ACADEMIC TASK FORCE

MATHEMATICS APPLICATIONS

UNITS 1 & 2

SHANAHAN MOONEY

ACADEMIC
TASK FORCE

WACE REVISION SERIES

MATHEMATICS APPLICATIONS

YEAR 11 ATAR COURSE
UNITS 1 & 2

SECOND EDITION

SHANAHAN MOONEY

About the Author
Shanahan Mooney has been an educator for 14 years and is currently the Head of Mathematics at a prominent Perth school.

Contents

This Mathematics Applications Year 11 Study Guide provides a concise review of the theory and a constructive set of review and exam level questions for the Mathematics Applications Year 11 ATAR Course. It is accompanied by a set of answers.

How to Use this Study Guide

- This book is designed to assist you in preparation for the Mathematics Applications Year 11 exams.

- It follows the current Western Australian syllabus which has adopted and adapted the approved Australian Curriculum for the WACE ATAR course.

- Use this book in conjunction with your textbook(s) and notes to help you revise throughout the year and at the end of your course.

- Background notes illustrate the key ideas and concepts.

- Worked examples help you further understand important content and demonstrate how to set out answers clearly.

- Revision questions - both resource-free and calculator assumed - reinforce your understanding.

- Where appropriate, Screenshots from the Casio ClassPad II © have been included.

- Tips and hints boost your performance.

- This guide also includes a 3 hour WACE style exam with solutions.

Practice exams help improve your timing and confidence and identify areas for improvement. They also provide you with the opportunity to assess the strength of your notes or "cheat sheets".

Shanahan Mooney

Chapter 1 – Formula Substitution

A formula is a special type of equation that shows the relationship between different variables.

Worked Example 1

Given $T = a + (n - 1)d$; evaluate T when $a = 12$, $n = 10$ and $d = 3$

$T = 12 + (10 - 1)3$

$T = 12 + 27$

$\underline{T = 39}$

Worked Example 2

Consider the rectangle shown opposite.

a)　　Write a formula for the perimeter of the rectangle.

　　　$P = 2x + y + 2x + y$

　　　$P = 4x + 2y$
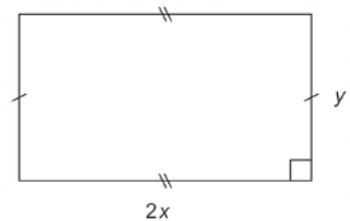
b)　　If $x = 4$ and $y = 5$, calculate the length of the perimeter.

　　　$P = 4x + 2y$

　　　$P = 4(4) + 2(5)$

　　　$\underline{P = 26\ units}$

Resource Free Questions

1.   Given that $S = \frac{D}{T}$ calculate;

    a) S, when $D = 56$ and $T = 4$

    b) D, when $S = 12$ and $T = 8$

    c) T, when $S = 20$ and $D = 4$

2.   If $v = u + at$,

    a)   Calculate the value of v when $u = 5$, $a = 4$ and $t = 10$

    b)   Calculate the value of v when $u = -7$, $a = 6$ and $t = -4$

    c)   Calculate the value of u when $v = 45$, $a = 3$ and $t = 5$

    d)   Calculate the value of a when $v = 70$, $u = 10$ and $t = 12$

3.   An electrician charges a call out fee of $85 plus $45 per hour for labour.

    a)   Write a formula for the cost (C) of the electrician in terms of the number of hours (h) taken.

    b)   Determine the cost of a 2 hour job.

    c)   If the electrician charged $310 for a job, what the length of time taken to complete the job?

4.   A child receives $10 for his birthday. He uses this money to purchase an unknown quantity of pencils at 22 cents each.

    a)   Write a formula for the change (C) he will receive from the purchase of x pencils.

    b)   Use this formula to determine the amount of money he will have remaining after he buys 17 pencils.

2

5.   Consider the rectangle shown below.

$x + 2$
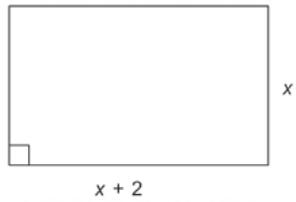
a)   Write a formula for the perimeter of the rectangle.

b)   Write a formula for the area of the rectangle.

c)   If $x = 7$, calculate the perimeter and area of the rectangle.

Resource Rich Questions

6.   Consider the roasting chart below.

	ROASTING CHART (minutes per 500g)		
	Rare	**Medium**	**Well Done**
Lamb	25	30	40
Beef	20	25	30
Veal	15	25	30

Add 15 minutes to cooking time to rest the meat before serving

a)   Explain why the cooking time (T), in minutes, for a medium rare lamb roast is given by the formula $T = \frac{30w}{500} + 15$, where w is the weight of the roast in grams.

b)   Give an expression for the cooking time for a rare beef roast.

c)   Calculate the total cooking time for 3 kg lamb roast cooked medium rare.

7.   Given $A = (1 + r)^n$,

a)   Find the value of A when $r = \frac{1}{4}$ and $n = -4$.

b)   Find the value of r when $A = 2$ and $n = 0.2$.

8.   The equation $r = \sqrt[3]{\frac{3V}{4\pi}}$ can be used to determine the radius of a sphere with a known volume. What is the radius of sphere with a volume of 300 cm^3 accurate to 2 decimal places?

9. A person's body mass index (B) is calculated using the formula $B = \frac{m}{h^2}$, where m represents mass in kilograms and h represents height in metres.

Use the formula to complete the table below. Give your answers to 2 decimal places where appropriate.

Patient	B	m	h
Tammy		50	162
Tamara	20		150
Tasha	19	46	

10. The velocity of a moving object is given by the formula $v = \sqrt{u^2 + 2as}$. Use the formula to complete the table below.

v	u	a	s
	13	7	4
20		10	12.8
9	7		1.6

Solutions

1.   a) $S = \dfrac{D}{T}$ b) $D = S \times T$ c) $T = \dfrac{D}{S}$

         $S = \dfrac{56}{4}$ $D = 12 \times 8$ $T = \dfrac{4}{20}$

         $S = 14$ $D = 96$ $T = \dfrac{1}{5}$

2.   a) $v = u + at = 5 + 4 \times 10 = 45$

     b) $v = u + at = -7 + 6 \times -4 = -31$

     c) $u = v - at = 45 - 3 \times 5 = 30$

     d) $a = \dfrac{v-u}{t} = \dfrac{(70-10)}{12} = 5$

3.   a) $C\ (\$) = 45h + 85$ b) $C = 45 \times 2 + 85$

                                              $\underline{C = \$175\ for\ a\ 2\ hr\ job}$

     c) $310 - 85 = 225$

         $\dfrac{225}{45} = 5\ hours$

4.   a) $C = 10 - 0.22x$ b) $C = 10 - 0.22 \times (17) = \$6.26$

5.   a) $P = 2 \times (x + x + 2)$ b) $A = (x + 2) \times x$

         $\underline{P = 4x + 4}$ $\underline{A = x^2 + 2x}$

     c) $P = 4x + 4$ $A = x^2 + 2x$

         $P = 4(7) + 4$ $A = (7)^2 + 2(7)$

         $\underline{P = 32\ units}$ $\underline{A = 63\ sq.units}$

6.   a)   30 mins per 500g of lamb plus 15 minutes rest time.

     b) $T = \dfrac{20w}{500} + 15$ c) $T = 30 \times \dfrac{3000}{500} + 15$

         or $T = 0.04w + 15$ $\underline{T = 195\ mins}$

7.   a) $A = 0.4096$ b) $r = 31$

Edit Action Interactive

$$\text{solve}\left(x = \left(1 + \frac{1}{4}\right)^{-4}, x\right)$$

$$\{x = 0.4096\}$$

$$\text{solve}\left(2 = (1+x)^{0.2}, x\right)$$

$$\{x = 31\}$$

8. $r = \sqrt[3]{\dfrac{3 \times v}{4 \times \pi}}$

        $r = \sqrt[3]{\dfrac{(3(300)}{(4\,\pi)}}$

        $\underline{r = 4.15\ (2\ dp.)}$

9. $B(Tammy) = \dfrac{50}{(1.62^2)} = 19.05(2\ dp.)$

        $m(Tamara) = 20 \times 1.5^2 = 45$

        $h(Tasha) = \sqrt{(46/19} = 1.56\ (2\ dp.)$

10. $v = 15$ $u = 12$ $a = 10$

Chapter 2 – Growth and Decay

Percentage Increase and Percentage Decrease

Finding a Percentage of an Amount

The 10, 5, 1 Method	Multiplication Method
Calculate 17% of $36	Calculate 17% of $36 i.e. 0.17×36
10% or $\frac{1}{10}$ of $36 is $3.60	$17\% = \frac{17}{100} = 0.17$
5% of $36 is 1.80	$\begin{array}{r} 36 \\ \times \quad .17 \\ \hline 2.52 \\ 3.60 \\ \hline = \quad 6.12 \end{array}$
1% of $36 is 0.36	
17% = 10% + 5% + 1% + 1%	
= 3.60 + 1.80 + 0.36 + 0.36	
= $6.12	

Percentage Increase and Decrease.

Percentage Increase	Percentage Decrease
Increase $80 by 12%	Decrease $70 by 15%.
100% + 12% = 112%	100% − 15% = 85%
$1.12 \times 80 = \$89.60$	$0.85 \times 70 = \$59.50$

Determining the Percentage Change

$$\frac{new\ quantity}{original\ quantity} \times 100 - 100 \quad or \quad \frac{increase/decrease}{orignal\ quantity} \times 100$$

Worked Example 1

The price of a jacket increases from $120 to $150. Determine the percentage change.

$\frac{150}{120} \times 100 - 100 = 25$ $\qquad$ *or* $\qquad$ $\frac{30}{120} \times 100 = 25$

∴ 25% increase in price. $\qquad\qquad\qquad$ ∴ 25% increase in price.

Worked Example 2

The battery life of a mobile phone falls from 400 standby hours to 350 standby hours. Determine the percentage change.

$\frac{350}{400} \times 100 - 100 = -12.5$ $\qquad$ *or* $\qquad$ $Decrease\ is\ 400 - 350 = 50\ hours$

∴ 12.5% decrease in battery life $\qquad\qquad$ $\frac{50}{400} \times 100 = 12.5$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ∴ 12.5% decrease in battery life

Inflation and Appreciation

Worked Example

A rare stamp is estimated to increase in value by 4% every year. If it is currently valued at $3500;

a)   What will its estimated value be after one year?

   $3500 \times 1.04 = \$3640$

b)   What will be its value after 6 years?

   $3500 \times 1.04^6 = 4428.62 \ (2dp)$

c)   Assuming the rate of appreciation has remained constant, what was the stamp's value 3 years ago?

$x \times 1.04^3 = \$3500$

$x = \dfrac{3500}{1.04^3}$

$x = \$3111.49 \ (2 \ dp)$

☼ Edit Action Interactive   ☒
[calculator toolbar icons]
solve(x×1.04^3=3500,x)
{x=3111.487255}
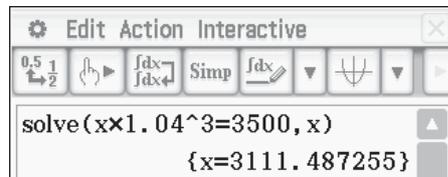
Depreciation

Worked Example

A new refrigerator costs $2000. Its value depreciates at 5% pa. Caluate its cost after 3 years.

Diminishing Value Method	Prime-Cost Method
Decrease by 5% is equivalent to multiplying by 0.95	Deprection is 5% of Prime Cost $0.05 \times 200 = \$100$
Value After 1 Year $= \$2000 \times 0.95 = \$1900$	Value After 1 Year $= \$2000 - 100 = \$1900$
Value After 2 Years $= \$1900 \times 0.95 = \$1805$	Value After 2 Years $= \$1900 - 100 = \$1800$
Value After 3 Years $= \$1805 \times 0.95 = \$1714.75$	Value After 3 Years $= \$1800 - 100 = \$1700$
$2000(0.95)^3 = \$1745.75$	$2000 - \dfrac{2000 \times 5 \times 3}{100} = \$1700$

Resource Free Questions

1. Complete the table below using the 10, 5, 1, method.

Amount	10%	5%	1%	7.5%	26%
$60					
$45					
$114					

2. Calculate 18% of the following values using the multiplication method.

   a)   8   b)   29   c)   156

3. Consider each of the following alterations in values. Express your answer and a percentage increase or a percentage decrease.

   a) $40 \; to \; 50$ b) $40 \; to \; 120$ c) $40 \; to \; 18$

4. A school with 570 students takes 30% on an excursion. How many students are left at school?

5. What is 40% of 60% of 180?

6. A company claims that their new product is now 15% bigger than the original. If the original product was 250 grams, what is the mass of the new product?

7. In an orchard, 91% of all apples grown are sold as whole fruit and the remainder is made into juice. If 63 500 apples are produced during the season, how many will be:

   a)   sold as whole fruit?   b)   made into juice?

8. A rare rookie card for a famous basketball player is currently valued at $240. An expert estimates that it will increase in value by 5% every year.

   a)   Determine the value of the card after one year.

   b)   Determine the value of the card after two years.

9.   A handyman's drill is purchased brand new for $380. If it is determined that its price
        depreciates at a rate of 16% a year, use the prime cost method to determine its value in

        a)   one year

        b)   four years.

10.   A new car costs $20 000. If it depreciates at a rate of 8% pa, use the diminishing value
        method to determine the cost of the vehicle after;

        a)   one year

        b)   two years.

Resource Rich Questions

11.   Adam decided to purchase a new computer tablet. He sees an advertisement for 15% off tablets at his local electronics store.

   a)   If Adam paid $884.00 for his tablet, determine the price prior to the discount.

   After a few weeks, Adam realises that the model he purchased does not suit his needs. He sells it to a friend and makes a loss of 8.25%.

   b)   How much does he sell it for?

   c)   If Adam wanted to make a 5% profit, how much should he have sold it to his friend for?

12.   Amy and Julia attend rival schools.

   At Amy's school, 450 sat the Human Biology exam with $16\frac{2}{3}\%$ of participants not passing.

   At Julia's school 249 out of the 296 candidates passed the exam.

   Which school performed better? Justify your answer.

13.   A study has shown that in Western Australia, approximately 14 000 children aged 6-14 years do not have internet access at home.

   a)   If this figure represents 4.375% of the 6 to14-year age group, estimate the total number of people aged 6-14 in WA.

   b)   If there are 2.4 million people in WA, use your answer to (a) to determine the percentage of the population that is between 6-14 years of age.

14.   Three brothers each purchased a video game from their local retailer.

   a)   Garry bought a game at 12% off the recommended retail price (RRP). He paid $47.74 for the game. What was the RRP of the game?

   b)   Harry bought a game at 18% off the RRP.

      i)   If the discount came to $22.86, what was the RRP of the game?

      ii)   How much did he pay for the game?

   c)   Larry selected a game with a price tag of $70.00 but was only required to pay $66.50.

      i)   How much money did Larry save?

      ii)   What was Larry's percentage discount?

15. A farmer had 4000 head of cattle on his farm.

   During January, 6% died due to drought. A few months later the herd was struck by disease, killing 15% of the population.

   a)   Show that 3196 head of cattle remain.

   b)   Express this as a percentage loss.

16. Jerry starts a new job and is paid a salary of $46 000 per annum in his first year.

   a)   At his old job, Jerry's annual salary was 15% less than what he is currently earning. What was his salary before starting his new job?

   b)   Under the terms of his contract, Jerry will receive an increase of 2% to his salary for every full year of service to the company. Determine Jerry's salary in his 6th year with the company.

17. Candice is currently paid a salary of $65 000.

   a)   Working the same job, she was earning $58 000 4 years ago. What is the annual rate of increase of her salary? Give your answer to 1 decimal place accuracy.

   b)   After negotiating with her boss, Candice now receives $69 600 annually. Compare this figure to her salary 4 years ago and express this change as a percentage increase.

18. According to the ATO, the claim on depreciating computer expenses is calculated using the formula

$$\frac{\text{asset's cost} \times \frac{\text{days held}}{365} \times \% \text{ of work use}}{\text{effective life of the tool}}$$

   Tom purchases a new desktop computer for $3800 on July 1, and has worked out that 45% of the time, he uses it for work purposes. How much can Tom claim for the decline in value of his computer in the first year if the effective life of a desktop PC is four years?

19. According to the ATO, the claim on depreciating computer expenses is calculated using the formula

$$\frac{\text{asset's cost} \times \frac{\text{days held}}{365} \times \% \text{ of work use}}{\text{effective life of the tool}}$$

   Tom purchases a new laptop for $2500 on July 6, and has worked out that 35% of the time, he uses it for work purposes. How much can Tom claim for the decline in value of his computer in the first year if the effective life of a laptop is four years?

20. A new motorbike cost $14 000. Its value depreciates at 8% pa.

   a)   Calcuate its value after 4 years using the diminishing value method.

   b)   Calcuate its value after 4 years using the prime cost method.

   c)   Give your answer to (b) as a percentage of your answer to (a).

Solutions

1.

Amount	10%	5%	1%	7.5%	26%
$60	6	3	0.6	4.5	15.6
$45	4.5	2.25	0.45	3.375	11.7
$114	11.40	5.7	1.14	8.55	29.64

2. a) 1.44 b) 5.22 c) 28.08

3. a) $\frac{50}{40} \times 100 = 125\%$ b) $\frac{120}{40} \times 100 = 300\%$ c) $\frac{18}{40} \times 100 = 45\%$

        25% increase   300% increase   55% decrease

4. $570 - (0.3 \times 570) = 399$

5. $= 0.4 \times 0.6 \times 180$

    $= 0.24 \times 180$

    $= 43.2$

6. $1.15 \times 250 = 287.5$ grams

7. a) $0.91 \times 63\,500 = 57\,785$ b) $63\,500 - 57785 = 5715$

8. a) $240 \times 1.05 = \$252$ b) $252 \times 1.05 = \$264.60$

9. a) $C = 380 - (0.16 \times 380)$ b) $C = 380 - (4 \times 60.80)$

        $C = 360 - 60.80 = \$319.20$ $C = \$136.80$

10. a) $20\,000 \times 0.92 = \$18\,400$ b) $18\,400 \times 0.92 = \$16\,928$

11. a) $1 - \frac{15}{100} = 0.85$ b) $100 - 8.25 = 91.75\%$

        $\frac{884}{0.85} = \$1040.00$ $0.9175 \times 884 = \$811.07$

   c) $100 + 5 = 105\%$

        $1.05 \times 884 = \$928.20$

12. At Amy's school, $83\frac{1}{3}\%$ of students passes the exam.

    At Julia's school $\frac{249}{296} = 84.1\%$ passed the exam.

    Julia's school performed marginally better on the exam.

13.   13.   a)   320 000

b) $13\frac{1}{3}\%$

$\frac{320000}{2400000} \times 100$

14.   a) $\frac{47.74}{0.88} = \$54.25$

b)   i) $\frac{22.86}{0.18} = \$127.00$ ii) $127 - 22.86 = \$104.14$

c)   i) $\$70 - 66.50 = \$3.50$ ii) $\frac{3.50}{70} \times 100 = 5\%$ discount

15.   a) $4000 \times 0.94 \times 0.85 = 3196$ b) $\frac{4000 - 3196}{4000} \times 100 = 20.1\%$

16.   a) $0.85 \times 46000 = \$39\,100.00$ b) $46000 \times 1.02^5 = \$50\,787.72$

17.   a) $58\,800 \times x^6 = 65000$

$x = 1.025378029$

$\therefore 2.5\%$ pa increase (1dp)

b) $\frac{69600 - 58000}{58000} \times 100 = 20$

$\therefore 20\% \ increase$

18. $V = \dfrac{3800 \times \frac{365}{365} \times 0.45}{4}$

$V = \$427.50$

19. $V = \dfrac{2500 \times \frac{360}{365} \times 0.35}{4}$

$V = \$215.75$

20.   a) $\$10\,029.50$

b) $\$9\,520$

c) 94.92%

Chapter 3 – Finance

Dividends

A dividend is a sum of money paid regularly (typically annually) by a company to its shareholders out of its profits. It is calculated by dividing the profit that is to be distributed by the number of shares in the company. The dividend is then declared on a per-share basis.

Worked Example

Consider a company with a share-capital of $80 000 000 and clear profits of $2 400 000. The company has 25 million shares.

The value of each share is $\frac{80\,000\,000}{25\,000\,000} = 3.2$ or $3.20 per share.

The dividend the company will declare is $\frac{2\,400\,000}{25\,000\,000} = 0.096$ or 9.6 cents per share.

The percentage dividend the company will declare is $\frac{2\,400\,000}{80\,000\,000} = 0.03$ or 3%.

A shareholder with 500 shares will receive a dividend of $0.096 \times 500 = \$48$.

Dividend Yield

Dividend yield refers to a share's annual dividend payments to shareholders, expressed as a percentage of the share's current price. It is calculated by taking the amount of dividends paid per share over the course of a year and dividing by the stock's share price.

$$\text{Dividend yield} = \frac{\text{declared dividend}}{\text{share price}} \times 100$$

Worked Example

If a stock pays out $2 in dividends over the course of a year and <u>trades</u> at $40, what is the company's dividend yield?

$$\frac{2}{40} \times 100 = 5$$

<u>The dividend yield is 5%</u>

Price to Earnings Ratio

The price to earnings ratio, P/E, is used to compare investments. It can be calculated using the following formula,

$$\text{Price to Earnings ratio} = \frac{\text{Market share price}}{\text{Earnings per share (Annual)}}$$

Worked Example

A stock has a market price of $18.00 and earnings of over the last 12 months were $1.50 per share. Determine the P/E ratio.

$$P/E = \frac{18}{1.50} = 12$$

Brokerage Fees

You have to pay a brokerage fee each time you buy or sell shares in a company. The brokerage fee is usually a percentage of your investment.

Worked Example

Larry buys 2000 shares at $3.00 a share. If the stockbroker charges a 0.75% brokerage fee, what will be the total cost of Larry's investment?

$$2000 \times \$3.00 = \$6000$$

$$6000 \times \frac{0.75}{100} = 45$$

Total cost is $6000 + 45 = \$6045$

Simple Interest

The simple interest formula is as follows:

I = P R T   where:

'I' - **'Interest'** is the total amount of interest paid,
'P' - **'Principal'** is the amount lent or borrowed,
'R' - **'Rate'** is the percentage of the principal charged as interest each year. The rate is expressed as a decimal, so percentages must be divided by 100. For example, if the rate is 15%, then use 15/100 or 0.15 in the formula.
'T' - **'Time'** is the time in years of the loan. (18 months = $\frac{18}{12}$ = 1.5years).

Compound Interest

The compound interest formula is as follows

$$A = P(1 + \frac{r}{n})^{nt} \text{ where;}$$

'A' is the total amount after the investment period,
'P' is the principal,
'r' is the interest rate (expressed as a decimal),
'n' is the number of compounding periods per year,
't' is the number of years for which the principal is invested.

