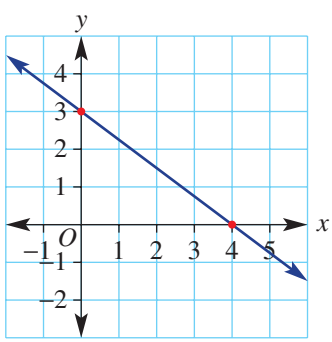
d



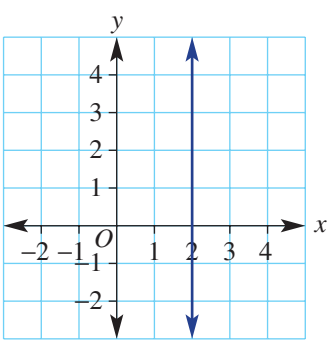
e



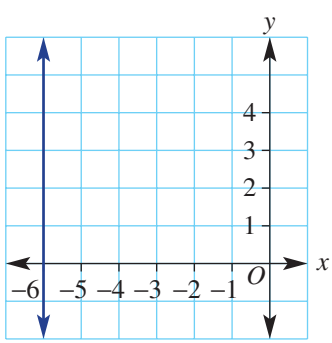
f



g

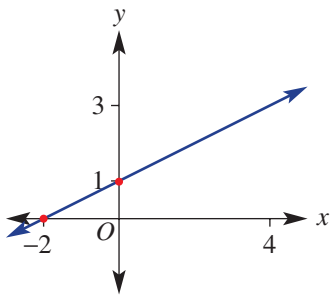


h

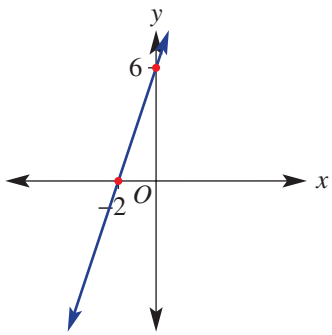


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

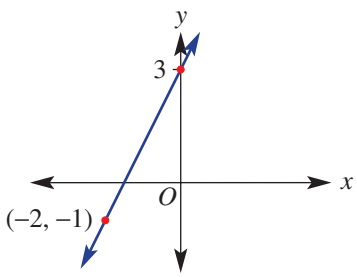
i



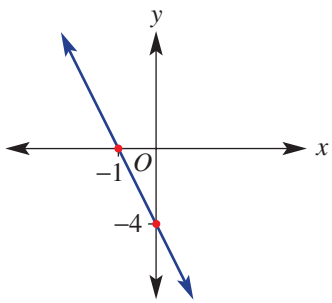
j



k



l

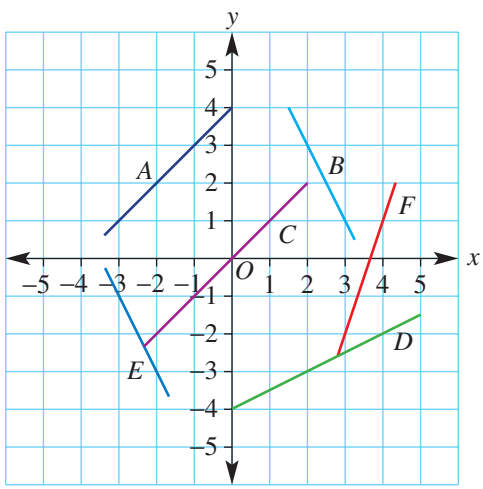


Example 8

4 Find the gradient of the lines joining the following pairs of points.

a $A(2, 3)$ and $B(3, 5)$
b $C(-2, 6)$ and $D(0, 8)$
c $E(-4, 1)$ and $F(4, -1)$
d $G(2, 1)$ and $H(5, 5)$
e $A(1, 5)$ and $B(2, 7)$
f $C(-2, 4)$ and $D(1, -2)$
g $E(-3, 4)$ and $F(2, -1)$
h $G(-1, 5)$ and $H(1, 6)$
i $A(-2, 1)$ and $B(-4, -2)$
j $C(3, -4)$ and $D(1, 1)$
k $E(3, 2)$ and $F(0, 1)$
l $G(-1, 1)$ and $H(-3, -4)$

5 Find the gradient of each line A–F on this graph and grid.



PROBLEM-SOLVING AND REASONING	6, 7, 10	6–8, 10, 11	7–9, 10, 11

6 Find the gradient corresponding to the following slopes.

a A road falls 10 m for every 200 horizontal metres.
b A cliff rises 35 metres for every 2 metres horizontally.
c A plane descends 2 km for every 10 horizontal kilometres.
d A submarine ascends 150 m for every 20 horizontal metres.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Find the missing number.

- **a** The gradient joining the points (0, 2) and (1, ☐) is 4.
- **b** The gradient joining the points (☐, 5) and (1, 9) is 2.
- **c** The gradient joining the points (−3, ☐) and (0, 1) is −1.
- **d** The gradient joining the points (−4, −2) and (☐, −12) is −4.

8 A train climbs a slope with gradient 0.05. How far horizontally has the train travelled after rising 15 metres?

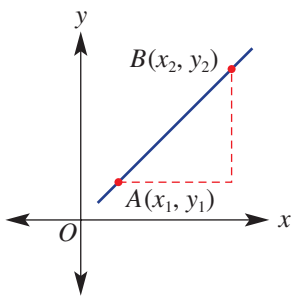
9 Complete this table showing the gradient, x-intercept and y-intercept for straight lines.

	a	b	c	d	e	f
Gradient	3	−1	$\frac{1}{2}$	$-\frac{2}{3}$	0.4	−1.25
***x*-intercept**	−3			6	1	
***y*-intercept**		−4	$\frac{1}{2}$			3

10 Give reasons why a line with gradient $\frac{7}{11}$ is steeper than a line with gradient $\frac{3}{5}$.

11 The two points A and B shown here have coordinates (x_1, y_1) and (x_2, y_2).

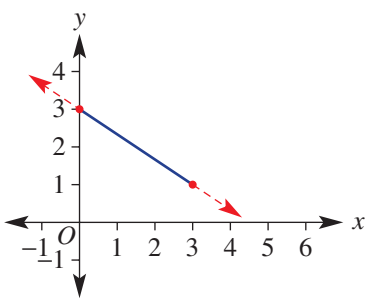
- **a** Write a rule for the run using x_1 and x_2.
- **b** Write a rule for the rise using y_1 and y_2.
- **c** Write a rule for the gradient m using x_1, x_2, y_1 and y_2.
- **d** Use your rule to find the gradient between these pairs of points.
  - **i** (1, 1) and (3, 4)
  - **ii** (0, 2) and (4, 7)
  - **iii** (−1, 2) and (2, −3)
  - **iv** (−4, −6) and (−1, −2)
- **e** Does your rule work for points which include negative coordinates? Explain why.



ENRICHMENT — — 12

Where does it hit?

12 The line here has gradient $\frac{-2}{3}$, which means that it falls 2 units for every 3 across. The y-intercept is (0, 3).



- **a** Use the gradient to find the y coordinate on the line where:
  - **i** $x = 6$
  - **ii** $x = 9$
- **b** What will be the x-intercept?
- **c** What would be the x-intercept if the gradient was changed to:
  - **i** $\frac{-1}{2}$
  - **ii** $\frac{-5}{4}$
  - **iii** $\frac{-7}{3}$
  - **iv** $\frac{-2}{5}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4E Gradient and direct proportion

Interactive
Widgets
HOTsheets
Walkthrough

The connection between gradient, rate problems and direct proportion can be illustrated through the use of linear rules and graphs. If two variables are directly related, then the rate of change of one variable with respect to the other is constant. This implies that the rule linking the two variables is linear and can be represented as a straight line graph passing through the origin. The amount of water squirting from a hose, for example, is directly proportional to the time since it was turned on. The gradient of the graph of *water volume* versus *time* will equal the rate at which water is squirting from the hose.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

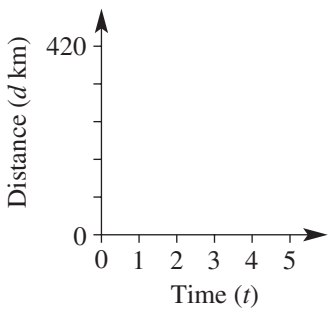


The volume of water squirting from a hose is directly proportional to the time since it was turned on.

Let's start: Average speed

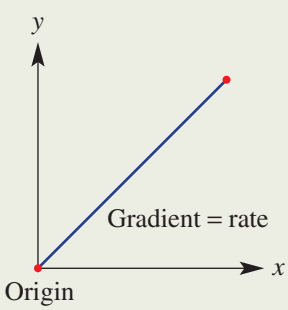
Over 5 hours, Sandy travels 420 km.

- What is Sandy's average speed for the trip?
- Is speed a rate? Discuss.
- Draw a graph of distance versus time, assuming a constant speed.
- Where does your graph intersect the axes and why?
- Find the gradient of your graph. What do you notice?
- Find a rule linking distance (d) and time (t).



Key ideas

- If two variables are **directly proportional**:
  - the rate of change of one variable with respect to the other is constant.
  - the graph is a straight line passing through the origin.
  - the rule is of the form $y = mx$ (or $y = kx$).
  - the gradient (m) of the graph equals the rate of change of y with respect to x.
  - the letter m (or k) is called the constant of proportionality.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 9 Exploring direct proportion

Water is poured into an empty tank at a constant rate. It takes 3 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leq t \leq 3$.

c Find the:

- **i** gradient of your graph
- **ii** rule for V

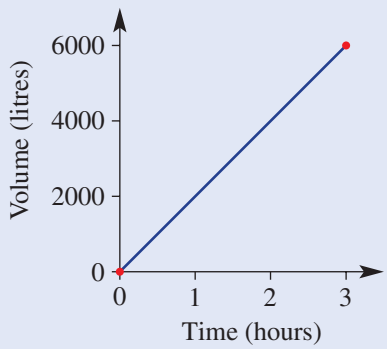
d Use your rule to find the:

- **i** volume after 1.5 hours
- **ii** time to fill 5000 litres

SOLUTION

a 6000 L in 3 hours = 2000 L/hour

b



c **i** $\text{gradient} = \dfrac{6000}{3} = 2000$

ii $V = 2000t$

d **i** $V = 2000t$
$= 2000 \times 1.5$
$= 3000$ litres

ii $V = 2000t$
$5000 = 2000t$
$2.5 = t$

$\therefore$ It takes $2\frac{1}{2}$ hours.

EXPLANATION

6000 L per 3 hours = 2000 L per 1 hour

Plot the two end points (0, 0) and (3, 6000), then join with a straight line.

The gradient is the same as the rate.

2000 L are filled for each hour.

Substitute $t = 1.5$ into your rule.

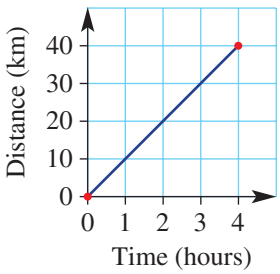
Substitute $V = 5000$ into the rule and solve for t.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4E

UNDERSTANDING AND FLUENCY | 1–5 | 2–6 | 3–6

1 This graph shows how far a bike travels over 4 hours.



a State how far the bike has travelled after:
- **i** 1 hour
- **ii** 2 hours
- **iii** 3 hours

b Write down the speed of the bike (rate of change of distance over time).

c Find the gradient of the graph.

d What do you notice about your answers from parts **b** and **c**?

2 The rule linking the height of a plant over time is given by $h = 5t$, where h is in millimetres and t is in days.

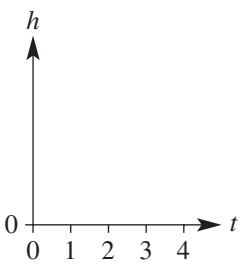
a Find the height of the plant after 3 days.

b Find the time for the plant to reach:
- **i** 30 mm
- **ii** 10 cm

c Complete this table.

t	0	1	2	3	4
h					

d Complete this graph.



e Find the gradient of the graph.

Example 9

3 A 300-litre fish tank takes 3 hours to fill from a hose.



a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leq t \leq 3$.

c Find the:
- **i** gradient of your graph
- **ii** rule for V

d Use your rule to find the:
- **i** volume after 1.5 hours
- **ii** time to fill 2000 litres

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 A solar-powered car travels 100 km in 4 hours.

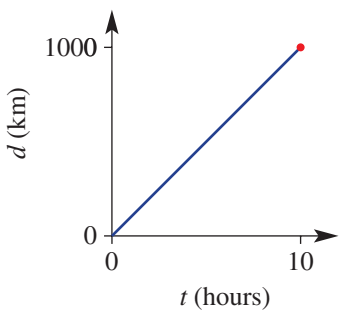
- **a** What is the rate of change of distance over time (i.e. speed)?
- **b** Draw a graph of distance (d km) vs time (t hours) using $0 \leq t \leq 4$.
- **c** Find the:
  - **i** gradient of your graph
  - **ii** rule for d
- **d** Use your rule to find the:
  - **i** distance after 2.5 hours
  - **ii** time to travel 40 km

5 Write down a rule linking the given variables.

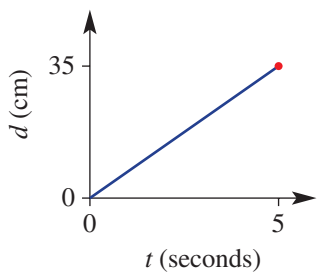
- **a** I travel 600 km in 12 hours. Use d for distance and t for time.
- **b** A calf grows 12 cm in 6 months. Use g for growth height and t for time.
- **c** The cost of petrol is \$100 for 80 litres. Use C for cost and n for the number of litres.
- **d** The profit is \$10 000 for 500 tonnes. Use P for profit and t for the number of tonnes.

6 Use the gradient to find the rate of change of distance over time (speed) for these graphs. Use the units given on each graph.

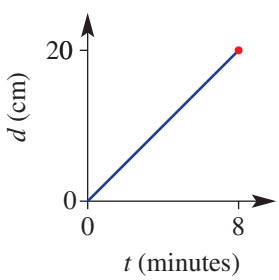
a



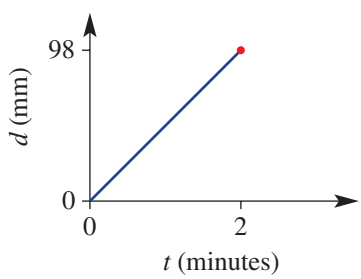
b



c



d



PROBLEM-SOLVING AND REASONING	7, 8, 11	7–9, 11, 12	9, 10, 12–14

7 A car's trip computer says that the fuel economy for a trip is 8.5 L per 100 km.

- **a** How many litres would be used for 120 km?
- **b** How many litres would be used for 850 km?
- **c** How many kilometres could be travelled if the car's petrol tank capacity was 68 L?

8 Who is travelling the fastest?

- Mick runs 120 m in 20 seconds.
- Sally rides 700 m in 1 minute.
- Udhav jogs 2000 m in 5 minutes.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 Which animal is travelling the slowest?

- A leopard runs 200 m in 15 seconds.
- A jaguar runs 2.5 km in 3 minutes.
- A panther runs 60 km in 1.2 hours.



10 An investment fund starts at \$0 and grows at a rate of \$100 per month. Another fund starts at \$4000 and reduces by \$720 per year. After how long will the funds have the same amount of money?

11 The circumference of a circle (given by $C = 2\pi r$) is directly proportional to its radius.

a Find the circumference for a circle with the given radius. Give an exact answer like 6π.

i $r = 0$

ii $r = 2$

iii $r = 6$

b Draw a graph of C against r for $0 \leq r \leq 6$. Use exact values for C.

c Find the gradient of your graph. What do you notice?

12 Is the area of a circle directly proportional to its radius? Give reasons.

13 The base length of a triangle is 4 cm but its height h cm is variable.

a Write a rule for the area of this triangle.

b What is the rate at which the area changes with respect to height h?

14 Over a given time interval (say 5 hours), is an object's speed directly proportional to the distance travelled? Give a rule for speed (s) in terms of distance (d).

ENRICHMENT	—	—	15, 16

Rate challenge

15 Hose A can fill a bucket in 2 minutes and hose B can fill the same bucket in 3 minutes. How long would it take to fill the bucket if both hoses were used at the same time?

16 A river is flowing downstream at rate of 2 km/h. Murray can swim at a rate of 3 km/h. Murray jumps in and swims downstream for a certain distance then turns around and swims upstream back to the start. In total it takes 30 minutes. How far did Murray swim downstream?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

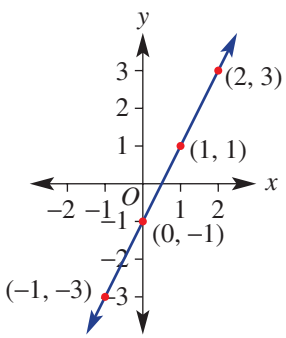
4F Gradient–intercept form

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

Shown here is the graph of the rule $y = 2x - 1$. It shows a gradient of 2 and a y-intercept of -1. The fact that these two numbers correspond to numbers in the rule is no coincidence. This is why rules written in this form are called gradient–intercept form. Other examples of rules in this form include:

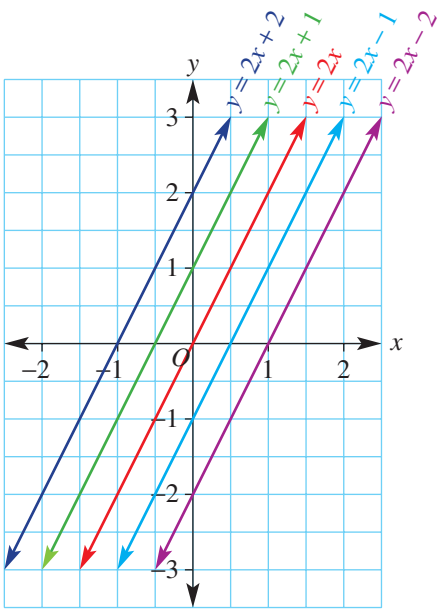
$$y = -5x + 2, y = \frac{1}{2}x - 0.5 \text{ and } y = \frac{x}{5} + 20.$$



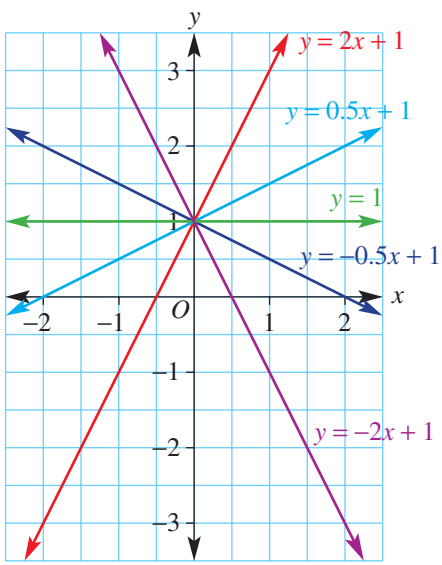
Let's start: Family traits

The graph of a linear relation can be sketched easily if you know the gradient and the y-intercept. If one of these is kept constant, we create a family of graphs.

Different y-intercepts and same gradient



Different gradients and same y-intercept



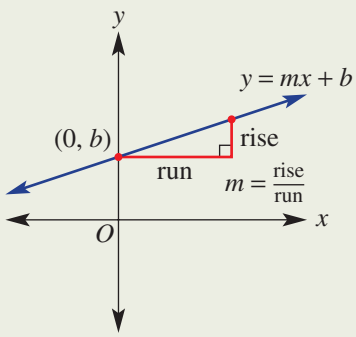
- For the first family, discuss the relationship between the y-intercept and the given rule for each graph.
- For the second family, discuss the relationship between the gradient and the given rule for each graph.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- The **gradient–intercept form** of a straight line equation.
  m = gradient, y-intercept $= b$
  $y = mx + b$ or $y = mx + c$
- If the y-intercept is zero, the equation becomes $y = mx$ and these graphs will therefore pass through the origin.
- To sketch a graph using the **gradient–intercept method**, locate the y-intercept and use the gradient to find a second point.

  For example: if $m = \frac{2}{5}$, move 5 across and 2 up.



Example 10 Stating the gradient and y-intercept

State the gradient and the y-intercept for the graphs of the following relations.

a $y = 2x + 1$ **b** $y = -3x$

SOLUTION

a $y = 2x + 1$
gradient $= 2$
y-intercept $= 1$

b $y = -3x$
gradient $= -3$
y-intercept $= 0$

EXPLANATION

The rule is given in gradient–intercept form.
The gradient is the coefficient of x.
The constant term is the y-intercept.

The gradient is the coefficient of x including the negative sign.
The constant term is not present so the y-intercept $= 0$

Example 11 Rearranging linear equations

Rearrange these linear equations into the form shown in the brackets.

a $4x + 2y = 10\,(y = mx + c)$ **b** $y = 4x - 7\,(ax + by = d)$

SOLUTION

a $4x + 2y = 10$
$2y = -4x + 10$
$y = -2x + 5$

b $y = 4x - 7$
$y - 4x = -7$
or $-4x + y = -7$
or $4x - y = 7$

EXPLANATION

Subtract $4x$ from both sides. Here $10 - 4x$ is better written as $-4x + 10$
Divide both sides by 2.

Subtract $4x$ from both sides.
Multiply both sides by -1 to convert between forms.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 12 Sketching linear graphs using the gradient and y-intercept

Find the value of the gradient and y-intercept for these relations and sketch their graphs.

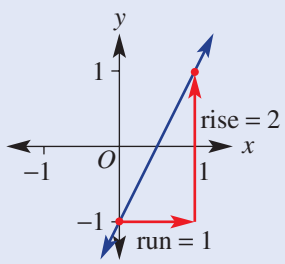
a $y = 2x - 1$ **b** $x + 2y = 6$

SOLUTION

a $y = 2x - 1$

y-intercept $= -1$

gradient $= 2 = \frac{2}{1}$



b $x + 2y = 6$

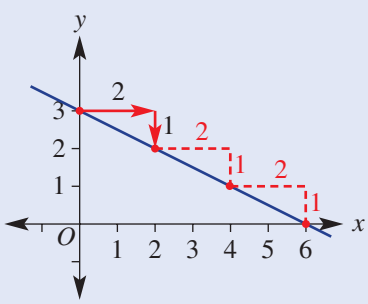
$2y = -x + 6$

$y = \frac{-x + 6}{2}$

$= -\frac{1}{2}x + 3$

So y-intercept is 3.

$m = -\frac{1}{2} = \frac{-1}{2}$



EXPLANATION

The rule is in gradient–intercept form so we can read off the gradient and the y-intercept.

Label the y-intercept at $(0, -1)$.

For the gradient $\frac{2}{1}$.
For every 1 across, move 2 up.
From $(0, -1)$ this gives a second point at $(1, 1)$.
Mark and join the points to form a line.

Rewrite in the form $y = mx + b$ to read off the gradient and y-intercept.
Make y the subject by subtracting x from both sides and then dividing both sides by 2.

Link the negative sign to the rise (-1) so the run is positive $(+2)$.

Mark the y-intercept $(0, 3)$ then from this point move 2 right and 1 down to give a second point at $(2, 2)$.

Note that the x-intercept will be 6. If the gradient is $\frac{-1}{2}$ then a run of 6 gives a fall of 3.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4F

UNDERSTANDING AND FLUENCY	1, 2, 3–7(½)	2, 3–7(½)	3–7(½)

1 Write a rule (in gradient–intercept form) for a straight line with the given properties.

a Gradient $= 2$, y-intercept $= 5$

b Gradient $= 3$, y-intercept $= -1$

c Gradient $= -2$, y-intercept $= 3$

d Gradient $= -1$, y-intercept $= -2$

e Gradient $= \frac{-1}{2}$, y-intercept $= -10$

f Gradient $= \frac{-2}{3}$, y-intercept $= \frac{5}{2}$

2 Substitute $x = -3$ to find the value of y for these rules.

a $y = x + 4$

b $y = x - 2$

c $y = 2x + 1$

d $y = 3x - 2$

e $y = -2x + 3$

f $y = -x - 1$

Example 10

3 State the gradient and y-intercept for the graphs of the following relations.

a $y = 3x - 4$ **b** $y = -5x - 2$ **c** $y = -2x + 3$ **d** $y = \frac{1}{3}x + 4$

e $y = -4x$ **f** $y = 2x$ **g** $y = 2.3x$ **h** $y = -0.7x$

Example 12a

4 Find the gradient and y-intercept for these relations and sketch their graphs.

a $y = x - 2$ **b** $y = 2x - 1$ **c** $y = \frac{1}{2}x + 1$ **d** $y = \frac{-1}{2}x + 2$

e $y = -3x + 3$ **f** $y = \frac{3}{2}x + 1$ **g** $y = \frac{-4}{3}x$ **h** $y = \frac{5}{3}x - \frac{1}{3}$

Example 11

5 Rearrange to make y the subject.

a $y - x = 7$

b $x + y = 3$

c $2y - 4x = 10$

Example 11

6 Rearrange these linear equations into the form shown in the brackets.

a $2x + y = 3$ $(y = mx + c)$ **b** $-3x + y = -1$ $(y = mx + c)$

c $6x + 2y = 4$ $(y = mx + c)$ **d** $-3x + 3y = 6$ $(y = mx + c)$

e $y = 2x - 1$ $(ax + by = d)$ **f** $y = -3x + 4$ $(ax + by = d)$

g $3y = x - 1$ $(ax + by = d)$ **h** $7y - 2 = 2x$ $(ax + by = d)$

Example 12b

7 Find the gradient and y-intercept for these relations and sketch their graphs. Rearrange each equation first.

a $x + y = 4$ **b** $x - y = 6$ **c** $x + 2y = 6$ **d** $x - 2y = 8$

e $2x - 3y = 6$ **f** $4x + 3y = 12$ **g** $x - 3y = -4$ **h** $2x + 3y = 6$

i $3x - 4y = 12$ **j** $x + 4y = 0$ **k** $x - 5y = 0$ **l** $x - 2y = 0$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

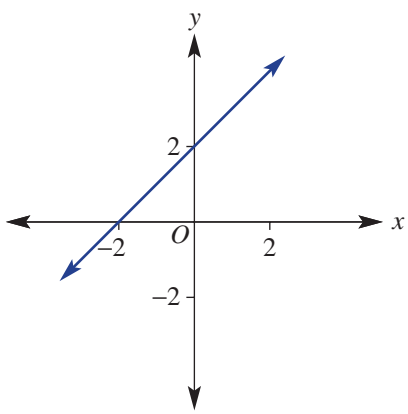
PROBLEM-SOLVING AND REASONING | 8, 12 | 8, 9–11(½), 12 | 10–11(½), 12–14

8 Match the following equations to the straight lines shown.

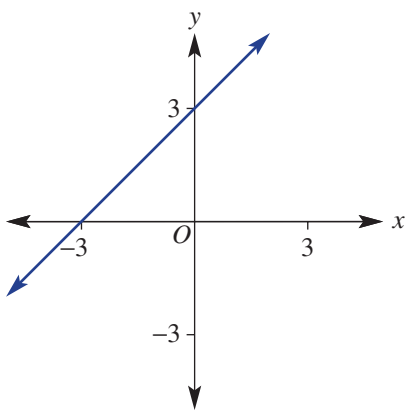
i $y = 3$ **ii** $y = -2x - 1$ **iii** $y = -2x - 4$

iv $y = x + 3$ **v** $x = 2$ **vi** $y = x + 2$

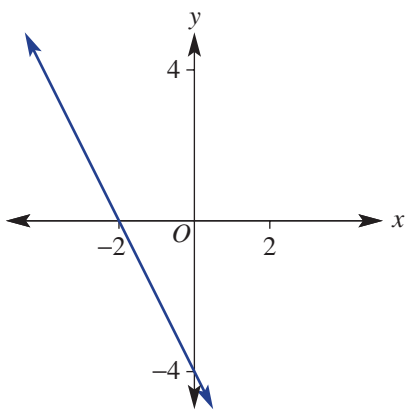
a



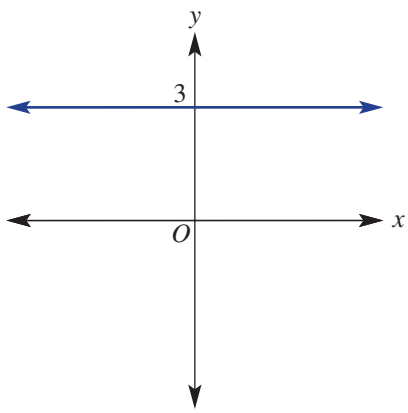
b



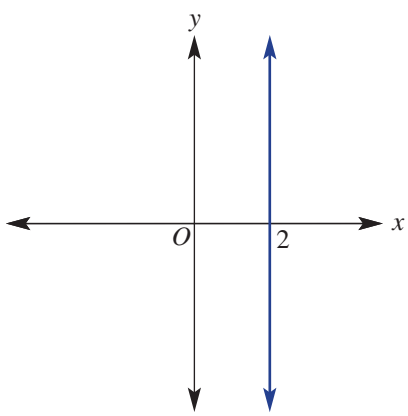
c



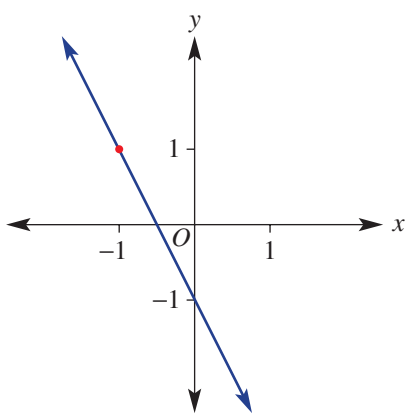
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 Sketch the graph of each of the following linear relations, by finding the gradient and y-intercept.

a $5x - 2y = 10$ **b** $y = 6$

c $x + y = 0$ **d** $y = 5 - x$

e $y = \frac{x}{2} - 1$ **f** $4y - 3x = 0$

g $4x + y - 8 = 0$ **h** $2x + 3y - 6 = 0$

10 Decide if the following points are on the line with equation $3x - y = 7$.

a $(1, -2)$ **b** $(-1, 4)$

c $(5, 8)$ **d** $(-2, -10)$

11 Which of these linear relations have a gradient of 2 and y-intercept of -3?

A $y = 2(x - 3)$ **B** $y = 3 - 2x$

C $y = \frac{3 - 2x}{-1}$ **D** $y = 2(x - 1.5)$

E $y = \frac{2x - 6}{2}$ **F** $y = \frac{4x - 6}{2}$

G $2y = 4x - 3$ **H** $-2y = 6 - 4x$

12 Jeremy says that the graph of the rule $y = 2(x + 1)$ has gradient 2 and y-intercept 1.

a Explain his error.

b What can be done to the rule to help show the y-intercept?

13 A horizontal line has gradient 0 and y-intercept at $(0, k)$. Using gradient–intercept form, write the equation for the line.

14 Write the equation $ax + by = d$ in gradient–intercept form. Then state the gradient m and the y-intercept.

ENRICHMENT — — 15

The missing y-intercept

15 This graph shows two points $(-1, 3)$ and $(1, 4)$ with a gradient of $\frac{1}{2}$. By considering the gradient, the y-intercept can be calculated to be 3.5 (or $\frac{7}{2}$) so $y = \frac{1}{2}x + \frac{7}{2}$.

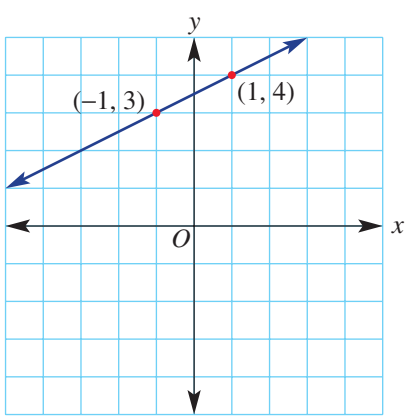
Use this approach to find the equation of the line passing through these points.

a $(-1, 1)$ and $(1, 5)$

b $(-2, 4)$ and $(2, 0)$

c $(-1, -1)$ and $(2, 4)$

d $(-3, 1)$ and $(2, -1)$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4G Finding the equation of a line using $y = mx + b$

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

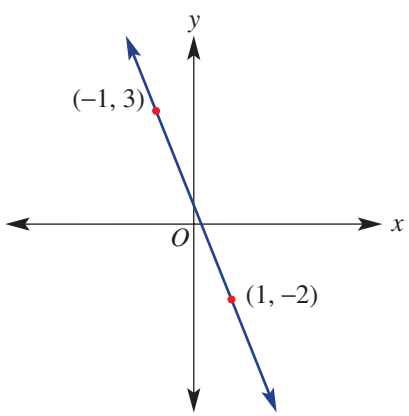
Using gradient–intercept form, the rule (or equation) of a line can be found by calculating the value of the gradient and the y-intercept. Given a graph, the gradient can be calculated using two points. If the y-intercept is known, then the value of the constant in the rule is obvious, but if not, another point can be used to help find its value.



Let's start: But we don't know the y-intercept!

A line with the rule $y = mx + b$ passes through two points $(-1, 3)$ and $(1, -2)$.

- Using the information given is it possible to find the value of m? If so, calculate its value.
- The y-intercept is not given on the graph. Discuss what information could be used to find the value of the constant b in the rule. Is there more than one way you can find the y-intercept?
- Write the equation for the line.



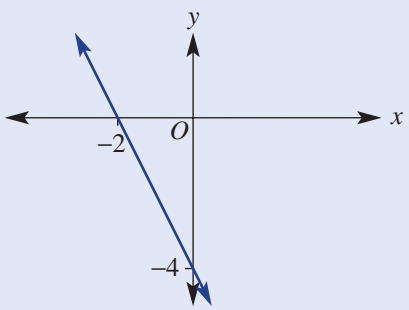
Key ideas

- To find the equation of a line in gradient–intercept form $y = mx + b$, you need to find the value of the:
  - gradient (m) using $m = \frac{\text{rise}}{\text{run}}$
  - constant (b), by observing the y-intercept or by substituting another point.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 13 Finding the equation of a line given the y-intercept and another point

Determine the equation of the straight line shown here.



SOLUTION

$y = mx + b$

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{2}$

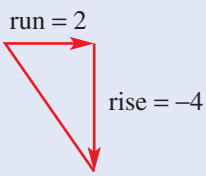
$= -2$

y-intercept $= -4$.

$\therefore y = -2x - 4$

EXPLANATION

Write down the general straight line equation



The y-intercept is -4.

Substitute $m = -2$ and the y-intercept into the equation.

Example 14 Finding the equation of a line given the gradient and a point

Find the equation of the line that has a gradient m of $\frac{1}{3}$ and passes through the point (9, 2).

SOLUTION

$y = mx + b$

$y = \frac{1}{3}x + b$

$2 = \frac{1}{3}(9) + b$

$2 = 3 + b$

$-1 = b$

$\therefore y = \frac{1}{3}x - 1$

EXPLANATION

Substitute $m = \frac{1}{3}$ into $y = mx + b$.

Since (9, 2) is on the line, it must satisfy the equation $y = \frac{1}{3}x + b$, hence substitute the point (9, 2) where $x = 9$ and $y = 2$ to find b. Simplify and solve for b.

Write the equation in the form $y = mx + b$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4G

UNDERSTANDING AND FLUENCY	1, 2, 3–4(½)	2–5(½)	3–5(½)

1 Substitute the given values of m and b into $y = mx + b$ to write the rule.

a $m = 2, b = 5$ **b** $m = 4, b = -1$

c $m = -2, b = 5$ **d** $m = -1, b = -\frac{1}{2}$

2 Substitute the point into the given rule and solve to find the value of b. For example, using (3, 4), substitute $x = 3$ and $y = 4$ into the rule.

a $(3, 4), y = x + b$ **b** $(1, 5), y = 2x + b$

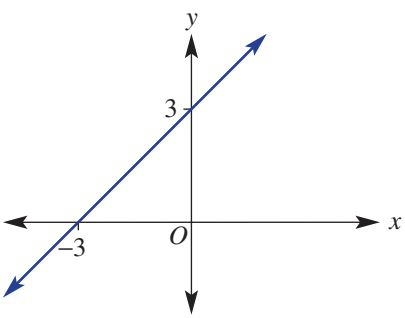
c $(-2, 3), y = 3x + b$ **d** $(-1, 6), y = 4x + b$

e $(3, -1), y = -2x + b$ **f** $(-2, 4), y = -x + b$

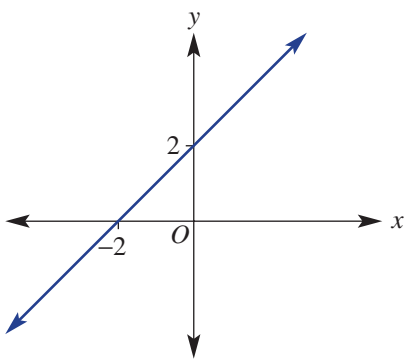
Example 13

3 Determine the equations of the following straight lines.

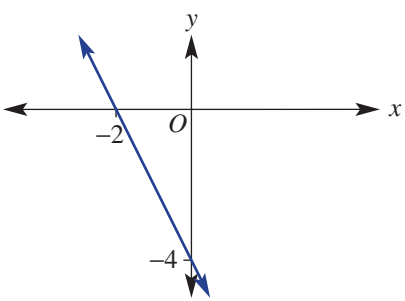
a



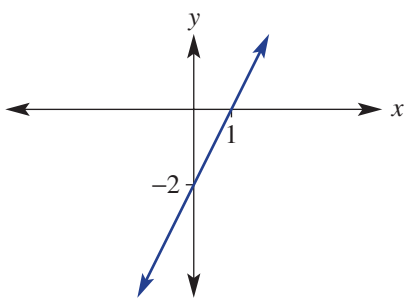
b



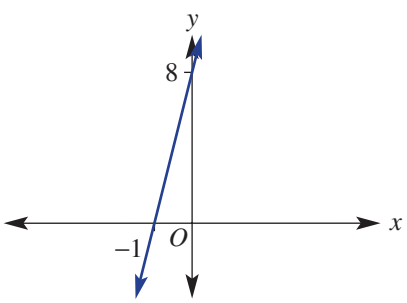
c



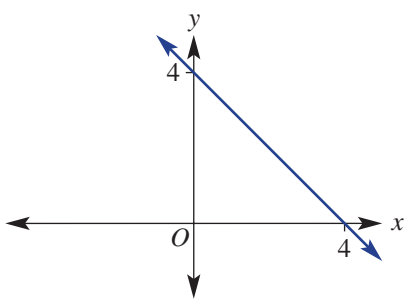
d



e



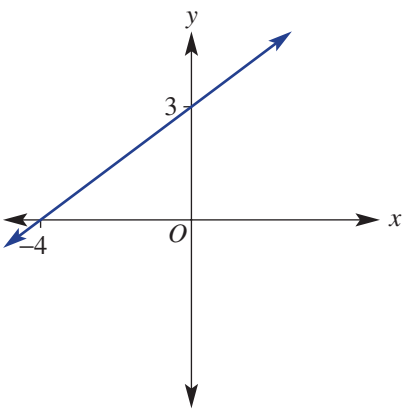
f



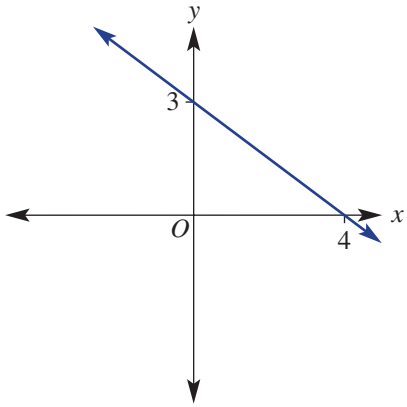
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 Find the equations of these straight lines, which have fractional gradients.

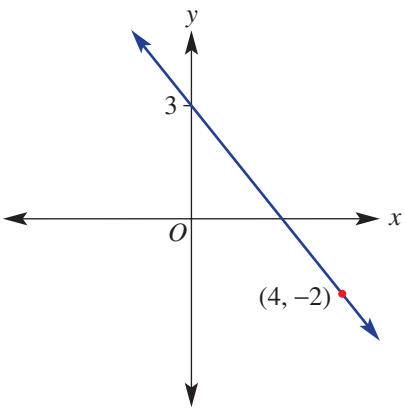
a



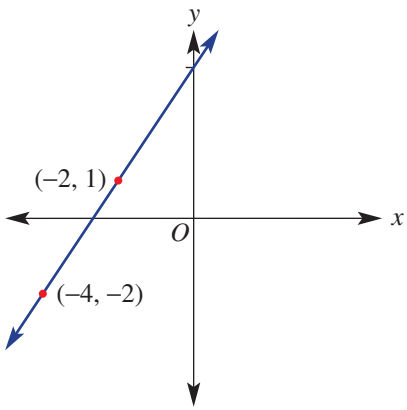
b



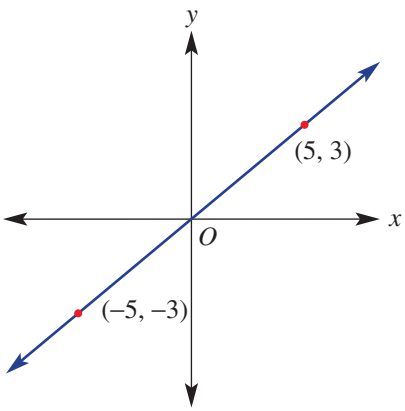
c



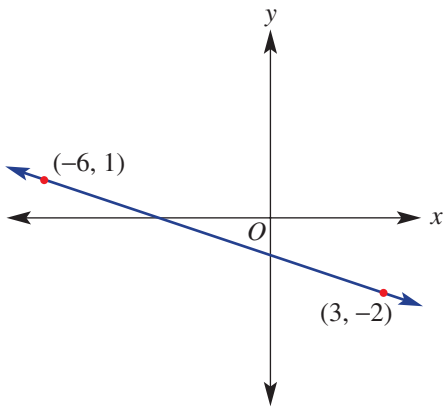
d



e



f



Example 14

5 Find the equation of the line that has a gradient of:

a 3 and passes through the point (1, 8)
b −2 and passes through the point (2, −5)
c −3 and passes through the point (2, 2)
d 1 and passes through the point (1, −2)
e −3 and passes through the point (−1, 6)
f 5 and passes through the point (2, 9)
g −1 and passes through the point (4, 4)
h −3 and passes through the point (3, −3)
i −2 and passes through the point (−1, 4)
j −4 and passes through the point (−2, −1)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING	6–8, 11	6–9, 11, 12	8–10, 12, 13

6 For the line connecting the following pairs of points, find the:
- **i** gradient
- **ii** equation

a (2, 6) and (4, 10)
b (−3, 6) and (5, −2)
c (1, 7) and (3, −1)
d (−4, −8) and (1, −3)

7 A line has gradient −2 and y-intercept 5. Find its x-intercept.

8 A line passes through the points (−1, −2) and (3, 3). Find its x- and y-intercepts.

9 Water is leaking from a tank. The volume of water in the tank is 100 L after 1 hour and 20 L after 5 hours. Assuming the relationship is linear, find a rule and then state the initial volume of water in the tank.

10 The coordinates (0, 0) mark the take-off point for a rocket constructed as part of a science class. The positive x direction from (0, 0) is considered to be east.
- **a** Find the equation of the rocket's path if it rises at a rate of 5 m vertically for every 1 m in an easterly direction.
- **b** A second rocket is fired 2 m vertically above from where the first rocket was launched. It rises at a rate of 13 m for every 2 m in an easterly direction. Find the equation describing its path.

11 A line has equation $y = mx - 2$. Find the value of m if the line passes through:

a (2, 0)
b (1, 6)
c (−1, 4)
d (−2, −7)

12 A line with rule $y = 2x + b$ passes through (1, 5) and (2, 7).
- **a** Find the value of b using the point (1, 5).
- **b** Find the value of b using the point (2, 7).
- **c** Does it matter which point you use? Explain.

13 A line passes through the origin and the point (a, b). Write its equation in terms of a and b.

ENRICHMENT	—	—	14

The general rule

14 To find the equation of a line between two points (x_1, y_1) and (x_2, y_2), some people prefer to use the rule:

$y - y_1 = m(x - x_1)$ where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Use this rule to find the equation of the line passing through these pairs of points. Write your answer in the form $y = mx + b$.

a (1, 2) and (3, 6)
b (0, 4) and (2, 0)
c (−1, 3) and (1, 7)
d (−4, 8) and (2, −1)
e (−3, −2) and (4, 3)
f (−2, 5) and (1, −8)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4H Midpoint and length of a line segment from diagrams

Interactive

Widgets

HOTsheets

Walkthrough

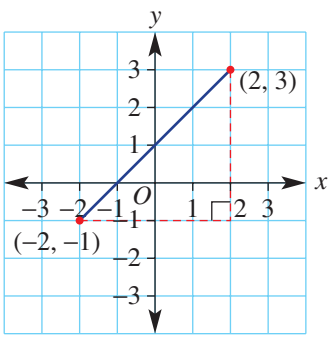
A line segment (or line interval) has a defined length and therefore must have a midpoint. Both the midpoint and length can be found by using the coordinates of the end points.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Choosing a method

This graph shows a line segment between the points at $(-2, -1)$ and $(2, 3)$.

- What is the horizontal distance between the two points?
- What is the vertical distance between the two points?
- What is the x-coordinate of the point halfway along the line segment?
- What is the y-coordinate of the point halfway along the line segment?
- Discuss and explain a method for finding the midpoint of a line segment.
- Discuss and explain a method for finding the length of a line segment.

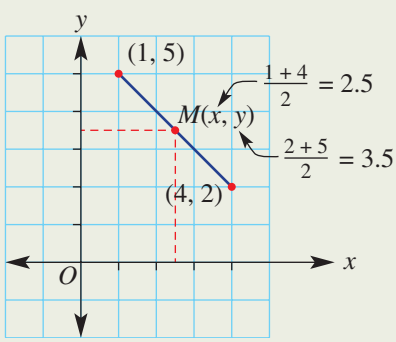


Using graphing software or dynamic geometry software, produce a line segment like the one shown above. Label the coordinates of the end points and the midpoint. Also find the length of the line segment. Now drag one or both of the end points to a new position.

- Describe how the coordinates of the midpoint relate to the coordinates of the end points. Is this true for all positions of the end points that you choose?
- Now use your software to calculate the vertical distance and the horizontal distance between the two end points. Then square these lengths. Describe these squared lengths compared to the square of the length of the line segment. Is this true for all positions of the end points that you choose?

Key ideas

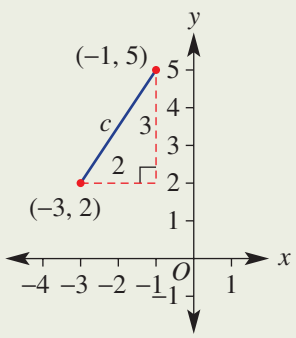
- The **midpoint** (M) of a line segment is the halfway point between the two end points.
  - The x coordinate is the average (mean) of the x coordinates of the two end points.
  - The y coordinate is the average (mean) of the y coordinates of the two end points.
  - $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- The **length of a line segment** (distance between two points) can be found using Pythagoras' theorem.
  - The line segment is the hypotenuse (longest side) of a right-angled triangle.
  - Find the horizontal distance by subtracting the lower x coordinate from the upper x coordinate.
  - Find the vertical distance by subtracting the lower y coordinate from the upper y coordinate.
  - The unit of length on the number plane is **units**.



$$\begin{aligned}\text{horizontal distance} &= -1 - (-3)\\ &= 2\\ \text{vertical distance} &= 5 - 2\\ &= 3\\ c^2 &= 2^2 + 3^2\\ &= 13\end{aligned}$$

$\therefore c = \sqrt{13}$ units

Example 15 Finding a midpoint

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (1, 0) and (4, 4)

b (−3, −2) and (5, 3)

SOLUTION

a $x = \dfrac{1+4}{2} = 2.5$

$y = \dfrac{0+4}{2} = 2$

$\therefore M = (2.5, 2)$

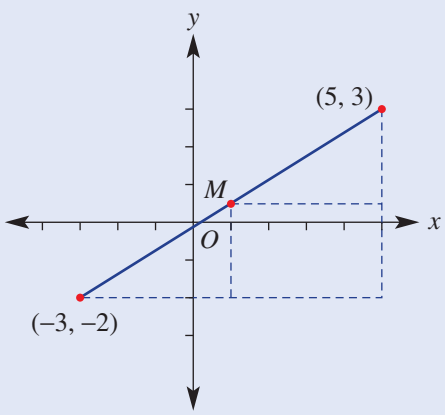
b $x = \dfrac{-3+5}{2} = 1$

$y = \dfrac{-2+3}{2} = 0.5$

$\therefore M = (1, 0.5)$

EXPLANATION

Find the average (mean) of the x coordinates and y coordinates for both points.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 16 Finding the length of a segment

Find the length of the segment joining $(-2, 2)$ and $(4, -1)$, correct to 2 decimal places.

SOLUTION

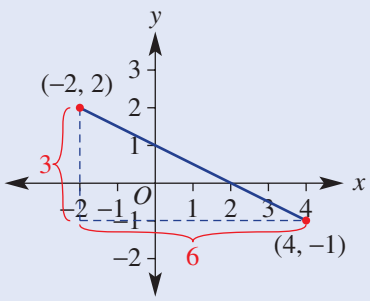
Horizontal length $= 4 - (-2)$
$= 6$

Vertical length $= 2 - (-1)$
$= 3$

$c^2 = 6^2 + 3^2$
$= 45$
$\therefore c = \sqrt{45}$

$\therefore$ length = 6.71 units (to 2 d.p.)

EXPLANATION



Apply Pythagoras' theorem $c^2 = a^2 + b^2$.
Round as required.

Exercise 4H

UNDERSTANDING AND FLUENCY 1–3, 4–5(½) 3, 4–5(½) 4–5(½)

1 Find the number that is halfway between these pairs of numbers.

a 1, 7 **b** 5, 11 **c** −2, 4 **d** −6, 0

2 Find the average (mean) of these pairs of numbers.

a 4, 7 **b** 0, 5 **c** −3, 0 **d** −4, −1

3 Evaluate c in the following correct to 2 decimal places.

a $c^2 = 1^2 + 2^2$ **b** $c^2 = 5^2 + 7^2$ **c** $c^2 = 10^2 + 2^2$

Example 15

4 Find the midpoint $M(x, y)$ of the interval joining these pairs of points.

a (0, 0) and (6, 6)
b (0, 0) and (4, 4)
c (0, 2) and (2, 8)
d (3, 0) and (5, 2)
e (−2, 0) and (0, 6)
f (−4, −2) and (2, 0)
g (1, 3) and (2, 0)
h (−1, 5) and (6, −1)
i (−3, 7) and (4, −1)
j (−2, −4) and (−1, −1)
k (−7, −16) and (1, −1)
l (−4, −3) and (5, −2)

Example 16

5 Find the length of the interval joining these pairs of points correct to 2 decimal places.

a (1, 1) and (2, 6)
b (1, 2) and (3, 4)
c (0, 2) and (5, 0)
d (−2, 0) and (0, −4)
e (−1, 3) and (2, 1)
f (−2, −2) and (0, 0)
g (−1, 7) and (3, −1)
h (−4, −1) and (2, 3)
i (−3, −4) and (3, −1)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING	6, 7, 10	6, 8, 10, 11	7–11

6 Find the missing coordinates in this table if M is the midpoint of points A and B.

A	B	M
(4, 2)		(6, 1)
	(0, −1)	(−3, 2)
	(4, 4)	(−1, 6.5)

7 A circle has centre (2, 1). Find the coordinates of the end point of a diameter if the other end point has these coordinates.

a (7, 1) **b** (3, 6) **c** (−4, −0.5)

8 Find the perimeter of these shapes correct to 1 decimal place.

a A triangle with vertices (−2, 0), (−2, 5) and (1, 3).

b A trapezium with vertices (−6, −2), (1, −2), (0, 4) and (−5, 4).

9 Find the coordinates of the four points which have integer coordinates and are a distance of $\sqrt{5}$ from the point (1, 2). *Hint*: $5 = 1^2 + 2^2$.

10 A line segment has two end points (x_1, y_1) and (x_2, y_2) and a midpoint $M(x, y)$.

a Write a rule for x, the x coordinate of the midpoint.

b Write a rule for y, the y coordinate of the midpoint.

c Test your rule to find the coordinates of M if $x_1 = -3$, $y_1 = 2$, $x_2 = 5$ and $y_2 = -3$.

11 A line segment has two end points (x_1, y_1) and (x_2, y_2). Assume $x_2 > x_1$ and $y_2 > y_1$.

a Write a rule for the:

i horizontal distance between the end points

ii vertical distance between the end points

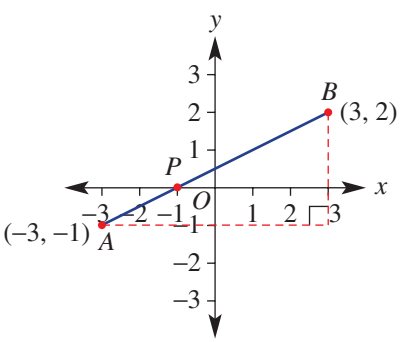
iii length of the segment

b Use your rule to show that the length of the segment joining (−2, 3) with (1, −3) is $\sqrt{45}$.

ENRICHMENT	—	—	12

Division by ratio

12 Looking from left to right, this line segment shows the point $P(-1, 0)$ which divides the segment in the ratio 1 : 2.



a What fraction of the horizontal distance between the end points is P from A?

b What fraction of the vertical distance between the end points is P from A?

c Find the coordinates of point P on the segment AB if it divides the segment in these ratios.

i 2 : 1 **ii** 1 : 5 **iii** 5 : 1

d Find the coordinates of point P that divides the segments with the given end points in the ratio 2 : 3.

i $A(-3, -1)$ and $B(2, 4)$ **ii** $A(-4, 9)$ and $B(1, -1)$

iii $A(-2, -3)$ and $B(4, 0)$ **iv** $A(-6, -1)$ and $B(3, 8)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4I Perpendicular lines and parallel lines

Interactive

Widgets

HOTsheets

Walkthrough

Perpendicular lines and parallel lines are commonplace in mathematics and in the world around us. Using parallel lines in buildings, for example, ensures that beams or posts point in the same direction. Perpendicular beams help to construct rectangular shapes, which are central in the building of modern structures.

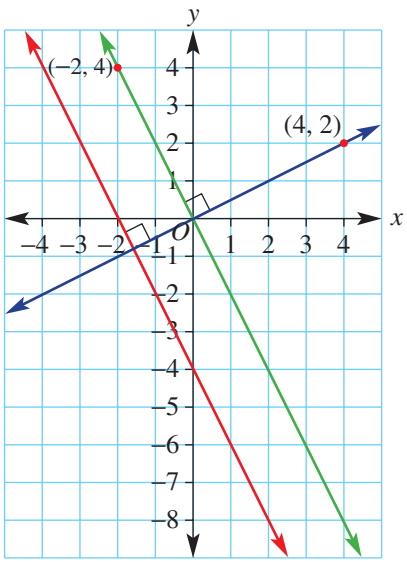
Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How are they related?

This graph shows a pair of parallel lines and a line perpendicular to the other two. Find the equation of all three lines.

- What do you notice about the equation for the pair of parallel lines?
- What do you notice about the gradient of the line that is perpendicular to the other two lines?
- Write down the equations of three other lines that are parallel to $y = -2x$.
- Write down the equations of three other lines that are perpendicular to $y = -2x$.



Key ideas

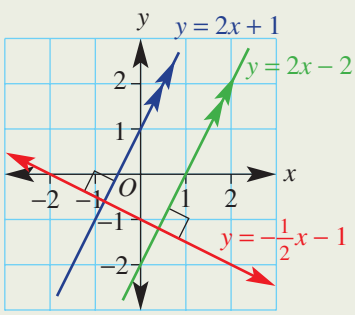
- If two lines are **parallel**, then they have the same gradient.
- If two **perpendicular** lines (at right angles) have gradients m_1 and m_2, then

  $m_1 \times m_2 = -1$ or $m_2 = \dfrac{-1}{m_1}$

  - The **negative reciprocal** of m_1 gives m_2.

    For example: If $m_1 = 4$, then $m_2 = \dfrac{-1}{4}$

    If $m_1 = -\dfrac{2}{3}$, then $m_2 = \dfrac{-1}{\left(-\frac{2}{3}\right)} = -1 \times \left(-\dfrac{3}{2}\right) = \dfrac{3}{2}$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 17 Finding the equation of a parallel line

Find the equation of a line which is parallel to $y = 3x - 1$ and passes through $(0, 4)$.

SOLUTION

$y = mx + b$
$m = 3$
$b = 4$
$\therefore y = 3x + 4$

EXPLANATION

Since it's parallel to $y = 3x - 1$, the gradient is the same so $m = 3$.
The y-intercept is given in the question so $b = 4$.

Example 18 Finding the equation of a perpendicular line

Find the equation of a line which is perpendicular to the line $y = 2x - 3$ and passes through $(0, -1)$.

SOLUTION

$y = mx + b$
$m = -\frac{1}{2}$
$b = -1$
$y = -\frac{1}{2}x - 1$

EXPLANATION

Since it is perpendicular to $y = 2x - 3$,
$m_2 = \frac{-1}{m_1} = -\frac{1}{2}$.
The y-intercept is given.

Exercise 4I

UNDERSTANDING AND FLUENCY	1–3, 4–5(½)	2, 3, 4–6(½)	4–6(½)

1 Decide if the pairs of lines with these equations are parallel (have the same gradient).

a $y = 3x - 1$ and $y = 3x + 4$
b $y = 2x - 1$ and $y = 2x - 3$
c $y = 7x - 2$ and $y = 2x - 7$
d $y = x + 4$ and $y = x - 3$
e $y = \frac{1}{2}x - 1$ and $y = -\frac{1}{2}x + 2$
f $y = -\frac{3}{4}x + 4$ and $y = -\frac{3}{4}x - 6$

2 Two perpendicular lines with gradients m_1 and m_2 are such that $m_2 = -\frac{1}{m_1}$. Find m_2 for the given values of m_1.

a $m_1 = 5$
b $m_1 = 10$
c $m_1 = -3$
d $m_1 = -6$

3 Decide if the pairs of lines with these equations are perpendicular.

a $y = 4x - 2$ and $y = -\frac{1}{4}x + 3$
b $y = 2x - 3$ and $y = \frac{1}{2}x + 4$
c $y = -\frac{1}{2}x + 6$ and $y = -\frac{1}{2}x - 2$
d $y = -\frac{1}{5}x + 1$ and $y = 5x + 2$

Example 17

4 Find the equation of the line that is parallel to the line with equation:

a $y = 2x + 3$ and passes through $(0, 1)$
b $y = 4x + 2$ and passes through $(0, 8)$
c $y = -x + 3$ and passes through $(0, 5)$
d $y = -2x - 3$ and passes through $(0, -7)$
e $y = \frac{2}{3}x + 6$ and passes through $(0, -5)$
f $y = -\frac{4}{5}x - 3$ and passes through $\left(0, \frac{1}{2}\right)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18

5 Find the equation of the line that is perpendicular to the line with equation:

a $y = 3x - 2$ and passes through $(0, 3)$

b $y = 5x - 4$ and passes through $(0, 7)$

c $y = -2x + 3$ and passes through $(0, -4)$

d $y = -x + 7$ and passes through $(0, 4)$

e $y = -7x + 2$ and passes through $\left(0, -\frac{1}{2}\right)$

f $y = x - \frac{3}{2}$ and passes through $\left(0, \frac{5}{4}\right)$.

6 **a** Write the equation of the line parallel to $y = 4$ and which passes through:

i $(0, 1)$ **ii** $(0, -3)$

iii $(1, 6)$ **iv** $(-3, -2)$

b Write the equation of the line parallel to $x = -2$ and which passes through:

i $(3, 0)$ **ii** $(-4, 0)$

iii $(1, 5)$ **iv** $(-3, -3)$

c Write the equation of the line perpendicular to $y = -3$ and which passes through:

i $(2, 0)$ **ii** $(-1, 0)$

iii $(0, 0)$ **iv** $(3, 5)$

d Write the equation of the line perpendicular to $x = 6$ and which passes through:

i $(0, 7)$ **ii** $\left(0 - \frac{1}{2}\right)$

iii $(1, 3)$ **iv** $\left(-2, \frac{1}{2}\right)$

PROBLEM-SOLVING AND REASONING 7, 10 | 7, 8, 10, 11 | 7–11

7 Find the equation of the line that is:

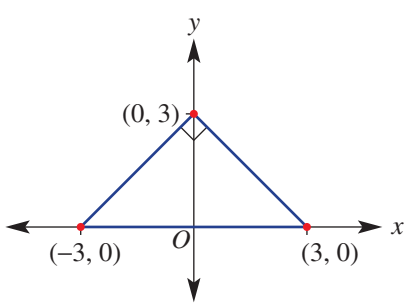
a parallel to the line with equation $y = -3x - 7$ and passes through $(3, 0)$; remember to substitute the point $(3, 0)$ to find the value of the y-intercept

b parallel to the line with equation $y = \frac{1}{2}x + 2$ and passes through $(1, 3)$

c perpendicular to the line with equation $y = 5x - 4$ and passes through $(1, 6)$

d perpendicular to the line with equation $y = -x - \frac{1}{2}$ and passes through $(-2, 3)$

8 A right-angled isosceles triangle has vertices at $(0, 3)$, $(3, 0)$ and $(-3, 0)$. Find the equation of each of the three sides.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 A parallelogram has two side lengths of 5 units. Three of its sides have equations $y = 0$, $y = 2$, $y = 2x$. Find the equation of the fourth side.

10 **a** Using $m_2 = -\frac{1}{m_1}$, find the gradient of a line perpendicular to the line with the given gradient.

i $m_1 = \frac{2}{3}$ **ii** $m_1 = \frac{1}{5}$

iii $m_1 = -\frac{1}{7}$ **iv** $m_1 = -\frac{3}{11}$

b If $m_1 = \frac{a}{b}$, find m_2 given $m_1 \times m_2 = -1$.

11 **a** Find the gradient of a line that is parallel to:

i $2x + 4y = 9$ **ii** $3x - y = 8$

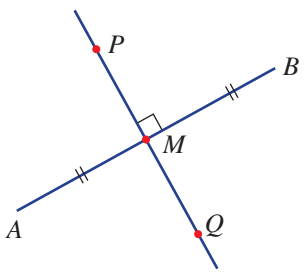
b Find the gradient of a line which is perpendicular to:

i $5x + 5y = 2$ **ii** $7x - y = -1$

ENRICHMENT — — 12

Perpendicular bisectors

12 If a line segment AB is cut by another line PQ at right angles at the midpoint (M) of AB, then PQ is called the perpendicular bisector. This is shown in the diagram below.



By first finding the midpoint of AB, find the equation of the perpendicular bisector of the segment connecting these points.

a $A(1, 1)$, $B(3, 5)$

b $A(0, 6)$, $B(4, 0)$

c $A(-2, 3)$, $B(6, -1)$

d $A(-6, -1)$, $B(0, 2)$

e $A(-1, 3)$, $B(2, -4)$

f $A(-6, -5)$, $B(4, 7)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4J Linear modelling FRINGE

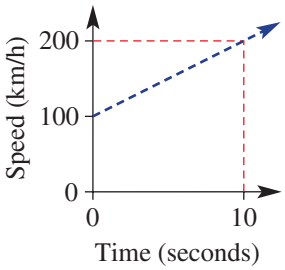








If a relationship between two variables is linear, the graph will be a straight line and the equation linking the two variables can be written in gradient–intercept form. The process of describing and using such line graphs and rules for the relationship between two variables is called linear modelling. A test car, for example, increasing its speed from 100 km/h to 200 km/h in 10 seconds with constant acceleration could be modelled by the rule $S = 10t + 100$. This rule could then be used to calculate the speed at different times in the test run.





Let's start: The test car

The above graph describes the speed of a racing car over a 10-second period.

- Explain why the rule is $S = 10t + 100$.
- Why might negative values of t not be considered for the graph?
- How could you accurately calculate the speed after 6.5 seconds?
- If the car continued to accelerate at the same rate, how could you accurately predict the car's speed after 13.2 seconds?

Key ideas

- Many situations can often be **modelled** by using a linear rule or graph. The key elements of linear modelling include:
  - finding the rule linking the two variables
  - sketching a graph
  - using the graph or rule to predict or estimate the value of one variable given the other
  - finding the rate of change of one variable with respect to the other variable, which is equivalent to finding the gradient.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 19 Applying linear relationships

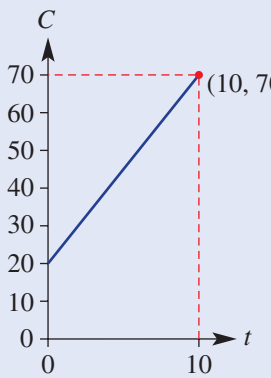
The deal offered by Netshare, an internet provider, to its new customers is a fixed charge of $20 per month plus $5 per hour of use.

a Write a rule for the total monthly cost, $\$C$, of using Netshare for t hours per month.

b Sketch the graph of C versus t using $0 \leq t \leq 10$.

c What is the total cost in a month when Netshare is used for 4 hours?

d If the monthly cost was $50, for how many hours was Netshare used during the month?

SOLUTION	EXPLANATION
a $C = 20 + 5t$	A fixed amount of $20 plus $5 for each hour.
b C-intercept is 20 At $t = 10$, $C = 20 + 5(10) = 70$ $\therefore$ End point is (10, 70).	Let $t = 0$ to find the C-intercept. Letting $t = 10$ gives $C = 70$ and this gives the other end point.
	Sketch the graph using the points (0, 20) and (10, 70).
c $C = 20 + 5t$ $= 20 + 5(4)$ $= 40$ The cost is $40.	Substitute $t = 4$ into the rule. Answer the question using the correct units.
d $C = 20 + 5t$ $50 = 20 + 5t$ $30 = 5t$ $t = 6$ Netshare was used for 6 hours.	Write the rule and substitute $C = 50$. Solve the resulting equation for t by subtracting 20 from both sides then dividing both sides by 5. Answer the question in words.

Exercise 4J FRINGE

UNDERSTANDING AND FLUENCY	1–6	3–6	4, 5, 7

1 A person gets paid $50 plus $20 per hour. Decide which rule describes the relationship between the pay $\$P$ and number of hours n.

A $P = 50 + n$ **B** $P = 50 + 20n$ **C** $P = 50n + 20$ **D** $P = 20 + 50$

2 The amount of money in a bank account is $1000 and is increasing by $100 per month.

a Find the amount of money in the account after:

i 2 months **ii** 5 months **iii** 12 months

b Write a rule for the amount of money A dollars after n months.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 If $d = 5t - 4$, find:
a d if $t = 10$ **b** d if $t = 1.5$ **c** t if $d = 6$ **d** t if $d = 11$

Example 19

4 A sales representative earns $400 a week plus $20 for each sale she makes.
a Write a rule that gives the total weekly wage, $\$W$, if the sales representative makes x sales.
b Draw a graph of W versus x using $0 \leq x \leq 40$.
c How much does the sales representative earn if, in a particular week, she makes 12 sales?
d If, in a particular week, the sales representative earns $1000, how many sales did she make?

5 A plumber charges a $40 fee up-front and $50 for each hour he works.
a Find a linear equation for the total charge, $\$C$, for n hours of work.
b What will a 4-hour job cost?
c If the plumber works on a job for two days and averages 6 hours per day, what will the total cost be?

6 A catering company charges $500 for the hire of a marquee, plus $25 per guest.
a Write a rule for the cost, $\$C$, of hiring a marquee and catering for n guests.
b Draw a graph of C versus n for $0 \leq n \leq 100$.
c How much would a party catering for 40 guests cost?
d If a party cost $2250, how many guests were catered for?

7 The cost, $\$C$, of recording a music CD is $300, plus $120 per hour of studio time.
a Write a rule for the cost, $\$C$, of recording a CD requiring t hours of studio time.
b Draw a graph of C versus t for $0 \leq t \leq 10$.
c How much does a recording requiring 6 hours of studio time cost?
d If a recording cost $660 to make, for how long was the studio used?

PROBLEM-SOLVING AND REASONING	8, 9, 11	9–11	9–12

8 A petrol tank holds 66 litres of fuel. If it contains 12 litres of petrol initially and the petrol pump fills it at 3 litres every 10 seconds, find:
a a linear equation for the amount of fuel (F litres) in the tank after t minutes
b how long it will take to fill the tank
c how long it will take to put 45 litres into the petrol tank

9 A tank is initially full with 4000 L of water and water is being used at a rate of 20 L per minute.
a Write a rule for the volume, V litres, of water after t minutes.
b Calculate the volume after 1.5 hours.
c How long will it take for the tank to be emptied?
d How long will it take for the tank to have only 500 L?

10 A spa pool contains 1500 litres of water. It is draining at the rate of 50 litres per minute.
a Draw a graph of the volume of water, V litres, remaining after t minutes.
b Write a rule for the volume of water at time t minutes.
c What does the gradient represent?
d What is the volume of water remaining after 5 minutes?
e After how many minutes is the pool half empty?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 The rule for distance travelled, d km, over a given time, t hours, for a moving vehicle is given by $d = 50 + 80t$.

- **a** What is the speed of the vehicle?
- **b** If the speed was actually 70 km per hour, how would this change the rule? Give the new rule.

12 The altitude, h metres, of a helicopter t seconds after it begins its descent is given by $h = 350 - 20t$.

- **a** At what rate is the helicopter altitude decreasing?
- **b** At what rate is the helicopter altitude increasing?
- **c** What is the helicopter's initial height?
- **d** How long will it take for the helicopter to reach the ground?
- **e** If instead the rule was $h = 350 + 20t$, describe what the helicopter would be doing.

ENRICHMENT	—	—	13, 14

Sausages and cars

13 Joanne organised a sausage sizzle to raise money for her science club. The hire of the barbecue cost Joanne $20, and the sausages cost 40c each.

- **a i** Write a rule for the total cost, $C, if Joanne buys and cooks n sausages.
- **ii** If the total cost was $84, how many sausages did Joanne buy?
- **b i** If Joanne sells each sausage for $1.20, write a rule to find her profit, $P, after buying and selling n sausages.
- **ii** How many sausages must she sell to start making a profit?
- **iii** If Joanne's profit was $76, how many sausages did she buy and sell?



14 The directors of a car manufacturing company believe that, in order to make a new component, they would need to spend $6700 on set-up costs, and each component would cost $10 to make. They make x components.

- **a** Write a rule for the total cost, $C, of producing x components.
- **b** Find the cost of producing 200 components.
- **c** How many components could be produced for $13 000?
- **d** Find the cost of producing 500 components.
- **e** If each component is able to be sold for $20, how many must they sell to make a profit?
- **f** Write a rule for the profit, $P, in terms of x.
- **g** Write a rule for the profit, $T, per component in terms of x.
- **h** Find x, the number of components, if the profit per component is to be $5.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4K Graphical solutions to linear simultaneous equations

Interactive
Widgets
HOTsheets
Walkthrough

To find a point that satisfies more than one equation involves finding the solution to simultaneous equations. An algebraic approach was considered in Chapter 2. Graphically this involves finding an intersection point.



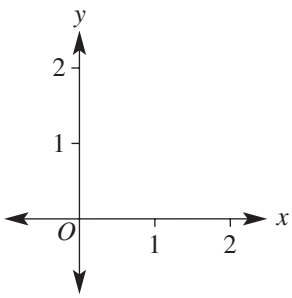
This is an example of an intersection of two linear features.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Accuracy counts

Two graphs have the rules $y = x$ and $y = 2 - 4x$.

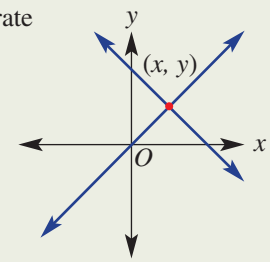
Accurately sketch the graphs of both rules on a large set of axes like the one shown.



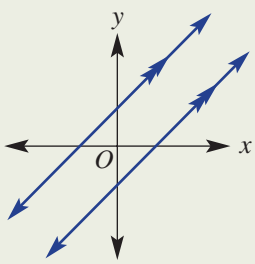
- State the x value of the point at the intersection of your two graphs.
- State the y value of the point at the intersection of your two graphs.
- Discuss how you could use the rules to check if your point is correct.
- If your point does not satisfy both rules, check the accuracy of your graphs and try again.

Key ideas

- When we consider two or more equations at the same time, they are called **simultaneous equations**.
- To determine the **point of intersection** of two lines, we can use an accurate graph and determine its coordinates (x, y). Two situations can arise.
  - The two graphs intersect at one point only and there is one solution (x, y).
  - The point of intersection is simultaneously on both lines and is the **solution** to the simultaneous equations.



  - The two lines are parallel and there is no intersection.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 20 Checking an intersection point

Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 2x + 3$ and $y = -x$ with point $(-1, 1)$

b $y = -2x$ and $3x + 2y = 4$ with point $(2, -4)$

SOLUTION	EXPLANATION
a Substitute $x = -1$	Substitute $x = -1$ into $y = 2x + 3$ to see if $y = 1$.
$y = 2x + 3$	If so, then $(-1, 1)$ is on the line.
$= 2 \times (-1) + 3$	
$= 1$	
So $(-1, 1)$ satisfies $y = 2x + 3$	
$y = -x$	Repeat for $y = -x$.
$= -(-1)$	
$= 1$	
So $(-1, 1)$ satisfies $y = -x$.	
$\therefore$ $(-1, 1)$ is the intersection point.	If $(-1, 1)$ is on both lines, then it must be the intersection point.
b Substitute $x = 2$	Substitute $x = 2$ into $y = -2x$ to see if $y = -4$.
$y = -2x$	
$= -2 \times (2)$	
$= -4$	If so, then $(2, -4)$ is on the line.
So $(2, -4)$ satisfies $y = -2x$	
$3x + 2y = 4$	Substitute $x = 2$ and $y = -4$ to see if $3x + 2y = 4$ is true.
$3(2) + 2(-4) = 4$	
$6 + (-8) = 4$	
$-2 = 4$ (false)	Clearly, the equation is not satisfied.
So $(2, -4)$ is not on the line.	
$\therefore$ $(2, -4)$ is not the intersection point.	Since the point is not on both lines, it cannot be the intersection point.

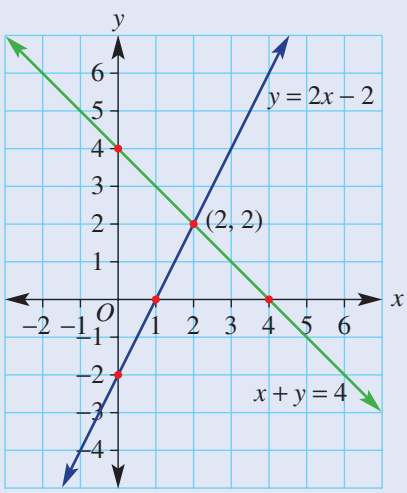


These planes need to make sure that their paths do not intersect at the same point in time.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 21 Solving simultaneous equations graphically

Solve the simultaneous equations $y = 2x - 2$ and $x + y = 4$ graphically.

SOLUTION	EXPLANATION
$y = 2x - 2$	Sketch each linear graph by first finding the y-intercept (substitute $x = 0$) and the x-intercept (substitute $y = 0$ and solve the resulting equation.)
y-intercept (let $x = 0$):	
$y = 2 \times (0) - 2$	
$y = -2$	Plot $(0, -2)$ on the y-axis.
x-intercept (let $y = 0$):	
$0 = 2x - 2$	
$2 = 2x$	
$x = 1$	Plot $(1, 0)$ on the x-axis.
$x + y = 4$	Repeat the process for the second equation.
y-intercept (let $x = 0$):	
$0 + y = 4$	
$y = 4$	Plot $(0, 4)$ on the y-axis.
x-intercept (let $y = 0$):	
$x + 0 = 4$	
$x = 4$	Plot $(4, 0)$ on the x-axis.
	Sketch both graphs on the same set of axes by marking in intercepts and joining in a straight line. Ensure that the axes are scaled accurately. Locate the intersection point and read off the coordinates. The point $(2, 2)$ simultaneously belongs to both lines.
The intersection point is $(2, 2)$.	
$\therefore$ The solution is $x = 2$, $y = 2$.	

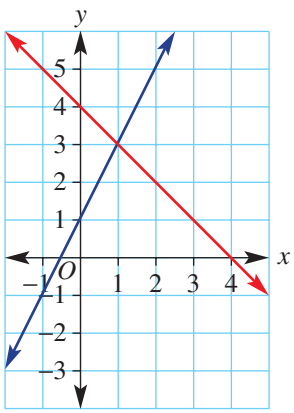
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 4K

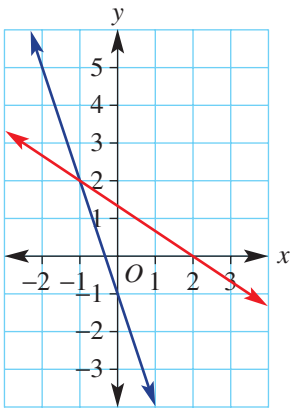
UNDERSTANDING AND FLUENCY	1–3, 4–5(½)	3, 4–5(½)	4–5(½)

1 Give the coordinates of the point of intersection for these pairs of lines.

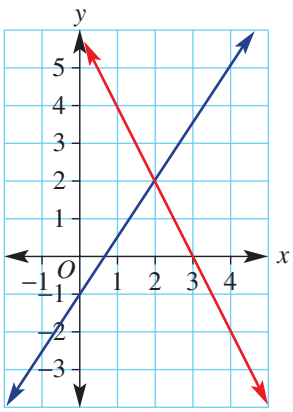
a



b



c



2 Decide if the point (1, 3) satisfies these equations. *Hint*: substitute (1, 3) into each equation to see if the equation is true or false.

a $y = 2x + 1$

b $y = -x + 5$

c $y = -2x - 1$

d $y = 4x - 1$

3 Consider the two lines with the rules $y = 5x$ and $y = 3x + 2$ and the point (1, 5).

a Substitute (1, 5) into $y = 5x$. Does (1, 5) sit on the line with equation $y = 5x$?

b Substitute (1, 5) into $y = 3x + 2$. Does (1, 5) sit on the line with equation $y = 3x + 2$?

c Is (1, 5) the intersection point of the lines with the given equations?

Example 20

4 Decide if the given point is at the intersection of the two lines with the given equations.

a $y = x + 2$ and $y = 3x$ with point (1, 3)

b $y = 3x - 4$ and $y = 2x - 2$ with point (2, 2)

c $y = -x + 3$ and $y = -x$ with point (2, 1)

d $y = -4x + 1$ and $y = -x - 1$ with point (1, −3)

e $x + 2y = 6$ and $3x - 4y = -2$ with point (2, 2)

f $x - y = 10$ and $2x + y = 8$ with point (6, −4)

g $2x + y = 0$ and $y = 3x + 4$ with point (−1, 2)

h $x - 3y = 13$ and $y = -x - 1$ with point (4, −3)

Example 21

5 Solve these pairs of simultaneous equations graphically by finding the coordinates of the intersection point.

a $2x + y = 6$
$x + y = 4$

b $3x - y = 7$
$y = 2x - 4$

c $y = x - 6$
$y = -2x$

d $y = 2x - 4$
$x + y = 5$

e $2x - y = 3$
$3x + y = 7$

f $y = 2x + 1$
$y = 3x - 2$

g $x + y = 3$
$3x + 2y = 7$

h $y = x + 2$
$y = 3x - 2$

i $y = x - 3$
$y = 2x - 7$

j $y = 3$
$x + y = 2$

k $y = 2x - 3$
$x = -1$

l $y = 4x - 1$
$y = 3$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING 6, 7, 9 | 6, 7, 9, 10 | 7–11

6 A company manufactures electrical components. The cost, $\$C$ (including rent, materials and labour), is given by the rule $C = n + 3000$, and its revenue, $\$R$, is given by the rule $R = 5n$, where n is the number of components produced.

- **a** Sketch the graphs of C and R on the same set of axes and determine the point of intersection.
- **b** State the number of components n where the costs $\$C$ are equal to the revenue $\$R$.

7 Dvdcom and Associates manufacture DVDs. Its costs, $\$C$, are given by the rule $C = 4n + 2400$, and its revenue, $\$R$, is given by the rule $R = 6n$, where n is the number of DVDs produced. Sketch the graphs of C and R on the same set of axes and determine the number of DVDs to be produced if the costs are equal to the revenue.

8 Two asteroids are 1000 km apart and are heading straight for each other. One asteroid is travelling at 59 km per second and the other at 41 km per second. How long does it take for them to collide?

9 Explain why the graphs of the rules $y = 3x - 7$ and $y = 3x + 4$ have no intersection point.

10 For the following families of graphs, determine their points of intersection (if any).

- **a** $y = x$, $y = 2x$, $y = 3x$
- **b** $y = x$, $y = -2x$, $y = 3x$
- **c** $y = x + 1$, $y = x + 2$, $y = x + 3$
- **d** $y = -x + 1$, $y = -x + 2$, $y = -x + 3$
- **e** $y = 2x + 1$, $y = 3x + 1$, $y = 4x + 1$
- **f** $y = 2x + 3$, $y = 3x + 3$, $y = 4x + 3$

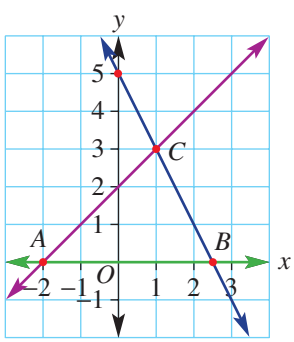
11 **a** If two lines have the equations $y = 3x + 1$ and $y = 2x + b$, find the value of b if the intersection point is at $x = 1$.

- **b** If two lines have the equations $y = mx - 4$ and $y = -2x - 3$, find the value of m if the intersection point is at $x = -1$.

ENRICHMENT — | — | 12, 13

Intersecting to find triangular areas

12 The three lines with equations $y = 0$, $y = x + 2$ and $y = -2x + 5$ are illustrated here.



- **a** State the coordinates of the intersection point of $y = x + 2$ and $y = -2x + 5$.
- **b** Use $A = \frac{1}{2}bh$ to find the area of the enclosed triangle ABC.

13 Use the method outlined in Question **12** to find the area enclosed by these sets of three lines.

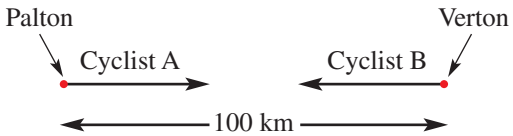
- **a** $y = 0$, $y = x + 3$ and $y = -2x + 9$
- **b** $y = 0$, $y = \frac{1}{2}x + 1$ and $y = -x + 10$
- **c** $y = 2$, $x - y = 5$ and $x + y = 1$
- **d** $y = -5$, $2x + y = 3$ and $y = x$
- **e** $x = -3$, $y = -3x$ and $x - 2y = -7$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Investigation

Coming and going

The distance between two towns, Palton and Verton, is 100 km. Two cyclists travel in opposite directions between the towns, starting their journeys at the same time. Cyclist A travels from Palton to Verton at a speed of 20 km/h while cyclist B travels from Verton to Palton at a speed of 25 km/h.



Measuring distance from Palton

a Using d_A km as the distance cyclist A is from Palton after t hours, explain why the rule connecting d_A and t is $d_A = 20t$

b Using d_B km as the distance cyclist B is from Palton after t hours, explain why the rule connecting d_B and t is $d_B = 100 - 25t$.

Technology – spreadsheet (alternatively use a graphics or CAS calculator – see parts e and f below)

c Instructions:

- Enter the time in hours into column A starting at 0 hours.
- Enter the formulas for the distances d_A and d_B into columns B and C.
- Use the Fill down function to fill in the columns. Fill down until the distances show that both cyclists have completed their journey.

	A	B	C
1	0	= 20 * A1	= 100 – 25 * A1
2	= A1 + 1		
3			

d **i** Determine how long it takes for cyclist A to reach Verton.

ii Determine how long it takes for cyclist B to reach Palton.

iii After which hour are the cyclists the closest?

Alternative technology – graphics or CAS calculator

e Instructions:

- Enter or define the formulas for the distances d_A and d_B.
- Go to the table and scroll down to view the distance for each cyclist at hourly intervals. You may need to change the settings so that t increases by 1 each time.

f **i** Determine how long it takes for cyclist A to reach Verton.

ii Determine how long it takes for cyclist B to reach Palton.

iii After which hour are the cyclists the closest?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Investigating the intersection

a Change the time increment to a smaller unit for your chosen technology.
- Spreadsheet: Try 0.5 hours using '= A1 + 0.5' or 0.1 hours using '= A1 + 0.1' in column A.
- Graphics or CAS calculator: Try changing the t increment to 0.5 or 0.1.

b Fill or scroll down to ensure that the distances show that both cyclists have completed their journey.

c Determine the time at which the cyclists are the closest.

d Continue altering the time increment until you are satisfied that you have found the time of intersection of the cyclists correct to 1 decimal place.

e **Extension**: Complete part **d** but give your answer correct to 3 decimal places.

The graph

a Sketch a graph of d_A and d_B on the same set of axes. Scale your axes carefully to ensure that the full journey for both cyclists is represented.

b Determine the intersection point as accurately as possible on your graph and hence estimate the time when the cyclists meet.

c Use technology (graphing calculator) to confirm the point of intersection and hence determine the time at which the cyclists meet correct to 3 decimal places.

Algebra and proof

a At the point of intersection it could be said that $d_A = d_B$.
This means that $20t = 100 - 25t$.
Solve this equation for t.

b Find the exact distance from Palton at the point where the cyclists meet.

Reflection

Write a paragraph describing the journey of the two cyclists. Comment on:
- the speeds of the cyclists
- their meeting point
- the difference in computer and algebraic approaches in finding the time of the intersection point.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Puzzles and challenges

1 Matchsticks are arranged by a student so that the first three diagrams in the pattern are:

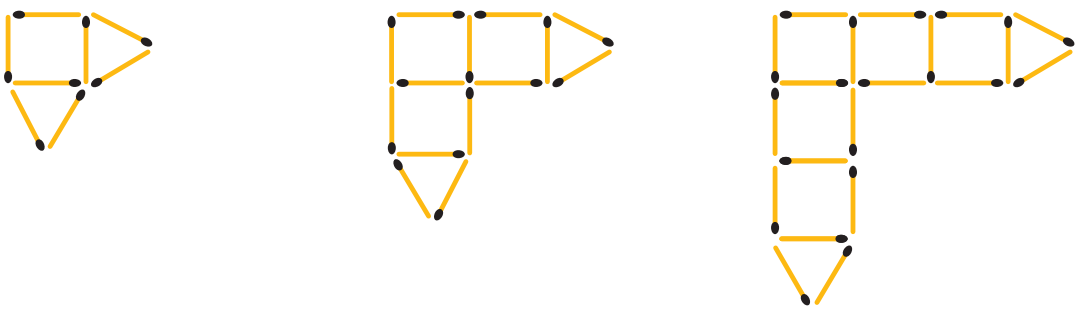


How many matchsticks are in the 50th diagram of the pattern?

2 Two cars travel towards each other on a 400 km stretch of road. One car travels at 90 km/h and the other at 70 km/h. How long does it take before they pass each other?

3 The points $(-1, 4)$, $(4, 6)$, $(2, 7)$ and $(-3, 5)$ are the vertices of a parallelogram. Find the midpoints of its diagonals. What do you notice?

4 The first three shapes in a pattern made with matchsticks are:



How many matchsticks make up the 100th shape?

5 Prove that the triangle with vertices at the points $A(-1, 3)$, $B(0, -1)$ and $C(3, 2)$ is isosceles.

6 Find the perimeter (to the nearest whole number) and area of the triangle enclosed by the lines with equations $x = -4$, $y = x$ and $y = -2x - 3$.

7 $ABCD$ is a parallelogram. A, B and C have coordinates $(5, 8)$, $(2, 5)$ and $(3, 4)$ respectively. Find the coordinates of D.

8 A tank with 520 L of water begins to leak at a rate of 2 L per day. At the same time, a second tank is being filled at a rate of 1 L per hour, starting at 0 L. How long does it take for the tanks to have the same volume?

9 Bill takes 3 days to paint a house, Rory takes 4 days to paint a house and Lucy takes 5 days to paint a house. How long would it take to paint a house (to the nearest hour) if all three of them worked together?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

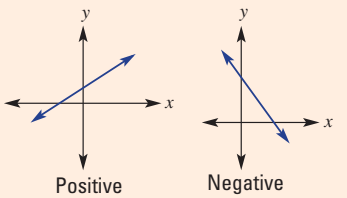
Linear relationships

Direct proportion

If two variables are directly proportional, their rule is of the form $y = mx$. The gradient of the graph represents the rate of change of y with respect to x, e.g. for a car travelling at 60 km/h for t hours, distance $= 60t$.

Gradient

This measures the slope of a line



Gradient $m = \frac{\text{rise}}{\text{run}}$

e.g. $m = \frac{4}{2} = 2$



Parallel and perpendicular lines

Parallel lines have the same gradient e.g. $y = 2x + 3$, $y = 2x - 1$ and $y = 2x$

Perpendicular lines are at right angles

Their gradients m_1 and m_2 are such that $m_1 m_2 = -1$, i.e. $m_2 = -\frac{1}{m_1}$

A line perpendicular to $y = 2x + 3$ has gradient $-\frac{1}{2}$.

Linear graphs

Sketch with two points. Often we find the x-intercept ($y = 0$) and y-intercept ($x = 0$)

e.g. $y = 2x - 4$

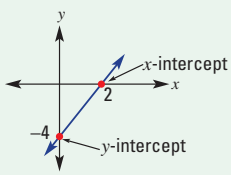
y-int: $x = 0$, $y = 2(0) - 4$

$y = -4$

x-int: $y = 0$, $0 = 2x - 4$

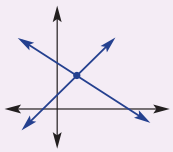
$2x = 4$

$x = 2$

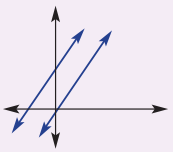


Graphical solutions of simultaneous equations

Graph each line and read off point of intersection.



Parallel lines have no intersection point



Straight line

$y = mx + b$

gradient m, y-intercept b

In the form $ax + by = d$, rearrange to $y = mx + b$ form to read off gradient and y-intercept.

A linear relationship is made up of points (x, y) that form a straight line when plotted.

Linear modelling

Define variables to represent the problem and write a rule relating the two variables. The rate of change of one variable with respect to the other is the gradient.

Finding the equation of a line

Gradient–intercept form ($y = mx + b$)

Require gradient and y-intercept or any other point to substitute, e.g. line with gradient 3 passes through point (2, 1).

$\therefore y = 3x + b$

Substitute $1 = 3(2) + b$

$\therefore b = 1 + 6 = -5$

$y = 3x - 5$

Midpoint and length of line segment

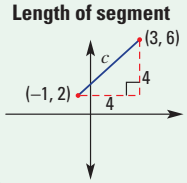
Midpoint is halfway between segment end points, e.g. midpoints of segment joining (–1, 2) and (3, 6)

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$x = \frac{-1 + 3}{2} = 1$

$y = \frac{2 + 6}{2} = 4$, i.e. (1, 4)

Length of segment



$c^2 = 4^2 + 4^2$

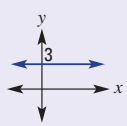
$c = \sqrt{32}$

$c = 5.7$ (to 1 d.p.)

Special lines

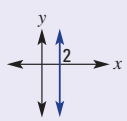
Horizontal line $y = b$

e.g. $y = 3$

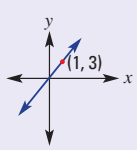


Vertical line $x = a$

e.g. $x = 2$



$y = mx$ passes through the origin, substitute $x = 1$ to find another point, e.g. $y = 3x$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter review

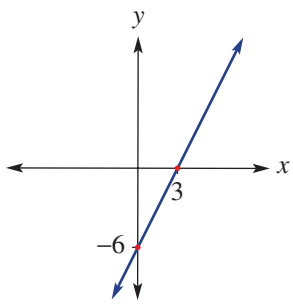
Multiple-choice questions

1 The x- and y-intercepts of the graph of $2x + 4y = 12$ are respectively:

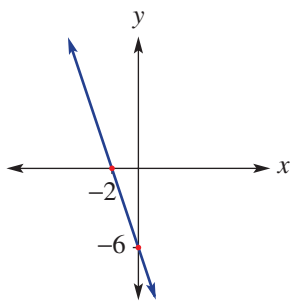
A 4 and 2 **B** 3 and 2 **C** 6 and 3
D 2 and 12 **E** 6 and 8

2 The graph of $y = 3x - 6$ is represented by:

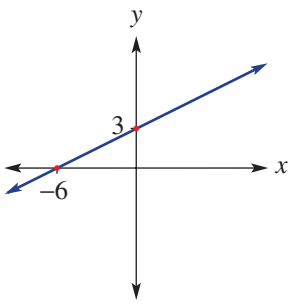
A



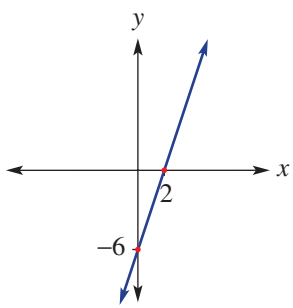
B



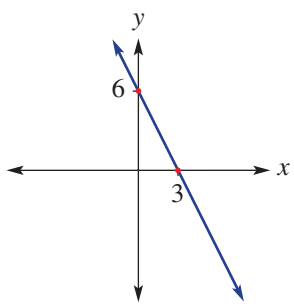
C



D



E

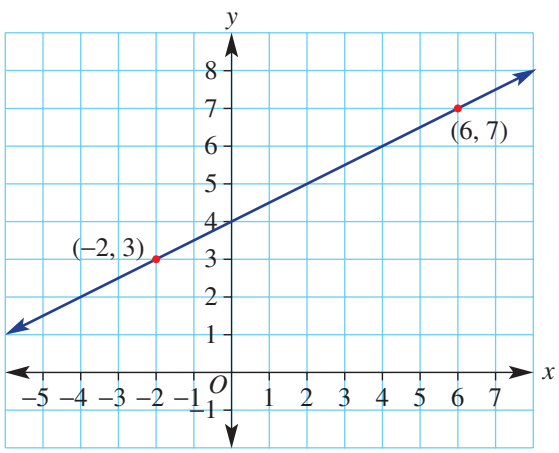


3 The point that is not on the straight line $y = -2x + 3$ is:

A (1, 1) **B** (0, 3) **C** (2, 0) **D** (−2, 7) **E** (3, −3)

4 The equation of the graph shown is:

A $y = x + 1$
B $y = \frac{1}{2}x + 4$
C $y = 2x + 1$
D $y = -2x - 1$
E $y = \frac{1}{2}x + 2$



5 The gradient of the line joining the points (1, −2) and (5, 6) is:

A 2 **B** 1 **C** 3 **D** −1 **E** $\frac{1}{3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 The linear graph that *does not* have a gradient of 3 is:

A $y = 3x + 7$ **B** $\frac{y}{3} = x + 2$ **C** $2y - 6x = 1$

D $y - 3x = 4$ **E** $3x + y = -1$

7 A straight line that has a gradient of -2 and passes through the point $(0, 5)$, has the equation:

A $2y = -2x + 10$ **B** $y = -2x + 5$ **C** $y = 5x - 2$

D $y - 2x = 5$ **E** $y = -2(x - 5)$

8 The length of the line segment joining the points $A(1, 2)$ and $B(4, 6)$ is:

A 5 **B** $\frac{4}{3}$ **C** 7 **D** $\sqrt{5}$ **E** $\sqrt{2}$

9 The gradient of a line perpendicular to $y = 4x - 7$ would be:

A $\frac{1}{4}$ **B** 4 **C** -4 **D** $-\frac{1}{4}$ **E** -7

10 The point of intersection of $y = 2x$ and $y = 6 - x$ is:

A $(6, 2)$ **B** $(-1, -2)$ **C** $(6, 12)$

D $(2, 4)$ **E** $(3, 6)$

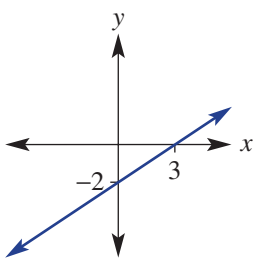
Short-answer questions

1 Read the x- and y-intercepts from the table and graph.

a

x	-2	-1	0	1	2
y	0	2	4	6	8

b



2 Sketch the following linear graphs labelling x- and y-intercepts.

a $y = 2x - 4$ **b** $y = 3x + 9$ **c** $y = -2x + 5$ **d** $y = -x + 4$

e $2x + 4y = 8$ **f** $4x - 2y = 10$ **g** $2x - y = 7$ **h** $-3x + 6y = 12$

3 Holly leaves her beach house by car and drives back to her home. Her distance d kilometres from her home after t hours is given by $d = 175 - 70t$.

a How far is her beach house from her home?

b How long does it take to reach her home?

c Sketch a graph of her journey between the beach house and her home.

4 Sketch the following lines.

a $y = 3$ **b** $y = -2$ **c** $x = -4$

d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 By first plotting the given points, find the gradient of the line passing through the points.

a (3, 1) and (5, 5) **b** (2, 5) and (4, 3) **c** (1, 6) and (3, 1)

d (−1, 2) and (2, 6) **e** (−3, −2) and (1, 6) **f** (−2, 6) and (1, −4)

6 An inflatable backyard swimming pool is being filled with water by a hose. It takes 4 hours to fill 8000 L.

a What is the rate at which water is poured into the pool?

b Draw a graph of volume (V litres) vs time (t hours) for $0 \leq t \leq 4$.

c By finding the gradient of your graph, give the rule for V in terms of t.

d Use your rule to find the time to fill 5000 L.

7 For each of the following linear relations, state the value of the gradient and the y-intercept and then sketch using the gradient–intercept method.

a $y = 2x + 3$ **b** $y = -3x + 7$ **c** $2x + 3y = 9$ **d** $2y - 3x - 8 = 0$

8 Give the equation of the straight line that has gradient:

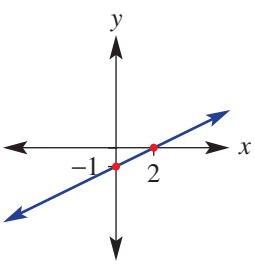
a 3 and passes through the point (0, 2)

b −2 and passes through the point (3, 0)

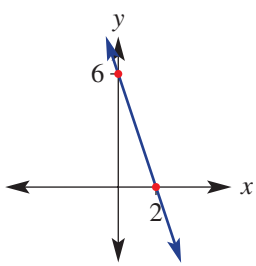
c $\frac{4}{3}$ and passes through the point (6, 3)

9 Find the equations of the linear graphs below.

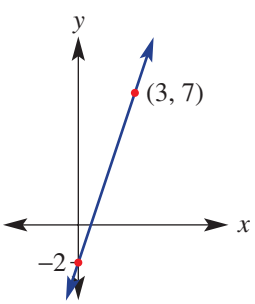
a



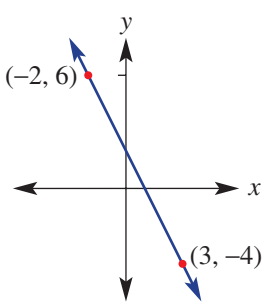
b



c



d



10 Determine the linear equation that is:

a parallel to the line $y = 2x - 1$ and passes through the point (0, 4)

b parallel to the line $y = -x + 4$ and passes through the point (0, −3)

c perpendicular to the line $y = 2x + 3$ and passes through the point (0, −1)

d perpendicular to the line $y = -\frac{1}{3}x - 2$ and passes through the point (0, 4)

e parallel to the line $y = 3x - 2$ and passes through the point (1, 4)

f parallel to the line $3x + 2y = 10$ and passes through the point (−2, 7)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 For the line segment joining the following pairs of points, find the:

i midpoint **ii** length (to 2 decimal places where applicable)

a (2, 4) and (6, 8) **b** (5, 2) and (10, 7) **c** (−2, 1) and (2, 7) **d** (−5, 7) and (−1, −2)

12 Find the missing coordinate n if the:

a line joining $(-1, 3)$ and $(2, n)$ has gradient 2

b line segment joining $(-2, 2)$ and $(4, n)$ has length 10, $n > 0$

c midpoint of the segment joining $(n, 1)$ and $(1, -3)$ is $(3.5, -1)$

13 Determine if the point (2, 5) is the intersection point of the graphs of the following pairs of equations.

a $y = 4x - 3$ and $y = -2x + 6$ **b** $3x - 2y = -4$ and $2x + y = 9$

14 Find the point of intersection of the following straight lines.

a $y = 2x - 4$ and $y = 6 - 3x$ **b** $2x + 3y = 8$ and $y = -x + 2$

Extended-response questions

1 Joe requires an electrician to come to his house to do some work. He is trying to choose between two electricians he has been recommended.

a The first electrician's cost $\$C$ is given by $C = 80 + 40n$ where n is the number of hours the job takes.

i State the hourly rate this electrician charges and his initial fee for coming to the house.

ii Sketch a graph of C vs n for $0 \leq n \leq 8$.

iii What is the cost of a job that takes 2.5 hours?

iv If the job costs \$280, how many hours did it take?

b The second electrician charges a call-out fee of \$65 to visit the house and then \$45 per hour thereafter.

i Give the equation for the cost C of a job that takes n hours.

ii Sketch the graph of part **b i** for $0 \leq n \leq 8$ on the same axes as the graph in part **a**.

c Determine the point of intersection of the two graphs.

d After how many hours does the first electrician become the cheaper option?

2 Abby has set up a small business making clay vases. She is trying to determine the selling price of these vases to ensure that she makes a weekly profit.

Abby has determined that the cost of producing 7 vases in a week is \$146 and the cost of producing 12 vases in a week is \$186.

a Find a linear rule relating the production cost, $\$C$, to the number of vases produced, v.

b Use your rule to state the:

i initial cost of materials each week

ii ongoing cost of production per vase

At a selling price of \$12 per vase Abby determines her weekly profit to be given by $P = 4v - 90$.

c How many vases must she sell in order to make a profit?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

5 Length, area, surface area and volume

What you will learn

5A Length and perimeter REVISION
5B Circumference and perimeter of sectors REVISION
5C Area REVISION
5D Perimeter and area of composite shapes
5E Surface area of prisms and pyramids
5F Surface area of cylinders
5G Volume of prisms
5H Volume of cylinders

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY
SUBSTRANDS: AREA AND SURFACE AREA (S4, 5.1, 5.2, 5.3) VOLUME (S4, 5.2)

Outcomes

A student calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms.
(MA5.1–8MG)

A student calculates the surface areas of right prisms, cylinders and related composite solids.
(MA5.2–11MG)

A student applies formulas to find the surface areas of right pyramids, right cones, spheres and related composite solids.
(MA5.3–13MG)

A student applies formulas to calculate the volumes of composite solids composed of right prisms and cylinders.
(MA5.2–12MG)

A student applies formulas to find the volumes of right pyramids, right cones, spheres and related composites solids.
(MA5.3–14MG)

© NESA, 2012

Body Surface Area

Body Surface Area (BSA) is measured in square metres and is the total surface area of a person's body. It is used in medicine to calculate the correct drug dose and the volume of fluids that are to be administered intravenously.

Calculating the surface area of prisms and pyramids is relatively easy, as we use the area formulas for each face, and then find the total. However, calculating the actual surface area of the irregular solid, such as a person, is a difficult task. To find the surface area of a person the following formula can be used.

$$\text{BSA } (\text{m}^2) = \sqrt{\frac{\text{body weight (kg)}}{\text{height (cm)}} \div 3600}$$

Note that the average BSA for a 12–13-year-old is 1.33 m^2 and 1.7 m^2 for the average adult.

Paediatricians need to know a child's BSA to use Clarke's formula for a child's dose to calculate the correct dose required by each patient. Clarke's formula states:

Child's dose

$$= \frac{\text{Surface area of child } (\text{m}^2) \times \text{Adult dose}}{\text{Average surface area of adult } (1.7\,\text{m}^2)}$$

Doctors and nurses need to be able to perform these calculations quickly and accurately in order to save lives.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.
Updated March 2022

Pre-test

1 Calculate the following.

a 2.3×10 **b** 0.048×1000 **c** $270 \div 100$

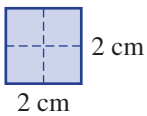
d $52\,134 \div 10\,000$ **e** $0.0005 \times 100\,000$ **f** $72\,160 \div 1000$

2 For these basic shapes find the:

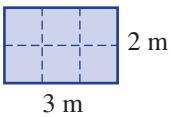
i number of squares inside the shape (area)

ii distance around the outside of the shape (perimeter)

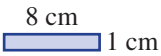
a



b

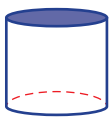


c



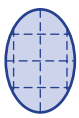
3 **a** Name this solid.

b What is the name of the shape (shaded) on top of the solid?



4 Estimate the area (number of squares) in these shapes.

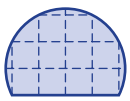
a



b



c

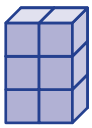


5 Count the number of cubes in these solids to find the volume.

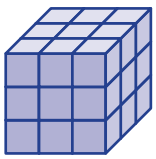
a



b



c



6 Convert the following to the units shown in the brackets.

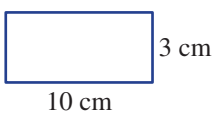
a 3 cm (mm) **b** 20 m (cm) **c** 1.6 km (m)

d 23 mm (cm) **e** 3167 m (km) **f** 72 cm (m)

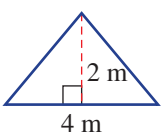
g 20 000 mm (m) **h** 0.03 km (cm)

7 Find the area of these basic shapes.

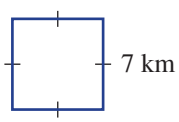
a



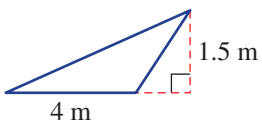
b



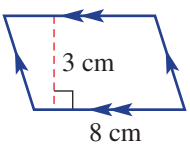
c



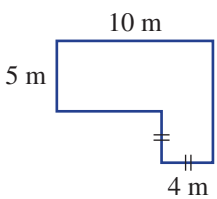
d



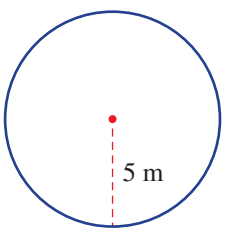
e



f



8 Find the circumference ($C = 2\pi r$) and area ($A = \pi r^2$) of this circle, rounding to 2 decimal places.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5A Length and perimeter REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Length is at the foundation of measurement from which the concepts of perimeter, circumference, area and volume are developed. From the use of the royal cubit (distance from tip of middle finger to the elbow) used by the ancient Egyptians to the calculation of pi (π) by modern computers, units of length have helped to create the world in which we live.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

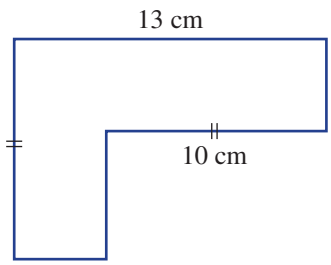


Units of length are essential in measuring distance, area and volume.

Let's start: Not enough information?

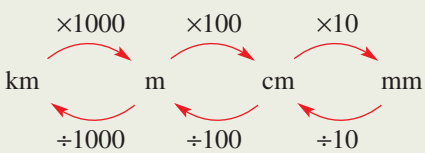
All the angles at each vertex in this shape are 90° and the two given lengths are 10 cm and 13 cm.

- Is there enough information to find the perimeter of the shape?
- If there is enough information, find the perimeter and discuss your method. If not, then say what information needs to be provided.

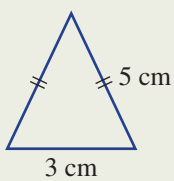


Key ideas

■ To convert between **metric units** of length, multiply or divide by the appropriate power of 10.



■ **Perimeter** is the distance around a closed shape.
- Sides with the same markings are of equal length.



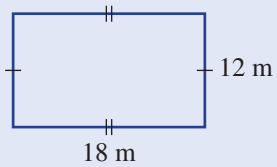
$P = 2 \times 5 + 3$
$= 13$ cm

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

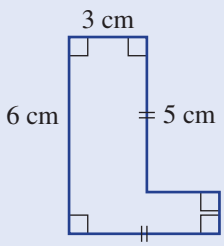
Example 1 Finding perimeters of simple shapes

Find the perimeter of each of the following shapes.

a



b



SOLUTION

a Perimeter $= 2 \times 12 + 2 \times 18$
$= 24 + 36$
$= 60$ m

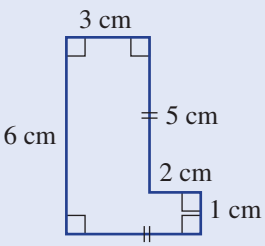
b Perimeter $= (2 \times 5) + 6 + 3 + 2 + 1$
$= 22$ cm

Alternative method:
Perimeter $= 2 \times 6 + 2 \times 5$

EXPLANATION

Two lengths of 12 m and two lengths of 18 m.

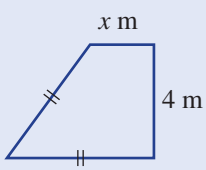
Missing sides are:
5 cm − 3 cm = 2 cm
6 cm − 5 cm = 1 cm



The horizontal sides are 5 cm each.
The vertical sides are 6 cm each.

Example 2 Finding missing sides given a perimeter

Find the unknown side length in this shape with the given perimeter.



Perimeter = 16 m

SOLUTION

$2 \times 5 + 4 + x = 16$
$14 + x = 16$
$x = 2$
$\therefore$ The missing length is 2 m

EXPLANATION

Add all the lengths and set equal to the given perimeter.
Solve for the unknown.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 5A REVISION

UNDERSTANDING AND FLUENCY 1–7 3, 4–5(½), 6, 7–8(½) 5(½), 6(c), 7–8(½)

1 Convert the following length measurements into the units given in the brackets.

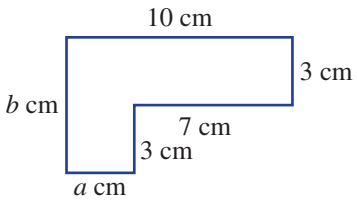
a 5 cm (mm) **b** 2.8 m (cm) **c** 521 mm (cm)

d 83.7 cm (m) **e** 4.6 km (m) **f** 2170 m (km)

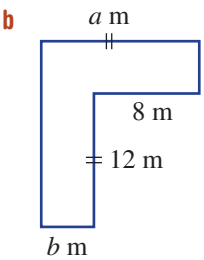
2 A steel beam is 8.25 m long and 22.5 mm wide. Write down the dimensions of the beam in centimetres.

3 Write down the values of the pronumerals in these shapes.

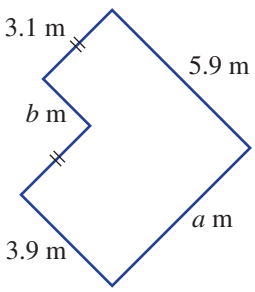
a



b



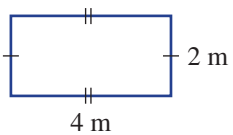
c



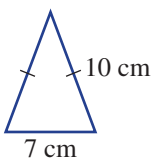
Example 1a

4 Find the perimeter of each of the following shapes.

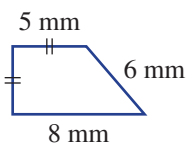
a



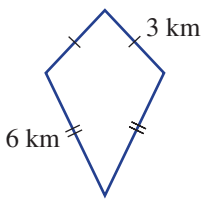
b



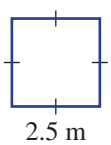
c



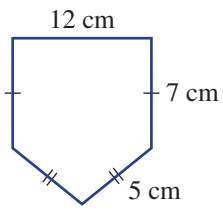
d



e



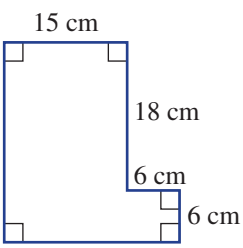
f



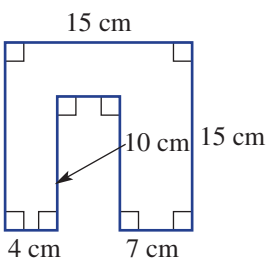
Example 1b

5 Find the perimeter of each of the following composite shapes.

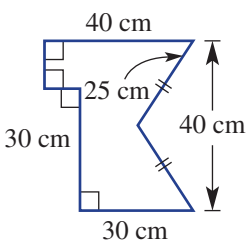
a



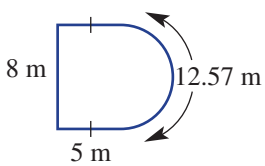
b



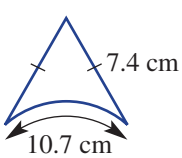
c



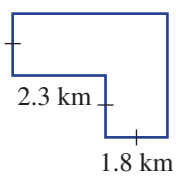
d



e



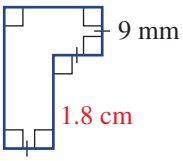
f



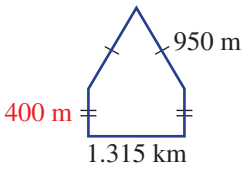
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Find the perimeter of each of these shapes. You will need to convert the measurements to the same units. Give your answers in the units given in red.

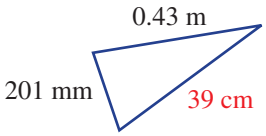
a



b



c



7 Convert the following measurements into the units given in the brackets.

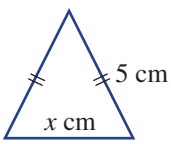
a 8 m (mm) **b** 110 000 mm (m) **c** 0.00001 km (cm)

d 0.02 m (mm) **e** 28 400 cm (km) **f** 62 743 000 mm (km)

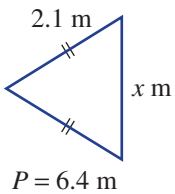
Example 2

8 Find the unknown side length in these shapes with the given perimeters.

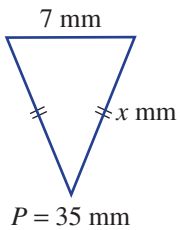
a



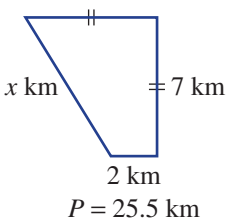
b



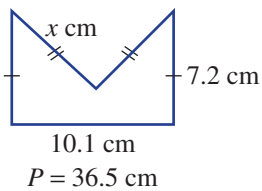
c



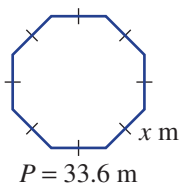
d



e



f

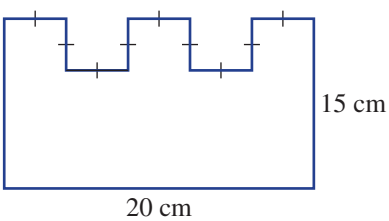


PROBLEM-SOLVING AND REASONING 9, 10, 13 | 9–11, 13, 14 | 10–12, 14, 15

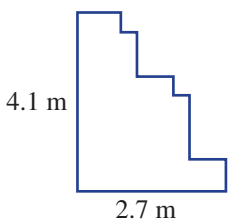
9 A lion enclosure is made up of five straight fence sections. Three sections are 20 m in length and the other two sections are 15.5 m and 32.5 m. Find the perimeter of the enclosure.

10 Find the perimeter of these shapes. Assume all angles are right angles.

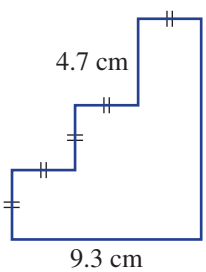
a



b

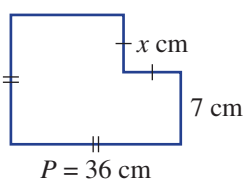


c

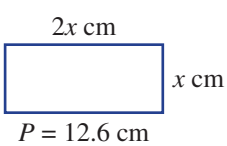


11 Find the value of x in these shapes with the given perimeter.

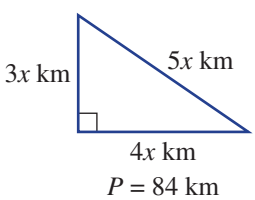
a



b



c

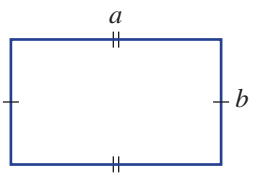


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

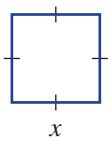
12 A photo 12 cm wide and 20 cm long is surrounded with a picture frame 3 cm wide. Find the outside perimeter of the framed picture.

13 Give the rule for the perimeter of these shapes using the given pronumerals, e.g. $P = 3a + 2b$.

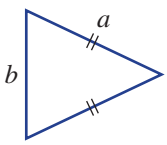
a



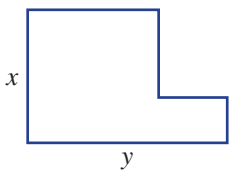
b



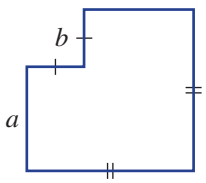
c



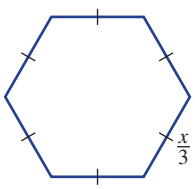
d



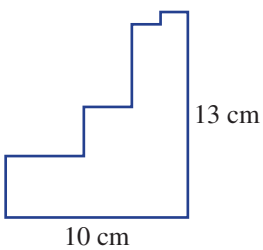
e



f



14 Explain why you do not need any more information to find the perimeter of this shape, assuming all angles are right angles.



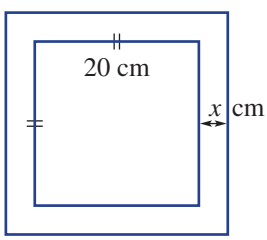
15 A piece of string 1 m long is divided into the given ratios. Find the length of each part.

a 1 : 3 **b** 2 : 3 **c** 5 : 3 **d** 1 : 2 : 3 : 4

ENRICHMENT — — 16

Picture framing

16 A square picture of side length 20 cm is surrounded by a frame of width x cm.



a Find the perimeter of the framed picture if:

i $x = 2$

ii $x = 3$

iii $x = 5$

b Write a rule for the perimeter P of the framed picture in terms of x.

c Use your rule to find the perimeter if:

i $x = 3.7$

ii $x = 7.05$

d Use your rule to find the value of x if the perimeter is:

i 90 cm

ii 102 cm

e Is there a value of x for which the perimeter is 75 cm? Explain.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5B Circumference and perimeter of sectors REVISION

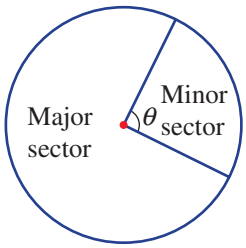








A portion of a circle enclosed by two radii and an arc is called a sector. The perimeter of a sector is made up of three components: two radii of the same length and the circular arc. Given an angle θ, it is possible to find the length of the arc using the rule for the circumference of a circle $C = 2\pi r$ or $C = \pi d$.



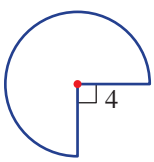
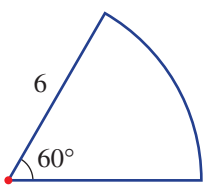
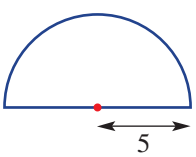
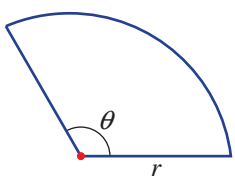
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Perimeter of a sector



A sector is formed by dividing a circle with two radius cuts. The angle between the two radii determines the size of the sector. The perimeter will therefore depend on both the radius length and the angle between them.

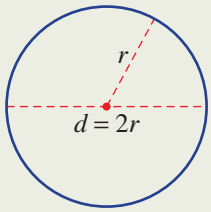
- Complete this table to see if you can determine a rule for the perimeter of a sector. Remember that the circumference C of a circle is given by $C = 2\pi r$ where r is the radius length.

Shape	Fraction of full circle	Working and answer
	$\frac{90}{360} = \frac{1}{4}$	$P = 2 \times 3 + \frac{1}{4} \times 2\pi \times 3 \approx 10.71$
	$\frac{270}{360} =$	$P =$
		$P =$
		$P =$
		$P =$

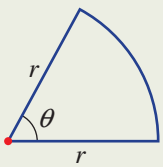
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- **Circumference** of a circle $C = 2\pi r$ or $C = \pi d$.
  - Use $\frac{22}{7}$ or 3.14 to approximate π or use technology for more precise calculations.



- A **sector** is a portion of a circle enclosed by two radii and an arc.
  Special sectors include the following.
  - A half circle is called a **semicircle**.
  - A quarter circle is called a **quadrant**.
- The perimeter of a sector is given by $P = 2r + \frac{\theta}{360} \times 2\pi r$.



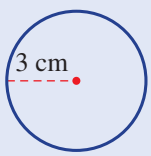
- The symbol for pi (π) can be used to write an answer exactly.
  For example: $C = 2\pi r$
  $= 2\pi \times 3$
  $= 6\pi$



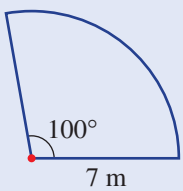
Example 3 Finding the circumference of a circle and perimeter of a sector

Find the circumference of this circle and perimeter of this sector correct to 2 decimal places.

a



b



SOLUTION

a $C = 2\pi r$
$= 2 \times \pi \times 3$
$= (6\pi)$ cm
$= 18.85$ cm (to 2 d.p.)

b $P = 2r + \frac{\theta}{360} \times 2\pi r$
$= 2 \times 7 + \frac{100}{360} \times 2 \times \pi \times 7$
$= \left(14 + \frac{35\pi}{9}\right)$m
$= 26.22$ m (to 2 d.p.)

EXPLANATION

Use the formula $C = 2\pi r$ or $C = \pi d$ and substitute $r = 3$ (or $d = 6$).
6π would be the exact answer and 18.85 is the rounded answer.

Write the formula.

The fraction of the circle is $\frac{100}{360}$ (or $\frac{5}{18}$).

$14 + \frac{35\pi}{9}$ is the exact answer.

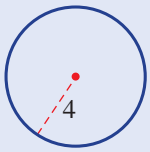
Use a calculator and round to 2 decimal places.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

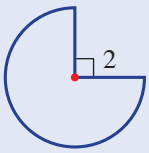
Example 4 Using exact values

Give the exact circumference or perimeter of these shapes.

a



b



SOLUTION

a $C = 2\pi r$
$= 2 \times \pi \times 4$
$= 8\pi$

EXPLANATION

Write the formula with $r = 4$.

$2 \times \pi \times 4 = 2 \times 4 \times \pi$

Write the answer exactly in terms of π.

b $P = 2 \times r + \frac{\theta}{360} \times 2\pi r$
$= 2 \times 2 + \frac{270}{360} \times 2\pi \times 2$
$= 4 + \frac{3}{4} \times 4\pi$
$= 4 + 3\pi$

Use the formula for the perimeter of a sector.

The angle inside the sector is 270° so the fraction is $\frac{270}{360} = \frac{3}{4}$.

$4 + 3\pi$ cannot be simplified further.

Exercise 5B REVISION

UNDERSTANDING AND FLUENCY	1–7	2(½), 4–6, 7–8(½)	5–8(½)

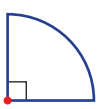
1 **a** What is the radius of a circle with diameter 5.6 cm?
b What is the diameter of a circle with radius 48 mm?

2 Simplify these expressions to give an exact answer. Do not evaluate with a calculator or round off. The first one is done for you.

a $2 \times 3 \times \pi = 6\pi$
b $6 \times 2\pi$
c $7 \times 2.5 \times \pi$
d $3 + \frac{1}{2} \times 4\pi$
e $2 \times 6 + \frac{1}{4} \times 12\pi$
f $2 \times 5 + \frac{2}{5} \times 2 \times \pi \times 5$
g $2 \times 4 + \frac{90}{360} \times 2 \times \pi \times 4$
h $3 + \frac{270}{360} \times \pi$
i $7 + \frac{30}{360} \times \pi$

3 Determine the fraction of a circle shown in these sectors. Write the fraction in simplest form.

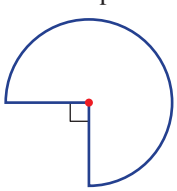
a



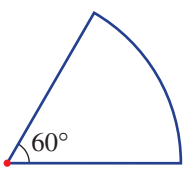
b



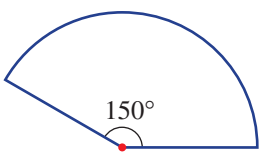
c



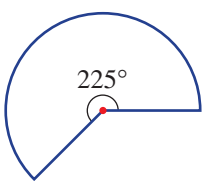
d



e



f

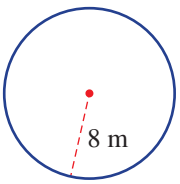


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

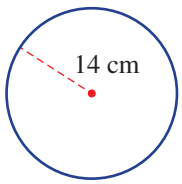
Example 3a

4 Find the circumference of these circles correct to 2 decimal places. Use a calculator for the value of π.

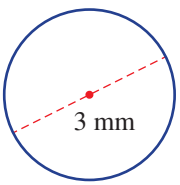
a



b



c



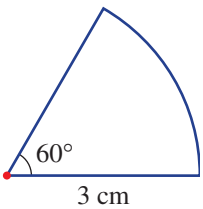
d



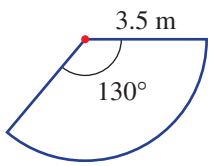
Example 3b

5 Find the perimeter of these sectors correct to 2 decimal places.

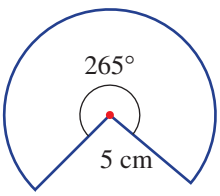
a



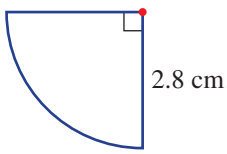
b



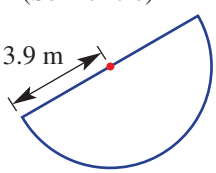
c



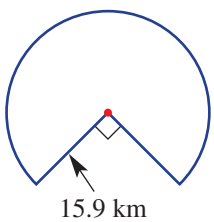
d (Quadrant)



e (Semicircle)

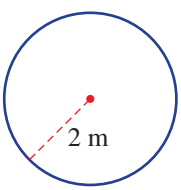


f



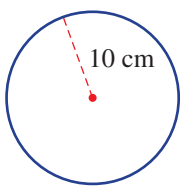
6 Find the circumference of these circles without a calculator, using the given approximation of π.

a



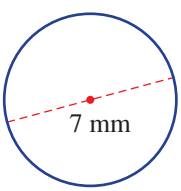
$\pi \approx 3.14$

b



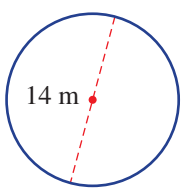
$\pi \approx 3.14$

c



$\pi \approx \frac{22}{7}$

d



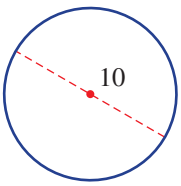
$\pi \approx \frac{22}{7}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

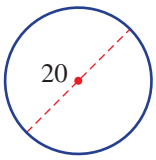
Example 4a

7 Give the exact circumference of these circles.

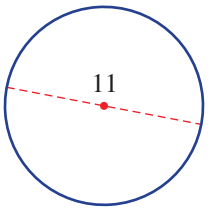
a



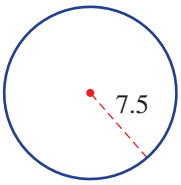
b



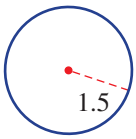
c



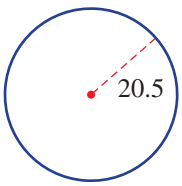
d



e



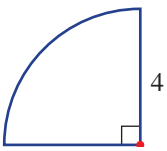
f



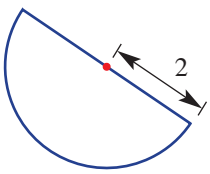
Example 4b

8 Give the exact perimeter of these sectors.

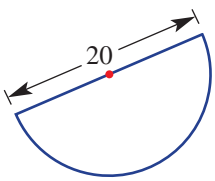
a



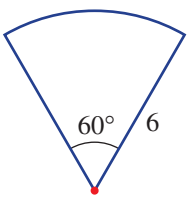
b



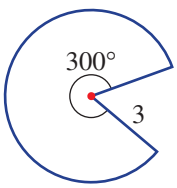
c



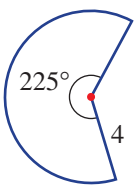
d



e



f



PROBLEM-SOLVING AND REASONING 9, 10, 13 | 9–11, 13 | 10–12, 13(½), 14

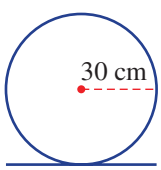
9 Find the distance around the outside of a circular pool of radius 4.5 m, correct to 2 decimal places.

10 Find the length of string required to surround the circular trunk of a tree that has a diameter of 1.3 m, correct to 1 decimal place.



11 The end of a cylinder has a radius of 5 cm. Find the circumference of the end of the cylinder, correct to 2 decimal places.

12 A wheel of radius 30 cm is rolled in a straight line.



a Find the circumference of the wheel correct to 2 decimal places.

b How far, correct to 2 decimal places, has the wheel rolled after completing:

i 2 rotations?

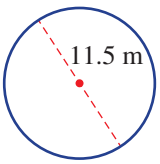
ii 10.5 rotations?

c Can you find how many rotations would be required to cover at least 1 km in length? Round to the nearest whole number.

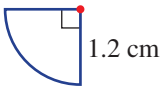
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 Give exact answers for the perimeter of these shapes. Express answers using fractions.

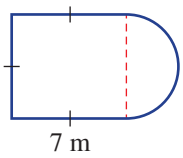
a



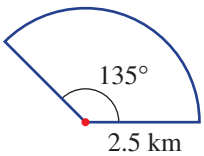
b



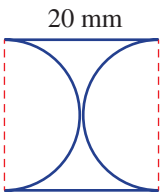
c



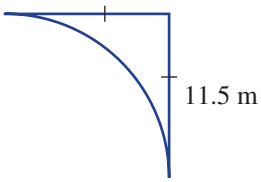
d



e



f



14 We know that the rule for the circumference of a circle is $C = 2\pi r$.

a Find a rule for r in terms of C.

b Find the radius of a circle to 1 decimal place if its circumference is:

i 10 cm **ii** 25 m

c Give the rule for the diameter of a circle in terms of its circumference C.

d After 1000 rotations a wheel has travelled 2.12 km. Find its diameter to the nearest centimetre.

ENRICHMENT — — 15

The ferris wheel

15 A large ferris wheel has a radius of 21 m. Round to 2 decimal places for these questions.

a Find the distance a person will travel on one rotation of the wheel.

b A ride includes 6 rotations of the wheel. What distance is travelled in one ride?

c How many rotations would be required to ride a distance of:

i 500 m?

ii 2 km?

d A ferris wheel has a sign that reads, 'One ride of 10 rotations will cover 2 km'. What must be the diameter of the wheel?



We can find the distance travelled during one rotation on a ferris wheel by calculating the circumference.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5C Area REVISION

Interactive

Widgets

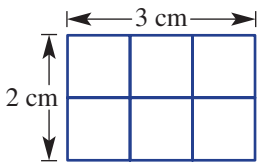
HOTsheets

Walkthrough

The number of square centimetres in this rectangle is 6; therefore the area is 6 cm^2.

A quicker way to find the number of squares is to note that there are two rows of three squares and hence the area is $2 \times 3 = 6$ cm^2. This leads to the formula $A = l \times b$ for the area of a rectangle.

For many common shapes, such as the parallelogram and trapezium, the rules for their area can be developed through consideration of simple rectangles and triangles. Shapes that involve circles or sectors rely on calculations involving pi (π).





How many tiles cover this area?

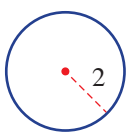
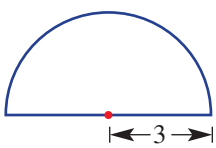
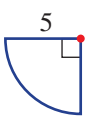
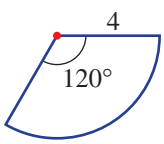
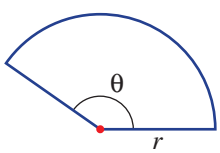
Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Formula for the area of a sector

We know that the area of a circle with radius r is given by the rule $A = \pi r^2$.

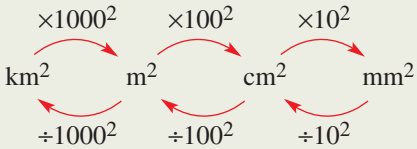
Complete this table of values to develop the rule for the area of a sector.

Shape	Fraction of full circle	Working and answer
	1	$A = \pi \times 2^2 \approx 12.57$ units^2
	$\frac{180}{360} =$	$A = \frac{1}{2} \times$
		$A =$
		$A =$
		$A =$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

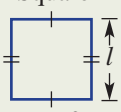
- Conversion of area units



$10^2 = 10 \times 10 = 100$
$100^2 = 100 \times 100 = 10\,000$
$1000^2 = 1000 \times 1000 = 1\,000\,000$

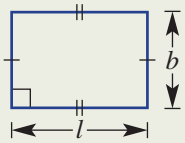
- The area of a two-dimensional shape is a measure of the space enclosed within its boundaries.

Square



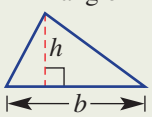
$A = l^2$

Rectangle



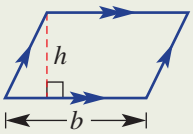
$A = lb$

Triangle



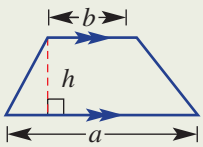
$A = \frac{1}{2}bh$

Parallelogram/Rhombus



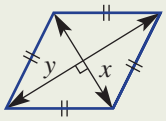
$A = bh$

Trapezium



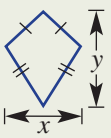
$A = \frac{1}{2}h(a + b)$

Rhombus



$A = \frac{1}{2}xy$

Kite



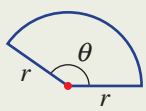
$A = \frac{1}{2}xy$

Circle



$A = \pi r^2$

Sector



$A = \frac{\theta}{360}\pi r^2$

Example 5 Converting units of area

Convert the following area measurements into the units given in the brackets.

a 859 mm² (cm²)

b 2.37 m² (cm²)

SOLUTION

a $859 \text{ mm}^2 = 859 \div 10^2 \text{ cm}^2$
$= 8.59 \text{ cm}^2$

b $2.37 \text{ m}^2 = 2.37 \times 100^2 \text{ cm}^2$
$= 23\,700 \text{ cm}^2$

EXPLANATION

cm² ← mm² ÷10² = 100

859.

×100² = 10 000

m² → cm²

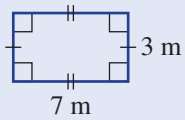
2.3700

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

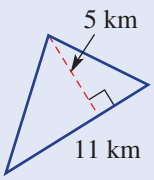
Example 6 Finding areas of rectangles, triangles and parallelograms

Find the area of each of the following plane figures.

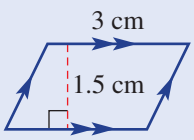
a



b



c

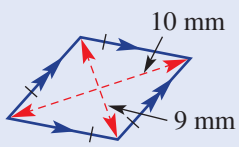


SOLUTION	EXPLANATION
a $A = l \times b$ $= 7 \times 3$ $= 21\,\text{m}^2$	Use the area formula for a rectangle. Substitute $l = 7$ and $b = 3$. Include the correct units.
b $A = \frac{1}{2} \times b \times h$	Use the area formula for a triangle.
$= \frac{1}{2} \times 11 \times 5$	Substitute $b = 11$ and $h = 5$.
$= 27.5\,\text{km}^2$	Include the correct units.
c $A = b \times h$ $= 3 \times 1.5$ $= 4.5\,\text{cm}^2$	Use the area formula for a parallelogram. Multiply the base length by the perpendicular height.

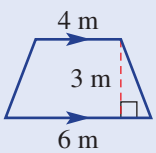
Example 7 Finding areas of rhombuses and trapeziums

Find the area of each of the following plane figures.

a



b



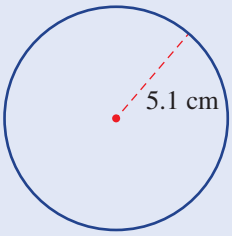
SOLUTION	EXPLANATION
a $A = \frac{1}{2} \times x \times y$	Use the area formula for a rhombus.
$= \frac{1}{2} \times 10 \times 9$	Substitute $x = 10$ and $y = 9$.
$= 45\,\text{mm}^2$	Include the correct units.
b $A = \frac{1}{2} \times h \times (a + b)$	Use the area formula for a trapezium.
$= \frac{1}{2} \times 3 \times (4 + 6)$	Substitute $a = 4$, $b = 6$ and $h = 3$.
$= 15\,\text{m}^2$	Include the correct units.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

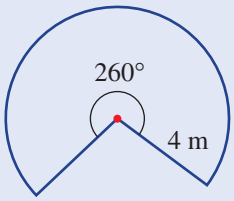
Example 8 Finding areas of circles and sectors

Find the area of this circle and sector correct to 2 decimal places.

a



b



SOLUTION

a $A = \pi r^2$
$= \pi \times (5.1)^2$
$= 81.71 \text{ cm}^2$ (to 2 d.p.)

b $A = \frac{\theta}{360} \times \pi r^2$
$= \frac{260}{360} \times \pi \times 4^2$
$= \frac{13}{18} \times \pi \times 16$
$= 36.30 \text{ m}^2$ (to 2 d.p.)

EXPLANATION

Write the rule and substitute $r = 5.1$.
81.7128 ... rounds to 81.71 since the third decimal place is 2.

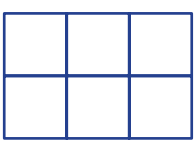
Use the sector formula.
The fraction of the full circle is $\frac{260}{360} = \frac{13}{18}$, so multiply this by πr^2 to get the sector area.
$\frac{104\pi}{9} \text{ m}^2$ would be the exact answer.

Exercise 5C REVISION

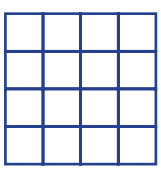
UNDERSTANDING AND FLUENCY	1–9	2, 4–9(½)	4–9(½)

1 Find the area of these shapes. Each square in each shape represents one square unit.

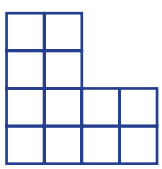
a



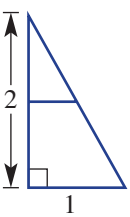
b



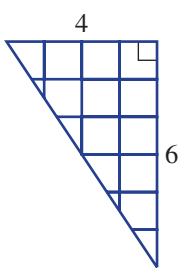
c



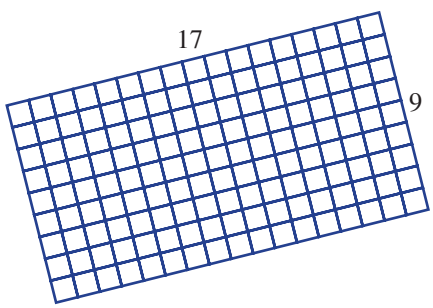
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Name the shape that has the given area formula.

a $A = lb$ **b** $A = \pi r^2$ **c** $A = \frac{1}{2}xy$ (2 shapes)

d $A = \frac{\theta}{360} \times \pi r^2$ **e** $A = \frac{1}{2}bh$ **f** $A = \frac{1}{2}h(a + b)$

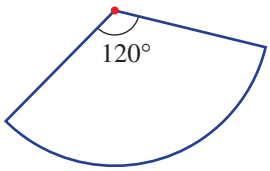
g $A = bh$ **h** $A = l^2$ **c** $A = \frac{1}{2}\pi r^2$

3 What fraction of a full circle is shown by these sectors? Simplify your fraction.

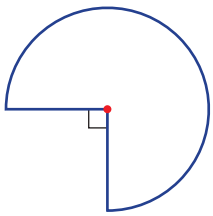
a



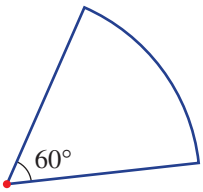
b



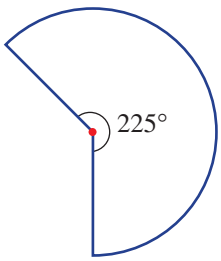
c



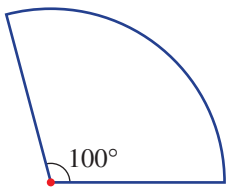
d



e



f



Example 5

4 Convert the following area measurements into the units given in the brackets.

a 2 cm^2 (mm^2)

b 500 mm^2 (cm^2)

c 2.1 m^2 (cm^2)

d 210 000 cm^2 (m^2)

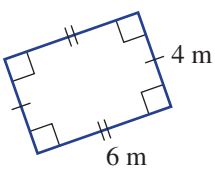
e 0.001 km^2 (m^2)

f 3 200 000 m^2 (km^2)

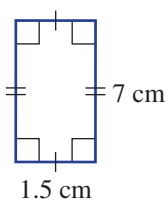
Example 6

5 Find the area of each of the following plane figures.

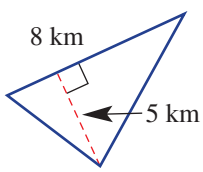
a



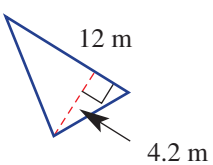
b



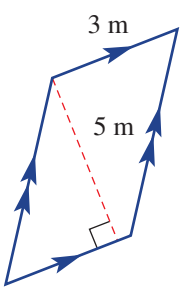
c



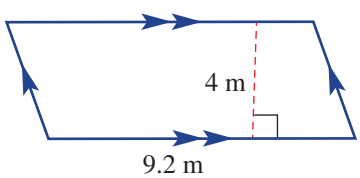
d



e



f

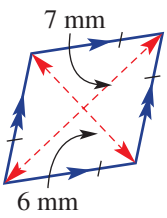


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

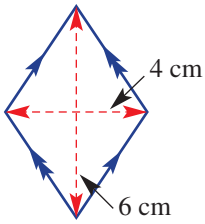
Example 7

6 Find the area of each of the following plane figures.

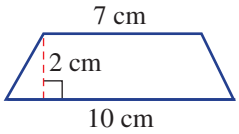
a



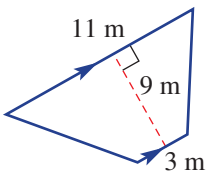
b



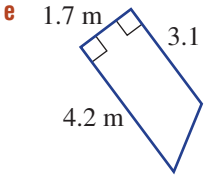
c



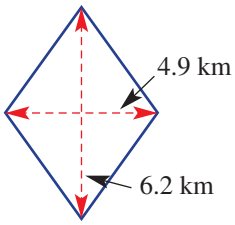
d



e

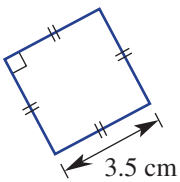


f

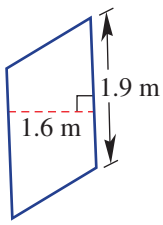


7 Find the area of each of the following mixed-plane figures.

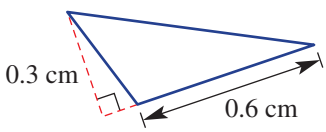
a



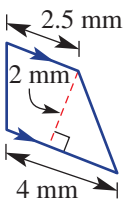
b



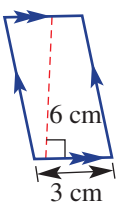
c



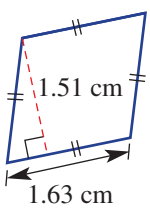
d



e



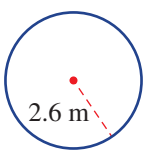
f



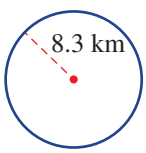
Example 8a

8 Find the area of these circles using the given value for pi (π). Round to 2 decimal places.

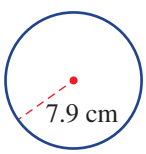
a $\pi = 3.14$



b $\pi = \frac{22}{7}$



c π (from calculator)

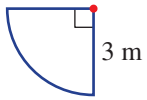


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

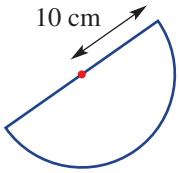
Example 8b

9 Find the area of these sectors, rounding to 2 decimal places.

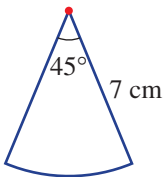
a



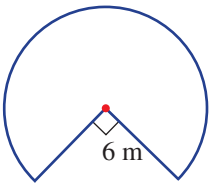
b



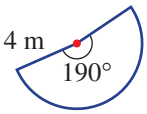
c



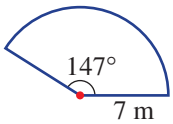
d



e



f



PROBLEM-SOLVING AND REASONING 10–12, 16 | 10, 11, 13, 14, 16 | 10, 13–17

10 Convert the following measurements into the units given in the brackets.

a 1.5 km^2 (cm^2) **b** 0.000005 m^2 (mm^2) **c** 75 000 mm^2 (m^2)

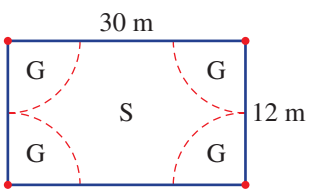
11 A valuer tells you that your piece of land has an area of half a square kilometre (0.5 km^2). How many square metres (m^2) do you own?

12 A rectangular park covers an area of 175 000 m^2. Give the area of the park in km^2.

13 An old picture frame that was once square now leans to one side to form a rhombus. If the distances between pairs of opposite corners are 85 cm and 1.2 m, find the area enclosed within the frame in m^2.

14 A pizza shop is considering increasing the diameter of its family pizza tray from 32 cm to 34 cm. Find the percentage increase in area, correct to 2 decimal places, from the 32 cm tray to the 34 cm tray.

15 A tennis court area is illuminated by four corner lights. The illumination of the sector area close to each light is considered to be good (G) while the remaining area is considered to be lit satisfactorily (S).



What percentage of the area is considered good? Round to the nearest per cent.

16 The rule for the area of a circle is given by $A = \pi r^2$.

a Rearrange this rule to find a rule for r in terms of A.

b Find the radius of a circle with the given areas. Round to 1 decimal place.

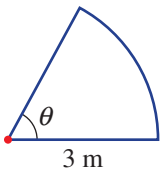
i 5 cm^2

ii 6.9 m^2

iii 20 km^2

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

17 A sector has a radius of 3 m.



a Find the angle θ, correct to the nearest degree, if its area is:

i 5 m^2

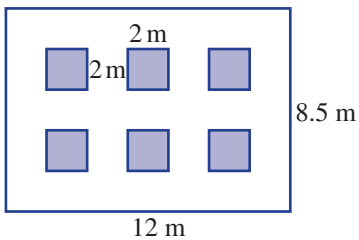
ii 25 m^2

b Explain why the area of the sector could not be 30 m^2.

ENRICHMENT — — 18, 19

Windows

18 Six square windows of side length 2 m are to be placed into a 12 m wide by 8.5 m high wall as shown. The windows are to be positioned so that the vertical spacing between the windows and the wall edges are equal. Similarly, the horizontal spacings are also equal.



a **i** Find the horizontal distance between the windows.

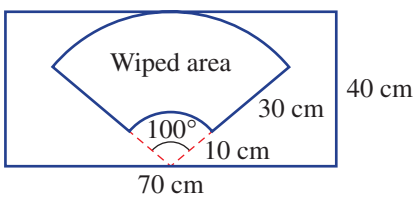
ii Find the vertical distance between the windows.

b Find the area of the wall not including the window spaces.

c If the wall included 3 rows of 4 windows (instead of 2 rows of 3), investigate if it would be possible to space all the windows so that the horizontal and vertical spacings are uniform (although not necessarily equal to each other).

19 A rectangular window is wiped by a wiper blade forming the given sector shape.

What percentage area is cleaned by the wiper blade? Round to 1 decimal place.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5D Perimeter and area of composite shapes

Interactive
Widgets
HOTsheets
Walkthrough

A composite shape can be thought of as a combination of more simplistic shapes such as triangles and rectangles. Finding perimeters and areas of such shapes is a matter of identifying the more basic shapes they consist of and combining any calculations in an organised fashion.

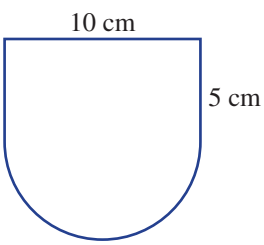
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



The area of glass in this glass structure can be calculated using the area of trapeziums, triangles and rectangles.

Let's start: Incorrect layout

Three students write their solution to finding the area of this shape on the board. Each student's working is shown in the table below.



Chris	Matt	Moira
$A = l \times b$ $= 50 + \frac{1}{2}\pi r^2$ $= \frac{1}{2}\pi \times 5^2$ $= 39.27 + 50$ $= 89.27 \text{ cm}^2$	$A = \frac{1}{2}\pi \times 5^2$ $= 39.27 + 10 \times 5$ $= 89.27 \text{ cm}^2$	$A = l \times b + \frac{1}{2}\pi r^2$ $= 10 \times 5 + \frac{1}{2}\pi \times 5^2$ $= 89.27 \text{ cm}^2$

- All three students have the correct answer but only one student receives full marks. Who is it?
- Explain what is wrong with the layout of the other two solutions.

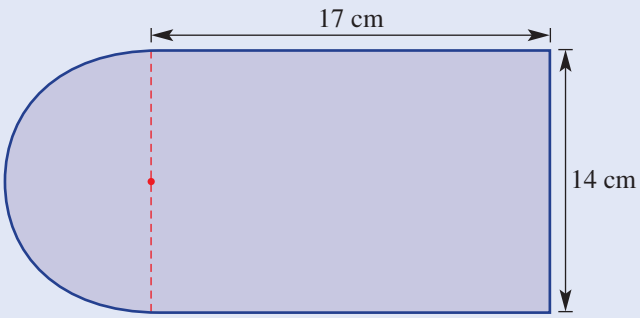
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- **Composite shapes** are made up of more than one basic shape.
- Addition and/or subtraction can be used to find areas and perimeters of composite shapes.
- The layout of the relevant mathematical working needs to make sense so that the reader of your work understands each step.

Example 9 Finding perimeters and areas of composite shapes

Find the perimeter and area of this composite shape, rounding answers to 2 decimal places.



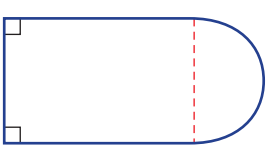
SOLUTION	EXPLANATION
$P = 2 \times l + b + \frac{1}{2} \times 2\pi r$	3 straight sides + semicircle
$= 2 \times 17 + 14 + \frac{1}{2} \times 2\pi \times 7$	Substitute $l = 17$, $b = 14$ and $r = 7$.
$= 34 + 14 + \pi \times 7$	Simplify.
$= 69.99$ cm (to 2 d.p)	Calculate and round to 2 decimal places.
$A = l \times b + \frac{1}{2}\pi r^2$	Area of rectangle + area of semicircle
$= 17 \times 14 + \frac{1}{2} \times \pi \times 7^2$	Substitute $l = 17$, $b = 14$ and $r = 7$.
$= 238 + \frac{1}{2} \times \pi \times 49$	Simplify.
$= 314.97$ cm^2 (to 2 d.p)	Calculate and round to 2 decimal places.

Exercise 5D

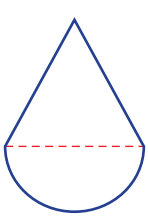
UNDERSTANDING AND FLUENCY	1–6	2, 3–7(½)	3–7(½)

1 Name the two different shapes that make up these composite shapes, e.g. square and semicircle.

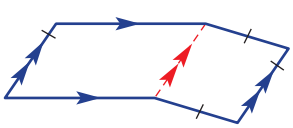
a



b



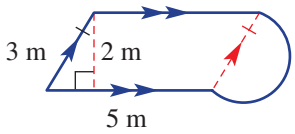
c



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Copy and complete the working to find the perimeter and area of these composite shapes. Round to 1 decimal place.

a



$P = 2 \times ___ + 3 + \frac{1}{2} \times 2\pi r$

$= ____ + 3 + ____$

$= _____$

$= ____$ m

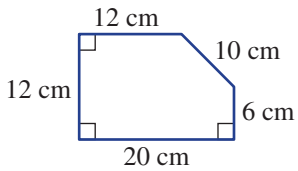
$A = bh + \frac{1}{2}____$

$= 5 \times ___ + \frac{1}{2} \times ____$

$= ____ + ____$

$= ____$ m^2

b



$P = 20 + 12 + ___ + ___ + ___$

$= ____$ cm

$A = lb - \frac{1}{2}bh$

$= 12 \times ___ - \frac{1}{2} \times 8 \times ____$

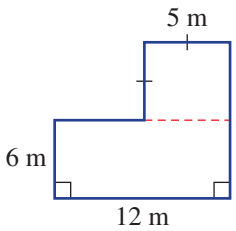
$= ____ - ____$

$= ____$ cm^2

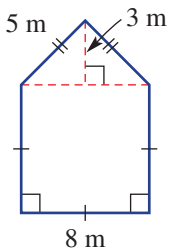
Example 9

3 Find the perimeter and the area of each of these simple composite shapes, rounding answers to 2 decimal places where necessary.

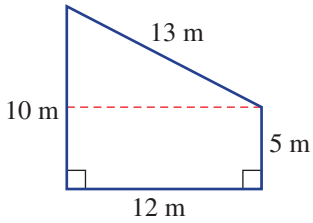
a



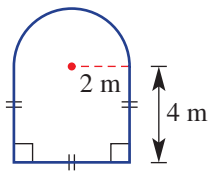
b



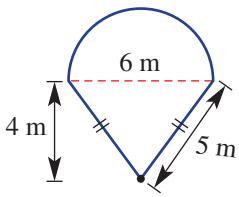
c



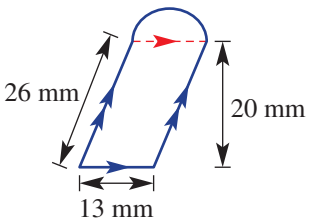
d



e

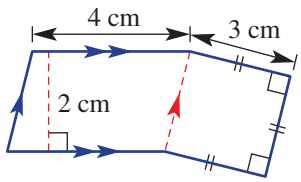


f

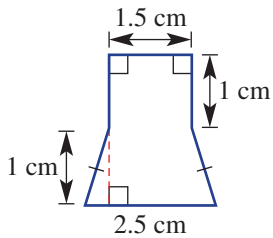


4 Find the area of each of the following composite shapes.

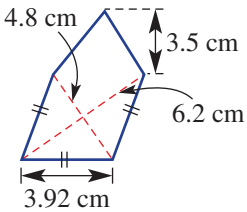
a



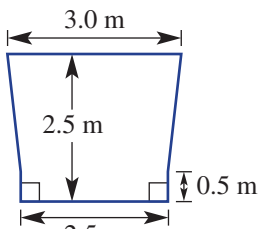
b



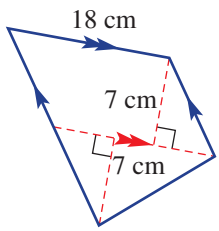
c



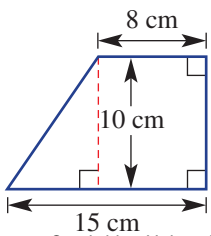
d



e



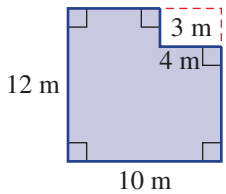
f



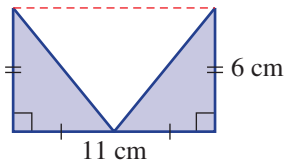
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.
Updated March 2022

5 Find the area of each of these composite shapes. *Hint*: use subtraction.

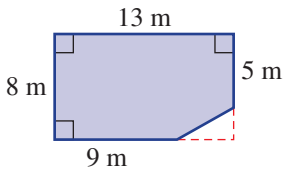
a



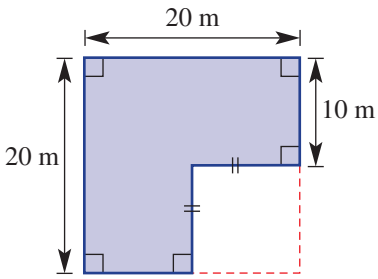
b



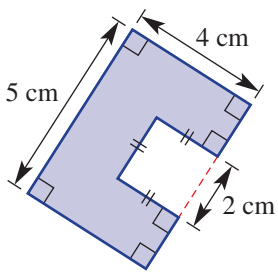
c



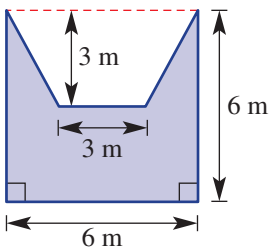
d



e

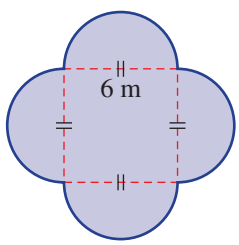


f

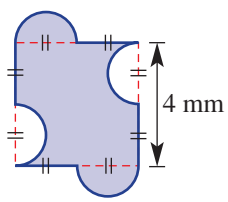


6 Find the perimeter and the area of each of the following composite shapes correct to 2 decimal places where necessary.

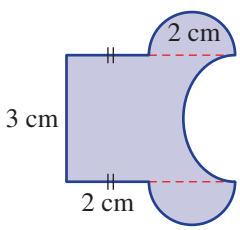
a



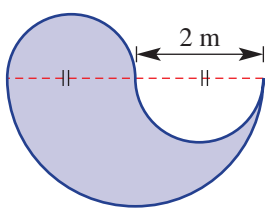
b



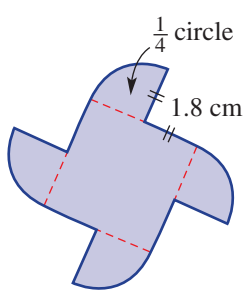
c



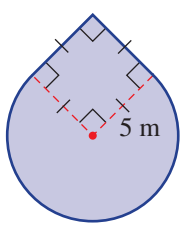
d



e



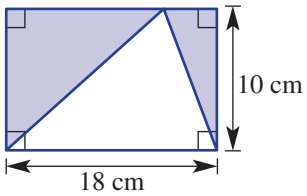
f



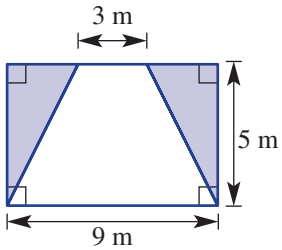
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Find the area of the shaded region of each of the following shapes by subtracting the area of the clear shape from the total area. Round to 1 decimal place where necessary.

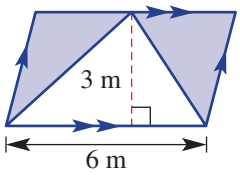
a



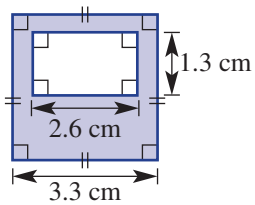
b



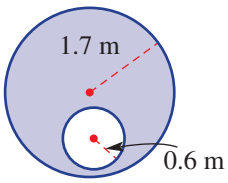
c



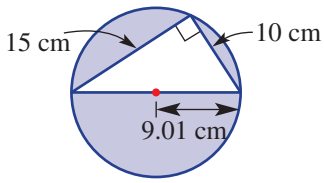
d



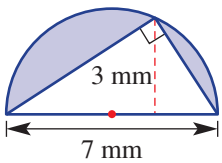
e



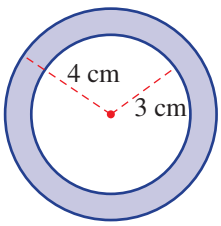
f



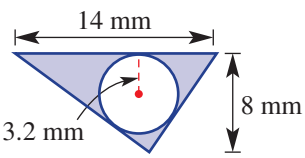
g



h



i



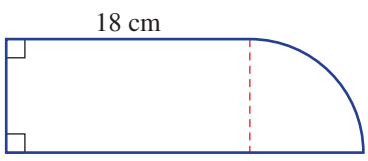
PROBLEM-SOLVING AND REASONING 8, 9, 12 | 9, 10, 12, 13 | 10, 11, 12(½), 13, 14

8 An area of lawn is made up of a rectangle measuring 10 m by 15 m and a semicircle of radius 5 m. Find the total area of lawn correct to 2 decimal places.

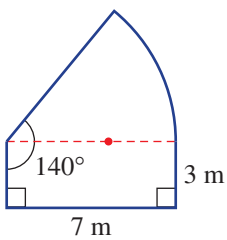
9 Twenty circular pieces of pastry, each of diameter 4 cm, are cut from a rectangular layer of pastry 20 cm long and 16 cm wide. What is the area, correct to 2 decimal places, of pastry remaining after the 20 pieces are removed?

10 These shapes include sectors. Find their area to 1 decimal place.

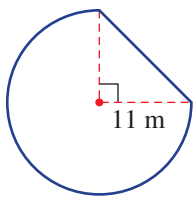
a



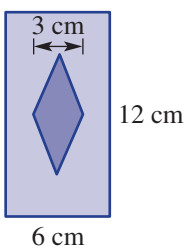
b



c

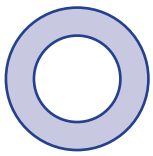


11 A new car manufacturer is designing a logo. It is in the shape of a diamond inside a rectangle. The diamond is to have a horizontal width of 3 cm and an area equal to one sixth of the area of the rectangle. Find the required height of the diamond.



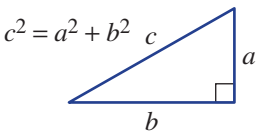
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 Using exact values (e.g.10 + 4π), find the area of the shapes given in Question **6**.

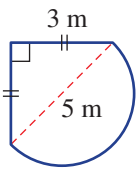


13 A circle of radius 10 cm has a hole cut out of its centre to form a ring. Find the radius of the hole if the remaining area is 50% of the original area. Round to 1 decimal place.

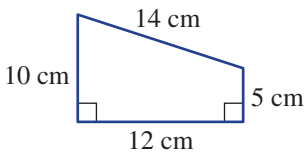
14 Use Pythagoras' theorem (illustrated in this diagram) to help explain why these composite shapes include incorrect information.



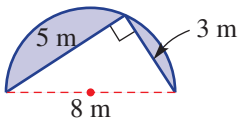
a



b



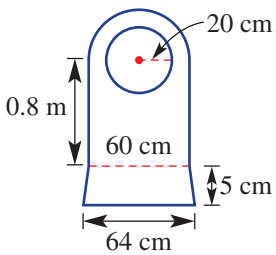
c



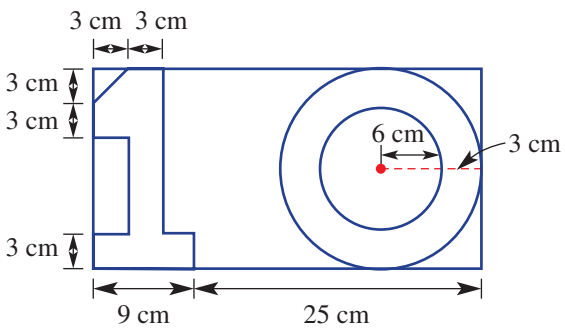
ENRICHMENT	—	—	15, 16

Construction cut-outs

15 The front of a grandfather clock consists of a timber board with dimensions as shown. A circle of radius 20 cm is cut from the board to form the clock face. Find the remaining area of the timber board correct to 1 decimal place.



16 The number 10 is cut from a rectangular piece of paper. The dimensions of the design are shown below.



a Find the length and width of the rectangular piece of paper shown.

b Find the sum of the areas of the two cut-out digits, 1 and 0, correct to 1 decimal place.

c Find the area of paper remaining after the digits have been removed (include the centre of the '0' in your answer) and round to 1 decimal place.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5E Surface area of prisms and pyramids

Interactive

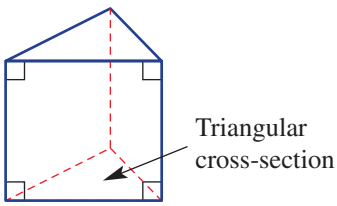
Widgets

HOTsheets

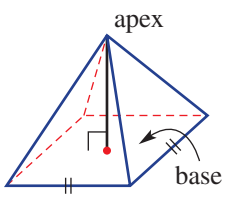
Walkthrough

Three-dimensional objects or solids have outside surfaces that together form the total surface area. Nets are very helpful for determining the number and shape of the surfaces of a three-dimensional object.

For this section we will deal with right prisms and pyramids. A right prism has a uniform cross-section with two identical ends and the remaining sides are rectangles. A right pyramid has its apex sitting above the centre of its base.



Right triangular prism



Right square-based pyramid

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

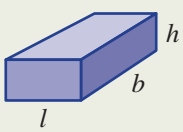
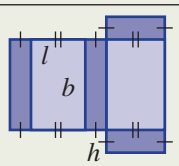
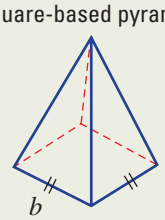
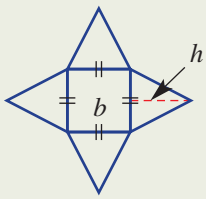
Let's start: Drawing prisms and pyramids

Prisms are named by the shape of their cross-section and pyramids are named by the shape of their base.

- Try to draw as many different right prisms and pyramids as you can.
- Describe the different kinds of shapes that make up the surface of your solids.
- Which solids are the most difficult to draw and why?

Key ideas

- The **surface area** (A) of a solid is the sum of the areas of all the surfaces.
- A **net** is a two-dimensional illustration of all the surfaces of a solid.
- A right **prism** is a solid with a uniform cross-section and remaining sides are rectangles.
  - They are named by the shape of their cross-section.
- The nets for a **rectangular prism** (**cuboid**) and square-based **pyramid** are shown here.

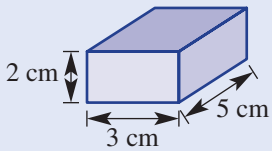
Solid	Net	Surface area
Rectangular prism 		$A = 2(lb + lh + hb)$
Square-based pyramid 		$A = b^2 + 4\left(\frac{1}{2}bh\right)$ $= b^2 + 2bh$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

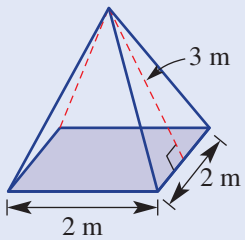
Example 10 Calculating surface area

Find the surface area of each of the following solid objects.

a



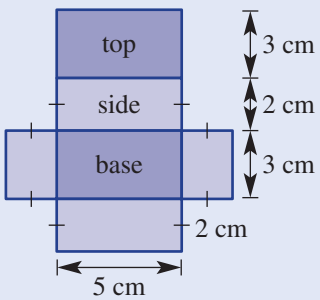
b



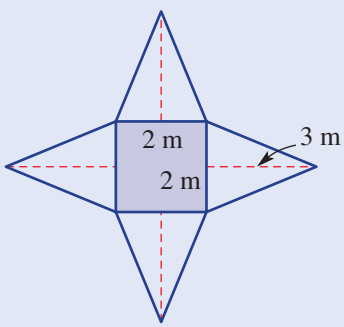
SOLUTION

a $A = 2 \times (5 \times 3) + 2 \times (5 \times 2) + 2 \times (2 \times 3)$
$= 30 + 20 + 12$
$= 62 \text{ cm}^2$

EXPLANATION



b $A = 2 \times 2 + 4 \times \frac{1}{2} \times 2 \times 3$
$= 4 + 12$
$= 16 \text{ m}^2$

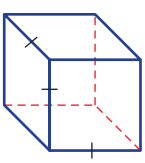


Exercise 5E

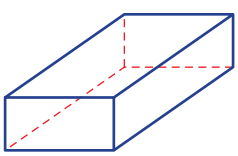
UNDERSTANDING AND FLUENCY	1–4, 6	2, 3–5(½), 6	3–5(½), 6

1 Draw a suitable net for these prisms and pyramids and name each solid.

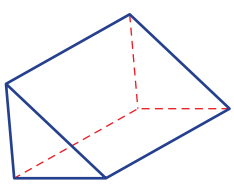
a



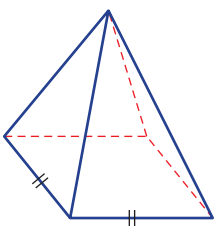
b



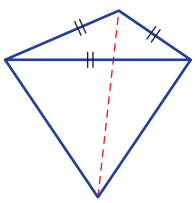
c



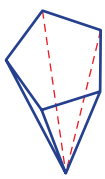
d



e



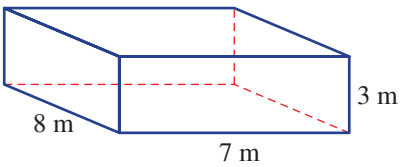
f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Copy and complete the working to find the surface area of these solids.

a

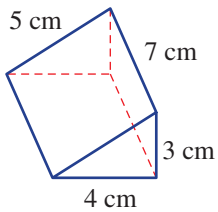


$A = 2 \times 8 \times 7 + 2 \times 8 \times __ + 2 \times __ \times __$

$= ___ + ___ + ___$

$= ____ \text{ m}^2$

b



$A = 2 \times \frac{1}{2} \times 4 \times __ + 5 \times 7 + 4 \times __ + __ \times __$

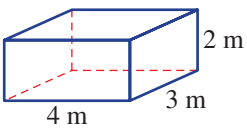
$= ___ + ___ + ___ + ___$

$= ____ \text{cm}^2$

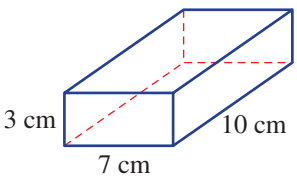
Example 10a

3 Find the surface area of the following rectangular prisms. Draw a net of the solid to help you.

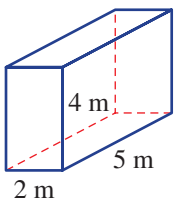
a



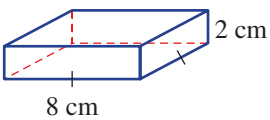
b



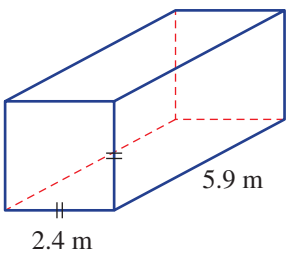
c



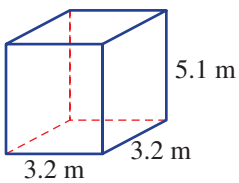
d



e



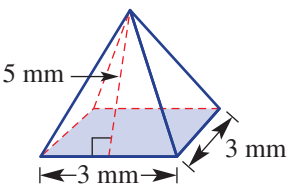
f



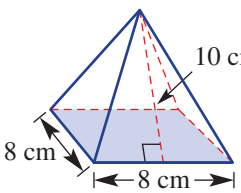
Example 10b

4 Find the surface area of each of these pyramids. Draw a net of the solid to help you.

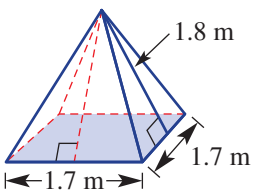
a



b

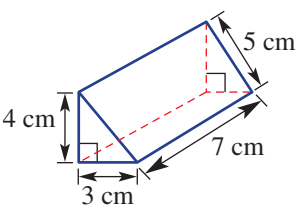


c

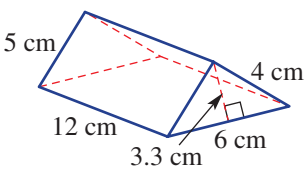


5 Find the surface area of each of the following solid objects.

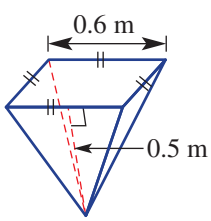
a



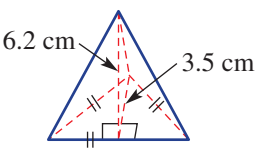
b



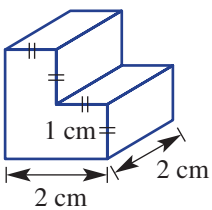
c



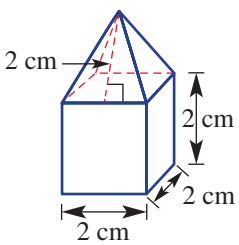
d



e



f



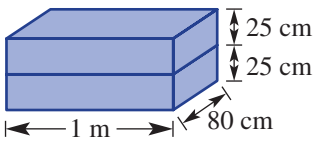
6 Find the surface area of a cube of side length 1 metre.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING 7, 8, 11 | 8, 9, 11 | 9–12

7 A rectangular box is to be covered in material. How much is required to cover the entire box if it has the dimensions 1.3 m, 1.5 m and 1.9 m?

