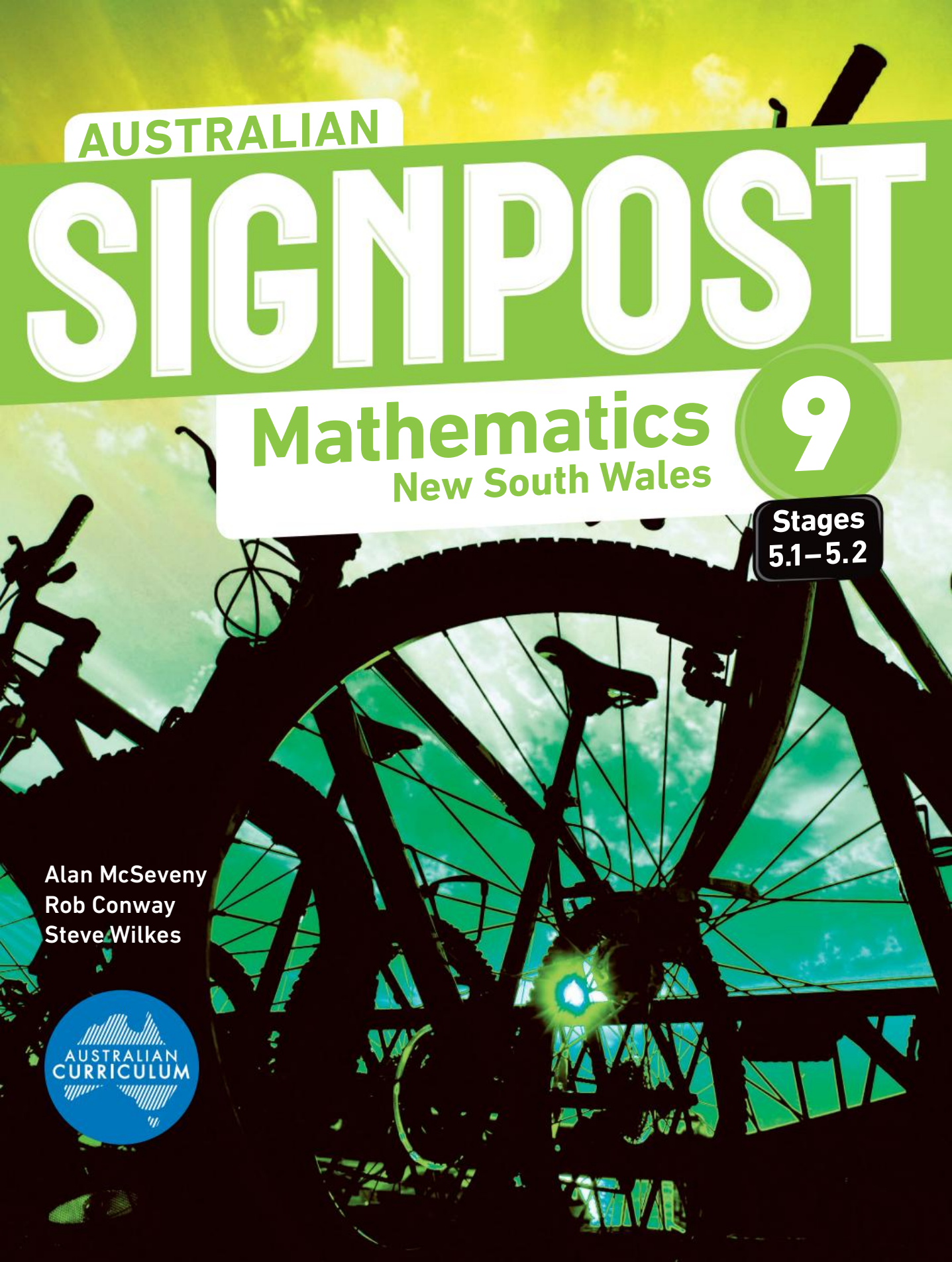
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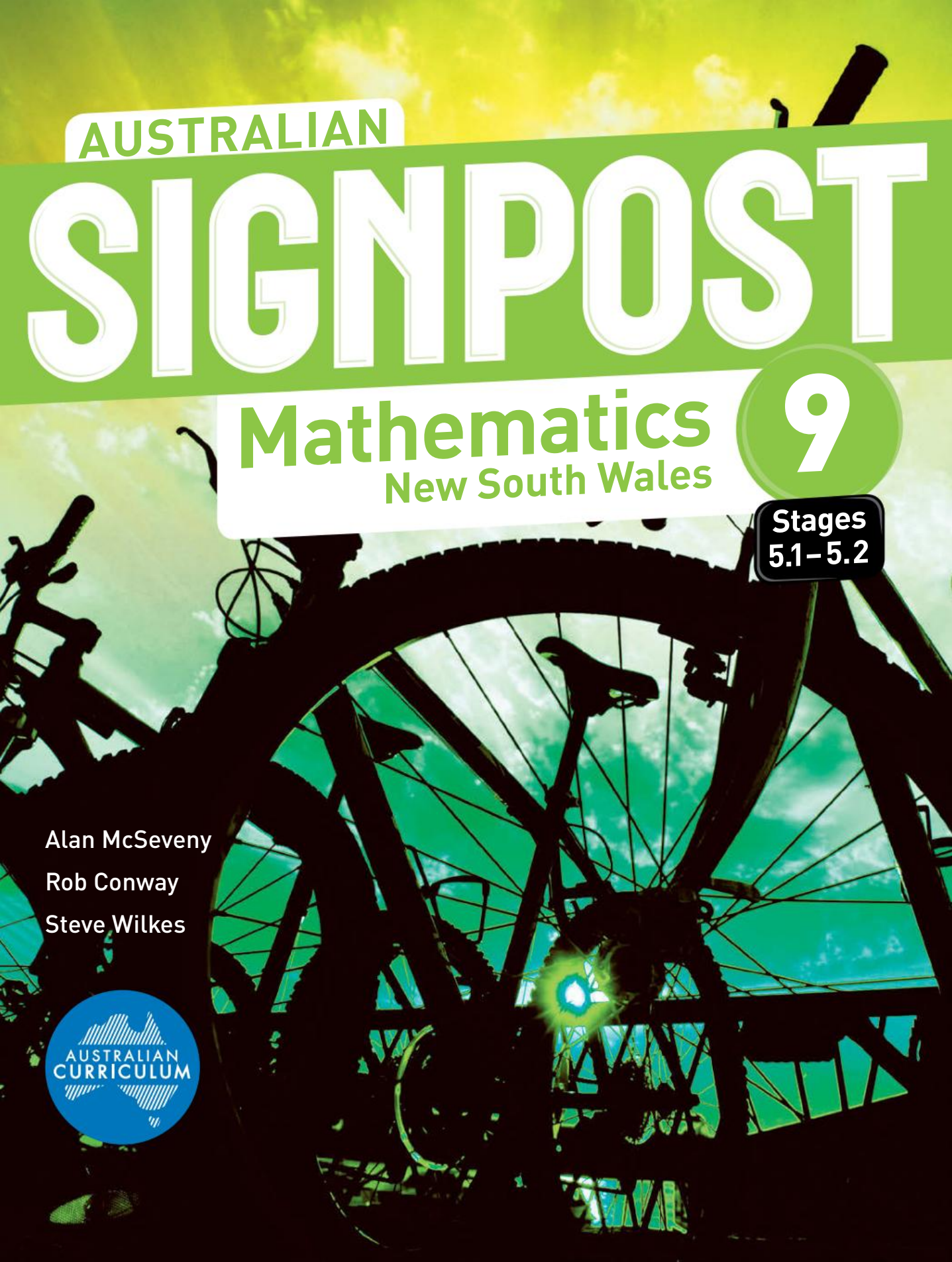
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Train a child in the way he should go, and
when he is old he will not turn from it.

Proverbs 22:6

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CONTENTS

How to use the Student Book viii
Treatment of syllabus content x
P Digital resources xiv
Metric equivalents xvi
The language of mathematics xvii
ID Card 1 (Metric units) xvii
ID Card 2 (Symbols) xvii
ID Card 3 (Language) xviii
ID Card 4 (Language) xix
ID Card 5 (Language) xx
ID Card 6 (Language) xxi
ID Card 7 (Language) xxii
Algebra card xxiii

Chapter 1 Number and measurement 1

1:01 The language of mathematics 2
1:02 Diagnostic check-ups 2
1:02A Integers 3
1:02B Fractions 3
1:02C Decimals 4
1:02D Percentages 5
Fun spot 1:02 What was the Prime Minister's name in 1998? 6
1:03 Rational numbers 7
1:04 Recurring decimals 8
Challenge 1:04 Try this with repeating decimals 10
Fun spot 1:04 Speedy addition 11
1:05 Ratios and rates 11
1:06 Significant figures 14
Investigation 1:06 A truck driver's memories 17
1:07 Approximations 18
Investigation 1:07 Take your medicine 22
1:08 Estimation 22
1:09 Using scientific notation 25
1:10 Units of measurement 28
1:11 Accuracy of measurements 30
Maths terms, Diagnostic test, Assignments 34

Chapter 2 Working mathematically 39

2:01 Solving routine problems 40
2:01A Rates 40
2:01B Ratios 42
2:01C Dividing a quantity in a given ratio 43
Challenge 2:01 Mixing drinks 45
2:01D Percentages 46
2:01E Measurement 48
Fun spot 2:01 Bouncing ball 50
2:02 Solving non-routine problems 51
Fun spot 2:02 What nationality is Santa Claus? 55
Investigation 2:02 Line marking 55
2:03 Using Venn diagrams 56
Fun spot 2:03 What type of breakfast takes an hour to finish? 59
Investigation 2:03 Comparing speeds 59
Maths terms, Diagnostic test, Assignments 59

Chapter 3 Algebra 65

3:01 Addition and subtraction in algebra 66
3:02 Multiplication in algebra 67
3:03 Division in algebra 68
3:04 Simplifying algebraic fractions 69
3:04A Addition and subtraction 69
3:04B Multiplication and division 70
3:05 Grouping symbols 71
Investigation 3:05 Checking algebraic simplification by substitution 73
3:06 Simplifying expressions with grouping symbols 73
3:07 Factorising using common factors 75
Fun spot 3:07 In the theatre, when do you tell someone to break a leg? 77
Maths terms, Diagnostic test, Assignments 78

Chapter 4 Probability 83

4:01 The language of probability 84
Investigation 4:01 Rolling dice 87
4:02 Experimental probability 87
Investigation 4:02 Tossing a coin 91
GeoGebra activity 4:02 Rolling a single dice a large number of times 92
4:03 Theoretical probability 92
Investigation 4:03 Chance experiments 97
4:04 Mutually and non-mutually exclusive events 98
Investigation 4:04 Chance happenings 101
4:05 Using diagrams and tables 102
Fun spot 4:05 What are Dewey decimals? 104
4:06 Two-step chance experiments 105
Investigation 4:06 Two-step chance experiments 110
Maths terms, Diagnostic test, Assignments 111

Chapter 5 Area, surface area and volume 116

5:01 Review of area 117
GeoGebra activity 5:01 Area formulas 118
Challenge 5:01 Areas of scale drawings 121
5:02 Areas of composite figures 122
GeoGebra activity 5:02 Areas of composite figures 126
5:03 Surface area of a prism 127
GeoGebra activity 5:03 Surface area of a rectangular prism 132
Fun spot 5:03 How did the boy know he had an affinity with the sea? 133
5:04 Surface area of a cylinder 134
Fun spot 5:04 Let's play with blocks 137
5:05 Volume of prisms and cylinders: A review 138
Investigation 5:05 Tank sizes 140
5:06 Volumes of other prisms 141
GeoGebra activity 5:06 Volume of a prism 146
Maths terms, Diagnostic test, Assignments 146

Chapter 6 Indices 152

6:01 Review of indices 153
Challenge 6:01 Family trees 155
6:02 The index laws 156
Investigation 6:02 Exploring index notation 159
6:03 Negative indices 160
Investigation 6:03 Zero and negative indices 163
6:04 Scientific (or standard) notation 164
Investigation 6:04 Multiplying and dividing by powers of 10 164
Fun spot 6:04 Why is a room full of married people always empty? 167
6:05 Scientific notation and the calculator 168
Investigation 6:05 Using scientific notation 170
6:06 Conversions using scientific notation 171
Maths terms, Diagnostic test, Assignments 173

Chapter 7 Equations and inequalities 178

7:01 Solving equations by inspection 179
7:02 Inverse operations 182
7:03 Solving simple equations 184
Investigation 7:03 Solving equations using a spreadsheet 188
7:04 Equations with pronumerals on both sides 189
GeoGebra activity 7:04 Equations with pronumerals on both sides 191
7:05 Equations with grouping symbols 191
GeoGebra activity 7:05 Equations with grouping symbols 193
Fun spot 7:05 If I have 7 apples in one hand and 4 in the other, what have I got? 193
7:06 Equations with fractions (1) 194
Fun spot 7:06 Who holds up submarines? 196
7:07 Equations with fractions (2) 196
Challenge 7:07 Solving equations by 'guess and check' 198
7:08 Graphing inequalities 199
7:09 Solving inequalities 200
Investigation 7:09 Operating on inequalities 201
Fun spot 7:09 Who 'dunnit'? 203
Maths terms, Diagnostic test, Assignments 204

Chapter 8 Financial mathematics 209

8:01 Earning money 210
8:02 Extra payments 214
8:03 Wage deductions 218
8:04 Income tax 221
Investigation 8:04 Income tax returns 224
Fun spot 8:04 What is brought to the table, cut, but never eaten? 225
8:05 Spending money 226
8:06 Ways of paying 234
Fun spot 8:06 The puzzle of the missing dollar 239
Maths terms, Diagnostic test, Assignments 240

Chapter 9 Coordinate geometry 246

9:01 Number plane review 247
9:02 The midpoint of an interval 251
9:03 The gradient of an interval 254
Investigation 9:03 Gradients and building 258
9:04 The length of an interval 259
GeoGebra activity 9:04 The midpoint, gradient and length of an interval 262
9:05 Graphing straight lines 263
Investigation 9:05 x- and y-intercepts 266
GeoGebra activity 9:05 Graphing lines by plotting points 266
9:06 Horizontal and vertical lines 267
9:07 The gradient–intercept form of a straight line: $y = mx + b$ 269
Investigation 9:07 What does $y = mx + b$ tell us? 269
GeoGebra activity 9:07 The gradient–intercept form of a line 274
Fun spot 9:07 Why did the banana go out with a fig? 275
9:08 Non-linear graphs 276
GeoGebra activity 9:08 Non-linear graphs 279
Maths terms, Diagnostic test, Assignments 280

Chapter 10 Similarity 286

10:01 Similar figures 287
GeoGebra activity 10:01 Similar figures 292
Investigation 10:01 Drawing enlargements and reductions 293
10:02 Finding unknown sides of similar figures 294
Investigation 10:02 Photo enlargements 299
10:03 Similar triangles 300
10:03A Matching angles 301
10:03B Ratios of matching sides 304
GeoGebra activitiy 10:03 Similar triangles 309
10:04 Using proportion to find unknown sides 310
GeoGebra activity 10:04 Finding an unknown side in similar triangles 315
10:05 Scale drawings 315
10:05A Calculating real sizes from scale drawings 316
10:05B Making a scale drawing 318
Investigation 10:05 Scale drawing 321
Fun spot 10:05 What happened to the mushroom that was double parked? 322
Maths terms, Diagnostic test, Assignments 323

Chapter 11 Formulas and problem solving 329

11:01 Generalised arithmetic 330
Fun spot 11:01 How much logic do you have? 333
11:02 Substitution 334
11:03 Formulas: Evaluating the subject 336
Investigation 11:03 Spreadsheet formulas 339
11:04 Formulas: Equations arising from substitution 340
11:05 Translating problems into equations 342
Fun spot 11:05 Read carefully (and think)! 345
11:06 Further problem solving using equations 345
Fun spot 11:06 Why are cooks cruel? 349
Maths terms, Diagnostic test, Assignments 350

Chapter 12 Trigonometry 355

12:01 Right-angled triangles 356
12:02 Similar right-angled triangles: The ratio of sides 358
GeoGebra activity 12:02 Ratios of the sides of similar right-angled triangles 360
12:03 Tangent ratio 361
Investigation 12:03 Tangent ratio 365
12:04 Finding an unknown side 366
GeoGebra activity 12:04 Finding an unknown side using the tangent ratio 368
12:05 Finding an unknown angle 369
12:06 Sine and cosine ratios 372
12:07 Finding unknown sides with sine and cosine 375
12:07A Finding a short side 375
GeoGebra activity 12:07A Finding an unknown side using the sine and cosine ratios 378
12:07B Finding the hypotenuse 378
GeoGebra activity 12:07B Finding the hypotenuse using the sine and cosine ratios 381
12:08 Using sine and cosine to find an unknown angle 381
GeoGebra activity 12:08 Using the trig ratios to find an angle 384
12:09 Miscellaneous exercises 384
12:09A Angles of elevation and depression 384
12:09B Compass bearings 386
12:09C Other topics 389
Maths terms, Diagnostic test, Assignments 390

Chapter 13 Statistics 395

13:01 Review of statistics 396
Investigation 13:01 Adding and averaging 404
13:02 Cumulative frequency 405
Fun spot 13:02 Which hand should you use to stir tea? 413
13:03 Measures of spread: Interquartile range 414
Fun spot 13:03 Why did the robber flee from the music store? 419
13:04 Box plots 420
Investigation 13:04 Code breaking and statistics 423
13:05 Comparing sets of data 424
Investigation 13:05 The ageing population 430
Maths terms, Diagnostic test, Assignments 431

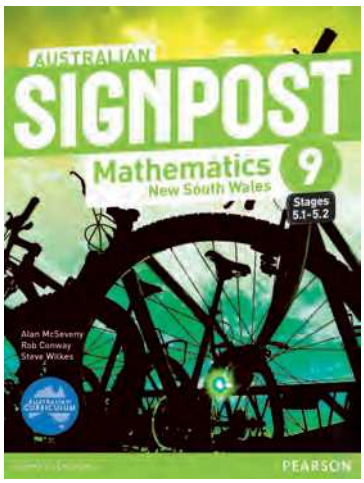
Chapter 14 Proportion 439

14:01 Review of rates 440
14:02 Direct proportion 443
14:03 Graphing direct proportion 448
Investigation 14:03 Proportional ups and downs! 452
14:04 Inverse proportion 453
Investigation 14:04 Graphing inverse proportion 457
Fun spot 14:04 How do poets write upside down? 458
Maths terms, Diagnostic test, Assignments 458

Answers 464

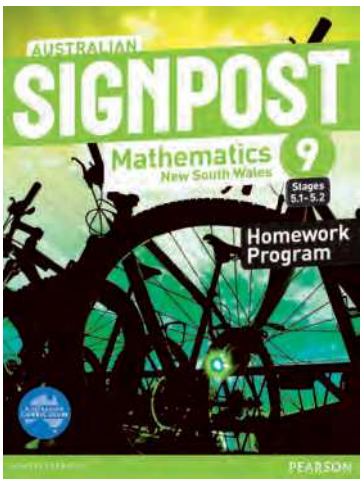
Australian Signpost Mathematics New South Wales 9 Stages 5.1–5.2

The **Australian Signpost Mathematics New South Wales** series has been completely updated to comprehensively cover the NSW Syllabus for the Australian Curriculum. Students and teachers are supported with a Student Book, an eBook, a Homework Program and a Teacher Companion for each stage of development. Features within each component, along with additional resources on the eBook, allow the creation of a personalised learning package for the individual. Improvements in technology, teaching resources and design allow students and teachers to approach the NSW Syllabus with confidence.



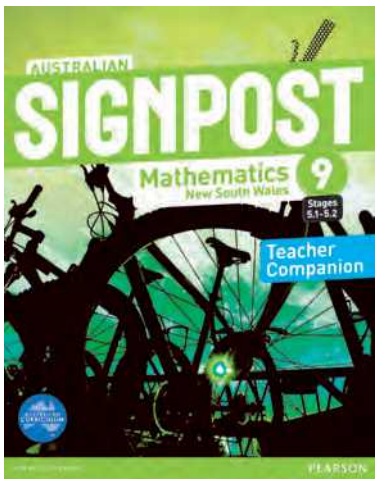
Student Book

The *Australian Signpost Mathematics New South Wales 9 Stages 5.1–5.2* and *Australian Signpost Mathematics New South Wales 9 Stages 5.1–5.3* learning package addresses Stage 5 of the NSW Syllabus for the Australian Curriculum. This new series continues the Signpost tradition of expanding students' ability in a broad range of mathematical skills while emphasising problem-solving and working mathematically.



Homework Program

The Homework Program consists of tear-out worksheets that mirror and supplement the content of the student book, providing opportunity for further practice and application of key skills.



Teacher Companion

The Teacher Companion is an invaluable resource that provides guidance and support to teachers using the **Australian Signpost Mathematics New South Wales** package. It is a practical resource featuring teaching strategies, activities, reference to Stage 5 outcomes and content statements, and links to digital learning materials.

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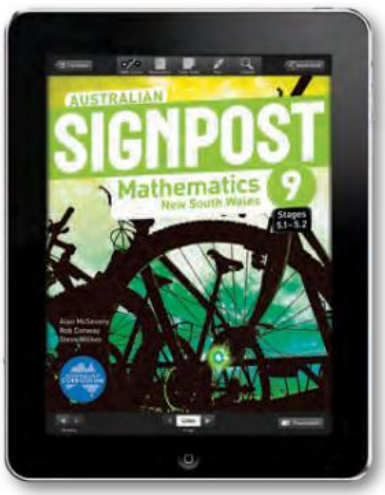

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Any device, every school

Australian Signpost Mathematics New South Wales 9 Stages 5.1–5.2 eBook 3.0 lets you use the Student Book online or offline on any device. It allows students to independently work through each exercise, linking to a range of technological applications designed specifically for Signpost users. The eBook will engage students while consolidating learning and providing opportunities for increased depth of understanding.

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HOW TO USE THE STUDENT BOOK

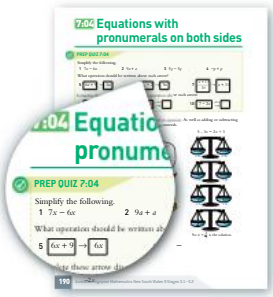
The **Australian Signpost Mathematics New South Wales 9 Stages 5.1–5.2** learning package covers Stage 5 outcomes of the NSW Syllabus for the Australian Curriculum.

Features of the Student Book

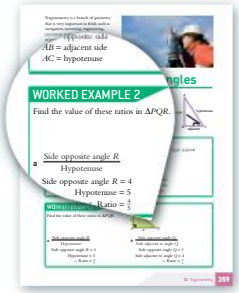
- a complete year's work with full coverage of the NSW Syllabus for the Australian Curriculum
- a flexible structure with carefully graded exercises, colour-coded to indicate level of difficulty
- technological applications with GeoGebra activities and Pearson eBook 3.0 resources
- integration of a broad range of mathematical skills, expanding students' ability to solve problems, work mathematically and investigate
- diagnostic tests, chapter reviews and cumulative revision components for every chapter

Chapter opener pages

summarise the key content and present the content statements addressed in each chapter. Features within each exercise such as GeoGebra activities, Fun spots and Investigations are listed here.

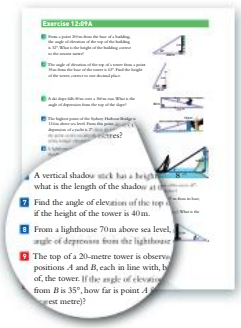


Prep quizzes review important skills and knowledge that students will need to complete an exercise.



Worked examples

take students through key processes with step-by-step demonstrations of questions similar to those they will encounter.



Well-graded exercises

indicate each question's level of difficulty.

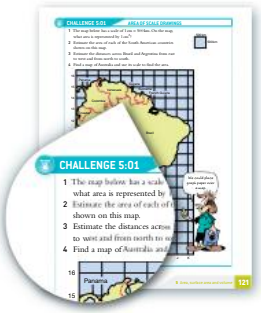
- 2 Foundation
- 7 Stages 5.1–5.2 level
- 12 Extension

GeoGebra activities

provide a deeper understanding of concepts covered within the exercise while providing the opportunity to integrate ICT into the classroom.

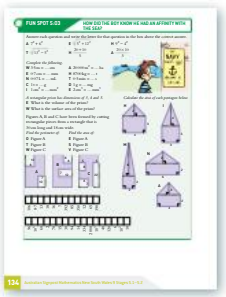
Investigations

encourage students to seek knowledge and develop research skills. The investigative tasks provide opportunities for students to interpret, question, model and communicate mathematically.

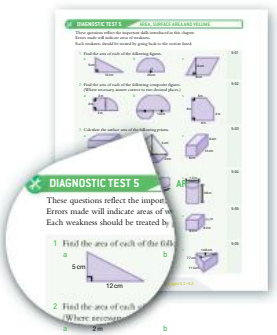


Challenge activities

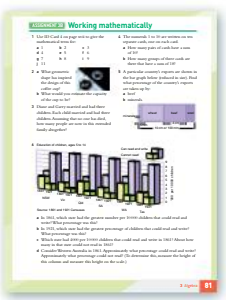
provide more difficult questions and investigations. They can be used to help students with a stronger understanding extend their knowledge.



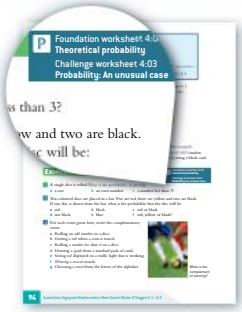
Fun spots provide amusement and interest while reinforcing concepts learnt within the chapter. They encourage creativity and divergent thinking, and show that mathematics is enjoyable.



Diagnostic tests at the end of each chapter test students' achievement of outcomes. Importantly, they also indicate student weaknesses that need to be addressed and provide links to the relevant sections in the text or digital resources.



Assignments are provided at the end of each chapter. They concentrate on revising the content of previous chapters and developing students' ability to work mathematically.

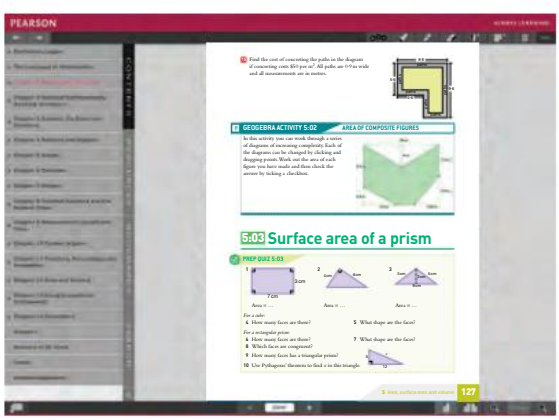


Foundation and Challenge worksheets

are indicated in the student book wherever a worksheet is available on the eBook. Foundation worksheets provide alternative exercises for consolidation of the fundamental content of the work being introduced. Challenge worksheets help students apply their learning in more advanced activities.

Pearson digital resources

A vast range of technology applications are available with the eBook and ProductLink to complement the content presented in the student book. Digital resources include interactive activities, Foundation and Challenge worksheets and appendices. From the eBook, students can also link directly to the GeoGebra activities referred to in the student book exercises.



The eBook aims to create a classroom environment that embraces mathematical technologies in a way that consolidates and extends student learning.

TREATMENT OF SYLLABUS CONTENT

The syllabus outcomes and statements relevant to Stages 5.1–5.2 of the NSW Syllabus for the Australian Curriculum are listed below. Chapters where the content is treated are shown.

Working Mathematically (encompassing five components: Communicating, Problem Solving, Reasoning, Understanding and Fluency) is an integral part of the structure of each chapter. Special attention is given to Working Mathematically in Chapter 2 of this Student Book and in the second Assignment of every chapter in all Year 9 and 10 Student Books.

Working Mathematically	
A student:	
> uses appropriate terminology, diagrams and symbols in mathematical contexts	**MA5.1-1WM**
> selects and uses appropriate strategies to solve problems	**MA5.1-2WM**
> provides reasoning to support conclusions that are appropriate to the context	**MA5.1-3WM**
> selects appropriate notations and conventions to communicate mathematical ideas and solutions	**MA5.2-1WM**
> interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems	**MA5.2-2WM**
> constructs arguments to prove and justify results	**MA5.2-3WM**
Number and Algebra	
A student:	
> solves financial problems involving earning, spending and investing money	**MA5.1-4NA**
> operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases	**MA5.1-5NA**
> determines the midpoint, gradient and length of an interval, and graphs linear relationships	**MA5.1-6NA**
> graphs simple non-linear relationships	**MA5.1-7NA**
> solves financial problems involving compound interest	**MA5.2-4NA ◊**
> recognises direct and indirect proportion, and solves problems involving direct proportion	**MA5.2-5NA**
> simplifies algebraic fractions, and expands and factorises quadratic expressions	**MA5.2-6NA**
> applies index laws to operate with algebraic expressions involving integer indices	**MA5.2-7NA**
> solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques	**MA5.2-8NA**
> uses the gradient-intercept form to interpret and graph linear relationships	**MA5.2-9NA**
> connects algebraic and graphical representations of simple non-linear relationships	**MA5.2-10NA ◊**
Measurement and Geometry	
A student:	
> calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms	**MA5.1-8MG**
> interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures	**MA5.1-9MG**
> applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression	**MA5.1-10MG**
> describes and applies the properties of similar figures and scale drawings	**MA5.1-11MG**
> calculates the surface areas of right prisms, cylinders and related composite solids	**MA5.2-11MG**
> applies formulas to calculate the volumes of composite solids composed of right prisms and cylinders	**MA5.2-12MG**
> applies trigonometry to solve problems, including problems involving bearings	**MA5.2-13MG ◊**
> calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar	**MA5.2-14MG**
Statistics and Probability	
A student:	
> uses statistical displays to compare sets of data, and evaluates statistical claims made in the media	**MA5.1-12SP**
> calculates relative frequencies to estimate probabilities of simple and compound events	**MA5.1-13SP**
> uses quartiles and box plots to compare sets of data, and evaluates sources of data	**MA5.2-15SP ◊**
> investigates relationships between two statistical variables, including their relationship over time	**MA5.2-16SP**
> describes and calculates probabilities in multi-step chance experiments	**MA5.2-17SP**

Stages 5.1–5.2 content

The Stage 5 content is divided into three strands:

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

Content statements are listed below with their Australian Curriculum code or NSW syllabus reference. The chapters that address each content statement are shown in the right-hand column.

Number and Algebra (5.1)	Reference	Chapter
Financial Mathematics		
• Solve problems involving earning money.	NSW (Stage 5.1)	Ch 8 (Y9)
• Solve problems involving simple interest.	ACMNA211 (Stage 5.1)	Ch 8 (Y9)
• Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.	ACMNA229 (Stage 5.1)	Ch 5 (Y10)
Indices		
• Extend and apply the index laws to variables, using positive-integer indices and the zero index.	ACMNA212 (Stage 5.1)	Ch 6 (Y9)
• Simplify algebraic products and quotients using index laws.	ACMNA231 (Stage 5.1)	Ch 6 (Y9)
• Apply index laws to numerical expressions with integer indices.	ACMNA209 (Stage 5.1)	Ch 6 (Y9)
Linear Relationships		
• Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.	ACMNA294 (Stage 5.1)	Ch 9 (Y9)
• Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software.	ACMNA214 (Stage 5.1)	Ch 9 (Y9)
• Sketch linear graphs using the coordinates of two points.	ACMNA215 (Stage 5.1)	Ch 7 (Y9) Ch 9 (Y9)
• Solve problems involving parallel lines.	ACMNA238 (Stage 5.1)	Ch 3 (Y10)
Non-linear Relationships		
• Graph simple non-linear relations, with and without use of digital technologies.	ACMNA296 (Stage 5.1)	Ch 9 (Y9)
• Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technologies as appropriate.	ACMNA239 (Stage 5.1)	Ch 7 (Y10)

Measurement and Geometry (5.1)	Reference	Chapter
Area and Surface Area		
• Calculate the areas of composite shapes.	ACMMG216 (Stage 5.1)	Ch 5 (Y9)
• Solve problems involving the surface areas of right prisms.	ACMMG218 (Stage 5.1)	Ch 5 (Y9)
Numbers of Any Magnitude		
• Investigate very small and very large time scales and intervals.	ACMMG219 (Stage 5.1)	Ch 1 (Y9)
• Express numbers in scientific notation.	ACMMG210 (Stage 5.1)	Ch 1 (Y9) Ch 6 (Y9)
Right-angled Triangles (Trigonometry)		
• Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles.	ACMMG223 (Stage 5.1)	Ch 12 (Y9)
• Apply trigonometry to solve right-angled triangle problems.	ACMMG224 (Stage 5.1)	Ch 12 (Y9)
• Solve right-angled triangle problems, including those involving angles of elevation and depression.	ACMMG245 (Stage 5.1)	Ch 12 (Y9)
Properties of Geometrical Figures		
• Use the enlargement transformation to explain similarity.	ACMMG220 (Stage 5.1)	Ch 10 (Y9)
• Solve problems using ratio and scale factors in similar figures.	ACMMG221 (Stage 5.1)	Ch 10 (Y9)

Statistics and Probability (5.1)	Reference	Chapter
Single Variable Data Analysis		
• Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources.	ACMSP228 (Stage 5.1)	Ch 8 (Y10)
• Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal'.	ACMSP282 (Stage 5.1)	Ch 13 (Y9)
• Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread.	ACMSP283 (Stage 5.1)	Ch 13 (Y9)
• Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data.	ACMSP253 (Stage 5.1)	Ch 8 (Y10)
Probability		
• Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.	ACMSP226 (Stage 5.1)	Ch 4 (Y9)

Number and Algebra (5.2)	Reference	Chapter
Financial Mathematics ◊		
• Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.	ACMNA229 (Stage 5.2)	Ch 6 (Y10)
Ratio and Rates		
• Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems.	ACMNA208 (Stage 5.2)	Ch 14 (Y9)
Algebraic Techniques		
• Apply the four operations to simple algebraic fractions with numerical denominators.	ACMNA232 (Stage 5.2)	Ch 3 (Y9)
• Apply the four operations to algebraic fractions with pronumerals in the denominator.	NSW (Stage 5.2)	Ch 2 (Y10)
• Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 (Stage 5.2)	Ch 3 (Y9)
• Factorise algebraic expressions by taking out a common algebraic factor.	ACMNA230 (Stage 5.2)	Ch 3 (Y9)
• Expand binomial products and factorise monic quadratic expressions using a variety of strategies.	ACMNA233 (Stage 5.2)	Ch 2 (Y10)
Indices		
• Apply index laws to algebraic expressions involving integer indices.	NSW (Stage 5.2)	Ch 6 (Y9)
Equations		
• Solve linear equations.	ACMNA215 (Stage 5.2)	Ch 7 (Y9)
• Solve linear equations involving simple algebraic fractions.	ACMNA240 (Stage 5.2)	Ch 7 (Y9)
• Solve simple quadratic equations using a range of strategies.	ACMNA241 (Stage 5.2)	Ch 2 (Y10)
• Substitute values into formulas to determine an unknown.	ACMNA234 (Stage 5.2)	Ch 11 (Y9)
• Solve problems involving linear equations, including those derived from formulas.	ACMNA235 (Stage 5.2)	Ch 11 (Y9)
• Solve linear inequalities and graph their solutions on a number line.	ACMNA236 (Stage 5.2)	Ch 7 (Y9)
• Solve linear simultaneous equations, using algebraic and graphical techniques, including with the use of digital technologies.	ACMNA237 (Stage 5.2)	Ch 5 (Y10)
Linear Relationships		
• Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line.	NSW (Stage 5.2)	Ch 9 (Y9)
• Solve problems involving parallel and perpendicular lines.	ACMNA238 (Stage 5.2)	Ch 3 (Y10)
Non-linear Relationships ◊		
• Graph simple non-linear relationships, with and without use of digital technologies, and solve simple related equations.	ACMNA296 (Stage 5.2)	Ch 7 (Y10)
• Explore the connection between algebraic and graphical representations of relationships such as simple quadratics, circles and exponentials using digital technologies as appropriate.	ACMNA239 (Stage 5.2)	Ch 7 (Y10)

Measurement and Geometry (5.2)	Reference	Chapter
Area and Surface Area		
• Calculate the surface areas of cylinders and solve related problems.	ACMMG217 (Stage 5.2)	Ch 5 (Y9)
• Solve problems involving surface area for a range of prisms, cylinders and composite solids.	ACMMG242 (Stage 5.2)	Ch 5 (Y9) Ch 8 (Y10)
Volume		
• Solve problems involving the volumes of right prisms.	ACMMG218 (Stage 5.2)	Ch 5 (Y9)
• Solve problems involving volume for a range of prisms, cylinders and composite solids.	ACMMG242 (Stage 5.2)	Ch 8 (Y10)
Right-angled Triangles (Trigonometry) ◊		
• Apply trigonometry to solve right-angled triangle problems.	ACMMG224 (Stage 5.2)	Ch 12 (Y9)
• Solve right-angled triangle problems, including those involving direction and angles of elevation and depression.	ACMMG245 (Stage 5.2)	Ch 12 (Y9)
Properties of Geometrical Figures		
• Formulate proofs involving congruent triangles and angle properties.	ACMMG243 (Stage 5.2)	Ch 3 (Y10)
• Use the enlargement transformations to explain similarity and to develop the conditions for triangles to be similar.	ACMMG220 (Stage 5.2)	Ch 10 (Y9)
• Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes.	ACMMG244 (Stage 5.2)	Ch 3 (Y10)

Statistics and Probability (5.2)	Reference	Chapter
Single Variable Data Analysis ◊		
• Determine quartiles and interquartile range.	ACMSP248 (Stage 5.2)	Ch 13 (Y9)
• Construct and interpret box plots and use them to compare data sets.	ACMSP249 (Stage 5.2)	Ch 13 (Y9)
• Compare shapes of box plots to corresponding histograms and dot plots.	ACMSP250 (Stage 5.2)	Ch 13 (Y9)
• Investigate reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians.	ACMSP227 (Stage 5.2)	Ch 9 (Y10)
Bivariate Data Analysis		
• Investigate and describe bivariate numerical data where the independent variable is time.	ACMSP252 (Stage 5.2)	Ch 9 (Y10)
• Use scatter plots to investigate and comment on relationships between two numerical variables.	ACMSP251 (Stage 5.2)	Ch 9 (Y10)
Probability		
• List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays: assign probabilities to outcomes and determine probabilities for events.	ACMSP225 (Stage 5.2)	Ch 4 (Y9)
• Describe the results of two- and three-step chance experiments, with and without replacement, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence.	ACMSP246 (Stage 5.2)	Ch 4 (Y10)
• Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.	ACMSP247 (Stage 5.2)	Ch 4 (Y10)

P DIGITAL RESOURCES

Chapter	Drag-and-drop activities **ProductLink only**	Technology activities **Excel (E)—ProductLink only** **GeoGebra (G)—eBook and ProductLink**	Worksheets **Foundation worksheets (F)** **Challenge worksheets (C)** **—eBook and ProductLink**
1 Number and measurement	Maths terms 1 Converting fractions to decimals Simplifying ratios Rates Significant figures	Coldest temperatures (E)	1:04 Decimals and fractions (F) 1:07 Approximation (F) 1:08 Estimation (F)
2 Working mathematically		Sharing the prize (E)	
3 Algebra	Maths terms 3 Addition and subtraction in algebra Multiplying and dividing in algebra Addition and subtraction of algebraic fractions Multiplication and division of algebraic fractions Grouping symbols Simplifying expressions with grouping symbols Factorising using common factors		3:01 Collecting like terms (F) 3:02 Multiplying pronumerals (F) 3:03 Algebraic division (F) 3:03 Algebraic simplifications (C) 3:04A Simplifying algebraic fractions (+/–) (F) 3:04B Simplifying algebraic fractions (×/÷) (F) 3:04B Algebraic fractions (C) 3:05 Grouping symbols (F) 3:06 Simplifications with expansions (F) 3:07 Factorising (F)
4 Probability	Maths terms 4 Pack of cards Rolling two dice	Theoretical and experimental probabilities (E) Chance experiments (E) Two-step chance experiments (E) Rolling a single dice a large number of times (G)	4:02 Experimental probability (F) 4:03 Theoretical probability (F) 4:03 Probability: An unusual case (C)
5 Area, surface area and volume	Area of sectors and composite figures Surface area of a prism Surface area of a cylinder	Area formulas (G) Areas of composite figures (G) Surface area of a rectangular prism (G) Volume of a prism (G)	5:02 Area (F) 5:03 Surface area of prisms (F) 5:04 Perimeter and area (F) 5:06 Volume of a prism (F)
6 Indices	Index laws Negative indices		6:02 The index laws (F) 6:03 Negative indices (F) 6:04 Scientific notation (F)
7 Equations and inequalities	Maths terms 7 Equations Equations with fractions Solving inequalities	Flowcharts (E) Solving equations using a spreadsheet (E) Equations with pronumerals on both sides 1 (G) Equations with pronumerals on both sides 2 (G) Equations with grouping symbols 1 (G) Equations with grouping symbols 2 (G)	7:01 Solving equations by inspection (F) 7:03 Solving simple equations (F) 7:04 Equations with pronumerals on both sides (F) 7:05 Equations with grouping symbols (F) 7:06 Equations with fractions 1 (F) 7:07 Equations with fractions 2 (F) 7:09 Solving inequalities (F)
8 Financial mathematics	Maths terms 8 Find the weekly wage GST Going shopping	Gross and net salaries (E)	8:02 Extra payments (F) 8:04 Taxation (F) 8:05 Goods and Services Tax (F) 8:06 Best buy, shopping lists, change (F)
9 Coordinate geometry	Maths terms 9 Number plane review Midpoint and gradient of an interval	Plotting points in four quadrants (E) Equation grapher (E) The midpoint, gradient and length of an interval (G)	9:01 Coordinates (F) 9:02 Midpoint (F) 9:03 Gradients (F) 9:04 Distance between points (F)

Chapter	Drag-and-drop activities **ProductLink only**	Technology activities **Excel (E)—ProductLink only** **GeoGebra (G)—eBook and ProductLink**	Worksheets **Foundation worksheets (F)** **Challenge worksheets (C)** **—eBook and ProductLink**
9 Coordinate geometry (continued)	Graphing lines *x*- and *y*-intercepts Horizontal and vertical lines The gradient–intercept form of a straight line	Graphing lines by plotting points (G) Finding the equation of a line (G) Equations of the form $y = mx + b$ (G) Graphing lines using the gradient and *y*-intercept (G) Matching equations of lines to their graphs (G) Graphing parabolas (G) Exponential curves (G) Matching parabolas to their equations (G)	9:05 Graphing lines (F) 9:06 Horizontal and vertical lines (F) 9:07 Gradient–intercept form (F)
10 Similarity	Maths terms 10 Finding lengths in similar figures Using the scale factor	Similar figures (G) Equal angles (G) Matching sides in the same ratio (G) Finding an unknown side in similar triangles (G)	10:04 Finding unknown sides in similar triangles (F)
11 Formulas and problem solving	Maths terms 11 Substitution Formulas Equations from formulas	Substitution (E) Substituting and transposing formulas (E) Spreadsheet formulas (E)	11:01 Generalised arithmetic (F) 11:02 Substitution (F) 11:05 Translating problems into equations (F)
12 Trigonometry	Maths terms 12 Finding angles The trigonometric ratios Finding sides Bearings 1 Bearings 2	Ratios of the sides of similar right-angled triangles (G) Finding an unknown side using the tangent ratio (G) Finding an unknown side using the sine and cosine ratios (G) Finding the hypotenuse using the sine and cosine ratios (G) Using the trig ratios to find an angle (G)	12:01 Sides of right-angled triangles (F) 12:03 The tangent ratio (F) 12:04 Finding unknown sides using the tangent ratio (F) 12:05 Finding unknown angles using the tangent ratio (F) 12:07A Finding unknown sides using sine and cosine (F) 12:07B Finding the hypotenuse (F) 12:09B Compass bearings (F)
13 Statistics	Maths terms 13 Box-and-whisker plots	Adding and averaging (E)	13:02 Frequency and cumulative frequency (F) 13:03 Interquartile range (F)
14 Proportion			

Teachers also have access to other digital resources on the eBook and ProductLink, including:

- syllabus overview document
- editable teaching programs
- answers to all worksheets
- answers to Prep quizzes and Investigations
- answers to the Homework Program.

Metric equivalents

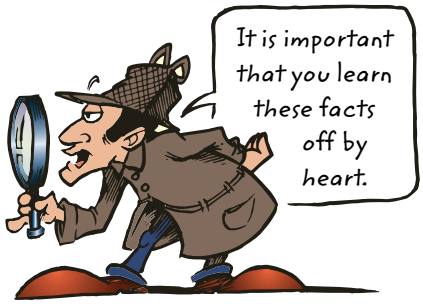
Length
$1\text{ m} = 1000\text{ mm}$ $= 100\text{ cm}$ $= 10\text{ dm}$ $1\text{ cm} = 10\text{ mm}$ $1\text{ km} = 1000\text{ m}$
Area
$1\text{ m}^2 = 10\,000\text{ cm}^2$ $1\text{ ha} = 10\,000\text{ m}^2$ $1\text{ km}^2 = 100\text{ ha}$
Mass
$1\text{ kg} = 1000\text{ g}$ $1\text{ t} = 1000\text{ kg}$ $1\text{ g} = 1000\text{ mg}$
Volume
$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$ $= 1000\text{ dm}^3$ $1\text{ L} = 1000\text{ mL}$ $1\text{ kL} = 1000\text{ L}$ $1\text{ m}^3 = 1\text{ kL}$ $1\text{ cm}^3 = 1\text{ mL}$ $1000\text{ cm}^3 = 1\text{ L}$
Time
$1\text{ min} = 60\text{ s}$ $1\text{ h} = 60\text{ min}$ $1\text{ day} = 24\text{ h}$ $1\text{ year} = 365\text{ days}$ $1\text{ leap year} = 366\text{ days}$

Months of the year

30 days each has September,
April, June and November.
All the rest have 31, except February alone,
Which has 28 days clear and 29 each leap year.

Seasons

Summer: December, January, February
Autumn: March, April, May
Winter: June, July, August
Spring: September, October, November



The language of mathematics

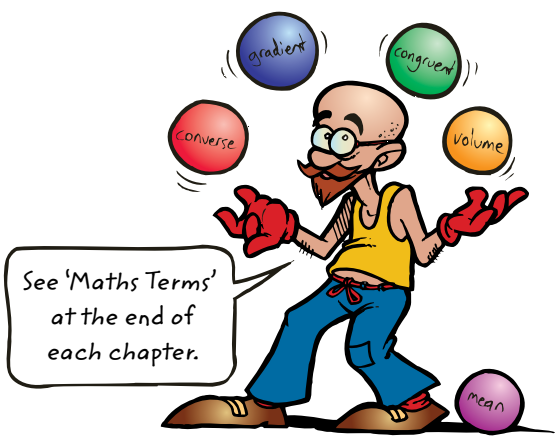
You should regularly test your knowledge by identifying the items on each card.

ID Card 1 (Metric units)			
1 m	2 dm	3 cm	4 mm
5 km	6 m^2	7 cm^2	8 km^2
9 ha	10 m^3	11 cm^3	12 s
13 min	14 h	15 m/s	16 km/h
17 g	18 mg	19 kg	20 t
21 L	22 mL	23 kL	24 °C

For answers see page 464.

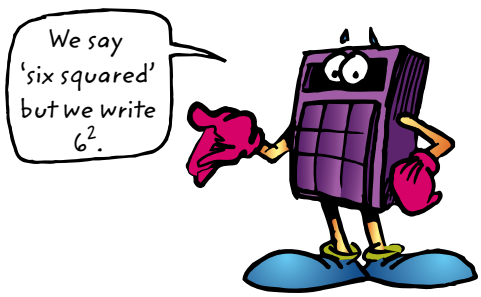
ID Card 2 (Symbols)			
1 $=$	2 $\div$ or $\approx$	3 $\neq$	4 $<$
5 $\leq$	6 $\not<$	7 $>$	8 $\geq$
9 4^2	10 4^3	11 $\sqrt{2}$	12 $\sqrt[3]{2}$
13	14 $\parallel$	15 $\equiv$	16 $\parallel\!\!\parallel$
17 %	18 $\therefore$	19 e.g.	20 i.e.
21 π	22 Σ	23 $\bar{x}$	24 $P(E)$

For answers see page 464.

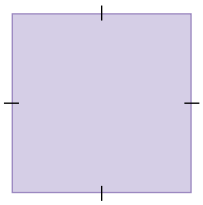
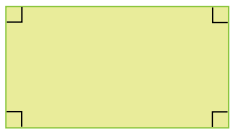
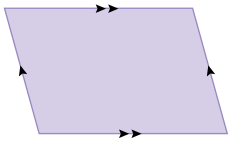
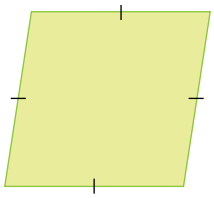
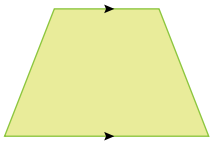
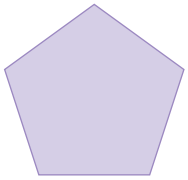
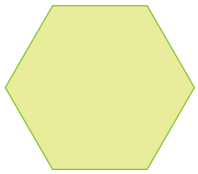
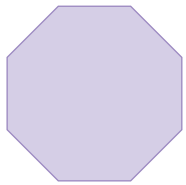
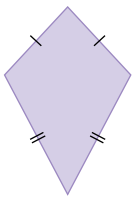
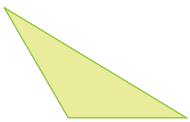
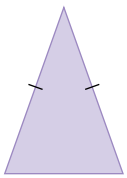
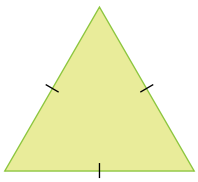
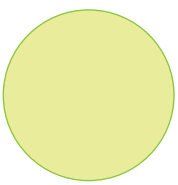
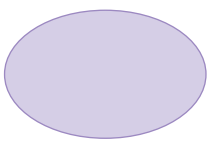
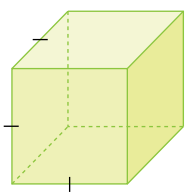
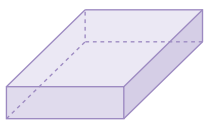
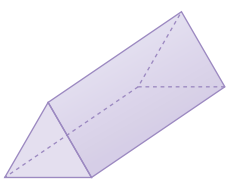
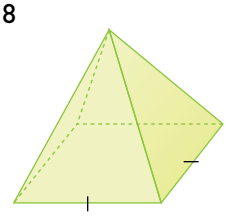
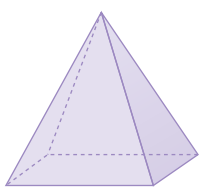
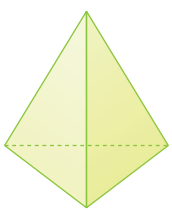
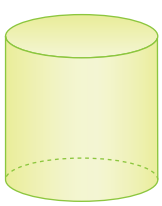
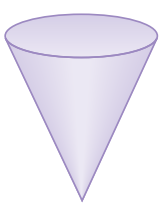
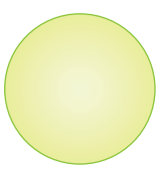
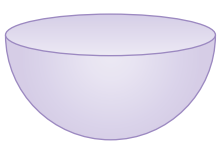


ID Card 3 (Language)			
1 6 minus 2	**2** the sum of 6 and 2	**3** divide 6 by 2	**4** subtract 2 from 6
5 the quotient of 6 and 2	**6** $2\overline{)6}$ the divisor is	**7** $2\overline{)6}$ the dividend is	**8** 6 lots of 2
9 decrease 6 by 2	**10** the product of 6 and 2	**11** 6 more than 2	**12** 2 less than 6
13 6 squared	**14** the square root of 36	**15** 6 take away 2	**16** multiply 6 by 2
17 average of 6 and 2	**18** add 6 and 2	**19** 6 to the power of 2	**20** 6 less 2
21 the difference between 6 and 2	**22** increase 6 by 2	**23** share 6 between 2	**24** the total of 6 and 2

For answers see page 464.



ID Card 4 (Language)

1 	2 	3 	4 
5 	6 	7 	8 
9 	10  All sides different	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 

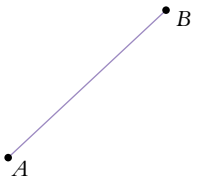
For answers see page 464.

ID Card 5 (Language)

1

.....

2



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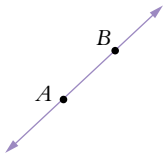

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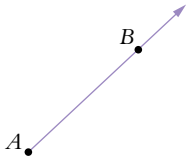
4



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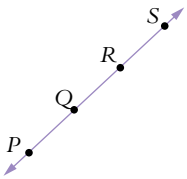
5



..... points

6

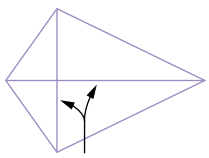
C is the .....

7

.....

.....

8



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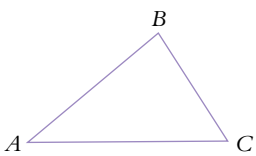
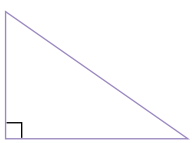
9



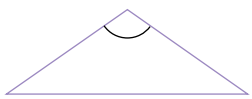
all angles less than 90°

10



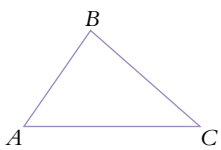
one angle 90°

11



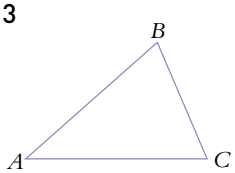
one angle greater than 90°

12



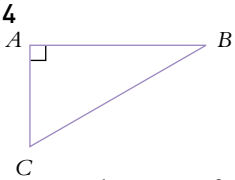
A, *B* and *C* are ..... of the triangle.

13



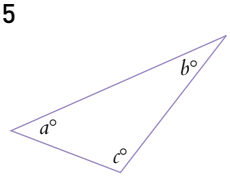
Use the vertices to name the Δ.

14



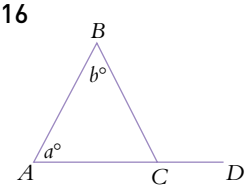
BC is the ..... of the right-angled Δ.

15



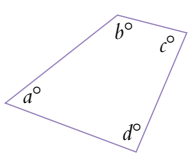
$a° + b° + c°$ = ...

16



$\angle BCD$ = ...

17

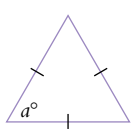


$a° + b° + c° + d°$ = ...

18

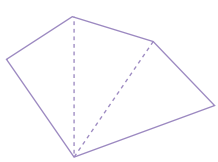
Which is true?
(a) $a° < b°$
(b) $a° = b°$
(c) $a° > b°$

19



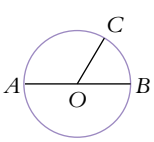
$a°$ = ...

20



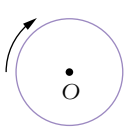
Angle sum = ...

21



AB is a .....
OC is a .....

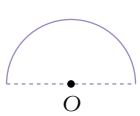
22



Name of distance around the circle.

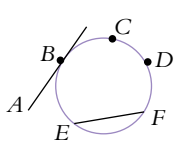
.....

23



s.....

24

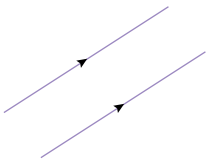


AB is a .....
CD is an .....
EF is a .....

For answers see page 464.

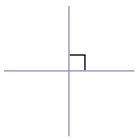
ID Card 6 (Language)

1



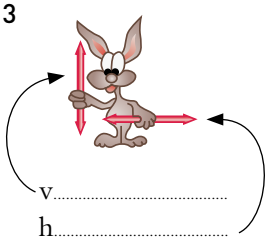
..... lines

2



..... lines

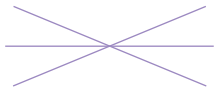
3



v.....

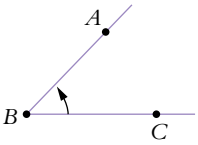
h.....

4



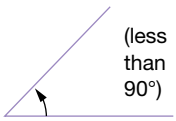
..... lines

5



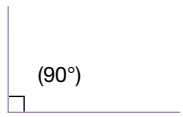
angle .....

6



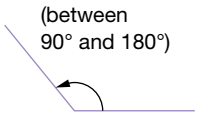
..... angle

7



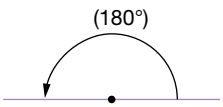
..... angle

8



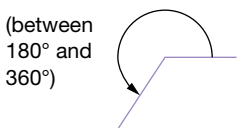
..... angle

9



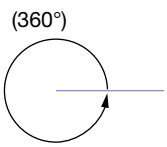
..... angle

10



..... angle

11



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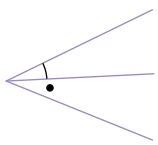
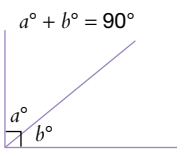
12



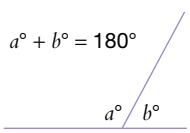
..... angles

13



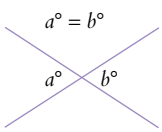
..... angles

14



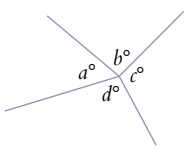
..... angles

15



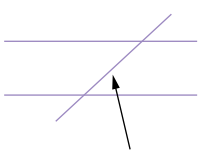
..... angles

16



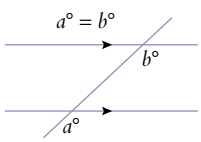
$a° + b° + c° + d° =$ ....

17



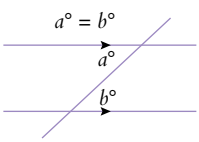
.....

18



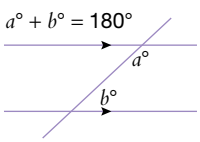
..... angles

19



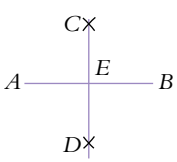
..... angles

20



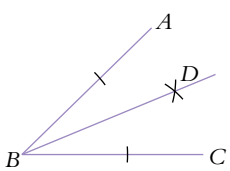
..... angles

21



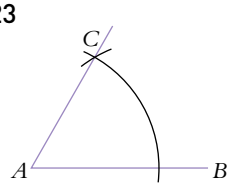
b..... an interval

22



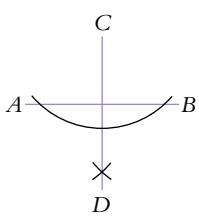
b..... an angle

23



$\angle CAB =$ .....

24



CD is p..... to AB.

For answers see page 465.

ID Card 7 (Language)

1

CE

C..... E.....

AD

A..... D.....

2

BCE

B..... C..... E.....

BC

B..... C.....

3

am

a..... m.....

means

b..... m.....

4

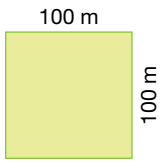
pm

p..... m.....

means

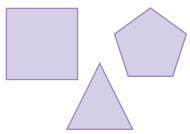
a..... m.....

5



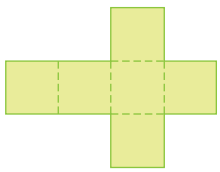
area is 1 .....

6



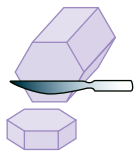
r..... shapes

7



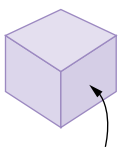
..... of a cube

8



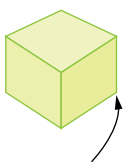
c.....-s.....

9



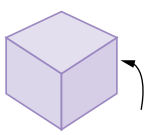
f.....

10



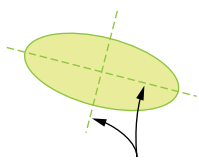
v.....

11



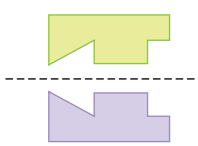
e.....

12



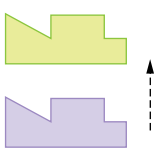
axes of .....

13



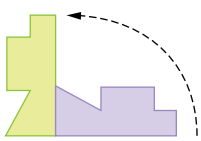
r.....

14



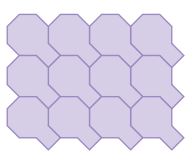
t.....

15



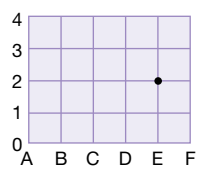
r.....

16



t.....

17



The c.....
of the dot are E2.

18

Cars sold

Mon	𝍸 I
Tues	𝍸 𝍸 𝍸 I
Wed	𝍸 𝍸 II
Thurs	𝍸 𝍸 𝍸
Fri	𝍸 𝍸 II

t.....

19

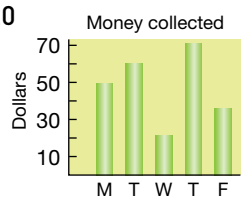
Money collected

Mon	● ● ● ●
Tues	● ● ● ◔
Wed	● ◔
Thurs	● ● ● ●
Fri	● ● ◔

● stands for $10

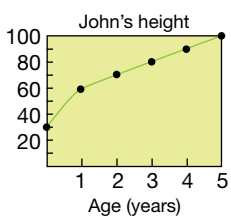
p..... graph

20



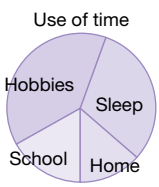
c..... graph

21



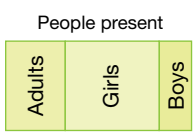
l..... graph

22



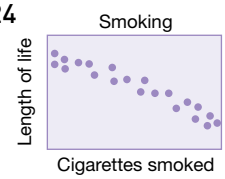
s..... graph

23



b..... graph

24



s..... d.....

For answers see page 465.

Algebra card

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	3	2·1	$\frac{1}{4}$	m	$\frac{2m}{3}$	$-3m$	$5m^2$	$-5x$	$\frac{x}{6}$	$-3x$	$-\frac{x}{2}$	$x + 2$	$x - 3$	$2x + 1$	$3x - 8$
2	−1	−0·4	$\frac{1}{8}$	$-4m$	$\frac{m}{4}$	$2m$	$-2m^3$	$3x$	$-\frac{x}{3}$	$5x^2$	$\frac{x}{4}$	$x + 7$	$x - 6$	$4x + 2$	$x - 1$
3	5	0·8	$\frac{1}{3}$	$10m$	$-\frac{m}{4}$	$-5m$	$8m^5$	$10x$	$-\frac{2x}{7}$	$-8x$	$\frac{2x}{5}$	$x + 5$	$x + 5$	$6x + 2$	$x - 5$
4	−2	1·5	$\frac{1}{20}$	$-8m$	$-\frac{3m}{2}$	$7m$	$6m^2$	$-15x$	$\frac{x}{10}$	$-4x^4$	$-\frac{x}{5}$	$x + 1$	$x - 9$	$3x + 3$	$2x + 4$
5	−8	−2·5	$\frac{3}{5}$	$2m$	$-\frac{m}{5}$	$10m$	m^2	$7x$	$\frac{2x}{3}$	$2x^3$	$\frac{x}{3}$	$x + 8$	$x + 2$	$3x + 8$	$3x + 1$
6	10	−0·7	$\frac{2}{7}$	$-5m$	$-\frac{3m}{7}$	$-6m$	$-9m^3$	$9x$	$-\frac{2x}{5}$	x^2	$\frac{3x}{5}$	$x + 4$	$x - 7$	$3x + 1$	$x + 7$
7	−6	−1·2	$\frac{3}{8}$	$8m$	$-\frac{m}{6}$	$9m$	$2m^6$	$-6x$	$\frac{5x}{6}$	$5x^2$	$\frac{2x}{3}$	$x + 6$	$x - 1$	$x + 8$	$2x - 5$
8	12	0·5	$\frac{9}{20}$	$20m$	$\frac{2m}{5}$	$-4m$	$-3m^3$	$-12x$	$\frac{3x}{4}$	$4x^3$	$-\frac{x}{7}$	$x + 10$	$x - 8$	$5x + 2$	$x - 10$
9	7	0·1	$\frac{3}{4}$	$5m$	$\frac{3m}{5}$	$-10m$	m^7	$5x$	$-\frac{3x}{7}$	$-3x^5$	$-\frac{3x}{7}$	$x + 2$	$x + 5$	$2x + 4$	$2x - 4$
10	−5	−0·6	$\frac{7}{10}$	$-9m$	$-\frac{4m}{5}$	$-7m$	$-8m^4$	$-3x$	$-\frac{x}{6}$	$-7x^5$	$\frac{2x}{9}$	$x + 1$	$x - 7$	$5x + 4$	$x + 7$
11	−11	−1·8	$\frac{1}{10}$	$-7m$	$\frac{m}{5}$	$-8m$	$-4m$	$-4x$	$\frac{x}{5}$	$-x^3$	$\frac{x}{3}$	$x + 9$	$x + 6$	$2x + 7$	$x - 6$
12	4	−1·4	$\frac{2}{5}$	$3m$	$\frac{m}{3}$	$12m$	$7m^2$	$-7x$	$-\frac{3x}{4}$	x^{10}	$\frac{x}{6}$	$x + 3$	$x - 10$	$2x + 3$	$2x + 3$

How to use this card

As an example, if the instruction given for the Algebra card is 'column D + column F' then you would write answers for the following problems.

1 $m + (-3m)$ **2** $(-4m) + 2m$ **3** $10m + (-5m)$ **4** $(-8m) + 7m$

5 $2m + 10m$ **6** $(-5m) + (-6m)$ **7** $8m + 9m$ **8** $20m + (-4m)$

9 $5m + (-10m)$ **10** $(-9m) + (-7m)$ **11** $(-7m) + (-8m)$ **12** $3m + 12m$

NUMBER AND MEASUREMENT

1

Contents

1:01 The language of mathematics
1:02 Diagnostic check-ups
1:02A Integers
1:02B Fractions
1:02C Decimals
1:02D Percentages
Fun spot 1:02 What was the Prime Minister's name in 1998?
1:03 Rational numbers
1:04 Recurring decimals
Challenge 1:04 Try this with repeating decimals
Fun spot 1:04 Speedy addition
1:05 Ratios and rates
1:06 Significant figures
Investigation 1:06 A truck driver's memories
1:07 Approximations
Investigation 1:07 Take your medicine
1:08 Estimation
1:09 Using scientific notation
1:10 Units of measurement
1:11 Accuracy of measurements
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Numbers of Any Magnitude*** [Stage 5.1]

- Investigate very small and very large time scales and intervals. (ACMMG219)
- Express numbers in scientific notation. (ACMNA210)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

1:01 The language of mathematics

Much of the language met so far is reviewed in the identification cards (ID Cards) found on pages xvii to xxii. These should be referred to throughout the Student Book. Make sure that you can identify every term.

Exercise 1:01

1 Write each symbol and its meaning in **ID Card 1** (Metric units) on page xvii.

2 Write each symbol and its meaning in **ID Card 2** (Symbols) on page xvii.

3 Write a number sentence, including the answer, for each expression in **ID Card 3** (Language) on page xviii. For example, for '6 minus 2' write '$6 - 2 = 4$'.

4 Write *true* or *false*, without using a calculator.

a $344 \times 1{\cdot}5 \neq 246$ **b** $830 \div 0{\cdot}9 > 830$ **c** $830 \div 1{\cdot}1 < 830$

d $199 \times 0{\cdot}9 > 199$ **e** $699 \div 0{\cdot}99 > 699$ **f** $699 \div 1{\cdot}01 < 699$

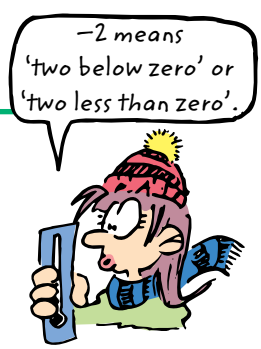
g $1{\cdot}1^2 \not< 1{\cdot}1^3$ **h** $\pi \div 3{\cdot}14$ **i** $0{\cdot}1\% = 0{\cdot}001$

j $\sqrt{1{\cdot}1} < 1{\cdot}1$ **k** $\sqrt[3]{1{\cdot}2} > 1{\cdot}2$ **l** $0{\cdot}8^2 < 0{\cdot}8$

m $\sqrt{2} > \sqrt[3]{2}$ **n** $\sqrt{1} > \sqrt[3]{1}$ **o** $\sqrt{0{\cdot}8} < \sqrt[3]{0{\cdot}8}$

Estimate the width of this bridge, in metres.

1:02 Diagnostic check-ups



Without obtaining help, complete the following diagnostic tests to determine which areas need attention. Errors made will indicate areas of weakness. If you are having trouble, refer to the stated Appendix, which can be found on the eBook and ProductLink. These include explanations and worked examples relating to these skills. **Do not use a calculator!**

Conversion facts you should know

a $10\% = 0{\cdot}1 = \frac{1}{10}$
Multiply each by 6.
$60\% = 0{\cdot}6 = \frac{6}{10}$

b $5\% = 0{\cdot}05 = \frac{1}{20}$
Multiply each by 7.
$35\% = 0{\cdot}35 = \frac{7}{20}$

c $20\% = 0{\cdot}2 = \frac{1}{5}$
Add 1 or 100% to each.
$120\% = 1{\cdot}2 = 1\frac{1}{5}$

d $12\frac{1}{2}\% = 0{\cdot}125 = \frac{1}{8}$
Add 1 or 100% to each.
$112\frac{1}{2}\% = 1{\cdot}125 = 1\frac{1}{8}$

Percentage	Decimal	Fraction
1%	0·01	$\frac{1}{100}$
5%	0·05	$\frac{1}{20}$
10%	0·1	$\frac{1}{10}$
$12\frac{1}{2}\%$	0·125	$\frac{1}{8}$
20%	0·2	$\frac{1}{5}$
25%	0·25	$\frac{1}{4}$
$33\frac{1}{3}\%$	$0{\cdot}\dot{3}$	$\frac{1}{3}$
50%	0·5	$\frac{1}{2}$
100%	1	1

CHECK-UP 1:02A INTEGERS

P Appendix 1:02A Sets A–D

Give the simplest answer for each of the following.

1 a $-7 + 14$ b $2 - 15$ c $-2 - 8$

2 a $3 - -6$ b $12 + (-5)$ c $6 - (3 - 8)$

3 a -3×2 b -5×-6 c $7 \times (-9)$

4 a $(-15) \div (-3)$ b $63 \div -9$ c $\frac{-156}{-3}$

5 a $14 - 7 \times 10$ b $(-3 + 6) \div 3$ c $(4 - 18) \div (-8 + 6)$

CHECK-UP 1:02B FRACTIONS

P Appendix 1:02B Sets A–L

1 Write these improper fractions as mixed numerals.
a $\frac{7}{4}$ b $\frac{13}{3}$ c $\frac{141}{10}$

2 Write these mixed numerals as improper fractions.
a $2\frac{1}{2}$ b $5\frac{3}{10}$ c $3\frac{1}{7}$

3 Simplify these fractions.
a $\frac{16}{24}$ b $\frac{100}{650}$ c $\frac{240}{3600}$

4 Complete the following to make equivalent fractions.
a $\frac{3}{4} = \frac{\square}{28}$ b $\frac{17}{20} = \frac{\square}{100}$ c $\frac{3}{8} = \frac{\square}{1000}$

3 of 4 equal parts
$\frac{3}{4}$ → Numerator
→ Denominator

Give the simplest answer for each of the following.

5 a $\frac{3}{8} + \frac{2}{8}$ b $\frac{9}{10} + \frac{3}{10}$ c $\frac{7}{9} + \frac{2}{9}$

6 a $\frac{9}{10}-\frac{7}{10}$ b $\frac{13}{14}-\frac{9}{14}$ c $\frac{37}{100}-\frac{11}{100}$

7 a $\frac{3}{4}+\frac{4}{5}$ b $\frac{3}{10}+\frac{2}{5}$ c $\frac{7}{100}+\frac{3}{40}$

8 a $\frac{7}{8}-\frac{3}{4}$ b $\frac{9}{10}-\frac{1}{4}$ c $\frac{5}{6}-\frac{3}{5}$

9 a $3\frac{1}{2}+4\frac{3}{5}$ b $6\frac{7}{10}+5\frac{3}{4}$ c $1\frac{5}{6}+\frac{7}{8}$

10 a $4\frac{1}{2}-1\frac{2}{9}$ b $10\frac{3}{4}-5\frac{1}{10}$ c $20\frac{3}{8}-\frac{1}{5}$

11 a $7\frac{1}{2}-\frac{7}{8}$ b $6\frac{3}{5}-2\frac{7}{10}$ c $3\frac{1}{2}-1\frac{5}{6}$

12 a $\frac{4}{5}\times\frac{3}{11}$ b $\frac{3}{10}\times\frac{7}{10}$ c $\frac{1}{10}\times\frac{3}{5}$

13 a $\frac{7}{8}\times\frac{3}{7}$ b $\frac{15}{38}\times\frac{19}{20}$ c $\frac{7}{10}\times\frac{5}{6}$

14 a $3\frac{1}{5}\times\frac{5}{7}$ b $1\frac{3}{10}\times1\frac{4}{5}$ c $5\frac{1}{4}\times2\frac{2}{3}$

15 a $4\times3\frac{4}{5}$ b $2\frac{1}{4}\times3$ c $5\times6\frac{3}{8}$

16 a $\frac{8}{10}\div\frac{2}{10}$ b $\frac{9}{20}\div\frac{3}{20}$ c $\frac{7}{10}\div\frac{7}{10}$

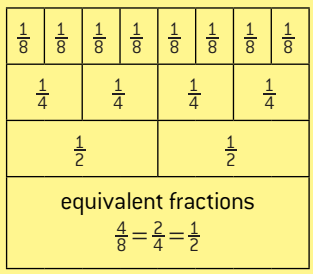
17 a $\frac{3}{5}\div\frac{1}{2}$ b $\frac{8}{9}\div\frac{3}{4}$ c $\frac{5}{8}\div\frac{4}{7}$

18 a $1\frac{7}{8}\div\frac{3}{4}$ b $3\frac{4}{7}\div2\frac{1}{2}$ c $3\frac{5}{8}\div2\frac{9}{10}$

19 a $7\frac{1}{2}\div3$ b $4\frac{9}{10}\div7$ c $6\frac{7}{8}\div5$

20 a $5\div\frac{1}{4}$ b $10\div\frac{1}{5}$ c $4\div\frac{1}{10}$

Fractions should always be expressed in lowest terms.
$\frac{4}{6}=\frac{2}{3}$



CHECK-UP 1:02C DECIMALS

P Appendix 1:02C Sets A–J

1 Place in order from smallest to largest.
a 0·505, 0·5, 0·55 b 8·4, 8·402, 8·41 c 1·01, 1·1, 1·011

Give the simplest answer for each of the following.

2 a 2·6 + 3·14 b 18·6 + 3 c 0·145 + 0·12

3 a 12·83 − 1·2 b 9 − 1·824 c 4·02 − 0·005

4 a 0·7 × 6 b $(0{\cdot}3)^2$ c 0·02 × 1·7

5 a 3·142 × 100 b 0·04 × 1000 c 0·065 × 10

6 a $2{\cdot}1\times10^4$ b $8{\cdot}04\times10^6$ c $1{\cdot}25\times10^2$

7 a 4·08 ÷ 2 b 12·1 ÷ 5 c 0·19 ÷ 4

8 Write answers as repeating decimals.
a 2·5 ÷ 6 b 5·32 ÷ 9 c 28 ÷ 3

Calculate each of the following.

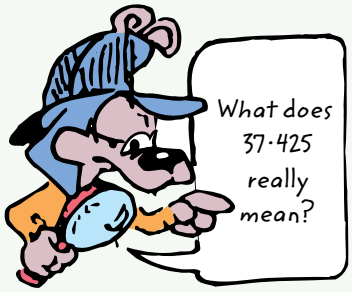
9 a 24·35 ÷ 10 b 6·7 ÷ 100 c 0·7 ÷ 1000

10 a 6·4 ÷ 0·2 b 0·824 ÷ 0·08 c 6·5 ÷ 0·05

11 Convert these decimals to fractions.
a 0·5 b 0·18 c 9·105

12 Convert these fractions to decimals.

a $\frac{4}{5}$ b $\frac{3}{8}$ c $\frac{5}{6}$



3 tens
7 units
4 tenths
2 hundredths
5 thousandths

3 7·4 2 5

10	1	·	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	7	·	4	2	5

CHECK-UP 1:02D PERCENTAGES

P Appendix 1:02D Sets A–H

1 Convert to fractions.

a 18% b 7% c 224%

2 Convert to fractions.

a 9·5% b $6\frac{1}{4}$% c 12·25%

3 Convert to percentages.

a $\frac{11}{20}$ b $\frac{5}{6}$ c $1\frac{1}{4}$

4 Convert to decimals.

a 9% b 16% c 110%

5 Convert to decimals.

a 23·8% b $12\frac{1}{2}$% c $4\frac{2}{3}$%

6 Convert to percentages.

a 0·51 b 0·085 c 1·8

7 Find:

a 35% of 600 m b 162% of $8

8 Find:

a 7% of 84·3 m b $6\frac{1}{4}$% of 44 tonnes

9 a 7% of my spending money was spent on a watch band that cost $4.20. How much spending money did I have?

b 30% of my weight is 18 kg. How much do I weigh?

10 a 5 kg of sugar, 8 kg of salt and 7 kg of flour were mixed accidentally. What is the percentage (by weight) of sugar in the mixture?

b Javis scored 24 runs out of the team's total of 60 runs. What percentage of runs did he score?

11 a Increase $60 by 15%. b Decrease $8 by 35%.

FUN SPOT 1:02 — WHAT WAS THE PRIME MINISTER'S NAME IN 1998?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Y Write $\frac{15}{4}$ as a mixed numeral.

M Change $1\frac{3}{4}$ to an improper fraction.

H Write $\frac{2}{5}$ as a percentage.

H Write 0·75 as a fraction.

I Increase 50 kg by 10%.

D 40% of my weight is 26 kg. How much do I weigh?

D What fraction is 125 g of 1 kg?

S Write $4 \div 9$ as a repeating (recurring) decimal.

S 10 cows, 26 horses and 4 goats are in a paddock. What is the percentage of animals that are horses?

S Increase \$5 by 20%.

S 600 kg is divided between Misal and Pratik so that Misal gets $\frac{3}{5}$ of the amount. How much does Pratik get?

Write the basic numeral for:

A $-8 + 10$ **A** $-7 - 3$ **A** -6×4 **A** $6 - (3 - 4)$ **A** $(-5)^2$

Write the simplest answer for:

I $\frac{37}{100} - \frac{12}{100}$ **I** $\frac{3}{8} + \frac{1}{3}$ **T** $\frac{4}{5} - \frac{2}{3}$ **T** $\frac{7}{8} \times \frac{8}{7}$

T $(\frac{1}{3})^2$ **T** $4\frac{3}{8} + \frac{5}{8}$ **N** $\frac{1}{2} \div \frac{1}{8}$ **N** $0{\cdot}05 + 3$

O $0{\cdot}3 - 0{\cdot}02$ **O** $0{\cdot}3 \times 5$ **E** $(0{\cdot}3)^2$ **T** $3{\cdot}142 \times 100$

E $6{\cdot}12 \div 6$ **E** $20{\cdot}08 \div 10$ **C** $1{\cdot}8 \div 0{\cdot}2$ **G** $\frac{3}{4}$ of 60 kg

H 5% of 80 kg

$\frac{1}{9}$	$\frac{3}{4}$	1·02	0·4	7	$\frac{7}{4}$	0·09	2	\$6	$\frac{1}{4}$	1	$\frac{17}{24}$	65%	$\frac{2}{15}$	1·5	$\frac{1}{8}$	25	$3\frac{3}{4}$

55 kg	314·2	4 kg	−10	240 kg	4	0·28	5	9	40%	−24	3·05	45 kg	2·008	65 kg	

1:03 Rational numbers

Integers, fractions, decimals and percentages (both positive and negative) are **rational numbers**.

e.g. $\frac{3}{4}$, 8, 52%, 0·186, $0\dot{3}$, −1·5, −10

Exercise 1:03 (Calculators are needed)

1 From the list on the right, choose two equivalent numbers for:

a $2\frac{1}{2}$ **b** 130%

c 2·8 **d** $1\frac{1}{4}$

125%	114%	$2\frac{4}{5}$	28%	280%
$2\frac{1}{8}$	1·4	2·5	208%	13
1·25	1·3	$1\frac{3}{10}$	250%	25%

2 Write each list in ascending order (from smallest to largest).

a 0·85, 0·805, 0·9, 1 **b** 87·5%, 100%, 104%, $12\frac{1}{4}$%

c $\frac{5}{8}, \frac{2}{7}, \frac{2}{3}, \frac{6}{100}$ **d** $1\frac{3}{4}$, 150%, 1·65, 2

e 1·42, $\sqrt{2}$, 1·41, 140% **f** $\pi, 3\frac{1}{4}, 3{\cdot}1, \sqrt{12}$

3 Find the number half-way between:

a 6·8 and 6·9 **b** 12% and 20%

c 1 and $\frac{1}{5}$ **d** 6·3 and 6·4.

4 Two unit fractions have a difference of $\frac{3}{8}$. What are they? ($\frac{1}{2}$ is one of the fractions.)

5 Write the next three numbers in these sequences.

a 0·2, 0·4, 0·8, 1·6, … **b** 2·4, 1·2, 0·6, 0·3, …

6 The average (i.e. mean) of five numbers is 5·8.

a What is the sum of these numbers?

b If four of the numbers are 5s, what is the other number?

7 What is meant by an interest rate of 8% p.a.?

8 An advertisement reads: '67% of flats in Tertiary Place already rented. Only one flat remaining for rent.' How many flats would you expect there to be in Tertiary Place?

9 Using a diameter growth rate of 4·3 mm per year, find the number of years it will take for a tree with a diameter of 20 mm to reach a diameter of 50 mm.

10 At the South Pole, the temperature dropped 15°C in 2 hours from −18°C. What was the temperature after that 2 hours?

11 Julius Caesar invaded Britain in 55 BCE and again a year later. What was the date of the second invasion?

12 Chub was playing 'Five Hundred'.

a His score was −150 points. He then gained 520 points. What was his new score?

b His score was 60 points. He then lost 180 points. What was his new score?

c His score was −120 points. He then lost 320 points. What was his new score?

13 What fraction would be displayed on a calculator as:

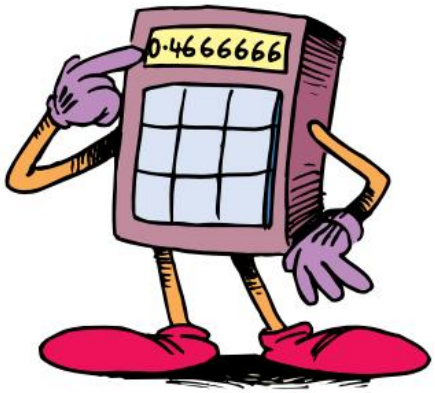
a 0.3333333 b 0.6666666

c 0.1111111 d 0.5555555

14 To change $\frac{7}{15}$ to a decimal approximation, press 7 ÷ 15 = on a calculator. Use this method to write the following as decimals, correct to five decimal places.

a $\frac{8}{9}$ b $\frac{2}{7}$ c $\frac{7}{13}$

d $\frac{20}{21}$ e $\frac{4}{11}$ f $\frac{5}{18}$



15 My income was $21 500 when I started work. If my income has increased by 200%, how much do I earn now?

16 What information is needed to complete the following questions?

a If Mary scored 40 marks in a test, what was her percentage?

b In a test out of 120, Nandor made only 3 mistakes. What was his percentage score?

c If 53% of cases of cancer occur after the age of 65, what is the chance per 10 000 of developing cancer after the age of 65?

1:04 Recurring decimals

PREP QUIZ 1:04

Write these fractions as decimals.

1 $\frac{1}{4}$ **2** $\frac{2}{5}$ **3** $\frac{1}{3}$ **4** $\frac{5}{6}$

0·639 749 749 74 … is written as $0{\cdot}63\dot{9}\,7\dot{4}$.

Rewrite these recurring decimals using the 'dot' notation.

5 0·444 4… **6** 0·631 631 631… **7** 0·166 666… **8** 0·726 969 69…

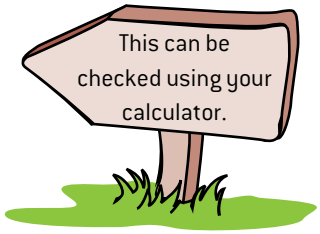
Rewrite these decimals in simplest fraction form.

9 0·75 **10** 0·875

To write fractions in decimal form we simply divide the numerator (top) by the denominator (bottom). This may result in either a 'terminating' or 'recurring' decimal.

For $\frac{3}{8}$: $8\overline{)3\cdot^{3}0^{6}0^{4}0}$ $0\cdot 3\ 7\ 5$

For $\frac{1}{6}$: $6\overline{)1\cdot^{1}0^{4}0^{4}0^{4}0}$ $0\cdot 1\ 6\ 6\ \ldots$



To rewrite a terminating decimal as a fraction, simply place the numbers in the decimal over the correct power of 10 (e.g. 10, 100, 1000) and then simplify.

For example: $0{\cdot}375 = \frac{\cancel{375}}{\cancel{1000}} \begin{matrix}\div 125\\ \div 125\end{matrix}$
$= \frac{3}{8}$

Recurring decimals are sometimes called repeating decimals.

Rewriting a recurring decimal as a fraction is a more difficult process. Carefully examine the two examples below and use the method shown in the following exercise.

WORKED EXAMPLE 1

When each number in the decimal part is repeated.
Write $0.\dot{6}\dot{3}$ as a fraction
Let $x = 0{\cdot}6363\ldots$

Multiply by 100 because two digits are repeated.
$100x = 63{\cdot}6363\ldots$

Subtract the two lines.
$100x - x = 63{\cdot}6363\ldots - 0{\cdot}6363\ldots$
$99x = 63$
$x = \frac{\cancel{63}}{\cancel{99}} \begin{matrix}\div 9\\ \div 9\end{matrix}$

Simplify this fraction.
$\therefore\ x = \frac{7}{11}$
$0.\dot{6}\dot{3} = \frac{7}{11}$

WORKED EXAMPLE 2

When only some digits are repeated.
Write $0.61\dot{7}$ as a fraction
Let $x = 0{\cdot}617\,77\ldots$

Multiply by 100 to move the non-repeating digits to the left of the decimal point.
$100x = 61{\cdot}777\ldots$

Multiply by 1000 to move one set of the repeating digits to the left of the decimal point.
$1000x = 617{\cdot}777$

Subtract the previous two lines.
$1000x - 100x = 617{\cdot}777 - 61{\cdot}777$
$900x = 556$
$x = \frac{\cancel{556}}{\cancel{900}} \begin{matrix}\div 4\\ \div 4\end{matrix}$

Simplifying this fraction using your calculator.
$\therefore\ x = \frac{139}{225}$

This answer can be checked by finding $139 \div 225$ using your calculator.
$0.61\dot{7} = \frac{139}{225}$

Exercise 1:04

P Foundation worksheet 1:04 Decimals and fractions

1 Write these fractions as terminating decimals.

a $\frac{3}{4}$ b $\frac{4}{5}$ c $\frac{5}{8}$ d $\frac{7}{10}$ e $\frac{7}{100}$

f $\frac{35}{20}$ g $\frac{4}{25}$ h $\frac{17}{50}$ i $\frac{19}{40}$ j $\frac{117}{125}$

2 Write these fractions as recurring decimals.

a $\frac{2}{3}$ b $\frac{5}{9}$ c $\frac{8}{9}$ d $\frac{2}{11}$ e $\frac{1}{7}$

f $\frac{1}{6}$ g $\frac{1}{15}$ h $\frac{7}{15}$ i $\frac{1}{24}$ j $\frac{17}{30}$

3 Write these terminating decimals as fractions.

a 0·47 b 0·16 c 0·125 d 0·85 e 0·035

4 Rewrite these recurring decimals as fractions, following Worked Example 1.

a 0·4444… b 0·575 757… c 0·173 173 173…

d $0\cdot\dot{7}$ e $0\cdot\dot{3}\dot{6}$ f $0\cdot\dot{1}23\dot{4}$

5 Determine the value of $0\cdot\dot{3}$.

6 Rewrite these recurring decimals as fractions, following Worked Example 2.

a 0·833 33… b 0·635 353 5… c 0·197 777…

d $0\cdot6\dot{4}$ e $0\cdot73\dot{6}$ f $0\cdot82\dot{4}\dot{9}$

g $0\cdot5\dot{1}2\dot{3}$ h $0\cdot527\dot{8}$ i $0\cdot64\dot{7}3\dot{4}$

CHALLENGE 1:04 TRY THIS WITH REPEATING DECIMALS

Here is a clever shortcut method for writing a repeating decimal as a fraction.
Follow the steps carefully.

	Example 1	Example 2
Step 1 (Numerator) Subtract the digits in front of the repeating digits from all the digits.	$0\cdot\dot{2}\dot{6}$ 26 – 0	$0\cdot32\dot{7}$ 327 – 32
Step 2 (Denominator) Write a 9 for each repeating digit and then a zero for each non-repeating digit in the decimal.	99	900
Step 3 Simplify the fraction if possible.	$0\cdot\dot{2}\dot{6} = \frac{26-0}{99}$ $= \frac{26}{99}$	$0\cdot32\dot{7} = \frac{327-32}{900}$ $= \frac{295}{900}$ $= \frac{59}{180}$

Try converting these repeating decimals to fractions using this method.

1 $0\cdot\dot{7}$ **2** $0\cdot6\dot{7}$ **3** $0\cdot3\dot{1}\dot{2}$ **4** $0\cdot1\dot{6}$ **5** $0\cdot32\dot{1}\dot{7}$

FUN SPOT 1:04 SPEEDY ADDITION

Rachel discovered an interesting trick.

1. She asked her father to write a five-digit number.
2. Rachel then wrote a five-digit number below her father's. She chose each digit of her number so that when she added it to the digit above, she got 9.
3. She then asked her father to write another five-digit number.
4. She then repeated Step 2.
5. She then asked her father to write one more five-digit number.
6. She now challenged her father to a race in adding these five numbers.

4 1 8 4 9
5 8 1 5 0
3 8 1 4 6
6 1 8 5 3
2 1 4 1 1
2 2 1 4 0 9

Put a 2 at the front. These three digits are the same as in the last number. Make this 2-digit number two less than the one above it.

1:05 Ratios and rates

PREP QUIZ 1:05

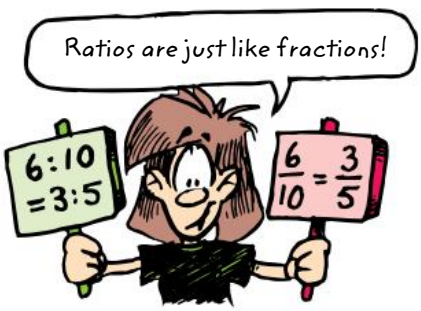
Simplify these fractions.

1 $\frac{50}{60}$ 2 $\frac{16}{20}$ 3 $\frac{72}{84}$ 4 $\frac{125}{625}$

What fraction is:

5 50c of $1
6 40c of 160c
7 8 kg of 10 kg
8 100 cm of 150 cm
9 1 m of 150 cm
10 $2 of $2.50?

A ratio is a comparison of like quantities.
e.g. Comparing 3 km to 5 km we write:
3 to 5 or 3 : 5 or $\frac{3}{5}$



A rate is a comparison of unlike quantities.
e.g. If 150 kg of flour is used every 10 minutes, the rate of use would be:

$\frac{150 \text{ kg}}{10 \text{ min}}$ or $\frac{15 \text{ kg}}{1 \text{ min}}$ or 15 kg/min.

Usually, we write how many of the first quantity corresponds to one of the second quantity.

e.g. 46 km in 2 hours is 23 km/h.

WORKED EXAMPLE 1

a Jan's height is 1 m and Dan's is 150 cm. Find the ratio of their heights.

b Of the class $\frac{3}{5}$ walk to school and $\frac{1}{4}$ ride bicycles. Find the ratio of those who walk to those who ride bicycles.

Solutions

a Jan's height to Dan's height
= 1 m to 150 cm
= 100 cm to 150 cm
= 100 : 150
Divide both terms by 50.
= 2 : 3 or $\frac{2}{3}$
From this ratio we can see that Jan is $\frac{2}{3}$ as tall as Dan.

Each term is expressed in the same units, then units are left out.

We may simplify ratios by dividing or multiplying each term by the same number.

b Those walking to those cycling = $\frac{3}{5} : \frac{1}{4}$
Multiply both terms by 20.
$= \frac{3}{\cancel{5}_1} \times \frac{\cancel{20}^{4}}{1} : \frac{1}{\cancel{4}_1} \times \frac{\cancel{20}^{5}}{1}$
= 12 : 5

To remove fractions multiply each term by the lowest common denominator.

WORKED EXAMPLE 2

Express the ratio 11 to 4 in the form:

a $X : 1$ **b** $1 : Y$

Solutions

a 11 to 4
= 11 : 4
Divide both terms by 4.
= $\frac{11}{4} : 1$
= $2\frac{3}{4} : 1$
This is in the form $X : 1$.

b 11 to 4
= 11 : 4
Divide both terms by 11.
= $1 : \frac{4}{11}$
This is in the form $1 : Y$

WORKED EXAMPLE 3

It cost me \$36 to purchase 20 kg of potting mix. What is the cost per kilogram?

Solution

Cost = $\frac{\$36}{20\,\text{kg}}$
= $\frac{36}{20}$ dollars per kilogram
= \$1.80/kg

Units must be shown.

$\frac{36}{20}$ means 36 ÷ 20.

Exercise 1:05

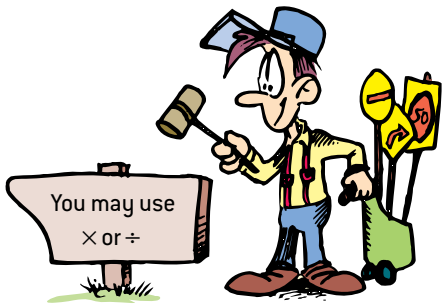
1 Express the first quantity as a fraction of the second.

Simplify the fractions.

a 7 men, 15 men	**b** 10 kg, 21 kg	**c** 3 cm, 4 cm
d \$5, \$50	**e** 8 m, 10 m	**f** 10 bags, 100 bags
g 75 g, 80 g	**h** 6 runs, 30 runs	**i** 25 goals, 120 goals

2 Simplify the ratios.

a 6 : 4	**b** 10 : 5	**c** 65 : 15
d 14 : 35	**e** 20 : 45	**f** 42 : 60
g 60 : 15	**h** 45 : 50	**i** 1000 : 5
j 1100 : 800	**k** 55 : 20	**l** 16 : 28
m 10 : 105	**n** 72 : 2	**o** 4 : 104
p $10 : \frac{1}{2}$	**q** $\frac{1}{2} : 2\frac{1}{2}$	**r** $2\frac{1}{2} : 2$
s $2\frac{3}{4} : 1$	**t** $2 : 3\frac{1}{4}$	**u** $6\frac{3}{4} : 3$
v $\frac{3}{4} : \frac{1}{2}$	**w** $\frac{11}{16} : \frac{1}{2}$	**x** $\frac{2}{3} : \frac{1}{2}$



3 In each, find the ratio of the first quantity to the second, giving your answers in simplest form.

a 7 men, 9 men	**b** 13 kg, 15 kg	**c** 7 cm, 8 cm
d \$8, \$12	**e** 16 m, 20 m	**f** 15 bags, 85 bags
g 90 g, 100 g	**h** 9 runs, 18 runs	**i** 50 goals, 400 goals
j 64 ha, 50 ha	**k** 25 m, 15 m	**l** $100\,m^2$, $40\,m^2$

4 Find the ratio of the first quantity to the second. Give answers in simplest form.

Are units the same?

a \$1, 50c	**b** \$5, \$2.50	**c** \$1.20, \$6
d 1 m, 60 cm	**e** 25 cm, 2 m	**f** 100 m, 1 km
g 600 mL, 1 L	**h** 1 L, 600 mL	**i** 1 L, 250 mL
j 2 h, 40 min	**k** 50 min, 1 h	**l** 2 h 30 min, 5 h

5 Express each ratio in the form $X : 1$.

a 2 : 5 **b** 3 : 4 **c** 7 : 10 **d** 5 : 3 **e** 20 : 9

6 Express each ratio in Question **5** in the form $1 : Y$.

7 **a** A recipe recommends the use of 2 parts sugar to 1 part flour and 1 part custard powder. What percentage of the mixture is each part?

b A mix for fixing the ridge-capping on a roof is given as 1 part cement to 5 parts sand and $\frac{1}{2}$ part of lime. What fraction of the mixture is each part?

c The ratio of a model to the real thing is called the scale factor. My model of an aeroplane is 40 cm long. If the real plane is 16 m long, what is the scale factor of my model?

d My father is 180 cm tall. If a photograph of him has a height of 9 cm, what is the scale of the photograph?

8 Write each pair of quantities as a rate in its simplest form.

a 42 kilometres, 7 hours	**b** \$50, 2 tonnes	**c** 200 grams, \$4
d \$2500, 10 days	**e** 12 g, 0·5 L	**f** 87 boys, 3 teachers

9 **a** If I walk at 3 km/h, how far can I walk in 3 hours?
b If bananas cost $3.60 per kg, what is the cost of 5 kg of bananas?
c If bananas cost $3.60/kg, how much can I buy for $36?
d Our run rate so far in the cricket match is 17·5 runs per wicket. How many runs have been scored if 4 wickets have been lost?

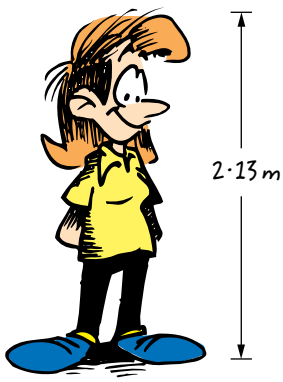
10 Complete these equivalent rates.

a 25 m/min = … m/h	**b** $40/m = … $/km	**c** 3 t/min = … kg/min
d 3000 g/kL = … g/L	**e** 3600 g/min = … g/h	**f** 120 beats/min = … beats/s

1:06 Significant figures

No matter how accurate measuring instruments are, a quantity such as length cannot be measured exactly. Any measurement is only an approximation.

- A measurement is only useful when one can be confident of its validity. To make sure that a measurement is useful, each digit in the number should be significant. For example, if the height of a person, expressed in significant figures, is written as 2·13 m it is assumed that only the last figure may be in error. Clearly any uncertainty in the first or second figure would remove all significance from the last figure. (If we are not sure of the number of metres, it is pointless to worry about the number of centimetres.)
- It is assumed that in the measurement 2·13 m we have measured the number of metres, the number of tenths of a metre and the number of hundredths of a metre. Three of the figures have been measured so there are three significant figures.
- To calculate the number of significant figures in a measurement we use the rules below.
- A significant figure is a number that we believe to be correct within some specific or implied limit of error.



Rules for determining significant figures

1 Coming from the left, the first non-zero digit is the first significant figure.

2 All figures following the first significant figure are also significant, unless the number is a whole number ending in zeros.

3 Final zeros in a whole number may or may not be significant
e.g. 0·001 20 has three significant figures, 8800 may have two, three or four.

1 Starting from the left, the first significant figure is the first non-zero digit.

e.g. 0·003 250 865 000 8·007

These are the first non-zero digits.

2 Final zeros in a whole number may or may not be significant.

e.g. 56 000 000 73 210 18 000

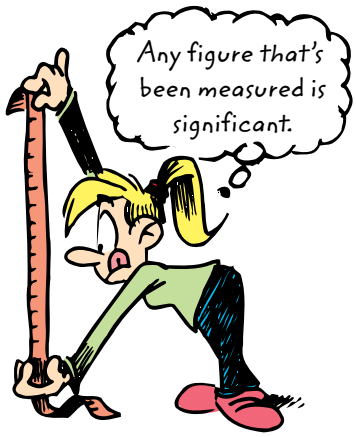
Unless we are told, we cannot tell whether these zeros are significant.

3 All non-zero digits are significant.

4 Zeros at the end of a decimal are significant.

e.g. 3·0 213·123 0 0·000 0100

These final zeros are significant.



Method for counting the number of significant figures

> Every figure between the first and last significant figure is significant.

Locate the first and last significant figures, then count the significant figures, including all digits between the first and last significant figures.

WORKED EXAMPLE 1

How many significant figures are in each of the following numbers?

a 316 000 000 (to nearest million)
b 316 000 000 (to nearest thousand)
c 42 007
d 31·0050
e 0·000 130 50

Solutions

a 316 000 000 (to nearest million)
last significant figure
first significant figure
Number of significant figures = 3

b 316 000 000 (to nearest thousand)
last significant figure (thousands)
first significant figure
Number of significant figures = 6

c 42 007
last significant figure
first significant figure
Number of significant figures = 5

d 31·0050
last significant figure
first significant figure
Number of significant figures = 6

e 0·000 130 50
last significant figure
first significant figure
Number of significant figures = 5

WORKED EXAMPLE 2

a The distance of the Earth from the Sun is 152 000 000 km.
In this measurement, the distance has been given to the nearest million kilometres. The zeros may or may not be significant but it seems that they are being used only to locate the decimal point. Hence, the measurement has three significant figures.

b An athlete said she ran 5000 m.
The zeros may or may not be significant. You would have to decide whether or not they were significant from the context of the statement.

Exercise 1:06

1 How many significant figures are in each of the following numbers?

a 2·1	**b** 1·76	**c** 9·05
d 0·62	**e** 7·305	**f** 0·104
g 3·6	**h** 3·60	**i** 0·002
j 0·056	**k** 0·04	**l** 0·40
m 0·004 71	**n** 3·040	**o** 0·5
p 304	**q** 7001	**r** 0·001 50
s 0·000 000 125	**t** 0·000 000 100	**u** 0·000 000 001

2 How many significant figures are in each of the following numbers?

a 2000 (to the nearest thousand)	**b** 2000 (to the nearest hundred)
c 53 000 (to the nearest thousand)	**d** 530 000 (to the nearest thousand)
e 25 000 (to the nearest ten)	**f** 26 300 (to the nearest hundred)
g 26 000 (to the nearest hundred)	**h** 8 176 530 (to the nearest ten)

3 State the number of significant figures in the following measurements.

a 4·1 L	**b** 12·62 m	**c** 7·08 t
d 0·5 cm	**e** 33·8 °C	**f** 0·070 L
g 0·003 km	**h** 2·5 ML	**i** 0·085 g

4 Give the number of significant figures in each of the following very large measurements. Assume the final zeros are not significant.

a 12 345 678 t	**b** 12 000 000 t
c 25 000 000 000 L	**d** 401 000 000 000 000 km

5 A newspaper article reported that 20 000 people attended Carols by Candlelight. How accurate would you expect this number to be? (That is, how many significant figures would the number have?)

INVESTIGATION 1:06 A TRUCK DRIVER'S MEMORIES

- Norm Richardson was a truck diver for 47 years.
- He estimated that he had travelled between 7 and 8 million kilometres driving trucks.
- He would often drive 1000 km in a day. For much of this time he carried coal from the Burragorang Valley to Port Kembla.

Questions

Assume that average values are being calculated.

Give your answers correct to two significant figures.

1 What distance would Norm have travelled each year?

2 If Norm worked 48 weeks each year, how many kilometres per week did he travel?

3 If he drove for 6 days in each week, what distance did he travel each day?

During this time he had two major accidents. The more serious one involved a 44-gallon drum of petrol falling from another truck onto the engine of his truck while he was travelling about 40 km/h around a bend in the road. The drum pushed the engine back into the cabin. He had no control over his vehicle so he jumped from his truck and hurtled into a wire fence. The truck burst into flames and was destroyed.

4 When Norm jumped from the truck, how many metres per second was he traveling?

5 If Norm hit the fence 2 seconds after jumping from his truck, how far was the truck from the fence when he jumped?

6 How many litres of petrol could a 44-gallon drum hold if 1 gallon = 4·546 litres? Round your answer to the nearest litre.

7 How much would the petrol in the drum weigh (to the nearest kg) if 1 litre of petrol weighs 0·711 kg?

1:07 Approximations

PREP QUIZ 1:07

How many significant figures do the following numbers have?

1 3·605 **2** 0·06 **3** 0·1050

Write one more term for each number sequence.

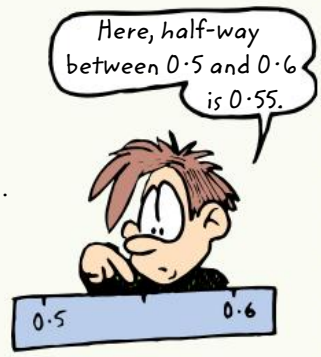
4 3·06, 3·07, … **5** 0·78, 0·79, … **6** 2·408, 2·409, …

7 Is 3·7 closer to 3 or 4?

8 Is 2·327 closer to 2·32 or 2·33?

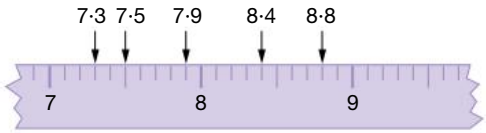
9 What number is half-way between 3·5 and 3·6?

10 What number is half-way between 0·06 and 0·07?



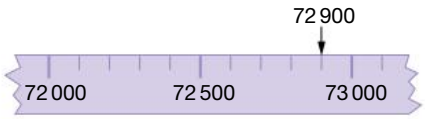
Discussion

To round a decimal to the nearest whole number, we write the whole number closest to it.



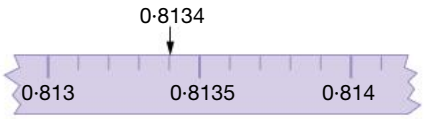
7·3 is closer to 7	7·9 is closer to 8	8·4 is closer to 8	8·8 is closer to 9
7·5 is exactly half-way between 7 and 8. In cases like this it is common to round up. We say 7·5 = 8, correct to the nearest whole number.			

To round 72 900 to the nearest thousand, we write the thousand closest to it.



72 900 is closer to 73 000 than to 72 000.
∴ 72 900 = 73 000 (nearest thousand)

To round 0·8134 to the nearest thousandth, we write the thousandth closest to it.



0·8134 is closer to 0·813 than to 0·814.
∴ 0·8134 = 0·813 (nearest thousandth)

To round (or approximate) a number correct to a given place:
- round up if the next digit is 5 or more
- round down if the next digit is less than 5.

WORKED EXAMPLE 1

Round:

a 56 700 000 to the nearest million
b 0·0851 to the nearest hundredth
c 86·149 to one decimal place
d 0·666 15 to four decimal places.

Solutions

a 56 700 000 has a 6 in the millions column.
The number after the 6 is 5 or more (i.e. 7).
∴ 56 700 000 = 57 000 000 (nearest million)

b 0·085 1 has an 8 in the hundredths column.
The number after the 8 is 5 or more (i.e. 5).
∴ 0·0851 = 0·09 (nearest hundredth)

c 86·149 has a 1 in the first decimal place.
The number after the 1 is less than 5 (i.e. 4).
∴ 86·149 = 86·1 (1 dec. pl.)

d 0·666 15 has a 1 in the fourth decimal place.
The number after the 1 is 5 or more (i.e. 5).
∴ 0·666 15 = 0·6662 (4 dec. pl.)

To approximate correct to a certain number of significant figures, we write the number that contains only the required number of significant figures and is closest in value to the given number.

WORKED EXAMPLE 2

Round:

a 507 000 000 to two significant figures
b 1·098 to three significant figures
c 0·006 25 to one significant figure
d 0·080 25 to three significant figures.

Solutions

a The second significant figure is the 0 between the 5 and 7. The number after the zero is 5 or more (i.e. 7).
∴ 507 000 000 = 510 000 000 (2 sig. fig.)

b The third significant figure is the 9. The number after the 9 is 8, so increase the 9 to 10. Put down the 0 and carry the 1.
∴ 1·098 = 1·10 (3 sig. fig.)

c The first significant figure is 6. The number after the 6 is less than 5 (i.e. 2).
∴ 0·006 25 = 0·006 (1 sig. fig.)

d The third significant figure is 2. The number after the 2 is 5 or more (i.e. 5).
∴ 0·080 25 = 0·0803 (3 sig. fig.)

Exercise 1:07

P Foundation worksheet 1:07 Approximation

1 Round these numbers to the nearest hundred.

a 7923 **b** 1099 **c** 67 314 **d** 853·461
e 609·99 **f** 350 **g** 74 932 **h** 7850

2 Round these numbers to the nearest whole number.

a 9·3	**b** 79·5	**c** 45·1	**d** 2·7
e 2·314	**f** 17·81	**g** 236·502	**h** 99·5

When you 'round' you are making an approximation.

3 Round these numbers to the nearest hundredth.

a 243·128	**b** 79·664	**c** 91·351	**d** 9·807
e 0·3046	**f** 0·085 2	**g** 0·097	**h** 1·991

4 Round these numbers, correct to one decimal place.

a 6·70	**b** 8·45	**c** 2·119	**d** 6·092
e 0·05	**f** 0·035	**g** 29·88	**h** 9·99

5 Round these numbers, correct to two significant figures.

a 8170	**b** 3504	**c** 655
d 849	**e** 14 580	**f** 76 399
g 49 788	**h** 76 500	

6 Round the numbers in Question **5**, correct to one significant figure.

7 Round these to three significant figures.

a 694·8	**b** 35·085	**c** 320·5	**d** 0·081 54
e 0·666 66	**f** 9·3333	**g** 10·085	**h** 9·095

8 To change $1\frac{7}{9}$ to a decimal, Gregory divided 16 by 9 using his calculator. Give the answer correct to:

a 1 dec. pl.	**b** 2 dec. pl.
c 3 dec. pl.	**d** 1 sig. fig.
e 2 sig. fig.	**f** 3 sig. fig.

'sig. fig.' is short for 'significant figures'
'dec. pl.' is short for 'decimal places'

9 Diane cut 60 cm of blue ribbon into 11 equal parts to make a suit for her new baby. After dividing, she got the answer $5.\dot{4}\dot{5}$. Give the length of one part correct to:

a the nearest centimetre	**b** the nearest millimetre	**c** 1 dec. pl.
d 2 dec. pl.	**e** 3 dec. pl.	**f** 1 sig. fig.
g 2 sig. fig.	**h** 3 sig. fig.	

10 The following calculator display represents an answer in cents. 14059.705
Give this answer correct to:

a the nearest dollar	**b** the nearest cent	**c** 1 dec. pl.
d 2 dec. pl.	**e** 3 dec. pl.	**f** 1 sig. fig.
g 2 sig. fig.	**h** 3 sig. fig.	

11 What level of accuracy do you think was used in each of these measurements and what would be the greatest error possible as a result of the approximation?

a The crowd size was 18 000.

b The nation's gross domestic product was $62 000 000 000.

12 What approximation has been made in each of these measurements and what would be the greatest error possible?

a 6·4 cm **b** 0·007 mg

13 A number is rounded to give 2·15. What could the number have been? What is the smallest the number could have been? Is it possible to write the largest number that can be rounded to give 2·15?

14 Write the measurement 37 648 708 t correct to:

a one significant figure **b** two significant figures

c three significant figures **d** five significant figures.

e Which of **a** to **d** would you use if you were comparing the steel production of countries?

15 Write the measurement 8·126 m correct to:

a one significant figure **b** two significant figures

c three significant figures **d** four significant figures.

e Which of **a** to **d** would you use on a building plan?

16 A surveyor measures a length as 42 063 mm.

a How many significant figures does this measurement have?

b Convert the measurement to metres.

c What is this measurement to the nearest metre?

d If this measurement were written to the nearest metre, how many significant figures would be needed?

e Round this number to the nearest centimetre.

f How many significant figures would be needed to write this number to the nearest centimetre?

42 063 mm
= 40 000 mm (1 sig. fig.)
= 42 000 mm (2 sig. fig.)
= 42 100 mm (3 sig. fig.)

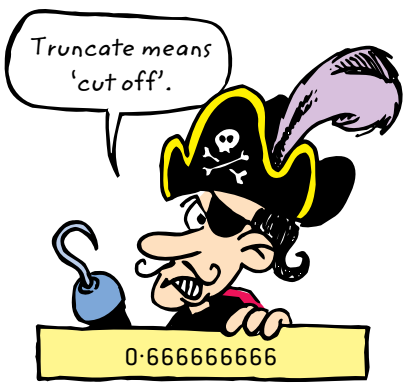
17 **a** An answer is given as 3 000 000, correct to one significant figure. What might the exact measure have been?

b Seven people decide to share a bill of $187.45 equally. How much should each person pay? What could be done with the surplus amount?

c The area of a room is needed to order floor tiles. The room dimensions, 2·49 m by 4·31 m, were rounded to 2 m by 4 m to calculate the area. What problems might arise?

18 A 10-digit calculator was used to change fractions into decimals. The truncating of the decimal produced an error. What error is present in the display after entering:

a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{5}{9}$?

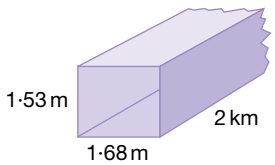


19 Find an approximation for $345{\cdot}45^2$ by first rounding the 345·45 correct to:

a 1 sig. fig. **b** 1 dec. pl. **c** 2 sig. fig.

d What is the difference between the answer to **a** and the real answer?

20 To find the volume of the tunnel drawn on the right, each measurement was rounded correct to one significant figure before calculation. What error in volume occurred?



Note: Truncating or rounding numbers before a calculation may produce unwanted errors or inaccuracy.

INVESTIGATION 1:07 TAKE YOUR MEDICINE

Many deaths have occurred because people have misread or not understood directions on medicine bottles. Instructions are often difficult to read and require sophisticated measuring instruments.

Before beginning the investigations listed below, use the picture to answer these questions. The information shown was on the label of a 100 mL bottle of a certain medicine.

Read the information carefully and answer the questions below.

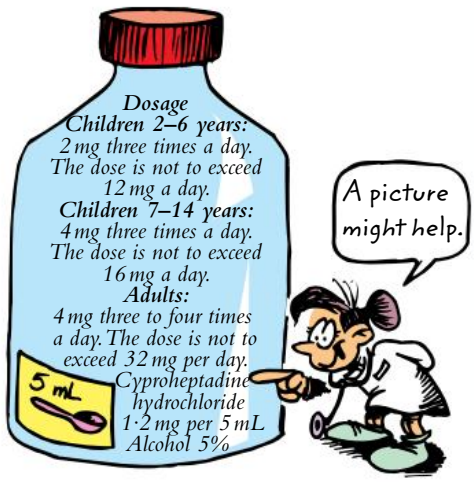
1 What is the dosage for an 8-year-old child and how often should it be taken?

2 A 5-year-old girl has a dosage of 2 mg. What is the usual number of times she should take the dose each day? What is the maximum number of doses she may take?

3 What is the youngest age for which the adult dose is recommended?

4 In a 100 mL bottle, how many millilitres of alcohol are present?

5 How many milligrams of the chemical cyproheptadine hydrochloride would be in a 100 mL bottle of this medicine?



a Investigate the labelling on medicine bottles and other medicinal preparations (e.g. pet worming tablets).

b Suggest ways in which directions could be given that would make them easier to understand.

c Redesign the label in the picture so that it reflects your answer to part **b**.

d Present your findings in the form of a written report.

1:08 Estimation

PREP QUIZ 1:08

1 Write 216 to the nearest hundred.

2 Write 17·68 to the nearest ten.

3 $15{\cdot}61 \times 10$

4 $15{\cdot}61 \times 100$

5 Is $\frac{0{\cdot}716}{3{\cdot}5}$ less than or greater than 1?

6 If $3 < 4$ and $5{\cdot}3 < 7{\cdot}8$, what sign ($<$ or $>$) can we put in the box? $3 \times 5{\cdot}3 \square 4 \times 7{\cdot}8$

7 *True* or *false*? $21{\cdot}68 \times 0{\cdot}716 < 21{\cdot}68 \times 1$

8 Which is the best approximation for $0{\cdot}316 \times 0{\cdot}81$?

A 2·5 **B** 0·25 **C** 0·025

9 *True* or *false*? $\frac{7{\cdot}6}{0{\cdot}25} > 7{\cdot}6$

10 *True* or *false*? $\frac{7{\cdot}6}{8{\cdot}3} > 1$

Like all machines, calculators only operate correctly if they are used correctly. Even when doing simple calculations it is still possible to press the wrong button. So it is essential that you learn how to estimate the size of the answer before the calculation is even started.

An estimate is a valuable way to check whether your calculator gives a sensible answer. If your estimate and the actual answer are different then it tells you that a mistake has been made either in your estimate or your calculation.

The following examples will show you how to estimate the size of an answer.

WORKED EXAMPLES

Estimate the size of each of the following calculations.

1 $14{\cdot}61 - 7{\cdot}15 + 3{\cdot}2$

2 $7{\cdot}56 \times 5{\cdot}173$

3 $0{\cdot}0253 \div 0{\cdot}45$

4 $\dfrac{21{\cdot}73 \times 0{\cdot}815}{7{\cdot}3}$

5 $\dfrac{\sqrt{86}}{2{\cdot}8 \times 16{\cdot}18}$

To simplify the question, multiply both numbers by 100.

Solutions

1 $14{\cdot}61 - 7{\cdot}15 + 3{\cdot}2$
$\approx 15 - 7 + 3$
≈ 11

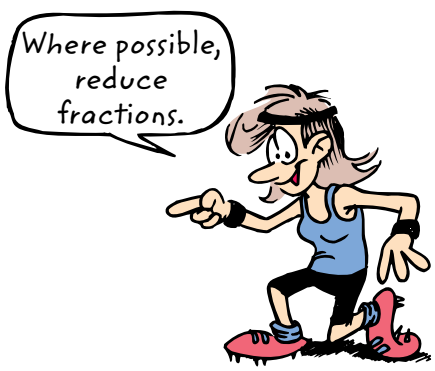
2 $7{\cdot}56 \times 5{\cdot}173$
$\approx 8 \times 5$
≈ 40

3 $0{\cdot}0253 \div 0{\cdot}45$
$= 2{\cdot}53 \div 45$
$\approx 3 \div 45$
$\approx \frac{1}{15}$ or $0{\cdot}07$

$\approx$ or $\doteqdot$ means 'is approximately equal to'.

4 $\dfrac{21{\cdot}73 \times 0{\cdot}815}{7{\cdot}3}$
$\doteqdot \dfrac{21 \times 1}{7}$
$\doteqdot 3$

5 $\dfrac{\sqrt{86}}{2{\cdot}8 \times 16{\cdot}18}$
$\doteqdot \dfrac{{}^{3}\cancel{9}}{{}_{1}\cancel{3} \times 16}$
$\doteqdot \frac{3}{15}$
$\doteqdot \frac{1}{5}$ or $0{\cdot}2$



- When estimating, look for numbers that are easy to work with, e.g. 1, 10, 100.
- Remember it's an estimate. When you approximate a number you don't have to take the nearest whole number.
- Try thinking of decimals as fractions. It often helps.
  e.g. $7{\cdot}6 \times 0{\cdot}518 \approx 8 \times \frac{1}{2} = 4$
- When dealing with estimates involving fraction bars, look for numbers that nearly cancel out.
  e.g. $\dfrac{{}^{2}\cancel{17{\cdot}68} \times 5{\cdot}8}{\cancel{8{\cdot}9}_{1}} \approx \dfrac{2 \times 6}{1} = 12$
- Check that your estimate makes sense.

Exercise 1:08

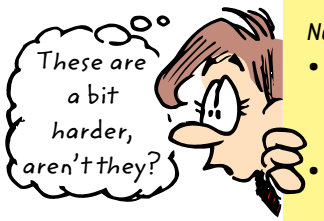
P Foundation worksheet 1:08 Estimation

1 Estimate the answers to the following calculations.

a $7{\cdot}9 + 0{\cdot}81 + 13{\cdot}56$
b $42{\cdot}56 - 15{\cdot}81 + 9{\cdot}2$
c $5{\cdot}6 \times (7{\cdot}2 + 5{\cdot}9)$
d $14{\cdot}31 \times 8{\cdot}97$
e $73{\cdot}95 \div 14{\cdot}2$
f $0{\cdot}73 \times 0{\cdot}05 \div 4{\cdot}53$
g $0{\cdot}916 \times 0{\cdot}032 \times 18{\cdot}34$
h $(15{\cdot}6 + 6{\cdot}82) \times 5{\cdot}31$
i $15{\cdot}6 + 6{\cdot}82 \times 5{\cdot}31$
j $(14{\cdot}56 + 3{\cdot}075) \div (0{\cdot}561 \times 20{\cdot}52)$

2 Estimate each of the following, giving your answer as an integer (i.e. a whole number).

a $\dfrac{7{\cdot}3 \times 9{\cdot}8}{15{\cdot}6 \times 3{\cdot}2}$
b $\dfrac{212 \times 71{\cdot}5}{15{\cdot}8 \times 0{\cdot}89}$
c $\dfrac{3{\cdot}6 \times 9{\cdot}7}{15{\cdot}8}$
d $\dfrac{7{\cdot}82 \times 5{\cdot}64}{9{\cdot}8 \times 2{\cdot}96}$
e $\sqrt{41{\cdot}6 + 39{\cdot}5}$
f $\dfrac{205{\cdot}3 - 76{\cdot}8}{41{\cdot}16 + 13{\cdot}7}$
g $6{\cdot}28 \times \sqrt{\dfrac{9{\cdot}78}{0{\cdot}53}}$
h $\dfrac{76{\cdot}5(3{\cdot}7 + 15{\cdot}6)}{2{\cdot}9 \times 15{\cdot}8}$
i $9{\cdot}6 \times 0{\cdot}75 + \dfrac{10{\cdot}6 \times (3{\cdot}5)^2}{2}$



Note:
- The fraction bar acts a little like grouping symbols. You work out the numerator and denominator separately.
- In $\sqrt{41{\cdot}6+39{\cdot}5}$ you must work out the addition first. The square root sign also acts like grouping symbols.

3 When estimating the size of a measurement, both the number and the unit must be considered. In each case, choose the most likely answer by estimation.

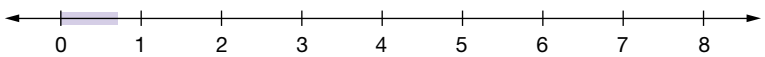
a The weight of the newborn baby was:
A 350 g B 7·8 kg C 3·1 kg D 50 pounds

b The length of a mature blue whale is about:
A 27 m B 3 km C 32 cm D 98 m

c 12% discount on a television set marked $2300 is:
A $86.60 B $866 C $276 D $27.60

d I just borrowed more than $80 000 from the bank at a rate of 8·5% p.a. In the next year, the interest on the loan is:
A $873 B $6800 C $185.60 D $21 140

4



Use the number line above to estimate each of the following as a whole number.

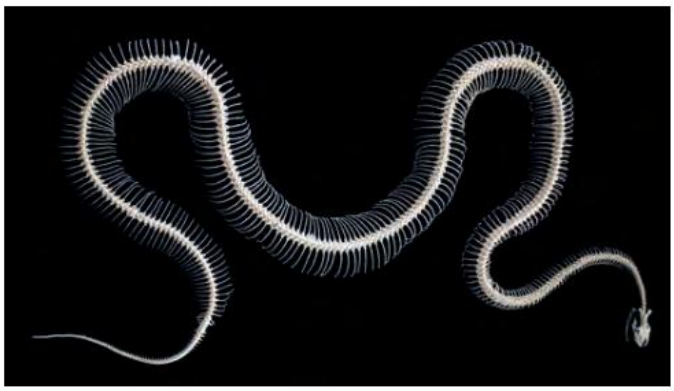
a four lots of 0·75
b 0·75 multiplied by 7
c $0{\cdot}75 \times 3$
d $0{\cdot}75 \times 5$
e how many 0·75 make 8
f how many 0·75 in 1
g $6 \div 0{\cdot}75$
h $8 \div 0{\cdot}75$

5 a A pile of paper is 3·2 cm thick. If there are 300 sheets in the pile, estimate the thickness of one sheet of paper.

b Paloma estimated that there were 80 people sitting in an area of 50 m^2 at a Carols by Candlelight concert. She estimated that about 2000 m^2 of area was similarly occupied by the crowd. To the nearest 100, what is her estimate of the crowd size?

c Would $8{\cdot}6 \times 84{\cdot}4$ be between 8×80 and 9×90? Explain your answer.

6 Two measurements were rounded correct to two significant figures and then multiplied to estimate an area. The working was: $92\,\text{m} \times 0{\cdot}81\,\text{m} = 74{\cdot}52\,\text{m}^2$
Between which two measurements would the real area lie? How many of the figures in this estimate are useful, given the possible spread of the area?



Estimate the number of vertebrae in this python skeleton. (The numerous vertebrae, or spinal-column segments, give the backbone great flexibility.)

1:09 Using scientific notation

PREP QUIZ 1:09

Write each number as a basic numeral.

1 3×10^2 **2** 7×10^3 **3** 4×10^6 **4** $7{\cdot}2 \times 100$
5 $7{\cdot}2 \times 10^2$ **6** $1{\cdot}6 \times 10^4$ **7** $2{\cdot}15 \times 10$ **8** $23{\cdot}15 \times 100$
9 $(7 \times 10\,000) + (4 \times 1000) + (9 \times 100) + (7 \times 10) + 6$
10 $(3 \times 1\,000\,000) + (4 \times 100\,000)$

Powers of 10

When multiplying by a power of 10, move the decimal point to the right.

WORKED EXAMPLE 1

a $2{\cdot}125 \times 10^2 = 2{\cdot}125 \times 100$
$= 212{\cdot}5$
100 has 2 zeros and the power of 10 has an index of 2, so move the decimal point 2 places to the right.

b $8{\cdot}7 \times 10^6 = 8{\cdot}7 \times 1\,000\,000$
$= 8\,700\,000$
1 000 000 has 6 zeros and the power of 10 has an index of 6, so move the decimal point 6 places to the right.

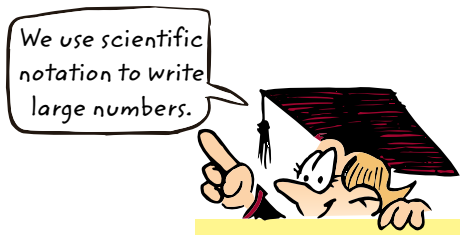
Expanded notation

Questions **9** and **10** of the Prep quiz show numbers written in expanded notation.

Using our knowledge of powers, we can write expanded notation using powers of 10.

1 $(7 \times 10\,000) + (4 \times 1000) + (9 \times 100) + (7 \times 10) + 6$
$= (7 \times 10^4) + (4 \times 10^3) + (9 \times 10^2) + (7 \times 10^1) + 6$
$= 74\,976$
$= 7{\cdot}4976 \times 10^4$ (Here, we moved the decimal point four places.)

2 $(3 \times 1\,000\,000) + (4 \times 100\,000)$
$= (3 \times 10^6) + (4 \times 10^5)$
$= 3\,400\,000$
$= 3{\cdot}4 \times 10^6$ (Here, we moved the decimal point six places.)



$10^3 = 1000$
(one thousand)

$10^6 = 1\,000\,000$
(one million)

$10^9 = 1\,000\,000\,000$
(one billion)

Scientific notation

$7{\cdot}4976 \times 10^4$ and $3{\cdot}4 \times 10^6$ are examples of scientific notation.

Using scientific notation, we write the number as the product of a number between 1 and 10 and a power of 10.

The number between 1 and 10 shows all the significant figures.

WORKED EXAMPLE 2

Write each number using scientific notation.

a $35\,000 = 3{\cdot}5 \times 10\,000$
$= 3{\cdot}5 \times 10^4$

To get a number between 1 and 10, we move the decimal point 4 places to the left so that 35 000 becomes 3·5. To make up for this, we multiply by 10^4.
(This number has two significant figures.)

b $278\,000\,000 = 2{\cdot}78 \times 100\,000\,000$
$= 2{\cdot}78 \times 10^8$

To get a number between 1 and 10, we move the decimal point 8 places to the left so that 278 000 000 becomes 2·78. To make up for this, we multiply by 10^8.
(This number has three significant figures.)

WORKED EXAMPLE 3

Write each of the numbers (written in scientific notation), as a basic numeral.

a $9{\cdot}03 \times 10^5 = 9{\cdot}03000 \times 100\,000$
$= 903\,000$

To multiply by 10^5, we move the decimal point 5 places to the right.
(This number has three significant figures.)

b $6{\cdot}1 \times 10^7 = 6{\cdot}1000000 \times 10\,000\,000$
$= 61\,000\,000$

To multiply by 10^7, we move the decimal point 7 places to the right.
(This number has two significant figures.)

Exercise 1:09

1 These are written in expanded notation. Write each as a basic numeral.

a $(9 \times 10\,000) + (2 \times 1000) + (3 \times 100) + (6 \times 10) + 5$

b $(7 \times 100\,000) + (4 \times 10\,000) + (3 \times 10)$

c $(3 \times 1000\,000) + (4 \times 100\,000) + (8 \times 1000) + 7$

d $(6 \times 10\,000) + (5 \times 1000) + (9 \times 100)$

e $(2 \times 10^4) + (8 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + 5$

f $(5 \times 10^3) + (9 \times 10^2) + (2 \times 10^1) + 4$

g $(6 \times 10^4) + (9 \times 10^3) + (7 \times 10^2) + (2 \times 10^1) + 7$

h $(3 \times 10^3) + (8 \times 10^2) + (4 \times 10^1) + 1$

i $(7 \times 10^6) + (5 \times 10^5) + (8 \times 10^4)$

j $(2 \times 10^4) + (7 \times 10^3)$

2 Complete each expression to write the number in scientific notation.

a $(1 \times 10^2) + (7 \times 10^1) + 3 = 1{\cdot}73 \times 10^{\cdots}$

b $(9 \times 10^2) + (1 \times 10^1) = 9{\cdot}1 \times 10^{\cdots}$

c $(3 \times 10^6) + (4 \times 10^5) + (5 \times 10^4) = 3{\cdot}45 \times 10^{\cdots}$

d $(6 \times 10^3) + (1 \times 10^2) = 6{\cdot}1 \times 10^{\cdots}$

e $(5 \times 10^7) + (2 \times 10^6) + (2 \times 10^5) = 5{\cdot}22 \times 10^{\cdots}$

f $(4 \times 10^9) + (5 \times 10^8) = 4{\cdot}5 \times 10^{\cdots}$

3 Write each number using scientific notation.

a 8600	**b** 856 000	**c** 47 000	**d** 37 000 000
e 6 000 000	**f** 230 000 000	**g** 101 000 000	**h** 8 125 000 000

4 These are written in scientific notation. Write each number as a basic numeral.

a $6{\cdot}86 \times 10^3$	**b** $1{\cdot}02 \times 10^2$	**c** $9{\cdot}5 \times 10^6$	**d** $4{\cdot}7 \times 10^5$
e $3{\cdot}96 \times 10^6$	**f** $4{\cdot}55 \times 10^7$	**g** $8{\cdot}05 \times 10^4$	**h** $6{\cdot}1 \times 10^9$

5 **a** Explain the difference between 7×10^3 and 7^3.

b *True* or *false*? There are more than 5×10^9 people on Earth.

c *True* or *false*? $\$5 \times 10^6$ is the same as 5 million dollars.

6 How many significant figures are in each number?

a $4{\cdot}05 \times 10^6$	**b** $7{\cdot}0 \times 10^4$	**c** $9{\cdot}3 \times 10^7$	**d** $8{\cdot}125 \times 10^3$

The mean radius of Earth is 6370 km, correct to three significant figures. Has this distance been measured to the nearest kilometre, nearest 10 kilometres, nearest 100 kilometres or nearest 1000 kilometres?

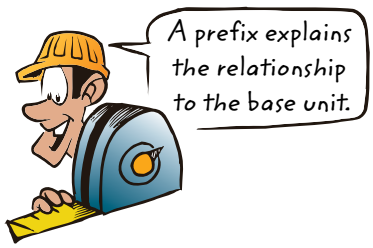
1:10 Units of measurement

Units of measurement are used to record measurements accurately. The most common units of measurement used in Australia are metric units. In the metric system:

- each measure has a base unit (e.g. for length it is the metre)
- other units are obtained by writing a prefix in front of the base unit (e.g. for length, some are millimetres, centimetres and kilometres).

Prefixes always have the same value as shown in the table below.

Prefix	Symbol	Value
nano	n	one billionth
micro	μ	one millionth
milli	m	one thousandth
centi	c	one hundredth
kilo	k	thousand
mega	M	million
giga	G	billion
tera	T	trillion



WORKED EXAMPLES

1 7 micrometres = 7 μm = $\frac{7}{1000000}$ m

2 5 gigabytes = 5 GB = 5 000 000 000 bytes

3 120 terabytes = 120 TB = 120 000 000 000 000 bytes

We must choose the most appropriate unit when measuring. For example, when measuring the distance between Sydney and Melbourne, we use kilometres rather than millimetres. In some situations more than one unit may be appropriate (e.g. the mass of a car can be measured in kilograms or tonnes).

1000 kilobytes (kB) = 1 megabyte (MB)
1000 megabytes (MB) = 1 gigabyte (GB)
1000 gigabytes (GB) = 1 terabyte (TB)
1000 nanoseconds (ns) = 1 microsecond (μs)
1000 microseconds (μs) = 1 milliseconds (ms)
1000 milliseconds (ms) = 1 second (s)

It is also important to be able to convert between different units. To do this we must know the value of the prefixes. This will tell us whether we need to multiply or divide when doing the conversion.

Tm	Gm	Mm	km	m	mm	μm	nm
10^{12} m	10^{9} m	10^{6} m	10^{3} m	1 m	$\frac{1}{10^{3}}$ m	$\frac{1}{10^{6}}$ m	$\frac{1}{10^{9}}$ m

In this table each unt is 1000 times as big as the one on its right.

PREP QUIZ 1:10

Write *true* or *false*.

1 1 μm = 1000 nm **2** 1 mm = 1000 μm **3** 1 m = 10 mm

4 1 Mm = 1000 km **5** 1 GB = 1000 MB **6** 1 TB = 1000 GB

Write the following measurements in words.

7 8 GB **8** 3 TB **9** 4 MB **10** 9 kB

WORKED EXAMPLES

1 Convert 5·4 MB (5·4 megabytes) to kilobytes (kB).

2 Convert 500 000 ns (nanoseconds) to microseconds (μs).

3 Convert 25 mm to μm

Solutions

1 $5{\cdot}4\text{ MB} = 5{\cdot}4 \times 1000\text{ kB}$ 1000 kB = 1 MB

$= 5400\text{ kB}$

OR $5{\cdot}4\text{ MB} = 5{\cdot}4 \times 10^6\text{ B}$ 1 million bytes = 1 megabyte

$= 5{\cdot}4 \times 10^6 \div 10^3\text{ kB}$ 1000 bytes = 1 kilobyte

$= 5{\cdot}4 \times 10^3\text{ kB}$

$= 5400\text{ kB}$

2 $500\,000\text{ ns} = 500\,000 \div 1000\text{ μs}$ 1000 ns = 1 μs

$= 500\text{ μs}$

3 $25\text{ mm} = 25 \times 1000\text{ μm}$ 1000 μm = 1 mm

Note: Some digital devices use different notations to display digital information. e.g. 40 kB could be shown as 40 K or 40 k or 40 KB.

Exercise 1:10

1 Convert the following.

a 7 km to m **b** 12 t to kg **c** 3 ML to L

d 800 cm to m **e** 9·3 m to mm **f** 3·45 L to mL

g 7·2 g to mg **h** 3845 mL to L **i** 500 mm to m

2 Convert the following.

a 5 Mm to m **b** 6 Gm to m **c** 5 Tm to m

d 6 m to mm **e** 9 m to μm **f** 3 m to nm

g 17·1 km to m **h** 1·8 Gm to m **i** 4·8 Mm to m

3 Convert the following.

a 4 MB to B **b** 4 MB to kB **c** 7 GB to MB

d 3 TB to GB **e** 3 μm to nm **f** 5000 nm to μm

g 3·2 MB to kB **h** 4·1 GB to MB **i** 0·6 TB to GB

4 Convert the following.

a 3000 m to km	**b** 8000 kB to MB	**c** 11 000 MB to GB
d 6500 GB to TB	**e** 3500 ns to μs	**f** 730 ms to μs
g 5 GB to kB	**h** 2 TB to MB	**i** 3 TB to kB

5 **a** How many years, to the nearest year, is the same as 1 gigasecond?

b How many minutes are the same as 1 billion microseconds?

c My computer has a capacity of one terabyte. I have used 100 gigabytes. How much available space do I have on my computer?

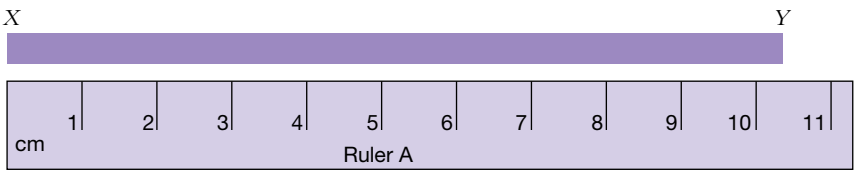
d Our computer network server has a capacity of 12 TB (12 terabytes). We have 9000 GB of available space left. How much space have we used?

1:11 Accuracy of measurements

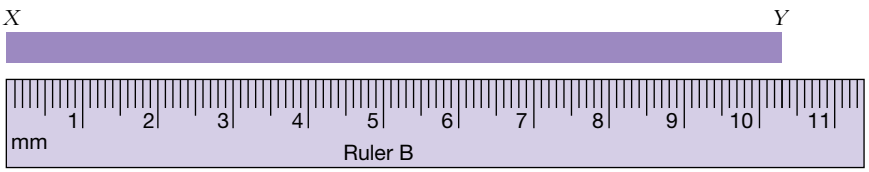
Consider the length of the rectangle above. If a group of students were asked to measure its length, how accurately could it be done? Could it be measured exactly or only to a given level of accuracy such as the nearest centimetre?

- It is not possible to give an exact measurement. When measuring, we must record the closest unit mark on the scale.

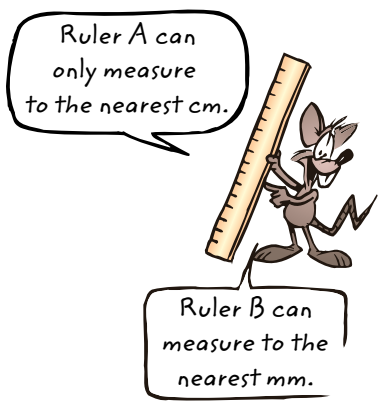
Therefore, the accuracy of a measuring instrument is limited by its scale. Measuring the interval XY with a ruler marked in millimetres would give a more accurate result than measuring it with a ruler marked only in centimetres.



Using ruler A, we measure XY to be 10 cm long.

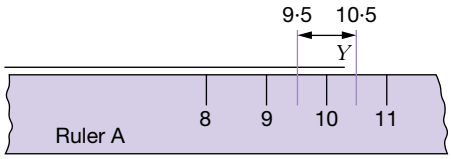


Using ruler B, we measure XY to be 10·3 cm long.



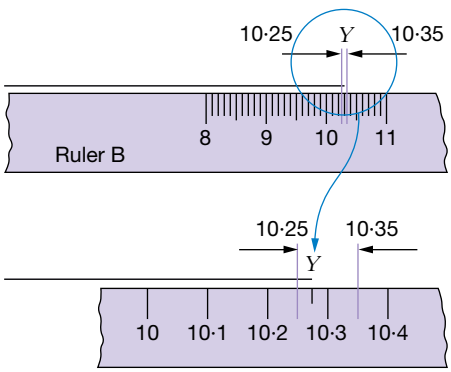
When using a measuring instrument (such as a ruler), the measurement taken cannot be more accurate than the smallest unit on the scale of the instrument. If measured in cm, the measure is correct to the nearest cm.

- Using ruler A we recorded that XY is 10 cm in length. Clearly, any length between 9·5 and 10·5 cm would be recorded as 10 cm. Hence, the greatest possible error when using a ruler marked in centimetres is $\frac{1}{2}$ of a centimeter.



So, if the length XY is recorded as 10 cm, then XY is between 9·5 cm and 10·5 cm or $10 \pm 0{\cdot}5$ cm. The greatest possible error or absolute error is 0·5 cm.

- Using ruler B we recorded that XY is 10·3 cm in length. This means that XY is between 10·25 cm and 10·35 cm as shown. (The measure could also be 10·25 cm, as this would round up to 10·3 cm.)



If the length XY is recorded as 10·3 cm, then XY is between 10·25 cm and 10·35 cm or $10{\cdot}3 \pm 0{\cdot}05$ cm. The greatest possible error, or absolute error, is 0·05 or $\frac{1}{2}$ of 0·1 cm.

> The greatest possible error, or absolute error, is equal to half of the smallest unit on the measuring scale.

WORKED EXAMPLE 1

For each of the following, find the absolute error.

a 8 g **b** 17·4 g **c** 27·625 L

Solutions

a 8 g is measured to the nearest gram. The smallest unit used is 1 gram.
The absolute error will be half of 1 g.
∴ Absolute error = 0·5 × 1 g
= 0·5 g

Is the smallest unit used g or 0·5 g?

b 17·4 g is measured to the nearest tenth of a gram. The smallest unit used is 0·1 grams.
The absolute error will be half of 0·1 g.
∴ Absolute error = 0·5 × 0·1 g
= 0·05 g

c 27·625 L is measured to the nearest thousandth of a litre. The smallest unit used is 0·001 litres.
The absolute error will be half of 0·001 L.
∴ Absolute error = 0·5 × 0·001 L
= 0·0005 L (or 0·5 mL)

WORKED EXAMPLE 2

Alana's bathroom scales are marked in tenths of a kilogram. She used them to measure her mass to be 63·7 kg. Find the absolute error of the measurement and her smallest and largest possible mass.

Solution

The smallest unit on the bathroom scales is a tenth of a kilogram.
The smallest unit used is 0·1 kilograms. The absolute error will be half of 0·1 kg.

$\therefore$ Absolute error $= 0{\cdot}5 \times 0{\cdot}1$ kg
$= 0{\cdot}05$ kg

Alana's mass is $(63{\cdot}7 \pm 0{\cdot}05)$ kg

$\therefore$ Alana's smallest possible mass $= (63{\cdot}7 - 0{\cdot}05)$ kg
$= 63{\cdot}65$ kg

$\therefore$ Alana's largest possible mass $= (63{\cdot}7 + 0{\cdot}05)$ kg
$= 63{\cdot}75$ kg

Exercise 1:11

1 Calculate the absolute error (or greatest possible error) for each measurement.

a 37 cm	**b** 165 g	**c** 450 L (measured to the nearest 10 L)
d 11·3 cm	**e** 6·18 m	**f** 9·0 L (measured to the nearest 0·1 L)
g 285·9 t	**h** 3·625 s	**i** 25 000 t (measured to the nearest 1000 t)

2 **a** A length of timber is measured as 584 cm (or 5·84 m) using a tape measure marked in centimetres.
- **i** What is the absolute error of this measurement?
- **ii** In what range would the actual measurement lie?

b The mass of a silver ingot is 47·29 g when measured on a digital scale.
- **i** To what accuracy is the scale measuring?
- **ii** What is the absolute error of the measurement?
- **iii** In what range would the true mass of the silver lie?

c The mass of a car is listed as 1·4 t.
- **i** To what degree of accuracy has the mass been measured?
- **ii** What is the absolute error of the measurement?
- **iii** In what range will the mass of the car lie?

3 Luke and Jo decided to fill the car's tank with petrol. Luke reported that it took 35 L to fill the tank. Jo reported that it took 34·9 L to fill the tank.

a What is the absolute error of Luke's measurement and within what range could Luke's measurement lie?

b What is the absolute error of Jo's measurement and within what range could Jo's measurement lie?

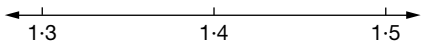
4 Phil and Naomi used a tape marked in centimetres to measure the width and length of a path. Phil measured the width as 1·48 m. Naomi measured the length as 120·36 m.

- **a** What was the absolute error of Phil's measurement? Within what range could Phil's reported measurement fall?
- **b** What was the absolute error of Naomi's measurement? Within what range could Naomi's reported measurement fall?
- **c** Are the absolute errors the same for the two measurements?
- **d** Which measurement would you consider to be the more accurate? Give a reason for your answer.

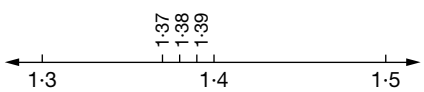
5 The thickness of a metal slab was measured using various measuring instruments. The results were:

Ruler (marked in cm)	1 cm
Ruler (marked in cm and mm)	1·4 cm
Vernier calipers	1·38 cm
Micrometer screw gauge	1·383 cm

- **a** Which instrument gives the measurement to:
  - **i** two significant figures
  - **ii** four significant figures?
- **b** What is the absolute error when the thickness was measured with:
  - **i** the ruler (marked in cm)
  - **ii** the ruler (marked in cm and mm)
  - **iii** Vernier calipers
  - **iv** the micrometer screw gauge?
- **c** In what range must the thickness of the slab lie if its measurement is recorded as:
  - **i** 1·4 cm



  - **ii** 1·38 cm?



The National Measurement Institute (NMI) ensures the accuracy of measuring instruments under fluctuating environmental conditions such as temperature and electromagnetic interference. Fuel pumps must be accurate to within ±0·3%.

MATHS TERMS 1

absolute error

- the greatest possible error due to the scale used
- half the smallest unit on the scale used,
  e.g. the absolute error of 6·2 cm is
  0·05 cm

approximate

- to replace a number with a less accurate one, often to make it simpler,
  e.g. 3·94 m might be approximated to 4 m

decimal place

- the position of a numeral after the decimal point, each position being $\frac{1}{10}$ of the one before it,
  e.g. 0·639 has three decimal places
  $0{\cdot}639 = \frac{6}{10} + \frac{3}{100} + \frac{9}{1000}$

estimate

- to calculate roughly (verb)
- a good guess or the result of calculating roughly (noun)

expanded notation

- a way of writing numerals to show the place value of each digit,
  e.g. 175 000
  $= (1 \times 100\,000) + (7 \times 10\,000) + (5 \times 1000)$
  $= (1 \times 10^5) + (7 \times 10^4) + (5 \times 10^3)$

fraction

- one or more parts of a whole expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$,
  e.g. $\frac{7}{8}$ ← numerator / denominator

integer

- a whole number which may be positive, negative or zero,
  e.g. 7, −23, 0

percentage

- a fraction that has a denominator of 100, written using the symbol %,
  e.g. $\frac{27}{100} = 27\%$

rational number

- a number which can be expressed in fraction form; this includes integers, percentages, terminating and recurring decimals,
  e.g. $\frac{4}{7}, -2\frac{1}{9}, 7, 16\%, -0{\cdot}69, 4{\cdot}6\dot{3}\dot{2}$

recurring decimal (repeating)

- a decimal for which the digits set up a repeating pattern,
  e.g. $0{\cdot}737\,373\ldots = 0{\cdot}\dot{7}\dot{3}$
  $0{\cdot}694\,44\ldots = 0{\cdot}69\dot{4}$

scientific (or standard) notation

- a useful way to write very large or very small numbers
- numbers are written as the product of a number between 1 and 10 and a power of 10
  e.g. $76\,000\,000 = 7{\cdot}6 \times 10^7$

significant figure

- a number that we believe to be correct within some limit of error
- to round a number to a number of significant figures is to specify the accuracy required from a calculation,
  e.g. 16·483 to 3 sig. fig. = 16·5
  0·004 75 to 1 sig. fig. = 0·005

terminating decimal

- a decimal number that has a limited number of decimal places,
  e.g. 0·6, 0·475, 0·0069

DIAGNOSTIC TEST 1 NUMBER AND MEASUREMENT

These questions reflect some of the important skills introduced in this chapter. Other skills have been covered in diagnostic tests within the chapter or previously in Year 8.
Errors made in this test will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Write these fractions as recurring decimals. **1:04**

a $\frac{2}{3}$ **b** $\frac{1}{6}$ **c** $\frac{7}{9}$ **d** $\frac{4}{15}$

2 Write these recurring decimals in fraction form. **1:04**

a 0·5555… **b** $0\cdot\dot{3}\dot{7}$ **c** 0·5777… **d** $0\cdot6\dot{8}\dot{9}$

3 How many significant figures has: **1:06**

a 316 000 000 (nearest thousand) **b** 42 007
c 0·000 130 50 **d** 6200

4 Round: **1:07**

a 56 700 000 to the nearest million **b** 0·666 15 (4 dec. pl.)
c 7·983 (1 dec. pl.) **d** 4·673 m (nearest cm)

5 Round: **1:07**

a 0·006 25 (1 sig. fig.) **b** 507 000 000 (2 sig. fig.)
c 0·060 75 (2 sig. fig.) **d** 93 784 231 (3 sig. fig.)

6 Estimate the size of each calculation to the nearest whole number. **1:08**

a $8\cdot73 \times 4\cdot132$ **b** $0\cdot0394 \div 0\cdot008\,12$
c $\frac{25\cdot4 \times 7\cdot98}{9\cdot93}$ **d** $\sqrt{65\cdot4} \times (4\cdot97)^2$

7 Write each number using scientific notation. **1:09**

a 85 000 **b** 960 000 000

Write each number as a basic numeral.

c $1\cdot06 \times 10^6$ **d** $8\cdot4 \times 10^4$

8 Convert: **1:10**

a 7·2 MB to kB **b** 40 000 ns to μs

9 Find the absolute error for: **1:11**

a 6 L **b** 35·9 kg

10 **a** What is the smallest possible mass that could be written as 3·26 g? **1:11**
b In what range will a mass lie if it is recorded as 9·4 kg?

ASSIGNMENT 1A Chapter review

1 Simplify:

a $15 - 3 \times 2$ **b** $24 \div 4 + 5 \times 2$

c $-6 \times (-5) \div (-10)$

2 Give the simplest answer to:

a $\frac{7}{10} + \frac{1}{10}$ **b** $\frac{5}{6} - \frac{2}{3}$

c $\frac{2}{5} \times \frac{1}{4}$ **d** $\frac{5}{9} \div \frac{2}{3}$

e $2\frac{3}{4} + 3\frac{1}{5}$ **f** $6\frac{1}{3} - 4\frac{3}{5}$

g $3\frac{1}{2} \times 2\frac{2}{3}$ **h** $5\frac{1}{4} \div \frac{7}{8}$

3 Simplify:

a $9{\cdot}2 - 4{\cdot}73$ **b** $6{\cdot}2 \times 0{\cdot}7$

c $24{\cdot}3 \div 0{\cdot}6$ **d** $(0{\cdot}3)^2$

4 Write these decimals as fractions.

a $0{\cdot}35$ **b** $0{\cdot}875$

c $0{\cdot}\dot{2}\dot{3}$ **d** $0{\cdot}3\dot{4}\dot{7}$

5 **a** Find $7\frac{1}{2}\%$ of \$350.

b What percentage is \$65 of \$325?

6 Simplify these ratios:

a 25 : 45 **b** \$6 : \$18

c 50 cm : 2 m **d** 15 min : $1\frac{1}{2}$ h

7 How many significant figures are there in each of these numerals?

a 5·27

b 0·006 04

c 6·90

d 93 000 (nearest thousand)

e $9{\cdot}30 \times 10^4$

f $6{\cdot}02 \times 10^7$

8 Round these numbers as indicated.

a 6·4472 (1 dec. pl.)

b 6·916 73 (nearest hundredth)

c 47 643 908 (nearest thousand)

d 647·542 (nearest whole number)

e 6·3942 (2 sig. fig.)

f 0·005 817 (3 sig. fig.)

g 47 649 503 (3 sig. fig.)

h 0·704 906 (2 sig. fig.)

9 Write each number as a basic numeral.

a $5{\cdot}82 \times 10^6$

b $9{\cdot}1 \times 10^3$

10 Write each number using scientific notation.

a 80 000 000 (to the nearest ten million)

b 80 000 000 (to the nearest million)

11 **a** Convert 2·1 terabytes (2·1 TB) to gigabytes (GB).

b Convert 8·5 gigabytes (8·5 GB) to kilobytes (kB).

c What is the absolute error of the measurement 3·7 km?

d In what range could the measurement 3·7 km lie?

How many notes are there in an octave on a piano? (There are more than eight.)
What fraction are white notes?
What fraction are black notes?

ASSIGNMENT 1B Working mathematically

1 Refer to **ID Card 1** on page xvii to identify the items numbered:

a 9 **b** 10 **c** 15 **d** 16
e 18 **f** 19 **g** 20 **h** 22
i 23 **j** 24

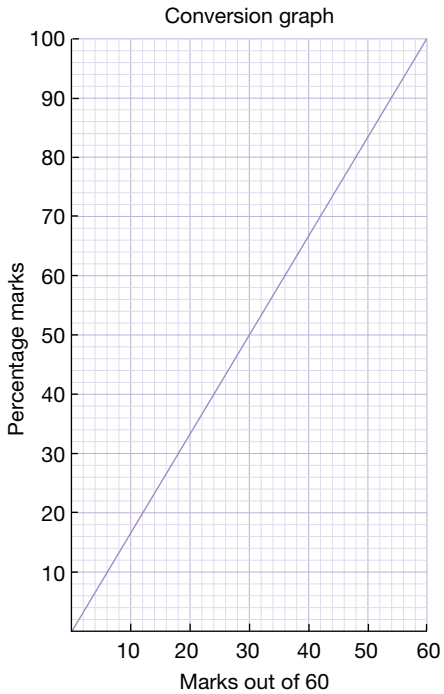
2 Refer to **ID Card 2** on page xvii to identify the items numbered:

a 2 **b** 5 **c** 10 **d** 11
e 14 **f** 15 **g** 16 **h** 18
i 19 **j** 23

3 Twelve schools participate in a knockout competition. How many matches must be played before the winner is decided?

4 How many different counting numbers less than one thousand could be formed if only the digits 5, 6 and 7 could be used? (These could be used more than once in a number.)

5 This graph converts marks out of 60 into percentages and vice versa.



a Use the graph to convert the following marks out of 60 into percentages. (Give your answer to the nearest whole per cent.)

i 30 **ii** 40
iii 12 **iv** 16

b Use the graph to convert the following percentage marks to marks out of 60. (Give your answer to the nearest integer.)

i 80% **ii** 90%
iii 56% **iv** 38%

c Two students score 30 and 40 marks out of 60. What is the difference in their percentage marks?

d Troy scores 80% and Marcus scores 60% in the same test. If the test was marked out of 60, by how many marks did Troy outscore Marcus?

e Is there a point on the graph corresponding to a mark of 66? Why or why not?

6 Use the petrol consumption graph below to answer these questions.

a How much petrol was in the tank at 9 am?

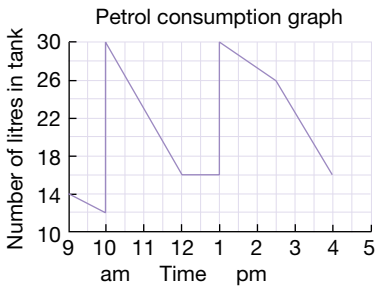
b How much petrol was used from 9 am to 10 am?

c When did the driver fill the tank?

d How much petrol was used from noon to 1 pm? What does this tell us about the car?

e How much petrol was used altogether?

f How much more petrol was in the tank at 4 pm than at 9 am?



ASSIGNMENT 1C Cumulative revision

1 Simplify:
a $-7 - 6 \times 4$ **b** $-216 - (-32)$ **c** $120 \div (-12)$ **d** $-10 \times (15 - 32)$

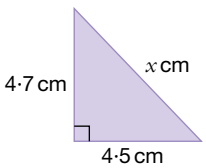
1:02A

2 Write each as a basic numeral.
a $(9 \times 1\,000\,000) + (9 \times 100\,000) + (9 \times 10)$
b $(5 \times 10^3) + (3 \times 10^2) + (7 \times 10^1) + 4$
c $(4 \times 10^9) + (2 \times 10^8) + (3 \times 10^7) + (1 \times 10^6)$
d $(9 \times 10^6) + (2 \times 10^5)$

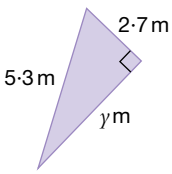
1:09

3 Use Pythagoras' theorem ($c^2 = a^2 + b^2$) to find the value of each pronumeral correct to two significant figures.

a



b

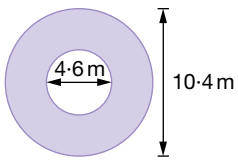


Year 8, Ch 3

4 **a** Simplify $3(8m + 7) + 2(5m - 6) + 2$. **b** Simplify $7(2y - 4) + 6(y - 1)$.
c Factorise $a^2 - 5a$. **d** Write y^4 in expanded form.

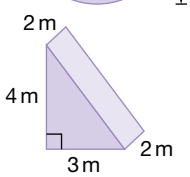
Year 8, Ch 5

5 **a** Find the area of the coloured annulus, correct to two decimal places.
b What is the difference between the circumferences of the two circles (3 sig. fig.)?



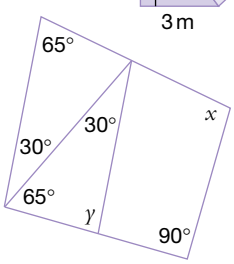
Year 8, Ch 12

6 **a** Use Pythagoras' theorem to find the length of the missing side of the triangular base of this prism.
b What is the volume of the prism?
c What is the surface area of the prism?



Year 8, Ch 3 and Ch 8

7 Use the angle sum of a quadrilateral and of a triangle to find the value of x and y.



Year 8, Ch 15

8 **a** EzyShaid is such a good art product that Alan bought 6 colour packs at \$30 a pack and paid \$8.46 in postage to have them delivered. How much did he pay altogether?
b Rhonda paid \$16.50 for 5 kg of bananas. What is the cost per kilogram?
c Change 180 beats/min into beats/s.

1:05

9 Use the table to find the rule connecting x and y.

x	8	9	10	11	12
y	17	19	21	23	25

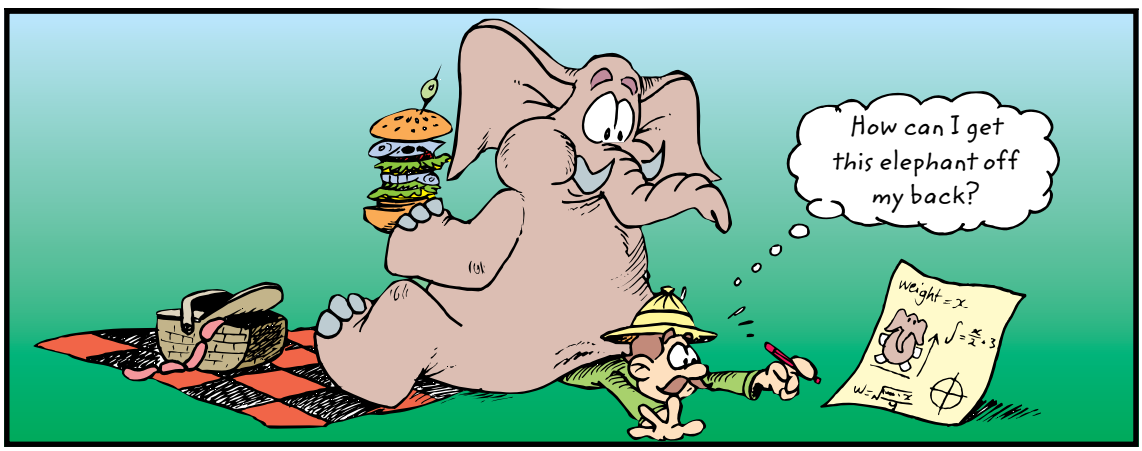
Year 8, Ch 5

10 Solve: **a** $7x - 3 = 18$ **b** $13x = 90 - 2x$

Year 8, Ch 9

WORKING MATHEMATICALLY

2



Contents

2:01 Solving routine problems
- 2:01A Rates
- 2:01B Ratios
- 2:01C Dividing a quantity in a given ratio

Challenge 2:01 Mixing drinks
- 2:01D Percentages
- 2:01E Measurement

Fun spot 2:01 Bouncing ball

2:02 Solving non-routine problems

Fun spot 2:02 What nationality is Santa Claus?

Investigation 2:02 Line marking

2:03 Using Venn diagrams

Fun spot 2:03 What type of breakfast takes an hour to finish?

Investigation 2:03 Comparing speeds

Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

The Working Mathematically strand is interwoven throughout the content strands **Number and Algebra**, **Measurement and Geometry**, **Statistics and Probability.**

Selections from ***Working Mathematically*** [Stages 5.1, 5.2]

- Uses appropriate terminology, diagrams and symbols in mathematical contexts. (MA5.1-1WM)
- Selects appropriate notations and conventions to communicate mathematical ideas and solutions. (MA5.2-1WM)
- Selects and uses appropriate strategies to solve problems. (MA5.1-2WM)
- Interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems. (MA5.2-2WM)
- Provides reasoning to support conclusions which are appropriate to the context. (MA5.1-3WM)
- Constructs arguments to prove and justify results. (MA5.2-3WM)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

Non-routine problems and problem solving involving work covered in Year 8 will be revised in this chapter. This includes rates, ratios, percentages, measurement and the use of Venn diagrams.

2:01 Solving routine problems

In mathematics, the learning of new skills and concepts is usually followed by the use of that newly acquired knowledge to solve problems.

These problems are generally routine in nature as the mathematical knowledge and skills needed are fairly obvious. The problem may still be hard to do but at least it is obvious what the problem is about, and which mathematical skills are required (e.g. percentages, measurement, geometry).

No matter what type of problem you are trying to solve, the following steps are important.

Steps for solving problems

Step 1 Read the question carefully.
Step 2 Decide what you are asked to find.
Step 3 Look for information that might be helpful.
Step 4 Decide on the method you will use.
Step 5 Set out your solution clearly.
Step 6 Make sure that your answer makes sense.

The problems in the following exercises are based on Stage 4 content.

2:01A Rates

WORKED EXAMPLES

1 A 20 kg box of tomatoes is sold for $33.

a What is the cost per kilogram?

b How much should 12 kg of these tomatoes cost?

20 kg for $33.
How much per kilogram?

Solutions

a 20 kg cost $33
1 kg costs $\$\frac{33}{20}$
$\therefore$ Cost is $1.65/kg

b 1 kg costs $1.65
12 kg cost $12 \times \$1.65$
$\therefore$ 12 kg cost $19.80

2 Wendy earns $16 per hour.

a How much would she earn in $7\frac{1}{2}$ h?

b How many hours would she have to work to earn $200?

Solutions

a Amount earned $= 7\frac{1}{2} \times \$16$
$= \$120$

b Number of hours $= \$200 \div \$16$
$= 12\frac{1}{2}$

Exercise 2:01A

1 **a** Apples cost $4.69/kg.
- i How much will 20 kg of apples cost?
- ii How many kilograms of apples can be bought for $30? Answer correct to one decimal place.

b I earn $9.60 per hour.
- i How much will I earn for 35 hours work?
- ii How many hours would I need to work to earn $2400?

c Sharnie bought 1000 manila folders for $40.
- i Express this as a rate in cents/folder.
- ii How much would she pay for 700 folders at this rate?

d The council rates on my block are 0·83 cents for every $1 on the value of the land. If my land is valued at $145 000, how much must I pay to the nearest dollar?

e If the exchange rate for one Australian dollar is 0·9845 American dollars:
- i how many American dollars would I get for 500 Australian dollars
- ii how many Australian dollars would I get for 500 American dollars? (Hint: 1 $US = 1 ÷ 0·9845 $AUD)

f A block of iron has a volume of 13 500 cm^3. If the density of iron is 7·5 g/cm^3 what is the mass of the block in kilograms?

g A car is travelling at an average speed of 80 km/h. At this rate find:
- i how far it will travel in $6\frac{3}{4}$ h
- ii how long it would take to travel 980 km.

2 On a trip, my car usually averages 8·4 L per 100 km. When carrying a heavy load it averages only 11·2 L per 100 km.
- **a** What is the difference in petrol consumption between these two conditions?
- **b** What is the difference between the distance travelled per litre of petrol used for the two conditions?

3 While on holiday in Hawaii, Taya bought a surfboard for US$580. If the exchange rate for the Australian dollar is 104·5 cents (US), what was the cost of the surfboard in Australian currency? (Answer correct to the nearest 5 cents.)

4 Sandy's pulse rate was calculated to be 84 beats/min, measured over 5 minutes. During this time, Sandy took 60 breaths. Calculate Sandy's rate of heartbeats/breath during this time.

5 Toni's average reaction time when driving was measured to be 0·8 seconds. How far would her car travel in this time if its speed is 60 km/h? (Answer correct to the nearest metre.)

2:01B Ratios

WORKED EXAMPLES

1 The ratio of John's weight to Andrew's is 5 : 4. If Andrew's weight is 56 kg, how much does John weigh?

2 The ratio of profit to cost price on sales at a store was 2 : 3. Find the cost price for an item if the store's profit is $600.

Solutions

1 John's weight : Andrew's weight is
5 lots : 4 lots
As Andrew's weight is 56 kg,
4 lots = 56 kg.
∴ 1 lot = 56 ÷ 4 kg
= 14 kg
Here John's weight is 5 lots.
∴ 5 lots = 14 kg × 5
= 70 kg
John's weight is 70 kg.

2 Profit : cost price is
2 lots : 3 lots
As the profit is $600,
2 lots = $600.
∴ 1 lot = $600 ÷ 2
= $300
Here the cost price is 3 lots.
∴ 3 lots = $300 × 3
= $900
The cost price is $900.

Exercise 2:01B

1 **a** The ratio of Jamie's weight to Alan's is 1 : 7. If Jamie's weight is 9 kg, what is Alan's weight?

b The ratio of time working to time playing is 5 : 1. If seven hours were spent playing, how much time was spent working?

c The ratio of balls caught to balls dropped was 9 : 2. If the number of balls dropped was 16, how many balls were caught?

d The ratio of the number of bananas to the number of apples used in a fruit punch is 2 : 5. Find the number of apples used if 6 bananas were used.

2 **a** In church on Sunday, the ratio of women to men was 5 : 4. How many men were present if 45 women were there?

b At the Entertainment Centre the ratio of basketball players to spectators was 3 : 500. How many spectators were there if there were 30 players?

c The ratio of Rhonda's savings to Alan's was 7 : 4. If Rhonda had saved $8708, how much had Alan saved?

d The ratio of Zebra finches to Gouldian finches in our aviaries is 13 : 2. If there are 234 zebra finches, how many Gouldian finches are there?

3 **a** Two streets on a map are 2·8 cm apart. If the scale of the map is 1 : 20 000, what is the distance between the streets?

b Our town hall is 1·8 km from the church. How far apart are they on the map referred to in part **a**?

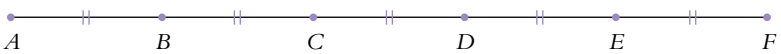
4 **a** On a holiday the ratio of the cost of accommodation to the cost of food was 3 : 7. Find the cost of accommodation if the food cost $420.

b For every $3 in tax that I pay, I get to keep $7. How much money do I keep if I pay $8400 tax?

c From Sharon's home, the ratio of the distance to the swimming pool to the distance to the tennis court is 3 : 8. How far is it to the pool if the court is 4 km away?

d Pete is trying to fill a bottle with water, but for every 5 mL of water he puts in the bottle, he wastes another 4 mL. How much water will he waste in filling a 600 mL bottle?

5



These points are equally spaced along the interval AF.

a Find the ratio of AC to CF.

b Find the ratio of AD to CF.

c If $CF = 108$ cm, find the length of AC.

2:01C Dividing a quantity in a given ratio

WORKED EXAMPLES

1 $9000 is divided between Tom and James in the ratio 4 : 5. How much does each receive?

2 The sizes of the angles of a triangle are in the ratio 2 : 3 : 5. Find the size of each angle.

Solutions

1 There are 9 parts; 4 for Tom and 5 for James.

Tom receives $\frac{4}{9}$ of $9000

$= \$\frac{4}{\cancel{9}_1} \times \frac{\cancel{9000}^{1000}}{1}$

$= \$4000$

James receives $\frac{5}{9}$ of $9000

$= \$\frac{5}{\cancel{9}_1} \times \frac{\cancel{9000}^{1000}}{1}$

$= \$5000$

(*Note:* The sum of the parts is equal to the whole $9000.)

2 There are 10 parts: 2, 3 and 5. The angle sum of a triangle is 180°.

First angle $= \frac{2}{10}$ of 180°

$= \frac{2}{\cancel{10}_1} \times \frac{\cancel{180}^{18}}{1}$

$= 36°$

Second angle $= \frac{3}{\cancel{10}_1} \times \frac{\cancel{180}^{18}}{1}$

$= 54°$

Third angle $= \frac{5}{10}$ of 180°

$= \frac{1}{2}$ of 180°

$= 90°$

(*Note:* 36° + 54° + 90° = 180°)

Exercise 2:01C

1 **a** $10 000 is divided between two charities, A and B, in the ratio 3 : 7. How much does each charity receive?

b During the cricket season Ganesh and Dev took 105 wickets. If the ratio of their wickets was 2 : 3, find how many wickets each took.

c A cordial is made by mixing syrup and water in the ratio 3 : 8. If 5·5 L of the mixture was made, how much syrup was used?

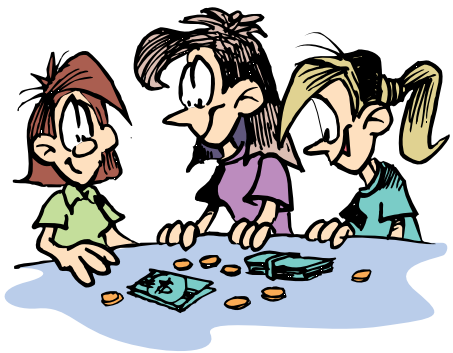
d Divide $770 between Mike and Joy in the ratio 3 : 4.

e A chocolate bar has 40 pieces. If Mai and Satsuki shared the pieces in the ratio 3 : 5, how many pieces did each get?

f John is paid $5 an hour, whereas his younger brother Zac is paid $3 an hour. They worked together for several hours and earned a total of $56. How much did each earn? How long did they work?

g Defence spending for the Army and Navy is allocated in the ratio of 9 : 4. If an additional $179 400 000 must be allocated, how much would each receive?

2 **a** $200 is to be divided between Rachel, Alana and Naomi in the ratio 3 : 5 : 2. How much does each receive?



b A band of three robbers divided their spoils so that the leader took three shares and the others took one share each. When caught they were given punishment in the same ratio as they shared the money. In total, the punishment was 15 years imprisonment. How many years was the leader given?

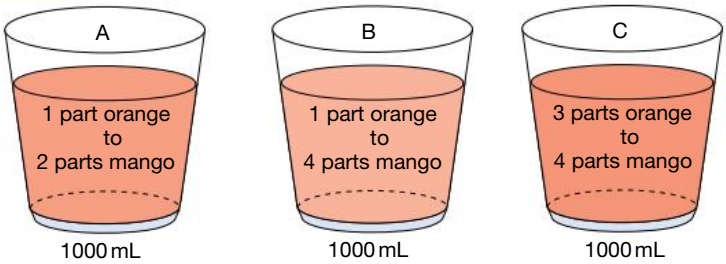
c An author wrote three novels that had their number of pages in the ratio 9 : 10 : 11. If the total number of pages is 900, how long is each novel?

d For every $100 I earn, the Taxation Department gets $38, living expenses account for $60 and I save $2. How much do I save in a year if my salary is $81 000 per year?

e The lengths of the three sides of a triangle are in the ratio 4 : 5 : 3. Find the length of each side if the perimeter of the triangle is 1·08 m.

If the ratio of petrol to oil is 50 : 1, how many litres of petrol need to be mixed with 80 mL of this oil?

CHALLENGE 2:01 MIXING DRINKS



- Each container above holds a mixture of orange and mango fruit juice.
- If the contents of containers A and B are mixed, would the ratio of orange to mango be the same as in container C? Explain why or why not.

Questions

1 Find to the nearest millilitre, the amount of orange juice in:
a A **b** B **c** C

2 Find to the nearest millilitre, the amount of mango juice in:
a A **b** B **c** C

3 What is the total amount of orange juice (to the nearest mL) in:
a A and B **b** B and C?

4 What is the total amount of mango juice (to the nearest mL) in:
a A and B **b** B and C?

5 What is the percentage of orange juice (to the nearest per cent) in the mixture of:
a A and B **b** B and C?

6 Use your answers to Question **5** to find the ratio of orange to mango juice in the mixture of:
a A and B **b** B and C.

7 Discuss the flavour of the fruit juices made from carrot, apple and celery juice in the following ratios.

Juice	Carrot	:	Apple	:	Celery
A	3	:	2	:	1
B	2	:	3	:	1
C	1	:	4	:	3
D	8	:	5	:	2

a Which juice would you expect to have the strongest carrot taste?
b Which juice would you expect to have the strongest celery taste?

Ratios are used to record the 'secret recipes' of fruit juice mixes. By varying the numbers in the ratio different tastes are created.

2:01D Percentages

WORKED EXAMPLE 1

Larry earns a commission of $3\frac{1}{2}\%$.
How much will he earn on a sale of $70 000?

Solution

Commission = $3\frac{1}{2}\%$ of $70 000

$= \frac{3{\cdot}5}{100} \times \$70\,000$

$= \$2450$

WORKED EXAMPLE 2

On New Year's Eve, 8000 drivers were given random breath tests. Of these, 160 were found to be over the limit of 0·05. What percentage of the drivers tested were over the limit?

What percentage is 160 of 8000?

Solution

160 drivers as a % of 8000

$= \frac{160}{8000} \times 100\%$

$= 2\%$

WORKED EXAMPLE 3

My salary was increased in February by 12%. This meant I would get an extra $110.40 per week. What is my new weekly salary?

Solution

12% of old salary = $110.40

$\therefore$ 1% of old salary = $110.40 ÷ 12

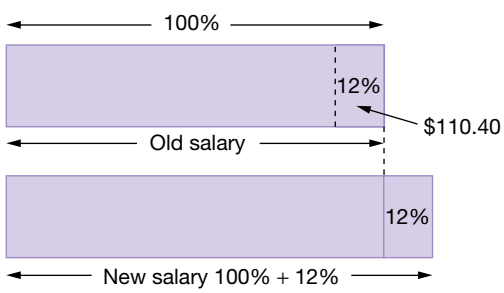
= $9.20

$\therefore$ 100% of old salary = $9.20 × 100

= $920

$\therefore$ New salary (112%) = $920 + $110.40

= $1030.40



Exercise 2:01D

1 **a** A theatre holds 1200 people when full. If it is 80% full, how many people are present?

b Larry has a trade discount of $15\frac{1}{2}\%$ on the selling price of all the articles at a hardware store. How much will he pay for a saw priced $275?

c A farmer lost 65% of his cattle in a prolonged drought. If he originally had 2300 cattle, how many does he have now?

d My car depreciated in value by 12% in 1 year. If its value was $15 800, what is its value now?

e Chiyo's present wage is $47 000 p.a. What will her wage be if it is increased by 4·5%?

2 **a** Wendy earns $220 interest on an investment of $4000. What is the interest as a percentage of her investment?

b Cliff scores 486 baskets in basketball from 573 attempts. What is his success rate as a percentage?

c Alan bought some land for $165 000 and later sold it for $210 000. Find:

i his profit

ii his profit as a percentage of the selling price.

d A town with a population of 30 689 has 3829 people in the 10–19 age group. Express the number in this age group as a percentage of the total population, to the nearest whole percent.

e Packets of food are checked for weight.
A machine records the following.

Overweight:	643
Underweight:	490
Correct weight:	24 608

What percentage of the packets weighed were underweight? Answer to the nearest whole percentage.

486 baskets from 573 attempts. What is the shooting %?

3 **a** A discount of 15% is given on a TV set. This reduces the price by $194.85. What was the original price?

b A town's population increased by 422 people in a year. If this represented an increase in the population of $2\frac{1}{2}\%$, what was the town's population at the start of the year?

c A property agent earns a commission of 3% on sales. If his commission for the year was $43 500, what was the value of his sales?

d Max's shares rose in value by 18%. If this increased their value by $14 040, what is their present value?

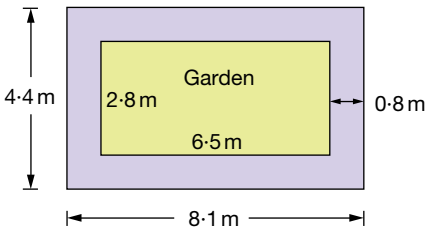
e A couple spend 78% of their monthly income on major expenses. If this amounts to $5616, what is their monthly income?

2:01E Measurement

WORKED EXAMPLE 1

A path 0·8 m wide surrounds a rectangular garden that is 6·5 m long and 2·8 m wide. What is the area of the path?

Solution



Area $= (8{\cdot}1 \times 4{\cdot}4) - (6{\cdot}5 \times 2{\cdot}8)$
$= 35{\cdot}64 - 18{\cdot}2$
$= 17{\cdot}44\ \text{m}^2$

WORKED EXAMPLE 2

A cylindrical tank has a diameter of 8 m. What is the height of the tank if it has a volume of $400\ \text{m}^3$? (Do not include the height of the stand.)

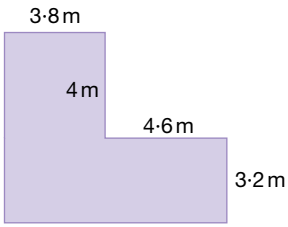
Solution

$V = \pi r^2 h$
$400 = (\pi \times 4^2) \times h$
$h = \dfrac{400}{(\pi \times 4^2)}$
$= 7{\cdot}96\ \text{m}$ (2 dec. pl.)

Exercise 2:01E

1 **a** Adam's trailer is 2·4 m long, 1·2 m wide and 0·4 m high. What is the maximum volume of soil it can carry?

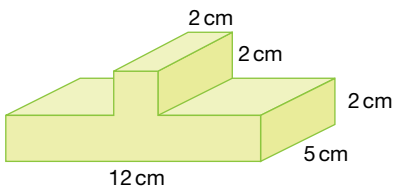
b A lawn is shaped as shown in the diagram. The lawn is to be covered to a depth of 4 cm with soil. What volume of soil is required?



c A water tank is cylindrical in shape. It is 9·8 m high and 12·4 m in diameter. Calculate its capacity if $1\text{ m}^3 = 1\text{ kL}$. (Give your answer to the nearest litre.)

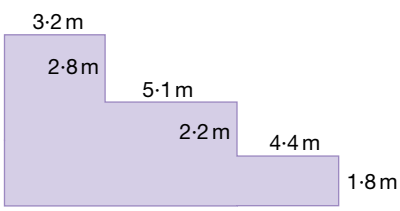
d A cylindrical cup has a diameter of 7 cm. How high must it be if it is to hold 250 mL? ($1\text{ cm}^3 = 1\text{ mL}$)

e Calculate the volume of the steel solid shown and calculate its mass if 1 cm^3 of steel weighs 7·8 g.

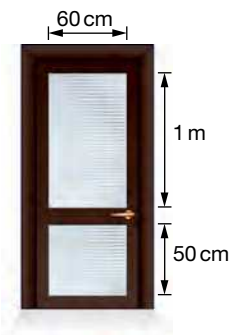


2 **a** The triangular gable shown in the photograph is to be painted. It has a base of 12·6 m and a perpendicular height of 1·8 m. Calculate the area to be painted and the cost at $\$30/\text{m}^2$.

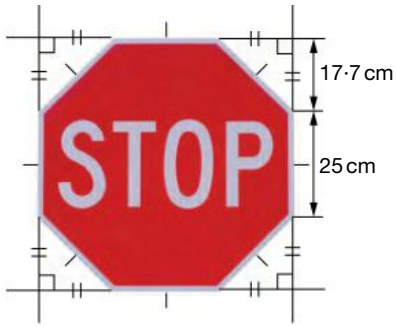
b The area shown below is to be tiled. Calculate the area and the cost of the tiles at $\$45/\text{m}^2$.



c Calculate the cost of replacing the glass in the door shown if glass costs $\$38/\text{m}^2$.



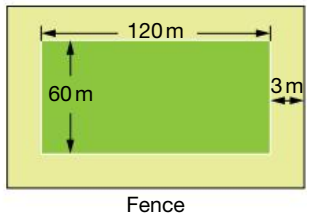
d Calculate the area of metal in the 'STOP' sign.



e A room is 7·6 m long, 6·4 m wide and 2·8 m high. Calculate the area of the four walls.

3 **a** A football field is 120 m long and 60 m wide. A fence is to be placed around the field 3 m away from the field. Calculate the length of fencing needed.

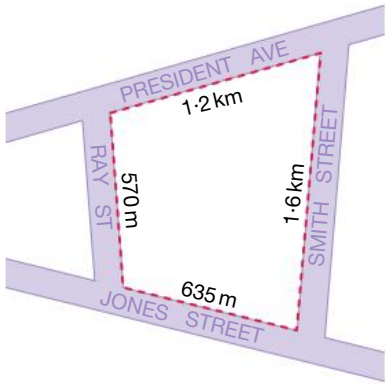
b A rectangular area is fenced off using 200 m of rope. If the rectangle is to be 65 m long, how wide will it be?



c The diagram shows the course followed by an athlete.

i If she runs the block bounded by the streets given, how far does she run?

ii How many complete laps would she need to run around this course if she wished to travel at least 20 km?



d A trundle wheel has a circumference of 1 m. What must its diameter be, to the nearest millimetre?

e The diameter of a car wheel is 40 cm. How many complete revolutions will this wheel need to make to travel 1 km?

FUN SPOT 2:01 BOUNCING BALL

A ball is known to rebound to half the height that it drops. If the ball is dropped from a height of 80 m, how far will it have travelled by the time it hits the ground for the:

a 1st time **b** 2nd time

c 3rd time **d** 4th time

e 5th time **f** 6th time?

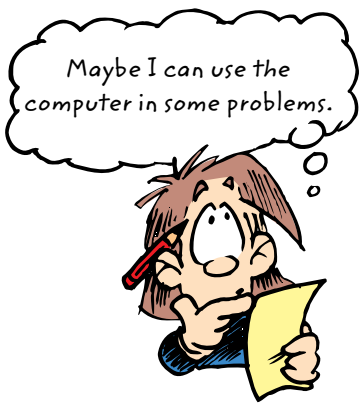
2:02 Solving non-routine problems

Often in mathematics (as well as in real life), we find a problem that is unlike any we have seen before. We need to reflect on what we already know and see how our existing knowledge can be used. Sometimes the problem will need us to develop new skills or we may need to look at the problem in a different way.

Applying strategies is one of the processes involved in Working Mathematically.

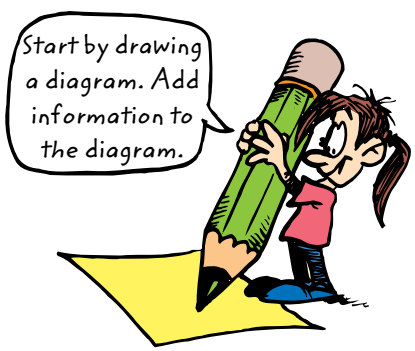
Problem-solving strategies

- Eliminating possibilities
- Working backwards
- Acting it out
- Looking for patterns
- Solving a simpler problem
- Trial and error
- Making a drawing, diagram or model
- Using algebra
- Using technology



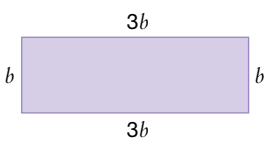
WORKED EXAMPLE 1

A rectangle has a perimeter of 44 m. If its length is three times as long as its breadth, find its dimensions.



Solution

Let the breadth be b.
The length then must be $3b$.
Now the perimeter is 44 m.



$$3b + b + 3b + b = 44$$
$$8b = 44$$
$$\therefore\ b = 5{\cdot}5$$

The breadth is 5·5 m and the length is 16·5 m.
(*Note:* This problem could also be solved using 'trial and error'.)

WORKED EXAMPLE 2

Screwdrivers come in four different sizes priced at $8.90, $7.80, $5.40 and $4.80. I bought seven screwdrivers. Which of the following amounts was the total cost?

A $48.40 **B** $34 **C** $50.65 **D** $61.40

Solution

Try to eliminate possibilities.