Worked Example

What is the compound interest on $2000 invested for 2 years at 10% p.a. if the interest is compounded quarterly?

r = 10% = 0.1   P = 2000   n = 4 (quarterly)   t =2

$$A = P(1 + \frac{r}{n})^{nt}$$

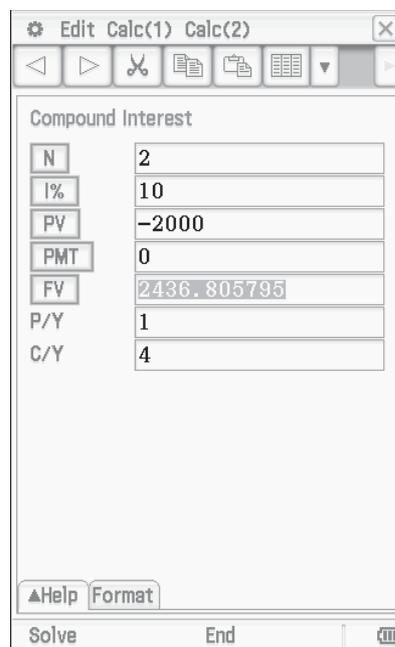$$A = 2000(1 + \frac{0.1}{4})^{4\times2}$$

$A = \$2436.81$, so the interest is $436.81

Using the ClassPad,

Number of Years [N] = 2

Interest Rate [I%] = 10

Principal [PV] = -2000

Number of Compounding Periods [C/Y] =4

Compound Interest

N	2
I%	10
PV	−2000
PMT	0
FV	2436.805795
P/Y	1
C/Y	4

Solve   End

Applications of the Compound Interest Formula

Finding the Principal

Worked Example

How much would you need to invest now to receive \$12 000 at an interest rate of 6% compounded bi-annually at the end of 10 years?

$A =$12 000 $P = x$ $r = 0.06$ $n = 2$ $t =10$

$12000 = x(1 + \frac{0.06}{2})^{2 \times 10}$

$12000 = x(1.03)^{20}$

$x = \frac{12000}{1.03^{20}}$

$\underline{x = \$6644.11}$

Edit Calc(1) Calc(2)	
Compound Interest	
N	10
I%	6
PV	−6644.10905
PMT	0
FV	12000
P/Y	1
C/Y	2

▲Help Format

Solve   End

Edit Action Interactive

$\text{solve}(12000 = x \cdot 1.03^{20}, x)$

$\{x = 6644.10905\}$

Alg   Decimal   Real   Deg

Finding the Interest Rate

Worked Example

After investing \$3800 at an interest rate of x% pa compounded quarterly, it grew to \$6119.24 after 8 years. Calculate the annual interest rate.

$A =$6119.24 $P = 3800$ $r = x$ $n = 4$ $t =8$

$6119.24 = 3800(1 + \frac{x}{4})^{4 \times 8}$

$6119.24 = 3800(1 + \frac{x}{4})^{32}$

$\frac{6119.24}{3800} = (1 + \frac{x}{4})^{32}$

$\sqrt[32]{\frac{6119.24}{3800}} = 1 + \frac{x}{4}$
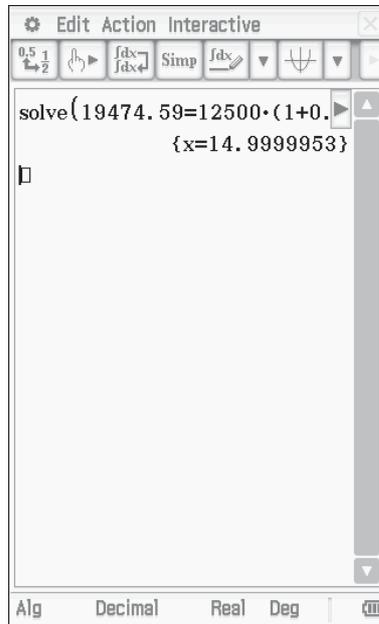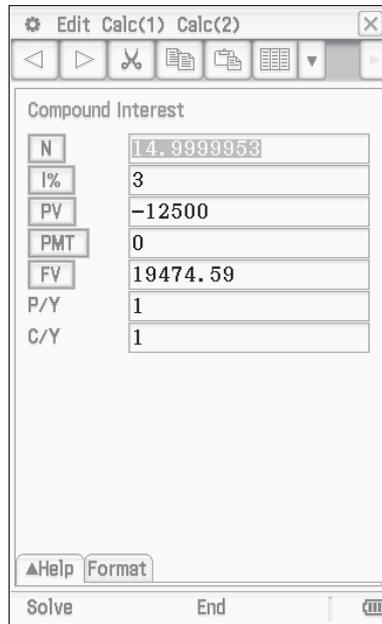
$0.015 = \frac{x}{4}$

$x = 0.015 \times 4$

$x = 0.06$

$\underline{\therefore R = 6\%}$

Edit Calc(1) Calc(2)	
Compound Interest	
N	8
I%	6.000015723
PV	−3800
PMT	0
FV	6119.24
P/Y	1
C/Y	4

▲Help Format

Solve   End

Edit Action Interactive

$\text{solve}\left(6119.24 = 3800 \cdot \left(1 + \frac{x}{4}\right)^{3}\right)$

0000157, x=0.06000015723}

Alg   Decimal   Real   Deg

Finding the Time Period

Worked Example

$12 500 is invested at 3% p.a. compounded annually. After a number of years, the final amount is $19 474.59. For how many years was the money invested for?

$A =$19 474.59 $P =$ 12 500   r = 3% $n = 1$ $t = x$

⚙ Edit Calc(1) Calc(2)	☒
◁ ▷ ✂ 📋 📋 ▦ ▼	▷

Compound Interest

N	14.9999953
I%	3
PV	−12500
PMT	0
FV	19474.59
P/Y	1
C/Y	1

▲Help Format

Solve   End

⚙ Edit Action Interactive ☒

solve$(19474.59=12500\cdot(1+0.$ ▶

$\{x=14.9999953\}$

Alg   Decimal   Real Deg

The money was invested for 15 years.

Finance Questions

Resource Free

1. $800 is invested at 3% pa. for 5 years.

     a)   Calculate 3% of $800.

     b)   Determine the simple interest earned on $800 over 3 years at an interest rate of 3% per annum.

     c)   Determine the simple interest earned on $800 over 5 years at an interest rate of 6% per annum.

2.   Calculate the total simple interest earned on each of the following investments.

     a) $8000 at 7% for 5 years.
     b) $600 at 4.5% for 3 years.

3. $8000 is invested for *x* years at a rate of 5% simple interest. If $2000 interest is earned, how long was the investment period?

4. $A = 5000(1.06)^3$ can be used to represent an investment of $5000 invested at 6% per annum for 3 years. Adjust the formula above for each of the following:

     a) $5000 invested at 12% p.a. compounded 6 monthly for 3 years.
     c) $5000 invested at 6% p.a. compounded quarterly for 3 years

5.   After 4 years an investment of $1000 earns $80 simple interest at x% pa. What is the annual interest rate?

6. $1000 is invested at 5% pa. compounded annually.

    a)   What will be the total value of the investment after 1 year?

    b)   What will the total value of the investment after 2 years?

7.   A company with 2.4 million shares posts after tax profits of $180 000. Determine the dividend that the company will declare.

8.   Last year, YF Enterprises declared a dividend of 75c. If there are a quarter of a million shares in this company, calculate the post-tax profit of YF Enterprises.

9.   a)   Calculate the dividend yield of a company with a share price of $68.00 and a declared dividend of 17c per share.

    b)   i)   A company on the Australian Stock Exchange is currently trading at $24 a share and its earnings over the last 12 months were $3 per share. Determine the price-to-earnings ratio for the stock.

        ii)   Calculate the current share price for a company with a P/E ratio of 22.4 and a dividend of 5 cents.

10.   Selina's stockbroker charges a 1.2% brokerage fee on the purchase of shares. JKL Enterprises lists its stock at 45 cents per share. If Selina buys 2000 shares, what will be the brokerage fee she will have to pay?

Calculator Assumed

11.   Which of the following scenarios would yield the maximum interest?

      a) $6000 invested at 6% per annum compounded annually for 6 years.

      b) $12000 invested at 3% per annum compounded annually for 3 years.

      c) $3000 invested at 6% per annum compounded annually for 12 years.

12.   Jenny takes out a personal loan of $3300.

      a)   Determine, to the nearest cent, the amount of money owed after 6 months if the loan
            is charged at 18% per annum compounded monthly.

      b)   Determine the amount of interest owed.

Jenny's mother is worried about the debt rising too quickly so she decides to pay out the
loan and all interest accrued.
Jenny must repay her mother in regular monthly instalments but will not be charged interest.

      c)   What amount must Jenny pay her mother each month to pay off the loan in exactly
            one year?

13.   Richard invests $4560 at an annual rate of 8.5% for 6 years.

      a)   Calculate the simple interest Richard will earn in this time period.

      b)   Calculate the compound interest Richard will earn in this time period if the interest is
            compounded annually.

      c)   Express the difference in the interest earned between the two investment types as a
            percentage of the original investment.

14.   Claire invests $14 000 in a government bond that earns 4.2% simple interest for a 16 month investment. Calculate the simple interest earned.

15.   a)   Alex invested $80 000 in a building society that pays 6.5% pa. simple interest. Over the years, the investment has paid him $57 200 in interest. How many years has he had the investment?

      b) $8000 is invested at 3% p.a. compounded annually. After a number of years, the final amount is $9552.42.

           i)   How much interest was earned?

           ii)   For how many years was the money invested for?

16.   A bank offers 8% pa. compounded daily. How much will need to be invested if the investment is to be worth $100 000 in 10 years?

17.   a) $243 000 is invested at 3.2% p.a. compounded quarterly. After a number of years, the final amount is $313 576.35. For how many years was the money invested?

      b)   When $12 000 earns simple interest at rate of 2.5% pa, it accrues $1200 in interest. For what period of time was simple interest accrued?

      c) $8000 is borrowed against a simple interest rate of at 3% p.a. After a number of years, the amount owing on the loan had grown to $9 920. Assume no repayments were made.

           i)   How much interest was accumulated over the period?

           ii)   For how many years did the loan accrue interest?

18.   Five years ago, Timmy invested $5000. His investment earned him $2500 interest.

      a)   If the investment was made under a simple interest arrangement, calculate the annual interest rate.

      b)   If the investment was made under a compound interest arrangement, calculate the annual interest rate;

           i)   if the investment was compounded annually.

           ii)   if the investment was compounded monthly.

           iii)   Explain why your answer to a) and bi) are different

           iv)   Explain why your answers to bi) and bii) are different.

19.   The table below shows the declared dividends of three companies.

Company	Share Price	Dividend
ROG Ltd	$ 6.40	71 c
DFV	$ 8.85	93 c
Gordon & Sons	$ 7.10	85 c

Compare the dividend yields of three companies to determine the best yielding stocks.

20.   After careful consideration, Lizzy decides to invest some of her money in shares. She
        decides to purchase 500 shares in a company known as TFB. Shares in TFB are valued at
        $4.45.

   a)   What is the value of Lizzy's shares?

   b)   Lizzy's stockbroker charges a 1.5% brokerage fee on the value of Lizzy's shares.
          What is the total fee?

   c)   What is Lizzy's total cost of investing in these 500 shares?

   d)   Over the last 12 months, TFB shares have had earnings of 50 cents per share.
          Determine the P/E ratio.

Solutions

1. a) $\dfrac{3 \times 800}{100} = 24$ b) $3\% \ of \ \$800 = \$24$

   $24 \times 3 = \$72$

   c) $3\% \ of \ \$800 = \$24$

   $6\% \ of \ \$800 = \$48$

   $48 \times 5 = \$240$

2. a) $SI = \dfrac{8000 \times 7 \times 5}{100}$ b) $SI = \dfrac{600 \times 4.5 \times 3}{100}$

   $SI = 80 \times 35$ $SI = 18 \times 4.5$

   $SI = \$2\,400$ $SI = \$81$

3. 2000 = 8000 × 0.05 × t

   $t = \dfrac{2000}{(8000 \times 0.05)}$

   $\underline{t = 5\ years}$

4. a) $A = 5000 \times (1.06)^6$ b) $A = 5000(\,1.015)^{12}$

5. $I = \dfrac{80}{4 \times 1000} \times 100 = 2\%$

6. a) $A = 1000(1.05) = \$1050$ b) 10% = $105
   5% = $52.50
   1050 + 52.50 = $1102.50

7. $\dfrac{180000}{2400000} = 0.075 \ ie. \ 7.5 \ cents \ per \ share.$

8. $ 0.75 × 250000 = $ 187 500

9. a) $\dfrac{0.17 \times 100}{68}$ b) i) $P/E = \dfrac{24}{3} = 8$

   $= \dfrac{17}{68}$ ii) $22.4 \times 0.05 = \$1.12$

   = 0.25% dividend yield

10. 2000 × 0.45 = $900
    0.012 × 900 = $10.80
    $10.80 Fee

11. a) $I = 6000 \times (1.06)^6 - 6000 = \$2511.11$

     b) $I = 12000 \times (1.03)^3 - 12\,000 = \$1112.72$

     c) $I = 3000 \times (1.06)^{12} - 3000 = \$3036.59$

     Option (c) yields the most interest $ 3036.59

12. a) $A = 3300\left(1 + \frac{0.18}{12}\right)^6$ b) $I = \$3608.36 - \$3300$

         $A = \$3608.36$ $I = \$308.36$

     c) $\frac{\$3608.36}{12} = \$300.70$

13. a) $SI = 4560 \times 0.085 \times 6 = \$2325.60$ b) $A = 4560 \times (1.085)^6 = \$7439.49$

                                                          Interest is $2879.49

     c) $2879.49 - $2325.60 = $553.89 $\frac{553.89}{4560} \times 100 = 12.15\%$

14. a) $14000 \times 0.042 \times \frac{16}{12} = \$784.00$ b) $1200 = 12000 \times 0.025 \times x$
                                                              $x = 4 \Rightarrow 4$ years

     c) i) $9920 - 8000 = \$1920$ ii) $1920 = 8000 \times 0.03 \times x$
                                                       $x = 8 \Rightarrow 8$ years

15. a) $t = \frac{57200}{(80000 \times 0.065)}$ t = 11 years

     b) i) $9552.42 - 8000 = \$1552.42$
         ii) $9552.42 = 8000(1.03)^x$
              $x = 6 \Rightarrow 6$ years

16. PV = $ 44 936.84

17.   a)   n = 7.99999 = 8 years

Compound Interest

N	7.9999999968
I%	3.2
PV	−243000
PMT	0
FV	313576.35
P/Y	1
C/Y	4

▼Help Format
Number of installment periods

Solve   End
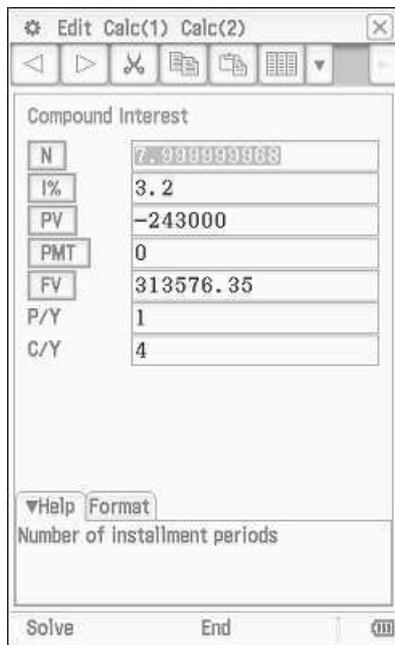
b) $1200 = 12000 \times \frac{2.5}{100} \times r$ c)   i) $9920 − 8000 = $1920

$r = 4$ ii) $1920 = 8000 \times \frac{3}{100} \times r$

4 years $r = 8$

8 years

18.   a)   Rate of interest $= \frac{2500}{(5000 \times 5)} \times 100 = 10\%$

b)   i)   7500 = 5000 × (1+R/100)^5 R = 8.447%p.a.

ii)   Rate = 8.136764314% p.a.

iii)   In simple interest calculations the annual interest remains constant.

iv)   Increasing the number of compounding periods, increases the effective interest rate.

19.   ROG Ltd $= \frac{0.71}{6.40} \times 100 = 11.09\%$ DFV $= \frac{.93}{8.85} \times 100 = 10.50\%$

Gordon & Sons $= \frac{0.85}{7.10} \times 100 = 11.97\%$ *Gordon & Sons are best yielding stocks*

20.   a)   500 × $ 4.45 = $ 2225   b)   Brokerage fee = 0.015 × 2225 = $ 33.38

c) $2225 + $ 33.38 = $ 2258.38   d) $P/E = \frac{4.45}{0.5} = 8.9$

Chapter 4 – Goods and Services

The Unitary Method

The unitary method involves scaling down one of the variables to a single unit then performing the operation necessary to alter it to the required units. The unitary method can be used for comparision purposes in "best buy" questions as well as situations involving exchange rates.

Worked Example 1

3 avocados cost $4.20. What will be the cost of 7 avocados?

Unitary Method	Ratio Method
1 avocado costs $\frac{4.20}{3} = \$1.40$ 7 avocados cost $7 \times \$1.40 = \$9.80$	$\frac{7}{3} \times 4.20 = \$9.80$

Worked Example 2

On the 3rd of June, 1 AUD (Australian Dollar) buys 0.74 EUR (Euro). How many Australian dollars could you buy for 500 EUR?

Unitary Method	Ratio Method
1 EUR buys $\frac{1}{0.74} = 1.\overline{351}$ AUD 500 EUR buys $1.\overline{351} \times 500$ 500 EUR buys 675.68AUD	$\frac{500}{0.74} \times 1 = 675.68$ AUD

Goods and Servies Tax - GST

GST is a broad-based consumption tax, generally charged at 10%. GST is a type of Value Added Tax (VAT) applied to most good and services sold or consumed in Australia.
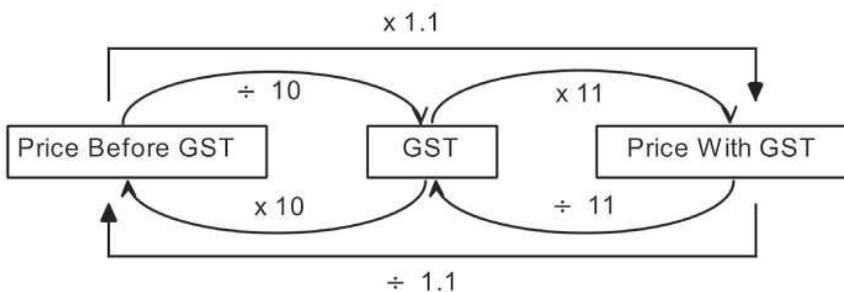
Resource Free Questions

1.   Find the cost of 1 kg if

  a)   4 kg cost $20

  b)   4 kg cost $18

  c)   4 kg cost $6.80

2.   Determine which of the following represents the best value for money.

  a)   3 kg of bananas for $8.25 or 8 kg of bananas for $22.16?

  b)   250 grams of venison for $8.85 or 600 g of venison for $20.80

  c)   150 mL of chilli sauce for $4.40 or 500 mL for $13.50.

3.   If 7% of an amount is 42, what is 20%?

4.   Sam and Kim both purchase three identical CDs each retailing at $21.00.

  Sam goes to his local music store where they have a promotion, "buy two CDs and get one free".

  At her local store, Kim receives a 30% discount when she buys two or more CDs.

  Who received the best deal and by how much?

5.   Jack is a plumber. He charges a client $440 plus GST to supply and install a toilet. What will be the total cost of the job to the client?

6.   As part of their "End of Financial Year" promotion, a car dealership covers the cost of the GST on all new cars. Calculate the pre-tax price of a car that costs $29 700, including GST.

7. Complete the table below to determine the total amount payable for items purchased from Con's Farmers Market.

Item	Cost per kilo	Cost
Apples - 4kg	$2.25	
Bananas - 3 kg	$12	$36.00
Broccoli - 2 kg	$4.98	
Tomatoes - 3 kg	$2.95	

Sub Total	$63.81
GST (10%)	

Total Amount Payable	

8. One morning, two friends exchanged their Australian dollars (AUD) at different locations while holidaying in India.

   One friend exchanged 9AUD and received 729 Indian Rupiahs (INR).

   The other friend exchanged 4 AUD for 328 INR

   a) Who obtained the better exchange rate? Justify your answer.

   b) If the friend who received the better rate had exchanged 520AUD, how many INR would have been received?

9. Upon arriving in Switzerland, Julia exchanged 500 Australian Dollars for 420 Swiss Francs (CHF).

   a) Determine the exchange rate for AUD to CHF.

   b) Julia was charged a 1.5% commission fee for the conversion. How much did Julia receive after the conversion and the deduction of this fee? Give your answer in CHF.

10. The price of a pillow increases from $15 to $18. Express this increase as a percentage of the old price.

Resource Rich Section

11.   If 13% of an amount is $6.76, what is 22.5%?

12.   If 350 AUD buys 241.58 EUR, how many EUR does 785 AUD buy?

13.   If 115% of an amount is 920, what is 27%?

14.   A restaurant bill has a total price of $113.56. What was the actual price of the meal, before the GST was added?

15.   On Christmas Day, a plumber is called out to attend to a broken hot water system. The plumber charges $350 in labour, $1237 for parts and fittings and a $30 charge for disposal of waste materials. As it was on a public holiday, the cost of labour incurs a 15% surcharge. The total cost has 10% GST added. What is the final balance?

16.   At the local delicatessen, a particular brand of soft drink is sold in three sizes

       340 mL of soft drink for $1.95, 600 ml of soft drink for $3.30 or 1.25 L of soft drink for $7.00

       a)   Which size represents the best value for money?

       b)   In a new promotion if a customer purchases two 340 mL sized bottles, the customer receives a 5% discount. Taking this promotion into consideration, which size represents the best value for money?

17.   Prior to October 2010, the VAT in New Zealand was 12.5%. After this date, the VAT increased to 15%.

       a)   Determine the VAT on a $2600 television in September 2010.

       b)   Determine the VAT on the same $2600 television in November 2010.

       c)   Without using the figure of $2600 in your calculation, show how your answer to (a) could be used to determine the answer to part (b).

18.   If 1 Australian dollar buys 0.69 Euro cents, i.e. 1AUD = 0.690 EUR;

    a)   How many AUD can be exchanged for 400 EUR?

    b)   If my credit card statement shows the cost of the accommodation on my holiday was $1942.03 AUD, what is the equivalent price in EUR?

19.   Consider the exchange rates below. Currently, 1 USD = 1.07 AUD

	AUD	USD	GBP
AUD	1	0.934	0.621
Inverse	1	1.07	1.61
USD	1.07	1	1.28
Inverse	0.934	1	0.78
GBP	1.61	1.28	1
Inverse	0.621	0.78	1

Garry searches online for the best deals on a particular model of mobile phone.

- Company A, based in Australian, sells the phone for $639.99 AUD.
- Company B, based in New York, has the phone for $585 USD.
- Company C, located in London, sells it for 350.77 GBP

    a)   Assuming that all three companies have free postage to Garry's home in Australia, which Company has the best price on this model of phone?

    b)   The London based company charges a 12% sales tax, and the New York Based company charges 3% for overseas transactions. Taking this information into consideration, which country should Garry purchase from? Justify your answer.

20.   Consider the conversion chart for the three currencies shown below.
       In this chart, 1 AUD = 0.628 USD

	Convert		
	AUD	**USD**	**GBP**
to AUD	1	1.592	1.984
Inverse	1	0.628	0.504
to USD	0.628	1	1.248
Inverse	1.592	1	0.801
to GBP	0.504	0.801	1
Inverse	1.984	1.248	1

a)   Use the chart above to convert 50AUD to USD then the USD to GBP.

b)   Use the chart above to convert 50AUD to GBP.

c)   Anna is flying from Sydney to London with a one hour stop at Los Angeles
        International Airport. She has $1000. Based on the table given, would you
        recommend that she convert her Australian Dollars (AUD) to USD in Los Angeles
        then convert her USD to GBP in London, or wait until she gets to London and convert
        her AUD to GBP? Explain.