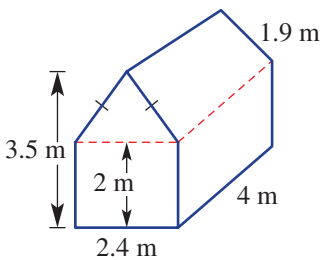
8 Two wooden boxes, both with dimensions 80 cm, 1 m and 25 cm, are placed on the ground, one on top of the other as shown. The entire outside surface is then painted. Find the area of the painted surface.



9 The four walls and roof of a barn (shown) are to be painted.

a Find the surface area of the barn, not including the floor.

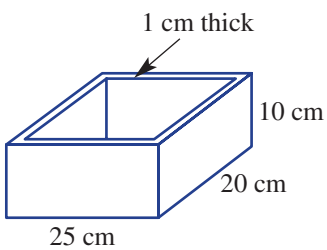
b If 1 litre of paint covers 10 m^2, find how many litres are required to complete the job.



10 An open top rectangular box 20 cm wide, 25 cm long and 10 cm high is made from wood 1 cm thick. Find the surface area:

a outside the box (do not include the top edge)

b inside the box (do not include the top edge)



11 Draw the stack of 1 cm cube blocks that gives the minimum outside surface area and state this surface area if there are:

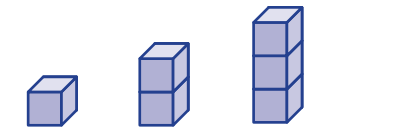
a 2 blocks

b 4 blocks

c 8 blocks

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 Cubes of side length one unit are stacked as shown.



a Complete this table.

Number of cubes (n)	1	2	3	4	5	6	7	8	9
Surface area (A)									

b Can you find the rule for the surface area (A) for n cubes stacked in this way? Write down the rule for A in terms of n.

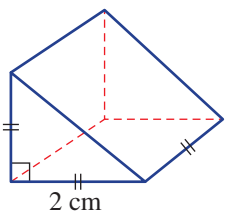
c Investigate other ways of stacking cubes and look for rules for surface area in terms of n, the number of cubes.

ENRICHMENT — — 13

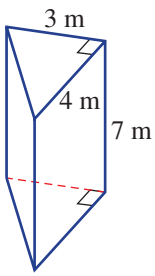
Pythagoras required

13 For prisms and pyramids involving triangles, Pythagoras' theorem ($c^2 = a^2 + b^2$) can be used. Apply the theorem to help find the surface area of these solids. Round to 1 decimal place.

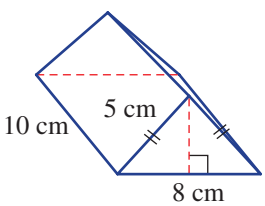
a



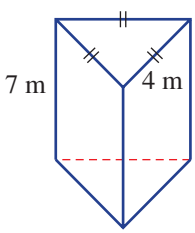
b



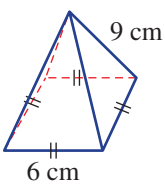
c



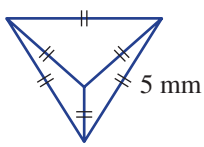
d



e



f



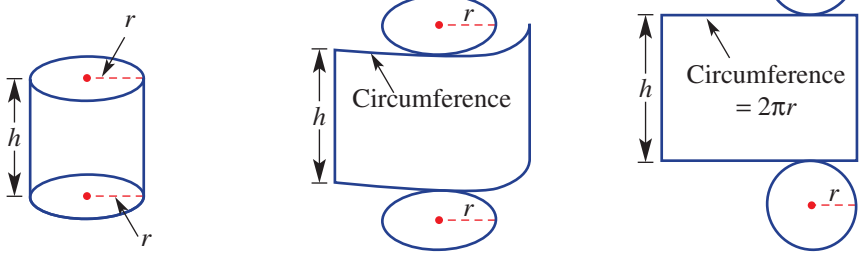
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5F Surface area of cylinders

Interactive
Widgets
HOTsheets
Walkthrough

The net of a cylinder includes two circles and one rectangle. The length of the rectangle is equal to the circumference of the circle.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



Let's start: Curved area

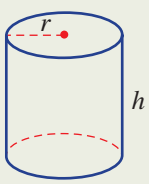
- Roll a piece of paper to form the curved surface of a cylinder.
- Do not stick the ends together so you can allow the paper to return to a flat surface.
- What shape is the paper when lying flat on a table?
- When curved to form the cylinder, what do the sides of the rectangle represent on the cylinder? How does this help to find the surface area of a cylinder?





These pipes are hollow cylinders, like the rolled-up paper sheets.

■ Surface area of a **cylinder** = 2 circles + 1 rectangle
$= 2 \times \pi r^2 + 2\pi r \times h$
$\therefore A = 2\pi r^2 + 2\pi rh$
2 circular ends — curved area
or $A = 2\pi r(r + h)$

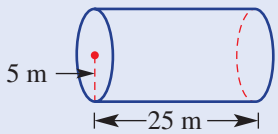


Key ideas

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11 Finding the surface area of a cylinder

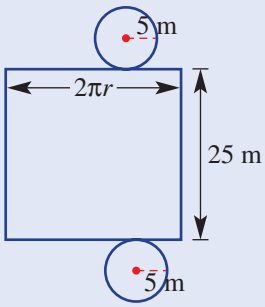
Find the surface area of this cylinder, rounding to 2 decimal places.



SOLUTION

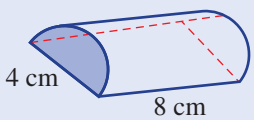
$A = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 5^2 + 2\pi \times 5 \times 25$
$= 50 \times \pi + 250 \times \pi$
$= 50\pi + 250\pi$
$= 300\pi$
$= 942.48 \text{ m}^2$ (to 2 d.p.)

EXPLANATION



Example 12 Finding surface areas of cylindrical portions

Find the surface area of this half-cylinder, rounding to 2 decimal places.

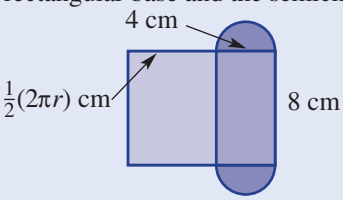


SOLUTION

$A = 2\left(\frac{1}{2}\pi r^2\right) + \frac{1}{2}(2\pi r) \times 8 + 4 \times 8$
$= 2 \times \frac{1}{2}\pi \times 2^2 + \frac{1}{2} \times 2 \times \pi \times 2 \times 8 + 32$
$= 20\pi + 32$
$= 94.83 \text{ cm}^2$ (to 2 d.p.)

EXPLANATION

In addition to the curved surface, include the rectangular base and the semicircular ends.

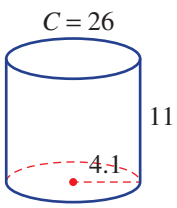


Exercise 5F

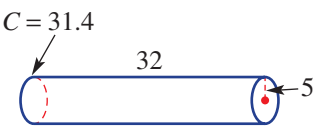
UNDERSTANDING AND FLUENCY	1–5	2–6	3(c), 4(c), 5, 6

1 Draw a net suited to these cylinders. Label the sides using the given measurements.

a



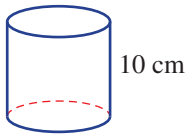
b



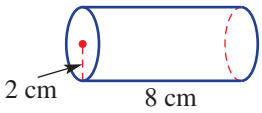
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 The curved surface of these cylinders is allowed to flatten out to form a rectangle. What would be the length and breadth of this rectangle in each cylinder? Round to 2 decimal places where necessary.

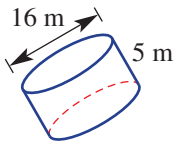
a



b



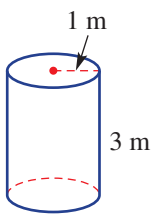
c



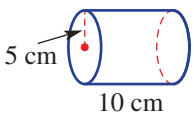
Example 11

3 Find the surface area of these cylinders, rounding to 2 decimal places. Use a net to help.

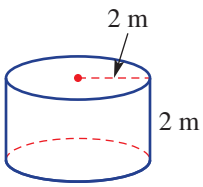
a



b

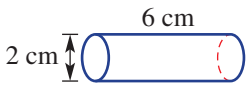


c

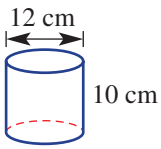


4 Find the surface area of these cylinders, rounding to 1 decimal place. Remember that the radius of a circle is half the diameter.

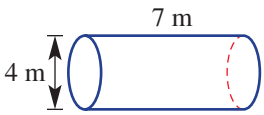
a



b



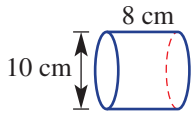
c



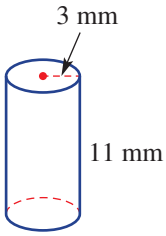
5 Find the surface area of a cylindrical plastic container of height 18 cm and with a circle of radius 3 cm at each end, correct to 2 decimal places.

6 Find the area of the curved surface only for these cylinders, correct to 2 decimal places.

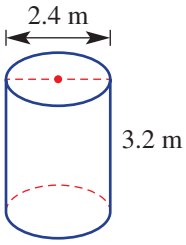
a



b



c

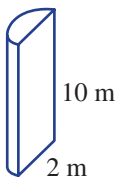


PROBLEM-SOLVING AND REASONING 7(½), 8, 11 | 7(½), 8, 9, 11, 12 | 7(½), 9–12

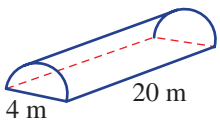
Example 12

7 Find the surface area of these solids, rounding to 2 decimal places.

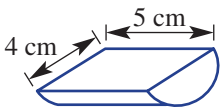
a



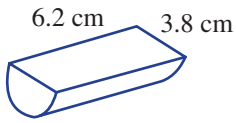
b



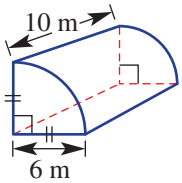
c



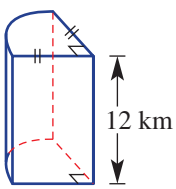
d



e

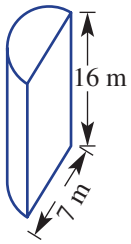


f

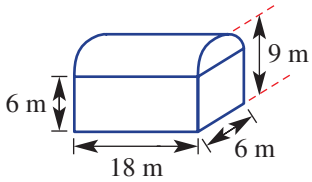


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

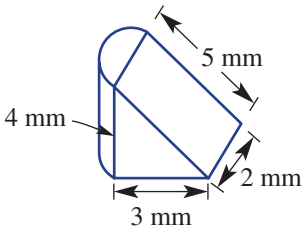
g



h

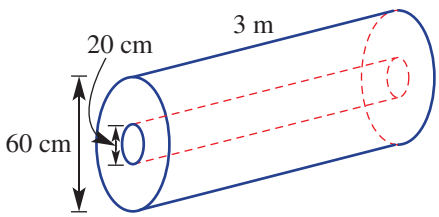


i

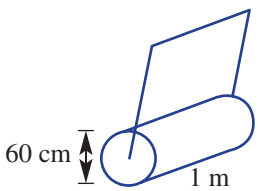


8 A water trough is in the shape of a half-cylinder. Its semicircular ends have diameter 40 cm and the trough length is 1 m. Find the outside surface area in cm^2 of the curved surface plus the two semicircular ends, correct to 2 decimal places.

9 A log with diameter 60 cm is 3 m in length. Its hollow centre is 20 cm in diameter.
Find the surface area of the log in cm^2, including the ends and the inside, correct to 1 decimal place.



10 A cylindrical roller is used to press crushed rock in preparation for a tennis court. The rectangular tennis court area is 30 m long and 15 m wide. The roller has a breadth of 1 m and diameter 60 cm.



a Find the surface area of the curved part of the roller in cm^2 correct to 3 decimal places.

b Find the area, in m^2 to 2 decimal places, of crushed rock that can be pressed after:

i 1 revolution **ii** 20 revolutions

c Find the minimum number of complete revolutions required to press the entire tennis court area.

11 It is more precise to give exact values for calculations involving π, e.g. 24π.
Give the exact answers to the surface area of the cylinders in Question **3**.

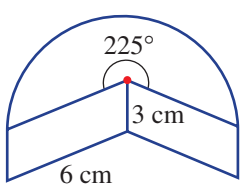
12 A cylinder cut in half gives half the volume but not half the surface area. Explain why.

ENRICHMENT — — 13

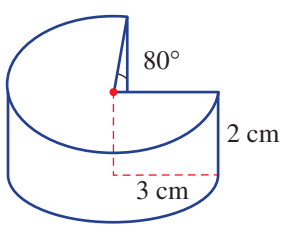
Solid sectors

13 The sector area rule $A = \frac{\theta}{360} \times \pi r^2$ can be applied to find the surface areas of solids that have ends that are sectors. Find the exact surface area of these solids.

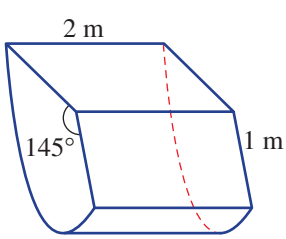
a



b



c



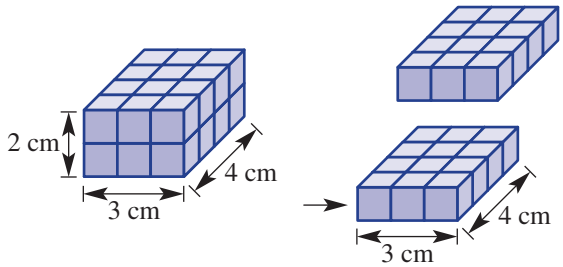
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5G Volume of prisms

Interactive
Widgets
HOTsheets
Walkthrough

Volume is the number of cubic units contained within a three-dimensional object.

To find the volume we can count 24 cubic centimetres (24 cm^3) *or* multiply $3 \times 4 \times 2 = 24\ cm^3$.



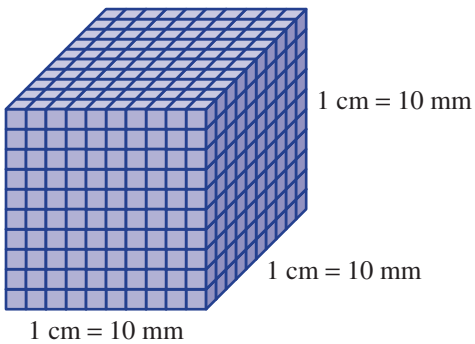
Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

We can see that the area of the base ($3 \times 4 = 12\ cm^2$) also gives the volume of the base layer 1 cm high. The number of layers equals the height, hence, multiplying the area of the base by the height will give the volume.

This idea can be applied to all right prisms provided a uniform cross-section can be identified. In such solids, the height length used to calculate the volume is the length of the edge running perpendicular to the base or cross-section.

Let's start: Cubic units



Consider this 1 cm cube divided into cubic millimetres.

- How many cubic mm sit on one edge of the 1 cm cube?
- How many cubic mm sit on one layer of the 1 cm cube?
- How many layers are there?
- How many cubic mm are there in total in the 1 cm cube?
- Complete this statement $1\ cm^3 =$ _____ mm^3
- Explain how you can find how many:
  - **a** cm^3 in $1\ m^3$
  - **b** m^3 in $1\ km^3$

Key ideas

- Common metric units for **volume** include **cubic kilometres** (km^3), **cubic metres** (m^3), **cubic centimetres** (cm^3) and **cubic millimetres** (mm^3).

$km^3 \xrightarrow{\times 1000^3} m^3 \xrightarrow{\times 100^3} cm^3 \xrightarrow{\times 10^3} mm^3$

$km^3 \xleftarrow{\div 1000^3} m^3 \xleftarrow{\div 100^3} cm^3 \xleftarrow{\div 10^3} mm^3$

$1000^3 = 1\,000\,000\,000$
$100^3 = 1\,000\,000$
$10^3 = 1000$

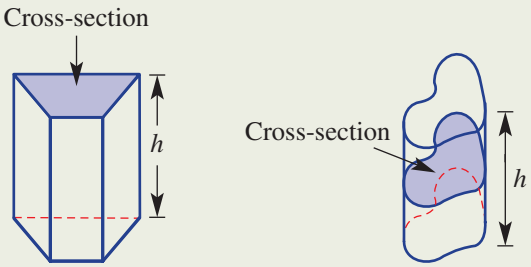
- For **capacity** common units include:
  - **megalitres** (ML) $1\ ML = 1000\ kL$
  - **kilolitres** (kL) $1\ kL = 1000\ L$
  - **litres** (L) $1\ L = 1000\ mL$
  - **millilitres** (mL)

Also $1\ cm^3 = 1\ mL$ so $1\ L = 1000\ cm^3$ and $1\ m^3 = 1000\ L$.

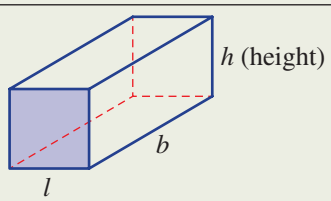
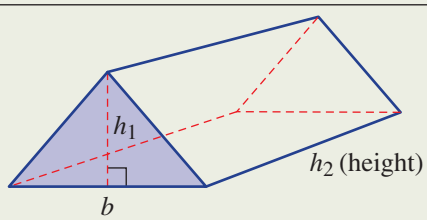
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Volume of solids with a uniform cross-section: $V =$ area of **cross-section** $(A) \times$ perpendicular height (h).
  $V = A \times h$
- The 'height' is the length of the edge that runs perpendicular to the cross-section.



- Some common formulas for volume include:

Rectangular prism (cuboid)	Triangular prism
 $V = A \times h$ $= l \times b \times h$ $= lbh$	 $V = A \times h$ $= \left(\frac{1}{2} \times b \times h_1\right) \times h_2$ $= \left(\frac{1}{2} bh_1\right) \times h_2$

Example 13 Converting units of volume

Convert the following volume measurements into the units given in the brackets.

a $2.5\ \text{m}^3\ (\text{cm}^3)$ **b** $458\ \text{mm}^3\ (\text{cm}^3)$

SOLUTION

a $2.5\ \text{m}^3 = 2.5 \times 100^3\ \text{cm}^3$
$= 2\,500\,000\ \text{cm}^3$

b $458\ \text{mm}^3 = 458 \div 10^3\ \text{cm}^3$
$= 0.458\ \text{cm}^3$

EXPLANATION

$\times 100^3 = 1\,000\,000$

m^3 cm^3 2.500000

cm^3 mm^3

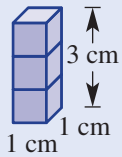
$\div 10^3 = 1000$ 458.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

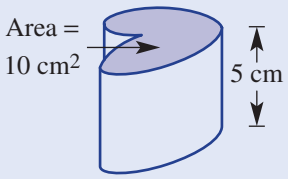
Example 14 Finding volumes of prisms and other solids

Find the volume of each of these three-dimensional objects.

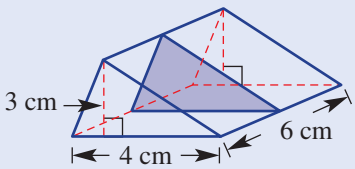
a



b



c



SOLUTION

a $V = l \times b \times h$
$= 1 \times 1 \times 3$
$= 3\ \text{cm}^3$

b V = area of cross-section × perp. height
$= 10 \times 5$
$= 50\ \text{cm}^3$

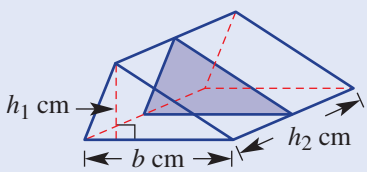
c V = area of cross-section × perp. height
$= \left(\frac{1}{2} \times b \times h_1\right) \times h_2$
$= \frac{1}{2}(4 \times 3) \times 6$
$= 36\ \text{cm}^2$

EXPLANATION

The solid is a rectangular prism.
Length = 1 cm, breadth = 1 cm and height = 3 cm

Substitute cross-sectional area = 10 and height = 5.

The cross-section is a triangle.

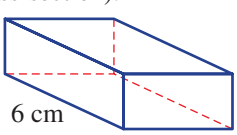


Exercise 5G

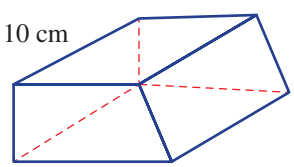
UNDERSTANDING AND FLUENCY	1–4, 5(½), 6, 7	2, 3(½), 4, 5(½), 6–8	3(½), 5(½), 6, 8

1 Draw the cross-sectional shape for these prisms and state the 'height' (perpendicular to the cross-section).

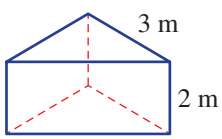
a



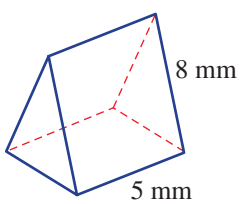
b



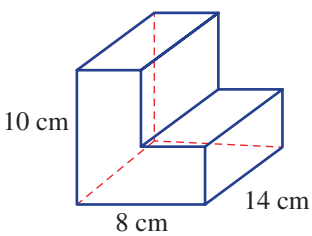
c



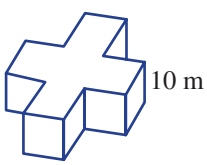
d



e



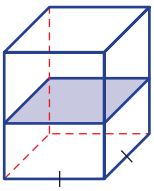
f



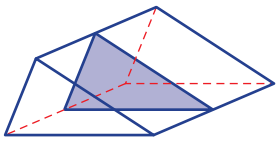
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 What is the name given to the shape of the shaded cross-section of each of the following solids?

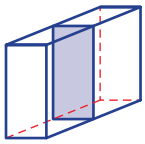
a



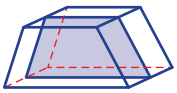
b



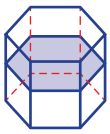
c



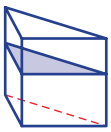
d



e



f



Example 13

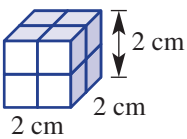
3 Convert the following volume measurements into the units given in the brackets.

a 3 cm^3 (mm^3)
b 2000 mm^3 (cm^3)
c 8.7 m^3 (cm^3)
d 5900 cm^3 (m^3)
e 0.00001 km^3 (m^3)
f 21700 m^3 (km^3)
g 3 L (mL)
h 0.2 kL (L)
i 3500 mL (L)
j 0.021 L (mL)
k 37000 L (kL)
l 42900 kL (ML)

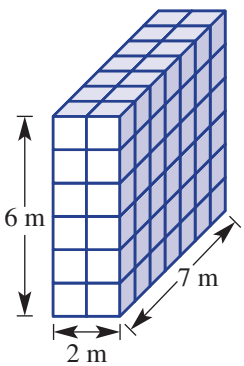
Example 14a

4 Find the volumes of these three-dimensional rectangular prisms.

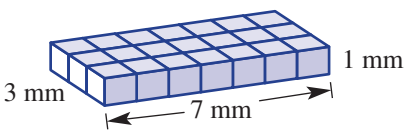
a



b



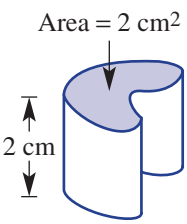
c



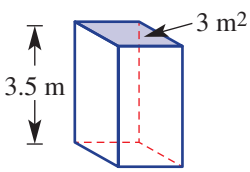
Example 14b

5 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

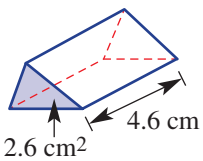
a



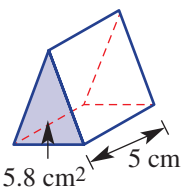
b



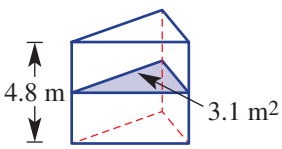
c



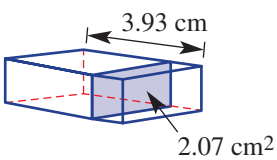
d



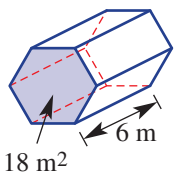
e



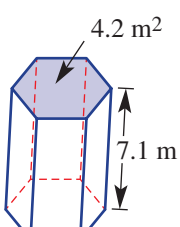
f



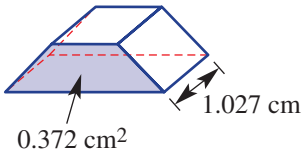
g



h



i

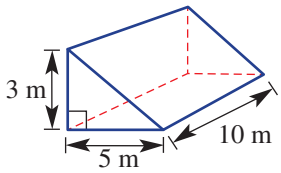


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

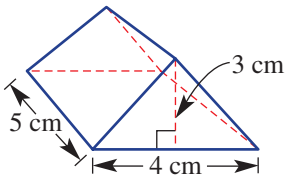
Example 14c

6 Find the volumes of these prisms.

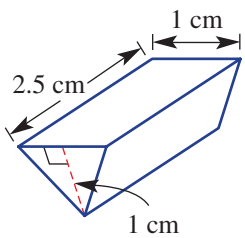
a



b

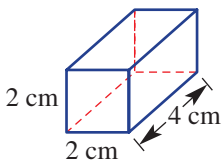


c

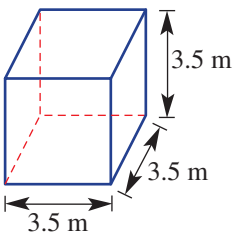


7 Find the volume of each of these rectangular prisms (cuboids).

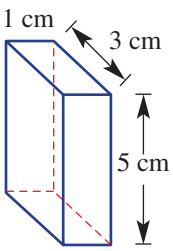
a



b

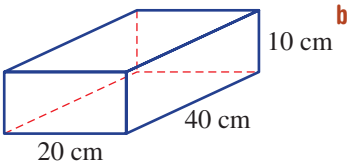


c

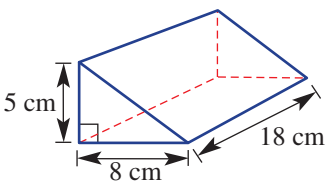


8 Find the volume of these solids converting your answer to litres.

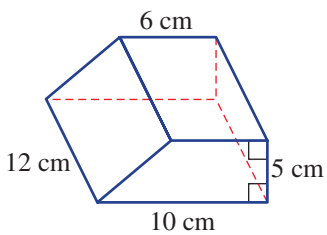
a



b



c



PROBLEM-SOLVING AND REASONING

9, 10, 15 | 10–12, 15 | 11–16

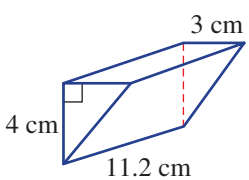
9 A brick is 10 cm wide, 20 cm long and 8 cm high. How much space would five of these bricks occupy?

10 How much air space is contained inside a rectangular cardboard box that has the dimensions 85 cm by 62 cm by 36 cm. Answer using cubic metres (m^3) correct to 2 decimal places.

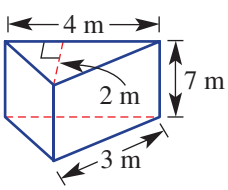
11 25 L of water is poured into a rectangular fish tank which is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

12 Find the volume of each of the following solids, rounding to 1 decimal place where necessary.

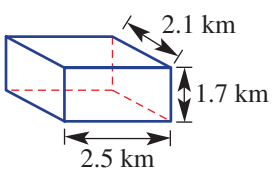
a



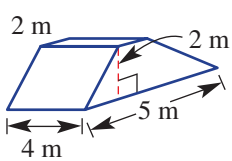
b



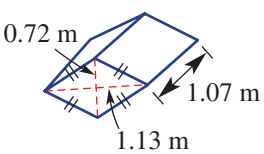
c



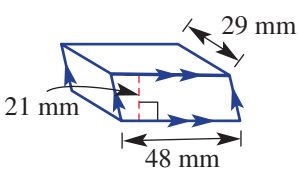
d



e



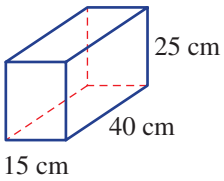
f



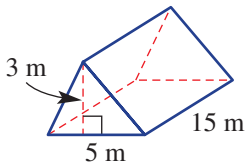
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 Use units for capacity to find the volume of these solids in litres.

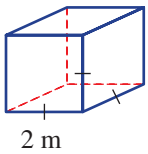
a



b



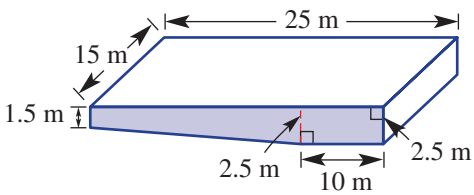
c



14 The given diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

a Find the area of one side of the pool (shaded).

b Find the volume of the pool in litres.



15 **a** What single number do you multiply by to convert from:

i L to cm^3?

ii L to m^3?

iii mL to mm^3?

b What single number do you divide by to convert from:

i mm^3 to L?

ii m^3 to ML?

iii cm^3 to kL?

16 Write rules for the volume of these solids using the given pronumerals.

a A rectangular prism with length = breadth = x and height h.

b A cube with side length s.

c A rectangular prism with a square base (with side length t) and height 6 times the side length of the base.

ENRICHMENT — — 17

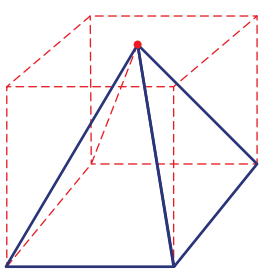
Volume of a pyramid

17 Earlier we looked at finding the surface area of a right pyramid like the one shown here. Imagine the pyramid sitting inside a prism with the same base.

a Make an educated guess as to what fraction of the prism's volume is the pyramid's volume.

b Use the internet to find the actual answer to part **a**.

c Draw some pyramids and find their volume using the results from part **b**.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5H Volume of cylinders

Technically, a cylinder is not a right prism because its sides are not rectangles. It does, however, have a uniform cross-section (a circle) and so a cylinder's volume can be calculated in a similar way to that of a right prism. Cylindrical objects are commonly used to store gases and liquids and so working out the volume of a cylinder is an important measurement calculation.



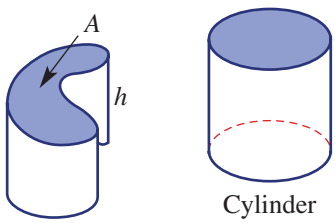
The volume of liquid in this tanker can be calculated using the volume formula for a cylinder.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Writing the rule

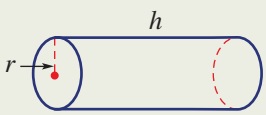
Previously we used the formula $V = A \times h$ to find the volume of solids with a uniform cross-section.

- Discuss any similarities between the two given solids.
- How can the rule $V = A \times h$ be developed further to find the rule for the volume of a cylinder?



Key ideas

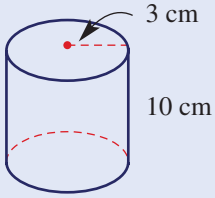
- The volume of a cylinder is given by:
  $V = \pi r^2 \times h$ or $V = \pi r^2 h$, where
  - r is the radius of the circular ends
  - h is the length or distance between the circular ends.



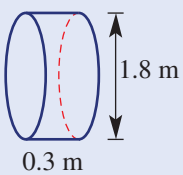
Example 15 Finding the volume of a cylinder

Find the volume of these cylinders correct to 2 decimal places.

a



b



SOLUTION

a $V = \pi r^2 h$
$= \pi \times (3)^2 \times 10$
$= 90\pi$
$= 282.74 \text{ cm}^3$ (to 2 d.p.)

b $V = \pi r^2 h$
$= \pi \times (0.9)^2 \times 0.3$
$= 0.76 \text{ m}^3$ (to 2 d.p.)

EXPLANATION

Substitute $r = 3$ and $h = 10$ into the rule.
$90\pi \text{ cm}^3$ would be the exact answer.

Include volume units.

The diameter is 1.8 m so $r = 0.9$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 16 Finding the capacity of a cylinder

Find the capacity in litres of a cylinder with radius 30 cm and height 90 cm. Round to the nearest litre.

SOLUTION

$V = \pi r^2 h$
$= \pi \times (30)^2 \times 90$
$= 254469 \text{ cm}^3$
$= 254 \text{ L}$ (to the nearest litre)

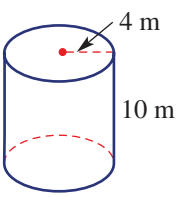
EXPLANATION

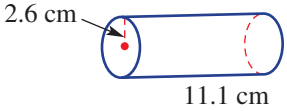
Substitute $r = 30$ and $h = 90$.
There are 1000 cm^3 in 1 L so divide by 1000 to convert to litres, then round to the nearest litre.

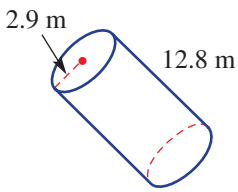
Exercise 5H

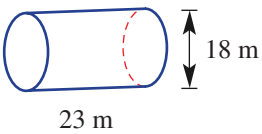
UNDERSTANDING AND FLUENCY | 1–3, 4–5(½) | 3, 4–5(½) | 4–5(½)

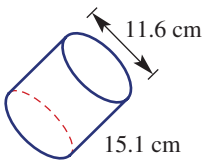
1 State the radius and the height of these cylinders.

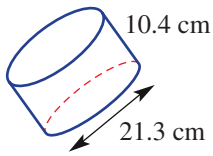
a


b


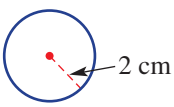
c


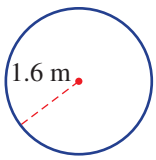
d


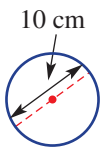
e


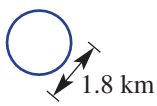
f


2 Find the area of these circles correct to 2 decimal places.

a


b


c


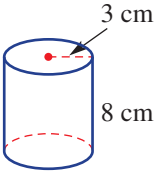
d


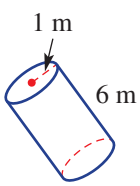
3 Convert the following into the units given in the brackets. Remember, 1 L = 1000 cm^3 and $1 \text{ m}^3 = 1000$ L.

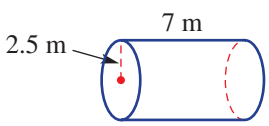
a 2000 cm^3 (L)
b 4.3 cm^3 (mL)
c 3.7 L (cm^3)
d 1 m^3 (L)
e 38000 L (m^3)
f 0.0002 m^3 (mL)

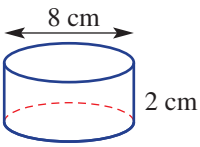
Example 15

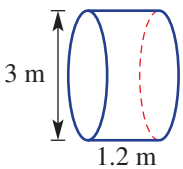
4 Find the volume of these cylinders correct to 2 decimal places.

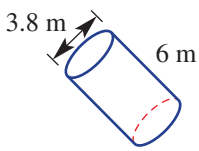
a


b


c


d


e


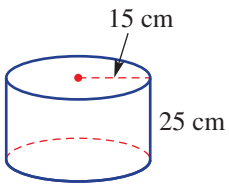
f


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

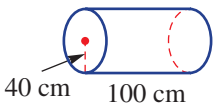
Example 16

5 Find the capacity in litres of these cylinders. Round to the nearest litre. Remember, $1\,\text{L} = 1000\,\text{cm}^3$ and $1\,\text{m}^3 = 1000\,\text{L}$.

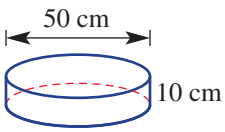
a



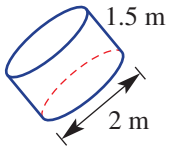
b



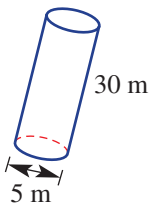
c



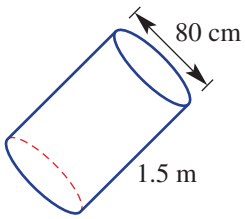
d



e



f



PROBLEM-SOLVING AND REASONING	6, 7, 10	7, 8, 9(½), 10, 11(½)	8–10, 11(½), 12

6 A cylindrical water tank has a radius of 2 m and a height of 2 m.

a Find its capacity in m^3 rounded to 3 decimal places.

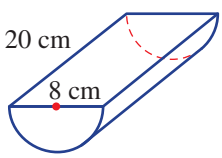
b Find its capacity in L rounded to the nearest litre.

7 How many litres of gas can a tanker carry if its tank is cylindrical with a 2 m diameter and is 12 m in length? Round to the nearest litre.

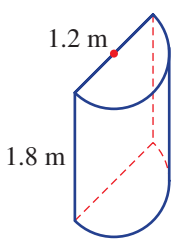
8 Which has a bigger volume and what is the difference in volume to 2 decimal places: a cube with side length 1 m or a cylinder with radius 1 m and height 0.5 m?

9 Find the volume of these cylindrical portions correct to 2 decimal places.

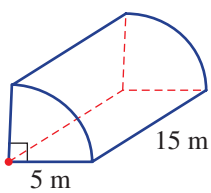
a



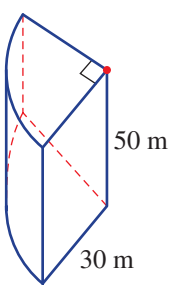
b



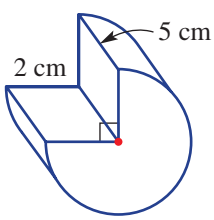
c



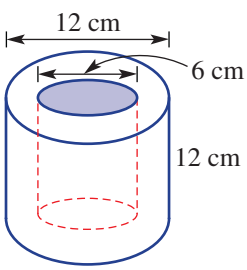
d



e



f



10 The rule for the volume of a cylinder is $V = \pi r^2 h$. Show how you could use this rule to find, correct to 3 decimal places:

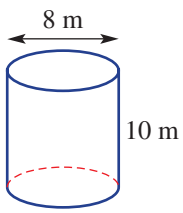
a h if $V = 20$ and $r = 3$

b r if $V = 100$ and $h = 5$

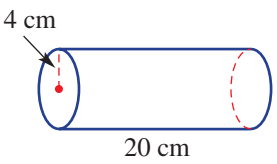
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Using exact values (e.g. 20π) find the volume of these solids.

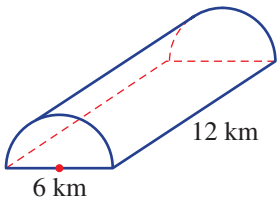
a



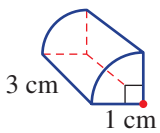
b



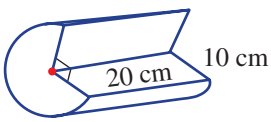
c



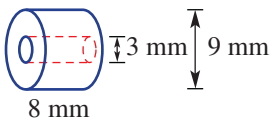
d



e



f

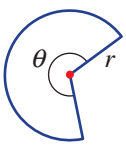


12 Draw a cylinder with its circumference equal to its height. Try to draw it to scale.

ENRICHMENT — — 13

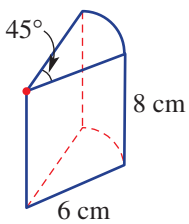
Solid sectors

13 You will recall that the area of a sector is given by $A = \frac{\theta}{360} \times \pi r^2$.

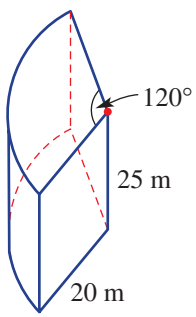


Use this fact to help find the volume of these solids correct to 2 decimal places.

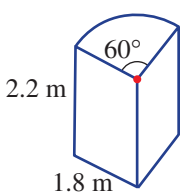
a



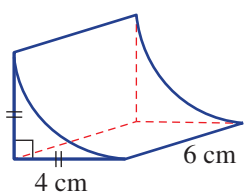
b



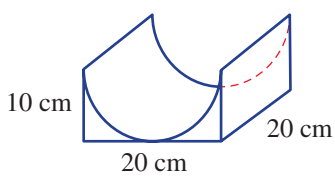
c



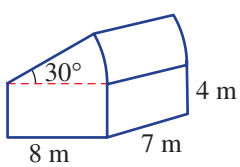
d



e



f



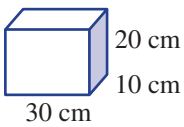
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Capacity and depth

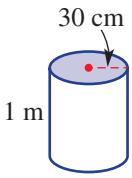
Finding capacity

Find the capacity in litres of these containers (i.e. find the total volume of fluid they can hold).

a



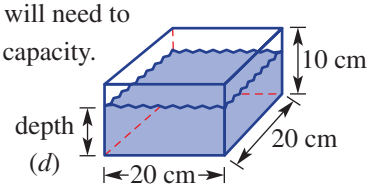
b



> Remember: 1 millilitre (mL) of fluid occupies 1 cm^3 of space; therefore, 1 litre (L) occupies 1000 cm^3 as there are 1000 mL in 1 litre.

Designing containers

Design a container with the given shape that has a 10-litre capacity. You will need to state the dimensions length, breadth, height, radius etc. and calculate its capacity.



a rectangular prism **b** cylinder

Finding depth

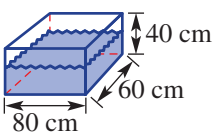
The depth of water in a prism can be found if the base (cross-sectional) area and volume of water are given.

Consider a cuboid, as shown, with 2.4 litres of water. To find the depth of water:

- Convert the volume to cm^3: $2.4\text{ L} = 2.4 \times 1000$
  $= 2400\text{ cm}^3$
- Find the depth: Volume = area of base $\times d$
  $2400 = 20 \times 20 \times d$
  $2400 = 400 \times d$
  $\therefore d = 6$
  $\therefore$ depth is 6 cm

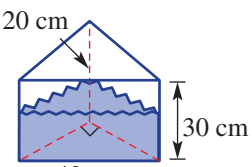
Use the above method to find the depth of water in these prisms.

a



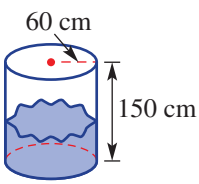
Volume = 96 L

b



Volume = 8 L

c



Volume = 500 L

Volumes of odd-shaped objects

Some solids may be peculiar in shape and their volume may be difficult to measure.

a A rare piece of rock is placed into a cylindrical jug of water and the water depth rises from 10 cm to 11 cm. The radius of the jug is 5 cm.

- **i** Find the area of the circular base of the cylinder.
- **ii** Find the volume of water in the jug before the rock is placed in the jug.
- **iii** Find the volume of water in the jug including the rock.
- **iv** Hence find the volume of the rock.

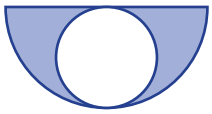
b Use the procedure outlined in part **a i–iv** above to find the volume of an object of your choice. Explain and show your working and compare your results with those of other students in your class if they are measuring the volume of the same object.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

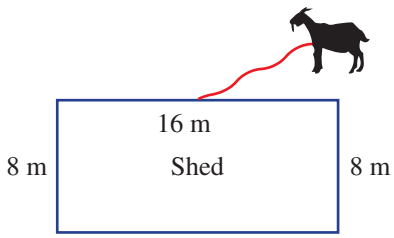
Puzzles and challenges

1 A 100 m^2 factory flat roof feeds all the water collected to a rainwater tank. If there is 1 mm of rainfall, how many litres of water go into the tank?

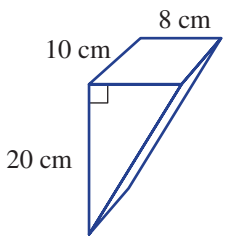
2 What is the relationship between the shaded and non-shaded regions in this circular diagram?



3 A goat is tethered to the centre of one side of a shed with a 10 m length of rope. What area of grass can the goat graze?



4 A rain gauge is in the shape of a triangular prism with dimensions as shown. What is the depth of water when it is half full?



5 A rectangular fish tank has base area 0.3 m^2 and height 30 cm and is filled with 80 L of water. Ten large fish with volume 50 cm^3 each are placed into the tank. By how much does the water rise?

6 If it takes 4 people 8 days to knit 2 rugs, how many days will it take for 1 person to knit 1 rug?

7 Find the rule for the volume of a cylinder in terms of r only if the height is equal to its circumference.

8 The surface area of a cylinder is 2π square units. Find a rule for h in terms of r.

9 Give the dimensions of a cylinder that can hold one litre of liquid. When designing containers for cylinders, what factors need to be considered?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Length, area, surface area and volume

Units

$\times 1000^3$ $\times 100^3$ $\times 10^3$

km^3 m^3 cm^3 mm^3

$\div 1000^3$ $\div 100^3$ $\div 10^3$

1 L = 1000 mL = 1000 cm^3

Volume

Prism or cylinders

$V = Ah$

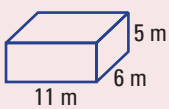
A = area of cross-section

h = height

$V = lbh$

$= 11 \times 6 \times 5$

$= 330$ m^3

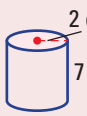


$V = \pi r^2 h$

$= \pi(2)^2 \times 7$

$= 28\pi$

$= 87.96$ cm^3



Circle circumference

$C = 2\pi r$ or πd

Length and perimeter

Sector



$P = 2r + \frac{\theta}{360} \times 2\pi r$

$= 2 \times 3 + \frac{90}{360} \times 2\pi \times 3$

$= 6 + 1.5\pi$ (exact)

$= 10.71$ cm (2 d.p.)

Area

Triangle

$A = \frac{1}{2}bh$

Quadrilaterals

Square $A = l^2$

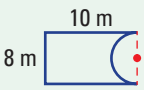
Rectangle $A = lb$

Parallelogram $A = bh$

Trapezium $A = \frac{1}{2}h(a + b)$

Rhombus or kite $A = \frac{1}{2}xy$

Composite shapes



$A = lb - \frac{1}{2}\pi r^2$

$= 10 \times 8 - \frac{1}{2}\pi \times 4^2$

$= 80 - 8\pi$

$= 54.87$ m^2

$P = 2 \times 10 + 8 + \frac{1}{2}\ 2\pi(4)$

$= 28 + 4\pi$

$= 40.57$ m

Circle

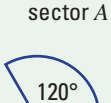
$A = \pi r^2$

sector $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{120}{360} \times \pi \times 5^2$

$= \frac{1}{3} \times \pi \times 25$

$= 26.18$ m^2



Units

$\times 1000^2$ $\times 100^2$ $\times 10^2$

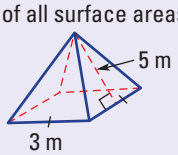
km^2 m^2 cm^2 mm^2

$\div 1000^2$ $\div 100^2$ $\div 10^2$

Surface area

Square-based prism or pyramid

Use a net to help find the sum of all surface areas.

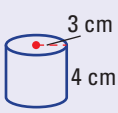


$A = b^2 + 4(\frac{1}{2}bh)$

$= 3^2 + 4 \times \frac{1}{2} \times 3 \times 5$

$= 39$ m^2

Cylinder



$A = 2\pi r^2 + 2\pi rh$

$= 2\pi(3)^2 + 2\pi(3)(4)$

$= 18\pi + 24\pi$

$= 42\pi$

$= 131.95$ cm^2

Chapter summary

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter review

Multiple-choice questions

1 If the area of a square field is 25 km^2, its perimeter is:

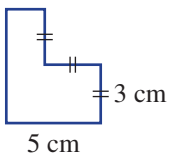
A 10 km **B** 20 km **C** 5 km **D** 50 km **E** 25 km

2 2.7 m^2 is the same as:

A 270 cm^2 **B** 0.0027 km^2 **C** 27000 cm^2 **D** 2700 mm^2 **E** 27 cm^2

3 The perimeter of this shape is:

A 21 cm
B 14 cm
C 24 cm
D 20 cm
E 22 cm

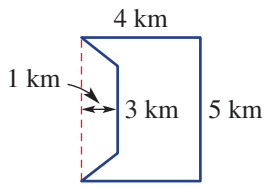


4 A parallelogram has area 10 m^2 and base 5 m. Its perpendicular height is:

A 50 m **B** 2 m **C** 50 m^2 **D** 2 m^2 **E** 0.5 m

5 This composite shape could be considered as a rectangle with an area in the shape of a trapezium removed. The shape's area is:

A 16 km^2
B 12 km^2
C 20 km^2
D 24 km^2
E 6 km^2

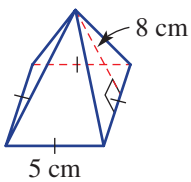


6 A semicircular goal area has diameter 20 m. Its perimeter correct to the nearest metre is:

A 41 m **B** 36 m **C** 83 m **D** 51 m **E** 52 m

7 The surface area of the pyramid shown is:

A 185 cm^2
B 105 cm^2
C 65 cm^2
D 100 cm^2
E 125 cm^2



8 The area of the curved surface only of a half-cylinder with radius 5 mm and height 12 mm is closest to:

A 942.5 mm^2 **B** 94.2 mm^2 **C** 377 mm^2
D 471.2 mm^2 **E** 188.5 mm^2



9 A prism's cross-sectional area is 100 m^2. If its volume is 6500 m^3, the prism's total height would be:

A 0.65 m **B** 650000 m **C** 65 m **D** 6.5 m **E** 650 m

10 The exact volume of a cylinder with radius 3 cm and height 10 cm is:

A 60π cm^2 **B** 80π cm **C** 45π cm^3 **D** 30π cm^3 **E** 90π cm^3

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

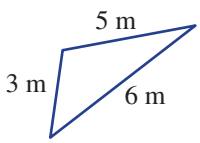
Short-answer questions

1 Convert the following measurements into the units given in brackets.

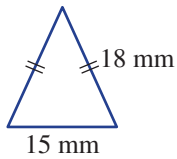
a 3.8 m (cm)
b 1.27 km (m)
c 273 mm^2 (cm^2)
d 5.2 m^2 (cm^2)
e 0.01 m^3 (cm^3)
f 53 100 mm^3 (cm^3)
g 3100 mL (L)
h 0.043 L (mL)
i 2.83 kL (L)

2 Find the perimeter of each of the following shapes.

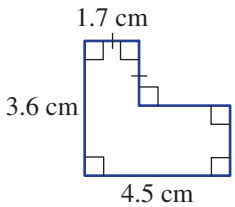
a



b

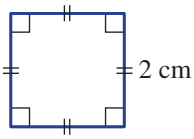


c

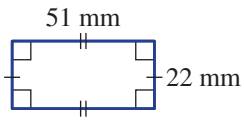


3 Find the area of each of the following plane figures.

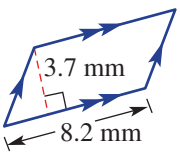
a



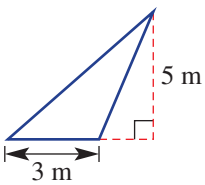
b



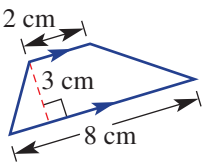
c



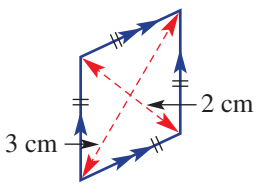
d



e



f

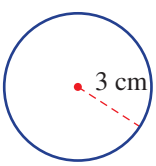


4 Inside a rectangular lawn area of length 10.5 m and width 3.8 m, a new garden bed is to be constructed. The garden bed is to be the shape of a triangle with base 2 m and height 2.5 m. With the aid of a diagram, find the area of the:

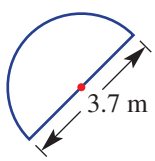
a garden bed
b lawn remaining around the garden bed

5 Find the area and circumference or perimeter of each of the following shapes correct to 2 decimal places.

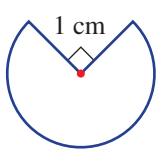
a



b



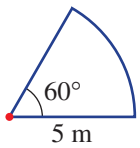
c



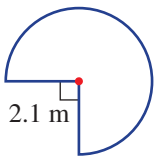
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Find the perimeter and area of each of these sectors. Round to 2 decimal places.

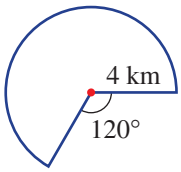
a



b

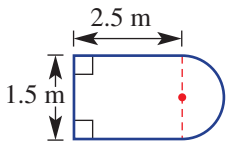


c

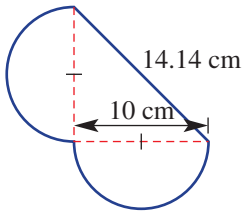


7 Find the perimeter and area of each of the following composite shapes correct to 2 decimal places.

a

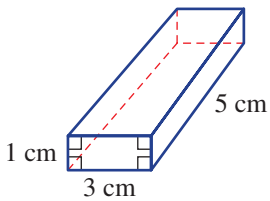


b

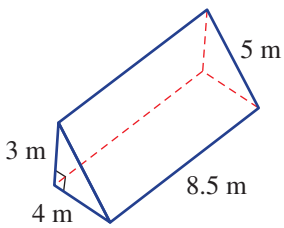


8 Find the surface area of each of the following solid objects.

a

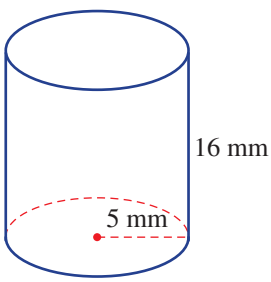


b

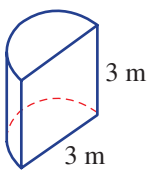


9 Find the surface area of each of the following solid objects correct to 2 decimal places.

a



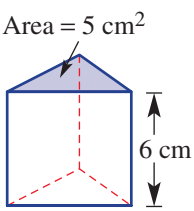
b



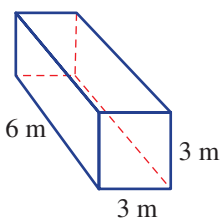
Half-cylinder

10 Find the volume of each of these solid objects, rounding to 2 decimal places where necessary.

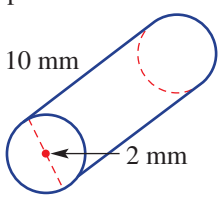
a



b



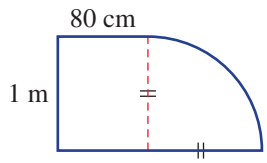
c



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Extended-response questions

1 An office receives five new desks with a bench shape made up of a rectangle and quarter-circle as shown below.
The edge of the bench is lined with a rubber strip at a cost of $2.50 per metre.



a Find the length of the rubber edging strip in centimetres for one desk correct to 2 decimal places.

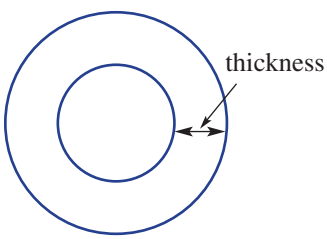
b By converting your answer in part **a** to metres, find the total cost of the rubber strip for the five desks. Round to the nearest dollar.

The manufacturer claims that the desk top area space is more than 1.5 m^2.

c Find the area of the desk top in cm^2 correct to 2 decimal places.

d Convert your answer to m^2 and determine whether or not the manufacturer's claim is correct.

2 Circular steel railing of diameter 6 cm is to be used to fence the side of a bridge. The railing is hollow and the radius of the hollow circular space is 2 cm.



a By adding the given information to this diagram of the cross-section of the railing, determine the thickness of the steel.

b Determine, correct to 2 decimal places, the area of steel in the cross-section.

Eight lengths of railing at 10 m each are required for the bridge.

c Using your result from part **b**, find the volume of steel required for the bridge in cm^3.

d Convert your answer in part **c** to m^3.

The curved outside surface of the steel railings is to be painted to help protect the steel from the weather.

e Find the outer circumference of the cross-section of the railing correct to 2 decimal places.

f Find the surface area in m^2 of the eight lengths of railing that are to be painted. Round to the nearest m^2.

The cost of the railing paint is $80 per m^2.

g Using your answer from part **f**, find the cost of painting the bridge rails to the nearest dollar.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 1: Computation and financial mathematics

Multiple-choice questions

1 $-3 + (4 + (-10)) \times (-2)$ is equal to:

A 21 **B** 9 **C** 18 **D** -15 **E** -6

2 The estimate of $221.7 \div 43.4 - 0.0492$ using 1 significant figure rounding is:

A 4.9 **B** 5.06 **C** 5.5 **D** 5 **E** 4.95

3 \$450 is divided in the ratio 4:5. The value of the smaller portion is:

A \$210 **B** \$250 **C** \$90 **D** \$200 **E** \$220

4 A book that costs \$27 is discounted by 15%. The new price is:

A \$20.25 **B** \$31.05 **C** \$4.05 **D** \$22.95 **E** \$25.20

5 Anna is paid a normal rate of \$12.10 per hour. If in a week she works 6 hours at the normal rate, 2 hours at time and a half and 3 hours at double time, how much does she earn?

A \$181.50 **B** \$145.20 **C** \$193.60 **D** \$175.45 **E** \$163.35

Short-answer questions

1 Evaluate the following.

a $\frac{3}{7} + \frac{1}{4}$ **b** $2\frac{1}{3} - 1\frac{5}{9}$ **c** $\frac{9}{10} \times \frac{5}{12}$ **d** $3\frac{3}{4} \div 2\frac{1}{12}$

2 Convert each of the following to a percentage.

a 0.6 **b** $\frac{5}{16}$ **c** 2 kg out of 20 kg **d** 75 c out of \$3

3 Write these rates and ratios in simplest form.

a Prize money is shared between two people in the ratio 60 : 36.

b Jodie travels 165 km in three hours.

c 3 mL of rain falls in $1\frac{1}{4}$ hours.

4 Jeff earns a weekly retainer of \$400 plus 6% of the sales he makes. If he sells \$8200 worth of goods, how much will he earn for the week?

Extended-response question

Husband and wife Jim and Jill are trialling new banking arrangements.

a Jill plans to trial a simple interest plan. Before investing her money she increases the amount in her account by 20% to \$21 000.

i What was the original amount in her account?

ii She invests the \$21 000 for 4 years at an interest rate of 3% p.a. How much does she have in her account at the end of the 4 years?

iii She continues with this same plan and after a certain number of years has obtained \$5670 interest. How many years has she had the money invested for?

iv What percentage increase does this interest represent on her initial investment?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b Jim is investing his $21 000 in an account that compounds annually at 3% p.a. How much does he have after 4 years to the nearest cent?

c **i** Who had the most money after 4 years and by how much? Round to the nearest dollar.

ii Who will have the most money after 10 years and by how much? Round to the nearest dollar.

Chapter 2: Expressions, equations and inequalities

Multiple-choice questions

1 The simplified form of $5ab + 6a \div 2 + a \times 2b - a$ is:

A $10ab$ **B** $10ab + 3a$ **C** $13ab - a$

D $5ab + 3a + 2b$ **E** $7ab + 2a$

2 The expanded form of $-2(3m - 4)$ is:

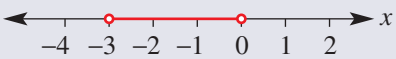
A $-6m + 8$ **B** $-6m + 4$ **C** $-6m - 8$ **D** $-5m - 6$ **E** $5m + 8$

3 The solution to $\frac{d}{4} - 7 = 2$ is:

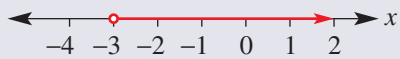
A $d = -20$ **B** $d = 15$ **C** $d = 36$ **D** $d = 1$ **E** $d = 30$

4 The solution to $1 - 3x < 10$ represented on a number line is:

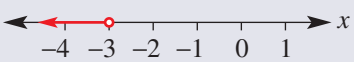
A



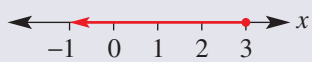
B



C



D



E



5 The formula $m = \sqrt{\frac{b-1}{a}}$ with b as the subject is:

A $b = a\sqrt{m} + 1$ **B** $b = \frac{m^2}{a} + 1$ **C** $b = a^2m^2 + 1$

D $b = am^2 + 1$ **E** $b = m^2\sqrt{a} + 1$

Short-answer questions

1 Solve the following equations and inequalities.

a $3x + 7 = 25$ **b** $\frac{2x - 1}{4} > 2$ **c** $4(2m + 3) = 15$

d $-3(2y + 4) - 2y = -4$ **e** $3(a + 1) \leq 4 - 8a$ **f** $3(2x - 1) = -2(4x + 3)$

2 Noah receives m dollars pocket money per week. His younger brother Jake gets half of three dollars less than Noah's amount. If Jake receives $6:

a write an equation to represent the problem

b solve the equation in part **a** to determine how much Noah receives each week

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 The formula $S = \frac{n}{2}(a + l)$ gives the sum S of a sequence of n numbers with first term a and last term l.

- **a** Find the sum of the sequence of 10 terms 2, 5, 8, …, 29.
- **b** Rearrange the formula to make l the subject.
- **c** If a sequence of 8 terms has a sum of 88 and a first term equal to 4, use your answer to part **b** to find the last term of this sequence.

4 Solve the following equations simultaneously.

a $x + 4y = 18$, $x = 2y$ **b** $7x - 2y = 3$, $y = 2x - 3$ **c** $2x + 3y = 4$, $x + y = 3$ **d** $3x + 4y = 7$, $5x + 2y = -7$

Extended-response question

a Chris referees junior basketball games on a Sunday. He is paid \$20 plus \$12 per game he referees. He is trying to earn more than \$74 one Sunday. Let x be the number of games he referees.

- **i** Write an inequality to represent the problem.
- **ii** Solve the inequality to find the minimum number of games he must referee.

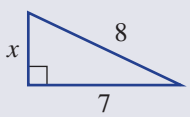
b Two parents support the game by buying raffle tickets and badges. One buys 5 raffle tickets and 2 badges for \$11.50 while the other buys 4 raffle tickets and 3 badges for \$12. Determine the cost of a raffle ticket and the cost of a badge by:

- **i** defining two variables
- **ii** setting up two equations to represent the problem
- **iii** solving your equations simultaneously

Chapter 3: Right-angled triangles

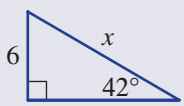
Multiple-choice questions

1 The exact value of x in the triangle shown is:



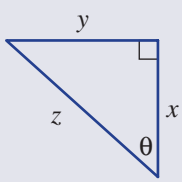
A 3.9 **B** $\sqrt{113}$ **C** $\sqrt{57}$ **D** $\sqrt{15}$ **E** 2.7

2 The correct expression for the triangle shown is:



A $x = \frac{6}{\sin 42°}$ **B** $x = 6 \tan 42°$ **C** $x = 6 \sin 42°$ **D** $x = \frac{6}{\cos 42°}$ **E** $x = \frac{\sin 42°}{6}$

3 The correct expression for the angle θ is:

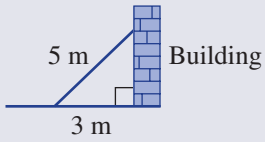


A $\tan \theta = \frac{x}{z}$ **B** $\sin \theta = \frac{x}{z}$ **C** $\tan \theta = \frac{y}{x}$ **D** $\cos \theta = \frac{z}{x}$ **E** $\cos \theta = \frac{x}{y}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

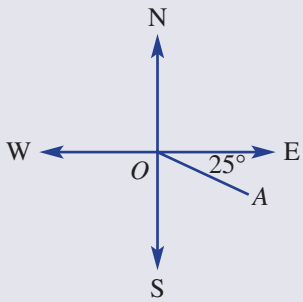
4 A 5-metre plank of wood is leaning up against a side of a building as shown. If the wood touches the ground 3 m from the base of the building, the angle the wood makes with the building is closest to:

A 36.9° **B** 59° **C** 53.1° **D** 31° **E** 41.4°



5 The bearing of A from O is:

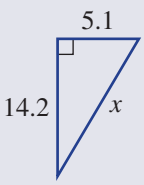
A 025°
B 125°
C 155°
D 065°
E 115°



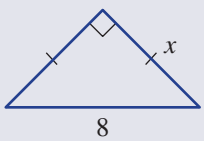
Short-answer questions

1 Find the value of each pronumeral, correct to 1 decimal place.

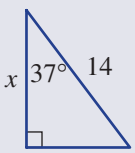
a



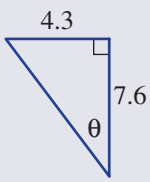
b



c

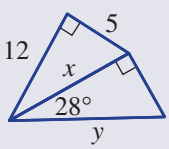


d

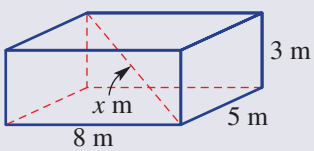


2 Find the value of the pronumerals. Round to 1 decimal place where necessary.

a



b



3 A wire is to be connected from the edge of the top of a 28 m high building to the edge of the top of a 16 m high building. The buildings are 15 m apart.

a What length of wire, to the nearest centimetre, is required?

b What is the angle of depression from the top of the taller building to the top of the smaller building? Round to 1 decimal place.

4 A yacht sails 18 km from its start location on a bearing of 295°.

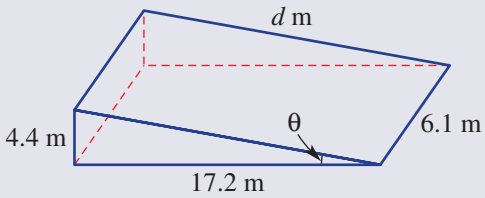
a How far east or west is it from its start location? Answer correct to 1 decimal place.

b On what bearing would it need to sail to return directly to its start location?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Extended-response question

A skateboard ramp is constructed as shown.



a Calculate the distance d metres up the ramp correct to 2 decimal places.

b What is the angle of inclination (θ) between the ramp and the ground correct to 1 decimal place?

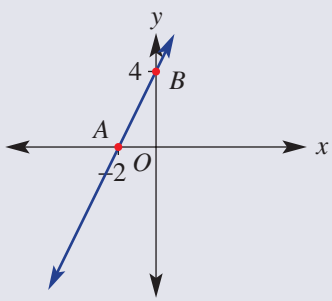
c **i** If the skateboarder rides from one corner of the ramp diagonally to the other corner, what distance would be travelled? Round to 1 decimal place.

ii If the skateboarder travels at an average speed of 10 km/h, how many seconds does it take to ride diagonally across the ramp? Answer correct to 1 decimal place.

Chapter 4: Linear relationships

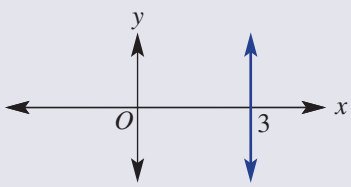
Multiple-choice questions

1 The coordinates of points A and B are:



A $(-2, 4)$ and $(4, -2)$
B $(0, 4)$ and $(-2, 0)$
C $(-2, 0)$ and $(0, 4)$
D $(4, 0)$ and $(0, -2)$
E $(2, 0)$ and $(0, -4)$

2 The graph shown has equation:



A $y = 3x$
B $y = 3$
C $y = x + 3$
D $x = 3$
E $x + y = 3$

3 If the point $(-1, 3)$ is on the line $y = 2x + b$, the value of b is:

A 1 **B** 5 **C** -7 **D** -5 **E** -1

4 The line passing through the points $(-3, -1)$ and $(1, y)$ has gradient 2. The value of y is:

A 3 **B** 5 **C** 7 **D** 1 **E** 4

5 The midpoint and length to 1 decimal place of the line segment joining the points $(-2, 1)$ and $(4, 6)$ are:

A (1, 3.5) and 7.8 **B** (3, 5) and 5.4 **C** (3, 3.5) and 6.1
D (1, 3.5) and 3.3 **E** (3, 3.5) and 3.6

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Short-answer questions

1 Sketch the following linear graphs labelling x- and y-intercepts.

a $y = 2x - 6$ **b** $3x + 4y = 24$ **c** $y = 4x$

2 Find the gradient of each of the following.

a The line passing through the points $(-1, 2)$ and $(2, 4)$

b The line passing through the points $(-2, 5)$ and $(1, -4)$

c The line with equation $y = -2x + 5$

d The line with equation $-4x + 3y = 9$

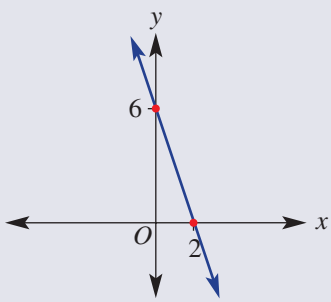
3 Give the equation of the following lines in gradient–intercept form.

a The line with the given graph

b The line with gradient 3 and passing through the point $(2, 5)$

c The line parallel to the line with equation $y = 2x - 1$ that passes through the origin

d The line perpendicular to the line with equation $y = 3x + 4$ that passes through the point $(0, 2)$



4 Solve the simultaneous equations $y = 2x - 4$ and $x + y = 5$ graphically by finding the coordinates of the point of intersection.

Extended-response question

Doug works as a labourer. He is digging a trench and has 180 kg of soil to remove. He has taken 3 hours to remove 36 kg.

a What is the rate at which he is removing the soil?

b If he maintains this rate, write a rule for the amount of soil, S (kg), remaining after t hours.

c Draw a graph of your rule.

d How long will it take to remove all of the soil?

e Doug is paid $40 for the job plus $25 per hour worked.

i Write a rule for his pay P dollars for working h hours.

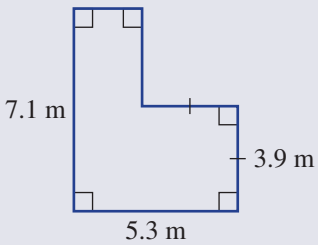
ii How much will he be paid to remove all the soil?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 5: Length, area, surface area and volume

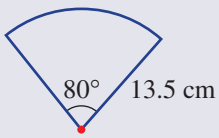
Multiple-choice questions

1 The perimeter and area of the figure shown are:



A 20.2 m, 22.42 m^2
B 24.8 m, 22.42 m^2
C 24.8 m, 25.15 m^2
D 20.2 m, 25.15 m^2
E 21.6 m, 24.63 m^2

2 The exact perimeter in centimetres of this sector is:

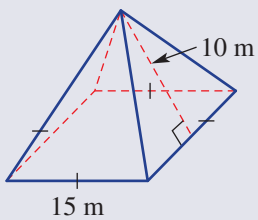


A 127.2 **B** $\frac{81\pi}{2} + 27$ **C** $12\pi + 27$
D 45.8 **E** $6\pi + 27$

3 420 cm^2 is equivalent to:

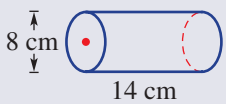
A 4.2 m^2 **B** 0.42 m^2 **C** 42000 m^2
D 0.042 m^2 **E** 0.0042 m^2

4 This square pyramid has a total surface area of:



A 525 m^2
B 300 m^2
C 750 m^2
D 450 m^2
E 825 m^2

5 The volume of the cylinder shown is closest to:

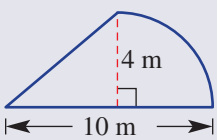


A 703.7 cm^3 **B** 351.9 cm^3 **C** 2814.9 cm^3
D 452.4 cm^3 **E** 1105.8 cm^3

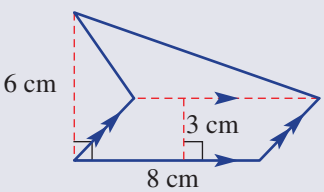
Short-answer questions

1 Find the area of each of the figures below. Round to 2 decimal places where necessary.

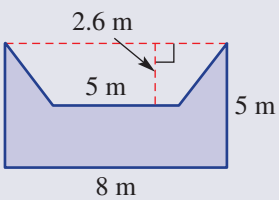
a



b



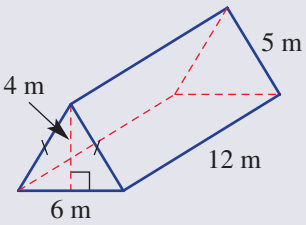
2 A tin of varnish for the timber (shaded) on the deck shown covers 6.2 square metres. How many tins will be required to completely varnish the deck?



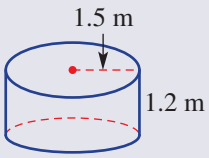
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 Find the surface area of these solid objects. Round to 2 decimal places where necessary.

a

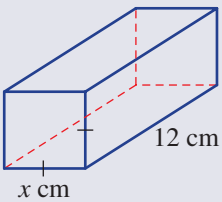


b



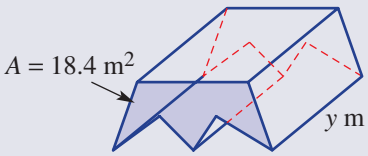
4 Find the value of the pronumeral for the given volume.

a



$V = 192 \text{ cm}^3$

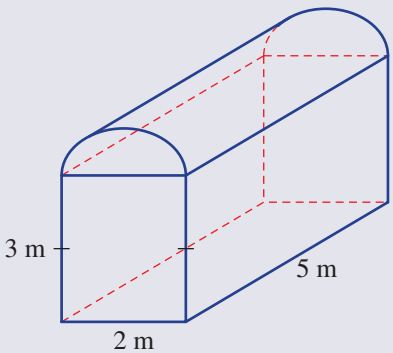
b



$V = 156.4 \text{ m}^3$

Extended-response question

A barn in the shape of a rectangular prism with a semicylindrical roof and with the dimensions shown is used to store hay.



a The roof and the two long side walls of the barn are to be painted. Calculate the surface area to be painted correct to 2 decimal places.

b A paint roller has a width of 20 cm and a radius of 3 cm.

i Find the area of the curved surface of the paint roller in m^2. Round to 4 decimal places.

ii Hence, state the area that the roller will cover in 100 revolutions.

c Find the minimum number of revolutions required to paint the area of the barn in part **a** with one coat.

d Find the volume of the barn correct to 2 decimal places.

e A rectangular bail of hay has dimensions 1 m by 40 cm by 40 cm. If there are 115 bails of hay in the barn, what volume of air space remains? Answer to 2 decimal places.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

6 Indices and surds

What you will learn

6A Index notation
6B Index laws for multiplying and dividing
6C The zero index and power of a power
6D Index laws extended
6E Negative indices
6F Scientific notation
6G Scientific notation using significant figures
6H Fractional indices and surds
6I Simple operations with surds

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: INDICES (S4, 5.1, 5.2) SURDS AND INDICES (S5.3§)

STRAND: MEASUREMENT AND GEOMETRY
SUBSTRAND: NUMBERS OF ANY MAGNITUDE (S5.1)

Outcomes

A student operates with positive-integer and zero indices of numerical bases.
(MA4–9NA)
A student operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases.
(MA5.1–5NA)
A student applies index laws to operate with algebraic expressions involving integer indices.
(MA5.2–7NA)
A student performs operations with surds and indices.
(MA5.3–6NA)
A student interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures.
(MA5.1–9MG)

© NESA, 2012

Musical notes and indices

Each octave on the piano has 12 notes or semi-tones with the highest note having double the frequency of the lowest note. The frequency of a piano's lowest C, for example, multiplied by 2, 2^2, 2^3, 2^4, 2^5, 2^6 calculates the higher C frequencies, each an octave apart. The highest C frequency multiplied by 2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, 2^{-5}, 2^{-6} gives the lower C frequencies.

Within an octave, each note's frequency increases by $\times 2^{\frac{1}{12}}$ because $\left(2^{\frac{1}{12}}\right)^{12} = 2^{\frac{1}{12}} \times 12 = 2$.

Hence each note's frequency can be found by multiplying the lowest note's frequency by $2^{\frac{1}{12}}, \left(2^{\frac{1}{12}}\right)^2, \left(2^{\frac{1}{12}}\right)^3, \ldots \left(2^{\frac{1}{12}}\right)^{11}, \left(2^{\frac{1}{12}}\right)^{12}$.

For example:

- **Middle C frequency is 261.6 Hz $\left(\text{hertz} = \text{cycles/s}\right)$**
- **C sharp frequency $= 261.6 \times 2^{\frac{1}{12}} = 277.2$ Hz**
- **D frequency $= 261.6 \times \left(2^{\frac{1}{12}}\right)^2 = 293.6$ Hz**
- **At the top of this octave:**
  C frequency $= 261.6 \times \left(2^{\frac{1}{12}}\right)^{12} = 261.6 \times 2 = 523.2$ Hz
- **Tuning a piano to concert pitch means that A above middle C is tuned to exactly 440 Hz.**
- **Each note above A is $\times 2^{\frac{1}{12}}$ the frequency of the note below. Each note below A is $\times 2^{-\frac{1}{12}}$ the frequency of the note above it. Middle C is 9 notes or semi-tones below A, giving:**
  Middle C frequency $= 440 \times \left(2^{-\frac{1}{12}}\right)^9 = 261.6$ Hz.
- **Note combinations that sound pleasing to the ear have close to whole number ratios of their frequencies.**

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Evaluate:

a 5^2 **b** 10^2 **c** 2^4 **d** 3^3 **e** $(-3)^2$

2 **a** List the factors of 24.

b List the factors of 45.

c List the prime factors of 24.

d List the prime factors of 45.

3 Write each of the following in index form (as a power).

a $a \times b \times b$

b $5b \times 5b \times 5a$

c $3x \times 3x$

d $2 \times a \times c \times 3 \times c \times c$

4 Write each of the following as 2 raised to a single power.

a $2^2 \times 2$ **b** $2^4 \times 2^2$ **c** $2^3 \times 2^2$

5 Simplify the following by removing brackets.

a $(3^2)^2$ **b** $(xy)^2$ **c** $(2a)^2$ **d** $\left(\frac{3}{4}\right)^2$

6 Evaluate:

a $\frac{1}{4^2}$ **b** $\frac{1}{2^3}$ **c** $\frac{1}{6^3}$ **d** $\frac{1}{\left(\frac{9}{4}\right)}$

7 Round the following to 2 decimal places.

a 3.732 **b** 24.6174 **c** 18.3654 **d** 4.3971

8 State the number of significant figures in each of the following.

a 23.102 **b** 30.05 **c** 0.0012 **d** 49 500

9 Complete the following.

a $3.8 \times 10 =$ _____ **b** $2.31 \times 1000 =$ _____

c $17.2 \div 100 =$ _____ **d** $0.18 \div 100 =$ _____

e $3827 \div$ _____ $= 3.827$ **f** $6.49 \times$ _____ $= 64\,900$

10 Complete the following.

a $15^2 = 225$ so $\sqrt{225} =$ _____ **b** $4^3 = 64$ so $\sqrt[3]{64} =$ _____

c $\sqrt[3]{125} = 5$ so $5^3 =$ _____ **d** $\sqrt[5]{32} = 2$ so $2^5 =$ _____

11 Simplify by collecting like terms.

a $3a + 7a - 4b$ **b** $5a - 2a + 3$

c $2ab + 8a - ab$ **d** $3a^2b + 2ab^2 - 4a^2b$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6A Index notation

Interactive
Widgets
HOTsheets
Walkthrough

When a product includes the repeated multiplication of the same factor, indices can be used to produce a more concise expression. For example, $5 \times 5 \times 5$ can be written as 5^3 and $x \times x \times x \times x \times x$ can be written as x^5. The expression 5^3 is a power and we can say '5 to the power of 3'. The 5 is called the base and the 3 is the index, exponent or power. Numbers written with indices are common in mathematics and can be applied to many types of problems. The mass of a 100 kg limestone block, for example, might decrease by 2% per year for 20 years. The mass after 20 years could be calculated by multiplying 100 by 0.98, 20 times. This is written as $100 \times (0.98)^{20}$.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

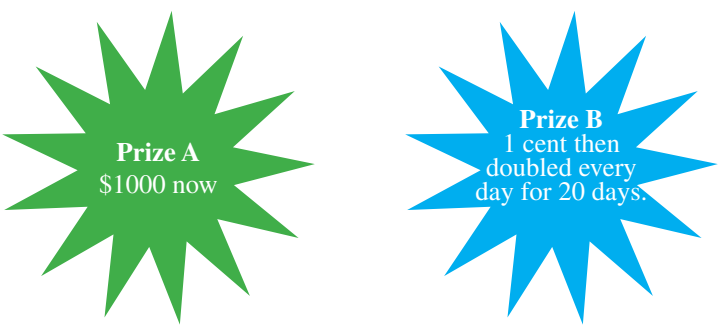


Index notation is a convenient way for expressing large numbers or carrying out calculations such as how much mass is lost over time from ancient stone monuments.

Let's start: Who has the most?

A person offers you one of two prizes.

- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use indices to help calculate the value of prize B?
- How can a calculator help to find the amount of prize B using the power button [^]?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

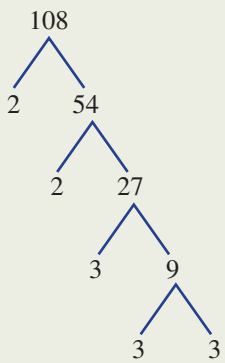
- **Indices** (plural of **index**) can be used to represent a product of the same factor.
- The **base** is the factor in the product.
- The index (**exponent or power**) is the number of times the factor (base number) is written.
- Note that $a^1 = a$.
  For example: $5^1 = 5$
- **Prime factorisation** involves writing a number as a product of its prime factors.

Expanded form: $\overbrace{2 \times 2 \times 2 \times 2 \times 2}^{} =$ Index form: $2^5 = 32$

base, index, basic numeral

$x \times x \times x \times x = x^4$

base, index



$108 = 2 \times 2 \times 3 \times 3 \times 3$
$= 2^2 \times 3^3$
prime factor form

Example 1 Writing in expanded form

Write each of the following in expanded form.

a a^3 b $(xy)^4$ c $2a^3b^2$

SOLUTION

a $a^3 = a \times a \times a$

b $(xy)^4 = xy \times xy \times xy \times xy$

c $2a^3b^2 = 2 \times a \times a \times a \times b \times b$

EXPLANATION

Factor a is written three times.

Factor xy is written four times.

Factor a is written three times and factor b is written twice. Factor 2 only appears once.

Example 2 Expanding and evaluating

Write each of the following in expanded form and then evaluate.

a 5^3 b $(-2)^5$ c $\left(\frac{2}{5}\right)^3$

SOLUTION

a $5^3 = 5 \times 5 \times 5$
$= 125$

EXPLANATION

Write in expanded form with 5 written three times and evaluate.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$
$= -32$

Write in expanded form with -2 written five times and evaluate.

c $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{8}{125}$

Write in expanded form.
Evaluate by multiplying numerators and denominators.

Example 3 Writing in index form

Write each of the following in index form.

a $6 \times x \times x \times x \times x$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

c $8 \times a \times a \times 8 \times b \times b \times a \times b$

SOLUTION

a $6 \times x \times x \times x \times x = 6x^4$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^3$

c $8 \times a \times a \times 8 \times b \times b \times a \times b$
$= 8 \times 8 \times a \times a \times a \times b \times b \times b$
$= 8^2a^3b^3$

EXPLANATION

Factor x is written 4 times, 6 only once.

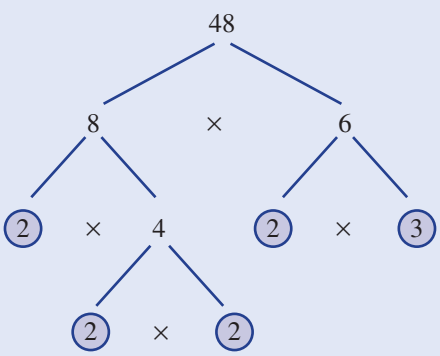
There are two groups of $\left(\frac{3}{7}\right)$ and three groups of $\left(\frac{4}{5}\right)$.

Group the numerals and like pronumerals and write in index form.

Example 4 Finding the prime factorisation

Express 48 as a product of prime factors in index form.

SOLUTION



$\therefore 48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

EXPLANATION

Choose a pair of factors of 48, for example 8 and 6.
Choose a pair of factors of 8, i.e. 2 and 4.
Choose a pair of factors of 6, i.e. 2 and 3.
Continue this process until the factors are all prime numbers.

Write the prime factorisation of 48.
Express in index notation.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 6A

UNDERSTANDING AND FLUENCY	1–10	4, 5–7(½), 8, 9, 10–11(½)	5–11(½)

1 Evaluate each of the following, without using a calculator.

a 5^2 **b** 2^3 **c** 3^3 **d** $(-4)^2$

e 10^3 **f** 2^4 **g** $(-2)^3$ **h** $(-2)^4$

2 Use a calculator, if necessary, to evaluate each of the following. Give answers **e** to **h** as fractions.

a 5^5 **b** $(-5)^5$ **c** 5^6 **d** $(-5)^6$

e $\left(\frac{1}{2}\right)^{10}$ **f** $\left(\frac{2}{3}\right)^5$ **g** $\left(\frac{3}{4}\right)^3$ **h** $\left(-\frac{3}{4}\right)^3$

3 Write the number that is the index in these expressions.

a 4^3 **b** 10^8 **c** $(-3)^7$ **d** $\left(\frac{1}{2}\right)^4$

e x^{11} **f** $(xy)^{13}$ **g** $\left(\frac{x}{2}\right)^9$ **h** $(1.3x)^2$

4 Write the prime factorisation of these numbers.

a 6 **b** 15 **c** 30 **d** 77

Example 1

5 Write each of the following in expanded form.

a a^4 **b** b^3 **c** x^3 **d** $(xp)^6$

e $(5a)^4$ **f** $(3y)^3$ **g** $4x^2y^5$ **h** $(pq)^2$

i $-3s^3t^2$ **j** $6x^3y^5$ **k** $5(yz)^6$ **l** $4(ab)^3$

Example 2

6 Write each of the following in expanded form and then evaluate.

a 6^2 **b** 2^4 **c** 3^5 **d** 12^1

e $(-2)^3$ **f** $(-1)^7$ **g** $(-3)^4$ **h** $(-5)^2$

i $\left(\frac{2}{3}\right)^3$ **j** $\left(\frac{3}{4}\right)^2$ **k** $\left(\frac{1}{6}\right)^3$ **l** $\left(\frac{5}{2}\right)^2$

m $\left(-\frac{2}{3}\right)^3$ **n** $\left(-\frac{3}{4}\right)^4$ **o** $\left(-\frac{1}{4}\right)^2$ **p** $\left(-\frac{5}{2}\right)^5$

Example 3a

7 Write each of the following in index form.

a $3 \times 3 \times 3$

b $8 \times 8 \times 8 \times 8 \times 8 \times 8$

c $y \times y$

d $3 \times x \times x \times x$

e $4 \times c \times c \times c \times c \times c$

f $5 \times 5 \times 5 \times d \times d$

g $x \times x \times y \times y \times y$

h $7 \times b \times 7 \times b \times 7$

Example 3b

8 Write each of the following in index form.

a $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ **b** $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

c $\frac{4}{7} \times \frac{4}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ **d** $\frac{7x}{9} \times \frac{7x}{9} \times \frac{y}{4} \times \frac{y}{4} \times \frac{y}{4}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 3c

9 Write each of the following in index form.

a $3 \times x \times y \times x \times 3 \times x \times 3 \times y$

b $3x \times 2y \times 3x \times 2y$

c $4d \times 2e \times 4d \times 2e$

d $6by(6by)(6y)$

e $3pq(3pq)(3pq)(3pq)$

f $7mn \times 7mn \times mn \times 7$

Example 4

10 Express each of the following as a product of prime factors in index form.

a 10 **b** 8 **c** 144

d 512 **e** 216 **f** 500

11 If $a = 3$, $b = 2$ and $c = -3$, evaluate these expressions.

a $(ab)^2$ **b** $(bc)^3$ **c** $\left(\frac{a}{c}\right)^4$ **d** $\left(\frac{b}{c}\right)^3$

e $(abc)^1$ **f** $c^2 + ab$ **g** ab^2c **h** c^2ab^3

PROBLEM-SOLVING AND REASONING 12, 13, 16, 17 | 12–14, 16, 17 | 13–17, 18(½)

12 Find the missing number.

a $3^{\square} = 81$ **b** $2^{\square} = 256$ **c** $\square^3 = 125$ **d** $\square^5 = 32$

e $\square^3 = -64$ **f** $\square^7 = -128$ **g** $\square^3 = \frac{1}{8}$ **h** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

13 Bacteria cells split in 2 every 5 minutes. New cells also continue splitting in the same way. Use a table to help answer the following.

a How long will it take for 1 cell to divide into:

i 4 cells?

ii 16 cells?

iii 64 cells?

b A single cell is set aside to divide for two hours. How many cells will there be after this time?

14 A sharebroker says he can triple your money every year, so you invest $1000 with him.

a How much should your investment be worth in 5 years?

a How many years should you invest for if you were hoping for a total of at least $100 000? Give a whole number of years.

15 A fat cat that was initially 12 kg reduces its weight by 10% each month. How long does it take for the cat to be at least 6 kg lighter than its original weight? Give your answer as a whole number of months.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

16 **a** Evaluate the following.

i 3^2 **ii** $(-3)^2$ **iii** $-(3)^2$ **iv** $-(-3)^2$

b Explain why the answers to parts **i** and **ii** are positive.

c Explain why the answers to parts **iii** and **iv** are negative.

17 **a** Evaluate the following.

i 2^3 **ii** $(-2)^3$ **iii** $-(2)^3$ **iv** $-(-2)^3$

b Explain why the answers to parts **i** and **iv** are positive.

c Explain why the answers to parts **ii** and **iii** are negative.

18 It is often easier to evaluate a decimal raised to a power by first converting the decimal to a fraction as shown below.

$$\begin{aligned}\left(0.5\right)^4 &= \left(\frac{1}{2}\right)^4 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16}\end{aligned}$$

Use this idea to evaluate these as a fraction.

a $(0.5)^3$ **b** $(0.25)^2$ **c** $(0.2)^3$

d $(0.5)^6$ **e** $(0.7)^2$ **f** $(1.5)^4$

g $(2.6)^2$ **h** $(11.3)^2$ **i** $(3.4)^2$

ENRICHMENT — — 19

LCM and HCF from prime factorisation

19 Last year you may have used prime factorisation to find the LCM (lowest common multiple) and the HCF (highest common factor) of two numbers. Here are the definitions.

- The LCM of two numbers in their prime factor form is the product of all the different primes raised to their highest power.
- The HCF of two numbers in their prime factor form is the product of all the common primes raised to their smallest power.

For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$

The prime factors 2 and 3 are common.

$\therefore$ LCM $= 2^2 \times 3 \times 5 = 60$ $\quad \therefore$ HCF $= 2 \times 3 = 6$

Find the LCM and HCF of these pairs of numbers by first writing them in prime factor form.

a 4, 6 **b** 42, 28

c 24, 36 **d** 10, 15

e 40, 90 **f** 100, 30

g 196, 126 **h** 2178, 1188

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6B Index laws for multiplying and dividing

Interactive
Widgets
HOTsheets
Walkthrough

An index law (or identity) is an equation that is true for all possible values of the variables in that equation. When multiplying or dividing numbers with the same base, index laws can be used to simplify the expression.

Consider $a^m \times a^n$.

Using expanded form: $a^m \times a^n = a \times \overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \overbrace{a \times a \times \ldots \times a}^{n \text{ factors of } a}$

$$= \overbrace{a \times a \times \ldots \times a}^{m+n \text{ factors of } a}$$

$$= a^{m+n}$$

So the total number of factors of a is $m + n$.

Also $a^m \div a^n = \dfrac{\overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a}}{\underbrace{a \times \ldots \times a \times a}_{n \text{ factors of } a}}$

$$= a^{m-n}$$

So the total number of factors of a is $m - n$.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Discovering laws for multiplying and dividing

Consider the two expressions $2^3 \times 2^5$ and $6^8 \div 6^6$.
Complete this working.

$$2^3 \times 2^5 = 2 \times \square \times \square \times 2 \times \square \times \square \times \square \times \square$$
$$= 2^{\square}$$

$$6^8 \div 6^6 = \frac{6 \times \square \times \square \times \square \times \square \times \square \times \square \times \square}{6 \times \square \times \square \times \square \times \square \times \square}$$
$$= \frac{6 \times 6}{1}$$
$$= 6^{\square}$$

- What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?
- Repeat the type of working given above and test your laws on these expressions.

a $3^2 \times 3^7$ **b** $4^{11} \div 4^8$



Index laws have applications in calculations involving very large (or very small) numbers.

Key ideas

- **Index law for multiplication**: $a^m \times a^n = a^{m+n}$
  - When multiplying terms with the same base, add the powers.
- **Index law for division**: $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$
  - When dividing terms with the same base, subtract the powers.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 5 Multiplying terms with numerical bases

Simplify, giving your answer in index form.

a $3^6 \times 3^4$ **b** $7^9 \div 7^5$

SOLUTION

a $3^6 \times 3^4 = 3^{10}$

b $7^9 \div 7^5 = 7^4$

EXPLANATION

$a^m \times a^n = a^{m+n}$ (add the powers)

$a^m \div a^n = a^{m-n}$ (subtract the powers)

Example 6 Multiplying algebraic expressions

Simplify each of the following using the law for multiplication.

a $x^4 \times x^5$ **b** $x^3y^4 \times x^2y$ **c** $3m^4 \times 2m^5$

SOLUTION

a $x^4 \times x^5 = x^{4+5}$
$= x^9$

b $x^3y^4 \times x^2y = x^{3+2}y^{4+1}$
$= x^5y^5$

c $3m^4 \times 2m^5 = 3 \times 2 \times m^{4+5}$
$= 6m^9$

EXPLANATION

The bases are the same so add the indices.

Add the indices corresponding to each different base. Recall $y = y^1$.

Multiply the numbers, then add the indices of the base m.

Example 7 Dividing algebraic expressions

Simplify each of the following using the law for division.

a $x^{10} \div x^2$ **b** $\dfrac{8a^6b^3}{12a^2b^2}$

SOLUTION

a $x^{10} \div x^2 = x^{10-2}$
$= x^8$

b $\dfrac{8a^6b^3}{12a^2b^2} = \dfrac{{}^{2}\cancel{8}a^{6-2}b^{3-2}}{{}^{3}\cancel{12}}$
$= \dfrac{2a^4b}{3}$

EXPLANATION

Subtract the indices.

Cancel the numbers using the highest common factor (4) and subtract the indices for each different base.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8 Combining index laws

Simplify each of the following using the index laws for multiplication and division.

a $x^2 \times x^3 \div x^4$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2}$

SOLUTION

a $x^2 \times x^3 \div x^4 = x^5 \div x^4$
$= x$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2} = \dfrac{16a^5b^4}{4a^4b^2}$
$= 4ab^2$

EXPLANATION

Add the indices for $x^2 \times x^3$.
Subtract the indices for $x^5 \div x^4$.

Multiply the numbers and add the indices for each different base in the numerator.
Subtract the indices of each different base and cancel the numbers.

Exercise 6B

UNDERSTANDING AND FLUENCY 1–3, 4–7(½) | 3, 4–8(½) | 4–8(½)

1 Write the missing words.

a Multiplying with common bases. If you _____ two terms with the same _____ you _____ the powers.

b Dividing with common bases. If you _____ two terms with the same _____ you _____ the powers.

2 Copy and complete to give an answer in index form. Use cancelling in parts **c**.

a $3^2 \times 3^4 = 3\times\square \times 3\square \times \square \times \square$
$= 3^{\square}$

b $6^4 \times 6^3 = 6 \times \square \times \square \times \square \times 6 \times \square \times \square$
$= 6^{\square}$

c $5^5 \div 5^3 = \dfrac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

d $9^4 \div 9^2 = \dfrac{9 \times \square \times \square \times \square}{9 \times \square}$
$= 9^{\square}$

e $x \times x^2 = \square \times \square \times \square$
$= x^{\square}$

f $x^5 \div x^2 = \dfrac{\square\times\square\times\square\times\square\times\square}{\square\times\square}$
$= \square$

3 Decide if these statements are true or false.

a $5 \times 5 \times 5 \times 5 = 5^4$

b $2^6 \times 2^2 = 2^{6+2}$

c $7^2 \times 7^4 = 7^{4-2}$

d $8^4 \div 8^2 = 8^{4+2}$

e $a \times a^2 = a^3$

f $a^5 \times a^2 = a^{5-2}$

Example 5

g $x^7 \div x = x^7$

h $b^4 \div b = b^3$

4 Simplify, giving your answers in index form.

a $2^4 \times 2^3$

b $5^6 \times 5^3$

c $7^2 \times 7^4$

d $8^9 \times 8$

e $3^4 \times 3^4$

f $6^5 \times 6^9$

g $3^7 \div 3^4$

h $6^8 \div 6^3$

i $5^4 \div 5$

j $10^6 \div 10^5$

k $9^9 \div 9^6$

l $(-2)^5 \div (-2)^3$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 6

5 Simplify each of the following using the law for multiplication.

a $x^4 \times x^3$ **b** $a^6 \times a^3$ **c** $t^5 \times t^3$ **d** $y \times y^4$

e $d^2 \times d$ **f** $y^2 \times y \times y^4$ **g** $b \times b^5 \times b^2$ **h** $q^6 \times q^3 \times q^2$

i $x^3y^3 \times x^4y^2$ **j** $x^7y^3 \times x^2y$ **k** $5x^3y^5 \times xy^4$ **l** $xy^4z \times 4xy$

m $3m^3 \times 5m^2$ **n** $4e^4f^2 \times 2e^2f^2$ **o** $5c^4d \times 4c^3d$ **p** $9yz^2 \times 2yz^5$

q $6m^{12} \times 3mn$ **r** $3x^4 \times 2x^3 \times 3x$ **s** $-9x^5 \times 3x^2$ **t** $(-3m^3n^2) \times (-4mn^7)$

Example 7

6 Simplify each of the following using the law for division.

a $a^6 \div a^4$ **b** $x^5 \div x^2$ **c** $\frac{q^{12}}{q^2}$ **d** $\frac{d^7}{d^6}$

e $\frac{8b^{10}}{4b^5}$ **f** $\frac{12d^{10}}{36d^5}$ **g** $\frac{4a^{14}}{2a^7}$ **h** $\frac{18y^{15}}{9y^7}$

i $9m^3 \div m^2$ **j** $14x^4 \div x$ **k** $5y^4 \div y^2$ **l** $6a^6 \div a^5$

m $\frac{3m^7}{12m^2}$ **n** $\frac{5w^2}{25w}$ **o** $\frac{4a^4}{20a^3}$ **p** $\frac{7x^5}{63x}$

q $\frac{16x^8y^6}{12x^2y^3}$ **r** $\frac{6s^6t^3}{14s^5t}$ **s** $\frac{8m^5n^4}{6m^4n^3}$ **t** $-\frac{5x^2y}{xy}$

Example 8

7 Simplify each of the following using the index laws.

a $b^5 \times b^2 \div b$ **b** $y^5 \times y^4 \div y^3$ **c** $c^4 \div c \times c^4$

d $x^4 \times x^2 \times x^5$ **e** $\frac{t^4 \times t^3}{t^6}$ **f** $\frac{p^2 \times p^7}{p^3}$

g $\frac{d^5 \times d^3}{d^2}$ **h** $\frac{x^9 \times x^2}{x}$ **i** $\frac{3x^3y^4 \times 8xy}{6x^2y^2}$

j $\frac{9b^4}{2g^3} \times \frac{4g^4}{3b^2}$ **k** $\frac{24m^7n^5}{5m^3n} \times \frac{5m^2n^4}{8mn^2}$ **l** $\frac{p^4q^3}{p^2q} \times \frac{p^6q^4}{p^3q^2}$

8 Simplify each of the following.

a $\frac{m^4}{n^2} \times \frac{m}{n^3}$ **b** $\frac{x}{y} \times \frac{x^3}{y}$ **c** $\frac{a^4}{b^3} \times \frac{b^6}{a}$

d $\frac{12a}{3c^3} \times \frac{6a^4}{4c^4}$ **e** $\frac{3f^2 \times 8f^7}{4f^3}$ **f** $\frac{4x^2b \times 9x^3b^2}{3xb}$

g $\frac{8k^4m^5}{5km^3} \times \frac{15km}{4k}$ **h** $\frac{12x^7y^3}{5x^4y} \times \frac{25x^2y^3}{8xy^4}$ **i** $-\frac{9m^5n^2 \times 4mn^3}{12mn \times m^4n^2} \times \frac{m^3n^2}{2m^2n}$

PROBLEM-SOLVING AND REASONING 9(½), 10, 12 | 9(½), 10, 12, 13 | 10, 11, 13, 14(½)

9 Write the missing number.

a $2^7 \times 2^{\square} = 2^{19}$ **b** $6^{\square} \times 6^3 = 6^{11}$ **c** $11^6 \div 11^{\square} = 11^3$

d $19^{\square} \div 19^2 = 19$ **e** $x^6 \times x^{\square} = x^7$ **f** $a^{\square} \times a^2 = a^{20}$

g $b^{13} \div b^{\square} = b$ **h** $y^{\square} \div y^9 = y^2$ **i** $\square \times x^2 \times 3x^4 = 12x^6$

j $15y^4 \div \square y^3 = y$ **k** $\square a^9 \div 4a = \frac{a^8}{2}$ **l** $13b^6 \div \square b^5 = \frac{b}{3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Evaluate without using a calculator.

a $7^7 \div 7^5$ **b** $10^6 \div 10^5$

c $13^{11} \div 13^9$ **d** $2^{20} \div 2^{17}$

e $101^5 \div 101^4$ **f** $200^{30} \div 200^{28}$

g $7 \times 31^{16} \div 31^{15}$ **h** $3 \times 50^{200} \div 50^{198}$

11 If m and n are positive integers, how many combinations of m and n satisfy the following?

a $a^m \times a^n = a^8$

b $a^m \times a^n = a^{15}$

12 The given answers are incorrect. Give the correct answer and explain the error made.

a $a^4 \times a = a^4$ **b** $x^7 \div x = x^7$

c $3a^5 \div 6a^3 = 2a^2$ **d** $5x^7 \div 10x^3 = \dfrac{1}{2x^4}$

e $2x^7 \times 3x^4 = 5x^{11}$ **f** $a^5 \div a^2 \times a = a^5 \div a^3 = a^2$

13 Given that $a = 2x$, $b = 4x^2$ and $c = 5x^3$, find expressions for:

a $2a$ **b** $3b$

c $2c$ **d** $-2a$

e abc **f** $\dfrac{c}{b}$

g $\dfrac{ab}{c}$ **h** $\dfrac{-2bc}{a}$

14 Simplify these expressions using the given variables.

a $2^x \times 2^y$ **b** $5^a \div 5^b$

c $t^x \times t^y$ **d** $3^x \div 3^y$

e $10^p \div 10^y$ **f** $t^x \div t^y$

g $2^p \times 2^q \div 2^r$ **h** $10^p \div 10^q \div 10^r$

i $2^a \times 2^{a+b} \times 2^{3a-b}$ **j** $a^{x-2}b^x \times a^{2x}b^3$

k $a^xb^y \times a^yb^x$ **l** $a^xb^y \div a^yb^x$

m $w^{x+2}b^x \div w^{2x} \times b^3$ **n** $\dfrac{a^x \times 3a^y}{3a^2}$

o $\dfrac{4p^a \times 5q^6}{20q^5}$ **p** $\dfrac{10k^xm^y}{8km^3} \div \dfrac{5k^xm^{2x}}{16k}$

ENRICHMENT — — 15, 16

Equal to *ab*

15 Show working to prove that these expressions simplify to ab.

a $\dfrac{5a^2b^7}{9a^3b} \times \dfrac{9a^4b^2}{5a^2b^7}$ **b** $\dfrac{3a^5bc^3}{6a^4c} \times \dfrac{4b^3}{2abc} \times \dfrac{2a^3b^2c}{2a^2b^4c^2}$

c $\dfrac{3a^4b^5}{a^5b^2} \div \dfrac{6b^3}{2a^2b}$ **d** $\dfrac{2a}{3a^2b^3} \times \dfrac{9a^4b^7}{ab^5} \div \dfrac{6a}{b^2}$

16 Make up your own expressions that simplify to ab. Test them on a friend.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6C The zero index and power of a power

Stage: 5.3# 5.3 5.3§ 5.2 5.2◊ 5.1 4

Interactive Widgets HOTsheets Walkthrough

Sometimes we find expressions already written in index form are raised to another power, such as $(2^3)^4$ or $(a^2)^5$.

Consider $(a^m)^n$.

Using expanded form: $(a^m)^n = \overbrace{a^m \times a^n \times \ldots \times a^m}^{n \text{ factors of } a^m}$

$$= \overbrace{\underbrace{a \times a \times \ldots \times a}_{m \text{ factors of } a} \times \underbrace{a \times a \times \ldots \times a}_{m \text{ factors of } a} \times \ldots \times \underbrace{a \times a \ldots \times a}_{m \text{ factors of } a}}^{m \times n \text{ factors of } a}$$

$$= a^{m \times n}$$

So the total number of factors of a is $m \times n$.

We also know that $a^m \div a^m = \dfrac{\cancel{a} \times \cancel{a} \times \cancel{a} \times \ldots \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \ldots \times \cancel{a}}$

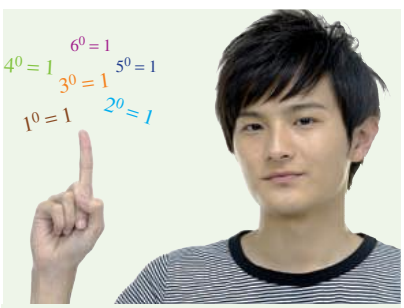
$$= \frac{1}{1}$$

$$= 1$$

But using index law number 2: $a^m \div a^m = a^{m-m}$

$$= a^0$$

This implies that $a^0 = 1$.



Any number (other than 0) raised to the power of zero is one.

Let's start: Power of a power and the zero index

Use the expanded form of 5^3 to simplify $(5^3)^2$ as shown.

$(5^3)^2 = 5 \times \square \times \square \times 5 \times \square \times \square$

$= 5^{\square}$

- Repeat these steps to also simplify $(3^2)^4$ and $(x^4)^2$.
- What do you notice about the given expression and answer in each case? Can you express this as a law or rule in words?

  Now complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0
Basic numeral	243	81				

- What pattern do you notice in the basic numerals?
- What conclusion do you come to regarding 3^0?

Key ideas

- **Index law for power of a power**: $(a^m)^n = a^{mn}$
  - When raising a term in index form to another power, retain the base and multiply the indices.
    For example: $(x^2)^3 = x^{2\times3} = x^6$.
- **The zero index**: $a^0 = 1$, where $a \neq 0$
  - Any term except 0 raised to the power of zero is 1.
    For example: $(2a)^0 = 1$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 9 Power of a power

Simplify each of the following.

a $(x^5)^4$ b $3(y^5)^2$

SOLUTION

a $(x^5)^4 = x^{5\times4}$
$= x^{20}$

b $3(y^5)^2 = 3y^{5\times2}$
$= 3y^{10}$

EXPLANATION

Retain x as the base and multiply the indices.

Retain y and multiply the indices. The power only applies to the bracketed term.

Example 10 Using the zero index

Apply the zero index rule to evaluate each of the following.

a $(-3)^0$ b $-(5x)^0$ c $2y^0 - (3y)^0$

SOLUTION

a $(-3)^0 = 1$

b $-(5x)^0 = -1$

c $2y^0 - (3y)^0 = 2 \times 1 - 1$
$= 2 - 1$
$= 1$

EXPLANATION

Any number raised to the power of 0 is 1.

Everything in the brackets is to the power of 0 so $(5x)^0$ is 1.

$2y^0$ has no brackets so the power applies to the y only so $2y^0 = 2 \times y^0 = 2 \times 1$ while $(3y)^0 = 1$.

Example 11 Combining index laws

Simplify each of the following by applying the various index laws.

a $(x^2)^3 \times (x^3)^5$ b $\dfrac{(m^3)^4}{m^7}$ c $\dfrac{4x^2 \times 3x^3}{6x^5}$

SOLUTION

a $(x^2)^3 \times (x^3)^5 = x^6 \times x^{15}$
$= x^{21}$

b $\dfrac{(m^3)^4}{m^7} = \dfrac{m^{12}}{m^7}$
$= m^5$

c $\dfrac{4x^2 \times 3x^3}{6x^5} = \dfrac{12x^5}{6x^5}$
$= 2x^0$
$= 2 \times 1$
$= 2$

EXPLANATION

Remove brackets first by multiplying indices, then add indices.

Remove brackets by multiplying indices, then simplify.

Simplify the numerator first by multiplying numbers and adding indices of base x.
Then cancel and subtract indices.
The zero index says $x^0 = 1$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 6C

UNDERSTANDING AND FLUENCY 1–3, 4–7(½) 3–8(½) 4–8(½)

1 Use your calculator to evaluate:

a 5^0 **b** 1^0 **c** 0^0

2 Write the missing numbers in these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

3 Copy and complete this working.

a $(4^2)^3 = \left(4 \times \square\right) \times \left(4 \times \square\right) \times \left(4 \times \square\right)$
$= 4^{\square}$

b $(12^3)^3 = \left(12 \times \square \times \square\right) \times \left(12 \times \square \times \square\right) \times \left(12 \times \square \times \square\right)$
$= 12^{\square}$

c $(x^4)^2 = \left(x \times \square \times \square \times \square\right) \times \left(x \times \square \times \square \times \square\right)$
$= x^{\square}$

d $(a^2)^5 = \left(a \times \square\right) \times \left(a \times \square\right) \times \left(a \times \square\right) \times \left(a \times \square\right) \times \left(a \times \square\right)$
$= a^{\square}$

Example 9

4 Simplify each of the following. Leave your answers in index form.

a $(y^6)^2$ **b** $(m^3)^6$ **c** $(x^2)^5$ **d** $(b^3)^4$
e $(3^2)^3$ **f** $(4^3)^5$ **g** $(3^5)^6$ **h** $(7^5)^2$
i $5(m^8)^2$ **j** $4(q^7)^4$ **k** $-3(c^2)^5$ **l** $2(j^4)^6$

Example 10

5 Evaluate each of the following.

a 5^0 **b** 9^0 **c** $(-6)^0$ **d** $(-3)^0$
e $-(4)^0$ **f** $\left(\frac{3}{4}\right)^0$ **g** $\left(-\frac{1}{7}\right)^0$ **h** $(4y)^0$
i $5m^0$ **j** $-3p^0$ **k** $6x^0 - 2x^0$ **l** $-5n^0 - (8n)^0$
m $(3x^4)^0$ **n** $1^0 + 2^0 + 3^0$ **o** $(1 + 2 + 3)^0$ **p** $100^0 - a^0$

Example 11a

6 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$ **b** $(3^4)^2 \times 3$ **c** $x \times (x^0)^5$ **d** $y^5 \times (y^2)^4$
e $b^5 \times (b^3)^3$ **f** $(a^2)^3 \times a^4$ **g** $(d^3)^4 \times (d^2)^6$ **h** $(y^2)^6 \times (y)^4$
i $z^4 \times (z^3)^2 \times (z^5)^3$ **j** $a^3f \times (a^4)^2 \times (f^4)^3$ **k** $x^2y \times (x^3)^4 \times (y^2)^2$ **l** $(s^2)^3 \times 5(r^0)^3 \times rs^2$

Example 11b

7 Simplify each of the following.

a $7^8 \div (7^3)^2$ **b** $(4^2)^3 \div 4^5$ **c** $(3^6)^3 \div (3^5)^2$
d $(m^3)^6 \div (m^2)^9$ **e** $(y^5)^3 \div (y^6)^2$ **f** $(h^{11})^2 \div (h^5)^4$
g $\frac{(b^2)^5}{b^4}$ **h** $\frac{(x^4)^3}{x^7}$ **i** $\frac{(y^3)^3}{y^3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11c

8 Simplify each of the following using various index laws.

a $\dfrac{3x^4 \times 6x^3}{9x^{12}}$ **b** $\dfrac{5x^5 \times 4x^2}{2x^{10}}$ **c** $\dfrac{24(x^4)^4}{8(x^4)^2}$

d $\dfrac{4(d^4)^3 \times (e^4)^2}{8(d^2)^5 \times e^7}$ **e** $\dfrac{6(m^3)^2(n^5)^3}{15(m^5)^0(n^2)^7}$ **f** $\dfrac{2(a^3)^4(b^2)^6}{16(a)^0(b^6)^2}$

PROBLEM-SOLVING AND REASONING	9, 12	9, 10, 12, 13	10, 11, 13, 14

9 There are 100 rabbits on Mt Burrow at the start of the year 2000. The rule for the number of rabbits N after t years (from the start of the year 2000) is $N = 100 \times 2^t$.

a Find the number of rabbits at:

i $t = 2$ **ii** $t = 6$ **iii** $t = 0$

b Find the number of rabbits at the beginning of:

i 2003 **ii** 2007 **iii** 2010

c How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

10 If m and n are positive integers, in how many ways can $(a^m)^n = a^{16}$?

11 Evaluate these without using a calculator.

a $(2^4)^8 \div 2^{30}$ **b** $(10^3)^7 \div 10^{18}$ **c** $(7^4)^9 \div 7^{36}$

d $((-1)^{11})^2 \div ((-1)^2)^{11}$ **e** $-2((-2)^3)^3 \div (-2)^8$ **f** $\dfrac{(5^2)^3}{(8^4)^7} \times \dfrac{(8^7)^4}{(5^3)^2}$

12 Explain the errors made in the following problems and then give the correct answers.

a $(a^4)^5 = a^9$ **b** $3(x^3)^2 = 9x^6$ **c** $(2x)^0 = 2$

13 **a** Simplify these by first working with the inner brackets. Leave your answers in index form.

i $((2^3)^4)^2$ **ii** $(((-2)^2)^5)^3$ **iii** $((x^6)^2)^7$ **iv** $(((a^2)^4)^3)^2$

b Simplify these expressions.

i $((2^a)^b)^c$ **ii** $((a^m)^n)^p$ **iii** $(x^{2y})^{3z}$

14 **a** Show that $\dfrac{5ab^2}{2ab^2} \div \dfrac{10a^4b^7}{4a^3b^8}$ is equal to 1.

b Make up your own expression like the one above for which the answer is equal to 1. Test it on a friend.

ENRICHMENT	—	—	15

Changing the base

15 The base of a number in index form can be changed using the power of a power.

For example: $8^2 = (2^3)^2$
$= 2^6$

Change the base numbers and simplify the following using the smallest possible base integer.

a 8^4 **b** 32^3 **c** 9^3

d 81^5 **e** 25^5 **f** 243^{10}

g 256^9 **h** 2401^{20} **i** $100\,000^{10}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6D Index laws extended

Stage

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

It is common to find expressions such as $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ in mathematical problems. These differ from most of the expressions in previous sections as they contain more than one single number or variable, connected by multiplication or division, raised to a power. These expressions can also be simplified using two index laws which effectively remove the brackets.

Consider $(a \times b)^m$.

Using expanded form: $(a \times b)^m = \overbrace{ab \times ab \times ab \times \ldots \times ab}^{m \text{ factors of } ab}$

$$= \overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \overbrace{b \times b \times \ldots \times b}^{m \text{ factors of } b}$$

$$= a^m \times b^m$$

So this becomes a product of m factors of a and m factors of b.

Also, $\left(\frac{a}{b}\right)^m = \overbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \ldots \times \frac{a}{b}}^{m \text{ factors of } \frac{a}{b}}$

$$= \frac{\overbrace{a \times a \times a \times \ldots \times a}^{m \text{ factors of } a}}{\underbrace{b \times b \times b \times \ldots \times b}_{m \text{ factors of } b}}$$

$$= \frac{a^m}{b^m}$$

So to remove the brackets, we can raise each of a and b to the power m.

Let's start: Extending the index laws

Use the expanded form of $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ to help simplify the expressions.

$(2x)^3 = 2x \times \square \times \square$

$= 2 \times 2 \times 2 \times \square \times \square \times \square$

$= 2^{\square} \times \square^{\square}$

$\left(\frac{x}{3}\right)^3 = \frac{x}{3} \times \square \times \square \times \square$

$= \frac{x \times \square \times \square \times \square}{3 \times \square \times \square \times \square}$

$= \frac{\square^{\square}}{\square^{\square}}$

- Repeat these steps to also simplify the expressions $(3y)^4$ and $\left(\frac{x}{2}\right)^5$.
- What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- $(a \times b)^m = (ab)^m = a^m b^m$
  - When multiplying two or more numbers or pronumerals raised to the power of m, raise each number in the brackets to the power of m. For example: $(2x)^2 = 2^2x^2 = 4x^2$.
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ and $b \neq 0$
  - When dividing two numbers or pronumerals raised to the power of m, raise each number in the brackets to the power of m. For example: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$.

Example 12 Using the power of a power result

Expand each of the following.

a $(5b)^3$ **b** $(-2x^3y)^4$ **c** $4(c^2d^3)^5$

SOLUTION

a $(5b)^3 = 5^3b^3$
$= 125b^3$

b $(-2x^3y)^4 = (-2)^4(x^3)^4y^4$
$= 16x^{12}y^4$

c $4(c^2d3)^5 = 4(c^2)^5(d^3)^5$
$= 4c^{10}d^{15}$

EXPLANATION

Raise each numeral and pronumeral in the brackets to the power of 3.
Evaluate $5^3 = 5 \times 5 \times 5$.

Raise each value in the brackets to the power of 4.
Evaluate $(-2)^4$ and simplify.

Raise each value in the brackets to the power of 5.
Note that the coefficient (4) is not raised to the power of 5. Simplify using index laws.

Example 13 Removing brackets from fractions

Simplify the following.

a $\left(\frac{6}{b}\right)^3$ **b** $\left(\frac{-2a^2}{3bc^3}\right)^4$ **c** $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4$

SOLUTION

a $\left(\frac{6}{b}\right)^3 = \frac{6^3}{b^3}$
$= \frac{216}{b^3}$

b $\left(\frac{-2a^2}{3bc^3}\right) = \frac{(-2)^4a^8}{3^4b^4c^{12}}$
$= \frac{16a^8}{81b^4c^{12}}$

c $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4 = \frac{x^6y^9}{c^3} \times \frac{x^4c^4}{y^4}$
$= \frac{x^{10}y^9c^4}{c^3y^4}$
$= x^{10}y^5c$

EXPLANATION

Raise each value in the brackets to the power of 3 and evaluate 6^3.

Raise each value in the brackets to the power of 4.
Recall $(a^2)^4 = a^{2\times4}$ and $(c^3)^4 = c^{3\times4}$.
Evaluate $(-2)^4$ and 3^4.

Rasie each value in the brackets to the power. Multiply the numerators, then divide by subtracting powers of the same base.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 6D

UNDERSTANDING AND FLUENCY 1, 2, 3–5(½) 2, 3–6(½) 3–6(½)

1 Copy and complete.

a $(a \times b)^m = a^m \times \square$ **b** $\left(\frac{a}{b}\right)^m = \frac{a^m}{\square}$

2 Copy and complete this working.

a $(5a)^3 = 5a \times \square \times \square$
$= 5 \times 5 \times 5 \times a \times \square \times \square$
$= 5^3 \times \square$

b $(ab)^4 = ab \times \square \times \square \times \square$
$= a \times \square \times \square \times \square \times b \times \square \times \square \times \square$
$= a^4 \times \square$

c $\left(\frac{x}{6}\right)^3 = \frac{x}{6} \times \square \times \square$
$= \frac{x \times \square \times \square}{6 \times \square \times \square}$
$= \frac{x^3}{\square}$

d $\left(\frac{a}{b}\right)^5 = \frac{a}{b} \times \square \times \square \times \square \times \square$
$= \frac{a \times \square \times \square \times \square \times \square}{b \times \square \times \square \times \square \times \square}$
$= \frac{a^5}{\square}$

Example 12

3 Expand each of the following.

a $(2x)^3$ **b** $(5y)^2$ **c** $(4a^2)^3$ **d** $(-3r)^2$
e $-(3b)^4$ **f** $-(7r)^3$ **g** $(-2h^2)^4$ **h** $(5c^2d^3)^4$
i $(2x^3y^2)^5$ **j** $9(p^2q^4)^3$ **k** $2(x^3y)^2$ **l** $(8t^2u^9v^4)^0$
m $(-3w^3y)^3$ **n** $-4(p^4qr)^2$ **o** $(-5s^7t)^2$ **p** $-(-2x^4yz^3)^3$

Example 13a, b

4 Expand the following.

a $\left(\frac{p}{q}\right)^3$ **b** $\left(\frac{x}{y}\right)^4$ **c** $\left(\frac{4}{y}\right)^3$ **d** $\left(\frac{5}{p^2}\right)^4$
e $\left(\frac{2}{r^3}\right)^2$ **f** $\left(\frac{s^3}{7}\right)^2$ **g** $\left(\frac{2m}{n}\right)^5$ **h** $\left(\frac{2a^2}{3}\right)^3$
i $\left(\frac{3n^3}{2m^4}\right)^3$ **j** $\left(\frac{-2r}{n}\right)^4$ **k** $\left(\frac{-3f}{2^3g^5}\right)^2$ **l** $\left(\frac{5w^4y}{2x^3}\right)^2$
m $\left(\frac{-3x}{2y^3g^5}\right)^2$ **n** $\left(\frac{3km^3}{4n^7}\right)^3$ **o** $-\left(\frac{-5w^4y}{2zx^3}\right)^2$ **p** $\left(-\frac{3x^2y^3}{2a^5b^3}\right)^2$

5 Simplify each of the following by applying the various index laws.

a $a(3b)^2$ **b** $a(3b^2)^3$ **c** $-3(2a^3b^4)^2a^2$ **d** $2(3x^2y^3)^3$
e $(-4b^2c^5d)^3$ **f** $a(2a)^3$ **g** $a(3a^2)^2$ **h** $5a^3(-2a^4b)^3$
i $-5(-2m^3pt^2)^5$ **j** $-(-7d^2f^4g)^2$ **k** $-2(-2^3x^4yz^3)^3$ **l** $-4a^2b^3(-2a^3b^2)^2$

Example 13c

6 Simplify each of the following.

a $((x^2)^3)^4$ **b** $((2x^3)^2)^4$ **c** $(a^3b^2)^3 \times (a^4b)^2$ **d** $(a^2b)^3 \times (ab^2)^4$
e $\frac{(2m^3n)^3}{m^4}$ **f** $\frac{3(2^2c^4d^5)^3}{(2cd^2)^4}$ **g** $\left(\frac{-3x^2y^0}{5a^5b^3}\right)^3$ **h** $\frac{-3(2^4a^4b^3)^3}{(-2^3a^2b)^4}$
i $\frac{-5(3^5m^3n^2)^2}{(-3^3m^2n)^3}$ **j** $\left(\frac{a^3b}{c}\right)^3 \times \left(\frac{ac^4}{b}\right)^2$ **k** $\left(\frac{x^2z}{y}\right)^4 \times \left(\frac{xy^2}{z}\right)^3$ **l** $\left(\frac{r^3s}{t}\right)^2 \div \left(\frac{s}{rt^4}\right)^3$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING	7, 9	7, 8(½), 9, 10	7–10

7 The rule for the number of seeds germinating in a glass house over a two-week period is given by $N = \left(\frac{t}{2}\right)^3$, where N is the number of germinating seeds and t is the number of days.

a Find the number of germinating seeds after:
i 4 days **ii** 10 days

b Use powers of fractions to rewrite the rule without brackets.

c Use your rule in part **b** to find the number of seeds germinating after:
i 6 days **ii** 4 days

d Find the number of days required to germinate:
i 64 seeds **ii** 1 seed

8 Find the value of a that makes these equations true, given $a > 0$.

a $\left(\frac{a}{3}\right)^2 = \frac{4}{9}$ **b** $\left(\frac{a}{2}\right)^4 = 16$

c $(5a)^3 = 1000$ **d** $(2a)^4 = 256$

e $\left(\frac{2a}{3}\right)^2 = \frac{4}{9}$ **f** $\left(\frac{6a}{7}\right)^3 = 1728$

9 Rather than evaluating $\frac{2^4}{4^4}$ as $\frac{16}{256} = \frac{1}{16}$, it is easier to evaluate $\frac{2^4}{4^4}$ in the following way (right).

$$\frac{2^4}{4^4} = \left(\frac{2}{4}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1^4}{2^4} = \frac{1}{16}$$

a Explain why this method is helpful.

b Use this idea to evaluate these without the use of a calculator.
i $\frac{6^3}{3^3}$ **ii** $\frac{10^4}{5^4}$ **iii** $\frac{4^4}{12^4}$ **iv** $\frac{3^3}{30^3}$

10 Decide if the following are true or false. Give reasons.

a $(-2x)^2 = -(2x)^2$ **b** $(-3x)^3 = -(3x)^3$

c $\left(\frac{-5}{x}\right)^5 = -\left(\frac{5}{x}\right)^5$ **d** $\left(\frac{-4}{x}\right)^4 = -\left(\frac{4}{x}\right)^4$

ENRICHMENT	—	—	11

False laws

11 Consider the equation $(a + b)^2 = a^2 + b^2$.

a Using $a = 2$ and $b = 3$, evaluate $(a + b)^2$.

b Using $a = 2$ and $b = 3$, evaluate $a^2 + b^2$.

c Would you say that the equation is true for all values of a and b?

d Now decide if $(a - b)^2 = a^2 - b^2$ for all values of a and b. Give an example to support your answer.

e Decide if these equations are true or false for all values of a and b.
i $(-ab)^2 = a^2b^2$ **ii** $-(ab)^2 = a^2b^2$

iii $\left(\frac{-a}{b}\right)^3 = \frac{-a^3}{b^3}$ **iv** $\left(\frac{-a}{b}\right)^4 = \frac{-a^4}{b^4}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6E Negative indices

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

We know that $2^3 = 8$ and $2^0 = 1$ but what about 2^{-1} or 2^{-6}? Such numbers written in index form using negative indices also have meaning in mathematics.

Consider $a^2 \div a^5$.

Method 1: Using index law for division

$$\frac{a^2}{a^5} = a^{2-5}$$
$$= a^{-3} \text{ (from index law for division)}$$

Method 2: By cancelling

$$\frac{a^2}{a^5} = \frac{a^1 \times a^1}{a \times a \times a \times a_1 \times a_1}$$
$$= \frac{1}{a^3}$$

$$\therefore a^{-3} = \frac{1}{a^3}$$

Also, using the index law for multiplication, we can write:

$$a^m \times a^{-m} = a^{m+(-m)}$$
$$= a^0$$
$$= 1$$

So dividing by a^m we have $a^{-m} = \frac{1}{a^m}$, or dividing by a^{-m} we have $a^m = \frac{1}{a^{-m}}$.

Let's start: Continuing the pattern

Explore the use of negative indices by completing this table.

Index form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Whole number or fraction	16	8					$\frac{1}{4} = \frac{1}{2^2}$	

÷ 2 ÷ 2 ÷ 2

- What do you notice about the numbers with negative indices in the top row in comparison to the fractions in the second row?
- Can you describe this connection formally in words?
- What might be another way of writing 2^{-7} or 5^{-4}?

Key ideas

- $a^{-m} = \frac{1}{a^m}$ For example: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ (2^{-3} is the reciprocal of 2^3)
- $\frac{1}{a^{-m}} = a^m$ For example: $\frac{1}{2^{-3}} = 2^3 = 8$ (2^3 is the reciprocal of 2^{-3})
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$ For example: $\left(\frac{x^2}{3}\right)^{-4} = \left(\frac{3}{x^2}\right)^4 = \frac{81}{x^8}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- a^{-1} is the reciprocal of a^1, which is $\frac{1}{a}$. For example: $5^{-1} = \frac{1}{5}$, which is the reciprocal of 5
- $\left(\frac{a}{b}\right)^{-1}$ is the reciprocal of $\frac{a}{b}$, which is $\frac{b}{a}$. For example: $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$, which is the reciprocal of $\frac{2}{3}$

Example 14 Writing expressions using positive indices

Express each of the following with positive indices only.

a x^{-2} b $3a^{-2}b^4$

SOLUTION

a $x^{-2} = \frac{1}{x^2}$

b $3a^{-2}b^4 = \frac{3}{1} \times \frac{1}{a^2} \times \frac{b^4}{1}$
$= \frac{3b^4}{a^2}$

EXPLANATION

$a^{-m} = \frac{1}{a^m}$

Rewrite a^{-2} using a positive power and multiply numerators and denominators.

Example 15 Using $\frac{1}{a^{-m}} = a^m$

Express each of the following using positive indices only.

a $\frac{1}{c^{-2}}$ b $\frac{x^{-3}}{y^{-5}}$ c $\frac{5}{x^3y^{-4}}$

SOLUTION

a $\frac{1}{c^{-2}} = c^2$

b $\frac{x^{-3}}{y^{-5}} = x^{-3} \times \frac{1}{y^{-5}}$
$= \frac{1}{x^3} \times \frac{y^5}{1}$
$= \frac{y^5}{x^3}$

c $\frac{5}{x^3y^{-4}} = \frac{5}{x^3} \times \frac{1}{y^{-4}}$
$= \frac{5y^4}{x^3}$

EXPLANATION

$\frac{1}{a^{-m}} = a^m$

Express x^{-3} and $\frac{1}{y^{-5}}$ with positive indices using $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$.

Express $\frac{1}{y^{-4}}$ as a positive power, y^4.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 16 Evaluating without a calculator

Express the following using positive powers only, then evaluate without using a calculator.

a 3^{-4} **b** $\frac{-5}{3^{-2}}$ **c** $\left(\frac{2}{3}\right)^{-4}$

SOLUTION

a $3^{-4} = \frac{1}{3^4}$
$= \frac{1}{81}$

b $\frac{-5}{3^{-2}} = -5 \times \frac{1}{3^{-2}}$
$= -5 \times 3^2$
$= -5 \times 9$
$= -45$

c $\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}}$
$= 2^{-4} \times \frac{1}{3^{-4}}$
$= \frac{3^4}{2^4}$
$= \frac{81}{16}$

EXPLANATION

Express 3^{-4} as a positive power and evaluate 3^4.

Express $\frac{1}{3^{-2}}$ as a positive power and simplify.

Apply the power to each numeral in the brackets.

Express 2^{-4} and $\frac{1}{3^{-4}}$ with positive indices and evaluate.

Exercise 6E

UNDERSTANDING AND FLUENCY	1, 2(a), 3–5(½)	2(b), 3–6(½)	3–6(½)

1 Write the following using positive indices. For example: $\frac{1}{8} = \frac{1}{2^3}$.

a $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{1}{125}$ **d** $\frac{1}{27}$

2 Complete the tables with the missing numbers.

a

Index form	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	81	27					$\frac{1}{9} = \frac{1}{3^2}$	

÷3 ÷3 ÷3

b

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	10000							$\frac{1}{1000} = \frac{1}{10^3}$

÷10 ÷10

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14

3 Express each of the following with positive indices only.

a x^{-1} **b** a^{-4} **c** b^{-6} **d** 5^{-2}

e 4^{-3} **f** 9^{-1} **g** $5x^{-2}$ **h** $4y^{-3}$

i $3m^{-5}$ **j** $p^{7}q^{-2}$ **k** mn^{-4} **l** $x^{4}y^{-4}$

m $2a^{-3}b^{-1}$ **n** $7r^{-2}s^{-3}$ **o** $5^{-1}u^{-8}v^{2}$ **p** $9^{-1}m^{-3}n^{-5}$

Example 15a

4 Express each of the following using positive indices only.

a $\frac{1}{y^{-1}}$ **b** $\frac{1}{b^{-2}}$ **c** $\frac{1}{m^{-5}}$ **d** $\frac{1}{x^{-4}}$

e $\frac{7}{q^{-1}}$ **f** $\frac{3}{t^{-2}}$ **g** $\frac{5}{h^{-4}}$ **h** $\frac{4}{p^{-4}}$

i $\frac{a}{b^{-2}}$ **j** $\frac{e}{d^{-1}}$ **k** $\frac{2n^{2}}{m^{-3}}$ **l** $\frac{y^{5}}{3x^{-2}}$

m $\frac{-3}{7y^{-4}}$ **n** $\frac{-2}{b^{-8}}$ **o** $\frac{-3g}{4h^{-3}}$ **p** $\frac{(-3u)^{2}}{5t^{-2}}$

Example 15b

5 Express each of the following using positive indices only.

a $\frac{a^{-3}}{b^{-3}}$ **b** $\frac{x^{-2}}{y^{-5}}$ **c** $\frac{g^{-2}}{h^{-3}}$ **d** $\frac{m^{-1}}{n^{-1}}$

e $\frac{5^{-1}}{7^{-3}}$ **f** $\frac{3^{-2}}{4^{-3}}$ **g** $\frac{5^{-2}}{6^{-1}}$ **h** $\frac{4^{-3}}{8^{-2}}$

Example 15c

6 Express each of the following using positive indices only.

a $\frac{7}{x^{-4}y^{3}}$ **b** $\frac{1}{u^{-3}v^{2}}$ **c** $\frac{a^{-3}5^{-1}}{y^{-3}}$ **d** $\frac{2a^{-4}}{b^{-5}c^{2}}$

e $\frac{5a^{2}c^{-4}}{6b^{-2}d}$ **f** $\frac{5^{-1}h^{3}k^{-2}}{4^{-1}m^{-2}p}$ **g** $\frac{4t^{-1}u^{-2}}{3^{-1}v^{2}w^{-6}}$ **h** $\frac{4^{-1}x^{2}y^{-5}}{4m^{-1}n^{-4}}$

PROBLEM-SOLVING AND REASONING 7(½), 10 | 7(½), 8, 10 | 7(½), 8–11

Example 16

7 Evaluate the following without using a calculator. *Hint*: write expressions using positive indices.

a 5^{-1} **b** 3^{-2} **c** $(-4)^{-2}$ **d** -5^{-2}

e 4×10^{-2} **f** -5×10^{-3} **g** -3×2^{-2} **h** $8 \times (2^{2})^{-2}$

i $6^{4} \times 6^{-6}$ **j** $8^{-7} \times (8^{2})^{3}$ **k** $(5^{2})^{-1} \times (2^{-2})^{-1}$ **l** $(3^{-2})^{2} \times (7^{-1})^{-1}$

m $\frac{1}{8^{-1}}$ **n** $\frac{1}{10^{-2}}$ **o** $\frac{-2}{5^{-3}}$ **p** $\frac{2}{2^{-3}}$

q $\frac{-5}{2^{-1}}$ **r** $\frac{2^{3}}{2^{-3}}$ **s** $\left(\frac{3}{8}\right)^{-2}$ **t** $\left(\frac{-4}{3}\right)^{-3}$

u $\frac{(-5)^{2}}{2^{-2}}$ **v** $\frac{(3^{-2})^{3}}{3^{-5}}$ **w** $\frac{(-2^{-3})^{-3}}{(2^{-2})^{-4}}$ **x** $\left(\frac{2^{-4}}{7^{-2}}\right)^{-1} \times \left(\frac{7^{-1}}{2^{-1}}\right)^{-4}$

8 The mass of a small insect is 2^{-9} kg. How many grams is this? Round to 2 decimal places.

9 Find the value of x in these equations.

a $2^{x} = \frac{1}{16}$ **b** $5^{x} = \frac{1}{625}$ **c** $(-3)^{x} = \frac{1}{81}$

d $(0.5)^{x} = 2$ **e** $(0.2)^{x} = 25$ **f** $3(2^{2x}) = 0.75$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Describe the error made in these problems then give the correct answer.

a $2x^{-2} = \dfrac{1}{2x^2}$ **b** $\dfrac{5}{a^4} = \dfrac{a^{-4}}{5}$ **c** $\dfrac{2}{(3b)^{-2}} = \dfrac{2b^2}{9}$

11 Consider the number $\left(\dfrac{2}{3}\right)^{-1}$.

a Complete this working: $\left(\dfrac{2}{3}\right)^{-1} = \dfrac{1}{\left(\frac{2}{3}\right)}$

$= 1 \div \square$

$= 1 \times \square$

$= \dfrac{3}{2}$

b Show similar working as in part **a** to simplify these.

i $\left(\dfrac{5}{4}\right)^{-1}$ **ii** $\left(\dfrac{2}{7}\right)^{-1}$ **iii** $\left(\dfrac{x}{3}\right)^{-1}$ **iv** $\left(\dfrac{a}{b}\right)^{-1}$

c What conclusion can you come to regarding the simplification of fractions raised to the power -1?

d Simplify these fractions.

i $\left(\dfrac{2}{3}\right)^{-2}$ **ii** $\left(\dfrac{4}{5}\right)^{-2}$ **iii** $\left(\dfrac{1}{2}\right)^{-5}$ **iv** $\left(\dfrac{7}{3}\right)^{-3}$

ENRICHMENT — — 12

Exponential equations

12 To find x in $2^x = 32$ you could use trial and error; however, the following approach is more useful.

$2^x = 32$
$2^x = 2^5$ (express 32 using a matching base)
$\therefore x = 5$

Use this idea to solve for x in these equations.

a $2^x = 16$ **b** $3^x = 81$ **c** $5^x = 25$

d $\left(\dfrac{1}{2}\right)^x = \dfrac{1}{8}$ **e** $\left(\dfrac{1}{7}\right)^x = \dfrac{1}{49}$ **f** $\left(\dfrac{2}{3}\right)^x = \dfrac{16}{81}$

g $4^{2x} = 64$ **h** $3^{x+1} = 243$ **i** $2^{3x-1} = 64$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6F Scientific notation

Interactive

Widgets

HOTsheets

Walkthrough

It is common in the practical world to be working with very large or very small numbers. For example, the number of cubic metres of concrete used to build the Hoover Dam in the United States was $3\,400\,000\,m^3$ and the mass of a molecule of water is 0.000000000000000000000000299 grams. Such numbers can be written more efficiently using powers of 10 with positive or negative indices. This is called scientific notation or standard form. The number is written using a number between 1 inclusive and 10 and this is multiplied by a power of 10. Such notation is also used to state very large and very small time intervals.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



At the time of construction, the Hoover Dam was the largest concrete structure in the world.

Let's start: Building scientific notation

Use the information given to complete the table.

Decimal form	Working	Scientific notation
2350000	2.35×1000000	2.35×10^6
502170		
314060000		
0.000298	$2.98 \div 10000 = \frac{2.98}{10^4}$	2.98×10^{-4}
0.000004621		
0.003082		

- Discuss how each number using scientific notation is formed.
- When are positive indices used and when are negative indices used?
- Where does the decimal point appear to be placed when using scientific notation?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Positive numbers written in **scientific notation** are expressed in the form $a \times 10^m$ where $1 \leq a < 10$ and m is an integer.
- Large numbers will use positive powers of 10.
  For example: 38 million years $= 38\,000\,000$ years
  $= 3.8 \times 10^7$ years
- Small numbers will use negative powers of 10.
  For example: 417 nanoseconds $= 0.000000417$ seconds
  $= 4.17 \times 10^{-7}$ seconds
- To write numbers using scientific notation, place the decimal point after the first non-zero digit and then multiply by a power of 10.
- Examples of units where very large or small numbers may be used:
  - 2178 km $= 2\,178\,000$ m $= 2.178 \times 10^6$ metres
  - 4517 centuries $= 451\,700$ years $= 4.517 \times 10^5$ years
  - 12 million years $= 12\,000\,000$ years $= 1.2 \times 10^7$ years
  - 2320 tonnes $= 2.32 \times 10^6$ kg
  - 27 microns (millionth of a metre) $= 0.000027$ m $= 2.7 \times 10^{-5}$ metres
  - 109 milliseconds (thousandths of a second) $= 0.109$ seconds $= 1.09 \times 10^{-1}$ seconds
  - 3.8 microseconds (millionth of a second) $= 0.0000038 = 3.8 \times 10^{-6}$ seconds
  - 54 nanoseconds (billionth of a second) $= 0.000000054 = 5.4 \times 10^{-8}$ seconds.



The big bang theory deals with measurements from microscopically small to astronomically large, all expressed conveniently in scientific notation.

Example 17 Writing numbers using scientific notation

Write the following using scientific notation.

a 4500000

b 0.0000004

SOLUTION

a $4500000 = 4.5 \times 10^6$

b $0.0000004 = 4 \times 10^{-7}$

EXPLANATION

Place the decimal point after the first non-zero digit (4) then multiply by 10^6 since decimal place has been moved 6 places to the left.

The first non-zero digit is 4. Multiply by 10^{-7} since decimal place has been moved 7 places to the right.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18 Writing numbers in decimal form

Express each of the following in decimal form.

a 9.34×10^5 **b** 4.71×10^{-5}

SOLUTION

a $9.34 \times 10^5 = 934000$

b $4.71 \times 10^{-5} = 0.0000471$

EXPLANATION

Move the decimal point 5 places to the right.

Move the decimal point 5 places to the left and insert zeros where necessary.

Exercise 6F

UNDERSTANDING AND FLUENCY	1–8	3, 4–8(½)	4–8(½)

1 Which of the numbers 1000, 10000 or 100000 completes each equation?

a $6.2 \times$ _____ $= 62000$
b $9.41 \times$ _____ $= 9410$
c $1.03 \times$ _____ $= 103000$
d $3.2 \div$ _____ $= 0.0032$
e $5.16 \div$ _____ $= 0.0000516$
f $1.09 \div$ _____ $= 0.000109$

2 Write the following as powers of 10.

a 100000 **b** 100 **c** 1000000000

3 If these numbers were written using scientific notation, would positive or negative indices be used?

a 2000 **b** 0.0004 **c** 19300 **d** 0.00101431

Example 17a

4 Write the following in scientific notation.

a 40000 **b** 2300000000000 **c** 16000000000
d −7200000 **e** −3500 **f** −8800000
g 52 hundreds **h** 3 million **i** 21 thousands

Example 17b

5 Write the following in scientific notation.

a 0.000003 **b** 0.0004 **c** −0.00876
d 0.00000000073 **e** −0.00003 **f** 0.000000000125
g −0.00000000809 **h** 0.000000024 **i** 0.0000345

6 Write each of the following numbers in scientific notation.

a 6000 **b** 720000 **c** 324.5
d 7869.03 **e** 8459.12 **f** 0.2
g 0.000328 **h** 0.00987 **i** −0.00001
j −460100000 **k** 17467 **l** −128

Example 18a

7 Express each of the following in decimal form.

a 5.7×10^4 **b** 3.6×10^6 **c** 4.3×10^8
d 3.21×10^7 **e** 4.23×10^5 **f** 9.04×10^{10}
g 1.97×10^8 **h** 7.09×10^2 **i** 6.357×10^5

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18b

8 Express each of the following in decimal form.

a 1.2×10^{-4} **b** 4.6×10^{-6} **c** 8×10^{-10}

d 3.52×10^{-5} **e** 3.678×10^{-1} **f** 1.23×10^{-7}

g 9×10^{-5} **h** 5×10^{-2} **i** 4×10^{-1}

PROBLEM-SOLVING AND REASONING 9–11(½), 15(½) | 9–11(½), 12, 13, 15(½) | 9–11(½), 13, 14, 15–16(½)

9 Express each of the following approximate numbers using scientific notation.

a The mass of Earth is 6 000 000 000 000 000 000 000 000 kg.

b The diameter of Earth is 40 000 000 m.

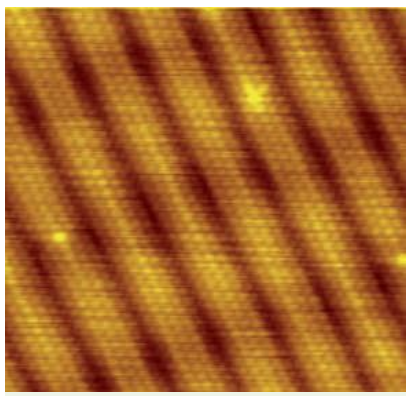
c The diameter of a gold atom is 0.0000000001 m.

d The radius of Earth's orbit around the Sun is 150 000 000 km.

e The universal constant of gravitation is 0.0000000000667 $N\,m^2/kg^2$.

f The half-life of polonium-214 is 0.00015 seconds.

g Uranium-238 has a half-life of 4 500 000 000 years.



This image of gold atoms has been formed by a very powerful electron microscope.

10 Express each of the following in decimal form.

a Neptune is approximately 4.6×10^9 km from Earth.

b A population of bacteria contains 8×10^{12} organisms.

c The Moon is approximately 3.84×10^5 km from Earth.

d A fifty-cent coin is approximately 3.8×10^{-3} m thick.

e The diameter of the nucleus of an atom is approximately 1×10^{-14} m.

f The population of a city is 7.2×10^5.



Earth is about 3.84×10^5 kilometres from the Moon.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Write the following using scientific notation in the units given in the brackets.

Recall: 1 second = 1000 milliseconds
1 millisecond = 1000 microseconds
1 microsecond = 1000 nanoseconds

a 3 million years (months)
b 0.03 million years (months)
c 492 milliseconds (seconds)
d 0.38 milliseconds (seconds)
e 2.1 microseconds (seconds)
f 0.052 microseconds (seconds)
g 4 nanoseconds (seconds)
h 139.2 nanoseconds (seconds)
i 39.5 centuries (years)
j 438 decades (years)
k 430 tonnes (kg)
l 0.5 kg (grams)
m 2.3 hours (milliseconds)
n 5 minutes (nanoseconds)

12 When Sydney was planning for the 2000 Olympic Games, the Olympic Organising Committee made the following predictions:

- The cost of staging the games would be \$1.7 billion ($\$1.7 \times 10^9$) (excluding infrastructure). In fact, \$140 million extra was spent on staging the games.
- The cost of constructing or upgrading infrastructure would be \$807 million.

Give each of the following answers in scientific notation.

a The actual total cost of staging the Olympic Games.
b The total cost of staging the games and constructing or upgrading the infrastructure.

13 Two planets are 2.8×10^8 km and 1.9×10^9 km from their closest Sun. What is the difference between these two distances in scientific notation?

14 Two particles weigh 2.43×10^{-2} g and 3.04×10^{-3} g. Find the difference in their weight in scientific notation.

15 The number 47×10^4 is not written using scientific notation since 47 is not a number between 1 and 10. The following shows how to convert to scientific notation.

$47 \times 10^4 = 4.7 \times 10 \times 10^4$
$= 4.7 \times 10^5$

Write these numbers using scientific notation.

a 32×10^3
b 41×10^5
c 317×10^2
d 5714×10^2
e 0.13×10^5
f 0.092×10^3
g 0.003×10^8
h 0.00046×10^9
i 61×10^{-3}
j 424×10^{-2}
k 1013×10^{-6}
l 490000×10^{-1}
m 0.02×10^{-3}
n 0.0004×10^{-2}
o 0.00372×10^{-1}
p 0.04001×10^{-6}

16 Use power of a power $(a^m)^n = a^{m \times n}$ and powers of products $(a \times b)^m = a^m \times b^m$ to simplify these numbers. Then write your answer in scientific notation where necessary.

a $(2 \times 10^2)^3$
b $(3 \times 10^4)^2$
c $(2.5 \times 10^{-2})^2$
d $(1.5 \times 10^{-3})^3$
e $(2 \times 10^{-3})^{-3}$
f $(5 \times 10^{-4})^{-2}$
g $\left(\frac{1}{3} \times 10^2\right)^{-2}$
h $\left(\frac{2}{5} \times 10^{-4}\right)^{-1}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

ENRICHMENT	—	—	17–19

Scientific notation with index laws

17 Use index laws to simplify these and write using scientific notation.

a $(3 \times 10^2) \times (2 \times 10^4)$
b $(4 \times 10^4) \times (2 \times 10^7)$
c $(8 \times 10^6) \div (4 \times 10^2)$
d $(9 \times 10^{20}) \div (3 \times 10^{11})$
e $(7 \times 10^2) \times (8 \times 10^2)$
f $(1.5 \times 10^3) \times (8 \times 10^4)$
g $(6 \times 10^4) \div (0.5 \times 10^2)$
h $(1.8 \times 10^6) \div (0.2 \times 10^3)$
i $(3 \times 10^{-4}) \times (3 \times 10^{-5})$
j $(15 \times 10^{-2}) \div (2 \times 10^6)$
k $(4.5 \times 10^{-3}) \div (3 \times 10^2)$
l $(8.8 \times 10^{-1}) \div (8.8 \times 10^{-1})$

18 Determine, using index laws, how long it takes for light to travel from the Sun to Earth in seconds given that Earth is 1.5×10^8 km from the Sun and the speed of light is 3×10^5 km/s.

19 Using index laws and the fact that the speed of light is equal to 3×10^5 km/s, determine:

a how far light travels in one nanosecond (1×10^{-9} seconds). Answer in scientific notation in km then convert your answer to cm.
b how long light takes to travel 300 kilometres. Answer in seconds.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6G Scientific notation using significant figures









The number of digits used to record measurements depends on how accurately the measurements can be recorded. The volume of Earth, for example, has been calculated as 1 083 210 000 000 km^3. This shows six significant figures and could be written using scientific notation as 1.08321×10^{12}. A more accurate calculation may include more non-zero digits in the last seven places.

The mass of a single oxygen molecule is known to be 0.000000000000000000000000053 g.
This shows two significant figures and is written using scientific notation as 5.3×10^{-26}. On many calculators you will notice that very large or very small numbers are automatically converted to scientific notation using a certain number of significant figures. Numbers can also be entered into a calculator using scientific notation.



The accuracy of a measurement of the volume of Earth is reflected in the number of significant figures shown.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Significant discussions

Begin a discussion regarding scientific figures by referring to these questions.

- Why is the volume of Earth given as 1 083 210 000 000 km^3 written using seven zeros at the end of the number? Wouldn't the exact volume of Earth include some other digits in these places?
- Why is the mass of an oxygen molecule given as 5.3×10^{-26} written using only two digits to the left of the power of 10? Wouldn't the exact mass of a water molecule include more decimal places?

Key ideas

- **Significant figures** are counted from left to right starting at the first non-zero digit. Zeros with no non-zero digit on their right are not counted. For example:
  - 38 041 has five significant figures
  - 0.0016 has two significant figures
  - 3.21×10^2 has three significant figures
  - 256 000 could have 3, 4, 5 or 6 significant figures.
- When using scientific notation, only the first significant figure sits to the left of the decimal point.
- Calculators can be used to work with scientific notation.
  - E or EE or EXP or $\times 10^x$ are common key names on calculators.
  - Pressing 2.37 EE 5 gives 2.37×10^5.
  - 2.37E5 means 2.37×10^5.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 19 Stating the number of significant figures

State the number of significant figures given in these numbers.

a 451 000 **b** 0.005012 **c** 3.2×10^{-7}

SOLUTION	EXPLANATION
a 3, 4, 5 or 6 significant figures	In a whole number with trailing zeros, those zeroes may or may not be significant.
b 4 significant figures	Start counting at the first non-zero digit.
c 2 significant figures	With scientific notation the first significant figure is to the left of the decimal point.

Example 20 Writing numbers in scientific notation using significant figures

Write these numbers using scientific notation and 3 significant figures.

a 2 183 000 **b** 0.0019482

SOLUTION	EXPLANATION
a $2\,183\,000 = 2.18 \times 10^6$ (to 3 sig. fig.)	Put the decimal point after the first non-zero digit. The decimal point has moved 6 places so multiply by 10^6. Round the third significant figure down since the following digit is less than 5.
b $0.0019482 = 1.95 \times 10^{-3}$ (to 3 sig. fig.)	Move the decimal point 3 places to the right and multiply by 10^{-3}. Round the third significant figure up to 5 since the following digit is greater than 4.

Example 21 Using a calculator with scientific notation

Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to 4 significant figures.

a $3.67 \times 10^5 \times 23.6 \times 10^4$ **b** $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$

SOLUTION	EXPLANATION
a $3.67 \times 10^5 \times 23.6 \times 10^4$ $= 8.661 \times 10^{10}$ (to 4 sig. fig.)	Write in scientific notation with 4 significant figures.
b $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$ $= 0.1625$ $= 1.625 \times 10^{-1}$ (to 4 sig. fig.)	Write in scientific notation with a number between 1 and 10.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 6G

UNDERSTANDING AND FLUENCY	1, 2, 3–6(½)	2–7(½)	3–7(½)

1 Complete the tables, rounding each number to the given number of significant figures.

a 57263

Significant figures	Rounded number
4	
3	57300
2	
1	

b 4170162

Significant figures	Rounded number
5	
4	
3	4170000
2	
1	

c 0.0036612

Significant figures	Rounded number
4	
3	
2	
1	0.004

d 24.8706

Significant figures	Rounded number
5	
4	
3	
2	25
1	

2 Decide if the following numbers are written using scientific notation with 3 significant figures. (Yes or No.)

a 4.21×10^{4} **b** 32×10^{-3} **c** 1800×10^{6}
d 0.04×10^{2} **e** 1.89×10^{-10} **f** 9.04×10^{-6}
g 5.56×10^{-14} **h** 0.213×10^{2} **i** 26.1×10^{-2}

Example 19

3 State the number of significant figures given in these numbers.

a 27200 **b** 1007 **c** 301010 **d** 190
e 0.0183 **f** 0.20 **g** 0.706 **h** 0.00109
i 4.21×10^{3} **j** 2.905×10^{-2} **k** 1.07×10^{-6} **l** 5.90×10^{5}

Example 20

4 Write these numbers using scientific notation and 3 significant figures.

a 242300 **b** 171325 **c** 2829 **d** 3247000
e 0.00034276 **f** 0.006859 **g** 0.01463 **h** 0.001031
i 23.41 **j** 326.042 **k** 19.618 **l** 0.172046

5 Write each number using scientific notation rounding to the number of significant figures given in the brackets.

a 47760 (3) **b** 21610 (2) **c** 4833160 (4)
d 37.16 (2) **e** 99.502 (3) **f** 0.014427 (4)
g 0.00201 (1) **h** 0.08516 (1) **i** 0.0001010 (1)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 21a

6 Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to 4 significant figures.

a 4^{-6}

b 78^{-3}

c $(-7.3 \times 10^{-4})^{-5}$

d $\dfrac{3.185}{7 \times 10^{4}}$

e $2.13 \times 10^{4} \times 9 \times 10^{7}$

f $5.671 \times 10^{2} \times 3.518 \times 10^{5}$

g $9.419 \times 10^{5} \times 4.08 \times 10^{-4}$

h $2.85 \times 10^{-9} \times 6 \times 10^{-3}$

i 12345^{2}

j 87.14^{8}

k $\dfrac{1.8 \times 10^{26}}{4.5 \times 10^{22}}$

l $\dfrac{-4.7 \times 10^{-2} \times 6.18 \times 10^{7}}{3.2 \times 10^{6}}$

Example 21b

7 Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to 5 significant figures.

a $\sqrt{8756}$

b $\sqrt{634 \times 7.56 \times 10^{7}}$

c $8.6 \times 10^{5} + \sqrt{2.8 \times 10^{-2}}$

d $-8.9 \times 10^{-4} + \sqrt{7.6 \times 10^{-3}}$

e $\dfrac{5.12 \times 10^{21} - 5.23 \times 10^{20}}{2 \times 10^{6}}$

f $\dfrac{8.942 \times 10^{47} - 6.713 \times 10^{44}}{2.5 \times 10^{19}}$

g $\dfrac{2 \times 10^{7} + 3 \times 10^{8}}{5}$

h $\dfrac{4 \times 10^{8} + 7 \times 10^{9}}{6}$

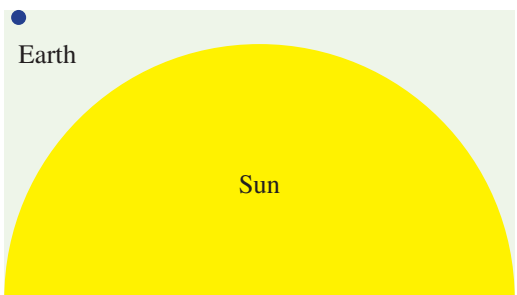
i $\dfrac{6.8 \times 10^{-8} + 7.5 \times 10^{27}}{4.1 \times 10^{27}}$

j $\dfrac{2.8 \times 10^{-6} - 2.71 \times 10^{-9}}{5.14 \times 10^{-6} + 7 \times 10^{-8}}$

PROBLEM-SOLVING AND REASONING 8, 9, 12 | 9–13 | 10, 11, 13, 14

8 The mass of Earth is approximately 6000000000000000000000000 kg. Given that the mass of the Sun is 330000 times the mass of Earth, find the mass of the Sun. Express your answer in scientific notation correct to 3 significant figures.

9 The diameter of Earth is approximately 12756000 m. If the Sun's diameter is 109 times that of Earth, compute its diameter in kilometres. Express your answer in scientific notation correct to 3 significant figures.



The size of the Sun and Earth compared (distance of Earth to Sun is not to scale)

10 Using the formula for the volume of a sphere, $V = \dfrac{4\pi r^3}{3}$, and, assuming Earth to be spherical, calculate the volume of Earth in km^3. Use the data given in Question **9**. Express your answer in scientific notation correct to 3 significant figures.

11 Write these numbers from largest to smallest.
2.41×10^{6}, 24.2×10^{5}, 0.239×10^{7}, 2421×10^{3}, 0.02×10^{8}

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 The following output is common on a number of different calculators and computers. Write down the number that you think they represent.

a 4.26E6
b 9.1E−3
c 5.04EXP11
d 1.931EXP−1
e 2.1^{06}
f 6.14^{-11}

13 Anton writes down $352\,000 \times 250\,000 = 8.8^{10}$. Explain his error.

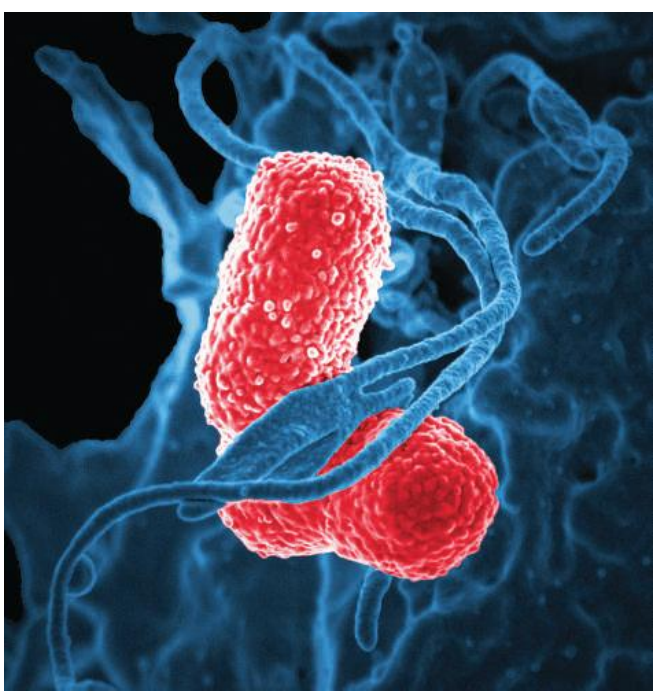
14 **a** Round these numbers to 3 significant figures. Retain the use of scientific notation.
i 2.302×10^2 **ii** 4.9045×10^{-2} **iii** 3.996×10^6
b What do you notice about the digit that is the third significant figure?
c Why do you think that it might be important to a scientist to show a significant figure at the end of a number which is a zero?

ENRICHMENT — — 15

Combining bacteria

15 A flask of type A bacteria contains 5.4×10^{12} cells and a flask of type B bacteria contains 4.6×10^8 cells. The two types of bacteria are combined in the same flask.
a How many bacterial cells are there in the flask?
b If type A bacterial cells double every 8 hours and type B bacterial cells triple every 8 hours, how many cells are in the flask after:
i one day?
ii a week?
iii 30 days?

Express your answers in scientific notation correct to 3 significant figures.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6H Fractional indices and surds

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

So far we have considered indices including positive and negative integers and zero. Numbers can also be expressed using fractional indices. Two examples are $9^{\frac{1}{2}}$ and $5^{\frac{1}{3}}$.

Using index law 1: $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9^1 = 9$

Since $\sqrt{9} \times \sqrt{9} = 9$ and $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$ then $9^{\frac{1}{2}} = \sqrt{9}$.

Similarly, $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^1 = 5$

Since $\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = 5$ and $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5$ then $5^{\frac{1}{3}} = \sqrt[3]{5}$.

This shows that numbers with fractional powers can be written using root signs. In the example above, $9^{\frac{1}{2}}$ is the square root of $9(\sqrt{9})$ and $5^{\frac{1}{3}}$ is the cubed root of $5\left(\sqrt[3]{5}\right)$.

You will have noticed that $9^{\frac{1}{2}} = \sqrt{9} = 3 = \frac{3}{1}$ and so $9^{\frac{1}{2}}$ is a rational number (a fraction) but $5^{\frac{1}{3}} = \sqrt[3]{5}$ does not appear to be able to be expressed as a fraction. In fact, $\sqrt[3]{5}$ is irrational and cannot be expressed as a fraction and is called a surd. As a decimal $\sqrt[3]{5} = 1.70997594668\ldots$, which is an infinite non-recurring decimal with no repeated pattern. This is a characteristic of all surds.

Let's start: A surd or not?

Surds are numbers with a root sign that cannot be expressed as a fraction. As a decimal they are infinite and non-recurring (with no pattern).

Use a calculator to help complete this table, then decide if you think the numbers are surds.

Index form	With root sign	Decimal	Surd (yes or no)
$2^{\frac{1}{2}}$	$\sqrt{2}$		
$4^{\frac{1}{2}}$	$\sqrt{4}$		
$11^{\frac{1}{2}}$			
$36^{\frac{1}{2}}$			
$\left(\frac{1}{9}\right)^{\frac{1}{2}}$			
$(0.1)^{\frac{1}{2}}$			
$3^{\frac{1}{3}}$	$\sqrt[3]{3}$		
$8^{\frac{1}{3}}$	$\sqrt[3]{8}$		
$15^{\frac{1}{3}}$			
$\left(\frac{1}{27}\right)^{\frac{1}{3}}$			
$5^{\frac{1}{4}}$			
$64^{\frac{1}{6}}$			

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Numbers written with **fractional indices** can also be written using a radical sign.
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $\sqrt[2]{a}$ is written $\sqrt{a}$

  For example: $3^{\frac{1}{2}} = \sqrt{3}$, $7^{\frac{1}{3}} = \sqrt[3]{7}$, $2^{\frac{1}{5}} = \sqrt[5]{2}$
- **Surds** are irrational numbers written with a radical sign.
  - Irrational numbers cannot be expressed as a fraction.
  - The decimal expansion is infinite and non-recurring with no pattern.

$\sqrt{2} = 1.41421356237\ldots$
$\sqrt[3]{10} = 2.15443469003\ldots$
$3^{1/2} = 1.73205080757\ldots$

Example 22 Writing numbers using a radical sign

Write these numbers using a radical sign.

a $6^{\frac{1}{2}}$ **b** $2^{\frac{1}{5}}$

SOLUTION	EXPLANATION
a $6^{\frac{1}{2}} = \sqrt{6}$	$a^{\frac{1}{m}} = \sqrt[m]{a}$ so $6^{\frac{1}{2}} = \sqrt[2]{6}$ (or $\sqrt{6}$) the square root of 6.
b $2^{\frac{1}{5}} = \sqrt[5]{2}$	$\sqrt[5]{2}$ is called the 5th root of 2.

Example 23 Evaluating numbers with fractional indices

Evaluate the following.

a $144^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$

SOLUTION	EXPLANATION
a $144^{\frac{1}{2}} = \sqrt{144}$ $= 12$	$a^{\frac{1}{m}} = \sqrt[m]{a}$ where $m = 2$ and the square root of 144 = 12 since $12^2 = 144$.
b $27^{\frac{1}{3}} = \sqrt[3]{27}$ $= 3$	The cubed root of 27 is 3 since $3^3 = 3 \times 3 \times 3 = 27$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 24 Using index laws

Use index laws to simplify these expressions.

a $a^{\frac{1}{2}} \times a^{\frac{3}{2}}$ **b** $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}$ **c** $(y^2)^{\frac{1}{4}}$

SOLUTION

a $a^{\frac{1}{2}} \times a^{\frac{3}{2}} = a^{\frac{1}{2}+\frac{3}{2}}$
$= a^{\frac{4}{2}}$
$= a^2$

EXPLANATION

When multiplying indices with the same base, add the powers.

b $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{2}-\frac{1}{3}}$
$= x^{\frac{1}{6}}$

When dividing indices with the same base, subtract the powers.
$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

c $(y^2)^{\frac{1}{4}} = y^{2\times\frac{1}{4}}$
$= y^{\frac{1}{2}}$

When raising a power to a power, multiply the indices.
$2 \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Exercise 6H

UNDERSTANDING AND FLUENCY 1–3, 4–7(½) 1, 4–8(½) 4–8(½)

1 Evaluate these numbers, rounding to 2 decimal places where necessary.

a $4^{\frac{1}{2}}$ and $\sqrt{4}$ **b** $8^{\frac{1}{3}}$ and $\sqrt[3]{8}$ **c** $9^{\frac{1}{2}}$ and $\sqrt{9}$

d $12^{\frac{1}{2}}$ and $\sqrt{12}$ **e** $\sqrt[3]{1000}$ and $1000^{\frac{1}{3}}$ **f** $\sqrt[5]{32}$ and $32^{\frac{1}{5}}$

2 State whether the following are true or false.

a All fractions are rational numbers.
b $\sqrt{2}$ is a rational number.
c A surd in decimal form will be infinite and non recurring (with no pattern).
d $\sqrt{3} = 3^{\frac{1}{2}}$
e $\sqrt{8} = 8^{\frac{1}{3}}$
f $5^{\frac{1}{3}} = \sqrt{5}$
g $10^{\frac{1}{6}} = \sqrt[10]{6}$

3 State whether the following are true or false.

a $8^{\frac{1}{2}} = 4$ **b** $16^{\frac{1}{2}} = 4$ **c** $27^{\frac{1}{3}} = 9$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 22

4 Write these numbers using a radical sign.

a $3^{\frac{1}{2}}$ **b** $7^{\frac{1}{2}}$ **c** $5^{\frac{1}{3}}$ **d** $12^{\frac{1}{3}}$

e $31^{\frac{1}{5}}$ **f** $18^{\frac{1}{7}}$ **g** $9^{\frac{1}{9}}$ **h** $3^{\frac{1}{8}}$

5 Write these numbers in index form.

a $\sqrt{8}$ **b** $\sqrt{19}$ **c** $\sqrt[3]{10}$ **d** $\sqrt[3]{31}$

e $\sqrt[4]{5}$ **f** $\sqrt[5]{9}$ **g** $\sqrt[8]{11}$ **h** $\sqrt[11]{20}$

Example 23

6 Without a calculator, evaluate these numbers with fractional indices.

a $25^{\frac{1}{2}}$ **b** $49^{\frac{1}{2}}$ **c** $81^{\frac{1}{2}}$ **d** $169^{\frac{1}{2}}$

e $8^{\frac{1}{3}}$ **f** $64^{\frac{1}{3}}$ **g** $125^{\frac{1}{3}}$ **h** $1000^{\frac{1}{3}}$

i $16^{\frac{1}{4}}$ **j** $81^{\frac{1}{4}}$ **k** $625^{\frac{1}{4}}$ **l** $32^{\frac{1}{5}}$

Example 24

7 Use index laws to simplify these expressions. Leave your answer in index form.

a $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ **b** $a^{\frac{1}{3}} \times a^{\frac{1}{3}}$ **c** $a^{\frac{2}{3}} \times a^{\frac{4}{3}}$ **d** $a^2 \times a^{\frac{1}{2}}$

e $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ **f** $\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$ **g** $\dfrac{x^{\frac{7}{6}}}{x^{\frac{2}{6}}}$ **h** $\dfrac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$

i $(y^2)^{\frac{1}{2}}$ **j** $(y^3)^{\frac{2}{3}}$ **k** $(y^{\frac{1}{2}})^3$ **l** $(x^{\frac{1}{2}})^{\frac{1}{2}}$

m $(x^{\frac{2}{3}})^4$ **n** $(a^{\frac{2}{5}})^{\frac{1}{3}}$ **o** $(a^{\frac{3}{4}})^{\frac{1}{2}}$ **p** $(n^{\frac{2}{5}})^{\frac{10}{3}}$

8 Use index laws to simplify these expressions.

a $a \times a^{\frac{1}{3}}$ **b** $a^{\frac{1}{2}} \times a^{\frac{1}{5}}$ **c** $a^{\frac{2}{3}} \times a^{\frac{3}{7}}$ **d** $a^5 \div a^{\frac{7}{3}}$ **e** $b^{\frac{2}{3}} \div b^{\frac{1}{2}}$ **f** $x^{\frac{4}{5}} \div x^{\frac{2}{3}}$

PROBLEM-SOLVING AND REASONING 9–10(½), 12 | 9–11(½), 12, 13 | 9–11(½), 13, 14

9 Evaluate the following without using a calculator. *Hint*: first rewrite each question using positive indices.

a $4^{-\frac{1}{2}}$ **b** $8^{-\frac{1}{3}}$ **c** $32^{-\frac{1}{5}}$ **d** $81^{-\frac{1}{4}}$

e $25^{-\frac{1}{2}}$ **f** $27^{-\frac{1}{3}}$ **g** $1000^{-\frac{1}{3}}$ **h** $256^{-\frac{1}{4}}$

10 Note that $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2$ using power of a power.

$8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2$

$= (\sqrt[3]{8})^2$ since $8^{\frac{1}{3}} = \sqrt[3]{8}$

$= 2^2$ $\sqrt[3]{8} = 2$ since $2^3 = 8$

$= 4$

Use the approach shown in the example above to evaluate these numbers.

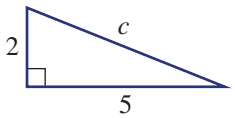
a $27^{\frac{2}{3}}$ **b** $64^{\frac{2}{3}}$ **c** $9^{\frac{3}{2}}$ **d** $25^{\frac{3}{2}}$

e $16^{\frac{5}{4}}$ **f** $4^{\frac{5}{2}}$ **g** $81^{\frac{3}{2}}$ **h** $125^{\frac{5}{3}}$

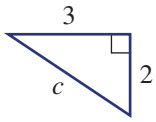
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Find the length of the hypotenuse (c) in these right-angled triangles. Use Pythagoras' theorem ($c^2 = a^2 + b^2$) and write your answer as a surd.