- $50.65 cannot be the answer as the costs of all screwdrivers are multiples of 10 cents. It's impossible to get the 65 cents.
- Look at the maximum cost.
  7 at $8.90 = $62.30
- $61.40 is less than $62.30 but close to it.
  6 at $8.90 and 1 at $7.80 = $61.20
  Since all other combinations would cost less than $61.20, $61.40 cannot be the answer.
- Look at the minimum cost.
  7 at $4.80 = $33.60
- $34 is not much more than $33.60.
  6 at $4.80 and 1 at $5.40 = $34.20
  All other combinations would cost more than $34.20, so $34 is not the answer.
- The only possibility remaining is $48.40, so **A** is the answer.

Note: We could continue to try different combinations of prices but we know that one of the possibilities was correct and we have eliminated the other three.
$(2 \times \$8.90) + (2 \times \$7.80) + (1 \times \$5.40) + (2 \times \$4.80) = \$48.40$

Exercise 2:02

1 Use problem-solving strategies to solve these problems.

a Vickie is expecting a baby. She must pick two given names (in order) for her child from the names she is considering. Girls' names being considered are Rachel, Jessye, Faith and Kate. Boys' names are Jason, Brent and Grant. She is also considering the name Sandy for both a girl or a boy. How many ways of naming the child are being considered?

b If tyres sell for $96.70, $113.50, $125.90 and $143.30, which of the following amounts could be the cost of five tyres?

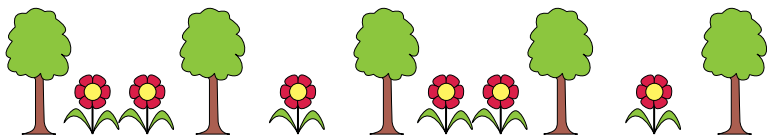
A $483.30 **B** $592.90 **C** $610 **D** $717.50

c Luke has 13 finches now but yesterday he lost $\frac{3}{4}$ of his finches when part of the roof blew off his aviary. The day before that he had given six zebra finches to his cousin and 2 days before that he had purchased two pairs of Gouldian finches. How many finches did Luke have before he bought the Gouldians?

d One 'move' involves turning three coins over. What is the least number of 'moves' needed to change these five 'tails' to five 'heads'?

e The difference of two numbers plus their sum is equal to 1. Write one of these two numbers.

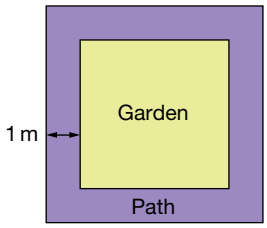
f A row of 865 trees was planted. Between the first and second trees, 2 flowers were planted. Between the second and third trees, 1 flower was planted. Between the third and fourth trees, 2 flowers were planted, and so on to the end of the row. How many flowers were planted in total?



g Particular text books have a mass 6 times as great as the summary books. If 5 text books and 10 summary books have a mass of 8 kg in total, what is the mass of 1 text book?

h What is the greatest number of points at which four circles can intersect?

2 **a** A square garden is surrounded by a path of width 1 m. What is the area of the garden if the outside perimeter of the path is 34 m? Report on the process you used to obtain a solution.



b A rectangle has a perimeter of 60 cm. If its length is four times as long as its breadth, find its dimensions. How could you check your answer?

c In my moneybox I have \$2.95. The 37 coins are all either 5-cent or 10-cent coins. How many 5-cent coins are in my moneybox? Report on the process you used to obtain a solution. How could you check your answer?

Compare and contrast your solution with those of other students.

3 For the chessboard on the right, how many squares are there of side length:

a 8 units **b** 7 units
c 6 units **d** 5 units
e 4 units **f** 3 units
g 2 units **h** 1 unit?

Write the total number of squares as the sum of eight square numbers.

4 a How many pieces of paper would you have if you cut a strip of paper:

i 1 time ii 2 times iii 3 times

iv 4 times v 15 times vi 186 times?

How is the number of pieces related to the number of cuts?

b If the strip of paper above is folded at the centre and the result folded at the centre again, 3 creases are made. On the second fold, 2 additional creases are made. If this folding process were to continue, how many additional creases would be made on the:

i 3rd fold ii 4th fold iii 5th fold

iv 6th fold v 7th fold vi 15th fold?

How is the number of additional folds related to the fold number?

5 a Which two numbers have a sum of 79 and a product of 1288?

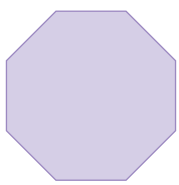
b Which two numbers have a difference of 11 and a product of 5226?

c Which two numbers have a sum of 1200 and a difference of 68?

6 In a basketball competition, each of the six teams must play the other five teams on two occasions. How many games must be played?

7 Plastic figures are needed for the doors on the fifth floor of a large hotel. If all of the numbers from 500 to 550 are needed, how many of each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 will be needed?

8 How many diagonals does an octagon have?



9 A pentomino is formed by joining five squares together so that each square is joined to another square along an edge. How many different pentominoes are there? (*Note:* They are the same if one can be turned into the other by turning it upside down or rotating.)

10 I own three ties, five shirts and two belts. How many different combinations of a tie, a shirt and a belt can I wear?

How many times in a day are the minute and hour hands of the clock at right angles?

FUN SPOT 2:02 WHAT NATIONALITY IS SANTA CLAUS?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Find:

H $3\frac{1}{2}\%$ of 1450

Simplify:

H $-8-(-16)$

I $(-4)^2$

L $64 \div (4-8)$

N $-1-1-1$

O $1{\cdot}25 \times 100$

O $30 \times 0{\cdot}2$

P 7% of 300

R 125% of 10

S $\frac{3}{4}$ of (-60)

T $\frac{4}{5} \times 1\frac{1}{4}$

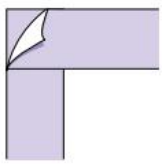
−3	125	12·5	1	50·75	21	6	−16	16	−45	8

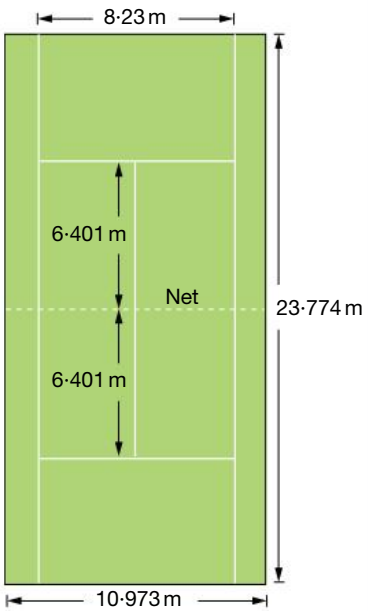
INVESTIGATION 2:02 LINE MARKING

Special adhesive tape is used to mark the lines on a tennis court.

Use the dimensions given to calculate the length of tape required, including the line underneath the net.

Assume the tape overlaps at each point of intersection.

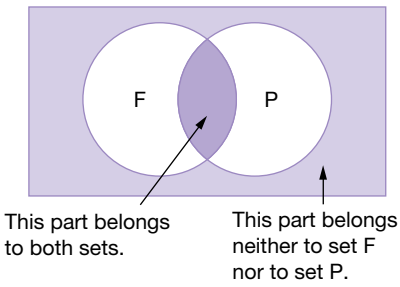




2:03 Using Venn diagrams

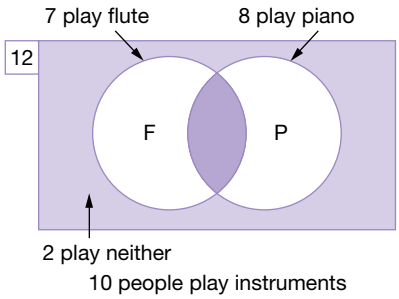
Drawing Venn diagrams is a useful problem-solving strategy.

- A rectangle is used to stand for the larger group out of which smaller sets are considered.
- Circles are used to represent those smaller sets.
- If the smaller sets have members in common, the circles will overlap.



WORKED EXAMPLE 1

Of 12 students, 7 play the flute and 8 play the piano. If 2 students play neither instrument, how many students play both instruments?



Solution

A Venn diagram has been drawn on the right.

- As 2 students play neither instrument, 10 students must be placed inside the circles.
- The coloured section is in both circles so, if we add the numbers playing each instrument, the students playing both are counted twice.

Number inside the circles = (number in F + number in P) − number in overlap

$10 = (7 + 8) - n$

$10 = 15 - n$

$n = 15 - 10$

$\therefore n = 5$

Therefore, 5 students play both instruments.

WORKED EXAMPLE 2

Of 30 students, 20 play the piano, 5 the flute and 6 the violin. 3 violinists also play the piano but only 1 of these 3 also play the flute. 5 students play both the piano and the flute. None of the students could play any other type of musical instrument. How many students played:

a the piano and the flute but not the violin

b only the piano

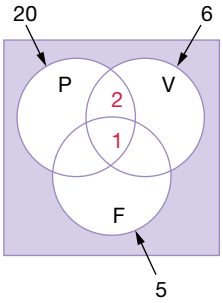
c only the flute

d only the violin

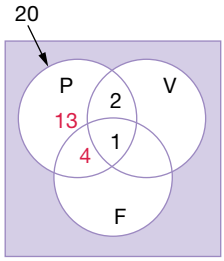
e no instrument at all?

Solution

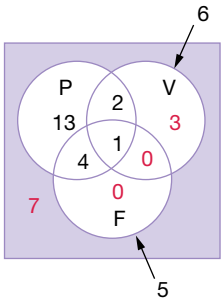
We need to put the information in the question onto the Venn diagram. As we enter this, we can deduce other information.



Of the 3 violinists who also play piano, 1 can play the flute. So only 1 person can play all three instruments.



5 students play both piano and flute, so 4 would play only the piano and flute. There are 20 piano players, so 13 must only play the piano.



There are only 5 flute players, so the other two sections of that circle will be zero and the remaining section of the violin circle will have 3 members (to make the total of violin players 6). Within the circles there are 23 students, so 7 must be outside. These 7 play no instrument.

The answers can now be read from the final Venn diagram above on the right.

a 4 **b** 13 **c** 0 **d** 3 **e** 7

Exercise 2:03

1 Of 30 people in our class, 9 can catch left-handed, 24 can catch right-handed and 6 can catch with both their left hands and their right hands.

- **a** How many can catch only with the left hand?
- **b** How many can catch only with the right hand?
- **c** How many cannot catch?

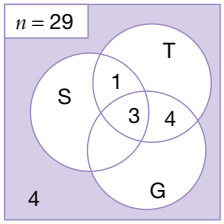
2 The *Tribute* was delivered to 75 homes and the *Bugle* was delivered to 68 homes. These papers were delivered to 110 homes.

- **a** How many homes received both papers?
- **b** How many homes received only the *Tribute*?
- **c** How many homes received only the *Bugle*?

3 Of the 30 students in 9M, 13 like surfing, 15 like hiking and 7 like neither.

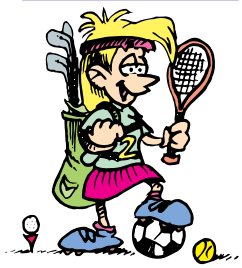
- **a** How many students like both surfing and hiking?
- **b** How many like surfing but not hiking?
- **c** How many like hiking but not surfing?

4 The Venn diagram on the right shows three intersecting sets: those who play soccer (S), those who play tennis (T) and those who play golf (G). A number in one part of the diagram shows the number of people in that part.

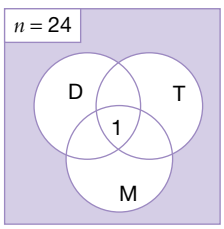


- Our class has 29 students and all but 4 students play at least one of these sports.
- 3 students play all three sports, one plays both soccer and tennis but not golf, 4 play tennis and golf but not soccer, a total of 5 play soccer and golf, a total of 16 play tennis and altogether 11 play soccer. How many play:

a soccer and golf but not tennis
b only tennis **c** only soccer **d** only golf
e tennis or soccer but not golf?



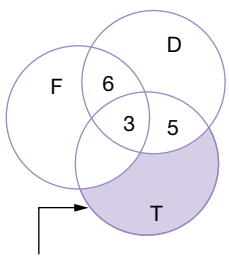
5 When 24 adults were asked which of Dubbo, Maitland and Terrigal they had visited, the following information was obtained.



- Only 1 person had visited all three places, 3 had visited Maitland and Dubbo but not Terrigal, 2 had visited Dubbo and Terrigal but not Maitland, 4 had visited Maitland, 16 had visited Dubbo and 3 of the adults had visited none of these places.

a How many people had visited only Dubbo?
b How many people had visited only Maitland?
c How many people had visited only Terrigal?
d How many people had visited Terrigal?
e Are those who have visited Maitland a part of those who have visited Dubbo?

6 This Venn diagram shows three intersecting sets: those friends of Alan that can drive a car (D), those who play tennis (T) and those who are female (F).



Play tennis but are not female and do not drive (male non-driving tennis players)

- Alan has 40 friends.
- 5 friends fit into none of the three categories, 3 friends fit all three categories, 6 who do not play tennis are female and drive, 5 are male tennis players who drive, 8 are female and play tennis, a total of 20 drive and altogether 17 are female.

Draw a Venn diagram and place numbers in specific sections to find how many friends are:

a male drivers who don't play tennis
b non-driving female tennis players
c females who neither drive nor play tennis
d male tennis players who don't drive
e tennis players
f female drivers
g either female or drivers
h either drivers or tennis players
i male.

Hi! I'm a female driver who doesn't play tennis. Does Alan have any more friends like me?

FUN SPOT 2:03 — WHAT TYPE OF BREAKFAST TAKES AN HOUR TO FINISH?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

E 4 L of mercury was shared between two schools in the ratio 3 : 7. What was the smaller share?

I Change 180 km/h into m/s.

10 g of salt is dissolved in 1 L of water.
How much salt would there be in:

I 500 mL **L** 2·5 L **M** 1·8 L
R 200 mL **S** 50 mL **A** 250 mL

3 L of water drips steadily from a tap in 5 hours.
How much water would be wasted in:

A 1 hour **T** 7 hours
A 4 hours **Y** 30 minutes
D 10 minutes **E** 7 minutes?

2·5 g	100 mL	600 mL	300 mL	4·2 L	50 m/s	18 g	70 mL	0·5 g	1·2 L	2 g	5 g	2·4 L	25 g

INVESTIGATION 2:03 — COMPARING SPEEDS

1 Find the Olympic record for:
   a men's 100 m, 200 m, 400 m, 800 m and marathon running
   b women's 100 m, 200 m, 400 m, 800 m and marathon running

2 For either men's or women's records, find the average speed for each distance in m/s and km/h.
(Give answers correct to three significant figures.)

3 Report on your findings. What conclusions can you draw?

MATHS TERMS 2

area
- the amount of space inside a 2D shape

percentage
- a fraction with a denominator of 100, written using the symbol %,
  e.g. $7\% = \frac{7}{100}$

perimeter
- the length of the boundary of a figure

rate
- a comparison of unlike quantities, usually expressed by writing how many of the first quantity corresponds to one of the second,
  e.g. 6000 mL in 10 minutes
  = 600 mL per 1 minute
  = 600 mL/min

ratio
- a comparison of numbers in a definite order

volume
- the amount of space inside a 3D shape

DIAGNOSTIC TEST 2 WORKING MATHEMATICALLY

Each part of this test has similar items that test a certain question type.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1	A 15 kg box of potatoes is sold for $12. a What is the cost per kilogram? b How much should 20 kg of these potatoes cost?	2:01A
2	The ratio of Mia's weight to Rex's is 3 : 5. If Rex's weight is 80 kg, how much does Mia weigh?	2:01B
3	The sizes of the angles of a triangle are in the ratio 7 : 2 : 3. Find the size of each angle.	2:01C
4	Ali earns a commission of $5\frac{1}{2}\%$ on all sales that he makes. What would be his commission on sales of $87 000?	2:01D
5	My salary was increased last week by 5%. This meant an increase of $56 per week. a How much was my weekly salary? b What is my new yearly salary if we take one year to be 52·143 weeks?	2:01D
6	A path 1·2 m wide is to be put around a rectangular garden that is 8·6 m long and 3·5 m wide. What is the area of the path?	2:01E
7	A rectangle has a perimeter of 66 m. If its length is 5 times as long as its breadth, find its dimensions.	2:02
8	I can buy four different types of plants. The cost of these four types are $17.80, $15.60, $10.80 and $9.60. I bought seven plants. Which of the following amounts could be the total cost? a $108.20 b $68 c $101.35 d $122.80	2:02
9	Of 20 students, 8 play tennis and 12 play cricket. If 6 students play neither sport, how many students play both sports?	2:03
10	Of 50 students, 16 can play the piano, 15 the flute and 6 the violin. All 6 violinists can play the piano, but only 2 of these can also play the flute. 5 students can play both the piano and the flute. No students can play any other type of musical instrument. How many students can play: a the piano and the flute but not the violin b no instrument at all?	2:03

ASSIGNMENT 2A Chapter review

1 **a** A motor mechanic charges $56/h for his labour. How much should he charge for a job that takes 75 minutes?

b Australia scored 265 runs off 50 overs in a one-day cricket match against New Zealand. Calculate the average run-rate per over. What should New Zealand's score be after 15 overs if they score at the same rate?

c Light travels at a speed of 300 000 km/s. How long will it take light to travel from the Sun to the Earth if the distance between the two is 150 000 000 km. (Answer in minutes and seconds.)

d A nurse has to set the drip rate for an IV drip for a patient. The drip has to empty a 500 mL flask in 8 hours. Work out the correct drip rate in drops/minute if it is known that 1 mL = 15 drops.

2 **a** The ratio of two angles α and θ is 7 : 4. If the size of θ is 92°, what is the size of α?

b An interval is divided in the ratio of 11 : 4. If the larger part is 143 mm long, what is the length of the smaller part? What was the length of the original interval?

c The ratio of the area of a triangle to the area of a rhombus is 6 : 11. Find the area of the rhombus if the area of the triangle is 126 cm^2.

d The ratio of the volume of a service station's underground petrol tank to the volume of the tank on a petrol tanker is 7 : 4. Find the volume of the service station's underground tank if the volume of the tank on a petrol tanker is 15·4 m^3.

3 **a** A type of solder is made by mixing lead and tin in the ratio 2 : 3. How much lead is needed to make 300 g of solder?

b Concrete is made by mixing sand, gravel and cement in the ratio 5 : 8 : 2 by volume. How many cubic metres of cement will be needed to make 4 m^3 of concrete. (Answer correct to one decimal place.)

c Heather shared a kilogram pack of grass seed between two plots in proportion to their areas. One plot had an area of 20 m^2, the other 30 m^2. How much seed was used for each plot?

d Two friends invested in a business in the ratio 3 : 2. The profit is to be distributed in proportion to their investments. What percentage of the profits does each receive?

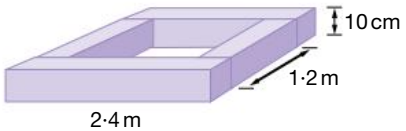
4 Using the ingredients below, Heather was able to make 40 vanilla slices. Make a list of the ingredients she would need to make 100 vanilla slices.

Vanilla slice

500 g of packaged puff pastry	1·25 litres milk
1 cup sugar	60 g butter
$\frac{3}{4}$ cup cornflower	2 egg yolks
$\frac{1}{2}$ cup custard powder	2 teaspoons vanilla

5 a A discount of 5% was given on a computer priced at $2750. How much was paid for the computer?
b A salesperson gave a discount of $120 on a computer with a selling price of $1890. What was the discount as a percentage of the selling price?
c A discount of 6% on the cost of a new computer saved me $168.
i What was the original price of the computer?
ii How much did I pay?

6 Four identical garden beds are to be made from treated pine sleepers. One bed is shown. The sleepers are rectangular prisms 2·4 m long, 20 cm wide and 10 cm high.



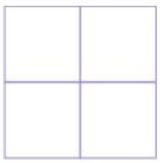
a How many sleepers will be needed to build the four beds?
b What volume of soil is needed to fill all four beds?

7 My salary has changed twice this year. At the start of the year it was $48 000 p.a. In February my salary increased by 12%, but in November my salary was decreased by 12%. What was my salary at the end of the year?

8 Six teams enter a round-robin competition where each team plays every other team. How many games will there be in the competition?

9 A section of fence is 12 m long. How far apart will the posts be if there are:
a 6 posts
b 9 posts?

10 Four non-zero numbers are placed in these four squares to make two two-digit numbers across, and two two-digit numbers down. Find all four two-digit numbers if they add to 237.



A cheetah can run at 80 km/h. At this rate, how long would it take a cheetah to run 100 km?

ASSIGNMENT 2B Working mathematically

1 Refer to **ID Card 4** on page xix to give the reference numbers identifying:

a the solids or 3D shapes
b the prisms
c the pyramids
d the 3D shapes with curved surfaces
e a triangular prism.

2 Refer to **ID Card 1** on page xvii to give the reference numbers identifying:

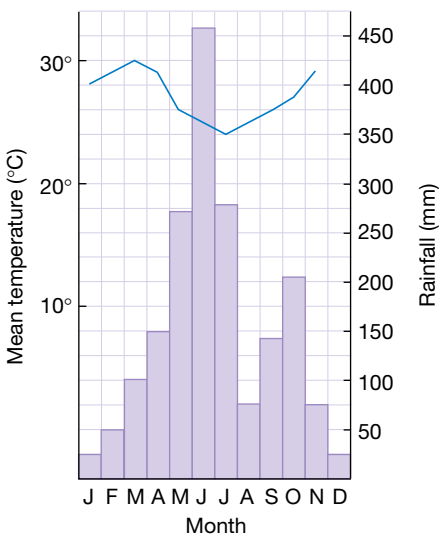
a the units that measure capacity
b the units that use 'kilo' in their name
c the units that measure time
d the unit for a hectare.

3 400 m of fencing encloses a square paddock of area one hectare. If 800 m of fencing were used to enclose a square paddock, how many hectares would be enclosed?

4 Mia is nine years older than Sandra. Alan is twice as old as Mia and Peter is three times as old as Sandra. If the sum of the ages of Alan and Peter is 88, how old is:

a Sandra **b** Mia
c Alan **d** Peter?

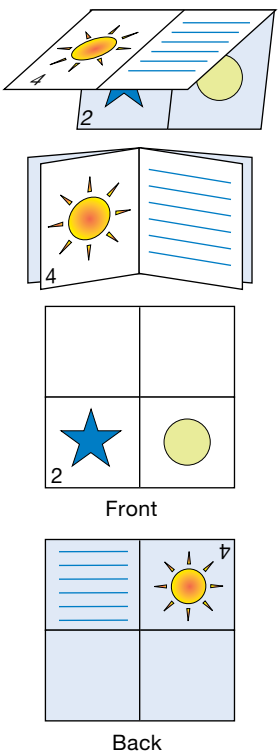
5 This graph is a combination line and column graph often used in geography.



There are two vertical axes. The left-hand temperature axis refers to the line graph. The right-hand rainfall axis refers to the column graph.

a In which month was the highest rainfall recorded?
b Which month had the lowest mean temperature? What was this temperature?
c How many months had a rainfall greater than 200 mm?
d The scale on the rainfall axis is 1 cm : 75 mm. Use this fact to find the rainfall for September.

6 One large sheet of paper was ruled up and folded. It was then cut along the fold shown in the diagram on the top to form an 8-page booklet. On the diagrams to the right, put the page number on each quarter (as has been done for 2 and 4).



ASSIGNMENT 2C Cumulative revision

1 Harry, Lester and Michelle bought a painting. Harry contributed $2000, Lester $5000 and Michelle $3000. They sold the painting 4 years later for $15 000. If they shared the proceeds in the same ratio as their investments, how much did each receive? 2:01B

2 Write *true* or *false*. 1:01

a $200 \times 1{\cdot}5 \neq 100$ **b** $300 \div 0{\cdot}9 > 300$ **c** $300 \div 1{\cdot}1 < 300$

d $45 \times 0{\cdot}9 > 45$ **e** $45 \div 0{\cdot}9 > 45$ **f** $45 \div 1{\cdot}01 < 45$

g $0{\cdot}05 = 50\%$ **h** $\pi \div 3{\cdot}14$ **i** $0{\cdot}1\% = 0{\cdot}001$

j $0{\cdot}5^2 < 0{\cdot}5$ **k** $\sqrt{9} > \sqrt[3]{9}$ **l** $\sqrt{1} > \sqrt[3]{1}$

3 1:02A, 1:02C

a $8 - (1 - 5)$ **b** -24×-3 **c** $3{\cdot}142 \times 100$

d $3{\cdot}142 \div 100$ **e** $1{\cdot}05 \times 10^4$ **f** $1{\cdot}05 \div 10^4$

4 1:02D

a Find 15% of $18.

b The 10% deposit on my bicycle was $23.50. How much did the bicycle cost?

c Peter scored 27 of his teams total of 60 runs. What percentage of the runs did he score?

d Increase $38.60 by 10%.

5 1:03

a What fraction would be displayed on a calculator as 0.444444444?

b What is the next term in this pattern?
2·4, 1·2, 0·6, 0·3, …

c 8% p.a. simple interest is paid on $200 over 2 years. How much interest was paid?

d Find the number half way between 0·05 and 10%.

6 1:05

a Write $\frac{2}{3}$ as a recurring decimal. **b** Simplify the ratio 25 : 15.

c Express the ratio 7 : 4 in the form x : 1. **d** Write 2500 m/h as km/h.

7 1:06, 1:07

a How many significant figures has 0·020 m?

b Write 80·05 L, correct to three significant figures.

c Round 23 670 000 to the nearest million.

d Round 3·142, correct to two decimal places.

8 1:09

a Write 35 000 000 in scientific notation with two significant figures.

b Write 35 000 000 in scientific notation with three significant figures.

c How many significant figures has $9{\cdot}0 \times 10^9$?

d Write $5{\cdot}08 \times 10^6$ as a basic numeral.

9 1:10

a Convert 2TB (terabytes) to GB (gigabytes).

b Convert 3000 kB (kilobytes) to MB (megabytes).

c Convert 25 000 GB to TB.

d Convert 8·5 MB to kB.

10 Find the absolute error of the measurement: 1:11

a 35 g **b** 0·25 m **c** 14·128 L

d In what range would a measurement recorded as 6·5 cm lie?

ALGEBRA

3

Contents

3:01 Addition and subtraction in algebra
3:02 Multiplication in algebra
3:03 Division in algebra
3:04 Simplifying algebraic fractions
- **3:04A** Addition and subtraction
- **3:04B** Multiplication and division

3:05 Grouping symbols
- Investigation 3:05 Checking algebraic simplification by substitution

3:06 Simplifying expressions with grouping symbols
3:07 Factorising using common factors
- Fun spot 13:07 In the theatre, when do you tell someone to break a leg?

Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Algebraic Techniques*** [Stage 5.2]

- Simplify algebraic expressions involving the four operations. [Stage 4]
- Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate. (ACMNA213)
- Apply the four operations to simple algebraic fractions with numerical denominators. (ACMNA232)
- Factorise algebraic expressions by taking out a common algebraic factor. (ACMNA230)

Working Mathematically

- Problem Solving • Reasoning • Understanding • Fluency • Communicating

3:01 Addition and subtraction in algebra

PREP QUIZ 3:01

Simplify the following.

1 $12 + 7$ **2** $8 - 12$ **3** $-4 + 5$ **4** $-3 - 8$ **5** $6 - (-9)$

Complete the following.

6 $5x + 2x = (\ldots + \ldots)x$ **7** $9y - 4y = (\ldots - \ldots)y$ **8** $a \times b = b \times \ldots$

Simplify the following.

9 $2^2 + 2^3$ **10** $a + a$

WORKED EXAMPLES

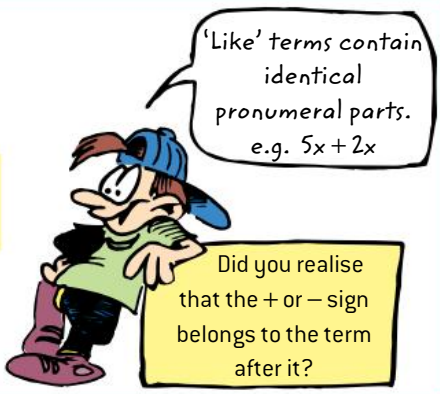
1 $5a + 2b - 3a + b = 5a - 3a + 2b + b$
$= 2a + 3b$

2 $5p^2 + 2p - 3p^2 = 5p^2 - 3p^2 + 2p$
$= 2p^2 + 2p$

Note: p^2 and p are not like terms.

3 $6ab - 4ba = 6ab - 4ab$
$= 2ab$

4 $6a - 2x + 5 + x - 2a - 7 = 6a - 2a - 2x + x + 5 - 7$
$= 4a - x - 2$



Only like terms can be added or subtracted.

Exercise 3:01

P Foundation worksheet 3:01 Collecting like terms

1 Simplify the following expressions.

a $3x + 2x$ b $8a + 5a$ c $10p + 21p$ d $7a - 4a$
e $9b - 3b$ f $11q - q$ g $3p + 5p - 6p$ h $4x + 2x + x$
i $10x - 9x + 3x$ j $3x^2 + 9x^2$ k $8a^2 - 5a^2$ l $y^2 + 3y^2 - 2y^2$
m $8y - 10y$ n $x - 3x$ o $2n - 4n + 5n$ p $14ab + 2ab$
q $6pq - 5pq$ r $10x^2y - 12x^2y$

2 Collect the like terms to simplify these expressions.

a $2a + b + 3a$ b $8x - 9 + 5x$ c $10 + 3a + 7$
d $5p + 4q - 6p$ e $3y + 2x - x$ f $m + 3n - 2n$
g $6q - 5q + 3p$ h $8h - 10h + k$ i $5m + n + 2m + 3n$

j $2a + 3b + a + 2b$
k $5t + 2u - u + 4t$
l $9q + 4p - 10q + p$
m $2a + p - a + 3p$
n $a + m - a + m$
o $8 + 2x - 5x - 7$
p $8y - 1 - 8y - 1$
q $x^2 + 2x + 2x^2 - x$
r $p^2 + 4p + 3p^2 + p$
s $3q^2 + 8q - 4q - q^2$
t $y^2 + y + y^2 - y$
u $7 - p^2 + p - 5$
v $2a + a^2 + 7 + a$
w $8x - 7 - 7x - 3x^2$
x $5ab - 7 + 3ba - 9$

3:02 Multiplication in algebra

PREP QUIZ 3:02

Simplify the following.

1 -6×4
2 -3×-2
3 7×-1
4 3^2
5 $(-2)^2$

Write in simplest form.

6 $7 \times a$
7 $x \times y$
8 $a \times a$

Simplify:

9 $2x + 2x + 2x$
10 $3 \times 2x$

WORKED EXAMPLES

1 $8 \times 7m = 8 \times 7 \times m$
$= 56 \times m$
$= 56m$

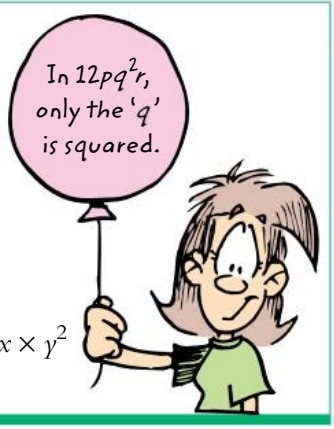
2 $5a \times 6b = 5 \times 6 \times a \times b$
$= 30ab$

3 $10x \times x = 10x \times 1x$
$= 10xx$
$= 10x^2$

4 $3pq \times 4qr = 12pqqr$
$= 12pq^2r$

5 $-4n \times 2y = -4 \times 2 \times n \times y$
$= -8ny$

6 $-7x \times -3xy^2 = -7 \times -3 \times x \times x \times y^2$
$= 21x^2y^2$



Multiply numbers first and then pronumerals.

Exercise 3:02

P Foundation worksheet 3:02 Multiplying pronumerals

1 Simplify these products.

a $5 \times 2x$
b $7 \times 3p$
c $8y \times 3$
d $9 \times a$
e $b \times 5$
f $9p \times 12$
g $x \times y$
h $3x \times 2b$
i $8x \times 3y$
j $3m \times 10n$
k $x \times 17y$
l $10a \times 5x$
m $\frac{1}{2} \times 4x$
n $10y \times \frac{1}{2}x$
o $\frac{1}{4}a \times 12$

2 Write these products in simplest form.

a $x \times x$	**b** $2y \times y$	**c** $a \times 3a$	**d** $4m \times 2m$
e $7p \times 2p$	**f** $8b \times 10b$	**g** $11t \times 11t$	**h** $10x \times 3x$
i $2a \times ab$	**j** $xy \times yz$	**k** $ab \times ac$	**l** $3pq \times 2p$
m $4mn \times 5np$	**n** $2ab \times 4ab$	**o** $xy \times 3yz$	**p** $4mn \times \frac{1}{2}m$
q $(-4x) \times 3y$	**r** $-2x \times -3x$	**s** $5a^2 \times (-7)$	**t** $9b \times a^2$
u $(-4a) \times (-7m)$	**v** $(-5p) \times 2pg$	**w** $(-ab) \times (-bc)$	**x** $14ab \times (-\frac{1}{2}ab)$

3 Simplify:

a $7 \times 2a \times 3$	**b** $5x \times 2y \times 3z$	**c** $2x \times 5 \times x$	**d** $ab \times bc \times ac$
e $x \times 2y \times 3z$	**f** $3a \times 5 \times 4a$	**g** $m \times n \times 7$	**h** $2k \times 3l \times k$
i $(-2) \times 7x \times (-5)$	**j** $10a \times (-3a) \times 4$	**k** $9x \times (-4y) \times (-5)$	**l** $\frac{1}{4}m \times 4n \times (-p)$

3:03 Division in algebra

PREP QUIZ 3:03

Simplify the following.

1 $72 \div 8$ **2** $(-12) \div 6$ **3** $20 \div (-5)$ **4** $(-24) \div (-6)$

Reduce these fractions to their simplest form.

5 $\frac{6}{2}$ **6** $\frac{10}{4}$ **7** $\frac{24}{10}$ **8** $\frac{100}{35}$

Simplify:

9 $5 \div 5$ **10** $x \div x$

WORKED EXAMPLES

1 $6a \div 2 = \dfrac{{}^{3}\cancel{6}\,a}{\cancel{2}_{1}}$ Divide top and bottom by 2.

$= 3a$

2 $15xy \div 3x = \dfrac{{}^{5}\cancel{15}\,\cancel{x}\,y}{{}_{1}\cancel{3}\,\cancel{x}}$ Divide top and bottom by 3 and x.

$= 5y$

3 $12ac \div 8ab = \dfrac{{}^{3}\cancel{12}\,\cancel{a}\,c}{{}_{2}\cancel{8}\,\cancel{a}\,b}$

$= \dfrac{3c}{2b}$

4 $-6x \div 18xy = \dfrac{-{}^{1}\cancel{6}\,\cancel{x}}{{}_{3}\cancel{18}\,\cancel{x}\,y}$

$= -\dfrac{1}{3y}$

$1 \div 3$ and $\frac{1}{3}$ are different ways of writing the same thing.

Express the division as a fraction and reduce it to its lowest terms.

Exercise 3:03

P Foundation worksheet 3:03
Algebraic division
Challenge worksheet 3:03
Algebraic simplifications

1 Reduce these fractions to their lowest terms.

a $\frac{3x}{6}$ b $\frac{12b}{30}$ c $\frac{8a}{8}$ d $\frac{5a}{10a}$ e $\frac{7x}{35x}$

f $\frac{12m}{4m}$ g $\frac{10ab}{5b}$ h $\frac{24a}{6b}$ i $\frac{7}{7t}$

2 Simplify these divisions.

a $12x \div 4$ b $16y \div 8$ c $5p \div 5$ d $15m \div 5$

e $2ab \div 2$ f $20xy \div 5$ g $9x^2 \div 3$ h $14x \div 7$

i $12a \div 3a$ j $8x \div 4x$ k $5x \div 5x$ l $36y \div 9y$

m $15m \div 10n$ n $18x \div 4x$ o $32a \div 12b$ p $5 \div 20x$

q $24xy \div 6x$ r $48ab \div 16a$ s $27x \div 21xy$ t $12a \div 10ba$

u $2 \div 18k$ v $a \div 3a$ w $5mn \div 10n$ x $45ab \div 20bc$

3 Simplify:

a $8p \div (-2)$ b $(-9x) \div (-3)$ c $(-20p) \div 4$ d $(-24x) \div (-12x)$

e $(-25) \div 15a$ f $(-xy) \div yz$ g $6ab \div 96$ h $(-54xy) \div (-9xy)$

i $14a \div (-a)$ j $20ab \div 12ba$ k $(-15x) \div 5xy$ l $-28mnp \div (-7mp)$

3:04 Simplifying algebraic fractions

3:04A Addition and subtraction

WORKED EXAMPLES

1 $\frac{2x}{5} + \frac{x}{5} = \frac{2x + x}{5}$

$= \frac{3x}{5}$

2 $\frac{7m}{3} - \frac{5m}{3} = \frac{7m - 5m}{3}$

$= \frac{2m}{3}$

If the denominators are the same, simply add or subtract the numerators.

3 $\frac{3a}{5} + \frac{2a}{4} = \frac{3a \times 4}{5 \times 4} + \frac{2a \times 5}{4 \times 5}$

$= \frac{12a}{20} + \frac{10a}{20}$

$= \frac{22a}{20}$

$= \frac{11a}{10}$

Cancel if possible.

4 $\frac{2n}{5} - \frac{n}{10} = \frac{2n \times 2}{5 \times 2} - \frac{n}{10}$

$= \frac{4n}{10} - \frac{n}{10}$

$= \frac{3n}{10}$

If the denominators are different, rewrite each fraction with a common denominator, then add or subtract the numerators.

Always look for the lowest common denominator!

Exercise 3:04A

P Foundation worksheet 3:04A
Simplifying algebraic fractions (+/−)

Simplify each addition or subtraction.

1 a $\frac{x}{3}+\frac{x}{3}$ b $\frac{3m}{7}+\frac{2m}{7}$ c $\frac{5n}{6}+\frac{2n}{6}$

d $\frac{3x}{5}-\frac{2x}{5}$ e $\frac{7t}{8}-\frac{2t}{8}$ f $\frac{4m}{7}-\frac{2m}{7}$

2 a $\frac{7p}{10}+\frac{p}{10}$ b $\frac{3t}{8}+\frac{t}{8}$ c $\frac{5y}{6}-\frac{y}{6}$

d $\frac{9x}{10}-\frac{3x}{10}$ e $\frac{7x}{12}+\frac{7x}{12}$ f $\frac{5m}{16}+\frac{9m}{16}$

3 a $\frac{x}{3}+\frac{x}{2}$ b $\frac{2a}{3}+\frac{a}{4}$ c $\frac{m}{2}+\frac{2m}{5}$ d $\frac{2t}{3}+\frac{t}{2}$ e $\frac{2x}{5}+\frac{2x}{3}$

f $\frac{a}{2}-\frac{a}{3}$ g $\frac{3w}{5}-\frac{w}{3}$ h $\frac{3n}{2}-\frac{2n}{5}$ i $\frac{2m}{3}-\frac{m}{2}$ j $\frac{3a}{4}-\frac{3a}{5}$

4 a $\frac{2a}{3}+\frac{a}{6}$ b $\frac{3y}{2}+\frac{5y}{6}$ c $\frac{2m}{5}+\frac{m}{10}$ d $\frac{4m}{3}-\frac{5m}{6}$ e $\frac{3t}{5}-\frac{3t}{10}$

f $\frac{m}{6}+\frac{m}{9}$ g $\frac{3a}{4}+\frac{a}{6}$ h $\frac{7x}{10}+\frac{x}{4}$ i $\frac{3n}{8}+\frac{5n}{6}$ j $3y-\frac{3y}{8}$

5 a $\frac{5}{6}m-\frac{1}{4}m$ b $\frac{7}{12}y-\frac{3}{8}y$ c $\frac{3}{4}x-\frac{3}{10}x$ d $5y+\frac{5}{4}y$ e $2m-\frac{4}{5}m$

Remember:

- $\frac{x}{3}$ is the same as $\frac{1}{3}x$ and $\frac{5m}{6}$ is the same as $\frac{5}{6}m$.
- $5y=\frac{5y}{1}$

3:04B Multiplication and division

WORKED EXAMPLES

Cancel any common factors, then multiply the 'tops' and then multiply the 'bottoms'.

1 $\frac{a}{2}\times\frac{b}{3}=\frac{a\times b}{2\times 3}$

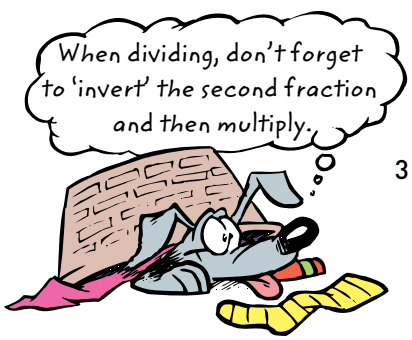
$=\frac{ab}{6}$

Multiply and divide in the same way as for numerical fractions.

2 $\frac{2x}{3}\times\frac{6}{5x}=\frac{2\,{}^{1}\!\cancel{x}}{{}_{1}\cancel{3}}\times\frac{\cancel{6}^{2}}{5\cancel{x}_{1}}$

$=\frac{2\times 2}{1\times 5}=\frac{4}{5}$

When dividing, don't forget to 'invert' the second fraction and then multiply.



3 $\frac{x}{5}\div\frac{x}{3}=\frac{{}^{1}\!\cancel{x}}{5}\times\frac{3}{\cancel{x}_{1}}$

$=\frac{1\times 3}{5\times 1}$

$=\frac{3}{5}$

4 $\frac{ab}{4}\div\frac{b}{2}=\frac{a\,{}^{1}\!\cancel{b}}{{}_{2}\cancel{4}}\times\frac{\cancel{2}^{1}}{\cancel{b}_{1}}$

$=\frac{a\times 1}{2\times 1}$

$=\frac{a}{2}$

Exercise 3:04B

P Foundation worksheet 3:04B
Simplifying algebraic fractions (×/÷)
Challenge worksheet 3:04B
Algebraic fractions

Simplify these products.

1 **a** $\frac{x}{2} \times \frac{y}{3}$ **b** $\frac{a}{5} \times \frac{b}{6}$ **c** $\frac{m}{3} \times \frac{n}{4}$ **d** $\frac{a}{3} \times \frac{2}{x}$ **e** $\frac{5}{m} \times \frac{n}{3}$ **f** $\frac{2p}{3} \times \frac{5}{q}$

2 **a** $\frac{3x}{2} \times \frac{5y}{4}$ **b** $\frac{4m}{3} \times \frac{2n}{3}$ **c** $\frac{7x}{2} \times \frac{y}{3}$ **d** $\frac{2a}{5} \times \frac{3b}{5}$ **e** $\frac{3x}{2} \times \frac{5x}{4}$

3 **a** $\frac{2m}{3} \times \frac{m}{4}$ **b** $\frac{3x}{2} \times \frac{y}{6}$ **c** $\frac{a}{10} \times \frac{5}{6}$ **d** $\frac{3x}{2} \times \frac{4y}{3}$ **e** $\frac{5t}{2} \times \frac{3t}{2}$

4 **a** $\frac{2x}{3} \times \frac{9}{x}$ **b** $\frac{m}{6} \times \frac{3}{2m}$ **c** $\frac{a}{b} \times \frac{b}{a}$ **d** $\frac{2x}{y} \times \frac{3y}{x}$ **e** $\frac{ab}{5} \times \frac{a}{b}$

5 **a** $10 \times \frac{y}{5}$ **b** $4 \times \frac{n}{6}$ **c** $\frac{5a}{2} \times 6$ **d** $6 \times \frac{2y}{3}$ **e** $\frac{3a}{8} \times 4$

Simplify these divisions.

6 **a** $\frac{m}{3} \div \frac{m}{2}$ **b** $\frac{x}{4} \div \frac{x}{3}$ **c** $\frac{p}{6} \div \frac{p}{5}$ **d** $\frac{t}{2} \div \frac{t}{5}$ **e** $m \div \frac{m}{5}$

f $\frac{a}{4} \div 2$ **g** $\frac{8x}{5} \div 4$ **h** $\frac{m}{2} \div \frac{t}{5}$ **i** $2m \div \frac{n}{5}$ **j** $\frac{2x}{5} \div 5y$

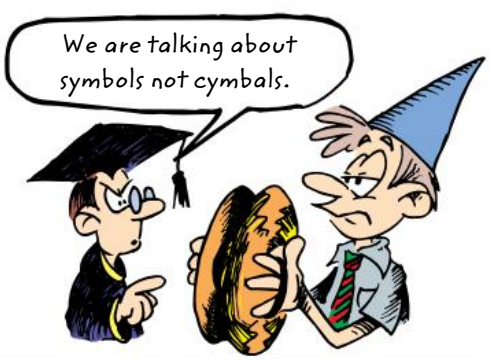
7 **a** $\frac{2x}{3} \div \frac{a}{5}$ **b** $\frac{3a}{4} \div \frac{b}{2}$ **c** $\frac{3p}{4} \div \frac{p}{2}$ **d** $\frac{4a}{5} \div \frac{2x}{5}$ **e** $\frac{5a}{3} \div 3a$

8 **a** $\frac{5}{x} \div \frac{x}{2}$ **b** $\frac{a}{3} \div \frac{2}{a}$ **c** $\frac{5m}{4} \div \frac{3}{2}$ **d** $\frac{ab}{3} \div \frac{ac}{2}$ **e** $\frac{2a}{b} \div \frac{3a}{c}$

3:05 Grouping symbols

The two most commonly used grouping symbols are:

parentheses	( )
brackets	[ ]



PREP QUIZ 3:05

Simplify:

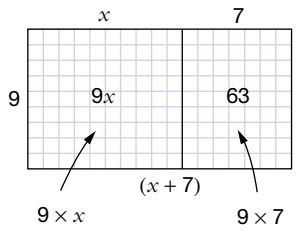
1 $5 \times 2a$ **2** $a \times 2a$ **3** $2x \times 5x$ **4** -4×-5

5 $-3 \times 2x$ **6** $-x \times x$ **7** $7p \times (-2p)$

Complete:

8 $5 \times 6 + 3 \times 6 = \square \times 6$ **9** $7 \times 2 - 4 \times 2 = \square \times 2$ **10** $3(8 + 5) = 3 \times \square + 3 \times \square$

Removing grouping symbols can be demonstrated using the area of a rectangle.



Area $= 9(x + 7)$
$= 9x + 63$

$a(b + c) = ab + ac$ or $a(b - c) = ab - ac$

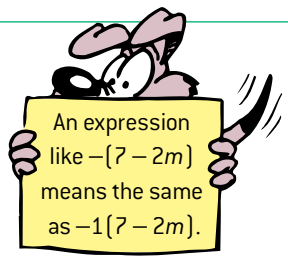
To expand an expression, such as $a(b + c)$, each term inside the grouping symbols is multiplied by the term outside.

WORKED EXAMPLES

1 $p(p + 3) = p \times p + p \times 3$
$= p^2 + 3p$

2 $3a(5 - 2a) = 3a \times 5 - 3a \times 2a$
$= 15a - 6a^2$

3 $-5(3x + 4) = (-5) \times 3x + (-5) \times 4$
$= -15x - 20$



4 $-(7 - 2m) = (-1) \times 7 - (-1) \times 2m$
$= -7 + 2m$

Exercise 3:05

P Foundation worksheet 3:05 Grouping symbols

1 Expand:

a $2(x + 3)$	b $3(a + 5)$	c $5(x - 1)$	d $10(a + 1)$
e $3(x - 2)$	f $8(y + 4)$	g $2(5a + 1)$	h $3(2x - 5)$
i $9(4x + 7)$	j $5(1 + x)$	k $4(3 - a)$	l $2(5 - 3m)$
m $2(3a + 2b)$	n $5(x + y)$	o $7(3x - 5y)$	

2 Remove the grouping symbols.

a $x(x + 7)$	b $a(a - 1)$	c $m(m + 10)$	d $n(n - 3)$
e $a(2a + 1)$	f $b(3b - 5)$	g $x(5 + x)$	h $y(3 - y)$
i $2x(x + 3)$	j $3y(y - 1)$	k $5z(2 + z)$	l $10m(1 - m)$
m $3a(2a - 1)$	n $9x(2x + 7)$	o $8p(2 - 5p)$	p $7q(3 + 2q)$
q $x(a + b)$	r $y(2y + x)$	s $2m(m + n)$	t $5a(2a + 3b)$

3 Expand by removing the parentheses.

a $-2(x + 3)$	b $-3(a + 5)$	c $-2(y - 1)$	d $-5(p - 3)$
e $-7(3a + 2)$	f $-5(2x - 1)$	g $-7(3 + 2m)$	h $-4(7 - x)$
i $-(a + 1)$	j $-(3x + 7)$	k $-(8 - 2p)$	l $-(3a + 2b)$
m $-x(x + 10)$	n $-y(5 - y)$	o $-3x(2x + 7)$	p $-10n(8m - n)$

4 Expand and simplify.

a $4\left(\frac{x}{2} + 3\right)$ b $6\left(\frac{2x}{3} - 1\right)$ c $4\left(\frac{3x}{2} + \frac{x}{2}\right)$ d $10\left(\frac{2y}{5} - \frac{y}{2}\right)$

INVESTIGATION 3:05 CHECKING ALGEBRAIC SIMPLIFICATION BY SUBSTITUTION

If two algebraic expressions are equal they will result in the same value when a number is substituted into a particular pronumeral in each expression. This gives an easy way to check algebraic simplifications.

Examples

1 A student thinks that $2(x + 5)$ is the same as $2x + 5$.
Substituting $x = 1$:

$2(x + 5) = 2(1 + 5)$
$= 2 \times 6$
$= 12$

$2x + 5 = 2 \times 1 + 5$
$= 2 + 5$
$= 7$

$12 \neq 7$, so the expressions are not equal.

2 Use substitution to see whether $(a + b)^2$ is the same as $a^2 + b^2$.
Substituting $a = 1$ and $b = 2$:

$(a + b)^2 = (1 + 2)^2$
$= 3^2$
$= 9$

$a^2 + b^2 = 1^2 + 2^2$
$= 1 + 4$
$= 5$

$9 \neq 5$, so the expressions are not equal.

Questions

Substitute a value for the pronumeral to show why the following pairs of expressions are not equal.

1 $2x + 3, \quad 5x$
2 $7a - 7, \quad a$
3 $6m \div 2m, \quad 3m$
4 $\frac{a}{3} + \frac{a}{2}, \quad \frac{2a}{5}$
5 $a + a, \quad a^2$
6 $3a + 4 - a + 2, \quad 2a + 2$
7 $\sqrt{a^2 + 1}, \quad a + 1$
8 $(a + 2)^2, \quad a^2 + 4$
9 $\frac{a + 3}{3}, \quad a + 1$

3:06 Simplifying expressions with grouping symbols

PREP QUIZ 3:06

Simplify:

1 $7x + 3x$
2 $4a^2 - a^2$
3 $4x + 3 + 2x + 5$
4 $2x + 7 - x - 5$
5 $3y^2 + 5y + 2y^2 - y$
6 $7 - 3a + 6 + 5a$

Expand:

7 $3(x - 7)$
8 $9(2 - 5y)$
9 $2a(a + 3)$
10 $-5(x + 7)$

WORKED EXAMPLES

1 $5x + 3 + 2(x + 3) = 5x + 3 + 2x + 6$
$= 7x + 9$

2 $x(x - 1) - x^2 + 5 = x^2 - x - x^2 + 5$
$= -x + 5$

3 $2a(a + b) - a(3a - 4b) = 2a^2 + 2ab - 3a^2 + 4ab$
$= 6ab - a^2$

Exercise 3:06

P Foundation worksheet 3:06
Simplifications with expansions

1 Simplify the following expressions.

a $6(x + 2) + 3$ **b** $4(a + 6) + 7$ **c** $6 + 2(y + 1)$
d $8 + 3(p + 2)$ **e** $6a + 3(a + 4)$ **f** $4m + 3(2m + 3)$
g $2(a + 3) + 5a + 2$ **h** $2x + 7 + 6(2x + 3)$ **i** $3(2a + 5) + 6a + 1$
j $4 + 4(3m + 1) + 6$

2 Expand all grouping symbols and then simplify each expression.

a $3(x + 2) + 2(x + 1)$ **b** $5(y + 2) + 3(y + 4)$ **c** $3(m + 5) + 2(m + 4)$
d $x(x + 3) + 3(x + 1)$ **e** $a(a + 5) + 5(a + 5)$ **f** $n(2n + 1) + 3(2n + 1)$

3 Expand and simplify.

a $2(a - 4) + 6$ **b** $7(a - 3) + 25$ **c** $4(m + 6) - 10$
d $3(x + 3) - 11$ **e** $4(m - 3) - 3$ **f** $8 + 3(2p - 5)$
g $5(y - 2) + 3y - 7$ **h** $4(a - 1) + 6a - 5$ **i** $2x + 7 + 5(2x - 1)$

4 Expand all grouping symbols, then simplify each expression.

a $8 - 2(a + 4)$ **b** $20 - 3(6 + x)$ **c** $6a - 3(2a + 1)$
d $3m + 10 - 2(m + 1)$ **e** $10a - 4 - 5(a + 4)$ **f** $12y - 3(2y + 5) + 15$
g $6(m + 1) - 3(m + 2)$ **h** $5(x + 7) - 3(x + 4)$ **i** $2a(2a + 3b) - b(3a + 2b)$

5 Expand all grouping symbols, then simplify each expression.

a $9k - 2(3 - 4k)$ **b** $11m - (5 - 2m)$ **c** $8a + 7 - 2(3a - 4)$
d $6 - 2(y - 4) + 4y$ **e** $20 - 4(x - 2) - 2x$ **f** $8x - 3(1 - 2x) - 10$
g $9(a + 5) - 7(a - 3)$ **h** $t(t - 5) - 4(t - 5)$ **i** $4x(x - 2) - 3(x - 2)$
j $2m(m - 3) - 5(m - 3)$ **k** $4(y - 5) - (y - 3)$ **l** $2a(3a + 1) - 3(2a - 3)$
m $2a(a + b) - a(a + b)$ **n** $x(x - y) - 3(2x - y)$ **o** $2x(3x - y) - y(3x - 2)$

If the width of one figure is x, what is the width of:
a 2 figures
b 4 figures
c half a figure?

3:07 Factorising using common factors

PREP QUIZ 3:07

Expand:

1 $2(x + 5)$ **2** $x(4x - 1)$

3 $3a(2a + 7)$ **4** $2m(m + 3n)$

Write the factors of:

5 12 **6** 20 **7** 30

Write the highest common factor (HCF) of:

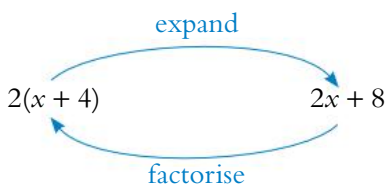
8 12 and 20 **9** 12 and 30 **10** 20 and 30

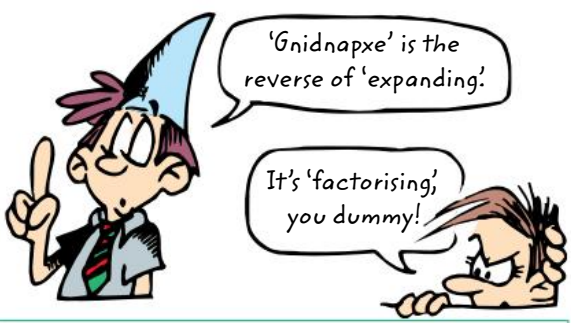
A factor of a given number is a number that divides into the given number with no remainder.
e.g. {1, 2, 3, 6, 9, 18} is the set of factors of 18.

To factorise an algebraic expression we must determine the highest common factor (HCF) of the terms and insert grouping symbols, usually parentheses.

If we expand the expression $2(x + 4)$, we obtain $2x + 8$. To factorise $2x + 8$ we simply reverse this procedure. The highest common factor of $2x$ and 8 is 2, so 2 is written outside the parentheses. The remainder is written inside the parentheses.

$2x + 8 = 2(x + 4)$





WORKED EXAMPLES

1 $2x + 6 = 2 \times x + 2 \times 3$ (HCF is 2)
$= 2(x + 3)$

2 $10x + 15y = 5 \times 2x + 5 \times 3y$ (HCF is 5)
$= 5(2x + 3y)$

3 $2mn + m = m \times 2n + m \times 1$ (HCF is m)
$= m(2n + 1)$

4 $9ax + 6a = 3a \times 3x + 3a \times 2$ (HCF is $3a$)
$= 3a(3x + 2)$

5 $5x^2 - 10xy = 5x \times x - 5x \times 2y$ (HCF is $5x$)
$= 5x(x - 2y)$

6 $-4a - 8 = -4 \times a + -4 \times 2$ (HCF is -4)
$= -4(a + 2)$

7 $-a^2 + 3a = -a \times a - (-a) \times 3$ (HCF is $-a$)
$= -a(a - 3)$

8 $4xy + 6x^2 - 2xz$ (HCF is $2x$)
$= 2x \times 2y + 2x \times 3x - 2x \times z$
$= 2x(2y + 3x - z)$

$ab + ac = a(b + c)$ and $ab - ac = a(b - c)$

Exercise 3:07

P Foundation worksheet 3:07 Factorising

1 Complete the following.

a $4x + 8 = 4(\quad)$ b $6a + 18 = 6(\quad)$ c $8a - 12 = 4(\quad)$

d $5x + 10y = 5(\quad)$ e $21x - 14y = 7(\quad)$ f $12pq - 15x = 3(\quad)$

g $mn - np = n(\quad)$ h $ab - ac = a(\quad)$ i $2ax + 4ay = 2a(\quad)$

j $y^2 - 5y = y(\quad)$ k $5at - 3a^2t = at(\quad)$ l $12m^2n - 6mn = 6mn(\quad)$

2 Factorise the following by taking out the HCF.

a $2x + 10$ b $6a + 4$ c $7y + 21$ d $28 + 4x$

e $27 - 3y$ f $24x + 6$ g $9x - 45$ h $16 - 12a$

i $9x + 3y$ j $5a + 10b$ k $15m - 20n$ l $4b - 6a$

m $mp + mn$ n $ax + ay$ o $x^2 + xy$ p $p^2 - pq$

q $ap + 3a$ r $5x + ax$ s $4m - mn$ t $xt - t$

3 Factorise completely.

a $3ax + 6ay$ b $5mn - 10mp$ c $4ab - 6bc$ d $9pq - 6qr$

e $5x^2 - 10xy$ f $3ab + 6a^2$ g $10m^2 - 4mn$ h $12x^2 + 4xy$

i $abc + bcd$ j $apq - bpq$ k $xyz + xy$ l $mn - mnp$

m $x^2a - xay$ n $5ax - 10xy$ o $a^2p - 5ap$ p $xy^2 + xyz$

q $10ab - 15bc$ r $5x^2y - 3xy^2$ s $ap^2 - a^2p$ t $5ab - a^2b^2$

4 Factorise the following by taking out the negative common factor.

a $-2a - 6$ b $-5x - 15$ c $-8m - 12$ d $-10x - 5$

e $-8x + 4$ f $-3n + 9$ g $-7y + 35$ h $-6a + 4$

i $-x^2 - 3x$ j $-m^2 - m$ k $-3x^2 + 2x$ l $-5y^2 + 10y$

m $-4p - p^2$ n $-3x - 2x^2$ o $-m + 7m^2$ p $-4a + 18a^2$

5 Factorise each of the following.

a $ab + ac + ad$ b $3x + xy + xz$ c $m^2 - 3m + mn$ d $7a - ab + a^2$

e $p^2 + pq - 5p$ f $2x + 4y - 6z$ g $10a - 5b + 15c$ h $9x^2 + 6x - 12$

i $8 - 4x + 6x^2$ j $25 + 15y - 20y^2$ k $x^2y - 3xy + x$ l $2ab - 4ac + 10a$

m $3x^2 + 6xy - 9x$ n $x^2y + xy + xy^2$ o $a^2b^2 + 3a^2b + 2ab^2$ p $mn + 4m^2n - 8mn^2$

6 Examine this example.

If $3(x + 2) - a(x + 2)$ is to be factorised then $(x + 2)$ is common to both terms, so it can be taken out as a common factor.

$3(x + 2) - a(x + 2) = (x + 2)(3 - a)$

Similarly:

$x(a + 1) + (a + 1) = x(a + 1) + 1(a + 1)$

$= (a + 1)(x + 1)$

Note:
$[a+1][x+1] = [x+1][a+1]$

Now factorise these similar types.

a $a(a + 2) + 3(a + 2)$ b $m(m + 2) + 4(m + 2)$ c $x(x - 1) + 5(x - 1)$

d $b(b + 1) - 5(b + 1)$ e $7(y - 2) - y(y - 2)$ f $t(t - 7) - 9(t - 7)$

g $4(2m - 3) + 3m(2m - 3)$ h $2x(7x + 1) - 5(7x + 1)$ i $x(a + 3) - (a + 3)$

j $y(2y - 1) - (2y - 1)$ k $p(p - 3) - 3(p - 3)$ l $(5x + 3) - x(5x + 3)$

FUN SPOT 3:07

IN THE THEATRE, WHEN DO YOU TELL SOMEONE TO 'BREAK A LEG'?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Simplify:

A $5a + 8a$	A $13x - x$	A $x^2 + x^2$
A $4a \times 3$	A $16x \div 8$	A $2a \times 6a$
A $3(a + b) - 3b$	B $7x \times 2y$	C $ab + ba$
C $a^2 \div a$	E $2a \times 2b$	E $x + x + x$
E $10x \div 2x$	G $-8x - 2x$	H $-2x \times -5$

Expand and simplify where possible:

I $3(7x + 4)$	I $2(7 - 3x)$
K $-4(x - 3)$	L $-4(x + 3)$
N $x(x + 7)$	N $2a(3a - 1)$
N $-a(2a + 3)$	O $x(x + y)$
O $3(2x - 4) + 5(3x - 5)$	R $x(x + 9) + 4(x + 9)$
R $a(a + 7) - 2(a + 7)$	S $1(x - 3) - x(x - 3)$

Factorise:

S $3x + 3y$	S $3x + 9$	T $a + ax$	T $x^2 + x$
U $6a - 12$	W $-3x - 3$	Y $-3x + 3$	Y $4a^2 - 3a$

$a(4a - 3)$	$x^2 + xy$	$6(a - 2)$	$3(x + y)$	$13a$	$-3(x - 1)$	$14xy$	$a^2 + 5a - 14$	$4ab$	$12x$	$-4x + 12$	$2x^2$	$-4x - 12$	$3x$	$-10x$	$-3(x + 1)$	$10x$	5	$x^2 + 7x$

$12a$	$-2a^2 - 3a$	$2x$	a	$x(x + 1)$	$21x - 37$	$x^2 + 13x + 36$	$21x + 12$	$3(x + 3)$	$14 - 6x$	$6a^2 - 2a$	$3a$	$2ab$	$12a^2$	$-x^2 + 4x - 3$	$a(1 + x)$

Algebra is important in the design and construction of buildings.

MATHS TERMS 3

algebra
- a branch of mathematics where numbers are represented by symbols

algebraic expression
- a group of terms and numbers that are joined by addition or subtraction signs

brackets
- the name given to these grouping symbols: [ ]

cancel
- to simplify a fraction by dividing the numerator and denominator by any common factor,

e.g. $\frac{\overset{7}{\cancel{21} \div 3}}{\underset{10}{\cancel{30} \div 3}}$ so $\frac{21}{30} = \frac{7}{10}$

collect like terms
- to simplify an algebraic expression containing many terms by addition and/or subtraction,

e.g. $5x + 3 + 7x - 4$
$= 12x - 1$

denominator
- the bottom number of a fraction

expand
- to remove grouping symbols by multiplying each term inside the grouping symbols by the term outside

factorise
- to write an expression as a product
- the reverse of expanding

like terms
- terms that have identical pronumeral parts, e.g. $6x$ and $3x$

numerator
- the top number of a fraction.

parentheses
- the name given to these grouping symbols: ( )

pronumeral
- a symbol used to represent a number

1 Three darts are thrown and all land in the '20' sector. What are the possible total scores for the three darts if all darts can land on either the 20, double 20 or triple 20?

2 Three darts are thrown and all land in the x sector. Write an algebraic expression for the possible total scores.

3 Three darts are thrown and all land in the same sector. The total score is 102. In what sector did the darts land?

DIAGNOSTIC TEST 3 ALGEBRA

Each part of this test has similar items that test a certain type of example.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1	Simplify:				3:01
	a $5a + 2b - 3a + b$		b $5p^2 + 2p - 3p^2$		
	c $6ab - 4ba$		d $6a - 2x + 5 + x - 2a - 7$		
2	Simplify:				3:02
	a $8 \times 7m$	b $5a \times 6b$	c $10y \times y$	d $-4n \times 2y$	
3	Simplify:				3:03
	a $6a \div 2$	b $15xy \div 3x$	c $12ac \div 8ab$	d $-6x \div 18xy$	
4	Simplify completely:				3:04A
	a $\frac{3a}{5} + \frac{a}{5}$	b $\frac{5m}{7} - \frac{3m}{7}$	c $\frac{x}{6} + \frac{2x}{6}$	d $\frac{7p}{10} - \frac{3p}{10}$	
5	Simplify:				3:04A
	a $\frac{a}{3} + \frac{a}{4}$	b $\frac{2m}{5} - \frac{m}{3}$	c $\frac{3x}{4} + \frac{x}{2}$	d $\frac{3n}{6} - \frac{n}{4}$	
6	Simplify completely:				3:04B
	a $\frac{x}{3} \times \frac{y}{4}$	b $\frac{a}{5} \times \frac{a}{2}$	c $\frac{2a}{5} \times \frac{10}{a}$	d $\frac{ab}{2x} \times \frac{x}{b}$	
7	Simplify completely:				3:04B
	a $\frac{m}{3} \div \frac{m}{4}$	b $\frac{a}{5} \div \frac{2}{a}$	c $\frac{3p}{4} \div \frac{p}{2}$	d $\frac{mn}{3} \div \frac{m}{n}$	
8	Expand:				3:05
	a $9(x + 7)$	b $6(5a - 2)$	c $p(p + 3)$	d $3a(5 - 2a)$	
9	Expand:				3:05
	a $-3(x + 2)$	b $-2(m - 8)$	c $-5(3x + 4)$	d $-(7 - 2m)$	
10	Simplify completely:				3:06
	a $2(a + 3) + 5$		b $x(2x - 7) + 3x^2$		
	c $9m - 2(1 - m)$		d $2(x + 5) + 4(x - 7)$		
11	Factorise completely:				3:07
	a $5m + 10$	b $x^2 - 3x$	c $6ab + 15a$	d $-8y - 12$	

ASSIGNMENT 3A Chapter review

1 Simplify:

a $3x + 2x$ **b** $3x \times 2x$

c $7x - x$ **d** $x^2 + x^2$

e $12x \div 6$ **f** $12x \div 6x$

g $2x + 3y$ **h** $2x \times 3y$

2 Simplify:

a $3ab \times 2b$ **b** $12a^2b \div 6a$

c $3ab + 2ab$ **d** $3a^2 - a$

e $4x + 3y + 2x$ **f** $12 + 6x + 3$

g $9a - 3 - 2a + 4$

h $x^2 - 3x + 2x + 3x^2$

3 Expand:

a $3(x + 5)$ **b** $2(x - 7)$

c $-3(x + 2)$ **d** $-4(1 - 3x)$

e $a(a + 3)$ **f** $x(y + z)$

g $3p(p - q)$ **h** $5x(2x - 7)$

4 Factorise fully:

a $3a + 15$ **b** $6m + 9$

c $15 - 5y$ **d** $ax - 3x$

e $2x + 6xy$ **f** $4x^2 - 2x$

g $9ab - 6bc$ **h** $6x^2 - 9x + 3xy$

5 Expand and simplify:

a $3(x + 3) + x + 4$

b $2(2y + 5) + 3(2y + 5)$

c $a - 4 + 2(a + 5)$

d $4(a - 3) + 2(1 - a)$

e $3(2m - 6) - (m - 4)$

f $3x(x - 2) - 2(x - 2)$

6 Simplify:

a $\frac{a}{3} + \frac{a}{3}$ **b** $\frac{a}{5} + \frac{a}{4}$

c $\frac{7m}{10} + \frac{m}{10}$ **d** $\frac{3y}{4} + \frac{2y}{3}$

e $\frac{5d}{4} - \frac{3d}{4}$ **f** $\frac{x}{2} - \frac{x}{5}$

g $\frac{3a}{4} - \frac{a}{2}$ **h** $\frac{3m}{5} - \frac{3m}{20}$

7 Simplify:

a $\frac{a}{4} \times \frac{a}{3}$ **b** $\frac{3a}{4} \times \frac{2b}{5}$

c $\frac{2mn}{3} \times \frac{9}{2n}$ **d** $\frac{ab}{4} \times \frac{a}{3b}$

8 Simplify:

a $\frac{m}{2} \div \frac{m}{6}$ **b** $\frac{A}{5} \div \frac{2A}{3}$

c $\frac{2a}{3} \div \frac{a}{6}$ **d** $\frac{m}{3} \div \frac{m}{12n}$

9 Determine whether each of the following simplifications is correct by substituting numbers for the pronumerals. (Finding just one number for which the statement is not true means the simplification is incorrect.)

For example: $3(x + 2) = 3x + 2$
For $x = 1$, $3(1 + 2) = 3 \times 1 + 2$
$9 = 5$
Hence, the simplification is incorrect.

a $5(a + 4) = 5a + 4$

b $-3(a - 2) = -3a - 6$

c $4(p - 3) + 2(p + 4) = 6p - 4$

d $5(a + 1) - 3(a - 2) = 2a - 1$

10 Find the error in each of the following expansions and rewrite each correctly.

a $4(3a - 4) = 12a - 4$

b $-3(x - 2) = -3x - 6$

c $2(a + 3) - 3(a + 4)$
$= 2a + 6 - 3a - 12$
$= a - 6$

d $4(3a - 2) - 3(2a + 1)$
$= 12a - 2 - 6a - 3$
$= 6a - 5$

ASSIGNMENT 3B Working mathematically

1 Refer to **ID Card 4** on page xix to identify the figures numbered:

a 1 **b** 2 **c** 3
d 4 **e** 5 **f** 6
g 7 **h** 8 **i** 9
j 11

2 **a** What geometric shape has inspired the design of these coffee cups?

b What would you estimate the capacity of the cup to be?

3 Diane and Garry married and had three children. Each child married and had three children. Assuming that no one has died, how many people are now in this extended family altogether?

4 The numerals 1 to 10 are written on ten separate cards, one on each card.

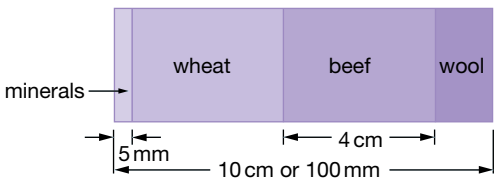
a How many pairs of cards have a sum of 10?

b How many groups of three cards are there that have a sum of 18?

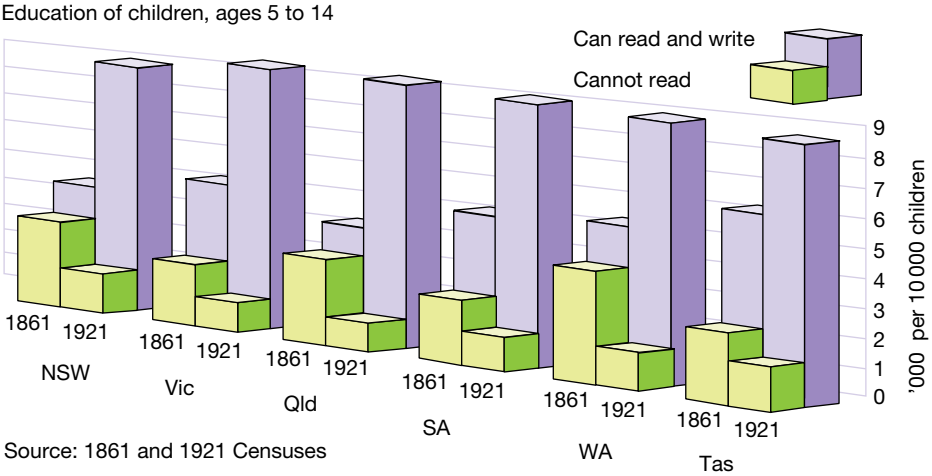
5 A particular country's exports are shown in the bar graph below (reduced in size). Find what percentage of the country's exports are taken up by:

a beef

b minerals.



6 Education of children, ages 5 to 14



Source: 1861 and 1921 Censuses

a In 1861, which state had the greatest number per 10 000 children who could read and write? What percentage was this?

b In 1921, which state had the greatest percentage of children who could read and write? What percentage was this?

c Which state had 4000 per 10 000 children who could read and write in 1861? About how many in that state could not read in 1861?

d Consider Western Australia in 1861. Approximately what percentage could read and write? Approximately what percentage could not read? (To determine this, measure the height of this column and measure this height on the scale.)

ASSIGNMENT 3C Cumulative revision

1 **a** Increase a wage of $900 by 6·5%. 1:02D, 2:01D

b In 1980 the population of a town was 42 000. By 2010 the population had decreased by 3·5%. What was the population of the town at that time?

c Which is larger?

i 15% or 20% **ii** $\frac{5}{12}$ or $\frac{2}{5}$ **iii** 0·65 or 0·0655

2 Change the following to decimals. 1:02C, 1:02D

a $\frac{7}{8}$ **b** 6·8% **c** $\frac{5}{12}$

3 State the number of significant figures in the following measurements. 1:06

a 8·8 L **b** 123·45 m **c** 6 km **d** 4·0°C

4 In what range would each of the measurements in Question **3** lie? 1:11

5 Estimate the answers to the following calculations. 1:07, 1:08

a $\dfrac{9{\cdot}89 \times 3{\cdot}123}{5{\cdot}089}$ **b** $4{\cdot}95^2 + 2{\cdot}13 \times 5{\cdot}237$

c $\dfrac{212{\cdot}3}{\sqrt{104{\cdot}2}}$ **d** $\dfrac{48{\cdot}24 \times 0{\cdot}888}{(12{\cdot}5 - 7{\cdot}056)^2}$

6 Convert the following. 1:10

a 3·2 ML to L **b** 45 GW to W

c 3TB to MB **d** 15 MHz to kHz

7 Car A uses petrol at the rate of 4·8 L/100 km, whereas car B uses it at a rate of 10 L/100 km. What will the difference in petrol costs be in a year in which both cars travel 30 000 km, if the petrol costs on average $1.45/L? 1:05, 2:01A

8 **a** A greengrocer buys 20 cases of oranges at a cost of $15 per case. Each case contains 10 kg of oranges. If he sells the oranges at $4/kg, how many kilograms must he sell before he makes a profit? If he sells all the oranges what will be his profit? 2:01

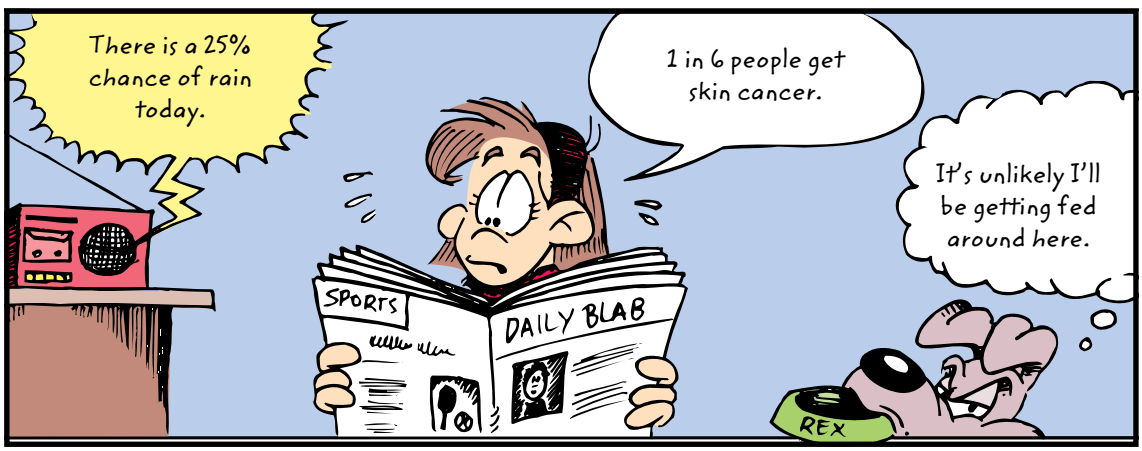
b Concrete is made by mixing volumes of cement, sand and gravel in the ratio 1 : 4 : 5. Jim calculates that he needs 2 m^3 of concrete to finish a job. How many cubic metres of sand and gravel does he need to order to make the concrete?

9 What is the last digit of the number 2^{2014}? 2:02

10 How many pairs of parallel edges are there in a rectangular prism? 2:02

PROBABILITY

4



Contents

4:01 The language of probability
Investigation 4:01 Rolling dice
4:02 Experimental probability
Investigation 4:02 Tossing a coin
GeoGebra activity 4:02 Rolling a single dice a large number of times
4:03 Theoretical probability
Investigation 4:03 Chance experiments
4:04 Mutually and non-mutually exclusive events
Investigation 4:04 Chance happenings
4:05 Using diagrams and tables
Fun spot 4:05 What are Dewey decimals?
4:06 Two-step chance experiments
Investigation 4:06 Two-step chance experiments
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Statistics and Probability

Selections from ***Probability*** [Stages 5.1, 5.2]

- Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'. (ACMSP226)
- List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays; assign probabilities to outcomes and determine probabilities for events. (ACMSP225)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

4:01 The language of probability

Terms used in probability

- **Sample space** — The set of all possible outcomes (e.g. 1, 2, 3, 4, 5, 6 on a normal dice).
- **Chance event** — A category in a chance experiment (e.g. rolling a 3 or 4 on a dice).
- **Probability** — The chance of an event happening (e.g. the chance of tossing a tail on a coin is 50%).
- **Trial** — One case of carrying out a chance experiment (e.g. spinning a spinner).
- **Equally likely outcomes** — A situation where each of the outcomes has the same chance of occurring.
- **Subjective probability** — Probabilities based on opinions or judgements.
- **Mutually exclusive events** — Events that have no elements in common (e.g. rolling an odd number and rolling an even number on a dice).
- **Non-mutually exclusive** — Events that have some elements in common (e.g. rolling an odd number and rolling a number less than 4 on a dice).

The sum of the probabilities of all possible outcomes is always 1, or 100%.

The **complement** of an event happening is that it does not happen.

Understanding the meaning when compound events are described

• Rolling at least 4 on a dice.	This means rolling 4, 5 or 6.
• Rolling at most 4 on a dice.	This means rolling 1, 2, 3 or 4.
• Rolling a number that is not 4 on a dice.	This means rolling 1, 2, 3, 5 or 6.
• Rolling a number that is even and larger than 2.	This means that both attributes must be present (i.e. 4 or 6).
• Choosing someone rich or famous but not both.	The person cannot be both rich and famous.
• Choosing someone rich or famous or both.	The person could be rich or famous or both rich and famous.

Choosing at random

- This is choosing in such a way that does not affect the likelihood of any outcome.
- If one student is chosen at random from a class, then each student has the same chance of being chosen.
- Random results can be obtained from coins, dice, spinners or digital generators.

For example, rolling two dice provides 36 different outcomes. '1 on Dice A and 6 on Dice B' would be one outcome and may be assigned to Student 1. Each of the students in the class would be assigned a different outcome. When the dice are rolled we select a student at random. (We may have to roll again if one of the outcomes that were not assigned is rolled.)

Relative frequency

- The **relative frequency** of an event is the fraction of times that event occurs. (This can also be written as a percentage.) This is the same as the experimental probability of the event occurring.

For example, if two coins were tossed 60 times and two heads occurred 14 times, then the relative frequency is $\frac{14}{60}$. This is called the experimental probability of tossing two heads in this experiment.

Note: If we were to repeat this experiment we may obtain a different relative frequency in the second experiment.

Exercise 4:01

1 Use the labels to the right to describe the chance that a dice, when rolled, will show:

a an odd number
b a 6
c a number greater than 6
d a zero
e a 1-digit number.

impossible | unlikely | even chance | likely | certain

2 A dice is rolled once. List the outcomes in each of these events.

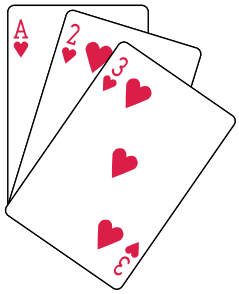
a A number less than 3 is rolled.
b A number less than 3 is not rolled.
c An even number is rolled.
d An even number is not rolled.
e A 5 is not rolled.
f A 7 is not rolled.

There are four suits in a standard pack of cards: hearts, diamonds, clubs and spades. There are 13 cards in each suit. Picture cards (or court cards) are the Jack, Queen and King of each suit.

3 Cards from a standard pack can be shown like this: 4H (4 of hearts), JD (Jack of diamonds) or AS (Ace of spades). List the cards from a standard pack that make up each of these events.

a a black Queen
b a red card between 4 and 8
c a red Jack
d a heart that is between 2 and 5
e a red picture card
f a card that is neither black nor a heart

4 Using a standard pack of cards, how many cards are in each event?



- **a** a number higher than 4
- **b** a number that is at least 4
- **c** a number that is a 7
- **d** a number no less than 4
- **e** not a picture card
- **f** not an even number
- **g** black or a number above 7
- **h** a number between 4 and 8
- **i** even or between 4 and 7 but not both
- **j** a number between Ace and 5 or a number greater than 5

5 In a raffle, tickets numbered 1 to 100 are sold. How many tickets are:

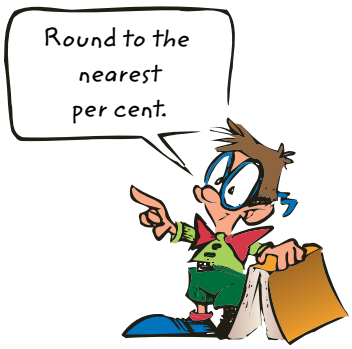
- **a** even and higher than 82
- **b** odd and higher than 83
- **c** between 37 and 47
- **d** between 37 and 38
- **e** not between 37 and 47
- **f** not between 37 and 38?

6
- **a** What are mutually exclusive events?
- **b** What are non-mutually exclusive events?
- **c** What is the sum of the probabilities of all possible outcomes in a chance experiment?
- **d** What is the complement of an event?
- **e** Explain what it means to choose at random.

7 Two coins were tossed 100 times. The results are shown in the table. Complete the relative frequency column.

Outcome	Frequency	Relative frequency (or experimental probability)
two heads	24	$\frac{24}{100}$ or 24%
head and tail	51	
two tails	25	
Total:	100	

8 A dice was rolled 60 times. The results are shown in the table. Complete the relative frequency column.



Outcome	Frequency	Relative frequency (or experimental probability)
1	8	$\frac{8}{60}$ or 13%
2	10	
3	9	
4	12	
5	10	
6	11	
Total:	60	

9 Design a four-coloured circular spinner where each sector has a relative frequency of 25%.

10 Design a device where the relative frequency of Alan being chosen is 50%, the relative fequency of Alana being chosen is 25% and the relative frequency of Heather being chosen is 25%.

INVESTIGATION 4:01 ROLLING DICE

Roll two dice and record the result. Carry out this experiment 50 times.

1 Use your results to complete the table below.
(*Note:* One result may be entered against several outcomes.)

2 Choose an expression from those in the list on the right that best describes the probability for each outcome

Outcome	Tally	Frequency
12		
2		
less than 12		
less than 6		
even		
1		
less than 10		

Probability

- impossible
- very unlikely
- unlikely
- even chance
- likely
- very likely
- certain

4:02 Experimental probability

PREP QUIZ 4:02

A dice was rolled several times and the following results were obtained.

5, 1, 2, 5, 6, 5, 2, 4, 6, 3

1 How many times was the dice rolled?
2 How many times was the result a 5?
3 What fraction of the rolls were 5s?
4 What fraction of the rolls were 2s?
5 What fraction of the rolls were odd numbers?
6 A coin was tossed 10 times, resulting in 4 heads. How many tails were there?
7 A coin was tossed several times and $\frac{2}{5}$ of the results were tails. What fraction of the tosses were heads?

Simplify:

8 $\frac{4}{12}$ 9 $\frac{16}{20}$ 10 $\frac{24}{50}$

One way of determining the chance of something happening is by observing what occurs in a sample 'experiment'. If simple equipment such as coins, dice, spinners, cards or random numbers are used to represent real events, then the 'experiment' is called a simulation.

The experimental probability is sometimes called the relative frequency.

Experimental probability formula

$$\text{The experimental probability of an event} = \frac{\text{number of times this event occurred}}{\text{total number in sample}}$$
$$= \text{relative frequency}$$

Data (also called empirical evidence) is collected continually, and it can be used to make predictions. Probabilities based on data are used to determine the cost of insurance, life expectancy and the likelihood of events occurring. These estimates are often called empirical probabilities and are a type of experimental probability. For example, if Australia had beaten England at the SCG four of the last five times they have played there, then it would be likely that Australia would win next time.

Experimental probabilities are usually based on an examination of a **sample** or **trial run** of the activity under examination.

WORKED EXAMPLES

1 A farmer collects 10 eggs and finds that 2 of them are bad. If he chooses another egg, what is the chance of getting another bad one?

2 The contents of 20 matchboxes were examined and the results recorded.

Number of matches	48	49	50	51	52	53
Number of boxes	1	5	8	3	2	1

If the contents of a similar box of matches were counted, what would be the experimental probability that it would contain 50 matches or more?

Solutions

1 As 2 of the first 10 eggs were bad, it seems that $\frac{2}{10}$, or $\frac{1}{5}$ of the farmer's eggs might be bad. So, *if* the first 10 eggs were truly representative of all the farmer's eggs *then* the chance of picking another bad one is $\frac{1}{5}$, or 1 out of 5.

2 In the sample, 14 of the 20 boxes had 50 or more matches.

$$\text{Experimental probability} = \frac{\text{number of times this event occurred}}{\text{total number in sample}}$$
$$= \frac{14}{20}$$

The chance of a similar box having 50 or more matches is $\frac{14}{20}$ or $\frac{7}{10}$.

- The probability of an event occurring in an experiment is the same as its 'relative frequency'.
- Relative frequency $= \frac{n(E)}{n(S)}$, where $n(E)$ = number of times this event occured and $n(S)$ = total number in sample.

Exercise 4:02

P Foundation worksheet 4:02
Experimental probability

1 The first 100 vehicles to pass a checkpoint gave the results in the table. If these figures truly represent the traffic at any time past this checkpoint, determine the experimental probability that the next vehicle will be:

a a car
b a motor cycle
c a bus
d not a car
e not a car or truck
f a car or a truck
g a bus or a truck
h a car or a bus.

Type of vehicle	Frequency	Relative frequency
Cars	70	$\frac{70}{100}$
Trucks	15	$\frac{15}{100}$
Motor cycles	10	$\frac{10}{100}$
Buses	5	$\frac{5}{100}$
Total:	100	1

2 We examined the contents of 37 packets of coloured lollies. The average number of each colour in a packet is shown in the table. One lolly is taken at random from a new packet. Use these results to determine:

a which colour is most likely to be picked
b which colour has the least chance of being picked
c the probability that it is red
d how many lollies are usually in a packet.

Colour	Frequency	Relative frequency
Brown	10	$\frac{10}{24}$
Green	5	$\frac{5}{24}$
Red	6	$\frac{6}{24}$
Yellow	3	$\frac{3}{24}$
Total:	24	1

3 A factory tested a sample of 100 light bulbs and 5 were found to be faulty. From these results, what is the probability of buying a faulty light bulb? What is the probability of buying a good bulb?

4 Sid Fowler recorded the number of eggs his chickens laid each day, for 6 weeks.

a If these results are typical for Sid's chickens at any time of the year find, as a fraction, the probability that on any particular day the number of eggs laid is:
  i 2
  ii 4 or 5
  iii 2 or more.

b Convert each of your answers to a percentage, correct to the nearest whole per cent.

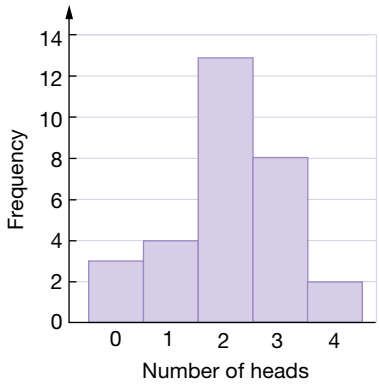
No. of eggs	Frequency	Relative frequency
0	8	$\frac{8}{42}$
1	13	$\frac{13}{42}$
2	10	$\frac{10}{42}$
3	6	$\frac{6}{42}$
4	3	$\frac{3}{42}$
5	2	$\frac{2}{42}$

5 A survey of 100 households was taken to determine how many used certain washing powders. Based on these results, what is the probability of a household chosen at random:

a using Foam brand
b using Supersoap or Pow
c not using Supersoap or Pow
d not using any of these four brands?

Brand	Frequency	Relative frequency
Foam	18	$\frac{18}{100}$
Suds-O	27	$\frac{27}{100}$
Supersoap	20	$\frac{20}{100}$
Pow	15	$\frac{15}{100}$

6 Jenny tossed four coins 30 times and the number of heads was recorded each time. The histogram shows the results.



a From this experiment, what is the probability of tossing:
   i no heads
   ii two heads
   iii at least three heads?

b If this experiment were to be repeated, would you expect the same results?

c As the number of trials increases, would the experimental probabilities be more likely to resemble the real probabilities?

7 A dice was rolled 50 times and the results were recorded. The table shows the results.

Number shown	Frequency	Relative frequency
1	7	$\frac{7}{50}$
2	5	$\frac{5}{50}$
3	5	$\frac{5}{50}$
4	10	$\frac{10}{50}$
5	9	$\frac{9}{50}$
6	14	$\frac{14}{50}$

a From this experiment, what is the probability of rolling:
   i a six
   ii a two
   iii an odd number?

b Would you expect to get seven ones every time a dice is rolled 50 times?

c If the dice was rolled 1000 times, how would you expect the experimental probabilities to change?

8 Of Year 9 students, $\frac{4}{20}$ have a shoe size greater than $10\frac{1}{2}$, but $\frac{19}{20}$ have a shoe size less than $11\frac{1}{2}$. What is the chance of a Year 9 student having a shoe size:

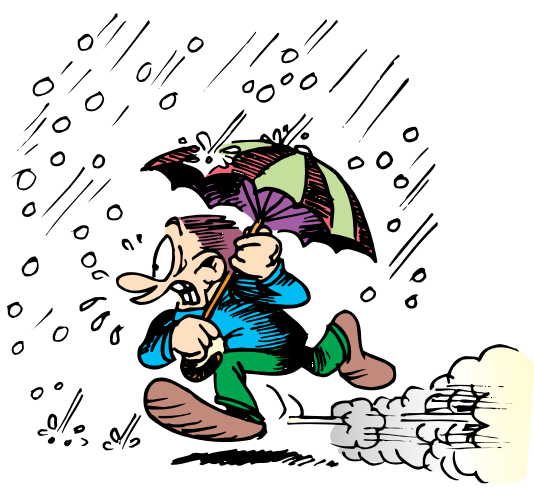
a less than or equal to $10\frac{1}{2}$

b $11\frac{1}{2}$ or larger

c between $10\frac{1}{2}$ and $11\frac{1}{2}$?
(*Note:* This is every size not in **a** and **b**.)

9 a Order the following events from least likely to most likely.
   A There will be a hail-storm tomorrow.
   B The next person to visit my home will be male.
   C My next maths teacher will be over 60 years of age.
   D I will see the principal of our school next week.
   E At least one member of our class will be married within 10 years.

b Estimate the probability of each event listed in part **a**, giving each on the scale 0 to 1.



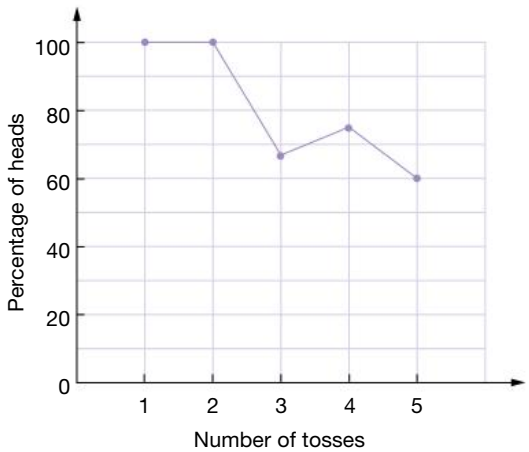
10 Explain the meaning of a probability of:

a 0 **b** $\frac{1}{2}$ **c** 1 **d** 0·5 **e** 100% **f** 50%

INVESTIGATION 4:02 TOSSING A COIN

When tossing a coin we assume that the probability of getting a head is $\frac{1}{2}$ or 50%, but is this true?

Luke tossed a coin five times and graphed the percentage of heads after each toss. He tossed: head, head, tail, head, tail. His graph is shown below.



- Percentage of heads after 3 tosses
  $= \frac{2}{3} \times 100\%$
  $= 66\frac{2}{3}\%$
- Percentage of heads after 4 tosses
  $= \frac{3}{4} \times 100\%$
  $= 75\%$

1 Toss a coin 10 times and graph the percentage of heads after each toss. Did the percentage get closer to 50% as the number of tosses increased?

2 Would this experiment be a reasonable simulation for the gender of babies born in a local hospital?

3 If you repeat this investigation, would you obtain the same graph? In what way would the second graph resemble the first?

- Is it reasonable to assume that the probability is $\frac{1}{2}$? Explain.
- Does any bias exist in the design of this experiment?

I often play my son Luke at chess. How could I estimate my chances of beating him in our next game?

P GEOGEBRA ACTIVITY 4:02 ROLLING A SINGLE DICE A LARGE NUMBER OF TIMES

Roll a single dice a large number of times and investigate the probability of each outcome.

Experimental probabilities (%)

	1	2	3	4	5	6
After 12 rolls	8.3	41.7	16.7	8.3	16.7	8.3
After 60 rolls	10	31.7	13.3	13.3	21.7	10
After 120 rolls	10.8	27.5	10	18.3	20.8	12.5
After 300 rolls	12.7	23.7	11	16.3	17.3	19

Results of first 12 rolls

1	2	2	5	6	2	2	3	5	2	4	3

Results of first 60 rolls

1	2	2	5	6	2	2	3	5	2	4	3
5	3	1	3	1	2	5	4	5	6	2	6
4	5	3	1	2	5	4	1	5	1	3	2
2	2	5	2	6	4	5	2	6	5	2	3
2	5	4	2	6	4	4	3	2	2	5	2

4:03 Theoretical probability

PREP QUIZ 4:03

Simplify:

1 $\frac{21}{24}$ **2** $\frac{34}{51}$ **3** $\frac{39}{91}$ **4** $1 - \frac{13}{16}$ **5** $1 - \frac{27}{30}$

A bag contains three white, five red and four black marbles. What fraction are:

6 white **7** black **8** not black?

9 What fraction of the letters of the alphabet are vowels?

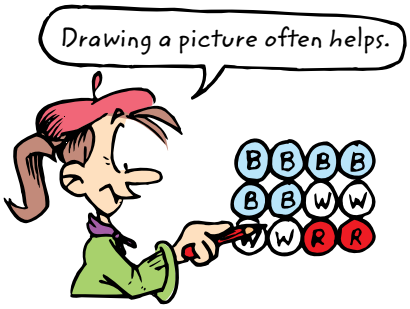
10 What fraction of integers from 1 to 50 inclusive are prime?

Performing an experiment will not always give a consistent result, or even a result we may think is most likely to occur.

In many cases we can work out the expected or theoretical probability of an event by considering the possible outcomes. For example, when tossing a coin there are two possible outcomes, a head or a tail.

Since there is only one head, the probability of tossing a head would be 1 out of 2 (i.e. $\frac{1}{2}$).

When calculating the probability of an event we shall assume that each possible outcome is equally likely (i.e. no two-headed coins or loaded dice).
Theoretical probability is the likelihood of outcomes occurring under ideal circumstances.



WORKED EXAMPLE 1

If a dice is rolled, what is the probability of getting:

a a six **b** an odd number **c** a number less than seven?

Solutions

The possible outcomes when rolling a dice are 1, 2, 3, 4, 5, 6. So the number of possible outcomes is 6.

a There is one 6 on a dice. So the probability of rolling a six is 1 out of 6. This can be written as:
$P(6) = \frac{1}{6}$

b There are three odd numbers on a dice. So the probability of rolling an odd number is 3 out of 6.
$P(\text{odd}) = \frac{3}{6}$
$= \frac{1}{2}$

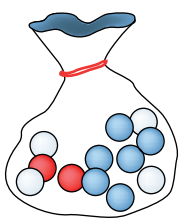
c All six numbers on a dice are less than 7, so the probability of rolling a number less than 7 is 6 out of 6.
$P(< 7) = \frac{6}{6}$
$= 1$

The probability of an event *certain* to happen is 1.
$P(\text{sure thing}) = 1$

WORKED EXAMPLE 2

A bag contains six blue marbles, four white marbles and two red marbles. What is the probability of choosing at random:

a a blue marble **b** a blue or white marble **c** a pink marble?



Solutions

The total number of marbles in the bag is 12. So the number of possible outcomes is 12.

a There are 6 blue marbles.
$\therefore P(\text{blue}) = \frac{6}{12}$
$= \frac{1}{2}$

b There are 10 blue or white marbles.
$\therefore P(\text{blue or white}) = \frac{10}{12}$
$= \frac{5}{6}$

c There are no pink marbles.
$\therefore P(\text{pink}) = \frac{0}{12}$
$= 0$

The probability of an event that *cannot happen* is 0.
$P(\text{impossibility}) = 0$

If each possible outcome is equally likely, then:

$$\text{Probability of an event, } P(E) = \frac{n(E)}{n(S)}$$

where $n(E)$ = number of ways the event can occur
$n(S)$ = number of ways all events can occur

(S is used to represent the sample space, which is the set of possible outcomes.)

The probability of any event occurring must lie in the range $0 \leq P(E) \leq 1$.

It should be pointed out that the probabilities of each possible event must add up to 1. As a consequence of this, if the probability of an event occurring is $P(E)$, then the probability of E *not* occurring is $1 - P(E)$.

$P(E')$ = Probability of E *not* occurring
$P(E') = 1 - P(E)$

E' is set notation for the 'complement' of E, i.e. those outcomes outside of E. For example:

- The complementary event for rolling an even number on a dice is rolling an odd number.
- The complement of drawing a red card from a standard pack of cards is drawing a black card.

Exercise 4:03

P Foundation worksheet 4:03 **Theoretical probability**
Challenge worksheet 4:03 **Probability: An unusual case**

1 A single dice is rolled. What is the probability of getting:

a a one **b** an even number **c** a number less than 3?

2 Ten coloured discs are placed in a hat. Five are red, three are yellow and two are black. If one disc is drawn from the hat, what is the probability that the disc will be:

a red **b** black **c** red or black
d not black **e** blue **f** red, yellow or black?

3 Write the complementary event for each of the following.

a Rolling an *odd number* on a dice.
b Getting a *tail* when a coin is tossed.
c Rolling a *number less than 6* on a dice.
d Drawing a *spade* from a standard pack of cards.
e Seeing *red* displayed on a traffic light that is working.
f *Winning* a soccer match.
g Choosing a *vowel* from the letters of the alphabet.

What is the complement of winning?

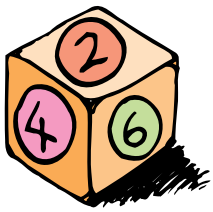
4 A card is drawn at random from a standard pack of 52 playing cards. What is the probability that the card is:

a the Ace of diamonds **b** a King **c** a red card
d a spade **e** a black Jack **f** a 7 or 8
g a picture card (Jack, Queen or King)?

5 The 26 letters of the alphabet are written on cards and placed in a box. If one card is picked at random from the box, what is the chance that the letter on it will be:

a X **b** a vowel
c M or N **d** a letter in the word *MATHEMATICS*?

6 Stickers were placed on a dice so that the faces showed three 2s, two 4s and a 6. If the dice is now rolled, what is the probability that the upper surface will be:



a a 2 **b** a 4 **c** a 6
d even **e** odd **f** less than 6?

7 Six students, John, Bob, Joan, Helen, Betty and Janet, each wrote their name on a piece of paper and put it in a hat. If one name was drawn at random, what is the probability that the name would:

a be Helen **b** start with J
c be a girl's name **d** have four letters in it?

8 A letter is chosen at random from the word *KATOOMBA*. What is the probability that the letter is:

a A **b** a consonant **c** one of the first four letters of the alphabet **d** Z?

9 The numbers from 1 to 25 are written on cards. If one card is chosen at random, what is the probability that the number on the card will be:

a an odd number **b** a multiple of 5
c a factor of 24 **d** a number that contains the digit 2?

10 A five-digit number is to be formed using the digits 1, 2, 3, 4, 5 (without repetition). What is the probability that the number will:

a be odd (end with 1, 3 or 5) **b** be even (end with 2 or 4)
c start with 5 **d** be greater than 30 000 (start with 3 or 4)
e be divisible by 3? (Hint: Is the sum of the digits divisible by 3?)

11 If the probability of an event is $\frac{1}{5}$, how many times, on average, would you expect it to occur in 20 trials? Can you say for certain how many times it will occur?

12 A bag contains five red balls numbered 1 to 5 and seven blue balls numbered 1 to 7. If a ball is chosen at random, what is the chance of choosing:

a a red ball **b** a ball numbered 3 **c** a ball numbered 6
d the blue ball numbered 1 **e** an even-numbered ball?

13 If a dice is rolled 12 times, how many times on average would you expect the result to be:

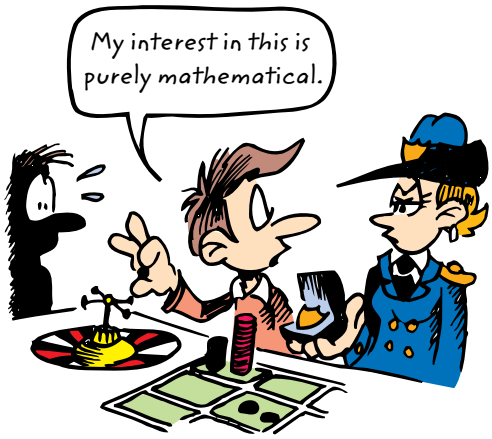
a a 6 **b** an odd number **c** a number greater than 2?

14 A survey of Kylie Crescent shows that 20% of the families have 1 child, 45% have 2 children, 15% have 3 children and 10% have more than 3 children. What is the probability that a family chosen at random will have:

a 2 children
b more than 2 children
c at least 2 children
d no children?

15 A roulette wheel is numbered from 0 to 36. Half of the numbers from 1 to 36 are red, the other half are black and the zero slot is green. Find the probability that the result of a spin will be:

a odd
b black
c 0
d not red
e a number from 1 to 6 inclusive
f a number greater than 25.



If the result is the green zero, the bank wins all wagers made.

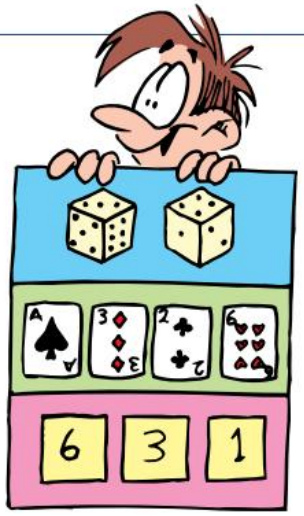
g How often would you expect a zero?

The payout for black or red is 'even money' (i.e. for a successful bet of $1, the payout is $1 plus the original $1 bet).

h If a gambler continued to bet the same amount on black, should he or she eventually win, lose or 'break even'?

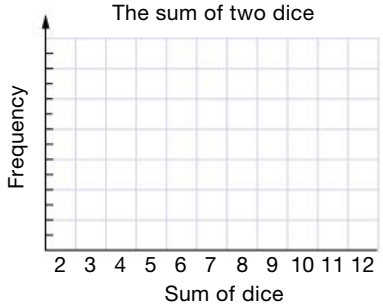
In what country did the game of roulette originate?

INVESTIGATION 4:03 CHANCE EXPERIMENTS



1 Carry out an experiment many times, and then estimate the experimental probability of one of the statements below. (In each case, state the number of trials you feel are necessary to get a good estimate.)
   - **a** A dice rolled twice will give two consecutive numbers.
   - **b** Four cards drawn at random from a standard pack will be of different suits.
   - **c** There are ten cards numbered 1 to 10. Three cards drawn at random, one after the other, will all be less than 7 if:
     - **i** each time the card is taken it is then replaced
     - **ii** each time the card is taken it is not replaced.

2 When two dice are rolled, the upper faces are added.
   - **a** Design an experiment to investigate the probability of each possible sum.
   - **b** Use a table to record your results. Graph these results as a column graph.
   - **c** Use the results of your experiment to estimate the probability of each sum.



The experiments in Questions **1a** and **1b** can be carried out using Microsoft® Excel® to record and calculate the data. This activity is available on ProductLink.

36 possible outcomes

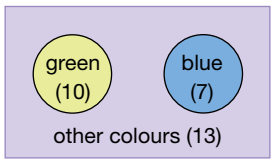
1st throw \ 2nd throw	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2 ,3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3 ,3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

The favourable outcomes are highlighted.

Theoretical probability = 5/36 ≈ 14%

4:04 Mutually and non-mutually exclusive events

A container holds 30 coloured counters (10 green, 7 blue and 13 of other colours).



If a counter is drawn at random from the container, what is the probability that it is:

- green
- blue
- green or blue
- neither green nor blue?

It is clear that the probability of drawing either a green or blue counter is equal to the probability of green plus the probability of blue.

$P(\text{green or blue}) = P(\text{green}) + P(\text{blue})$

A counter cannot be both green and blue at the same time so these two events are called mutually exclusive events.

> Mutually exclusive events are events that cannot happen at the same time.
>
> For example:
>
> - Selecting a boy and selecting a girl.
> - Rolling an even number on a dice and rolling a three.
>
> If two events, A and B are mutually exclusive then $P(\text{either } A \text{ or } B) = P(A) + P(B)$.
>
> (If the events were not mutually exclusive, we would need to subtract the probability that both events would happen at the same time.)

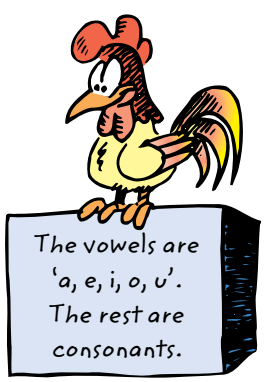
WORKED EXAMPLE

A different letter of the alphabet was placed on each of 26 cards. One of these cards was then drawn at random. What is the probability that the card drawn is:

a either a vowel or a consonant

b either a letter of the word *FINAL* or a letter of the word *METHOD*

c either a letter of the word *THICK* or a letter of the word *PICK*?



Solutions

a There are 5 vowels out of 26 letters and there are 21 consonants.

$\therefore P(\text{either vowel or consonant}) = \frac{5}{26} + \frac{21}{26}$

$= \frac{26}{26}$

$= 1$

$\therefore$ The card is certain to be either a vowel or a consonant.

b $P(\text{letter in the word } FINAL) = \frac{5}{26}$
$P(\text{letter in the word } METHOD) = \frac{6}{26}$
No letters are in both *FINAL* and *METHOD*.
$\therefore P(\text{either a letter in } FINAL \text{ or } METHOD) = \frac{5}{26} + \frac{6}{26}$
$= \frac{11}{26}$

c $P(\text{letter in the word } THICK) = \frac{5}{26}$
$P(\text{letter in the word } PICK) = \frac{4}{26}$
Some letters are in both *THICK* and *PICK*.
These events are not mutually exclusive.
$\therefore P(\text{either a letter in thick or pick}) \neq \frac{5}{26} + \frac{4}{26}$
The number of different letters in these two words is only 6 (*T*, *H*, *I*, *C*, *K*, *P*).
$\therefore P(\text{either a letter in thick or pick}) = \frac{6}{26}$
$= \frac{3}{13}$

Exercise 4:04

1 Write 'mutually exclusive' (no outcomes in common) or 'non-mutually exclusive' (at least one outcome in common) for the two events. In each case, the events relate to choosing a card from a standard pack.

a A red card will be chosen. / A picture card will be chosen.
b A King or Queen will be chosen. / An Ace will be chosen.
c A 10 will be chosen. / A heart will be chosen.

2 In a survey on censorship of violence and sex on TV, the following information was collected.

Description	Percentage of total
male adult	20
female adult	22
male teenager	27
female teenager	31

Response selected	Percentage of total
strong censorship needed	57
some censorship needed	25
little censorship needed	13
no censorship needed	5

What is the probability that a person chosen at random from the group surveyed:

a is female **b** is male **c** is an adult **d** is a teenager
e selected either *little censorship needed* or *no censorship needed*
f did not *select strong censorship needed* **g** is not a male adult
h is not a male teenager?

3 A pack of cards has four suits: hearts and diamonds (both red), and spades and clubs (both black). Each suit has 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The Jack, Queen and King are called picture cards.

A card is drawn from a standard pack.
What is the probability that the card is:

a red
b not red
c a 6
d not a 6
e a picture card
f a red Ace
g a spade
h a red 13
i either a red 5 or a 10
j either a heart or a black Ace
k either a blue 5 or a 7
l either a heart or a black card

In each of the following cases, the events might not be mutually exclusive.

m either a picture card or a diamond
n either a number larger than 2 or a club
o either a heart or a 5
p either a Queen or a black picture card
q either a number between 2 and 8 or an even-numbered heart?

There are 4 suits with 13 cards in each suit, so there are 52 cards in a standard pack. (In some games a Joker is also used.)

4 Using the results of past tests, the probabilities of getting the highest score on the next test (assuming no equal firsts) were:

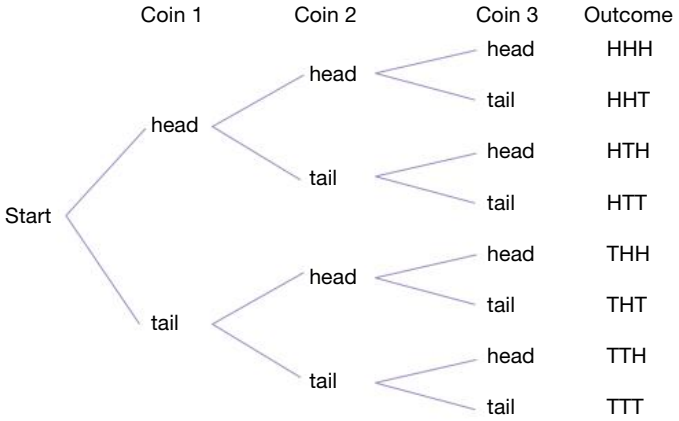
Amy $\frac{3}{10}$, Greg $\frac{1}{4}$, Rachel $\frac{2}{10}$ and Luke $\frac{1}{10}$

The probability that a girl would get the highest score was $\frac{6}{10}$

Based on these results, what is the probability that the person with the highest score is:

a either Amy or Rachel
b either Amy or Luke
c either Rachel or Luke
d a boy
e someone other than Amy
f someone other than Rachel
g either a boy or a girl
h either Greg or Luke
i a girl other than Amy
j a girl other than Rachel
k a boy other than Luke
l a boy other than Greg
m Amy, Rachel or Luke
n anyone but Amy, Rachel or Luke
o Amy, Rachel, Luke or Greg
p someone other than the four listed?

5 Three coins are tossed. The tree diagram below shows all possible outcomes.



a What is the probability of tossing:
   i exactly 2 heads ii exactly 2 tails?

b Is it possible that in one toss of the coins we could get 2 heads as well as 2 tails?

c What is the probability that we would get either 2 heads or 2 tails?

d *True* or *false*:
$P(\text{either 2 heads or 2 tails}) = P(\text{2 heads}) + P(\text{2 tails})$

e What is the probability of tossing:
   i exactly one tail ii more than one head?

f Is it possible that in one toss of the coins we could get exactly 1 tail as well as more than 1 head?

g *True* or *false*:
$P(\text{either one tail or more than one head}) = P(\text{one tail}) + P(\text{more than one head})$

h *True* or *false*:
If A and B are two mutually exclusive events in the same experiment then $P(\text{either } A \text{ or } B) = P(A) + P(B)$.

INVESTIGATION 4:04 CHANCE HAPPENINGS

1 Probability device
Design and test a probability device to produce results with probabilities of $\frac{1}{5}$, $\frac{2}{3}$ and $\frac{2}{15}$.

2 Media language
Investigate the use of chance language in the printed media. Collect examples and organise these in order, from most likely to least likely.

3 Birthdays
Investigate the probability of students in your class sharing the same birthday.

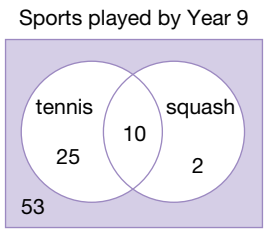
4 Travel
Investigate which mode of travel is safest.

4:05 Using diagrams and tables

Venn diagrams and two-way tables can be used to calculate the probability of events.

WORKED EXAMPLE 1

The Venn diagram shows two sports played by 90 Year 9 students.



What is the probability that a randomly chosen student plays:

a both tennis and squash

b neither tennis nor squash

c tennis

d tennis or squash but not both?

(Give answers as a fraction and as a percentage, correct to the nearest percent.)

Solutions

a This is the part where the circles overlap.
10 students play both tennis and squash.
$P(\text{both tennis and squash}) = \frac{10}{90}$ or 11%

b This is the part outside both circles.
53 students play neither tennis nor squash.
$P(\text{neither tennis nor squash}) = \frac{53}{90}$ or 59%

c This is both parts inside the tennis circle.
$25 + 10 = 35$
35 students play tennis.
$P(\text{tennis}) = \frac{35}{90}$ or 39%

d This is the part inside either circle but not in both.
$25 + 2 = 27$
27 students play tennis or squash but not both.
$P(\text{tennis or squash but not both}) = \frac{27}{90}$ or 30%

WORKED EXAMPLE 2

The two-way table shows how many females and males among 85 Year 10 students are right-handed or left-handed. What is the probability that a randomly chosen student is:

a both female and left-handed

b right-handed

c male

d a right-handed male?

	left-handed	right-handed	total
female	4	41	45
male	7	33	40
total	11	74	85

(Give answers as a fraction and as a percentage correct to the nearest percent.)

Solutions

a 4 students are both female and left-handed.
$P(\text{both female and left-handed}) = \frac{4}{85}$ or 5%

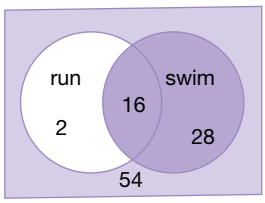
b 74 students are right-handed.
$P(\text{right-handed}) = \frac{74}{85}$ or 87%

c 40 students are male.
$P(\text{male}) = \frac{40}{85}$ or 47%

d 33 students are right-handed males.
$P(\text{right-handed male}) = \frac{33}{85}$ or 39%

Exercise 4:05

1 The same information about Year 8 students in KV High School is found in the two-way table and Venn diagram below.

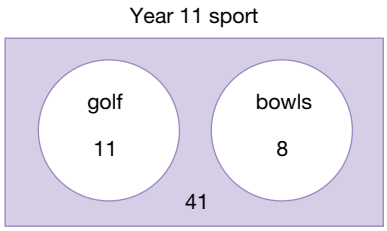


	swim	do not swim	total
run	16	2	18
do not run	28	54	82
total	44	56	100

What is the probability that a randomly chosen student:

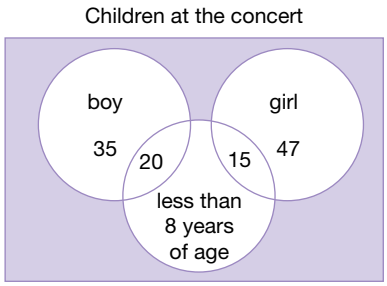
a runs **b** swims **c** does not swim
d does not run **e** runs and swims **f** runs but does not swim
g neither runs nor swims **h** swims but does not run **i** runs or swims or both?

2 The students of Year 11 were asked whether they played golf and/or bowls. The results are shown in the Venn diagram. What is the probability that a randomly chosen Year 11 student:



a plays golf **b** plays bowls
c plays golf or bowls **d** does not play golf
e plays neither golf nor bowls **f** does not play bowls
g plays both golf and bowls?

3 We surveyed all children at the concert and asked them their age. The data was used to produce this Venn diagram.



a How many children were at the concert?
b How many children were less than 8 years of age?

If one child is selected at random, what is the chance, written to the nearest percent, that the child will be:

c less than 8 years of age **d** a boy
e a girl younger than 8 **f** a girl 8 or older
g 8 years of age or older **h** a boy 8 or older?

4 We held a census of all students in our school to find out who owned a mobile phone. The results are shown in the two-way table.

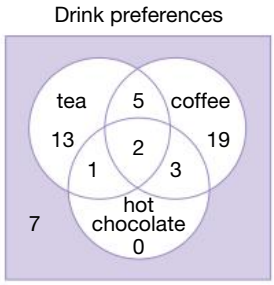
	owns a mobile phone	does not own a mobile phone	total
female	32	25	57
male	29	39	68
total	61	64	125

If we choose a student at random from our school, what is the probability that the student:

a owns a mobile phone
b does not own a mobile phone
c is a male who owns a mobile phone
d is a male who does not own a mobile phone
e is a female who does not own a mobile phone?

5 50 people were asked which of tea, coffee or hot chocolate they drink. The results are shown in the Venn diagram. If we choose one of these people at random, what is the chance that the person:

a does not drink any of the three drinks
b drinks all three
c drinks tea and coffee but not hot chocolate
d drinks tea but not coffee or hot chocolate
e drinks hot chocolate
f drinks hot chocolate but not tea or coffee?



FUN SPOT 4:05 WHAT ARE DEWEY DECIMALS?

Work out the answer to each part and write the letter for that part in any box that is above the correct answer.

One mechanic, six teachers and three nurses are in a waiting room. One of these people is chosen at random. What is the probability that the person chosen is:

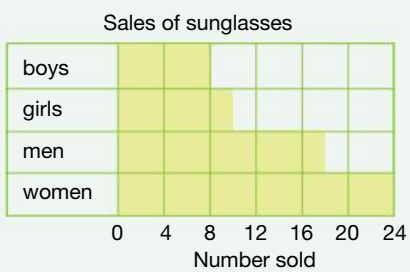
A a teacher
B a doctor
D a teacher or a nurse
E not a nurse
F either a male or a female
G neither a teacher nor a nurse?

One card is taken at random from a standard pack. What is the probability that it is:

H a heart
I black
K not a heart
L the 6 of clubs
M not the 6 of clubs
N a Jack
O not a Jack
R a 4 or 5
S a number between 3 and 10?

The graph shows the number of boys, girls, men and women who purchased sunglasses at our store last week. Use this information to find the experimental probability that the next person to buy sunglasses will be:

T a boy
U a man
V a woman
W not a girl
Y female



4:06 Two-step chance experiments

We can use an array or a tree diagram to show all possible outcomes when two events occur at random, one after the other, in a chance experiment.

Random selections without replacement

- 'Without replacement' means that an item taken as the first choice cannot be taken as the second choice.

WORKED EXAMPLE 1

There are one red, two green and three yellow counters in a container. Show all possible outcomes when two counters are take from the container, one after the other, without replacing the first counter. (This is the same as simply taking two counters together.)

Using an array

There are two green and three yellow counters so we will call the counters red R; green G1 and G2; yellow Y1, Y2 and Y3. Remember that the same counter cannot be chosen in both choices. In the first table, the possible outcomes are coloured.

Second choice (without replacement)

First choice	R	G1	G2	Y1	Y2	Y3
R						
G1						
G2						
Y1						
Y2						
Y3						

OR

Second choice (without replacement)

First choice	R	G1	G2	Y1	Y2	Y3
R		RG	RG	RY	RY	RY
G1	GR		GG	GY	GY	GY
G2	GR	GG		GY	GY	GY
Y1	YR	YG	YG		YY	YY
Y2	YR	YG	YG	YY		YY
Y3	YR	YG	YG	YY	YY	

There are 30 possible outcomes (6 rows of 5).

Using the array, we can now count the number of ways each outcome can occur.

- red and green (RG and GR) occurs 4 times (twice in the first row and twice in the first column)
- red and yellow (RY and YR) occurs 6 times.
- green and yellow (GY and YG) occurs 12 times.
- two reds is not possible because the first counter is not replaced.
- green and green (GG) occurs 2 times.
- yellow and yellow (YY) occurs 6 times.

This gives a total of 30 possibilities.

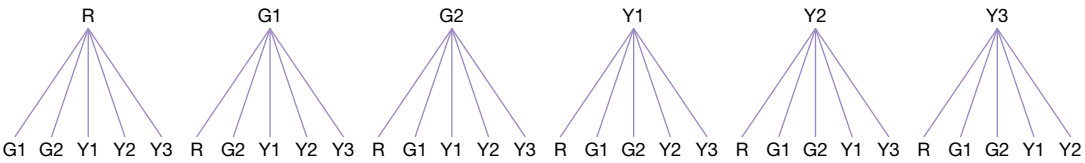
The table shows the probability of each event.

Event	Number of selections	Probability
red and green	4	$\frac{4}{30}$
red and yellow	6	$\frac{6}{30}$
green and yellow	12	$\frac{12}{30}$
both red	0	$\frac{0}{30}$
both green	2	$\frac{2}{30}$
both yellow	6	$\frac{6}{30}$
Total:	30	1

Using a tree diagram

We can also show all possible choices using a tree diagram.

(without replacement)



The array and the tree diagram are useful ways of showing all possible outcomes.

In the previous example, if the counter taken as the first choice is returned to the container before the second choice is made, then the array, the tree diagram and the table would be different. This would be called 'choosing two counters *with replacement*'.

Random selections with replacement

- 'With replacement' means that an item taken as the first choice can also be taken as the second choice.

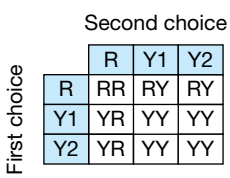
WORKED EXAMPLE 2

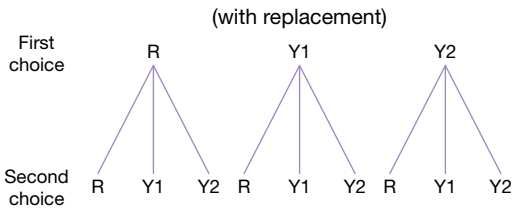
Consider a simpler case where we have only three counters: one red and two yellow.

Show all possible outcomes when two counters are taken from the container, one after the other, returning the first choice to the container before making the second choice (i.e. with replacement).

The array and the tree diagram would look like this.

- The number of possible outcomes is 9 (i.e. 3×3).

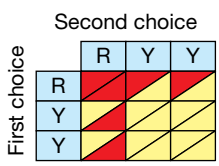




As the first chosen counter is replaced before the second is chosen, it is possible to choose two reds.

Using the array, we can count the number of ways each event could occur.

- Red and yellow could occur in 4 possible ways.
- Both red could occur in 1 possible way.
- Both yellow could occur in 4 possible ways.



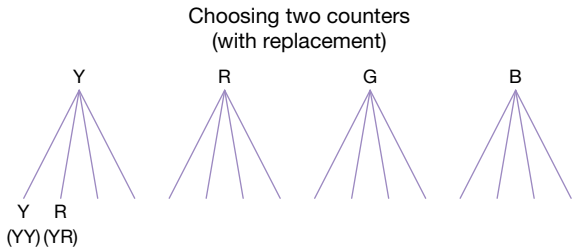
Event	Number of selections	Probability
red and yellow	4	$\frac{4}{9}$
both red	1	$\frac{1}{9}$
both yellow	4	$\frac{4}{9}$
Total:	9	1

This gives a total of 9 possibilities.

Exercise 4:06

1 A counter is taken at random from this box. It is then returned and another counter is taken at random.

a Complete the tree diagram and the array to show all possible outcomes.



Second choice (with replacement)

First choice	Y	R	G	B
Y	YY			
R	RY			
G				
B				

b How many possible outcomes are shown?

c How many of the outcomes show a red and a yellow?

d What is the probability that a red and a yellow will be taken?

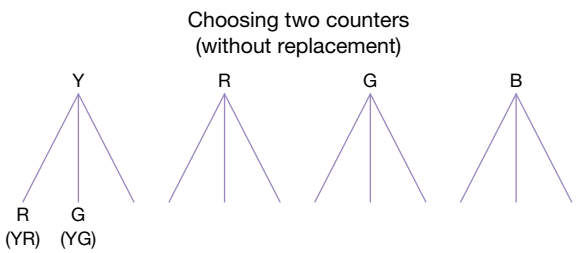
e What is the probability that a red and then a yellow will be taken?

f What is the probability that two yellows will be taken?

g What is the probability that two yellows will not be taken?

2 A counter is taken at random from the box in Question 1. It is not returned. A second counter is then taken at random.

a Complete this tree diagram and this array to show all possible outcomes.



Second choice (without replacement)

First choice	Y	R	G	B
Y		YR	YG	
R				
G				
B				

b How many possible outcomes are shown?

c How many of the outcomes show a red and a yellow?

d What is the probability that a red and a yellow will be taken?

e What is the probability that a red then a yellow will be taken?

f What is the probability that two yellows will be taken?

g What is the probability that two yellows will not be taken?

h Complete this table showing the probability of each event listed.

Event	Number of selections	Probability
yellow and red	2	$\frac{2}{12}$
yellow and green		
yellow and blue		
both yellow		

Event	Number of selections	Probability
red and green		
red and blue		
green and blue		
both red		

3 In a two-step experiment, a coin is tossed and a dice is rolled.

a Use an array and a tree diagram to show all possible outcomes.

b How many possible outcomes are there?

c What is the probability of:

i tossing a head and rolling a 6

ii tossing a head

iii tossing a tail and a rolling a number less than 3

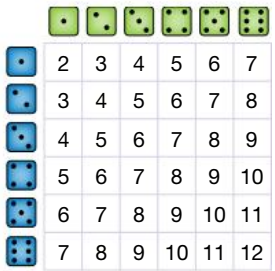
iv rolling a 5 or 6

v tossing a head and not rolling a 6

vi not rolling a 6?

d Does the tossing of the coin in any way affect the outcome of rolling the dice?

4 In a two-step experiment, two dice are rolled and the total is calcuated.



	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

a How many outcomes are possible?

b How many different totals are possible?

Use the array to find the probability of rolling a total of:

c 12 d 2 e 7

f 4 g 10 h 9

i 6 j 8 k 5

What is the probability of rolling a total:

l of 7 or 11 m less than 7 n of 2 or 12

o that is even p that is not 7 q that is a prime number?

5 In a two-step experiment, two cards are chosen at random (one after the other) from the five cards shown on the right. The first card is *not replaced* before the second is chosen.

(without replacement)

The suits of playing cards are hearts, diamonds, clubs and spades.

a Make a list of all possible outcomes
e.g. (3C, 6H), (3C, 7H), (3C, 8D), …

b How many different outcomes are there?

Use your list to find the probability that the outcome is:

c a 3 and a 6

d a 3 then a 6

e two clubs

f the 3 of clubs followed by a heart

g the second card will be a heart

h a club and a heart

i a club then a heart.

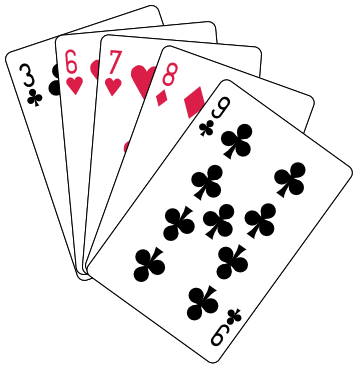
What is the probability that the numbers on the two cards have a total:

j of 15 k of 12 l more than 14

m that is even n that is not 15 o less than 12?

6 The experiment in Question 5 is repeated, but this time the first card is *replaced* before the second is chosen.

(with replacement)



a Make a list of all possible outcomes if the first card is replaced before the second card is chosen
e.g. (3C, 3C), (3C, 6H), (3C, 7H), …

b How many different outcomes are there?

Use your list to find the probability that the outcome is:

c a 3 and a 6 **d** a 3 then a 6 **e** two clubs

f the 3 of clubs followed by a heart

g the second card will be a heart

h a club and a heart

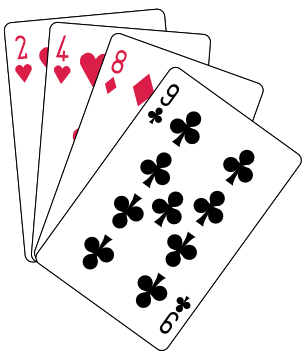
i a club then a heart.

What is the probability that the numbers on the two cards have a total:

j of 15 **k** of 12 **l** more than 14

m that is even **n** that is not 15 **o** less than 12?

7 In a two-step experiment, two cards are chosen at random (one after the other) from the four cards shown on the right.



a Here, the first card is *not replaced* before the second is chosen. If we have already chosen the 2 of hearts as our first card, what is the probability that we will end up with:

i two hearts **ii** two even numbers

iii a heart and a diamond **iv** a total greater than 8

v an even total **vi** a heart?

b Here, the first card is *replaced* before the second is chosen. If we have already chosen the 2 of hearts as our first card, what is the probability that we will end up with:

i two hearts **ii** two even numbers **iii** a heart and a diamond

iv a total greater than 8 **v** an even total **vi** a heart?

c Explain how knowing the result of the first step in parts **a** and **b** affected the probability of the events in the two-step chance experiment.

8 A representative of our school is to speak at a Dawn Service in town and at the school Anzac Day service after lunch. The choices must be made from Alan, Luke, Naomi and Rachel.

a A different person is to speak at each service.

i In how many different ways can the speaker be selected?

ii If the two people are to be chosen at random, what is the probability that the two boys will be chosen?

b In this case, the same person may speak at both services.

i In how many different ways can the speakers be selected?

ii If the speaker for each is to be chosen at random, what is the probability that there will be a boy speaking at both services?

9 Make up a two-step chance experiment of your own. Describe the experiment and write and then answer a set of questions about the probability of events relating to your experiment.

INVESTIGATION 4:06 TWO-STEP CHANCE EXPERIMENTS

1 On four pieces of identical paper or cardboard, write 'red 1', 'red 2, 'green' and 'purple'. Place these in a container and draw out two of them at random, one at a time, without replacement. Use the array below to keep a tally as you repeat this experiment 50 times. Record the results of this experiment by filling in the table, writing the probabilities as percentages.

Second choice (without replacement)

First choice	R1	R2	G	P
R1				
R2				
G				
P				

Tallies

Event	Number of selections	Experimental probability
2 red		%
red and green		%
red and purple		%
2 green		%
green and purple		%
2 purple		%
Total:		%

2 Repeat the experiment above, but this time replace the first choice before making the second. Use the array below to keep a tally as you repeat this experiment 50 times. Record the results of this experiment by filling in the table, writing the probabilities as percentages.

Second choice (with replacement)

First choice	R1	R2	G	P
R1				
R2				
G				
P				

Tallies

Event	Number of selections	Experimental probability
2 red		%
red and green		%
red and purple		%
2 green		%
green and purple		%
2 purple		%
Total:		%

3 Compare the results of the two experiments above. What difference does replacing the first choice make? Would you expect there to be a difference? Why? Write a report of at least 100 words on your investigation of these two cases.

The experiments in Investigation 4:06 can be carried out using Excel to record and calculate the data. This Excel activity can be accessed on ProductLink.

Event	Number of selections	Experimental probability (%)
2 red		
red and green		
red and purple		
2 green		
green and purple		
2 purple		
Total:		100%

MATHS TERMS 4

complementary event

- the complement of an event is all possible outcomes outside that event
  e.g. the complementary event to 'rolling a six' is 'not rolling a six'
- the probabilities of an event and its complement add up to 1; if $P(E)$ is the probability that E will occur, then the probability that E won't occur is $P(E') = 1 - P(E)$

experimental probability

- the chance of an event occurring as calculated by observing what happens in a sample experiment
  Experimental probability $= \dfrac{\text{no. of times event occurred}}{\text{total number of trials}}$

mutually exclusive events

- events that cannot occur at the same time,
  e.g. rolling an even number and rolling a 3 with a dice

outcomes

- the possible results when calculating a probability,
  e.g. the outcomes when tossing a coin are heads or tails

probability

- the calculated chance of an event happening

random

- without predetermination
- a way of choosing that does not affect the likelihood of outcomes

sample space

- the list of possible outcomes,
  e.g. when rolling a dice the sample would be 1, 2, 3, 4, 5, 6

simulation

- an experiment which uses simple equipment such as cards or coins to represent a real event,
  e.g. using heads or tails to represent the birth of boys or girls

survey

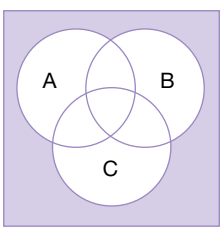
- to gather data or information from which conclusions might be drawn,
  e.g. to count the number of trucks in a line of traffic or to question a group of students about their favourite music

theoretical probability

- the probability of an event that is determined by considering the possible outcomes,
  e.g. the probability of rolling a '6' with a dice is $\frac{1}{6}$, because there are 6 equally likely outcomes

Venn diagram

- a diagram that uses circles to show the relationship between events



DIAGNOSTIC TEST 4 PROBABILITY

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

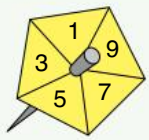
4:01

1 List all possible outcomes of each experiment below.

a A dice is rolled.

b A coin is tossed.

c The spinner on the right is spun.



4:02

2 My last 44 scores on our golf course have been entered in this table.

My score	90–93	94–97	98–101	102–105	106–109
Frequency	3	5	7	20	9

I am about to play another game. What is the experimental probability that my score will be:

a lower than 94 b higher than 93 c higher than 109?

Explain why the experimental probability that my score is higher than 109 is not the real probability.

4:03

3 A bag contains six blue marbles, four white marbles and two red marbles. A marble is chosen at random. What is the probability of choosing:

a a blue marble b a blue or a white marble

c a pink marble d anything but a white marble?

4:04

4 A card is drawn at random from a standard pack. What is the probability that the card is:

a either a 4 or a 10 b either a red 4 or a 10

c either the 5 of spades or an Ace d either a heart or a 5

e either a red card or a Jack f anything except either a red card or a Jack?

4:05

5 Use this Venn diagram to find the probability that a randomly chosen student plays:

a tennis

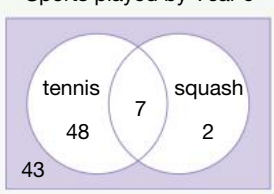
b neither tennis nor squash

c both tennis and squash

d tennis or squash but not both.

(Give answers as a fraction and as a percentage.)

Sports played by Year 9



4:05

6 Use this table to find the probability that a randomly chosen student is:

a both female and left-handed

b right-handed

c male

d is a right-handed male.

(Give answers as a fraction and as a percentage.)

Students in Year 12

	left-handed	right-handed	total
female	3	20	23
male	4	23	27
total	7	43	50

ASSIGNMENT 4A Chapter review

1 A dice is rolled. What is the probability that it will:

a show a 6
b show a 7
c show a number less than 3
d show an odd number
e show a number less than 7?

2 Presuming that the figures shown in the table are typical of the people in a certain town, what is the probability of a person chosen at random from this town being:

Hair type	Number
Brunette	55
Blonde	35
Red	10

a a blonde
b a red-head
c not a blonde?

3 A box contains ten apples, eight oranges and two lemons. If a piece of fruit is picked at random, what is the chance of getting:

a an orange **b** a lemon
c an orange or a lemon **d** a banana?

4 One red (R1), one green (G1) and two yellow counters (Y1 and Y2) are in a container.

a Show all possible outcomes when two counters are taken from the container, one after the other, **without replacing the first counter**. Use your list or diagram to find the probability of randomly choosing 2 counters of the same colour.

b Show all possible outcomes when two counters are take from the container, one after the other, **replacing the first counter before the second is drawn**. Use your list or diagram to find the probability of randomly choosing 2 counters of the same colour.

5 Bobby Striker played 20 games of tenpin bowls and gained the following scores.

169	148	173	171	156
161	159	183	137	164
152	199	145	168	155
142	175	187	140	167

a Going on his past performances, what is the probability that Bobby's next score will be:
i greater than 170 **ii** less than 150?

b Bobby did not score over 200. Does this mean that the probability of him scoring a 200 game is zero? Comment on your answer.

6 When two teams play football there are three possible results. Each team could win or the game could be drawn. Therefore, the probability that a particular team wins is $\frac{1}{3}$. Is this statement correct? Justify your answer.

In poker, the probability of being dealt a royal flush is $\frac{1}{649\,740}$.

ASSIGNMENT 4B Working mathematically

1 Refer to **ID Card 5** on page xx to identify the figures numbered:

a 6	**b** 7
c 8	**d** 12
e 14	**f** 15
g 17	**h** 18
i 19	**j** 22

2 Using the **Algebra card** on page xxiii:

a add column M to column N

b subtract column M from column N

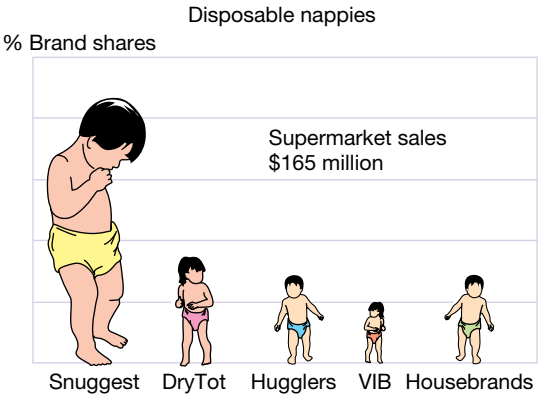
c add columns N and O

d find the value of the term in J if $x = -2$.

3 Through how many degrees does the spoke of a wheel turn in 5 minutes if the wheel is turning 700 revolutions per minute?

4 Indu is 8 years younger than John, but John is twice her age. How old is Indu?

5 The graph below shows the percentage brand shares of all disposable nappies sold in supermarkets in a year. The scale on the vertical axes has been removed.



Estimate the percentage of the sales belonging to Snuggest if:

a the *height* of each picture is the significant measure

b the *area* of each picture is the significant measure

c the *volume* of the child represented in each picture is the significant measure.

- One of these keys is the one I want. If I choose one of them at random, what is the probability that it is the one I want?
- If the first one I chose was not the one I wanted, what would be the probability that I will choose the right one on my second choice?
- What would be the probability of choosing the correct key on either my first or second selection?

ASSIGNMENT 4C Cumulative revision

1 **a** $3 - (-4 \times 7)$ **b** $18 - 8 \times 9$ **c** $(-7)^2 - (-8)^2$ 1:02A

2 **a** 30% of my wages was taken as tax. This was $186. What was my wage? 1:02D
b I was able to reduce my weight by 4·8 kg. This was 6% of my original weight. What was my original weight?

3 **a** I scored 7 of our teams 40 points. What percentage of the points did I score? 1:02D
b A box of pens contained 16 blue, 8 red and 25 black. What percentage (2 dec. pl.) of the pens were red?

4 **a** Write $\frac{3}{8}$ as a decimal. **b** Write $\frac{1}{6}$ as a decimal. 1:04
c Write 0·636 363 … as a fraction in its simplest form.

5 **a** Find the ratio of Rob's height (2 m) to Alan's height (180 cm). 1:05, 1:06
b What was my average rate of speed if I travelled 1210 km in 5 h 30 min?
c How many significant figures has 0·0050 m?

6 **a** Write $6{\cdot}4 \times 10^6$ as a basic numeral. 1:09, 1:10
b Write 95 000 000 in scientific notation using two significant figures.
c Convert 2·3 MB to kB.

7 Simplify: 3:01, 3:02
a $12y - y$ **b** $8m - 7p + 3p$ **c** $4a^2 - 2a + 3a^2 + 3a$
d $4a \times 7$ **e** $3t \times 4t$ **f** $7a \times 6b \times 10$

8 Simplify: 3:03, 3:04
a $\frac{16a}{4}$ **b** $8xy \div 2x$ **c** $-10m \div 20mn$
d $\frac{m}{10} + \frac{3m}{10}$ **e** $\frac{8m}{5} - \frac{7m}{10}$ **f** $\frac{5a}{6} + \frac{3a}{5}$
g $\frac{m}{5} \times \frac{m}{4}$ **h** $\frac{5b}{3} \times \frac{9}{10b}$ **i** $\frac{xy}{10} \div \frac{y}{5}$

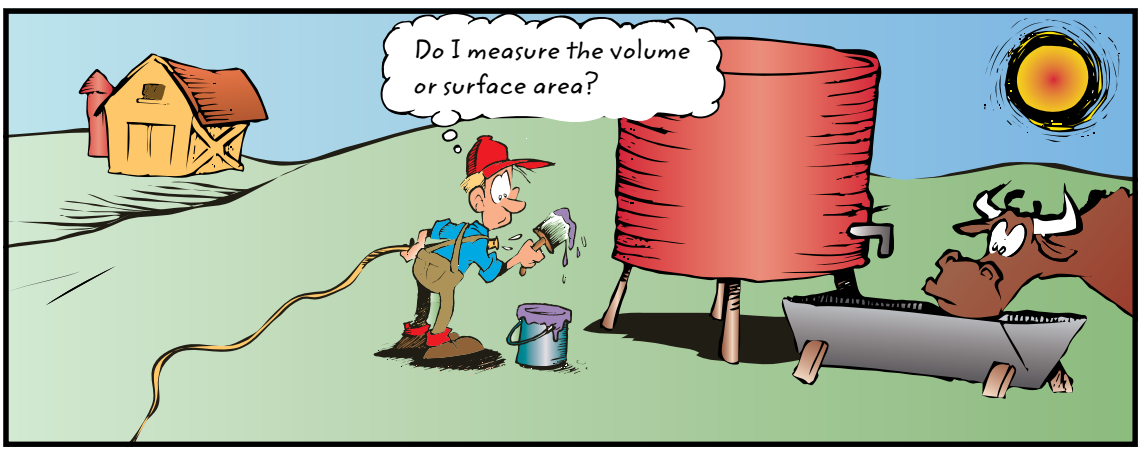
9 Expand and simplify where possible. 3:05, 3:06
a $x(3x + 7)$ **b** $6y(7 - 2y)$ **c** $-2(5m - 5)$
d $11m + 15 + 6(m - 9)$ **e** $4a(a - b) - a(7a - 4b)$

10 Factorise: 3:07
a $8m + 12$ **b** $4x^2 - 6x$ **c** $12ab - b$

11 Illustrate each problem using a Venn diagram and then answer the question. 2:03
a Of 15 people, 6 had been to Turkey and 10 had been to China. If 3 people had been to neither country, how many had been to both Turkey and China?
b Of 25 students, 16 had been to Scotland, 9 had been to Wales, and 8 had been to Ireland. 5 had been to both Wales and Scotland but only one of these 5 had been to Ireland as well. 2 students had been to Ireland and Wales but not Scotland and 4 students had been to both Scotland and Ireland. How many students had been to none of these countries?

AREA, SURFACE AREA AND VOLUME

5



Contents

5:01 Review of area
GeoGebra activity 5:01 Area formulas
Challenge 5:01 Areas of scale drawings
5:02 Areas of composite figures
GeoGebra activity 5:02 Areas of composite figures
5:03 Surface area of a prism
GeoGebra activity 5:03 Surface area of a rectangular prism
Fun spot 5:03 How did the boy know he had an affinity with the sea?
5:04 Surface area of a cylinder
Fun spot 5:04 Let's play with blocks
5:05 Volume of prisms and cylinders: A review
Investigation 5:05 Tank sizes
5:06 Volumes of other prisms
GeoGebra activity 5:06 Volume of a prism
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Area and Surface Area*** [Stages 5.1, 5.2]

- Calculate the areas of composite shapes. (ACMMG216)
- Calculate the surface areas of cylinders and solve related problems. (ACMMG217)
- Solve problems involving the surface areas of right prisms. (ACMMG218)
- Solve problems involving surface area for a range of prisms, cylinders and composite solids. (ACMMG242)

Selections from ***Volume*** [Stage 5.2]

- Solve problems involving the volumes of right prisms. (ACMMG218)
- Solve problems involving volume for a range of prisms, cylinders and composite solids. (ACMMG242)

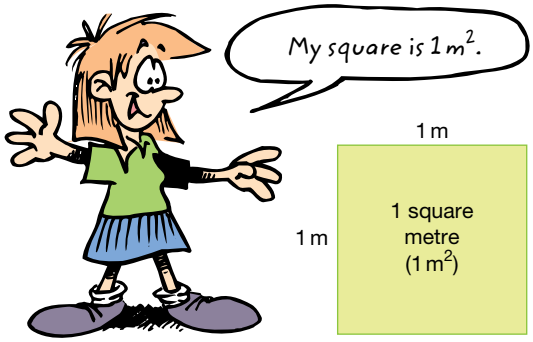
Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

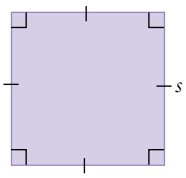
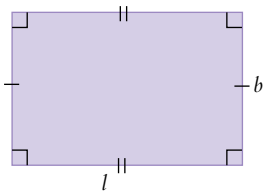
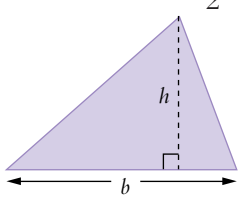
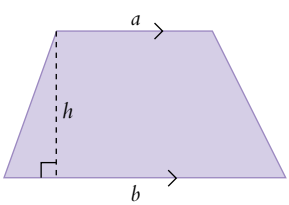
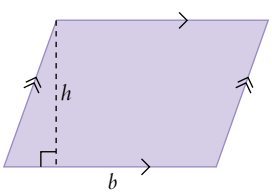
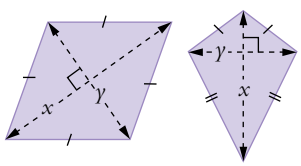
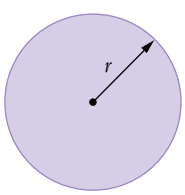
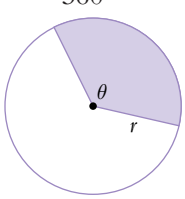
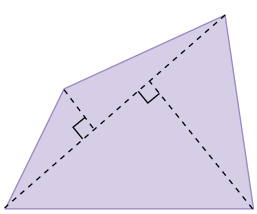
5:01 Review of area

Measurement of area

- The area of a plane figure is the amount of space it occupies.
- Area is measured by calculating how many squares it would take to cover the figure.
  Small squares are used to measure small areas and large squares are used to measure large areas.
  It should not be surprising then that the units for measuring area are called square units.
- $1\,cm^2$ is the area within a square with 1 cm sides.
  $1\,m^2$ is the area within a square with 1 m sides.
  1 ha is the area within a square with 100 m sides.
  $1\,km^2$ is the area within a square with 1 km sides.
- Area is calculated using a formula.



Area formulas

Square $A = s^2$ 	Rectangle $A = lb$ 	Triangle $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$ 
Trapezium $A = \frac{1}{2}h(a+b)$ 	Parallelogram $A = bh$ 	Rhombus and Kite $A = \frac{1}{2}xy$  x and y are the lengths of the diagonals
Circle $A = \pi r^2$ 	Sector $A = \frac{\theta}{360} \times \pi r^2$ 	Quadrilateral  There is no formula. Divide into two triangles and add the areas.

PREP QUIZ 5:01

1 $\frac{1}{2} \times 10{\cdot}6 \times 48$ **2** $\dfrac{3{\cdot}4 \times 1{\cdot}2}{2}$ **3** $\frac{1}{2} \times 4{\cdot}6 \times (10{\cdot}7 + 3{\cdot}5)$ **4** Simplify $l \times b$.

Evaluate:

5 lb if $l = 4$ and $b = 5$ **6** $\frac{1}{2}bh$ if $b = 5$ and $h = 8$

7 $\dfrac{bh}{2}$ if $b = 5$ and $h = 8$ **8** ab^2 if $a = 3$ and $b = 4$

Complete the following:

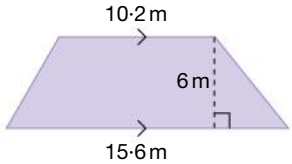
9 1 m = … cm **10** 1 cm = … mm

WORKED EXAMPLES

1 a Find the area of a square with sides of 3·1 cm.

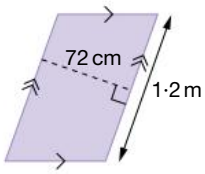
b Find the area of a circle that has a radius of 5 m (use $\pi = 3{\cdot}14$).

c Find the area of this trapezium.

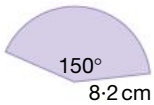


2 Find the area of the following figures.

a



b



Solutions

1 a $A = s^2$
$= (3{\cdot}1)^2$
$= 9{\cdot}61 \text{ cm}^2$

b $A = \pi r^2$
$= 3{\cdot}14 \times 5^2$
$= 3{\cdot}14 \times 25$
$= 78{\cdot}5 \text{ m}^2$

c $A = \frac{1}{2}h(a+b)$
$= \frac{1}{2} \times 6(10{\cdot}2 + 15{\cdot}6)$
$= 3(25{\cdot}8)$
$= 77{\cdot}4 \text{ m}^2$

2 a $A = bh$
$b = 1{\cdot}2 \text{ m}$
$h = 72 \text{ cm}$
$= 0{\cdot}72 \text{ m}$
$\therefore A = 1{\cdot}2 \times 0{\cdot}72$
$= 0{\cdot}864 \text{ m}^2$

OR

$b = 120 \text{ cm}$
$h = 72 \text{ cm}$
$A = 120 \times 72$
$= 8640 \text{ cm}^2$

b $A = \dfrac{\theta}{360} \times \pi r^2$
$= \dfrac{150}{360} \times \pi \times 8{\cdot}2^2$
$= 88 \text{ cm}^2$ (2 sig. fig.)

Make sure all dimensions are measured in the same units.

P GEOGEBRA ACTIVITY 5:01 AREA FORMULAS

In this activity you can test your knowledge of the basic area formulas. After calculating the areas of given figures, the figures can be changed. A checkbox is available to show the area calculations.

Exercise 5:01

1 Find the area of squares that have the following sides.

a 7 cm **b** 12·7 m **c** 0·75 m

2 Find the area of rectangles that have the following lengths and breadths.

a $l = 15$ m, $b = 4$ m **b** $l = 17$ cm, $b = 5{\cdot}2$ cm **c** $l = 25{\cdot}6$ m, $b = 7{\cdot}9$ m

d $l = 1{\cdot}5$ m, $b = 50$ cm **e** $l = 54$ cm, $b = 2{\cdot}2$ m **f** $l = 7{\cdot}1$ cm, $b = 25$ mm

3 Find the area of circles that have the following radii or diameters. Give your answers correct to two decimal places.

a $r = 6$ cm **b** $r = 3{\cdot}5$ cm **c** $r = 2{\cdot}6$ m

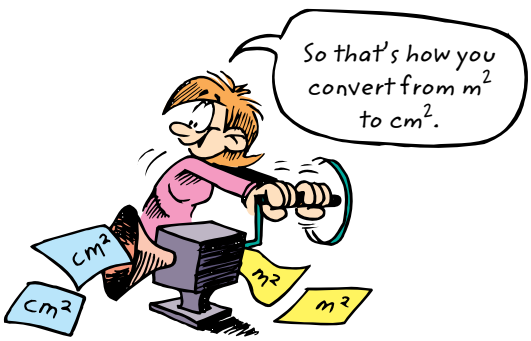
d $d = 7$ m **e** $d = 9{\cdot}1$ m **f** $d = 0{\cdot}9$ m

4 **a** Find the area of a square with side 1 m. (Answer in m^2.)

b Find the area of a square with side 100 cm. (Answer in cm^2.)

c Are the squares in parts **a** and **b** the same size?

d From your answers to parts **a**, **b** and **c** complete: $1\ m^2 = \ldots\ cm^2$

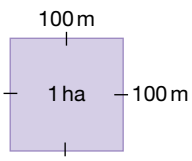


5 **a** A square of side 100 m has an area of 1 hectare (ha). How many square metres does it take to make 1 ha?

b A rectangle has a length of 700 m and a width of 300 m. What is the area of this rectangle in hectares?

c A rectangle is 750 m long and 420 m wide. Find its area in hectares.

d The length and width of a rectangle are 220 m and 75 m respectively. Calculate the area of this rectangle in square metres and hectares.

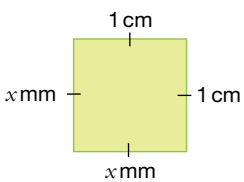


6 **a** What is the value of x?

b Calculate the area of the square in cm^2.

c Calculate the area of the square in mm^2.

d Complete the conversion: $1\ cm^2 = \ldots\ mm^2$

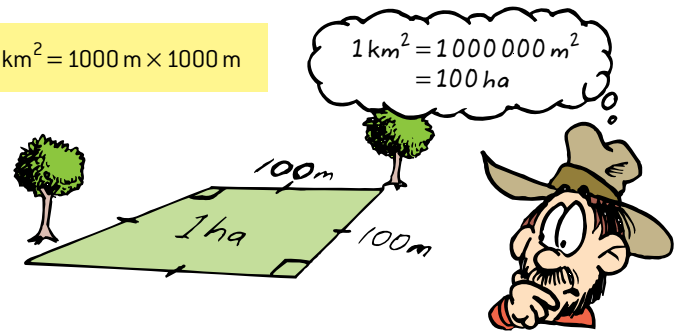


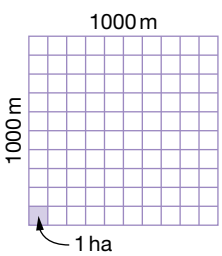
7 Complete the following.

a $1\ km^2 = \ldots\ m^2$

b $1\ km^2 = \ldots$ ha

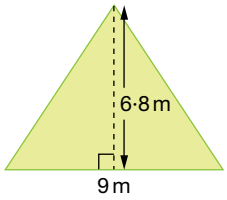
$1\ km^2 = 1000\ m \times 1000\ m$



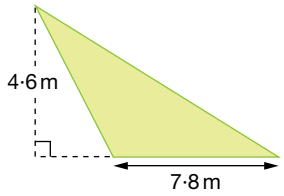


8 Calculate the areas of the following figures.

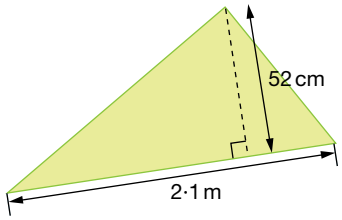
a



b

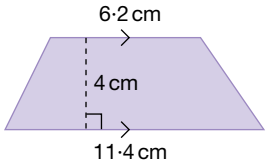


c

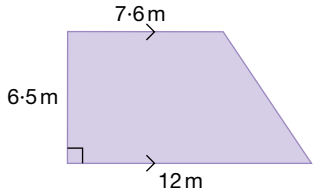


9 Find the area of each of the following trapeziums.

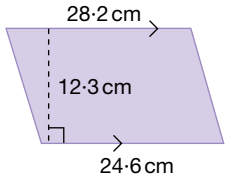
a



b

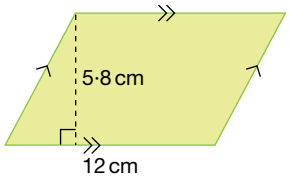


c

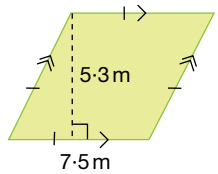


10 Calculate the area of each of the following figures.

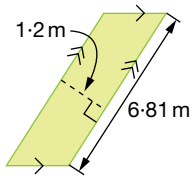
a



b

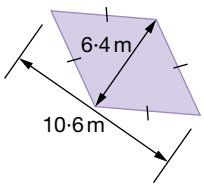


c

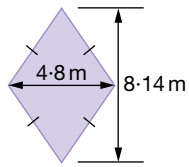


11 Find the area of each rhombus or kite.

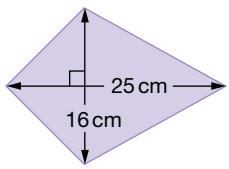
a



b

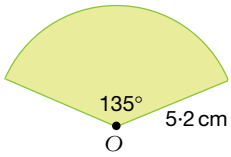


c

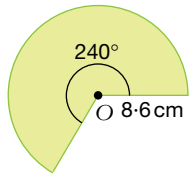


12 Find the area of each of the following sectors, correct to one decimal place.

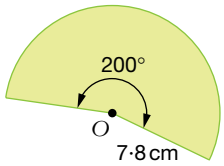
a



b

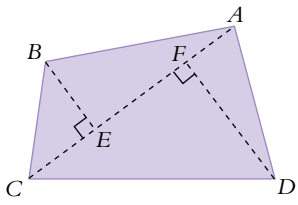


c



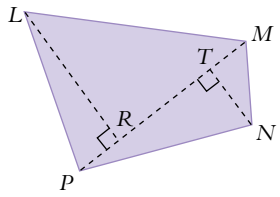
13 Work out the area of each of the following quadrilaterals.

a



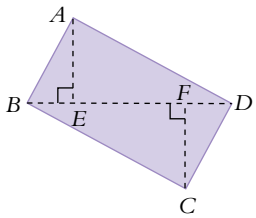
$AC = 22\,\text{cm}$
$BE = 6\,\text{cm}$
$FD = 16\,\text{cm}$

b



$MP = 16{\cdot}8\,\text{m}$
$LR = 7{\cdot}4\,\text{m}$
$TN = 2{\cdot}3\,\text{m}$

c



$BD = 8{\cdot}6\,\text{m}$
$AE = 3{\cdot}7\,\text{m}$
$CF = 6{\cdot}85\,\text{m}$

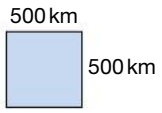
CHALLENGE 5:01 AREAS OF SCALE DRAWINGS

1 The map below has a scale of 1 cm = 500 km. On the map, what area is represented by 1 cm^2?

2 Estimate the area of each of the South American countries shown on this map.

3 Estimate the distances across Brazil and Argentina from east to west and from north to south.

4 Find a map of Australia and use its scale to find the area.



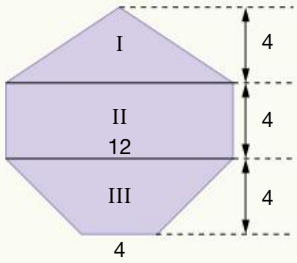
5:02 Areas of composite figures

PREP QUIZ 5:02

Three shapes have been used to form a heptagon.

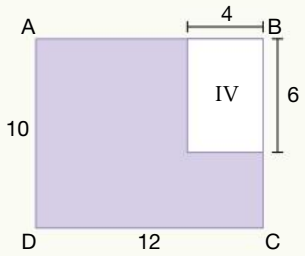
What is the name of:

1 shape I
2 shape II
3 shape III?



Find the area of:

4 shape I
5 shape II
6 shape III
7 the heptagon.



What is the area:

8 of rectangle ABCD
9 of rectangle IV
10 shaded purple?

The area of composite figures can be calculated by either of two methods.

Method 1 (by addition of parts)

We imagine that smaller regular figures have been joined to form the figure, as in Figures 1 and 2.

1 Copy the figure.
2 Divide the figure up into simpler parts.
Each part is a shape whose area can be calculated directly.
e.g. square or rectangle
3 Calculate the area of the parts separately.
4 Add the area of the parts to give the area of the figure.

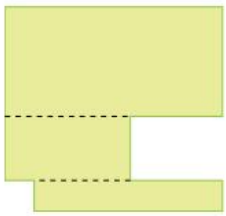


Figure 1

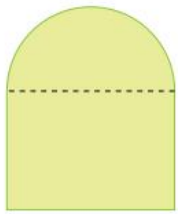
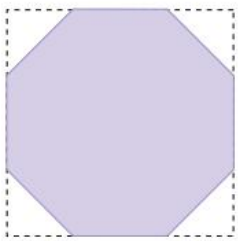


Figure 2

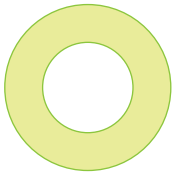
Method 2 (by subtraction)

We imagine the figure is formed by cutting away simple shapes from a larger complete figure, as shown.

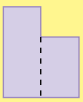
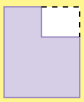
1 Copy the figure and mark in the original larger figure from which it has been cut.
2 Calculate the area of the larger original figure.
3 Calculate the area of the parts that have been removed.
4 Area of figure = (area of original figure) − (area of parts that have been removed).



This method is also used when the inside of a figure is removed. The figure shown is an **annulus**. To find the area we subtract the area of the small circle from the area of the large circle.



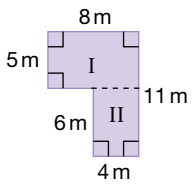
Think carefully before deciding which method to use.

Addition  OR Subtraction 

WORKED EXAMPLE 1

Calculate the shaded area of each figure.

a

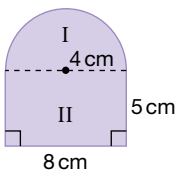


Area of rectangle I
$= 8 \times 5$
$= 40\,\text{m}^2$

Area of rectangle II
$= 6 \times 4$
$= 24\,\text{m}^2$

$\therefore$ Area of figure
$= 40 + 24$
$= 64\,\text{m}^2$

b

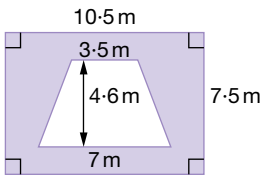


Area of semicircle I
$= \frac{1}{2}\pi r^2$
$= \frac{1}{2} \times \pi \times 16$
$= 25{\cdot}12\,\text{cm}^2$

Area of rectangle II
$= 8 \times 5$
$= 40\,\text{cm}^2$

$\therefore$ Area of figure
$= 40 + 25{\cdot}12$
$= 65{\cdot}12\,\text{cm}^2$ (2 dec. pl.)

c

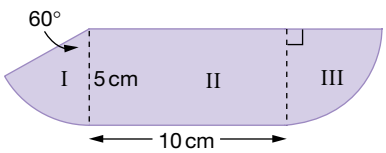


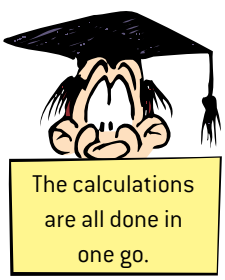
Area of rectangle
$= 10{\cdot}5 \times 7{\cdot}5$
$= 78{\cdot}75\,\text{m}^2$

Area of trapezium
$= \frac{1}{2} \times 4{\cdot}6 \times (3{\cdot}5 + 7)$
$= 24{\cdot}15\,\text{m}^2$

$\therefore$ Shaded area
$= 78{\cdot}75 - 24{\cdot}15$
$= 54{\cdot}6\,\text{m}^2$

d Calculate the shaded area of the figure.





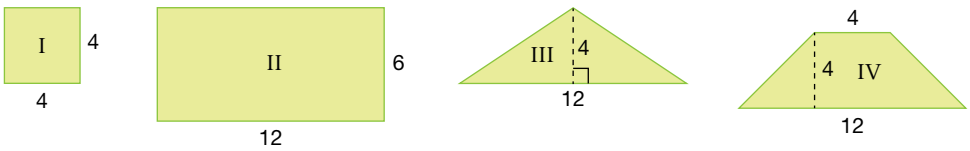
Area of figure = area of sector I + area of rectangle II + area of quadrant III
$= (\frac{60}{360} \times \pi \times 5^2) + (5 \times 10) + (\frac{1}{4} \times \pi \times 5^2)$
$= 82{\cdot}7\,\text{m}^2$ (1 dec. pl.)

Exercise 5:02

P Foundation worksheet 5:02 Area

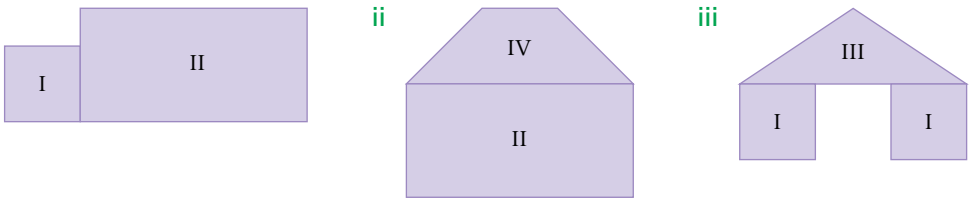
1 a Which of the shapes below have the following area?

i 24 unit^2 ii 72 unit^2 iii 16 unit^2 iv 32 unit^2



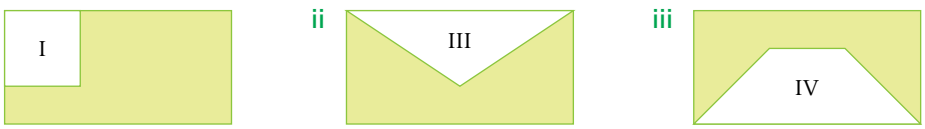
b Each of the following figures has been made from the shapes I to IV above. Calculate the area of each figure by adding the area of the appropriate shapes.

i ii iii



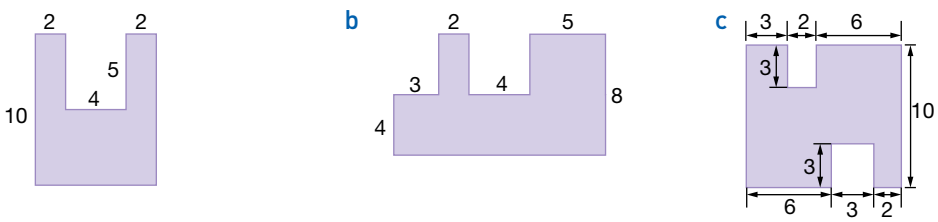
c The following shapes have been made by cutting one of the other shapes above from shape II. What is the area of the coloured region?

i ii iii



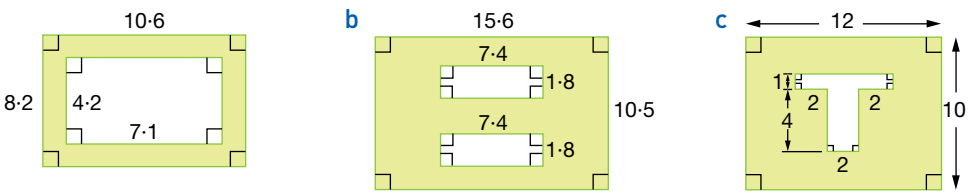
2 Calculate the area of each of the following figures. All measurements are in centimetres and all angles are right angles.

a b c



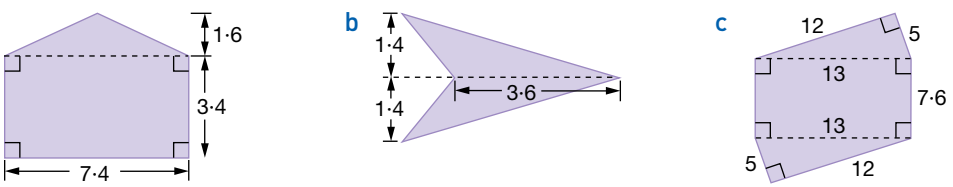
3 Calculate the shaded area of the following figures. All measurements are in metres.

a b c

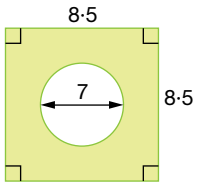


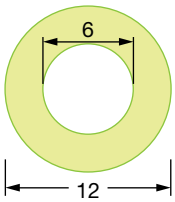
4 Calculate the area of each figure. All measurements are in metres.

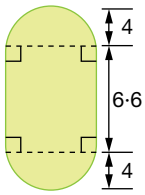
a b c



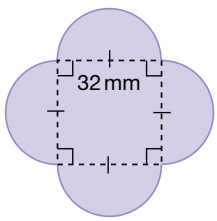
5 Calculate the shaded area of each of the following figures. All measurements are in centimetres. Give answers correct to two decimal places.

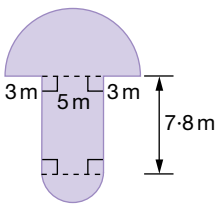
a 

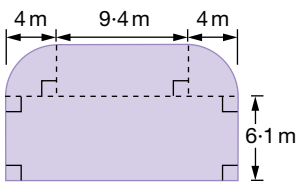
b 

c 

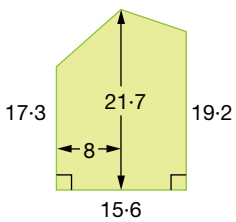
6 Find the area of each of the following composite figures. Give answers correct to one decimal place.

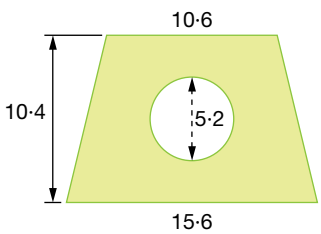
a 

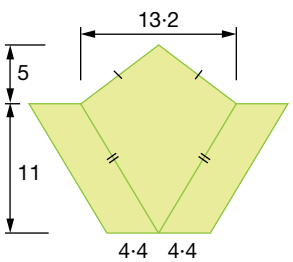
b 

c 

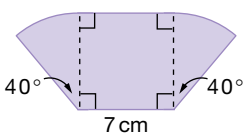
7 Calculate the shaded area of the following figures. All measurements are in metres. Give answers correct to one decimal place, where necessary.

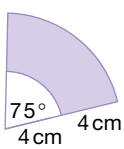
a 

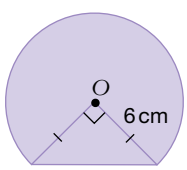
b 

c 

8 Find the shaded area of these figures, correct to the nearest square centimetre.

a 

b 

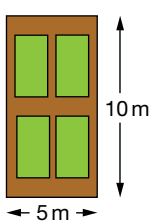
c 

9 The diagram shows four garden beds, each 3·4 m long and 1·8 m wide, surrounded by paving bricks. Find:

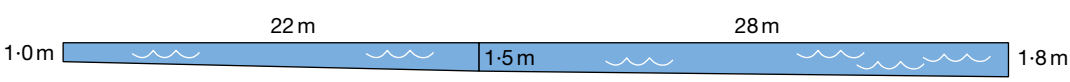
a the area of the garden beds

b the paved area

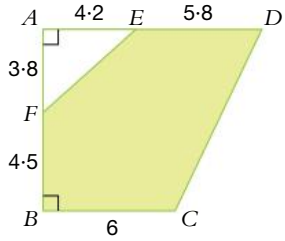
c the number of bricks needed to pave the area (rounding up to the next hundred) if a brick is 225 mm by 112 mm.



10 The diagram shows the wall of a swimming pool that is made from two trapeziums. Calculate the area of this wall.

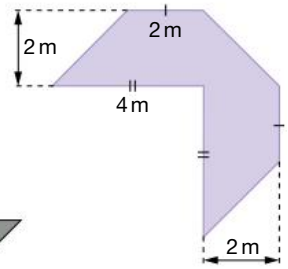


11 In the diagram shown, *BCDEF* is to be covered with tiles at a cost of $90 per square metre. (All measurements are in metres.)



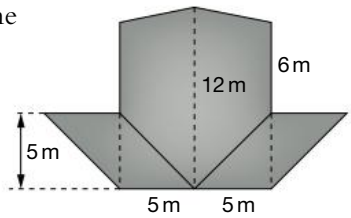
- **a** Calculate the areas of:
  - **i** trapezium *ABCD*
  - **ii** triangle *AEF*.
- **b** Use the answers from part **a** to find the area of *BCDEF*.
- **c** What is the cost of tiling the floor?

12 The diagram shows a balcony built on the corner of an apartment block. Calculate the area of the balcony.



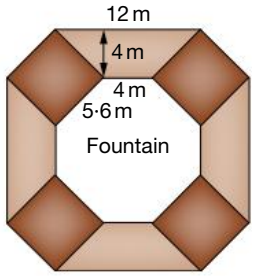
13 A metal sign is being made to attach to the side of a large building. Calculate:

- **a** the area of the sign in square metres
- **b** the cost of the metal and construction of the sign at $100 per square metre.

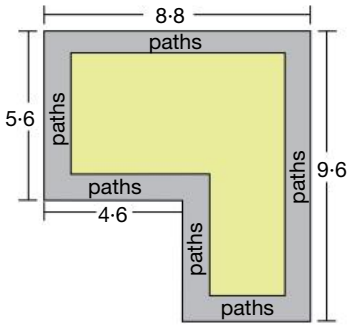


14 A fountain is to be surrounded by a paved area consisting of bricks of two different colours. One colour will form squares, whereas the other will form trapeziums. Calculate:

- **a** the area of concrete needed to form the base of the fountain
- **b** the area of bricks needed to make the squares
- **c** the area of bricks needed to make the trapeziums
- **d** the total cost of the job if the paving costs $60 per m^2 and the concreting costs $50 per m^2.

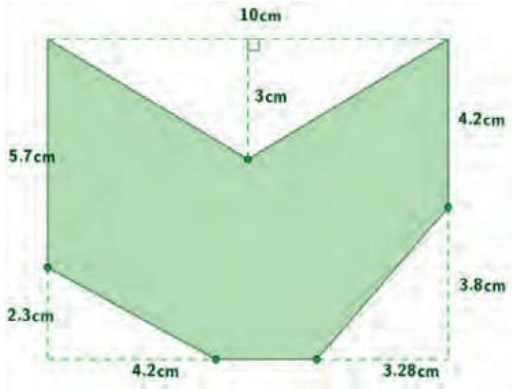


15 Find the cost of concreting the paths in the diagram if concreting costs $50 per m^2. All paths are 0·9 m wide and all measurements are in metres.



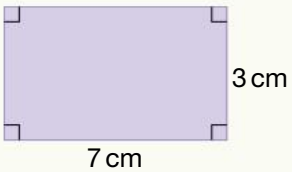
P GEOGEBRA ACTIVITY 5:02 AREAS OF COMPOSITE FIGURES

In this activity you can work through a series of diagrams of increasing complexity. Each of the diagrams can be changed by clicking and dragging points. Work out the area of each figure you have made and then check the answer by ticking a checkbox.

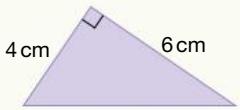


5:03 Surface area of a prism

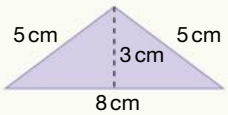
PREP QUIZ 5:03

1 

Area = …

2 

Area = …

3 

Area = …

For a cube:

4 How many faces are there?

5 What shape are the faces?

For a rectangular prism:

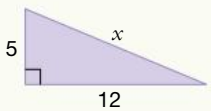
6 How many faces are there?

7 What shape are the faces?

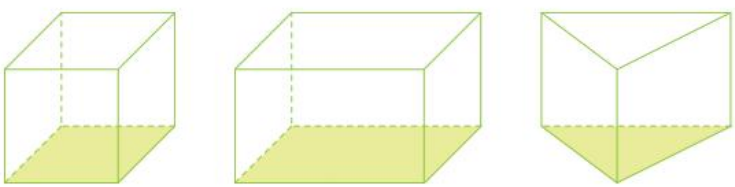
8 Which faces are congruent?

9 How many faces has a triangular prism?

10 Use Pythagoras' theorem to find x in this triangle.



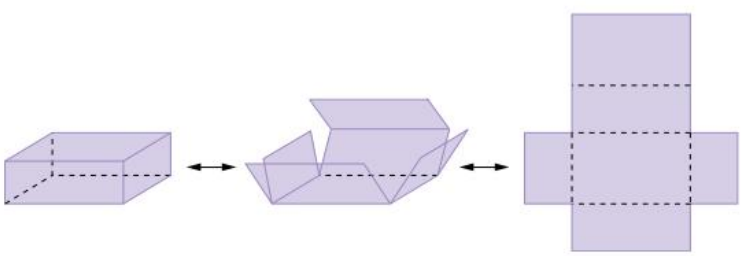
If we look at solid shapes such as those pictured below, we can see that the faces of these solids are plane shapes.



The surface area of a solid is the sum of the areas of its faces.

To calculate the surface area of a solid, you must know the number of faces and the shapes of the faces.

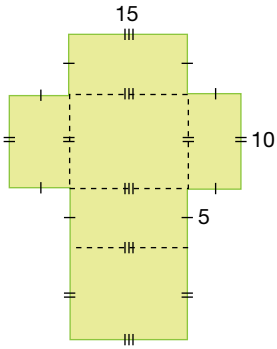
A net of a solid is a two-dimensional drawing that shows how the faces of the solid would appear if they were unfolded to make a two-dimensional shape. As the net consists of all the faces of the solid, the area of the net is equal to the surface area of the solid.



WORKED EXAMPLE 1

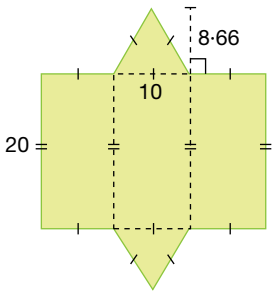
Use the nets to find the surface area of the following prisms. (All measurements are in centimetres).

a



Surface area
= area of net
$= 2 \times (10 \times 5) + 2 \times (15 \times 5) + 2 \times (15 \times 10)$
$= 550\,\text{cm}^2$

b

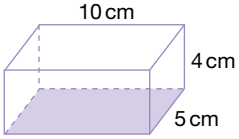


Surface area
= area of net
$= 3 \times (10 \times 20) + 2 \times \frac{1}{2} \times (10 \times 8{\cdot}66)$
$= 686{\cdot}6\,\text{cm}^2$

WORKED EXAMPLE 2

Find the surface area of the following prisms.

a



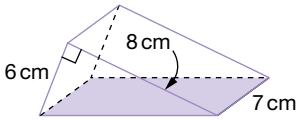
Area of top and bottom $= (10 \times 5) + (10 \times 5)$
$= 50 + 50$
$= 100\,\text{cm}^2$

Area of sides $= (5 \times 4) + (5 \times 4)$
$= 20 + 20$
$= 40\,\text{cm}^2$

Area of front and back $= (10 \times 4) + (10 \times 4)$
$= 40 + 40$
$= 80\,\text{cm}^2$

$\therefore$ Surface area $= 100 + 40 + 80$
$= 220\,\text{cm}^2$

b



Calculate the width of the bottom using Pythagoras' theorem.

Now $x^2 = 6^2 + 8^2$
$\therefore x^2 = 100$
$\therefore x = 10$

Area of front $= \dfrac{6 \times 8}{2}$
$= 24\,\text{cm}^2$

$\therefore$ Area of back $= 24\,\text{cm}^2$

Area of sides $= (8 \times 7) + (6 \times 7)$
$= 56 + 42$
$= 98\,\text{cm}^2$

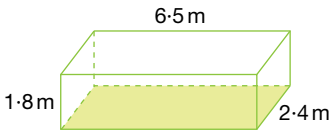
Area of bottom $= 10 \times 7$
$= 70\,\text{cm}^2$

$\therefore$ Surface area $= 24 + 24 + 98 + 70$
$= 216\,\text{cm}^2$

WORKED EXAMPLE 3

Find the surface area of a rectangular prism that is 6·5 m long, 2·4 m wide and 1·8 m high, if the prism is:

a open **b** closed



Solutions

a Surface area of closed prism

$= 2 \times (6{\cdot}5 \times 1{\cdot}8) + 2 \times (2{\cdot}4 \times 1{\cdot}8) + 2 \times (6{\cdot}5 \times 2{\cdot}4)$

(front and back) + (sides) + (top and bottom)

$= 23{\cdot}4 + 8{\cdot}64 + 31{\cdot}2$

$= 63{\cdot}24\ \text{m}^2$

b Surface area of open prism

$= 2 \times (6{\cdot}5 \times 1{\cdot}8) + 2 \times (2{\cdot}4 \times 1{\cdot}8) + (6{\cdot}5 \times 2{\cdot}4)$

(front and back) + (sides) + (bottom)

$= 23{\cdot}4 + 8{\cdot}64 + 15{\cdot}6$

$= 47{\cdot}64\ \text{m}^2$

Closed prisms have a top.
Open prisms have no top.

Tips on finding surface area

1. Make a sketch of the solid. Find all necessary dimensions.
2. Calculate the area of the faces. Be systematic. Do front and back, top and bottom, and then the sides.
3. Add the area of the faces.
4. Check to make sure that no face has been left out.

Exercise 5:03

P Foundation worksheet 5:03 Surface area of prisms

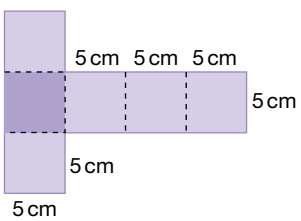
1 For each of the following nets find:

i its area

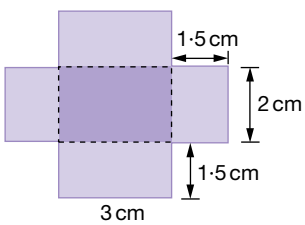
ii the name of the solid that it produces when folded along the dotted lines

iii whether the prism is open or closed.

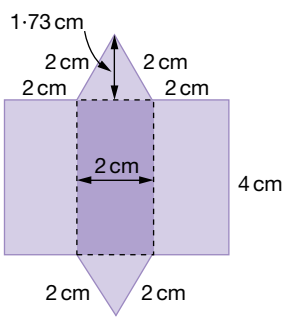
a



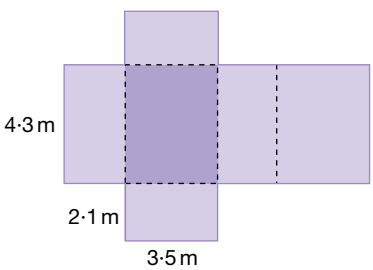
b



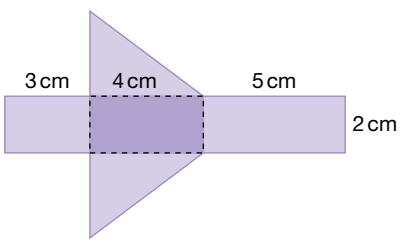
c



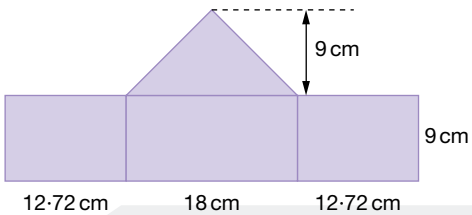
d



e

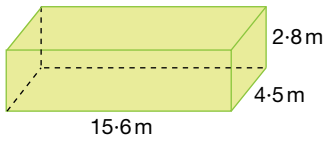


f



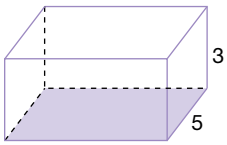
2 Copy and complete the following for the rectangular prism on the right.

Area of top and bottom = …
Area of sides = …
Area of front and back = …
Total surface area = …

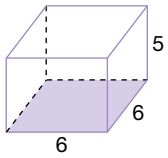


3 Find the surface area of each of the following rectangular prisms.
All measurements are in centimetres.

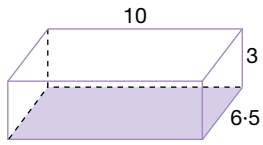
a



b

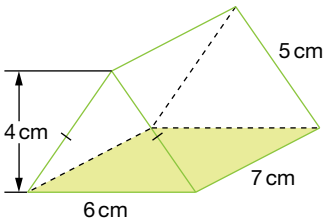


c

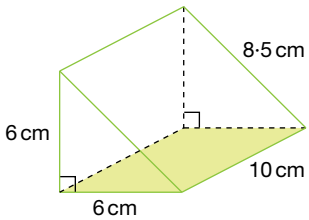


4 Find the surface area of each of these triangular prisms.

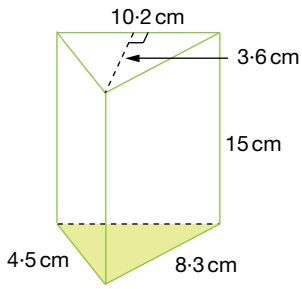
a



b



c

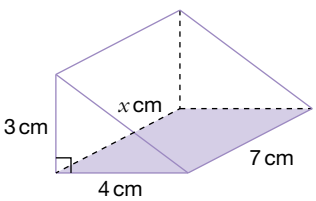


5 Find the surface areas of the following prisms.

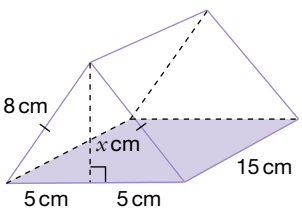
a cube with an edge length of 10 cm
b rectangular prism 25 cm long, 20 cm wide and 15 cm high
c square prism with dimensions 2 m, 2 m and 0·75 m

6 In each of the following, use Pythagoras' theorem to calculate the unknown length, x, correct to two decimal places, and then calculate the surface area of the solid.