Solutions

1. a)   20 ÷ 4 = $5 per kg   b) 18 ÷ 4 = $4.50 per kg

    c)   6.8 ÷ 4 = $1.70 per kg

2. a)   cents/gram = 825 ÷ 3000 = 0.275   or   2216 ÷ 8000 = 0.277

         <u>3kg is better value</u>

    b)   885 ÷ 250 = 3.53 c/g or 2080 ÷ 600 = 3.4667 c/g

         <u>600g for $20.80 is better buy</u>

    c)   440 ÷ 150 = 2.93 c/ml or 1350 ÷ 500 = 2.7 c/ml

         <u>500 mL is the best buy</u>

3. If 7% is 42 $\frac{20}{7} \times 42$

    then, 1% is 6   OR $= \frac{20}{1} \times 6$

    then, 20% is 120 $= 120$

4. Sam pays $42 for 3 cds & Kim pays $\frac{70}{100} \times 63 = 44.10 for 3 cds.

    <u>Sam receives better deal by $2.10 ($44.10 – $42).</u>

5. $\frac{10}{100} \times 440 = 44$

    $440 + 44 = 484.00

6. $110\% \; is \; 29700

    $10\% \; is \; \frac{29700}{11} = 2700

    $100\% \; is \; 27000

    $\therefore $27000 \; is \; the \; pre-tax \; price \; of \; the \; car$

7.

Item	Cost per kilo	Cost
Apples - 4kg	$2.25	$9.00
Bananas - 3 kg	$12	$36.00
Broccoli - 2 kg	$4.98	$9.96
Tomatoes - 3 kg	$2.95	$8.85
	Sub Total	$63.81
	GST (10%)	$6.38
	Total Amount Payable	$70.19

8   a)   729 ÷ 9 = 81 or 328 ÷ 4 =   82

4AUD for 328 INR is better rate

b) $520 × 82 =   42640 INR

9   a) $\frac{420}{500} =$ 0.84

b) $\frac{15}{1000} \times 420 = 6.3 \; CHF$ 1% is 4.2 CHF

$420 - 6.3 = 413.7$ OR   0.5 is 2.1 CHF

Julie received 413.7 CHF $420 - (4.2 + 2.1) = 413.7$

Julie received 413.7 CHF

10. $\frac{6.76}{13} \times 22.4 = \$11.7 \; (2dp.)$

11. $\frac{22.5}{13} \times 6.76 = 11.7$

12. $\frac{350}{241.58} = \frac{785}{x}$

$\frac{x}{1} = \frac{785 \times 241.58}{350}$

$\underline{x \quad = 541.83 \; EUR}$

13. $1.15x = 920$ $\frac{27}{115} \times 920$

$x = \frac{920}{1.15}$ OR $= 216$

$\underline{x = 800 \; \times 0.27 = 216}$

14. $1.1 \; x = 113.56$

$x = \frac{113.56}{1.1}$

$\underline{x = \$103.24}$

15.   Cost = 350 × 1.15 + 1237 + 30 = <u>$1669.50</u>

G.S.T. = 10 % $1.1 \times 1669.5 = \$1836.45$

16.   a)   Cost Per mL $\frac{195}{340} = 0.57$ or $\frac{330}{600} = 0.55$ or $\frac{700}{1250} = 0.56$

<u>The best buy is 600ml for $3.30</u>

b) $1.95 × 0.95 = $1.85 for discounted 340ml but 2 must be bought.

$1.85 ÷ 340 = 0.54 cents per mL.

<u>$\therefore$ the 340 mL $size$ at a 5% $discounted$ $price$ is the $best$ $value.$</u>

17.   a) $2600 \times 1.125 - 2600 = \$325$ b) $2600 \times 1.15 - 2600 = \$390$

c) $\frac{325}{12.5} \times 15 = 390$ OR $\frac{15}{12.5} \times 325 = 390$

18.   a)   400 ÷ 0.69 = 579.71 AUD   b) $\$1942.03 \times 0.62 = 1340$ EUR

19.   a)   NY $625.95AUD, <u>LND $564.74AUD</u> or   AUS $629.99AUD

London based price is best.

b)   With sales tax added on NY $644.73, LND$632.51, AUS$629.99
<u>AUS is best.</u>

20.   a)   AUD>USD>GBP $50 \times 0.628 \times 0.801 = 25.15$ GBP

b)   AUD>GBP 50×0.504 $= 25.20$ GBP

c)   Anna should wait until she arrives in London.
She will be approximately 1 GBP better off.

Chapter 5 – Income, Taxes & Household Expenses

Salary Pay

A salary is a fixed amount and does not change regardless of the number of hours the employee works. Therefore, an employee would not normally receive overtime or penalty rates.

$$Gross\ Pay = Pay\ per\ Time\ Period \times Number\ of\ Time\ Periods\ Worked$$

Wages

Wages are paid at an hourly (base) rate.

$$Gross\ Pay = Hourly\ Rate \times Number\ of\ Hours\ Worked$$

Overtime Rates

Overtime is when an employee works extra time outside their regular hours, or works beyond the agreed number of hours. Award or penalty rates apply when overtime is incurred.

$Time\text{-}and\text{-}a\text{-}half = 1.5 \times Hourly\ Rate$ $Double\ Time = 2 \times Hourly\ Rate$

Commission

An employee paid by results (sales) as opposed to an hourly or annual pay rate. Commission payment is calculated at a percentage of the employee's total sales.

Standard Commission

$$Commission = \frac{Rate(\%)}{100} \times Total\ Sales$$

Graduated Commission

An employee's rate of commission changes with the volume of sales.

Worked Example

A real estate agent sells two houses, one for $745 000 and the other for 1.2 million. She is paid by commission and her entitlements are displayed in the following table.

Settlement Price	Commission
Up to $500 000	1.4%
$500 000 to 1 million	1.8%
More than 1 million	1.7%

Calculate her commission on these sales.

$0.018 \times 745\,000 + 0.017 \times 1\,200\,000 = \$33\,810$

Income Tax

Income tax is applied to all forms of income including wages, salaries, commission, dividends and interest. In Australia, the government uses a progressive tax system whereby higher income earners pay a higher tax rate than those earning lower incomes.

Taxable income	Tax On This Income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

https://www.ato.gov.au/individuals/income-and-deductions/how-much-income-tax-you-pay/individual-income-tax-rates/

Worked Example

Determine the tax owing on a taxable income of $48 560

$T = 3\,572 + \frac{32.5}{100} \times (48560 - 37000)$

$T = 3572 + (0.325 \times 11\,560)$

$T = \$7329$

Government Allowances

The government offers a range of social and health-related payments and services to individuals or families that meet the eligibility requirements. Some of these schemes include Youth Allowance, Austudy, Abstudy and Seniors Cards.

<u>**Youth Allowance**</u>

Criteria	Maximum fortnightly payment
single, with no children, under 18 years, and living at parental home	$226.80
single, with no children, under 18 years, and required to live away from parental home to study undertake training or look for work	$414.40
single, with no children, 18 years or more, and living at parental home	$272.80
single, with no children, 18 years or more, and required to live away from parental home	$414.40
single, with children	$542.90
member of a couple, with no children	$414.40
member of a couple, with children	$455.00

Source: http://www.humanservices.gov.au/customer/services/centrelink/youth-allowance

<u>**Austudy**</u>

If you are	Maximum payment per fortnight
single	$414.40
single, with children	$542.90
couple, no children	$414.40
couple, with children	$455.00

Source: http://www.humanservices.gov.au/customer/services/centrelink/austudy

Resource Free Questions

1.   Geoff earns $ 31 200 per year, calculate his wage per:

   a)   Month   b)   Fortnight   c)   Week

2.   (a)   Abdul gets time and a half for the first two hours over his regular eight hour shift, and double time for any extra hours on top of that. If his base rate is $14.00 per hour how much does he earn on a day where his shift lasts 11 hours?

   (b)   Mounith works as a kitchen hand in a restaurant and his normal rate of pay is $10.60 per hour. If he works more than 5 hours in a weekday, any hours beyond that are paid at time and a half. On Sundays, Mounith is paid double time.
   Calculate the total amount he is paid in a week where he works 5 hours on Monday, 9 hours on Wednesday and 7 hours on Sunday.

3.   Tammy is 17, lives in the family home and receives a Youth Allowance payment of $226.80 per fortnight.

   She pays her mother $55 per week for food and utilities, her mobile phone plan costs her $59 a month and her transport costs are $37.40 per week.

   a)   What are her fortnightly expenses?

   b)   How much money does Tammy have left remaining each fortnight?

4.   Roger is a real estate agent and is paid by commission only. His entitlements are displayed in the following table.

Settlement Price	Commission
Up to $500 000	2.5%
$500 000 to 1 million	2%
More than 1 million	1.5%

   a)   Roger sells a house for $750 000. What is his commission on this sale?

   b)   Roger received a commission of $30 000 for the sale of a mansion in an exclusive beachside suburb. What was the settlement price of the home?

5.   Claire earns a salary of $3500 per month

    a)   What is her annual salary?

    b)   Use the table below to determine the tax payable on her annual salary.

Taxable income	Tax On This Income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

https://www.ato.gov.au/individuals/income-and-deductions/how-much-income-tax-you-pay/individual-income-tax-rates/

    c)   Claire's older sister has a taxable income of exactly $80 000. What would be the tax owing on this income?

Resource Rich Questions

6.   Steven lives with his best mate and studies full time. Each week, he does two 6 hour night fill shifts at the local deli for $21.80 an hour. He also receives an Austudy allowance of $414.40 per fortnight from the Government. The spreadsheet below breaks down Steven's expenses over a year (52 weeks).

Expense	Cost	Frequency	Annual Cost
Rent	$195.00	Weekly	$10,140.00
Electricity	$180.00	Quarterly	$720.00
Water	$70.00	Quarterly	$280.00
Phone / Internet	$35.49	Monthly	$425.88
Groceries	$165.00	Weekly	$8,580.00
Transport	$40.00	Weekly	$2,080.00

Total	$22,225.88

a)   What are his average weekly expenses?

b)   What is Steven's total annual income?

c)   Steven's best friend is always playing games on the television, so Steven decides to get a tv for himself. He sees an advert where he can rent a 60" television for $41.95 per week. Can Steven afford to add this extra expense into this budget? Justify your answer.

7.   Cameron and his girlfriend have a combined annual net income of $68 759. They are planning to take a holiday in twelve months' time and attempt to estimate their annual expenditures to determine how much money they can save over the coming year.

   a)   Complete the spreadsheet below to determine the couple's total expenditures for the year.

Expense	Cost	Frequency	Annual Cost
Rent	$450.00	weekly	$23,400.00
Electricity	$375.00	quarter	$1,500.00
Water	$165.00	quarterly	$660.00
Gas	$120.00	quarterly	
Phone / Internet	$79.95	monthly	
Parking	$4,500.00	annually	
House Insurance	$600.00	annually	$600.00
Car Insurance	$650.00	annually	$650.00
Fuel	$70.00	weekly	$3,640.00
Groceries	$400.00	weekly	$20,800.00
Entertainment	$150.00	weekly	$7,800.00

Total   [   ]

   b)   If the couple's estimates are correct, how much money will they be able to put aside for their holiday?

   c)   Express the couple's total expenditures as a percentage of their combined post tax earnings.

   d)   If rent was increased by 6% per week, how will this effect the answer to part (b)?

8.   Consider the tax table shown below.

Taxable income	Tax On This Income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$x plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

   a)   Determine the value of x.

   a)   Calculate the tax payable on an income of $95 600.

   b)   If I paid $28 647 in tax last financial year, what was my taxable income?

9.   The table below shows the general rates of Transfer (Stamp) Duty on the purchase of an
      established home.

Purchase Price	Generate Rate of Transfer Duty
0 - $80 000	$1.90 per 100 or part thereof
$80 001 - $100 000	$1 520 plus $2.85 per 100 or part thereof above $80 000
$100 001 - $250 000	$2 090 plus $.380 per 100 or part thereof above $100 000
$250 001 - $500 000	$7 790 plus $4.75 per 100 or part thereof above $250 000
$500 001 and upwards	$19 665 plus $5.15 per 100 or part thereof above $500 000

   ** adapted from http://www.finance.wa.gov.au/cms/uploadedFiles/_State_Revenue/Duties/Transfer_Duty_Overview.pdf

   a)   Calculate the stamp duty on a home with a purchase price of $480 000.

   b)   The stamp duty on the purchase of property was $34 085. What the sale price of the
         property?

10.   Mandy takes out a car loan for the purchase price of $34 990. She agrees to fixed monthly repayments of $686.21 for a term of 5 years.

Using the information provided by a motoring body, she creates a spreadsheet with the average weekly cost of running her particular model of vehicle. She bases her figures on a 52-week period per year.

Category	Weekly Cost
Loan Repayments	
Registration / Insurance	$22.08
Petrol	$48.60
Parking	$90.00
Servicing	$17.25
Tyres	$3.06

a)   Fill in the missing entry in Mandy's spreadsheet.

b)   After tax, Mandy has net annual income of $71 903. What is Mandy's weekly net Income?

c)   Including loan repayments, what is the total weekly cost of owning and maintaining her vehicle?

d)   Express this cost as percentage of her weekly pay.

Solutions

1. a) $31200 \div 12 = \$2600 \ per \ month$ b) $31200 \div 26 = \$1200 \ per \ f/n$

    c) $31200 \div 52 = \$600 \ per \ week$

2. a) $Gross = (8 \times 14) + (2 \times 14 \times 1.5) + (1 \times 14 \times 2)$

    $\underline{Gross = \$182.00}$

    b) $5 \times 10.60 + 5 \times 10.60 + 1.5 \times 4 \times 10.60 + 7 \times 2 \times 10.60 = \$318.00$

    Mounith earns \$318 for that week

3. a) $Expenses = 55 \times 2 + \frac{59}{2} + 37.40 \times 2$ b) $226.80 - 214.30 = \$12.50$

    $\underline{Expenses = \$214.30}$

4. a) $\frac{2}{100} \times 750\ 000 = \$15\ 000$ b) 1.5% = \$30 000

    1% = \$20 000
    100% = \$2 000 000
    ∴ settlement price is 2 million dollars

5. a) $3\ 500 \times 12 = \$42\ 000$

    b) $42\ 000 - 37\ 000 = \$5000$ $Tax \ Payable = 1625 + 3572$

    $\frac{32.5}{100} \times 5000$ $\underline{Tax \ Payable = \$5\ 197}$

    $= 325 \times 5$

    $= 1625$

    c) \$17 547 (the base figure in the next tax bracket)

6. a) \$427.42   b) Annual Income $= (2 \times 6 \times 21.80 + 26 \times 414.40)$

    Annual Income $= \$24\ 377.60$

    c) $\frac{24\ 377.60}{52} = \$468.80$ per week including Austudy

    $468.8 - 427.42 = \$41.38$ *He cannot afford to rent the tv unless he makes changes to his expenses or his income.*

7. a)

Expense	Cost	Frequency	Annual Cost
Rent	$450.00	Weekly	$23,400.00
Electricity	$375.00	Quarter	$1,500.00
Water	$165.00	Quarterly	$660.00
Gas	$120.00	Quarterly	$480.00
Phone / Internet	$79.95	monthly	$959.40
Parking	$4,500.00	annually	$4,500.00
House Insurance	$600.00	annually	$600.00
Car Insurance	$650.00	annually	$650.00
Fuel	$70.00	weekly	$3,640.00
Groceries	$400.00	weekly	$20,800.00
Entertainment	$150.00	weekly	$7,800.00

<div align="center">

Total $64 989.40

</div>

b) $3,769.60   c) $\frac{64\,989.40}{68\,759} \times 100 = 94.52\%\ (2\ dp.)$

d) $0.06 \times 450 = \$27$

   $3\,769.60 - (52 \times 27) = 2365.60$

   The couple will be able to save $2 365.60 after the rent increase of 6%

8. a) $x = 0.19(37\,000 - 18\,200)$

   $x = \$3,572$

b) $T = 17547 + 0.37(95600 - 80000)$

   $T = \$23\,319$

c)   *Interactive > Advanced > Solve*

   $17547 + 0.37(x - 80000) = \$28\,647$

   $x = \$110\,000$

9. a) $Stamp\ Duty = 7790 + \frac{4.75}{100}(480000 - 250000)$

   $Stamp\ Duty = \$18\,715.00$

b)   *Interactive > Advanced > Solve*

   $19665 + 0.0515(x - 500\,000) = 34\,085$

   $x = \$780\,000$
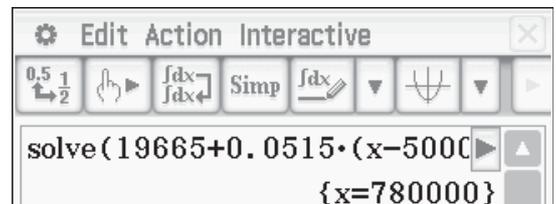
10. a) $171.55   b) $1382.75   c) $352.54

d) $\frac{352.54}{1382.75} = 25.50\%\ (2dp.)$

Chapter 6 – Matrices

Describing Matrices

In Mathematics the term matrix can be used to represent or denote a rectangular array of numbers. The order of matrix is determined by the number of rows and columns. A 3×2 matrix has 3 rows ($\Leftrightarrow$)and 2 columns ($\Updownarrow$).

Matrices are categorized according to the configurations of their elements.

Example	Description
$\begin{bmatrix} 7 & 2 & 5 \end{bmatrix}$	1×3 Row Matrix
$\begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$	3×1 Column Matrix
$\begin{bmatrix} 2 & 1 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	3×3 Square Matrix
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	3×3 Identity Matrix or I_3
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	2×2 Zero Matrix

We generally name Matrices using capital letters and lower case letters and subscripts are used to denote the entries or elements of the matrix.

Consider the 2 x 3 matrix is shown below:

$$A = \begin{bmatrix} 25 & 9 & -4 \\ -5 & 2 & 21 \end{bmatrix} \quad \text{where the elements of } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Thus:

$a_{12} = 9.$ This means that 9 is in the 1st row and 2nd column

Adding and Subtracting Matrices

Add or subtract each corresponding element of the two (or more) matrices. The matrices must be of the same order, i.e. possess the same number of rows and columns.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

Worked Example

Find $\begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 3 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+4 & 4+5 \\ 2+3 & -1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 \\ 5 & 2 \end{bmatrix}$$

$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 3 & 3 \end{bmatrix}$

$\begin{bmatrix} 5 & 8 \\ 5 & 2 \end{bmatrix}$

☐

Multiplication by Scalar

A matrix can be multiplied by a constant, k, (also known as a scalar).

$$k \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

Worked Example

If $B = \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$, determine 2B

$$2B = \begin{bmatrix} 2 \times 4 & 2 \times 5 \\ 2 \times 3 & 2 \times 1 \end{bmatrix}$$

$$2B = \begin{bmatrix} 8 & 10 \\ 6 & 2 \end{bmatrix}$$

$2 \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$

$\begin{bmatrix} 8 & 10 \\ 6 & 2 \end{bmatrix}$

☐

Determining the Order of a Product Matrix

Not all matrices can be multiplied by each other. The number of columns of the first matrix must be the same as the number of rows of the second matrix. Write down the dimensions of the two matrices to be multiplied and check if the "inside numbers" are the same. If so, the two matrices can be multiplied. The order of the resulting matrix will be the outside numbers.

A	B	AB
m x r	r x n	m x n

inside

outside
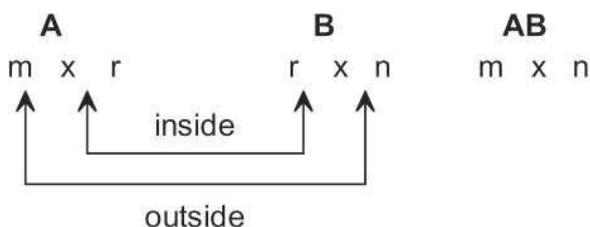
For instance, when a **3** x 3 matrix is multiplied by 3 x **2** matrix, the result is a 3 x 2 matrix

In a situation where a 3×3 matrix is multiplied by a 2×3 matrix, there are not sufficient elements in the second matrix to complete the multiplication. In this situation, the product of the two matrices is said to be undefined.

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 2 \\ 3 & 5 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$$

ERROR!   ☒

Invalid Dimension

OK
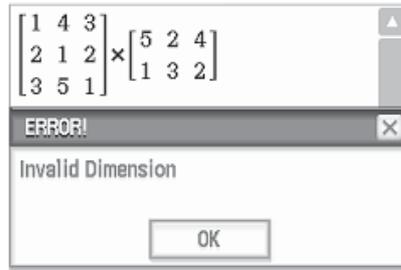
Matrix Multiplication

Once it has been established that the product of the two matrices can be defined, every row in the first matrix must be multiplied by every column in the second matrix. When multiplying a pair of 2×2 matrices, A and B, multiplying each column by each row gives;

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11 \times}b_{11} + a_{12} + b_{21} & a_{11} \times b_{12} + a_{12} \times b_{22} \\ a_{21 \times}b_{11} + a_{22} + b_{21} & a_{21} \times b_{12} + a_{22} \times b_{22} \end{bmatrix}$$

Worked Example

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$, find AB.

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \times 2 + 2 \times 4 & 1 \times 5 + 2 \times 1 \\ 3 \times 2 + 4 \times 4 & 3 \times 5 + 4 \times 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & 7 \\ 22 & 19 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 7 \\ 22 & 19 \end{bmatrix}$$

▯
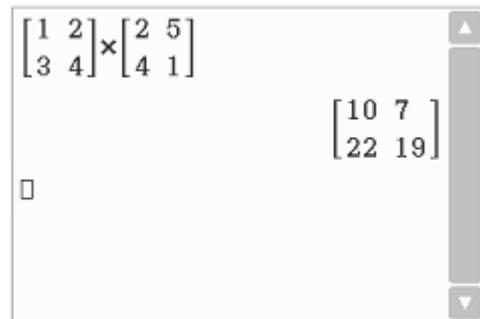
Adjacency Matrices for Undirected Networks

In all undirected networks, the matrices are symmetrical about the diagonal (all zeros). Each element in the matrix represents the number of direct paths between the nodes.

Worked Example

The diagram below shows the familiarity between four people at a wedding table.
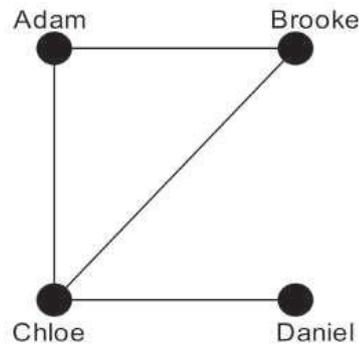
Construct an adjacency matrix to show the relationship between the four people at the table.

Adam and Chloe are friends, and as such, AC and CA are 1.

Adam and Brooke are friends, and as such, AB and BA are 1.

Adam and Daniel are not friends, and as such, AD and DA are 0.

Brooke and Chloe are firiends, and such BC and CD are 1.

Brooke and Daniel are not friends, and as such, BD and DB are 0.

Chloe and Daniel are friends, and as such, CD and DC are 1.