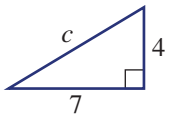
a



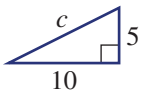
b



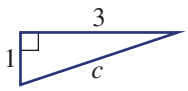
c



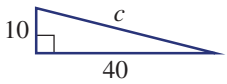
d



e



f



12 Show working to prove that the answers to all these questions simplify to 1. Remember $a^0 = 1$.

a $a^{\frac{1}{2}} \times a^{-\frac{1}{2}}$ **b** $a^{\frac{2}{3}} \times a^{-\frac{2}{3}}$ **c** $a^{\frac{4}{7}} \times a^{-\frac{4}{7}}$

d $a^{\frac{5}{6}} \div a^{\frac{5}{6}}$ **e** $\left(a^{\frac{1}{2}}\right)^{\frac{1}{2}} \div a^{\frac{1}{4}}$ **f** $a^2 \div (a^3)^{\frac{2}{3}}$

13 A student tries to evaluate $9^{\frac{1}{2}}$ on a calculator and types 9^1/2 and gets 4.5. But you know that $9^{\frac{1}{2}} = \sqrt{9} = 3$. What has the student done wrong? (Note: ^ on some calculators is x^y.)

14 **a** Evaluate the following.

i $\sqrt{3^2}$ **ii** $\sqrt{5^2}$ **iii** $\sqrt{10^2}$

b Simplify $\sqrt{a^2}$.

c Use fractional indices to show that $\sqrt{a^2} = a$ if $a \geq 0$.

d Evaluate the following.

i $(\sqrt{4})^2$ **ii** $(\sqrt{9})^2$ **iii** $(\sqrt{36})^2$

e Simplify $(\sqrt{a})^2$.

f Use fractional indices to show that $(\sqrt{a})^2 = a$. Assume $a \geq 0$.

g Simplify:

i $\sqrt[3]{a^3}$ **ii** $\sqrt[5]{a^5}$ **iii** $(\sqrt[3]{a})^3$ **iv** $(\sqrt[6]{a})^6$

ENRICHMENT — — 15, 16

Fractions raised to fractions

15 Note, for example, that $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. Now evaluate the following.

a $\left(\frac{16}{25}\right)^{\frac{1}{2}}$ **b** $\left(\frac{9}{49}\right)^{\frac{1}{2}}$ **c** $\left(\frac{4}{81}\right)^{\frac{1}{2}}$ **d** $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

e $\left(\frac{64}{125}\right)^{\frac{1}{3}}$ **f** $\left(\frac{16}{81}\right)^{\frac{1}{4}}$ **g** $\left(\frac{256}{625}\right)^{\frac{1}{4}}$ **h** $\left(\frac{1000}{343}\right)^{\frac{1}{3}}$

16 Note that $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{4}{9}\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{4}{9}}} = \frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$. Now evaluate the following.

a $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$ **b** $\left(\frac{49}{144}\right)^{-\frac{1}{2}}$ **c** $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ **d** $\left(\frac{1296}{625}\right)^{-\frac{1}{4}}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6I Simple operations with surds

Interactive

Widgets

HOTsheets

Walkthrough

Since surds, such as $\sqrt{2}$ and $\sqrt{7}$, are numbers, they can be added, subtracted, multiplied or divided. Expressions with surds can also be simplified but this depends on the surds themselves and the types of operations that sit between them.

$\sqrt{2} + \sqrt{3}$ cannot be simplified since $\sqrt{2}$ and $\sqrt{3}$ are not 'like' surds. This is like trying to simplify $x + y$.

However, $\sqrt{2} + 5\sqrt{2}$ simplifies to $6\sqrt{2}$ and this is like simplifying $x + 5x = 6x$. Subtraction of surds is treated in the same manner.

Products and quotients involving surds can also be simplified as in these examples:

$\sqrt{11} \times \sqrt{2} = \sqrt{22}$ and $\sqrt{30} \div \sqrt{3} = \sqrt{10}$

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Rules for multiplication and division

Use a calculator to find a decimal approximation for each of the expressions in these pairs.

- $\sqrt{2} \times \sqrt{3}$ and $\sqrt{6}$
- $\sqrt{10} \times \sqrt{5}$ and $\sqrt{50}$

What does this suggest about the simplification of $\sqrt{a} \times \sqrt{b}$?

Repeat the above exploration for these.

- $\sqrt{6} \div \sqrt{2}$ and $\sqrt{\frac{6}{2}}$
- $\sqrt{80} \div \sqrt{8}$ and $\sqrt{\frac{80}{8}}$

What does this suggest about the simplification of $\sqrt{a} \div \sqrt{b}$?

Key ideas

- **Surds** can be simplified using addition or subtraction if they are 'like' surds.
  - $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$
  - $11\sqrt{7} - 2\sqrt{7} = 9\sqrt{7}$
  - $\sqrt{3} + \sqrt{5}$ cannot be simplified.
- $(\sqrt{a})^2 = a$

  For example: $(\sqrt{7})^2 = 7$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

  For example: $\sqrt{5} \times \sqrt{3} = \sqrt{5 \times 3} = \sqrt{15}$
- $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

  For example: $\sqrt{10} \div \sqrt{5} = \sqrt{\frac{10}{5}} = \sqrt{2}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 25 Adding and subtracting surds

Simplify each of the following.

a $2\sqrt{5} + 6\sqrt{5}$ b $\sqrt{3} - 5\sqrt{3}$ c $3\sqrt{7} - \sqrt{6} + 2\sqrt{7}$

SOLUTION

a $2\sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$

b $\sqrt{3} - 5\sqrt{3} = -4\sqrt{3}$

c $3\sqrt{7} - \sqrt{6} + 2\sqrt{7} = 5\sqrt{7} - \sqrt{6}$

EXPLANATION

This is like simplifying $2x + 6x = 8x$, $2\sqrt{5}$ and $6\sqrt{5}$ contain like surds.

This is similar to $x - 5x = -4x$ in algebra.

This is similar to $3x - y + 2x = 5x - y$, where $3\sqrt{7}$ and $2\sqrt{7}$ contain like surds.

Example 26 Multiplying and dividing surds

Simplify each of the following.

a $\sqrt{3} \times \sqrt{10}$ b $\sqrt{24} \div \sqrt{8}$

SOLUTION

a $\sqrt{3} \times \sqrt{10} = \sqrt{3 \times 10}$
$= \sqrt{30}$

b $\sqrt{24} \div \sqrt{8} = \sqrt{\frac{24}{8}}$
$= \sqrt{3}$

EXPLANATION

Use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

Use $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.

Exercise 6I

UNDERSTANDING AND FLUENCY	1–4	2, 3–4(½)	3–4(½)

1 Decide if the following pairs of numbers contain 'like' surds.

a $3\sqrt{2}, 4\sqrt{2}$ b $5\sqrt{3}, 2\sqrt{3}$ c $4\sqrt{2}, 5\sqrt{7}$ d $\sqrt{3}, 2\sqrt{5}$

e $6\sqrt{6}, 3\sqrt{3}$ f $\sqrt{8}, 3\sqrt{8}$ g $19\sqrt{2}, -\sqrt{2}$ h $-3\sqrt{6}, 3\sqrt{5}$

2 a Use a calculator to find a decimal approximation for both $\sqrt{5} \times \sqrt{2}$ and $\sqrt{10}$. What do you notice?

b Use a calculator to find a decimal approximation for both $\sqrt{7} \times \sqrt{3}$ and $\sqrt{21}$. What do you notice?

c Use a calculator to find a decimal approximation for both $\sqrt{15} \div \sqrt{5}$ and $\sqrt{3}$. What do you notice?

d Use a calculator to find a decimal approximation for both $\sqrt{60} \div \sqrt{10}$ and $\sqrt{6}$. What do you notice?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 25

3 Simplify by collecting like surds.

a $3\sqrt{7} + 5\sqrt{7}$ **b** $2\sqrt{11} + 6\sqrt{11}$

c $\sqrt{5} + 8\sqrt{5}$ **d** $3\sqrt{6} + \sqrt{6}$

e $3\sqrt{3} + 2\sqrt{5} + 4\sqrt{3}$ **f** $5\sqrt{7} + 3\sqrt{5} + 4\sqrt{7}$

g $3\sqrt{5} - 8\sqrt{5}$ **h** $6\sqrt{7} - 10\sqrt{7}$

i $3\sqrt{7} - 2\sqrt{7} + 4\sqrt{7}$ **j** $5\sqrt{14} + \sqrt{14} - 7\sqrt{14}$

k $3\sqrt{2} - \sqrt{5} + 4\sqrt{2}$ **l** $6\sqrt{3} + 2\sqrt{7} - 3\sqrt{3}$

Example 26

4 Simplify the following.

a $\sqrt{5} \times \sqrt{6}$ **b** $\sqrt{3} \times \sqrt{7}$

c $\sqrt{10} \times \sqrt{7}$ **d** $\sqrt{8} \times \sqrt{2}$

e $\sqrt{12} \times \sqrt{3}$ **f** $\sqrt{2} \times \sqrt{11}$

g $\sqrt{3} \times \sqrt{3}$ **h** $\sqrt{12} \times \sqrt{12}$

i $\sqrt{36} \div \sqrt{12}$ **j** $\sqrt{20} \div \sqrt{2}$

k $\sqrt{60} \div \sqrt{20}$ **l** $\sqrt{45} \div \sqrt{5}$

m $\sqrt{32} \div \sqrt{2}$ **n** $\sqrt{49} \div \sqrt{7}$

PROBLEM-SOLVING AND REASONING 5–8(½) 5–8(½) 5–9(½)

5 Simplify the following.

a $2 - \sqrt{3} + 6 - 2\sqrt{3}$ **b** $\sqrt{2} - \sqrt{3} + 5\sqrt{2}$

c $7\sqrt{5} - \sqrt{2} + 1 + \sqrt{2}$ **d** $\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{2}$

e $\frac{\sqrt{7}}{2} + \frac{\sqrt{7}}{5}$ **f** $\frac{2\sqrt{6}}{7} - \frac{\sqrt{6}}{2}$

g $\sqrt{10} - \frac{\sqrt{10}}{3}$ **h** $5 - \frac{2\sqrt{3}}{3} + \sqrt{3}$

i $\frac{2\sqrt{8}}{7} - \frac{5\sqrt{8}}{8}$

6 Note, for example, that $2\sqrt{3} \times 5\sqrt{2} = 2 \times 5 \times \sqrt{3} \times \sqrt{2}$
$= 10\sqrt{6}$

Now simplify the following.

a $5\sqrt{2} \times 3\sqrt{3}$ **b** $3\sqrt{7} \times 2\sqrt{3}$

c $4\sqrt{5} \times 2\sqrt{6}$ **d** $2\sqrt{6} \times 5\sqrt{3}$

e $10\sqrt{6} \div 5\sqrt{2}$ **f** $18\sqrt{12} \div 6\sqrt{2}$

g $20\sqrt{28} \div 5\sqrt{2}$ **h** $6\sqrt{14} \div 12\sqrt{7}$

7 Expand and simplify.

a $2\sqrt{3}(3\sqrt{5} + 1)$ **b** $\sqrt{5}(\sqrt{2} + \sqrt{3})$

c $5\sqrt{6}(\sqrt{2} + 3\sqrt{5})$ **d** $7\sqrt{10}(2\sqrt{3} - \sqrt{10})$

e $\sqrt{13}(\sqrt{13} - 2\sqrt{3})$ **f** $\sqrt{5}(\sqrt{7} - 2\sqrt{5})$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 Using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ the surd $\sqrt{18}$ can be simplified as shown.

$$\begin{aligned}\sqrt{18} &= \sqrt{9 \times 2} \\ &= \sqrt{9} \times \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

This simplification is possible because 18 has a factor that is a perfect square (9). Use this technique to simplify these surds.

a $\sqrt{8}$ **b** $\sqrt{12}$
c $\sqrt{27}$ **d** $\sqrt{45}$
e $\sqrt{75}$ **f** $\sqrt{200}$
g $\sqrt{60}$ **h** $\sqrt{72}$

9 Building on the idea discussed in Question **8**, expressions like $\sqrt{8} - \sqrt{2}$ can be simplified as shown:

$$\begin{aligned}\sqrt{8} - \sqrt{2} &= \sqrt{4 \times 2} - \sqrt{2} \\ &= 2\sqrt{2} - \sqrt{2} \\ &= \sqrt{2}\end{aligned}$$

Now simplify these expressions.

a $\sqrt{8} + 3\sqrt{2}$ **b** $3\sqrt{2} - \sqrt{8}$
c $\sqrt{18} + \sqrt{2}$ **d** $5\sqrt{3} - 2\sqrt{12}$
e $4\sqrt{8} - 2\sqrt{2}$ **f** $\sqrt{27} + 2\sqrt{3}$
g $3\sqrt{45} - 7\sqrt{5}$ **h** $6\sqrt{12} - 8\sqrt{3}$

ENRICHMENT — — 10

Binomial products

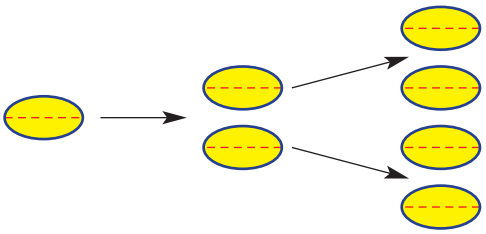
10 Simplify the following by using the rule $(a + b)(c + d) = ac + ad + bc + bd$.

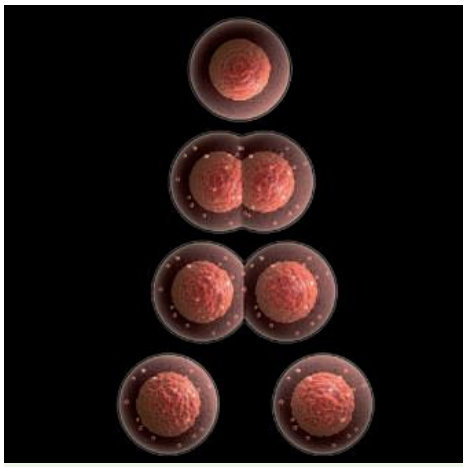
a $(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{5})$
b $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{2})$
c $(2\sqrt{5} - 1)(3\sqrt{2} + 4)$
d $(1 - 3\sqrt{7})(2 + 3\sqrt{2})$
e $(2 - \sqrt{3})(2 + \sqrt{3})$
f $(\sqrt{5} - 1)(\sqrt{5} + 1)$
g $(3\sqrt{2} + \sqrt{3})(3\sqrt{2} - \sqrt{3})$
h $(8\sqrt{2} + \sqrt{5})(8\sqrt{2} - \sqrt{5})$
i $(1 + \sqrt{2})^2$
j $(\sqrt{6} - 3)^2$
k $(2\sqrt{3} - 1)^2$
l $(\sqrt{2} + 2\sqrt{5})^2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Cell growth

Many cellular organisms reproduce by a process of subdivision. A single cell, for example, may divide into two every hour as shown at right. After another hour, the single starting cell has become four:





A cell dividing in two

Dividing into two

A single cell divides into two every hour.

a How many cells will there be after the following number of hours? Explain how you obtained your answers.

i 1 **ii** 2 **iii** 5

b Complete the table showing the number of cells after n hours.

n hours	0	1	2	3	4	5	6
Number of cells, N	1	2	4				
N in index form	2^0	2^1	2^2				

c Write a rule for the number of cells N after n hours.

d Use your rule from part **c** to find the number of cells after:

i 8 hours **ii** 12 hours **iii** 2 days

e Find how long it takes for a single cell to divide into a total of:

i 128 cells **ii** 1024 cells **iii** 65 536 cells

Dividing into three or more

a Complete a table similar to the table in the previous section for a cell that divides into 3 every hour.

b Write a rule for N in terms of n if a cell divides into 3 every hour. Then use the rule to find the number of cells after:

i 2 hours **ii** 4 hours **iii** 8 hours

c Write a rule for N in terms of n if a single cell divides into the following number of cells every hour.

i 4 **ii** 5 **iii** 10

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Cell cycle times

a If a single cell divides into two every 20 minutes, investigate how many cells there will be after 4 hours.

b If a single cell divides into three every 10 minutes, investigate how many cells there will be after 2 hours.

c Use the internet to research the cell cycle time and the types of division for at least two different types of cells. Describe the cells and explain the reproductive process.

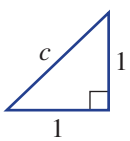
Constructing surds

Since surds are not fractions, it is difficult to precisely measure a length representing a surd. Pythagoras' theorem can however be used to construct lengths which represent surds.

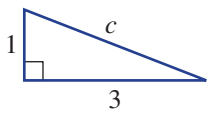
Using Pythagoras' theorem

Use Pythagoras' theorem to find the hypotenuse (c) in these triangles.

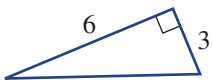
a



b



c



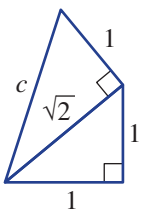
Constructing surds

Show how you can use a single triangle to construct a hypotenuse with the following lengths. The lengths of the shorter sides have to be whole numbers.

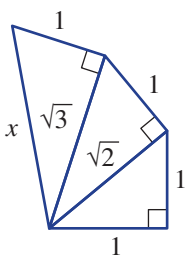
a $\sqrt{5}$ **b** $\sqrt{13}$ **c** $\sqrt{26}$

Combined triangles

The diagram below shows how you can construct the surd $\sqrt{3}$



$$\begin{aligned} c^2 &= 1^2 + (\sqrt{2})^2 \\ &= 1 + 2 \\ &= 3 \\ c &= \sqrt{3} \end{aligned}$$



a Copy the diagram on the right to find the value of x.

b Show how you can add other triangles to construct a line segment with the following lengths.

i $\sqrt{5}$ **ii** $\sqrt{6}$ **iii** $\sqrt{7}$

c Using compasses, draw exact right angles and transfer exact lengths to a number line. Mark these exact lengths on a number line.

i $\sqrt{2}$ **ii** $\sqrt{3}$ **iii** $-\sqrt{5}$ **iv** $-\sqrt{7}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1 Determine the last digit of each of the following without using a calculator.

a 2^{222} **b** 3^{300} **c** 6^{87}

2 Determine the smallest value of n such that:

a $24n$ is a square number

b $750n$ is a square number

3 Simplify $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$ without a calculator.

4 If $2^x = t$, express the following in terms of t.

a 2^{2x+1} **b** 2^{1-x}

5 A single cell divides in two every 5 minutes and each new cell continues to divide every 5 minutes. How long does it take for the cell population to reach at least 1 million?

6 Find the value of x if $3^{3x-1} = \frac{1}{27}$.

7 **a** Write the following in index form.

i $\sqrt{2\sqrt{2}}$

ii $\sqrt{2\sqrt{2\sqrt{2}}}$

iii $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$

b What value do your answers to part **a** appear to be approaching?

8 Determine the highest power of 2 that divides exactly into 2000000.

9 Simplify these surds.

a $5\sqrt{8} - \sqrt{18}$

b $\frac{1}{\sqrt{2}} + \sqrt{2}$

c $(\sqrt{2} + 3\sqrt{5})^2 - (\sqrt{2} - 3\sqrt{5})^2$

10 Prove that:

a $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b $\frac{3}{\sqrt{3}} = \sqrt{3}$

c $\frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

11 Solve for x. There are two solutions for each.

a $2^{2x} - 3 \times 2^x + 2 = 0$

b $3^{2x} - 12 \times 3^x + 27 = 0$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Indices and surds

Index laws extended

Power of a power: $(ab)^m = a^m b^m$

e.g. $(3x^2)^3 = 3^3 (x^2)^3$
$= 27x^6$

Powers of fractions: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. $\left(\frac{x}{2}\right)^4 = \frac{x^4}{2^4}$
$= \frac{x^4}{16}$

Zero index

$a^0 = 1$
Any number (except 0) to the power 0 equals 1.
e.g. $3x^0 = 3 \times 1$
$= 3$
$(3x)^0 = 1$

Index law for power of a power

$(a^m)^n = a^{mn}$
To expand brackets, multiply indices.
e.g. $(x^2)^5 = x^{2\times 5}$
$= x^{10}$

Index form

$a^m \leftarrow$ index
base
e.g. $2^3 = 2 \times 2 \times 2$
$3^4 = 3 \times 3 \times 3 \times 3$
$a \times b \times a \times b \times b = a^2 b^3$

Negative indices

$a^{-m} = \frac{1}{a^m}$

e.g. $2x^{-3} = \frac{2}{1} \times \frac{1}{x^3}$
$= \frac{2}{x^3}$

$\frac{1}{a^{-m}} = a^m$

e.g. $\frac{1}{(2x)^{-2}} = (2x)^2$
$= 4x^2$

Index laws for multiplying and dividing

$a^m \times a^n = a^{m+n}$
When multiplying terms with the same base, add indices.
e.g. $x^3 \times x^5 = x^{3+5}$
$= x^8$
$a^m \div a^n = a^{m-n}$
When dividing terms with the same base, subtract indices.
e.g. $\frac{x^7}{x^4} = x^{7-4}$
$= x^3$

Scientific notation

In scientific notation, very large and very small positive numbers are written in the form
$a \times 10^m$ where $1 \leq a < 10$
Large numbers will use positive powers of 10.
e.g. 2 350 000 kg $= 2.35 \times 10^6$ kg
Small numbers will use negative powers of 10.
e.g. 16 nanoseconds
$= 0.000000016$ s
$= 1.6 \times 10^{-8}$ seconds

Fractional indices

$a^{\frac{1}{n}} = \sqrt[n]{a}$
e.g. $a^{\frac{1}{2}} = \sqrt{a}$
$a^{\frac{1}{3}} = \sqrt[3]{a}$
e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$
since $5^2 = 25$
$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
since $2^3 = 8$

Operations with surds

'Like' surds can be added or subtracted.
$3\sqrt{7}$ and $5\sqrt{7}$ are 'like surds'
e.g. $3\sqrt{7} + 4\sqrt{2} + 5\sqrt{7} = 8\sqrt{7} + 4\sqrt{2}$
$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
e.g. $\sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$
$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
e.g. $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}}$
$= \sqrt{4} = 2$

Surds

A surd is a number written with a root sign, which has a decimal expansion that is infinite and non-recurring with no pattern.
e.g. $\sqrt{2} = 1.41421356 \ldots$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Multiple-choice questions

1 $3x^7 \times 4x^4$ is equivalent to:

A $12x^7$ **B** $12x^{28}$ **C** $7x^{11}$ **D** $12x^{11}$ **E** $7x^3$

2 $3(2y^2)^0$ simplifies to:

A 6 **B** 3 **C** $6y^2$ **D** $3y$ **E** 12

3 $(2x^2)^3$ expands to:

A $2x^5$ **B** $2x^6$ **C** $6x^6$ **D** $8x^5$ **E** $8x^6$

4 $2x^3y \times \dfrac{x^5y^2}{4x^2y}$ simplifies to:

A $\dfrac{x^6y^2}{2}$ **B** $2x^8y$ **C** $2x^6y^2$ **D** $\dfrac{x^4y^2}{2}$ **E** $8x^6y$

5 $\left(\dfrac{-x^2y}{3z^4}\right)^3$ is equal to:

A $\dfrac{x^6y^3}{3z^{12}}$ **B** $\dfrac{-x^5y^4}{9z^7}$ **C** $\dfrac{-x^6y^3}{27z^{12}}$ **D** $\dfrac{-x^2y^3}{3z^{12}}$ **E** $\dfrac{x^6y^3}{9z^{12}}$

6 $2x^{-3}y^4$ expressed with positive indices is:

A $\dfrac{y^4}{2x^3}$ **B** $\dfrac{2y^4}{x^3}$ **C** $-2x^3y^4$ **D** $\dfrac{2}{x^3y^4}$ **E** $\dfrac{y^4}{8x^3}$

7 $\dfrac{3}{(2x)^{-2}}$ is equivalent to:

A $\dfrac{-3}{(2x)^2}$ **B** $6x^2$ **C** $\dfrac{3x^2}{2}$ **D** $12x^2$ **E** $\dfrac{-3x^2}{4}$

8 The weight of a cargo crate is 2.32×10^4 kg. In expanded form this weight in kilograms is:

A 2320000 **B** 232 **C** 23200 **D** 0.000232 **E** 2320

9 0.00032761 in scientific notation rounded to 3 significant figures is:

A 328×10^{-5} **B** 3.27×10^{-4} **C** 3.28×10^4 **D** 3.30×10^4 **E** 3.28×10^{-4}

10 $36^{\frac{1}{2}}$ is equal to:

A 18 **B** 6 **C** 1296 **D** 9 **E** 81

11 The simplified form of $2\sqrt{7} - 3\sqrt{5} + 4\sqrt{7}$ is:

A $-2\sqrt{7} - 3\sqrt{5}$ **B** $3\sqrt{2}$ **C** $6\sqrt{7} - 3\sqrt{5}$ **D** $-2 - 3\sqrt{5}$ **E** $3\sqrt{5}$

12 $\sqrt{3} \times \sqrt{7}$ is equivalent to:

A $\sqrt{21}$ **B** $\sqrt{10}$ **C** $2\sqrt{10}$ **D** $10\sqrt{21}$ **E** $21\sqrt{10}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Short-answer questions

1 Express each of the following in index form.

a $3 \times 3 \times 3 \times 3$ **b** $2 \times x \times x \times x \times y \times y$

c $3 \times a \times a \times a \times \frac{b}{a} \times b$ **d** $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{1}{7} \times \frac{1}{7}$

2 Write the following as a product of prime factors in index form.

a 45 **b** 300

3 Simplify the following.

a $x^3 \times x^7$ **b** $2a^3b \times 6a^2b^5c$ **c** $3m^2n \times 8m^5n^3 \times \frac{1}{2}m^{-3}$

d $a^{12} \div a^3$ **e** $x^5y^3 \div x^2y$ **f** $\frac{5a^6b^3}{10a^8b}$

4 Simplify the following.

a $(m^2)^3$ **b** $(3a^4)^2$ **c** $(-2a^2b)^5$

d $3a^0b$ **e** $2(3m)^0$ **f** $\left(\frac{a^2}{3}\right)^3$

5 Express each of the following with positive indices.

a x^{-3} **b** $4t^{-3}$ **c** $(3t)^{-2}$

d $\frac{2}{3}x^2y^{-3}$ **e** $5\left(\frac{x^2}{y^{-1}}\right)^{-3}$ **f** $\frac{5}{m^{-3}}$

6 Fully simplify each of the following.

a $\frac{5x^8y^{-12}}{x^{10}} \times \frac{(x^2y^5)^2}{10}$ **b** $\left(\frac{(3x)^0}{3x^0y^2}\right)^4 \times \frac{9y^{10}}{x^{-3}}$ **c** $\frac{(4m^2n^3)^2}{2m^5n^4} \div \frac{mn^5}{(m^3n^2)^3}$

7 Arrange the following numbers in ascending order:
$2.35,\ 0.007 \times 10^2,\ 0.0012,\ 3.22 \times 10^{-1},\ 0.4,\ 35.4 \times 10^{-3}$.

8 Write the following numbers that are in scientific notation in decimal form.

a 3.24×10^2 **b** 1.725×10^5 **c** 2.753×10^{-1} **d** 1.49×10^{-3}

9 Write each of the following values in scientific notation using 3 significant figures.

a The population of Australia during 2010 was projected to be 22475056.

b The area of the USA is 9629091 km^2.

c The time taken for light to travel 1 metre (in a vacuum) is 0.00000000333564 seconds.

d The wavelength of ultraviolet light from a fluorescent lamp is 0.000000294 m.

10 Write each of the following values using scientific notation in the units given in brackets.

a 25 years (hours) **b** 12 milliseconds (seconds)

c 432 nanoseconds (seconds) **d** 5 tonnes (grams)

11 Use a calculator to evaluate the following, giving your answer in scientific notation correct to 2 significant figures.

a $m_S \times m_E$ where m_S (mass of Sun) $= 1.989 \times 10^{30}$ kg and m_E(mass of Earth) $= 5.98 \times 10^{24}$ kg

b The speed, v, in m/s of an object of mass $m = 2 \times 10^{-3}$ kg and kinetic energy $E = 1.88 \times 10^{-12}$ joules, where $v = \sqrt{\frac{2E}{m}}$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 Evaluate without using a calculator.

a $\sqrt[4]{16}$ **b** $\sqrt[3]{125}$ **c** $49^{\frac{1}{2}}$

d $81^{\frac{1}{4}}$ **e** $27^{-\frac{1}{3}}$ **f** $121^{-\frac{1}{2}}$

13 Simplify the following, expressing all answers in positive index form.

a $\sqrt[3]{s^6}$ **b** $\sqrt{81t^3}$ **c** $3x^{\frac{1}{2}} \times 5x^2$

d $(3m^{\frac{1}{2}}n^2)^2 \times m^{-\frac{1}{4}}$ **e** $\dfrac{t}{2\sqrt{t}}$ **f** $\dfrac{4}{a^{\frac{1}{3}}} \times \dfrac{(a^{\frac{1}{2}})^4}{a}$

14 Simplify the following operations with surds.

a $8\sqrt{7} - \sqrt{7}$ **b** $2\sqrt{3} + 5\sqrt{2} - \sqrt{3} + 4\sqrt{2}$ **c** $\sqrt{8} \times \sqrt{8}$

d $\sqrt{5} \times \sqrt{3}$ **e** $2\sqrt{7} \times \sqrt{2}$ **f** $3\sqrt{2} \times 5\sqrt{11}$

g $\sqrt{42} \div \sqrt{7}$ **h** $2\sqrt{75} \div \sqrt{3}$ **i** $\sqrt{50} \div (2\sqrt{10})$

Extended-response questions

1 Simplify each of the following using a combination of index laws.

a $\dfrac{(4x^2y)^3 \times x^2y}{12(xy^2)^2}$ **b** $\dfrac{2a^3b^4}{(5a^3)^2} \times \dfrac{20a}{3b^{-4}}$

c $\dfrac{(5m^4n^{-3})^2}{m^{-1}n^2} \div \dfrac{5(m^{-1}n)^{-2}}{mn^{-4}}$ **d** $\dfrac{(8x^4)^{\frac{1}{3}}}{2(y^3)^0} \times \dfrac{(3x^{\frac{1}{3}})^2}{3(x^2)^{\frac{1}{2}}}$

2 The law of gravitational force is given by $F = \dfrac{Gm_1m_2}{d^2}$, where F is the magnitude of the gravitational force (in newtons, N) between two objects of mass m^1 and m^2 (in kilograms) a distance d (metres) apart. G is the universal gravitational constant which is approximately $6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$.

a If two objects of masses 2 kg and 4 kg are 3 m apart, calculate the gravitational force F between them. Answer in scientific notation correct to 3 significant figures.

b The average distance between Earth and the Sun is approximately 149 597 870 700 m.

i Write this distance in scientific notation with 3 significant figures.

ii Hence, if the mass of Earth is approximately 5.98×10^{24} kg and the mass of the Sun is approximately 1.99×10^{30}, calculate the gravitational force between them in scientific notation to 3 significant figures.

c The universal gravitational constant, G, is constant throughout the universe. However, acceleration due to gravity (a, units m s^{-2}) varies according to where you are in the solar system. Using the formula $a = \dfrac{Gm}{r^2}$ and the following table, work out and compare the acceleration due to gravity on Earth and on Mars. Answer to 3 significant figures.

Planet	Mass, m	Radius, r
Earth	5.98×10^{24} kg	6.375×10^6 m
Mars	6.42×10^{23} kg	3.37×10^6 m

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

7 Properties of geometrical figures

What you will learn

7A Angles and triangles REVISION
7B Parallel lines REVISION
7C Quadrilaterals and other polygons
7D Congruent triangles REVISION
7E Using congruence in proof
7F Enlargement and similar figures
7G Similar triangles
7H Proving and applying similar triangles

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY
SUBSTRAND: PROPERTIES OF GEOMETRICAL FIGURES
(S4, 5.1, 5.2, 5.3§)

Outcomes

A student classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles.
(MA4–17MG)

A student identifies and uses angle relationships, including those related to transversals on sets of parallel lines.
(MA4–18MG)

A student describes and applies the properties of similar figures and scale drawings.
(MA5.1–11MG)

A student calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar.
(MA5.2–14MG)

A student proves triangles are similar, and uses formal geometric reasoning to establish properties of triangles and quadrilaterals.
(MA5.3–16MG)

© NESA, 2012

Fortifications of Vauban

The fortifications of Sebastian Le Prestre de Vauban (1633–1707) is a collection of 12 fortified buildings in France. It is listed on the UNESCO (United Nations Educational, Scientific and Cultural Organisation) world heritage list.

Vauban was a military engineer of King Louis XIV. As an engineer and cadet, he had first-hand experience of the needs of a fortress and of a fortified city. The picture shown here is of the Citadel of Neuf-Brisach, in France. Its octagonal layout, which extends out to form the shape of a star, can be seen from this aerial view. The city is divided into 48 blocks, with the most important buildings located in the centre. Vauban understood the symmetry of octagons and squares and how they could be related to each other.

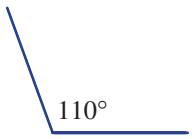
The designs of Vauban, and the star fortress design, have been used by architects since to improve the workings of forts. Vauban's designs show an understanding not just of the workings of geometry but how geometry could be used to ensure his fortifications could withstand attack.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

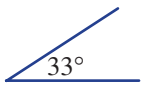
Pre-test

1 Are the following angles acute, right, obtuse or reflex?

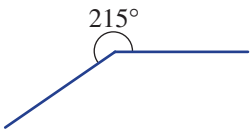
a



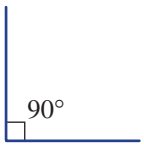
b



c

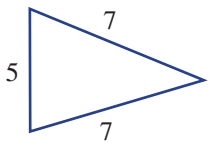


d

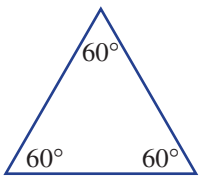


2 Identify the following triangles as equilateral, isosceles, scalene or right angled.

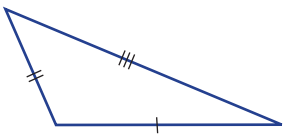
a



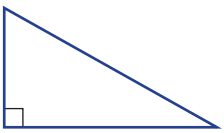
b



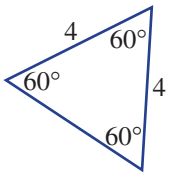
c



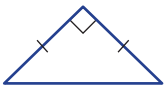
d



e

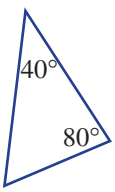


f

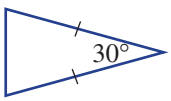


3 Find any missing angles in these triangles.

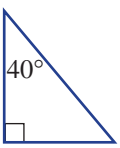
a



b



c



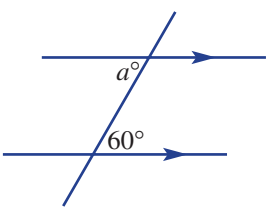
4 Solve the following equations.

a $x + 35 = 90$ **b** $x + 80 = 180$ **c** $x + 20 + 50 = 180$

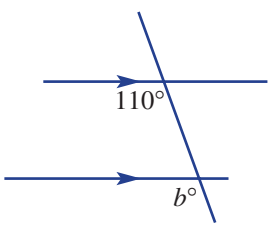
d $2x + 90 = 180$ **e** $x + 2x + 240 = 360$ **f** $5x = 540$

5 Find the value of the pronumerals in each of the following.

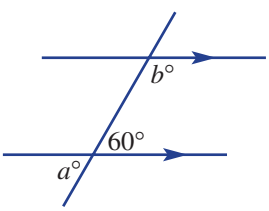
a



b

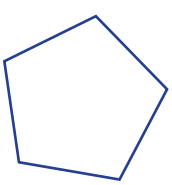


c

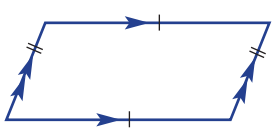


6 Name the following shapes.

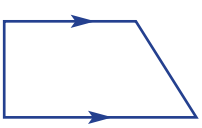
a



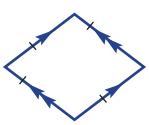
b



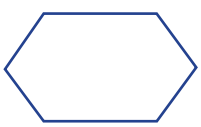
c



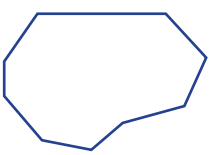
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7A Angles and triangles REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Geometry is all around us. The properties of the shape of a window, doorway or roofline, depend on their geometry. When lines meet, angles are formed and it is these angles which help define the shape of an object. Fundamental to geometry are the angles formed at a point and the three angles in a triangle. Lines meeting at a point and triangles have special properties and will be revised in this section.



Modern architecture using geometric shapes: the spire of the Arts Centre, Melbourne

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

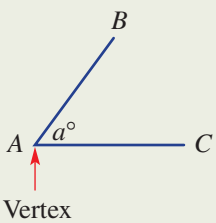
Let's start: Impossible triangles

Triangles are classified either by their side lengths or by their angles.

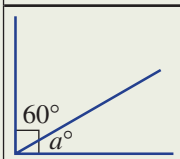
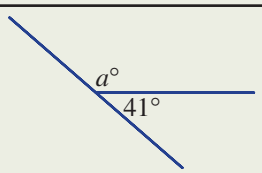
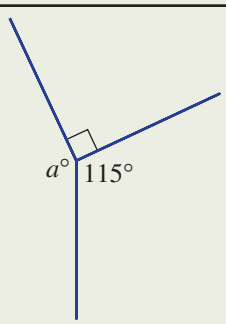
- First, write the list of three triangles that are classified by their side lengths and the three triangles that are classified by their angles.
- Now try to draw a triangle for each of these descriptions. Then decide which are possible and which are impossible.
  - acute scalene triangle
  - obtuse scalene triangle
  - obtuse isosceles triangle
  - right isosceles triangle
  - right equilateral triangle
  - acute equilateral triangle

Key ideas

- When two **rays, lines** or **line segments** meet at a point, an **angle** is formed.
  - This angle is named $\angle A$ or $\angle BAC$ or $\angle CAB$



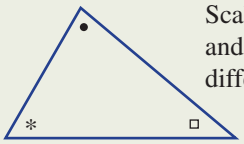
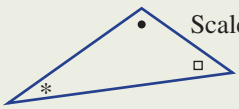
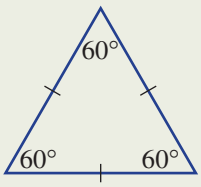
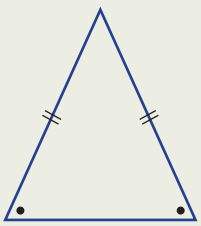
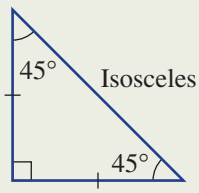
- The pronumeral a represents the number of degrees in the angle.
- Angle types
  - **Acute** between 0° and 90°
  - **Obtuse** between 90° and 180°
  - **Reflex** between 180° and 360°
  - **Right** 90°
  - **Straight** 180°
  - **Revolution** 360°
- Angles at a point

• **Complementary** (sum to 90°)	• **Supplementary** (sum to 180°)	• **Revolution** (sum to 360°)	• **Vertically opposite** (are equal)
 $a + 60 = 90$	 $a + 41 = 180$	 $a + 90 + 115 = 360$	

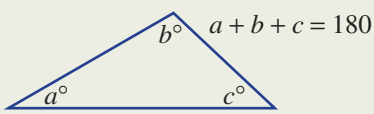
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

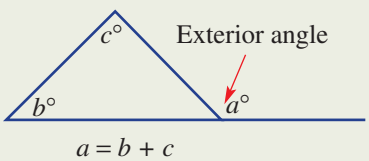
- Types of triangles

Acute angled (all angles $< 90°$)	Right angled (includes a $90°$ angle)	Obtuse angled (one angle $> 90°$)
 Scalene (all sides and angles are different sizes)	 Scalene	 Scalene
 Equilateral (all angles $60°$ and all sides equal)  Isosceles (two angles equal and two sides equal)		 Isosceles

- The sum of the angles in a triangle is $180°$.



- An **exterior angle** is formed by extending one side of a shape.
  - **Exterior angle theorem of a triangle**. The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



Example 1 Finding supplementary and complementary angles

For the angle $47°$ determine the size of its:

a supplementary angle

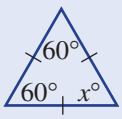
b complementary angle

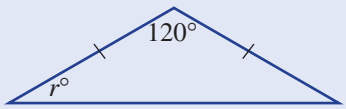
SOLUTION	EXPLANATION
a $180° - 47° = 133°$	Supplementary angles sum to $180°$.
b $90° - 47° = 43°$	Complementary angles sum to $90°$.

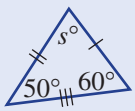
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 2 Finding unknown angles in triangles

Name the types of triangles shown here and determine the values of the pronumerals.

a 

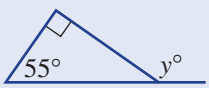
b 

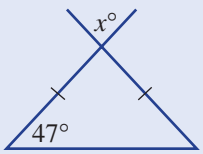
c 

SOLUTION	EXPLANATION
a Equilateral triangle $x = 60$	All sides are equal, therefore all angles are equal.
b Obtuse isosceles triangle $2r + 120 = 180$ $2r = 60$ $r = 30$	One angle is more than 90° and two sides are equal. Angles in a triangle add to 180°. Subtract 120° from both sides and then divide both sides by 2.
c Acute scalene triangle $s + 50 + 60 = 180$ $s + 110 = 180$ $s = 70$	All angles are less than 90° and all sides are of different length. Angles in a triangle add to 180°. Simplify and solve for s.

Example 3 Finding exterior angles

Find the value of each pronumeral. Give reasons for your answers.

a 

b 

SOLUTION

a Let a be the unknown angle.

$a + 90 + 55 = 180$ (Angle sum of a triangle)

$a = 35$

$y + 35 = 180$

$y = 145$

Alternative method

$y = 90 + 55$ (The exterior angle is equal to the sum of the two opposite interior angles)

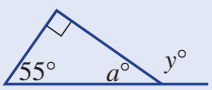
$= 145$

b $x + 47 + 47 = 180$ (Angle sum of a triangle)

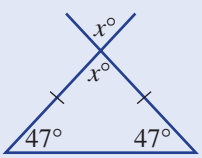
$x + 94 = 180$

$x = 86$

EXPLANATION



Angles in a triangle sum to 180°.
Angles in a straight line are supplementary (sum to 180°).
Alternatively, use the exterior angle theorem.



Note the isosceles triangle and vertically opposite angles. Angles in a triangle add to 180° and vertically opposite angles are equal. Simplify and solve for x.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 7A REVISION

UNDERSTANDING AND FLUENCY	1–8	2, 4, 5–9(½)	6–9(½)

1 Choose a word or number to complete each sentence.

- **a** A 90° angle is called a _____ angle.
- **b** A _____ angle is called a straight angle.
- **c** A 360° angle is called a _____.
- **d** _____ angles are between 90° and 180°.
- **e** _____ angles are between 0° and 90°.
- **f** Reflex angles are between ____ and 360°.
- **g** Complementary angles sum to _____.
- **h** _____ angles sum to 180°.
- **i** The three angles in a triangle add to _____.
- **j** Vertically opposite angles are _____.

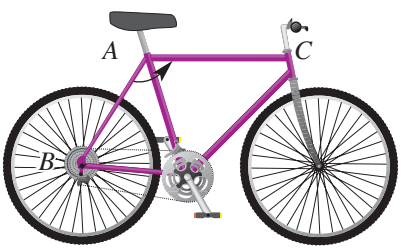
2 What type of triangle has:

- **a** a pair of equal length sides?
- **b** one obtuse angle?
- **c** all angles 60°?
- **d** one pair of equal angles?
- **e** all angles acute?
- **f** all sides of different length?

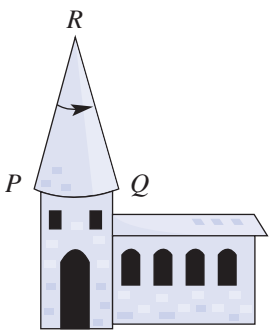
3 For each diagram:

- **i** name the angle shown (e.g. $\angle ABC$)
- **ii** state the type of angle given
- **iii** estimate the size of the angle
- **iv** measure the angle using a protractor

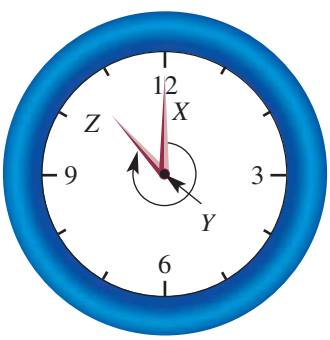
a



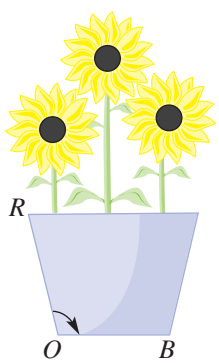
b



c



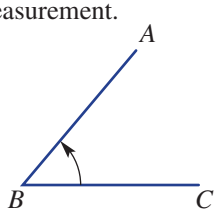
d



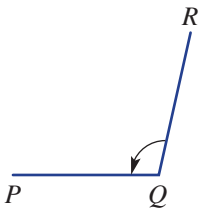
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 Estimate the size of each of the following angles and use your protractor to determine an accurate measurement.

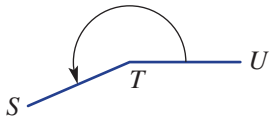
a



b



c



Example 1

5 For each of the following angles determine the:

i supplementary angle

ii complementary angle

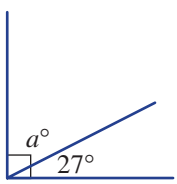
a 55°
b 31°
c 74°
d 10°
e 89°
f 22°
g 38°
h 65°
i 47°
j 77°

6 State whether each of the following pairs of angles is supplementary (S), complementary (C) or neither (N).

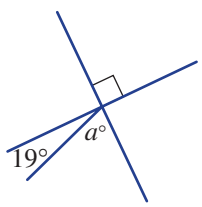
a 30°, 60°
b 45°, 135°
c 100°, 90°
d 50°, 40°
e 70°, 110°
f 14°, 66°
g 137°, 43°
h 24°, 56°

7 Find the value of a in these diagrams, giving reasons.

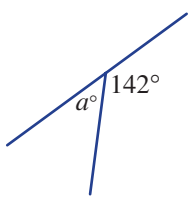
a



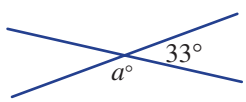
b



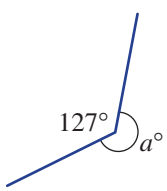
c



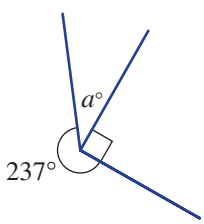
d



e



f

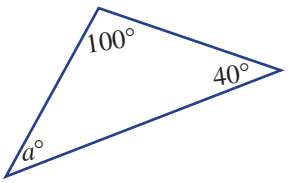


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

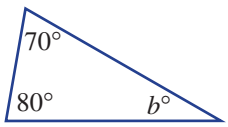
Example 2

8 Name the types of triangles shown here and determine the values of the pronumerals.

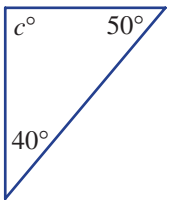
a



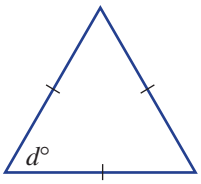
b



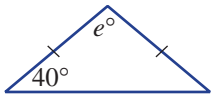
c



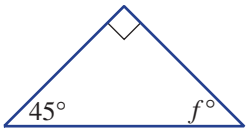
d



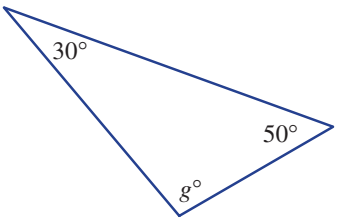
e



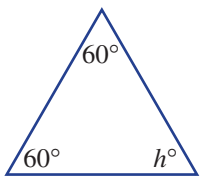
f



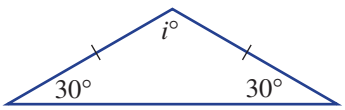
g



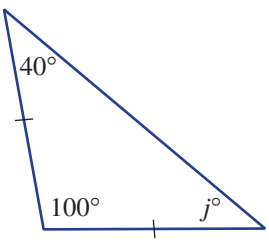
h



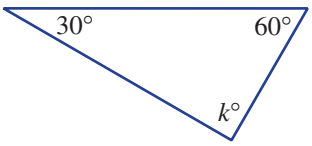
i



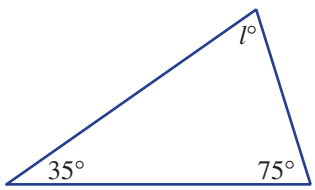
j



k



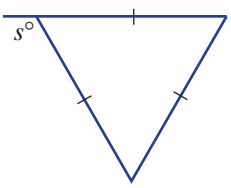
l



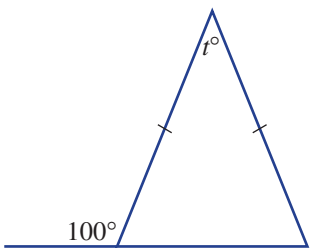
Example 3

9 Find the value of each pronumeral.

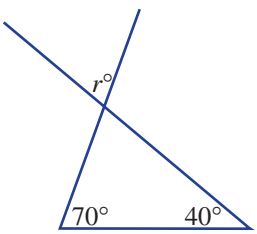
a



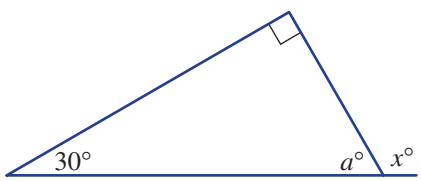
b



c

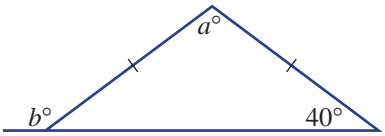


d

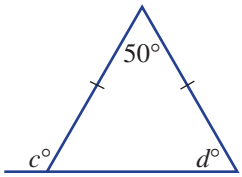


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

e



f



PROBLEM-SOLVING AND REASONING 10, 11, 13, 14 | 10(½), 11, 12(½), 13, 14 | 11, 12(½), 14–16

10 Calculate how many degrees the minute hand of a clock rotates in:

a 1 hour

b $\frac{1}{4}$ of an hour

c 10 minutes

d 15 minutes

e 72 minutes

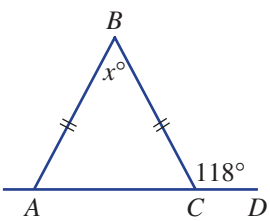
f 1 minute

g 2 hours

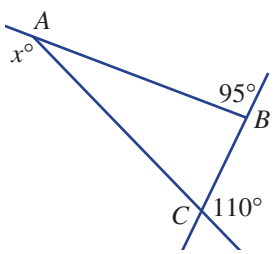
h 1 day

11 Find the value of x in these diagrams, giving reasons.

a



b



12 Find the acute (to obtuse) angle between the hour and minute hands at these times. Remember to consider how the hour hand moves between each whole number.

a 3:00 p.m.

b 5:00 a.m.

c 6:30 p.m.

d 11:30 p.m.

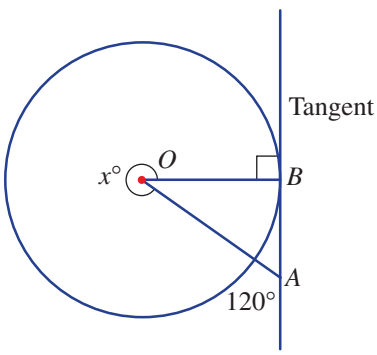
e 3:45 a.m.

f 1:20 a.m.

g 4:55 a.m.

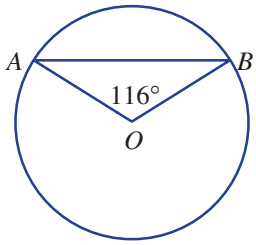
h 2:42 a.m.

13 A tangent to a circle is 90° to its radius. Explain why $x = 330$ in this diagram.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

14 Explain why $\angle OAB$ is 32° in this circle if O is the centre of the circle.

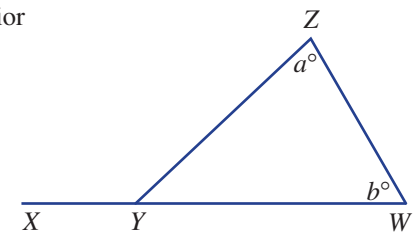


15 In this diagram, $\angle XYZ$ is an exterior angle. *Do not* use the exterior angle theorem in the following.

a If $a = 85$ and $b = 75$, find $\angle XYZ$.

b If $a = 105$ and $b = 60$, find $\angle XYZ$.

c Now using the pronumerals a and b, prove that $\angle XYZ = a° + b°$.



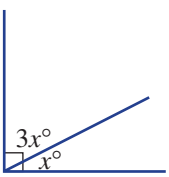
16 Prove that the three exterior angles of a triangle sum to 360°. Use the fact that the three interior angles sum to 180°.

ENRICHMENT — — 17

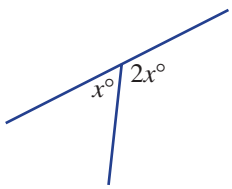
Algebra in geometry

17 Write an equation for each diagram and solve it to find x, giving reasons.

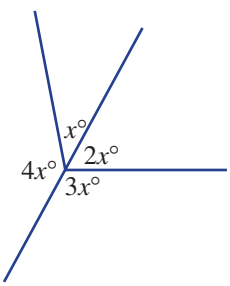
a



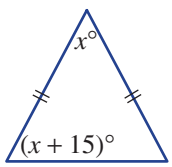
b



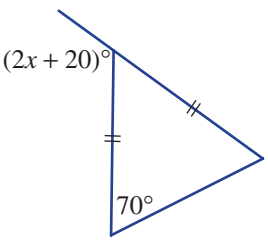
c



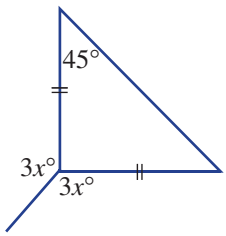
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7B Parallel lines REVISION

Interactive
Widgets
HOTsheets
Walkthrough

A line crossing two or more other lines (called a transversal) creates a number of special pairs of angles. If the transversal cuts two parallel lines, then these special pairs of angles will be either equal or supplementary.



Double lines on straight roads are parallel.

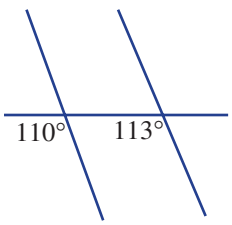
Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

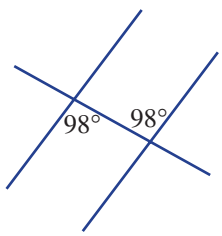
Let's start: Are they parallel?

Here are three diagrams including a transversal crossing two other lines.

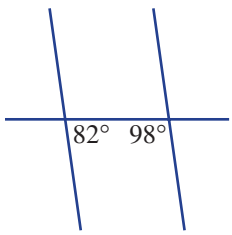
1



2



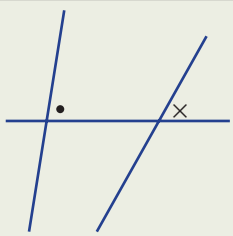
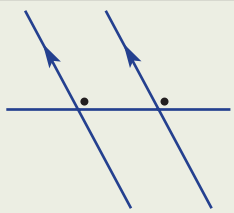
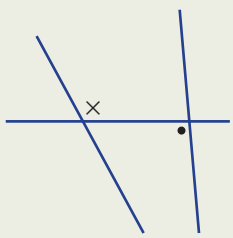
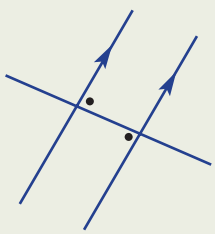
3



- Decide if each diagram contains a pair of parallel lines. Give reasons for your answer.
- What words do you remember regarding the name given to each pair of angles shown in the diagrams?

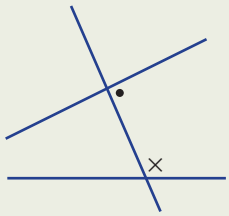
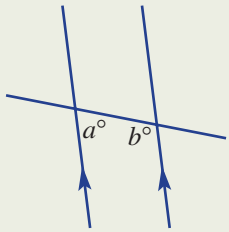
Key ideas

■ A **transversal** is a line crossing two or more other lines.

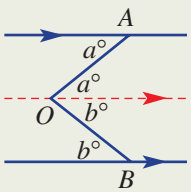
	Non-parallel lines	Parallel lines
Corresponding angles • If lines are parallel, then corresponding angles are equal.		
Alternate angles • If lines are parallel, then alternate angles are equal.		

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

	Non-parallel lines	Parallel lines
Co-interior angles • If lines are parallel, then co-interior angles are supplementary.		 $a + b = 180$

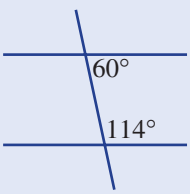
- If a line AB is parallel to a line CD, we write $AB \parallel CD$.
- A parallel line can be added to diagrams to help find other angles.

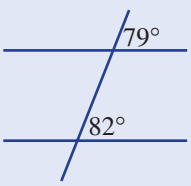


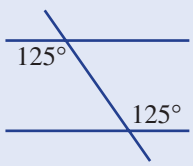
$\angle AOB = a° + b°$

Example 4 Deciding if lines are parallel

Decide if each diagram contains a pair of parallel lines. Give a reason.

a 

b 

c 

SOLUTION

a No. The two co-interior angles are not supplementary.

b No. The two corresponding angles are not equal.

c Yes. The two alternate angles are equal.

EXPLANATION

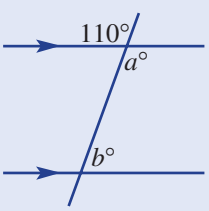
$60° + 114° = 174° \neq 180°$

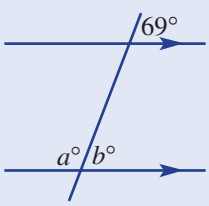
$79° \neq 82°$

If alternate angles are equal, then the lines are parallel.

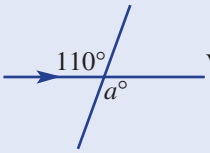
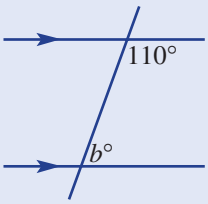
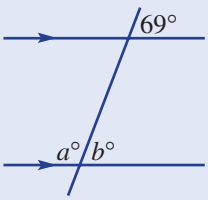
Example 5 Finding angles in parallel lines

Find the value of each of the pronumerals. Give reasons for your answers.

a 

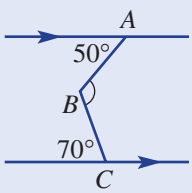
b 

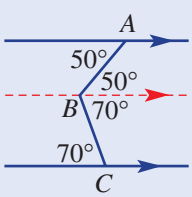
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
a $a = 110$ (vertically opposite angles)	 Vertically opposite angles are equal.
$b + 110 = 180$ $b = 70$ (co-interior angles in parallel lines)	 Co-interior angles add to 180° in parallel lines.
b $b = 69$ (corresponding angles in parallel lines) $a + b = 180$ $a + 69 = 180$ $a = 111$ (angles on a straight line)	 Corresponding angles are equal in parallel lines.

Example 6 Adding a third parallel line

Add a third parallel line to help find $\angle ABC$ in this diagram.



SOLUTION	EXPLANATION
	Add a third parallel line through B to create two pairs of equal alternate angles.
$\angle ABC = 50° + 70°$ $= 120°$	Add 50° and 70° to give the size of $\angle ABC$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 7B REVISION

UNDERSTANDING AND FLUENCY | 1–4 | 2–4(½) | 3–4(½)

1 Use the word *equal* or *supplementary* to complete these sentences.

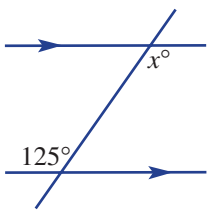
a If two lines are parallel, corresponding angles are _____.

b If two lines are parallel, alternate angles are _____.

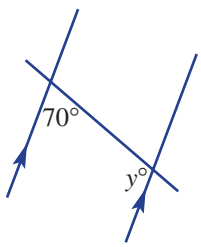
c If two lines are parallel, co-interior angles are _____.

2 Find the values of the pronumerals. Give reasons for your answers.

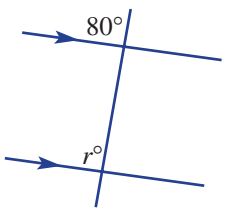
a



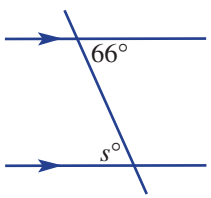
b



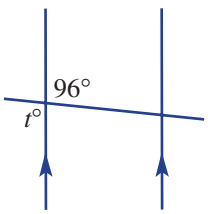
c



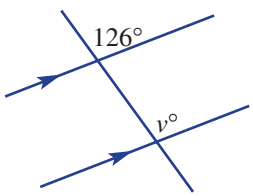
d



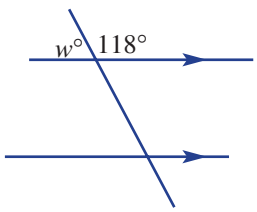
e



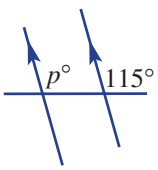
f



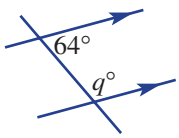
g



h



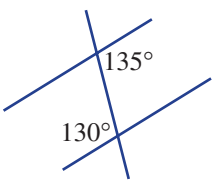
i



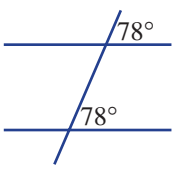
Example 4

3 Decide if each diagram contains a pair of parallel lines. Give a reason.

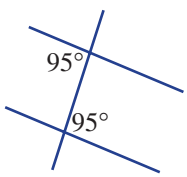
a



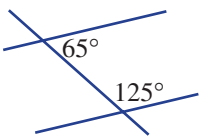
b



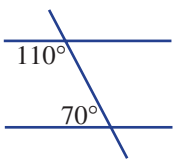
c



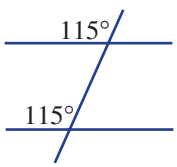
d



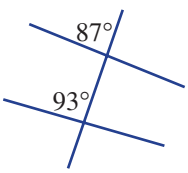
e



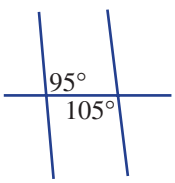
f



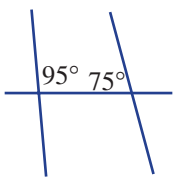
g



h



i

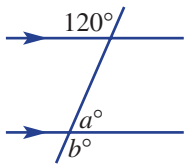


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

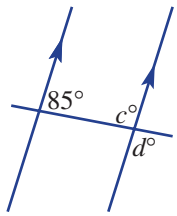
Example 5

4 Find the value of each pronumeral. Give reasons for your answers.

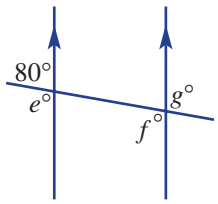
a



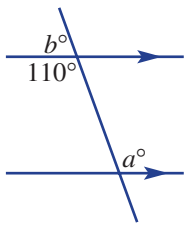
b



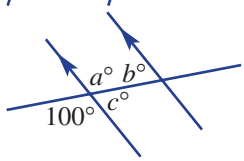
c



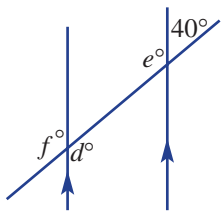
d



e



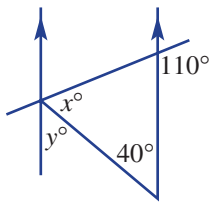
f



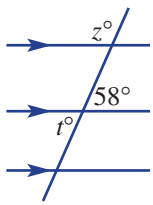
PROBLEM-SOLVING AND REASONING	5, 8	5–6(½), 8(½), 9	5–6(½), 7, 9, 10

5 Find the value of each pronumeral.

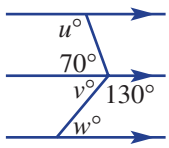
a



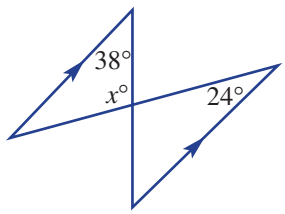
b



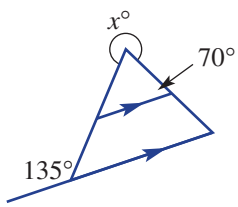
c



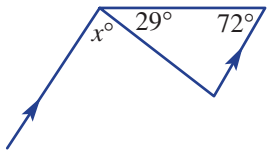
d



e



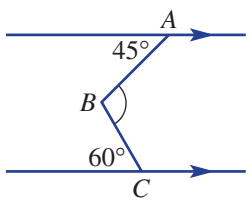
f



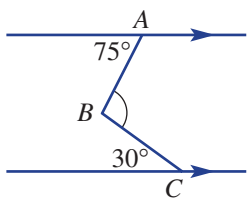
Example 6

6 Add a third parallel line to help find $\angle ABC$ in these diagrams.

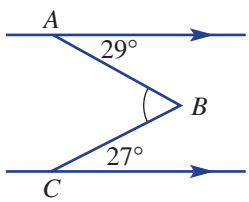
a



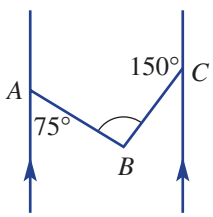
b



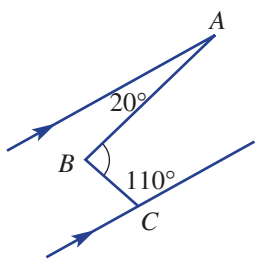
c



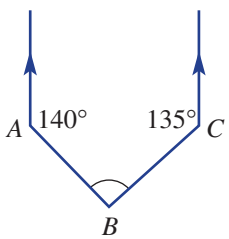
d



e



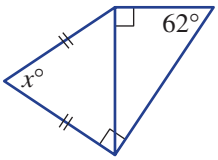
f



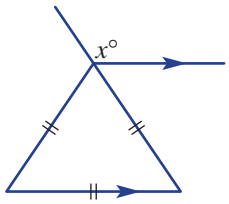
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Find the value of x.

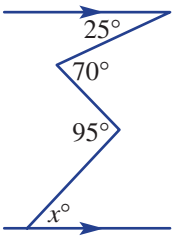
a



b

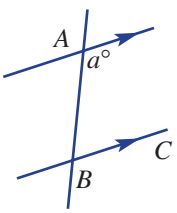


c

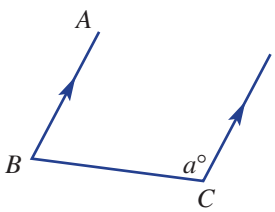


8 Write an expression (for example, $180° - a°$) for $\angle ABC$ in these diagrams.

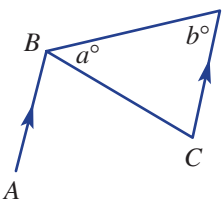
a



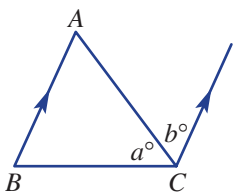
b



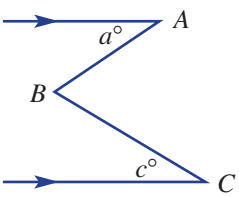
c



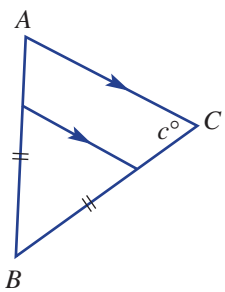
d



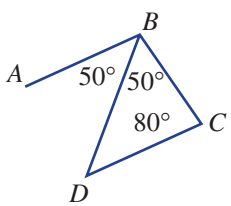
e



f

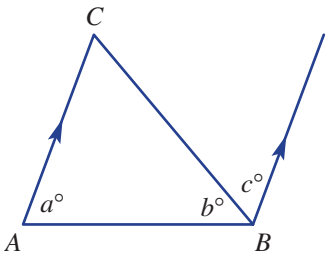


9 Give reasons why $AB \parallel DC$ (AB is parallel to DC) in this diagram.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 This diagram includes a triangle and a pair of parallel lines.



a Using the parallel lines, explain why $a + b + c = 180$.

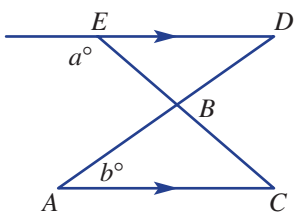
b Explain why $\angle ACB = c°$.

c Explain why this diagram helps to prove that the angle sum of a triangle is 180°.

ENRICHMENT — — 11

Proof in geometry

11 Here is a written proof showing that $\angle ABC = a° - b°$.



$\angle BED = 180° - a°$ (Supplementary angles)

$\angle BCA = 180° - a°$ (Alternate angles and $ED \parallel AC$)

$\angle ABC = 180° - b° - (180 - a)°$ (Angle sum of a triangle)

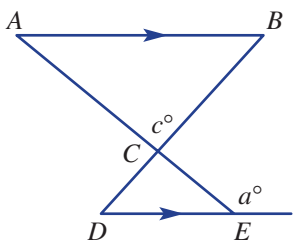
$= 180° - b° - 180° + a°$

$= -b° + a°$

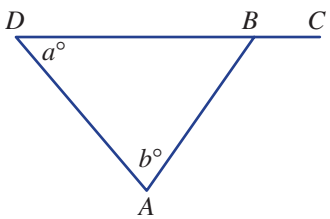
$= a° - b°$

Write proofs similar to the above for each of the following.

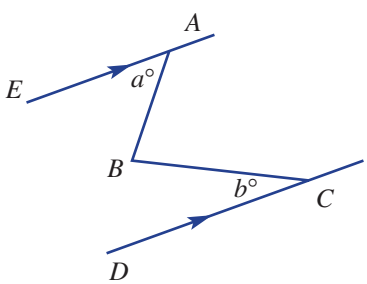
a $\angle ABC = a° - c°$



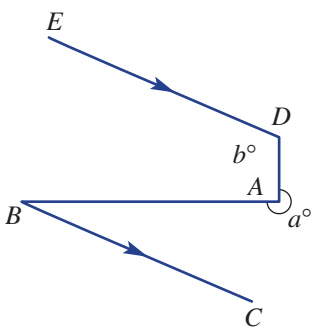
b $\angle ABC = a° + b°$



c $\angle ABC = a° + b°$



d $\angle ABC = 180° + b° - a°$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7C Quadrilaterals and other polygons

Interactive

Widgets

HOTsheets

Walkthrough

Closed two-dimensional shapes with straight sides are called polygons and are classified by their number of sides. Quadrilaterals have four sides and are classified further by their special properties.



Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Draw that shape

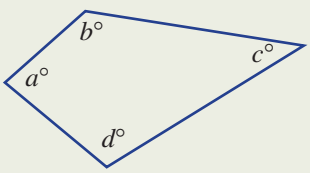
Use your knowledge of polygons to draw each of the following shapes. Mark any features, including parallel sides and sides of equal length.

- convex quadrilateral
- square, rectangle, rhombus and parallelogram
- non-convex pentagon
- kite and trapezium
- regular hexagon

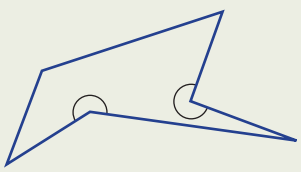
Compare the properties of each shape to ensure you have indicated each property on your drawings.

Key ideas

- **Convex polygons** have all interior angles less than 180°. A **non-convex polygon** has at least one interior angle greater than 180°.

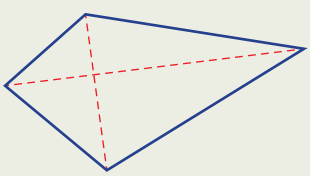


Convex quadrilateral

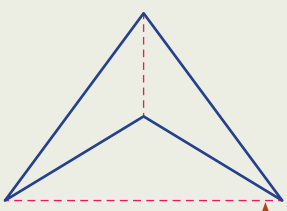


Non-convex hexagon

- The diagonals of a convex polygon lie *inside* the figure.



Convex quadrilateral



Non-convex quadrilateral — This diagonal is outside the figure.

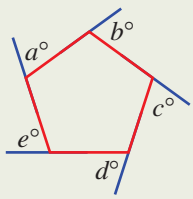
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

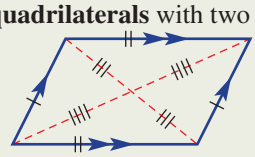
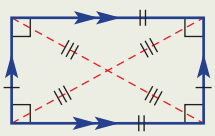
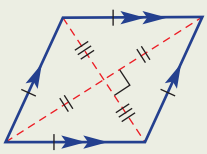
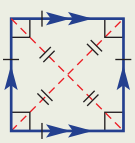
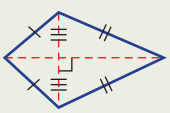
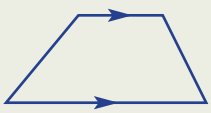
Key ideas

- The **sum of the interior angles**, $S°$, in a polygon with n sides is given by $\boldsymbol{S = 180(n - 2)}$.

Polygon	Number of sides (n)	Angle sum ($S°$)
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Heptagon	7	900°
Octagon	8	1080°
Nonagon	9	1260°
Decagon	10	1440°
Undecagon	11	1620°
Dodecagon	12	1800°
n-gon	n	$180(n - 2)°$

- The diagram on the right shows the *exterior* angles of a pentagon.



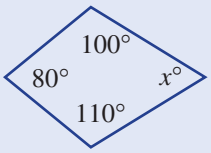
- In *every* polygon, the sum of the exterior angles is 360°.
- **Regular polygons** have equal length sides and equal interior angles.
- **Parallelograms** are **quadrilaterals** with two pairs of parallel sides. They include:
  - Parallelogram 
  - Rectangle 
  - Rhombus 
  - Square 
- The kite and trapezium are also special quadrilaterals.
  - Kite 
  - Trapezium 
- Rhombuses and squares are kites.
- A trapezium is a quadrilateral with at least one pair of parallel sides. So parallelograms, including rhombuses, rectangles and squares, are trapezia.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

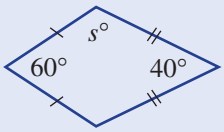
Example 7 Finding unknown angles in quadrilaterals

Find the value of the pronumeral in each of these quadrilaterals, giving reasons.

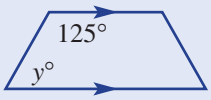
a



b



c



SOLUTION

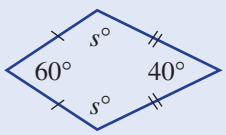
a $x + 80 + 100 + 110 = 360$ (angle sum of a quadrilateral)
$x + 290 = 360$
$x = 70$

b $2s + 60 + 40 = 360$ (angle sum of a quadrilateral)
$2s + 100 = 360$
$2s = 260$
$s = 130$

c $y + 125 = 180$ (co-interior angles in parallel lines)
$y = 55$

EXPLANATION

The angles in a quadrilateral add to 360°.
Simplify and solve for x.



The angles in a quadrilateral add to 360° and the opposite angles (s°) are equal in a kite.

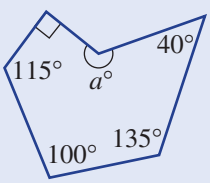
Simplify and solve for s.

Co-interior angles inside parallel lines are supplementary.

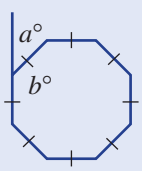
Example 8 Finding unknown angles in polygons

For each polygon find the angle sum using $S = 180(n - 2)$, then find the value of the pronumerals.

a



b



SOLUTION

a $n = 6$ and $S = 180(n - 2)$
$= 180(6 - 2)$
$= 720$

$a + 90 + 115 + 100 + 135 + 40 = 720$
$a + 480 = 720$
$a = 240$

b $n = 8$ and $S = 180(n - 2)$
$= 180(8 - 2)$
$= 1080$
$8b = 1080$
$b = 135$
$a + 135 = 180$
$a = 45$

EXPLANATION

The shape is a hexagon with 6 sides so $n = 6$.

The sum of all angles is 720°. Simplify and solve for a.

The regular octagon has 8 sides so use $n = 8$.

Each interior angle is equal so $8b$° makes up the angle sum.

a° is an exterior angle and a° and b° are supplementary.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 7C

UNDERSTANDING AND FLUENCY	1–4, 5–7(½)	3, 4, 5–7(½)	5–7(½)

1 What is the name given to shapes with:

a five equal sides **b** ten equal sides **c** eight sides

2 Copy and complete the table.

Polygon	Sides	Interior angle sum	Interior angles	Exterior angle sum	Exterior angles
a Equilateral triangle					
b Square					
c Regular hexagon					
d Regular octagon					

3 Write the word to complete these sentences.

a A parallelogram has two pairs of _____ sides.

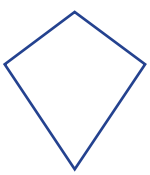
b The diagonals in a rhombus intersect at _____ angles.

c A _____ has one pair of parallel sides.

d The diagonals in a rectangle are _____ in length.

4 Name each of these shapes as convex or non convex.

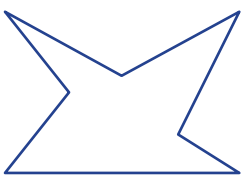
a



b



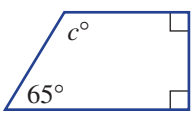
c



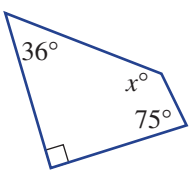
Example 7a

5 Find the values of the pronumerals.

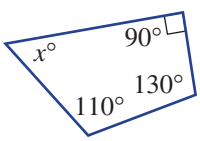
a



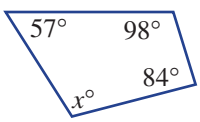
b



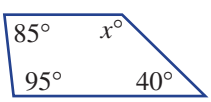
c



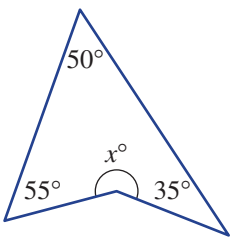
d



e



f

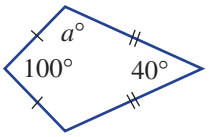


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

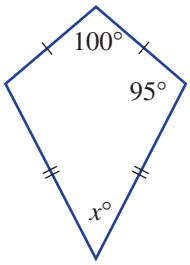
Example 7b, c

6 Find the values of the pronumerals.

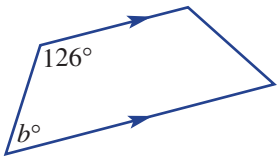
a



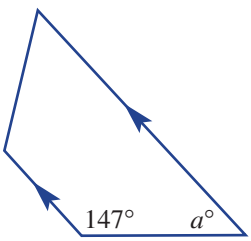
b



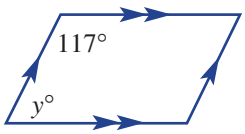
c



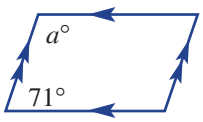
d



e



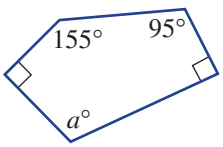
f



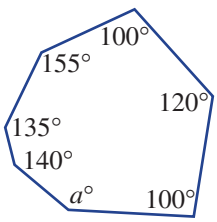
Example 8

7 For each polygon find the angle sum using $S = 180(n - 2)$, then find the value of a.

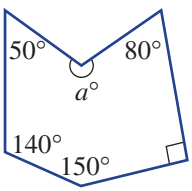
a



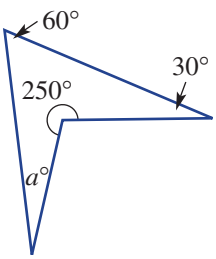
b



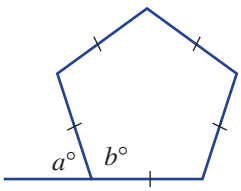
c



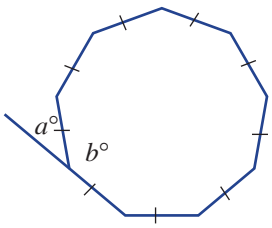
d



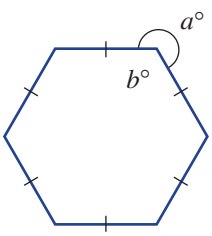
e



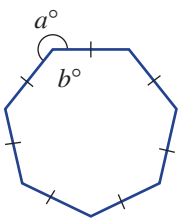
f



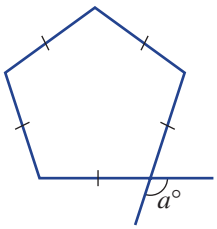
g



h



i



PROBLEM-SOLVING AND REASONING	8, 11, 13	8, 9, 10(½), 11–13	9, 10, 12–14

8 List all the quadrilaterals that have these properties.

a 2 pairs of equal length sides

b All interior angles 90°

c Diagonals of equal length

d Diagonals intersecting at right angles

9 Calculate the number of sides if a polygon has the given angle sum. *Hint*: use the rule $S = 180(n - 2)$.

a 2520°

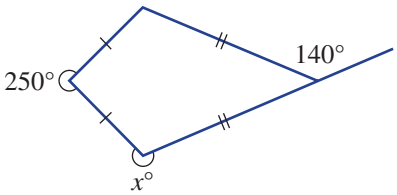
b 4140°

c 18 000°

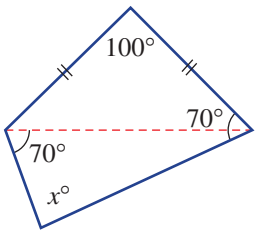
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Find the value of x in these diagrams.

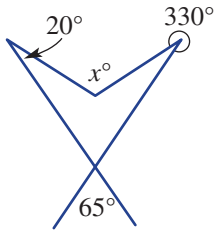
a



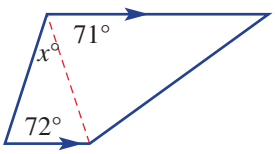
b



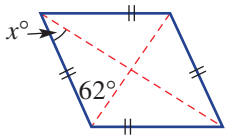
c



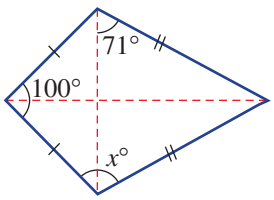
d



e



f



11 Explain why a rectangle, square and rhombus are all parallelograms.

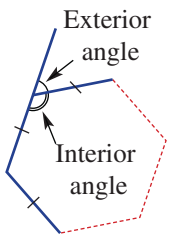
12 For a regular polygon with n sides:

a write the rule for the sum of interior angles (S)

b write the rule for the size of each interior angle (I)

c write the rule for the size of each exterior angle (E)

d use your rule from part **c** to find the size of the exterior angle of a regular decagon.



13 A 50-cent piece has 12 equal sides and 12 equal angles. Calculate the size of each:

a exterior angle **b** interior angle

14 Recall that a non-convex polygon has at least one reflex interior angle.

a What is the maximum number of interior reflex angles possible for these polygons?

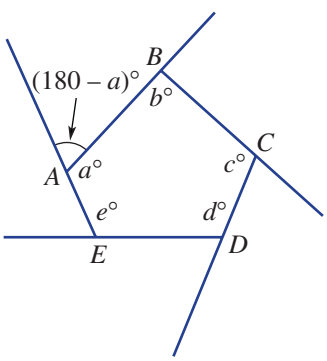
i Quadrilateral **ii** Pentagon **iii** Octagon

b Write an expression for the maximum number of interior reflex angles for a polygon with n sides.

ENRICHMENT — — 15

Angle sum proof

15 Note that if you follow the path around this pentagon starting and finishing at point A (provided you finish by pointing in the same direction as you started) you will have turned a total of 360°.



Complete this proof of the angle sum of a pentagon (540°).

$(180 - a) + (180 - b) + (\quad) + (\quad) + (\quad) =$ _____

(Sum of exterior angles is _____)

$180 + 180 +$ ___ + ___ + ___ $-$ ($a + b +$ __ + __ + __) $= 360$

_____ $-$ ( ) $= 360$

( ) = ___

Now complete a similar proof for the angle sum of these polygons.

a Hexagon **b** Heptagon

Extension: Try to complete a similar proof for a polygon with n sides.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7D Congruent triangles REVISION

Interactive
Widgets
HOTsheets
Walkthrough

When two objects have the same shape and size, we say they are congruent. Matching sides will be the same length and matching angles will be the same size. The area of congruent shapes will also be equal. However, not every property of a pair of shapes needs to be known in order to determine their congruence. This is highlighted in the study of congruent triangles where four tests can be used to establish congruence.



The Petronas Twin Towers in Kuala Lumpur look congruent.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

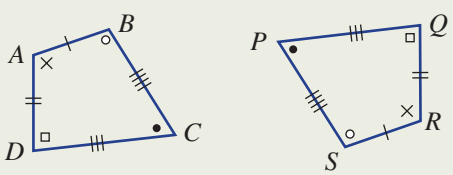
Let's start: Constructing congruent triangles

To complete this task you will need a ruler, pencil and protractor. (For accurate constructions you may wish to use compasses.) Divide these constructions up equally among the members of the class. Each group is to construct their triangle with the given properties.

1. Triangle ABC with $AB = 8$ cm, $AC = 5$ cm and $BC = 4$ cm
2. Triangle DEF with $DE = 7$ cm, $DF = 6$ cm and $\angle EDF = 40°$
3. Triangle GHI with $GH = 6$ cm, $\angle IGH = 50°$ and $\angle IHG = 50°$
4. Triangle JKL with $\angle JKL = 90°$, $JL = 5$ cm and $KL = 4$ cm
   - Compare all triangles with the vertices ABC. What do you notice? What does this say about two triangles that have three pairs of equal side lengths?
   - Compare all triangles with the vertices DEF. What do you notice? What does this say about two triangles that have two pairs of equal side lengths and the included angles equal?
   - Compare all triangles with the vertices GHI. What do you notice? What does this say about two triangles that have two equal matching angles and one matching equal length side?
   - Compare all triangles with the vertices JKL. What do you notice? What does this say about two triangles that have one right angle, the hypotenuse and one other matching equal length side?

Key ideas

- **Congruent figures** have the same shape and size. They are identical in every way.
  - If two figures are congruent, one of them can be transformed by using rotation, reflection and/or translation to match the other figure exactly.

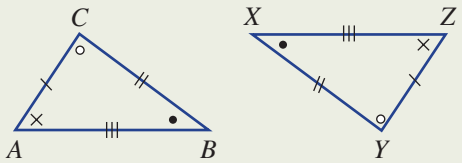


- In the above diagram, AB and RS are called 'matching sides' or 'corresponding sides'.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Consider these congruent triangles.



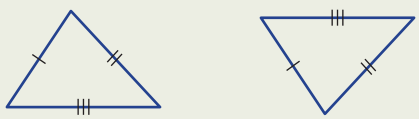
Matching (corresponding) sides	Matching (corresponding) angles
$AB = ZX$	$\angle A = \angle Z$
$BC = XY$	$\angle B = \angle X$
$AC = ZY$	$\angle C = \angle Y$

We write $\Delta ABC \equiv \Delta ZXY$ (not $\Delta ABC \equiv \Delta XYZ$)

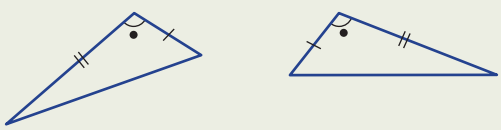
This is called a **congruence statement**. Note the order of the letters in the statement.

- **Corresponding** sides are opposite equal corresponding angles.

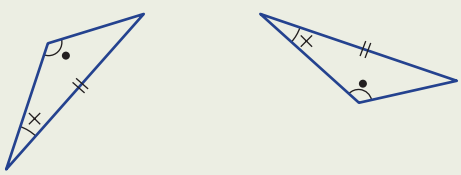
- Tests for triangle congruence.
  - **Side, Side, Side (SSS) test**
    If the three sides of a triangle are respectively equal to the three sides of another triangle, then the two triangles are congruent.



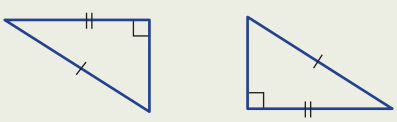
  - **Side, Angle, Side (SAS) test**
    If two sides and the included angle of a triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent.



  - **Angle, Angle, Side (AAS) test**
    If two angles and one side of a triangle are respectively equal to two angles and the matching side of another triangle, then the two triangles are congruent.



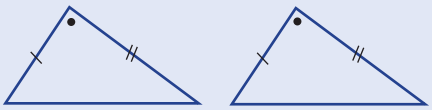
  - **Right angle, Hypotenuse, Side (RHS) test**
    If the hypotenuse and second side of a right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent.

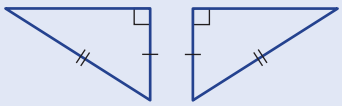


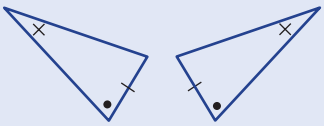
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

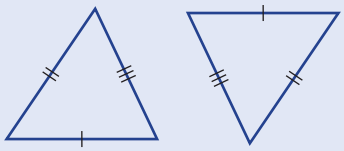
Example 9 Choosing the appropriate congruence test

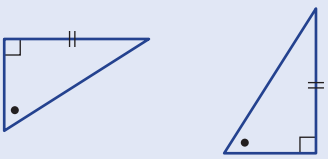
Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

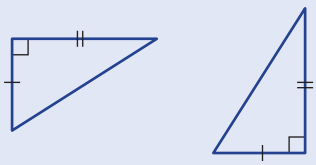
a 

b 

c 

d 

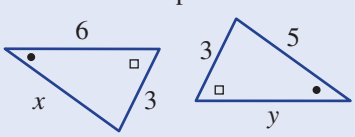
e 

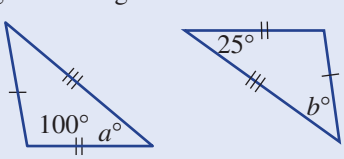
f 

SOLUTION	EXPLANATION
a SAS	Two pairs of matching sides and the included angles are equal.
b RHS	A right angle, hypotenuse and one pair of matching sides are equal.
c AAS	Two pairs of angles and a pair of matching sides are equal.
d SSS	Three pairs of matching sides are equal.
e AAS	Two pairs of angles and a pair of matching sides are equal.
f SAS	Two pairs of matching sides and the included angles are equal.

Example 10 Finding unknown side lengths and angles using congruence

Find the values of the pronumerals in these pairs of congruent triangles.

a 

b 

SOLUTION	EXPLANATION
a $x = 5$	The side of length x and the side of length 5 are in matching positions (opposite the □).
$y = 6$	The longest side on both triangles must be equal.
b $a = 25$	The angle marked $a°$ matches the 25° angle in the other triangle. The angle marked $b°$ correspands to the missing angle in the first triangle. The sum of three angles in a triangle is 180°.
$b = 180 - 100 - 25$	
$= 55$	

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 7D REVISION

UNDERSTANDING AND FLUENCY	1–5	2, 4–5(½)	4–5(½)

1 Copy and complete these sentences.

a Congruent figures are exactly the same shape and _____.

b If triangle ABC is congruent to triangle STU then we write $\Delta ABC \square \Delta STU$.

c The abbreviated names of the four congruence tests for triangles are SSS, ____, ____ and _____.

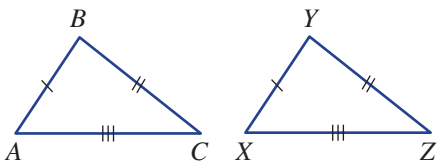
2 These two triangles are congruent.

a Name the side on ΔXYZ that matches:

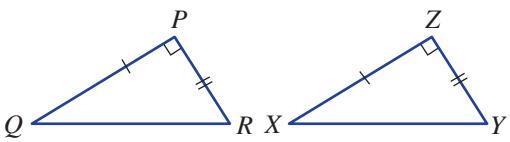
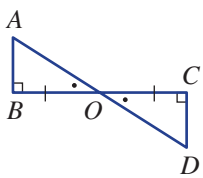
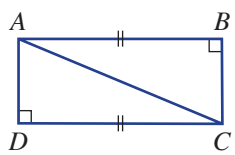
i AB **ii** AC **iii** BC

b Name the angle in ΔABC that matches:

i $\angle X$ **ii** $\angle Y$ **iii** $\angle Z$



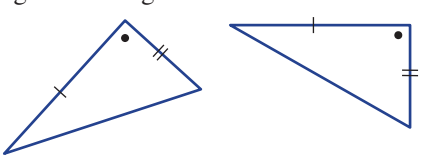
3 Copy and completed the table.

	Triangles	Matching sides	Matching angles	Congruence statement	Test
a		= = =	$\angle = \angle$ $\angle = \angle$ $\angle = \angle$	Δ___ $\equiv \Delta$___	
b		= = =	$\angle = \angle$ $\angle = \angle$ $\angle = \angle$	Δ___ $\equiv \Delta$___	
c		= = =	$\angle = \angle$ $\angle = \angle$ $\angle = \angle$	Δ___ $\equiv \Delta$___	

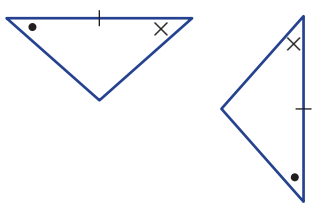
Example 9

4 Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

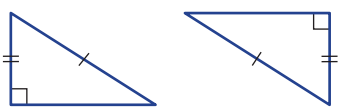
a



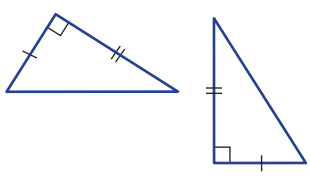
b



c

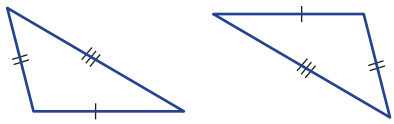


d

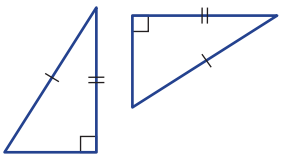


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

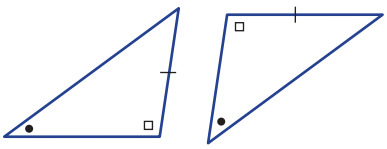
e



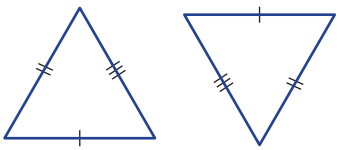
f



g



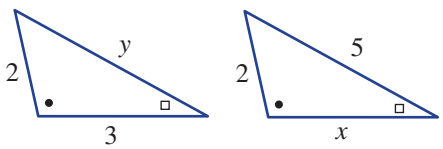
h



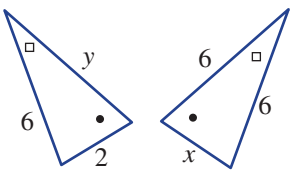
Example 10

5 Find the values of the pronumerals in these pairs of congruent triangles.

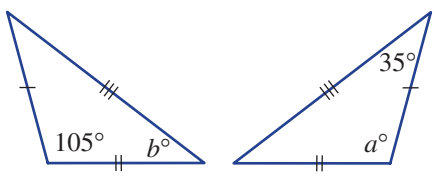
a



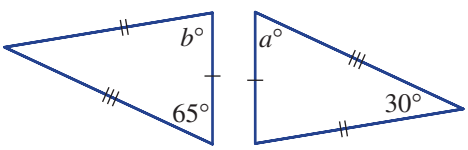
b



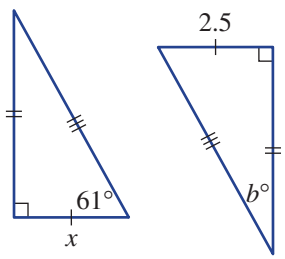
c



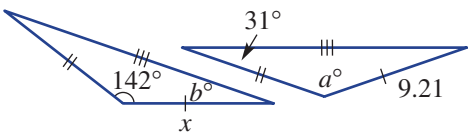
d



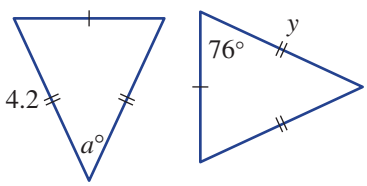
e



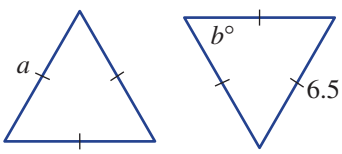
f



g



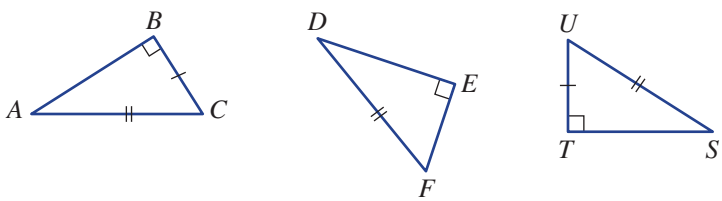
h



PROBLEM-SOLVING AND REASONING 6, 7, 9, 10 | 6, 7, 9–11 | 6–8, 10–12

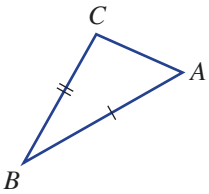
6 For each set of three triangles, choose the two that are congruent. Choose the appropriate test (SSS, SAS, AAS or RHS) and write a congruence statement (e.g. $\Delta ABC \equiv \Delta FGH$).

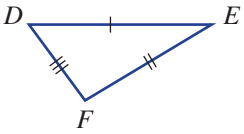
a

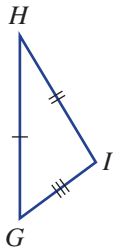


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

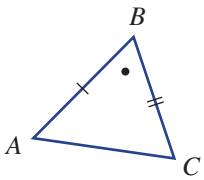
b

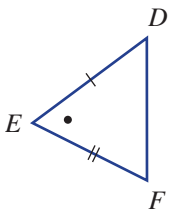


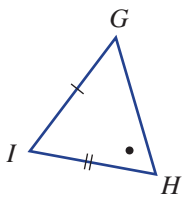




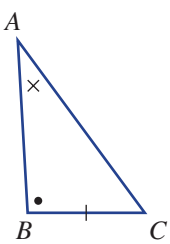
c

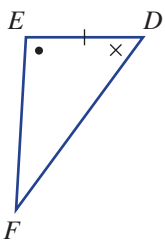


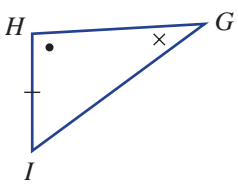




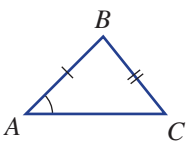
d

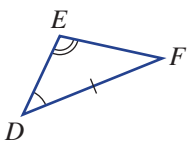


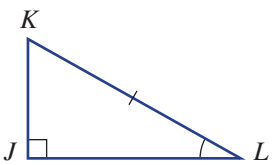


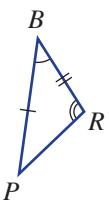


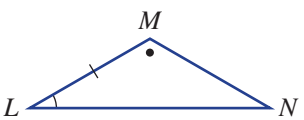
7 Identify all pairs of congruent triangles from those below. Angles with the same mark are equal.

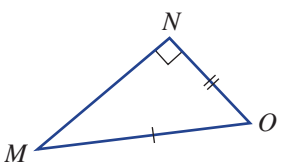


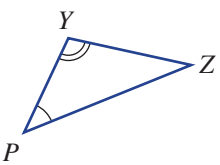


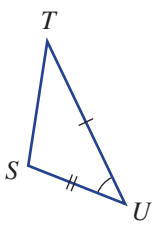


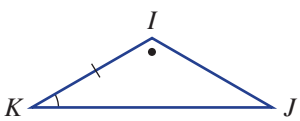


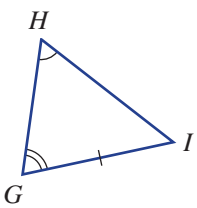


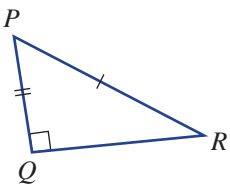


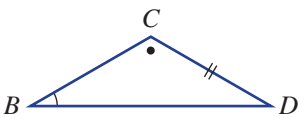


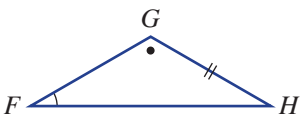








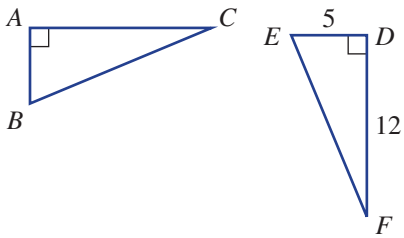




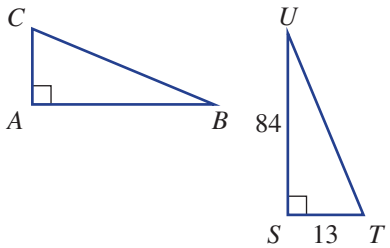
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 Use Pythagoras' theorem to help find the length BC in these pairs of congruent triangles.

a

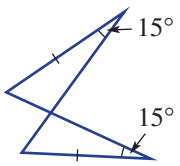


b



9 Are all triangles with three pairs of equal matching angles congruent? Explain why or why not.

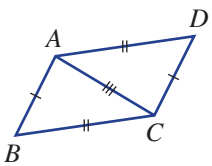
10 Consider this diagram including two triangles.



a Explain why there are two pairs of equal matching angles.

b Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.

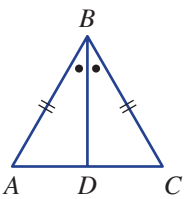
11 $ABCD$ is a parallelogram.



a Give the reason why $\Delta ABC \equiv \Delta CDA$.

b What does this say about $\angle B$ and $\angle D$?

12 Consider this diagram.



a Explain why there are two pairs of matching sides of equal length for the two triangles.

b Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.

c Write a congruence statement.

d Explain why AC is perpendicular (90°) to DB.

ENRICHMENT — — 13

Why not ASS?

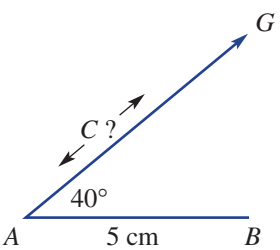
13 Angle, Side, Side (ASS) is not a test for congruence of triangles. Complete these tasks to see why.

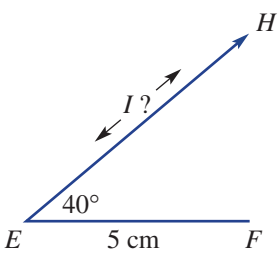
a Draw two line segments AB and EF both 5 cm long.

b Draw two rays AG and EH so that both $\angle A$ and $\angle E$ are 40°.

c Now place a point C on ray AG so that $BC = 4$ cm.

d Place a point I on ray EH so that FI is 4 cm but place it in a different position so that ΔABC is not congruent to ΔEFI.





e Show how you could use compasses to find the two different places you could put the points C or I so that BC and FI are 4 cm.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7E Using congruence in proof

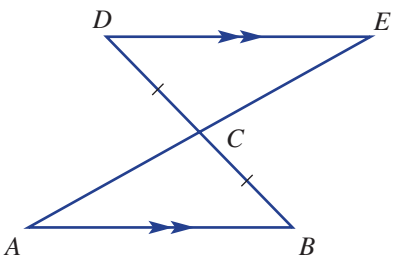
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

A mathematical proof is a sequence of correct statements that leads to a result. It should not contain any big 'leaps' and should provide reasons at each step. The proof that two triangles are congruent should list all the corresponding pairs of sides and angles. Showing that two triangles are congruent can then lead to the proof of other geometrical results.

Let's start: Complete the proof

Help complete the proof that $\Delta ABC \equiv \Delta EDC$ for this diagram. Give the missing reasons and congruent triangle in the final statement.



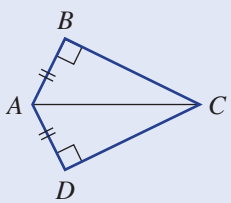
In ΔABC and ΔEDC
$\angle DCE = \angle BCA$ (_____)
$\angle ABC = \angle EDC$(_____)
$BC = DC$ (given equal sides)
$\therefore \Delta ABC \equiv$ _____ (AAS)

Key ideas

- Prove that two triangles are congruent by listing all the matching equal angles and sides.
  - Give a reason for each statement.
  - Conclude by writing a congruence statement and the congruence test (SSS, SAS, AAS or RHS).
  - Vertex labels are written in matching order.
- Other geometrical results can be proved by using the properties of congruent triangles.

Example 11 Proving that two triangles are congruent

Prove that $\Delta ABC \equiv \Delta ADC$.



SOLUTION

In ΔABC and ΔADC
$\angle B = \angle D = 90°$ (given equal angles) (R)
AC is common (H)
$AB = AD$ (given equal sides) (S)
$\therefore \Delta ABC \equiv \Delta ADC$ (RHS)

EXPLANATION

Both triangles have a right angle.
AC is common to both triangles.
AB and AD are marked as equal.

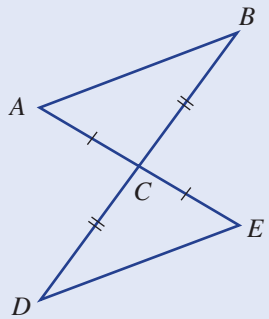
Write the congruence statement and the abbreviated reason. Write the vertex labels in matching order.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 12 Proving geometrical results using congruence

a Prove that $\Delta ABC \equiv \Delta EDC$.

b Hence prove that $AB \parallel DE$ (AB is parallel to DE).

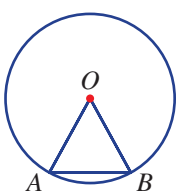


SOLUTION	EXPLANATION
a In ΔABC and ΔEDC $AC = EC$ (given equal sides) (S) $BC = DC$ (given equal sides) (S) $\angle ACB = \angle ECD$ (vertically opposite angles) (A) $\therefore \Delta ABC \equiv \Delta EDC$ (SAS)	List the given pairs of equal length sides and the vertically opposite angles. The included angle is between the given sides, hence SAS.
b $\angle BAC = \angle DEC$ (matching angles in congruent triangles)	All matching angles are equal.
$\therefore AB \parallel DE$ (alternate angles are equal)	If alternate angles are equal, then AB and DE must be parallel.

Exercise 7E

UNDERSTANDING AND FLUENCY	1–4, 5(½)	3, 4, 5	5(½)

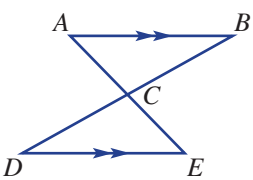
1 The diagram shows a cricle with centre O.



a Which two sides of ΔOAB are equal? Why?

b Which two angles are equal? Why?

2 Consider the diagram shown below.



a Which angle is vertically opposite to $\angle ACB$?

b Which angle is alternate to $\angle CDE$?

c Which angle is alternate to $\angle BAC$?

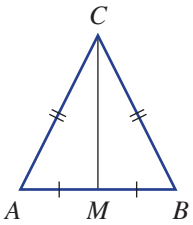
d Do we have enough information to conclude that $\Delta ABC \equiv \Delta DCE$?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 For this isosceles triangle, CM is common to ΔAMC and ΔBMC.

a Which congruence test would be used to prove that $\Delta AMC \equiv \Delta BMC$?

b Which angle in ΔBMC corresponds to $\angle AMC$ in ΔAMC?



Example 11

4 $ABCD$ is a rectangle. Copy and complete the proof.

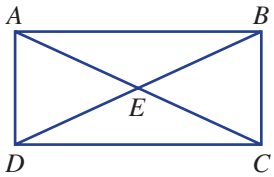
In ΔAEB and ΔDEC

$AB =$ ___ (___ sides of a rectangle) (S)

$\angle AEB = \angle$___ (___ ___ angles) (A)

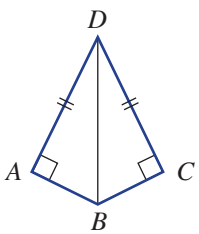
$\angle ABE = \angle$___ (___ angles in parallel lines) (A)

$\therefore \Delta$___ $\equiv \Delta$___ (___)

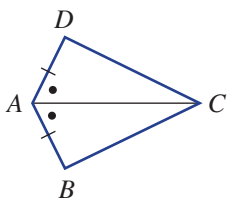


5 Prove that each pair of triangles is congruent. List your reasons and give the abbreviated congruence test.

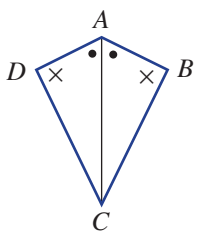
a



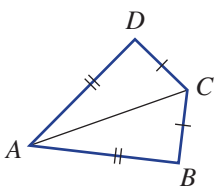
b



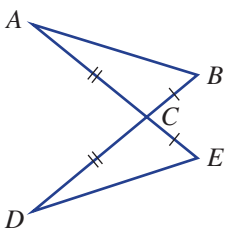
c



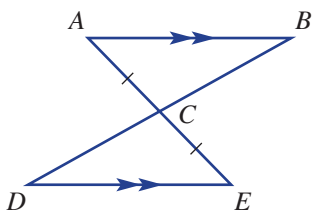
d



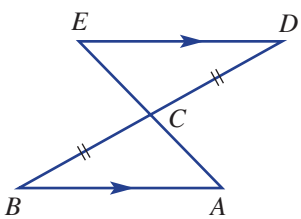
e



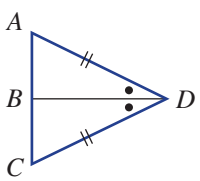
f



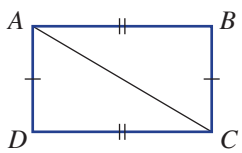
g



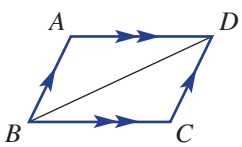
h



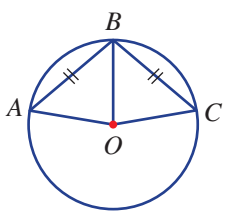
i



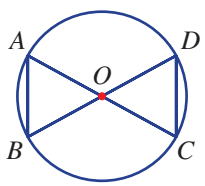
j



k



l

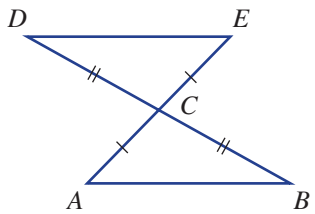


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

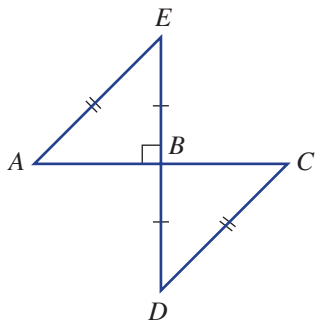
PROBLEM-SOLVING AND REASONING | 6, 7, 12 | 6–9, 12, 13 | 8–15

Example 12

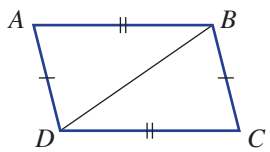
6 **a** Prove $\Delta ABC \equiv \Delta EDC$.
b Hence, prove $AB \parallel DE$.



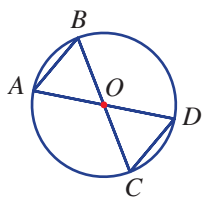
7 **a** Prove $\Delta ABE \equiv \Delta CBD$.
b Hence, prove $AE \parallel DC$.



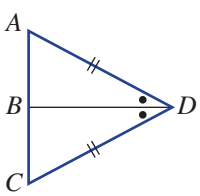
8 **a** Prove $\Delta ABD \equiv \Delta CDB$.
b Hence, prove $AD \parallel BC$.



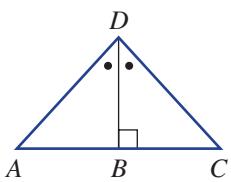
9 **a** Prove that $\Delta AOB \equiv \Delta DOC$. ($O$ is the centre of the circle)
b Hence, prove that $AB \parallel CD$.



10 **a** Prove that $\Delta ABD \equiv \Delta CBD$.
b Hence, prove that AC is perpendicular to BD. ($AC \perp BD$)

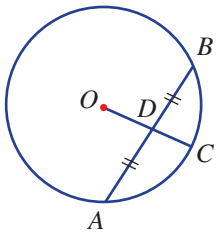


11 **a** Prove that $\Delta ABD \equiv \Delta CBD$.
b Hence, prove that ΔACD is isosceles.

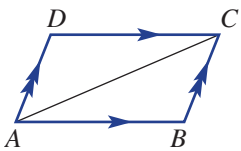


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

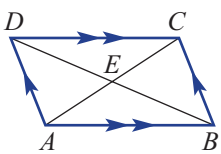
12 Use congruence to explain why OC is perpendicular to AB in this diagram.



13 Use congruence to explain why $AD = BC$ and $AB = DC$ in this parallelogram.



14 Use ΔABE and ΔCDE to explain why $AE = CE$ and $BE = DE$ in this parallelogram.

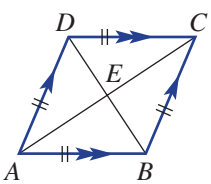


15 Use congruence to show that the diagonals of a rectangle are equal in length.

ENRICHMENT	—	—	16, 17

Extended proofs

16 $ABCD$ is a rhombus.



Prove that AC bisects BD at 90°. Show all steps.

a Prove that $\Delta ABE \equiv \Delta CDE$.

b Hence prove that AC bisects BD at 90°.

17 Use congruence to prove that the three angles in an equilateral triangle (given three equal side lengths) are all 60°.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7F Enlargement and similar figures

Interactive
Widgets
HOTsheets
Walkthrough

Similar figures have the same shape but not necessarily the same size. Two figures are similar if an enlargement of one is identical (congruent) to the other.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

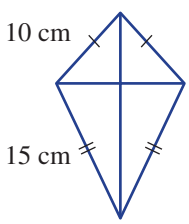


Photographic images that are reduced or enlarged to any given size are similar figures.

Let's start: Enlarging a kite

After drawing a kite design, Mandy cuts out a larger shape to make the actual kite. The actual kite shape is to be similar to the design drawing. The 10 cm length on the drawing matches a 25 cm length on the kite.

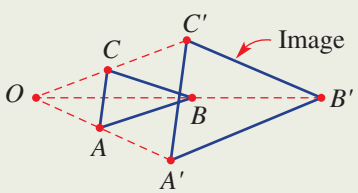
- How should the interior angles compare between the drawing and the actual kite?
- By how much has the drawing been enlarged, i.e. what is the scale factor? Explain your method to calculate the scale factor.
- What length on the kite matches the 15 cm length on the drawing?



Key ideas

- **Enlargement** is a transformation that involves the increase or decrease in size of an object.
  - The 'shape' of the object is unchanged.
  - Enlargement uses a **centre of enlargement** and an **enlargement factor** or **scale factor**.

Centre of enlargement



Scale factor

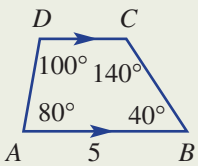
$$\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

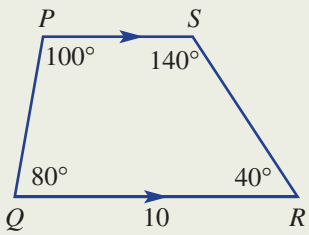
- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Matching angles are equal.
  - Pairs of matching sides are in the same proportion or ratio.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$
  - If the scale factor is between 0 and 1, the image will be smaller than the original.
  - If the scale factor is greater than 1, the image will be larger than the original.
  - If the scale factor is 1, the image and the original are similar *and* congruent.
- The symbol ||| is used to describe similarity.
  - For example, $ABCD \mid\mid\mid QRSP$.
  - The letters are written in matching order.





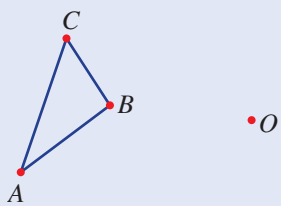
  - Scale factor $= \dfrac{QR}{AB} = \dfrac{10}{5} = 2$

Example 13 Enlarging figures using a scale factor

Copy the given diagram using plenty of space and use the given centre of enlargement (O) and these scale factors to enlarge ΔABC.

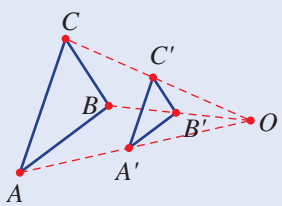
a Scale factor $\frac{1}{2}$

b Scale factor 3

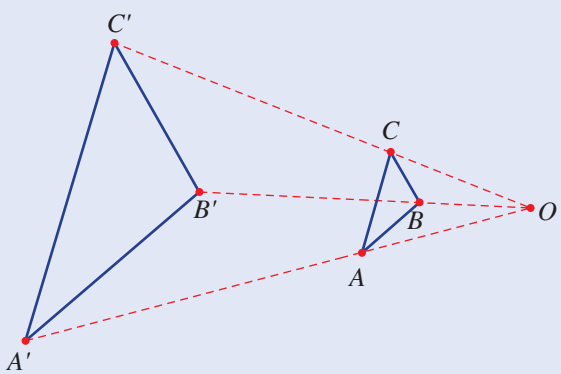


SOLUTION

a



b



EXPLANATION

Connect dashed lines between O and the vertices A, B and C.
Since the scale factor is $\frac{1}{2}$, place A' so that OA' is half Of OA.
Repeat for B' and C'. Join vertices A', B' and C'.

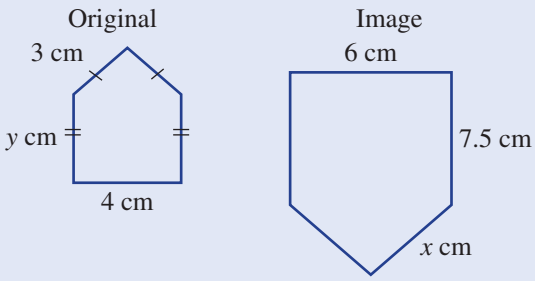
Draw dashed lines from O through A, B and C. Place A' so that $OA' = 3OA$. Repeat for B' and C' and form $\Delta A'B'C'$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14 Using the scale factor to find unknown sides

These figures are similar.

a Find a scale factor.
b Find the value of x.
c Find the value of y.



SOLUTION

a Scale factor $= \frac{6}{4} = 1.5$

b $x = 3 \times 1.5$
$= 4.5$

c $y = 7.5 \div 1.5$
$= 5$

EXPLANATION

Choose two corresponding sides and use
scale factor $= \frac{\text{image length}}{\text{original length}}$

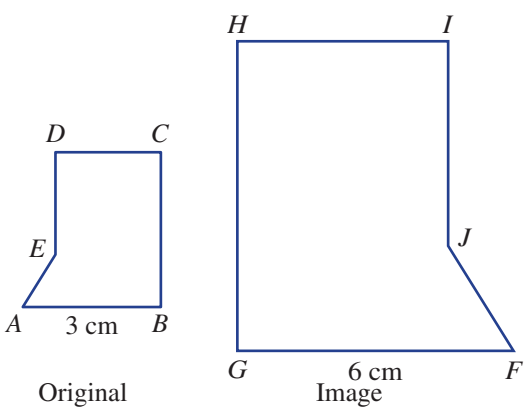
Multiply the side lengths on the original by the scale factor to get the length of the corresponding side on the image.

Divide the side lengths on the image by the scale factor to get the length of the corresponding side on the original.

Exercise 7F

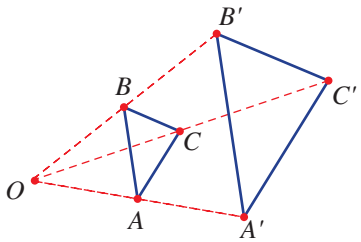
UNDERSTANDING AND FLUENCY	1–5, 7(½)	3–5, 7(½)	6, 7(½)

1 The two figures below are similar.
a Name the angle in the larger figure which corresponds to $\angle A$.
b Name the angle in the smaller figure which corresponds to $\angle I$.
c Name the side in the larger figure which corresponds to BC.
d Name the side in the smaller figure which corresponds to FJ.
e Use FG and AB to find the scale factor.



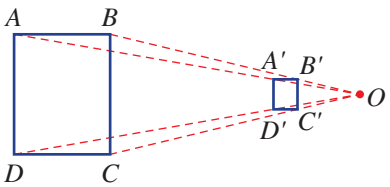
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 This diagram shows ΔABC enlarged to give the image $\Delta A'B'C'$.



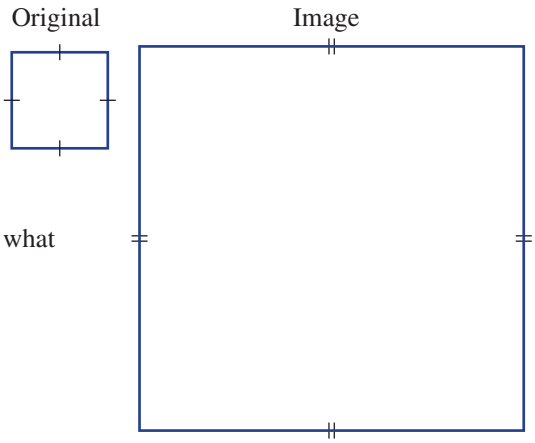
- **a** Measure the lengths OA and OA'. What do you notice?
- **b** Measure the lengths OB and OB'. What do you notice?
- **c** Measure the lengths OC and OC'. What do you notice?
- **d** What is the scale factor?
- **e** Is $A'B'$ twice the length of AB? Measure to check.

3 This diagram shows rectangle $ABCD$ enlarged (in this case reduced) to rectangle $A'B'C'D'$.



- **a** Measure the lengths OA and OA'. What do you notice?
- **b** Measure the lengths OD and OD'. What do you notice?
- **c** What is the scale factor?
- **d** Compare the lengths AD and $A'D'$. Is $A'D'$ one quarter of the length of AD?

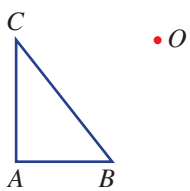
4 A square is enlarged by a scale factor of 4.



- **a** Are the internal angles the same for both the original and the image?
- **b** If the side length of the original square was 2 cm, what would be the side length of the image square?
- **c** If the side length of the image square was 100 m, what would be the side length of the original square?

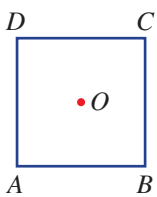
Example 13

5 Copy the given diagram leaving plenty of space around it and use the given centre of enlargement (O) and given scale factors to enlarge ΔABC.



- **a** Scale factor $\frac{1}{3}$
- **b** Scale factor 2

6 This diagram includes a square with centre O and vertices $ABCD$.



- **a** Copy the diagram, leaving plenty of space around it.
- **b** Enlarge square $ABCD$ by these scale factors and draw the image. Use O as the centre of enlargement.
  - **i** $\frac{1}{2}$
  - **ii** 1.5

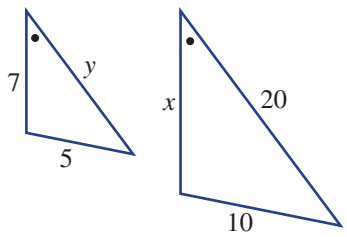
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14

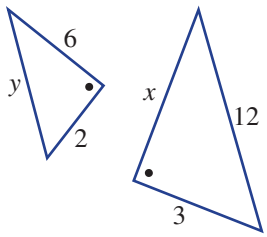
7 Each of the pairs of figures shown here are similar. For each pair find:

i a scale factor **ii** the value of x **iii** the value of y

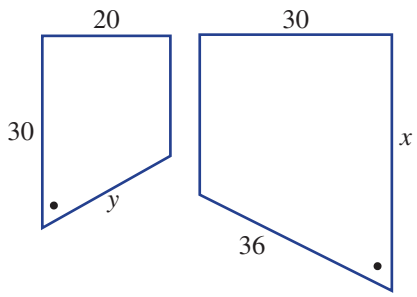
a



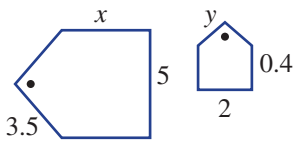
b



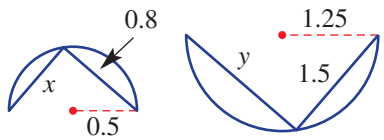
c



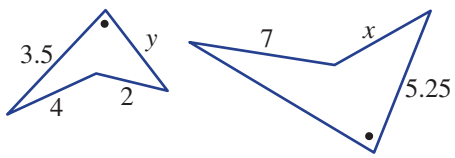
d



e



f



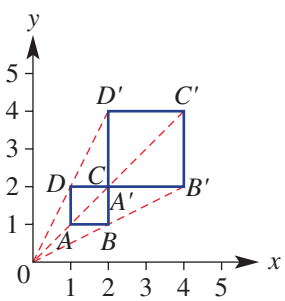
PROBLEM-SOLVING AND REASONING 8, 10–12 | 8(½), 9, 11–13 | 9, 10, 12–14

8 These diagrams show a shape and its image after enlargement. For each part, find the:

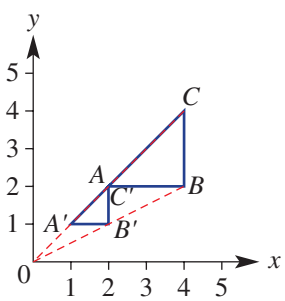
i scale factor

ii coordinates (x, y) of the centre of enlargement

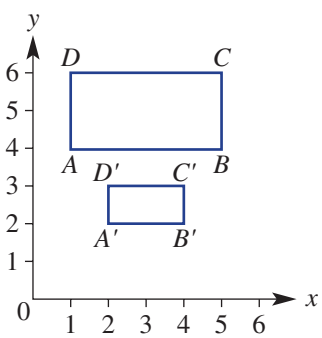
a



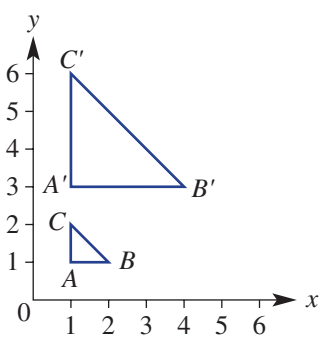
b



c

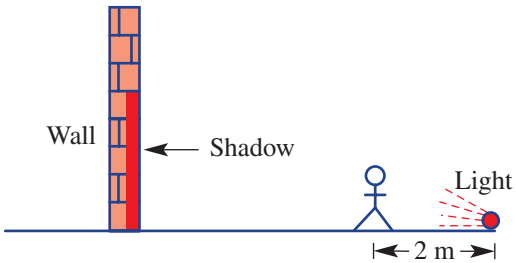


d



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 A person 1.8 m tall stands in front of a light that sits on the floor.
The person casts a shadow on the wall behind them.



a How tall will the shadow be if the distance between the wall and the light is:
- **i** 4 m?
- **ii** 10 m?
- **iii** 3 m?

b How tall will the shadow be if the distance between the wall and the person is:
- **i** 4 m?
- **ii** 5 m?

c Find the distance from the wall to the person if the shadow is the following height.
- **i** 5.4 m
- **ii** 7.2 m

10 This truck is 12.7 m long.
- **a** Measure the length of the truck in the photo.
- **b** Measure the height of the truck in the photo.
- **c** Estimate the actual height of the truck.



11 A figure is enlarged by a scale factor of a where $a > 0$. For what values of a will the image be:
- **a** larger than the original figure?
- **b** smaller than the original figure?
- **c** congruent to the original figure?

12 Explain why any two:
- **a** squares are similar
- **b** equilateral triangles are similar
- **c** rectangles are not necessarily similar
- **d** isosceles triangles are not necessarily similar

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 An object is enlarged by a factor of k. What scale factor should be used to reverse this enlargement?

14 A map has a scale ratio of 1 : 50000.

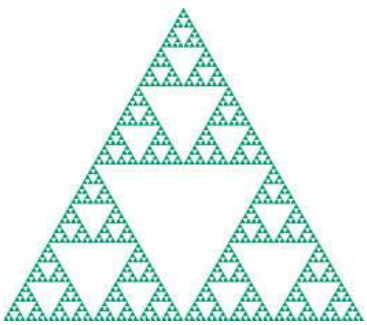
a What length on the ground is represented by 2 cm on the map?

b What length on the map is represented by 12 km on the ground?

ENRICHMENT — — 15

The Sierpinski triangle

15 The Sierpinski triangle shown is a mathematically generated pattern.



It is created by repeatedly enlarging triangles by a factor of $\frac{1}{2}$.
The steps are:

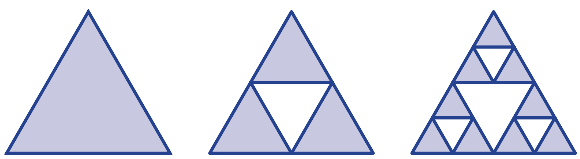
1 Start with an equilateral triangle as in figure 1.

2 Enlarge the triangle by a factor $\frac{1}{2}$.

3 Arrange three copies of the image as in figure 2.

4 Continue repeating steps 2 and 3 with each triangle.

Figure 1 Figure 2 Figure 3



a Make a large copy of figures 1 to 3 then draw the next two figures in the pattern.

b If the original triangle (figure 1) had side length l, find the side length of the smallest triangle in:

i figure 2

ii figure 3

iii figure 8 (assuming figure 8 is the 8th diagram in the pattern)

c What fraction of the area is shaded in:

i figure 2?

ii figure 3?

iii figure 6 (assuming figure 6 is the 6th diagram in the pattern)?

d The Sierpinski triangle is one where the process of enlargement and copying is continued forever. What is the area of a Sierpinski triangle?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7G Similar triangles

Interactive
Widgets
HOTsheets
Walkthrough

Many geometric problems can be solved by using similar triangles. Shadows, for example, can be used to determine the height of a tall mast where the shadows form the base of two similar triangles. Solving such problems involves the identification of two triangles and an explanation as to why they are similar. As with congruence of triangles, there is a set of minimum conditions to establish similarity in triangles.



Similar triangles can be used to calculate distance in the natural world.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Are they similar?

Each point below describes two triangles. Accurately draw each pair and decide if they are similar (same shape but of different size).

- ΔABC with $AB = 2$ cm, $AC = 3$ cm and $BC = 4$ cm
  ΔDEF with $DE = 4$ cm, $DF = 6$ cm and $EF = 8$ cm
- ΔABC with $AB = 3$ cm, $AC = 4$ cm and $\angle A = 40°$
  ΔDEF with $DE = 6$ cm, $DF = 8$ cm and $\angle D = 50°$
- ΔABC with $\angle A = 30°$ and $\angle B = 70°$
  ΔDEF with $\angle D = 30°$ and $\angle F = 80°$
- ΔABC with $\angle A = 90°$, $AB = 3$ cm and $BC = 5$ cm
  ΔDEF with $\angle D = 90°$, $DE = 6$ cm and $EF = 9$ cm

Which pairs are similar and why? For the pairs that are not similar, what measurements could be changed so that they are similar?

Key ideas

- Two triangles are **similar** if:
  - matching angles are equal
  - matching sides are in proportion (the same ratio).
- The **similarity statement** for two similar triangles ΔABC and ΔDEF is:
  - $\Delta ABC \Vert\Vert \Delta DEF$

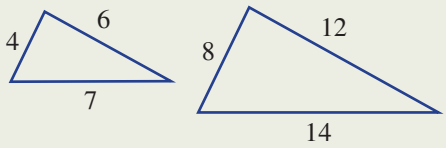
  Letters are usually written in matching order so $\angle A$ corresponds to $\angle D$ etc.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

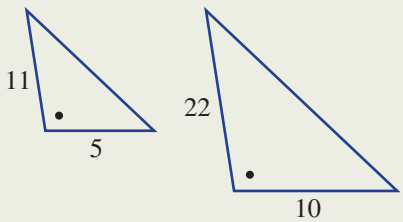
■ **Tests for similar triangles**. (Not to be confused with the congruence tests for triangles.)

- If the three sides of a triangle are proportional to the three sides of another triangle, then the two triangles are similar.



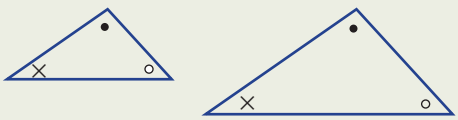
$\frac{12}{6} = \frac{8}{4} = \frac{14}{7} = 2$ (i.e. scale factor is 2)

- If two sides of a triangle are proportional to two sides of another triangle, and the included angles are equal, then the two triangles are similar.

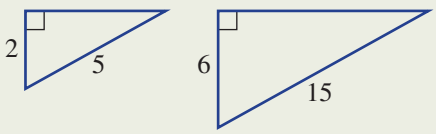


$\frac{22}{11} = \frac{10}{5} = 2$ (i.e. scale factor is 2)

- If two angles of a triangle are equal to two angles of another triangle, then the two triangles are similar.
(Note: if there are two equal pairs, then the third pair must be equal.)



- If the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the two triangles are similar.



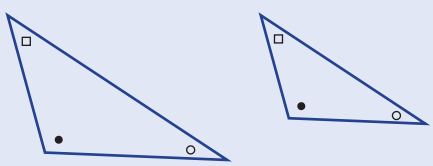
$\frac{15}{5} = \frac{6}{2} = 3$ (i.e. scale factor is 3)

■ There are no abbreviations such as AAA in the NSW Syllabus for these tests of similarity.

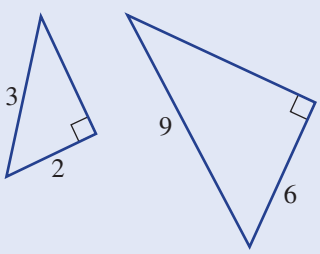
Example 15 Choosing the appropriate similarity test for triangles

Choose the similarity test that proves that these pairs of triangles are similar. For the purpose of this exercise we shall call them tests 1, 2, 3 and 4 in the order of the Key Ideas.

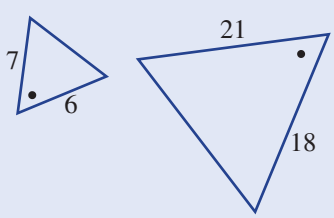
a



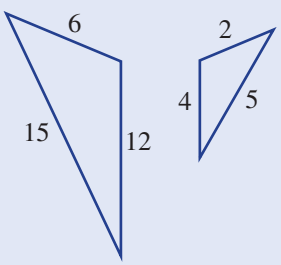
b



c



d

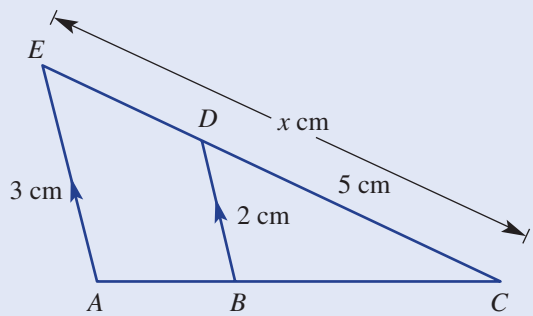


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
a Test 3	Three pairs of matching angles are equal.
b Test 4	Both are right-angled triangles and the hypotenuses and another pair of sides are in the same ratio $\left(\frac{9}{3} = \frac{6}{2}\right)$.
c Test 2	Two pairs of corresponding sides are in the same ratio $\left(\frac{21}{7} = \frac{18}{6}\right)$ and the included angles are equal.
d Test 1	Three pairs of corresponding sides are in the same ratio $\left(\frac{15}{5} = \frac{12}{4} = \frac{6}{2}\right)$.

Example 16 Finding an unknown length using similarity

For this pair of triangles:



a Explain why the two triangles are similar.

b Find the value of x.

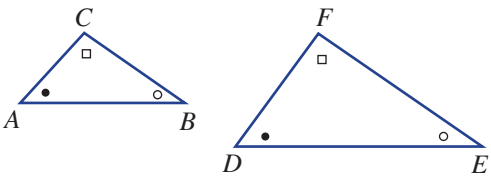
SOLUTION	EXPLANATION
a $AE \parallel BD$ (given). $\therefore \angle EAC = \angle DBC$ (corresponding angles in parallel lines) Similarly $\angle AEC = \angle BDC$ $\therefore$ The triangles contain two pairs of matching angles. $\therefore$ The triangles are similar.	$\angle EAC = \angle DBC$ since AE is parallel to BD and $\angle C$ is common to both triangles. (Also $\angle AEC = \angle BDC$ since AE is parallel to BC.)
b Scale factor $= \frac{3}{2} = 1.5$. $\therefore x = 5 \times 1.5$ $= 7.5$	$\frac{AE}{BD} = \frac{3}{2}$ Multiply CD by the scale factor to find the length of the corresponding side CE.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 7G

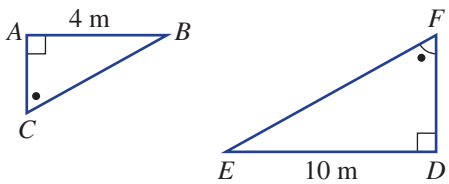
UNDERSTANDING AND FLUENCY | 1–6 | 3, 4–5(½), 6–8 | 4–5(½), 6, 8, 9

1 These two triangles are similar.



- **a** Which vertex on ΔDEF corresponds to (matches) vertex B on ΔABC?
- **b** Which vertex on ΔABC corresponds to (matches) vertex F on ΔDEF?
- **c** Which side on ΔDEF corresponds to (matches) side AC on ΔABC?
- **d** Which side on ΔABC corresponds to (matches) side EF on ΔDEF?
- **e** Which angle on ΔABC corresponds to (matches) $\angle D$ on ΔDEF?
- **f** Which angle on ΔDEF corresponds to (matches) $\angle B$ on ΔABC?
- **g** Complete the similarity statement for these triangles $\Delta ACB \;|||\; \Delta____$

2 What is the scale factor on this pair of similar triangles which enlarges ΔABC to ΔDEF?



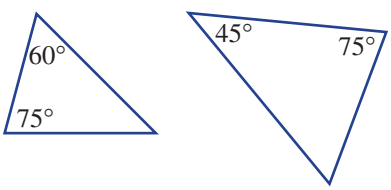
3 Copy and complete the following sentences.

- **a** There are _____ tests for similarity.
- **b** Similar figures have the same _____ but are not necessarily the same _____.

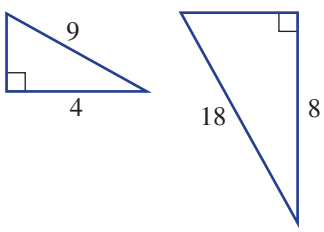
Example 15

4 Choose the similarity test that proves that these pairs of triangles are similar. For the purpose of this exercise they are called tests 1, 2, 3 and 4 in the order of the Key Ideas.

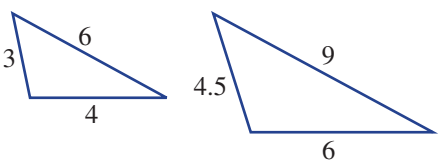
a



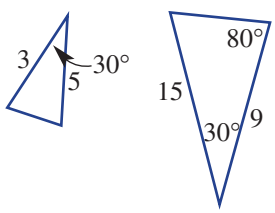
b



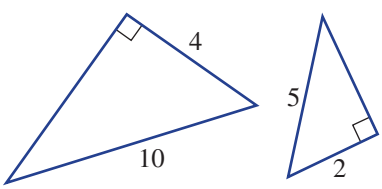
c



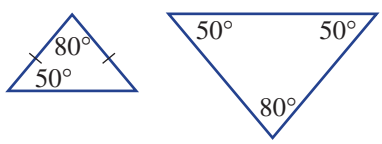
d



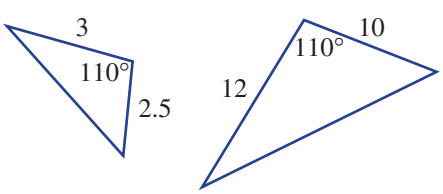
e



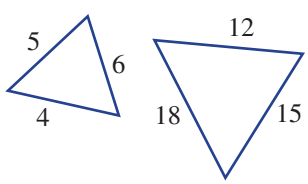
f



g



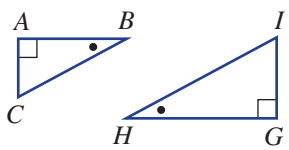
h



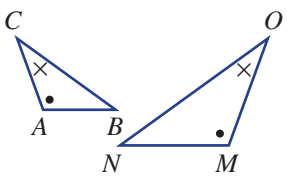
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 Write similarity statements for these pairs of similar triangles. Write letters in matching order.

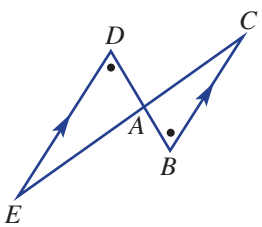
a



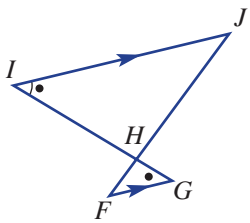
b



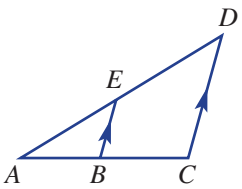
c



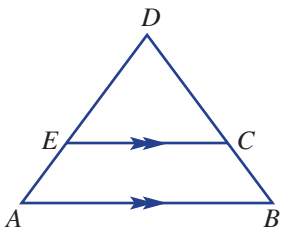
d



e



f

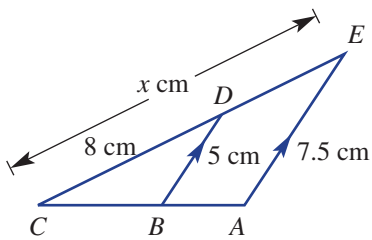


Example 16

6 For this pair of triangles:

a explain why the two triangles are similar

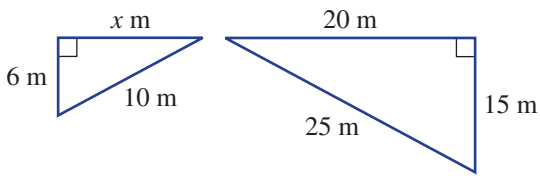
b find the value of x



7 For this pair of triangles:

a explain why the two triangles are similar

b find the value of x

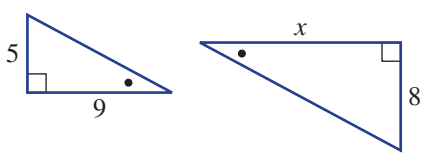


8 Each given pair of triangles is similar. For each pair find the:

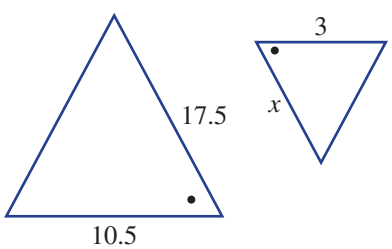
i enlargement factor (scale factor) that enlarges the smaller triangle to the larger triangle

ii value of x

a



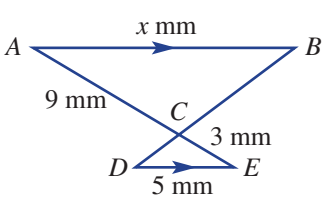
b



9 For this pair of triangles:

a explain why the two triangles are similar

b find the value of x



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

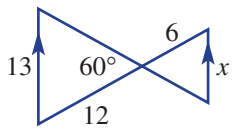
PROBLEM-SOLVING AND REASONING | 10, 13 | 10, 11, 13, 14 | 10–12, 14–16

10 For each pair of similar triangles:

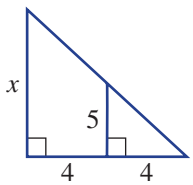
i explain why the two triangles are similar

ii find the value of x

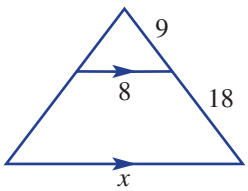
a



b

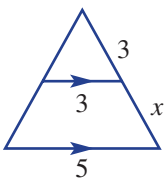


c

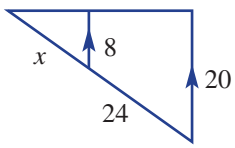


11 Find the value of x in these triangles.

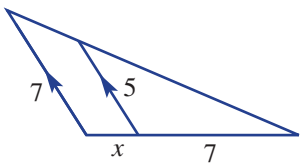
a



b

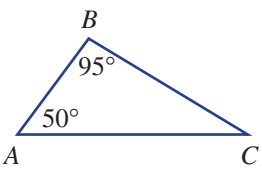


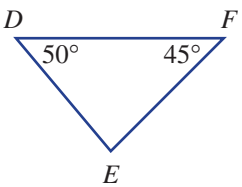
c

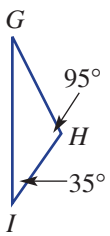


12 Name the triangle that is not similar to the other two in each group of three triangles.

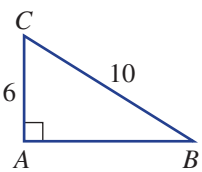
a

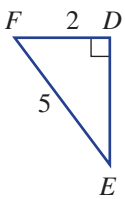


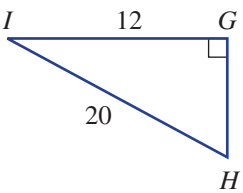




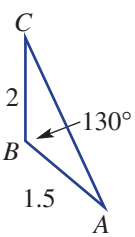
b

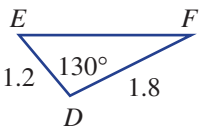


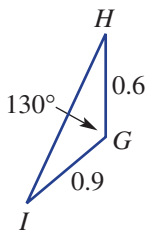




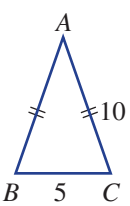
c

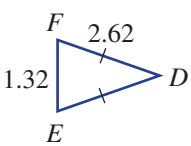


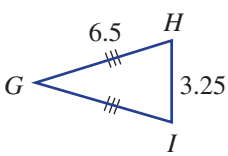




d

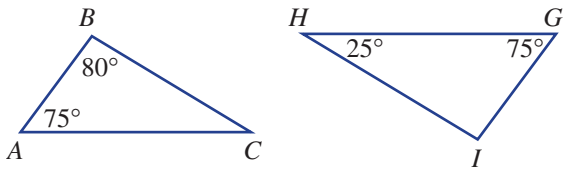




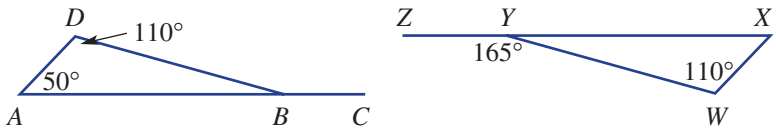


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

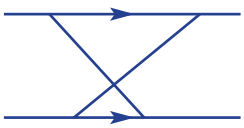
13 Explain why these two triangles are similar.



14 Give reasons why the two triangles in these diagrams are not similar.



15 When two intersecting transversals join parallel lines, two triangles are formed. Explain why these two triangles are similar.

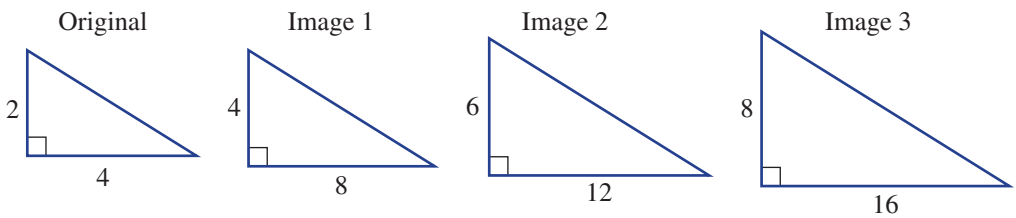


16 The four tests for similarity closely resemble the tests for congruence. Which similarity test closely matches the AAS congruence test? Explain the difference.

ENRICHMENT — — 17

Area ratio

17 Consider these three similar triangles (not drawn to scale).



a Complete this table, comparing each image to the original.

Triangle	Original	Image 1	Image 2	Image 3
Length scale factor	1	2		
Area				
Area scale factor	1			

b What do you notice about the area scale factor compared to the length scale factor?

c What would be the area scale factor if the length scale factor is n?

d What would be the area scale factor if the length scale factor is:

i 10?

ii 20?

iii 100?

e What would be the area scale factor if the length scale factor is $\frac{1}{2}$?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7H Proving and applying similar triangles

Interactive
Widgets
HOTsheets
Walkthrough

Similar triangles can be used in many mathematical and practical problems. If two triangles are proved to be similar, then the properties of similar triangles can be used to find missing lengths or unknown angles. Finding the approximate height of a tall object, or the width of a projected image, for example, can be found using similar triangles.



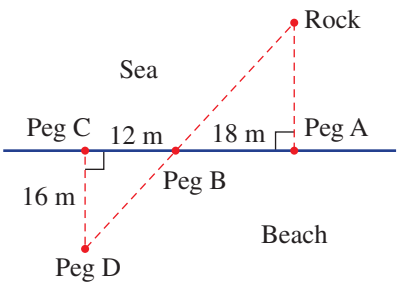
The height of even the tallest building in the world, the Burj Khalifa in Dubai, can be verified using similar triangles.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How far is the rock?

Ali is at the beach and decides to estimate how far an exposed rock is from seashore. He places four pegs in the sand as shown and measures the distance between them.



- Why do you think Ali has placed the four pegs in the way that is shown in the diagram?
- Why are the two triangles similar? Which test could be used and why?
- How would Ali use the similar triangles to find the distance from the beach at peg A to the rock?

Key ideas

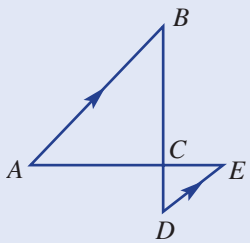
- To prove triangles are similar, list any pairs of matching angles or sides in a given ratio.
  - Give reasons at each step.
  - Write a similarity statement, for example, $\Delta ABC \Vert\Vert\ \Delta DEF$.
  - Describe the triangle similarity test.
    - Two pairs of equal matching angles
    - Sides adjacent to equal angles are in proportion
    - Three pairs of sides in proportion
    - Hypotenuse and another side in one right-angled triangle in proportion to matching sides in another
- To apply similarity in practical problems:
  - prove two triangles are similar using one of the four tests
  - find a scale factor
  - find the value of any unknowns.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

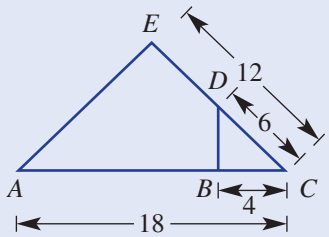
Example 17 Proving two triangles are similar

Prove that each pair of triangles is similar.

a



b



SOLUTION

a In ΔABC and ΔEDC

$\angle BAC = \angle DEC$ (alternate angles and $DE \parallel AB$)

$\angle ABC = \angle EDC$ (alternate angles and $DE \parallel AB$)

$\therefore \Delta ABC \,|||\, \Delta EDC$ (two pairs of equal matching angles)

b $\dfrac{AC}{DC} = \dfrac{18}{6} = 3$

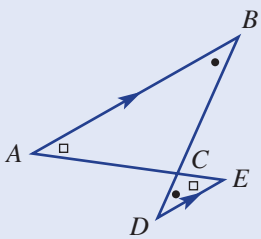
$\angle ACE = \angle DCB$ (common)

$\dfrac{EC}{BC} = \dfrac{12}{4} = 3$

$\therefore \Delta ACE \,|||\, \Delta DCB$ (sides adjacent to equal angles are in proportion)

EXPLANATION

Parallel lines cut by a transversal will create a pair of equal alternate angles. Vertically opposite angles are also equal but not required. Write the similarity statement and the abbreviated reason.



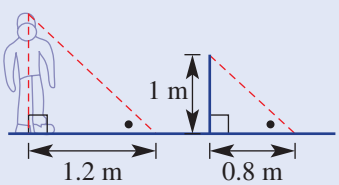
Note that there is a common angle and two pairs of corresponding sides. Find the scale factor for both pairs of sides to see if they are equal. Complete the proof with a similarity statement.

Example 18 Applying similarity

Chris' shadow is 1.2 m long while a 1 m vertical stick has a shadow 0.8 m long.

a Give a reason why the two triangles are similar.

b Determine Chris' height.



SOLUTION

a Two pairs of matching angles.

b Scale factor $= \dfrac{1.2}{0.8} = 1.5$

$\therefore$ Chris' height $= 1 \times 1.5$

$= 1.5$ m

EXPLANATION

The sun's rays will pass over Chris and the stick and hit the ground at approximately the same angle.

First find the scale factor.

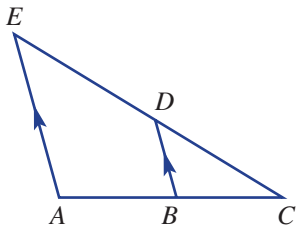
Multiply the height of the stick by the scale factor to find Chris' height.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 7H

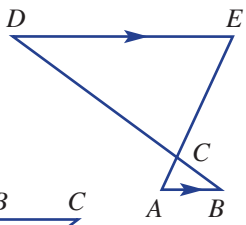
UNDERSTANDING AND FLUENCY | 1–3, 4(½) | 3, 4(½), 5 | 4(½), 5

1 In this diagram:



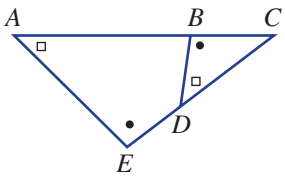
- **a** use three letters to name the common angle
- **b** why is $\angle EAC = \angle DBC$?
- **c** explain why we have enough information to conclude that the triangles are similar
- **d** complete the similarity statement: $\Delta BCD \;|||\; \Delta____$

2 In this diagram:



- **a** name the pair of vertically opposite angles
- **b** name the two pairs of equal alternate angles
- **c** complete the similarity statement: $\Delta ABC \;|||\; \Delta____$

3 In this diagram name the:

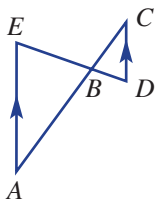


- **a** common angle for the two triangles
- **b** side that corresponds to side:
  - **i** DC
  - **ii** AE

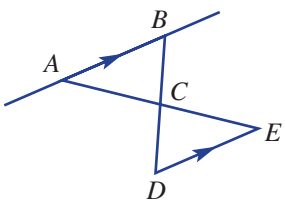
Example 17a

4 Prove that each pair of triangles is similar.

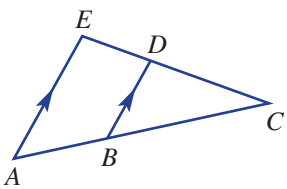
a



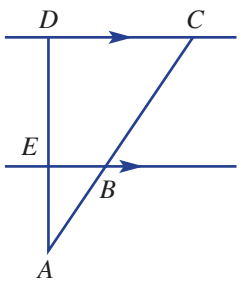
b



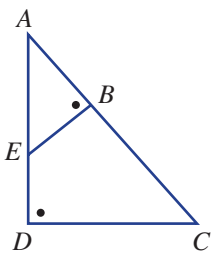
c



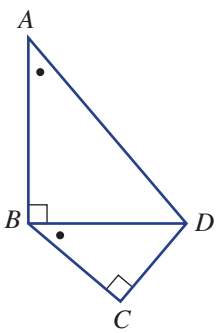
d



e



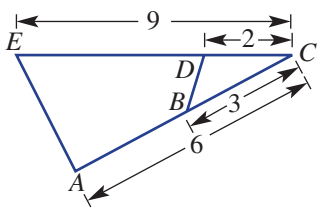
f



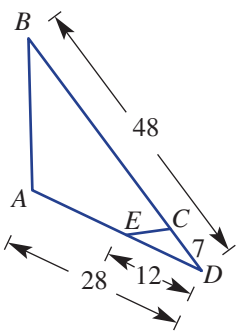
Example 17b

5 Prove that each pair of triangles is similar.

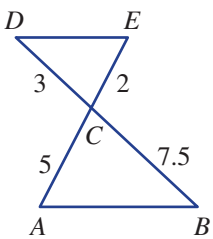
a



b



c



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING 6–9, 12 6–10, 12 8–13

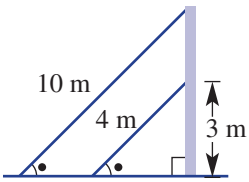
Example 18

6 A tree's shadow is 20 m long, while a 2 m vertical stick has a shadow 1 m long.

a Give a reason why the two triangles contained within the objects and their shadows are similar.

b Find the height of the tree.

7 Two cables support a steel pole at the same angle as shown. The two cables are 4 m and 10 m in length while the shorter cable reaches 3 m up the pole.

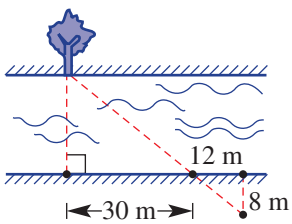


a Give a reason why the two triangles are similar.

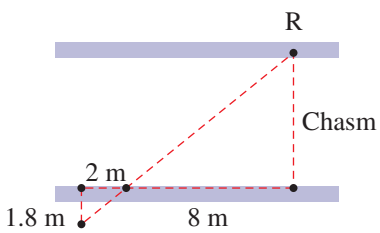
b Find the height of the pole.

8 John stands 6 m from a lamp post and casts a 2 m shadow. The shadow from the pole and from John end at the same place. Determine the height of the lamp post if John is 1.5 m tall.

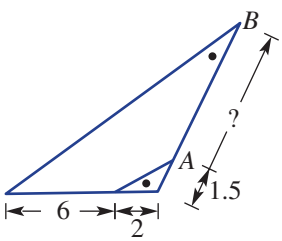
9 Joanne wishes to determine the width of the river shown without crossing it. She places four pegs as shown. Calculate the river's width.



10 A deep chasm has a large rock (R) sitting on its side as shown. Find the width of the chasm.

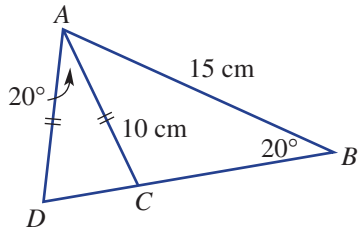


11 Find the length AB in this diagram if the two triangles are similar.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

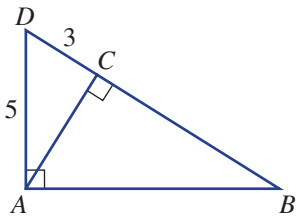
12 In this diagram ΔADC is isosceles.



a There are two triangles that are similar. Identify them and complete a proof. You may need to find another angle first.

b Find the lengths DC and CB, expressing your answers as fractions.

13 In this diagram AC is perpendicular to BD and ΔABD is right angled.



a Prove that ΔABD is similar to:

i ΔCBA

ii ΔCAD

b Find the lengths:

i BD

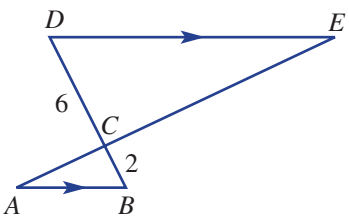
ii AC

iii AB

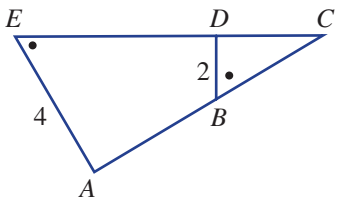
ENRICHMENT — — 14

Extended proofs

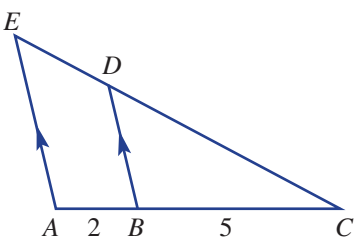
14 a Prove $AE = 4AC$.



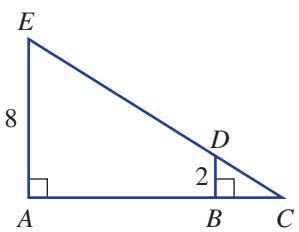
b Prove $BC = \frac{1}{2}CE$.



c Prove $CE = \frac{7}{5}CD$.



d Prove $AB = 3BC$.



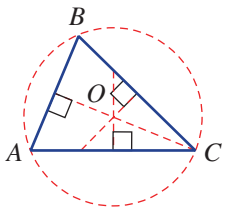
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Technology

Use a computer dynamic geometry package such as Geometers Sketchpad or Cabri Geometry to construct the shapes in each of the following questions.

The circumcentre of a triangle

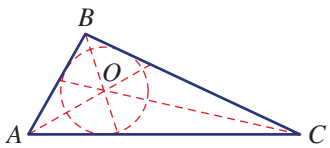
The point at which all perpendicular bisectors of the sides of a triangle meet is called the circumcentre.



- **a** Draw any triangle.
- **b** Label the vertices A, B and C.
- **c** Draw a perpendicular bisector for each side.
- **d** Label the intersection point of the bisectors O. This is the circumcentre of the triangle.
- **e** Using O as the centre, construct a circle that touches the vertices of the triangle.
- **f** Drag any of the vertices and describe what happens to your construction.

The incentre of a triangle

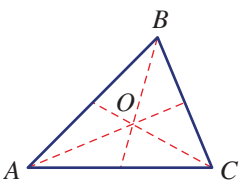
The point at which all angle bisectors of a triangle meet is called the incentre.



- **a** Draw any triangle.
- **b** Label the vertices A, B and C.
- **c** Draw the three angle bisectors, through the vertices.
- **d** Label the intersection point of the bisectors O. This is the incentre of the triangle.
- **e** Using O as your centre, construct a circle that touches the sides of the triangle.
- **f** Drag any of the vertices and describe what happens to your construction.

The centroid of a triangle

The point of intersection of the three medians of a triangle is called the centroid. It can also be called the centre of gravity.



- **a** Draw a triangle and label the vertices A, B and C.
- **b** Find the midpoint of each line and draw a line segment from each midpoint to its opposite vertex. (This is a median.)
- **c** Label the intersection point of these lines O. This is the centroid of the triangle.
- **d** Show your teacher the final construction and print it. Cut out the triangle. Balance the triangle at the centroid on a sharp pencil. The triangle should balance perfectly.

The equilateral triangle: the special triangle

- **a** Construct an equilateral triangle. Determine its incentre, circumcentre and centroid.
- **b** What do you notice?

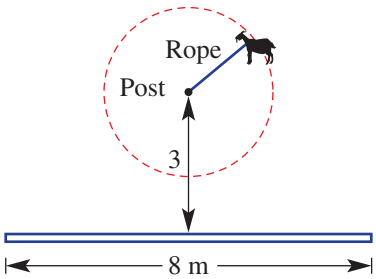
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

The tethered goat

A goat is tied to a post by a rope. The area over which the goat can graze will vary in shape depending on where the post is placed or the length of the rope.

Fixed distance from a fence

Assume the post is 3 m from the centre of a high fence 8 m long.



If the rope is quite short as shown in the diagram, the area the goat can graze in is circular in shape. For longer lengths of the rope, the shape of the accessible area is different.

a On a sheet of paper draw a scale diagram of the location of the post and fence shown above.

b On your scale diagram use a compass (or a string and drawing pins) to help you trace out the shape of the area accessible to the goat if the length of the rope is:

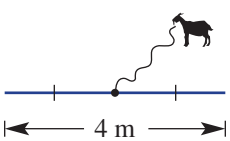
i 2 m **ii** 3 m **iii** 4 m **iv** 5 m
v 6 m **vi** 9 m **vii** 11 m **viii** 13 m

Be careful! Think about what will happen when the goat reaches either end of the fence.

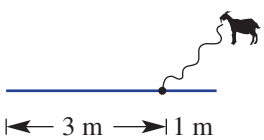
Fixed length of rope

For the following situations the goat is tied to a post on the fence by a 3 m length of rope. Draw a scale diagram of each one and determine the shape of the accessible area.

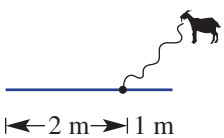
a



b

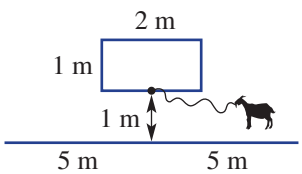


c



Shed problem

In this diagram the goat is tied to a post of a shed, which is 2 m long and 1 m wide, by a 3 m length of rope.



a Draw a scale diagram and determine the shape of the accessible area.

b Investigate other situations in which the goat is tied to other positions on the shed. Clearly show your diagrams and post position.

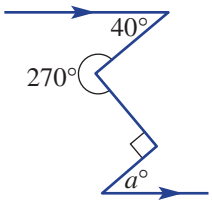
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1 Use 12 matchsticks to make 6 equilateral triangles.

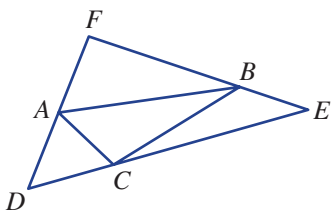
2 How many acute angles are there in this diagram?



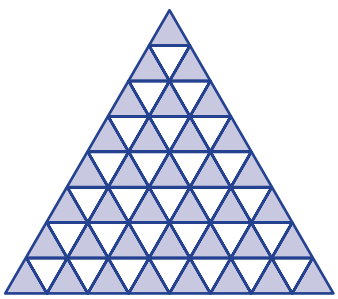
3 Find the value of a.



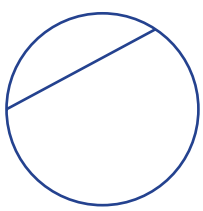
4 Explore (using dynamic geometry) where the points A, B and C should be on the sides of ΔDEF so that the perimeter of ΔABC is a minimum.



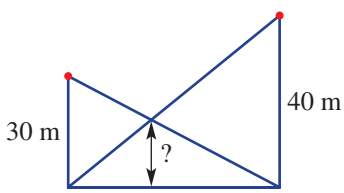
5 How many triangles are there in the diagram on the right?



6 A circle is divided using chords (one chord is shown here). What is the maximum number of regions that can be formed if the circle is divided with 4 chords?



7 Two poles are 30 m and 40 m high. Cables connect the top of each vertical pole to the base of the other pole. How high is the intersection point of the cables above the ground?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Properties of geometrical figures

Angles
Acute $0° < \theta < 90°$
Right $90°$
Obtuse $90° < \theta < 180°$
Straight $180°$
Reflex $180° < \theta < 360°$
Revolution $360°$
Complementary angles sum to $90°$
Supplementary angles sum to $180°$

Parallel lines
If the lines are parallel:

Corresponding angles are equal

Alternate angles are equal

Vertically opposite angles are equal

Co-interior angles are supplementary
$a + b = 180$

Quadrilaterals
Four-sided figures – sum of the interior angles is $360°$

Parallelogram

Rhombus

Rectangle

Square

Kite

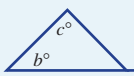
Trapezium

Congruent triangles
These are identical in shape and size. They may need to be rotated or reflected. We write $\Delta ABC \equiv \Delta DEF$
($\equiv$: congruent to)

Tests for congruence:
(SSS) – three pairs of matching sides equal
(SAS) – two pairs of matching sides and included angle are equal
(AAS) – two angles and any pair of matching sides are equal
(RHS) – a right angle, hypotenuse and one other pair of matching sides are equal

Polygons
The sum of the interior angles in a polygon with n sides is $S = 180(n - 2)$. Sum of the exterior angles is $360°$. Regular polygons have all sides and angles equal.

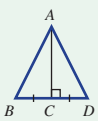
Triangles
Sum of angles is $180°$.
Exterior angle equals sum of two opposite interior angles



$a = b + c$
exterior angle

Types:
Acute angled – all angles $< 90°$
Obtuse angled – 1 angle $> 90°$
Right angled – 1 angle $90°$
Equilateral – all angles $60°$ – all sides are equal
Isosceles – 2 angles and 2 sides are equal
Scalene – all sides and angles are different sizes

Proving congruence/similarity
List matching pairs of equal angles and pairs of sides that are equal/in same ratio. Give reasons for each pair. Write a congruence/similarity statement giving the abbreviated reason.
e.g.



AC (common)
$BC = DC$ (given)
$\angle ACD = \angle ACB = 90°$ (given)
$\therefore \Delta ACD \equiv \Delta ACB$ (SAS)

Similar figures
These are the same shape but different size.
Two figures are similar if one can be enlarged to be congruent to the other.
Enlargement uses a scale factor.
Scale factor $= \frac{\text{image length}}{\text{original length}}$

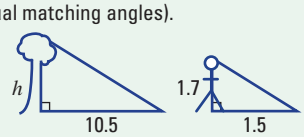
Similar triangles
Similar triangles have matching angles equal and matching sides in the same ratio.
We say $\Delta ABC \,|||\, \Delta DEF$
($|||$: similar to)

Tests for similar triangles:
– two pairs of equal matching angles
– sides adjacent to equal angles in proportion
– three pairs of sides in proportion
– hypotenuse and another side in one right-angled triangle in proportion to matching sides in another.

Applying similar triangles
In practical problems, look to identify and prove pairs of similar triangles. Find a scale factor and use this to find the value of any unknowns, e.g. shadow cast by a tree is 10.5 m while a person 1.7 m tall has a 1.5 m shadow. How tall is the tree?

Similar (two pairs of equal matching angles).

Scale factor $= \frac{10.5}{1.5} = 7$
$\therefore h = 1.7 \times 7$
$= 11.9$ m
Tree is 11.9 m tall.