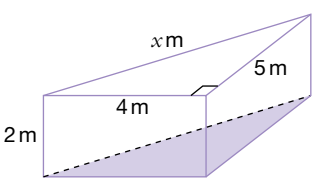
a



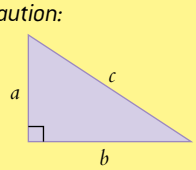
b



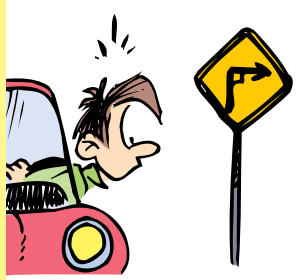
c



Caution:

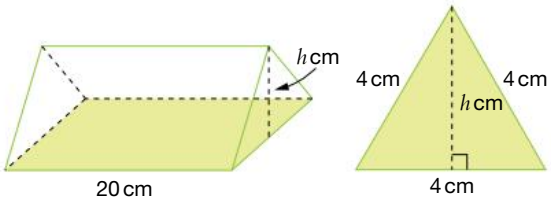


$c^2 = a^2 + b^2$
Right-angled triangles!

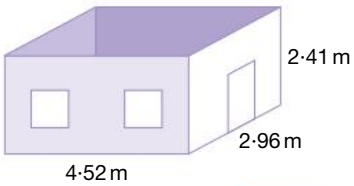


7 A lap pool is in the shape of a rectangular prism. It is 25 m long, 10 m wide and has a constant depth of 1·5 m. Find the surface area of the pool and the cost of tiling it at $60 per square metre.

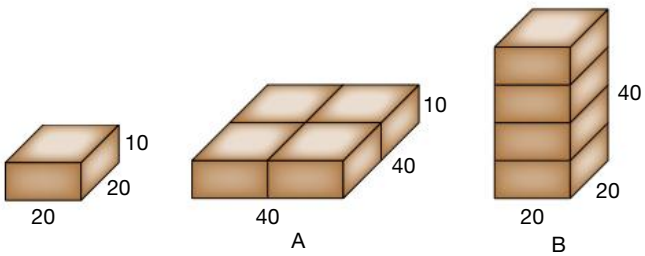
8 A chocolate package is in the shape of a triangular prism. The triangle is equilateral with a side edge of 4 cm. What is the area of cardboard used to make the package? (*Note:* You will need to use Pythagoras' theorem to find the value of *h*.)



9 The diagram shows a room in the shape of a rectangular prism. The room has two windows and a door that have a combined area of 5·52 m^2. Calculate the area of the internal walls that need to be painted.



10 Four small cartons are to be stacked and wrapped in cardboard for shipping. The small carton is a square prism with dimensions 20 cm, 20 cm and 10 cm. Which of the two stacks shown will use the smaller amount of cardboard?

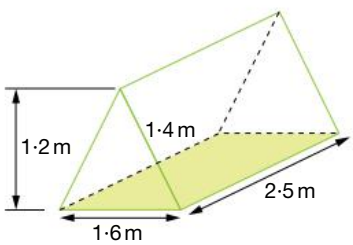


11 A tent has the shape of a triangular prism with dimensions as shown in the diagram.

a Find the area of material needed to make this tent. (Include the floor area.)

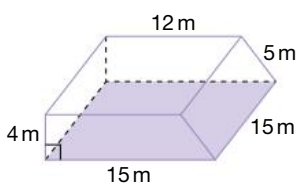
b If the material comes in rolls that are 3·7 m wide, what length of material must be purchased so that the tent can be made without any joins except those at the edges? (Hint: Consider the net of the solid.)

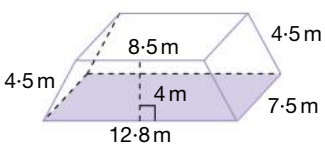
c If special joining tape is needed to strengthen each join, what length of tape will be needed?

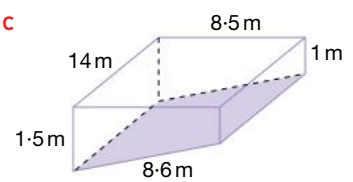


12 Draw a net or complete the table shown below, to find the surface area of the following trapezoidal prisms. (*Note:* trapezoidal prisms have two trapezoidal and four rectangular faces.)

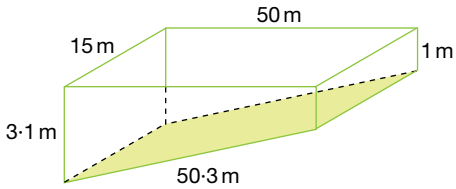
Face	Trapezium	Rectangle A	Rectangle B	Rectangle C	Rectangle D
Area					
Surface area = 2 × area of trapezium + area of rectangles A, B, C and D = …					

a


b


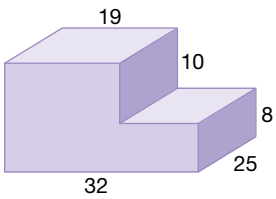
c


13 A swimming pool has the shape of a trapezoidal prism as shown in the diagram. If this pool is to be tiled, find the cost of tiling it at $45 per square metre.

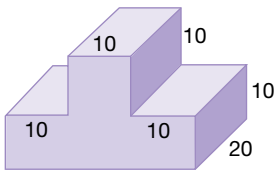


14 Find the surface areas of the following prisms. All measurements are in centimetres.

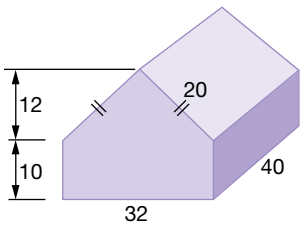
a



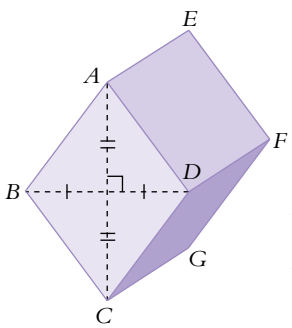
b



c

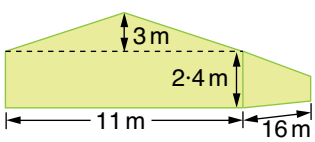


d



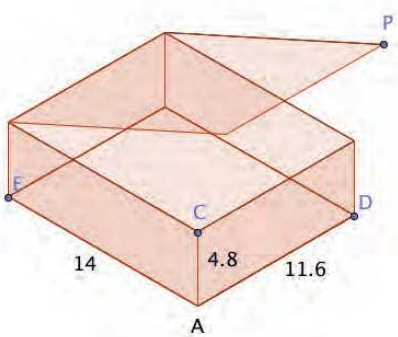
$AC = 4{\cdot}8$
$BD = 3{\cdot}6$
$AE = 5{\cdot}2$

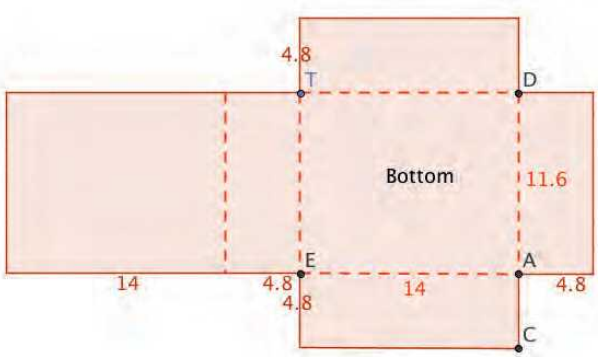
15 A marquee is in the shape of a pentagonal prism. Use the dimensions shown to calculate the surface area. (There is no floor.)



P GEOGEBRA ACTIVITY 5:03 SURFACE AREA OF A RECTANGULAR PRISM

Use a 3D view and the net of a prism to calculate its surface area. Tick a checkbox to show the solution. Click and drag points to change the dimensions of the prism.





FUN SPOT 5:03

HOW DID THE BOY KNOW HE HAD AN AFFINITY WITH THE SEA?

Answer each question and write the letter for that question in the box above the correct answer.

A $7^2 + 6^2$

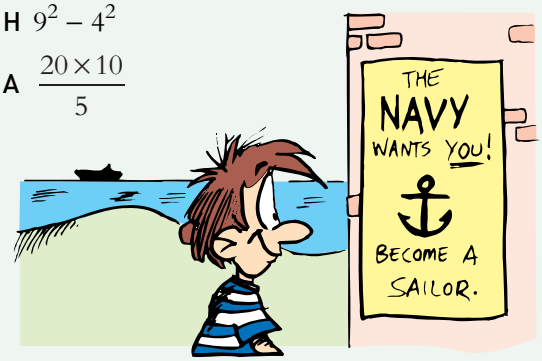
T $\sqrt{13^2 - 5^2}$

E $\sqrt{5^2 + 12^2}$

H $\dfrac{20 + 10}{5}$

H $9^2 - 4^2$

A $\dfrac{20 \times 10}{5}$



Complete the following.

W 3·5 m = … cm

E 0·7 cm = … mm

N 0·07 L = … mL

C 1 t = … g

I 1 cm^2 = … mm^2

A 20 000 m^2 = … ha

H 8700 kg = … t

T 0·5 min = … s

D 1 g = … mg

E 2 cm^3 = … mm^3

A rectangular prism has dimensions of 3, 4 and 5.

E What is the volume of the prism?

W What is the surface area of the prism?

Figures A, B and C have been formed by cutting rectangular pieces from a rectangle that is 30 cm long and 18 cm wide.

Find the perimeters of:

O Figure A

T Figure B

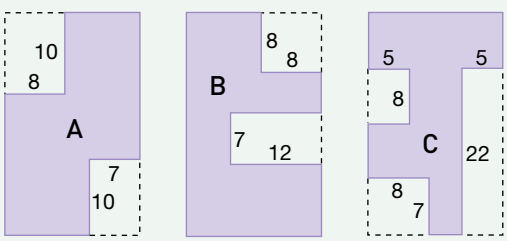
W Figure C

Find the areas of:

E Figure A

S Figure B

V Figure C

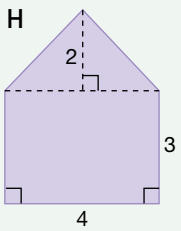


106	8·7	13	18	16	7	392	85	350	12	65	390

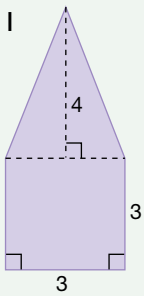
96	10^6	60	2	70	15	30	94	14	334	2000	10^3	40	120	6	10^2	10

Calculate the area of each pentagon below.

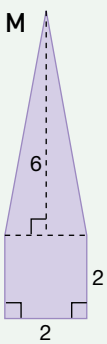
H



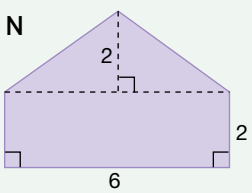
I



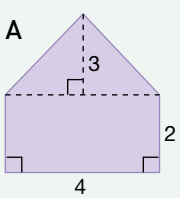
M



N



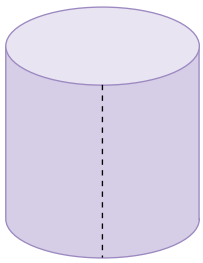
A

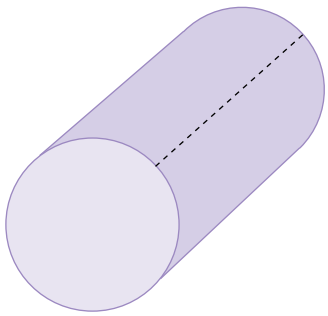


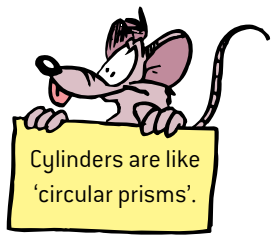
5:04 Surface area of a cylinder

In all of the solids encountered so far the faces have been plane figures such as squares, rectangles, triangles and trapeziums.

With the cylinder, this is no longer the case. The cylinder's surface area is made up of a curved surface and two circles.

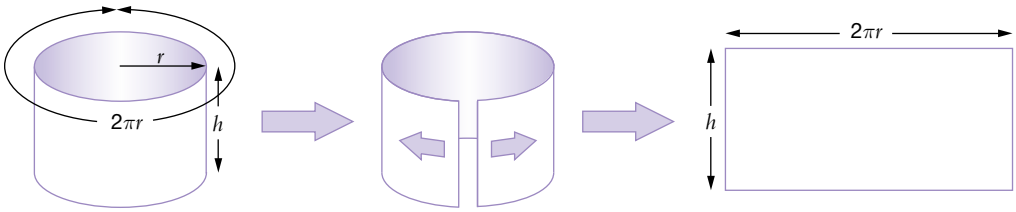






To calculate the area of the curved surface imagine that the cylinder is hollow. If we cut the curved surface along the dotted line and flattened it out it would form a rectangle.

The area of this rectangle would be the same as the area of the curved surface.

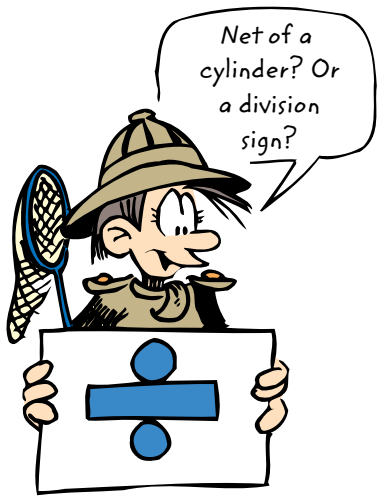


From the series of diagrams above, we see that the curved surface area is equivalent to a rectangle that has a length equal to the circumference of the circle and a width equal to the height of the cylinder. Using the formula for the area of a rectangle we obtain the following formula.

Curved surface area $= 2\pi rh$

To find the surface area of the cylinder we add the area of the two circular ends.

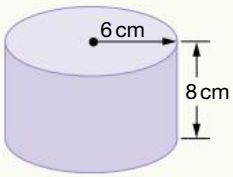
Surface area = curved surface area + area of circles
$= 2\pi rh + 2\pi r^2$

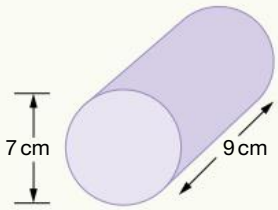


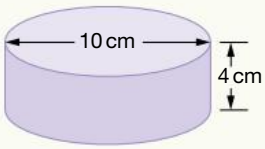
PREP QUIZ 5:04

1 Evaluate πr^2, correct to two decimal places, if $r = 1{\cdot}6$.
2 Evaluate $2\pi rh$, correct to three significant figures, if $r = 15{\cdot}6$ and $h = 3{\cdot}8$.
3 If the diameter of a circle is 16 cm, what is the radius?
4 If the radius of a circle is 6·5 cm, what is the diameter?

Questions **5–10** refer to these cylinders.





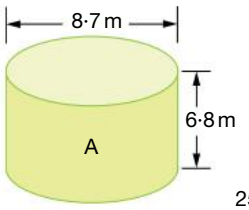


5 Radius = …
6 Height = …
7 Radius = …
8 Height = …
9 r = …
10 h = …

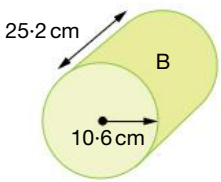
WORKED EXAMPLES

1 Find the surface area of a cylinder that has a radius of 8 cm and a height of 9·5 cm. Give your answer correct to two decimal places.

2 For cylinder A, find:
a the curved surface area
b the area of the circular ends
c the surface area.
Give the answers correct to three significant figures.



3 Find the curved surface area of cylinder B, correct to one decimal place.



Solutions

1 Surface area $= 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 8^2 + 2 \times \pi \times 8 \times 9{\cdot}5$
$= 879{\cdot}65\text{ cm}^2$ (2 dec. pl.)

2 **a** Curved surface area
$= 2\pi rh$
$= 2\pi \times 4{\cdot}35 \times 6{\cdot}8$
$= 59{\cdot}16\pi$
$= 186\text{ m}^2$ (3 sig. fig.)

b Area of circular ends
$= 2\pi r^2$
$= 2\pi \times (4{\cdot}35)^2$
$= 37{\cdot}845\pi$
$= 119\text{ m}^2$ (3 sig. fig.)

c Surface area
$= 59{\cdot}16\pi + 37{\cdot}845\pi$
$= 305\text{ m}^2$ (3 sig. fig.)

3 Curved surface area
$= 2\pi rh$
$= 2 \times \pi \times 10{\cdot}6 \times 25{\cdot}2$
$= 1678{\cdot}4\text{ cm}^2$ (1 dec. pl.)

Exercise 5:04

P Foundation worksheet 5:04
Perimeter and area

1 Find the area of the curved surfaces of the following cylinders. Give your answers correct to two decimal places.

a radius = 8 cm, height = 10 cm
b radius = 4 cm, height = 15 cm
c radius = 6·5 cm, height = 11·3 cm
d radius = 16·3 m, height = 8·3 m

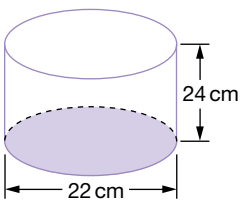
2 Find the curved surface areas of the following cylinders. Give your answers correct to three significant figures.

a diameter = 20 cm, height = 7 cm
b diameter = 10 m, height = 5·6 m
c diameter = 3·6 m, height = 1·8 m
d diameter = 15·7 cm, height = 18·6 cm

3 For each of the following cylinders find, correct to two decimal places:

i the curved surface area
ii the area of the circular ends
iii the total surface area.

a



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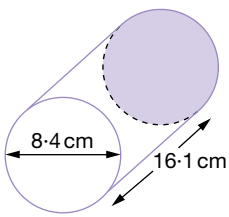

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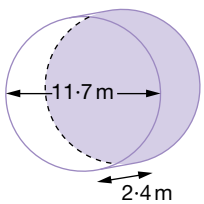

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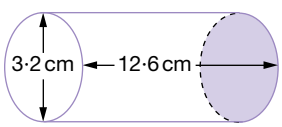

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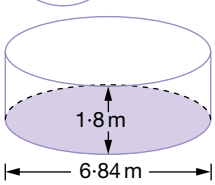

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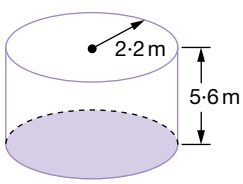


4 Find the surface areas of the following cylinders, giving your answers correct to one decimal place.

a radius = 6 cm, height = 9 cm
b radius = 4·8 m, height = 1·8 m
c radius = 2·1 m, height = 10·1 m
d diameter = 10 m, height = 11·4 m
e diameter = 14·6 cm, height = 21·5 cm
f diameter = 1·6 m, height = 4·2 m
g radius = 50 cm, height = 1·5 m
h diameter = 2·4 m, height = 750 mm

Remember:
All measurements must be in the same units.

5 A swimming pool is cylindrical. It has a diameter of 7 m and a depth of 1·2 m. Find the area of the curved surface. Give your answer correct to one decimal place.

6 A cylindrical paint tin has a diameter of 25 cm and a height of 30 cm. Calculate the area of metal needed to make the tin, including the lid.

7 A silo is cylindrical. It is 25 m high and has a diameter of 7·4 m. Calculate the cost of spraying the curved surface of the silo with a spray-on concrete mixture if it costs $30 per square metre.

8 A cylindrical water tank (with top) has a radius of 2·1 m and a height of 3·2 m. The tank is to be painted inside and outside with rust-preventive paint. Find:

a the total area to be painted, correct to one decimal place

b the cost of the paint, if it is $25 per square metre.

9 Ten concrete cylinders are used as fishponds. Each cylinder is 1·5 m high and has an internal diameter of 2 m. The inside of each cylinder is to be treated with a waterproofing paint. Find:

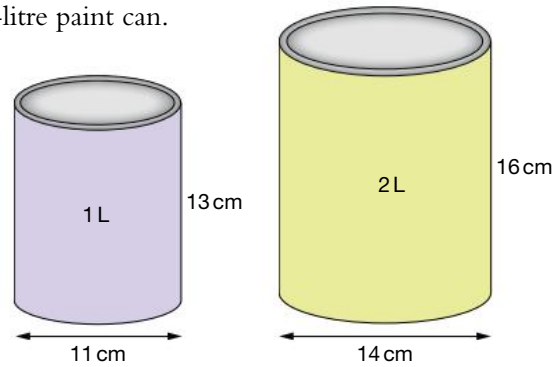
a the total area that needs to be covered, to the nearest square metre

b the cost of the treatment if the charge is $70 per square metre.

10 The diagrams show a 1-litre paint can and a 2-litre paint can.

a Calculate the surface area of each can, correct to one decimal place.

b Based on the surface area of metal used, is it cheaper to sell 2 litres of paint in one 2 L can or two 1 L cans?

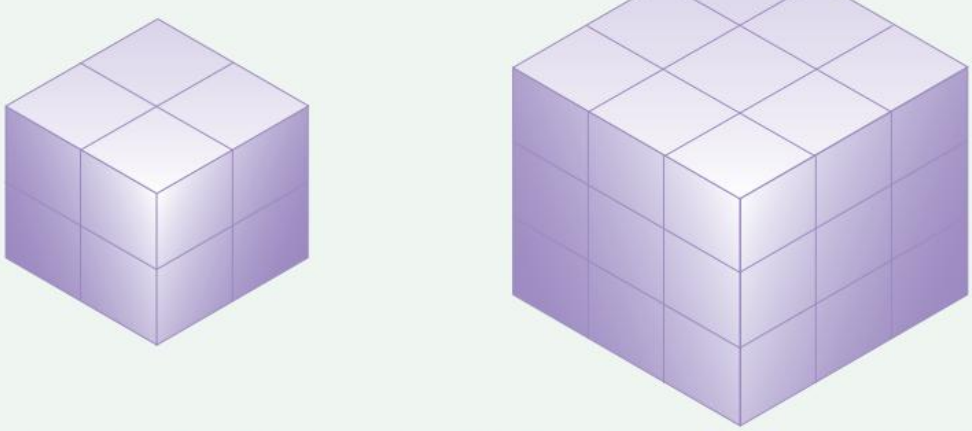


FUN SPOT 5:04 LET'S PLAY WITH BLOCKS

Eight blocks have been stacked together to form the first cube below. If the outside of the cube were painted, how many sides of each block would be painted?

How many blocks make up the second cube? If this cube were painted, how many blocks would have three sides, two sides, one side or even no sides painted?

What would be the result of painting a cube that had four blocks along each edge?



5:05 Volume of prisms and cylinders: A review

The volume of an object is the space it occupies.

The capacity of a container is the volume of liquid it can hold.

Units of volume:
- cubic millimetre (mm^3)
- cubic centimetre (cm^3)
- cubic metre (m^3)

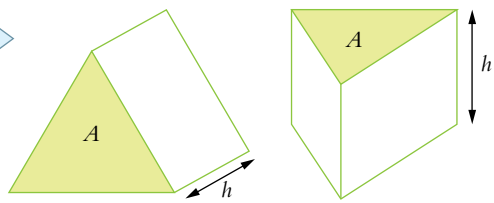
Units of capacity:
- millilitre (mL)
- litre (L)
- kilolitre (kL)

Conversion of units:
$1\,cm^3 = 1\,mL$
$1\,m^3 = 1\,kL$

In Stage 4, the following formulas were derived and used to calculate the volumes of simple prisms and cylinders. These are reviewed in the worked examples.

The volume of all prisms is given by the formula:

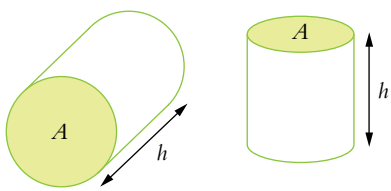
$V = Ah$, where V = volume
A = cross-sectional area
h = height of the prism



The volume of a cylinder is given by the formula:

$V = \pi r^2 h$, where r = radius of the cylinder
h = height of the cylinder

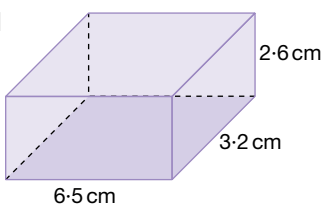
Note: For a cylinder, $A = \pi r^2$.



WORKED EXAMPLES

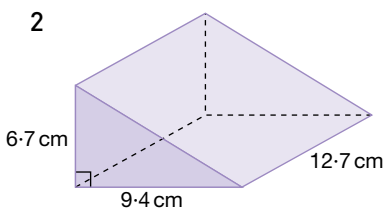
Find the volume of the following solids. Give answers correct to one decimal place.

1



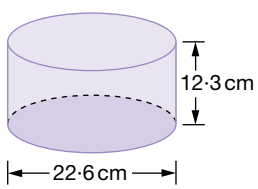
$V = Ah$
$A = 6{\cdot}5 \times 3{\cdot}2\,cm^2$
$h = 2{\cdot}6\,cm$
$\therefore\ V = 6{\cdot}5 \times 3{\cdot}2 \times 2{\cdot}6$
$= 54{\cdot}1\,cm^3$

2



$V = Ah$
$A = \dfrac{6{\cdot}7 \times 9{\cdot}4}{2}$
$= 31{\cdot}49\,cm^2$
$h = 12{\cdot}7\,cm$
$\therefore\ V = 31{\cdot}49 \times 12{\cdot}7$
$= 399{\cdot}9\,cm^3$

3

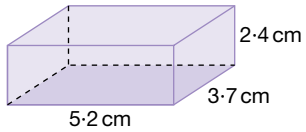


$V = \pi r^2 h$
$r = 22{\cdot}6 \div 2$
$= 11{\cdot}3\,cm$
$h = 12{\cdot}3\,cm$
$\therefore\ V = \pi \times (11{\cdot}3)^2 \times 12{\cdot}3$
$= 4934{\cdot}1\,cm^3$

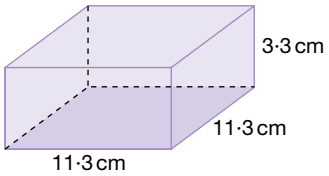
Exercise 5:05

1 Find the volume of each of the following rectangular prisms. Give answers correct to one decimal place.

a



b



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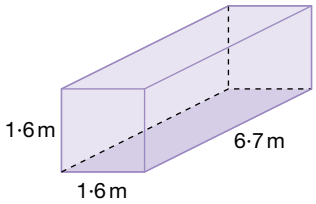
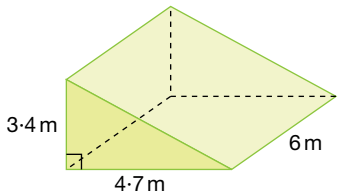


2 Calculate the volumes of the following triangular prisms.

a



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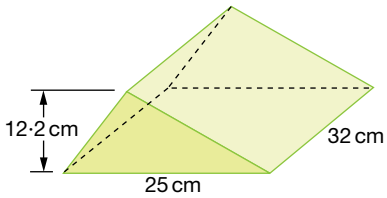

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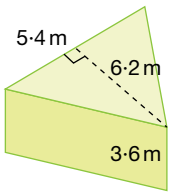

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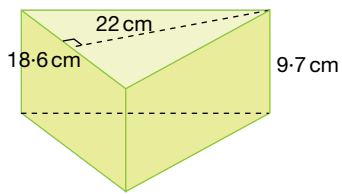

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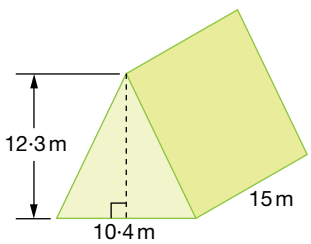

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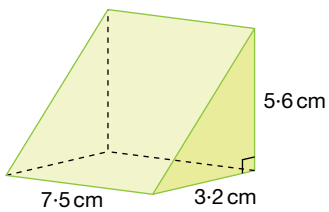
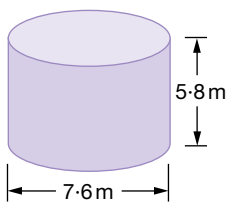


3 Calculate the volumes of the following cylinders. Give answers correct to one decimal place.

a



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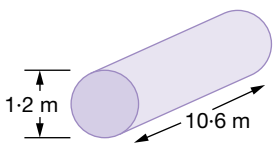

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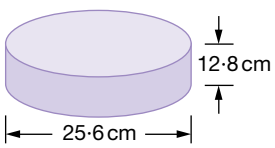
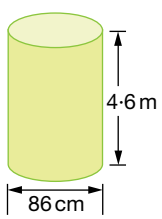


4 Calculate the volumes of the following cylinders. Give answers correct to three significant figures.

a



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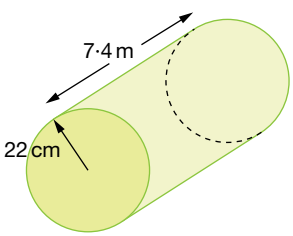

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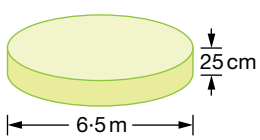
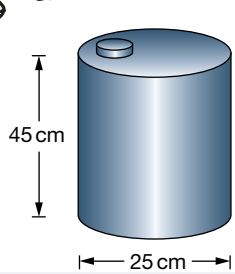


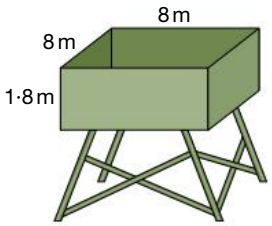
5 a Calculate the volume of the cylindrical tin, correct to the nearest cubic centimetre. (Ignore the volume of the cap.)

b Using the fact that $1\text{ cm}^3 = 1\text{ mL}$, find the capacity of the tin to the nearest litre.

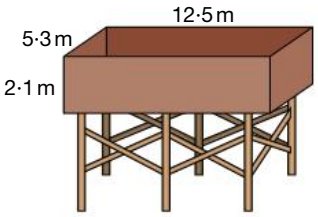


6 Find the volume of each of the tanks below in cubic metres, correct to one decimal place. Then find the capacity of each tank using $1\,m^3 = 1000$ litres.

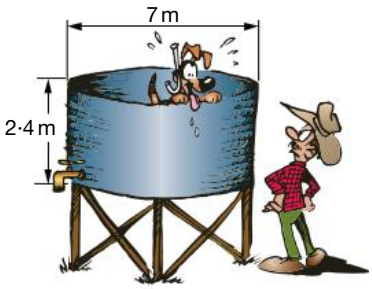
a



b



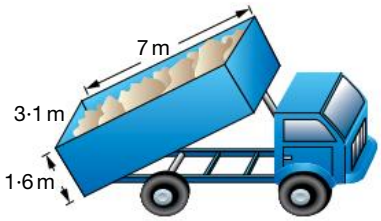
c



7 The tipper of a truck is a rectangular prism. It is 7 m long, 3·1 m wide and 1·6 m high.

a Calculate the volume of the tipper.

b If the truck carries sand and $1\,m^3$ of sand weighs 1·6 tonnes, find the weight of sand carried when the truck is three-quarters full.



8 A concrete slab is to be 3000 mm by 4000 mm by 100 mm.

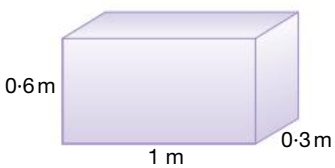
a How many cubic metres of concrete mix would be needed for the slab?

b What would be the cost of the concrete mix if it is $300 a cubic metre?

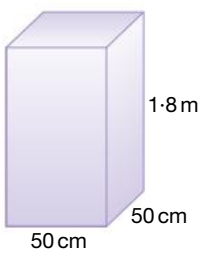
INVESTIGATION 5:05 TANK SIZES

1 Use the fact that a tank with a volume of $1\,m^3$ has a capacity of 1 kL (or 1000 L) to calculate the capacity, in litres, of the three tanks shown.

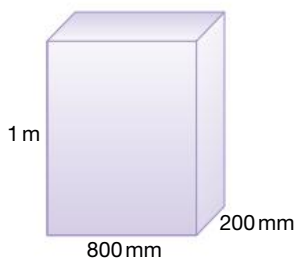
a



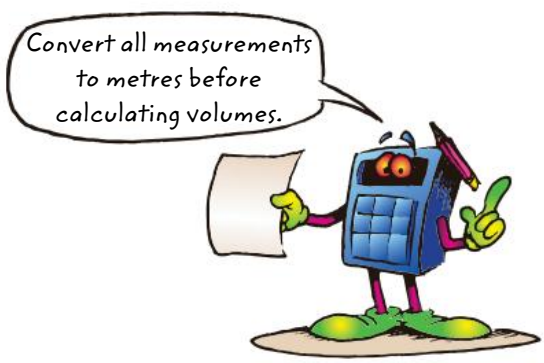
b



c



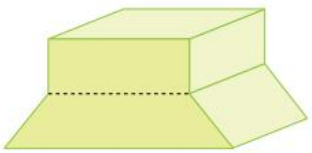
2 A tank is to hold 1000 L. Give the dimensions of three tanks with this capacity. Assume that all the tanks are rectangular prisms.



5:06 Volumes of other prisms

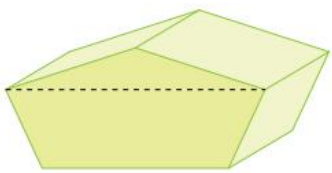
So far we have dealt with prisms that have special names like a rectangular prism, a triangular prism or a trapezoidal prism, but prisms with a cross-section that is a composite figure do not have special names.

The volumes of these prisms can be calculated by either:
- adding or subtracting the volumes of simpler prisms, or
- using the formula $V = Ah$.



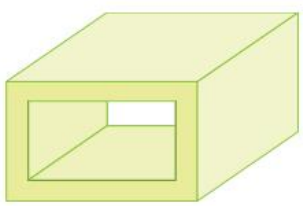
A

Prism A could have its volume calculated by adding the volume of the rectangular prism to the volume of the trapezoidal prism.



B

Prism B could have its volume calculated by adding the volume of the trapezoidal prism to the volume of the triangular prism.



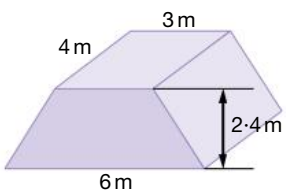
C

Prism C could have its volume calculated by subtracting the volume of the hole (a rectangular prism) from the volume of the complete solid.

WORKED EXAMPLE 1

Calculate the volume of the following prisms that have a trapezium and a kite as their cross-sections.

a



$V = Ah$

$A = \frac{1}{2}h(a+b)$

$= \frac{1}{2} \times 2{\cdot}4(6+3)$

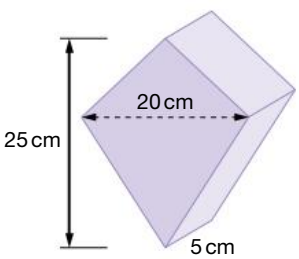
$= 10{\cdot}8\,\text{m}^2$

$h = 4\,\text{m}$

$\therefore\ V = 10{\cdot}8 \times 4$

$= 43{\cdot}2\,\text{m}^3$

b



$V = Ah$

$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 25 \times 20$

$= 250\,\text{cm}^2$

$h = 5\,\text{cm}$

$\therefore\ V = 250 \times 5$

$= 1250\,\text{cm}^3$

$V = Ah$

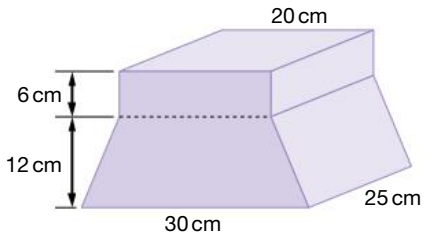
$A = \frac{1}{2}h(a+b)$

Be careful! h in the first formula is not the same as h in the second formula.

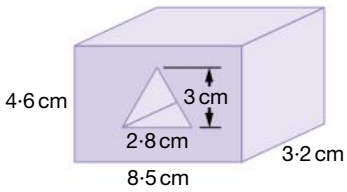
WORKED EXAMPLE 2

Calculate the volumes of the following composite prisms.

a



b



Method 1

a $V = Ah$

A = area of rectangle + area of trapezium

$= (20 \times 6) + \frac{1}{2} \times 12 \times (30 + 20)$

$= 120 + 300$

$= 420\text{ cm}^2$

$h = 25\text{ cm}$

$\therefore\ V = 420 \times 25$

$= 10\,500\text{ cm}^3$

b $V = Ah$

A = area of rectangle − area of triangle

$= (8{\cdot}5 \times 4{\cdot}6) - \left(\dfrac{2{\cdot}8 \times 3}{2}\right)$

$= 39{\cdot}1 - 4{\cdot}2$

$= 34{\cdot}9\text{ cm}^2$

$h = 3{\cdot}2\text{ cm}$

$\therefore\ V = 34{\cdot}9 \times 3{\cdot}2$

$= 111{\cdot}68\text{ cm}^3$

Method 2

a Volume = volume of rectangular prism + volume of trapezoidal prism

Volume of rectangular prism $= Ah$

$A = 20 \times 25$

$= 500\text{ cm}^2$

$h = 6\text{ cm}$

$\therefore\ V = 500 \times 6$

$= 3000\text{ cm}^3$

Volume of trapezoidal prism $= Ah$

$A = \frac{1}{2} \times 12 \times (30 + 20)$

$= 300\text{ cm}^2$

$h = 25\text{ cm}$

$\therefore\ V = 300 \times 25$

$= 7500\text{ cm}^3$

$\therefore$ Volume of solid $= 3000 + 7500$

$= 10\,500\text{ cm}^3$

b Volume = volume of rectangular prism − volume of triangular prism

Volume of rectangular prism $= Ah$

$A = 8{\cdot}5 \times 4{\cdot}6$

$= 39{\cdot}1\text{ cm}^2$

$h = 3{\cdot}2\text{ cm}$

$\therefore\ V = 39{\cdot}1 \times 3{\cdot}2$

$= 125{\cdot}12\text{ cm}^3$

Volume of triangular prism $= Ah$

$A = \dfrac{2{\cdot}8 \times 3}{2}$

$= 4{\cdot}2\text{ cm}^2$

$h = 3{\cdot}2\text{ cm}$

$\therefore\ V = 4{\cdot}2 \times 3{\cdot}2$

$= 13{\cdot}44\text{ cm}^3$

$\therefore$ Volume of solid $= 125{\cdot}12 - 13{\cdot}44$

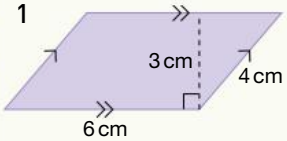
$= 111{\cdot}68\text{ cm}^3$

Sometimes one method is easier than another.

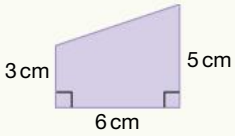
PREP QUIZ 5:06

Calculate the areas of the shapes in Questions **1** to **4**.

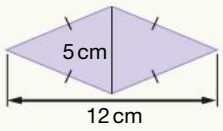
1



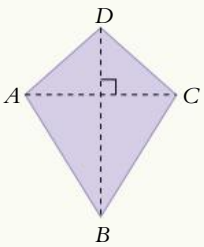
2



3



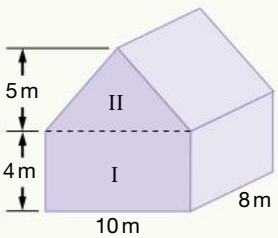
4



$AC = 8\,\text{cm}$
$BD = 10\,\text{cm}$

For the prism pictured find:

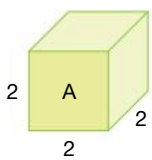
5 the cross-sectional area, A
6 the height of the prism, h
7 the value of Ah
8 the volume of the rectangular prism I
9 the volume of the triangular prism II
10 the volume of the prism.

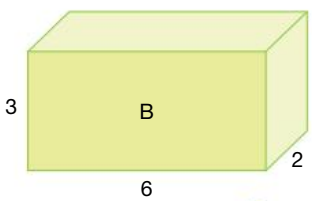


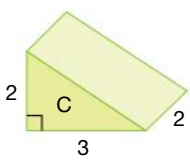
Exercise 5:06

P Foundation worksheet 5:06
Volume of a prism

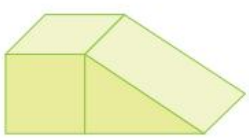
1 Calculate the volumes of the prisms A, B and C and use the answers to find the volumes of each of the composite prisms in **a** to **f**.



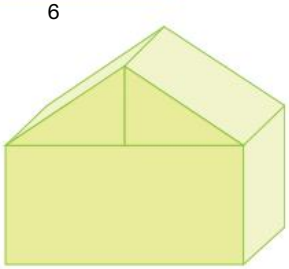




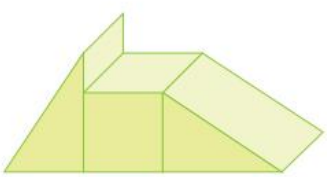
a



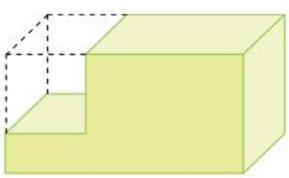
b



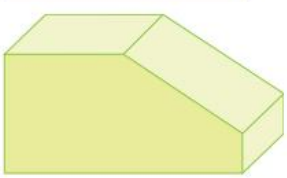
c



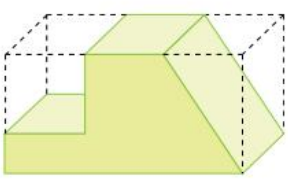
d



e

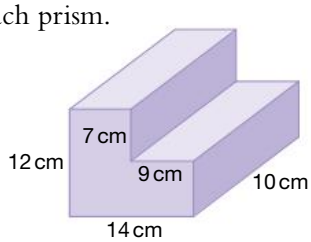


f

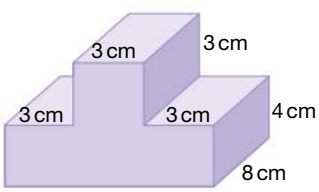


2 Each of the following prisms can be formed from rectangular prisms. Calculate the volume of each prism.

a



b



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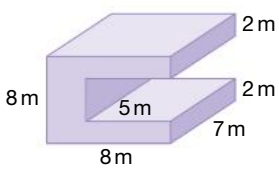
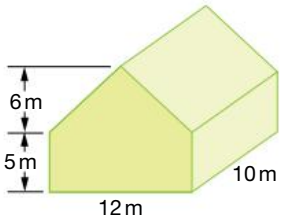
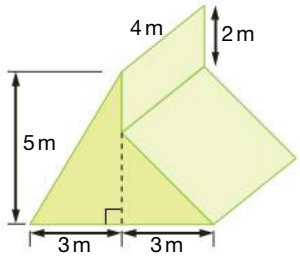


3 Each of the following prisms has been made by joining rectangular and triangular prisms. Find the volume of each prism.

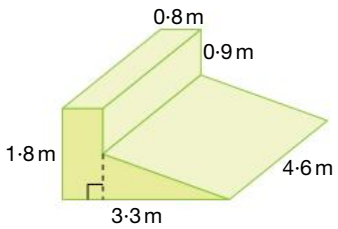
a



b

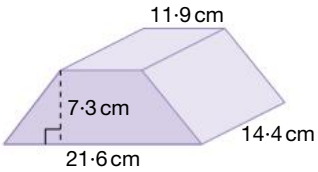


c

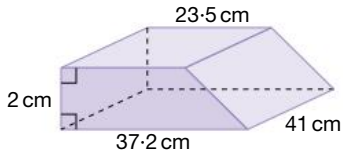


4 Calculate the volumes of the following trapezoidal prisms.

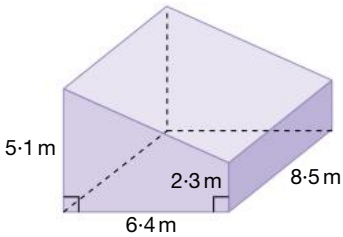
a



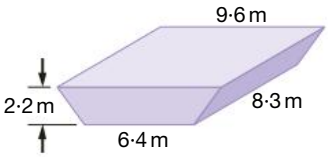
b



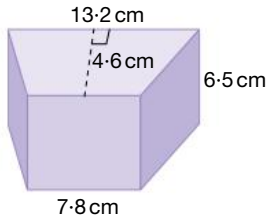
c



d

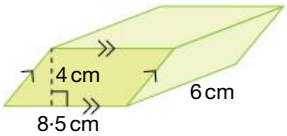


e

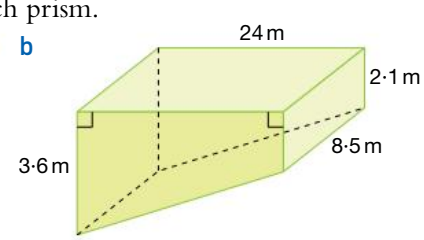


5 These prisms have a parallelogram, a trapezium and a rhombus as their cross-sections. Calculate the volume of each prism.

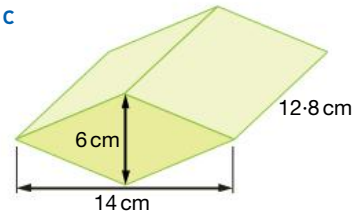
a



b

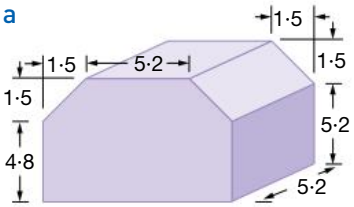


c

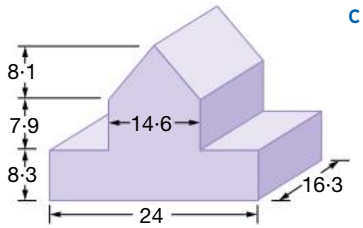


6 Calculate the volumes of the following prisms. All measurements are in cm. Give answers correct to one decimal place.

a



b



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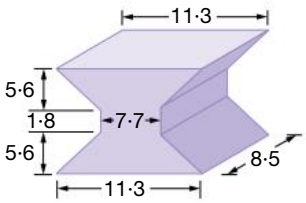
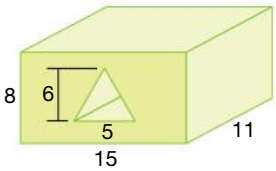


7 These rectangular prisms have had prism-shaped pieces cut from them. Calculate the volume of the remaining solid. All measurements are in centimetres.

a



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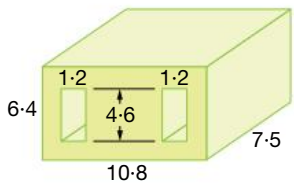

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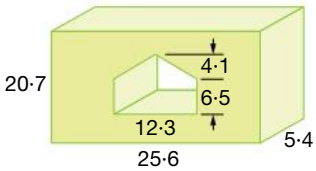
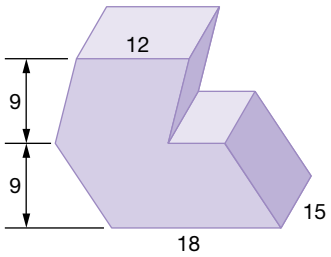


8 Calculate the volume of the following prisms. All measurements are in centimetres.

a



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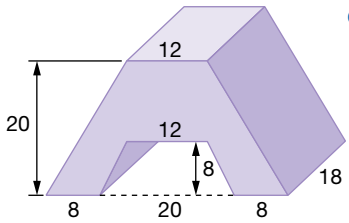

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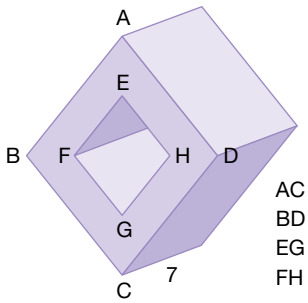
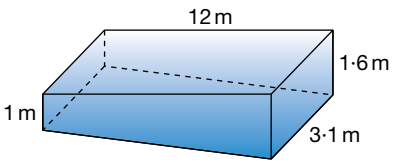
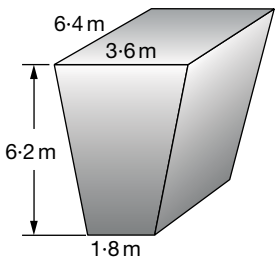


9 Calculate the capacity of the swimming pool shown. Give your answer correct to three significant figures.



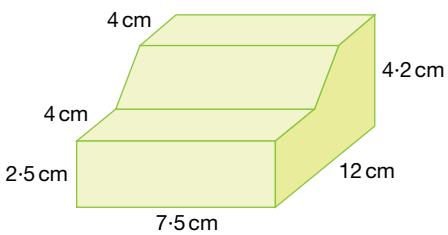
10 A bridge is to be supported by concrete supports. Calculate the volume of concrete needed for each support. The supports are trapezoidal prisms, as pictured in the diagram. Give the answer to the nearest cubic metre.



11 The solid pictured is formed by filling a mould with molten metal.

a Calculate the volume of the metal needed to make this object.

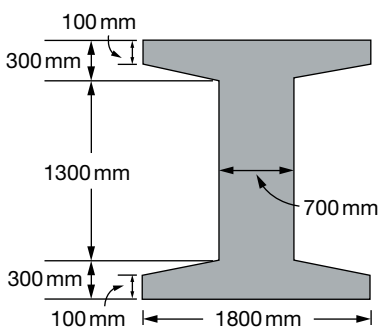
b Find its mass if 1 cm^3 of metal has a mass of 11·4 g.



12 A concrete beam with a cross-section as shown in the diagram is 10 m long.

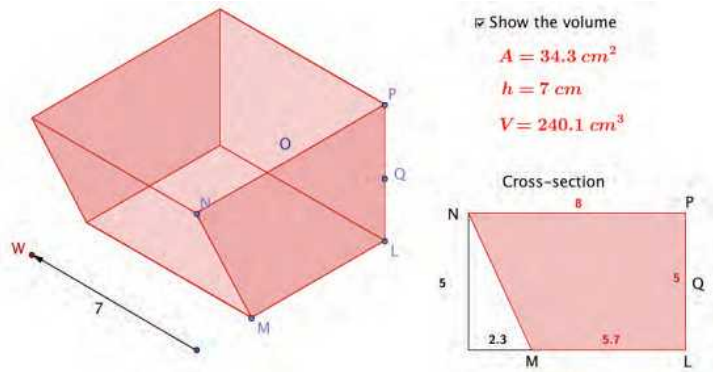
a Calculate the volume of concrete in the beam.

b Calculate the mass of this beam if 1 m^3 of the concrete has a mass of 2·5 tonnes.



GEOGEBRA ACTIVITY 5:06 VOLUME OF A PRISM

Click and drag points to change the cross-sectional shape of a prism and then calculate the volume of the prism. Tick a checkbox to check the answer.

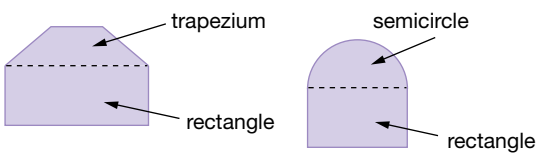


MATHS TERMS 5

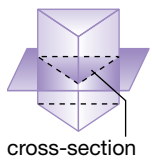
area
- the amount of space inside a two-dimensional shape
- units of area:
  square millimetre (mm^2)
  square centimetre (cm^2)
  square metre (m^2)
  hectare (ha)
  square kilometre (km^2)
- formulas are used to calculate the area of the common plane figures

capacity
- the amount of fluid that can be held by a container
- units of capacity:
  litre (L)
  millilitre (mL)
  kilolitre (kL)

composite figure
- a figure that is formed by joining simple figures

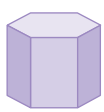


cross-section
- the shape on the face where a solid has been sliced



cylinder
- a prism-like solid with a circular cross-section
- it has two circular ends and a curved surface

prism
- a solid that has two identical ends joined by rectangular faces
- cross-sections parallel to the ends are identical to the ends

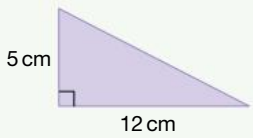


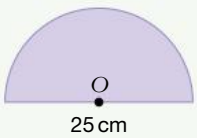
surface area
- the sum of the areas of the faces (or surfaces) of a three-dimensional figure (or solid)

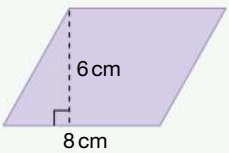
volume
- the amount of space inside a three-dimensional shape
- units of volume:
  cubic centimetre (cm^3)
  cubic metre (m^3)
- formulas are used to calculate the volume of common solids (three-dimensional shapes)

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

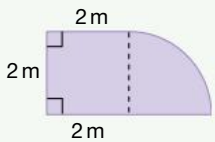
1 Find the area of each of the following figures. 5:01

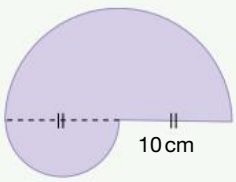
a 

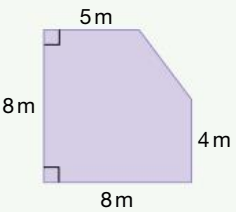
b 

c 

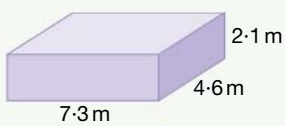
2 Find the area of each of the following composite figures. 5:02
(Where necessary, answer correct to two decimal places.)

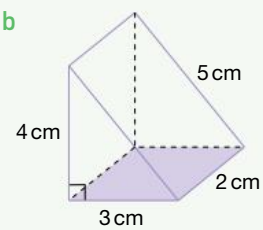
a 

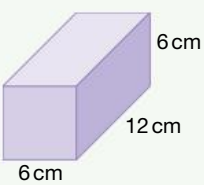
b 

c 

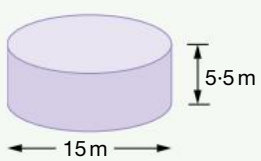
3 Calculate the surface areas of the following prisms. 5:03

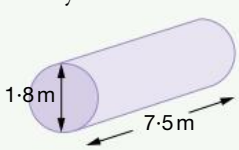
a 

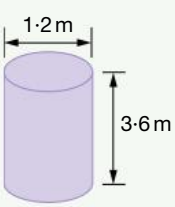
b 

c 

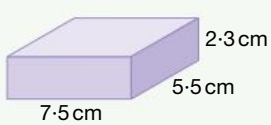
4 Calculate the surface areas of these cylinders. 5:04

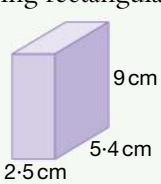
a 

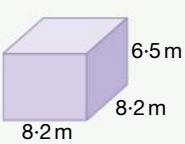
b 

c 

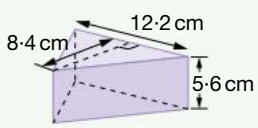
5 Find the volumes of the following rectangular prisms. 5:05

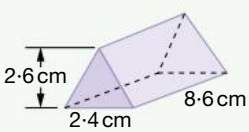
a 

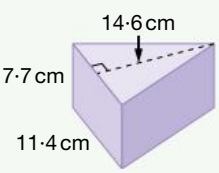
b 

c 

6 Find the volumes of the following triangular prisms. 5:05

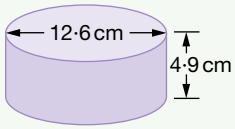
a 

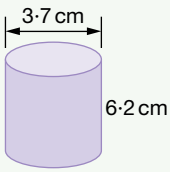
b 

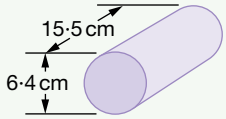
c 

5:05

7 Calculate the volumes of the following cylinders, correct to one decimal place.

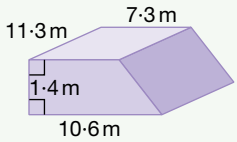
a 

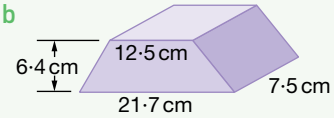
b 

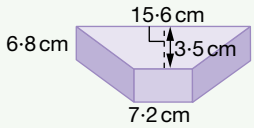
c 

5:06

8 Find the volumes of the following trapezoidal prisms.

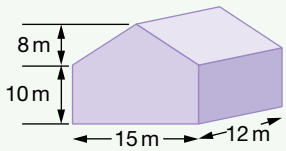
a 

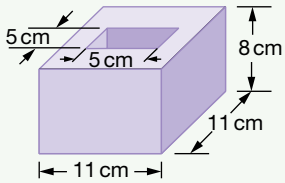
b 

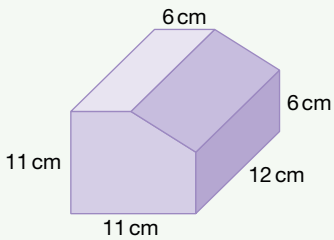
c 

5:06

9 Calculate the volumes of the following solids, correct to one decimal place.

a 

b 

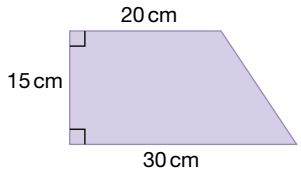
c 

How could the mass of this large-diameter cylindrical pipe be estimated?

ASSIGNMENT 5A Chapter review

1 Find the areas of the following trapeziums.

a



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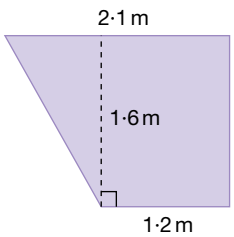
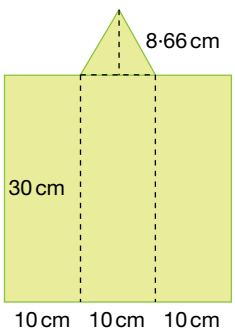
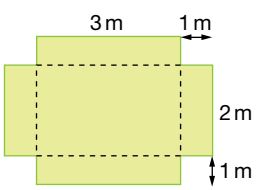


2 The nets of two prisms are shown. Find the surface area of each prism and state whether the prism is open or closed.

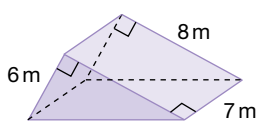
a



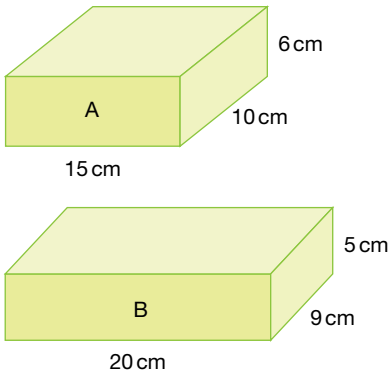
b



3 Find the surface area of this prism.



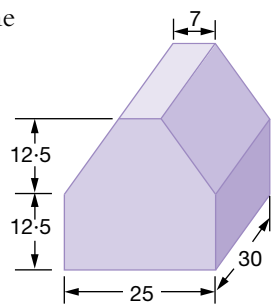
4 The two prisms shown both have a volume of 900 cm^3. Which prism has the smaller surface area?



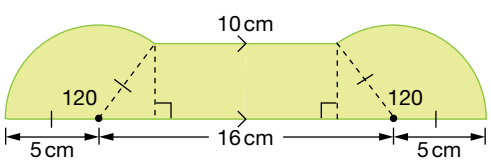
5 **a** Calculate the surface area of a cylindrical can that has a height of 11 cm and a diameter of 7 cm. Give your answer correct to one decimal place.

b An inground concrete swimming pool is cylindrical. It has a height of 1·2 m and a diameter of 3·6 m. The floor and walls of the swimming pool are to be cement rendered. Calculate the area that needs to be rendered, correct to two decimal places. Find the cost if the cement costs $50 per square metre.

6 Calculate the volume of the prism.



7 Calculate the area of this figure, correct to one decimal place.



ASSIGNMENT 5B Working mathematically

1 **a** How many bricks are in this stack?
b What is the weight of the stack in kilograms, if each brick weighs 3·1 kg?

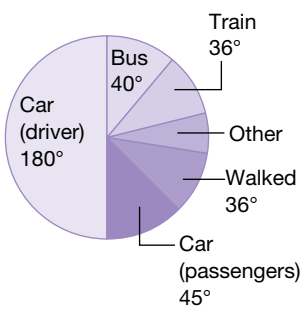
2 If each of the bricks in the stack above is 230 mm long and 108 mm wide, estimate the length, width and height of the stack. Use these dimensions to calculate the volume of the stack.

3 Heather is 7 years younger than Rachel. Ester is six times as old as Heather. Kuan is 7 years older than Ester. If Kuan is 43, how old is Rachel?

4 Every male bee has only one parent, a female. Every female bee has two parents, a male and a female. In the 8th generation back, how many ancestors has a male bee? (Assume that no ancestor occurs more than once.)

5 Four different playing cards are dealt into two piles: left first, then right, then left, and then right. The left pile is then placed on top of the right pile. How many times must this process be repeated before the cards return to their original positions? How many times would the process need to be repeated if there had been eight cards?

6 This sector graph shows the method of travelling to work for all people in a workforce.



a What percentage of the workforce caught a train to work?
b What percentage of the workforce was driven to work by car, bus or train?
c What is the size of the sector angle for 'other' means of transport? Do not use a protractor.
d What percentage of the workforce used a car to get to work?

ASSIGNMENT 5C Cumulative revision

1 Expand and simplify: 3:06

a $2(x + 5) + 3(2x - 1)$ **b** $2(x + 5) - 3(2x - 1)$ **c** $2x + 5 - 3(2x - 1)$

2 Factorise: 3:07

a $5x + 15$ **b** $5x + 15y$ **c** $5x - 15xy$ **d** $25x + 15x^2$

3 The measurements 6 m, 6·0 m and 6·00 m all have the same numerical value but they have different levels of accuracy. Between what two numbers would each of these measurements lie? 1:11

4 Give the answer to $\frac{16{\cdot}94 \times 32{\cdot}15}{24{\cdot}2}$ correct to: 1:06

a one decimal place **b** two decimal places

c one significant figure **d** three significant figures.

5 A set of cards is numbered from 1 to 20. The cards are shuffled and one card is chosen at random. Find the probability that the card is: 4:03

a an even number

b a number less than 10

c an even number and a number less than 10

d an even number or a number less than 10

e a number that is odd and greater than 10.

6 Sharon used a random number generator on a computer to investigate how many sixes would appear when three dice were rolled. She repeated the experiment 100 times and her results are shown in the table. 4:02

Number of sixes	Frequency
0	57
1	36
2	7
3	0

a Using these results, find the probability as a percentage of obtaining:

i 0 sixes **ii** 1 six **iii** 2 sixes.

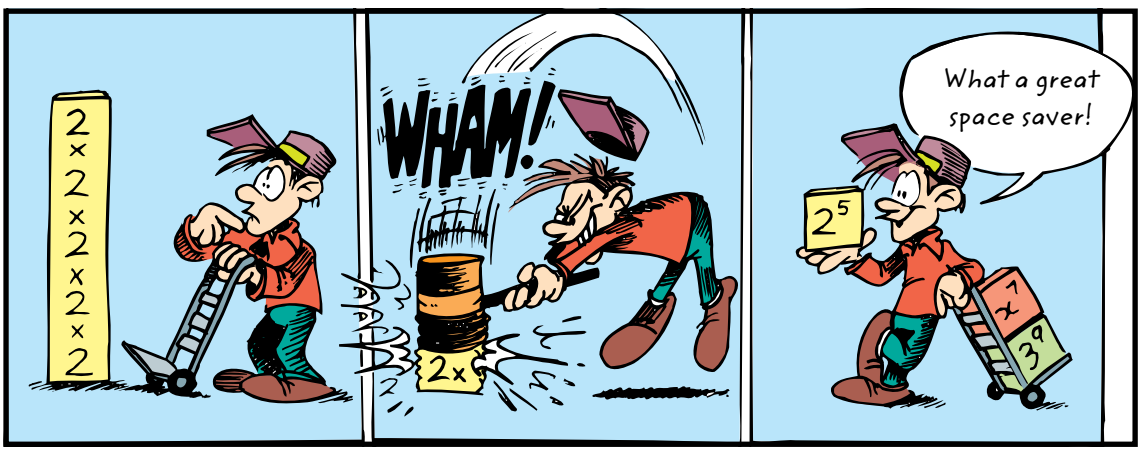
b It is possible to obtain 3 sixes. On the basis of these results, what would you estimate the probability of obtaining 3 sixes to be?

c How could you improve the accuracy of these experimental probabilities?

If one dice can land in 6 different ways and two dice can land in 36 different ways, in how many ways can three dice land?

INDICES

6



Contents

6:01 Review of indices
Challenge 6:01 Family trees
6:02 The index laws
Investigation 6:02 Exploring index notation
6:03 Negative indices
Investigation 6:03 Zero and negative indices
6:04 Scientific (or standard) notation
Investigation 6:04 Multiplying and dividing by powers of 10
Fun spot 6:04 Why is a room full of married people always empty?
6:05 Scientific notation and the calculator
Investigation 6:05 Using scientific notation
6:06 Conversions using scientific notation
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Indices*** [Stages 5.1 and 5.2]

- Extend and apply the index laws to variables, using positive-integer indices and the zero index. (ACMNA212)
- Simplify algebraic products and quotients using index laws. (ACMNA231)
- Apply index laws to numerical expressions with integer indices. (ACMNA209)
- Apply index laws to algebraic expressions involving integer indices (NSW)

Measurement and Geometry [Stage 5.1]

Selections from ***Numbers of Any Magnitude***

- Express numbers in scientific notation. (ACMNA210)

Working Mathematically

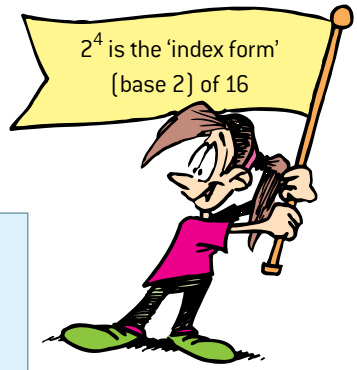
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

6:01 Review of indices

$2 \times 2 \times 2 \times 2 = 16$

$2^4 = 16$

- 2 is the base.
- 4 is the index.
- 16 is the basic numeral.



2^4 is called a power of 2.

$x^n = \underbrace{x \times x \times x \times x \times \ldots \times x \times x}_{n \text{ factors}}$ (where n is a positive integer)

For x^n: x is the base
n is the index

WORKED EXAMPLE 1

a Express the following in index form.

i $3 \times 3 \times 3 \times 3$ ii 6×6

iii $m \times m \times m$

b Rewrite in expanded form.

i 2^5 ii 7^3 iii a^5

c Find the basic numeral for:

i 4^3 ii 13^5 iii $(-4)^2$

Solutions

a i $3 \times 3 \times 3 \times 3 = 3^4$

ii $6 \times 6 = 6^2$

iii $m \times m \times m = m^3$

b i $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

ii $7^3 = 7 \times 7 \times 7$

iii $a^5 = a \times a \times a \times a \times a$

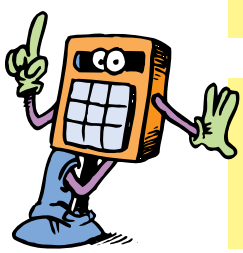
c i $4^3 = 4 \times 4 \times 4$
$= 64$

ii $13^5 = 13 \times 13 \times 13 \times 13 \times 13$
Use a calculator to evaluate 13^5.
Press 13 $x^{\square}$ 5 =
$13^5 = 371\,293$

iii $(-4)^2 = -4 \times -4$
$= 16$

What is the largest power of 2 that can be displayed on a 10-digit screen?

Another name for an index is an exponent.



Remember the index key. Enter the base x first, press $x^{\square}$, then enter the index 5 and press =.

WORKED EXAMPLE 2

Express as a product of its prime factors:

a 72 **b** 3500

Solutions

To find the prime factors of a number, we continue to break down each set of factors into further factors until each one is a prime number. Examine these solutions carefully.

a 72

$= 9 \times 8$

$= 3 \times 3 \times 4 \times 2$

$= 3 \times 3 \times 2 \times 2 \times 2$

This product of prime numbers can be written as $3^2 \times 2^3$.

$72 = 3^2 \times 2^3$

b 3500

$= 35 \times 100$

$= 7 \times 5 \times 10 \times 10$

$= 7 \times 5 \times 5 \times 2 \times 5 \times 2$

$3500 = 7 \times 5^3 \times 2^2$

Exercise 6:01

1 Write each expression in index form.

a $2 \times 2 \times 2 \times 2$ **b** 3×3

c $5 \times 5 \times 5$ **d** $7 \times 7 \times 7 \times 7 \times 7$

e $10 \times 10 \times 10$ **f** $9 \times 9 \times 9 \times 9$

g $x \times x$ **h** $a \times a \times a \times a$

i $n \times n \times n$ **j** $m \times m \times m \times m \times m$

k $p \times p \times p \times p \times p \times p$ **l** $y \times y$

m $4 \times 4 \times 4$ **n** $t \times t \times t \times t$

o $x \times x \times x \times x \times x \times x$

2 Rewrite in expanded form.

a 2^3 **b** 4^2 **c** 6^5 **d** 10^4

e 7^3 **f** 3^1 **g** a^3 **h** x^4

i y^2 **j** m^5 **k** n^7 **l** p^3

3 Determine the basic numeral.

a 2^5 **b** 3^4 **c** 7^2 **d** 11^2

e 10^4 **f** 6^3 **g** 2^8 **h** 5^7

i 8^5 **j** 9^5 **k** 4^{12} **l** 3^{15}

m $2^6 \times 5^3$ **n** $3^4 \times 7^3$ **o** $9^3 \times 5^4$ **p** $12^3 \times 7^3$

4 Write as powers of 10.

a 1000 **b** 100 000 **c** one million **d** one billion

5 Write as powers of 2.

a 64 **b** 256 **c** 16 384 **d** 1 048 576

6 Write as powers of 3.

a 27 **b** 243 **c** 59 049 **d** 4 782 969

7 Express the following integers as products of their prime factors in index form.

a 36 **b** 200 **c** 196 **d** 80

8 Find the value of x in each of the following exponential equations.

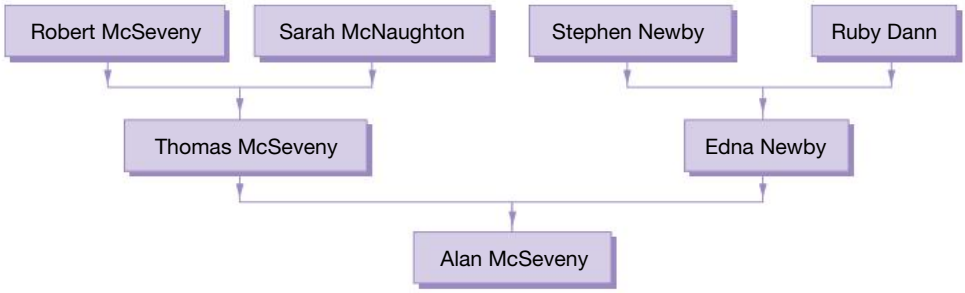
a $8^x = 64$ **b** $3^x = 27$ **c** $10^x = 100\,000$ **d** $2^x = 16$

9 As a reward for service, a man asked his king for the amount of rice that would be needed to place one grain of rice on the first square of a chessboard, 2 on the second, 4 on the third, 8 on the fourth, doubling each time, until the 64th square is considered. How many grains of rice would be needed for the 32nd square?

10 For Question 9, would the sum of the grains of rice on the first 31 squares be more or less than the number of grains of rice on the 32nd square? (Hint: Consider the rectangle below to represent the rice on the 32nd square. Colour the fraction of the rectangle equal to the amount of rice on the 31st square, then the 30th square, etc.)

CHALLENGE 6:01 FAMILY TREES

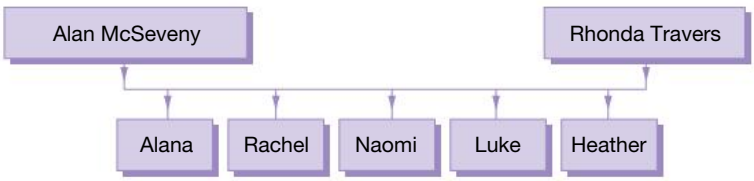
A part of Alan's family tree is drawn below.



1 How many great-grandparents would Alan have had?

2 Estimate the number of generations you would need to go back to before over one million boxes would be required to show that generation. Would this mean that in that generation there would be over 1 000 000 different ancestors?

3 Estimate Alan's total number of ancestors in the previous 20 generations.



It is much harder to estimate the number of descendants Alan will have in any one generation, as a lot of assumptions will need to be made. Will all of his five children marry? Will there be wars, diseases or population control in the future?

4 Estimate how many generations would be needed before Alan has a total number of descendants in excess of 1 000 000.

6:02 The index laws

PREP QUIZ 6:02

Write in index form.
1 $5 \times 5 \times 5 \times 5$ **2** $a \times a \times a$

Rewrite as a product.
3 2^4 **4** m^5

Evaluate the following.
5 2^2 **6** 2^3 **7** $2^2 \times 2^3$
8 3^3 **9** $3^3 \div 3^2$ **10** $3^3 \div 3^3$

Multiplication using indices

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$
$= 3^6 \ [= 3^{4+2}]$

$x^5 \times x^3 = (x \times x \times x \times x \times x) \times (x \times x \times x)$
$= x^8 \ [= x^{5+3}]$

Index law 1 When multiplying terms, **add** the indices: $x^m \times x^n = x^{m+n}$

Division using indices

$3^5 \div 3^2 = \dfrac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}}$
$= 3^3 \ [= 3^{5-2}]$

$x^4 \div x^3 = \dfrac{x \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x}}$
$= x^1 \ [= x^{4-3}]$

Index law 2 When dividing terms, **subtract** the indices: $x^m \div x^n = x^{m-n}$

Powers of indices

$(3^3)^2 = 3^3 \times 3^3$
$= 3^{3+3}$ Using index law 1
$= 3^6 \ [= 3^{3 \times 2}]$

$(x^5)^4 = x^5 \times x^5 \times x^5 \times x^5$
$= x^{5+5+5+5}$ Using index law 1
$= x^{20} \ [= x^{5 \times 4}]$

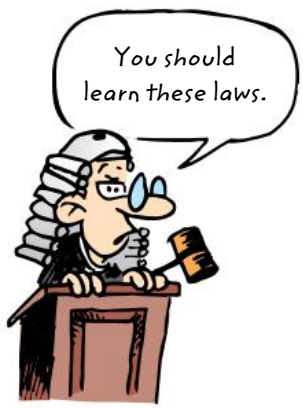
Index law 3 For powers of a power, **multiply** the indices: $(x^m)^n = x^{mn}$

If we simplify the division $x^n \div x^n$, using the second law above:
$x^n \div x^n = x^{n-n}$
$= x^0$

But any expression divided by itself must equal 1.
$x^n \div x^n = 1$

Therefore, x^0 must be equal to 1.
$x^0 = 1$

Index law 4 A term with a zero index is 1: $x^0 = 1$



WORKED EXAMPLES

1 Simplify:

a $3^2 \times 3^5$ **b** $x^3 \times x^2$ **c** $6m^2n \times mn^4$

2 Simplify:

a $x^7 \div x^2$ **b** $15a^5 \div 3a^2$ **c** $20a^3b^2 \div 10ab$

3 Simplify:

a $(a^4)^2$ **b** $(2a^4)^3$ **c** $(p^4)^3 \div (p^2)^4$

4 Simplify:

a 7^0 **b** $18x^3 \div 6x^3$ **c** $(2y^3)^4 \div (4y^6)^2$

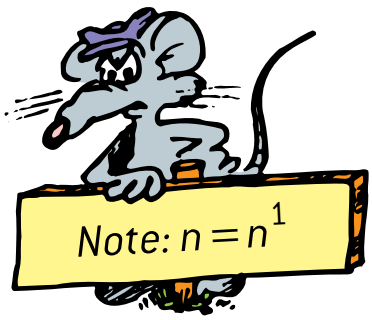
Solutions

1 Using index law 1

a $3^2 \times 3^5 = 3^{2+5}$
$= 3^7$

b $x^3 \times x^2 = x^{3+2}$
$= x^5$

c $6m^2n \times mn^4 = 6 \times m^{2+1} \times n^{1+4}$
$= 6m^3n^5$



2 Using index law 2

a $x^7 \div x^2 = x^{7-2}$
$= x^5$

b $15a^5 \div 3a^2 = \dfrac{15a^5}{3a^2}$
$= \dfrac{15}{3} \times \dfrac{a^5}{a^2}$
$= 5 \times a^{5-2}$
$= 5a^3$

c $20a^3b^2 \div 10ab$
$= \dfrac{20a^3b^2}{10ab}$
$= \dfrac{20}{10} \times \dfrac{a^3}{a} \times \dfrac{b^2}{b}$
$= 2a^2b$

3 Using index law 3

a $(a^4)^2 = a^{4 \times 2}$
$= a^8$

b $(2a^4)^3 = 2^3 \times (a^4)^3$
$= 8 \times a^{4 \times 3}$
$= 8a^{12}$

c $(p^4)^3 \div (p^2)^4 = p^{12} \div p^8$
$= p^4$

4 Using index law 4

a $7^0 = 1$

b $18x^3 \div 6x^3$
$= \dfrac{18x^3}{6x^3}$
$= 3 \times x^{3-3}$
$= 3x^0$
$= 3 \times 1$
$= 3$

c $(2y^3)^4 \div (4y^6)^2$
$= \dfrac{2^4 \times (y^3)^4}{4^2 \times (y^6)^2}$
$= \dfrac{16}{16} \times \dfrac{y^{12}}{y^{12}}$
$= 1 \times y^0$
$= 1$

With practice, many of the steps in the above solutions can be left out.

Exercise 6:02

P Foundation worksheet 6:02 The index laws

1 Use a calculator to complete the following tables.

Power	11^6	6^8	8^{10}	5^6	8^7	7^8	5^4	3^5
Answer								

Expression	$7^3 \times 7^5$	$5^8 \div 5^2$	$(11^3)^2$	56^0	$3^{10} \div 3^5$	$8^2 \times 8^5$	$(13)^0$	$(6^2)^4$
Answer								

2 Use the tables in Question 1 to help you write *true* or *false* for:

a $7^3 \times 7^5 = 7^8$ b $5^8 \div 5^2 = 5^4$ c $5^8 \div 5^2 = 5^6$

d $(11^3)^2 = 11^6$ e $56^0 = 1$ f $8^2 \times 8^5 = 8^7$

g $3^{10} \div 3^5 = 3^5$ h $3^{10} \div 3^5 = 3^2$ i $(6^2)^4 = 6^8$

3 Simplify these products, writing answers in index form.

a $10^2 \times 10^3$ b 10×10^2 c $10^3 \times 10^3$

d $5^2 \times 5^4$ e $2^3 \times 2^2$ f $7^5 \times 7$

g $3^2 \times 3^3 \times 3^4$ h $2 \times 2^2 \times 2^5$ i $10^7 \times 10 \times 10^2$

4 Simplify these quotients, writing answers in index form.

a $10^2 \div 10$ b $10^6 \div 10^3$ c $10^5 \div 10^4$

d $5^8 \div 5^4$ e $7^5 \div 7^2$ f $3^{10} \div 3$

g $2^3 \div 2^2$ h $5^3 \div 5^3$ i $2^8 \div 2^2$

5 Simplify these powers, writing answers in index form.

a $(10^2)^3$ b $(10^3)^3$ c $(10^6)^2$ d $(2^3)^4$

e $(2^2)^2$ f $(2^7)^5$ g $(3^4)^2$ h $(5^3)^5$

i $(7^2)^4$ j $(2^3 \times 3^2)^2$ k $(7^3 \times 11^4)^2$ l $(3 \times 2^2)^4$

6 Simplify:

a $x^3 \times x^2$ b $y^4 \times y^2$ c $m^3 \times m^3$ d $m \times m^4$

e $p^5 \times p^5$ f $a \times a$ g $y^3 \times y^4$ h $x^2 \times x$

i $m^2 \times m^5$ j $3y^4 \times y$ k $m^6 \times 3$ l $5x^3 \times 3$

7 Simplify:

a $x^6 \div x^3$ b $x^6 \div x^2$ c $x^6 \div x$ d $m^3 \div m$

e $y^5 \div y^2$ f $m^4 \div m^2$ g $y^6 \div y^6$ h $x^5 \div x^5$

i $y^8 \div y^2$ j $6m^6 \div 3$ k $4y^8 \div 2$ l $20x^5 \div 5$

8 Simplify:

a $(x^2)^3$ b $(y^4)^2$ c $(a^3)^5$ d $(m^2)^0$

e $(x^0)^3$ f $(a^7)^0$ g $(y^3)^2$ h $(a^6)^3$

i $(x^3)^3$ j $(2x)^3$ k $(3x^2)^2$ l $(5m^2)^4$

9 Simplify:

a $8x^4 \times x^3$ b $5a^2 \times a$ c $4m^6 \times m^4$ d $8x^4 \div x^3$ e $5a^2 \div a$

f $4m^6 \div m^4$ g $10y^3 \times 5y$ h $16m^2 \times 2m^2$ i $8a^5 \times 4a^4$ j $10y^3 \div 5y$

k $16m^2 \div 2m^2$ l $8a^5 \div 4a^4$ m $12x^5 \times 6x^3$ n $9a^7 \times 3a^2$ o $18y^6 \times 6y$

p $\frac{12x^5}{6x^3}$ q $\frac{9a^7}{3a^2}$ r $\frac{18y^6}{6y}$ s $\frac{3a^5}{a^3}$ t $\frac{10x^5}{5}$ u $\frac{42a^7}{21a^6}$

10 Simplify:

a $6a^0$ b $6(a^3)^0$ c $(6a^3)^0$

d ab^0 e x^0y^3 f m^5n^0

g $(2m^2)^3$ h $(4n^3)^2$ i $(2p^3)^4$

j $x^2y^3 \times x^3$ k $a^2b^5 \times b^4$ l $xy^3 \times x^4$

m $x^2y^2 \times xy^2$ n $a^3b \times a^2b$ o $m^2n \times mn^3$

p $(x^2y^3)^2$ q $(abc)^2$ r $(pq^3)^3$

s $5x^2y \times 2xy$ t $4a^2b^2 \times 7ab^3$ u $11a^3 \times 4a^2b^2$

v $a \times a \times a \times 3a$ w $3a \times 2a \times -4a$ x $5c^4 \times 4c^2 \div 10c^5$

y $12x^2 - 5x^3 + 10x^2$ z $4x(3x + 2) - (x - 1)$

11 Simplify:

a $5x^2 \times 2x^3 \times 3x$ b $5 \times 2a \times 4a^2$ c $5y \times y^2 \times xy$

d $4x^4 \div 8x$ e $7y^3 \div 49y^2$ f $100x^3 \div 10x^4$

g $(x^2)^3 \times x^2$ h $(a^4)^2 \times a^5$ i $(y^7)^3 \times y^5$

j $(a^2)^3 \div a^4$ k $(m^3)^4 \div m^{10}$ l $n^8 \div (n^2)^3$

m $(y^5)^2 \times (y^3)^3$ n $(2a^4)^3 \times (a^3)^2$ o $(b^4)^3 \div (b^2)^5$

p $(x^4 \times x^7) \div x^9$ q $(4a^3 \times 5a^4) \div 10a^5$ r $7p^7q^5 \div (p^2q)^3$

s $\frac{5x^3 \times 4x^7}{10x^5}$ t $\frac{(3x^3)^2 \times 4x^5}{6x^4 \times x}$ u $\frac{x^2 \times (xy)^3}{(2x)^4}$

INVESTIGATION 6:02 EXPLORING INDEX NOTATION

Searching for patterns is part of mathematics and being able to explain concepts is important if our ideas are to be shared.

- Find pairs of terms that can be multiplied to give 2^7. Explain the relationship between the members of any pair.
- Find pairs of terms that can be divided to give $3x^2$. Explain the difference between the members of any pair.
- Explain the difference between $7x^0$ and $(7x)^0$.
- Explain why $3^2 \times 3^4 \neq 9^6$.
- List all the pairs of expressions that could be multiplied together to give $10xy^2$. (Use only whole numbers.)
- Fold a sheet of A4 paper in half as many times as you can to create regions bounded by the folds. How does the number of regions increase with each new fold? Write a formula for the number of regions R for n folds.

6:03 Negative indices

All the indices seen so far have been positive integers or zero.

Using the second index law: $2^3 \div 2^5 = 2^{3-5}$
i.e. $2^3 \div 2^5 = 2^{-2}$

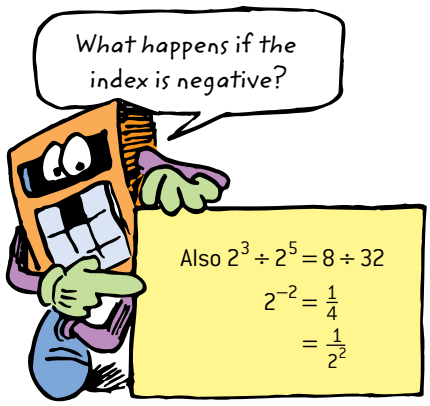
But this could also be written in this way:

$$\frac{2^3}{2^5} = \frac{{}^{1}\cancel{2} \times {}^{1}\cancel{2} \times {}^{1}\cancel{2}}{{}^{1}\cancel{2} \times {}^{1}\cancel{2} \times {}^{1}\cancel{2} \times 2 \times 2}$$

$$= \frac{1}{2 \times 2}$$

$$= \frac{1}{2^2}$$

So $2^{-2} = \frac{1}{2^2}$



In general, the meaning of a negative index can be summarised by the rules:

$$x^{-m} = \frac{1}{x^m}, (x \neq 0)$$

x^{-m} is the reciprocal of x^m, because $x^m \times x^{-m} = 1$

$x^{-3} = \frac{1}{x^3}$

$x^3 \times x^{-3} = x^0$
$= 1$

WORKED EXAMPLE 1

Simplify each of the following.

a $3^{-2} = \frac{1}{3^2}$
$= \frac{1}{9}$

b $5^{-1} = \frac{1}{5^1}$
$= \frac{1}{5}$

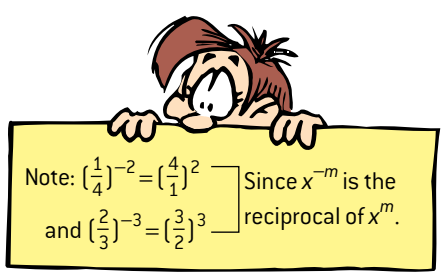
c $x^7 \times x^{-3}$
$= x^{7+(-3)}$
$= x^4$

d $6x^2 \div 3x^4$
$= 2x^{2-4}$
$= 2x^{-2}$
$= \frac{2}{x^2}$

e $(\frac{1}{4})^{-2} = \frac{1}{(\frac{1}{4})^2}$
$= \frac{1}{(\frac{1}{16})}$
$= 16$

$\frac{1}{(\frac{1}{16})} = 1 \div \frac{1}{16} = 1 \times \frac{16}{1} = 16$

f $(\frac{2}{3})^{-3} = \frac{1}{(\frac{2}{3})^3}$
$= \frac{1}{(\frac{8}{27})}$
$= \frac{27}{8}$
$= 3\frac{3}{8}$



WORKED EXAMPLE 2

Evaluate, using a calculator.

a 2^{-3} **b** $(\frac{1}{3})^{-2}$

Solutions

The key $x^{\blacksquare}$ can also be used for negative indices. Examine the following:

a Press: 2 $x^{\blacksquare}$ (–) 3 = Answer: $2^{-3} = \frac{1}{8} = 0{\cdot}125$

b Press: 1 $\square\!\!\square$ 3 ▶ $x^{\blacksquare}$ (–) 2 = Answer: $(\frac{1}{3})^{-2} = (\frac{3}{1})^2 = 9$

Exercise 6:03

P Foundation worksheet 6:03 Negative indices

1 Write the value of each of the following.

a 3^{-1} **b** 5^{-1} **c** 2^{-1} **d** 6^{-2} **e** 4^{-2}

f 10^{-3} **g** 2^{-4} **h** 10^{-4} **i** 5^{-2}

2 Write each with a negative index.

a $\frac{1}{11}$ **b** $\frac{1}{3}$ **c** $\frac{1}{5}$ **d** $\frac{1}{7}$ **e** $\frac{1}{3^3}$ **f** $\frac{1}{5^4}$

g $\frac{1}{2^8}$ **h** $\frac{1}{7^2}$ **i** $\frac{1}{10^2}$ **j** $\frac{1}{10^3}$ **k** $\frac{1}{10^6}$ **l** $\frac{1}{10^5}$

3 Write *true* or *false* for:

a $1024 = 2^{10}$ **b** $8 = 2^4$ **c** $3^{-2} = \frac{1}{9}$ **d** $2(3)^2 = 36$

e $2(3)^{-1} = \frac{1}{6}$ **f** $4^{-1} = \frac{1}{2}$ **g** $2^{-1} < 1$ **h** $-2^8 = (-2)^8$

4 Simplify, leaving answers as powers of ten.

a $10^2 \div 10^5$ **b** $10^{-2} \times 10^3$ **c** $10 \div 10^6$ **d** $10^{-1} \times 10^{-2}$

e $\frac{10^2 \times 10^3}{10^6}$ **f** $\frac{(10^{-2})^2}{10^2}$ **g** $\frac{10}{10^2 \div 10^7}$ **h** $\frac{10^4 \times 10^{-2}}{(10^3)^2}$

5 Write each without a negative index.

a a^{-1} **b** x^{-1} **c** m^{-1} **d** y^{-1} **e** x^{-3} **f** y^{-2}

g x^{-4} **h** m^{-6} **i** $2x^{-1}$ **j** $5a^{-3}$ **k** $10y^{-2}$ **l** $36q^{-4}$

6 Rewrite each using a negative index to avoid having a fraction.

a $\frac{1}{x}$ **b** $\frac{1}{x^2}$ **c** $\frac{1}{x^3}$ **d** $\frac{1}{x^4}$ **e** $\frac{5}{y^2}$ **f** $\frac{3}{a}$

g $\frac{10}{m^4}$ **h** $\frac{75}{m^3}$ **i** $\frac{x}{y^2}$ **j** $\frac{m}{a^2}$ **k** $\frac{3a}{b^2}$ **l** $\frac{4x}{y}$

7 Write each as an integer, fraction or mixed number.

a $\left(\frac{1}{2}\right)^{-1}$ b $\left(\frac{1}{3}\right)^{-1}$ c $\left(\frac{2}{3}\right)^{-1}$ d $\left(\frac{1}{10}\right)^{-1}$

e $\left(\frac{1}{2}\right)^{-2}$ f $\left(\frac{1}{3}\right)^{-2}$ g $\left(\frac{2}{3}\right)^{-2}$ h $\left(\frac{3}{10}\right)^{-2}$

$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

8 Rewrite each expression with a positive index.

a x^{-2} b a^{-5} c $3x^{-1}$ d $5m^{-2}$

e $(x+1)^{-2}$ f $(3+a)^{-1}$ g $(6x)^{-2}$ h $4(x+2)^{-1}$

9 Evaluate the following, using your calculator. Leave your answers in decimal form.

a 2^{-3} b 4^{-2} c 5^{-2} d 8^{-2}

e 2^{-6} f 4^{-3} g $(0{\cdot}5)^{-3}$ h $(0{\cdot}2)^{-2}$

i $(0{\cdot}05)^{-2}$ j $(2{\cdot}5)^{-3}$ k $(0{\cdot}1)^{-5}$ l $(0{\cdot}625)^{-2}$

10 Simplify, writing your answers without negative indices.

a $x^3 \times x^{-2}$ b $a^{-2} \times a^5$ c $m^4 \times m^{-1}$ d $n^5 \times n^{-5}$

e $3a^2 \times a^{-1}$ f $6x^{-2} \times 5x^4$ g $a^{-2} \times 5a^3$ h $15m^{-1} \times 2m^3$

i $x^{-4} \times x$ j $2a^{-2} \times a^{-3}$ k $4y \times 2y^{-2}$ l $15m^{-4} \times 2m^{-1}$

11 Simplify, writing your answers without negative indices.

a $m^4 \div m^{-1}$ b $x^2 \div x^{-2}$ c $y^{-6} \div y^{-8}$ d $x^3 \div x^{-1}$

e $a^{-2} \div a^2$ f $y^{-1} \div y^3$ g $y^{-2} \div y$ h $x^{-3} \div x^{-1}$

i $6x^2 \div 2x^{-1}$ j $10a^3 \div 5a^7$ k $24a^{-2} \div a^3$ l $18n^{-1} \div 9n^{-2}$

12 Simplify, writing your answers without negative indices.

a $(a^{-3})^{-2}$ b $(x^2)^{-1}$ c $(y^{-3})^2$ d $(m^{-2})^{-2}$

e $(2x^2)^{-1}$ f $(3x)^{-2}$ g $(5x^{-1})^2$ h $(7x^{-2})^2$

i $(abc)^{-1}$ j $(a^2b^2c^2)^{-1}$ k $(2a^2b)^{-1}$ l $2(a^2b)^{-1}$

13 If $x = 2$, $y = 3$ and $z = \frac{1}{2}$, evaluate:

a $x^{-1} + y^{-1}$ b $(xy)^{-1}$ c $(xz)^{-1}$ d $x^{-1}y^{-1}z$

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where $\pi \approx 3{\cdot}142$ and r is the radius of the sphere.

INVESTIGATION 6:03 ZERO AND NEGATIVE INDICES

1 What does it mean to have a zero or negative index? Complete these tables, writing answers less than 1 as fractions. Divide each answer by 10 to reduce the power.

Power of 10	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Answer	10 000	1000							

$\div 10$

- As the power of 10 decreases, does the answer decrease?

$10^4 \div 10 = 10^3$
$10^3 \div 10 = 10^2$
$10^2 \div 10 = 10^1$
$10^1 \div 10 = 10^0$
$10^0 \div 10 = 10^{-1}$
$\ldots \div 10 = \ldots$

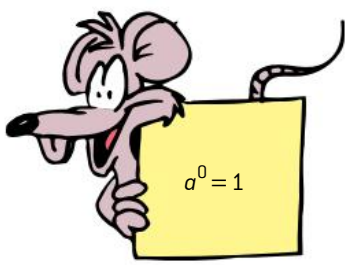
Divide each answer by 5 to reduce the power.

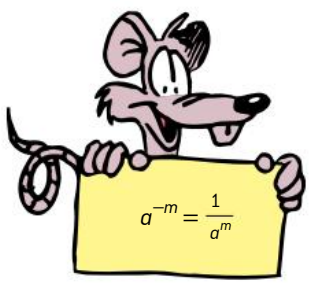
Power of 5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
Answer	625								

$\div 5$

- As the power of 5 decreases, does the answer decrease?

$5^4 \div 5 = 5^3$
$5^3 \div 5 = 5^2$
$5^2 \div 5 = 5^1$
$5^1 \div 5 = 5^0$
$5^0 \div 5 = 5^{-1}$
$\ldots \div 5 = \ldots$





2 Use the tables above to write *true* or *false* for each of the following.

a $10^{-1} = \frac{1}{10}$ **b** $5^0 = 1$ **c** $5^{-1} = \frac{1}{5}$

d $10^{-2} = \frac{1}{10^2}$ **e** $5^{-3} = \frac{1}{5^3}$ **f** $10^0 = 1$

g $10^{-4} = \frac{1}{10^4}$ **h** $5^{-2} = \frac{1}{5^2}$ **i** $10^{-3} = \frac{1}{10^3}$

6:04 Scientific (or standard) notation

INVESTIGATION 6:04 MULTIPLYING AND DIVIDING BY POWERS OF 10

Use the $x^{\blacksquare}$ button on your calculator to answer these questions.

Look for a connection between questions and answers and then fill in the rules at the end of the investigation.

Exercises

1 **a** $1{\cdot}8 \times 10^1$ **b** $1{\cdot}8 \times 10^2$ **c** $1{\cdot}8 \times 10^3$
d $4{\cdot}05 \times 10^1$ **e** $4{\cdot}05 \times 10^2$ **f** $4{\cdot}05 \times 10^3$
g $6{\cdot}2 \times 10^4$ **h** $6{\cdot}2 \times 10^5$ **i** $6{\cdot}2 \times 10^6$
j $3{\cdot}1416 \times 10^2$ **k** $3{\cdot}1416 \times 10^3$ **l** $3{\cdot}1416 \times 10^4$

To multiply by 10^n move the decimal point … places to the …

2 **a** $1{\cdot}8 \div 10^1$ **b** $1{\cdot}8 \div 10^2$ **c** $1{\cdot}8 \div 10^3$
d $968{\cdot}5 \div 10^2$ **e** $968{\cdot}5 \div 10^3$ **f** $968{\cdot}5 \div 10^4$

To divide by 10^n move the decimal point … places to the…

The investigation above should have reminded you that:

- when we multiply a decimal by 10, 100 or 1000, we move the decimal point 1, 2 or 3 places to the right
- when we divide a decimal by 10, 100 or 1000, we move the decimal point 1, 2 or 3 places to the left.

When expressing numbers in scientific (or standard) notation, each number is written as the product of a number between 1 and 10, and a power of 10.

$6{\cdot}1 \times 10^5$

- This number is written in scientific notation (or standard form).
- The first part is between 1 and 10.
- The second part is a power of 10.

'Scientific notation' is sometimes called 'standard notation' or 'standard form'.

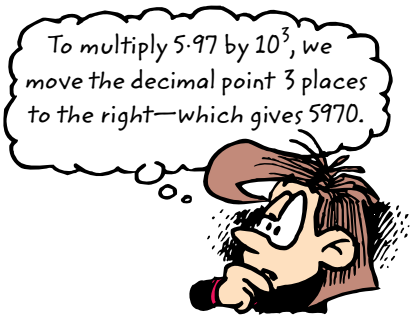
Scientific notation is useful when writing very large or very small numbers.

Numbers greater than 1

$5970. = 5{\cdot}97 \times 10^3$

To write 5970 in standard form:

- place a decimal point after the first digit
- count the number of places you have to move the decimal point to the left from its original position.



WORKED EXAMPLE 1

Express the following in scientific notation.

a $243 = 2{\cdot}43 \times 100$
$= 2{\cdot}43 \times 10^2$

$10^6 = 1$ million
$10^9 = 1$ billion

b $60\,000 = 6 \times 10\,000$
$= 6 \times 10^4$

If end zeros are significant, write them in your answer.
e.g. $60\,000 = 6{\cdot}00 \times 10^4$ (to nearest 100)

c $93\,800\,000 = 9{\cdot}38 \times 10\,000\,000$
$= 9{\cdot}38 \times 10^7$

We have moved the decimal point 7 places from its original position.

WORKED EXAMPLE 2

Write each of the following as a basic numeral.

a $1{\cdot}3 \times 10^2 = 1{\cdot}30 \times 100$
$= 130$ (1·3 hundreds)

b $2{\cdot}431 \times 10^2 = 2{\cdot}431 \times 100$
$= 243{\cdot}1$ (2·431 hundreds)

c $4{\cdot}63 \times 10^7 = 4{\cdot}6300000 \times 10\,000\,000$
$= 46\,300\,000$ (4·63 tens of millions)

To multiply by 10^7, move the decimal point 7 places right.

Numbers less than 1

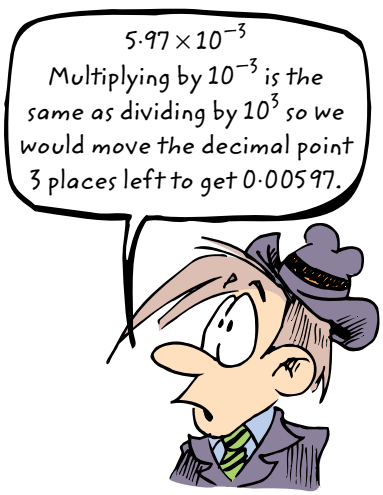
$0{\cdot}005\,97 = 5{\cdot}97 \times 10^{-3}$ (5·97 thousandths)

To write 0·005 97 in scientific notation:

- place a decimal point after the first non-zero digit
- count the number of places you have moved the decimal point to the right from its original position.

This will show the negative number needed as the power of 10.

$5{\cdot}97 \times 10^{-3}$ is the same as $5{\cdot}97 \div 10^3$.



WORKED EXAMPLE 1

Express each number in scientific notation.

a $0{\cdot}043 = 4{\cdot}3 \div 100$
$= 4{\cdot}3 \times 10^{-2}$

b $0{\cdot}000\,059\,7 = 5{\cdot}97 \div 100\,000$
$= 5{\cdot}97 \times 10^{-5}$

c $0{\cdot}004 = 4 \div 1000$
$= 4 \times 10^{-3}$

Short-cut method

- In 0·043, how many places do we move the decimal point for scientific notation? [Answer: 2 places]
- Is 0·043 bigger or smaller than 1? [Answer: smaller]
- So, the power of 10 is '–2'.
  $\therefore\ 0{\cdot}043 = 4{\cdot}3 \times 10^{-2}$

WORKED EXAMPLE 2

Write each as a basic numeral.

a $2{\cdot}9 \times 10^{-2} = 002{\cdot}9 \div 100$
$= 0{\cdot}029$

b $9{\cdot}38 \times 10^{-5} = 000\,009{\cdot}38 \div 100\,000$
$= 0{\cdot}000\,093\,8$

c $1{\cdot}004 \times 10^{-3} = 001{\cdot}004 \div 1000$
$= 0{\cdot}001\,004$

Exercise 6:04

P Foundation worksheet 6:04 Scientific notation

1 **a** Explain the difference between 2×10^4 and 2^4.
b Explain the difference between 5×10^{-2} and 5^{-2}.
c How many seconds are in 50 000 years?
d Have you lived $8{\cdot}2 \times 10^4$ hours?
e Order the following, from smallest to largest.

$3{\cdot}24 \times 10^3$	6	$9{\cdot}8 \times 10^{-5}$	$5{\cdot}6 \times 10^{-2}$
$1{\cdot}2 \times 10^4$	2·04	$5{\cdot}499 \times 10^2$	0·0034

f Write the thickness of a sheet of paper in scientific notation if 500 sheets of paper have a thickness of 3·8 cm.
g Estimate the thickness of the cover of this book. Write your estimate in scientific notation.

2 Write as a basic numeral.

a $2{\cdot}1 \times 10^1$	**b** $2{\cdot}1 \div 10^1$	**c** $2{\cdot}1 \times 10^{-1}$
d $7{\cdot}04 \times 10^2$	**e** $7{\cdot}04 \div 10^2$	**f** $7{\cdot}04 \times 10^{-2}$
g $1{\cdot}375 \times 10^3$	**h** $1{\cdot}375 \div 10^3$	**i** $1{\cdot}375 \times 10^{-3}$

3 Express in scientific notation. (Assume that final zeros are not significant.)

a 470	**b** 2600	**c** 53 000	**d** 700	**e** 50 000
f 700 000	**g** 65	**h** 342	**i** 90	**j** 4970
k 63 500	**l** 2 941 000	**m** 297·1	**n** 69·3	**o** 4976·5
p 9 310 000	**q** 67 000 000	**r** 190 100	**s** 600 000	**t** 501 700
u 100 000				

4 Express in scientific notation.

a 0·075	**b** 0·0063	**c** 0·59	**d** 0·08	**e** 0·0003
f 0·009	**g** 0·3	**h** 0·0301	**i** 0·000 529	**j** 0·426
k 0·001	**l** 0·000 009 7	**m** 0·000 06	**n** 0·000 907	**o** 0·000 000 004

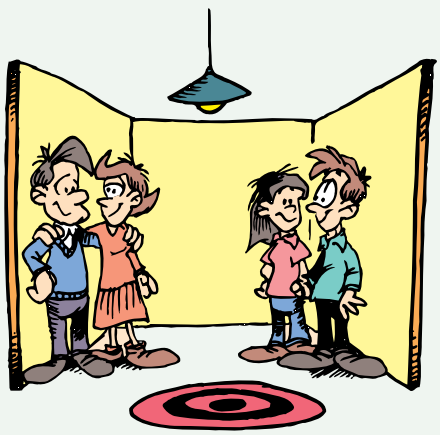
5 Write as a basic numeral.

a $2{\cdot}3 \times 10^{2}$	**b** $9{\cdot}4 \times 10^{4}$	**c** $3{\cdot}7 \times 10^{3}$	**d** $2{\cdot}95 \times 10^{2}$
e $8{\cdot}74 \times 10^{1}$	**f** $7{\cdot}63 \times 10^{5}$	**g** $1{\cdot}075 \times 10^{3}$	**h** $2{\cdot}0 \times 10^{4}$
i 8×10^{1}	**j** $2{\cdot}9 \times 10^{-2}$	**k** $1{\cdot}9 \times 10^{-3}$	**l** $9{\cdot}5 \times 10^{-1}$
m $3{\cdot}76 \times 10^{-3}$	**n** $4{\cdot}63 \times 10^{-4}$	**o** $1{\cdot}07 \times 10^{-2}$	**p** 7×10^{-2}
q $8{\cdot}0 \times 10^{-1}$	**r** 5×10^{-6}	**s** $9{\cdot}73 \times 10^{5}$	**t** $6{\cdot}3 \times 10^{-3}$
u $4{\cdot}7 \times 10^{7}$	**v** $9{\cdot}142 \times 10^{2}$	**w** $1{\cdot}032 \times 10^{-2}$	**x** $1{\cdot}0 \times 10^{8}$

FUN SPOT 6:04

WHY IS A ROOM FULL OF MARRIED PEOPLE ALWAYS EMPTY?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.



Write in index form.

E 10×10 **E** $8 \times 8 \times 8$ **E** $yyyyaa$

Find the value of:

E 3^2 **E** 2^3 **E** $5^2 - 5$

A 10^3 **A** $2^3 - 3^2$

Find the value of x in:

I $2^x = 16$ **I** $3^x = 9$

Write as a basic numeral.

I $1{\cdot}7 \times 10^2$ **I** $6{\cdot}04 \div 10$

Evaluate:

O x^0 **O** $7^0 + 5$ **U** $11y^0$ **N** 2^{-2} **N** 10^{-3}

N $(\frac{2}{3})^{-1}$ **N** $7a^2 \times 3a^{-2}$ **B** $5m^{-3} \times 2m^3$ **C** $7a \times a^{-1}$

H To fill a jar in 6 minutes, Jan doubled the number of peanuts in the jar every minute. After how many minutes was the jar half-full?

Simplify:

S $3x - x$ **S** $3x \times x$ **T** $5 \times 5x$ **R** $10x \div 10$

T $x^2 \times x^3$ **T** $x^{10} \div x^2$ **S** $(x^2)^3$ **L** $60x^3 \div 5x^3$

S $\dfrac{4aab}{ab}$ **R** $x^{10} \times x^{-3}$ **P** $x^3 \div x^{-1}$ **G** $\dfrac{1}{10^2}$

10	10^2	7	-1	11	$2x$	9	x^5	5	8^3	x	8	2	$3x^2$	$\frac{1}{4}$	1	$25x$	

1000	$4a$	0·604	$1\frac{1}{2}$	0·01	12	y^4a^2	x^4	20	x^7	x^6	6	21	4	$\frac{1}{1000}$	170	x^8	

6:05 Scientific notation and the calculator

PREP QUIZ 6:05

Write in scientific notation.

1 690 **2** 4000 **3** 963·2 **4** 0·073 **5** 0·0003

Rewrite as basic numerals.

6 $2{\cdot}9 \times 10^{3}$ **7** $8{\cdot}0 \times 10^{5}$ **8** $4{\cdot}6 \times 10^{-2}$ **9** 5×10^{-7} **10** $8{\cdot}14 \times 10^{-1}$

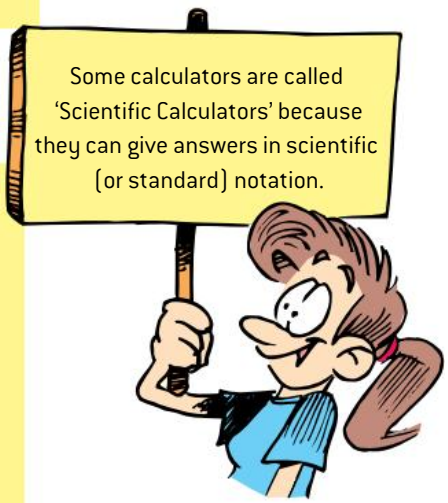
To enter scientific notation, press:

- 5.517 [×10^x] 12 to enter $5{\cdot}517 \times 10^{12}$
- 3.841 [×10^x] [(–)] 6 to enter $3{\cdot}841 \times 10^{-6}$

On some calculators:

- $5{\cdot}517 \times 10^{12}$ is shown as 5.517 12
- $3{\cdot}841 \times 10^{-6}$ is shown as 3.841 -06

On some calculators the [×10^x] key is shown as [EXP].



To convert calculator answers into decimal form:

$2{\cdot}16 \times 10^{-3}$

First part — Index

1. Locate the decimal point in the first part of the number (the part between 1 and 10).
2. Look at the sign of the index. This tells you in which direction to move the decimal point.
   If it is negative the point moves to the left.
   If it is positive the point moves to the right.
3. Look at the size of the index. This tells you how many places the decimal point has to be moved.
4. Move the decimal point to its new position, filling in any gaps, where necessary, with zeros.

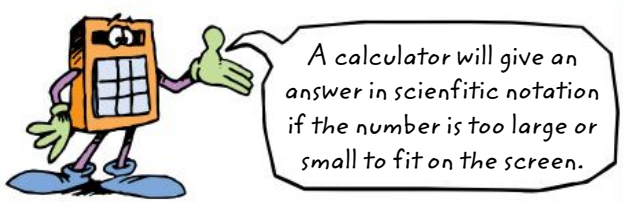
0·002 16

WORKED EXAMPLES

Use a calculator to find the answers.

1 $630\,000 \times (47\,000)^2$
$= 1{\cdot}391\,67 \times 10^{15}$
$= 1\,391\,670\,000\,000\,000$

2 $45 \div (8614)^3$
$= 7{\cdot}040\,409\,359 \times 10^{-11}$
$= 0{\cdot}000\,000\,000\,070\,404\,093\,59$



The answers to 1 and 2 are too long to fit on the screen.

3 $(8{\cdot}4 \times 10^6) + (3{\cdot}8 \times 10^7)$
Press: 8.4 [×10^x] 6 [+] 3.8 [×10^x] 7 [=]
$= 46\,400\,000$

4 $\sqrt{1{\cdot}44 \times 10^{-6}}$
Press: [√■] 1.44 [×10^x] [(–)] 6 [=]
$= 0{\cdot}0012$

Note: Not all calculators work the same way.

Exercise 6:05

1 Enter each of these on your calculator using the $\times 10^x$ key, and copy the readout.

a $6{\cdot}3 \times 10^4$ **b** $1{\cdot}4 \times 10^{-3}$ **c** $9{\cdot}25 \times 10^7$

2 Rewrite these calculator answers in scientific notation using powers of 10.

a 56000 **b** 43000000 **c** 763000

d 0.0004 **e** 0.0000029 **f** 0.0000731

g 1500 **h** 0.00278 **i** 6090000

j Explain why a calculator entry of 2×10^4 has a different value to 2^4.

$1{\cdot}402 \times 10^7$ has four significant figures, as four figures are used in the decimal part.

3 Place the nine answers in Question **2** in order of size from smallest to largest.

4 Give the answers to these in scientific notation, correct to five significant figures.

a $38\,14^4$ **b** $0{\cdot}0004 \div 84\,00^2$

c $(0{\cdot}000\,7)^5$ **d** $93\,000\,000 \div 0{\cdot}000\,13$

e $(65 \times 847)^3$ **f** $(0{\cdot}0045)^3 \times (0{\cdot}0038)^2$

g $\dfrac{9865 \times 8380}{0{\cdot}000\,021}$ **h** $\dfrac{6800}{(0{\cdot}0007)^5}$

Use index laws to check the size of your answer.

5 Use a calculator to answer correct to four significant figures, and then use the index laws to check your answer.

a $13{\cdot}85 \times (2{\cdot}3 \times 10^4)$ **b** $(8{\cdot}14 \times 10^{-2})^2$ **c** $(2{\cdot}1 \times 10^8) \div (8{\cdot}6 \times 10^8)$

d $(3{\cdot}8 \times 10^{-3})^2$ **e** $468 \times (1{\cdot}8 \times 10^{-5})$ **f** $(9{\cdot}1 \times 10^4) + (6{\times}8 \times 10^5)$

g $\sqrt{7{\cdot}45 \times 10^9}$ **h** $\sqrt[3]{9{\cdot}1 \times 10^{-8}}$ **i** $\sqrt[3]{6{\cdot}714 \times 10^{-12}}$

6 **a** An American reported that the diameter of the Sun is approximately $8{\cdot}656 \times 10^5$ miles. Write this in kilometres using scientific notation correct to four significant figures. (There are 1·609 km in a mile.) If the Sun's diameter is 109 times that of Earth, what is Earth's diameter, correct to three significant figures?

b The distance to the Sun varies from $1{\cdot}47 \times 10^8$ km in January to $1{\cdot}52 \times 10^8$ km in July. This is because Earth's orbit is an ellipse. What is the difference between these distances?

c If we use the average distance to the Sun ($1{\cdot}50 \times 10^8$ km), how long would it take light travelling at $3{\cdot}0 \times 10^8$ m/s to reach Earth? (Answer correct to the nearest minute.)

d The mass of Earth is approximately 6×10^{21} tonnes. The Sun's mass is about 333 400 times greater than the mass of Earth. What is the mass of the Sun correct to one significant figure?

e We belong to the galaxy known as the Milky Way. It contains about 1×10^{11} stars. If the Sun is taken to have average mass (see part **d**), what is the total mass, correct to one significant figure, of the stars in the Milky Way?

INVESTIGATION 6:05 USING SCIENTIFIC NOTATION

1 The speed of light is $3{\cdot}0 \times 10^8$ m/s. Use reference materials (such as books or the internet) and your calculator to complete this table for five stars of your choice (e.g. Vega, Polaris, Betelgeuse, Antares, Sirius).

Name of star	Distance from Earth	Time taken for light to travel to Earth
The Sun	$1{\cdot}5 \times 10^8$ km	
Alpha Centauri	$4{\cdot}1 \times 10^{13}$ km	

10^6 is 1 million.
10^9 is 1 billion.

Order the distances of the five stars from Earth, from smallest to largest.

2 Research nanotechnology, which involves the use of very small machine parts. Parts are often measured in micrometres. Make comparisons between the sizes of components.

Distances are measured in light years, which is the distance that light travels in a year.
A light year is approximately $9{\cdot}5 \times 10^{12}$ km.

6:06 Conversions using scientific notation

Very large and very small units of measurement were introduced in Section 1:10.

In these tables, each unit is 1000 times as big as the one on its right.

Tm	Gm	Mm	km	m	mm	μm	nm
10^{12} m	10^{9} m	10^{6} m	10^{3} m	1 m	10^{-3} m	10^{-6} m	10^{-9} m

1 terabyte = 1000 gigabytes (1 TB = 1000 GB)
1 gigabyte = 1000 megabytes (1 GB = 1000 MB)
1 megabyte = 1000 kilobytes (1 MB = 1000 kB)

TB	GB	MB	kB	B
10^{12} B	10^{9} B	10^{6} B	10^{3} B	1 B

1 second = 1000 milliseconds (1 s = 1000 ms)
1 millisecond = 1000 microseconds (1 ms = 1000 μs)
1 microsecond = 1000 nanoseconds (1 μs = 1000 ns)

s	ms	μs	ns
1 s	10^{-3} s	10^{-6} s	10^{-9} s

- When using scientific notation, all significant figures are shown in the first part of the number.
  e.g. 7×10^{12} B has 1 sig. fig. $7{\cdot}000 \times 10^{-3}$ s has 4 sig. fig.
  $3{\cdot}4 \times 10^{-3}$ s has 2 sig. fig. $3{\cdot}40 \times 10^{-3}$ s has 3 sig. fig.

PREP QUIZ 6:06

Convert:

1. 4 terabytes (TB) to gigabytes (GB)
2. 7 gigabytes (GB) to megabytes (MB)
3. 6 megabytes (MB) to kilobytes (kB)
4. 9 kilobytes (kB) to bytes (B)
5. 2 metres (m) to millimetres (mm)
6. 3 millimetres (mm) to micrometres (μm)
7. 5 micrometres (μm) to nanometres (nm)
8. 4 megametres (Mm) to metres (m)
9. 8 metres (m) to micrometres (μm)
10. 7 metres (m) to nanometres (nm)

WORKED EXAMPLES

1 Use scientific notation to write 3·4 terabytes as bytes.
1 TB = 10^{12} B, so
3·4 TB = $3{\cdot}4 \times 10^{12}$ B

2 Use scientific notation to write 7·8 microseconds as seconds.
1 μs = 10^{-6} s, so
7·8 μs = $7{\cdot}8 \times 10^{-6}$ s

3 Use scientific notation to write 850 MB as bytes.
1 MB = 10^{6} B, so
850 MB = 850×10^{6} B
(However, the first part needs to be a number between 1 and 10.)
= $(8{\cdot}5 \times 10^{2}) \times 10^{6}$ B
= $8{\cdot}5 \times 10^{8}$ B

4 Use scientific notation to write 67 ns as seconds.
1 ns = 10^{9} s, so
67 ns = 67×10^{-9} s
(However, the first part needs to be a number between 1 and 10.)
= $(6{\cdot}7 \times 10^{1}) \times 10^{-9}$ s
= $6{\cdot}7 \times 10^{-8}$ s

Exercise 6:06

1 How many significant figures do each of these measurements have?

a $9{\cdot}1 \times 10^{7}$ B **b** $4{\cdot}125 \times 10^{-6}$ s **c** $3{\cdot}00 \times 10^{-5}$ s **d** 9×10^{8} L

2 Use scientific notation to write each of these as bytes (B).

a 8000 bytes **b** 6 kilobytes **c** 4 megabytes **d** 8 gigabytes
e 4·7 terabytes **f** 2·4 megabytes **g** 6·15 kilobytes **h** 8·0 gigabytes
i 9·2TB **j** 7·70 MB **k** 5·15 kB **l** 8·0 GB
m 15TB **n** 103 MB **o** 852 kB **p** 605 GB
q 80 000 kB (3 sig.fig.) **r** 4000 GB (nearest 100 gigabytes)

3 Use scientific notation to write each of these as seconds (s).

a 8 milliseconds **b** 7 microseconds **c** 2 nanoseconds
d 9 milliseconds **e** 3·60 milliseconds **f** 8·4 microseconds
g 7·0 nanoseconds **h** 9·56 milliseconds **i** 1·25 ns
j 7·7 µs **k** 8·125 ms **l** 8·0 µs
m 15 ms **n** 103 µs **p** 605 µs
q 80 000 µs (2 sig.figs.) **r** 20 000 ns (nearest 100 nanoseconds)

4 The capacity of Warragamba Dam is $2{\cdot}031 \times 10^{12}$ L. Write this in ML, GL and TL.

5 In biology, the sizes of viruses and bacteria are measured in nanometres (nm) and micrometres (µs). A nanometre is one thousand-millionth of a metre.
How many nanometres are in:

a a metre
b a millimetre
c a micrometre?

6 In physics, the frequency of electromagnetic radiation is measured in kilohertz (kHz) and gigahertz (GHz). A gigahertz is one thousand million hertz.
Convert 1 gigahertz into:

a kilohertz **b** megahertz

7 The hard disk of a computer was reported by the manufacturer to have a capacity of about 400 gigabytes. However, the operating system of the computer reported that the hard disk had a capacity of 372 GB. This confusion occurs because capacity within a hard drive is always measured as a power of 2 bytes.
Find the difference, correct to four significant figures, between 1 GB and 2^{30} bytes (which is sometimes referred to as a gigabyte).

MATHS TERMS 6

base

- the term that is operated on by the index,
  e.g. for x^n, x is the base
  for 5^3, 5 is the base

exponent

- another term for a power or index
- equations that involve a power are called exponential equations,
  e.g. $3x = 2^7$

index

- a number indicating how many of a base term need to be multiplied together,
  e.g. for x^n, n is the index

$$x^n = \underbrace{x \times x \times x \times x \ldots \times x}_{n \text{ factors}}$$

- the plural of index is indices

negative indices

- indicate the reciprocal of a term,
  e.g. $x^{-1} = \frac{1}{x}, x^{-n} = \frac{1}{x^n}$

  i.e. $5^{-1} = \frac{1}{5}, 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

power

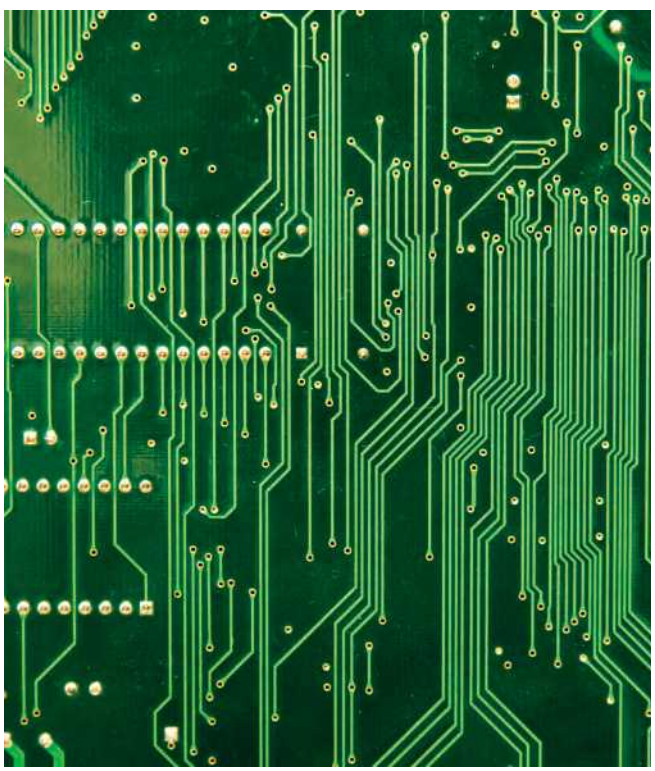
- another term for an index or exponent

scientific (standard) notation

- a useful way to write very big or very small numbers
- numbers are written as the product of a number between 1 and 10 and a power of 10,
  e.g. $76\,000\,000 = 7{\cdot}6 \times 10^7$
  $0{\cdot}000\,0054 = 5{\cdot}4 \times 10^{-6}$

zero index

- a term or number with a zero index is equal to 1,
  e.g. $x^0 = 1, 4^0 = 1$



A computer's memory is measured in gigabytes. In computing, a gigabyte is 2^{30} bytes. Write this number in scientific notation correct to one significant figure.

DIAGNOSTIC TEST 6 INDICES

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Express in index form:		6:01
a $3 \times 3 \times 3 \times 3$	b 5×5	c $m \times m \times m$
2 Evaluate:		6:01
a 3^2	b 2^4	c 10^3
3 Simplify:		6:02
a $3^2 \times 3^5$	b $x^3 \times x^2$	c $6m^2n \times mn^4$
4 Simplify:		6:02
a $x^7 \div x^2$	b $15a^5 \div 3a^2$	c $20a^3b^2 \div 10ab$
5 Simplify:		6:02
a $(a^4)^2$	b $(x^3)^4$	c $(2a^4)^3$
6 Simplify:		6:03
a 7^0	b $5p^0$	c $18x^3 \div 6x^3$
7 Simplify:		6:03
a 3^{-2}	b 5^{-1}	c $(\frac{2}{3})^{-3}$
8 Simplify, writing answers without negative indices.		6:04
a $x^7 \times x^{-3}$	b $6x^2 \div 3x^4$	c $(3x^{-1})^2$
9 Express in scientific notation.		6:04
a 243	b 67 000	c 93 800 000
10 Write as a basic numeral.		6:04
a $1{\cdot}3 \times 10^2$	b $2{\cdot}431 \times 10^2$	c $4{\cdot}63 \times 10^7$
11 Express in scientific notation.		6:04
a 0·043	b 0·000 0597	c 0·004
12 Write as a basic numeral.		6:05
a $2{\cdot}9 \times 10^{-2}$	b $9{\cdot}38 \times 10^{-5}$	c $1{\cdot}004 \times 10^{-3}$
13 Simplify, giving answers in scientific notation.		6:05
a $(3{\cdot}1 \times 10^8)^2$	b $(8{\times}4 \times 10^6) + (3{\times}8 \times 10^7)$	
c $\sqrt{1{\cdot}96 \times 10^{24}}$	d $\sqrt{1{\cdot}44 \times 10^{-6}}$	
14 Write each of these as bytes (B), using scientific notation.		6:06
a 270 000 B	b 25 kilobytes (kB)	
c 4·3 terabytes (TB)	d 12 GB	
15 Convert these quantities, using scientific notation.		6:06
a 5 s = … μs	b 500 g = … mg	c 3 ns = … s d 5 μg = … g

ASSIGNMENT 6A Chapter review

1 Simplify, writing the answers in index form:

a $a^2 \times a^3$ **b** $3a^2 \times 4a^3$

c $a^2b \times ab$ **d** $3a^2b \times 4ab^2$

e $3^2 \times 3^3$ **f** $a^6 \div a^3$

g $7m^2 \div m$ **h** $12y^6 \div 3y^2$

i $20a^4b^3 \div 10a^2b^2$ **j** $4^7 \div 4^3$

k $(3^2)^4$ **l** $(x^2)^3$

m $(a^3)^2 \times a^5$ **n** $m^7 - (m^2)^3$

o $\dfrac{12x^2}{6x}$ **p** $\dfrac{4a^3}{8a}$

2 Express in simplest form:

a $(2x^2)^0$ **b** $6x^0$

c $(5x^3)^3$ **d** $(10a^2)^3$

e $(4x^2)^3 \div 8x^5$

3 Write each of the following in standard form (scientific notation).

a 21 600 **b** 125

c 0·000 07 **d** 0·000 156

4 Write each of the following as a basic numeral.

a $8{\cdot}1 \times 10^5$ **b** $1{\cdot}267 \times 10^3$

c $3{\cdot}5 \times 10^{-2}$ **d** $1{\cdot}06 \times 10^{-4}$

5 Use your calculator to evaluate:

a 2^{10} **b** 3^{12}

c $5^5 \times 6^6$ **d** $7^3 \times 4^5$

6 Find the value of n, if:

a $2^n = 128$ **b** $3^n = 243$

c $10^n = 100\,000\,000$

7 Simplify and evaluate:

a $3^2 \times 3^5$ **b** $10^7 \div 10^4$ **c** $(2^4)^2$

8 Simplify:

a $\dfrac{m^7 \times m^6}{m^{10}}$ **b** $(2a^3)^3 \times (3a^4)^2$

c $\dfrac{8x^7 \times 9x^4}{6x^6 \times 6x^5}$

9 Evaluate (writing answers in fraction form):

a 5^{-1} **b** 2^{-4}

c 3^{-2} **d** 10^{-3}

10 Simplify, giving your answers in scientific notation correct to two significant figures.

a 3579^5

b $(0{\cdot}0075)^{10}$

c $987\,654\,321 \times 123\,456\,789$

d $\dfrac{13\,579 \times 8642}{0{\cdot}000\,456}$

e $(2{\cdot}3 \times 10^7) \times (5{\cdot}6 \times 10^5)$

f $(2{\cdot}3 \times 10^7) + (5{\cdot}6 \times 10^5)$

g $(2{\cdot}3 \times 10^7) \div (5{\cdot}6 \times 10^5)$

h $(2{\cdot}3 \times 10^7) - (5{\cdot}6 \times 10^5)$

The inventor of the game of chess is said to have asked for 18 400 000 000 000 000 000 grains of wheat as a reward. What is this number in scientific notation?

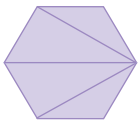
ASSIGNMENT 6B Working mathematically

1 Refer to **ID Card 7** on page xxii to identify:

a 5 **b** 8 **c** 17 **d** 18
e 19 **f** 20 **g** 21 **h** 22
i 23 **j** 24

2 Refer to **ID Card 6** on page xxi to identify the items numbered 1 to 12.

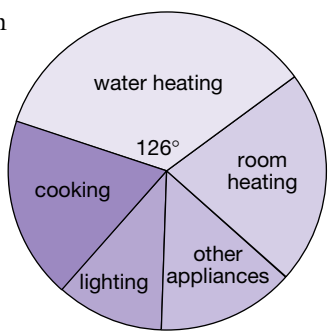
3 **a** How many diagonals can be drawn from one vertex of a regular hexagon? How many vertices has a hexagon?



b Each diagonal joins two vertices and a diagonal cannot be drawn from a vertex to the two adjacent vertices or to itself. The number of diagonals of a hexagon is $\frac{6(6-3)}{2}$. How many diagonals has:

i a regular octagon
ii a regular decagon
iii a regular polygon that has 30 sides?

4 **a** The sector graph shows the usage of electricity in the average household. What percentage of electricity is used in heating water?



b Use a protractor to measure each angle on the graph.

c Construct a column graph to represent the information shown in this graph.

5 Tom was given a cheque for an amount between $31 and $32. The bank teller made a mistake and exchanged dollars and cents on the cheque. Tom took the money without examining it and gave 5 cents to his son.

He now found that he had twice the value of the original cheque. If he had no money before entering the bank, what was the amount of the cheque?

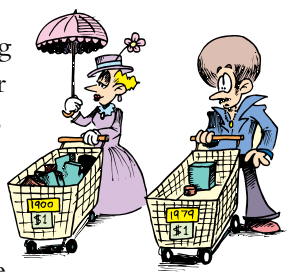
6 Refer to the graph below.

a In which year did the dollar have its greatest purchasing power? What was the dollar called at that time?

b How many times more purchasing power did the dollar have in 1900 than in 1978/79?

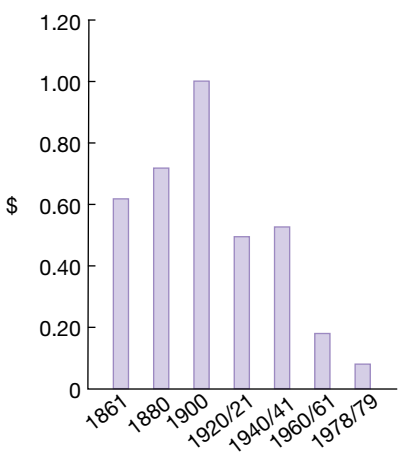
c Why doesn't $1 buy as much now as in the past?

d Give some reasons why the purchasing power of the dollar may have been less before 1900 than in 1900.



e Compared to 1900, what was the purchasing power of the dollar in:

i 1940/41 **ii** 1960/61
iii 1920/21?



THE DOLLAR IN YOUR POCKET
(Base: 1900 = $1.00)
The dollar, valued in terms of the basket of goods it could buy in 1900, was only worth 6c in 1978/79, so the goods that could be purchased for $1.00 in 1900 would cost about $16.70 in 1978/79.

ASSIGNMENT 6C Cumulative revision

1:04

1 Write the following decimals as fractions.

a $0{\cdot}8$ **b** $0{\cdot}125$ **c** $0{\cdot}002$

d $0{\cdot}\dot{5}$ **e** $0{\cdot}\dot{1}\dot{5}$ **f** $0{\cdot}6\dot{8}$

2:01D

2 The population of a small town was 9500. After the discovery of copper, the town's population increased in the first year by 20%. In the second year, it increased by another 10%.

a What was the population of the town at the end of the 2-year period?

b What was the percentage increase in the population over the 2-year period?

3:01

3 Simplify:

a $2a + 3 - a + 3$ **b** $2a + 3 - a - 3$ **c** $2a - 3 + a + 3$ **d** $2a - 3 - a - 3$

3:03

4 Simplify:

a $10a \div 5$ **b** $10a \div 5a$ **c** $10ab \div 5a$ **d** $10ab \div 20b$

4:06

5 Two big bars of chocolate are offered as prizes by a maths teacher in a quiz contest. Three boys and two girls all end in a tie and it is decided to break the tie in true mathematical fashion by drawing two names one after the other from the hat. What is the probability that:

a the two girls names are chosen

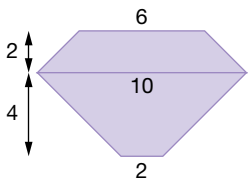
b two boys names are chosen

c at least one of the chocolates is won by a girl?

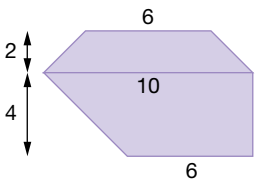
5:02

6 Two trapeziums have been used to form the hexagons shown. What is the area of each hexagon? (All measurements are in centimetres.)

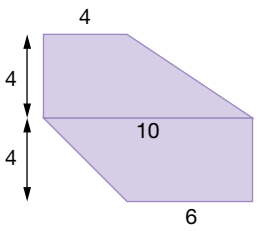
a



b

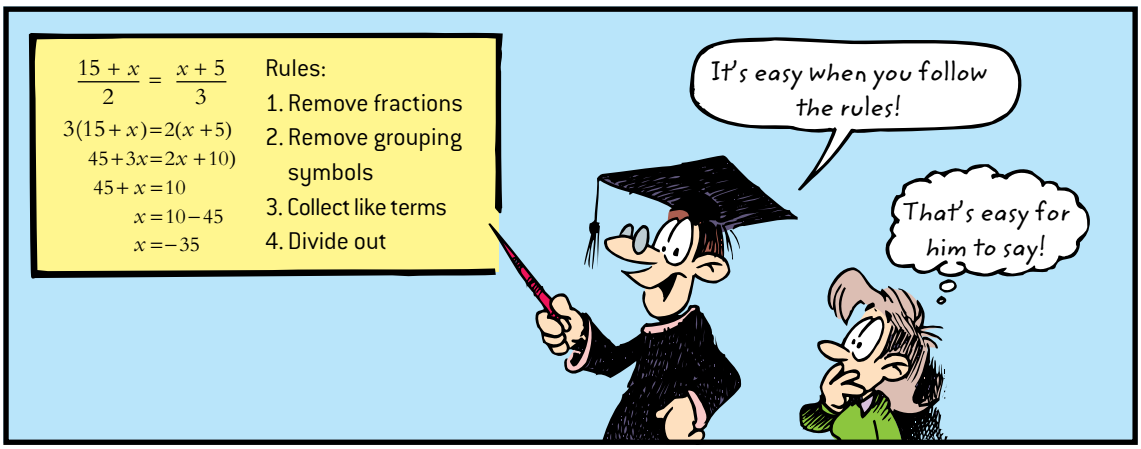


c



EQUATIONS AND INEQUALITIES

7



Contents

7:01 Solving equations by inspection
7:02 Inverse operations
7:03 Solving simple equations
Investigation 7:03 Solving equations using a spreadsheet
7:04 Equations with pronumerals on both sides
GeoGebra activity 7:04 Equations with pronumerals on both sides
7:05 Equations with grouping symbols
GeoGebra activity 7:05 Equations with grouping symbols
Fun spot 7:05 If I have 7 apples in one hand and 4 in the other, what have I got?
7:06 Equations with fractions (1)
Fun spot 7:06 Who holds up submarines?
7:07 Equations with fractions (2)
Challenge 7:07 Solving equations by 'guess and check'
7:08 Graphing inequalities
7:09 Solving inequalities
Investigation 7:09 Operating on inequalities
Fun spot 7:09 Who 'dunnit'?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Equations*** [Stage 5.2]

- Solve linear equations. (ACMNA215)
- Solve linear equations involving simple algebraic fractions. (ACMNA240)
- Solve linear inequalities and graph their solutions on a number line. (ACMNA236)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

7:01 Solving equations by inspection

- Equations are number sentences where one or more of the numbers is missing or unknown. Because it is unknown, the number is represented by a pronumeral.
- When we solve an equation, we are trying to find the numerical value of the pronumeral that makes the sentence true. With some equations it is easy to find this value or solution. With harder equations more work has to be done before the solution is found.
- A solution is correct if it gives a true number sentence when it replaces the pronumeral in the equation. We say that the solution satisfies the equation.

> An equation is a number sentence where one or more of the numbers have been replaced by a pronumeral.

Often we can just 'see' the answer to an equation. When we write the answer we are solving the equation by inspection.

PREP QUIZ 7:01

Make the number sentences true by finding the missing number.

1 $\square + 6 = 8$ **2** $5 - \triangle = 2$ **3** $3 \times \oplus = 36$ **4** $10 \div \star = 2$

Find another way of writing:

5 $x \div 6$ **6** $5 \times y$ **7** $\frac{1}{2}y$ **8** $\frac{p}{4}$

Which of the following values makes the sentence true?

9 $\square + 3 = 15$ $\{11, 12, 13\}$ **10** $9 - \triangle = 5$ $\{4, 5, 2\}$

WORKED EXAMPLE 1

Find the value of the pronumeral in each of the following.

a $x + 6 = 8$ **b** $6a = 18$ **c** $\frac{p}{4} = 3$ **d** $y - 3 = 7$

Solution

a

x	+	6	=	8	
2	+	6	=	8	
		$\therefore x$	=	2	

b

6	×	a	=	18	
6	×	3	=	18	
		$\therefore a$	=	3	

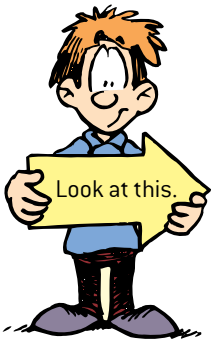
c

p	÷	4	=	3	
12	÷	4	=	3	
		$\therefore p$	=	12	

d

y	−	3	=	7	
10	−	3	=	7	
		$\therefore y$	=	10	

Therefore $\therefore$



- Notice how we compare the equation with an equivalent number sentence.
- The parts in the boxes are equal.
- Often these can be solved mentally. There is no need to write the number sentence underneath the equation.
- Always check your solution by substituting back into the equation.
- If in trouble, see whether you can express the equation in words.
  e.g. $\frac{p}{4} = 3$ means 'What number does 4 go into 3 times?'

WORKED EXAMPLE 2

An equation and a solution are given. Check to see whether each solution is correct or incorrect by substituting the value into the equation.

a $x + 9 = 11$
$x = 3$

b $5x = 35$
$x = 7$

c $y - 3 = 11$
$y = 8$

d $\frac{a}{6} = 9$
$a = 54$

Solution

a $x + 9 = 11$
If $x = 3$ then
$x + 9 = 3 + 9$
$= 12$
But $12 \neq 11$
$\therefore x = 3$
is incorrect

b $5x = 35$
If $x = 7$ then
$5x = 5 \times 7$
$= 35$
Now $35 = 35$
$\therefore x = 7$
is correct

c $y - 3 = 11$
If $y = 8$ then
$y - 3 = 8 - 3$
$= 5$
But $5 \neq 11$
$\therefore y = 8$
is incorrect

d $\frac{a}{6} = 9$
If $a = 54$ then
$\frac{a}{6} = \frac{54}{6}$
$= 9$
Now $9 = 9$
$\therefore a = 54$
is correct

Exercise 7:01

P Foundation worksheet 7:01
Solving equations by inspection

1 Solve the following equations.

a $x + 5 = 10$ **b** $y + 7 = 20$ **c** $12 + y = 16$ **d** $9 = 3 + x$
e $x - 1 = 5$ **f** $7 - y = 1$ **g** $6 = 8 - p$ **h** $x - 10 = 20$
i $5 \times y = 30$ **j** $10p = 100$ **k** $7x = 28$ **l** $120 = 6m$
m $x \div 6 = 3$ **n** $\frac{y}{4} = 5$ **o** $8 \div p = 4$ **p** $\frac{6}{p} = 2$

2 Find the value of the pronumeral in each of the following equations.

a $6x = 66$ **b** $\frac{m}{3} = 9$ **c** $y - 7 = 7$ **d** $6 + p = 13$
e $3 \div b = 3$ **f** $a + 3 = 3$ **g** $\frac{1}{3}x = 5$ **h** $2y = 3$
i $14 - p = 6$ **j** $\frac{12}{x} = 1$ **k** $20 = 16 + m$ **l** $12 = 20 - t$
m $\frac{x}{3} = 8$ **n** $\frac{1}{2}p = 7$ **o** $30 - x = 15$ **p** $5 + a = 12$

3 Each question contains an equation and a solution. Check to see whether the solution is correct or incorrect by substituting the solution into the equation. (If it's correct you will obtain a true number sentence.)

a $x + 9 = 14$
$x = 5$

b $y + 7 = 11$
$y = 5$

c $12 = a + 7$
$a = 6$

d $16 + b = 20$
$b = 4$

e $p - 3 = 6$
$p = 9$

f $3 - x = 4$
$x = 2$

g $10 = m - 7$
$m = 3$

h $a - 4 = 4$
$a = 0$

i $3x = 18$
$x = 6$

j $5y = 45$
$y = 8$

k $60 = 5p$
$p = 10$

l $4a = 1$
$a = \frac{1}{4}$

m $\frac{1}{2}m = 7$
$m = 14$

n $\frac{y}{3} = 9$
$y = 3$

o $\frac{b}{6} = 2$
$b = \frac{1}{3}$

p $4a = 2$
$a = 2$

4 Under each equation is a set of solutions. Only one is correct. Find the correct solution by substitution.

a $x + 27 = 53$
$\{25, 26, 27\}$

b $6 + y = 4$
$\{2, 0, -2\}$

c $12 = p - 4$
$\{8, 16, 10\}$

d $3b = 69$
$\{24, 23, 30\}$

e $\frac{x}{4} = 8$
$\{2, 32, 16\}$

f $\frac{6}{x} = 3$
$\{18, \frac{1}{2}, 2\}$

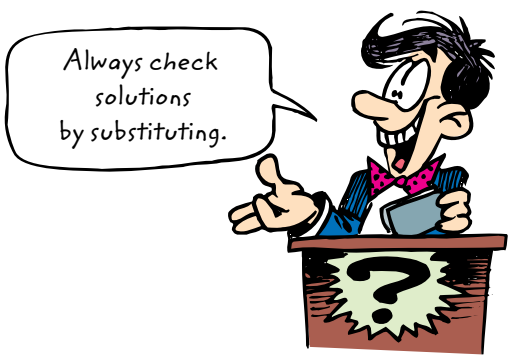
g $138 = m + 69$
$\{70, 45, 69\}$

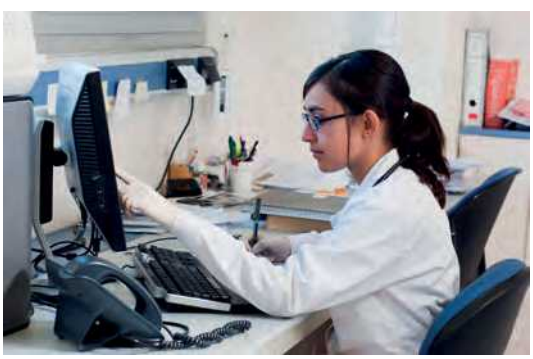
h $\frac{1}{2}a = 7$
$\{3\frac{1}{2}, 14, 7\}$

5 The left-hand column gives a set of equations, **a** to **h**. The right-hand column gives the solutions to these equations, **A** to **H**. Match each equation with its solution.

a $x + 7 = 10$
b $7y = 28$
c $\frac{p}{6} = 10$
d $12 \div x = 12$
e $30 = 32 - p$
f $16 - m = 5$
g $7 + p = 12$
h $\frac{1}{2}p = 20$

A 3
B 40
C 1
D 60
E 2
F 4
G 11
H 5





Computers are used to solve many equations.

7:02 Inverse operations

PREP QUIZ 7:02

Write an algebraic expression for the following.

1 add 7 to x
2 multiply c by 4
3 divide y by 3
4 take 9 away from a
5 multiply m by 4 and add 8
6 subtract 5 from x and multiply the result by 4

What is the opposite of:

7 adding 6
8 subtracting 11
9 multiplying by 3
10 dividing by 7?

Arrow diagrams can be used to show how an expression has been built from a pronumeral. The operation that has been applied is written above the arrow.

When solving equations we need to reverse the steps to get back to the pronumeral. We need to do the 'inverse' operations. For example, if:

$x \xrightarrow{+3} x+3$ then $x+3 \xrightarrow{-3} x$

$x \xrightarrow{-3} x-3$ then $x-3 \xrightarrow{+3} x$

$x \xrightarrow{\times 3} 3x$ then $3x \xrightarrow{\div 3} x$

$x \xrightarrow{\div 3} \frac{x}{3}$ then $\frac{x}{3} \xrightarrow{\times 3} x$

'Inverse' means opposite.

$2 \xrightarrow{+3} 5$

$5 \xrightarrow{-3} 2$

Adding 3 and subtracting 3 are inverse operations.

WORKED EXAMPLES

1 Note how each expression has been built from the pronumeral.

a $y \xrightarrow{-7} y-7$

b $a \xrightarrow{\times 3} 3a \xrightarrow{+2} 3a+2$

c $q \xrightarrow{+5} q+5 \xrightarrow{\times 3} 3(q+5)$

d $m \xrightarrow{\div 2} \frac{m}{2} \xrightarrow{-1} \frac{m}{2}-1$

e $n \xrightarrow{-5} n-5 \xrightarrow{\div 3} \frac{n-5}{3}$

2 Note the inverse operations used to get back to the pronumeral.

a $7m \xrightarrow{\div 7} m$

b $5x+2 \xrightarrow{-2} 5x \xrightarrow{\div 5} x$

c $2(y+3) \xrightarrow{\div 2} y+3 \xrightarrow{-3} y$

d $\frac{a-2}{4} \xrightarrow{\times 4} a-2 \xrightarrow{+2} a$

Exercise 7:02

1 Copy and complete the following arrow diagrams by writing the correct operation above the arrow.

a $x \xrightarrow{\quad} 5x$

b $a \xrightarrow{\quad} a + 7$

c $q \xrightarrow{\quad} q - 4$

d $m \xrightarrow{\quad} \frac{m}{3}$

e $a \xrightarrow{\quad} 4a \xrightarrow{\quad} 4a + 1$

f $x \xrightarrow{\quad} x - 8 \xrightarrow{\quad} 3(x - 8)$

g $y \xrightarrow{\quad} -7y \xrightarrow{\quad} -7y + 2$

h $m \xrightarrow{\quad} m + 3 \xrightarrow{\quad} \frac{m+3}{10}$

i $a \xrightarrow{\quad} \frac{a}{7} \xrightarrow{\quad} \frac{a}{7} + 6$

j $x \xrightarrow{\quad} x + 9 \xrightarrow{\quad} -6(x + 9)$

2 Complete these arrow diagrams, showing how to get back to the pronumeral.

a $6n \xrightarrow{\quad} n$

b $\frac{a}{2} \xrightarrow{\quad} a$

c $x + 9 \xrightarrow{\quad} x$

d $p - 4 \xrightarrow{\quad} p$

e $8n - 7 \xrightarrow{\quad} 8n \xrightarrow{\quad} n$

f $2 + 9y \xrightarrow{\quad} 9y \xrightarrow{\quad} y$

g $4 - 5x \xrightarrow{\quad} -5x \xrightarrow{\quad} x$

h $\frac{m}{4} - 1 \xrightarrow{\quad} \frac{m}{4} \xrightarrow{\quad} m$

i $\frac{x+2}{4} \xrightarrow{\quad} x + 2 \xrightarrow{\quad} x$

j $5 + \frac{a}{4} \xrightarrow{\quad} \frac{a}{4} \xrightarrow{\quad} a$

3 Use arrow diagrams to show how the following were made from the pronumeral.

a $7m$ **b** $3y - 5$ **c** $8a + 1$ **d** $-3a + 2$

e $5(p + 4)$ **f** $9 + 4m$ **g** $10(x - 7)$ **h** $7 - 5c$

i $\frac{x}{4} + 1$ **j** $\frac{m+6}{2}$ **k** $\frac{a-8}{7}$ **l** $6 + \frac{c}{3}$

4 Complete the following arrow diagrams.

a $x \xrightarrow{+4} \square$

b $m \xrightarrow{\times 5} \square$

c $t \xrightarrow{\div 2} \square$

d $y \xrightarrow{-4} \square$

e $a \xrightarrow{\times 2} \square \xrightarrow{+1} \square$

f $x \xrightarrow{+2} \square \xrightarrow{\times 9} \square$

g $n \xrightarrow{\times(-6)} \square \xrightarrow{-7} \square$

h $y + 6 \xrightarrow{-6} \square$

i $\boxed{a-3} \xrightarrow{+3} \boxed{}$

j $\boxed{9m} \xrightarrow{\div 9} \boxed{}$

k $\boxed{\frac{x}{4}} \xrightarrow{\times 4} \boxed{}$

l $\boxed{3+7x} \xrightarrow{-3} \boxed{} \xrightarrow{\div 7} \boxed{}$

m $\boxed{\frac{a+3}{5}} \xrightarrow{\times 5} \boxed{} \xrightarrow{-3} \boxed{}$

n $\boxed{5-3n} \xrightarrow{-5} \boxed{} \xrightarrow{\div(-3)} \boxed{x}$

5 What operations must be performed on x to get the following expressions?

a $2x+7$ **b** $-6x-2$ **c** $9x-1$ **d** $4(x-6)$ **e** $7(x+5)$ **f** $3(4+x)$

6 What must be done to these to get back to a?

a $3a+10$ **b** $15-11a$ **c** $20a-200$ **d** $\frac{a}{3}$ **e** $6+\frac{a}{5}$ **f** $\frac{a+3}{4}$

7 What order of inverse operations must be performed to get back to the pronumeral?

a $\frac{6a-1}{4}$ **b** $10(7x+2)$ **c** $\frac{7-2a}{3}$ **d** $8(4-5x)$

e $\frac{5b}{3}-4$ **f** $6+\frac{10m}{11}$ **g** $\frac{7(4-3a)}{10}$ **h** $\frac{6(2a-3)}{5}$

7:03 Solving simple equations

PREP QUIZ 7:03

Simplify the following.

1 $9x \div 9$ **2** $x+2-2$ **3** $a-5+5$ **4** $\frac{m}{3} \times 3$

What is the 'inverse' of:

5 multiplying by 7 **6** adding 8 **7** subtracting 1 **8** dividing by 4?

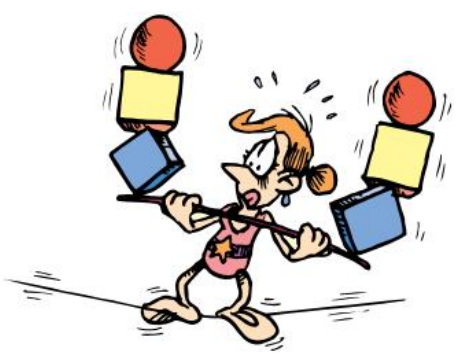
Complete these arrow diagrams.

9 $\boxed{7m+10} \rightarrow \boxed{7m} \rightarrow \boxed{m}$

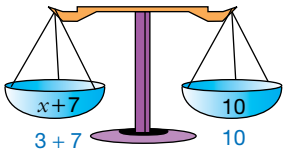
10 $\boxed{3a-2} \rightarrow \boxed{3a} \rightarrow \boxed{a}$

If one equation can be changed into another by performing the same operation on both sides, then the equations are said to be equivalent.

Solving equations is like balancing scales. One side of an equation is equal to the other. The solution of the equation is the value of the pronumeral that 'balances' the equation.

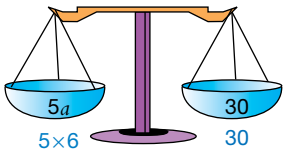


$x + 7 = 10$



$x = 3$ balances the scale,
so $x = 3$ is the solution.

$5a = 30$

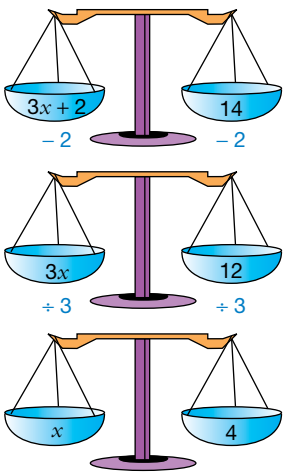


$a = 6$ balances the scale,
so $a = 6$ is the solution.

Solving an equation often requires us to change the equation into a simpler one. We can do this by adding (+), subtracting (–), multiplying (×) or dividing (÷) both sides of the equation by the same number.

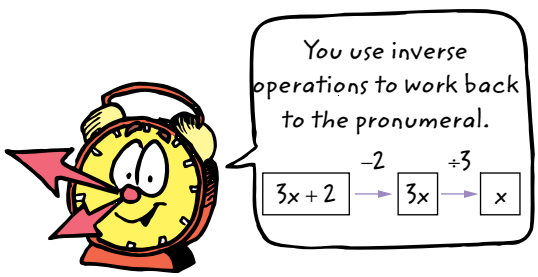
Study the solution to the following equations. Note that the sides remain balanced because we perform the same operation on both sides.

$3x + 2 = 14$

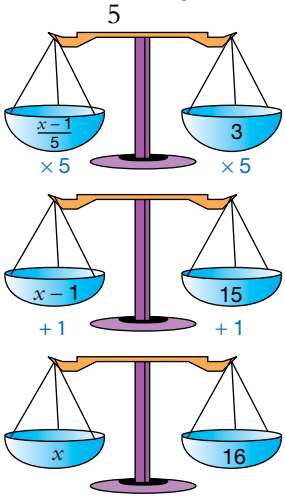


$x = 4$ is the solution.

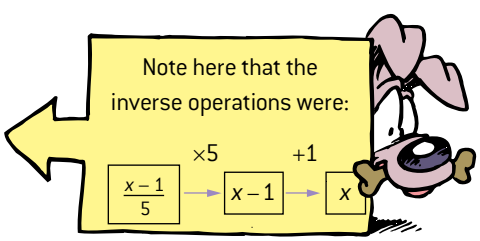
We need to perform operations that will leave only the pronumeral on one side of the equation.

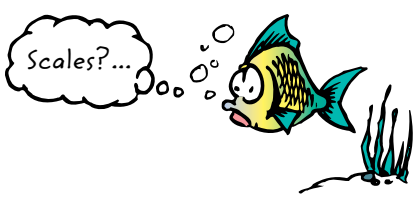


$\dfrac{x-1}{5} = 3$



$x = 16$ is the solution.





Inverse operations have been used to solve the equations in the following examples.

WORKED EXAMPLE 1

These solutions involve only one step.

a $m + 17 = 28$

$-17 \quad -17$

$m = 28 - 17$

$\therefore m = 11$

b $a - 13 = 31$

$+13 \quad +13$

$a = 31 + 13$

$\therefore a = 44$

c $3p = 5$

$\div 3 \quad \div 3$

$p = \frac{5}{3}$

$\therefore p = 1\frac{2}{3}$

d $\frac{x}{4} = 8$

$\times 4 \quad \times 4$

$x = 8 \times 4$

$\therefore x = 32$

WORKED EXAMPLE 2

These solutions involve two steps.

a $5m - 4 = 16$

$+4 \quad +4$

$5m = 20$

$\div 5 \quad \div 5$

$\frac{5m}{5} = \frac{20}{5}$

$\therefore m = 4$

b $8a + 6 = 15$

$-6 \quad -6$

$8a = 9$

$\div 8 \quad \div 8$

$\frac{8a}{8} = \frac{9}{8}$

$\therefore a = 1\frac{1}{8}$

c $1 - 3b = 7$

$-1 \quad -1$

$-3b = 6$

$\div -3 \quad \div -3$

$\frac{-3b}{-3} = \frac{6}{-3}$

$\therefore b = -2$

d $12 = 5x + 4$

$-4 \quad -4$

$8 = 5x$

$\div 5 \quad \div 5$

$\frac{8}{5} = \frac{5x}{5}$

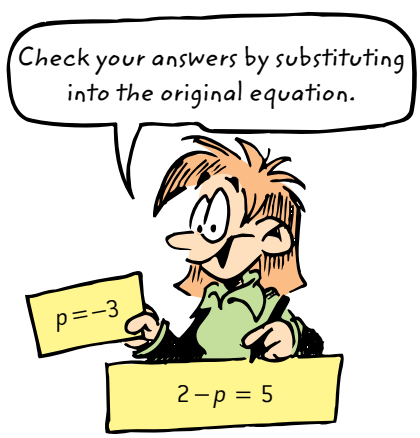
$\therefore x = 1\frac{3}{5}$

Exercise 7:03

P Foundation worksheet 7:03
Solving simple equations

1 Solve the following equations.

a $x + 72 = 138$
b $y + 37 = 68$
c $72 + m = 145$
d $725 = a + 473$
e $p - 64 = 237$
f $55 = x - 96$
g $x - 125 = 716$
h $a + 412 = 917$
i $\frac{x}{6} = 32$
j $\frac{p}{14} = 20$
k $\frac{a}{7} = 92$
l $\frac{m}{50} = 13$
m $10m = 96$
n $7m = 58$
o $36y = 728$



2 The answer to an equation can be checked by substituting it into the equation. Check to see whether the answer to each question below is correct.

a $x + 96 = 123$
$x = 27$

b $18 + y = 21$
$y = 5$

c $m - 13 = 26$
$m = 13$

d $2 - q = 0$
$q = 2$

e $100 - y = 76$
$y = 24$

f $14m = 20$
$m = \frac{10}{7}$

g $-4y = 12$
$y = -3$

h $\frac{x}{2} = 9$
$x = 18$

i $\frac{m}{2} = 1$
$m = 2$

$136 - x = 72$
$x = 64$
LHS $= 136 - 64$
$= 72$
$=$ RHS

3 Solving these equations involves two steps. Clearly show each step in your working. (All the answers are integers.)

a $4x + 1 = 21$
b $3a + 2 = 32$
c $6m + 7 = 31$
d $3n - 8 = 19$
e $5k - 1 = 44$
f $2t - 4 = 196$
g $11 + 5a = 26$
h $10 + 3w = 25$
i $12 + 4q = 16$
j $10 - 3x = 10$
k $15 - 2m = 15$
l $20 - 5q = 0$
m $10 = 2x - 6$
n $7 = 5y - 28$
o $-6 = 2 - 4a$
p $6 - x = -7$
q $-3 - x = -2$
r $3 = 4 - x$

4 The solutions to these equations involve fractions.

a $4x + 1 = 4$
b $8a + 5 = 10$
c $2m + 4 = 5$
d $7n + 2 = 8$
e $3p - 3 = 2$
f $2q - 1 = 2$
g $5n - 5 = 4$
h $6y - 3 = 1$
i $5 + 3k = 12$
j $1 + 3x = 9$
k $9 + 3a = 10$
l $4 + 2a = 4$
m $4m + 6 = 3$
n $5p + 7 = 4$
o $1 - 2a = 6$
p $8 - 3a = 1$

$3a + 2 = 6$
$-2 \quad -2$
$3a = 4$
$\div 3 \quad \div 3$
$a = \frac{4}{3}$ or $1\frac{1}{3}$

5 Try this set of equations that has one-step and two-step types.

a $7x = 35$
b $m + 9 = 24$
c $11x + 1 = 89$
d $12 - n = 0$
e $3a + 4 = 40$
f $a - 6 = -7$
g $y + 3 = 1$
h $3m - 1 = 5$
i $-8x = 16$
j $5 + 3n = 10$
k $3x = 2$
l $7 - q = 10$
m $6 - 2a = 12$
n $3p + 5 = -4$
o $420 - 2x = 20$
p $10 = 7 - 2p$

You must perform the same operation on both sides of an equation.

INVESTIGATION 7:03 SOLVING EQUATIONS USING A SPREADSHEET

A spreadsheet such as Microsoft® Excel® can be used to solve equations using the 'formula bar'. To solve an equation such as $2x + 5 = 11$ we need to enter the numbers into the appropriate 'cells' and tell the spreadsheet how to arrive at the answer for x.

- The equation $2x + 5 = 11$ is of the form $ax + b = c$.
- Enter the letters a, x, b and c in the first row as shown. These will act as headings.
- The numbers 2, 5 and 11 are then placed in cells A2, C2 and D2.
- To solve $2x + 5 = 11$ we would need to complete these steps:

$$2x + 5 - 11$$

$$2x - 11 - 5$$

$$x - \frac{11-5}{2}$$

- Matching the 'cells' with these numbers we would have:

$$x = \frac{D2 - C2}{A2}$$

- In cell B2 we type '=(D2–C2)/A2'. This also shows in the formula bar.
- When we press ENTER the answer 3 appears in cell B2.

B2 *fx* =(D2–C2)/A2

	A	B	C		D
1	a	x	b	c	
2	2	3	5		11
3					
4					

	A	B	C	D
1	a	x	b	c
2	2	3	5	11
3				
4				

We can now change any of the values for a, b or c and the value for x (i.e. the answer) will automatically change.

Try entering other values including negative numbers and decimals.

7:04 Equations with pronumerals on both sides

PREP QUIZ 7:04

Simplify the following.

1 $7x - 6x$ **2** $9a + a$ **3** $5y - 5y$ **4** $-p + p$

What operation should be written above each arrow?

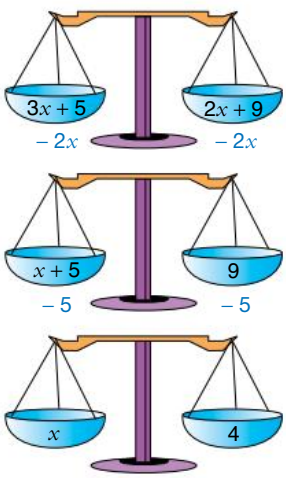
5 $6x + 9 \xrightarrow{\square} 6x$ **6** $5x - 2 \xrightarrow{\square} 5x$ **7** $\frac{x+11}{10} \xrightarrow{\square} x + 11$

Complete these arrow diagrams using the operation above each arrow.

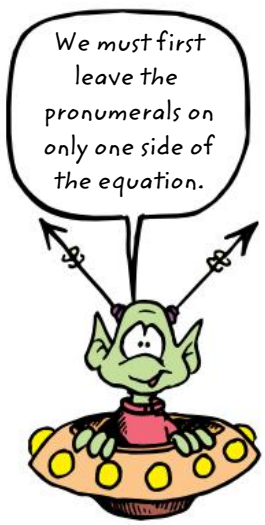
8 $14 + 2x \xrightarrow{-2x} \square$ **9** $3x + 4 \xrightarrow{-3x} \square$ **10** $7 - 2x \xrightarrow{+2x} \square$

It is possible to have pronumerals on both sides of an equation. As well as adding or subtracting numerals, we may also need to add or subtract pronumerals.

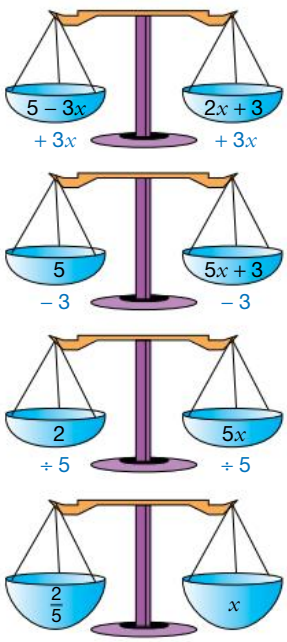
$3x + 5 = 2x + 9$



So $x = 4$ is the solution.



$5 - 3x = 2x + 3$



So $x = \frac{2}{5}$ is the solution.

Hint: Leave the pronumerals on the side where the term will be positive.

WORKED EXAMPLES

1 $3a + 5 = 2a + 7$
$-2a \quad -2a$
$a + 5 = 7$
$-5 \quad -5$
$\therefore\ a = 2$

2 $5x - 3 = 2x + 9$
$-2x \quad -2x$
$3x - 3 = 9$
$+3 \quad +3$
$3x = 12$
$\div 3 \quad \div 3$
$\therefore\ x = 4$

3 $2 - 2x = 8 + x$
$+2x \quad +2x$
$2 = 8 + 3x$
$-8 \quad -8$
$-6 = 3x$
$\div 3 \quad \div 3$
$-2 = x$
$\therefore\ x = -2$

4 By substituting the given solution, see whether it is correct or incorrect.

$5a - 9 = 3 - a$
$a = 5$

Solution

$\text{LHS} = 5a - 9$
$= 5 \times 5 - 9$
$= 16$

$\text{RHS} = 3 - a$
$= 3 - 5$
$= -2$

$\therefore\ \text{LHS} \neq \text{RHS}$
$\therefore\ a = 5$ is incorrect.

LHS means left-hand side.

RHS means right-hand side.

Exercise 7:04

P Foundation worksheet 7:04
Equations with pronumerals on both sides

1 Solve the following equations. (The answers are all integers.)

a $3a + 2 = 2a + 9$
b $3x + 3 = 2x + 10$
c $5m + 7 = 4m + 8$
d $4p - 8 = 3p + 2$
e $6q - 3 = 5q + 5$
f $9n - 5 = 8n + 1$
g $8n = 6n + 12$
h $9x = 6x + 3$
i $6x = x + 10$
j $7m + 8 = 8m - 4$
k $a + 11 = 2a + 5$
l $9q + 4 = 8q$
m $8x + 3 = x + 31$
n $6m + 1 = 3m + 16$
o $10x - 5 = x + 4$
p $8y - 5 = y + 9$
q $3x + 9 = 7x + 1$
r $6p + 1 = 9q - 5$
s $x + 8 = 3x - 10$
t $2m + 11 = 6m + 3$
u $x + 5 = x - 1$
v $3x + 4 = 9 - 2x$
w $15 - 2m = 6m - 1$
x $9 - 5m = m - 21$

2 Substitute the given solution to see whether it is correct or incorrect.

a $9m - 5 = 4m + 10$, $m = 3$
b $2p + 6 = 9p + 10$, $p = 5$
c $3q + 3 = 2q + 1$, $q = -2$
d $11a + 8 = 5a + 20$, $a = 2$
e $8q - 3 = 5q + 9$, $q = 4$
f $x + 7 = 5x - 4$, $x = 2$
g $2n + 1 = n - 2$, $n = -3$
h $2m + 5 = 8m - 7$, $m = 2$
i $8a + 2 = 3a - 3$, $a = -1$
j $5x = 12x - 14$, $x = 2$
k $11a + 8 = 13a$, $a = 4$
l $27x = 50 + 2x$, $x = 2$

3 Solve these equations.

a $5x + 10 = x + 2$	**b** $m + 8 = 3m + 5$	**c** $4p + 7 = p + 11$
d $6m = 2m + 1$	**e** $4a = a - 1$	**f** $8n + 6 = n + 6$
g $9a - 2 = 3a + 5$	**h** $7n - 9 = 9n + 1$	**i** $7q - 5 = 5q + 4$
j $8n + 7 = 4n + 17$	**k** $11a + 1 = 5a + 6$	**l** $4k - 7 = k - 10$
m $2a + 7 = 4a + 15$	**n** $x + 16 = 7x + 3$	**o** $3q + 7 = 6q - 1$
p $3m = 7m - 3$	**q** $4a = 11 - 3a$	**r** $x - 4 = 7x + 4$
s $2y = 5y + 60$	**t** $5 - 2x = 3 + x$	**u** $11 - m = 24 - 4m$

GEOGEBRA ACTIVITY 7:04 EQUATIONS WITH PRONUMERALS ON BOTH SIDES

Choose from three different types of equations and then use the sliders to fix the numbers in each equation. Solve the equation and then tick the checkbox to see a worked solution.

$4x + 5 = 3x + 3$
$1x + 5 = 3$
$x = -2$

7:05 Equations with grouping symbols

PREP QUIZ 7:05

Rewrite these expressions without grouping symbols.

1 $7(x + 4)$ **2** $2(a - 3)$ **3** $5(4a + 9)$ **4** $6(2p - 7)$ **5** $-3(x - 4)$

Solve these one-step equations.

6 $x + 9 = 4$ **7** $x - 8 = -2$ **8** $3p = -27$ **9** $10p = 5$ **10** $6 + x = -1$

If you remember how to expand grouping symbols, these equations are no harder than those you have already seen.

WORKED EXAMPLE 1

Expand the grouping symbols and then solve the equation.

a $2(x + 3) = 8$
$2x + 6 = 8$
$-6 \quad -6$
$2x = 2$
$\div 2 \quad \div 2$
$\therefore x = 1$

b $5(a - 3) = 3$
$5a - 15 = 3$
$+15 \quad +15$
$5a = 18$
$\div 5 \quad \div 5$
$\therefore a = \frac{18}{5}$ or $3\frac{3}{5}$

c $3(2m - 4) = 4m - 6$
$6m - 12 = 4m - 6$
$-4m \quad -4m$
$2m - 12 = -6$
$+12 \quad +12$
$2m = 6$
$\div 2 \quad \div 2$
$\therefore m = 3$

WORKED EXAMPLE 2

Expand each set of grouping symbols and then solve the equations.

a $3(a + 7) = 4(a - 2)$

$3a + 21 = 4a - 8$

$-3a \quad -3a$

$21 = a - 8$

$+8 \quad +8$

$29 = a$

$\therefore\ a = 29$

b $3(x + 4) + 2(x + 5) = 4$

$3x + 12 + 2x + 10 = 4$

Collect like terms.

$5x + 22 = 4$

$-22 \quad -22$

$5x = -18$

$\div 5 \quad \div 5$

$\therefore\ x = -3\frac{3}{5}$

The solution can be checked by substituting the value back into the original equation.

Exercise 7:05

P Foundation worksheet 7:05
Equations with grouping symbols

1 Expand the grouping symbols and then solve each equation. (Answers are all integers.)

a $3(a + 2) = 18$ **b** $2(x + 4) = 10$ **c** $5(6 + m) = 45$

d $6(x - 4) = 6$ **e** $4(x - 1) = 20$ **f** $7(x - 3) = 70$

g $5(2y + 1) = 25$ **h** $3(2p - 7) = 9$ **i** $10(6 + 5x) = 10$

j $8(1 + x) = 9x + 4$ **k** $2(5a + 3) = 8a + 10$ **l** $7(2a + 3) = 10a + 33$

m $9(2x - 1) = x + 42$ **n** $6(m - 4) = 2m + 8$ **o** $2(7y - 5) = 15y - 16$

p $2a + 5 = 3(a - 1)$ **q** $12x + 6 = 6(x + 2)$ **r** $8 - 2x = 3(x - 4)$

2 Solve each equation.

a $5(a + 1) = 8$ **b** $2(m - 3) = -3$ **c** $4(x - 7) = 3$

d $3(3 - x) = 12$ **e** $5(7 - n) = 15$ **f** $3(7 - y) = 9$

g $5(2a + 3) = 20$ **h** $2(5x - 1) = 15$ **i** $3(4x - 5) = 1$

j $4(x - 2) = x - 2$ **k** $5(n + 3) = 3n + 11$ **l** $3(x - 10) = x + 20$

m $2 - x = 3(2x - 1)$ **n** $8 + 7x = 3(3x - 1)$ **o** $8n - 16 = 4(3n - 7)$

p $7a = 6(2a - 1)$ **q** $2n = 3(4n - 10)$ **r** $5x = 4(2x + 9)$

3 Find the solution to each equation by first expanding all grouping symbols.

a $3(a + 2) = 2(a + 1)$ **b** $5(x - 1) = 4(x + 2)$

c $5(p - 2) = 4(p + 2)$ **d** $3(q + 2) = 2(q + 5)$

e $3(m + 1) = 5(m - 1)$ **f** $6(x + 2) = 4(x + 6)$

g $2(a - 7) = 5(a - 4)$ **h** $7(t + 2) = 4(t + 5)$

i $3(2a + 1) = 5(a + 2)$ **j** $4(3p - 1) = 5(2p + 1)$

k $6(t + 7) = 4(t + 10) + 8$ **l** $5(2a - 1) = 3(a + 6) - 7$

m $3(2 + m) = 5(2 - m) + 6m$ **n** $6(p + 3) = 5(2 - p) + 7p - 12$

4 Solve each equation. Use Worked Example 2b as a guide.

a $3(a + 2) + a + 5 = 15$
b $5(m - 1) + 2m = 2$
c $2(m + 3) + 5(m + 2) = 23$
d $3(x + 2) + 2(x - 3) = 10$
e $5(p + 1) + 2(p + 4) = 20$
f $4(t - 2) + 2(t + 5) = 14$
g $4(2a + 3) + 2(a - 5) = 22$
h $2(2m + 3) + 3(m - 5) = 5$
i $5(a - 3) + 3(2 + 3a) = 19$
j $7(a + 5) + 2(6 - 3a) = 1$

5 Try solving these equations, but first read the warning sign.

a $3(a + 2) - 2(a + 1) = 6$
b $5(m + 3) - 4(m + 2) = 10$
c $5(n + 4) - 3(n - 2) = 30$
d $6(a + 2) - 4(a - 1) = 20$
e $4(a + 3) - (a + 2) = 13$
f $4(p + 5) - (p + 3) = 23$
g $5(2a + 1) - 2(a - 4) = 2$
h $6(2x + 5) - 5(3x + 2) = 10$

Warning!
Remember how to expand with a negative term.
$-2[x + 4] = -2x - 8$
OR
$-3[a - 1] = -3a + 3$

P GEOGEBRA ACTIVITY 7:05 EQUATIONS WITH GROUPING SYMBOLS

Choose from five types of equations with varying degrees of difficulty. Use sliders to fix the numbers in each equation. Solve the equation and then tick the checkbox to see a worked solution.

$5(3x - 4) = 25$
$15x - 20 = 25$
$15x = 45$
$x = 3$

FUN SPOT 7:05 IF I HAVE 7 APPLES IN ONE HAND AND 4 IN THE OTHER, WHAT HAVE I GOT?

I $x + 12 = 7$
A $5x = 2$
H $x - 7 = -2$
D $\frac{x}{10} = 5$
N $5x + 30 = 15$
G $15 - 3x = 2x + 5$
B $\frac{5x + 21}{8} = 3$
S $5(3x + 8) + 6(10 - 2x) = 109$

$x = \frac{3}{5}$	$x = -5$	$x = 2$	$x = 5$	$x = \frac{2}{5}$	$x = -3$	$x = 50$	$x = 3$

7:06 Equations with fractions (1)

PREP QUIZ 7:06

Simplify:

1 $-4+4$ **2** -3×-1 **3** $-7+4$ **4** $\frac{15}{4}$ **5** $16\div -5$

6 $6\times\frac{p}{6}$ **7** $\frac{12p}{p}$ **8** $4\times\frac{(x+7)}{4}$ **9** $4\times\frac{5m}{4}$ **10** $4(3-\frac{p}{4})$

To remove a fraction, multiply both sides of an equation by the denominator of the fraction.

WORKED EXAMPLES

Find the value of the pronumeral in each of the following equations.

1 $\frac{y}{6}-1=3$ **2** $\frac{x+7}{4}=8$ **3** $\frac{3x-1}{5}=7$

Solutions

1 $\frac{y}{6}-1=3$

$\times 6 \quad \times 6$

$6(\frac{y}{6}-1)=3\times 6$

$\frac{6y}{6}-6=18$

$y-6=18$

$\therefore y=24$

2 $\frac{x+7}{4}=8$

$\times 4 \quad \times 4$

$\frac{4(x+7)}{4}=8\times 4$

$x+7=32$

$\therefore x=25$

3 $\frac{3x-1}{5}=7$

$\times 5 \quad \times 5$

$\frac{5(3x-1)}{5}=7\times 5$

$3x-1=35$

$3x=36$

$\therefore x=12$

1 Make sure you multiply both sides of the equation by the same number.

2 Make sure you expand the parentheses correctly.

$4(3+2)=4\times 3+4\times 2$

so

$4(x+2)=4\times x+4\times 2$

$=4x+8$

Exercise 7:06

P Foundation worksheet 7:06
Equations with fractions 1

1 Solve the following equations.

a $\frac{y}{4}+3=9$ **b** $\frac{y}{5}+3=7$ **c** $4+\frac{p}{6}=10$

d $7=\frac{m}{5}+4$ **e** $\frac{a}{4}-1=2$ **f** $13-\frac{m}{4}=6$

g $16=\frac{m}{2}-9$ **h** $5=8-\frac{p}{6}$ **i** $\frac{5p}{2}=4$

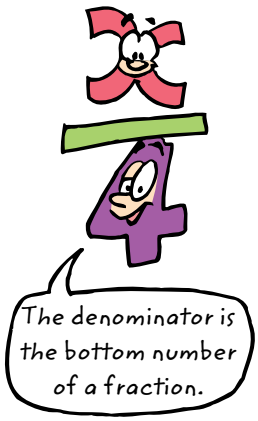
j $\frac{3m}{4}=6$ **k** $15=\frac{5p}{3}$ **l** $\frac{3}{5}m=7$

m $\frac{p+3}{4}=8$ **n** $\frac{5m+6}{3}=12$ **o** $\frac{7+x}{4}=2$

p $\frac{15-x}{4}=3$ **q** $\frac{3m+4}{2}=5$ **r** $\frac{6+5p}{4}=9$

s $\frac{12a-3}{5}=6$ **t** $\frac{7-3x}{3}=1$ **u** $\frac{5a-2}{4}=-3$

v $\frac{-x-2}{4}=5$ **w** $\frac{10+3p}{7}=9$



2 Check to see whether the given solution is correct or incorrect.

a $\frac{x}{4}+7=8$, $x=4$ **b** $\frac{y}{3}-4=2$, $y=10$ **c** $6+\frac{m}{4}=9$, $m=12$ **d** $4=\frac{m}{6}-1$, $m=30$

e $\frac{5m}{4}=3$, $m=3$ **f** $\frac{m+4}{2}=5$, $m=3$ **g** $\frac{p-5}{4}=3$, $p=17$ **h** $\frac{15-p}{4}=-1$, $p=19$

i $\frac{3m+2}{4}=1$, $m=\frac{2}{3}$ **j** $\frac{4m-6}{3}=8$, $m=6\frac{1}{2}$ **k** $\frac{1+3p}{2}=-1$, $p=-1$ **l** $\frac{9-5p}{5}=8$, $p=-8$

3 Solve these equations that have pronumerals on both sides.

a $\frac{2m}{3}-4=m$ **b** $\frac{x}{5}+3=2x$ **c** $p-5=\frac{p}{2}$

d $3x-6=\frac{3x}{4}$ **e** $\frac{x}{4}=3-x$ **f** $\frac{3n}{2}=2n-5$

g $\frac{a+3}{2}=a$ **h** $\frac{2x-1}{3}=3x$ **i** $\frac{2y+3}{3}=4y$

j $\frac{1-3p}{6}=p$ **k** $\frac{4-x}{5}=x+2$ **l** $\frac{3b-1}{4}=2-b$

Solve equations one step at a time.

FUN SPOT 7:06 — WHO HOLDS UP SUBMARINES?

Solve each equation and write the letter for that part in the box above the correct answer.

I	$\frac{m}{4} = 8$	**I**	$\frac{x}{5} = 4$
U	$\frac{2n}{3} = 5$	**E**	$\frac{5y}{3} = 2$
Y	$\frac{x}{4} + 1 = 5$	**S**	$\frac{a}{4} + 7 = 8$
L	$\frac{x}{2} - 8 = 5$	**H**	$\frac{m}{6} - 5 = 3$
L	$\frac{a+2}{3} = 4$	**D**	$\frac{n-3}{5} = 2$
T	$\frac{3y+1}{5} = 2$	**Q**	$\frac{7m-4}{2} = 5$
B	$\frac{12+3x}{4} = 6$		

4	20	10	26	16	3	48	1·2	6	2	7·5	32	13	

7:07 Equations with fractions (2)

PREP QUIZ 7:07

What are the first three multiples of:

1 2 **2** 5?

What is the lowest common multiple of:

3 2 and 6 **4** 3, 4 and 5 **5** x and $3x$?

Simplify:

6 $5 \times \frac{a}{5}$ **7** $10 \times \frac{3x}{2}$ **8** $\frac{3(2x+1)}{3}$ **9** $\frac{4(2x-1)}{2}$ **10** $5 \times 3\left(\frac{2x-1}{5}\right)$

In the previous section, the equations involved only one fraction with a single denominator. The equations in this section have more than one denominator. To simplify these equations we must multiply by the lowest common multiple of all the denominators. (In other words, we must multiply by some number that will cancel out every denominator.)

WORKED EXAMPLES

Solve the following equations.

1 $\frac{3x}{5} - \frac{x}{4} = 1$ **2** $\frac{a}{5} - 1 = \frac{3a-1}{2} + 4$ **3** $\frac{m+2}{3} = \frac{m-4}{5}$

Solutions

1 $\frac{3x}{5} - \frac{x}{4} = 1$

Multiply both sides by 20.

$20\left(\frac{3x}{5} - \frac{x}{4}\right) = 1 \times 20$

$\frac{60x}{5} - \frac{20x}{4} = 20$

$12x - 5x = 20$

$7x = 20$

$x = \frac{20}{7}$

$\therefore x = 2\frac{6}{7}$

2 $\frac{a}{5} - 1 = \frac{3a-1}{2} + 4$

$\frac{a}{5} = \frac{3a-1}{2} + 5$

Multiply both sides by 10.

$10\left(\frac{a}{5}\right) = 10\left(\frac{3a-1}{2}\right) + 5 \times 10$

$2a = 5(3a-1) + 50$

$2a = 15a - 5 + 50$

$-13a = 45$

$a = \frac{45}{-13}$

$\therefore a = -3\frac{6}{13}$

3 $\frac{m+2}{3} = \frac{m-4}{5}$

Multiply both sides by 15.

$\frac{15(m+2)}{3} = \frac{15(m-4)}{5}$

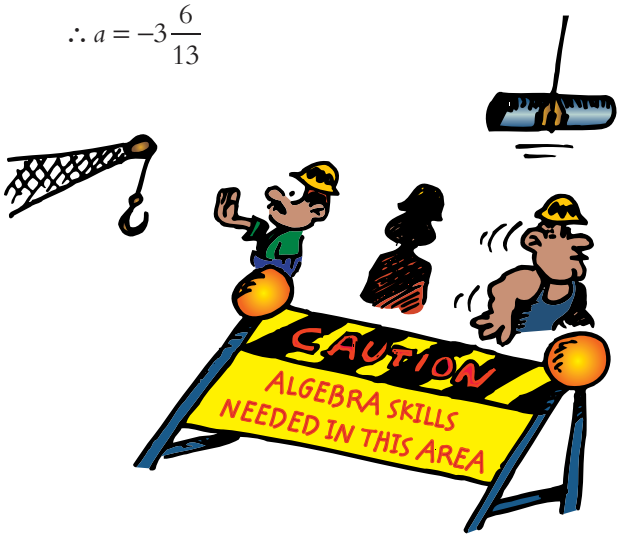
$\frac{{}^{5}\cancel{15}(m+2)}{\cancel{3}_{1}} = \frac{{}^{3}\cancel{15}(m-4)}{\cancel{5}_{1}}$

$5(m+2) = 3(m-4)$

$5m + 10 = 3m - 12$

$2m = -22$

$\therefore m = -11$



Exercise 7:07

P Foundation worksheet 7:07
Equations with fractions 2

1 Solve the following equations. All solutions are integers. Check your solutions by substituting into the equation.

a $\frac{x}{2} + \frac{x}{3} = 5$ **b** $\frac{p}{6} + \frac{p}{2} = 8$ **c** $\frac{m}{4} + \frac{m}{6} = 20$ **d** $\frac{y}{2} - \frac{y}{4} = 3$

e $\frac{m}{4} - \frac{m}{5} = 1$ **f** $\frac{x}{3} - \frac{x}{5} = 4$ **g** $\frac{3k}{2} + \frac{k}{4} = 14$ **h** $\frac{5x}{3} + \frac{4x}{5} = 37$

i $\frac{3p}{2}+\frac{p}{3}=11$ j $\frac{x}{2}-\frac{2x}{5}=3$ k $\frac{5m}{6}-\frac{m}{4}=7$ l $4a-\frac{2a}{5}=18$

m $\frac{m}{4}=\frac{m}{3}-2$ n $\frac{3p}{10}-4=\frac{p}{2}-8$ o $\frac{x}{2}+\frac{x}{10}=\frac{2x}{5}+4$ p $\frac{z}{3}+\frac{1}{6}=\frac{z}{2}-\frac{2}{3}$

2 Solve:

a $\frac{x+3}{2}=\frac{x}{3}$ b $\frac{b+4}{2}=\frac{b+10}{3}$

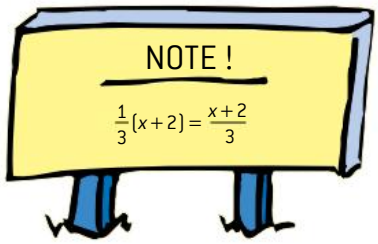
c $\frac{k-1}{4}=\frac{k-5}{2}$ d $\frac{2x+5}{3}=\frac{x-1}{4}$

e $\frac{m+6}{3}=\frac{2m+3}{4}$ f $\frac{a-3}{2}=\frac{a-4}{5}$

g $\frac{4x-3}{6}=\frac{2x-1}{4}$ h $\frac{4x}{7}=\frac{2x+3}{4}$

i $\frac{t+2}{4}=\frac{2-t}{3}$ j $\frac{1}{3}(y+2)=\frac{1}{4}(y+3)$

k $\frac{1}{2}(x+5)=\frac{2}{5}(x+1)$ l $\frac{3}{4}(a-2)=\frac{2}{3}(a-1)$



CHALLENGE 7:07 SOLVING EQUATIONS BY 'GUESS AND CHECK'

Example

If we do not know how to solve an equation, we could use guess and check.