The adjacency matrix for the diagram is as follows.

$$
\begin{array}{c}
 \\
A \\
B \\
C \\
D
\end{array}
\begin{array}{cccc}
A & B & C & D \\
\end{array}
\begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

Resource Free Questions

1. What is the order of the following matrices.

    a) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 5 & 8 \\ 3 & 4 & 2 \end{bmatrix}$

2. Consier the matrix below.

    $$D = \begin{bmatrix} 7 & 3 & 5 \\ 1 & 2 & 4 \\ 6 & 0 & 3 \end{bmatrix}$$

    What element is found in

    a) D_{23} b)   Row 1, column 2

3. Suppose that A , B ,C , D and E are the matrices with following sizes :

    A   B   C   D   E

    (4 x 5)   (4 x 5)   (5 x 2)   (4 x 2)   (5 x 4)

    Determine if the following matrix expressions are defined and state the order of the resulting matrix for those which are defined.

    a)   BA   b)   AC + D   c)   AE + B   d)   AB + B

    e)   E (A+B)   f)   E (AC)   g) A^2 h)   2A(3E)

4. Consider the matrices

    $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 8 \\ 5 & 9 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 8 & 2 \\ 3 & 0 \end{bmatrix}$

    Find the following (where possible):

    a)   B + A   b)   B – A   c)   3A   d)   (-2C)

    e)   2(C+D)   f)   3D – 4I   g)   2C-A   h)   D-3C

5. Let $A = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 0 & 4 \\ 0 & 4 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 7 & 1 \\ -2 & 9 & 3 \end{bmatrix}$

a)   Find the first row of AB

b)   Find the third row of AB

c)   Find the second column of AB

d)   Find the second column of BA

6. Consider the sum of the two matrices shown below. Solve for x and y.

$$\begin{bmatrix} 2 & -3 & 4 \\ x & 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 6 & 8 \\ 2 & y & 2 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 12 \\ 7 & -3 & 4 \end{bmatrix}$$

7. Consider the product of the two matrices shown below. Solve for x.

$$[x \quad 7 \quad 9] \times \begin{bmatrix} 4 & 1 \\ 3 & 2 \\ 6 & 4 \end{bmatrix} = [91 \quad 54]$$

8. a)   If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ show that $AB \neq BA$

b)   Determine the values of x, y & z if $2A - \begin{bmatrix} x & 7 \\ -1 & y \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ z & 0 \end{bmatrix}$

9. The undirected network below shows "Who has been for lunch with whom?" between four friends, Joe, Kate, Paul and Gary.
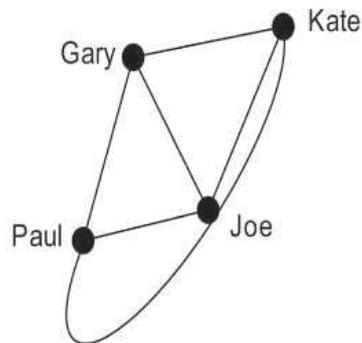
a)   Why are there no "one-way paths" in the diagram?

b)   State the adjacency matrix for this network: (Use zeros to show if a pair has not been to lunch together)

10. a) In the following diagram, the paths represent roads between four towns A, B ,C & D. Construct a route matrix in the blank matrix shown below to show the connections of roads between the 4 towns.
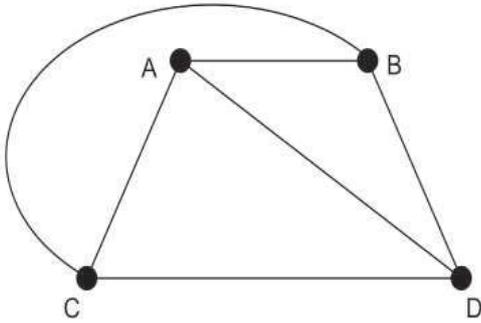
$$\begin{array}{c c c c c} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \left[\begin{array}{c c c c} & & & \\ & & & \\ & & & \\ & & & \end{array} \right] \end{array}$$

b) If we call the matrix from part (a) matrix $\mathbf{P}$ and calculate $\mathbf{P^2}$ we can use this matrix to find out how many 2-way paths exist from B to A and from C to C. Determine the number of paths and list these paths.

Resource Rich Questions

11.   Given the following Matrices

$$A = \begin{bmatrix} -3 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -1 & -1 \\ 5 & 3 \end{bmatrix}$$

Find the following

a)   AB   b)   CA + B   c)   AC + B   d)   AB + B

e)   C (A+B)   f)   B (CA)   g) A^2 h)   2A(3B)

12.   Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 8 \\ 5 & 9 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 8 & 2 \\ 3 & 0 \end{bmatrix}$$

a)   AB   b)   BA   c)   (2C)D   d)   (2A)D

e)   (CD)A   f)   -3(C-D)   g) A^2

13.   A sushi train has different coloured plates denoting the price of the food on the plate. A red plate costs $3, a blue plate costs $4 and a green plate costs $5.50.

Table 1 had 5 red plates, 2 blue plates and 3 green plates.

Table 2 had 3 red plates, 5 blue plates and 2 green plates.

a)   Represent the cost of each plate as a 1×3 matrix.

b)   Represent the number of each type of plate taken by each table as a 3×2 matrix.

c)   Find the product of the two matrices to find the total cost of the sushi for each table.

14.   Carol runs a small jewellery stall at the weekend markets. She sells three products:
        necklaces for $28, pairs of earrings for $17 and anklets for $14. On Saturday, she sold 3
        necklaces, 8 pairs of earrings and 2 anklets. On Sunday she sold 14 necklaces, 11 pairs of
        earrings and 5 anklets.

        a)   Give the breakdown of pieces sold 2×3 matrix. Label this matrix A.

        b)   Write the cost of each piece as a 3×1 cost matrix. Label this matrix B.

        c)   Calculate AB. Describe the resulting matrix.

15.   A financial planner is asked to set up a share portfolio for 3 clients involving 3 different
        companies in a 3 x 3 matrix shown below:

	Company 1	Company 2	Company 3
Client A	700	200	350
Client B	300	500	100
Client C	250	450	150

At the start each share was worth

Company 1	$4.50
Company 2	$1.20
Company 3	$0.80

Three years later each share was worth

Company 1	$6.00
Company 2	$3.00
Company 3	$1.50

        Use Matrix multiplication to calculate the value of each client's portfolio at the start
and three years later according to the share values specified above.

Solutions

1. a) 2×1 b) 1×3 c) 2×3

2. a) 4   b)   3

3. a) Undefined   b)   Yes 4×2 c)   Undefined

    d) Undefined   e)   Yes 5×5 f)   Yes 5×2

    g) Undefined   h)   Yes 4×4

4. a) $B + A = \begin{bmatrix} 7 & 3 & 2 \\ 2 & 1 & 10 \\ 9 & 10 & 2 \end{bmatrix}$ b) $B - A = \begin{bmatrix} 5 & -1 & 4 \\ -4 & 1 & -6 \\ -1 & -8 & 4 \end{bmatrix}$

    c) $3A = \begin{bmatrix} 3 & 6 & -3 \\ 9 & 0 & 24 \\ 15 & 27 & -3 \end{bmatrix}$ d) $-2C = \begin{bmatrix} -8 & 2 \\ -2 & -6 \end{bmatrix}$

    e) $2(C + D) = \begin{bmatrix} 24 & 2 \\ 8 & 6 \end{bmatrix}$ f) $3D - 4I = 3\begin{bmatrix} 8 & 2 \\ 3 & 0 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 6 \\ 9 & -4 \end{bmatrix}$

    g) Undefined   h) $D - 3C = \begin{bmatrix} -4 & 5 \\ 0 & -9 \end{bmatrix}$

5. a) $[11 \quad 11 \quad -1]$ b) $[8 \quad 46 \quad 10]$

    c) $\begin{bmatrix} 11 \\ 48 \\ 46 \end{bmatrix}$ d) $\begin{bmatrix} 2 \\ 10 \\ 8 \end{bmatrix}$

6. $x + 2 = 7$ $2 + y = -3$

    $x = 5$ $y = -5$

7. $4x + (7 \times 3) + (9 \times 6) = 91$

    $4x + 75 = 91$

    $4x = 16$

    $\underline{x = 4}$

8. a) $AB = \begin{bmatrix} 17 & 14 \\ 11 & 12 \end{bmatrix}$ $BA = \begin{bmatrix} 7 & 8 \\ 13 & 22 \end{bmatrix}$

    b) $\begin{bmatrix} 2 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} x & 7 \\ -1 & y \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ z & 0 \end{bmatrix}$

Therefore:

   $2 - x = -1$ and $4 - y = 0$ and $6 - (-1) = z$

   $2 + 1 = x$ $4 = y$ $7 = z$

      $3 = x$

9. a) The network indicates that each person had lunch with at least one other person i.e.

nobody had lunch alone. Thus, there are no one-way paths.

$$\begin{array}{c} \quad G \quad P \quad K \quad J \\ \begin{array}{c} Gary \\ Paul \\ Kate \\ Joe \end{array} \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \end{array}$$

b)

10. a)

$$\begin{array}{c} \quad\; A \quad B \quad C \quad D \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 1 & 1 & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & \mathbf{0} \end{bmatrix} \end{array}$$

b) $$P^2 = \begin{vmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{vmatrix}$$

From B to A there are two options $B - C - A$ and $B - D - A$

From C to C there are 3 options $C - D - C,\ C - A - C$ and $C - B - C$

11. a) $AB = \begin{bmatrix} 0 & -9 \\ 5 & -7 \end{bmatrix}$ b) $CA + B = \begin{bmatrix} 4 & -10 \\ -11 & 37 \end{bmatrix}$

c) $AC + B = \begin{bmatrix} 35 & 20 \\ 15 & 6 \end{bmatrix}$ d) $AB + B = \begin{bmatrix} 2 & -10 \\ 6 & -9 \end{bmatrix}$

e) $C(A + B) = \begin{bmatrix} -1 & -6 \\ 1 & 28 \end{bmatrix}$ f) $B(CA) = \begin{bmatrix} 16 & -57 \\ 26 & -87 \end{bmatrix}$

g) $A^2 = \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}$ h) $2A(3B) = \begin{bmatrix} 0 & -54 \\ 30 & -42 \end{bmatrix}$

12. a) $AB = \begin{bmatrix} 0 & 2 & 4 \\ 50 & 11 & 33 \\ 17 & 13 & 30 \end{bmatrix}$ b) $BA = \begin{bmatrix} 24 & 39 & -1 \\ 12 & 16 & 7 \\ 22 & 35 & 1 \end{bmatrix}$

c) $(2C)D = \begin{bmatrix} 58 & 16 \\ 34 & 4 \end{bmatrix}$ d) Undefined   e)   Undefined

f) $-3(C - D) = \begin{bmatrix} 12 & 9 \\ 6 & -9 \end{bmatrix}$ g) $A^2 = \begin{bmatrix} 2 & -7 & 16 \\ 43 & 78 & -11 \\ 27 & 1 & 68 \end{bmatrix}$

13.   a) $[3 \quad 4 \quad 5.5]$ b) $\begin{bmatrix} 5 & 3 \\ 2 & 5 \\ 3 & 2 \end{bmatrix}$

$[3 \ 4 \ 5.5] \times \begin{bmatrix} 5 & 3 \\ 2 & 5 \\ 3 & 2 \end{bmatrix}$

$[39.5 \ 40]$

□

c) $[3 \quad 4 \quad 5.5] \times \begin{bmatrix} 5 & 3 \\ 2 & 5 \\ 3 & 2 \end{bmatrix} = [39.5 \quad 40]$
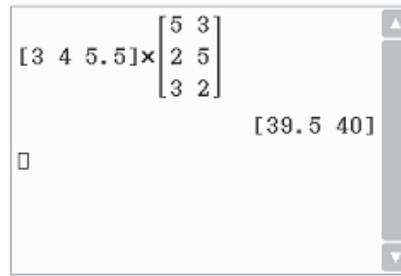
14.   a) $A = \begin{bmatrix} 3 & 8 & 2 \\ 14 & 11 & 5 \end{bmatrix}$ b) $B = \begin{bmatrix} 28 \\ 17 \\ 14 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 8 & 2 \\ 14 & 11 & 5 \end{bmatrix} \times \begin{bmatrix} 28 \\ 17 \\ 14 \end{bmatrix} = \begin{bmatrix} 248 \\ 649 \end{bmatrix}$

On Saturday she sold $248 of jewellery and on Sunday $649.

15.   Start of the portfolio   Client A $\begin{bmatrix} \$3670 \\ \$2030 \\ \$1785 \end{bmatrix}$ Client B   Client C

3 years later   Client A $\begin{bmatrix} \$5325 \\ \$3450 \\ \$3075 \end{bmatrix}$ Client B   Client C

Chapter 7 – Pythagoras' Theorem

The length of the square of the hypotenuse (c)
of a right-angled triangle is equal to the sum of the
lengths of the squares of the other two (shorter) sides.

a

c (hypotenuse)

b
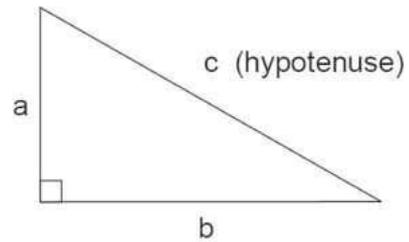
Finding the Length of the Hypotenuse

x cm

6 cm

8 cm

not to scale
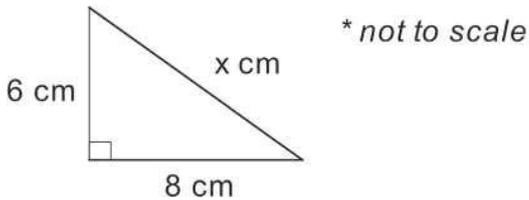
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$100 = c^2$$
$$\sqrt{100} = c$$
$$\underline{c = 10 \text{ cm}}$$

Finding the length of a Shorter Side

15 cm

x cm

12 cm

not to scale
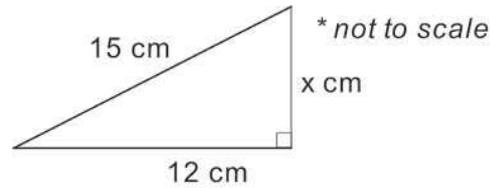
$$a^2 + b^2 = c^2$$
$$a^2 + 12^2 = 15^2$$
$$a^2 = 15^2 - 12^2$$
$$a = \sqrt{81}$$
$$\underline{a = 9 \text{ cm}}$$

Isosceles Triangles

14 cm
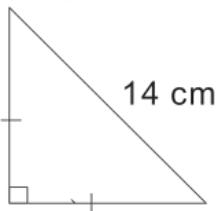
$$a^2 + a^2 = 14^2$$
$$2a^2 = 196$$
$$a^2 = \frac{196}{2}$$
$$a = \sqrt{98}$$
$$\underline{a = 9.90 \text{ cm (2 d.p.)}}$$

Resource Free Questions

1.   For the triangle shown opposite;

        a)   Name the hypotenuse

        b)   Name the right angle.
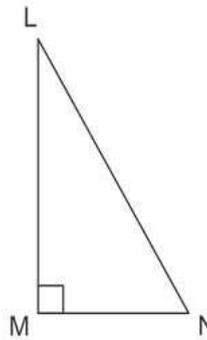
2.   Consider the triangle shown below.
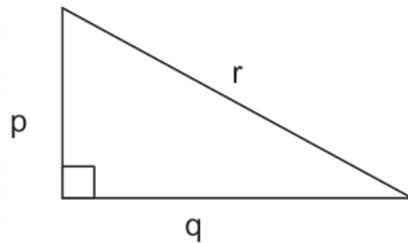
        Circle TRUE or FALSE for each of the following equations as they relate to the diagram above.

        a) $r^2 = p^2 + q^2$ b) $p^2 = q^2 - r^2$ c) $q^2 = r^2 - p^2$

                TRUE / FALSE   TRUE / FALSE   TRUE / FALSE

        d) $q^2 = \sqrt{r^2 - p^2}$ e) $\sqrt{r} = \sqrt{p} + \sqrt{q}$

                TRUE / FALSE   TRUE / FALSE

3.   A triangle has sides 13 cm, 16 cm and 20 cm. Is this triangle right angled?

        Provide evidence to support your answer.

4.   A triangle has dimensions 9 cm, 40 cm and 41 cm. Prove that the triangle is right-angled.
        Provide evidence to support your answer.

5.   George normally travels the direct route from his home (H) to school (S).
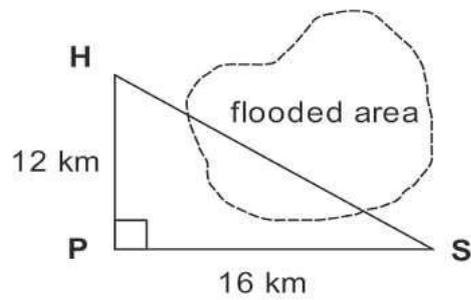
a)   What is this distance?

b)   After a weekend of heavy rain, George learns that he will have to take an alternate route home via the post office (P). How much further must he travel?

c)   Express the longer distance as a percentage of the shorter, direct distance. Interpret your answer.

Resource Rich Questions

6.   Determine the value of x, correct to two decimal places.

2.8 m

x

300 cm
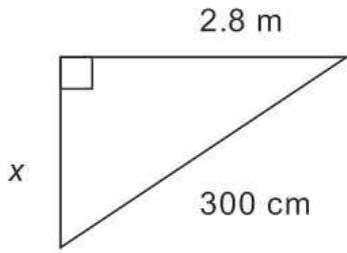
7.   Determine the length of the hypotenuse of this isosceles triangle.

13 cm
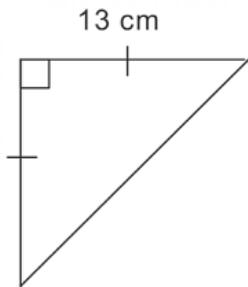
8.   An isosceles triangle has a base length of 18 cm and a perpendicular height of 22 cm.

a)   Calculate the length of one of the sloping sides.

b)   What is the perimeter of the triangle?

9.   Calculate the diagonal length of a square with sides of 8 cm.

10.   Consider the diagram below. Determine the lengths of x and y. Note that the diagram is not drawn to scale.

13 m

x

y

5 m   84 m
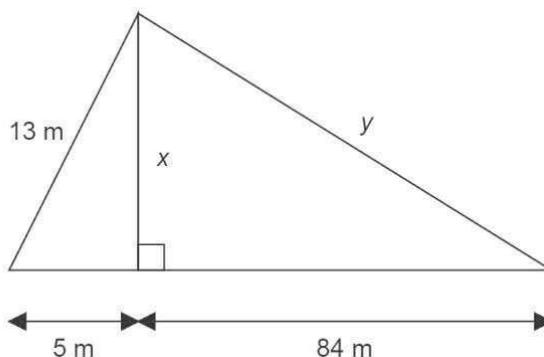
11.   Two friends are standing side by side in a park. One friend runs due north for 8 km and one
       friend runs due west for 11 km. What is the direct distance between the two friends?

12.   Two boats leave the harbour. Boat A sails for 9 km on a bearing of 47°, while Boat B sails for
       16 km on bearing of 137°. What is the direct distance between the two boats?

13.   A ladder 7ft long is placed with its end 3 feet away from the base of the wall.

       a)   How far up the wall will the ladder reach? Give your answer to 2 decimal place
              accuracy.

       b)   If 1 foot is equivalent to 30.48 cm, give your answer to (a) to the nearest centimetre.

14.   Consider the square-based pyramid ADCDE shown opposite.
       Point M is the midpoint of BE.
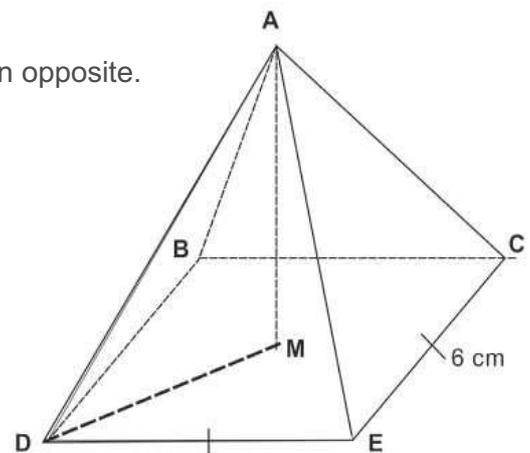
       a)   What is the length DM?

       b)   If AM, the perpendicular height of the pyramid, is 9 cm, what is the length of the
              slanted edge AD?

       c)   If the formula for the volume of a pyramid is $V = \frac{b^2 h}{3}$, determine the volume of the
              pyramid.

15.   Alana has a small storage box underneath her bed. Its dimensions are shown below.
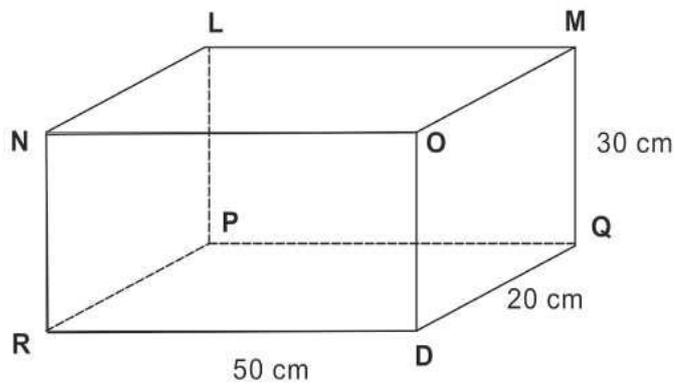
a)   Calculate the length of RQ

b)   When repainting her room, Alana rolls up her favourite poster into a cylinder 55 cm in length. Will she be able to place this into the box without bending the poster, or having part of the poster stick out from the box?

Solutions

1. a) $\overline{LN}$ or LN b) $\angle LMN$ or $\angle NML$

2. a)   True   b)   False   c)   True   d)   False

    e)   False

3. $13^2 + 16^2 = 425$ $20^2 \neq 425$ $\therefore$ *This is NOT a right triangle.*

4. $9^2 + 40^2 = 1681$ $41^2 = 1681$ $\therefore$ *This is IS a right triangle.*

5. a) $HS^2 = 12^2 + 16^2$ b) $12 + 16 = 28\ km$ c) $\frac{28}{20} = \frac{140}{100} = 140\%$

    $HS = \sqrt{400}$ George's journey is 40% longer

    $\underline{HS = 20\ km}$

6. $x = \sqrt{3^2 - 2.8^2}$
    $x = 1.08\ m\ (2\ dp.)$

7. $h = \sqrt{(13^2 + 13^2)}\ = 18.39\ cm\ (2\ dp.)$

8. a) $\sqrt{(22^2 + 9^2)} = 23.77\ (2\ dp.)$

    b)   Perimeter = 23.77 × 2 + 18 = 65.54 cm (2 dp.)

9. $d = \sqrt{(8^2 + 8^2)}\ =\ 11.314\ cm = 11.3\ cm$

10. $x = \sqrt{13^2 - 5^2}$ $y = \sqrt{84^2 + 12^2}$
    $\underline{x = 12\ m}$ $\underline{y = 84.85m\ (2\ dp.)}$

11. $d = \sqrt{11^2 + 8^2}$
    $\underline{d = 13.601km}$

12. $d = \sqrt{9^2 + 16^2}$
    $\underline{d = 18.36\ km\ (2\ dp.)}$

13. a) $d = \sqrt{7^2 - 3^2}$ b) $6.32 \times 30.48 = 193\ cm\ (nearest\ cm)$
          $\underline{d = 6.32\ ft\ (2\ dp.)}$

14. a) $DM = (\sqrt{6^2 + 6^2})\ \div 2$ b) $AD = \sqrt{4.24^2 + 9^2}$
          $\underline{DM = 4.24\ cm\ (2\ dp.)}$ $\underline{AD = 9.95\ cm\ (2\ dp.)}$

    c) $V = \frac{36 \times 9}{3}$
          $V = 108\ cm^3$

15. a) $RQ = \sqrt{50^2 + 20^2}$ b) Yes, diagonally from R to M.

$\underline{RQ = 53.85\ cm\ (2\ dp.)}$ $RM = \sqrt{53.85^2 + 30^2}$

$\underline{RM = 61.64\ cm\ (2\ dp.)}$

Chapter 8 – Perimeter and Area

Perimeter is the distance around a closed shape or figure. It is measured in the units of length: mm, cm, m, km and other less common measurements.