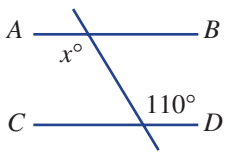
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Multiple-choice questions

1 The angle that is supplementary to an angle of 55° is:

A 35° **B** 55° **C** 95°
D 125° **E** 305°

2 What is the value of x if AB is parallel to CD?

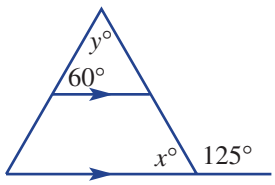


A 110 **B** 70
C 20 **D** 130
E 120

3 If two angles in a triangle are complementary then the third angle is:

A acute **B** a right angle
C obtuse **D** reflex
E supplementary

4 The values of x and y in the diagram are:



A $x = 35, y = 85$
B $x = 45, y = 45$
C $x = 50, y = 60$
D $x = 55, y = 65$
E $x = 65, y = 55$

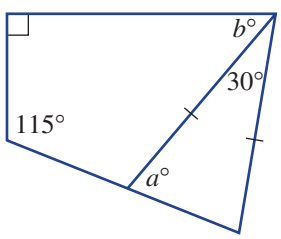
5 The sum of the interior angles in a hexagon is:

A 1078° **B** 360° **C** 720°
D 900° **E** 540°

6 The quadrilateral with all sides equal, two pairs of opposite parallel sides and no right angles is a:

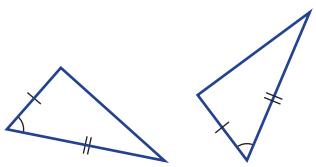
A kite **B** trapezium **C** parallelogram
D rhombus **E** square

7 The values of a and b in the diagram are:



A $a = 85, b = 60$
B $a = 75, b = 80$
C $a = 80, b = 55$
D $a = 70, b = 55$
E $a = 75, b = 50$

8 The abbreviated reason for congruence in the two triangles shown is:

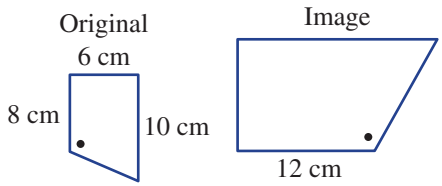


A AA
B SAS
C SSS
D AAS
E RHS

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

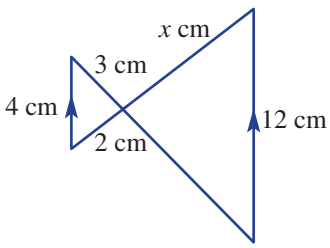
9 The scale factor in the two similar figures that enlarges the original figure to its image is:

A $\frac{2}{3}$
B 2
C 1.2
D 1.5
E 0.5



10 The value of x in the diagram is:

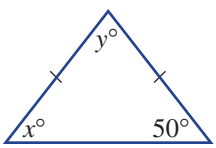
A 6
B 9
C 10
D 8
E 7.5



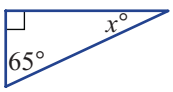
Short-answer questions

1 Name the following triangles and find the value of the pronumerals.

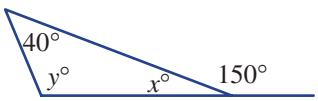
a



b

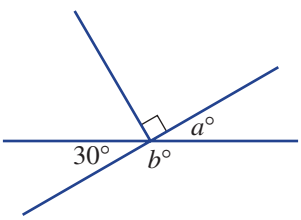


c

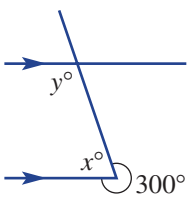


2 Find the value of each pronumeral in the diagrams. Give reasons for your answers.

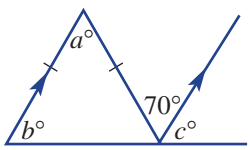
a



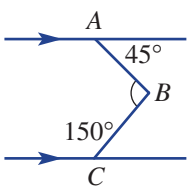
b



c



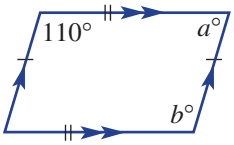
3 By adding a third parallel line to the diagram, find $\angle ABC$.



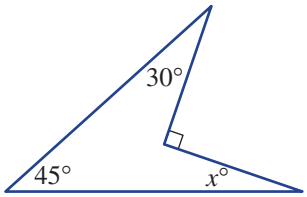
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 Find the value of each pronumeral in the following polygons.

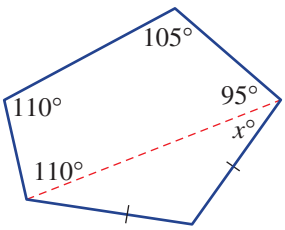
a



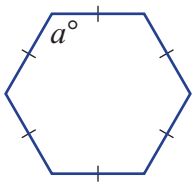
b



c

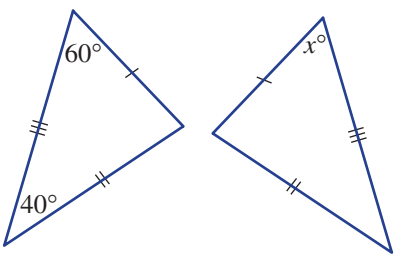


d

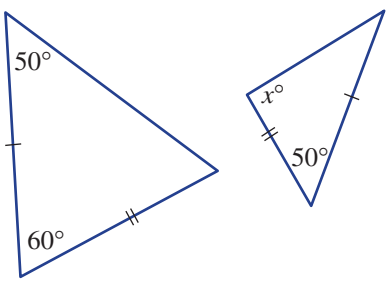


5 Determine if each pair of triangles is congruent. If congruent, give the abbreviated reason and state the value of any pronumerals.

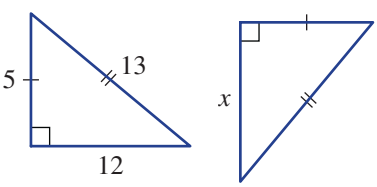
a



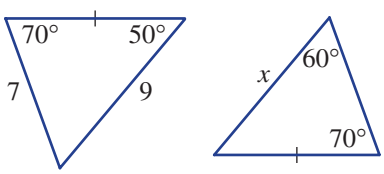
b



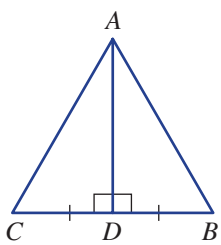
c



d

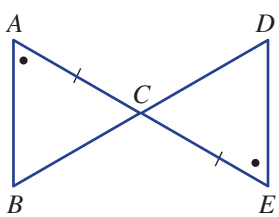


6 a Prove that $\Delta ADB \equiv \Delta ADC$. List your reasons and give the abbreviated congruence test.



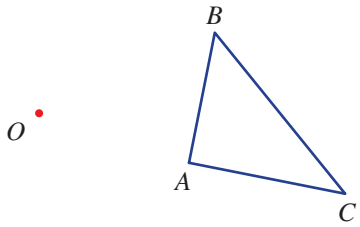
b i Prove that $\Delta ACB \equiv \Delta ECD$. List your reason and give the abbreviated congruence test.

ii Hence, prove that $AB \parallel DE$.



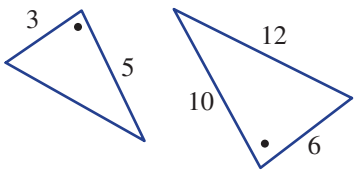
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Copy the given diagram using plenty of space. Using the centre of enlargement (O) and a scale factor of 3, enlarge ΔABC.

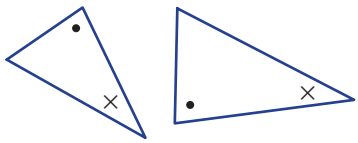


8 Determine if the following pairs of triangles are similar, and state the similarity test that proves this.

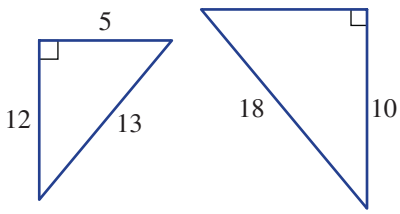
a



b

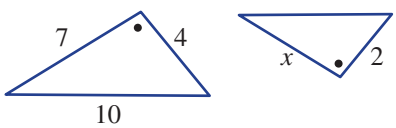


c

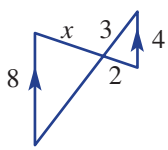


9 For the following pairs of similar triangles find the value of x.

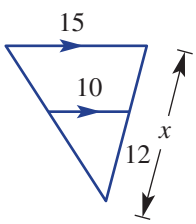
a



b



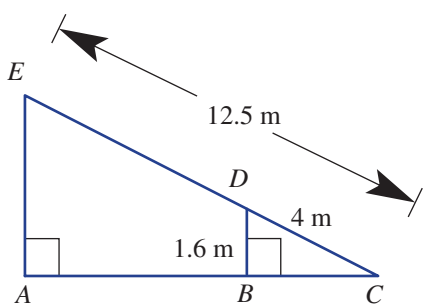
c



10 A conveyor belt loading luggage onto a plane is 12.5 m long. A vertical support 1.6 m high is placed under the conveyor belt so that it is 4 m along the conveyor belt as shown.

a Prove that $\Delta BCD \;|||\; \Delta ACE$.

b Find the height (AE) of the luggage door above the ground.

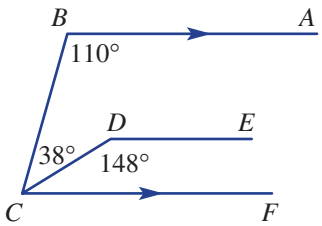


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

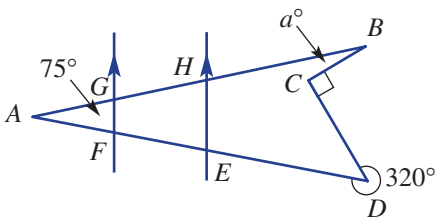
Extended-response questions

1 Complete the following.

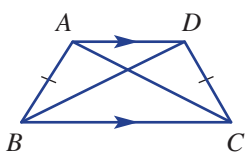
a Prove that $DE \parallel CF$.



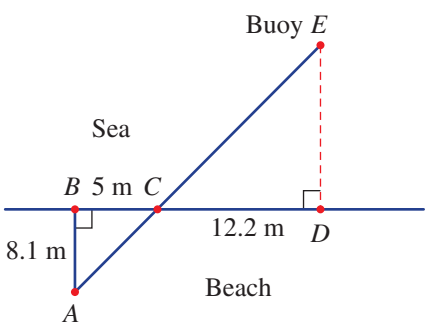
b Show, with reasons, that $a = 20$.



c Use congruence to prove that $AC = BD$ in the diagram, given $AB = CD$ and $\angle ABC = \angle DCB$.



2 A buoy (E) is floating in the sea at an unknown distance from the beach as shown. The points A, B, C and D are measured and marked out on the beach as shown.

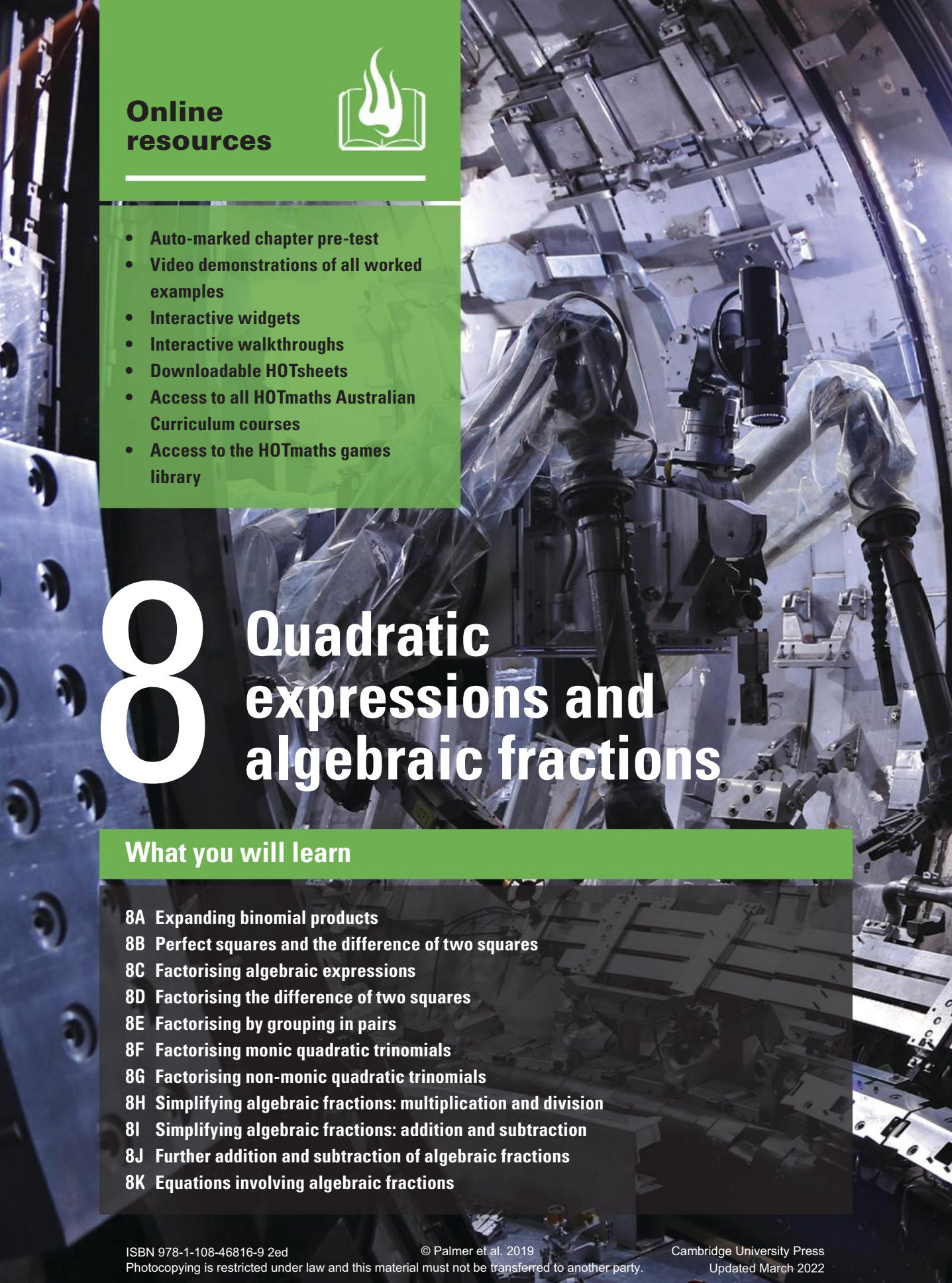


a Name the angle that is vertically opposite to $\angle ACB$.

b Explain, with reasons, why $\Delta ABC \parallel\!\parallel\!\parallel \Delta EDC$.

c Find the distance from the buoy to the beach (ED) to 1 decimal place.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

8 Quadratic expressions and algebraic fractions

What you will learn

8A Expanding binomial products
8B Perfect squares and the difference of two squares
8C Factorising algebraic expressions
8D Factorising the difference of two squares
8E Factorising by grouping in pairs
8F Factorising monic quadratic trinomials
8G Factorising non-monic quadratic trinomials
8H Simplifying algebraic fractions: multiplication and division
8I Simplifying algebraic fractions: addition and subtraction
8J Further addition and subtraction of algebraic fractions
8K Equations involving algebraic fractions

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: ALGEBRAIC TECHNIQUES (S5.2, 5.3§)
EQUATIONS (S5.2, 5.3§)

Outcomes
A student simplifies algebraic fractions, and expands and factorises quadratic expressions.
(MA5.2–6NA)
A student solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques.
(MA5.2–8NA)
A student selects and applies appropriate algebraic techniques to operate with algebraic expressions.
(MA5.3–5NA)
A student solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations.
(MA5.3–7NA)

© NESA, 2012

Nuclear fusion for sustainable energy

One of the most famous equations of all time is Einstein's mass (m) energy (E) equivalence $E = mc^2$ where E is in Joules, m is in kg and c is the speed of light (3×10^8m/s). Einstein's equation can calculate energy created from mass lost when two hydrogen nuclei are fused to form helium.

In the sun's core, at 15 million °C and compressed by immense gravity, nuclear fusion reactions convert 600 million tonnes of hydrogen per second into helium, releasing enormous quantities of heat and light energy.

From small nuclear fusion experiments, scientists have shown that 1 to 10 million times more energy is produced per kilogram of nuclear fuel compared to oil, coal or gas. 'Heavy' hydrogen nuclear fuel is readily available from sea water.

A group of 35 countries working together are currently constructing the International Thermonuclear Experimental Reactor (ITER) in France. The large, doughnut shaped chamber, called a Tokamak, is where fuel will reach 150 million °C, the temperature needed for fusion reactions on earth.

Nuclear fusion power offers a safe, reliable and unlimited source of clean energy. Development of technology to enable a fusion reaction to produce more energy than it uses is one of the world's greatest challenges for mathematicians, scientists and engineers to solve.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Expand the following.

a $2(x + 3)$

b $3(a - 5)$

c $4x(3 - 2y)$

d $-3(2b - 1)$

2 Evaluate the following.

a 9^2

b 4^2

c 1^2

d $(2x)^2$ if $x = 4$

e $(5a)^2$ if $a = -1$

f b if $b^2 = 49$ and $b > 0$

g y if $y^2 = 9$ and $y > 0$

h m if $m^2 = 36$ and $m < 0$

3 Write down the highest common factor of:

a 4 and 6 **b** 12 and 18 **c** $2x$ and $4x$

d $3xy$ and $9y$ **e** $10x$ and $15x^2$ **f** $3a^2b$ and $4ab^2$

4 Factorise by taking out the highest common factor.

a $2a + 6$ **b** $3x + 12y$ **c** $5x^2 - 15x$ **d** $4m - 6mn$

5 List the pairs of whole numbers that multiply to give each of the following.

a 6 **b** 12 **c** -10 **d** -27

6 Simplify:

a $\frac{2}{7} + \frac{2}{3}$ **b** $\frac{7}{9} - \frac{1}{6}$ **c** $\frac{3}{8} \times \frac{2}{9}$ **d** $\frac{5}{8} \div \frac{5}{12}$

7 Expand and simplify.

a $3(x - 1) + 5$

b $4(1 - x) + 5x$

c $-2(5 + x) - x$

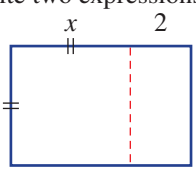
d $4(2x + 1) - 3(x + 2)$

8 Solve each of the following equations.

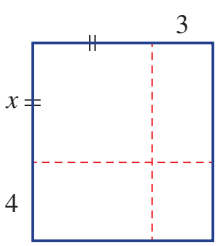
a $2x - 3 = 9$ **b** $\frac{x + 1}{3} = 4$ **c** $2(x - 1) = 3(1 - 2x)$

9 Write two expressions for the area of these rectangles, one with brackets and one without.

a



b



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8A Expanding binomial products

Interactive
Widgets
HOTsheets
Walkthrough

A binomial is an expression with two terms such as $x + 5$ or $x^2 + 3$. You will recall from Chapter 2 that we looked at the product of a single term with a binomial expression, for example $2(x - 3)$ or $x(3x - 1)$. The product of two binomial expressions can also be expanded using the distributive law. This involves multiplying every term in one expression by every term in the other expression.

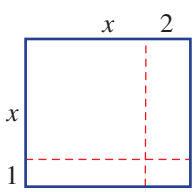
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



Expanding the product of two binomial expressions can be applied to problems involving the expansion of rectangular areas such as a farmer's paddock.

Let's start: Rectangular expansions

If $(x + 1)$ and $(x + 2)$ are the side lengths of a rectangle as shown, the total area can be found as an expression in two different ways.



- Write an expression for the total area of the rectangle using length $= (x + 2)$ and width $= (x + 1)$.
- Now find the area of each of the four parts of the rectangle and combine to give an expression for the total area.
- Compare your two expressions above and complete this equation:

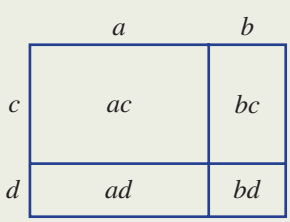
  $(x + 2)(____) = x^2 + ____ + ____$.
- Can you explain a method for expanding the left-hand side to give the right-hand side?

Key ideas

- Expanding **binomial products** uses the **distributive law**.

$$(a + b)(c + d) = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$

- Diagrammatically $(a + b)(c + d) = ac + ad + bc + bd$



For example: $(x + 1)(x + 5) = x^2 + 5x + x + 5$
$= x^2 + 6x + 5$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 1 Expanding binomial products

Expand the following.

a $(x + 3)(x + 5)$ **b** $(x - 4)(x + 7)$

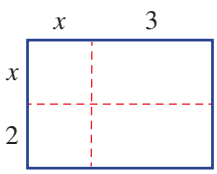
c $(2x - 1)(x - 6)$ **d** $(5x - 2)(3x + 7)$

SOLUTION	EXPLANATION
a $(x+3)(x+5) = x^2 + 5x + 3x + 15$ $= x^2 + 8x + 15$	Use the distributive law to expand the brackets and then collect the like terms $5x$ and $3x$.
b $(x-4)(x+7) = x^2 + 7x - 4x - 28$ $= x^2 + 3x - 28$	After expanding to get the four terms, collect the like terms $7x$ and $-4x$.
c $(2x-1)(x-6) = 2x^2 - 12x - x + 6$ $= 2x^2 - 13x + 6$	Remember $2x \times x = 2x^2$ and $-1 \times (-6) = 6$.
d $(5x-2)(3x+7) = 15x^2 + 35x - 6x - 14$ $= 15x^2 + 29x - 14$	Recall $5x \times 3x = 5 \times 3 \times x \times x = 15x^2$.

Exercise 8A

UNDERSTANDING AND FLUENCY 1–3, 4–5(½) 3–6(½) 4–6(½)

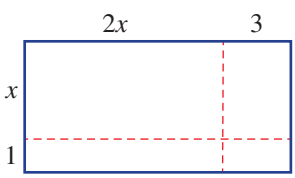
1 The given diagram shows the area $(x + 2)(x + 3)$.



a Write down an expression for the area of each of the four regions inside the rectangle.

b Copy and complete. $(x + 2)(x + 3) = ___ + 3x + ___ + 6$
$= ___ + 5x + ___$

2 The given diagram shows the area $(2x + 3)(x + 1)$.



a Write down an expression for the area of each of the four regions inside the rectangle.

b Copy and complete. $(2x + 3)(____) = 2x^2 + ____ + 3x + ____$
$= ____ + ____ + ____$

3 Copy and complete these expansions.

a $(x + 1)(x + 5) = ___ + 5x + ___ + 5$
$= ___ + 6x + ___$

b $(x - 3)(x + 2) = ___ + ___ - 3x - ___$
$= ___ - x - ___$

c $(3x - 2)(7x + 2) = ___ + 6x - ___ - ___$
$= ___ - ___ - ___$

d $(4x - 1)(3x - 4) = ___ - ___ - 3x + ___$
$= ___ - 19x + ___$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 1a

4 Expand the following.

a $(x + 2)(x + 5)$	**b** $(b + 3)(b + 4)$	**c** $(t + 8)(t + 7)$
d $(p + 6)(p + 6)$	**e** $(x + 9)(x + 6)$	**f** $(d + 15)(d + 4)$
g $(a + 1)(a + 7)$	**h** $(y + 10)(y + 2)$	**i** $(m + 4)(m + 12)$

Example 1b,c,d

5 Expand the following.

a $(x + 3)(x - 4)$	**b** $(x + 5)(x - 2)$	**c** $(x + 4)(x - 8)$
d $(x - 6)(x + 2)$	**e** $(x - 1)(x + 10)$	**f** $(x - 7)(x + 9)$
g $(x - 2)(x + 7)$	**h** $(x - 1)(x - 2)$	**i** $(x - 4)(x - 5)$
j $(4x + 3)(2x + 5)$	**k** $(3x + 2)(2x + 1)$	**l** $(3x + 1)(5x + 4)$
m $(2x - 3)(3x + 5)$	**n** $(8x - 3)(3x + 4)$	**o** $(3x - 2)(2x + 1)$
p $(5x + 2)(2x - 7)$	**q** $(2x + 3)(3x - 2)$	**r** $(4x + 1)(4x - 5)$
s $(3x - 2)(6x - 5)$	**t** $(5x - 2)(3x - 1)$	**u** $(7x - 3)(3x - 4)$

6 Expand these binomial products.

a $(a + b)(a + c)$	**b** $(a - b)(a + c)$	**c** $(b - a)(a + c)$
d $(x - y)(y - z)$	**e** $(y - x)(z - y)$	**f** $(1 - x)(1 + y)$
g $(2x + y)(x - 2y)$	**h** $(2a + b)(a - b)$	**i** $(3x - y)(2x + y)$
j $(2a - b)(3a + 2)$	**k** $(4x - 3y)(3x - 4y)$	**l** $(xy - yz)(z + 3x)$

PROBLEM-SOLVING AND REASONING	7, 10(½)	7, 8, 10(½), 11	8, 9, 10(½), 11

7 A room in a house with dimensions 4 m by 5 m is to be extended. Both the length and width are to be increased by x m.

a Find an expanded expression for the area of the new room.

b If $x = 3$:

i find the area of the new room

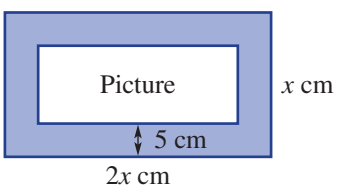
ii determine by how much the area has increased



8 A picture frame 5 cm wide has a length that is twice the width x cm. Find an expression:

a for the total area of the frame and picture

b in expanded form for the area of the picture only

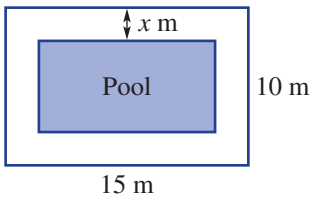


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 The outside edge of a path around a rectangular swimming pool is 15 m long and 10 m wide. The path is x metres wide.

a Find an expression for the area of the pool in expanded form.

b Find the area of the pool if $x = 2$.



10 Write the missing terms in these expansions.

a $(x + 2)(x + __) = x^2 + 5x + 6$

b $(x + __)(x + 5) = x^2 + 7x + 10$

c $(x + 1)(x + __) = x^2 + 7x + __$

d $(x + __)(x + 9) = x^2 + 11x + __$

e $(x + 3)(x - __) = x^2 + x - __$

f $(x - 5)(x + __) = x^2 - 2x - __$

g $(x + 1)(__ + 3) = 2x^2 + __ + __$

h $(__ - 4)(3x - 1) = 9x^2 - __ + __$

i $(x + 2)(__ + __) = 7x^2 + __ + 6$

j $(__ - __)(2x - 1) = 6x^2 - __ + 4$

11 Consider the binomial product $(x + a)(x + b)$. Find the possible integer values of a and b if:

a $(x + a)(x + b) = x^2 + 5x + 6$

b $(x + a)(x + b) = x^2 - 5x + 6$

c $(x + a)(x + b) = x^2 + x - 6$

d $(x + a)(x + b) = x^2 - x - 6$

ENRICHMENT — — 12, 13

Trinomial expansions

12 Using the distributive law $(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$.
Use this knowledge to expand and simplify these products. Note: $x \times x^2 = x^3$.

a $(x + 1)(x^2 + x + 1)$

b $(x - 2)(x^2 - x + 3)$

c $(2x - 1)(2x^2 - x + 4)$

d $(x^2 - x + 1)(x + 3)$

e $(5x^2 - x + 2)(2x - 3)$

f $(2x^2 - x + 7)(4x - 7)$

g $(x + a)(x^2 - ax + a)$

h $(x - a)(x^2 - ax - a^2)$

i $(x + a)(x^2 - ax + a^2)$

j $(x - a)(x^2 + ax + a^2)$

13 Now try to expand $(x + 1)(x + 2)(x + 3)$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8B Perfect squares and the difference of two squares

Interactive
Widgets
HOTsheets
Walkthrough

There are two special types of binomial products that involve perfect squares. $2^2 = 4$, $15^2 = 225$, x^2 and $(a + b)^2$ are all examples of perfect squares. To expand $(a + b)^2$ we multiply $(a + b)$ by $(a + b)$ and use the distributive law:

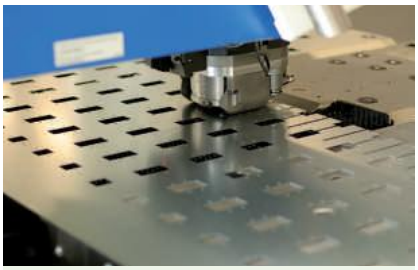
$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

A similar result is obtained for the square of $(a - b)$:

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Another type of expansion involves the case that deals with the product of the sum and difference of the same two terms. The result is the difference of two perfect squares:

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ba - b^2\\ &= a^2 - b^2 \text{ (since } ab = ba \text{ the two middle terms cancel each other out)}\end{aligned}$$



Binomial products can be used to calculate the most efficient way to cut the shapes required for a fabrication out of a metal sheet.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Seeing the pattern

Using $(a + b)(c + d) = ac + ad + bc + bd$, expand and simplify the binomial products in the two sets below.

Set A

$(x + 1)(x + 1) = x^2 + x + x + 1$
$\quad\quad\quad

Example 2 Expanding perfect squares

Expand each of the following.

a $(x - 2)^2$ **b** $(2x + 3)^2$

SOLUTION

a $(x - 2)^2 = (x - 2)(x - 2)$
$= x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

EXPLANATION

Write in expanded form.
Use the distributive law.
Collect like terms.

Alternative method

$(x - 2)^2 = x^2 - 2 \times x \times 2 + 2^2$
$= x^2 - 4x + 4$

Expand using $(a - b)^2 = a^2 - 2ab + b^2$ where $a = x$ and $b = 2$.

b $(2x + 3)^2 = (2x + 3)(2x + 3)$
$= 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Write in expanded form.
Use the distributive law.
Collect like terms.

Alternative method

$(2x + 3)^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$
$= 4x^2 + 12x + 9$

Expand using $(a + b)^2 = a^2 + 2ab + b^2$ where $a = 2x$ and $b = 3$. Recall $(2x)^2 = 2x \times 2x = 4x^2$.

Example 3 Forming a difference of two squares

Expand and simplify the following.

a $(x + 2)(x - 2)$ **b** $(3x - 2y)(3x + 2y)$

SOLUTION

a $(x + 2)(x - 2) = x^2 - \cancel{2x} + \cancel{2x} - 4$
$= x^2 - 4$

EXPLANATION

Expand using the distributive law.
$-2x + 2x = 0$

Alternative method

$(x + 2)(x - 2) = (x)^2 - (2)^2$
$= x^2 - 4$

$(a + b)(a - b) = a^2 - b^2$. Here $a = x$ and $b = 2$.

b $(3x - 2y)(3x + 2y) = 9x^2 + \cancel{6xy} - \cancel{6xy} - 4y^2$
$= 9x^2 - 4y^2$

Expand using the distributive law.
$6xy - 6xy = 0$

Alternative method

$(3x - 2y)(3x + 2y) = (3x)^2 - (2y)^2$
$= 9x^2 - 4y^2$

$(a + b)(a - b) = a^2 - b^2$ with $a = 3x$ and $b = 2y$ here.

Exercise 8B

UNDERSTANDING AND FLUENCY	1–3, 4–7(½)	1, 3, 4–8(½)	4–8(½)

1 Complete these expansions.

a $(x + 3)(x + 3) = x^2 + 3x + __ + __$
$= _____$

b $(x + 5)(x + 5) = x^2 + 5x + __ + __$
$= _____$

c $(x - 2)(x - 2) = x^2 - 2x - __ + __$
$= _____$

d $(x - 7)(x - 7) = x^2 - 7x - __ + __$
$= _____$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Match the equivalent expressions.

a	$x(x + 4)$	**A**	$x^2 - 8x + 16$
b	$(x + 1)(x + 4)$	**B**	$x^2 + 5x + 4$
c	$(x + 4)^2$	**C**	$x^2 + 4x$
d	$(x - 4)^2$	**D**	$x^2 - 16$
e	$(x + 4)(x - 4)$	**E**	$x^2 + 8x + 16$

3 Complete these expansions.

a $(x + 4)(x - 4) = x^2 - 4x + __ - __$
$= _____$

b $(x - 10)(x + 10) = x^2 + 10x - __ - __$
$= _____$

c $(2x - 1)(2x + 1) = 4x^2 + __ - __ - __$
$= _____$

d $(3x + 4)(3x - 4) = 9x^2 - __ + __ - __$
$= _____$

Example 2a

4 Expand each of the following perfect squares.

a	$(x + 1)^2$	**b**	$(x + 3)^2$	**c**	$(x + 2)^2$	**d**	$(x + 5)^2$
e	$(x + 4)^2$	**f**	$(x + 9)^2$	**g**	$(x + 7)^2$	**h**	$(x + 10)^2$
i	$(x - 2)^2$	**j**	$(x - 6)^2$	**k**	$(x - 1)^2$	**l**	$(x - 3)^2$
m	$(x - 9)^2$	**n**	$(x - 7)^2$	**o**	$(x - 4)^2$	**p**	$(x - 12)^2$

Example 2b

5 Expand each of the following perfect squares.

a	$(2x + 1)^2$	**b**	$(2x + 5)^2$	**c**	$(3x + 2)^2$	**d**	$(3x + 1)^2$
e	$(5x + 2)^2$	**f**	$(4x + 3)^2$	**g**	$(7 + 2x)^2$	**h**	$(5 + 3x)^2$
i	$(2x - 3)^2$	**j**	$(3x - 1)^2$	**k**	$(4x - 5)^2$	**l**	$(2x - 9)^2$
m	$(3x + 5y)^2$	**n**	$(2x + 4y)^2$	**o**	$(7x + 3y)^2$	**p**	$(6x + 5y)^2$
q	$(4x - 9y)^2$	**r**	$(2x - 7y)^2$	**s**	$(3x - 10y)^2$	**t**	$(4x - 6y)^2$

6 Expand each of the following perfect squares.

a	$(3 - x)^2$	**b**	$(5 - x)^2$	**c**	$(1 - x)^2$	**d**	$(6 - x)^2$
e	$(11 - x)^2$	**f**	$(4 - x)^2$	**g**	$(7 - x)^2$	**h**	$(12 - x)^2$
i	$(8 - 2x)^2$	**j**	$(2 - 3x)^2$	**k**	$(9 - 2x)^2$	**l**	$(10 - 4x)^2$

Example 3a

7 Expand and simplify the following to form a difference of two squares.

a	$(x + 1)(x - 1)$	**b**	$(x + 3)(x - 3)$	**c**	$(x + 8)(x - 8)$	**d**	$(x + 4)(x - 4)$
e	$(x + 12)(x - 12)$	**f**	$(x + 11)(x - 11)$	**g**	$(x - 9)(x + 9)$	**h**	$(x - 5)(x + 5)$
i	$(x - 6)(x + 6)$	**j**	$(5 - x)(5 + x)$	**k**	$(2 - x)(2 + x)$	**l**	$(7 - x)(7 + x)$

Example 3b

8 Expand and simplify the following.

a	$(3x - 2)(3x + 2)$	**b**	$(5x - 4)(5x + 4)$	**c**	$(4x - 3)(4x + 3)$	**d**	$(7x - 3y)(7x + 3y)$
e	$(9x - 5y)(9x + 5y)$	**f**	$(11x - y)(11x + y)$	**g**	$(8x + 2y)(8x - 2y)$	**h**	$(10x - 9y)(10x + 9y)$
i	$(7x - 5y)(7x + 5y)$	**j**	$(6x - 11y)(6x + 11y)$	**k**	$(8x - 3y)(8x + 3y)$	**l**	$(9x - 4y)(9x + 4y)$

PROBLEM-SOLVING AND REASONING	9, 11	9, 10, 12, 13	9, 10, 12, 13

9 Lara is x years old and her two best friends are $(x - 2)$ and $(x + 2)$ years old.

a Write an expression for the:

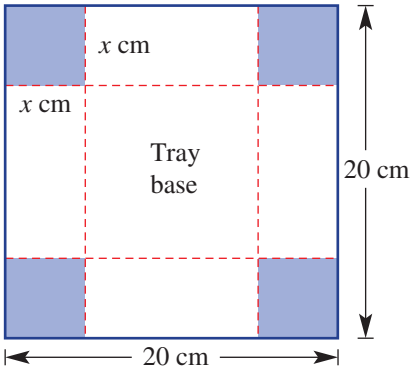
i square of Lara's age

ii product of the ages of Lara's best friends

b Are the answers from parts **a i** and **ii** equal? If not, by how much do they differ?

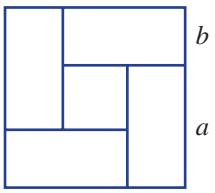
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 A square piece of tin of side length 20 cm has four squares of side length x cm removed from each corner. The sides are folded up to form a tray. The centre square forms the tray base.



- **a** Write an expression for the side length of the base of the tray.
- **b** Write an expression for the area of the base of the tray. Expand your answer.
- **c** Find the area of the tray base if $x = 3$.
- **d** Find the volume of the tray if $x = 3$.

11 Four tennis courts are arranged as shown with a square storage area in the centre. Each court area has the same dimensions $a \times b$.

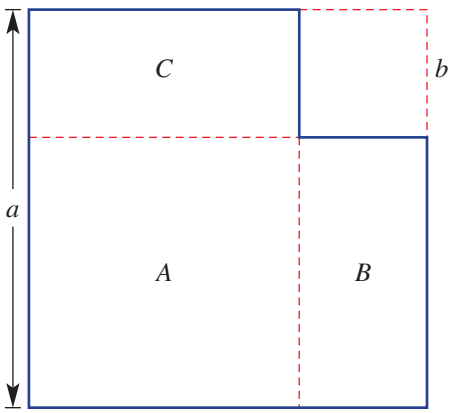


- **a** Write an expression for the side length of the total area.
- **b** Write an expression for the area of the total area.
- **c** Write an expression for the side length of the inside storage area.
- **d** Write an expression for the area of the inside storage area.
- **e** Subtract your answer to part **d** from your answer to part **b** to find the area of the four courts.
- **f** Find the area of one court. Does your answer confirm that your answer to part **e** is correct?

12 A square of side length x has one side reduced by 1 unit and the other increased by 1 unit.

- **a** Find an expanded expression for the area of the resulting rectangle.
- **b** Is the area of the original square the same as the area of the resulting rectangle? Explain why/why not.

13 A square of side length b is removed from a square of side length a.



- **a** Using subtraction write down an expression for the remaining area.
- **b** Write expressions for the area of the regions:
  - **i** A **ii** B **iii** C
- **c** Add all the expressions from part **b** to see if you get your answer from part **a**.

ENRICHMENT — — 14

Extended expansions

14 Expand and simplify these expressions.

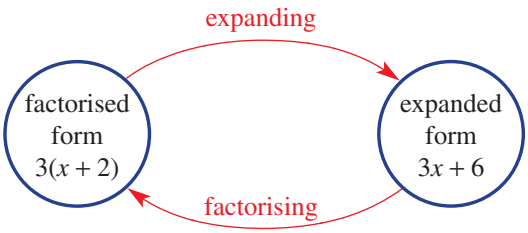
- **a** $(x + 2)^2 - 4$
- **b** $(2x - 1)^2 - 4x^2$
- **c** $(x + 3)(x - 3) + 6x$
- **d** $1 - (x + 1)^2$
- **e** $x^2 - (x + 1)(x - 1)$
- **f** $(x + 1)^2 - (x - 1)^2$
- **g** $(3x - 2)(3x + 2) - (3x + 2)^2$
- **h** $(5x - 1)^2 - (5x + 1)(5x - 1)$
- **i** $(x + y)^2 - (x - y)^2 + (x + y)(x - y)$
- **j** $(2x - 3)^2 + (2x + 3)^2$
- **k** $(2 - x)^2 - (2 + x)^2$
- **l** $(3 - x)^2 + (x - 3)^2$
- **m** $2(3x - 4)^2 - (3x - 4)(3x + 4)$
- **n** $2(x + y)^2 - (x - y)^2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8C Factorising algebraic expressions

Interactive
Widgets
HOTsheets
Walkthrough

The process of factorisation is a key step in the simplification of many algebraic expressions and in the solution of equations. It is the reverse process of expansion and involves writing an expression as a product of its factors.





Factorising is a key mathematical skill required in many diverse occupations, such as in business, science, technology and engineering.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Which factorised form?

The product $x(4x + 8)$ when expanded gives $4x^2 + 8x$.

- Write down three other products that when expanded give $4x^2 + 8x$. (Do not use fractions.)
- Which of your products uses the highest common factor of $4x^2$ and $8x$? What is this highest common factor?

Key ideas

- When **factorising** expressions with common factors, take out the highest common factor (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral
    For example: $x^2 + 5x = x(x + 5)$
  - the product of numbers and pronumerals
    For example: $2x^2 + 10x = 2x(x + 5)$
- A factorised expression can be checked by using expansion.
  For example: $2x(x + 5) = 2x^2 + 10x$

Example 4 Finding the HCF of an expression

Determine the HCF of the following.

a $6a$ and $8ab$ **b** $3x^2$ and $6xy$

SOLUTION

a $2a$

b $3x$

EXPLANATION

HCF of 6 and 8 is 2.
HCF of a and ab is a.

HCF of 3 and 6 is 3.
HCF of x^2 and xy is x.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 5 Factorising expressions

Factorise the following.

a $40 - 16b$ **b** $-8x^2 - 12x$

SOLUTION

a $40 - 16b = 8(5 - 2b)$

b $-8x^2 - 12x = -4x(2x + 3)$

EXPLANATION

The HCF of 40 and $16b$ is 8. Place 8 in front of the brackets and divide each term by 8.

The HCF of the terms is $-4x$, including the common negative. Place the factor in front of the brackets and divide each term by $-4x$.

Example 6 Taking out a binomial factor

Factorise the following.

a $3(x + y) + x(x + y)$ **b** $(7 - 2x) - x(7 - 2x)$

SOLUTION

a $3(x + y) + x(x + y)$
$= (x + y)(3 + x)$

b $(7 - 2x) - x(7 - 2x)$
$= 1(7 - 2x) - x(7 - 2x)$
$= (7 - 2x)(1 - x)$

EXPLANATION

HCF $= (x + y)$. The second pair of brackets contains what remains when $3(x + y)$ and $x(x + y)$ are divided by $(x + y)$.

Insert 1 in front of the first bracket.
HCF $= (7 - 2x)$. The second bracket must contain $1 - x$ after dividing $(7 - 2x)$ and $x(7 - 2x)$ by $(7 - 2x)$.

Exercise 8C

UNDERSTANDING AND FLUENCY	1–3, 4–7(½)	3, 4–8(½)	4–8(½)

1 Write down the highest common factor (HCF) of these pairs of numbers.

a 8, 12 **b** 10, 20 **c** 5, 60 **d** 24, 30
e 3, 5 **f** 100, 75 **g** 16, 24 **h** 36, 72

2 Write down the missing factor.

a $5 \times ___ = 5x$ **b** $7 \times ___ = 7x$ **c** $2a \times ___ = 2a^2$
d $5a \times ___ = 10a^2$ **e** $___ \times 3y = -6y^2$ **f** $__ \times 12x = -36x^2$
g $-3 \times ___ = 6x$ **h** $-2x \times ___ = 20x^2$ **i** $__ \times 7xy = -14x^2y$

3 Copy and complete these factorisations.

a $7x + 7 = 7(___ + ___)$ **b** $7x + 14 = 7(___ + ___)$
c $14x + 7 = 7(___ + ___)$ **d** $14x - 7 = 7(___ - ___)$
e $-9x + 9 = -9(___ - ___)$ **f** $-9x + 9 = 9(___ + ___)$
g $6 - 12x = 6(___ - ___)$ **h** $12 - 6x = 6(___ - ___)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 4

4 Determine the HCF of the following.

a $6x$ and $14xy$	**b** $12a$ and $18a$	**c** $10m$ and 4	**d** $12y$ and 8
e $15t$ and $6s$	**f** 15 and p	**g** $9x$ and $24xy$	**h** $6n$ and $21mn$
i $10y$ and $2y$	**j** $8x^2$ and $14x$	**k** $4x^2y$ and $18xy$	**l** $5ab^2$ and $15a^2b$

Example 5a

5 Factorise the following.

a $7x + 7$	**b** $3x + 3$	**c** $4x - 4$	**d** $5x - 5$
e $4 + 8y$	**f** $10 + 5a$	**g** $3 - 9b$	**h** $6 - 2x$
i $12a + 3b$	**j** $6m + 6n$	**k** $10x - 8y$	**l** $4a - 20b$
m $x^2 + 2x$	**n** $a^2 - 4a$	**o** $y^2 - 7y$	**p** $x - x^2$
q $3p^2 + 3p$	**r** $8x - 8x^2$	**s** $4b^2 + 12b$	**t** $6y - 10y^2$
u $12a - 15a^2$	**v** $9m + 18m^2$	**w** $16xy - 48x^2$	**x** $7ab - 28ab^2$

Example 5b

6 Factorise the following using a negative factor.

a $-8x - 4$	**b** $-4x - 2$	**c** $-10x - 5y$	**d** $-7a - 14b$
e $-9x - 12$	**f** $-6y - 8$	**g** $-10x - 15y$	**h** $-4m - 20n$
i $-3x^2 - 18x$	**j** $-8x^2 - 12x$	**k** $-16y^2 - 6y$	**l** $-5a^2 - 10a$
m $-6x - 20x^2$	**n** $-6p - 15p^2$	**o** $-16b - 8b^2$	**p** $-9x - 27x^2$

Example 6

7 Factorise the following, which involve a binomial common factor.

a $4(x + 3) + x(x + 3)$	**b** $3(x + 1) + x(x + 1)$	**c** $7(m - 3) + m(m - 3)$
d $x(x - 7) + 2(x - 7)$	**e** $8(a + 4) - a(a + 4)$	**f** $5(x + 1) - x(x + 1)$
g $y(y + 3) - 2(y + 3)$	**h** $a(x + 2) - x(x + 2)$	**i** $t(2t + 5) + 3(2t + 5)$
j $m(5m - 2) + 4(5m - 2)$	**k** $y(4y - 1) - (4y - 1)$	**l** $(7 - 3x) + x(7 - 3x)$

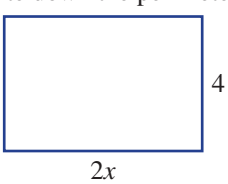
8 Factorise these mixed expressions.

a $6a + 30$	**b** $5x - 15$	**c** $8b + 18$
d $x^2 - 4x$	**e** $y^2 + 9y$	**f** $a^2 - 3a$
g $x^2y - 4xy + xy^2$	**h** $6ab - 10a^2b + 8ab^2$	**i** $m(m + 5) + 2(m + 5)$
j $x(x + 3) - 2(x + 3)$	**k** $b(b - 2) + (b - 2)$	**l** $x(2x + 1) - (2x + 1)$
m $y(3 - 2y) - 5(3 - 2y)$	**n** $(x + 4)^2 + 5(x + 4)$	**o** $(y + 1)^2 - 4(y + 1)$

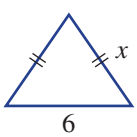
PROBLEM-SOLVING AND REASONING 9, 12 | 9, 10, 12, 13 | 9(½), 10, 11, 13, 14(½)

9 Write down the perimeter of these shapes in factorised form.

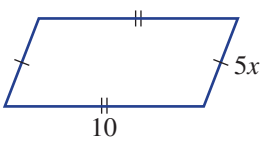
a



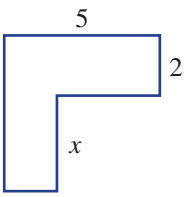
b



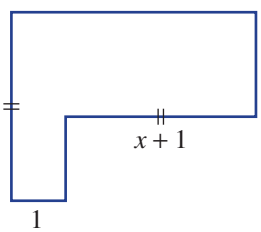
c



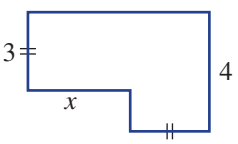
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 The expression for the area of a rectangle is $(4x^2 + 8x)$. Find an expression for its width if the length is $(x + 2)$.

11 The height, in metres, of a ball thrown in the air is given by $5t - t^2$, where t is the time in seconds.

a Write an expression for the ball's height in factorised form.

b Find the ball's height at these times:

i $t = 0$

ii $t = 2$

iii $t = 4$

c How long does it take for the ball's height to return to 0 metres? Use trial and error if required.

12 $7 \times 9 + 7 \times 3$ can be evaluated by first factorising to $7(9 + 3)$. This gives $7 \times 12 = 84$. Use a similar technique to evaluate the following.

a $9 \times 2 + 9 \times 5$ **b** $6 \times 3 + 6 \times 9$ **c** $-2 \times 4 - 2 \times 6$

d $-5 \times 8 - 5 \times 6$ **e** $23 \times 5 - 23 \times 2$ **f** $63 \times 11 - 63 \times 8$

13 Common factors can also be removed from expressions with more than two terms.

For example: $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$

Factorise these expressions by taking out the HCF.

a $3a^2 + 9a + 12$ **b** $5z^2 - 10z + zy$ **c** $x^2 - 2xy + x^2y$

d $4by - 2b + 6b^2$ **e** $-12xy - 8yz - 20xyz$ **f** $3ab + 4ab^2 + 6a^2b$

14 Sometimes we can choose to factor out a negative or a positive HCF. Both factorisations are correct. For example:

$$\begin{aligned} & -13x + 26 = -13(x - 2) \quad \text{(HCF is } -13\text{)} \\ \text{or} \quad & -13x + 26 = 13(-x + 2) \quad \text{(HCF is 13)} \\ & \qquad\quad = 13(2 - x) \end{aligned}$$

Factorise in two different ways by first factoring out a negative and then a positive HCF.

a $-4x + 12$ **b** $-3x + 9$ **c** $-8n + 8$ **d** $-3b + 3$

e $-5m + 5m^2$ **f** $-7x + 7x^2$ **g** $-5x + 5x^2$ **h** $-4y + 22y^2$

i $-8n + 12n^2$ **j** $-8y + 20$ **k** $-15mn + 10$ **l** $-15x + 45$

ENRICHMENT — — 15

Factoring out a negative

15 Using the fact that $a - b = -(b - a)$ you can factorise $x(x - 2) - 5(2 - x)$ by following these steps.

$$\begin{aligned} x(x - 2) - 5(2 - x) &= x(x - 2) + 5(x - 2) \\ &= (x - 2)(x + 5) \end{aligned}$$

Use this idea to factorise these expressions.

a $x(x - 4) + 3(4 - x)$ **b** $x(x - 5) - 2(5 - x)$ **c** $x(x - 3) - 3(3 - x)$

d $3x(x - 4) + 5(4 - x)$ **e** $3(2x - 5) + x(5 - 2x)$ **f** $2x(x - 2) + (2 - x)$

g $-4(3 - x) - x(x - 3)$ **h** $x(x - 5) + (10 - 2x)$ **i** $x(x - 3) + (6 - 2x)$

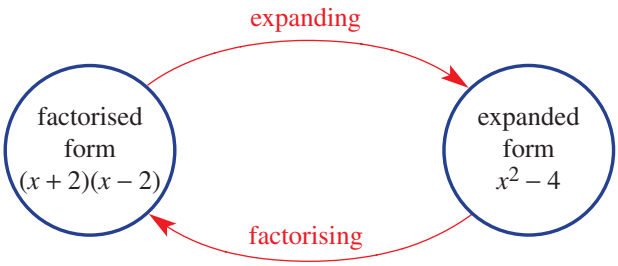
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8D Factorising the difference of two squares

Interactive
Widgets
HOTsheets
Walkthrough

Recall that a difference of two squares is formed when expanding the product of the sum and difference of two terms. For example, $(x + 2)(x - 2) = x^2 - 4$. Reversing this process means that a difference of two squares can be factorised into two binomial expressions of the form $(a + b)$ and $(a - b)$.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4



Let's start: Expanding to understand factorising

Complete the steps in these expansions and then write the conclusion.

- $(x + 3)(x - 3) = x^2 - 3x + __ - __$
  $= x^2 - ___$
  $\therefore x^2 - 9 = (__ + __)(__ - __)$
- $(2x - 5)(2x + 5) = 4x^2 + 10x - __ - __$
  $= ___ - ___$
  $\therefore 4x^2 - ___ = (__ + __)(__ - __)$
- $(a + b)(a - b) = a^2 - ab + __ - __$
  $= ___ - ___$
  $\therefore a^2 - __ = (__ + __)(__ - __)$

Key ideas

- Factorising the difference of two squares uses the rule $a^2 - b^2 = (a + b)(a - b)$.
  - $x^2 - 16 = x^2 - 4^2$
    $= (x + 4)(x - 4)$
  - $9x^2 - 100 = (3x)^2 - 10^2$
    $= (3x + 10)(3x - 10)$
  - $25 - 4y^2 = 5^2 - (2y)^2$
    $= (5 + 2y)(5 - 2y)$
- First take out common factors where possible.
  - $2x^2 - 18 = 2(x^2 - 9) = 2(x + 3)(x - 3)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 7 Factorising the difference of two squares

Factorise each of the following.

a $x^2 - 4$ **b** $9a^2 - 25$
c $81x^2 - y^2$ **d** $2b^2 - 32$
e $(x + 1)^2 - 4$

SOLUTION	EXPLANATION
a $x^2 - 4 = x^2 - 2^2$ $= (x + 2)(x - 2)$	Write as a difference of two squares (4 is the same as 2^2). Write in factorised form $a^2 - b^2 = (a + b)(a - b)$. Here $a = x$ and $b = 2$.
b $9a^2 - 25 = (3a)^2 - 5^2$ $= (3a + 5)(3a - 5)$	Write as a difference of two squares. $9a^2$ is the same as $(3a)^2$. Write in factorised form.
c $81x^2 - y^2 = (9x)^2 - y^2$ $= (9x + y)(9x - y)$	$81x^2 = (9x)^2$ Use the result $a^2 - b^2 = (a + b)(a - b)$
d $2b^2 - 32 = 2(b^2 - 16)$ $= 2(b^2 - 4^2)$ $= 2(b + 4)(b - 4)$	First, factor out the common factor of 2. Write as a difference of two squares and then factorise.
e $(x + 1)^2 - 4 = (x + 1)^2 - 2^2$ $= (x + 1 + 2)(x + 1 - 2)$ $= (x + 3)(x - 1)$	Write as a difference of two squares. In $a^2 - b^2$ here, a is the expression $x + 1$ and $b = 2$. Write in factorised form and simplify.

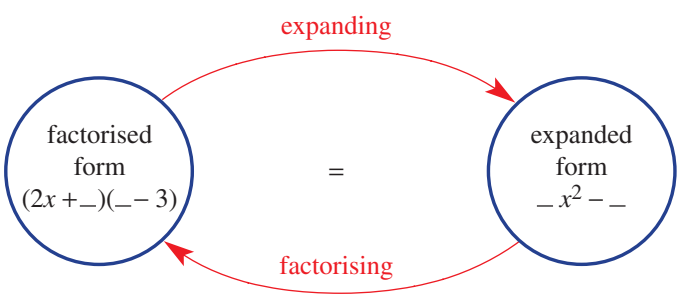
Exercise 8D

UNDERSTANDING AND FLUENCY	1	3, 4–7(½)	4–7(½)

1 Expand these binomial products to form a difference of two squares.

a $(x + 2)(x - 2)$ **b** $(x - 7)(x + 7)$ **c** $(2x - 1)(2x + 1)$
d $(x + y)(x - y)$ **e** $(3x - y)(3x + y)$ **f** $(a + b)(a - b)$

2 Copy and complete this diagram.



3 Complete these factorisations.

a $x^2 - 16 = x^2 - 4^2$
$= (x + 4)(__ - __)$

b $x^2 - 144 = x^2 - (__)^2$
$= (__ + 12)(x - __)$

c $16x^2 - 1 = (__)^2 - (__)^2$
$= (4x + __)(__ - 1)$

d $9a^2 - 4b^2 = (__)^2 - (__)^2$
$= (3a + __)(__ - 2b)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 7a

4 Factorise each of the following.

a $x^2 - 9$ **b** $y^2 - 25$
c $y^2 - 1$ **d** $x^2 - 64$
e $x^2 - 16$ **f** $b^2 - 49$
g $a^2 - 81$ **h** $x^2 - y^2$
i $a^2 - b^2$ **j** $16 - a^2$
k $25 - x^2$ **l** $1 - b^2$
m $36 - y^2$ **n** $121 - b^2$
o $x^2 - 400$ **p** $900 - y^2$

Example 7b,c

5 Factorise each of the following.

a $4x^2 - 25$ **b** $9x^2 - 49$
c $25b^2 - 4$ **d** $4m^2 - 121$
e $100y^2 - 9$ **f** $81a^2 - 4$
g $1 - 4x^2$ **h** $25 - 64b^2$
i $16 - 9y^2$ **j** $36x^2 - y^2$
k $4x^2 - 25y^2$ **l** $64a^2 - 49b^2$
m $4p^2 - 25q^2$ **n** $81m^2 - 4n^2$
o $25a^2 - 49b^2$ **p** $100a^2 - 9b^2$

Example 7d

6 Factorise each of the following by first taking out the common factor.

a $3x^2 - 108$ **b** $10a^2 - 10$
c $6x^2 - 24$ **d** $4y^2 - 64$
e $98 - 2x^2$ **f** $32 - 8m^2$
g $5x^2y^2 - 5$ **h** $3 - 3x^2y^2$
i $63 - 7a^2b^2$

Example 7e

7 Factorise each of the following.

a $(x + 5)^2 - 9$ **b** $(x + 3)^2 - 4$
c $(x + 10)^2 - 16$ **d** $(x - 3)^2 - 25$
e $(x - 7)^2 - 1$ **f** $(x - 3)^2 - 36$
g $49 - (x + 3)^2$ **h** $4 - (x + 2)^2$
i $81 - (x + 8)^2$

PROBLEM-SOLVING AND REASONING 8, 10 | 8–11 | 9–12

8 The height above ground (in metres) of an object thrown off the top of a building is given by $36 - 4t^2$, where t is in seconds.

a Factorise the expression for the height of the object by first taking out the common factor.

b Find the height of the object:

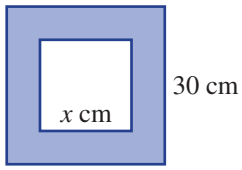
i initially ($t = 0$)

ii at 2 seconds ($t = 2$)

c How long does it take for the object to hit the ground? Use trial and error if you wish.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 This 'multisize' square picture frame has side length 30 cm and can hold a square picture with any side length less than 26 cm.



a If the side length of the picture is x cm, write an expression for the area of the:

i picture

ii frame (in factorised form)

b Use your result from part **a ii** to find the area of the frame if:

i $x = 20$

ii the area of the picture is 225 cm^2

10 Initially it may not appear that an expression such as $-4 + 9x^2$ is a difference of two squares. However, swapping the position of the two terms makes $-4 + 9x^2 = 9x^2 - 4$, which can be factorised to $(3x + 2)(3x - 2)$. Use this idea to factorise these difference of two squares.

a $-9 + x^2$

b $-121 + 16x^2$

c $-25a^2 + 4$

d $-y^2 + x^2$

e $-25a^2 + 4b^2$

f $-36a^2b^2 + c^2$

g $-16x^2 + y^2z^2$

h $-900a^2 + b^2$

11 Olivia factorises $16x^2 - 4$ to get $(4x + 2)(4x - 2)$ but the answer says $4(2x + 1)(2x - 1)$.

a What should Olivia do to get from her answer to the actual answer?

b What should Olivia have done initially to avoid this issue?

12 Find and explain the error in this working and correct it.

$$9 - (x - 1)^2 = (3 + x - 1)(3 - x - 1)$$
$$= (2 + x)(2 - x)$$

ENRICHMENT — — 13

Factorising with fractions and powers of 4

13 Some expressions with fractions or powers of 4 can be factorised in a similar way. Factorise these.

a $x^2 - \frac{1}{4}$

b $x^2 - \frac{4}{25}$

c $25x^2 - \frac{9}{16}$

d $\frac{x^2}{9} - 1$

e $\frac{a^2}{4} - \frac{b^2}{9}$

f $\frac{5x^2}{9} - \frac{5}{4}$

g $\frac{7a^2}{25} - \frac{28b^2}{9}$

h $\frac{a^2}{8} - \frac{b^2}{18}$

i $x^4 - y^4$

j $2a^4 - 2b^4$

k $21a^4 - 21b^4$

l $\frac{x^4}{3} - \frac{y^4}{3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8E Factorising by grouping in pairs

Interactive
Widgets
HOTsheets
Walkthrough

When a simplified expression contains four terms, such as $x^2 + 2x - ax - 2a$, it may be possible to factorise it into a product of two binomial terms like $(x - a)(x + 2)$. In such situations the method of grouping is often used.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Two methods – same result

The four-term expression $x^2 - 3x - 3 + x$ is written on the board.
Tommy chooses to rearrange the terms to give $x^2 - 3x + x - 3$ then factorises by grouping.
Sharon chooses to rearrange the terms to give $x^2 + x - 3x - 3$ then also factorises by grouping.

- Complete Tommy and Sharon's factorisation working.

Tommy

$x^2 - 3x + x - 3 = x(x - 3) + 1(___)$
$= (x - 3)(___)$

Sharon

$x^2 + x - 3x - 3 = x(___) - 3(___)$
$= (x + 1)(___)$

- Discuss the differences in the methods. Is there any difference in their answers?
- Whose method do you prefer?

Key ideas

■ **Factorisation by grouping** is a method that is often used to factorise a four-term expression.

- Terms are grouped into pairs and factorised separately.
- The common binomial factor is then taken out to complete the factorisation.
- Terms can be rearranged to assist in the search of a common factor.

$x^2 + 3x - 2x - 6$
$= x(x + 3) - 2(x + 3)$
$= (x + 3)(x - 2)$

Which could also be written as
$(x - 2)(x + 3)$

Example 8 Factorising by grouping in pairs

Use the method of grouping to factorise these expressions.

a $x^2 + 2x + 3x + 6$ **b** $x^2 + 3x - 5x - 15$

SOLUTION

a $x^2 + 2x + 3x + 6 = (x^2 + 2x) + (3x + 6)$
$= x(x + 2) + 3(x + 2)$
$= (x + 2)(x + 3)$

EXPLANATION

Group the first and second pair of terms.
Factorise each group.
Take the common factor $(x + 2)$ out of both groups.

b $x^2 + 3x - 5x - 15 = (x^2 + 3x) + (-5x - 15)$
$= x(x + 3) - 5(x + 3)$
$= (x + 3)(x - 5)$

Group the first and second pair of terms.
Factorise each group.
Take out the common factor $(x + 3)$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 9 Rearranging an expression to factorise by grouping in pairs

Factorise $2x^2 - 9 - 18x + x$ using grouping.

SOLUTION

$$\begin{aligned} 2x^2 - 9 - 18x + x &= 2x^2 + x - 18x - 9 \\ &= x(2x + 1) - 9(2x + 1) \\ &= (2x + 1)(x - 9) \end{aligned}$$

Alternative method

$$\begin{aligned} 2x^2 - 9 - 18x + x &= 2x^2 - 18x + x - 9 \\ &= 2x(x - 9) + 1(x - 9) \\ &= (x - 9)(2x^2 + 1) \end{aligned}$$

EXPLANATION

Rearrange so that each group has a common factor.

Factorise each group then take out $(2x + 1)$.

Alternatively, you can group in another order where each group has a comman factor. Then factorise.

The answer will be the same.

Exercise 8E

UNDERSTANDING AND FLUENCY	1–3, 4–5(½)	3, 4–6(½)	4–6(½)

1 Copy and then fill in the missing information.

a $2(x + 1) + a(x + 1) = (x + 1)(___)$
b $3(x + 3) - a(x + 3) = (x + 3)(___)$
c $5(x + 5) - a(x + 5) = (x + 5)(___)$
d $a(x + 7) + 4(x + 7) = (x + 7)(___)$
e $a(x - 3) + (x - 3) = (x - 3)(___)$
f $a(x + 4) - (x + 4) = (x + 4)(___)$
g $(x - 3) - a(x - 3) = (x - 3)(___)$
h $(4 - x) + 2a(4 - x) = (4 - x)(___)$

2 Take out the common binomial term to factorise each expression.

a $x(x - 3) - 2(x - 3)$
b $x(x + 4) + 3(x + 4)$
c $x(x - 7) + 4(x - 7)$
d $3(2x + 1) - x(2x + 1)$
e $4(3x - 2) - x(3x - 2)$
f $2x(2x + 3) - 3(2x + 3)$
g $3x(5 - x) + 2(5 - x)$
h $2(x + 1) - 3x(x + 1)$
i $x(x - 2) + (x - 2)$

3 Copy and complete

a
$$\begin{aligned} x^2 + 3x + 4x + 12 &= (x^2 + 3x) + (___ + ___) \\ &= ___(x + 3) + ___(x + 3) \\ &= (x + ___)\,(x + ___) \end{aligned}$$

b
$$\begin{aligned} x^2 + 3x - 4x - 12 &= (x^2 + 3x) - (___ + ___) \\ &= x(___ + ___) - 4(___ + ___) \\ &= (_____)(_____) \end{aligned}$$

Example 8

4 Use the method of grouping to factorise these expressions.

a $x^2 + 3x + 2x + 6$
b $x^2 + 4x + 3x + 12$
c $x^2 + 7x + 2x + 14$
d $x^2 - 6x + 4x - 24$
e $x^2 - 4x + 6x - 24$
f $x^2 - 3x + 10x - 30$
g $x^2 + 2x - 18x - 36$
h $x^2 + 3x - 14x - 42$
i $x^2 + 4x - 18x - 72$
j $x^2 - 2x - xa + 2a$
k $x^2 - 3x - 3xc + 9c$
l $x^2 - 5x - 3xa + 15a$

5 Use the method of grouping to factorise these expressions.

a $3ab + 5bc + 3ad + 5cd$
b $4ab - 7ac + 4bd - 7cd$
c $2xy - 8xz + 3wy - 12wz$
d $5rs - 10r + st - 2t$
e $4x^2 + 12xy - 3x - 9y$
f $2ab - a^2 - 2bc + ac$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Factorise these expressions. Remember to use a factor of 1 where necessary, for example, $x^2 - ax + x - a = x(x - a) + 1(x - a)$.

a $x^2 - bx + x - b$ **b** $x^2 - cx + x - c$ **c** $x^2 + bx + x + b$
d $x^2 + cx - x - c$ **e** $x^2 + ax - x - a$ **f** $x^2 - bx - x + b$

PROBLEM-SOLVING AND REASONING	7(½), 10	7–8(½), 10	7–9(½), 10, 11

Example 9

7 Factorise these expressions by first rearranging the terms.

a $2x^2 - 7 - 14x + x$ **b** $5x + 2x + x^2 + 10$ **c** $2x^2 - 3 - x + 6x$
d $3x - 8x - 6x^2 + 4$ **e** $11x - 5a - 55 + ax$ **f** $12y + 2x - 8xy - 3$
g $6m - n + 3mn - 2$ **h** $15p - 8r - 5pr + 24$ **i** $16x - 3y - 8xy + 6$

8 What expanded expression factorises to the following?

a $(x - a)(x + 4)$ **b** $(x - c)(x - d)$ **c** $(x + y)(2 - z)$ **d** $(x - 1)(a + b)$
e $(3x - b)(c - b)$ **f** $(2x - y)(y + z)$ **g** $(3a + b)(2b + 5c)$ **h** $(m - 2x)(3y + z)$

9 Note that $x^2 + 5x + 6 = x^2 + 2x + 3x + 6$ which can be factorised by grouping. Use a similar method to factorise the following.

a $x^2 + 7x + 10$ **b** $x^2 + 8x + 15$ **c** $x^2 + 10x + 24$
d $x^2 - x - 6$ **e** $x^2 + 4x - 12$ **f** $x^2 - 11x + 18$

10 $xa - 21 + 7a - 3x$ could be rearranged in two different ways before factorising.

Method 1

$xa + 7a - 3x - 21 = a(x + 7) - 3(____)$
$= _____$

Method 2

$xa - 3x + 7a - 21 = x(a - 3) + 7(____)$
$= _____$

a Copy and complete both methods for the above expression.
b Use different arrangements of the four terms to complete the factorisation of the following in two ways. Show working using both methods.

i $xb - 6 - 3b + 2x$ **ii** $xy - 8 + 2y - 4x$
iii $4m^2 - 15n + 6m - 10mn$ **iv** $2m + 3n - mn - 6$
v $4a - 6b^2 + 3b - 8ab$ **vi** $3ab - 4c - b + 12ac$

11 Make up at least three of your own four-term expressions that factorise to a binomial product. Describe the method that you used to make up each four-term expression.

ENRICHMENT	—	—	12

Grouping with more than four terms

12 Factorise the following by grouping.

a $2(a - 3) - x(a - 3) - c(a - 3)$ **b** $b(2a + 1) + 5(2a + 1) - a(2a + 1)$
c $x(a + 1) - 4(a + 1) - ba - b$ **d** $3(a - b) - b(a - b) - 2a^2 + 2ab$
e $c(1 - a) - x + ax + 2 - 2a$ **f** $a(x - 2) + 2bx - 4b - x + 2$
g $a^2 - 3ac - 2ab + 6bc + 3abc - 9bc^2$ **h** $3x - 6xy - 5z + 10yz + y - 2y^2$
i $8z - 4y + 3x^2 + xy - 12x - 2xz$ **j** $-ab - 4cx + 3aby + 2abx + 2c - 6cy$

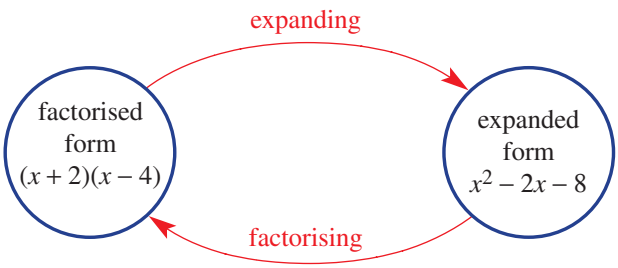
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8F Factorising monic quadratic trinomials

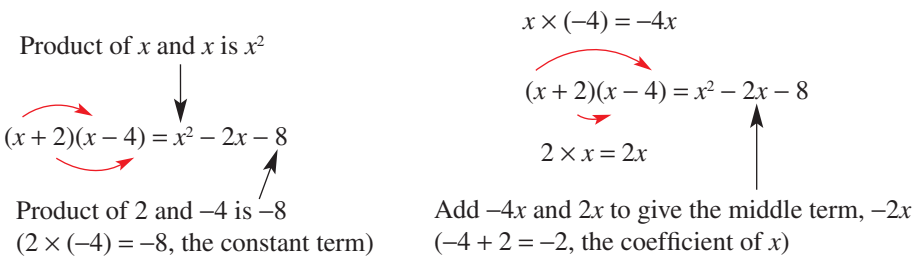
Interactive
Widgets
HOTsheets
Walkthrough

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

An expression that takes the form $x^2 + bx + c$, where b and c are constants, is an example of a monic quadratic trinomial that has the coefficient of x^2 equal to 1.
To factorise a quadratic expression, we need to use the distributive law in reverse. Consider the following.



If we examine the diagram above, we can see how each term of the product is formed.



Let's start: So many choices

Mia says that since $-2 \times 3 = -6$ then $x^2 + 5x - 6$ must equal $(x - 2)(x + 3)$.

- Expand $(x - 2)(x + 3)$ to see if Mia is correct.
- What other pairs of numbers multiply to give -6?
- Which pair of numbers should Mia choose to correctly factorise $x^2 + 5x - 6$?
- What advice can you give Mia when trying to factorise these types of trinomials?

Key ideas

- To factorise a **quadratic trinomial** of the form $x^2 + bx + c$, find two numbers which:
  - multiply to give c and
  - add to give b.

  For example, consider the expression $x^2 - 3x - 10$
  Find two numbers that:
  - multiply to give -10
  - add to give -3.

  Choose -5 and $+2$ because
  - $-5 \times 2 = -10$
  - $-5 + 2 = -3$

- Use expansion to check your factorisation.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- The answer can be written as $(x - 5)(x + 2)$ or $(x + 2)(x - 5)$.

  $\therefore x^2 - 3x - 10 = (x - 5)(x + 2)$

- Use expansion to check your factorisation.

  $(x - 5)(x + 2) = x^2 - 3x - 10$

- If c is a negative number, the signs in the factorised expression will be different from each other.

  For example: $x^2 + 5x \textcircled{-} 6 = (x \textcircled{+} 6)(x \textcircled{-} 1)$

  (c is negative; different signs)

- If c is a positive number, the signs in the factorised expression will be the same as each other.

  Example 1: $x^2 + 5x \textcircled{+} 6 = (x \textcircled{+} 2)(x \textcircled{+} 3)$

  (c is positive; same signs)

  Example 2: $x^2 - 5x \textcircled{+} 6 = (x \textcircled{-} 2)(x \textcircled{-} 3)$

Example 10 Factorising monic quadratic trinomials

Factorise each of the following quadratic expressions.

a $x^2 + 7x + 10$ **b** $x^2 + 2x - 8$ **c** $x^2 - 7x + 10$

SOLUTION	EXPLANATION
a $x^2 + 7x + 10 = (x + 5)(x + 2)$ or $x^2 + 7x + 10 = (x + 2)(x + 5)$	$c = 10$, which is positive, so look for two numbers with the same sign. Factors of 10 are (10, 1)(−10, −1), (5, 2) and (−5, −2). The pair that adds to positive 7 is (5, 2).
b $x^2 + 2x - 8 = (x + 4)(x - 2)$ or $x^2 + 2x - 8 = (x - 2)(x + 4)$	$c = -8$, which is negative, so look for two numbers with different signs. Factors of −8 are (−8, 1), (8, −1), (4, −2) or (−4, 2) and $4 + (-2) = 2$ so choose (4, −2).
c $x^2 - 7x + 10 = (x - 2)(x - 5)$ or $x^2 - 7x + 10 = (x - 5)(x - 2)$	$c = 10$, which is positive, so look for two numbers with the same sign. Factors of 10 are (10, 1), (−10, −1), (5, 2) and (−5, −2). To add to a negative (−7), both factors must then be negative: $-5 + (-2) = -7$ so choose (−5, −2).

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11 Factorising with a common factor

Factorise the quadratic expression $2x^2 - 2x - 12$.

SOLUTION

$2x^2 - 2x - 12 = 2(x^2 - x - 6)$
$= 2(x - 3)(x + 2)$

EXPLANATION

First take out common factor of 2.
$c = -6$, which is negative, so look for two numbers with different signs.
Factors of -6 are $(-6, 1)$, $(6, -1)$, $(-3, 2)$ and $(3, -2)$.
$-3 + 2 = -1$ so choose $(-3, 2)$.
$2(x + 2)(x - 3)$ is another correct answer.

Exercise 8F

UNDERSTANDING AND FLUENCY	1–6(½)	3–8(½)	3–8(½)

1 Expand these binomial products.

a $(x + 1)(x + 3)$
b $(x + 2)(x + 7)$
c $(x - 3)(x + 11)$
d $(x - 5)(x + 6)$
e $(x + 12)(x - 5)$
f $(x + 13)(x - 4)$
g $(x - 2)(x - 6)$
h $(x - 20)(x - 11)$
i $(x - 9)(x - 1)$

2 Decide what two numbers multiply to give the first number and add to give the second number.

a 6, 5
b 10, 7
c 12, 13
d 20, 9
e −5, 4
f −7, −6
g −15, 2
h −30, −1
i 6, −5
j 18, −11
k 40, −13
l 100, −52

Example 10a

3 Factorise each of the following quadratic expressions in which c is positive.

a $x^2 + 3x + 2$
b $x^2 + 4x + 3$
c $x^2 + 7x + 6$
d $x^2 + 10x + 9$
e $x^2 + 8x + 7$
f $x^2 + 15x + 14$
g $x^2 + 6x + 8$
h $x^2 + 7x + 12$
i $x^2 + 10x + 16$
j $x^2 + 8x + 15$
k $x^2 + 9x + 20$
l $x^2 + 11x + 24$

Example 10b

4 Factorise each of the following quadratic expressions in which c is negative.

a $x^2 + 3x - 4$
b $x^2 + x - 2$
c $x^2 + 4x - 5$
d $x^2 + 5x - 14$
e $x^2 + 2x - 15$
f $x^2 + 8x - 20$
g $x^2 + 3x - 18$
h $x^2 + 7x - 18$
i $x^2 + x - 12$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 10c

5 Factorise each of the following quadratic expressions in which c is positive.

a $x^2 - 6x + 5$
b $x^2 - 2x + 1$
c $x^2 - 5x + 4$
d $x^2 - 9x + 8$
e $x^2 - 4x + 4$
f $x^2 - 8x + 12$
g $x^2 - 11x + 18$
h $x^2 - 10x + 21$
i $x^2 - 5x + 6$

6 Factorise each of the following quadratic expressions in which c is negative.

a $x^2 - 7x - 8$
b $x^2 - 3x - 4$
c $x^2 - 5x - 6$
d $x^2 - 6x - 16$
e $x^2 - 2x - 24$
f $x^2 - 2x - 15$
g $x^2 - x - 12$
h $x^2 - 11x - 12$
i $x^2 - 4x - 12$

Example 11

7 Factorise each of the following quadratic expressions by first taking out a common factor, c.

a $2x^2 + 10x + 8$
b $2x^2 + 22x + 20$
c $3x^2 + 18x + 24$
d $2x^2 + 14x - 60$
e $2x^2 - 14x - 36$
f $4x^2 - 8x + 4$
g $2x^2 + 2x - 12$
h $6x^2 - 30x - 36$
i $5x^2 - 30x + 40$
j $3x^2 - 33x + 90$
k $2x^2 - 6x - 20$
l $3x^2 - 3x - 36$

8 Factorise the following.

a $x^2 + 5x + 6$
b $x^2 - 5x + 6$
c $x^2 + 5x - 6$
d $x^2 - 5x - 6$
e $x^2 + 7x + 12$
f $x^2 - 7x + 12$
g $x^2 - x - 12$
h $x^2 + x - 12$
i $x^2 - 11x - 12$

PROBLEM-SOLVING AND REASONING 9, 11(½) 9(½), 10, 11 9(½), 10, 11(½), 12, 13

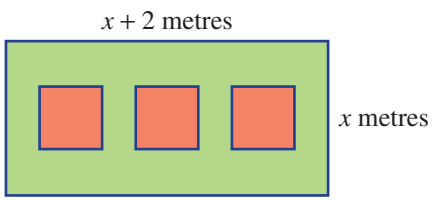
9 Find the missing term in these trinomials if they are to factorise using integers. For example: the missing term in $x^2 + \square + 10$ could be $7x$ because $x^2 + 7x + 10$ factorises to $(x + 5)(x + 2)$ and 5 and 2 are integers. There may be more than one answer in each case.

a $x^2 + \square + 5$
b $x^2 - \square + 9$
c $x^2 - \square - 12$
d $x^2 + \square - 12$
e $x^2 + \square + 18$
f $x^2 - \square + 18$
g $x^2 - \square - 16$
h $x^2 + \square - 25$

10 A backyard, rectangular in area, has a length 2 metres more than its width (x metres). Inside the rectangle are three square paved areas each of area $5\,m^2$ as shown. The remaining area is lawn.

a Find an expression for the:
- **i** total backyard area
- **ii** area of lawn in expanded form
- **iii** area of lawn in factorised form

b Find the area of lawn if:
- **i** $x = 10$
- **ii** $x = 7$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 The expression $x^2 - 6x + 9$ factorises to $(x - 3)(x - 3) = (x - 3)^2$, which is a perfect square. Factorise these perfect squares.

a $x^2 + 8x + 16$ **b** $x^2 + 10x + 25$ **c** $x^2 + 30x + 225$
d $x^2 - 2x + 1$ **e** $x^2 - 14x + 49$ **f** $x^2 - 26x + 169$
g $2x^2 + 4x + 2$ **h** $5x^2 - 30x + 45$ **i** $-3x^2 + 36x - 108$

12 Look at Question **11a**: $x^2 + 8x + 16$

$\div 2$ $\sqrt{\ }$

④ = ④ so $x^2 + 8x + 16 = (x + 4)^2$

Try this with Question **11a–i**.
Now make up 5 perfect squares of your own and factorise them.

13 Sometimes it is not possible to factorise quadratic trinomials using integers. Decide which of the following cannot be factorised using integers.

A $x^2 - x - 56$ **B** $x^2 + 5x - 4$
C $x^2 + 7x - 6$ **D** $x^2 + 3x - 108$
E $x^2 + 3x - 1$ **F** $x^2 + 12x - 53$

ENRICHMENT — — 14

Completing the square

14 It is useful to be able to write a simple quadratic trinomial in the form $(x + b)^2 + c$. This involves adding (and subtracting) a special number to form the first perfect square. This procedure is called completing the square. Here is an example.

$$\left(-\frac{6}{2}\right)^2 = 9$$

$$x^2 - 6x - 8 = \underbrace{x^2 - 6x + 9}\ \underbrace{- 9 - 8}$$
$$= (x - 3)(x - 3) - 17$$
$$= (x - 3)^2 - 17$$

Complete the square for these trinomials.

a $x^2 - 2x - 8$ **b** $x^2 + 4x - 1$
c $x^2 + 10x + 3$ **d** $x^2 - 16x - 3$
e $x^2 + 18x + 7$ **f** $x^2 - 32x - 11$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8G Factorising non-monic quadratic trinomials

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

So far we have factorised quadratic trinomials where the coefficient of x^2 is 1, such as $x^2 - 3x - 40$. These are called monic trinomials. We will now consider non-monic trinomials where the coefficient of x is not equal to 1 and is also not a common factor to all three terms, such as in $6x^2 + x - 15$.

Let's start: How the grouping method works

Consider the trinomial $2x^2 + 9x + 10$.

- First write $2x^2 + 9x + 10 = 2x^2 + 4x + 5x + 10$ then factorise by grouping.
- Note that $9x$ was split to give $4x + 5x$ and the product of $2x^2$ and $10 = 20x^2$. Describe the link between the pair of numbers $\{4, 5\}$ and the pair of numbers $\{2, 10\}$.
- Why was $9x$ split to give $4x + 5x$ and not, say, $3x + 6x$?
- Describe how the $13x$ should be split in $2x^2 + 13x + 15$ so it can be factorised by grouping.
- Now try your method for $2x^2 - 7x - 15$.



Factorisation is a key step in the solution of equations relating to motion, including flight.

Key ideas

- To factorise a trinomial of the form $ax^2 + bx + c$, there are several different methods which can be used. Three methods are shown in this section.
- Method 1 involves grouping in pairs.

For example:

$5x^2 + 13x - 6$	This expression has no common factors.
$= 5x^2 - 2x + 15x - 6$	c is negative, so look for two numbers with different signs.
$= x(5x - 2) + 3(5x - 2)$	Multiply a and c to give -30. Factors of -30 are $(-1, 30)$, $(1, -30)$, $(-2, 15)$, $(2, -15)$ etc.
$\therefore 5x^2 + 13x - 6 = (5x - 2)(x + 3)$	Look for two numbers that multiply to give -30 and add to give $+13$.
or	Those numbers are $(-2, 15)$.
$5x^2 + 13x - 6 = (x + 3)(5x - 2)$	Split the middle term into two terms.
	Factorise the pairs and look for a common factor.
	Take out the common factor. Check the answer by expanding.

$5x^2$ $\quad -6$

$(5x - 2)(x + 3) = 5x^2 + 13x - 6$

$-2x$ $\quad 15x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- Method 2 involves the use of an algebraic fraction. For example:

$5x^2 + 13x - 6$ — This expression has no common factors.

c is negative, so the factors contain different signs.

$= \dfrac{(5x - 2)\ (5x + 15)}{5}$ — Multiply a and c to give -30. Factors of -30 include $(-1, 30)$, $(1, -30)$, $(-2, 15)$, $(2, -15)$ etc.

$= \dfrac{(5x - 2)\ (5^1x + 15^3)}{5^1}$ — Look for two numbers that multiply to give -30 and add to give $+13$.

Those numbers are $(-2, 15)$.

Set up a fraction and use the value of a in three places.

$\dfrac{(5x\ \ )\ (5x\ \ )}{5}$

Insert the two numbers, then try to simplify the fraction.

It should always be possible to simplify the denominator to 1.

It might require two steps.

$\therefore 5x^2 + 13x - 6 = (5x - 2)(x + 3)$ — Check the answer by expanding (same as method 1).

or $5x^2 + 13x - 6 = (x + 3)(5x - 2)$

- Method 3 is often called the 'cross method'. It is explained in the Enrichment section at the end of Exercise 8G.

Example 12 Factorising trinomials of the form $ax^2 + bx + c$

Factorise $2x^2 + 7x + 3$.

SOLUTION

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + x + 6x + 3 \\ &= x(2x + 1) + 3(2x + 1) \\ &= (2x + 1)(x + 3) \end{aligned}$$

EXPLANATION

$a \times c = 2 \times 3 = 6$ then ask what factors of this number (6) add to 7. The answer is 1 and 6, so split $7x = x + 6x$. Then factorise by grouping.

Example 13 Factorising trinomials with negative numbers

Factorise the quadratic trinomials.

a $10x^2 + 9x - 9$

b $6x^2 - 17x + 12$

SOLUTION

a
$$\begin{aligned} 10x^2 + 9x - 9 &= 10x^2 + 15x - 6x - 9 \\ &= 5x(2x + 3) - 3(2x + 3) \\ &= (2x + 3)(5x - 3) \end{aligned}$$

b
$$\begin{aligned} 6x^2 - 17x + 12 &= 6x^2 - 9x - 8x + 12 \\ &= 3x(2x - 3) - 4(2x - 3) \\ &= (2x - 3)(3x - 4) \end{aligned}$$

EXPLANATION

$10 \times (-9) = -90$ so ask what factors of -90 add to give 9. Choose 15 and -6. Then complete the factorisation by grouping.

$6 \times 12 = 72$ so ask what factors of 72 add to give -17. Choose -9 and -8.

Complete a mental check.

$(2x - 3)(3x - 4)$

$-9x$, $-8x$

$-9x - 8x = -17x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 8G

UNDERSTANDING AND FLUENCY	1–4	2, 3–4(½)	3–4(½)

1 List the two numbers that satisfy each part.

- **a** Multiply to give 6 and add to give 5
- **b** Multiply to give 12 and add to give 8
- **c** Multiply to give −10 and add to give 3
- **d** Multiply to give −24 and add to give 5
- **e** Multiply to give 18 and add to give −9
- **f** Multiply to give 35 and add to give −12
- **g** Multiply to give −30 and add to give −7
- **h** Multiply to give −28 and add to give −3

2 Copy and complete.

a $2x^2 + 7x + 5 = 2x^2 + 2x + ____ + 5$
$= 2x(___) + 5(___)$
$= (___)(___)$

b $3x^2 + 8x + 4 = 3x^2 + 6x + ____ + 4$
$= 3x(___) + 2(___)$
$= (___)(___)$

c $2x^2 - 7x + 6 = 2x^2 - 3x - ____ + 6$
$= x(___) - 2(___)$
$= (___)(___)$

d $5x^2 + 9x - 2 = 5x^2 + 10x - ____ - 2$
$= 5x(___) - 1(___)$
$= (___)(___)$

e $4x^2 + 11x + 6 = 4x^2 + ___ + 3x + 6$
$= __(x + 2) + 3(___)$
$= (___)(___)$

f $6x^2 - 7x - 3 = 6x^2 - 9x + ____ - 3$
$= __(___) + 1(___)$
$= (___)(___)$

Example 12

3 Factorise these quadratic trinomials.

- **a** $2x^2 + 9x + 4$
- **b** $3x^2 + 7x + 2$
- **c** $2x^2 + 7x + 6$
- **d** $3x^2 + 8x + 4$
- **e** $5x^2 + 12x + 4$
- **f** $2x^2 + 11x + 12$
- **g** $6x^2 + 13x + 5$
- **h** $4x^2 + 5x + 1$
- **i** $8x^2 + 14x + 5$

4 Factorise these non-monic quadratic trinomials.

- **a** $2x^2 + 13x + 15$
- **b** $2x^2 - 13x + 15$
- **c** $2x^2 + 7x - 15$
- **d** $2x^2 - 7x - 15$
- **e** $2x^2 + 11x + 15$
- **f** $2x^2 + x - 15$
- **g** $2x^2 - 11x + 15$
- **h** $2x^2 - x - 15$
- **i** $4x^2 + 4x - 15$

PROBLEM-SOLVING AND REASONING	5–6(½), 9	5–7(½), 9	5–7(½), 8–10

Example 13

5 Factorise these quadratic trinomials.

- **a** $3x^2 + 2x - 5$
- **b** $5x^2 + 6x - 8$
- **c** $8x^2 + 10x - 3$
- **d** $6x^2 - 13x - 8$
- **e** $10x^2 - 3x - 4$
- **f** $5x^2 - 11x - 12$
- **g** $4x^2 - 16x + 15$
- **h** $2x^2 - 15x + 18$
- **i** $6x^2 - 19x + 10$
- **j** $12x^2 - 13x - 4$
- **k** $4x^2 - 12x + 9$
- **l** $7x^2 + 18x - 9$
- **m** $9x^2 + 44x - 5$
- **n** $3x^2 - 14x + 16$
- **o** $4x^2 - 4x - 15$

6 Factorise these quadratic trinomials.

- **a** $10x^2 + 27x + 11$
- **b** $15x^2 + 14x - 8$
- **c** $20x^2 - 36x + 9$
- **d** $18x^2 - x - 5$
- **e** $25x^2 + 5x - 12$
- **f** $32x^2 - 12x - 5$
- **g** $27x^2 + 6x - 8$
- **h** $33x^2 + 41x + 10$
- **i** $54x^2 - 39x - 5$
- **j** $12x^2 - 32x + 21$
- **k** $75x^2 - 43x + 6$
- **l** $90x^2 + 33x - 8$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Factorise by firstly taking out a common factor.

a $30x^2 - 14x - 4$ **b** $12x^2 + 18x - 30$ **c** $27x^2 - 54x + 15$

d $21x^2 - 77x + 42$ **e** $36x^2 + 36x - 40$ **f** $50x^2 - 35x - 60$

8 Factorise these trinomials. Begin by removing a negative factor and arranging the terms in the form $ax^2 + bx + c$.

a $-2x^2 + 7x - 6$ **b** $-5x^2 - 3x + 8$ **c** $-6x^2 + 13x + 8$

d $18 - 9x - 5x^2$ **e** $16x - 4x^2 - 15$ **f** $14x - 8x^2 - 5$

9 When splitting the $3x$ in $2x^2 + 3x - 20$, you could write:

A $2x^2 + 8x - 5x - 20$ or **B** $2x^2 - 5x + 8x - 20$

a Complete the factorisation using **A**.

b Complete the factorisation using **B**.

c Does the order matter when you split the $3x$?

d Factorise these trinomials twice each. Factorise once by grouping then repeat but reverse the order of the two middle terms in the first line of working.

i $3x^2 + 5x - 12$

ii $5x^2 - 3x - 14$

iii $6x^2 + 5x - 4$

10 Make up five non-monic trinomials with the coefficient of x^2 not equal to 1 that factorise using the above method. Explain your method in finding these trinomials.

ENRICHMENT — — 11

The cross method

11 The cross method is another way to factorise trinomials of the form $ax^2 + bx + c$. It involves finding factors of ax^2 and factors of c then choosing pairs of these factors that add to bx.

For example: Factorise $6x^2 - x - 15$.

Factors of $6x^2$ include $(x, 6x)$ and $(2x, 3x)$.

Factors of -15 include $(15, -1)$, $(-15, 1)$, $(5, -3)$ and $(-5, 3)$.

We arrange a chosen pair of factors vertically then cross-multiply and add to get $-1x$.

x 15
$6x$ -1
$x \times (-1) + 6x \times 15$
$= 89x \neq -1x$

x 5
$6x$ -3
$x \times (-3) + 6x \times 5$
$= 27x \neq -1x$

$2x$ 3 ← $(2x + 3)$
$3x$ -5 ← $(3x - 5)$
$2x \times (-5) + 3x \times 3$
$= -1x$

You will need to continue until a particular combination works. The third cross-product gives a sum of $-1x$ so choose the factors $(2x + 3)$ and $(3x - 5)$ so:

$6x^2 - x - 15 = (2x + 3)(3x - 5)$

Try this method on the trinomials from Questions **4** and **5**.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8H Simplifying algebraic fractions: multiplication and division

Interactive
Widgets
HOTsheets
Walkthrough

With a numerical fraction such as $\frac{6}{9}$, the highest common factor of 6 and 9 is 3, which can be cancelled.

$$\frac{6}{9} = \frac{{}^{1}\cancel{3} \times 2}{\cancel{3}_{1} \times 3} = \frac{2}{3}$$

For algebraic fractions the process is the same. If expressions are in a factorised form, common factors can be easily identified and cancelled.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Correct cancelling

Consider this cancelling attempt:

$$\frac{5x + {}^{1}\cancel{10}}{\cancel{20}_{2}} = \frac{5x + 1}{2}$$

- Substitute $x = 6$ into the left-hand side to evaluate $\frac{5x + 10}{20}$.
- Substitute $x = 6$ into the right-hand side to evaluate $\frac{5x + 1}{2}$.
- What do you notice about the two answers to the above? How can you explain this?
- Decide how you might correctly cancel the expression on the left-hand side. Show your steps and check by substituting a value for x.

Key ideas

- Simplify **algebraic fractions** by factorising and cancelling common factors.

Incorrect

$$\frac{2x + \cancel{4}^{2}}{\cancel{2}^{1}} = 2x + 2$$

Correct

$$\frac{2x + 4}{2} = \frac{{}^{1}\cancel{2}(x + 2)}{\cancel{2}^{1}} = x + 2$$

- To multiply algebraic fractions:
  - factorise expressions where possible
  - cancel if possible
  - multiply the numerators and denominators together.
- To divide algebraic fractions:
  - multiply by the reciprocal of the fraction following the division sign
  - the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$
  - follow the rules for multiplication.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14 Simplifying algebraic fractions

Simplify the following by cancelling.

a $\dfrac{3(x+2)(x-4)}{6(x-4)}$ **b** $\dfrac{20-5x}{8-2x}$ **c** $\dfrac{x^2-4}{x+2}$ **d** $\dfrac{x^2-x-20}{x-5}$

SOLUTION

a $\dfrac{{}^{1}\cancel{3}(x+2)\cancel{(x-4)}^{1}}{{}^{2}\cancel{6}\cancel{(x-4)}^{1}} = \dfrac{x+2}{2}$

b $\dfrac{20-5x}{8-2x} = \dfrac{5\cancel{(4-x)}^{1}}{2\cancel{(4-x)}^{1}}$

$= \dfrac{5}{2} = 2\dfrac{1}{2}$

c $\dfrac{x^2-4}{x+2} = \dfrac{{}^{1}\cancel{(x+2)}(x-2)}{\cancel{(x+2)}_{1}}$

$= x-2$

d $\dfrac{x^2-x-20}{x-5} = \dfrac{{}^{1}\cancel{(x-5)}(x+4)}{\cancel{(x-5)}_{1}}$

$= x+4$

EXPLANATION

Cancel the common factors $(x-4)$ and 3.

Factorise the numerator and denominator, then cancel common factor of $(4-x)$.

Factorise the difference of squares in the numerator then cancel the common factor.

Factorise the quadratic trinomial in the numerator, then cancel the common factor.

Example 15 Multiplying and dividing algebraic fractions

Simplify the following.

a $\dfrac{3(x-1)}{(x+2)} \times \dfrac{4(x+2)}{9(x-1)(x-7)}$ **b** $\dfrac{(x-3)(x+4)}{x(x+7)} \div \dfrac{3(x+4)}{x+7}$ **c** $\dfrac{x^2-4}{25} \times \dfrac{5x+5}{x^2-x-2}$

SOLUTION

a $\dfrac{{}^{1}\cancel{3}\cancel{(x-1)}^{1}}{\cancel{(x+2)}_{1}} \times \dfrac{4\cancel{(x+2)}^{1}}{{}_{3}\cancel{9}\cancel{(x-1)}_{1}(x-7)}$

$= \dfrac{1 \times 4}{1 \times 3(x-7)}$

$= \dfrac{4}{3(x-7)}$

b $\dfrac{(x-3)(x+4)}{x(x+7)} \div \dfrac{3(x+4)}{x+7}$

$= \dfrac{(x-3)\cancel{(x+4)}^{1}}{x\cancel{(x+7)}_{1}} \times \dfrac{\cancel{x+7}^{1}}{3\cancel{(x+4)}^{1}}$

$= \dfrac{x-3}{3x}$

c $\dfrac{x^2-4}{25} \times \dfrac{5x+5}{x^2-x-2}$

$= \dfrac{{}^{1}\cancel{(x-2)}(x+2)}{{}^{5}\cancel{25}} \times \dfrac{{}^{1}\cancel{5}^{1}\cancel{(x+1)}^{1}}{{}_{1}\cancel{(x-2)(x+1)}_{1}}$

$= \dfrac{x+2}{5}$

EXPLANATION

First cancel any factors in the numerator with a common factor in the denominators. Then multiply the numerators and denominators.

Multiply by the reciprocal of the fraction after the division sign.
Cancel common factors and multiply remaining numerators and denominators.

First factorise all the algebraic expressions. Note that x^2-4 is a difference of two squares. Then cancel as normal.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 8H

UNDERSTANDING AND FLUENCY	1–4, 5–8(½)	3, 4–9(½)	5–9(½)

1 Simplify these fractions by cancelling.

a $\frac{5}{15}$ **b** $\frac{24}{16}$ **c** $\frac{5x}{10}$ **d** $\frac{42x}{12}$

e $\frac{24}{8x}$ **f** $\frac{9}{18x}$ **g** $\frac{3(x+1)}{6}$ **h** $\frac{22(x-4)}{11}$

2 Factorise these by taking out common factors.

a $3x + 6$

b $20 - 40x$

c $x^2 - 7x$

d $6x^2 + 24x$

3 Copy and complete.

a $\frac{2x-4}{8} = \frac{2(\quad)}{8}$

$= \frac{\quad}{4}$

b $\frac{12-18x}{2x-3x^2} = \frac{6(\quad)}{x(\quad)}$

$= \frac{6}{x}$

c $\frac{x-1}{x+3} \div \frac{2(x-1)}{(x+3)(x+2)} = \frac{x-1}{x+3} \times \frac{\quad}{\quad}$

$= \frac{x+2}{2}$

d $\frac{x^2-9}{2x-6} = \frac{(\quad)(\quad)}{2(\quad)}$

$= \frac{\quad}{2}$

Example 14a

4 Simplify the following by cancelling.

a $\frac{3(x+2)}{4(x+2)}$ **b** $\frac{x(x-3)}{3x(x-3)}$

c $\frac{20(x+7)}{5(x+7)}$ **d** $\frac{(x+5)(x-5)}{(x+5)}$

e $\frac{6(x-1)(x+3)}{9(x+3)}$ **f** $\frac{8(x-2)}{4(x-2)(x+4)}$

Example 14b

5 Simplify the following by factorising and then cancelling.

a $\frac{5x-5}{5}$ **b** $\frac{4x-12}{10}$ **c** $\frac{2x-4}{3x-6}$ **d** $\frac{12-4x}{6-2x}$

e $\frac{x^2-3x}{x}$ **f** $\frac{4x^2+10x}{5x}$ **g** $\frac{3x+3y}{2x+2y}$ **h** $\frac{4x-8y}{3x-6y}$

Example 14c, d

6 Simplify by first factorising.

a $\frac{x^2-x-6}{x-3}$ **b** $\frac{x^2+8x+16}{x+4}$

c $\frac{x^2-7x+12}{x-4}$ **d** $\frac{x-2}{x^2+x-6}$

e $\frac{x+7}{x^2+5x-14}$ **f** $\frac{x-9}{x^2-19x+90}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 15a

7 Simplify the following by cancelling.

a $\frac{2x(x-4)}{4(x+1)} \times \frac{x+1}{x}$

b $\frac{(x+2)(x-3)}{x-5} \times \frac{x-5}{x+2}$

c $\frac{x-3}{x+2} \times \frac{3(x+4)(x+2)}{x+4}$

d $\frac{2(x+3)(x+4)}{(x+1)(x-5)} \times \frac{x+1}{4(x+3)}$

Example 15b

8 Simplify the following by cancelling.

a $\frac{x(x+1)}{x+3} \div \frac{x+1}{x+3}$

b $\frac{x+3}{x+2} \div \frac{x+3}{2(x-2)}$

c $\frac{x-4}{(x+3)(x+1)} \div \frac{x-4}{4(x+3)}$

d $\frac{4x}{x+2} \div \frac{8x}{x-2}$

e $\frac{3(4x-9)(x+2)}{2(x+6)} \div \frac{9(x+4)(4x-9)}{4(x+2)(x+6)}$

f $\frac{5(2x-3)}{(x+7)} \div \frac{(x+2)(2x-3)}{x+7}$

9 Simplify the following. These expressions involve difference of two squares.

a $\frac{x^2-100}{x+10}$

b $\frac{x^2-49}{x+7}$

c $\frac{x^2-25}{x+5}$

d $\frac{2(x-20)}{x^2-400}$

e $\frac{5(x-6)}{x^2-36}$

f $\frac{3x+27}{x^2-81}$

PROBLEM-SOLVING AND REASONING 10(½), 12(½) | 10, 11(½) | 10–12(½)

Example 15c

10 These expressions involve a combination of trinomials, difference of perfect squares and simple common factors. Simplify by first factorising where possible.

a $\frac{x^2+5x+6}{x+5} \div \frac{x+3}{x^2-25}$

b $\frac{x^2+6x+8}{x^2-9} \div \frac{x+4}{x-3}$

c $\frac{x^2+x-12}{x^2+8x+16} \times \frac{x^2-16}{x^2-8x+16}$

d $\frac{x^2+12x+35}{x^2-25} \times \frac{x^2-10x+25}{x^2+9x+14}$

e $\frac{9x^2-3x}{6x-45x^2}$

f $\frac{x^2-4x}{3x-x^2}$

g $\frac{3x^2-21x+36}{2x^2-32} \times \frac{2x+10}{6x-18}$

h $\frac{2x^2-18x+40}{x^2-x-12} \times \frac{3x+15}{4x^2-100}$

11 The expression $\frac{5-2x}{2x-5}$ can be written in the form $\frac{-1(-5+2x)}{2x-5} = \frac{-1(2x-5)}{2x-5}$, which can be cancelled to -1.

Use this idea to simplify these algebraic fractions.

a $\frac{7-3x}{3x-7}$

b $\frac{4x-1}{1-4x}$

c $\frac{8x+16}{-2-x}$

d $\frac{x+3}{-9-3x}$

e $\frac{5-3x}{18x-30}$

f $\frac{x^2-9}{3-x}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 Just as $\frac{a^2}{2a}$ can be cancelled to $\frac{a}{2}$, $\frac{(a+5)^2}{2(a+5)}$ cancels to $\frac{a+5}{2}$. Use this idea to cancel these fractions.

a $\frac{(a+1)^2}{(a+1)}$

b $\frac{5(a-3)^2}{(a-3)}$

c $\frac{7(x+7)^2}{14(x+7)}$

d $\frac{3(x-1)(x+2)^2}{18(x-1)(x+2)}$

e $\frac{x^2+6x+9}{2x+6}$

f $\frac{11x-22}{x^2-4x+4}$

ENRICHMENT — — 13

All in together

13 Use your knowledge of factorisation and the ideas in Questions **11** and **12** to simplify these algebraic fractions.

a $\frac{2x^2-2x-24}{16-4x}$

b $\frac{x^2-14x+49}{21-3x}$

c $\frac{x^2-16x+64}{64-x^2}$

d $\frac{4-x^2}{x^2+x-6} \times \frac{2x+6}{x^2+4x+4}$

e $\frac{2x^2-18}{x^2-6x+9} \times \frac{6-2x}{x^2+6x+9}$

f $\frac{x^2-2x+1}{4-4x} \div \frac{1-x^2}{3x^2+6x+3}$

g $\frac{4x^2-9}{x^2-5x} \div \frac{6-4x}{15-3x}$

h $\frac{x^2-4x+4}{8-4x} \times \frac{-2}{4-x^2}$

i $\frac{(x+2)^2-4}{(1-x)^2} \times \frac{x^2-2x+1}{3x+12}$

j $\frac{2(x-3)^2-50}{x^2-11x+24} \div \frac{x^2-4}{3-x}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

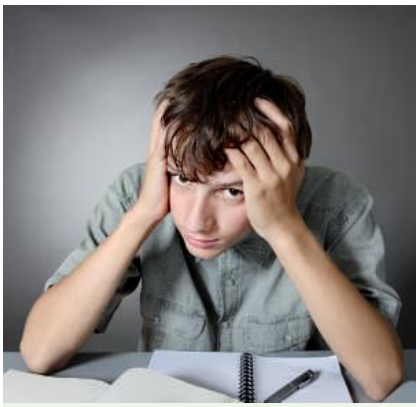
8I Simplifying algebraic fractions: addition and subtraction

Interactive
Widgets
HOTsheets
Walkthrough

The process required for adding or subtracting algebraic fractions is similar to that used for fractions without pronumerals.

To simplify $\frac{2}{3} + \frac{4}{5}$, for example, you would find the lowest common multiple of the denominators (15) then express each fraction with this denominator. Adding the numerators completes the task.

The same process is applied to $\frac{2x}{3} + \frac{4x}{5}$



Although algebraic fractions seem abstract, performing operations on them and simplifying them is essential to many calculations in real-life mathematical problems.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Compare the working

Here is the working for the simplification of the sum of a pair of numerical fractions and the sum of a pair of algebraic fractions.

$$\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} \qquad \frac{2x}{5} + \frac{3x}{4} = \frac{8x}{20} + \frac{15x}{20}$$
$$= \frac{23}{20} \qquad = \frac{23x}{20}$$

- What type of steps were taken to simplify the algebraic fractions that are the same as for the numerical fractions?
- Write down the steps required to add (or subtract) algebraic fractions.

Key ideas

■ To add or subtract algebraic fractions:
- determine the lowest common denominator (LCD)
- express each fraction using the LCD
- add or subtract the numerators.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 16 Adding and subtracting with numerical denominators

Simplify the following.

a $\frac{x}{4} - \frac{2x}{5}$ **b** $\frac{7x}{3} + \frac{x}{6}$ **c** $\frac{x+3}{2} + \frac{x-2}{5}$

SOLUTION

a $\frac{x}{4} - \frac{2x}{5} = \frac{5x}{20} - \frac{8x}{20}$

$= -\frac{3x}{20}$

b $\frac{7x}{3} + \frac{x}{6} = \frac{14x}{6} + \frac{x}{6}$

$= \frac{15x}{6}$

$= \frac{5x}{2}$

c $\frac{x+3}{2} + \frac{x-2}{5} = \frac{5(x+3)}{10} + \frac{2(x-2)}{10}$

$= \frac{5x + 15 + 2x - 4}{10}$

$= \frac{7x + 11}{10}$

EXPLANATION

Determine the LCD of 4 and 5, i.e. 20. Express each fraction as an equivalent fraction with a denominator of 20. $2x \times 4 = 8x$. Then subtract numerators.

Note: the LCD of 3 and 6 is 6 not $3 \times 6 = 18$.

$\frac{15}{6}$ simplifies to $\frac{5}{2}$
Leave it as an improper fraction.

The LCD of 2 and 5 is 10, write as equivalent fractions with denominator 10.
Expand the brackets and simplify the numerator by adding and collecting like terms.

Example 17 Adding and subtracting with algebraic denominators

Simplify the following.

a $\frac{2}{x} - \frac{5}{2x}$ **b** $\frac{2}{x} + \frac{3}{x^2}$

SOLUTION

a $\frac{2}{x} - \frac{5}{2x} = \frac{4}{2x} - \frac{5}{2x}$

$= -\frac{1}{2x}$

b $\frac{2}{x} + \frac{3}{x^2} = \frac{2x}{x^2} + \frac{3}{x^2}$

$= \frac{2x + 3}{x^2}$

EXPLANATION

The LCD of x and $2x$ is $2x$, so rewrite the first fraction in an equivalent form with a demominator also of $2x$.

The LCD of x and x^2 is x^2 so rewrite the first fraction in an equivalent form so its denominator is also x^2, then add numerators.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 8I

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	3, 4, 5–7(½)	5–7(½)

1 Copy and complete the following:

a $\frac{2w}{12} + \frac{3w}{12} = \frac{\quad}{12}$ **b** $\frac{2x+1}{10} + \frac{x+4}{10} = \frac{\quad}{10}$

c $\frac{10x}{11} - \frac{7x}{11} = \frac{\quad}{11}$ **d** $\frac{2}{5} - \frac{x}{5} = \frac{\quad}{5}$

2 Write equivalent fractions by stating the missing expression.

a $\frac{2x}{5} = \frac{\square}{10}$ **b** $\frac{7x}{3} = \frac{\square}{9}$ **c** $\frac{x+1}{4} = \frac{\square(x+1)}{12}$

d $\frac{3x+5}{11} = \frac{\square(3x+5)}{22}$ **e** $\frac{4}{x} = \frac{\square}{2x}$ **f** $\frac{30}{x+1} = \frac{\square}{3(x+1)}$

3 Copy and complete these simplifications.

a $\frac{x}{4} + \frac{2x}{3} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{12}$ **b** $\frac{5x}{7} - \frac{2x}{5} = \frac{\square}{35} - \frac{\square}{35} = \frac{\square}{35}$

c $\frac{x+1}{2} + \frac{2x+3}{4} = \frac{\square(x+1)}{4} + \frac{2x+3}{4} = \frac{\square + 2x + 3}{4} = \frac{\square}{4}$

4 Write down the LCD for these pairs of fractions.

a $\frac{x}{3}, \frac{2x}{5}$ **b** $\frac{3x}{7}, \frac{x}{2}$ **c** $\frac{-5x}{4}, \frac{x}{8}$ **d** $\frac{2x}{3}, \frac{-5x}{6}$ **e** $\frac{7x}{10}, \frac{-3x}{5}$

Example 16a,b

5 Simplify the following.

a $\frac{x}{7} + \frac{x}{2}$ **b** $\frac{x}{3} + \frac{x}{15}$ **c** $\frac{x}{4} - \frac{x}{8}$ **d** $\frac{x}{9} + \frac{x}{5}$

e $\frac{y}{7} - \frac{y}{8}$ **f** $\frac{a}{2} + \frac{a}{11}$ **g** $\frac{b}{3} - \frac{b}{9}$ **h** $\frac{m}{3} - \frac{m}{6}$

i $\frac{m}{6} + \frac{3m}{4}$ **j** $\frac{a}{4} + \frac{2a}{7}$ **k** $\frac{2x}{5} + \frac{x}{10}$ **l** $\frac{p}{9} - \frac{3p}{7}$

m $\frac{b}{2} - \frac{7b}{9}$ **n** $\frac{9y}{8} + \frac{2y}{5}$ **o** $\frac{4x}{7} - \frac{x}{5}$ **p** $\frac{3x}{4} - \frac{x}{3}$

Example 16c

6 Simplify the following.

a $\frac{x+1}{2} + \frac{x+3}{5}$ **b** $\frac{x+3}{3} + \frac{x-4}{4}$ **c** $\frac{a-2}{7} + \frac{a-5}{8}$

d $\frac{y+4}{5} + \frac{y-3}{6}$ **e** $\frac{m-4}{8} + \frac{m+6}{5}$ **f** $\frac{x-2}{12} + \frac{x-3}{8}$

g $\frac{2b-3}{6} + \frac{b+2}{8}$ **h** $\frac{3x+8}{6} + \frac{2x-4}{3}$ **i** $\frac{2y-5}{7} + \frac{3y+2}{14}$

j $\frac{2t-1}{8} + \frac{t-2}{16}$ **k** $\frac{4-x}{3} + \frac{2-x}{7}$ **l** $\frac{2m-1}{4} + \frac{m-3}{6}$

Example 17a

7 Simplify the following.

a $\frac{3}{x} + \frac{5}{2x}$ **b** $\frac{7}{3x} - \frac{2}{x}$ **c** $\frac{7}{4x} - \frac{5}{2x}$ **d** $\frac{4}{3x} + \frac{2}{9x}$

e $\frac{3}{4x} - \frac{2}{5x}$ **f** $\frac{2}{3x} + \frac{1}{5x}$ **g** $\frac{-3}{4x} - \frac{7}{x}$ **h** $\frac{-5}{3x} - \frac{3}{4x}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING | 8(½), 11 | 8–9(½), 11, 12 | 8–10(½), 11, 12

Example 17b

8 Simplify the following.

a $\frac{3}{x}+\frac{2}{x^2}$ **b** $\frac{5}{x^2}+\frac{4}{x}$ **c** $\frac{7}{x}+\frac{3}{x^2}$ **d** $\frac{4}{x}-\frac{5}{x^2}$

e $\frac{3}{x^2}-\frac{8}{x}$ **f** $-\frac{4}{x^2}+\frac{1}{x}$ **g** $\frac{3}{x}-\frac{7}{2x^2}$ **h** $-\frac{2}{3x}+\frac{3}{x^2}$

9 Simplify these mixed algebraic fractions.

a $\frac{2}{x}+\frac{x}{4}$ **b** $\frac{-5}{x}+\frac{x}{2}$ **c** $\frac{-2}{x}-\frac{4x}{3}$ **d** $\frac{3}{2x}-\frac{5x}{4}$

e $\frac{3x}{4}-\frac{5}{6x}$ **f** $\frac{1}{3x}-\frac{x}{9}$ **g** $-\frac{2}{5x}+\frac{3x}{2}$ **h** $-\frac{5}{4x}-\frac{3x}{10}$

10 Find the missing algebraic fraction. The fraction should be in simplest form.

a $\frac{x}{2}+\frac{\square}{\square}=\frac{5x}{6}$ **b** $\frac{\square}{\square}+\frac{x}{4}=\frac{3x}{8}$ **c** $\frac{2x}{5}+\frac{\square}{\square}=\frac{9x}{10}$

d $\frac{2x}{3}-\frac{\square}{\square}=\frac{7x}{15}$ **e** $\frac{\square}{\square}-\frac{x}{3}=\frac{5x}{9}$ **f** $\frac{2x}{3}-\frac{\square}{\square}=\frac{5x}{12}$

11 Find and describe the error in each set of working. Then give the correct answer.

a $\frac{4x}{5}-\frac{x}{3}=\frac{3x}{2}$

b $\frac{x+1}{5}+\frac{x}{2}=\frac{2x+1}{10}+\frac{5x}{10}$

$=\frac{7x+1}{10}$

c $\frac{5x}{3}+\frac{x-1}{2}=\frac{10x}{6}+\frac{3x-1}{6}$

$=\frac{13x-1}{6}$

d $\frac{2}{x}-\frac{3}{x^2}=\frac{2}{x^2}-\frac{3}{x^2}$

$=\frac{-1}{x^2}$

12 A student thinks that the LCD to use when simplifying $\frac{x+1}{2}+\frac{2x-1}{4}$ is 8.

a Complete the simplification using a common denominator of 8.

b Now complete the simplification using the actual LCD of 4.

c How does your working for parts **a** and **b** compare? Which method is preferable and why?

ENRICHMENT | — | — | 13

More than two fractions!

13 Simplify by finding the LCD.

a $\frac{2x}{5}-\frac{3x}{2}-\frac{x}{3}$ **b** $\frac{x}{4}-\frac{2x}{3}+\frac{5x}{6}$ **c** $\frac{5x}{8}-\frac{5x}{6}+\frac{3x}{4}$

d $\frac{x+1}{4}+\frac{2x-1}{3}-\frac{x}{5}$ **e** $\frac{2x-1}{3}-\frac{2x}{7}+\frac{x-3}{6}$ **f** $\frac{1-2x}{5}-\frac{3x}{8}+\frac{3x+1}{2}$

g $\frac{2}{3x}+\frac{5}{x}-\frac{1}{x}$ **h** $-\frac{1}{2x}+\frac{2}{x}-\frac{4}{3x}$ **i** $-\frac{4}{5x}-\frac{1}{2x}+\frac{3}{4x}$

j $\frac{4}{x^2}+\frac{3}{2x}-\frac{5}{3x}$ **k** $\frac{5}{x}-\frac{3}{2x^2}-\frac{5}{7x}$ **l** $\frac{2}{x^2}-\frac{4}{9x}-\frac{5}{3x^2}$

m $\frac{2}{x}+\frac{x}{5}-\frac{x}{3}$ **n** $\frac{3x}{2}-\frac{1}{2x}+\frac{x}{3}$ **o** $-\frac{4x}{9}+\frac{2}{5x}+\frac{2x}{5}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8J Further addition and subtraction of algebraic fractions

Stage

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

More complex addition and subtraction of algebraic fractions involves expressions such as:

$\frac{2x-1}{3} - \frac{x+4}{4}$ and $\frac{2}{x-3} - \frac{5}{(x-3)^2}$.

In such examples, take care at each step in the working to avoid common errors.

HOTsheets

Let's start: Three critical errors

Walkthrough

The following simplification of algebraic fractions has three critical errors.
Can you find them?

$$\frac{2x+1}{3} - \frac{x+2}{2} = \frac{2x+1}{6} - \frac{3(x+2)}{6}$$
$$= \frac{2x+1-3x+6}{6}$$
$$= \frac{x+7}{6}$$

The correct answer is $\frac{x-4}{6}$.

Fix the solution to produce the correct answer.

Key ideas

- When combining algebraic fractions that involve subtraction signs, recall that the:
  - product of two numbers of opposite sign is a negative number
  - product of two negative numbers is a positive number.

  For example:

$$\frac{2(x-1)}{6} - \frac{3(x+2)}{6} = \frac{2x-2-3x-6}{6}$$

  and

$$\frac{5(1-x)}{8} - \frac{2(x-1)}{8} = \frac{5-5x-2x+2}{8}$$

- A common denominator can be a product of two binomial terms.

  For example:

$$\frac{2}{x+3} + \frac{3}{x-1} = \frac{2(x-1)}{(x+3)(x-1)} + \frac{3(x+3)}{(x+3)(x-1)}$$
$$= \frac{2x-2+3x+9}{(x+3)(x-1)}$$
$$= \frac{5x+7}{(x+3)(x-1)}$$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18 Simplifying with more complex numerators

Simplify the following.

a $\frac{x-1}{3} - \frac{x+4}{5}$ **b** $\frac{2x-3}{6} - \frac{3-x}{5}$

SOLUTION

a
$$\frac{x-1}{3} - \frac{x+4}{5} = \frac{5(x-1)}{15} - \frac{3(x+4)}{15}$$
$$= \frac{5x-5-3x-12}{15}$$
$$= \frac{2x-17}{15}$$

b
$$\frac{2x-3}{6} - \frac{3-x}{5} = \frac{5(2x-3)}{30} - \frac{6(3-x)}{30}$$
$$= \frac{10x-15-18+6x}{30}$$
$$= \frac{16x-33}{30}$$

EXPLANATION

The LCD of 3 and 5 is 15. Insert brackets around each numerator.
Note: $-3(x+4) = -3x-12$ not $-3x+12$.

Determine the LCD and express as equivalent fractions. Insert brackets.
Expand the brackets, recall $-6 \times (-x) = 6x$ and then simplify the numerator.

Example 19 Simplifying with more complex denominators

Simplify the following.

a $\frac{4}{x+1} + \frac{3}{x-2}$ **b** $\frac{3}{(x-1)^2} - \frac{2}{x-1}$

SOLUTION

a
$$\frac{4}{x+1} + \frac{3}{x-2} = \frac{4(x-2)}{(x+1)(x-2)} + \frac{3(x+1)}{(x+1)(x-2)}$$
$$= \frac{4x-8+3x+3}{(x+1)(x-2)}$$
$$= \frac{7x-5}{(x+1)(x-2)}$$

b
$$\frac{3}{(x-1)^2} - \frac{2}{x-1} = \frac{3}{(x-1)^2} - \frac{2(x-1)}{(x-1)^2}$$
$$= \frac{3-2x+2}{(x-1)^2}$$
$$= \frac{5-2x}{(x-1)^2}$$

EXPLANATION

The LCD for $(x+1)$ and $(x-2)$ is $(x+1)(x-2)$. Rewrite each fraction with this denominator then add numerators.

Just as the LCD of 3^2 and 3 is 3^2, the LCD of $(x-1)^2$ and $x-1$ is $(x-1)^2$.
Remember that $-2(x-1) = -2x+2$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 8J

UNDERSTANDING AND FLUENCY 1, 2, 3–5(½) 2–5(½) 3–5(½)

1 Expand the following.

a $-2(x+3)$ **b** $-5(x+1)$ **c** $-7(2+3x)$

d $-3(x-1)$ **e** $-10(3-2x)$ **f** $-16(1-4x)$

2 Write the LCD for these pairs of fractions.

a $\frac{1}{3}, \frac{5}{9}$ **b** $\frac{3}{16}, \frac{1}{8}$ **c** $\frac{3}{x}, \frac{5}{x^2}$ **d** $-\frac{5}{2x}, \frac{3}{2}$

e $\frac{3}{x-1}, \frac{2}{x+1}$ **f** $\frac{7}{x-2}, \frac{3}{x+3}$ **g** $\frac{4}{2x-1}, \frac{-1}{x-4}$ **h** $\frac{5}{(x+1)^2}, \frac{4}{x+1}$

Example 18a **3** Simplify the following.

a $\frac{x+3}{4} - \frac{x+2}{3}$ **b** $\frac{x-1}{3} - \frac{x+3}{5}$ **c** $\frac{x-4}{3} - \frac{x+1}{6}$

d $\frac{3-x}{5} - \frac{x+4}{2}$ **e** $\frac{5x-1}{4} - \frac{2+x}{8}$ **f** $\frac{3x+2}{14} - \frac{x+4}{4}$

g $\frac{1+3x}{4} - \frac{2x+3}{6}$ **h** $\frac{2-x}{5} - \frac{3x+1}{3}$ **i** $\frac{2x-3}{6} - \frac{4+x}{15}$

Example 18b **4** Simplify the following.

a $\frac{x+5}{3} - \frac{x-1}{2}$ **b** $\frac{x-4}{5} - \frac{x-6}{7}$ **c** $\frac{3x-7}{4} - \frac{x-1}{2}$

d $\frac{5x-9}{7} - \frac{2-x}{3}$ **e** $\frac{3x+2}{4} - \frac{5-x}{10}$ **f** $\frac{9-4x}{6} - \frac{2-x}{8}$

g $\frac{4x+3}{3} - \frac{5-2x}{9}$ **h** $\frac{2x-1}{4} - \frac{1-3x}{14}$ **i** $\frac{3x-2}{8} - \frac{4x-3}{7}$

Example 19a **5** Simplify the following.

a $\frac{3}{x-1} + \frac{4}{x+1}$ **b** $\frac{5}{x+4} + \frac{2}{x-3}$ **c** $\frac{3}{x-2} + \frac{4}{x+3}$

d $\frac{3}{x-4} + \frac{2}{x+7}$ **e** $\frac{7}{x+2} - \frac{3}{x+3}$ **f** $\frac{3}{x+4} - \frac{2}{x-6}$

g $\frac{-1}{x+5} + \frac{2}{x+1}$ **h** $\frac{-2}{x-3} - \frac{4}{x-2}$ **i** $\frac{3}{x-5} - \frac{5}{x-6}$

PROBLEM-SOLVING AND REASONING 6(½), 8 6–7(½), 8, 9(½) 6–7(½), 9, 10(½)

Example 19b **6** Simplify the following.

a $\frac{4}{(x+1)^2} - \frac{3}{x+1}$ **b** $\frac{2}{(x+3)^2} - \frac{4}{x+3}$ **c** $\frac{3}{x-2} + \frac{4}{(x-2)^2}$

d $\frac{-2}{x-5} + \frac{8}{(x-5)^2}$ **e** $\frac{-1}{x-6} + \frac{3}{(x-6)^2}$ **f** $\frac{2}{(x-4)^2} - \frac{3}{x-4}$

g $\frac{5}{(2x+1)^2} + \frac{2}{2x+1}$ **h** $\frac{9}{(3x+2)^2} - \frac{4}{3x+2}$ **i** $\frac{4}{(1-4x)^2} - \frac{5}{1-4x}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Simplify the following.

a $\frac{3x}{(x-1)^2} + \frac{2}{x-1}$ **b** $\frac{3x+2}{3x} + \frac{7}{12}$ **c** $\frac{2x-1}{4} + \frac{2-3x}{10x}$

d $\frac{2x}{x-5} - \frac{x}{x+1}$ **e** $\frac{3}{4-x} - \frac{2x}{x-1}$ **f** $\frac{5x+1}{(x-3)^2} + \frac{x}{x-3}$

g $\frac{3x-7}{(x-2)^2} - \frac{5}{x-2}$ **h** $\frac{-7x}{2x+1} + \frac{3x}{x+2}$ **i** $\frac{x}{x+1} - \frac{5x+1}{(x+1)^2}$

8 One of the most common errors made when subtracting algebraic fractions is hidden in the working shown here.

$$\begin{aligned}\frac{7x}{2} - \frac{x-2}{5} &= \frac{35x}{10} - \frac{2(x-2)}{10} \\ &= \frac{35x - 2x - 4}{10} \\ &= \frac{33x - 4}{10}\end{aligned}$$

a What is the error and in which step is it made?

b By correcting the error how does the answer change?

9 Simplify the following.

a $\frac{1}{(x+3)(x+4)} + \frac{2}{(x+4)(x+5)}$ **b** $\frac{3}{(x+1)(x+2)} - \frac{5}{(x+1)(x+4)}$

c $\frac{4}{(x-1)(x-3)} - \frac{6}{(x-1)(2-x)}$ **d** $\frac{5x}{(x+1)(x-5)} - \frac{2}{x-5}$

e $\frac{3}{x-4} + \frac{8x}{(x-4)(3-2x)}$ **f** $\frac{3x}{(x+4)(2x-1)} - \frac{x}{(x+4)(3x+2)}$

10 Use the fact that $a - b = -1(b - a)$ to help simplify these.

a $\frac{3}{1-x} - \frac{2}{x-1}$ **b** $\frac{4x}{5-x} + \frac{3}{x-5}$ **c** $\frac{2}{7x-3} - \frac{7}{3-7x}$

d $\frac{1}{4-3x} + \frac{2x}{3x-4}$ **e** $\frac{-3x}{5-3x} - \frac{5}{3x-5}$ **f** $\frac{4}{x-6} + \frac{4}{6-x}$

ENRICHMENT — — 11

Factorise first

11 Factorising a denominator before further simplification is a useful step. Simplify these by first factorising the denominators if possible.

a $\frac{3}{x+2} + \frac{5}{2x+4}$ **b** $\frac{7}{3x-3} - \frac{2}{x-1}$

c $\frac{3}{8x-4} - \frac{5}{1-2x}$ **d** $\frac{4}{x^2-9} - \frac{3}{x+3}$

e $\frac{5}{2x+4} + \frac{2}{x^2-4}$ **f** $\frac{10}{3x-4} - \frac{7}{9x^2-16}$

g $\frac{7}{x^2+7x+12} + \frac{2}{x^2-2x-15}$ **h** $\frac{3}{(x+1)^2-4} - \frac{2}{x^2+6x+9}$

i $\frac{3}{x^2-7x+10} - \frac{2}{10-5x}$ **j** $\frac{1}{x^2+x} - \frac{1}{x^2-x}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8K Equations involving algebraic fractions

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

For equations with more than one fraction it is often best to try to simplify the equation by dealing with all the denominators at once. This involves finding and multiplying both sides by the lowest common denominator.

Let's start: Why use the LCD?

For this equation follow each instruction.

$$\frac{x+1}{3} + \frac{x}{4} = 1$$

- Multiply every term in the equation by 3. What effect does this have on the fractions on the left-hand side?
- Starting with the original equation, multiply every term in the equation by 4. What effect does this have on the fractions on the left-hand side?
- Starting with the original equation, multiply every term in the equation by 12 and simplify.

Which instruction above does the best job in simplifying the algebraic fractions? Why?

Key ideas

- For equations with more than one fraction multiply both sides by the **lowest common denominator (LCD)**.
  - Multiply every term on both sides, not just the fractions.
  - Simplify the fractions and solve the equation using the methods learned earlier.
- Alternatively, convert all the expressions in the equation to fractions with common denominators, then equate the numerators.

Example 20 Solving equations involving algebraic fractions

Solve each of the following equations.

a $\frac{2x}{3} + \frac{x}{2} = 7$

b $\frac{x-2}{5} - \frac{x-1}{3} = 1$

c $\frac{5}{2x} - \frac{4}{3x} = 2$

d $\frac{3}{x+1} = \frac{2}{x+4}$

SOLUTION

a $\frac{2x}{3} + \frac{x}{2} = 7$

$\frac{2x}{3_1} \times 6^2 + \frac{x}{2_1} \times 6^3 = 7 \times 6$

$4x + 3x = 42$

$7x = 42$

$x = 6$

EXPLANATION

Multiply each term by the LCD (LCD of 3 and 2 is 6) and cancel.

Simplify and solve for x.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

OR

$\frac{2x}{3} + \frac{x}{2} = 7$ — The LCD is 6.

$\frac{4x}{6} + \frac{3x}{6} = \frac{42}{6}$ — Equate the numerators (i.e. ignore the common denominators).

$7x = 42$

$x = 6$

b

$\frac{x-2}{5} - \frac{x-1}{3} = 1$

$\frac{15^{3}(x-2)}{5_{1}} - \frac{15^{5}(x-1)}{3_{1}} = 15 \times 1$ — Multiply each term on both sides by 15 (LCD of 3 and 5 is 15) and cancel.

$3(x-2) - 5(x-1) = 15$ — Expand the brackets and simplify by combining like terms.

$3x - 6 - 5x + 5 = 15$

Note: $-5(x-1) = -5x + 5$ not $-5x - 5$.

$-2x - 1 = 15$

$-2x = 16$

$x = -8$

OR

$\frac{x-2}{5} - \frac{x-1}{3} = 1$ — LCD is 15.

$\frac{3(x-2)}{15} - \frac{5(x-1)}{15} = \frac{15}{15}$

$3x - 6 - 5x + 5 = 15$ — Equate the numerators.

$-2x - 1 = 15$

$-2x = 16$

$x = -8$

c

$\frac{5}{2x} - \frac{4}{3x} = 2$ — LCD of $2x$ and $3x$ is $6x$.

$\frac{5}{2x_{1}} \times 6x^{3} - \frac{4}{3x_{1}} \times 6x^{2} = 2 \times 6x$ — Multiply each term by $6x$. Cancel and simplify.

$15 - 8 = 12x$ — Solve for x, leaving the answer in fraction form.

$7 = 12x$

$\frac{7}{12} = x$

$x = \frac{7}{12}$

OR

$\frac{5}{2x} - \frac{4}{3x} = 2$ — LCD is $6x$.

$\frac{15}{6x} - \frac{8}{6x} = \frac{12x}{6x}$

$7 = 12x$ — Equate the numerators.

$12x = 7$

$x = \frac{7}{12}$

Continued on the next page

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Continued from previous page

d
$$\frac{3}{x+1} = \frac{2}{x+4}$$
$$\frac{3\cancel{(x+1)}(x+4)}{\cancel{x+1}} = \frac{2(x+1)\cancel{(x+4)}}{\cancel{x+4}}$$

Multiply each term by the common denominator $(x+1)(x+4)$.

$$3(x+4) = 2(x+1)$$
$$3x + 12 = 2x + 2$$
$$x + 12 = 2$$
$$x = -10$$

Expand the brackets.
Subtract $2x$ from both sides to gather x terms on one side then subtract 12 from both sides.

OR

$$\frac{3}{x+1} = \frac{2}{x+4}$$

The LCD is $(x+1)(x+4)$.

$$\frac{3(x+4)}{(x+1)(x+4)} = \frac{2(x+1)}{(x+1)(x+4)}$$

Equate the numerators.

$$3(x+4) = 2(x+1)$$
$$3x + 12 = 2x + 2$$
$$x + 12 = 2$$
$$x = -10$$

Exercise 8K

UNDERSTANDING AND FLUENCY 1–5(½) 2–7(½) 3–7(½)

1 Write down the lowest common denominator of all the fractions in these equations.

a $\frac{x}{3} - \frac{2x}{5} = 1$ **b** $\frac{x}{2} - \frac{3x}{4} = 3$ **c** $\frac{x+1}{3} - \frac{x}{6} = 5$

d $\frac{2x-1}{7} + \frac{5x+2}{4} = 6$ **e** $\frac{x}{3} + \frac{x}{2} = \frac{1}{5}$ **f** $\frac{x-1}{4} - \frac{3x}{8} = \frac{1}{8}$

2 Simplify the fractions by cancelling.

a $\frac{12x}{3}$ **b** $\frac{21x}{7}$ **c** $\frac{4(x+3)}{2}$

d $\frac{5(2x+5)}{5}$ **e** $\frac{15x}{5x}$ **f** $\frac{-7(x+1)(x+2)}{7(x+1)}$

g $\frac{36(x-7)(x-1)}{9(x-7)}$ **h** $\frac{18(3-2x)(1-x)}{9(3-2x)}$ **i** $\frac{-8(2-3x)(2x-1)}{-8(2-3x)}$

Example 20a

3 Solve each of the following equations.

a $\frac{x}{2} + \frac{x}{5} = 7$ **b** $\frac{x}{2} + \frac{x}{3} = 10$ **c** $\frac{y}{3} + \frac{y}{4} = 14$

d $\frac{x}{2} - \frac{3x}{5} = -1$ **e** $\frac{5m}{3} - \frac{m}{2} = 1$ **f** $\frac{3a}{5} - \frac{a}{3} = 2$

g $\frac{3x}{4} - \frac{5x}{2} = 14$ **h** $\frac{8a}{3} - \frac{2a}{5} = 34$ **i** $\frac{7b}{2} + \frac{b}{4} = 15$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 20b

4 Solve each of the following equations.

a $\frac{x-1}{2}+\frac{x+2}{3}=11$ **b** $\frac{b+3}{2}+\frac{b-4}{3}=1$ **c** $\frac{n+2}{3}+\frac{n-2}{2}=1$

d $\frac{a+1}{5}-\frac{a+1}{6}=2$ **e** $\frac{x+5}{2}-\frac{x-1}{4}=3$ **f** $\frac{x+3}{2}-\frac{x+1}{3}=2$

g $\frac{m+4}{3}-\frac{m-4}{4}=3$ **h** $\frac{2a-8}{2}+\frac{a+7}{6}=1$ **i** $\frac{2y-1}{4}-\frac{y-2}{6}=-1$

5 Solve each of the following equations.

a $\frac{x+1}{2}=\frac{x}{3}$ **b** $\frac{x-2}{3}=\frac{x}{2}$ **c** $\frac{n+3}{4}=\frac{n-1}{2}$

d $\frac{a+2}{3}=\frac{a+1}{2}$ **e** $\frac{3+y}{2}=\frac{2-y}{3}$ **f** $\frac{2m+4}{4}=\frac{m+6}{3}$

Example 20c

6 Solve each of the following equations.

a $\frac{3}{4x}-\frac{1}{2x}=4$ **b** $\frac{2}{3x}-\frac{1}{2x}=2$ **c** $\frac{4}{2m}-\frac{2}{5m}=3$

d $\frac{1}{2x}-\frac{1}{4x}=9$ **e** $\frac{1}{2b}+\frac{1}{b}=2$ **f** $\frac{1}{2y}+\frac{1}{3y}=4$

g $\frac{1}{3x}+\frac{1}{2x}=2$ **h** $\frac{2}{3x}-\frac{1}{x}=2$ **i** $\frac{7}{2a}-\frac{2}{3a}=1$

Example 20d

7 Solve each of the following equations.

a $\frac{3}{x+1}=\frac{1}{x+2}$ **b** $\frac{2}{x+3}=\frac{3}{x+2}$ **c** $\frac{2}{x+5}=\frac{3}{x-2}$

d $\frac{1}{x-3}=\frac{1}{2x+1}$ **e** $\frac{2}{x-1}=\frac{1}{2x+1}$ **f** $\frac{1}{x-2}=\frac{2}{3x+2}$

PROBLEM-SOLVING AND REASONING 8, 11, 12 | 8, 9(½), 11, 12 | 9(½), 10, 12, 13

8 Half of a number (x) plus one third of twice the same number is equal to 4.

a Write an equation describing the situation.

b Solve the equation to find the number.

9 Use your combined knowledge of all the methods learned earlier to solve these equations with algebraic fractions.

a $\frac{2x+3}{1-x}=4$ **b** $\frac{5x+2}{x+2}=3$ **c** $\frac{3x-2}{x-1}=2$

d $\frac{2x}{3}+\frac{x-1}{4}=2x-1$ **e** $\frac{3}{x^2}-\frac{2}{x}=\frac{5}{x}$ **f** $\frac{1-3x}{x^2}+\frac{3}{2x}=\frac{4}{x}$

g $\frac{x-1}{2}+\frac{3x-2}{4}=\frac{2x}{3}$ **h** $\frac{4x+1}{3}-\frac{x-3}{6}=\frac{x+5}{6}$ **i** $\frac{1}{x+2}-\frac{2}{x-3}=\frac{5}{(x+2)(x-3)}$

10 Molly and Billy each have the same number of computer games (x computer games each). Hazel takes one third of Molly's computer games and a quarter of Billy's computer games to give her a total of 77 computer games.

a Write an equation describing the total number of computer games for Hazel.

b Solve the equation to find how many computer games Molly and Billy each had.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 A common error when solving equations with algebraic fractions is made in this working. Find the error and explain how to avoid it.

$$\frac{3x-1}{4}+2x=\frac{x}{3} \qquad (\text{LCD}=12)$$
$$\frac{12(3x-1)}{4}+2x=\frac{12x}{3}$$
$$3(3x-1)+2x=4x$$
$$9x-3+2x=4x$$
$$7x=3$$
$$x=\frac{3}{7}$$

12 Another common error is made in this working. Find and explain how to avoid this error.

$$\frac{x}{2}-\frac{2x-1}{3}=1 \qquad (\text{LCD}=6)$$
$$\frac{6x}{2}-\frac{6(2x-1)}{3}=6$$
$$3x-2(2x-1)=6$$
$$3x-4x-2=6$$
$$-x=8$$
$$x=-8$$

13 A similar technique to that used in Questions **11** and **12** can be used to solve equations with decimals. Here is an example.

$0.8x-1.2=2.5$

$8x-12=25$ Multiply both sides by 10 to remove all decimals.

$8x=37$

$x=\frac{37}{8}$

Solve these decimal equations using the same idea. For parts **d**–**f** you will need to multiply by 100.

a $0.4x+1.4=3.2$ **b** $0.3x-1.3=0.4$ **c** $0.5-0.2x=0.2$

d $1.31x-1.8=2.13$ **e** $0.24x+0.1=3.7$ **f** $2-3.25x=8.5$

ENRICHMENT — — 14

Literal equations

14 Solve each of the following equations for x in terms of the other pronumerals.

Hint: you may need to use factorisation.

a $\frac{x}{a}-\frac{x}{2a}=b$ **b** $\frac{ax}{b}-\frac{cx}{2}=d$ **c** $\frac{x-a}{b}=\frac{x}{c}$

d $\frac{x+a}{b}=\frac{d+e}{c}$ **e** $\frac{ax+b}{4}=\frac{x+c}{3}$ **f** $\frac{x+a}{3b}+\frac{x-a}{2b}=1$

g $\frac{2a-b}{a}+\frac{a}{x}=a$ **h** $\frac{1}{a}-\frac{1}{x}=\frac{1}{c}$ **i** $\frac{a}{x}=\frac{b}{c}$

j $\frac{a}{x}+b=\frac{c}{x}$ **k** $\frac{ax-b}{x-b}=c$ **l** $\frac{cx+b}{x+a}=d$

m $\frac{2a+x}{a}=b$ **n** $\frac{1}{x-a}=\frac{1}{ax+b}$ **o** $\frac{a}{b}-\frac{a}{a+x}=1$

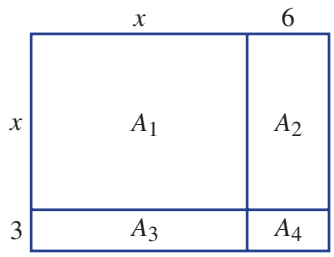
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Expanding quadratics using area

Consider the expansion of the quadratic $(x + 3)(x + 6)$. This can be represented by finding the area of the diagram shown.

Total area $= A_1 + A_2 + A_3 + A_4$
$= x^2 + 6x + 3x + 18$

Therefore:
$(x + 3)(x + 6) = x^2 + 9x + 18$



Expanding with positive signs

a Draw a diagram and calculate the area to determine the expansion of the following quadratics.

i $(x + 4)(x + 5)$ **ii** $(x + 7)(x + 8)$

iii $(x + 3)^2$ **iv** $(x + 5)^2$

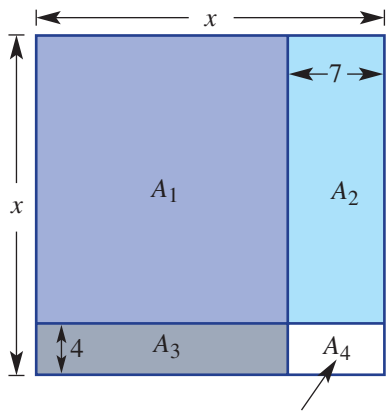
b Using the same technique, establish the rule for expanding $(a + b)^2$.

Expanding with negative signs

Consider the expansion of $(x - 4)(x - 7)$.

Area required = total area $- (A_2 + A_3 + A_4)$
$= x^2 - [(A_2 + A_4) + (A_3 + A_4) - A_4]$
$= x^2 - (7x + 4x - 28)$
$= x^2 - 11x + 28$

Therefore:
$(x - 4)(x - 7) = x^2 - 11x + 28$



a Draw a diagram and calculate the area to determine the expansion of the following quadratics.

i $(x - 3)(x - 5)$

ii $(x - 6)(x - 4)$

iii $(x - 4)^2$

iv $(x - 2)^2$

b Using the same technique, establish the rule for expanding $(a - b)^2$.

Difference of two squares

Using a diagram to represent $(a - b)(a + b)$, determine the appropriate area and establish a rule for the expansion of $(a - b)(a + b)$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Numerical applications of perfect squares

The expansion and factorisation of perfect squares and difference of perfect squares can be applied to the mental calculation of some numerical problems.

Evaluating a perfect square

The perfect square 32^2 can be evaluated using $(a + b)^2 = a^2 + 2ab + b^2$.

$32^2 = (30 + 2)^2$ (Let $a = 30$, $b = 2$)
$= 30^2 + 2(30)(2) + 2^2$
$= 900 + 120 + 4$
$= 1024$

a Use the same technique to evaluate these perfect squares.

i 22^2 **ii** 21^2 **iii** 33^2 **iv** 51^2
v 1.2^2 **vi** 3.2^2 **vii** 6.1^2 **viii** 9.01^2

Similarly, the perfect square 29^2 can be evaluated using $(a - b)^2 = a^2 - 2ab + b^2$.

$29^2 = (30 - 1)^2$ (Let $a = 30$, $b = 1$)
$= 30^2 - 2(30)(1) + 1^2$
$= 900 - 60 + 1$
$= 841$

b Use the same technique to evaluate these perfect squares.

i 19^2 **ii** 39^2 **iii** 98^2 **iv** 87^2
v 1.9^2 **vi** 4.7^2 **vii** 8.8^2 **viii** 3.96^2

Evaluating the difference of perfect squares

The difference of perfect squares $14^2 - 9^2$ can be evaluated using $a^2 - b^2 = (a + b)(a–b)$.

$14^2 - 9^2 = (14 + 9)(14 - 9)$ (Let $a = 14$, $b = 9$)
$= 23 \times 5$
$= 115$

a Use the same technique to evaluate these difference of perfect squares.

i $13^2 - 8^2$ **ii** $25^2 - 23^2$ **iii** $42^2 - 41^2$ **iv** $85^2 - 83^2$
v $1.4^2 - 1.3^2$ **vi** $4.9^2 - 4.7^2$ **vii** $1001^2 - 1000^2$ **viii** $2.01^2 - 1.99^2$

The expansion $(a + b)(a - b) = a^2 - b^2$ can also be used to evaluate some products. Here is an example:

$31 \times 29 = (30 + 1)(30 - 1)$ (Let $a = 30$, $b = 1$)
$= 30^2 - 1^2$
$= 900 - 1$
$= 899$

b Use the same technique to evaluate these products.

i 21×19 **ii** 32×28 **iii** 63×57 **iv** 105×95
v 2.1×1.9 **vi** 7.4×6.6 **vii** 520×480 **viii** 915×885

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1 **a** The difference between the squares of two consecutive numbers is 97.
What are the two numbers?

b The difference between the squares of two consecutive odd numbers is 136.
What are the two numbers?

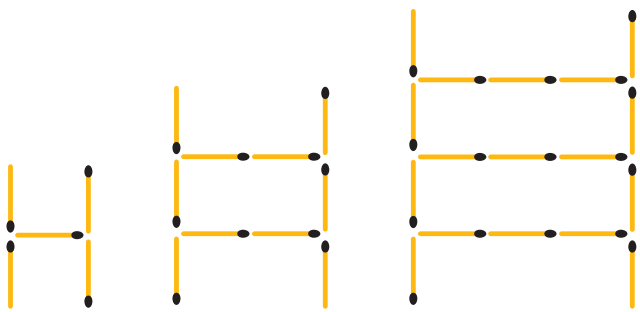
c The difference between the squares of two consecutive multiples of 3 is 81.
What are the two numbers?

2 **a** If $x^2 + y^2 = 6$ and $(x + y)^2 = 36$, find the value of xy.

b If $x + y = 10$ and $xy = 2$, find the value of $\frac{1}{x} + \frac{1}{y}$.

3 Find the values of the different digits a, b, c and d if the four digit number $abcd \times 4 = dcba$.

4 **a** Find the quadratic rule that relates the width n to the number of matchsticks in the pattern below.



$n = 1$ $n = 2$ $n = 3$

b Draw a possible pattern for these rules.

i $n^2 + 3$

ii $n(n - 1)$

5 Factorise $n^2 - 1$ and use the factorised form to explain why when n is prime and greater than 3, $n^2 - 1$ is:

a divisible by 4

b divisible by 3

c thus divisible by 12

6 Prove that this expression is equal to 1.

$$\frac{2x^2 - 8}{5x^2 - 5} \div \frac{x - 2}{5x - 5} \div \frac{2x^2 - 10x - 28}{x^2 - 6x - 7}$$

7 Prove that $4x^2 - 4x + 1 \geq 0$ for all x.

8 In a race over 4 km Ryan ran at a constant speed. Sophie, however, ran the first 2 km at a speed 1 km/h more than Ryan and ran the second 2 km at a speed 1 km/h less than Ryan. Who won the race?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Quadratic expressions and algebraic fractions

Monic quadratic trinomials

These are of the form:
$x^2 + bx + c$
Require two numbers that multiply to c and add to b
e.g. $x^2 - 7x - 18 = (x - 9)(x + 2)$
since $-9 \times 2 = -18$
and $-9 + 2 = -7$

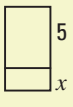
Difference of two squares

$a^2 - b^2 = (a - b)(a + b)$
e.g. $x^2 - 16 = x^2 - 4^2$
$= (x - 4)(x + 4)$
e.g. $4x^2 - 9 = (2x)^2 - 3^2$
$= (2x - 3)(2x + 3)$

Expansion

The process of removing brackets

e.g. $2(x + 5) = 2x + 10$



$(x + 3)(2x - 4) = x(2x - 4) + 3(2x - 4)$
$= 2x^2 - 4x + 6x - 12$
$= 2x^2 + 2x - 12$

Special cases

Difference of two squares
$(a - b)(a + b) = a^2 - b^2$
e.g. $(x - 5)(x + 5) = x^2 + 5x - 5x - 25$
$= x^2 - 25$

Perfect squares
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
e.g. $(2x + 3)^2 = (2x + 3)(2x + 3)$
$= 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Factorisation

The reverse process of expansion. Always remove the highest common factor first.

e.g. $2ab + 8b$ HCF $= 2b$
$= 2b(a + 4)$
e.g. $3(x + 2) + y(x + 2)$ HCF $= x + 2$
$= (x + 2)(3 + y)$

By grouping

If there are four terms, we may be able to group into two binomial terms.
e.g. $5x + 10 - 3x^2 - 6x$
$= 5(x + 2) - 3x(x + 2)$
$= (x + 2)(5 - 3x)$

Add/subtract

Must find lowest common denominator (LCD) before applying operation

e.g. $\frac{3x}{4} + \frac{x + 1}{3}$

$= \frac{9x}{12} + \frac{x - 1}{12}$

$= \frac{9x + 4(x - 1)}{12}$

$= \frac{13x - 4}{12}$

Algebraic fractions

These involve algebraic expressions in the numerator and/or denominator

e.g. $\frac{2x}{x - 7}, \frac{3}{5x}, \frac{7}{(x - 1)^2}$

Multiply/divide

To divide, multiply by the reciprocal. Factorise all expressions, cancel and then multiply.

e.g. $\frac{3}{x^2 - 9} \div \frac{15}{2x + 6}$

$= \frac{\cancel{3}^{1}}{(x - 3)\cancel{(x + 3)}^{1}} \times \frac{2\cancel{(x + 3)}^{1}}{\cancel{15}^{5}}$

$= \frac{2}{5(x - 3)}$

Trinomials of the form $ax^2 + bx + c$ (Ext)

Can be factorised using grouping also

e.g. $3x^2 + 7x - 6$,	$3 \times (-6) = -18$
$= 3x^2 + 9x - 2x - 6$,	Use 9 and -2
$= 3x(x + 3) - 2(x + 3)$	Since
$= (x + 3)(3x - 2)$	$9 \times (-2) = -18$ and $9 + (-2) = 7$

Solving equations with algebraic fractions

Find lowest common denominator (LCD) and multiply every term by the LCD.
e.g. LCD of $2x$ and $3x$ is $6x$
LCD of $(x - 1)$ and $(x + 3)$ is $(x - 1)(x + 3)$
LCD of $2x$ and x^2 is $2x^2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Multiple-choice questions

1 $(2x - 1)(x + 5)$ in expanded and simplified form is:

A $2x^2 + 9x - 5$ **B** $x^2 + 11x - 5$ **C** $4x^2 - 5$
D $3x^2 - 2x + 5$ **E** $2x^2 + 4x - 5$

2 $(3a + 2b)^2$ is equivalent to:

A $9a^2 + 6ab + 4b^2$ **B** $9a^2 + 4b^2$ **C** $3a^2 + 6ab + 2b^2$
D $3a^2 + 12ab + 2b^2$ **E** $9a^2 + 12ab + 4b^2$

3 $16x^2 - 49$ in factorised form is:

A $(4x - 7)^2$ **B** $(16x - 49)(16x + 49)$ **C** $(2x - 7)(8x + 7)$
D $(4x - 7)(4x + 7)$ **E** $4(4x^2 - 49)$

4 The factorised form of $b^2 + 3b - 2b - 6$ is:

A $(b - 3)(b + 2)$ **B** $b - 2(b + 3)^2$ **C** $(b + 3)(b - 2)$
D $b - 2(b + 3)$ **E** $b(b + 3) - 2$

5 If $(x - 2)$ is a factor of $x^2 + 5x - 14$ the other factor is:

A x **B** $x + 7$ **C** $x - 7$
D $x - 16$ **E** $x + 5$

6 The factorised form of $3x^2 + 10x - 8$ is:

A $(3x + 1)(x - 8)$ **B** $(x - 4)(3x + 2)$ **C** $(3x + 2)(x + 5)$
D $(3x - 2)(x + 4)$ **E** $(x + 1)(3x - 8)$

7 The simplified form of $\dfrac{3x + 6}{x^2 + 6x + 5} \times \dfrac{x + 5}{x + 2}$ is:

A $\dfrac{3}{x + 1}$ **B** $\dfrac{15}{2x^2 + 5}$ **C** $x + 3$
D $\dfrac{5}{x + 2}$ **E** $3(x + 5)$

8 $\dfrac{x + 2}{5} + \dfrac{2x - 1}{3}$ written as a single fraction is:

A $\dfrac{11x + 1}{15}$ **B** $\dfrac{11x + 9}{8}$ **C** $\dfrac{3x + 1}{8}$ **D** $\dfrac{11x + 7}{15}$ **E** $\dfrac{13x + 1}{15}$

9 The LCD of $\dfrac{3x + 1}{2x}$ and $\dfrac{4}{x + 1}$ is:

A $8x$ **B** $2x(x + 1)$ **C** $(x + 1)(3x + 1)$
D $8x(3x + 1)$ **E** $x(x + 1)$

10 The solution to $\dfrac{3}{1 - x} = \dfrac{4}{2x + 3}$ is:

A $x = -\dfrac{1}{2}$ **B** $x = -\dfrac{9}{11}$ **C** $x = \dfrac{1}{7}$ **D** $x = 2$ **E** $x = -\dfrac{4}{5}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Short-answer questions

1 Expand the following binomial products.

a $(x-3)(x+4)$ **b** $(x-7)(x-2)$

c $(2x-3)(3x+2)$ **d** $3(x-1)(3x+4)$

2 Expand the following.

a $(x+3)^2$ **b** $(x-4)^2$ **c** $(3x-2)^2$

d $(x-5)(x+5)$ **e** $(7-x)(7+x)$ **f** $(11x-4)(11x+4)$

3 Write the following in fully factorised form by removing the highest common factor.

a $4a+12b$ **b** $6x-9x^2$ **c** $-5x^2y-10xy$

d $3(x-7)+x(x-7)$ **e** $x(2x+1)-(2x+1)$ **f** $(x-2)^2-4(x-2)$

4 Factorise the following expressions.

a x^2-100 **b** $3x^2-48$ **c** $25x^2-y^2$

d $49-9x^2$ **e** $(x-3)^2-81$ **f** $1-x^2$

5 Factorise the following by grouping.

a $x^2-3x+2xy-6y$ **b** $4ax+10x-2a-5$ **c** $3x-8b+2bx-12$

6 Factorise the following trinomials.

a $x^2+8x+15$ **b** $x^2-3x-18$ **c** x^2-7x+6 **d** $3x^2+15x-42$

e $2x^2+16x+32$ **f** $5x^2+17x+6$ **g** $4x^2-4x-3$ **h** $6x^2-17x+12$

7 Simplify the following.

a $\dfrac{3x+12}{3}$ **b** $\dfrac{2x-16}{3x-24}$ **c** $\dfrac{x^2-9}{5(x+3)}$

8 Simplify the following algebraic fractions by first factorising and cancelling where possible.

a $\dfrac{3}{2x}\times\dfrac{x}{6}$ **b** $\dfrac{x(x-4)}{8(x+1)}\times\dfrac{4(x+1)}{x}$ **c** $\dfrac{(x^2+3x)}{3x+6}\times\dfrac{x+2}{x+3}$

d $\dfrac{2x}{5x+20}\div\dfrac{x}{x+4}$ **e** $\dfrac{x^2+5x+6}{x+3}\div\dfrac{x^2-4}{4x-8}$ **f** $\dfrac{4x^2-9}{10x^2}\div\dfrac{10x-15}{x}$

9 Simplify the following by first finding the lowest common denominator.

a $\dfrac{x}{4}+\dfrac{2x}{3}$ **b** $\dfrac{x-1}{6}-\dfrac{x+3}{8}$ **c** $\dfrac{3}{4x}+\dfrac{1}{2x}$

d $\dfrac{7}{x}-\dfrac{2}{x^2}$ **e** $\dfrac{3}{x+1}+\dfrac{5}{x+2}$ **f** $\dfrac{7}{(x-4)^2}-\dfrac{2}{x-4}$

10 Solve the following equations involving fractions.

a $\dfrac{x}{4}+\dfrac{2x}{5}=13$ **b** $\dfrac{4}{x}-\dfrac{2}{3x}=20$

c $\dfrac{x+3}{2}+\dfrac{x-4}{3}=6$ **d** $\dfrac{4}{1-x}=\dfrac{5}{x+4}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Extended-response questions

1 A pig pen for a small farm is being redesigned. It is originally a square of side length x m.

a In the planning the length is initially kept as x m and the width altered so that the area of the pen is $(x^2 + 3x)$ square metres. What is the new width?

b Instead, it is determined that the original length will be increased by 1 metre and the original width will be decreased by 1 metre.

i What effect does this have on the perimeter of the pig pen compared with the original size?

ii Determine an expression for the new area of the pig pen in expanded form. How does this compare to the original area?

c The final set of dimensions requires an extra 8 m of fencing to go around the pen compared with the original pen. If the length of the pen has been increased by 7 m, then the width of the pen must decrease. Find:

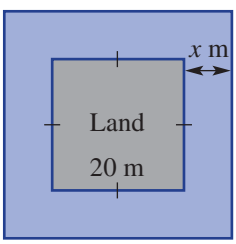
i the change that has been made to the width of the pen

ii the new area enclosed by the pen

iii what happens when $x = 3$



2 The security tower for a palace is on a small square piece of land 20 m by 20 m with a moat of width x metres the whole way around it as shown.



a State the area of the piece of land.

b **i** Give expressions for the length and the width of the combined moat and land.

ii Find an expression, in expanded form, for the entire area occupied by the moat and the land.

c If the tower occupies an area of $(x + 10)^2$ m^2, what fraction of the total area in part **b ii** is this?

d Use your answers to parts **a** and **b** to give an expression for the area occupied by the moat alone, in factorised form.

e Use trial and error to find the value of x so that the area of the moat alone is 500 m^2.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

9 Probability and single variable data analysis

What you will learn

9A Probability review REVISION
9B Venn diagrams and two-way tables
9C Using set notation FRINGE
9D Using arrays for two-step experiments
9E Using tree diagrams
9F Using relative frequencies to estimate probabilities
9G Mean, median and mode REVISION
9H Stem-and-leaf plots
9I Grouping data into classes FRINGE
9J Measures of spread: range and interquartile range
9K Box plots

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



NSW syllabus

STRAND: STATISTICS AND PROBABILITY
SUBSTRANDS: PROBABILITY (S5.1, 5.2) SINGLE VARIABLE DATA ANALYSIS (S5.1, 5.2◊)

Outcomes

A student calculates relative frequencies to estimate probabilites of simple and compound events.
(MA5.1–13SP)

A student uses statistical displays to compare sets of data, and evaluates statistical claims made in the media.
(MA5.1–12SP)

A student describes and calculates probabilities in multistep chance experiments.
(MA5.2–17SP)

A student uses quartiles and box plots to compare sets of data, and evaluates sources of data.
(MA5.2–15SP)

© NESA, 2012

Are lotteries worth it?

Imagine this. There are 45 balls in a bucket. They are numbered from 1 to 45. Six of them will be chosen at random from the bucket. For a few dollars, you get four chances to predict which six numbers will be chosen. Would you play?

There are 8 145 060 different ways to choose six numbers from 45. That gives you four chances in 8 145 060. That is roughly one chance in 2 million! If you did this every day, you could expect to win this game once every 5578 years (but you will have spent more than the prize money).

Every week thousands of people play this game. Almost all of them lose their money week after week. In the year 2011, the 'average' Australian spent over $1600 on gambling activities.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Write the following as decimals.

a $\frac{1}{10}$ **b** $\frac{2}{8}$ **c** 30% **d** 85% **e** 23.7%

2 Express the following in simplest form.

a $\frac{4}{8}$ **b** $\frac{7}{21}$ **c** $\frac{20}{30}$ **d** $\frac{100}{100}$ **e** $\frac{0}{4}$

f $\frac{12}{144}$ **g** $\frac{36}{72}$ **h** $\frac{36}{58}$ **i** $\frac{72}{108}$ **j** $\frac{2}{7}$

3 A six-sided die is tossed.

a List the possible outcomes.

b How many of the outcomes are:

i even? **ii** less than 3? **iii** less than or equal to 3?

iv at least 2? **v** not a 6? **vi** not odd?

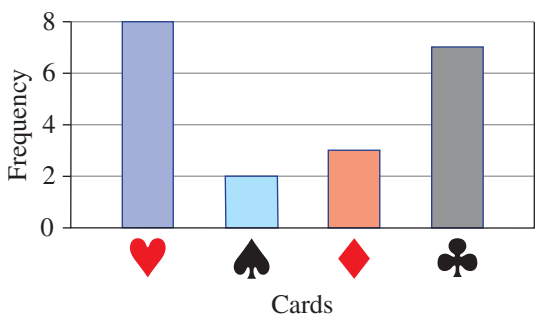
4 One number is selected from the group of the first 10 integers $\{1, 2, \ldots, 10\}$. How many of the numbers are:

a odd? **b** less than 8?

c greater than or equal to 5? **d** no more than 7?

e prime? **f** not prime?

5 Several cards were randomly selected from a pack of playing cards. The suit of each card was noted, the card was replaced and the pack was shuffled. The frequency of each suit is shown in the column graph below.



a How many times was a heart selected?

b How many times was a card selected in total?

c In what fraction of the trials was a diamond selected?

6 Consider this simple data set: 1, 2, 3, 5, 5, 7, 8, 10, 13.

a Find the mean.

b Find the median (the middle value).

c Find the mode (most common value).

d Find the range (difference between highest and lowest value).

e Find the probability of randomly selecting a:

i 5

ii number that is not 5

iii number that is no more than 5

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9A Probability review REVISION

Interactive
Widgets
HOTsheets
Walkthrough

The mathematics associated with describing chance is called probability. We can calculate the chance of some events occurring, such as rolling a sum of 12 from two dice or flipping 3 heads when a coin is tossed 5 times. To do this we need to know how many outcomes there are in total and how many of the outcomes are favourable (i.e. match the result we are interested in). The number of favourable outcomes in comparison to the total number of outcomes will determine how likely it is that the favourable event will occur.



Using probability we can find the likelihood of rolling a particular total score with two dice.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Choose an event

As a class group, write down and discuss at least three events that have the following chance of occurring.

- impossible chance
- very low chance
- medium to low chance
- even (50–50) chance
- medium to high chance
- very high chance
- certain chance

What does 'Buckley's chance' mean?

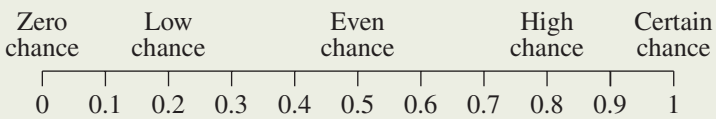
Key ideas

- A chance experiment is an activity that may produce a variety of different results, which occur randomly. Rolling a die is a chance experiment.
- The **sample space** is the list of all possible outcomes of an experiment.
- An **event** is a collection of outcomes resulting from an experiment.

  For example: rolling a die is a random experiment with six possible outcomes: 1, 2, 3, 4, 5 and 6. The event 'rolling a number greater than 4' includes the outcomes 5 and 6 and is an example of a compound event because it includes more than one of the outcomes from the sample space.
- The probability of an event where all outcomes are **equally likely** is given by:

$$P(\text{event}) = \frac{\text{number of outcomes where event occurs}}{\text{total number of outcomes}}$$

- Probabilities are numbers between 0 and 1 and can be written as a decimal, fraction or percentage. For example: 0.55 or $\frac{11}{20}$ or 55%
- For all events, $0 \leq P(\text{event}) \leq 1$.



- The **complement** of an event A is the event where A does not occur.

$$P(\text{not } A) = 1 - P(A)$$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 1 Finding probabilities of events

This spinner has five congruent sections.

a List the sample space using the given numbers.
b Find $P(3)$.
c Find $P(\text{not a } 3)$.
d Find $P(\text{a 3 or a 7})$.
e Find $P(\text{a number which is at least a 3})$.



SOLUTION

a $\{1, 2, 3, 3, 7\}$

b $P(3) = \frac{2}{5}$ or 0.4

c $P(\text{not a } 3) = 1 - P(3)$
$= 1 - \frac{2}{5}$ or $1 - 0.6$
$= \frac{3}{5}$ or 0.4

d $P(\text{a 3 or a 7}) = \frac{2}{5} + \frac{1}{5}$
$= \frac{3}{5}$

e $P(\text{at least a 3}) = \frac{3}{5}$

EXPLANATION

Use set brackets and list all the possible outcomes in any order.

$P(3) = \frac{\text{number of sections labelled 3}}{\text{number of equal sections}}$

'Not a 3' is the complementary event of obtaining a 3.

Alternatively, count the number of sectors that are not 3.

There are two 3s and one 7 in the five sections.

Three of the sections have the numbers 3 or 7, which are 3 or more.

Example 2 Choosing letters from a word

A letter is randomly chosen from the word PROBABILITY. Find the following probabilities.

a $P(\text{L})$
b $P(\text{not L})$
c $P(\text{vowel})$
d $P(\text{consonant})$
e $P(\text{vowel or a B})$
f $P(\text{vowel or consonant})$

SOLUTION

a $P(\text{L}) = \frac{1}{11}$

b $P(\text{not L}) = 1 - \frac{1}{11}$

c $P(\text{vowel}) = \frac{4}{11}$

d $P(\text{consonant}) = 1 - \frac{4}{11}$
$= \frac{7}{11}$

e $P(\text{vowel or a B}) = \frac{6}{11}$

f $P(\text{vowel or consonant}) = 1$

EXPLANATION

One of the 11 letters in PROBABILITY is an L.

The event 'not L' is the complement of the event selecting an L. Complementary events sum to 1.

There are 4 vowels: O, A and two letter Is.

The events 'vowel' and 'consonant' are complementary.

There are 4 vowels and 2 letter Bs.

This event includes all possible outcomes.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9A REVISION

UNDERSTANDING AND FLUENCY | 1–5, 7, 8(½) | 3, 4–6(½), 7, 8(½) | 4–6(½), 7, 8(½)

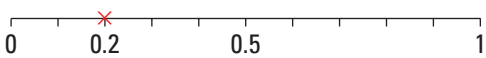
1 Jim believes that there is a 1 in 4 chance that the flower on his prized rose will bloom tomorrow.

a Write the chance '1 in 4' as a:

i fraction **ii** decimal **iii** percentage

b Draw a number line from 0 to 1 and mark the level of chance described by Jim.

2 Copy and complete this table.

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	
b	25%			
c			$\frac{3}{4}$	
d				
e		0.6		
f			$\frac{17}{20}$	

3 Ten people make the following guesses of the chance that they will get a salary bonus this year.

$0.7, \frac{2}{5}, 0.9, \frac{1}{3}$, 2 in 3, $\frac{3}{7}$, 1 in 4, $0.28, \frac{2}{9}, 0.15$

Order their chances from lowest to highest. *Hint*: change each into a decimal.

Example 1

4 The spinners below have congruent sections. Complete the following for each spinner.

i List the sample space using the given numbers. **ii** Find $P(2)$.

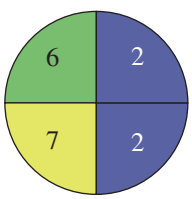
iii Find P(not a 2). **iv** Find P(a 2 or a 3).

v Find P(a number that is at least a 2).

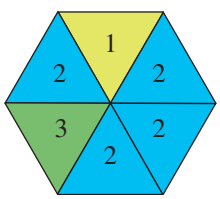
a



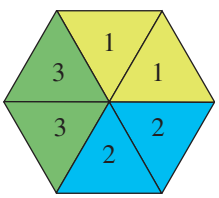
b



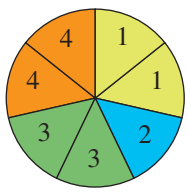
c



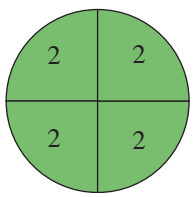
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 Find the probability of obtaining a blue ball if a ball is selected at random from a box that contains:

- **a** 4 blue balls and 4 red balls
- **b** 3 blue balls and 5 red balls
- **c** 1 blue ball, 3 red balls and 2 white balls
- **d** 8 blue balls, 15 black balls and 9 green balls
- **e** 15 blue balls only
- **f** 5 yellow balls and 2 green balls

6 Find the probability of not selecting a blue ball if a ball is selected at random from a box containing the balls described in Question **5** parts **a–f**.

7 If a swimming pool has eight lanes and each of eight swimmers has an equal chance of being placed in lane 1, find the probability that a particular swimmer will:

a swim in lane 1 **b** not swim in lane 1

Example 2

8 A letter is chosen at random from the word ALPHABET. Find the following probabilities.

a $P(\text{L})$ **b** $P(\text{A})$ **c** $P(\text{A or L})$
d $P(\text{vowel})$ **e** $P(\text{consonant})$ **f** $P(\text{vowel or consonant})$
g $P(\text{Z})$ **h** $P(\text{A or Z})$ **i** $P(\text{not an A})$
j $P(\text{letter from the first half of the alphabet})$

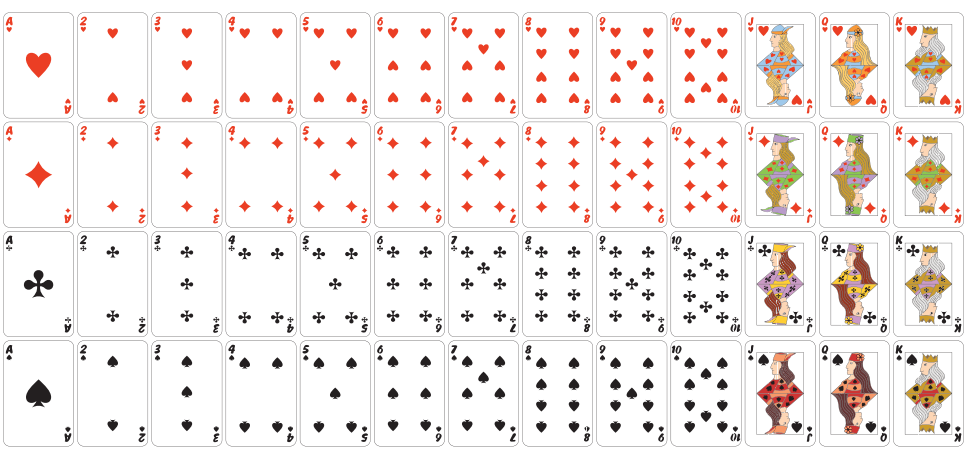
PROBLEM-SOLVING AND REASONING 9, 10, 13, 14 | 10, 11, 13, 14 | 10–12, 14, 15

9 The school captain is to be chosen at random from four candidates. Two are girls (Hayley and Alisa) and two are boys (Rocco and Stuart).

- **a** List the sample space.
- **b** Find the probability that the school captain will be:
  - **i** Hayley **ii** male **iii** neither Stuart nor Alisa

10 From a pack of 52 playing cards a card is drawn at random. The pack includes 13 black spades, 13 black clubs, 13 red hearts and 13 red diamonds. This includes four of each of ace, king, queen, jack, 2, 3, 4, 5, 6, 7, 8, 9 and 10. Find the probability that the card will be:

a the queen of diamonds **b** an ace **c** a red king
d a red card **e** a jack or a queen **f** any card except a 2
g any card except a jack or a black queen **h** not a black ace



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 A six-sided die is tossed and the upper-most face is observed and recorded. Find the following probabilities.

a $P(6)$ **b** $P(3)$ **c** $P(\text{not a } 3)$
d $P(1 \text{ or } 2)$ **e** $P(\text{a number less than } 5)$ **f** $P(\text{even number or odd number})$
g $P(\text{square number})$ **h** $P(\text{not a prime number})$ **i** $P(\text{a number greater than } 1)$

12 A letter is chosen at random from the word PROBABILITY. Find the probability that the letter will be:

a a B
b not a B
c a vowel
d not a vowel
e a consonant
f a letter belonging to one of the first five letters in the alphabet
g a letter from the word RABBIT
h a letter that is not in the word RABBIT

13 Amanda selects a letter from the word SOLO and writes $P(\text{S}) = \frac{1}{3}$. Explain her error.

14 A six-sided die is rolled. Which of the following events have a probability equal to $\frac{1}{3}$?

A More than 4 **B** At least 4
C Less than or equal to 3 **D** No more than 2
E At most 4 **F** Less than 3

15 A number is selected from the set $\{1, 2, 3, \ldots, 25\}$. Find the probability that the number chosen is:

a a multiple of 2 **b** a factor of 24 **c** a square number
d a prime number **e** divisible by 3 **f** divisible by 3 or 2
g divisible by 3 and 2 **h** divisible by 2 or 3 or 7 **i** divisible by 13 and 7

ENRICHMENT — — 16

Faulty CD player

16 A compact disc (CD) contains eight tracks. The time length for each track is as shown in the table on the right.
The CD is placed in a faulty CD player, which begins playing randomly at an unknown place somewhere on the CD, not necessarily at the beginning of a track.

Track	Time (minutes)
1	3
2	4
3	4
4	5
5	4
6	3
7	4
8	4

a Find the total number of minutes of music available on the CD.
b Find the probability that the CD player will begin playing somewhere on track 1.
c Find the probability that the CD player will begin somewhere on:

i track 2 **ii** track 3 **iii** a track that is 4 minutes long
iv track 4 **v** track 7 or 8 **vi** a track that is not 4 minutes long

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9B Venn diagrams and two-way tables

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

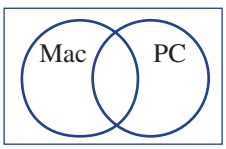
When the results of an experiment involve overlapping categories, it can be very helpful to organise the information into a Venn diagram or two-way table. Probabilities of particular events can easily be calculated from these types of diagrams.