Solve $x(x-1)=6$		
x	$x-1$	$x(x-1)$
4	3	12
2	1	2
3	2	6

- $x = 4$ gives 12, and $x = 2$ gives 2, so an answer will probably lie between $x = 2$ and $x = 4$.
- As $3(3 - 1) = 6$, $x = 3$ is a solution.
- For some equations there may be more than one solution.

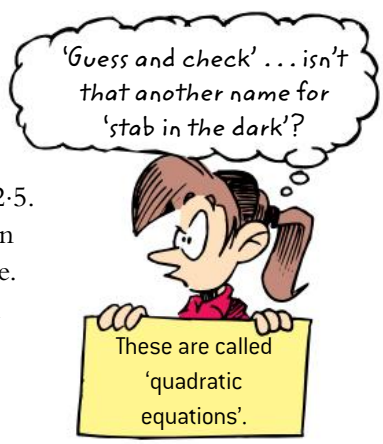
Exercises

1 Use this table to solve $x(x + 3) = 14{\cdot}56$.

$x(x+3)=14{\cdot}56$		
x	$x+3$	$x(x+3)$
1	4	4
3	6	18
2·5	5·5	…
…	…	…

18 is too big, so try $x = 2{\cdot}5$.

- The solution you obtain here is not the only one. The other lies between -3 and -8. Find this other solution also.



2 Use the above technique to find two solutions for:

a $x(x - 14) = -40$ b $x(x - 1{\cdot}7) = 12{\cdot}6$ c $x(x + 8) = 34{\cdot}41$

7:08 Graphing inequalities

PREP QUIZ 7:08

1 3×-2 **2** -4×-3 **3** $-12 \div 6$

4 $-6 \div -3$ **5** $-6 > 2$, *true* or *false*? **6** $-3 < -4$, *true* or *false*?

7 What is the meaning of '>'? **8** Graph the set $\{1, 3, 5\}$ on a number line.

Insert either < or > to make each of the following sentences true.

9 $-2 \square -4$ **10** $-4 \square 6$

An inequality is a number sentence where an inequality sign is used instead of an equals sign. The most common inequality signs are shown below.

$>$	$<$	$\geq$	$\leq$
'is greater than'	'is less than'	'is greater than or equal to'	'is less than or equal to'

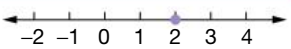
Inequalities, unlike equations, usually have more than one solution.
For example:

- The equation $x + 6 = 10$ has one solution, namely $x = 4$.
- The inequality $x + 6 > 10$ has an infinite number of solutions.

The numbers $4\frac{1}{2}$, 8, 9·5, 30 are some solutions.
The full set of solutions is written as $x > 4$.

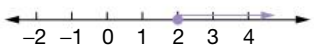
WORKED EXAMPLES

1



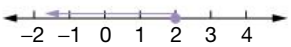
This shows the solution $x = 2$.

2



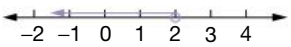
This shows the solution $x \geq 2$.

3



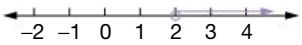
This shows the solution $x \leq 2$.

4

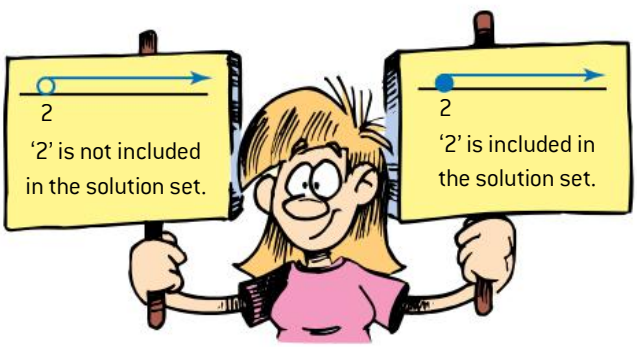


This shows the solution $x < 2$.

5

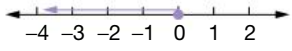


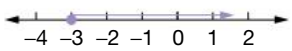
This shows the solution $x > 2$.

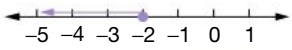


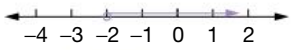
Exercise 7:08

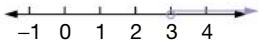
1 State the set of values of x that have been graphed below.

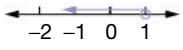
a 

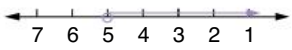
b 

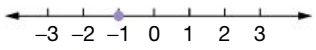
c 

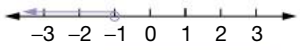
d 

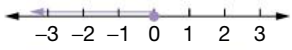
e 

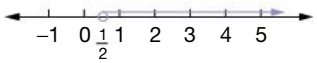
f 

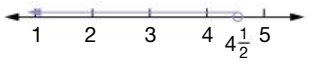
g 

h 

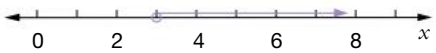
i 

j 

k 

l 

2 Write an inequality using $<, >, \leq$ or $\geq$ to describe each graph.

a 

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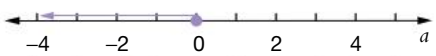
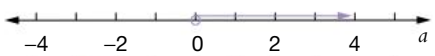
c 

d 

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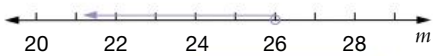
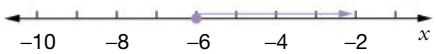
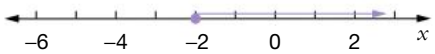
f 

g 

h 

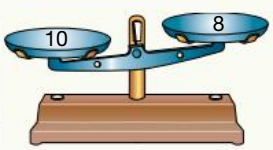
3 Graph each of the following inequalities on a separate number line.

a $x > 2$ **b** $x \leq -1$ **c** $x < 10$ **d** $x \geq 7$ **e** $x \leq 12$ **f** $x > 15$

g $x \geq -20$ **h** $x < 32$ **i** $a \geq -5$ **j** $a < 6$ **k** $m > -3$ **l** $m \leq -8$

7:09 Solving inequalities

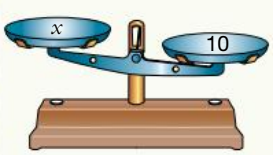
PREP QUIZ 7:09



$10 > 8$

The scales on the left show that $10 > 8$. Would the scales stay as they are if:

1 the same amount is added to both sides
2 the same amount is subtracted from both sides
3 the mass on each side is doubled
4 the mass on each side is halved?



$x < 10$

The scales on the left show that $x < 10$. Would the scales stay as they are if:

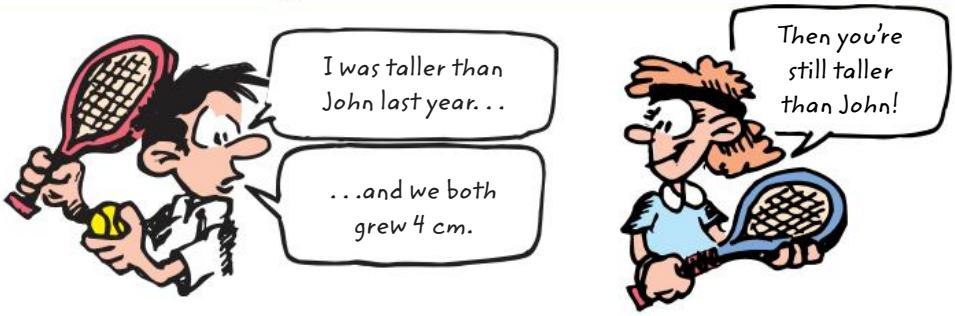
5 the same amount is added to both sides
6 the same amount is subtracted from both sides
7 the mass on each side is doubled
8 the mass on each side is halved?

What sign must be placed in the box to make the final statements true?

9
$8 > 4$
$\div -2 \quad \div -2$
$-4 \square -2$

10
$2 < 10$
$\times -3 \quad \times -3$
$-6 \square -30$

INVESTIGATION 7:09 OPERATING ON INEQUALITIES



- When we solved equations we saw that performing the same operation on both sides gave an equivalent equation.
- Use the examples below to investigate what happens with inequalities.

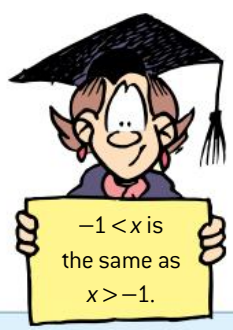
Each of the following inequalities is true. Perform the operation indicated on both sides of each inequality and state whether the inequality remains true.

1	**a** $6 > 2$ add 4	**b** $-6 < -4$ add 4	**c** $6 > -4$ add 4	**d** $-2 < 4$ add 4
2	**a** $6 > 2$ subtract 4	**b** $-6 < -4$ subtract 4	**c** $6 > -4$ subtract 4	**d** $-2 < 4$ subtract 4
3	**a** $6 > 2$ multiply by 2	**b** $-6 < -4$ multiply by 2	**c** $6 > -4$ multiply by 2	**d** $-2 < 4$ multiply by 2
4	**a** $6 > 2$ divide by 2	**b** $-6 < -4$ divide by 2	**c** $6 > -4$ divide by 2	**d** $-2 < 4$ divide by 2
5	**a** $6 > 2$ multiply by -3	**b** $-6 < -4$ multiply by -3	**c** $6 > -4$ multiply by -3	**d** $-2 < 4$ multiply by -3
6	**a** $6 > 2$ divide by -2	**b** $-6 < -4$ divide by -2	**c** $6 > -4$ divide by -2	**d** $-2 < 4$ divide by -2

- Write the results of your investigation. Compare these with the results obtained by others in your class.

From the investigation, you should have found that multiplying or dividing a true inequality by a negative numeral did not produce a true inequality. This is the very important difference between equations and inequalities.

To allow for this, the inequality sign must be reversed when an inequality is multiplied or divided by a negative numeral.



When solving inequalities, we can use the *same rules* as for equations except for *one important exception:*

- When multiplying or dividing an inequality by a negative number, the inequality sign must be reversed to obtain an equivalent inequality.
  e.g. $<$ is changed to $>$ and $\leq$ is changed to $\geq$.

WORKED EXAMPLES

Solve the following inequalities.

1 $2x + 3 < 6$

2 $\frac{x}{2} - 3 \leq 7$

3 $5 - 3x > 6$

4 $-\frac{1}{3}x < 5$

5 $2(1 - 2x) \leq 6$

Solutions

1 $2x + 3 < 6$

$-3 \quad -3$

$2x < 3$

$\div 2 \quad \div 2$

$x < \frac{3}{2}$

$\therefore x < 1\frac{1}{2}$

2 $\frac{x}{2} - 3 \leq 7$

$+3 \quad +3$

$\frac{x}{2} \leq 10$

$\times 2 \quad \times 2$

$2 \times \frac{x}{2} \leq 10 \times 2$

$\therefore x \leq 20$

3 $5 - 3x > 6$

$-5 \quad -5$

$-3x > 1$

$\div -3 \quad \div -3$

(reverse sign)

$\therefore x < -\frac{1}{3}$

4 $-\frac{1}{3}x < 5$

$\times -3 \quad \times -3$

(reverse sign)

$-\frac{1}{3}x \times (-3) > 5 \times (-3)$

$\therefore x > -15$

5 $2(1 - 2x) \leq 6$

$2 - 4x \leq 6$

$-2 \quad -2$

$-4x \leq 4$

$\div -4 \quad \div -4$

(reverse sign)

$\therefore x \geq -1$

OR

$2(1 - 2x) \leq 6$

$2 - 4x \leq 6$

$+4x \quad +4x$

$2 \leq 6 + 4x$

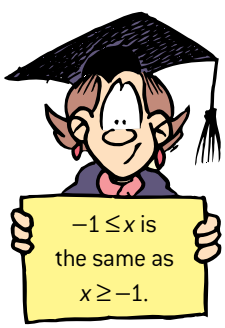
$-6 \quad -6$

$-4 \leq 4x$

$\div 4 \quad \div 4$

$-1 \leq x$

$\therefore x \geq -1$



Exercise 7:09

Foundation worksheet 7:09
Solving inequalities

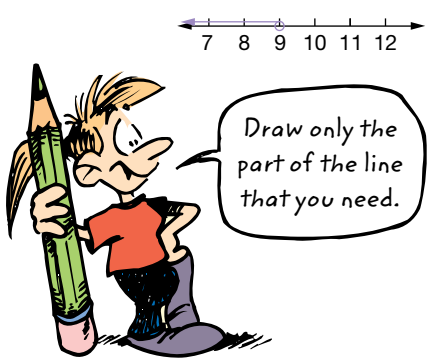
1 Solve the following inequalities and graph each solution on a number line.

a $x + 6 < 15$ **b** $y + 7 > 9$ **c** $m + 6 < 4$

d $m + 10 > 5$ **e** $5 + p \leq 2$ **f** $15 \geq m + 6$

g $y - 3 > 2$ **h** $m - 6 < 1$ **i** $x - 5 \geq 5$

j $12 < m - 6$ **k** $3 > p - 2$ **l** $-1 \leq x - 1$



2 Solve the following inequalities.

a $3m > 21$ **b** $6p \leq 42$ **c** $12m < 24$

d $5y \geq -42$ **e** $15 > 4x$ **f** $-20 < 10x$

g $\frac{x}{3} < 5$ **h** $\frac{y}{2} > 6$ **i** $\frac{m}{4} \geq 1$

j $\frac{x}{5} < -10$ **k** $3 > \frac{x}{2}$ **l** $-4 < \frac{x}{3}$

3 Solve the following inequalities.

a $-3m < 24$	**b** $-4x > 16$	**c** $-5p \leq 20$
d $-6x < -12$	**e** $15 < -6x$	**f** $10 \geq -3x$
g $-x > 4$	**h** $-x < 3$	**i** $5 \geq -x$
j $-\frac{1}{2}x > 3$	**k** $-\frac{1}{5}x > 1$	**l** $-\frac{1}{4}x \geq 2$
m $-\frac{x}{2} < 1$	**n** $-\frac{x}{3} > 2$	**o** $-\frac{x}{4} > 3$

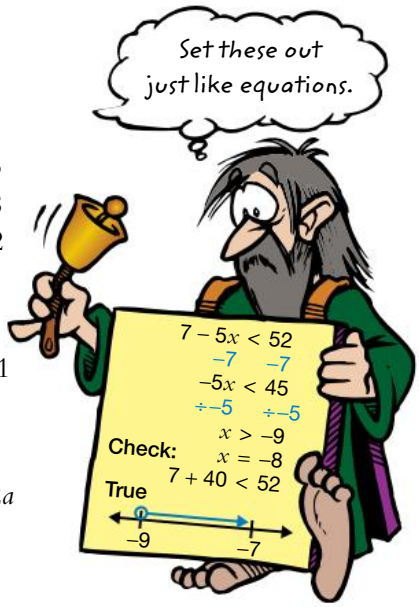
Remember:
Change '<' to '>', or vice versa, if you multiply or divide by a negative number.

4 Solve:

a $2x + 5 > 11$	**b** $4m + 3 < 19$	**c** $2p + 1 \geq 7$	**d** $10 + 3p \leq 7$
e $4 + 5p \geq 10$	**f** $13 + 2x < 6$	**g** $3x - 8 \leq 4$	**h** $5p - 1 \geq 9$
i $4y - 3 < 2$	**j** $12 > 5x - 3$	**k** $3 < 2x - 1$	**l** $3 \geq 8x - 9$
m $2(x + 3) < 14$	**n** $3(m + 2) > 15$	**o** $3(2x - 5) \leq 6$	**p** $4(x - 3) \geq 5$
q $4 < 2(2m - 3)$	**r** $2(5p - 4) > 22$		

5 Solve:

a $5 - 3m > 11$	**b** $10 - 2y < 14$	**c** $12 \leq 6 - 4x$
d $10 - x \leq 12$	**e** $2 - m \geq -4$	**f** $17 - 2m \leq 1$
g $9 - 4p < 7$	**h** $2(1 - x) < 6$	**i** $3(4 - y) > 15$
j $3(4 - 2x) \geq 18$	**k** $2(3 - 5y) \leq -4$	**l** $2(3 - 2p) > 8$
m $4 \geq 3(1 - 2x)$	**n** $2(5 - 2a) \leq -5$	**o** $3(1 - 2x) > 2$



6 Solve each inequality, check the solution and then graph the solution on a number line.

a $8 - 2x \leq 4$	**b** $9 - 6x > 25$	**c** $-6x + 3 \leq 21$
d $-3x - 8 > 11$	**e** $-x - 5 < 14$	**f** $8(4 - x) \geq 8$

7 Find the solution of each of the following inequalities.

a $5x + 6 > x + 18$	**b** $3x - 5 < x + 6$	**c** $3 - a \leq 5 - 2a$
d $12 - b \geq 2b + 21$	**e** $\frac{x}{2} + 1 < 6$	**f** $\frac{p}{3} - 1 > 4$
g $5 - \frac{2y}{3} \leq 6$	**h** $\frac{p-1}{4} < 2$	**i** $\frac{4-x}{3} \geq 1$

FUN SPOT 7:09 — WHO 'DUNNIT'?

Four boys playing cricket in the park were having a great game when suddenly one of them hit a mighty six, right through Mr Yorker's window. Mr Yorker stormed out and demanded to know who did it. Their responses were:

Bob: 'Steve did it.'
Steve: 'Mark did it.'
Alan: 'I didn't do it.'
Mark: 'Steve lied when he said I did it.'

If only one of the boys is not telling the truth, who hit the ball through the window?

On the other hand, if only one of the four statements is true, who hit the ball?

MATHS TERMS 7

equation

- a number sentence where one or more of the numbers is missing or unknown
- the unknown number is represented by a pronumeral,

  e.g. $x + 5 = 8$, $\frac{3x+1}{7} = \frac{x-5}{2}$

expression

- an algebraic expression consists of one or more terms joined together by operation signs,

  e.g. $a + 5$, $x^2 - x + 4$, $\frac{3m-1}{7}$
- an expression does not have an 'equals' sign like an equation

grouping symbols

- the most common types are:

  parentheses ( )
  brackets [ ]
  braces { }
- used to 'group' a number of terms together in an expression,

  e.g. $5(x + 3)$

inequality signs

- $>$ greater than, $<$ less than
- $\geq$ greater than or equal to
- $\leq$ less than or equal to,

  e.g. $x + 3 < 4$ means that
  $x + 3$ is less than 4

inequality

- an equation where an inequality sign is used instead of an equals sign,

  e.g. $4x - 1 > 5$ or $\frac{x}{3} \leq 4$

inverse operation

- the operation that will reverse or 'undo' a previous operation,

  e.g. addition is the inverse operation of subtraction
  division is the inverse operation of multiplication

pronumeral

- a symbol used to represent a number
- usually a letter such as x

solution

- method of finding the answer to a problem or the answer to a problem
- the solution to an equation is the number or numbers that satisfy the equation or make it a true sentence,

  e.g. $x = 3$ is the solution to $x + 2 = 5$

solve

- find the solution or answer to a problem or equation

substitute

- to replace a pronumeral with a numeral,

  e.g. Substitute 3 for a in the expression $4a - 2$:
  $4(3) - 2$
  $= 12 - 2$
  $= 10$

DIAGNOSTIC TEST 7 EQUATIONS AND INEQUALITIES

Each part of this test has similar items that test a certain type of question.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Solve: **7:01**

a $x + 7 = 13$ **b** $y + 10 = 19$ **c** $p - 4 = 10$ **d** $m - 7 = 3$

2 Solve: **7:01**

a $5x = 30$ **b** $6x = 48$ **c** $\frac{y}{3} = 9$ **d** $\frac{y}{4} = 5$

3 Solve: **7:01**

a $6 + p = 24$ **b** $30 - x = 10$ **c** $\frac{12}{x} = 2$ **d** $12 = 24 - t$

4 Check the given solution to see whether it is correct or incorrect. **7:01**

a $a + 3 = 5$, $a = 12$ **b** $y - 3 = 9$, $y = 6$ **c** $6x = 42$, $x = 6$ **d** $\frac{x}{3} = 6$, $x = 18$

5 Solve: **7:03**

a $4p + 3 = 31$ **b** $2m - 7 = 17$ **c** $9p = 76$ **d** $25 = 5 - 2m$

6 Solve: **7:04**

a $3x + 5 = 2x + 1$ **b** $5a - 7 = 3a - 1$ **c** $4b + 7 = b - 8$

7 Solve: **7:05**

a $3(x + 1) = 9$ **b** $4(a - 3) = 24$ **c** $6(x - 3) + 4x = 8$

8 Solve: **7:06**

a $\frac{y}{2} + 1 = 7$ **b** $\frac{m}{2} - 1 = 5$ **c** $4 = 13 - \frac{p}{2}$ **d** $\frac{3m}{5} = 6$

9 Solve: **7:06**

a $\frac{m+3}{4} = 2$ **b** $\frac{m-6}{3} = 1$ **c** $\frac{3p-7}{2} = 7$ **d** $\frac{5+3x}{5} = 5$

10 Solve: **7:06**

a $\frac{m}{5} + 2 = m$ **b** $2x + 1 = \frac{3x}{2}$ **c** $\frac{n+7}{4} = 2n$ **d** $\frac{5a-2}{3} = 3 - a$

11 Solve: **7:07**

a $\frac{a}{3} + \frac{a}{2} = 10$ **b** $\frac{2m}{3} - \frac{m}{2} = 4$ **c** $\frac{x}{4} = \frac{x}{5} - 3$ **d** $\frac{2y+1}{4} = \frac{3y-4}{3}$

12 Graph the following on a number line. **7:08**

a $x > 3$ **b** $x \leq -1$ **c** $x \geq 0$ **d** $x < 5$

13 Solve: **7:09**

a $2x + 5 \geq 6$ **b** $\frac{2x-1}{3} < 6$ **c** $3x - 7 > x + 3$

14 Solve: **7:09**

a $5 - 3x > 11$ **b** $-\frac{1}{3}x > 21$ **c** $3 - 4a > 2 - a$

ASSIGNMENT 7A Chapter review

1 Solve these one-step equations.

a $4x = 13$ **b** $\frac{a}{2} = 6$

c $m + 73 = 117$ **d** $n - 61 = 27$

2 Solve:

a $5m - 7 = 8$ **b** $3y + 7 = 4$

c $6m - 1 = 17$ **d** $4n + 10 = 2$

e $3p + 6 = 5$ **f** $2k - 7 = 10$

g $5y - 7 = 2$ **h** $9q + 2 = 5$

3 Solve for x.

a $5x + 3 = 4x + 7$ **b** $9x - 3 = 7x + 5$

c $8x + 6 = 5x - 3$ **d** $6x - 4 = 4x + 5$

e $3x + 7 = 5x - 4$ **f** $x - 5 = 6x + 3$

g $10 + 4x = 3x - 1$ **h** $8 - 3x = 7 + 2x$

i $12 - 5x = 10 - 3x$

4 Solve these equations that involve grouping symbols.

a $5(x + 7) = 30$

b $7(a - 3) = 21$

c $8(m - 1) = 4$

d $4(x + 3) = 3(x + 2)$

e $5(n - 2) = 3(n + 4)$

f $10(x - 7) = 7(x - 10)$

g $4(2a + 3) + 3(a - 3) = 5$

h $5(3n + 4) + 2(5 - 2n) = 7$

i $6(m + 4) - 5(m + 3) = 6$

j $7(4x + 3) - 3(8x - 5) = 0$

5 Solve:

a $\frac{x}{4} + 3 = 5$ **b** $\frac{m}{5} - 2 = 1$

c $\frac{2x}{3} - 4 = 2$ **d** $\frac{a+7}{4} = 6$

e $\frac{y-5}{3} = 1$ **f** $\frac{3p+1}{5} = 2$

g $\frac{2m}{3} - 1 = m$ **h** $\frac{5m-1}{4} = 2m$

i $2n + 5 = \frac{n-1}{3}$ **j** $\frac{x+5}{2} = \frac{x-3}{4}$

k $\frac{a}{5} - \frac{a}{2} = 7$ **l** $\frac{5q+1}{2} = \frac{q}{3} - \frac{q}{4}$

6 Solve and graph each solution on a number line.

a $m + 7 \geq 5$

b $2x - 1 < 7$

c $5n + 1 > 3$

d $3x + 7 \leq x + 10$

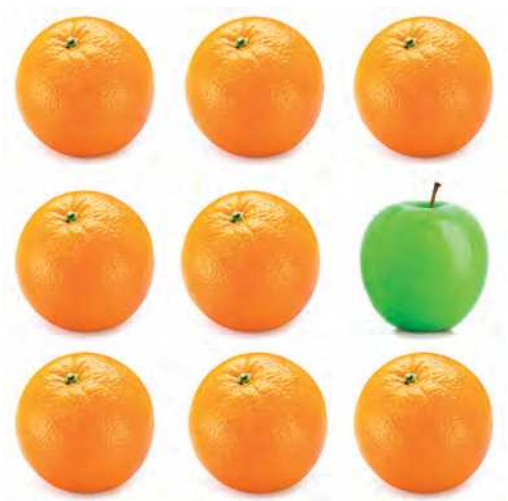
e $y - 5 > 3y - 8$

f $4n + 7 \geq 7n - 4$

g $6 - 2x > 14$

h $-\frac{2x}{3} \geq 6$

i $10 - 3a \leq 7 - a$



An apple costs 20c more than an orange. The cost of 8 oranges and 1 apple is $6.50. An equation that represents this information is $8x + (x + 20) = 650$.

ASSIGNMENT 7B Working mathematically

1 Ten students lined up at a tap for a drink of water. Each person took 1 minute.

a How long did it take for all students to have a drink?

b What was the total time spent waiting and drinking by all 10 students?

2 Find the basic numeral for:

$(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})$

$+(\frac{1}{4}-\frac{1}{5})+(\frac{1}{5}-\frac{1}{6})+(\frac{1}{6}-\frac{1}{7})$

$+(\frac{1}{7}-\frac{1}{8})+(\frac{1}{8}-\frac{1}{9})+(\frac{1}{9}-\frac{1}{10})$

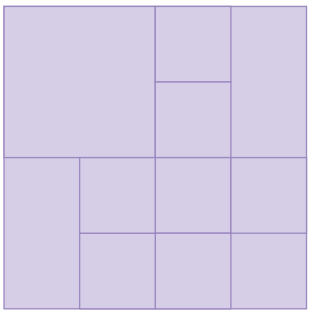
3 Uncle Pete was sensitive about his age. When asked how old he was he said he was only 40, not counting the weekends. What was his actual age?

4 We know that a prime number has exactly two factors, itself and one. Which numbers, less than 100, have exactly three factors?

5 Casey spent one-third of what was in her purse, and then spent one-third of what was left. If she spent $25 altogether, how much did she originally have in her purse?

6 Lucky Louis liked going to the horse races. Each time he went he had to pay a $10 entrance fee and he bought a $10 snack on the way out. Each day for 4 days he lost half of the money he had left after paying his entrance fee. At the end of the fourth day he went home with $10. How much did Louis have at the start of the first day?

7 How many squares are contained in this figure?



Designing an aeroplane involves solving equations.

ASSIGNMENT 7C Cumulative revision

1 Write the following percentages as decimals. 1:03

a 8% **b** 8·5% **c** 125% **d** 0·25%

2 **a** A business estimates that its expenses will rise next year by 4·8%. If its expenses this year were $110 500, what do they estimate that they will be next year? 2:01D

b Wages are the largest part of a company's expenses. If their expenses were $110 500 and they paid $54 000 in wages, what percentage were the wages of the expenses?

3 An athlete runs 21 km in 63 min. 1:05, 2:01A

a Express this rate as a rate in:

i km/min **ii** km/h **iii** min/km

b At this rate how long does it take to run 100 m?

4 A company interested in starting up a gym surveyed people in the local area and obtained the following results. 4:05

	Exercises regularly	No regular exercise	Totals
Male	85	115	200
Female	82	68	150
Totals	167	183	350

Based on these results, find the probability (expressed to the nearest whole %) that a person chosen at random:

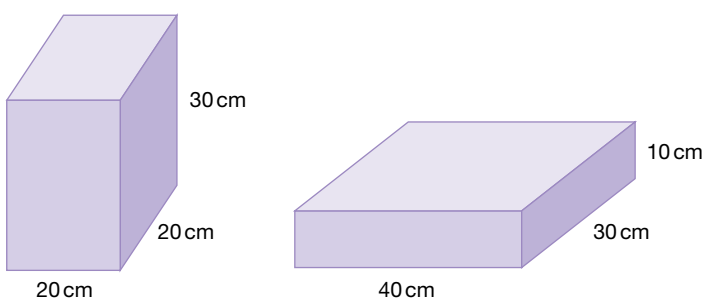
a exercises regularly

b is a female who exercises regularly

c exercises regularly, if we know that the person chosen is male

d is male, if we know that the person chosen exercises regularly.

5 Each of the following boxes is a rectangular prism with the same volume. Calculate the area of cardboard needed to make each box. Which one uses less cardboard? 5:03

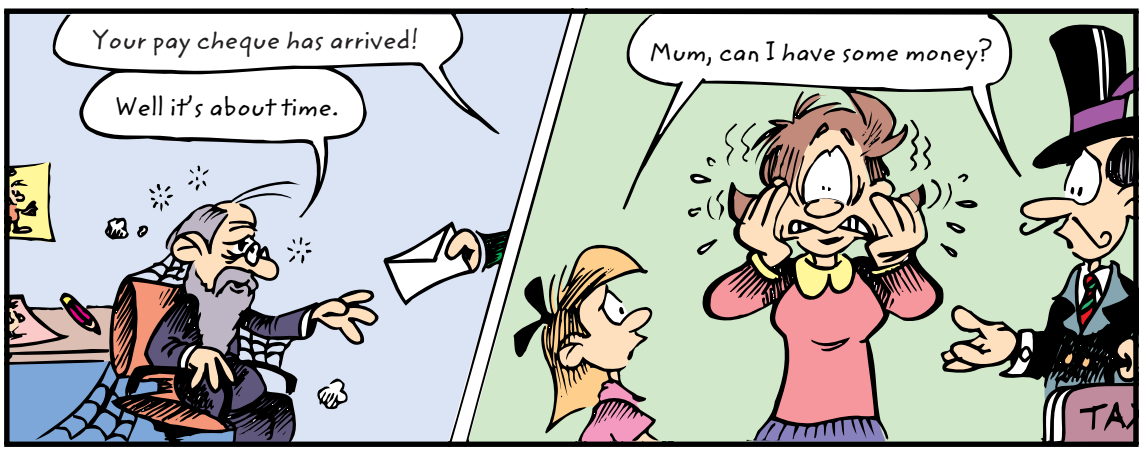


6 Simplify each of the following, writing your answer as a power of 10. 6:02, 6:03

a $10^4 \times 10^6$ **b** $10^4 \div 10^6$ **c** $\dfrac{10^4 \times 10^{-6}}{10^{-4}}$ **d** $(10^{-2})^3 \div (10^6)^{-2}$

FINANCIAL MATHEMATICS

8



Contents

8:01 Earning money
8:02 Extra payments
8:03 Wage deductions
8:04 Income tax
Investigation 8:04 Income tax returns
Fun spot 8:04 What is brought to the table, cut, but never eaten?
8:05 Spending money
8:06 Ways of paying
Fun spot 8:06 The puzzle of the missing dollar
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Financial Mathematics*** [Stage 5.1]

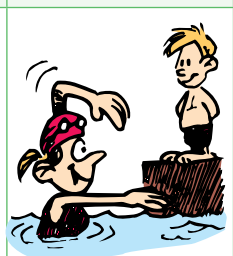
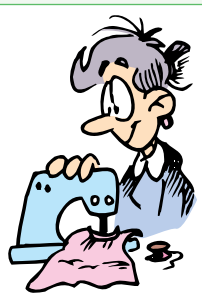
- Solve problems involving earning money. (NSW)
- Solve problems involving simple interest. (ACMNA211)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

8:01 Earning money

Some people work for themselves and charge a fee for their services or sell for a profit, but most people work for others to obtain an income. In the chart below, the main ways of earning an income from an employer are introduced.

Employment				
Salary	**Wage**	**Casual**	**Commission**	**Piece work**
Meaning				
A fixed amount is paid for the year's work even though it may be paid weekly or fortnightly.	Usually paid weekly to a permanent employee and based on an hourly rate, for an agreed number of hours per week.	A fixed rate is paid per hour. The person is not permanent but is employed when needed.	This payment is usually a percentage of the value of goods sold.	The worker is paid a fixed amount for each piece of work completed.
Advantages				
Permanent employment. Holiday and sick pay. Superannuation is paid. A bonus may be given as an incentive or time off for working outside normal working hours.	Permanent employment. Holiday and sick pay. Superannuation is paid. If additional hours are worked, additional money is earned, sometimes at a higher hourly rate of pay.	A higher rate of pay is given as a compensation for other benefits lost. Part-time work may suit some or casual work may be a second job. Superannuation is paid.	The more you sell the more you are paid. Some firms pay a low wage plus a commission to act as an incentive.	The harder you work, the more you earn. You can choose how much work you do and in some cases the work may be done in your own home.
Disadvantages				
During busy periods, additional hours might be worked, without additional pay. There may be little flexibility in working times e.g. 9 am–5 pm.	There is little incentive to work harder, since your pay is fixed to time not effort. There may be little flexibility in working times e.g. 9 am–5 pm.	No holiday or sick pay. No permanency of employment. Few fringe benefits.	There may be no holiday or sick pay. If you sell nothing you may be paid nothing. Your security depends on the popularity of your product.	No holiday or sick pay. No fringe benefits. No permanency of employment in most piece work.
Salary	**Wage**	**Casual**	**Commission**	**Piece work**
				
teachers	mechanics	swimming instructors	sales people	dressmakers

WORKED EXAMPLES

1 Use the information on the right to answer these questions.

POSITIONS VACANT
5 people required to promote our nationally known building product in the suburbs.
Pay: $500 p.w. and 2% commission.
Please phone YRU-POOR during business hours.

a How much would an employee earn in a week if no sales were made?

b If Jane sold $18 000 worth of building products in a week, how much would she earn?

c If Peter sold $24 000 worth of materials in a week and $5000 worth in the next, find his average weekly income for the 2 weeks.

2 Luke has a casual job from 4:00 pm until 5:30 pm Monday to Friday. He also works from 9 am until 12:30 pm on Saturdays. Find his weekly income if his casual rate is $18.80 per hour Monday to Friday, and $28.20 an hour on Saturdays.

Solutions

1 a Week's earnings = $500 + 2% of $0
= $500 + $0

∴ Employee making no sales is paid $500.

b Jane's earnings = $500 + 2% of $18 000
= $500 + 0·02 × $18 000
= $860 in the week

c Week 1
Peter's earnings = $500 + 2% of $24 000
= $500 + 0·02 × $24 000
∴ Earnings week 1 = $980

Week 2
Peter's earnings = $500 + 2% of $5000
= $500 + 0·02 × $5000
∴ Earnings week 2 = $600

∴ Peter's average weekly wage = ($980 + $600) ÷ 2
= $790

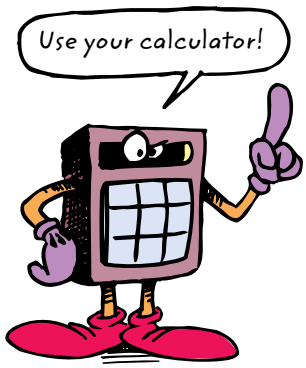
2 Luke's weekly income = (hours, Mon–Fri) × $18.80 + (hours, Sat) × $28.20
= $(1\frac{1}{2} \times 5) \times \$18.80 + 3\frac{1}{2} \times \$28.20$
= 1·5 × 5 × $18.80 + 3·5 × $28.20
= $239.70

Exercise 8:01

1 Write answers in your own words.

- **a** What are the advantages of working for a wage?
- **b** What is piece work?
- **c** What is a salary?
- **d** What form of payment gives the worker a percentage of the value of goods sold?
- **e** What advantages are there in casual work?
- **f** What are the disadvantages of being on a salary?
- **g** What is a wage?
- **h** Which forms of payment depend on success or the amount of work completed?
- **i** What are the disadvantages of casual work?
- **j** Which two forms of payment are often combined in determining a worker's pay?

2 **a** A man is paid $28.50 an hour for a 35-hour week. What is his normal weekly wage?



- **b** A boy is paid a wage based on $16.15 an hour. How much is he paid for an 8-hour day of work?
- **c** For a 38-hour working week a woman is paid $1079.20. Find her hourly rate of pay?
- **d** Adam is paid $16.05 an hour for a 35-hour week. Luke receives $15.75 an hour for a 38-hour week. Who has the higher weekly wage and by how much?
- **e** Irene is paid $999.70 for a 38-hour week, whereas Shireen is paid $906.50 for a 35-hour week. Who has the higher rate of pay and by how much?
- **f** A painter works a 38-hour week for an hourly rate of $29.65. An extra height allowance of $1.95 per hour is paid. Find his total weekly wage.
- **g** A woman is paid a salary of $46 089 per year. How much would she receive each week if it is calculated on 52·178 weeks in a year? (Answer to nearest dollar.)
- **h** Find the weekly income (assuming there are 52·178 weeks in the year) for a salary of:
  i $43 000 **ii** $76 400 **iii** $138 950 **iv** $58 200
  (Give answers to the nearest cent.)
- **i** Find the yearly salary of a person whose monthly income is:
  i $4600 **ii** $3150.50 **iii** $5305 **iv** $8950
- **j** Two jobs are advertised: one with a salary of $55 000, the other a salary with a fortnightly payment of $2165.60. Which is the greater weekly salary and by how much? Use '1 year = 52·178 weeks.' (Give your answer correct to the nearest cent.)

It is assumed that each day of the year, the salaried person earns $\frac{1}{365\frac{1}{4}}$ of their annual salary.
There are $365\frac{1}{4}$ days, on average, in a year.
∴ On average 52·178 (approximately) weeks are in each year.

k Two jobs are advertised. One is based on 37 hours per week at $30.15 an hour, the other is a yearly salary of $59 400. If 1 year is taken to be 52·178 weeks, which weekly income is higher and by how much? (Answer to the nearest cent.)

3 a i Does this job guarantee an income?

ii If you have never heard of the products of this company, is it likely that you will sell much of their product?

iii Is any compensation mentioned for petrol used or provision of a vehicle?

iv Find the commission paid on sales of:

1 $3000 2 $7430

3 $16 580 4 $920

CLEAN-U-UP PTY LTD
Selling cleaning machinery equipment and chemicals.
Sales people required to sell on total commission of 23% of sales.
Great potential!
Excellent reward.
Ring: Ugo Broke. YRU-000.

b Janice is offered a sales position with a retainer (guaranteed wage) of $740 plus a commission of 7% on sales.

i How much could she make in a week for the following weekly sales?

1 $800 2 $3500 3 $4865 4 $5213

ii She is told that the average weekly sales per person is $6300. What is the average weekly income?

iii If Janice is a shy person who has no previous sales experience, is it likely that she will succeed in this job?

c John works as a sales assistant receiving $800 per week plus 10% commission on sales in excess of $5000 for the week. Find his income for the following weekly sales.

i $3400 ii $5700 iii $8424 iv $6129.50

4 a Heather works in a supermarket on a casual basis. She is paid $16.60 an hour from Monday to Friday and $20.85 an hour on Saturdays. Find her week's income if she works from 3:30 pm until 5:30 pm, Monday to Friday, and from 8:30 am until 1:00 pm on Saturday.

b Edward works as a waiter from 6:00 pm until 1:30 am for 4 days in 1 week. His hourly rate of pay is $18.35 and he gets an average of $6.50 as tips per working night. Find his income for the week. (A 'tip' is a payment in appreciation of good service.)

c An electrician charged $54 per hour for labour. Find the charge for labour if he works from 11:20 am until 1:50 pm.

5 a A factory worker was paid $4.16 for each garment completed. How much would be earned if 240 garments were completed?

b A doctor charges each patient $65 for a consultation. If she works for 5 hours during one day and sees an average of six patients per hour, find the amount of money received that day. Her costs per day are $730. What was her profit for the day?

c Smokey and Smiley were two shearers who were paid $5.40 for each sheep shorn. By how much was Smokey's pay greater than Smiley's, if Smokey sheared 673 sheep and Smiley sheared only 489?

d Flo works at home altering dresses for a dress shop. She is paid $24.95 for a major alteration and $16.80 for a small alteration. In the week before Christmas, she completed 13 major alterations and 27 small alterations. Find her income for the week. If she spent 39 hours working on the alterations, what was her hourly rate of pay? (Answer to the nearest cent.)

8:02 Extra payments

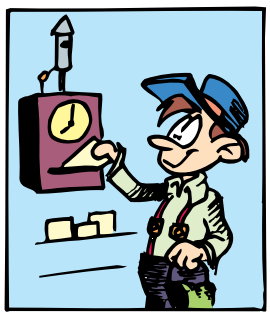
There are several additional payments that may add to a person's income. Terms needed are listed below.

1 **Overtime:** This is time worked in excess of a standard day or week. Often rates of $1\frac{1}{2}$ or 2 times the normal rate of pay are paid for overtime.

2 **Bonus:** This is money, or an equivalent, given in addition to an employee's usual income.

3 **Holiday loading:** A payment calculated as a fixed percentage of the normal pay over a fixed number of weeks. It may be paid at the beginning of annual holidays to meet the increased expenses often occurring then.

4 **Superannuation:** This is a payment made by an employer into an investment fund nominated by the employee which provides benefits for the employee upon retirement. Employees are eligible for superannuation payments if they are between the ages of 18 and 69 inclusive and earn at least $450 per month. These payments must be at least 9.25% of the employee's 'ordinary time' earnings. Extra contributions may also be made by the employee.



Time cards or time sheets

These can be used to record the number of hours worked in a week. A time clock can be used to stamp times on to the card. Therefore an employee 'clocks on' at the beginning of a shift and 'clocks off' at the end. The record of hours worked will also indicate any overtime hours worked.

No. 53 **TIME CARD** **Whit. Pty Ltd**
Name: Tom McSeveny

Week ending	Fri 21 Jan		Fri 28 Jan		Fri 4 Feb		Fri 11 Feb		Fri 18 Feb	
Day	**IN**	**OUT**	**IN**	**OUT**	**IN**	**OUT**	**IN**	**OUT**	**IN**	**OUT**
Sat	–	–	–	–	8:00	10:02	8:00	12:00	8:02	11:30
Sun	–	–	–	–	–	–	–	–	–	–
Mon	7:57	4:00	8:00	4:04	7:59	4:00	8:00	4:02	7:57	3:59
Tues	7:58	4:02	7:55	3:59	7:56	4:02	8:00	4:05	8:00	4:05
Wed	8:00	4:01	8:00	4:02	8:03	4:01	7:56	3:02	7:58	4:07
Thu	8:02	4:05	7:58	7:00	7:58	4:03	8:01	4:02	7:55	6:00
Fri	8:00	4:00	8:00	4:00	8:00	4:01	8:02	6:31	7:59	6:30

Hourly rate: $26.20
Lunch: 12 noon until 1:00 pm (unpaid)
Normal hours: Mon-Fri, 8:00 a.m. – 4:00 p.m.
Overtime: 'Time-and-a-half' is paid and 'double-time' for overtime in excess of 3 hours (on any one day)

Note:

1 In the week ending 21 Jan, no overtime was worked.
Total of hours worked = (8 hours × 5) − 5 hours for lunch
= 35 hours

2 In the week ending 11 Feb, only 34 normal hours were worked, as Tom left work 1 hour early on Wednesday. However, $2\frac{1}{2}$ hours overtime was worked on Friday and 4 hours on the Saturday. Three of the hours worked on Saturday are time-and-a-half and one (that in excess of 3 h) was at double-time.

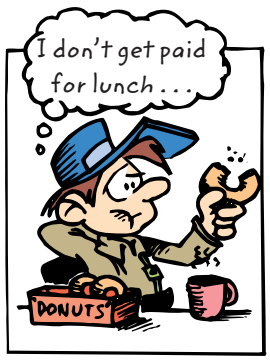
A few minutes variation from the hour or half-hour will not be considered in determining hours worked.

WORKED EXAMPLE 1

a During one week Peter worked 35 hours at the normal rate of $17.60 per hour. He also worked 6 hours overtime: 4 at 'time-and-a-half' and 2 at 'double-time'.
How much did he earn?

b Use the time card on the previous page to calculate Tom McSeveny's wage for the week ending Friday, 18 February.

c Calculate Diane's holiday loading if she is given $17\frac{1}{2}\%$ of 4 weeks' salary and she earns $2980 per fortnight.



Solutions

time-and-a-half ↓ double time ↓

a Peter's earnings = (35 h at $17.60) + (4 h at $17.60 × $1\frac{1}{2}$) + (2 h at $17.60 × 2)
= (35 × 17·6) + (4 × 17·6 × 1·5) + (2 × 17·6 × 2) dollars
= $792 (using a calculator)

b For the week ending Friday, 18 February, Tom worked:

Normal hours: 35 hours (8–4, Mon–Fri with 1 hour lunch)

Time-and-a-half: $7\frac{1}{2}$ hours (8–11 on Sat, 4–6 on Thur, 4–6:30 on Fri)

Double-time: $\frac{1}{2}$ hour (11–11:30 on Sat as double-time is paid only after 3 hours)

∴ Tom's earnings = (35 h at $26.20) + ($7\frac{1}{2}$ h at $26.20 × $1\frac{1}{2}$) + ($\frac{1}{2}$ h at $26.20 × 2)
= (35 × 26·2) + (7·5 × 26·2 × 1·5) + (0·5 × 26·2 × 2) dollars
= $1237.95 (using a calculator)

c Diane's holiday loading = $17\frac{1}{2}\%$ of 4 weeks' salary
= $17\frac{1}{2}\%$ of ($2980 × 2)
= 0·175 × (2980 × 2) dollars
= $1043

$17\frac{1}{2}\% = \frac{17{\cdot}5}{100}$
= 17·5 ÷ 100
= 0·175

WORKED EXAMPLE 2

For each of the 4 weeks in June, Paul's gross wage was $1375. What amount did Paul's employer have to contribute to his superannuation fund for the month if it is 9.25% of his monthly earnings?

Solution

Paul's earnings for the month = $1375 × 4
= $5500

∴ Superannuation payment = 9.25% of $5500
= 0.0925 × $5500
= $508.75

Superannuation = 9.25% of earnings
This will increase to 12% by 2019.

Exercise 8:02

P Foundation worksheet 8:02
Extra payments

1 **a** Bill earns $9.60 per hour. Calculate his wages for the week if he worked 35 hours at the normal rate and 5 hours overtime at 'time-and-a-half'.

b At Bigfoot Enterprises a wage rate of $12.70 per hour is paid on the first 37 hours and 'time-and-a-half' after that. What is the wage for a 42-hour week?

c Each day Pauline receives $18.10 per hour for the first 7 hours, 'time-and-a-half' for the next 2 hours, and 'double-time' thereafter. Find her wage for:

i a 6-hour day **ii** a 9-hour day **iii** an $8\frac{1}{2}$-hour day **iv** an 11-hour day

d An electrician earns a wage of $22.40 per hour for a 35-hour week and 'time-and-a-half' after that. How much would he earn in a week in which he works:

i 30 hours **ii** 37 hours **iii** $41\frac{1}{2}$ hours **iv** $45\frac{1}{2}$ hours

e A pipe factory asks a labourer to work 8 hours on Saturday at 'time-and-a-half' for the first 3 hours and 'double-time' after that. If his normal rate of pay is $20.40, how much is he paid for the day's work?

f Brian earns $17.60 an hour, whereas his boss earns $23.20 an hour. How much more than Brian is the boss paid for a 7-hour day? If Brian gets time-and-a-half for overtime, how many additional hours would he need to work in a day to get the same wage as his boss?

g By referring to the time card on page 214, complete the summary below.

The tricky parts of the time card on page 214 are shaded.

No. 53 Name: Tom McSeveny	**Time card summary**			Whit. Pty Ltd. Rate: $26.20 p/h
Week ending	Number of hours at:			Wage
	normal rates	time-and-a-half	double-time	
21 Jan				
28 Jan				
4 Feb				
11 Feb				
18 Feb				

2 **a** If $17\frac{1}{2}\%$ holiday loading is given on 4 weeks' normal pay, find the holiday loading for:

- **i** John, who earns $4000 in 4 weeks
- **ii** Mary, who earns $2820 in a fortnight
- **iii** Wilkes, who earns $1495 a week
- **iv** McNally, who earns $29.80 an hour (for a 35-hour week).
  (Assume that in each case no extra payments are included.)

b When June was given her holiday pay she received 4 weeks' pay and a $17\frac{1}{2}\%$ holiday loading. If her normal wage is $1427 per week, how much holiday pay did she receive?

c Fred works a 35-hour week at a rate of $27.60 per hour. Calculate his holiday loading if $17\frac{1}{2}\%$ is given on 4 weeks' wage.

d Mr Bigsuccess earns a salary of $96 000 per year. At the end of the year he is given a bonus equal to 80% of one month's pay. How much did he earn in the year?

e Alana managed a small business for a salary of $56 400. At the end of a successful year in which the business made a profit of $211 000, she was given a bonus of 1·2% of the profits. What was her bonus and what was her income for the year?

f Luke works for a mining company at a wage rate of $42.80 per hour. If he works underground he is paid a penalty rate of $13.65 per hour in addition to his normal pay. Find his weekly wage if during the normal 38 hours he works underground for 16 hours.

g Mary works in a food-processing plant at a wage rate of $25.95 per hour. From time to time she is required to work beside ovens where the temperature is uncomfortable. When this is necessary, she is paid an additional $2.95 an hour. Calculate her wage for a normal working week of 36 hours where 5 hours were beside the ovens.

h Lyn received a $17\frac{1}{2}\%$ holiday loading on 4 weeks' normal wages. (She works a 36-hour week.) Find her normal weekly wage if the total of 4 weeks' wages and the holiday loading was:

i $4700 **ii** $6298

i Sundeep received a holiday-loading payment of $868 that represented $17\frac{1}{2}\%$ on 4 weeks' wages. What is his weekly wage? What percentage of his total income for the year (containing 52 weeks) does the holiday loading represent? (Answer correct to two decimal places.)

3 **a** Calculate the 9.25% superannuation contribution owing for the following monthly earnings.

i $3200 **ii** $1260 **iii** $5340 **iv** $7335

b For a month in which there are four weekly wage payments, calculate the employer superannuation contribution at 9.25% for these weekly wages.

i $765 **ii** $1170 **iii** $1520 **iv** $2136

c Libby works 40 hours a week for $27.40 per hour. For a month in which there are five wage payments, calculate Libby's superannuation contribution at 9.25%?

d Jo works occasionally at the local cafe when they need her. For the last 6 months her monthly earnings were: $580, $390, $760, $1020, $350 and $840. What would have been the total amount in superannuation payments at 9.25% the employer would have had to pay for this 6 months?

e What would have been the total monthly earnings corresponding to superannuation contributions of 9.25% of:

i $370 ii $536.50 iii $320.05 iv $654.16

Note: No super payment is required if monthly earnings are less than $450.

8:03 Wage deductions

A person's weekly wage or salary is referred to as the weekly gross pay. After deductions have been made the amount actually received is called the weekly net pay.

Possible deductions

Income tax The Commonwealth government takes a part of all incomes earned to finance federal, state and local government activities. Employers deduct this tax on the government's behalf at the end of each pay period. It is called PAYG (pay-as-you-go). The rate of tax varies according to the amount of money earned and the number of dependants.

Superannuation This is a form of insurance or investment. Often the employee, as well as the employer, contributes to this fund. It provides for an income or lump-sum payment upon retirement, and in the event of the employee's death, it provides a pension for the family.

Miscellaneous Other deductions could be for medical insurance, life insurance, home payments, credit union savings and union membership fees.

Here is a pay advice slip representing 2 weeks' pay.

Peter Newby	Serial No.	Dept.	Location	Gross Salary or Wage Rate (1)	Super Units		Fortnight Ended	Net Pay (5)	Pay Advice No.
					Entld.	Held			
	6552750	AA	8436	76880	113	113	4/11/13	1998.70	12381

*** Reasons for Adjustments**

A – increment **B** – award agreement **C** – national wage **D** – promotion
E – L.W.O.P. **F** – allowance **G** – termination **H** – resumed duty
I – reduction in salary **Z** – combination of reasons

Deductions this fortnight				Pay this fortnight			
Taxation (7)	S'annuation (8)	M'laneous	Total (6)	Normal Pay (2)	Adjustments (3)	Overtime	Gross Earnings (4)
636.80	147.50	163.80	948.10	2946.80	–	–	2946.80

Peter Newby has a gross yearly salary or wage (1) of $76 880, earns $2946.80 (2) per fortnight and in the fortnight ending 4/11/13 had no adjustments or overtime (3). This means that his gross earnings were $2946.80 (4). His net pay (5) was only $1998.70 because his total deductions (6) were $948.10.

Peter paid $636.80 in tax (7) on an income of $2946.80. This is about 22%. His contribution to superannuation (8) was $147.50.

WORKED EXAMPLE

a Find the net pay for the week if John earns $923.60, is taxed $138.50, pays $36.90 for superannuation and has miscellaneous deductions totalling $76.30.

b What percentage of his gross pay did he pay in tax?

Solution

a Total deductions = Tax + Superannuation + Miscellaneous
= $138.50 + $36.90 + $76.30
= $251.70

∴ John's net pay = $923.60 − $251.70
= $671.90

b John's tax payment = $138.50

$$\text{Tax as a percentage of gross pay} = \frac{\text{tax}}{\text{gross pay}} \times 100\%$$
$$= \frac{\$138.50}{\$923.60} \times 100\%$$
$$= 15\%$$

Exercise 8:03

1 Find the net pay if:

- **a** gross pay is $315.60 and total deductions are $115
- **b** gross pay is $810.20, tax is $99.70, superannuation is $20.70 and miscellaneous is $94.80
- **c** gross pay is $461.55, tax is $22.20, superannuation is $4.80 and union fees are $2.60
- **d** gross pay (for a fortnight) is $1930.40, superannuation is $48.50, miscellaneous is $174.70 and tax is $303.55
- **e** gross salary is $902.10, superannuation is $31.90, medical insurance is $21.60, life insurance is $4.10, house payment is $76.50, credit union savings are $11.00, union fees are $2.45 and tax is $131.60.

2 a Vicki Turner receives a yearly salary of $72 096, pays 21% of her weekly gross salary (calculated on 52·18 weeks in the year) in income tax, pays 5% of her weekly gross to her superannuation fund and has $86 in miscellaneous deductions each week. Find her:

i weekly gross salary
ii weekly tax deductions
iii weekly superannuation payment
iv weekly net pay.

'Net pay' is what you take home.

b Use Question 2a to complete this pay advice slip.

Turner, Vicki	Serial No.	Gross Salary or Wage Rate	Super Units		Week Ended	Net Pay	Pay Advice No.
			Entld.	Held			
	6841672		98	98	18/11/11		11364

*** Reasons for Adjustments**

A – increment **B** – award agreement **C** – national wage **D** – promotion
E – L.W.O.P. **F** – allowance **G** – termination **H** – resumed duty
I – reduction in salary **Z** – combination of reasons

Deductions this week				Pay this week			
Taxation	S'annuation	M'laneous	Total	Normal Pay	Adjustments	Overtime	Gross Earnings
					–	–	

3 Find the net pay and the tax as a percentage of the gross pay for each person.

	Name	Gross Pay	Tax	Net Pay	Tax as % of Gross Pay
a	R. Collison	$385.70	$6.90		
b	G. Foster	$1450.00	$308.70		
c	B. Jones	$947.50	$145.40		
d	R. Sinclair	$2591.60	$727.10		

4 a Upon retirement, Joe Simmons received annual superannuation payments of 68% of his final year's salary. If his salary at that time was $62 600, how much is his annual superannuation (before tax)?

b Ellen's annual superannuation is 63% of her finishing wage of $52 186. What is her monthly income? (Answer correct to the nearest cent.)

c John earns $82 490 a year. If he were to die, his widow would receive 65% of this figure in superannuation payments each year. What would be her weekly income from the superannuation payments (before tax) taking 1 year to be 52·18 weeks?

d Jim has just retired. He has the option of receiving a monthly payment of $2450 or $330 000 as a 'lump sum' (a final single payment).

- **i** Find the yearly superannuation payment.
- **ii** What yearly income would result if the lump sum could be invested at 7% p.a.?
- **iii** Which option seems most attractive and why?
- **iv** What would his yearly income be if he elected to receive 30% of the monthly payment, and 70% of the lump sum of which he invested 40% at 7% p.a.?

e Janice retired in 1995 on a fixed income of $2400 per month. How many toothbrushes costing $1.80 could she buy with a month's income?

One year later, inflation had caused the cost of toothbrushes to rise by 8%. How many toothbrushes could she buy with a month's income after the rise? (As years pass, inflation greatly affects the purchasing power of people on fixed incomes.)

Challenge

1. Expected returns in superannuation, based on average life span, could be calculated.
2. Effects of inflation (with or without indexation) on these expected returns could be considered.

8:04 Income tax

- The annual Income Tax Return is a form, filled out each year, to determine the exact amount of tax that has to be paid for the preceding 12 months. Since most people have been paying tax as they have earned their income, this exercise may mean that a *tax rebate* is given.
- Some expenses, such as those necessary in the earning of our income, are classified as tax deductions, and the tax we have paid on this money will be returned to us. On the other hand, if we have additional income (such as interest on savings) that has not yet been taxed, additional taxes will have to be paid.

  The tax deductions are subtracted from the total income to provide the *taxable income*.
- The tax to be paid on the *taxable income* can be calculated from the table below (2013 scale).
- If your taxable income is more than $22 828, your Medicare levy is 1·5% of that taxable income. This covers you for basic medical costs and is a separate payment in addition to tax.

Taxable income	Tax on this income
$0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

The above rates do not include the Medicare levy of 1·5%.

WORKED EXAMPLE

Albert received a salary of $47 542 and a total from other income (investments) of $496. His total tax deductions were $1150. During the year he had already paid tax instalments amounting to $10 710.75. Find:

a his total income
b his taxable income
c how much Albert must pay as his Medicare levy
d the tax payable on his taxable income
e his refund due or balance payable when the Medicare levy is included
f how much extra Albert would receive each week if he is given a wage rise of $10 per week.

Solutions

a Albert's total income
= $47 542 + $496
= $48 038

b Albert's taxable income
= total income − tax deductions
= $48 038 − $1150
= $46 888

c Medicare levy
= 1·5% of the taxable income
= 1·5% of $46 888
= $703.32

d Taxable income = $46 888 (or $37 000 + $9888)
Tax on $37 000 = $3572 (from the tax table) (A)
Tax on $9888 at 32.5 cents = $3213.60 (32.5c/$ for amount over $37 000) (B)
∴ Tax on $46 888 = (A) + (B)
= $3572 + $3213.60
= $6785.60

e Tax on $46 888 + Medicare levy
= $6785.60 + $703.32
= $7488.92
Tax instalments paid = $10 710.75
∴ Refund = $10 710.75 − $7488.92
= $3221.82

f For salaries over $37 000 and less than $80 001, for each additional $1 earned you pay 32.5 cents tax and a Medicare levy of 1·5%.
∴ Tax on an extra $10 per week
= 10 × $0.325 + 1·5% of $10
= $3.25 + $0.15
∴ Amount left after tax
= $10 − $3.40
= $6.60 per week

Exercise 8:04

P Foundation worksheet 8:04 Taxation

1 Use the tax table on page 221 to determine the tax payable, not including the Medicare levy, on a taxable income of:

a $13 963 **b** $22 587 **c** $41 460 **d** $97 346 **e** $284 914

2 Mrs Short has a salary of $33 600, receives income from other sources of $342, has tax deductions of $655, and has paid PAYG tax instalments throughout the year of $6570. Find:

a her total income
b her taxable income
c tax payable on her taxable income
d her refund or balance payable after the Medicare levy of 1·5% is paid.

PAYG stands for:
'Pay As You Go'
i.e. you pay tax instalments as you earn your income

3 When Joy left school she had a weekly wage of $988. During the financial year there were 52 weeks' pay received. (*Note:* In a normal year of 52 weeks and one day, there could be 53 paydays.) She had no extra income and calculated her tax deductions to be $217. Find the tax payable on her taxable income.

4 Karl earned $82 850 as a tiler. His employer deducted tax payments of $18 650. However, Karl earned a further $4156 on weekends and during his holidays. His tax deductions came to $2096 for expenses in earning the additional income. Find:

a Karl's total income
b Karl's taxable income
c the tax on his taxable income
d additional tax payable or refund, including the Medicare levy of 1·5%.

5 Eight workers in a factory were each given a pay rise of $2000 per annum. How much of the $2000 would each have received after tax and the Medicare levy of $30? Their yearly taxable incomes are listed below. (Use the tax table on page 221.)

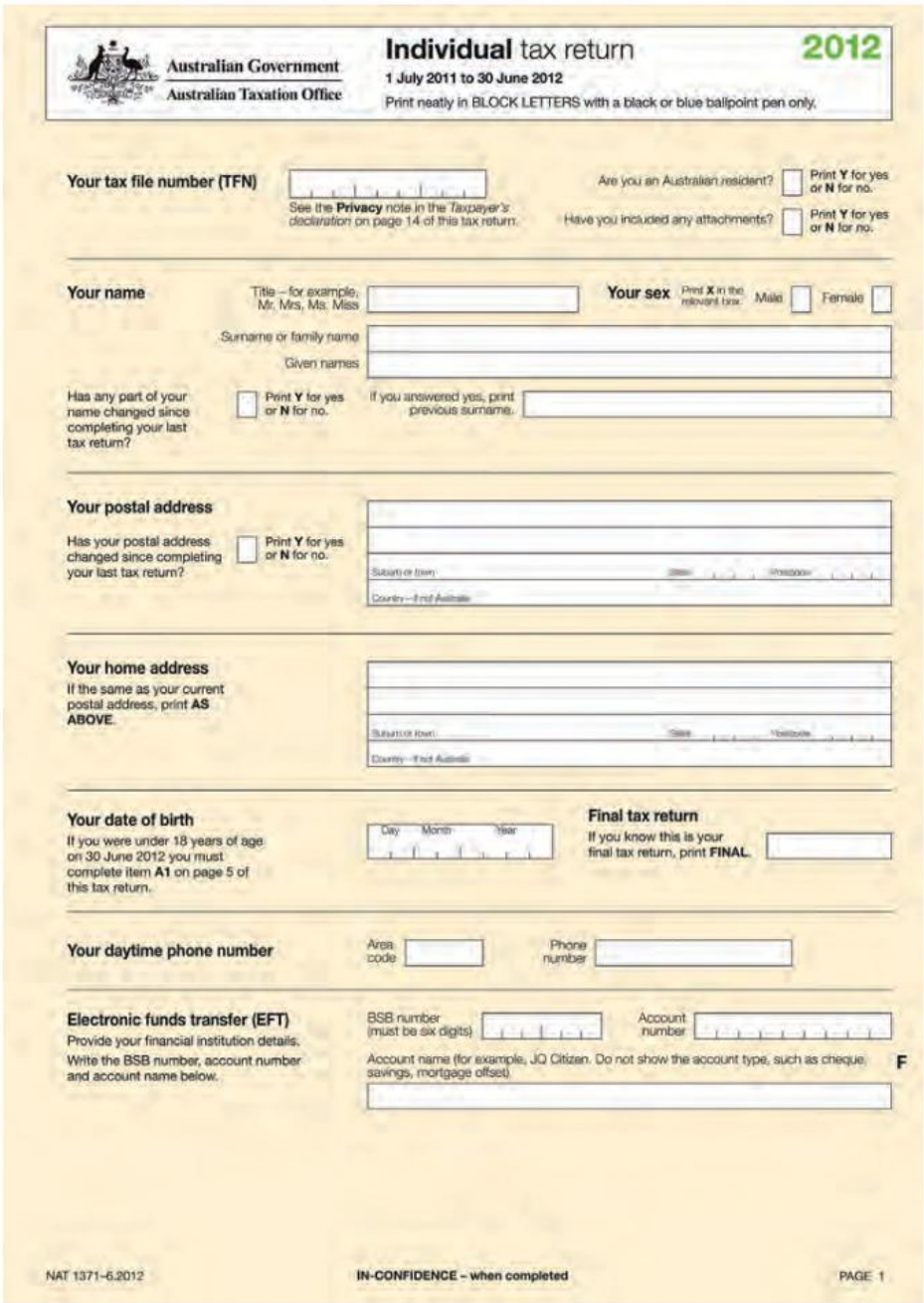
a M. Callow $13 900
b A. Smith $25 500
c P. Farmer $45 200
d M. Awad $83 720
e R. Sissi $193 000
f P. Mifsud $36 000
g R. Ringe $79 500
h S. Sze $179 200

Note:
The Medicare levy is not paid if your taxable income is less than $19 404.

INVESTIGATION 8:04 INCOME TAX RETURNS

1 **a** Obtain copies of an Income Tax Return form.
b Choose an occupation, an income figure and a number of dependants.
c Make a list of tax deductions (money spent to produce the income). Estimate an amount for each of these, trying to be as realistic as possible.
d Complete the Income Tax Return form using the estimates above.

2 **a** Using class data, investigate rates of pay from students' part-time jobs, or jobs advertised.
b Calculate the tax to be paid on five of these jobs.



Australian Government
Australian Taxation Office

Individual tax return **2012**

1 July 2011 to 30 June 2012

Print neatly in BLOCK LETTERS with a black or blue ballpoint pen only.

Your tax file number (TFN)

See the **Privacy** note in the *Taxpayer's declaration* on page 14 of this tax return.

Are you an Australian resident? Print **Y** for yes or **N** for no.

Have you included any attachments? Print **Y** for yes or **N** for no.

Your name

Title – for example, Mr, Mrs, Ms, Miss

Surname or family name

Given names

Your sex Print **X** in the relevant box. Male Female

Has any part of your name changed since completing your last tax return? Print **Y** for yes or **N** for no.

If you answered yes, print previous surname.

Your postal address

Has your postal address changed since completing your last tax return? Print **Y** for yes or **N** for no.

Suburb or town State Postcode

Country – if not Australia

Your home address

If the same as your current postal address, print **AS ABOVE**.

Suburb or town State Postcode

Country – if not Australia

Your date of birth

If you were under 18 years of age on 30 June 2012 you must complete item **A1** on page 5 of this tax return.

Day Month Year

Final tax return

If you know this is your final tax return, print **FINAL**.

Your daytime phone number

Area code

Phone number

Electronic funds transfer (EFT)

Provide your financial institution details.

Write the BSB number, account number and account name below.

BSB number (must be six digits)

Account number

Account name (for example, JQ Citizen. Do not show the account type, such as cheque, savings, mortgage offset)

F

NAT 1371-6.2012 **IN-CONFIDENCE – when completed** PAGE 1

FUN SPOT 8:04

WHAT IS BROUGHT TO THE TABLE, CUT, BUT NEVER EATEN?

Work out the answer to each question and write the letter for that part in the box that is above the correct answer.

Express as a percentage:

A 0·76 **A** 0·125 **A** $\frac{3}{5}$

O 12% of \$81 250 **C** \$13.50 × 38 + \$20.25 × 6

C Sid is paid \$22.80 an hour for a 40-hour week. What is his normal weekly wage?

D For a 38-hour week Naomi is paid \$744.80. Find her hourly rate of pay.

F My salary is \$31 306.80 per year. How much would I receive each week if it is calculated on 52·178 weeks in a year?

S Alana has a casual job from 4:00 pm until 5:30 pm Monday to Friday. What is her weekly income if her casual rate is \$9.20 per hour?

P A tiler charges \$28.50 per square metre to lay tiles. How much would he charge to lay an area of 8 square metres?

K Find the net pay for a week if Anna earns \$520.50, is taxed \$104.10, pays \$8 for superannuation and has miscellaneous deductions totalling \$77.40.

R In the previous question, what percentage of Anna's gross pay was paid in tax?

60%	\$228	76%	\$634.50	\$331	\$9750	\$600	\$912	12.5%	20%	\$19.60	\$69

It is important to balance your budget.

8:05 Spending money

PREP QUIZ 8:05

Find:

1 10% of $60 **2** 10% of $31 **3** 10% of $43.50 **4** 10% of $6.73

Calculate the following:

5 $55 ÷ 11 **6** $67.10 ÷ 11 **7** $93.50 ÷ 11

Find the price per kg if:

8 4 kg costs $10 **9** 7 kg costs $32.90 **10** 15 kg costs $43.20

This section is a review of work covered in Year 8. This included determining 'best buys', calculating discounts, finding the GST included in prices and determining a percentage profit or loss. The worked examples below will remind you of the skills involved.

Best buys

When we know the cost of a certain amount of a product we usually find the cost of 1 unit so we can compare prices. This is called the **unitary method**.

WORKED EXAMPLES

1 A bag of 12 apples costs $3.60 and a bag of 20 costs $5.00.
What is the price per apple for each bag?
Which is the better buy?

2 A 130 mL can of spaghetti costs 55 cents and a 420 mL can costs $1.60.
Which is the better buy?

Solutions

1 To find the 'unit' price of one apple in each bag we divide by the number in each bag.
For the bag of 12, the unit cost = $3.60 ÷ 12
= 0.30c
For the bag of 20, the unit cost = $5.00 ÷ 20
= 0.25c
The bag of 20 is the better buy.

2 For this example we need to use a calculator.
We can divide the cost of the 130 mL can by 130 to find the cost of 1 mL (1 unit) and then multiply this figure by 420 to find the cost of 420 mL at this price.
Press: 0.55 ÷ 130 × 420 =
The answer is $1.78, to the nearest cent. This is dearer than the cost of the 420 mL can at $1.60. So the 420 mL can is the better buy.

Discounts

Often when selling goods a retailer will offer a discount on the advertised price to encourage a buyer to purchase. Sometimes a discount might be given to favoured or regular customers.

WORKED EXAMPLES

1 a Greg was given $12\frac{1}{2}\%$ discount on a rug with a marked price of $248. How much did he pay?

b A television marked at $2240 was eventually sold for $2128. What was the discount and what was the percentage discount given on the marked price?

c After a discount of 14% was given, I paid $5848 for my yellow Holden. What was the original marked price?

2 Brenda bought a TV priced at $1200 after it was discounted by 10%. Brenda received a further 5% discount because she was a member of staff. How much did she pay for the TV?

Solutions

1 a Discount on rug = 12·5% of $248
= 0·125 × $248
= $31
Amount paid = $248 − $31
= $217

b Discount on TV = $2240 − $2128
= $112
Percentage discount = ($112 ÷ $2240) × 100%
= 5%

c Price paid = (100 − 14)% of marked price
= 86% of marked price
1% of marked price = $5848 ÷ 86
= $68
100% of marked price = $6800

2 Price after original 10% discount = (100 − 10)% of $1200
= 90% of $1200
= $1080

This is an example of successive discounts.

Price after a further 5% discount = (100 − 5)% of $1080
= 95% of $1080
= $1026

(*Note:* This is not the same as a 15% discount off the original price i.e. 85% of $1200 = $1020)

GST (Goods and Services Tax)

The GST is a tax of 10% on most goods and services. It is included in the price you pay. It is simple, however, to calculate the GST included in any price by dividing by 11, since the base price has been increased by 10%, or $\frac{1}{10}$. This means that the price that includes the GST is $\frac{11}{10}$ times the original price.

To calculate the GST to add on to a price, simply find 10% of the price.

To find the GST included in a price, divide the price by 11.

WORKED EXAMPLES

1 Find the GST that needs to be applied to a price of $325.

2 What is the retail price of a DVD player worth $325 after the GST has been applied?

3 How much GST is contained in a price of $357.50?

4 What was the price of an item retailing at $357.50 before the GST was applied?

Solutions

1 The GST is 10% of the price.
$\therefore$ GST = $325 × 10%
= $32.50

2 The GST is added on to get the retail price.
$\therefore$ Retail price = $325 + $32.50
= $357.50

Note: The retail price can also be calculated by multiplying the original price by 110% (or 1·1) since 10% is added on.

Retail price = $325 × 1·1
= $357.50

3 To find the GST contained in a price, we divide it by 11. (If the original price is increased by $\frac{1}{10}$, then the retail price, including the GST, is $\frac{11}{10}$ of the original price.)
$\therefore$ GST = $357.50 ÷ 11
= $32.50

4 To find the original price, simply subtract the GST from the retail price.
$\therefore$ Original price = $357.50 − $32.50
= $325

Note: The original price can also be found by multiplying the retail price by $\frac{10}{11}$.

Original price = $357.50 × $\frac{10}{11}$
= $325

Profit and loss

Sometimes we need to consider whether our purchases, or sales, have been worthwhile.

We may do this by calculating, and comparing, the percentage profit, or loss, we may have made on a given transaction.

- When buying and selling:

  Selling price = Cost price + Profit

  or

  Profit = Selling price − Cost price

  If the profit is *negative*, then we have made a *loss*.
- When calculating money made:

  Profit = Money received − Expenses

WORKED EXAMPLES

1 A book was bought online for $16 and sold for $20. Find the profit as a percentage of the:

a cost price

b selling price.

2 The cost price of a car is $7800. If the dealer wants to make a profit of 15% on the cost price, what must be the selling price of the car?

3 A selling price of $392 includes a 12% profit margin on the original cost price. What was the cost price?

Solutions

1 Profit = Selling price − Cost price
= $20 − $16
= $4

∴ Percentage profit on:

a Cost price = $\frac{\$4}{\$16} \times 100\%$
= 25%

b Selling price = $\frac{\$4}{\$20} \times 100\%$
= 20%

2 Profit = Cost price × 15%
= $7800 × 15%
= $1170

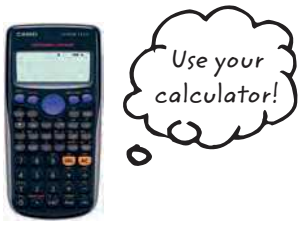
∴ Selling price = Cost price + Profit
= $7800 + $1170
= $8970

3 Selling price = Cost price + 12% of Cost price
= 112% of Cost price
= $\frac{112}{100} \times$ Cost price

∴ Cost price = Selling price × $\frac{100}{112}$
= $392 \times \frac{100}{112}$
= $350

Exercise 8:05

P Foundation worksheet 8:05
Goods and Services Tax



1
a If 3 books cost $5, find the cost of 12 books.
b If 5 bananas cost $2, find the cost of 8 bananas.
c If 4 cakes cost $10, find the cost of 7 cakes.
d If 8 stamps cost $4.80, find the cost of 15 stamps.
e If 15 bolts cost $5.25, find the cost of 24 bolts.
f If 24 lollies cost $3.60, find the cost of 75 lollies.

2 Find the cost of 1 'unit' in each of the examples below and then determine the 'best buy'.

Use the unitary method

a	i 3 apples cost $1.20	ii 5 apples cost $1.50
b	i 4 bags cost $3.00	ii 7 bags cost $5.60
c	i 8 kg cost $10.40	ii 12 kg cost $15.00
d	i 10 comics cost $26	ii 8 comics cost $22.40
e	i 12 litres cost $15.60	ii 15 litres cost $19.95
f	i 24 plants cost $105.60	ii 32 plants cost $134.40

3 Find the discount on the following prices at the given percentages. Also find the discounted price.

a $50, 10%	b $20, 20%	c $100, 8%
d $24, 25%	e $40, 50%	f $50, 15%
g $30, 40%	h $28, 75%	i $200, 90%
j $25, 12%	k $45, 60%	l $80, 35%

4 For each marked price below, find the discount that was given to arrive at the final selling price. Also find what percentage this discount was of the marked price.

a Marked price = $40, Selling price = $30	b Marked price = $50, Selling price = $45
c Marked price = $100, Selling price = $75	d Marked price = $60, Selling price = $48
e Marked price = $36, Selling price = $18	f Marked price = $25, Selling price = $23
g Marked price = $24, Selling price = $21	h Marked price = $36, Selling price = $24

5 Find the GST that must be added to the following prices. Also state the final retail price.

GST = 10%

a $50	b $20	c $100
d $25	e $8	f $12
g $56	h $37	i $83
j $5.50	k $9.60	l $16.40
m $5.26	n $23.42	o $71.96

6 Find the GST that is contained in the following retail prices. (Answer to the nearest cent, when necessary.)

Divide by 11!

a $44	b $77	c $132
d $24.20	e $92.40	f $12.65
g $8.14	h $21.34	i $81.73
j $50	k $24	l $145
m $9.50	n $21.90	o $67.45

7 Complete the table, filling in the missing cost price, profit or selling price.

	Cost price	Profit	Selling price
a	$400	$100	
b	$240		$300
c		$70	$420
d	$840	$42	
e	$750		$1000

8 For each of the examples in the table in Question **7**, find the profit as a percentage of:
a the cost price **b** the selling price.
Answer to the nearest whole percentage, when necessary.

9 Find the 'best buy' for each of the alternatives below.
a 4 tyres for $450 OR 5 tyres for $550
b 6 ice creams for $20.40 OR 10 ice creams for $35
c 2 litres of paint for $43.60 OR 5 litres for $92.50
d 250 mL soft drink for $2.30 OR 600 mL soft drink for $5.20
e 375 g can for $4.20 OR 650 g can for $7.50
f 4 lollies for $4.40 OR 5 lollies for $5.70
g 250 mL shampoo for $5.20 OR 375 mL shampoo for $7.65
h 2.5 kg of potatoes for $3.40 OR 20 kg of potatoes for $25

10 Find the best buy for each of the following.
a 250 g for $4.20 OR 420 g for $6.60 OR 600 g for $9.20
b 3 litres for $44 OR 5 litres for $72 OR 8 litres for $120
c 275 mL bottle for $3.75 OR 450 mL bottle for $6.30 OR 1 L bottle for $12.90

11 **a** Rachel was given a discount of 45% on a T-shirt with a marked price of $32. How much did she pay?
b Katherine was given a 7% discount on all purchases because she was a store employee. If the total of her purchases came to $85, how much did she pay?
c After bargaining, I purchased a chess set for $240 that was originally advertised for $320. What discount was I given and what is this as a percentage of the advertised price?
d My airline ticket was to cost me $550 but, because I booked more than 6 weeks ahead, I was given a discount price of $385. What percentage discount was I given?
e After a discount of 10% was given, Luke paid $585 for an additional hard disk drive for his computer. What was the original price of this hard disk drive?
f My mother paid $21.70 to attend an exhibit. She had been given a discount of 65% because she was a pensioner. What was the original price?
g Because Helen was prepared to pay cash for a yellow car with a marked price of $9200, she was given a discount of $1012. What percentage was the discount of the marked price?
h Greg was given a discount of 10% on the marked price of a kitchen table. If the discount was $22, how much did he pay?

12 Calculate the final price if successive discounts of 20% and 10% were applied to a price of $100.

13 A bookstore discounted all its books by 15%. A further discount of 10% was given to the local school. How much did they pay for books that originally sold for a total of $1750?

14 Apply the following successive discounts to the given prices.

a 10%, 10%, $500

b 15%, 25%, $720

c 5%, 20%, $260

d $12\frac{1}{2}\%$, 15%, $965

e $7\frac{1}{2}\%$, $17\frac{1}{2}\%$, $2230

15 When retailers sell a mixture of goods where the GST is only applied to some, they normally indicate which items on the bill include the GST. This bill indicates the items that include the GST with an asterisk (*). Determine the amount of GST that would be included in the total on this bill.

SHOPQUICK

Sauce	$ 1.20
Biscuits*	$ 1.35
Bread	$ 3.60
Toothbrush*	$ 3.10
Soap*	$ 1.93
Total	$ 11.18

GST incl. = ?

16 A retailer indicates the GST items with a percentage sign (%). Determine the GST included in these bills.

a

% Magazine	$ 2.85
% Deodorant	$ 4.00
Milk	$ 5.60
Muffins	$ 2.60
% Biscuits	$ 2.17
Total	$ 17.22

GST incl. = ?

b

Bread	$ 2.70
Flour	$ 0.90
% Ice-cream	$ 4.48
% Soap	$ 0.98
% Tissues	$ 5.10
Total	$ 14.16

GST incl. = ?

c

Bananas	$ 3.50
% Soap	$ 3.92
% Detergent	$ 3.31
Tea bags	$ 4.46
Eggs	$ 2.89
% Bath salts	$ 6.05
Rice	$ 1.54
% Biscuits	$ 2.56
Total	$ 28.23

GST incl. = ?

17 What was the original price before the GST of 10% was added to each of these retail prices?

a $77 **b** $132 **c** $198

d $275 **e** $15.40 **f** $28.60

g $106.70 **h** $126.50 **i** $13.86

j $41.25 **k** $237.05 **l** $1238.38

18 Luke bought a microphone for $28 and sold it for $50. Find:

a his profit from the sale

b the profit as a percentage of the cost price

c the profit as a percentage of the selling price.

19 When selling products in a store, a percentage mark-up is added to the cost price to obtain the marked (or selling) price. If sporting gear has a mark-up of 60%, toys 40% and clothing 45%, what will be the marked price of:

a a baseball bat with a cost price of $24

b a doll with a cost price of $15

c a T-shirt with a cost price of $8.60?

20 A store marks up everything by 40%. If the selling price of a tennis racquet is $308, what is the cost price and the profit on the racquet?

21 A travelling salesman marks up each item by 110%. Find the cost price and the profit on an item with a selling price of:

a $630 **b** $8.40 **c** $127.05

22 John travelled to a factory for a 'direct buy' purchase of 60 litres of paint for $690. He noticed that the same paint was being sold at a local store for a 'sale' price of $46.60 for 4 litres. Which was the less expensive and by how much? If John spent an hour of travelling and $4.60 worth of petrol, which do you think was the better buy? Why?

23 Jane bought a new tyre for $160, Robyn bought one for $130 and Diane bought a retread for $76. If Jane's tyre lasted 32 000 km, Robyn's 27 500 km, and Diane's 16 000 km, which was the best buy? (Assume that safety and performance for the tyres are the same.)

24 A radio on sale for $50 is reduced in price by 30%. Later the discounted price is increased by 30%. What is the final price? By what percentage (to the nearest per cent) must the first discounted price be increased to give the original price?

25 Stephen bought a computer that had a marked price of $960. He received a discount of 15%. He paid a deposit of $81.60 and monthly repayments of $67.30 for 1 year. Find the interest paid expressed as a percentage of the money borrowed.

26 John had to pay interest on his credit card. He pays 1·7% per month on the greatest amount owing during each month. The greatest amount owing in April was $166, in May $294 and, in June, $408. Find the total interest charged for the three months.

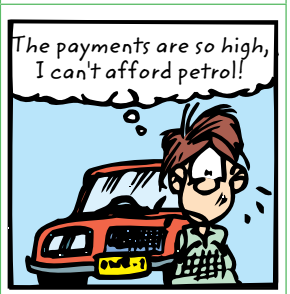
27 Heather works for a toy store where the percentage mark-up is 40% of the cost price. She is offered 10% discount on any item and can have this on the cost price before mark-up occurs, or on the marked price. Use each method to find the discount price Heather would have to pay on a game that has a cost price of $32. What do you notice?

We shop online!

8:06 Ways of paying

When buying the things we need, we can pay cash (or by cheque or by electronic fund transfer), buy on terms or use credit cards.

EFTPOS stands for electronic funds transfer at point of sale.

Using money		
Paying cash or transferring funds	**Buying with credit card**	**Buying on terms**
Meaning		
A payment with cheque, electronic funds transfer (EFTPOS) card or money.	A readily acceptable method of making credit purchases. 'Buy now pay later'.	A way of having the item and spreading the payment over a period of time. (Hire purchase)
Advantages		
Paying cash may help you get a discount. Money is accepted anywhere. You own the item. You keep out of debt. It doesn't encourage impulse buying. With cheque or EFTPOS card you don't have to carry a lot of money.	Convenient. Safer than carrying large sums of money. Useful in meeting unexpected costs. If payment is made promptly the charge is small. Many stores accept credit cards.	You can buy essential items and make use of them as you pay. Buying a house on terms saves rent. The item bought may be used to generate income. Little immediate cost.
Disadvantages		
Carrying large sums of money can be dangerous (risk of loss) and some shops won't accept cheques or EFTPOS cards. You may miss out on a good buy if you don't carry much money with you.	There is a tendency to overspend, buy on impulse and not out of need. The interest charged on the debt is high. Hidden costs (bank charges on stores) generally lift prices.	Relies on a regular income and, if you cannot continue payments, the item can be repossessed, sold and, if its value has depreciated (dropped), you still may owe money. High interest rates. You are in debt.
		

When transferring funds a 'Debit Card' can be used. The money comes from an account in which you have sufficient funds.

When we buy on terms (hire purchase), or we don't 'pay off' a credit card each month, we are charged interest that is calculated as a percentage of the amount we owe. For the examples in this exercise, we will be using simple interest, which is calculated as an annual percentage of the amount borrowed multiplied by the number of years taken to repay the amount.

For example, if \$5000 is borrowed at an interest rate of 7% p.a. and the loan is repaid over 3 years, the amount of interest charged would be calculated as:

Amount borrowed is called the **principal.**
p.a. stands for 'per annum' (or per year).

$$\begin{aligned}\text{Interest} &= \text{Principal} \times \text{Rate of Interest} \times \text{No. of years} \\ &= \$5000 \times 7\% \times 3 \\ &= \$5000 \times 0.07 \times 3 \\ &= \$1050\end{aligned}$$

$$I = PRN$$

Interest = **P**rincipal × **R**ate of interest × **N**umber of terms

Therefore, the total amount that will have to be paid will be the original amount borrowed (the principal) plus the interest.

$$\begin{aligned}\text{Amount to be repaid} &= \text{Principal} + \text{Interest} \\ &= \$5000 + \$1050 \\ &= \$6050\end{aligned}$$

Amount paid may be divided into weekly or fortnighly repayments.

So if payments are to be made weekly, then:

$$\begin{aligned}\text{Weekly repayment} &= \$6050 \div 52 \\ &= \$116.35\end{aligned}$$

WORKED EXAMPLE 1

Bryn bought a car on terms of \$100 deposit and 60 monthly repayments of \$179.80. The price of the car was \$5000.

a How much did she pay for the car?
b How much interest did she pay on the money borrowed?
c How much money did she borrow?

Solutions

a $\begin{aligned}\text{Total payments for car} &= \text{Deposit} + \text{Payments} \\ &= \$100 + 60 \times \$179.80 \\ &= \$10\,888\end{aligned}$

b $\begin{aligned}\text{Interest} &= \text{Extra money paid} \\ &= \$10\,888 - \$5000 \\ &= \$5888\end{aligned}$

c $\begin{aligned}\text{Amount borrowed} &= \text{Price of car} - \text{Deposit} \\ &= \$5000 - \$100 \\ &= \$4900\end{aligned}$

WORKED EXAMPLE 2

Malak bought a home theatre package for $3000 by paying a 10% deposit and the balance over 2 years at an interest rate of 8% p.a.

a How much was the deposit?
b How much was the balance owing after paying the deposit?
c How much interest was charged on this balance?
d What was the total owing over the 2 years?
e If the repayments were monthly, how much was each repayment?
f How much more did Malak pay for the package than if he had paid by cash?

Solutions

a Deposit = $3000 × 10%
= $300

b Balance owing = Price − Deposit
= $3000 − $300
= $2700

c Interest = Balance owing × Interest rate × No. of years
= $2700 × 8% × 2
= $2700 × 0.08 × 2
= $432

d Total amount owing = Balance owing + Interest
= $2700 + $432
= $3132

e Repayment = Total amount owing ÷ No. of weeks
= $3132 ÷ (52 × 2)
= $3132 ÷ 104
= $30.12 (to the nearest cent)

f Total amount paid for the package = Deposit + Repayments
= $300 + $30.12 × 104
= $3432.48

∴ Amount Malak paid more than cash = $3432.48 − $3000
= $432.48

The slight difference between the extra amount Malak paid and the interest calculation is due to the rounding of the repayment amount to the nearest cent.

WORKED EXAMPLE 3

Sylvia's bank charges 1.5% per month interest on balances owing on her credit card. At the end of May, Sylvia has a balance of $960 on her credit card. If she makes a repayment of $200, how much interest will she be charged next month?

Solutions

Amount owing on Sylvia's credit card after May payment = $960 − $200
= $760

∴ Interest charged in June on amount owing from May = $760 × 1.5%
= $760 × 0.015
= $11.40

Exercise 8:06

P Foundation worksheet 8:06
Best buy, shopping lists, change

1 Use the table on page 234 to answer these questions in your own words.

- **a** What are the advantages of paying cash?
- **b** What are the advantages of buying on terms?
- **c** What are the disadvantages of buying on terms?
- **d** What are the disadvantages of buying with credit cards?

2 **a** Find the amount John will pay for a fishing line worth $87 if he pays $7 deposit and $5.70 per month for 24 months. How much extra will he pay in interest charges?

b How much will Ingrid pay for her wedding dress worth $1290 if she pays $90 deposit and $74 per month for 2 years? How much interest will she pay on the money borrowed?

c When Robyn said that she could buy an item marked at $640 for $610 at another store, the salesman offered her the item for $601.60 if she bought it immediately. She bought it on terms, paying a deposit of $20 and monthly repayments of $23.30 for 4 years. How much did she pay all together? How much extra did she pay in interest charges?

d Joshua wants to buy a tent with a marked price of $730. He wants to pay it off on terms. For each of the following, work out how much he pays altogether and the interest charged.

- **i** Deposit of $100 and monthly payments of $64.50 for 1 year.
- **ii** Deposit of $100 and monthly payments of $38.30 for 2 years.
- **iii** Deposit of $100 and monthly payments of $29.60 for 3 years.
- **iv** Deposit of $100 and monthly payments of $25.20 for 4 years.
- **v** Deposit of $100 and monthly payments of $22.60 for 5 years.
- **vi** Deposit of $100 and monthly payments of $17.35 for 10 years.

e A man bought a car for $8250 paying a deposit of $250, $130 in extra charges, and paying $280 per month over 5 years. How much would he have to pay altogether? After paying four payments he found it was too expensive. The car was repossessed and sold for $6400. The finance company then sent him a bill for $1066.70 being the difference between what he owed and what they received for the car. He paid the bill and had no car. How much money had he paid altogether?

When borrowing money:
1 Read the terms
2 Work out the costs
3 Be prepared to say NO!

3 Find the amount of interest charged on the following principal amounts for the given interest rate and time period.

- **a** $1000, 5% p.a., 4 years
- **b** $2500, 7% p.a., 3 years
- **c** $5600, 6% p.a., 2 years
- **d** $900, 8% p.a., 3 years
- **e** $2000, 7.5% p.a., 2 years
- **f** $2500, 8.5% p.a., 4 years
- **g** $10 000, 9% p.a., 2 years
- **h** $12 500, 7·25% p.a., 15 months

4 For each example in Question **3** find the balance owing by adding the interest to the principal. Then divide the balance owing by the number of weeks in each example to calculate the correct weekly repayment. (Assume there are 52 weeks in a year.)

5 A racing bicycle priced at $8600 was bought on hire purchase over 3 years, paying an interest rate of 9% p.a. No deposit was paid and repayments were made monthly.

- **a** Find the amount of interest charged.
- **b** What is the total amount paid for the bicycle?
- **c** Find the amount of each monthly repayment.

6 Peggy wanted to buy a sports car worth $55 900. The car dealer gave her $12 400 for her old car as a trade-in. She used this as a deposit. She agreed to pay the balance off over 5 years at an interest rate of 7.9% p.a.

- **a** Find the balance owing after paying the deposit.
- **b** How much interest is charged on the balance owing?
- **c** What is the total amount that has to be repaid over the 5 years?
- **d** Find the amount of each monthly repayment.

7 Ivan and Ivy bought a furniture package deal. The value of the furniture was $12 400. They agreed to pay a 10% deposit and the balance off over 4 years via fortnightly repayments. The interest rate charged was 8·6% p.a.

- **a** Find the amount of the deposit.
- **b** What is the balance owing after paying the deposit?
- **c** How much interest is charged on the balance owing?
- **d** What is the total amount that has to be paid over the 4 years?
- **e** Find the amount of each fortnightly repayment. (Assume there are 26 fortnights in a year.)

8 Find the amount of monthly interest charged on the following credit card balances at the given interest rate. Where daily interest is given, assume the month has 30 days.

- **a** $100, 2% per month
- **b** $250, 1·8% per month
- **c** $560, 1·78% per month
- **d** $978, 1·92% per month
- **e** $200, 18% p.a.
- **f** $2500, 20·24% p.a.
- **g** $1000, 0·06% per day
- **h** $1250, 0·055 45% per day

9 Terry's bank charges 1·7% per month interest on balances owing on his credit card. At the end of April Terry has a balance of $680 on his credit card. If he makes a repayment of $180, how much interest will he be charged next month?

10 Wendy's bank charges 21% per annum interest on balances owing on her credit card. At the end of June Wendy has a balance of $1450 on her credit card.

- **a** If she forgets to make a payment for the month, how much interest will she be charged next month?
- **b** Wendy is also charged a penalty of $25 for not making a payment. If she makes no further purchases in July, what will be the balance owing at the end of July on her credit card?
- **c** Wendy does make a payment of $500 at the end of July. What will be the amount of interest charged on the remaining balance?

11 Meryl bought a second-hand caravan, priced at $10 050, on terms of $300 deposit and 60 monthly repayments of $260.

- **a** Find the total amount paid for the caravan.
- **b** What amount of interest was paid?
- **c** What was the amount of interest paid each year?
- **d** What percentage of the amount borrowed was this yearly interest? (This is the annual simple interest rate that was charged on the money borrowed.)

12 Ken bought a motor bike on hire purchase. He paid a 20% deposit and monthly repayments of $331.52 for 4 years. The price of the bike was $14 800.

- **a** Find the balance owing on the bike after the deposit had been paid.
- **b** How much did Ken pay in repayments?
- **c** How much interest was charged?
- **d** Determine the annual rate of interest charged on the amount borrowed.

13 Allyson's credit card charges a daily interest rate of 0·049% on balances owing at the end of each month. At the end of the month Allyson has a balance of $840 and she makes a payment of $200.

- **a** What is the yearly interest rate as a percentage, correct to one decimal place?
- **b** At the end of the next month (which has 30 days) how much interest will be charged on the balance owing from the previous month?
- **c** If Allyson makes a further payment of $400 after 10 days into the next month, how much interest will she save herself from paying at the end of the month?

FUN SPOT 8:06 THE PUZZLE OF THE MISSING DOLLAR

Three men had lunch in a busy restaurant. When it came time to pay their bill, each of the men gave the waiter $30. So, the waiter received $90 altogether.

When the waiter added up the bill, he found it only came to $85. Knowing that $5 would not divide among the three men evenly, the waiter decided to give each of the men $1 change and put the remaining $2 in his pocket.

It now appears that each man has paid $29 for the meal, and the waiter has $2; a total of $89.

Where is the missing dollar?

MATHS TERMS 8

commission
- income usually calculated as a percentage of the value of the goods sold

discount
- to reduce the price of goods sold (v)
- the amount or percentage a price is reduced by (n)

GST
- Goods and Services Tax
- 10% of a base price is added on to the cost of most goods and services and included in the advertised retail price

gross pay
- the amount of pay before any deductions such as income tax are subtracted

income tax
- tax paid to the government which is based on the amount of income received

interest
- fee charged on borrowed money
- usually charged as a percentage of the amount borrowed

net pay
- the amount of pay an employee receives after deductions such as income tax have been subtracted

overtime
- time worked by an employee in excess of a standard day or week
- usually rates of pay $1\frac{1}{2}$ or 2 times the normal rate of pay are paid for overtime

profit
- the gain when a good is sold for a higher price than its cost price
- if the selling price is lower a negative profit, or loss, is made

salary
- a fixed amount paid for a year's employment. It may be paid weekly or fortnightly

simple interest
- interest charged at a given rate on the original amount borrowed (principal), multiplied by the length of time of the loan
  $I = PRN$, where I = Interest, P = Principal, R = Rate of interest, N = Number of terms

superannuation
- an investment fund usually contributed to by both employer and employee on the employee's behalf
- it provides benefits for employees upon retirement, or for relatives if the employee dies

taxable income
- amount after allowable deductions are subtracted from the gross pay
- the income tax payable is calculated on this amount

wage
- pay given to an employee often based on an agreed hourly rate
- usually paid weekly or fortnightly

DIAGNOSTIC TEST 8 FINANCIAL MATHEMATICS

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 a John sells cars for a living. He is paid a retainer (a base wage) of $550 a week as well as 2% commission on sales made. Find his income for the week, if in one week he sells cars to the value of:
i $80 000 ii $96 500
b Luke has a casual job from 4:00 pm until 5:30 pm Monday to Friday. He also works from 9 am until 12:30 pm on Saturdays. Find his weekly income if his casual rate is $19.80 per hour Monday to Friday, and $24.70 an hour on Saturdays. 8:01

2 a During one week Petra worked 35 hours at the normal rate of $22.60 per hour. She also worked 6 hours overtime: 4 at 'time-and-a-half' and 2 at 'double-time'. How much did she earn?
b Calculate Diane's holiday loading if she is given $17\frac{1}{2}\%$ of 4 weeks' salary and she earns $1860 per fortnight. 8:02

3 a Find the net pay for the week if John earns $1586.80, is taxed $365.20, pays $73.50 into his superannuation and has miscellaneous deductions totalling $79.40.
b What percentage of John's gross pay did he pay in tax? 8:03

4 Alana received a salary of $73 465 and other income (investments) of $965 during the year. Her total tax deductions were $2804. She had already paid tax instalments amounting to $15 634 for the year.
a What is her total income for the year?
b What is her taxable income for the year?
c Calculate her Medicare levy (1·5%).
d Find her tax payable on her taxable income using the table on page 221.
e Find her refund due or balance payable after the Medicare levy is included.
f How much extra would Alana receive each week if she is given a wage rise of $100 per week? (Don't forget to subtract $1.50 for Medicare payments.) 8:04

5 a A lawn fertiliser comes in three sizes: 20 kg (for $11.60), 50 kg (for $24.80) and 110 kg (for $56.60). Which size is the best buy?
b Rich Red strawberry flavouring can be purchased at 240 mL for $1.70, 660 mL for $3.75, or 1 L for $6.25. Which buy represents the best value? 8:06

6 a Determine the GST that needs to be added to a base price of $73.70.
b Determine the retail price after 10% GST is added to a base price of $53.90.
c How much GST is contained in a retail price of $32.45?
d What was the base price before 10% GST was added to give a retail price of $21.45? 8:05

7 a Naomi was given $12\frac{1}{2}\%$ discount on a rug with a marked price of $460. How much did she pay?

8:05

b A television marked at $4200 was eventually sold for $3612. What was the discount and what was the percentage discount given on the marked price?

c After a discount of 13% was given, I paid $27 840 for my yellow Holden. What was the original marked price?

8 a Jane bought a desk with a marked price of $650. She was given a discount of 10% for paying cash, and then received a further 10% off the discounted price because the desk was scratched. How much did she pay?

8:05

b What is the final price if successive discounts of 15% and 20% are applied to a retail price of $1250?

9 a Rachel bought a painting for $250 and sold it for $575. Find the profit as a percentage of the cost.

8:05

b We held a games night to raise money for The House with No Steps. We charged each of the 287 people who came an entrance fee of $17.50. Hire of the hall cost $110, decorations cost $63, prizes $185.60, food $687, cleaning $96 and advertising $240.
How much money did we make and what percentage is this of the money received?

10 a Jim bought a car with a marked price of $3000. He paid a deposit of $100 and 36 monthly payments of $136.20. How much did he pay? How much more than the marked price did he pay?

8:06

b Peta installed a new kitchen that cost $14 350. She agreed to pay for it by paying monthly instalments for 2 years. She was charged 9% p.a. interest. Find:
- i the amount of interest charged
- ii the total amount paid over the 2 years
- iii the amount of each instalment.

c Bhupesh bought a truck worth $82 000 by paying 20% deposit and the balance off over 5 years at an interest rate of 7.8%. Find:
- i the amount of the deposit
- ii the balance owing after the deposit was paid
- iii the amount of interest charged
- iv the amount of each repayment if paid monthly.

ASSIGNMENT 8A Chapter review

1 **a** A woman works for a wage of $16.80 per hour. How much will she earn in a week in which she works:
- **i** 40 hours at normal time
- **ii** 40 hours of normal time and 5 hours of overtime if overtime is paid at $1\frac{1}{2}$ times the normal rate of pay?

b A salesman works for a wage of $500 per week plus 3% commission. How much will he earn in a week if he sells $4500 worth of goods?

c A factory worker is paid a wage of $540 a week. The factory has a special bonus system which enables a worker to be paid an extra 25c per article for every article in excess of the weekly quota of 5000. How much will the worker earn in a week in which 7200 articles are made?

d How much holiday pay will a girl receive if she is to be paid 4 weeks' holiday pay plus a holiday loading of $17\frac{1}{2}\%$ of 4 weeks' pay? Her weekly wage is $452.

2 **a** Fibreglass resin comes in the following sizes: 1 kg for $9.80; 5 kg for $26.80, and 21 kg for $59.60.
- **i** What is the best value for money?
- **ii** What is the most economical way of buying 17 kg?

b A TV set with a cash price of $680 is bought for a deposit of $68 and 48 monthly payments of $15.68. Find the difference between the cash price and the price paid.

c Calculate the amount of GST included in items that retailed for $736, $245 and $579.

3 Mary-Ann is paid an annual salary of $53 350. Her allowable tax deductions total $1340. During the year her employer paid income tax instalments on her behalf of $12 732.
- **a** What is Mary-Ann's taxable income?
- **b** How much income tax should she pay for the year?
- **c** What is the amount of her refund from the tax office?

4 Jeremy added 10% GST onto the price of a book valued at $29.90 to get its retail price. He then discounted the retail price by 10% to get a sales price.
- **a** What is the retail price?
- **b** What is the sales price?
- **c** Is the sales price the same as the original value of the book?
- **d** By what percentage should Jeremy have discounted the retail price to get back to the original value of the book?

5 **a** Vicki sold azaleas in her nursery for $15.90. She bought them for $11.35. What percentage profit does she make?

b Michael sold a bike he bought for $350 to a friend 2 years later for $230. What percentage loss is this?

6 A boat priced at $17 300 was bought by paying 15% deposit with the balance being paid off over 3 years at 8·5% p.a. interest, by fortnightly repayments. Find:
- **a** the amount of the deposit
- **b** the balance owing after the deposit was paid
- **c** the amount of interest charged
- **d** the amount of each repayment
- **e** the total amount paid for the boat.

ASSIGNMENT 8B Working mathematically

1 Refer to **ID Card 2** on page xvii to identify:

a 10 **b** 12 **c** 17 **d** 18 **e** 19
f 20 **g** 21 **h** 22 **i** 23 **j** 24

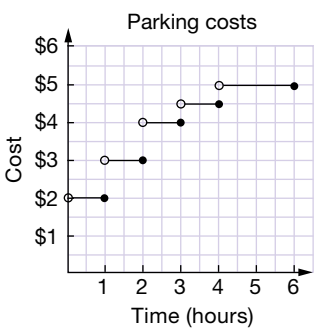
2 Refer to **ID Card 4** on page xix to identify:

a 1 **b** 2 **c** 3 **d** 4 **e** 5
f 6 **g** 7 **h** 8 **i** 9 **j** 13

3 Through how many degrees does the hour hand of a clock turn in half an hour?

4 At a meeting, each person who arrived shook the hand of every other person present. How many handshakes would have occurred if 8 people were present?

5 The parking charges for a parking station are shown in the graph.

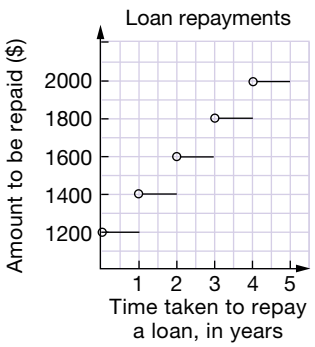


a Find:
- **i** the cost of parking for 2 h
- **ii** the cost of parking for $1\frac{1}{2}$ h.

b What is the longest time a car can be parked for $4?

c What is the cost of parking a car from Monday to Friday if it is parked for periods of $1\frac{1}{2}$ h, $2\frac{1}{2}$ h, 3 h, $\frac{1}{2}$ h and 2 h on those days?

6 A loan of $1000 is to be repaid at an interest rate of 20% p.a. The faster the loan is repaid, the less interest is charged. The graph shows how the amount to be repaid varies according to the time taken to repay the loan.



a How much has to be repaid if $3\frac{1}{2}$ years is taken to repay the loan?

b If a person wished to repay the loan in 2 years what amount would have to be repaid?

c How much must be paid monthly if this loan is to be repaid in 4 years?

ASSIGNMENT 8C Cumulative revision

1 Find the absolute error in a measurement of: 1:11

a 5 cm **b** 8·5 cm **c** 5·254 m

2 **a** Write correct to two significant figures: 1:06–1:08

i 27·1608 **ii** 0·0716

b Estimate the answers to the following questions. Then use a calculator to evaluate each answer, correct to two significant figures.

i $\dfrac{21{\cdot}7 \times \sqrt{47{\cdot}62}}{6{\cdot}9}$ **ii** $\sqrt{\dfrac{23{\cdot}5 \times 6{\cdot}73}{(3{\cdot}2 + 1{\cdot}967)}}$

3 **a** What percentage is 25 g of 0·45 kg? 2:01D

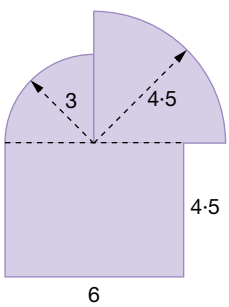
b My wage increases from \$850 per week to \$900 per week. What is the percentage increase?

c A woman buys 500 shares for \$1.55 each. She sells them later for \$2.10 each. What profit does she make and what is the profit as a percentage of the purchase price?

4 Simplify: 3:04

a $\dfrac{m}{4} - \dfrac{m}{6}$ **b** $\dfrac{m}{4} + \dfrac{m}{6}$ **c** $\dfrac{m}{4} \times \dfrac{m}{6}$ **d** $\dfrac{m}{4} \div \dfrac{m}{6}$

5 Find the area of the following figure that is formed from two quadrants and a rectangle. Measurements are in metres. Give your answer correct to one decimal place. 5:02



6 Solve: 7:06

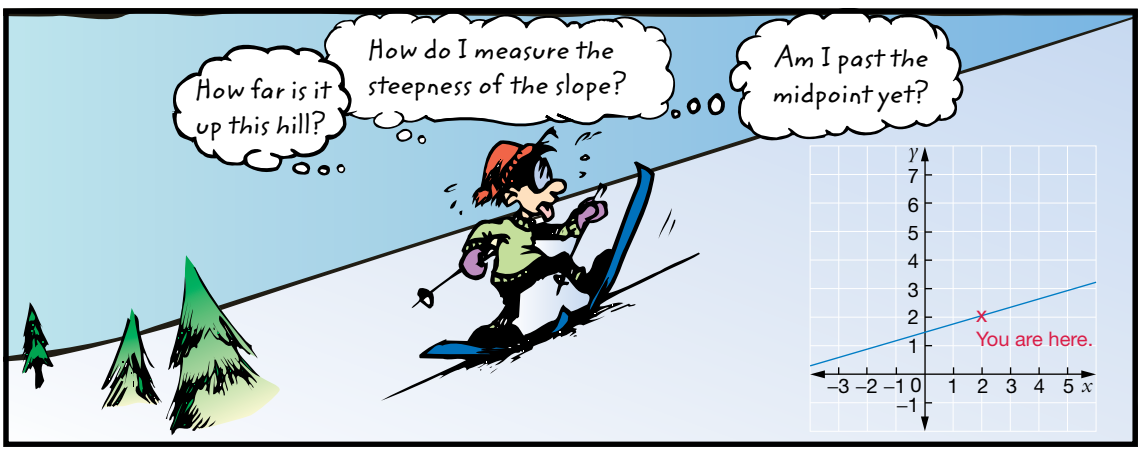
a $\dfrac{a}{4} - 6 = 8$ **b** $\dfrac{3a}{8} + 5 = 8$ **c** $\dfrac{3a + 5}{8} = 8$

7 Write the following numbers in scientific notation. 6:04

a 213 000 **b** 215 000 000 **c** 0·005 67 **d** 0·000 000 12

COORDINATE GEOMETRY

9



Contents

9:01 Number plane review
9:02 The midpoint of an interval
9:03 The gradient of an interval
Investigation 9:03 Gradients and building
9:04 The length of an interval
GeoGebra activity 9:04 The midpoint, gradient and length of an interval
9:05 Graphing straight lines
Investigation 9:05 *x*- and *y*-intercepts
GeoGebra activity 9:05 Graphing lines by plotting points
9:06 Horizontal and vertical lines
9:07 The gradient–intercept form of a straight line: $y = mx + b$
Investigation 9:07 What does $y = mx + b$ tell us?
GeoGebra activity 9:07 The gradient–intercept form of a line
Fun spot 9:07 Why did the banana go out with a fig?
9:08 Non-linear graphs
GeoGebra activity 9:08 Non-linear graphs
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Linear Relationships*** [Stages 5.1, 5.2]

- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software. (ACMNA294)
- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software. (ACMNA214)
- Sketch linear graphs using the coordinates of two points. (ACMNA215)
- Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line. (NSW)

Selections from ***Non-linear Relationships*** [Stages 5.1, 5.2 ◊]

- Graph simple non-linear relations, with and without the use of digital technologies. (ACMNA296)

Working Mathematically

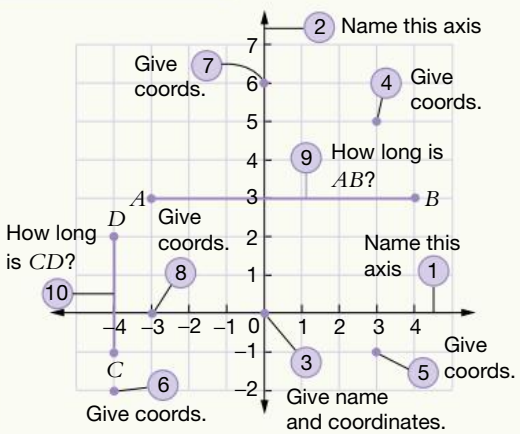
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

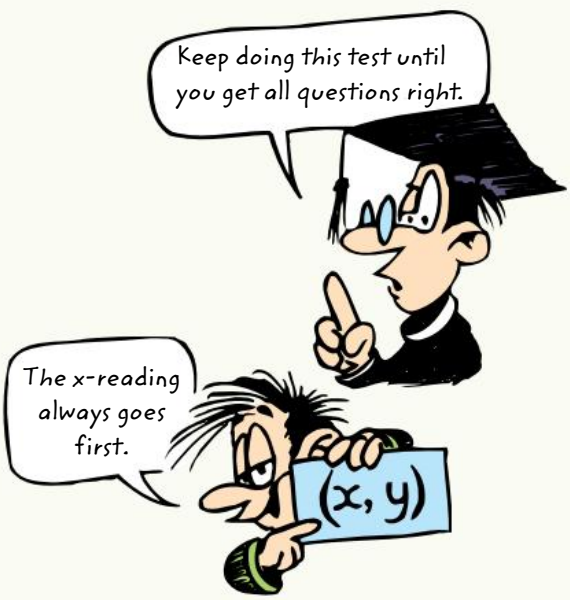
9:01 Number plane review

The French mathematician René Descartes first introduced the number plane. He realised that using two sets of lines to form a square grid allowed the position of a point in the plane to be recorded using a pair of numbers or coordinates.

The basic ideas behind his system should have been met in Years 7 and 8. Check your knowledge by doing the prep quiz below.

PREP QUIZ 9:01

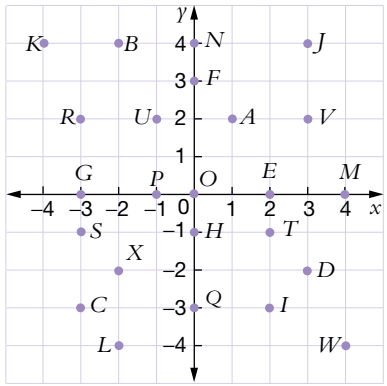




Exercise 9:01

P Foundation worksheet 9:01 Coordinates

1 Write the letter naming the point with these coordinates.



a (3, 2) b (−2, 4) c (3, 4)
d (2, −1) e (0, 3) f (−2, −2)
g (0, 0) h (−3, 0) i (3, −2)

2 Write the coordinates of these points.

a A b B c C d D
e E f F g G h H
i I j J k K l L

3 Using the number plane above, write the points:

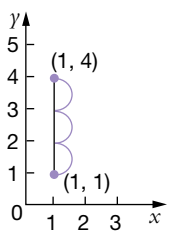
a with a y-coordinate of 2
b with an x-coordinate of −3.

4 Plot each set of points on a number plane. What shapes are formed if the points are joined in the given order?

a (−3, 1), (3, 1), (3, −1), (−3, −1), (−3, 1)
b (−1, −1), (0, 4), (1, −1), (−1, −1)
c (2, 3), (6, 3), (5, 0), (1, 0), (2, 3)
d (1, 3), (3, 3), (4, 1), (−2, 1), (1, 3)

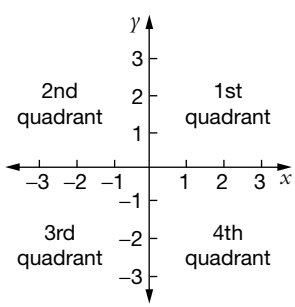
5 Find the distance between:

a (2, 0) and (5, 0) b (0, 1) and (0, 4)
c (−1, 0) and (1, 0) d (0, −3) and (0, 1)
e (−5, 0) and (−1, 0) f (0, −4) and (0, −2)
g (3, 0) and (−1, 0) h (2, 0) and (−2, 0)
i (0, 5) and (0, −2) j (1, 2) and (4, 2)
k (−2, −1) and (1, −1) l (4, 3) and (−2, 3)
m (3, 1) and (3, 4) n (−2, −3) and (−2, 1)



6 In which quadrant does each point fall?

a (3, 2) b (−1, 2) c (−2, −2)
d (1, −2) e (−3, −2) f (1, 1)
g (−1, −2) h (−3, 1) i (3, −1)
j (−1, 1) k (−2, −1) l (1, −1)
m (2, 2) n (2, −2) o (−2, 2)



7 What can you say about the coordinates of a point that lies on:

a the x-axis b the y-axis?

8 **a** Write the letter naming the points:

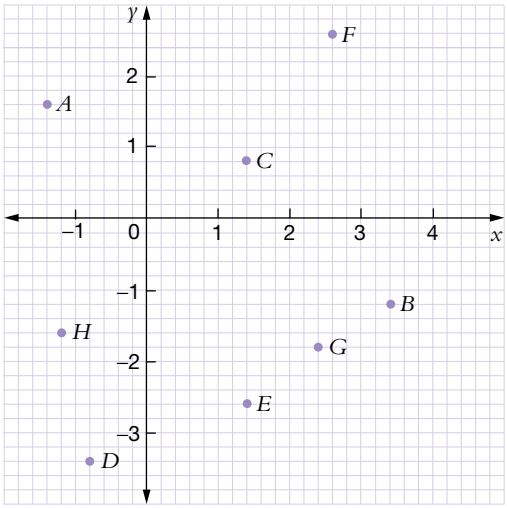
i $(-1{\cdot}2, -1{\cdot}6)$ **ii** $(3{\cdot}4, -1{\cdot}2)$

iii $(1{\cdot}4, -2{\cdot}6)$ **iv** $(2{\cdot}4, -1{\cdot}8)$

b Write the coordinates of:

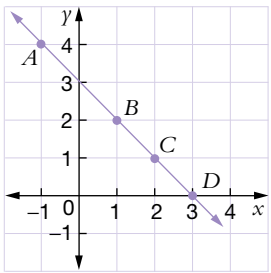
i C **ii** A

iii E **iv** D

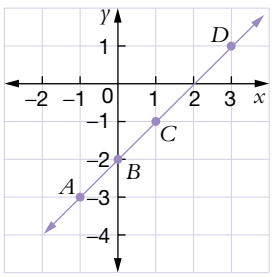


9 Write the coordinates of the points A, B, C and D on each of the following lines.

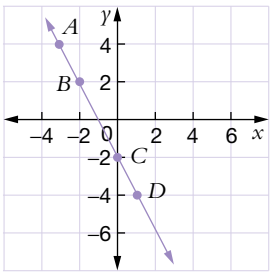
a



b



c



10 **a** The x-coordinates for some of the points on the lines A to D are given. Find the missing y-coordinates.

Line A

x	-3	-2	0	1
y				

Line B

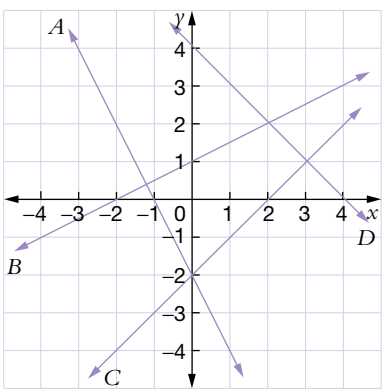
x	-4	-2	0	2
y				

Line C

x	-2	-1	2	3
y				

Line D

x	-1	0	2	4
y				

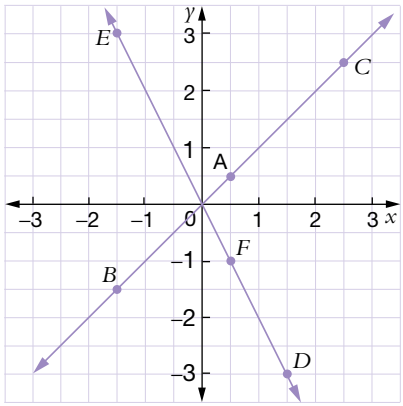


b If line C was extended it would pass through the points $(5, a)$ and $(-4, b)$. What are the values of a and b?

11 **a** Write the coordinates of three points that lie on the line AB between the points A and C.

b Write the coordinates of three points on the line ED between F and D.

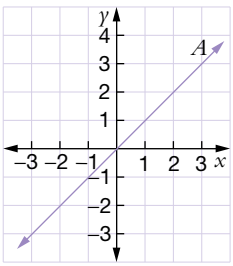
c If the line ED was extended it would pass through the points $(2, a)$, $(-2, b)$ and $(-2\frac{1}{2}, c)$. Find the values of a, b and c.

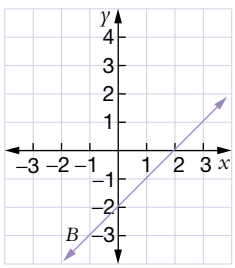


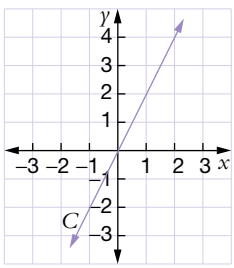
12 The points on a line can be described by an equation. Which of the lines A to F are best described by the following equations.

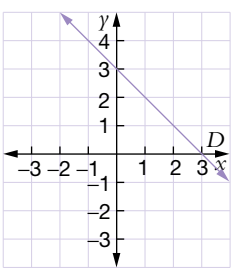
a $y = 2x$ **b** $y = x - 2$ **c** $2x + y = 0$

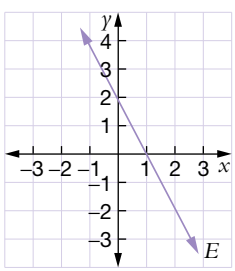
d $y = x$ **e** $x + y = 3$ **f** $2x + y = 2$

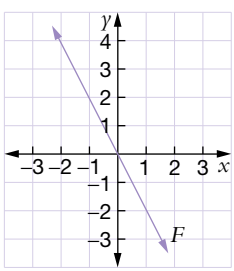












13 Which of the lines A, B, C or D in Question 10 could be described by the following equations?

a $x - y = 2$ **b** $x + y = 4$ **c** $2x + y + 2 = 0$ **d** $x - 2y + 2 = 0$

Roger has four different pizza toppings.
How many different pizzas could be made using:

- one topping
- two toppings
- any number of toppings?

9:02 The midpoint of an interval

The number plane is the basis of coordinate geometry, which is an important branch of mathematics with many technological applications. Three of the basic tools of coordinate geometry are the midpoint, gradient and length of an interval joining two points.

PREP QUIZ 9:02

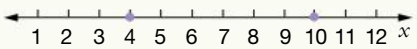
1 $\frac{4+10}{2} = ?$

2 $\frac{-2+4}{2} = ?$

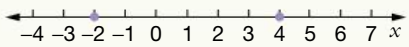
3 What is the average of 4 and 10?

4 What is the average of −2 and 4?

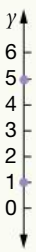
5 What number is half-way between 4 and 10?



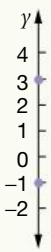
6 What number is half-way between −2 and 4?



7 What number is half-way between 1 and 5?



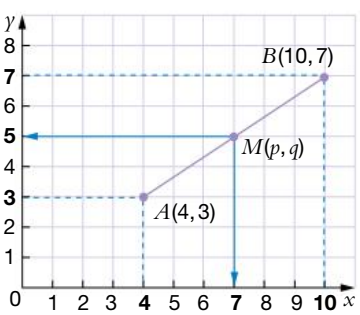
8 What number is half-way between −1 and 3?

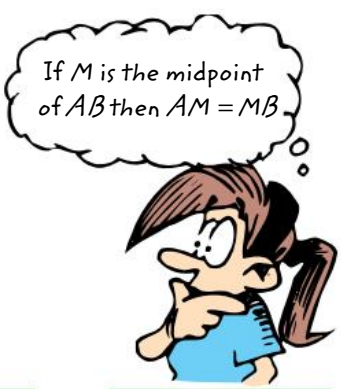


9 $\frac{1+5}{2} = ?$

10 $\frac{-1+3}{2} = ?$

- The midpoint of an interval is the half-way position. If M is the midpoint of AB then it is half-way between A and B.





Consider the x-coordinates.

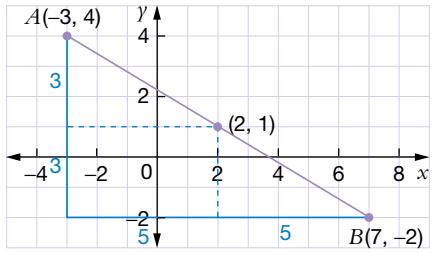
7 is half-way between 4 and 10.	→ The average of 4 and 10 is 7.	→ $\frac{4+10}{2} = 7$	→ $p = 7$

Consider the y-coordinates.

5 is half-way between 3 and 7.	→ The average of 3 and 7 is 5.	→ $\frac{3+7}{2} = 5$	→ $q = 5$

Hence, M is the point (7, 5).

- Drawing a right-angled triangle with AB as the hypotenuse is helpful when finding the midpoint. By drawing a vertical line from the upper point and a horizontal line from the lower point, a triangle is formed.
  Finding the middle of the horizontal and vertical sides will give the x- and y-coordinates of the midpoint.
  When a diagram is not suitable the averaging method gives the same result.



Midpoint $= \left(\frac{-3+7}{2}, \frac{4+-2}{2}\right) = (2, 1)$

> If M is the midpoint of the interval AB, its coordinates are found by taking the averages of the x- and y-coordinates of the points A and B.

WORKED EXAMPLES

1 Find the midpoint of the interval joining (2, 6) and (8, 10).

2 Find the midpoint of interval AB, if A is the point (–3, 5) and B is (4, –2).

Solutions

1 Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2+8}{2}, \frac{6+10}{2}\right)$

$= (5, 8)$

2 Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3+4}{2}, \frac{5+-2}{2}\right)$

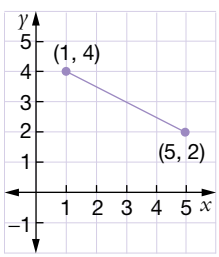
$= (\frac{1}{2}, \frac{3}{2})$ or $(\frac{1}{2}, 1\frac{1}{2})$

Exercise 9:02

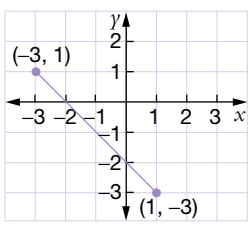
P Foundation worksheet 9:02 Midpoint

1 Use the graph to find the midpoint of each interval.

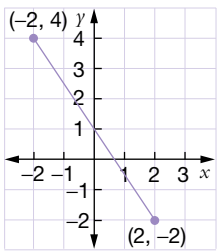
a



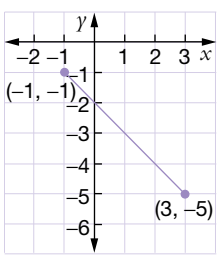
b



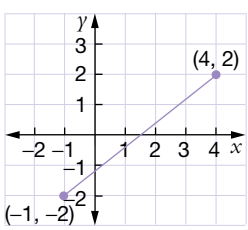
c



d

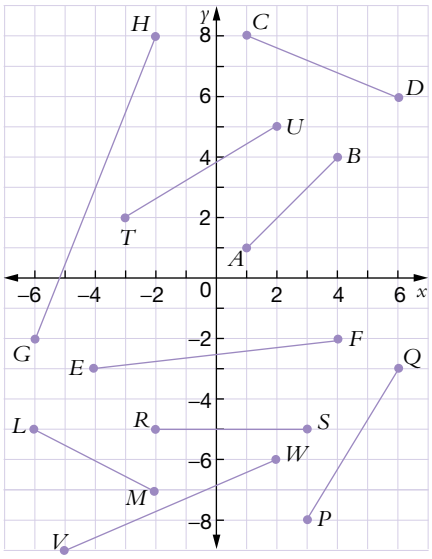


e



2 Use the graph to find the midpoints of the intervals:

a AB **b** CD **c** GH
d EF **e** LM **f** PQ
g RS **h** TU **i** VW



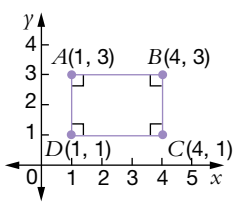
3 Find the midpoint of each interval AB if:

a A is $(2, 4)$, B is $(6, 10)$ **b** A is $(1, 8)$, B is $(5, 6)$ **c** A is $(4, 1)$, B is $(8, 7)$
d A is $(0, 0)$, B is $(4, 2)$ **e** A is $(4, 8)$, B is $(0, 0)$ **f** A is $(0, 0)$, B is $(6, 6)$
g A is $(3, 4)$, B is $(8, 1)$ **h** A is $(4, 0)$, B is $(1, 7)$ **i** A is $(7, 2)$, B is $(4, 7)$
j A is $(1, 1)$, B is $(5, 6)$ **k** A is $(7, 1)$, B is $(2, 5)$ **l** A is $(3, 2)$, B is $(7, 5)$
m A is $(0, 0)$, B is $(-4, 2)$ **n** A is $(-8, -6)$, B is $(0, 0)$ **o** A is $(0, 0)$, B is $(0, -10)$
p A is $(-2, -6)$, B is $(4, 2)$ **q** A is $(-1, 0)$, B is $(5, 4)$ **r** A is $(-3, -7)$, B is $(1, 3)$

4 Find the midpoint of the interval joining:

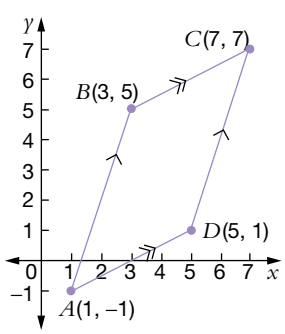
a $(-3, -3)$ and $(2, -3)$ **b** $(8, -1)$ and $(7, -1)$ **c** $(5, 5)$ and $(5, -5)$
d $(6, -7)$ and $(-7, 6)$ **e** $(0, -4)$ and $(-4, 0)$ **f** $(6, -6)$ and $(5, -5)$
g $(111, 98)$ and $(63, 42)$ **h** $(68, -23)$ and $(72, -29)$ **i** $(400, 52)$ and $(124, 100)$
j $(-82, 41)$ and $(-51, 63)$ **k** $(-71, 42)$ and $(23, -17)$ **l** $(49, -7)$ and $(19, -22)$

5 **a** **i** Find the midpoint of AC.
ii Find the midpoint of BD.
iii Are the answers for **i** and **ii** the same?
iv What property of a rectangle does this result demonstrate?



b If $(4, 6)$ and $(2, 10)$ are points at opposite ends of a diameter of a circle, what are the coordinates of the centre?

c **i** Find the midpoint of AC.
ii Find the midpoint of BD.
iii Are the answers for **i** and **ii** the same?
iv What property of a parallelogram does this result demonstrate?



6 **a** If the midpoint of $(3, k)$ and $(13, 6)$ is $(8, 3)$, find the value of k.

b If the midpoint of (c, d) and $(10, 2)$ is $(8, 5)$, find the value of c and d.

c If the midpoint of $(x, 1)$ and $(7, y)$ is $(9, 6)$, find the value of x and y.

d If the midpoint of $(-2, -6)$ and (b, c) is $(-5, 1)$, find the value of b and c.

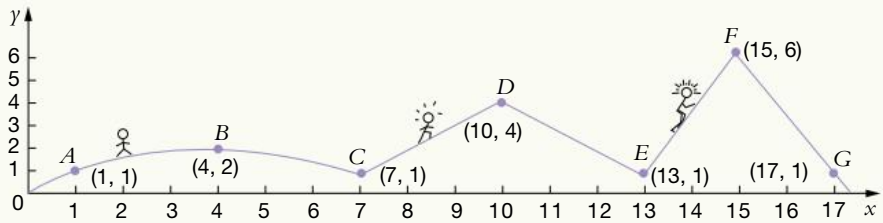
e The midpoint of AB is $(7, -3)$. Find the value of d and e if A is the point $(d, 0)$ and B is $(-1, e)$.

f The midpoint of AB is $(-6, 2)$. If A is the point $(4, 4)$, what are the coordinates of B?

g A circle with centre $(3, 4)$ has a diameter of AB. If A is the point $(-1, 6)$ what are the coordinates of B?

9:03 The gradient of an interval

PREP QUIZ 9:03



1 Which is steepest, AB or EF?

If I travel from left to right, between which two points am I travelling upwards:

2 for the first time **3** for the second time **4** for the last time?

If I travel from left to right, between which two points am I travelling downwards:

5 for the first time **6** for the second time **7** for the last time?

Is the hill sloping up, down, or not at all, at these points?

8 A **9** G **10** F

The gradient slope of an interval (or line) is a measure of *how steep* it is.

Negative gradient

Steep (positive gradient)

Steeper

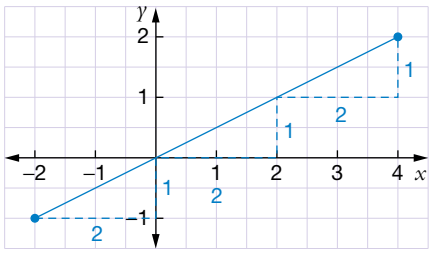
If we move from left to right:

- an interval going down is said to have a negative gradient (or slope)
- an interval going up is said to have a positive gradient (or slope)
- a horizontal interval (not going up or down) has zero gradient.

We find the gradient of an interval by comparing its rise (change in y) with its run (change in x).

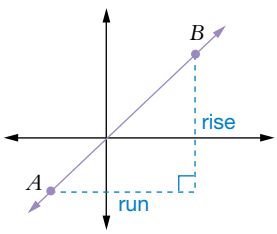
- So, a gradient of $\frac{1}{2}$ means that for every run of 2 there is a rise of 1 (or for every 2 you go across, you go up 1).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$



Finding the gradient of a line

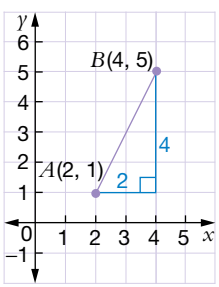
1. Select any two points on the line.
2. Join the points and form a right-angled triangle by drawing a vertical line from the higher point and a horizontal side from the lower point.
3. Find the change in the y-coordinates (rise) and the change in the x-coordinates (run).
4. Use the formula above to find the gradient.



WORKED EXAMPLES

Use the points A and B to find the gradient of the interval AB in each case.

1



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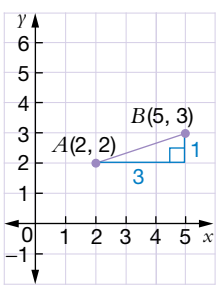

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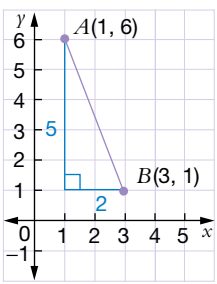


Solutions

1 Gradient

$$= \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{up } 4}{\text{across } 2}$$

$$= \frac{4}{2}$$

$$= 2$$

2 $m = \frac{\text{change in } y}{\text{change in } x}$

$$= \frac{\text{up } 1}{\text{across } 3}$$

$$= \frac{1}{3}$$

m is used for 'gradient'

3 $m = \frac{\text{change in } y}{\text{change in } x}$

$$= \frac{\text{down } 5}{\text{across } 2}$$

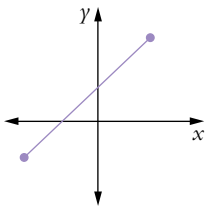
$$= \frac{-5}{2}$$

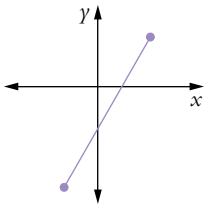
$$= -2\tfrac{1}{2}$$

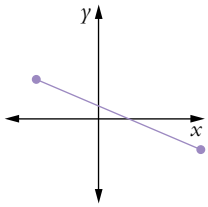
Exercise 9:03

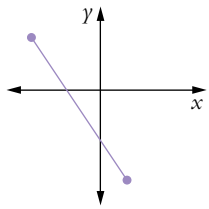
P Foundation worksheet 9:03 Gradients

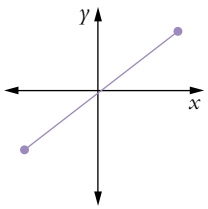
1 State whether each interval below has a positive or negative gradient.

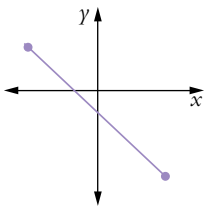
a 

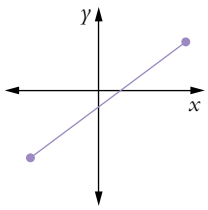
b 

c 

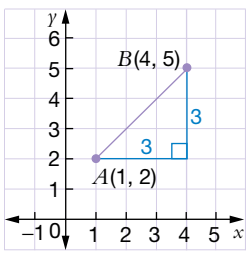
d 

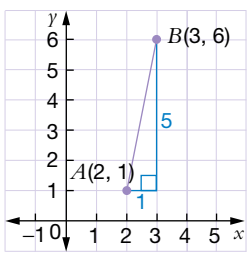
e 

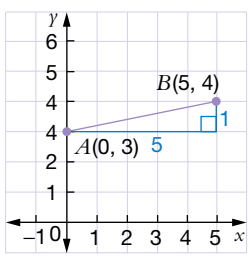
f 

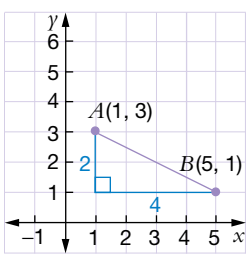
g 

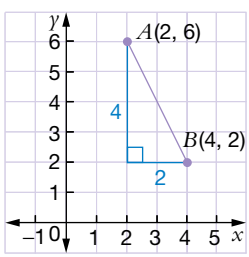
2 Find the gradient of the interval AB.

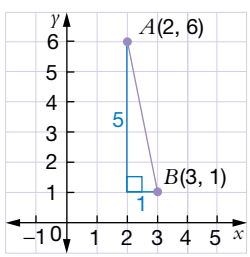
a 

b 

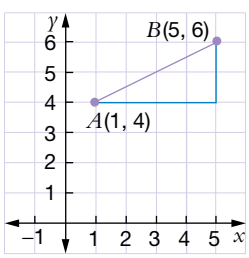
c 

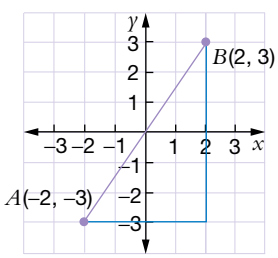
d 

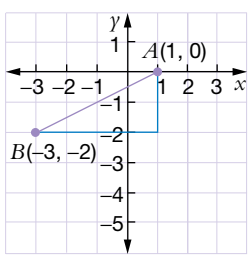
e 

f 

3 Find the gradient of the interval AB.

a 

b 

c 

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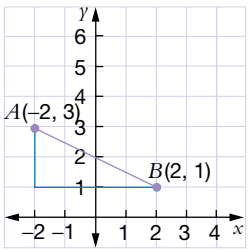

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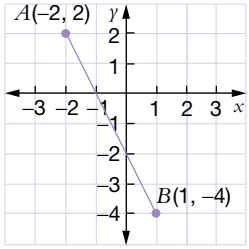

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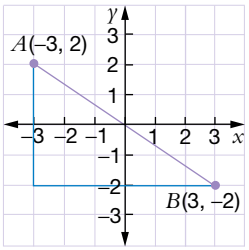
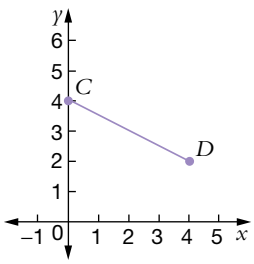


4 Use the points C and D to find the slope of the line.

a



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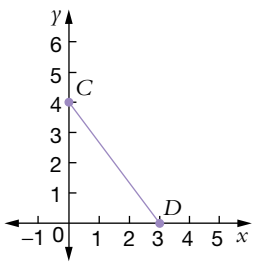

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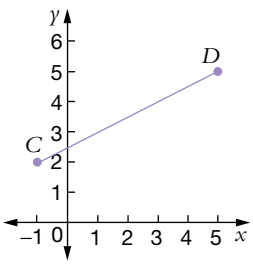

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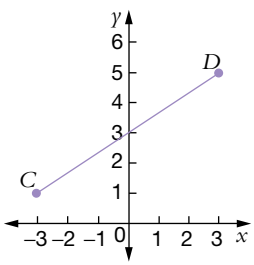

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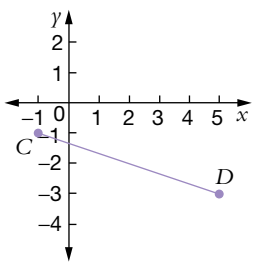

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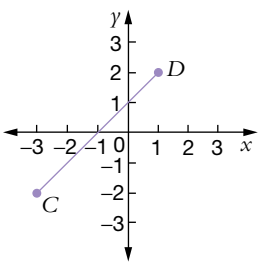
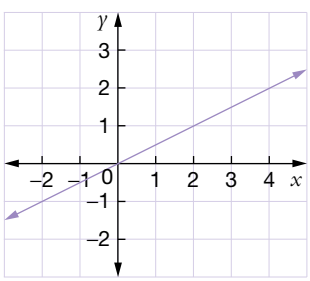
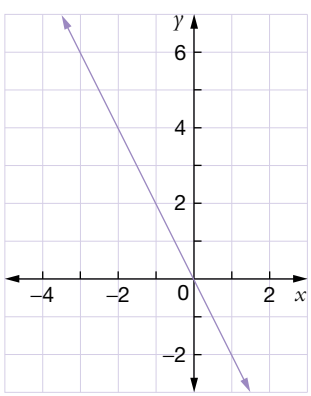


5 In each of the following, use any two points on the line to find its slope.

a



b



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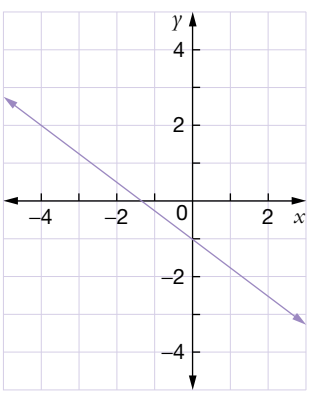


6 Find the slope of the line that passes through each pair of points.

a (1, 0) and (3, 4)
b (–2, 1) and (0, 3)
c (1, –3) and (3, –2)
d (–1, 4) and (1, 2)
e (–2, –2) and (0, –5)
f (0, 4) and (2, –2)

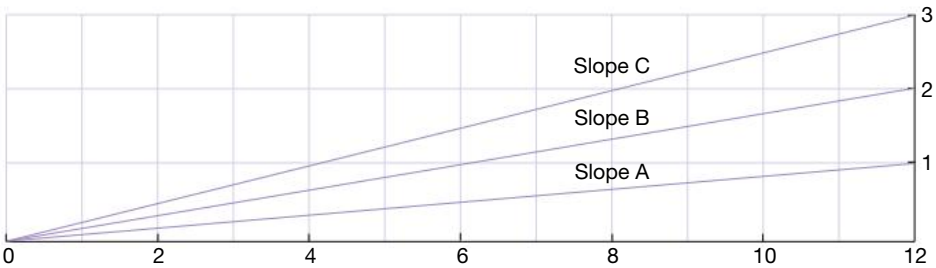
INVESTIGATION 9:03 GRADIENTS AND BUILDING

- Engineers, architects and builders need to understand and calculate gradients when designing and constructing ramps, driveways and roads.
- They usually refer to a gradient in ratio form.

  e.g. A driveway can have a maximum gradient of 1 in 4 (1 : 4). This means a run of 4 units will produce a rise (or fall) of 1 unit. We would say the gradient is $\frac{1}{4}$.

1 The graph below shows three slopes. Which of the slopes A, B or C is:

a 1 in 4 **b** 1 in 6 **c** 1 in 12?



2 A slope is to be 1 in 4 (1 : 4). What would the run have to be if the rise is:

a 2 m **b** 1·4 m **c** 3·2 m?

3 Use the graph above (or ratio) to find the rise for a:

a 1 in 6 slope, if the run is 9 m **b** 1 in 12 slope, if the run is 6 m?

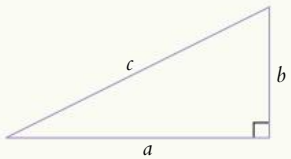
4 A builder has to construct a wheelchair ramp from the front of the house to the front of the property, a run of 10 m. He calculates the fall to be 0·8 m. Building regulations state that the maximum gradient of a ramp is 1 in 14. Is it possible for the builder to build a straight ramp? If not, how can he build it and satisfy the regulations?

9:04 The length of an interval

PREP QUIZ 9:04

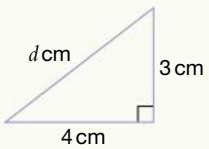
1 Which of the following is the correct statement of Pythagoras' theorem for the triangle shown?

A $a^2 = b^2 + c^2$ **B** $b^2 = a^2 + c^2$ **C** $c^2 = a^2 + b^2$



For Questions **2** to **4** use Pythagoras' theorem to find the value of d.

2



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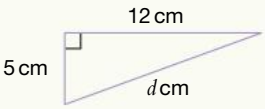

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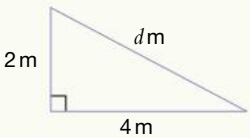
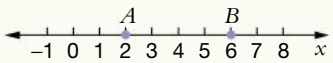
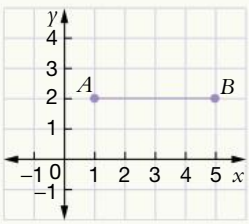


5 Distance AB = … units.

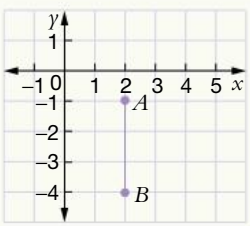


6 Distance AB = … units.

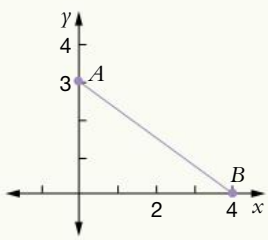
7 Distance AB = … units.



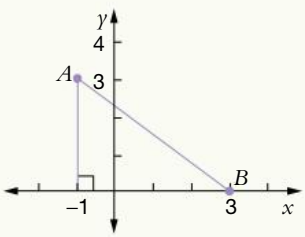
8 Distance AB = … units.



9 Find the distance AB.

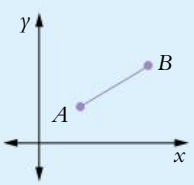


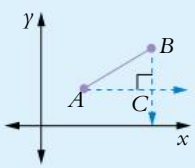
10 Find the distance AB.

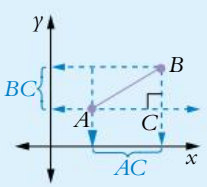


Pythagoras' theorem can be used to find the distance between two points on the number plane.

By drawing a right-angled triangle, we can use Pythagoras' theorem to find the distance between any two points on the number plane.







$c^2 = a^2 + b^2$

$AB^2 = AC^2 + BC^2$

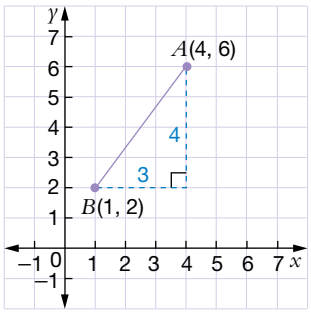
WORKED EXAMPLES

1 Find the distance between the points (1, 2) and (4, 6).

2 If A is (4, 5) and B is (−2, 2) find the length of AB.

Solutions

1



$c^2 = a^2 + b^2$

$AB^2 = AC^2 + BC^2$

$= 4^2 + 3^2$

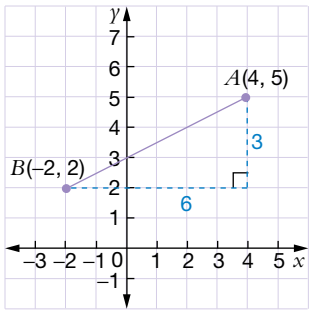
$= 16 + 9$

$= 25$

$\therefore AB = \sqrt{25}$

$\therefore$ Length of AB is 5 units.

2



$c^2 = a^2 + b^2$

$AB^2 = AC^2 + BC^2$

$= 3^2 + 6^2$

$= 9 + 36$

$= 45$

$\therefore AB = \sqrt{45}$

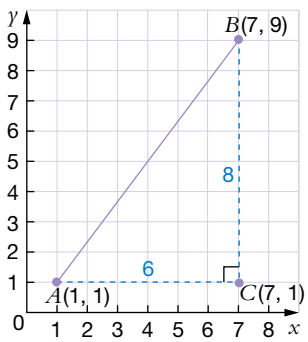
$\therefore$ Length of AB is $\sqrt{45}$ units.

Exercise 9:04

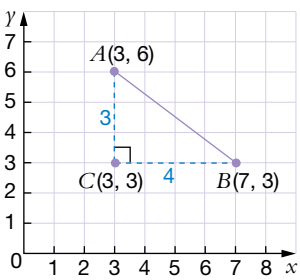
P Foundation worksheet 9:04 Distance between points

1 Use Pythagoras' theorem to find the length of each of the following. (Leave your answer as a surd, where necessary.)

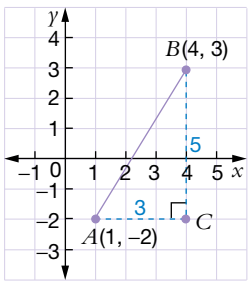
a



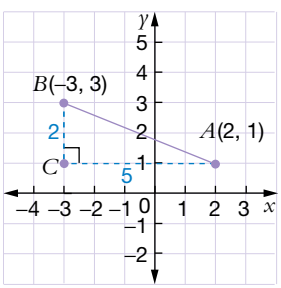
b



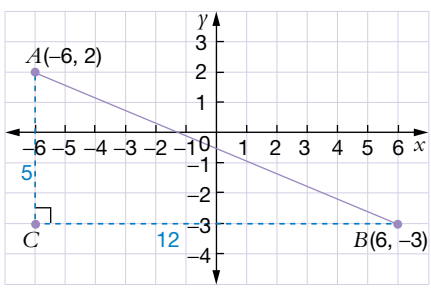
c



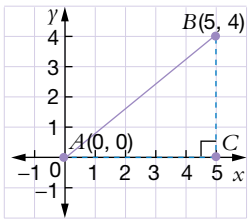
d

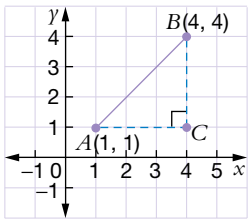


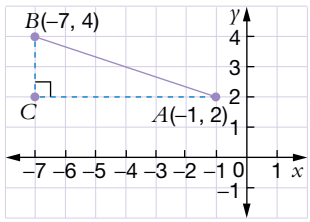
e

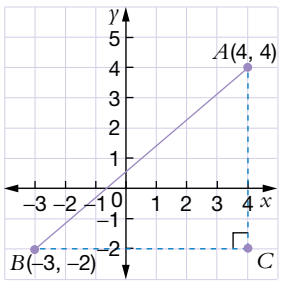


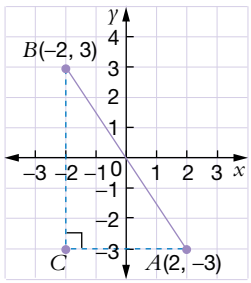
2 Find the lengths BC and AC and use these to find the lengths of AB. (Leave your answers in surd form.)

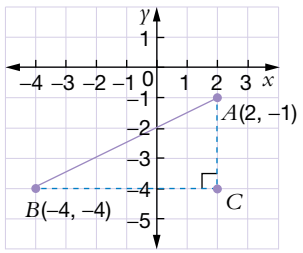
a 

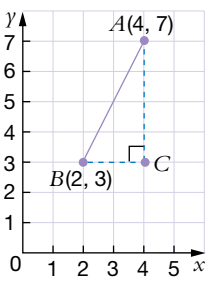
b 

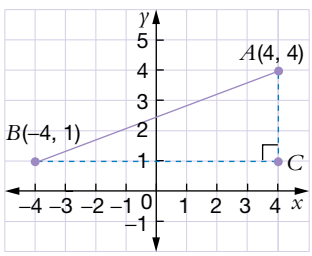
c 

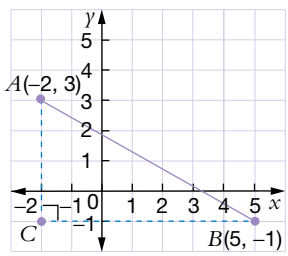
d 

e 

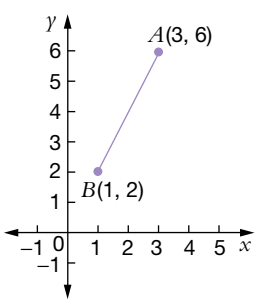
f 

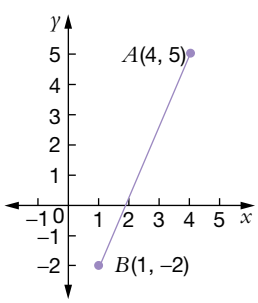
g 

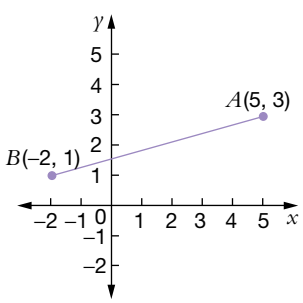
h 

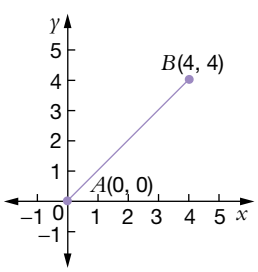
i 

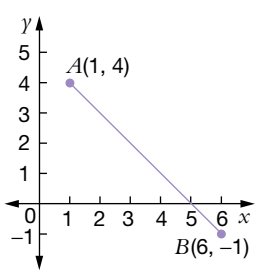
3 Use Pythagoras' theorem to find the length of interval AB in each of the following. (Leave your answers in surd form.)

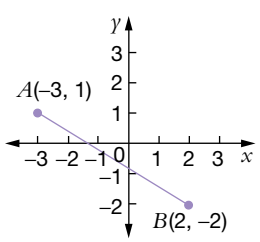
a 

b 

c 

d 

e 

f 

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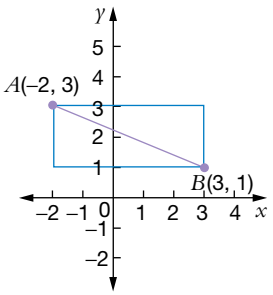

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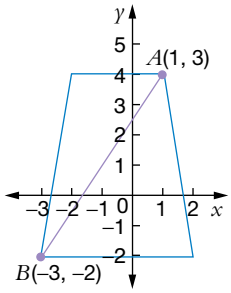

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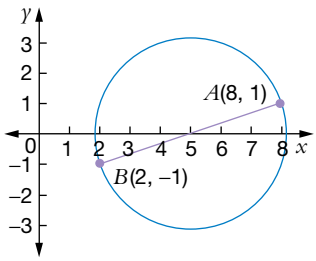


4 Find the distance AB for each of the following. (Leave your answers in surd form.)

a $A(1, 2), B(3, 8)$ **b** $A(-2, 0), B(2, 4)$ **c** $A(0, 0), B(4, 3)$

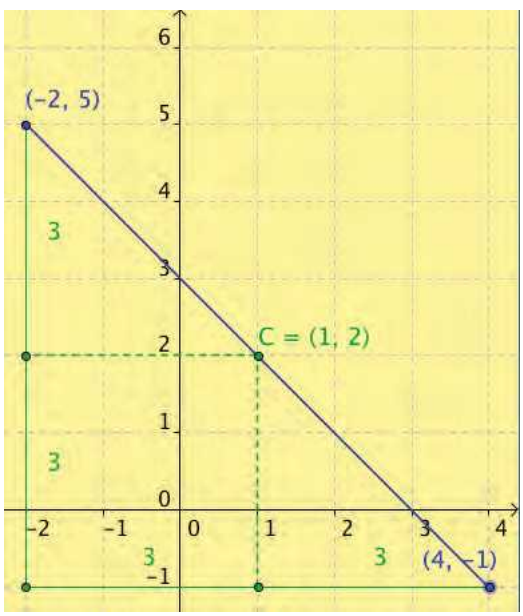
d $A(-2, -3), B(6, 5)$ **e** $A(4, 5), B(5, 6)$ **f** $A(1, 4), B(4, 7)$

g $A(-1, 4), B(2, -2)$ **h** $A(4, 3), B(2, -4)$ **i** $A(0, 0), B(-2, -2)$

j $A(-6, 4), B(6, -1)$ **k** $A(-1, -1), B(-4, -5)$ **l** $A(0, 6), B(4, 0)$

P GEOGEBRA ACTIVITY 9:04 THE MIDPOINT, GRADIENT AND LENGTH OF AN INTERVAL

In this activity you can click and drag points to change the length and gradient of an interval. Calculate the midpoint, gradient and length of the interval, and then use a checkbox to see the solution.



Architectural design often requires an understanding of gradients (slopes).

9:05 Graphing straight lines

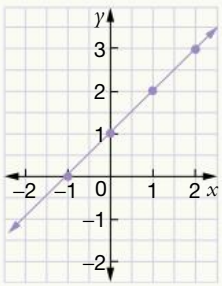
PREP QUIZ 9:05

If $y = 2x + 1$ find y when x is: **1** 0 **2** 1 **3** −2

If $x + y = 3$ find y when x is: **4** 1 **5** −2

If $x - y = 3$ find y when x is: **6** 1 **7** −2

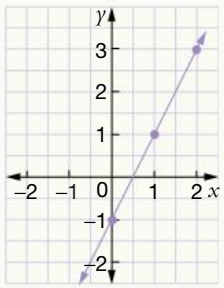
The tables below give the coordinates of three points on a line. Which of the graphs corresponds to each table?



Graph A

8

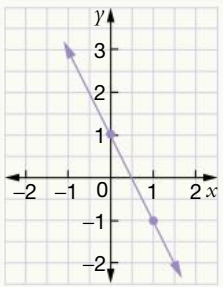
x	−1	0	1
y	3	1	−1



Graph B

9

x	−1	0	1
y	0	1	2



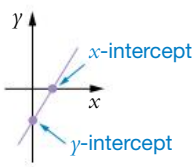
Graph C

10

x	0	1	2
y	−1	1	3

To graph a straight line we need:

- an equation to allow us to calculate the x- and y-coordinates for each point on the line
- a table to store at least two sets of coordinates
- a number plane on which to plot the points



Two important points on a line are:

- the x-intercept (where the line crosses the x-axis)
- the y-intercept (where the line crosses the y-axis)

WORKED EXAMPLES

Draw the graph of each straight line. From the graph write the line's x- and y-intercepts.

1 $x + y = 5$ **2** $y = 3x - 2$ **3** $4x + y = 2$

Solutions

1 $x + y = 5$

x	0	1	2
y	5	4	−3

2 $y = 3x - 2$

x	0	1	2
y	−2	1	4

3 $4x + y = 2$

x	0	1	2
y	−1	−2	−6

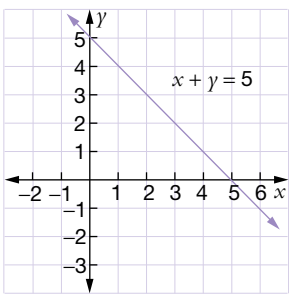
Note: We may choose any three values for x, but it is usually best to use 0, 1 and 2.

Only two points are required to graph a line, however, it is usual to use three. This is done to check that no errors have been made with the calculation of the points. If a mistake is made with one of the calculations it is usually impossible to draw a straight line through the three points.

When $x = 0$,
$0 + y = 5$
$\therefore y = 5$

When $x = 1$,
$1 + y = 5$
$\therefore y = 4$

When $x = 2$,
$2 + y = 5$
$\therefore y = 3$

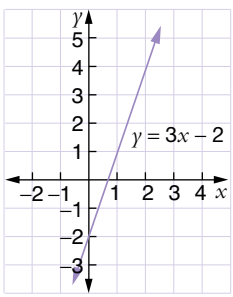


x-intercept is 5
y-intercept is 5

When $x = 0$,
$y = 3 \times 0 - 2$
$= -2$

When $x = 1$,
$y = 3 \times 1 - 2$
$\therefore y = 1$

When $x = 2$,
$y = 3 \times 2 - 2$
$\therefore y = 4$

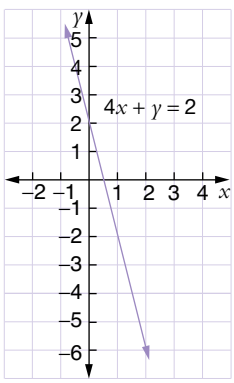


x-intercept is $\frac{2}{3}$
y-intercept is -2

When $x = 0$,
$4 \times 0 + y = 2$
$0 + y = 2$
$\therefore y = 2$

When $x = 1$,
$4 \times 1 + y = 2$
$4 + y = 2$
$\therefore y = -2$

When $x = 2$,
$4 \times 2 + y = 2$
$8 + y = 2$
$\therefore y = -6$



x-intercept is $\frac{1}{2}$
y-intercept is 2

Exercise 9:05

P Foundation worksheet 9:05 Graphing lines

1 Complete the table for each equation.

a $y = 3x$

x	0	1	2
y			

b $y = x + 1$

x	0	1	2
y			

c $y = 5 - x$

x	0	1	2
y			

d $y = 6x - 7$

x	2	3	4
y			

e $y = 18 - 5x$

x	2	3	4
y			

f $y = 7x + 2$

x	2	3	4
y			

g $y = x - 7$

x	0	1	2
y			

h $y = 1 - x$

x	0	1	2
y			

2 Draw the graph of each of the following lines. Use axes numbered from −3 to 7. In each case complete the table first.

a $y = x + 1$

x	0	1	2
y			

b $y = 2x$

x	0	1	2
y			

c $y = -x + 1$

x	0	1	2
y			

d $y = 3x + 1$

x	0	1	2
y			

e $y = 6 - 2x$

x	0	1	2
y			

f $y = x + 3$

x	0	1	2
y			

3 Graph the lines described by these equations.

a $x + y = 3$ **b** $x + y = 5$ **c** $x + y = 2$
d $y = x + 1$ **e** $y = x + 4$ **f** $y = x + 2$
g $y = 2x - 1$ **h** $y = 3x - 1$ **i** $y = 2x - 2$

4 On the same number plane, draw the graphs of these equations.

a $y = 2x - 1$ **b** $y = 2x$ **c** $y = 2x + 3$

How are these lines alike?

5 Graph these equations on one number plane.

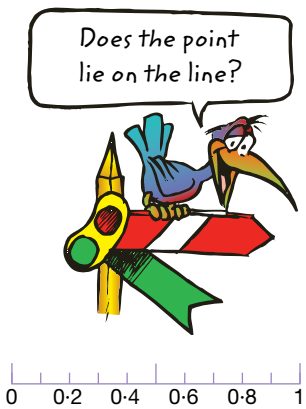
a $y = x$ **b** $y = 2x$ **c** $y = 3x$

What do these lines have in common?

6 On one number plane, graph these equations.

a $y = 2x - 1$ **b** $x + y = 2$ **c** $y = x$

What do these lines have in common?



A point lies on a line if its coordinates satisfy the equation of the line.

Example

Does the point $(6, 7)$ lie on the line $y = 2x - 5$?

Solution

Substitute $x = 6$ and $y = 7$ into $y = 2x - 5$.

$y = 2x - 5$
$7 = 2 \times 6 - 5$
$7 = 7$
which is true.
$\therefore$ $(6, 7)$ lies on the line $y = 2x - 5$.

7 On which of the following lines does the point $(6, 7)$ lie?

a $y = x + 1$ **b** $x + 2y = 20$ **c** $y = 3x - 4$
d $4x - 3y = 3$ **e** $x - y = 1$ **f** $y = x - 1$

8 **a** Does $(4, 0)$ lie on the line $4x + 3y = 16$?
b Does the line $y = 2x - 3$ pass through the point $(7, 11)$?
c Which of the points $(7, 2)$ and $(7, -2)$ lie on the line $y = x - 9$?
d Which of the points $(5, 0)$ and $(0, 5)$ lie on the line $x + 2y = 5$?

9 Draw the graph of each line and use it to find the x- and y-intercepts.

a $2x + y = 2$ **b** $3x + y = 6$ **c** $2x + y = 4$

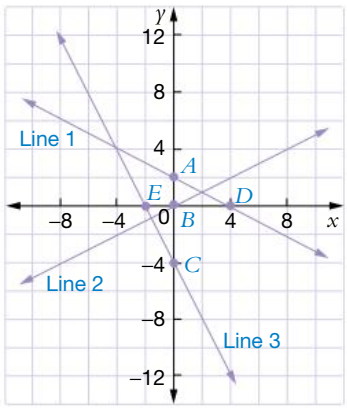
d $2x - y = 4$ **e** $3x - y = 3$ **f** $4x - y = 2$

10 Draw the graph of each equation.

a $y = \frac{3x}{2}$ **b** $y = \frac{x+1}{2}$ **c** $y = \frac{x-1}{2}$

d $3x + 2y = 7$ **e** $5x - 2y - 6 = 0$ **f** $2x - 3y - 5 = 0$

INVESTIGATION 9:05 — x- AND y-INTERCEPTS



1 From the points A, B, C, D and E choose the x-intercept of:
a line 1 **b** line 2 **c** line 3

2 From the points A, B, C, D and E choose the y-intercept of:
a line 1 **b** line 2 **c** line 3

3 Write the coordinates of the x-intercepts.
What do you notice?

4 Write the coordinates of the y-intercepts.
What do you notice?

5 What can you say about the y-coordinate of every point on the x-axis?

6 What can you say about the x-coordinate of every point on the y-axis?

7 How can you find the x-intercept from the line's equation?

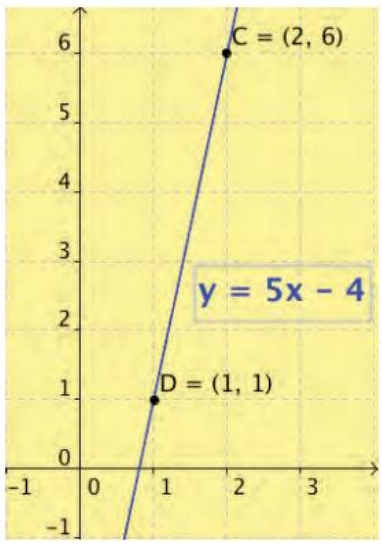
8 How can you find the y-intercept from the line's equation?

9 Find the x- and y-intercepts of the lines in Questions 3 and 10 of Exercise 9:05.

GEOGEBRA ACTIVITY 9:05 — GRAPHING LINES BY PLOTTING POINTS

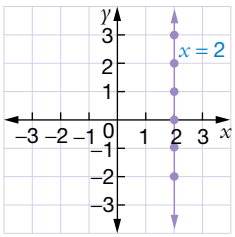
Choose one of the three sets of equations and work out the coordinates of two points on the line. Then click and drag two given points to these positions. If you have calculated correctly, the given line will appear. You can then repeat the process with the other two lines.
The degree of difficulty of calculating the points varies from line to line.

New sets of three equations can be generated randomly.



9:06 Horizontal and vertical lines

The line on the right is vertical.

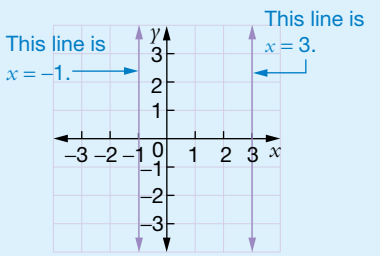


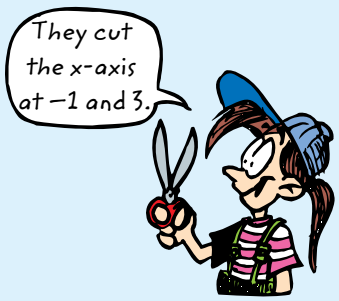
- The following table of values shows the points marked on the line.

x	2	2	2	2	2	2
y	–2	–1	0	1	2	3

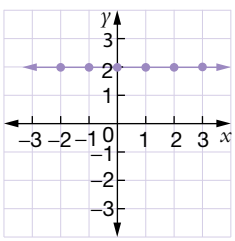
- There seems to be no connection between x and y. However, x is always 2. So the equation is $x = 2$.

Vertical lines have equations of the form $x = a$, where a is the x-intercept.





The line on the right is horizontal.

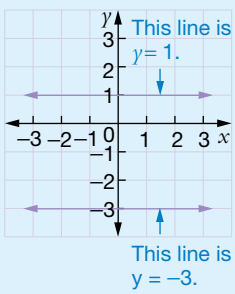


- The following table of values shows the points marked on the line.

x	–2	–1	0	1	2	3
y	2	2	2	2	2	2

- There seems to be no connection between x and y. However, y is always 2. So the equation is $y = 2$.

Horizontal lines have equations of the form $y = b$, where b is the y-intercept.

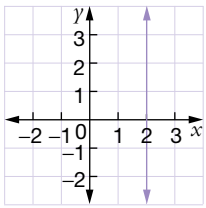


Exercise 9:06

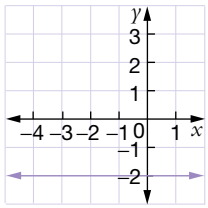
P Foundation worksheet 9:06
Horizontal and vertical lines

1 Write the equation of each line.

a

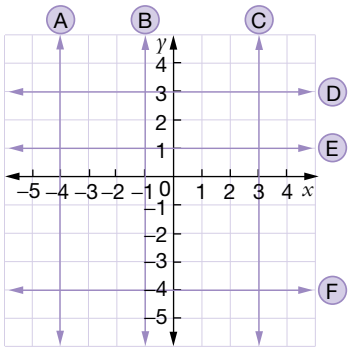


b

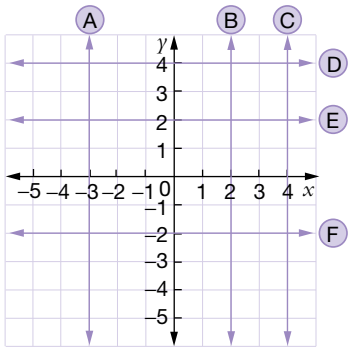


2 For each number plane write the equations of the lines A to F.

a



b

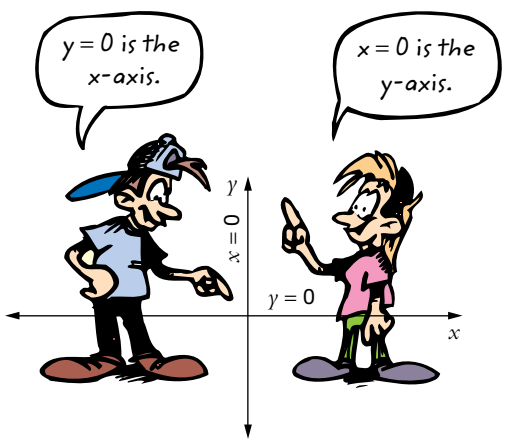


3 **a** What is the equation of the y-axis?
b What is the equation of the x-axis?

4 Using values from -5 to 5 on each axis, draw the graphs of the following straight lines. Use a new diagram for each part.

a $y = 4$, $x = 5$, $y = -1$, $x = 0$
b $x = 1$, $y = 0$, $x = 2$, $y = 3$
c $y = 4$, $x = 2$, $y = -2$, $x = -4$
d $x = 5$, $y = -5$, $x = 2$, $y = 2$
e $y = -2$, $y = 0$, $x = 0$, $x = 3$

Which of these encloses a square region?



5 Give the point where each pair of lines cross. (You can use the graphs in Question 2.)

a $x = 3$ and $y = 1$
b $x = -4$ and $y = 3$
c $x = -1$ and $y = -4$
d $x = 2$ and $y = -2$
e $x = -3$ and $y = 2$
f $x = 4$ and $y = 4$
g $x = 0$ and $y = 3$
h $x = 2$ and $y = 0$
i $x = 0$ and $y = 0$

9:07 The gradient–intercept form of a straight line: $y = mx + b$

PREP QUIZ 9:07

If $x = 0$, what is the value of:

1 $2x$ **2** $mx + b$?

If $x = 0$, what is the value of y when:

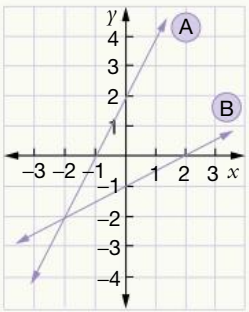
3 $y = 3x + 2$ **4** $y = 4x - 1$?

What is the gradient of:

5 line A **6** line B?

What are the coordinates of the y-intercept of:

7 line A **8** line B?

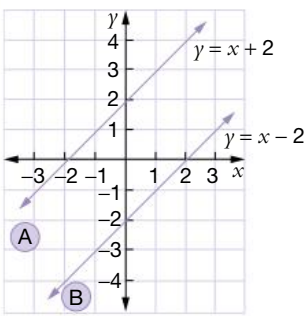


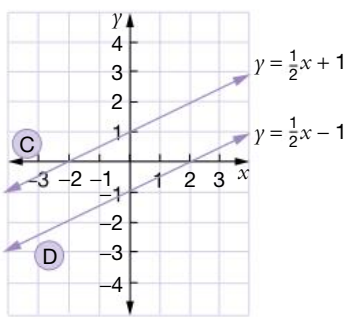
9 Does every point on the y-axis have an x-coordinate of 0?

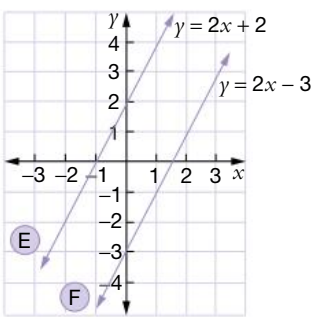
10 Can the y-intercept of a line be found by substituting $x = 0$?

- The equation of a line can be written in several ways. For example:
  $x - y - 4 = 0$, $y = x - 4$ and $x - y = 4$ are different ways of writing the same equation.
- $x - y - 4 = 0$ is said to be in *general form*.
- $y = x - 4$ allows us to get information about the line directly from the equation.

INVESTIGATION 9:07 WHAT DOES $y = mx + b$ TELL US?





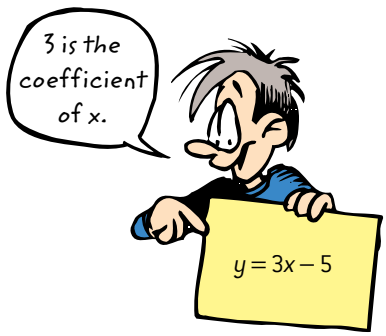


1 Use the graphs above to complete the table on the following page.

Line	Equation	Gradient	y-intercept
A	$y = x + 2$		
B	$y = x - 2$		
C	$y = \frac{1}{2}x + 1$		
D	$y = \frac{1}{2}x - 1$		
E	$y = 2x + 2$		
F	$y = 2x - 3$		

In an equation like $y = 3x - 5$:

- the number 3, in front of x, tells us how many of x we have and is called the coefficient of x
- similarly, the number in front of y is called the coefficient of y. In this case the coefficient of y is 1
- the number -5 is called the constant.



2 Complete the table below.

Line	Equation	Coefficient of x	Constant
A	$y = x + 2$		
B	$y = x - 2$		
C	$y = \frac{1}{2}x + 1$		
D	$y = \frac{1}{2}x - 1$		
E	$y = 2x + 2$		
F	$y = 2x - 3$		

3 From the tables in Questions **1** and **2**, state how the gradient and y-intercept of a line are related to its equation.

4 The graphs of equations $2x + y - 3 = 0$ and $y = -2x + 3$ are the same line. The gradient is -2 and the y-intercept is 3. Which form of the equation gives this information directly?

5 What does the form $y = mx + b$ tell us about a line?

- When an equation of a line is written in the form $y = mx + b$:
  m gives the gradient
  b gives the y-intercept
- Clearly, lines with the same gradient are parallel.
  (See the pairs of lines in Investigation 9:07.)

WORKED EXAMPLE 1

Write the gradient and y-intercept of these lines.

a $y = 3x - 5$

Here, $m = 3$, $b = -5$.
The gradient is 3 and the y-intercept is -5.

b $y = -2x$

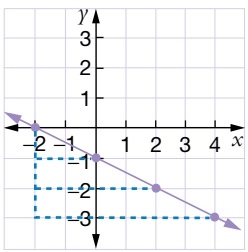
Here, $m = -2$, $b = 0$.
The gradient is -2 and the y-intercept is 0.

c $y = 4 - 3x$

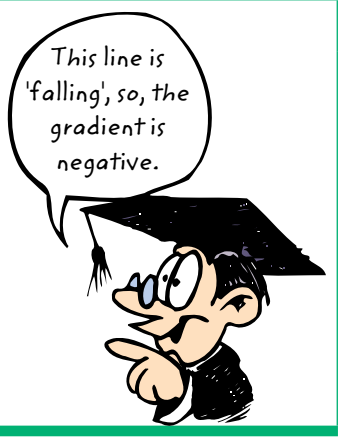
Here $m = -3$, $b = 4$.
The gradient is -3 and the y-intercept is 4.

WORKED EXAMPLE 2

Find the gradient and y-intercept from the graph and write the equation of the line.



From the graph:
For every run of 2 there is a fall of 1,
gradient $= -\frac{1}{2}$
y-intercept $= -1$
$\therefore$ Equation of the line is $y = -\frac{1}{2}x - 1$.



WORKED EXAMPLE 3

Change the equation into the form $y = mx + b$ to find the gradient and y-intercept of the following.

a $2y = 4x + 3$
$\div 2 \quad \div 2 \quad \div 2$
$y = 2x + 1\frac{1}{2}$

$\therefore$ gradient $= 2$
y-intercept $= 1\frac{1}{2}$

b $3x + y = 1$
$-3x \qquad -3x$
$y = 1 - 3x$

$\therefore$ gradient $= -3$
y-intercept $= 1$

c $2x - 3y + 6 = 0$
$+3y \qquad +3y$
$2x + 6 = 3y$
$\div 3 \quad \div 3 \quad \div 3$
$\frac{2}{3}x + 2 = y$

$\therefore$ gradient $= \frac{2}{3}$
y-intercept $= 2$

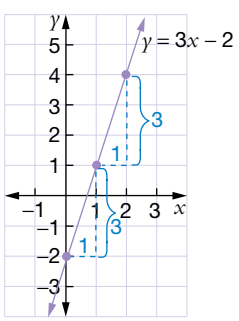
WORKED EXAMPLE 4

Use the y-intercept and gradient to graph the line $y = 3x - 2$.

Start at the y-intercept of -2.

Now gradient $= \dfrac{\text{rise}}{\text{run}} = 3 = \dfrac{3}{1}$

$\therefore$ For every run of 1 there is a rise of 3.

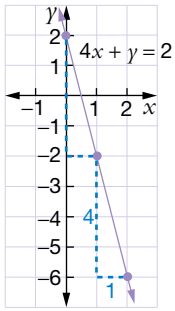


WORKED EXAMPLE 5

Use the y-intercept and gradient to graph the line $4x + y = 2$.
First, rearrange the equation.

$4x + y = 2$
$-4x \quad -4x$
$y = -4x + 2$

Then graph the line using the y-intercept of 2 and gradient of -4.



Exercise 9:07

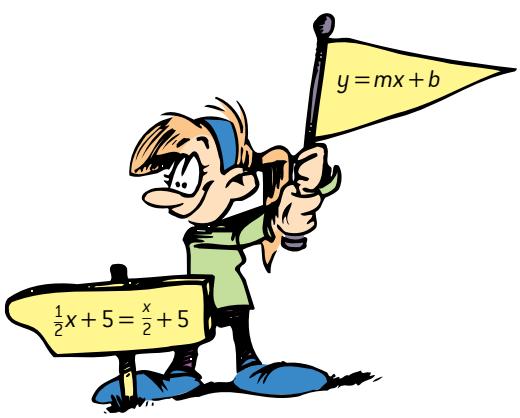
P Foundation worksheet 9:07 Gradient–intercept form

1 What are the gradient and y-intercept of each of the following lines?

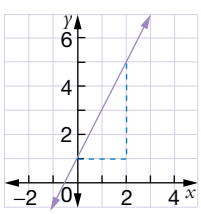
a $y = 2x + 3$ **b** $y = 5x + 1$ **c** $y = 3x + 2$ **d** $y = 1x + 6$
e $y = 4x + 0$ **f** $y = x$ **g** $y = x - 2$ **h** $y = 5x - 1$
i $y = 6x - 4$ **j** $y = -2x + 3$ **k** $y = -x - 2$ **l** $y = -3x + 1$
m $y = \frac{1}{2}x + 4$ **n** $y = \frac{3}{4}x - 2$ **o** $y = \frac{x}{3} + 5$ **p** $y = 4 - 3x$
q $y = -2 + 4x$ **r** $y = 3 - \frac{1}{2}x$

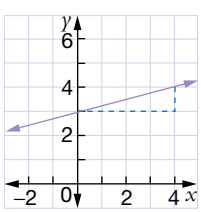
2 Find the equation of the line that has:

a a gradient of 4 and a y-intercept of 9
b a gradient of -2 and a y-intercept of 3
c a gradient of 7 and a y-intercept of -1
d a gradient of -5 and a y-intercept of -2
e a gradient of $\frac{1}{2}$ and a y-intercept of 5
f a gradient of $\frac{2}{3}$ and a y-intercept of -4
g a y-intercept of 1 and a gradient of 3
h a y-intercept of -3 and a gradient of 2
i a y-intercept of $\frac{1}{2}$ and a gradient of 5
j a gradient of 1 and a y-intercept of $1\frac{1}{2}$



3 For each diagram find the gradient and the y-intercept and use these to write the equation of each line.

a


b


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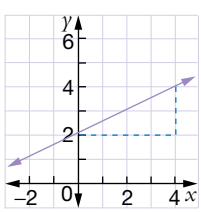

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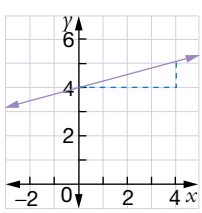

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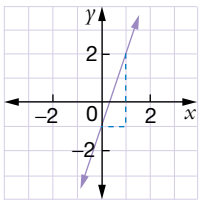

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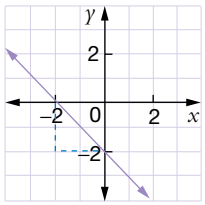

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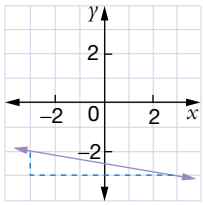
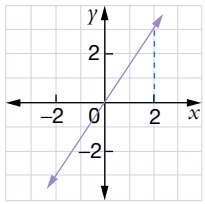


h



4 Match the graphs A to G with the equations below.

a $y = x$

b $y = 6 - 2x$

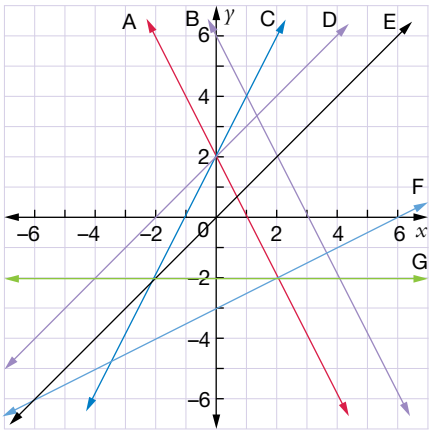
c $y = x + 2$

d $y = -2$

e $y = \frac{1}{2}x - 3$

f $y = -2x + 2$

g $y = 2x + 2$



5 By first changing the equation into the form $y = mx + b$, find the gradient and y-intercept of the following.

a $2y = 4x + 10$

$\div 2 \quad \div 2$

$y = \ldots\ldots$

$\therefore$ gradient is ……

y-intercept is ……

b $y + 3 = 3x$

$- 3 \quad - 3$

$y = \ldots\ldots$

$\therefore$ gradient is ……

y-intercept is ……

c $\frac{y}{2} = x + 2$

$\times 2 \quad \times 2$

$y = \ldots\ldots$

$\therefore$ gradient is ……

y-intercept is ……

d $3y = 6x - 3$

e $y - 8 = 5x$

f $\frac{y}{3} = 2x - 4$

g $4y = 12x - 8$

h $y + 4 = -2x$

i $y - 2x = 7$

j $x + y = 4$

k $2y = x + 3$

l $3x + y = 5$

m $6x + 2y = 4$

$- 6x \quad - 6x$

$2y = \ldots\ldots$

$\div 2 \quad \div 2$

$y = \ldots\ldots$

$\therefore$ gradient is ……

y-intercept is ……

n $3x + 2y = 6$

$- 3x \quad - 3x$

$2y = \ldots\ldots$

$\div 2 \quad \div 2$

$y = \ldots\ldots$

$\therefore$ gradient is ……

y-intercept is ……

o $2x - 5y + 1 = 0$

$+ 5y \quad + 5y$

$\ldots\ldots = 5y$

$\div 5 \quad \div 5$

$\ldots\ldots = y$

$\therefore$ gradient is ……

y-intercept is ……

6 Lines with the same gradient are parallel. Are the following pairs of lines parallel or not?

a $y = 3x + 2$ and $y = 3x - 1$

b $y = 5x - 2$ and $y = 2x - 5$

c $y = x + 7$ and $y = x + 1$

d $y = x - 3$ and $y = 1x + 2$

7 **a** Which of the following lines are parallel to $y = 2x + 3$?

$y = 3x + 2$ $\qquad$ $2x - y + 6 = 0$

$2y = x + 3$ $\qquad$ $y = 2x - 3$

b Two of the following lines are parallel. Which are they?

$y = x - 3$ $\quad$ $x + y = 3$ $\quad$ $y = 3x$ $\quad$ $3y = x$ $\quad$ $y = -x + 8$

If two lines are parallel, they have the same gradient.