The perimeter of a circle is referred to as circumference.

Converting Units of Length
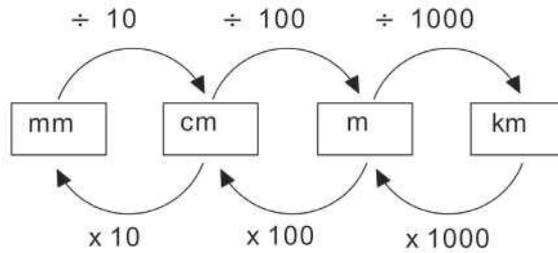
Converting Units of Area
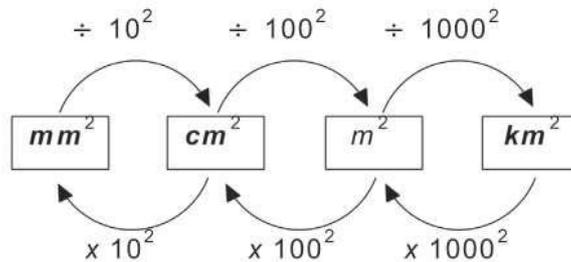
Area of Common Shapes

Square	Rectangle	Parallelogram	Triangle
$A = l^2$	$A = lw$	$A = bh$	$A = 0.5bh$

*** These three shapes are not outlined in the syllabus, but it may be useful to memorise these formulae to assist you with calculating the area of composite shapes.

Trapezium		Rhombus	Kite
a h b	a h b	y x	b h
$A = 0.5h(a + b)$		$A = 0.5xy$	$A = 0.5bh$
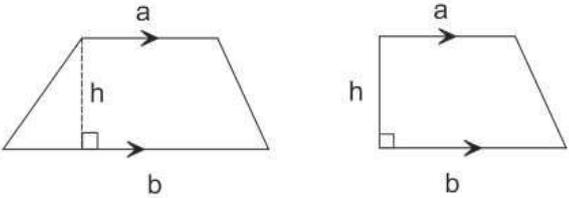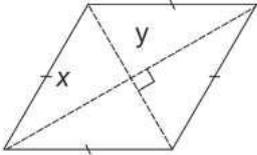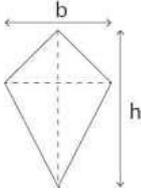
Circles and Sectors

		Perimeter	Area
5 cm		$r = 5cm, then\ D = 10cm$ $C = \pi D$ $C = \pi \times 10$ $\underline{C = 31.42\ cm\ (2\ d.p.)}$	$A = \pi r^2$ $A = \pi \times 5^2$ $\underline{A = 78.54\ cm^2\ (2\ d.p.)}$
8 cm		$r = 8cm, then\ D = 16cm$ $P = (\pi \times D \times \frac{90}{360}) + 2r$ $P = (\pi \times 16 \times \frac{90}{360}) + 16$ $\underline{P = 28.57\ cm\ (2\ d.p.)}$	$A = \pi r^2 \times \frac{90}{360}$ $A = \pi \times 8^2 \times \frac{90}{360}$ $\underline{A = 50.27\ cm^2\ (2\ d.p.)}$
115 ° 3 cm		$r = 3\ cm, then\ D = 6\ cm$ $P = (\pi \times D \times \frac{115}{360}) + 2r$ $P = (\pi \times 6 \times \frac{115}{360}) + 6$ $\underline{P = 12.02\ cm\ (2\ d.p.)}$	$A = \pi r^2 \times \frac{115}{360}$ $A = \pi \times 3^2 \times \frac{115}{360}$ $\underline{A = 9.03\ cm^2\ (2\ d.p.)}$
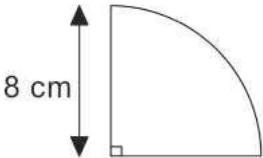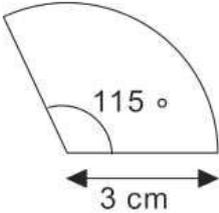
Resource Free Questions

1.   Calculate the perimeter of the following objects.

a)   b)   c)
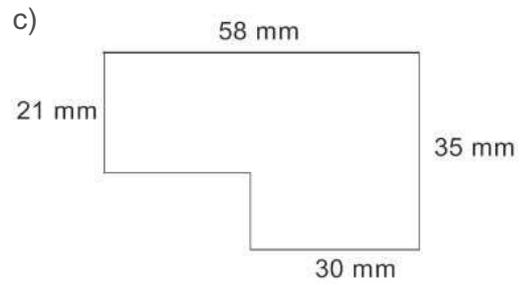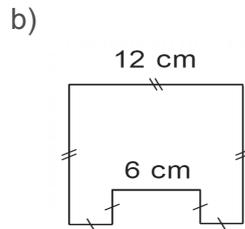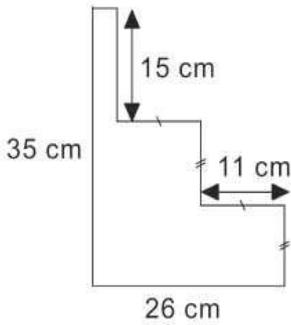
2.   Complete each of the following conversions.

a)   32.3 cm =   mm   b)   72.05 km =   m   c)   222 mm =   cm
d)   0.068 km =   m   e)   15.1 cm =   m   f)   931 m =   km
g) $5\frac{1}{2}$ cm =   mm   h) $3\frac{2}{5}$ km =   m   i) $\frac{3}{5}$ m =   cm

j)   3 cm² =   mm²   k)   25 m² =   cm²   l)   5 m² =   mm²

3.   Calculate the shaded area of the following objects.

a)   b)
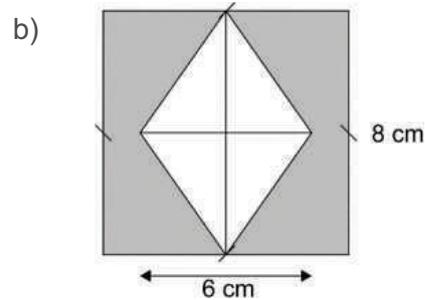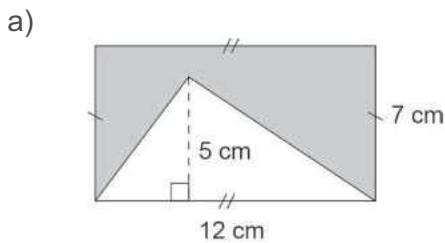
4.   Use $\pi = 3.141592654$ to calculate the perimeter of the objects below. Give your answers to one decimal place accuracy.

a)   b)
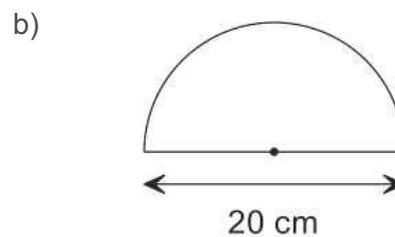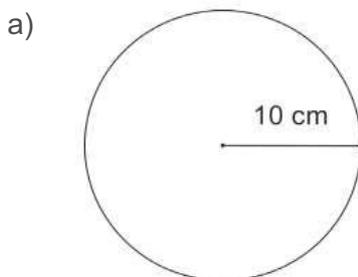
5.   Laura rides her bike to and from her house (H) to school (S) every day. She normally cycles the direct route through a park that leads from the back of her house to the school.

    a)   What is the direct distance from the school to Laura's house?

H

First Av.
(600 m)

S

Lord St. (800 m)
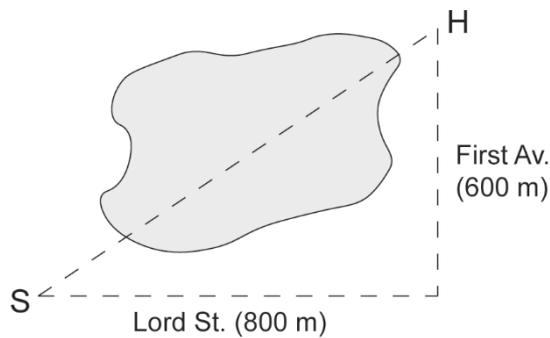
    A heavy rainstorm over the weekend has left the park flooded. Laura is forced to take an alternate route to school. She now must travel south down First Avenue and turn right down Lord Street to get to school.

    b)   How much further must Laura travel to get to school and return home each day?

    c)   If the park remains flooded for the entire school week, how much further will Laura have to cycle over the five days?

6.   Tom's lounge room has an unusual shape. He wants to lay carpet over the entire room. What is the total area to be carpeted?

2 m

2 m

4 m

6 m
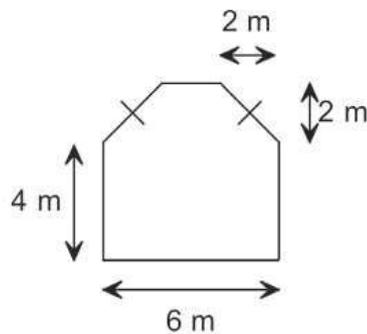
7.   A regular hexagon with sides of 21 mm is shown below. What is the area of the hexagon?

10 mm

38 mm
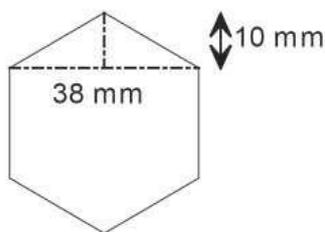
8.   Write expressions for the perimeter and area of the shapes shown below.

a)

2x

b)

(x + 5)

c)

y

x

d)

(4x − 2)

x

(2x + 6)
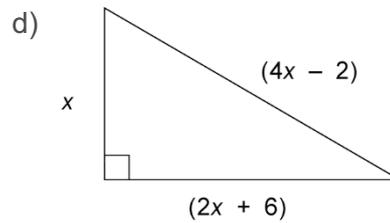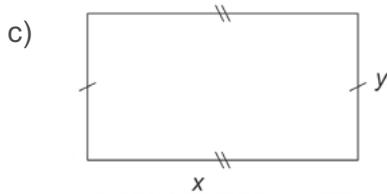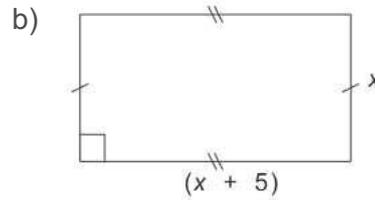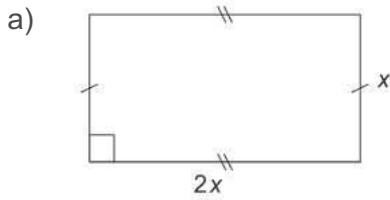
9.   A trapezium can be formed by combining shapes together. Calculate the area of the trapezium shown below:

7 cm

12 cm
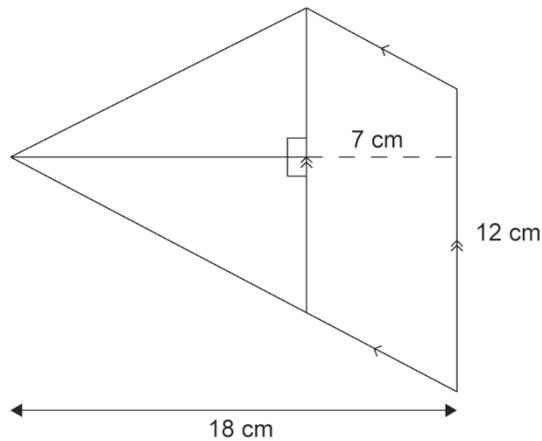
18 cm

10.   Give the dimensions of the shapes described below.

a)   A rectangle that has an area of 20 cm² and a perimeter of 18 cm.
b)   A rectangle that has an area of 20 cm² and a perimeter of 42 cm.
c)   A rectangle that has an area of 25 cm² and a perimeter of 25 cm.
d)   A triangle with a base length of 6 cm and an area of 30 cm².
e)   A triangle with a perpendicular height of 8 cm and an area of 24 cm².
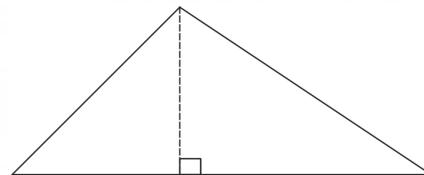
Calculator Assumed Questions

11.   The diagram below shows the dimensions of an outdoor patio that needs to be paved.

4 m

5.5 m
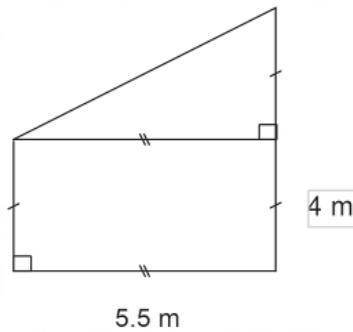
a)   Calculate the area of the patio.

b)   If a paver has dimension 225 x 75 mm how many pavers will be needed to cover the entire area? Assume that pavers can be cut to size and give your answer to the nearest whole.

12.   The area of the both the objects shown below is 98.52 cm^2. Calculate the length of the radius of each of the shapes correct to 1 decimal place accuracy.

a)

x cm

b)

x cm
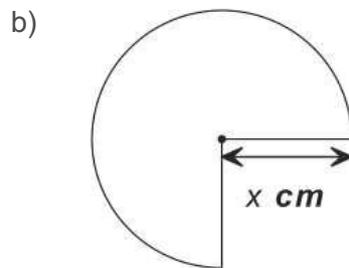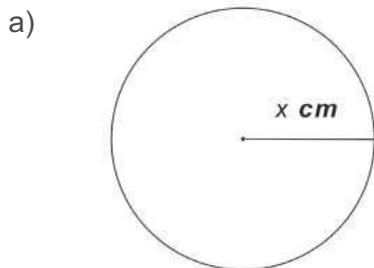
13.   Calculate the perimeter and area of the composite shape shown below.

20 cm

16 cm
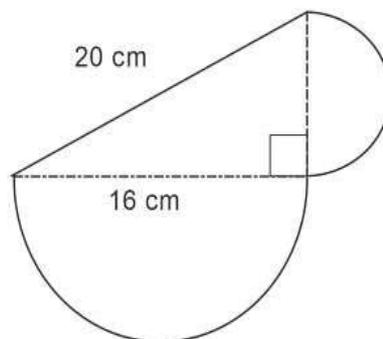
14. The diagram below shows a running track in a small sports stadium.
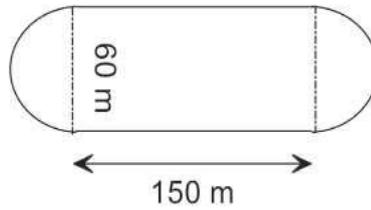
An athlete runs along the innermost lane represented by the solid lines.

a) Calcuate the distance covered by one complete circuit of the track. Give your answer to one decimal place accuracy.

b) If an athlete wants to run the equivalent of 12 km, how many laps of the track must he complete?

c) A gardener has been employed to cover the area enclosed by the track with fake grass. Calcuate this area to one decimal place accuracy.

d) If the gardener charges $22 per square metre for fake grass, what will be the total cost of covering the internal area?

15. A rubber washer has an internal radius of 11 mm and an external radius of 15 mm. Calcuate the area of the rubber washer.
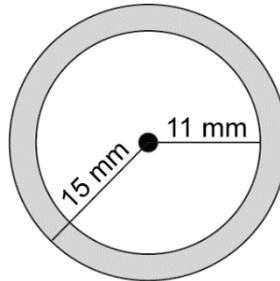
16. The diameter of the circle shown below is 8 cm. Find the area of the shaded region.

17.   Six identical cupcakes sit in a rectangular tray in such a way that the edges of each cupcake touch the walls of the tray. If the dimensions of the tray are 27 cm by 18 cm, what is the area of the tray not covered by cupcakes?

18.   A machine is used to cut semicircular pieces from rectangular plates of aluminium with dimensions of 10 by 4 cm.

4 cm

10 cm

a)   Calculate the area of material that is wasted.

b)   Express the wasted material as a percentage of the original piece. Give your answer to one decimal place accuracy.

19.   If the minute hand of clock extends from the centre of the clock to the edge of the clock face, how far does the minute hand travel from;

12

5 cm

a)   09:15 to 9:30?   b)   17:17 to 17:37?

20.   A pendulum is attached to a piece of string 11 cm in length. What is the total distance the pendulum travels over two full swings? Give your answer to the nearest mm.

75°

11 cm

Solutions

1.   a) $P = 122\ cm$ b) $P = 54\ cm$ c) $P = 186\ mm$

2.   a)   323 mm   b)   72050 m   c)   22.2 cm

     d)   68 m   e)   0.151 m   f)   0.931 km

     g)   55 mm   h)   3400 m   i)   60 cm

     j)   300 mm^2 k)   250 000 cm^2 l)   5 000 000 mm^2

3.   a) $A = 12 \times 7 - 0.5 \times 12 \times 5$ b) $A = 8^2 - 0.5 \times 6 \times 8$

          $\underline{A = 54\ cm^2}$ $\underline{A = 40\ cm^2}$

4.   a) $P = 2 \times \pi \times 10 = 62.8 cm$ b) $P = \frac{(2 \times \pi \times 10)}{2} + 20 cm = 51.4 cm$

5   a)   1000m   b)   800m   c) $8 \times 400 = 3200\ m\ or\ 3.2\ km$

6. $Area = 36 - 4 = 32 m^2$

7. $Area = 0.5 \times 38 \times 10 \times 2 + 38 \times 21$

     $\underline{Area = 1178\ mm^2}$

8.   a) $P = 2x + 2x + x + x = 6x$ b) $P = 2(x + 5) + x + x = 4x + 10$

          $A = 2x^2$ $A = x^2 + 5x$

     c) $P = x + x + y + y = 2x + 2y$ d) $P = x + 2x + 6 + 4x - 2 = 7x + 4$

          $A = xy$ $A = x^2 + 3x$

9. $A\ (triangle) = \frac{1}{2} \times 11 \times 12 = 66\ cm^2$

     $A\ (parallelogram) = 12 \times 7 = 84\ cm^2$

     $\therefore A\ (trapezium) = 150\ cm^2$

10.   a) $5cm \times 4cm$ b) $20cm \times 1\ cm$ c) $10cm \times 2.5cm$

      d) $Base = 6cm\ h = 10cm$ e) $Base = 6cm\ h = 8cm$

11.   a) $A = (5.5 \times 4) + (0.5 \times 5.5 \times 4)$
              $A = 33\ m^2$

       b) $33\ m^2 = 33\ 000\ 000\ mm^2$

            $Number\ of\ Pavers = \frac{33\ 000\ 000}{(225 \times 75)}$

            $Number\ of\ Pavers = 1\ 955.55$
            $\underline{Number\ of\ Pavers = 1956\ (nearest\ whole)}$

12.   a) $98.52 = \pi \times r^2$ b) $0.75 \times \pi \times r^2 = 98.52$

            $98.52 \div \pi = r^2$ $r^2 = 98.52 \div (0.75 \times \pi)$

            $\underline{r = 5.6\ cm}$ $\underline{r = \ \ 6.5\ cm}$

13. $20^2 - 16^2 = x^2$
       $x = 12\ cm$

       $A(triangle) = 0.5 \times 16 \times 12$ $P(semicircle1) = \frac{12\pi}{2}$

       $A(triangle) = 96\ cm^2$ $P(semicircle1) = 18.85\ cm$

       $A(semicircle\ 1) = \frac{\pi \times 6^2}{2}$ $P(semicircle2) = \frac{16\pi}{2}$
       $A(semicircle\ 1) = 56.55\ cm^2\ (2dp)$ $P(semicircle2) = 25.13\ cm$

       $A(semicircle\ 2) = \frac{\pi \times 8^2}{2}$ $P(total) = 20 + 18.85 + 25.13$
       $A(semicircle\ 1) = 100.53\ cm^2\ (2dp)$ $\underline{P = 63.98\ cm\ (2\ dp.)}$

       $\underline{Area(total) = 253.08\ cm^2}$

14.   a) $D = 2 \times \pi \times 30 + 300$ b) $Laps = 12000 \div 488.5$

            $\underline{D = 488.5m}$ $\underline{=\ 24.56\ laps\ \approx 25\ laps}$

       c) $A = 150 \times 60 + \pi \times 302$ d) $C = 11827.4 \times \$22$

            $A = \ 9000 + 2827$ $\underline{C = \$\ 260202.80}$

            $\underline{A = \ 11827.4m^2}$

15. $Area = \pi \times 152 - \pi \times 112$

       $\underline{Area = \ 326.73mm^2\ (to\ 2\ d.p.)}$

16. $radius\ of\ the\ circle\ = 4\ cm$ $Alternative\ Approach$

$Length\ (square)\qquad = \sqrt{4^2 + 4^2}$ $Area(shaded) = \pi \times 42 - 32$

$\qquad\qquad\qquad\quad = \sqrt{32}\ cm$ $\underline{Area(shaded) = \ 18.27cm^2(2\ dp.)}$

$Area\ (square) = \sqrt{32} \times \sqrt{32}$

$\qquad\qquad\qquad = 32\ cm^2$

$Area\ (circle)\ = \pi \times 4^2$

$\qquad\qquad\qquad = 50.26548246\ cm^2$

$Area\ (shaded) = 50.27 - 32$

$\underline{Area\ (shaded) = 18.27cm^2(to\ 2\ dp.)}$

17. $Area\ = \ 27 \times 18 - \pi \times 4.52 \times 6$

$\underline{Area\ = \ 104.29cm^2(to\ 2\ d.p.)}$

18. a) $A = \ 40 - \dfrac{(\pi \times 4)^2}{2}$ b) $\dfrac{14.87}{40} \times 100$

$\underline{A = \ 14.87\ cm^2(to\ 2\ d.p.)}$ $\underline{= 37.2\%}$

19. a) $P = \dfrac{90}{360} \times 2 \times \pi \times 5$ b) $P = \dfrac{120}{360} \times 2 \times \pi \times 5$

$\underline{P = 7.85\ cm\ (2dp)}$ $\underline{P = 10.47\ cm\ (2dp)}$

20. $\left(\dfrac{75}{360} \times 2 \times \pi \times 11\right) \times 2 = 28.79cm$

$\underline{Distance = 288\ mm\ (nearest\ mm)}$

Chapter 9 – Surface Area and Volume

Shape	Surface Area	Volume
Cube	TSA = area of 6 squares $\quad= 6x^2$	V = base area ×height $\quad= (x \times x) \times x$ $\quad= x^3$
Rectangular Prism	TSA = area of 6 rectangles $\quad= 2(lw + wh + lh)$	V = base area ×height $\quad= (l \times w) \times h$
Cylinder	TSA = area 2 circles + curved surface $\quad= 2\pi r^2 + 2\pi rh$ $\quad= 2\pi r(r + h)$	V = base area ×height $\quad= (\pi r^2) \times h$ $\quad= \pi r^2 h$
Cone $*r^2 + h^2 = s^2$	TSA = area of circle + curved surface $\quad= \pi r^2 + \pi rs$ $\quad= \pi r(r + s)$	$V = \frac{1}{3} \times$base area×height $V = \frac{\pi r^2 h}{3}$
Sphere	TSA = $4\pi r^2$	$V = \frac{4\pi r^3}{3}$
Hemisphere	TSA = ½ a sphere + area of circle TSA = $\frac{4\pi r^2}{2} + \pi r^2$ TSA = $2\pi r^2 + \pi r^2$ TSA = $3\pi r^2$	$V = \frac{2\pi r^3}{3}$
Square-based Pyramid	TSA = base area + area 4 sides $\quad= b^2 + 4 \times 0.5bh$ $\quad= b^2 + 2bh$	$V = \frac{1}{3} \times$base area×height $V = \frac{b^2 h}{3}$
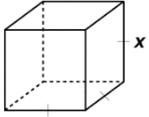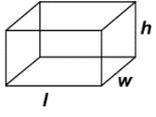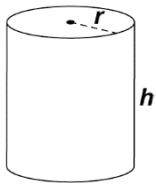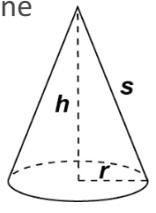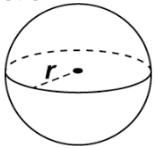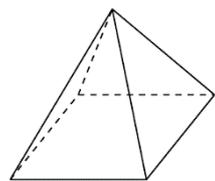
Surface Area

Worked Example 1

$x = \sqrt{3^2 + 4^2}$

$x = 5cm$

$A_{base} = 3 \times 2 \ cm = 6 \ cm^2$

$A_{left} = 4 \times 2 \ cm = 8 \ cm^2$

$A_{front} = 0.5 \times 3 \times 4 \ cm = 6 \ cm^2$

$A_{back} = 0.5 \times 3 \times 4 \ cm = 6 \ cm^2$

$A_{right} = 2 \times 5 \ cm = 10 \ cm^2$

$\underline{TSA = 10 + 8 + 3(6) = 36 \ cm^2}$
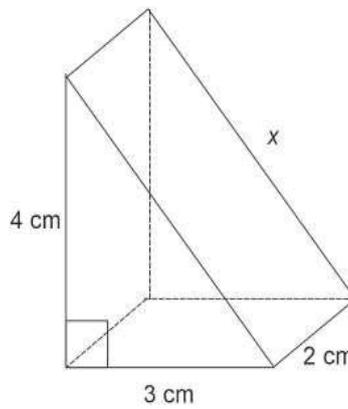
Worked Example 2

$r = \sqrt{13^2 - 12^2}$

$r = 5 \ mm$

$SA = \pi r(r + s)$

$SA = \pi \times 5(5 + 13)$

$\underline{SA = 282.74 \ cm^2 \ (2 \ dp.)}$
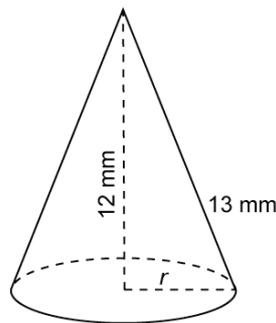
Worked Example 3

The surface area of a sphere is $1256.64 \ cm^2 \ (2 \ dp.)$. Determine the length of the radius.