Let's start: Mac or PC

Twenty people were surveyed to find out whether they owned a Mac or a PC at home. The survey revealed that 8 people owned a Mac and 15 people owned a PC. All people surveyed owned at least one type of computer.

- Do you think some people owned both a Mac and a PC? Discuss.
- Use these diagrams to help organise the number of people who own Macs and PCs.
- Use your diagrams to describe the proportion (fraction) of people owning Macs and/or PCs for all the different areas in the diagrams.

Venn diagram



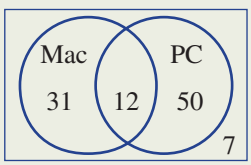
Two-way table

	Mac	No Mac	Total
PC			
No PC			
Total			

Key ideas

■ A **Venn diagram** and a **two-way table** help to organise outcomes into different categories. This example shows the type of computers owned by 100 people.

Venn diagram



Two-way table

	Mac	No Mac	Total
PC	12	50	62
No PC	31	7	38
Total	43	57	100

These diagrams show, for example:

- 12 people own both a Mac and a PC
- 62 people own a PC
- 57 people do not own a Mac
- $P(\text{Mac}) = \frac{43}{100}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- $P(\text{only Mac}) = \frac{31}{100}$
- $P(\text{Mac or PC}) = \frac{93}{100}$
- $P(\text{Mac and PC}) = \frac{12}{100} = \frac{3}{25}$

Example 3 Using a Venn diagram

A survey of 30 people found that 21 like AFL and 12 like soccer. Also 7 people like both AFL and soccer and 4 like neither AFL nor soccer.



a Construct a Venn diagram for the survey results.

b How many people:

i like AFL or soccer?

ii do not like soccer?

iii like only AFL?

c If one of the 30 people was randomly selected, find:

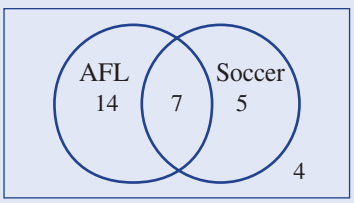
i $P(\text{like AFL and soccer})$

ii $P(\text{like neither AFL nor soccer})$

iii $P(\text{like only soccer})$

SOLUTION

a



b i 26

ii $30 - 12 = 18$

iii 14

c i $P(\text{like AFL and soccer}) = \frac{7}{30}$

ii $P(\text{like neither AFL nor soccer}) = \frac{4}{30} = \frac{2}{15}$

iii $P(\text{like soccer only}) = \frac{5}{30} = \frac{1}{6}$

EXPLANATION

Place the appropriate number in each category ensuring that:

- the total that like AFL is 21
- the total that like soccer is 12
- there are 30 in total

The total number of people that like AFL, soccer or both is $14 + 7 + 5 = 26$.

12 like soccer so 18 do not.

21 like AFL but 7 of these also like soccer.

7 out of 30 people like AFL and soccer.

The 4 people who like neither AFL nor soccer sit outside both categories.

5 people like soccer but not AFL.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 4 Using a two-way table

At a car yard, 24 cars are tested for fuel economy. Eighteen of the cars run on petrol, 8 cars run on gas and 3 cars can run on both petrol and gas.

a Illustrate the situation using a two-way table.

b How many of the cars:

i do not run on gas? **ii** run on neither petrol nor gas?

c Find the probability that a randomly selected car:

i runs on gas **ii** runs on only gas **iii** runs on gas or petrol

SOLUTION

a

	Gas	Not gas	Total
Petrol	3	15	18
Not petrol	5	1	6
Total	8	16	24

EXPLANATION

Set up a table as shown and enter the numbers (in black) from the given information.
Fill in the remaining numbers (in red), ensuring that each column and row adds to the correct total.

b i 16

The total at the base of the 'Not gas' column is 16.

ii 1

The number at the intersection of the 'Not gas' column and the 'Not petrol' row is 1.

c i $P(\text{gas}) = \frac{8}{24} = \frac{1}{3}$

8 cars in total run on gas out of the 24 cars.

ii $P(\text{only gas}) = \frac{5}{24}$

Of the 8 cars that run on gas, 5 of them do not also run on petrol.

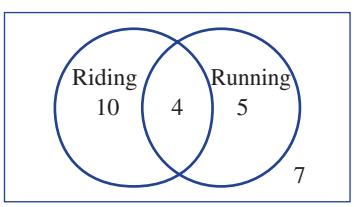
iii $P(\text{gas or petrol}) = \frac{15 + 5 + 3}{24} = \frac{23}{24}$

Of the 24 cars, some run on petrol only (15), some run on gas only (5) and some run on gas and petrol (3).

Exercise 9B

UNDERSTANDING AND FLUENCY	1–4, 5(a), 6, 7(a)	2–4, 5(a), 6, 7(a)	5(b), 6, 7(b)

1 This Venn diagram shows the number of people who enjoy riding and running.



a How many people in total are represented by this Venn diagram?

b How many people enjoy:

i only riding?
ii riding (in total)?
iii only running?
iv running (in total)?
v both riding and running?
vi neither riding nor running?
vii riding or running?

c How many people do not enjoy:

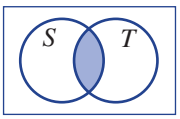
i riding? **ii** running?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

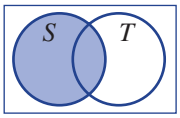
2 Match the diagrams **A**, **B**, **C** or **D** with the given description.

a S **b** S only **c** S and T **d** S or T

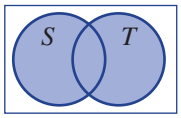
A



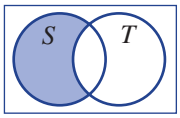
B



C



D



3 Fill in the missing numbers in these two-way tables.

a

	A	Not A	Total
B	7	8	
Not B		1	
Total	10		

b

	A	Not A	Total
B	2		7
Not B		4	
Total			20

Example 3

4 In a class of 30 students, 22 carried a phone and 9 carried an iPod. Three carried both a phone and an iPod and 2 students carried neither.

a Represent the information using a Venn diagram.

b How many people:

i carried a phone or an iPod (includes carrying both)?

ii do not carry an iPod?

iii carry only an iPod?

c If one of the 30 people were selected at random, find the following probabilities.

i P(carry a phone and an iPod)

ii P(carry neither a phone nor an iPod)

iii P(carry only a phone)

5 For each simple Venn diagram, find the following probabilities. You will need to calculate the total number in the sample first.

i $P(A)$

ii $P(A$ only$)$

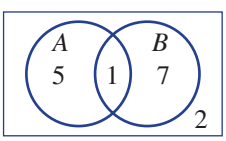
iii $P($not $B)$

iv $P(A$ and $B)$

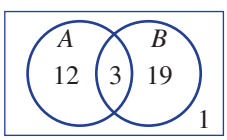
v $P(A$ or $B)$

vi $P($neither A nor $B)$

a



b



Example 4

6 From 50 desserts served at a restaurant one evening, 25 were served with ice-cream, 21 were served with cream and 5 were served with both cream and ice-cream.

a Illustrate the situation using a two-way table.

b How many of the desserts:

i did not have cream?

ii had neither cream nor ice-cream?

c Find the probability that a randomly chosen dessert had:

i cream **ii** only cream **iii** cream or ice-cream

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Find the following probabilities using each of the given tables. First fill in the missing numbers.

- **i** $P(A)$
- **ii** $P(\text{not } A)$
- **iii** $P(A \text{ and } B)$
- **iv** $P(A \text{ or } B)$
- **v** $P(B \text{ only})$
- **vi** $P(\text{neither } A \text{ nor } B)$

a

	A	Not A	Total
B	3	1	
Not B	2		4
Total			

b

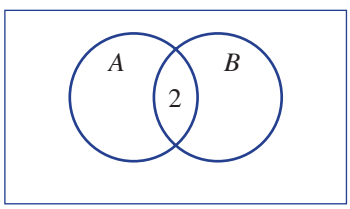
	A	Not A	Total
B		4	15
Not B	6		
Total			26

PROBLEM-SOLVING AND REASONING 8–10, 13 | 8–11, 13, 14 | 8, 10, 12, 14–16

8 For each two-way table, fill in the missing numbers then transfer the information to a Venn diagram.

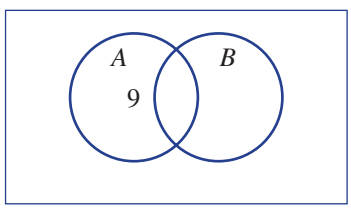
a

	A	Not A	Total
B	2		8
Not B			
Total		7	12



b

	A	Not A	Total
B		4	
Not B	9		13
Total	12		



9 In a group of 10 people, 6 rented their house, 4 rented a car and 3 did not rent either a car or their house.

- **a** Draw a Venn diagram.
- **b** How many people rented both a car and their house?
- **c** Find the probability that one of them rented only a car.

10 One hundred citizens are surveyed regarding their use of water for their garden. 23 say that they use rain water, 48 say that they use tap water and 41 say that they do not use water at all.

- **a** Represent this information in a two-way table.
- **b** How many people use both rain and tap water?
- **c** What is the probability that one of the people uses only tap water?
- **d** What is the probability that one of the people uses tap water or rain water?

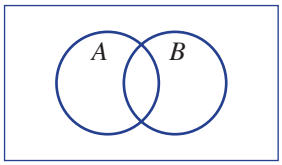
11 All members of a ski club enjoy either skiing and/or snowboarding. Seven enjoy only snowboarding, 16 enjoy skiing and 4 enjoy both snowboarding and skiing. How many people are in the ski club?

12 Of a group of 30 cats, 24 eat tinned or dry food, 10 like dry food and 5 like both tinned and dry food. Find the probability that a selected cat likes only tinned food.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 Complete the two-way table and transfer the information to a Venn diagram using the pronumerals w, x, y and z.

	A	Not A	Total
B	x	y	
Not B	z	w	
Total			



14 The total number of people in a survey is T. The number of males in the survey is x and the number of doctors is y. The number of doctors that are male is z. Write algebraic expressions for the following using any of the variables x, y, z and T.

- **a** The number who are neither male nor a doctor
- **b** The number who are not male
- **c** The number who are not doctors
- **d** The number who are male but not a doctor
- **e** The number who are a doctor but not male
- **f** The number who are female and a doctor
- **g** The number who are female or a doctor

15 Explain what is wrong with this two-way table. Try to complete it to find out.

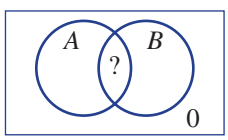
	A	Not A	Total
B		12	
Not B			7
Total	11		19

16 How many numbers need to be given in a two-way table so that all numbers in the table can be calculated?

ENRICHMENT — — 17

Finding a rule for A and B

17 Two overlapping events, A and B, include 20 elements with 0 elements in the 'neither A nor B' region.



- **a** Draw a Venn diagram for the following situations.
  - **i** The number in A is 12 and the number in B is 10.
  - **ii** The number in A is 15 and the number in B is 11.
  - **iii** The number in A is 18 and the number in B is 6.
- **b** If the total number in A or B is now 100 (not 20), complete a Venn diagram for the following situations.
  - **i** The number in A is 50 and the number in B is 60.
  - **ii** The number in A is 38 and the number in B is 81.
  - **iii** The number in A is 83 and the number in B is 94.
- **c** Now describe a method that finds the number in the common area for A and B. Your method should work for all the above examples.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9C Using set notation FRINGE

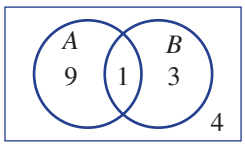
Interactive
Widgets
HOTsheets
Walkthrough

Using symbols to describe different sets of objects can make the writing of mathematics more efficient and easier to read. For example, *the probability that a randomly chosen person likes both apples and bananas* could be written $P(A \cap B)$ provided the events A and B are clearly defined.



Let's start: English language meaning to mathematical meaning

The number of elements in two events called A and B are illustrated in this Venn diagram. Use your understanding of the English language meaning of the given words to match with one of the mathematical terms and a number from the Venn diagram. They are in jumbled order.



English	Mathematical	Number in Venn diagram
a not A	A A union B	i 1
b A or B	B sample space	ii 7
c A and B	C complement of A	iii 13
d anyone	D A intersection B	iv 17

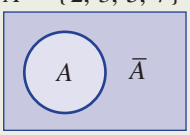
Key ideas

- The **sample space** (list of all possible outcomes) is sometimes called the universal set and is given the symbol S, Ω, U or ξ.

  For example:
  $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

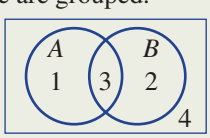
- A is a particular **subset** ($\subset$) of the sample space if all the elements in A are contained in the sample space. For example, $\{2, 3, 5, 7\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

  $A = \{2, 3, 5, 7\}$



  $\overline{A} = \{1, 4, 6, 8, 9, 10\}$

- $\overline{A}$ or A' or A^c is the complement of A and contains the elements not in A.
- $5 \in A$ means that 5 is **an element of** A.
- $\emptyset$ is the **null** or **empty** set and contains no elements. $\therefore \emptyset = \{\ \}$
- $n(A)$ is the **cardinal number** of A and means the number of elements in A. $n(A) = 4$
- A Venn diagram can be used to illustrate how different subsets in the sample space are grouped.
  For example: $A = \{2, 3, 5, 7\}$
  $B = \{1, 3, 5, 7, 9\}$



	A	not A	Total
B	3	2	5
not B	1	4	5
Total	4	6	10

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

- $A \cap B$ means A **and** B, which means the **intersection** of A and B and includes the elements in common with both sets. $\therefore A \cap B = \{3, 5, 7\}$
- $A \cup B$ means A **or** B, which means the **union** of A and B and includes the elements in either A or B or both. $\therefore A \cup B = \{1, 2, 3, 5, 7, 9\}$

■ A **only** is the elements in A but not in B. $\therefore A$ only $= \{2\}$ and $n(A \text{ only}) = 1$

Example 5 Using set notation

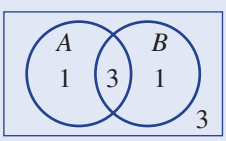
A number is chosen from the set of positive integers between 1 and 8 inclusive. If A is the set of odd numbers between 1 and 8 inclusive and B is the set of prime numbers between 1 and 8 inclusive:

a list the sets:
i the sample space **ii** A **iii** B

b draw a Venn diagram

c list the sets:
i $A \cap B$ **ii** $A \cup B$ **iii** $\overline{A}$ **iv** B only

d find:
i $n(A)$ **ii** $P(A)$ **iii** $n(A \cap B)$ **iv** $P(A \cap B)$

SOLUTION	EXPLANATION
a i $\{1, 2, 3, 4, 5, 6, 7, 8\}$	List all the numbers, using set brackets.
ii $A = \{1, 3, 5, 7\}$	A includes all the odd numbers.
iii $B = \{2, 3, 5, 7\}$	B includes all the prime numbers. 1 is not prime.
b 	Place each cardinal number into the appropriate region, i.e. there are 3 numbers common to sets A and B so 3 is placed in the overlapping region.
c i $A \cap B = \{3, 5, 7\}$	$\{3, 5, 7\}$ are common to both A and B.
ii $A \cup B = \{1, 2, 3, 5, 7\}$	$\{1, 2, 3, 5, 7\}$ are in either A or B or both.
iii $\overline{A} = \{2, 4, 6, 8\}$	$\overline{A}$ means the elements not in A.
iv B only $= \{2\}$	B only means the elements in B but not in A.
d i $n(A) = 4$	$n(A)$ is the cardinal number of A. There are four elements in A.
ii $P(A) = \frac{4}{8} = \frac{1}{2}$	$P(A)$ means the chance that the element will belong to A. There are 4 numbers in A compared with 8 in the sample space.
iii $n(A \cap B) = 3$	There are three elements in $A \cap B$.
iv $P(A \cap B) = \frac{3}{8}$	Three out of eight elements are in $A \cap B$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9C FRINGE

UNDERSTANDING AND FLUENCY | 1–6 | 1–4, 6, 7 | 5–7

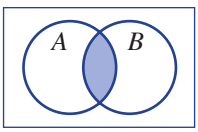
1 Match each of the terms in the first list with the symbols in the second list.

a	Complement	**A**	$\cap$
b	Union	**B**	$n(A)$
c	Element of	**C**	$\overline{A}$
d	Intersection	**D**	$\cup$
e	Empty or null set	**E**	$\in$
f	Number of elements	**F**	$\emptyset$

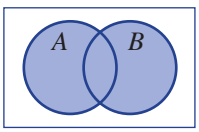
2 Choose a diagram that matches each of these sets.

a $A \cup B$ **b** $\overline{A}$ **c** $A \cap B$ **d** B only

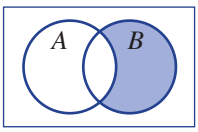
A



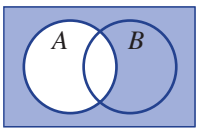
B



C



D



3 Using the given two-way table, find the following.

a $n(A \cap B)$

b $n(B)$

c $n(A)$

d $n(\overline{A})$

	A	$\overline{A}$	Total
B	2	8	10
$\overline{B}$	5	1	6
Total	7	9	16

Example 5

4 A number is chosen from the set of positive integers between 1 and 10 inclusive. If A is the set of odd numbers between 1 and 10 inclusive and B is the set of prime numbers between 1 and 10 inclusive:

a list the sets:

i the sample space **ii** A **iii** B

b draw a Venn diagram

c list the sets:

i $A \cap B$ **ii** $A \cup B$ **iii** $\overline{A}$ **iv** B only

d find:

i $n(A)$ **ii** $P(A)$ **iii** $n(A \cap B)$ **iv** $P(A \cap B)$

5 A number is chosen from the set of positive integers between 1 and 20 inclusive. If A is the set of multiples of 3 less than 20 and B is the set of factors of 15:

a draw a Venn diagram

b list the sets:

i $A \cap B$ **ii** $A \cup B$ **iii** $\overline{A}$ **iv** B only

c find:

i $n(B)$ **ii** $P(B)$ **iii** $n(A \cap B)$

iv $P(A \cap B)$ **v** $n(A \cup B)$ **vi** $P(A \cup B)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Consider the sample space $\{1, 2, 3, 4, 5, 6\}$ and the sets $A = \{1, 4, 7\}$ and $B = \{2, 3, 5\}$.

a State whether the following are true or false.

i $A \subset$ the sample space
ii $B \subset$ the sample space
iii $3 \in A$
iv $5 \in B$
v $n(B) = 3$
vi $n(\overline{B}) = 2$
vii $A \cap B = \emptyset$
viii $A \cup B = \emptyset$

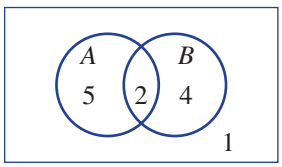
b Find the following probabilities.

i $P(B)$ **ii** $P(\overline{B})$ **iii** $P(A \cap B)$

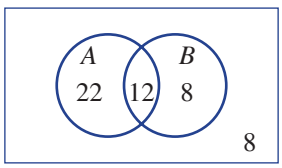
7 For each diagram, find the following probabilities.

i $P(A \cap B)$
ii $P(A \cup B)$
iii $P(\overline{A})$

a



b



c

	A	$\overline{A}$	Total
B	5	2	
$\overline{B}$			
Total	9		15

d

	A	$\overline{A}$	Total
B		8	13
$\overline{B}$	4		
Total		12	

PROBLEM-SOLVING AND REASONING 8–11, 13 8–13 9, 10, 12–15

8 Four students have the names FRED, RON, RACHEL and HELEN. The sets A, B and C are defined by:

$A = \{$students with a name including the letter R$\}$
$B = \{$students with a name including the letter E$\}$
$C = \{$students with a name including the letter Z$\}$

a List the sets:

i A **ii** B **iii** C **iv** $A \cap B$

b If a student is chosen at random from the group, find these probabilities.

i $P(A)$ **ii** $P(\overline{A})$ **iii** $P(C)$
iv $P(\overline{C})$ **v** $P(A \cap B)$ **vi** $P(A \cup B)$

9 Consider all the letters of the alphabet. Let $A = \{$the set of vowels$\}$ and $B = \{$letters of the word MATHEMATICS$\}$. Find:

a $n(\text{sample space})$ **b** $n(A)$
c $n(A \cap B)$ **d** $n(\overline{B})$
e $P(A)$ **f** $P(\overline{A})$
g $P(A \cap B)$ **h** $P(A \cup B)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 From 50 people who all have at least a cat or a dog, let A be the set of all pet owners who have a dog and B be the set of all pet owners who have a cat. If $n(A) = 32$ and $n(B) = 29$, find the following.

a $n(A \cap B)$ **b** $n(A \text{ only})$ **c** $P(A \cup B)$ **d** $P(\overline{A})$

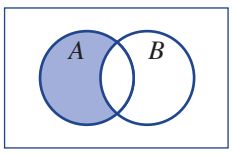
11 From Question **10**, A is the set of dog owners and B is the set of cat owners. Write a brief description of the following groups of people.

a $A \cap B$ **b** $A \cup B$ **c** $\overline{B}$ **d** B only

12 **a** Is $n(A) + n(B)$ ever equal to $n(A \cup B)$? If so, show an example using a Venn diagram.

b Is $n(A) + n(B)$ ever less than $n(A \cap B)$? Explain.

13 The region A only could be thought of as the intersection of A and the complement of B so A only $= A \cap \overline{B}$. Use set notation to describe the region 'B only'.



14 The four inner regions of a two-way table can be described using intersections. The A only region is described by $n(A \cap \overline{B})$. Describe the other three regions using intersections.

	A	$\overline{A}$	Total
B			$n(B)$
$\overline{B}$	$n(A \cap \overline{B})$		$n(\overline{B})$
Total	$n(A)$	$n(\overline{A})$	$n(\text{sample space})$

15 Research what it means if we say that two events A and B are **mutually exclusive**. Give a brief description.

ENRICHMENT — — 16

How are $\overline{A} \cap \overline{B}$ and $\overline{A \cup B}$ related?

16 Consider the set of integers $\{1, 2, 3, \ldots, 20\}$. Let $A = \{\text{prime numbers less than } 20\}$ and $B = \{\text{factors of } 12\}$.

a List:

i A **ii** B **iii** $A \cap B$
iv $A \cup B$ **v** $\overline{A} \cap B$ **vi** $A \cap \overline{B}$
vii $\overline{A} \cup B$ **viii** $A \cup \overline{B}$ **ix** $\overline{A} \cup \overline{B}$

b Find the following probabilities.

i $P(A \cup B)$ **ii** $P(A \cap B)$ **iii** $P(\overline{A} \cap B)$ **iv** $P(\overline{A} \cap \overline{B})$
v $P(A \cup \overline{B})$ **vi** $P(\overline{A \cap B})$ **vii** $P(\overline{A} \cup \overline{B})$ **viii** $P(\overline{A \cup B})$

c Draw Venn diagrams to shade the regions $\overline{A} \cap \overline{B}$ and $\overline{A \cup B}$. What do you notice?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9D Using arrays for two-step experiments

Interactive

Widgets

HOTsheets

Walkthrough

When an experiment consists of two steps, such as rolling two dice or selecting two people from a group, we can use an array (or a table) to systematically list the sample space.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Is it a 1 in 3 chance?

Billy tosses two coins on the kitchen table at home and asks what the chance is of getting two tails.

- Dad says that there are 3 outcomes: two heads, two tails, or one of each, so there is a 1 in 3 chance.
- Mum says that with coins, all outcomes have a 1 in 2 chance of occurring.
- Billy's sister Betty says that there are 4 outcomes so it's a 1 in 4 chance.

Can you explain who is correct and why?



Tossing one coin gives a 1 in 2 chance of heads or tails, but what are the chances of getting two tails with two coins?

Key ideas

- Example of two-step experiments include:
  - tossing two coins (or tossing one coin twice)
  - choosing two socks from a drawer full of socks
  - rolling two dice
  - choosing two cards from a pack
  - rolling a die and then tossing a coin.
- An array (table) is often used to list the sample space of a two-step experiment.
- When listing outcomes it is important to be consistent with the order for each outcome. For example: the outcome (heads, tails) is different to the outcome (tails, heads).
- Some experiments are conducted **without replacement**, which means some outcomes that may be possible **with replacement** are not possible.
  For example: Two letters are chosen from the word CAT.

With replacement

		1st		
		C	A	T
2nd	C	(C, C)	(A, C)	(T, C)
	A	(C, A)	(A, A)	(T, A)
	T	(C, T)	(A, T)	(T, T)

9 outcomes

Without replacement

		1st		
		C	A	T
2nd	C	×	(A, C)	(T, C)
	A	(C, A)	×	(T, A)
	T	(C, T)	(A, T)	×

6 outcomes

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 6 Finding the sample space for events with replacement

Two coins are tossed.

a Draw a table to list the sample space.

b Find the probability of obtaining (H, T).

c Find $P(1\text{ head})$.

SOLUTION

a

		Toss 1	
		H	T
Toss 2	H	(H, H)	(T, H)
	T	(H, T)	(T, T)

Sample space $= \{(H, H), (H, T), (T, H), (T, T)\}$

b $P(H, T) = \frac{1}{4}$

c $P(1\text{ head}) = \frac{2}{4} = \frac{1}{2}$

EXPLANATION

Represent the results of each coin toss.

The table shows four possible outcomes.

One of the four outcomes is (H, T).

Two outcomes have one head: (H, T), (T, H).

Example 7 Finding the sample space for events without replacement

Two letters are chosen from the word TREE without replacement.

a List the outcomes in a table.

b Find the probability that the two letters chosen are both E.

c Find the probability that at least one of the letters is an E.

SOLUTION

a

		1st			
		T	R	E	E
2nd	T	×	(R, T)	(E, T)	(E, T)
	R	(T, R)	×	(E, R)	(E, R)
	E	(T, E)	(R, E)	×	(E, E)
	E	(T, E)	(R, E)	(E, E)	×

b $P(E, E) = \frac{2}{12}$
$= \frac{1}{6}$

c $P(\text{at least one E}) = \frac{10}{12}$
$= \frac{5}{6}$

EXPLANATION

List all the outcomes, maintaining a consistent order. Note that the same letter cannot be chosen twice. Both Es need to be listed so that each outcome in the sample space is equally likely.

Since there are 2 Es in the word TREE, it is still possible to obtain the outcome (E, E) in two ways.

10 of the 12 outcomes contain at least one E.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9D

UNDERSTANDING AND FLUENCY	1–6	2, 5–7	4, 6, 7

1 Fill in these tables to show all the outcomes then count the total number of outcomes. Parts **a** and **b** are *with replacement* and parts **c** and **d** are *without replacement*.

a

		1st		
		1	2	3
2nd	1	(1, 1)	(2, 1)	
	2			
	3			

b

		Die					
		1	2	3	4	5	6
Coin	H	(1, H)					
	T			(3, T)			

c

		1st		
		A	B	C
2nd	A	×	(B, A)	(C, A)
	B		×	
	C			×

d

		1st		
		•	∘	∘
2nd	•	×	(∘, •)	
	∘		×	(∘, ∘)
	∘			×

2 These two tables list the outcomes for the selection of two letters from the word MAT.

Table A

	M	A	T
M	(M, M)	(A, M)	(T, M)
A	(M, A)	(A, A)	(T, A)
T	(M, T)	(A, T)	(T, T)

Table B

	M	A	T
M	×	(A, M)	(T, M)
A	(M, A)	×	(T, A)
T	(M, T)	(A, T)	×

a Which table shows selection where replacement is allowed (with replacement)?

b Which table shows selection where replacement is not allowed (without replacement)?

c What is the probability of choosing the outcome (T, M) from:

i table A? **ii** table B?

d How many outcomes include the letter A using:

i table A? **ii** table B?

3 Two dot circles are selected, one from each of the sets A and B where $A = \{\bullet, \circ\}$ and $B = \{\bullet, \circ, \circ\}$.

a Copy and complete this table, showing all the possible outcomes.

b State the total number of outcomes.

c Find the probability that the outcome will:

i be (•, ∘)

ii contain one black dot

iii contain two of the same dots

		A	
		•	∘
B	•	(•, •)	(∘, •)
	∘		
	∘		

Example 6

4 Two four-sided dice (numbered 1, 2, 3, 4) are rolled.

a Complete a table like the one shown and list the sample space.

b Find the probability of obtaining (2, 3).

c Find P(double). A double is an outcome with two of the same number.

		1st			
		1	2	3	4
2nd	1	(1, 1)	(2, 1)		
	2				
	3				
	4				

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 A six-sided die is rolled twice.

a Complete a table like the one shown.

b What is the total number of outcomes?

c Find the probability that the outcome is:

i (1, 1)

ii a double

iii (3, 1), (2, 2) or (1, 3)

iv any outcome containing a 1 or a 6

		1st					
		1	2	3	4	5	6
2nd	1	(1, 1)	(2, 1)				
	2						
	3						
	4						
	5						
	6						

Example 7

6 Two letters are chosen from the word DOG *without replacement*.

a Complete the given table.

b Find the probability of obtaining the (G, D) outcome.

c Find the probability of obtaining an outcome with an O in it.

		1st		
		D	O	G
2nd	D	×	(O, D)	(G, D)
	O		×	
	G			×

7 Two digits are selected *without replacement* from the set $\{1, 2, 3, 4\}$.

a Draw a table to show the sample space. Remember doubles like (1, 1), (2, 2) etc. are not allowed.

b Find:

i $P(1, 2)$

ii $P(4, 3)$

c Find the probability that:

i both numbers will be at least 3

ii the outcome will contain a 1 or a 4

iii the outcome will contain a 1 and a 4

iv the outcome will not contain a 3

PROBLEM-SOLVING AND REASONING 8, 9, 11, 12 | 8, 9, 12–14 | 9, 10, 13–15

8 The total sum is recorded from rolling two four-sided dice.

a Copy and complete this table, showing all possible totals that can be obtained.

b Find the probability that the total sum is:

i 2

ii 2 or 3

iii less than or equal to 4

iv more than 6

v at most 6

		1st			
		1	2	3	4
2nd	1	2	3		
	2				
	3				
	4				

9 Jill guesses the answers to two multiple-choice questions with options A, B, C, D or E.

a Copy and complete this table, showing all possible guesses that can be obtained.

b Find the probability that she will guess:

i (D, A)

ii the same letter

iii different letters

c Find the probability that Jill will get:

i exactly one of her answers correct

ii both of her answers correct

		Guess 1				
		A	B	C	D	E
Guess 2	A	(A, A)	(B, A)			
	B					
	C					
	D					
	E					

10 Many board games involve the rolling of two six-sided dice.

a Use a table to help find the probability that the sum of the two dice is:

i 12

ii 2 or 3

iii 11 or 12

iv less than or equal to 7

v less than 7

vi at least 10

vii at most 4

viii 1

b Which total sum has the highest probability and what is the probability of rolling that sum?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 A letter is chosen at random from the word MATHEMATICIAN. A second letter is then chosen and placed to the right of the first letter.

a How many outcomes sit in the sample space if selections are made:

i with replacement? **ii** without replacement?

b How many of the outcomes contain the same letter if selections are made:

i with replacement? **ii** without replacement?

12 Two letters are chosen from the word WOOD without replacement. Is it possible to obtain the outcome (O, O)? Explain why.

13 In a bag are five counters each of a different colour: green (G), yellow (Y), red (R), blue (B) and purple (P).

a If one counter is drawn from the bag and replaced and then a second is selected, find the probability that a green counter then a blue counter is selected. That is, find $P(\text{G, B})$.

b If the first counter selected is not replaced before the second is selected, find the probability that a green counter then a blue counter is selected.

14 A six-sided die and a ten-sided die have been tossed simultaneously. What total sum(s) has the highest probability?

15 A spinner numbered 1 to 50 is spun twice. Find the probability that the total from the two spins is:

a 100 **b** 51 **c** 99 **d** 52 **e** 55

ENRICHMENT — — 16

Two cards from the deck

16 Two cards are dealt to you from a pack of playing cards, which includes four of each of $\{2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J}, \text{Q}, \text{K}, \text{A}\}$. You keep both cards.

a Does this situation involve with replacement or without replacement?

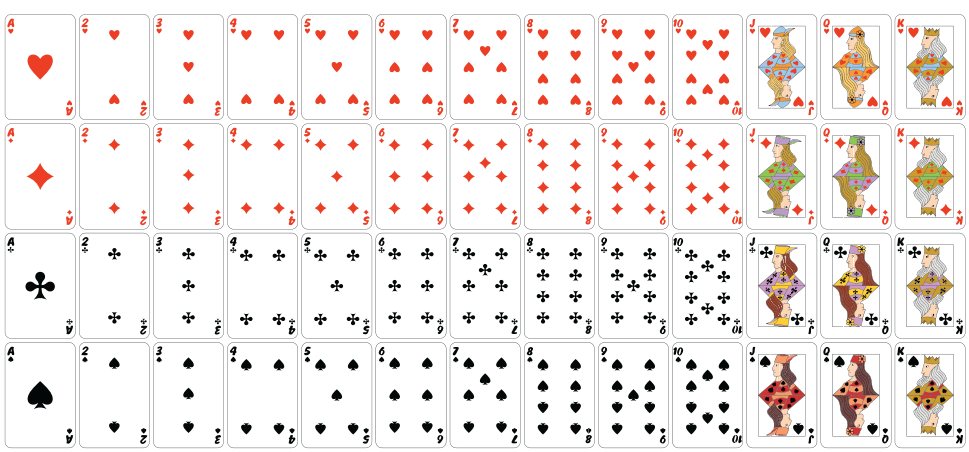
b How many outcomes would there be in your sample space (all possible selections of two cards)?

c What is the probability of receiving an ace of diamonds and an ace of hearts?

d Find the probability of obtaining two cards which are both:

i twos

ii hearts



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9E Using tree diagrams

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

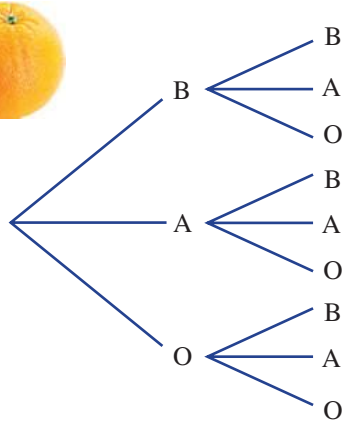
When experiments involve two or more steps, a tree diagram can be used to list the sample space. While tables are often used for two-step experiments, a tree diagram can be extended for experiments with any number of steps.

Interactive
Widgets
HOTsheets
Walkthrough

Let's start: What's the difference?

You are offered a choice of two pieces of fruit from a banana, an apple and an orange. You choose two at random. This tree diagram shows selection with replacement.





- How many outcomes will there be?
- How many of the outcomes contain two round fruits?
- How would the tree diagram change if the selection was completed without replacement? Would there be any difference in the answers to the above two questions? Discuss.

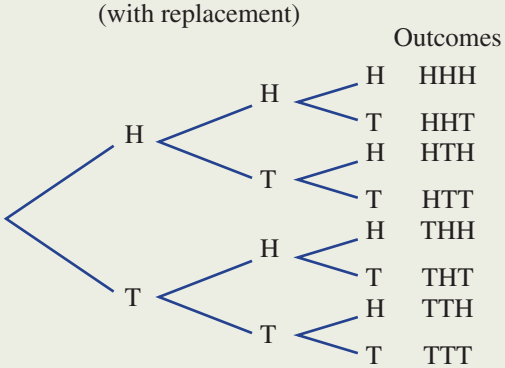
Key ideas

■ **Tree diagrams** are used to list the sample space for experiments with two or more steps.

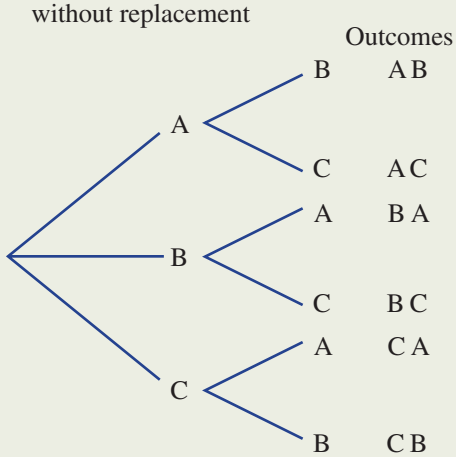
- The outcomes for each stage of the experiment are listed vertically and each stage is connected with branches.

For example:

Tossing a coin 3 times (with replacement)



Selecting 2 letters from {A, B, C} without replacement



In these examples, each set of branches produces outcomes that are all equally likely.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 8 Constructing a tree diagram

An experiment involves tossing two coins.

a Complete a tree diagram to show all possible outcomes.

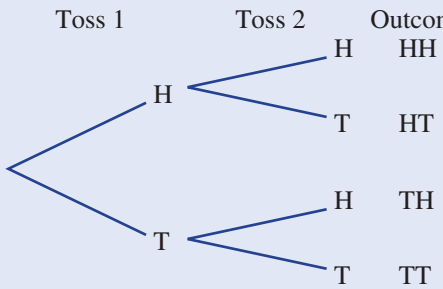
b What is the total number of outcomes?

c Find the probability of tossing:

i two tails **ii** one tail **iii** at least one head

SOLUTION

a



b The total number of outcomes is 4

c **i** $P(\text{TT}) = \frac{1}{4}$

ii $P(1\,\text{tail}) = \frac{2}{4} = \frac{1}{2}$

iii $P(\geq 1\,\text{head}) = \frac{3}{4}$

EXPLANATION

The tree diagram shows two coin tosses one after the other resulting in $2 \times 2 = 4$ outcomes.

There are four possibilities in the outcomes column.

One out of the four outcomes is TT.

Two outcomes have one tail: {HT, TH}

Three outcomes have at least one head: {HH, HT, TH}



A tree diagram is structured in a similar way to how tree branches are formed. Each branch of the tree represents a possible outcome of the experiment.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 9 Constructing a tree diagram without replacement

Two people are selected without replacement from a group of three: Annabel (A), Brodie (B) and Chris (C).

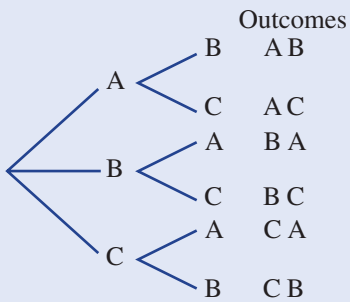
a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

i Annabel and Brodie **ii** Chris **iii** Chris or Brodie

SOLUTION

a



EXPLANATION

On the first choice, there are three options (A, B or C) but on the second choice there are only two remaining.

b i $P(\text{Annabel and Brodie}) = \frac{2}{6} = \frac{1}{3}$

2 of the 6 outcomes contain Annabel and Brodie (A, B) and (B, A).

ii $P(\text{Chris}) = \frac{4}{6} = \frac{2}{3}$

4 out of the 6 outcomes contain Chris.

iii $P(\text{Chris or Brodie}) = \frac{6}{6} = 1$

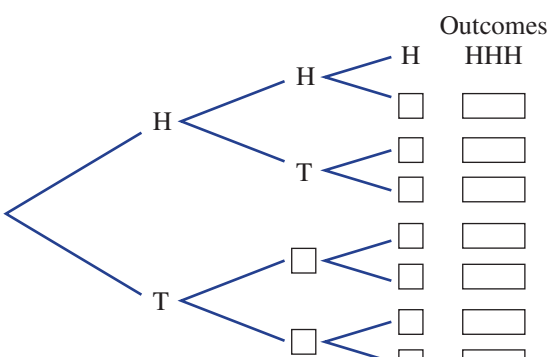
All of the outcomes contain at least one of Chris or Brodie.

Exercise 9E

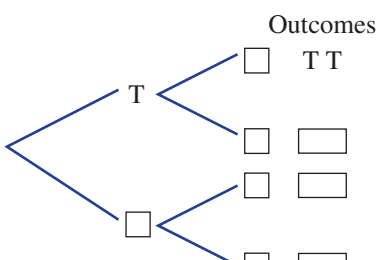
UNDERSTANDING AND FLUENCY	1–6	1–2(b), 3, 5–7	4, 6, 7

1 Find the total number of outcomes from these experiments *with replacement*. First, complete each tree diagram.

a Tossing a coin 3 times



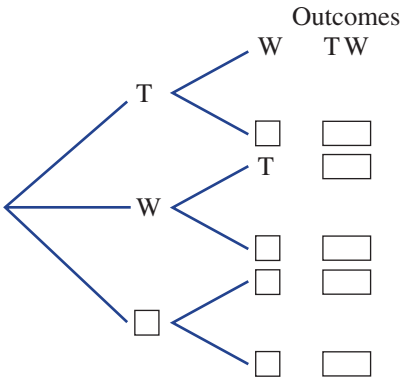
b Selecting 2 letters from the word TO



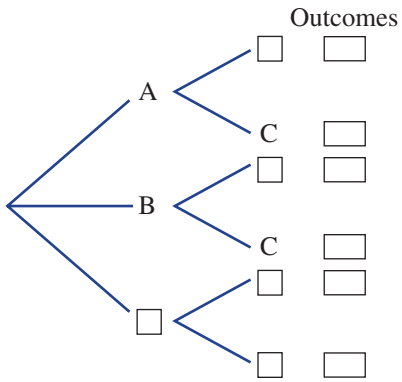
ISBN 978-1-108-46816-9 2ed © Palmer et al. 2019 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party. Updated March 2022

2 By first completing each tree diagram, find the total number of outcomes from these experiments *without replacement*.

a Selecting two letters from the word TWO



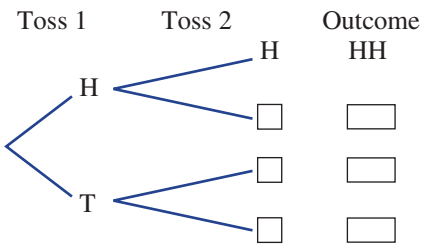
b Selecting two people from a group of three (A, B and C).



Example 8

3 A coin is tossed twice.

a Complete the tree diagram below to show all the possible outcomes.



b What is the total number of outcomes?

c Find the probability of obtaining:

i two heads **ii** one head **iii** at least one head **iv** at least one tail

4 A spinner with three numbers, 1, 2 and 3, is spun twice.

a List the set of possible outcomes, using a tree diagram.

b What is the total number of possible outcomes?

c Find the probability of spinning:

i two 3s

ii at least one 3

iii no more than one 2

iv two odd numbers

Example 9

5 Two people are selected without replacement from a group of three: Donna (D), Elle (E) and Fernando (F).

a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:

i Donna and Elle

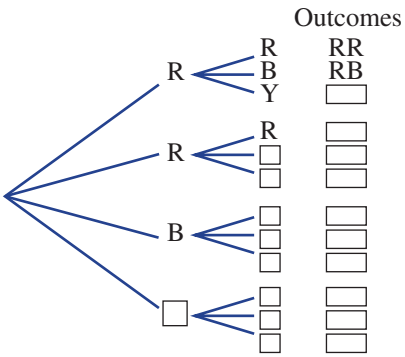
ii Fernando

iii Fernando or Elle

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 A drawer contains 2 red socks (R), 1 blue sock (B) and 1 yellow sock (Y). Two socks are selected at random without replacement.

a Complete the tree diagram below.



b Find the probability of obtaining:

i a red sock and a blue sock
ii two red socks
iii any pair of socks of the same colour
iv any pair of socks of different colour

7 A student who has not studied for a multiple-choice test decides to guess the answers for every question. There are three questions, and three choices of answer (A, B and C) for each question. If only one of the possible choices (A, B or C) is correct for each question, find the probability that the student guesses:

a 1 correct answer **b** 2 correct answers **c** 3 correct answers **d** 0 correct answers

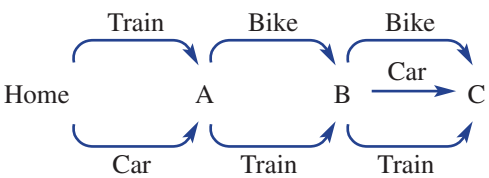
PROBLEM-SOLVING AND REASONING 8, 10 9, 10, 11 9, 11, 12

8 A discount supermarket shelf contains a large number of tomato tins and peach tins with no labels. There are an equal number of both types of tin all mixed together on the same shelf. You select four tins in a hurry. Use a tree diagram to help find the probability of selecting the correct number of tins of tomatoes and/or peaches for each of these recipe requirements. Assume that the shelves are continually refilled so that probabilities remain constant.



a You need four tins of tomatoes for a stew.
b You need four tins of peaches for a peach crumble.
c You need at least three tins of tomatoes for a bolognaise.
d You need at least two tins of peaches for a fruit salad.
e You need at least one tin of tomatoes for a vegetable soup.

9 Michael needs to deliver parcels to three places (A, B and C in order) in the city. This diagram shows the different ways that he can travel.



a Draw a tree diagram showing all the possible arrangements of transportation.
b What is the total number of possible outcomes?
c If Michael randomly chooses one of these outcomes, what is the probability he will use:
i the train all three times
ii the train exactly twice
iii his bike exactly once
iv different transport each time
v a car at least once



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 If a coin is tossed four times, use a tree diagram to find the probability that you receive:

- **a** 0 tails
- **b** 1 tail
- **c** 2 tails
- **d** 3 tails
- **e** 4 tails

11 **a** A coin is tossed 5 times. How many outcomes will there be?

b A coin is tossed n times. Write a rule for the number of outcomes in the sample space.

12 Use a tree diagram to investigate the probability of selecting two counters from a bag of 3 black and 2 white counters if the selection is drawn:

- **a** with replacement
- **b** without replacement

Is there any difference?

ENRICHMENT — — 13

Selecting matching clothes

13 A man randomly selects a tie from his collection of one green and two red ties, a shirt from a collection of one red and two white, and either a red or black hat. Use a tree diagram to help find the probability that the man selects a tie, shirt and hat according to the following descriptions.

- **a** A red tie, red shirt and black hat
- **b** All three items red
- **c** One item red
- **d** Two items red
- **e** At least two items red
- **f** Green hat
- **g** Green tie and a black hat
- **h** Green tie or a black hat
- **i** Not a red item
- **j** Red tie or white shirt or black hat



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9F Using relative frequencies to estimate probabilities

Interactive
Widgets
HOTsheets
Walkthrough

In some situations it may not be possible to list all the outcomes in the sample space and find theoretical probabilities. If this is the case, then an experiment, survey or simulation can be conducted to find relative frequencies. Provided the experiment includes a sufficient number of trials, these probabilities can be used to estimate the chance of particular events. Experimental probability is frequently used in science experiments and for research in medicine and economics.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Newspaper theories

A tabloid newspaper reports that of 10 people interviewed in the street 5 had a dose of the flu. At a similar time, a medical student tested 100 people and found that 21 had the flu.

- What is the experimental probability of having the flu, according to the newspaper's survey?
- What is the experimental probability of having the flu, according to the medical student's results?
- Which of the two sets of results would be most reliable and why? Discuss the reasons.
- Using the results from the medical student, how many people would you expect to have the flu in a group of 1000 and why?



Key ideas

- **Experimental probability** is calculated using relative frequencies from the results of an experiment or survey.

  $$\text{Experimental probability} = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in the experiment}} = \text{relative frequency}$$

- The **long-run proportion** is the experimental probability for a sufficiently large number of trials.
- The **expected number of occurrences** = probability × number of trials.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 10 Finding the experimental probability

A box contains an unknown number of coloured balls. A ball is drawn from the box and then replaced. The procedure is repeated 100 times and the colour of the ball drawn is recorded each time. Twenty-five red balls were recorded.

a Find the experimental probability for the number of red balls.

b Find the expected number of red balls drawn if the procedure was repeated 500 times.

SOLUTION

a $P(\text{red balls}) = \frac{25}{100}$
$= 0.25$

b Expected number of red balls in 500
$= 0.25 \times 500$
$= 125$

EXPLANATION

$P(\text{red balls}) = \frac{\text{number of red balls drawn}}{\text{total number of balls drawn}}$

There are 25 red balls and 100 balls in total.

Expected number of occurrences
= probability × number of trials

Exercise 9F

UNDERSTANDING AND FLUENCY 1–5 3–6 5, 6

1 This table shows the results of three different surveys of how many people in Perth use public transport.

Survey	Number who use public transport	Survey size	Experimental probability (relative frequency)
A	2	10	$\frac{2}{10} = 0.2$
B	5	20	_____
C	30	100	_____

a What are the two missing numbers in the experimental probability list?

b Which survey should be used to estimate the probability that a person uses public transport and why?

2 The experimental probability of Jess hitting a bullseye on a dartboard is 0.05 (or $\frac{5}{100}$).

How many bullseyes would you expect Jess to get if he threw the following number of darts?

a 100 darts

b 200 darts

c 1000 darts

d 80 darts

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 The results of tossing a drawing pin and observing how many times the pin lands with the spike pointing up are shown in the table. Results are recorded at different stages of the experiment.

Number of throws	Frequency (spike up)	Experimental probability
1	1	1.00
5	2	0.40
10	5	0.50
20	9	0.45
50	18	0.36
100	41	0.41

Which experimental probability would you choose to best represent the probability that the pin will land spike up? Why?

Example 10

4 A bag contains an unknown number of counters, and a counter is selected from the bag and then replaced. The procedure is repeated 100 times and the colour of the counter is recorded each time. Sixty of the counters drawn were blue.

a Find the experimental probability for the number of blue counters.

b Find the expected number of blue counters drawn if the procedure was repeated:

i 100 times **ii** 200 times **iii** 600 times

5 In an experiment involving 200 people chosen at random, 175 people said that they owned a home computer.

a Calculate the experimental probability of choosing a person who owns a home computer.

b Find the expected number of people who own a home computer from the following group sizes.

i 400 people **ii** 5000 people **iii** 40 people

6 By calculating the experimental probability, estimate the chance that each of the following events will occur.

a Nat will walk to work today, given that she walked to work five times in the last 75 working days.

b Mike will win the next game of cards if, in the last 80 games, he has won 32.

c Brett will hit the bullseye on the dartboard with his next attempt if, in the last 120 attempts, he was successful 22 times.



PROBLEM-SOLVING AND REASONING 7, 8, 11, 12 | 8, 9, 11–13 | 8–10, 12–14

7 A six-sided die is rolled 120 times. How many times would you expect the following events to occur?

a 6

b 1 or 2

c a number less than 4

d a number that is at least 5

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 The colour of cars along a highway was noted over a short period of time and summarised in this frequency table.

Colour	White	Silver	Blue	Green
Frequency	7	4	5	4

a How many cars had their colour recorded?

b Find the experimental probability that a car's colour is:

i blue

ii white

c If the colour of 100 cars was recorded, find the expected number of:

i blue cars

ii green cars

iii blue or green cars



9 The letters from a three-letter word are written separately onto cards. A card is chosen at random and replaced and this is repeated 100 times. The results are shown in the table.

Letter	E	S
Frequency	64	36

a Find the experimental probability of obtaining an E.

b Based on these results, what is the three-letter word likely to be?

10 A spinner is divided into three regions not necessarily of equal size. The regions are numbered 1, 2 and 3 and the spinner is spun 50 times. The table shows the results.

Number	1	2	3
Frequency	26	11	13

a Find the experimental probability of obtaining:

i a 1 **ii** at least a 2 **iii** a 1 or a 3

b Based on these results, how many 3s would you expect if the spinner is spun 300 times?

c In fact, the spinner is divided up using simple and common fractions. Give a likely example describing how the regions might be divided.

11 Phil tosses a fair six-sided die 10 times and receives 9 sixes.

a Find the experimental probability of rolling a six.

b Is it likely that Phil will receive the same number of sixes if he tosses the die 10 more times? Explain.

12 Do you think that a coin is fair or biased given the following experimental results? Give reasons.

a 47 tails out of 100 tosses

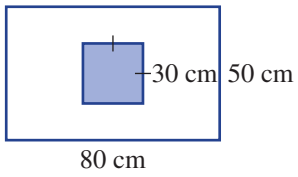
b 23 heads out of 24 tosses

c 1 tail out of 1 toss

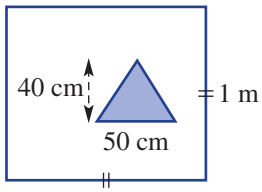
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 One hundred darts are randomly thrown at the given dartboards. No darts miss the dartboard entirely. How many darts do you expect to hit the blue shaded region? Give reasons.

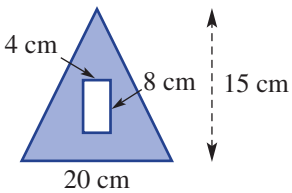
a



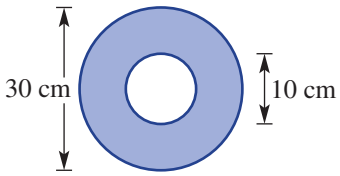
b



c



d



14 Decide if the following statements are true or false.

- **a** The experimental probability is always equal to the theoretical probability.
- **b** The experimental probability can be greater than the theoretical probability.
- **c** If the experimental probability is zero, then the theoretical probability is zero.
- **d** If the theoretical probability is zero, then the experimental probability is zero.

ENRICHMENT	—	—	15, 16

More than a guessing game

15 A bag of 10 counters includes counters with four different colours. The results from drawing and replacing one counter at a time for 80 trials are shown in this table.
Use the given information to find out how many counters of each colour were likely to be in the bag.

Colour	Total
Blue	26
Red	17
Green	29
Yellow	8

16 A box of 12 chocolates, all of which are the same size and shape, include five different centres. The results from selecting and replacing one chocolate at a time for 60 trials are shown in this table.
Use the given information to find out how many chocolates of each type were likely to be in the box.

Centre	Total
Strawberry	11
Caramel	14
Coconut	9
Nut	19
Mint	7

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9G Mean, median and mode REVISION

Interactive
Widgets
HOTsheets
Walkthrough

The discipline of statistics involves collecting and summarising data. It also involves drawing conclusions and making predictions, which is why many of the decisions we make today are based on statistical analysis. The type and amount of product stocked on supermarket shelves, for example, is determined by the sales statistics and other measures such as average cost and price range.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Game purchase

Arathi purchases 7 computer games at a sale. Three games cost $20 each, 2 games cost $30, 1 game costs $50 and the last game costs $200.

- Recall and discuss the meaning of the words 'mean', 'median' and 'mode'.
- Can you work out the mean, median or mode for the cost of Arathi's games?
- Which of the mean, median or mode gives the best 'average' for the cost of Arathi's games?
- Why is the mean greater than the median in this case?

Key ideas

- Statisticians use **summary statistics** to highlight important aspects of a data set. These are summarised below.
- Some summary statistics are called **measures of location**.
  - The two most commonly used measures of location are the **mean** and the **median**. These are also called '**measures of centre**' or '**measures of central tendency**'. Mean and median can only be applied to numerical data.
  - The **mean** is sometimes called the '**arithmetic mean**' or the '**average**'. The formula used for calculating the mean, $\bar{x}$, is:

$$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

For example, in the following data set

5	7	2	5	1

the mean is $\frac{5+7+2+5+1}{5} = 20 \div 5 = 4$

  - The **median** divides an ordered data set into two sets, each of which contain the same number of data values. It is often called the 'middle value'. The median is found by firstly ensuring the data values are in ascending order, then selecting the 'middle' value.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

If the number of values is odd, simply choose the one in the middle.

2 2 3 4 6 9 9

median = 4

If the number of values is even, find the average of the two in the middle.

2 2 3 4 7 9 9 9

Median = (4 + 7) ÷ 2 = 5.5

- The **mode** of a data set is the most frequently occurring data value. There can be more than one mode. When there are two modes, the data is said to be **bimodal**. The mode can be very useful for categorical data. It can also be used for numerical data, but it may not be an accurate measure of centre. For example, in the data set below the mode is 10, but it is the largest data value.

1	2	3	10	10

- An **outlier** is a data value which is significantly greater than or less than the other data values in the data set. The inclusion of an outlier in a data set can inflate or deflate the mean and give a false impression of the 'average'.
  For example:
  In the data set 1, 2, 3, 4, 5, the mean is 3 and median is 3.
  If the data value 5 is changed to 20, it is now an outlier.
  In the data set 1, 2, 3, 4, 20, the median is still 3, but the mean is now 6, which is greater than four of the five data values. In this case, the median is a better measure of centre than the mean.

Example 11 Finding measures of centre

For the given data sets, find:

i the mean ii the median iii the mode

a 5 2 4 10 6 1 2 9 6

b 17 13 26 15 9 10

SOLUTION

a i $\text{Mean} = \dfrac{5 + 2 + 4 + 10 + 6 + 1 + 2 + 9 + 6}{9}$

$= 5$

ii 1 2 2 4 (5) 6 6 9 10

Median = 5

iii Mode = 2 and 6

EXPLANATION

Find the sum of all the numbers and divide by the number of values.

First, order the data. The median is the middle value.

The data set is bimodal since there are two numbers with the highest frequency.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b i $\text{Mean} = \dfrac{17 + 13 + 26 + 15 + 9 + 10}{6}$
$= 15$

The sum is 90 and there are 6 values.

ii 9 10 13 15 17 26
$\underbrace{\quad\quad}_{14}$

$\text{Median} = \dfrac{13 + 15}{2}$
$= 14$

First, order the data.
Since there are two values in the middle, find the mean of them.

iii No mode

None of the values are repeated so there is no mode. The mode is not a useful tool for very small sets of data.

Example 12 Finding a data value for a required mean

The hours a shop assistant spends cleaning the store in eight successive weeks are:
8, 9, 12, 10, 10, 8, 5, 10.

a Calculate the mean for this set of data.

b How many hours would the shop assistant need to clean in the ninth week for the mean to equal 10?

SOLUTION

a $\text{Mean} = \dfrac{8 + 9 + 12 + 10 + 10 + 8 + 5 + 10}{8}$
$= 9$

EXPLANATION

Sum of the 8 data values is 72.

b Let a be the number of hours in the ninth week.

$\text{Require } \dfrac{72 + a}{8 + 1} = 10$
$\dfrac{72 + a}{9} = 10$
$72 + a = 90$
$a = 18$

$\therefore$ She needs to work 18 hours.

OR

Currently 8 data values with sum 72.
1 more makes 9
$9 \times 10 = 90$
$90 - 72 = 18$
$\therefore$ She needs to work 18 hours.

$72 + a$ is the total of the new data and $8 + 1$ is the new total number of data values. Set this equal to the required mean of 10.
Solve for a.

Write the answer.

Add the given data.

$\left(\begin{matrix}\text{number of} \\ \text{data values}\end{matrix}\right) \times (\text{mean}) = (\text{sum of data})$

(new sum) $-$ (old sum) $=$ extra data value

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9G REVISION

UNDERSTANDING AND FLUENCY	1–5	1, 3–5(½)	3–5(½)

1 Write the missing word.

a The mode is the most _____ value.

b The median is the _____ value.

c To calculate the _____, add up all the values and divide by the number of values.

2 Find the mean, median and mode for these simple ordered data sets.

a 1 2 2 2 4 4 6

b 1 4 8 8 9 10 10 10 12

c 1 5 7 7 8 10 11

d 3 3 6 8 10 12

e 7 11 14 18 20 20

f 2 2 2 4 10 10 12 14

Example 11

3 For the given data sets, find the:

i mean **ii** median **iii** mode

a 7 2 3 8 5 9 8

b 6 13 5 4 16 10 3 5 10

c 12 9 2 5 8 7 2 3

d 10 17 5 16 4 14

e 3.5 2.1 4.0 8.3 2.1

f 0.7 3 2.9 10.4 6 7.2 1.3 8.5

g 6 0 −3 8 2 −3 9 5

h 3 −7 2 3 −2 −3 4

4 These data sets include an outlier. Write down the outlier, then calculate the mean and the median. Include the outlier in your calculations.

a 5 7 7 8 12 33

b 1.3 1.1 1.0 1.7 1.5 1.6 −1.1 1.5

c −58 −60 −59 −4 −64

5 Decide if the following data sets are bimodal.

a 2 7 9 5 6 2 8 7 4

b 1 6 2 3 3 1 5 4 1 9

c 10 15 12 11 18 13 9 16 17

d 23 25 26 23 19 24 28 26 27

PROBLEM-SOLVING AND REASONING	6–8, 11, 12	7–9, 11–13	7–10, 12–14

6 In three races Paula recorded the times 25.1 seconds, 24.8 seconds and 24.1 seconds.

a What is the mean time of the races? Round to 2 decimal places.

b Find the median time.

Example 12

7 A netball player scored the following number of goals in her 10 most recent games:
15 14 16 14 15 12 16 17 16 15

a What is her mean number of goals per game?

b What number of goals does she need to score in the next game for the mean of her scores to be 16?

8 Stevie obtained the following scores on her first five Maths tests:
92 89 94 82 93

a What is her mean test score?

b If there is one more test left to complete, and she wants to achieve an average of at least 85, what is the lowest score Stevie can obtain for her final test?

9 Seven numbers have a mean of 8. Six of the numbers are 9, 7, 6, 4, 11 and 10. Find the seventh number.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Write down a set of 5 numbers which has the following values:

a mean of 5, median of 6 and mode of 7

b mean of 5, median of 4 and mode of 8

c mean of 4, median of 4 and mode of 4

d mean of 4.5, median of 3 and mode of 2.5

e mean of 1, median of 0 and mode of 5

f mean of 1, median of $1\frac{1}{4}$ and mode of $1\frac{1}{4}$

11 This data contains six houses prices in Darwin.

$324 000 $289 000 $431 000 $295 000 $385 000 $1 700 000

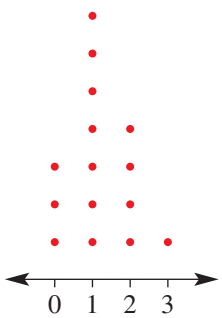
a Which price would be considered the outlier?

b If the outlier was removed from the data set, by how much would the median change? (First work out the median for each case.)

c If the outlier was removed from the data set, by how much would the mean change, to the nearest dollar? (First work out the mean for each case.)

12 Explain why outliers significantly affect the mean but not the median.

13 This dot plot shows the frequency of households with 0, 1, 2 or 3 pets.



a How many households were surveyed?

b Find the mean number of pets correct to 1 decimal place.

c Find the median number of pets.

d Find the mode.

e Another household with 7 pets is added to the list. Does this change the median? Explain.

14 This simple data set contains nine numbers.

1 2 2 2 2 2 3 4 5

a Find the median.

b How many numbers greater than 5 need to be added to the list to change the median? (Give the least number.)

c How many numbers less than 1 need to be added to the list to change the median? (Give the least number.)

ENRICHMENT — — 15

Formula to get an A

15 A school awards grades in Mathematics each semester according to this table.

Average score	Grade
90–100	A +
80 –	A
70 –	B +
60 –	B
50 –	C +
0 –	C

Ryan has scored the following results for four topics this semester and has one topic to go.

75 68 85 79

a What is Ryan's mean score so far? Round to 1 decimal place.

b What grade will Ryan get for the semester if his fifth score is:

i 50? **ii** 68? **iii** 94?

c Find the maximum average score Ryan can receive for the semester. Is it possible for him to get an A+?

d Find the least score that Ryan needs in his fifth topic for him to receive an average of:

i B+ **ii** A

e Write a rule for the mean score M for the semester if Ryan receives a mark of m in the fifth topic.

f Write a rule for the mark m in the fifth topic if he wants an average of M for the semester.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9H Stem-and-leaf plots

Interactive
Widgets
HOTsheets
Walkthrough

Stem-and-leaf plots (or stem plots) are commonly used to display a single data set or two related data sets. They help to show how the data is distributed like a histogram but retain all the individual data elements so no detail is lost. The median and mode can be easily read from a stem-and-leaf plot because all the data sits in order.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

For this data below, digits representing the stem give the tens and digits representing the leaves give the units.

Let's start: Ships vs Chops

At a school, Ms Ships' class and Mr Chops' class sit the same exam. The marks are displayed using this back-to-back stem-and-leaf plot. Discuss the following.

- Which class had the most students?
- What were the lowest and highest marks from each class?
- What were the median marks from each class?
- Which class could be described as symmetrical and which as skewed?
- Which class had the better results?

Ms Ships' class		Mr Chops' class
31	5	011357
8875	6	2355799
64421	7	899
743	8	03
6	9	1

7 | 8 means 78

Key ideas

- Stem-and-leaf plots are very useful for displaying two-digit numbers, such as ages or test results.
- A **stem-and-leaf plot** uses a stem number and leaf number to represent data.
  - The data is shown in two parts: a stem and a leaf.
  - The 'key' tells you how the plot is to be read.
  - The graph is similar to a histogram on its side or a bar graph with class intervals, but there is no loss of detail of the original data.

Ordered stem-and-leaf plot

Stem	Leaf
1	26
2	2347
3	124789
4	23458
5	79

2 | 4 represents 24 people

A key is added to show the place value of the stems and leaves

- **Back-to-back stem-and-leaf plots** can be used to compare two sets of data. The stem is drawn in the middle with the leaves on either side.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	45889
1	8	003367
753	9	1236
8441	10	39
950	11	1
1	12	

10 | 9 represents 109

81 lowest winning score

Symmetrical

Skewed

111 highest losing score

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- A **cluster** is a set of data values that are similar to each other. They form the peak of the distribution. In the data sets above, the losing scores are clustered around the high 70s and low 80s.
- **Symmetrical data** will produce a graph that is evenly distributed above and below the central cluster.
- **Skewed data** will produce a graph that includes data bunched to one side of the centre.

Example 13 Constructing and using a stem-and-leaf plot

Consider the following set of data.

0.3	2.5	4.1	3.7	2.0	3.3	4.8	3.3	4.6	0.1	4.1	7.5	1.4	2.4
5.7	2.3	3.4	3.0	2.3	4.1	6.3	1.0	5.8	4.4	0.1	6.8	5.2	1.0

a Organise the data into an ordered stem-and-leaf plot.
b Find the median.
c Find the mode.
d Describe the data as symmetrical or skewed.

SOLUTION

a

Stem	Leaf
0	1 1 3
1	0 0 4
2	0 3 3 4 5
3	0 3 3 4 7
4	1 1 1 4 6 8
5	2 7 8
6	3 8
7	5

3 | 4 means 3.4

EXPLANATION

The minimum is 0.1 and the maximum is 7.5 so stems range from 0 to 7. Place leaves in order from smallest to largest. Since some numbers appear more than once, e.g. 0.1, their leaf (1) appears the same number of times.

b $\text{Median} = \dfrac{3.3 + 3.4}{2}$
$= 3.35$

There are 28 data values. The median is the average of the two middle values (the 14th and 15th values).

c Mode is 4.1.

The most common value is 4.1.

d Data is approximately symmetrical.

The distribution of numbers is approximately symmetrical about the stem containing the median.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 14 Constructing back-to-back stem-and-leaf plots

A shop owner has two jeans shops. The daily sales in each shop over a 16-day period are monitored and recorded as follows.

Shop A

3 12 12 13 14 14 15 15 21 22 24 24 24 26 27 28

Shop B

4 6 6 7 7 8 9 9 10 12 13 14 14 16 17 27

a Draw a back-to-back stem-and-leaf plot with an interval of 10.

b Compare and comment on differences between the sales made by the two shops.

SOLUTION

a

Shop A		Shop B
3	0	4 6 6 7 7 8 9 9
5 5 4 4 3 2 2	1	0 2 3 4 4 6 7
8 7 6 4 4 4 2 1	2	7

1 | 3 means 13

EXPLANATION

The data for each shop is already ordered. Stems are in intervals of 10. Record leaf digits for Shop A on the left and for Shop B on the right.

b Shop A has the highest number of daily sales. Its sales are generally between 12 and 28, with one day of very low sales of 3. Shop B sales are generally between 4 and 17 with only one high sale day of 27.

Look at both sides of the plot for the similarities and differences.

Exercise 9H

UNDERSTANDING AND FLUENCY 1, 2, 3(½), 4, 5(½), 6 | 2, 3(½), 5(½), 6 | 3(½), 5(½), 7

1 This stem-and-leaf plot shows the number of minutes Alexis spoke on her phone for a number of calls.

Stem	Leaf
0	8
1	5 9
2	1 1 3 7
3	4 5

2 | 1 means 21 minutes

a How many calls are represented by the stem-and-leaf plot?

b What is the length of the:

- **i** shortest phone call?
- **ii** longest phone call?

c What is the mode (the most common call time)?

d What is the median call time (middle value)?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 This back-to-back stem-and-leaf plot shows the thickness of tyre tread on a selection of cars from the city and country.

City		Country
8731000	0	6889
86310	1	045569
1	2	344

1 | 3 means 13 mm

a How many car tyres were tested altogether?

b What was the smallest tyre tread thickness in the:
i city? **ii** country?

c What was the largest tyre tread thickness in the:
i city? **ii** country?

d Find the median tyre tread thickness for tyres in the:
i city **ii** country

e Is the distribution of tread thickness for city cars more symmetrical or skewed?

f Is the distribution of tread thickness for country cars more symmetrical or skewed?

Example 13

3 For each of the following data sets:
i organise the data into an ordered stem-and-leaf plot
ii find the median
iii find the mode
iv describe the data as symmetrical or skewed

a 41 33 28 24 19 32 54 35 26 28 19 23 32 26 28

b 31 33 23 35 15 23 48 50 35 42 45 15 21 45
51 31 34 23 42 50 26 30 45 37 39

c 34.5 34.9 33.7 34.5 35.8 33.8 34.3 35.2 37.0 34.7
35.2 34.4 35.5 36.5 36.1 33.3 35.4 32.0 36.3 34.8

d 167 159 159 193 161 164 167 157 158 175 177 185
177 202 185 187 159 189 167 159 173 198 200

4 The number of vacant rooms in a motel each week over a 20-week period is shown below.
12 8 11 10 21 12 6 11 12 16 14 22 5 15 20 6 17 8 14 9

a Draw a stem-and-leaf plot of this data.

b In how many weeks were there fewer than 12 vacant rooms?

c Find the median number of vacant rooms.

Example 14

5 For each of the following sets of data:
i draw a back-to-back stem-and-leaf plot
ii compare and comment on the difference between the two data sets

a Set A: 46 32 40 43 45 47 53 54 40 54 33 48 39 43
Set B: 48 49 31 40 43 47 48 41 49 51 44 46 53 44

b Set A: 1 43 24 26 48 50 2 2 36 11 16 37 41 3 36
6 8 9 10 17 22 10 11 17 29 30 35 4 23 23
Set B: 9 18 19 19 20 21 23 24 27 28 31 37 37 38 39 39 39
40 41 41 43 44 44 45 47 50 50 51 53 53 54 54 55 56

c Set A: 0.7 0.8 1.4 8.8 9.1 2.6 3.2 0.3 1.7 1.9 2.5 4.1 4.3 3.3 3.4
3.6 3.9 3.9 4.7 1.6 0.4 5.3 5.7 2.1 2.3 1.9 5.2 6.1 6.2 8.3
Set B: 0.1 0.9 0.6 1.3 0.9 0.1 0.3 2.5 0.6 3.4 4.8 5.2 8.8 4.7 5.3
2.6 1.5 1.8 3.9 1.9 0.1 0.2 1.2 3.3 2.1 4.3 5.7 6.1 6.2 8.3

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 **a** Draw back-to-back stem-and-leaf plots for the final scores of St Kilda and Collingwood in the 24 AFL games given here.

St Kilda:	126	68	78	90	87	118
	88	125	111	117	82	82
	80	66	84	138	109	113
	122	80	94	83	106	68
Collingwood:	104	80	127	88	103	95
	78	118	89	82	103	115
	98	77	119	91	71	70
	63	89	103	97	72	68

b In what percentage of games did each team score more than 100 points?

c Comment on the distribution of the data for each team.

7 The data below gives the maximum temperature each day for a three-week period in spring.

18	18	15	17	19	17	21
20	15	17	15	18	19	19
20	22	19	17	19	15	17

Use a stem-and-leaf plot to determine:

a how many days the temperature was higher than 18°C

b the median temperature

c the difference in the minimum and maximum temperatures

PROBLEM-SOLVING AND REASONING 8, 10, 11 | 8, 10–12 | 8–10, 12, 13

8 This stem-and-leaf plot shows the time taken, in seconds, by Helena to run 100 m in her last 25 races.

Stem	Leaf
14	9
15	4 5 6 6 7 7 7 8 9
16	0 0 1 1 2 2 3 4 4 5 5 5 7 7
17	2

14 | 9 represents 14.9 seconds

a Find Helena's median time.

b What is the difference between the slowest and fastest time?

c If in her 26th race Helena's time was 14.8 seconds and this was added to the stem-and-leaf plot, would her median time change? If so, by how much?

9 Two brands of batteries were tested to determine their lifetime in hours. The data below shows the lifetime of 20 batteries of each brand.

Brand A:	7.3	8.2	8.4	8.5	8.7	8.8	8.9	9.0	9.1	9.2
	9.3	9.4	9.4	9.5	9.5	9.6	9.7	9.8	9.9	9.9
Brand B:	7.2	7.3	7.4	7.5	7.6	7.8	7.9	7.9	8.0	8.1
	8.3	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.8	9.8

a Draw a back-to-back stem-and-leaf plot for this data.

b How many batteries from each brand lasted more than 9 hours?

c Compare the two sets of data and comment on any similarities or differences.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 This ordered stem-and-leaf plot has some unknown digits.

a What is the value of c?

b What is the smallest number in the data set?

c What values could the following pronumerals take?

i a **ii** b

0	238
1	15a8
c	0b66
3	259

3 | 5 means 0.35

11 The back-to-back stem-and-leaf plot below shows the birth weight in kilograms of babies born to mothers who do or don't smoke.

Birth weight of babies

Smoking mothers		Non-smoking mothers
4322	2	4
99876655	2*	89
432111000	3	001223
655	3*	55566778
1	4	
	4*	556

2 | 4 means 2.4

2* | 5 means 2.5

a What percentage of babies born to smoking mothers have a birth weight of less than 3 kg?

b What percentage of babies born to non-smoking mothers have a birth weight of less than 3 kg?

c Compare and comment on the differences between the birth weights of babies born to mothers who smoke and those born to mothers who don't smoke.

12 Explain why in a symmetrical distribution the mean is close to the median.

13 Find the median if all the data in each back-to-back stem-and-leaf plot was combined.

a

5	3	89
9771	4	022368
86522	5	3379
740	6	14

4 | 2 means 42

b

3	16	033679
9661	17	011488
875540	18	2267
2	19	01

16 | 3 means 16.3

ENRICHMENT — — 14

How skewed?

14 Skewness can be positive or negative. If the tail of the distribution is pointing up in a stem-and-leaf plot (towards the smaller numbers), then we say the data is negatively skewed. If the tail is pointing in the reverse direction, then the data is positively skewed.

1	3
2	14
3	027
4	113899
5	04457

Negatively Skewed data

3 | 2 means 32

a Find the mean (correct to 2 decimal places) and the median for the above data.

b Which of the mean or median is higher for the given data? Can you explain why?

c Which of the mean or median would be higher for a set of positively skewed data? Why?

d What type of distribution would lead to the median and mean being quite close?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9I Grouping data into classes FRINGE







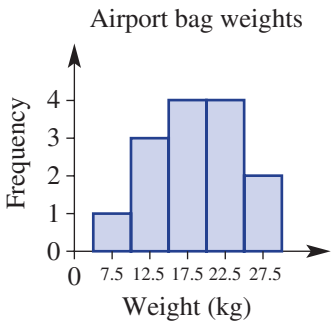


For some data, especially measurements, it makes sense to group the data and then record the frequency for each group to produce a frequency table. For numerical data, a graph generated from a frequency table gives a histogram. Like a stem-and-leaf plot, a histogram shows how the data is distributed across the full range of values. A histogram, for example, is used to display the level of exposure of the pixels in an image in digital photography. It uses many narrow columns to show how the luminance values are distributed across the scale from black to white.

Let's start: Baggage check

This histogram shows the distribution of the weight of a number of bags checked at the airport.

- How many bags had a weight in the range 10–15 kg?
- How many bags were checked in total?
- Is it possible to determine the exact mean, median or mode of the weight of the bags by looking at the histogram? Discuss.
- Describe the distribution of checked bag weights for the given graph.



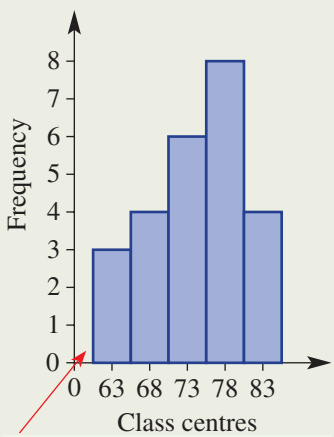
Key ideas

- A **frequency table** shows the number of values within a set of categories or **class intervals**.
- Grouped numerical data can be illustrated using a **histogram**.
  - The height of a column corresponds to the frequency of values in that class interval.
  - There are usually no gaps between columns.
  - The scales are evenly spread with each bar spreading across the boundaries of the class interval.
  - A **percentage frequency histogram** shows the frequencies as percentages of the total.
- Like a stem-and-leaf plot, a histogram can show if the data is **skewed** or **symmetrical**.

Frequency table

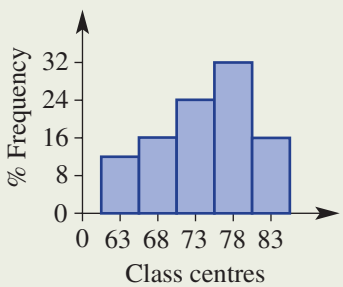
Class	Class centre	Frequency	Percentage frequency
61–65	63	3	12
66–70	68	4	16
71–75	73	6	24
76–80	78	8	32
81–85	83	4	16
	Total	25	100

Frequency histogram



This gap should be half as wide as the columns

Percentage frequency histogram



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 15 Constructing frequency tables and histograms

The data below shows the number of hamburgers sold each hour by a 24-hour fast-food store during a 50-hour period.

1	10	18	14	20	11	19	10	17	21
5	16	7	15	21	15	10	22	11	18
12	12	3	12	8	12	6	5	14	14
14	4	9	15	17	19	6	24	16	17
14	11	17	18	19	19	19	18	18	20

a Set up and complete a grouped frequency table, using class intervals 0–4, 5–9 etc.

b Construct a frequency histogram.

c How many hours did the fast-food store sell:

i fewer than 10 hamburgers?

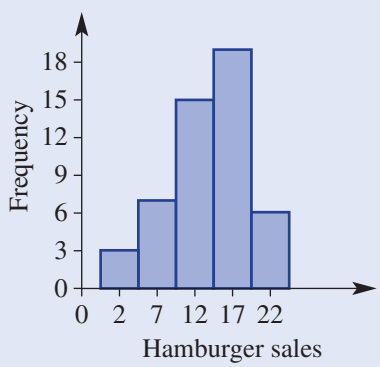
ii at least 15 hamburgers?

SOLUTION

a

Class	Class centre	Frequency
0–4	2	3
5–9	7	7
10–14	12	15
15–19	17	19
20–24	22	6
	Total	50

b



c **i** $3 + 7 = 10$ hours

ii $19 + 6 = 25$ hours

EXPLANATION

Create classes and class centres. Record the number of data values in each interval in the frequency column.

Create a frequency histogram with frequency on the vertical axis and the class intervals on the horizontal axis. The height of the column shows the frequency of that interval.

Fewer than 10 hamburgers covers the 0–4 and 5–9 intervals.

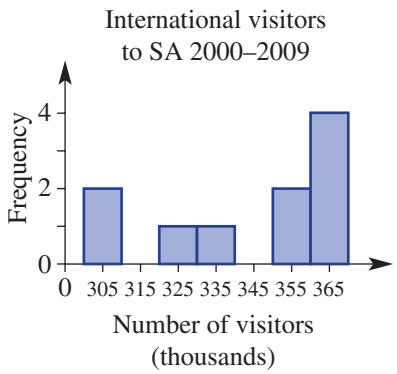
At least 15 hamburgers covers the 15–19 and 20–24 intervals.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9I FRINGE

UNDERSTANDING AND FLUENCY	1–5, 7	3, 4, 6, 7	4, 6, 7

1 This frequency histogram shows in how many years the number of international visitors to South Australia is within a given range for the decade from 2000 to 2009.



a How many years in the decade were there less than 330 000 international visitors?

b Which range of visitor numbers had the highest frequency?

2 Write down the missing numbers in these frequency tables, i.e. find the values of the pronumerals.

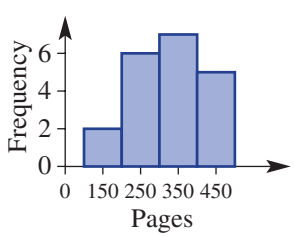
a

Class interval	Frequency	Percentage frequency
0–4	1	10
5–9	3	c
10–14	4	d
a–19	b	e
Total	10	f

b

Class interval	Frequency	Percentage frequency
40–49	20	20
a–59	28	b
60–69	12	c
70–79	d	40
Total	100	e

3 This frequency histogram shows in how many books the number of pages is within a given interval for some textbooks selected from a school library.



a How many textbooks had between 100 and 200 pages?

b How many textbooks were selected from the library?

c What percentage of textbooks had between:

i 200 and 300 pages?

ii 200 and 400 pages?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 15

4 The data below shows the number of ice-creams sold from an ice-cream van over a 50-day period.

0	5	0	35	14	15	18	21	21	36	45	2	8
2	2	3	17	3	7	28	35	7	21	3	46	47
1	1	3	9	35	22	7	18	36	3	9	2	
11	37	37	45	11	12	14	17	22	1	2	2	



a Set up and complete a grouped frequency table using class intervals 0–9, 10–19 etc.

b Construct a frequency histogram.

c How many days did the ice-cream van sell:

i fewer than 20 ice-creams?

ii at least 30 ice-creams?

d What percentage of days were 20 or more ice-creams sold?

5 The data below shows the mark out of 100 on the science exam for 60 Year 9 students.

50	67	68	89	82	81	50	50	89	52	60	82	52	60	87	89	71	73	75	83
86	50	52	71	80	95	87	87	87	74	60	60	61	63	63	65	82	86	96	88
50	94	87	64	64	72	71	72	88	86	89	69	71	80	89	92	89	89	60	83

a Set up and complete a grouped frequency table, using class intervals 50–59, 60–69 etc. Include a percentage frequency column rounding to 2 decimal places where necessary.

b Construct a frequency histogram.

c **i** How many marks were less than 70 out of 100?

ii What percentage of marks were at least 80 out of 100? Give your answer correct to 1 decimal place.

6 The number of goals kicked by a country footballer in each of his last 30 football matches is given below.

8	9	3	6	12	14	8	3	4	5	2	5	6	4	13
8	9	12	11	7	12	14	10	9	8	12	10	11	4	5

a Organise the data into a grouped frequency table using a class interval width of 3 starting at 0.

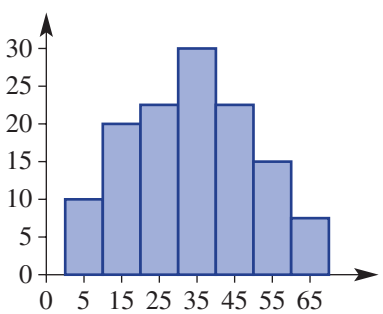
b Draw a frequency histogram for the data.

c In how many games did the player kick fewer than six goals?

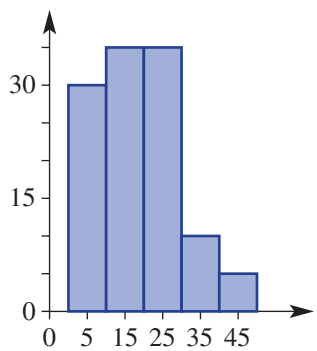
d In how many games did he kick more than 11 goals?

7 Which one of these histograms illustrates a symmetrical data set and which one shows a skewed data set?

a



b



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING 8, 9, 11 8, 9, 11, 12 8, 10, 11, 13

8 Find the unknown numbers (pronumerals) in these frequency tables.

a

Class interval	Frequency	Percentage frequency
10–14	a	15
15–19	11	b
20–24	7	c
25–29	d	10
30–34	e	f
Total	40	g

b

Class interval	Frequency	Percentage frequency
0–2	a	2
3–5	9	b
6–8	c	16
9–11	12	d
12–14	e	f
Total	50	g

9 This percentage frequency histogram shows the heights of office towers in a city.

a What percentage of office towers are:

i between 50 m and 100 m?

ii less than 150 m?

iii no more than 200 m?

iv at least 100 m?

v between 100 m and 150 m or greater than 200 m?

b If the city had 100 office towers, how many have a height of:

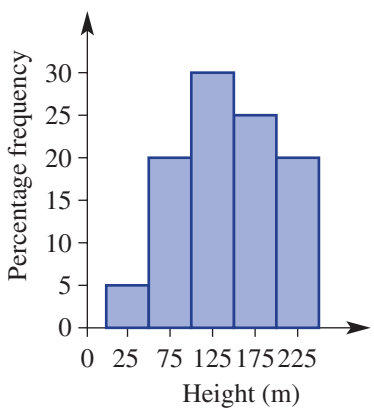
i between 100 m and 150 m?

ii at least 150 m?

c If the city had 40 office towers, how many have a height of:

i between 0 m and 50 m?

ii no more than 150 m?



10 The data below shows the length of overseas phone calls (in minutes) made by a particular household over a six-week period.

1.5	1	1.5	1	4.8	4	4	10.1	9.5	1	3
8	5.9	6	6.4	7	3.5	3.1	3.6	3	4.2	4.3
4	12.5	10.2	10.3	4.5	4.5	3.4	3.5	3.5	5	3.5
3.6	4.5	4.5	12	11	12	14	14	12	13	10.8
12.1	2.4	3.8	4.2	5.6	10.8	11.2	9.3	9.2	8.7	8.5

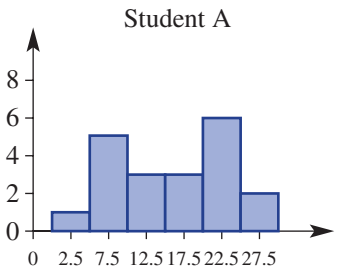
What percentage of phone calls were more than 3 minutes in length? Answer to 1 decimal place.

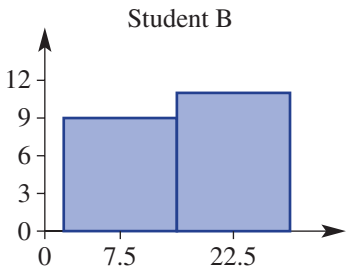
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Explain why you cannot work out the exact value for the mean, median and mode for a set of data presented in a histogram.

12 What can you work out from a frequency histogram that you cannot work out from a percentage frequency histogram? Completing Question **9** will provide a clue.

13 Two students show different histograms for the same set of data.





a Which histogram is more useful in helping to analyse the data?

b What would you advise student B to do differently when constructing the histogram?

ENRICHMENT — — 14

The distribution of weekly wages

14 The data below shows the weekly wages of 50 people in dollars.

400	500	552	455	420	424	325	204	860	894	464	379	563	230
940	384	370	356	345	380	720	540	654	678	628	656	670	725
740	750	730	766	760	700	700	768	608	576	890	920	874	860
450	674	725	612	605	600	548	670						

a What is the minimum weekly wage and the maximum weekly wage?

b **i** Organise the data into about 10 class intervals.

ii Draw a frequency histogram for the data.

c **i** Organise the data into about five class intervals.

ii Draw a frequency histogram for the data.

d Discuss the shapes of the two graphs. Which graph represents the data better and why?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9J Measures of spread: range and interquartile range

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

The mean, median and mode are three numbers that help define the centre of a data set; however, it is also important to describe the spread. Two teams of swimmers from different countries, for example, might have similar mean race times but the spread of race times for each team could be very different.

Let's start: Swim times

Two Olympic swimming teams are competing in the 4 × 100 m relay. The 100 m personal best times for the four members of each team are:

Team A 48.3 s, 48.5 s, 48.9 s, 49.2 s

Team B 47.4 s, 48.2 s, 49.0 s, 51.2 s

- Find the mean for each team.
- Which team's times are more spread out?
- What does the difference in the range of values for each team tell you about the spread?



Key ideas

- The **range** is the difference between the maximum and minimum values
  - Range = maximum value − minimum value
- If a set of numerical data is placed in order, from smallest to largest, then:
  - the middle number of the lower half is called the **lower quartile** (Q_1)
  - the middle number of the data is called the median (Q_2)
  - the middle number of the upper half is called the **upper quartile** (Q_3)
  - the difference between the upper quartile and lower quartile is called the **interquartile range** (IQR).
    $IQR = Q_3 - Q_1$
  - if there is an odd number of values, disregard the middle value (the median) before calculating Q_1 and Q_3.
- The range and interquartile range are measures of spread. They describe the dispersion of data in a data set.
- An **outlier** is a value that is not in the vicinity of the rest of the data. It is significantly higher or lower than the other data values.

Example 16 Finding the range and quartiles for an odd number of data values

The following data values are the results for a school mathematics test.

67 96 62 85 73 56 79 19 76 23 68 89 81

a List the data in order, from smallest to largest.

b Find the range.

c Find the:

i median (Q_2)

ii lower quartile (Q_1)

iii upper quartile (Q_3)

iv interquartile range (IQR)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION

a 19 23 56 62 67 68 73 76 79 81 85 89 96

Q_1 = 59, Q_2 = 73, Q_3 = 83

b Range = 96 − 19
= 77

c **i** $Q_2 = 73$

ii $Q_1 = \frac{56 + 62}{2} = 59$

iii $Q_3 = \frac{81 + 85}{2} = 83$

iv IQR = 83 − 59 = 24

EXPLANATION

Order the data.

Range = maximum value − minimum value

The median is 73. Since there is an odd number of values, exclude the number 73 before finding Q_1 and Q_3.

Q_1 (the middle value of the lower half) is halfway between 56 and 62.

Q_3 (the middle value of the upper half) is halfway between 81 and 85.

IQR = $Q_3 - Q_1$

Example 17 Finding quartiles for an even number of data values

Here is a set of measurements, collected by measuring the lengths, in metres, of 10 long-jump attempts.

6.7 9.2 8.3 5.1 7.9 8.4 9.0 8.2 8.8 7.1

a List the data in order, from smallest to largest.

b Find the range.

c Find the:

i median (Q_2)

ii lower quartile (Q_1)

iii upper quartile (Q_3)

iv interquartile range (IQR)

d Interpret the IQR.

SOLUTION

a 5.1 6.7 7.1 7.9 8.2 8.3 8.4 8.8 9.0 9.2

Q_1 = 6.7, Q_2 = 8.2, Q_3 = 8.8

b Range = 9.2 − 5.1
= 4.1

c **i** $Q_2 = \frac{8.2 + 8.3}{2}$
= 8.25 m

ii Q_1 = 7.1 m

iii Q_3 = 8.8 m

iv IQR = 8.8 − 7.1
= 1.7 m

d The middle 50% of jumps differed by less than 1.7 m.

EXPLANATION

Order the data to locate Q_1, Q_2 and Q_3.

Range = highest value − lowest value

Q_2 is halfway between 8.2 and 8.3.

The middle value of the lower half is 7.1.
The middle value of the upper half is 8.8.
IQR is the difference between Q_1 and Q_3.

The IQR is the range of the middle 50% of the data.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9J

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4	2, 3(½), 4	3(½), 4

1 This ordered data set shows the number of hours of sleep 11 people had the night before.
3 5 5 6 7 7 7 8 8 8 9

a Find the range (the difference between the highest and lowest values).

b Find the median (Q_2, the middle number).

c Disregard the median then find the:

i lower quartile (Q_1, the middle of the lower half)

ii upper quartile (Q_3, the middle of the upper half)

d Find the interquartile range (IQR, the difference between the upper quartile and lower quartile).

2 This ordered data set shows the number of fish Daniel caught in the 12 weekends that he went fishing during the year.
1 2 3 3 4 4 6 7 7 9 11 13

a Find the range (the difference between the highest and lowest values).

b Find the median (Q_2, the middle number).

c Split the data in half, then find:

i the lower quartile (Q_1, the middle of the lower half)

ii the upper quartile (Q_3, the middle of the upper half)

d Find the interquartile range (IQR, the difference between the upper quartile and lower quartile).



Example 16

3 For each of the following sets of data:

i list the set of data in order, from smallest to largest

ii find the range

iii find the median (Q_2)

iv find the lower quartile (Q_1)

v find the upper quartile (Q_3)

vi find the interquartile range (IQR)

a 5 7 3 6 2 1 9 7 11 9 0 8 5

b 38 36 21 18 27 41 29 35 37 30 30 21 26

c 180 316 197 176 346 219 183 253 228

d 256 163 28 520 854 23 367 64 43 787 12 343 76 3 28

e 1.8 1.9 1.3 1.2 2.1 1.2 0.9 1.7 0.8

f 10 35 0.1 2.3 23 12 0.02

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 17

4 The running time, in minutes, of 16 movies at the cinema were as follows:
123 110 98 120 102 132 112 140 120 139 42 96 152 115 119 128

a Find the range.

b Find the:

i median (Q_2)

ii lower quartile (Q_1)

iii upper quartile (Q_3)

iv interquartile range (IQR)

c Interpret the IQR.

PROBLEM-SOLVING AND REASONING 5, 6, 8 | 5–9 | 6, 7, 9, 10

5 The following data set represents the sale price, in thousands of dollars, of 14 vintage cars.
89 46 76 41 12 52 76 97 547 59 67 76 78 30

a For the 14 vintage cars, find the:

i lowest price paid

ii highest price paid

iii median price

iv lower quartile of the data

v upper quartile of the data

vi IQR

b Interpret the IQR for the price of the vintage cars.

c If the price of the most expensive vintage car increased, what effect would this have on Q_1, Q_2 and Q_3? What effect would it have on the mean price?



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 Find the interquartile range for the data in these stem-and-leaf plots.

a

Stem	Leaf
3	4 8 9
4	1 4 8 8
5	0 3 6 9
6	2 6

5 | 2 means 52

b

Stem	Leaf
17	5 8
18	0 4 6 7
19	1 1 2 9 9
20	4 4 7 8
21	2 6 8

21 | 3 means 21.3

7 Over a period of 30 days, Lara records how many fairies she sees in the garden each day. The data is organised into this frequency table.

Number of fairies each day	0	1	2	3	4	5
Frequency	7	4	8	4	6	1

a Find the median number of fairies seen in the 30 days.

b Find the interquartile range.

c If Lara changes her mind about the day she saw 5 fairies and now says that she saw 10 fairies, would this change the IQR? Explain.

8 Two data sets have the same range. Does this mean they have the same lowest and highest values? Explain.

9 A second-hand car yard has more than 10 cars listed for sale. One expensive car is priced at $600 000 while the remaining cars are all priced near $40 000. Later the salesperson realises that there was an error in the price for the expensive car – there was one too many zeros printed onto the price.

a Does the change in price for the expensive car change the value of the range? Give reasons.

b Does the change in price for the expensive car change the value of the median? Give reasons.

c Does the change in price for the expensive car change the value of the IQR? Give reasons.

10 **a** Is it possible for the range to equal the IQR? If so, give an example.

b Is it possible for the IQR to equal zero? If so, give an example.

ENRICHMENT — — 11

How many lollies in the jar?

11 The following two sets of data represent the number of jelly beans found in 10 jars purchased from two different confectionery stores, A and B.

Shop A: 25 26 24 24 28 26 27 25 26 28

Shop B: 22 26 21 24 29 19 25 27 31 22

a Find Q_1, Q_2 and Q_3 for:

i shop A

ii shop B

b The top 25% of the data is above which value for shop A?

c The lowest 25% of the data is below which value for shop B?

d Find the interquartile range (IQR) for the number of jelly beans in 10 jars from:

i shop A

ii shop B

e By looking at the given sets of data, why should you expect there to be a significant difference between the IQR of shop A and the IQR of shop B?

f Which shop offers greater consistency in the number of jelly beans in each jar it sells?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9K Box plots

Interactive
Widgets
HOTsheets
Walkthrough

A box plot is a commonly used graph for a data set showing the maximum and minimum values, the median and the upper and lower quartiles. Box plots are often used to show how a data set is distributed and how two sets compare. Box plots are used, for example, to compare a school's examination performance against the performance of all schools in a state. They are also used to show medical test results before and after treatment.



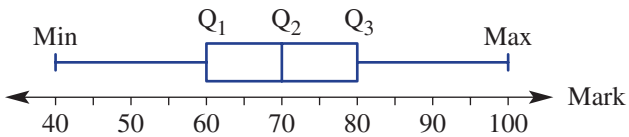
Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

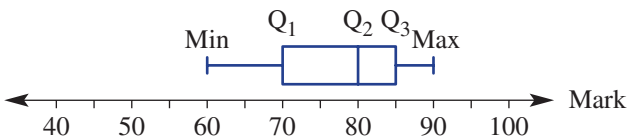
Let's start: School performance

These two box plots show the performance of two schools on their English exams.

Box High School



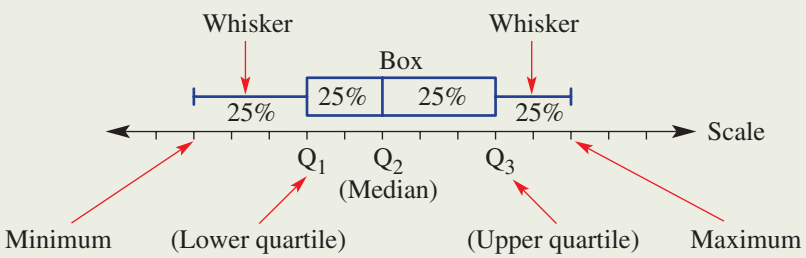
Plot Secondary College



- Which school produced the highest mark?
- Which school produced the highest median (Q_2)?
- Which school produced the highest IQR?
- Which school produced the highest range?
- Describe the performance of Box High School against Plot Secondary College. Which school has better overall results and why?

Key ideas

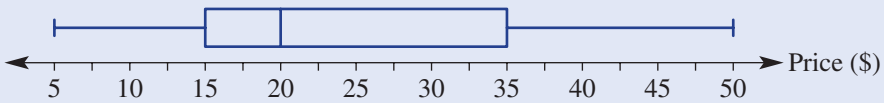
- A **box plot** (also called a box-and-whisker plot) is a graph that shows:
  - maximum and minimum values
  - median (Q_2)
  - lower and upper quartiles (Q_1 and Q_3).
- Approximately 25% of the data is spread across in each of the four sections of the graph.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 18 Interpreting a box plot

This box plot summarises the price of all the books in a book shop.



a State the minimum and maximum book prices.
b Find the range of the book prices.
c State the median book price.
d Find the interquartile range.
e Fifty per cent of the books are priced below what amount?
f Twenty five per cent of the books are priced above what amount?
g If there were 1000 books in the store, how many would be priced below $15?

SOLUTION

a Minimum book price = $5
Maximum book price = $50

b Range = $50 – $5
= $45

c Median book price = $20

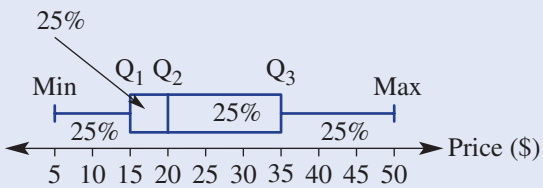
d Interquartile range = $35 – $15
= $20

e $20

f $35

g 0.25 × 1000 = 250
250 books would be below $15

EXPLANATION



Range = maximum price – minimum price

The median is Q_2.

Interquartile range = $Q_3 - Q_1$

50% of books are below Q_2.

The top 25% of books are above Q_3.

25% of books are priced below $15.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 19 Constructing a box plot

Consider the following set of data representing 11 scores resulting from rolling two dice and adding their scores.

7 10 7 12 8 9 6 6 5 4 8

a Find the:

- **i** minimum value
- **ii** maximum value
- **iii** median
- **iv** lower quartile
- **v** upper quartile

b Draw a box plot to represent the data.

SOLUTION

a Q_1 Q_2 Q_3

4 5 (6) 6 7 (7) 8 8 (9) 10 12

i Min.value = 4

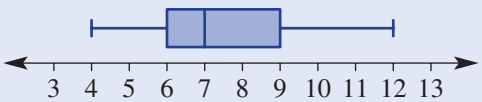
ii Max.value = 12

iii $Q_2 = 7$

iv $Q_1 = 6$

v $Q_3 = 9$

b Box plot: Rolling two dice



EXPLANATION

Order the data.

Determine the minimum and maximum value.

The median is the middle value.

Disregard the middle value, then locate the lowerquartile and upper quartile.

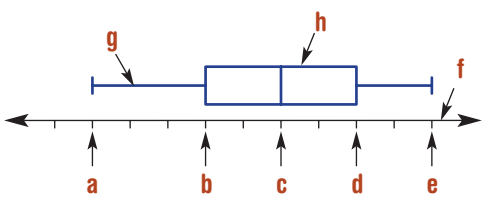
Draw a scaled horizontal axis.

Place the box plot above the axis marking in the five key statistics from part **a**.

Exercise 9K

UNDERSTANDING AND FLUENCY	1–5, 6(½)	1, 3, 4, 6(½)	5, 6(½)

1 Write down the names of all the features labelled **a** to **h** on this box plot.



2 Write the missing words.

a To find the range, you subtract the _____ from the _____.

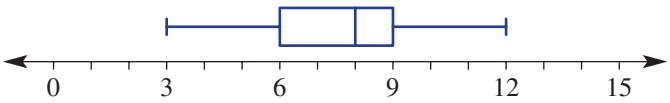
b To find the interquartile range, you subtract the _____ from the _____.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

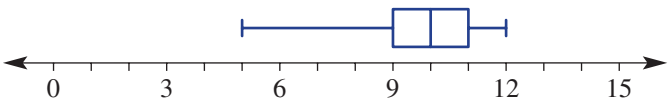
3 Choose the correct box plot for a data set that has all these measures:

- minimum = 3
- maximum = 12
- median = 9
- lower quartile = 5
- upper quartile = 11

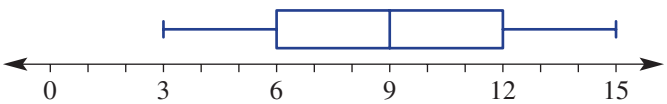
A



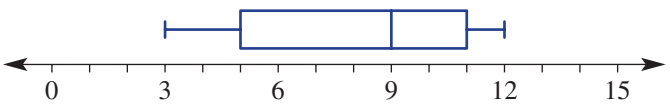
B



C

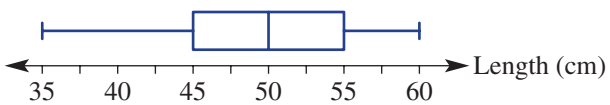


D



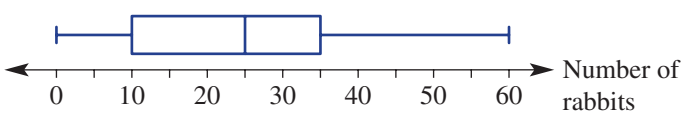
Example 18

4 This box plot summarises the length of babies born in a particular week in a hospital.



a State the minimum and maximum baby lengths.
b Find the range of the length of the babies.
c State the median baby length.
d Find the interquartile range.
e Fifty per cent of the baby lengths are below what amount?
f Twenty-five per cent of the babies are born longer than what amount?
g If there were 80 babies born in the week, how many would be expected to be less than 45 cm in length?

5 This box plot summarises the number of rabbits spotted per day in a paddock over a 100-day period.



a State the minimum and maximum number of rabbits spotted.
b Find the range of the number of rabbits spotted.
c State the median number of rabbits spotted.
d Find the interquartile range.
e Seventy-five per cent of the days the number of rabbits spotted is below what amount?
f Fifty per cent of the days the number of rabbits spotted is more than what amount?
g How many days was the number of spotted rabbits less than 10?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 19

6 For each of the sets of data below:

- **i** state the minimum value
- **ii** state the maximum value
- **iii** find the median (Q_2)
- **iv** find the lower quartile (Q_1)
- **v** find the upper quartile (Q_3)
- **vi** draw a box plot to represent the data

a 2 2 3 3 4 6 7 7 7 8 8 8 8 9 11 11 13 13 13

b 43 21 65 45 34 42 40 28 56 50 10 43 70 37 61 54 88 19

c 435 353 643 244 674 364 249 933 523 255 734

d 0.5 0.7 0.1 0.2 0.9 0.5 1.0 0.6 0.3 0.4 0.8 1.1 1.2 0.8 1.3 0.4 0.5

PROBLEM-SOLVING AND REASONING 7, 8, 10 | 7, 8, 10–12 | 8, 9, 11–13

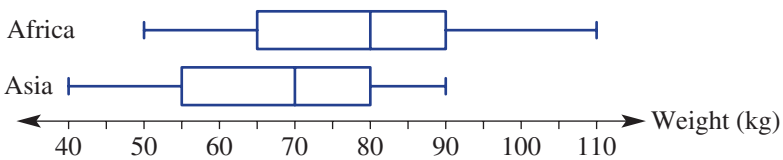
7 The following set of data describes the number of cars parked in a street on 18 given days.
14 26 39 46 13 30 5 46 37 26 39 8 8 9 17 48 29 27

a Represent the data as a box plot.

b On what percentage of days were the number of cars parked on the street between:

- **i** 5 and 48?
- **ii** 13 and 39?
- **iii** 5 and 39?
- **iv** 39 and 48?

8 The weights of two samples of adult leopards from Africa and Asia are summarised in these box plots.



a Which leopard population sample has the highest minimum weight?

b What is the difference between the ranges for both population samples?

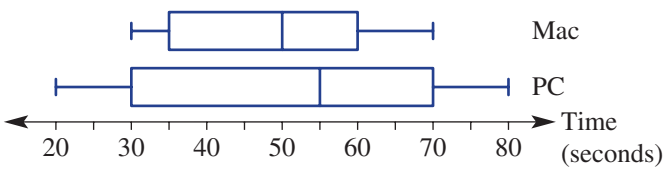
c Is the IQR the same for both leopard samples? If so, what is it?

d What percentage of leopards have a weight less than 80 kg for:

- **i** African leopards?
- **ii** Asian leopards?

e A leopard has a weight of 90 kg. Is it likely to be an Asian or African leopard?

9 The time that it takes for a sample of computers to start up is summarised in these box plots.



a What type of computer has the lowest median?

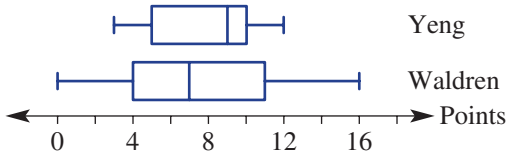
b What percentage of Mac computers loaded in less than 1 minute?

c What percentage of PC computers took longer than 55 seconds to load?

d What do you notice about the range for Mac computers and the IQR for PC computers? What does this mean?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 The number of points per game for two basketball players over a season is summarised in these box plots.



a Which player has the highest maximum?
b Which player has the highest median?
c Which player has the smallest IQR?
d Which player is a more consistent basketball scorer? Give reasons.
e Which player most likely scored the greatest number of points? Give reasons.

11 Give an example of a small data set that has the following.
a Maximum = upper quartile
b Median = lower quartile

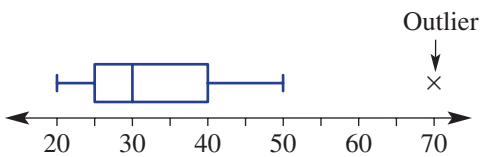
12 Does the median always sit exactly in the middle of a box on a box plot? Explain.

13 Could the mean of a data set be greater than Q_3? Explain.

ENRICHMENT — — 14

Outliers and battery life

14 Outliers on box plots are shown as a separate point.



The life in months of a particular kind of battery used in a special type of high-powered calculator is shown in this data set.
3 3 3 4 4 5 6 6 6 7 8 8 9 17
a Use all the values to calculate Q_1, Q_2 and Q_3 for the data set.
b Do any of the values appear to be outliers?
c Not including the outlier, what is the highest value?
d Draw a box plot for the data using a cross (×) to show the outlier.
e Can you give a logical reason for the outlier?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

How many in the bag?

For this activity you will need:

- a bag or large pocket
- different-coloured counters
- paper and pen.

Five counters

a Form pairs, and then without watching, have a third person (e.g. a teacher) place five counters of two different colours into the bag or pocket. An example of five counters could be two red and three blue, but at this point in the activity you will not know this.

b One person selects a counter from the bag without looking, while the other person records its colour. Replace the counter in the bag.

c Repeat part **b** for a total of 100 trials. Record the results in a table similar to this one.

Colour	Tally	Frequency
Red	𝍸 ||| …	…
Blue	𝍸 𝍸||| …	…
Total	100	100

d Find the experimental probability for each colour. For example, if 42 red counters were recorded then the experimental probability $= \frac{42}{100} = 0.42$.

e Use these experimental probabilities to help estimate how many of each colour of counter are in the bag. For example: 0.42 is close to $0.4 = \frac{2}{5}$, therefore guess two red and three blue.

Use this table to help.

Colour	Frequency	Experimental probability	Closest multiple of 0.2, e.g.0.2, 0.4, …	Guess of how many counters of this colour
Total	**100**	**1**	**1**	**5**

Now take the counters out of the bag to see if your estimate is correct.

More colours and counters

a Repeat the steps above, but use three colours and 8 counters.

b Repeat the steps above, but use four colours and 12 counters.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

How do Australians get to work?

The collection of data is an important part of statistics. There are a number of ways that data may be collected: a **census** includes the whole population, a **survey** includes only a sample of the population, a **controlled experiment** records cause and effect, and an **observational study** records data without a controlled experiment.
If a survey is to be used as the method of data collection, it is important that it is well designed. If the data is to be representative of the *population*, the selection of the sample that is surveyed must be thought through carefully.

Sampling

Different methods are used to generate a random sample (a subset of the population) to survey.
Write down some of the problems with the following methods of selecting a sample from a population for the given topic.

a A morning survey of the people on a train about their current employment
b A survey of the people on the electoral roll about their favourite music
c A phone poll of every 10th person in the phone book about their mobile phone plan
d A survey of people in a shopping centre on their yearly income
e A survey of the people in your year level on Australia's most popular television shows
f A call for volunteers in a newspaper advertisement for a medical trial
g A phone poll on a television news program about capital punishment

Census data

The major collection of data about the population of Australia occurs in the national census. In the census, a person in each household completes a range of questions about the people staying in their house that night. Census data can be accessed via the Australian Bureau of Statistics (ABS) website.

a Access the census data and list four topics of interest that contain data.
b **i** Find the data on 'Method of Travel to Work' for 2011.
ii Use the data to calculate the proportion of people in each category. Show your results in a table and a graph.
iii Design a survey to collect this same information. Include at least three questions. Use this to survey a class on their parents' method of travel to work.
iv Work out the proportion of people in each category for your data in part **iii**. Show your results in a table and a graph.
v Compare your answers in parts **ii** and **iv** for each category. Comment on any similarities or differences. What can you say about the quality of your sample based on these results? Discuss some of the limitations of your sample and how it could be improved.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

1 A fair coin is tossed 5 times.
- **a** How many outcomes are there?
- **b** Find the probability of obtaining at least 4 tails.
- **c** Find the probability of obtaining at least 1 tail.

2 Three cards, A, B and C, are randomly arranged in a row. Find the probability that the:
- **a** cards will be arranged in the order A B C from left to right
- **b** B card will take the right-hand position

3 Four students, Rick, Belinda, Katie and Chris, are the final candidates for the selection of school captain and vice-captain. Two of the four students will be chosen at random to fill the positions.
- **a** Find the probability that Rick will be chosen for:
  - **i** captain
  - **ii** captain or vice-captain
- **b** Find the probability that Rick and Belinda will be chosen for the two positions.
- **c** Find the probability that Rick will be chosen for captain and Belinda will be chosen for vice-captain.
- **d** Find the probability that the two positions will be filled by two boys or two girls.

4 State what would happen to the mean, median and range of a data set in these situations.
- **a** Five is added to each value in the data set.
- **b** Each value in the data set is doubled.
- **c** Each value in the data set is doubled and then decreased by 1.

5 Three pieces of fruit have an average weight of m grams. After another piece of fruit is added, the average weight doubles. Find the weight of the extra piece of fruit in terms of m.



6
- **a** Five different data values have a range and median equal to 7. If two of the values are 3 and 5, what are the possible combinations of values?
- **b** Four data values have a range of 10, a mode of 2 and a median of 5. What are the four values?

7 Five integer scores out of 10 are all greater than 0. If the median is x, the mode is one more than the median and the mean is one less than the median, find all the possible sets of values if $x < 7$.

8 Thomas works in an office. He is given four addressed letters and four addressed envelopes. What is the probability that Thomas, who puts the letters into random envelopes, places none of the four letters into their correct envelope?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

Probability and single variable data analysis

Experiments with two or more steps

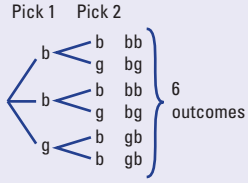
These can be represented by tables or tree diagrams. They involve more than one component and can occur with or without replacement.
e.g. a bag contains 2 blue counters and 1 green, 2 are selected at random.
(I) Table with replacement.

Pick 2 \ Pick 1	b	b	g
b	(b, b)	(b, b)	(g, b)
b	(b, b)	(b, b)	(g, b)
g	(b, g)	(b, g)	(g, g)

Sample space of 9 outcomes

P(2 of same colour) $= \frac{5}{9}$

(II) Tree without replacement



$P(2 \text{ of same colour}) = \frac{2}{6} = \frac{1}{3}$

Set notation

Within a sample space are a number of subsets.
$\overline{A}$ = not A



$A \cup B$ means A or B; the union of A and B



$A \cap B$ means A and B; the intersection of A and B



A only is the elements in A but not in B



$n(A)$ is the number of elements in A
$\emptyset$ is the empty or null set
e.g. $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 4, 6, 8\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$
$A \cap B = \{2, 4\}$ $n(B) = 4$
B only $= \{6, 8\}$
$3 \in A$ means 3 is an element of A

Venn diagrams and two-way tables

These organise data from two or more categories



	A	$\overline{A}$	
B	7	3	10
$\overline{B}$	5	6	11
	12	9	21

Venn diagram
i.e. 6 in neither category, 7 in both categories

$P(A) = \frac{12}{21} = \frac{4}{7}$

$P(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$

Probability review

The sample space is the list of all possible outcomes of an experiment.
For all possible outcomes:
$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
e.g. roll a normal six-sided die
$P(>4) = \frac{2}{6} = \frac{1}{3}$
$0 \le P(\text{event}) \le 1$
$P(\text{not } A) = 1 - P(A)$

Experimental probability (relative frequency)

This is calculated from results of experiment or survey.
Experimental probability $= \frac{\text{number of times event occurs}}{\text{total number of trials}}$
Expected number of occurrences = probability × number of trials

Stem-and-leaf plots

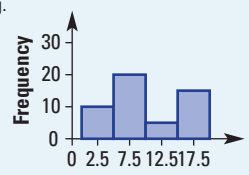
These display all the data values using a stem and a leaf.
An ordered back-to-back stem-and-leaf plot.

Leaf	Stem	Leaf	
9 8 7 2	1	0 3	
7 4 3 3	2	2 2 4	
5 2 1	3	3 6 7 8	
7	4	4 5 9	
0	5	0	
skewed			symmetrical

3 | 5 means 35
key

Grouped data

Data values can be grouped into class intervals, e.g. 0–4, 5–9, etc. and recorded in a frequency table.
The frequency or percentage frequency of each interval can be recorded in a histogram.
e.g.



Summarising data: Measures of centre

Mode is the most common value (there can be more than one). Two modes means data is bimodal.
Mean = average $= \frac{\text{sum of all values}}{\text{number of values}}$
Median is the middle value of data that is ordered.

odd data set: 2 4 (7) 10 12 — median
even data set: 2 4 6 10 15 18 — median = 8

An outlier is a value that is not in the vicinity of the rest of the data.

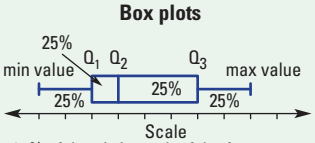
Measures of spread

Range = maximum value − minimum value
Interquartile range (IQR) = upper quartile (Q_3) − lower quartile (Q_1)
IQR is the range of the middle 50% of data.
e.g. Finding Q_1 and Q_3. First, locate median (Q_2) then find middle values of upper and lower half.

e.g. 1 2 (5) 6 7 | 9 10 (12) 12 15
Q_1 $Q_2 = 8$ Q_3

For odd number of values exclude median from each half.

Box plots



25% of data is in each of the four sections.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Multiple-choice questions

1 A letter is randomly chosen from the word XYLOPHONE. The probability that it is an O is:

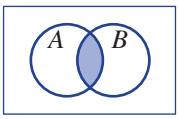
A $\frac{1}{8}$ **B** $\frac{2}{9}$ **C** $\frac{1}{4}$ **D** $\frac{1}{9}$ **E** $\frac{1}{3}$

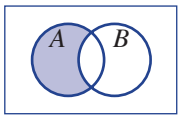
2 The values of x and y in the two-way table are:

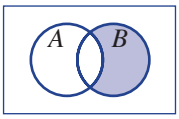
A $x = 12, y = 8$
B $x = 12, y = 11$
C $x = 16, y = 4$
D $x = 10, y = 1$
E $x = 14, y = 6$

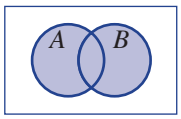
	A	Not A	Total
B		5	9
Not B	8	y	
Total	x		25

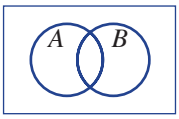
3 Which shaded region represents A and B?

A


B


C


D


E


4 A bag contains 2 green balls and 1 red ball. Two balls are randomly selected *without replacement*. The probability of selecting one of each colour is:

A $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** $\frac{1}{3}$ **E** $\frac{3}{4}$

5 From rolling a biased die, a class finds an experimental probability of 0.3 of rolling a 5. From 500 rolls of the die, the expected number of 5s would be:

A 300 **B** 167 **C** 180 **D** 150 **E** 210

6 The median of the data in this stem-and-leaf plot is:

A 74 **B** 71 **C** 86
D 65 **E** 70

Stem	Leaf
5	3 5 8
6	1 4 7
7	0 2 4 7 9
8	2 6 6

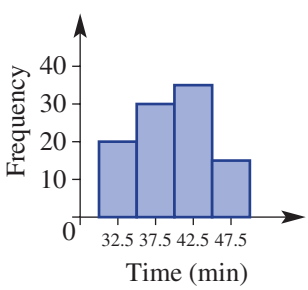
7 | 4 means 74

7 If Jacob achieved scores of 12, 9, 7 and 12 on his last four language vocabulary tests, what score must he get on the fifth test to have a mean of 11?

A 16 **B** 14 **C** 11
D 13 **E** 15

8 This frequency histogram shows the times of finishers in a fun run. The percentage of competitors that finished in better than 40 minutes was:

A 55% **B** 85% **C** 50%
D 62.5% **E** 60%



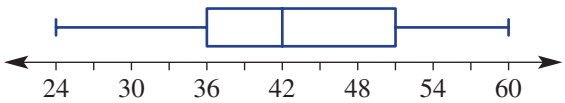
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 The interquartile range of the set of ordered data below is:

1.1 2.3 2.4 2.8 3.1 3.4 3.6 3.8 3.8 4.1 4.5 4.7 4.9

A 2–5 **B** 1.7 **C** 3.8 **D** 1.3 **E** 2.1

10 Choose the *incorrect* statement about the box plot below.



A The range is 36.
B Fifty per cent of values are between 36 and 51.
C The median is 42.
D Twenty-five per cent of values are below 36.
E The interquartile range is 20.

Short-answer questions

1 Determine the probability of each of the following.
a Rolling more than 2 on a normal six-sided die
b Selecting a vowel from the word EDUCATION
c Selecting a pink or white jelly bean from a packet containing 4 pink, 2 white and 4 black jelly beans

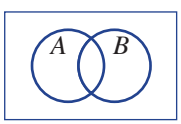
2 From a survey of 50 people, 30 have the newspaper delivered, 25 read it online, 10 do both and 5 do neither.
a Construct a Venn diagram for the survey results.
b How many people only read the newspaper online?
c If one of the 50 people were randomly selected, find:
i P(have paper delivered and read it online)
ii P(don't have it delivered)
iii P(only read it online)



3 **a** Copy and complete this two-way table.

	A	Not A	Total
B		16	
Not B	8		20
Total	17		

b Convert the information into a Venn diagram as shown.



c Find the following probabilities.
i $P(\text{not } B)$
ii $P(A \text{ and } B)$
iii $P(A \text{ only})$
iv $P(A \text{ or } B)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 A spinner with equal areas of red, green and blue is spun and a four-sided die numbered 1 to 4 is rolled.

		Die			
		1	2	3	4
Spinner	red	(red, 1)	(red, 2)		
	green				
	blue				

- **a** Complete a table like the one shown and state the number of outcomes in the sample space.
- **b** Find the probability that the outcome:
  - **i** is red and an even number
  - **ii** is blue or green and a 4
  - **iii** does not involve blue

5 Libby randomly selects two coins from her pocket *without replacement*. Her pocket contains a $1 coin, and two 10-cent coins.

- **a** List all the possible combinations using a tree diagram.
- **b** If a chocolate bar costs $1.10, find the probability that Libby can hand over the two coins to pay for it.

6 A quality controller records the frequency of the types of chocolates from a sample of 120 off its production line.

Centre	Soft	Hard	Nut
Frequency	50	22	48



- **a** What is the experimental probability of randomly selecting a nut centre?
- **b** In a box of 24 chocolates, how many would be expected to have a non-soft centre?

7 Claudia records the number of emails she receives each week day for two weeks as follows.

30 31 33 23 29 31 21 15 24 23

Find the:

- **a** mean
- **b** median
- **c** mode

8 Two mobile-phone salespeople are aiming for a promotion to be the new assistant store manager. The best salesperson over a 15-week period will achieve the promotion. The number of mobile phones they sold each week is recorded below.

Employee 1: 21 34 40 38 46 36 23 51 35 25 39 19 35 53 45
Employee 2: 37 32 29 41 24 17 28 20 37 48 42 38 17 40 45

- **a** Draw an ordered back-to-back stem-and-leaf plot for the data.
- **b** For each employee, find the:
  - **i** median number of sales
  - **ii** mean number of sales
- **c** By comparing the two sets of data, state, with reasons, who you think should get the promotion.
- **d** Describe each employee's data as approximately symmetrical or skewed.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 The data below represents the finish times, in minutes, of 25 competitors in a local car rally race.

134 147 162 164 145 159 151 143 136 155 163 157 168
171 152 128 144 161 158 136 178 152 167 154 161

a Record the above data in a frequency table in class intervals of 10 minutes. Include a percentage frequency column.

b Construct a frequency histogram.

c Determine the:

i number of competitors that finished in less than 140 minutes

ii percentage of competitors that finished between 130 and 160 minutes

10 Scott scores the following runs in each of his innings across the course of a cricket season.

20 5 34 42 10 3 29 55 25 37 51 12 34 22

a Find the range.

b Construct a box plot to represent the data by first finding the quartiles.

c From the box plot, 25% of his innings were above what number of runs?

Extended-response questions

1 The local Sunday market has a number of fundraising activities.

a For $1 you can spin a spinner numbered 1–5 twice. If you spin two even numbers, you receive $2 (your dollar back plus an extra dollar). If you spin two odd numbers, you get your dollar back. Otherwise you lose your dollar.

i Complete the table shown to list the sample space.

ii What is the probability of losing your dollar?

iii What is the probability of making a dollar profit?

iv In 50 attempts, how many times would you expect to lose your dollar?

v If you start with $100 and have 100 attempts, how much money would you expect to end up with?

		First spin				
		1	2	3	4	5
Second spin	1	(1, 1)	(2, 1)			
	2					
	3					
	4					
	5					

b Forty-five people were surveyed as they walked through a market about whether they bought a sausage and/or a drink from the sausage sizzle. Twenty-five people bought a sausage, 30 people bought a drink, and 15 bought both. Let S be the set of people who bought a sausage and D the set of people who bought a drink.

i Construct a Venn diagram to represent this information.

ii How many people bought neither a drink nor a sausage?

iii How many people bought a sausage only?

iv If a person was randomly selected from the 45, what is the probability they bought a drink but not a sausage?

v Find $P(\overline{S})$.

vi Find $P(\overline{S} \cup D)$ and state what this probability represents.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 The data below represents the data collected over a month of 30 consecutive days of the delay time (in minutes) of the flight departure of the same evening flight of two rival airlines.

Airline A

2	11	6	14	18	1	7	4	12	14	9	2	13	4	19
13	17	3	52	24	19	12	14	0	7	13	18	1	23	8

Airline B

6	12	9	22	2	15	10	5	10	19	5	12	7	11	18
21	15	10	4	10	7	18	1	18	8	25	4	22	19	26

a Does the data for airline A appear to have any outliers (numbers not near the majority of data elements)?

b By removing any outliers listed in part **a**, find the following for airline A.

i Median (Q_2)

ii Lower quartile (Q_1)

iii Upper quartile (Q_3)

c Hence, complete a box plot of the delay times for airline A.

d Airline A reports that half its flights for that month had a delay time of less than 10 minutes. Is this claim correct? Explain.

e On the same axis as in part **c**, construct a box plot for airline B's delay times.

f By finding the range and interquartile range of the two airlines' data, comment on the spread of the delay times for each company.

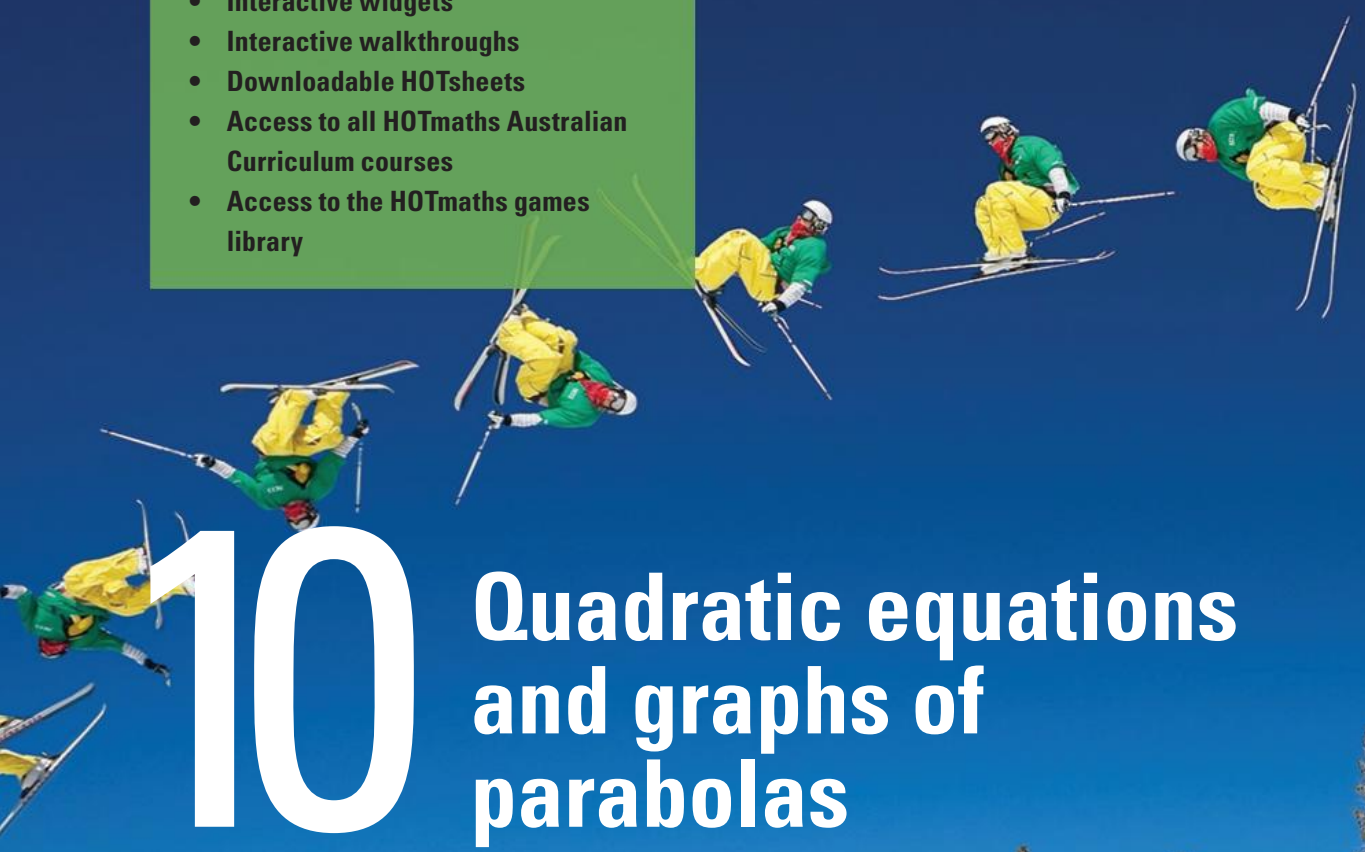
g Based on the data provided, would you choose airline A or airline B? Use your answers from parts **a–f** to justify and explain your choice of airline.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

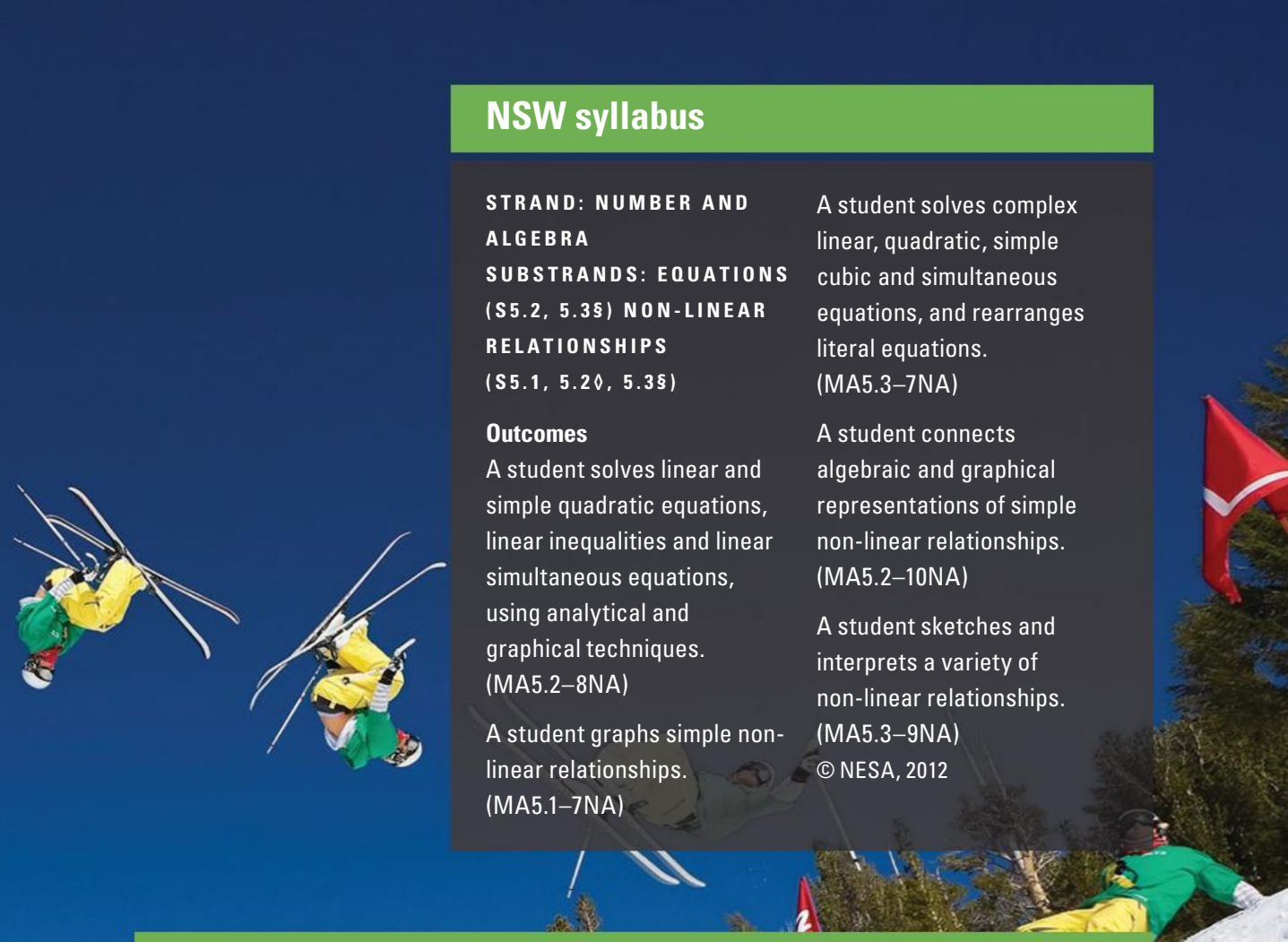


10 Quadratic equations and graphs of parabolas

What you will learn

10A Quadratic equations
10B Solving $ax^2 + bx = 0$ and $x^2 - d^2 = 0$ by factorising
10C Solving $x^2 + bx + c = 0$ by factorising
10D Using quadratic equations to solve problems
10E The parabola
10F Sketching $y = ax^2$ with dilations and reflections
10G Translations of $y = x^2$
10H Sketching parabolas using intercept form

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022



NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: EQUATIONS (S5.2, 5.3§) NON-LINEAR RELATIONSHIPS (S5.1, 5.2◊, 5.3§)

Outcomes

A student solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques. (MA5.2–8NA)

A student graphs simple non-linear relationships. (MA5.1–7NA)

A student solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations. (MA5.3–7NA)

A student connects algebraic and graphical representations of simple non-linear relationships. (MA5.2–10NA)

A student sketches and interprets a variety of non-linear relationships. (MA5.3–9NA)

© NESA, 2012

Flight paths in sport

Objects launched into the air follow a parabola-shaped curve due to the effects of gravity. The flight path will be a wider or thinner parabola shape depending on the force applied and the launching angle. Generally, a ball thrown or hit at an angle of 45° will have maximum range (i.e. covers the longest horizontal distance). However, the ideal launching angle varies with the specific conditions of each sport.

For a basketball throw, launch angles are about 70° for a steep, thin parabola required when close to the goal; 50° at the free-throw line; and 45° for a three-point shot needing a longer parabolic path. Shorter players use steeper launching angles than taller players.

Snowboarders and skiers follow a parabolic flight path when jumping. To reduce landing impact, engineers can use quadratic equations to design curved landing slopes that extend the parabolic shape of the jump.

In many ball sports, such as tennis or soccer, ball spin will modify the parabolic pathway causing the ball to drop steeply and/or curve sideways. Famous soccer goals, such as Brazilian Roberto Carlos' 1997 'banana' goal against France, show how a powerful kick and spin can drive the ball into a sideways curve, out of reach of the goal keeper.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Pre-test

1 Evaluate the following by substituting each of the four values of x into each expression.

i $x = 0$

ii $x = 2$

iii $x = -1$

iv $x = -3$

a x^2 **b** $2x^2 - 3x$ **c** $-x^2 + 3$ **d** $x^2 + 2x - 1$

2 Does $x = 1$ satisfy the following equations? Substitute to find out.

a $x^2 - 4x + 3 = 0$ **b** $x^2 - 2x = 0$ **c** $1 - x^2 = 0$

3 Solve the following for x.

a $2x = 0$ **b** $x - 3 = 0$ **c** $x + 5 = 0$

d $2x + 6 = 0$ **e** $3x - 12 = 0$ **f** $3x - 4 = 0$

4 Rearrange the following to produce 0 on the right-hand side.

a $a^2 = 2a$

b $a^2 = 3a - 1$

5 State the highest common factor of the following pairs of terms.

a $4x$ and $2xy$ **b** x^2 and $2x$ **c** $3x$ and $9x^2$

d $2x^2$ and $4x$ **e** $4x^2$ and 36 **f** $5x^2$ and 20

6 Factorise the following expressions.

a $x^2 - 9$ **b** $x^2 - 25$ **c** $x^2 - 64$ **d** $x^2 - 16$

7 Factorise the following quadratic trinomials.

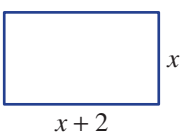
a $x^2 + 8x + 12$ **b** $x^2 + 5x + 4$ **c** $x^2 - 3x - 28$ **d** $x^2 - 9x + 20$

8 Expand these expressions.

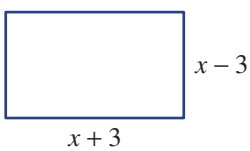
a $2(x - 3)$ **b** $x(2 - x)$ **c** $(x + 1)(x - 2)$ **d** $(x - 3)(x - 4)$

9 Write expanded expressions for the areas of the following rectangles.

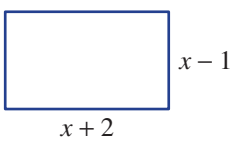
a



b



c



10 Find the x- and y-intercepts of the straight lines with the given rules.

a $y = 2x - 4$

b $3x + 6y = 12$

11 Give the x coordinate of the point halfway between the following pairs of points.

a (0, 0) and (10, 0)

b (2, 0) and (4, 0)

c (−1, 0) and (3, 0)

d (3, 0) and (8, 0)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10A Quadratic equations

Interactive
Widgets
HOTsheets
Walkthrough

Quadratic equations are commonplace in theoretical and practical applications of mathematics. They are used to solve problems in geometry and measurement as well as in number theory and physics. The path that a projectile takes while flying through the air, for example, can be analysed using quadratic equations.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

A quadratic equation can be written in the form $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$. Examples include $x^2 - 2x + 1 = 0$, $5x^2 - 3 = 0$ and $-0.2x^2 + 4x = 0$. Unlike a linear equation, which has a single solution, a quadratic equation can have zero, one or two solutions. For example, $x = 2$ and $x = -1$ are solutions to the quadratic equation $x^2 - x - 2 = 0$ since $2^2 - 2 - 2 = 0$ and $(-1)^2 - (-1) - 2 = 0$. Recall that we solved simple quadratic equations of the form $ax^2 = c$ in Chapter 2. Another method for finding the solutions to quadratic equations involves the use of the Null Factor Law where each factor of a factorised quadratic expression is equated to zero.

Let's start: Exploring the Null Factor Law

$x = 1$ is not a solution to the quadratic equation $x^2 - x - 12 = 0$ since $1^2 - 1 - 12 \neq 0$.

- Use trial and error to find at least one of the two numbers that are solutions to $x^2 - x - 12 = 0$.
- Rewrite the equation in factorised form.
  $x^2 - x - 12 = 0$ becomes $(\quad)(\quad) = 0$
- Now repeat the trial-and-error process to find solutions to the equation using the factorised form.
- Was the factorised form easier to work with? Discuss.

Key ideas

- A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$.
  - This is called **standard form**.
  - a, b and c are constants and $a \neq 0$.
- The **Null Factor Law** states that if the product of two numbers is zero, then either or both of the numbers is zero.
  - If $a \times b = 0$, then $a = 0$ or $b = 0$.

  For example: If $x^2 - 2x - 3 = 0$
  then $(x + 1)(x - 3) = 0$
  so $x + 1 = 0$ or $x - 3 = 0$
  so $x = -1$ or $x = 3$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 1 Writing in standard form

Write these quadratic equations in standard form.

a $x^2 = 2x + 7$ **b** $2(x^2 - 3x) = 5$ **c** $2x - 7 = -3x^2$

SOLUTION

a $x^2 = 2x + 7$
$x^2 - 2x - 7 = 0$

b $2(x^2 - 3x) = 5$
$2x^2 - 6x = 5$
$2x^2 - 6x - 5 = 0$

c $2x - 7 = -3x^2$
$3x^2 + 2x - 7 = 0$

EXPLANATION

We require the form $ax^2 + bx + c = 0$. Subtract $2x$ and 7 from both sides to move all terms to the left-hand side.

First expand brackets then subtract 5 from both sides.

Add $3x^2$ to both sides.

Example 2 Testing for a solution

Substitute the given x value into the equation and say whether or not it is a solution.

a $x^2 + x - 6 = 0$ $(x = 2)$ **b** $2x^2 + 5x - 3 = 0$ $(x = -4)$

SOLUTION

a $x^2 + x - 6 = 2^2 + 2 - 6$
$= 6 - 6$
$= 0$
$\therefore x = 2$ is a solution

b $2x^2 + 5x - 3 = 2(-4)^2 + 5(-4) - 3$
$= 2 \times 16 + (-20) - 3$
$= 32 - 20 - 3$
$= 9$, not 0
$\therefore x = -4$ is not a solution

EXPLANATION

Substitute $x = 2$ into the equation to see if the left-hand side equals zero.
$x = 2$ satisfies the equation so $x = 2$ is a solution.

Substitute $x = -4$. Recall that $(-4)^2 = 16$ and $5 \times (-4) = -20$.

The equation is not satisfied so $x = -4$ is not a solution.

Example 3 Using the Null Factor Law

Use the Null Factor Law to solve these equations.

a $x(x + 2) = 0$ **b** $(x - 1)(x + 5) = 0$ **c** $(2x - 1)(5x + 3) = 0$

SOLUTION

a $x(x + 2) = 0$
$x = 0$ or $x + 2 = 0$
$x = 0$ or $x = -2$

EXPLANATION

The factors are x and $(x + 2)$. Let each linear factor be equal to zero.
Check each solution by substituting ino the original equation.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b $(x - 1)(x + 5) = 0$
$x - 1 = 0$ or $x + 5 = 0$
$x = 1$ or $x = -5$

Let the factors $(x - 1)$ and $(x + 5)$ be equal to zero, then solve. Check your solutions using substitution.

c $(2x - 1)(5x + 3) = 0$
$2x - 1 = 0$ or $5x + 3 = 0$
$2x = 1$ or $5x = -3$
$x = \frac{1}{2}$ or $x = -\frac{3}{5}$

The two factors are $(2x - 1)$ and $(5x + 3)$. Each one results in a two-step linear equation.
Check your solutions using substitution.

Exercise 10A

UNDERSTANDING AND FLUENCY 1–2(½), 3, 4, 5–6 (½), 7, 9(½) | 2(½), 4, 5–6(½), 7, 9(½) | 5–6(½), 8, 9–10(½)

1 Evaluate these quadratic expressions by substituting the given x value.

a $x^2 + 3$ $(x = 3)$ **b** $x^2 - 5$ $(x = 1)$ **c** $x^2 + x$ $(x = 2)$
d $x^2 + 2x$ $(x = -1)$ **e** $x^2 - x$ $(x = -4)$ **f** $2x^2 - x + 1$ $(x = -1)$
g $3x^2 - x + 2$ $(x = 5)$ **h** $5x^2 + 2x - 3$ $(x = -1)$ **i** $-2x^2 + 2x - 3$ $(x = 4)$

2 Decide if the following equations are quadratics.

a $x + 1 = 0$ **b** $1 - 2x = 0$ **c** $x^2 + 3x + 1 = 0$
d $5x^2 - x + 2 = 0$ **e** $x^2 - 1 = 0$ **f** $x^3 + x^2 - 2 = 0$
g $x^5 + 1 - x = 0$ **h** $x^2 = x + 2$ **i** $x + 2 = 4x^2$

3 Solve these linear equations.

a $x - 1 = 0$ **b** $x + 3 = 0$ **c** $x + 11 = 0$
d $2x + 4 = 0$ **e** $3x - 9 = 0$ **f** $5x + 30 = 0$

4 Copy and complete the following working, which uses the Null Factor Law.

a $x(x - 5) = 0$
$x = 0$ or _____ $= 0$
$x = 0$ or $x =$ _____

b $(2x + 1)(x - 3) = 0$
_____ $= 0$ or $x - 3 = 0$
$2x =$ _____ or $x =$ _____
$x =$ _____ or $x =$ _____

Example 1 **5** Write these quadratic equations in standard form $(ax^2 + bx + c = 0)$.

a $x^2 + 2x = 5$ **b** $x^2 - 5x = -2$ **c** $x^2 = 4x - 1$
d $x^2 = 7x + 2$ **e** $2(x^2 + x) + 1 = 0$ **f** $3(x^2 - x) = -4$
g $4 = -3x^2$ **h** $3x = x^2 - 1$ **i** $5x = 2(-x^2 + 5)$

Example 2 **6** Substitute the given x value into the quadratic equation and say whether or not it is a solution.

a $x^2 - 1 = 0$ $(x = 1)$ **b** $x^2 - 25 = 0$ $(x = 5)$ **c** $x^2 - 4 = 0$ $(x = 1)$
d $2x^2 + 1 = 0$ $(x = 0)$ **e** $x^2 - 9 = 0$ $(x = -3)$ **f** $x^2 + 2x + 1 = 0$ $(x = -1)$
g $x^2 - x - 12 = 0$ $(x = 5)$ **h** $2x^2 - x + 3 = 0$ $(x = -1)$ **i** $5 - 2x + x^2 = 0$ $(x = -2)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 Substitute $x = -2$ and $x = 5$ into the equation $x^2 - 3x - 10 = 0$. What do you notice?

8 Substitute $x = -3$ and $x = 4$ into the equation $x^2 - x - 12 = 0$. What do you notice?

Example 3a,b

9 Use the Null Factor Law to solve these equations.

a $x(x + 1) = 0$ **b** $x(x + 5) = 0$ **c** $x(x - 2) = 0$

d $x(x - 7) = 0$ **e** $(x + 1)(x - 3) = 0$ **f** $(x - 4)(x + 2) = 0$

g $(x + 7)(x - 3) = 0$ **h** $\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right) = 0$ **i** $2x(x + 5) = 0$

j $5x\left(x - \frac{2}{3}\right) = 0$ **k** $\frac{x}{3}\left(x + \frac{2}{3}\right) = 0$ **l** $\frac{2x}{5}(x + 2) = 0$

Example 3c

10 Use the Null Factor Law to solve these equations.

a $(2x - 1)(x + 2) = 0$ **b** $(x + 2)(3x - 1) = 0$ **c** $(5x + 2)(x + 4) = 0$

d $(x - 1)(3x - 1) = 0$ **e** $(x + 5)(7x + 2) = 0$ **f** $(3x - 2)(5x + 1) = 0$

g $(11x - 7)(2x - 13) = 0$ **h** $(4x + 9)(2x - 7) = 0$ **i** $(3x - 4)(7x + 1) = 0$

PROBLEM-SOLVING AND REASONING 11, 14 | 11, 12(½), 14, 15 | 12–13(½), 15, 16

11 Find the numbers that satisfy the given condition.

a The product of x and a number 3 more than x is zero.

b The product of x and a number 7 less than x is zero.

c The product of a number 1 less than x and a number 4 more than x is zero.

d The product of a number 1 less than twice x and 6 more than x is zero.

e The product of a number 3 more than twice x and 1 less than twice x is zero.

12 Write these equations as quadratics in standard form. Remove any brackets and fractions.

a $5x^2 + x = x^2 - 1$ **b** $2x = 3x^2 - x$

c $3(x^2 - 1) = 1 + x$ **d** $2(1 - 3x^2) = x(1 - x)$

e $\frac{x}{2} = x^2 - \frac{3}{2}$ **f** $\frac{4x}{3} - x^2 = 2(1 - x)$

g $\frac{5}{x} + 1 = x$ **h** $\frac{3}{x} + \frac{5}{2} = 2x$

13 These quadratic equations have two integer solutions between -5 and 5. Use trial and error to find them.

a $x^2 - x - 2 = 0$ **b** $x^2 - 4x + 3 = 0$ **c** $x^2 - 4x = 0$

d $x^2 + 3x = 0$ **e** $x^2 + 3x - 4 = 0$ **f** $x^2 - 16 = 0$

14 Consider the quadratic equation $(x + 2)^2 = 0$.

a Write the equation in the form (_____)(_____).

b Use the Null Factor Law to find the solutions to the equation. What do you notice?

c Now solve these quadratic equations.

i $(x + 3)^2 = 0$

ii $(x - 5)^2 = 0$

iii $(2x - 1)^2 = 0$

iv $(5x - 7)^2 = 0$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 Consider the equation $3(x - 1)(x + 2) = 0$.

- **a** First divide both sides of the equation by 3. Write down the new equation.
- **b** Solve the equation using the Null Factor Law.
- **c** Compare the given original equation with the equation found in part **a**. Explain why the solutions are the same.
- **d** Solve these equations.
  - **i** $7(x + 2)(x - 3) = 0$
  - **ii** $11x(x + 2) = 0$
  - **iii** $\dfrac{(x + 1)(x - 3)}{4} = 0$

16 Consider the equation $(x - 3)^2 + 1 = 0$.

- **a** Substitute these x values to decide if they are solutions to the equation.
  - **i** $x = 3$
  - **ii** $x = 4$
  - **iii** $x = 0$
  - **iv** $x = -2$
- **b** Do you think the equation will have a solution? Explain why.

ENRICHMENT — — 17, 18

Polynomials

17 Polynomials are sums of integer powers of x.

Example	Polynomial name
2	Constant
$2x + 1$	Linear
$x^2 - 2x + 5$	Quadratic
$x^3 - x^2 + 6x - 1$	Cubic
$7x^4 + x^3 + 2x^2 - x + 4$	Quartic
$4x^5 - x + 1$	Quintic

Name these polynomial equations.

- **a** $3x - 1 = 0$
- **b** $x^2 + 2 = 0$
- **c** $x^5 - x^4 + 3 = 0$
- **d** $5 - 2x + x^3 = 0$
- **e** $3x - 2x^4 + x^2 = 0$
- **f** $5 - x^5 = x^4 + x$

18 Solve these polynomial equations using the Null Factor Law.

- **a** $(x + 1)(x - 3)(x + 2) = 0$
- **b** $(x - 2)(x - 5)(x + 11) = 0$
- **c** $(2x - 1)(3x + 2)(5x - 1) = 0$
- **d** $(3x + 2)(5x + 4)(7x + 10)(2x - 13) = 0$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10B Solving $ax^2 + bx = 0$ and $x^2 - d^2 = 0$ by factorising

Interactive

Widgets

HOTsheets

Walkthrough

In Chapter 2 we solved equations such as $x^2 = 9$ and $2x^2 - 5 = 0$.

In this section we will use the Null Factor Law to solve equations such as $2x^2 + 8x = 0$ and $x^2 - 9 = 0$.

When using the Null Factor Law, we notice that equations must first be expressed as a product of two factors. Hence, any equation not in this form must first be factorised. Two types of quadratic equations are studied here. The first is of the form $ax^2 + bx = 0$, where x is a common factor. The second is of the form $x^2 - d^2 = 0$, which is a difference of two perfect squares.



Remember: 'null' means 'zero'.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Which factorisation technique?

These two equations may look similar but they are not the same: $x^2 - 9x = 0$ and $x^2 - 9 = 0$.

- Discuss how you could factorise each expression on the left-hand side of the equations.
- How does the factorised form help to solve the equations? What are the solutions? Are the solutions the same for both equations?

Key ideas

- When solving an equation of the form $ax^2 + bx = 0$, factorise by taking out common factors including x.

$$2x^2 - 8x = 0$$
$$2x(x - 4) = 0$$
$$2x = 0 \text{ or } x - 4 = 0$$
$$x = 0 \text{ or } x = 4$$

- When solving an equation of the form $a(x^2 - d^2) = 0$, look for common factors and a difference of two squares.
  - Recall: $a^2 - d^2 = (a + d)(a - d)$
  - The constant a in $a(x^2 - d^2) = 0$ has no bearing on the solutions for x.

$$5x^2 - 20 = 0$$
$$x^2 - 4 = 0$$
$$(x + 2)(x - 2) = 0$$
$$x + 2 = 0 \text{ or } x - 2 = 0$$
$$x = -2 \text{ or } x = 2$$

Example 4 Solving when x is a common factor

Solve each of the following equations.

a $x^2 + 4x = 0$ b $2x^2 = 8x$

SOLUTION	EXPLANATION
a $x^2 + 4x = 0$	Factorise by taking out the common factor x.
$x(x + 4) = 0$	Using the Null Factor Law, set each factor, x and $(x + 4)$,
$x = 0$ or $x + 4 = 0$	equal to 0. Solve for x.
$x = 0$ or $x = -4$	Check your solutions using substitution.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

SOLUTION	EXPLANATION
b $2x^2 = 8x$	Make the right-hand side equal to zero by subtracting $8x$ from both sides.
$2x^2 - 8x = 0$	
$x^2 - 4x = 0$	Divide LHS and RHS by 2.
$x(x - 4) = 0$	Factorise by taking out the common factor of x and apply the Null Factor Law to solve.
$x = 0$ or $x - 4 = 0$	
$x = 0$ or $x = 4$	Check your solutions by substitution.

Example 5 Solving with a difference of two squares

Solve each of these equations.

a $x^2 = 16$ **b** $3x^2 = 18$

SOLUTION	EXPLANATION
a $x^2 = 16$	Take the square root of both sides. $x = 4$ or -4 since $4^2 = 16$ and $(-4)^2 = 16$.
$x = 4$ or $x = -4$	
Altenative solution:	
$x^2 = 16$	Rearrange into standard form.
$x^2 - 16 = 0$	
$(x + 4)(x - 4) = 0$	Factorise using $a^2 - b^2 = (a + b)(a - b)$, then use the Null Factor Law to find the solutions.
$x + 4 = 0$ or $x - 4 = 0$	
$x = -4$ or $x = 4$	This could be written as $x = \pm 4$
b $3x^2 = 18$	Divide both sides by 3 and then take the square root of both sides.
$x^2 = 6$	
$x = \sqrt{6}$ or $x = -\sqrt{6}$	Since 6 is not a square number leave answers in exact form as $\sqrt{6}$ and $-\sqrt{6}$.
$x = \pm\sqrt{6}$	
Alternative solution:	
$3x^2 = 18$	Alternatively, express in standard form and then note the common factor of 3.
$3x^2 - 18 = 0$	Divide both sides by 3.
$x^2 - 6 = 0$	Treat $x^2 - 6 = x^2 - (\sqrt{6})^2$ as a difference of squares.
$(x + \sqrt{6})(x - \sqrt{6}) = 0$	
$x + \sqrt{6} = 0$ or $x - \sqrt{6} = 0$	Apply the Null Factor Law and solve for x.
$x = -\sqrt{6}$ or $x = \sqrt{6}$	

Exercise 10B

UNDERSTANDING AND FLUENCY	1–2(½), 3, 4–5(½), 6	2–8(½)	4(½), 6–8(½)

1 Write down the highest common factor of these pairs of terms.

a $2x$ and 4 **b** $5x$ and 10

c $4x$ and 6 **d** $16x$ and 24

e x^2 and $2x$ **f** x^2 and $7x$

g $3x^2$ and $6x$ **h** $9x^2$ and $15x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Factorise these expressions fully by first taking out the highest common factor.

a $2x^2 - 8$ **b** $4x^2 - 36$ **c** $3x^2 - 75$ **d** $12x^2 - 12$
e $x^2 - 3x$ **f** $x^2 + 7x$ **g** $2x^2 - 4x$ **h** $5x^2 - 15x$
i $6x^2 + 4x$ **j** $9x^2 - 27x$ **k** $4x - 16x^2$ **l** $14x - 21x^2$

3 Use the Null Factor Law to write down the solutions to these equations.

a $x(x - 3) = 0$
b $4x(x + 1) = 0$
c $-7x(x + 2) = 0$
d $(x + 1)(x - 1) = 0$
e $(x + 5)(x - 5) = 0$
f $(2x + 1)(2x - 1) = 0$

Example 4a

4 Solve each of these equations.

a $x^2 + 3x = 0$ **b** $x^2 + 7x = 0$
c $x^2 + 4x = 0$ **d** $x^2 - 5x = 0$
e $x^2 - 8x = 0$ **f** $x^2 - 2x = 0$
g $x^2 + \frac{1}{3}x = 0$ **h** $x^2 - \frac{1}{2}x = 0$

5 Solve these equations by first taking out the highest common factor.

a $2x^2 - 6x = 0$ **b** $3x^2 - 12x = 0$
c $4x^2 + 20x = 0$ **d** $6x^2 - 18x = 0$
e $-5x^2 + 15x = 0$ **f** $-2x^2 - 8x = 0$
g $x^2 + 9x = 0$ **h** $x^2 - 9x = 0$
i $2x^2 - 2x = 0$ **j** $5x - x^2 = 0$
k $4x^2 + 2x = 0$ **l** $3x^2 + 12x = 0$
m $3x^2 - 12x = 0$ **n** $-x^2 + 2x = 0$
o $-x^2 - 4x = 0$ **p** $-8x^2 - 16x = 0$

Example 4b

6 Solve each of the following equations.

a $x^2 = 3x$ **b** $5x^2 = 10x$
c $4x^2 = 16x$ **d** $3x^2 = -9x$
e $2x^2 = -8x$ **f** $7x^2 = -21x$

Example 5a

7 Solve each of the following equations, noting the difference of two squares.

a $x^2 - 9 = 0$ **b** $x^2 - 16 = 0$
c $x^2 - 25 = 0$ **d** $x^2 - 144 = 0$
e $x^2 - 81 = 0$ **f** $x^2 - 400 = 0$

Example 5b

8 Solve each of the following equations by taking out a common factor first.

a $7x^2 - 28 = 0$ **b** $5x^2 - 45 = 0$
c $2x^2 - 50 = 0$ **d** $6x^2 - 24 = 0$
e $2x^2 = 8$ **f** $3x^2 = 12$
g $5x^2 = 80$ **h** $8x^2 = 72$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING	9(½), 12	9(½), 11, 12	10–13

9 Rearrange these equations then solve them.

a $4 = x^2$ **b** $-x^2 + 25 = 0$ **c** $-x^2 = -100$

d $-3x^2 = 21x$ **e** $-5x^2 + 35x = 0$ **f** $1 - x^2 = 0$

g $12x = 18x^2$ **h** $2x^2 + x = 7x^2 - 2x$ **i** $9 - 2x^2 = x^2$

10 Remove brackets or fractions to help solve these equations.

a $x - \frac{4}{x} = 0$ **b** $\frac{36}{x} - x = 0$

c $\frac{3}{x^2} = 3$ **d** $5(x^2 + 1) = 3x^2 + 7$

e $x(x - 3) = 2x^2 + 4x$ **f** $3x(5 - x) = x(7 - x)$

11 Write an equation and solve it to find the number.

a The square of the number is 7 times the same number.

b The difference between the square of a number and 64 is zero.

c 3 times the square of a number is equal to -12 times the number.

12 Consider the equation $x^2 + 4 = 0$.

a Explain why it cannot be written in the form $(x + 2)(x - 2) = 0$.

b Are there any solutions to the equation $x^2 + 4 = 0$? Why/why not?

13 An equation of the form $ax^2 + bx = 0$ will always have two solutions if a and b are not zero.

a Explain why one of the solutions will always be $x = 0$.

b Write the rule for the second solution in terms of a and b.

ENRICHMENT	—	—	14, 15

Tougher examples involving difference of two squares

14 Note for example that $4x^2 - 9 = 0$ factorises to $(2x + 3)(2x - 3) = 0$. Now solve these equations.

a $9x^2 - 16 = 0$

b $25x^2 - 36 = 0$

c $4 - 100x^2 = 0$

d $81 - 25x^2 = 0$

e $64 = 121x^2$

f $-49x^2 = -144$

15 Note for example that $(x - 1)^2 - 4 = 0$ factorises to $((x - 1) + 2)((x - 1) - 2) = 0$, which is equivalent to $(x + 1)(x - 3) = 0$. Now solve these equations.

a $(x - 2)^2 - 9 = 0$

b $(x + 5)^2 - 16 = 0$

c $(2x + 1)^2 - 1 = 0$

d $(5x - 3)^2 - 25 = 0$

e $(4 - x)^2 = 9$

f $(3 - 7x)^2 = 100$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10C Solving $x^2 + bx + c = 0$ by factorising

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

In Chapter 8 we learned to factorise quadratic trinomials with three terms. For example, $x^2 + 5x + 6$ factorises to $(x + 2)(x + 3)$. This means that the Null Factor Law can be used to solve equations of the form $x^2 + bx + c = 0$.

Let's start: Remembering how to factorise quadratic trinomials

First expand these quadratics using the distributive law:

$(a + b)(c + d) = ac + ad + bc + bd$

- $(x + 1)(x + 2)$
- $(x - 3)(x + 4)$
- $(x - 5)(x - 2)$

Now factorise these expressions.

- $x^2 + 5x + 6$
- $x^2 - 5x + 6$
- $x^2 + x - 6$
- $x^2 - x - 6$

Discuss your method for finding the factors of each quadratic above.

Key ideas

- Solve quadratics of the form $x^2 + bx + c = 0$ by factorising the quadratic trinomial.
  - Ask 'What factors of c add to give b?'
  - Then use the Null Factor Law.
  - $x^2 + bx + c$ is called a monic quadratic since the coefficient of x^2 is 1.

$$x^2 - 3x - 28 = 0$$
$$(x - 7)(x + 4) = 0$$
$$x - 7 = 0 \text{ or } x + 4 = 0$$
$$x = 7 \text{ or } x = -4$$

and $-7 \times 4 = -28$, $-7 + 4 = -3$

- An equation involving a perfect square will give only one solution.

$$x^2 - 6x + 9 = 0$$
$$(x - 3)(x - 3) = 0$$
$$x - 3 = 0$$
$$x = 3$$

Example 6 Solving equations with quadratic trinomials

Solve these quadratic equations.

a $x^2 + 7x + 12 = 0$ **b** $x^2 - 2x - 8 = 0$ **c** $x^2 - 8x + 15 = 0$

SOLUTION

a $x^2 + 7x + 12 = 0$
$(x + 3)(x + 4) = 0$
$x + 3 = 0$ or $x + 4 = 0$
$x = -3$ or $x = -4$

EXPLANATION

Factors of 12 which add to 7 are 3 and 4.
$3 \times 4 = 12$, $3 + 4 = 7$
Use the Null Factor Law to solve the equation.
Check solutions using substitution.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b $x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x - 4 = 0$ or $x + 2 = 0$
$x = 4$ or $x = -2$

Factors of -8 which add to -2 are -4 and 2.
$-4 \times 2 = -8$, $-4 + 2 = -2$,
Finish using the Null Factor Law.
Check solutions.

c $x^2 - 8x + 15 = 0$
$(x - 5)(x - 3) = 0$
$x - 5 = 0$ or $x - 3 = 0$
$x = 5$ or $x = 3$

The factors of 15 must add to give -8. $-5 \times (-3) = 15$ and $-5 + (-3) = -8$, so -5 and -3 are the two numbers.
Check solutions.

Example 7 Solving with two squares and other trinomials

Solve these quadratic equations.

a $x^2 - 8x + 16 = 0$ **b** $x^2 = x + 6$

SOLUTION

a $x^2 - 8x + 16 = 0$
$(x - 4)(x - 4) = 0$
$x - 4 = 0$
$x = 4$

b $x^2 = x + 6$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x - 3 = 0$ or $x + 2 = 0$
$x = 3$ or $x = -2$

EXPLANATION

Factors of 16 which add to -8 are -4 and -4.
$(x - 4)(x - 4) = (x - 4)^2$ is a perfect square so there is only one solution.
Check solutions.

First make the right-hand side equal zero by subtracting x and 6 from both sides. This is standard form. Factors of -6 which add to -1 are -3 and 2.
Check solutions.

Exercise 10C

UNDERSTANDING AND FLUENCY	1, 2, 3–4(½)	2, 3–5(½)	3–5(½)

1 Find two numbers which add to give the first number and multiply to give the second.

a 6, 5 **b** 8, 6 **c** 2, -3 **d** 10, -7
e -2, 1 **f** -5, 4 **g** -12, -1 **h** -12, -4

2 Copy and complete the working to solve each equation.

a $x^2 + 9x + 20 = 0$
$(x + 5)(____) = 0$
$x + 5 = 0$ or $____ = 0$
$x = ____$ or $x = ____$

b $x^2 - 2x - 24 = 0$
$(x - 6)(____) = 0$
$x - 6 = 0$ or $____ = 0$
$x = ____$ or $x = ____$

c $x^2 + 4x - 45 = 0$
$(x + 9)(____) = 0$
$x + 9 = 0$ or $____ = 0$
$x = ____$ or $x = ____$

d $x^2 - 10x + 16 = 0$
$(x - 8)(____) = 0$
$x - 8 = 0$ or $____ = 0$
$x = ____$ or $x = ____$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 6

3 Solve these quadratic equations.

a $x^2 + 8x + 12 = 0$ **b** $x^2 + 11x + 24 = 0$ **c** $x^2 + 7x + 10 = 0$
d $x^2 + 5x - 14 = 0$ **e** $x^2 + 4x - 12 = 0$ **f** $x^2 + 7x - 30 = 0$
g $x^2 - 12x + 32 = 0$ **h** $x^2 - 9x + 18 = 0$ **i** $x^2 - 10x + 21 = 0$
j $x^2 - 2x - 15 = 0$ **k** $x^2 - 6x - 16 = 0$ **l** $x^2 - 4x - 45 = 0$
m $x^2 - 10x + 24 = 0$ **n** $x^2 - x - 42 = 0$ **o** $x^2 + 5x - 84 = 0$
p $x^2 + 4x + 3 = 0$ **q** $x^2 - 6x - 27 = 0$ **r** $x^2 - 12x + 20 = 0$

Example 7a

4 Solve these quadratic equations, which include perfect squares.

a $x^2 + 6x + 9 = 0$ **b** $x^2 + 4x + 4 = 0$ **c** $x^2 + 14x + 49 = 0$
d $x^2 + 24x + 144 = 0$ **e** $x^2 - 10x + 25 = 0$ **f** $x^2 - 16x + 64 = 0$
g $x^2 - 12x + 36 = 0$ **h** $x^2 - 18x + 81 = 0$ **i** $x^2 - 20x + 100 = 0$

Example 7b

5 Solve these quadratic equations by first rearranging to standard form.

a $x^2 = 3x + 10$ **b** $x^2 = 7x - 10$ **c** $x^2 = 6x - 9$
d $x^2 = 4 - 3x$ **e** $14 - 5x = x^2$ **f** $x^2 + 16 = 8x$
g $x^2 - 12 = -4x$ **h** $6 - x^2 = 5x$ **i** $15 = 8x - x^2$
j $16 - 6x = x^2$ **k** $-6x = x^2 + 8$ **l** $-x^2 - 7x = -18$

PROBLEM-SOLVING AND REASONING 6(½), 9, 10 | 6–8(½), 9–11 | 6–8(½), 9, 11, 12

6 Solve these equations by first taking out a common factor.

a $2x^2 - 2x - 12 = 0$ **b** $3x^2 + 24x + 45 = 0$ **c** $4x^2 - 24x - 64 = 0$
d $4x^2 - 20x + 24 = 0$ **e** $2x^2 - 8x + 8 = 0$ **f** $3x^2 + 6x + 3 = 0$
g $7x^2 - 70x + 175 = 0$ **h** $2x^2 + 12x = -18$ **i** $5x^2 = 35 - 30x$

7 Remove brackets, decimals or fractions and write in standard form to help solve these equations.

a $x^2 = 5(x - 1.2)$
b $2x(x - 3) = x^2 - 9$
c $3(x^2 + x - 10) = 2x^2 - 5(x + 2)$
d $x - 2 = \frac{35}{x}$
e $2 + \frac{1}{x} = -x$
f $\frac{x}{4} = 1 - \frac{1}{x}$

8 Write down a quadratic equation in standard form which has the following solutions.

a $x = 1$ and $x = 2$ **b** $x = 3$ and $x = -2$ **c** $x = -4$ and $x = 1$
d $x = -3$ and $x = 10$ **e** $x = 5$ only **f** $x = -11$ only

9 The temperature in °C inside a room at a scientific base in Antarctica after 10 a.m. is given by the expression $t^2 - 9t + 8$, where t is in hours. Find the times in the day when the temperature is 0°C.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 Consider this equation and solution.

$$x^2 - x - 12 = -6$$
$$(x - 4)(x + 3) = -6$$
$$x - 4 = -6 \text{ or } x + 3 = -6$$
$$x = -2 \text{ or } x = -9$$

a Explain what is wrong with the solution.

b Find the correct solutions to the equation.

11 Explain why $x^2 - 2x + 1 = 0$ only has one solution.

12 Write down a quadratic equation that has these solutions. Write in factorised form.

a $x = a$ only **b** $x = a$ and $x = b$

ENRICHMENT — — 13

Coefficient of x^2 other than 1

13 Many quadratic equations have a coefficient of x^2 not equal to 1. These are called non-monic quadratics. Factorisation can be trickier but is still possible. You may have covered this in Chapter 8. Here is an example.

$$2x^2 + 5x - 3 = 0 \qquad \text{(factors of } -6 \text{ that add to give 5 are } -1 \text{ and 6.)}$$
$$2x^2 - x + 6x - 3 = 0 \qquad (6x - 1x = 5x)$$
$$x(2x - 1) + 3(2x - 1) = 0$$
$$(2x - 1)(x + 3) = 0$$
$$2x - 1 = 0 \text{ or } x + 3 = 0$$
$$x = \frac{1}{2} \text{ or } x = -3$$

Solve these quadratic equations.

a $5x^2 + 16x + 3 = 0$

b $3x^2 + 4x - 4 = 0$

c $6x^2 + x - 1 = 0$

d $10x^2 + 5x - 5 = 0$

e $2x^2 = 15 - 7x$

f $4x^2 = 12x - 9$

g $x = 3x^2 - 14$

h $17x = 12 - 5x^2$

i $25 = -9x^2 - 30x$

j $17x - 6x^2 = 5$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10D Using quadratic equations to solve problems

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

When using mathematics to solve problems, we often arrive at a quadratic equation. In solving the quadratic equation, we obtain the solutions to the problem. Setting up the original equation and then interpreting the solution are important parts of the problem-solving process.

Let's start: Solving for the unknown number

The product of a positive number and 6 more than the same number is 16.

- Using x as the unknown number, write an equation describing the given condition.
- Solve your equation for x.
- Are both solutions feasible (allowed)?
- Discuss how this method compares to the method of trial and error.



Surveyors work on many problems that involve quadratic equations.

Key ideas

When using quadratic equations to solve problems, follow these steps.

- Define your variable. Write 'Let x be … '
  For example: Let x be the height
- Write an equation describing the situation.
- Solve your equation using the Null Factor Law.
- Check that your solutions are feasible.
  - Some problems may not allow solutions that are negative numbers or fractions.
- Answer the original question in words.
- Read the question again to make sure the solution is reasonable.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

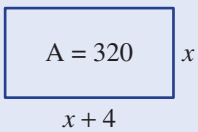
Example 8 Solving area problems

The length of a book is 4 cm more than its width and the area of the face of the book is 320 cm^2. Find the dimensions of the face of the book.

SOLUTION

Let x cm be the width of the book face.
Length $= (x + 4)$ cm

Area: $x(x + 4) = 320$
$x^2 + 4x - 320 = 0$
$(x + 20)(x - 16) = 0$
$x + 20 = 0$ or $x - 16 = 0$
$x = -20$ or $x = 16$
$\therefore x = 16$ (-20 is not valid)
$\therefore$ width $= 16$ cm
length $= 20$ cm



EXPLANATION

Define a variable for width, then write the length in terms of the width.