8 Draw the graph of each line on a separate number plane by following these steps. (Look at Worked Example 4.)

- Mark the y-intercept (the value of b) on the y-axis.
- 'Count' the gradient (the value of m) from this point, and mark a second point.
- Draw a line through this second point and the y-intercept.

a $y = 2x + 1$ **b** $y = x + 2$ **c** $y = 3x - 1$

d $y = -2x - 1$ **e** $y = -x + 1$ **f** $y = -3x - 3$

g $y = \frac{1}{2}x + 2$ **h** $y = \frac{1}{3}x - 1$ **i** $y = \frac{3}{2}x$

j $y = -\frac{1}{2}x + 1$ **k** $y = -\frac{1}{3}x$ **l** $y = -\frac{3}{2}x - 1$

9 **a** Graph lines with a y-intercept of 1 and a gradient of 0, 1, 2 and −1 on the same number plane.

b Graph lines with a slope of −1 and y-intercepts of −2, 0, 1 and 2 on the same number plane.

10 The equation of each line has been given in general form. Find the gradient of each line by rearranging the equation into gradient–intercept form.

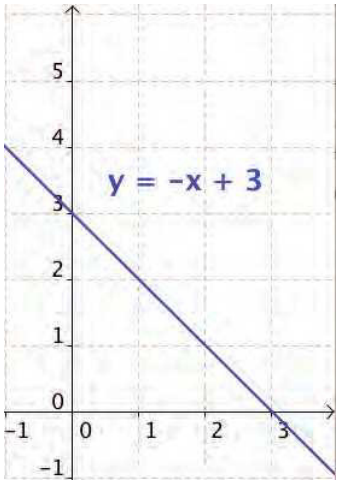
a $2x + y + 6 = 0$ **b** $4x - 2y + 5 = 0$ **c** $x + 2y + 1 = 0$

d $4x - 3y + 6 = 0$ **e** $4x - y + 3 = 0$ **f** $x + 3y - 6 = 0$

P GEOGEBRA ACTIVITY 9:07 THE GRADIENT–INTERCEPT FORM OF A LINE

1 Finding the equation of a line

In this activity a line is generated randomly. Find the gradient and y-intercept of the line and use these to write its equation in the form $y = mx + b$. Tick a checkbox to see if the equation is correct.



2 Equations of the form $y = mx + b$

Use sliders to vary the values of m and b on a blue line. As these values are varied you can analyse the effect that each has on the shape of the graph. You can then find the equation of a randomly produced red line. Use the sliders to change the values of m and b on the blue line until it coincides with the red line.

3 Graphing lines using the gradient and y-intercept

In this activity you are given an equation and asked to graph the line by first dragging a point to the y-intercept. A gradient triangle and a line appears when this is done correctly. The gradient triangle is used to find another point on the line. When this is successful, the line changes colour.

4 Matching equations of lines to their graphs

The graphs and equations of four different lines are given for you to match each equation to its graph. You can then check the solutions and generate a new set of lines and equations.

FUN SPOT 9:07 — WHY DID THE BANANA GO OUT WITH A FIG?

Answer each question and write the letter for that question in the box above the correct answer.

For the points $A(2, 5)$, $B(-1, 3)$, $C(4, 2)$, $D(0, 6)$, find:

E distance AC | **E** distance CD | **E** distance BD

A slope of AB | **A** slope of AD | **T** slope of BC

T midpoint of DC | **N** midpoint of AC | **I** midpoint of AB

What is the gradient of the following lines?

C $y = 2x - 1$ | **U** $2y = x - 5$ | **G** $2x + y + 1 = 0$

What is the y-intercept of the following lines?

D $y = 2x - 1$ | **T** $2y = x - 5$ | **L** $2x + y - 1 = 0$

Find the equation of the line with:

C gradient of 2 and a y-intercept of -1

T y-intercept of 4 and a slope of -2

O slope of 3 and a y-intercept of 5

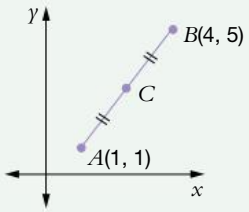
S y-intercept of -2 and a slope of 4.

Write in the form $y = mx + b$.

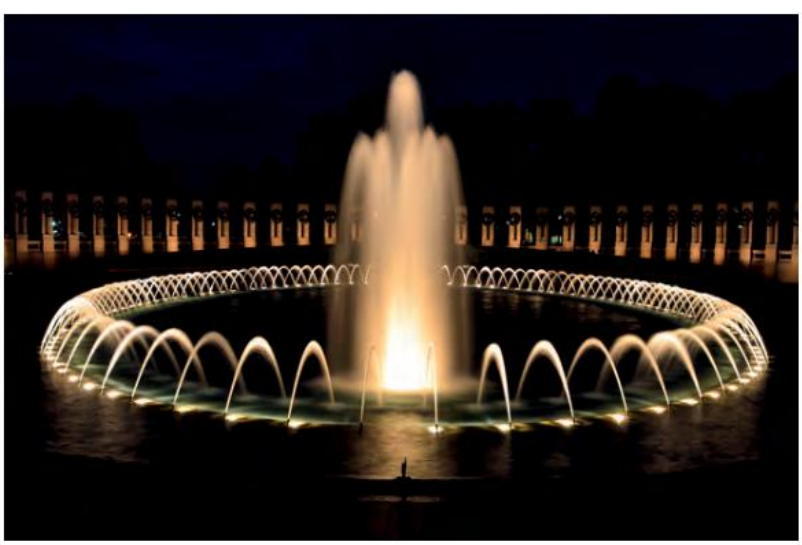
B $2x - y - 5 = 0$ | **E** $2x - y + 5 = 0$ | **A** $x = \frac{1}{2}y - 2$

In the diagram, find:

D the slope of AB | **U** coordinates of C.



$y = 2x - 5$	$y = 2x + 5$	2	$-\frac{1}{2}$	$\frac{1}{2}$	$y = 4x - 2$	$\sqrt{13}$	$(\frac{1}{2}, 4)$	$(2, 4)$	$y = 2x - 1$	$y = 3x + 5$	$(2\frac{1}{2}, 3)$	1	-1	$(3, 3\frac{1}{2})$	$-\frac{1}{5}$	-2	$\sqrt{10}$	$-2\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{3}$	$y = 2x + 4$	$y = 4 - 2x$	$\sqrt{32}$



Not all graphs are straight lines. A parabola is one type of curve that is non-linear.

9:08 Non-linear graphs

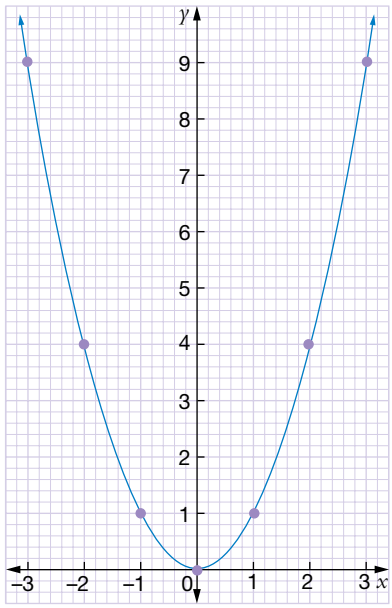
Up to this point, all the graphs have been straight lines. There are many types of mathematical curves and in this section we will look at two of them, the parabola and the exponential curve.

Parabolas

- The equations of parabolas can be identified because they have x^2 as the highest power of x.
  The simplest equation of a parabola is $y = x^2$.
- As with the straight line, the equation is used to find the points on the curve. Some of these are shown in the table.

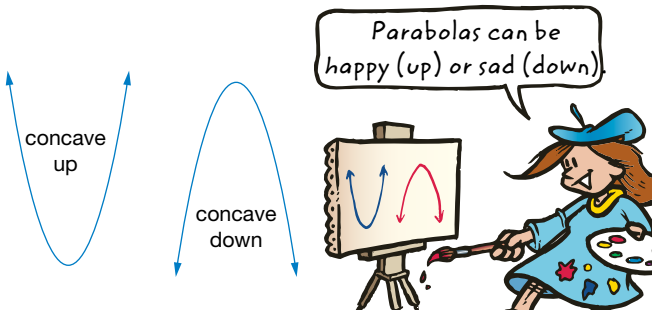
$y = x^2$

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



For an accurate graph, many points would have to be plotted.

- From the graph, we can see that the parabola has a turning point, or vertex, which is the minimum value of y on $y = x^2$.
- The y-axis is an axis of symmetry of the curve, so the right side of the curve is a reflection of the left side. This can be seen when points on either side of the axis are compared.
- The parabolas that we will consider have two basic shapes. The one on the left is said to be concave up, whereas the one on the right is said to be concave down.

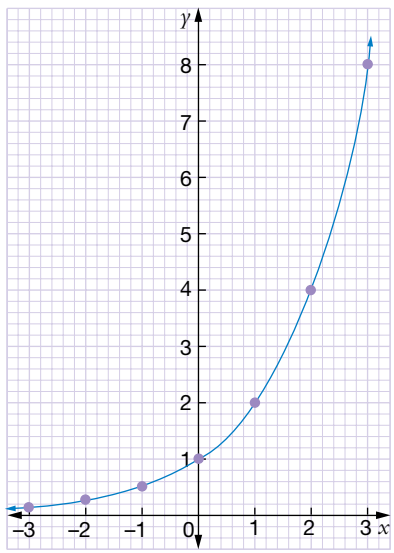


Exponential graphs

- The simplest exponential equation is $y = 2^x$.
  Notice that in this equation x is a power, and hence there is a connection with the topic of Indices (see Chapter 6).
  Again, a table of values is used to find points that lie on the curve.

$y = 2^x$

x	−3	−2	−1	0	1	2	3
y	0.125	0.25	0.5	1	2	4	8



- When plotting points it is often necessary to round the y-values to one decimal place.
- Notice what happens to the y-values of the points as the x-values increase and decrease.

Exercise 9:08

1 Copy and complete the following tables. By comparing the values obtained with the given graphs, find which of the graphs A to C would result from plotting each set of points.

a $y = x^2 + 1$

x	−3	−2	−1	0	1	2	3
y							

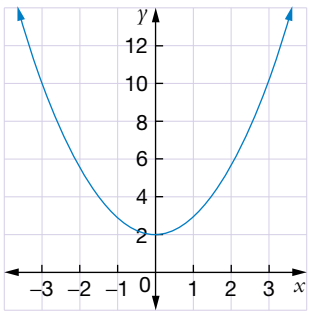
b $y = x^2 - 1$

x	−3	−2	−1	0	1	2	3
y							

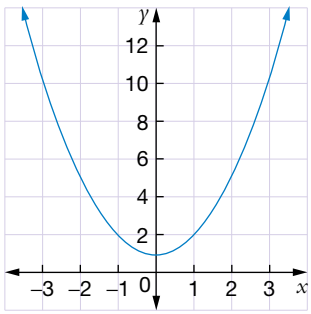
c $y = x^2 + 2$

x	−3	−2	−1	0	1	2	3
y							

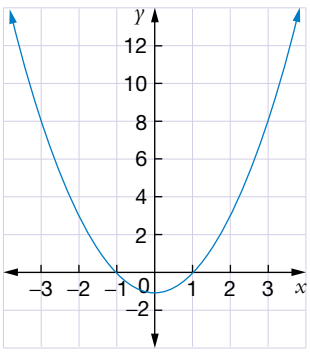
A



B



C



2 Complete a table of values ranging from $x = -3$ to $x = 3$ for the parabola $y = x^2 - 2$ and use the values obtained to graph the equation.

3 **a** What are the coordinates of the vertex for each of the parabolas in question 1?

b Is the vertex also the y-intercept of the parabola?

c Are the parabolas symmetrical?

d Does the vertex lie on the axis of symmetry?

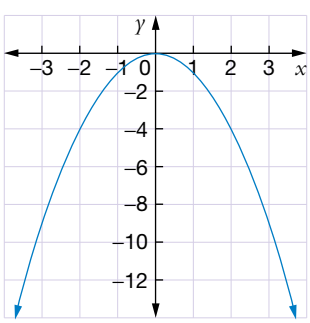
4 Complete a table of values ranging from $x = -3$ to $x = 3$ for each of the following equations. State which of the graphs E, F or G would be obtained by plotting those points.

a $y = -x^2$

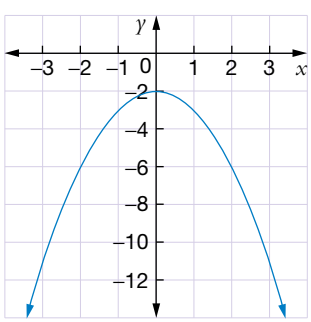
b $y = -x^2 + 1$

c $y = -x^2 - 2$

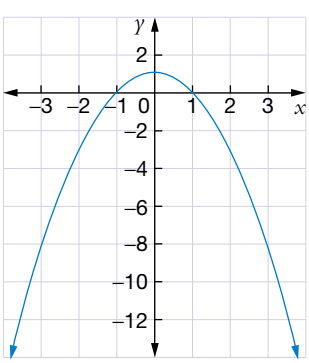
E



F



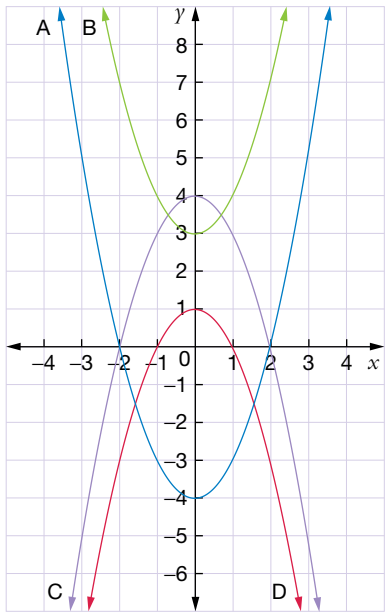
G



5 a Write the coordinates of the vertex for each of the parabolas in question **4**.
b Is the vertex also the y-intercept of the parabola?
c Are the parabolas symmetrical?
d Does the vertex lie on the axis of symmetry?

6 For each of the following, complete a table of values and use it to state the graph (A, B, C or D) that matches the equation.

a $y = x^2 + 3$
b $y = 1 - x^2$
c $y = x^2 - 4$
d $y = -x^2 + 4$



7 Using the results from Questions **1** to **6**, how can you use the equation of a parabola to tell if the parabola is concave up or concave down?

8 Using a table of values ranging from −3 to 3, graph each of the following equations.

a $y = 3 - x^2$
b $y = x^2 + 4$
c $y = -x^2 - 1$

9 Copy and complete each of the following table of values. Which of the exponential graphs E, F G or H would result from plotting that set of points?

a $y = 2^x$

x	−3	−2	−1	0	1	2	3
y							

b $y = 2^{-x}$

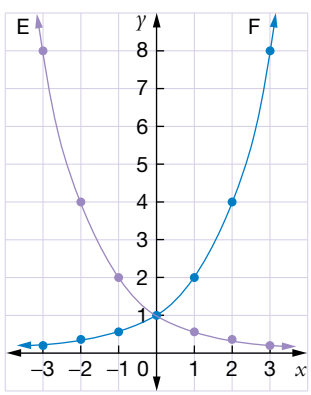
x	−3	−2	−1	0	1	2	3
y							

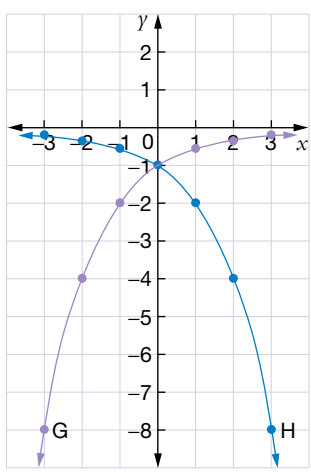
c $y = -2^x$

x	−3	−2	−1	0	1	2	3
y							

d $y = -2^{-x}$

x	−3	−2	−1	0	1	2	3
y							





10 By finding suitable points, match each equation with one of the exponential curves A to D.

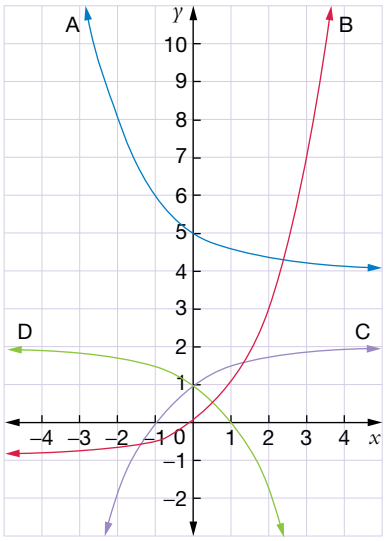
a $y = -1 + 2^x$

b $y = 4 + 2^{-x}$

c $y = -2^x + 2$

d $y = 2 - 2^{-x}$

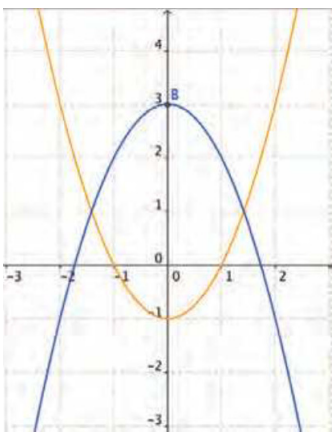
Hint: Use the equations to find the y-intercept for each curve. If these are the same, find another point.



P GEOGEBRA ACTIVITY 9:08 NON-LINEAR GRAPHS

1 Graphing parabolas

The equations of two parabolas that are given can be varied by using sliders. After completing a table of values the points can be plotted via the Input Bar. Tick the checkbox to reveal the graph and show whether the points have been plotted accurately.

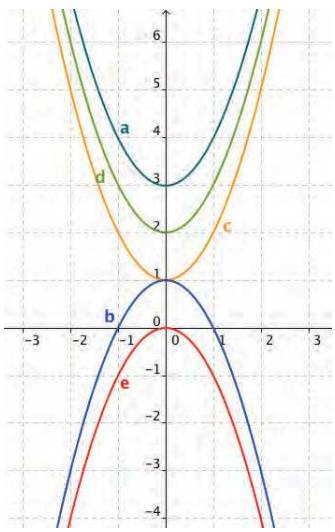


2 Exponential curves

This activity is similar to Activity 1 but uses exponential curves rather than parabolas.

3 Matching parabolas to their equations

The equations of five parabolas and their graphs are given and you are asked to match each parabola to its equation. Tick a checkbox to show the correct answer. A new set of equations and parabolas can be randomly generated.



MATHS TERMS 9

coefficient

- the number that multiplies a pronumeral in an equation,
  e.g. $3x - 5y = 6$
  coefficient of x is 3
  coefficient of y is -5

coordinates

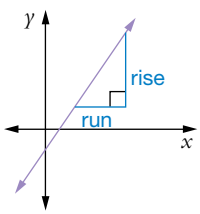
- a pair of numbers that gives the position of a point in a number plane relative to the origin; see $(3, 2)$, $(3, -2)$, $(-3, 2)$, $(-2, -3)$ and $(0, -1)$ in number plane

constant

- the number part of an equation or expression,
  e.g. $3x - 5y = 6$
  The constant is 6

gradient

- the slope of a line or interval; it can be measured using the formula:
  $\text{Gradient} = \frac{\text{rise}}{\text{run}}$



gradient-intercept form

- a way of writing the equation of a line,
  e.g. $y = 2x - 5$, $y = \frac{1}{2}x + 2$

  when an equation is rearranged and written in the form $y = mx + b$ then m is the gradient and b is the y-intercept

graph (noun)

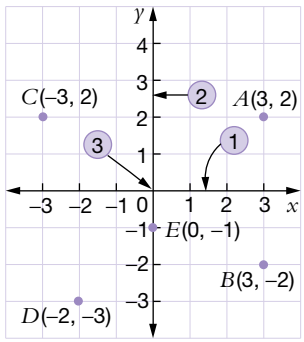
- a diagram showing the relationship between two variable quantities, usually shown on a number plane

graph (verb)

- to plot the points that lie on a line or curve

number plane

- a rectangular grid that allows the position of points in a plane to be identified by an ordered pair of numbers



origin

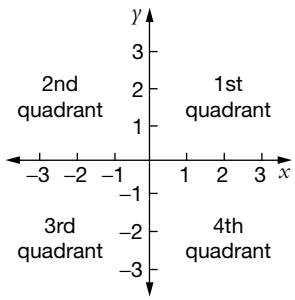
- the point where the x-axis and y-axis intersect, $(0, 0)$; see 3 in number plane

plot

- to mark the position of a point on the number plane

quadrants

- the four quarters that the number plane is divided into by the x- and y-axes



x-axis

- the horizontal number line in a number plane; see 1 in number plane

x-coordinate

- the first number in a pair of coordinates; it tells how far right (or left) the point is from the origin

x-intercept

- the point where a line or curve crosses the x-axis

y-axis

- the vertical number line in a number plane; see 2 in number plane

y-coordinate

- the second number in a pair of coordinates; it tells how far the point is above (or below) the origin

y-intercept

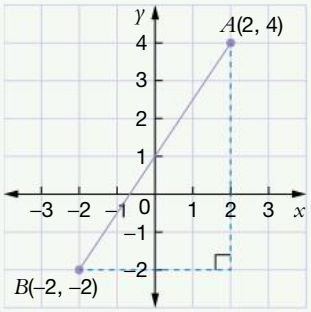
- the point where a line or curve crosses the y-axis

DIAGNOSTIC TEST 9 COORDINATE GEOMETRY

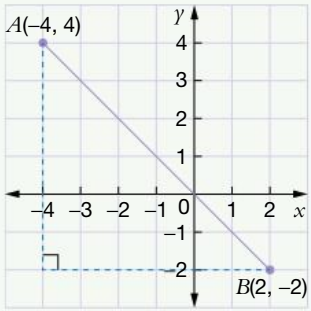
These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Find the midpoint of the interval AB in each of the following. 9:02

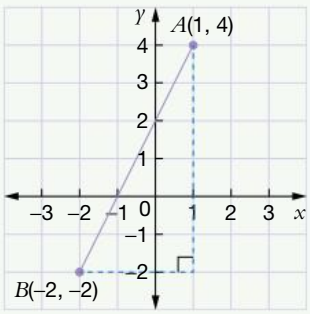
a



b



c



2 For each of the following find the midpoint of the interval AB. 9:02

a $A(1, 2), B(7, 10)$ b $A(3, 0), B(5, 3)$ c $A(-3, -2), B(1, -3)$

3 For each diagram in Question 1, find the gradient of the interval AB. 9:03

4 Find the gradient of the line that passes through: 9:03

a $(1, 3), (2, 7)$ b $(-2, 8), (4, 5)$ c $(0, 3), (3, 5)$

5 For each diagram in Question 1, find the length of the interval AB. (Leave answers in surd form). 9:04

6 Find the distance between the points A and B if: 9:04

a A is $(1, 2)$, B is $(7, 10)$ b A is $(3, 0)$, B is $(5, 3)$ c A is $(-3, -2)$, B is $(1, -3)$

7 Graph the lines with the following equations: 9:05

a $y = 2x + 1$ b $y = 2 - x$ c $y = 2x$

8 a Does the point $(3, 2)$ lie on the line $x + y = 5$? 9:05

b Does the point $(-1, 3)$ lie on the line $y = x + 2$?

c Does the point $(2, -2)$ lie on the line $y = x - 4$?

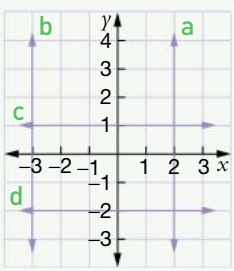
9 State the x-intercept of each line: 9:05

a $y = x - 3$ b $x + y = 4$ c $x + 2y = 4$

10 State the y-intercept of each line: 9:05

a $y = x + 4$ b $x + y = -1$ c $x - y = 5$

11 Write the equation of each line in this diagram. 9:06



12 Write the equation of the line that has: 9:07

a a gradient of 3 and a y-intercept of 2

b a gradient of $\frac{1}{2}$ and a y-intercept of -3

c a y-intercept of 3 and a gradient of -1

d a y-intercept of $-\frac{1}{2}$ and a gradient of $\frac{1}{3}$

13 State the gradient and y-intercept of each line: 9:07

a $y = 2x + 3$ **b** $y = \frac{1}{2}x - 3$ **c** $y = 3 - 2x$ **d** $y = 4 - x$

14 Write the equation of each of the lines AB in Question 1. 9:07

15 Rearrange these equations into gradient–intercept form. 9:07

a $4x - y + 6 = 0$ **b** $2x + 3y - 3 = 0$

c $5x + 2y + 1 = 0$ **d** $x - 2y + 5 = 0$

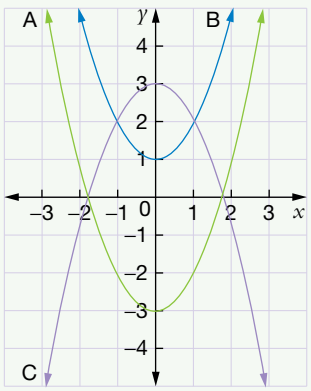
16 Graph the lines in Question 12. 9:07

17 Match each equation with the corresponding parabola in the diagram. 9:08

a $y = -x^2 + 3$

b $y = x^2 + 1$

c $y = x^2 - 3$

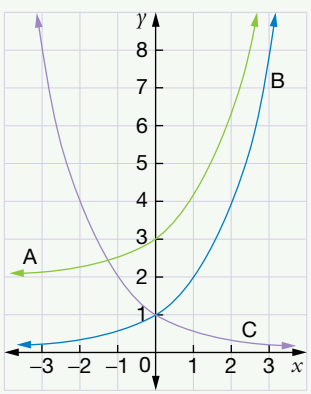


18 Match each equation with the corresponding exponential curve in the diagram. 9:08

a $y = 2^x$

b $y = 2^{-x}$

c $y = 2 + 2^x$



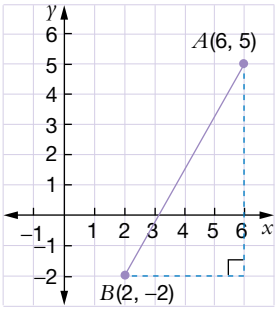
ASSIGNMENT 9A Chapter review

1 Find:

a the length AB as a surd

b the slope of AB

c the midpoint of AB.



2 A is the point $(2, 5)$ and B is the point $(7, 17)$.

a What is the length AB (as a surd)?

b What is the slope of AB?

c What is the midpoint of AB?

3 The equation of line A is $y = 2x - 4$ and the equation of line B is $y = 4 - 2x$.

a Which line has a negative gradient?

b Which line has a y-intercept of 4?

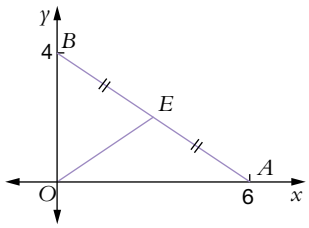
c Which line is parallel to the line $2x + y + 4 = 0$?

d Find the point of intersection of the lines.

4 **a** A line has an x-intercept of 3 and a gradient of 1. Find where the line crosses the y-axis, and hence write its equation.

b A line has a slope of $-\frac{1}{2}$ and a y-intercept of 6. What is its equation? What is its x-intercept?

5 A right-angled triangle OAB is shown.



a Find the coordinates of E, the midpoint of AB.

b Find the length of OE.

c Find the length of EA.

d What can you say about the distance of E from O, A and B?

6 Complete the table of values below for each of the following equations and draw the graphs on the same number plane.

x	–3	–2	–1	0	1	2	3
y							

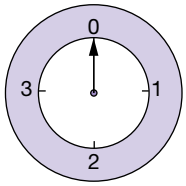
a $y = 2x$ **b** $y = x^2$ **c** $y = 2^x$

I have 20 Australian banknotes totalling $720. Show how this could be done with:

- 2 types of notes
- 3 types of notes
- 4 types of notes

ASSIGNMENT 9B Working mathematically

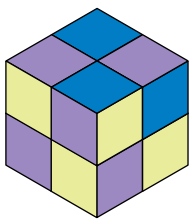
1 The diagram shows a 4-minute timer.



a If this timer was started with the pointer on zero, what number would it be pointing to after 17 minutes?

b At what times between 30 minutes and 1 hour will the pointer be pointing at number 3?

2 The faces of a cube are divided into 4 squares. If each square on each face is to be painted, what is the minimum number of colours needed if no squares that share an edge can be the same colour?



3 John is given $200 each month for a year. Anna is given $1 for the first month, $2 for the second, $4 for the third, and so on until the twelfth month. Who is given the most in the year and by how much is the amount bigger?

4 Four friends decide to play tennis. Find out how many different:

a singles matches can be played. (A singles match is one player against another player.)

b doubles matches can be played. (A doubles match is two players against two players.)

5 The table below has been prepared from information supplied by the Federal Office of Road Safety, for side-impact accidents in which at least one occupant was killed. It gives the number of car occupants in various seating positions and the number killed as a direct result of an impact to the side of the car. The figures exclude 38 occupants as there was insufficient information to determine their exact seating position at the time of the crash.

	Seating position						All occupants	
	Left side (passenger's side)		Centre seat		Right side (driver's side)			
	Dead	Total	Dead	Total	Dead	Total	Dead	Total
Front seat	114	187	0	1	199	316	313	504
Rear seat	29	63	2	19	20	44	51	126
All occupants	143	250	2	20	219	360	364	630

a How many occupants were in:

i the driver's seat

ii the front left side seat

iii a front seat

iv a passenger's side seat?

b What percentage of drivers were killed?

c What percentage of all the occupants who were killed were drivers?

d For each of the six seating positions, calculate the number of deaths as a percentage of the total number of people in that seat. Can you infer anything about the relative safety of the various seating positions from this information? Give reasons for your answer.

ASSIGNMENT 9C Cumulative revision

1 Solve: 7:07

a $\frac{3x}{4} - 5 = \frac{x}{2} - 3$ **b** $\frac{3x-5}{4} = \frac{x}{2} - 3$ **c** $\frac{3x-5}{4} = \frac{x-3}{2}$

2 Simplify each of the following. Give your answer in scientific notation correct to two significant figures. 6:05

a $(1{\cdot}65 \times 10^{3}) \times (7{\cdot}2 \times 10^{-6})$ **b** $\frac{1{\cdot}06 \times 10^{-5}}{5{\cdot}1 \times 10^{-9}}$

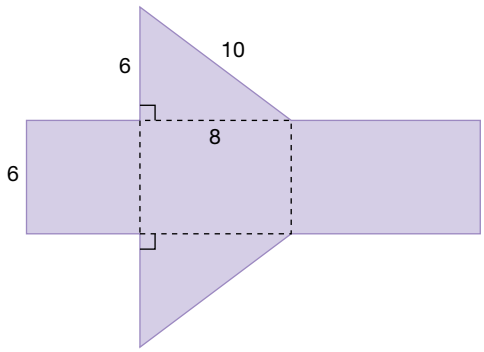
c $(6{\cdot}03 \times 10^{-3})^{2} \div (4{\cdot}7 \times 10^{-10})$

3 **a** Jim earns $18 an hour. Calculate his wages for a week in which he works 35 hours at the normal rate , 6 hours overtime at 'time-and-a-half' and two hours overtime at 'double-time'. 8:02

b Ralph is a tailor. He charges $15 for a minor alteration and $25 for a major alteration. How much will he earn in a week in which he does 32 minor alterations and 23 major alterations? What is his hourly rate if it takes him a total of 38 hours to do the work?

4 **a** Use the net of the triangular prism to calculate its surface area. (All measurements are in cm.) 5:03

b Calculate the surface area and volume of a cylinder that has a diameter of 6 cm and a height of 6 cm.



5 A card is chosen at random from a standard pack of cards. What is the chance that it is: 4:03

a a King **b** red **c** red or a King

d red and a King **e** not an Ace, King, Queen or Jack?

6 The 2009–10 Census produced the following figures on the involvement of people aged 15 years and over in organised and non-organised sporting and physical recreational activities. If a person over 15 is chosen at random, what is the chance that they: 4:02

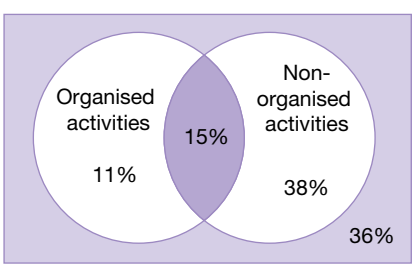
a do not participate in sport or physical recreation

b participate only in organised activities

c participate in only non-organised activities

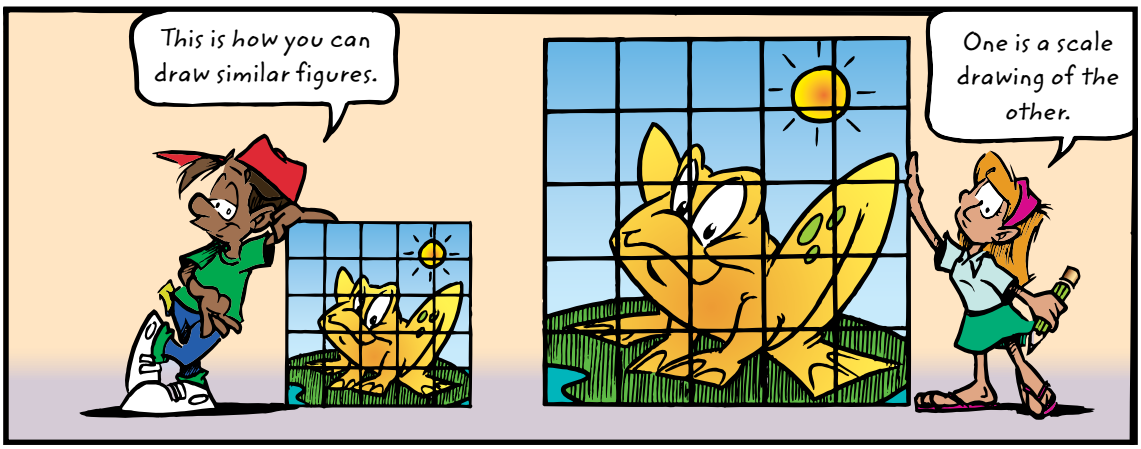
d participate in an organised activity

e participate in both organised and non-organised activites?



SIMILARITY

10



Contents

10:01 Similar figures
GeoGebra activity 10:01 Similar figures
Investigation 10:01 Drawing enlargements and reductions
10:02 Finding unknown sides of similar figures
Investigation 10:02 Photo enlargements
10:03 Similar triangles
10:03A Matching angles
10:03B Ratios of matching sides
GeoGebra activity 10:03 Similar triangles
10:04 Using proportion to find unknown sides
GeoGebra activity 10:04 Finding an unknown side in similar triangles
10:05 Scale drawings
10:05A Calculating real sizes from scale drawings
10:05B Making a scale drawing
Investigation 10:05 Scale drawing
Fun spot 10:05 What happened to the mushroom that was double parked?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Properties of Geometrical Figures*** [Stages 5.1, 5.2]

- Use the enlargement transformation to explain similarity. (ACMMG220)
- Solve problems using ratio and scale factors in similar figures. (ACMMG221)
- Use the enlargement transformations to explain similarity and to develop the conditions for triangles to be similar. (ACMMG220)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

10:01 Similar figures

In mathematics, the word similar does not mean 'almost equal' or 'nearly the same' but actually means 'the same shape'.

- Two figures are similar when one figure can be enlarged and superimposed on the other so that they coincide exactly.
- Similar figures have the same shape, but different size.

We use similar figures in house plans, when enlarging photographs, when using projectors, when making models, in scale drawings, and when using maps.

If two figures are similar, one can be thought of as the enlargement or reduction of the other. The amount of the enlargement or reduction is determined by the scale factor. The scale factor can be found by dividing any pair of matching sides.

- Figure 2 is an enlargement of Figure 1.
  The scale factor = $\frac{6}{2} = 3$.
  Each length in Figure 2 is 3 times the matching length in Figure 1.
- Figure 1 is a reduction of Figure 2.
  The scale factor = $\frac{2}{6} = \frac{1}{3}$.
  Each length in Figure 1 is $\frac{1}{3}$ the matching length in Figure 2.

When a figure is enlarged or reduced, the sizes of matching angles do not change.

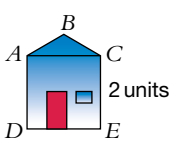


Figure 1

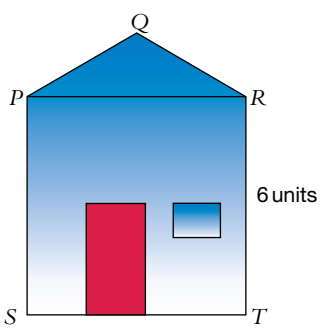
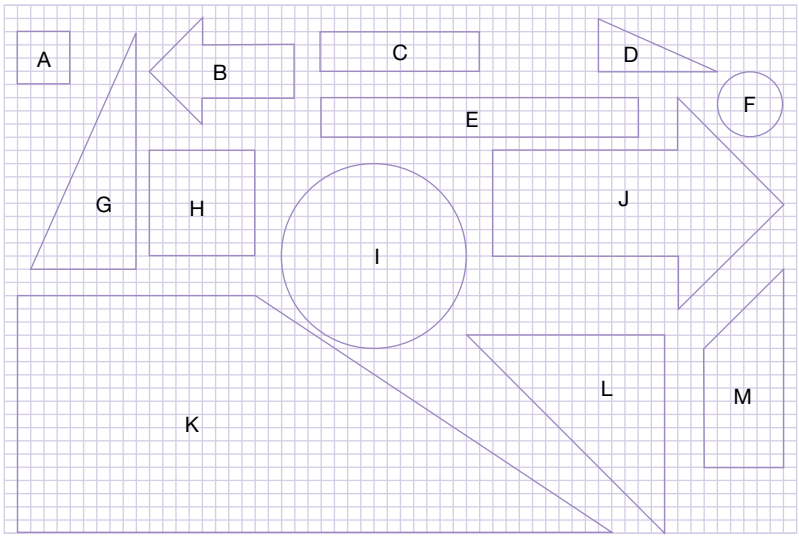


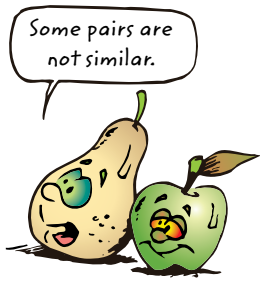
Figure 2

When naming similar figures, the vertices should be listed in matching order.
In the figures shown, we would write $ABCED \mid\mid\mid PQRTS$.
The symbol $\mid\mid\mid$ means 'is similar to'.

Discussion

- List the pairs of similar figures below.
- Find the enlargement (or scale) factor for each pair of similar figures.



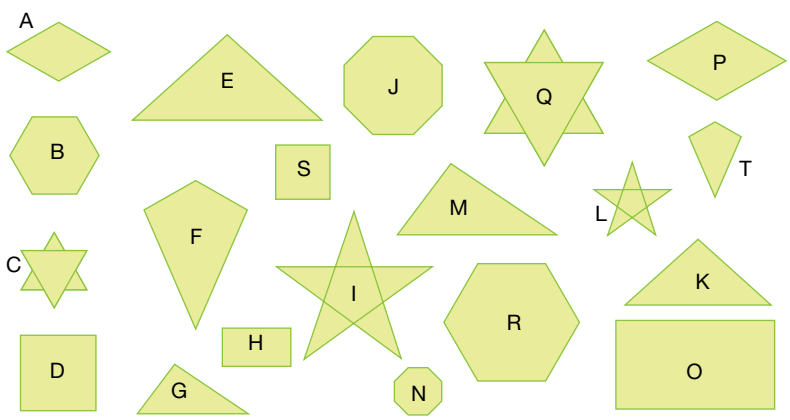


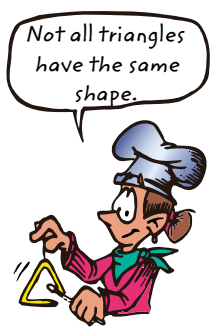
In similar figures:
- matching angles are equal
- the ratio of the lengths of matching sides is constant.

The ratio of matching lengths is the **scale factor**.

Exercise 10:01

1

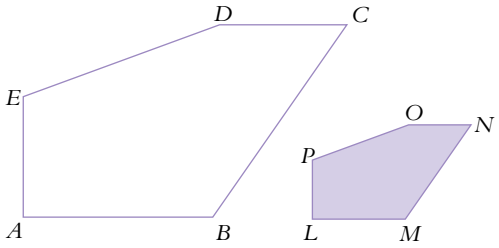


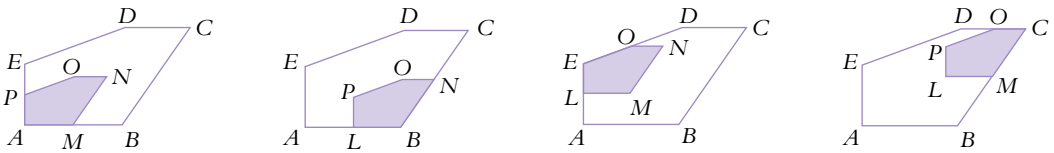


Select the figure that has the same shape as:

a A **b** B **c** C **d** D **e** E
f F **g** G **h** H **i** I **j** J

2 The figures drawn on the right are similar. If the smaller figure were traced and cut out it could be moved around to show matching angles, as shown in the diagrams.





a Which angle in the figure *PLMNO* is in the same position as:

i $\angle A$ **ii** $\angle B$ **iii** $\angle C$ **iv** $\angle D$ **v** $\angle E$?

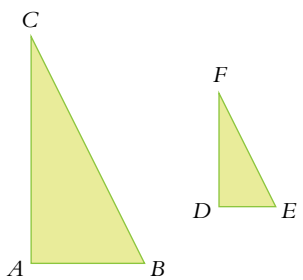
b Which side in the figure *PLMNO* is in the same position as:

i AE **ii** DC **iii** CB **iv** ED **v** AB?

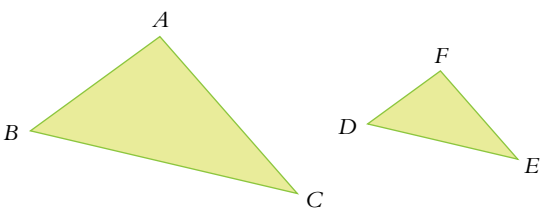
- Angles that are in corresponding positions are called matching angles.
- Sides that are in corresponding positions are called matching sides.

3 For each of the following pairs of similar figures, list the pairs of matching angles and write a similarity statement like $ABCD \parallel\!\parallel LMNP$.

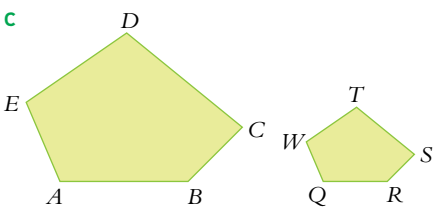
a



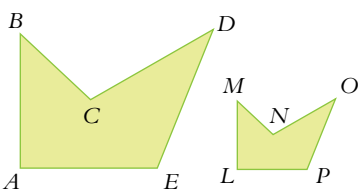
b



c



d



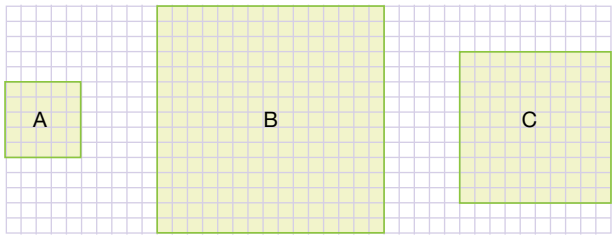
For two figures to be similar, all matching angles must be equal.

List the vertices of similar figures in matching order.

4 For each pair of figures in Question 3, write the pairs of matching sides.

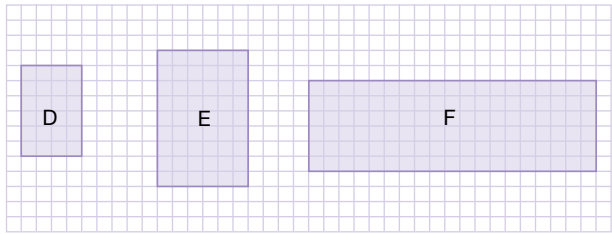
5 For squares A and B:

- **a** are matching angles equal
- **b** are the ratios of pairs of matching sides equal
- **c** are squares A and B similar
- **d** are all the squares similar?



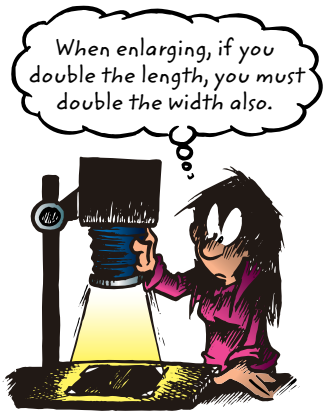
6 **a** Find the ratio of the length of rectangle E to the length of rectangle D.

- **b** Find the ratio of the width of rectangle E to the width of rectangle D.
- **c** Are the rectangles D and E similar?
- **d** Are all of the rectangles similar?



7 **a** A photograph is 8 cm long and 4 cm wide. If the photograph is to be enlarged so that the length is 16 cm, what will the width be?

- **b** A photograph is enlarged. If the length is tripled, what happens to its width?
- **c** When a photograph is enlarged, is it possible to increase the length without changing the width?
- **d** Could rectangle A be enlarged to give rectangle B?

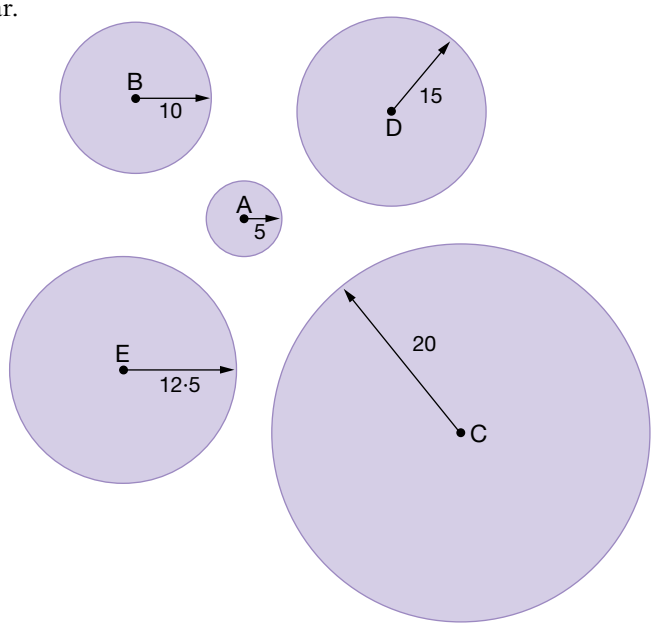


8 Write *true* or *false* for the following, and give a reason for each answer.

- **a** Any two circles are similar.
- **b** Any two equilateral triangles are similar.
- **c** Any two isosceles triangles are similar.

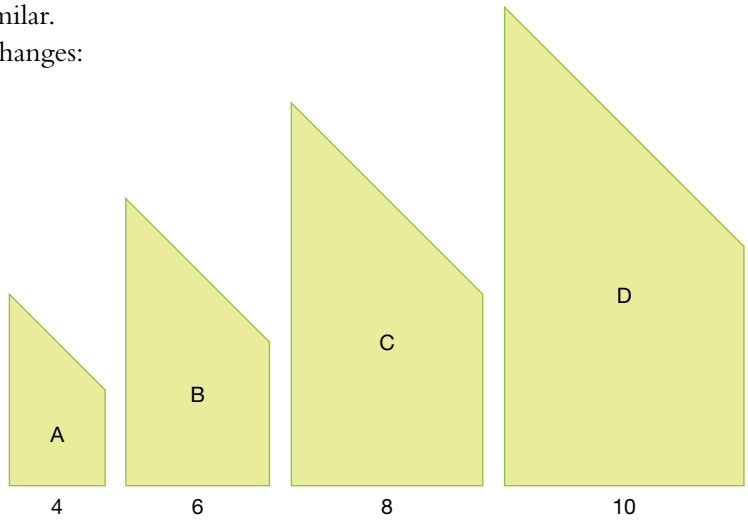
9 **a** Are all of these circles similar?

- **b** If all the other circles are copies of circle A, what is the enlargement factor for:
  - **i** B **ii** C
  - **iii** D **iv** E?
- **c** If all the other circles are copies of C, what is the reduction factor for:
  - **i** D **ii** A
  - **iii** B **iv** E?

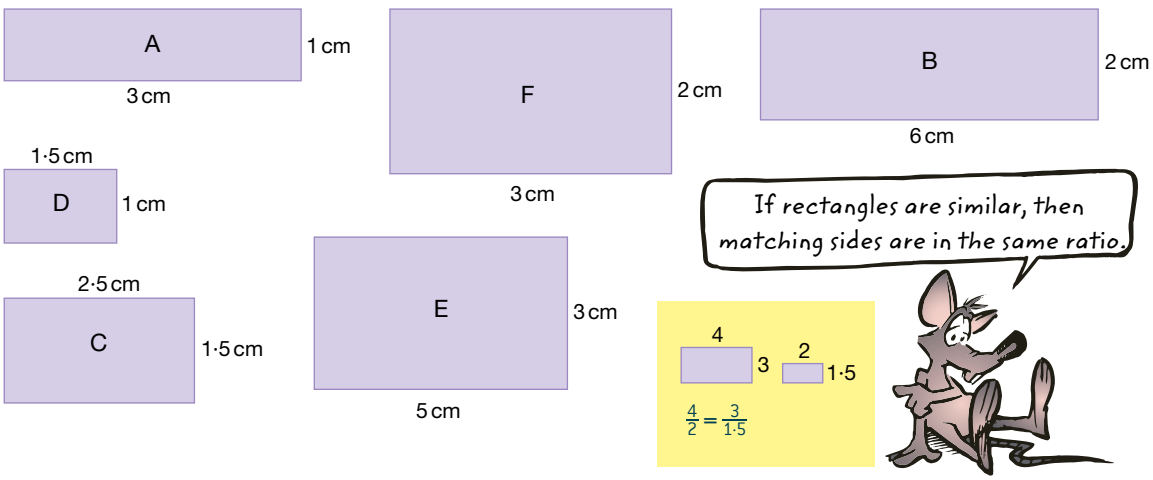


10 The four figures shown are similar.
What is the scale factor that changes:

a A to C
b C to A
c A to B
d B to C
e A to D
f D to B?



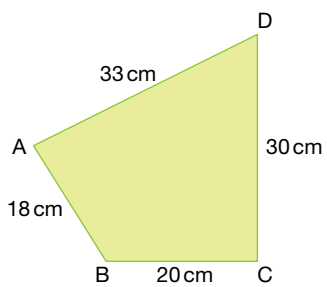
11 Each of the rectangles shown is similar to one of the other rectangles. Find the pairs of rectangles that are similar.



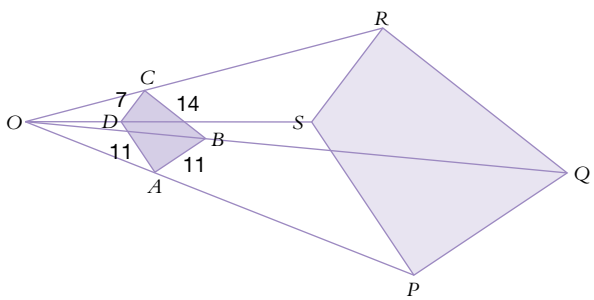
12 Calculate the dimensions of the quadrilateral shown if it is:

a enlarged using a scale factor of 2
b enlarged using a scale factor of 3·2
c reduced using a scale factor of 0·8
d reduced using a scale factor of 0·45.

In each case, list the sides from smallest to largest.



13 The diagram shows the construction for the enlargement of quadrilateral *ABCD* using an enlargement factor of 3.
What are the lengths of the sides of the quadrilateral *PQRS*? (*Note: OR* is three times the length of *OC*.)



14 On a photocopier, a document can be enlarged or reduced using the zoom function. On most photocopiers the scale factor has to be entered as a percentage.

a What percentage would be used for a scale factor of:

i 2 **ii** $\frac{1}{2}$?

b What percentage would need to be entered for a scale factor of:

i $2\frac{1}{4}$ **ii** $\frac{3}{4}$?

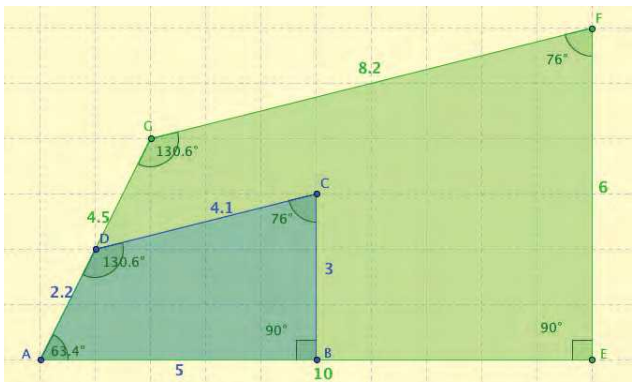
c A rectangle that is 10 cm long and 6 cm wide is to be copied on a photocopier. What would the dimensions of the copied rectangle be if it is copied using a scale factor of:

i 160% **ii** 80%?

P GEOGEBRA ACTIVITY 10:01 SIMILAR FIGURES

This activity allows you to see that when a figure is enlarged or reduced:

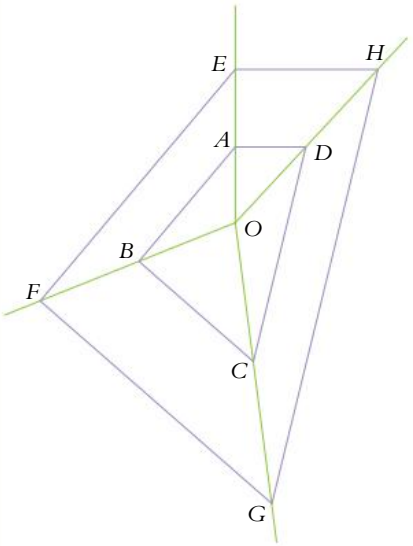
- the matching angles do not change
- the ratios of matching sides are equal.



The larger photo is twice as long and twice as wide as the original photo. Could you say it is twice as big? Explain your answer.

INVESTIGATION 10:01 DRAWING ENLARGEMENTS AND REDUCTIONS

The following questions illustrate a simple way of constructing similar figures.



$ABCD$ is similar to $EFGH$.

a Which angle corresponds to:

i $\angle A$ **ii** $\angle B$ **iii** $\angle C$ **iv** $\angle D$?

b Which side corresponds to:

i AB **ii** BC **iii** CD **iv** AD?

c Measure the side lengths and then calculate each ratio.

i $\frac{EF}{AB}$ **ii** $\frac{FG}{BC}$ **iii** $\frac{GH}{CD}$ **iv** $\frac{HE}{DA}$

d **i** How far is E from O?

ii How far is A from O?

iii Is $EO = 2 \times AO$?

e **i** Is $FO = 2 \times BO$?

ii Is $GO = 2 \times CO$?

iii Is $HO = 2 \times DO$?

Each point has doubled its distance from *O*.

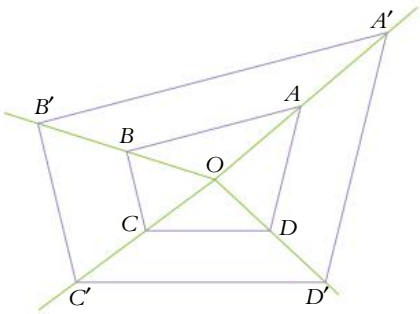
- To draw a figure similar to $ABCD$ by enlargement, follow these steps.

Step 1 Select a point O to be the centre of enlargement.

Step 2 Draw rays from O through each of the vertices A, B, C and D.

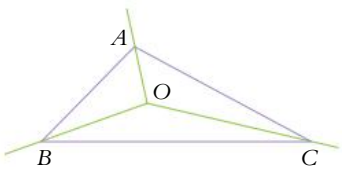
Step 3 Decide on an enlargement factor, say 2.

Step 4 Move each vertex to a new position on its ray, which is twice its present distance from O. For example, if A is 18 mm from O, then the new position is 36 mm from O.

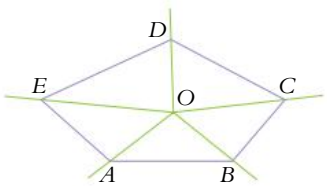


- Draw a figure like those below and then make a similar figure using the method above. Try using different scale factors.

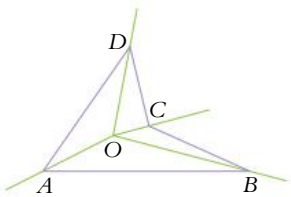
a



b



c



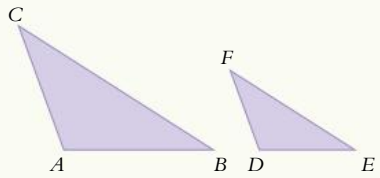
- Measure the angles of each figure and check that matching angles are equal. Measure the sides of each figure and find the ratio of each pair of matching sides. These should be equal.
- If your construction is accurate you should also find that each pair of matching sides is parallel.

10:02 Finding unknown sides of similar figures

PREP QUIZ 10:02

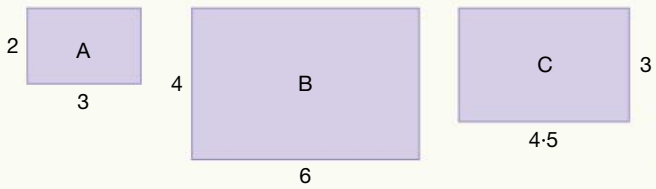
The two triangles are similar.
What side matches:

1 AB **2** AC **3** BC?



What is the scale factor that changes:

4 A to B **5** A to C
6 B to C **7** C to B
8 C to A **9** B to A?



10 A triangle with sides of 3 cm, 4 cm and 5 cm is enlarged using a scale factor of 3. In the enlarged triangle, what will be the length of the:
a longest side **b** shortest side?

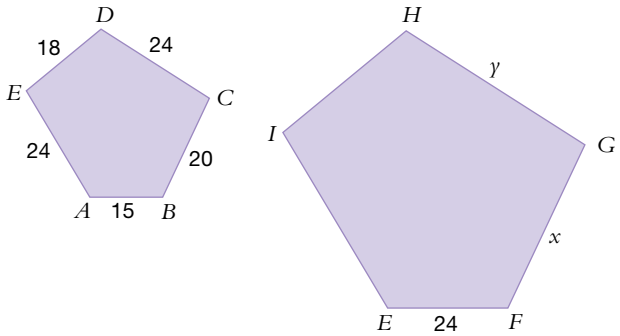
Discussion

A teacher drew a rectangle with dimensions 10 cm by 4 cm and then projected it onto a screen using an overhead projector. The image of the rectangle had a side length of 50 cm.

- Is the image on the screen similar to the rectangle drawn by the teacher?
- What is the enlargement factor?
- What is the width of the image on the screen?

WORKED EXAMPLES

1 The two figures shown are similar. Calculate the enlargement factor and use it to find the value of the pronumerals.



Solution

Enlargement factor = ratio of matching sides

$$= \frac{EF}{AB} = \frac{24}{15} = 1{\cdot}6$$

$x = 1{\cdot}6 \times 20$
$= 32$

$y = 1{\cdot}6 \times 24$
$= 38{\cdot}4$

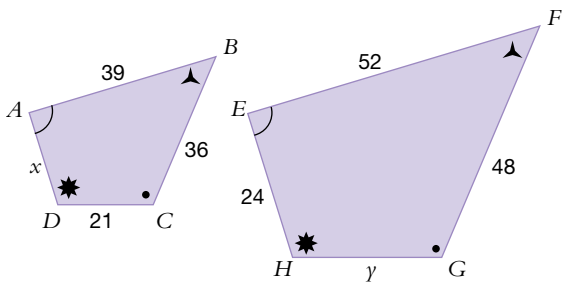
Follow these steps:

Step 1 Use a pair of known matching sides to calculate the enlargement or reduction factor.

Step 2 Find the unknown side and its matching side.

Step 3 Multiply the known side by the scale factor.

2 *ABCD* is similar to *EFGH*. Find the values of x and y.



Solution

The scale factor can be found by dividing any pair of matching sides.

$\therefore$ Reduction factor $\quad \frac{AB}{EF} = \frac{39}{52} = \frac{3}{4}$

$\therefore$ Enlargement factor $\quad \frac{EF}{AB} = \frac{52}{39} = \frac{4}{3}$

To find x, use a reduction factor of $\frac{3}{4}$.
$x = \frac{3}{4} \times 24$
$= 18$

To find y, use and enlargement factor of $\frac{4}{3}$.
$y = \frac{4}{3} \times 21$
$= 28$

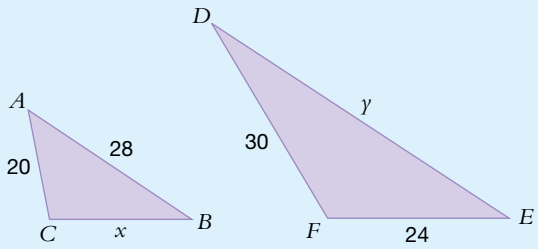
If the scale factor of similar figures is 2 : 3, then:

- the *reduction factor* is $\frac{2}{3}$
- the *enlargement factor* is $\frac{3}{2}$

e.g. In these triangles the scale factor is 20 : 30 or 2 : 3.

$\therefore\ x = \frac{2}{3} \times 24$

$y = \frac{3}{2} \times 28$



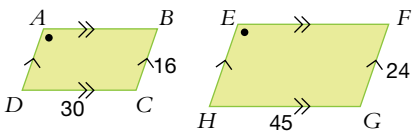
ΔABC is similar to ΔDEF is written as:

$\Delta ABC \,|||\, \Delta DEF$

Exercise 10:02

1 In each case, give the reduction factor and the enlargement factor of these similar figures.

a

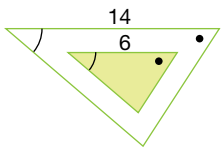


Scale factor = 30 : 45 = 2 : 3

$\therefore$ Reduction factor = $\frac{\square}{\square}$

Enlargement factor = $\frac{\square}{\square}$

b

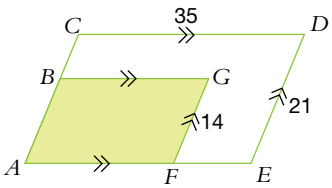


Scale factor = 14 : 6 = 7 : 3

$\therefore$ Reduction factor = $\frac{\square}{\square}$

Enlargement factor = $\frac{\square}{\square}$

c

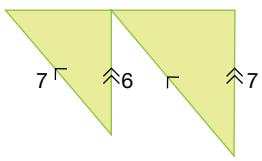


Scale factor = 21 : 14 = 3 : 2

$\therefore$ Reduction factor = $\frac{\square}{\square}$

Enlargement factor = $\frac{\square}{\square}$

d



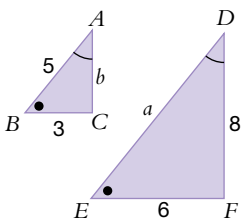
Scale factor = 6 : 7

$\therefore$ Reduction factor = $\frac{\square}{\square}$

Enlargement factor = $\frac{\square}{\square}$

2 In each part, the figures are similar. Find the value of each pronumeral.

a



Scale factor = 3 : 6 = 1 : 2

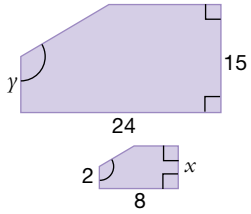
$\therefore$ Enlargement factor = 2

$\therefore$ Reduction factor = $\frac{1}{2}$

$\therefore\ a = \ldots \times 5$ $\quad\therefore\ b = \ldots \times 8$

$\therefore\ a = \ldots$ $\quad\therefore\ b = \ldots$

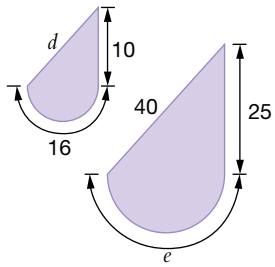
b



Scale factor = 24 : 8 = 3 : 1
$\therefore$ Enlargement factor = 3
$\therefore$ Reduction factor = $\frac{1}{3}$

$\therefore x = \ldots \times 15$ $\quad$ $\therefore y = \ldots \times 2$
$\therefore x = \ldots$ $\quad$ $\therefore y = \ldots$

c



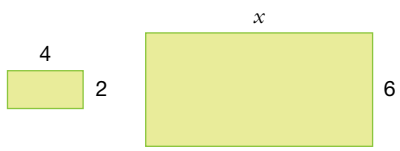
Scale factor = 10 : 25 = 2 : 5
$\therefore$ Enlargement factor = $\frac{5}{2}$
$\therefore$ Reduction factor = $\frac{2}{5}$

$\therefore d = \ldots \times 40$ $\quad$ $\therefore e = \ldots \times 16$
$\therefore d = \ldots$ $\quad$ $\therefore e = \ldots$

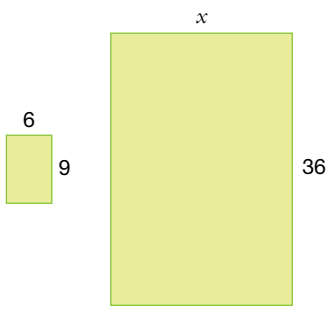
||| means 'is similar to'
e.g. $\Delta ABC \,|||\, \Delta DEF$

3 Each of the following pairs of figures are similar. Find the enlargement factor and use it to find the value of each pronumeral.

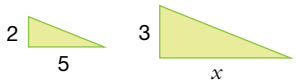
a



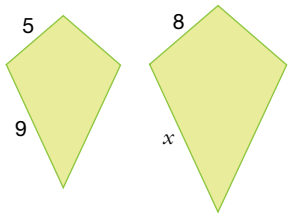
b



c

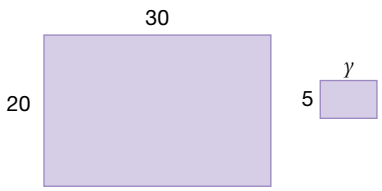


d

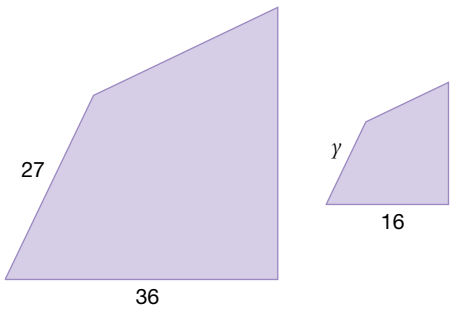


4 Each of the following pairs of figures are similar. Find the reduction factor and use it to find the value of each pronumeral.

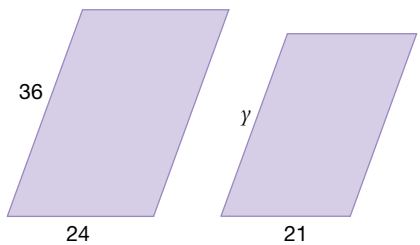
a



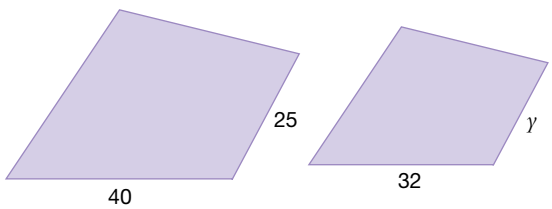
b



c



d

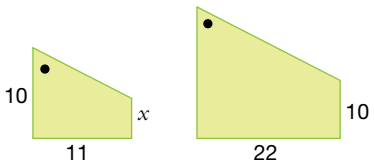


5 A triangle has sides of length 9 cm, 12 cm and 15 cm.

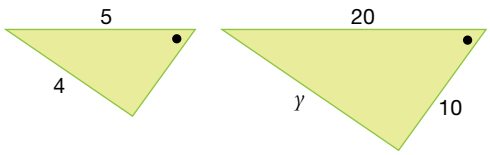
- **a** If the triangle is enlarged using a scale factor of 4, what will be the length of the shortest side?
- **b** If the triangle is reduced using a scale factor of $\frac{2}{3}$, what will be the length of the longest side?
- **c** If the triangle is enlarged so that the longest side becomes 21 cm long, what would the scale factor be? How long would the shortest side of the enlarged triangle be?
- **d** A rectangular photograph is 15 cm long and 10 cm wide. If this is enlarged on a photocopier so that the length becomes 18 cm, what would the width become?

6 The figures in each part are similar. Find the scale factor and the value of the pronumeral for each part.

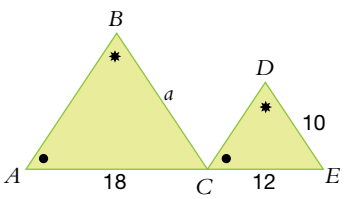
a



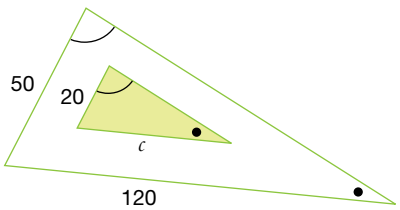
b



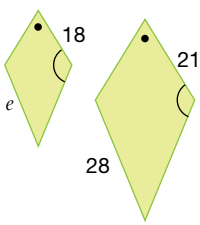
c



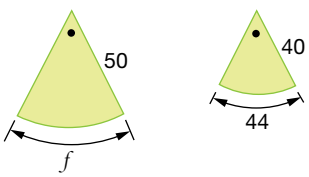
d



e

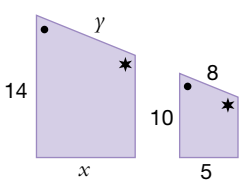


f

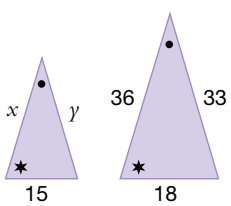


7 For these similar figures, write each scale factor and find the value of the pronumerals, correct to one decimal place.

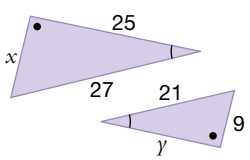
a



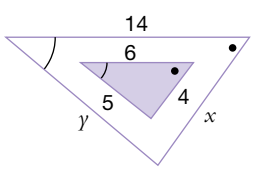
b



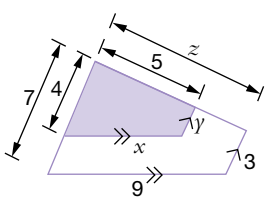
c



d



e



From the scale factor you can write the reduction or enlargement factor.

INVESTIGATION 10:02 PHOTO ENLARGEMENTS

A photographic store offers to print any combination of three sizes of the same photo on a single sheet of paper that is 30·5 cm by 21 cm. The different sizes are used in the combination below. The sizes are all similar and have lengths of 25 cm, 18 cm and 12 cm. The width of the largest photo is 20 cm.

1 Calculate the widths of each of the two smaller photo sizes.

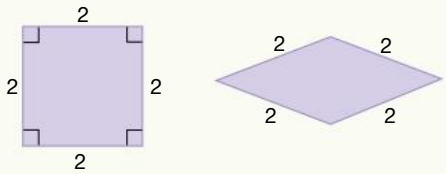
2 Investigate what combinations of the three photo sizes are possible on the 30·5 cm by 21 cm sheet. Describe how you would go about it.

The mathematics of similar figures is used in building models.

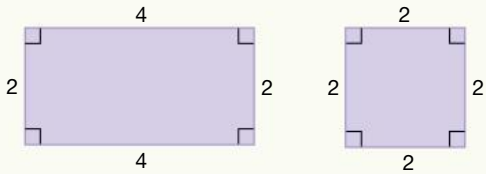
10:03 Similar triangles

PREP QUIZ 10:03

1 Are these two figures similar?



2 Are these two figures similar?



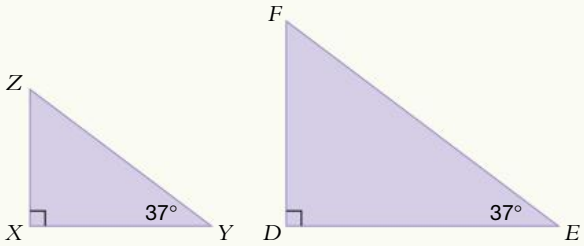
True or *false*?

3 If two figures have matching angles equal, then they are similar.

4 If the ratio of matching sides in two figures are equal, then the figures are similar.

5 Two angles in a triangle measure 80° and 60°. What is the size of the third angle?

6 Does $\angle Z$ equal $\angle F$?

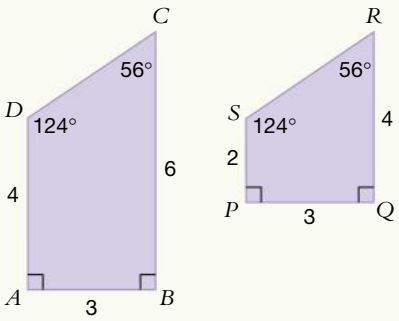


7 What is the value of the ratio $\frac{AD}{PS}$?

8 What is the value of the ratio $\frac{BC}{QR}$?

9 Are the ratios of matching sides equal?

10 Is $ABCD$ similar to $PQRS$?



The prep quiz should have reminded you that for two *figures* to be similar you need to show that:

- matching angles are equal **and**
- matching sides are in the same ratio.

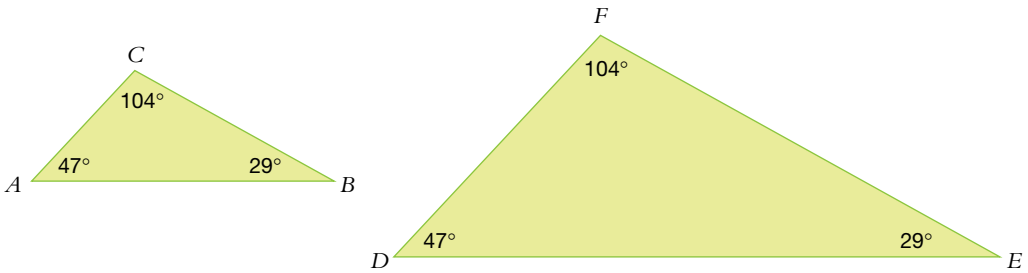
The following exercises investigate the conditions that are necessary for two triangles to be similar.

10:03A Matching angles

Similar triangles have matching angles that are equal.

Exercise 10:03A

1 The two triangles have the same-sized angles.



a Measure the side lengths to the nearest millimetre and complete the table below.

Side	AB	AC	BC	DE	DF	EF
Length						

b Use your answers above to calculate each of the following side ratios, correct to one decimal place.

i $\frac{DE}{AB}$ ii $\frac{DF}{AC}$ iii $\frac{EF}{BC}$

c *True* or *false*? $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

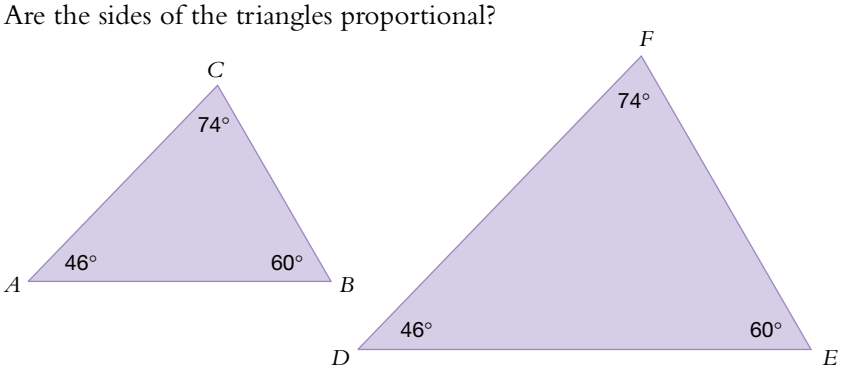
> If the ratios of matching sides are equal, we say that the three sides of one triangle are proportional to the three sides of the other triangle. More simply, the sides of the triangles are proportional.

2 **a** Measure the lengths of the sides of ΔABC and ΔDEF to the nearest millimetre.

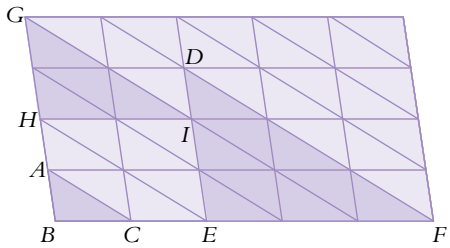
b Calculate the value of the following ratios, correct to one decimal place:

i $\frac{DE}{AB}$ ii $\frac{EF}{BC}$ iii $\frac{DF}{AC}$

c Are the sides of the triangles proportional?



3 The three triangles shown are drawn on a grid of parallel lines. All of the small triangles are congruent.



a Consider ΔABC and ΔDEF.
- i Are the matching angles equal?
- ii What is the value of each of the matching side ratios $\frac{DE}{AB}, \frac{EF}{BC}, \frac{DF}{AC}$?
- iii Are the sides of the two triangles proportional?

b Consider ΔGHI and ΔDEF.
- i Name the pairs of matching angles. Are the matching angles equal?
- ii What is the value of each of the following ratios: $\frac{DE}{GH}, \frac{EF}{HI}, \frac{DF}{IG}$?
- iii Are the triangles similar?

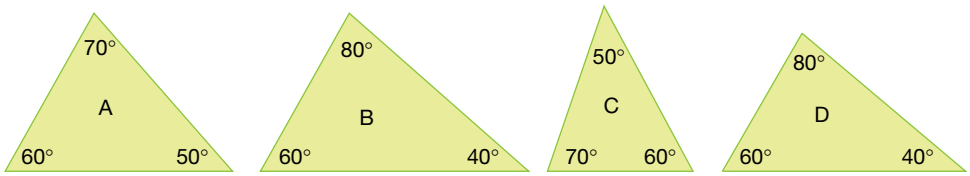
c Consider ΔABC and ΔGHI.
- i Are the matching angles equal?
- ii Are the sides of the two triangles proportional (i.e. are the ratios of matching sides equal)?
- iii Are the triangles similar?

Questions 1 to 3 illustrate the following result.

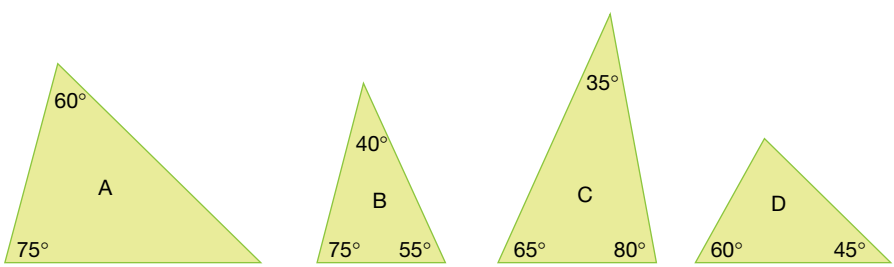
Test 1
If two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar.

4 Identify the triangles that are similar in each of the following.

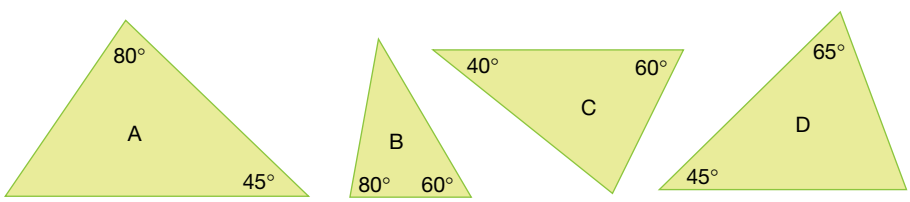
a



b

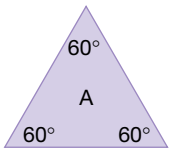


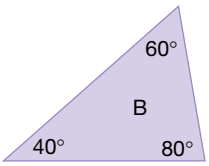
c

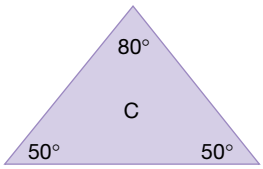


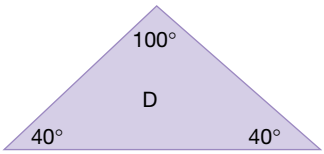
5 Below are 10 pairs of similar triangles. Each of the triangles A to J is similar to one of the triangles L to W. Select the triangle that is similar to triangle:

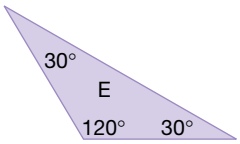
a A **b** B **c** C **d** D **e** E
f F **g** G **h** H **i** I **j** J

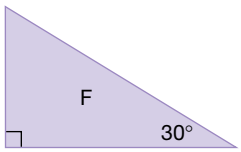


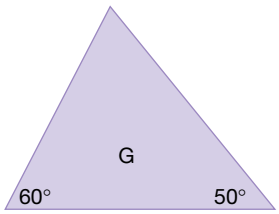


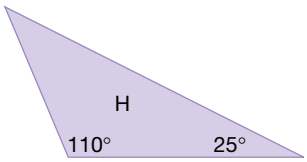


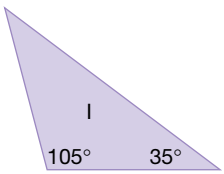


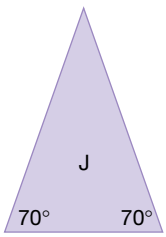


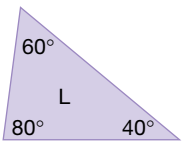


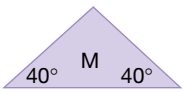


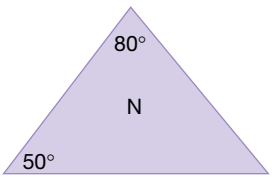


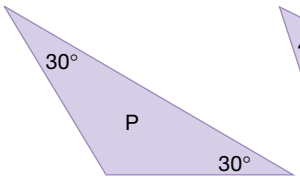


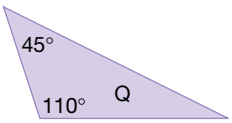


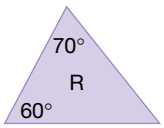


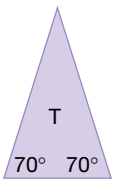


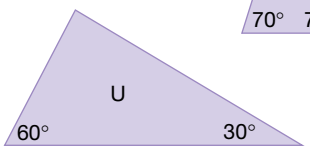


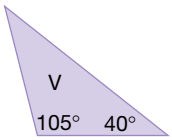


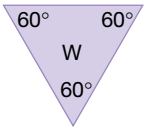


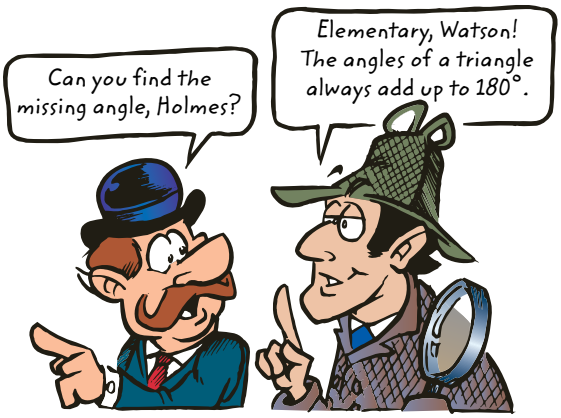








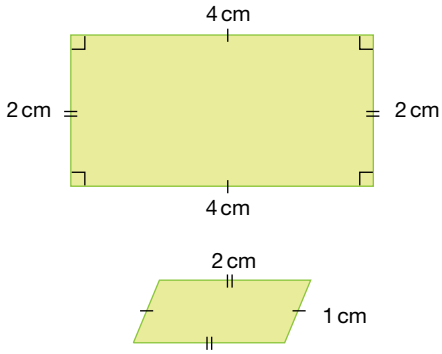




10:03B Ratios of matching sides

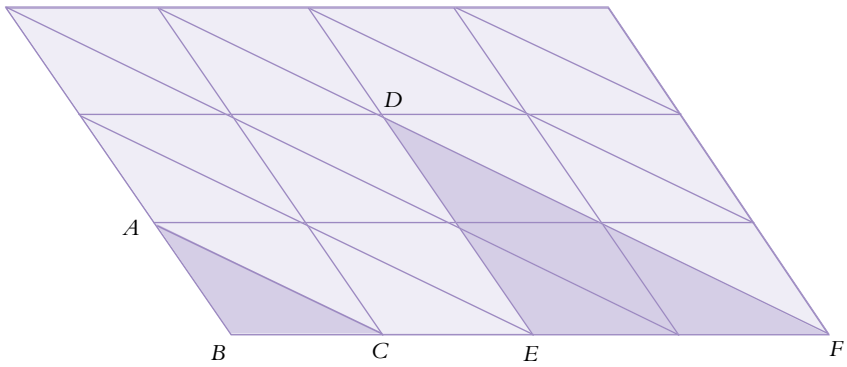
The rectangle and parallelogram shown here are a reminder that if the ratios of matching sides are equal then the figures are not necessarily similar. The matching angles must also be equal.

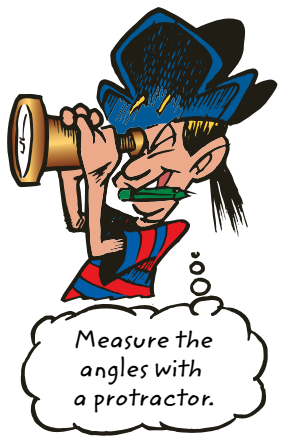
In this exercise we investigate pairs of triangles that have matching sides in the same ratio.



Exercise 10:03B

1 Triangle ABC has been used to form the grid that has three sets of parallel lines.





a Are all the sides of ΔDEF twice as long as the sides of ΔABC?

b What is the value of the following ratios?

i $\frac{DF}{AC}$ ii $\frac{DE}{AB}$ iii $\frac{EF}{BC}$

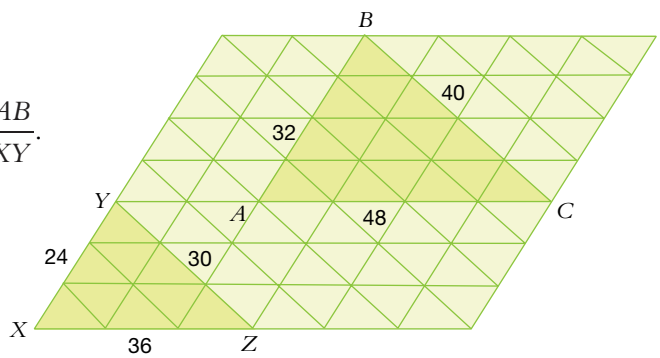
c Are the sides of the triangles proportional?

d Are the matching angles equal?

e Are the triangles similar?

2 **a** Evaluate the ratios $\frac{BC}{YZ}$, $\frac{AC}{XZ}$ and $\frac{AB}{XY}$.

Are the three sides of ΔABC proportional to the three sides of ΔXYZ?



b Are the matching angles equal?

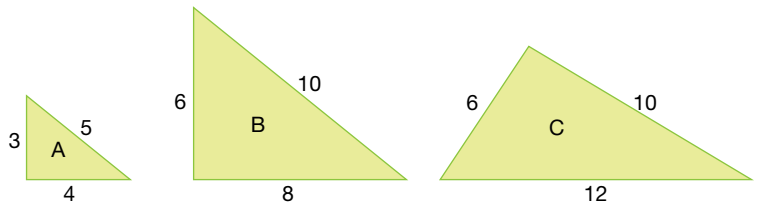
c Are the two triangles similar?

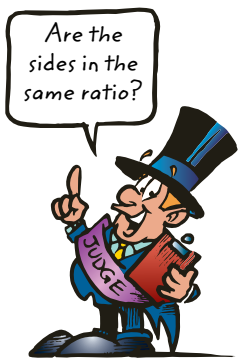
Questions 1 and 2 illustrate the following result.

Test 2
If the three sides of one triangle are proportional to the three sides of another triangle, then the triangles are similar. This means that matching angles are also equal.

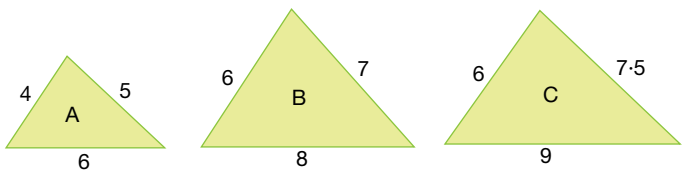
3 Identify the triangles that are similar in each of the following.

a

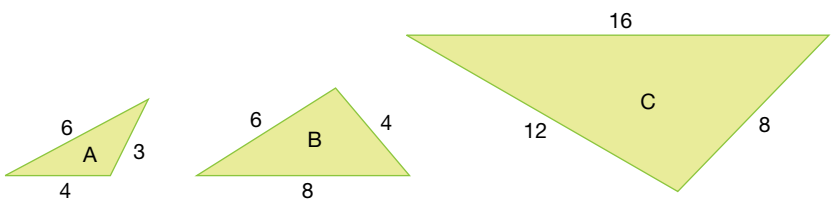




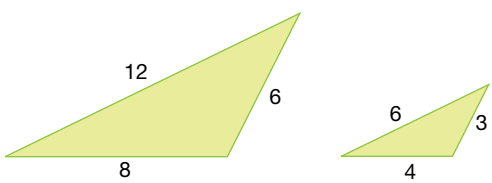
b

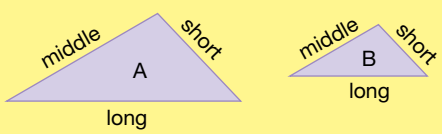


c



d





Triangle A is similar to triangle B if:

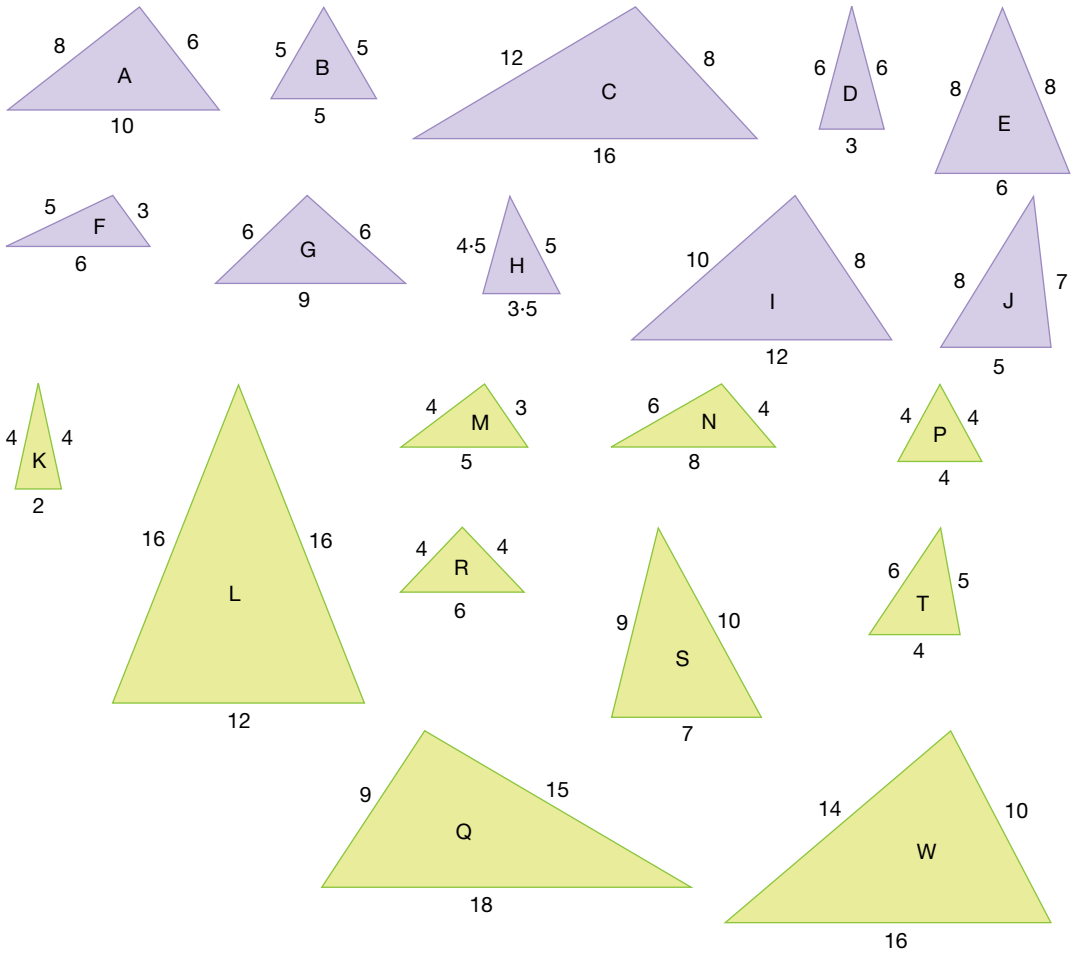
$$\frac{\text{long side of A}}{\text{long side of B}} = \frac{\text{middle side of A}}{\text{middle side of B}} = \frac{\text{short side of A}}{\text{short side of B}}$$

If these ratios are equal, then the sides of the triangles are said to be in proportion (or proportional).

4 Below are ten pairs of similar triangles. Each of the triangles A to J is similar to one of the triangles *K* to *W*. By checking the ratios of matching sides, find the triangle that is similar to:

a A **b** B **c** C **d** D **e** E

f F **g** G **h** H **i** I **j** J

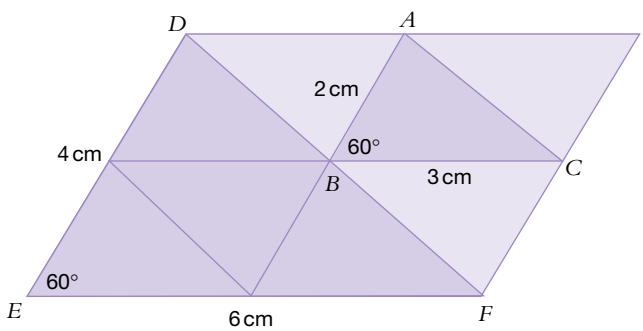


5 In ΔABC and ΔDEF:

$\frac{DE}{AB} = \frac{EF}{BC} = 2$ and $\angle B = \angle E$.

a Does $\frac{DF}{AC} = 2$?

b Are the two triangles similar?

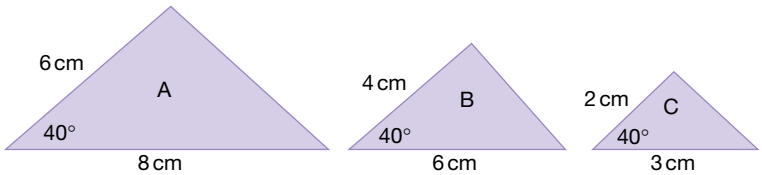


Question 5 illustrates the third condition for two triangles to be similar.

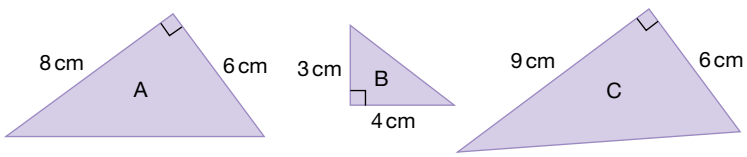
Test 3
If two sides of one triangle are proportional to two sides of another triangle, and the included angles are equal, then the two triangles are similar.

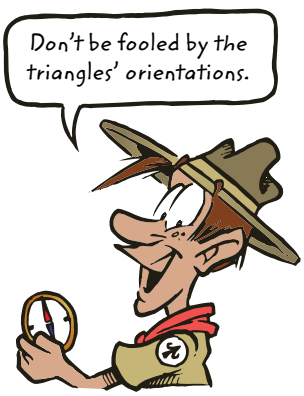
6 Identify the pair of similar triangles in each of the following.

a

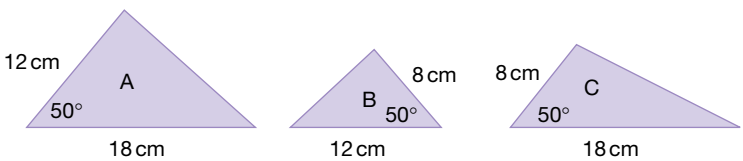


b

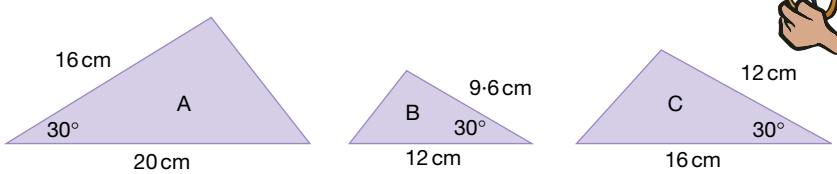




c



d



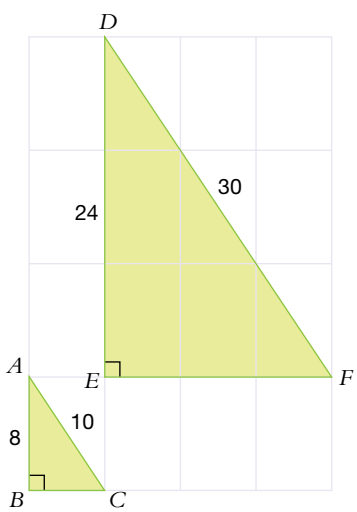
7 In ΔABC and ΔDEF:

$\frac{DF}{AC} = \frac{DE}{AB} = 3$

$\angle ABC$ and $\angle DEF$ are both right angles.

a Does $\frac{EF}{BC} = 3$?

b Are the two triangles similar?

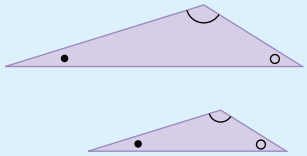


Question 7 illustrates the fourth condition for two triangles to be similar.

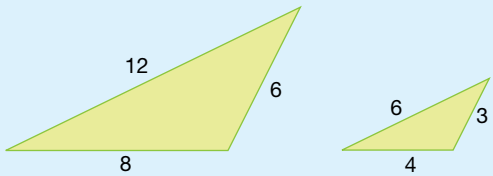
Test 4
If the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the two triangles are similar.

Conditions for similar triangles

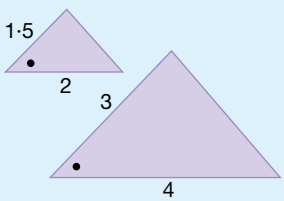
1 If two angles in one triangle are equal to two angles in another triangle (all three matching angles are equal), then the triangles are similar.



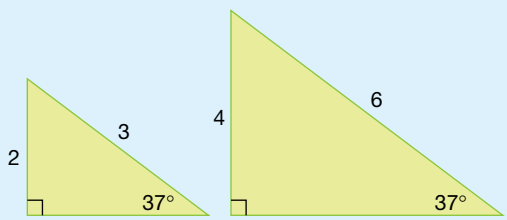
2 If three sides of one triangle are proportional to the three sides of another triangle, then the triangles are similar.



3 If two sides of one triangle are proportional to two sides of another triangle and the included angles are equal, then the triangles are similar.

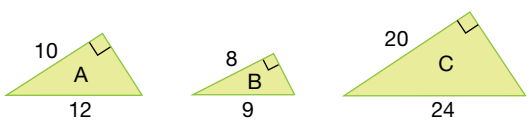


4 If the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the triangles are similar.

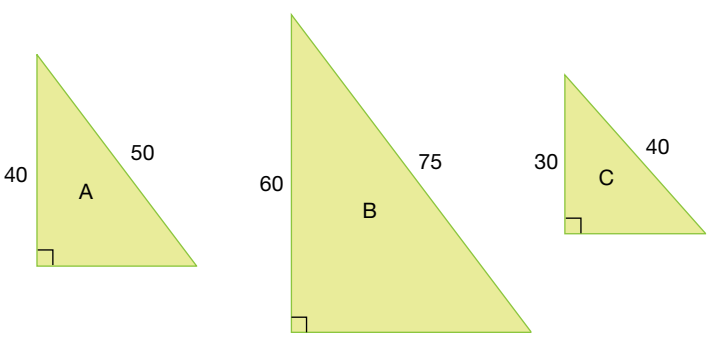


8 Identify the pair of similar triangles in each of the following.

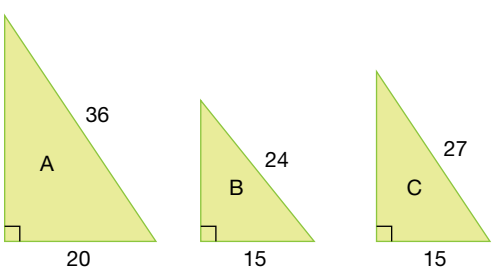
a



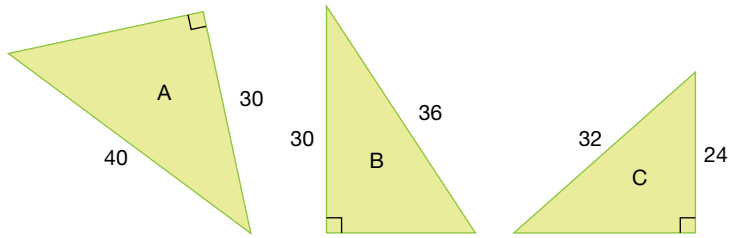
b



c

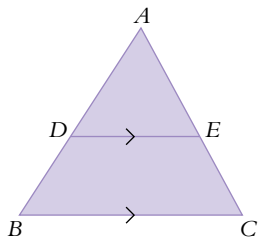


d

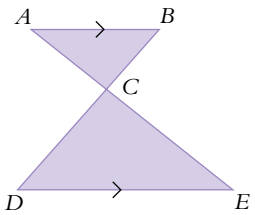


9 Find two similar triangles in each of the following. State which condition could be used to show that the triangles are similar. (Triangles may overlap.)

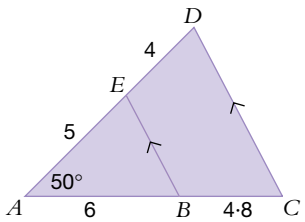
a



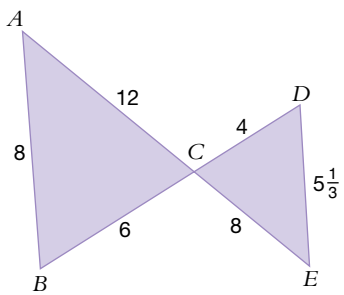
b



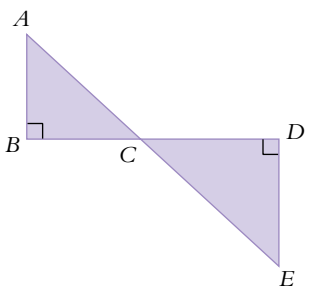
c

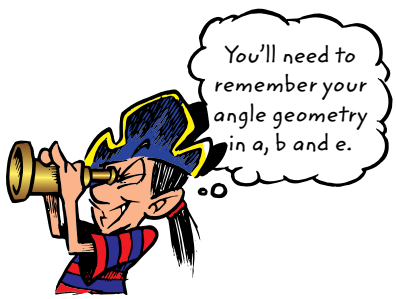


d



e





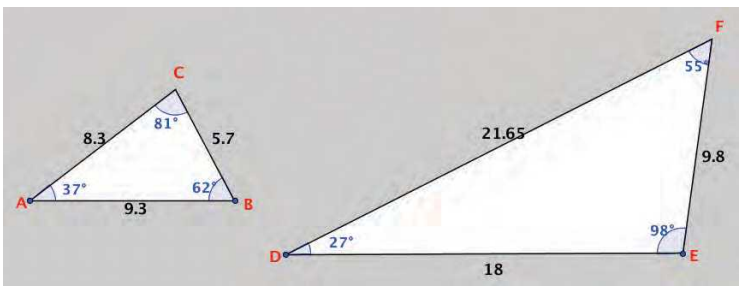
P GEOGEBRA ACTIVITY 10:03 SIMILAR TRIANGLES

1 Equal angles

Use sliders to change the sizes of the angles in a pair of triangles. You can see that when matching angles are equal, the ratios of matching sides are also equal. Hence, the two triangles are similar.

2 Matching sides in the same ratio

Use sliders to change the lengths of the sides. As the side lengths change, the ratios of the matching sides also change. You can see that when the ratios of matching sides are equal, the matching angles are also equal. Hence, the triangles are similar.



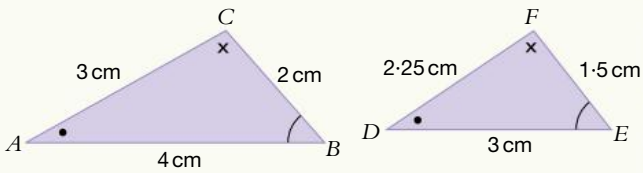
10:04 Using proportion to find unknown sides

PREP QUIZ 10:04

Complete the following.

1 In any triangle, the longest side is opposite the … angle.
2 In any triangle, the smallest side is opposite the … angle.
3 If two triangles are similar, the two longest sides are matching. *True* or *false*?
4 If two triangles are similar, the two shortest sides are matching. *True* or *false*?
5 Matching sides are opposite matching angles. *True* or *false*?

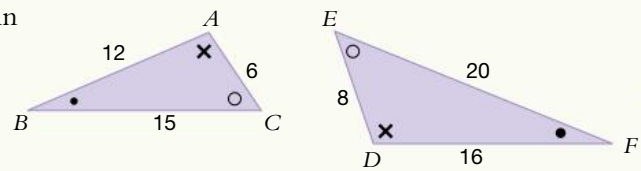
The triangles on the right are similar.



6 Which is the matching side to AB?
7 Complete the proportion statement:

$$\frac{1{\cdot}5}{2} = \frac{2{\cdot}25}{?} = \frac{3}{?}$$

In the triangles on the right, which side in ΔEDF is in a matching position to:



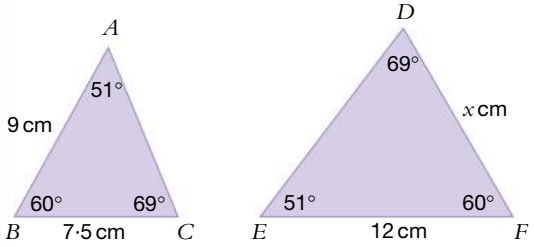
8 BC 9 AC?
10 Complete the proportion statement:

$$\frac{8}{?} = \frac{16}{?} = \frac{20}{?}$$

The unknown sides in similar triangles can be found using the scale factor method (as shown in Section 10:02), or by using a proportion statement (i.e. a statement that matching sides are in the same ratio).

WORKED EXAMPLE 1

Find the value of the pronumeral.



The symbol ||| means 'is similar to'.

Solution

$\Delta ABC ||| \Delta EFD$ (matching angles are equal)
The matching sides are:
x and 7·5 (both opposite 51° angle)
12 and 9 (both opposite the 69° angle)

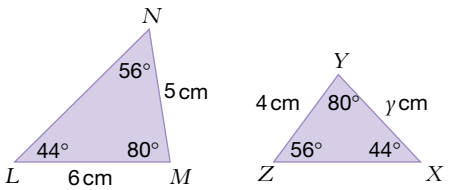
$$\frac{x}{7{\cdot}5} = \frac{12}{9}$$ (sides are in proportion)

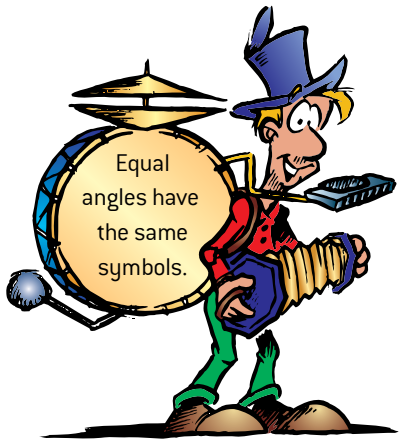
$$x = 7{\cdot}5 \times \frac{12}{9}$$

$$x = 10 \text{ cm}$$

WORKED EXAMPLE 2

a Find the value of the pronumeral.





Solution

$\Delta LMN \| \| \| XYZ$ (matching angles are equal)

The matching sides are:

y and 6 (both opposite 56° angle)

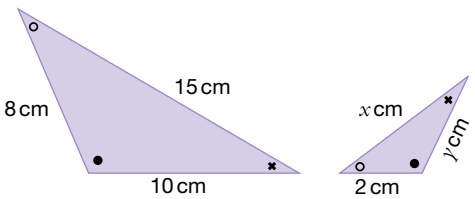
4 and 5 (both opposite the 44° angle)

$$\frac{y}{6} = \frac{4}{5}$$ (sides are in proportion)

$$y = 6 \times \frac{4}{5}$$

$$y = 4{\cdot}8 \text{ cm}$$

b Find the value of the pronumeral.



In any proportion statement such as this:

$$\frac{x}{15} = \frac{y}{10} = \frac{2}{8}$$

- the sides on the top all come from the same triangle
- the matching sides underneath these all come from the other triangle.

When finding an unknown side, check your answer by asking yourself: 'Should the unknown side be bigger or smaller than its matching side?'

Solution

The triangles are similar as the matching angles are equal. As matching sides are opposite matching angles, the matching pairs are:

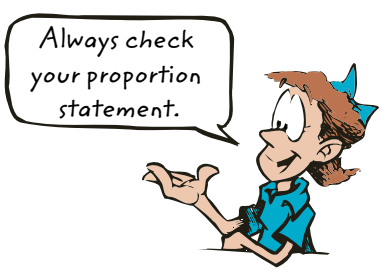
x and 15, y and 10 and 2 and 8

$$\therefore \frac{x}{15} = \frac{y}{10} = \frac{2}{8}$$ (sides are in proportion)

$$\frac{x}{15} = \frac{2}{8} \quad \text{and} \quad \frac{y}{10} = \frac{2}{8}$$

$$x = 15 \times \frac{2}{8} \qquad y = 10 \times \frac{2}{8}$$

$$= 3{\cdot}75 \qquad = 2{\cdot}5$$

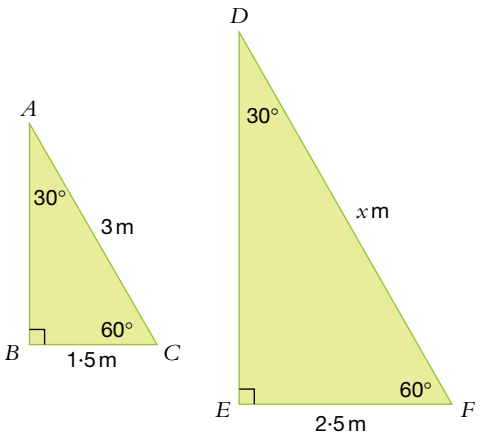


Exercise 10:04

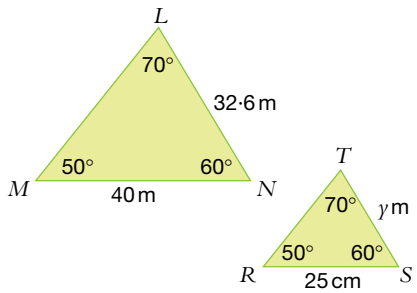
P Foundation worksheet 10:04
Finding unknown sides in similar triangles

1 For each of the following, copy and complete the working.

a



b



$\Delta ABC \| \| \| \Delta DEF$

AB matches with …

BC matches with …

AC matches with …

$\therefore \frac{DF}{\ldots} = \frac{EF}{\ldots}$

$\therefore \frac{x}{\ldots} = \frac{2{\cdot}5}{\ldots}$

$\therefore x = \ldots \times \frac{2{\cdot}5}{\ldots}$

$\therefore x = \ldots$

$\Delta LMN \| \| \| \Delta TRS$

LM matches with …

LN matches with …

MN matches with …

$\therefore \frac{TS}{\ldots} = \frac{RS}{\ldots}$

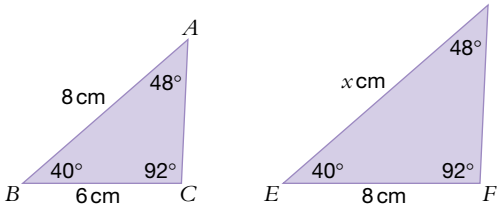
$\therefore \frac{\gamma}{\ldots} = \frac{25}{\ldots}$

$\therefore \gamma = \ldots \times \frac{25}{\ldots}$

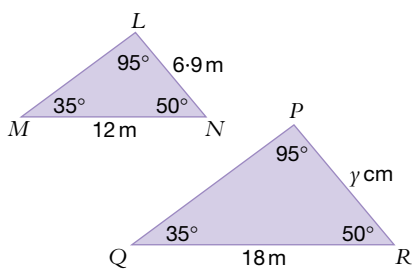
$\therefore \gamma = \ldots$

2 Find the values of the pronumerals, correct to one decimal place where necessary.

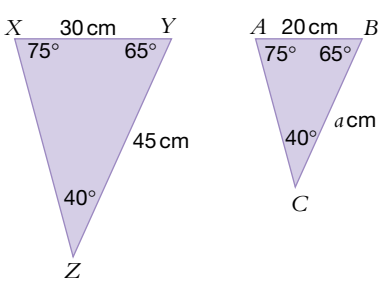
a



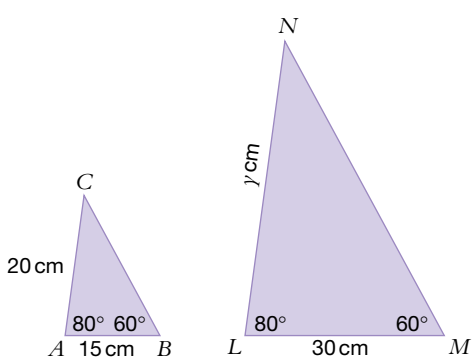
b



c

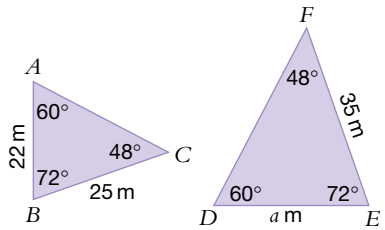


d

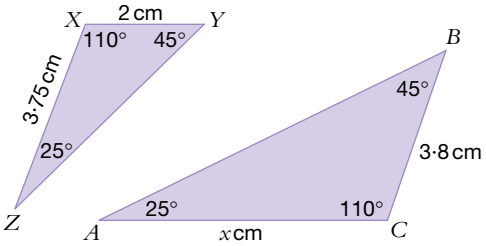


3 Find the values of the pronumerals, correct to one decimal place where necessary.

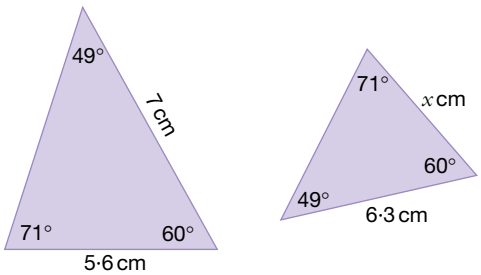
a



b



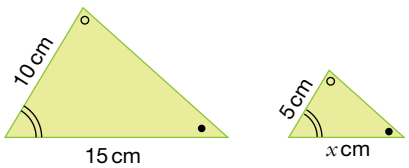
c



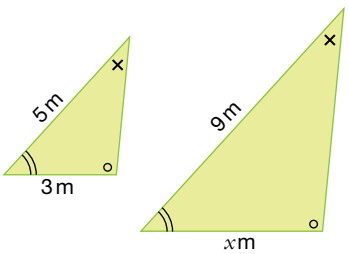
Matching sides are opposite equal angles.

4 Find the value of the pronumeral in each of the following.
(Equal angles are marked with the same symbols.)

a



b



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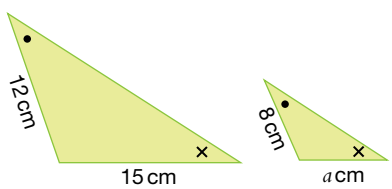
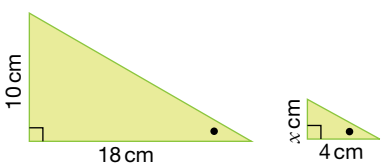
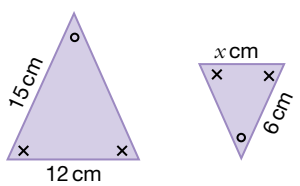


d

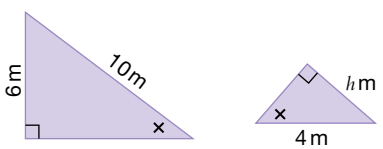


5 Find the value of the pronumeral in each of the following.

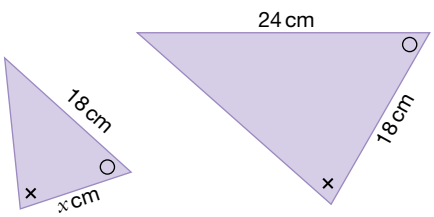
a



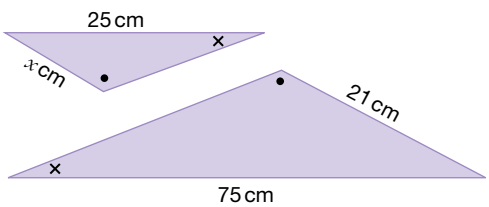
b



c

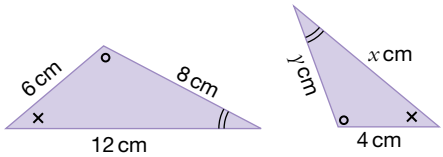


d

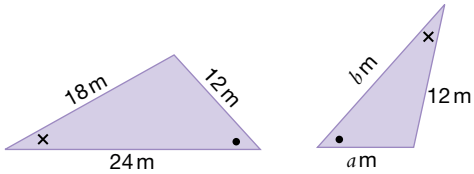


6 Find the values of the pronumerals in each of the following.

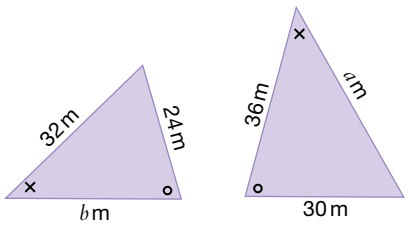
a



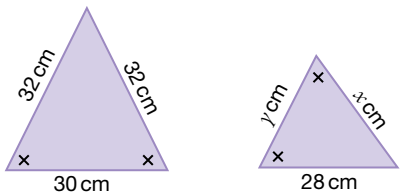
b



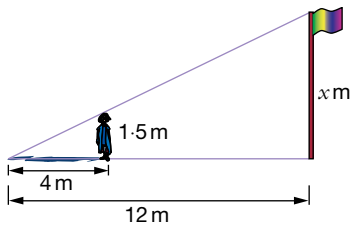
c



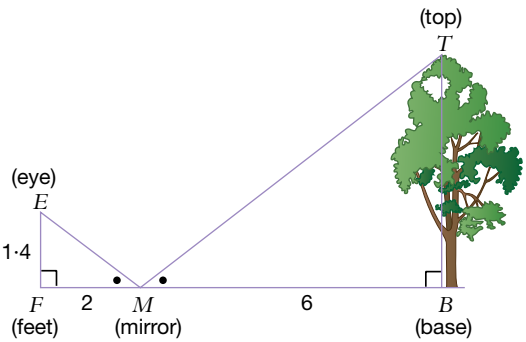
d



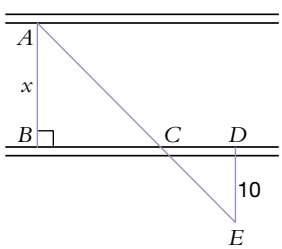
7 **a** A girl found that her shadow was 4 m long when the shadow of a flagpole was 12 m long. If her height is 1·5 m, what is the height of the flagpole?



b To find the height of a tree, Greg used a mirror placed on the ground. He moved it until he could see the top of the tree in the mirror. At this point, his eye was 1·4 m from the ground and the mirror was 2 m from him, whereas the tree was 6 m from the mirror. Find the height of the tree.

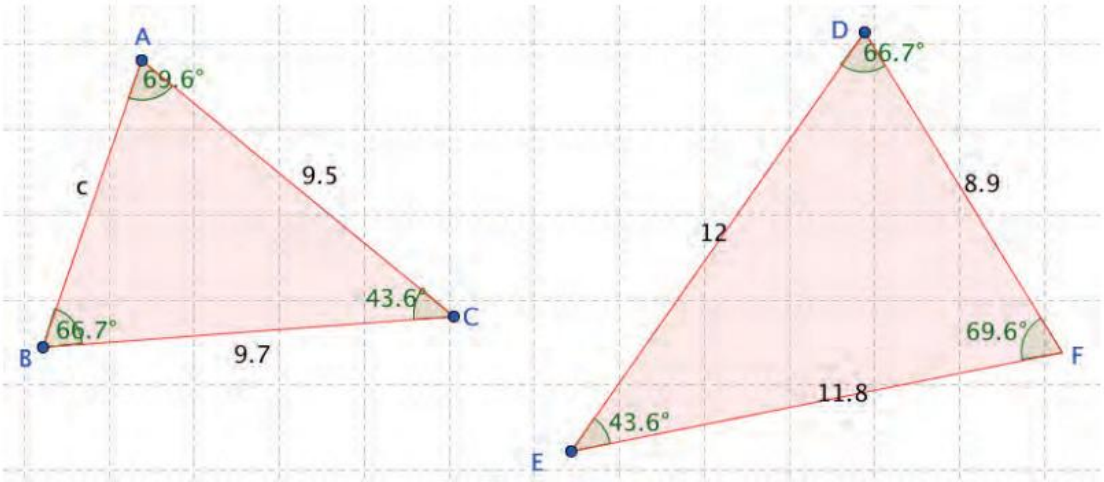


c The distance across a storm channel was discovered by choosing a point A on the opposite bank and finding the point B that was directly opposite A. Point E was chosen so that it was 10 m from the bank D, and by sighting from E to A, the point C was found. The distances DC and CB were measured to be 9 m and 18 m, respectively. How wide was the channel?



P GEOGEBRA ACTIVITY 10:04 FINDING AN UNKNOWN SIDE IN SIMILAR TRIANGLES

Click and drag the vertices of two triangles to change the shape of the triangles as well as their size and orientation. Then complete the matching sides ratio and calculate the length of the missing side. Tick a checkbox to check your answer.



10:05 Scale drawings

PREP QUIZ 10:05

Complete the following:

1 1 m = … cm **2** 1 m = … mm

Convert:

3 20 m to cm **4** 25 000 mm to m

Simplify the ratios:

5 1 cm : 1 m **6** 1 mm : 1 m

A line is 6 cm long. How long would it be if it was:

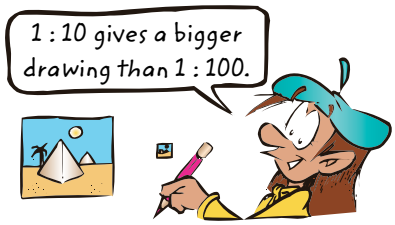
7 reduced using a reduction factor of $\frac{1}{10}$

8 enlarged using an enlargement factor of 10?

Complete the following:

9 1 : 100 = 5 : … **10** 1 : 100 = … : 400

A scale drawing is the same shape as the object it represents but it has a different size.



- A scale of 1 : 100 means that one unit on the drawing represents 100 units on the original object.
- A scale of 1 : 100 gives a drawing that is smaller than the original object.
- The lengths on the drawing are $\frac{1}{100}$ the lengths on the original.
- A scale of 100 : 1 gives a drawing that is larger than the original.
- The lengths on the drawing are 100 times longer than the original lengths.
- The scale determines the size of the drawing. So a 1 : 10 scale drawing would be bigger than a 1 : 100 scale drawing of the same object.

> A scale drawing of an object is the same shape as the object but a different size.
> Scale = length on drawing : length on the real object
> A scale of 1 : 100 means the drawing is smaller than the real object.
> A scale of 100 : 1 means the drawing is larger than the real object.

There are two types of problems that involve scale drawings:
- calculating the real size of an object from a scale drawing
- making a scale drawing.

10:05A Calculating real sizes from scale drawings

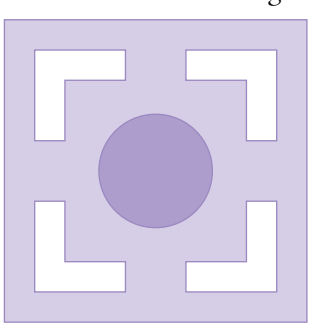
WORKED EXAMPLES

A landscape gardener made a scale drawing of a formal garden as shown below.

1 What real distance is represented by a distance of 1 cm on the drawing?

2 **a** What is the length of the square that forms the outer boundary of the garden?

b What is the diameter of the circular pond in the middle of the garden?



Scale 1 : 400

Solutions

1 As the scale is 1 : 400, the real length is 400 times bigger than the length on the drawing.

$\therefore$ Real length = 1 cm × 400
= 400 cm
= 4 m

2 **a** The length of the side of the square is 4 cm on the drawing.

$\therefore$ Real length = 4 × 4 m
= 16 m

b Diameter of pond is 1·5 cm on drawing.

$\therefore$ Real diameter = 1·5 × 4 m
= 6 m

Exercise 10:05A

1 Copy and complete each of the following:

a 1 cm : 1 m = 1 : ☐ **b** 1 m : 1 km = 1 : ☐ **c** 1 mm : 1 m = 1 : ☐

2 The scale on a map is 1 : 1000. Calculate the real distance between two points if they are the following distance apart on the map.

a 3 cm **b** 4·6 cm **c** 23 mm

3 The scale on a drawing is 1 cm to 5 m. Calculate the real distance between two points if they are the following distance apart on the drawing.

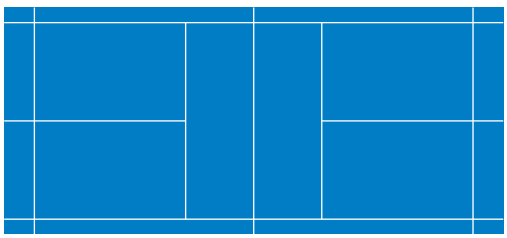
a 7 cm **b** 1 mm **c** 2·4 cm

4 A scale drawing of a badminton court is shown.

a Measure the length and width of the court on the drawing in millimetres.

b Use the scale to calculate the length and width of the court in millimetres.

c Convert the length and width into metres.



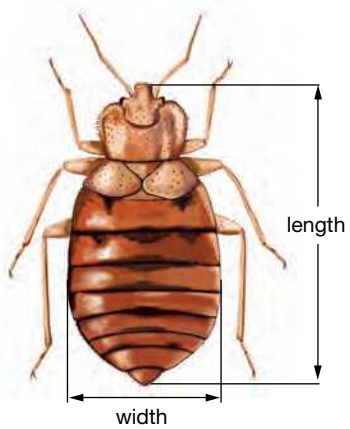
Scale 1 : 200

5 A scale drawing of an insect is shown.

a Measure the length and width of the body of the insect.

b Convert the measurements in part **a** into real measurements if the scale of the drawing is:

i 10 : 1 **ii** 20 : 1



6 The diagram shows a set of scale drawings of a car. The real length of the car is 3700 mm.

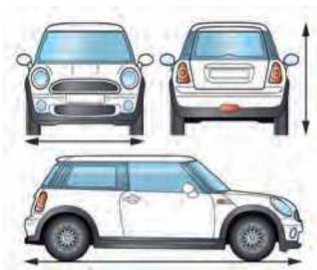
a What is the length of the car on the drawing?

b Use the real length and your answer to **a** to find the scale of the drawing.

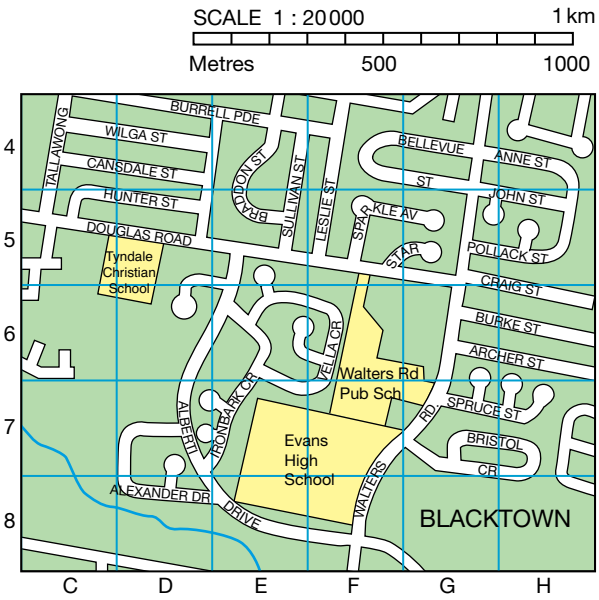
c By measurement and calculation find:

i the real width of the car, excluding mirrors

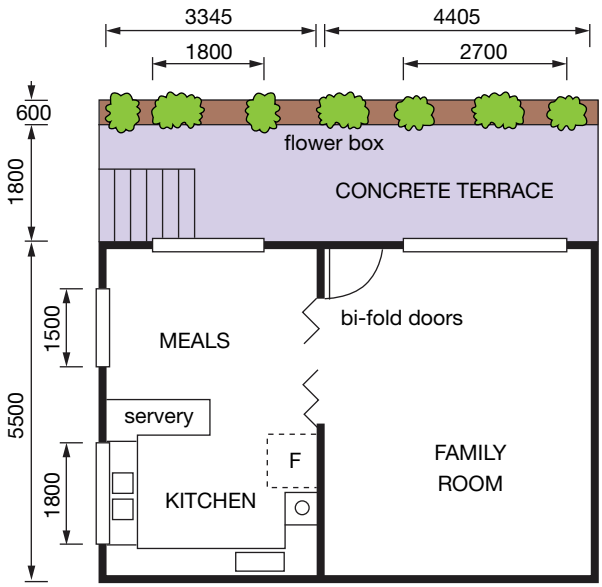
ii the real height of the car.



7 a What real distance is represented by 1 cm on this map?
b What is the length of Wilga St?
c What is the length of Douglas Road from Tallawong Avenue to Walters Road?
d Calculate the cost of repairing this section of Douglas Road at $15 000 per 100 metres.
e Estimate the area of land occupied by Evans High School.



8 The measurements on this plan are in millimetres.
a What length on the plan stands for 5500 mm?
b What distance would be represented by 1 mm on the plan?
c What is the scale of the plan?
d Calculate the distance from one corner of the family room to the opposite corner.
e What is the distance between the window in the kitchen and the adjacent window in the meals area?



10:05B Making a scale drawing

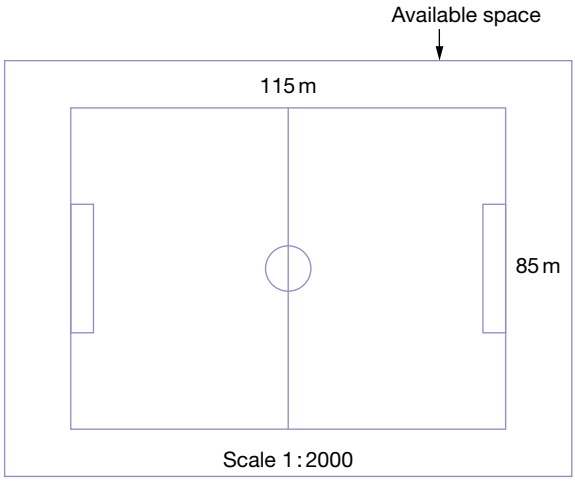
To make a scale drawing we need to:
1 find the real lengths involved
2 decide how large the drawing will be
3 choose an appropriate scale
4 transfer the real lengths into scale lengths using the scale.

To decide on a scale, find the ratio of the length available for the drawing to the length of the real object.
Scale = length of drawing : length of object

Discussion

A scale drawing of a particular soccer field is to be drawn. The finished drawing must fit inside a rectangle that is 7·5 cm long and 5·5 cm wide. Carefully examine the steps used to do this.

- By measurement or research, we discover that the dimensions of the soccer field are 115 m by 85 m.
- We choose a length that will fit the available space (a rectangle 7·5 cm by 5·5 cm).
  Let's make the length about 6 cm.
  Scale = length of drawing : length of field
  = 6 cm : 115 m
  = 6 : 11 500
  ÷ 1 : 1917
  Let the scale be 1 : 2000.
- Find the dimensions of the diagram.
  Scaled length = 115 m ÷ 2000
  = 0·0575 m
  = 5·75 cm
  Scaled width = 85 m ÷ 2000
  = 0·0425 m
  = 4·25 cm
- Draw the soccer field.



Exercise 10:05B

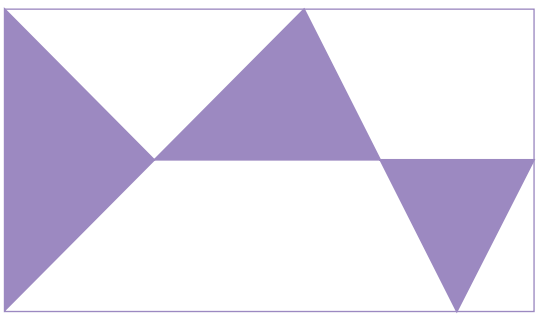
1 A scale drawing is to be made using a scale of 1 : 4. This means that the distance on the drawing is $\frac{1}{4}$ of the real distance. What length on the drawing represents a real distance of:

a 100 cm **b** 80 cm **c** 1·2 m **d** 68 mm **e** 668 mm

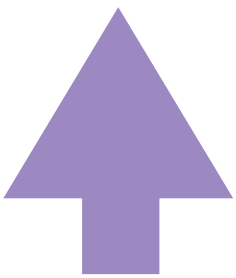
2 The scale of a drawing is 4 : 1. This means that the distance on the drawing is 4 times greater than the real distance. What length on the drawing represents a real distance of:

a 4 cm **b** 8 mm **c** 3·5 mm **d** 0·8 mm **e** 1·3 cm

3 **a** Use a scale of 1 : 2 to make a scale drawing of this picture.



b Use a scale of 3 : 1 to make a scale drawing of this shape.



4 What scale has been used to make the copy?

a Original

Copy

Scale = 1 : □

b Original

Copy

Scale = △ : □

c Original

Copy

Scale = □ : 1

d Original

Copy

Scale = □ : △

5 **a** In Question **4a**, what fraction of the original height is the height of the copy?

b In Question **4b**, what fraction of the original width is the width of the copy?

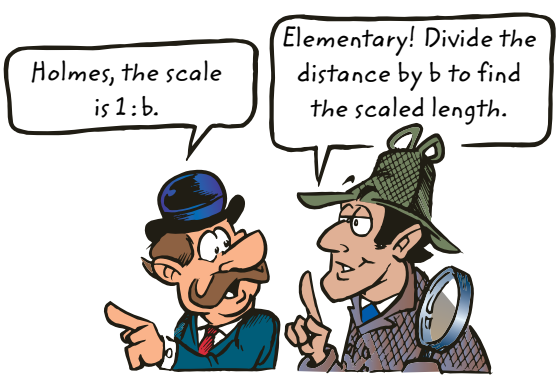
c In Question **4c**, how many times as wide as the original is the copy?

d In Question **4d**, how many times as high as the original is the copy?

6 A scale drawing uses a scale of 1 : 100. What length on the drawing would represent a distance of:

a 100 cm **b** 200 cm **c** 800 cm

d 8 m **e** 3·5 m **f** 2000 mm?



7 What is the scale of a diagram that represents a real length of 20 m by a length of:

a 20 cm **b** 10 cm **c** 2 cm

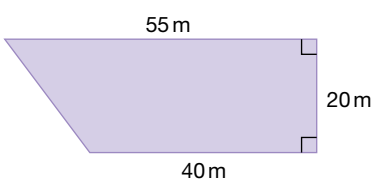
d 20 mm **e** 50 cm **f** 5 mm?

8 A field is 40 m long and 20 m wide. What would these dimensions be on a scale drawing if the scale is:

a 1 : 40 **b** 1 : 200 **c** 1 : 500 **d** 1 : 1000?

9 The sketch on the right shows a block of land that has the shape of a trapezium. Use the sketch to make a scale drawing of the block of land using a scale of:

a 1 : 1000 **b** 1 : 2000 **c** 1 : 5000



10 A scale drawing is to be made of a soccer field that is 110 m long and 80 m wide. What would be an appropriate scale to use for a scale drawing if it had to fit inside a rectangle with dimensions:

a 24 cm by 18 cm **b** 30 cm by 21 cm **c** 70 mm by 45 mm?

11 Make a scale drawing of a tennis court, choosing a scale that will allow the drawing to fit comfortably on your page. (*Note:* The outside dimensions of a tennis court are 23·77 m by 10·97 m.)

12 Make a scale drawing of one of your hands so that the drawing fits into a square of side length 10 cm.

13 **a** A house is 21 500 mm long and 14 800 mm wide. What lengths on a scale drawing would you use to represent these dimensions if the scale is 1 : 100?

b A 1 : 30 scale model of a tower is to be built. If the tower is 90 m high, how high will the model be?

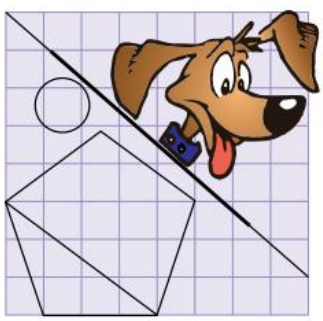
c A company is asked to make a model of a shower block that has dimensions 8000 mm by 4200 mm. What would be the dimensions of the model if the scale is 1 : 20?

d A plan of a house is drawn using a scale of 1 : 100. What would be the scaled dimensions of the kitchen if its real dimensions are 4800 mm by 3600 mm? Calculate the scaled length of the sink top if the real length is 120 cm.

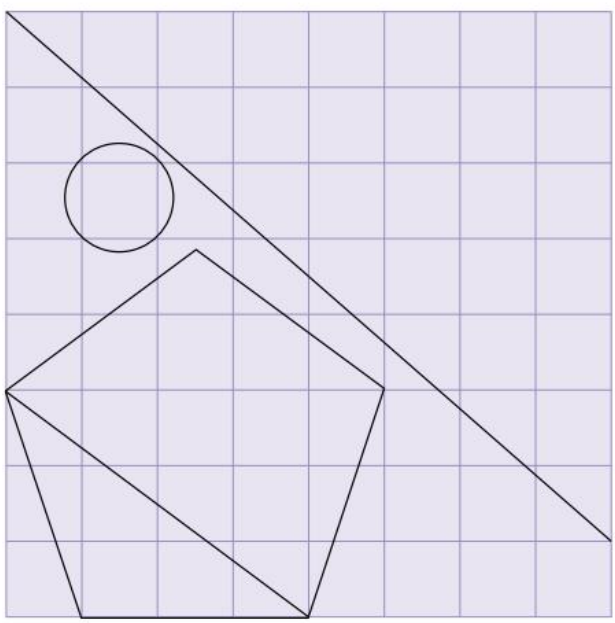
INVESTIGATION 10:05 SCALE DRAWING

Grids with different-sized squares can be used to make scale drawings of pictures and designs.

- Complete the scale drawing begun here.
- Are matching angles equal?
- Each small square in Picture 1 matches a large square in Picture 2.



Picture 1



Picture 2

- Investigate the relationship between matching distances and pictures.
- Compare as many matching lengths as you can.
- What is the ratio of matching lengths?

Length in Picture 2 (L_2)	Length in Picture 1 (L_1)	Ratio $L_2 : L_1$

FUN SPOT 10:05

WHAT HAPPENED TO THE MUSHROOM THAT WAS DOUBLE PARKED?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

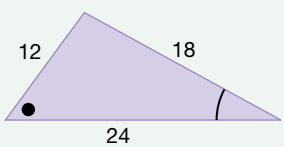
Simplify each ratio:

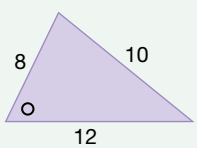
A 20 : 4 **A** 8 : 16 **B** 8 : 6 **C** 1 : 0·5 **D** $\frac{8}{10}$ **E** $\frac{9}{6}$

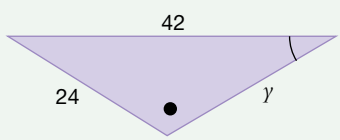
Solve:

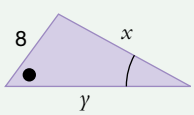
E $\frac{x}{2} = \frac{3}{2}$ **E** $\frac{x}{3} = \frac{1}{3}$ **I** $\frac{x}{3} = 9$ **L** $\frac{x}{2} = \frac{3}{5}$

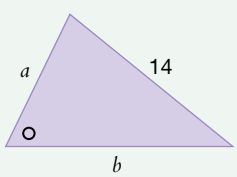
Find the values below for each pair of triangles:

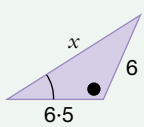












M Reduction factor =
S x =
W y =

O Enlargement factor =
T a =
O b =

O Enlargement factor =
T x =
T y =

$x = 27$	26	4 : 3	$\frac{3}{2}$	2 : 1	5 : 1	$\frac{2}{3}$	$x = 1$	1 : 2	11 : 2	16 : 8	16	$x = 3$	$\frac{4}{5}$	$x = 12$	10 : 5	1 : 4	4	$x = 1{\cdot}2$

MATHS TERMS 10

dimensions

- size measured in particular directions, usually length, breadth and height

enlargement

- a bigger version of the original

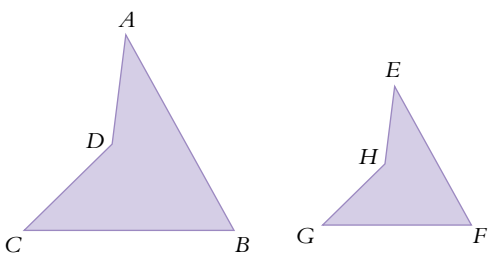
enlargement factor

- the factor by which each length of the original has been enlarged to produce the similar figure

enlargement factor = 3

matching

- in the same (or corresponding) position
- matching sides and angles



matching sides	matching angles
• AB and EF	• $\angle A$ and $\angle E$
• BC and FG	• $\angle B$ and $\angle F$
• CD and GH	• $\angle C$ and $\angle G$
• DA and HE	• $\angle D$ and $\angle H$

ratio

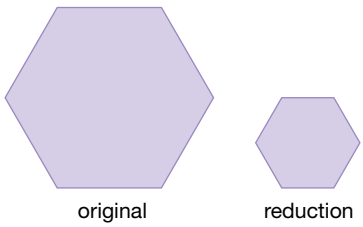
- a comparison of two like quantities or numbers written in a definite order,
  e.g. 1 cm to 1 m = 1 cm : 100 cm
  = 1 : 100

reduction

- a smaller version of the original

reduction factor

- the factor by which each length of the original has been reduced to produce the similar figure



reduction factor = $\frac{1}{2}$

scale (scale factor)

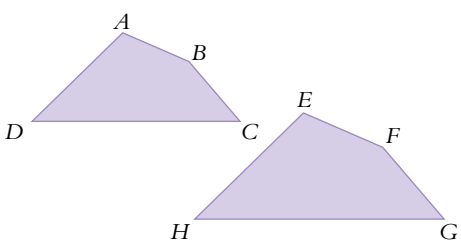
- the ratio of distance on a map or drawing to the real or original distance,
  e.g. a scale of 1 : 10 000 means that lengths on the drawing are $\frac{1}{10\,000}$ of the length of the original

scale drawing

- a drawing of an object in which all dimensions have been enlarged or reduced by the same factor.

similar figures

- figures that have the same shape but a different size
- matching angles are equal
- the enlargement or reduction factor gives us the ratio of matching sides



- $ABCD$ is similar to $EFGH$,
  i.e. $ABCD \,|||\, EFGH$
  $\angle A = \angle E, \angle B = \angle F, \angle C = \angle G,$
  $\angle D = \angle H$
- the enlargement factor = $\frac{EH}{AD}$

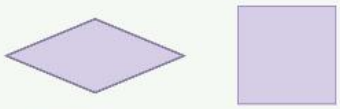
superimpose

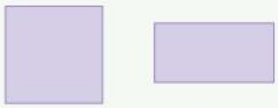
- to place one figure upon another in such a way that the parts of one coincide with the parts of the other

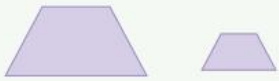
Each section of the test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

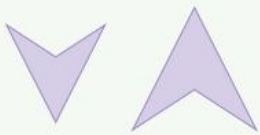
10:01

1 State whether each pair of figures below are similar.

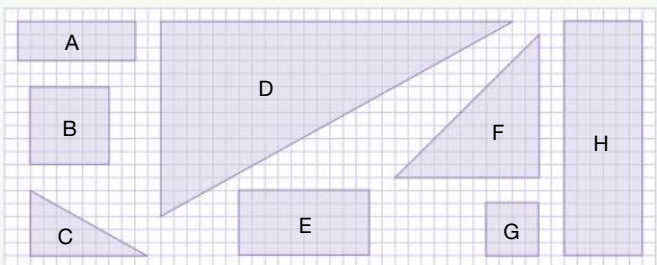
a 

b 

c 

d 

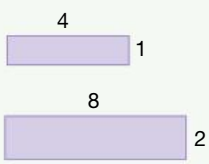
10:01

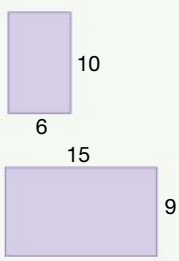
2 

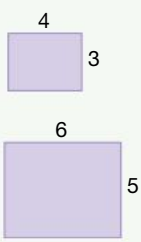
a Name three pairs of similar figures from those shown above.
b For each pair of similar figures, assume that the larger is an enlargement of the smaller. In each case, write the enlargement factor.

10:01

3 Are the rectangles in the following pairs similar?

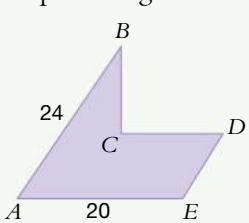
a 

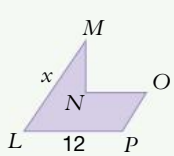
b 

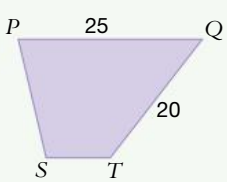
c 

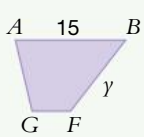
10:02

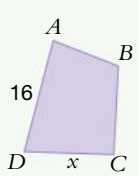
4 Each pair of figures is similar. Find the value of each pronumeral.

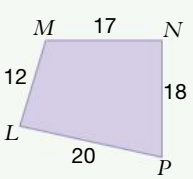
a 



b 



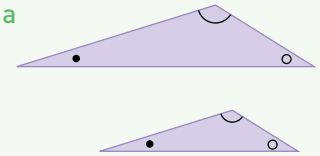
c 



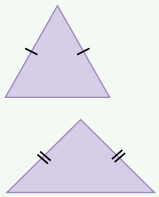
10:03

5 Are the following pairs of triangles similar?

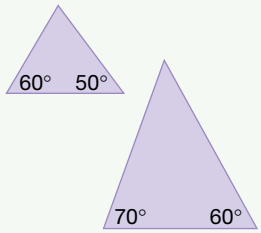
a



b



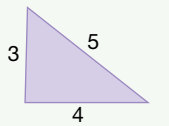
c

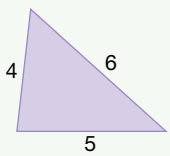


10:03

6 Are the following pairs of triangles similar?

a





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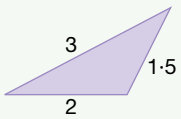
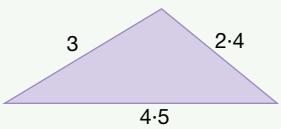
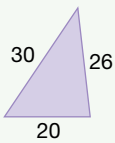
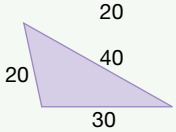




c

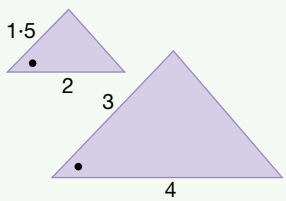




10:03

7 Are the following pairs of triangles similar?

a



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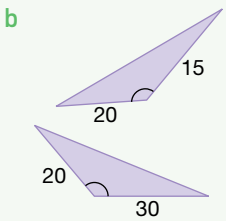
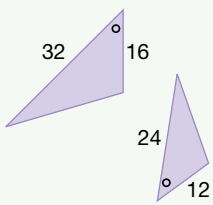


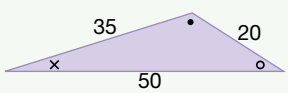
c



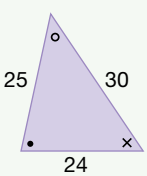
10:04

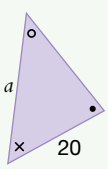
8 Find the value of the pronumeral in each of the following.

a

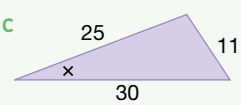


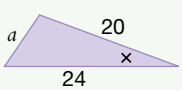
b





c





10:05

9 A scale drawing is to be made of a soccer field that is 110 m long and 80 m wide. Calculate the dimensions of the scale drawing if it is made using a scale of:

a 1 : 1000 **b** 1 : 2000 **c** 1 : 500

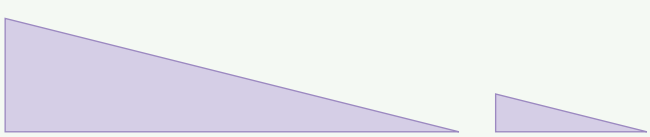
10:05

10 **a** Convert the scale 1 cm : 40 m into the form 1 : x.

b A scale drawing uses a scale of 1 : 100. What length on the drawing would represent a distance of 4 m?

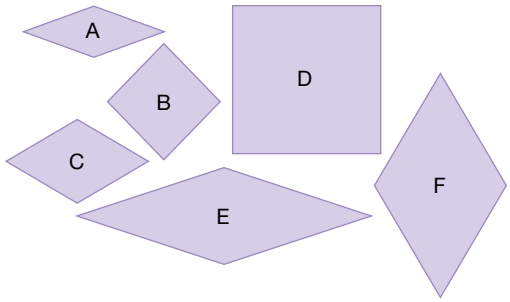
c A scale drawing has a scale of 1 : 4000. What real distance would be represented by 2 cm?

d The small triangle is a scale drawing of the large triangle. What scale is used?



ASSIGNMENT 10A Chapter review

1



Look carefully at the angles in each figure and then choose which figure is similar to:

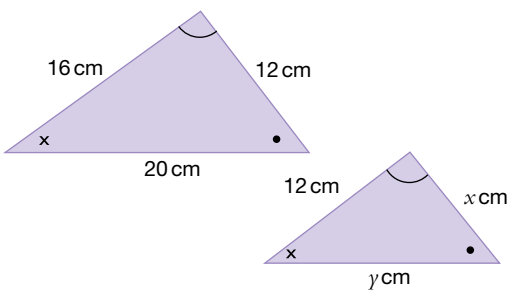
a A **b** B **c** C

2 **a** A rectangle 12·6 cm long and 6·8 cm wide is to be enlarged using an enlargement factor of 1·5. What will be the dimensions of the new rectangle?

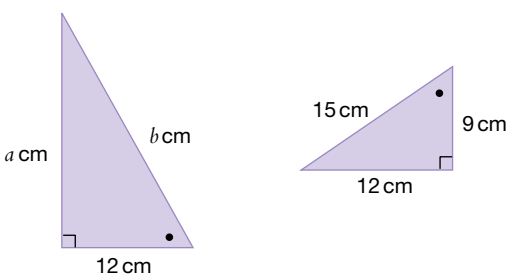
b A rectangle 15 cm long and 6 cm wide is enlarged so that the width is 13·2 cm. What should the length be?

3 Each pair of triangles is similar. Find the value of the pronumerals.

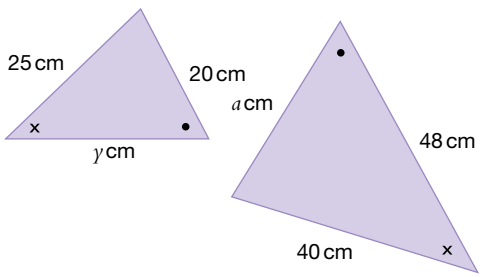
a



b

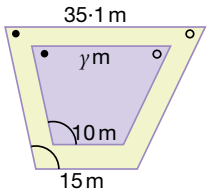


c

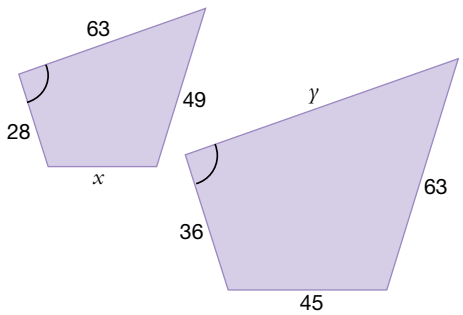


4 In each part, the quadrilaterals are similar. Find the value of each pronumeral.

a



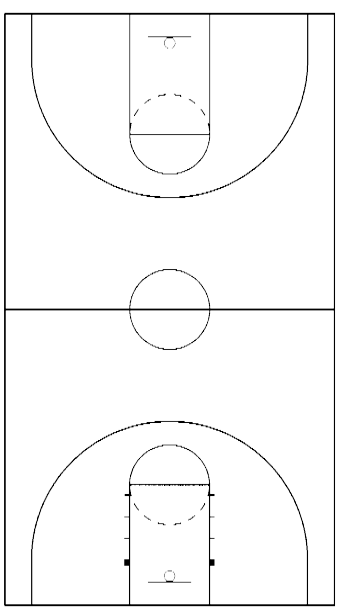
b



5 A basketball court is rectangular with length 28 m and width 15 m. A scale drawing is to be made of the court.

a Calculate the dimensions of the scale drawing if the scale used is 1:100.

b What scale should be chosen if the length of the scale drawing is to be 7 cm? What would be the width of the drawing?



How could you estimate the scale of this drawing?

ASSIGNMENT 10B Working mathematically

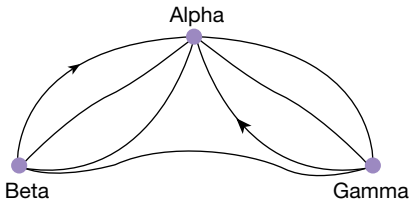
1 On a dartboard, it is possible with one dart to score:
- a number from 1 to 20
- the double or triple of a number from 1 to 20
- 25 or 50 for an outer or inner bullseye.

Hence, the largest score possible with one dart is 60 and the smallest is 1. What scores between 1 and 60 are impossible?

2 In a recent class test, the average was 74·5%. If the 11 girls in the class averaged 80%, what was the boys' average mark if there were 28 students in the class?

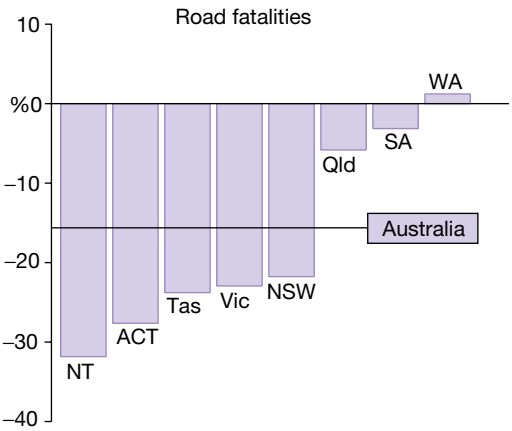
3 When a glass of juice is full, the total weight of the glass and juice is 400 g. When the glass is one-third full of juice, its weight is 280 g. How much does the glass weigh?

4 The following diagram shows the roads that run between Alpha, Beta and Gamma. How many ways are there to travel from:
- **a** Alpha to Beta
- **b** Beta to Gamma
- **c** Gamma to Beta and back again
- **d** Gamma to Beta and back again, without using any road twice?



Arrows indicate one way only

5



This graph shows the percentage change in road fatalities in Australian regions for a 12-month period compared with the previous 5-year average.
- **a** What does the dark line labelled 'Australia' refer to?
- **b** What percentage change has occurred in:
  i WA **ii** Vic **iii** NT?
- **c** How would you describe this year's statistics compared with those of the previous 5 years?

ASSIGNMENT 10C Cumulative revision

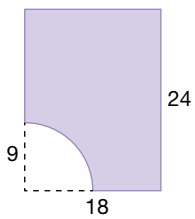
1 Simplify:
a $12x^3 \div 4x^2$ **b** $12x^3 \div 4x^{-2}$ **c** $12x^{-3} \div 4x^2$ **d** $4x^{-2} \div 12x^3$ 6:03

2 Use a power of 10 to complete each of the following. 6:06
a 1 km = ... m **b** 1 mm = ... m **c** 1 GL = ... L **d** 1 μm = ... m

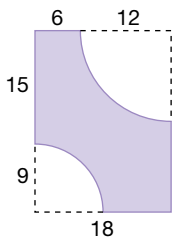
3 Simplify: 8:06
a $\frac{a}{4} + \frac{a}{5}$ **b** $\frac{a}{4} - \frac{a}{5}$ **c** $\frac{a}{4} \times \frac{a}{5}$ **d** $\frac{a}{4} \sqrt{\frac{a}{5}}$

4 Calculate the areas of these figures, correct to one decimal place. They have been formed from a rectangle by subtracting quadrants of different sizes. (All measurements are in centimetres.) 5:02

a



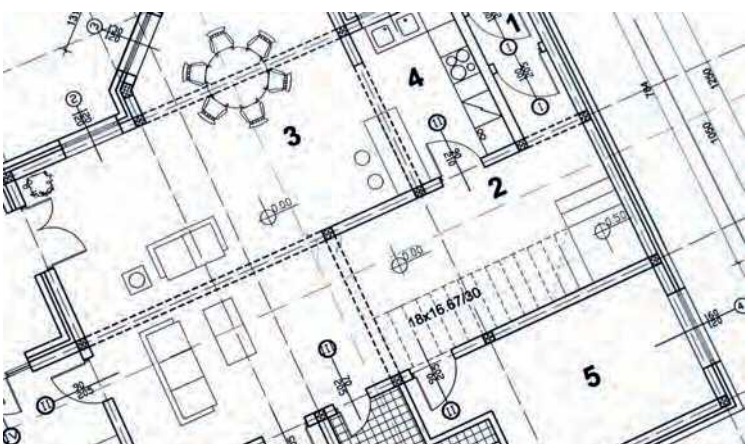
b



5 A salesperson earns $600 per week plus a commission of 5·5% on their sales. 8:01
a How much will they earn in a week when they sell $10 000 worth of goods?
b Calculate their annual income based on average weekly sales of $8000. (Assume that there are 52 weeks in a year).

6 Graph the following lines on the same number plane. 9:05
$x + y = 6$, $y = 4$, $y = 2x - 3$

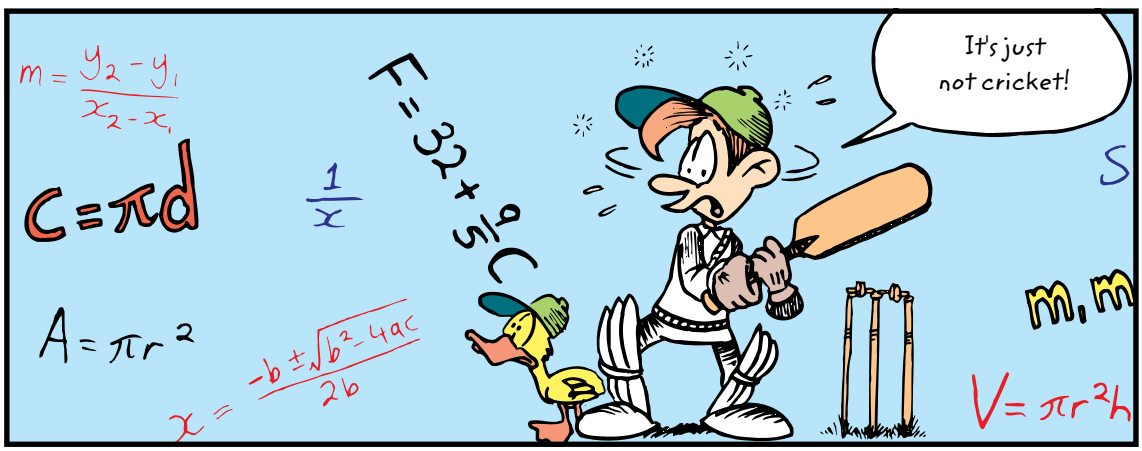
7 Bag A contains 2 red balls and 1 white ball. Bag B contains 1 red and 1 white ball. One ball is selected from each bag. 4:06
a Use a list, table or tree diagram to record the possible outcomes.
b Use your answer in part **a** to find the probability that the balls are:
i both red **ii** both white **iii** different colours.



The plans and drawings made by architects and engineers are scale drawings.

FORMULAS AND PROBLEM SOLVING

11



Contents

11:01 Generalised arithmetic
Fun spot 11:01 How much logic do you have?
11:02 Substitution
11:03 Formulas: Evaluating the subject
Investigation 11:03 Spreadsheet formulas
11:04 Formulas: Equations arising from substitution
11:05 Translating problems into equations
Fun spot 11:05 Read carefully (and think)!
11:06 Further problem solving using equations
Fun spot 11:06 Why are cooks cruel?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details)

Number and Algebra

Selection from ***Equations*** [Stage 5.2]

- Substitute values into formulas to determine an unknown. (ACMNA234)
- Solve problems involving linear equations, including those derived from formulas. (ACMNA235)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

11:01 Generalised arithmetic

PREP QUIZ 11:01

Find:

1 the sum of 7 and 5
2 the difference between 9 and 2
3 the number 8 less than 25
4 the quotient of 48 and 6
5 the product of 7 and 3
6 12 more than 8
7 the average of 41 and 47
8 the total of 13 and 21
9 the number of times 23 can be taken from 138
10 the number 8 less than the product of 4 and 5.

In mathematics, the method of solving a problem is sometimes hard to express in words. In cases like this, pronumerals are often used. The result could be a simple formula.

- Some numbers in a pattern are known. How can we find the others?
  For example: 9, 8, 7, 6, … or 3, 5, 7, 9, …
  Patterns like these can be written in a table of values, where n represents the position of the number in the pattern, and T the actual number (or term).

n	1	2	3	4
T	9	8	7	6

Here we can see that:

$T = 10 - n$

So, an algebraic expression that represents this pattern would be:

$10 - n$

n	1	2	3	4
T	3	5	7	9

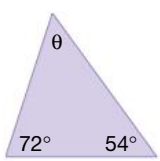
Here we can see that:

$T = 2n + 1$

So, an algebraic expression that represents this pattern would be:

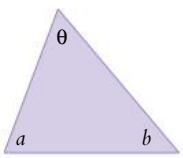
$2n + 1$

- Two angles of a triangle are known. How can we find the third?
  Consider a numerical example.



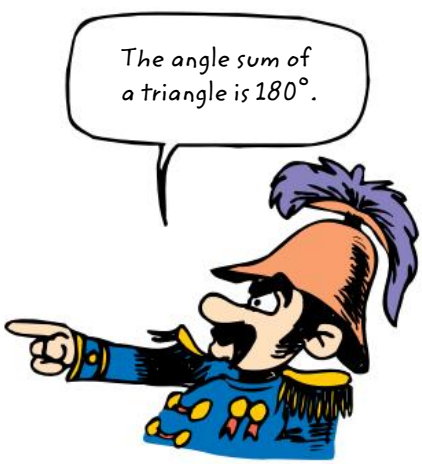
$\theta = 180° - (72 + 54)°$
$= 54°$

Show the general result.



$\theta = 180° - (a + b)°$
OR $180° - a° - b°$

$180° - (a + b)°$ is called the general case.



WORKED EXAMPLES

1 The sum of 8 and 12 $= 8 + 12$
So, the sum of x and y $= x + y$

2 The cost of 6 books at 30c each $= 6 \times 30\text{c}$
So, the cost of x books at 30c each $= x \times 30$
$= 30x$ cents

3 The average of 9 and 13 $= \dfrac{9+13}{2}$

So, the average of a and b $= \dfrac{a+b}{2}$

4 The change from \$10 after buying 3 books at \$2 each
$= 10 - (2 \times 3)$ dollars
So, the change from \$10 after buying x books at \$2 each
$= 10 - 2 \times x$
$= 10 - 2x$ dollars

The aim of 'generalised arithmetic' is to write an algebraic expression that shows the steps to be taken, no matter which numbers are involved.

Exercise 11:01

P Foundation worksheet 11:01 Generalised arithmetic

1 Match each table of values with the correct algebraic expression from the given list to complete the statement $T = \ldots$

a

n	1	2	3	4
T	3	4	5	6

b

n	1	2	3	4
T	-2	-1	0	1

c

n	1	2	3	4
T	3	6	9	16

d

n	1	2	3	4
T	5	8	11	14

e

n	1	2	3	4
T	1	4	9	16

f

n	1	2	3	4
T	1	3	5	7

List	
A	$3n$
B	n^2
C	$n + 2$
D	$3n + 2$
E	$2n - 1$
F	$n - 3$

2 Write an algebraic expression that represents each pattern of numbers, using n to represent the position of each number in the pattern.

a 2, 4, 6, 8, …
b 4, 5, 6, 7, …
c 7, 6, 5, 4, …
d 5, 7, 9, 11, …
e $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$
f $-3, -1, 1, 3, \ldots$

A table of values may be helpful.

For Questions **3** to **8**, write an expression to represent each quantity.

3 **a** the sum of 5 and 7
b the sum of 5 and y
c the sum of x and y

4 **a** the product of 3 and 7
b the product of a and 7
c the product of a and b

5 **a** the difference between 8 and 3
b the difference between 8 and p
c the difference between q and p

6 **a** the average of 8 and 12
b the average of 8 and x
c the average of w and x

7 **a** the cost of 5 books at 75c each
b the cost of a books at 75c each
c the cost of a books at b cents each

8 **a** dividing 30 cm into 5 equal lengths
b dividing 30 cm into t equal lengths
c dividing A cm into t equal lengths

9 **a** If Steve is 15 years old, how old will he be in 6 years?
b How old will Steve be in y years?

10 **a** If a car travels at 60 km/h for 3 hours, how far does it travel?
b If the same car travels at the same speed for h hours, how far does it travel?

11 **a** If three lengths of rope, each 2 m long, are cut from a piece of rope 10 m long, what length is left?
b If two lengths of rope, each x m long, are cut from a piece of rope X m long, what length is left?

12 Bob and Tom have \$1 between them. If Bob has x cents, how much has Tom?

13 **a** What is the next even number after 6?
b What is the next even number after y, if y is an even number?
c What is the largest odd number less than y if y is even?

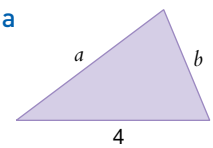
Try it with numbers first.

14 **a** How far will a person walk at m km/h in h hours?
b What is the average speed of a car that travels k km in h hours?
c How long will it take to travel k km at m km/h?

15 **a** A TV set is bought for \$$P$. If it is sold for \$$R$, what is the profit?
b If a gain of \$$G$ is to be made, what should the selling price of the TV be?

16 Write an expression for the perimeter of each figure below. All measurements are in centimetres.

a



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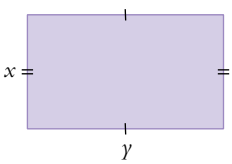

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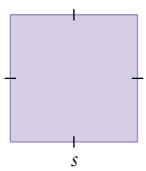

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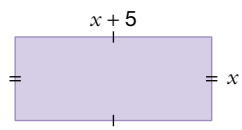
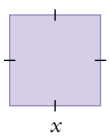


17 Write an expression for the area of each figure.

a



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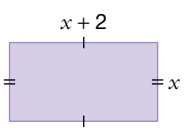

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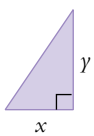

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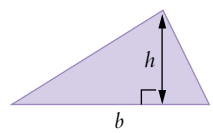
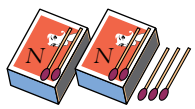


18 If N is the number of matches in a matchbox, then 4 boxes plus 5 loose matches would be represented by the expression $4N + 5$. The diagram is shown on the right.

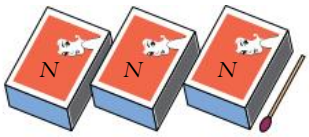
The expression $2(N + 2) + 3$ represents 2 boxes, each with 2 extra matches, plus 3 loose matches. The diagram is shown on the right.



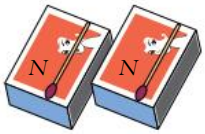
a Write expressions for the following diagrams.

i

ii



iii



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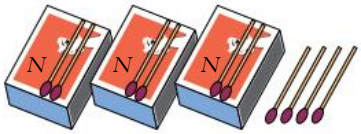

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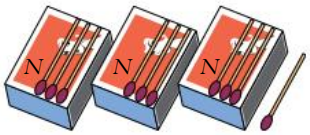
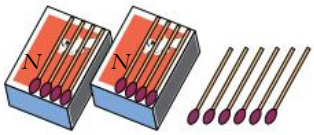


vi



b How could a diagram be drawn for an expression like $N - 3$?

19 Translate these algebraic expressions into words. There may be more than one way of doing this. For example, $5 - 2a$ may be written as:

'Subtract the product of two and a from five.' OR 'Five take away two times a.'

a $3m + 4$ **b** $2y - 5$ **c** $3 - 4x$

d $\frac{a}{2} + 1$ **e** $\frac{a+1}{2}$ **f** $4 - \frac{a}{2}$

g $2(m + 1)$ **h** $4(3 - a)$ **i** $9(a + b)$

j $5(a + 3) + 4$ **k** $2(x - 1) - 3$ **l** $10 - 3(a + 5)$

FUN SPOT 11:01 HOW MUCH LOGIC DO YOU HAVE?

1 What is the next letter in this sequence? $O, T, T, F, F, S, S, \ldots$?

2 A man passing a pilot at an airport exclaimed, 'I am that pilot's father!' But the pilot was not the man's son. How can this be?

3 Two guards are guarding two sacks. One guard always tells the truth, but the other guard always lies, but you do not know which guard is which. One of the sacks is full of gold; the other is full of peanuts. You are permitted to take one of the sacks but you are not sure which one contains the gold. You are also allowed to ask one of the guards just one question.

What question should you ask to ensure you get the sack of gold?

11:02 Substitution

PREP QUIZ 11:02

Write the following expressions in their simplest form.

1 $l + l + b + b$ **2** $l \times b$ **3** $\pi \times r \times r$ **4** $2 \times r + 2 \times w$

Find the value of:

5 $2 + 4 \times 3$ **6** $3 \times 4 + 2 \times 5$ **7** 4×5^2

8 $3(6 - 10)$ **9** $\frac{1}{2} \times 6 - 4$ **10** $\frac{8-2}{2}$

A value for an algebraic expression can be obtained if a number is substituted for each of the pronumerals.

WORKED EXAMPLE 1

Find the value of the following expressions given that $a = 10$, $b = 4$, $x = 5$, $y = 3$.

a ab **b** $3a + 2b$ **c** $x^2 + y^2$ **d** $\frac{1}{2}ab^2$ **e** $\frac{1}{a} + \frac{1}{b}$

Solutions

a ab
$= a \times b$
$= 10 \times 4$
$= 40$

b $3a + 2b$
$= 3 \times a + 2 \times b$
$= 3 \times 10 + 2 \times 4$
$= 30 + 8$
$= 38$

c $x^2 + y^2$
$= x \times x + y \times y$
$= 5 \times 5 + 3 \times 3$
$= 25 + 9$
$= 34$

d $\frac{1}{2}ab^2$
$= \frac{1}{2} \times a \times b \times b$
$= \frac{1}{2} \times 10 \times 4 \times 4$
$= \frac{1}{2} \times 160$
$= 80$

e $\frac{1}{a} + \frac{1}{b}$
$= \frac{1}{10} + \frac{1}{4}$
$= \frac{2}{20} + \frac{5}{20}$
$= \frac{7}{20}$

WORKED EXAMPLE 2

a Find the value of $mx + c$ if $m = 2$, $x = 6$ and $c = 1$.

b Find the value of $\frac{h}{2}(a+b)$ if $h = 10$, $a = 3$ and $b = 6$.

c Evaluate πr^2 if $\pi = 3{\cdot}1$ and $r = 10$.

Solutions

a $mx + c$
$= 2 \times 6 + 1$
$= 12 + 1$
$= 13$

b $\frac{h}{2}(a+b)$
$= \frac{10}{2}(3+6)$
$= 5 \times 9$
$= 45$

c πr^2
$= 3{\cdot}1 \times 10^2$
$= 3{\cdot}1 \times 100$
$= 310$

Exercise 11:02

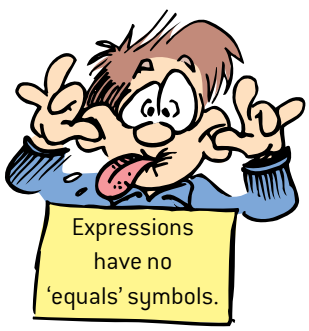
P Foundation worksheet 11:02 Substitution

1 Evaluate the following expressions if $a = 6$ and $b = 10$.

a $3a$	**b** $4b$	**c** $2a + 5$	**d** $3b - 6$	**e** $4(a + 2)$
f $3(12 - b)$	**g** $\frac{a}{2}$	**h** $\frac{b}{5}$	**i** $\frac{1}{2}b$	**j** $\frac{1}{3}a$
k $\frac{1}{2}a + 4$	**l** $10 + \frac{1}{4}b$	**m** $\frac{a+10}{4}$	**n** $\frac{b-5}{5}$	**o** $\frac{1}{2}(a+b)$
p $3(a + b)$	**q** a^2	**r** b^2	**s** $4a^2$	**t** $3b^3$
u $\frac{5b}{4}$	**v** $\frac{2}{3}a$	**w** $\frac{a}{8} + \frac{b}{10}$	**x** $4(3a + 2)$	

2 Evaluate the following expressions if $x = 3$, $y = 4$ and $z = 8$.

a $x + y$	**b** $3x + 2y$	**c** $2x - y$
d x^2y	**e** $(x + y)^2$	**f** $x^2 + y^2$
g $z(x + y)$	**h** $xz - 10$	**i** $xy - xz$
j $\frac{x+y}{2}$	**k** $\frac{x}{2} + \frac{y}{4}$	**l** $z^2 - z$
m $xy^2 + x^2y$	**n** $xz - yz$	**o** $4(x - 2y)$
p $x^2(y^2 - z^2)$	**q** $x - 3y$	**r** $y^2 - z^2$
s $z - xy$	**t** $\frac{x}{y} + \frac{y}{x}$	**u** $\frac{x+y}{xy}$
v $\frac{x+y}{x-y}$	**w** $\frac{1}{2}xyz^2$	**x** $\sqrt{3xy}$



3 **a** Find the value of $mx + c$ if:

i $m = 3, x = 10, c = 1$	**ii** $m = 5, x = 4, c = -3$	**iii** $m = -2, x = 7, c = -4$

b Find the value of $\frac{h}{2}(a+b)$ if:

i $h = 4, a = 7, b = 5$	**ii** $h = 7, a = 4, b = 6$	**iii** $h = 3{\cdot}4, a = 9{\cdot}2, b = 3{\cdot}7$

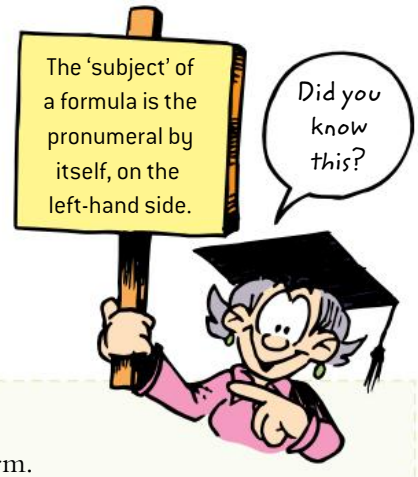
c Find the value of πr^2 if:

i $\pi = 3{\cdot}1, r = 8$	**ii** $\pi = 3{\cdot}14, r = 2{\cdot}5$	**iii** $\pi = 3{\cdot}142, r = 100$

11:03 Formulas: Evaluating the subject

Formulas are special types of equations. A formula represents a relationship between physical quantities. For example, the formula $A = l \times b$ represents the connection between the area of a rectangle and its length and breadth.

A formula is different from an equation in that it will always have more than one pronumeral. However, to find the value of a pronumeral in a formula we must be told the values of every other pronumeral.



PREP QUIZ 11:03

Write the following algebraic expressions in their simplest form.

1 $2 \times a \times b$ **2** $3 \times a \times b \times b$ **3** $3 \times a + 2 \times b$

Write the following expressions in expanded form.

4 $4xy^2$ **5** $(x + y)^2$ **6** $x^2 + y^2$

True or *false*?

7 $\frac{2a}{3} = \frac{2}{3}a$

If $a = 6$ and $b = 10$, evaluate:

8 $\frac{1}{2}ab^2$ **9** $\frac{a+b}{2}$ **10** $3(a - b)$

WORKED EXAMPLE 1

a Given that $I = \frac{PRN}{100}$, find I when $P = 500$, $R = 12$ and $N = 4$.

b If $V = \frac{1}{3}Ah$, find V when $A = 15$ and $h = 4$.

c Given that $a = 4$ and $b = 3$, find c when $c = \sqrt{a^2 + b^2}$.

Solutions

a $P = 500$, $R = 12$ and $N = 4$

$$I = \frac{PRN}{100}$$
$$= \frac{^{5}\cancel{500} \times 12 \times 4}{\cancel{100}_{1}}$$
$$= 5 \times 12 \times 4$$
$$\therefore I = 240$$

b $A = 15$ and $h = 4$

$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times 15 \times 4$$
$$= \frac{1}{\cancel{3}_{1}} \times \cancel{60}^{20}$$
$$\therefore V = 20$$

c $a = 4$ and $b = 3$

$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$\therefore c = 5$$

WORKED EXAMPLE 2

If $K = \frac{1}{2}mv^2$, find K when $m = 5$ and $v = 6$.

Solution

$m = 5$ and $v = 6$

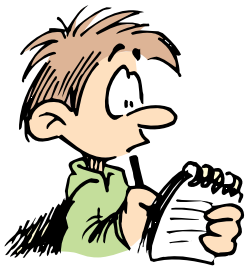
$K = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 5 \times 6^2$

$= \frac{1}{2} \times 5 \times 36$

$= \frac{1}{\cancel{2}_1} \times \cancel{180}^{90}$

$\therefore K = 90$



Remember:

- Replace the pronumerals with the given numerals.
- To find the value of one of the pronumerals you must be given the value of every other pronumeral.

Exercise 11:03

1 Match each formula with its title taken from the list beneath the table.

a $A = lb$	b $A = bh$	c $A = \frac{1}{2}bh$
d $A = s^2$	e $A = \frac{1}{2}xy$	f $A = \pi r^2$
g $A = \frac{1}{2}h(a+b)$	h $C = \pi d$	i $V = lbh$
j $V = Ah$	k $P = 2l + 2b$	l $c^2 = a^2 + b^2$
m $\alpha + \beta + \gamma = 180°$	n $\alpha + \beta + \gamma + \delta = 360°$	o $F + V = E + 2$

A Euler's formula
B Area of a circle
C Pythagoras' theorem
D Volume of a prism
E Area of a rhombus
F Area of a square
G Angle sum of a triangle
H Circumference of a circle
I Area of a rectangle
J Area of a parallelogram
K Perimeter of a rectangle
L Angle sum of a quadrilateral
M Area of a trapezium
N Area of a triangle
O Volume of a rectangular prism

2 a If $A = lb$, find A when $l = 3{\cdot}6$ and $b = 2{\cdot}4$.

b If $A = bh$, find A when $b = 15$ and $h = 3{\cdot}6$.

c Given that $A = \frac{1}{2}bh$, find A when $b = 15$ and $h = 3{\cdot}6$.

d Given that $A = s^2$, find A when $s = 3{\cdot}5$.

e If $A = \frac{1}{2}xy$, find A when $x = 8$ and $y = 11$.

f If $A = \pi r^2$, find A correct to one decimal place if $r = 3{\cdot}4$.

g Given that $A = \frac{1}{2}h(a+b)$, find A when $h = 8$, $a = 11$ and $b = 9$.

h Given $C = \pi d$, find C correct to one decimal place if $d = 6{\cdot}8$.

i If $V = lbh$, find V if $l = 8{\cdot}1$, $b = 7{\cdot}2$ and $h = 4{\cdot}5$.

j If $V = Ah$, find V if $A = 341{\cdot}6$ and $h = 6{\cdot}15$.

k Given that $P = 2l + 2b$, find P when $l = 3{\cdot}75$ and $b = 2{\cdot}45$.

l Given that $c = \sqrt{a^2 + b^2}$, find c when $a = 5$ and $b = 12$.

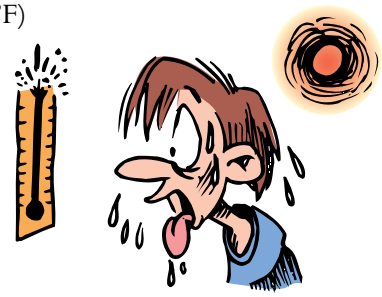
m If $\alpha = 180° - \beta - \gamma$, find α when $\beta = 30°$ and $\gamma = 45°$.

n If $\alpha = 360° - \beta - \gamma - \delta$, find α when $\beta = 37°$, $\gamma = 51°$ and $\delta = 132°$.

o Given that $E = F + V - 2$, find the number of edges (E) when the number of faces (F) is 7 and the number of vertices (V) is 10.

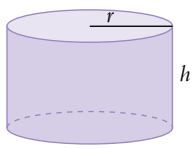
3 a The formula $C = \frac{5}{9}(F - 32)$ converts degrees Fahrenheit (°F) to degrees Celsius (°C). Find C when:

i $F = 212$ ii $F = 32$ iii $F = 104$



b The kinetic energy (E) of a particle is given by the formula $E = \frac{1}{2}mv^2$ where m and v are the mass and velocity of the particle, respectively. Find E if $m = 6$ and $v = 2{\cdot}5$.

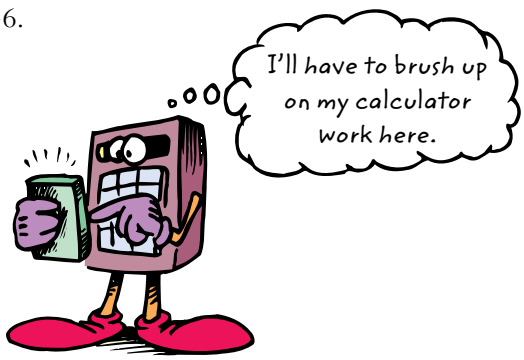
c The surface area (S) of a cylinder is given by the formula $S = 2\pi r^2 + 2\pi rh$. Evaluate S when $\pi = 3{\cdot}14$, $r = 1{\cdot}50$ and $h = 3{\cdot}25$. Give the answer correct to 3 significant figures.



d The volume (V) of a cylinder is given by the formula $V = \pi r^2 h$. Find the volume correct to the nearest cubic centimetre if the radius (r) is 8 cm and the height (h) is 10 cm.

4 a If $P = RI^2$, evaluate P when $R = 2{\cdot}5$ and $I = 0{\cdot}6$.

b It is known that $F = \frac{mp}{r^2}$. If $r = 0{\cdot}2$, $m = 3{\cdot}6$ and $p = 9{\cdot}2$, find the value of F.



c $T = 2m\sqrt{\frac{l}{g}}$. Find T if $m = 3{\cdot}6$, $l = 2{\cdot}5$ and $g = 0{\cdot}1$.

d Given the formula $P = \frac{nkT}{V}$, find P when $n = 2{\cdot}4$, $k = 7{\cdot}6$, $T = 6$ and $V = 0{\cdot}15$.

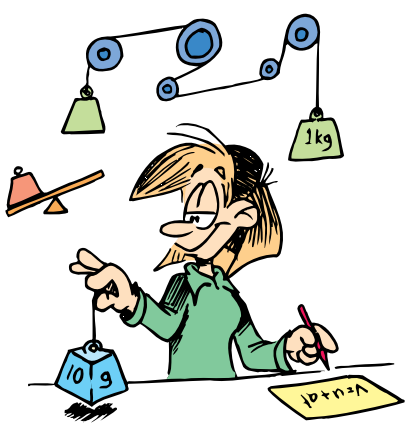
e If $S = \frac{n}{2}[2a + (n-1)d]$, find S when $n = 20$, $a = -4$ and $d = 0{\cdot}4$.

f Given the formula $S = ut + \frac{1}{2}at^2$, find S when $u = 8$, $t = 4$ and $a = -10$.

g If $I = \frac{PRN}{100}$ find I when $P = 450$, $R = 12$ and $N = 3{\cdot}5$.

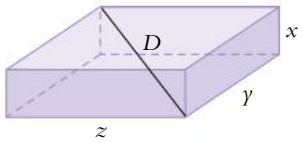
h If $S = \frac{a(r^n - 1)}{r - 1}$, find S when $a = 5{\cdot}2$, $r = 1{\cdot}4$ and $n = 2$.

i If $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, find x when $b = 3$, $a = 2$ and $c = -10$. Give the answer correct to two decimal places.