$SA = 4\pi r^2$

$1256.64 = 4\pi r^2$

$r = \sqrt{\dfrac{1256.64}{4\pi}}$

$\underline{r = 10 \ cm}$

solve$(4 \cdot \pi \cdot x^2 = 1256.64, x)$

$\{x = -10.00001169, x = 10.00001169\}$

Worked Example 4

$TSA = SA_{cube} + \frac{SA_{sphere}}{2} - A_{circle}$

$TSA = 6(7 \times 7) + \frac{4 \times \pi \times 3^2}{2} - \pi \times 3^2$

$\underline{TSA = 322.27 \ cm^2 \ (2 \ dp.)}$
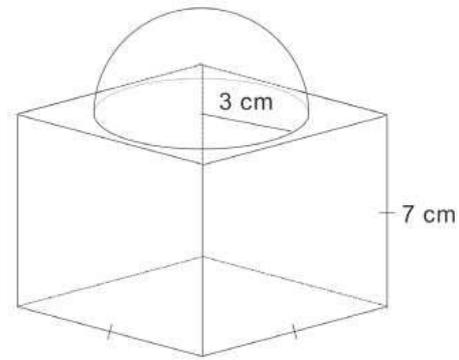
Effect of Scale Factor on Surface Area
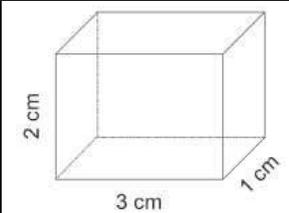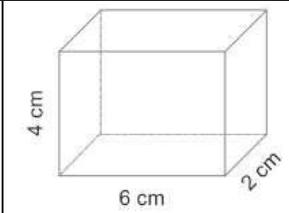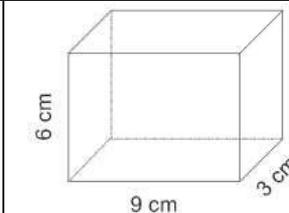
	Original	Dimensions Doubled $(k = 2)$	Dimensions Tripled $(k = 3)$
$A_{base} =$	$3 \times 1 = 3 \ cm^2$	$6 \times 2 = 12 \ cm^2$	$9 \times 3 = 27 \ cm^2$
$A_{top} =$	$3 \times 1 = 3 \ cm^2$	$6 \times 2 = 12 \ cm^2$	$9 \times 3 = 27 \ cm^2$
$A_{left} =$	$2 \times 1 = 2 \ cm^2$	$4 \times 2 = 8 \ cm^2$	$6 \times 3 = 18 \ cm^2$
$A_{right} =$	$2 \times 1 = 2 \ cm^2$	$4 \times 2 = 8 \ cm^2$	$6 \times 3 = 18 \ cm^2$
$A_{back} =$	$3 \times 2 = 6 \ cm^2$	$6 \times 4 = 24 \ cm^2$	$9 \times 6 = 54 \ cm^2$
$A_{front} =$	$3 \times 2 = 6 \ cm^2$	$6 \times 4 = 24 \ cm^2$	$9 \times 6 = 54 \ cm^2$
$TSA =$	$22 \ cm^2$	$88 \ cm^2$	$198 \ cm^2$

If the lengths in a shape or solid are all multiplied by a scale factor of k, then the areas will be multiplied by a scale factor of k^2

$$\frac{88}{22} = 4 = 2^2 \qquad\qquad \frac{198}{22} = 9 = 3^2$$

Volume

Worked Example 1

$x = \sqrt{10^2 - 8^2}$

$x = 6\ cm$

$A_{triangle} = 0.5 \times 6 \times 8\ cm$

$A_{triangle} = 24\ cm^2$

$V = AH$

$V = 24 \times 3\ cm$

$\underline{V = 72\ cm^3}$
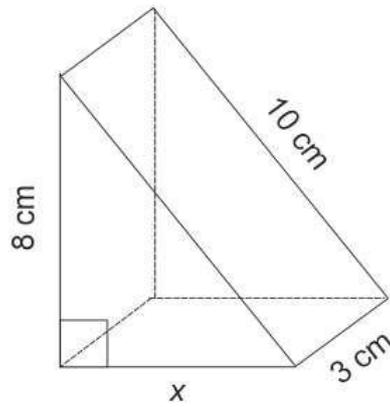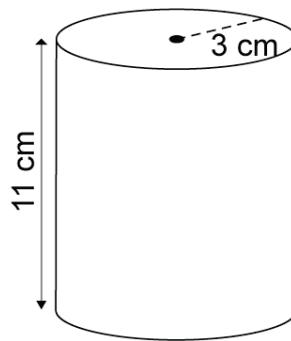
Worked Example 2

$A_{circle} = \pi \times 3^2$

$A_{circle} = 28.27433388\ cm^2$

$V = 28.27433388 \times 11 cm$

$\underline{V = 311.02\ cm^3\ (2\ dp.)}$

Worked Example 3

$h = \sqrt{13^2 - 5^2}$

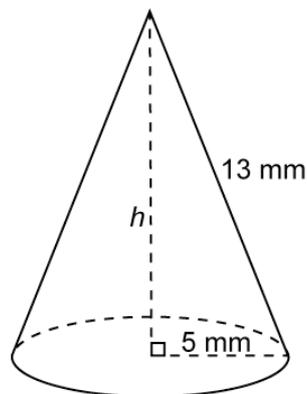$h = 12\ mm$

$V = \frac{\pi r^2 h}{3}$

$V = \frac{\pi \times 5^2 \times 12}{3}$

$\underline{V = 314.16\ cm^3\ (2\ dp.)}$

Worked Example 4

The volume of sphere is $523.60\ cm^3$ ($2\ dp.$). Determine the radius of the sphere to the nearest centimetre

$$523.60 = \frac{4\pi r^3}{3}$$

$$\sqrt[3]{\frac{423.60(3)}{4\pi}} = r$$

$\underline{r = 5\ cm\ (nearest\ cm)}$

solve$\left(523.60 = \frac{4 \cdot \pi \cdot x^3}{3}, x\right)$

$\{x = 5.000003897\}$
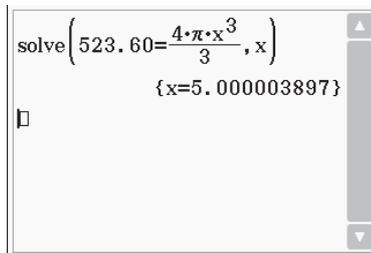
Worked Example 5

$V = V_{cube} - 0.5 \times V_{hemisphere}$

$TSA = (7 \times 7) \times 7 - (0.5 \times \frac{4 \times \pi \times 3^3}{3})$

$\underline{TSA = 324.15\ cm^3\ (2\ dp.)}$

3 cm

7 cm
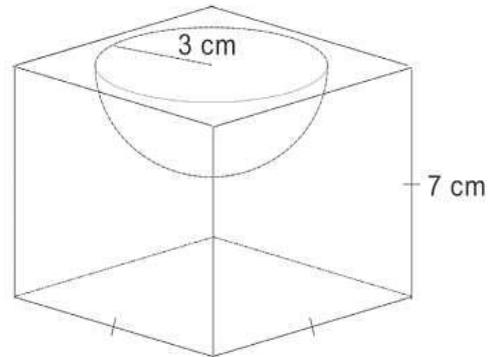
Effect of Scale Factor on Volume

| Original	Dimensions Doubled ($k = 2$)	Dimensions Tripled ($k = 3$)
$V = (3 \times 1) \times 2$	$V = (6 \times 2) \times 4$	$V = (9 \times 3) \times 6$
$V = 6\ cm^3$	$V = 48\ cm^3$	$V = 162\ cm^3$

If the lengths in a shape or solid are all multiplied by a scale factor of k, then the volume will be multiplied by a scale factor of k^3

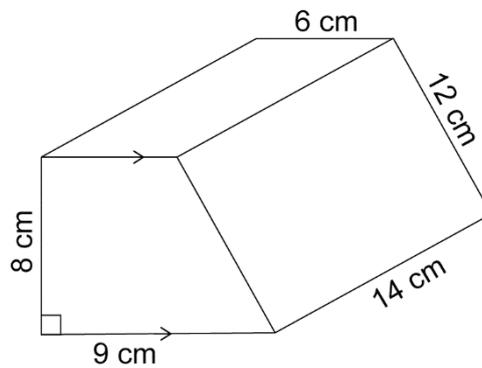$$\frac{48}{6} = 8 = 2^3 \qquad\qquad \frac{162}{6} = 27 = 3^3$$

Resource Free Questions

1.   Find the volume of a rectangular prism with the following dimensions.

        a)   8 metres wide, 6 metres long and 3 metres deep.

        b)   5 metres wide, 2 metres long and 150 cm deep.

2.   If the volume of cube is 12 cm^3, give its volume in mm^3.

3.   What is the volume of the prism shown below?
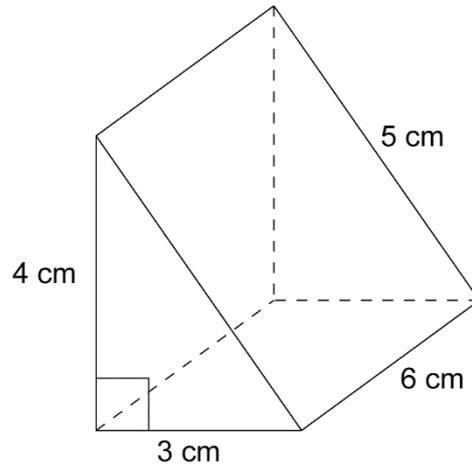
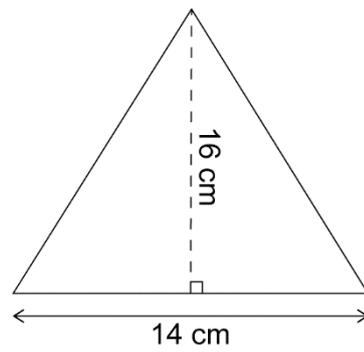

4.   Calculate the surface area of the prism shown below.

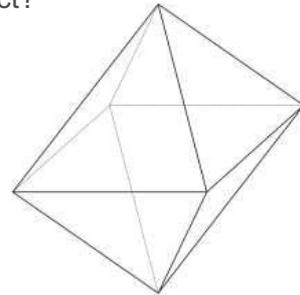5.   If the volume of a cube is 125 cm^3, what is the length of one its sides?

6.   The triangle shown below, is one side of a square-based pyramid.

   a)   Calculate the area of the triangle.
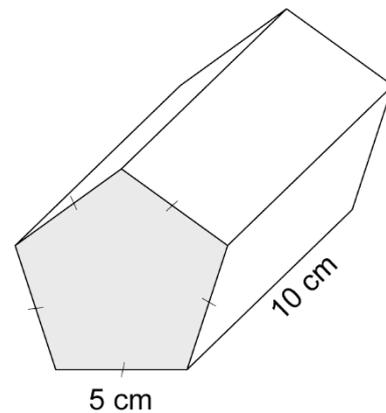

16 cm

14 cm

   b)   If two square based pyramids with identical dimensions are connected by their bases, what is the surface area of the resulting object?

7.   Consider the pentagonal prism shown below. The area of the shaded pentagon is 43 cm².

   a)   Calculate the volume of this prism.
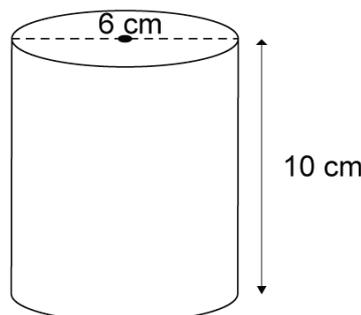
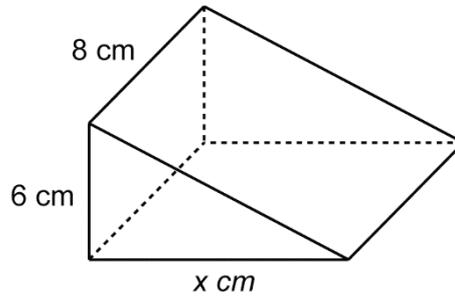   b)   Calculate the surface area of this prism.

10 cm

5 cm

8.   Consider the cylinder shown below.

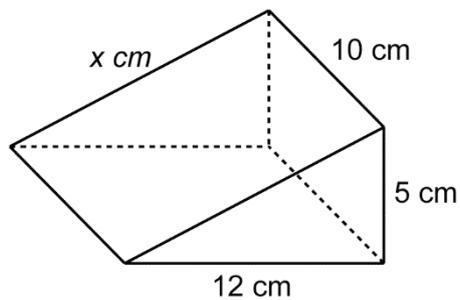   a)   Show how the volume of this cylinder can be expressed as 90π.

   b)   If the dimensions were doubled, the volume of the cylinder could be expressed as $x\pi$. Determine the value of x.

6 cm

10 cm

9.   If the volume of the prism shown below is 240 cm³, what is the length marked x?

8 cm

6 cm

x cm

10.   Calculate the surface area and volume of the prism shown below.

x cm   10 cm
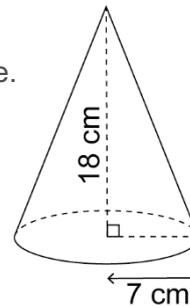
5 cm

12 cm

    a)   Calculate the volume of the prism.

    b)   Calculate the total surface area of the prism.

    c)   If the dimensions of the prism were tripled, compare the surface area of the larger
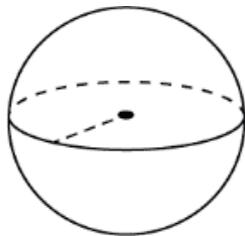   prism to the one above.

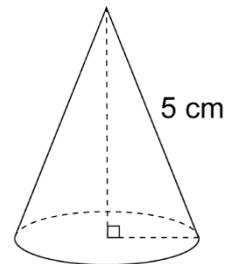## Calculator Assumed

11.   Consider the cone shown below.

   a)   Calculate the length of s, the slanted side of the cone.

   b)   Determine the total surface area of the cone.

18 cm

7 cm

12.   If the volume of sphere is 5276.67 cm³, what is the length of its radius?
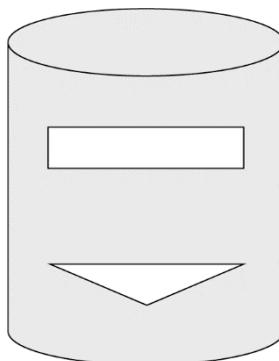

13.   If the volume of a cone is 41.06 cm³, what is the length of the radius?

5 cm

14.   If the surface area of 8 cm tall cylinder is 239.26 cm², what is the cylinder's radius?

15.   For a dress up party, a child asks his parents to make him robot helmet. A rough sketch, not to scale, is pictured below.

   The length and height of both the eye piece and the mouth pieces are identical (9 cm x 4 cm). The width of the helmet is 15 cm and the height of the helmet is 24 cm. There is no base to the helmet to allow for the entry of the head.

   What is the total area of material required to construct this helmet?

16.   A new observatory has been built on the top of the highest point in town.
   If the height of the tower is 18 metres and it is 12 metres at its widest point.
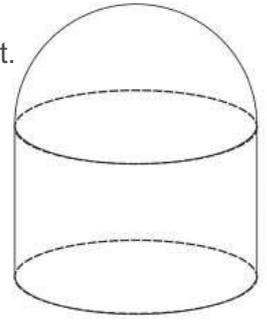
       a)   What is the total surface area of all visible surfaces
               of the observatory?

       b)   In order to blend in with the surrounding, it is decided that all visible outer surfaces
               be painted a sandstone colour. If the paint costs $8.30 per square metre, what is
               the total cost of the paint required to cover all visible outer surfaces of the
               observatory?

17.   Consider the plans for a shed shown below.

       a)   Determine the area of the front of the shed marked 'F'.

       b)   Determine the width of the roof
               panel marked 'x'

       c)   Determine the total surface area of the shed.
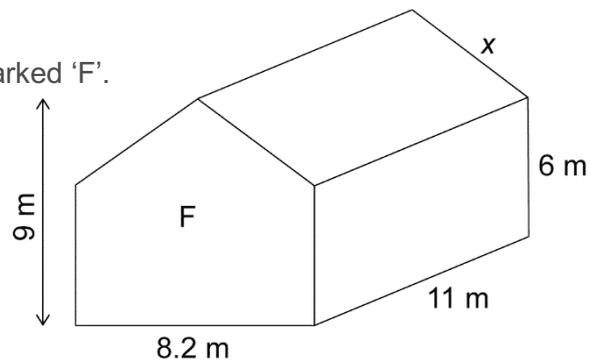
18.   The diagram below shows a cylindrical shaped hole removed from a solid cube. Determine
       the total surface area.
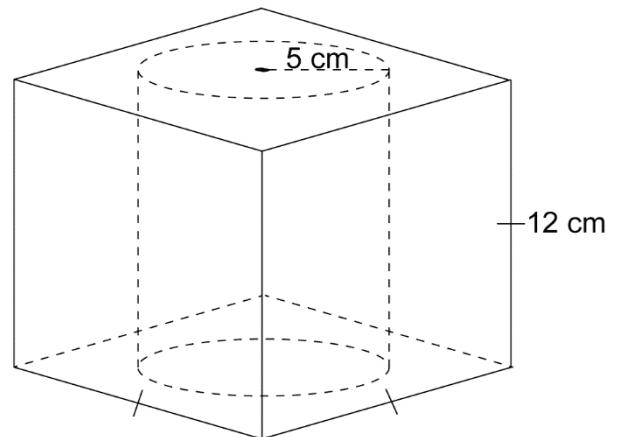
19.   The diagram below shows the dimensions of a concrete slab to be laid for a shed.

       a)   Calculate the area of the slab.

       b)   If the slab is uniformly 18 cm thick, calculate the
               volume of concrete that will be required for the slab.

       c)   If a contractor quotes $68.50 per cubic metre for
               concrete, what is the total cost?
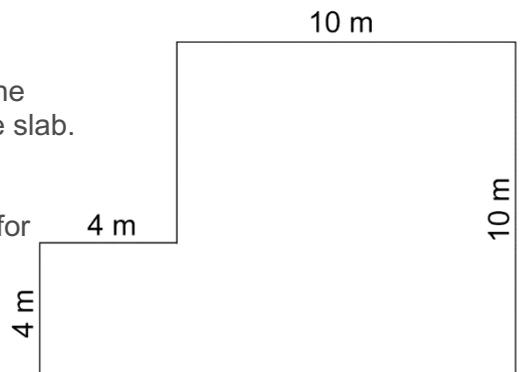
20. The town of Isseka wishes to place a giant concrete "I" in the centre of town to mark the anniversary of the town's founding.
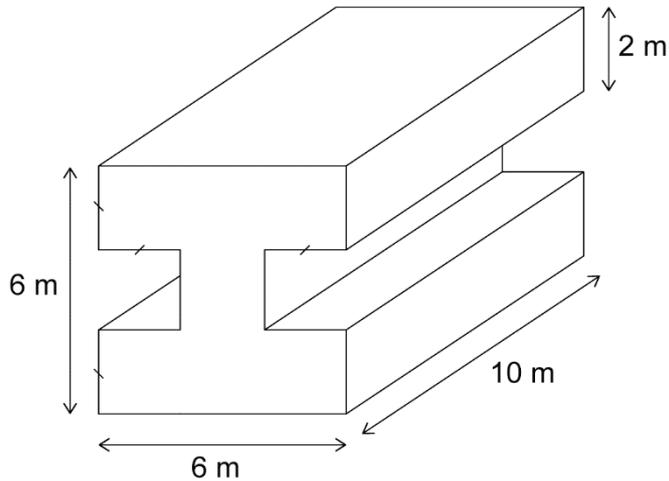

a) What volume of concrete will be needed to constuct this solid structure?

b) Unhappy with the plain grey colour of the concrete, the town council votes to paint the stucture a bright red colour for maximum visibility.

   i) Assuming the base of the stucture is inaccessible, what is the surface area of the structure that needs to be painted?

   ii) A local trademan quotes a rate of $26 per square metre to paint the structure. How much will this cost the town council?

Solutions

1. a) $V = 6 \times 8 \times 3 = 144m^3$ b) $V = 5 \times 2 \times 1.5 = 15m^3$

2. $1cm^3 = 1\,000\,mm^3$ $Therefore$ $12cm^3 = 12\,000\,mm^3$

3. $.V = 0.5 \times 3 \times 4 \times 6 = 36cm^3$

4.