Write an equation to suit the given situation.
Expand and subtract 320 from both sides.

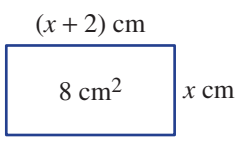
Solve using the Null Factor Law but note that a width of -20 cm is not feasible.

Finish by writing the dimensions width and length as required.
Length $= x + 4 = 16 + 4 = 20$ cm
Read the question again to make sure the solution is reasonable.

Exercise 10D

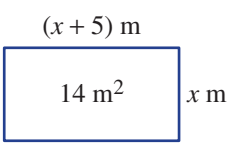
UNDERSTANDING AND FLUENCY	1, 3–7	2, 3, 5, 7, 8	6–8

1 This rectangle has an area of 8 cm^2 and a length which is 2 cm more than its width.



- **a** Using length × width = area, write an equation.
- **b** Solve your equation by expanding and subtracting 8 from both sides. Then use the Null Factor Law.
- **c** Which of your two solutions is feasible for the width of the rectangle?
- **d** Write down the dimensions (width and length) of the rectangle.

2 This rectangle has an area of 14 m^2 and a length that is 5 m more than its width.



- **a** Using length × width = area, write an equation.
- **b** Solve your equation by expanding and subtracting 14 from both sides. Then use the Null Factor Law.
- **c** Which of your two solutions is feasible for the width of the rectangle?
- **d** Write down the dimensions (width and length) of the rectangle.

3 Solve these equations for x by first expanding and producing a zero on the right-hand side.

- **a** $x(x + 3) = 18$
- **b** $x(x - 1) = 20$
- **c** $(x - 1)(x + 4) = 6$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 The product of a number and 2 more than the same number is 48. Write an equation and solve to find the two possible solutions.

5 The product of a number and 7 less than the same number is 60. Write an equation and solve to find the two possible solutions.

6 The product of a number and 13 less than the same number is 30. Write an equation and solve to find the two possible solutions.

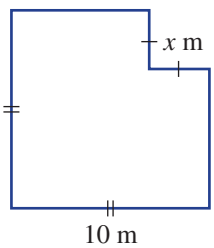
Example 8

7 The length of a rectangular brochure is 5 cm more than its width, and the area of the face of the brochure is 36 cm^2. Find the dimensions of the face of the brochure.

8 The length of a small rectangular kindergarten play area is 20 metres less than its width and the area is 69 m^2. Find the dimensions of the kindergarten play area.

PROBLEM-SOLVING AND REASONING | 9, 10, 13 | 9–11, 13, 14 | 9–12, 14–16

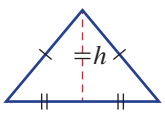
9 A square of side length 10 metres has a square of side length x metres removed from one corner.



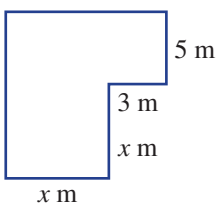
a Write an expression for the area remaining after the square of side length x metres is removed. *Hint*: use subtraction.

b Find the value of x if the area remaining is to be 64 m^2.

10 An isosceles triangle has height (h) equal to half its base length. Find the value of h if the area is 25 square units.



11 A farm shed (3 m by 5 m) is to be extended to form an L shape.

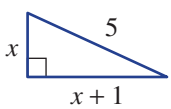


a Write an expression for the total area of the extended farm shed.

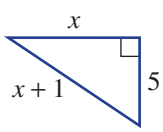
b Find the value of x if the total area is to be 99 m^2.

12 Use Pythagoras' theorem to find the value of x in these right-angled triangles.

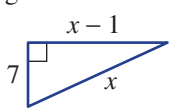
a



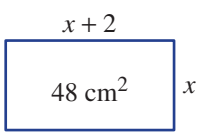
b



c



13 The equation for this rectangle is $x^2 + 2x - 48 = 0$, which has solutions $x = -8$ and $x = 6$. Which solution do you ignore and why?



14 The product of an integer and one less than the same integer is 6. The equation for this is $x^2 - x - 6 = 0$. How many different solutions are possible and why?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 This table shows the sum of the first n positive integers.
If $n = 3$, then the sum is $1 + 2 + 3 = 6$.

n	1	2	3	4	5	6
Sum	1	3	6			

a Write the sum for $n = 4$, $n = 5$ and $n = 6$.

b The expression for the sum is given by $\frac{n(n+1)}{2}$. Use this expression to find the sum if:

i $n = 7$

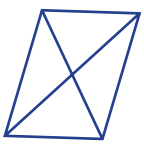
ii $n = 20$

c Use the expression to find n for these sums. Write an equation and solve it.

i sum $= 45$

ii sum $= 120$

16 The number of diagonals in a polygon with n sides is given by $\frac{n}{2}(n-3)$. Shown here are the diagonals for a quadrilateral and a pentagon.





n	4	5	6	7
Diagonals	2	5		

a Use the given expression to find the two missing numbers in the table.

b Find the value of n if the number of diagonals is:

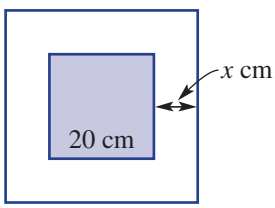
i 20

ii 54

ENRICHMENT — — 17, 18

Picture frames

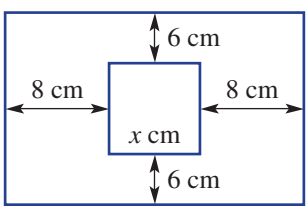
17 A square picture is to be edged with a border of width x cm. The inside picture has side length of 20 cm.



a Write an expression for the total area.

b Find the width of the frame if the total area is to be 1600 cm^2.

18 A square picture is surrounded by a rectangular frame as shown. The total area is to be 320 cm^2. Find the side length of the picture.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10E The parabola

Interactive
Widgets
HOTsheets
Walkthrough

In Chapter 4, we graphed linear relationships (i.e. straight lines), such as $y = 5x - 3$, on the Cartesian plane. In this chapter, non-linear relationships such as $y = x^2$, $y = 2x^2 - 3$ and $y = 3x^2 + 2x - 4$,will be graphed. These produce curves called parabolas. Parabolic shapes can be seen in many modern objects or situations such as the arches of bridges, the paths of projectiles and the surface of reflectors.



Jets of water angled away from the vertical show parabolic trajectories.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

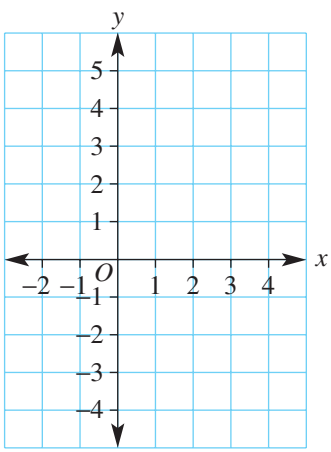
Let's start: Finding features

A quadratic is given by the equation $y = x^2 - 2x - 3$. Complete these tasks to discover its graphical features.

- Use the rule to complete this table of values.

x	−2	−1	0	1	2	3	4
y							

- Plot your points on a copy of the axes shown at right and join them to form a smooth curve.
- Describe the:
  - minimum turning point
  - axis of symmetry
  - coordinates of the y-intercept
  - coordinates of the x-intercepts.

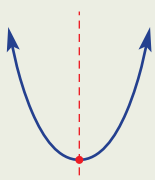


Key ideas

- The graph of a quadratic relation is called a **parabola**. Its basic shape is as shown here.
  - The basic quadratic rule is $y = x^2$.

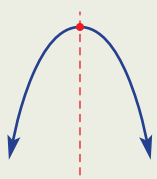


- The general equation of a quadratic is $y = ax^2 + bx + c$.
- A parabola is symmetrical about a line called the **axis of symmetry** and it has one **turning point** (a vertex), which may be a **maximum** or a **minimum**.
- If a is positive the parabola is concave up.
- If a is negative the parabola is concave down.



Axis of symmetry

Vertex (minimum turning point) ($a > 0$)



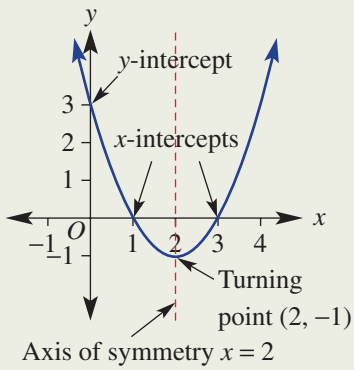
Vertex (maximum turning point)

Axis of symmetry ($a < 0$)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

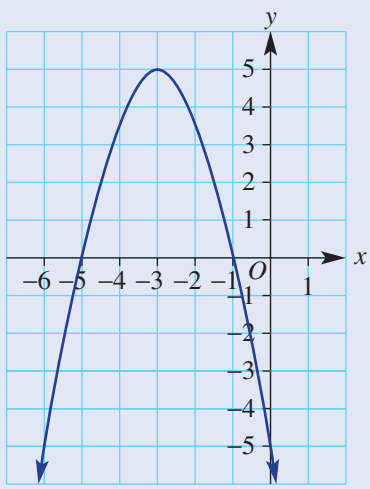
- This is an example of a parabola with equation $y = x^2 - 4x + 3$, showing all the key features.



Example 9 Identifying features

For this graph state the:

a equation of the axis of symmetry
b type of turning point
c coordinates of the turning point
d x-intercepts
e y-intercept



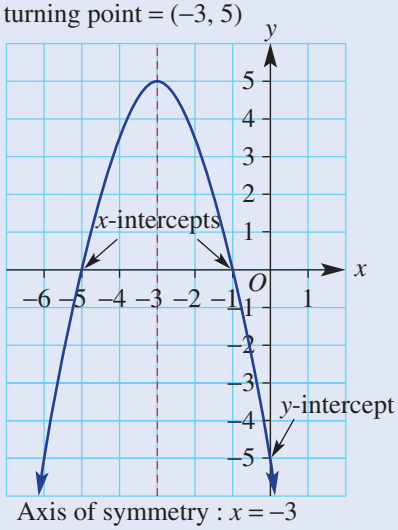
SOLUTION

a $x = -3$
b maximum turning point
c turning point is $(-3, 5)$
d x-intercepts are -5 and -1
e y-intercept is -5

EXPLANATION

Graph is symmetrical about the vertical line $x = -3$. Graph has its highest y coordinate at the turning point, so it is a maximum point.

Highest point:
turning point = $(-3, 5)$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 10 Plotting a parabola

Use the quadratic rule $y = x^2 - 4$ to complete these tasks.

x	−3	−2	−1	0	1	2	3
y							

a Complete this table of values.

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Axis of symmetry

iii Coordinates of the turning point

iv The y-intercept

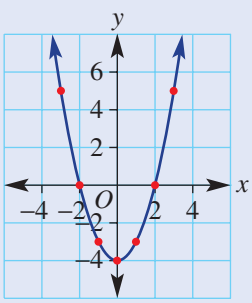
v The x-intercepts

SOLUTION

a

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5

b



c **i** minimum turning point

ii axis of symmetry: $x = 0$

iii turning point $(0, -4)$

iv y-intercept: -4

v x-intercept: -2 and 2

EXPLANATION

Substitute each x value into the rule to find each y value, e.g. $x = -3$, $y = (-3)^2 - 4$
$= 9 - 4 = 5$

Plot each coordinate pair and join to form a smooth curve.

The turning point at $(0, -4)$ has the lowest y coordinate for the entire graph.
The vertical line $x = 0$ divides the graph like a mirror line. The y-intercept is at $x = 0$.
The x-intercepts are at $y = 0$ on the x-axis.

Exercise 10E

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4, 6	3(½), 4–6	4, 6, 7

1 Choose a word from this list to complete each sentence.
lowest, parabola, vertex, highest, intercepts, zero

a A maximum turning point is the _____ point on the graph.

b The graph of a quadratic is called a _____.

c The x-_____ are the points where the graph cuts the x-axis.

d The axis of symmetry is a vertical line passing through the _____.

e A minimum turning point is the _____ point on the graph.

f The y-intercept is at x equals _____.

2 Write down the equation of a vertical line (e.g. $x = 2$) that passes through these points.

a $(3, 0)$ **b** $(1, 5)$ **c** $(-2, 4)$ **d** $(-5, 0)$

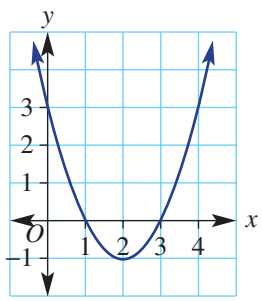
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 9

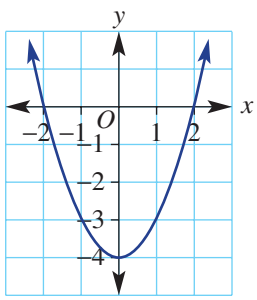
3 For each of the following graphs, state the:

i equation of the axis of symmetry
ii coordinates of the turning point
iii type of turning point (maximum or minimum)
iv x-intercepts
v y-intercept

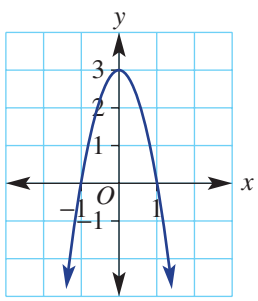
a



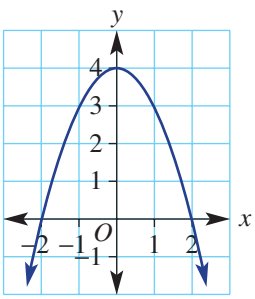
b



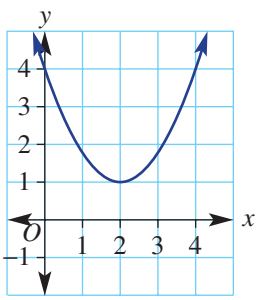
c



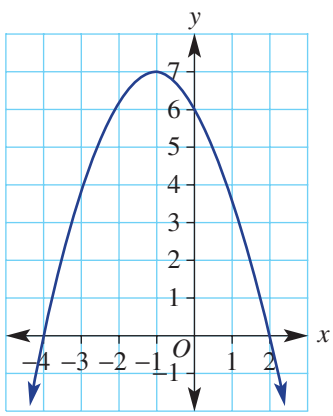
d



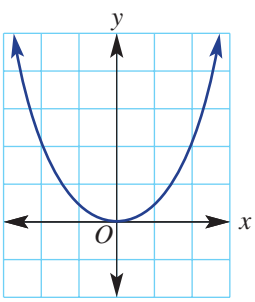
e



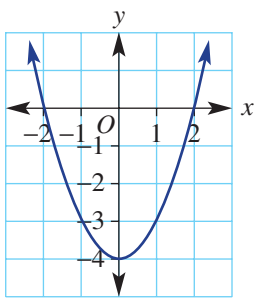
f



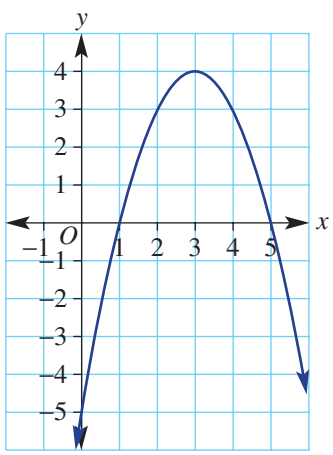
g



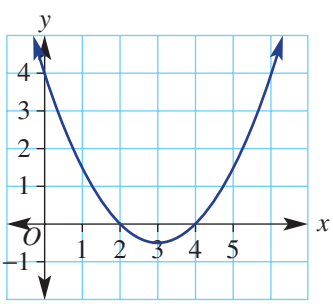
h



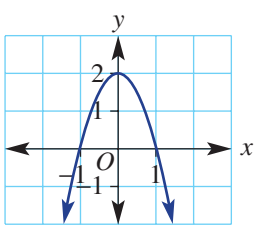
i



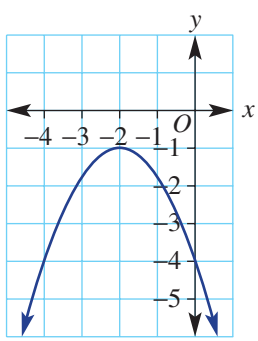
j



k



l



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 10

4 Use the quadratic rule $y = x^2 - 1$ to complete these tasks.

a Complete the table of values.

x	−2	−1	0	1	2
y					

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Coordinates of the turning point

iii Axis of symmetry

iv The y-intercept

v The x-intercepts

5 Use the quadratic rule $y = 9 - x^2$ to complete these tasks.

a Complete the table of values.

x	−3	−2	−1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Coordinates of the turning point

iii Axis of symmetry

iv The y-intercept

v The x-intercepts

6 Use the quadratic rule $y = x^2 + 2x - 3$ to complete these tasks.

a Complete the table of values.

x	−4	−3	−2	−1	0	1	2
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Coordinates of the turning point

iii Axis of symmetry

iv The y-intercept

v The x-intercepts

7 Use the quadratic rule $y = -x^2 + x + 2$ to complete these tasks. Recall that $-x^2 = -1 \times x^2$.

a Complete the table of values.

x	−2	−1	0	1	2	3
y						

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Coordinates of the turning point

iii Axis of symmetry

iv The y-intercept

v The x-intercepts

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

PROBLEM-SOLVING AND REASONING	8, 9, 13, 14	8, 10, 11–15	10, 14–16

8 This graph shows the height of a cricket ball, y metres, as a function of time t seconds.

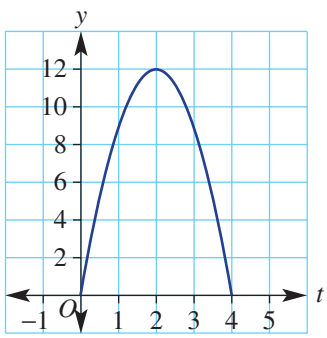
a **i** At what times is the ball at a height of 9 m?

ii Why are there two different times?

b **i** At what time is the ball at its greatest height?

ii What is the greatest height the ball reaches?

iii After how many seconds does it hit the ground?



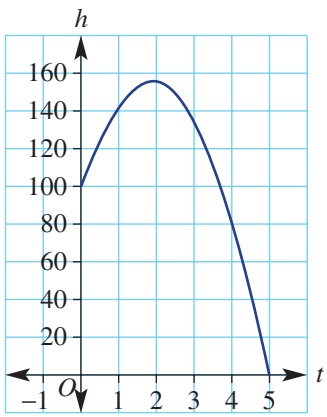
9 The graph gives the height, h m, at time t seconds, of a rocket that is fired up in the air.

a From what height is the rocket launched?

b What is the approximate maximum height that the rocket reaches?

c For how long is the rocket in the air?

d What is the difference in time for when the rocket is going up and when it is going down?



10 A parabola has two x-intercepts at -2 and 4. The y coordinate of the turning point is -3.

a What is the equation of its axis of symmetry?

b What are the coordinates of the turning point?

11 A parabola has a turning point at (1, 3) and an x-intercept at 0.

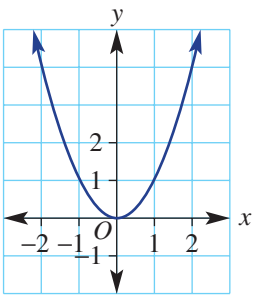
a What is the equation of its axis of symmetry?

b What is the other x-intercept?

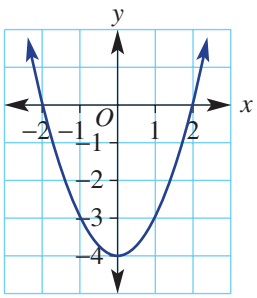
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 Write the rule for these parabolas. Use trial and error to help and check your rule by substituting a known point.

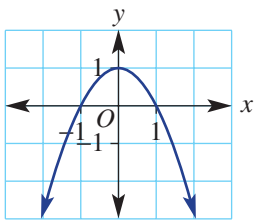
a



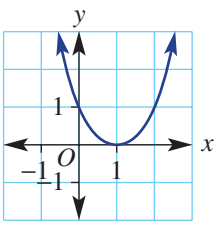
b



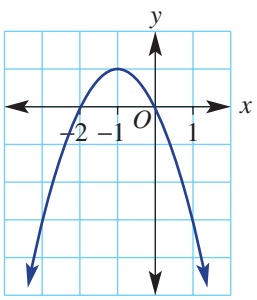
c



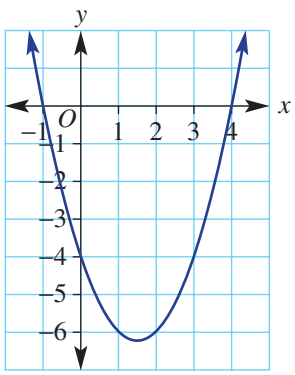
d



e



f



13 Is it possible to draw a parabola with the following properties? If yes, draw an example.

a Two x-intercepts

b One x-intercept

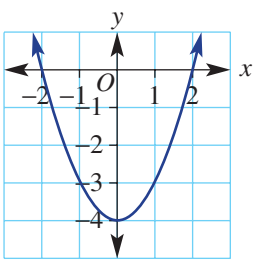
c No x-intercepts

d No y-intercept

14 **a** Mal calculates the y value for $x = 2$ using $y = -x^2 + 2x$ and gets $y = 8$. Explain his error.

b Mai calculates the y value for $x = -3$ using $y = x - x^2$ and gets $y = 6$. Explain her error.

15 This graph shows the parabola for $y = x^2 - 4$.



a For what values of x is $y = 0$?

b For what value of x is $y = -4$?

c How many values of x give a y value that is:

i greater than -4?

ii equal to -4?

iii less than -4?

16 This table corresponds to the rule $y = x^2 - 2x$.

x	−1	0	1	2	3	4
y	3	0	−1	0	3	8

a Use this table to solve these equations.

i $0 = x^2 - 2x$

ii $3 = x^2 - 2x$

b How many solutions would there be to the equation $8 = x^2 - 2x$? Why?

c How many solutions would there be to the equation $-1 = x^2 - 2x$? Why?

d How many solutions would there be to the equation $-2 = x^2 - 2x$? Why?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

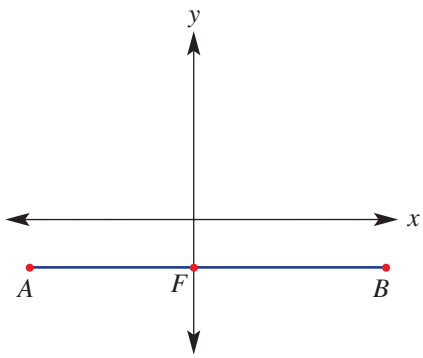
ENRICHMENT — — 17

Using software to construct a parabola

17 Follow the steps below to construct a parabola using a dynamic geometry package.

Step 1. Show the coordinate axes system by selecting Show Axes from the Draw toolbox.

Step 2. Construct a line which is parallel to the x-axis and passes through a point F on the y-axis near the point $(0, -1)$.

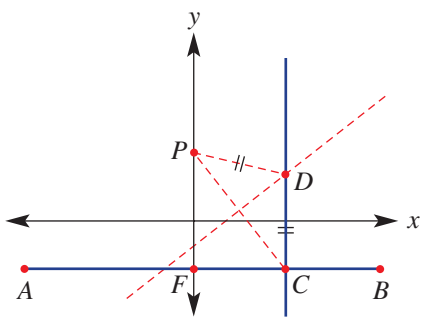


Step 3. Construct a line segment AB on this line as shown in the diagram.

Step 4. Hide the line AB and then construct a point:
- C on the line segment AB
- P on the y-axis near the point $(0, 1)$.

Step 5. Construct a line that passes through the point C and is perpendicular to AB.

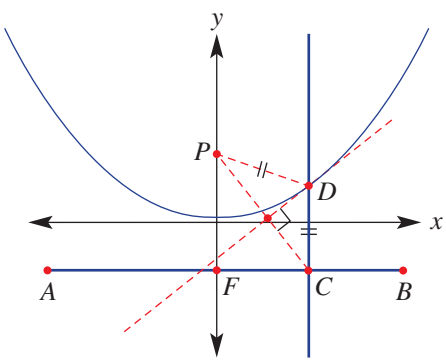
Step 6. Construct the point D which is equidistant from point P and segment AB. *Hint*: use the perpendicular bisector of PC.



Step 7. Select Trace from the Display toolbox and click on the point D.

Step 8. Animate point C and observe what happens.

Step 9. Select Locus from the Construct toolbox and click at D and then at C.



Step 10. Drag point P and/or segment AB (by dragging F). (Clear the trace points by selecting Refresh drawing from the Edit menu.) What do you notice?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10F Sketching $y = ax^2$ with dilations and reflections

Stage

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

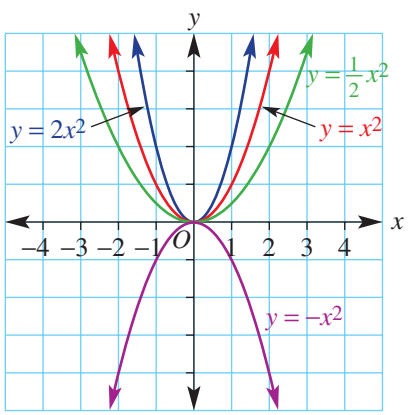
Graphing parabolas using a table of values is a slow process. Some parabolas, such as $y = -2x^2$, can be drawn quickly *without* a table of values.

In geometry we know that shapes can be transformed by applying reflections, rotations, translations and dilations (enlargement). The same types of transformations can also be applied to graphs including parabolas. Altering the value of a in $y = ax^2$ causes both dilations and reflections.

Let's start: What is the effect of a?

This table and graph show a number of examples of $y = ax^2$ with varying values of a. They could also be produced using technology.

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 2x^2$	8	2	0	2	8
$y = \frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2
$y = -x^2$	-4	-1	0	-1	-4

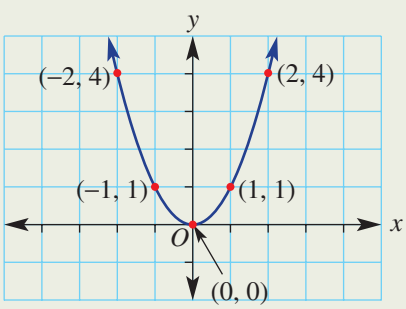


- Discuss how the different values of a affect the y values in the table.
- Discuss how the different values of a affect the shape of the graph.
- Why does $y = -x^2$ look like a graph of $y = x^2$ reflected in the x-axis?
- How would the graphs of the following rules compare to the graphs shown above?

a $y = 3x^2$ **b** $y = \frac{1}{4}x^2$ **c** $y = -\frac{1}{2}x^2$

Key ideas

■ The most basic parabola is the graph of $y = x^2$. It can be graphed quickly using the five key points below.



If you memorise these five points, you can use them to draw many other parabolas.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

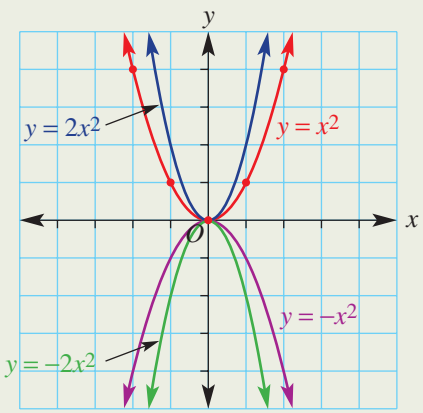
Key ideas

- The equation $y = ax^2$ is used to describe the family of parabolas that includes $y = x^2$, $y = 2x^2$, $y = \frac{1}{2}x^2$, $y = -x^2$ etc.

- All the parabolas in this family possess the following features.
  - The vertex (or turning point) is (0, 0).
  - The axis of symmetry is $x = 0$.
  - If $a > 0$, the graph is **upright** (or **concave up**) and has a minimum turning point.
  - If $a < 0$, the graph is **inverted** (or **concave down**) and has a maximum turning point.

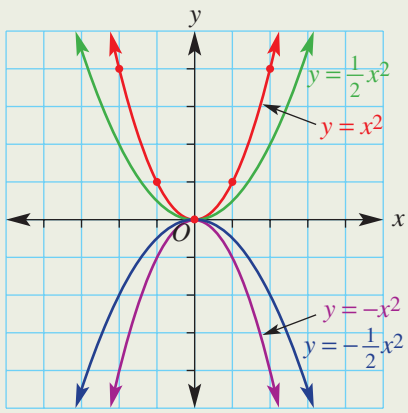
- If $a > 1$ or $a < -1$:
  the graph appears narrower than either $y = x^2$ or $y = -x^2$.

  For example: $y = 2x^2$ or $y = -2x^2$

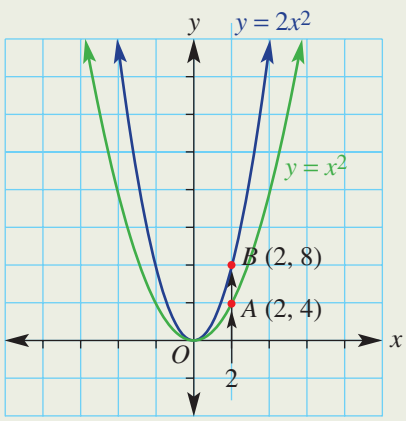


- If $-1 < a < 1$:
  the graph appears wider than either $y = x^2$ or $y = -x^2$.

  For example: $y = \frac{1}{2}x^2$ or $y = -\frac{1}{2}x^2$

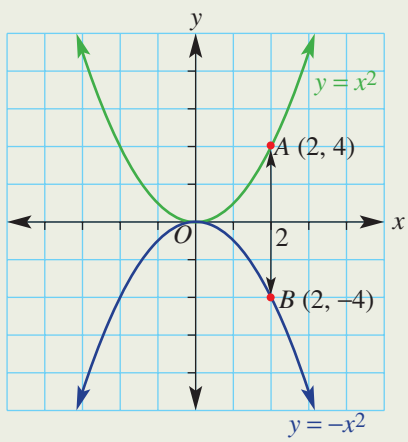


- For $y = 2x^2$ we say that the graph of $y = x^2$ is **dilated** from the x-axis by a factor of 2.
  For example: the point $A(2, 4)$ on $y = x^2$ is dilated to the point $B(2, 8)$ on $y = 2x^2$.



This dilation makes $y = 2x^2$ appear narrower than $y = x^2$.

- For $y = -x^2$ we say that the graph of $y = x^2$ is **reflected** in the x-axis.
  For example: the point $A(2, 4)$ on $y = x^2$ is reflected across the x-axis to $B(2, -4)$ on $y = -x^2$.



$y = x^2$ is concave up and $y = -x^2$ is concave down.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 11 Comparing graphs of $y = ax^2$, $a > 0$

Complete the following for $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$.

a Draw up and complete a table of values for $-2 \leq x \leq 2$.

b Plot their graphs on the same set of axes.

c Write down the the equation of the axis of symmetry and the coordinates of the turning point.

d **i** Does the graph of $y = 2x^2$ appear wider or narrower than $y = x^2$?

ii Does the graph of $y = \frac{1}{2}x^2$ appear wider or narrower than $y = x^2$?

SOLUTION

a $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

$y = 2x^2$

x	-2	-1	0	1	2
y	8	2	0	2	8

$y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

EXPLANATION

Substitute each x value into $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$.

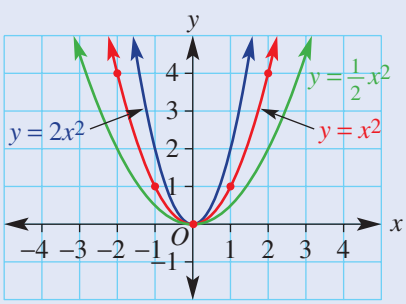
For example:

For $y = 2x^2$,

if $x = 2$, $y = 2(2)^2$
$= 2(4)$
$= 8$

if $x = -1$, $y = 2(-1)^2$
$= 2(1)$
$= 2$

b



Plot the points for each graph using the coordinates from the tables and join them with a smooth curve.

c axis of symmetry: y-axis ($x = 0$)
turning point: minimum at (0, 0)

Look at graphs to see symmetry about the y-axis and a minimum turning point at the origin.

d **i** The graph of $y = 2x^2$ appears narrower than the graph of $y = x^2$.

For each value of x, $2x^2$ is twice that of x^2; hence, the graph (y values) of $2x^2$ rises more quickly.

ii The graph of $y = \frac{1}{2}x^2$ appears wider than the graph of $y = x^2$.

For each value of x, $\frac{1}{2}x^2$ is half that of x^2; hence, the graph of $\frac{1}{2}x^2$ rises more slowly.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 12 Comparing graphs of $y = ax^2$, $a < 0$

Complete the following for $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.

a Draw up and complete a table of values for $-2 \leq x \leq 2$.
b Plot their graphs on the same set of axes.
c Write down the the equation of the axis of symmetry and the coordinates of the turning point.
d i Does the graph of $y = -3x^2$ appear wider or narrower than $y = -x^2$?
ii Does the graph of $y = -\frac{1}{2}x^2$ appear wider or narrower than $y = -x^2$?

SOLUTION

a $y = -x^2$

x	-2	-1	0	1	2
y	-4	-1	0	-1	-4

$y = -3x^2$

x	-2	-1	0	1	2
y	-12	-3	0	-3	-12

$y = -\frac{1}{2}x^2$

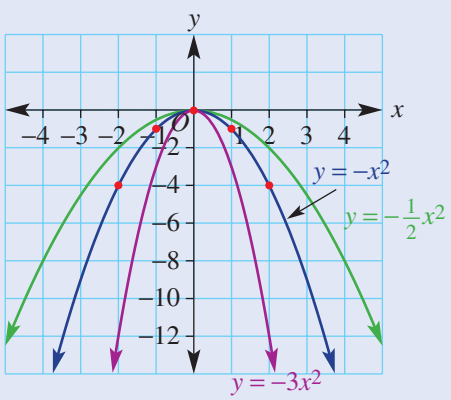
x	-2	-1	0	1	2
y	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2

EXPLANATION

Substitute each x value into $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.

For example:
For $y = -3x^2$,
if $x = 2$, $y = -3(2)^2$
$= -3(4)$
$= -12$
if $x = -1$, $y = -3(-1)^2$
$= -3(1)$
$= -3$

b



Plot the coordinates for each graph from the tables and join them with a smooth curve. The negative sign indicates that these graphs are concave down.

c axis of symmetry: y-axis ($x = 0$)
turning point: maximum at (0, 0)

Graphs are symmetrical about the y-axis with a maximum turning point at the origin.

d i The graph of $y = -3x^2$ appears narrower than the graph of $y = -x^2$.

For each value of x, $-3x^2$ is three times that of $-x^2$; hence, the graph of $-3x^2$ gets larger in the negative direction more quickly.

ii The graph of $y = -\frac{1}{2}x^2$ appears wider than the graph of $y = -x^2$.

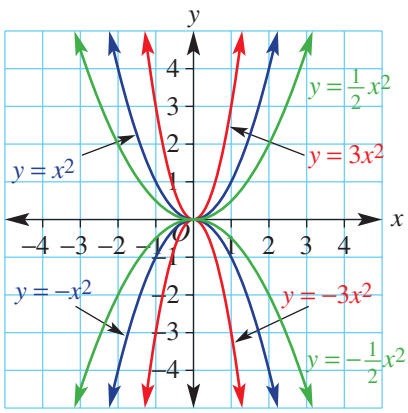
For each value of x, $-\frac{1}{2}x^2$ is half that of $-x^2$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 10F

UNDERSTANDING AND FLUENCY	1–6	2, 4, 5–6(½)	4, 6

1 Shown here are the graphs of $y = x^2$, $y = 3x^2$, $y = \frac{1}{2}x^2$, $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.



- **a** Write the rules of the three graphs that have a minimum turning point.
- **b** Write the rules of the three graphs that have a maximum turning point.
- **c** What are the coordinates of the turning point for all the graphs?
- **d** What is the equation of the axis of symmetry for all the graphs?
- **e** Write the rule of the graph that is:
  - **i** upright (concave up) and narrower than $y = x^2$
  - **ii** upright (concave up) and wider than $y = x^2$
  - **iii** inverted (concave down) and narrower than $y = -x^2$
  - **iv** inverted (concave down) and wider than $y = -x^2$
- **f** Write the rule of the graph that is a reflection of:
  - **i** $y = x^2$ in the x-axis
  - **ii** $y = 3x^2$ in the x-axis
  - **iii** $y = -\frac{1}{2}x^2$ in the x-axis

2 Select the word *positive* or *negative* to suit each sentence.

- **a** The graph of $y = ax^2$ will be upright (concave up) with a minimum turning point if a is ___.
- **b** The graph of $y = ax^2$ will be inverted (concave down) with a maximum turning point if a is ___.

3

- **a** Write the rule of a graph that is a reflection in the x-axis of the graph of the rule $y = x^2$.
- **b** Write the rule of a graph that is a reflection in the x-axis of the graph of the rule $y = 4x^2$.
- **c** Write the rule of a graph that is a reflection in the x-axis of the graph of the rule $y = -5x^2$.
- **d** Write the rule of a graph that is a reflection in the x-axis of the graph of the rule $y = -\frac{1}{3}x^2$.

Example 11

4 Complete the following for $y = x^2$, $y = 3x^2$ and $y = \frac{1}{3}x^2$.

- **a** Draw up and complete a table of values for $-2 \leq x \leq 2$.
- **b** Plot their graphs on the same set of axes.
- **c** Write down the coordinates of the turning point and the equation of the axis of symmetry.
- **d**
  - **i** Does the graph of $y = 3x^2$ appear wider or narrower than $y = x^2$?
  - **ii** Does the graph of $y = \frac{1}{3}x^2$ appear wider or narrower than $y = x^2$?

5 For the equations given below, complete these tasks.

- **i** Draw up and complete a table of values for $-2 \leq x \leq 2$.
- **ii** Plot the graphs of the equations on the same set of axes.
- **iii** Write down the coordinates of the turning point and the equation of the axis of symmetry.
- **iv** Determine whether the graphs of the equations each appear wider or narrower than the graph of $y = x^2$.

a $y = 4x^2$ **b** $y = 5x^2$ **c** $y = \frac{1}{4}x^2$ **d** $y = \frac{1}{5}x^2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 12

6 For the equations given below, complete these tasks.

i Draw up and complete a table of values for $-2 \leq x \leq 2$.

ii Plot the graphs of the equations on the same set of axes.

iii List the key features for each graph, such as the axis of symmetry, turning point, x-intercept and y-intercept.

iv Determine whether the graphs of the equations each appear wider or narrower than the graph of $y = -x^2$.

a $y = -2x^2$ **b** $y = -3x^2$ **c** $y = -\frac{1}{2}x^2$ **d** $y = -\frac{1}{3}x^2$

PROBLEM-SOLVING AND REASONING 7, 9 7–10 8, 10–12

7 Here are eight quadratics of the form $y = ax^2$.

A $y = 6x^2$ **B** $y = -7x^2$ **C** $y = 4x^2$ **D** $y = \frac{1}{9}x^2$

E $y = \frac{x^2}{7}$ **F** $y = 0.3x^2$ **G** $y = -4.8x^2$ **H** $y = -0.5x^2$

a Which rule would give a graph that is upright (concave up) and the narrowest?

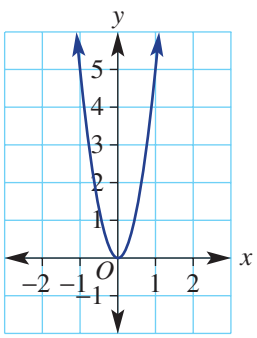
b Which rule would give a graph that is inverted (concave down) and the widest?

8 Match each of the following parabolas with the appropriate equation from the list below. Do a mental check by substituting the coordinates of a known point other than (0, 0).

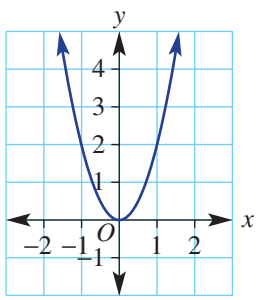
i $y = 3x^2$ **ii** $y = -x^2$ **iii** $y = 5x^2$

iv $y = \frac{1}{2}x^2$ **v** $y = -5x^2$ **vi** $y = 2x^2$

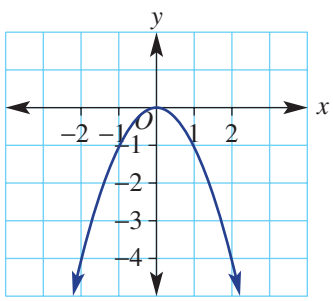
a



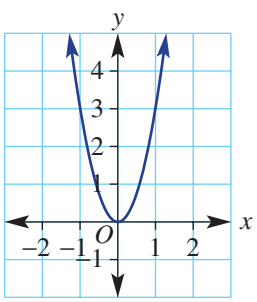
b



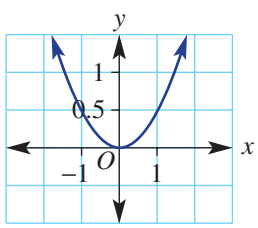
c



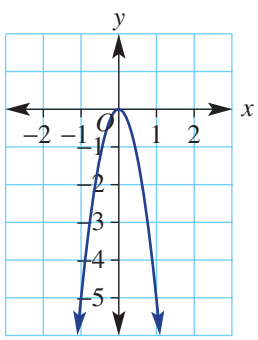
d



e



f



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 The graph of $y = -2x^2$ can be obtained from $y = x^2$ by conducting these transformations.

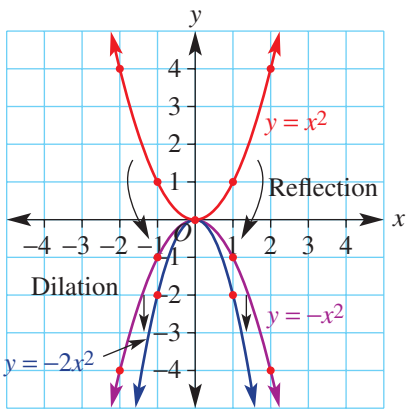
- Reflecting in the x-axis
- Dilating by a factor of 2 from the x-axis

In the same way as above, describe the two transformations that take:

a $y = x^2$ to $y = -3x^2$ **b** $y = x^2$ to $y = -6x^2$

c $y = x^2$ to $y = -\frac{1}{2}x^2$ **d** $y = -x^2$ to $y = 2x^2$

e $y = -x^2$ to $y = 3x^2$ **f** $y = -x^2$ to $y = \frac{1}{3}x^2$



10 Write the rule for the graph after each set of transformations.

a The graph of $y = x^2$ is reflected in the x-axis, then dilated by a factor of 4 from the x-axis.

b The graph of $y = -x^2$ is reflected in the x-axis, then dilated by a factor of $\frac{1}{3}$ from the x-axis.

c The graph of $y = 2x^2$ is reflected in the x-axis, then dilated by a factor of 2 from the x-axis.

d The graph of $y = \frac{1}{3}x^2$ is reflected in the x-axis, then dilated by a factor of 4 from the x-axis.

11 The graph of $y = ax^2$ is reflected in the x-axis and dilated from the x-axis by a given factor. Does it matter which transformation is completed first? Explain.

12 The graph of the rule $y = ax^2$ is reflected in the y-axis. What is the new rule of the graph?

ENRICHMENT — — 13, 14

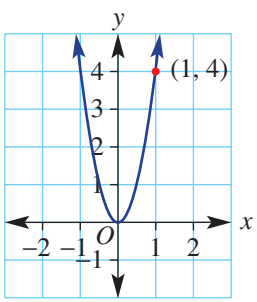
Substitute to find the rule

13 If a rule is of the form $y = ax^2$ and it passes through a point, say (1, 4), we can substitute this point to find the value of a.

So $y = ax^2$

$4 = a \times 12$

$\therefore a = 4$ and $y = 4x^2$

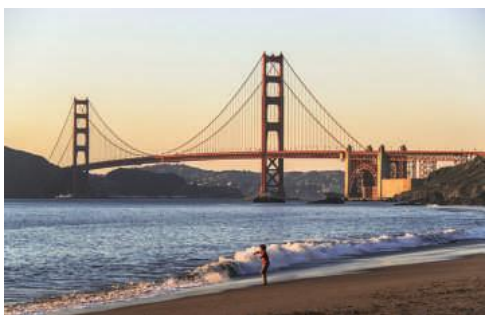


Use this method to determine the equation of a quadratic relation if it has an equation of the form $y = ax^2$ and passes through:

a (1, 5) **b** (1, 7) **c** (−1, 1) **d** (−2, 7)

e (−5, 4) **f** (3, 26) **g** (4, 80) **h** (−1, −52)

14 This photo shows the parabolic-like cables of the Golden Gate Bridge. The rule of the form $y = ax^2$ models the shape of the cables. If the cable is centred at (0, 0) and the top of the right pylon has the coordinates (492, 67), find a possible equation that describes this shape. The numbers given are in metres.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10G Translations of $y = x^2$

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive
Widgets
HOTsheets
Walkthrough

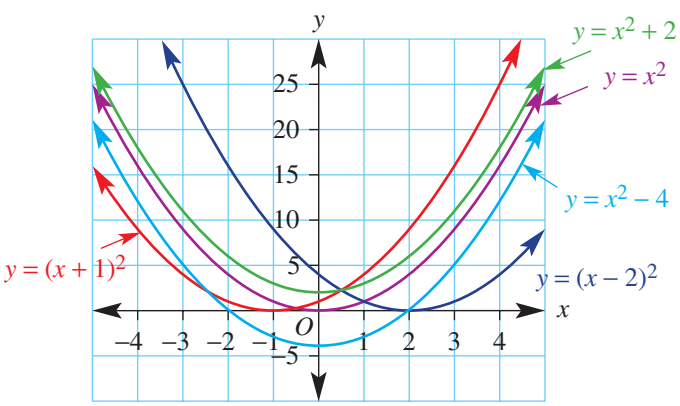
Added to reflection and dilation is a third type of transformation called translation. This involves a shift of every point on the graph horizontally and/or vertically. Unlike reflections and dilations, a translation alters the coordinates of the turning point. The shape of the curve is unchanged but a horizontal shift changes the equation of the axis of symmetry.

Let's start: Which way – left, right, up or down?

This table and graph shows the quadratics $y = x^2$, $y = (x - 2)^2$, $y = (x + 1)^2$, $y = x^2 - 4$ and $y = x^2 + 2$. The table could also be produced using technology.

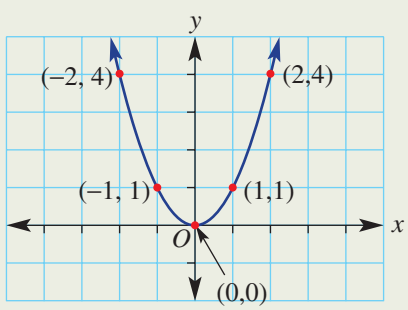
- Discuss what effect the different numbers in the rules had on the y values in the table.
- Also discuss what effect the numbers in the rules have on each graph. How are the coordinates of the turning point changed?
- What conclusions can you draw about the effect of h in the rule $y = (x - h)^2$?
- What conclusions can you draw about the effect of k in the rule $y = x^2 + k$?
- What if the rule was $y = -x^2 + 2$ or $y = -(x + 1)^2$? Describe how the graphs would look.

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = (x - 2)^2$	25	16	9	4	1	0	1
$y = (x + 1)^2$	4	1	0	1	4	9	16
$y = x^2 - 4$	5	0	−3	−4	−3	0	5
$y = x^2 + 2$	11	6	3	2	3	6	11



Key ideas

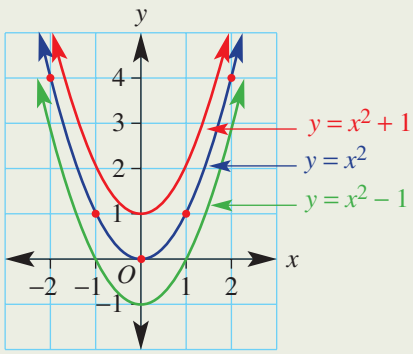
- A **translation** of a graph involves a shift of every point horizontally and/or vertically.
- Recall that the most basic parabola ($y = x^2$) passes through these five key points.
  - Translating this graph up or down will change the y values of these five key points. For example: the point (2, 4) on $y = x^2$ is translated to (2, 3) on $y = x^2 - 1$.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

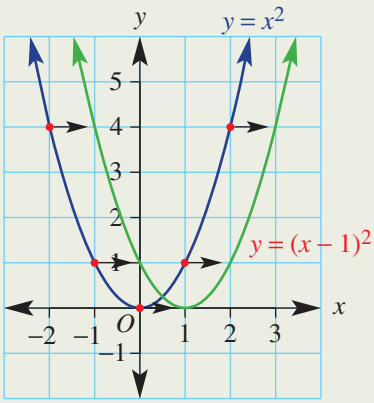
- Vertical translations: $y = x^2 + k$
  - If $k > 0$, the graph is translated k units up (red curve).
  - If $k < 0$, the graph is translated k units down (green curve).
  - In the diagram the five key points move up and down.
  - The turning point is $(0, k)$ for all curves.
  - The axis of symmetry is the line $x = 0$ for all curves. The y-intercept is k for all curves.



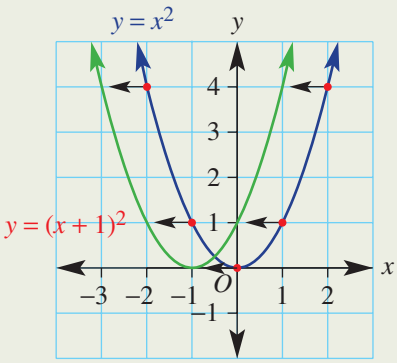
- Horizontal translations: $y = (x - h)^2$

In the diagrams, the five key points all move 1 unit to the right or 1 unit to the left.

- If $h > 0$, the graph is translated h units to the right.



- If $h < 0$, the graph is translated h units to the left.



- The turning point is $(h, 0)$ in both cases.
- The axis of symmetry is the line $x = h$ in both cases.
- The y-intercept is h^2 in both cases.

- The **turning point form** of a quadratic is given by:

$y = a(x - h)^2 + k$

$a > 0$ upright (concave up) graph ∪

$a < 0$ graph inverted (concave down) ∩

Translates the graph up or down

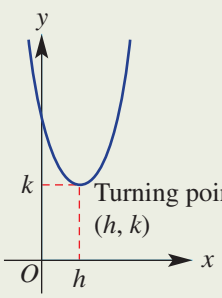
$k > 0$ ↑ $k < 0$ ↓

Translates the graph left or right

$h > 0$ →

$h < 0$ ←

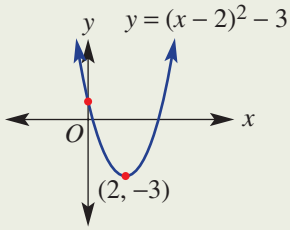
Axis of symmetry: $x = h$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- To sketch a graph of a quadratic equation in turning point form, follow these steps.
  - Draw and label a set of axes.
  - Identify important points including the turning point and y-intercept.
  - Sketch the curve connecting the key points and making the curve symmetrical.



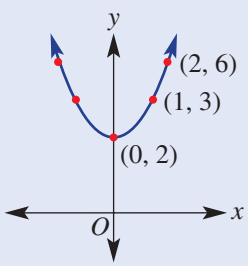
Example 13 Sketching with horizontal and vertical translations

Sketch the graphs of these rules, showing the y-intercept and the coordinates of the turning point.

a $y = x^2 + 2$ **b** $y = -x^2 - 1$ **c** $y = (x - 3)^2$ **d** $y = -(x + 2)^2$

SOLUTION

a $y = x^2 + 2$
Turning point is (0, 2)
y-intercept: $y = 0^2 + 2 = 2$

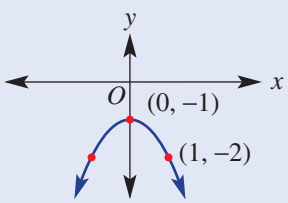


EXPLANATION

For $y = x^2 + k$, $k = 2$ so the graph of $y = x^2$ is translated 2 units up.

The point (0, 0) shifts to (0, 2).
The five key points all move up 2 units.

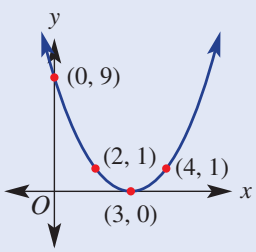
b $y = -x^2 - 1$
Turning point is (0, −1)
y-intercept: $y = -0^2 - 1 = -1$



The graph of $y = -x^2$ is a reflection of the graph of $y = x^2$ in the x-axis, i.e. key point (1, 1) becomes (1, −1).

For $y = -x^2 + k$, $k = -1$ so the graph of $y = -x^2$ is translated down 1 unit. This changes the y values of the key points, i.e. key point (0, 0) becomes (0, −1), key point (1, −1) becomes (1, −2).

c $y = (x - 3)^2$
Turning point is (3, 0)
y-intercept: $y = (0 - 3)^2$
$= (-3)^2$
$= 9$



For $y = (x - h)^2$, $h = 3$ so the graph of $y = x^2$ is translated 3 units to the right. This changes the x values of the key points, i.e. key point (0, 0) becomes (3, 0), key point (1, 1) becomes (4, 1), key point (−1, 1) becomes (2, 1).
The y-intercept is found by substituting $x = 0$ into the rule.

Continued on the next page

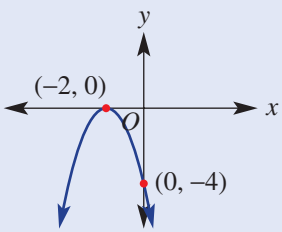
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.
Updated March 2022

Continued from previous page

d $y = -(x + 2)^2$

Turning point is $(-2, 0)$

y-intercept: $y = -(0 + 2)^2$
$= -2^2$
$= -4$



The negative sign in front means the graph is reflected across the x-axis and will be concave down.

For $y = -(x - h)^2$, $h = -2$ since $-(x - (-2))^2 = -(x + 2)^2$. So the graph of $y = -x^2$ is translated 2 units to the left.
This changes the x values of the key points. Visualise the five key points moving 2 units to the left, then being reflected across the x-axis.
The y-intercept is found by substituting $x = 0$.

Example 14 Sketching with combined translations

Sketch these graphs on the same set of axes showing the y-intercept and the coordinates of each turning point.

a $y = (x - 2)^2 - 1$ **b** $y = -(x + 3)^2 + 2$

SOLUTION

a $y = (x - 2)^2 - 1$

Turning point is $(2, -1)$

y-intercept: $y = (0 - 2)^2 - 1$
$= 4 - 1$
$= 3$

EXPLANATION

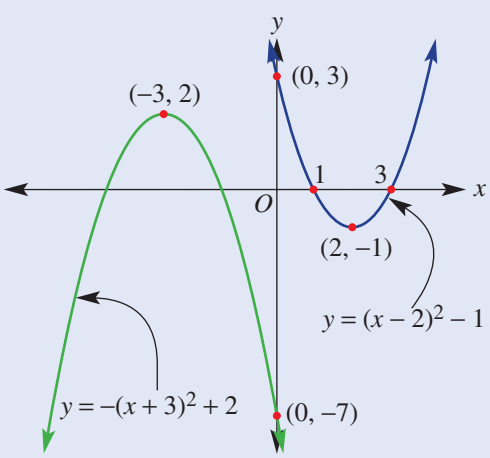
For $y = (x - h)^2 + k$, $h = 2$ and $k = -1$ so the five key points are shifted 2 to the right and 1 down:
$(0, 0) \rightarrow (2, 0) \rightarrow (2, -1)$
$(1, 1) \rightarrow (3, 1) \rightarrow (3, 0)$
$(-1, 1) \rightarrow (1, 1) \rightarrow (1, 0)$
Substitute $x = 0$ for the y-intercept.

b $y = -(x + 3)^2 + 2$

Turning point is $(-3, 2)$

y-intercept: $y = -(0 + 3)^2 + 2$
$= -9 + 2$
$= -7$

This parabola is concave down.
For $y = -(x - h)^2 + k$, $h = -3$ and $k = 2$ so the graph of $y = x^2$ is shifted 3 to the left and 2 up:
$(0, 0) \rightarrow (-3, 0) \rightarrow (-3, 2)$



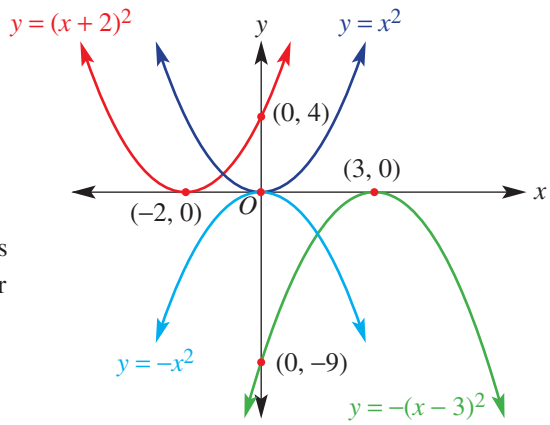
First, position the coordinates of the turning point and y-intercept then join to form each curve.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 10G

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	3, 4, 5–7(½)	5–7(½)

1 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = (x + 2)^2$ and $y = -(x - 3)^2$.



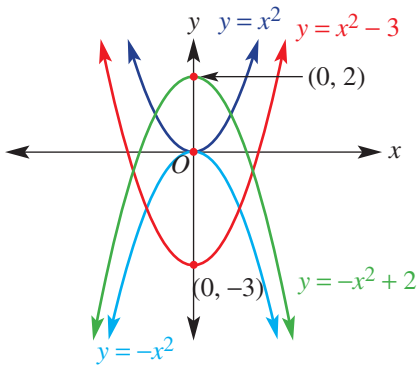
a State the turning point of the graph of:
- **i** $y = (x + 2)^2$
- **ii** $y = -(x - 3)^2$

b State the y-intercept for the graph of:
- **i** $y = (x + 2)^2$
- **ii** $y = -(x - 3)^2$

c Compared to the graph of $y = x^2$, which way has the graph of $y = (x + 2)^2$ been translated (left or right)?

d Compared to the graph of $y = -x^2$, which way has the graph of $y = -(x - 3)^2$ been translated (left or right)?

2 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = x^2 - 3$ and $y = -x^2 + 2$.



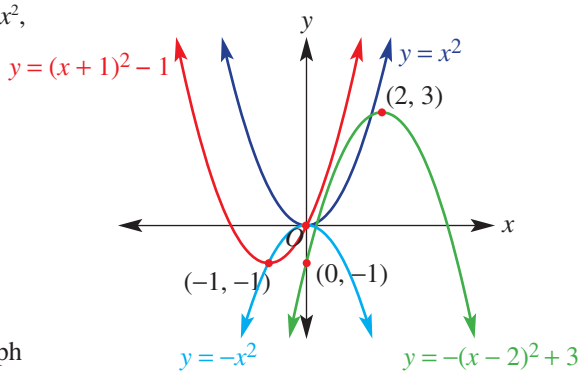
a State the turning point of the graph of:
- **i** $y = x^2 - 3$
- **ii** $y = -x^2 + 2$

b State the y-intercept for the graph of:
- **i** $y = x^2 - 3$
- **ii** $y = -x^2 + 2$

c Compared to the graph of $y = x^2$, which way has the graph of $y = x^2 - 3$ been translated (up or down)?

d Compared to the graph of $y = -x^2$, which way has the graph of $y = -x^2 + 2$ been translated (up or down)?

3 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = -(x - 2)^2 + 3$ and $y = (x + 1)^2 - 1$.



a State the turning point of the graph of:
- **i** $y = -(x - 2)^2 + 3$
- **ii** $y = (x + 1)^2 - 1$

b State the y-intercept for the graph of:
- **i** $y = -(x - 2)^2 + 3$
- **ii** $y = (x + 1)^2 - 1$

c State the missing words and numbers.
- **i** Compared to the graph of $y = x^2$, the graph of $y = (x + 1)^2 - 1$ has to be translated _____ unit to the _____ and _____ unit _____.
- **ii** Compared to the graph of $y = -x^2$, the graph of $y = -(x - 2)^2 + 3$ has to be translated _____ units to the _____ and _____ units _____.

4 Substitute $x = 0$ to find the y-intercept for these equations.

a $y = x^2 + 3$ **b** $y = -x^2 - 4$ **c** $y = -(x - 2)^2$ **d** $y = (x + 5)^2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 13a,b

5 Sketch the graphs of these equations, showing the coordinates of the turning point and y-intercept.

a $y = x^2 + 1$ **b** $y = x^2 + 3$ **c** $y = x^2 - 2$

d $y = -x^2 + 4$ **e** $y = -x^2 + 1$ **f** $y = -x^2 - 5$

Example 13c,d

6 Sketch the graphs of these equations, showing the coordinates of the turning point and y-intercept.

a $y = (x - 2)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 3)^2$

d $y = -(x - 3)^2$ **e** $y = -(x + 2)^2$ **f** $y = -(x + 6)^2$

Example 14

7 Sketch each graph showing the coordinates of the turning point and the y-intercept.

a $y = (x - 3)^2 + 2$ **b** $y = (x - 1)^2 - 1$ **c** $y = (x + 2)^2 - 3$

d $y = (x + 1)^2 + 7$ **e** $y = -(x - 2)^2 + 1$ **f** $y = -(x - 5)^2 + 3$

g $y = -(x + 3)^2 - 4$ **h** $y = -(x + 1)^2 - 5$ **i** $y = -(x - 3)^2 - 6$

PROBLEM-SOLVING AND REASONING 8, 11 | 8, 9, 11, 12 | 8–10, 12, 13

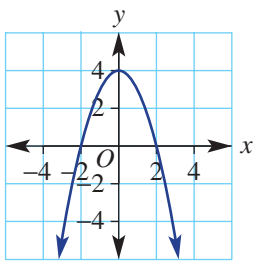
8 Match each of the parabolas with the appropriate equation from the list below.

i $y = x^2$ **ii** $y = (x + 2)^2$ **iii** $y = x^2 - 4$ **iv** $y = -x^2 - 3$

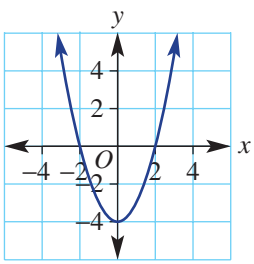
v $y = (x - 2)^2$ **vi** $y = x^2 + 4$ **vii** $y = 4 - x^2$ **viii** $y = (x + 3)^2$

ix $y = (x - 5)^2$ **x** $y = x^2 + 3$ **xi** $y = -(x + 3)^2$ **xii** $y = -x^2 + 3$

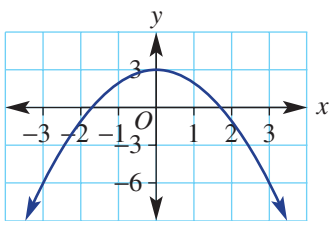
a



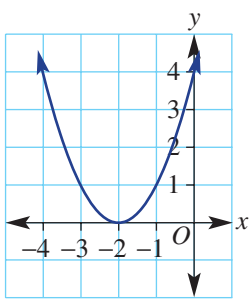
b



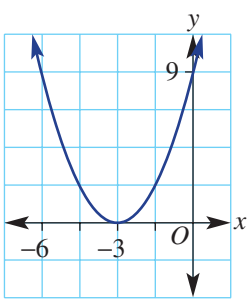
c



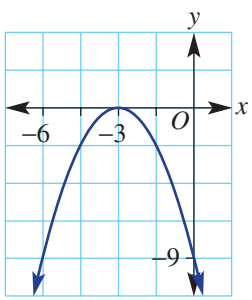
d



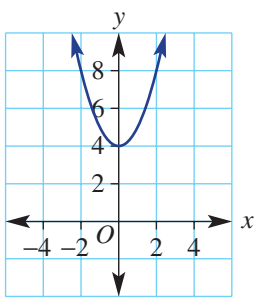
e



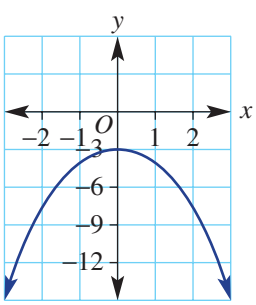
f



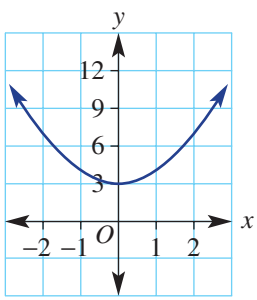
g



h

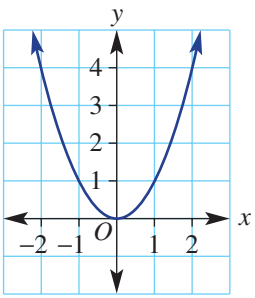


i

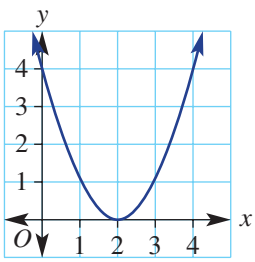


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

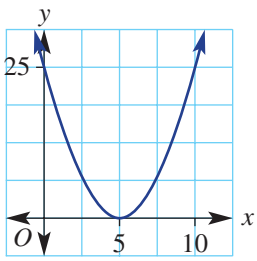
j



k

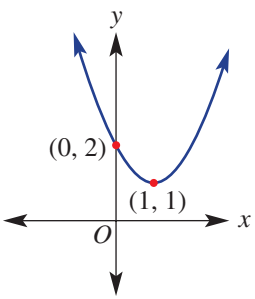


l

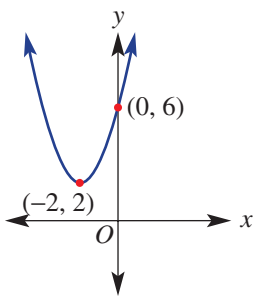


9 Write the rule for each graph in turning point form.

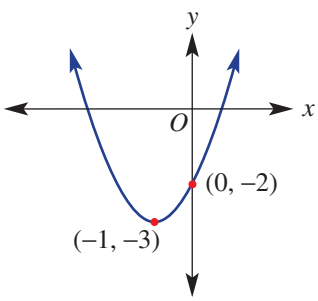
a



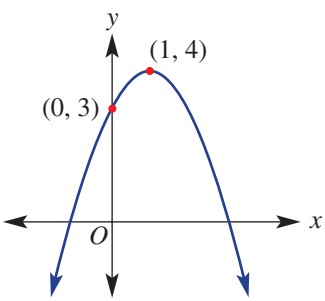
b



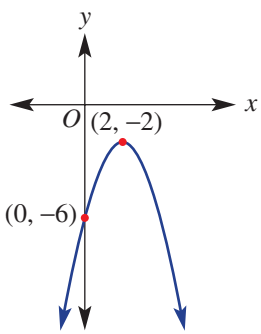
c



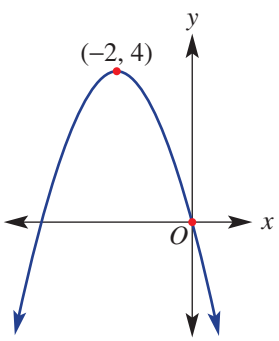
d



e



f



10 A bike track can be modelled approximately by combining two different quadratic equations.
The first part of the bike path can be modelled by the equation $y = -(x - 2)^2 + 9$ for $-2 \leqslant x \leqslant 5$
The second part of the bike track can be modelled by the equation $y = (x - 7)^2 - 4$ for $5 \leqslant x \leqslant 10$.

a Find the turning point of the graph of each quadratic equation.

b Sketch each graph on the same set of axes. On your sketch of the bike path you need to show the coordinates of the start and finish of the track and where it crosses the x-axis.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 Written in the form $y = a(x - h)^2 + k$, the equation $y = 4 - (x + 2)^2$ could be rearranged to give $y = -(x + 2)^2 + 4$.

a Rearrange these equations and write in the form $y = a(x - h)^2 + k$.

i $y = 3 - (x + 1)^2$

ii $y = 4 + (x + 3)^2$

iii $y = -3 + (x - 1)^2$

iv $y = -7 - (x - 5)^2$

v $y = -2 - x^2$

vi $y = -6 + x^2$

b Write down the coordinates of the turning point for each of the above quadratics.

12 A quadratic has the equation $y = a(x - h)^2 + k$.

a What are the coordinates of the turning point?

b Write an expression for the y-intercept.

13 Investigate and explain how the graph of:

a $y = (2 - x)^2$ compares to the graph of $y = (x - 2)^2$

b $y = (1 - x)^2$ compares to the graph of $y = (x - 1)^2$

ENRICHMENT — — 14–17

Finding rules

14 Find the equation of the quadratic relationship that is of the form $y = x^2 + c$ and passes through:

a (1, 4) **b** (3, 5)

c (2, 1) **d** (2, −1)

15 Find the equation of the quadratic relationship that is of the form $y = -x^2 + c$ and passes through:

a (1, 3) **b** (−1, 3)

c (3, 15) **d** (−2, 6)

16 Find the possible equations of each of the following quadratics if their equation is of the form $y = (x - h)^2$ and their graph passes through the point:

a (1, 16) **b** (3, 1)

c (−1, 9) **d** (3, 9)

17 Find the rule for each of these graphs with the given turning point and y-intercept.

a Turning point = (1, 1), y-intercept = 0

b Turning point = (−2, 0), y-intercept = 4

c Turning point = (3, 0), y-intercept = −9

d Turning point = (−3, 2), y-intercept = −7

e Turning point = (−1, 4), y-intercept = 5

f Turning point = (3, −9) , y-intercept = 0

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10H Sketching parabolas using intercept form

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Interactive

Widgets

HOTsheets

Walkthrough

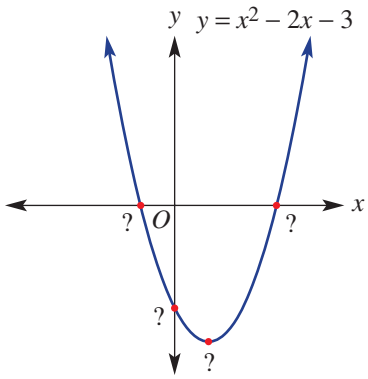
So far we have sketched parabolas using rules of the form $y = a(x - h)^2 + k$ where the coordinates of the turning point can be determined directly from the rule. An alternative method for sketching parabolas uses the factorised form of the quadratic rule and the Null Factor Law to find the x-intercepts. The turning point can be found by considering the axis of symmetry, which sits halfway between the two x-intercepts.

Let's start: From x-intercepts to turning point

This graph has the rule $y = x^2 - 2x - 3$ but all its important features are not shown.

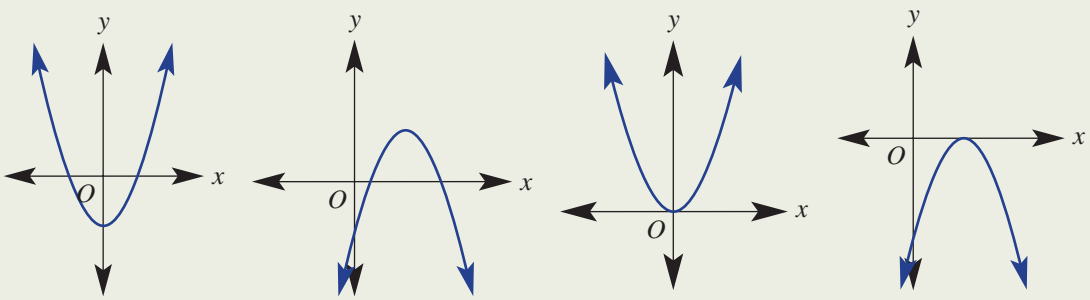
Find the features by discussing these questions.

- Can the quadratic rule be factorised?
- What is true about the coordinates of both x-intercepts?
- How can the factorised form of the rule help to find the x-intercepts?
- How does the x coordinate of the turning point relate to the x-intercepts?
- Discuss how the y coordinate of the turning point can be found.
- Finish by finding the y-intercept.



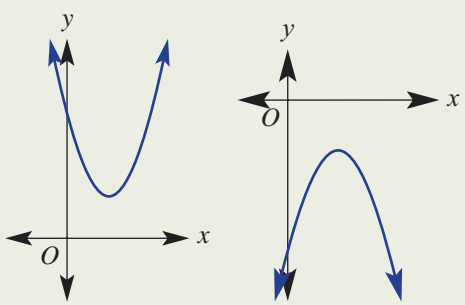
Key ideas

■ All parabolas have one y-intercept and can have two, one or zero x-intercepts.



Two x-intercepts

One x-intercept

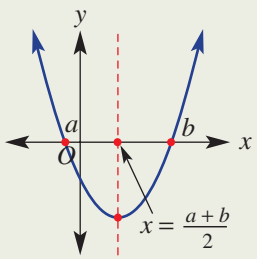


Zero x-intercepts

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Key ideas

- x-intercepts can be found by substituting $y = 0$ and using the Null Factor Law.
  $y = (x - a)(x - b)$
  $0 = (x - a)(x - b)$
  $x - a = 0$ or $x - b = 0$
  $x = a$ or $x = b$
- If the graph has two x-intercepts (a and b), the turning point can be found by:
  - calculating the x coordinate of the turning point, the midpoint of a and b; that is, $x = \frac{a + b}{2}$
  - calculating the y value of the turning point by substituting the x coordinate into the rule for the quadratic.



Example 15 Finding intercepts

For each of the following quadratic equations, find the:

i x-intercepts ii y-intercept

a $y = x(x + 1)$ b $y = 2(x + 2)(x - 3)$

SOLUTION

a i x-intercepts (let $y = 0$):
$x(x + 1) = 0$
$x = 0$ or $x + 1 = 0$
$x = 0$ or $x = -1$
x-intercepts $= 0$ and -1

EXPLANATION

Let $y = 0$ to find the x-intercepts.

Apply the Null Factor Law to set each factor equal to 0 and solve.

ii y-intercept (let $x = 0$):
$y = 0(0 + 1)$
$y = 0$
y-intercept $= 0$

Let $x = 0$ to find the y-intercept.

b i $y = 2(x + 2)(x - 3)$ has two x-intercepts
x-intercepts (let $y = 0$):
$2(x + 2)(x - 3) = 0$
$x + 2 = 0$ or $x - 3 = 0$
$x = -2$ or $x = 3$
x-intercepts $= -2$ and 3

There are two different factors.

Let $y = 0$ to find the x-intercepts. Set each factor equal to 0 and solve.

ii y-intercept (let $x = 0$):
$y = 2(2)(-3)$
$y = -12$
y-intercept $= -12$

Let $x = 0$ to find the y-intercept.

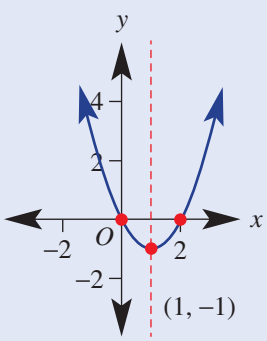
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 16 Sketching using intercept form

For the quadratic equation $y = x^2 - 2x$:

- **a** factorise the relation
- **b** find the y-intercept
- **c** find the x-intercepts
- **d** find the axis of symmetry
- **e** find the turning point
- **f** sketch the graph, clearly showing all the key features

SOLUTION	EXPLANATION
a $y = x^2 - 2x$ $= x(x - 2)$	Take out common factor of x.
b y-intercept (let $x = 0$): $y = 0$	Let $x = 0$ to find the y-intercept $0^2 - 2 \times 0 = 0$.
c x-intercepts (let $y = 0$): $0 = x(x - 2)$ $x = 0$ or $x - 2 = 0$ $x = 0$ or $x = 2$	Let $y = 0$ to find the x-intercepts. Set each factor equal to 0 and solve.
d Axis of symmetry: $x = \frac{0 + 2}{2} = 1$	The axis of symmetry is halfway between the x-intercepts.
e Turning point occurs when $x = 1$. When $x = 1$, $y = 1^2 - 2(1)$ $y = -1$ $\therefore$ There is a minimum turning point at $(1, -1)$.	Substitute $x = 1$ into $y = x^2 - 2x$ to find the y coordinate. $x = 1$ and $y = -1$
f Graph of $y = x^2 - 2x$	The coefficient of x^2 is positive; therefore, the parabola is concave up. ∪ Plot the important points and draw a smooth curve.

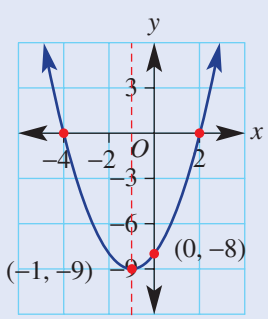


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Example 17 Sketching more challenging parabolas

For the quadratic relationship $y = x^2 + 2x - 8$:

a factorise the equation
b find the y-intercept
c find the x-intercepts
d find the axis of symmetry
e find the turning point
f sketch the graph clearly showing all the key features

SOLUTION	EXPLANATION
a $y = x^2 + 2x - 8$ $= (x + 4)(x - 2)$	Factorise the quadratic trinomial. $4 \times (-2) = -8$ and $4 + (-2) = 2$.
b y-intercept (let $x = 0$): $y = -8$	Let $x = 0$ to find the y-intercept.
c x-intercepts (let $y = 0$): $0 = (x + 4)(x - 2)$ $x + 4 = 0$ or $x - 2 = 0$ $x = -4$ or $x = 2$	Let $y = 0$ to find the x-intercepts.
d Axis of symmetry: $x = \frac{-4 + 2}{2} = -1$	The axis of symmetry is halfway between the x-intercepts.
e Turning point occurs when $x = -1$. When $x = -1$, $y = (-1)^2 + 2(-1) - 8$ $= -9$ $\therefore$ There is a minimum turning point at $(-1, -9)$.	Substitute $x = -1$ into $y = x^2 + 2x - 8$ to find the y coordinate. $x = -1$ and $y = -9$
f 	The coefficient of x^2 is positive; therefore, the parabola is concave up ∪. Plot the important points and draw a smooth curve.

Exercise 10H

UNDERSTANDING AND FLUENCY 1–6(½) 3–7(½) 4–7(½)

1 Factorise these quadratics.

a $x^2 + 2x$	**b** $x^2 - 3x$	**c** $5x^2 - 10x$	**d** $x^2 - 9$
e $x^2 - 1$	**f** $x^2 - 49$	**g** $x^2 + 3x + 2$	**h** $x^2 + 5x + 6$
i $x^2 - x - 12$	**j** $x^2 - 3x - 28$	**k** $x^2 - 4x + 4$	**l** $x^2 + 10x + 25$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 For these factorised quadratics, use the Null Factor Law to solve for x, then find the x value halfway between.

a $0 = (x - 2)(x + 2)$ **b** $0 = (x - 5)(x + 5)$ **c** $0 = (x + 1)(x + 5)$
d $0 = (x - 6)(x - 10)$ **e** $0 = (x - 1)(x + 3)$ **f** $0 = (x - 3)(x + 5)$
g $0 = (x - 2)(x + 3)$ **h** $0 = (x - 10)(x + 1)$ **i** $0 = (x - 15)(x + 3)$

3 Use substitution to find the y coordinate of the turning point of these quadratics. The x coordinate of the turning point is given.

a $y = x^2 - 4, x = 0$ **b** $y = x^2 + 2x, x = -1$ **c** $y = x^2 - 6x, x = 3$
d $y = x^2 + 4x + 3, x = -2$ **e** $y = x^2 + 2x - 8, x = -1$ **f** $y = x^2 - 4x - 5, x = 2$

Example 15

4 For each of the following quadratic equations, find the:

i x-intercepts
ii y-intercept

a $y = x(x + 7)$ **b** $y = x(x + 3)$ **c** $y = x(x + 4)$
d $y = (x - 4)(x + 2)$ **e** $y = (x + 2)(x - 5)$ **f** $y = (x - 7)(x + 3)$
g $y = 2(x + 3)(x - 1)$ **h** $y = 3(x + 4)(x + 1)$ **i** $y = (2 - x)(3 - x)$

Example 16

5 For each of the following quadratic equations:

i factorise the relation
ii find the y-intercept
iii find the x-intercepts
iv find the axis of symmetry
v find the turning point
vi sketch each graph, clearly showing all the key features

a $y = x^2 - 5x$ **b** $y = x^2 + x$ **c** $y = x^2 - 3x$
d $y = 2x + x^2$ **e** $y = 5x + x^2$ **f** $y = 3x - x^2$
g $y = -x^2 - 8x$ **h** $y = -2x - x^2$ **i** $y = -x^2 + x$

Example 17

6 For each of the following quadratic equations:

i factorise the relation
ii find the y-intercept
iii find the x-intercepts
iv find the axis of symmetry
v find the turning point
vi sketch each graph, clearly showing all the key features

a $y = x^2 - 3x + 2$ **b** $y = x^2 - 4x + 3$ **c** $y = x^2 + 2x - 3$
d $y = x^2 + 4x + 4$ **e** $y = x^2 + 2x + 1$ **f** $y = x^2 + 2x - 8$

7 For each of the following relationships, sketch the graph, clearly showing the x- and y-intercepts and the turning point.

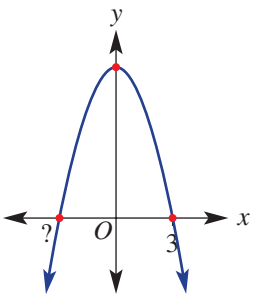
a $y = x^2 - 1$ **b** $y = 9 - x^2$ **c** $y = x^2 + 5x$
d $y = 2x^2 - 6x$ **e** $y = 4x^2 - 8x$ **f** $y = x^2 - 3x - 4$
g $y = x^2 + x - 12$ **h** $y = x^2 + 2x + 1$ **i** $y = x^2 - 6x + 9$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

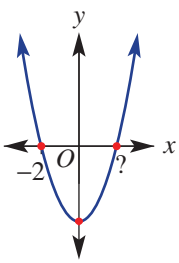
PROBLEM-SOLVING AND REASONING | 8, 11 | 8, 9, 11, 12 | 8(½), 9, 10, 12, 13

8 State the missing number in these quadratic graphs.

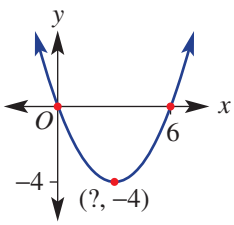
a



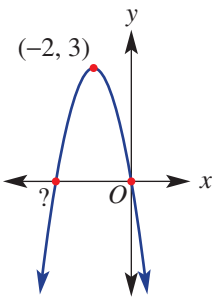
b



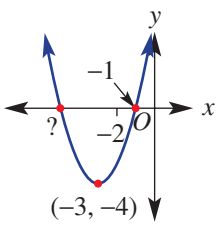
c



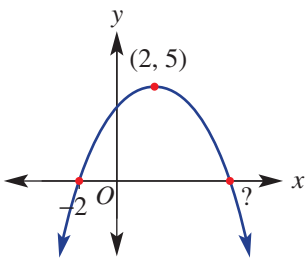
d



e



f



9 A golf ball's path is given by the rule $y = 30x - x^2$, where y is the height in metres above the ground and x is the horizontal distance in metres. Find:

a how far the ball travels horizontally

b how high the ball reaches mid-flight



10 A test rocket is fired and follows a path described by $y = 0.1x(200 - x)$. The height is y metres above ground and x is the horizontal distance in metres.

a How far does the rocket travel horizontally?

b How high does the rocket reach mid-flight?

11 Explain why the coordinates of the x-intercept and the turning point for $y = (x - 2)^2$ are the same.

12 Write down an expression for the y-intercept for these quadratics.

a $y = ax^2 + bx + c$ **b** $y = (x - a)(x - b)$

13 $y = x^2 - 2x - 15$ can also be written in the form $y = (x - 1)^2 - 16$.

a Use the second rule to state the coordinates of the turning point.

b Use the first rule to find the x-intercepts then the turning point. Check you get the same result.

c $y = x^2 - 4x - 45$ can be written in the form $y = (x - h)^2 + k$. Find the value of h and k.

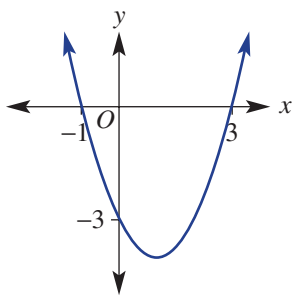
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

ENRICHMENT — — 14

Rule finding

14 Find the rule for these graphs using intercept form. The first one has been done for you.

a



$y = a(x + 1)(x - 3)$

Sub $(0, -3)$ $-3 = a(0 + 1)(0 - 3)$

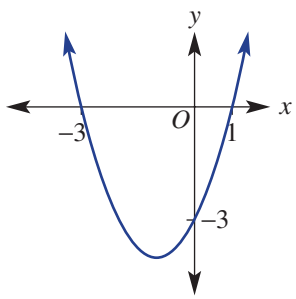
$-3 = a(1) \times (-3)$

$-3 = -3a$

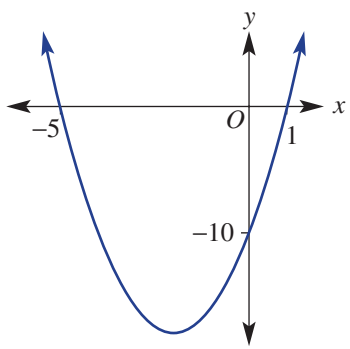
$a = 1$

$\therefore y = (x + 1)(x - 3)$

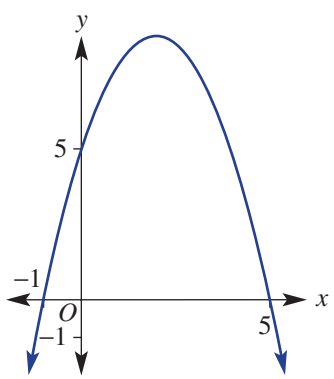
b



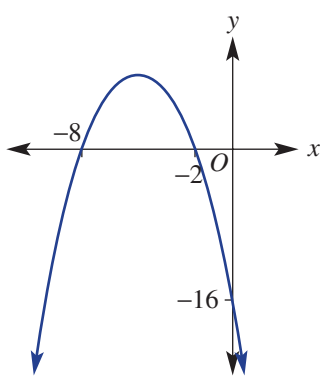
c



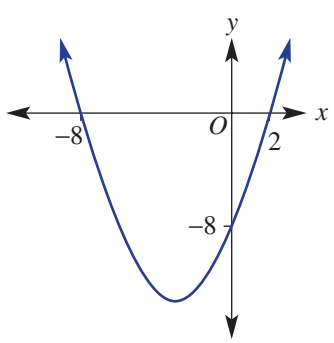
d



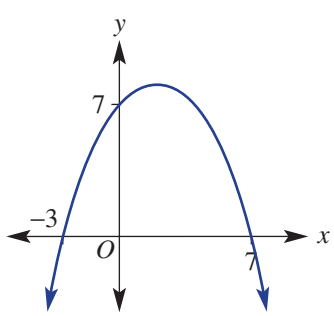
e



f



g



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Investigation

Investigating non-linear relationships

The graph of a quadratic relationship, called a parabola, was thoroughly investigated in this chapter. Many other relationships also produce graphs that are non-linear. Two examples are the circle and the hyperbola.

Investigating rules and graphs of circles

The general equation of a circle is given by the rule $(x - a)^2 + (y - b)^2 = r^2$, where a, b and r are constants. Examples include: $x^2 + y^2 = 1$, $x^2 + y^2 = 25$, $(x - 1)^2 + (y + 3)^2 = 9$ and $(x + 3)^2 + y^2 = 100$.

Use graphing software to investigate graphs of circles with the rule $x^2 + y^2 = r^2$. (Note: for some software you may have to enter the rule for a circle in two parts: $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$).

a Sketch the graph of the relationship $x^2 + y^2 = r^2$ for:

i $r = 1$ **ii** $r = 2$ **iii** $r = 3$ **iv** $r = 7$

b Describe how the value of r relates to the graphs of the circles.

c Sketch by hand the graphs of these circles.

i $x^2 + y^2 = 16$ **ii** $x^2 + y^2 = 25$ **iii** $x^2 + y^2 = 10$

d **Extension**: Investigate the effect of the values of a and b in the graph of the rule of $(x - a)^2 + (y - b)^2 = r^2$. Write a brief report showing your rules and graphs and describing your conclusions.

Investigating rules and graphs of hyperbolas

The basic form of the rule for a hyperbola is given by $y = \dfrac{a}{x - b} + c$, where a, b and c are constants.

Examples include $y = \dfrac{1}{x}$, $y = -\dfrac{5}{x}$, $y = \dfrac{3}{x} - 1$ and $y = \dfrac{-2}{x - 3} + 2$.

Use graphing software to investigate graphs of hyperbolas with the rule $y = \dfrac{a}{x - b} + c$.

a Sketch the graph of the relationship $y = \dfrac{a}{x}$ for:

i $a = 1$ **ii** $a = 2$ **iii** $a = -1$ **iv** $a = -2$

b Describe the key features of the graphs including the asymptotes. (Research the meaning of the word *asymptote* if you are unsure.) Also describe how changing the value of a changes the shape of your graph.

c Investigate how changing the value of c in $y = \dfrac{1}{x} + c$ changes the graph. Show your graphs for your chosen values of c and describe the effect.

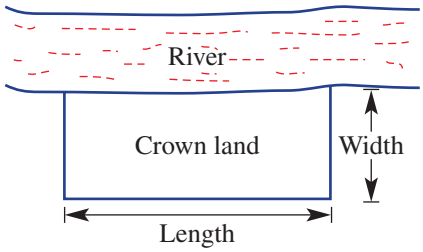
d Investigate how changing the value of b in $y = \dfrac{1}{x - b}$ changes the graph. Show your graphs for your chosen values of b and describe the effect.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Grazing Crown land

Land along the side of rivers is usually owned by the government and is sometimes called Crown land. Farmers can often lease this land to graze their sheep or cattle.

A farmer has a permit to fence off a rectangular area of land alongside the river. She has 400 m of fencing available and does not need to fence along the river.



A given width

a Find the length of the rectangular area if the width is:
- **i** 50 m
- **ii** 120 m
- **iii** 180 m

b Find the area of the land if the width is:
- **i** 50 m
- **ii** 120 m
- **iii** 180 m

c Which width from part **b** above gave the most area? Explain why the area decreases for small and large values of width.

The variable width

a Using x metres to represent the width, write an expression for the length, showing working.

b Write an expression for the area of the land in terms of x.

c Use your area expression from part **b** to find the area of the land if x is:
- **i** 20
- **ii** 80
- **iii** 160

The graph

a Using your expression from part **b** above, sketch a graph of area (A) vs x. You should find the following to help complete the graph.
- **i** y-intercept
- **ii** x-intercepts
- **iii** axis of symmetry
- **iv** turning point

b What value of x gives the maximum area of land for grazing? Explain your choice and give the dimensions of the rectangular area of land.

General observations

a What do you notice about the width and the length when there is a maximum area?

b See if the same is true if the farmer had 600 m of fencing instead. Show your expressions and graph.

c **Extension**: Prove your observation to parts **a** and **b** above by finding the x value that gives a maximum area using k metres of fencing. *Hint*: use $A = x(k - 2x)$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Puzzles and challenges

1 **a** What do you notice about the sums of these numbers?

i $1 + 3$

ii $1 + 3 + 5$

iii $1 + 3 + 5 + 7$

iv $1 + 3 + 5 + 7 + 9$

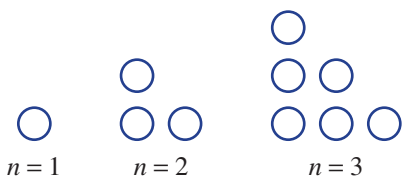
b Find the sum of the first 100 odd integers.

2 Solve these equations.

a $6x^2 = 35 - 11x$ **b** $\frac{2}{x^2} = 1 - \frac{1}{x}$ **c** $\sqrt{x - 1} = \frac{2}{x - 1}$

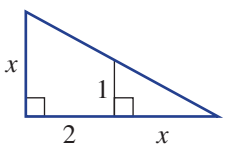
3 A projectile's height (in metres) above ground is given by the expression $t(14 - t)$, where time t is in seconds. How long is the projectile above a height of 40 m?

4 Find the quadratic rule that relates the number of balls to the term number (n) in the following pattern.

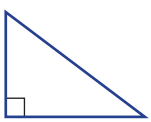


If 66 balls are in the pattern, what term number is it?

5 Find the value of x in this diagram.



6 **a** A right-angled tiangle's shortest side is of length x, the hypotenuse is 9 units longer than the shortest side while the other side is 1 unit longer than the shortest side. Find the side lengths of the triangle.



a The area of a right-angled triangle is 60 square units and the lengths of the two shorter sides differ by 7 units. Find the length of the hypotenuse.

7 Given $a > 0$, for what values of k does $y = a(x - h)^2 + k$ have:

a two x-intercepts?

b one x-intercept?

c no x-intercepts?

8 For the following equations, list the possible values of a that will give integer solutions for x.

a $x^2 + ax + 24 = 0$

b $x^2 + ax - 24 = 0$

9 Solve the equation $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$ for all values of x. *Hint:* let $a = x^2 - 3x$.

10 Solve the equation $x^4 - 10x^2 + 9 = 0$ for all values of x. *Hint:* let $a = x^2$.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter summary

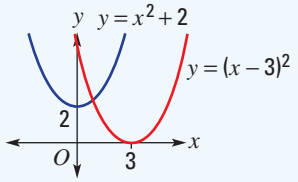
Quadratic equations and graphs of parabolas

Turning point form
$y = a(x-h)^2 + k$
turning point (h, k)
e.g. $y = 2(x+1)^2 + 3$
Turning point: $(-1, 3)$
y-intercept: $x = 0$, $y = 2(0+1)^2 + 3$
$= 2(1)^2 + 3$
$= 5$

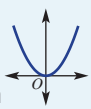
Forms
A quadratic is of the form
$y = ax^2 + bx + c$, $a \neq 0$

Translation
$y = x^2 + k$ $k > 0$, graph shifts up k units
$k < 0$, graph shifts down k units
$y = (x-h)^2$ $h > 0$, graph shifts h units right
$h < 0$, graph shift h units left

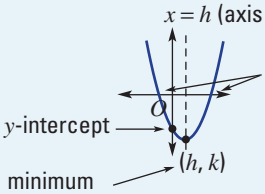
e.g.



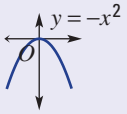
Parabola
Basic form $y = x^2$



Turning point form
$y = a(x-h)^2 + k$
$x = h$ (axis of symmetry)

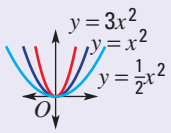


Dilation and reflection
$y = ax^2$
$a < 0$, graph is reflected in x-axis

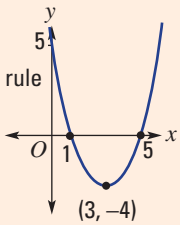


$-1 < a < 1$:
graph is wider than
$y = x^2$ or $y = -x^2$

$a > 1$ or $a < -1$:
graph is narrower than
$y = x^2$ or $y = -x^2$
e.g.



Sketching $y = ax^2 + bx + c$
Find x- and y-intercepts and turning point then sketch.
e.g. $y = x^2 - 6x + 5$
y-intercept: $x = 0$, $y = 0^2 - 6(0) + 5$
$= 5$
x-intercept: $y = 0$, $0 = x^2 - 6x + 5$
$0 = (x-1)(x-5)$
$x - 1 = 0$ or $x - 5 = 0$
$x = 1$ or $x = 5$
Turning point: halfway between the x-intercepts:
$x = \frac{1+5}{2} = 3$
substitute $x = 3$ into rule
$y = 3^2 - 6(3) + 5$
$= -4$
$(3, -4)$ minimum turning point



Solving problems
Define variable.
Set up equation.
Solve using Null Factor Law.
Check answer makes sense.
Answer the question in words.

Solving quadratic equations
Rewrite equation in standard form, i.e. $ax^2 + bx + c = 0$
Factorise
Apply Null Factor Law:
If $ab = 0$, $a = 0$ or $b = 0$,
solve remaining equation
e.g. $x^2 + 5x = 0$
$x(x+5) = 0$
$x = 0$ or $x + 5 = 0$
$x = 0$ or $x = -5$
e.g. $x^2 - 12 = 4x$
$x^2 - 4x - 12 = 0$
$(x-6)(x+2) = 0$
$x - 6 = 0$ or $x + 2 = 0$
$x = 6$ or $x = -2$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter review

Multiple-choice questions

1 (1, 3) is a point on which curve?

A $y = x^2$ **B** $y = (x - 3)^2$ **C** $y = x^2 + 2x - 3$
D $y = x^2 + 2$ **E** $y = 6 - x^2$

2 The solution(s) to $2x(x - 3) = 0$ is/are:

A $x = -3$ **B** $x = 0$ or $x = 3$ **C** $x = 2$ or $x = 3$
D $x = 0$ or $x = -3$ **E** $x = 0$

3 The quadratic equation $x^2 = 7x - 12$ in standard form is:

A $x^2 - 7x - 12 = 0$ **B** $-x^2 + 7x + 12 = 0$ **C** $x^2 - 7x + 12 = 0$
D $x^2 + 7x - 12 = 0$ **E** $x^2 + 7x + 12 = 0$

4 The solution(s) to $x(x + 2) = 2x + 9$ are:

A $x = -3$ or $x = 3$ **B** $x = 0$ or $x = -2$ **C** $x = 3$
D $x = 9$ or $x = -1$ **E** $x = 9$ or $x = 0$

The following applies to Questions **5** and **6**.
The height, h metres, of a toy rocket above ground t seconds after launch is given by $h = 6t - t^2$.

5 The rocket returns to ground level after:

A 5 seconds
B 3 seconds
C 12 seconds
D 6 seconds
E 8 seconds

6 The rocket reaches its maximum height after:

A 6 seconds
B 3 seconds
C 10 seconds
D 4 seconds
E 9 seconds

7 The turning point of $y = (x - 2)^2 - 4$ is a:

A maximum at (2, 4)
B minimum at (−2, 4)
C maximum at (2, 4)
D minimum at (−2, −4)
E minimum at (2, −4)

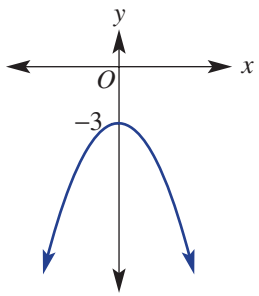
8 The transformation of the graph of $y = x^2$ to $y = x^2 - 2$ is described by a:

A translation of 2 units to the left
B translation of 2 units to the right
C translation of 2 units down
D translation of 2 units up
E translation of 2 units right and 2 units down

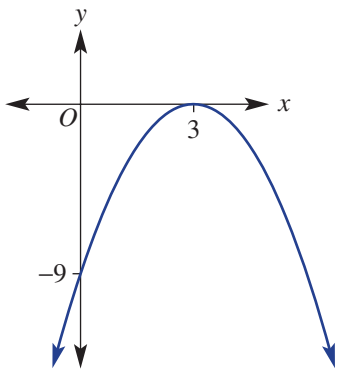
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 The graph of $y = -(x - 3)^2$ is:

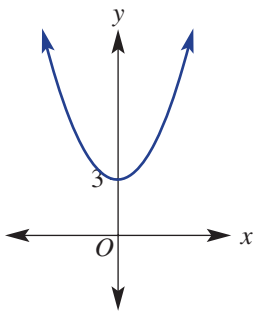
A



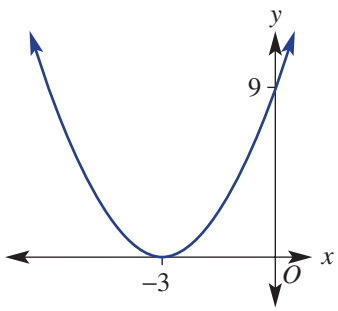
B



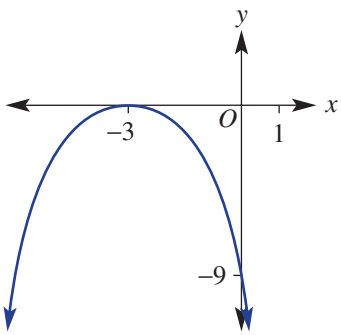
C



D



E



10 Compared to $y = x^2$, the graph that appears the narrowest is:

A $y = 5x^2$ **B** $y = 0.2x^2$ **C** $y = 2x^2$ **D** $y = \frac{1}{2}x^2$ **E** $y = 3.5x^2$

Short-answer questions

1 Consider the quadratic $y = x^2 - 2x - 3$.

a Complete this table of values for the equation.

x	−3	−2	−1	0	1	2	3
y							

b Plot the points in part **a** on a Cartesian plane and join in a smooth curve.

2 Use the Null Factor Law to solve the following equations.

a $x(x + 2) = 0$ **b** $3x(x - 4) = 0$ **c** $(x + 3)(x - 7) = 0$

d $(x - 2)(2x + 4) = 0$ **e** $(x + 1)(5x - 2) = 0$ **f** $(2x - 1)(3x - 4) = 0$

3 Solve the following quadratic equations by first factorising.

a $x^2 + 3x = 0$ **b** $2x^2 - 8x = 0$ **c** $x^2 - 25 = 0$

d $x^2 - 81 = 0$ **e** $5x^2 - 20 = 0$ **f** $3x^2 - 48 = 0$

g $x^2 + 10x + 21 = 0$ **h** $x^2 - 3x - 40 = 0$ **i** $x^2 - 8x + 16 = 0$

4 Write the following quadratic equations in standard form and solve for x.

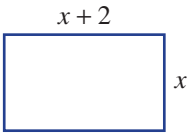
a $x^2 = 5x$ **b** $3x^2 = 18x$ **c** $x^2 + 12 = -8x$

d $2x + 15 = x^2$ **e** $x^2 + 15 = 8x$ **f** $4 - x^2 = 3x$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

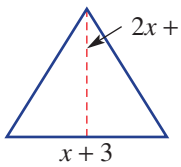
5 Set up and solve a quadratic equation to determine the value of x that gives the specified area of the shapes below.

a



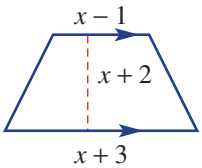
$A = 80$ sq. units

b



$A = 30$ sq. units

c



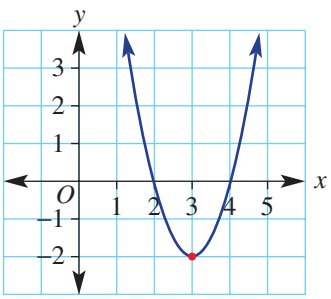
$A = 30$ sq. units

6 For the following graphs state the:

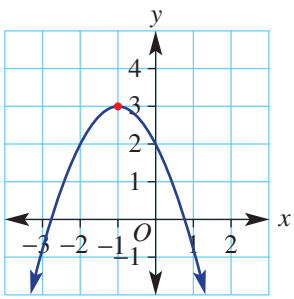
i axis of symmetry

ii turning point and its type

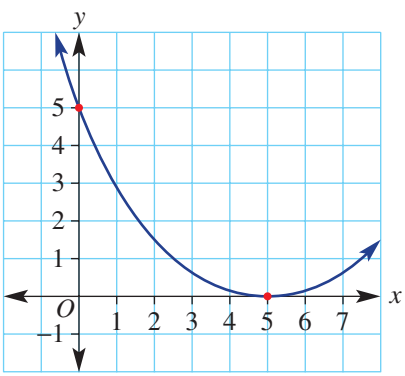
a



b



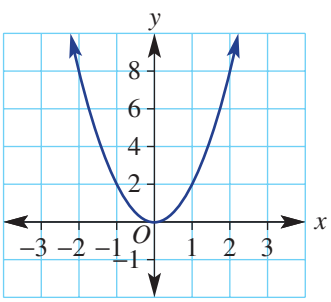
c



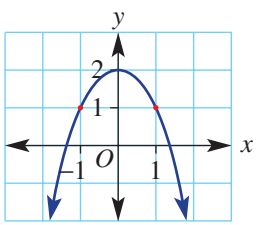
7 Match the following quadratic equations with their graphs.

$y = x^2 - 4$, $y = 2x^2$, $y = (x + 1)^2$, $y = (x - 3)^2$, $y = -x^2 + 2$

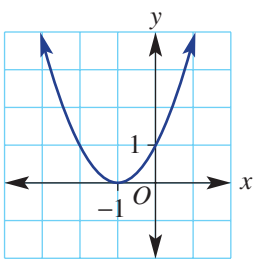
a



b

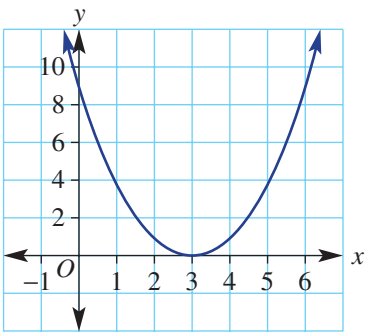


c

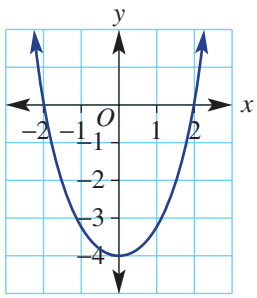


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

d



e



8 Sketch the following graphs, labelling the turning point and y-intercept.

a $y = x^2 + 2$ **b** $y = -x^2 - 5$ **c** $y = (x + 2)^2$

d $y = -(x - 3)^2$ **e** $y = (x - 1)^2 + 3$ **f** $y = 2(x + 2)^2 - 4$

9 State the transformations that take $y = x^2$ to each of the graphs in Question **8**.

10 Sketch the following graphs, labelling the y-intercept, turning point and x-intercepts.

a $y = x^2 - 8x + 12$ **b** $y = x^2 + 10x + 16$

c $y = x^2 + 2x - 15$ **d** $y = x^2 + 4x - 5$

Extended-response questions

1 A sail of a yacht is in the shape of a right-angled triangle. It has a base length of $2x$ metres and its height is 5 metres more than half its base.

a Write an expression for the height of the sail.

b Give an expression for the area of the sail in expanded form.

c If the area of the sail is 14 m^2, find the value of x.

d Hence, state the dimensions of the sail.

2 Connor and Sam are playing in the park with toy rockets that they have made. They each launch their rockets at the same time to see whose is better.

a The path of Sam's rocket is modelled by the equation $h = 12t - 2t^2$, where h is the height of the rocket in metres after t seconds.

i Find the axis intercepts.

ii Find the turning point.

iii Sketch a graph of the height of Sam's rocket over time.

b The path of Connor's rocket is modelled by the equation $h = -(t - 4)^2 + 16$ where h is the height of the rocket in metres after t seconds.

i Find the h-intercept.

ii State the turning point.

iii Use the answers from parts **b i** and **ii** to state the two t-intercepts.

iv Sketch a graph of the height of Connor's rocket over time.

c **i** Whose rocket was in the air longest?

ii Whose rocket reached the greatest height and by how much?

iii How high was Sam's rocket when Connor's was at its maximum height?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 6: Indices and surds

Multiple-choice questions

1 $3a^2b^3 \times 4ab^2$ is equivalent to:

A $12a^2b^6$ **B** $7a^3b^5$ **C** $12a^3b^5$ **D** $12a^4b^5$ **E** $7a^2b^6$

2 $\left(\frac{2x}{5}\right)^3$ is equivalent to:

A $\frac{6x^3}{5}$ **B** $\frac{8x^3}{125}$ **C** $\frac{2x^3}{5}$ **D** $\frac{2x^4}{15}$ **E** $\frac{2x^3}{125}$

3 4^{-2} can be expressed as:

A $\frac{1}{4^{-2}}$ **B** $\frac{1}{8}$ **C** -16 **D** $\frac{1}{16}$ **E** -8

4 $3x^{-4}$ written with positive indices is:

A $-3x^4$ **B** $\frac{1}{3x^4}$ **C** $-\frac{3}{x^4}$ **D** $\frac{1}{3x^{-4}}$ **E** $\frac{3}{x^4}$

5 0.00371 in scientific notation is:

A 0.371×10^{-3} **B** 3.7×10^{-2} **C** 3.71×10^{-3} **D** 3.71×10^3 **E** 371×10^3

Short-answer questions

1 Use index laws to simplify the following.

a $\frac{9a^6b^3}{18a^4b^2}$ **b** $\frac{(-3x^4y^2)^2 \times 6xy^2}{27x^6y}$ **c** $(2x^2)^3 - 3x^0 + (5x)^0$

2 Write each of the following using positive indices and simplify.

a $\frac{5}{m^{-2}}$ **b** $\frac{4a^6b^{-4}}{6a^{-2}b}$ **c** $3(x^{\frac{1}{2}})^3y^{-\frac{1}{3}} \times x^{\frac{1}{2}}y^{-\frac{2}{3}}$

3 Convert these numbers to the units given in brackets. Write your answer in scientific notation using 3 significant figures.

a 30.71 g (kg)
b 4236 tonnes (kg)
c 3.4 hours (seconds)
d 235 nanoseconds (seconds)

4 Simplify the following.

a $144^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $2\sqrt{3} + 3\sqrt{5} - \sqrt{3} + 4\sqrt{5}$ **d** $\sqrt{35} \div \sqrt{5}$

Extended-response question

The average human body contains about 74 billion cells.

a Write this number of cells in:

i decimal form **ii** scientific notation

b If the population of a particular city is 2.521×10^6, how many human cells are there in the city? Give your answer in scientific notation correct to 3 significant figures.

c If the average human weighs 64.5 kilograms, what is the average mass of one cell in grams? Give your answer in scientific notation correct to 3 significant figures.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 7: Properties of geometrical figures

Multiple-choice questions

1 The supplementary angle to 55° is:

A 55° **B** 35° **C** 125° **D** 135° **E** 70°

2 A quadrilateral with all four sides equal and opposite sides parallel is best described by a:

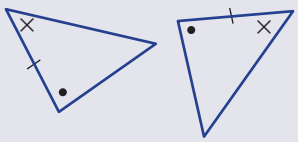
A parallelogram **B** rhombus **C** rectangle **D** trapezium **E** kite

3 The size of the interior angle in a regular pentagon is:

A 108° **B** 120° **C** 96° **D** 28° **E** 115°

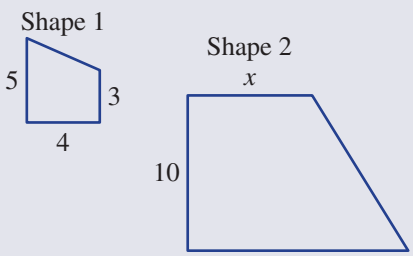
4 The test that proves congruence in these two triangles is:

A SAS **B** RHS **C** AAA
D SSS **E** AAS



5 The scale factor that enlarges shape 1 to shape 2 in these similar figures, and the value of x are:

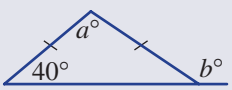
A 2 and $x = 8$ **B** 2.5 and $x = 7.5$
C 3.33 and $x = 13.33$ **D** 2.5 and $x = 12.5$
E 2 and $x = 6$



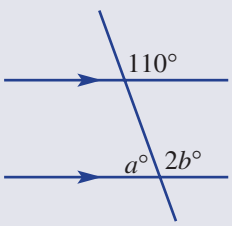
Short-answer questions

1 Find the value of each pronumeral in the following.

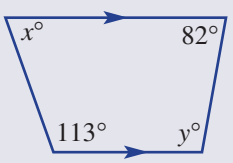
a



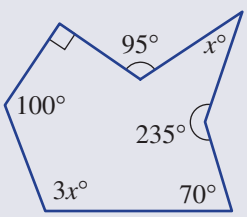
b



c

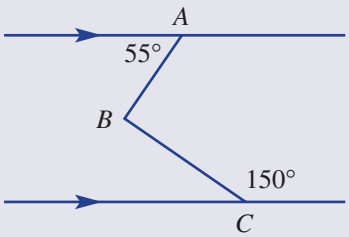


d

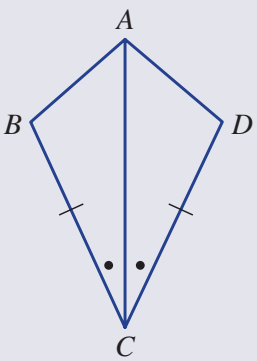


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Find the value of $\angle ABC$ by adding a third parallel line.



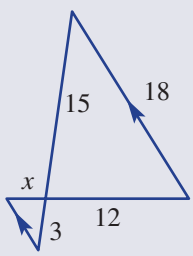
3 Prove $\Delta ABC \equiv \Delta ADC$.



4 For this pair of triangles:

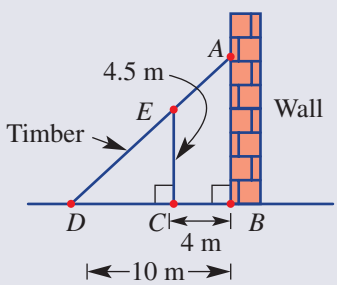
a Explain why the two triangles are similar.

b Find the value of x.



Extended-response question

A vertical wall is being supported by a piece of timber that touches the ground 10 m from the base of the wall. A vertical metal support 4.5 m high is placed under the timber support 4 m from the wall.



a Prove $\Delta ABD \text{ III } \Delta ECD$.

b Find how far the timber reaches up the wall.

c How far above the ground is the point halfway along the timber support?

d The vertical metal support is moved so that the timber support is able to reach one metre higher up the wall. If the piece of timber now touches the ground 9.2 m from the wall, find how far the metal support is from the wall. Give your answer correct to 1 decimal place.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 8: Quadratic expressions and algebraic fractions

Multiple-choice questions

1 The expanded form of $(x + 4)(3x - 2)$ is:

A $3x^2 + 12x - 8$ **B** $4x^2 + 14x - 8$ **C** $3x^2 + 12x - 10$
D $3x^2 - 8$ **E** $3x^2 + 10x - 8$

2 $2(x + 2y) - x(x + 2y)$ factorises to:

A $2x(x + 2y)$ **B** $(x + 2y)^2(2 - x)$ **C** $(2 + x)(x + 2y)$
D $(x + 2y)(2 - x)$ **E** $(x + 2y)(2 - x^2 - 2xy)$

3 $x^2 - 2x - 24$ in factorised form is:

A $(x - 2)(x + 12)$ **B** $(x - 6)(x + 4)$ **C** $(x - 8)(x + 3)$
D $(x + 6)(x - 4)$ **E** $(x - 6)(x - 4)$

4 $\dfrac{3x + 6}{(x - 2)(x - 4)} \times \dfrac{x^2 - 4}{(x + 2)^2}$ is equivalent to:

A $\dfrac{3}{x - 4}$ **B** $\dfrac{3(x - 2)}{(x - 4)(x + 2)}$ **C** $\dfrac{3x - 2}{(x + 1)^2}$ **D** $\dfrac{3}{(x + 2)(x - 2)}$ **E** $\dfrac{3x^2}{(x - 2)(x + 2)}$

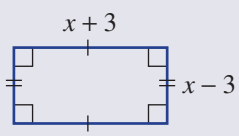
5 $\dfrac{7}{(x + 1)^2} - \dfrac{4}{x + 1}$ simplifies to:

A $\dfrac{3x + 1}{(x + 1)^2}$ **B** $\dfrac{6 - 4x}{(x + 1)^2}$ **C** $\dfrac{-9}{(x + 1)^2}$ **D** $\dfrac{3 - 4x}{(x + 1)^2}$ **E** $\dfrac{11 - 4x}{(x + 1)^2}$

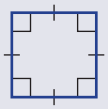
Short-answer questions

1 Find the area of the following shapes in expanded form.

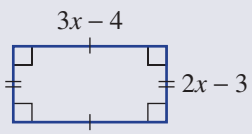
a



b



c



2 Factorise each of the following fully.

a $8ab + 2a^2b$ **b** $9m^2 - 25$ **c** $3b^2 - 48$
d $(a + 7)^2 - 9$ **e** $x^2 + 6x + 9$ **f** $x^2 + 8x - 20$
g $2x^2 - 16x + 30$ **h** $2x^2 - 11x + 12$ **i** $6x^2 + 5x - 4$

3 Factorise the following by grouping.

a $x^2 - 3ax - x + 3a$ **b** $2ax - 10b - 5bx + 4a$

4 **a** Simplify these algebraic fractions.

i $\dfrac{3x + 24}{2x + 16}$ **ii** $\dfrac{12}{x^2 - 9} \div \dfrac{3}{x - 3}$ **iii** $\dfrac{x + 3}{2} + \dfrac{3x}{7}$

iv $\dfrac{2}{x} - \dfrac{5}{3x}$ **v** $\dfrac{7}{x + 1} + \dfrac{4}{x - 2}$ **vi** $\dfrac{5}{x - 5} - \dfrac{3}{x + 2}$

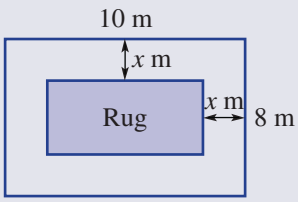
b Solve these equations involving algebraic fractions.

i $\dfrac{5}{2x} + \dfrac{1}{3x} = 2$ **ii** $\dfrac{4}{x - 5} = \dfrac{2}{x + 3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Extended-response question

A room that is 10 metres long and 8 metres wide has a rectangular rug in the middle of it that leaves a border, x metres wide, all the way around it as shown.



a Write expressions for the length and the breadth of the rug.

b Write an expression for the area of the rug in expanded form.

c What is the area of the rug when $x = 1$?

d Fully factorise your expression in part **b** by first removing the common factor.

e What happens when $x = 4$?

f Use trial and error to find the value of x that gives an area of 15 m^2.

Chapter 9: Probability and single variable data analysis

Multiple-choice questions

1 The probability of not rolling a number less than three on a normal six-sided die is:

A $\frac{1}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** 3

2 From the two-way table, $P(A \text{ and } B)$ is:

A $\frac{1}{5}$ **B** 4 **C** $\frac{9}{20}$ **D** $\frac{1}{4}$ **E** 16

	A	$\overline{A}$	Total
B		7	
$\overline{B}$	5		
Total		11	20

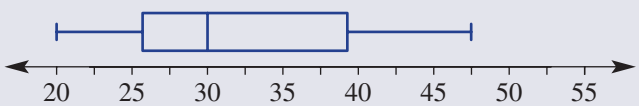
3 In the selection of 40 marbles, 28 were blue. The experimental probability of the next one selected being blue is:

A 0.28 **B** 0.4 **C** 0.7 **D** 0.54 **E** 0.75

4 The median, mean and range of the data set 12, 3, 1, 6, 10, 1, 5, 18, 11, 15 are, respectively:

A 5.5, 8.2, 1–18 **B** 8, 8.2, 17 **C** 5.5, 8, 17 **D** 8, 8.2, 1–18 **E** 8, 74.5, 18

5 The interquartile range of the data in the box plot shown is:



A 10 **B** 27 **C** 13 **D** 30 **E** 17

Short-answer questions

1 In a survey of 30 people, 18 people drink coffee during the day, 14 people drink tea and 8 people drink both. Let C be the set of people who drink coffee and T the set of people who drink tea.

a Construct a Venn diagram for the survey results.

b If one of the 30 people was randomly selected, find:

i $P(\text{drinks neither coffee nor tea})$ **ii** $P(C \text{ only})$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 Two ice-creams are randomly selected without replacement from a box containing one vanilla (V), two strawberry (S) and one chocolate (C) flavoured ice-creams.

- **a** Draw a tree diagram to show each of the possible outcomes.
- **b** What is the probability of selecting:
  - **i** a vanilla and a strawberry-flavoured ice-cream?
  - **ii** two strawberry-flavoured ice-creams?
  - **iii** no vanilla-flavoured ice-cream?

3 The data below shows the number of aces served by a player in each of their grand-slam tennis matches for the year.

15 22 11 17 25 25 12 31 26 18 32 11 25 32 13 10

- **a** Construct a stem-and-leaf plot for the data.
- **b** From the stem-and-leaf plot, find the mode and median number of aces.
- **c** Is the data symmetrical or skewed?

4 The frequency table shows the number of visitors, in intervals of fifty, to a theme park each day in April.

- **a** Complete the frequency table shown. Round to 1 decimal place where necessary.
- **b** Construct a frequency histogram.
- **c** **i** How many days were there fewer than 100 visitors?
  - **ii** What percentage of days had between 50 and 200 visitors?

Class interval	Class centre	Frequency	Percentage frequency
0–49		2	
50–99		4	
100–149		5	
150–199		9	
200–249			
250–299		3	
Total		30	

Extended-response question

A game at a school fair involves randomly selecting a green ball and a red ball each numbered 1, 2 or 3.

- **a** List the outcomes in a table.
- **b** What is the probability of getting an odd and an even number?
- **c** Participants win $1 when they draw each ball showing the same number.
  - **i** What is the probability of winning $1?
  - **ii** If someone wins six times, how many games are they likely to have played?
- **d** The ages of those playing the game in the first hour are recorded and are shown below.

12 16 7 24 28 9 11 17 18 18 37 9 40 16 32 42 14

  - **i** Draw a box plot to represent the data.
  - **ii** Twenty-five per cent of the participants are below what age?
  - **iii** If this data is used as a model for the 120 participants throughout the day, how many would be expected to be aged less than 30?

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 10: Quadratic equations and graphs of parabolas

Multiple-choice questions

1 The solution(s) to $3x(x + 5) = 0$ is/are:

A $x = -5$ **B** $x = 0, -5$ **C** $x = 3, 5$ **D** $x = 5$ **E** $x = 0, 5$

2 $x^2 = 3x - 2$ is the same as the equation:

A $x^2 + 3x - 2 = 0$ **B** $x^2 - 3x + 2 = 0$ **C** $x^2 - 3x - 2 = 0$
D $-x^2 + 3x + 2 = 0$ **E** $x^2 + 3x + 2 = 0$

3 Choose the *incorrect* statement about the graph of $y = 2x^2$.

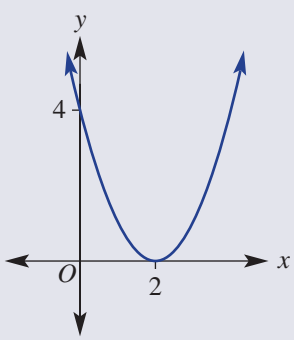
A The graph is the shape of a parabola.
B The point $(-2, 8)$ is on the graph.
C Its turning point is at $(0, 0)$.
D It has a minimum turning point.
E The graph is wider than the graph of $y = x^2$.

4 The type and coordinates of the turning point of the graph of $y = -(x + 3)^2 + 2$ are a:

A minimum at $(3, 2)$
B maximum at $(3, 2)$
C maximum at $(-3, 2)$
D minimum at $(-3, 2)$
E minimum at $(3, -2)$

5 The graph shown has the equation:

A $y = 4x^2$
B $y = x^2 + 4$
C $y = (x + 2)^2$
D $y = x^2 + 2$
E $y = (x - 2)^2$



Short-answer questions

1 Solve the following quadratic equations.

a $(x + 5)(x - 3) = 0$ **b** $(2x - 1)(3x + 5) = 0$ **c** $4x^2 + 8x = 0$
d $5x^2 - 45 = 0$ **e** $x^2 - 9x + 14 = 0$ **f** $8x = -x^2 - 16$

2 The length of a rectangular swimming pool is x m and its breadth is 7 m less than its length. If the area occupied by the pool is 120 m^2, solve an appropriate equation to find the dimensions of the pool.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 Sketch the following graphs, showing the y-intercept and turning point and state the transformations that have taken place from the graph of $y = x^2$.

a $y = x^2 + 3$ **b** $y = -(x + 4)^2$ **c** $y = (x - 2)^2 + 5$

4 Consider the quadratic relationship $y = x^2 - 4x - 12$.

a Find the y-intercept.

b Factorise the relationship and find the x-intercepts.

c Find the coordinates of the turning point.

d Sketch the graph.

Extended-response question

The flight path of a soccer ball kicked upwards from the ground is given by the equation $y = 120x - 20x^2$, where y is the height of the ball above the ground in centimetres at any time, x seconds.

a Find the x-intercepts to determine when the ball lands on the ground.

b Find the coordinates of the turning point and state:

i the maximum height reached by the ball

ii after how many seconds the ball reaches this maximum height

c At what times was the ball at a height of 160 cm?

d Sketch a graph of the path of the ball until it returns to the ground.



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Answers

Chapter 1

Pre-test

1 **a** 17 **b** 11 **c** 3 **d** 8
e −4 **f** −1 **g** −6 **h** 2

2 **a** $2\frac{1}{5}$ **b** 2.2

3 **a** 9 **b** 5 **c** 16 **d** 8

4 $\frac{7}{9} > \frac{3}{4}$

5 **a** 2.654, 2.645, 2.564, 2.465
b 0.654, 0.564, 0.456, 0.0456

6 **a** 7.99 **b** 10.11 **c** 7.11

7 **a** 1.4 **b** 0.06 **c** 3.68
d 16.38 **e** 3.7 **f** 180

8 **a** 34.5 **b** 374 000 **c** 0.03754 **d** 0.00003754

9 **a** 15 **b** 12 **c** 10

10 **a** $\frac{5}{7}$ **b** 1 **c** $\frac{1}{2}$ **d** $\frac{1}{4}$

11 **a** 13 **b** 60 **c** 8.1

Exercise 1A

1 **a** 1, 2, 4, 8, 16 **b** 1, 2, 4, 7, 8, 14, 28, 56
c HCF = 8 **d** 3, 6, 9, 12, 15, 18, 21
e 5, 10, 15, 20, 25, 30 **f** LCM = 15
g 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
h 83, 89, 97, 101, 103, 107, 109

2 **a** 121 **b** 225 **c** 12 **d** 20
e 27 **f** 125 **g** 2 **h** 4

3 **a** −5 **b** −8 **c** −1 **d** 9
e −1 **f** −16 **g** 15 **h** 9
i −6 **j** −84 **k** 22 **l** 42
m −9 **n** −6 **o** 10 **p** 19

4 **a** 2 **b** 2 **c** 10 **d** 16
e −9 **f** −3 **g** −4 **h** 10
i −11 **j** 2 **k** −3 **l** 4
m −23 **n** 10 **o** 0 **p** 3
q −9 **r** 7 **s** −1 **t** 4

5 **a** 28 **b** 24 **c** 187 **d** 30

6 **a** 4 **b** 5 **c** 1 **d** 23

7 **a** 4 **b** 23 **c** −3
d 2 **e** −2 **f** −18
g −6 **h** −1 **i** 1

8 **a** −2 **b** −38 **c** −8 **d** 27
e 1 **f** 24 **g** 21 **h** 0

9 **a** $-2 \times [11 + (-2)] = -18$ **b** $[-6 + (-4)] \div 2 = -5$
c $[2 - 5] \times (-2) = 6$ **d** $-10 \div [3 + (-5)] = 5$
e $3 - [(-2) + 4] \times 3 = -3$ **f** $[(-2)^2 + 4] \div (-2) = -2^2$

10 4 **11** 252 days

12 **a** 7 and −2 **b** −5 and 2

13 8

14 **a** **i** 16 **ii** 16 **b** $a = \pm 4$ **c** $a = 3$
d The square of a negative number has negative signs occurring in pairs and will create a positive answer.
e −3
f As the squaring of any number produces a positive answer
g **i** −4 **ii** −125 **iii** −9 **iv** −16
h No **i** Yes
j Prime numbers have only two factors – itself and one, therefore the only common factor for any pair of prime numbers is 1.
k Again as there are only 2 factors of any prime, the LCM must be the multiple of primes.

15 **a** False **b** False **c** True
d True **e** False **f** True

16 **a** **i** 1 + 2 + 3 = 6
ii 28 (1 + 2 + 4 + 7 + 14 = 28)
iii 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496
b **i**



ii 28, 36
c **i** 0, 1, 1, 2, 3, 5, 8, 13, 21, 34
ii −21, 13, −8, 5, −3, 2, −1, 1, 0, 1...
∴ −21, −8, −3, −1

Exercise 1B

1 **a** 40 **b** 270 **c** 7.9 **d** 0.04

2 **a** 32 100 **b** 432 **c** 5.89
d 0.443 **e** 0.001 97

3 **a** 17.96 **b** 11.08 **c** 72.99 **d** 47.86
e 63.93 **f** 23.81 **g** 804.53 **h** 500.57
i 821.27 **j** 5810.25 **k** 1005.00 **l** 2650.00

4 **a** 7 **b** 73 **c** 130 **d** 36 200

5 **a** 0.333 **b** 0.286 **c** 1.182 **d** 13.793

6 **a** 18 **b** 11 **c** 73 **d** 48
e 64 **f** 24 **g** 800 **h** 500
i 820 **j** 5800 **k** 1000 **l** 2600

7 **a** 20 **b** 10 **c** 70 **d** 50
e 60 **f** 20 **g** 800 **h** 500
i 800 **j** 6000 **k** 1000 **l** 3000

8 **a** 60 **b** 57.375 **c** 2.625

9 **a** 3600, 3693 **b** 760, 759.4
c 4000, 4127.16 **d** 3000, 3523.78
e 0, 0.722 16 **f** 4, 0.716 245
g 0.12, 0.1186 **h** 0.02, 0.02254
i 10, 8.4375 **j** 1600, 1683.789156
k 0.08, 0.074957 ... **l** 11, 10.25538 ...

10 **a** A: 54.3, B: 53.8, 0.5 **b** A: 54.28, B: 53.79, 0.49
c A: 54, B: 54, 0 **d** A: 50, B: 50, 0

11 **a** Each to 1 significant figure
b Each to 2 significant figures
c Each to the nearest whole number

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 8.33 m **13** 0.143 tonnes

14 2.14999 is closer to 2.1 correct to 1 decimal place ∴ round down

15 As magnesium in this case would be zero if rounded to 2 decimal places rather than 2 significant figures

16 a i 50 **ii** 624 **b** 50 **c** 600

d The addition is the same as the original but the multiplication is lower: $20 \times 30 < 24 \times 26$.

17 a 0.18181818

b i 8 **ii** 1 **iii** 8

c 0.1428571428571

d i 4 **ii** 2 **iii** 8

e Not possible

Exercise 1C

1 a $1\frac{2}{5}$ **b** $4\frac{1}{3}$ **c** $4\frac{4}{11}$ **d** $6\frac{8}{53}$

2 a $\frac{11}{7}$ **b** $\frac{16}{3}$ **c** $\frac{19}{2}$ **d** $\frac{238}{13}$

3 a $\frac{2}{5}$ **b** $\frac{4}{29}$ **c** $4\frac{1}{6}$ **d** $-72\frac{1}{8}$

4 a 9 **b** 24 **c** 21 **d** 35

e 3 **f** 7 **g** 10 **h** 6

5 a 2.75 **b** 0.35 **c** 3.4 **d** 1.875

e 2.625 **f** 3.8 **g** 2.3125 **h** 0.218 75

6 a $0.\overline{27}$ **b** $0.\dot{7}$ **c** $1.\overline{285714}$ **d** $0.41\dot{6}$

e $1.\dot{1}$ **f** $3.8\dot{3}$ **g** $7.2\dot{6}$ **h** $2.\overline{63}$

7 a $\frac{7}{20}$ **b** $\frac{3}{50}$ **c** $3\frac{7}{10}$ **d** $\frac{14}{25}$

e $1\frac{7}{100}$ **f** $\frac{3}{40}$ **g** $3\frac{8}{25}$ **h** $7\frac{3}{8}$

i $2\frac{1}{100}$ **j** $10\frac{11}{250}$ **k** $6\frac{9}{20}$ **l** $2\frac{101}{1000}$

8 a $\frac{5}{6}$ **b** $\frac{13}{20}$ **c** $\frac{7}{10}$ **d** $\frac{5}{12}$

e $\frac{7}{16}$ **f** $\frac{11}{14}$ **g** $\frac{19}{30}$ **h** $\frac{11}{27}$

9 a $\frac{5}{12}, \frac{7}{18}, \frac{3}{8}$ **b** $\frac{5}{24}, \frac{3}{16}, \frac{1}{6}$ **c** $\frac{7}{12}, \frac{23}{40}, \frac{8}{15}$

10 a $\frac{9}{20}$ **b** $\frac{3}{20}$ **c** $\frac{32}{45}$ **d** $\frac{23}{75}$

11 a $\frac{11}{6}, \frac{7}{3}$ **b** $\frac{2}{5}, \frac{2}{15}$ **c** $\frac{11}{12}, 1$ **d** $\frac{5}{7}, \frac{11}{14}$

12 Weather forecast

13 a $\frac{3}{5}$ **b** $\frac{5}{9}$ **c** $\frac{8}{13}$ **d** $\frac{23}{31}$

14 a 31, 32 **b** 36, 37, 38, . . . , 55

c 4, 5 **d** 2, 3

e 43, 44, 45, . . . , 55 **f** 4, 5, 6

15 $\frac{ac+b}{c}$

16 a Yes, e.g. $\frac{7}{14} = \frac{1}{2}$ and 7 is prime

b No, as a and b will have no common factors other than one.

c No, as then a factor of 2 can be used to cancel.

d Yes, e.g. $\frac{5}{7}$

17 a $\frac{8}{9}$ **b** $1\frac{2}{9}$ **c** $\frac{81}{99}$ **d** $3\frac{43}{99}$

e $9\frac{25}{33}$ **f** $\frac{44}{333}$ **g** $2\frac{917}{999}$ **h** $13\frac{8125}{9999}$

Exercise 1D

1 a 6 **b** 63 **c** 65 **d** 30

e 4 **f** 33 **g** 60 **h** 87

2 a $\frac{7}{3}$ **b** $\frac{39}{5}$ **c** $\frac{41}{4}$ **d** $\frac{137}{6}$

3 a $\frac{9}{6} + \frac{8}{6} = \frac{17}{6}$ **b** $\frac{4}{3} - \frac{2}{5} = \frac{20}{15} - \frac{6}{15} = \frac{14}{15}$ **c** $\frac{5}{3} \times \frac{7}{2} = \frac{35}{6}$

4 a $\frac{3}{5}$ **b** $\frac{4}{9}$ **c** $1\frac{2}{7}$ **d** $\frac{19}{20}$

e $\frac{19}{21}$ **f** $1\frac{7}{40}$ **g** $\frac{7}{10}$ **h** $\frac{17}{27}$

5 a 5 **b** $3\frac{2}{5}$ **c** $5\frac{1}{7}$

d $6\frac{11}{15}$ **e** $7\frac{17}{63}$ **f** $17\frac{13}{16}$

6 a $\frac{2}{5}$ **b** $\frac{1}{45}$ **c** $\frac{11}{20}$ **d** $\frac{1}{10}$

e $\frac{1}{18}$ **f** $\frac{1}{8}$ **g** $\frac{13}{72}$ **h** $\frac{5}{48}$

7 a $1\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{13}{20}$

d $\frac{29}{40}$ **e** $\frac{5}{6}$ **f** $1\frac{16}{77}$

8 a $\frac{6}{35}$ **b** $\frac{1}{2}$ **c** $\frac{1}{12}$ **d** $\frac{4}{9}$

e $4\frac{1}{2}$ **f** $5\frac{1}{3}$ **g** $7\frac{1}{2}$ **h** 5

i 15 **j** 26 **k** $\frac{2}{3}$ **l** $\frac{5}{6}$

m $2\frac{1}{4}$ **n** $3\frac{1}{2}$ **o** 6 **p** $1\frac{1}{3}$

9 a $\frac{1}{3}$ **b** $\frac{7}{5}$ **c** 8 **d** $\frac{9}{13}$

10 a $\frac{20}{21}$ **b** $1\frac{1}{8}$ **c** $\frac{45}{56}$ **d** $\frac{27}{28}$

e $1\frac{1}{3}$ **f** $1\frac{1}{2}$ **g** 6 **h** $\frac{7}{8}$

i 18 **j** 9 **k** 16 **l** 64

m $\frac{1}{10}$ **n** $\frac{1}{12}$ **o** $\frac{4}{27}$ **p** $3\frac{1}{3}$

q 4 **r** $\frac{1}{6}$ **s** $1\frac{1}{2}$ **t** $\frac{7}{8}$

11 a $\frac{2}{7}$ **b** $\frac{8}{15}$ **c** 16

d $\frac{66}{85}$ **e** $2\frac{10}{21}$ **f** $\frac{3}{13}$

12 $\frac{7}{8}$ tonnes **13** $5\frac{29}{56}$ tonnes **14** $\frac{5}{12}$ hours (25 min)

15 7 truckloads **16** 3 hours

17 $\frac{5}{6}$, problem is the use of negatives in the method since $\frac{1}{3} < \frac{1}{2}$

18 a $\frac{b}{a}$ **b** $\frac{c}{ac+b}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

19 **a** 1 **b** $\frac{a^2}{b^2}$ **c** 1
d $\frac{c}{a}$ **e** a **f** $\frac{c}{b}$

20 **a** $-\frac{2}{3}$ **b** $\frac{5}{4}$ **c** $\frac{83}{10}$ **d** 1 **e** $\frac{50}{31}$
f $\frac{81}{400}$ **g** $\frac{329}{144}$ **h** $\frac{969}{100}$ **i** $\frac{583}{144}$ **j** $\frac{5}{11}$

Exercise 1E

1 **a** 4 **b** 12 **c** 24 **d** 72
e 3 **f** 9 **g** 11 **h** 3

2 **a** 9 **b** $\frac{4}{9}$ **c** $\frac{5}{9}$ **d** 8 **e** 10

3 **a** **i** 240 km **ii** 40 km **iii** 520 km
b **i** 5 h **ii** $4\frac{1}{2}$ h **iii** 15 min

4 **a** \$4 **b** \$3 **c** \$2.50

5 **a** 1 : 5 **b** 2 : 5 **c** 3 : 2 **d** 4 : 3
e 9 : 20 **f** 45 : 28 **g** 3 : 14 **h** 22 : 39
i 1 : 3 **j** 1 : 5 **k** 20 : 7 **l** 10 : 3

6 **a** 1 : 10 **b** 1 : 5 **c** 2 : 3 **d** 7 : 8
e 25 : 4 **f** 1 : 4 **g** 1 : 4 **h** 24 : 5
i 4 : 20 : 5 **j** 4 : 3 : 10 **k** 5 : 72 **l** 3 : 10 : 40

7 **a** \$200, \$300 **b** \$150, \$350
c \$250, \$250 **d** \$175, \$325

8 126 g

9 **a** \$10, \$20, \$40 **b** \$14, \$49, \$7 **c** \$40, \$25, \$5

10 **a** 15 km/h **b** 2000 rev/min
c 45 strokes/min **d** 14 m/s
e 8 mL/h **f** 92 beats/min

11 **a** 55 km **b** $16\frac{1}{2}$ km **c** $5\frac{1}{2}$ km

12 **a** 3 kg deal **b** red delicious
c 2.4 L **d** 0.7 GB

13 **a** Coffee A: \$3.60, coffee B: \$3.90. Therefore, coffee A is the best buy.
b Pasta A: \$1.25, pasta B: \$0.94. Therefore, pasta B is the best buy.
c Cereal A: \$0.37, cereal B: \$0.40. Therefore, cereal A is the best buy.

14 120

15 \$3000, \$1200, \$1800 respectively

16 \$15.90

17 108 L

18 \$3600, \$1200, \$4800 respectively

19 \$1.62

20 1 : 4

21 36°, 72°, 108°, 144°

22 Find cost per kilogram or number of grams per dollar. Cereal A is the best buy.

23 **a** $16\frac{2}{3}$ False **b** False **c** True **d** True

24 **a** $a + b$ **b** $\frac{a}{a+b}$ **c** $\frac{b}{a+b}$

25 **a** **i** 100 mL **ii** 200 mL
b **i** 250 mL **ii** 270 mL
c **i** 300 mL **ii** 1 : 4
d **i** 1 : 3 **ii** 7 : 19 **iii** 26 : 97 **iv** 21 : 52
e Jugs 3 and 4

Exercise 1F

1 **a** $\frac{3}{100}$ **b** $\frac{11}{100}$ **c** $\frac{7}{20}$ **d** $\frac{2}{25}$

2 **a** 0.04 **b** 0.23 **c** 0.86 **d** 0.463

3 **a** 50% **b** 60% **c** 25% **d** 90%
e 75% **f** 50% **g** 20% **h** 12.5 %

4 **a** 34% **b** 40% **c** 6% **d** 70%
e 100% **f** 132% **g** 109% **h** 310%

5 **a** 0.67 **b** 0.3 **c** 2.5 **d** 0.08
e 0.0475 **f** 0.10625 **g** 0.304 **h** 0.4425

6 **a** $\frac{67}{100}$ **b** $\frac{3}{10}$ **c** $2\frac{1}{2}$ **d** $\frac{2}{25}$
e $\frac{19}{400}$ **f** $\frac{17}{160}$ **g** $\frac{38}{125}$ **h** $\frac{177}{400}$

7

Percentage	Fraction	Decimal
10%	$\frac{1}{10}$	0.1
50%	$\frac{1}{2}$	0.5
5%	$\frac{1}{20}$	0.05
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
12.5%	$\frac{1}{8}$	0.125
1%	$\frac{1}{100}$	0.01
$11\frac{1}{9}$%	$\frac{1}{9}$	$0.\dot{1}$
$22\frac{2}{9}$%	$\frac{2}{9}$	$0.\dot{2}$
75%	$\frac{3}{4}$	0.75
15%	$\frac{3}{20}$	0.15
90%	$\frac{9}{10}$	0.9
37.5%	$\frac{3}{8}$	0.375
$33\frac{1}{3}$%	$\frac{1}{3}$	$0.\dot{3}$
$66\frac{2}{3}$%	$\frac{2}{3}$	$0.\dot{6}$
62.5%	$\frac{5}{8}$	0.625
$16\frac{2}{3}$%	$\frac{1}{6}$	$0.1\dot{6}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 **a** 25% **b** $33\frac{1}{3}\%$ **c** 16%
d 200% **e** 2800% **f** 25%
9 **a** \$36 **b** \$210 **c** 48 kg
d 30 km **e** 15 apples **f** 350 m
g 250 people **h** 200 cars **i** \$49
10 **a** \$120 **b** \$700 **c** \$300
d \$7 **e** \$0.20 **f** \$400
11 **a** \$540 **b** \$600 **c** \$508
d \$1250 **e** \$120 **f** \$40
12 $16\frac{2}{3}\%$ **13** $6\frac{1}{4}\%$
14 48 kg **15** 15 students
16 9 students **17** \$1150
18 **a** $P = 100$ **b** $P > 100$ **c** $P < 100$
19 **a** $x = 2y$ **b** $x = 5y$
c $x = \frac{3}{5}y$ (or $5x = 3y$) **d** $14x = 5y$
20 **a** 72 **b** $\frac{10}{11}$ **c** 280%
d $3\frac{1}{4}$ **e** 150%

Exercise 1G

1 **a** 1.4 **b** 1.26 **c** 60% **d** 21%
e 0.8 **f** 0.27 **g** 6% **h** 69%
2 **a** \$30 **b** 25%
3 **a** 12 kg **b** 11.1%
4 **a** \$52.50 **b** 37.8 min (37 min 48 s)
c 375 mL **d** 1.84 m **e** 27.44 kg
f 36 watts **g** \$13 585 **h** \$1322.40
5 **a** 19.2 cm **b** 24.5 cm **c** 39.06 kg
d 48.4 min (48 min 24 s) **e** \$78.48
f 202.4 mL **g** 18°C **h** \$402.36
6 50% **7** 44%
8 28% **9** 4%
10 **a** 22.7% **b** 26.7% **c** 30.9% **d** 38.4 %
11 \$21.50 **12** 30 068
13 \$14 895 **14** 193 474 ha
15 \$10.91 **16** \$545.45
17 **a** \$900 **b** \$990
c As 10% of 1000 = 100 but 10% of 900 = 90
18 25% **19** 100% **20** 42.86%
21 **a** \$635.58 **b** \$3365.08 **c** \$151.20 **d** \$213.54
22 **a** 79.86 g **b** \$97 240.50
c \$336 199.68 **d** 7.10 cm

Exercise 1H

1 **a**

Cost price (\$)	7	18	24.80	7.30	460.95	3250
Selling price (\$)	10	15.50	26.20	11.80	395	2070
Profit/Loss (\$)	3 profit	2.50 loss	1.40 profit	4.50 profit	65.95 loss	1180 loss
Profit/Loss (%)	42.9	13.9	5.6	61.6	14.3	36.3

b

Cost price (\$)	30.95	99.95	199.95	\$799.95	18 799	18 000
Mark up (\$)	10	80	\$395.95	700	16 700	8995
Selling price (\$)	40.95	179.95	595.90	1499.95	35 499	26 995
Mark-up (%)	32.3	80.0	198.0	87.5	88.8	50.0

c

Original price (\$)	100	49.95	29.95	199	345.50	2215
New price (\$)	72	40.90	22.70	176	299.95	2037
Discount (\$)	28	9.05	7.25	23	45.55	178
Discount (%)	28.0	18.1	24.2	11.6	13.2	8.0

2 **a** 90% **b** 80% **c** 85% **d** 92%
3 **a** **i** \$2 **ii** 20% **b** **i** \$5 **ii** 25%
c **i** \$16.80 **ii** 14% **d** **i** \$2450 **ii** 175%
4 166.67% **5** 40%
6 92.5% **7** \$37.50
8 \$1001.25 **9** 28%
10 42.3% **11** \$148.75
12 \$760.50 **13** \$613.33
14 \$333 333 **15** increased by 4%
16 12.5%
17 **a** \$54.75 **b** 128%
18 No, either way it gives the same price.
19 \$2100
20 **a** **i** \$54 187.50 **ii** \$33 277.90 **b** 10 years
21 **a** \$34 440 **b** \$44 000 **c** \$27 693.75
d \$32 951.10 **e** \$62 040 **f** \$71 627.10

Exercise 1I

1 **a** \$3952 **b** \$912 **c** \$24
2 **a** \$79.80 **b** \$62.70 **c** \$91.20 **d** \$102.60
3 **a** \$200 **b** \$56 **c** \$900 **d** \$145.10
4 \$1663
5 **a** **i** \$19.50 **ii** \$27 **iii** \$42.50
b **i** \$30 201.60 **ii** \$44 044 **iii** \$20 134.40
6 \$3760
7 **a** \$82.80 **b** \$119.60 **c** \$184
d \$276 **e** \$257.60 **f** \$404.80
8 **a** 7 **b** 18 **c** 33
d 25 **e** 37 **f** 40
9 \$14.50 per hour **10** \$12.20 per hour
11 \$490 **12** \$4010
13 **a** **i** \$40 035 **ii** 17.0%
b **i** \$53 905.80 **ii** 20.1%
c **i** \$41 218.20 **ii** 15%
d **i** \$30 052.56 **ii** 22.2%
14 Cate, Adam, Ed, Diana, Bill **15** \$839.05
16 \$1239.75 **17** 4.58% **18** \$437.50 **19** \$0.06
20 **a** 12 hours
b 12 and 0, 9 and 2, 6 and 4, 3 and 6 or 0 and 8
21 **a** **i** \$920 **ii** \$1500

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b i $A = 0.02x$ **ii** $A = 1200 + 0.025(x - 60000)$

22 a $5500

b Choose plan A if you expect that you will sell less than $5500 worth of jewellery in a week or plan B if you expect to sell more than $5500.

23 a i Superannuation is a system in which part of your salary is set aside for your retirement.

ii It is compulsory in the hope that when workers retire they will have enough funds for a comfortable retirement and will not require the age pension, which is funded by taxpayers.

iii 9%

iv It is invested by the people who manage superannuation funds, in the hope that the funds will grow rapidly over time.

b Yes **c** $8312

d 27.02% (to 2 decimal places) **e** 27

Exercise 1J

1 Taxable income = **gross** income minus **tax deductions**.

2 False

3 Anything from $0 to $18 200

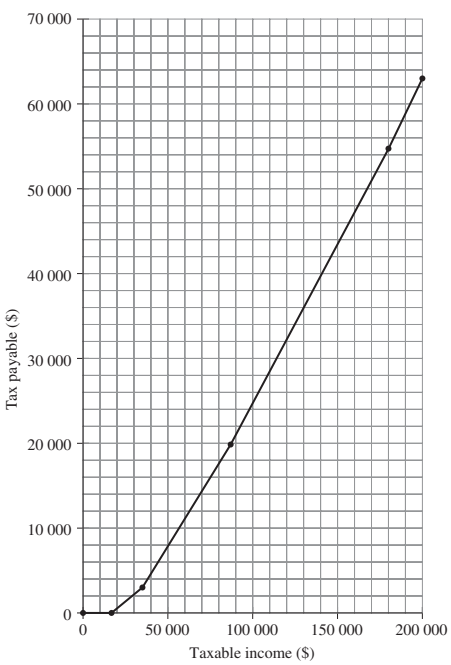
4 37 cents

5 a $2242 **b** $11 047 **c** $43 132 **d** $63 232

6 a

Taxable income	$0	$18 200	$37 000	$87 000	$180 000	$200 000
Tax payable	$0	$0	$3572	$19 822	$54 232	$63 232

b



7 $6172

8 a $65 625 **b** $12 875.13

c $1312.50 **d** $14 187.63

e 21.6% (to 1 decimal place)

f Not enough tax paid. Owes $1117.63.

9 $87 500

10 $172.84 (to 2 decimal places)

11 $95 000

12 Gross income is the total income earned before tax is deducted. Taxable income is found by subtracting tax deductions from gross income.

13 If a person pays too much tax during the year, they will receive a tax refund. If they do not pay enough tax during the year, they will have a tax liability to pay.

14 They only pay 45 cents for every dollar over $180 000.

15 a The tax-free threshold has been increased from $6000 to $18 200. In the second tax bracket, the rate has changed from 15 cents to 19 cents. In the third tax bracket, the rate has changed from 30 cents to 32.5 cents.

b

	2011–12	**2012–13**	
i Ali	$0	$0	No change
ii Bill	$1350	$0	$1350 less tax to pay
iii Col	$3600	$2242	$1358 less tax to pay
iv Di	$8550	$7797	$753 less tax to pay

16

	Resident	**Non-resident**	
a Ali	$0	$1625	Non-residents pay a lot more tax than residents.
b Bill	$0	$4875	
c Col	$2242	$9750	
d Di	$7797	$16 250	

17 a Answers will vary.

b i $17 547

ii $32.50, so this means the $100 donation really only cost you $67.50.

Exercise 1K

1 a $180, $1180 **b** $360, $1360 **c** $180, $1180

d $360, $1360 **e** $400, $1400 **f** $1600, $2600

2 a $12 000 **b** 6% p.a. **c** 3.5 years

d $720 **e** $1440 **f** $2520

3 a $3000 **b** $3600 **c** $416 **d** $315

4 $1980, $23 980

5 $2560

6 9 months

7 16 months

8 $2083.33

9 Choice 2

10 a $14 400

b $240

11 10%

12 a $\$P$

b 12.5%

c i 20 years **ii** 40 years **iii** Double

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 a 15% **b** $1907
14 a $51 000 **b** 4 years
15 a $P = \frac{I}{RN}$ **b** $N = \frac{I}{PR}$ **c** $R = \frac{I}{PN}$
16 a $1750 a month **b** $18 000
c $6000 **d** 2%

Exercise 1L

1 a $200 **b** $2200 **c** $220
d $2420 **e** $242 **f** $2662
2 a 2731.82 **b** 930.44
c 2731.82 **d** 930.44
3 a $\$4000 \times (1.2)^3$ **b** $\$15\,000 \times (1.07)^6$
c $\$825 \times (1.11)^4$ **d** $\$825 \times (0.89)^4$
4 a $6515.58 **b** $10 314.68
c $34 190.78 **d** $5610.21
5 $293 865.62
6 a 21.7% **b** 19.1% **c** 136.7% **d** 33.5%
7 $33 776
8 a $23 558 **b** $33 268 **c** $28 879 **d** $25 725
9 a $480 **b** $22 185 **c** $9844
10 $543 651 **11** 6142 people **12** 6.54 kg
13 Trial and error gives 12 years
14 Trial and error gives 5 years
15 a 35% **b** 40.26%
c As it calculates each years' interest on the original $400, not the accumulated total that compound interest uses
16 a 15.76% **b** 25.44% **c** 24.02%
d 86.96% **e** $\left(\left(1 + \frac{r}{100}\right)^t - 1\right) \times 100\%$
17 a $7509.25 **b** 9.39% p.a.
18 a 70.81% **b** 7.08%
19 a 5.39% p.a. **b** 19.28% p.a.

Exercise 1M

1 C
2

Year	Value at start of year	10% depreciation	Value at end of year
1	$20 000	$2000	$18 000
2	$18 000	$1800	$16 200
3	$16 200	$1620	$14 580
4	$14 580	$1458	$13 122

3 C
4 a i $R = 0.12, n = 1$ **ii** $R = 0.01, n = 12$
iii $R = 0.03, n = 4$ **iv** $R = -0.1, n = 1$
b ii
5 a $5909.82 **b** $2341.81 **c** $31 737.49
d $24 363.96 **e** $1270.29 **f** $399.07
g $87 045.75 **h** $2 704 813.83
6 a $6448.90 **b** $5635.80 **c** $19 629.68
d $1119.92 **e** $100 231.95 **f** $35 905.58
7 a $116 622 **b** $18 946 **c** $9884
d $27 022 **e** $18 117 **f** $18 301
8 a i $1236 **ii** $1255.09 **iii** $1264.93
iv $1271.60 **v** $1274.86
b The more compounding periods, the more interest is earned over time.
9 a $48 **b** $49.44 **c** $50.86
10 a $555.13 **b** 5.7 years **c** 7.4% p.a.
11 9.01% p.a.
12 a $3.83 **b** $4.10 **c** $2
d $133.05 **e** $8.86
13

P ($)	*R*	*n*	*A* ($)
250	0.1	5	402.63
600	0.08	6	952.12
15 000	0.05	10	24 433.42
1	0.2	2	1.44
100	0.3	3	219.70
8500	0.075	8	15 159.56

14 $20 887.38
15 a $2155.06
b $4310.13, yes
c $2100, no
d Doubling the principal doubles the interest.
e Doubling the interest rate and halving the time period does not produce the same amount of interest on the investment.
16 a i $60 **ii** $60.90 **iii** $61.36
iv $61.68 **v** $61.76 **vi** $61.83
b As the interest is being added back onto the principal at a faster rate, then the more compounding periods used, the more interest is earned over time.
17 Answers can vary – some examples of possible scenarios are shown here.
a $300 at 7% p.a. for 12 years compounded annually or $300 at 84% p.a. compounded monthly for 1 year
b $300 at 2% p.a. compounded annually for 36 years or $300 at 24 %p.a. compounded monthly for 3 years or $300 at 8% p.a. compounded quarterly for 9 years
c $1000 at 0.5% p.a. compounded annually for 48 years or $1000 at 6% p.a. compounded monthly for 4 years
d $1000 depreciating at 5% p.a. for 5 years
e $9000 depreciating at 20% p.a. for 3 years
18 a

n	1	2	3	4	5
$(1.09)^n$	1.09	1.188	1.295	1.412	1.539
	6	7	8	9	
	1.677	1.828	1.993	2.17	

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b **i** Approx. 9 years
ii Approx. 6 years
iii Approx. 4 years
c 26% p.a. compounded annually
d 72 (it is known as the rule of 72)
e **i** 36% p.a **ii** 16% p.a. **iii** 7.2% p.a.
f years × rate = 144
g The rule of 72 still applies.

Puzzles and challenges

1 Discuss with classmates as more than one answer for each may be possible. Some suggestions are given below (be creative).

$(4 - 4) \times (4 + 4) = 0$ $(4 - 4) + (4 \div 4) = 1$
$4 \div 4 + 4 \div 4 = 2$ $\sqrt{4 \times 4} - (4 \div 4) = 3$
$4 + (4 - 4) \times 4 = 4$ $\sqrt{4 \times 4} + 4 \div 4 = 5$
$(4 + 4 + 4) \div \sqrt{4} = 6$ $4 + 4 - 4 \div 4 = 7$
$\sqrt{4} + \sqrt{4} + \sqrt{4} + \sqrt{4} = 8$ $\sqrt{4} \times \sqrt{4} \times \sqrt{4} + \sqrt{4} = 10$

2 12
3 **a** $\frac{4}{7}$ **b** $\frac{14}{17}$
4 125 mL **5** 40.95% reduction
6 7.91% p.a.
7 **a** 44% **b** 10%
8 200 000 cm^2 (20 m^2)
9 9, 7, 2, 14, 11, 5, 4, 12, 13, 3, 6, 10, 15, 1, 8

Multiple-choice questions

1 D **2** B **3** C **4** A
5 E **6** E **7** D **8** E
9 C **10** A **11** C **12** B

Short-answer questions

1 **a** −16 **b** 2 **c** 0
d 10 **e** −23 **f** 1
2 **a** 21.5 **b** 29 100 **c** 0.153 **d** 0.00241
3 **a** 200 **b** 60 **c** 2
4 **a** 2.125 **b** $0.8\overline{3}$ **c** $1.\overline{857142}$
5 **a** $\frac{3}{4}$ **b** $1\frac{3}{5}$ **c** $2\frac{11}{20}$
6 **a** $\frac{1}{2}$ **b** $2\frac{1}{6}$ **c** $\frac{7}{24}$
d 2 **e** $3\frac{3}{4}$ **f** $2\frac{19}{28}$
7 **a** 5 : 2 **b** 16 : 9 **c** 75 : 14
8 **a** 50, 30 **b** 25, 55 **c** 10, 20, 50
9 **a** Store A: \$2.25/kg; store B: \$2.58/kg ∴ A is best buy.
b Store A: 444 g/\$; store B: 388 g/\$

10

Decimal	Fraction	Percentage
0.6	$\frac{3}{5}$	60%
$0.\dot{3}$	$\frac{1}{3}$	$33\frac{1}{3}\%$
0.0325	$\frac{13}{400}$	$3\frac{1}{4}\%$
0.75	$\frac{3}{4}$	75%
1.2	$1\frac{1}{5}$	120%
2	2	200%

11 **a** \$77.50 **b** 1.65
12 **a** 150 **b** 25
13 **a** 72 **b** 1.17 **c** 20%
14 12.5 kg **15** \$1800
16 **a** \$25 **b** $16\frac{2}{3}\%$
17 **a** \$18.25 **b** \$14.30
18 \$50 592 **19** \$525 **20** $4\frac{1}{2}$ years
21 \$63 265.95 **22** \$39 160

Extended-response questions

1 **a** \$231 **b** \$651
c **i** \$63 **ii** \$34.65
2 **a** **i** \$26 625 **ii** \$46 928.44
b 87.71% **c** \$82 420 **d** 26.26% **e** 7.4% p.a.

Chapter 2

Pre-test

1 **a** $x + 3$ **b** ab **c** $2y - 3$ **d** $\frac{x+2}{3}$
2 **a** 1 **b** −2 **c** 5 **d** −6
3 **a** $3x$ **b** $6y + xy$ **c** 0
d $17y$ **e** $10a + a^2$ **f** $5xy - 7y$
4 **a** $6a$ **b** $-21xy$ **c** $4b$ **d** $\frac{3m}{2}$
5 $4 \times 5 + 3 \times 5 = 35$ or $(4 + 3) \times 5 = 35$
6 **a** $2x + 6$ **b** $3a - 15$ **c** $12x - 8xy$ **d** $-6b + 3$
7 B, C
8 **a** 9 **b** 10 **c** 4 **d** 21
9 **a** True **b** False **c** True
d True **e** False **f** True
10 **a** $y = 7$ **b** $y = 1$ **c** $y = 9$ **d** $y = 6$
11 A **12** E

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 2A

1 **a** 4 **b** 5 **c** $\frac{1}{4}$ **d** 9

2 A c, B d, C b, D a, E f, F e

3 **a** 5 **b** -2 **c** $\frac{1}{3}$ **d** $-\frac{2}{5}$

4 **a** **i** $4 + r$ **ii** $t + 2$ **iii** $b + g$ **iv** $x + y + z$

b **i** $6P$ **ii** $10n$ **iii** $2D$ **iv** $5P + 2D$

c $\frac{500}{C}$

5 **a** $2 + x$ **b** $ab + y$ **c** $x - 5$ **d** $3x$

e $3x - 2y$ or $2y - 3x$ **f** $3p$ **g** $2x + 4$

h $\frac{x + y}{5}$ **i** $4x - 10$ **j** $(m + n)^2$ **k** $m^2 + n^2$

l $\sqrt{x + y}$ **m** $a + \frac{1}{a}$ **n** $(\sqrt{x})^3$

6 **a** -31 **b** -25 **c** -33 **d** -19

e $\frac{1}{2}$ **f** 4 **g** 1 **h** 85

7 **a** $1\frac{1}{6}$ **b** $4\frac{1}{4}$ **c** $\frac{1}{6}$ **d** $-1\frac{1}{3}$

8 **a** 60 m^2

b Length $= 12 + x$, width $= 5 - y$

c $A = (12 + x)(5 - y)$

9 **a** 18 square units **b** 1, 2, 3, 4, 5

10 **a** $\frac{P}{10}$ **b** $\frac{nP}{10}$

11 **a** **i** $P = 2x + 2y$ **ii** $A = xy$

b **i** $P = 4p$ **ii** $A = p^2$

c **i** $P = x + y + 5$ **ii** $A = \frac{5x}{2}$

12 **a** A: $2(x + y)$ B: $2x + y$, different

b Same

13 **a** B

b A: $c^2 = (a + b)^2$

14 **a** $\frac{n(n + 1)}{2}$ **b** **i** 10 **ii** 55

c $\frac{n^2}{2} + \frac{n}{2}$ **d** 10, 55

e Half the sum of n and the square of n.

Exercise 2B

1 **a** True **b** True **c** True **d** False

2 **a** ab **b** $6a$ **c** $8a$

d $4ab$ **e** $10mn$ **f** $9x^2$

3 **a** Like **b** Unlike **c** Unlike

d Unlike **e** Like **f** Like

g Like **h** Unlike **i** Like

4 **a** $10m$ **b** $12b$ **c** $15p$ **d** $6xy$

e $18pr$ **f** $16mn$ **g** $-14xy$ **h** $-15mn$

i $-12cd$ **j** $30ab$ **k** $-24rs$ **l** $-40jk$

5 **a** $24n^2$ **b** $-3q^2$ **c** $10s^2$

d $21a^2b$ **e** $-15mn^2$ **f** $18gh^2$

g $12x^2y^2$ **h** $8a^2b^2$ **i** $-6m^2n^2$

6 **a** $4b$ **b** $-\frac{a}{3}$ **c** $\frac{2ab}{3}$ **d** $\frac{m}{2}$

e $-\frac{x}{4}$ **f** $\frac{5s}{3}$ **g** uv **h** $\frac{5rs}{8}$

i $\frac{5ab}{9}$ **j** $\frac{7}{y}$

7 **a** $\frac{2x}{5}$ **b** $\frac{4}{3a}$ **c** $\frac{11mn}{3}$ **d** $6ab$

e $-\frac{5}{gh}$ **f** 8 **g** -3 **h** $\frac{7n}{3}$

i $-\frac{9q}{2}$ **j** $3b$ **k** $-5x$ **l** $\frac{m}{2}$

8 **a** $\frac{4x}{y}$ **b** $\frac{5p}{2}$ **c** $-6ab$ **d** $-\frac{3a}{2b}$

e $-\frac{7n}{5m}$ **f** $\frac{10s}{t}$ **g** $8n$ **h** $3y$

i $4b$ **j** $6xy$ **k** $5m$ **l** $3pq^2$

9 **a** $7x$ **b** $5x$ **c** $-5x$

d $2x$ **e** 0 **f** $-x$

g $8x$ **h** $3 + 5x$ **i** $7 + x$

j $10a$ **k** $-4a$ **l** $4a$

m $11x$ **n** $3ab$ **o** $7mn$

p $y + 8$ **q** $3x + 5$ **r** $7xy + 4y$

s $12ab + 3$ **t** $2 - 6m$ **u** $4 - x$

10 **a** $5a + 9b$ **b** $6x + 5y$ **c** $4t + 6$ **d** $11x + 4$

e $5xy + 4x$ **f** $7mn - 9$ **g** $5ab - a$ **h** 0

11 **a** xy^2 **b** $7a^2b$ **c** $3m^2n$ **d** p^2q^2

e $7x^2y - 4xy^2$ **f** $13rs^2 - 6r^2s$ **g** $-7x - 2x^2$

h $4a^2b - 3ab^2$ **i** $7pq^2 - 8pq$ **j** $8m^2n^2 - mn^2$

12 **a** $x + y$ **b** $4x + 2y$

13 **a** $8x$ **b** $3x^2$

14 **a** $30x$ cm **b** $30x^2$ cm^2

15 $\frac{20x + 75}{21}$

16 **a** True **b** False **c** True

d False **e** False **f** True

17 **a** $4x + 4y$ **b** $8x + 4$ **c** $2x - 2$

18 **a** a^3 **b** $\frac{b^2}{3}$ **c** $\frac{b^2}{3}$ **d** $\frac{3ab}{8}$

e $-\frac{2a^3}{3}$ **f** $-\frac{2}{5a^2}$ **g** $\frac{2}{5a^5}$ **h** $\frac{a^2}{4b^3}$

i $3a^3b$ **j** $\frac{4b^3}{a}$ **k** $\frac{a^3}{3b}$ **l** $-\frac{a}{2b^3}$

Exercise 2C

1 **a** **i** $5x$ **ii** 10

b $5x + 10$ **c** $x + 2$ **d** $5(x + 2)$ **e** $5x + 10$

2 **a** **i** -16 **ii** -4

b No, $-2x - 6$

c **i** -27 **ii** -33

d No, $-3x + 3$

3 **a** $2x + 6$ **b** $5x + 60$ **c** $2x - 14$

d $7x - 63$ **e** $6 + 3x$ **f** $21 - 7x$

g $28 - 4x$ **h** $2x - 12$ **i** $12x + 6$

j $8 - 12w$ **k** $18w + 27h$ **l** $12d - 8w - 4$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 **a** $-3x - 6$ **b** $-2x - 22$ **c** $-5x + 15$
d $-6x + 36$ **e** $-8 + 4x$ **f** $-65 - 13x$
g $-180 - 20x$ **h** $-300 + 300x$ **i** $-10w + 15$
j $-3w - 24$ **k** $-6x + 14$ **l** $-w + x - 9$

5 **a** $2a + 2b$ **b** $5a - 10$ **c** $3m - 12$
d $-16x - 40$ **e** $-12x - 15$ **f** $-4x^2 + 8xy$
g $-18ty + 27t$ **h** $3a^2 + 4a$ **i** $2d^2 - 5d$
j $-6b^2 + 10b$ **k** $8x^2 + 2x$ **l** $5y - 15y^2$

6 **a** $2x + 11$ **b** $6x - 14$ **c** $15x - 3$
d $1 + 3x$ **e** $4x - 5$ **f** $2x + 1$
g $-3x - 4$ **h** $-5x - 19$ **i** $11 - x$
j $12 - x$ **k** $2 - 2x$ **l** $6 - 3x$

7 **a** $5x + 12$ **b** $4x - 8$ **c** $11x - 2$
d $17x - 7$ **e** $-4x - 6$ **f** $x - 7$
g $2x - 4$ **h** $-27x - 4$ **i** $-4x - 18$
j $-13x - 7$ **k** $11x - 7$ **l** $2x - 15$

8 $A = x^2 + 4x$

9 **a** $2x + 2$ **b** $2x^2 - 3x$ **c** $6x^2 - 2x$
d $2x^2 + 4x$ **e** $x^2 + 2x + 6$ **f** $20 + 8x - x^2$

10 $20n - 200$ **11** $0.2x - 2000$

12 **a** $2x + 12$ **b** $x^2 - 4x$ **c** $-3x - 12$
d $-7x + 49$ **e** $19 - 2x$ **f** $x - 14$

13 **a** $6(50 + 2) = 312$ **b** $9(100 + 2) = 918$
c $5(90 + 1) = 455$ **d** $4(300 + 26) = 1304$
e $3(100 - 1) = 297$ **f** $7(400 - 5) = 2765$
g $9(1000 - 10) = 8910$ **h** $6(900 - 21) = 5274$

14 **a** $a = \$6000$, $b = \$21000$
b **i** \$3000 **ii** \$12600 **iii** \$51000
c **i** 0 **ii** $0.2x - 4000$ **iii** $0.3x - 9000$
iv $0.5x - 29000$

Exercise 2D

1 **a** $x = 4$ **b** $x = 8$ **c** $x = 4$ **d** $x = -4$
e $x = 3$ **f** $x = \frac{1}{3}$ **g** $x = 12$ **h** $x = 3$

2 **a** $x = 3$ **b** $x = 2$ **c** $x = 2$ **d** $x = 12$
e $x = 12$ **f** $x = -5$ **g** $x = 6$ **h** $x = 1$

3 A, D, E, F, H

4 **a** $x = 2$ **b** $a = 1$ **c** $m = 4$ **d** $x = -1$
e $n = -3$ **f** $x = -6$ **g** $b = -4$ **h** $y = -3\frac{2}{3}$
i $a = 1\frac{2}{3}$ **j** $b = 4\frac{1}{2}$ **k** $x = \frac{1}{2}$ **l** $x = 1\frac{1}{3}$
m $y = -\frac{5}{7}$ **n** $a = -\frac{1}{8}$ **o** $n = -\frac{3}{20}$

5 **a** $x = 8$ **b** $x = 2$ **c** $b = 12$ **d** $t = -6$
e $a = -6$ **f** $y = 30$ **g** $x = -15$ **h** $s = -8$
i $x = 12$ **j** $m = 20$ **k** $y = -5$ **l** $x = -10$

6 **a** $x = -3$ **b** $x = -1$ **c** $x = 2$ **d** $x = 8$
e $x = -1\frac{2}{5}$ **f** $x = -2\frac{5}{7}$ **g** $x = \frac{3}{8}$ **h** $x = 1\frac{3}{4}$

7 **a** $b = 9$ **b** $x = 6$ **c** $x = -6\frac{3}{4}$ **d** $x = -7\frac{1}{2}$
e $x = \frac{2}{3}$ **f** $n = -\frac{4}{25}$ **g** $x = 12$ **h** $x = 12$
i $x = -9$ **j** $d = -8$ **k** $f = 6$ **l** $z = -1\frac{1}{2}$
m $x = -6$ **n** $x = 0$ **o** $p = 10$ **p** $p = 0$

8 **a** $x = 11$ **b** $x = 6$ **c** $y = -10$ **d** $b = -12$
e $a = -5$ **f** $x = -1$ **g** $m = 7$ **h** $x = 5$
i $x = 4\frac{2}{7}$ **j** $b = 5\frac{1}{3}$ **k** $y = -7$ **l** $t = 3$
m $x = -3$ **n** $w = -12$ **o** $w = 4$ **p** $w = 0$

9 **a** 26 **b** 28 **c** 3 **d** 28
e 8 **f** $3\frac{3}{7}$ **g** 7 **h** \$900

10 **a** Should have $+1$ before $\div 2$
b Should have $\times 3$ before -2
c Need to $\div -1$ as $-x = 7$
d Should have $+4$ before $\times 3$

11 **a** **i** $x = 5$ **ii** $x = -3$ **iii** $x = \frac{1}{5}$
iv $x = 3$ **v** $x = -\frac{5}{6}$ **vi** $x = \frac{3}{10}$
b When the common factor divides evenly into the RHS

12 **a** $a = b + c$ **b** $a = \frac{c - b}{2}$ **c** $a = \frac{c - 2d}{b}$
d $a = c(b + d)$ **e** $a = -\frac{cd}{b}$ **f** $a = \frac{b}{2c}$
g $a = \frac{c(3 - d)}{2b}$ **h** $a = \frac{2d(b - 3)}{c}$ **i** $a = cd - b$
j $a = b + cd$ **k** $a = \frac{2be + 6c}{d}$ **l** $a = \frac{d - 3ef}{4c}$

Exercise 2E

1 **a** $4x - 12$ **b** 2 **c** $5x - 9$
d $3 - 9x$ **e** $3x - 7$ **f** $24 - 12x$

2 **a** $2x + 6 = 5$ **b** $5 + 2x - 2 = 7$
c $2x + 1 = -6$ **d** $2x - 3 = 1$

3 **a** $x = 2\frac{1}{2}$ **b** $a = -1\frac{2}{5}$ **c** $m = 6\frac{1}{3}$
d $y = 4\frac{3}{5}$ **e** $p = -3\frac{3}{4}$ **f** $k = 9\frac{1}{2}$
g $b = \frac{1}{2}$ **h** $m = -5\frac{1}{2}$ **i** $x = -\frac{4}{5}$
j $a = \frac{1}{14}$ **k** $x = 3\frac{1}{6}$ **l** $n = \frac{2}{3}$
m $x = \frac{9}{10}$ **n** $y = 1\frac{1}{6}$ **o** $a = -\frac{1}{8}$

4 **a** $x = 1$ **b** $x = -10$ **c** $x = 1$
d $x = -2$ **e** $x = 5$ **f** $x = 0$
g $x = 1$ **h** $x = 5$ **i** $x = 3$
j $x = 1$

5 **a** $b = 1$ **b** $a = -4$ **c** $t = 2$
d $m = 8$ **e** $x = 8$ **f** $a = 3$
g $x = 4$ **h** $y = -1$ **i** $m = \frac{5}{11}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

6 **a** $x = -1$ **b** $a = 3\frac{1}{2}$ **c** $y = -9$ **d** $x = -14$
e $b = -16$ **f** $m = 3$ **g** $a = 19$ **h** $x = -3$
i $x = -13$ **j** $n = -26$ **k** $a = -2\frac{2}{3}$ **l** $x = -1\frac{1}{10}$

7 **a** $\frac{1}{2}$ **b** $2\frac{2}{3}$ **c** $1\frac{1}{2}$ **d** $-\frac{5}{6}$
e $\frac{2}{5}$ **f** $1\frac{1}{2}$ **g** -6

8 \$5/hour **9** 11 marbles

10 **a** $x = 5$ **b** $x = 5$
c Dividing both sides by 3 is faster because $9 \div 3$ is a whole number.