5 The distance **D** from the top corner of a rectangular box to the opposite bottom corner is given by the formula:

$D = \sqrt{x^2 + y^2 + z^2}$



where **x**, **y** and **z** are the dimensions of the box.
Find **D** if the dimensions **x**, **y** and **z** are:

a 3 cm, 4 cm, 5 cm
b 7 cm, 11 cm, 15 cm
c 2·3 cm, 4·2 cm, 4·9 cm
d 11·6 cm, 13·2 cm, 17·8 cm

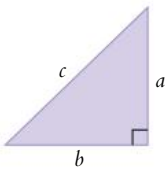
Answer correct to one decimal place.

INVESTIGATION 11:03 SPREADSHEET FORMULAS

In spreadsheet programs such as Microsoft® Excel®, a formula can be entered in the 'formula bar' that will operate on selected 'cells'.

The example below shows a simple example that uses Pythagoras' theorem to find the hypotenuse of a right triangle.

For this triangle, $c = \sqrt{a^2 + b^2}$.



You can see this formula in the formula bar written as:

=SQRT(A2^2+B2^2)

SQRT means 'square root'.
^ means 'to the power of'.

This means that the number in cell C2 will be equal to the square root of (cell A2 squared plus cell B2 squared).

If the numbers in cells A2 or B2 are changed, cell C2 will also change automatically.

- Try this example yourself and then experiment with other formulas you know.

(You can alter the number of decimal places by selecting Cells > Number in the Format menu, or by clicking on the 'change decimal places' button in the ribbon.)

Formula bar

C2		=SQRT(A2^2+B2^2)			
	A	B	C	D	E	F
1	a	b	c			
2	3	6	6.71			
3						
4						
5						

Spreadsheets can be very useful mathematical tools. Can you set up spreadsheets to automatically calculate the formulas in Exercise 11:03 Question 1?

11:04 Formulas: Equations arising from substitution

Consider the following formulas.

$$S = \pi rl \qquad T = 2\pi\sqrt{\frac{l}{g}} \qquad D = \frac{M}{V} \qquad S = ut + \tfrac{1}{2}at^2$$

- The pronumeral on the left-hand side of each formula is the subject. In all the exercises so far, you have been asked to find the subject of the formula.
- We often know the value of the subject and are asked to find the value of one of the other pronumerals. To find the value of this pronumeral, we need to solve an equation.

PREP QUIZ 11:04

Solve:

1 $x + 152 = 315$ **2** $y - 73 = 149$ **3** $7x = 343$

4 $\frac{x}{10} = 37$ **5** $110 = 14 + x$ **6** $96 = y - 87$

7 $112 = 4p$ **8** $18 = \frac{m}{3}$ **9** $2a + 1 = 11$

10 $50 = 16 + 17t$

WORKED EXAMPLE 1

Given that $V = \frac{Ah}{3}$, find h when $V = 12$ and $A = 5$.

Solution

$V = 12$ and $A = 5$

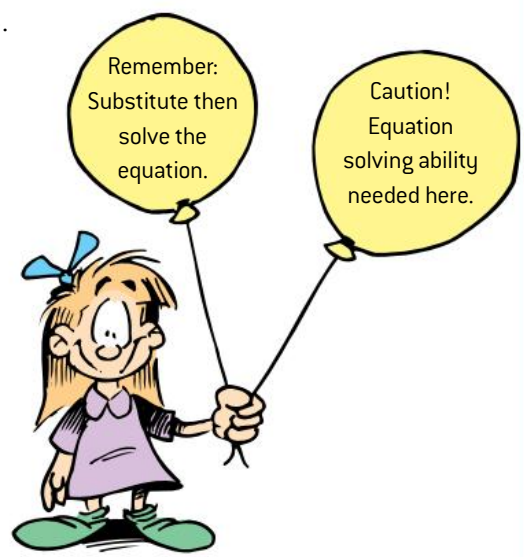
$$V = \frac{Ah}{3}$$

$$\therefore\ 12 = \frac{5h}{3} \qquad (\times \text{ both sides by } 3)$$

$$36 = 5h \qquad (\div \text{ both sides by } 5)$$

$$\therefore\ h = \tfrac{36}{5}$$

$$= 7\tfrac{1}{5}$$



WORKED EXAMPLE 2

a $A = \frac{1}{2}h(x + y)$. Find the value of x, correct to one decimal place, if $A = 11$, $h = 3{\cdot}6$ and $y = 4{\cdot}5$.

b If $S = \dfrac{a}{1 - r}$, find r when $S = 10$ and $a = 1{\cdot}5$.

Solutions

a $A = 11$, $h = 3{\cdot}6$ and $y = 4{\cdot}5$

$$A = \tfrac{1}{2}h(x + y)$$
$$\therefore\ 11 = \frac{1}{2_1} \times 3{\cdot}6^{1{\cdot}8}(x + 4{\cdot}5)$$
$$11 = 1{\cdot}8(x + 4{\cdot}5)$$
$$= 1{\cdot}8x + 1{\cdot}8 \times 4{\cdot}5$$
$$11 = 1{\cdot}8x + 8{\cdot}1$$
$$11 - 8{\cdot}1 = 1{\cdot}8x$$
$$2{\cdot}9 = 1{\cdot}8x$$
$$\tfrac{2{\cdot}9}{1{\cdot}8} = x$$
$$\therefore\ x = 1{\cdot}6 \text{ (1 dec. pl.)}$$

b $S = 10$ and $a = 1{\cdot}5$

$$S = \frac{a}{1 - r}$$
$$\therefore\ 10 = \frac{1{\cdot}5}{1 - r}$$
$$10(1 - r) = 1{\cdot}5$$
$$10 - 10r = 1{\cdot}5$$
$$-10r = -8{\cdot}5$$
$$\therefore\ r = 0{\cdot}85$$

Exercise 11:04

1 If $v = u + at$, find:

a u when $v = 25$, $a = 3$, $t = 5$
b a when $v = 40$, $u = 10$, $t = 2$
c t when $v = 16$, $u = 4$, $a = 5$
d u when $v = 27{\cdot}2$, $a = 4$, $t = 3{\cdot}5$
e a when $v = 15$, $u = 5$, $t = \frac{1}{4}$
f t when $v = 26$, $u = -16$, $a = 7$
g u when $v = 12{\cdot}68$, $a = 4{\cdot}2$, $t = 1{\cdot}5$
h a when $v = 16$, $u = -4$, $t = 0{\cdot}75$

2 Given that $V = \dfrac{Ah}{3}$, find:

a h when $V = 5$ and $A = 3$
b h when $V = 12$ and $A = 9$
c h when $V = 16$ and $A = 10$
d h when $V = 15{\cdot}5$ and $A = 3{\cdot}1$
e A when $V = 6$ and $h = 5$
f A when $V = 15$ and $h = 12$
g A when $V = 7$ and $h = 0{\cdot}7$
h A when $V = 3{\cdot}6$ and $h = 0{\cdot}4$

3 For the formula $K = \frac{1}{2}mv^2$, find:

a m when $K = 60$, $v = 2$
b m when $K = 15$, $v = 3$
c m when $K = 3{\cdot}2$, $v = 4$
d m when $K = 7$, $v = \frac{1}{2}$
e v when $K = 25$, $m = 2$
f v when $K = 36$, $m = 8$
g v when $K = 216$, $m = 3$
h v when $K = \frac{3}{8}$, $m = 3$

4 $A = \frac{1}{2}h(x + y)$. Find:

a x when $A = 10$, $h = 4$, $y = 3$
b x when $A = 20$, $h = 8$, $y = 2$
c y when $A = 6$, $h = 3$, $x = 4$
d y when $A = 45$, $h = 6$, $x = 3$
e h when $A = 3$, $x = 4$, $y = 6$
f h when $A = 18$, $x = 5$, $y = 13$
g h when $A = 7{\cdot}8$, $x = 1{\cdot}7$, $y = 0{\cdot}9$
h h when $A = 5{\cdot}16$, $x = 4{\cdot}6$, $y = 8{\cdot}3$

5 **a** If $A = lb$, find l when $A = 9{\cdot}6$ and $b = 2{\cdot}4$.

b If $V = RI$, find I when $V = 15$ and $R = 0{\cdot}6$.

c Given that $V = Ah$, find h, correct to one decimal place, if $V = 12{\cdot}6$ and $A = 4{\cdot}1$.

d It is known that $V = lbh$. Find b correct to two decimal places, if $V = 60$, $l = 3{\cdot}4$ and $h = 2{\cdot}6$.

e Calculate the value of r in the formula $A = 2\pi rh$ if $A = 75$, $\pi = 3{\cdot}14$ and $h = 7{\cdot}6$. (Give your answer correct to one decimal place.)

6 **a** If $S = \dfrac{a}{1-r}$, find a when $S = 5{\cdot}2$ and $r = 0{\cdot}3$.

b For the formula $S = ut + \frac{1}{2}at^2$ evaluate a, given that $S = 15$, $u = 2$ and $t = 5$.

c $X = \dfrac{x + ky}{1 + k}$. Find y when $X = 10$, $k = 2$ and $x = 3$.

d Given that $T = \dfrac{ab}{6} + C$, find b when $T = 15$, $a = 5$, $C = 4$.

e It is known that $F = 32 + \frac{9}{5}C$. Find C if $F = 212$.

7 In each of the following, give your answer correct to two decimal places.

a If $A = lb$, find b given that $A = 15$ and $l = 4{\cdot}8$.

b Given that $A = \pi r^2$, find r when $A = 10$ and $\pi = 3{\cdot}14$.

c If $T = a + (n - 1)d$, find d given $T = 19{\cdot}6$, $a = 3{\cdot}6$, $n = 12$.

d If $P = RI^2$, find R if $P = 100$ and $I = 3{\cdot}6$.

e If $F = 32 + \dfrac{9C}{5}$, find C if $F = 100$.

8 **a** The area of a circle can be found using the formula $A = \dfrac{\pi d^2}{4}$. Use this formula to find the area of a circle with a diameter of 12 cm.

b Use the formula $A = \pi r^2$ to find the area of a circle with a radius of 6 cm. Is your answer the same as in part **a**?

11:05 Translating problems into equations

PREP QUIZ 11:05

What is the sum of:

1 3 and 4 **2** x and 3 **3** x and y?

What is the product of:

4 3 and 4 **5** x and 3 **6** x and y?

7 I have \$50. I spend \$$x$. How much do I have left?

8 There are x books and each one costs \$5. What is the total cost of the books?

9 There are x cars with y people in each car. How many people are there altogether?

10 A man is x years old. How old will he be in 5 years' time?

Consider the following simple problem.

'I think of a number. If I add 7 to the number the result is 22. What is the number?'

- This problem can be solved by forming an equation. If the missing number is represented by the pronumeral x, then the equation $x + 7 = 22$ represents the information given in the problem. Solving the equation then yields the answer to the original problem.
- Although this problem is quite simple it illustrates how 'word sentences' or problems can be translated into number sentences. When solving problems we have to be able to translate the words into equations.

WORKED EXAMPLES

Translate the following into number sentences, using x to represent the unknown number.

We often use 'x' to represent an unknown number.

1 I multiply a number by 2 and the result is 50.

$2 \times x$ $= 50$

The equation is $2x = 50$.

2 If I add 6 to a number the answer is 11.

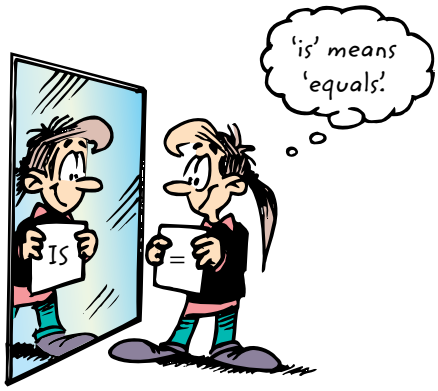
$x + 6$ $= 11$

The equation is $x + 6 = 11$.

3 I subtract a number from 6 and the answer is 2.

$6 - x$ $= 2$

The equation is $6 - x = 2$.



4 A certain number is multiplied by 3, then 6 is added and the result is 17.

$x \times 3$ $+ 6$ $= 17$

The equation is $3x + 6 = 17$.

Exercise 11:05

P Foundation worksheet 11:05 Translating problems into equations

For Questions 1 to 5, form an equation and then solve it.

1 **a** I multiply a number by 8 and the result is 32.
b If I multiply a number by 4 the answer is 100.
c When a certain number is multiplied by 7, the result is 42.
d Multiplying a certain number by 12 gives an answer of 78.
e If a certain number is multiplied by 6 the result is 15·6.

You can use any letter to represent the number in these questions.

2 a If I add 6 to a number the result is 10.
b If 5 is added to a number the answer is 22.
c When 22 is added to a number the result is 50.
d Adding 17 to a number gives an answer of 12.
e What must be added to 12 to give 38?

3 a If I subtract 3 from a certain number the result is 10.
b 5 is subtracted from a number and the answer is 22.
c When 25 is subtracted from a certain number the answer is 10.
d Subtracting 17 from a certain number gives an answer of 18.
e If a number is subtracted from 6, the result is 4.

'I subtract 3 from a certain number' translates to $x - 3$.

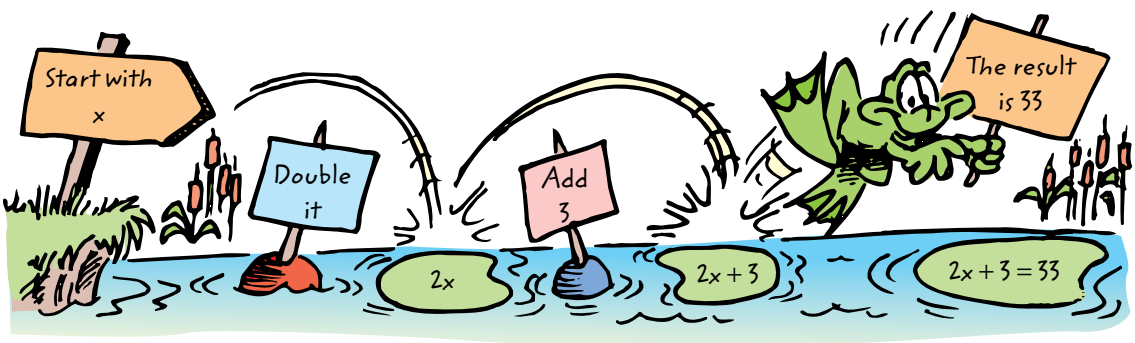
'I subtract a certain number from 3' translates to $3 - x$.

$x - 3$ is not the same as $3 - x$.

4 a If a number is divided by 5, the answer is 4.
b Dividing a certain number by 8 gives an answer of 7.
c One-tenth of a certain number is 8.
d What number does 6 go into 7 times?
e If 10 is divided by a certain number the result is 2.

5 a A number is multiplied by 2, then 6 is added and the result is 14.
b Three times a certain number is added to 5 and the result is 20.
c A certain number is multiplied by 5, then 8 is subtracted and the result is 22.
d If 5 is added to a certain number and the result is multiplied by 4, the answer is 56.
e When 5 is subtracted from half of a number the result is 3.

6 For each of the following problems, form an equation and then solve it.
a I think of a number, double it, add 3 and the result is 33. What is the number?



b I think of a number and multiply it by 4. If I then subtract 3, the answer is 25. Find the number.
c I think of a number, add 3 and then double that result. If the answer is 22, find the number.
d I think of a number. After dividing it by 4 and subtracting 7 the result is 1. What is the number?
e I think of a number. If I add 4 and then divide the result by 3, the answer is 8. Find the number.

7 Solve each of the following problems by first forming an equation.

- **a** If 5 is added to 3 times a certain number, the result is 38. What is the number?
- **b** If I subtract 6 from 5 times a certain number the result is 29. What is the number?
- **c** If 5 is subtracted from a certain number and that result is then halved, the answer is 6. What is the number?
- **d** A number is doubled and then 5 is added. When this is divided by 3, the result is 7. What is the number?

FUN SPOT 11:05 READ CAREFULLY (AND THINK)!

1 Which weighs more: 3 kg of lead or 3 kg of feathers?

2 A company has 5943 employees. All but 742 of them attended the company picnic. How many did not attend?

3 There are 27 students in a class, of which 18 were boys. How many were not girls?

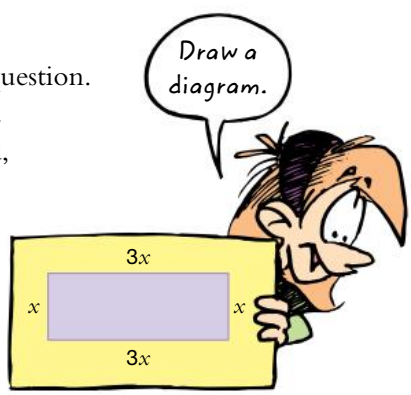
4 Thelma left for work at 7:45 am, half an hour earlier than normal. She arrived at work at 8:25 am, worked for 8 hours and then went home. The trip home took 10 minutes longer than the trip to work. What time did Thelma usually leave for work in the morning?

11:06 Further problem solving using equations

To use equations to solve problems we must be able to analyse a written problem, translate it into an equation and then solve it.

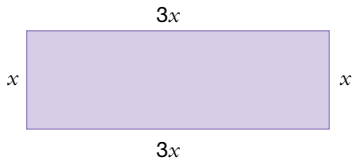
Approach

- Read the problem carefully, examining the wording of the question.
- Establish what is to be found and what information is given.
- Ask yourself whether any other information can be assumed, e.g. that a pack of cards mentioned is a standard pack.
- Try to connect the given information to form an equation. This will often require a knowledge of a formula or the meaning of mathematical terms.



WORKED EXAMPLE 1

A rectangle is three times longer than it is wide. If it has a perimeter of 192 m, what are its dimensions?



Solution

Let the breadth be x m.

$\therefore$ The length $= 3 \times x$ m
$= 3x$ m

$P = 192$ m

Perimeter is the the sum of the lengths of the sides (or the distance around the outside of the figure).

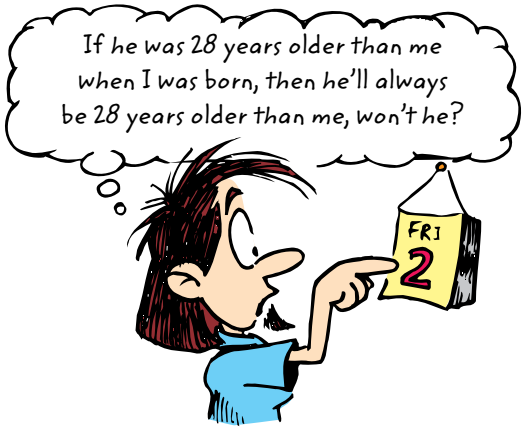
$P = 3x + x + 3x + x$
$8x = 192$
$\therefore\ x = 24$
$\therefore\ 3x = 3 \times 24 = 72$

The breadth is 24 m and the length is 72 m.

The first line of each solution should include a pronumeral that represents the quantity you wish to find.

WORKED EXAMPLE 2

My father was 28 years old when I was born. If he is now three times as old as I am, what are our present ages?



Solution

Let my present age be x years.
$\therefore$ My father's present age is $3 \times x$ years.
When I was born my father was 28.
$\therefore$ The difference in our ages is 28 years.
Father's age $-$ my age $= 28$ years

$3x - x = 28$
$2x = 28$
$\therefore\ x = 14$
$\therefore\ 3x = 3 \times 14 = 42$

I am 14 years old and my father is 42 years old.

WORKED EXAMPLE 3

Car A left Sydney for Melbourne at 6:00 am and travelled at an average speed of 80 km/h. At 7:30 am car B left Sydney for Melbourne. If car B travels at an average speed of 100 km/h, at what time will it catch up to car A?

Solution

Car B will catch up to car A when both have travelled the same distance.
Distance travelled = average speed × time

Now let car B catch up to car A t hours after it starts.
Car B has been travelling for t hours.
Car A has been travelling for $(t + 1\frac{1}{2})$ hours (since it started at 6 am).

$\therefore$ Distance travelled by car A $= 80 \times (t + 1\frac{1}{2})$

$\therefore$ Distance travelled by car B $= 100 \times t$

Distance travelled by car A = Distance travelled by car B

$$80(t + 1\tfrac{1}{2}) = 100t$$
$$80t + 80 \times 1\tfrac{1}{2} = 100t$$
$$120 = 20t$$
$$\therefore\ t = 6$$

$\therefore$ Car B will catch up to car A 6 hours after it starts, at 1:30 pm.

Car A
Speed = 80 km/h
Starts at 6:00 am
Travels for $(t + 1\frac{1}{2})$ hours
Distance travelled = $80(t + 1\frac{1}{2})$

Car B
Speed = 100 km/h
Starts at 7:30 am
Travels for t hours
Distance travelled = $100t$

Exercise 11:06

1 **a** My father is three times as old as I am. If he is 26 years older than I, what are our ages?

b Two men have $560 between them. If one man has six times as much money as the other, how much has each man?

c Joan has $7 less than Anne. Together they have $43. How much does each girl have?

d Prize money of $500 is divided between Alan and Jim so that Alan receives $170 more than Jim. How much does each receive?

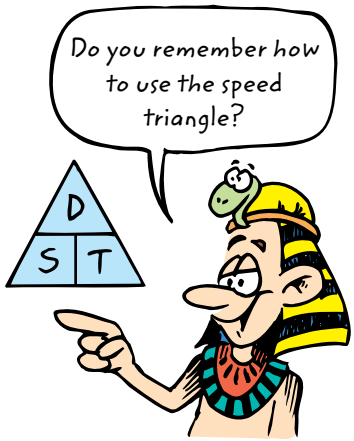
e If a father is five times as old as his son at present, how old is he if he was 32 years old when his son was born?

2 **a** In a class of 32 students, it is known that there are 6 more boys than girls. How many girls are there in the class?

b A rectangle is 6 cm longer than it is wide. Find its dimensions if its perimeter is 64 cm.

c If a quarter of the weight of a roast is lost in roasting, what weight of roast should be bought in order to have 3 kg of roasted meat?

d Town B is between towns A and C. Town B is five times as far from C as it is from A. The distance from A to C is 144 km. How far is it from A to B?



3 **a** A blend of tea is made from 6 kg of an inferior tea and 3 kg of a superior quality tea that costs \$2 a kilogram more. The total price of the mixture is \$24. What was the price of the inferior tea?

b Two bike riders X and Y both start at 2 pm riding towards each other from 40 km apart. X rides at 30 km/h, Y at 20 km/h. If they meet after t hours, find when and where they meet.

c A man is twice as old as his son. Find their present ages, if 9 years ago, the sum of their ages was 66 years.

d A tank is half-full. If 600 litres were emptied from the tank, it would be one-third full. How much does the tank hold when full?

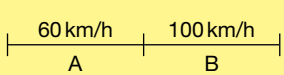
4 **a** Fred is 25 years older than Bill and, in 5 years, he will be twice as old as Bill. Find their present ages.

b A bank teller has some 5-cent and 10-cent coins. If she has a total of 50 coins that have a value of \$4.20, how many of each must she have?

c A tennis player has won 36 out of 54 matches. Her sponsor says that she must win 60% of her total number of matches to qualify for a bonus. If there are 26 matches remaining on the tour, how many more must she win to collect the bonus?

d One tank holds 300 litres more than another. If the smaller is two-thirds full, it holds as much as the larger when it is half full. What is the capacity of each?

e A certain journey took 40 min to complete. If half the distance was travelled at an average speed of 100 km/h and the other half at an average speed of 60 km/h, what was the length of the journey?



Time for A + Time for B = 40 min

40 min = $\frac{2}{3}$ h

FUN SPOT 11:06 WHY ARE COOKS CRUEL?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Simplify:

E $9a - 5a$
E $4 \times 2y$
E $-3a + 9a$
E $-a - 5a$
A $7x + a + x$
A $-8 \times \frac{1}{2}x$
I $ax \times 5a$
U $25x^2 \div 5x - 3x$
T $16a^2 \div (-2a^2)$
E $10x - x$
E $18m \div 2$
E $(-3a)^2$
A $9x + x$
A $20m \div m$
A $5x^2 - x - x$
I $\frac{3}{a} \times \frac{a}{3}$
T $6a^2 - a^2 - 5$
T $8x^2 \div 4x \times 2x$

Write an expression for:

T one more than x
B the average of x and a
B the product of a and b
B the difference between x and a

Find the value of $x^2 - 4x + 4$ *if*:

S $x = 7$
S $x = 10$
S $x = 9$
C $x = -1$

Solve these equations:

H $x + 8 = 5$
H $x - 9 = 4$
F $\frac{-13}{x} = 1$
C $\frac{x}{3} = 1$

$l = 6$, $b = 8$, $h = 10$, $a = 5$, $b = 9$ and $s = 7$, find A if:

R $A = lb$
M $A = \frac{1}{2}bh$
N $A = s^2$
D $A = \frac{1}{2}h(a + b)$

Evaluate:

Y $\frac{a}{2}$ if $a = \frac{1}{2}$
G $\frac{1}{M}$ if $M = \frac{1}{2}$
G $90 - \theta$ if $\theta = 17°$
H $3{\cdot}14r^2$ if $r = 10$
R $\frac{8}{1-r}$ if $r = 0{\cdot}5$
P $\frac{9C}{5}$ if $C = 20$
W $ut + \frac{1}{2}at^2$ if $u = 12$, $a = 9{\cdot}8$ and $t = 10$

ab	$4a$	9	20	$2x$	64	$9x$	-8	314	$8y$	$\frac{-1}{4}$	$x - a$	$9m$	$10x$	$4x^2$	$6a$	73°	2	25

610	$x = 13$	$5a^2x$	36	$x = 3$	$A = 48$	$-6a$	$-4x$	$A = 40$	$5x^2 - 2x$	$A = 49$	$A = 70$	$\frac{x+a}{2}$	$8x + a$	$5a^2 - 5$	$x + 1$	$9a^2$	16	$x = -13$	1	49	$x = -3$

MATHS TERMS 11

equation

- a number sentence where one or more of the numbers is missing or unknown; the unknown number is represented by a pronumeral,
  e.g. $x + 5 = 8$, $\frac{x+1}{7} = \frac{x-5}{2}$

evaluate

- to find the value of a required expression,
  e.g. find the value of $3a + 7$ if $a = 3$

formula (plural: formulas or formulae)

- a formula represents a relationship between physical quantities; it will always have more than one pronumeral,
  e.g. $A = l \times b$ represents the relationship between the area (A) of a rectangle and its length (l) and breadth (b)

pronumeral

- a symbol used to represent a number, such as the letter x

subject

- the subject of a formula is the pronumeral by itself, on the left-hand side,
  e.g. v is the subject of the formula
  $v = u + at$

substitution

- the replacing of a pronumeral with a numeral in a formula or expression,
  e.g. substituting 3 for a in the expression $4a - 2$ would give:
  $4 \times 3 - 2$
  $= 12 - 2$
  $= 10$

Formulas are used extensively in physics and other sciences.

DIAGNOSTIC TEST 11 FORMULAS AND PROBLEM SOLVING

Each part of this test has similar items that test a certain question type.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 For each table of values, find the expression in x that completes the rule $y = \ldots$ 11:01

a

x	0	1	2	3
y	0	4	8	12

b

x	5	6	7	8
y	12	14	16	18

c

x	1	2	3	4
y	1	4	9	16

2 Write an algebraic expression for each of the following. 11:01

a the sum of x and y
b the average of 5 and m
c the cost of b books at p dollars each
d \$2 was shared between Sue and Jenny. If Sue received x cents, how many cents did Jenny receive?

3 If $m = 2$, $x = 6$, $c = 1$, $h = 10$, $a = 3$, $b = 6$, $\pi = 3{\cdot}1$ and $r = 10$, evaluate: 11:02

a $mx + c$ **b** $\frac{h}{2}(a+b)$ **c** πr^2

4 If $v = u + at$, find v when: 11:03

a $u = 6$, $a = 3$, $t = 4$ **b** $u = 6{\cdot}8$, $a = 9{\cdot}8$, $t = 3$ **c** $u = 5$, $a = 4{\cdot}6$, $t = 0{\cdot}6$

5 If $C = 2\pi r^2$, find C when: 11:03

a $\pi = 3{\cdot}14$, $r = 5$ **b** $\pi = 3{\cdot}142$, $r = 1{\cdot}6$ **c** $\pi = \frac{22}{7}$, $r = 0{\cdot}75$

6 If $M = 2m + 3n$, find m when: 11:04

a $M = 12$, $n = 1$ **b** $M = 17{\cdot}5$, $n = 0{\cdot}5$ **c** $M = 29$, $n = 3{\cdot}75$

7 If $V = \frac{Ah}{3}$, find h when: 11:04

a $V = 15$, $A = 2$ **b** $V = 14{\cdot}8$, $A = 4$ **c** $V = 12$, $A = 1{\cdot}8$

8 Form an equation from the given data for each of the following. (In each case let a represent the unknown number.) 11:05

a I think of a number, multiply it by 2, add 7 and the result is 10.
b I think of a number, divide it by 3, subtract 4 and the result is 4.
c I think of a number, add 6, then multiply by 3 and the result is 32.

9 Form an equation for each question. (Let the unknown quantity be x.) 11:06

a The sum of a brother's and sister's ages is 57 years. If the brother is 5 years older than his sister, find their ages.
b A rectangle is three times longer than it is wide. If its perimeter is 48 cm, find its length and width.
c A father is presently three times as old as his son. In 10 years he will be twice as old as his son. Find their present ages.

ASSIGNMENT 11A Chapter review

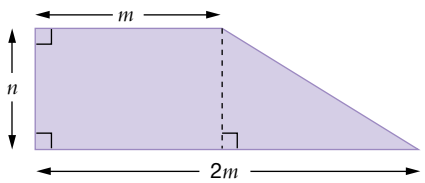
1 Write an algebraic expression for each of the following.

a the product of $3a$ and $4b$

b the total cost of x books at \$7 each and y books at \$9 each.

c the time it will take a car to travel d kilometres at s km/h

d the area of this figure



2 **a** If $a = 5$ and $b = 6$, evaluate:

i $a + b$ **ii** $ab - 10$

b If $x = 1{\cdot}6$ and $y = 2{\cdot}7$, evaluate:

i $2x + 3y$ **ii** $x^2 - x$

c Given that $m = 9$ and $n = 4{\cdot}5$, evaluate:

i $\frac{mn}{2}$ **ii** $\frac{m+n}{2}$

3 **a** If $v = u + at$, find v when:

i $u = 7, a = 3, t = 5$

ii $u = 9, a = 7, t = \frac{1}{2}$

iii $u = -10, a = 9{\cdot}8, t = 3{\cdot}7$

iv $u = 7{\cdot}25, a = -9{\cdot}8, t = 7{\cdot}5$

b Given that $P = 2l + 2b$, find b when:

i $P = 10, l = 2$

ii $P = 7, l = 1{\cdot}5$

iii $P = 77, l = 14$

iv $P = 68{\cdot}4, l = 14{\cdot}2$

v $P = 42, l = 6{\cdot}85$

c If $D = \frac{M}{V}$ find:

i D when $M = 16, V = 4$

ii D when $M = 20, V = 1{\cdot}5$

iii M when $D = 2, V = 5$

iv M when $D = 1{\cdot}12, V = 3{\cdot}6$

v V when $D = 2, M = 6$

vi V when $D = 1{\cdot}5, M = 0{\cdot}5$

4 **a** Given that $S = ut + \frac{1}{2}at^2$, find S when $u = 7, t = 3, a = 10$.

b If $C = 2mr$, find m when $C = 17{\cdot}6$ and $r = 1{\cdot}1$.

c If $P = \frac{1}{a} + \frac{1}{b}$, find P when $a = 0{\cdot}4$ and $b = 0{\cdot}625$.

5 Write an equation for each of the following and then solve it.

a A number is multiplied by 3, then 7 is added and the result is 15.

b Nine is subtracted from a number and the result is multiplied by five to equal thirty.

c Eight times a certain number plus ten is equal to twelve times the same number minus seven.

Formulas have an important role in engineering.

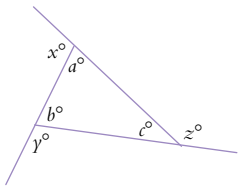
ASSIGNMENT 11B Working mathematically

1 **a** Find three numbers greater than 1 that multiply together to give 4301.

b How many sets of three numbers are there that result in this product? Justify your answer.

2 Alf left $1200 to be divided between his five grandchildren for their next birthdays. Each grandchild was to receive $40 more than the next youngest grandchild. How much did the oldest grandchild receive?

3 If the exterior angles x, y and z of a triangle are in the ratio $4:5:6$, what is the ratio of the interior angles a, b and c?



4 The average of five numbers is 11. A sixth number is added and the new average is 12. What is the sixth number?

5 A bottle containing 20 identical tablets has a mass of 180 grams. When 5 tablets are removed the mass is 165 grams. What is the mass of the bottle?

6 To win a trifecta, you have to pick 1st, 2nd and 3rd places in the correct order. In how many different ways can a trifecta be chosen in:

a a 3-horse race

b a 4-horse race

c a 5-horse race

d a 10-horse race?

ASSIGNMENT 11C Cumulative revision

1 Solve these inequalities. 7:09

a $2a - 7 > 10$ **b** $-3x < 12$ **c** $-\frac{3m}{5} \geq 12$

2 Under a fringe benefits tax agreement, employees can reduce their taxable income by paying 30% of their salary into an expense account. If their salary is $55 200 per annum, find: 2:01D

a how much can be paid into the expense account

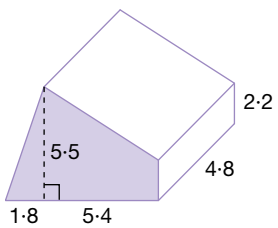
b the tax saving, if every dollar that goes into the expense account reduces the tax payable by 32·5 cents.

3 **a** Find the length of the interval joining the points $(1, -1)$ to $(-3, 2)$. 9:04, 9:03, 9:05

b Find the x- and y-intercepts of the line $3x + 2y = 9$.

c What is the slope of the line that has an x-intercept of -2 and a y-intercept of -4?

4 Calculate the volume of the following prism. (Measurements are in centimetres.) 5:06



5 Jim buys 2000 shares in the WeDiGiT mining company at a cost of $2.10 per share. After 3 months he sells 500 shares at $2.50 per share. He then waits for the market to improve and then sells another 750 shares for $3.20 per share. Find Jim's profit and the percentage return on the original cost of those 1250 shares. 8:05

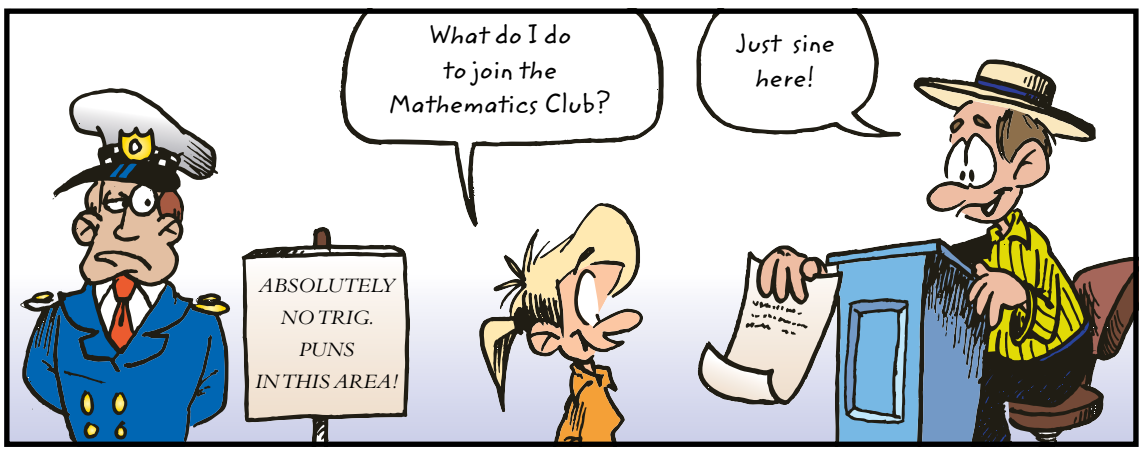
6 Hannah earns $34.75 per hour. 8:03

a Find her gross weekly wage if she works a 38 hour week.

b What is her net weekly wage if she has the following deductions: PAYG $287, Superannuation $223, Union Membership $5.20, Health Insurance $69.75, Credit Union Deduction (savings) $200?

TRIGONOMETRY

12



Contents

12:01 Right-angled triangles
12:02 Similar right-angled triangles: The ratio of sides
GeoGebra activity 12:02 Ratios of the sides of similar right-angled triangles
12:03 Tangent ratio
Investigation 12:03 Tangent ratio
12:04 Finding an unknown side
GeoGebra activity 12:04 Finding an unknown side using the tangent ratio
12:05 Finding an unknown angle
12:06 Sine and cosine ratios
12:07 Finding unknown sides with sine and cosine
12:07A Finding a short side
GeoGebra activity 12:07A Finding an unknown side using the sine and cosine ratios
12:07B Finding the hypotenuse
GeoGebra activity 12:07B Finding the hypotenuse using the sine and cosine ratios
12:08 Using sine and cosine to find an unknown angle
GeoGebra activity 12:08 Using the trig ratios to find an angle
12:09 Miscellaneous exercises
12:09A Angles of elevation and depression
12:09B Compass bearings
12:09C Other topics
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Right-Angled Triangles (Trigonometry)*** [Stages 5.1, 5.2$^{\lozenge}$]

- Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles. (ACMMG223)
- Apply trigonometry to solve right-angled triangle problems. (ACMMG224)
- Solve right-angled triangle problems, including those involving direction and angles of elevation and depression. (ACMMG245)

Working Mathematically

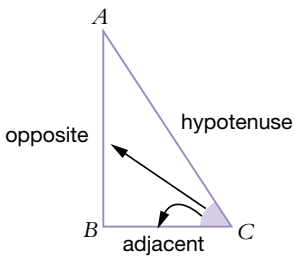
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

Trigonometry is a branch of geometry that is very important in fields such as navigation, surveying, engineering, astronomy and architecture. Basic trigonometry is used to find unknown sides and angles in right-angled triangles. The word 'trigonometry' is actually derived from two Greek words: *trigonon* (triangle) and *metron* (measurement).

12:01 Right-angled triangles

Before introducing trigonometry, we need to be aware of some further information concerning right-angled triangles.

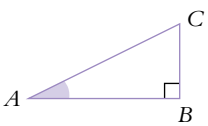
From Pythagoras' theorem, we know that the longest side in a right-angled triangle is called the hypotenuse. The other two sides also have names that refer to one of the acute angles in the triangle. The side farthest from the angle is the opposite side, and the side next to the angle is the adjacent side.



WORKED EXAMPLE 1

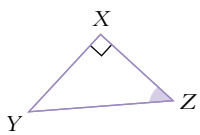
Name the sides in these two right-angled triangles with reference to the angle marked.

a



BC = opposite side
AB = adjacent side
AC = hypotenuse

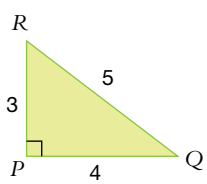
b



XY = opposite side
XZ = adjacent side
YZ = hypotenuse

WORKED EXAMPLE 2

Find the value of these ratios in ΔPQR.



a $\dfrac{\text{Side opposite angle } R}{\text{Hypotenuse}}$

Side opposite angle $R = 4$
Hypotenuse $= 5$
$\therefore$ Ratio $= \frac{4}{5}$

b $\dfrac{\text{Side opposite angle } Q}{\text{Side adjacent to angle } Q}$

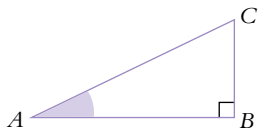
Side opposite angle $Q = 3$
Side adjacent to angle $Q = 4$
$\therefore$ Ratio $= \frac{3}{4}$

Exercise 12:01

P Foundation worksheet 12:01
Sides of right-angled triangles

1 Name the side opposite the marked angle in each triangle.

a



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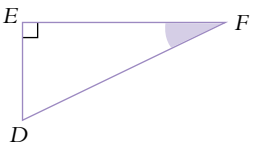
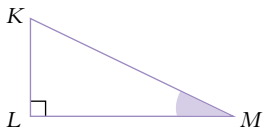


c



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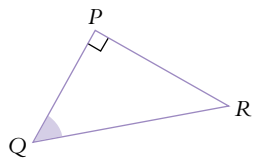
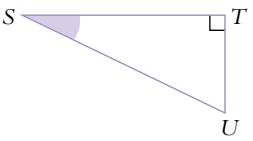


e



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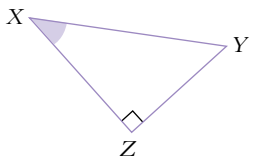


2 Name the adjacent side in each of the triangles in Question 1.

3 Name the hypotenuse in each triangle in Question 1.

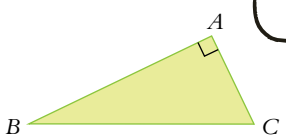
4 In triangle ABC:

a which side is opposite angle B

b which side is adjacent to angle C

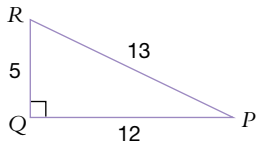
c which angle is opposite side AB

d which angle is adjacent to side AC?

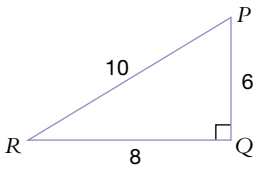


5 Find the value of the ratio $\frac{\text{Side opposite angle } P}{\text{Side adjacent to angle } P}$ in these triangles.

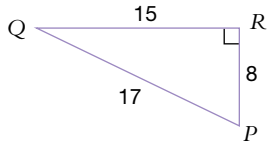
a



b



c



6 Find the value of the ratio $\frac{\text{Side adjacent to angle } P}{\text{Hypotenuse}}$ for each triangle in Question 5.

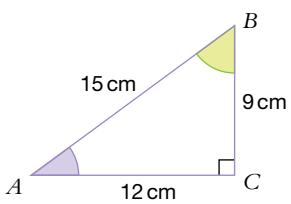
7 Find the value of each of the following ratios for the triangle shown.

a $\frac{\text{Side opposite angle } A}{\text{Hypotenuse}}$

b $\frac{\text{Side opposite angle } B}{\text{Hypotenuse}}$

c $\frac{\text{Side adjacent to angle } A}{\text{Hypotenuse}}$

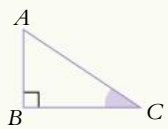
d $\frac{\text{Side opposite angle } B}{\text{Side adjacent to angle } B}$

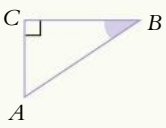


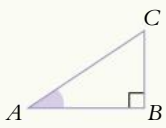
12:02 Similar right-angled triangles: The ratio of sides

PREP QUIZ 12:02

For each triangle state whether AB is opposite the angle marked, adjacent to the angle marked, or is the hypotenuse.

1 

2 

3 

Complete the following statements.

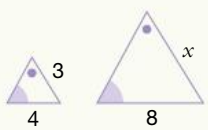
4 If a triangle is enlarged, the original triangle and the enlarged triangle are said to be …

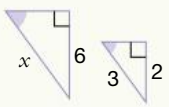
5 When a triangle is enlarged, the angles do not …

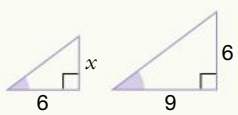
6 When a triangle is enlarged, the ratios of matching sides are …

7 If a 4 cm side is enlarged using a scale factor of 3, the length of the enlarged side is …

In the pairs of triangles below, find x.

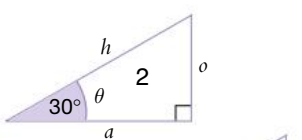
8 

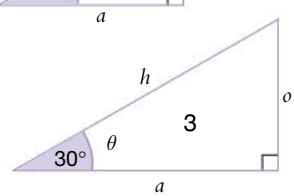
9 

10 

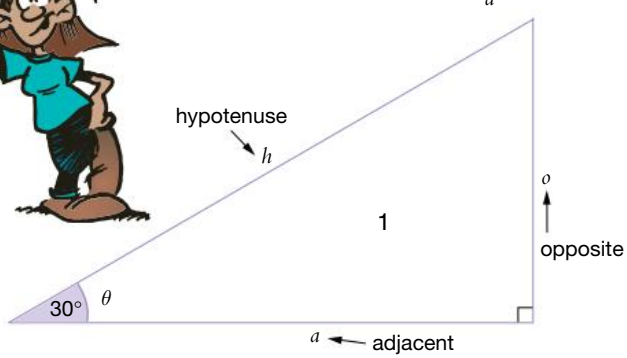
Exercise 12:02

1 a Three similar right-angled triangles are shown. Measure the lengths of the sides to the nearest millimetre and then copy and complete the following table.



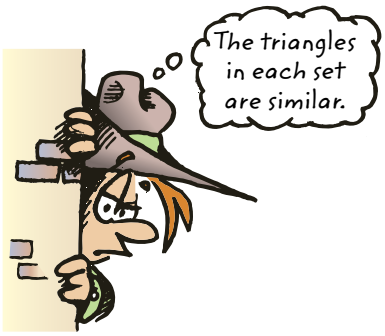


	θ	$\frac{o}{h}$	$\frac{a}{h}$	$\frac{o}{a}$
1	30°			
2	30°			
3	30°			

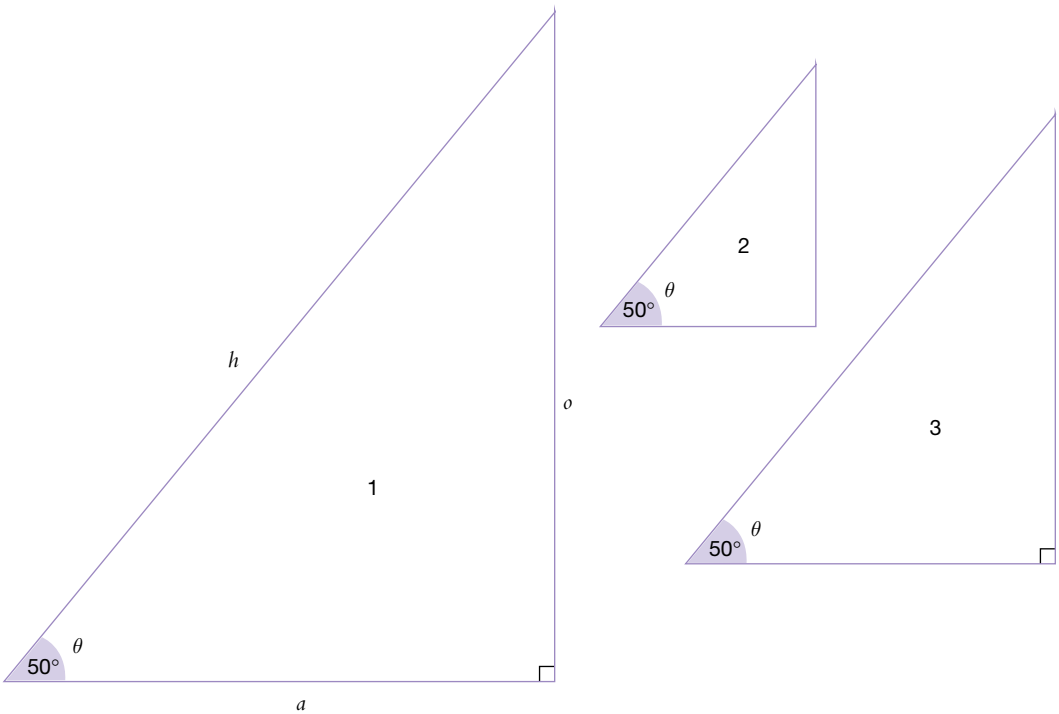


b What conclusion can you draw from the ratios $\frac{o}{h}, \frac{a}{h}, \frac{o}{a}$ in each triangle?

2 **a** Copy and complete the following table using the three similar triangles below. (Give your answers correct to one decimal place.)



	θ	$\frac{o}{h}$	$\frac{a}{h}$	$\frac{o}{a}$
1	50°			
2	50°			
3	50°			



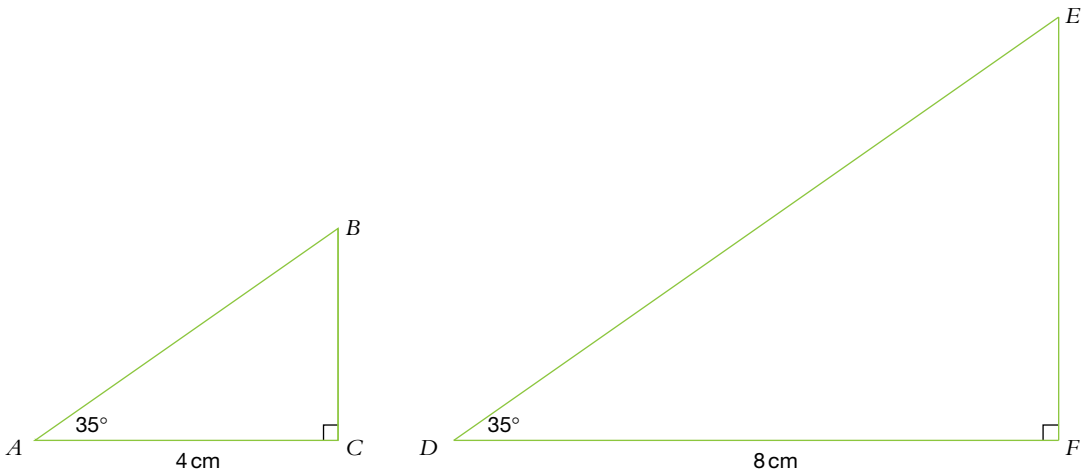
b What conclusion can you draw from the ratios $\frac{o}{h}, \frac{a}{h}, \frac{o}{a}$ in each triangle?

3 **a** Measure the sides of ΔABC on the following page and then calculate the value of the following ratios, correct to one decimal place.

i $\frac{o}{h}$ ii $\frac{a}{h}$ iii $\frac{o}{a}$

b Measure the sides of ΔDEF on the following page and calculate the value of each ratio, correct to one decimal place.

i $\frac{o}{h}$ ii $\frac{a}{h}$ iii $\frac{o}{a}$



c What conclusions can you draw from the results in parts **a** and **b**?

4 Triangle ABC is similar to triangle DEF (they have two pairs of matching angles equal).

a What enlargement factor has been used to change ΔABC into ΔDEF?

b Use the fact that the ratios of matching sides are equal to find the values of x and y.

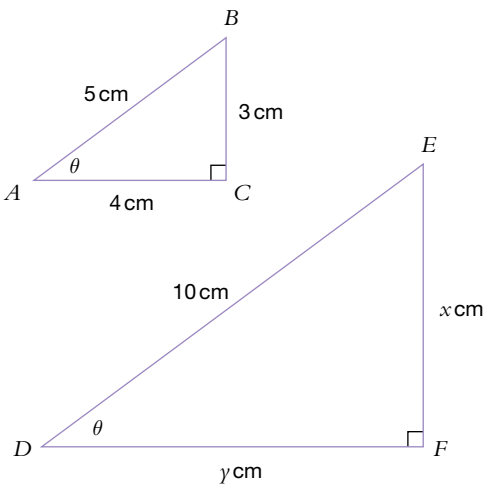
c For ΔABC, write the values of the ratios:

i $\frac{o}{h}$ ii $\frac{a}{h}$ iii $\frac{o}{a}$

d For ΔDEF, write the values of the ratios:

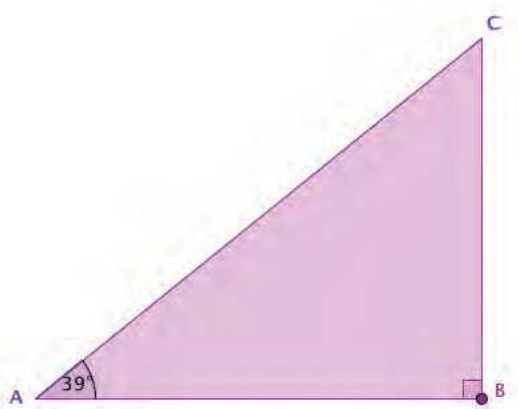
i $\frac{o}{h}$ ii $\frac{a}{h}$ iii $\frac{o}{a}$

e Compare the values of the ratios in the two triangles. What conclusions can you draw?



P GEOGEBRA ACTIVITY 12:02 RATIOS OF THE SIDES OF SIMILAR RIGHT-ANGLED TRIANGLES

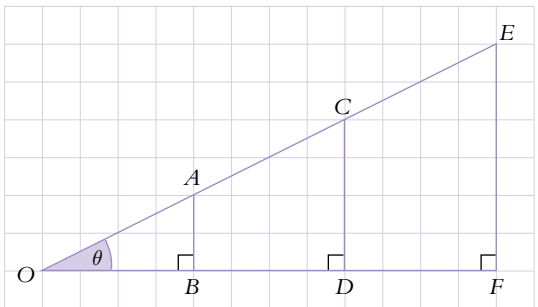
In this activity you can see that the values of the ratios $\frac{o}{h}$, $\frac{a}{h}$ and $\frac{o}{a}$ for a given angle in a right-angled triangle are constant, regardless of the size of the triangle.



12:03 Tangent ratio

In Chapter 10 we saw that two triangles are similar if one triangle is the enlargement of the other. When two triangles are similar they have three pairs of equal angles and the ratios of their matching sides are equal. (We say the sides are in proportion.)

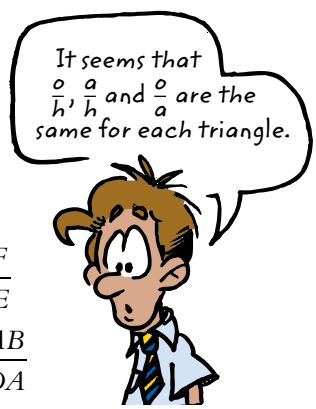
Similar triangles can always be superimposed to produce a diagram like the one below. Measure the sides and check the results in the table. (Answers are correct to one decimal place.)



	θ	$\frac{o}{h}$	$\frac{a}{h}$	$\frac{o}{a}$
ΔAOB	27°	0·4	0·9	0·5
ΔCOD	27°	0·4	0·9	0·5
ΔEOF	27°	0·4	0·9	0·5

- The answers in the table suggest that, in a series of similar right-angled triangles, the ratios $\frac{o}{h}$, $\frac{a}{h}$ and $\frac{o}{a}$ are equal for a particular angle. This is, in fact, true and can be proved in the following way.

In the diagram, ΔCOD and ΔEOF are enlargements of ΔAOB.
The enlargement factors are 2 and 3 respectively.



Hence:

$OD = 2OB$, $CD = 2AB$ and $OC = 2OA$
$OE = 3OA$, $EF = 3AB$ and $OF = 3OB$

Now in ΔAOB: $\frac{o}{h} = \frac{AB}{OA}$

ΔCOD: $\frac{o}{h} = \frac{CD}{OC} = \frac{2AB}{2OA} = \frac{AB}{OA}$

ΔEOF: $\frac{o}{h} = \frac{EF}{OE} = \frac{3AB}{3OA} = \frac{AB}{OA}$

$\therefore$ $\frac{o}{h}$ has the same value in all three triangles.

- This method can also be used to show that the ratios $\frac{a}{h}$ and $\frac{o}{a}$ also have the same values in all three triangles.

- Now let's look at the ratio $\frac{o}{a}$.

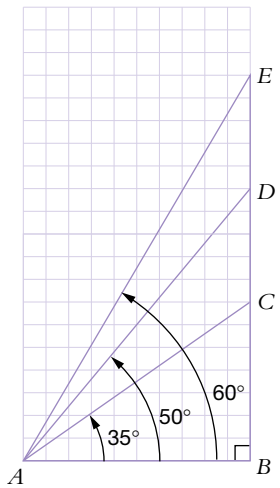
$\frac{o}{a}$ is an abbreviation for $\frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$

In ΔAOB: $\frac{o}{a} = \frac{2}{4}$
$= \frac{1}{2}$
$= 0{\cdot}5$

ΔCOD: $\frac{o}{a} = \frac{4}{8}$
$= \frac{1}{2}$
$= 0{\cdot}5$

ΔEOF: $\frac{o}{a} = \frac{6}{12}$
$= \frac{1}{2}$
$= 0{\cdot}5$

The ratio $\frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$ is called the tangent ratio of angle θ and is abbreviated to $\tan\theta$.

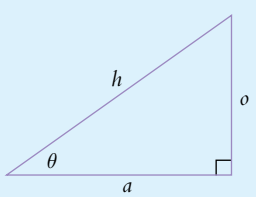


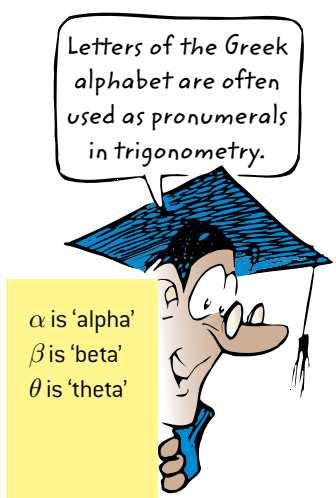
- Note that although the side lengths are not equal, the tangent ratio has a constant value for a given angle θ, regardless of how big the right-angled triangle might be.
- Furthermore, the diagram to the right suggests that each angle must have a different tangent ratio. As the angle A changes, so does the length of the opposite side and hence, so does the value of $\tan A$.
- The value of the tangent ratio can be estimated by graphical means but this has limited accuracy.
  e.g. The graph shows $\tan 35° \div 0{\cdot}7$.
  The value of the tangent ratio is usually obtained from a calculator.
- You would be used to measuring angles in degrees. When greater accuracy is required, angles can be measured in degrees, minutes and seconds. The relationships between the units are:
  1 degree = 60 minutes and 1 minute = 60 seconds
- Angles will be given in either degrees (e.g. 40°) or degrees and minutes (e.g. 40°20′). Note the symbol for minutes.
- Modern calculators have a Degrees, Minutes, Seconds key (° ′ ″) for entering angles given in degrees and minutes. The use of this key is explained in the examples below.

In a right-angled triangle, the tangent ratio of angle θ is:

$\tan\theta = \frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$

$\tan\theta = \frac{o}{a}$



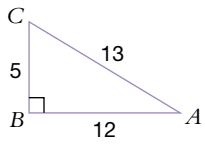


α is 'alpha'
β is 'beta'
θ is 'theta'

WORKED EXAMPLES

1 Find $\tan A$ in these triangles.

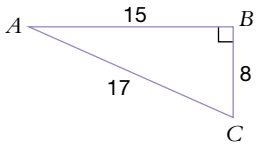
a



$$\tan A = \frac{\text{side opposite } A}{\text{side adjacent to } A}$$

$$= \frac{5}{12} \text{ (or approx. 0·417 as a decimal)}$$

b



$$\tan A = \frac{\text{side opposite } A}{\text{side adjacent to } A}$$

$$= \frac{8}{15} \text{ (or approx. 0·533 as a decimal)}$$

The keying instructions shown in this chapter are for the Casio fx-82AU PLUS. Note that calculations can often be done in more than one way and that usually only one solution is given.

2 Use your calculator to find the value of each tangent ratio, correct to three decimal places.

a $\tan 25°$

[tan] [25] [=]

The calculator gives 0·466, correct to three decimal places.

b $\tan 40°25'$

[tan] [40] [°'"] [25] [°'"] [=]

The calculator gives 0·852, correct to three decimal places.

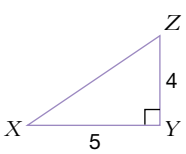
The calculator must be in 'degrees' mode.

Exercise 12:03

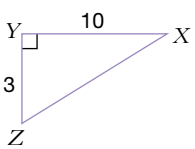
P Foundation worksheet 12:03 The tangent ratio

1 Find the tangent ratio for angle X in each of the following.

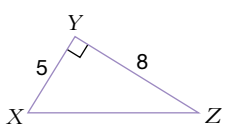
a



b



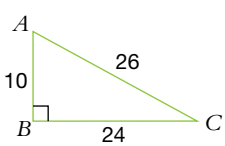
c



2 Find $\tan Z$ for each triangle in Question 1.

3 Find the tangent ratios in each of the following triangles.

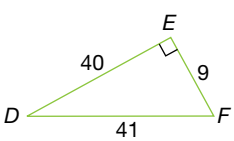
a



$\tan A =$

$\tan C =$

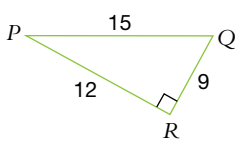
b



$\tan D =$

$\tan F =$

c



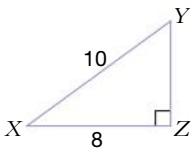
$\tan P =$

$\tan Q =$

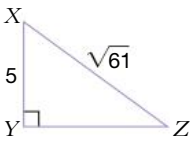
Pythagoras' theorem is used to find an unknown side in a right-angled triangle when the other two sides are known. Trigonometry is used when only one side and one of the acute angles is known.

4 Use Pythagoras' theorem to find the length of YZ, then state $\tan X$.

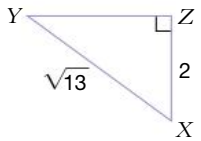
a



b

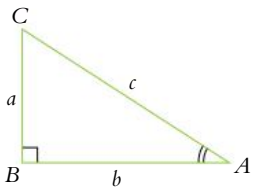


c

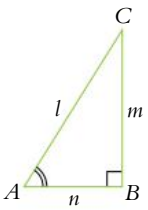


5 Find $\tan A$ for each triangle.

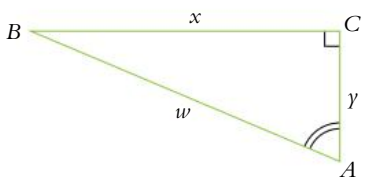
a



b



c

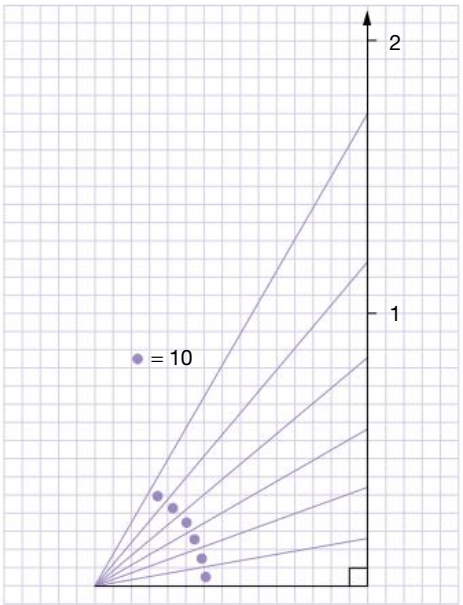


6 **a** Use the grid on the diagram to estimate the value of the following.

- **i** $\tan 10°$
- **ii** $\tan 40°$
- **iii** $\tan 60°$

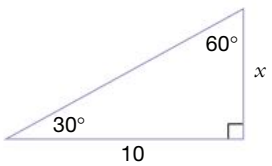
b Use the diagram to estimate the size of the angle with a tangent ratio of:

- **i** 0·25
- **ii** 0·7
- **iii** 1·4

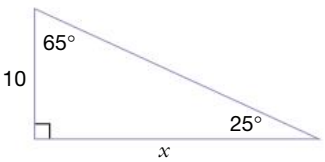


7 In each of the following, select which angle has a tangent ratio of $\frac{x}{10}$.

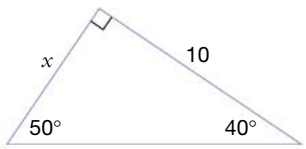
a



b

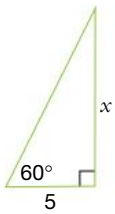


c



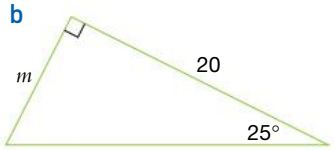
8 Complete each of the following.

a



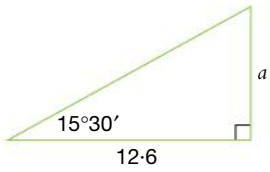
$\tan 60° = \ldots$

b



$\tan 25° = \ldots$

c



$\tan 15°30' = \ldots$

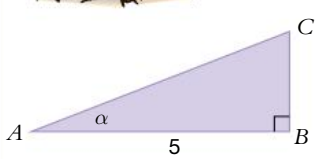
9 Use a calculator to find the value of the tangent ratio for the following angles. (Answer correct to three decimal places.)

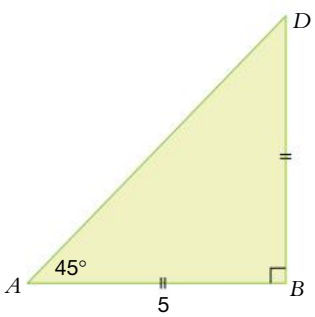
a 10° **b** 20° **c** 30° **d** 60° **e** 48·6°
f 25°10′ **g** 46°40′ **h** 72°6′ **i** 15°30′ **j** 62°33′

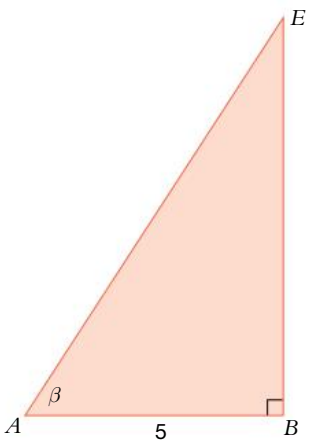
10 Use a calculator to find each of the following, correct to three decimal places.

a tan 25° **b** tan 37° **c** tan 63° **d** tan 45° **e** tan 16·5°
f tan 14°8′ **g** tan 28°16′ **h** tan 70°6′ **i** tan 30°18′ **j** tan 55°55′

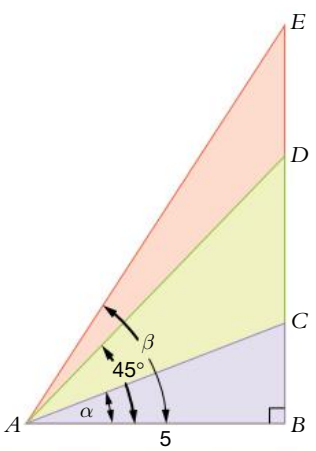
INVESTIGATION 12:03 TANGENT RATIO







The three triangles each have a base of 5 units.
They have been superimposed to produce the figure below.



1 What type of triangle is ΔABD?
2 What is the value of $\tan 45°$?
3 How does $\tan \alpha$ compare with $\tan 45°$?
4 How does $\tan \beta$ compare with $\tan 45°$?
5 What happens to α as BC becomes smaller?
6 What happens to β as BE becomes larger?
7 What happens to $\tan \alpha$ as α approaches 0°?
8 What happens to $\tan \beta$ as β approaches 90°?

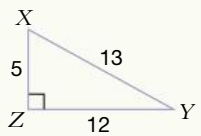
12:04 Finding an unknown side

PREP QUIZ 12:04

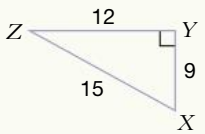
1 Use a calculator to find tan 40°20′, correct to three decimal places.

State the value of tan X.

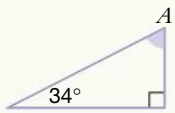
2



3

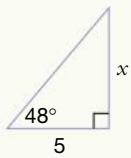


4 Find the size of angle A.



5 What is the value of tan 48°, correct to three decimal places?

6 Find a value for the ratio $\frac{x}{5}$ for this triangle, correct to three decimal places.



7 Calculate the value of $5 \times \tan 48°$.

8 If $\frac{x}{5} = \tan 48°$, find x.

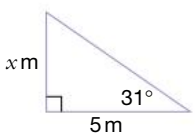
9 If $\tan 25° = \frac{4}{x}$, then $\frac{1}{\tan 25°} = \ldots$

10 If $\frac{7}{y} = \tan 85°$, then $\frac{y}{7} = \ldots$

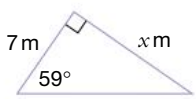
WORKED EXAMPLE 1

Find x in these triangles, correct to two decimal places.

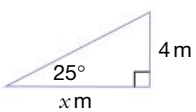
a



b



c



Solutions

a $\frac{x}{5} = \tan 31°$

$\therefore x = 5 \times \tan 31°$

$= 3{\cdot}00$ (2 dec. pl.)

b $\frac{x}{7} = \tan 59°$

$\therefore x = 7 \times \tan 59°$

$= 11{\cdot}65$ (2 dec. pl.)

For $7 \times \tan 59°$ press:
7 × tan 59 =

c For a question like this, there are two methods.

Method 1

$\frac{4}{x} = \tan 25°$

$\frac{x}{4} = \frac{1}{\tan 25°}$

$x = \frac{4}{\tan 25°}$

$= 8{\cdot}58$

For $\frac{4}{\tan 25°}$ press:
4 ÷ tan 25 =

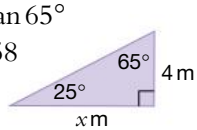
Method 2

(finding the angle opposite x)

$\frac{x}{4} = \tan 65°$

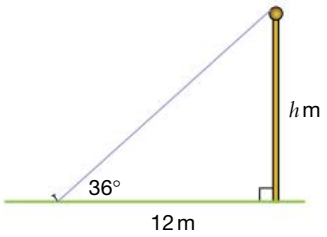
$x = 4 \tan 65°$

$= 8{\cdot}58$



WORKED EXAMPLE 2

A flagpole has a guy rope from the top of the pole to a point on the ground 12 m from the base of the pole. If the rope makes an angle of 36° with the ground, find the height of the pole.



Solution

First, draw a diagram with the height of the flagpole, h.

From the diagram: $\frac{h}{12} = \tan 36°$

$\therefore\ h = 12 \tan 36°$

$= 8{\cdot}7$ (1 dec. pl.)

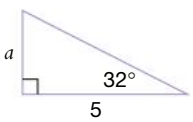
$\therefore$ The height of the flagpole is 8·7 m.

Exercise 12:04

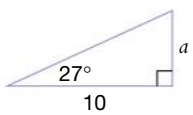
P Foundation worksheet 12:04
Finding unknown sides using the tangent ratio

1 Find the value of a, correct to one decimal place, for each triangle.

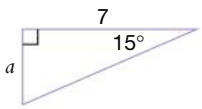
a



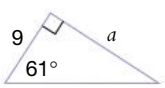
b



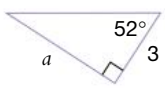
c



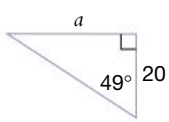
d



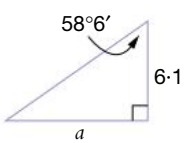
e



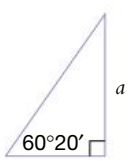
f



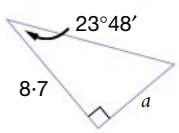
g



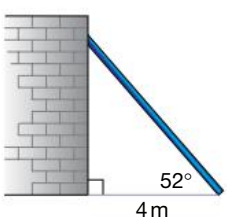
h



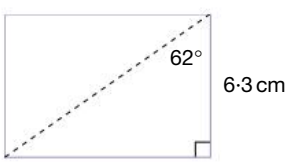
i



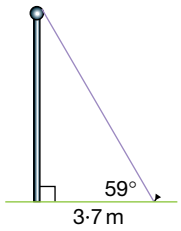
2 A ladder leans against a wall so that the angle it makes with the ground is 52°. Its base is 4 m from the wall. How far does the ladder reach up the wall? Give your answer correct to the nearest centimetre.



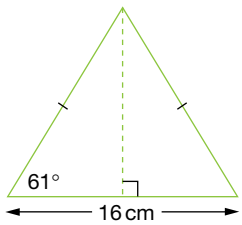
3 Find the length of a rectangle if the diagonal makes an angle of 62° with the breadth, which is 6·3 cm. (Give your answer correct to one decimal place.)



4 How high is this flagpole (to the nearest cm) if a supporting wire from the top makes an angle of 59° with the ground and the wire meets the ground 3·7 m from the base of the flagpole?

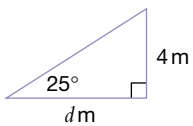


5 Find the height of this isosceles triangle. It has a base of 16 cm and base angles of 61°. Give your answer correct to one decimal place.

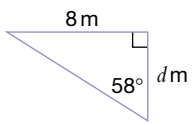


6 Determine the value of d, correct to one decimal place. (See Worked Example **1c**)

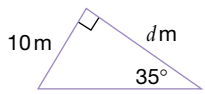
a



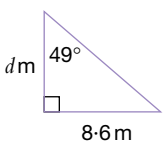
b



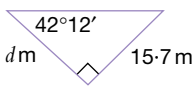
c



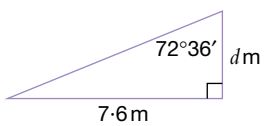
d



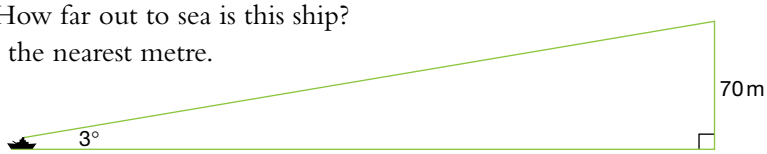
e



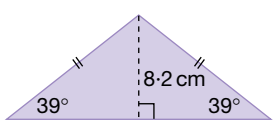
f



7 A ship out to sea observes a lighthouse on the top of a 70 m cliff at an angle of 3°. How far out to sea is this ship? Give your answer correct to the nearest metre.

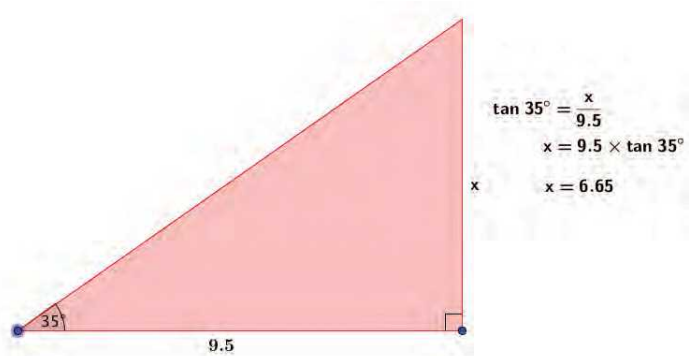


8 Find the base of an isosceles triangle if the height is 8·2 cm and the base angles are each 39°. Give your answer correct to the nearest millimetre.



P GEOGEBRA ACTIVITY 12:04 FINDING AN UNKNOWN SIDE USING THE TANGENT RATIO

This activity allows you to practise finding an unknown side using the tangent ratio. The shape of the triangle can be changed and the length of an unknown side has to be calculated. Two different types of exercises can be done and a checkbox can be ticked to show the solution.

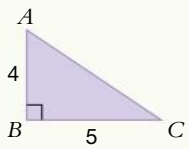


12:05 Finding an unknown angle

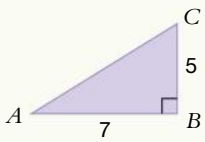
PREP QUIZ 12:05

State the value of $\tan A$, as a fraction, for these triangles.

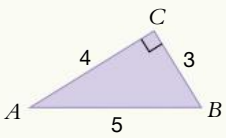
1



2



3



Write these fractions as decimals.

4 $\frac{4}{5}$ **5** $\frac{2}{3}$ **6** $\frac{5}{12}$

The table gives the value of the tan ratio for some angles.

From the table, find:

7 $\tan 5°$
8 $\tan 48°$

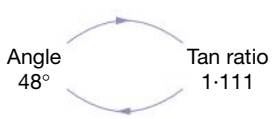
From the table, find the angle for which the tangent is:

9 0·158
10 1·192

Degrees	Tan	Degrees	Tan
0	0·000	45	1·000
1	0·018	46	1·035
2	0·035	47	1·072
3	0·052	48	1·111
4	0·070	49	1·150
5	0·088	50	1·192
6	0·105	51	1·235
7	0·123	52	1·280
8	0·141	53	1·327
9	0·158	54	1·376

In the previous sections, all the problems involved finding the length of the unknown side.

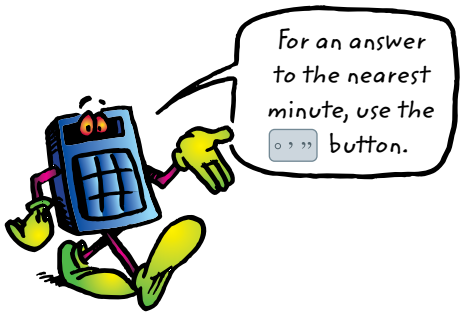
We can also use the tan ratio to find an unknown angle in a right-angled triangle. Because every angle has a unique tan ratio, this allows us to find an angle when we know its tan ratio.



This relationship is easily seen when the information is shown in a table (as in the prep quiz above) but it can't be seen inside a calculator.

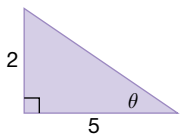
We need a new keying strategy.

If $\tan\theta = 1{\cdot}5$ press: SHIFT tan 1.5 =
The calculator gives 56·309 … or 56° to the nearest degree. For an answer to the nearest minute, press ° ′ ″. The calculator gives 56°18′35·76″ or 56°19′ to the nearest minute.

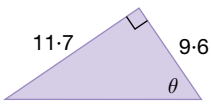


WORKED EXAMPLES

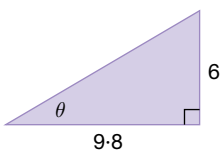
1 Find the size of angle θ correct to the nearest degree.



2 Find the size of angle θ, correct to the nearest minute.



3 Find the value of θ in degrees, correct to one decimal place.



Solutions

1 In this triangle, $\tan\theta = \frac{2}{5}$
$= 0{\cdot}4$

Press:

SHIFT tan 0.4 =

$\theta = 21{\cdot}80\ \ldots$
$= 22°$ (nearest degree)

2 Here, $\tan\theta = \frac{11{\cdot}7}{9{\cdot}6}$

Press:

SHIFT tan 11.7 ÷ 9.6 = ° ′ ″

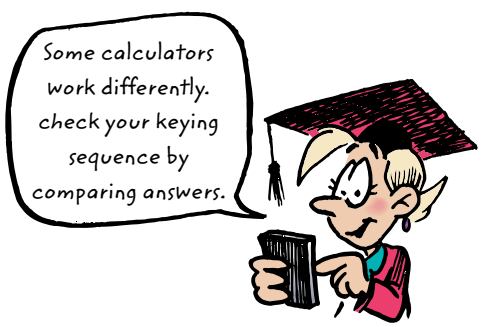
$\theta = 50°37'50{\cdot}46$
$= 50°38'$ (nearest minute)

3 $\tan\theta = \frac{6}{9{\cdot}8}$

Press:

SHIFT tan 6 ÷ 9.8 =

$\theta = 31{\cdot}47\ \ldots$
$= 31{\cdot}5°$ (1 dec. pl.)



Exercise 12:05

P Foundation worksheet 12:05
Finding unknown angles using the tangent ratio

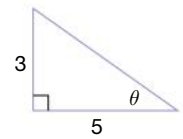
1 Find the size of θ, to the nearest degree, if:

a $\tan\theta = \frac{4}{5}$ **b** $\tan\theta = \frac{9}{4}$ **c** $\tan\theta = \frac{1}{2}$ **d** $\tan\theta = \frac{1{\cdot}8}{2{\cdot}5}$ **e** $\tan\theta = \frac{6{\cdot}5}{4{\cdot}2}$

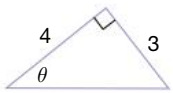
f $\tan\theta = \frac{11{\cdot}6}{20}$ **g** $\tan\theta = 0{\cdot}41$ **h** $\tan\theta = 0{\cdot}25$ **i** $\tan\theta = 4{\cdot}1$

2 Find the size of the angle marked θ in these triangles. Give answers to the nearest minute.

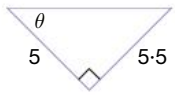
a



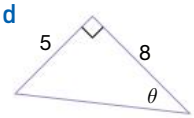
b



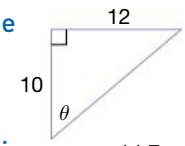
c



d



e



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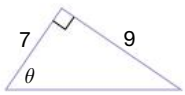

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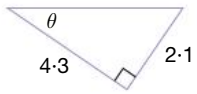

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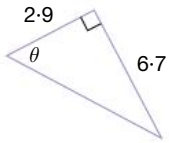
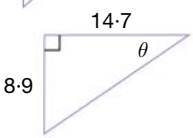


i



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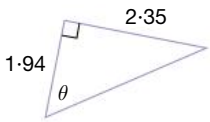
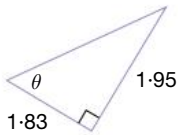
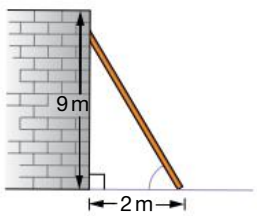


k



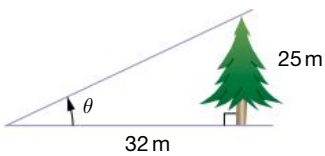
In the following questions, give all your answers correct to the nearest degree.

3 A ladder reaches 9 m up a wall. The foot of the ladder is 2 m from the base of the wall. What angle does the ladder make with the ground?

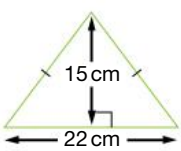


4 A road is inclined so that it rises 1 m for each horizontal distance of 8 m. What angle does the road make with the horizontal?

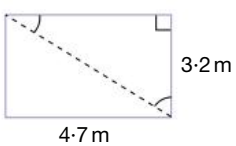
5 At a certain time of the day, a tree 25 m high casts a shadow 32 m long. At this time of day, what angle do the rays of the Sun make with the ground?



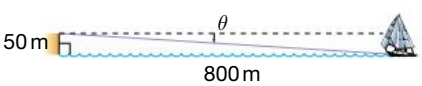
6 Find the base angles of an isosceles triangle if the height of the triangle is 15 cm and the length of the base is 22 cm.



7 What angles does a diagonal of a rectangle make with each of the sides if the dimensions of the rectangle are 4·7 m by 3·2 m?

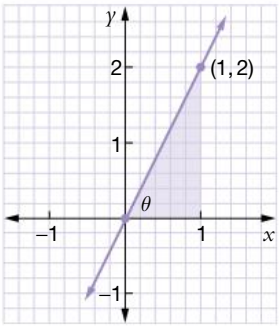


8 Find the angle of depression (marked θ in the diagram) from the top of a cliff 50 m high to a boat 800 m from the base of the cliff.

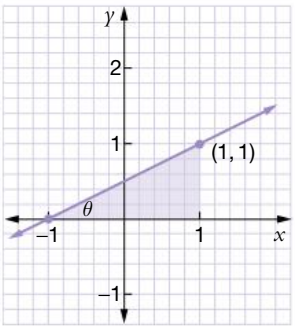


9 Trigonometry can be used in number plane work to find angles. Use the coloured triangle to find the value of θ (to the nearest degree) in each of the following.

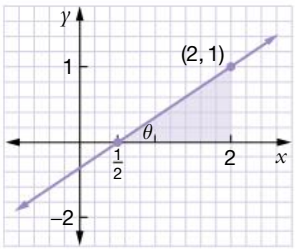
a



b

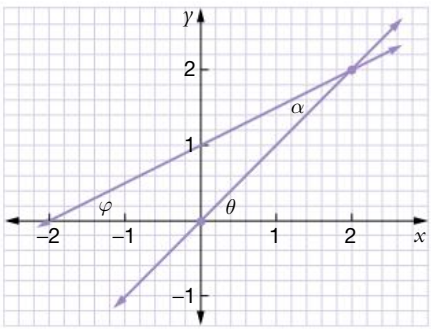


c



10 **a** Find the angles α, θ and φ in the diagram.

b A line has an x-intercept of 1 and passes through the point (3, 1). What acute angle does this line make with the x-axis?

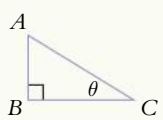


12:06 Sine and cosine ratios

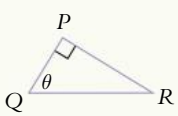
PREP QUIZ 12:06

Name the side opposite angle θ in each triangle.

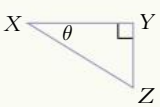
1



2

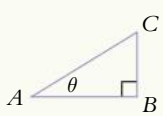


3

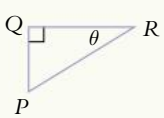


Name the side adjacent to angle θ in these triangles.

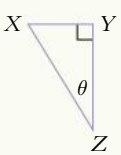
4



5



6



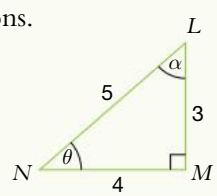
For the triangle LMN, form the following fractions.

7 $\dfrac{\text{side opposite } \theta}{\text{side adjacent to } \theta}$

8 $\dfrac{\text{side opposite } \theta}{\text{hypotenuse}}$

9 $\dfrac{\text{side adjacent to } \theta}{\text{hypotenuse}}$

10 $\dfrac{\text{side opposite } \alpha}{\text{side adjacent to } \alpha}$



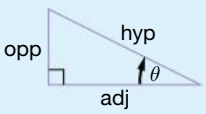
In Exercise 12:03, you were introduced to three ratios: $\frac{o}{h}$, $\frac{a}{h}$ and $\frac{o}{a}$. The last three exercises concentrated on the tangent ratio $\frac{o}{a}$. We now look at the sine ratio $\frac{o}{h}$ and the cosine ratio $\frac{a}{h}$.

The sine ratio of angle $\theta = \dfrac{\text{side opposite } \theta}{\text{hypotenuse}}$ OR $\sin\theta = \frac{o}{h}$

The cosine ratio of angle $\theta = \dfrac{\text{side adjacent to } \theta}{\text{hypotenuse}}$ OR $\cos\theta = \frac{a}{h}$

As tangent is abbreviated to tan, so sine is abbreviated to sin, and cosine to cos.

Just as $\tan\theta$ had a constant value for a given angle θ in any right-angled triangle, so do $\sin\theta$ and $\cos\theta$. The values of these ratios can be obtained from a calculator.

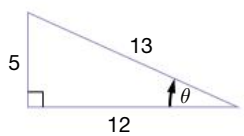


$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

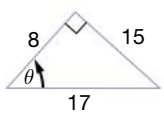
WORKED EXAMPLE 1

State $\sin\theta$ and $\cos\theta$ for these triangles.

a



b



Solutions

a $\sin\theta = \dfrac{\text{side opposite } \theta}{\text{hypotenuse}}$

$= \frac{5}{13}$

$\cos\theta = \dfrac{\text{side adjacent to } \theta}{\text{hypotenuse}}$

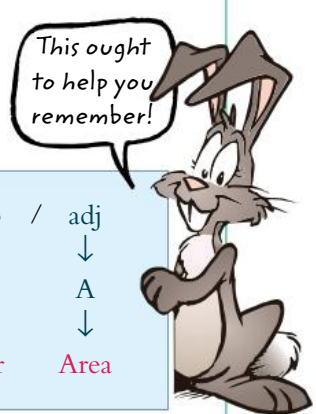
$= \frac{12}{13}$

b $\sin\theta = \dfrac{\text{side opposite } \theta}{\text{hypotenuse}}$

$= \frac{15}{17}$

$\cos\theta = \dfrac{\text{side adjacent to } \theta}{\text{hypotenuse}}$

$= \frac{8}{17}$



sin	=	opp	/	hyp	cos	=	adj	/	hyp	tan	=	opp	/	adj
↓		↓		↓	↓		↓		↓	↓		↓		↓
S		O		H	C		A		H	T		O		A
↓		↓		↓	↓		↓		↓	↓		↓		↓
Some		Old		Hare	Came		A		Hopping	Through		Our		Area

WORKED EXAMPLE 2

Use your calculator to find the values of the following ratios, correct to four decimal places.

a $\sin 25°$ **b** $\cos 67°15'$

Solutions

a Press:

sin 25 =

The calculator gives 0·4226. (4 dec. pl.)

b Press:

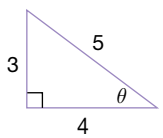
cos 67 ° ' " 15 ° ' " =

The calculator gives 0·3867. (4 dec. pl.)

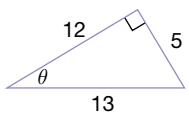
Exercise 12:06

1 Find $\sin\theta$ and $\cos\theta$ in these triangles (as a simple fraction).

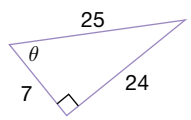
a



b

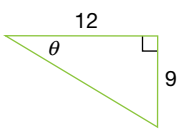


c

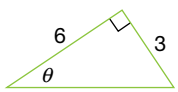


2 Find the unknown side using Pythagoras' theorem and then find $\sin\theta$ and $\cos\theta$ in decimal form, correct to three decimal places.

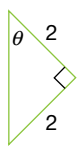
a



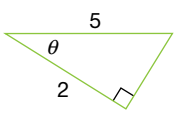
b



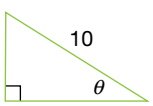
c



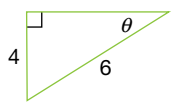
d



e



f



3 Use your calculator to find the value of the following, correct to four decimal places.

a $\sin 30°$	**b** $\cos 30°$	**c** $\sin 71°$	**d** $\cos 58°$
e $\sin 7°$	**f** $\cos 85°$	**g** $\cos 29°$	**h** $\sin 61°$
i $\sin 17°$	**j** $\cos 50°20'$	**k** $\sin 40°7'$	**l** $\cos 6°58'$

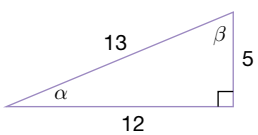
4 **a** What happens to $\sin A$ as A increases from 0° to 90°?

b What happens to $\cos A$ as A increases from 0° to 90°?

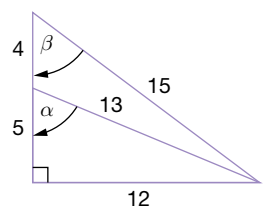
5 For each of the following diagrams find the values of:

i $\sin\beta$ **ii** $\cos\beta$ **iii** $\sin\alpha$ **iv** $\cos\alpha$

a



b



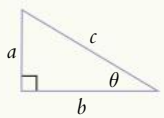
12:07 Finding unknown sides with sine and cosine

12:07A Finding a short side

PREP QUIZ 12:07A

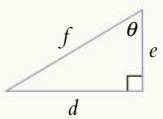
Write *true* or *false* for the sin or cos values given for each of these triangles.

1



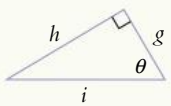
$\sin\theta = \frac{c}{a}$

2



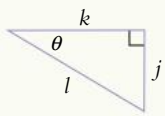
$\cos\theta = \frac{e}{f}$

3



$\cos\theta = \frac{g}{i}$

4



$\sin\theta = \frac{j}{l}$

Find these correct to three decimal places.

5 $\sin 69°$ **6** $\cos 19°$ **7** $\sin 45°$

Calculate:

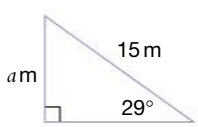
8 $8 \times \sin 69°$ **9** $10 \times \cos 19°$ **10** $6{\cdot}7 \times \sin 45°$

The method used in Section 12:05 to find an unknown side is now repeated using the sine or cosine ratio instead of the tangent ratio. The first step is to decide which ratio should be used.

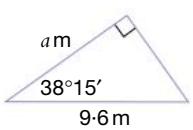
WORKED EXAMPLE 1

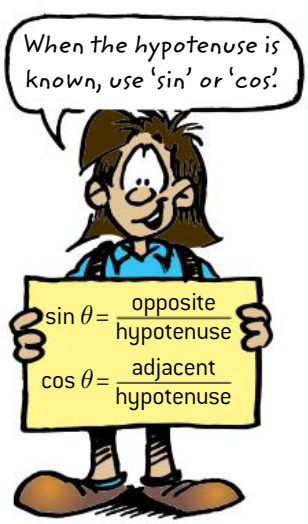
Find a in these triangles, correct to one decimal place.

a



b





Solutions

a $\frac{a}{15} = \sin 29°$

$\therefore\ a = \sin 29° \times 15$

$= 7{\cdot}272\,144\,3$

$= 7{\cdot}3$ (1 dec. pl.)

For $15 \times \sin 29°$ press:

15 × sin 29 =

b $\frac{a}{9{\cdot}6} = \cos 38°15'$

$\therefore\ a = \cos 38°15' \times 9{\cdot}6$

$= 7{\cdot}539\,042\,5$

$= 7{\cdot}5$ (1 dec. pl.)

For $9{\cdot}6 \times \cos 38°15'$ press:

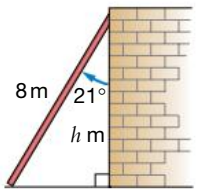
9.6 × cos 38 ° ′ ″ 15 ° ′ ″ =

WORKED EXAMPLE 2

A ladder, 8 m long, leans against a wall at an angle of 21°.
How far does the ladder reach up the wall, to the nearest centimetre?

Solution

From the information in the question, a diagram like the one below can be drawn. Let the height of the wall be h metres.



$$\frac{h}{8} = \cos 21°$$
$$h = 8 \times \cos 21°$$
$$= 7{\cdot}468\,643\,4$$
$$= 7{\cdot}47 \text{ (nearest cm)}$$

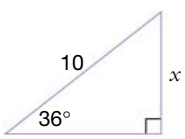
The ladder reaches 7·47 m up the wall.

Exercise 12:07A

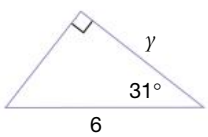
P Foundation worksheet 12:07A
Finding unknown sides using sine and cosine

1 Find the value of the pronumeral in each triangle, correct to one decimal place.

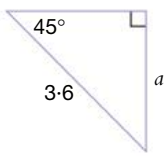
a



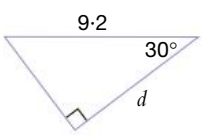
b



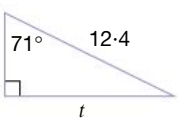
c



d



e



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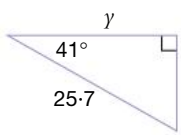

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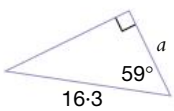

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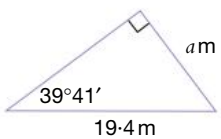
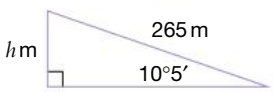
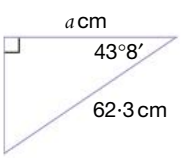


i

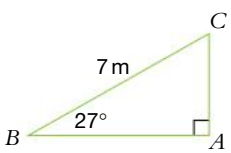


j

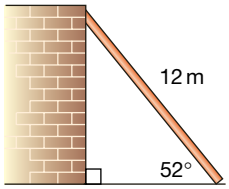


Use the sine or cosine ratio to solve the following problems.

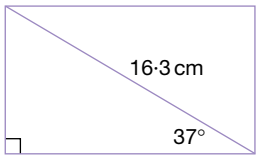
2 In a right-angled triangle ABC, $\angle A = 90°$, $\angle B = 27°$, $BC = 7$ m. Find the length of side AC, correct to one decimal place.



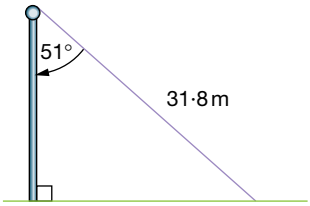
3 A ladder is 12 m long. If it leans against a wall, making an angle of 52° with the ground, how far will it reach up the wall (to the nearest centimetre)?



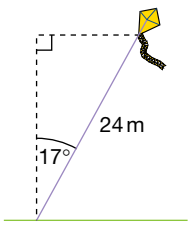
4 The diagonal of a rectangle is 16·3 cm long and makes an angle with the longest side of 37°. Find the length of the rectangle, to the nearest centimetre.



5 A rope is stretched from the top of a pole to a point on the ground so that the rope is inclined at an angle of 51° to the vertical. How high is the pole, to the nearest metre?

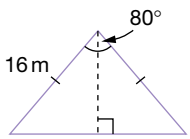


6 A kite is flying on the end of a string that is 24 m long. If the string makes an angle of 17° with the vertical, find the height of the kite above the ground. Give your answer correct to the nearest metre.

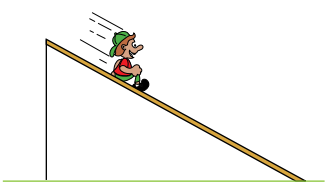


7 The diagonal of a square is 21·2 cm. Find the length of each side (to the nearest millimetre).

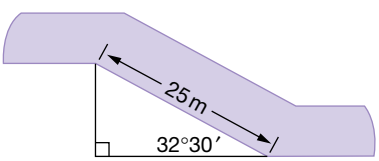
8 The equal sides of an isosceles triangle are 16 m long and the apex angle is 80°. Find, to the nearest centimetre, the length of the base.



9 If the length of a child's slippery-dip is 3·4 m and one end makes an angle of 38°42′ with the ground, how high above the ground is the other end? (Answer to the nearest centimetre.)

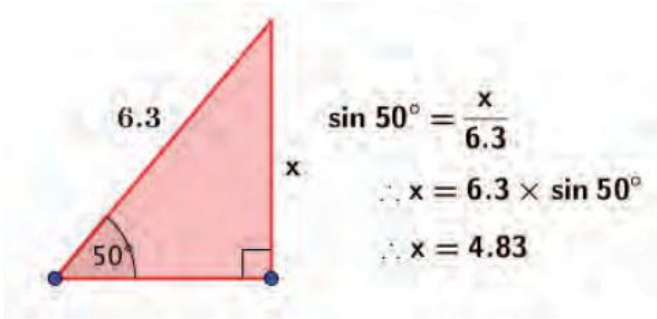


10 An escalator at an airport slopes at an angle of 32°30′ and is 25 m long. What vertical height would a person be lifted when travelling on the escalator?



GEOGEBRA ACTIVITY 12:07A FINDING AN UNKNOWN SIDE USING THE SINE AND COSINE RATIOS

This activity allows you to practise finding a short side of a right-angled triangle using either the sine or cosine ratio. You can control the shape of the triangle by using a slider and dragging the vertices of the triangle. After calculating the length of the missing side you can check the answer by ticking a checkbox.

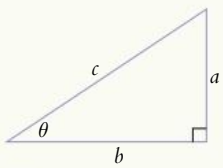


12:07B Finding the hypotenuse

PREP QUIZ 12:07B

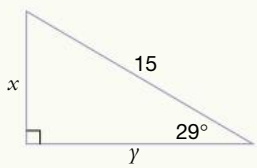
For the triangle on the right, write an expression for:

1 $\sin\theta$ **2** $\cos\theta$



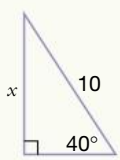
For the triangle on the right, write an expression for:

3 $\sin 29°$ **4** $\cos 29°$



Complete the calculation under each of these triangles.

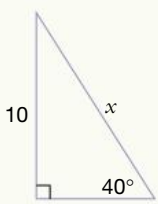
5



$\sin 40° = \frac{\text{opp}}{\text{hyp}}$

$\sin 40° = \ldots$

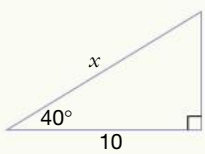
6



$\sin 40° = \frac{\text{opp}}{\text{hyp}}$

$\sin 40° = \ldots$

7



$\cos 40° = \frac{\text{adj}}{\text{hyp}}$

$\cos 40° = \ldots$

8 If $\frac{8}{1} = \frac{5}{x}$, then $\frac{1}{8} = \ldots$

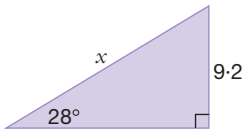
9 If $\frac{\sin 40°}{1} = \frac{5}{x}$, then $\frac{1}{\sin 40°} = \ldots$

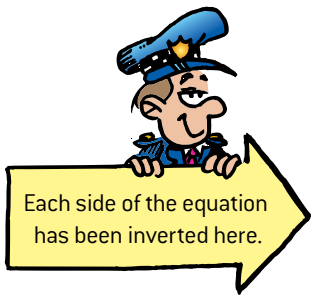
10 Evaluate $\frac{5}{\sin 40°}$, correct to two decimal places.

When solving all the problems in the last exercise, you should have noticed that the unknown side was always the numerator of the ratio involved. This is not true for all trigonometry problems, particularly when the hypotenuse is the unknown side.

In this triangle, the two sides involved are the opposite and hypotenuse, so the sine ratio must be used.

$$\sin = \frac{\text{opp}}{\text{hyp}} \quad \text{so} \quad \frac{9{\cdot}2}{x} = \sin 28°$$





This type of problem is then solved as follows.

$$\frac{9{\cdot}2}{x} = \sin 28°$$

$$\frac{x}{9{\cdot}2} = \frac{1}{\sin 28°}$$

$$x = \frac{9{\cdot}2}{\sin 28°}$$

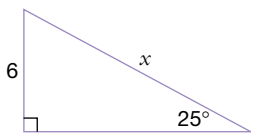
$$= 19{\cdot}6 \text{ (1 dec. pl.)}$$

Press: 9.2 ÷ sin 28 =

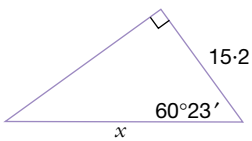
WORKED EXAMPLE 1

Find x, correct to one decimal place.

a



b



Solutions

a

$$\frac{6}{x} = \sin 25°$$

$$\text{so } \frac{x}{6} = \frac{1}{\sin 25°}$$

$$x = \frac{6}{\sin 25°}$$

$$= 14{\cdot}2 \text{ (1 dec. pl.)}$$

b

$$\frac{15{\cdot}2}{x} = \cos 60°23'$$

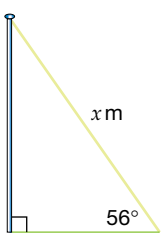
$$\text{so } \frac{x}{15{\cdot}2} = \frac{1}{\cos 60°23'}$$

$$x = \frac{15.2}{\cos 60°23'}$$

$$= 30{\cdot}8 \text{ (1 dec. pl.)}$$

WORKED EXAMPLE 2

Find the length of a guy rope that must be used to secure a pole 12·5 m high, if the angle the guy rope makes with the ground is 56°.



Solution

Let the length of the rope be x metres.

$$\frac{12{\cdot}5}{x} = \sin 56°$$

$$\frac{x}{12{\cdot}5} = \frac{1}{\sin 56°}$$

$$x = \frac{12{\cdot}5}{\sin 56°}$$

$$= 15{\cdot}08 \text{ (nearest cm)}$$

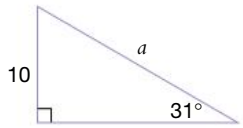
$\therefore$ The rope is 15·08 m long.

Exercise 12:07B

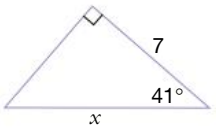
P Foundation worksheet 12:07B
Finding the hypotenuse

1 Determine the values of the pronumerals, correct to one decimal place.

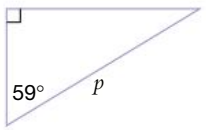
a



b



c



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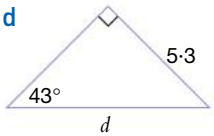

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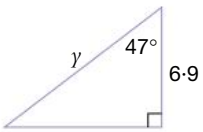

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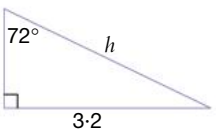

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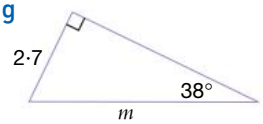

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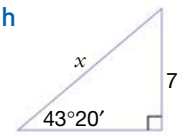

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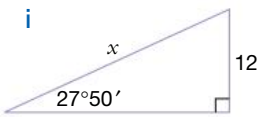
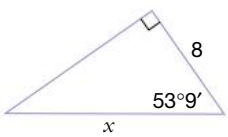
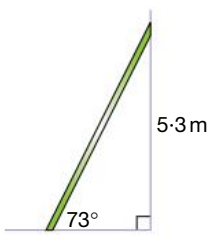


j

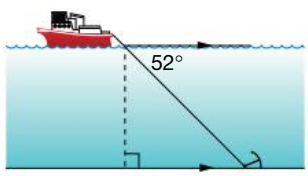


When checking your answer, the hypotenuse must be the longest side.

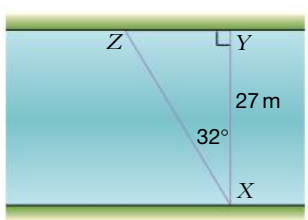
2 A ladder leaning against a wall reaches 5·3 m up the wall when the angle between the ground and the ladder is 73°. How long, to the nearest centimetre, is the ladder?



3 A boat is anchored in a river that is 3·2 m deep. If the anchor rope makes an angle of 52° with the surface of the water, how long is the rope from the surface of the water to the anchor? (Answer correct to the nearest centimetre.)

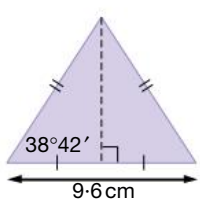


4 X and Y are points on opposite banks of a river 27 m wide. A boy swims from X across the river but, because of the current, ends up at point Z, where $\angle YXZ = 32°$. Find the distance the boy actually swam. (Answer correct to three significant figures.)

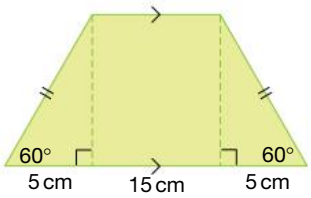


5 Find the length of the diagonal of a rectangle if the length of the rectangle is 7·5 cm and the diagonal makes an angle of 25° with each of the longer sides. (Answer correct to the nearest millimetre.)

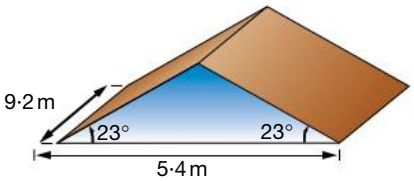
6 The base of an isosceles triangle is 9·6 cm long and each of the base angles is 38°42′. Find the length of each of the equal sides. (Answer correct to to three significant figures.)



7 Find the perimeter of the trapezium.



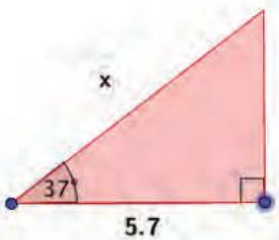
8



Calculate the sloping area of this roof that needs to be tiled, given that the width of the roof is 5·4 m and its length is 9·2 m. Each roof section is pitched at an angle of 23°. (Answer correct to the nearest square metre.)

P GEOGEBRA ACTIVITY 12:07B FINDING THE HYPOTENUSE USING THE SINE AND COSINE RATIOS

This activity is similar to GeoGebra activity 12:07A. It allows you to practise finding the length of the hypotenuse.



$$\cos 37° = \frac{5.7}{x}$$

$$\therefore \frac{1}{\cos 37°} = \frac{x}{5.7}$$

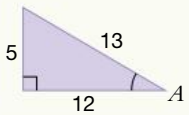
$$\therefore x = \frac{5.7}{\cos 37°}$$

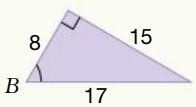
$$\therefore x = 7.14$$

12:08 Using sine and cosine to find an unknown angle

PREP QUIZ 12:08

Complete the ratios below for each triangle.





1 $\sin A =$
2 $\cos A =$
3 $\tan A =$
4 $\sin B =$
5 $\cos B =$
6 $\tan B =$

7 Write $\frac{15}{8}$ in decimal form.

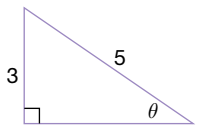
For each of the following, find the size of θ correct to the nearest degree.

8 $\tan\theta = 0{\cdot}6$
9 $\tan\theta = 1{\cdot}5$
10 $\tan\theta = \frac{15}{8}$

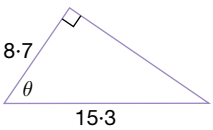
To find an unknown angle using the sine or cosine ratio, use the same method as for the tangent ratio in Section 12:05.

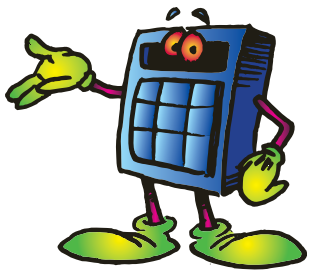
WORKED EXAMPLES

1 Find the size of angle θ, correct to the nearest degree.



2 Find the size of angle θ, correct to the nearest minute.





Solutions

Determine which ratio to use.

1 3 is opposite θ.
5 is the hypotenuse
$\therefore$ use $\sin\theta$.

$\sin\theta = \frac{3}{5}$
$\theta = 36{\cdot}86\ldots$ (from the calculator)
$\theta = 37°$ (nearest degree)

2 8·7 is the adjacent side to θ.
15·3 is the hypotenuse
$\therefore$ use $\cos\theta$.

$\cos\theta = \frac{8{\cdot}7}{15{\cdot}3}$
$\theta = 55°20'43{\cdot}55$ (from the calculator)
$\theta = 55°21'$ (nearest minute)

Using your calculator

1 $\sin\theta = \frac{3}{5}$

Press:
SHIFT sin 3 ÷ 5 =

$\therefore\ \theta = 36{\cdot}869\,898°$
So $\theta = 37°$ (nearest degree)

2 $\cos\theta = \frac{8{\cdot}7}{15{\cdot}3}$

Press:
SHIFT cos 8.7 ÷ 15.3 = ° ' "

$\therefore\ \theta = 55°20'43{\cdot}55$
So $\theta = 55°21'$ (nearest minute)

Exercise 12:08

1 Find the size of θ (to the nearest degree) when:

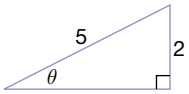
a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\sin\theta = \frac{5}{6}$

d $\cos\theta = \frac{4{\cdot}8}{6}$ **e** $\sin\theta = \frac{11{\cdot}2}{15{\cdot}5}$ **f** $\cos\theta = \frac{20{\cdot}8}{35{\cdot}6}$

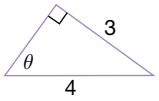
g $\sin\theta = 0{\cdot}412$ **h** $\cos\theta = 0{\cdot}619$ **i** $\sin\theta = 0{\cdot}866$

2 Find the size of the angle marked θ in each triangle. Give your answers correct to the nearest minute.

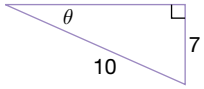
a



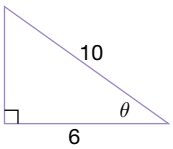
b



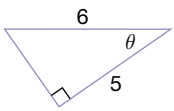
c



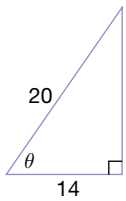
d



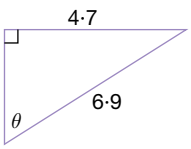
e



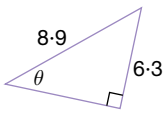
f



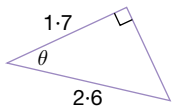
g



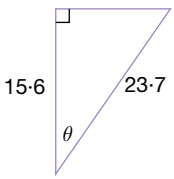
h



i



j



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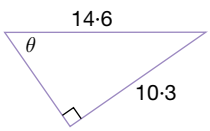
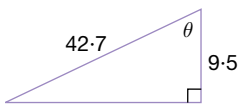
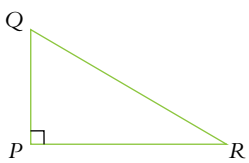


l

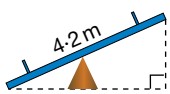


In each of the following problems, give your answer correct to the nearest degree.

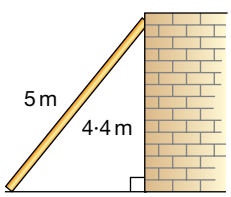
3 In a right-angled triangle PQR, $\angle P$ is the right angle, $PQ = 5$ cm and QR is 8 cm. Find the size of $\angle R$.



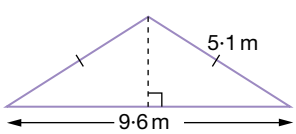
4 The beam of a see-saw is 4·2 m long. If one end is 1·2 m above the ground when the other end is resting on the ground, find the angle the beam makes with the ground.



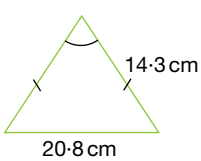
5 What angle will a 5 m ladder make with the ground if it is to reach 4·4 m up a wall?



6 The cross-section of a roof is an isosceles triangle. Find the pitch of the roof (the angle it makes with the horizontal) if the width of the roof is 9·6 m and the length of one of the pitched sections is 5·1 m.



7 Find the apex angle of an isosceles triangle if the length of each of the equal sides is 14·3 cm and the length of the base is 20·8 cm.
(Hint: Construct a perpendicular bisector to the base.)



P GEOGEBRA ACTIVITY 12:08 USING THE TRIGONOMETRIC RATIOS TO FIND AN ANGLE

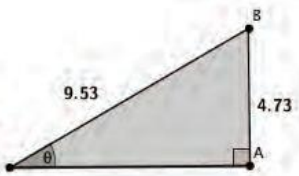
In this activity the lengths of two sides of a right-angled triangle are given and you have to determine the correct ratio to find the required angle. The given side lengths are chosen randomly so that you can practise using all three ratios. A checkbox can be used to show the solution.

$\sin\theta = \frac{4.73}{9.53}$

$\therefore \theta = 29.8°$

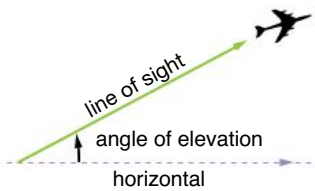
$\therefore \theta = 29°48'$ to the nearest minute

$\therefore \theta = 30°$ to the nearest degree

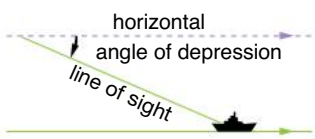


12:09 Miscellaneous exercises

12:09A Angles of elevation and depression



- When looking upwards towards an object, the angle of **elevation** is defined as the angle between the line of sight and the horizontal.
- When looking downwards towards an object, the angle of **depression** is defined as the angle between the line of sight and the horizontal.

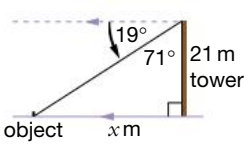


WORKED EXAMPLE 1

The angle of depression of an object on a level plain is observed to be 19° from the top of a 21 m tower. How far from the foot of the tower is the object? Answer correct to the nearest centimetre.

Solution

The diagram would look like the one drawn on the right.
Let the distance from the object to the foot of the tower be x m.
The angle between the line of sight and the tower must be 71° (complementary angle to 19°).



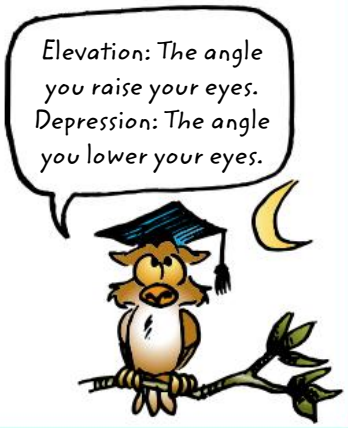
$$\frac{x}{21} = \tan 71°$$

$$x = (\tan 71°) \times 21$$

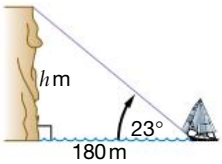
$\therefore$ Distance from tower = 60·99 m (nearest cm)

WORKED EXAMPLE 2

The angle of elevation of the top of a vertical cliff is observed to be 23° from a boat 180 m from the base of the cliff. What is the height of the cliff? Answer correct to one decimal place.



Solution



Let the height of the cliff be h m.

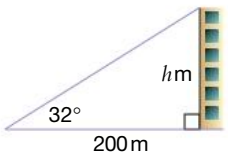
$$\frac{h}{180} = \tan 23°$$

$$h = (\tan 23°) \times 180$$

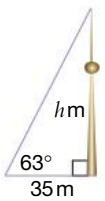
$\therefore$ Height of cliff = 76·4 m (1 dec. pl.)

Exercise 12:09A

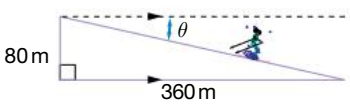
1 From a point 200 m from the base of a building, the angle of elevation of the top of the building is 32°. What is the height of the building correct to the nearest metre?



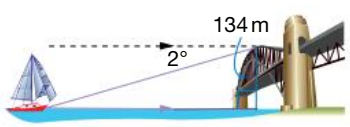
2 The angle of elevation of the top of a tower from a point 35 m from the base of the tower is 63°. Find the height of the tower, correct to one decimal place.



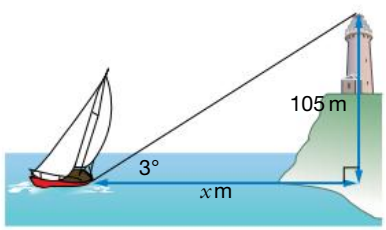
3 A ski slope falls 80 m over a 360 m run. What is the angle of depression from the top of the slope to the bottom of the run?



4 The highest point of the Sydney Harbour Bridge is 134 m above sea level. From this point, the angle of depression of a yacht is 2°. How far does the yacht have to sail to reach the bridge? (Answer to the nearest 10 m.)



5 A lighthouse is 105 m above sea level. The navigator on a boat observes that the angle of elevation of the light is 3°. How far is the boat from the point directly below the lighthouse, to the nearest ten metres?

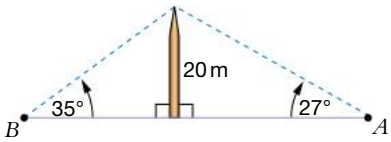


6 A vertical shadow stick has a height of 1·8 m. If the angle of elevation of the Sun is 42°, what is the length of the shadow at that time, correct to one decimal place?

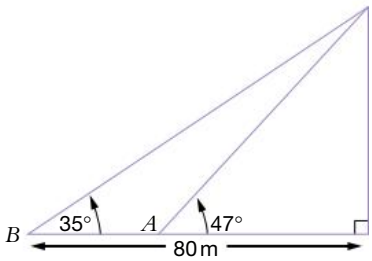
7 Find the angle of elevation of the top of a vertical tower from a point 25 m from its base, if the height of the tower is 40 m.

8 From a lighthouse 70 m above sea level, a ship, 1·2 km out to sea, is observed. What is the angle of depression from the lighthouse to the ship?

9 The top of a 20-metre tower is observed from two positions A and B, each in line with, but on opposite sides of, the tower. If the angle of elevation from A is 27° and from B is 35°, how far is point A from point B (to the nearest metre)?

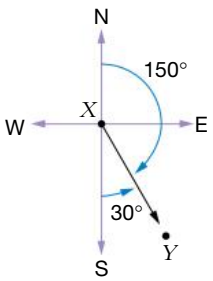


10 A and B are in line with the foot of a building and on the same side of it. B is 80 m from the base of the building and the angle of elevation of the top of the building from B is 35°. The angle of elevation from A is 47°. How far is A from the base of the building?



12:09B Compass bearings

- The direction of a point Y from point X is called the bearing of Y from X.
- A bearing is usually expressed in one of two ways. These are illustrated in the examples below.



Method 1:

This method measures the size of the angle between the line XY and north. The angle is always measured in a clockwise direction. This is the method used in geography and navigation. These are called three-figure bearings.

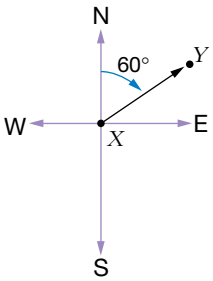
The bearing of Y from X is 150°.

Method 2:

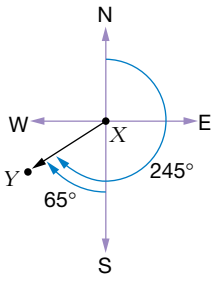
This method measures how far the line XY has rotated to the east or west of the north-south line. In this example, it has rotated 30° to the east from south.

∴ The bearing of Y from X is S30°E.

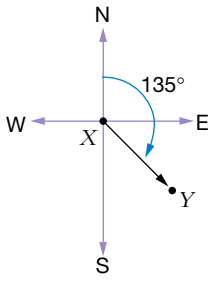
Here are three more examples.



060° or N60°E



245° or S65°W



135° or SE

Sometimes, as shown above, only letters are used. For example, SE (or south-east) is half-way between south (180°) and east (090°); that is, 135° or S45°E.

For SSW (or south-south-west), the direction is halfway between south (180°) and south-west (225°); that is, 202·5° or S22$\frac{1}{2}$°W.

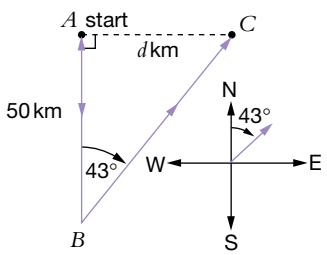
WORKED EXAMPLES

1 A ship sails south for 50 km, then N43°E until it is due east of its starting point. How far is the ship from its starting point (to the nearest metre)?

2 If the town of Bartley is 5 km north and 3 km west of Kelly Valley, find the bearing of Bartley from Kelly Valley.

Solutions

1 The ship sails from point A in the diagram, south to point B and then N43°E to a point C, which is due east of A.



Let the distance AC be d km.

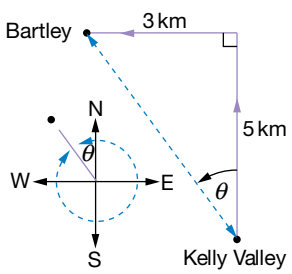
$$\frac{d}{50} = \tan 43$$

$$d = (\tan 43°) \times 50$$

$$= 46{\cdot}625\,754$$

$\therefore$ The distance (AC) from start is 46·626 km (nearest metre).

2 The diagram shows the positions of Bartley and Kelly Valley.



Let the angle indicated in the diagram be θ.

$$\tan\theta = \frac{3}{5}$$

$$= 0{\cdot}6$$

$$\theta = 31° \text{ (nearest degree)}$$

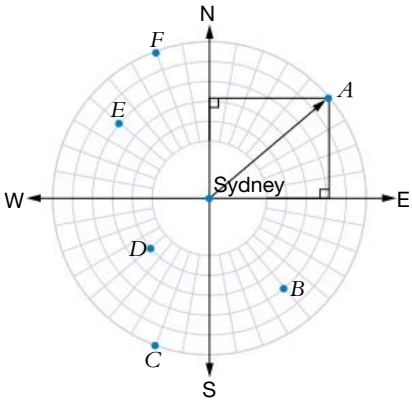
$\therefore$ The bearing of Bartley from Kelly Valley is 329° or N31°W.

Exercise 12:09B

P Foundation worksheet 12:09B
Compass bearings

1 Write the three-figure bearings from Sydney of:

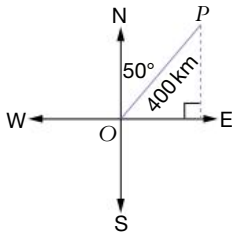
a A b B c C
d D e E f F



2 Repeat Question 1, writing the bearings as shown in Method 2.

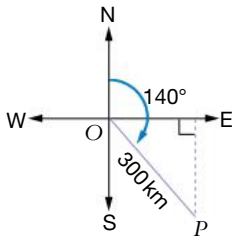
3 A plane leaves Sydney and flies 400 km on a bearing of 050°.

a How far east of Sydney is the plane at this time?
b How far north of Sydney is the plane at this time?



4 A ship leaves Sydney and sails 300 km on a bearing of 140°.

a How far east of Sydney is the ship at this time?
b How far south of Sydney is the ship at this time?



5 A ship sails due north, then turns and sails 50 km due west. The ship is then N67°W of its starting point. For how many kilometres did the ship sail due north? (Give your answer to the nearest kilometre.)

6 After cycling due east, turning and cycling due south, a cyclist is 5 km S25°E from his starting point. How far did he cycle in a southerly direction, to the nearest tenth of a kilometre?

7 An aircraft flies 10 km south and then 6 km west. What is its bearing from its starting point?

8 A ship travels 45 km in a direction with a bearing of 132°. How far south of its starting point is the ship (correct to the nearest km)?

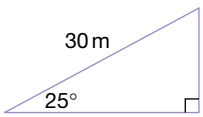
9 Two towns, A and B, are 9 km apart and the bearing of B from A is 320°. Find how far B is west of A (correct to the nearest kilometre).

10 Two cars leave from the same starting point, one in a direction due west, the second in a direction with a bearing of S15°W. After travelling 15 km, the second car is due south of the first. How far has the first car travelled (to the nearest kilometre)?

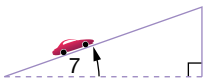
12:09C Other topics

Exercise 12:09C

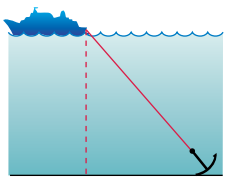
1 An escalator slopes at an angle of 25° and is 30 m long. Through what height (to the nearest centimetre) would a person be lifted when travelling on the escalator?



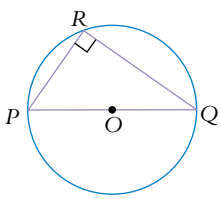
2 Through what vertical height does a car rise when it travels 5 km along a road that slopes at an angle of 7°? (Answer correct to the nearest metre.)



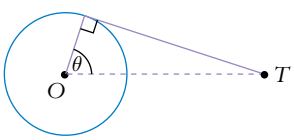
3 The length of rope from a boat to its anchor is 10·6 m. Because of the current, the rope makes an angle of 47° with the horizontal. How deep is the water (correct to the nearest metre)?



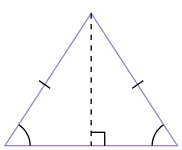
4 PQ is a diameter of the circle with centre O, as shown, and $\angle PRQ = 90°$. If the radius of the circle is 6 cm find to the nearest millimetre, the length of the chord PR, given that $\angle PQR = 40°$.



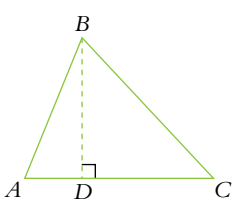
5 A tangent of length 16 cm is drawn to a circle of radius 7·5 cm from an external point T. What is the angle, marked θ in the diagram, that this tangent subtends at the centre of the circle?



6 The base of an isosceles triangle is 8·3 cm long. If the height of the triangle is 9·6 cm, find the size of the base angles of the triangle.

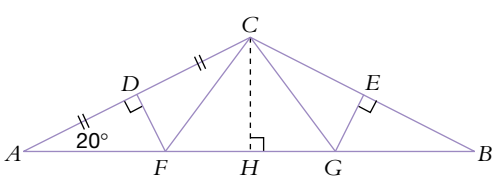


7 In ΔABC, BD is perpendicular to AC. Given that $AB = 13$ m, $BD = 11$ m and $DC = 10$ m, find, to the nearest degree, the size of $\angle ABC$.



8 The diagram shows a roof truss in the shape of an isosceles triangle. Use trigonometry to find the following lengths to the nearest millimetre.

a CH **b** AD **c** AF **d** FD

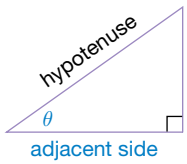


$AB = 12\,400$ mm

MATHS TERMS 12

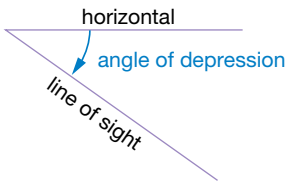
adjacent side (to a given angle)

- the side of a triangle that together with the hypotenuse forms the arms of a given angle



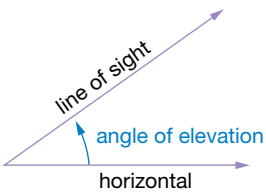
angle of depression

- when looking down, the angle between the line of sight and the horizontal



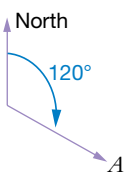
angle of elevation

- when looking up, the angle between the line of sight and the horizontal



bearing

- an angle used to measure the direction of a line from north
- bearings can be recorded in two different ways, e.g. 120° or S60°E



cosine ratio (of an angle θ)

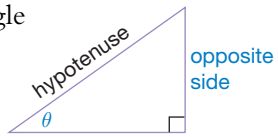
- $\dfrac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$
- abbreviated to $\cos\theta$

hypotenuse

- the longest side in a right-angled triangle
- the side that is not one of the arms of the right angle in a right-angled triangle

opposite side (to a given angle)

- the side of a triangle that is not one of the arms of the given angle



similar triangles

- two triangles that have the same shape but a different size
- triangles that can be changed into each other by either an enlargement or reduction
- triangles that have matching angles equal
- triangles where the ratio of matching sides is constant

sine ratio (of an angle θ)

- $\dfrac{\text{side opposite angle } \theta}{\text{hypotenuse}}$
- abbreviated to $\sin\theta$

tangent ratio (of an angle θ)

- $\dfrac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$
- abbreviated to $\tan\theta$

trigonometric (trig.) ratios

- a collective name for different ratios of the side lengths of right-angled triangles
- the ratios have constant values for any particular angle

trigonometry

- a branch of mathematics, part of which deals with the calculation of the sides and angles of triangles

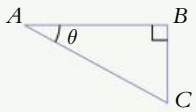
DIAGNOSTIC TEST 12 TRIGONOMETRY

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

12:01

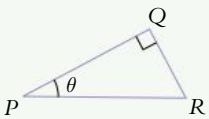
1 Name the side asked for in each triangle with respect to θ.

a



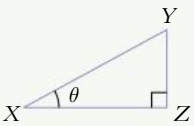
adjacent side

b



hypotenuse

c

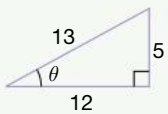


opposite side

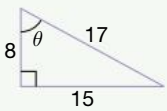
12:03

2 State, as a fraction, the value of $\tan\theta$.

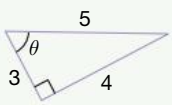
a



b



c

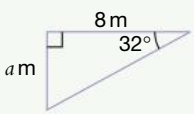


12:04

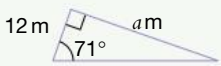
3 Find the value of a, correct to one decimal place.

a

b



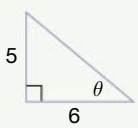
c



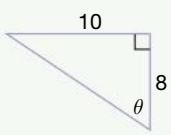
12:05

4 Find θ to the nearest degree.

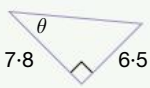
a



b



c



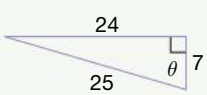
12:06

5 State the value of each stated ratio as a fraction.

a

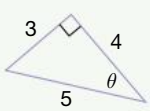
$\sin\theta$

b



$\cos\theta$

c

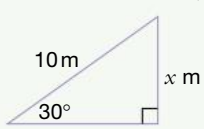


$\sin\theta$

12:07A

6 Find the value of x, correct to one decimal place.

a



b

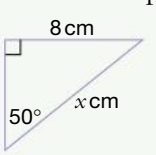
c

12:07B

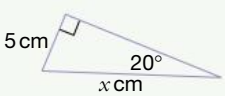
7 Find the value of x, correct to one decimal place.

a

b



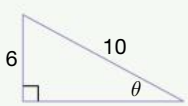
c



12:08

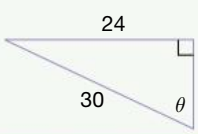
8 Find θ, correct to the nearest minute.

a



b

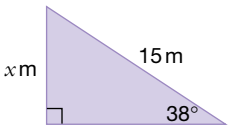
c



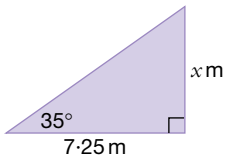
ASSIGNMENT 12A Chapter review

1 Find the value of the pronumerals, correct to two decimal places.

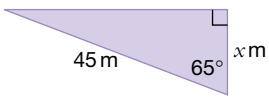
a



b

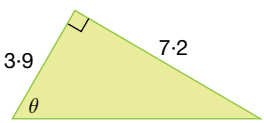


c



2 Find the value of θ, to the nearest minute.

a



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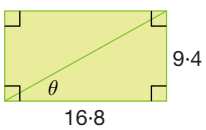

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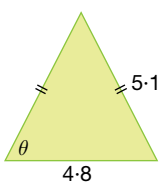
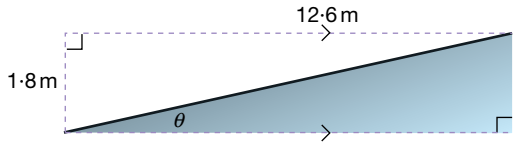


3 **a** What is the slope of the driveway?

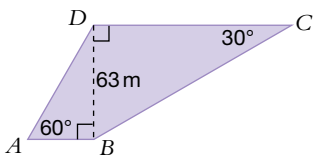


b A plane flies at a speed of 650 km/h. It starts from town A and flies on a bearing of 120° for 3 hours. At that time, how far is it:

i south of A

ii east of A?

4 Find the perimeter of $ABCD$, to the nearest metre.



5 A plane leaves Sydney and flies in a north-easterly direction for 250 km. It then changes course and flies in a northerly direction for 400 km. At this point:

a how far east is it of Sydney

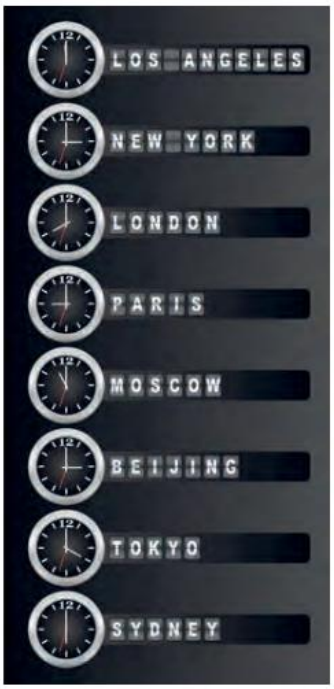
b how far north is it of Sydney

c what is its distance and bearing from Sydney?

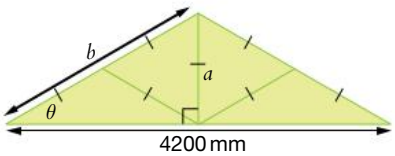
Trigonometry is used in many branches of science, including astronomy.

ASSIGNMENT 12B Working mathematically

1 When it is 12 midnight in New York it is 6 am in Paris. A plane leaves New York at 8 pm. If it takes 8 hours to fly to Paris, what will the time be in Paris when it arrives?

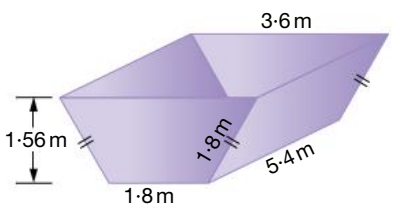


2 A builder is asked to make the window shown. What is the value of θ, a and b?



3 Decrease \$360 by 20% and then increase the result by 20%. What is the difference between \$360 and your final answer?

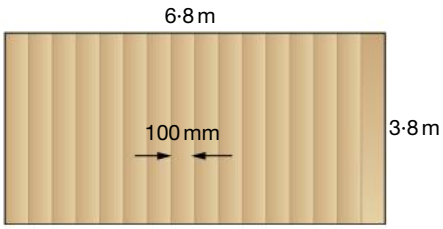
4 A tank is made by welding five sheets of metal together. Three of the sheets are rectangles and two are trapeziums.



a Calculate the length of welds needed to join the sheets.

b Calculate the area of metal that needs to be painted if both the inside and outside are to be painted.

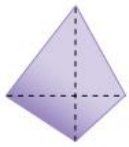
5 The diagram shows a rectangular floor 6·8 m long by 3·8 m wide. It is to be covered with floorboards 100 mm wide. How many lineal metres of floorboard need to be bought to cover the floor?



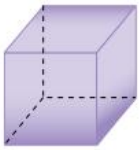
6 **a** Use this formula to calculate the number of edges on the solids shown below.

$$E = \frac{F \times N}{2}$$

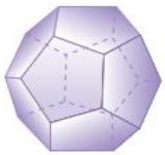
E = Number of edges
F = Number of faces
N = Number of edges on each face



Regular tetrahedron



Cube (or regular hexahedron)



Regular dodecahedron (12 faces)

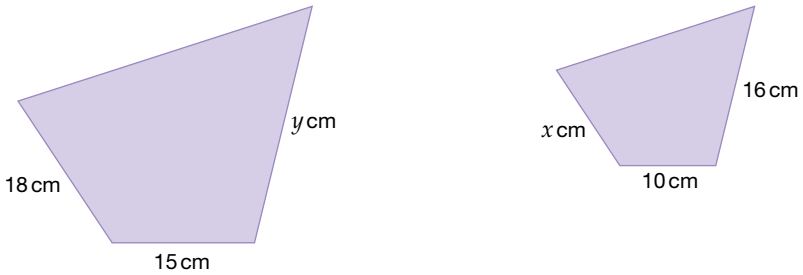
Regular icosahedron (20 faces)

b How could the formula in part **a** be changed to find the number of vertices on each solid?

c Find the number of vertices on each solid.

ASSIGNMENT 12C Cumulative revision

1 The two figures are similar. Find the values of the pronumerals. 10:02



2 Find the gradient of each of these lines: 9:07

a $y = 2x + 5$ **b** $x + y = 6$ **c** $x - 2y = 6$

3 Naomi bought a computer system for $2300. She paid a deposit of $300. She decided to pay off the balance plus the interest (calculated at 9% p.a.) in 24 monthly instalments. 8:06

a How much did she pay in total for the computer?

b What was the size of her monthly instalment?

4 Solve the equations. 7:05

a $2x - 5 = 3x + 6$ **b** $2(x - 5) = 3x + 6$ **c** $2(x - 5) = 3(x - 6)$

5 The names of three girls and two boys are placed in a hat. Two names are drawn from the hat without replacement. 4:06

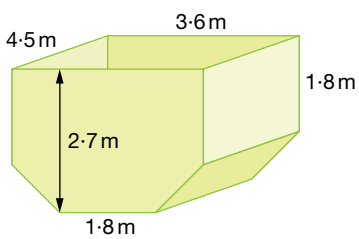
a In how many different ways can the two names be drawn?

b What is the probability of drawing the names of:

i two girls **ii** two boys?

6 **a** Calculate the volume of the tank shown. 5:06

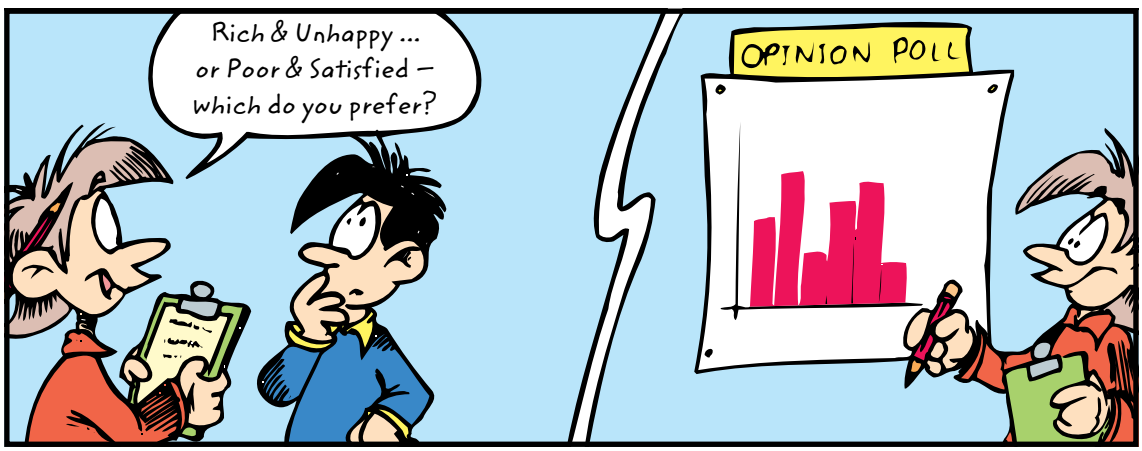
b What is the maximum capacity of the tank if it can only be filled to 20 cm from the top?



Trigonometry is used in the building industry to determine the length of sides and the size of angles.

STATISTICS

13



Contents

13:01 Review of statistics
Investigation 13:01 Adding and averaging
13:02 Cumulative frequency
Fun spot 13:02 Which hand should you use to stir tea?
13:03 Measures of spread: Interquartile range
Fun spot 13:03 Why did the robber flee from the music store?
13:04 Box plots
Investigation 13:04 Code breaking and statistics
13:05 Comparing sets of data
Investigation 13:05 The ageing population
Maths Terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details)

Statistics and Probability

Selections from ***Single Variable Data Analysis*** [Stages 5.1, 5.2◊]

- Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal'. (ACMSP282)
- Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread. (ACMSP283)
- Determine quartiles and interquartile range. (ACMSP248)
- Construct and interpret box plots and use them to compare data sets. (ACMSP249)
- Compare shapes of box plots to corresponding histograms and dot plots. (ACMSP250)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

13:01 Review of statistics

In Years 7 and 8 the work in statistics concentrated on the classification, collection, organisation and analysis of data.

The topics covered included:

- types of data: categorical and numerical (discrete or continuous)
- frequency tables and their graphs
- analysing data: range, mode, mean, median
- stem-and-leaf plots and dot plots
- sources of data: primary and secondary sources
- sampling a population: random, systematic and stratified sampling
- grouped data.

Here is a reminder of the statistical measures used and their definitions.

The **range** = highest score – lowest score.
The **mode** is the outcome that occurs the most.
The **median** is the middle score for an odd number of scores.
The **median** is the average of the middle two scores for an even number of scores.
The **mean** is the arithmetic average.

$$\text{Mean} = \frac{\text{sum of the scores}}{\text{total number of scores}} = \frac{\text{sum of } fx \text{ column}}{\text{sum of } f \text{ column}}$$

The Greek letter Σ (sigma) is used for 'the sum of'.
The symbol $\bar{x}$ (x bar) is used for 'the mean'.
So, a compact definition of the mean is $\bar{x} = \frac{\Sigma x}{\Sigma f}$.

WORKED EXAMPLE 1

Find the range, mode, median and mean of each set of scores.

a 4 4 4 12 9 6 10

b 15 36 40 23 18 46 21 28 32 36

Solutions

a Range = highest score – lowest score
= 12 – 4
= 8

Mode = score occurring most
= 4

Median = middle score
= 6

$$\text{Mean} = \frac{\text{sum of the scores}}{\text{total number of scores}} = \frac{4+4+4+12+9+6+10}{7} = 7$$

b Range = highest score – lowest score
= 46 – 15
= 31

Mode = score occurring most
= 36

Median = average of two middle scores
$$= \frac{28+32}{2} = 30$$

$$\text{Mean} = \frac{295}{10} = 29{\cdot}5$$

WORKED EXAMPLE 2

The following marks out of ten were obtained in a class quiz.

5 3 8 6 7 5 7 3 4 5 7 8 5 5 4 6 6 6 3 6 6 3 6 4 5 3 7 5 6

- Organise these scores into a frequency distribution table.
- Use the table to calculate the mode, median and mean mark for the quiz.

Solution

Mark (x)	Tally	Frequency (f)	fx
3	𝍸	5	15
4	|||	3	12
5	𝍸 ||	7	35
6	𝍸 |||	8	48
7	||||	4	28
8	||	2	16
	Total:	29	154

- The mode is the score that occurs the most. Here, the mode is 6.
- The median is the middle score when the scores are arranged in order. As there are 29 scores, the 15th score will be the middle score. Counting down the frequency column, it can be seen that the 15th score is a 5. Hence, the median is 5.
- The mean $= \dfrac{\text{sum of } fx \text{ column}}{\text{sum of } f \text{ column}} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{154}{29}$

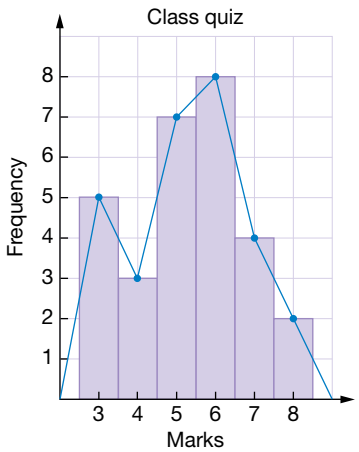
$= 5{\cdot}3$ (1 dec. pl.)

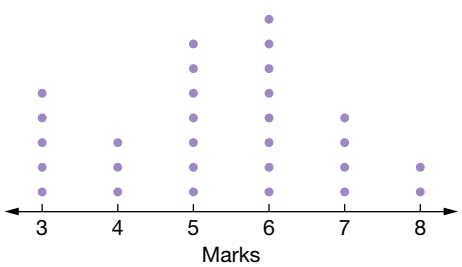
WORKED EXAMPLE 3

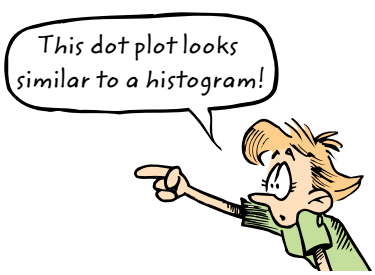
Draw a frequency histogram and polygon for the data in Worked Example 2.

Solution

- The frequency histogram is a column graph.
- The graph has a title and the axes are labelled.
- The first column begins one-half of a column width in from the vertical axis.
- The frequency polygon is a line graph.
- The first non-zero dot is one unit in from the vertical axis.
- The dots showing the data are joined by straight lines and joined to the horizontal axis as shown.







WORKED EXAMPLE 4

The scores for a group of 45 students on a spelling test out of 60 were:

50	41	34	25	18	8	35	45	54	14	59	28	39	42	53
34	51	38	47	21	50	9	54	57	46	10	48	34	11	40
52	8	23	42	52	46	37	27	55	17	32	41	30	25	11

a Sort this set of data into a grouped frequency distribution table with groupings of 0–9, 10–19, etc.

b Find the modal class, median class and an estimate for the mean using the class centres for each group.

c Construct a stem-and-leaf plot and use this to find the mode, median and mean. Compare these measures with those for the grouped data.

Solutions

a

Class	Class Centre (*c.c.*)	Tally	Frequency (f)	$f \times c.c.$
0–9	4·5	|||	3	13·5
10–19	14·5	𝍸 |	6	87
20–29	24·5	𝍸 |	6	147
30–39	34·5	𝍸 ||||	9	310·5
40–49	44·5	𝍸 𝍸	10	445
50–59	54·5	𝍸 𝍸 |	11	599·5
		Total:	$\Sigma f = 45$	$\Sigma(f \times c.c.) = 1602{\cdot}5$

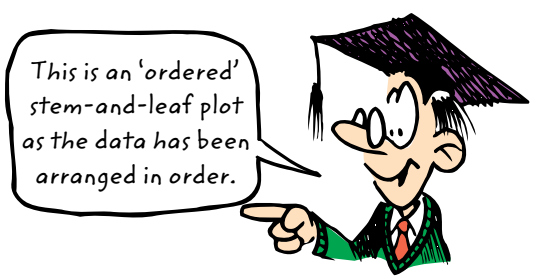
b Modal class = 50–59 (highest frequency of 11)

Median class = 30–39 (count down the frequency column to the middle score; 23rd out of 45)

$$\text{Mean} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{1602{\cdot}5}{45} = 35{\cdot}6 \text{ (1 dec. pl.)}$$

c The stem-and-leaf plot looks like this:

Stem	Leaf
0	8 8 9
1	0 1 1 4 7 8
2	1 3 5 5 7 8
3	0 2 4 4 4 5 7 8 9
4	0 1 1 2 2 5 6 6 7 8
5	0 0 1 2 2 3 4 4 5 7 9



The mode is 34 as it is the score that occurs the most.

The median is 38, the 23rd score (middle score of 45 scores).

$$\text{The mean} = \frac{\Sigma x}{\Sigma f} = \frac{1593}{45} = 35{\cdot}4$$

Comparing these measures with those of the grouped table shows the mode and modal class to be very different, however, the median lies within the median class and the mean is only slightly different.
If this data is compared with the group's results for a previous test using a back-to-back stem-and-leaf plot, simple comparisons can be made.

Previous test		Latest test
Leaf	Stem	Leaf
9 7 6 5 4	0	8 8 9
9 8 6 5 5 4 3 3 2 1	1	0 1 1 4 7 8
9 8 6 5 4 4 4 2 1 0 0	2	1 3 5 5 7 8
9 9 8 6 6 5 3 3 2 1	3	0 2 4 4 4 5 7 8 9
9 8 7 5 3 2 2 0	4	0 1 1 2 2 5 6 6 7 8
8	5	0 0 1 2 2 3 4 4 5 7 9

The mode for the previous test was 24.
The median was 22.
The mean $= \frac{\Sigma x}{\Sigma f} = \frac{1206}{45} = 26{\cdot}8$
There was also an outlier score of 58.

In particular, the median and mean indicated significant overall improvement in the scores. This can also be seen by comparing the distribution of the scores in the back-to-back plot.

Reminder: How to use a calculator to find the mean

- For sets of individual scores, simply add the scores in the calculator and then divide this sum by the number of scores.

The following instructions are for a Casio fx-82AU PLUS. Consult your calculator's manual if you have a different brand or model. Most calculators have a statistics mode that enables the use of various statistical functions. The 'mean' function, which is usually labelled as $\bar{x}$, is used here.

To find the mean for grouped scores:

Outcome	Frequency
0	2
1	3
2	4
3	7
4	3
5	1

- Firstly press SHIFT then SET UP and toggle to the second screen of functions. Select 3:STAT. Then select 1:ON to choose a frequency column.
- Press MODE then select 2:STAT to enter statistics mode.
- Select 1:1-VAR and two columns should appear on the screen.
- Enter the scores in the 'Outcome' column labelled 'x' using the equals key.
- Toggle across to the second column labelled '**FREQ**' and up to the top of the column and enter the frequency for each score.

Now this step is IMPORTANT!

Press the AC (All Clear) key to remove the table from the screen.

- Press SHIFT then STAT (located above the 1 key).
- Select 4: VAR. You will be presented with four choices.
- Select 2: $\bar{x}$ and then press the equals key. This gives you the mean of 2·45.

Note: If you don't enter any numbers in the frequency column the calculator assumes there is only one of each score.

Exercise 13:01

1 Determine the: i range, ii mode, iii median and iv mean for each set of scores.

a 5, 9, 2, 7, 5, 8, 4

b 5, 8, 5, 7, 8, 5, 9, 7

c 21, 24, 19, 25, 24

d 1·3, 1·5, 1·1, 1·5, 1·6, 1·4, 1·7, 1·9

2 Use your calculator to evaluate $\bar{x}$ for each set of scores.

a 6, 9, 7, 8, 5, 4, 9
6, 5, 4, 3, 8, 8, 5
7, 6, 5, 7, 5, 4

b 61, 47, 56, 87, 91, 44, 59
65, 77, 73, 49, 39, 82, 60
51, 84, 73, 67, 65, 55

c 8, 8, 8, 8, 8, 8, 6, 6
6, 6, 6, 6, 7, 7, 7, 7
7, 7, 9, 9, 9, 9, 3, 3

d

Outcome	Frequency
1	6
2	9
3	11
4	7
5	2

e

Outcome	Frequency
48	6
49	11
50	27
51	15
52	8
53	3

f

Outcome	Frequency
12	2
13	15
14	43
15	67
16	27
17	8

3 **a** Barbara's bowling average after 9 games is 178. If her next three games are 190, 164 and 216 what is her new average (mean) based on the 12 games she has bowled?

b Rob bowls in a competition where 4 games are bowled in each night of competition. His average (mean) is 187. In his first two games he scores 191 and 163. What must he total in the last two games if he wants his average to stay at 187?

4 **a** Copy and complete this table and then determine the mode, mean and median for this set of data.

b How many scores were greater than the mean? How many were less than the mean?

c What is the range of the scores?

d Draw a frequency histogram and polygon for this data.

Outcome	Frequency	fx
5	1	
6	4	
7	6	
8	7	
9	5	
10	2	

Total:

5 This table shows the number of each type of car that passed by Jenny's house in an hour. This is an example of **categorical data**. (The other questions in this exercise involve **numerical data**.)

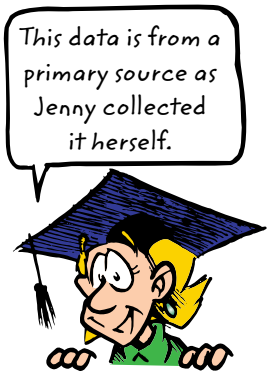
a What type of car is the 'mode', i.e. the one that had the highest frequency?

b Can you determine the mean type of car?

c Can you determine the median?

d What type of graph would best represent this data?

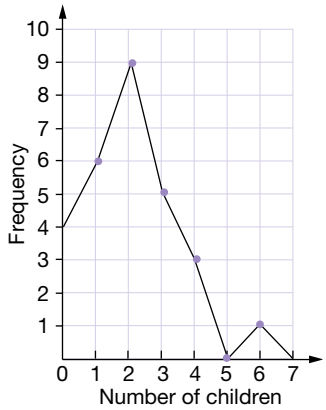
Type of car	Frequency
Ford	12
Holden	15
Hyundai	13
Mazda	20
Nissan	9
Toyota	17



6 This frequency polygon represents a survey of all families in Allyson Street.

a Using this data, complete the table.

Children	Frequency	fx
0		
1		
2		
3		
4		
5		
6		



b How many families were surveyed in Allyson Street?

c How many children lived in Allyson Street?

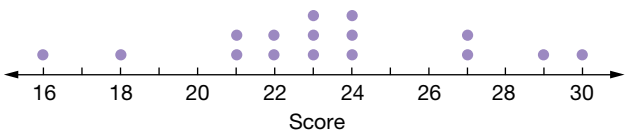
d What was the most common number of children per family (mode)?

e If the national average number of children per family is 2·0, is the average number of children per family in Allyson Street above or below this national average?

7 For each set of data below determine the:

i range ii mode iii median iv mean

a



b

Stem	Leaf
3	8
4	2 6 9
5	3 3 6 7
6	0 1 3 3 4 7 8
7	4 6 6 8 9
8	3 4 8 9

8 Each golfer in a tournament completed three rounds of golf. The scores for each round were tabulated in this grouped frequency distribution table.

Class	Class Centre (*c.c.*)	Frequency (f)	$f \times c.c.$
64–66	65	25	
67–69		74	
70–72		73	
73–75		24	
76–78		2	
	Total:	…	…

- **a** Copy and complete this table.
- **b** How many golfers competed in this tournament?
- **c** What was the maximum possible range?
- **d** What is the modal class?
- **e** Use the class centres to find an approximation for the mean.
- **f** Determine the median class.
- **g** Construct a frequency histogram.

9 A test of 20 spelling words was given to one hundred students. These marks are the results.

Number correct									
17	19	15	13	20	19	15	16	14	20
18	19	15	17	19	12	9	20	19	16
12	14	14	18	16	19	20	19	18	14
10	9	18	16	15	11	15	16	19	20
20	13	14	17	17	19	18	14	15	17
18	20	20	17	19	12	11	17	16	19
17	20	19	16	13	17	15	15	20	20
12	14	20	19	17	18	14	18	18	12
9	16	17	19	20	17	19	17	20	19
17	15	14	20	18	13	14	15	19	18

- **a** Tabulate this data in a frequency distribution table.
- **b** Draw a frequency histogram.
- **c** Determine the mean number of spelling words answered correctly.
- **d** If Kylie spelt 17 words correctly, was she above or below the mean?
- **e** If this information is to be displayed as a grouped frequency distribution, would it be best to use a class interval of 2, 5 or 10?
- **f** Display the information above as a grouped frequency distribution using class intervals of:
  i 2 (9–10, 11–12, …, 19–20) **ii** 5 (6–10, 11–15, 16–20) **iii** 10 (1–10, 11–20)
  Explain how changing the grouping of the data has changed the shape of the display.

10 Year 9 students decided to sell chocolates to earn sufficient money to buy a Christmas present for each patient in the local nursing home. The list on the right shows the number of chocolates sold by each student.

Number sold				
7	0	35	14	22
17	30	11	5	29
26	20	12	24	15
10	16	32	39	28
19	28	11	24	30
21	32	18	21	4
30	19	6	20	35
38	26	23	8	37

- **a** Prepare a grouped frequency table using a class interval of:
  i 2 **ii** 5 **iii** 10
- **b** Draw a frequency histogram using the table with class interval:
  i 2 **ii** 5 **iii** 10
- **c** From your results so far, choose the most appropriate class interval to display these scores. Give reasons for your choice.

11 Mario took a systematic sample of students from Year 9. He identified every tenth student on the alphabetical roll and recorded their English and Maths percentage test results.

Maths: 72, 63, 87, 94, 55, 46, 66, 81, 62, 84, 97, 59, 75, 77, 49, 57, 68, 77, 51, 70

English: 61, 39, 52, 45, 79, 59, 51, 63, 71, 75, 66, 60, 53, 48, 59, 68, 61, 72, 46, 59

The Maths marks have already been entered into this ordered back-to-back stem-and-leaf plot.

a Copy the plot and enter the English marks. (Remember to order them first.)

b For each set of marks, determine the:

i range ii median iii mean

c One particular student scored 77 for Maths but only 75 for English. Can you say this student is better at Maths than English? Justify your answer.

(Maths) Leaf	Stem	Leaf (English)
	3	
9 6	4	
9 7 5 1	5	
8 6 3 2	6	
7 7 5 2 0	7	
7 4 1	8	
7 4	9	

12 Year 9's exam results have been organised into an ordered stem-and-leaf plot using a class size of 5 as shown.

a Complete the frequency column and use it to determine the modal class.

b Complete the 'class centre' and 'frequency times class centre' columns. Use the totals to calculate an approximation for the mean.

c Determine the actual median for these results using the leaf column.

d Determine the median class using the frequency column. How does this compare with the answer to part c?

Stem	Leaf	Freq	*c.c.*	$f \times c.c.$
$6^{(5)}$	8 8 9 9 9		67	
$7^{(0)}$	0 0 0 0 1 2 2 3 4			
$7^{(5)}$	5 5 5 5 6 6 7 7 7 7 8 8			
$8^{(0)}$	0 0 0 1 1 1 1 2 3 4 4			
$8^{(5)}$	5 8			
$9^{(0)}$	0 0 0 1 2 3 3 4			
$9^{(5)}$	7 8 9			
	Totals:	…		…

13 For this question you will generate the data yourself by using the random number generator on your calculator. Each element of data will be a random number squared.

a Press the following keys:

SHIFT Ran# = x^2 =

This will give a 6-digit decimal such as 0·283 024

Now take the first two decimal places and record this as a number between 00 and 99 in the table. Do this a total of 100 times.

b Determine:

i the modal class

ii the median class

iii an approximate value for the mean using the class centres.

c Construct a frequency polygon.

d Compare the shape of your graph with others in the class. Do you notice any consistent pattern? What reasons are there for any pattern in the data?

Class	*c.c.*	Tally	Freq	$f \times c.c.$
0–9	4·5			
10–19	14·5			
20–29	24·5			
30–39	34·5			
40–49	44·5			
50–59	54·5			
60–69	64·5			
70–79	74·5			
80–89	84·5			
90–99	94·5			
	Total:		…	…

B10 = =SUM(B2:B8)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	***DAY***	***TAKINGS***		***DAY***	***TAKINGS***		***DAY***	***TAKINGS***		***DAY***	***TAKINGS***		***DAY***	***TAKINGS***	
2	27-Oct	$2,490		3-Nov	$3,260		10-Nov	$2,800		17-Nov	$2,570		24-Nov	$2,170	
3	28-Oct	$4,360		4-Nov	$4,040		11-Nov	$4,690		18-Nov	$2,920		25-Nov	$3,640	
4	29-Oct	$1,440		5-Nov	$1,420		12-Nov	$1,520		19-Nov	$2,360		26-Nov	$1,420	
5	30-Oct	$1,660		6-Nov	$1,960		13-Nov	$1,340		20-Nov	$1,100		27-Nov	$1,350	
6	31-Oct	$1,370		7-Nov	$1,180		14-Nov	$1,900		21-Nov	$1,170		28-Nov	$1,480	
7	1-Nov	$1,430		8-Nov	$1,230		15-Nov	$1,440		22-Nov	$1,700		29-Nov	$1,350	
8	2-Nov	$1,860		9-Nov	$1,510		16-Nov	$1,660		23-Nov	$1,550		30-Nov	$1,440	
9															
10		$14,610													
11															
12															

The calculating power of a spreadsheet makes it an extremely useful statistical tool.

Part of a spreadsheet is shown above. It shows the daily takings for the Lazy Lizard Café for the 5-week period from 27 October until the 30 November.

- Enter this information into a spreadsheet.

Note the formula =SUM(B2:B8) has been added in cell B10. This gives the weekly takings by adding the numbers in cells B2 to B8.

1 Write formulas to give the weekly takings for the other 4 weeks.

2 Now write a formula that could give you the total takings for the 5-week period.

Each row of the spreadsheet gives the takings for the same day of the week. For example, the days are all Saturdays in row 2 and all Sundays in row 3.

The formula =AVERAGE(B2,E2,H2,K2,N2) will calculate the mean of the numbers in cells B2, E2, H2, K2 and N2. Typing this formula into cell O2 will give the average sales for Saturday.

- Copy this formula into cells O3 to O8. Use the results to find the average sales for each day of the week from Saturday to Friday.

3 Which day of the week has the highest average takings?

4 Which day has the lowest average takings?

13:02 Cumulative frequency

The previous section reminded you of how to sort data into a frequency distribution table, and how to analyse the data using various statistical measures. The data could also be displayed in the form of a frequency histogram or polygon.

A further column that may be attached to the frequency distribution table is the 'cumulative frequency' or *c.f.* column. This column gives the progressive total of the outcomes.

This column has been added to the frequency distribution table below.

WORKED EXAMPLE

Outcome (x)	Tally	Frequency (f)	Cumulative frequency ($c.f.$)
3	|||	3	3
4	|||	3	6
5	||||	4	10
6	𝍸	5	15
7	||||	4	19
8	||	2	21
9	|||	3	24
10	||	2	26
	Total:	26	

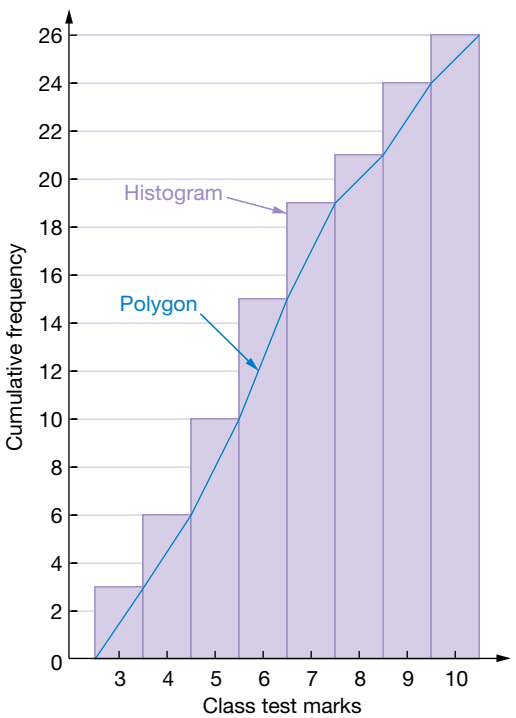
For a class of 26 students the following marks out of 10 were obtained in a test.

- 15 students scored 6 or less. Since 4 students scored 7, the cumulative frequency of 7 is 15 + 4 or 19.
- The last figure in the *c.f.* column must be equal to the sum of the frequencies, as all students are on or below the highest outcome.

The cumulative frequency can also be displayed in the form of a histogram or polygon. However, there are some differences, as noted below.

- The histogram progressively steps upwards to the right.
- The polygon is obtained by joining the top right-hand corner of each column. (Why is it drawn this way?)
- Imagine that the column before the '3' column has zero height.

Another name for the cumulative frequency polygon is the 'ogive'.



Finding the median from a frequency distribution table

The cumulative frequency can be used to find the median of a set of scores.

WORKED EXAMPLES

1

Outcome (x)	Frequency (f)	Cumulative frequency
3	5	5
4	3	8
5	7	15
6	8	23
7	4	27
8	2	29

The middle score is the 15th score (14 above it and 14 below it).
The 15th score is a 5.
$\therefore$ Median = 5

2

Outcome (x)	Frequency (f)	*c.f.*
5	2	2
6	4	6
7	3	9
8	7	16
9	5	21
10	1	22

Here, there is an even number of scores (22) so the middle two scores are the 11th and 12th scores.
From the *c.f.* column it can be seen that each of these scores is 8.
$\therefore$ Median = 8

3

Outcome (x)	Frequency (f)	*c.f.*
5	6	6
6	9	15
7	5	20
8	4	24
9	3	27
10	3	30

Here, there is also an even number of scores (30) so the middle two scores are the 15th and 16th scores.
In this example, the 15th score is 6 and the 16th score is 7. The median is the average of these two scores.
$\therefore$ Median = 6·5 (or $6\frac{1}{2}$)

Finding the median from an ogive

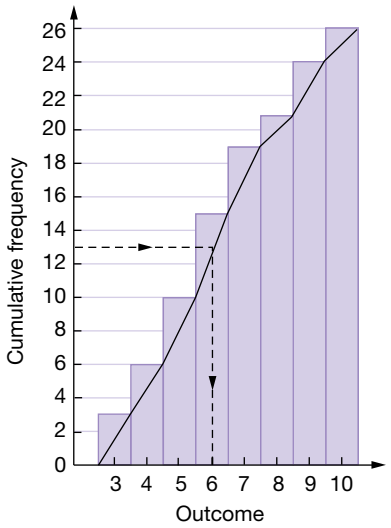
The cumulative frequency polygon, or ogive, can be used to find the median. Note the method used is different from that used for the table.

WORKED EXAMPLE 1

To find the median follow these steps.

- Find the halfway point ($\frac{1}{2} \times 26 = 13$).
- Draw a horizontal line from this point to the ogive.
- Then draw a vertical line to meet the horizontal axis.
- This meets the horizontal axis within the '6' column.

$\therefore$ The median is 6.



For **grouped data**, the outcomes are grouped into classes with a representative class centre (c.c.). The horizontal axis usually shows these class centres and the median class is found as above. Alternatively, an estimate for a median could be read from the horizontal axis, as shown below.

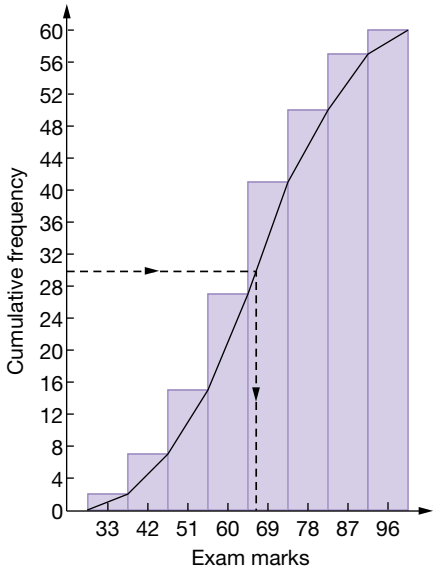
WORKED EXAMPLE 2

The percentage results for 60 students in an examination are given in this table.

Class	Class centre (*c.c.*)	Tally	Frequency (f)	$f \times c.c.$	*c.f.*
29–37	33	||	2	66	2
38–46	42	𝍸	5	210	7
47–55	51	𝍸 |||	8	408	15
56–64	60	𝍸 𝍸 ||	12	720	27
65–73	69	𝍸 𝍸 ||||	14	966	41
74–82	78	𝍸 ||||	9	702	50
83–91	87	𝍸 ||	7	609	57
92–100	96	|||	3	288	60
		Total:	60	3969	

When constructing frequency diagrams for grouped data, the only point to note is that the columns are indicated on the horizontal axis by the class centres. The diagram for the worked example above would look like this.

- The cumulative frequency polygon can be drawn by joining the top right corners of each column.
- There are 60 scores altogether so to find the median class we come across from 30 until we meet the polygon and then down to the horizontal axis.
- Clearly the median class is 65–73.
- An estimate of the median mark can be read from the horizontal axis, i.e. 67.



The cumulative frequency of an outcome gives the number of outcomes equal to, or less than, that particular outcome.

Exercise 13:02

P Foundation worksheet 13:02
Frequency and cumulative frequency

1 Calculate the total of the frequency column and complete the cumulative frequency column in each of these tables.

a

x	f	c.f.
0	3	3
1	8	11
2	11	22
3	17	
4	9	
5	2	

Total:

b

x	f	c.f.
9	1	
10	13	
11	22	
12	30	
13	21	
14	13	

Total:

c

x	f	c.f.
0	1	
1	0	
2	3	
3	8	
4	14	
5	20	
6	31	
7	32	
8	28	
9	11	
10	5	

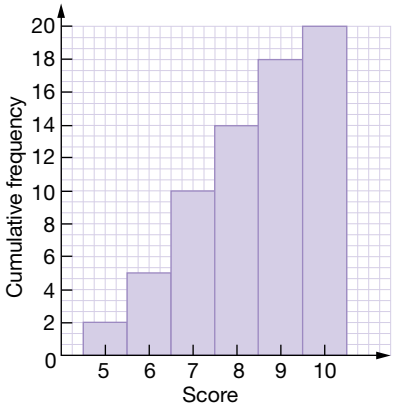
Total:

d

x	f	c.f.
15	4	
20	8	
25	3	
30	9	
35	7	
40	10	
45	15	
50	8	
55	10	
60	2	
65	4	

Total:

2 Use the cumulative frequency histogram shown to complete the table below.

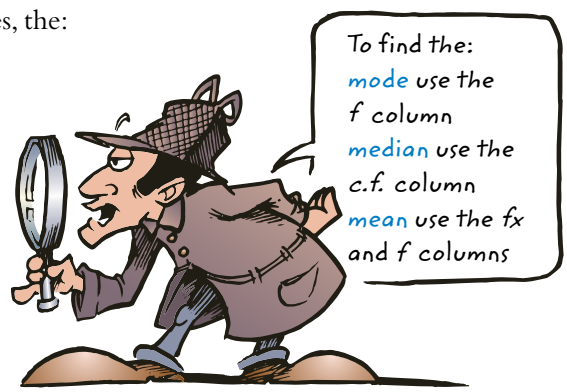


Score (x)	Frequency (f)	fx	c.f.
5			
6			
7			
8			
9			
10			

Total: $\Sigma f =$ $\Sigma fx =$

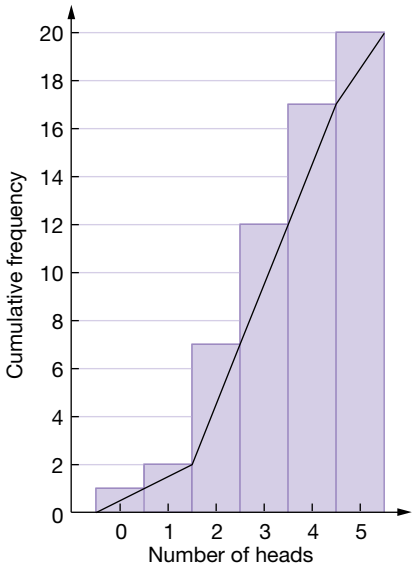
From the table, determine for this set of scores, the:

a mode
b mean
c median
d range



3 Five coins were tossed many times and the number of heads recorded. The cumulative frequency for each number of heads was calculated and a cumulative frequency graph was drawn.

- **a** How many times did zero heads occur?
- **b** How many times did 1 or less heads occur?
- **c** How many times did 2 or less heads occur?
- **d** How many times did 3 or less heads occur?
- **e** Think about your answers to **c** and **d**. How many times did 3 heads occur?
- **f** How many times were the 5 coins tossed?



4 Sharon organises her family's football tipping competition. Each week Dad, Sharon, Adam and Bron have to pick the results of the 7 rugby league matches played. The table below shows the results for rounds 1 to 5.

	Sharon		Adam		Dad		Bron	
	Score	Prog. Total	Score	Prog. Total	Score	Prog. Total	Score	Prog. Total
Round 1	4	4	6	6	4		4	4
Round 2	3	7	3	9	4		3	7
Round 3	6	13	4	13	6		5	12
Round 4	4	17	4	17	2		3	15
Round 5	5	22	4	21	5		4	19

- **a** 'Prog. Total' is short for 'Progressive Total'. It is like a cumulative frequency column. Complete Dad's Prog. Total column.
- **b** Who was leading the competition at the end of:
  - **i** round 1
  - **ii** round 3
  - **iii** round 5?
- **c** What has been the highest score achieved in a round? How many times has this happened?
- **d** Who has had the lowest score in a round?
- **e** In round 6 the scores were: Sharon 6, Adam 6, Dad 7, Bron 5. Use these results to add the next line in the table.

5 Two dice were rolled one hundred times and the total showing on the two upper faces was recorded to obtain this set of scores.

5	7	6	12	10	2	4	5	7	9	7	6	4	3	5	8	6	3	5	6
5	8	7	9	6	8	9	4	8	7	8	4	8	4	8	7	6	7	10	5
9	5	6	5	2	9	5	9	11	10	6	7	7	7	10	6	11	10	7	8
8	3	9	3	5	8	7	12	10	9	7	8	7	5	6	4	5	8	9	11
10	6	9	6	7	8	9	10	11	3	6	4	7	2	4	8	8	4	6	7

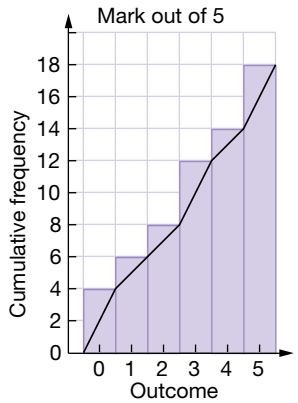
a By completing a frequency distribution table, determine the mode and mean of the set of scores.

b Complete a cumulative frequency column and use it to find the median.

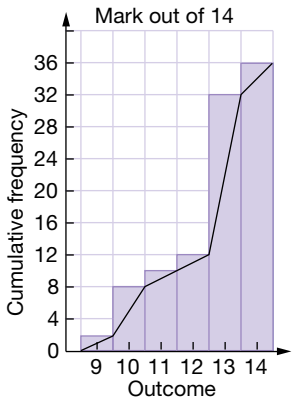
c Look at the cumulative frequency column to determine how many scores were less than the median.

6 Use the ogive to find the median from each graph.

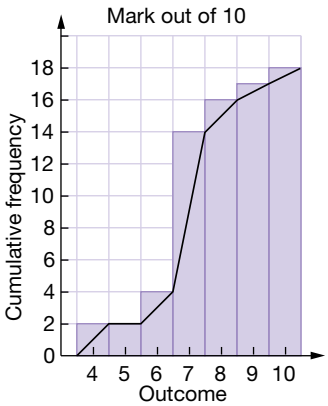
a



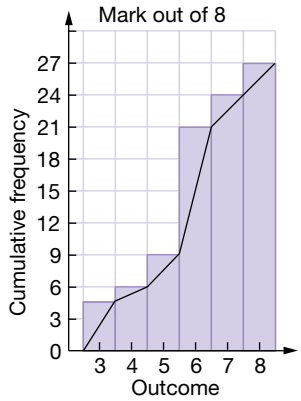
b



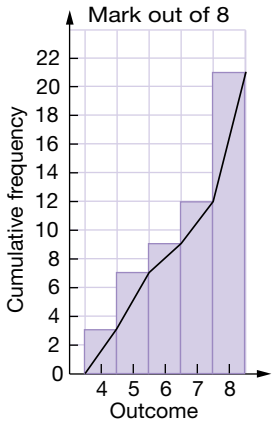
c



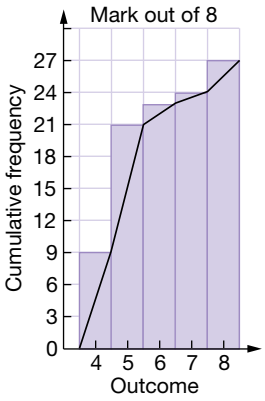
d



e



f



7 The number of cans of drink sold by a shop each day is shown in this table of grouped data.

a Construct a cumulative frequency histogram and ogive.

b Determine the median class.

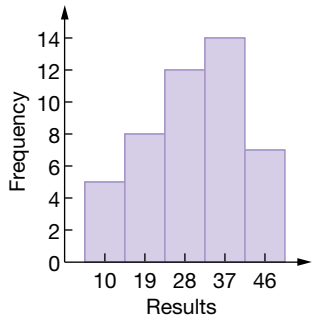
No. of cans sold (x)	*c.c.*	Frequency (f)	*c.f.*
16–22	19	7	7
23–29	26	18	25
30–36	33	18	43
37–43	40	15	58
44–50	47	8	66
51–57	54	4	70

8 From this frequency histogram:

a determine what the class groupings must have been if the data are discrete whole numbers

b construct a cumulative frequency histogram and ogive

c determine the median class.



9 From this cumulative frequency diagram:

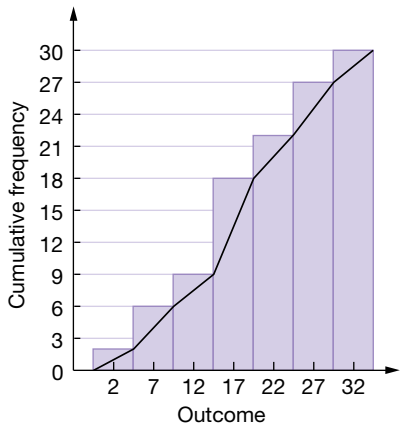
a determine the median class

b complete the table below to show the frequency of each class

c.c.	2	7	12	17	22	27	32
f							

c determine the modal class

d calculate the mean.



10 The exam results for Year 9 students have been collated in this grouped frequency distribution table.

Class	*c.c.*	Tally	Frequency (f)	$f \times c.c.$	*c.f.*
1–10	5·5	𝍸 ||||	9	49·5	9
11–20	15·5	𝍸 𝍸 𝍸	15	232·5	24
21–30	25·5	𝍸 𝍸 𝍸 ||||	19	484·5	43
31–40	35·5	𝍸 𝍸 𝍸 ||	17	603·5	60
41–50	45·5	𝍸 𝍸 𝍸 𝍸	20	910	80
51–60	55·5	𝍸 𝍸 𝍸 ||	17	943·5	97
61–70	65·5	𝍸 𝍸 𝍸 𝍸 |	21	1375·5	118
71–80	75·5	𝍸 𝍸 𝍸 |	16	1208	134
81–90	85·5	𝍸 𝍸 𝍸 |||	18	1539	152
91–100	95·5	𝍸 |||	8	764	160
		Total:	$\Sigma f = 160$	$\Sigma(f \times c.c.) = 8110$	

a What is the greatest possible range for this data?

b What is the modal class?

c Calculate a value for the mean.

d Construct a cumulative frequency histogram and ogive.

e From your graph, determine the median class.

f Considering part e, what would be a reasonable single numerical value for the median?

g What percentage of students obtained: i more than 80 ii 20 or less iii more than 50?

FUN SPOT 13:02 WHICH HAND SHOULD YOU USE TO STIR TEA?

Work out each part and write the letter for that part in the box above the correct answer.

What is the probability of:

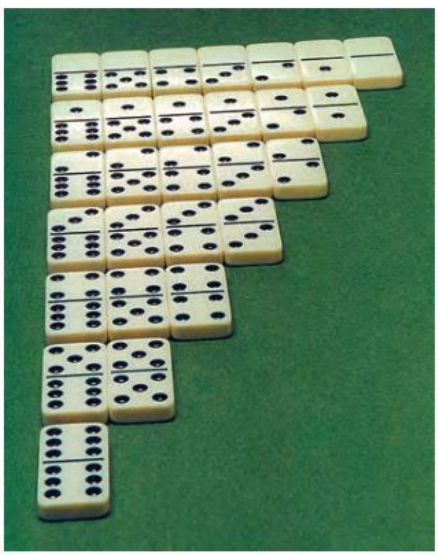
A rolling a 7 on one dice
E rolling a sum less than 14 on two dice
E rolling a number less than 6 on one dice
E selecting a club from a standard pack of cards
H selecting a 4 from a standard pack
I selecting a picture card from a standard pack
N selecting a King or a heart from a standard pack
N getting an even number when a dice is rolled?

Use this frequency distribution table to find:

O the total frequency
O the frequency of 3
P the range of the scores
R the mode of the scores
S the median of the scores
S the cumulative frequency of 1
T the cumulative frequency of 4
U the mean, correct to one decimal place.

x	f	*c.f.*
0	1	
1	7	
2	6	
3	3	
4	9	
5	1	

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{13}$	26	$\frac{1}{13}$	$\frac{5}{6}$	4	2.5	2	1	0	8	5	3	27	$\frac{4}{13}$



The photo shows a set of double-six dominoes. Each domino consists of two squares with a number of dots from 0 to 6.

A set of double-six dominoes goes from double blank (0, 0) to double six (6, 6) with each pair occurring only once. So a (3, 4) is the same as a (4, 3).

How many dots are there on this double-six set of dominoes?

There are 28 dominoes in a double-six set. How many dominoes would there be in a double-eight set?

13:03 Measures of spread: Interquartile range

PREP QUIZ 13:03

Find the range of the scores in:

1 Set A **2** Set B **3** Set C

4 Which set contains an outlier?

5 Is the range affected by an outlier?

Set A:	8, 8, 8, 8, 8	Set D:	8, 8, 8, 8, 8
Set B:	7, 8, 8, 8, 8	Set E:	7, 8, 8, 8, 9
Set C:	1, 8, 8, 8, 8	Set F:	6, 7, 8, 9, 10

Find the mean in:

6 Set D **7** Set E **8** Set F

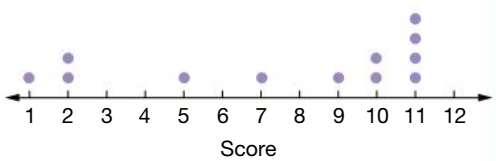
9 Which set of scores is the least spread out?

10 Which set of scores is the most spread out?

- So far we have concentrated on finding the measures of central tendency: mode, mean and median. These values tell us how the scores tend to cluster.
- We have used the range as a measure of spread. But, as seen in the prep quiz above, the range is easily affected by an outlier.
- A much better measure of spread than the range is the interquartile range. This is the range of the middle 50% of scores.

WORKED EXAMPLE 1

Find the interquartile range of the scores:
1, 2, 2, 5, 7, 9, 10, 10, 11, 11, 11, 11



Method 1

- Make sure that the scores are in ascending order.
- Divide the scores into four equal groups. (This is not always possible. See Worked Example 2.)

1, 2, 2, | 5, 7, 9, | 10, 10, 11, | 11, 11, 11,

1st quartile (25th percentile)	Median (50th percentile)	3rd quartile (75th percentile)
$Q_1 = \frac{2+5}{2} = 3{\cdot}5$	$Q_2 = \frac{9+10}{2} = 9{\cdot}5$	$Q_3 = \frac{11+11}{2} = 11$

The 1st quartile (Q_1) is 3·5, which lies half-way between 2 and 5.
The 2nd quartile (median) is 9·5, which lies half-way between 9 and 10.
The 3rd quartile (Q_3) is 11.

- The interquartile range is the difference between the 3rd and 1st quartiles.

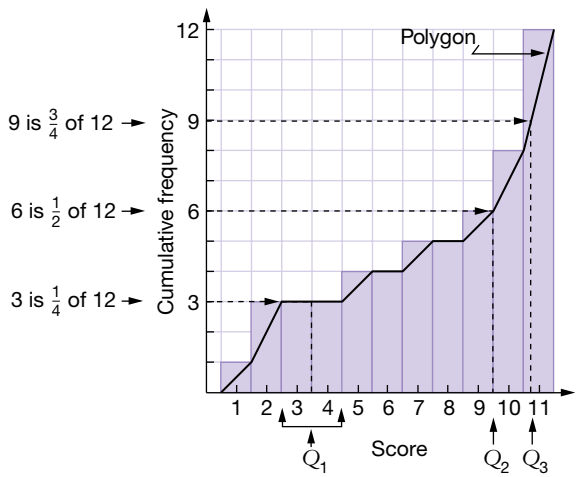
Interquartile range $= Q_3 - Q_1$
$= 11 - 3{\cdot}5$
$= 7{\cdot}5$

Method 2

- Construct a cumulative frequency polygon.
- Come across from the vertical axis to the polygon from positions representing 25%, 50% and 75% of the scores. Take the readings on the horizontal axis to obtain 1st quartile, median and 3rd quartile.

x	f	$c.f.$
1	1	1
2	2	3
3	0	3
4	0	3
5	1	4
6	0	4
7	1	5
8	0	5
9	1	6
10	2	8
11	4	12

$\Sigma f = 12$



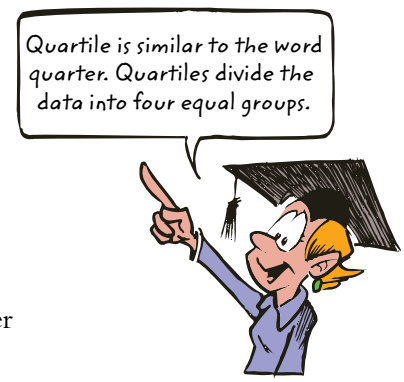
The 1st quartile (Q_1) is somewhere in the range 2·5 to 4·5, as the polygon has the height 3 for those values. We resolve this problem by taking the average of 2·5 and 4·5.

$$\therefore\ Q_1 = \frac{2{\cdot}5 + 4{\cdot}5}{2}$$
$$= 3{\cdot}5$$

The median (Q_2) is seen to be 9·5.