Surface	Dimensions	Area (cm^2)
Front + Back	0.5×(6+9) ×8 ×2=	120
LHS + RHS	8×14 + 14×12=	112 + 168 = 280
Bottom	9×14=	126
Top	6×14=	84

$Total\ surface\ area = 120\ +\ 280\ +\ 126\ +\ 84 = 610cm^2$

5. $l = \sqrt[3]{125}$

$\underline{l = 5cm}$

6.   a) $A = 0.5 \times 14 \times 16$ b) $TSA = 112 \times 8$

$\underline{A = 112\ cm^2}$ $\underline{TSA = 896\ cm^2}$

7.   a) $v = 43\ cm^2 \times 10\ cm$ b) $TSA = (2 \times 43) + (5 \times 5 \times 10)$

$v = 430\ cm^3$ $\underline{TSA = 336\ cm^2}$

8.   a) $V = \pi r^2 h$ b) $V = 90\pi \times 2^3$

$V = \pi \times 3^2 \times 10$ $V = 720\pi$

$\underline{V = 90\pi}$ $\therefore x = 720$

9. $0.5x \times 6 \times 8 = 240$

$24x = 240$

$x = \frac{240}{24}$

$\underline{x = 10\ cm}$

10.   a)   Volume = $0.5 \times 12 \times 5 \times 10 = 300$ cm³

b)

Surface	Dimensions	Area (cm^2)
Front + Back	$0.5 \times 12 \times 5 \times 2 =$	60
Right hand side	$10 \times 5 =$	50
Bottom	$12 \times 10 =$	120
Top	$10 \times \sqrt{(5^2 + 12^2)} = 10 \times 13$	130

$\underline{Total\ surface\ area = 60 + 50 + 120 + 130 = 360cm^2}$

c)   The surface area would be 9 times that of the original.

11.   a) $S = \sqrt{7^2 + 18^2} = 19.31\ cm\ (to\ 2\ d.p.)$ b) $TSA = \pi r^2 + \pi rs$

$TSA = \pi \times 7^2 + \pi \times 7 \times 19.31$

$\underline{TSA = 578.59\ cm^2\ (to\ 2\ d.p.)}$

12. $r = 10.80\ cm$

solve$\left(5276.67=(4 \cdot \pi \cdot x^3)/3, x\right)$
    $\{x=10.80000049\}$

13. $r = 2.80$

solve$\left(41.06=\pi \cdot x^2 \cdot \dfrac{5}{3}, x\right)$
    $\{x=-2.800336115, x=2.8003\blacktriangleright$

14. $r = 3.35\ cm\ (2dp.)$

solve$(239.26=2 \cdot \pi \cdot x \cdot (x+8), x)$
    $\blacktriangleleft.35387052, x=3.353870524\}$

15. $TSA = \pi r^2 + 2\pi rh - rectangle - triangle$

$TSA = \pi \times 7.5^2 + 2 \times \pi \times 7.5 \times 24 - (9 \times 4) - (0.5 \times 9 \times 4)$

$\underline{TSA = 1253.69\ cm^2\ (2\ dp.)}$

16. $TSA = \dfrac{4\pi r^2}{2} + 2\pi rh$ b) $C = 904.78 \times 8.3$

$TSA = \dfrac{4\pi 6^2}{2} + 2\pi \times 6 \times 18$ $\underline{C = \$7509.66}$

$\underline{TSA = 904.78\ m^2}$

17. $Area(F) = (8.2 \times 6) + (0.5 \times 8.2 \times 3)$ b) $x = \sqrt{4.1^2 + 3^2}$

        $\underline{Area(F) = 61.5\ m^2}$ $\underline{x = 5.08\ m}$ (2 dp.)

18. $TSA = SA_{cube} - 2 \times A_{circle} + A_{inside\ cylinder}$

        $TSA = 6(12^2) - 2(\pi \times 5^2) + (2 \times \pi \times 5 \times 12)$

        $TSA = 1083.91\ cm^2\ (2\ dp.)$

19.   a) $A = 4 \times 4 + 10 \ \times 10 = 116 m^2$ b) $0.18 \ \times 116 = 20.88 m^3$

        c) $68.50 \times 20.88 = \$1430.28$

20.   a) $V = 6 \times 6 \times 10 - 2 \times 2 \times 10 \times 2$

               $\underline{V = 280\ m^3}$

        b)   i) $TSA = 10 \times 2 \times 10 \ + \ 6 \times 10 + 2 \times (\ 36 - 4 - 4\ )$

                        $\underline{TSA = 316\ m^2}$

   ii)   Cost to paint = 316 × 26 = \$8 216

Chapter 10 – Similarity and Scale Factor

**Similar Triangles**

There are three tests for similar triangles.

Test	Description
AAA	If the three sets of corresponding angles of two triangles are the same, then the triangles are similar.
SSS	If the three sets of corresponding sides of two triangles are in proportion, then the triangles are similar.
SAS	If an angle of one triangle is equal to the corresponding angle of the other and the lengths of the sides including the angle are in proportion (included angle), then the triangles are similar.
RHS	If the hypotenuses of right-angled triangles and another set of corresponding sides are in the same ratio, then the triangles are similar.

When triangles are similar, their corresponding sides are in proportion i.e. $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = 2$
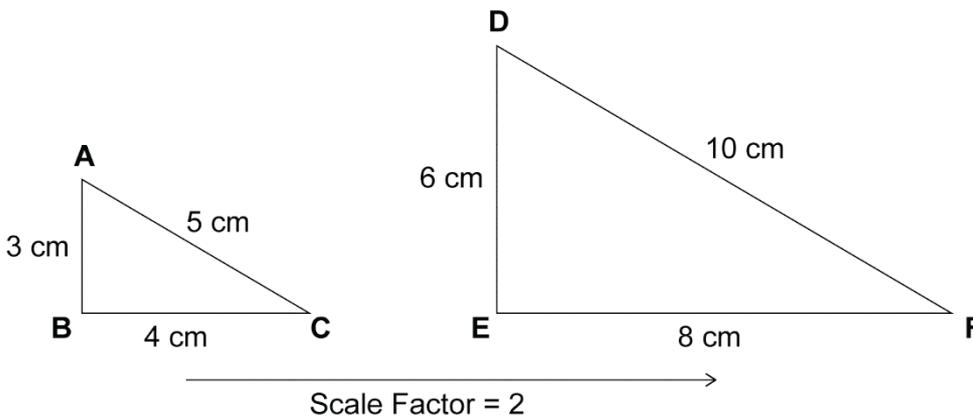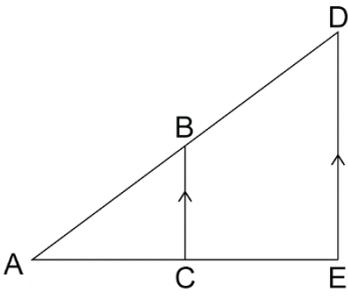

Scale Factor = 2

Worked Example 1

Prove that $\Delta ABC \sim \Delta ADE$
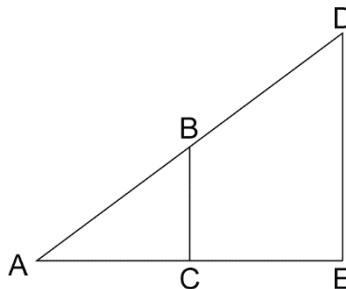
*** Note that line BC is parallel to line DE

$\angle BAC = \angle DAE$ (common angle)

$\angle BCA = \angle DEA$ (corresponding angles)

$\angle ABC = \angle ADE$ (corresponding angles)

$\therefore \Delta ABC \sim \Delta ADE$ (AAA)

Worked Example 2

Find the value of x if $\Delta DEF \sim \Delta PQR$.
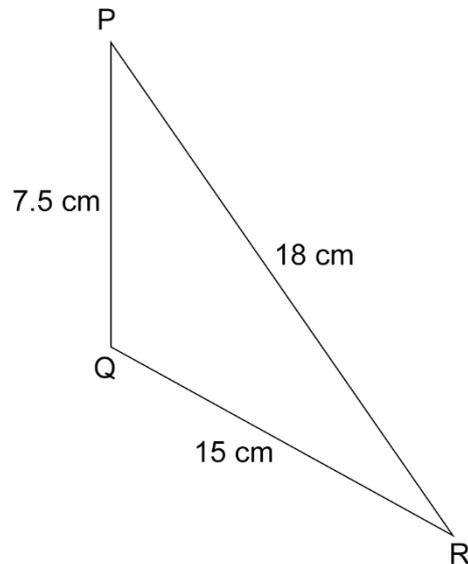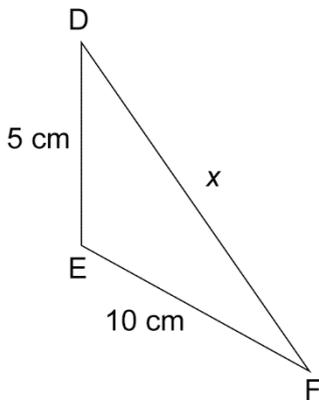
Establish the scale factor.

$\frac{PQ}{DE} = \frac{7.5}{5} = 1.5$

$\frac{QR}{EF} = \frac{15}{10} = 1.5$ $\therefore$ Scale factor (k) is 1.5

$\therefore \frac{PR}{DF} = 1.5$

$\therefore \frac{18}{x} = 1.5$

$x = \frac{18}{1.5}$

$\underline{x = 12}$

Similarity and Area

If a shape is enlarged by a scale factor of k, then the area of the similar shape can be determined by multiplying the area of the original shape by k^2.

Worked Example 1
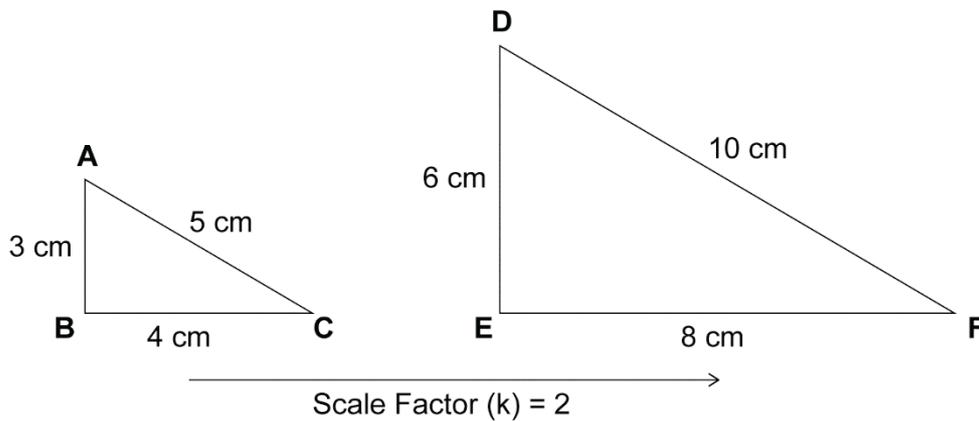
$Area(\Delta ABC) = 0.5 \times 4 \times 3 = 6\ cm^2$ $\frac{24}{6} = 4 = 2^2$

$Area(\Delta DEF) = 0.5 \times 8 \times 6 = 24\ cm^2$

Worked Example 2

A rectangle with an area of $12\ cm^2$ is enlarged by a scale factor of 3. Determine the area of the enlarged rectangle.

Scale factor $(k) = 3$

$12 \times 3^2 = 108\ cm^2$

Resource Free Questions

1.   Determine the conditions for which the pairs of triangles are similar. If the triangles are not similar, write "not similar".

a)

b)

c)

d)
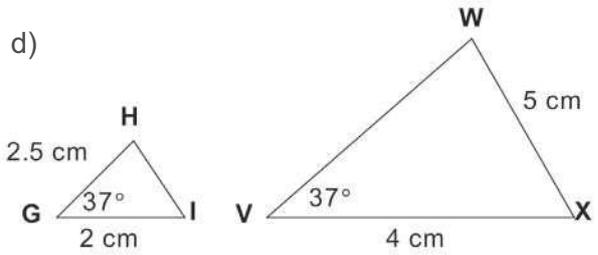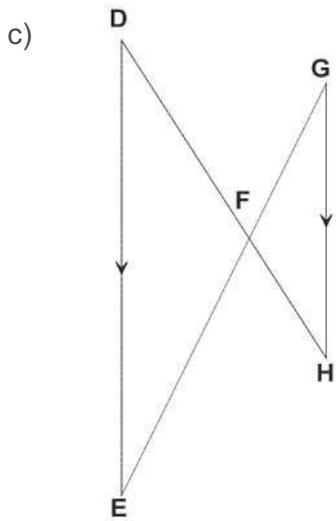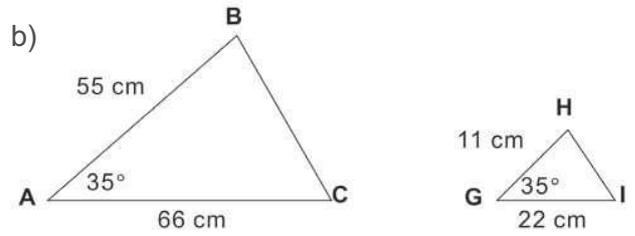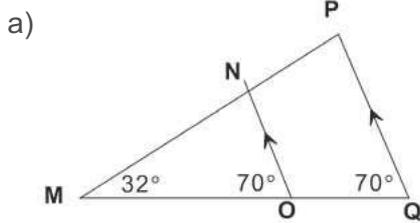
2.   For each of the following pairs of triangles, explain why they are similar.

a)

b)
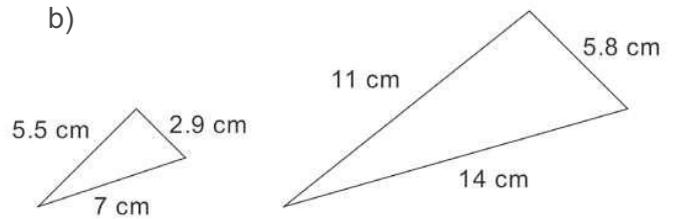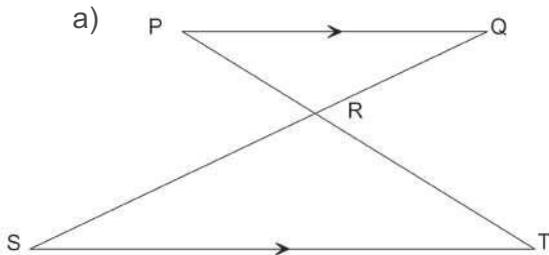
3.   Is triangle ABC similar to triangle GHI? Provide evidence to support your answer.
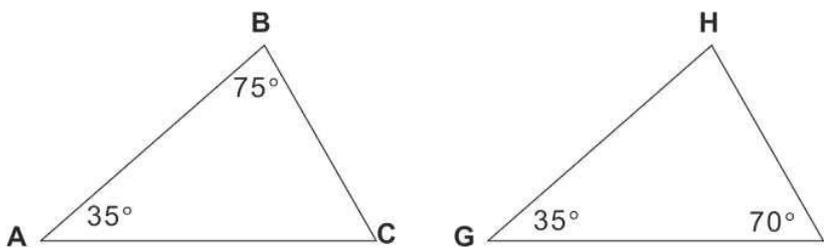
4.   Prove triangle ABC is similar to triangle ADE.
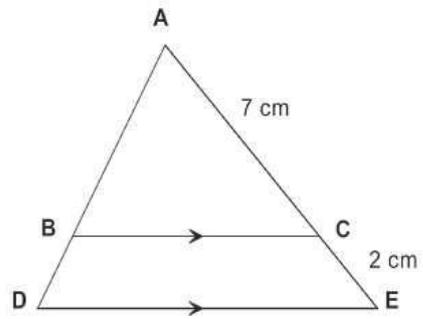
5.   Calculate the value of the unknowns in each of these similar triangles.
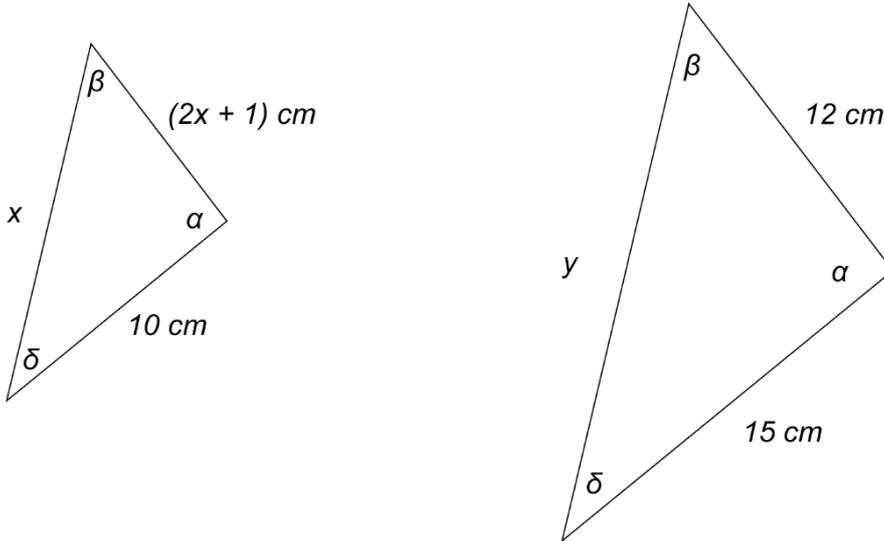
6.   The triangles shown below are similar. Calculate the values of *x* and *y*.
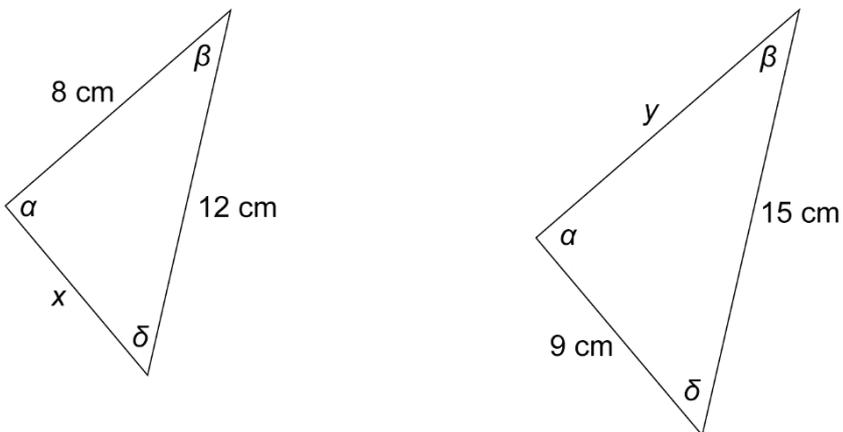
7.   If $\triangle ABC \sim \triangle ADE$, calculate the value of the pronumerals.
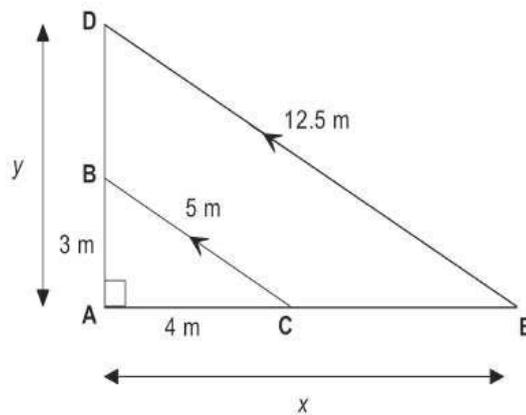
8. Find the base and height of the new triangle formed when the triangle below undergoes the following transformations.

6 cm

8 cm
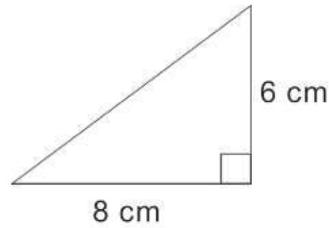
Transformation	Base	Height
Scale factor of $\frac{1}{2}$		
Scale factor of 3		
Scale factor of $\frac{3}{2}$		
Scale factor of $\frac{1}{3}$		

9. The scale factor on a map is given as 1 cm = 6 km. If the distance between the two towns on the map is 2.2 cm, what is the actual distance between the two towns?

10. The scale on a set of house plans is given as 1 : 180.

   a) Calculate the length of the master bedroom if it is shown as 3.2 cm on the plans.

   b) The length of the balcony is 9 metres. What is this length on the house plans?

11. Shapes A and B are similar figures. In comparing A with B;

   a) what is the ratio of the sides?

   b) what is the enlargement factor?

.

A

18 cm

B

27 cm
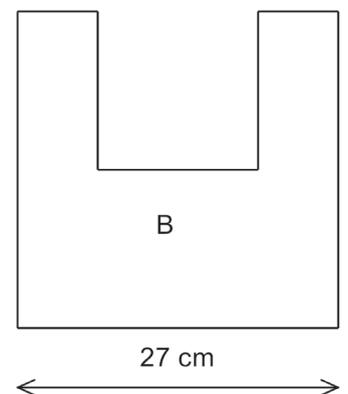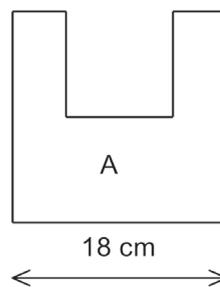
12.   Consider the four triangles A, B, C and D shown below.
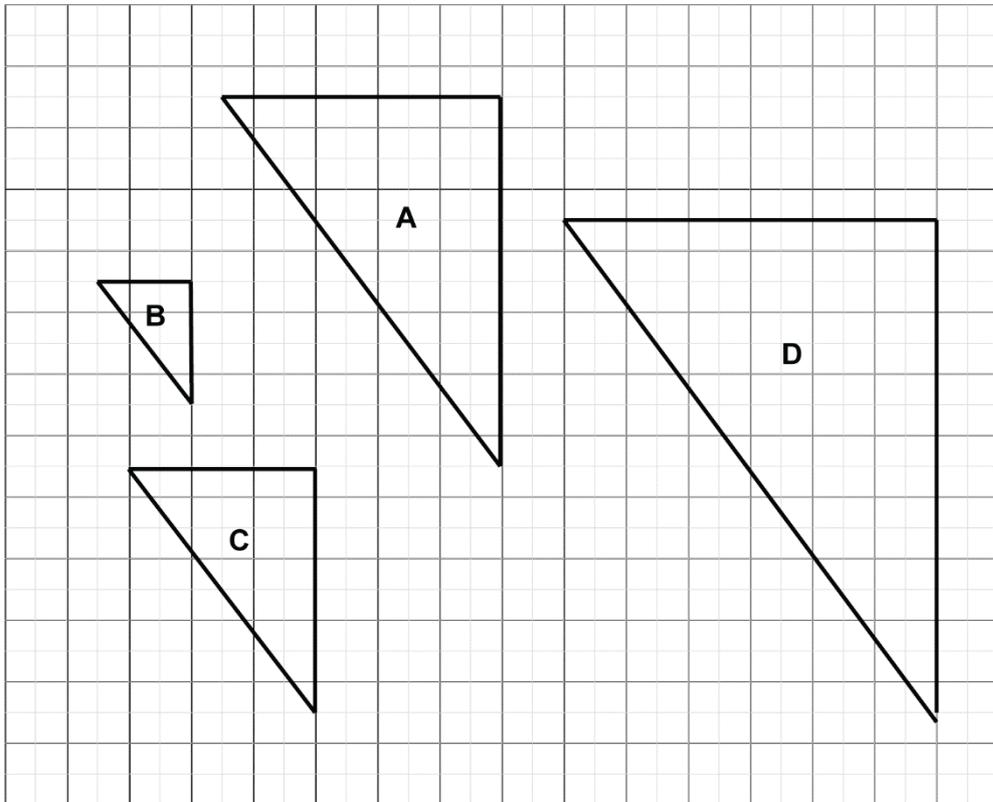
a)   If A is an enlargement of C, determine the scale factor.

b)   If B is a reduction of D, determine the scale factor.

c)   If C is an enlargement of B, determine the scale factor.

d)   If D is an enlargement of A, determine the scale factor.

e)   If A is an enlargement of B, determine the scale factor.