11 **a** $x = 4\frac{1}{3}$ **b** $x = 4\frac{1}{3}$
c Expanding the brackets is faster because $7 \div 3$ gives a fraction answer.

12 **a** $x = 4$ **b** $x = 4$
c Method **a**: don't have to deal with negatives
Final step in method **a**: divide both sides by a positive number
Final step in method **b**: divide both sides by a negative number

13 **a** $x = \frac{d}{a-b}$ **b** $x = \frac{2}{a-b}$ **c** $x = \frac{c}{5a-b}$
d $x = -\frac{6}{3a-4b}$ or $\frac{6}{4b-3a}$ **e** $x = -c$
f $x = b$ **g** $x = \frac{d+bd+c}{a-b}$
h $x = -\frac{ab+bc}{a-b+1}$ or $\frac{ab+bc}{b-a-1}$

Exercise 2F

1 **a** $x - 3 = 4x - 9$ **b** $3(x + 7) = 9$
c $4(x - 9) = 12$ **d** $2x = x + 5$
e $x - 8 = 3x + 2$

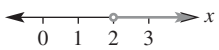
2 **a** Let e be the number of goals for Emma.
b $e + 8$ **c** $e + e + 8 = 28$ **d** $e = 10$
e Emma scored 10 goals, Leonie scored 18 goals

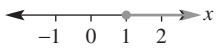
3 **a** Let b be the breadth in centimetres.
b length $= 4b$ **c** $2b + 2(4b) = 560$
d $b = 56$ **e** length = 224 cm, breadth = 56 cm

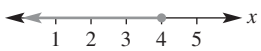
4 6 days **5** 10 km, 20 km **6** \$360, \$640
7 15, 45 **8** 19 km
9 A\$102.50, B\$175, C\$122.50
10 4 fiction, 8 non-fiction books
11 I am 10 years old.
12 Eric is 18 years old now.
13 First leg = 54 km, second leg = 27 km, third leg = 18 km, fourth leg = 54 km
14 2 hours **15** 8 p.m.
16 Rectangle $l = 55$ m, $w = 50$ m. Triangle side = 70 m.
17 **a** 27, 28, 29
b **i** $x, x + 2, x + 4$ **ii** 4, 6, 8
c **i** $x, x + 2, x + 4$ **ii** 15, 17, 19
d **i** $x, x + 3, x + 6$ **ii** 24, 27, 30
18 **a** $T = 8x + 7200$
b 300 **c** $R = 24x$ **d** $x = 350$ **e** 3825
19 \$15 200 **20** 438 km
21 Anna 6; Henry 4; Chloe 12; twins 11

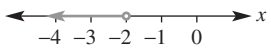
Exercise 2G

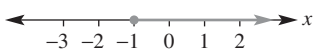
1 **a** D **b** A **c** C **d** B

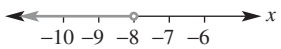
2 **a** 

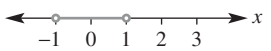
b 

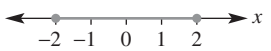
c 

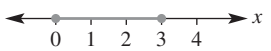
d 

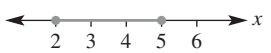
e 

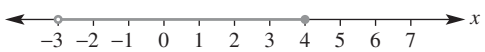
f 

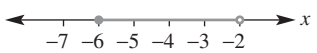
g 

h 

i 

j 

k 

l 

3 **a** $x < 3$ **b** $b > 5$ **c** $y > 6$ **d** $m < 5$
e $x \geq 3$ **f** $t > -5$ **g** $x \geq 12$ **h** $y \geq -10$
i $m < 6$ **j** $a \geq 1\frac{1}{2}$ **k** $x < 1$ **l** $x > 8$

4 **a** $x < 4$ **b** $n \leq -1$ **c** $x \geq \frac{3}{5}$
d $a \geq 4$ **e** $x \geq -6$ **f** $x \geq 10$
g $x < -6$ **h** $t \leq -2$ **i** $m > -4\frac{5}{6}$

5 **a** $x \leq 16$ **b** $x \leq -9$ **c** $x \leq 20$
d $x < 11$ **e** $x > -2\frac{2}{3}$ **f** $x \geq -2$

6 **a** $x < 1$ **b** $a < -8$ **c** $x \leq -2$
d $x < 2\frac{1}{2}$ **e** $y < -3\frac{1}{5}$ **f** $x < -\frac{4}{7}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

7 **a** $x \geq 2\frac{1}{2}$ **b** $t > -\frac{3}{5}$ **c** $y \leq \frac{3}{8}$
d $a < 1\frac{1}{5}$ **e** $m \geq 6$ **f** $b < 5\frac{1}{2}$

8 Less than 18 **9** $b < 13$

10 **a** 3 **b** 2 **c** 4 **d** 0

11 $x = 3$ or $x = 4$ or $x = 5$ **12** 399 km

13 **a** **i** −0.9, 0, 0.5, 1, 1.8 etc.
ii Numbers must be less than 2.
b **i** −4, −2, 0, 1, 5 etc.
ii Numbers must be greater than −5.
c **i** $x < a$ **ii** $x > -a$

14 **a** $x < 3$ **b** $x < 3$
c Reverse inequality sign when dividing by a negative number.

15 **a** $x < -13$ **b** $x \geq -3$ **c** $x > \frac{4}{7}$
d $x \leq \frac{13}{5}$ **e** $x > \frac{10}{17}$ **f** $x \geq \frac{3}{4}$

16 **a** $x > \frac{b-c}{a}$ **b** $x \geq b - a$ **c** $x \leq a(b+c)$
d $x \leq \frac{ac}{b}$ **e** $x < \frac{cd-b}{a}$ **f** $x \geq \frac{b-cd}{2}$
g $x < \frac{c}{a} - b$ **h** $x > \frac{b-cd}{a}$ **i** $x < b - \frac{c}{a}$
j $x \leq \frac{b+c}{1-a}$ **k** $x < \frac{b+1}{b-a}$ **l** $x \leq \frac{c-b}{b-a}$

Exercise 2H

1 **a** A **b** D **c** M **d** A

2 **a** 21 **b** 24 **c** 2 **d** 6
e 452.16 **f** 33.49 **g** 25.06 **h** 14.95
i 249.86 **j** 80

3 **a** 36 **b** 5 **c** 20 **d** 4.14
e 3.39 **f** 18.67 **g** 0.06

4 **a** $r = \frac{A}{2\pi h}$ **b** $r = \frac{100I}{Pt}$ **c** $n = \frac{p}{m} - x$
d $x = \frac{cd-a}{b}$ **e** $r = \sqrt{\frac{V}{\pi h}}$ **f** $v = \sqrt{PR}$
g $h = \frac{S - 2\pi r^2}{2\pi r}$ **h** $p = -q \pm \sqrt{A}$ **i** $g = \frac{4\pi^2 l}{T^2}$
j $A = (4C - B)^2$

5 **a** 88.89 km/h
b **i** $d = st$ **ii** 285 km

6 **a** **i** 212°F **ii** 100.4°F
b $C = \frac{5}{9}(F - 32)$
c **i** −10°C **ii** 36.7°C

7 **a** 35 m/s **b** 2 s

8 **a** Decrease **b** 3988 L
c 6 hours 57 minutes **d** 11 hours 7 minutes

9 **a** $D = \frac{c}{100}$ **b** $d = 100e$
c $D = 0.7M$ **d** $V = 1.15P$
e $C = 50 + 18t$ **f** $d = 42 - 14t$ **g** $C = \frac{c}{b}$

10 **a** $a = \frac{P}{4}$ **b** $a = 180 - b$ **c** $a = 90 - b$
d $a = \frac{180 - b}{2}$ **e** $a = \sqrt{c^2 - b^2}$ **f** $a = \sqrt{\frac{4A}{\pi}}$

11 **a** 73 **b** 7 **c** 476.3

Exercise 2I

1 **a** $y = 5$ **b** $x = -3$ **c** $x = 10$

2 **a** A **b** C

3 **a** Yes **b** No **c** No **d** Yes

4 **a** $x = 1, y = 2$ **b** $x = 5, y = 1$ **c** $x = \frac{1}{2}, y = \frac{3}{2}$
d $x = -2, y = -1$ **e** $x = 2, y = 6$ **f** $x = -3, y = 9$

5 **a** $x = 3, y = 9$ **b** $x = -1, y = 3$ **c** $x = 1, y = 0$
d $x = 6, y = 11$ **e** $x = 4, y = 3$ **f** $x = 12, y = -3$
g $x = -18, y = -4$ **h** $x = -2, y = 10$ **i** $x = 2, y = -4$

6 **a** $x = 2, y = 1$ **b** $x = 1, y = 3$ **c** $x = 0, y = 4$
d $x = 3, y = -2$ **e** $x = 4, y = 1$ **f** $x = -1, y = 4$

7 17, 31

8 10 tonnes, 19 tonnes

9 Breadth $= 1\frac{2}{3}$ cm, length $= 3\frac{5}{6}$ cm

10 $x - (3x - 1) = x - 3x + 1$, to avoid sign error use brackets when substituting

11 **a** $x = \frac{1}{3}, y = 2\frac{1}{3}$ **b** $x = -\frac{5}{6}, y = 6\frac{1}{3}$ **c** $x = -22, y = -7$

12 **a** $x = \frac{b}{a+b}, y = \frac{b^2}{a+b}$ **b** $x = \frac{b}{a+1}, y = \frac{1}{a+1}$
c $x = \frac{a-b}{2}, y = \frac{a+b}{2}$ **d** $x = \frac{a-ab}{a-b}, y = \frac{a-ab}{a-b} - a$
e $x = \frac{2a}{a-b}, y = \frac{2ab}{a-b} + a$
f $x = \frac{2a+ab}{a-b} - b, y = \frac{2a+ab}{a-b}$

Exercise 2J

1 **a** **i** $5x = 16$ **ii** $7x = -3$ **iii** $4y = 18$
b **i** $4y = 4$ **ii** $x = 4$ **iii** $2x = 5$

2 **a** **i** subtraction **ii** addition **iii** subtraction
b **i** addition **ii** subtraction **iii** addition

3 **a** $x = 1, y = 1$ **b** $x = 10, y = 2$ **c** $x = 1, y = 3$
d $x = 1, y = 1$ **e** $x = 2, y = 2$ **f** $x = 2, y = -1$

4 **a** $x = 2, y = 4$ **b** $x = 8, y = -1$ **c** $x = -2, y = 6$

5 **a** $x = -3, y = -13$ **b** $x = 2, y = 3$ **c** $x = 1, y = 3$

6 **a** $x = -3, y = 4$ **b** $x = 2, y = 1$ **c** $x = 2, y = 1$
d $x = -2, y = 3$ **e** $x = 7, y = -5$ **f** $x = 5, y = 4$
g $x = 5, y = -5$ **h** $x = -3, y = -2$ **i** $x = -1, y = -3$

7 **a** $x = 3, y = -5$ **b** $x = 2, y = -3$ **c** $x = 2, y = 4$
d $x = 3, y = 1$ **e** $x = -1, y = 4$ **f** $x = 3, y = -2$
g $x = 5, y = -3$ **h** $x = -2, y = -2$ **i** $x = 2, y = 1$
j $x = -5, y = -3$ **k** $x = -3, y = 9$ **l** $x = -1, y = -2$

8 21 and 9

9 102, 78

10 $L = 261.5$ m, $W = 138.5$ m

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 11 mobile phones, 6 iPods

12 a $x = 2, y = 1$ **b** $x = 2, y = 1$

c Method **b** is preferable as it avoids the use of a negative coefficient.

13 Rearrange one equation to make x or y the subject.

a $x = 4, y = 1$ **b** $x = -1, y = -1$

14 a no solution **b** no solution

15 a $x = \frac{a+b}{2}, y = \frac{a-a}{2}$ **b** $x = \frac{b}{2a}, y = -\frac{b}{2}$

c $x = -\frac{a}{2}, y = -\frac{3a}{2b}$ **d** $x = 0, y = b$

e $x = \frac{1}{3}, y = \frac{b}{3a}$ **f** $x = -\frac{4}{a}, y = 6$

g $x = \frac{3b}{7a}, y = \frac{b}{7}$ **h** $x = \frac{3b}{7a}, y = -\frac{b}{7}$

i $x = 0, y = \frac{b}{a}$ **j** $x = -\frac{c}{a}, y = c$

k $x = \frac{b-4}{ab-4}, y = \frac{1-a}{ab-4}$ **l** $x = 1, y = 0$

m $x = \frac{c-bd}{a(b+1)}, y = \frac{c+d}{b+1}$ **n** $x = \frac{a+2b}{a-b}, y = \frac{3a}{a-b}$

o $x = \frac{3b}{a+3b}, y = \frac{3b-2a}{a+3b}$ **p** $x = -\frac{b}{a-bc}, y = \frac{bc-2a}{a-bc}$

q $x = \frac{c+f}{a+d}, y = \frac{cd-af}{b(a+d)}$ **r** $x = \frac{c-f}{a-d}, y = \frac{af-cd}{b(a-d)}$

Exercise 2K

1 a $x + y = 42, x - y = 6$

b $x = 24, y = 18$

c One number is 18, the other number is 24

2 a $l = b + 5, 2l + 2b = 84$

b $l = 23.5, b = 18.5$

c length = 23.5 cm, breadth = 18.5 cm

3 a $l = 3b, 2l + 2b = 120$

b $l = 45, b = 15$

c length = 45 m, breadth = 15 m

4 a Let $\$m$ be the cost of milk.
Let $\$c$ be the cost of chips.

b $3m + 4c = 17, m + 5c = 13$

c $m = 3, c = 2$

d A bottle of milk costs \$3 and a bag of chips \$2.

5 a Let $\$g$ be the cost of lip gloss.
Let $\$e$ be the cost of eye shadow.

b $7g + 2e = 69,\ 4g + 3e = 45$

c $g = 9, e = 3$

d Lip gloss \$9, eye shadow \$3

6 Cricket ball \$12, tennis ball \$5

7 4 chips, 16 hot dogs

8 300 adults, 120 children

9 Potatoes 480, corn 340

10 11 five-cent and 16 twenty-cent coins

11 Michael is 35 years old now.

12 Jenny \$100, Kristy \$50

13 160 adults and 80 children

14 5 hours

15 Jogging 3 km/h, cycling 9 km/h

16 a Malcolm is 14 years old.

b The second digit of Malcolm's age is 3 more than the first digit.

c 6: 14, 25, 36, 47, 58 or 69

17 Original number is 37

18 Any two-digit number that has the first digit 2 more than the second (e.g. 42 or 64)

Exercise 2L

1 a 4 **b** 0 **c** None

2 a $2x^2 - 16 = 0$
$2x^2 = 16$
$x^2 = 8$
$x = \pm\sqrt{8}$

b $2x^2 + 16 = 16$
$2x^2 = 0$
$x^2 = 0$
$x = 0$

c $2x^2 + 16 = 0$
$2x^2 = -16$
$x^2 = -8$
No solutions

3 a Two **b** One **c** None

d Two **e** None **f** One

4 a $x = \pm 3$ **b** $x = \pm 11$ **c** $x = \pm 1$

d $x = \pm 20$ **e** $x = \pm 2$ **f** $x = \pm 10$

g $x = \pm 5$ **h** $x = \pm 7$ **i** $x = \pm 6$

j $x = \pm 4$ **k** $x = \pm 8$ **l** $x = \pm 9$

m $x = \pm 11$ **n** $x = \pm 4$ **o** $x = \pm\frac{1}{12}$

p $x = \pm\frac{1}{10}$

5 a **i** $x = \pm\sqrt{15}$ **ii** $x = \pm\sqrt{10}$ **iii** $x = \pm\sqrt{42}$

iv $x = \pm\sqrt{3}$ **v** $x = \pm\sqrt{7}$ **vi** $x = \pm\sqrt{11}$

vii $x = \pm\sqrt{34}$ **viii** $x = \pm\sqrt{39}$

b **i** $x = \pm 2.2$ **ii** $x = \pm 4.1$ **iii** $x = \pm 6.3$

iv $x = \pm 11.8$

6 a $x = \pm 7$ **b** $x = \pm 6$ **c** $x = \pm\sqrt{13}$

d No solutions **e** No solutions **f** $x = \pm\sqrt{22}$

g $x = 5$ **h** $x = \sqrt{2}$ **i** $x = \pm\sqrt{15}$

j $x = -2$ **k** $x = -\sqrt{22}$ **l** No solution

7 a $x = \pm 3$ **b** $x = \pm 4$ **c** $x = \pm 2$

d $x = \pm 8$ **e** $x = \sqrt{6}$ **f** No solution

g $x = -1$ **h** $x = -\sqrt{2}$

8 9 or –9

9 a $x^2 = 14, x = \pm 3.7$

b Side length must be positive so the value of x will be 3.7 m.

10 a 2.8 m since $r > 0$ **b** 3.8 cm since $r > 0$

c 2 m since $r > 0$

11 a **i** $c = 10$ **ii** $a = \sqrt{7}$

iii $b = \sqrt{56}\ (= 2\sqrt{14})$

b $x = \sqrt{32}\ (= 4\sqrt{2})$

12 8.5 m

13 a 15 m **b** 3.46 seconds **c** 2 seconds

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

14 x^2 is positive for all values of x, since a positive × a positive = a positive and a negative × a negative = a positive.

15 a $c > 0$ **b** $c = 0$ **c** $c < 0$

16 $x^2 \geq 0$ for all values of x so $x^2 + 10 \leq 10$ for all values of x so it cannot equal 6.

17 a i $x = \pm 4$ **ii** $x = 4$

b Different since in part ii, just $+\sqrt{16}$ is required not $-\sqrt{16}$ also.

18 10 seconds **19** 6.4 seconds

Puzzles and challenges

1 $x = 3$

4	9	2
3	5	7
8	1	6

2 \$140

3 8

4 a i > **ii** < **iii** > **iv** <

b c, b, a, d

5 $x = 1, y = -2, z = 5$

6 a $x = \dfrac{ab}{a - b}$ **b** $x = \dfrac{10}{3}$ **c** $x = -\dfrac{7}{11}$ **d** $x = \dfrac{29}{4}$

Multiple-choice questions

1 C **2** D **3** D **4** C **5** B **6** A
7 A **8** E **9** B **10** A **11** B **12** D

Short-answer questions

1 a $7m$ **b** $2(x + y)$ **c** $3m$ **d** $\dfrac{n}{4} - 3$

2 a -7 **b** 7 **c** 24 **d** 8

3 a $8mn$ **b** $\dfrac{xy}{3}$ **c** $6b^2$ **d** $4 - 3b$

e $2mn + 2m - 1$ **f** $2p + 4q$

4 a $2x + 14$ **b** $-6x - 15$ **c** $6x^2 - 8x$

d $-10a + 8a^2$ **e** $13 - 4x$ **f** $27x - 10$

5 a $x = 9$ **b** $x = 26$ **c** $x = 15$

d $x = 1$ **e** $x = 2$ **f** $x = -9$

g $x = -10$ **h** $x = \dfrac{5}{7}$

6 a $2n + 3 = 21, n = 9$

b $\dfrac{l - 5}{3} = 7, l = 26$

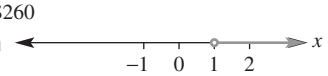
c $\dfrac{x}{4} - 5 = 0, x = 20$

7 a $x = 5$ **b** $x = 1\dfrac{5}{6}$ **c** $x = 4$

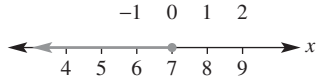
d $x = -5$ **e** $x = 1$ **f** $x = 4$

8 \$260

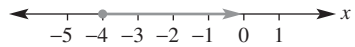
9 a



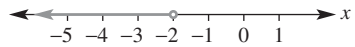
b



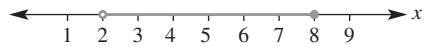
c



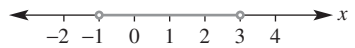
d



e



f



10 a $x < 12$ **b** $m > -1$ **c** $y \geq -6$

d $x < 17$ **e** $a > 4$ **f** $x \geq 2$

11 n(sales) ≥ \$24 000

12 a $E = 60$ **b** $a = 5$ **c** $h = 10$

13 a $x = \dfrac{v^2 - u^2}{2a}$ **b** $\theta = \dfrac{2A}{r^2}$

c $I = \sqrt{\dfrac{P}{R}}$ **d** $a = \dfrac{2S}{n} - l$ or $a = 2S - \dfrac{nl}{n}$

14 a $x = 8, y = 2$ **b** $x = -\dfrac{3}{5}, y = -1\dfrac{3}{5}$

c $x = 4, y = 11$ **d** $x = 11, y = 4$

e $x = -3, y = -5$ **f** $x = 4, y = 1$

15 3 show bags, 6 rides

16 a $x = \pm 2$ **b** $x = \pm\sqrt{7}$

c $x = \pm\sqrt{10}$ **d** $x = -1$

Extended-response questions

1 a $0 < b \leq 10$ **b** $b = \dfrac{2A}{h} - a$ **c** 8 m

d $h = \dfrac{2A}{a + b}$ **e** 8 m

2 a \$5 per ride

b i Let \$$c$ be the cost of chips.
Let \$$d$ be the cost of a drink.

ii $2d + c = 11, 3d + 2c = 19$

iii $d = 3, c = 5$

iv Chips cost \$5 per bucket and drinks cost \$3 each.

Chapter 3

Pre-test

1 a 9 **b** 400 **c** 20 **d** 900

2 a 5.92 **b** 15.36 **c** 4.86 **d** 8.09

3 a $x = 2$ **b** $x = 4$ **c** $x = 5$ **d** $x = 13$

4 a $x = 3$ **b** $x = 4$ **c** $x = 5$ **d** $x = 8$

5 a 0.4568 **b** 0.3457 **c** 0.0456 **d** 0.2800

6 a 4.23 **b** 5.68 **c** 76.90 **d** 23.90

7 a $x = 2$ **b** $x = 3$ **c** $x = 12$ **d** $x = 6$

e $x = 12$ **f** $x = 30$ **g** $x = 28$ **h** $x = 182$

i $x = 9$ **j** $x = 2\dfrac{1}{2}$ **k** $x = 12$ **l** $x = 8$

8 a $x = 15.04$ **b** $x = 15.60$ **c** $x = 1.38$ **d** $x = 6.30$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 **a** $x = 0.6$ **b** $x = 0.6$ **c** $x = 2.1$ **d** $x = 0.5$
e $x = 0.6$ **f** $x = 2.0$ **g** $x = 9.1$ **h** $x = 7.1$
10 **a** $x = 64$ **b** $x = 28$ **c** $x = 108$
d $x = 60$ **e** $x = 50$ **f** $x = 55$

Exercise 3A

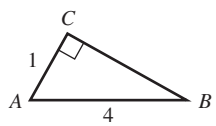
1 **a** 17 **b** 50 **c** $\sqrt{8}$
2 **a** $c^2 = a^2 + b^2$ **b** $x^2 = y^2 + z^2$ **c** $j^2 = k^2 + l^2$
3 **a** 81 **b** 10.24 **c** 13
d 106 **e** 6 **f** 10
g 4.90 **h** 3.61
4 **a** 10 **b** 13 **c** 17
d 15 **e** 25 **f** 41
g 50 **h** 30 **i** 25
5 **a** 4.47 **b** 3.16 **c** 15.62
d 11.35 **e** 7.07 **f** 0.15
6 **a** $\sqrt{5}$ **b** $\sqrt{58}$ **c** $\sqrt{34}$
d $\sqrt{37}$ **e** $\sqrt{109}$ **f** $\sqrt{353}$
7 **a** 21.63 mm **b** 150 mm **c** 50.99 mm
d 155.32 cm **e** 1105.71 m **f** 0.02 m
8 **a** 8.61 m **b** 5.24 m **c** 13.21 cm
d 0.19 m **e** 17.07 mm **f** 10.93 cm
9 42 units **10** 4.4 m **11** 250 m
12 495 m **13** 2.4 m
14 **a** 5 **b** $\frac{25}{3}$
15 5.83 m
16 **a** No **b** No **c** Yes
d No **e** Yes **f** Yes
17 **a** 5.66 cm **b** 5.66 cm, 8 cm **c** Yes
18 **a** 77.78 cm **b** 1.39 m
c Reduce it by 7.47 cm **d** 43.73 cm
19 42.43 cm, 66.49 cm

Exercise 3B

1 **a** 14 **b** 11 **c** 12
d 20 **e** 4 **f** 24
g 8 **h** 8
2 **a** True **b** False **c** False
d True **e** True **f** False
3 **a** 16 **b** 24 **c** 6
d 21 **e** 60 **f** 27
4 **a** 8.66 **b** 11.31 **c** 5.11
d 17.55 **e** 7.19 **f** 0.74
5 **a** 30 cm **b** 149.67 cm **c** 52.92 cm
d 2.24 km **e** 12 cm **f** 1.65 mm
6 **a** $\sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}}$ **b** $\sqrt{8}$ **c** $\sqrt{\frac{1521}{200}} = \frac{39}{\sqrt{200}} = \frac{39}{10\sqrt{2}}$
7 **a** $\sqrt{187}$ **b** $\sqrt{567}$ **c** 40
8 14.2 m **9** 5.3 m **10** 49 cm **11** 1.86 m
12 **a** $\sqrt{\frac{81}{5}} = \frac{9}{\sqrt{5}}$ **b** $\sqrt{5}$ **c** $\sqrt{\frac{5}{2}}$
13 **a** $\sqrt{\frac{5}{2}}$, $3\sqrt{\frac{5}{2}}$ **b** $\frac{10}{\sqrt{13}}$, $\frac{15}{\sqrt{13}}$ **c** $\frac{25}{\sqrt{74}}$, $\frac{35}{\sqrt{74}}$
14 **a** The side c **b** $\sqrt{6}$ **c** No, c^2 is enough
d Error due to rounding, x would not be exact
15 **a** **i** $\sqrt{8}$ **ii** $\sqrt{7}$ **iii** $\sqrt{6}$
b OB: 2.6, OC: 2.79, OD: 2.96
c Differ by 0.04; the small difference is the result of rounding errors

Exercise 3C

1 **a** C, II **b** A, III **c** B, I
2 3.0 m **3** 142.9 m **4** 3823 mm
5 1060 m
6 **a** 6.4 m **b** 5.7 cm **c** 6.3 m **d** 6.0 m
7 466.18 m
8 **a**



b 3.9 cm
9 **a** 27 m **b** 118.3 m
10 171 cm
11 **a** $\frac{5}{\sqrt{2}}$ **b** $6 + \frac{5}{\sqrt{2}}$ **c** 10.2
12 **a** 1.1 km **b** 1.1 km **c** 3.8 km
13 **a** 28.28 cm
b **i** 80 cm **ii** 68.3 cm **iii** 48.3 cm

Exercise 3D

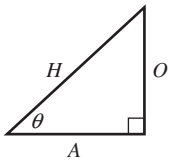
1 **a** Yes **b** Yes **c** No
d Yes **e** Yes **f** No
g Yes **h** No **i** Yes
2 **a** 11.18 **b** 0.34 **c** 19.75
3 **a** 3.6 **b** 2.5 **c** 3.1
4 **a** 18.0 cm **b** 7 mm **c** 0.037 m
5 **a** $\sqrt{2}$ cm **b** 1.7 cm
6 **a** $\sqrt{208}$ cm **b** 15.0 cm
7 84 m **8** 2.86 cm **9** $1 : \frac{2}{\sqrt{3}}$
10 **a** **i** $\sqrt{8}$ cm **ii** 3.74 cm
b **i** $\sqrt{50}$ cm **ii** 5.45 cm
11 **a** $\sqrt{65}$ **b** 8.06 **c** $\sqrt{69}$, 8.31 **d** 8.30
e Rounding errors have accumulated
12 **a** **i** 11.82 m **ii** 12.15 m **iii** 11.56 m **iv** 11.56 m
b Shortest is 10.44 m

Exercise 3E

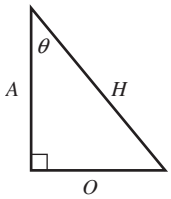
1 **a** hypotenuse **b** opposite **c** adjacent
d opposite **e** hypotenuse **f** adjacent

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

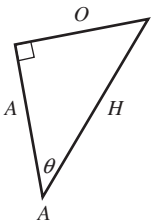
2 a



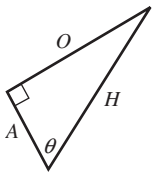
b



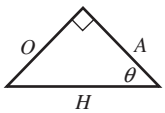
c



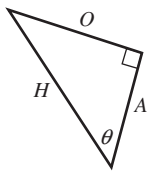
d



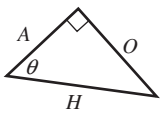
e



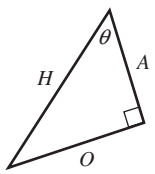
f



g



h



3 a 5 **b** 4 **c** 3 **d** 3 **e** 4

4 a $\sin\theta = \frac{4}{7}$ **b** $\tan\theta = \frac{5}{4}$ **c** $\cos\theta = \frac{3}{5}$

d $\sin\theta = \frac{2}{3}$ **e** $\tan\theta = 1$ **f** $\cos\theta = \frac{x}{y}$

g $\tan\theta = \frac{4}{5}$ **h** $\cos\theta = \frac{a}{2b}$ **i** $\tan\theta = \frac{5y}{3x}$

5 a i $\frac{5}{13}$ **ii** $\frac{5}{13}$ **iii** The same

b i $\frac{12}{13}$ **ii** $\frac{12}{13}$ **iii** The same

c i $\frac{5}{12}$ **ii** $\frac{5}{12}$ **iii** The same

6 a $\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

b $\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$ $\tan\theta = \frac{5}{12}$

c $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

7 a $\frac{4}{5}$ **b** $\frac{3}{5}$ **c** $\frac{3}{5}$

d $\frac{3}{4}$ **e** $\frac{4}{5}$ **f** $\frac{4}{3}$

8 $\frac{3}{4}$

9 a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$

10 a i 5

ii $\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

b i 25

ii $\sin\theta = \frac{7}{25}$

$\cos\theta = \frac{24}{25}$

$\tan\theta = \frac{7}{24}$

c i 15

ii $\sin\theta = \frac{9}{15} = \frac{3}{5}$

$\cos\theta = \frac{12}{15} = \frac{4}{5}$

$\tan\theta = \frac{9}{12} = \frac{3}{4}$

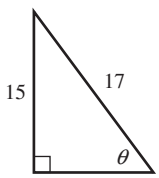
d i 10

ii $\sin\theta = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{8}{6} = \frac{4}{3}$

11 a



b 8

c $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

12 a i $\frac{1}{2}$ **ii** $\frac{\sqrt{3}}{2}$ **iii** $\frac{1}{\sqrt{3}}$

iv $\frac{\sqrt{3}}{2}$ **v** $\frac{1}{2}$ **vi** $\sqrt{3}$

b i They are equal.

ii They are equal.

13 a Answers may vary.

b i 0.766 **ii** 0.643 **iii** 0.839

iv 0.766 **v** 1.192 **vi** 0.643

c $\sin 40° = \cos 50°$, $\sin 50° = \cos 40°$, $\tan 50° = \frac{1}{\tan 40°}$

$\tan 40° = \frac{1}{\tan 50°}$

14 a Yes, any isosceles right-angled triangle

b No, as it would require the hypotenuse (the longest side) to equal the opposite

c No, adjacent side can't be zero

d No, as the numerator < denominator for sin and cos as the hypotenuse is the longest side

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 a $\sin\theta = \frac{3}{5}$ $\tan\theta = \frac{3}{4}$

b i $\cos\theta = \frac{\sqrt{3}}{2}$ $\tan\theta = \frac{1}{\sqrt{3}}$

ii $\tan\theta = \sqrt{3}$ $\sin\theta = \frac{\sqrt{3}}{2}$

iii $\sin\theta = \frac{1}{\sqrt{2}}$ $\cos\theta = \frac{1}{\sqrt{2}}$

c Equals one

d $(\sin\theta)^2 + (\cos\theta)^2 = 1$ (the Pythagorean identity)

Exercise 3F

1 a A **b** O **c** H

2 a sin **b** tan **c** cos

3 a 0.34 **b** 0.80 **c** 2.05
d 0.73 **e** 0.10 **f** 0.25
g 0.46 **h** 0.24

4 a 3.06 **b** 18.94 **c** 5.03
d 0.91 **e** 1.71 **f** 9.00
g 2.36 **h** 4.79 **i** 7.60

5 a 5.95 **b** 0.39 **c** 13.38
d 3.83 **e** 8.40 **f** 1.36
g 29.00 **h** 1.62 **i** 40.10
j 4.23 **k** 14.72 **l** 13.42
m 17.62 **n** 5.48 **o** 10.02
p 1.02

6 1.12 m **7** 44.99 m **8** 10.11 m

9 a 20.95 m **b** 10 cm

10 a 65° **b** 1.69 **c** 1.69

11 a i 80° **ii** 62° **iii** 36° **iv** 9°

b i Both 0.173… **ii** Both 0.469…
iii Both 0.587… **iv** Both 0.156…

c $\sin\theta = \cos(90 - \theta)$

d i 70° **ii** 31° **iii** 54° **iv** 17°

12 a $\sqrt{2}$

b i $\frac{1}{\sqrt{2}}$ **ii** $\frac{1}{\sqrt{2}}$ **iii** 1

c $\sqrt{3}$

d i $\frac{1}{2}$ **ii** $\frac{\sqrt{3}}{2}$ **iii** $\frac{1}{\sqrt{3}}$
iv $\frac{\sqrt{3}}{2}$ **v** $\frac{1}{2}$ **vi** $\sqrt{3}$

Exercise 3G

1 a $x = 2$ **b** $x = 5$ **c** $x = 3$
d $x = 4$ **e** $x = 7$ **f** $x = 4$
g $x = 0.5$ **h** $x = 0.2$

2 a 4.10 **b** 6.81 **c** 37.88
d 0.98 **e** 12.80 **f** 14.43
g 9.52 **h** 114.83 **i** 22.05

3 a 13.45 **b** 16.50 **c** 57.90
d 26.33 **e** 15.53 **f** 38.12
g 9.15 **h** 32.56 **i** 21.75
j 48.28 **k** 47.02 **l** 28.70

4 a $x = 7.5$, $y = 6.4$ **b** $a = 7.5$, $b = 10.3$
c $a = 6.7$, $b = 7.8$ **d** $x = 9.5$, $y = 12.4$
e $x = 12.4$, $y = 9.2$ **f** $x = 20.8$, $y = 18.4$
g $m = 56.9$, $n = 58.2$ **h** $x = 15.4$, $y = 6.0$

5 40 m **6** 3848 m

7 a 26 m **b** 97 m

8 a 23.7 m **b** 124.9 m

9 a B as student B did not use an approximation in their working out.

b Do not round sin 31° during working.

c i Difference of 0.42 **ii** Difference of 0.03

10 a i 10.990 **ii** 11.695

b i 0.34 **ii** 0.94 **iii** 0.36

c Equal to tan 20°

d i $\frac{b}{c}$ **ii** $\frac{a}{c}$ **iii** $\frac{b}{a}$ **iv** $\frac{b}{a}$

e Same as $\tan\theta$

Exercise 3H

1 a 6 **b** 12 **c** 15
d 30 **e** 37, 30 **f** 40

2 a 25°17′ **b** 45°10′ **c** 67°29′

3 a 47° **b** 12° **c** 18°
d 51° **e** 24° **f** 42°
g 79° **h** 13° **i** 3°

4 a sine **b** sine **c** cosine **d** tan

5 a 30° **b** 60° **c** 45°
d 30° **e** 45° **f** 30°
g 90° **h** 50° **i** 90°
j 55° **k** 0° **l** 70°

6 a 34.85° **b** 19.47° **c** 64.16°
d 75.52° **e** 36.87° **f** 38.94°
g 30.96° **h** 57.99° **i** 85.24°

7 a 43°3′ **b** 31°9′ **c** 40°36′
d 15°36′ **e** 55°18′ **f** 50°29′
g 48°35′ **h** 41°25′

8 17° **9** 23.13° **10** 25.4° **11** 26°38'

12 a 128.7° **b** 72.5° **c** 27.3°

13 a 90°, 37°, 53° **b** 90°, 23°, 67° **c** 90°, 16°, 74°

14 45°

15 $\angle ACM = 18.4°$, $\angle ACB = 33.7°$, no it is not half

16 a 18° **b** 27° **c** 45°
d 5.67 m **e** up to 90°

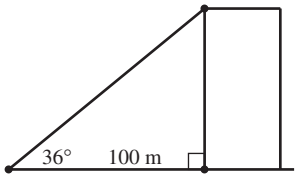
Exercise 3I

1 $a = 65$, $b = 25$

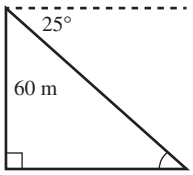
2 a 22° **b** 22°

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

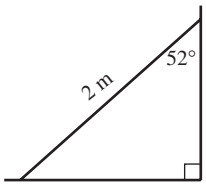
3 a



b



c



4 29 m **5** 16 m **6** 157 m **7** 38 m
8 90 m **9** 37° **10** 6° **11** 10°
12 Yes, by 244.8 m
13 a 6° **b** 209 m
14 4634 mm **15** 15°, 4319 mm **16** 1.25 m
17 a 6.86 m **b** 26.6°
c i $m = h + y$
ii $y = x \tan \theta$
iii $m = h + x \tan \theta$
18 $A = \frac{1}{2}a^2 \tan \theta$
19 a i 47.5 km **ii** 16.25 km
b No, $\sin(2 \times 20°) < 2 \times \sin 20°$
c Yes, $\sin\left(\frac{1}{2}20°\right) > \frac{1}{2}\sin 20°$
20 a Yes **b** 12.42 km
c No, after 10 minutes the plane will be above 4 km.
d 93 km/h or more
21 a 1312 m **b** 236.16 km/h

Exercise 3J

1 a 0° **b** 045° **c** 090° **d** 135°
e 180° **f** 225° **g** 270° **h** 315°
2 a 070° **b** 130° **c** 255° **d** 332°
3 a i 040° **ii** 220° **b i** 142° **ii** 322°
c i 210° **ii** 030° **d i** 288° **ii** 108°
e i 125° **ii** 305° **f i** 067° **ii** 247°
g i 330° **ii** 150° **h i** 228° **ii** 048°
i i 206° **ii** 026°
4 3.28 km **5** 59.45 km **6** 3.4 km **7** 11 km
8 a 39 km **b** 320°
9 a 11.3 km **b** 070°
10 310 km **11** 3.6 km **12** 6.743 km
13 a $180 + a$ **b** $a - 180$
14 a 320° **b** 245° **c** 065°
d 238° **e** 278°
15 a 620 km **b** 606 km **c** 129 km
16 a i 115 km **ii** 96 km
b 158 km **c i** 68 km **ii** 39.5 min

Puzzles and challenges

1 $P = 4 + 2\sqrt{8}$ **2** 10 m^2
3 15 cm **4** 010°
5 Round peg in square hole
6 122°
7 a i 3, 4, 5 **ii** 5, 12, 13 **b** $m^2 + n^2$

Multiple-choice questions

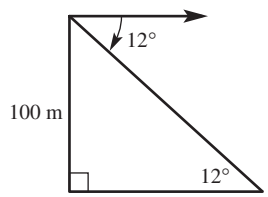
1 A **2** B **3** A **4** C **5** B
6 C **7** D **8** C **9** A **10** B

Short-answer questions

1 a 37 **b** $\sqrt{12}$ **c** $\sqrt{50}$
2 4.49 m **3 a** 13 cm **b** 13.93 cm
4 19 m
5 a 4.91 m **b** $x = 2.83$, $h = 2.65$
6 a 0.64 **b** 2.25 **c** 0.72
7 a 11.33 **b** 48.14 **c** 50.71
8 28.01 m **9** 25 m
10 a 59.45 km **b** 53.53 km
11 177.91 m **12** 053.13° **13** 63.2 m
14 5.3 m **15 a** 52.5 km **b** 13.59 km

Extended-response questions

1 a 2.15 m **b** 0.95 m **c i** 3.05 m **ii** yes
2 a



b 470 m **c** 3° **d** 1530 m

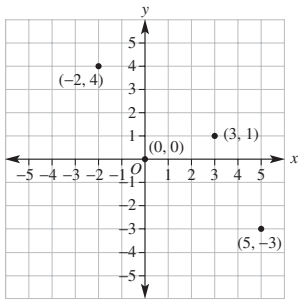
Chapter 4

Pre-test

1 a 3 **b** −4 **c** 4 **d** −3
2 a 1 **b** 7 **c** −5 **d** −8
3 a $x = 4$ **b** $x = -2$ **c** $x = 4$
d $x = -4$ **e** $x = 5$ **f** $x = 6$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4



5 **a** 7 **b** 6.5 **c** 3 **d** −5

6 **a** 5 **b** 4 **c** 5 **d** 3

7 **a** 4 **b** 5 **c** 7 **d** 3

8 **a** $y = 2x + 5$ **b** $y = -3x + 2$

c $y = 2x + 3$ **d** $y = 6x - 8$

9 **a** False **b** True **c** True **d** False

10 **a** 2 **b** −5 **c** −3 **d** 9

Exercise 4A

1 **a** A (4, 1), B (2, 3), C (0, 3), D (−2, 2)
E (−3, 1), F (−1, 0), G (−3, −2), H (−2, −4)
I (0, −3), J (2, −2), K (3, −4), L (2, 0)

b **i** *F*, *L* **ii** *C*, *I* **c** **i** *D*, E **ii** *J*, *K*

d *F* **e** *K*

2 **a** 2 **b** −1 **c** −4 **d** −7

3 **a** 1 **b** −5 **c** 3 **d** 21

4 **a** $y = x - 1$

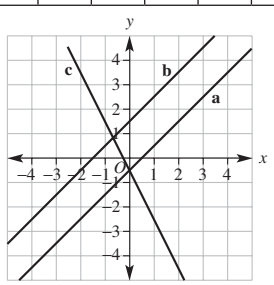
x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

b $y = x + 3$

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

c $y = -2x - 1$

x	−3	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5	−7



d $y = 2x - 3$

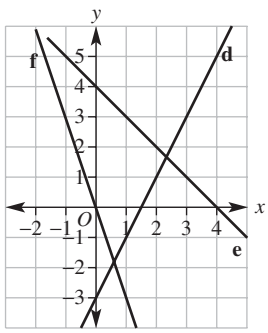
x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3

e $y = -x + 4$

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

f $y = -3x$

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9



5 **a** 1 and 1 **b** −2 and 2 **c** 4 and 8 **d** −5 and 10

e 2 and 3 **f** 7 and −3 **g** −11 and 5 **h** −2 and −5

6 **a** Yes **b** No **c** Yes **d** Yes **e** No **f** No

7 **a** No **b** Yes **c** No **d** No **e** Yes **f** No

8 **a** $y = x + 2$ **b** $y = 2x$

c $y = 2x + 1$ **d** $y = -x + 2$

9 Ac, Bd, Cb, Da

10 **a** False **b** True **c** True **d** False

11 **a** The graph passes through (1, 1) and (2, −1).
The midpoint of those two points is $\left(\frac{3}{2}, 0\right)$. Therefore, the x-intercept is $\frac{3}{2}$.

b The graph passes through (0, −1) and (3, 1). The midpoint of those two points is $\left(\frac{3}{2}, 0\right)$. Therefore, the x-intercept is $\frac{3}{2}$.

12 **a** Yes **b** No **c** No **d** Yes

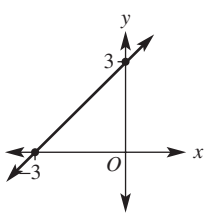
13 **a** $y = 2x + 7$ **b** $y = -x + 20$ **c** $y = -3x - 10$

d $y = -5x + 4$ **e** $y = \frac{1}{2}x + \frac{1}{2}$ **f** $y = -\frac{1}{2}x - \frac{3}{2}$

Exercise 4B

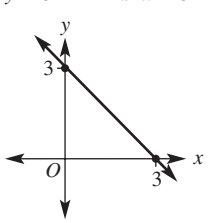
1 **a** $y = 3$ **b** $x = -3$ **c** (0, 3) **d** (−3, 0)

e



2 **a** $y = 3$ **b** $x = 3$ **c** (0, 3) **d** (3, 0)

e

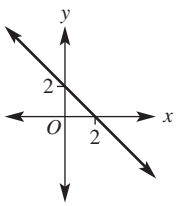


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

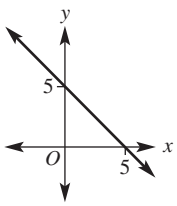
3 **a** 4 **b** −5 **c** 3 **d** −4

4 **a** 5 **b** −2 **c** 3 **d** 2

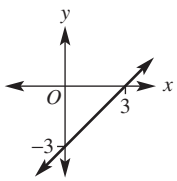
5 **a**



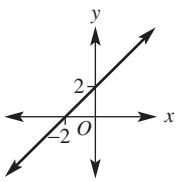
b



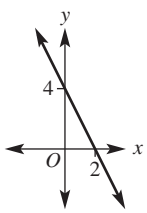
c



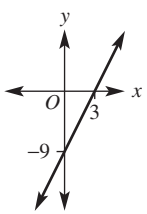
d



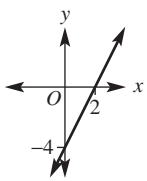
e



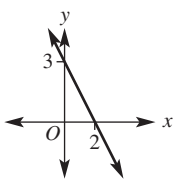
f



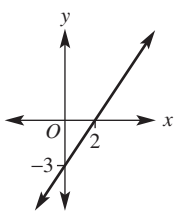
g



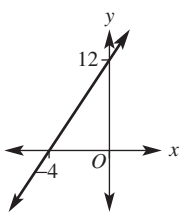
h



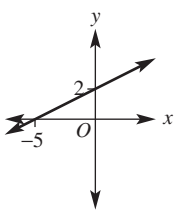
i



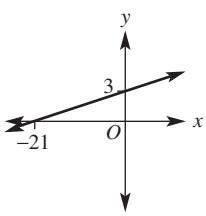
j



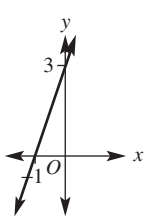
k



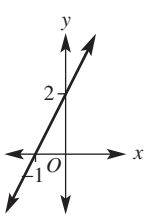
l



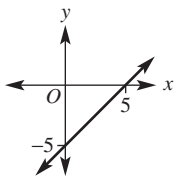
6 **a**



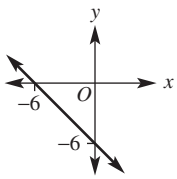
b



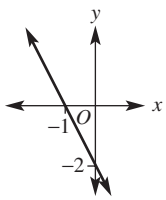
c



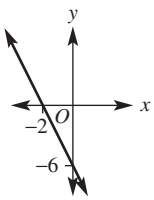
d



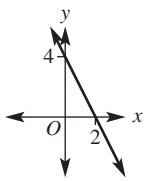
e



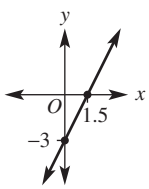
f



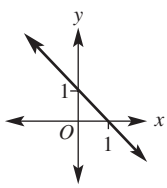
g



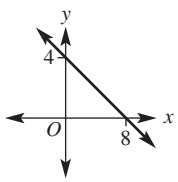
h



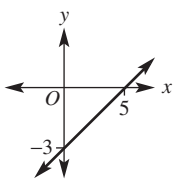
i



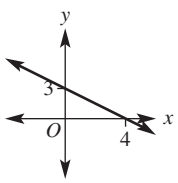
7 **a**



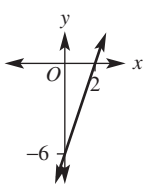
b



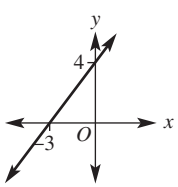
c



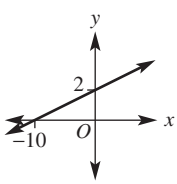
d



e

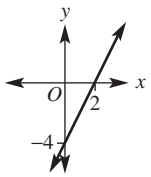


f

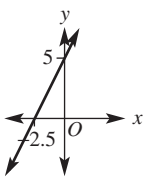


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

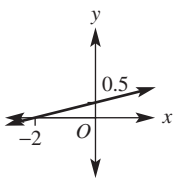
g



h

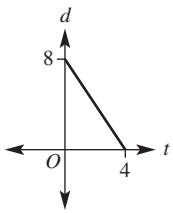


i



8 a 8 m **b** 4 seconds

c



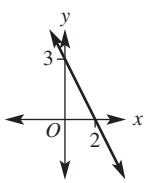
9 a 100 m **b** 12.5 s

10 a $1\frac{2}{3}$ and $-2\frac{1}{2}$ **b** -7 and $-1\frac{2}{5}$

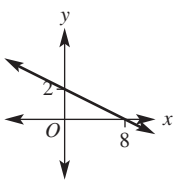
c $6\frac{1}{2}$ and -13 **d** $-\frac{1}{2}$ and -1

e $1\frac{2}{3}$ and $-1\frac{1}{2}$ **f** $\frac{1}{3}$ and $-\frac{1}{7}$

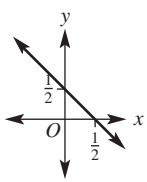
11 a



b



c



12 (0, 0) is the x- and y-intercept for all values of a and b.

13 a $x + y = 4$ **b** $x + y = 2$ **c** $x - y = 3$

d $x - y = -1$ **e** $x + y = k$ **f** $x + y = -k$

14 a $\frac{c}{a}$ and $\frac{c}{b}$ **b** $-\frac{cb}{a}$ and c **c** $\frac{c}{a}$ and $-\frac{c}{b}$

d $\frac{c}{b}$ and $-\frac{c}{a}$ **e** $-\frac{c}{b}$ and $\frac{c}{a}$ **f** $\frac{bc}{a}$ and $\frac{bc}{a}$

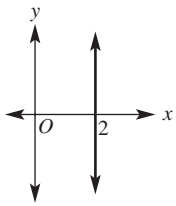
Exercise 4C

1 a 2 **b** -3 **c** 0

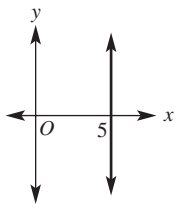
2 a 3 **b** -2 **c** 0 **d** 0

3 a 5 **b** $\frac{1}{3}$ **c** -4 **d** -0.1

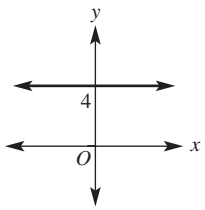
4 a



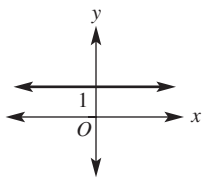
b



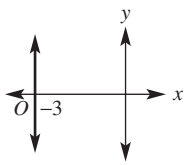
c



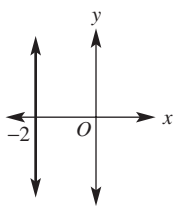
d



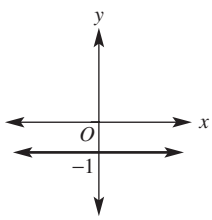
e



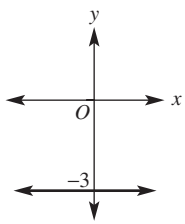
f



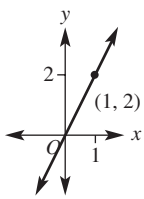
g



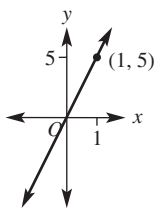
h



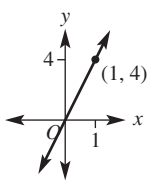
5 a



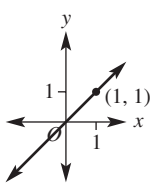
b



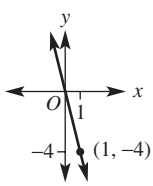
c



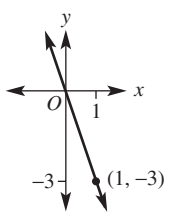
d



e

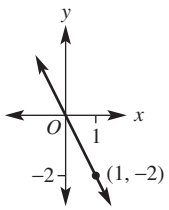


f

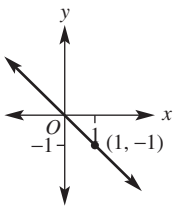


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

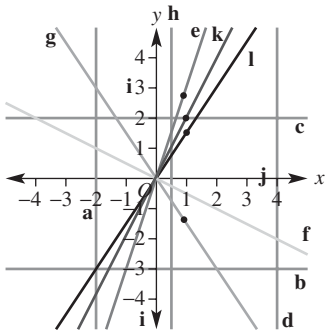
g



h



6



7 **a** $y = -2$ **b** $y = 4$ **c** $x = -2$
d $x = 5$ **e** $y = 1.5$ **f** $x = -6.7$

8 **a** $y = 3$ **b** $x = 5$ **c** $x = -2$ **d** $y = 0$

9 **a** $y = 250$ **b** $y = -45$

10 **a** (1, 2) **b** (−3, 5) **c** (0, −4)
d (4, 0) **e** (0, 0) **f** (1, 3)
g (3, −27) **h** (5, 40) **i** (3, 15)

11 **a** $A = 15$ square units **b** $A = 68$ square units

12 **a** **i** $y = 1$ or $y = -5$ **ii** $y = 0$ or $y = -4$
iii $y = 3\frac{1}{2}$ or $y = -7\frac{1}{2}$
b **i** $y = 1$ or $y = -5$ **ii** $y = 7$ or $y = -11$
iii $y = 9\frac{1}{2}$ or $y = -13\frac{1}{2}$

13 **a** $y = 2x$ **b** $y = -x$ **c** $y = 3x$ **d** $y = -3x$

14 **a** $y = 3x$ **b** $y = 4x$ **c** $y = -5x$ **d** $y = -2x$

15 $x = -1$, $y = -5$, $y = 5x$

16 **a** (a, b) **b** $m = \frac{b}{a}$

17 $m = \frac{1}{2}$

Exercise 4D

1 **a** 2 **b** $\frac{1}{2}$ **c** 3
d −1 **e** −3 **f** $-\frac{1}{4}$

2 **a** Zero **b** Negative **c** Positive
d Undefined

3 **a** Positive, 1 **b** Positive, 2 **c** Zero
d Zero **e** Negative, $-\frac{2}{3}$ **f** Negative, $-\frac{3}{4}$
g Undefined **h** Undefined **i** Positive, $\frac{1}{2}$
j Positive, 3 **k** Positive, 2 **l** Negative, −4

4 **a** 2 **b** 1 **c** $-\frac{1}{4}$ **d** $\frac{4}{3}$
e 2 **f** −2 **g** −1 **h** $\frac{1}{2}$
i $\frac{3}{2}$ **j** $-\frac{5}{2}$ **k** $\frac{1}{3}$ **l** $\frac{5}{2}$

5 A 1, B −2, C 1, D $\frac{1}{2}$, E −2, F 3

6 **a** $-\frac{1}{20}$ **b** $\frac{35}{2}$ **c** $-\frac{1}{5}$ **d** $\frac{15}{2}$

7 **a** 6 **b** −1 **c** 4 **d** $-1\frac{1}{2}$

8 300 m

9 **a** 9 **b** −4 **c** −1 **d** 4
e −0.4 **f** 2.4

10 $\frac{7}{11} = \frac{35}{55}$, $\frac{3}{5} = \frac{33}{55}$ hence $\frac{7}{11} > \frac{3}{5}$ so $\frac{7}{11}$ is steeper

11 **a** run $= x_2 - x_1$ **b** rise $= y_2 - y_1$ **c** $m = \frac{y_2 - y_1}{x_2 - x_1}$
d **i** $m = \frac{3}{2}$ **ii** $m = \frac{5}{4}$ **iii** $m = -\frac{5}{3}$ **iv** $m = \frac{4}{3}$
e Yes, the rise and the run work out regardless.

12 **a** **i** −1 **ii** −3 **b** 4.5
c **i** 6 **ii** 2.4 **iii** $1\frac{2}{7}$ **iv** 7.5

Exercise 4E

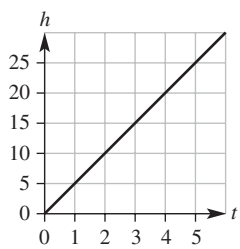
1 **a** **i** 10 km **ii** 20 km **iii** 30 km
b 10 km/h **c** 10 **d** They are the same.

2 **a** 15 mm **b** **i** 6 days **ii** 20 days

c

t	0	1	2	3	4
h	0	5	10	15	20

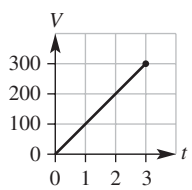
d



e $m = 5$

3 **a** 100 L/h

b



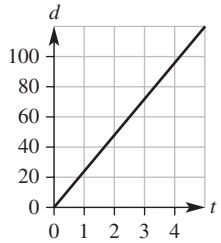
c **i** 100
ii $V = 100t$

d **i** $V = 150$ L
ii $t = 20$ hours

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

4 **a** 25 km/h

b



c **i** 25 **ii** $d = 25t$

d **i** 62.5 km **ii** 1.6 hours

5 **a** $d = 50t$ **b** $g = 2t$

c $C = 1.25n$ **d** $P = 20t$

6 **a** 100 km/h **b** 7 cm/s

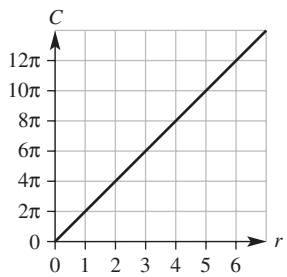
c 2.5 cm/minute **d** 49 mm/min

7 **a** 10.2 L **b** 72.25 L **c** 800 km

8 Sally **9** Leopard **10** 25 months

11 **a** **i** 0 **ii** 4π **iii** 12π

b



c Gradient is 2π, the coefficient of r

12 No, $A = \pi r^2$ so area (A) is proportional to square of radius (r^2).

13 **a** $A = 2h$

b 2 cm^2 increase for each 1 cm increase in height

14 Yes, for any fixed time, e.g. $t = 5$, $s = \frac{d}{5}$

15 1.2 min or 72 seconds

16 $\frac{5}{12}$km

Exercise 4F

1 **a** $y = 2x + 5$ **b** $y = 3x - 1$ **c** $y = -2x + 3$

d $y = -x - 2$ **e** $y = -\frac{1}{2}x - 10$ **f** $y = -\frac{2}{3}x + \frac{5}{2}$

2 **a** 1 **b** -5 **c** -5

d -11 **e** 9 **f** 2

3 **a** gradient = 3, y-intercept = -4

b gradient = -5, y-intercept = -2

c gradient = -2, y-intercept = 3

d gradient = $\frac{1}{3}$, y-intercept = 4

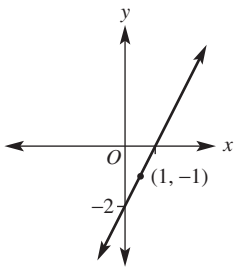
e gradient = -4, y-intercept = 0

f gradient = 2, y-intercept = 0

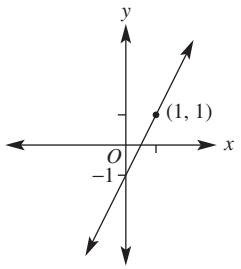
g gradient = 2.3, y-intercept = 0

h gradient = -0.7, y-intercept = 0

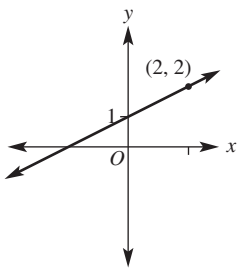
4 **a** 1, -2,



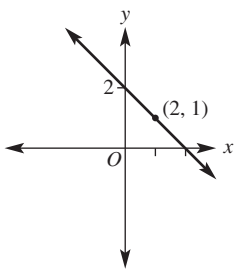
b 2, -1,



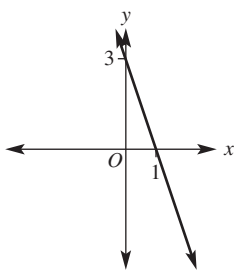
c $\frac{1}{2}$, 1,



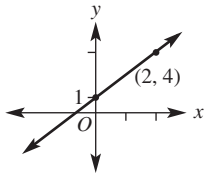
d $-\frac{1}{2}$, 2,



e -3, 3,

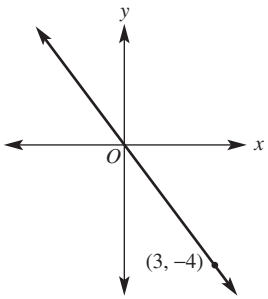


f $\frac{3}{2}$, 1,

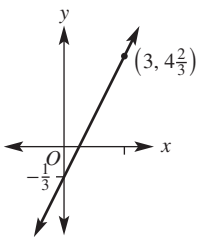


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

g $-\frac{4}{3}$, 0,



h $\frac{5}{3}$, $-\frac{1}{3}$



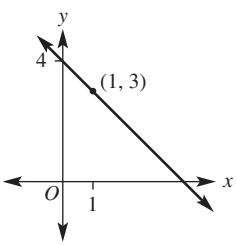
5 **a** $y = x + 7$ **b** $y = -x + 3$ **c** $y = 2x + 5$

6 **a** $y = -2x + 3$ **b** $y = 3x - 1$ **c** $y = -3x + 2$

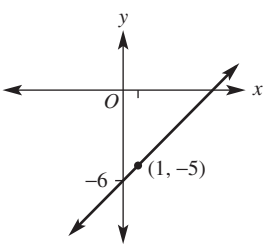
d $y = x + 2$ **e** $2x - y = 1$ **f** $3x + y = 4$

g $x - 3y = 1$ **h** $2x - 7y = -2$

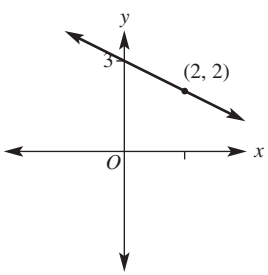
7 **a** -1, 4,



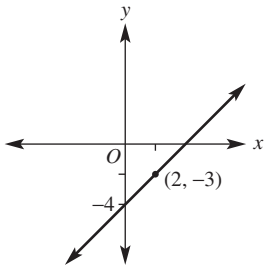
b 1, -6,



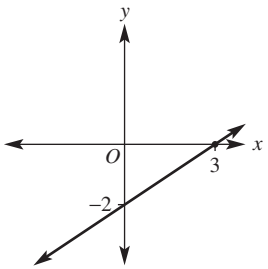
c $-\frac{1}{2}$, 3,



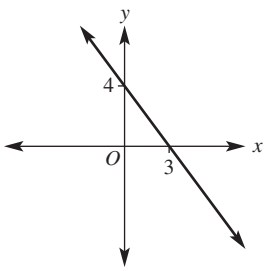
d $\frac{1}{2}$, -4,



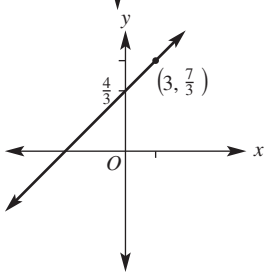
e $\frac{2}{3}$, -2,



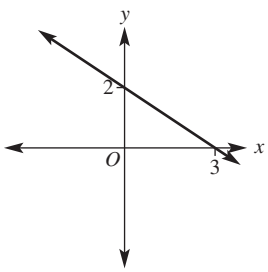
f $-\frac{4}{3}$, 4,



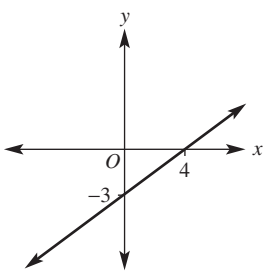
g $\frac{1}{3}$, $\frac{4}{3}$,



h $-\frac{2}{3}$, 2,

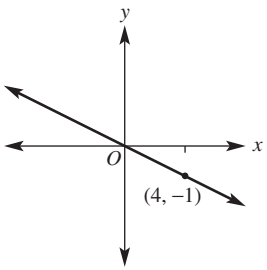


i $\frac{3}{4}$, -3,

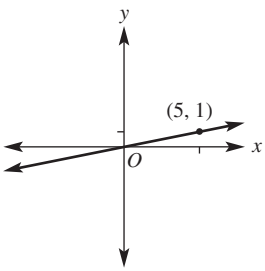


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

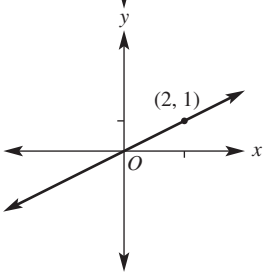
j $-\frac{1}{4}$, 0,



k $\frac{1}{5}$, 0,

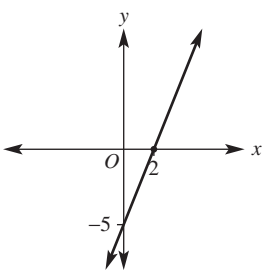


l $\frac{1}{2}$, 0,

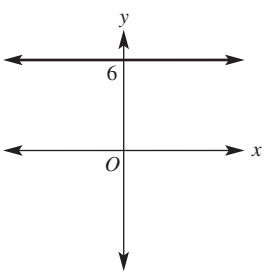


8 **i** d **ii** f **iii** c
iv b **v** e **vi** a

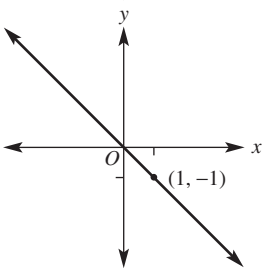
9 **a**



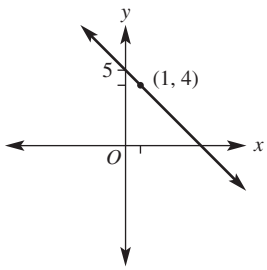
b



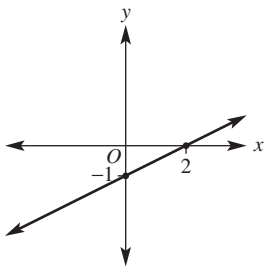
c



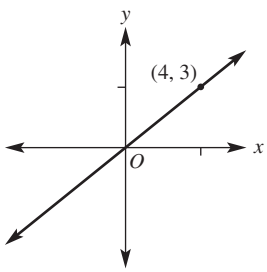
d



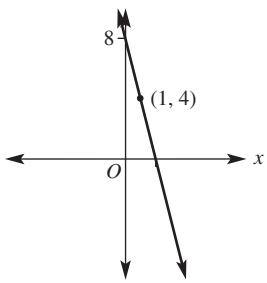
e



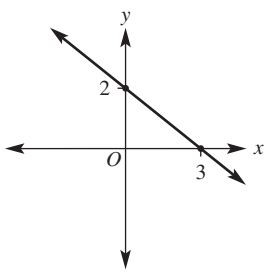
f



g



h



10 **a** No **b** No **c** Yes **d** No

11 C, D, F, H

12 **a** $y = 2x + 2$, y-intercept is 2

b Expand brackets and simplify

13 $y = k$

14 $y = -\frac{a}{b}x + \frac{d}{b}$, $m = -\frac{a}{b}$, $c = \frac{d}{b}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 a $y = 2x + 3$ **b** $y = -x + 2$
c $y = \frac{5}{3}x + \frac{2}{3}$ **d** $y = -\frac{2}{5}x - \frac{1}{5}$

Exercise 4G

1 a $y = 2x + 5$ **b** $y = 4x - 1$
c $y = -2x + 5$ **d** $y = -x - \frac{1}{2}$

2 a 1 **b** 3 **c** 9
d 10 **e** 5 **f** 2

3 a $y = x + 3$ **b** $y = x + 2$ **c** $y = -2x - 4$
d $y = 2x - 2$ **e** $y = 8x + 8$ **f** $y = -x + 4$

4 a $y = \frac{3}{4}x + 3$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -\frac{5}{4}x + 3$
d $y = \frac{3}{2}x + 4$ **e** $y = \frac{3}{5}x$ **f** $y = -\frac{1}{3}x - 1$

5 a $y = 3x + 5$ **b** $y = -2x - 1$ **c** $y = -3x + 8$
d $y = x - 3$ **e** $y = -3x + 3$ **f** $y = 5x - 1$
g $y = -x + 8$ **h** $y = -3x + 6$ **i** $y = -2x + 2$
j $y = -4x - 9$

6 a i 2 **ii** $y = 2x + 2$
b i -1 **ii** $y = -x + 3$
c i -4 **ii** $y = -4x + 11$
d i 1 **ii** $y = x - 4$

7 $x = 2\frac{1}{2}$

8 $\frac{3}{5}$ and $-\frac{3}{4}$

9 $V = -20t + 120$, $V = 120$ L initially

10 a $y = 5x$ **b** $y = 6.5x + 2$

11 a 1 **b** 8 **c** -6 **d** $\frac{5}{2}$

12 a $b = 3$ **b** $b = 3$
c No, y-intercept is fixed for any given line.

13 $y = \frac{b}{a}x$

14 a $y = 2x$ **b** $y = -2x + 4$ **c** $y = 2x + 5$
d $y = -\frac{3}{2}x + 2$ **e** $y = \frac{5}{7}x + \frac{1}{7}$ **f** $y = -\frac{13}{3}x - \frac{11}{3}$

Exercise 4H

1 a 4 **b** 8 **c** 1 **d** -3

2 a 5.5 **b** 2.5 **c** -1.5 **d** -2.5

3 a 2.24 **b** 8.60 **c** 10.20

4 a (3, 3) **b** (2, 2) **c** (1, 5) **d** (4, 1)
e $(-1, 3)$ **f** $(-1, -1)$ **g** (1.5, 1.5) **h** (2.5, 2)
i (0.5, 3) **j** $(-1.5, -2.5)$ **k** $(-3, -8.5)$ **l** $(0.5, -2.5)$

5 a 5.10 **b** 2.83 **c** 5.39
d 4.47 **e** 3.61 **f** 2.83
g 8.94 **h** 7.21 **i** 6.71

6 $B(8, 0), A(-6, 5), A(-6, 9)$

7 a $(-3, 1)$ **b** $(1, -4)$ **c** (8, 2.5)

8 a 12.8 **b** 24.2

9 (0, 0), (0, 4), (2, 0), (2, 4)

10 a $x = \frac{x_1 + x_2}{2}$ **b** $y = \frac{y_1 + y_2}{2}$ **c** $M(1, -0.5)$

11 a i $x_2 - x_1$ **ii** $y_2 - y_1$ **iii** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

12 a $\frac{1}{3}$ **b** $\frac{1}{3}$
c i (1, 1) **ii** $(-2, -0.5)$ **iii** (2, 1.5)
d i $(-1, 1)$ **ii** $(-2, 5)$
iii (0.4, -1.8) **iv** $(-2.4, 2.6)$

Exercise 4I

1 a Yes **b** Yes **c** No
d Yes **e** No **f** Yes

2 a $-\frac{1}{5}$ **b** $-\frac{1}{10}$ **c** $\frac{1}{3}$ **d** $\frac{1}{6}$

3 a Yes **b** No **c** No **d** Yes

4 a $y = 2x + 1$ **b** $y = 4x + 8$ **c** $y = -x + 5$
d $y = -2x - 7$ **e** $y = \frac{2}{3}x - 5$ **f** $y = -\frac{4}{5}x + \frac{1}{2}$

5 a $y = -\frac{1}{3}x + 3$ **b** $y = -\frac{1}{5}x + 7$ **c** $y = \frac{1}{2}x - 4$
d $y = x + 4$ **e** $y = \frac{1}{7}x - \frac{1}{2}$ **f** $y = -x + \frac{5}{4}$

6 a i $y = 1$ **ii** $y = -3$ **iii** $y = 6$ **iv** $y = -2$
b i $x = 3$ **ii** $x = -4$ **iii** $x = 1$ **iv** $x = -3$
c i $x = 2$ **ii** $x = -1$ **iii** $x = 0$ **iv** $x = 3$
d i $y = 7$ **ii** $y = -\frac{1}{2}$ **iii** $y = 3$ **iv** $y = \frac{1}{2}$

7 a $y = -3x + 9$ **b** $y = \frac{1}{2}x + \frac{5}{2}$
c $y = -\frac{1}{5}x + 6\frac{1}{5}$ **d** $y = x + 5$

8 $y = 0, y = x + 3, y = -x + 3$

9 $y = 2x - 10$ or $y = 2x + 10$

10 a i $-\frac{3}{2}$ **ii** -5 **iii** 7 **iv** $\frac{11}{3}$
b $-\frac{b}{a}$

11 a i $-\frac{1}{2}$ **ii** 3
b i 1 **ii** $-\frac{1}{7}$

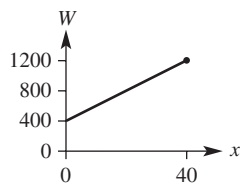
12 a $y = -\frac{1}{2}x + 4$ **b** $y = \frac{2}{3}x + 1\frac{2}{3}$ **c** $y = 2x - 3$
d $y = -2x - 5\frac{1}{2}$ **e** $y = \frac{3}{7}x - \frac{5}{7}$ **f** $y = -\frac{5}{6}x + \frac{1}{6}$

Exercise 4J

1 B

2 a i \$1200 **ii** \$1500 **iii** \$2200
b $A = 1000 + 100n$

3 a 46 **b** 3.5 **c** 2 **d** 3

4 a $W = 20x + 400$
b


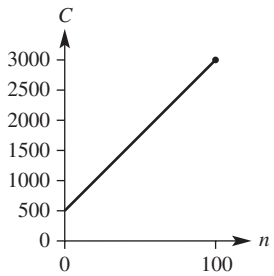
c \$640 **d** 30

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 **a** $C = 50n + 40$ **b** \$240 **c** \$640

6 **a** $C = 25n + 500$

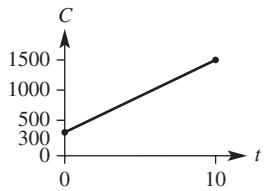
b



c \$1500 **d** 70

7 **a** $C = 120t + 300$

b



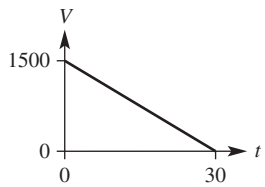
c \$1020 **d** 3 hours

8 **a** $F = 18t + 12$ **b** 3 minutes **c** 2.5 minutes

9 **a** $V = 4000 - 20t$ **b** 2200 L **c** 200 minutes

d 2 hours 55 minutes or 175 minutes

10 **a**



b $V = 1500 - 50t$

c The number of litres drained per minute

d 1250 L **e** 15 minutes

11 **a** 80 km/h

b Rate changes, i.e. new gradient $= 70$, $d = 50 + 70t$

12 **a** 20 m/s **b** -20 m/s **c** 350 m **d** 17.5 s

e Increasing altitude at rate of 20 m/s

13 **a** **i** $C = 0.4n + 20$ **ii** 160

b **i** $P = 0.8n - 20$ **ii** 25 **iii** 120

14 **a** $C = 10x + 6700$ **b** \$8700 **c** 630

d \$11 700 **e** 670 **f** $P = 10x - 6700$

g $T = \dfrac{10x - 6700}{x}$ **h** 1340

Exercise 4K

1 **a** (1, 3) **b** (−1, 2) **c** (2, 2)

2 **a** True **b** False **c** False **d** True

3 **a** Yes **b** Yes **c** Yes

4 **a** Yes **b** Yes **c** No **d** No

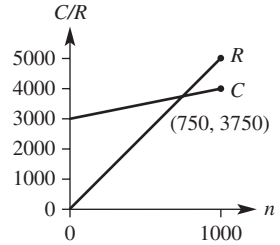
e Yes **f** Yes **g** No **h** No

5 **a** (2, 2) **b** (3, 2) **c** (2, −4) **d** (3, 2)

e (2, 1) **f** (3, 7) **g** (1, 2) **h** (2, 4)

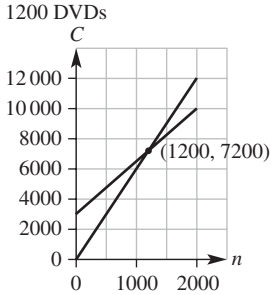
i (4, 1) **j** (−1, 3) **k** (−1, −5) **l** (1, 3)

6 **a**



b 750

7 1200 DVDs



8 10 seconds

9 Parallel lines, i.e. same gradient

10 **a** (0, 0) **b** (0, 0)

c No intersection **d** No intersection

e (0, 1) **f** (0, 3)

11 **a** 2 **b** −3

12 **a** (1, 3) **b** $A = 6.75$ units^2

13 **a** 18.75 units^2 **b** 24 units^2

c 16 units^2 **d** 27 units^2

e 7 units^2

Puzzles and challenges

1 101 **2** 2.5 hours

3 (0.5, 5.5), diagonals intersect at their midpoint

4 602 **5** length AB = length AC

6 $A = 13.5$ square units, $P = 20$ units

7 (6, 7) **8** 20 days **9** 31 hours

Multiple-choice questions

1 C **2** D **3** C **4** B **5** A

6 E **7** B **8** A **9** D **10** D

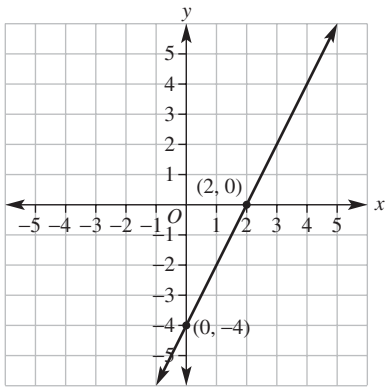
Short-answer questions

1 **a** −2 and 4

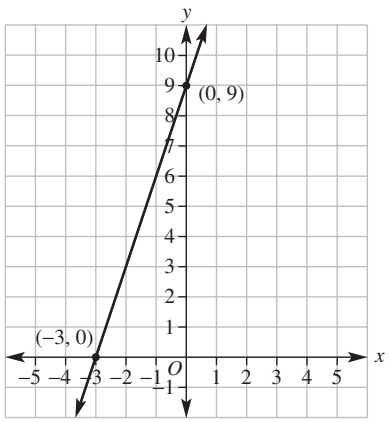
b 3 and −2

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

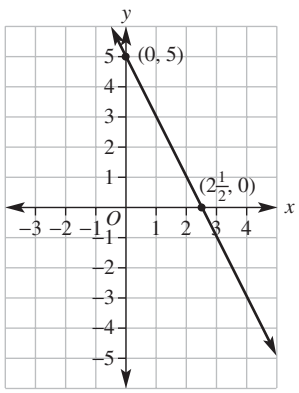
2 a



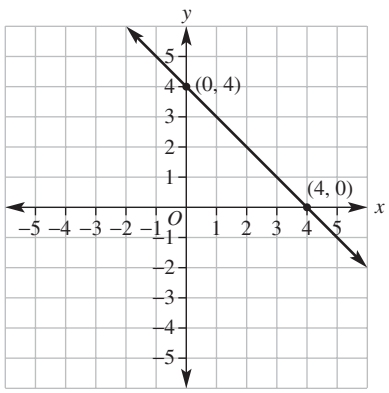
b



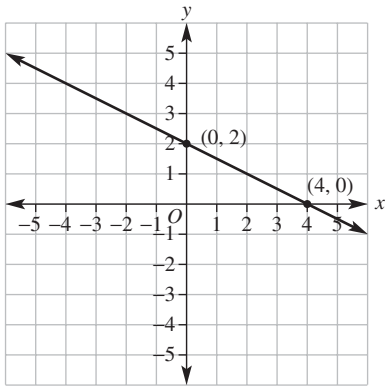
c



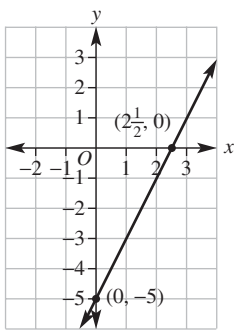
d



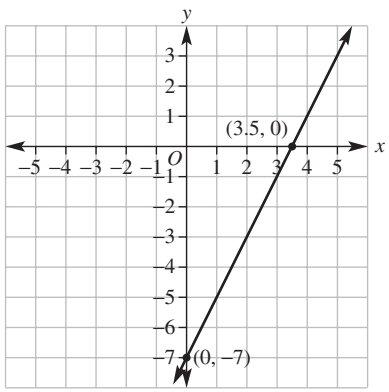
e



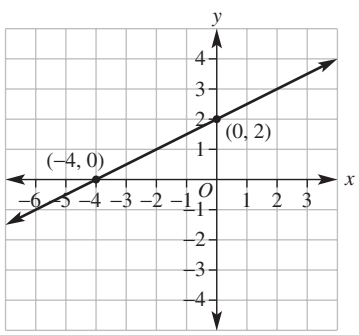
f



g



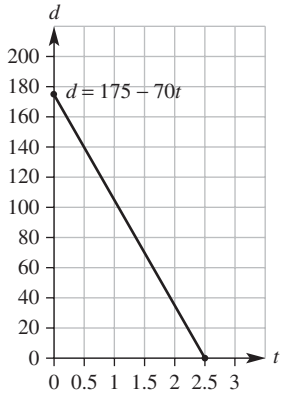
h



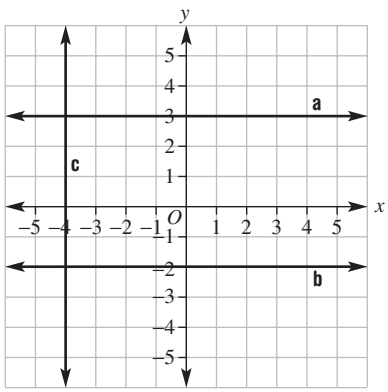
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 **a** 175 km **b** 2.5 hours

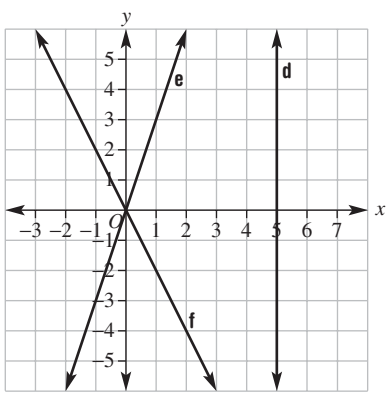
c



4 **a** $y = 3$ **b** $y = -2$ **c** $x = -4$



d $x = 5$ **e** $y = 3x$ **f** $y = -2x$



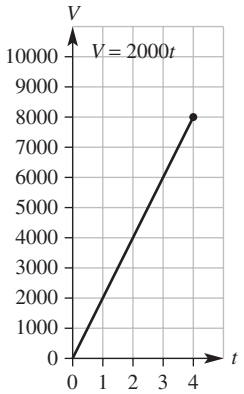
5 **a** 2 **b** -1

c $-\frac{5}{2}$ **d** $\frac{4}{3}$

e 2 **f** $-\frac{10}{3}$

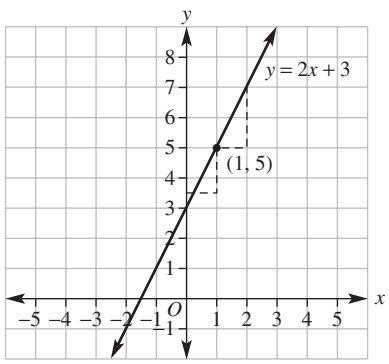
6 **a** 2000 L/h

b

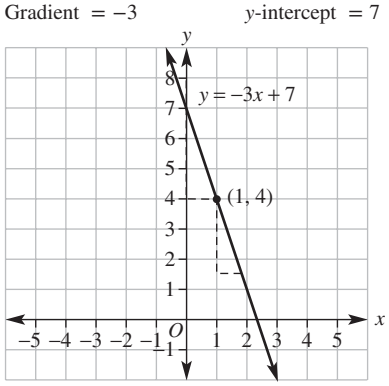


c $V = 2000t$ **d** 2.5 hours

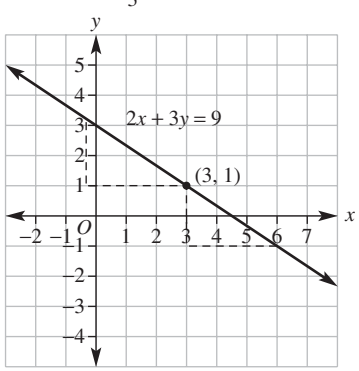
7 **a** Gradient $= 2$ y-intercept $= 3$



b Gradient $= -3$ y-intercept $= 7$

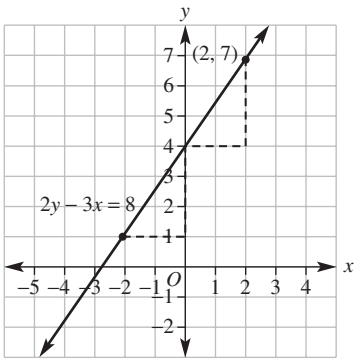


c Gradient $= -\frac{2}{3}$ y-intercept $= 3$



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

d Gradient $\frac{3}{2}$ y-intercept $= 4$



8 **a** $y = 3x + 2$ **b** $y = -2x + 6$ **c** $y = \frac{4}{3}x - 5$

9 **a** $y = \frac{x}{2} - 1$ **b** $y = -3x + 6$

c $y = 3x - 2$ **d** $y = -2x + 2$

10 **a** $y = 2x + 4$ **b** $y = -x - 3$ **c** $y = -\frac{1}{2}x - 1$

d $y = 3x + 4$ **e** $y = 3x + 1$

f $3x + 2y = 8$ or $y = -\frac{3x}{2} + 4$

11 **a** **i** $M(4, 6)$ **ii** 5.66

b **i** $M(7.5, 4.5)$ **ii** 7.07

c **i** $M(0, 4)$ **ii** 7.21

d **i** $M(-3, 2.5)$ **ii** 9.85

12 **a** $n = 9$ **b** $n = 10$ **c** $n = 6$

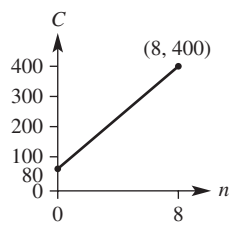
13 **a** No **b** Yes

14 **a** (2, 0) **b** (−2, 4)

Extended-response questions

1 **a** **i** \$40/h, \$80

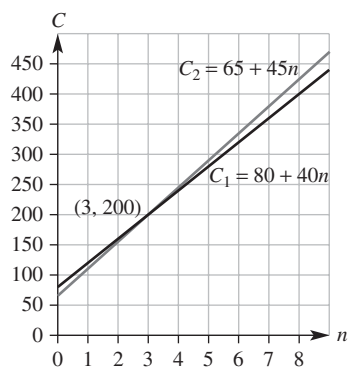
ii



iii \$180 **iv** 5 hours

b **i** $C = 65 + 45n$

ii



c (3, 200) **d** 3 hours

2 **a** $C = 8v + 90$

b **i** \$90 **ii** \$8 per vase

c 23 vases

Chapter 5

Pre-test

1 **a** 23 **b** 48 **c** 2.7

d 5.2134 **e** 50 **f** 72.16

2 **a** **i** $A = 4\,\text{cm}^2$ **ii** $P = 8\,\text{cm}$

b **i** $A = 6\,\text{m}^2$ **ii** $P = 10\,\text{m}$

c **i** $A = 8\,\text{cm}^2$ **ii** $P = 18\,\text{cm}$

3 **a** Cylinder **b** Circle

4 **a** 7 **b** 4.5 **c** 12

5 **a** 3 **b** 6 **c** 27

6 **a** 30 mm **b** 2000 cm **c** 1600 m **d** 2.3 cm

e 3.167 km **f** 0.72 m **g** 20 m **h** 3000 cm

7 **a** $30\,\text{cm}^2$ **b** $4\,\text{m}^2$ **c** $49\,\text{km}^2$

d $3\,\text{m}^2$ **e** $24\,\text{cm}^2$ **f** $66\,\text{m}^2$

8 $C = 31.42$ m, $A = 78.54\,\text{m}^2$

Exercise 5A

1 **a** 50 mm **b** 280 cm **c** 52.1 cm

d 0.837 m **e** 4600 m **f** 2.17 km

2 825 cm, 2.25 cm

3 **a** $a = 3, b = 6$ **b** $a = 12, b = 4$ **c** $a = 6.2, b = 2$

4 **a** 12 m **b** 27 cm **c** 24 mm

d 18 km **e** 10 m **f** 36 cm

5 **a** 90 cm **b** 80 cm **c** 170 cm

d 30.57 m **e** 25.5 cm **f** 15.4 km

6 **a** 9 cm **b** 4015 m **c** 102.1 cm

7 **a** 8000 mm **b** 110 m **c** 1 cm

d 20 mm **e** 0.284 km **f** 62.743 km

8 **a** $x = 4$ **b** $x = 2.2$ **c** $x = 14$

d $x = 9.5$ **e** $x = 6$ **f** $x = 4.2$

9 108 m

10 **a** 86 cm **b** 13.6 m **c** 40.4 cm

11 **a** $x = 2$ **b** $x = 2.1$ **c** $x = 7$

12 88 cm

13 **a** $P = 2a + 2b$ **b** $P = 4x$ **c** $P = 2a + b$

d $P = 2x + 2y$ **e** $P = 4(a + b)$ **f** $P = 2x$

14 All vertical sides add to 13 cm and all horizontal sides add to 10 cm.

15 **a** 25 cm, 75 cm **b** 40 cm, 60 cm

c 62.5 cm, 37.5 cm **d** 10 cm, 20 cm, 30 cm, 40 cm

16 **a** **i** 96 cm **ii** 104 cm **iii** 120 cm

b $P = 4(20 + 2x)$ $\therefore P = 8x + 80$

c **i** 109.6 cm **ii** 136.4 cm

d **i** $x = 1.25$ **ii** $x = 2.75$

e No, as with no frame the picture has a perimeter of 80 cm.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 5B

1 **a** 2.8 cm **b** 96 mm

2 **a** 6π **b** 12π **c** $\frac{35\pi}{2}$ **d** $3 + 2\pi$
e $12 + 3\pi$ **f** $10 + 4\pi$ **g** $8 + 2\pi$ **h** $3 + \frac{3\pi}{4}$
i $7 + \frac{\pi}{12}$

3 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
d $\frac{1}{6}$ **e** $\frac{5}{12}$ **f** $\frac{5}{8}$

4 **a** 50.27 m **b** 87.96 cm **c** 9.42 mm **d** 12.57 km

5 **a** 9.14 cm **b** 14.94 m **c** 33.13 cm
d 10.00 cm **e** 20.05 m **f** 106.73 km

6 **a** 12.56 m **b** 62.8 cm **c** 22 mm **d** 44 m

7 **a** 10π **b** 20π **c** 11π
d 15π **e** 3π **f** 41π

8 **a** $8 + 2\pi$ **b** $4 + 2\pi$ **c** $10\pi + 20$
d $12 + 2\pi$ **e** $5\pi + 6$ **f** $5\pi + 8$

9 28.27 m **10** 4.1 m **11** 31.42 cm

12 **a** 188.50 cm **b** **i** 376.99 cm **ii** 1979.20 cm
c 531

13 **a** $\frac{23\pi}{2}$m **b** $2.4 + 0.6\pi$ **c** $21 + \frac{7\pi}{2}$
d $5 + \frac{15\pi}{8}$ **e** $40 + 20\pi$ **f** $23 + \frac{23\pi}{4}$

14 **a** $r = \frac{C}{2\pi}$ **b** **i** 1.6 cm **ii** 4.0 m
c $d = \frac{C}{\pi}$ **d** 67 cm

15 **a** 131.95 m **b** 791.68 m
c **i** 3.79 **ii** 15.16 **d** 63.66 m

Exercise 5C

1 **a** 6 **b** 16 **c** 12 **d** 1 **e** 12 **f** 153

2 **a** Rectangle **b** Circle **c** Rhombus/kite
d Sector of circle **e** Triangle **f** Trapezium
g Parallelogram **h** Square **i** Semicircle

3 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$ **d** $\frac{1}{6}$ **e** $\frac{5}{8}$ **f** $\frac{5}{18}$

4 **a** 200 mm^2 **b** 5 cm^2 **c** 21 000 cm^2
d 21 m^2 **e** 1000 m^2 **f** 3.2 km^2

5 **a** 24 m^2 **b** 10.5 cm^2 **c** 20 km^2
d 25.2 m^2 **e** 15 m^2 **f** 36.8 m^2

6 **a** 21 mm^2 **b** 12 cm^2 **c** 17 cm^2
d 63 m^2 **e** 6.205 m^2 **f** 15.19 km^2

7 **a** 12.25 cm^2 **b** 3.04 m^2 **c** 0.09 cm^2
d 6.5 mm^2 **e** 18 cm^2 **f** 2.4613 cm^2

8 **a** 21.23 m^2 **b** 216.51 km^2 **c** 196.07 cm^2

9 **a** 7.07 m^2 **b** 157.08 cm^2 **c** 19.24 cm^2
d 84.82 m^2 **e** 26.53 m^2 **f** 62.86 m^2

10 **a** 1.5×10^{10} (15 000 000 000) cm^2 **b** 5 mm^2 **c** 0.075 m^2

11 500 000 m^2 **12** 0.175 km^2 **13** 0.51 m^2

14 12.89% **15** 31%

16 **a** $r = \sqrt{\frac{A}{\pi}}$ **b** **i** 1.3 cm **ii** 1.5 m **iii** 2.5 km

17 **a** **i** 64° **ii** 318°
b As angle would be greater than 360°, which is not possible (28.3 m^2 is the largest area possible, i.e. full circle)

18 **a** **i** 1.5 m **ii** 1.5 m
b 78 m^2 **c** Yes

19 46.7%

Exercise 5D

1 **a** Semicircle and rectangle **b** Triangle and semicircle
c Rhombus and parallelogram

2 **a** $P = 2 \times 5 + 3 + \frac{1}{2} \times 2\pi r$
$= 10 + 3 + 1.5\pi$
$= 13 + 1.5\pi$
$= 17.7$ m

$A = bh + \frac{1}{2}\pi r^2$
$= 5 \times 2 + \frac{1}{2} \times \pi \times 1.5^2$
$= 10 + 1.125\pi$
$= 13.5$ m^2

b $P = 20 + 12 + 12 + 10 + 6$
$= 60$ cm

$A = lb - \frac{1}{2}bh$
$= 12 \times 20 - \frac{1}{2} \times 8 \times 6$
$= 240 - 24$
$= 216$ cm^2

3 **a** 46 m, 97 m^2 **b** 34 m, 76 m^2
c 40 m, 90 m^2 **d** 18.28 m, 22.28 m^2
e 19.42 m, 26.14 m^2 **f** 85.42 mm, 326.37 mm^2

4 **a** 17 cm^2 **b** 3.5 cm^2 **c** 21.74 cm^2
d 6.75 m^2 **e** 189 cm^2 **f** 115 cm^2

5 **a** 108 m^2 **b** 33 cm^2 **c** 98 m^2
d 300 m^2 **e** 16 cm^2 **f** 22.5 m^2

6 **a** 37.70 m, 92.55 m^2 **b** 20.57 mm, 16 mm^2
c 18.00 cm, 11.61 cm^2 **d** 12.57 m, 6.28 m^2
e 25.71 cm, 23.14 cm^2 **f** 33.56 m, 83.90 m^2

7 **a** 90 cm^2 **b** 15 m^2 **c** 9 m^2
d 7.51 cm^2 **e** 7.95 m^2 **f** 180.03 cm^2
g 8.74 mm^2 **h** 21.99 cm^2 **i** 23.83 mm^2

8 189.27 m^2 **9** 68.67 cm^2

10 **a** 136.3 cm^2 **b** 42.4 m^2 **c** 345.6 m^2

11 8 cm

12 **a** $36 + 18\pi$ **b** 16 **c** $12 - \frac{\pi}{8}$
d 2π **e** $12.96 + 3.24\pi$ **f** $25 + \frac{75\pi}{4}$

13 7.1 cm

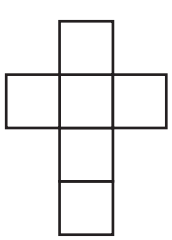
14 **a** Hypotenuse (diameter) would equal 4.24 not 5
b Hypotenuse (sloped edge) should be 13 cm not 14 cm
c Hypotenuse (diameter) should be 5.83 not 8 m

15 5267.1 cm^2

16 **a** 34 cm, 18 cm **b** 226.9 cm^2 **c** 385.1 cm^2

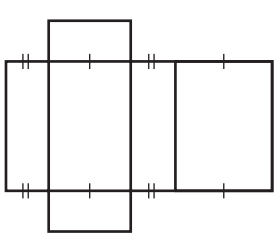
Exercise 5E

1 **a**



Cube

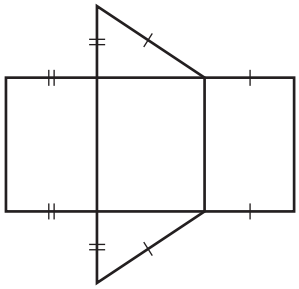
b



Rectangular prism

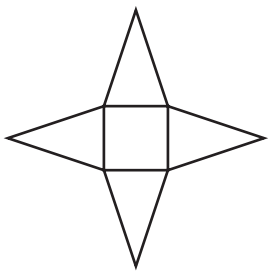
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

c



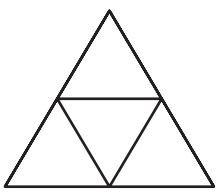
Triangular prism

d



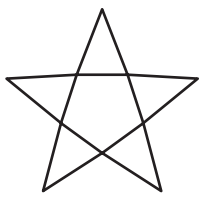
Square pyramid

e



Tetrahedron
(triangular pyramid)

f



Pentagonal pyramid

2 a $A = 2 \times 8 \times 7 + 2 \times 8 \times 3 + 2 \times 7 \times 3$
$= 112 + 48 + 42$
$= 202\,\text{m}^2$

b $A = 2 \times \frac{1}{2} \times 4 \times 3 + 5 \times 7 + 4 \times 7 + 3 \times 7$
$= 12 + 35 + 28 + 21$
$= 96\,\text{cm}^2$

3 a $52\,\text{m}^2$ **b** $242\,\text{cm}^2$ **c** $76\,\text{m}^2$
d $192\,\text{cm}^2$ **e** $68.16\,\text{m}^2$ **f** $85.76\,\text{m}^2$

4 a $39\,\text{mm}^2$ **b** $224\,\text{cm}^2$ **c** $9.01\,\text{m}^2$

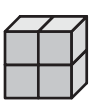
5 a $96\,\text{cm}^2$ **b** $199.8\,\text{cm}^2$ **c** $0.96\,\text{m}^2$
d $44.2\,\text{m}^2$ **e** $22\,\text{cm}^2$ **f** $28\,\text{cm}^2$

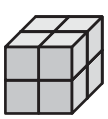
6 $6\,\text{m}^2$ **7** $14.54\,\text{m}^2$ **8** $34\,000\,\text{cm}^2$

9 a $44.4\,\text{m}^2$ **b** 4.44 L

10 a $1400\,\text{cm}^2$ **b** $1152\,\text{cm}^2$

11 a $10\,\text{cm}^2$ 

b $16\,\text{cm}^2$ 

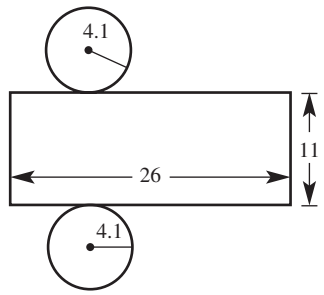
c $24\,\text{cm}^2$ 

12 a [6, 10, 14, 18, 22, 26, 30, 34, 38] **b** $A = 4n + 2$

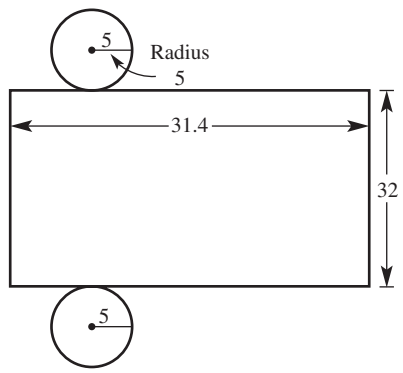
13 a $17.7\,\text{cm}^2$ **b** $96\,\text{m}^2$ **c** $204\,\text{cm}^2$
d $97.9\,\text{m}^2$ **e** $137.8\,\text{cm}^2$ **f** $43.3\,\text{mm}^2$

Exercise 5F

1 a



b



2 a 22 cm by 10 cm **b** 12.57 cm by 8 cm
c 50.27 m by 5 m

3 a $25.13\,\text{m}^2$ **b** $471.24\,\text{cm}^2$ **c** $50.27\,\text{m}^2$

4 a $44.0\,\text{cm}^2$ **b** $603.2\,\text{cm}^2$ **c** $113.1\,\text{m}^2$

5 $395.84\,\text{cm}^2$

6 a $251.33\,\text{cm}^2$ **b** $207.35\,\text{mm}^2$ **c** $24.13\,\text{m}^2$

7 a $54.56\,\text{m}^2$ **b** $218.23\,\text{m}^2$ **c** $63.98\,\text{cm}^2$
d $71.91\,\text{cm}^2$ **e** $270.80\,\text{m}^2$ **f** $313.65\,\text{km}^2$
g $326.41\,\text{m}^2$ **h** $593.92\,\text{m}^2$ **i** $43.71\,\text{mm}^2$

8 $7539.82\,\text{cm}^2$ **9** $80\,424.8\,\text{cm}^2$

10 a $18\,849.556\,\text{cm}^2$
b i $1.88\,\text{m}^2$ **ii** $37.70\,\text{m}^2$ **c** 239

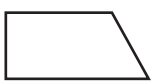
11 a $8\pi\,\text{m}^2$ **b** $150\pi\,\text{cm}^2$ **c** $16\pi\,\text{m}^2$

12 Half cylinder is more than half surface area as it includes new rectangular surface.

13 a $\left(\frac{135\pi}{2} + 36\right)\text{cm}^2$ **b** $\left(\frac{70\pi}{3} + 12\right)\text{cm}^2$ **c** $\left(\frac{29\pi}{12} + 4\right)\text{m}^2$

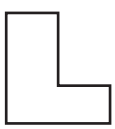
Exercise 5G

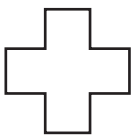
1 a  , 6 cm

b  , 10 cm

c  , 2 m

d  , 5 mm

e  , 14 cm

f  , 10 m

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

2 **a** Square **b** Triangle **c** Rectangle
d Trapezium **e** Hexagon **f** Triangle
3 **a** 3000 mm^3 **b** 2 cm^3 **c** 8 700 000 cm^3
d 0.0059 m^3 **e** 10 000 m^3 **f** 0.000 0217 km^3
g 3000 mL **h** 200 L **i** 3.5 L
j 21 mL **k** 37 kL **l** 42.9 ML
4 **a** 8 cm^3 **b** 84 m^3 **c** 21 mm^3
5 **a** 4 cm^3 **b** 10.5 m^3 **c** 11.96 cm^3
d 29 cm^3 **e** 14.88 m^3 **f** 8.1351 cm^3
g 108 m^3 **h** 29.82 m^3 **i** 0.382 044 cm^3
6 **a** 75 m^3 **b** 30 cm^3 **c** 1.25 cm^3
7 **a** 16 cm^3 **b** 42.875 m^3 **c** 15 cm^3
8 **a** 8 L **b** 0.36 L **c** 0.48 L
9 8000 cm^3 **10** 0.19 m^3
11 Yes, the tank only holds 20 L.
12 **a** 67.2 cm^3 **b** 28 m^3 **c** 8.9 km^3
d 28 m^3 **e** 0.4 m^3 **f** 29232 mm^3
13 **a** 15 L **b** 112 500 L **c** 8000 L
14 **a** 55 m^2 **b** 825 m^3
15 **a** **i** 1000 **ii** $\frac{1}{1000}$ **iii** 1000
b **i** 1 000 000 **ii** 1000
iii 1 000 000 or 1000^2
16 **a** $V = x^2h$ **b** $V = s^3$ **c** $V = 6t^3$
17 **a** $\frac{1}{3}$

Exercise 5H

1 **a** $r = 4$ m, $h = 10$ m **b** $r = 2.6$ cm, $h = 11.1$ cm
c $r = 2.9$ m, $h = 12.8$ m **d** $r = 9$ m, $h = 23$ m
e $r = 5.8$ cm, $h = 15.1$ cm **f** $r = 10.65$ cm, $h = 10.4$ cm
2 **a** 12.57 cm^2 **b** 8.04 m^2 **c** 78.54 cm^2 **d** 2.54 km^2
3 **a** 2 L **b** 4.3 mL **c** 3700 cm^3
d 1000 L **e** 38 m^3 **f** 200 mL
4 **a** 226.19 cm^3 **b** 18.85 m^3 **c** 137.44 m^3
d 100.53 cm^3 **e** 8.48 m^3 **f** 68.05 m^3
5 **a** 18 L **b** 503 L **c** 20 L
d 4712 L **e** 589 049 L **f** 754 L
6 **a** 25.133 m^3 **b** 25 133 L
7 37 699 L **8** Cylinder by 0.57 m^3
9 **a** 502.65 cm^3 **b** 1.02 m^3 **c** 294.52 m^3
d 35 342.92 m^3 **e** 47.12 cm^3 **f** 1017.88 cm^3
10 **a** 0.707 **b** 2.523
11 **a** 160π m^3 **b** 320π cm^3 **c** 54π km^3
d $\frac{3\pi}{4}$ cm^3 **e** 1500π cm^3 **f** 144π mm^3
12 A number of answers. Require $h = 2\pi r$.
13 **a** 113.10 cm^3 **b** 10471.98 m^3 **c** 3.73 m^3
d 20.60 cm^3 **e** 858.41 cm^3 **f** 341.29 m^3

Puzzles and challenges

1 100 L **2** Non-shaded is half the shaded area
3 163.4 m^2 **4** $\sqrt{200}$ cm = 14.14 cm **5** $\frac{1}{6}$ cm
6 16 days **7** $V = 2\pi^2r^3$ **8** $h = \frac{1 - r^2}{r}$
9 Answers can vary but $1000 = \pi r^2 h$ needs to hold true for r and h in centimetres. Designers need to consider production costs, material costs and keeping the surface area to a minimum so that they can maximise profits as well as the ability to use, stack and market their products. If the container is for cold storage then fitting into a standard fridge door is also a consideration.

Multiple-choice questions

1 B **2** C **3** E **4** B **5** A
6 D **7** B **8** E **9** C **10** E

Short-answer questions

1 **a** 380 cm **b** 1270 m **c** 2.73 cm^2
d 52 000 cm^2 **e** 10 000 cm^3 **f** 53.1 cm^3
g 3.1 L **h** 43 mL **i** 2830 L
2 **a** 14 m **b** 51 mm **c** 16.2 cm
3 **a** 4 cm^2 **b** 1122 mm^2 **c** 30.34 mm^2
d 7.5 m^2 **e** 15 cm^2 **f** 3 cm^2
4 **a** 2.5 m^2 **b** 37.4 m^2
5 **a** $A = 28.27$ cm^2, $P = 18.85$ cm
b $A = 5.38$ m^2, $P = 9.51$ m
c $A = 2.36$ cm^2, $P = 6.71$ cm
6 **a** $P = 15.24$ m, $A = 13.09$ m^2
b $P = 14.10$ m, $A = 10.39$ m^2
c $P = 24.76$ km, $A = 33.51$ km^2
7 **a** 8.86 m, 4.63 m^2 **b** 45.56 cm, 128.54 cm^2
8 **a** 46 cm^2 **b** 114 m^2
9 **a** 659.73 mm^2 **b** 30.21 m^2
10 **a** 30 cm^3 **b** 54 m^3 **c** 31.42 mm^3

Extended-response questions

1 **a** 517.08 cm **b** $65
c 15 853.98 cm^2 **d** 1.59 m^2, claim is correct
2 **a** 1 cm **b** 15.71 cm^2 **c** 125 680 cm^3 **d** 0.125 68 m^3
e 18.85 cm **f** 15 m^2 **g** $1200

Semester review 1

Computation and financial mathematics
Multiple-choice questions

1 B **2** E **3** D **4** D **5** A

Short-answer questions

1 **a** $\frac{19}{28}$ **b** $\frac{7}{9}$ **c** $\frac{3}{8}$ **d** $1\frac{4}{5}$
2 **a** 60% **b** 31.25% **c** 10% **d** 25%
3 **a** 5:3 **b** 55 km/h **c** 2.4 mL/h
4 $892

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Extended-response question

a i \$17 500 **ii** \$23 520 **iii** 9 years **iv** 27%
b \$23635.69
c i Jim by \$116 **ii** Jim by \$922

Expressions, equations and inequalities
Multiple-choice questions

1 E **2** A **3** C **4** B **5** D

Short-answer questions

1 a $x = 6$ **b** $x > \frac{9}{2}$ **c** $m = \frac{3}{8}$
d $y = -1$ **e** $a \leq \frac{1}{11}$ **f** $x = -\frac{3}{14}$
2 a $\frac{m-3}{2} = 6$ **b** Noah gets \$15 pocket money.
3 a 155 **b** $l = \frac{2S}{n} - a$ **c** 18
4 a $x = 6, y = 3$ **b** $x = -1, y = -5$
c $x = 5, y = -2$ **d** $x = -3, y = 4$

Extended-response question

a i $12x + 20 > 74$ **ii** 5 games
b i Let \$$x$ be the cost of a raffle ticket and \$$y$ the cost of a badge.
ii $5x + 2y = 11.5$ and $4x + 3y = 12$
iii A raffle ticket costs \$1.50 and a badge costs \$2.

Right-angled triangles
Multiple-choice questions

1 D **2** A **3** C **4** A **5** E

Short-answer questions

1 a $x = 15.1$ **b** $x = 5.7$ **c** $x = 11.2$ **d** $\theta = 29.5$
2 a $x = 13, y = 14.7$ **b** $x = 9.9$
3 a 19.21 m **b** 38.7°
4 a 16.3 km west **b** 115°

Extended-response question

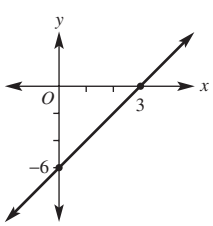
a 17.75 m **b** 14.3° **c i** 18.8 m **ii** 6.8 s

Linear relationships
Multiple-choice questions

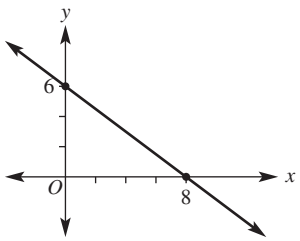
1 C **2** D **3** B **4** C **5** A

Short-answer questions

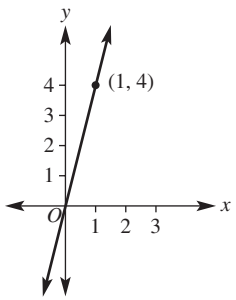
1 a



b



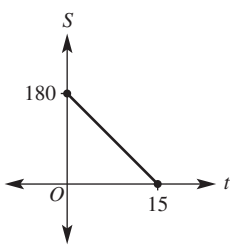
c



2 a $\frac{2}{3}$ **b** -3 **c** -2 **d** $\frac{4}{3}$
3 a $y = -3x + 6$ **b** $y = 3x - 1$
c $y = 2x$ **d** $y = -\frac{1}{3}x + 2$
4 (3, 2)

Extended-response question

a 12 kg/h **b** $S = 180 - 12t$
c



d 15 hours **e i** $P = 40 + 25h$ **ii** \$415

Length, area, surface area and volume
Multiple-choice questions

1 C **2** E **3** D **4** A **5** A

Short-answer questions

1 a 24.57 m^2 **b** 36 cm^2
2 4 tins
3 a 216 m^2 **b** 25.45 m^2
4 a $x = 4$ **b** $y = 8.5$

Extended-response question

a 45.71 m^2 **b i** 0.0377 m^2 **ii** 3.77 m^2
c 1213 **d** 37.85 m^3 **e** 19.45 m^3

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 6

Pre-test

1 **a** 25 **b** 100 **c** 16
d 27 **e** 9

2 **a** 24, 1, 12, 2, 8, 3, 6, 4 **b** 45, 1, 15, 3, 9, 5
c 2, 3 **d** 3, 5

3 **a** ab^2 **b** 5^3ab^2 **c** 3^2x^2 **d** $6ac^3$

4 **a** 2^3 **b** 2^6 **c** 2^5

5 **a** 3^4 **b** x^2y^2 **c** 2^2a^2 **d** $\frac{3^2}{4^2}$

6 **a** $\frac{1}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{216}$ **d** $\frac{4}{9}$

7 **a** 3.73 **b** 24.62 **c** 18.37 **d** 4.40

8 **a** 5 **b** 4 **c** 2 **d** 3

9 **a** 38 **b** 2310 **c** 0.172
d 0.0018 **e** 1000 **f** 10000

10 **a** 15 **b** 4 **c** 125 **d** 32

11 **a** $10a - 4b$ **b** $3a + 3$ **c** $ab + 8a$ **d** $2ab^2 - a^2b$

Exercise 6A

1 **a** 25 **b** 8 **c** 27 **d** 16
e 1000 **f** 16 **g** -8 **h** 16

2 **a** 3125 **b** -3125 **c** 15625 **d** 15625
e $\frac{1}{1024}$ **f** $\frac{32}{243}$ **g** $\frac{27}{64}$ **h** $-\frac{27}{64}$

3 **a** 3 **b** 8 **c** 7 **d** 4
e 11 **f** 13 **g** 9 **h** 2

4 **a** 2, 3 **b** 3, 5 **c** 2, 3, 5 **d** 7, 11

5 **a** $a \times a \times a \times a$ **b** $b \times b \times b$ **c** $x \times x \times x$
d $x \times p \times x \times p \times x \times p \times x \times p \times x \times p \times x \times p$
e $5 \times a \times 5 \times a \times 5 \times a \times 5 \times a$
f $3 \times y \times 3 \times y \times 3 \times y$
g $4 \times x \times x \times y \times y \times y \times y \times y$
h $p \times q \times p \times q$
i $-3 \times s \times s \times s \times t \times t$
j $6 \times x \times x \times x \times y \times y \times y \times y \times y$
k $5 \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z \times y$
l $4 \times a \times b \times a \times b \times a \times b$

6 **a** 36 **b** 16 **c** 243 **d** 12
e -8 **f** -1 **g** 81 **h** 25
i $\frac{8}{27}$ **j** $\frac{9}{16}$ **k** $\frac{1}{216}$ **l** $\frac{25}{4}$
m $-\frac{8}{27}$ **n** $\frac{81}{256}$ **o** $\frac{1}{16}$ **p** $-\frac{3125}{32}$

7 **a** 3^3 **b** 8^6 **c** y^2 **d** $3x^3$
e $4c^5$ **f** 5^3d^2 **g** x^2y^3 **h** 7^3b^2

8 **a** $\left(\frac{2}{3}\right)^4$ **b** $\left(\frac{3}{5}\right)^5$ **c** $\left(\frac{4}{7}\right)^2\left(\frac{1}{5}\right)^4$ **d** $\left(\frac{7x}{9}\right)^2\left(\frac{y}{4}\right)^3$

9 **a** $3^3x^3y^2$ **b** $(3x)^2(2y)^2$ or $3^2 2^2 x^2 y^2$
c $(4d)^2(2e)^2$ or $4^2 2^2 d^2 e^2$ **d** $6^3b^2y^3$
e $(3pq)^4$ or $3^4p^4q^4$ **f** $(7mn)^3$ or $7^3m^3n^3$

10 **a** 2×5 **b** 2^3 **c** $2^4 \times 3^2$ **d** 2^9
e $2^3 \times 3^3$ **f** $2^2 \times 5^3$

11 **a** 36 **b** -216 **c** 1 **d** $-\frac{8}{27}$
e -18 **f** 15 **g** -36 **h** 216

12 **a** 4 **b** 8 **c** 5 **d** 2
e -4 **f** -2 **g** $\frac{1}{2}$ **h** 4

13 **a** **i** 10 min **ii** 20 min **iii** 30 min
b $2^{24} = 16\,777\,216$ cells

14 **a** $1000 \times 3^5 = \$243\,000$ **b** 5 years

15 7 months

16 **a** **i** 9 **ii** 9 **iii** -9 **iv** -9
b Same signs give positive when multiplying
c A positive answer is multiplied by the negative one out the front

17 **a** **i** 8 **ii** -8 **iii** -8 **iv** 8
b **i** a positive cubed is positive, **ii** a negative answer is multiplied by negative
c **i** a negative number cubed will be negative, **ii** a positive answer is multiplied by negative one

18 **a** $\frac{1}{8}$ **b** $\frac{1}{16}$ **c** $\frac{1}{125}$
d $\frac{1}{64}$ **e** $\frac{49}{100}$ **f** $\frac{81}{16}$
g $\frac{169}{25}$ **h** $\frac{12769}{100}$ **i** $\frac{289}{25}$

19 **a** LCM = 12, HCF = 2 **b** LCM = 84, HCF = 14
c LCM = 72, HCF = 12 **d** LCM = 30, HCF = 5
e LCM = 360, HCF = 10 **f** LCM = 300, HCF = 10
g LCM = 1764, HCF = 14 **h** LCM = 13068, HCF = 198

Exercise 6B

1 **a** multiply, base, add **b** divide, base, subtract

2 **a** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
b $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^7$
c $\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5^2$ **d** $\frac{9 \times 9 \times 9 \times 9}{9 \times 9} = 9^2$
e $x + x + x = x^3$ **f** $\frac{x + x + x + x + x}{x + x} = x^3$

3 **a** True **b** True **c** False **d** False
e True **f** False **g** False **h** True

4 **a** 2^7 **b** 5^9 **c** 7^6 **d** 8^{10}
e 3^8 **f** 6^{14} **g** 3^3 **h** 6^5
i 5^3 **j** 10 **k** 9^3 **l** $(-2)^2$

5 **a** x^7 **b** a^9 **c** t^8 **d** y^5
e d^3 **f** y^7 **g** b^8 **h** q^{11}
i x^7y^5 **j** x^9y^4 **k** $5x^4y^9$ **l** $4x^2y^5z$
m $15m^5$ **n** $8e^6f^4$ **o** $20c^7d^2$ **p** $18y^2z^7$
q $18m^{13}n$ **r** $18x^8$ **s** $-27x^7$ **t** $12m^4n^9$

6 **a** a^2 **b** x^3 **c** q^{10} **d** d
e $2b^5$ **f** $\frac{d^5}{3}$ **g** $2a^7$ **h** $2y^8$
i $9m$ **j** $14x^3$ **k** $5y^2$ **l** $6a$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

m $\frac{m^5}{4}$ **n** $\frac{w}{5}$ **o** $\frac{a}{5}$ **p** $\frac{x^4}{9}$

q $\frac{4x^6y^3}{3}$ **r** $\frac{3st^2}{7}$ **s** $\frac{4mn}{3}$ **t** $-5x$

7 a b^6 **b** y^6 **c** c^7 **d** x^{11}

e t **f** p^6 **g** d^6 **h** x^{10}

i $4x^2y^3$ **j** $6b^2g$ **k** $3m^5n^6$ **l** p^5q^4

8 a $\frac{m^5}{n^5}$ **b** $\frac{x^4}{y^2}$ **c** a^3b^3

d $\frac{6a^5}{c^7}$ **e** $6f^6$ **f** $12x^4b^2$

g $6k^3m^3$ **h** $\frac{15x^4y}{2}$ **i** $-\frac{3m^2n^3}{2}$

9 a 12 **b** 8 **c** 3 **d** 3

e 1 **f** 18 **g** 12 **h** 11

i 4 **j** 15 **k** 2 **l** 39

10 a $7^2 = 49$ **b** 10 **c** $13^2 = 169$

d $2^3 = 8$ **e** 101 **f** $200^2 = 40000$

g $7 \times 31 = 217$ **h** $3 \times 50^2 = 7500$

11 a 7 ways **b** 14 ways

12 a a^5, power of one not added

b x^6, power of one not subtracted

c $\frac{a^2}{2}$, $3 \div 6$ is $\frac{1}{2}$ not 2

d $\frac{x^4}{2}$, numerator power is larger, hence x^4 in numerator

e $6x^{11}$, multiply coefficients not add

f $a^3 \times a = a^4$, order of operations done incorrectly

13 a $4x$ **b** $12x^2$ **c** $10x^3$ **d** $-4x$

e $40x^6$ **f** $\frac{5x}{4}$ **g** $\frac{8}{5}$ **h** $-20x^4$

14 a 2^{x+y} **b** 5^{a-b} **c** t^{x+y} **d** 3^{x-y}

e 10^{p-y} **f** t^{x-y} **g** 2^{p+q-r} **h** 10^{p-q-r}

i 2^{5a} **j** $a^{3x-2}b^{x+3}$ **k** $a^{x+y}b^{x+y}$ **l** $a^{x-y}b^{y-x}$

m $w^{2-x}b^{x+3}$ **n** a^{x+y-2} **o** p^aq **p** $4m^{y-3-2x}$

Exercise 6C

1 a 1 **b** 1 **c** 0^0 is undefined

2 a 16, 8, 4, 2, 1 **b** 64, 16, 4, 1

3 a $(4 \times 4) \times (4 \times 4) \times (4 \times 4) = 4^6$

b $(12 \times 12 \times 12) \times (12 \times 12 \times 12) \times (12 \times 12 \times 12) = 12^9$

c $(x \times x \times x \times x \times x) \times (x \times x \times x \times x \times x) = x^8$

d $(a \times a) \times (a \times a) \times (a \times a) \times (a \times a) \times (a \times a) = a^{10}$

4 a y^{12} **b** m^{18} **c** x^{10} **d** b^{12}

e 3^6 **f** 4^{15} **g** 3^{30} **h** 7^{10}

i $5m^{16}$ **j** $4q^{28}$ **k** $-3c^{10}$ **l** $2j^{24}$

5 a 1 **b** 1 **c** 1 **d** 1

e -1 **f** 1 **g** 1 **h** 1

i 5 **j** -3 **k** 4 **l** -6

m 1 **n** 3 **o** 1 **p** 0

6 a 4^7 **b** 3^9 **c** x **d** y^{13}

e b^{14} **f** a^{10} **g** d^{24} **h** y^{16}

i z^{25} **j** $a^{11}f^{13}$ **k** $x^{14}y^5$ **l** $5rs^8$

7 a 7^2 **b** 4 **c** 3^8

d 1 **e** y^3 **f** h^2

g b^6 **h** x^5 **i** y^6

8 a $\frac{2}{x^5}$ **b** $\frac{10}{x^3}$ **c** $3x^8$

d $\frac{d^2e}{2}$ **e** $\frac{2m^6n}{5}$ **f** $\frac{a^{12}}{8}$

9 a i 400 **ii** 6400 **iii** 100

b i 800 **ii** 12 800 **iii** 102 400

c 13 years

10 5 ways

11 a 4 **b** 1000 **c** 1

d 1 **e** 4 **f** 1

12 a 4×5 not $4 + 5$, a^{20}

b Power of 2 only applies to x^3, $3x^6$

c Power zero applies to whole bracket, 1

13 a i 2^{24} **ii** $(-2)^{30} = 2^{30}$

iii x^{84} **iv** a^{48}

b i 2^{abc} **ii** a^{mnp} **iii** x^{6yz}

15 a 2^{12} **b** 2^{15} **c** 3^6

d 3^{20} **e** 5^{10} **f** 3^{50}

g 2^{72} **h** 7^{80} **i** 10^{50}

Exercise 6D

1 a $a^m \times b^m$ **b** $\frac{a^m}{b^m}$

2 a $5a \times 5a \times 5a$

$= 5 \times 5 \times 5 \times a \times a \times a$

$= 5^3 \times a^3$

b $ab \times ab \times ab \times ab$

$= a \times a \times a \times a \times b \times b \times b \times b$

$= a^4 \times b^4$

c $\frac{x}{6} \times \frac{x}{6} \times \frac{x}{6}$

$= \frac{x \times x \times x}{6 \times 6 \times 6}$

$= \frac{x^3}{6^3}$

d $\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$

$= \frac{a \times a \times a \times a \times a}{b \times b \times b \times b \times b}$

$= \frac{a^5}{b^5}$

3 a $8x^3$ **b** $25y^2$ **c** $64a^6$

d $9r^2$ **e** $-81b^4$ **f** $-343r^3$

g $(-2)^4h^8 = 16h^8$ **h** $625c^8d^{12}$ **i** $32x^{15}y^{10}$

j $9p^6q^{12}$ **k** $2x^6y^2$ **l** 1

m $-27w^9y^3$ **n** $-4p^8q^2r^2$ **o** $25s^{14}t^2$

p $8x^{12}y^3z^9$

4 a $\frac{p^3}{q^3}$ **b** $\frac{x^4}{y^4}$ **c** $\frac{64}{y^3}$

d $\frac{625}{p^8}$ **e** $\frac{4}{r^6}$ **f** $\frac{s^6}{49}$

g $\frac{32m^5}{n^5}$ **h** $\frac{8a^6}{27}$ **i** $\frac{27n^9}{8m^{12}}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

j $\frac{16r^4}{n^4}$ **k** $\frac{9f^2}{64g^{10}}$ **l** $\frac{25w^8y^2}{4x^6}$

m $\frac{9x^2}{4y^6g^{10}}$ **n** $\frac{27k^3m^9}{64n^{21}}$ **o** $-\frac{25w^8y^2}{4z^2x^6}$

p $\frac{9x^4y^6}{4a^{10}b^8}$

5 a $9ab^2$ **b** $27ab^6$ **c** $-12a^8b^8$

d $54x^6y^9$ **e** $-64b^6c^{15}d^3$ **f** $8a^4$

g $9a^5$ **h** $-40a^{15}b^3$ **i** $160m^{15}p^5t^{10}$

j $-49d^4f^8g^2$ **k** $1024x^{12}y^3z^9$ **l** $-16a^8b^7$

6 a x^{24} **b** $256x^{24}$ **c** $a^{17}b^8$

d $a^{10}b^{11}$ **e** $8m^5n^3$ **f** $12c^8d^7$

g $\frac{-27x^6}{125a^{15}b^9}$ **h** $-3a^4b^5$ **i** $15n$

j $a^{11}bc^5$ **k** $x^{11}y^2z$ **l** $\frac{r^9t^{10}}{s}$

7 a i 8 **ii** 125

b $N = \frac{t^3}{8}$ **c i** 27 **ii** 8

d i 8 **ii** 2

8 a 2 **b** 4 **c** 2

d 2 **e** 1 **f** 14

9 a By simplifying, there are smaller numbers to raise to powers.

b i 8 **ii** 16 **iii** $\frac{1}{81}$ **iv** $\frac{1}{1000}$

10 a False, $(-2)^2 = +(2)^2$ **b** True, $(-3)^3 = -(3)^3$

c True, $(-5)^5 = -(5)^5$ **d** False, $(-4)^4 = +(4)^4$

11 a 25 **b** 13 **c** No

d No, $(3 - 2)^2 \neq 3^2 - 2^2$

e i True **ii** False **iii** True **iv** False

Exercise 6E

1 a $\frac{1}{2^2}$ **b** $\frac{1}{3^2}$ **c** $\frac{1}{5^3}$ **d** $\frac{1}{3^3}$

2 a

Index form	3^4	3^3	3^2	3^1
Whole number or fraction	81	27	9	3

Index form	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	1	$\frac{1}{3}$	$\frac{1}{9} = \frac{1}{3^2}$	$\frac{1}{27} = \frac{1}{3^3}$

b

Index form	10^4	10^3	10^2	10^1
Whole number or fraction	10000	1000	100	10

Index form	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	1	$\frac{1}{10}$	$\frac{1}{100} = \frac{1}{10^2}$	$\frac{1}{1000} = \frac{1}{10^3}$

3 a $\frac{1}{x}$ **b** $\frac{1}{a^4}$ **c** $\frac{1}{b^6}$ **d** $\frac{1}{25}$

e $\frac{1}{64}$ **f** $\frac{1}{9}$ **g** $\frac{5}{x^2}$ **h** $\frac{4}{y^3}$

i $\frac{3}{m^5}$ **j** $\frac{p^7}{q^2}$ **k** $\frac{m}{n^4}$ **l** $\frac{x^4}{y^4}$

m $\frac{2}{a^3b}$ **n** $\frac{7}{r^2s^3}$ **o** $\frac{v^2}{5u^8}$ **p** $\frac{1}{9m^3n^5}$

4 a y **b** b^2 **c** m^5 **d** x^4

e $7q$ **f** $3t^2$ **g** $5h^4$ **h** $4p^4$

i ab^2 **j** de **k** $2m^3n^2$ **l** $\frac{x^2y^5}{3}$

m $-\frac{3y^4}{7}$ **n** $-2b^8$ **o** $-\frac{3gh^3}{4}$ **p** $\frac{9u^2t^2}{5}$

5 a $\frac{b^3}{a^3}$ **b** $\frac{y^5}{x^2}$ **c** $\frac{h^3}{g^2}$ **d** $\frac{n}{m}$

e $\frac{343}{5}$ **f** $\frac{64}{9}$ **g** $\frac{6}{25}$ **h** 1

6 a $\frac{7x^4}{y^3}$ **b** $\frac{u^3}{v^2}$ **c** $\frac{y^3}{5a^3}$ **d** $\frac{2b^5}{a^4c^2}$

e $\frac{5a^2b^2}{6c^4d}$ **f** $\frac{4h^3m^2}{5k^2p}$ **g** $\frac{12w^6}{tu^2v^2}$ **h** $\frac{mn^4x^2}{16y^5}$

7 a $\frac{1}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{16}$ **d** $-\frac{1}{25}$

e $\frac{1}{25}$ **f** $-\frac{1}{200}$ **g** $-\frac{3}{4}$ **h** $\frac{1}{2}$

i $\frac{1}{36}$ **j** $\frac{1}{8}$ **k** $\frac{4}{25}$ **l** $\frac{7}{81}$

m 8 **n** 100 **o** -250 **p** 16

q -10 **r** 64 **s** $\frac{64}{9}$ **t** $-\frac{27}{64}$

u 100 **v** $\frac{1}{3}$ **w** -2 **x** 49

8 1.95 g

9 a -4 **b** -4 **c** -4

d -1 **e** -2 **f** -1

10 a Negative power only applies to x, $\frac{2}{x^2}$

b $5 = 5^1$ has a positive power, $5a^{-4}$

c $\frac{2}{3^{-2}b^{-2}} = 2 \times 3^2 \times b^2 = 18b^2$

11 a $1 \div \frac{2}{3} = 1 \times \frac{3}{2}$

b i $\frac{4}{5}$ **ii** $\frac{7}{2}$ **iii** $\frac{3}{x}$ **iv** $\frac{b}{a}$

c (fraction)$^{-1}$ = reciprocal of fraction

d i $\frac{9}{4}$ **ii** $\frac{25}{16}$ **iii** 32 **iv** $\frac{27}{343}$

12 a 4 **b** 4 **c** 2

d 3 **e** 2 **f** 4

g $\frac{3}{2}$ **h** 4 **i** $\frac{7}{3}$

Exercise 6F

1 a 10000 **b** 1000 **c** 100000

d 1000 **e** 100000 **f** 10000

2 a 10^5 **b** 10^2 **c** 10^9

3 a Positive **b** Negative **c** Positive **d** Negative

4 a 4×10^4 **b** 2.3×10^{12} **c** 1.6×10^{10}

d -7.2×10^6 **e** -3.5×10^3 **f** -8.8×10^6

g 5.2×10^3 **h** 3×10^6 **i** 2.1×10^4

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 **a** 3×10^{-6} **b** 4×10^{-4} **c** -8.76×10^{-3}
d 7.3×10^{-10} **e** -3×10^{-5} **f** 1.25×10^{-10}
g -8.09×10^{-9} **h** 2.4×10^{-8} **i** 3.45×10^{-5}

6 **a** 6×10^{3} **b** 7.2×10^{5} **c** 3.245×10^{2}
d 7.86903×10^{3} **e** 8.45912×10^{3} **f** 2×10^{-1}
g 3.28×10^{-4} **h** 9.87×10^{-3} **i** -1×10^{-5}
j -4.601×10^{8} **k** 1.7467×10^{4} **l** -1.28×10^{2}

7 **a** 57000 **b** 3600000 **c** 430000000
d 32100000 **e** 423000 **f** 90400000000
g 197000000 **h** 709 **i** 635700

8 **a** 0.00012 **b** 0.0000046 **c** 0.0000000008
d 0.0000352 **e** 0.3678 **f** 0.000000123
g 0.00009 **h** 0.05 **i** 0.4

9 **a** 6×10^{24} **b** 4×10^{7} **c** 1×10^{-10}
d 1.5×10^{8} **e** 6.67×10^{-11} **f** 1.5×10^{-4}
g 4.5×10^{9}

10 **a** 4600000000 **b** 8000000000000 **c** 384000
d 0.0038 **e** 0.00000000000001 **f** 720000

11 **a** 3.6×10^{7} **b** 3.6×10^{5} **c** 4.92×10^{-1}
d 3.8×10^{-4} **e** 2.1×10^{-6} **f** 5.2×10^{-8}
g 4×10^{-9} **h** 1.392×10^{-7} **i** 3.95×10^{3}
j 4.38×10^{3} **k** 4.3×10^{5} **l** 5×10^{2}
m 8.28×10^{6} **n** 3×10^{11}

12 **a** $\$1.84 \times 10^{9}$ **b** $\$2.647 \times 10^{9}$

13 1.62×10^{9} km **14** 2.126×10^{-2} g

15 **a** 3.2×10^{4} **b** 4.1×10^{6} **c** 3.17×10^{4}
d 5.714×10^{5} **e** 1.3×10^{4} **f** 9.2×10^{1}
g 3×10^{5} **h** 4.6×10^{5} **i** 6.1×10^{-2}
j 4.24 **k** 1.013×10^{-3} **l** 4.9×10^{4}
m 2×10^{-5} **n** 4×10^{-6} **o** 3.72×10^{-4}
p 4.001×10^{-8}

16 **a** 8×10^{6} **b** 9×10^{8} **c** 6.25×10^{-4}
d 3.375×10^{-9} **e** 1.25×10^{8} **f** 4×10^{6}
g 9×10^{-4} **h** 2.5×10^{4}

17 **a** 6×10^{6} **b** 8×10^{11} **c** 2×10^{4}
d 3×10^{9} **e** 5.6×10^{5} **f** 1.2×10^{8}
g 1.2×10^{3} **h** 9×10^{3} **i** 9×10^{-9}
j 7.5×10^{-8} **k** 1.5×10^{-5} **l** 1

18 $5 \times 10^{2} = 500$ seconds

19 **a** 3×10^{-4} km $= 30$ cm
b 1×10^{-3} seconds (one thousandth of a second)
$= 0.001$ seconds

Exercise 6G

1 **a** 57260, 57300, 57000, 60000
b 4170200, 4170000, 4170000, 4200000, 4000000
c 0.003661, 0.00366, 0.0037, 0.004
d 24.871, 24.87, 24.9, 25, 20

2 **a** Yes **b** No **c** No
d No **e** Yes **f** Yes
g Yes **h** No **i** No

3 **a** 3, 4 or 5 **b** 4 **c** 5 or 6
d 2 or 3 **e** 3 **f** 2
g 3 **h** 3 **i** 3
j 4 **k** 3 **l** 3

4 **a** 2.42×10^{5} **b** 1.71×10^{5} **c** 2.83×10^{3}
d 3.25×10^{6} **e** 3.43×10^{-4} **f** 6.86×10^{-3}
g 1.46×10^{-2} **h** 1.03×10^{-3} **i** 2.34×10^{1}
j 3.26×10^{2} **k** 1.96×10^{1} **l** 1.72×10^{-1}

5 **a** 4.78×10^{4} **b** 2.2×10^{4} **c** 4.833×10^{6}
d 3.7×10^{1} **e** 9.95×10^{1} **f** 1.443×10^{-2}
g 2×10^{-3} **h** 9×10^{-2} **i** 1×10^{-4}

6 **a** 2.441×10^{-4} **b** 2.107×10^{-6} **c** -4.824×10^{15}
d 4.550×10^{-5} **e** 1.917×10^{12} **f** 1.995×10^{8}
g 3.843×10^{2} **h** 1.710×10^{-11} **i** 1.524×10^{8}
j 3.325×10^{15} **k** 4.000×10^{3} **l** -9.077×10^{-1}

7 **a** 9.3574×10^{1} **b** 2.1893×10^{5} **c** 8.6000×10^{5}
d 8.6288×10^{-2} **e** 2.2985×10^{15} **f** 3.5741×10^{28}
g 6.4000×10^{7} **h** 1.2333×10^{9} **i** 1.8293
j 5.3691×10^{-1}

8 1.98×10^{30} kg **9** 1.39×10^{6} km **10** 1.09×10^{12} km^3

11 2421×10^{3}, 24.2×10^{5}, 2.41×10^{6}, 0.239×10^{7}, 0.02×10^{8}

12 **a** 4.26×10^{6} **b** 9.1×10^{-3} **c** 5.04×10^{11}
d 1.931×10^{-1} **e** 2.1×10^{6} **f** 6.14×10^{-11}

13 Should be 8.8×10^{10}

14 **a** **i** 2.30×10^{2} **ii** 4.90×10^{-2} **iii** 4.00×10^{6}
b It is zero.
c It clarifies the precision of the number.

15 **a** 5.40046×10^{12}
b **i** 4.32×10^{13} **ii** 1.61×10^{19} **iii** 4.01×10^{51}

Exercise 6H

1 **a** 2 **b** 2 **c** 3
d 3.46 **e** 10 **f** 2

2 **a** True **b** False **c** True
d True **e** False **f** False
g False

3 **a** False **b** True **c** False

4 **a** $\sqrt{3}$ **b** $\sqrt{7}$ **c** $\sqrt[3]{5}$ **d** $\sqrt[3]{12}$
e $\sqrt[5]{31}$ **f** $\sqrt[7]{18}$ **g** $\sqrt[9]{9}$ **h** $\sqrt[8]{3}$

5 **a** $8^{\frac{1}{2}}$ **b** $19^{\frac{1}{2}}$ **c** $10^{\frac{1}{3}}$ **d** $31^{\frac{1}{3}}$
e $5^{\frac{1}{4}}$ **f** $9^{\frac{1}{5}}$ **g** $11^{\frac{1}{8}}$ **h** $20^{\frac{1}{11}}$

6 **a** 5 **b** 7 **c** 9 **d** 13
e 2 **f** 4 **g** 5 **h** 10
i 2 **j** 3 **k** 5 **l** 2

7 **a** a **b** $a^{\frac{2}{3}}$ **c** a^{2} **d** $a^{\frac{5}{2}}$
e $x^{\frac{1}{3}}$ **f** x **g** $x^{\frac{5}{6}}$ **h** x
i y **j** y^{2} **k** $y^{\frac{3}{2}}$ **l** $x^{\frac{1}{4}}$
m $x^{\frac{8}{3}}$ **n** $a^{\frac{2}{15}}$ **o** $a^{\frac{3}{8}}$ **p** $n^{\frac{4}{3}}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 **a** $a^{\frac{4}{3}}$ **b** $a^{\frac{7}{10}}$ **c** $a^{\frac{23}{21}}$ **d** $a^{\frac{8}{3}}$
e $b^{\frac{1}{6}}$ **f** $x^{\frac{2}{15}}$

9 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$
e $\frac{1}{5}$ **f** $\frac{1}{3}$ **g** $\frac{1}{10}$ **h** $\frac{1}{4}$

10 **a** 9 **b** 16 **c** 27 **d** 125
e 32 **f** 32 **g** 729 **h** 3125

11 **a** $\sqrt{29}$ **b** $\sqrt{13}$ **c** $\sqrt{65}$ **d** $\sqrt{125}$
e $\sqrt{10}$ **f** $\sqrt{1700}$

12 **a** $a^{\frac{1}{2}+\left(-\frac{1}{2}\right)} = a^0 = 1$ **b** $a^{\frac{2}{3}+\left(-\frac{2}{3}\right)} = a^0 = 1$
c $a^{\frac{4}{7}-\frac{4}{7}} = a^0 = 1$ **d** $a^{\frac{5}{6}-\frac{5}{6}} = a^0 = 1$
e $a^{\frac{1}{4}} \times a^{-\frac{1}{4}} = a^{\frac{1}{4}+\left(-\frac{1}{4}\right)} = a^0 = 1$
f $a^2 \div a^2 = a^{2-2} = a^0 = 1$

13 Brackets needed for fractional power, $9 \wedge (1/2) = 3$

14 **a** **i** 3 **ii** 5 **iii** 10
b a **c** $(a^2)^{\frac{1}{2}} = a^{2\times\frac{1}{2}} = a$
d **i** 4 **ii** 9 **iii** 36
e a **f** $(a^{\frac{1}{2}})^2 = a^{\frac{1}{2}\times 2} = a$
g **i** a **ii** a **iii** a **iv** a

15 **a** $\frac{4}{5}$ **b** $\frac{3}{7}$ **c** $\frac{2}{9}$ **d** $\frac{2}{3}$
e $\frac{4}{5}$ **f** $\frac{2}{3}$ **g** $\frac{4}{5}$ **h** $\frac{10}{7}$

16 **a** $\frac{2}{3}$ **b** $\frac{12}{7}$ **c** $\frac{5}{2}$ **d** $\frac{5}{6}$

Exercise 6I

1 **a** Like **b** Like **c** Unlike **d** Unlike
e Unlike **f** Like **g** Like **h** Unlike

2 **a** Both = 3.162 **b** Both = 4.583
c Both = 1.732 **d** Both = 2.449

3 **a** $8\sqrt{7}$ **b** $8\sqrt{11}$ **c** $9\sqrt{5}$ **d** $4\sqrt{6}$
e $7\sqrt{3} + 2\sqrt{5}$ **f** $9\sqrt{7} + 3\sqrt{5}$ **g** $-5\sqrt{5}$ **h** $-4\sqrt{7}$
i $5\sqrt{7}$ **j** $-\sqrt{14}$ **k** $7\sqrt{2} - \sqrt{5}$ **l** $3\sqrt{3} + 2\sqrt{7}$

4 **a** $\sqrt{30}$ **b** $\sqrt{21}$ **c** $\sqrt{70}$ **d** 4
e 6 **f** $\sqrt{22}$ **g** 3 **h** 12
i $\sqrt{3}$ **j** $\sqrt{10}$ **k** $\sqrt{3}$ **l** 3
m 4 **n** $\sqrt{7}$

5 **a** $8 - 3\sqrt{3}$ **b** $6\sqrt{2} - \sqrt{3}$ **c** $7\sqrt{5} + 1$
d $\frac{5\sqrt{2}}{6}$ **e** $\frac{7\sqrt{7}}{10}$ **f** $-\frac{3\sqrt{6}}{14}$
g $\frac{2\sqrt{10}}{3}$ **h** $5 + \frac{\sqrt{3}}{3}$ **i** $-\frac{19\sqrt{8}}{56}$

6 **a** $15\sqrt{6}$ **b** $6\sqrt{21}$ **c** $8\sqrt{30}$ **d** $10\sqrt{18}$
e $2\sqrt{3}$ **f** $3\sqrt{6}$ **g** $4\sqrt{14}$ **h** $\frac{\sqrt{2}}{2}$

7 **a** $6\sqrt{15} + 2\sqrt{3}$ **b** $\sqrt{10} + \sqrt{15}$ **c** $5\sqrt{12} + 15\sqrt{30}$
d $14\sqrt{30} - 70$ **e** $13 - 2\sqrt{39}$ **f** $\sqrt{35} - 10$

8 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $3\sqrt{3}$ **d** $3\sqrt{5}$
e $5\sqrt{3}$ **f** $10\sqrt{2}$ **g** $2\sqrt{15}$ **h** $6\sqrt{2}$

9 **a** $5\sqrt{2}$ **b** $\sqrt{2}$ **c** $4\sqrt{2}$ **d** $\sqrt{3}$
e $6\sqrt{2}$ **f** $5\sqrt{3}$ **g** $2\sqrt{5}$ **h** $4\sqrt{3}$

10 **a** $2 + \sqrt{10} + \sqrt{6} + \sqrt{15}$
b $3 + \sqrt{6} - \sqrt{15} - \sqrt{10}$
c $6\sqrt{10} + 8\sqrt{5} - 3\sqrt{2} - 4$
d $2 + 3\sqrt{2} - 6\sqrt{7} - 9\sqrt{14}$
e 1 **f** 4 **g** 15 **h** 123
i $3 + 2\sqrt{2}$ **j** $15 - 6\sqrt{6}$
k $13 - 4\sqrt{3}$ **l** $22 + 4\sqrt{10}$

Puzzles and challenges

1 **a** 4 **b** 1 **c** 6

2 **a** 6 **b** 30

3 $\frac{9}{4}$ **4** **a** $2t^2$ **b** $\frac{2}{t}$

5 100 minutes **6** $-\frac{2}{3}$

7 **a** **i** $2^{\frac{3}{4}}$ **ii** $2^{\frac{7}{8}}$ **iii** $2^{\frac{15}{16}}$ **b** 2

8 2^7

9 **a** $7\sqrt{2}$ **b** $\frac{3}{\sqrt{2}}$ **c** $12\sqrt{10}$

11 **a** $x = 0$ or 1 **b** $x = 1$ or 2

Multiple-choice questions

1 D **2** B **3** E **4** A **5** C **6** B
7 D **8** C **9** E **10** B **11** C **12** A

Short-answer questions

1 **a** 3^4 **b** $2x^3y^2$ **c** $3a^2b^2$
d $\left(\frac{3}{5}\right)^3 \times \left(\frac{1}{7}\right)^2$

2 **a** $3^2 \times 5$ **b** $2^2 \times 3 \times 5^2$

3 **a** x^{10} **b** $12a^5b^6c$ **c** $12m^4n^4$
d a^9 **e** x^3y^2 **f** $\frac{b^2}{2a^2}$

4 **a** m^6 **b** $9a^8$ **c** $-32a^{10}b^5$
d $3b$ **e** 2 **f** $\frac{a^6}{27}$

5 **a** $\frac{1}{x^3}$ **b** $\frac{4}{t^3}$ **c** $\frac{1}{9t^2}$
d $\frac{2x^2}{3y^3}$ **e** $\frac{5}{x^6y^3}$ **f** $5m^3$

6 **a** $\frac{x^2}{2y^2}$ **b** $\frac{x^3y^2}{9}$ **c** $8m^7n^3$

7 $0.0012,\ 35.4 \times 10^{-3},\ 3.22 \times 10^{-1},\ 0.4,\ 0.007 \times 10^2,\ 2.35$

8 **a** 324 **b** 172 500 **c** 0.2753 **d** 0.00149

9 **a** 2.25×10^7 people **b** 9.63×10^6 km^2
c 3.34×10^{-9} seconds **d** 2.94×10^{-7} m

10 **a** 2.19×10^5 **b** 1.2×10^{-2}
c 4.32×10^{-7} **d** 5×10^6

11 **a** 1.2×10^{55} **b** 4.3×10^{-5}

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 **a** 2 **b** 5 **c** 7
d 3 **e** $\frac{1}{3}$ **f** $\frac{1}{11}$

13 **a** s^2 **b** $9t^{\frac{3}{2}}$ **c** $15x^{\frac{5}{2}}$
d $9m^{\frac{3}{4}}n^4$ **e** $\frac{t^{\frac{1}{2}}}{2}$ **f** $4a^{\frac{2}{3}}$

14 **a** $7\sqrt{7}$ **b** $\sqrt{3}+9\sqrt{2}$ **c** 8
d $\sqrt{15}$ **e** $2\sqrt{14}$ **f** $15\sqrt{22}$
g $\sqrt{6}$ **h** 10 **i** $\frac{\sqrt{5}}{2}$

Extended-response questions

1 **a** $\frac{16x^6}{3}$ **b** $\frac{8b^8}{15a^2}$ **c** $\frac{5m^8}{n^{10}}$ **d** $3x$

2 **a** 5.93×10^{-11} N
b **i** 1.50×10^{11} m
ii 3.53×10^{22} N
c Earth $a = 9.81\,\text{m s}^{-2}$, Mars $a = 3.77\,\text{m s}^{-2}$
Acceleration due to gravity on Earth is more than $2\frac{1}{2}$ times that on Mars.

Chapter 7

Pre-test

1 **a** Obtuse **b** Acute **c** Reflex **d** Right

2 **a** Isosceles **b** Equilateral
c Scalene **d** Right-angled
e Equilateral **f** Right-angled, isosceles triangle

3 **a** 60° **b** 75° each **c** 50°

4 **a** $x = 55$ **b** $x = 100$ **c** $x = 110$ **d** $x = 45$
e $x = 40$ **f** $x = 108$

5 **a** $a = 60$ **b** $b = 110$ **c** $a = 60$, $b = 120$

6 **a** Pentagon **b** Parallelogram **c** Trapezium
d Rhombus **e** Hexagon **f** Nonagon

Exercise 7A

1 **a** right **b** 180° **c** revolution **d** obtuse
e acute **f** 180° **g** 90°
h supplementary **i** 180° **j** equal

2 **a** Isosceles triangle **b** Obtuse-angled triangle
c Equilateral triangle **d** Isosceles triangle
e Acute-angled triangle **f** Scalene triangle

3 **a** **i** $\angle BAC$ **ii** Obtuse **iii, iv** 120°
b **i** $\angle PRQ$ **ii** Acute **iii, iv** 30°
c **i** $\angle XYZ$ **ii** Reflex **iii, iv** 310°
d **i** $\angle ROB$ **ii** Obtuse **iii, iv** 103°

4 **a** 50° **b** 101° **c** 202°

5 **a** **i** 125° **ii** 35° **b** **i** 149° **ii** 59°
c **i** 106° **ii** 16° **d** **i** 170° **ii** 80°
e **i** 91° **ii** 1° **f** **i** 158° **ii** 68°
g **i** 142° **ii** 52° **h** **i** 115° **ii** 25°
i **i** 133° **ii** 43° **j** **i** 103° **ii** 13°

6 **a** C **b** S **c** N **d** C
e S **f** N **g** S **h** N

7 **a** $a + 27 = 90$ (angles in a right angle)
$a = 63$
b $a + 19 = 90$ (vertically opposite angles)
$a = 71$
c $a + 142 = 180$ (angles on a straight line)
$a = 38$
d $a + 33 = 180$ (angles on a straight line)
$a = 147$
e $a + 127 = 360$ (angles in a revolution
$a = 233$
f $a + 90 + 237 = 360$ (angles in a revolution)
$a = 33$

8 **a** Obtuse isosceles, $a = 40$
b Acute scalene, $b = 30$
c Right-angled scalene, $c = 90$
d Equilateral, $d = 60$
e Obtuse isosceles, $e = 100$
f Right-angled isosceles, $f = 45$
g Obtuse scalene, $g = 100$
h Equilateral, $h = 60$
i Obtuse isosceles, $i = 120$
j Obtuse isosceles, $j = 40$
k Right-angled scalene, $k = 90$
l Acute scalene, $l = 70$

9 **a** $s = 120$ **b** $t = 20$
c $r = 70$ **d** $a = 60$, $x = 120$
e $a = 100$, $b = 140$ **f** $c = 115$, $d = 65$

10 **a** 360° **b** 90° **c** 60° **d** 90°
e 432° **f** 6° **g** 720° **h** 8640°

11 **a** $\angle BCA = 180° - 118°$ (angles an a straight line)
$= 62°$
$\therefore \angle BAC = 62°$ (equal angles are opposite equal sides)
$\therefore x = 180 - 2 \times 62$ (angle sum of a triangle)
$= 56$
b $\angle ABC = 180° - 95°$ (angles an a straight line)
$= 85°$
$\angle ACB = 180° - 110°$ (angles an a straight line)
$= 70°$
$\therefore x = 85 + 70$ (exterior angle equals sum of two opposite interior angles)
$= 155$

12 **a** 90° **b** 150° **c** 15° **d** 165°
e 157.5° **f** 80° **g** 177.5° **h** 171°

13 $\angle AOB + \angle ABO = 120°$ (exterior angle of triangle)
$\angle AOB = 30°$
$x = 330$ (angles in a revolution)

14 AO = BO (radii)
ΔAOB is isoceles, 2 sides equal, $\angle AOB = 116°$
$\therefore \angle OAB = 32°$, base angles of isosceles triangle

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

15 a 160° **b** 165°

c $\angle WYZ + a^\circ + b^\circ = 180^\circ$ angle sum of a triangle

$\angle XYZ + \angle WYZ = 180^\circ$ straight line

$\therefore \angle XYZ = a^\circ + b^\circ$

16 Let the interior angles of any triangle be a, b and c

Now $a + b + c = 180^\circ$

The exterior angles become,

$180^\circ - a^\circ$, $180^\circ - b^\circ$, $180^\circ - c^\circ$ (straight line)

Exterior sum $= (180 - a) + (180 - b) + (180 - c)$

$= 540 - a - b - c$

$= 540 - (a + b + c)$

$= 540 - 180$

$= 360^\circ$

17 a $4x = 90$, $x = 22.5$ **b** $3x = 180$, $x = 60$

c $10x = 360$, $x = 36$ **d** $2(x + 15) + x = 180$, $x = 50$

e $2x + 20 = 140$, $x = 60$ **f** $6x + 90 = 360$, $x = 45$

Exercise 7B

1 a Equal

b Equal

c Supplementary

2 a $x = 125$, alternate angles in ∥ lines

b $y = 110$, co-interior angles in ∥ lines

c $r = 80$, corresponding angles in ∥ lines

d $s = 66$, alternate angles in ∥ lines

e $t = 96$, vertically opposite angles

f $v = 126$, corresponding angles on ∥ lines

g $w = 62$, angles on straight line

h $p = 115$, corresponding angles on ∥ lines

i $q = 116$, co-interior angles on ∥ lines

3 a No, alternate angles are not equal.

b Yes, corresponding angles are equal.

c Yes, alternate angles are equal.

d No, co-interior angles don't add to 180°.

e Yes, co-interior angles add to 180°.

f Yes, corresponding angles are equal.

g No, corresponding angles are not equal.

h No, alternate angles are not equal.

i No, co-interior angles do not add to 180°.

4 a $a = 60$, $b = 120$

b $c = 95$, $d = 95$

c $e = 100$, $f = 100$, $g = 100$

d $a = 110$, $b = 70$

e $a = 100$, $b = 80$, $c = 80$

f $e = 140$, $f = 140$, $d = 140$

5 a $x = 70$, $y = 40$ **b** $t = 58$, $z = 122$

c $u = 110$, $v = 50$, $w = 50$ **d** $x = 118$

e $x = 295$ **f** $x = 79$

6 a 105° **b** 105° **c** 56°

d 105° **e** 90° **f** 85°

7 a 56 **b** 120 **c** 50

8 a $180^\circ - a^\circ$ **b** $180^\circ - a^\circ$

c $180^\circ - (a^\circ + b^\circ)$ **d** $180^\circ - (a^\circ + b^\circ)$

e $a^\circ + c^\circ$ **f** $180^\circ - 2c^\circ$

9 $\angle ABC = 100^\circ$

$\angle BCD = 80^\circ$

$\therefore AB \parallel DC$ as co-interior angles are supplementary

$\angle ABC + \angle BCD = 180^\circ$

10 a Co-interior angles on parallel lines add to 180°.

b Alternate angles are equal, on parallel lines.

c $\Delta ABC \Rightarrow a + b + c = 180$ and these are the three angles of the triangle.

11 a $\angle BAE = 180^\circ - a^\circ$ (alternate angles and AB ∥ DE)

$\angle ABC = 180^\circ - c^\circ - (180 - a)^\circ$ (angle sum of a triangle)

$= 180^\circ - c^\circ - 180^\circ + a^\circ$

$= -c^\circ + a^\circ$

$= a^\circ - c^\circ$

b $\angle ABD = 180^\circ - (a^\circ + b^\circ)$ (angle sum of triangle ΔABD)

$\angle ABC + \angle ABD = 180^\circ$ (straight line)

$\therefore \angle ABC = a^\circ + b^\circ$

c Construct XY through B parallel to AE.

$\therefore \angle ABY = a^\circ$ (alternate angles, $AE \parallel XY$)

$\therefore \angle CBY = b^\circ$ (alternate angles, $DC \parallel XY$)

$\therefore \angle ABC = a^\circ + b^\circ$

d Construct XY through A parallel to ED.

Now

$\angle XAD = 180^\circ - b^\circ$ (co-interior angles, $ED \parallel XY$)

$\angle DAB = 360^\circ - a^\circ$ (revolution)

$\therefore \angle XAB = 360^\circ - a^\circ - (180^\circ - b^\circ)$

$= 180^\circ + b^\circ - a^\circ$

$\angle ABC = \angle XAB$ (alternate angles and $XY \parallel BC$)

Exercise 7C

1 a regular pentagons **b** regular decagons

c octagons

2 a 3, 180°, 60°, 360°, 120° **b** 4, 360°, 90°, 360°, 90°

c 6, 720°, 120°, 360°, 60° **d** 8, 1080°, 135°, 360°, 45°

3 a parallel **b** right

c trapezium **d** equal

4 a Convex quadrilateral **b** Non-convex hexagon

c Non-convex heptagon

5 a 115 **b** 159 **c** 30

d 121 **e** 140 **f** 220

6 a 110 **b** 70 **c** 54

d 33 **e** 63 **f** 109

7 a 110 **b** 150 **c** 210 **d** 20

e $b = 108$, $a = 72$ **f** $a = 40$, $b = 140$

g $b = 120$, $a = 240$ **h** $b = 128\frac{4}{7}$, $a = 231\frac{3}{7}$

i 108

8 a Parallelogram, rectangle, kite

b Rectangle, square

c Square, rectangle

d Square, rhombus, kite

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

9 **a** 16 **b** 25 **c** 102
10 **a** 255 **b** 80 **c** 115
d 37 **e** 28 **f** 111
11 A parallelogram has opposite sides parallel and equal and rectangles, squares and rhombi have these properties (and more) and are therefore all parallelograms.
12 **a** $S = 180(n - 2)$ **b** $I = \frac{180(n-2)}{n}$
c $E = \frac{360}{n}$ **d** 36°
13 **a** 30° **b** 150°
14 **a** **i** one **ii** two **iii** five
b $(n - 3)$
15 $(180-a)+(180-b)+(180-c)+(180-d)+(180-e) = 360$ (sum of exterior angles is 360°)
$180+180+180+180+180 - (a + b + c + d + e) = 360$
$900 - (a + b + c + d + e) = 360$
$a + b + c + d + e = 540$
a $a + b + c + d + e + f = 720$
b $a + b + c + d + e + f + g = 900$

Exercise 7D

1 **a** size
b ≡
c SAS, RHS, AAS
2 **a** **i** XY **ii** XZ **iii** YZ
b **i** $\angle A$ **ii** $\angle B$ **iii** $\angle C$

3 **a**	$PQ = ZX$ $PR = ZY$ $QR = XY$	$\angle P = \angle Z$ $\angle Q = \angle X$ $\angle R = \angle Y$	$\Delta PQR \equiv \Delta ZXY$	SAS
b	$AB = CD$ $BO = OC$ $AO = OD$	$\angle A = \angle D$ $\angle B = \angle C$ $\angle AOB = \angle COD$	$\Delta ABO \equiv \Delta DCO$	AAS
c	$AD = BC$ $AB = DC$ $AC = AC$	$\angle D = \angle B$ $\angle DAC = \angle BCA$ $\angle DCA = \angle BAC$	$\Delta ABC \equiv \Delta CDA$	RHS

4 **a** SAS **b** AAS **c** RHS **d** SAS
e SSS **f** RHS **g** AAS **h** SSS
5 **a** $x = 3, y = 5$ **b** $x = 2, y = 6$
c $a = 105, b = 40$ **d** $a = 65, b = 85$
e $x = 2.5, b = 29$ **f** $a = 142, x = 9.21, b = 7$
g $y = 4.2, a = 28$ **h** $a = 6.5, b = 60$
6 **a** $\Delta ABC \equiv \Delta STU$, RHS **b** $\Delta DEF \equiv \Delta GHI$, SSS
c $\Delta ABC \equiv \Delta DEF$, SAS **d** $\Delta ABC \equiv \Delta GHI$, AAS
7 $\Delta PBR \equiv \Delta FDE$
$\Delta LMN \equiv \Delta KIJ$
$\Delta FGH \equiv \Delta BCD$
$\Delta MNO \equiv \Delta RQP$
8 **a** $BC = 13$ **b** $BC = 85$
9 No, they can all be different sizes, one might have all sides 2 cm and another all sides 5 cm.
10 **a** One given, the other pair are vertically opposite
b AAS
11 **a** SSS **b** Equal
12 **a** One given ($BA = BC$) and side BD is common
b SAS **c** $\Delta ABD \equiv \Delta CBD$
d $\angle ADB = \angle CDB$ (corresponding angles in congruent triangles)
but $\angle ADB + \angle CDB = 180°$ (straight angle)
$\therefore \angle ADB = \angle CDB = 90°$
and AC is perpendicular to DB

Exercise 7E

1 **a** $OA = OB$ because they are radii of the circle
b $\angle OAB = \angle OBA$ because equal angles are opposite equal sides
2 **a** $\angle ECD$ **b** $\angle CBA$ **c** $\angle DEC$
d No. We have three pairs of matching angles but no matching sides. AAA is not a congruence test.
3 **a** SSS **b** $\angle BMC$
4 $AB = CD$, opposite, CED, vertically opposite, $\angle CDE$, alternate, $\Delta AEB \equiv \Delta CED$ (AAS)
5 **a** $AD = CD$ (given)
$\angle DAB = \angle DCB = 90°$ (given)
DB is common.
$\therefore \Delta ABD \equiv \Delta CBD$ (RHS)
b AC is common.
$AD = AB$ (given)
$\angle DAC = \angle BAC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (SAS)
c AC is common.
$\angle ADC = \angle ABC$ (given)
$\angle DAC = \angle BAC$ (given)
$\Delta ADC \equiv \Delta ABC$ (AAS)
d AC is common.
$AD = AB$ (given)
$DC = BC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (SSS)
e $AC = DC$ (given)
$BC = EC$ (given)
$\angle ACB \equiv \angle DCE$ (vertically opposite)
$\therefore \Delta ABC \equiv \Delta DEC$ (SAS)
f $AC = EC$ (given)
$\angle CAB = \angle CED$ (alternate angles, $AB \parallel DE$)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ABC \equiv \Delta EDC$ (AAS)
g $DC = BC$ (given)
$\angle EDC = \angle ABC$ (alternate angles, $DE \parallel AB$)
$\angle DCE = \angle BCA$ (vertically opposite)
$\Delta CDE \equiv \Delta CBA$ (AAS)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

h BD is common.
$AD = CD$ (given)
$\angle ADB = \angle CDB$ (given)
$\Delta ABD \equiv \Delta CBD$ (SAS)

i AC is common.
$AB = CD$ (given)
$BC = DA$ (given)
$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)

j BD is common.
$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)
$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)
$\therefore \Delta ABD \equiv \Delta CDB$ (AAS)

k $OA = OC$ (radii)
OB is common.
$AB = CB$ given
$\therefore \Delta AOB \equiv \Delta COB$ (SSS)

l $OA = OD$ and $OB = OC$ (radii)
$\angle AOB = \angle COD$ (vertically opposite)
$\Delta AOB \equiv \Delta COD$ (SAS)

6 a $DC = BC$ (given)
$EC = AC$ (given)
$\angle DCE = \angle BCA$ (vertically opposite)
$\therefore \Delta ABC \equiv \Delta EDC$ (SAS)

b $\angle EDC = \angle ABC$ (corresponding angles in congruent triangles)
$\therefore AB \parallel DE$ (alternate angles are equal)

7 a $AE = CD$ (given)
$BE = BD$ (given)
$\angle ABE = \angle CBD$ (vertically opposite with $\angle ABE$ given 90°)
$\therefore \Delta ABE \equiv \Delta CBD$ (RHS)

b $\angle EAB = \angle DCB$ (corresponding angles in congruent triangles)
$\therefore AE \parallel CD$ (alternate angles equal)

8 a DB is common.
$AB = CD$ (given)
$AD = CB$ (given)
$\therefore \Delta ABD \equiv \Delta CDB$ (SSS)

b $\angle ADB = \angle CBD$ (corresponding angles in congruent triangles)
$\therefore AD \parallel BC$ (alternate angles equal)

9 a $OB = OC$ (radii)
$OA = OD$ (radii)
$\angle AOB = \angle DOC$ (vertically opposite)
$\therefore \Delta AOB \equiv \Delta DOC$ (SAS)

b $\angle ABO = \angle DCO$ (corresponding angles in congruent triangles)
$\therefore AB \parallel CD$ (alternate angles equal)

10 a BD is common.
$AD = CD$ (given)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (SAS)

b $\angle ABD = \angle CBD$ (corresponding angles in congruent triangles) and $\angle ABD + \angle CBD = 180°$ (straight line)
$\therefore \angle ABD = \angle CBD = 90°$ and AC is perpendicular to BD

11 a DB is common.
$\angle ABD = \angle CBD$ (given 90°)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (AAS)

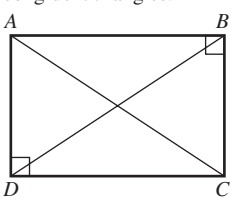
b $AD = CD$ (corresponding side in congruent triangles)
$\therefore \Delta ACD$ is isosceles (2 equal sides)

12 Consider ΔOAD and ΔOBD.
OD is common.
$OA = OB$ (radii)
$AD = BD$ (given)
$\therefore \Delta OAD \equiv \Delta OBD$ (SSS)
$\angle ODA = \angle ODB = 90°$ (corresponding angles in congruent triangles are equal and supplementary to a straight line)
$\therefore OC \perp AB$

13 Consider ΔADC and ΔCBA.
AC is common.
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
$\angle DCA = \angle BAC$ (alternate angles, $DC \parallel AB$)
$\therefore \Delta ADC \equiv \Delta CBA$ (AAS)
So $AD = BC$, $AB = DC$ are equal corresponding sides in congruent triangles.

14 $AB = DC$ (opposite sides of parallelogram)
$\angle AEB = \angle CED$ (vertically opposite)
$\angle BAE = \angle DCE$ (alternate angles $DC \parallel AB$)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)
So $AE = CE$ and $BE = DE$, corresponding sides in congruent triangles.

15



Consider ΔADC and ΔBCD.
$AD = CB$ and DC is common (opposite sides of a rectangle are equal)
$\angle ADC = \angle BCD = 90°$ (angles of a rectangle)
$\therefore \Delta ADC \equiv \Delta BCD$ (SAS)
So $AC = BD$ (corresponding sides in congruent triangles)
$\therefore$ The diagonals of a rectangle are equal

16 a Consider ΔABE and ΔCDE.
$AB = CD$ (sides of a rhombus)
$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel CD$)
$\angle BAE = \angle DCE$ (alternate angles, $AB \parallel CD$)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)

b Consider ΔDCE and ΔBCE.
CE is common.
$DC = BC$ (sides of a rhombus)
$DE = BE$ (corresponding sides in congruent triangles)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

$\therefore \Delta DCE \equiv \Delta BCE$ (SSS)

$\therefore DE = BE$ (corresponding sides in congruent triangles)

$\angle DEC = \angle BEC$ (corresponding angles in congruent triangles)

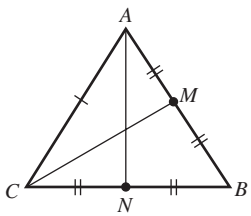
$\angle DEC + \angle BEC = 180°$ (straight line)

$\therefore \angle DEC = \angle BEC = 90°$

and $AE = CE$ (corresponding sides in congruent triangles)

$\therefore AC$ bisects BD at 90°

17



Let ΔABC be any equilateral triangle $AB = CB = AC$

Step 1

Join C to M, the midpoint of AB

Prove $\Delta CAM \equiv \Delta CBM$ (SSS)

$\therefore \angle CAM = \angle CBM$ (corresponding angles in congruent triangles)

Step 2

Join A to N, the midpoint of CB.

Prove $\Delta ANC \equiv \Delta ANB$

$\therefore \angle ACN = \angle ABN$ (corresponding angles in congruent triangles)

Now $\angle CAB = \angle ABC = \angle ACB$

and as $\angle CAB + \angle ABC + \angle ACB = 180°$ (angle sum of ΔABC)

$\angle CAB = \angle ABC = \angle ACB = 60°$

Exercise 7F

1 **a** $\angle F$ **b** $\angle D$ **c** GH
d AE **e** 2

2 **a** Double **b** Double **c** Double
d 2 **e** Yes

3 **a** OA' is a quarter of OA. **b** OD' is a quarter of OD.
c $\frac{1}{4}$ **d** Yes

4 **a** Yes **b** 8 cm **c** 25 m

5 Drawings
a $A'B'C'$ should have sides $\frac{1}{3}$ that of ABC
b $A'B'C'$ should have sides double that of ABC

6 **b** **i** $A'B'C'D'$ should have sides lengths $\frac{1}{2}$ that of $ABCD$.
ii $A'B'C'D'$ should have side lengths 1.5 times that of $ABCD$.