The 3rd quartile (Q_3) is seen to be 11 (or 10·75 if we consider the horizontal scale to be continuous rather than discrete).

The interquartile range $= Q_3 - Q_1$
$= 11 - 3{\cdot}5$
$= 7{\cdot}5$



The interquartile range is more useful when the number of scores is large. When the number of scores is small (e.g. 7), it is hard to define 'the middle half of the scores'.

The **interquartile range** is:
- the range of the middle 50% of the scores
- the difference between the points below which 75% and 25% of scores fall (the difference between the third and first quartiles)
- the median of the upper half of the scores minus the median of the lower half.

WORKED EXAMPLE 2

Find the interquartile range for the following sets of scores.
Set A: 1, 2, 2, 5, 7, 9, 10, 10, 11, 11, 11
Set B: 1, 2, 2, 5, 7, 9, 10, 10, 11, 11

Solution

When the number of scores in a set is not a multiple of 4, they cannot be divided into 4 equal groups.

Set A has 11 scores. Hence, the middle score, 9, is the median (Q_2).

The middle score of the bottom 5 scores is Q_1.

The middle score of the top 5 scores is Q_3.

1, 2, (2), 5, 7, (9), 10, 10, (11), 11, 11

1st quartile $Q_1 = 2$ | Median $Q_2 = 9$ | 3rd quartile $Q_3 = 11$

The interquartile range $= Q_3 - Q_1$
$= 11 - 2$
$= 9$

Set B has 10 scores. Hence, the median is between the 5th and 6th scores.

This divides the scores into two groups of 5 scores.

The middle scores of the bottom and top groups are Q_1 and Q_3 respectively.

1, 2, (2), 5, 7, 9, 10, (10), 11, 11

8

1st quartile | Median | 3rd quartile

$Q_1 = 2$ $\quad Q_2 = \frac{7+9}{2} = 8$ $\quad Q_3 = 10$

The interquartile range $= Q_3 - Q_1$
$= 10 - 2$
$= 8$

Exercise 13:03

P Foundation worksheet 13:03 Interquartile range

1 Use Method 1 on page 414 to find the interquartile range of each set of scores. (Rewrite the scores in order as the first step in each case.)

a 6, 4, 3, 8, 5, 4, 2, 7
b 1, 5, 2, 6, 3, 8, 7, 5, 4, 5, 7, 9
c 60, 84, 79, 83, 94, 88, 92, 99, 80, 90, 95, 78
d 15, 43, 30, 22, 41, 30, 27, 25, 28, 20, 19, 22, 25, 24, 33, 31, 41, 40, 49, 37
e 56, 83, 60, 72, 61, 52, 73, 24, 88, 70, 57, 63, 60, 48, 36, 53, 65, 49, 62, 65

2 The scores of 32 students have been used to graph this cumulative frequency histogram and polygon.
Use the graph to find:

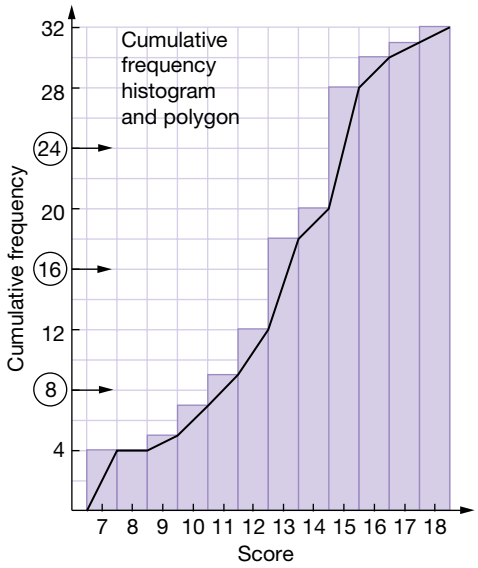
a the median, Q_2

b the 1st quartile, Q_1

c the 3rd quartile, Q_3

d the interquartile range, $Q_3 - Q_1$.

(*Note:* Here the answers are whole numbers.)



3 The same 32 students sat for a second test.
The results have been used to draw this graph.
Use the graph to find:

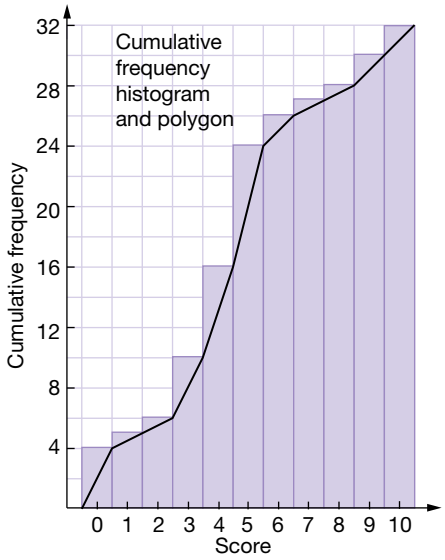
a the median, Q_2

b the 1st quartile, Q_1

c the 3rd quartile, Q_3

d the interquartile range, $Q_3 - Q_1$.

(*Note:* Here some answers will involve decimals.)



4 Make up a frequency distribution table for these scores.

7 8 6 9 4 6 5 5 4 2 3 7 6 6 5 8 4 5 6 4
7 6 8 5 3 4 8 9 6 5 4 5 7 3 6 6 5 5 5 6

Use your frequency distribution table to find:

a the interquartile range using Method 1 **b** the interquartile range using Method 2.

5 Use a cumulative frequency polygon to find the interquartile range for each of the following.

a

Marks	Frequency
16	3
17	4
18	5
19	5
20	3

b

Times	Frequency
35	3
36	4
37	7
38	10
39	18
40	18

6 Find the interquartile ranges of the following sets of scores.

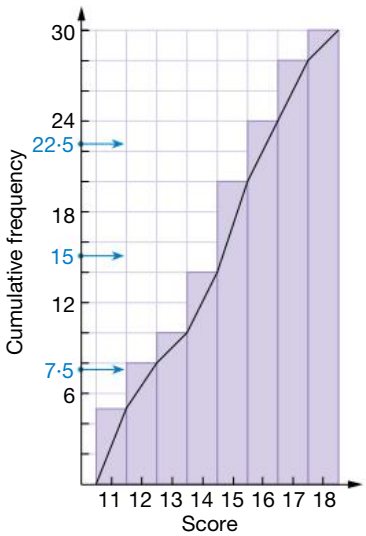
a 25, 45, 46, 50, 58, 58, 65, 66, 70, 90

b 25, 25, 26, 26, 26, 28, 29, 30, 30, 32, 32

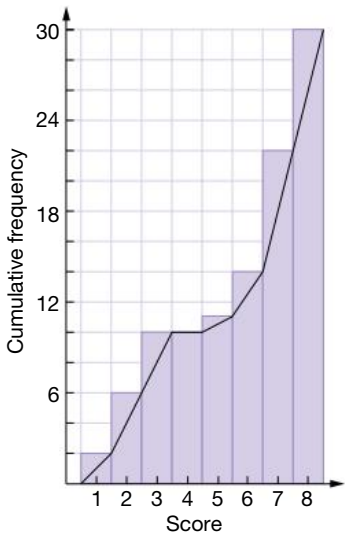
c 45, 45, 56, 56, 58, 59, 59, 59, 80

7 Use the cumulative frequency polygons to find the interquartile range of each set of scores.

a



b



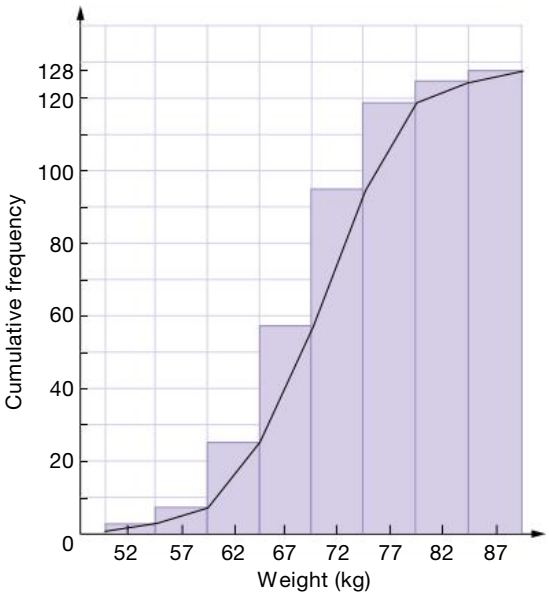
8 A cumulative frequency polygon (ogive) can also be used to obtain the interquartile range for grouped data. The weights of 128 boys were measured to the nearest kilogram and grouped in classes of 50–54 kg, 55–59 kg up to 85–89 kg.

Use the ogive to estimate the following.

a 1st quartile

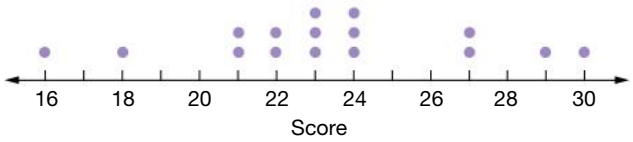
b 3rd quartile

c interquartile range



9 Find the quartiles for each of the following sets of data and then find the interquartile range. (Note that in both the dot plot and the stem-and-leaf plot, the scores have already been arranged in order.)

a



b

Stem	Leaf
3	8
4	2 6 9
5	3 3 6 7
6	0 1 3 3 4 7 8
7	4 6 6 8 9
8	3 4 8 9

FUN SPOT 13:03 — WHY DID THE ROBBER FLEE FROM THE MUSIC STORE?

Work out the answer to each part and write the letter for that part in any box that is above the correct answer.

In a coordination test, 12 students were rated 1 (*poor*) to 6 (*outstanding*). The results are shown in the frequency table.

x	f	*c.f.*
1	4	
2	2	
3	1	
4	2	
5	1	
6	2	

What is:

A the mode
E the range
F the mean
F the cumulative frequency of 5
E the median
E the highest score
R the fraction of scores that are 6
H the cumulative frequency of 6?

How many people:

H were rated as outstanding
H were rated higher than 6
I were rated less than 4
L were rated poor?

One of these students is selected at random.
What is the probability that the student's rank is:

N 3
O 1
T less than 3
U less than 5
W anything but 3?

2	5	$\frac{1}{6}$	1	$\frac{1}{12}$	13	3	10

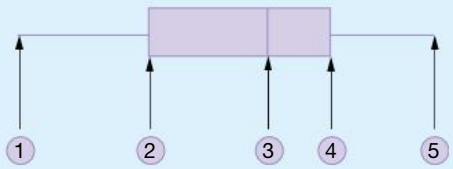
$\frac{11}{12}$	7	$\frac{1}{2}$	12	$\frac{1}{2}$	0	2.5	4	$\frac{3}{4}$	$\frac{1}{2}$	6	

13:04 Box plots

In Stage 4, the dot plot and stem-and-leaf plot were used to illustrate certain aspects of a set of scores or distribution.

Another type of display is the box-and-whisker plot or more simply, box plot. This is drawn using a **five-point summary** of the data as shown below.

1 The minimum score
2 The first quartile, Q_1
3 The median, Q_2
4 The third quartile, Q_3
5 The maximum score



In a box plot:

- the box shows the middle 50% (the interquartile range) between Q_1 and Q_3
- the whiskers extend from the box to the highest and lowest scores
- the whiskers show the range of the scores.

WORKED EXAMPLE 1

The scores in an assessment task for a class were as follows.

40 71 74 20 43 63 83 57 63 26 43 87 74 89 66 63

Find the five-point summary of these marks and use it to construct a box plot.

Solution

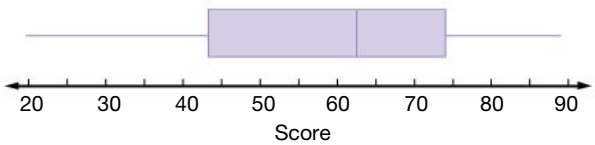
Rearrange the scores in order, and find Q_2, then Q_1 and Q_3.

20 26 40 43 43 57 63 63 63 66 71 74 74 83 87 89

Q_1 Q_2 Q_3

The five-point summary is (20, 43, 63, 74, 89).

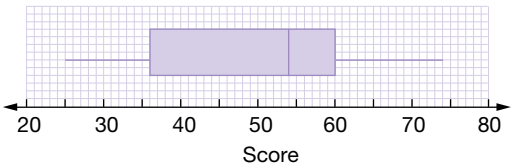
Use the five-point summary and a suitable scale (1 mark = 1 mm) to construct the box-and-whisker diagram or box plot.



WORKED EXAMPLE 2

Use the box plot to find the:

a range
b interquartile range
c median
d percentage of scores above 60
e percentage of scores below 36.



Solution

a Range = maximum score − minimum score
= 74 − 25
= 49

b Interquartile range = $Q_3 - Q_1$
= 60 − 36
= 24

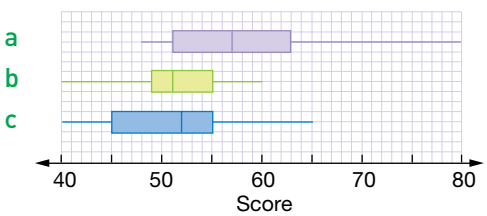
c Median = 54

d As Q_3 = 60, 25% of the scores are above 60.

e As Q_1 = 36, 25% of the scores are below 36.

Exercise 13:04

1 Use each box plot to find the:
i median ii range iii interquartile range.

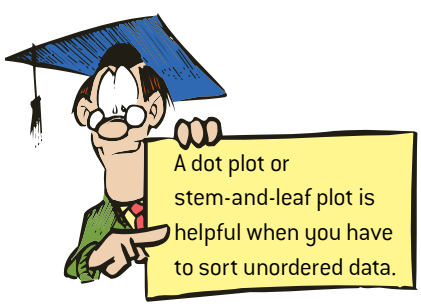


2 Find the five-point summary for each of the following sets of data and use it to construct a box plot.

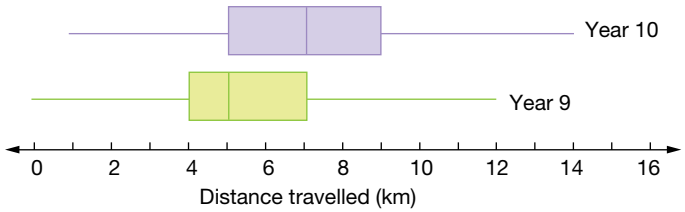
a 7, 7, 8, 8, 8, 9, 9, 9, 10, 12, 12, 12
b 16, 24, 25, 25, 26, 28, 28, 28, 28, 30, 32, 33, 34, 34, 37, 38
c 14, 19, 29, 36, 40, 43, 43, 44, 46, 46, 47, 49

3 Find the five-point summary for each of the following sets of data and use it to construct a box plot.

a 43, 37, 42, 48, 39, 39, 40, 40, 44, 47, 45, 44
b 75, 78, 63, 59, 68, 72, 74, 83, 87, 86, 59, 75, 82, 82, 84, 85, 77, 76, 70, 83



4 These double box plots represent the distance travelled to school by members of Year 9 and Year 10.



a What percentage of Year 9 students travel:
- **i** further than 7 km
- **ii** further than 5 km?

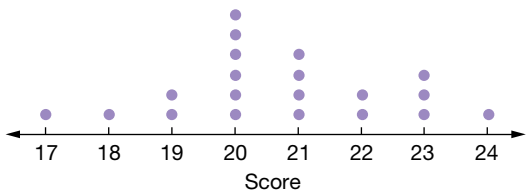
b What percentage of Year 10 students travel:
- **i** further than 7 km
- **ii** further than 5 km?

c Find the interquartile range for:
- **i** Year 9
- **ii** Year 10

d Which group does more travelling?

5 **a** Use the dot plot to find the five-point summary for the scores.

b Construct a box plot for the scores.



6 The marks of 24 students in a half-yearly test are recorded in the stem-and-leaf plot.

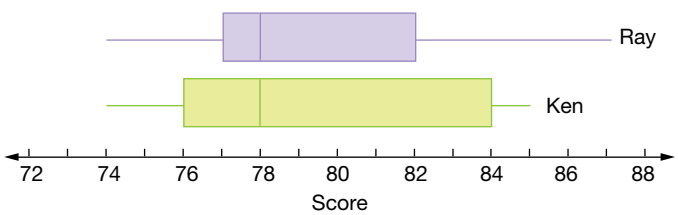
a Find the five-point summary for these marks.

b Construct a box plot for the marks.

Test scores

Stem	Leaf
2	6 7 8
3	5 8 8 9
4	0 1 3 8 9
5	2 5 6 7 7
6	7 9 9
7	5 5
8	2 2

7 Ray and Ken play 40 games of golf over a 1-year period. Their scores are shown on the double box plots below.



a What is the five-point summary for Ken's scores?

b Which golfer's scores have the smaller range?

c Which golfer's scores have the smaller interquartile range?

d Given your answers to **b** and **c**, which golfer do you think is the most consistent? Give a reason for your answer.

8 Rick recorded how long it took him to drive to work over 28 consecutive days. The times taken to the nearest minute are shown in the frequency table.

Time (minutes)	38	39	40	41	42	43	44	52
Frequency	1	2	6	7	5	4	2	1

One year later, after the addition of traffic lights and other traffic management measures, Rick repeated the process and obtained the following results.

Time (minutes)	38	39	40	41	42	43	45
Frequency	1	4	8	9	4	1	1

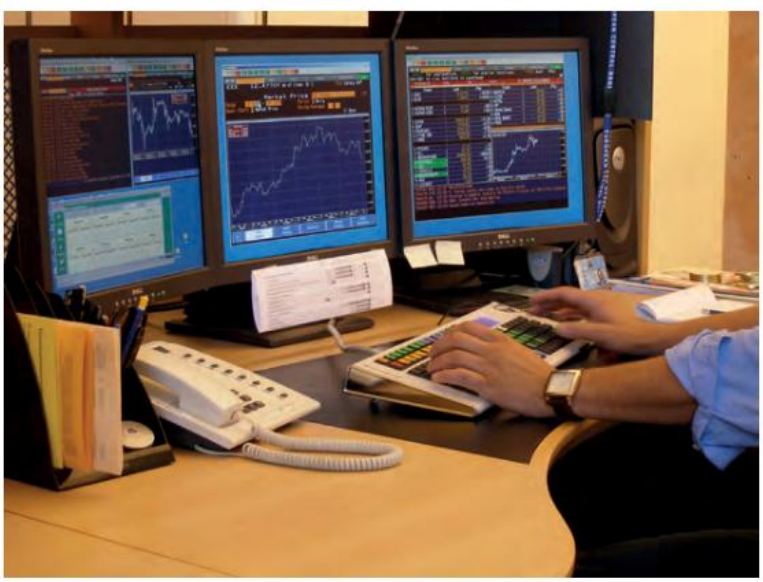
Draw double box plots to illustrate the before and after results and use it to comment on the effectiveness of the traffic changes.

INVESTIGATION 13:04 CODE BREAKING AND STATISTICS

Codebreakers use statistics to help decipher codes. It is based on the fact that certain letters are more common than others. What letter of the alphabet appears most often? What is the most common vowel? What consonant occurs most often?

1 Write what you think the answers are to the three questions above.

2 Use the statement above to do an alphabetic analysis. Were your answers in 1 supported by the statistics?



As well as deciphering codes, mathematicians are often employed to devise security codes to prevent access by unauthorised users. In particular, cryptographers are employed to stop computer hackers from accessing computer records.

On one side we have mathematicians trying to break codes, and on the other side we have mathematicians trying to design codes that cannot be broken.

13:05 Comparing sets of data

Statistics are often used to look at the similarities and differences between sets of data. Here are some examples.

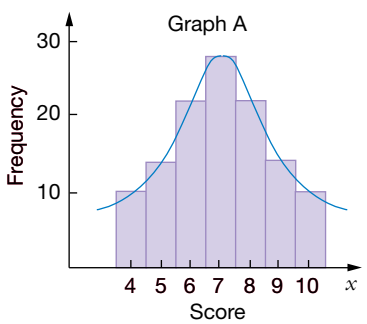
- Teachers are often interested in comparing the marks of a class on different topics or comparing the marks of different classes on the same topic.
- Medical researchers could compare the heart rates of different groups of people after exercise.
- Coaches might compare the performances of different players over a season or the same player over different seasons.
- Managing directors of companies could compare sales and profits over different periods.

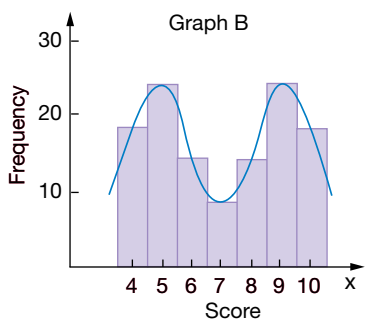
As well as calculating the measures of cluster (the mean, median and mode) and the measures of spread (the range and interquartile range), a comparison would usually involve using graphical methods. Back-to-back stem-and-leaf plots, double-column graphs, double box plots and histograms are useful ways of comparing sets of data.

Shape of a distribution

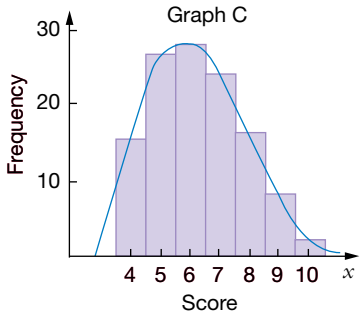
A significant feature of a set of data is its shape. This is most easily seen using a histogram or stem-and-leaf plot. For some data sets with many scores and a large range, the graph is often shown as a curve.

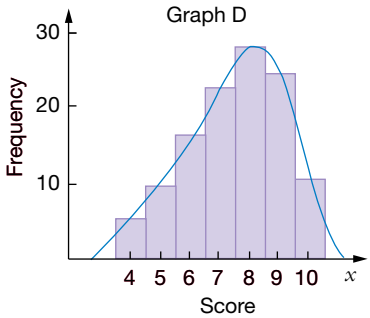
The graphs below show the results of 120 students on four different problem-solving tests.





- Graphs A and B are examples of **symmetric distributions**.
- Graph A has one mode; it is said to be unimodal.
- Graph B has two modes; it is **bi-modal**.
- A unimodal symmetric distribution is quite common in statistics and is called a **normal distribution**.
- Symmetric distributions are evenly distributed about the mean.





- Graphs C and D are examples of **skewed distributions**.
- If most of the scores are at the low end, the skew is said to be positive. See Graph C.
- If most of the scores are at the high end, the skew is said to be negative. See Graph D.

WORKED EXAMPLE

Our class was given a topic test in which we performed poorly. Our teacher decided to give a similar test one week later, after a thorough revision of the topic. The results are shown on this back-to-back stem-and-leaf plot. (This is an ordered display.)

Compare the results of the class on the two tests.

Test scores (4|1 represents 41)

Test 1	Stem	Test 2
9 8 6 6 0	3	
9 7 7 3 1 1 1	4	3 6 6 8 8
8 8 5 3 3 0	5	1 7 9 9
9 8 7 5 3	6	3 8 9
	7	0 5 5 5 8 9
	8	2 6 7 7
0	9	0 0 1 3

Solution

- The improvement in the second test is clear to see. The medians, which are easily found, verify this, as do the means.

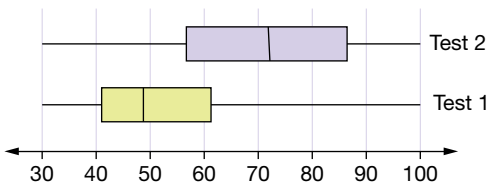
	Test 1	Test 2
Median	49·5	72·5
Mean	51·5	69·8

- The spread of the scores in Test 1 is smaller than in Test 2. The interquartile range confirms this.

	Test 1	Test 2
Interquartile range	19·5	29
Range	60	50

The presence of the outlier in Test 1 had made the range an unreliable measure of spread.

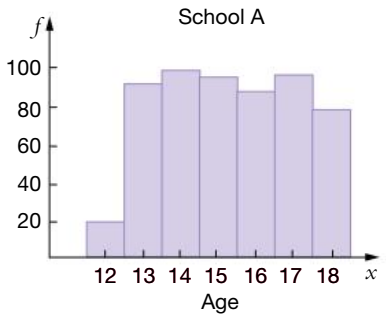
The double box plots clearly show these features.

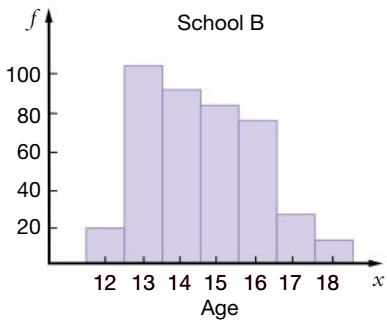


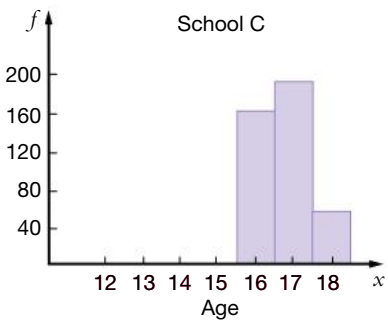
Test 1 is positively skewed as more scores are at the 'low end' indicated by the median being lower than the mean.
Test 2 is negatively skewed as more scores are at the 'high end', indicated by the median being greater than the mean.

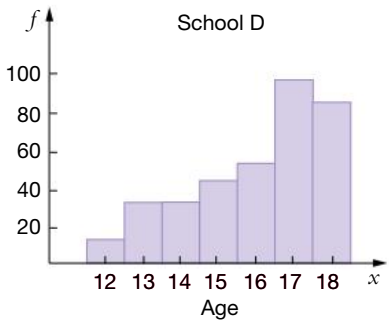
Exercise 13:05

1 The age distributions of students in four high schools are shown below.









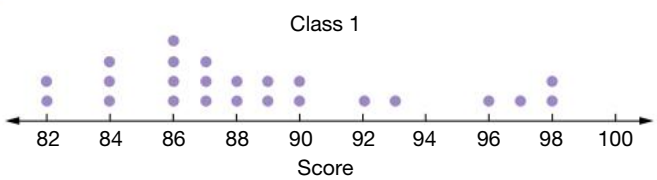
a Which schools' age distributions are skewed? What causes the skew?

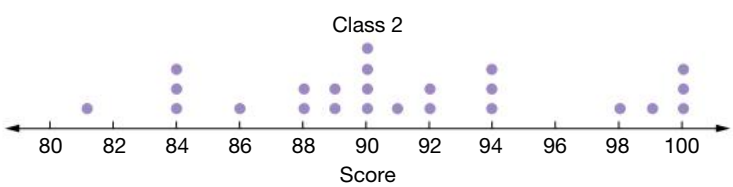
b Which school's age distribution is closest to being distributed evenly?

c In which school would the mean age of a student be:

i closest to 15 **ii** below 15 **iii** over 15 **iv** the largest?

2 The marks for two classes on the same test are shown in the dot plot below.





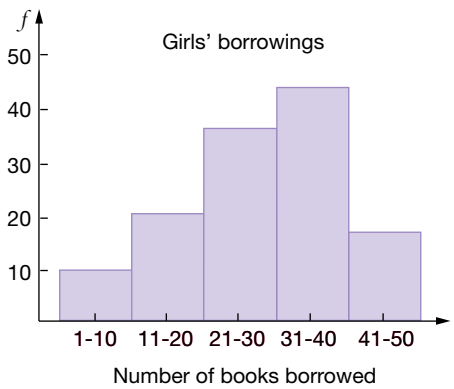
a Which set of results is more skewed?

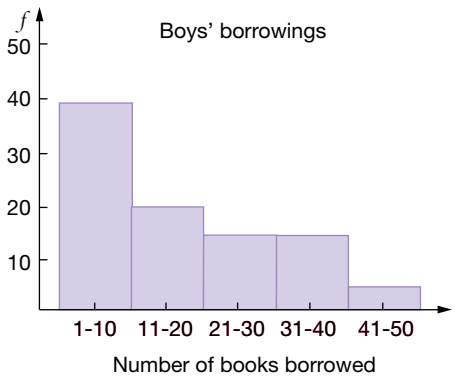
b By just looking at the dot plots, estimate which class has:

i the higher mean **ii** the greater spread of scores.

c Check your answer to part **b** by calculating the mean and range for each set of scores.

3 A school librarian was interested in comparing the number of books borrowed by boys and girls. At the end of the year, she looked at the number of books borrowed by each child and prepared the following graphs.





a Describe the shape of the distribution for:
  i the girls' borrowings
  ii the boys' borrowings.

b What is the first impression that the shapes of the distributions give about the boys' and girls' borrowings?

c Why do you think two grouped frequency histograms were used to display the results instead of a back-to-back stem-and-leaf plot?

d What sort of distribution would result if the librarian combined the boys' and girls' results?

4 The stem-and-leaf plot shows the marks of a class on two different topic tests.

a Which set of marks is nearly symmetric?

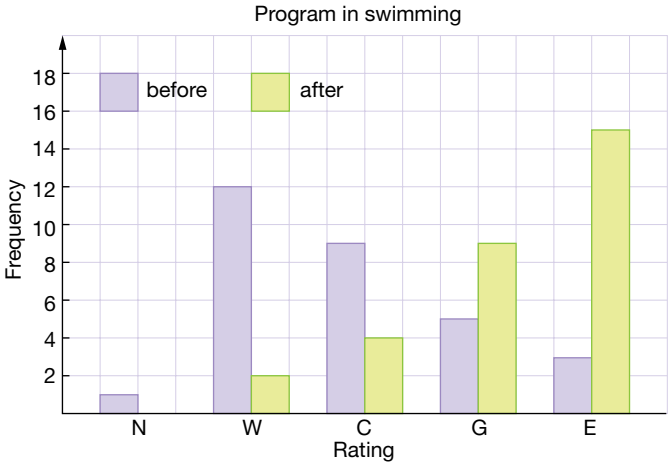
b Which set of marks has the smaller spread? What measures of spread can you use to support your answer?

c Calculate the median and mean for each set of marks. What do they suggest about the class performance on the two tests?

To check the shape of the distribution turn the stem-and-leaf plot side on.

Class tests		
Topic 1	Stem	Topic 2
6 5	3	
9 3 0	4	4 6 7 9
9 8 4 3 0	5	4
9 8 8 6 0	6	2 3 7
9 8 4 4	7	0 8 8 9
6 5	8	3 5 8 8 8 9 9
	9	5

5 Thirty students entered a swimming program hoping to improve their swimming. Before and after the program, students were rated as non-swimmer (N), weak swimmer (W), competent swimmer (C), good swimmer (G) or excellent swimmer (E).



a What was the mode rating before the program?

b What was the mode rating after the program?

c Before the program, what percentage of students were rated either non-swimmers or weak swimmers?

d After the program, what percentage of students were rated good or excellent swimmers?

e How would you describe the success of the program?

6 In two problem-solving tests, five questions were given to a class. The scores are shown below.

Problem test 1

5 1 3 4 3 4 1 1 2 2 5 3 3 1 1 2 4 3 2 4 4 3 2

Problem test 2

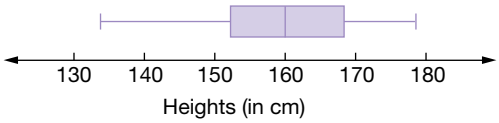
0 2 4 4 2 2 3 0 0 2 4 4 1 3 3 2 2 3 2 2 0 2 2

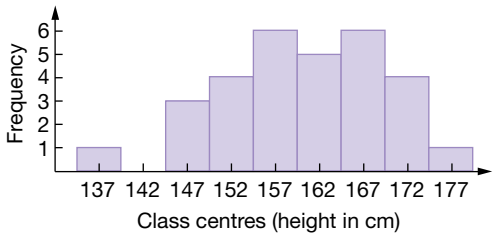
a Arrange the scores into a frequency distribution table and use frequency histograms to display the data.

b Calculate the mean and median for each test. What do they suggest about the difficulty of the tests?

c Both sets of scores have a range of 4. Which set of scores has the greater spread? Give a reason for your answer.

7 This box plot represents the height of 30 Year 10 students. The histogram also represents the heights of 30 students.



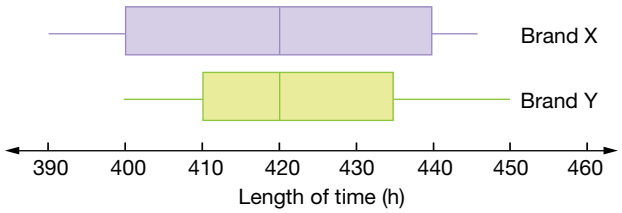


a What information is shown on the box plot?

b Could the information in the histogram represent the same 30 Year 10 students who are represented in the box plot? Explain.

8 A researcher tested two different brands of batteries to see how long they lasted. Her results are shown in the double box plots below.

Use the double box plots to compare the performance of Brand X and Brand Y.



9 Two groups of adults underwent a simple fitness test. One minute after undergoing a period of strenuous exercise, their heart rates were measured.

The results are shown in the back-to-back stem-and-leaf plot.

a What does the shape of the stem-and-leaf plot suggest about the data?

b Calculate the median and interquartile range for each group and use them to compare the results for each group.

Heart rates

Group 1	Stem	Group 2
7 5 5	11	
9 9 9 9 8 8 7 7 6	12	5 5 6
6 6 4 4 0	13	4 6 8 8
4 2 0	14	2 3 6 6 7 8 8
	15	3 5 5 7
	16	2 4

10 A local council was interested in speeding up the time it took to approve applications to build a house. It looked at the time taken in days to process 40 applications. After reviewing its procedures and monitoring processes, it then looked at another 40 applications. The results are shown below.

Before

44	53	38	39	52	41	40	41
43	43	42	57	47	45	50	50
68	50	45	42	58	48	40	39
44	46	52	45	46	53	54	40
48	47	43	38	43	42	54	55

After

40	39	43	42	54	48	46	44
51	52	44	38	40	40	51	52
39	46	46	49	52	51	42	43
40	45	44	39	50	43	48	40
52	53	38	40	47	44	47	42

a Discuss an appropriate way to organise and display the data.

b What measures of cluster and spread would you use to describe the data?

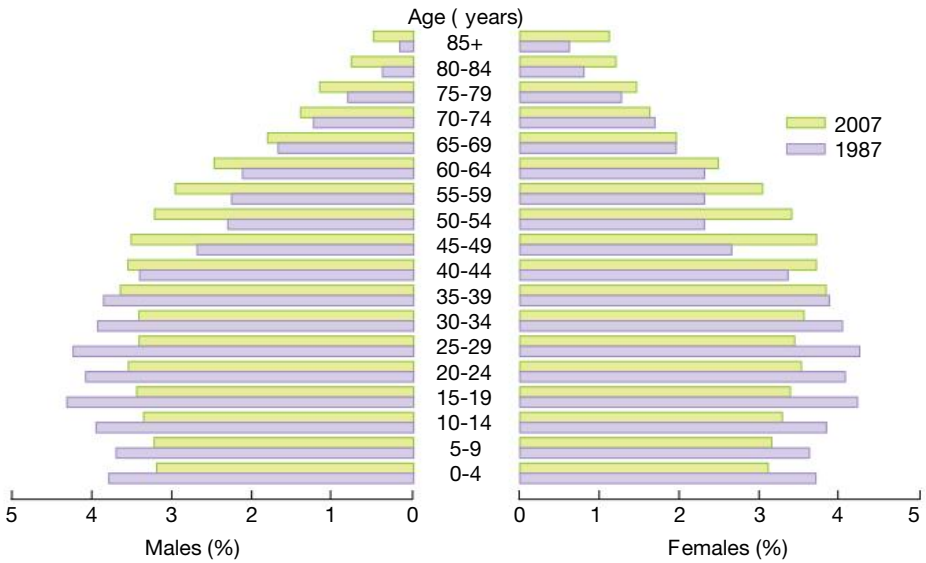
c How effective have the council's review procedures been in reducing the approval time?

INVESTIGATION 13:05 THE AGEING POPULATION

The Australian Bureau of Statistics collects information on a wide variety of topics. This information is used by governments to help formulate social policy.

The gradual aging of Australia's population has caused a rethinking of the government's policy towards pensions, superannuation and caring for the aged.

POPULATION STRUCTURE, by Age and Sex – 1987 and 2007



Where your answers are percentages, give them correct to one decimal place.

1 In 1987, what percentage of Australians were males aged:
a 10–14 **b** 40–44 **c** 70–74?

2 In 2007, what percentage of Australians were males aged:
a 10–14 **b** 40–44 **c** 70–74?

3 What do your answers to Questions **1** and **2** suggest about the changes in the male population over the 20 years from 1987 to 2007? Does this trend also occur in the female population?

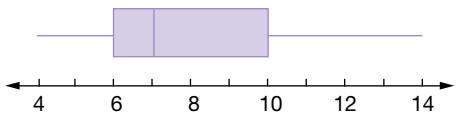
4 **a** What percentage of Australians were aged 70 and over in:
i 1987 **ii** 2007?
b If there were 1 920 200 people aged over 70 in 2007, calculate the total population of Australia in that year. Give your answer to the nearest thousand.

5 The approximate number of Australian females aged 65–69 was 300 100 in 1987 and 401 200 in 2007 and yet looking at the diagram, the percentage of females in the 65–69 age group was relatively similar in 1987 and 2007. How is this possible?

MATHS TERMS 13

box-and-whisker plot (box plot)

- a diagram obtained from the five-point summary
- the box shows the middle 50% of scores (the interquartile range)
- the whiskers show us the extent of the bottom and top quartiles as well as the range



class centre

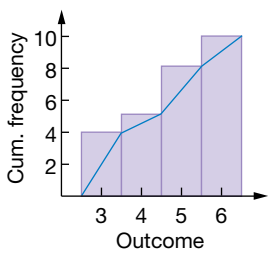
- the middle outcome of a class,
  e.g. the class 1–5 has a class centre of 3

class interval

- the size of the groups into which the data is organised,
  e.g. 1–5 (5 scores); 11–20 (10 scores)

cumulative frequency histogram (and polygon)

- these show the outcomes and their cumulative frequencies



dot plot

- a graph that uses one axis and a number of dots above the axis

five-point summary

- a set of numbers consisting of the minimum score, the three quartiles and the maximum score

frequency

- the number of times an outcome occurs in the data,
  e.g. for the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 the outcome 5 has a frequency of 3

frequency distribution table

- a table that shows all the possible outcomes and their frequencies (it usually is extended by adding other columns such as the cumulative frequency),

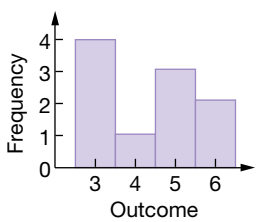
e.g.

Outcome	Frequency	Cumulative frequency
3	4	4
4	1	5
5	3	8
6	2	10

frequency histogram

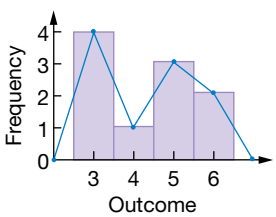
- a type of column graph showing the outcomes and their frequencies,

e.g.



frequency polygon

- a line graph formed by joining the midpoints of the top of each column; to complete the polygon the outcomes immediately above and below those present are used. The heights of these columns is zero



grouped data

- the organisation of data into groups or classes

interquartile range

- interquartile range = $Q_3 - Q_1$
- the range of the middle 50% of scores
- the median of the upper half of scores minus the median of the lower half of scores

mean

- the number obtained by 'evening out' all the scores until they are equal,
  e.g. if the scores 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 were 'evened out', the number obtained would be 4·3
- to obtain the mean, use the formula:

$$\text{Mean} = \frac{\text{sum of the scores}}{\text{total number of scores}}$$

median

- the middle score for an odd number of scores or the mean of the middle two scores for an even number of scores

median class

- in grouped data, the class that contains the median

mode (modal class)

- the outcome or class that contains the most scores

ogive

- this is another name for the cumulative frequency polygon

outcome

- a possible value of the data

outlier

- a score that is separated from the main body of scores

quartiles

- the points that divide the scores up into quarters
- the second quartile, Q_2, divides the scores into halves (Q_2 = median)
- the first quartile, Q_1, is the median of the lower half of scores
- the third quartile, Q_3, is the median of the upper half of scores

4 5 6 | 6 7 7 | 7 9 9 | 11 12 15

$Q_1 = 6$ $Q_2 = 7$ $Q_3 = 10$

range

- the difference between the highest and lowest scores

shape (of a distribution)

- a set of scores can be symmetric or skewed (see pages 424–425)

statistics

- the collection, organisation and interpretation of numerical data

stem-and-leaf plot

- a graph that shows the spread of scores without losing the identity of the data

DIAGNOSTIC TEST 13 STATISTICS

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

13:01, 13:02

1 The students of class 9M were given a reading test and rated from 0 (a non-reader) to 5 (an excellent reader).
The results are given below.

4	1	0	2	3	3	3	2	2	1
0	2	2	4	3	5	3	2	1	3
2	0	3	1	3	4	5	1	0	2

Outcome (x)	Tally	f	$c.f.$
0			
1			
2			
3			
4			
5			

Total:

a Complete this frequency distribution table.
b What is the frequency of 5?
c How many students were given a rating less than 4?
d On the same diagram, draw the frequency histogram and the frequency polygon.
e On the same diagram draw the cumulative frequency histogram and the cumulative frequency polygon.
f What is the range of these scores?
g Find the mode, median and mean for these scores.

13:01

2 Use your calculator to evaluate the mean for the scores in the following frequency tables. Give your answer correct to two decimal places.

a

Outcome	Freq.
27	18
28	50
29	23
30	9

b

Outcome	4·1	4·2	4·3	4·4	4·5	4·6	4·7
Freq.	7	11	16	8	12	7	3

13:01, 13:02

3 These are the scores gained by each team competing in the Lithgow car rally this year.

27	18	0	45	63	49	50	31	9	26
4	41	38	20	69	38	17	43	16	37
28	14	58	52	37	43	38	51	44	33
25	38	11	43	40	56	62	48	53	22

a Draw a grouped frequency table using classes 0–9, 10–19, etc. Use the columns: class, class centre, tally, frequency and cumulative frequency.
b Prepare a stem-and-leaf plot for the scores above.

4 Find the median for each of the following.

a

Outcome (x)	*c.f.*
4	3
5	7
6	10
7	16
8	23

b

Outcome (x)	*c.f.*
11	7
12	15
13	33
14	53
15	62

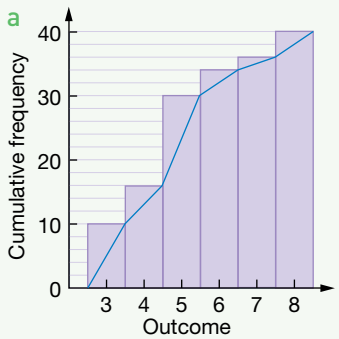
c

Outcome (x)	*c.f.*
1	24
2	37
3	44
4	47
5	50

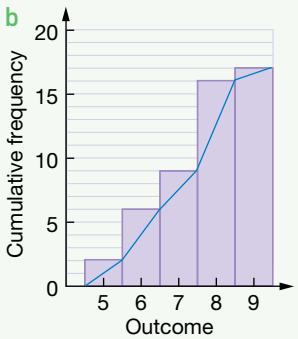
13:02

5 Use the following graphs to calculate the median.

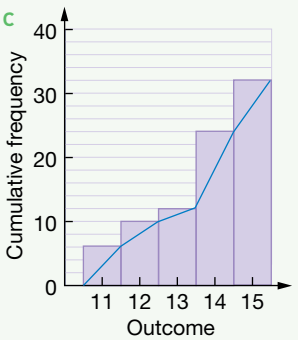
a



b



c



13:03

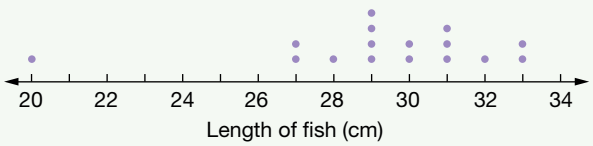
6 **a** Find the interquartile range of the scores:

1, 2, 2, 5, 7, 9, 10, 10, 11, 11, 11, 11

b Draw a cumulative frequency polygon using the frequency distribution table below and use it to find the interquartile range of the scores

x	10	11	12	13	14	15	16	17	18	19
f	2	0	5	4	5	6	5	6	3	4

c The lengths of 16 fish caught were measured. The results are shown on this dot plot. What is the interquartile range?



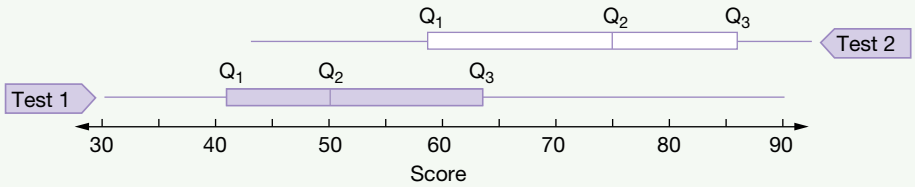
d What is the interquartile range of the times shown in the stem-and-leaf plot?

Time	
Stem	Leaf
3	8 8
4	0 3 4 5 8
5	1 3 4 4
6	0

7 Find the five-point summary for each set of data in Question 6. 13:04

8 Draw box plots for the data in Question 6 a, b and c. 13:04

9 These double box plots were drawn to compare the results of Year 10 on two tests. 13:05



a By how much was the median for test 2 higher than the median of test 1?

b What was the range and interquartile range of test 1?

10

Test 1 scores

12	17	19	12	15	10
9	22	24	11	18	8
25	15	18	20	18	18

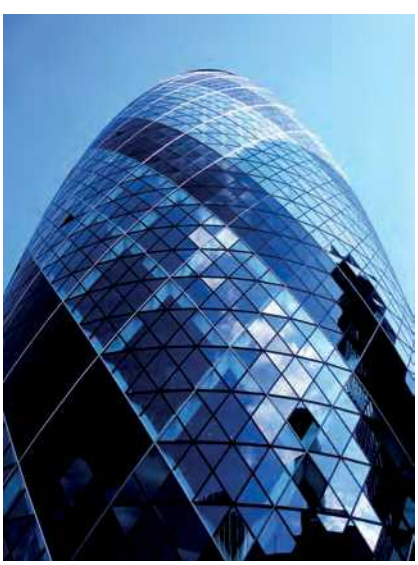
Test 2 scores

21	15	18	7	11	16
20	12	23	12	10	13
12	19	12	14	20	9

a Draw a dot plot for the scores on Test 1.

b Draw a back-to-back stem-and-leaf plot to compare the scores on Test 1 and Test 2.

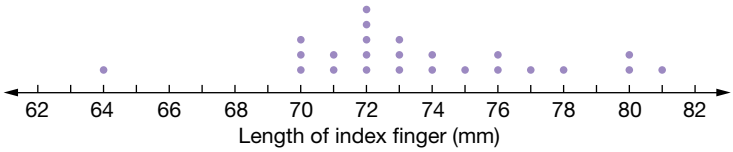
c Draw double box plots to compare the scores on Tests 1 and 2.



This building, nicknamed the Gherkin, is the first skyscraper in London built using a sustainable design. Sensors on the building collect data, which is used to control the lighting, ventilation and heating, and reduce energy usage.

ASSIGNMENT 13A Chapter review

1 The average length of the two index fingers of 24 teachers was recorded. The results are shown on this dot plot.



a Are any outliers present in this data?

b Find the five-point summary for the data if the outlier is:

i included **ii** omitted.

c What is the interquartile range if the outlier is:

i included **ii** omitted?

d Comment on the shape of the distribution (ignore the outlier in this case).

2 A group of Year 9 students sat for a college reading test and the results were entered in this stem-and-leaf plot.

Test scores (7|3 represents 73)

Stem	Leaf
3	1 8 2
4	6 3 3 1 3
5	4 1 1 5
6	2 2 8 4 7 7 1
7	6 3 4 6 5 5 1 4
8	2 6 6 7 3 3 1
9	2 9 9 1 0 0

a How many scored in the 50s?

b What was the mode of the results?

c How many students scored less than 50?

d How many students were tested?

e What was the median of the results?

f If 65 and above is considered to be a passing grade, how many passed?

g Make a grouped frequency table for this data using classes of 30–39, 40–49, …

3 Year 3 and Year 4 students were tested on their knowledge of multiplication tables. The results are shown in this back-to-back stem-and-leaf plot.

Test scores (5|1 represents 51)

Year 3	Stem	Year 4
9 8 3 0	3	5
8 8 7 4 2 2 2	4	
8 7 7 4 0	5	1 4 6 6 9
7 6 3	6	0 2 2 5 8
1	7	3 6 8 8 9 9 9
	8	0 0

Compare the results of Year 3 and Year 4 on this test. (You will need to refer to at least one measure of cluster and one measure of spread.)

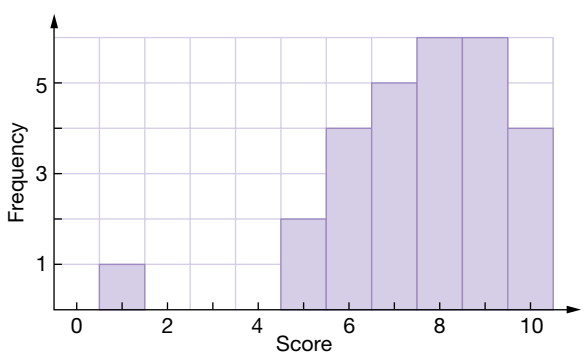
4 a For the data shown in this histogram, determine the:

i range

ii median

iii interquartile range

iv the five-point summary.



b Use this information to construct a box plot.

5 After the Year 8 semester exam the maths staff organised the 99 marks into a grouped frequency distribution. The results are shown in the table.

a Copy and complete the grouped frequency distribution table.
Use it to find:
i the modal class **ii** the mean.

b Construct an ogive and use it to find the median class.

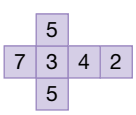
Class	Class centre (*c.c.*)	Freq. (f)	Cumu. Freq.	$f \times c.c.$
10–19		2		
20–29		9		
30–39		10		
40–49		8		
50–59		16		
60–69		20		
70–79		13		
80–89		14		
90–99		7		

ASSIGNMENT 13B Working mathematically

1 Give the names of the shapes found in this leadlight window.

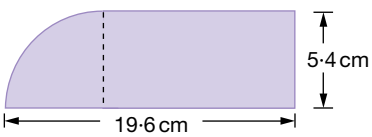
2 The sum of Peter and Paul's ages is 91. Peter is now twice as old as Paul was when Peter was as old as Paul is now. How old is Peter?

3 When this net is folded to form a cube, three faces meet at each vertex and are multiplied together. What is the smallest product possible?



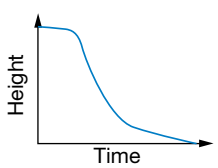
4 In a 360-minute tennis match, 4 players were always on the court. Altogether there were 12 players (6 from each team). Each of the players was on the court for the same length of time. How many minutes did each player spend on the court?

5 Find the perimeter and area of this figure.



6 Choose the heading from the list below that would best fit each graph.

a



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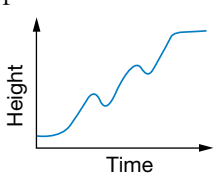

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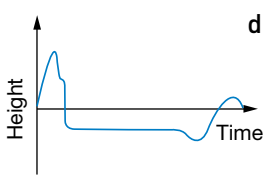

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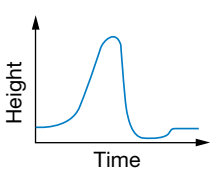
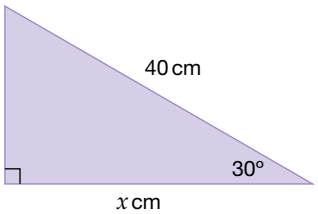


A The hook of a fishing line while fishing.
B An arrow fired into the air.
C Flying a kite.
D Position of my head while pole vaulting.
E A parachute jump.
F Position of my foot as I kick a ball.

ASSIGNMENT 13C Cumulative revision

1 **a** Find the value of x, correct to one decimal place.
b The two short sides of a right-angled triangle are 5 m and 8 m in length. Find the sizes of the angles in the triangle to the nearest degree.

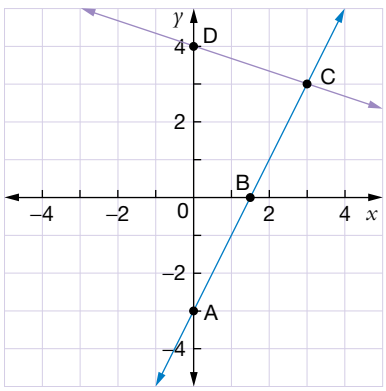


12:07A

2 On a photocopier the enlargement and reduction factors are given as percentages. Find the enlargement and reduction factor for the following.
a Enlarge a photograph that is 18 cm long and 13 cm wide so that it is 28·8 cm long and 20·8 cm wide.
b Reduce a drawing that is a square of side 16 cm to a square of side 12 cm.

10:01, 10:02

3 **a** Find the gradient and y-intercept of the line AB and use it to write its equation.
b Find the equation of the line DC and use it to find the x-intercept of the line.
c What is the slope (or gradient) of the line $2x + y = 4$?



9:07, Invest 9:07

4 Bag A contains 32 red and 48 green marbles. Bag B contains 18 red and 26 green marbles.
a Calculate the probability of choosing a red marble from Bag A.
b What is the probability of choosing a red marble from Bag B?
c Which bag gives the best chance of choosing a red marble?

4:03

5 At the end of the financial year Hannah's total income is $71 325. She has allowable tax deductions of $3327 and throughout the year she has made PAYG deductions totalling $14 924. Calculate:
a her taxable income
b the tax payable on her taxable income if she has to pay $4650 plus 30c for each dollar over $37 000
c the Medicare levy, which is 1·5% of her taxable income
d the refund due or the amount of tax still payable.

8:04

PROPORTION

14

Contents

14:01 Review of rates
14:02 Direct proportion
14:03 Graphing direct proportion
Investigation 14:03 Proportional ups and downs!
14:04 Inverse proportion
Investigation 14:04 Graphing inverse proportion
Fun spot 14:04 How do poets write upside down?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details)

Number and Algebra

Selections from ***Ratios and Rates*** [Stage 5.2]

- Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems. (ACMNA208)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

14:01 Review of rates

PREP QUIZ 14:01

If Wendy earns $16 per hour, how much would she earn in:

1 2 hours **2** 3 hours **3** 5 hours **4** half an hour?

Complete:

5 1 kg = … g **6** 1 tonne = … kg **7** 1 hour = … min

8 1 cm = … mm **9** 1 m^2 = … cm^2 **10** 1·5 litres = … millilitres

A rate is a comparison of unlike quantities.

e.g. If I travel 180 km in 3 hours my average rate of speed is $\frac{180\text{ km}}{3\text{ h}}$ or 60 km/h or 60 km per hour.

We usually write how many of the first quantity correspond to one of the second quantity.
e.g. 60 kilometres per one hour = 60 km/h

WORKED EXAMPLE 1

a 84 km in 2 hours

Divide each term by 2.

= 42 km in 1 hour

= 42 km/h

Units must be shown.

OR 84 km in 2 hours

$= \frac{84\text{ km}}{2\text{ h}}$

$= \frac{84}{2}\frac{\text{km}}{\text{h}}$

= 42 km/h

Example **a** is an average rate, because when you travel, your speed may vary from moment to moment.
Example **b** is a constant rate, because each kg will cost the same.

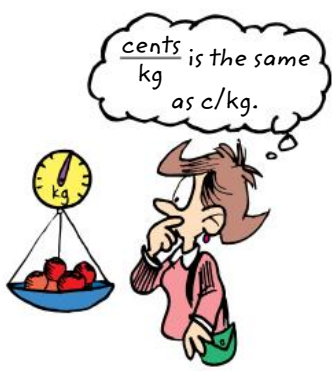
b 16 kg of tomatoes are sold for $10.
What is the cost per kilogram?

$\text{Cost} = \frac{\$10}{16\text{ kg}}$

$= \frac{1000}{16}\frac{\text{cents}}{\text{kg}}$

$= \frac{125}{2}\frac{\text{cents}}{\text{kg}}$

= 62·5 cents/kg



c A plumber charges a householder $64 per hour to fix the plumbing in a house.
Find the cost if it takes him $4\frac{1}{2}$ hours.

Rate = $64 per 1 hour

= $288 per $4\frac{1}{2}$ hours Multiply both terms by $4\frac{1}{2}$.

∴ Cost = $288

WORKED EXAMPLE 2

a Change 1·5 km/min into km/h.

$1{\cdot}5\,\text{km/min} = (1{\cdot}5 \times 60)\,\text{km/h}$

$= 90\,\text{km/h}$

b Change 25 000 mL/h into L/h.

$25\,000\,\text{mL/h} = (25\,000 \div 1000)\,\text{L/h}$

$= 25\,\text{L/h}$

c Convert 72 litres per hour into cm^3 per second.

$$72\,\text{L per h} = \frac{72\,\text{L}}{1\,\text{h}}$$

$$= \frac{(72 \times 1000)\,\text{mL}}{(60 \times 60)\,\text{s}}$$

$$= \frac{72\,000\,\text{cm}^3}{3600\,\text{s}}$$

$$= 20\,\text{cm}^3/\text{s}$$

Remember: $1\,\text{mL} = 1\,\text{cm}^3$

d Convert 10 m/s into km/h.

$$10\,\text{m per s} = \frac{10\,\text{m}}{1\,\text{s}}$$

$$= \frac{10\,\text{m} \times 60 \times 60}{1\,\text{h}}$$

$$= \frac{36\,000\,\text{m}}{1\,\text{h}}$$

$$= \frac{(36\,000 \div 1000)\,\text{km}}{1\,\text{h}}$$

$$= 36\,\text{km/h}$$

Exercise 14:01

1 Write each pair of quantities as a rate in its simplest form.

a 6 km, 2 h	**b** 10 kg, $5	**c** 500c, 10 kg
d 100 mL, $100\,\text{cm}^3$	**e** 160 L, 4 h	**f** $100, 5 h
g $315, 7 days	**h** 70 km, 10 L	**i** 20 degrees, 5 min
j 7000 g, 100 cm	**k** 50 t, 2 blocks	**l** 60 km, $\frac{1}{2}$ h
m 88 runs, 8 wickets	**n** 18 children, 6 mothers	**o** 75 g, $10\,\text{cm}^3$

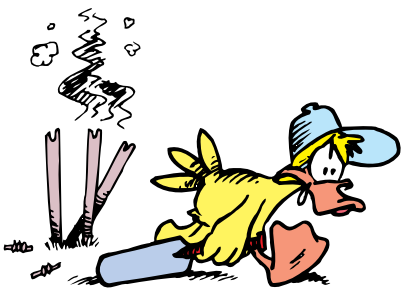
2 **a** I walk at 5 km/h. How far can I walk in 3 hours?

b Nails cost $2.45 per kg. What is the cost of 20 kg?

c I can buy four exercise books for $5. How many books can I buy for $20?

d I earn $8.45 per hour. How much am I paid for 12 hours work?

e The run rate per wicket in a cricket match has been 37·5 runs per wicket. How many runs have been scored if 6 wickets have been lost?



f The energy value of milk is measured as 670 kilojoules per cup. What is the energy value of 3 cups of milk?

g If the rate of exchange for one British pound is 1·60 American dollars, find the value of 10 British pounds in American currency.

h The density of iron is $7{\cdot}5\,\text{g/cm}^3$. What is the mass of $1000\,\text{cm}^3$ of iron? (Density is mass per unit of volume.)

i If light travels at 300 000 km/s, how far would it travel in one minute?

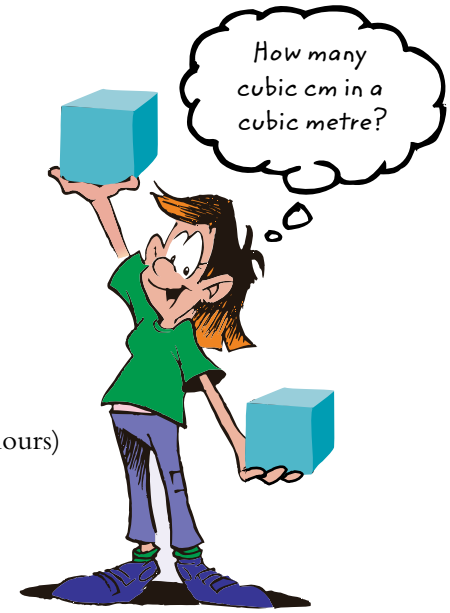
j If I am taxed 1·6c for every $1 on the value of my $50 000 block of land, how much must I pay?

3 Complete these equivalent rates.

a 1 km/min = … km/h
b 25 cents/g = … cents/kg
c 40 000 m/h = … km/h
d $5/kg = … c/kg
e 3 m/s = … m/min
f 7 m/s = … mm/s
g 6·5 km/L = … m/L
h 7200 m/L = … m/mL
i 120 beats/min = … beats/s
j $70/day = … $/week
k 30 c/m^2 = … c/ha
l 8500 kg/day = … t/day

4 Complete these conversions that involve changing both quantities to smaller units.

a 60 km/h = … m/min
b 72 km/h = … m/s
c 24 m/min = … cm/s
d 1·2 m/min = … mm/s
e $50/kg = … c/g
f 144 L/h = … mL/s
g 60 km/h = … m/s
h 7 km/L = … m/mL
i 3 t/h = … kg/min
j $50/$m^2$ = … c/cm^2



5 Complete these conversions that involve changing both quantities to larger units.

a 100 m/min = … km/h
b 25 m/s = … km/h
c 5 mm/s = … cm/min
d 150 cm/s = … km/h
e 30 mm/s = … km/h
f 95 c/h = … $/week (40 hours)
g 800 kg/h = … t/day
h 50 c/m^2 = … $/ha
i 20 mL/m = … L/km
j 5 g/cm^3 = … kg/m^3

6 In 1999, Noah Ngeny of Kenya ran 1000 m in 2 minutes, 11·96 seconds. What was his average speed in km/h?

7 Usain Bolt of Jamaica set a new record for running 100 m in 9·58 seconds. What average speed is this in km/h, correct to one decimal place?

8 In 1998, Takahiro Sunada of Japan ran 100 km in 6 hours, 13 minutes and 33 seconds. What was his average speed in km/h, correct to one decimal place?

9 A car is approaching a pedestrian crossing at 108 km/h. If it is 90 m from the crossing, how much time do you have to cross before the car reaches the crossing, assuming the car does not slow down?

14:02 Direct proportion

PREP QUIZ 14:02

Complete:

1 20 m/s = … m/min **2** 7000 m/h = … km/h **3** 360 g/min = … g/s

4 200 m in 10 s = … m/s **5** 12 L in 1 hour = … mL/min **6** $24 for 10 kg = … $/kg

7 What will 20 pencils cost if 4 pencils cost $1.36?

8 If 3 folders cost $2.40, how much will 8 cost?

9 Jenny measures 23 heart beats in 12 seconds. How many beats per minute is this?

10 Rob jogs at an average speed of 5 minutes and 35 seconds per km. How long will it take him to complete a fun run with a distance of 14 km?

If all parts of a shape or object are increased or decreased by the same ratio or fraction, then lengths on the original shape or object and lengths on the new shape or object are said to be in *direct proportion*. Any increase or decrease in one variable will result in a proportional change in the other variable (they will vary at the same rate).

For example, if a photo and its enlargement are in direct proportion, then we could determine unknown measurements using the enlargement factor. If the original photo measures 15 cm by 12 cm and the length of the enlargement is 25 cm, then its width w can be found using the following ratio statement.

$$\frac{w}{12} = \frac{25}{15}$$

$$w = \frac{25}{15} \times 12$$

$$\therefore w = 20 \text{ cm}$$

Ratio of the lengths = Ratio of the widths

We can approach this problem in a different way.

If any length (L) on the enlargement is proportional to the corresponding length (l) on the original photo, we can write this as:

$L \propto l$

The symbol $\propto$ means 'is proportional to'.

To be in direct proportion, all lengths are multiplied by the same factor, so we can write:

$L = kl$ where k is the **constant of proportionality** (or the proportional constant)

Now substitute corresponding values that are known. In this case, $L = 25$ cm and $l = 15$ cm.

$$L = kl$$

$$25 = k \times 15$$

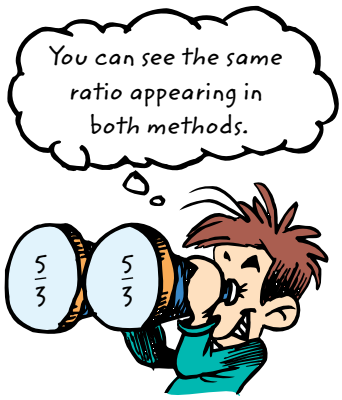
$$\therefore k = \frac{25}{15} \text{ or } \frac{5}{3}$$

We know that for any length on the photo (l), the corresponding length on the enlargement (L) is given by:

$$L = \frac{5l}{3}$$

So if the width of the photo is 12 cm, then the corresponding width on the enlargement is given by:

$$L = \frac{5}{3} \times 12$$
$$= 20\,\text{cm}$$



WORKED EXAMPLE 1

10 litres of paint covers 75 m^2 of wall. The area of wall covered is directly proportional to the amount of paint used.

a Write an equation expressing this relationship and find the constant of proportionality.

b Use this to find the area of wall covered by 7 litres of paint.

c How many litres of paint would be needed to cover 180 m^2 of wall?

Solutions

A is the area of wall covered and p is the amount of paint used.

a $A \propto p$

So $A = kp$

$\therefore 75 = k \times 10$ Substituting the corresponding known values.

so $k = \frac{75}{10}$ or 7·5

$\therefore$ The equation expressing this direct proportion is $A = 7{\cdot}5p$.

b To find the area covered by 7 litres of paint, substitute $p = 7$ into the equation to find the corresponding value of A.

$A = 7{\cdot}5p$

$= 7{\cdot}5 \times 7$

$= 52{\cdot}5$

So, 7 litres of paint covers 52·5 m^2.

c Similarly, by substituting $A = 180$ into the equation, we can find the corresponding value of p.

$A = 7{\cdot}5p$

$180 = 7{\cdot}5p$

$P = \frac{180}{7{\cdot}5}$

$= 24$

So, 24 litres of paint covers 180 m^2.

WORKED EXAMPLE 2

A 36 000 kJ survival pack is enough to sustain 5 hikers for a period of 4 days.

a How many kilojoules would be needed for 7 hikers to last 3 days?

b How many days would a 129 600 kJ pack last 4 hikers?

c How many hikers could survive on a 108 000 kJ pack for 3 days?

Solutions

Consider the supply needed by 5 hikers (h) for 4 days (d) as 20 rations. (5×4)

Then a 36 000 kJ pack (P) will supply 20 rations. ($h \times d$)

So, $P = k(hd)$

Finding k: $36\,000 = k \times 20$

$k = 1800$

$\therefore P = 1800hd$

a To supply 7 hikers for 3 days:

$P = 1800 \times 7 \times 3$

$= 37\,800$

So, 7 people for 3 days would need 37 800 kJ.

b If $P = 129\,600$:

$P = 1800hd$

$129\,600 = 1800hd$

$hd = 129\,600 \div 1800$

$= 72$

Now $h = 4$, so $4d = 72$

$\therefore d = 18$

So 129 600 kJ divided among 4 people would last 18 days.

c Similarly, if $P = 108\,000$:

$P = 1800hd$

$108\,000 = 1800hd$

$hd = 60$

Now $d = 3$, so $3h = 60$

$\therefore h = 20$

So, 108 000 kJ divided over 3 days would sustain 20 people.

Exercise 14:02

1 A is directly proportional to m.

a Write an equation relating A to m.

b If $A = 15$ when $m = 6$, find the constant of proportionality.

c What will be the value of A if $m = 10$?

d If $A = 50$ what is the corresponding value of m?

2 Y is directly proportional to x.

a Write an equation relating Y and x.

b If $Y = 20$ when $x = 25$, find the constant of proportionality.

c What will be the value of Y when $x = 10$?

d If $Y = 52$ what is the corresponding value of x?

3 The height (H) of a plant is directly proportional to the number of days (d) it has been growing.

a Write a proportional statement linking H and d.

b Find the proportional constant if after 4 days the height of the plant is 30 cm.

c What will be the height of the plant after 7 days?

d After how many days will the height of the plant reach 75 cm?

4 The quantity of paint (p) required for a job is directly proportional to the area (a) that is to be painted.

a Write an equation expressing this proportional relationship.

b If 4 litres of paint will cover 35 m^2, find the proportional constant.

c How many litres of paint will be needed to cover an area of 60 m^2. Answer to the nearest litre.

d What area will 15 litres of paint cover? Answer to the 'next' whole square metre.

5 P is directly proportional to m. When $m = 8{\cdot}6$, $P = 15{\cdot}05$.

a Find an equation relating P and m.

b What is the value of P when m is $1{\cdot}8$?

c What is the value of m when P is $20{\cdot}65$?

6 Y is directly proportional to x^2. When $x = 7$, $Y = 24{\cdot}5$.

a Find an equation relating Y and x^2.

b What is the value of Y when $x = 9$?

c What is the positive value of x when $Y = 8$?

7 The velocity V (in m/s) of a falling object is directly proportional to the time t (in seconds) after release. After 2 seconds the velocity is 19·6 m/s.

a Find an equation relating velocity and time.

b What will be the velocity after 3 seconds?

c When will the velocity be 10 m/s? Give your answer correct to two decimal places.

8 As a general rule, a minimum of 50 m^2 is required for every 4 people working in an office.

a How much space would be needed for 10 people?

b What would be the maximum number of people that could work in an office space of 180 m^2?

9 A mobile phone call lasting 5 minutes and 20 seconds costs $3.50. If the cost is proportional to the length of the call:

- **a** What would be the cost of an 8-minute call? (Hint: Express the time in seconds.)
- **b** How long a call could be made for $8? Give your answer to the nearest second.

10 In a gas fire, the amount X of oxygen used is directly proportional to the amount G of natural gas burned. The burning of 35 kg of natural gas uses 56 kg of oxygen. How many kilograms of oxygen are needed to burn 21 kg of natural gas?

For Questions **11** to **13**, look at Worked Example 2.

11 Fourteen cans of food can feed 10 cats for 3 days.

- **a** Find an equation relating the number of cans of food (F) to the number of cats (c) and number of days (y).
- **b** For how many days would 21 cans of food feed 10 cats?
- **c** How many cans would be needed to feed 10 cats for 9 days?
- **d** How many cans would be needed to feed 15 cats for 3 days?
- **e** How many cans would be needed to feed 15 cats for 9 days?

12 The length of a ditch is directly proportional to the number of men and the amount of time they spend digging.

- **a** Find an equation relating the length of the ditch (D) to the number of men (n) and the number of hours (h) they spend digging it.
- **b** If it takes 6 men 4 hours to dig a ditch 10 m long, find the constant of proportionality for this equation.
- **c** What length of ditch could 9 men dig in 4 hours?
- **d** What length of ditch could 6 men dig in 6 hours?
- **e** What length of ditch could 9 men dig in 6 hours?
- **f** How many men would it take to dig a ditch 25 m long in 5 hours?

13 For a certain species of tree, 10 mature trees can produce the same amount of oxygen inhaled by 500 people over a 4-year period.

- **a** Over a 4-year period, how many people inhale the oxygen produced by 16 mature trees?
- **b** How many trees would be needed to produce the same amount of oxygen used by 200 people, over a 4-year period?
- **c** How many trees would be needed to produce the oxygen used by 800 people over a 10-year period?

14 The force F needed to stretch a spring is directly proportional to the amount y the spring is stretched. A force of 7·5 newtons stretches a spring by 3 cm. Find the amount the spring will be stretched by a force of 8 newtons.

15 The distance D that an object falls is directly proportional to the square of the time t of the fall. That is, $D \propto t^2$. If an object falls 19·6 m in 2 seconds, find:

a an equation relating D and t

b how far will it fall in 4 seconds.

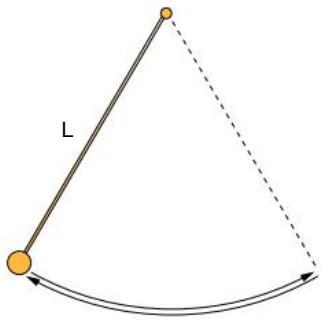
16 The volume V of a sphere and the cube of its radius are directly proportional. For a sphere of radius 1 cm, the volume is 4·189 cm^3.

a Find an equation relating V and r.

b What is the volume of a sphere of radius 3 cm?

c If the volume is 4189 cm^3, what is the radius?

17 The period of a simple pendulum is the time it takes to swing through one complete cycle. The period T (in seconds) of a simple pendulum is directly proportional to the square root of its length L (in metres). It is known that the period is 2·7 s when the length of the pendulum is 1·8 m.



a Find an equation relating T and L, writing the value of the proportional constant correct to three decimal places.

b What is the period of a pendulum of length 0·9 m? (Answer correct to one decimal place.)

c What is the length of a pendulum that has a period of 1 second? (Answer correct to the nearest centimetre.)

14:03 Graphing direct proportion

PREP QUIZ 14:03

These equations represent straight lines in the number plane. What is the gradient of each line?

1 $y = 3x$ **2** $y = x$ **3** $y = \frac{x}{2}$ **4** $y = \frac{5x}{3}$ **5** $y = 0{\cdot}25x$

6 If $y = 5x$, find y if $x = 7$.

7 If $y = \frac{x}{3}$, find y if $x = 1{\cdot}26$.

8 If $y = \frac{7x}{9}$, find y if $x = 4\frac{1}{2}$.

9 If $y = 3x$, find x if $y = 27$.

10 If $y = 0{\cdot}4x$, find x if $y = 1$.

In Chapter 9 we saw that equations like those in the prep quiz represent straight lines, when graphed on the number plane. Also, they all pass through the origin because $x = 0$ when $y = 0$. Each line has a constant gradient. The measure of this gradient (or slope) is given by the coefficient of x.

For the line $y = mx$, m is the gradient.

This equation is of the same form as the equation for direct proportion: $y = kx$, where x and y are the quantities in proportion and k is the constant of proportionality.

So, if we graphed the relationship between two quantities that are in direct proportion we would expect to see a straight line that passes through the origin with a gradient equal to the proportional constant k.

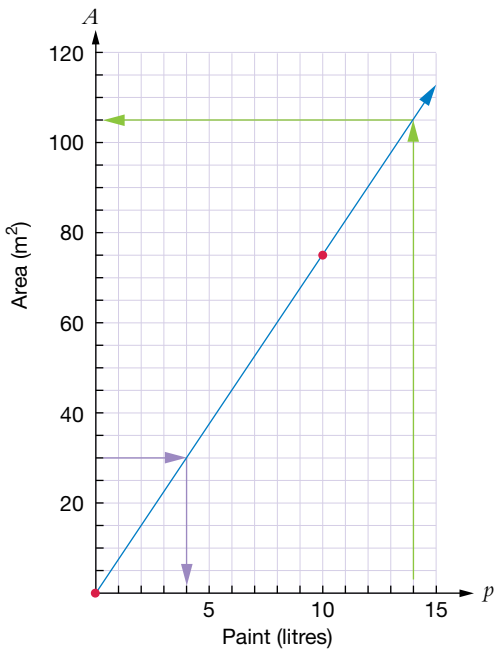
WORKED EXAMPLE 1

If 10 litres of paint (p) covers an area (A) of 75 m^2, this gives the equation of direct proportion: $A = 7{\cdot}5p$ (See Worked Example **1** in Section 14:02.)
Graph this equation and use it to find:

a the area covered by 14 litres of paint **b** the paint required to cover 30 m^2.

Solution

The graph can be simply drawn by plotting the two known pairs of values (i.e. when $p = 10$, $A = 75$ and of course, when $p = 0$, $A = 0$). This is shown by the red dots on the graph.
A straight line is then drawn from (0,0) passing through (10,75). The gradient of the line can be measured as 7·5, that is, the value of the constant of proportionality.
Any other values required can now be read from the graph.



a So, if $p = 14$, reading from 14 on the horizontal axis up to the line and across to the vertical axis, we find $A = 105$. Therefore, 14 litres of paint will cover 105 m^2.

b If we read from $A = 30$ on the vertical axis, across and down to the horizontal axis, we find the corresponding value for p is $p = 4$. So, to cover an area of 30 m^2 we need 4 litres of paint.

WORKED EXAMPLE 2

The currency exchange rate says that 1 Australian dollar will buy 1·3 New Zealand dollars.

a Use this fact to draw a graph showing this proportional relationship.

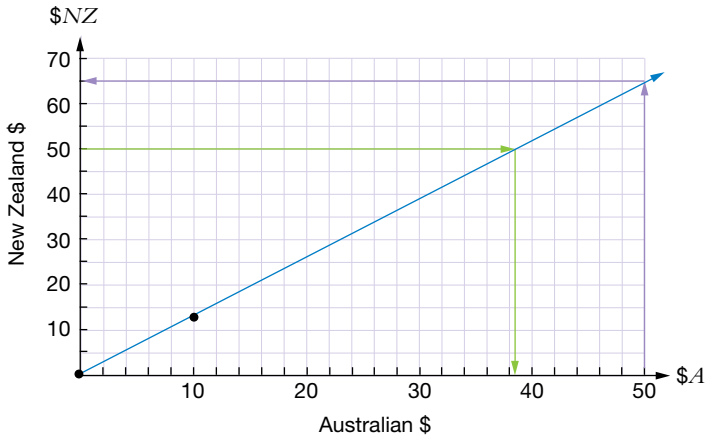
b What is the gradient of this linear graph and hence, the constant of proportionality?

c How many $NZ will $A50 buy?

d How many $A will $NZ50 buy?

Solutions

a If $A1 = $NZ1.30, then
$A10 = $NZ13
This is a more convenient pair of values to graph.
Plotting (0,0) and (10,13) and drawing the line gives the graph shown here.



b The gradient (and the value of the constant of proportionality) is 1·3.

c Following the purple arrows on the graph, we see that $A50 will buy $NZ65.

d Following the green arrows shows that $NZ50 will buy slightly more than $A38.
A more exact answer is difficult to read from the graph but by using the equation:
$NZ = 1·3$A
50 = 1·3$A (substituting $NZ = 50)
So, $A = 50 ÷ 1·3
= $38.46 to the nearest cent.
Therefore, $50 will buy $A38.46

Exercise 14:03

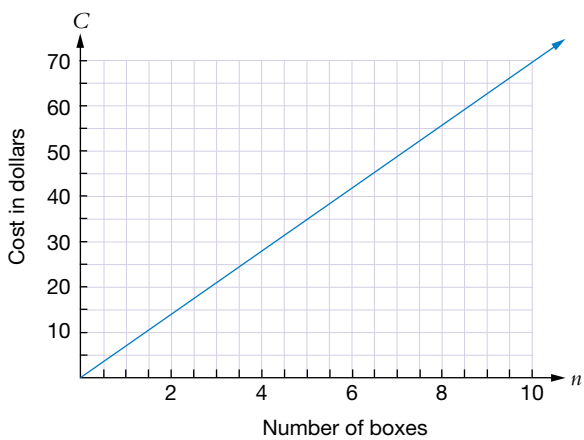
1 This graph shows the market for apples. The cost (C) is directly proportional to the number (n) of boxes bought.

a Determine the cost per box of apples.

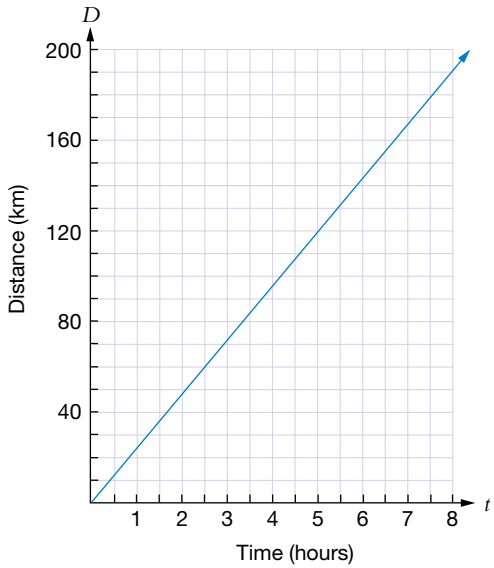
b Hence, determine the value of the constant of proportionality k in the equation: $C = kn$

c What would be the cost of 50 boxes?

d How many whole boxes could be bought for $100?



2 If a car travels at a constant speed then the distance travelled (D) is directly proportional to the time taken (t). This graph shows this relationship.



- **a** How far has the car travelled in 5 hours?
- **b** Therefore, what is the speed in km/h?
- **c** Hence, what is the value of the proportional constant in the equation $D = kt$?
- **d** How far would the car travel, at the same speed, in 7 hours?
- **e** How long would it take to travel 200 km, at the same speed? (Answer in hours and minutes.)

3 The exchange rate between Australian and British currency is \$A1 = £0.65 (or 65 British pence).

- **a** How many British pounds (£) would 100 Australian dollars buy?
- **b** Construct a line graph with Australian dollars on the horizontal axis, up to \$A100, and British pounds, up to £65, on the vertical axis.
- **c** The equation for this straight line would be of the form $£B = k\$A$. What is the value of k?
- **d** How many British pounds would 500 Australian dollars buy?
- **e** How many Australian dollars would 500 British pounds buy?

4 A casual worker earns \$22.50 per hour.

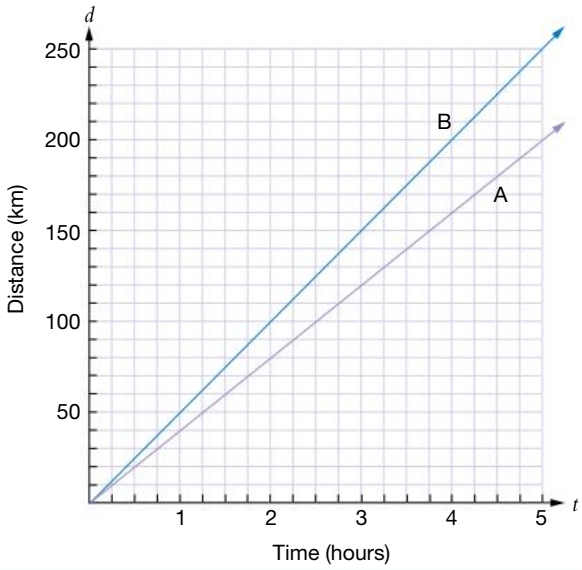
- **a** How much would be earned by working 4 hours.
- **b** Use your answer to part **a** to draw a graph with the horizontal axis showing the hours worked and the vertical axis showing the amount earned.
- **c** Write an equation that relates the amount earned (A) to the number of hours worked (h).
- **d** How many whole hours must be worked to earn \$200?

5 A mix of petrol and oil is used for some lawn mowers. Every 5 L of petrol requires 100 mL of oil.

- **a** If $O = kP$, where P = litres of petrol and O = millilitres of oil, find the value of k.
- **b** Draw the graph of this relationship with petrol showing on the horizontal axis.
- **c** How many litres of oil would be required to mix with 50 L of petrol?
- **d** If $P = KL$, where P = litres of petrol and L = litres of oil, find the value of K. (Hint: Use your answer to part **c**.)
- **e** Now draw the graph of the equation in part **d**, with oil being on the horizontal axis.

6 Two trucks, A and B, are travelling at two different constant speeds. Therefore, the distance travelled and the time taken are in direct proportion.

- **a** Given the equation $d = kt$, find the value of k for each truck.
- **b** How much farther has truck B travelled than truck A after 4 hours?
- **c** After what time period are the two trucks 50 km apart?
- **d** After 5 hours, truck A increased its speed so that it caught up to truck B after another 5 hours. To what speed did it increase?



INVESTIGATION 14:03 PROPORTIONAL UPS AND DOWNS!

Consider the following examples and write your answers in the table below.

1 If 4 men take 12 hours to build a brick fence, how long would it take:
a 1 man **b** 6 men?

2 If 3 women can paint a wall in 4 hours, how long would it take:
a 1 woman **b** 4 women?

3 For 6 days, 12 cans of dog food will feed 3 dogs. For how long will the same amount of dog food feed:
a 1 dog **b** 9 dogs?

4 It takes 3 men a time of 4 hours to dig a hole. To dig a hole the same size, how long would it take:
a 1 man **b** 5 men?

1	Time for 4 men	Time for 1 man	Time for 6 men
	12 hours		
2	Time for 3 women	Time for 1 woman	Time for 4 women
	4 hours		
3	Time for 3 dogs	Time for 1 dog	Time for 9 dogs
	6 days		
4	Time for 3 men	Time for 1 man	Time for 5 men
	4 hours		

5 What do you notice about the answers for the time in each case as the number of men, women or dogs:
a decreases **b** increases?

14:04 Inverse proportion

In Investigation 14:03, you should have seen that as the number of one quantity increased, the other decreased. This is the reverse of what happens with examples of direct proportion.

For example, if 5 painters take 6 days to paint a house, it should obviously take 3 painters longer to do the same job. That is, if the number of painters decreases, the time taken increases. Likewise, more painters would take less time.

Logically, if it takes 5 painters 6 days to do the job, it should take 1 painter 5 times as long.

So 1 painter should take 5 × 6 days, which is 30 days.

Likewise, if it takes 1 painter 30 days to do the job, then 3 painters should divide the time taken by 3.

So 3 painters should take 30 days ÷ 3, which is 10 days.

This is an example of **inverse proportion**, sometimes called **indirect proportion**.

We say that the time taken (T) is inversely proportional to the number of painters (p).

We write this as: $T \propto \frac{1}{p}$

or $T = \frac{k}{p}$ where k is the **constant of proportionality** (or the proportional constant)

Now if we know 5 painters take 6 days, then substituting these values will determine the value of k.

$$T = \frac{k}{p}$$

So $6 = \frac{k}{5}$

$\therefore$ $k = 30$ (6×5)

So now to paint this house, the equation $T = \frac{30}{p}$ represents the relationship between the number of painters used and the time taken to complete the job.

Now consider the following worked examples.

WORKED EXAMPLE 1

If 10 harvesters take 12 days to complete the harvesting, how long would it take 15 harvesters?

Solution

There are two methods for solving these problems involving inverse proportion.

Method 1 Unitary method

We know that 10 harvesters take 12 days.
We then find how long it takes 1 harvester.
1 harvester would take 10 times as long.
$12 \times 10 = 120$
1 harvester would take 120 days.
15 harvesters would divide the time by 15.
$120 \div 15 = 8$
So, 15 harvesters would take 8 days.

Method 2 Inverse proportion equation

$T = \frac{k}{H}$ where T = time in days and H = no. of harvesters

When $H = 10$, $T = 12$:

$12 = \frac{k}{10} \therefore k = 12 \times 10$
$= 120$

So, $T = \frac{120}{H}$

When $H = 15$, $T = \frac{120}{15} = 8$

So, 15 harvesters would take 8 days.

WORKED EXAMPLE 2

If I travel at 50 km/h it will take me 4 hours to get home. How long will it take if I travel at an average speed of 60 km/h?

Solution

Method 1 Unitary method

A speed of 50 km/h takes 4 hours.
A speed of 1 km/h will take 50 times as long.

Time taken for the trip at 1 km/h
$= 4 \times 50$
$= 200$ hours

A speed of 60 km/h will take $\frac{1}{60}$ of the time.
So, 200 hours would be divided by 60.

Time taken for the trip at 60 km/h
$= 200 \div 60$
$= 3\frac{1}{3}$ hours or 3 hours, 20 minutes.

Method 2 Inverse proportion equation

$T = \frac{k}{S}$ where T = time in hours and S = speed in km/h

When $S = 50$, $T = 4$:

$4 = \frac{k}{50} \quad \therefore k = 4 \times 50$
$= 200$

So, $T = \frac{200}{S}$

When $S = 60$, $T = \frac{200}{60}$
$= 3\frac{1}{3}$

So, the time taken at 60 km/h is 3 hours, 20 minutes.

Exercise 14:04

1 Four boys can mow a paddock in 3 hours.
- **a** How long would it take 1 boy?
- **b** How long would it take 6 boys?

2 Five people can eat 3 large pizzas in 20 minutes.
- **a** How long would it take 1 person to eat the 3 pizzas?
- **b** How long would it take 4 people?

3 Three people can survive on the provisions in a raft for 16 days. For how long could 8 people survive on the same provisions?

4 A team of 15 fruit pickers can clear 6 hectares of fruit in a week.
- **a** How many days would it take 21 fruit pickers?
- **b** If the 6 hectares had to be cleared in 2 days, how many fruit pickers would be needed?

5 A swimmer covers a distance of 300 m in 5 minutes.
- **a** What is the average speed of the swimmer in metres per second?
- **b** At what speed would the distance be covered in 4 minutes?
- **c** How long would it take for the distance to be covered at a speed of 1·5 m/s?

6 N is inversely proportional to P. When $N = 25$, $P = 12$.
- **a** Determine an equation relating N and P.
- **b** When $P = 10$, what is the value of N?
- **c** What is the value of P when $N = 100$?

7 X and Y are inversely proportional. When $X = 20$, $Y = 10$.
- **a** Find an equation showing the relationship between X and Y.
- **b** What is the value of Y when $X = 5$?
- **c** what is the value of X when $Y = 5$?

8 The time T taken to empty a water tank is inversely proportional to the cross-sectional area A of the drainage pipe. A drainage pipe with a cross-sectional area of 40 cm^2 takes 16 minutes to empty a full tank.
- **a** Find an equation relating T and A.
- **b** How long would it take to empty the full tank if the drainage pipe has a cross-sectional area of 30 cm^2?
- **c** What must be the cross-sectional area of the pipe if the tank is emptied in 10 minutes?

9 The rate of vibration R of a string under constant tension is inversely proportional to the length L of the string. The string vibrates 120 times per second when the string has a length of 124 cm.
- **a** Find an equation relating R and L.
- **b** What is the rate of vibration when the length is 80 cm?
- **c** What is the length of a string that vibrates 480 times per second?

10 At a given temperature, the pressure P of a gas is inversely proportional to the volume V. In a container of volume 2·3 L, a gas exerts a pressure of 130 units. How much pressure would the gas exert if the volume were 0·7 L? (Answer to the nearest whole unit.)

11 X is inversely proportional to the square of Y. When $Y = 8$, $X = 100$.

- **a** Find an equation relating X and Y.
- **b** What is the value of X when Y is 5?
- **c** Find the value of Y when X is 4.

12 The weight W of an object is inversely proportional to the square of its distance D from the centre of the Earth. A man weighs 70 kg on the surface of the Earth. Take the radius of the Earth to be 6400 km.

- **a** Find an equation relating W and D.
- **b** How much would this man weigh when 200 km above the surface of the Earth? Give your answer correct to two decimal places.

13 If it takes 5 cats 2 days to catch 6 mice, how many days will it take 3 cats to catch 9 mice? (Hint: It might be useful to set up a table with the headings as shown here.)

Cats	Days	Mice

14 If 3 boys take 2 hours to mow 3 lawns, how many boys can mow 6 lawns in 3 hours?

15 A 60 piece orchestra takes 12 minutes to play the Radetsky March. How long would it take an 80 piece orchestra?

Gymnasts often find themselves in inverse positions.

INVESTIGATION 14:04 GRAPHING INVERSE PROPORTION

If two quantities are in inverse proportion then as one quantity increases, the other decreases. So if we graphed an equation representing an inverse proportion relationship we would expect the graph to slope downwards to the right on the number plane. That is, we would expect it to have a negative gradient.

This is true, but unlike the graph of a direct proportion, it is not a straight line.

Consider the example mentioned in Section 14:04 about painting a house.

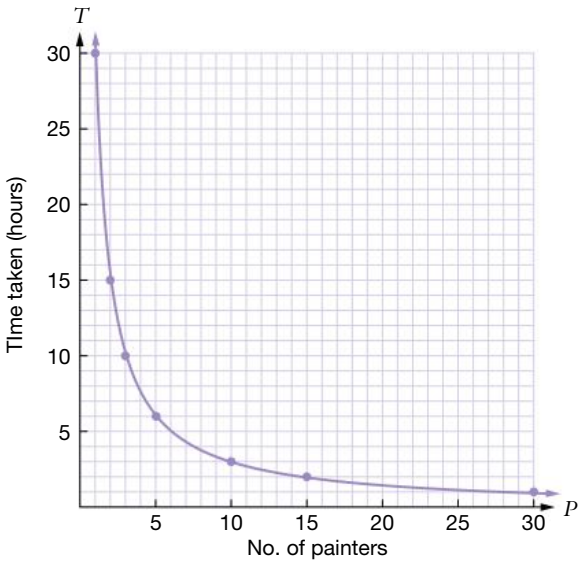
The equation was $T = \frac{30}{P}$.

A table of values for this equation would be:

P	1	2	3	5	10	15	30
T	30	15	10	6	3	2	1

Plotting these points would produce the pattern shown by the dots in this graph. Connecting the points results in a curve called a **hyperbola**. (The graph is said to be hyperbolic.)

Of course not all the points on the curve in this example have meaning, because you can't have a fraction of a painter.



Exercises

1 Complete the table of values for the equation $y = \frac{12}{x}$ and sketch its graph on a number plane.

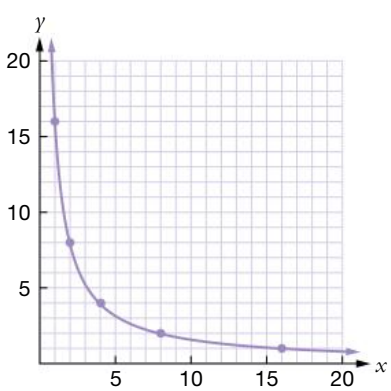
x	1	2	3	4	6	12
y						

2 Using a table of values, or otherwise, draw a graph for each of these equations.

a $y = \frac{20}{x}$ **b** $b = \frac{50}{n}$ **c** $S = \frac{100}{t}$

3 Determine the equation of each graph by considering points that lie on the curve.

a



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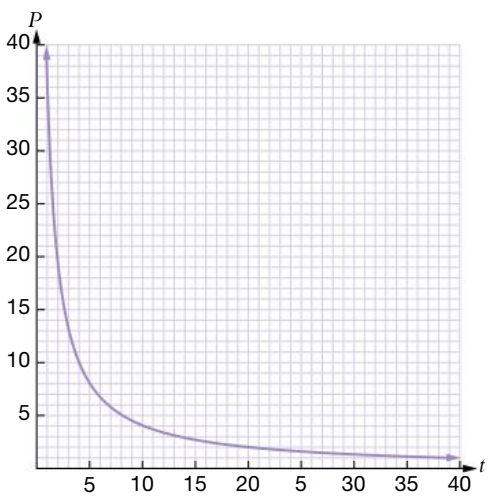


FUN SPOT 14:04 HOW DO POETS WRITE UPSIDE DOWN?

Answer each question and write the letter for that question in the box above the correct answer.

E 15 kg of potatoes costs $10. How many kg of potatoes can I buy for $12?

S Baking cakes for 8 people needs 12 eggs. How many eggs are needed to bake cakes for 20 people?

N 20 litres of petrol costs $28. How many litres can I buy for $63?

E A tap leaks 2·25 L of water in 30 minutes. How many millilitres of water will leak in 1 minute?

R A car travelling at 60 km/h takes 2 hours to complete a journey. At what speed, in km/h, will a car take 3 hours to complete the same journey?

I If 5 bricklayers take 6 hours to build a wall, how many bricklayers would build the wall in two hours?

V If 4 boys can eat 16 pies in 10 minutes, how many minutes would it take 6 boys to eat 24 pies?

		–								
15	45		10	18	40	30	75			

MATHS TERMS 14

constant of proportionality

- the numerical factor by which both quantities are increased or decreased in a given ratio

direct proportion

- when both related quantities are increased (or decreased) in a given ratio, i.e. both quantities are multiplied by the same value.
- two quantities Y and X in direct proportion are related by the equation $Y = kX$, where k is the constant of proportionality

inverse proportion

- when one related quantity increases as the other decreases in a given ratio
- two quantities Y and X in inverse proportion are related by the equation $Y = \frac{k}{X}$, where k is the constant of proportionality

proportion

- when all measures of an object or shape are increased, or decreased, by the same factor

rate

- a comparison of unlike quantities written in a definite order, e.g. 100 km in 5 hours or 20 km/h
- units are used in this comparison, e.g. 8 people per car

DIAGNOSTIC TEST 14 PROPORTION

These questions reflect some of the important skills introduced in this chapter.
Errors made in this test will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Write each rate in its simplest terms. 14:01

a 200 km in 4 hours = … km/h
b \$56 for 7 kg = \$… /kg
c 60 litres in $2\frac{1}{2}$ hours = … L/h
d 250 g per 10 cm^3 = … g/cm^3

2 Convert these rates. 14:01

a 6 km/h = … m/min
b 120 000 mL/min = … L/s
c 15 000 kg/day = … t/day
d \$100 per hectare = … c/m^2

3 For each example below, state whether it is an example of direct or indirect proportion. 14:02, 14:04

a The ingredients for making pizzas and the number of people who are going to eat them.
b The number of people eating pizzas and the time taken to eat them.
c The speed at which a car travels and the distance covered in a given time.
d The speed at which a car travels and the time taken to cover a certain distance.

4 If M is directly proportional to t, and when $t = 6$, $M = 30$. 14:02

a Given that $M = kt$, find the value of k.
b Find the value of M when $t = 17$.
c Find the value of t when $M = 174$.

5 I need 8 eggs to bake cakes for 3 people. 14:02

a How many eggs do I need to make cakes for 15 people?
b If I have 3 dozen eggs, for how many people can I make cakes?

6 Travelling at a constant average speed I can travel a distance D in a certain time T. If I travel 120 km in 5 hours: 14:02

a Determine an equation for this relationship in the form $D = kT$.
b What is the constant of proportionality?
c How far can I travel in 8 hours?
d How long would it take me to travel 228 km?

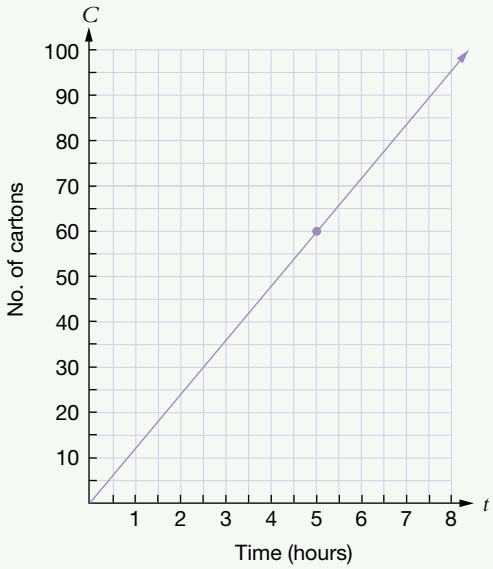
14:03

7 This graph shows the number of cartons a factory can produce in a given time.

a By determining the gradient of the line, find the constant of proportionality for this example of direct proportion.

b How many cartons could be produced in 22 hours?

c How many hours would be needed to produce 200 cartons? Give your answer in hours and minutes.



14:04

8 The time taken (t) to clean the windows of a building is inversely proportional to the number of cleaners employed (n). It takes 8 hours for 6 cleaners to clean the windows.

a Find an equation in the form $t = \frac{k}{n}$ for this relationship.

b What is the constant of proportionality?

c How long would it take 12 cleaners to do the same job?

d How many cleaners are needed to do the job in 3 hours?

14:04

9 The time taken (T) for Henry to drive home is inversely proportional to the average speed (S) at which he drives. If his speed is 40 km/h he gets home in 3 hours.

a What is the proportional constant for this relationship?

b How long does he take if he increases his average speed to 50 km/h? (Give your answer in hours and minutes.)

c At what speed must he travel to get home in 2 hours?

14:04

10 The frequency of a note produced on a string is inversely proportional to the length of the string. A string of length 50 cm produces a frequency of 600 Hz.

a What frequency is produced by a string of length 40 cm?

b What length of string would produce a frequency of 800 Hz?

ASSIGNMENT 14A Chapter review

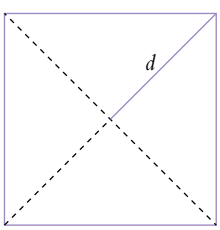
1 Convert this rate: 1 mL/s = … L/h

2 British athlete Mohamed Farah won the gold medal at the 2012 Olympic Games for the men's 10 000 m in a time of just over 27 minutes and 30 seconds.
- **a** What was his average time for 1 kilometre?
- **b** What average speed is this in km/h? (Answer correct to one decimal place.)

3 If the exchange rate between Australian dollars ($A) and European euros (€) is $1 = €0.85, find:
- **a** how many euros could be exchanged for $A400
- **b** how many whole $A could be bought for €400.

4 In a compound, 7 kg of chemical x must be mixed with 4 kg of chemical y.
- **a** How much of chemical y is needed if 59·5 kg of chemical x is used?
- **b** How much of chemical x is needed if 59·5 kg of chemical y is used?

5 The diagonals of any square intersect at the middle of the square. The perimeter (P) of a square is directly proportional to the distance d from the middle to one of the corners. When the perimeter is 16 cm, the distance from the middle to a corner is 2·83 cm.



- **a** Find the distance from the middle to a corner, correct to two decimal places, if the perimeter is 25 cm.
- **b** Find the perimeter if the distance from the middle to a corner is 15 cm. Give your answer to the nearest centimetre.

6 If 36 men can do a job in 8 days:
- **a** how long should it take 12 men to do the same job
- **b** how many men should it take to do the job in 6 days?

7 It was found that a 3-metre length of timber (L) can support a maximum weight (W) of 230 kg.
- **a** Find an equation relating W and L.
- **b** Find the weight that can be supported safely when the length of the timber is 3·5 metres. (Give your answer in kg, correct to one decimal place.)
- **c** Find the longest length of timber that can safely support a weight of 180 kg. (Give your answer correct to the nearest cm.)

8 In a stadium 1500 seats are possible when 60 cm is allowed for each person.
- **a** If 1800 seats were needed at 60 cm per person, how much more length of seating would be required?
- **b** If 1800 seats were needed, without being able to extend the stadium, how much would be allowed for each person? Is this reasonable?

ASSIGNMENT 14B Working mathematically

1 Naomi bought a computer system for Luke. The marked price was $2300. She paid a deposit of $1200 and 12 monthly payments of $115.

a How much did she pay?

b How much more than the marked price did she pay?

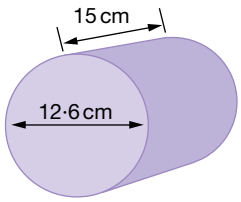
c What percentage was the extra money paid of the amount owing after the deposit was paid? (Give the percentage correct to one decimal place.)

2 For this cylinder find:

a the area of the base

b the curved surface area

c the total surface area.



3 A salesman's wages are $230 per week plus a commission of $4\frac{1}{2}\%$ on his sales. How much will he earn in a week when he sells $7000 worth of goods?

4 The graph below shows the 'normal' weight for girls aged 0 to 3 years. The numbers on the right side of the graph are percentages. (Only 3% of 3-year-old girls have a weight less than 11·3 kg.)

a Why are there two 3s on the horizontal axis?

b What is the median weight for girls of age:

i 3 months **ii** 1 year **iii** 19 months

iv 2 years 3 months **v** 1 year 2 months?

c What percentage of 3-year-old girls have a weight between 13·9 kg and 16 kg?

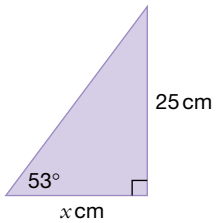
d What weights would be considered 'normal' for a girl of age 2 months?

e What weights would be considered 'normal' at birth for a girl?

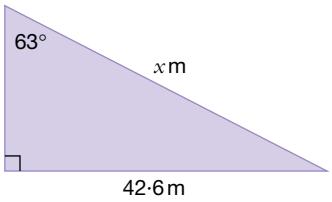
ASSIGNMENT 14C Cumulative revision

1 Find the value of the pronumeral in each of the following, correct to one decimal place. 12:03, 12:07B

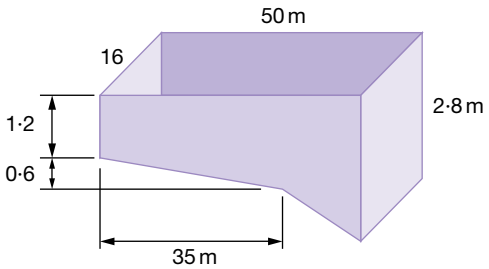
a



b



2 A swimming pool is in the shape of a prism. Its cross-section is formed from two trapeziums. Calculate the volume of the pool in cubic metres and its capacity in litres. (Measurements are in metres.) 5:06



3 The points $A(-4, 0)$, $B(6, 0)$ and $C(0, 6)$ form a triangle. D and E are the midpoints of AC and BC respectively. Find: 9:02, 9:04

a the length of AB

b the coordinates of D and E

c the length of DE.

4 The formula $S = \dfrac{n + n^2}{2}$ can be used to find the sum (S) of a long series of numbers like $S = 1 + 2 + 3 + 4 + \ldots + n$. 11:03, 11:04

a Find the value of S when $n = 4$. Check your answer by adding $1 + 2 + 3 + 4$.

b Find the value of S when $n = 100$.

c Use the formula twice, to find the sum $101 + 102 + 103 + 104 + \ldots + 200$.

d Use guess-and-check to find the value of n for which $S = 131\,328$.

5 Two normal dice are taken and the numbers are changed. On dice A the '1' is replaced by a '6'. On dice B the numbers '5' and '6' are replaced by the number '3'. 4:06

a Use a list, array or tree diagram to write all the possible outcomes when the two dice are rolled.

b What is the probability that a '3' will appear when the two dice are rolled?

c When the two dice are rolled is it more likely to get a 6 to appear or a 4 to appear?

ANSWERS

ID Cards

ID Card 1 (Metric units)

1 metres
2 decimetres
3 centimetres
4 millimetres
5 kilometres
6 square metres
7 square centimetres
8 square kilometres
9 hectares
10 cubic metres
11 cubic centimetres
12 seconds
13 minutes
14 hours
15 metres per second
16 kilometres per hour
17 grams
18 milligrams
19 kilograms
20 tonnes
21 litres
22 millilitres
23 kilolitres
24 degrees Celsius

ID Card 2 (Symbols)

1 is equal to
2 is approximately equal to
3 is not equal to
4 is less than
5 is less than or equal to
6 is not less than
7 is greater than
8 is greater than or equal to
9 4 squared
10 4 cubed
11 the square root of 2
12 the cube root of 2
13 is perpendicular to
14 is parallel to
15 is congruent to
16 is similar to
17 per cent
18 therefore
19 for example
20 that is
21 pi
22 the sum of
23 the mean
24 probability of event E

ID Card 3 (Language)

1 $6 - 2 = 4$
2 $6 + 2 = 8$
3 $6 \div 2 = 3$
4 $6 - 2 = 4$
5 $6 \div 2 = 3$
6 2
7 6
8 $6 \times 2 = 12$
9 $6 - 2 = 4$
10 $6 \times 2 = 12$
11 $2 + 6 = 8$
12 $6 - 2 = 4$
13 $6^2 = 36$
14 $\sqrt{36} = 6$
15 $6 - 2 = 4$
16 $6 \times 2 = 12$
17 $(6 + 2) \div 2 = 4$
18 $6 + 2 = 8$
19 $6^2 = 36$
20 $6 - 2 = 4$
21 $6 - 2 = 4$
22 $6 + 2 = 8$
23 $6 \div 2 = 3$
24 $6 + 2 = 8$

ID Card 4 (Language)

1 square
2 rectangle
3 parallelogram
4 rhombus (or diamond)
5 trapezium
6 regular pentagon
7 regular hexagon
8 regular octagon
9 kite
10 scalene triangle
11 isosceles triangle
12 equilateral triangle
13 circle
14 oval (or ellipse)
15 cube
16 rectangular prism
17 triangular prism
18 square pyramid
19 rectangular pyramid
20 triangular pyramid
21 cylinder
22 cone
23 sphere
24 hemisphere

ID Card 5 (Language)

1 point A
2 interval AB
3 line AB
4 ray AB
5 collinear points
6 midpoint
7 number line
8 diagonals
9 acute-angled triangle
10 right-angled triangle
11 obtuse-angled triangle
12 vertices
13 ΔABC
14 hypotenuse
15 $180°$
16 $a° + b°$
17 $360°$
18 [b] $a° = b°$
19 $a° = 60°$
20 $3 \times 180° = 540°$
21 AB is a diameter, OC is a radius
22 circumference
23 semicircle
24 AB is a tangent, CD is an arc, EF is a chord

ID Card 6 (Language)

1 parallel lines
2 perpendicular lines
3 vertical, horizontal
4 concurrent lines
5 angle ABC or CBA
6 acute angle
7 right angle
8 obtuse angle
9 straight angle
10 reflex angle
11 revolution
12 adjacent angles
13 complementary angles
14 supplementary angles
15 vertically opposite angles
16 360°
17 transversal
18 corresponding angles
19 alternate angles
20 co-interior angles
21 bisecting an interval
22 bisecting an angle
23 $\angle CAB = 60°$
24 CD is perpendicular to AB

ID Card 7 (Language)

1 Common Era, Anno Domini
2 Before Common Era, Before Christ
3 ante meridiem (before midday)
4 post meridiem (after midday)
5 hectare
6 regular shapes
7 net of a cube
8 cross-section
9 face
10 vertex
11 edge
12 axes of symmetry
13 reflection (or flip)
14 translation (or slide)
15 rotation (or turn)
16 tessellation
17 coordinates
18 tally
19 picture graph
20 column graph
21 line graph
22 sector (or pie) graph
23 bar graph
24 scatter diagram

Chapter 1

Exercise 1:01

1 1 metres
2 decimetres
3 centimetres
4 millimetres
5 kilometres
6 square metres
7 square centimetres
8 square kilometres
9 hectares
10 cubic metres
11 cubic centimetres
12 seconds
13 minutes
14 hours
15 metres per second
16 kilometres per hour
17 grams
18 milligrams
19 kilograms
20 tonnes
21 litres
22 millilitres
23 kilolitres
24 degrees Celsius

2 1 is equal to
2 is approximately equal to
3 is not equal to
4 is less than
5 is less than or equal to
6 is not less than
7 is greater than
8 is greater than or equal to
9 4 squared
10 4 cubed
11 the square root of 2
12 the cube root of 2
13 is perpendicular to
14 is parallel to
15 is congruent to
16 is similar to
17 per cent
18 therefore
19 for example
20 that is
21 pi
22 the sum of
23 the mean
24 probability of event E

3 1 $6 - 2 = 4$
2 $6 + 2 = 8$
3 $6 \div 2 = 3$
4 $6 - 2 = 4$
5 $6 \div 2 = 3$
6 2
7 6
8 $6 \times 2 = 12$
9 $6 - 2 = 4$
10 $6 \times 2 = 12$
11 $2 + 6 = 8$
12 $6 - 2 = 4$
13 $6^2 = 36$
14 $\sqrt{36} = 6$
15 $6 - 2 = 4$
16 $6 \times 2 = 12$
17 $(6 + 2) \div 2 = 4$
18 $6 + 2 = 8$
19 $6^2 = 36$
20 $6 - 2 = 4$
21 $6 - 2 = 4$
22 $6 + 2 = 8$
23 $6 \div 2 = 3$
24 $6 + 2 = 8$

4 a true
b true
c true
d false
e true
f true
g false
h true
i true
j true
k false
l true
m true
n false
o true

Diagnostic check-up 1:02A

1 a 7
b −13
c −10
2 a 9
b 7
c 11
3 a −6
b 30
c −63
4 a 5
b −7
c 52
5 a −56
b 1
c 7

Diagnostic check-up 1:02B

1 a $1\frac{3}{4}$
b $4\frac{1}{3}$
c $14\frac{1}{10}$
2 a $\frac{5}{2}$
b $\frac{53}{10}$
c $\frac{22}{7}$
3 a $\frac{2}{3}$
b $\frac{2}{13}$
c $\frac{1}{15}$
4 a 21
b 85
c 375
5 a $\frac{5}{8}$
b $1\frac{1}{5}$
c 1
6 a $\frac{1}{5}$
b $\frac{2}{7}$
c $\frac{13}{50}$
7 a $1\frac{11}{20}$
b $\frac{7}{10}$
c $\frac{29}{200}$
8 a $\frac{1}{8}$
b $\frac{13}{20}$
c $\frac{7}{30}$
9 a $8\frac{1}{10}$
b $12\frac{9}{20}$
c $2\frac{17}{24}$
10 a $3\frac{5}{18}$
b $5\frac{13}{20}$
c $20\frac{7}{40}$
11 a $6\frac{5}{8}$
b $3\frac{9}{10}$
c $1\frac{2}{3}$

12 a $\frac{12}{55}$ b $\frac{21}{100}$ c $\frac{3}{50}$

13 a $\frac{3}{8}$ b $\frac{3}{8}$ c $\frac{7}{12}$

14 a $2\frac{2}{7}$ b $2\frac{17}{50}$ c 14

15 a $15\frac{1}{5}$ b $6\frac{3}{4}$ c $31\frac{7}{8}$

16 a 4 b 3 c 1

17 a $1\frac{1}{5}$ b $1\frac{5}{27}$ c $1\frac{3}{32}$

18 a $2\frac{1}{2}$ b $1\frac{3}{7}$ c $1\frac{1}{4}$

19 a $2\frac{1}{2}$ b $\frac{7}{10}$ c $1\frac{3}{8}$

20 a 20 b 50 c 40

Diagnostic check-up 1:02C

1 a 0·5, 0·505, 0·55 b 8·4, 8·402, 8·41
c 1·01, 1·011, 1·1

2 a 5·74 b 21·6 c 0·265

3 a 11·63 b 7·176 c 4·015

4 a 4·2 b 0·09 c 0·034

5 a 314·2 b 40 c 0·65

6 a 21 000 b 8 040 000 c 125

7 a 2·04 b 2·42 c 0·0475

8 a $0{\cdot}41\dot{6}$ b $0{\cdot}59\dot{1}$ c $9{\cdot}\dot{3}$

9 a 2·435 b 0·067 c 0·0007

10 a 32 b 10·3 c 130

11 a $\frac{1}{2}$ b $\frac{9}{50}$ c $9\frac{21}{200}$

12 a 0·8 b 0·375 c $0.8\dot{3}$

Diagnostic check-up 1:02D

1 a $\frac{9}{50}$ b $\frac{7}{100}$ c $2\frac{6}{25}$

2 a $\frac{19}{200}$ b $\frac{1}{16}$ c $\frac{49}{400}$

3 a 55% b $83\frac{1}{3}\%$ c 125%

4 a 0·09 b 0·16 c 1·1

5 a 0·238 b 0·125 c $0{\cdot}04\dot{6}$

6 a 51% b $8\frac{1}{2}\%$ c 180%

7 a 210 m b \$12.96

8 a 5·901 m b 2·75 t

9 a \$60 b 60 kg

10 a 25% b 40%

11 a \$69 b \$5.20

Exercise 1:03

1 a 250%, 2·5 b 1·3, $1\frac{3}{10}$
c 280%, $2\frac{4}{5}$ d 125%, 1·25

2 a 0·805, 0·85, 0·9, 1
b $12\frac{1}{4}\%$, 87·5%, 100%, 104%
c $\frac{6}{100}, \frac{4}{7}, \frac{5}{8}, \frac{2}{3}$
d 150%, 1·65, $1\frac{3}{4}$, 2
e 140%, 1·41, $\sqrt{2}$, 1·42
f 3·1, π, $3\frac{1}{4}$, $\sqrt{12}$

3 a 6·85 b 16% c $\frac{3}{5}$ or 0·6 d 6·35

4 $\frac{1}{2}$ and $\frac{1}{8}$

5 a 3·2, 6·4, 12·8 b 0·15, 0·075, 0·0375

6 a 29 b 9

7 Each year, \$8 is paid in interest for every \$100 invested.

8 The building can house three tenants.

9 7 years **10** −33° **11** 54 BC

12 a 370 b −120 c −440

13 a $\frac{1}{3}$ b $\frac{2}{3}$ c $\frac{1}{9}$ d $\frac{5}{9}$

14 a 0·888 89 b 0·285 71 c 0·538 46
d 0·952 38 e 0·363 64 f 0·277 78

15 \$64 500

16 a The maximum possible score.
b The number of marks lost for those 3 mistakes.
c The number who have cancer and the number over 65.

Prep quiz 1:04

1 0·25 **2** 0·4 **3** $0{\cdot}\dot{3}$ **4** $0{\cdot}8\dot{3}$

5 $0{\cdot}\dot{4}$ **6** $0{\cdot}6\dot{3}\dot{1}$ **7** $0{\cdot}1\dot{6}$ **8** $0{\cdot}72\dot{6}\dot{9}$

9 $\frac{3}{4}$ **10** $\frac{7}{8}$

Exercise 1:04

1 a 0·75 b 0·8 c 0·625 d 0·7
e 0·07 f 1·75 g 0·16 h 0·34
i 0·475 j 0·936

2 a $0{\cdot}\dot{6}$ b $0{\cdot}\dot{5}$ c $0{\cdot}\dot{8}$ d $0{\cdot}\dot{1}\dot{8}$
e $0{\cdot}\dot{1}42857\dot{}$ f $0{\cdot}1\dot{6}$ g $0{\cdot}0\dot{6}$ h $0{\cdot}4\dot{6}$
i $0{\cdot}041\dot{6}$ j $0{\cdot}5\dot{6}$

3 a $\frac{47}{100}$ b $\frac{4}{25}$ c $\frac{1}{8}$ d $\frac{17}{20}$
e $\frac{7}{200}$

4 a $\frac{4}{9}$ b $\frac{19}{33}$ c $\frac{173}{999}$ d $\frac{7}{9}$
e $\frac{4}{11}$ f $\frac{1234}{9999}$

5 $\frac{1}{3}$

6 a $\frac{5}{6}$ b $\frac{629}{990}$ c $\frac{89}{450}$ d $\frac{29}{45}$
e $\frac{221}{300}$ f $\frac{8167}{9900}$ g $\frac{853}{1665}$ h $\frac{4751}{9000}$
i $\frac{6467}{9990}$

Challenge 1:04

1 $\frac{7}{9}$ **2** $\frac{67}{99}$ **3** $\frac{104}{333}$ **4** $\frac{1}{6}$ **5** $\frac{637}{1980}$

Prep quiz 1:05

1 $\frac{5}{6}$ **2** $\frac{4}{5}$ **3** $\frac{6}{7}$ **4** $\frac{1}{5}$ **5** $\frac{1}{2}$

6 $\frac{1}{4}$ **7** $\frac{4}{5}$ **8** $\frac{2}{3}$ **9** $\frac{2}{3}$ **10** $\frac{4}{5}$

Exercise 1:05

1 a $\frac{7}{15}$ b $\frac{10}{21}$ c $\frac{3}{4}$ d $\frac{1}{10}$ e $\frac{4}{5}$
f $\frac{1}{10}$ g $\frac{15}{16}$ h $\frac{1}{5}$ i $\frac{5}{24}$

2 a 3 : 2 b 2 : 1 c 13 : 3 d 2 : 5
e 4 : 9 f 7 : 10 g 4 : 1 h 9 : 10
i 200 : 1 j 11 : 8 k 11 : 4 l 4 : 7
m 2 : 21 n 36 : 1 o 1 : 26 p 20 : 1
q 1 : 5 r 5 : 4 s 11 : 4 t 8 : 13
u 9 : 4 v 3 : 2 w 11 : 8 x 4 : 3

3 **a** 7:9 **b** 13:15 **c** 7:8 **d** 2:3 **e** 4:5
f 3:17 **g** 9:10 **h** 1:2 **i** 1:8 **j** 32:25
k 5:3 **l** 5:2

4 **a** 2:1 **b** 2:1 **c** 1:5 **d** 5:3
e 1:8 **f** 1:10 **g** 3:5 **h** 5:3
i 4:1 **j** 3:1 **k** 5:6 **l** 1:2

5 **a** $\frac{2}{5}:1$ **b** $\frac{3}{4}:1$ **c** $\frac{7}{10}:1$ **d** $1\frac{2}{3}:1$ **e** $2\frac{2}{9}:1$

6 **a** $1:\frac{5}{2}$ **b** $1:\frac{4}{3}$ **c** $1:\frac{10}{7}$ **d** $1:\frac{3}{5}$ **e** $1:\frac{9}{20}$

7 **a** The mixture would be 50% sugar, 25% flour and 25% custard powder.
b The mixture would be $\frac{2}{13}$ cement, $\frac{10}{13}$ sand and $\frac{1}{13}$ lime.
c 1:40 **d** 1:20

8 **a** 6 km/h **b** $25/t **c** 50 g/$
d $250/day **e** 24 g/L **f** 29 boys/teacher

9 **a** 9 km **b** $18 **c** 10 kg
d 70 (runs)

10 **a** 1500 m/h **b** $40 000 $/km
c 3000 kg/min **d** 3 g/L
e 216 000 g/h **f** 2 beats/s

Exercise 1:06

1 **a** 2 **b** 3 **c** 3 **d** 2 **e** 4
f 3 **g** 2 **h** 3 **i** 1 **j** 2
k 1 **l** 2 **m** 3 **n** 4 **o** 1
p 3 **q** 4 **r** 3 **s** 3 **t** 3
u 1

2 **a** 1 **b** 2 **c** 2 **d** 3 **e** 4
f 3 **g** 3 **h** 6

3 **a** 2 **b** 4 **c** 3 **d** 1 **e** 3
f 2 **g** 1 **h** 2 **i** 2

4 **a** 8 **b** 2 **c** 2 **d** 3

5 1 or 2 significant figures (i.e. to the nearest 10 000 or the nearest 1000)

Investigation 1:06

The answers use two significant figures.

1 between 150 000 km/year and 170 000 km/year
2 between 3100 km/week and 3500 km/week
3 between 520 km/day and 590 (or 580) km/day
4 11 m/s **5** 22 m **6** 200 L **7** 142 kg

Prep quiz 1:07

1 4 **2** 1 **3** 4 **4** 3·08 **5** 0·80
6 2·410 **7** 4 **8** 2·33 **9** 3·55 **10** 0·065

Exercise 1:07

1 **a** 7900 **b** 1100 **c** 67 300 **d** 900
e 600 **f** 400 **g** 74 900 **h** 7900

2 **a** 9 **b** 80 **c** 45 **d** 3
e 2 **f** 18 **g** 237 **h** 100

3 **a** 243·13 **b** 79·66 **c** 91·35 **d** 9·81
e 0·30 **f** 0·09 **g** 0·10 **h** 1·99

4 **a** 6·7 **b** 8·5 **c** 2·1 **d** 6·1
e 0·1 **f** 0·0 **g** 29·9 **h** 10·0

5 **a** 8200 **b** 3500 **c** 660 **d** 850
e 15 000 **f** 76 000 **g** 50 000 **h** 77 000

6 **a** 8000 **b** 4000 **c** 700 **d** 800
e 10 000 **f** 80 000 **g** 50 000 **h** 80 000

7 **a** 695 **b** 35·1 **c** 321 **d** 0·0815
e 0·667 **f** 9·33 **g** 10·1 **h** 9·10

8 **a** 1·8 **b** 1·78 **c** 1·778 **d** 2
e 1·8 **f** 1·78

9 **a** 5 cm **b** 55 mm **c** 5·5 cm **d** 5·45 cm
e 5·455 cm **f** 5 cm **g** 5·5 cm **h** 5·45 cm

10 **a** $141 **b** $140·60
c 14 059·7 cents **d** 14 059·71 cents
e 14 059·705 cents **f** $100
g $140 **h** $141

11 **a** nearest thousand, 500
b nearest billion (i.e. $1 000 000 000), 500 000 000

12 **a** nearest tenth, 0·05 cm
b nearest thousandth, 0·0005 mg

13 Any number between 2·145 and 2·155 or equal to 2·145. No because the number can have any number of decimal places and must be less than 2·155.

14 **a** 40 000 000 t **b** 38 000 000 t
c 37 600 000 t **d** 37 649 000 t
e Answers will vary. Most people would choose part **b** or part **a**.

15 **a** 8 m **b** 8·1 m **c** 8·13 **d** 8·126 m
e Parts **d** or **c**, as builders use mm on building plans.

16 **a** 5 **b** 42·063 m **c** 42 m
d 2 **e** 4206 cm **f** 4

17 **a** Any number between 2 500 000 and 3 500 000 or equal to 2 500 000
b $26.80, Answers will vary but the 15c over could be given as a tip.
c There will be insufficient tiles. Because parts of tiles are required in some places, you must round up, so that the entire floor can be covered using parts of tiles and you must also allow for breakages as the tiles are cut.

18 **a** $0·000 000 000 \dot{3}$ **b** $0·000 000 000 \dot{6}$
c $0·000 000 000 \dot{5}$

19 **a** 90 000 **b** 119 370·25
c 122 500 **d** 29 335·7025

20 2859·2 m^3

Investigation 1:07

1 4 mg three times a day **2** three times a day, 6
3 15 years **4** 5 mL **5** 24 mg

Prep quiz 1:08

1 200 **2** 20 **3** 156·1 **4** 1561 **5** < 1
6 < **7** true **8** B **9** true **10** false

Exercise 1:08

For all these exercises it is possible to have more than one answer.

1 **a** 22 **b** 36 **c** 78 **d** 140 **e** 5
f 0·02 **g** 0·6 **h** 100 **i** 50 **j** 1·7

2 **a** 1 **b** 1000 **c** 2 **d** 2 **e** 9
f 2 **g** 25 **h** 32 **i** 60

3 **a** C or 3·1 kg **b** A or 27 m
c C or $276 **d** B or $9400

4 **a** 3 **b** 5 **c** 2 **d** 4 **e** 11
f 1 **g** 8 **h** 11

5 **a** 0·01 cm or 0·1 mm **b** 3200
c Yes, because in 8 × 80 both numbers have been rounded down and in 9 × 90 both have been rounded up.

6 73·6575 and 75·3875
(i.e. 73·6575 ≤ the measurement < 75·3875)

Prep quiz 1:09

1 300 **2** 7000 **3** 4 000 000 **4** 720
5 720 **6** 16 000 **7** 21·5 **8** 2315
9 74 976 **10** 3 400 00

Exercise 1:09

1 **a** 92 365 **b** 740 030 **c** 3 408 007
d 65 900 **e** 28 415 **f** 5924
g 69 727 **h** 3841 **i** 7 580 000
j 27 000

2 **a** $1{\cdot}73 \times 10^2$ **b** $9{\cdot}1 \times 10^2$ **c** $3{\cdot}45 \times 10^6$
d $6{\cdot}1 \times 10^3$ **e** $5{\cdot}22 \times 10^7$ **f** $4{\cdot}5 \times 10^9$

3 **a** $8{\cdot}6 \times 10^3$ **b** $8{\cdot}56 \times 10^5$ **c** $4{\cdot}7 \times 10^4$
d $3{\cdot}7 \times 10^7$ **e** 6×10^6 **f** $2{\cdot}3 \times 10^8$
g $1{\cdot}01 \times 10^8$ **h** $8{\cdot}125 \times 10^9$

4 **a** 6860 **b** 102 **c** 9 500 000
d 470 000 **e** 3 960 000 **f** 45 500 000
g 80 500 **h** 6 100 000 000

5 **a** 7×10^3 means 7 × 10 × 10 × 10 and 7^3 means 7 × 7 × 7
b True **c** True

6 **a** 3 **b** 2 **c** 2 **d** 4

Prep quiz 1:10

1 true **2** true **3** false **4** true
5 true **6** true
7 eight gigabytes **8** three terabytes
9 four megabytes **10** nine kilobytes

Exercise 1:10

1 **a** 7000 m **b** 12 000 kg **c** 3 000 000 L
d 8 m **e** 9300 mm **f** 3450 mL
g 7200 mg **h** 3·845 L **i** 0·5 m

2 **a** 5 000 000 m **b** 6 000 000 000 m
c 5 000 000 000 000 m **d** 6000 mm
e 9 000 000 μm **f** 3 000 000 000 nm
g 17 100 m **h** 1 800 000 000 m
i 4 800 000 m

3 **a** 4 000 000 B **b** 4000 kB **c** 7000 MB
d 3000 GB **e** 3000 nm **f** 5 μm
g 3200 kB **h** 4100 MB **i** 600 GB

4 **a** 3 km **b** 8 MB
c 11 GB **d** 6·5TB
e 3·5 μs **f** 730 000 μs
g 5 000 000 kB **h** 2 000 000 MB
i 3 000 000 000 kB

5 **a** 32 years **b** 17 min
c 900 gigabytes (i.e. 900 GB)
d 3000 GB (or 3TB)

Exercise 1:11

1 **a** 0·5 cm **b** 0·5 g **c** 5 L
d 0·05 cm **e** 0·005 m **f** 0·05 L
g 0·05 t **h** 0·0005 s **i** 500 t

2 **a** **i** 0·5 cm (or 5 mm)
ii 583·5 cm up to 584·5 cm (but excluding 584·5 cm)
b **i** to the nearest hundredth of a gram (i.e. 0·01 g)
ii 0·005 g
iii 47·285 g up to 47·295 g (but excluding 47·295 g)
c **i** to the nearest tenth of a tonne (i.e. 0·1 t)
ii 0·05 t (or 50 kg)
iii 1·35 t up to 1·45 t (but excluding 1·45 t)

3 **a** 0·5 L, 34·5 L up to 35·5 L (but excluding 35·5 L)
b 0·05 L, 34·85 L up to 34·95 L (but excluding 34·95 L)

4 **a** 0·005 m (or 5 mm), 1·475 m up to 1·485 m (but excluding 1·485 m)
b 0·005 m (or 5 mm), 120·355 m up to 120·365 m (but excluding 120·365 m)
c yes
d Generally, Naomi's measurement would be considered more accurate, as her greatest possible error (0·005 m) is approximately 0·0042% of her measurement. Phil's greatest possible error (0·005 m) is approximately 0·34% of his measurement.

5 **a** **i** the ruler marked in cm and mm
ii the micrometer screw gauge
b **i** 0·5 cm (or 5 mm) **ii** 0·05 cm
iii 0·005 cm **iv** 0·0005 cm
c **i** 1·35 cm up to 1·45 cm (but excluding 1·45 cm)
ii 1·375 cm to 1·385 cm (but excluding 1·385 cm)

Diagnostic test 1

1 **a** $0.\dot{6}$ **b** $0.1\dot{6}$ **c** $0.\dot{7}$ **d** $0.2\dot{6}$

2 **a** $\frac{5}{9}$ **b** $\frac{37}{99}$ **c** $\frac{26}{45}$ **d** $\frac{683}{990}$

3 **a** 6 **b** 5 **c** 5
d can't tell, may be 2, 3 or 4

4 **a** 57 000 000 **b** 0·6662 **c** 8·0
d 4·67 m

5 **a** 0·006 **b** 510 000 000 **c** 0·061
d 93 800 000

6 Answers may vary slightly.
a 36 **b** 5 **c** 20 **d** 200
7 **a** $8{\cdot}5 \times 10^4$ **b** $9{\cdot}6 \times 10^8$
c 1 060 000 **d** 84 000
8 **a** 7200 kB **b** 40 μs
9 **a** 0·5 L **b** 0·05 kg
10 **a** 3·255 g
b 9·35 kg up to 9·45 kg (but excluding 9·45 kg)

ASSIGNMENT 1A

1 **a** 9 **b** 16 **c** −3
2 **a** $\frac{4}{5}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10}$ **d** $\frac{5}{6}$
e $5\frac{19}{20}$ **f** $1\frac{11}{15}$ **g** $9\frac{1}{3}$ **h** 6
3 **a** 4·47 **b** 4·34 **c** 40·5 **d** 0·09
4 **a** $\frac{7}{20}$ **b** $\frac{7}{8}$ **c** $\frac{23}{99}$ **d** $\frac{172}{495}$
5 **a** $26.25 **b** 20%
6 **a** 5 : 9 **b** 1 : 3 **c** 1 : 4 **d** 1 : 6
7 **a** 3 **b** 3 **c** 3 **d** 2
e 3 **f** 3
8 **a** 6·4 **b** 6·92
c 47 644 000 **d** 648
e 6·4 **f** 0·005 82
g 47 600 000 **h** 0·70
9 **a** 5 820 000 **b** 9100
10 **a** 8×10^7 **b** $8{\cdot}0 \times 10^7$
11 **a** 2100 GB **b** 8 500 000 kB
c 0·05 km (or 50 m)
d 3·65 km up to 3·75 km (but excluding 3·75 km)

ASSIGNMENT 1B

1 **a** hectares **b** cubic metres
c metres per second **d** kilometres per hour
e milligrams **f** kilograms
g tonnes **h** millilitres
i kilolitres **j** degrees Celsius
2 **a** is approximately equal to
b is less than or equal to
c 4 cubed **d** the square root of 2
e is parallel to **f** is congruent to
g is similar to **h** therefore
i for example **j** the mean
3 11 **4** 39
5 **a** **i** 50 **ii** 67 **iii** 20 **iv** 27
b **i** 48 **ii** 54 **iii** 34 **iv** 23
c 17% **d** 12
e No. You cannot score more than 60 marks for a test out of 60.
6 **a** 14 litres **b** 2 litres
c 10:00 am and 1:00 pm
d No petrol; the car was stationary
e 30 litres **f** 2 litres

ASSIGNMENT 1C

1 **a** −31 **b** −184 **c** −10 **d** 170
2 **a** 9 900 090 **b** 5374
c 4 231 000 000 **d** 9 200 000
3 **a** $x = 6{\cdot}5$ **b** $y = 4{\cdot}6$
4 **a** $34m + 11$ **b** $20y - 34$
c $a(a - 5)$ **d** $y \times y \times y \times y$
5 **a** $68{\cdot}33\ m^2$ **b** 18·2 m
6 **a** 5 m **b** $12\ m^3$ **c** $36\ m^2$
7 $x = 110°$, $Y = 85°$
8 **a** $188.46 **b** $3.30 **c** 3 beats/s
9 $y = 2x + 1$
10 **a** $x = 3$ **b** $x = 6$

Chapter 2

Exercise 2:01A

1 **a** **i** $93.80 **ii** 6·4
b **i** $336 **ii** 250
c **i** 4c/folder **ii** $28
d $1204
e **i** $492.25 **ii** $507.87
f 101·25
g **i** 540 km **ii** 12·25 h
2 **a** 2·8 L/100 km **b** about 2·98 km
3 $555.00 **4** 7 heartbeats/breath **5** 13 m

Exercise 2:01B

1 **a** 63 kg **b** 35 hours **c** 72 **d** 15
2 **a** 36 **b** 5000 **c** $4976 **d** 36
3 **a** 560 m **b** 9 cm
4 **a** $180 **b** $19 600 **c** 1·5 km **d** 480 mL
5 **a** 2 : 3 **b** 1 : 1 **c** 72 cm

Exercise 2:01C

1 **a** *A*, $3000; *B*, $7000 **b** *G*, 42 : *D*, 63
c 1·5 L **d** *M*, $330 : *J*, $440
e *M*, 15 : *S*, 25 **f** *J*: $35, *Z*: $21, 7 hours
g Army, $124 200 000 : Navy, $55 200 000
2 **a** *R*, $60; *A*, $100; *N*, $40
b 9 **c** 270, 300, 330
d $1620 **e** 36 cm, 45 cm, 27 cm

Challenge 2:01

No, orange is only $\frac{1}{3}$ part of *A* and only $\frac{1}{5}$ part of *B*. Adding the two mixtures would give a mixture where the fraction of orange is between $\frac{1}{3}$ and $\frac{1}{5}$. Container *C* has $\frac{3}{7}$ orange.

1 **a** 333 mL **b** 200 mL **c** 429 mL
2 **a** 667 mL **b** 800 mL **c** 571 mL
3 **a** 533 mL **b** 629 mL
4 **a** 1467 mL **b** 1371 mL
5 **a** 27% **b** 31%
6 **a** 27 : 73 **b** 31 : 69
7 **a** Juice D **b** Juice C

Exercise 2:01D

1 **a** 960 **b** $232.38 **c** 805
d $13 904 **e** $49 115

2 **a** 5·5%
b 85% (to nearest whole %)
c **i** $45 000
ii 21·4% (1 dec. pl.)
d 12% **e** 2%

3 **a** $1299 **b** 16 880 **c** $1 450 000
d $78 000 **e** $7200

Exercise 2:01E

1 **a** $1{\cdot}152\,m^3$
b $1{\cdot}68\,m^3$ (2 dec. pl.)
c 1183 L (to the nearest litre)
d 6·5 cm (1 dec. pl.)
e Volume = $140\,cm^3$ Mass = 1092 g

2 **a** Area = $11{\cdot}34\,m^2$; Cost = $340.20
b Area = $50{\cdot}08\,m^2$; Cost = $2253.60
c $34.20
d $3022\,cm^2$ (to the nearest cm^2)
e $78{\cdot}4\,m^2$

3 **a** 384 m **b** 35 m
c **i** 4·005 km **ii** 5
d 318 mm **e** 796 revolutions

Fun spot 2:01

a 80 m **b** 160 m **c** 200 m
d 220 m **e** 230 m **f** 235 m

Exercise 2:02

1 **a** 32 **b** B **c** 54 **d** 3
e The first number must be $\frac{1}{2}$. If so, the other can be any number.
f 1296 **g** 1·2 kg **h** 12

2 **a** $42{\cdot}25\,m^2$. One quarter of the perimeter of the outside of the square path gives the length of one side (i.e. 8·5 m). The length of the inside of the path is therefore 2 m less (i.e. 6·5 m). The area of the garden is the area of the square bounded by the inside of the path. The area of the garden is $6{\cdot}5 \times 6{\cdot}5\,m^2$.
b length = 24 cm, breadth = 6 cm. Let the breadth be x cm. The length is therefore $4x$ cm. The perimeter, 60 cm = $2(x) + 2(4x)$. Solve this equation to find the breadth, x (i.e. $x = 6$). The length is four times the breadth (i.e. 24 cm).
c 15. This could have been done by *guess and check*. However, you could let the number of 5-cent coins be x. The number of 10-cent coins is then $37 - x$. The total, 295 cents = $x \times 5$ cents + $(37 - x) \times 10$ cents. Solving the equation $295 = 5x + 10(37 - x)$, we get $x = 15$. There are 15 five-cent coins. Check your answer by working out the value of 15×5 cents + $(37 - 15) \times 10$ cents.

3 **a** 1 **b** 4 **c** 9 **d** 16
e 25 **f** 36 **g** 49 **h** 64
Total = $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$

4 **a** **i** 2 **ii** 3 **iii** 4 **iv** 5
v 16 **vi** 187
The number of pieces is always one more than the number of cuts.
b **i** 4 **ii** 8 **iii** 16 **iv** 32
v 64 **vi** 16 384
The number of additional folds can be found by raising 2 to a power that is 1 less than the fold number.

5 **a** 56, 23 **b** 67, 78 **c** 634, 566

6 30

7

Number	0	1	2	3	4	5	6	7	8	9
Number needed	16	15	15	15	15	57	5	5	5	5

8 20 **9** 12 **10** 30

Investigation 2:02

146·304 m

Exercise 2:03

1 **a** 3 **b** 18 **c** 3
2 **a** 33 **b** 42 **c** 35
3 **a** 5 **b** 8 **c** 10
4 **a** 2 **b** 8 **c** 5 **d** 2 **e** 14
5 **a** 10 **b** 0 **c** 5 **d** 8 **e** yes
6 **a** 6 **b** 5 **c** 3 **d** 7 **e** 20
f 9 **g** 28 **h** 32 **i** 23

Diagnostic test 2

1 **a** 80c/kg **b** $16
2 48 kg **3** $105^o, 30^o, 45^o$ **4** $4785
5 **a** $1120 **b** $61 320.15 (to nearest 5c)
6 $34{\cdot}8\,m^2$
7 length = 27·5 m, breadth = 5·5 m
8 $108.20 **9** 6 play both sports.
10 **a** 3 **b** 24

ASSIGNMENT 2A

1 **a** $70 **b** 5·3 runs/over; 79 or 80 runs
c 8 min 20 s **d** 15 or 16 drops/min
2 **a** 161° **b** 52 mm, 195 mm
c $231\,cm^2$ **d** $26{\cdot}95\,m^3$
3 **a** 120 g **b** $0{\cdot}5\,m^3$
c 400 g in the smaller plot and 600 g in the larger plot
d 60%, 40%
4 1250 g of packaged puff pastry, $2\frac{1}{2}$ cups sugar, $1\frac{7}{8}$ cups cornflower, $1\frac{1}{4}$ cups custard powder, 3·125 litres milk, 150 g butter, 5 egg-yolks, 5 teaspoons vanilla
5 **a** $2612.50 **b** 6·3%
c **i** $2800 **ii** $2632

6 **a** 12 **b** $0{\cdot}96\,m^3$

7 $47 308.80 **8** 15 games

9 **a** 2·4 m **b** 1·5 m

10

6	3
6	9

ASSIGNMENT 2B

1 **a** 15 to 24 **b** 15, 16, 17

c 18, 19, 20 **d** 21, 22, 23, 24

e 17

2 **a** 21, 22, 23 **b** 5, 16, 19, 23

c 12, 13, 14 **d** 9

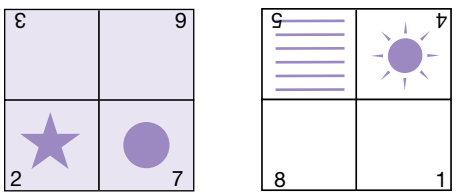
3 4 hectares

4 **a** 14 **b** 23 **c** 46 **d** 42

5 **a** June **b** July, 24°C

c 4 **d** 148 mm

6



Front Back

ASSIGNMENT 2C

1 Harry would receive $3000, Lester would receive $7500 and Michelle would receive $4500.

2 **a** true **b** true **c** true **d** false

e true **f** true **g** false **h** true

I true **j** true **k** true **l** false

3 **a** 12 **b** 72 **c** 314·2 **d** 0·031 42

e 10 500 **f** 0·000 105

4 **a** $2.70 **b** $235 **c** 45% **d** $42.46

5 **a** $\frac{4}{9}$ **b** 0·15

c $32 **d** 0·075 (or 7·5%)

6 **a** $0.\dot{6}$ **b** 5 : 3

c 1·75 : 1 (or $1\frac{3}{4}$: 1) **d** 2·5 km/h

7 **a** 2 **b** 80·1 L

c 24 000 000 **d** 3·14

8 **a** $3{\cdot}5 \times 10^7$ **b** $3{\cdot}50 \times 10^7$

c 2 **d** 5 080 000

9 **a** 2000 GB **b** 3 MB **c** 25 TB **d** 8500 kB

10 Find the absolute error of the measurement:

a 0·5 g **b** 0·005 m **c** 0·0005 L

d 6·45 up to 6·55 (but not including 6·55)

Chapter 3

Prep quiz 3:01

1 19 **2** −4 **3** 1 **4** −11 **5** 15

6 5 + 2 **7** 9 − 4 **8** a **9** 12 **10** $2a$

Exercise 3:01

1 **a** $5x$ **b** $13a$ **c** $31p$ **d** $3a$ **e** $6b$

f $10q$ **g** $2p$ **h** $7x$ **i** $4x$ **j** $12x^2$

k $3a^2$ **l** $2y^2$ **m** $-2y$ **n** $-2x$ **o** $3n$

p $16ab$ **q** pq **r** $-2x^2y$

2 **a** $5a + b$ **b** $13x - 9$ **c** $17 + 3a$

d $4q - p$ **e** $3y + x$ **f** $m + n$

g $q + 3p$ **h** $k - 2h$ **i** $7m + 4n$

j $3a + 5b$ **k** $9t + u$ **l** $5p - q$

m $a + 4p$ **n** $2m$ **o** $1 - 3x$

p -2 **q** $3x^2 + x$ **r** $4p^2 + 5p$

s $2q^2 + 4q$ **t** $2y^2$ **u** $2 - p^2 + p$

v $3a + a^2 + 7$ **w** $x - 7 - 3x^2$ **x** $8ab - 16$

Prep quiz 3:02

1 −24 **2** 6 **3** −7 **4** 9 **5** 4

6 $7a$ **7** xy **8** a^2 **9** $6x$ **10** $6x$

Exercise 3:02

1 **a** $10x$ **b** $21p$ **c** $24y$ **d** $9a$

e $5b$ **f** $108p$ **g** xy **h** $6xb$

i $24xy$ **j** $30mn$ **k** $17xy$ **l** $50ax$

m $2x$ **n** $5xy$ **o** $3a$

2 **a** x^2 **b** $2y^2$ **c** $3a^2$ **d** $8m^2$

e $14p^2$ **f** $80b^2$ **g** $121t^2$ **h** $30x^2$

i $2a^2b$ **j** xy^2z **k** a^2bc **l** $6p^2q$

m $20mn^2p$ **n** $8a^2b^2$ **o** $3xy^2z$ **p** $2m^2n$

q $-12xy$ **r** $6x^2$ **s** $-35a^2$ **t** $9a^2b$

u $28am$ **v** $-10p^2g$ **w** ab^2c **x** $-7a^2b^2$

3 **a** $42a$ **b** $30xyz$ **c** $10x^2$ **d** $a^2b^2c^2$

e $6xyz$ **f** $60a^2$ **g** $7mn$ **h** $6k^2l$

i $70x$ **j** $-120a^2$ **k** $180xy$ **l** $-mnp$

Prep quiz 3:03

1 9 **2** −2 **3** −4 **4** 4 **5** 3

6 $2\frac{1}{2}$ **7** $2\frac{2}{5}$ **8** $2\frac{6}{7}$ **9** 1 **10** 1

Exercise 3:03

1 **a** $\frac{x}{2}$ **b** $\frac{2b}{5}$ **c** a **d** $\frac{1}{2}$ **e** $\frac{1}{5}$

f 3 **g** $2a$ **h** $\frac{4a}{b}$ **i** $\frac{1}{t}$

2 **a** $3x$ **b** $2y$ **c** p **d** $3m$ **e** ab

f $4xy$ **g** $3x^2$ **h** $2x$ **i** 4 **j** 2

k 1 **l** 4 **m** $\frac{3m}{2n}$ **n** $\frac{9}{2}$ or $4\frac{1}{2}$

o $\frac{8a}{3b}$ **p** $\frac{1}{4x}$ **q** $4y$ **r** $3b$ **s** $\frac{9}{7y}$

t $\frac{6}{5b}$ **u** $\frac{1}{9k}$ **v** $\frac{1}{3}$ **w** $\frac{m}{2}$ **x** $\frac{9a}{4c}$

3 **a** $-4p$ **b** $3x$ **c** $-5p$ **d** 2 **e** $\frac{-5}{3a}$

f $\frac{-x}{z}$ **g** $\frac{ab}{16}$ **h** 6 **i** -14 **j** $\frac{5}{3}$

k $\frac{-3}{y}$ **l** $4n$

Exercise 3:04A

1 **a** $\frac{2x}{3}$ **b** $\frac{5m}{7}$ **c** $\frac{7n}{6}$ **d** $\frac{x}{5}$ **e** $\frac{5t}{8}$

f $\frac{2m}{7}$

2 **a** $\frac{4p}{5}$ **b** $\frac{t}{2}$ **c** $\frac{2y}{3}$ **d** $\frac{3x}{5}$ **e** $\frac{7x}{6}$

f $\frac{7m}{8}$

3 **a** $\frac{5x}{6}$ **b** $\frac{11a}{12}$ **c** $\frac{9m}{10}$ **d** $\frac{7t}{6}$ **e** $\frac{16x}{15}$

f $\frac{a}{6}$ **g** $\frac{4w}{15}$ **h** $\frac{11n}{10}$ **i** $\frac{m}{6}$ **j** $\frac{3a}{20}$

4 **a** $\frac{5a}{6}$ **b** $\frac{7y}{3}$ **c** $\frac{m}{2}$ **d** $\frac{m}{2}$ **e** $\frac{3t}{10}$

f $\frac{5m}{18}$ **g** $\frac{11a}{12}$ **h** $\frac{19x}{20}$ **i** $\frac{29n}{24}$ **j** $\frac{21y}{8}$

5 **a** $\frac{7}{12}m$ **b** $\frac{5}{24}y$ **c** $\frac{9}{20}x$ **d** $\frac{25}{4}y$ **e** $\frac{6}{5}m$

Exercise 3:04B

1 **a** $\frac{xy}{6}$ **b** $\frac{ab}{30}$ **c** $\frac{mn}{12}$ **d** $\frac{2a}{3x}$ **e** $\frac{5n}{3m}$

f $\frac{10p}{3q}$

2 **a** $\frac{15xy}{8}$ **b** $\frac{8mn}{9}$ **c** $\frac{7xy}{6}$ **d** $\frac{6ab}{25}$ **e** $\frac{15x^2}{8}$

3 **a** $\frac{m^2}{6}$ **b** $\frac{xy}{4}$ **c** $\frac{a}{12}$ **d** $2xy$ **e** $\frac{15t^2}{4}$

4 **a** 6 **b** $\frac{1}{4}$ **c** 1 **d** 6 **e** $\frac{a^2}{5}$

5 **a** $2y$ **b** $\frac{2n}{3}$ **c** $15a$ **d** $4y$ **e** $\frac{3a}{2}$

6 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{5}{6}$ **d** $\frac{5}{2}$ **e** 5

f $\frac{a}{8}$ **g** $\frac{2x}{5}$ **h** $\frac{5m}{2t}$ **i** $\frac{10m}{n}$ **j** $\frac{2x}{25y}$

7 **a** $\frac{10x}{3a}$ **b** $\frac{3a}{2b}$ **c** $\frac{3}{2}$ **d** $\frac{2a}{x}$ **e** $\frac{5}{9}$

8 **a** $\frac{10}{x^2}$ **b** $\frac{a^2}{6}$ **c** $\frac{5m}{6}$ **d** $\frac{2b}{3c}$ **e** $\frac{2c}{3b}$

Prep quiz 3:05

1 $10a$ **2** $2a^2$ **3** $10x^2$ **4** 20 **5** $-6x$

6 $-x^2$ **7** $-14p^2$ **8** 8 **9** 3 **10** 8 and 5

Exercise 3:05

1 **a** $2x + 6$ **b** $3a + 15$ **c** $5x - 5$

d $10a + 10$ **e** $3x - 6$ **f** $8y + 32$

g $10a + 2$ **h** $6x - 15$ **i** $36x + 63$

j $5 + 5x$ **k** $12 - 4a$ **l** $10 - 6m$

m $6a + 4b$ **n** $5x + 5y$ **o** $21x - 35y$

2 **a** $x^2 + 7x$ **b** $a^2 - a$ **c** $m^2 + 10m$

d $n^2 - 3n$ **e** $2a^2 + a$ **f** $3b^2 - 5b$

g $5x + x^2$ **h** $3y - y^2$ **i** $2x^2 + 6x$

j $3y^2 - 3y$ **k** $10z + 5z^2$ **l** $10m - 10m^2$

m $6a^2 - 3a$ **n** $18x^2 + 63x$ **o** $16p - 40p^2$

p $21q + 14q^2$ **q** $ax + bx$ **r** $2y^2 + xy$

s $2m^2 + 2mn$ **t** $10a^2 + 15ab$

3 **a** $-2x - 6$ **b** $-3a - 15$ **c** $-2y + 2$

d $-5p + 15$ **e** $-21a - 14$ **f** $-10x + 5$

g $-21 - 14m$ **h** $-28 + 4x$ **i** $-a - 1$

j $-3x - 7$ **k** $-8 + 2p$ **l** $-3a - 2b$

m $-x^2 - 10x$ **n** $-5y + y^2$ **o** $-6x^2 - 21x$

p $-80mn + 10n^2$

4 **a** $2x + 12$ **b** $4x - 6$ **c** $8x$ **d** $-y$

Prep quiz 3:06

1 $10x$ **2** $3a^2$ **3** $6x + 8$

4 $x + 2$ **5** $5y^2 + 4y$ **6** $13 + 2a$

7 $3x - 21$ **8** $18 - 45y$ **9** $2a^2 + 6a$

10 $-5x - 35$

Exercise 3:06

1 **a** $6x + 15$ **b** $4a + 31$ **c** $2y + 8$

d $3p + 14$ **e** $9a + 12$ **f** $10m + 9$

g $7a + 8$ **h** $14x + 25$ **i** $12a + 16$

j $12m + 14$

2 **a** $5x + 8$ **b** $8y + 22$ **c** $5m + 23$

d $x^2 + 6x + 3$ **e** $a^2 + 10a + 25$ **f** $2n^2 + 7n + 3$

3 **a** $2a - 2$ **b** $7a + 4$ **c** $4m + 14$

d $3x - 2$ **e** $4m - 15$ **f** $6p - 7$

g $8y - 17$ **h** $10a - 9$ **i** $12x + 2$

4 **a** $-2a$ **b** $2 - 3x$ **c** -3

d $m + 8$ **e** $5a - 24$ **f** $6y$

g $3m$ **h** $2x + 23$ **i** $4a^2 + 3ab - 2b^2$

5 **a** $17k - 6$ **b** $13m - 5$ **c** $2a + 15$

d $14 + 2y$ **e** $28 - 6x$ **f** $14x - 13$

g $2a + 66$ **h** $t^2 - 9t + 20$ **i** $4x^2 - 11x + 6$

j $2m^2 - 11m + 15$ **k** $3y - 17$

l $6a^2 - 4a + 9$ **m** $a^2 + ab$

n $x^2 - xy - 6x + 3y$ **o** $6x^2 - 5xy + 2y$

Prep quiz 3:07

1 $2x + 10$ **2** $4x^2 - x$

3 $6a^2 + 21a$ **4** $2m^2 + 6mn$

5 $\{1, 2, 3, 4, 6, 12\}$ **6** $\{1, 2, 4, 5, 10, 20\}$

7 $\{1, 2, 3, 5, 6, 10, 15, 30\}$ **8** 4

9 6 **10** 10

Exercise 3:07

1 **a** $x + 2$ **b** $a + 3$ **c** $2a - 3$
d $x + 2y$ **e** $3x - 2y$ **f** $4pq - 5x$
g $m - p$ **h** $b - c$ **i** $x + 2y$
j $y - 5$ **k** $5 - 3a$ **l** $2m - 1$

2 **a** $2(x + 5)$ **b** $2(3a + 2)$ **c** $7(y + 3)$
d $4(7 + x)$ **e** $3(9 - y)$ **f** $6(4x + 1)$
g $9(x - 5)$ **h** $4(4 - 3a)$ **i** $3(3x + y)$
j $5(a + 2b)$ **k** $5(3m - 4n)$ **l** $2(2b - 3a)$
m $m(p + n)$ **n** $a(x + y)$ **o** $x(x + y)$
p $p(p - q)$ **q** $a(p + 3)$ **r** $x(5 + a)$
s $m(4 - n)$ **t** $t(x - 1)$

3 **a** $3a(x + 2y)$ **b** $5m(n - 2p)$ **c** $2b(2a - 3c)$
d $3q(3p - 2r)$ **e** $5x(x - 2y)$ **f** $3a(b + 2a)$
g $2m(5m - 2n)$ **h** $4x(3x + y)$ **i** $bc(a + d)$
j $pq(a - b)$ **k** $xy(z + 1)$ **l** $mn(1 - p)$
m $xa(x - y)$ **n** $5x(a - 2y)$ **o** $ap(a - 5)$
p $xy(y + z)$ **q** $5b(2a - 3c)$ **r** $xy(5x - 3y)$
s $ap(p - a)$ **t** $ab(5 - ab)$

4 **a** $-2(a + 3)$ **b** $-5(x + 3)$ **c** $-4(2m + 3)$
d $-5(2x + 1)$ **e** $-4(2x - 1)$ **f** $-3(n - 3)$
g $-7(y - 5)$ **h** $-2(3a - 2)$ **i** $-x(x + 3)$
j $-m(m + 1)$ **k** $-x(3x - 2)$ **l** $-5y(y - 2)$
m $-p(4 + p)$ **n** $-x(3 + 2x)$ **o** $-m(1 - 7m)$
p $-2a(2 - 9a)$

5 **a** $a(b + c + d)$ **b** $x(3 + y + z)$
c $m(m - 3 + n)$ **d** $a(7 - b + a)$
e $p(p + q - 5)$ **f** $2(x + 2y - 3z)$
g $5(2a - b + 3c)$ **h** $3(3x^2 + 2x - 4)$
i $2(4 - 2x + 3x^2)$ **j** $5(5 + 3y - 4y^2)$
k $x(xy - 3y + 1)$ **l** $2a(b - 2c + 5)$
m $3x(x + 2y - 3)$ **n** $xy(x + 1 + y)$
o $ab(ab + 3a + 2b)$ **p** $mn(1 + 4m - 8n)$

6 **a** $(a + 2)(a + 3)$ **b** $(m + 2)(m + 4)$
c $(x - 1)(x + 5)$ **d** $(b + 1)(b - 5)$
e $(y - 2)(7 - y)$ **f** $(t - 7)(t - 9)$
g $(2m - 3)(4 + 3m)$ **h** $(7x + 1)(2x - 5)$
i $(a + 3)(x - 1)$ **j** $(2y - 1)(y - 1)$
k $(p - 3)(p - 3)$ **l** $(5x + 3)(1 - x)$

Diagnostic test 3

1 **a** $2a + 3b$ **b** $2p^2 + 2p$
c $2ab$ **d** $4a - x - 2$

2 **a** $56m$ **b** $30ab$ **c** $10y^2$ **d** $-8ny$

3 **a** $3a$ **b** $5y$ **c** $\frac{3c}{2b}$ **d** $\frac{-1}{3y}$

4 **a** $\frac{4a}{5}$ **b** $\frac{2m}{7}$ **c** $\frac{x}{2}$ **d** $\frac{2p}{5}$

5 **a** $\frac{7a}{12}$ **b** $\frac{m}{15}$ **c** $\frac{5x}{4}$ **d** $\frac{n}{4}$

6 **a** $\frac{xy}{12}$ **b** $\frac{a^2}{10}$ **c** 4 **d** $\frac{a}{2}$

7 **a** $\frac{4}{3}$ **b** $\frac{a^2}{10}$ **c** $\frac{3}{2}$ **d** $\frac{n^2}{3}$

8 **a** $9x + 63$ **b** $30a - 12$
c $p^2 + 3p$ **d** $15a - 6a^2$

9 **a** $-3x - 6$ **b** $-2m + 16$
c $-15x - 20$ **d** $-7 + 2m$

10 **a** $2a + 11$ **b** $5x^2 - 7x$
c $11m - 2$ **d** $6x - 18$

11 **a** $5(m + 2)$ **b** $x(x - 3)$
c $3a(2b + 5)$ **d** $-4(2y + 3)$

ASSIGNMENT 3A

1 **a** $5x$ **b** $6x^2$ **c** $6x$ **d** $2x^2$
e $2x$ **f** 2 **g** $2x + 3y$ **h** $6xy$

2 **a** $6ab^2$ **b** $2ab$ **c** $5ab$ **d** $3a^2 - a$
e $6x + 3y$ **f** $15 + 6x$ **g** $7a + 1$ **h** $4x^2 - x$

3 **a** $3x + 15$ **b** $2x - 14$ **c** $-3x - 6$ **d** $-4 + 12x$
e $a^2 + 3a$ **f** $xy + xz$
g $3p^2 - 3pq$ **h** $10x^2 - 35x$

4 **a** $3(a + 5)$ **b** $3(2m + 3)$
c $5(3 - y)$ **d** $x(a - 3)$
e $2x(1 + 3y)$ **f** $2x(2x - 1)$
g $3b(3a - 2c)$ **h** $3x(2x - 3 + y)$

5 **a** $4x + 13$ **b** $10y + 25$
c $3a + 6$ **d** $2a - 10$
e $5m - 14$ **f** $3x^2 - 8x + 4$

6 **a** $\frac{2a}{3}$ **b** $\frac{9a}{20}$ **c** $\frac{4m}{5}$ **d** $\frac{17y}{12}$
e $\frac{d}{2}$ **f** $\frac{3x}{10}$ **g** $\frac{a}{4}$ **h** $\frac{9m}{20}$

7 **a** $\frac{a^2}{12}$ **b** $\frac{3ab}{10}$ **c** $3m$ **d** $\frac{a^2}{12}$

8 **a** 3 **b** $\frac{3}{10}$ **c** 4 **d** $4n$

9 **a** incorrect **b** incorrect
c correct **d** incorrect

10 **a** $4(3a - 4) = 12a - 16$ **b** $-3(x - 2) = -3x + 6$
c $2(a + 3) - 3(a + 4)$ **d** $4(3a - 2) - 3(2a + 1)$
$= 2a + 6 - 3a - 12$ $= 12a - 8 - 6a - 3$
$= -a - 6$ $= 6a - 11$

ASSIGNMENT 3B

1 **a** square **b** rectangle **c** parallelogram
d rhombus **e** trapezium **f** pentagon
g hexagon **h** octagon **i** kite
j isosceles triangle

2 **a** An octagonal prism **b** 200 mL

3 17

4 **a** 4 **b** 10

5 **a** 40% **b** 5%

6 **a** Tasmania; 60% **b** Victoria; over 90%
c Queensland, just under 3000 per 10 000
d About 50%; between 30% and 40%.

ASSIGNMENT 3C

1 **a** $958.50 **b** 40 530
c **i** 20% **ii** $\frac{5}{12}$ **iii** 0·65
2 **a** 0·875 **b** 0·068 **c** $0.41\dot{6}$
3 **a** 2 **b** 5 **c** 1 **d** 2
4 **a** 8·75 m to 8·85. **b** 123·445 to 123·455
c 5·5 km to 6·5 km **d** 3·95°C to 4·05°C
5 **a** 6 **b** 35 **c** 21 **d** 2
6 **a** 3 200 000 L **b** 45 000 000 000 W
c 3 000 000 MB **d** 15 000 kHz
7 $2262
8 **a** 75kg; $500
b Volume of sand = 0·8 m^3; Volume of gravel = 1 m^3
9 4 **10** 18

Chapter 4

Exercise 4:01

1 **a** even chance **b** unlikely **c** impossible
d impossible **e** certain
2 **a** 1, 2 **b** 3, 4, 5, 6 **c** 2, 4, 6
d 1, 3, 5 **e** 1, 2, 3, 4, 6 **f** 1, 2, 3, 4, 5, 6
3 **a** QS, QC **b** 5D, 5H, 6D, 6H, 7D, 7H
c JD, JH **d** 3H, 4H
e JD, JH, QD, QH, KD, KH
f AD, 2D, 3D, 4D, 5D, 6D, 7D, 8D, 9D, 10D, JD, QD, KD
4 **a** 24 **b** 28 **c** 4 **d** 28 **e** 40
f 32 **g** 32 **h** 12 **i** 20 **j** 32
5 **a** 9 **b** 8 **c** 9 **d** 0 **e** 91
f 100
6 **a** Mutually exclusive events are events that have no element in common.
b Non-mutually exclusive events are events that have one or more elements in common.
c 1
d The complement of an event is every element in the sample space that is not in that event.
e Choosing at random means choosing in a way that does not affect the likelihood of outcomes.

7

Outcome	Frequency	Relative frequency (or experimental probability)
two heads	24	$\frac{24}{100}$ or 24%
head and tail	51	$\frac{51}{100}$ or 51%
two tails	25	$\frac{25}{100}$ or 25%
Total:	100	

8

Outcome	Frequency	Relative frequency (or experimental probability)
1	8	$\frac{8}{60}$ or about 13%
2	10	$\frac{10}{60}$ or about 17%
3	9	$\frac{9}{60}$ or about 15%
4	12	$\frac{12}{60}$ or about 20%
5	10	$\frac{10}{60}$ or about 17%
6	11	$\frac{11}{60}$ or about 18%
Total:	60	

9 Four different-coloured sectors, each with an angle of 90° at the centre, should be drawn on the spinner.
10 Answers will vary. A spinner with 4 quadrants could be used, with 'Alan' written in two quadrants and 'Alana' and 'Heather' each written in one quadrant.

Prep quiz 4:02

1 10 **2** 3 **3** $\frac{3}{10}$ **4** $\frac{1}{5}$ **5** $\frac{1}{2}$
6 6 **7** $\frac{3}{5}$ **8** $\frac{1}{3}$ **9** $\frac{4}{5}$ **10** $\frac{12}{25}$

Exercise 4:02

1 **a** $\frac{70}{100}$ (or $\frac{7}{10}$) **b** $\frac{10}{100}$ (or $\frac{1}{10}$) **c** $\frac{5}{100}$ (or $\frac{1}{20}$)
d $\frac{30}{100}$ (or $\frac{3}{10}$) **e** $\frac{15}{100}$ (or $\frac{3}{20}$) **f** $\frac{85}{100}$ (or $\frac{17}{20}$)
g $\frac{20}{100}$ (or $\frac{1}{5}$) **h** $\frac{75}{100}$ (or $\frac{3}{4}$)
2 **a** brown **b** yellow **c** $\frac{6}{24}$ (or $\frac{1}{4}$)
d 24
3 faulty: $\frac{5}{100}$ (or $\frac{1}{20}$), good: $\frac{95}{100}$ (or $\frac{19}{20}$)
4 **a** **i** $\frac{10}{42}$ (or $\frac{5}{21}$) **ii** $\frac{5}{42}$
iii $\frac{21}{42}$ (or $\frac{1}{2}$)
b 24%, 12%, 50%
5 **a** $\frac{18}{100}$ (or $\frac{9}{50}$) **b** $\frac{35}{100}$ (or $\frac{7}{20}$) **c** $\frac{65}{100}$ (or $\frac{13}{20}$)
d $\frac{20}{100}$ (or $\frac{1}{5}$)
6 **a** **i** $\frac{3}{30}$ (or $\frac{1}{10}$) **ii** $\frac{13}{30}$
iii $\frac{10}{30}$ (or $\frac{1}{3}$)
b no, but the graph should be similar in shape
c yes
7 **a** **i** $\frac{14}{50}$ **ii** $\frac{5}{50}$ **iii** $\frac{21}{50}$
b no
c The experimental probabilities of each number should be much closer to the real probabilities (i.e. $16\frac{2}{3}$%).
8 **a** $\frac{16}{20}$ (or $\frac{4}{5}$ or 80%) **b** $\frac{1}{20}$ (or 5%)
c $\frac{3}{20}$ (or 15%)
9 **a** A, C, D, B, E (Answers may vary.)
b (Answers will vary.) One possibility is $P(A) \div 0{\cdot}01$, $P(B) \div 0{\cdot}5$, $P(C) \div 0{\cdot}1$, $P(D) \div 0{\cdot}9$, $P(E) \div 0{\cdot}99$.

10 a impossible b even chance c certain
d even chance e certain f even chance

Prep quiz 4:03

1 $\frac{7}{8}$ 2 $\frac{2}{3}$ 3 $\frac{3}{7}$ 4 $\frac{3}{16}$ 5 $\frac{1}{10}$
6 $\frac{1}{4}$ 7 $\frac{1}{3}$ 8 $\frac{2}{3}$ 9 $\frac{5}{26}$ 10 $\frac{3}{10}$

Exercise 4:03

1 a $\frac{1}{6}$ b $\frac{1}{2}$ c $\frac{1}{3}$
2 a $\frac{1}{2}$ b $\frac{1}{5}$ c $\frac{7}{10}$
d $\frac{4}{5}$ e 0 f 1

3 a getting an even number
b getting a head c getting a 6
d drawing a club, diamond or heart
e seeing green or orange
f losing or playing a draw
g choosing a consonant

4 a $\frac{1}{52}$ b $\frac{1}{13}$ c $\frac{1}{2}$ d $\frac{1}{4}$ e $\frac{1}{26}$
f $\frac{2}{13}$ g $\frac{3}{13}$

5 a $\frac{1}{26}$ b $\frac{5}{26}$ c $\frac{1}{13}$ d $\frac{4}{13}$

6 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{1}{6}$ d 1 e 0
f $\frac{5}{6}$

7 a $\frac{1}{6}$ b $\frac{1}{2}$ c $\frac{2}{3}$ d $\frac{1}{3}$

8 a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{3}{8}$ d 0

9 a $\frac{13}{25}$ b $\frac{1}{5}$ c $\frac{8}{25}$ d $\frac{8}{25}$

10 a $\frac{3}{5}$ b $\frac{2}{5}$ c $\frac{1}{5}$ d $\frac{3}{5}$ e 1

11 4, no

12 a $\frac{5}{12}$ b $\frac{1}{6}$ c $\frac{1}{12}$ d $\frac{1}{12}$ e $\frac{5}{12}$

13 a 2 b 6 c 8

14 a $\frac{9}{20}$ b $\frac{1}{4}$ c $\frac{7}{10}$ d $\frac{1}{10}$

15 a $\frac{18}{37}$ b $\frac{18}{37}$ c $\frac{1}{37}$ d $\frac{19}{37}$ e $\frac{6}{37}$
f $\frac{11}{37}$ g Once every 37 spins
h Lose, because of the wheel's zero slot.

Exercise 4:04

1 a non-mutually exclusive
b mutually exclusive
c non-mutually exclusive

2 a $\frac{53}{100}$ or 53% b $\frac{47}{100}$ or 47% c $\frac{21}{50}$ or 42%
d $\frac{29}{50}$ or 58% e $\frac{9}{50}$ or 18% f $\frac{43}{100}$ or 43%
g $\frac{4}{5}$ or 80% h $\frac{73}{100}$ or 73%

3 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{1}{13}$
d $\frac{12}{13}$ e $\frac{3}{13}$ f $\frac{2}{52}$ or $\frac{1}{26}$
g $\frac{1}{4}$ h 0 i $\frac{6}{52}$ or $\frac{3}{26}$
j $\frac{15}{52}$ k $\frac{1}{13}$ l $\frac{3}{4}$
m $\frac{22}{52}$ or $\frac{11}{26}$ n $\frac{37}{52}$ o $\frac{16}{52}$ or $\frac{4}{13}$
p $\frac{8}{52}$ or $\frac{2}{13}$ q $\frac{23}{52}$

4 a $\frac{1}{2}$ b $\frac{2}{5}$ c $\frac{3}{10}$ d $\frac{2}{5}$
e $\frac{7}{10}$ f $\frac{4}{5}$ g 1 h $\frac{7}{20}$
i $\frac{3}{10}$ j $\frac{2}{5}$ k $\frac{3}{10}$ l $\frac{3}{20}$
m $\frac{3}{5}$ n $\frac{2}{5}$ o $\frac{17}{20}$ p $\frac{3}{20}$

5 a i $\frac{3}{8}$ ii $\frac{3}{8}$
b No c $\frac{3}{4}$ d True
e i $\frac{3}{8}$ ii $\frac{1}{2}$
f Yes g False h True

Exercise 4:05

1 a $\frac{18}{100}$ (or 18%) b $\frac{44}{100}$ (or 44%) c $\frac{56}{100}$ (or 56%)
d $\frac{82}{100}$ (or 82%) e $\frac{16}{100}$ (or 16%) f $\frac{2}{100}$ (or 2%)
g $\frac{54}{100}$ (or 54%) h $\frac{28}{100}$ (28%) i $\frac{46}{100}$ (or 46%)

2 a $\frac{11}{60}$ b $\frac{8}{60}$ (or $\frac{2}{15}$) c $\frac{19}{60}$
d $\frac{49}{60}$ e $\frac{41}{60}$ f $\frac{52}{60}$ (or $\frac{26}{30}$)
g 0 (0% chance)

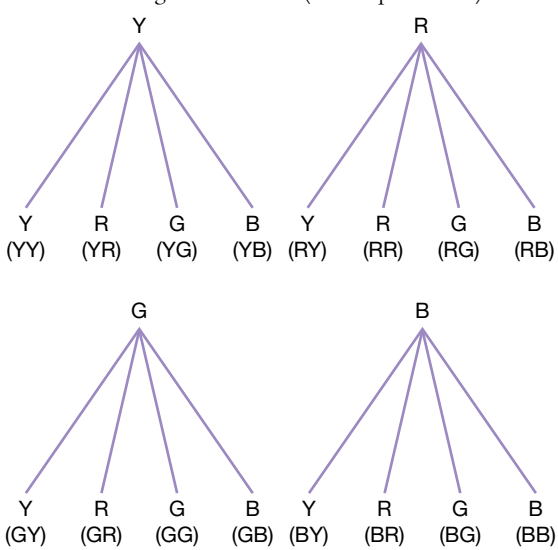
3 a 117 b 35 c 30%
d 47% e 13% f 40%
g 70% h 30%

4 a $\frac{61}{125}$ b $\frac{64}{125}$ c $\frac{29}{125}$
d $\frac{39}{125}$ e $\frac{25}{125}$ (or $\frac{1}{5}$)

5 a $\frac{7}{50}$ b $\frac{2}{50}$ (or $\frac{1}{25}$) c $\frac{5}{50}$ (or $\frac{1}{10}$)
d $\frac{13}{50}$ e $\frac{6}{50}$ (or $\frac{3}{25}$) f 0(0% chance)

Exercise 4:06

1 a Choosing two counters (with replacement)



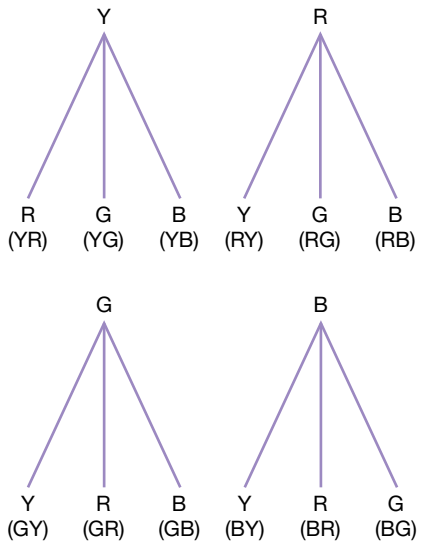
Second choice (with replacement)

First choice	Y	R	G	B
Y	YY	YR	YG	YB
R	RY	RR	RG	RB
G	GY	GR	GG	GB
B	BY	BR	BG	BB

b 16 **c** 2 **d** $\frac{2}{16}$

e $\frac{1}{16}$ **f** $\frac{1}{16}$ **g** $\frac{15}{16}$

2 a Choosing two counters (without replacement)



Second choice (without replacement)

First choice	Y	R	G	B
Y		YR	YG	YB
R	RY		RG	RB
G	GY	GR		GB
B	BY	BR	BG	

b 12 **c** 2 **d** $\frac{2}{12}$

e $\frac{1}{12}$ **f** 0 **g** 1

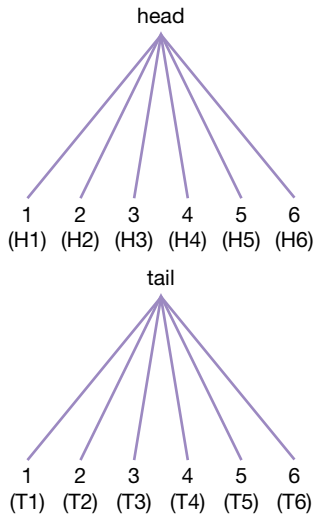
h

Event	Number of times	Probability
Yellow and Red	2	$\frac{2}{12}$
Yellow and Green	2	$\frac{2}{12}$
Yellow and Blue	2	$\frac{2}{12}$
2 Yellow	0	$\frac{0}{12}$ (or 0)

Event	Number of times	Probability
Red and Green	2	$\frac{2}{12}$
Red and Blue	2	$\frac{2}{12}$
Green and Blue	2	$\frac{2}{12}$
2 Red	0	$\frac{0}{12}$ (or 0)

3 a

Dice	Coin H	Coin T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6



b 12

c i $\frac{1}{12}$ ii $\frac{1}{2}$ iii $\frac{2}{12}$

iv $\frac{4}{12}$ v $\frac{5}{12}$ vi $\frac{10}{12}$

d no

4 a 36 **b** 11 **c** $\frac{1}{36}$ **d** $\frac{1}{36}$ **e** $\frac{6}{36}$

f $\frac{3}{36}$ **g** $\frac{3}{36}$ **h** $\frac{4}{36}$ **i** $\frac{5}{36}$ **j** $\frac{5}{36}$

k $\frac{4}{36}$ **l** $\frac{8}{36}$ **m** $\frac{15}{36}$ **n** $\frac{2}{36}$ **o** $\frac{18}{36}$

p $\frac{30}{36}$ **q** $\frac{15}{36}$

5 a (3C, 6H), (3C, 7H), (3C, 8D), (3C, 9C), (6H, 3C), (6H, 7H), (6H, 8D), (6H, 9C), (7H, 3C), (7H, 6H), (7H, 8D), (7H, 9C), (8D, 3C), (8D, 6H), (8D, 7H), (8D, 9C), (9C, 3C), (9C, 6H), (9C, 7H)), (9C, 8D)

b 20 **c** $\frac{2}{20}$ **d** $\frac{1}{20}$ **e** $\frac{2}{20}$ **f** $\frac{2}{20}$

g $\frac{8}{20}$ **h** $\frac{8}{20}$ **i** $\frac{4}{20}$ **j** $\frac{4}{20}$ **k** $\frac{2}{20}$

l $\frac{8}{20}$ **m** $\frac{8}{20}$ **n** $\frac{16}{20}$ **o** $\frac{6}{20}$

6 a (3C, 3C), (3C, 6H), (3C, 7H), (3C, 8D), (3C, 9C), (6H, 3C), (6H, 6H), (6H, 7H), (6H, 8D), (6H, 9C), (7H, 3C), (7H, 6H), (7H, 7H), (7H, 8D), (7H, 9C), (8D, 3C), (8D, 6H), (8D, 7H), (8D, 8D), (8D, 9C), (9C, 3C), (9C, 6H), (9C, 7H), (9C, 8D), (9C, 9C)

b 25 **c** $\frac{2}{25}$ **d** $\frac{1}{25}$ **e** $\frac{4}{25}$ **f** $\frac{2}{25}$

g $\frac{10}{25}$ **h** $\frac{8}{25}$ **i** $\frac{4}{25}$ **j** $\frac{4}{25}$ **k** $\frac{3}{25}$

l $\frac{10}{25}$ **m** $\frac{13}{25}$ **n** $\frac{21}{25}$ **o** $\frac{7}{25}$

7 a i $\frac{1}{3}$ ii $\frac{2}{3}$ iii $\frac{1}{3}$ iv $\frac{2}{3}$
v $\frac{2}{3}$ vi $\frac{3}{3}$ or 1

b i $\frac{2}{4}$ ii $\frac{3}{4}$ iii $\frac{1}{4}$ iv $\frac{2}{4}$
v $\frac{3}{4}$ vi $\frac{4}{4}$ or 1

c Knowing that the 2 of hearts has been chosen removes the uncertainty that the 2 will be chosen. The probability of the two-stage event can then be calculated as the possibility of drawing the required card in the second choice to complete the event.

8 a i 12 ii $\frac{2}{12}$ **b** i 16 ii $\frac{4}{16}$

9 Answers will vary.

Investigation 4:06

We would expect the results of the experiments to be different. A report of at least 100 words should be written.

Diagnostic test 4

1 a 1, 2, 3, 4, 5, 6 **b** head, tail **c** 1, 3, 5, 7, 9

2 a $\frac{3}{44}$ **b** $\frac{41}{44}$

c 0. It is possible for me to play worse than ever before and so score more than 109, so the real probability is greater than zero.

3 a $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** 0 **d** $\frac{2}{3}$

4 a $\frac{2}{13}$ **b** $\frac{3}{26}$ **c** $\frac{5}{52}$ **d** $\frac{16}{52}$ or $\frac{4}{13}$
e $\frac{28}{52}$ or $\frac{7}{13}$ **f** $\frac{24}{52}$ or $\frac{6}{13}$

5 a $\frac{55}{100}$ (or $\frac{11}{20}$), 55% chance

b $\frac{43}{100}$, 43% chance **c** $\frac{7}{100}$, 7% chance

d $\frac{50}{100}$ (or $\frac{1}{2}$), 50% chance

6 a $\frac{3}{50}$, 6% chance **b** $\frac{43}{50}$, 86% chance

c $\frac{27}{50}$, 54% chance **d** $\frac{23}{50}$, 46% chance

ASSIGNMENT 4A

1 a $\frac{1}{6}$ **b** 0 **c** $\frac{1}{3}$ **d** $\frac{1}{2}$ **e** 1

2 a $\frac{7}{20}$ **b** $\frac{1}{10}$ **c** $\frac{13}{20}$

3 a $\frac{2}{5}$ **b** $\frac{1}{10}$ **c** $\frac{1}{2}$ **d** 0

4 a (R1, G1) (R1,Y1) (R1,Y2) (G1, R1) (G1,Y1) (G1,Y2) (Y1, R1) (Y1, G1) (Y1,Y2) (Y2, R1) (Y2, G1) (Y2,Y1)

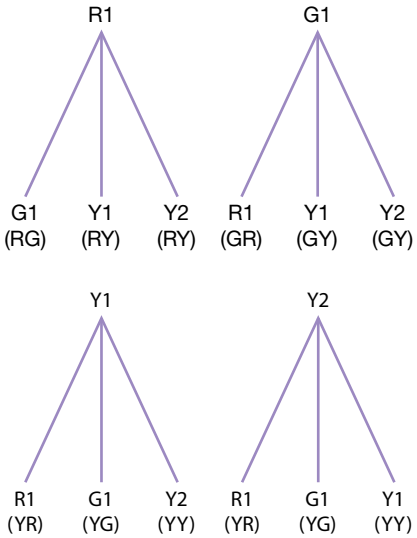
or

Second choice (without replacement)

First choice	R1	G1	Y1	Y2
R1		RG	RY	RY
G1	GR		GY	GY
Y1	YR	YG		YY
Y2	YR	YG	YY	

or

(without replacement)



P(the same colour) = $\frac{2}{12}$ or $\frac{1}{6}$

b (R1, R1) (R1, G1) (R1,Y1) (R1,Y2) (G1, R1) (G1, G1) (G1,Y1) (G1,Y2) (Y1, R1) (Y1, G1) (Y1,Y1) (Y1,Y2) (Y2, R1) (Y2, G1) (Y2,Y1) (Y2,Y2)

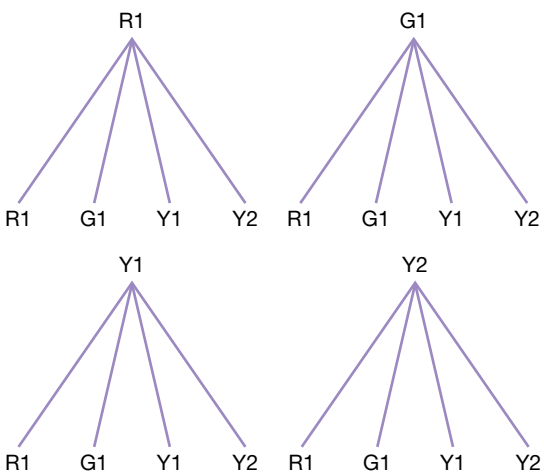
or

second choice (with replacement)

First choice	R1	G1	Y1	Y2
R1	RR	RG	RY	RY
G1	GR	GG	GY	GY
Y1	YR	YG	YY	YY
Y2	YR	YG	YY	YY

or

(with replacement)



P(the same colour) = $\frac{6}{12}$ or $\frac{1}{2}$

5 **a** i $\frac{3}{10}$ ii $\frac{1}{4}$

b No; the scores recorded so far are a limited sample. He theoretically could score any result from zero to 300. (Here the experimental probability is zero.)

6 No; the result of the game depends on the relative strengths of each team. Depending on the type of football, a draw might be much less likely than either team winning.