7 **a** **i** 2 **ii** 14 **iii** 10
b **i** $1\frac{1}{2}$ **ii** 9 **iii** 8
c **i** $1\frac{1}{2}$ **ii** 45 **iii** 24
d **i** $\frac{2}{5}$ **ii** 1 **iii** 1.4
e **i** 2.5 **ii** 0.6 **iii** 2
f **i** 1.75 **ii** 3.5 **iii** 3

8 **a** **i** 2 **ii** (0, 0)
b **i** $\frac{1}{2}$ **ii** (0, 0)
c **i** $\frac{1}{2}$ **ii** (3, 0)
d **i** 3 **ii** (1, 0)

9 **a** **i** 3.6 m **ii** 9 m **iii** 2.7 m
b **i** 5.4 m **ii** 6.3 m
c **i** 4 m **ii** 6 m

10 **a** 12.7 cm **b** 3 cm **c** 3 m

11 **a** $a > 1$ **b** $a < 1$ **c** $a = 1$

12 **a** All angles of any square equal 90°, with only 1 side length.
b All angles in any equilateral triangle equal 60° with only 1 side length.
c The length and width might be multiplied by different numbers.
d Two isosceles triangles do not have to have the same size equal angles.

13 $\frac{1}{k}$

14 **a** 100 000 cm = 1 km **b** 24 cm

15 **b** **i** $\frac{l}{2}$ **ii** $\frac{l}{4}$ **iii** $\frac{l}{128}$
c **i** $\frac{3}{4}$ **ii** $\frac{9}{16}$ **iii** $\frac{243}{1024}$
d Zero

Exercise 7G

1 **a** E **b** C **c** DF **d** BC
e $\angle A$ **f** $\angle E$ **g** DFE

2 2.5

3 **a** 4 **b** shape, size

4 **a** 3 **b** 4 **c** 1 **d** 2
e 4 **f** 3 **g** 2 **h** 1

5 **a** $\Delta ABC \,|||\, \Delta GHI$ **b** $\Delta ABC \,|||\, \Delta MNO$
c $\Delta ABC \,|||\, \Delta ADE$ **d** $\Delta HFG \,|||\, \Delta HJI$
e $\Delta ADC \,|||\, \Delta AEB$ **f** $\Delta ABD \,|||\, \Delta ECD$

6 **a** $AE \parallel BD$
$\therefore \angle EAC = \angle DBC$ and $\angle AEC = \angle BDC$
$\therefore$ Two pairs of matching angles
$\therefore \Delta CDB \,|||\, \Delta CEA$
b 12

7 **a** Both triangles have right angles.
$\frac{25}{10} = \frac{15}{6} = 2.5$
$\therefore$ Hypotenuses and another pair of sides are proportional.
$\therefore$ The triangles are similar.
b 8

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 a i 1.6 ii 14.4 **b** i 3.5 ii 5

9 a $AB \parallel DE$

$\therefore \angle BAC = \angle DEC$ and $\angle ABC = \angle EDC$

$\therefore$ Two pairs of matching angles

$\therefore \Delta ABC \,|||\, \Delta EDC$

b 15

10 a i Two pairs of equal matching angles ii 6.5

b i Two pairs of equal matching angles ii 10

c i Two pairs of equal matching angles ii 24

11 a 2 **b** 16 **c** 2.8

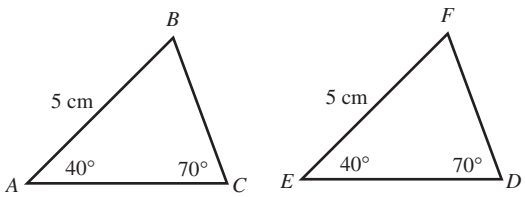
12 a ΔDEF **b** ΔDEF **c** ΔABC **d** ΔDEF

13 $\angle ACB = 25°$, two pairs of equal matching angles

14 $\angle WXY = 55°$, not similar as angles not equal

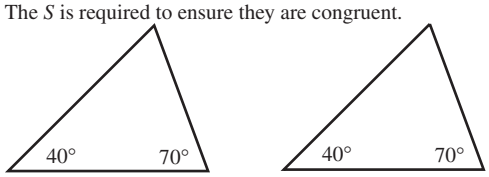
15 Two pairs of equal alternate angles are always formed.

16



$\Delta ABC \,|||\, \Delta EFD$ (*AAS*)

The *S* is required to ensure they are congruent.



If two angles and no sides are known, the triangles are certainly similar (and might also be congruent).

17 a

Triangle	Original	Image 1	2	3
Length scale factor	1	2	3	4
Area	4	16	36	64
Area scale Factor	1	4	9	16

b Area scale factor = (length scale factor)^2

c n^2

d i 100 ii 400 iii 10 000 **e** $\frac{1}{4}$

Exercise 7H

1 a $\angle BCD$ (or $\angle ACE$)

b They are corresponding angles on parallel lines

c We have two pairs of matching angles

d $\Delta BCD \,|||\, \Delta ACE$

2 a $\angle ACB$ and $\angle ECD$

b $\angle BAC = \angle DEC$ and $\angle CBA = \angle CDE$

c $\Delta ABC \,|||\, \Delta EDC$

3 a $\angle C$ **b** i AC ii DB

4 a $\angle AEB = \angle CDB$ (alternate angles, $EA \parallel DC$)

$\angle EAB = \angle DCB$ (alternate angles, $EA \parallel DC$)

$\angle EBA = \angle DBC$ (vertically opposite)

$\therefore \Delta AEB \,|||\, \Delta CDB$ (two pairs of equal matching angles)

b $\angle BAC = \angle DEC$ (alternate angles, $AB \parallel DE$)

$\angle ABC = \angle EDC$ (alternate angles, $AB \parallel DE$)

$\angle ACB = \angle ECD$ (vertically opposite)

$\therefore \Delta ACB \,|||\, \Delta ECD$ (two pairs of equal matching angles)

c $\angle C$ is common.

$\angle CDB = \angle CEA$ (corresponding angles, $AE \parallel BD$)

$\angle CBD = \angle CAE$ (corresponding angles, $AE \parallel BD$)

$\therefore \Delta CBD \,|||\, \Delta CAE$ (two pairs of equal matching angles)

d $\angle A$ is common.

$\angle AEB = \angle ADC$ (corresponding angles, $EB \parallel DC$)

$\angle ABE = \angle ACD$ (corresponding angles, $EB \parallel DC$)

$\therefore \Delta AEB \,|||\, \Delta ADC$ (two pairs of equal matching angles)

e $\angle A$ is common.

$\angle ABE = \angle ADC$ (given)

$\therefore \Delta ABE \,|||\, \Delta ADC$ (two pairs of equal matching angles)

f $\angle ABD = \angle BCD$ (given 90°)

$\angle BAD = \angle CBD$ (given)

$\therefore \Delta ABD \,|||\, \Delta BCD$ (two pairs of equal matching angles)

5 a $\angle C$ is common.

$\frac{CA}{CD} = \frac{6}{2} = \frac{3}{1} = 3$

$\frac{CE}{CB} = \frac{9}{3} = 3$

$\therefore \Delta CDB \,|||\, \Delta CAE$ (sides adjacent to equal angles are in proportion)

b $\angle D$ is common.

$\frac{AD}{CD} = \frac{28}{7} = 4$

$\frac{DB}{DE} = \frac{48}{12} = 4$

$\therefore \Delta ABD \,|||\, \Delta CED$ (sides adjacent to equal angles are in proportion)

c $\angle DCE = \angle BCA$ (vertically opposite)

$\frac{EC}{AC} = \frac{2}{5}$

$\frac{DC}{BC} = \frac{3}{7.5} = \frac{2}{5}$

$\therefore \Delta DCE \,|||\, \Delta BCA$ (sides adjacent to equal angles are in proportion)

6 a Two pairs of equal matching angles

b 40 m

7 a Two pairs of equal matching angles

b 7.5 m

8 6 m **9** 20 m **10** 7.2 m **11** $\frac{55}{6}$

12 a First, $\angle ADC = \angle ACD = 80°$ (base angles of isosceles ΔADC)

$\angle ACB = 100°$ (straight angle)

$\angle CAB = 60°$ (angle sum of ΔACB)

Now $\angle DAB = 80°$

Proof $\angle DAC = \angle DBA$ (given 20°)

$\angle D$ is common.

$\angle ACD = \angle BAD$ (both 80°)

$\therefore \Delta ACD \,|||\, \Delta BAD$ (two pairs of equal matching angles)

b $DC = \frac{20}{3}$ $CB = \frac{25}{3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

13 a i $\angle B$ is common.
$\angle DAB = \angle ACB$ (given 90°)
$\therefore \Delta ABD \,|||\, \Delta CBA$ (two pairs of equal matching angles)

ii $\angle D$ is common.
$\angle DCA = \angle DAB$ (given 90°)
$\therefore \Delta ABD \,|||\, \Delta CAD$ (two pairs of equal matching angles)

b i $BD = \frac{25}{3}$ **ii** $AC = 4$ **iii** $AB = \frac{20}{3}$

14 a $\angle ACB = \angle ECD$ (vertically opposite)
$\angle CAB = \angle CED$ (alternate angles, $DE \parallel BA$)
$\therefore \Delta ABC \,|||\, \Delta EDC$ (two pairs of equal matching angles)
$\therefore \frac{DC}{BC} = \frac{EC}{AC}$ (ratio of matching sides in similar triangles)
$\frac{6}{2} = \frac{EC}{AC}$
$\therefore 3AC = CE$
as $AC + CE = AE$
$AE = 4AC$

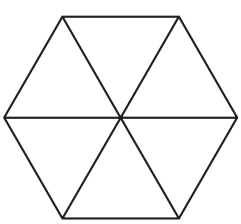
b $\angle C$ is common.
$\angle DBC = \angle AEC$ (given)
$\therefore \Delta CBD \,|||\, \Delta CEA$ (two pairs of equal matching angles)
$\therefore \frac{DB}{AE} = \frac{2}{4} = \frac{BC}{CE}$ (ratio of matching sides in similar triangles)
$\therefore 4BC = 2CE$
$BC = \frac{1}{2}CE$

c $\angle C$ is common.
$\angle CBD = \angle CAE$ (matching angles, $BD \parallel AE$)
$\therefore \Delta CBD \,|||\, \Delta CAE$ (two pairs of equal matching angles)
$\therefore \frac{CB}{CA} = \frac{CD}{CE}$ (ratio of matching sides in similar triangles)
$\therefore \frac{5}{7} = \frac{CD}{CE}$
$\therefore 5CE = 7CD$
$CE = \frac{7}{5}CD$

d $\angle C$ is common.
$\angle CBD = \angle CAE$ (given 90°)
$\therefore \Delta CBD \,|||\, \Delta CAE$ (two pairs of equal matching angles)
$\therefore \frac{BD}{AE} = \frac{CB}{CA}$ (ratio of matching sides in similar triangles)
$\frac{2}{8} = \frac{CB}{CA}$
$\frac{1}{4} = \frac{CB}{CB + AB}$
$CB + AB = 4CB$
$\therefore AB = 3CB$

Puzzles and challenges

1



2 15 **3** 40° **4** Midpoints
5 170 **6** 11 **7** $\frac{120}{7}$

Multiple-choice questions

1 D **2** A **3** B **4** D **5** C
6 D **7** E **8** B **9** D **10** A

Short-answer questions

1 a Isosceles, $x = 50$, $y = 80$ **b** Right angled, $x = 25$
c Obtuse angled, $x = 30$, $y = 110$

2 a $a = 30$ (vertically opposite) $b = 150$ (straight angle)
b $x = 60$ (revolution) $y = 120$ (co-interior angles in parallel lines)
c $a = 70$ (alternate angles and parallel lines)
$b = 55$ (angle sum of isosceles triangle)
$c = 55$ (corresponding angles and parallel lines)

3 $\angle ABC = 75°$

4 a $a = 70$, $b = 110$ **b** $x = 15$
c $x = 30$ **d** $a = 120$

5 a SSS, $x = 60$ **b** not congruent
c RHS, $x = 12$ **d** AAS, $x = 9$

6 a AD is common.
$\angle ADC = \angle ADB$ (given 90°)
$CD = BD$ (given)
$\therefore \Delta ADC \equiv \Delta ADB$ (SAS)

b i $AC = EC$ (given)
$\angle BAC = \angle DEC$ (given)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ACB \equiv \Delta ECD$ (AAS)

ii as $\angle BAC = \angle DEC$ (alternate angles are equal)
$\therefore AB \parallel DE$

7 For image $\Delta A'B'C'$, $OA' = 3OA'$, $OB' = 3OB$, $OC' = 3OC$

8 a Yes, sides adjacent to equal angles in proportion
b Yes, two pairs of equal matching angles
c Not similar

9 a 3.5 **b** 4 **c** 18

10 a $\angle C$ is common.
$\angle DBC = \angle EAC = 90°$ (given)
$\therefore \Delta BCD \,|||\, \Delta ACE$ (two pairs of equal matching angles)

b 5 m

Extended-response questions

1 a $\angle ABC + \angle BCF = 180°$ (co-interior angles, $AB \parallel CF$)
$\therefore \angle BCF = 70°$
$\therefore \angle DCF = 32°$
Now $\angle EDC + \angle DCF = 32° + 148°$
$= 180°$
$\therefore DE \parallel CF$ as co-interior angles add to 180°

b $\angle CDA = 40°$ (revolution)
Reflex $\angle BCD = 270°$ (revolution)
$\angle DAB = 30°$ (angle sum in isosceles triangle)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

$BADC$ quadrilateral angle sum 360°
$\therefore a = 360 - (30 + 270 + 40)$
$= 20$
or other proof.

c $AB = CD$ (given)
$\angle ABC = \angle DCB$ (given)
BC is common.
$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)
$\therefore AC = BD$ (matching sides in congruent triangles)

2 a $\angle ECD$

b $\angle ABC = \angle EDC$ (given 90°)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ABC \,|||\, \Delta EDC$ (two pairs of equal matching angles)

c 19.8 m

Chapter 8

Pre-test

1 a $2x + 6$ **b** $3a - 15$ **c** $12x - 8xy$ **d** $3 - 6b$

2 a 81 **b** 16 **c** 1 **d** 64
e 25 **f** 7 **g** 3 **h** −6

3 a 2 **b** 6 **c** $2x$ **d** $3y$
e $5x$ **f** ab

4 a $2(a + 3)$ **b** $3(x + 4y)$ **c** $5x(x - 3)$ **d** $2m(2 - 3n)$

5 a (1, 6), (2, 3), (−1, −6), (−2, −3)
b (1, 12), (2, 6), (3, 4), (−1, −12), (−2, −6), (−3, −4)
c (−1, 10), (1, −10), (−2, 5), (2, −5)
d (−1, 27), (1, −27), (−3, 9), (3, −9)

6 a $\frac{20}{21}$ **b** $\frac{11}{18}$ **c** $\frac{1}{12}$ **d** $1\frac{1}{2}$

7 a $3x + 2$ **b** $x + 4$ **c** $-10 - 3x$ **d** $5x - 2$

8 a $x = 6$ **b** $x = 11$ **c** $x = \frac{5}{8}$

9 a $x(x + 2)$, $x^2 + 2x$ **b** $(x + 3)(x + 4)$, $x^2 + 7x + 12$

Exercise 8A

1 a x^2, $2x$, $3x$, 6 **b** $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

2 a $2x^2$, $2x$, $3x$, 3
b $(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$

3 a $x^2 + 5x + x + 5 = x^2 + 6x + 5$
b $x^2 + 2x - 3x - 6 = x^2 - x - 6$
c $21x^2 + 6x - 14x - 4 = 21x^2 - 8x - 4$
d $12x^2 - 16x - 3x + 4 = 12x^2 - 19x + 4$

4 a $x^2 + 7x + 10$ **b** $b^2 + 7b + 12$
c $t^2 + 15t + 56$ **d** $p^2 + 12p + 36$
e $x^2 + 15x + 54$ **f** $d^2 + 19d + 60$
g $a^2 + 8a + 7$ **h** $y^2 + 12y + 20$
i $m^2 + 16m + 48$

5 a $x^2 - x - 12$ **b** $x^2 + 3x - 10$
c $x^2 - 4x - 32$ **d** $x^2 - 4x - 12$
e $x^2 + 9x - 10$ **f** $x^2 + 2x - 63$
g $x^2 + 5x - 14$ **h** $x^2 - 3x + 2$
i $x^2 - 9x + 20$ **j** $8x^2 + 26x + 15$
k $6x^2 + 7x + 2$ **l** $15x^2 + 17x + 4$
m $6x^2 + x - 15$ **n** $24x^2 + 23x - 12$
o $6x^2 - x - 2$ **p** $10x^2 - 31x - 14$
q $6x^2 + 5x - 6$ **r** $16x^2 - 16x - 5$
s $18x^2 - 27x + 10$ **t** $15x^2 - 11x + 2$
u $21x^2 - 37x + 12$

6 a $a^2 + ac + ab + bc$ **b** $a^2 + ac - ab - bc$
c $ab + bc - a^2 - ac$ **d** $xy - xz - y^2 + yz$
e $yz - y^2 - xz + xy$ **f** $1 + y - x - xy$
g $2x^2 - 3xy - 2y^2$ **h** $2a^2 - ab - b^2$
i $6x^2 + xy - y^2$ **j** $6a^2 + 4a - 3ab - 2b$
k $12x^2 - 25xy + 12y^2$ **l** $3x^2y - yz^2 - 2xyz$

7 a $x^2 + 9x + 20$
b **i** 56 m² **ii** 36 m²

8 a $2x^2$ **b** $2x^2 - 30x + 100$

9 a $150 - 50x + 4x^2$ **b** 66 m²

10 a 3 **b** 2 **c** 6, 6 **d** 2, 18
e 2, 6 **f** 3, 15 **g** $2x$, $5x$, 3 **h** $3x$, $15x$, 4
i $7x$, 3, $17x$ **j** $3x$, 4, $11x$

11 a $a = 3, b = 2$ or $a = 2, b = 3$
b $a = -3, b = -2$ or $a = -2, b = -3$
c $a = 3, b = -2$ or $a = -2, b = 3$
d $a = 2, b = -3$ or $a = -3, b = 2$

12 a $x^3 + 2x^2 + 2x + 1$ **b** $x^3 - 3x^2 + 5x - 6$
c $4x^3 - 4x^2 + 9x - 4$ **d** $x^3 + 2x^2 - 2x + 3$
e $10x^3 - 17x^2 + 7x - 6$ **f** $8x^3 - 18x^2 + 35x - 49$
g $x^3 + ax - a^2x + a^2$ **h** $x^3 - 2ax^2 + a^3$
i $x^3 + a^3$ **j** $x^3 - a^3$

13 $x^3 + 6x^2 + 11x + 6$

Exercise 8B

1 a $x^2 + 3x + 3x + 9 = x^2 + 6x + 9$
b $x^2 + 5x + 5x + 25 = x^2 + 10x + 25$
c $x^2 - 2x - 2x + 4 = x^2 - 4x + 4$
d $x^2 - 7x - 7x + 49 = x^2 - 14x + 49$

2 a C **b** B **c** E **d** A **e** D

3 a $x^2 + 10x + 4x - 16 = x^2 - 16$
b $x^2 + 4x - 10x - 100 = x^2 - 100$
c $4x^2 + 2x - 2x - 1 = 4x^2 - 1$
d $9x^2 - 12x + 12x - 16 = 9x^2 - 16$

4 a $x^2 + 2x + 1$ **b** $x^2 + 6x + 9$
c $x^2 + 4x + 4$ **d** $x^2 + 10x + 25$
e $x^2 + 8x + 16$ **f** $x^2 + 18x + 81$
g $x^2 + 14x + 49$ **h** $x^2 + 20x + 100$
i $x^2 - 4x + 4$ **j** $x^2 - 12x + 36$
k $x^2 - 2x + 1$ **l** $x^2 - 6x + 9$
m $x^2 - 18x + 81$ **n** $x^2 - 14x + 49$
o $x^2 - 8x + 16$ **p** $x^2 - 24x + 144$

5 a $4x^2 + 4x + 1$ **b** $4x^2 + 20x + 25$
c $9x^2 + 12x + 4$ **d** $9x^2 + 6x + 1$
e $25x^2 + 20x + 4$ **f** $16x^2 + 24x + 9$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

g $49 + 28x + 4x^2$ **h** $25 + 30x + 9x^2$
i $4x^2 - 12x + 9$ **j** $9x^2 - 6x + 1$
k $16x^2 - 40x + 25$ **l** $4x^2 - 36x + 81$
m $9x^2 + 30xy + 25y^2$ **n** $4x^2 + 16xy + 16y^2$
o $49x^2 + 42xy + 9y^2$ **p** $36x^2 + 60xy + 25y^2$
q $16x^2 - 72xy + 81y^2$ **r** $4x^2 - 28xy + 49y^2$
s $9x^2 - 60xy + 100y^2$ **t** $16x^2 - 48xy + 36y^2$

6 a $9 - 6x + x^2$ **b** $25 - 10x + x^2$
c $1 - 2x + x^2$ **d** $36 - 12x + x^2$
e $121 - 22x + x^2$ **f** $16 - 8x + x^2$
g $49 - 14x + x^2$ **h** $144 - 24x + x^2$
i $64 - 32x + 4x^2$ **j** $4 - 12x + 9x^2$
k $81 - 36x + 4x^2$ **l** $100 - 80x + 16x^2$

7 a $x^2 - 1$ **b** $x^2 - 9$ **c** $x^2 - 64$
d $x^2 - 16$ **e** $x^2 - 144$ **f** $x^2 - 121$
g $x^2 - 81$ **h** $x^2 - 25$ **i** $x^2 - 36$
j $25 - x^2$ **k** $4 - x^2$ **l** $49 - x^2$

8 a $9x^2 - 4$ **b** $25x^2 - 16$ **c** $16x^2 - 9$
d $49x^2 - 9y^2$ **e** $81x^2 - 25y^2$ **f** $121x^2 - y^2$
g $64x^2 - 4y^2$ **h** $100x^2 - 81y^2$ **i** $49x^2 - 25y^2$
j $36x^2 - 121y^2$ **k** $64x^2 - 9y^2$ **l** $81x^2 - 16y^2$

9 a i x^2 **ii** $x^2 - 4$
b No, they differ by 4.

10 a $20 - 2x$
b $(20 - 2x)(20 - 2x) = 400 - 80x + 4x^2$
c 196 cm^2 **d** 588 cm^3

11 a $a + b$ **b** $(a + b)(a + b) = a^2 + 2ab + b^2$
c $a - b$ **d** $(a - b)(a - b) = a^2 - 2ab + b^2$
e $4ab$ **f** ab so yes four courts area is $4ab$

12 a $x^2 - 1$ **b** No, area of rectangle is 1 square unit less

13 a $a^2 - b^2$
b i $(a - b)^2 = a^2 - 2ab + b^2$
ii $b(a - b) = ab - b^2$
iii $b(a - b) = ab - b^2$
c Yes, $a^2 - 2ab + b^2 + ab - b^2 + ab - b^2 = a^2 - b^2$

14 a $x^2 + 4x$ **b** $-4x + 1$
c $x^2 + 6x - 9$ **d** $-x^2 - 2x$
e 1 **f** $4x$
g $-12x - 8$ **h** $-10x + 2$
i $x^2 + 4xy - y^2$ **j** $8x^2 + 18$
k $-8x$ **l** $2x^2 - 12x + 18$
m $9x^2 - 48x + 48$ **n** $x^2 + 6xy + y^2$

Exercise 8C

1 a 4 **b** 10 **c** 5 **d** 6
e 1 **f** 25 **g** 8 **h** 36

2 a x **b** x **c** a
d $2a$ **e** $-2y$ **f** $-3x$
g $-2x$ **h** $-10x$ **i** $-2x$

3 a $7(x + 1)$ **b** $7(x + 2)$ **c** $7(2x + 1)$
d $7(2x - 1)$ **e** $-9(x - 1)$ **f** $9(-x + 1)$
g $6(1 - 2x)$ **h** $6(2 - x)$

4 a $2x$ **b** $6a$ **c** 2 **d** 4
e 3 **f** 1 **g** $3x$ **h** $3n$
i $2y$ **j** $2x$ **k** $2xy$ **l** $5ab$

5 a $7(x + 1)$ **b** $3(x + 1)$ **c** $4(x - 1)$
d $5(x - 1)$ **e** $4(1 + 2y)$ **f** $5(2 + a)$
g $3(1 - 3b)$ **h** $2(3 - x)$ **i** $3(4a + b)$
j $6(m + n)$ **k** $2(5x - 4y)$ **l** $4(a - 5b)$
m $x(x + 2)$ **n** $a(a - 4)$ **o** $y(y - 7)$
p $x(1 - x)$ **q** $3p(p + 1)$ **r** $8x(1 - x)$
s $4b(b + 3)$ **t** $2y(3 - 5y)$ **u** $3a(4 - 5a)$
v $9m(1 + 2m)$ **w** $16x(y - 3x)$ **x** $7ab(1 - 4b)$

6 a $-4(2x + 1)$ **b** $-2(2x + 1)$ **c** $-5(2x + y)$
d $-7(a + 2b)$ **e** $-3(3x + 4)$ **f** $-2(3y + 4)$
g $-5(2x + 3y)$ **h** $-4(m + 5n)$ **i** $-3x(x + 6)$
j $-4x(2x + 3)$ **k** $-2y(8y + 3)$ **l** $-5a(a + 2)$
m $-2x(3 + 10x)$ **n** $-3p(2 + 5p)$ **o** $-8b(2 + b)$
p $-9x(1 + 3x)$

7 a $(x + 3)(4 + x)$ **b** $(x + 1)(3 + x)$ **c** $(m - 3)(7 + m)$
d $(x - 7)(x + 2)$ **e** $(a + 4)(8 - a)$ **f** $(x + 1)(5 - x)$
g $(y + 3)(y - 2)$ **h** $(x + 2)(a - x)$ **i** $(2t + 5)(t + 3)$
j $(5m - 2)(m + 4)$ **k** $(4y - 1)(y - 1)$ **l** $(7 - 3x)(1 + x)$

8 a $6(a + 5)$ **b** $5(x - 3)$
c $2(4b + 9)$ **d** $x(x - 4)$
e $y(y + 9)$ **f** $a(a - 3)$
g $xy(x - 4 + y)$ **h** $2ab(3 - 5a + 4b)$
i $(m + 5)(m + 2)$ **j** $(x + 3)(x - 2)$
k $(b - 2)(b + 1)$ **l** $(2x + 1)(x - 1)$
m $(3 - 2y)(y - 5)$ **n** $(x + 4)(x + 9)$
o $(y + 1)(y - 3)$

9 a $4(x + 2)$ **b** $2(x + 3)$ **c** $10(x + 2)$
d $2(x + 7)$ **e** $2(2x + 3)$ **f** $2(x + 7)$

10 $4x$

11 a $t(5 - t)$
b i 0 m **ii** 6 m **iii** 4 m
c 5 seconds

12 a 63 **b** 72 **c** -20 **d** -70
e 69 **f** 189

13 a $3(a^2 + 3a + 4)$ **b** $z(5z - 10 + y)$
c $x(x - 2y + xy)$ **d** $2b(2y - 1 + 3b)$
e $-4y(3x + 2z + 5xz)$ **f** $ab(3 + 4b + 6a)$

14 a $-4(x - 3) = 4(3 - x)$ **b** $-3(x - 3) = 3(3 - x)$
c $-8(n - 1) = 8(1 - n)$ **d** $-3(b - 1) = 3(1 - b)$
e $-5m(1 - m) = 5m(m - 1)$ **f** $-7x(1 - x) = 7x(x - 1)$
g $-5x(1 - x) = 5x(x - 1)$
h $-2y(2 - 11y) = 2y(11y - 2)$
i $-4n(2 - 3n) = 4n(3n - 2)$ **j** $-4(2y - 5) = 4(5 - 2y)$
k $-5(3mn - 2) = 5(2 - 3mn)$ **l** $-15(x - 3) = 15(3 - x)$

15 a $(x - 4)(x - 3)$ **b** $(x - 5)(x + 2)$
c $(x - 3)(x + 3)$ **d** $(x - 4)(3x - 5)$
e $(2x - 5)(3 - x)$ **f** $(x - 2)(2x - 1)$
g $(x - 3)(4 - x)$ **h** $(x - 5)(x - 2)$
i $(x - 3)(x - 2)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 8D

1 **a** $x^2 - 4$ **b** $x^2 - 49$ **c** $4x^2 - 1$
d $x^2 - y^2$ **e** $9x^2 - y^2$ **f** $a^2 - b^2$

2 $(2x + 3)(2x - 3) = 4x^2 - 9$

3 **a** $(x + 4)(x - 4)$
b $x^2 - 12^2 = (x + 12)(x - 12)$
c $(4x)^2 - 1^2 = (4x + 1)(4x - 1)$
d $(3a)^2 - (2b)^2$

4 **a** $(x + 3)(x - 3)$ **b** $(y + 5)(y - 5)$
c $(y + 1)(y - 1)$ **d** $(x + 8)(x - 8)$
e $(x + 4)(x - 4)$ **f** $(b + 7)(b - 7)$
g $(a + 9)(a - 9)$ **h** $(x + y)(x - y)$
i $(a + b)(a - b)$ **j** $(4 + a)(4 - a)$
k $(5 + x)(5 - x)$ **l** $(1 + b)(1 - b)$
m $(6 + y)(6 - y)$ **n** $(11 + b)(11 - b)$
o $(x + 20)(x - 20)$ **p** $(30 + y)(30 - y)$

5 **a** $(2x + 5)(2x - 5)$ **b** $(3x + 7)(3x - 7)$
c $(5b + 2)(5b - 2)$ **d** $(2m + 11)(2m - 11)$
e $(10y + 3)(10y - 3)$ **f** $(9a + 2)(9a - 2)$
g $(1 + 2x)(1 - 2x)$ **h** $(5 + 8b)(5 - 8b)$
i $(4 + 3y)(4 - 3y)$ **j** $(6x + y)(6x - y)$
k $(2x + 5y)(2x - 5y)$ **l** $(8a + 7b)(8a - 7b)$
m $(2p + 5q)(2p - 5q)$ **n** $(9m + 2n)(9m - 2n)$
o $(5a + 7b)(5a - 7b)$ **p** $(10a + 3b)(10a - 3b)$

6 **a** $3(x + 6)(x - 6)$ **b** $10(a + 1)(a - 1)$
c $6(x + 2)(x - 2)$ **d** $4(y + 4)(y - 4)$
e $2(7 + x)(7 - x)$ **f** $8(2 + m)(2 - m)$
g $5(xy + 1)(xy - 1)$ **h** $3(1 + xy)(1 - xy)$
i $7(3 + ab)(3 - ab)$

7 **a** $(x + 8)(x + 2)$ **b** $(x + 5)(x + 1)$
c $(x + 14)(x + 6)$ **d** $(x + 2)(x - 8)$
e $(x - 6)(x - 8)$ **f** $(x + 3)(x - 9)$
g $(10 + x)(4 - x)$ **h** $-x(x + 4)$
i $(17 + x)(1 - x)$

8 **a** $4(3 + t)(3 - t)$
b **i** 36 m **ii** 20 m
c 3 seconds

9 **a** **i** x^2 **ii** $(30 + x)(30 - x)$
b **i** 500 cm^2 **ii** 675 cm^2

10 **a** $(x + 3)(x - 3)$ **b** $(4x + 11)(4x - 11)$
c $(2 + 5a)(2 - 5a)$ **d** $(x + y)(x - y)$
e $(2b + 5a)(2b - 5a)$ **f** $(c + 6ab)(c - 6ab)$
g $(yz + 4x)(yz - 4x)$ **h** $(b + 30a)(b - 30a)$

11 **a** Factorise each binomial
$(4x + 2)(4x - 2) = 2(2x + 1)\,2(2x - 1) = 4(2x + 1)(2x - 1)$
b Take out common factor of 4

12 $9 - (x - 1)^2$
$= (3 + x - 1)(3 - (x - 1))$ insert brackets when subtracting a binomial
$= (2 + x)(3 - x + 1)$ remember $-1 \times -1 = +1$
$= (2 + x)(4 - x)$

13 **a** $\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$ **b** $\left(x + \frac{2}{5}\right)\left(x - \frac{2}{5}\right)$
c $\left(5x + \frac{3}{4}\right)\left(5x - \frac{3}{4}\right)$ **d** $\left(\frac{x}{3} + 1\right)\left(\frac{x}{3} - 1\right)$
e $\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} - \frac{b}{3}\right)$ **f** $5\left(\frac{x}{3} + \frac{1}{2}\right)\left(\frac{x}{3} - \frac{1}{2}\right)$
g $7\left(\frac{a}{5} + \frac{2b}{3}\right)\left(\frac{a}{5} - \frac{2b}{3}\right)$ **h** $\frac{1}{2}\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} - \frac{b}{3}\right)$
i $(x + y)(x - y)(x^2 + y^2)$ **j** $2(a+b)(a - b)(a^2+b^2)$
k $21(a + b)(a - b)(a^2 + b^2)$ **l** $\frac{1}{3}(x + y)(x - y)(x^2 + y^2)$

Exercise 8E

1 **a** $2 + a$ **b** $3 - a$ **c** $5 - a$ **d** $a + 4$
e $a + 1$ **f** $a - 1$ **g** $1 - a$ **h** $1 + 2a$

2 **a** $(x - 3)(x - 2)$ **b** $(x + 4)(x + 3)$
c $(x - 7)(x + 4)$ **d** $(2x + 1)(3 - x)$
e $(3x - 2)(4 - x)$ **f** $(2x + 3)(2x - 3)$
g $(5 - x)(3x + 2)$ **h** $(x + 1)(2 - 3x)$
i $(x - 2)(x + 1)$

3 **a** $x^2 + 3x + 4x + 12 = (x^2 + 3x) + (4x + 12)$
$= x(x + 3) + 4(x + 3)$
$= (x + 3)(x + 4)$
b $x^2 + 3x - 4x - 12 = (x^2 + 3x) - (4x + 12)$
$= x(x + 3) - 4(x + 3)$
$= (x + 3)(x - 4)$

4 **a** $(x + 3)(x + 2)$ **b** $(x + 4)(x + 3)$
c $(x + 7)(x + 2)$ **d** $(x - 6)(x + 4)$
e $(x - 4)(x + 6)$ **f** $(x - 3)(x + 10)$
g $(x + 2)(x - 18)$ **h** $(x + 3)(x - 14)$
i $(x + 4)(x - 18)$ **j** $(x - 2)(x - a)$
k $(x - 3)(x - 3c)$ **l** $(x - 5)(x - 3a)$

5 **a** $(3a + 5c)(b + d)$ **b** $(4b - 7c)(a + d)$
c $(y - 4z)(2x + 3w)$ **d** $(s - 2)(5r + t)$
e $(x + 3y)(4x - 3)$ **f** $(2b - a)(a - c)$

6 **a** $(x - b)(x + 1)$ **b** $(x - c)(x + 1)$
c $(x + b)(x + 1)$ **d** $(x + c)(x - 1)$
e $(x + a)(x - 1)$ **f** $(x - b)(x - 1)$

7 **a** $(x - 7)(2x + 1)$ **b** $(x + 5)(x + 2)$
c $(x + 3)(2x - 1)$ **d** $(1 - 2x)(3x + 4)$
e $(x - 5)(11 + a)$ **f** $(3 - 2x)(4y - 1)$
g $(n + 2)(3m - 1)$ **h** $(3 - r)(5p + 8)$
i $(2 - y)(8x + 3)$

8 **a** $x^2 + 4x - ax - 4a$ **b** $x^2 - dx - cx + cd$
c $2x - xz + 2y - yz$ **d** $ax + bx - a - b$
e $3cx - 3bx - bc + b^2$ **f** $2xy + 2xz - y^2 - yz$
g $6ab + 15ac + 2b^2 + 5bc$ **h** $3my + mz - 6xy - 2xz$

9 **a** $(x + 2)(x + 5)$ **b** $(x + 3)(x + 5)$
c $(x + 4)(x + 6)$ **d** $(x - 3)(x + 2)$
e $(x + 6)(x - 2)$ **f** $(x - 9)(x - 2)$

10 **a** Method 1: $a(x + 7) - 3(x + 7) = (x + 7)(a - 3)$
Method 2: $x(a - 3) + 7(a - 3) = (a - 3)(x + 7)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b **i** $(x-3)(b+2)$ **ii** $(x+2)(y-4)$
iii $(2m+3)(2m-5n)$ **iv** $(2-n)(m-3)$
v $(1-2b)(4a+3b)$ **vi** $(3a-1)(b+4c)$

11 Answers will vary.

12 **a** $(a-3)(2-x-c)$ **b** $(2a+1)(b+5-a)$
c $(a+1)(x-4-b)$ **d** $(a-b)(3-b-2a)$
e $(1-a)(c-x+2)$ **f** $(x-2)(a+2b-1)$
g $(a-3c)(a-2b+3bc)$ **h** $(1-2y)(3x-5z+y)$
i $(x-4)(3x+y-2z)$ **j** $(ab-2c)(2x+3y-1)$

Exercise 8F

1 **a** x^2+4x+3 **b** $x^2+9x+14$ **c** $x^2+8x-33$
d x^2+x-30 **e** $x^2+7x-60$ **f** $x^2+9x-52$
g $x^2-8x+12$ **h** $x^2-31x+220$ **i** $x^2-10x+9$

2 **a** 3, 2 **b** 5, 2 **c** 12, 1
d 4, 5 **e** 5, −1 **f** −7, 1
g 5, −3 **h** −6, 5 **i** −3, −2
j −9, −2 **k** −5, −8 **l** −50, −2

3 **a** $(x+2)(x+1)$ **b** $(x+3)(x+1)$ **c** $(x+6)(x+1)$
d $(x+9)(x+1)$ **e** $(x+7)(x+1)$ **f** $(x+14)(x+1)$
g $(x+4)(x+2)$ **h** $(x+3)(x+4)$ **i** $(x+8)(x+2)$
j $(x+5)(x+3)$ **k** $(x+4)(x+5)$ **l** $(x+8)(x+3)$

4 **a** $(x+4)(x-1)$ **b** $(x+2)(x-1)$ **c** $(x+5)(x-1)$
d $(x+7)(x-2)$ **e** $(x+5)(x-3)$ **f** $(x+10)(x-2)$
g $(x+6)(x-3)$ **h** $(x+9)(x-2)$ **i** $(x+4)(x-3)$

5 **a** $(x-5)(x-1)$ **b** $(x-1)(x-1)$ **c** $(x-1)(x-4)$
d $(x-8)(x-1)$ **e** $(x-2)(x-2)$ **f** $(x-6)(x-2)$
g $(x-9)(x-2)$ **h** $(x-7)(x-3)$ **i** $(x-3)(x-2)$

6 **a** $(x-8)(x+1)$ **b** $(x-4)(x+1)$ **c** $(x-6)(x+1)$
d $(x-8)(x+2)$ **e** $(x-6)(x+4)$ **f** $(x-5)(x+3)$
g $(x-4)(x+3)$ **h** $(x-12)(x+1)$ **i** $(x-6)(x+2)$

7 **a** $2(x+4)(x+1)$ **b** $2(x+10)(x+1)$ **c** $3(x+2)(x+4)$
d $2(x+10)(x-3)$ **e** $2(x-9)(x+2)$ **f** $4(x-1)(x-1)$
g $2(x-2)(x+3)$ **h** $6(x-6)(x+1)$ **i** $5(x-4)(x-2)$
j $3(x-5)(x-6)$ **k** $2(x-5)(x+2)$ **l** $3(x-4)(x+3)$

8 **a** $(x+3)(x+2)$ **b** $(x-3)(x-2)$ **c** $(x+6)(x-1)$
d $(x-6)(x+1)$ **e** $(x+3)(x+4)$ **f** $(x-3)(x-4)$
g $(x+3)(x-4)$ **h** $(x-3)(x+4)$ **i** $(x-12)(x+1)$

9 **a** $6x$ **b** $6x$ or $10x$ **c** x, $4x$, $11x$
d x, $4x$, $11x$ **e** $9x$, $11x$, $19x$ **f** $9x$, $11x$, $19x$
g $0x$, $6x$, $15x$ **h** $0x$, $24x$

10 **a** **i** $x(x+2)$ **ii** $x^2+2x-15$ **iii** $(x+5)(x-3)$
b **i** $105\,\text{m}^2$ **ii** $48\,\text{m}^2$

11 **a** $(x+4)^2$ **b** $(x+5)^2$ **c** $(x+15)^2$
d $(x-1)^2$ **e** $(x-7)^2$ **f** $(x-13)^2$
g $2(x+1)^2$ **h** $5(x-3)^2$ **i** $-3(x-6)^2$

12 Answers will vary.

13 B, C, E, F

14 **a** $(x-1)^2-9$ **b** $(x+2)^2-5$ **c** $(x+5)^2-22$
d $(x-8)^2-67$ **e** $(x+9)^2-74$ **f** $(x-16)^2-267$

Exercise 8G

1 **a** 2, 3 **b** 2, 6 **c** 5, −2 **d** 8, −3
e −6, −3 **f** −5, −7 **g** −10, 3 **h** −7, 4

2 **a** $= 2x^2+2x+5x+5$
$= 2x(x+1)+5(x+1)$
$= (x+1)(2x+5)$
b $= 3x^2+6x+2x+4$
$= 3x(x+2)+2(x+2)$
$= (x+2)(3x+2)$
c $= 2x^2-3x-4x+6$
$= x(2x-3)-2(2x-3)$
$= (2x-3)(x-2)$
d $= 5x^2+10x-x-2$
$= 5x(x+2)-1(x+2)$
$= (x+2)(5x-1)$
e $= 4x^2+8x+3x+6$
$= 4x(x+2)+3(x+2)$
$= (x+2)(4x+3)$
f $= 6x^2-9x+2x-3$
$= 3x(2x-3)+1(2x-3)$
$= (2x-3)(3x+1)$

3 **a** $(2x+1)(x+4)$ **b** $(3x+1)(x+2)$
c $(2x+3)(x+2)$ **d** $(3x+2)(x+2)$
e $(5x+2)(x+2)$ **f** $(2x+3)(x+4)$
g $(3x+5)(2x+1)$ **h** $(4x+1)(x+1)$
i $(4x+5)(2x+1)$

4 **a** $(2x+3)(x+5)$ **b** $(2x-3)(x-5)$
c $(2x-3)(x+5)$ **d** $(2x+3)(x-5)$
e $(x+3)(2x+5)$ **f** $(x+3)(2x-5)$
g $(x-3)(2x-5)$ **h** $(x-3)(2x+5)$
i $(2x-3)(2x+5)$

5 **a** $(3x+5)(x-1)$ **b** $(5x-4)(x+2)$
c $(2x+3)(4x-1)$ **d** $(2x+1)(3x-8)$
e $(2x+1)(5x-4)$ **f** $(x-3)(5x+4)$
g $(2x-5)(2x-3)$ **h** $(x-6)(2x-3)$
i $(2x-5)(3x-2)$ **j** $(3x-4)(4x+1)$
k $(2x-3)(2x-3)$ **l** $(x+3)(7x-3)$
m $(x+5)(9x-1)$ **n** $(x-2)(3x-8)$
o $(2x-5)(2x+3)$

6 **a** $(2x+1)(5x+11)$ **b** $(3x+4)(5x-2)$
c $(2x-3)(10x-3)$ **d** $(2x+1)(9x-5)$
e $(5x-3)(5x+4)$ **f** $(4x+1)(8x-5)$
g $(3x+2)(9x-4)$ **h** $(3x+1)(11x+10)$
i $(6x-5)(9x+1)$ **j** $(2x-3)(6x-7)$
k $(3x-1)(25x-6)$ **l** $(6x-1)(15x+8)$

7 **a** $2(3x-2)(5x+1)$ **b** $6(x-1)(2x+5)$
c $3(3x-5)(3x-1)$ **d** $7(x-3)(3x-2)$
e $4(3x-2)(3x+5)$ **f** $5(2x-3)(5x+4)$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

8 **a** $-(x-2)(2x-3)$ **b** $-(x-1)(5x+8)$
c $-(2x+1)(3x-8)$ **d** $-(x+3)(5x-6)$
e $-(2x-5)(2x-3)$ **f** $-(2x-1)(4x-5)$

9 **a** $(x+4)(2x-5)$ **b** $(2x-5)(x+4)$
c No, you get the same result.
d **i** $(x+3)(3x-4)$ **ii** $(x-2)(5x+7)$
iii $(2x-1)(3x+4)$

10 Answers will vary.

11 See answers to Questions **4** and **5**.

Exercise 8H

1 **a** $\frac{1}{3}$ **b** $\frac{3}{2}$ **c** $\frac{x}{2}$ **d** $\frac{7x}{2}$
e $\frac{3}{x}$ **f** $\frac{1}{2x}$ **g** $\frac{x+1}{2}$ **h** $2(x-4)$

2 **a** $3(x+2)$ **b** $20(1-2x)$ **c** $x(x-7)$ **d** $6x(x+4)$

3 **a** $\frac{2(x-2)}{8} = \frac{x-2}{4}$ **b** $\frac{6(2-3x)}{x(2-3x)}$
c $\frac{(x+3)(x+2)}{2(x-1)}$ **d** $\frac{(x+3)\cancel{(x-3)}}{2\cancel{(x-3)}} = \frac{x+3}{2}$

4 **a** $\frac{3}{4}$ **b** $\frac{1}{3}$ **c** 4
d $x-5$ **e** $\frac{2(x-1)}{3}$ **f** $\frac{2}{x+4}$

5 **a** $x-1$ **b** $\frac{2(x-3)}{5}$ **c** $\frac{2}{3}$ **d** 2
e $x-3$ **f** $\frac{2(2x+5)}{5}$ **g** $\frac{3}{2}$ **h** $\frac{4}{3}$

6 **a** $x+2$ **b** $x+4$ **c** $x-3$ **d** $\frac{1}{x+3}$
e $\frac{1}{x-2}$ **f** $\frac{1}{x-10}$

7 **a** $\frac{x-4}{2}$ **b** $x-3$ **c** $3(x-3)$ **d** $\frac{x+4}{2(x-5)}$

8 **a** x **b** $\frac{2(x-2)}{x+2}$ **c** $\frac{4}{x+1}$
d $\frac{x-2}{2(x+2)}$ **e** $\frac{2(x+2)^2}{3(x+4)}$ **f** $\frac{5}{x+2}$

9 **a** $x-10$ **b** $x-7$ **c** $x-5$
d $\frac{2}{x+20}$ **e** $\frac{5}{x+6}$ **f** $\frac{3}{x-9}$

10 **a** $(x+2)(x-5)$ **b** $\frac{x+2}{x+3}$
c $\frac{x-3}{x-4}$ **d** $\frac{x-5}{x+2}$
e $\frac{3x-1}{2-15x}$ **f** $\frac{x-4}{3-x} = -\frac{x-4}{x-3}$
g $\frac{x+5}{2(x+4)}$ **h** $\frac{3}{2(x+3)}$

11 **a** -1 **b** -1 **c** -8
d $-\frac{1}{3}$ **e** $-\frac{1}{6}$ **f** $-(x+3)$

12 **a** $a+1$ **b** $5(a-3)$ **c** $\frac{x+7}{2}$
d $\frac{x+2}{6}$ **e** $\frac{x+3}{2}$ **f** $\frac{11}{x-2}$

13 **a** $-\frac{x+3}{2}$ **b** $-\frac{x-7}{3}$ **c** $-\frac{x-8}{x+8}$ **d** $-\frac{2}{x+2}$
e $-\frac{4}{x+3}$ **f** $\frac{3(x+1)}{4}$ **g** $\frac{3(2x+3)}{2x}$ **h** $-\frac{1}{2(x+2)}$
i $\frac{x}{3}$ **j** $-\frac{2}{x-2}$

Exercise 8I

1 **a** $\frac{5w}{12}$ **b** $\frac{3x+5}{10}$ **c** $\frac{3x}{11}$ **d** $\frac{2-x}{5}$

2 **a** $4x$ **b** $21x$ **c** 3
d 2 **e** 8 **f** 90

3 **a** $\frac{3x}{12} + \frac{8x}{12} = \frac{11x}{12}$ **b** $\frac{25x}{35} - \frac{14x}{35} = \frac{11x}{35}$
c $\frac{2(x+1)}{4} + \frac{(2x+3)}{4} = \frac{2x+2+2x+3}{4} = \frac{4x+5}{4}$

4 **a** 15 **b** 14 **c** 8
d 6 **e** 10

5 **a** $\frac{9x}{14}$ **b** $\frac{2x}{5}$ **c** $\frac{x}{8}$ **d** $\frac{14x}{45}$
e $\frac{y}{56}$ **f** $\frac{13a}{22}$ **g** $\frac{2b}{9}$ **h** $\frac{m}{6}$
i $\frac{11m}{12}$ **j** $\frac{15a}{28}$ **k** $\frac{x}{2}$ **l** $-\frac{20p}{63}$
m $-\frac{5b}{18}$ **n** $\frac{61y}{40}$ **o** $\frac{13x}{35}$ **p** $\frac{5x}{12}$

6 **a** $\frac{7x+11}{10}$ **b** $\frac{7x}{12}$ **c** $\frac{15a-51}{56}$ **d** $\frac{11y+9}{30}$
e $\frac{13m+28}{40}$ **f** $\frac{5x-13}{24}$ **g** $\frac{11b-6}{24}$ **h** $\frac{7x}{6}$
i $\frac{7y-8}{14}$ **j** $\frac{5t-4}{16}$ **k** $\frac{34-10x}{21}$ **l** $\frac{8m-9}{12}$

7 **a** $\frac{11}{2x}$ **b** $\frac{1}{3x}$ **c** $-\frac{3}{4x}$ **d** $\frac{14}{9x}$
e $\frac{7}{20x}$ **f** $\frac{13}{15x}$ **g** $-\frac{31}{4x}$ **h** $-\frac{29}{12x}$

8 **a** $\frac{3x+2}{x^2}$ **b** $\frac{5+4x}{x^2}$ **c** $\frac{7x+3}{x^2}$ **d** $\frac{4x-5}{x^2}$
e $\frac{3-8x}{x^2}$ **f** $\frac{x-4}{x^2}$ **g** $\frac{6x-7}{2x^2}$ **h** $\frac{9-2x}{3x^2}$

9 **a** $\frac{8+x^2}{4x}$ **b** $\frac{x^2-10}{2x}$ **c** $\frac{-6-4x^2}{3x}$ **d** $\frac{6-5x^2}{4x}$
e $\frac{9x^2-10}{12x}$ **f** $\frac{3-x^2}{9x}$ **g** $\frac{15x^2-4}{10x}$ **h** $\frac{-25-6x^2}{20x}$

10 **a** $\frac{x}{3}$ **b** $\frac{x}{8}$ **c** $\frac{x}{2}$
d $\frac{x}{5}$ **e** $\frac{8x}{9}$ **f** $\frac{x}{4}$

11 **a** Didn't make a common denominator, $\frac{7x}{15}$
b Didn't use brackets: $2(x+1) = 2x+2$, $\frac{7x+2}{10}$
c Didn't use brackets: $3(x-1) = 3x-3$, $\frac{3x-3}{6}$
d Didn't multiply numerator in $\frac{2}{x}$ by x as well as denominator, $\frac{2x-3}{x^2}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

12 **a** $\frac{8x+2}{8} = \frac{2(4x+1)}{8} = \frac{4x+1}{4}$ **b** $\frac{4x+1}{4}$

c Using denominator 8 does not give answer in simplified form and requires extra steps. Preferable to use actual LCD.

13 **a** $-\frac{43x}{30}$ **b** $\frac{5x}{12}$ **c** $\frac{13x}{24}$ **d** $\frac{43x-5}{60}$

e $\frac{23x-35}{42}$ **f** $\frac{29x+28}{40}$ **g** $\frac{14}{3x}$ **h** $\frac{1}{6x}$

i $-\frac{11}{20x}$ **j** $\frac{24-x}{6x^2}$ **k** $\frac{60x-21}{14x^2}$ **l** $\frac{3-4x}{9x^2}$

m $\frac{30-2x^2}{15x}$ **n** $\frac{11x^2-3}{6x}$ **o** $\frac{18-2x^2}{45x}$

Exercise 8J

1 **a** $-2x-6$ **b** $-5x-5$ **c** $-14-21x$

d $-3x+3$ **e** $-30+20x$ **f** $-16+64x$

2 **a** 9 **b** 16 **c** x^2 **d** $2x$

e $(x-1)(x+1)$ **f** $(x-2)(x+3)$

g $(2x-1)(x-4)$ **h** $(x+1)^2$

3 **a** $\frac{1-x}{12}$ **b** $\frac{2x-14}{15}$ **c** $\frac{x-9}{6}$

d $\frac{-7x-14}{10}$ **e** $\frac{9x-4}{8}$ **f** $\frac{-x-24}{28}$

g $\frac{5x-3}{12}$ **h** $\frac{1-18x}{15}$ **i** $\frac{8x-23}{30}$

4 **a** $\frac{13-x}{6}$ **b** $\frac{2x+2}{35}$ **c** $\frac{x-5}{4}$

d $\frac{22x-41}{21}$ **e** $\frac{17x}{20}$ **f** $\frac{30-13x}{24}$

g $\frac{14x+4}{9}$ **h** $\frac{20x-9}{28}$ **i** $\frac{10-11x}{56}$

5 **a** $\frac{7x-1}{(x-1)(x+1)}$ **b** $\frac{7x-7}{(x+4)(x-3)}$

c $\frac{7x+1}{(x-2)(x+3)}$ **d** $\frac{5x+13}{(x-4)(x+7)}$

e $\frac{4x+15}{(x+2)(x+3)}$ **f** $\frac{x-26}{(x+4)(x-6)}$

g $\frac{x+9}{(x+5)(x+1)}$ **h** $\frac{16-6x}{(x-3)(x-2)}$

i $\frac{7-2x}{(x-5)(x-6)}$

6 **a** $\frac{1-3x}{(x+1)^2}$ **b** $\frac{-4x-10}{(x+3)^2}$ **c** $\frac{3x-2}{(x-2)^2}$

d $\frac{18-2x}{(x-5)^2}$ **e** $\frac{9-x}{(x-6)^2}$ **f** $\frac{14-3x}{(x-4)^2}$

g $\frac{4x+7}{(2x+1)^2}$ **h** $\frac{1-12x}{(3x+2)^2}$ **i** $\frac{20x-1}{(1-4x)^2}$

7 **a** $\frac{5x-2}{(x-1)^2}$ **b** $\frac{19x+8}{12x}$ **c** $\frac{10x^2-11x+4}{20x}$

d $\frac{x^2+7x}{(x-5)(x+1)}$ **e** $\frac{2x^2-5x-3}{(4-x)(x-1)}$ **f** $\frac{x^2+2x+1}{(x-3)^2}$

g $\frac{3-2x}{(x-2)^2}$ **h** $\frac{-x^2-11x}{(2x+1)(x+2)}$ **i** $\frac{x^2-4x-1}{(x+1)^2}$

8 **a** second line $-2\times(-2) = +4$ not -4 **b** $\frac{33x+4}{10}$

9 **a** $\frac{3x+11}{(x+3)(x+4)(x+5)}$ **b** $\frac{2-2x}{(x+1)(x+2)(x+4)}$

c $\frac{26-10x}{(x-1)(x-3)(2-x)}$ **d** $\frac{3x-2}{(x+1)(x-5)}$

e $\frac{2x+9}{(x-4)(3-2x)}$ **f** $\frac{7x^2+7x}{(x+4)(2x-1)(3x+2)}$

10 **a** $\frac{5}{1-x}$ **b** $\frac{4x-3}{5-x}$ **c** $\frac{9}{7x-3}$

d $\frac{1-2x}{4-3x}$ **e** 1 **f** 0

11 **a** $\frac{11}{2(x+2)}$ **b** $\frac{1}{3(x-1)}$

c $\frac{23}{4(2x-1)}$ **d** $\frac{13-3x}{(x+3)(x-3)}$

e $\frac{5x-6}{2(x+2)(x-2)}$ **f** $\frac{30x+33}{(3x-4)(3x+4)}$

g $\frac{9x-27}{(x+3)(x+4)(x-5)}$ **h** $\frac{x+11}{(x-1)(x+3)^2}$

i $\frac{2x+5}{5(x-2)(x-5)}$ **j** $\frac{-2}{x(x-1)(x+1)}$

Exercise 8K

1 **a** 15 **b** 4 **c** 6

d 28 **e** 30 **f** 8

2 **a** $4x$ **b** $3x$ **c** $2(x+3)$

d $2x+5$ **e** 3 **f** $-(x+2)$

g $4(x-1)$ **h** $2(1-x)$ **i** $2x-1$

3 **a** 10 **b** 12 **c** 24

d 10 **e** $\frac{6}{7}$ **f** $7\frac{1}{2}$

g -8 **h** 15 **i** 4

4 **a** 13 **b** 1 **c** $1\frac{3}{5}$

d 59 **e** 1 **f** 5

g 8 **h** $3\frac{2}{7}$ **i** $-3\frac{1}{4}$

5 **a** -3 **b** -4 **c** 5

d 1 **e** -1 **f** 6

6 **a** $\frac{1}{16}$ **b** $\frac{1}{12}$ **c** $\frac{8}{15}$

d $\frac{1}{36}$ **e** $\frac{3}{4}$ **f** $\frac{5}{24}$

g $\frac{5}{12}$ **h** $-\frac{1}{6}$ **i** $2\frac{5}{6}$

7 **a** $-\frac{5}{2}$ **b** -5 **c** -19

d -4 **e** -1 **f** -6

8 **a** $\frac{x}{2}+\frac{2x}{3}=4$ **b** $x=3\frac{3}{7}$

9 **a** $\frac{1}{6}$ **b** 2 **c** 0

d $\frac{9}{13}$ **e** $\frac{3}{7}$ **f** $\frac{2}{11}$

g $1\frac{5}{7}$ **h** 0 **i** -12

10 **a** $\frac{x}{3}+\frac{x}{4}=77$ **b** 132 games

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

11 On the second line, not every term has been multiplied by 12. The $2x$ should be $24x$ to give an answer $x = \frac{3}{29}$.

12 On third line of working, $-2 \times (-1) = +2$ not -2, giving answer $x = -4$

13 **a** $4\frac{1}{2}$ **b** $5\frac{2}{3}$ **c** $1\frac{1}{2}$
d 3 **e** 15 **f** -2

14 **a** $x = 2ab$ **b** $x = \frac{2bd}{2a - bc}$
c $x = \frac{ac}{c - b}$ **d** $x = \frac{bd + be - ac}{c}$
e $x = \frac{4c - 3b}{3a - 4}$ **f** $x = \frac{6b + a}{5}$
g $x = -\frac{a^2}{2a - b - a^2}$ **h** $x = \frac{ac}{c - a}$
i $x = \frac{ac}{b}$ **j** $x = \frac{c - a}{b}$
k $x = \frac{b - bc}{a - c}$ **l** $x = \frac{ad - b}{c - d}$
m $x = ab - 2a$ **n** $x = \frac{a + b}{1 - a}$
o $x = \frac{2ab - a^2}{a - b}$

Puzzles and challenges

1 **a** 48, 49 **b** 33, 35 **c** 12, 15
2 **a** 15 **b** 5
3 $a = 2$, $b = 1$, $c = 7$ and $d = 8$
4 **a** $(n + 1)^2 + 1$ or $n^2 + 2n + 2$
b **i, ii** A number of answers
5 $(n - 1)(n + 1)$
a n is prime and greater than 3, so n is odd. Therefore, $n - 1$ and $n + 1$ are even. Therefore, $n - 1$ or $n + 1$ must be divisible by 4. Therefore, $(n - 1) \times (n + 1)$ is divisible by 4.
b Since $n - 1$, n and $n + 1$ are 3 consecutive numbers,one of them must be divisible by 3. Since n is prime, it must be $n - 1$ or $n + 1$, so their product is divisible by 3.
c $n^2 - 1$ is divisible by 3 and 4 and since they have no common factor it must also be divisible by $3 \times 4 = 12$.
6 Factorise each expression and cancel.
7 $4x^2 - 4x + 1 = (2x - 1)^2$, which is always greater than or equal to zero
8 Ryan

Multiple-choice questions

1 A **2** E **3** D **4** C **5** B
6 D **7** A **8** E **9** B **10** A

Short-answer questions

1 **a** $x^2 + x - 12$ **b** $x^2 - 9x + 14$
c $6x^2 - 5x - 6$ **d** $9x^2 + 3x - 12$
2 **a** $x^2 + 6x + 9$ **b** $x^2 - 8x + 16$
c $9x^2 - 12x + 4$ **d** $x^2 - 25$
e $49 - x^2$ **f** $121x^2 - 16$
3 **a** $4(a + 3b)$ **b** $3x(2 - 3x)$
c $-5xy(x + 2)$ **d** $(x - 7)(x + 3)$
e $(2x + 1)(x - 1)$ **f** $(x - 2)(x - 6)$
4 **a** $(x + 10)(x - 10)$ **b** $3(x + 4)(x - 4)$
c $(5x + y)(5x - y)$ **d** $(7 + 3x)(7 - 3x)$
e $(x - 12)(x + 6)$ **f** $(1 - x)(1 + x)$
5 **a** $(x - 3)(x + 2y)$
b $(2a + 5)(2x - 1)$
c $(x - 4)(3 + 2b)$
6 **a** $(x + 3)(x + 5)$ **b** $(x - 6)(x + 3)$
c $(x - 6)(x - 1)$ **d** $3(x + 7)(x - 2)$
e $2(x + 4)^2$ **f** $(5x + 2)(x + 3)$
g $(2x - 3)(2x + 1)$ **h** $(3x - 4)(2x - 3)$
7 **a** $x + 4$ **b** $\frac{2}{3}$ **c** $\frac{x - 3}{5}$
8 **a** $\frac{1}{4}$ **b** $\frac{x - 4}{2}$ **c** $\frac{x}{3}$
d $\frac{2}{5}$ **e** 4 **f** $\frac{2x + 3}{50x}$
9 **a** $\frac{11x}{12}$ **b** $\frac{x - 13}{24}$ **c** $\frac{5}{4x}$
d $\frac{7x - 2}{x^2}$ **e** $\frac{8x + 11}{(x + 1)(x + 2)}$ **f** $\frac{15 - 2x}{(x - 4)^2}$
10 **a** $x = 20$ **b** $x = \frac{1}{6}$ **c** $x = 7$
d $x = -1\frac{2}{9}$

Extended-response questions

1 **a** $(x + 3)$ m
b **i** No change
ii $(x^2 - 1)$ m^2, 1 square metre less in area
c **i** width $= (x - 3)$ m, decreased by 3 metres
ii $A = (x + 7)(x - 3) = (x^2 + 4x - 21)$ m^2
iii $A = 0$ m^2
2 **a** 400 m^2
b **i** $L = W = (20 + 2x)$ m
ii $(4x^2 + 80x + 400)$ m^2
c $\frac{1}{4}$
d $4x(x + 20)$ m^2
e $x = 5$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 9

Pre-test

1 **a** 0.1 **b** 0.25 **c** 0.3 **d** 0.85 **e** 0.237

2 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** 1 **e** 0

f $\frac{1}{12}$ **g** $\frac{1}{2}$ **h** $\frac{18}{29}$ **i** $\frac{2}{3}$ **j** $\frac{2}{7}$

3 **a** 1, 2, 3, 4, 5 or 6

b **i** 3 **ii** 2 **iii** 3

iv 5 **v** 5 **vi** 3

4 **a** 5 **b** 7 **c** 6 **d** 7 **e** 4 **f** 6

5 **a** 8 **b** 20 **c** $\frac{3}{20}$

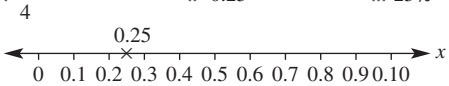
6 **a** 6 **b** 5 **c** 5 **d** 12

e **i** $\frac{2}{9}$ **ii** $\frac{7}{9}$ **iii** $\frac{5}{9}$

Exercise 9A

1 **a** **i** $\frac{1}{4}$ **ii** 0.25 **iii** 25%

b



2

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	0 0.5 1 x
b	25%	0.25	$\frac{1}{4}$	0 0.25 0.5 0.75 1 x
c	75%	0.75	$\frac{3}{4}$	0 0.25 0.5 0.75 1 x
d	20%	0.2	$\frac{1}{5}$	0 0.2 0.4 0.6 0.8 1 x
e	60%	0.6	$\frac{3}{5}$	0 0.2 0.4 0.6 0.8 1 x
f	85%	0.85	$\frac{17}{20}$	0 0.85 1 x

3 0.15, $\frac{2}{9}$, 1 in 4, 0.28, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, 2 in 3, 0.7, 0.9

4 **a** **i** {1, 2, 3, 4, 5, 6, 7} **ii** $\frac{1}{7}$

iii $\frac{6}{7}$ **iv** $\frac{2}{7}$ **v** $\frac{6}{7}$

b **i** {2, 2, 6, 7} **ii** $\frac{1}{2}$

iii $\frac{1}{2}$ **iv** $\frac{1}{2}$ **v** 1

c **i** {1, 2, 2, 2, 2, 3} **ii** $\frac{2}{3}$

iii $\frac{1}{3}$ **iv** $\frac{5}{6}$ **v** $\frac{5}{6}$

d **i** {1, 1, 2, 2, 3, 3} **ii** $\frac{1}{3}$

iii $\frac{2}{3}$ **iv** $\frac{2}{3}$ **v** $\frac{2}{3}$

e **i** {1, 1, 2, 3, 3, 4, 4} **ii** $\frac{1}{7}$

iii $\frac{6}{7}$ **iv** $\frac{3}{7}$ **v** $\frac{5}{7}$

f **i** {2, 2, 2, 2} **ii** 1 **iii** 0

iv 1 **v** 1

5 **a** $\frac{1}{2}$ **b** $\frac{3}{8}$ **c** $\frac{1}{6}$ **d** $\frac{1}{4}$ **e** 1 **f** 0

6 **a** $\frac{1}{2}$ **b** $\frac{5}{8}$ **c** $\frac{5}{6}$ **d** $\frac{3}{4}$ **e** 0 **f** 1

7 **a** $\frac{1}{8}$ **b** $\frac{7}{8}$

8 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{3}{8}$ **e** $\frac{5}{8}$ **f** 1

g 0 **h** $\frac{1}{4}$ **i** $\frac{3}{4}$ **j** $\frac{3}{4}$

9 **a** {Hayley, Alisa, Rocco, Stuart}

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$

10 **a** $\frac{1}{52}$ **b** $\frac{1}{13}$ **c** $\frac{1}{26}$ **d** $\frac{1}{2}$ **e** $\frac{2}{13}$

f $\frac{12}{13}$ **g** $\frac{23}{26}$ **h** $\frac{25}{26}$

11 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{3}$ **e** $\frac{2}{3}$

f 1 **g** $\frac{1}{3}$ **h** $\frac{1}{2}$ **i** $\frac{5}{6}$

12 **a** $\frac{2}{11}$ **b** $\frac{9}{11}$ **c** $\frac{4}{11}$ **d** $\frac{7}{11}$ **e** $\frac{7}{11}$

f $\frac{3}{11}$ **g** $\frac{7}{11}$ **h** $\frac{4}{11}$

13 The sample space has four elements as the letter O appears twice {S, L, O, O}. Amanda has only considered the name of the letter, not the total number of elements in the sample space. $P(\text{S}) = \frac{1}{4}$.

14 A, D, F

15 **a** $\frac{12}{25}$ **b** $\frac{8}{25}$ **c** $\frac{1}{5}$ **d** $\frac{9}{25}$ **e** $\frac{8}{25}$

f $\frac{16}{25}$ **g** $\frac{4}{25}$ **h** $\frac{17}{25}$ **i** 0

16 **a** 31 min **b** $\frac{3}{31}$

c **i** $\frac{4}{31}$ **ii** $\frac{4}{31}$ **iii** $\frac{20}{31}$

iv $\frac{5}{31}$ **v** $\frac{8}{31}$ **vi** $\frac{11}{31}$

Exercise 9B

1 **a** 26

b **i** 10 **ii** 14 **iii** 5 **iv** 9

v 4 **vi** 7 **vii** 19

c **i** 12 **ii** 17

2 **a** B **b** D **c** A **d** C

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

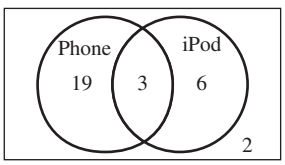
3 a

	A	Not *A*	Total
B	7	8	15
Not *B*	3	1	4
Total	10	9	19

b

	A	Not *A*	Total
B	2	5	7
Not *B*	9	4	13
Total	11	9	20

4 a



b i 28 ii 21 iii 6

c i $\frac{1}{10}$ ii $\frac{1}{15}$ iii $\frac{19}{30}$

5 a i $\frac{2}{5}$ ii $\frac{1}{3}$ iii $\frac{7}{15}$

iv $\frac{1}{15}$ v $\frac{13}{15}$ vi $\frac{2}{15}$

b i $\frac{3}{7}$ ii $\frac{12}{35}$ iii $\frac{13}{35}$

iv $\frac{3}{35}$ v $\frac{34}{35}$ vi $\frac{1}{35}$

6 a

	Cream	Not cream	Total
Ice-cream	5	20	25
Not ice-cream	16	9	25
Total	21	29	50

b i 29 ii 9

c i $\frac{21}{50}$ ii $\frac{8}{25}$ iii $\frac{41}{50}$

7 a i $\frac{5}{8}$ ii $\frac{3}{8}$ iii $\frac{3}{8}$

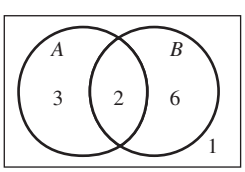
iv $\frac{3}{4}$ v $\frac{1}{8}$ vi $\frac{1}{4}$

b i $\frac{17}{26}$ ii $\frac{9}{26}$ iii $\frac{11}{26}$

iv $\frac{21}{26}$ v $\frac{2}{13}$ vi $\frac{5}{26}$

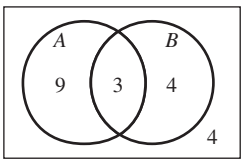
8 a

	A	Not *A*	Total
B	2	6	8
Not *B*	3	1	4
Total	5	7	12

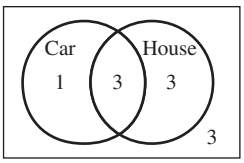


b

	A	Not *A*	Total
B	3	4	7
Not *B*	9	4	13
Total	12	8	20



9 a



b 3 **c** $\frac{1}{10}$

10 a

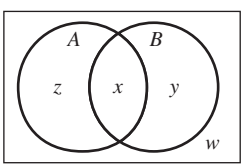
	Rain water	No rain water	Total
Tap water	12	36	48
No tap water	11	41	52
Total	23	77	100

b 12 **c** $\frac{9}{25}$ **d** $\frac{59}{100}$

11 23 **12** $\frac{7}{15}$

13

	A	Not *A*	Total
B	x	y	$x + y$
Not *B*	z	w	$w + z$
Total	$x + z$	$y + w$	$w + x + y + z$



14 a $T - x - y + z$ **b** $T - x$

c $T - y$ **d** $x - z$

e $y - z$ **f** $y - z$

g $T - x + z$

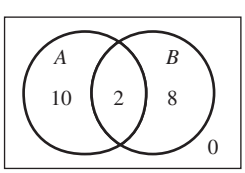
15

	A	Not *A*	Total
B	⓪	12	12
Not *B*	11	−4	7
Total	11	⑧	19

Filling in table so totals add up requires a negative number, which is impossible!

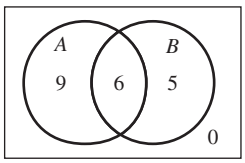
16 4

17 a i

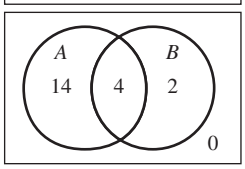


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

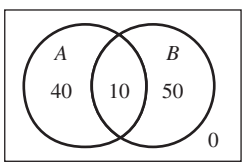
ii



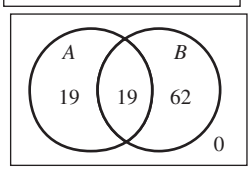
iii



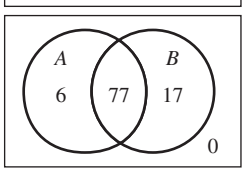
b i



ii



iii



c Overlap = $(A + B)$ − total

Exercise 9C

1 **a** C **b** D **c** E **d** A **e** F **f** B

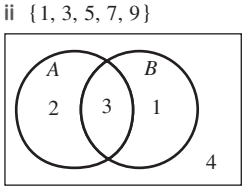
2 **a** B **b** D **c** A **d** C

3 **a** 2 **b** 10 **c** 7 **d** 9

4 **a** **i** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

ii {1, 3, 5, 7, 9} **iii** {2, 3, 5, 7}

b

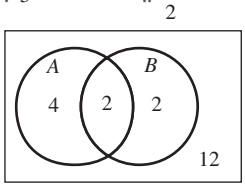


c **i** {3, 5, 7} **ii** {1, 2, 3, 5, 7, 9}

iii {2, 4, 6, 8, 10} **iv** {2}

d **i** 5 **ii** $\frac{1}{2}$ **iii** 3 **iv** $\frac{3}{10}$

5 **a**



b **i** {3, 15}

ii {1, 3, 5, 6, 9, 12, 15, 18}

iii {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}

iv {1, 5}

c **i** 4 **ii** $\frac{1}{5}$ **iii** 2 **iv** $\frac{1}{10}$ **v** 8 **vi** $\frac{2}{5}$

6 **a** **i** False **ii** True **iii** False **iv** True

v True **vi** False **vii** True **viii** False

b **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** 0

7 **a** **i** $\frac{1}{6}$ **ii** $\frac{11}{12}$ **iii** $\frac{5}{12}$

b **i** $\frac{6}{25}$ **ii** $\frac{21}{25}$ **iii** $\frac{8}{25}$

c **i** $\frac{1}{3}$ **ii** $\frac{11}{15}$ **iii** $\frac{2}{5}$

d **i** $\frac{5}{21}$ **ii** $\frac{17}{21}$ **iii** $\frac{4}{7}$

8 **a** **i** {Fred, Ron, Rachel} **ii** {Fred, Rachel, Helen}

iii {} **iv** {Fred, Rachel}

b **i** $\frac{3}{4}$ **ii** $\frac{1}{4}$ **iii** 0

iv 1 **v** $\frac{1}{2}$ **vi** 1

9 **a** 26 **b** 5 **c** 3 **d** 18

e $\frac{5}{26}$ **f** $\frac{21}{26}$ **g** $\frac{3}{26}$ **h** $\frac{5}{13}$

10 **a** 11 **b** 21 **c** 1 **d** $\frac{9}{25}$

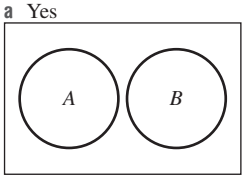
11 **a** Own both a dog and a cat

b Own dogs or cats or both

c Does not own a cat

d Owns a cat but no dogs

12 **a** Yes



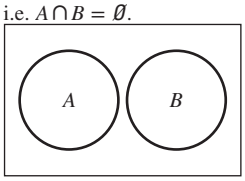
b No, as $A \cup B$ includes only elements from sets A and B

13 B only $= B \cap \overline{A}$

14

	A	$\overline{A}$	Total
B	$n(A \cap B)$	$n(B \cap \overline{A})$	$n(B)$
$\overline{B}$	$n(A \cap \overline{B})$	$n(\overline{A} \cap \overline{B})$	$n(\overline{B})$
Total	$n(A)$	$n(\overline{A})$	n(sample space)

15 Mutually exclusive events have no common elements, i.e. $A \cap B = \emptyset$.



16 **a** **i** {2, 3, 5, 7, 11, 13, 17, 19}

ii {1, 2, 3, 4, 6, 12}

iii {2, 3}

iv {1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 17, 19}

v {1, 4, 6, 12} **vi** {5, 7, 11, 13, 17, 19}

vii {1, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20}

viii {2, 3, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20}

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

ix {1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

b **i** $\frac{3}{5}$ **ii** $\frac{1}{10}$ **iii** $\frac{1}{5}$ **iv** $\frac{2}{5}$
v $\frac{4}{5}$ **vi** $\frac{9}{10}$ **vii** $\frac{9}{10}$ **viii** $\frac{2}{5}$

c They are equal.

Exercise 9D

1 **a** 9 outcomes

	1	**2**	**3**
1	(1, 1)	(2, 1)	(3, 1)
2	(1, 2)	(2, 2)	(3, 2)
3	(1, 3)	(2, 3)	(3, 3)

b 12 outcomes

	1	**2**	**3**	**4**	**5**	**6**
H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

c 6 outcomes

	A	**B**	**C**
A	×	(B, A)	(C, A)
B	(A, B)	×	(C, B)
C	(A, C)	(B, C)	×

d 6 outcomes

			1st	
		•	∘	∘
2nd	•	×	(∘, •)	(∘, •)
	∘	(•, ∘)	×	(∘, ∘)
	∘	(•, ∘)	(∘, ∘)	×

2 **a** Table A **b** Table B

c **i** $\frac{1}{9}$ **ii** $\frac{1}{6}$

d **i** 5 **ii** 4

3 **a**

		A	
		•	∘
B	•	(•, •)	(•, ∘)
	∘	(•, ∘)	(∘, ∘)
	∘	(•, ∘)	(∘, ∘)

b 6

c **i** $\frac{1}{3}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$

4 **a**

	1	**2**	**3**	**4**
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

Sample space = {(1, 1), (2, 1), … (4, 4)} i.e. all pairs from table.

b $\frac{1}{16}$ **c** $\frac{1}{4}$

5 **a**

	1	**2**	**3**	**4**	**5**	**6**
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

b 36

c **i** $\frac{1}{36}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{12}$ **iv** $\frac{5}{9}$

6 **a**

	D	**O**	**G**
D	×	(O, D)	(G, D)
O	(D, O)	×	(G, O)
G	(D, G)	(O, G)	×

b $\frac{1}{6}$ **c** $\frac{2}{3}$

7 **a**

				1st	
		1	**2**	**3**	**4**
2nd	**1**	×	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	×	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	×	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	×

b **i** $\frac{1}{12}$ **ii** $\frac{1}{12}$

c **i** $\frac{1}{6}$ **ii** $\frac{5}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

8 **a**

	1	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b **i** $\frac{1}{16}$ **ii** $\frac{3}{16}$ **iii** $\frac{3}{8}$
iv $\frac{3}{16}$ **v** $\frac{13}{16}$

9 **a**

	A	**B**	**C**	**D**	**E**
A	(A, A)	(B, A)	(C, A)	(D, A)	(E, A)
B	(A, B)	(B, B)	(C, B)	(D, B)	(E, B)
C	(A, C)	(B, C)	(C, C)	(D, C)	(E, C)
D	(A, D)	(B, D)	(C, D)	(D, D)	(E, D)
E	(A, E)	(B, E)	(C, E)	(D, E)	(E, E)

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b i $\frac{1}{25}$ ii $\frac{1}{5}$ iii $\frac{4}{5}$

c i $\frac{8}{25}$ ii $\frac{1}{25}$

10 a i $\frac{1}{36}$ ii $\frac{1}{12}$ iii $\frac{1}{12}$ iv $\frac{7}{12}$

v $\frac{5}{12}$ vi $\frac{1}{6}$ vii $\frac{1}{6}$ viii 0

b 7, $\frac{1}{6}$

11 a i 169 ii 156

b i 25 ii 12

12 Yes, because if one O is removed, another remains to be used.

13 a $\frac{1}{25}$ **b** $\frac{1}{20}$

14 7, 8, 9, 10, 11

15 a $\frac{1}{2500}$ **b** $\frac{1}{50}$ **c** $\frac{1}{1250}$ **d** $\frac{49}{2500}$ **e** $\frac{23}{1250}$

16 a Without replacement

b 2652 **c** $\frac{1}{1326}$

d i $\frac{1}{221}$ ii $\frac{1}{17}$

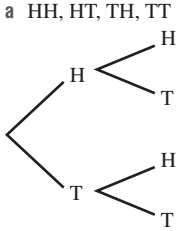
Exercise 9E

1 a HHH, HHT, HTH, HTT, THH, THT, TTH, TTT (8 outcomes)

b TT, TO, OT, OO (4 outcomes)

2 a 6 outcomes **b** 6 outcomes

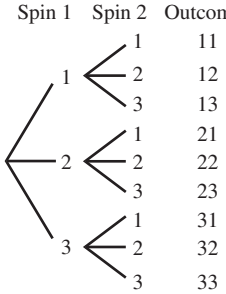
3 a HH, HT, TH, TT



b 4

c i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{3}{4}$ iv $\frac{3}{4}$

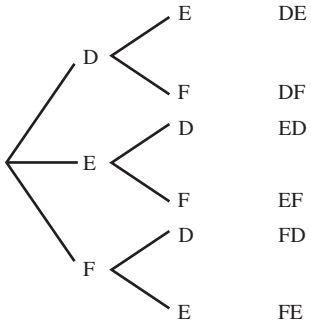
4 a Spin 1 Spin 2 Outcomes



b 9

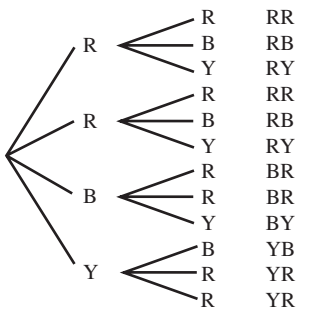
c i $\frac{1}{9}$ ii $\frac{5}{9}$ iii $\frac{8}{9}$ iv $\frac{4}{9}$

5 a Person 1 Person 2 Outcomes



b i $\frac{1}{3}$ ii $\frac{2}{3}$ iii 1

6 a Sock 1 Sock 2 Outcomes

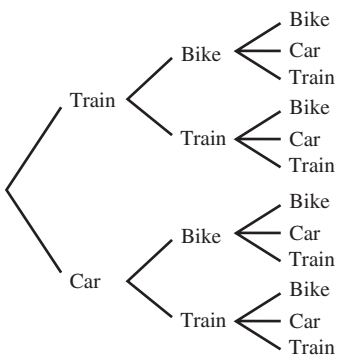


b i $\frac{1}{3}$ ii $\frac{1}{6}$ iii $\frac{1}{6}$ iv $\frac{5}{6}$

7 a $\frac{4}{9}$ **b** $\frac{2}{9}$ **c** $\frac{1}{27}$ **d** $\frac{8}{27}$

8 a $\frac{1}{16}$ **b** $\frac{1}{16}$ **c** $\frac{5}{16}$ **d** $\frac{11}{16}$ **e** $\frac{15}{16}$

9 a To A To B To C



b 12

c i $\frac{1}{12}$ ii $\frac{1}{3}$ iii $\frac{1}{2}$ iv $\frac{1}{4}$ v $\frac{2}{3}$

10 a $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{1}{4}$ **e** $\frac{1}{16}$

11 a 32 **b** 2^n

12 Yes, there is a difference. Probability of obtaining two of the same colour is lower without replacement.

13 a $\frac{1}{9}$ **b** $\frac{1}{9}$ **c** $\frac{7}{18}$ **d** $\frac{7}{18}$ **e** $\frac{1}{2}$

f 0 **g** $\frac{1}{6}$ **h** $\frac{2}{3}$ **i** $\frac{1}{9}$ **j** $\frac{17}{18}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 9F

1 **a** B: $\frac{5}{20} = 0.25$ C: $\frac{30}{100} = 0.3$

b C, as it is a larger sample size.

2 **a** 5 **b** 10 **c** 50 **d** 4

3 0.41, from the 100 throws as the more times an experiment is carried out, the closer the experimental probability becomes to the actual/theoretical probability.

4 **a** 0.6

b **i** 60 **ii** 120 **iii** 360

5 **a** $\frac{7}{8}$

b **i** 350 **ii** 4375 **iii** 35

6 **a** $\frac{1}{15}$ **b** $\frac{2}{5}$ **c** $\frac{11}{60}$

7 **a** 20 **b** 40 **c** 60 **d** 40

8 **a** 20

b **i** $\frac{1}{4}$ **ii** $\frac{7}{20}$

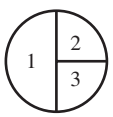
c **i** 25 **ii** 20 **iii** 45

9 **a** 0.64 **b** SEE

10 **a** **i** 0.52 **ii** 0.48 **iii** 0.78

b 78

c $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$



11 **a** $\frac{9}{10}$

b No; as the number of throws increases, the experiment should produce results closer to the theoretical probability $\left(\frac{1}{6}\right)$.

12 **a** Fair, close to 0.5 chance of tails

b Biased, nearly all results are heads

c Can't determine on such a small sample

13 **a** $\frac{\text{Shaded area}}{\text{Total area}} = 0.225 \quad \therefore 100 \text{ shots} \approx 23$

b $\frac{1}{10} \times 100 = 10$

c $\frac{150 - 32}{150} \times 100 \approx 79$

d $\frac{225\pi - 25\pi}{225\pi} \times 100 \approx 89$

14 **a** False **b** True **c** False **d** True

15 3 blue, 2 red, 4 green, 1 yellow

16 2 strawberry, 3 caramel, 2 coconut, 4 nut, 1 mint

Exercise 9G

1 **a** common **b** middle **c** mean

2

	Mean	Median	Mode
a	3	2	2
b	8	9	10
c	7	7	7
d	7	7	3
e	15	16	20
f	7	7	2

3

	Mean	Median	Mode
a	6	7	8
b	8	6	5, 10
c	6	6	2
d	11	12	none
e	4	3.5	2.1
f	5	4.5	none
g	3	3.5	−3
h	0	2	3

4 **a** Outlier = 33, mean = 12, median = 7.5

b Outlier = −1.1, mean = 1.075, median = 1.4

c Outlier = −4, mean = −49, median = −59

5 **a** Yes **b** No **c** No **d** Yes

6 **a** 24.67 s **b** 24.8 s

7 **a** 15 **b** 26

8 **a** 90 **b** 60

9 9

10 Answers may vary.

a 1, 4, 6, 7, 7 is a set **b** 2, 3, 4, 8, 8 is a set

c 4, 4, 4, 4, 4 is a set **d** 2.5, 2.5, 3, 7, 7.5 is a set

e −3, −2, 0, 5, 5 is a set **f** $0, \frac{1}{2}, 1\frac{1}{4}, 1\frac{1}{4}, 2$ is a set

11 **a** $1 700 000

b ($354 500 and $324 000) drops $30 500

c ($570 667 and $344 800) drops $225 867

12 An outlier has a large impact on the addition of all the scores and therefore significantly affects the mean. An outlier does not move the middle of the group of scores significantly.

13 **a** 15 **b** 1.2 **c** 1 **d** 1

e No, as it will still lie in the 1 column.

14 **a** 2 **b** 3 **c** 7

15 **a** 76.75

b **i** 71.4, B^+ **ii** 75, B^+ **iii** 80.2, A

c 81.4, he cannot get an A^+

d **i** 43 **ii** 93

e $\frac{307 + m}{5}$

f $m = 5M - 307$

Exercise 9H

1 **a** 9

b **i** 8 min **ii** 35 min

c 21 min **d** 21 min

2 **a** 26

b **i** 0 mm **ii** 6 mm

c **i** 21 mm **ii** 24 mm

d **i** 8 mm **ii** 15 mm

e Skewed **f** Symmetrical

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

3 a i

Stem	Leaf
1	9 9
2	3 4 6 6 8 8 8
3	2 2 3 5
4	1
5	4

2 | 3 means 23

ii 28 **iii** 28 **iv** Skewed

b i

Stem	Leaf
1	5 5
2	1 3 3 3 6
3	0 1 1 3 4 5 5 7 9
4	2 2 5 5 5 8
5	0 0 1

5 | 0 means 50

ii 35 **iii** 23, 45

iv Almost symmetrical

c i

Stem	Leaf
32	0
33	3 7 8
34	3 4 5 5 7 8 9
35	2 2 4 5 8
36	1 3 5
37	0

34 | 3 means 34.3

ii 34.85 **iii** 34.5, 35.2

iv Almost symmetrical

d i

Stem	Leaf
15	7 8 9 9 9 9
16	1 4 7 7 7
17	3 5 7 7
18	5 5 7 9
19	3 8
20	0 2

16 | 1 means 161

ii 173 **iii** 159 **iv** Skewed

4 a

Stem	Leaf
0	5 6 6 8 8 9
1	0 1 1 2 2 2 4 4 5 6 7
2	0 1 2

1 | 2 means 12

b 9 **c** 12

5 a i

Set A		Set B
3 2	3	1
9	3	
3 3 0 0	4	0 1 3 4 4
8 7 6 5	4	6 7 8 8 9 9
4 4 3	5	1 3

3 | 2 means 32

ii Set A has values spread between 32 and 54 while Set B has most of its values between 40 and 53 with an outlier at 31.

b i

Set A		Set B
9 8 6 4 3 2 2 1	0	9
7 7 6 1 1 0 0	1	8 9 9
9 6 4 3 3 2	2	0 1 3 4 7 8
7 6 6 5 0	3	1 7 7 8 9 9 9
8 3 1	4	0 1 1 3 4 4 5 7
0	5	0 0 1 3 3 4 4 5 6

1 | 8 means 18

ii Set A has values between 1 and 50 with the frequency decreasing as the numbers increase whereas Set B has values between 9 and 56 with the frequency increasing as the numbers increase.

c i

Set A		Set B
8 7 4 3	0	1 1 1 2 3 6 6 9 9
9 9 7 6 4	1	2 3 5 8 9
6 5 3 1	2	1 5 6
9 9 6 4 3 2	3	3 4 9
7 3 1	4	3 7 8
7 3 2	5	2 3 7
2 1	6	1 2
8 3	8	3 8
1	9	

1 | 4 means 1.4

ii Set A and set B are similar. The frequency decreases as the numbers increase.

6 a

Collingwood		St Kilda
8 3	6	6 8 8
8 7 2 1 0	7	8
9 9 8 2 0	8	0 0 2 2 3 4 7 8
8 7 5 1	9	0 4
4 3 3 3	10	6 9
9 8 5	11	1 3 7 8
7	12	2 5 6
	13	8

7 | 8 means 78 points

b Collingwood $33\frac{1}{3}\%$, St Kilda $41\frac{2}{3}\%$

c Collingwood is almost symmetrical data and based on these results seem to be consistent. St Kilda has groups of similar scores and while less consistent, they have higher scores.

7 a 9 days **b** 18°C **c** 7°C

8 a 16.1 s **b** 2.3 s

c Yes, 0.05 lower

9 a

Battery lifetime

Brand A		Brand B
3	7	2 3 4
	7	5 6 8 9 9
4 2	8	0 1 3
9 8 7 5	8	
4 4 3 2 1 0	9	0 1 2 3 4
9 9 8 7 6 5 5	9	5 6 8 8

8 | 3 represents 8.3 hours

b Brand A, 12; brand B, 8

c Brand A consistently performs better than brand B.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 a $c = 2$ **b** 0.02

c i 5, 6, 7 or 8 **ii** 0, 1, 2, 3, 4, 5 or 6

11 a 48% **b** 15%

c In general, birth weights of babies are lower for mothers who smoke.

12 In symmetrical data, the mean is close to the median as the data is spread evenly from the centre with an even number of data values with a similar difference from the median above and below it.

13 a 52 **b** 17.8

14 a Mean = 41.06, median = 43

b Median because more of the scores exist in the higher stems but a few low scores lower the mean.

c The mean is higher for positively skewed data because the majority of the scores are in the lower stems. However, a few high scores increase the mean.

d Symmetrical data

Exercise 9I

1 a 3 **b** 360000–370000

2 a $a = 15$ $b = 2$ $c = 30\%$ $d = 40\%$ $e = 20\%$ $f = 100\%$

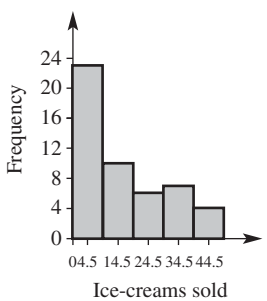
b $a = 50$ $b = 28\%$ $c = 12\%$ $d = 40$ $e = 100\%$

3 a 2 **b** 20 **c i** 30% **ii** 65%

4 a

Class	Class centre	Frequency	Percentage frequency
0–9	4.5	23	46
10–19	14.5	10	20
20–29	24.5	6	12
30–39	34.5	7	14
40–49	44.5	4	8
		50	100

b

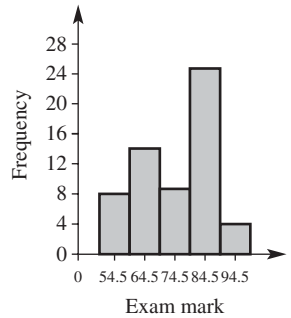


c i 33 **ii** 11 **d** 34%

5 a

Class	Class centre	Frequency	Percentage frequency
50–59	54.5	8	13.33
60–69	64.5	14	23.33
70–79	74.5	9	15
80–89	84.5	25	41.67
90–99	94.5	4	6.67
		60	100

b

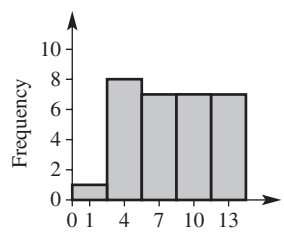


c i 22 **ii** 48.3%

6 a

Number of goals	Class centre	Frequency
0–2	1	1
3–5	4	8
6–8	7	7
9–11	10	7
12–14	13	7
		30

b



c 9 **d** 7

7 a Symmetrical data **b** Skewed data

8 a $a = 6$ $b = 27.5$ $c = 17.5$ $d = 4$ $e = 12$ $f = 30$ $g = 100$

b $a = 1$ $b = 18$ $c = 8$ $d = 24$ $e = 20$ $f = 40$ $g = 100$

9 a i 20% **ii** 55% **iii** 80%

iv 75% **v** 50%

b i 30 **ii** 45 **c i** 2 **ii** 22

10 85.5%

11 Because you only have the number of scores in the class interval not the individual scores

12 The number of data items within each class

13 a Student A

b Make the intervals for their groups of data smaller so that the graph conveys more information.

14 a Minimum wage \$204, maximum wage \$940

b i

Weekly wages ($)	Class centre	Frequency
200–299	249.5	2
300–399	349.5	7
400–499	449.5	6
500–599	549.5	6
600–699	649.5	11
700–799	749.5	11
800–899	849.5	5
900–999	949.5	2

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

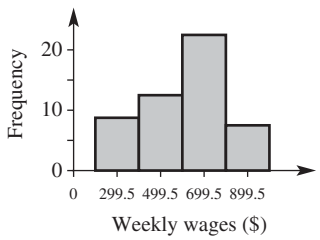
ii



c i

Weekly wages	Class centre	Frequency
200–399	299.5	9
400–599	499.5	12
600–799	699.5	22
800–999	899.5	7

ii



d More intervals shows greater detail. Since first graph has each pair of intervals quite similar, these two graphs are quite similar.

Exercise 9J

1 a 6 **b** 7 **c i** 5 **ii** 8 **d** 3

2 a 12 **b** 5 **c i** 3 **ii** 8 **d** 5

3

	Range	Q_2	Q_1	Q_3	IQR
a	11	6	2.5	8.5	6
b	23	30	23.5	36.5	13
c	170	219	181.5	284.5	103
d	851	76	28	367	339
e	1.3	1.3	1.05	1.85	0.8
f	34.98	10	0.1	23	22.9

4 a 110 min

b i 119.5 min **ii** 106 min

iii 130 min **iv** 24 min

c The middle 50% of videos rented varied in length by 24 min.

5 a i $12 000 **ii** $547 000 **iii** $71 500

iv $46 000 **v** $78 000 **vi** $32 000

b The middle 50% of prices differs by less than $32 000.

c No effect on Q_1, Q_2 or Q_3 but the mean would increase.

6 a 17.5 **b** 2.1

7 a 2 **b** 2

c No, only the highest value has changed so no impact on IQR.

8 No, because the range is the difference in the highest and lowest score and different sets of two numbers can have the same difference ($10 - 8 = 2$, $22 - 20 = 2$).

9 a Yes, the lowering of the highest price reduces the range.

b No, the middle price will not change.

c No, because only one value, the highest has changed, yet it still remains the highest, Q_1 and Q_3 remain unchanged.

10 a Yes (3, 3, 3, 4, 4, 4; IQR $= 4 - 3 = 1$; range $= 1$)

b Yes (4, 4, 4, 4, 4 has IQR $= 0$)

11 a i $Q_1 = 25; Q_2 = 26; Q_3 = 27$

ii $Q_1 = 22; Q_2 = 24.5; Q_3 = 27$

b 27 jelly beans **c** 22 jelly beans

d i IQR $= 2$ **ii** IQR $= 5$

e Shop B is less consistent than shop A and its data is more spread out.

f Shop A

Exercise 9K

1 a Minimum value **b** Lower quartile, Q_1

c Median, Q_2 **d** Upper quartile, Q_3

e Maximum value **f** Scale

g Whisker **h** Box

2 a minimum, maximum

b lower quartile Q_1, upper quartile Q_3

3 D

4 a min $= 35$ cm, max $= 60$ cm

b 25 cm **c** 50 cm

d 10 cm **e** 50 cm

f 55 cm **g** 20 babies

5 a min $= 0$ rabbits, max $= 60$ rabbits

b 60 **c** 25

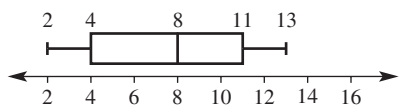
d 25 **e** 35

f 25 **g** 25

6 a i 2 **ii** 13 **iii** 8

iv 4 **v** 11

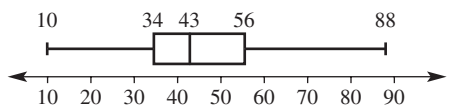
vi



b i 10 **ii** 88 **iii** 43

iv 34 **v** 56

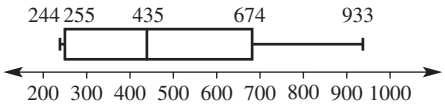
vi



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

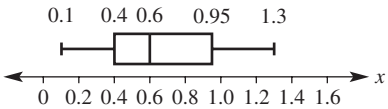
c **i** 244 **ii** 933 **iii** 435 **iv** 255 **v** 674

vi

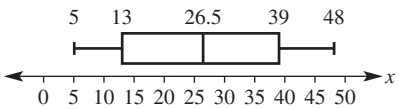


d **i** 0.1 **ii** 1.3 **iii** 0.6 **iv** 0.4 **v** 0.95

vi



7 **a**



b **i** 100% **ii** 50% **iii** 75% **iv** 25%

8 **a** Africa **b** 10 kg **c** Yes, 25 kg

d **i** 50% **ii** 75% **e** African

9 **a** Mac **b** 75% **c** 50%

d They are the same. Mac has 100% of times within same range as the middle 50% of PC start up times. Mac is more consistent.

10 **a** Waldren **b** Yeng **c** Yeng

d Yeng, smaller range and IQR

e Yeng, higher median

11 **a** For example: 1, 3, 5, 6, 6 **b** 1, 4, 4, 4, 5, 6

12 No, the median is anywhere within the box, including at times at its edges.

13 Yes, one reason is if an outlier is the highest score, then the mean > Q_3 [e.g. 1, 1, 3, 5, 6, 7, 256].

14 **a** $Q_1 = 4$

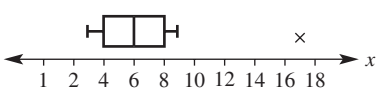
$Q_2 = 6$

$Q_3 = 8$

b Yes, 17

c 9

d



e Answers will vary.

Puzzles and challenges

1 **a** $2^5 = 32$ **b** $\frac{3}{16}$ **c** $\frac{31}{32}$

2 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$

3 **a** **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$

b $\frac{1}{6}$ **c** $\frac{1}{12}$ **d** $\frac{1}{3}$

4 **a** Mean and median increase by 5, range unchanged

b Mean, median and range all double

c Mean and median double and decrease by 1, range doubles

5 $5m$

6 **a** 3, 5, 7, 8, 10 or 3, 5, 7, 9, 10 **b** 2, 2, 8, 12

7 1, 4, 6, 7, 7; 2, 3, 6, 7, 7 and 1, 2, 5, 6, 6

8 $\frac{3}{8}$

Multiple-choice questions

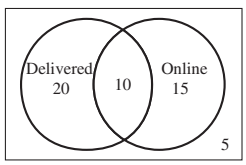
1 B **2** A **3** A **4** B **5** D

6 B **7** E **8** C **9** B **10** E

Short-answer questions

1 **a** $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{3}{5}$

2 **a**



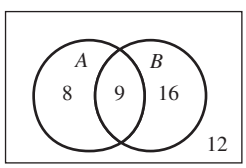
b 15

c **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{10}$

3 **a**

	A	$\overline{A}$	**Total**
B	9	16	25
$\overline{B}$	8	12	20
Total	17	28	45

b



c **i** $\frac{4}{9}$ **ii** $\frac{1}{5}$ **iii** $\frac{8}{45}$ **iv** $\frac{33}{45}$

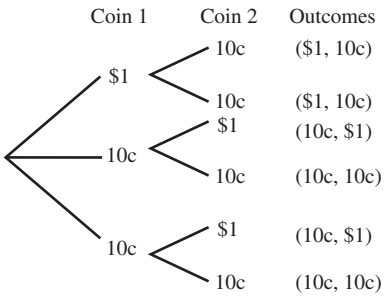
4 **a** 12 outcomes

	1	**2**	**3**	**4**
Red	(red, 1)	(red, 2)	(red, 3)	(red, 4)
Green	(green, 1)	(green, 2)	(green, 3)	(green, 4)
Blue	(blue, 1)	(blue, 2)	(blue, 3)	(blue, 4)

b **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{2}{3}$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

5 a



b $\frac{2}{3}$

6 a $\frac{48}{120} = \frac{2}{5}$ **b** 14

7 a 26 **b** 26.5 **c** 23, 31

8 a

Employee 1		Employee 2
9	1	7 7
5 3 1	2	0 4 8 9
9 8 6 5 5 4	3	2 7 7 8
6 5 0	4	0 1 2 5 8
3 1	5	

2 | 4 means 24

b **i** Employee 1, 36; employee 2, 37

ii Employee 1, 36; employee 2, 33

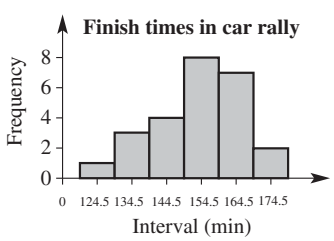
c Employee 1, they have a higher mean and more sales at the high end.

d Employee 1 symmetrical, employee 2 skewed

9 a

Class interval	Class centre	Frequency	Percentage frequency
120–129	124.5	1	4
130–139	134.5	3	12
140–149	144.5	4	16
150–159	154.5	8	32
160–169	164.5	7	28
170–179	174.5	2	8
Total		25	100

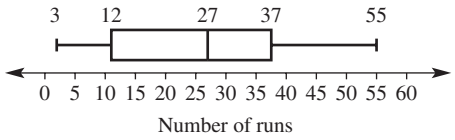
b



c **i** 4 **ii** 60%

10 a 52 runs

b



c 37 runs

Extended-response questions

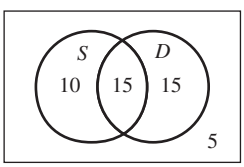
1 a **i**

		1st spin				
		1	**2**	**3**	**4**	**5**
2nd spin	**1**	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)

ii $\frac{12}{25}$ **iii** $\frac{4}{25}$

iv 24 **v** \$68

b **i**



ii 5 **iii** 10

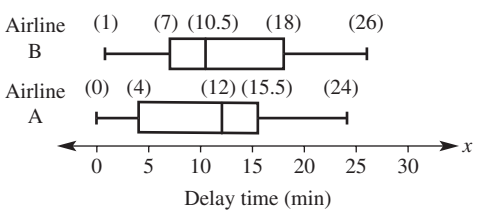
iv $\frac{1}{3}$ **v** $\frac{4}{9}$

vi $\frac{7}{9}$, the probability they didn't buy a drink or they did buy a sausage.

2 a Yes, 52 mins

b **i** 12 **ii** 4 **iii** 15.5

c, e



d No, the median time is 12 mins so half the flights have less than 12 minute delay.

f Airline A: range = 24, IQR = 11.5
Airline B: range = 25, IQR = 11
Both airlines have a very similar spread of their data when the outlier is removed.

g There is not much difference when airline A's outlier is removed. It has a marginally better performance with 75% of flights delayed less than 15.5 min compared with 18 min for airline B.

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Chapter 10

Pre-test

1 a i 0 ii 4 iii 1 iv 9
b i 0 ii 2 iii 5 iv 27
c i 3 ii −1 iii 2 iv −6
d i −1 ii 7 iii −2 iv 2

2 a Yes **b** No **c** Yes

3 a 0 **b** 3 **c** −5
d −3 **e** 4 **f** $\frac{4}{3}$

4 a $a^2 - 2a = 0$ **b** $a^2 - 3a + 1 = 0$

5 a $2x$ **b** x **c** $3x$
d $2x$ **e** 4 **f** 5

6 a $(x + 3)(x - 3)$ **b** $(x + 5)(x - 5)$
c $(x + 8)(x - 8)$ **d** $(x + 4)(x - 4)$

7 a $(x + 6)(x + 2)$ **b** $(x + 4)(x + 1)$
c $(x - 7)(x + 4)$ **d** $(x - 5)(x - 4)$

8 a $2x - 6$ **b** $2x - x^2$
c $x^2 - x - 2$ **d** $x^2 - 7x + 12$

9 a $A = x^2 + 2x$ **b** $A = x^2 - 9$
c $A = x^2 + x - 2$

10 a 2 and −4 **b** 4 and 2

11 a $x = 5$ **b** $x = 3$ **c** $x = 1$ **d** $x = 5.5$

Exercise 10A

1 a 12 **b** −4 **c** 6
d −1 **e** 20 **f** 4
g 72 **h** 0 **i** −27

2 a No **b** No **c** Yes
d Yes **e** Yes **f** No
g No **h** Yes **i** Yes

3 a 1 **b** −3 **c** −11
d −2 **e** 3 **f** −6

4 a $x - 5 = 0, x = 5$
b $2x + 1 = 0, 2x = -1$ or $x = 3, x = -\frac{1}{2}$ or $x = 3$

5 a $x^2 + 2x - 5 = 0$ **b** $x^2 - 5x + 2 = 0$
c $x^2 - 4x + 1 = 0$ **d** $x^2 - 7x - 2 = 0$
e $2x^2 + 2x + 1 = 0$ **f** $3x^2 - 3x + 4 = 0$
g $3x^2 + 4 = 0$ **h** $x^2 - 3x - 1 = 0$
i $2x^2 + 5x - 10 = 0$

6 a Yes **b** Yes **c** No
d No **e** Yes **f** Yes
g No **h** No **i** No

7 Both are solutions. **8** Both are solutions.

9 a $x = 0, x = -1$ **b** $x = 0, x = -5$ **c** $x = 0, x = 2$
d $x = 0, x = 7$ **e** $x = -1, x = 3$ **f** $x = 4, x = -2$
g $x = -7, x = 3$ **h** $x = -\frac{1}{2}, x = \frac{1}{2}$ **i** $x = 0, x = -5$
j $x = 0, x = \frac{2}{3}$ **k** $x = 0, x = -\frac{2}{3}$ **l** $x = 0, x = -2$

10 a $x = \frac{1}{2}, x = -2$ **b** $x = -2, x = \frac{1}{3}$ **c** $x = -\frac{2}{5}, x = -4$
d $x = 1, x = \frac{1}{3}$ **e** $x = -5, x = \frac{2}{7}$ **f** $x = \frac{2}{3}, x = -\frac{1}{5}$
g $x = \frac{7}{11}, x = \frac{13}{2}$ **h** $x = -\frac{9}{4}, x = \frac{7}{2}$ **i** $x = \frac{4}{3}, x = -\frac{1}{7}$

11 a $x = 0, x = -3$ **b** $x = 0, x = 7$ **c** $x = 1, x = -4$
d $x = \frac{1}{2}, x = -6$ **e** $x = -\frac{3}{2}, x = \frac{1}{2}$

12 a $4x^2 + x + 1 = 0$ **b** $3x^2 - 3x = 0$
c $3x^2 - x - 4 = 0$ **d** $5x^2 + x - 2 = 0$
e $2x^2 - x - 3 = 0$ **f** $3x^2 - 10x + 6 = 0$
g $x^2 - x - 5 = 0$ **h** $4x^2 - 5x - 6 = 0$

13 a $x = -1, x = 2$ **b** $x = 1, x = 3$ **c** $x = 0, x = 4$
d $x = 0, x = -3$ **e** $x = -4, x = 1$ **f** $x = -4, x = 4$

14 a $(x + 2)(x + 2) = 0$
b Both solutions are the same, $x = -2$.
c i $x = -3$ ii $x = 5$ iii $x = \frac{1}{2}$ iv $x = \frac{7}{5}$

15 a $(x - 1)(x + 2) = 0$ **b** $x = 1, x = -2$
c Multiplying by a constant doesn't change a zero value.
d i $x = -2, x = 3$ ii $x = 0, x = -2$ iii $x = -1, x = 3$

16 a i No ii No iii No iv No
b It has no solutions as $(x - 3)^2$ is always ≥ 0, so $(x - 3)^2 + 1 \geq 1$.

17 a Linear **b** Quadratic **c** Quintic
d Cubic **e** Quartic **f** Quintic

18 a $x = -2, x = -1, x = 3$ **b** $x = -11, x = 2, x = 5$
c $x = -\frac{2}{3}, x = \frac{1}{5}, x = \frac{1}{2}$
d $x = -\frac{10}{7}, x = -\frac{4}{5}, x = -\frac{2}{3}, x = \frac{13}{2}$

Exercise 10B

1 a 2 **b** 5 **c** 2 **d** 8
e x **f** x **g** $3x$ **h** $3x$

2 a $2(x - 2)(x + 2)$ **b** $4(x - 3)(x + 3)$
c $3(x - 5)(x + 5)$ **d** $12(x - 1)(x + 1)$
e $x(x - 3)$ **f** $x(x + 7)$
g $2x(x - 2)$ **h** $5x(x - 3)$
i $2x(3x + 2)$ **j** $9x(x - 3)$
k $4x(1 - 4x)$ **l** $7x(2 - 3x)$

3 a $x = 0, x = 3$ **b** $x = 0, x = -1$
c $x = 0, x = -2$ **d** $x = -1, x = 1$
e $x = -5, x = 5$ **f** $x = -\frac{1}{2}, x = \frac{1}{2}$

4 a $x = 0, x = -3$ **b** $x = 0, x = -7$
c $x = 0, x = -4$ **d** $x = 0, x = 5$
e $x = 0, x = 8$ **f** $x = 0, x = 2$
g $x = 0, x = -\frac{1}{3}$ **h** $x = 0, x = \frac{1}{2}$

5 a $x = 0, x = 3$ **b** $x = 0, x = 4$ **c** $x = 0, x = -5$
d $x = 0, x = 3$ **e** $x = 0, x = 3$ **f** $x = 0, x = -4$
g $x = 0, x = -9$ **h** $x = 0, x = 9$ **i** $x = 0, x = 1$
j $x = 0, x = 5$ **k** $x = 0, x = -\frac{1}{2}$ **l** $x = 0, x = -4$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

m $x = 0, x = 4$ **n** $x = 0, x = 2$ **o** $x = 0, x = -4$
p $x = 0, x = -2$

6 a $x = 0, x = 3$ **b** $x = 0, x = 2$ **c** $x = 0, x = 4$
d $x = 0, x = -3$ **e** $x = 0, x = -4$ **f** $x = 0, x = -3$

7 a $x = 3, x = -3$ **b** $x = 4, x = -4$
c $x = 5, x = -5$ **d** $x = 12, x = -12$
e $x = 9, x = -9$ **f** $x = 20, x = -20$

8 a $x = 2, x = -2$ **b** $x = 3, x = -3$ **c** $x = 5, x = -5$
d $x = 2, x = -2$ **e** $x = 2, x = -2$ **f** $x = 2, x = -2$
g $x = 4, x = -4$ **h** $x = 3, x = -3$

9 a $x = 2, x = -2$ **b** $x = 5, x = -5$
c $x = 10, x = -10$ **d** $x = 0, x = -7$
e $x = 0, x = 7$ **f** $x = -1, x = 1$
g $x = 0, x = \frac{2}{3}$ **h** $x = 0, x = \frac{3}{5}$
i $x = -\sqrt{3}, x = \sqrt{3}$

10 a $x = -2, x = 2$ **b** $x = -6, x = 6$ **c** $x = -1, x = 1$
d $x = -1, x = 1$ **e** $x = 0, x = -7$ **f** $x = 0, x = 4$

11 a 0 or 7 **b** 8 or -8 **c** 0 or -4

12 a $(x + 2)(x - 2) = x^2 - 4$, not $x^2 + 4$
b No, $\sqrt{-4}$ is not a real number.

13 a $ax^2 + bx = x(ax + b) = 0$, $x = 0$ is always one solution.
b $x = -\frac{b}{a}$

14 a $x = -\frac{4}{3}, x = \frac{4}{3}$ **b** $x = -\frac{6}{5}, x = \frac{6}{5}$
c $x = -\frac{1}{5}, x = \frac{1}{5}$ **d** $x = -\frac{9}{5}, x = \frac{9}{5}$
e $x = -\frac{8}{11}, x = \frac{8}{11}$ **f** $x = -\frac{12}{7}, x = \frac{12}{7}$

15 a $x = -1, x = 5$ **b** $x = -1, x = -9$
c $x = -1, x = 0$ **d** $x = -\frac{2}{5}, x = \frac{8}{5}$
e $x = 1, x = 7$ **f** $x = -1, x = \frac{13}{7}$

Exercise 10C

1 a 3, 2 **b** 4, 2 **c** $-2, -1$ **d** $-5, -2$
e $2, -1$ **f** $5, -1$ **g** $-4, 3$ **h** $-6, 2$

2 a $(x + 5)(x + 4) = 0$
$x + 5 = 0$ or $x + 4 = 0$
$x = -5$ or $x = -4$
b $(x - 6)(x + 4) = 0$
$x - 6 = 0$ or $x + 4 = 0$
$x = 6$ or $x = -4$
c $(x + 9)(x - 5) = 0$
$x + 9 = 0$ or $x - 5 = 0$
$x = -9$ or $x = 5$
d $(x - 8)(x - 2) = 0$
$x - 8 = 0$ or $x - 2 = 0$
$x = 8$ or $x = 2$

3 a $x = -6, x = -2$ **b** $x = -8, x = -3$ **c** $x = -5, x = -2$
d $x = -7, x = 2$ **e** $x = -6, x = 2$ **f** $x = -10, x = 3$
g $x = 4, x = 8$ **h** $x = 3, x = 6$ **i** $x = 3, x = 7$
j $x = -3, x = 5$ **k** $x = -2, x = 8$ **l** $x = -5, x = 9$
m $x = 4, x = 6$ **n** $x = -6, x = 7$ **o** $x = -12, x = 7$
p $x = -3, x = -1$ **q** $x = -3, x = 9$ **r** $x = 2, x = 10$

4 a $x = -3$ **b** $x = -2$ **c** $x = -7$
d $x = -12$ **e** $x = 5$ **f** $x = 8$
g $x = 6$ **h** $x = 9$ **i** $x = 10$

5 a $x = -2, x = 5$ **b** $x = 2, x = 5$ **c** $x = 3$
d $x = -4, x = 1$ **e** $x = -7, x = 2$ **f** $x = 4$
g $x = -6, x = 2$ **h** $x = -6, x = 1$ **i** $x = 3, x = 5$
j $x = -8, x = 2$ **k** $x = -4, x = -2$ **l** $x = -9, x = 2$

6 a $x = -2, x = 3$ **b** $x = -5, x = -3$ **c** $x = -2, x = 8$
d $x = 2, x = 3$ **e** $x = 2$ **f** $x = -1$
g $x = 5$ **h** $x = -3$ **i** $x = -7, x = 1$

7 a $x = 2, x = 3$ **b** $x = 3$ **c** $x = -10, x = 2$
d $x = -5, x = 7$ **e** $x = -1$ **f** $x = 2$

8 a $x^2 - 3x + 2 = 0$ **b** $x^2 - x - 6 = 0$
c $x^2 + 3x - 4 = 0$ **d** $x^2 - 7x - 30 = 0$
e $x^2 - 10x + 25 = 0$ **f** $x^2 + 22x + 121 = 0$

9 11 a.m., 6 p.m.

10 a Equation is not written in standard form $x^2 + bx + c = 0$ so cannot apply Null Factor Law in this form.
b $x = -2, x = 3$

11 It is a perfect square, $(x - 1)(x - 1)$, $(x - 1)^2 = 0, x = 1$

12 a $(x - a)(x - a) = 0$ or $(x - a)^2 = 0$
b $(x - a)(x - b) = 0$

13 a $x = -3$ or $x = -\frac{1}{5}$ **b** $x = \frac{2}{3}$ or $x = -2$
c $x = \frac{1}{3}$ or $x = -\frac{1}{2}$ **d** $x = \frac{1}{2}$ or $x = -1$
e $x = \frac{3}{2}$ or $x = -5$ **f** $x = \frac{3}{2}$
g $x = \frac{7}{3}$ or $x = -2$ **h** $x = \frac{3}{5}$ or $x = -4$
i $x = -\frac{5}{3}$ **j** $x = \frac{1}{3}$ or $x = \frac{5}{2}$

Exercise 10D

1 a $x(x + 2) = 8$ **b** $x = 2, x = -4$
c 2, width > 0 **d** $L = 4$ cm, $W = 2$ cm

2 a $x(x + 5) = 14$ **b** $x = 2, x = -7$
c 2, width > 0 **d** $L = 7$ m, $W = 2$ cm

3 a $x = -6, x = 3$ **b** $x = 5, x = -4$ **c** $x = 2, x = -5$

4 $-8, 6$ **5** $-5, 12$ **6** $-2, 15$

7 $L = 9$ cm, $W = 4$ cm

8 $L = 3$ m, $W = 23$ m

9 a $A = 100 - x^2$ **b** $x = 6$

10 $h = 5$

11 a $A = x^2 + 5x + 15$ **b** $x = 7$

12 a $x = 3$ (-4 not valid) **b** $x = 12$
c $x = 25$

13 -8 not valid because dimensions must be >0

14 $x = -2$ or $x = 3$ both valid as both are integers

15 a 10, 15, 21
b i 28 **ii** 210
c i 9 **ii** 15

16 a 9, 14
b i 8 **ii** 12

17 a $A = (20 + 2x)^2 = 4x^2 + 80x + 400$
b 10 cm

18 4 cm

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

Exercise 10E

1 **a** highest **b** parabola **c** intercepts
d vertex **e** lowest **f** zero

2 **a** $x = 3$ **b** $x = 1$ **c** $x = -2$ **d** $x = -5$

3

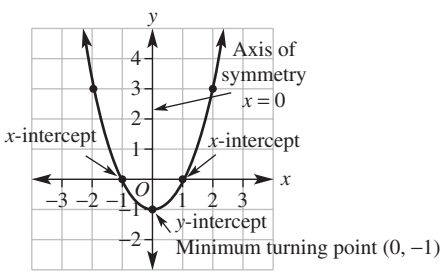
	i aixs of symmetry	ii turning point	iii type of turning point	iv x-intercepts	v y-intercepts
a	$x = 2$	$(2, -1)$	minimum	1 & 3	3
b	$x = 0$	$(0, -4)$	minimum	-2 & 2	-4
c	$x = 0$	$(0, 3)$	maximum	-1 & 1	3
d	$x = 0$	$(0, 4)$	maximum	-2 & 2	4
e	$x = 2$	$(2, 1)$	minimum	none	4
f	$x = -1$	$(-1, 7)$	maximum	-4 & 2	6
g	$x = 0$	$(0, 0)$	minimum	0	0
h	$x = 0$	$(0, -4)$	minimum	-2 & 2	-4
i	$x = 3$	$(3, 4)$	maximum	1 & 5	-5
j	$x = 3$	$\left(3, -\frac{1}{2}\right)$	minimum	2 & 4	4
k	$x = 0$	$(0, 2)$	maximum	-1 & 1	2
l	$x = -2$	$(-2, -1)$	maximum	none	-4

4 **a**

x	-2	-1	0	1	2
y	3	0	-1	0	3

$y = x^2 - 1$

b, c

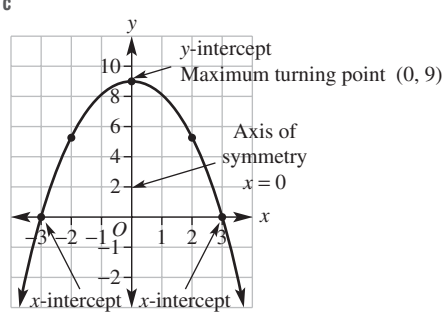


5 **a**

x	-3	-2	-1	0	1	2	3
y	0	5	8	9	8	5	0

$y = 9 - x^2$

b and c

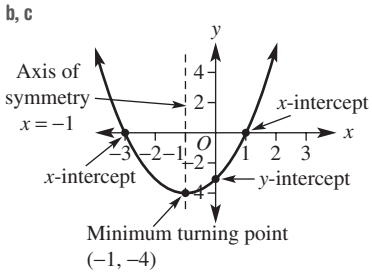


6 **a**

x	-4	-3	-2	-1	0	1	2
y	5	0	-3	-4	-3	0	5

$y = x^2 + 2x - 3$

b, c

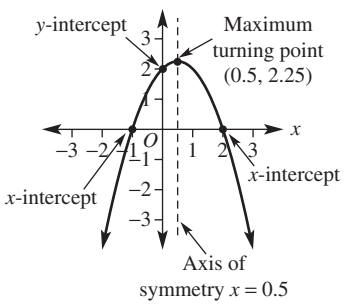


7 **a**

x	-2	-1	0	1	2	3
y	-4	0	2	2	0	-4

$y = -x^2 + x + 2$

b, c



8 **a** **i** 1 s, 3 s
ii One time is on the way up and the other is on the way down.
b **i** 2 s **ii** 12 m **iii** 4 s

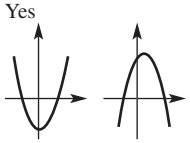
9 **a** 100 m **b** 155 m **c** 5 s
d Journey takes 1 s longer to go down to the ground.

10 **a** $x = 1$ **b** $(1, -3)$

11 **a** $x = 1$ **b** 2

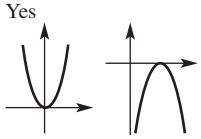
12 **a** $y = x^2$ **b** $y = x^2 - 4$
c $y = 1 - x^2$ **d** $y = x^2 - 2x + 1$
e $y = -x^2 - 2x$ **f** $y = x^2 - 3x - 4$

13 **a** Yes



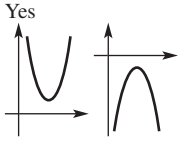
Two x-intercepts

b Yes



One x-intercept

c Yes



Zero x-intercepts

d No

14 **a** $-2^2 = -1 \times 2^2 = -4$. It is not $(-2)^2 = 4$, so correct solution is $y = -2^2 + 2 \times 2 = -4 + 4 = 0$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

b $-(-3)^2 = -1 \times 9 = -9$. It is not $--3^2 = +9$, so correct solution is $y = -3 - (-3)^2 = -3 - 9 = -12$

15 a $x = -2,\ x = 2$ **b** $x = 0$

c **i** Infinite **ii** One **iii** None

16 a **i** $x = 0, x = 2$ **ii** $x = -1, x = 3$

b Two, a parabola is symmetrical.

c One, this is the minimum turning point.

d None, -1 is the minimum y-value so there are no values of y less than -1.

Exercise 10F

1 a $y = 3x^2, y = x^2, y = \frac{1}{2}x^2$ **b** $y = -3x^2, y = -x^2, y = -\frac{1}{2}x^2$

c (0, 0) **d** $x = 0$

e **i** $y = 3x^2$ **ii** $y = \frac{1}{2}x^2$ **iii** $y = -3x^2$ **iv** $y = -\frac{1}{2}x^2$

f **i** $y = -x^2$ **ii** $y = -3x^2$ **iii** $y = \frac{1}{2}x^2$

2 a Positive **b** Negative

3 a $y = -x^2$ **b** $y = -4x^2$ **c** $y = 5x^2$ **d** $y = \frac{1}{3}x^2$

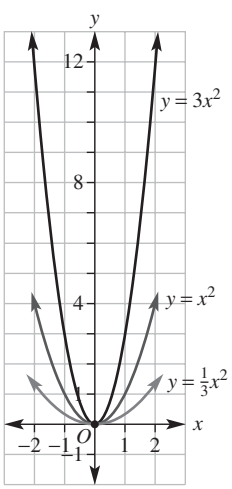
4 a

x	-2	-1	0	1	2
y	4	1	0	1	4

x	-2	-1	0	1	2
y	12	3	0	3	12

x	-2	-1	0	1	2
y	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$

b



c For all 3 graphs, the turning point is a minimum at (0, 0) and the axis of symmetry is $x = 0$.

d **i** Narrower **ii** Wider

5 a **i**

x	-2	-1	0	1	2
y	16	4	0	4	16

iii Axis of symmetry: $x = 0$; turning point: (0, 0)

iv Narrower

b **i**

x	-2	-1	0	1	2
y	20	5	0	5	20

iii Axis of symmetry: $x = 0$; turning point: (0, 0); x- and y-intercept = 0

iv Narrower

c **i**

x	-2	-1	0	1	2
y	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1

iii Axis of symmetry: $x = 0$; turning point: (0, 0)

iv Wider

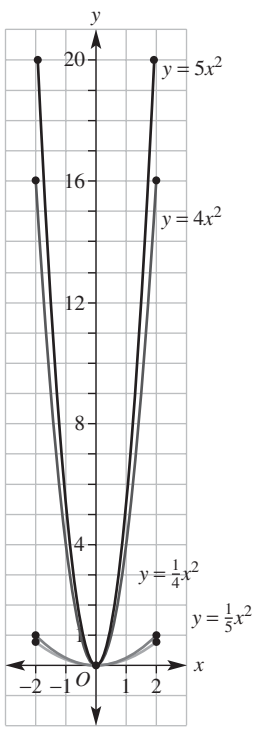
d **i**

x	-2	-1	0	1	2
y	$\frac{4}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{4}{5}$

iii Axis of symmetry: $x = 0$; turning point: (0, 0)

iv Wider

a–d **ii**



6 a **i**

x	-2	-1	0	1	2
y	-8	-2	0	-2	-8

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

iii Axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept $= 0$

iv Narrower

b i

x	-2	-1	0	1	2
y	-12	-3	0	-3	-12

iii Axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept $= 0$

iv Narrower

c i

x	-2	-1	0	1	2
y	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2

iii Axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept $= 0$

iv Wider

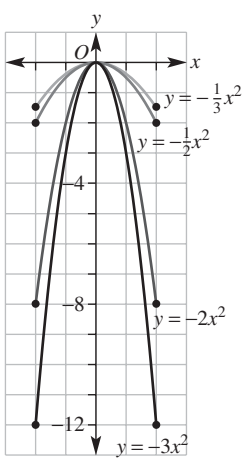
d i

x	-2	-1	0	1	2
y	$-\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{4}{3}$

iii Axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept $= 0$

iv Wider

a–d ii



7 a A **b** H

8 i d **ii** c **iii** a **iv** e **v** f **vi** b

9 a Reflection in the x-axis, dilating by a factor of 3 from the x-axis

b Reflection in the x-axis, dilating by a factor of 6 from the x-axis

c Reflection in the x-axis, dilating by a factor of $\frac{1}{2}$ from the x-axis

d Reflection in the x-axis, dilating by a factor of 2 from the x-axis

e Reflection in the x-axis, dilating by a factor of 3 from the x-axis

f Reflection in the x-axis, dilating by a factor of $\frac{1}{3}$ from the x-axis

10 a $y = -4x^2$ **b** $y = \frac{1}{3}x^2$ **c** $y = -4x^2$ **d** $y = -\frac{4}{3}x^2$

11 No because both transformations are multiplying to 'a' and multiplication is commutative: $bc = cb$

12 $y = ax^2$, has y-axis as axis of symmetry so it is symmetrical about the y-axis

13 a $y = 5x^2$ **b** $y = 7x^2$ **c** $y = x^2$ **d** $y = \frac{7}{4}x^2$

e $y = \frac{4}{25}x^2$ **f** $y = \frac{26}{9}x^2$ **g** $y = 5x^2$ **h** $y = -52x^2$

14 $y = \frac{67}{242064}x^2$

Exercise 10G

1 a i $(-2, 0)$ **ii** $(3, 0)$ **b i** 4 **ii** -9

c Left **d** Right

2 a i $(0, -3)$ **ii** $(0, 2)$ **b i** -3 **ii** 2

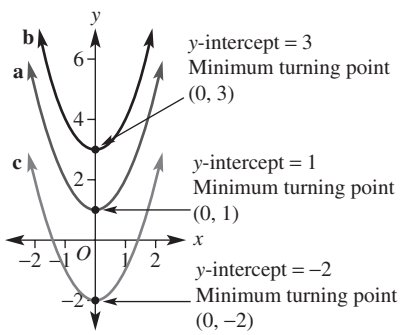
c Down **d** Up

3 a i $(2, 3)$ **ii** $(-1, -1)$ **b i** -1 **ii** 0

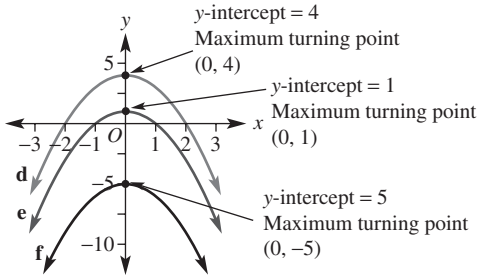
c i one, left, one, down **ii** two, right, three, up

4 a 3 **b** -4 **c** -4 **d** 25

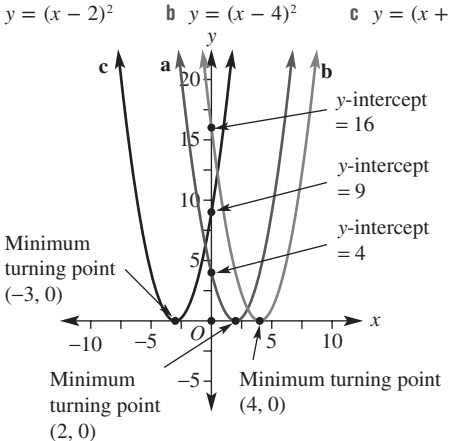
5 a $y = x^2 + 1$ **b** $y = x^2 + 3$ **c** $y = x^2 - 2$



d $y = -x^2 + 4$ **e** $y = -x^2 + 1$ **f** $y = -x^2 - 5$

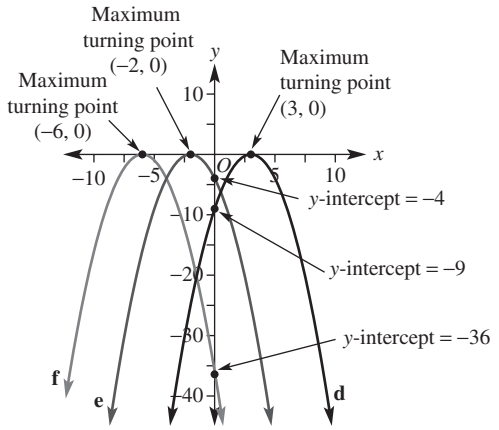


6 a $y = (x - 2)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 3)^2$

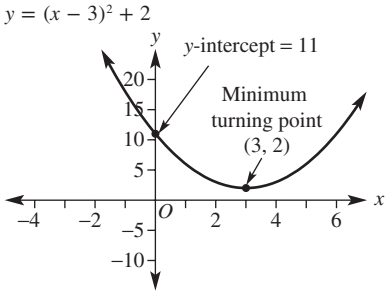


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

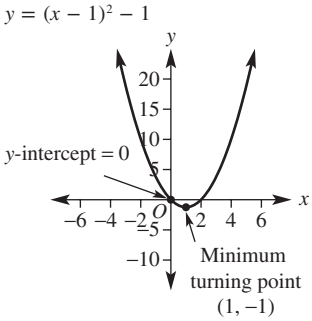
d $y = -(x - 3)^2$ **e** $y = -(x + 2)^2$ **f** $y = -(x + 6)^2$



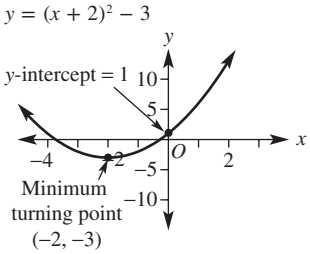
7 a $y = (x - 3)^2 + 2$



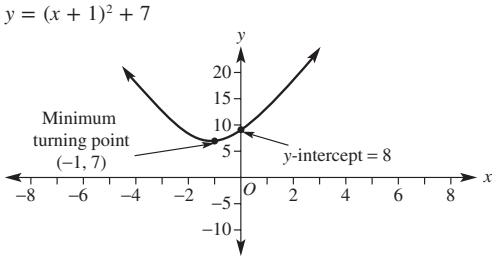
b $y = (x - 1)^2 - 1$



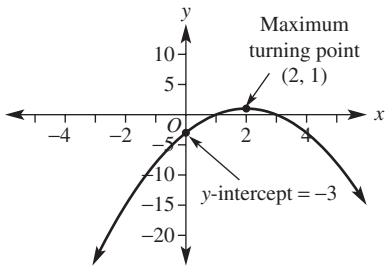
c $y = (x + 2)^2 - 3$



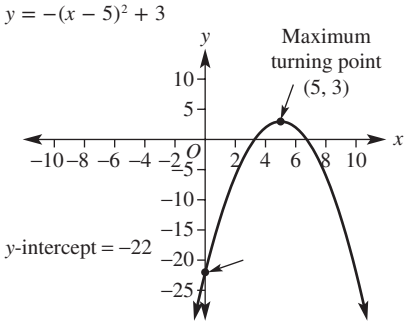
d $y = (x + 1)^2 + 7$



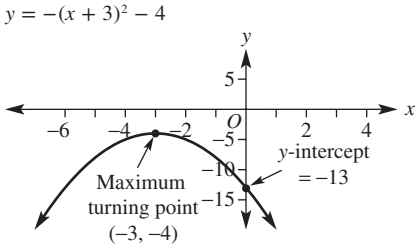
e $y = -(x - 2)^2 + 1$



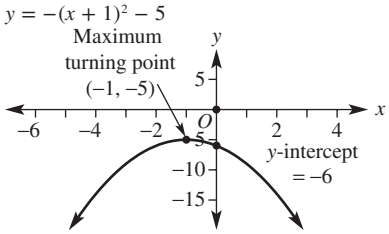
f $y = -(x - 5)^2 + 3$



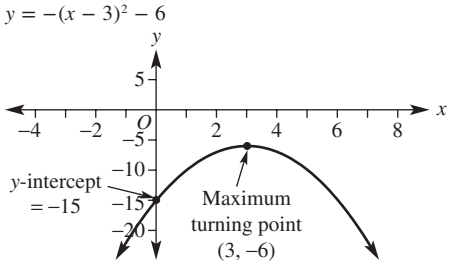
g $y = -(x + 3)^2 - 4$



h $y = -(x + 1)^2 - 5$



i $y = -(x - 3)^2 - 6$



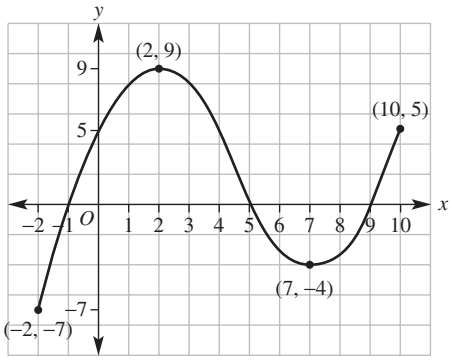
8 **i** j **ii** d **iii** b
iv h **v** k **vi** g
vii a **viii** e **ix** 1
x i **xi** f **xii** c

9 **a** $y = (x - 1)^2 + 1$ **b** $y = (x + 2)^2 + 2$
c $y = (x + 1)^2 - 3$ **d** $y = -(x - 1)^2 + 4$
e $y = -(x - 2)^2 - 2$ **f** $y = -(x + 2)^2 + 4$

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

10 a $y = -(x-2)^2 + 9$, turning point $(2, 9)$
$y = (x-7)^2 - 4$, turning point $(7, -4)$

b



11 a i $y = -(x+1)^2 + 3$ **ii** $y = (x+3)^2 + 4$
iii $y = (x-1)^2 - 3$ **iv** $y = -(x-5)^2 - 7$
v $y = -x^2 - 2$ **vi** $y = x^2 - 6$

b i $(-1, 3)$ **ii** $(-3, 4)$ **iii** $(1, -3)$
iv $(5, -7)$ **v** $(0, -2)$ **vi** $(0, -6)$

12 a (h, k) **b** $y = ah^2 + k$

13 a $(2-x)^2 = (-1(x-2))^2 = (-1)^2(x-2)^2 = (x-2)^2$, so graphs are the same.

b Graphs are the same.

14 a $y = x^2 + 3$ **b** $y = x^2 - 4$
c $y = x^2 - 3$ **d** $y = x^2 - 5$

15 a $y = -x^2 + 4$ **b** $y = -x^2 + 4$
c $y = -x^2 + 24$ **d** $y = -x^2 + 10$

16 a $y = (x+3)^2$ or $y = (x-5)^2$
b $y = (x-2)^2$ or $y = (x-4)^2$
c $y = (x+4)^2$ or $y = (x-2)^2$
d $y = x^2$ or $y = (x-6)^2$

17 a $y = -(x-1)^2 + 1$ **b** $y = (x+2)^2$
c $y = -(x-3)^2$ **d** $y = -(x+3)^2 + 2$
e $y = (x+1)^2 + 4$ **f** $y = (x-3)^2 - 9$

Exercise 10H

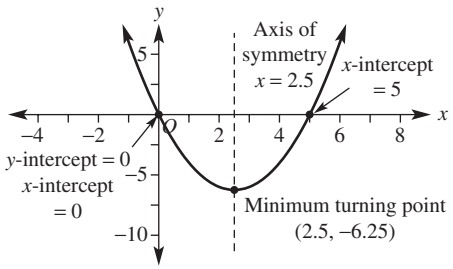
1 a $x(x+2)$ **b** $x(x-3)$
c $5x(x-2)$ **d** $(x+3)(x-3)$
e $(x+1)(x-1)$ **f** $(x+7)(x-7)$
g $(x+2)(x+1)$ **h** $(x+3)(x+2)$
i $(x-4)(x+3)$ **j** $(x-7)(x+4)$
k $(x-2)(x-2) = (x-2)^2$ **l** $(x+5)(x+5) = (x+5)^2$

2 a 0 **b** 0 **c** −3
d 8 **e** −1 **f** −1
g −0.5 **h** 4.5 **i** 6

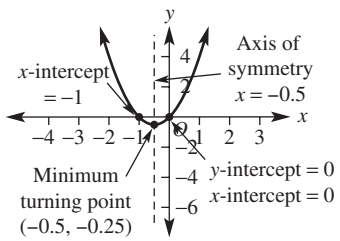
3 a −4 **b** −1 **c** −9
d −1 **e** −9 **f** −9

4 a i 0 and −7 **ii** 0 **b i** 0 and −3 **ii** 0
c i 0 and −4 **ii** 0 **d i** 4 and −2 **ii** −8
e i −2 and 5 **ii** −10 **f i** 7 and −3 **ii** −21
g i −3 and 1 **ii** −6 **h i** −4 and −1 **ii** 12
i i 2 and 3 **ii** 6

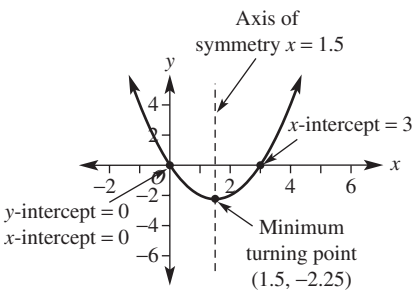
5 a i–vi $y = x(x-5)$



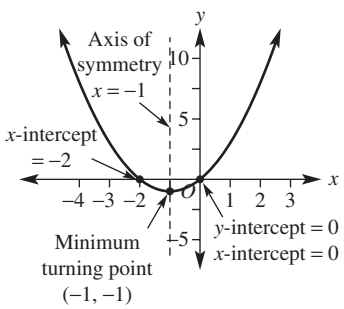
b i–vi $y = x(x+1)$



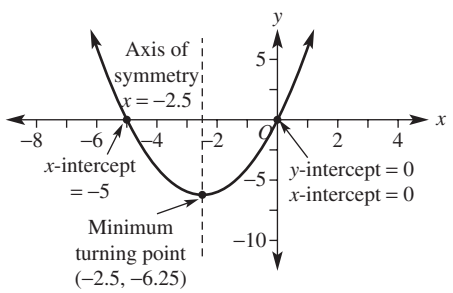
c i–vi $y = x(x-3)$



d i–vi $y = x(2+x)$

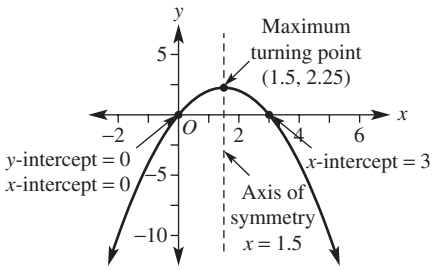


e i–vi $y = x(5+x)$

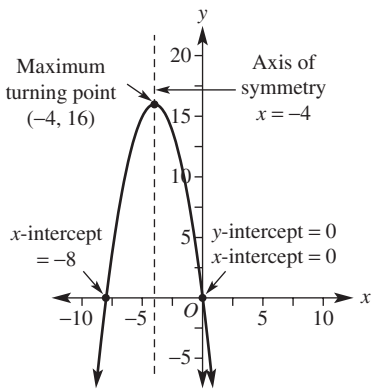


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

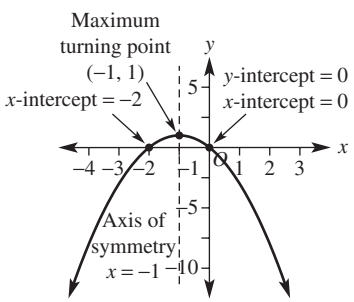
f i–vi $y = x(3 - x)$



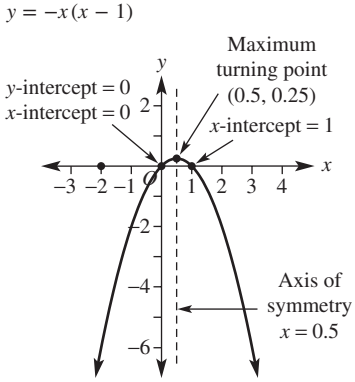
g i–vi $y = -x(x + 8)$



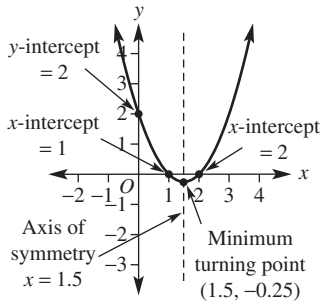
h i–vi $y = -x(2 + x)$



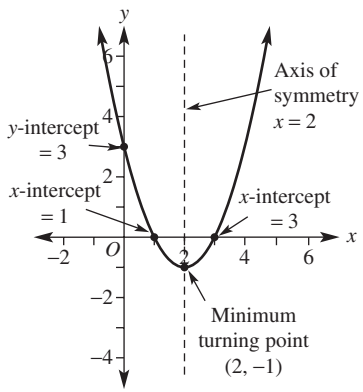
i i–vi $y = -x(x - 1)$



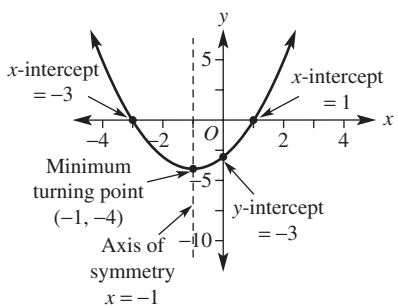
6 a i–vi $y = (x - 2)(x - 1)$



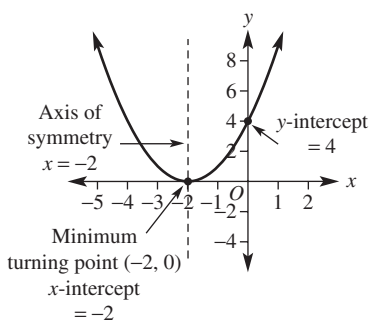
b i–vi $y = (x - 3)(x - 1)$



c i–vi $y = (x + 3)(x - 1)$

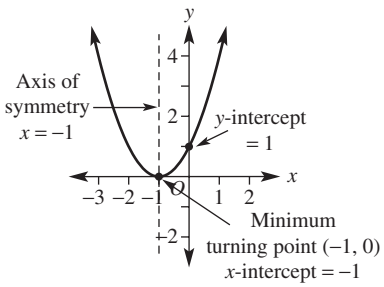


d i–vi $y = (x + 2)^2$

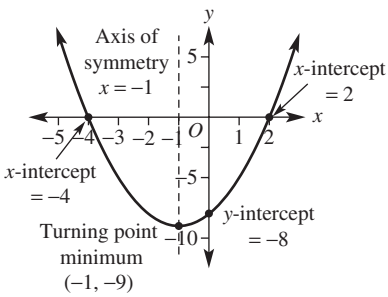


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

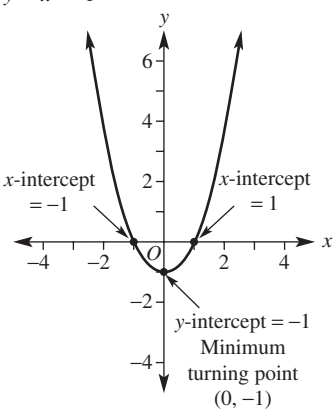
e i–vi $y = (x + 1)^2$



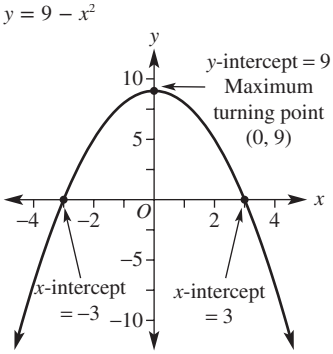
f i–vi $y = (x + 4)(x - 2)$



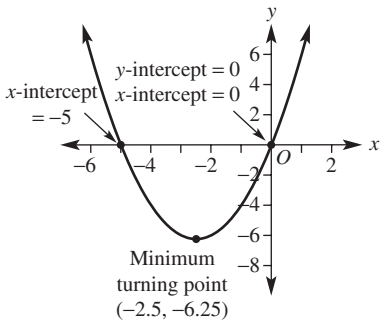
7 a $y = x^2 - 1$



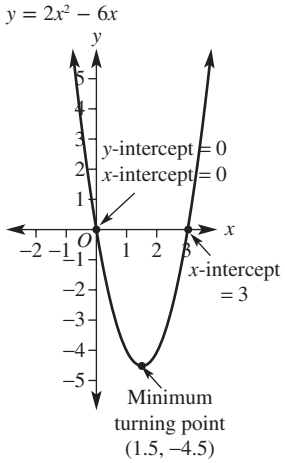
b $y = 9 - x^2$



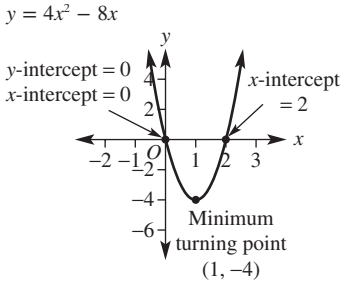
c $y = x^2 + 5x$



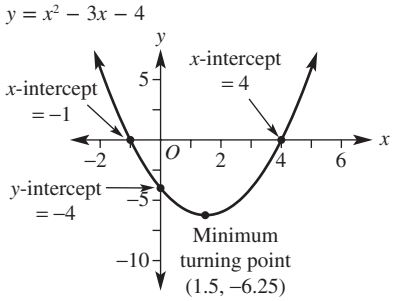
d $y = 2x^2 - 6x$



e $y = 4x^2 - 8x$

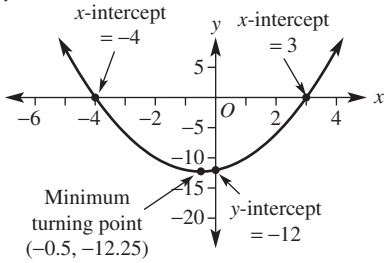


f $y = x^2 - 3x - 4$

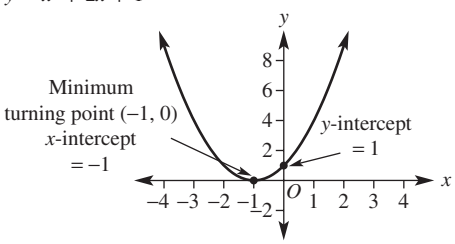


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

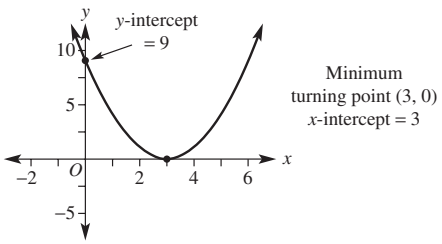
g $y = x^2 + x - 12$



h $y = x^2 + 2x + 1$



i $y = x^2 - 6x + 9$



8 **a** −3 **b** 2 **c** 3 **d** −4 **e** −5 **f** 6

9 **a** 30 m **b** 225 m **10** **a** 200 m **b** 1000 m

11 There is only one x-intercept, so this must be the turning point.

12 **a** $y = c$ **b** $y = ab$

13 **a** $(1, -16)$

b $(5, 0)\ (-3, 0)$ turning point $\left(\frac{5 + -3}{2}, -16\right) = (1, -16)$

c $y = (x - 2)^2 - 49$, $h = 2$, $k = -49$

14 **b** $y = (x + 3)(x - 1)$ **c** $y = 2(x + 5)(x - 1)$

d $y = -(x + 1)(x - 5)$ **e** $y = -(x + 8)(x + 2)$

f $y = \frac{1}{2}(x + 8)(x - 2)$ **g** $y = -\frac{1}{3}(x + 3)(x - 7)$

Puzzles and challenges

1 **a** They are square numbers. **b** $100^2 = 10\,000$

2 **a** $\frac{5}{3}, -\frac{7}{2}$ **b** $2, -1$ **c** $2^{\frac{2}{3}} + 1$

3 6 seconds **4** $\frac{n(n+1)}{2}$; $n = 11$ **5** $x = 2$

6 **a** 20 units, 21 units, 29 units

b 17 units

7 **a** $k < 0$ **b** $k = 0$ **c** $k > 0$

8 **a** 10, −10, 11, −11, 14, −14, 25, −25

b 2, −2, 5, −5, 10, −10, 23, −23

9 $x = -3, 1, 2, 6$

10 $x = -3, -1, 1, 3$

Multiple-choice questions

1 D **2** B **3** C **4** A **5** D

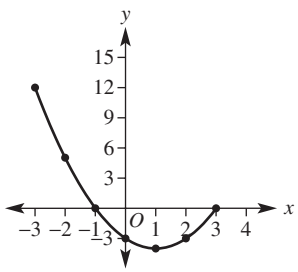
6 B **7** E **8** C **9** B **10** A

Short-answer questions

1 **a** $y = x^2 - 2x - 3$

x	−3	−2	−1	0	1	2	3
y	12	5	0	−3	−4	−3	0

b



2 **a** $x = 0, x = -2$ **b** $x = 0, x = 4$ **c** $x = -3, x = 7$

d $x = 2, x = -2$ **e** $x = -1, x = \frac{2}{5}$ **f** $x = \frac{1}{2}, x = \frac{4}{3}$

3 **a** $x = 0, x = -3$ **b** $x = 0, x = 4$ **c** $x = -5, x = 5$

d $x = -9, x = 9$ **e** $x = -2, x = 2$ **f** $x = -4, x = 4$

g $x = -3, x = -7$ **h** $x = -5, x = 8$ **i** $x = 4$

4 **a** $x^2 - 5x = 0; x = 0, x = 5$

b $3x^2 - 18x = 0; x = 0, x = 6$

c $x^2 + 8x + 12 = 0; x = -6, x = -2$

d $x^2 - 2x - 15 = 0; x = -3, x = 5$

e $x^2 - 8x + 15 = 0; x = 3, x = 5$

f $x^2 + 3x - 4 = 0; x = -4, x = 1$

5 **a** $x^2 + 2x - 80 = 0; x = 8$ units

b $x^2 + 5x - 24 = 0; x = 3$ units

c $x^2 + 3x - 28 = 0; x = 4$ units

6 **a** **i** $x = 3$ **ii** Minimum $(3, -2)$

b **i** $x = -1$ **ii** Maximum $(-1, 3)$

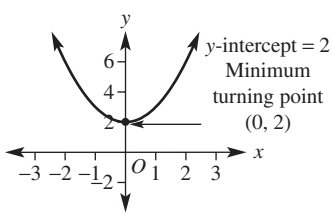
c **i** $x = 5$ **ii** Minimum $(5, 0)$

7 **a** $y = 2x^2$ **b** $y = -x^2 + 2$

c $y = (x + 1)^2$ **d** $y = (x - 3)^2$

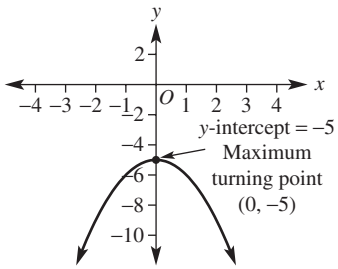
e $y = x^2 - 4$

8 **a** $y = x^2 + 2$

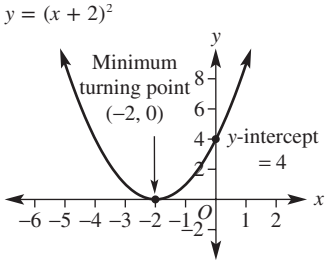


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

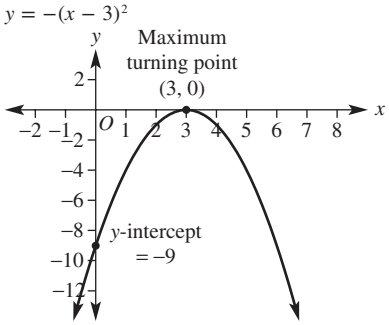
b $y = -x^2 - 5$



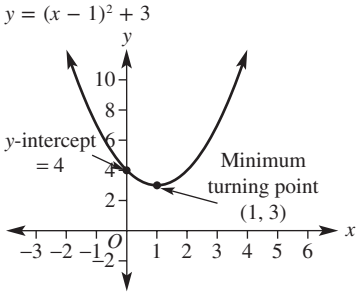
c $y = (x + 2)^2$



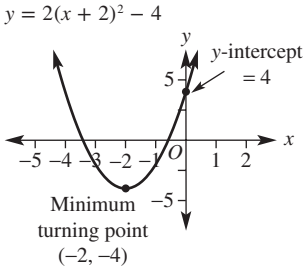
d $y = -(x - 3)^2$



e $y = (x - 1)^2 + 3$



f $y = 2(x + 2)^2 - 4$



9 a $y = x^2$ translated up 2 units

b $y = x^2$ reflected in the x-axis and translated down 5 units

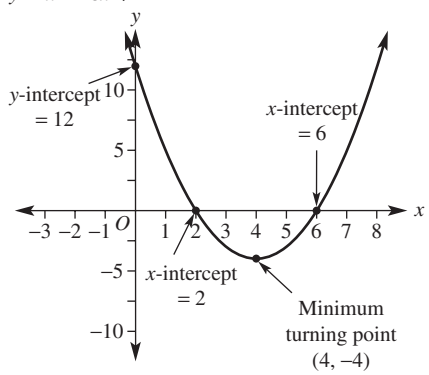
c $y = x^2$ translated 2 units left

d $y = x^2$ reflected in the x-axis then translated 3 units right

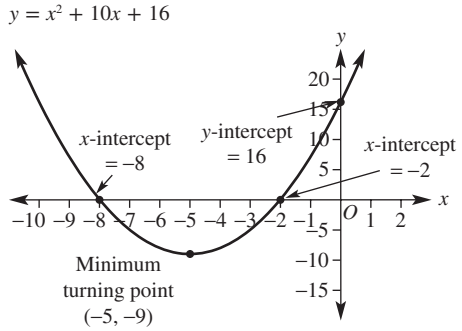
e $y = x^2$ translated 1 unit right then 3 units up

f $y = x^2$ dilated by a factor of 2 from the x-axis, translated 2 units left and then down 4 units

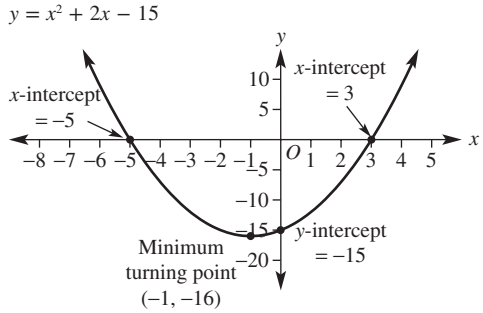
10 a $y = x^2 - 8x + 12$



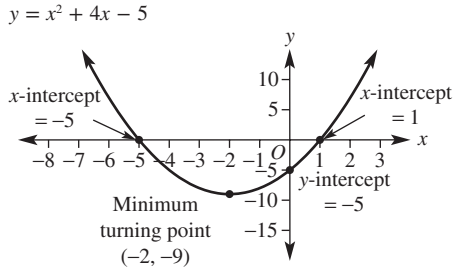
b $y = x^2 + 10x + 16$



c $y = x^2 + 2x - 15$



d $y = x^2 + 4x - 5$

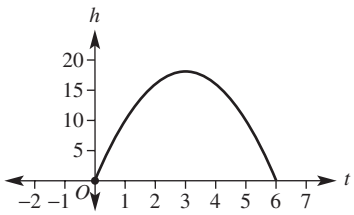


ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

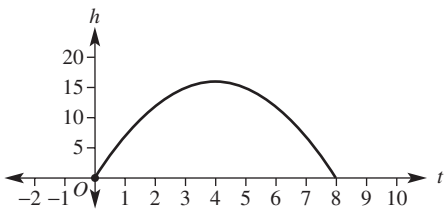
Extended-response questions

1 **a** $x + 5$ **b** $A = x^2 + 5x$
c $x = 2$ ($x = -7$ not valid as $x > 0$)
d Base = 4 m, height = 7 m

2 **a** **i** (0, 0), (6, 0) **ii** Maximum at (3, 18)
iii



b **i** (0, 0) **ii** (4, 16) **iii** (0, 0), (8, 0)
iv



c **i** Connor's rocket, 8 s
ii Sam's rocket, 2 m higher
iii 16 m, the same height

Semester review 2

Indices and surds
Multiple-choice questions

1 C **2** B **3** D **4** E **5** C

Short-answer question

1 **a** $\frac{a^2b}{2}$ **b** $2x^3y^5$ **c** $8x^6 - 2$
2 **a** $5m^2$ **b** $\frac{2a^8}{3b^5}$ **c** $\frac{3x^2}{y}$
3 **a** 3.07×10^{-2} kg **b** 4.24×10^6 kg
c 1.22×10^4 seconds **d** 2.35×10^{-7} seconds
4 **a** 12 **b** 3
c $\sqrt{3} + 7\sqrt{5}$ **d** $\sqrt{7}$

Extended-response question

a **i** 74000000000 **ii** 7.4×10^{10}
b 1.87×10^{17} **c** 8.72×10^{-7}

Properties of geometrical figures
Multiple-choice questions

1 C **2** B **3** A **4** E **5** B

Short-answer questions

1 **a** $a = 100$, $b = 140$ **b** $a = 70$, $b = 55$
c $x = 67$, $y = 98$ **d** $x = 35$
2 85°
3 $CB = CD$ (given equal sides)
$\angle ACB = \angle ACD$ (given equal angles)
AC is common.
$\therefore \Delta ABC \equiv \Delta ADC$ (SAS)
4 **a** Two pairs of equal alternate angles
b 2.4

Extended-restponse question

a $\angle ABD = \angle ECD$ (given right angles)
$\angle ADB = \angle EDC$ (common angle)
$\therefore \Delta ABD \equiv \Delta ECD$ (two pairs of equal angles)
b 7.5 m **c** 3.75 m **d** 4.3 m

Quadratic expressions and algebraic fractions
Multiple-choice questions

1 E **2** D **3** B **4** A **5** D

Short-answer questions

1 **a** $x^2 - 9$ **b** $x^2 + 4x + 4$ **c** $6x^2 - 17x + 12$
2 **a** $2ab(4 + a)$ **b** $(3m - 5)(3m + 5)$
c $3(b - 4)(b + 4)$ **d** $(a + 4)(a + 10)$
e $(x + 3)^2$ **f** $(x - 2)(x + 10)$
g $2(x - 3)(x - 5)$ **h** $(2x - 3)(x - 4)$
i $(2x - 1)(3x + 4)$
3 **a** $(x - 3a)(x - 1)$ **b** $(x + 2)(2a - 5b)$
4 **a** **i** $\frac{3}{2}$ **ii** $\frac{4}{x + 3}$
iii $\frac{13x + 21}{14}$ **iv** $\frac{1}{3x}$
v $\frac{11x - 10}{(x + 1)(x - 2)}$ **vi** $\frac{2x + 25}{(x - 5)(x + 2)}$
b **i** $x = \frac{17}{12}$ **ii** $x = -11$

Extended-response question

a $10 - 2x$ and $8 - 2x$
b $(10 - 2x)(8 - 2x) = 80 - 36x + 4x^2$
c 48 m^2
d $4(x - 4)(x - 5)$
e Area of rug is 0 as it has no width.
f $x = 2.5$

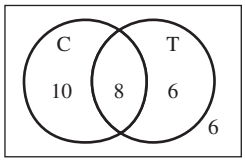
Probability and single variable data analysis
Multiple-choice questions

1 D **2** A **3** C **4** B **5** C

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

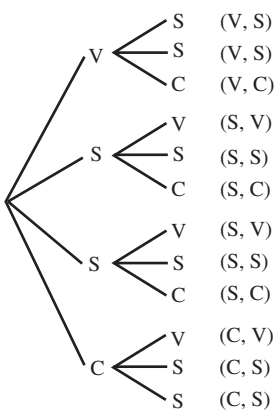
Short-answer questions

1 a



b i $\frac{1}{5}$ **ii** $\frac{1}{3}$

2 a



b i $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{2}$

3 a

Stem	Leaf
1	0 1 1 2 3 5 7 8
2	2 5 5 5 6
3	1 2 2

1|3 means 13 aces

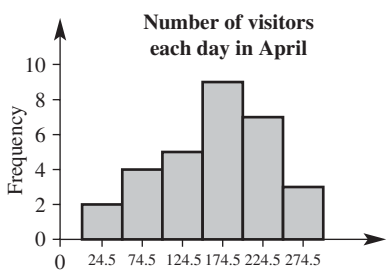
b Mode = 25, median = 20

c Skewed

4 a

Class interval	Class centre	Frequency	Percentage frequency
0–49	24.5	2	6.7
50–99	74.5	4	13.3
100–149	124.5	5	16.7
150–199	174.5	9	30
200–249	244.5	7	23.3
250–299	274.5	3	10
Total		30	100

b



c i 6 **ii** 60%

Extended-response question

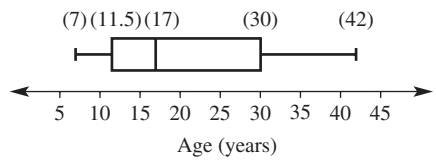
a

		Red		
		1	**2**	**3**
Green	**1**	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b $\frac{4}{9}$

c i $\frac{1}{3}$ **ii** 18

d i



ii 11.5 years **iii** 90

Quadratic equations and graphs of parabolas

Multiple-choice questions

1 B **2** B **3** E **4** C **5** E

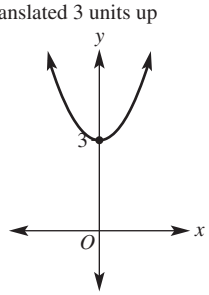
Short-answer questions

1 a $x = -5,\ x = 3$ **b** $x = \frac{1}{2},\ x = -\frac{5}{3}$ **c** $x = 0,\ x = -2$

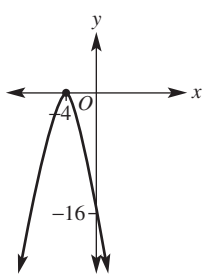
d $x = -3,\ x = 3$ **e** $x = 2,\ x = 7$ **f** $x = -4$

2 15 m by 8 m

3 a Translated 3 units up

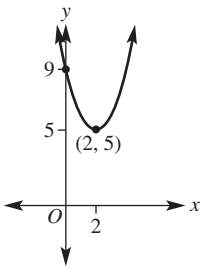


b Reflected in the x-axis and translated 4 units left



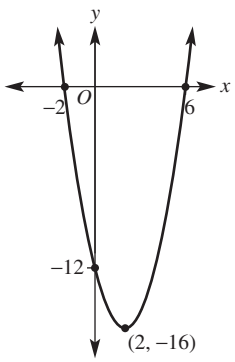
ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

c Translated 2 units right and 5 units up



4 **a** $y = -12$ **b** $x = -2,\ x = 6$ **c** $(2, -16)$

d



Extended-response question

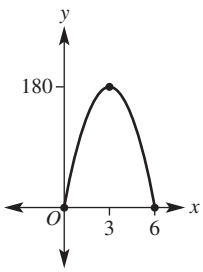
a Lands after 6 seconds

b **i** 180 cm

ii 3 seconds

c After 2 seconds and 4 seconds

d



ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

acute angle 391
adjacent 168
algebraic fraction 483
alternate 399
alternate angle 186
angle 391
angle of depression 186
angle of elevation 186
area 287
array 527
axis of symmetry 600

back-to-back stem-and-leaf plot 548
base 337
bearing 192
best buys 24
bimodal 544
binomial product 455
box plot 565

capacity 309
cardinal number 522
Cartesian plane 209
centre of enlargement 424
circumference 281
class interval 554
closed circle 108
cluster 549
coefficient 83
co-interior angle 400
commission 44
common denominator 15
complement 511
complementary angle 391
composite number 6
composite shape 294
compound interest 62
compound interest formula 67
concave down 609
concave up 609
congruence statement 413
congruent figure 412
constant term 83
convex polygon 406
coordinate geometry 209
coordinate pair 209
corresponding angle 399
corresponding sides 413
cosine (cos) 168
critical digit 10
cross method 480
cross-section 300, 309
cube 6
cubic centimetre 309
cubic kilometre 309
cubic metre 309
cubic millimetre 309
cylinder 305

decimal place 10
degree mode 173
denominator 15, 19, 178
depreciation 62, 67
dilation 609
difference of two squares 459
digit 10
directly proportional 231
discount 38
distributive law 92, 455
double time 44

element (of a subset) 522
elimination 121
empty set 522
enlargement 424
enlargement factor 424
equally likely 511
equilateral triangle 392
equivalent equation 96
equivalent fraction 14, 19
evaluated 83
event 511
exact 152
exact value 146
expansion 463
expected number of occurrences 538
experimental probability 538
exponent 338
expression 83
exterior angle 392
exterior angle theorem of a triangle 392

factorisation 463
factorisation by grouping 471
formula 112
fractional indices 373
frequency table 554

general equation of a quadratic 600
gradient 225
gradient–intercept form 237
gradient–intercept method 237
gross income 44

highest common factor (HCF) 6
histogram 554
hypotenuse 146

Index

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

improper fraction 14
index 338
index law for division 343
index law for multiplication 343
index law for power of a power 348
inequality 108
infinite 108
interest 56
interest rate 67
integer 5
intercept 210
interquartile range 560
intersection 523
inverse cosine 182
inverse sine 182
inverse tangent 182
inverted graph 609
investments 51
irrational numbers 14
isosceles triangle 392

kilolitre 309

leave loading 44
length of a line segment 248
like surd 377
like term 88
linear modelling 255
line 391
line segment 391
linear equation 96
linear relationship 209
litre 309
long-run proportion 538
lower quartile 560
lowest common denominator (LCD) 496
lowest common multiple (LCM) 6

mark-up 38
maximum (of parabola) 600
maximum value (of box plot) 565
mean 543
median 543
megalitre 309
metric unit 275
midpoint 247
millilitre 309
minimum (of parabola) 600
minimum value (of box plot) 565
mixed number 14
mode 543

negative reciprocal 251
net 300
net income 44
non-convex polygon 406
non-parallel line 399
non-significant digit 10
non-terminating 14
non-terminating and non-recurring decimal 14
non-zero digit 10
null set 522
Null Factor Law 583
numerator 15

obtuse angle 391
open circle 108
operation 6
opposite 168
order of operations 6
origin 209
outlier 543, 560
overtime 44

p.a. (per annum) 62
parabola 600
parallel 186, 251
parallel line 399
parallelogram 407
PAYG 50
payment summary 51
percentage 29
percentage change 34
percentage profit 38
perfect square 459
perimeter 275
perpendicular 251
pi(π) 281
piecework 44
point of intersection 259
positive integer 5
percentage frequency histogram 554
prime factorisation 338
prime number 6
principal 56
prism 300
profit 38
pronumeral 83
pronumeral factor 88
proper fraction 14
proportion 424
pyramid 300
Pythagoras' theorem 146

quadrant 209, 281
quadratic equation 130, 583
quadratic trinomial 474
quadrilateral 407

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022

range 560
rate 24
rate of change 231
ratio 24
rational numbers (fractions) 14
ray 391
real number 14
reciprocal 19
rectangular prism (cuboid) 300
recurring decimal 14
reflection 609
reflex angle 391
regular polygon 407
relative frequency 538
retainer 44
revolution angle 391
right angle 391
right-angled triangle 146
rotation 412
rounding down 10
rounding up 10

salary 44
sample space 511, 522
scale factor 424
scalene triangle 392
scientific notation 362
sector 281
semicircle 281
significant figure 10, 367
similar figures 424
simple interest 56
simultaneous equation 117, 259
sine (sin) 168
skewed data 549, 554
SOHCAHTOA 168
solution 259
square 6
standard form 583
stem-and-leaf plot 548
straight angle 391
subject (of a formula) 112
subset 522
substitution 117
sum 34
supplementrary angle 391
sum of the interior angle 407
surd 130, 146, 373, 377
surface area 300
symmetrical data 549, 554

tangent (tan) 168
tax bracket 51
tax liability 51
tax payable 51
tax refund 51
tax return 51
tax withheld 51
taxable income 51
taxations 44
term 83
terminating decimal 14
three-dimensional solid 162
time and a half 44
translation 412, 615
transposition (of formulas) 112
transversal 399
tree diagram 532
trigonometry 186
turning point 600
turning point form 616
two-way table 516

undefined gradient 225
union 523
unit 248
unitary method 24, 29
unlike term 88
upper and lower bound 108
upper quartile 560
upright graph 609

variable 596
Venn diagram 516
vertex 600
vertically opposite angle 391
volume 309

wage 44
whole number 19
with replacement (experiment) 527
without replacement (experiment) 551
word problem 104

x-intercept 210

y-intercept 210

zero power 348

ISBN 978-1-108-46816-9 2ed
© Palmer et al. 2019
Photocopying is restricted under law and this material must not be transferred to another party.
Cambridge University Press
Updated March 2022