ASSIGNMENT 4B

1 **a** midpoint **b** number line **c** diagonals
d vertices **e** hypotenuse **f** 180°
g 360° **h** [b] $a° = b°$ **i** $a = 60°$
j circumference

2 **a** (1) $3x - 2$ (2) $5x - 4$ (3) $7x + 7$
(4) $4x - 6$ (5) $4x + 10$ (6) $4x - 6$
(7) $2x + 7$ (8) $6x - 6$ (9) $3x + 9$
(10) $6x - 3$ (11) $3x + 13$ (12) $3x - 7$

b (1) $x + 4$ (2) $3x + 8$ (3) $5x - 3$
(4) $2x + 12$ (5) $2x + 6$ (6) $2x + 8$
(7) 9 (8) $4x + 10$ (9) $x - 1$
(10) $4x + 11$ (11) $x + 1$ (12) $x + 13$

c (1) $5x - 7$ (2) $5x + 1$ (3) $7x - 3$
(4) $5x + 7$ (5) $6x + 9$ (6) $4x + 8$
(7) $3x + 3$ (8) $6x - 8$ (9) $4x$
(10) $6x + 11$ (11) $3x + 1$ (12) $4x + 6$

d (1) 6 (2) 20 (3) 16
(4) −64 (5) −16 (6) 4
(7) 20 (8) −32 (9) 96
(10) 224 (11) 8 (12) 1024

3 1 260 000 degrees

4 8 years

5 Estimates should be reasonably close to these.
a 44% **b** 70% **c** 87%

ASSIGNMENT 4C

1 **a** 31 **b** −54 **c** −15

2 **a** \$620 **b** 80 kg

3 **a** 17·5% **b** 16·33%

4 **a** 0·375 **b** $1{\cdot}1\dot{6}$ **c** $\frac{7}{11}$ (or $\frac{63}{99}$)

5 **a** 10 : 9 **b** 200 km/h **c** 2

6 **a** 6 400 000 **b** $9{\cdot}5 \times 10^7$ **c** 2300 kB

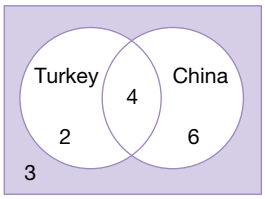
7 **a** $11y$ **b** $8m - 4p$ **c** $7a^2 + a$
d $28a$ **e** $12t^2$ **f** $420ab$

8 **a** $4a$ **b** $4y$ **c** $-\frac{1}{2n}$
d $\frac{2m}{5}$ (or $\frac{4m}{10}$) **e** $\frac{9m}{10}$ **f** $\frac{43a}{30}$
g $\frac{m^2}{20}$ **h** $1\frac{1}{2}$ **i** $\frac{x}{2}$

9 **a** $3x^2 + 7x$ **b** $42y - 12y^2$ **c** $-10m + 10$
d $17m - 39$ **e** $-3a^2$

10 **a** $4(2m + 3)$ **b** $2x(2x - 5)$ **c** $b(12a - 1)$

11 **a**



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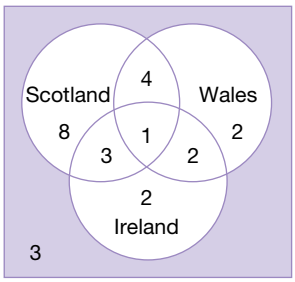
b



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Chapter 5:

Prep quiz 5:01

1 254·4 **2** 2·04 **3** 32·66 **4** *LB* **5** 20
6 20 **7** 20 **8** 48 **9** 100 **10** 10

Exercise 5:01

1 **a** 49 cm^2 **b** 161·29 m^2 **c** 0·5625 m^2

2 **a** 60 m^2 **b** 88·4 cm^2 **c** 202·24 m^2
d 0·75 m^2 or 7500 cm^2
e 1·188 m^2 or 11 880 cm^2
f 1775 mm^2 or 17·75 cm^2

3 **a** 113·10 cm^2 **b** 38·48 cm^2 **c** 21·24 m^2
d 38·48 m^2 **e** 65·04 m^2 **f** 0·64 m^2

4 **a** 1 m^2 **b** 10 000 cm^2 **c** Yes
d 1 m^2 = 10 000 cm^2

5 **a** 10 000 **b** 21 **c** 31·5
d 16 500 m^2 or 1·65 ha

6 **a** 10 **b** 1 cm^2 **c** 100 mm^2
d 1 cm^2 = 100 mm^2

7 **a** 1 km^2 = 1 000 000 m^2
b 1 km^2 = 100 ha

8 **a** 30·6 m^2 **b** 17·94 m^2 **c** 0·546 m^2

9 **a** 35·2 cm^2 **b** 63·7 m^2 **c** 324·72 cm^2

10 **a** 69·6 cm^2 **b** 39·75 m^2 **c** 8·172 m^2

11 **a** 33·92 m^2 **b** 19·536 m^2 **c** 200 cm^2

12 **a** 31·9 cm^2 **b** 154·9 cm^2 **c** 106·2 cm^2

13 **a** 242 cm^2 **b** 81·48 m^2 **c** 45·365 m^2

Challenge 5:01

1 250 000 km^2

2 Columbia 1 100 000 km^2, Venezuela 900 000 km^2, Guyana 200 000 km^2, Surinam 150 000 km^2, French Guiana 100 000 km^2, Brazil 8 500 000 km^2, Uruguay 150 000 km^2, Argentina 2 500 000 km^2, Paraguay 400 000 km^2, Chile 750 000 km^2, Bolivia 1 000 000 km^2, Peru 1 250 000 km^2, Ecuador 250 000 km^2

3 Brazil 4250 km (EW), 4000 km (NS) Argentina 1000 km (EW), 3500 km (NS)
4 Australia ~ 7 700 000 km^2

Prep quiz 5:02

1 triangle 2 rectangle 3 trapezium
4 24 unit^2 5 48 unit^2 6 32 unit^2
7 104 unit^2 8 120 unit^2 9 24 unit^2
10 96 unit^2

Exercise 5:02

1 a i III ii II iii I iv IV
b i 88 unit^2 ii 104 unit^2 iii 64 unit^2
c i 56 unit^2 ii 48 unit^2 iii 40 unit^2
2 a 60 cm^2 b 84 cm^2 c 95 cm^2
3 a 57·1 m^2 b 137·16 m^2 c 106 m^2
4 a 31·08 m^2 b 5·04 m^2 c 158·8 m^2
5 a 33·77 cm^2 b 84·82 cm^2 c 103·07 cm^2
6 a 2632·5 mm^2 b 96·3 m^2 c 168·9 m^2
7 a 311·4m^2 b 115·0 m^2 c 202·4 m^2
8 a 83 cm^2 b 31 cm^2 c 103 cm^2
9 a 24·48 m^2 b 25·52 m^2
c 1100 bricks would be needed
10 73·7 m^2
11 a i 66·4 m^2 ii 7·98 m^2
b 58·42 m^2 c $5257.80
12 14 m^2
13 a 140 m^2 b $14 000
14 a 112 m^2 b 125·44 m^2
c 128 m^2 d $20 806.40
15 $1494

Prep quiz 5:03

1 21 cm^2 2 12 cm^2 3 12 cm^2
4 6 5 squares 6 6
7 rectangles 8 opposite faces are congruent
9 5 10 13

Exercise 5:03

1 a i 150 cm^2 ii cube iii closed
b i 21 cm^2 ii rectangular prism
iii open
c i 27·46 cm^2 ii triangular prism
iii closed
d i 62·86 m^2 ii rectangular prism
iii closed
e i 36 cm^2 ii triangular prism
iii closed
f i 471·96 cm^2 ii triangular prism
iii open
2 Area of top and bottom = 140·4 m^2
Area of sides = 25·2 m^2
Area of front and back = 87·36 m^2
Total surface area = 252·96 m^2
3 a 142 cm^2 b 192 cm^2 c 229 cm^2
4 a 136 cm^2 b 241 cm^2 c 381·72 cm^2
5 a 600 cm^2 b 2350 cm^2 c 14 m^2
6 a x = 5 cm; S.A. = 96 cm^2
b x = 6·24 cm; S.A. = 452·4 cm^2
c x = 6·40 m; S.A. = 50·8 m^2
7 S.A. = 355 m^2; Cost = $21 300
8 253·9 cm^2 (1 dec. pl.)
9 30·53 m^2 (1 dec. pl.)
10 Stack B
11 a 12·92 m^2 b 3·5 m c 16·3 m
12 a 648 m^2 b 312·45 m^2 c 295·65 m^2
13 $45 945
14 a 3392 cm^2 b 2800 cm^2 c 4704 cm^2
d 79·68 cm^2
15 363·1 m^2 (1 dec. pl.)

Prep quiz 5:04

1 8·04 2 372 3 8 cm 4 13 cm
5 6 cm 6 8 cm 7 3·5 cm 8 9 cm
9 5 cm 10 4 cm

Exercise 5:04

1 a 502·65 cm^2 b 376·99 cm^2
c 461·50 cm^2 d 850·05 m^2
2 a 440 cm^2 (to nearest one)
b 176 m^2 c 20·4 m^2 d 917 cm^2
3 a i 1658·76 cm^2 ii 760·27 cm^2 iii 2419·03 cm^2
b i 424·87 cm^2 ii 110·84 cm^2 iii 535·71 cm^2
c i 88·22 m^2 ii 215·03 m^2 iii 303·25 m^2
d i 126·67 cm^2 ii 16·08 cm^2 iii 142·75 cm^2
e i 38·68 m^2 ii 73·49 m^2 iii 112·17 m^2
f i 77·41 m^2 ii 30·41 m^2 iii 107·82 m^2
4 a 565·5 cm^2 b 199·1 m^2 c 161·0 m^2
d 515·2 m^2 e 1321·0 cm^2 f 25·1 m^2
g 6·3 m^2 h 14·7 m^2
5 Curved surface area = 26·4 m^2
6 3338 cm^2 (nearest square cm)
7 Cost = $17 436 (nearest dollar)
8 a 139·9 m^2 b $3497.50
9 a 126 m^2 b $8820
10 a S.A. of 1 L can = 639·3 cm^2;
S.A. of 2 L can = 1011·6 cm^2
b It is cheaper to sell it in one 2 L can.

Exercise 5:05

1 a 46·2 cm^3 b 421·4 cm^3 c 17·2 m^3
2 a 47·94 m^3 b 4880 cm^3 c 60·264 m^3
d 1984·62 cm^3 e 959·4 m^3 f 67·2 cm^3
3 a 263·1 m^3 b 12·0 m^3 c 6588·4 cm^3
4 a 2·67 m^3 b 1·13 m^3 c 8·30 m^3
5 a 22 089 cm^3 b 22 L
6 a 115·2 m^3; 115 200 L b 139·1 m^3; 139 100 L
c 92·4 m^3; 92 400 L
7 a 34·72 m^3 b 41·66 t
8 a 1·2 m^3 b $360

Investigation 5:05

1 **a** 180 L **b** 450 L **c** 160 L

2 The three dimensions (measured in metres) have to have a product of 1. To find some, use fractions and then convert to metres (e.g. $\frac{5}{4} \times \frac{1}{2} \times \frac{8}{5} = 1$). Hence a possible set of dimensions is 1·25 m, 0·5 m, 1·6 m.

Prep quiz 5:06

1 18 cm^2 **2** 24 cm^2 **3** 30 cm^2 **4** 40 cm^2
5 65 m^2 **6** 8 m **7** 520 **8** 320 m^3
9 200 m^3 **10** 520 m^3

Exercise 5:06

1 Vol. A = 8 cubic units; Vol. B = 36 cubic units; Vol. C = 6 cubic units
a 14 $unit^3$ **b** 48 $unit^3$ **c** 20 $unit^3$
d 28 $unit^3$ **e** 30 $unit^3$ **f** 22 $unit^3$

2 **a** 1050 cm^3 **b** 360 cm^3 **c** 308 cm^3

3 **a** 960 cm^3 **b** 48 cm^3 **c** 11·799 cm^3

4 **a** 1760·76 cm^3 **b** 2488·7 cm^3 **c** 201·28 m^3
d 146·08 m^3 **e** 313·95 cm^3

5 **a** 204 cm^3 **b** 581·4 m^3 **c** 537·6 cm^3

6 **a** 256·9 cm^3 **b** 6090·8 cm^3 **c** 1022·2 cm^3

7 **a** 1155 cm^3 **b** 435·6 cm^3 **c** 2293·677 cm^3

8 **a** 4050 cm^3 **b** 6336 cm^3 **c** 1890 cm^3

9 48·4 kL **10** 107 m^3

11 **a** 301·5 cm^3 **b** 3437·1 g

12 **a** 17·7 m^3 **b** 44·25 t

Diagnostic test 5

1 **a** 30 cm^2 **b** 245·4 cm^2 (1 dec. pl.) **c** 48 cm^2

2 **a** 7·14 cm^2 **b** 196·35 cm^2 **c** 58 cm^2

3 **a** 117·14 cm^2 **b** 36 cm^2 **c** 360 cm^2

4 **a** 612·6 cm^2 **b** 47·5 cm^2 **c** 15·8 cm^2

5 **a** 94·875 cm^3 **b** 121·5 cm^3 **c** 437·06 cm^3

6 **a** 286·944 cm^3 **b** 26·832 cm^3 **c** 640·794 cm^3

7 **a** 611·0 cm^3 **b** 66·7 cm^3 **c** 498·6 cm^3

8 **a** 141·589 cm^3 **b** 820·8 cm^3 **c** 271·32 cm^3

9 **a** 2520 cm^3 **b** 768 cm^3 **c** 1302 cm^3

ASSIGNMENT 5A

1 **a** 375 cm^2 **b** 2·64 m^2

2 **a** 943·3 cm^2; open **b** 16 m^2; open

3 216 m^2 **4** A

5 **a** 318·9 cm^2 **b** 23·75 m^2; $1187.50

6 15 375 cm^3 **7** 104·4 cm^2

ASSIGNMENT 5B

1 **a** 336 **b** 1041·6 kg

2 Length = 1150 mm, Width = 690 mm, Height = 648 mm; Volume = 0·51 m^3 (2 dec. pl.)

3 Rachel is 13 years old.

4 34

5 4 times, 3 times

6 **a** 10% **b** $33\frac{11}{18}$% **c** 23° **d** 62·5%

ASSIGNMENT 5C

1 **a** $8x + 7$ **b** $-4x + 13$ **c** $-4x + 8$

2 **a** $5(x + 3)$ **b** $5(x + 3y)$ **c** $5x(1 - 3y)$
d $5x(5 + 3x)$

3 6 m is between 5·5 m and 6·5 m; 6·0 m is between 5·95 m and 6·05 m; 6·00 m is between 5·995 m and 6·005 m.

4 **a** 22·5 **b** 22·51 **c** 20 **d** 22·5

5 **a** $\frac{1}{2}$ **b** $\frac{9}{20}$ **c** $\frac{1}{5}$ **d** $\frac{3}{4}$ **e** $\frac{1}{4}$

6 **a** **i** 57% **ii** 36% **iii** 7%
b It would be less than 1%
c To improve the accuracy you could use a much larger number of trials than 100.

Chapter 6

Exercise 6:01

1 **a** 2^4 **b** 3^2 **c** 5^3 **d** 7^5 **e** 10^3
f 9^4 **g** x^2 **h** a^4 **i** n^3 **j** m^5
k p^6 **l** y^2 **m** 4^3 **n** t^4 **o** x^5

2 **a** $2 \times 2 \times 2$ **b** 4×4
c $6 \times 6 \times 6 \times 6 \times 6$ **d** $10 \times 10 \times 10 \times 10$
e $7 \times 7 \times 7$ **f** 3
g $a \times a \times a$ **h** $x \times x \times x \times x \times x$
i $y \times y$ **j** $m \times m \times m \times m \times m$
k $n \times n \times n \times n \times n \times n \times n$
l $p \times p \times p$

3 **a** 32 **b** 81 **c** 49
d 121 **e** 10 000 **f** 216
g 256 **h** 78 125 **i** 32 768
j 59 049 **k** 16 777 216 **l** 14 348 907
m 8000 **n** 27 783 **o** 455 625
p 592 704

4 **a** 10^3 **b** 10^5 **c** 10^6 **d** 10^9

5 **a** 2^6 **b** 2^8 **c** 2^{14} **d** 2^{20}

6 **a** 3^3 **b** 3^5 **c** 3^{10} **d** 3^{14}

7 **a** $2^2 \times 3^2$ **b** $2^3 \times 5^2$ **c** $2^2 \times 7^2$ **d** $2^4 \times 5$

8 **a** $x = 2$ **b** $x = 3$ **c** $x = 5$ **d** $x = 4$

9 2 147 483 648 **10** less

Challenge 6:01

1 8

2 20. No, the names of some people would be repeated.

3 The estimate should be less than 2 000 000 and probably less than 1 000 000.

4 Answers will vary but at least 12 generations would be needed and perhaps many more.

Prep quiz 6:02

1 5^4 **2** a^3 **3** $2 \times 2 \times 2 \times 2$
4 $m \times m \times m \times m \times m$
5 4 **6** 8 **7** 32
8 27 **9** 3 **10** 1

Exercise 6:02

1

Power	Answer
11^6	1 771 561
6^8	1 679 616
8^{10}	1 073 741 824
5^6	15 625
8^7	2 097 152
7^8	5 764 801
5^4	625
3^5	243

Expression	Answer
$7^3 \times 7^5$	5 764 801
$5^8 \div 5^2$	15 625
$(11^3)^2$	1 771 561
56^0	1
$3^{10} \div 3^5$	243
$8^2 \times 8^5$	2 097 152
$(13)^0$	1
$(6^2)^4$	1 679 616

2 **a** true **b** false **c** true **d** true **e** true
f true **g** true **h** false **i** true

3 **a** 10^5 **b** 10^3 **c** 10^6 **d** 5^6 **e** 2^5
f 7^6 **g** 3^9 **h** 2^8 **i** 10^{10}

4 **a** 10^1 **b** 10^3 **c** 10^1 **d** 5^4 **e** 7^3
f 3^9 **g** 2^1 **h** 5^0 or 1 **i** 2^6

5 **a** 10^6 **b** 10^9 **c** 10^{12} **d** 2^{12}
e 2^4 **f** 2^{35} **g** 3^8 **h** 5^{15}
i 7^8 **j** $2^6 \times 3^4$ **k** $7^6 \times 11^8$ **l** $3^4 \times 2^8$

6 **a** x^5 **b** y^6 **c** m^6 **d** m^5 **e** p^{10}
f a^2 **g** y^7 **h** x^3 **i** m^7 **j** $3y^5$
k $3m^6$ **l** $15x^3$

7 **a** x^3 **b** x^4 **c** x^5 **d** m^2 **e** y^3
f m^2 **g** 1 **h** 1 **i** y^6 **j** $2m^6$
k $2y^8$ **l** $4x^5$

8 **a** x^6 **b** y^8 **c** a^{15} **d** 1 **e** 1
f 1 **g** y^6 **h** a^{18} **i** x^9 **j** $8x^3$
k $9x^4$ **l** $625m^8$

9 **a** $8x^7$ **b** $5a^3$ **c** $4m^{10}$ **d** $8x$ **e** $5a$
f $4m^2$ **g** $50y^4$ **h** $32m^4$ **i** $32a^9$ **j** $2y^2$
k 8 **l** $2a$ **m** $72x^8$ **n** $27a^9$ **o** $108y^7$
p $2x^2$ **q** $3a^5$ **r** $3y^5$ **s** $3a^2$ **t** $2x^5$
u $2a$

10 **a** 6 **b** 6 **c** 1 **d** a
e y^3 **f** m^5 **g** $8m^6$ **h** $16n^6$
i $16p^{12}$ **j** x^5y^3 **k** a^2b^9 **l** x^5y^3
m x^3y^4 **n** a^5b^2 **o** m^3n^4 **p** x^4y^6
q $a^2b^2c^2$ **r** p^3q^9 **s** $10x^3y^2$ **t** $28a^3b^5$
u $44a^5b^2$ **v** $3a^4$ **w** $-24a^3$ **x** $2c$
y $22x^2 - 5x^3$ **z** $12x^2 + 7x + 1$

11 **a** $30x^6$ **b** $40a^3$ **c** $5xy^4$ **d** $\frac{x^3}{2}$
e $\frac{y}{7}$ **f** $\frac{10}{x}$ **g** x^8 **h** a^{13}
i y^{26} **j** a^2 **k** m^2 **l** n^2
m y^{19} **n** $8a^{18}$ **o** b^2 **p** x^2
q $2a^2$ **r** $7pq^2$ **s** $2x^5$ **t** $6x^6$
u $\frac{xy^3}{16}$

Exercise 6:03

1 **a** $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{1}{2}$ **d** $\frac{1}{36}$ **e** $\frac{1}{16}$
f $\frac{1}{1000}$ **g** $\frac{1}{16}$ **h** $\frac{1}{10\,000}$ **i** $\frac{1}{25}$

2 **a** 11^{-1} **b** 3^{-1} **c** 5^{-1} **d** 7^{-1} **e** 3^{-3}
f 5^{-4} **g** 2^{-8} **h** 7^{-2} **i** 10^{-2} **j** 10^{-3}
k 10^{-6} **l** 10^{-5}

3 **a** true **b** false **c** true **d** false **e** false
f false **g** true **h** false

4 **a** 10^{-3} **b** 10^1 **c** 10^{-5} **d** 10^{-3} **e** 10^{-1}
f 10^{-6} **g** 10^6 **h** 10^{-4}

5 **a** $\frac{1}{a}$ **b** $\frac{1}{x}$ **c** $\frac{1}{m}$ **d** $\frac{1}{y}$ **e** $\frac{1}{x^3}$
f $\frac{1}{y^2}$ **g** $\frac{1}{x^4}$ **h** $\frac{1}{m^6}$ **i** $\frac{2}{x}$ **j** $\frac{5}{a^3}$
k $\frac{10}{y^2}$ **l** $\frac{36}{q^4}$

6 **a** x^{-1} **b** x^{-2} **c** x^{-3} **d** x^{-4} **e** $5y^{-2}$
f $3a^{-1}$ **g** $10m^{-4}$ **h** $75x^{-3}$ **i** xy^{-2} **j** ma^{-3}
k $3ab^{-2}$ **l** $4xy^{-1}$

7 **a** 2 **b** 3 **c** $1\frac{1}{2}$ **d** 10
e 4 **f** 9 **g** $2\frac{1}{4}$ **h** $11\frac{1}{9}$

8 **a** $\frac{1}{x^2}$ **b** $\frac{1}{a^5}$ **c** $\frac{3}{x}$
d $\frac{5}{m^2}$ **e** $\frac{1}{(x+1)^2}$ **f** $\frac{1}{(3+a)^1}$
g $\frac{1}{(6x)^2}$ or $\frac{1}{36x^2}$ **h** $\frac{4}{(x+2)^1}$ or $\frac{4}{x+2}$

9 **a** 0·125 **b** 0·0625 **c** 0·04
d 0·015 625 **e** 0·015 625 **f** 0·015 625
g 8 **h** 25 **i** 400
j 0·064 **k** 100 000 **l** 2·56

10 **a** x **b** a^3 **c** m^3
d 1 **e** $3a$ **f** $30x^2$
g $5a$ **h** $30m^2$ **i** $\frac{1}{x^3}$
j $\frac{2}{a^5}$ **k** $\frac{8}{y}$ **l** $\frac{30}{m^5}$

11 **a** m^5 **b** x^4 **c** y^2
d x^4 **e** $\frac{1}{a^4}$ **f** $\frac{1}{y^4}$
g $\frac{1}{y^3}$ **h** $\frac{1}{x^2}$ **i** $3x^3$
j $\frac{2}{a^4}$ **k** $\frac{24}{a^5}$ **l** $2n$

12 **a** a^6 **b** $\frac{1}{x^2}$ **c** $\frac{1}{y^6}$
d m^4 **e** $\frac{1}{2x^2}$ **f** $\frac{1}{9x^2}$

g $\frac{25}{x^2}$ h $\frac{49}{x^4}$ i $\frac{1}{abc}$

j $\frac{1}{a^2b^2c^2}$ k $\frac{1}{2a^2b}$ l $\frac{2}{a^2b}$

13 a $\frac{5}{6}$ or $0{\cdot}8\dot{3}$ b $\frac{1}{6}$ or $0{\cdot}1\dot{6}$ c 1

d $\frac{1}{12}$ or $0{\cdot}08\dot{3}$

Investigation 6:03

1

Power of 10	Answer
10^4	10 000
10^3	1000
10^2	100
10^1	10
10^0	1
10^{-1}	$\frac{1}{10}$
10^{-2}	$\frac{1}{100}$
10^{-3}	$\frac{1}{1000}$
10^{-4}	$\frac{1}{10000}$

Power of 5	Answer
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	$\frac{1}{5}$
5^{-2}	$\frac{1}{25}$
5^{-3}	$\frac{1}{125}$
5^{-4}	$\frac{1}{625}$

- Yes
- Yes

2 a true b true c true d true e true
f true g true h true i true

Investigation 6:04

1 a 18 b 180 c 1800
d 40·5 e 405 f 4050
g 62 000 h 620 000 i 6 200 000
j 314·16 k 3141·6
l 31 416. To multiply by 10^n move the decimal point n places to the right.

2 a 0·18 b 0·018 c 0·0018 d 9·685
e 0·9685
f 0·096 85. To divide by 10^n move the decimal point n places to the left.

Exercise 6:04

1 a $2 \times 10^4 = 2 \times 10 \times 10 \times 10 \times 10 = 20\,000$,
$2^4 = 2 \times 2 \times 2 \times 2 = 16$

b $5 \times 10^{-2} = 5 \times \frac{1}{10^2} = 5 \times \frac{1}{100} = \frac{5}{100}$,
$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

c $1{\cdot}577\,88 \times 10^{12}$ or 1 577 800 000 000 (taking 1 year = 365·25 days). However, since years that are multiples of 100 are not leap years unless they are multiples of 400, the answer could be 1 577 847 600 000 or $1{\cdot}577\,847\,6 \times 10^{12}$

d Yes (if you are older than 9 years 4 months 8 days)

e $9{\cdot}8 \times 10^{-5}$, 0·0034, $5{\cdot}6 \times 10^{-2}$, 2·04, 6, $5{\cdot}499 \times 10^2$, $3{\cdot}24 \times 10^3$, $1{\cdot}2 \times 10^4$

f $7{\cdot}6 \times 10^{-3}$ cm

g Answers may vary. It would be about 0·5 mm, or 5×10^{-1} mm, or 5×10^{-2} cm.

2 a 21 b 0·21 c 0·21
d 704 e 0·0704 f 0·0704
g 1375 h 0·001 375 i 0·001 375

3 a $4{\cdot}7 \times 10^2$ b $2{\cdot}6 \times 10^3$ c $5{\cdot}3 \times 10^4$
d 7×10^2 e 5×10^4 f 7×10^5
g $6{\cdot}5 \times 10^1$ h $3{\cdot}42 \times 10^2$ i 9×10^1
j $4{\cdot}97 \times 10^3$ k $6{\cdot}35 \times 10^4$ l $2{\cdot}941 \times 10^6$
m $2{\cdot}971 \times 10^2$ n $6{\cdot}93 \times 10^1$ o $4{\cdot}9765 \times 10^3$
p $9{\cdot}31 \times 10^6$ q $6{\cdot}7 \times 10^7$ r $1{\cdot}901 \times 10^5$
s 6×10^5 t $5{\cdot}017 \times 10^5$ u 1×10^5

4 a $7{\cdot}5 \times 10^{-2}$ b $6{\cdot}3 \times 10^{-3}$ c $5{\cdot}9 \times 10^{-1}$
d 8×10^{-2} e 3×10^{-4} f 9×10^{-3}
g 3×10^{-1} h $3{\cdot}01 \times 10^{-2}$ i $5{\cdot}29 \times 10^{-4}$
j $4{\cdot}26 \times 10^{-1}$ k 1×10^{-3} l $9{\cdot}7 \times 10^{-6}$
m 6×10^{-5} n $9{\cdot}07 \times 10^{-4}$ o 4×10^{-9}

5 a 230 b 94 000 c 3700
d 295 e 87·4 f 763 000
g 1075 h 20 000 i 80
j 0·029 k 0·0019 i 0·95
m 0·003 76 n 0·000 463 o 0·0107
p 0·07 q 0·80 r 0·000 005
s 973 000 t 0·0063 u 47 000 000
v 914·2 w 0·010 32 x 100 000 000

Prep quiz 6:05

1 $6{\cdot}9 \times 10^2$ 2 4×10^3 3 $9{\cdot}632 \times 10^2$
4 $7{\cdot}3 \times 10^{-2}$ 5 3×10^{-4} 6 2900
7 800 000 8 0·046 9 0·000 000 5
10 0·814

Exercise 6:05

1 a 63 000 b 0·0014 c 92 500 000

2 a $5{\cdot}6 \times 10^4$ b $4{\cdot}3 \times 10^7$ c $7{\cdot}63 \times 10^5$
d $4{\cdot}0 \times 10^{-4}$ e $2{\cdot}9 \times 10^{-6}$ f $7{\cdot}31 \times 10^{-5}$
g $1{\cdot}5 \times 10^3$ h $2{\cdot}78 \times 10^{-3}$
i $6{\cdot}09 \times 10^6$
j Explanation: $2 \times 10^4 = 20\,000$ whereas $2^4 = 2 \times 2 \times 2 \times 2 = 16$

3 $2{\cdot}9 \times 10^{-6}$, $7{\cdot}31 \times 10^{-5}$, $4{\cdot}0 \times 10^{-4}$, $2{\cdot}78 \times 10^{-3}$, $1{\cdot}5 \times 10^3$, $5{\cdot}6 \times 10^4$, $7{\cdot}63 \times 10^5$, $6{\cdot}09 \times 10^6$, $4{\cdot}3 \times 10^7$

4 a $2{\cdot}1160 \times 10^{14}$ b $5{\cdot}6689 \times 10^{-12}$
c $1{\cdot}6807 \times 10^{-16}$ d $7{\cdot}1538 \times 10^{11}$
e $1{\cdot}6687 \times 10^{14}$ f $1{\cdot}3158 \times 10^{-12}$
g $3{\cdot}9366 \times 10^{12}$ h $4{\cdot}0459 \times 10^{19}$

5 a 318 600 b 0·006 626
c 0·2442 d 0·000 014 44
e 0·008 424 f 771 000
g 86 310 h 0·004 498
i 0·000 188 7

6 a $1{\cdot}394 \times 10^6$, 12 800 km or $1{\cdot}28 \times 10^4$ km
b 5 000 000 km or 5×10^6 km
c 8 min d 2×10^{27} tonnes
e 2×10^{38}

Prep quiz 6:06

1 4000 GB 2 7000 MB 3 6000 kB
4 9000 B 5 2000 mm 6 3000 μm
7 5000 nm 8 4 000 000 m 9 8 000 000 μm
10 7 000 000 000 nm

Exercise 6:06

1 a 2 b 4 c 3 d 1
2 a 8×10^3 B b 6×10^3 B
c 4×10^6 B d 8×10^9 B
e $4{\cdot}7 \times 10^{12}$ B f $2{\cdot}4 \times 10^6$ B
g $6{\cdot}15 \times 10^3$ B h $8{\cdot}0 \times 10^9$ B
i $9{\cdot}2 \times 10^{12}$ B j $7{\cdot}70 \times 10^6$ B
k $5{\cdot}15 \times 10^3$ B i $8{\cdot}0 \times 10^9$ B
m $1{\cdot}5 \times 10^{13}$ B n $1{\cdot}03 \times 10^8$ B
o $8{\cdot}52 \times 10^5$ B p $6{\cdot}05 \times 10^{11}$ B
q $8{\cdot}00 \times 10^7$ B r $4{\cdot}0 \times 10^{12}$ B
3 a 8×10^{-3} s b 7×10^{-6} s
c 2×10^{-9} s d 9×10^{-3} s
e $3{\cdot}60 \times 10^{-3}$ s f $8{\cdot}4 \times 10^{-6}$ s
g $7{\cdot}0 \times 10^{-9}$ s h $9{\cdot}56 \times 10^{-3}$ s
i $1{\cdot}25 \times 10^{-9}$ s j $7{\cdot}7 \times 10^{-6}$ s
k $8{\cdot}125 \times 10^{-3}$ s i $8{\cdot}0 \times 10^{-6}$ s
m $1{\cdot}5 \times 10^{-2}$ s n $1{\cdot}03 \times 10^{-4}$ s
o $1{\cdot}852 \times 10^{-6}$ s p $6{\cdot}05 \times 10^{-4}$ s
q $8{\cdot}0 \times 10^{-5}$ s r $2{\cdot}00 \times 10^{-5}$ s
4 Warragamba Dam's capacity is $2{\cdot}031 \times 10^6$ ML or $2{\cdot}031 \times 10^3$ GL or 2·031 TL.
5 a 10^9 or 1 000 000 000
b 10^6 or 1 000 000 c 10^3 or 1000
6 a 10^6 kHz or 1 000 000 kHz
b 10^3 MHz or 1000 MHz
7 73 740 000 bytes

Diagnostic test 6

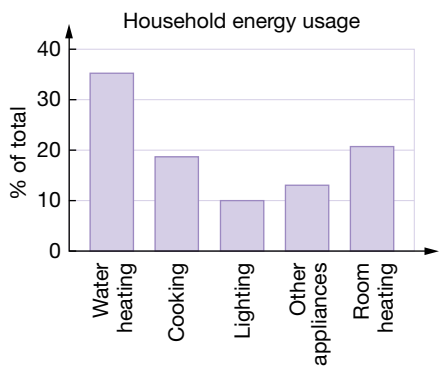
1 a 3^4 b 5^2 c m^3
2 a 9 b 16 c 1000
3 a 3^7 b x^5 c $6m^3n^5$
4 a x^5 b $5a^3$ c $2a^2b$
5 a a^8 b x^{12} c $8a^{12}$
6 a 1 b 5 c 3
7 a $\frac{1}{9}$ b $\frac{1}{5}$ c $\frac{27}{8}$ or $3\frac{3}{8}$
8 a x^4 b $\frac{2}{x^2}$ c $\frac{9}{x^2}$
9 a $2{\cdot}43 \times 10^2$ b $6{\cdot}7 \times 10^4$ c $9{\cdot}38 \times 10^7$
10 a 130 b 243·1 c 46 300 000
11 a $4{\cdot}3 \times 10^{-2}$ b $5{\cdot}97 \times 10^{-5}$ c 4×10^3
12 a 0·029 b 0·000 093 8 c 0·001 004
13 a $9{\cdot}61 \times 10^{16}$ b $4{\cdot}64 \times 10^7$ c $1{\cdot}4 \times 10^{12}$
d $1{\cdot}2 \times 10^{-3}$
14 a $2{\cdot}7 \times 10^5$ b $2{\cdot}5 \times 10^4$ c $4{\cdot}3 \times 10^{12}$
d $1{\cdot}2 \times 10^{10}$
15 a 5×10^6 b $5{\cdot}0 \times 10^5$ c 3×10^{-9}
d 5×10^{-6}

ASSIGNMENT 6A

1 a a^5 b $12a^5$ c a^3b^2 d $12a^3b^3$ e 3^5
f a^3 g $7m$ h $4y^4$ i $2a^2b$ j 4^4
k 3^8 l x^6 m a^{11} n m o $2x$
p $\frac{a^2}{2}$
2 a 1 b 6 c $125x^9$
d $1000a^6$ e $8x$
3 a $2{\cdot}16 \times 10^4$ b $1{\cdot}25 \times 10^2$ c $7{\cdot}0 \times 10^{-5}$
d $1{\cdot}56 \times 10^{-4}$
4 a 810 000 b 1267 c 0·035
d 0·000 106
5 a 1024 b 531 441 c 145 800 000
d 351 232
6 a 7 b 5 c 8
7 a $3^7 = 2187$ b $10^3 = 1000$ c $2^8 = 256$
8 a m^3 b $72a^{17}$ c 2
9 a $\frac{1}{5}$ b $\frac{1}{16}$ c $\frac{1}{9}$ d $\frac{1}{1000}$
10 a $1{\cdot}3 \times 10^7$ b $5{\cdot}6 \times 10^{-22}$ c $1{\cdot}2 \times 10^{17}$
d $2{\cdot}6 \times 10^{11}$ e $1{\cdot}3 \times 10^{13}$ f $2{\cdot}4 \times 10^7$
g $4{\cdot}1 \times 10^1$ h $2{\cdot}2 \times 10^7$

ASSIGNMENT 6B

1 a *hectare* b *cross-section*
c *coordinates* d *tally*
e *picture* graph f *column* graph
g *line* graph h *sector* (or *pie*) graph
i bar graph j scatter diagram
2 (1) *parallel* lines (2) *perpendicular* lines
(3) vertical, horizontal (4) concurrent lines
(5) *angle ABC* or *CBA* (6) *acute* angle
(7) *right* angle (8) *obtuse* angle
(9) *straight* angle (10) *reflex* angle
(11) *revolution* (12) *adjacent* angles
3 a 3, 6
b i 20 ii 35 iii 405
4 a 35%
b cooking 66°, lighting 39°, other appliances 50°, room heating 79°
c



5 $31.63
6 a 1900, ten shillings b 15·7
c Inflation has caused the value of the dollar to drop.

d Australia became a nation in 1901. Before this the prices may have been kept high by British taxes or the high cost of imports. There could have been many reasons.

e i 52c (or 52%)
ii 18c (or 18%) approx.
iii 48c (or 48%)

ASSIGNMENT 6C

1 **a** $\frac{4}{5}$ **b** $\frac{1}{8}$ **c** $\frac{1}{500}$ **d** $\frac{5}{9}$
e $\frac{5}{33}$ **f** $\frac{31}{45}$

2 **a** 12540 **b** 32%

3 **a** $a + 6$ **b** a **c** $3a$ **d** $a - 6$

4 **a** $2a$ **b** 2 **c** $2b$ **d** $\frac{a}{2}$

5 **a** $\frac{1}{10}$ **b** $\frac{3}{10}$ **c** $\frac{7}{10}$

6 **a** $40\,\text{cm}^2$ **b** $48\,\text{cm}^2$ **c** $60\,\text{cm}^2$

Chapter 7

Prep quiz 7:01

1 2 **2** 3 **3** 12 **4** 5
5 $\frac{x}{6}$ **6** $5y$ **7** $\frac{y}{2}$ or $y \div 2$
8 $p \div 4$ or $\frac{1}{4}p$ **9** 12 **10** 4

Exercise 7:01

1 **a** 5 **b** 13 **c** 4 **d** 6 **e** 6
f 6 **g** 2 **h** 30 **i** 6 **j** 10
k 4 **l** 20 **m** 18 **n** 20 **o** 2
p 3

2 **a** 11 **b** 27 **c** 14 **d** 7 **e** 1
f 0 **g** 15 **h** $1\frac{1}{2}$ **i** 8 **j** 12
k 4 **l** 8 **m** 24 **n** 14 **o** 15
p 7

3 **a** correct **b** incorrect **c** incorrect
d correct **e** correct **f** incorrect
g incorrect **h** incorrect **i** correct
j incorrect **k** incorrect **l** correct
m correct **n** incorrect **o** incorrect
p incorrect

4 **a** 26 **b** −2 **c** 16 **d** 23 **e** 32
f 2 **g** 69 **h** 14

5 **a** A **b** F **c** D **d** C **e** E
f G **g** H **h** B

Prep quiz 7:02

1 $x + 7$ **2** $4c$ **3** $\frac{y}{3}$
4 $a - 9$ **5** $4m + 8$ **6** $4(x - 5)$
7 subtracting 6 **8** adding 11 **9** dividing by 3
10 multiplying by 7

Exercise 7:02

1 **a** ×5 **b** +7 **c** −4
d ÷3 **e** ×4; +1 **f** −8; ×3
g × −7; +2 **h** +3; ÷10 **i** ÷7, +6
j +9; × −6

2 **a** ÷6 **b** ×2 **c** −9
d +4 **e** +7; ÷8 **f** −2; ÷9
g −4; ÷ −5 **h** +1; ×4 **i** ×4; −2
j −5; ×4

3 **a** $m \xrightarrow{\times 7} 7m$

b $y \xrightarrow{\times 3} 3y \xrightarrow{-5} 3y - 5$

c $a \xrightarrow{\times 8} 8a \xrightarrow{+1} 8a + 1$

d $a \xrightarrow{\times(-3)} -3a \xrightarrow{+2} -3a + 2$

e $p \xrightarrow{+4} p + 4 \xrightarrow{\times 5} 5(p + 4)$

f $m \xrightarrow{\times 4} 4m \xrightarrow{+9} 9 + 4m$

g $x \xrightarrow{-7} x - 7 \xrightarrow{\times 10} 10(x - 7)$

h $c \xrightarrow{\times(-5)} -5c \xrightarrow{+7} 7 - 5c$

i $x \xrightarrow{\div 4} \frac{x}{4} \xrightarrow{+1} \frac{x}{4} + 1$

j $m \xrightarrow{+6} m + 6 \xrightarrow{\div 2} \frac{m+6}{2}$

k $a \xrightarrow{-8} a - 8 \xrightarrow{\div 7} \frac{a-8}{7}$

l $c \xrightarrow{\div 3} \frac{c}{3} \xrightarrow{+6} 6 + \frac{c}{3}$

4 **a** $x + 4$ **b** $5m$ **c** $\frac{t}{2}$
d $y - 4$ **e** $2a$; $2a + 1$ **f** $x + 2$; $9(x + 2)$
g $-6n$; $-6n - 7$ **h** y **i** a
j m **k** x **l** $7x$, x
m $a + 3$; a **n** $-3n$; n

5 **a** ×2; +7 **b** ×(−6), −2 **c** ×9; −1
d −6; ×4 **e** +5; ×7 **f** +4; ×3

6 **a** −10; ÷3 **b** −15; ÷[−11] **c** +200; ÷20
d ×3 **e** −6; ×5 **f** ×4; −3

7 **a** ×4; +1; ÷6 **b** ÷10; −2; ÷7
c ×3; −7; ÷[−2] **d** ÷8; −4; ÷[−5]
e +4; ×3; ÷5 **f** −6; ×11; ÷10
g ×10; ÷7; −4; ÷[−3] **h** ×5; ÷6; +3; ÷2

Prep quiz 7:03

1 x **2** x
3 a **4** m
5 dividing by 7 **6** subtracting 8
7 adding 1 **8** multiplying by 4
9 −10; ÷7 **10** +2; ÷3

Exercise 7:03

1 **a** 66 **b** 31 **c** 73 **d** 252 **e** 301
f 151 **g** 841 **h** 505 **i** 192 **j** 280
k 644 **l** 650 **m** $9\frac{3}{5}$ **n** $8\frac{2}{7}$ **o** $20\frac{2}{9}$

2 **a** correct **b** incorrect **c** incorrect
d correct **e** correct **f** correct
g correct **h** correct **i** correct

3 **a** 5 **b** 10 **c** 4 **d** 9 **e** 9
f 100 **g** 3 **h** 5 **i** 1 **j** 0
k 0 **l** 4 **m** 8 **n** 7 **o** 2
p 13 **q** −1 **r** 1

4 **a** $\frac{3}{4}$ **b** $\frac{5}{8}$ **c** $\frac{1}{2}$ **d** $\frac{6}{7}$ **e** $1\frac{2}{3}$
f $1\frac{1}{2}$ **g** $1\frac{4}{5}$ **h** $\frac{2}{3}$ **i** $2\frac{1}{3}$ **j** $2\frac{2}{3}$
k $\frac{1}{3}$ **l** 0 **m** $-\frac{3}{4}$ **n** $-\frac{3}{5}$ **o** $-2\frac{1}{2}$
p $2\frac{1}{3}$

5 **a** 5 **b** 15 **c** 8 **d** 12 **e** 12
f −1 **g** −2 **h** 2 **i** −2 **j** $1\frac{2}{3}$
k $\frac{2}{3}$ **l** −3 **m** −3 **n** −3 **o** 200
p $-\frac{3}{2}$

Prep quiz 7:04

1 x **2** $10a$ **3** 0 **4** 0 **5** −9
6 +2 **7** ×10 **8** 14 **9** 4 **10** 7

Exercise 7:04

1 **a** 7 **b** 7 **c** 1 **d** 10 **e** 8
f 6 **g** 6 **h** 1 **i** 2 **j** 12
k 6 **l** −4 **m** 4 **n** 5 **o** 1
p 2 **q** 2 **r** 2 **s** 9 **t** 2
u no solution **v** 1 **w** 2 **x** 5

2 **a** correct **b** incorrect **c** correct
d correct **e** correct **f** incorrect
g correct **h** correct **i** correct
j correct **k** correct **l** correct

3 **a** −2 **b** $1\frac{1}{2}$ **c** $1\frac{1}{3}$ **d** $\frac{1}{4}$ **e** $-\frac{1}{3}$
f 0 **g** $1\frac{1}{6}$ **h** −5 **i** $4\frac{1}{2}$ **j** $2\frac{1}{2}$
k $\frac{5}{6}$ **l** −1 **m** −4 **n** $2\frac{1}{6}$ **o** $2\frac{2}{3}$
p $\frac{3}{4}$ **q** $1\frac{4}{7}$ **r** $-\frac{4}{3}$ **s** −20 **t** $\frac{2}{3}$
u $4\frac{1}{3}$

Prep quiz 7:05

1 $7x + 28$ **2** $2a - 6$ **3** $20a + 45$
4 $12p - 42$ **5** $-3x + 12$ **6** −5
7 6 **8** −9 **9** $\frac{1}{2}$ **10** −7

Exercise 7:05

1 **a** 4 **b** 1 **c** 3 **d** 5 **e** 6
f 13 **g** 2 **h** 5 **i** −1 **j** 4
k 2 **l** 3 **m** 3 **n** 8 **o** 6
p 8 **q** 1 **r** 4

2 **a** $\frac{3}{5}$ **b** $1\frac{1}{2}$ **c** $7\frac{3}{4}$ **d** −1 **e** 4
f 4 **g** $\frac{1}{2}$ **h** $1\frac{7}{10}$ **i** $1\frac{1}{3}$ **j** 2
k −2 **l** 25 **m** $\frac{5}{7}$ **n** $5\frac{1}{2}$ **o** 3
p $1\frac{1}{5}$ **q** 3 **r** −12

3 **a** −4 **b** 13 **c** 18 **d** 4 **e** 4
f 6 **g** 2 **h** 2 **i** 7 **j** $4\frac{1}{2}$
k 3 **l** $2\frac{2}{7}$ **m** 2 **n** −5

4 **a** 1 **b** 1 **c** 1 **d** 2 **e** 1
f 2 **g** 2 **h** 2 **i** 2 **j** −46

5 **a** 2 **b** 3 **c** 2 **d** 2 **e** 1
f 2 **g** $-1\frac{3}{8}$ **h** $3\frac{1}{3}$

Prep quiz 7:06

1 0 **2** 3 **3** −3 **4** $3\frac{3}{4}$ **5** $-3\frac{1}{5}$
6 p **7** 12 **8** $x + 7$ **9** $5m$ **10** $12 - p$

Exercise 7:06

1 **a** 24 **b** 20 **c** 36 **d** 15 **e** 12
f 28 **g** 50 **h** 18 **i** $1\frac{3}{5}$ **j** 8
k 9 **l** $11\frac{2}{3}$ **m** 29 **n** 6 **o** 1
p 3 **q** 2 **r** 6 **s** $2\frac{3}{4}$ **t** $1\frac{1}{3}$
u −2 **v** −22 **w** $17\frac{2}{3}$

2 **a** correct **b** incorrect **c** correct
d correct **e** incorrect **f** incorrect
g correct **h** correct **i** correct
j incorrect **k** correct **l** incorrect

3 **a** −12 **b** $\frac{15}{9}$ **c** 10 **d** $\frac{8}{3}$ **e** $\frac{12}{5}$
f 10 **g** 3 **h** $-\frac{1}{7}$ **i** $\frac{3}{10}$ **j** $\frac{1}{9}$
k −1 **l** $\frac{9}{7}$

Prep quiz 7:07

1 2, 4, 6 **2** 5, 10, 15 **3** 6
4 60 **5** $3x$ **6** a
7 $15x$ **8** $2x + 1$ **9** $4x - 2$
10 $6x - 3$

Exercise 7:07

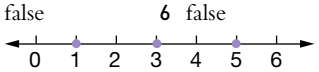
1 **a** 6 **b** 12 **c** 48 **d** 12 **e** 20
f 30 **g** 8 **h** 15 **i** 6 **j** 30
k 12 **l** 5 **m** 24 **n** 20 **o** 20
p 5

2 **a** −9 **b** 8 **c** 9 **d** $-4\frac{3}{5}$ **e** $7\frac{1}{2}$
f $-3\frac{1}{2}$ **g** $-4\frac{3}{5}$ **h** $1\frac{1}{2}$ **l** $\frac{2}{7}$ **j** 1
k −21 **l** 10

Challenge 7:07

1 2·6, −5·6
2 **a** 4, 10 **b** 4·5, −2·8 **c** 3·1, −11·1

Prep quiz 7:08

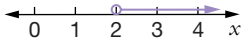
1 −6 **2** 12 **3** −2
4 2 **5** false **6** false
7 'greater than' **8** 

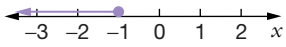
9 −2 > −4 **10** −4 < 6

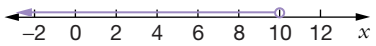
Exercise 7:08

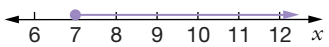
1 **a** $x \leq 0$ **b** $x \geq -3$ **c** $x \leq -2$
d $x > -2$ **e** $x > 3$ **f** $x < 1$
g $x > 5$ **h** $x = -1$ **i** $x < -1$
j $x \leq 0$ **k** $x > \frac{1}{2}$ **l** $x < 4\frac{1}{2}$

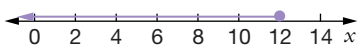
2 **a** $x > 3$ **b** $x \leq 7$ **c** $a \leq 0$
d $a > 0$ **e** $m < 28$ **f** $m < 26$
g $x \geq -6$ **h** $x \geq -2$

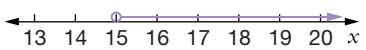
3 **a**


b


c


d


e


f


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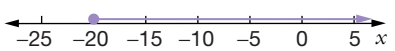
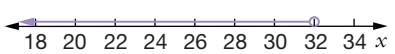


h


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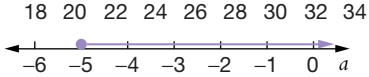

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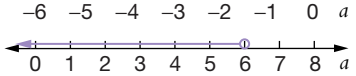
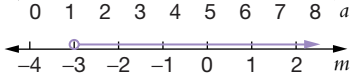
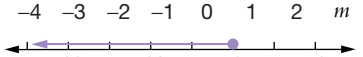


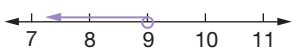
k


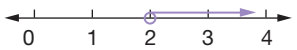
l


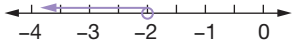
Prep quiz 7:09

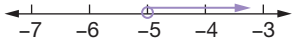
1 yes **2** yes **3** yes **4** yes
5 yes **6** yes **7** yes **8** yes
9 < **10** >

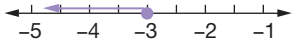
Exercise 7:09

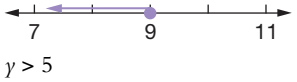
1 **a** $x < 9$


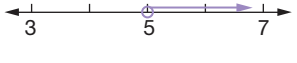
b $y > 2$


c $m < -2$


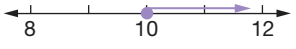
d $m > -5$


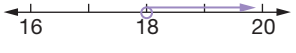
e $p \leq -3$


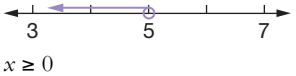
f $m \leq 9$


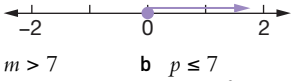
g $y > 5$


h $m < 7$

i $x \geq 10$


j $m > 18$


k $p < 5$


l $x \geq 0$


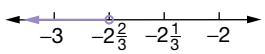
2 **a** $m > 7$ **b** $p \leq 7$ **c** $m < 2$
d $y \geq -8\frac{2}{5}$ **e** $x < 3\frac{3}{4}$ **f** $x > -2$
g $x < 15$ **h** $y > 12$ **i** $m \geq 4$
j $x < -50$ **k** $x < 6$ **l** $x > -12$

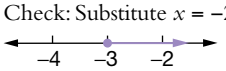
3 **a** $m > -8$ **b** $x < -4$ **c** $p \geq -4$
d $x > 2$ **e** $x < -2\frac{1}{2}$ **f** $x \geq -3\frac{1}{3}$
g $x < -4$ **h** $x > -3$ **i** $x \geq -5$
j $x < -6$ **k** $x < -5$ **l** $x \leq -8$
m $x > -2$ **n** $x < -6$ **o** $x < -12$

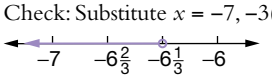
4 **a** $x > 3$ **b** $m < 4$ **c** $p \geq 3$
d $p \leq -1$ **e** $p \geq 1\frac{1}{5}$ **f** $x < -3\frac{1}{2}$
g $x \leq 4$ **h** $p \geq 2$ **i** $y < 1\frac{1}{4}$
j $x < 3$ **k** $x > 2$ **l** $x \leq 1\frac{1}{2}$
m $x < 4$ **n** $m > 3$ **o** $x \leq 3\frac{1}{2}$
p $x \geq 4\frac{1}{2}$ **q** $m > 2\frac{1}{2}$ **r** $p > 3$

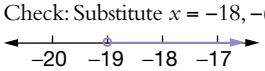
5 **a** $m < -2$ **b** $y > -2$ **c** $x \leq -1\frac{1}{2}$
d $x \geq -2$ **e** $m \leq 6$ **f** $m \geq 8$
g $p > \frac{1}{2}$ **h** $x > -2$ **i** $y < -1$
j $x \leq -1$ **k** $y \geq 1$ **l** $p < -\frac{1}{2}$
m $x \geq -\frac{1}{6}$ **n** $a \geq 3\frac{3}{4}$ **o** $x < \frac{1}{6}$

6 **a** $x \geq 2$
Check: Substitute $x = 3$, $8 - (2 \times 3)$, $2 \leq 4$, true.

b $x < -2\frac{2}{3}$
Check: Substitute $x = -3$, $9 - 6(-3)$, $27 > 25$, true.


c $x \geq -3$
Check: Substitute $x = -2$, $-6(-2) + 3$, $15 \leq 21$, true.


d $x < -6\frac{1}{3}$
Check: Substitute $x = -7$, $-3(-7) - 8$, $13 > 11$, true.


e $x > -19$
Check: Substitute $x = -18$, $-(-18) - 5$, $13 < 14$, true.


f $x \leq 3$

Check: Substitute $x = 2$, $8(4 - 2)$, $16 \geq 8$, true.

7 **a** $x > 3$ **b** $x < 5\frac{1}{2}$ **c** $a \leq 2$

d $b \leq -3$ **e** $x < 10$ **f** $p > 15$

g $y \geq -1\frac{1}{2}$ **h** $p < 9$ **i** $x \leq 1$

Fun spot 7:09

a Steve **b** Alan

Diagnostic test 7

1 **a** $x = 6$ **b** $y = 9$ **c** $p = 14$ **d** $m = 10$

2 **a** $x = 6$ **b** $x = 8$ **c** $y = 27$ **d** $y = 20$

3 **a** $p = 18$ **b** $x = 20$ **c** $x = 6$ **d** $t = 12$

4 **a** incorrect **b** incorrect **c** incorrect

d correct

5 **a** $p = 7$ **b** $m = 12$ **c** $p = 8\frac{4}{9}$ **d** $m = -10$

6 **a** $x = -4$ **b** $a = 3$ **c** $b = -5$

7 **a** $x = 2$ **b** $a = 9$ **c** $x = 2{\cdot}6$

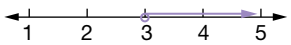
8 **a** $y = 12$ **b** $m = 12$ **c** $p = 18$ **d** $m = 10$

9 **a** $m = 5$ **b** $m = 9$ **c** $p = 7$ **d** $x = 6\frac{2}{3}$

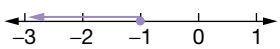
10 **a** $m = \frac{5}{2}$ **b** $x = -2$ **c** $n = 1$ **d** $a = \frac{11}{8}$

11 **a** $a = 12$ **b** $m = 24$ **c** $x = -60$ **d** $y = \frac{19}{6}$

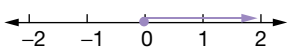
12 **a**



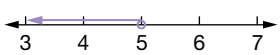
b



c



d



13 **a** $x \geq \frac{1}{2}$ **b** $x < 9\frac{1}{2}$ **c** $x > 5$

14 **a** $x < -2$ **b** $x < -63$ **c** $a < \frac{1}{3}$

ASSIGNMENT 7A

1 **a** $x = \frac{13}{4}$ **b** $a = 12$ **c** $m = 44$ **d** $n = 88$

2 **a** $m = 3$ **b** $y = -1$ **c** $m = 3$ **d** $n = -2$

e $p = -\frac{1}{3}$ **f** $k = \frac{17}{2}$ **g** $y = \frac{9}{5}$ **h** $q = \frac{1}{3}$

3 **a** $x = 4$ **b** $x = 4$ **c** $x = -3$ **d** $x = \frac{9}{2}$

e $x = \frac{11}{2}$ **f** $x = \frac{-8}{5}$ **g** $x = -11$ **h** $x = \frac{1}{5}$

i $x = 1$

4 **a** $x = -1$ **b** $a = 6$ **c** $m = 1\frac{1}{2}$ **d** $x = -6$

e $n = 11$ **f** $x = 0$ **g** $a = \frac{2}{11}$ **h** $n = -\frac{23}{11}$

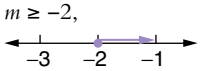
i $m = -3$ **j** $x = -9$

5 **a** $x = 8$ **b** $m = 15$ **c** $x = 9$ **d** $a = 17$

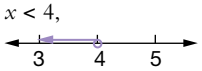
e $y = 8$ **f** $p = 3$ **g** $m = -3$ **h** $m = -\frac{1}{3}$

i $n = -\frac{16}{5}$ **j** $x = -13$ **k** $a = -\frac{70}{3}$ **l** $q = -\frac{6}{29}$

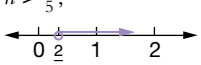
6 **a** $m \geq -2$,



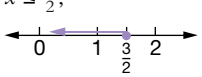
b $x < 4$,



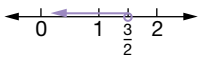
c $n > \frac{2}{5}$,



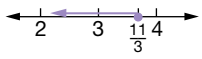
d $x \leq \frac{3}{2}$,



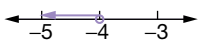
e $y < \frac{3}{2}$,



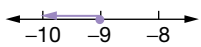
f $n \leq \frac{11}{3}$,



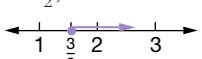
g $x < -4$,



h $x \leq -9$,



i $a \geq \frac{3}{2}$,



ASSIGNMENT 7B

1 **a** 10 minutes **b** 55 minutes

2 $\frac{9}{10}$ **3** 56 **4** 4, 9, 25, 49

5 \$45 **6** \$610 **7** 14

ASSIGNMENT 7C

1 **a** 0·08 **b** 0·085 **c** 1·25 **d** 0·0025

2 **a** \$115 804 **b** 48·9% (1 dec. pl.)

3 **a** **i** $0{\cdot}\dot{3}$ km/min **ii** 20 km/h **iii** 3 min/km

b 18 s

4 **a** 48% **b** 23% **c** 43% **d** 51%

5 SA of box A = 3200 cm^2; SA of box B = 3800 cm^2.

Box A uses the least amount.

6 **a** 10^{10} **b** 10^{-2} **c** 10^{2} **d** 10^{6}

Chapter 8

Exercise 8:01

1 **a** See page 210. **b** See page 210.

c See page 210. **d** commission

e See page 210. **f** See page 210.

g See page 210. **h** commission and piece work

i See page 210. **j** wages and commission

2 **a** \$997.50 **b** \$129.20

c \$28.40 **d** Luke, by \$36.75

e Irene by 40 cents **f** \$1200.80

g \$883 per week

h **i** \$824.10 **ii** \$1464.22

iii \$2663.00 **iv** \$1115.41

i **i** \$55 200 **ii** \$37 806

iii \$63 660 **iv** \$107 400

j The second salary is greater by \$28.72 per week

k The second income is higher by \$22.86 per week.

3 **a** **i** no **ii** no **iii** no

iv (1) \$690 (2) \$1708.90

(3) \$3813.40 (4) \$211.60

b **i** (1) \$796 (2) \$985

(3) \$1080.55 (4) \$1104.91

ii \$1181 **iii** no

c **i** \$800 **ii** \$870 **iii** \$1142.40

iv \$912.95

4 **a** \$259.83 **b** \$576.50 **c** \$135

5 **a** \$998.40 **b** \$1950, \$1220 profit

c \$993.60 **d** \$777.95, \$19.95

Exercise 8:02

1 a $408 b $565.15
c i $108.60 ii $181
iii $167.43 (nearest cent)
iv $253.40
d i $672 ii $851.20 iii $1002.40
iv $1136.80
e $295.80 f $39.20, 1·5 hours

g

No. 53 Time card summary Whit. Pty Ltd. Name: Tom McSeveny Rate: $26.20 p/h.				
Week ending	Number of hours at:			Wage
	normal rates	time-and-a-half	double-time	
21 Jan	35	0	0	$917
28 Jan	35	3	0	$1034.90
4 Feb	35	2	0	$995.60
11 Feb	34	5·5	1	$1159.35
18 Feb	35	7·5	0·5	$1237.95

2 a i $700 ii $987 iii $1046.50
iv $730.10
b $6706.90 c $676.20
d $102 400 e $2532, $58 932
f $1844.80 g $948.95
h i $1000 ii $1340
i $1240, 1.35%

3 a i $296 ii $116.55 iii $493.95
iv $678.49
b i $283.05 ii $432.90 iii $562.40
iv $790.32
c $506.90 d $364.45
e i $4000 ii $5800 iii $3460
iv $7072

Exercise 8:03

1 a $200.60 b $595 c $431.95
d $1403.65 e $622.95

2 a i $1381.68 ii $290.15
iii $69.08 iv $936.45

b

Turner, Vicki	Serial No.	Gross Salary or Wage Rate	Super Units		Week Ended	Net Pay	Pay Advice No.
			Entld.	Held			
	6841672	$7209.60	98	98	29/11/13	$936.45	11364

Deductions this week				Pay this week			
Taxation	S'annuation	M'laneous	Total	Normal Pay	Adjustments	Overtime	Gross Earnings
$290.15	$69.08	$86.00	$445.23	$1381.68	–	–	$1381.68

3 a $378.80, 1·8% b $1141.30, 21·3%
c $802.10, 15·3% d $1864.50, 28·1%

4 a $42 568 b $2739.77 c $1027.57
d i $29 400 ii $23 100
iii Answers will differ, but the 'lump sum' option is attractive since you have the money as well.
iv $24 990
e 1333, 1234

Exercise 8:04

1 a Nil b $833.53 c $5021.50
d $23 965.02 e $101 758.30

2 a $33 942 b $33 287 c $2866.53
d $3204.16

3 $8173.68

4 a $87 006 b $84 910 c $19 363.70
d Need to pay $1987.35

5 a $2000 b $1590 c $1320
d $1230 e $1070 f $1455
g $1282.50 h $1134

Prep quiz 8:05

1 $6 **2** $3.10 **3** $4.35 **4** $0.67 **5** $5
6 $6.10 **7** $8.50 **8** $2.50 **9** $4.70 **10** $2.88

Exercise 8:05

1 **a** $20 **b** $3.20 **c** $17.50
d $9.00 **e** $8.40 **f** $11.25

2 **a** **i** $0.40 **ii** $0.30 **ii** is better buy
b **i** $0.75 **ii** $0.80 **i** is better buy
c **i** $1.30 **ii** $1.25 **ii** is better buy
d **i** $2.60 **ii** $2.80 **i** is better buy
e **i** $1.30 **ii** $1.33 **i** is better buy
f **i** $4.40 **ii** $4.20 **ii** is better buy

3 **a** $5, $45 **b** $4, $16 **c** $8, $92
d $6, $18 **e** $20, $20 **f** $7.50, $42.50
g $12, $18 **h** $21, $7 **i** $180, $20
j $3, $22 **k** $27, $18 **l** $28, $52

4 **a** $10, 25% **b** $5, 10% **c** $25, 25%
d $12, 20% **e** $18, 50% **f** $2, 8%
g $3, 12·5% **h** $12, 33·3̇%

5 **a** $5, $55 **b** $2, $22
c $10, $110 **d** $2.50, $27.50
e 80c, $8.80 **f** $1.20, $13.20
g $5.60, $61.50 **h** $3.70, $40.70
i $8.30, $91.30 **j** 55c, $6.05
k 96c, $410.56 **l** $1.64, $18.04
m 53c, $5.79 **n** $2.34, $25.76
o $7.20, $79.16

6 **a** $4 **b** $7 **c** $12 **d** $2.20
e $8.40 **f** $1.15 **g** 74c **h** $1.94
i $7.43 **j** $4.55 **k** $2.18 **l** $13.18
m 86c **n** $1.99 **o** $6.13

7

	Cost price	Profit	Selling price
a	$400	$100	$500
b	$240	$60	$300
c	$350	$70	$420
d	$840	$42	$882
e	$750	$250	$1000

8 **a** (a) 25% (b) 25% (c) 20%
(d) 5% (e) 33·3̇%
b (a) 20% (b) 20% (c) 16·6̇%
(d) 4·8% (to 1 dec. pl.) (e) 25%

9 **a** 5 tyres for $55
b 6 ice creams for $20.40
c 5 litres for $92.50
d 600 mL soft drink for $5.20
e 375 g can for $4.20
f 4 lollies for $4.40
g 375 mL shampoo for $7.65
h 20 kg of potatoes for $25

10 **a** 600 g for $9.20 **b** 5 litres for $72
c 1 L bottle for $12.90

11 **a** $17.60 **b** $79.05 **c** $80 discount, 25%
d 30% **e** $650 **f** $62
g 11% **h** $198

12 $72 **13** $1338.75

14 **a** $405 **b** $459 **c** $197.60 **d** $717.72
e $1701.77

15 $0.58

16 **a** $0.82 **b** $0.96 **c** $1.44

17 **a** $70 **b** $120 **c** $180 **d** $250
e $14 **f** $26 **g** $97 **h** $115
i $12.60 **j** $37.50 **k** $215.50 **l** $1125.80

18 **a** $22 **b** 78·6% **c** 44%

19 **a** $38.40 **b** $21 **c** $12.47

20 C: S.P. = 100 : 140
∴ cost = $220, profit = $88

21 **a** $300, $330 **b** $4, $4.40 **c** $60.50, $66.55

22 The first 'direct buy' price by $9. The second 'sale' price if 1 hour of time and wear and tear on the car is valued at more than $4.40.

23 Robyn's for $130. **24** $45.50, 43%

25 9% **26** $14.76

27 The discount price is $40.32 in each case.

Exercise 8:06

1 **a** See page 234 **b** See page 234
c See page 234 **d** See page 234

2 **a** $143.80, $56.80 **b** $1866, $576
c $1138.40, $536.80
d **i** $874, $144 **ii** $1019.20, $289.20
iii $1165.60, $435.60 **iv** $1309.60, $579.60
v $1456, $726 **vi** $2182, $1452
e $17 180, $2566.70

3 **a** $200 **b** $525 **c** $672 **d** $216
e $300 **f** $850 **g** $2250 **h** $1132.81

4 **a** $1200, $5.77 **b** $3025, $19.39
c $6272, $60.31 **d** $1116, $7.15
e $2300, $22.12 **f** $3350, $16.11
g $12 250, $94.23 **h** $13 632.81, $209.74

5 **a** $2322 **b** $10 922 **c** $303.39

6 **a** $43 500 **b** $17 182.50 **c** $60 682.50
d $1011.38

7 **a** $1240 **b** $11 160 **c** $3839.04
d $14 999.04 **e** $144.22

8 **a** $2 **b** $4.50 **c** $9.97 **d** $18.78
e $3 **f** $42.17 **g** $18 **h** $20.79

9 $8.50

10 **a** $25.38 **b** $1500.38 **c** $17.51

11 **a** $15 900 **b** $5850 **c** $1170 **d** 12%

12 **a** $11 840 **b** $15 912.96 **c** $4072.96
d 8.6%

13 **a** 17.9% **b** $9.41 **c** $3.92

Diagnostic test 8

1 **a** **i** $2150 **ii** $2480
b $234.95

2 **a** $1017 **b** $651
3 **a** $1068.70 **b** 23%
4 **a** $74 430 **b** $71 626 **c** $1074.39
d $15 899.84 including Medicare
e $265.84 **f** $66
5 **a** 50 kg size **b** 660 mL for $3.75
6 **a** $7.37 **b** $59.29 **c** $2.95 **d** $19.50
7 **a** $402.50 **b** $588, 14% **c** $32 000
8 **a** $526.50 **b** $850
9 **a** 130% **b** $3640.90, 72·5%
10 **a** $5003.20, $2003.20
b **i** $2583 **ii** $16 933 **iii** $705.54
c **i** $16 400 **ii** $65 600 **iii** $25 584
iv $1519.73

ASSIGNMENT 8A

1 **a** **i** $672 **ii** $798
b $635 **c** $1090 **d** $2124.40
2 **a** **i** 21 kg for $59.60 **ii** 21 kg pack for $59.60
b $140.64 **c** $141.82
3 **a** $52 010 **b** $8450.25 **c** $3501.60
4 **a** $32.89 **b** $29.60 **c** no
d approx. 9·1%
5 **a** 40% **b** approx. 34%
6 **a** $2595 **b** $14 705 **c** $3749.78
d $236.60 **e** $21 049.80

ASSIGNMENT 8B

1 **a** four cubed **b** the cube root of 2
c per cent **d** therefore
e for example **f** that is
g pi **h** the sum of
i the mean **j** probability of event E
2 **a** square **b** rectangle **c** parallelogram
d rhombus **e** trapezium **f** pentagon
g hexagon **h** octagon **i** kite
j circle
3 15° **4** 28
5 **a** **i** $3 **ii** $3
b 3 h **c** $16
6 **a** $1800 **b** $1400 **c** $37.50

ASSIGNMENT 8C

1 **a** 0·5 cm **b** 0·05 cm **c** 0·0005 m
2 **a** **i** 27 **ii** 0·072
b **i** 22 **ii** 5· 5
3 **a** $5.\dot{5}\%$ **b** 5·9% **c** $275; 35%
4 **a** $\frac{m}{12}$ **b** $\frac{5m}{12}$ **c** $\frac{m^2}{24}$ **d** $\frac{3}{2} = 1\frac{1}{2}$
5 50·0 cm^2
6 **a** 56 **b** 8 **c** $19\frac{2}{3}$
7 **a** $2{\cdot}13 \times 10^5$ **b** $2{\cdot}15 \times 10^8$
c $5{\cdot}67 \times 10^{-3}$ **d** $1{\cdot}2 \times 10^{-7}$

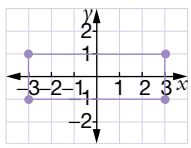
Chapter 9

Prep quiz 9:01

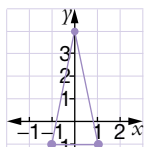
1 x-axis **2** y-axis **3** origin (0, 0)
4 (3, 5) **5** (3, −1) **6** (−4, −2)
7 (0, 6) **8** (−3, 0) **9** 7 units
10 3 units

Exercise 9:01

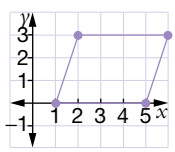
1 **a** V **b** B **c** J **d** T **e** F
f X **g** O **h** G **i** D
2 **a** (1, 2) **b** (−2, 4) **c** (−3, −3) **d** (3, −2)
e (2, 0) **f** (0, 3) **g** (−3, 0) **h** (0, −1)
i (2, −3) **j** (3, 4) **k** (−4, 4) **l** (−2, −4)
3 **a** R, U, A, V **b** C, S, G, R
4 **a**



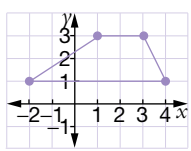
b



c



d



5 **a** 3 **b** 3 **c** 2 **d** 4 **e** 4
f 2 **g** 4 **h** 4 **i** 7 **j** 3
k 3 **l** 6 **m** 3 **n** 4
6 **a** 1st **b** 2nd **c** 3rd **d** 4th
e 3rd **f** 1st **g** 3rd **h** 2nd
i 4th **j** 2nd **k** 3rd **l** 4th
m 1st **n** 4th **o** 2nd
7 **a** It has a y-coordinate of 0.
b It has an x-coordinate of 0.
8 **a** **i** H **ii** B **iii** E **iv** G
b **i** (1·4, 0·8) **ii** (−1·4, 1·6)
iii (1·4, −2·6) **iv** (−0·8, −3·4)
9 **a** $A(-1, 4)$; $B(1, 2)$; $C(2, 1)$; $D(3, 0)$
b $A(-1, -3)$; $B(0, -2)$; $C(1, -1)$; $D(3, 1)$
c $A(-3, 4)$; $B(-2, 2)$; $C(0, -2)$ $D(1, -4)$
10 **a** Line A

x	−3	−2	0	1
y	4	2	−2	−4

Line B

x	−4	−2	0	2
y	−1	0	1	2

Line C

x	−2	−1	2	3
y	−4	−3	0	1

Line D

x	−1	0	2	4
y	5	4	2	0

b $a = 3, b = -6$
11 **a** (1, 1), $(1\frac{1}{2}, 1\frac{1}{2})$, (2, 2) are three of an infinite number of answers.
b $(\frac{3}{4}, -1\frac{1}{2})$, (1, −2), $(1\frac{1}{4}, -2\frac{1}{2})$

c $a = -4, b = 4, c = 5$

12 **a** C **b** B **c** F **d** A **e** D
f E

13 **a** C **b** D **c** A **d** B

Prep quiz 9:02

1 7 **2** 1 **3** 7 **4** 1 **5** 7
6 1 **7** 3 **8** 1 **9** 3 **10** 1

Exercise 9:02

1 **a** $(3, 3)$ **b** $(-1, -1)$ **c** $(0, 1)$
d $(1, -3)$ **e** $(1\frac{1}{2}, 0)$

2 **a** $(2\frac{1}{2}, 2\frac{1}{2})$ **b** $(3\frac{1}{2}, 7)$ **c** $(-4, 3)$
d $(0, -2\frac{1}{2})$ **e** $(-4, -6)$ **f** $(4\frac{1}{2}, -5\frac{1}{2})$
g $(\frac{1}{2}, -5)$ **h** $(-\frac{1}{2}, 3\frac{1}{2})$ **i** $(-1\frac{1}{2}, -7\frac{1}{2})$

3 **a** $(4, 7)$ **b** $(3, 7)$ **c** $(6, 4)$
d $(2, 1)$ **e** $(2, 4)$ **f** $(3, 3)$
g $(5\frac{1}{2}, 2\frac{1}{2})$ h $(2\frac{1}{2}, 3\frac{1}{2})$ **i** $(5\frac{1}{2}, 4\frac{1}{2})$
j $(3, 3\frac{1}{2})$ **k** $(4\frac{1}{2}, 3)$ **l** $(5, 3\frac{1}{2})$
m $(-2, 1)$ **n** $(-4, -3)$ **o** $(0, -5)$
p $(1, -2)$ **q** $(2, 2)$ **r** $(-1, -2)$

4 **a** $(-\frac{1}{2}, -3)$ **b** $(7\frac{1}{2}, -1)$ **c** $(5, 0)$
d $(-\frac{1}{2}, -\frac{1}{2})$ **e** $(-2, -2)$ **f** $(5\frac{1}{2}, -5\frac{1}{2})$
g $(87, 70)$ **h** $(70, -26)$ **i** $(262, 76)$
j $(-66\frac{1}{2}, 52)$ **k** $(-24, 12\frac{1}{2})$ **l** $(34, -14\frac{1}{2})$

5 **a** **i** $(2\frac{1}{2}, 2)$ **ii** $(2\frac{1}{2}, 2)$ **iii** yes
iv The diagonals bisect each other.
b $(3, 8)$
c **i** $(4, 3)$ **ii** $(4, 3)$ **iii** yes
iv The diagonals bisect each other.

6 **a** $k = 0$ **b** $c = 6, d = 8$
c $x = 11, y = 11$ **d** $b = -8, c = 8$
e $d = 15, e = -6$ **f** $(-16, 0)$
g $(7, 2)$

Prep quiz 9:03

1 EF **2** A and B **3** C and D
4 E and F **5** B and C **6** D and E
7 F and G **8** up **9** down
10 not at all

Exercise 9:03

1 **a** positive **b** positive **c** negative
d negative **e** positive **f** negative
g positive

2 **a** 1 **b** 5 **c** $\frac{1}{5}$ **d** $-\frac{1}{2}$ **e** -2
f -5

3 **a** $\frac{1}{2}$ **b** $\frac{3}{2}$ **c** $\frac{1}{2}$ **d** $-\frac{1}{2}$ **e** -2
f $-\frac{2}{3}$

4 **a** $-\frac{1}{2}$ **b** $-\frac{4}{3}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $-\frac{1}{3}$
f 1

5 **a** $\frac{1}{2}$ **b** -2 **c** $-\frac{3}{4}$

6 **a** 2 **b** 1 **c** $\frac{1}{2}$ **d** -1 **e** $-\frac{3}{2}$
f -3

Investigation 9:03

1 **a** C **b** B **c** A
2 **a** 8 m **b** 5·6 m **c** 12·8 m
3 **a** $1\frac{1}{2}$ m **b** $\frac{1}{2}$ m
4 No, he would need a run of 11·2 m. He could curve the rampway across the slope.

Prep quiz 9:04

1 C **2** 5 **3** 13
4 $\sqrt{20} \div 4{\cdot}47$ (2 dec. pl.) **5** 4 **6** 3
7 4 **8** 3 **9** 5 units **10** 5 units

Exercise 9:04

1 **a** 10 **b** 5 **c** $\sqrt{34}$ **d** $\sqrt{29}$ **e** 13

2 **a** $BC = 4; AC = 5; AB = \sqrt{41}$
b $BC = 3; AC = 3; AB = \sqrt{18}$
c $BC = 2; AC = 6; AB = \sqrt{40}$
d $BC = 7; AC = 6; AB = \sqrt{85}$
e $BC = 6; AC = 4; AB = \sqrt{52}$
f $BC = 6; AC = 3; AB = \sqrt{45}$
g $BC = 2; AC = 4; AB = \sqrt{20}$
h $BC = 8; AC = 3; AB = \sqrt{73}$
i $BC = 7; AC = 4; AB = \sqrt{65}$

3 **a** $\sqrt{20}$ **b** $\sqrt{58}$ **c** $\sqrt{53}$ **d** $\sqrt{32}$
e $\sqrt{50}$ **f** $\sqrt{34}$ **g** $\sqrt{29}$ **h** $\sqrt{41}$
i $\sqrt{40}$

4 **a** $\sqrt{40}$ **b** $\sqrt{32}$ **c** 5 **d** $\sqrt{128}$
e $\sqrt{2}$ **f** $\sqrt{18}$ **g** $\sqrt{45}$ **h** $\sqrt{53}$
i $\sqrt{8}$ **j** 13 **k** 5 **l** $\sqrt{52}$

Prep quiz 9:05

1 1 **2** 3 **3** -3 **4** 2 **5** 5
6 -2 **7** -5 **8** C **9** A **10** B

Exercise 9:05

1 **a** $y = 3x$

x	0	1	2
y	0	3	6

b $y = x + 1$

x	0	1	2
y	1	2	3

c $y = 5 - x$

x	0	1	2
y	5	4	3

d $y = 6x - 7$

x	2	3	4
y	5	11	17

e $y = 18 - 5x$

x	2	3	4
y	8	3	-2

f $y = 7x + 2$

x	2	3	4
y	16	23	30

g $y = x - 7$

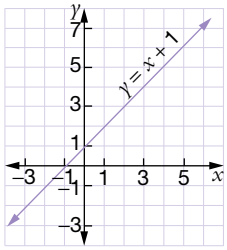
x	0	1	2
y	-7	-6	-5

h $y = 1 - x$

x	0	1	2
y	1	0	-1

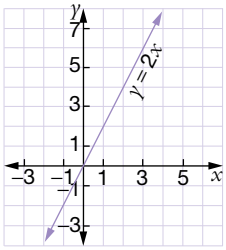
2 a $y = x + 1$

x	0	1	2
y	1	2	3



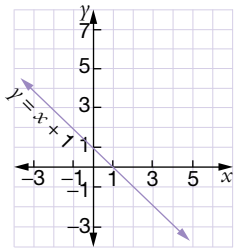
b $y = 2x$

x	0	1	2
y	0	2	4



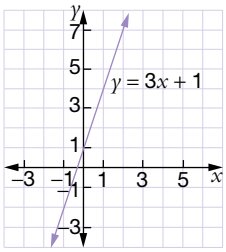
c $y = -x + 1$

x	0	1	2
y	1	0	−1



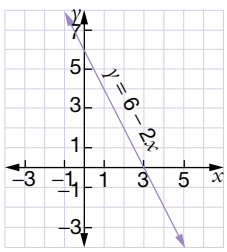
d $y = 3x + 1$

x	0	1	2
y	1	4	7



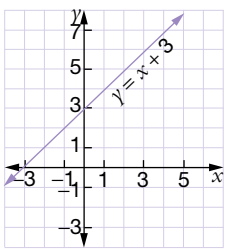
e $y = 6 - 2x$

x	0	1	2
y	6	4	2

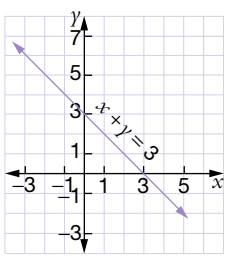


f $y = x + 3$

x	0	1	2
y	3	4	5



3 a



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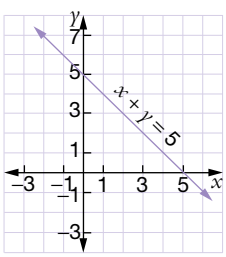

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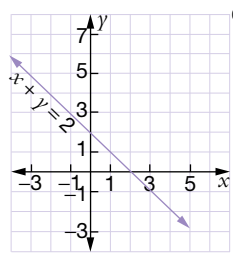

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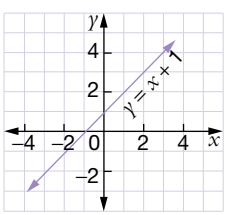
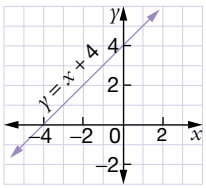


e



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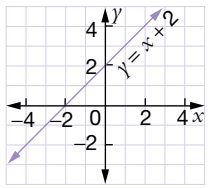

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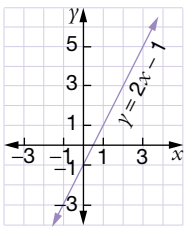

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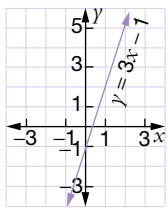

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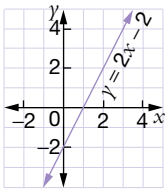

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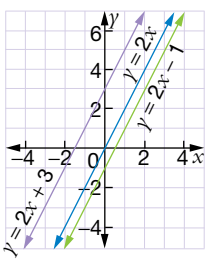
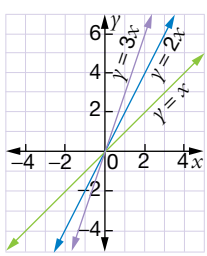


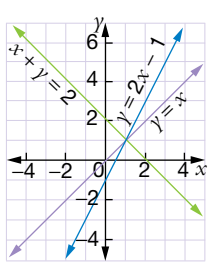
The lines are parallel.

5



The lines all pass through the origin.

6

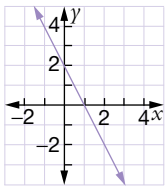


The lines are concurrent at (1, 1).

7 a, b, d

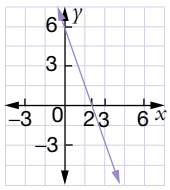
8 **a** yes **b** yes **c** (7, −2) **d** (5, 0)

9 **a**



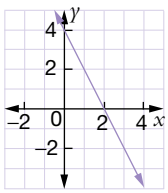
x-intercept is 1
y-intercept is 2

b



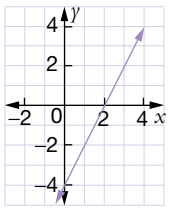
x-intercept is 2
y-intercept is 6

c



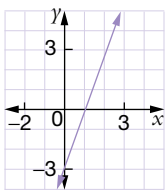
x-intercept is 2
y-intercept is 4

d



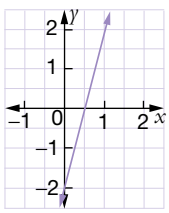
x-intercept is 2
y-intercept is −4

e



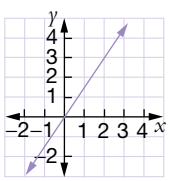
x-intercept is 1
y-intercept is −3

f



x-intercept is $\frac{1}{2}$
y-intercept is −2

10 **a**



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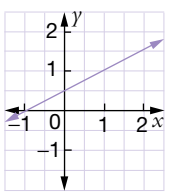

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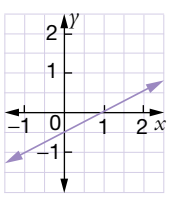

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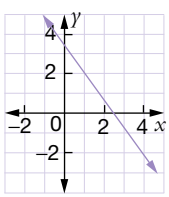

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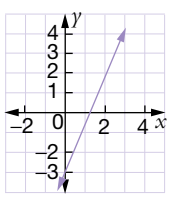

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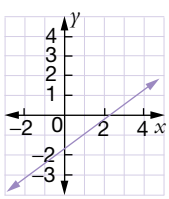


Investigation 9:05

1 **a** *D* **b** *B* **c** *E*

2 **a** *A* **b** *B* **c** *C*

3 (4, 0), (0, 0), (−2, 0); the *y*-coordinate is zero for each point.

4 (0, 2), (0, 0), (0, −4); the *x*-coordinate is zero for each point.

5 The *y*-coordinate is always zero.

6 The *x*-coordinate is always zero.

7 Substitute $y = 0$ into the equation and solve for x.

8 Substitute $x = 0$ into the equation and solve for y.

9 Lines in question 3

	a	b	c	d	e	f	g	h	i
x-intercept	3	5	2	−1	−4	−2	$\frac{1}{2}$	$\frac{1}{3}$	1
y-intercept	3	5	2	1	4	2	−1	−1	−2

Lines in question 10

	a	b	c	d	e	f
x-intercept	0	−1	1	$2\frac{1}{3}$	$1\frac{1}{5}$	$2\frac{1}{2}$
y-intercept	0	$\frac{1}{2}$	$-\frac{1}{2}$	$3\frac{1}{2}$	−3	$-1\frac{2}{3}$

Exercise 9:06

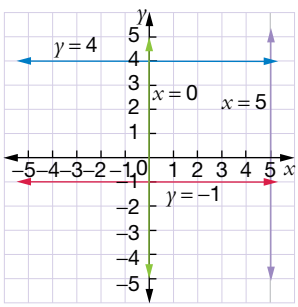
1 **a** $x = 2$ **b** $y = -2$

2 **a** A: $x = -4$, B: $x = -1$, C: $x = 3$, D: $y = 3$, E: $y = 1$, F: $y = -4$

b A: $x = -3$, B: $x = 2$, C: $x = 4$, D: $y = 4$, E: $y = 2$, F: $y = -2$

3 **a** $x = 0$ **b** $y = 0$

4 **a**



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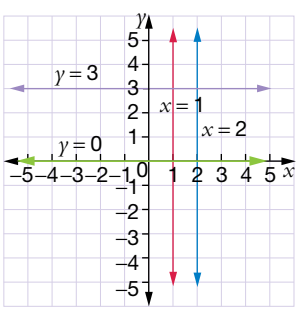
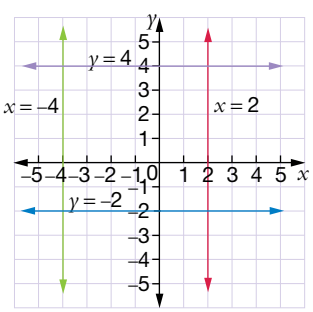


c



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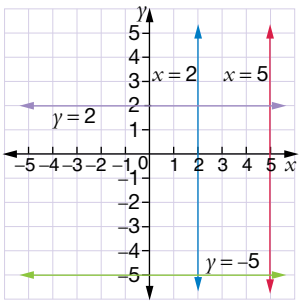

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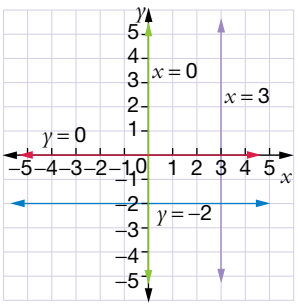


The lines in **a** and **c** enclose square regions.

5 **a** (3, 1) **b** (−4, 3) **c** (−1, −4) **d** (2, −2)
e (−3, 2) **f** (4, 4) **g** (0, 3) **h** (2, 0)
i (0, 0)

Prep quiz 9:07

1 0 **2** b **3** 2 **4** −1
5 2 **6** $\frac{1}{2}$ **7** (0, 2) **8** (0, −1)
9 yes **10** yes

Investigation 9:07

1

Line	Equation	Gradient	y-intercept
A	$y = x + 2$	1	2
B	$y = x - 2$	1	−2
C	$y = \frac{1}{2}x + 1$	$\frac{1}{2}$	1
D	$Y = \frac{1}{2}x - 1$	$\frac{1}{2}$	−1
E	$y = 2x + 2$	2	2
F	$y = 2x - 3$	2	−3

2

Line	Equation	Coefficient of x	Constant
A	$y = x + 2$	1	2
B	$y = x - 2$	1	−2
C	$y = \frac{1}{2}x + 1$	$\frac{1}{2}$	1
D	$y = \frac{1}{2}x - 1$	$\frac{1}{2}$	−1
E	$y = 2x + 2$	2	2
F	$y = 2x - 3$	2	−3

3 The gradient of the line is the same as the coefficient of x. The y-intercept of the line is the same as the constant.
4 $y = -2x + 3$
5 It tells us that m is the gradient and b is the y-intercept.

Exercise 9:07

1 The gradient is stated first.
a 2; 3 **b** 5; 1 **c** 3; 2 **d** 1; 6
e 4; 0 **f** 1; 0 **g** 1; −2 **h** 5; −1
i 6; −4 **j** −2; 3 **k** −1; −2 **l** −3; 1
m $\frac{1}{2}$; 4 **n** $\frac{3}{4}$; −2 **o** $\frac{1}{3}$; 5 **p** −3; 4
q 4; −2 **r** $-\frac{1}{2}$; 3

2 **a** $y = 4x + 9$ **b** $y = -2x + 3$
c $y = 7x - 1$ **d** $y = -5x - 2$
e $y = \frac{1}{2}x + 5$ **f** $y = \frac{2}{3}x - 4$
g $y = 3x + 1$ **h** $y = 2x - 3$
i $y = 5x + \frac{1}{2}$ **j** $y = x + 1\frac{1}{2}$

3 **a** 2; 1; $y = 2x + 1$ **b** $\frac{1}{4}$; 3; $y = \frac{x}{4} + 3$
c $\frac{1}{2}$; 2; $y = \frac{1}{2}x + 2$ **d** $\frac{1}{4}$; 4; $y = \frac{1}{4}x + 4$
e 3; −1; $y = 3x - 1$ **f** −1; −2; $y = -x - 2$
g $-\frac{1}{6}$; $-2\frac{1}{2}$; $y = -\frac{x}{6} - 2\frac{1}{2}$
h $\frac{3}{2}$; 0; $y = \frac{3x}{2}$

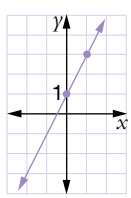
4 **a** E **b** B **c** D **d** G **e** F
f A **g** C

5 (In each part the gradient is stated first.)
a 2; 5 **b** 3; −3 **c** 2; 4 **d** 2; −1
e 5; 8 **f** 6; −12 **g** 3; −2 **h** −2; −4
i 2; 7 **j** −1; 4 **k** $\frac{1}{2}$; $1\frac{1}{2}$ **l** −3; 5
m −3; 2 **n** $-\frac{3}{2}$; 3 **o** $\frac{2}{5}$; $\frac{1}{5}$

6 **a** yes **b** no **c** yes **d** yes

7 **a** $2x - y + 6 = 0$; $y = 2x - 3$
b $x + y = 3$ and $y = -x + 8$

8 **a**



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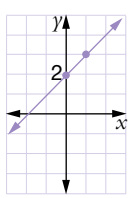

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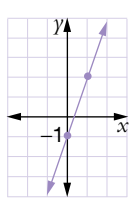

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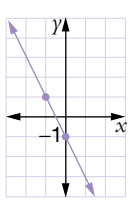

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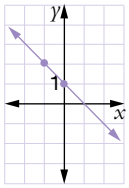

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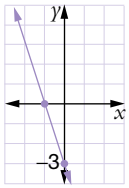

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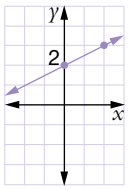

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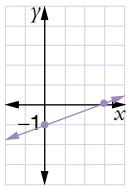

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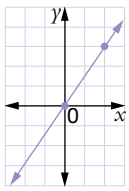

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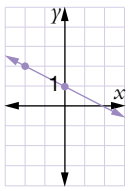

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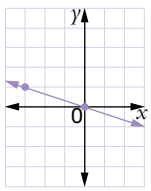

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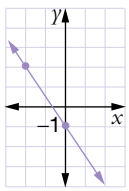
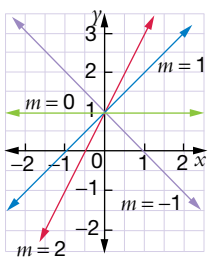
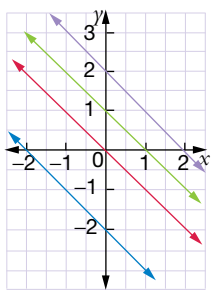


9 a



b



10 a –2 **b** 2 **c** $-\frac{1}{2}$ **d** $\frac{4}{3}$
e 4 **f** $-\frac{1}{3}$

Exercise 9:08

1 a Graph B

x	–3	–2	–1	0	1	2	3
y	10	5	2	1	2	5	10

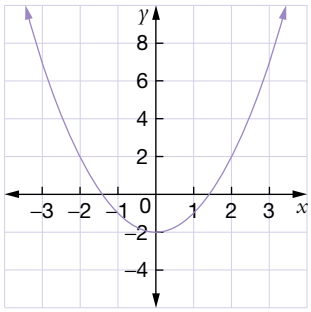
b Graph C

x	–3	–2	–1	0	1	2	3
y	8	3	0	–1	0	3	8

c Graph A

x	–3	–2	–1	0	1	2	3
y	11	6	3	2	3	6	11

2



x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7

3 a A (0, 2) B (0, 1) C (0, –1)
b yes **c** yes **d** yes

4 a Graph E

x	–3	–2	–1	0	1	2	3
y	–9	–4	–1	0	–1	–4	–9

b Graph G

x	–3	–2	–1	0	1	2	3
y	–8	–3	0	1	0	–3	–8

c Graph F

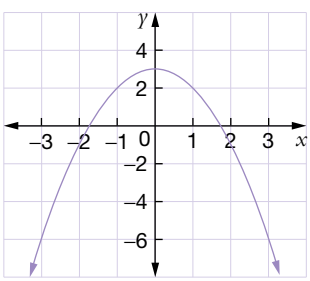
x	–3	–2	–1	0	1	2	3
y	–11	–6	–3	–2	3	–6	11

5 a E (0, 0) F (0, –2) G (0, 2)
b yes **c** yes **d** yes

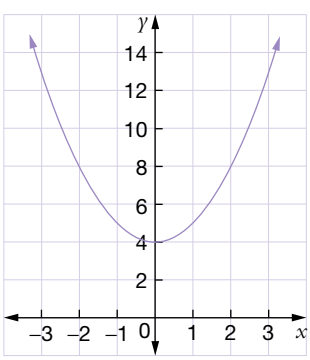
6 a B **b** D **c** A **d** C

7 The parabola is concave up if the coefficient of x^2 is positive. It is concave down if the coefficient of x^2 is negative.

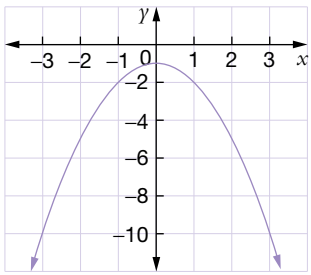
8 a



b



c



9 a Graph F

x	−3	−2	−1	0	1	2	3
y	0·125	0·25	0·5	1	2	4	8

b Graph E

x	−3	−2	−1	0	1	2	3
y	8	4	2	1	0·5	0·25	0·125

c Graph H

x	−3	−2	−1	0	1	2	3
y	−0·125	−0·25	−0·5	−1	−2	−4	−8

d Graph G

x	−3	−2	−1	0	1	2	3
y	−8	−4	−2	−1	−0·5	−0·25	−0·125

10 a B **b** A **c** D **d** C

Diagnostic test 9

1 a $(0, 1)$ **b** $(-1, 1)$ **c** $(-\frac{1}{2}, 1)$

2 a $(4, 6)$ **b** $(4, 1\frac{1}{2})$ **c** $(-1, -2\frac{1}{2})$

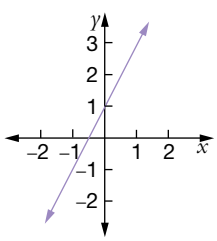
3 a $1\frac{1}{2}$ **b** -1 **c** 2

4 a 4 **b** $-\frac{1}{2}$ **c** $\frac{2}{3}$

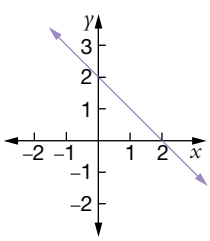
5 a $\sqrt{52}$ **b** $\sqrt{72}$ **c** $\sqrt{45}$

6 a 10 **b** $\sqrt{13}$ **c** $\sqrt{17}$

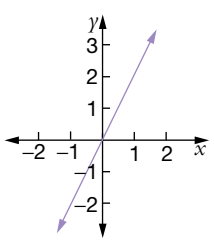
7 a



b



c



8 a yes **b** no **c** yes

9 a 3 **b** 4 **c** 4

10 a 4 **b** −1 **c** −5

11 a $x = 2$ **b** $x = -3$ **c** $y = 1$ **d** $y = -2$

12 a $y = 3x + 2$ **b** $y = \frac{1}{2}x - 3$

c $y = -x + 3$ **d** $y = \frac{1}{3}x - \frac{1}{2}$

13 a $m = 2, b = 3$ **b** $m = \frac{1}{2}, b = -3$

c $m = -2, b = 3$ **d** $m = -1, b = 4$

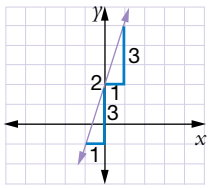
14 a $y = \frac{3}{2}x + 1$ **b** $y = -x$

c $y = 2x + 2$

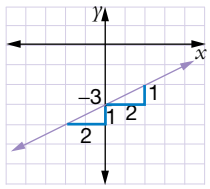
15 a $y = 4x + 6$ **b** $y = -\frac{2}{3}x + 1$

c $y = -\frac{5}{2}x - \frac{1}{2}$ **d** $y = \frac{1}{2}x + \frac{5}{2}$

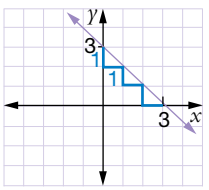
16 a



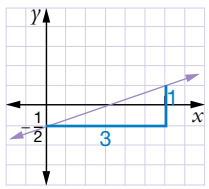
b



c



d



17 a C **b** B **c** A

18 a B **b** C **c** A

ASSIGNMENT 9A

1 a $\sqrt{65}$ **b** $\frac{7}{4}$ **c** $(4, 1\frac{1}{2})$

2 a 13 **b** $\frac{12}{5}$ **c** $(4\frac{1}{2}, 11)$

3 a B **b** B **c** B **d** $(2, 0)$

4 a $y = x - 3$ **b** $y = -\frac{1}{2}x + 6; 12$

5 a $(3, 2)$ **b** $\sqrt{13}$ **c** $\sqrt{13}$

d E is equidistant from the points O, A and B.

6 a

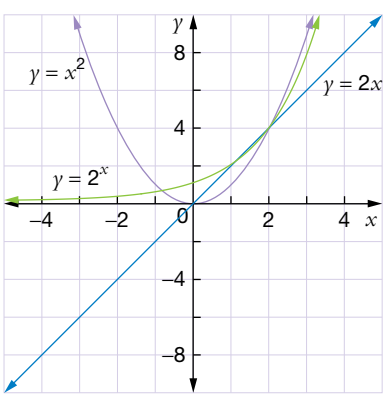
x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

b

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

c

x	−3	−2	−1	0	1	2	3
y	0·125	0·25	0·5	1	2	4	8



ASSIGNMENT 9B

1 **a** 1 **b** 31, 35, 39, 43, 47, 51, 55, 59 minutes
2 4 **3** Anna, $1695
4 **a** 6 **b** 3
5 **a** **i** 316 **ii** 187 **iii** 504 **iv** 250
b 63% **c** 55%
d Front left: 61%, front centre: 0%, driver's seat (front right): 63%, rear left: 46%, rear centre: 11%, rear right: 45%. There were too few occupants in the front centre to consider that spot. The safest spot seems to be the back centre, followed by the rear right and left. There isn't much difference between the driver's seat and the front left.

ASSIGNMENT 9C

1 **a** 8 **b** −7 **c** −1
2 **a** $1{\cdot}2 \times 10^{-2}$ **b** $2{\cdot}1 \times 10^{3}$ **c** $7{\cdot}7 \times 10^{4}$
3 **a** $864 **b** $1055; $27.76
4 **a** 192 cm^2 **b** S.A. = 169·6 cm^2 (1 dec. pl.); V = 169·6 cm^3 (1 dec. pl.)
5 **a** $\frac{1}{13}$ **b** $\frac{1}{4}$ **c** $\frac{15}{52}$ **d** $\frac{1}{26}$ **e** $\frac{9}{13}$
6 **a** 36% **b** 11% **c** 38% **d** 26% **e** 15%

Chapter 10

Exercise 10:01

1 **a** P **b** R **c** Q **d** S **e** K
f T **g** M **h** O **i** L **j** N
2 **a** **i** $\angle L$ **ii** $\angle M$ **iii** $\angle N$ **iv** $\angle O$ **v** $\angle P$
b **i** LP **ii** ON **iii** NM **iv** PO **v** LM
3 **a** angles A and D, B and E, C and F; $\Delta ABC \,|||\, \Delta DEF$
b angles A and F, B and D, C and E; $\Delta ABC \,|||\, \Delta DEF$
c angles A and Q, B and R, C and S, D and T, E and W; $ABCDE \,|||\, QRSTW$
d angles A and L, B and M, C and N, D and O, E and P; $ABCDE \,|||\, LMNOP$
4 **a** AB and DE, BC and EF, AC and DF
b AB and FD, AC and FE, BC and DE
c AB and QR, BC and RS, CD and ST, DE and TW, EA and WQ
d AE and LP, ED and PO, DC and ON, CB and NM, BA and ML
5 **a** yes **b** yes **c** yes **d** yes
6 **a** 3 : 2 **b** 3 : 2 **c** yes **d** no
7 **a** 8 cm **b** it is tripled **c** no **d** no
8 **a** true **b** true **c** false
9 **a** yes
b **i** 2 **ii** 4 **iii** 3 **iv** 2·5
c **i** 0·75 **ii** 0·25 **iii** 0·5 **iv** 0·625
10 **a** 2 **b** 0·5 **c** 1·5 **d** $1\frac{1}{3}$
e 2·5 **f** 0·6
11 A and B, C and E, D and F
12 **a** 66 cm, 60 cm, 40 cm, 36 cm
b 105·6 cm, 96 cm, 64 cm, 57·6 cm
c 26·4 cm, 24 cm, 16 cm, 14·4 cm
d 14·85 cm, 13·5 cm, 9 cm, 8·1 cm
13 RS = 21, RQ = 42, QP = 33, PS = 33
14 **a** **i** 200% **ii** 50%
b **i** 225% **ii** 75%
c **i** 16 cm long 9·6 cm wide
ii 8 cm long, 4·8 cm wide

Investigation 10:01

a **i** $\angle E$ **ii** $\angle F$ **iii** $\angle G$ **iv** $\angle H$
b **i** EF **ii** FG **iii** GH **iv** EH
c **i** 2 **ii** 2 **iii** 2 **iv** 2
d **i** 20 mm **ii** 10 mm **iii** 2
e **i** yes **ii** yes **iii** yes

Prep quiz 10:02

1 DE **2** DF **3** EF **4** 2 **5** 1·5
6 $\frac{3}{4}$ **7** $1\frac{1}{3}$ **8** $\frac{2}{3}$ **9** $\frac{1}{2}$
10 **a** 15 cm **b** 9 cm

Exercise 10:02

1 **a** $\frac{2}{3}, \frac{3}{2}$ **b** $\frac{3}{7}, \frac{7}{3}$ **c** $\frac{2}{3}, \frac{3}{2}$ **d** $\frac{6}{7}, \frac{7}{6}$
2 **a** a = 10, b = 4 **b** x = 5, y = 6
c d = 16, e = 40
3 **a** enlargement factor = 3; x = 12
b enlargement factor = 4; x = 24
c enlargement factor = 1·5; x = 7·5
d enlargement factor = 1·6; x = 14·4
4 **a** reduction factor = 0·25; y = 7·5
b reduction factor = $\frac{4}{9}$; y = 12
c reduction factor = $\frac{7}{8}$; y = 31·5
d reduction factor = 0·8; y = 20
5 **a** 36 cm **b** 10 cm
c 1·4; 12·6 cm **d** 12 cm
6 **a** 1 : 2, x = 5 **b** 1 : 4, y = 16 **c** 3 : 2, a = 15
d 5 : 2, c = 48 **e** 6 : 7, e = 24 **f** 5 : 4, f = 55
7 **a** 7 : 5, x = 7, y = 11·2 **b** 5 : 6, x = 30, y = 27·5
c 9 : 7, x = 11·6, y = 19·4 **d** 7 : 3, x = 9·3, y = 11·7
e 7 : 4, x = 5·1, y = 1·7, z = 8·8

Investigation 10:02

Photo widths are 14·4 cm and 9·6 cm. There are many combinations possible.

Prep quiz 10:03

1 no **2** no **3** false **4** false **5** 40°
6 yes **7** 2 **8** 1·5 **9** no **10** no

Exercise 10:03A

1 a

Side	AB	AC	BC	DE	DF	EF
Length	40	20	30	80	40	60

b i 2 ii 2 iii 2

c true

2 a $AB = 4$ cm, $BC = 3$ cm, $CA = 3{\cdot}6$ cm; $DE = 6{\cdot}0$ cm, $EF = 4{\cdot}5$ cm, $FD = 5{\cdot}4$ cm

b i 1·5 ii 1·5 iii 1·5

c yes

3 a i yes ii 3 iii yes

b i angles G and D, H and E, I and F; yes ii 1·5 iii yes

c i yes ii yes iii yes

4 a A, B and D **b** A and D **c** B and C

5 a W **b** L **c** N **d** M **e** P **f** U **g** R **h** Q **i** V **j** T

Exercise 10:03B

1 a yes

b i 2 ii 2 iii 2

c yes **d** yes **e** yes

2 a the ratios are all equal to $\frac{4}{3}$; yes

b yes **c** yes

3 a A and B **b** A and C **c** B and C **d** A and C

4 a M **b** P **c** N **d** K **e** L **f** Q **g** R **h** S **i** T **j** W

5 a yes **b** yes

6 a B and C **b** A and B **c** A and B **d** A and B

7 a yes **b** yes

8 a A and C **b** A and B **c** A and C **d** A and C

9 a ΔABC and ΔADE (3 equal angles)

b ΔABC and ΔEDC (3 equal angles)

c ΔACD and ΔABE (sides adjacent to equal angles in same ratio)

d ΔCED and ΔCAB (matching sides in same ratio)

e ΔABC and ΔEDC (3 equal angles)

Prep quiz 10:04

1 largest **2** smallest **3** true **4** true

5 true **6** DE **7** $\frac{1{\cdot}5}{2} = \frac{2{\cdot}25}{3} = \frac{3}{4}$

8 EF **9** ED **10** $\frac{8}{6} = \frac{16}{12} = \frac{20}{15}$

Exercise 10:04

1 a DE, EF, DF

$\therefore \frac{DF}{AC} = \frac{EF}{BC}$

$\therefore \frac{x}{3} = \frac{2{\cdot}5}{1{\cdot}5}$

$\therefore x = 3 \times \frac{2{\cdot}5}{1{\cdot}5}$

$\therefore x = 5$

b TR, TS, RS

$\therefore \frac{TS}{LN} = \frac{RS}{MN}$

$\therefore \frac{y}{32{\cdot}6} = \frac{25}{40}$

$\therefore y = 32{\cdot}6 \times \frac{25}{40}$

$\therefore y = 20{\cdot}375$ m

2 a $x = 10{\cdot}7$ **b** $y = 10{\cdot}4$ **c** $a = 30$ **d** $y = 40$

3 a $a = 30{\cdot}8$ **b** $x = 7{\cdot}1$ **c** $x = 5{\cdot}0$

4 a $x = 7{\cdot}5$ **b** $x = 5{\cdot}4$ **c** $a = 10$ **d** $x = 2\frac{2}{9}$

5 a $x = 4{\cdot}8$ **b** $h = 2{\cdot}4$ **c** $x = 13{\cdot}5$ **d** $x = 7$

6 a $x = 8$, $y = 5\frac{1}{3}$ **b** $a = 8$, $b = 16$

c $a = 40$, $b = 28{\cdot}8$ **d** $x = 28$, $y = 26{\cdot}25$

7 a 4·5 m **b** 4·2 m **c** 20 m

Prep quiz 10:05

1 100 cm **2** 1000 mm **3** 2000 cm **4** 25 m

5 1 : 100 **6** 1 : 1000 **7** 0·6 cm **8** 60 cm

9 500 **10** 4

Exercise 10:05A

1 a 100 **b** 1000 **c** 1000

2 a 30 m **b** 46 m **c** 23 m

3 a 35 m **b** 0·5 m **c** 12 m

4 a 66 mm, 30 mm **b** 13 200 mm, 6000 mm

c 13·2 m, 6 m

5 a 40 mm, 20 mm

b i 4 mm, 2mm ii 2 mm, 1 mm

6 a 37 mm **b** 1 : 100

c i 1600 mm ii 1500 mm

7 a 20 000 cm or 200 m **b** 380 m

c 1080 m or 1·08 km **d** about $160 000

e about 94 500 m^2 or 9·45 ha

8 a 4·5 cm **b** about 122 mm

c 1 : 122 **d** about 6800 mm or 6·8 m

e about 1220 mm

Exercise 10:05B

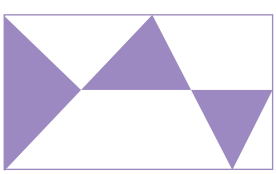
1 a 25 cm **b** 20 cm **c** 0·3 m

d 17 mm **e** 167 mm

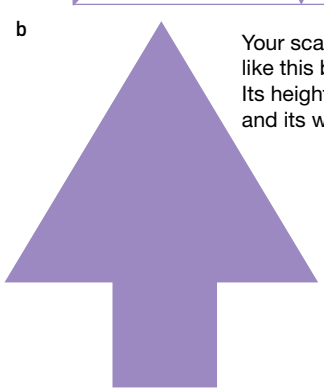
2 a 16 cm **b** 32 mm **c** 14 mm

d 3·2 mm **e** 5·2 cm

3 a



b



Your scale drawing will look like this but will be larger. Its height will be 10·5 cm and its width will be 9 cm.

4 a 1 : 2 **b** 1 : 3 **c** 2 : 1 **d** 3 : 1

5 a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** 2 times **d** 3 times

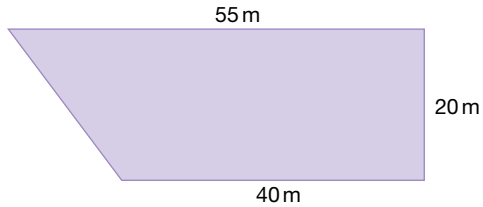
6 a 1 cm **b** 2 cm **c** 8 cm **d** 8 cm

e 3·5 cm **f** 2 cm

7 **a** 1 : 100 **b** 1 : 200 **c** 1 : 1000 **d** 1 : 1000
e 1 : 40 **f** 1 : 4000

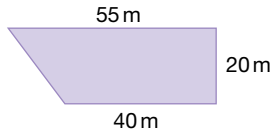
8 **a** 1 m long and 0·5 m wide (i.e. 1 m and 50 cm)
b 0·2 m long and 0·1 m wide (i.e. 20 cm and 10 cm)
c 8 cm long and 4 cm wide
d 4 cm long and 2 cm wide

9 **a**



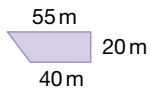
Scale 1:1000

b



Scale 1:2000

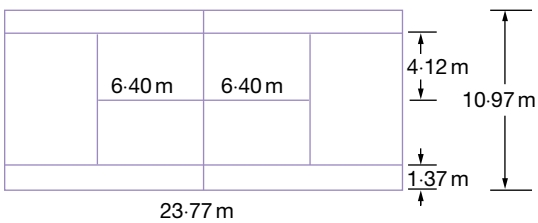
c



Scale 1:5000

10 **a** 1 : 500 **b** 1 : 400
c 1 : 1800 (Other answers are possible)

11



(Measurements are correct to two decimal places.)

12 Your scale drawing will show a hand drawn within a 10 cm square.

13 **a** 215 mm long and 148 mm wide
b 3 m
c 40 cm by 21 cm (or 400 mm by 210 mm)
d 48 mm by 36 mm, sink top is 12 mm long

Diagnostic test 10

1 **a** no **b** no **c** yes **d** yes

2 **a** A and H, B and G, C and D
b Factor for H and A is 2 : 1. Factor for B and G is 3 : 2. Factor for D and C is 3 : 1.

3 **a** yes **b** yes **c** no

4 **a** 14·4 **b** 12 **c** 14·4

5 **a** yes **b** no **c** yes

6 **a** no **b** no **c** no

7 **a** yes **b** no **c** yes

8 **a** 21 **b** 25 **c** 8·8

9 **a** 110 mm long, 80 mm wide
b 55 mm long, 40 mm wide
c 220 mm long, 160 mm wide

10 **a** 1 : 4000 **b** 4 cm **c** 80 m **d** 1 : 3

ASSIGNMENT 10A

1 **a** E **b** D **c** F

2 **a** 18·9 cm by 10·2 cm **b** 33 cm

3 **a** $x = 9, y = 15$ **b** $a = 16, b = 20$
c $x = 5{\cdot}7, y = 6{\cdot}9$

4 **a** 23·4 m **b** $x = 35, y = 81$

5 **a** 28 cm by 15 cm
b 1 cm : 4 m or 1 : 400; width 3·75 cm

ASSIGNMENT 10B

1 23, 29, 31, 35, 37, 41, 43, 44, 46, 47, 49, 52, 53, 55, 56, 58, 59

2 70·9% (1 dec. pl.)

3 220 g

4 **a** 4 **b** 7 **c** 49 **d** 16

5 **a** The percentage change for Australia as a whole.
b **i** about 1% increase **ii** about 23% decrease
iii about 32% decrease
c The statistics show an average decrease across Australia in the number of road fatalities of approximately 16%. All states and territories had a reduction in the number of fatalities except WA.

ASSIGNMENT 10C

1 **a** $3x$ **b** $3x^5$ **c** $3x^{-5}$
d $\frac{x^{-5}}{3}$ or $\frac{1}{3}x^{-5}$

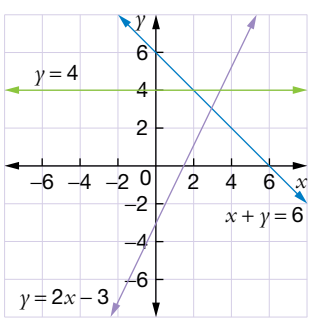
2 **a** 10^3 **b** 10^{-3} **c** 10^9 **d** 10^{-6}

3 **a** $\frac{9a}{20}$ **b** $\frac{a}{20}$ **c** $\frac{a^2}{20}$ **d** $\frac{5}{4} = 1\frac{1}{4}$

4 **a** 368·4 cm^2 **b** 255·3 cm^2

5 **a** \$1150 **b** \$54 080

6



7 **a**

		Bag B	
		R	W
Bag A	R_1	R_1R	R_1W
	R_2	R_2R	R_2W
	W	WR	WW

b **i** $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{2}$

Chapter 11

Prep quiz 11:01

1 $7 + 5 = 12$ **2** $9 - 2 = 7$
3 $25 - 8 = 17$ **4** $48 \div 6 = 8$
5 $7 \times 3 = 21$ **6** $8 + 12 = 20$
7 $(41 + 47) \div 2 = 44$ **8** $13 + 21 = 34$
9 $138 \div 23 = 6$ **10** $4 \times 5 - 8 = 12$

Exercise 11:01

1 **a** C **b** F **c** A **d** D **e** B **f** E

2 **a** $2n$ **b** $n + 3$ **c** $8 - n$ **d** $2n + 3$ **e** $\frac{n}{2}$ or $\frac{1}{2}n$ **f** $2n - 5$

3 **a** $5 + 7$ **b** $5 + y$ **c** $x + y$

4 **a** 3×7 **b** $7a$ **c** ab

5 **a** $8 - 3$ **b** $8 - p$ **c** $q - p$

6 **a** $\frac{8+12}{2} = 10$ **b** $\frac{8+x}{2}$ **c** $\frac{w+x}{2}$

7 **a** 5×75 cents **b** $75a$ cents **c** ab cents

8 **a** $\frac{30}{5}$ cm **b** $\frac{30}{t}$ cm **c** $\frac{A}{t}$ cm

9 **a** $15 + 6 = 21$ years old **b** $15 + y$

10 **a** $60 \times 3 = 180$ km **b** $60h$ km

11 **a** $10 - 3 \times 2 = 4$ m **b** $(X - 2x)$ m

12 $(100 - x)$ cents

13 **a** 8 **b** $y + 2$ **c** $y - 1$

14 **a** hm km **b** $\frac{k}{h}$ km/h **c** $\frac{k}{m}$ hours

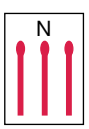
15 **a** $\$(R - P)$ **b** $\$(P + G)$

16 **a** $(a + b + 4)$ cm **b** $(2x + 2y)$ cm
c $4s$ cm **d** $(4x + 10)$ cm

17 **a** x^2 cm^2 **b** $x(x + 2)$ cm^2
c $\frac{xy}{2}$ cm^2 **d** $\frac{bh}{2}$ cm^2

18 **a** **i** $N + 3$ **ii** $3N + 1$
iii $2(N + 1)$ **iv** $3(N + 2) + 4$
v $3(N + 3) + 1$ **vi** $2(N + 4) + 6$

b You could possibly use 'red' to indicate a negative number of matches, like where the three matches are drawn in red.



$= N - 3$

19 **a** Add four to the product of three and m.
b Subtract five from the product of two and y.
c Three subtract the product of four and x.
d Add one to half of a.
e Half of the sum of a and one.
f Subtract half of a from four.
g Twice the sum of m and one.
h Subtract a from three and multiply the result by four.
i Multiply the sum of a and b by nine.
j Multiply the sum of a and three by five and then add four.
k Take away one from x, multiply the result by two and then subtract three.
l Subtract the product of three and the sum of a and five, from ten.

Fun spot 11:01

1 E, the first letter of the word Eight. Each letter is the first letter in the names of the numbers: One, Two, Three etc.
2 The pilot was the man's daughter.
3 Ask either guard: 'Which sack will the other guard tell me has the gold?'

Prep quiz 11:02

1 $2l + 2b$ **2** lb **3** πr^2 **4** $2r + 2w = 2(r + w)$
5 14 **6** 22 **7** 100 **8** -12
9 -1 **10** 3

Exercise 11:02

1 **a** 18 **b** 40 **c** 17 **d** 24 **e** 32
f 6 **g** 3 **h** 2 **i** 5 **j** 2
k 7 **l** 12·5 **m** 4 **n** 1 **o** 8
p 48 **q** 36 **r** 100 **s** 144 **t** 3000
u 12·5 **v** 4 **w** $1\frac{3}{4}$ **x** 80

2 **a** 7 **b** 17 **c** 2 **d** 36 **e** 49
f 25 **g** 56 **h** 14 **i** -12 **j** 3·5
k 2·5 **l** 56 **m** 84 **n** -8 **o** -20
p -432 **q** -9 **r** -48 **s** -4 **t** $2\frac{1}{12}$
u $\frac{7}{12}$ **v** -7 **w** 384 **x** 6

3 **a** **i** 31 **ii** 17 **iii** -18
b **i** 24 **ii** 35 **iii** 21·93
c **i** 198·4 **ii** 19·625 **iii** 31 420

Prep quiz 11:03

1 $2ab$ **2** $3ab^2$ **3** $3a + 2b$
4 $4 \times x \times y \times y$ **5** $(x + y) \times (x + y)$
6 $x \times x + y \times y$ **7** true **8** 300
9 8 **10** -12

Exercise 11:03

1 **a** I **b** J **c** N **d** F **e** E
f B **g** M **h** H **i** O **j** D
k K **l** C **m** G **n** L **o** A

2 **a** $A = 8{\cdot}64$ **b** $A = 54$ **c** $A = 27$
d $A = 12{\cdot}25$ **e** $A = 44$ **f** $A = 36{\cdot}3$
g $A = 80$ **h** $C = 21{\cdot}4$ **i** $V = 262{\cdot}44$
j $V = 2100{\cdot}84$ **k** $P = 12{\cdot}4$ **l** $c = 13$
m $\alpha = 105°$ **n** $\alpha = 140°$ **o** $E = 15$

3 **a** **i** $C = 100°$ **ii** $C = 0°$ **iii** $C = 40°$
b $E = 18{\cdot}75$ **c** $S = 44{\cdot}7$ **d** $V = 2011$ cm^3

4 **a** 0·9 **b** 828 **c** 36 **d** 729·6 **e** −4
f −48 **g** 189 **h** 12·48 **i** 1·61
5 **a** 7·1 **b** 19·9 **c** 6·9 **d** 25·0

Prep quiz 11:04

1 $x = 163$ **2** $y = 222$ **3** $x = 49$ **4** $x = 370$
5 $x = 96$ **6** $y = 183$ **7** $p = 28$ **8** $m = 54$
9 $a = 5$ **10** $t = 2$

Exercise 11:04

1 **a** 10 **b** 15 **c** 2·4 **d** 13·2 **e** 40
f 6 **g** 6·38 **h** $26\frac{2}{3}$
2 **a** 5 **b** 4 **c** 4·8 **d** 15 **e** 3·6
f 3·75 **g** 30 **h** 27
3 **a** 30 **b** $3\frac{1}{3}$ **c** 0·4 **d** 56 **e** ±5
f ±3 **g** ±12 **h** $\pm\frac{1}{2}$
4 **a** 2 **b** 3 **c** 0 **d** 12 **e** 0·6
f 2 **g** 6 **h** 0·8
5 **a** 4 **b** 25 **c** 3·1 **d** 6·79 **e** 1·6
6 **a** 3·64 **b** 0·4 **c** 13·5 **d** 13·2 **e** 100
7 **a** 3·13 **b** 1·78 **c** 1·45 **d** 7·72 **e** 37·78
8 **a** 113·097 cm^2 (3 dec. pl.) **b** 113·097 cm^2, yes

Prep quiz 11:05

1 7 **2** $x + 3$ **3** $x + y$ **4** 12
5 $3x$ **6** xy **7** $\$(50 - x)$ **8** $\$5x$
9 xy **10** $x + 5$ years

Exercise 11:05

1 **a** $8x = 32; x = 4$ **b** $4x = 100; x = 25$
c $7x = 42; x = 6$ **d** $12x = 78; x = 6\frac{1}{2}$
e $6x = 15{\cdot}6; x = 2{\cdot}6$
2 **a** $x + 6 = 10; x = 4$ **b** $x + 5 = 22; x = 17$
c $x + 22 = 50; x = 28$ **d** $x + 17 = 12; x = -5$
e $12 + x = 38; x = 26$
3 **a** $x - 3 = 10; x = 13$ **b** $x - 5 = 22; x = 27$
c $x - 25 = 10; x = 35$ **d** $x - 17 = 18; x = 35$
e $6 - x = 4; x = 2$
4 **a** $\frac{x}{5} = 4; x = 20$ **b** $\frac{x}{8} = 7; x = 56$
c $\frac{1}{10}x = 8; x = 80$ **d** $\frac{x}{6} = 7; x = 42$
e $\frac{10}{x} = 2; x = 5$
5 **a** $2x + 6 = 14; x = 4$ **b** $5 + 3x = 20; x = 5$
c $5x - 8 = 22; x = 6$ **d** $4(x + 5) = 56; x = 9$
e $\frac{1}{2}x - 5 = 3; x = 16$
6 **a** $2x + 3 = 33; x = 15$ **b** $4x - 3 = 25; x = 7$
c $2(x + 3) = 22; x = 8$ **d** $\frac{x}{4} - 7 = 1; x = 32$
e $\frac{x + 4}{3} = 8; x = 20$
7 **a** 11 **b** 7 **c** 17 **d** 8

Fun spot 11:05

1 Both weigh the same (3 kg)
2 742 **3** 18 **4** 8:15 am

Exercise 11:06

1 **a** 13 years, 39 years **b** \$480, \$80
c Anne \$25, Joan \$18 **d** Jim \$165, Alan \$335
e 40 years
2 **a** 13 **b** 19 cm by 13 cm **c** 4 kg **d** 24 km
3 **a** \$2 per kg
b They meet at 2:48 pm when X has travelled 24 km and Y has travelled 16 km
c The son is 28 years old and his father is 56 years old
d 3600 litres
4 **a** Bill is 20 years old and Fred is 45 years old
b 16 five-cent coins, 34 ten-cent coins
c 12
d small tank 900 L; large tank 1200 L
e 50 km

Diagnostic test 11

1 **a** $y = 4x$ **b** $y = 2x + 2$ **c** $y = x^2$
2 **a** $x + y$ **b** $\frac{5 + m}{2}$ or $(5 + m) \div 2$
c $\$pb$ **d** $200 - x$
3 **a** 13 **b** 45 **c** 310
4 **a** 18 **b** 36·2 **c** 7·76
5 **a** 157 **b** 16·09 **c** 3·54
6 **a** 4·5 **b** 8 **c** 8·875
7 **a** 22·5 **b** 11·1 **c** 20
8 **a** $2a + 7 = 10$ **b** $\frac{a}{3} - 4 = 4$
c $3(a + 6) = 32$
9 **a** $x + (x + 5) = 57$ **b** $3x + x + 3x + x = 48$
c $3x + 10 = 2(x + 10)$

ASSIGNMENT 11A

1 **a** $12ab$ **b** $7x + 9y$ **c** $\frac{d}{s}$
d $mn + \frac{1}{2}mn = \frac{3mn}{2}$
2 **a** **i** 11 **ii** 20
b **i** 11·3 **ii** 0·96
c **i** 20·25 **ii** 6·75
3 **a** **i** 22 **ii** $12\frac{1}{2}$ **iii** 26·26 **iv** −66·25
b **i** 3 **ii** 2 **iii** 24·5 **iv** 20
v 14·15
c **i** 4 **ii** 13·3 **iii** 10 **iv** 4·032
v 3 **vi** 0·3
4 **a** 66 **b** 8 **c** 4·1
5 **a** $3x + 7 = 15, x = \frac{8}{3}$ **b** $5(x - 9) = 30, x = 15$
c $8x + 10 = 12x - 7, x = \frac{17}{4}$

ASSIGNMENT 11B

1 **a** $11 \times 17 \times 23$
b Only 1, because the three numbers are all prime numbers.
2 \$320 **3** 84 : 60 : 36 or 7 : 5 : 3
4 17 **5** 120 g
6 **a** 6 **b** 24 **c** 60 **d** 720

ASSIGNMENT 11C

1 **a** $a > 8{\cdot}5$ **b** $x > -4$ **c** $m \leq -20$
2 **a** \$16 560 **b** \$5382
3 **a** 5 **b** x-int = 3; y-int = 4·5 **c** −2
4 123·552 cm^3 **5** \$1025; 39%
6 **a** \$1320.50 **b** \$535.55

Chapter 12

Exercise 12:01

1 **a** BC **b** DE **c** KL **d** PR
e TU **f** YZ
2 **a** AB **b** EF **c** LM **d** PQ
e ST **f** XZ
3 **a** AC **b** DF **c** KM **d** QR
e SU **f** XY
4 **a** AC **b** AC **c** $\angle C$ **d** $\angle C$
5 **a** $\frac{5}{12}$ **b** $\frac{8}{6}$ **c** $\frac{15}{8}$
6 **a** $\frac{12}{13}$ **b** $\frac{6}{10}$ **c** $\frac{8}{17}$
7 **a** $\frac{9}{15} = \frac{3}{5}$ **b** $\frac{12}{15} = \frac{4}{5}$ **c** $\frac{12}{15} = \frac{4}{5}$ **d** $\frac{12}{9} = \frac{4}{3}$

Prep quiz 12:02

1 opposite **2** hypotenuse **3** adjacent **4** similar
5 change size **6** equal **7** 12 cm
8 6 **9** 9 **10** 4

Exercise 12:02

1 **a**

	θ	$\frac{o}{h}$	$\frac{a}{h}$	$\frac{o}{a}$
1	30°	0·5	0·9	0·6
2	30°	0·5	0·9	0·6
3	30°	0·5	0·9	0·6

b The results for $\frac{o}{h}$, $\frac{a}{h}$ and $\frac{o}{a}$ are the same for each triangle.

2 **a**

	θ	$\frac{o}{h}$	$\frac{a}{h}$	$\frac{o}{a}$
1	50°	0·8	0·7	1·2
2	50°	0·8	0·7	1·2
3	50°	0·8	0·7	1·2

b The results for $\frac{o}{h}$, $\frac{a}{h}$ and $\frac{o}{a}$ are the same for each triangle.

3 **a** **i** $\frac{2{\cdot}8}{4{\cdot}9} \div 0{\cdot}6$ **ii** $\frac{4}{4{\cdot}9} \div 0{\cdot}8$ **iii** $\frac{2{\cdot}8}{4} = 0{\cdot}7$
b **i** $\frac{5{\cdot}6}{9{\cdot}8} \div 0{\cdot}6$ **ii** $\frac{8}{9{\cdot}8} \div 0{\cdot}8$ **iii** $\frac{5{\cdot}6}{8} = 0{\cdot}7$
c The results for $\frac{o}{h}$, $\frac{a}{h}$ and $\frac{o}{a}$ are the same for each triangle.
4 **a** 2 **b** $x = 6, y = 8$
c **i** $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$
d **i** $\frac{6}{10} = \frac{3}{5}$ **ii** $\frac{8}{10} = \frac{3}{5}$ **iii** $\frac{6}{8} = \frac{3}{4}$
e The values for each ratio are the same in both triangles.

Exercise 12:03

1 **a** $\frac{4}{5}$ **b** $\frac{3}{10}$ **c** $\frac{8}{5}$
2 **a** $\frac{5}{4}$ **b** $\frac{10}{3}$ **c** $\frac{5}{8}$
3 **a** $\tan A = \frac{24}{10}$; $\tan C = \frac{10}{24}$
b $\tan D = \frac{9}{40}$; $\tan F = \frac{40}{9}$
c $\tan P = \frac{9}{12}$; $\tan Q = \frac{12}{9}$
4 **a** $YZ = 6$; $\tan X = \frac{6}{8}$ **b** $YZ = 6$; $\tan X = \frac{6}{5}$
c $YZ = 3$; $\tan X = \frac{3}{2}$
5 **a** $\frac{a}{b}$ **b** $\frac{m}{n}$ **c** $\frac{x}{y}$
6 **a** **i** 0·17 **ii** 0·83 **iii** 1·73
b **i** 14° **ii** 35° **iii** 54°
7 **a** 30° **b** 65° **c** 40°
8 **a** $\frac{x}{5}$ **b** $\frac{m}{20}$ **c** $\frac{a}{12{\cdot}6}$
9 **a** 0·176 **b** 0·364 **c** 0·577 **d** 1·732
e 1·134 **f** 0·470 **g** 1·060 **h** 3·096
i 0·277 **j** 1·925
10 **a** 0·466 **b** 0·754 **c** 1·963 **d** 1
e 0·296 **f** 0·252 **g** 0·538 **h** 2·762
i 0·584 **j** 1·478

Investigation 12:03

1 isosceles **2** 1
3 $\tan\alpha < \tan 45°$ **4** $\tan\beta > \tan 45°$
5 α gets closer to 0° **6** β gets closer to 90°
7 $\tan\alpha$ gets closer to 0
8 $\tan\beta$ becomes infinitely large.

Prep quiz 12:04

1 0·849 **2** $\frac{12}{5}$ **3** $\frac{12}{9}$ **4** 56°
5 1·111 **6** 1·111 **7** 5·553 **8** 5·553
9 $\frac{x}{4}$ **10** $\frac{1}{\tan 85°}$

Exercise 12:04

1 **a** 3·1 **b** 5·1 **c** 1·9 **d** 16·2 **e** 3·8
f 23·0 **g** 9·8 **h** 22·1 **i** 3·8
2 5·12 m **3** 11·8 cm **4** 6·16 m **5** 14·4 cm

6 **a** 8·6 **b** 5·0 **c** 14·3 **d** 7·5 **e** 17·3
f 2·4
7 1336 m **8** 20·3 cm

Prep quiz 12:05

1 $\frac{5}{4}$ **2** $\frac{5}{7}$ **3** $\frac{3}{4}$ **4** 0·8 **5** $0.\dot{6}$
6 $0.41\dot{6}$ **7** 0·088 **8** 1·111 **9** 9° **10** 50°

Exercise 12:05

1 **a** 39° **b** 66° **c** 27° **d** 36° **e** 57°
f 30° **g** 22° **h** 14° **i** 76°
2 **a** 30°58′ **b** 36°52′ **c** 47°44′ **d** 32°0′
e 50°12′ **f** 52°8′ **g** 26°2′ **h** 66°36′
i 31°12′ **j** 50°28′ **k** 46°49′
3 77° **4** 7° **5** 38° **6** 54°
7 56°, 34° **8** 4°
9 **a** 63° **b** 27° **c** 34°
10 **a** $\theta = 45°, \phi = 26°34', \alpha = 18°26'$ **b** 26°34′

Prep quiz 12:06

1 AB **2** PR **3** YZ **4** AB **5** QR
6 YZ **7** $\frac{3}{4}$ **8** $\frac{3}{5}$ **9** $\frac{4}{5}$ **10** $\frac{4}{3}$

Exercise 12:06

1 **a** $\sin\theta = \frac{3}{5}; \cos\theta = \frac{4}{5}$ **b** $\sin\theta = \frac{5}{13}; \cos\theta = \frac{12}{13}$
c $\sin\theta = \frac{24}{25}; \cos\theta = \frac{7}{25}$
2 **a** 15, 0·6, 0·8 **b** $\sqrt{45}$, 0·447, 0·894
c $\sqrt{8}$, 0·707, 0·707 **d** $\sqrt{21}$, 0·917, 0·4
e $\sqrt{51}$, 0·714, 0·7 **f** $\sqrt{20}$, 0·667, 0·745
3 **a** 0·5 **b** 0·8660 **c** 0·9455
d 0·5299 **e** 0·1219 **f** 0·0872
g 0·8746 **h** 0·8746 **i** 0·2924
j 0·6383 **k** 0·6443 **l** 0·9926
4 **a** $\sin A$ increases in value from 0 to 1.
b $\cos A$ decreases in value from 1 to 0.
5 **a** **i** $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{5}{13}$ **iv** $\frac{12}{13}$
b **i** $\frac{12}{15}$ **ii** $\frac{9}{15}$ **iii** $\frac{12}{13}$ **iv** $\frac{5}{13}$

Prep quiz 12:07A

1 false **2** true **3** true **4** true **5** 0·934
6 0·946 **7** 0·707 **8** 7·47 **9** 9·46 **10** 4·74

Exercise 12:07A

1 **a** 5·9 **b** 5·1 **c** 2·5 **d** 8·0 **e** 11·7
f 19·4 **g** 8·4 **h** 12·4 **i** 46·4 **j** 45·5
2 3·2 m **3** 9·46 m **4** 13 cm
5 20 m **6** 23 m **7** 15·0 cm
8 20·57 m **9** 2·13 m **10** 13·43 m

Prep quiz 12:07B

1 $\frac{a}{c}$ **2** $\frac{b}{c}$ **3** $\frac{x}{15}$ **4** $\frac{y}{15}$ **5** $\frac{x}{10}$
6 $\frac{10}{x}$ **7** $\frac{10}{x}$ **8** $\frac{x}{5}$ **9** $\frac{x}{5}$ **10** 7·78

Exercise 12:07B

1 **a** 19·4 **b** 9·3 **c** 48·5 **d** 7·8 **e** 10·1
f 3·4 **g** 4·4 **h** 10·2 **i** 25·7 **j** 13·3
2 5·54 m **3** 4·06 m **4** 31·8 m **5** 8·3 cm
6 6·15 m **7** 60 cm **8** 54 m^2

Prep quiz 12:08

1 $\frac{5}{13}$ **2** $\frac{12}{13}$ **3** $\frac{5}{12}$ **4** $\frac{15}{17}$
5 $\frac{8}{17}$ **6** $\frac{15}{8}$ **7** 1·875 **8** 31°
9 56° **10** 62°

Exercise 12:08

1 **a** 30° **b** 60° **c** 56° **d** 37° **e** 46°
f 54° **g** 24° **h** 52° **i** 60°
2 **a** 23°35′ **b** 48°35′ **c** 44°26′ **d** 53°8′
e 33°33′ **f** 45°34′ **g** 42°56′ **h** 45°4′
i 49°10′ **j** 48°50′ **k** 44°52′ **l** 77°9′
3 39° **4** 17° **5** 62° **6** 20° **7** 93°

Exercise 12:09A

1 125 m **2** 68·7 m **3** 12°32′ **4** 3840 m
5 2000 m **6** 2·0 m **7** 58° **8** 3°
9 68 m **10** 52·2 m (1 dec. pl.)

Exercise 12:09B

1 **a** 050° **b** 140° **c** 200°
d 230° **e** 310° **f** 340°
2 **a** N50°E **b** S40°E **c** S20°W
d S50°W **e** N50°W **f** N20°W
3 **a** 306·4 km **b** 257·1 km
4 **a** 192·8 km **b** 229·8 km
5 21 km **6** 4·5 km **7** 211°
8 30 km **9** 6 km **10** 4 km

Exercise 12:09C

1 12·68 m **2** 609 m **3** 8 m **4** 7·7 cm
5 65° **6** 67° **7** 74°
8 **a** 2257 mm **b** 3299 mm
c 3511 mm **d** 1201 mm

Diagnostic test 12

1 **a** AB **b** PR **c** YZ
2 **a** $\frac{5}{12}$ **b** $\frac{15}{8}$ **c** $\frac{4}{3}$
3 **a** 2·5 **b** 5·0 **c** 34·9
4 **a** 40° **b** 51° **c** 40°
5 **a** $\frac{24}{25}$ **b** $\frac{7}{25}$ **c** $\frac{3}{5}$
6 **a** 5 **b** 6·9 **c** 6·5
7 **a** 7·1 **b** 10·4 **c** 14·6
8 **a** 36°52′ **b** 62°11′ **c** 53°8′

ASSIGNMENT 12A

1 **a** 9·23 **b** 5·08 **c** 19·02
2 **a** 61°33′ **b** 29°14′ **c** 61°56′
3 **a** 8° (nearest degree)
b **i** 975 km **ii** 1689 km
4 344 m
5 **a** 176·8 km **b** 576·8 km **c** 603·3 km; N17°E

ASSIGNMENT 12B

1 10 am
2 $\theta = 30°$; $a = 1212$ mm; $b = 2425$ mm
3 The final answer is \$14.40 less than \$360.
4 **a** 21·6 m **b** 75·168 m^2
5 258·4 m
6 **a** tetrahedron 6; cube 12; dodecahedron 30; icosahedron 30
b no. of vertices = (no. of faces × no. of vertices on each face) ÷ (no. of faces that share a vertex)
c tetrahedron 4; cube 8; dodecahedron 20; icosahedron 12

ASSIGNMENT 12C

1 $x = 12$ cm, $y = 24$ cm
2 **a** 2 **b** −1 **c** $\frac{1}{2}$
3 **a** \$2660 **b** \$98.34
4 **a** −11 **b** −16 **c** 8
5 **a** 20
b **i** $\frac{6}{20} = \frac{3}{10}$ **ii** $\frac{1}{20}$
6 **a** 40·095 m^3 **b** 36·855 kL

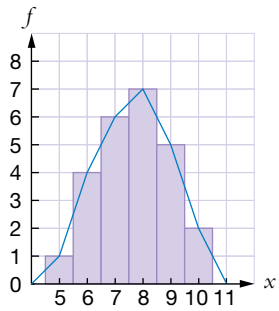
Chapter 13

Exercise 13:01

1 **a** **i** 7 **ii** 5 **iii** 5 **iv** 5·714
b **i** 4 **ii** 5 **iii** 7 **iv** 6·75
c **i** 6 **ii** 24 **iii** 24 **iv** 22·6
d **i** 0·8 **ii** 1·5 **iii** 1·5 **iv** 1·5
2 **a** 6·05 **b** 64·25 **c** 7 **d** 2·71
e 50·24 **f** 14·78
3 **a** 181 **b** 394
4 **a** mode = 8; mean = 7·68; median = 8

Outcome	Freq	fx
5	1	5
6	4	24
7	6	42
8	7	56
9	5	45
10	2	20
Total:	25	192

b 14; 11 **c** 5 **d**



5 **a** Mazda **b** no **c** no
d a column graph or a sector graph

6 **a**

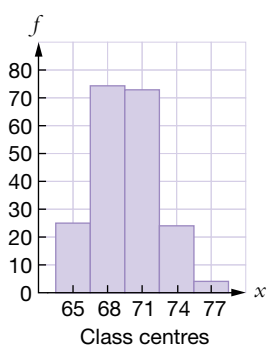
Children	Freq	fx
0	4	0
1	6	6
2	9	18
3	5	15
4	3	12
5	0	0
6	1	6
Totals:	28	57

b 28 **c** 57 **d** 2 **e** above
7 **a** **i** 14 **ii** 23 and 24
iii 23 **iv** 23·375
b **i** 51 **ii** 53, 63 and 76
iii 63·5 **iv** 65·3 (2 dec. pl.)

8 **a**

Class	*c.c.*	Freq	$f \times c.c.$
64–66	65	25	1 625
67–69	68	74	5 032
70–72	71	73	5 183
73–75	74	24	1 776
76–78	77	2	154
	Total:	198	13 770

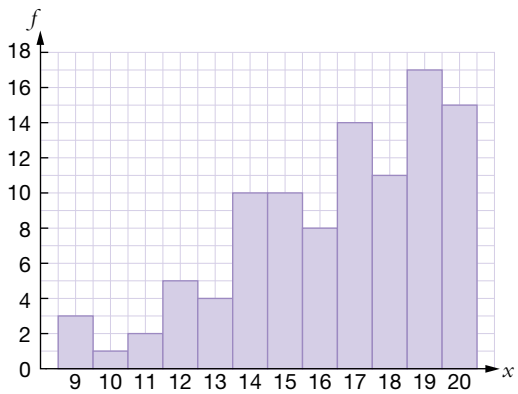
b 66 **c** 14 **d** 67–69 **e** 69·5
f median lies between 67–69 and 70–72
g



9 a

Number correct	Frequency (f)	fx	c.f.
9	3	27	3
10	1	10	4
11	2	22	6
12	5	60	11
13	4	52	15
14	10	140	25
15	10	150	35
16	8	128	43
17	14	238	57
18	11	198	68
19	17	323	85
20	15	300	100
Total:	100	1648	

b



c 16·48 **d** above

e 2 is the best interval; 5 and 10 give too few classes.

f i

Class	Class centre	f
9–10	9·5	4
11–12	11·5	7
13–14	13·5	14
15–16	15·5	18
17–18	17·5	25
19–20	19·5	32
	Total:	100

ii

Class	Class centre	f
6–10	8	4
11–15	13	31
16–20	18	65
	Total:	100

iii

Class	Class centre	f
1–10	5·5	4
11–20	15·5	96
	Total:	100

Six classes are sufficient to show the characteristics (shape) of the data. As the number of classes is reduced to 3 and then 2, the characteristics (or shape) of the data are lost.

10 a i

Class	f
0–1	1
2–3	0
4–5	2
6–7	2
8–9	1
10–11	3
12–13	1
14–15	2
16–17	2
18–19	3
20–21	4
22–23	2
24–25	2
26–27	2
28–29	3
30–31	3
32–33	2
34–35	2
36–37	1
38–39	2

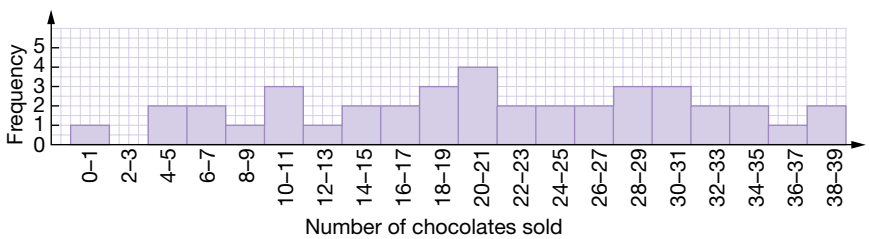
ii

Class	f
0–4	2
5–9	4
10–14	5
15–19	6
20–24	8
25–29	5
30–34	5
35–39	5

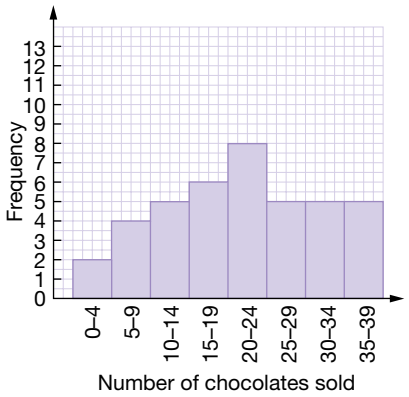
iii

Class	f
0–9	6
10–19	11
20–29	13
30–39	10

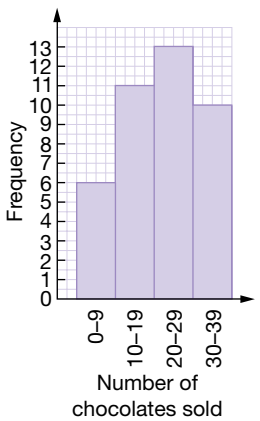
b i



ii



iii



c Answers may vary, but here, a class interval of 5 gives a good view of trends without losing too much detail.

11 a

Leaf (Maths)	Stem	Leaf (English)
	3	9
9 6	4	5 6 8
9 7 5 1	5	1 2 3 9 9 9
8 6 3 2	6	0 1 1 3 6 8
7 7 5 2 0	7	1 2 5 9
7 4 1	8	
7 4	9	

b Maths

i 51 **ii** 69 **iii** 69·5;

English

i 40 **ii** 59·5 **iii** 59·35

c No, actually 75 is a better mark for English than 77 is for Maths compared against the group results.

12 a

Stem	Leaf	Frequency	Class centre	$f \times c.c.$
$6^{(5)}$	8 8 9 9 9	5	67	335
$7^{(0)}$	0 0 0 0 1 2 2 3 4	9	72	648
$7^{(5)}$	5 5 5 5 6 6 7 7 7 7 8 8	12	77	924
$8^{(0)}$	0 0 0 1 1 1 1 2 3 4 4	11	82	902
$8^{(5)}$	5 8	2	87	174
$9^{(0)}$	0 0 0 1 2 3 3 4	8	92	736
$9^{(5)}$	7 8 9	3	97	291
Total:		50		4010

b modal class is 75–79 mean = 80·2

c median = 78

d median class is 75–79; includes answer to part c

13 Answers will vary due to experimental data.

Exercise 13:02

1 a

x	f	$c.f.$
0	3	3
1	8	11
2	11	22
3	17	39
4	9	48
5	2	50
Total:	50	

b

x	f	$c.f.$
9	1	1
10	13	14
11	22	36
12	30	66
13	21	87
14	13	100
Total:	100	

c

x	f	$c.f.$
0	1	1
1	0	1
2	3	4
3	8	12
4	14	26
5	20	46
6	31	77
7	32	109
8	28	137
9	11	148
10	5	153
Total:	153	

d

x	f	$c.f.$
15	4	4
20	8	12
25	3	15
30	9	24
35	7	31
40	10	41
45	15	56
50	8	64
55	10	74
60	2	76
65	4	80
Total:	80	

2

Score	Frequency	fx	$c.f.$
5	2	10	2
6	3	18	5
7	5	35	10
8	4	32	14
9	4	36	18
10	2	20	20

a 7 **b** 7·55 **c** 7·5 **d** 5

3 a 1 **b** 2 **c** 7 **d** 12

e 5 **f** 3

4 a

Dad
Prog. Total
4
8
14
16
21

b **i** Adam **ii** Dad **iii** Sharon

c 6; 3 times **d** Dad

e

	Sharon		Adam		Dad		Bron	
	Score	Prog. Total	Score	Prog. Total	Score	Prog. Total	Score	Prog. Total
Round 6	6	28	6	27	7	28	5	24

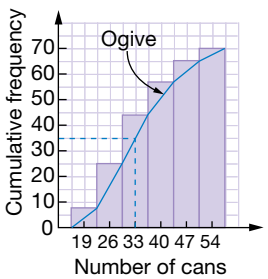
5

Number (x)	f	fx	$c.f.$
2	3	6	3
3	5	15	8
4	9	36	17
5	12	60	29
6	14	84	43
7	17	119	60
8	15	120	75
9	11	99	86
10	8	80	94
11	4	44	98
12	2	24	100
Total:	100	687	

a mode = 7; mean = 6·87 **b** 7 **c** 43

6 a 3 **b** 13 **c** 7 **d** 6 **e** 7 **f** 5

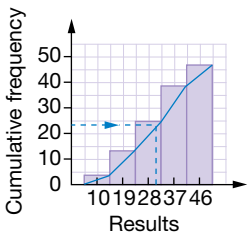
7 a



b 30 – 36

8 a 6–14, 15–23, 24–32, 33–41, 42–50

b



c The median class is 24–32.

9 a 15–19

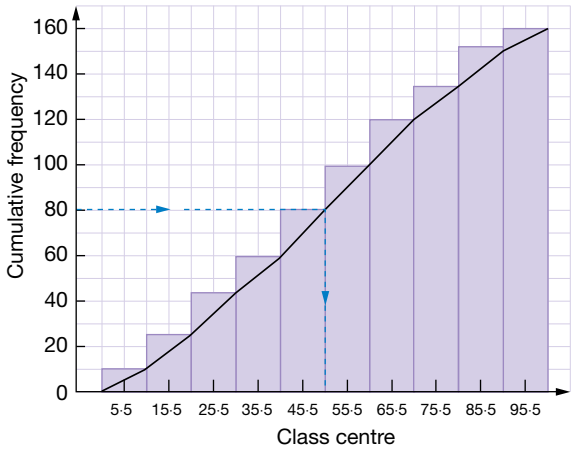
b

$c.c.$	2	7	12	17	22	27	32
f	2	4	3	9	4	5	3

c 15–19 **d** 18

10 a 99 **b** 61–70 **c** 50·7 (1 dec. pl.)

d



e The median involves two classes: 41–50 and 51–60.

f The median = 50·5

g **i** 16·25% **ii** 15% **iii** 50%

Prep quiz 13:03

1 0 **2** 1 **3** 7 **4** Set C **5** yes

6 8 **7** 8 **8** 8 **9** Sets A and D **10** Set C

Exercise 13:03

1 a 3 **b** 3·5 **c** 13·5 **d** 15·5 **e** 15

2 a 13 **b** 11 **c** 15 **d** 2

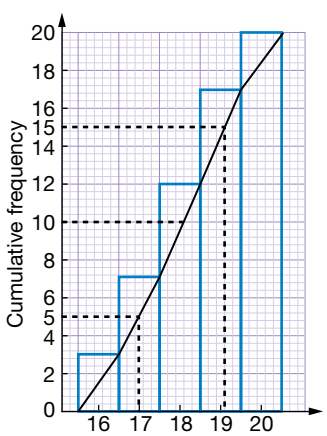
3 a 4·5 **b** 3 **c** 5·5 **d** 2·5

4

Score	Freq.	Cumulative freq.
2	1	1
3	3	4
4	6	10
5	10	20
6	10	30
7	4	34
8	4	38
9	2	40

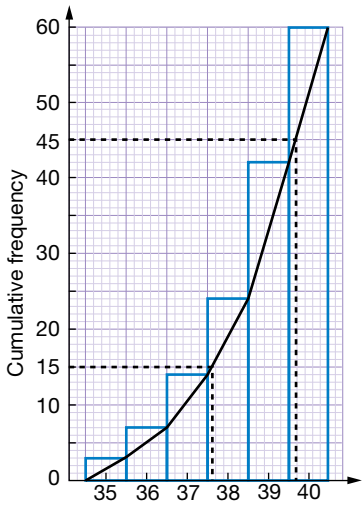
IQR = 2 (using either method)

5 a



IQR = 19 – 7
= 2

b



IQR = 40 − 38
= 2

6 **a** 20 **b** 4 **c** 8·5

7 **a** 4 **b** 5

8 **a** 65 **b** 75 **c** 10

9 **a** $Q_1 = 21{\cdot}5$, $Q_2 = 23$, $Q_3 = 25{\cdot}5$, IQR = 4

b $Q_1 = 54{\cdot}5$, $Q_2 = 63{\cdot}5$, $Q_3 = 77$, IQR = 22·5

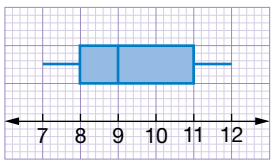
Exercise 13:04

1 **a** **i** 57 **ii** 32 **iii** 12

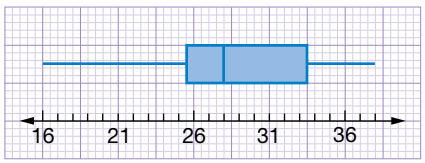
b **i** 51 **ii** 20 **iii** 6

c **i** 52 **ii** 25 **iii** 10

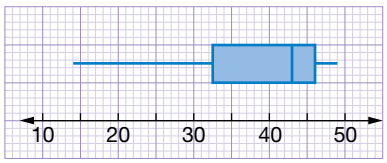
2 **a** (7, 8, 9, 11, 12)



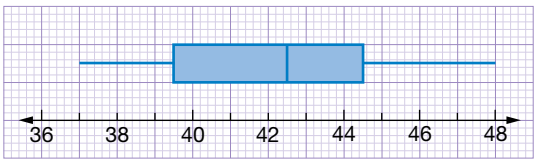
b (16, 25·5, 28, 33·5, 38)



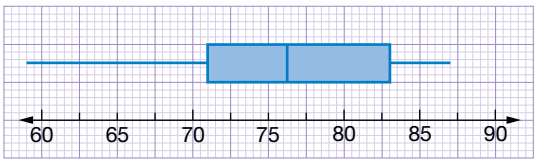
c (14, 32·5, 43, 46, 49)



3 **a** (37, 39·5, 42·5, 44·5, 48)



b (59, 71, 76·5, 83, 87)



4 **a** **i** 25% **ii** 50%

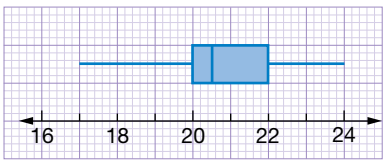
b **i** 50% **ii** 75%

c **i** 3 **ii** 4

d Year 10

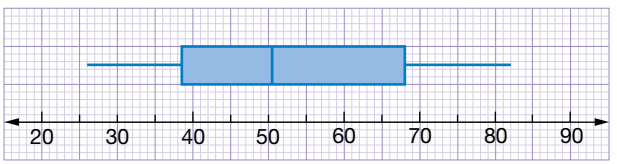
5 **a** (17, 20, 20·5, 22, 24)

b



6 **a** (26, 38·5, 50·5, 68, 82)

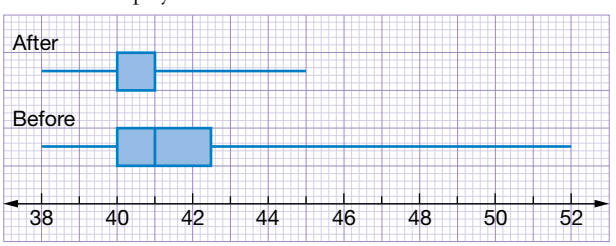
b



7 **a** (74, 76, 78, 84, 85) **b** Ken **c** Ray

d As the range can be easily affected by an outlier, it is not a good measure of consistency. Ray's lower interquartile range would suggest he is the most consistent player.

8



The traffic changes have slightly reduced the median time for Rick's trip, and have resulted in more consistent trip times, as indicated by the reduced interquartile range. In fact, 75% of the trip times were less than 41 minutes compared to 50% before the changes. This would suggest that the traffic changes had been effective in reducing the variability in the trip times.

Exercise 13:05

1 **a** B and D. The skew is caused because of the unevenness of the number of students in each age group. School B has a small senior school, whereas school D has a smaller junior school.

b A

c **i** A **ii** B **iii** C or D **iv** C

2 **a** class 1

b **i** class 2 **ii** class 2

c class 1: mean = 88·7, range = 16;
class 2: mean = 91·1, range = 19

3 **a** **i** negatively skewed
ii positively skewed

b Girls borrow more books than boys.

c Grouped frequency histograms allow a scale to be used to represent large frequencies. If a back-to-back stem-and-leaf plot were used, you would need a large number of leaves in the girls 30–39 column and the boys 1–9 column. This would make the stem-and-leaf plot very wide.

d The distribution would be more uniform. The lowness of the 41–50 column compared to the other columns would make the distribution slightly positively skewed.

4 **a** Topic 1

b Topic 1 has the smallest spread of scores. It has a lower standard deviation (15·1) and interquartile range (24·5), than that of topic 2.

c Topic 1: mean = 61, median = 60
Topic 2: mean = 72, median = 78
The results suggest that the class performed much better on topic test 2. This could be because the class was better prepared or perhaps because the test was easier.

5 **a** weak swimmer **b** excellent swimmer

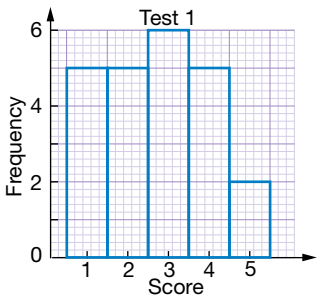
c $43\frac{1}{3}\%$ **d** 80%

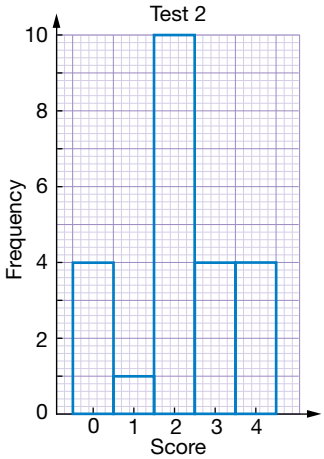
e The program was very successful.

6 **a**

Test 1	
Score	Frequency
1	5
2	5
3	6
4	5
5	2

Test 2	
Score	Frequency
0	4
1	1
2	10
3	4
4	4





b Test 1 Test 2
Mean = 2·7 Mean = 2·1
Median = 3 Median = 2
Both the mean and median for Test 2 are lower than that of Test 1. This suggests that test 2 was more difficult than Test 1.

c The interquartile range for Test 1 is 2, whereas for Test 2 is 1. This would suggest that the scores for Test 1 have a greater spread than those of Test 2.

7 **a** This shows that the median is 160 cm, the middle half of the scores lie in the range 153 to 168 cm and the 30 heights lie in the range 135 cm to 178 cm.

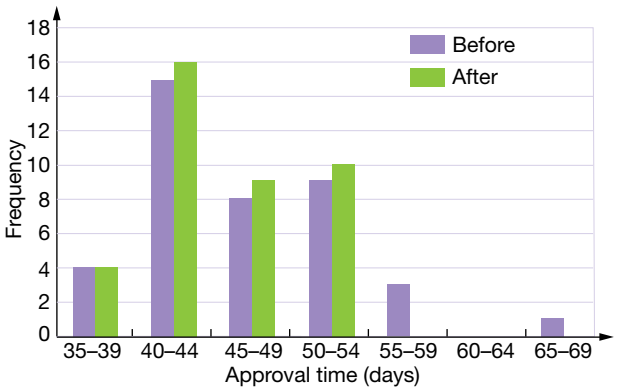
b Yes. The median class is 160–164 and this contains the median score of the box-and-whiskers graph and it is possible for the middle half of the scores to be in the range 153 cm to 168 cm and the range to be 135 cm to 178 cm.

8 Both brands have the same median, but brand X has a larger interquartile range, a lower minimum and a lower maximum. This would suggest that brand Y will be the best performer.

9 **a** The data for group 1 is positively skewed, whereas that for group 2 is close to normally distributed. The mean for group 1 would be in the 120s, whereas group 2 is in the 140s. The shape suggests that group 1 is fitter than group 2.

b Group 1: median = 129, IQR = 135 − 127 = 8
Group 2: median = 146, IQR = 154 − 137 = 17
Group 1 has a lower median and its scores are not as spread. Hence group 1 is overall a fitter group than group 2.

10 a The data could be organised into a grouped frequency distribution table using classes of 35–39, 40–44, 45–49, etc. This would give 7 classes. Using class intervals of 30–39, 40–49 etc. would result in only four classes and would not show the differences between the two sets of data. Stem-and-leaf plots would probably not be appropriate because of the number of leaves needed on the 4 stem (40–49) or the 4^0 stem (40–44). Either stem-and-leaf plot would probably be too wide. The best way to display the data is a double-column graph with one column for 'before' and another column for 'after'.



b Either the mean or median could be used as measures of cluster, whereas the interquartile range is probably the best measure of spread to use even though the spread of both sets of data is similar.

c Council's new procedures have reduced the average time taken to approve applications. The median time has been reduced from 45 days to 44 days. The 'after' application times also have a smaller spread than the 'before' times, indicating that the scores are closer to the mean. This would indicate that the new procedures have resulted in more consistent approval times.

Investigation 13:05

1 a 3·8% **b** 3·3% **c** 1·2%

2 a 3·3% **b** 3·5% **c** 1·4%

3 On the whole, the male population has aged over this period with a decrease in the younger age group and an increase in the older age groups. The same trend is noted in the female population.

4 a **i** 6·9% **ii** 9·1%

b 21 101 000

5 The population in 2007 was greater than the population in 1987.

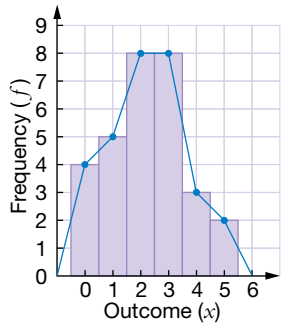
Diagnostic test 13

1 a

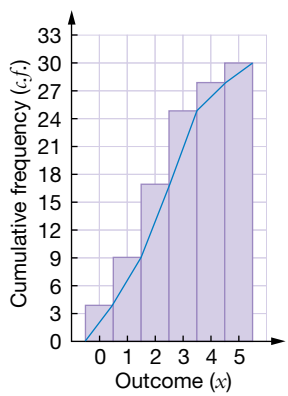
Outcome (x)	f	$c.f.$
0	4	4
1	5	9
2	8	17
3	8	25
4	3	28
5	2	30
	30	

b 2 **c** 25

d



e



f Range is 5

g Modes are 2 and 3 (bi-modal), median is 2, mean is 2·23 (2 dec. pl.)

2 a 28·23 **b** 4·36 (2 dec. pl.)

3 a

Class	Class centre	Freq.	$c.f.$
0–9	4·5	3	3
10–19	14·5	5	8
20–29	24·5	6	14
30–39	34·5	8	22
40–49	44·5	9	31
50–59	54·5	6	37
60–69	64·5	3	40

b Stem and leaf plot or Ordered stem and leaf plot

Stem	Leaf
0	0 9 4
1	8 7 6 4 1
2	7 6 0 8 5 2
3	1 8 8 7 7 8 3 8
4	5 9 1 3 3 4 3 0 8
5	0 8 2 1 6 3
6	3 9 2

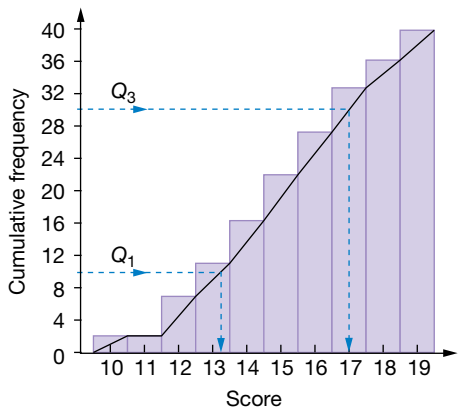
Stem	Leaf
0	0 4 9
1	1 4 6 7 8
2	0 2 5 6 7 8
3	1 3 7 7 8 8 8 8
4	0 1 3 3 3 4 5 8 9
5	0 1 2 3 6 8
6	2 3 9

4 **a** 7 **b** 13 **c** 2

5 **a** 5 **b** 7 **c** 14

6 **a** 7·5

b



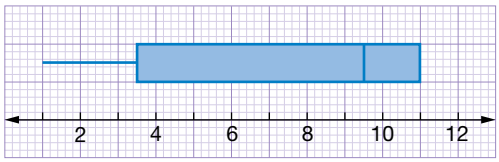
Interquartile range = $Q_3 - Q_1 = 17 - 13 = 4$

c 2·5 **d** 12

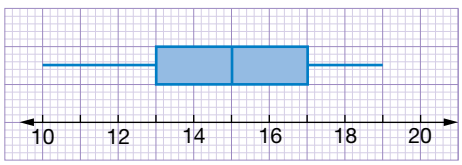
7 **a** (1, 3·5, 9·5, 11, 11) **b** (10, 13, 15, 17, 19)

c (20, 28·5, 29·5, 31, 33) **d** (38, 41·5, 46·5, 53·5, 60)

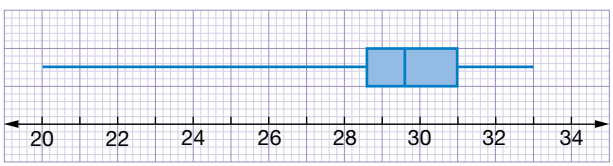
8 **a**



b

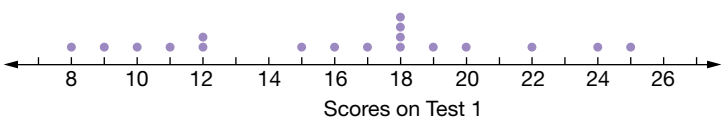


c



9 **a** 25 **b** range = 60, interquartile range = 22

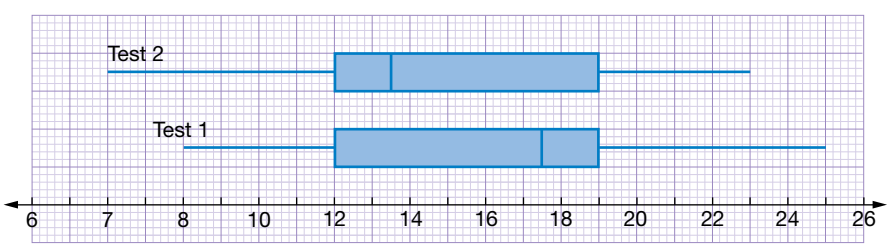
10 **a**



b

Scores		
Test 1 Leaf	Stem	Test 2 Leaf
9 8	0	7 9
9 8 8 8 8 7 5 5 2 2 1 0	1	0 1 2 2 2 2 3 4 5 6 8 9
5 4 2 0	2	0 0 1 3

c



ASSIGNMENT 13A

1 **a** Yes, 64 is an outlier.

b **i** (64, 71·5, 73, 76, 81) **ii** (70, 72, 73, 76 81)

c **i** 4·5 **ii** 4

d It is positively skewed.

2 **a** 4 **b** 43 **c** 8 **d** 40 **e** 72

f 24

g

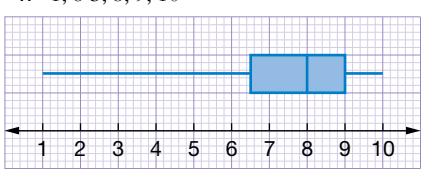
Class	*c.c.*	*f*
30–39	34·5	3
40–49	44·5	5
50–59	54·5	4
60–69	64·5	7
70–79	74·5	8
80–89	84·5	7
90–99	94·5	6

3 Year 4 have clearly performed better. The mean and median are both 66·5 compared to the mean and median for year 3, which are 49·8 and 48 respectively. Both sets of results have a similar spread, especially if the score of 35 in the Year 4 results is ignored, as it is an outlier. The Year 3 results are positively skewed, whereas the Year 4 results are reasonably symmetric. All of the Year 4 results, with the exception of the outlier, are above the Year 3 mean and median.

4 **a** **i** 9 **ii** 8 **iii** 9 − 6·5 = 2·5

iv 1, 6·5, 8, 9, 10

b

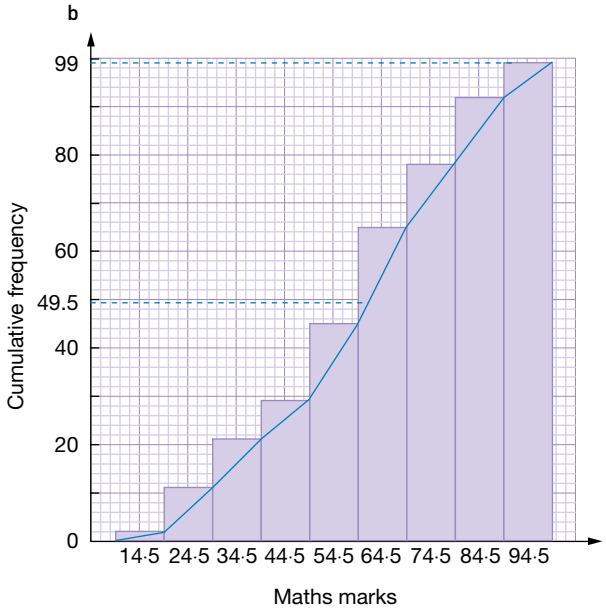


5 **a**

Class	Class centre (*c.c.*)	Frequency (*f*)	Cumu. frequency	*f* × *c.c.*
10–19	14·5	2	2	29
20–29	24·5	9	11	220·5
30–39	34·5	10	21	345
40–49	44·5	8	29	356
50–59	54·5	16	45	872
60–69	64·5	20	65	1290
70–79	74·5	13	78	968·5
80–89	84·5	14	92	1183
90–99	94·5	7	99	661·5

i Modal class 60–69 **ii** Mean 59·9 (1 dec. pl.)

b



The median class is 60–69.

ASSIGNMENT 13B

1 In green, there are four rectangles, two trapeziums and two pentagons. There are two clear squares and one pink square. Around the perimeter of the window are squares and rectangles.

2 Peter is 52 **3** 40 **4** 120 minutes

5 Perimeter = 47·7 cm (1 dec. pl.)

Area = 99·6 cm^2 (1 dec. pl.)

6 **a** E **b** C **c** A **d** D

ASSIGNMENT 13C

1 **a** 34·6 cm **b** 32°, 58°, 90°

2 **a** 160% **b** 75%

3 **a** gradient = 2, y-intercept = −3; $y = 2x - 3$

b $y = -0{\cdot}5x + 4$,(8,0) **c** −2

4 **a** Bag A = $\frac{32}{80}$ = 40% **b** Bag B = $\frac{18}{44} \cong 41\%$

c Bag B gives the better chance.

5 **a** $67 998 **b** $13 949.40 **c** $1019.97

d Amount owing = $45.37

Chapter 14

Prep quiz 14:01

1 \$32 **2** \$48 **3** \$80 **4** \$8
5 1000 g **6** 1000 kg **7** 60 min **8** 10 mm
9 10 000 cm^2 **10** 1500 mL

Exercise 14:01

1 **a** 3 km/h **b** 2 kg/\$
c 50c/kg **d** 1 mL/cm^3
e 40 L/h **f** \$20/h
g \$45/day **h** 7 km/L
i 4 deg/min **j** 70 g/cm
k 25 t/block **l** 120 km/h
m 11 runs/wicket **n** 3 children/mother
o 7·5 g/cm^3
2 **a** 15 km **b** \$49 **c** 16
d \$101.40 **e** 225 **f** 2010 kJ
g \$16 **h** 7·5 kg **i** 18 000 000 km
j \$800
3 **a** 60 **b** 25 000 **c** 40
d 500 **e** 180 **f** 7000
g 6500 **h** 7·2 **i** 2
j 490 **k** 300 000 **l** 8·5
4 **a** 1000 **b** 20 **c** 40 **d** 20 **e** 5
f 40 **g** $16.\dot{6}$ **h** 7 **i** 50 **j** 0·5
5 **a** 6 **b** 90 **c** 30 **d** 5·4 **e** 0·108
f 38 **g** 19·2 **h** 5000 **i** 20 **j** 5000
6 27·28 km/h **7** 37·6 km/h **8** 16·1 km/h
9 3 seconds

Prep quiz 14:02

1 1200 **2** 7 **3** 6 **4** 20
5 200 **6** \$2.40 **7** \$6.80 **8** \$6.40
9 115 **10** 78 min, 10 s

Exercise 14:02

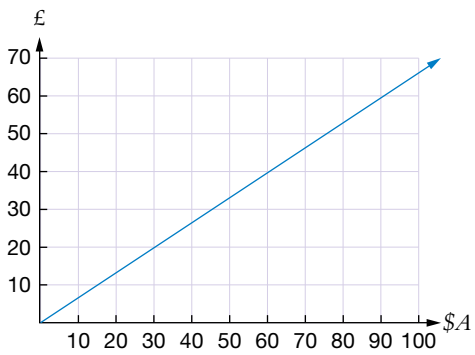
1 **a** $A = km$ **b** 2·5 **c** 25 **d** 20
2 **a** $Y = kx$ **b** 0·8 **c** 8 **d** 65
3 **a** $H = kd$ **b** 7·5 **c** 52·5 cm **d** 10
4 **a** $a = kp$ **b** 8·75 **c** 7 **d** 132
5 **a** $P = 1{\cdot}75$ m **b** 3·15 **c** 11·8
6 **a** $Y = 0{\cdot}5x^2$ **b** 40·5 **c** 4
7 **a** $V = 9{\cdot}8t$ **b** 29·4 m/s **c** 1·02 s
8 **a** 125 m^2 **b** 14
9 **a** \$5.25 **b** 12 min, 11 s
10 33·6
11 **a** $F = \frac{7}{15}cy$ **b** 4·5 **c** 42
d 21 **e** 63
12 **a** $D = knh$ **b** $k = \frac{5}{12}$ **c** 15 m
d 15 m **e** 22·5 m **f** 12
13 **a** 800 **b** 4 **c** 40
14 3·2 cm
15 **a** $D = 4{\cdot}9t^2$ **b** 78·4 m
16 **a** $V = 4{\cdot}189r^3$ **b** 113·103 cm^3 **c** 10 cm
17 **a** $T = 2{\cdot}012\sqrt{L}$ **b** 1·9 s
c 25 cm or 0·25 m

Prep quiz 14:03

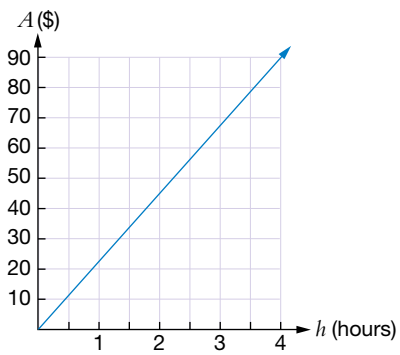
1 3 **2** 1 **3** $\frac{1}{2}$ **4** $\frac{5}{3}$ **5** 0·25
6 35 **7** 0·42 **8** 3·5 **9** 9 **10** 2·5

Exercise 14:03

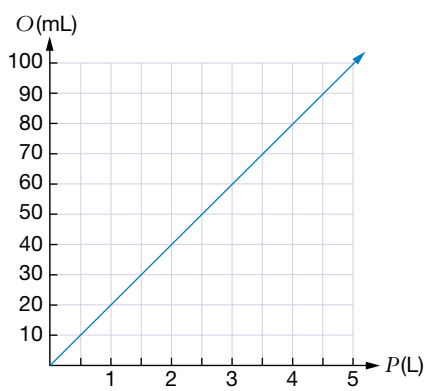
1 **a** \$7 **b** $k = 7$ **c** \$350 **d** 14
2 **a** 120 km **b** 24 km/h **c** $k = 24$
d 168 km **e** 8 h, 20 min
3 **a** 65
b



c 0·65 **d** £325 **e** \$769.23
4 **a** \$90
b

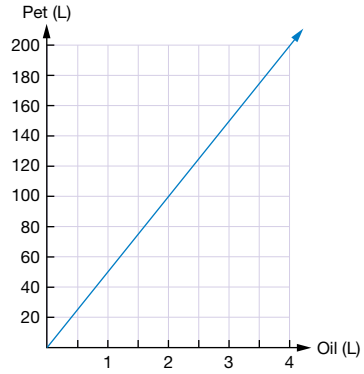


c $A = 22{\cdot}5$ h **d** 9
5 **a** $k = 20$
b



c 1 L **d** $K = 50$

e



6 **a** for A, $k = 40$; for B, $k = 50$

b 40 km **c** 5 hours **d** 60 km/h

Investigation 14:03

1

Time for 4 men	Time for 1 man	Time for 6 men
12 hours	48 hours	8 hours

2

Time for 3 women	Time for 1 woman	Time for 4 women
4 hours	12 hours	3 hours

3

Time for 3 dogs	Time for 1 dog	Time for 9 dogs
6 days	18 days	2 days

4

Time for 3 men	Time for 1 man	Time for 5 men
4 hours	12 hours	2 hours, 24 min

5 **a** the time increases **b** the time decreases

Exercise 14:04

1 **a** 12 h **b** 2 h

2 **a** 100 min **b** 25 min

3 6 full days

4 **a** 5 days

b 53 (answer to the next 'whole' fruit picker)

5 **a** 1 m/s **b** 1·25 m/s **c** $3\frac{1}{3}$ minutes

6 **a** $N = \frac{300}{P}$ **b** $N = 30$ **c** $P = 3$

7 **a** $X = \frac{200}{Y}$ **b** $Y = 40$ **c** $X = 40$

8 **a** $T = \frac{640}{A}$ **b** 21 min, 20 seconds

c 64 cm^2

9 **a** $R = \frac{14\,880}{L}$ **b** 186 times per second

c 31 cm

10 427 units

11 **a** $X = \frac{6400}{Y^2}$ **b** 256 **c** 40

12 **a** $W = \frac{2\,867\,200\,000}{D^2}$

b 65·82 kg (*Note:* $D = 6600$ km)

13 5 days

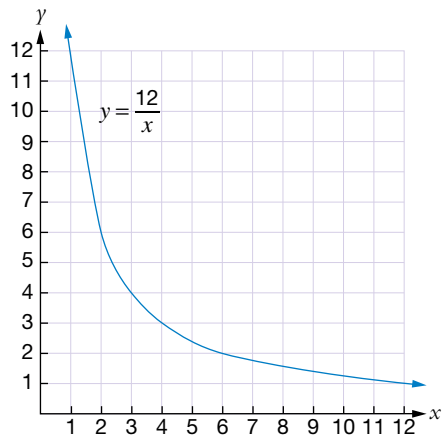
14 4 boys

15 This is a trick question! The Radetsky March would be the same length. The number of musicians and the length of the musical piece are not proportionally related.

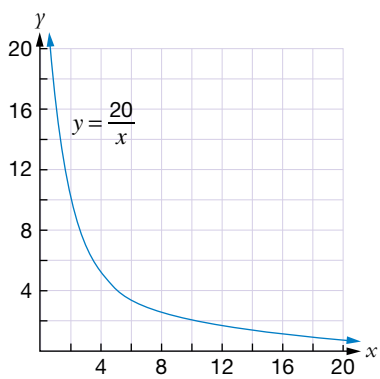
Investigation 14:04

1

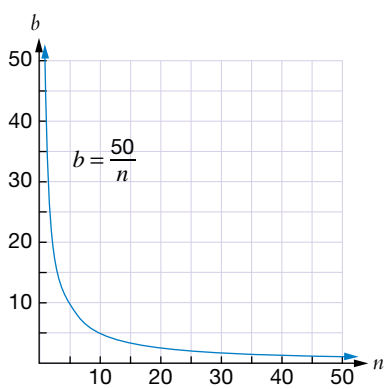
x	1	2	3	4	6	12
y	12	6	4	3	2	1



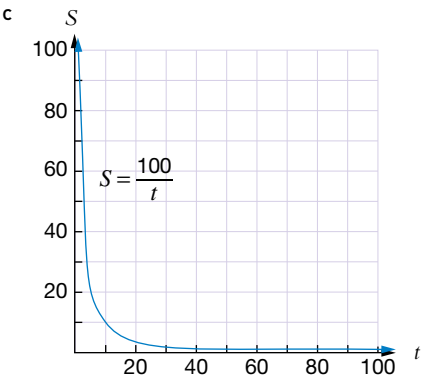
2 **a**



b



c



3 **a** $y = \frac{16}{x}$ **b** $P = \frac{40}{t}$

Diagnostic test 14

1 **a** 50 **b** 8 **c** 24 **d** 25
2 **a** 100 **b** 2 **c** 15 **d** 1
3 **a** direct **b** indirect **c** direct **d** indirect
4 **a** $k = 5$ **b** 85 **c** 34·8
5 **a** 40
b 13 (no. of people must be a whole number.)
6 **a** $D = 24T$ **b** $k = 24$
c 192 km **d** 9 h, 30 min
7 **a** $k = 12$ **b** 264 **c** 16 h, 40 min
8 **a** $t = \frac{48}{n}$ **b** $k = 48$ **c** 4 hours **d** 16
9 **a** $k = 120$ **b** 2 h, 24 min **c** 60 km/h
10 **a** 750 Hz **b** 37·5 cm

ASSIGNMENT 14A

1 3·6 L/h
2 **a** 2 min, 45 s **b** 21·8 km/h
3 **a** 340 **b** 470
4 **a** 34 kg **b** 104·125 kg
5 **a** 4·42 cm **b** 85 cm
6 **a** 24 days **b** 48 days
7 **a** $W = \frac{690}{L}$ **b** 197·1 kg **c** 3·83 m
8 **a** 180 m
b 50 cm; this would be a tight squeeze for most adults.

ASSIGNMENT 14B

1 **a** \$2580 **b** \$280 **c** 25·5%
2 **a** 124·7 cm^2 **b** 593·8 cm^2 **c** 843·2 cm^2
3 \$545
4 **a** The first stands for 3 months, the second for 3 years.
b **i** 5·9 kg **ii** 10 kg **iii** 11·7 kg
iv 12·9 kg **v** 10·6 kg
c 40% **d** From 4·3 kg to 6·2 kg.
e From about 2·5 kg to 4 kg.

ASSIGNMENT 14C

1 **a** 18·8 **b** 47·8
2 1392 m^3, 1 392 000 L
3 **a** 10 **b** D is $(-2, 3)$, E is $(3, 3)$ **c** 5
4 **a** 10 **b** 5050 **c** 15 050 **d** 512
5 **a**

Dice B

Dice A	Outcome	1	2	3	4	3	3
	6	61	62	63	64	63	63
	2	21	22	23	24	23	23
	3	31	32	33	34	33	33
	4	41	42	43	44	43	43
	5	51	52	53	54	53	53
	6	61	62	63	64	63	63

b $\frac{21}{36} = \frac{7}{12} \cong 58\%$
c $P(6) = \frac{12}{36}$, $P(4) = \frac{11}{36}$;
6 has more chance of appearing than a 4.