13.   The two quadrilaterals shown below are similar. If the area of the shape labelled "A" is

       20 cm^2, determine the area of quadrilateral "B".
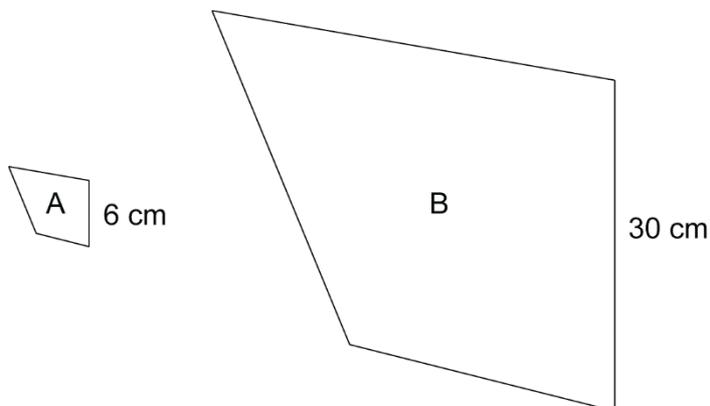
14.   Consider the quadrilaterals labelled A, B and C below.
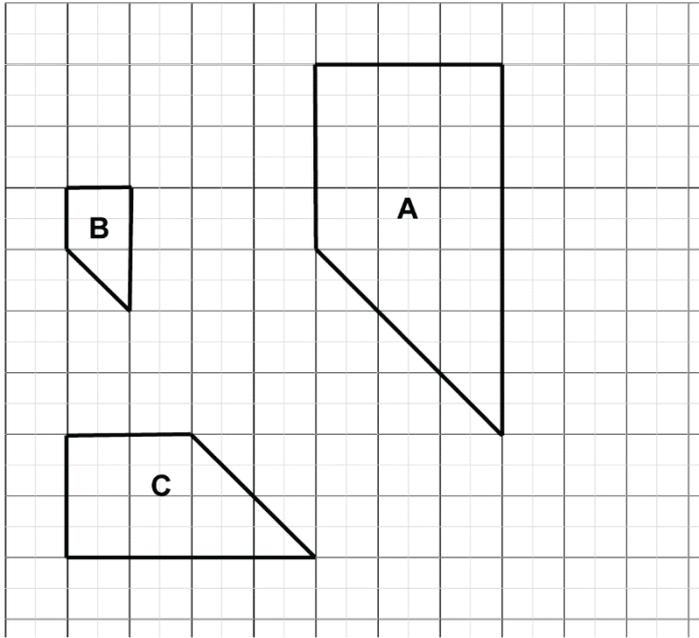
a)   If A is an enlargement of B, determine the scale factor.

b)   If C is an enlargement of B, determine the scale factor.

c)   If B is a reduction of C, determine the scale factor.

d)   If C is a reduction of A, determine the scale factor.

15.   Use the grid paper provided for the following.

a)   Draw an enlargement of shape G, scale factor 2 and label it H.

b)   Draw an enlargement of shape G, scale factor ½ and label it J.
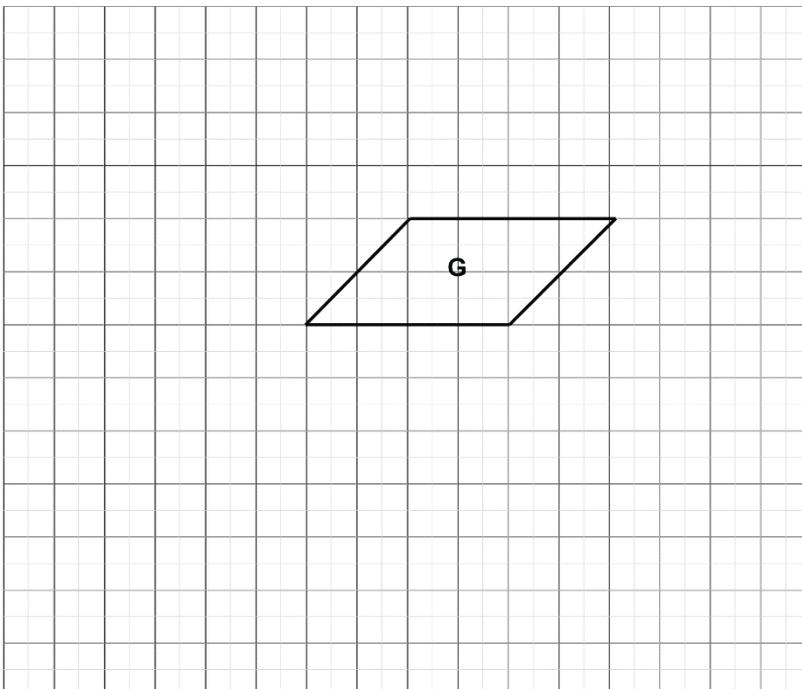
Resource Rich Questions

16.   A right-angled triangle has shorter sides of lengths 9 cm and 40 cm. A similar triangle has a hypotenuse of 369 cm. What is the scale factor?

17.   Consider the pair of triangles shown below
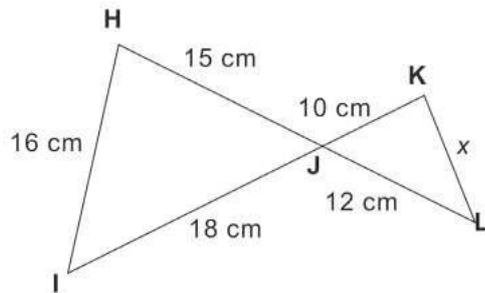
a)   Show that ΔHIJ~ ΔJKL

b)   Find the value of x.

18.   For the following pair of triangles, explain why they are similar.
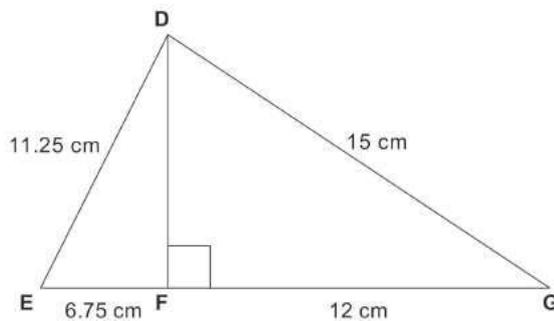
19.   When a tree casts a 20 m shadow, a man who is 1.8 metres tall casts a 2.4 m shadow. Use your knowledge of similar triangles to calculate the height of the tree.
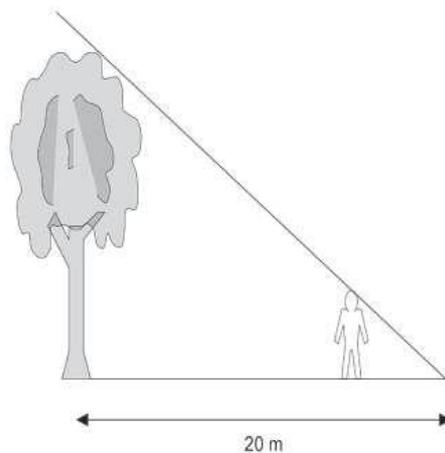
20.   A nine centimetre toy ladder leans against a vertical wall 6 centimetres above the ground. At what height would a 15 centimetre ladder touch the wall if both ladders form the same angle with the ground?

Solutions

1. a) $\angle NMO = \angle PMQ$ (common angle)   b)   Not Similar
          $\angle NOM = \angle PQM$ (corresponding angles)

          $\angle ONM = \angle QPM$ (corresponding angles)

          $\therefore \Delta MNO \sim \Delta MPQ$ [AAA]

    c) $\angle DFE = \angle HFG$ (vertically opposite angles)   d)   Not Similar

          $\angle FDE = \angle FHG$ (alternate angle)

          $\angle DEF = \angle HGF$ (alternate angle)

          $\therefore \Delta FDE \sim \Delta FHG$ [AAA]

2. a   AAA   b)   SSS

3. $\angle BAC = \angle HGI$ (corresponding angles)

    $\angle BCA = \angle HIG$ (corresponding angles)

    $\angle ABC = \angle GHI$ (corresponding angles)

    $\therefore \Delta ABC \sim \Delta GHI$ [AAA]

4. $\angle A \ is \ common$

    $\angle ABC = \angle ADE$ (corresponding angles)

    $\angle ACB = AED$ (corresponding angles)

    $\therefore \Delta ABC \sim \Delta ADE$ (AAA)

5. $\frac{15}{10} = 1.5$ $\frac{y}{15} = \frac{x}{10}$

    $\frac{12}{1.5} = 8$ $y = \frac{15 \times 3.5}{10}$

    $2x + 1 = 8$ $\underline{y = 5.25}$

    $\underline{x = 3.5}$

6. For y: $\frac{8}{y} = \frac{12}{15}$ For x: $\frac{12}{15} = \frac{x}{9}$

    $\frac{(8 \times 15)}{12} = \frac{y}{1}$ $\frac{12 \times 9}{15} = \frac{x}{1}$

    $\underline{\therefore y = 10cm}$ $\underline{\therefore x = 7.2cm}$

7. $$\frac{12.5}{5} = \frac{AE}{4}$$ $$\frac{12.5}{5} = \frac{AD}{3}$$

   $$AE = \frac{(12.5 \times 4)}{5}$$ $$AD = \frac{(12.5 \times 3)}{5}$$

   $$\underline{x = 10\ m}$$ $$\underline{y = 7.5\ m}$$

8.

Transformation	Base	Height
Scale factor of ½	4cm	3cm
Scale factor of 3	24cm	18cm
Scale factor of $^3/_2$	12cm	9cm
Scale factor of $^1/_3$	8/3 cm	2cm

9.   2.2 × 6m = 13.2m

10.   a) $Length = 180 \times 3.2\ cm$ b) $900cm \div 180 = 5\ cm\ on\ the\ house\ plans$

       $\underline{Length = 576cm = 5.76\ m}$

11   a)   18 : 27 = 2 : 3   b)   1.5

12.   a) $\frac{3}{2}$ b) $\frac{1}{4}$ c)   2   d) $\frac{4}{3}$ e)   3

13.   Scale factor x 5 so area of big quadrilateral is $5 \times 5 \times 20 = 500\ cm^2$

14.   a)   3   b)   2   c) $\frac{1}{2}$ d) $\frac{2}{3}$

15.
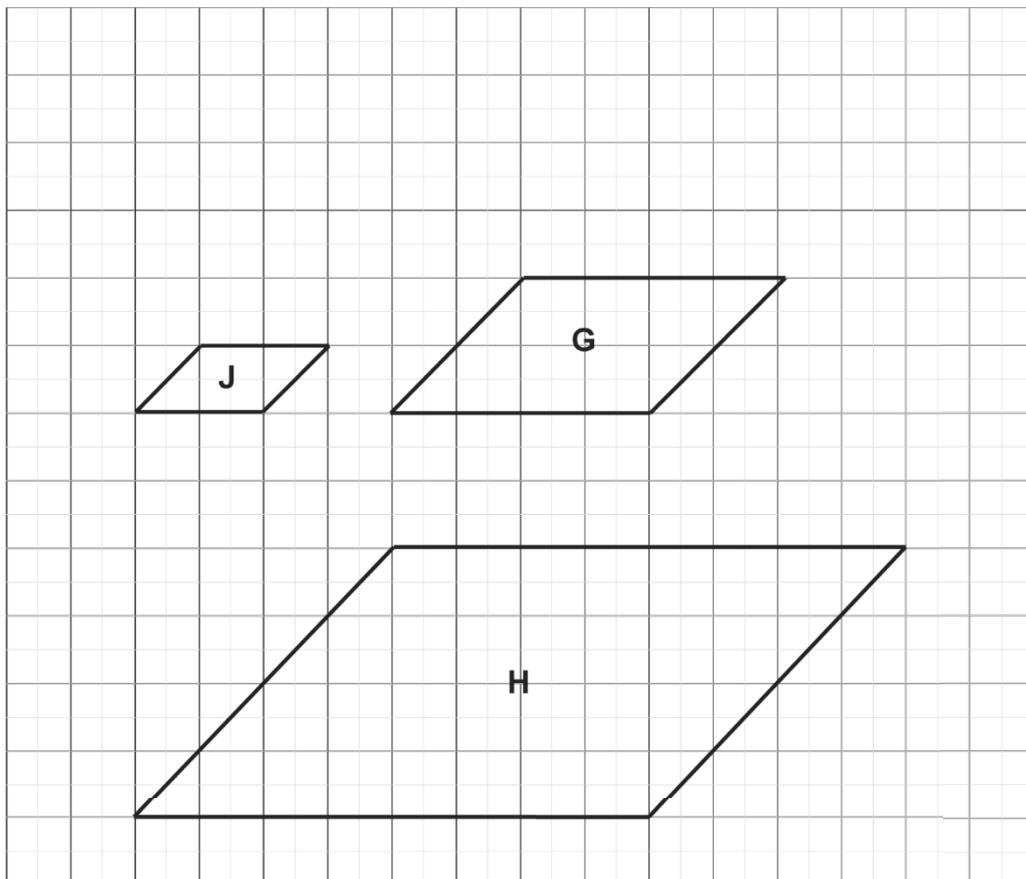
16. $369 \div 41 = 9$ Scale factor = <u>9 or 9 :1</u>

17.   a) $\angle KJL = \angle HJI$ (vertically opposite angles)

$\frac{HJ}{KJ} = \frac{IJ}{LJ}$ (surrounding sides are in proportion)

$\therefore \Delta JKL \sim \Delta HIJ$ [SAS]

b) $1.5x = 16$

$x = \frac{16}{1.5}$

$x = 10\frac{2}{3}\ cm$

18. $DF = \sqrt{15^2 - 12^2}$

$DF = 9\ cm$

$\frac{GD}{DE} = \frac{FG}{FD} = \frac{DF}{EF}$ $(common\ ratio\ of\ \frac{4}{3} for\ all\ corresponding\ sides)$

$\therefore \Delta DFE \sim \Delta FGD$ [SSS]

19. $\frac{x}{1.8} = \frac{20}{2.4}$ $\therefore\ \frac{x}{1} = \frac{(20 \times 1.8)}{2.4}$ $\therefore\ x = 15m$

20. $\frac{x}{6} = \frac{15}{9}$ $\therefore$ $\frac{x}{1} = \frac{(15 \times 6)}{9}$ $\therefore\ x = 10cm$

Chapter 11 - Univariate Data

Measures of Central Tendency – Averages

There are actually three different types of averages used in statistics. They are the **mean**, the **mode** and the **median**.

Mean $(\bar{x})$ = $\dfrac{\text{Sum of the values}}{\text{Total number of values}}$ = $\dfrac{\sum x}{n}$

The mode is the value that occurs most often. (There can be more than one mode or no mode at all).

The median is the middle value, when the set of numbers are placed in ascending order. If there are two numbers in the middle, the median is the *mean* of those two numbers.

Measures of Dispersion or Spread

The centre of a set of observations is one of the two most important characteristics. The other is its variability or spread, i.e. the extent to which the observations deviate from the central value.

The range

The range gives a rough idea of how the scores are spread. It is defined as the highest score (maximum) minus the lowest score (minimum)

Range = maximum score – minimum score

Interquartile range

The interquartile range is a number that describes the spread of the middle 50% of the data in an ordered data set. It is used to see how closely the data is grouped around the median. The data is organised into quartiles each containing 25% of the data. The lower quartile (Q_L) is the median of the lower set of values and the upper quartile (Q_U) is the median of the upper set of data.

Standard deviation

A very important measure of spread is the standard deviation. It is a sort of average of the squares of the deviations of the data values from the mean. The greater the spread of the scores from the mean, the larger the value of the standard deviation. It should be noted that the standard deviation, like the mean, is influenced by outlying or extreme observations. And, like the median, the interquartile range is not affected by extreme observations.

Worked Example

Consider the following set of scores 7, 8, 7, 11, 7, 10, 6

Mean = $\frac{56}{7}$ = 8   The range is 11 – 6 = 5

Mode is 7   The interquartile range is 10 – 7 = 3.

Median   6, 7, 7, (7), 8, 10, 11   The standard deviation is 1.69 (2 dp.)

Analysing Data With The ClassPad

Weighted Mean

The weighted mean is similar to the arithmetic mean, where instead of each data point contributing equally to the final average, some data points contribute more than others.

Consider the example below.

Score out 10	9	4	6	3
Weighting	3	2	1	4

Weighted mean $= \dfrac{(9\times3)+(4\times2)+(6\times1)+(3\times4)}{10}$

$= \dfrac{53}{10}$

$= 5.3$

Univariate Data Questions

Resource Free

1.   For the dot plot below, calculate the mean, mode and median of the scores.
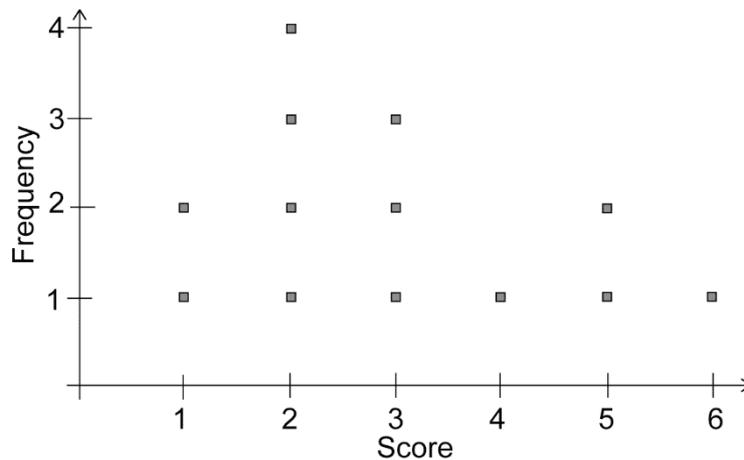
2.   Of a set of 7 numbers, 6 are given below.

        4   4   5   5   5   6

Note: The missing value (7th number) is a whole number greater than 4.

What must the seventh number be so that for the seven values:

a)   the mean is equal to the median?

b)   the mean is equal to the mode?

3.   Determine the median and range for data displayed in the stem-and-leaf plot shown below.

```
1  | 5 5 6
2  | 2 3 3 3 7
3  | 4 8
4  | 1 1 4 9
```
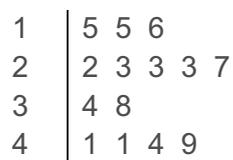
4.   The table below shows the sales of men's shoes at a local market.

Shoe Size	9	10	11	12	13	14
Frequency	3	5	7	1	2	2

a)   Calculate the mean shoe size.
b)   Calculate the median shoe size.
c)   Calculate the modal shoe size.
d)   Which measure of central tendency is of most value to the owner of the shoe store?

5.   Consider the following set of scores. 14, 23, 39, 41, 47, 51, 59.

a)   If the score of 23 were removed, describe how the mean and median would be affected.

b)   If you had a list of 6 scores and only 3 were known, would it still be possible to determine the;

i)   mean?
ii)   mode?
iii)   median?
iv)   range?

Calculator Assumed

6.   Five people have a mean weight of 67 kg. If a child weighing 43 kg was added to the group, what would the mean weight be?

7.   During a term, a student has an average of 56% after the first four tests and his average for the next six tests is 44%. If the student does not have an overall mean of at least 50% he will have to repeat the unit. Will this student have to repeat? Provide evidence to support your answer.

8.   Consider the data shown below.

Mobile Bill ($)	No. Students
0 - 19	3
20 - 39	7
40 - 59	5
60 - 79	9
80 - 99	d

a)   If $d = 3$, find the mean and standard deviation of the scores above.

c)   If the mean is 57, find the value of 'd'.

9.   In the student elections for the position of student council president, the two candidates, Meagan and Teagan both received 50 votes each. Unable to decide between the two girls, the year coordinator gives the girls scores out of 10 in four categories attendance, punctuality, public speaking and confidence. He then picks a winner based on the weighted average the two girls achieve from the four categories.

	Categories			
	Attendance	Punctuality	Public Speaking	Confidence
Meagan	6	9	8	7
Teagan	8	6	9	7
Weighting	4	5	3	4

Calculate the weighted mean for both girls and determine who wins the election.

10.   George owns a small independent supermarket. Last year he paid himself $325 000 and his employees were paid as follows:

Position	Number of Employees	Annual Salary
Owner	1	$325 000
Manager	1	$85 000
Assistant Managers	3	$65 000
Accountant	1	$58 000
Marketing Manager	1	$52 000
Fresh Produce Supervisors	3	$38 500
Butcher	2	$34 000
Retail Assistants	10	$31 000
Security Guard	2	$29 500

a)   Determine the mean, mode and median salary of the workers in this company.

b)   One of the employees argues that the workers receive a poor average wage compared to other companies of a similar size. Which "average" figure would he use to support his argument? Explain

c)   The owner of the company argues that the workers have an excellent "average" pay. Which figure would he quote to support his argument?

d)   Which measure of central tendency gives the "fairest" measure of the average pay?

Solutions

1. $\bar{x} = \dfrac{\Sigma x}{n}$

    $\bar{x} = \dfrac{(1\times2)+(2\times4)+(3\times3)+4+(5\times2)+6}{13}$

    $\bar{x} = 3$

    <u>Median = 3</u>

    <u>Mode = 2</u>

2. a) The median must be 5.

    $\bar{x} = \dfrac{\Sigma x}{n}$

    $5 = \dfrac{4+4+5+5+5+6+x}{7}$

    $x = 6$

    b) If $x > 4$, then the mode is 5.

    $5 = \dfrac{4+4+5+5+5+6+x}{7}$

    $\underline{x = 6}$

3. a) The median is the mean of the 7th and 8th score. $\dfrac{(23+27)}{2} = \underline{25}$

    b) Range is $49 - 15 = \underline{34}$

4. a) $\bar{x} = \dfrac{220}{20} = 11$

    b) Median = 11

    c) Mode = 11

    d) The mode, as this needs to be taken into account when restocking.

5. a) The mean and the median would both be higher.

    b) i)   no

        ii)   no

        iii) Only if the 3rd and 4th highest scores are known.

        iv)   Only if the highest and lowest scores are known.

6. $\bar{x} = \dfrac{(5\times67)+43}{6}$

    $\bar{x} = \dfrac{378}{6}$

    $\underline{\bar{x} = 63}$

7. $\overline{x} = \frac{(56\times4)+(44\times6)}{10}$

        $\overline{x} = 48.8$

        <u>The student must repeat the unit.</u>

8.   a)   Use interval midpoints

	list1	list2	list3
1	9.5	3	
2	29.5	7	
3	49.5	5	
4	69.5	9	
5	89.5	3	
6			

$\overline{x} = 50.98$ (2d.p.)
$s = 24.30$ (2d.p.)

        b) $\overline{x} = \frac{\Sigma x}{n}$

                $57 = \frac{(9.5 \times 3) + (29.5 \times 7) + (49.5 \times 5) + (69.5 \times 9) + (89.5 \times d)}{(3 + 7 + 5 + 9 + d)}$

                $d = 8$

9.   Meagan   Teagan

$\overline{x} = \frac{(6\times4)+(9\times5)+(8\times3)+(7\times4)}{4+5+3+4}$ $\overline{x} = \frac{(8\times4)+(6\times5)+(9\times3)+(7\times4)}{4+5+3+4}$

$\overline{x} = 7.56$ (2 d.p.) $\overline{x} = 7.31$ (2 d.p.)

<u>Meagan wins the election with a weighted mean of 7.56.</u>

10.   a) $\overline{x} =$ \$52 812.50 (2 d.p.)   Median = \$32 500   Mode = \$31 000

        b)   This employee would quote the lowest figure, the mode.

        c)   The owner would quote the highest figure, the mean.

        d)   The median gives the best measure of the average wage of the workers.

Chapter 12 – Collecting, Representing and Interpreting Data

Types of Data

Variable

Categorical

Nominal
- *Ordering Does Not Exist*
- *eye colour, gender, which brand?*

Ordinal
- *Ordering/Ranking Exists*
- *rating scale, army rank*

Numerical

Discrete
- *Obtained by Counting*
- *Values are integers*
- *how many?*

Continuous
- *Obtained through measurement*
- *height, weight, km/hr*
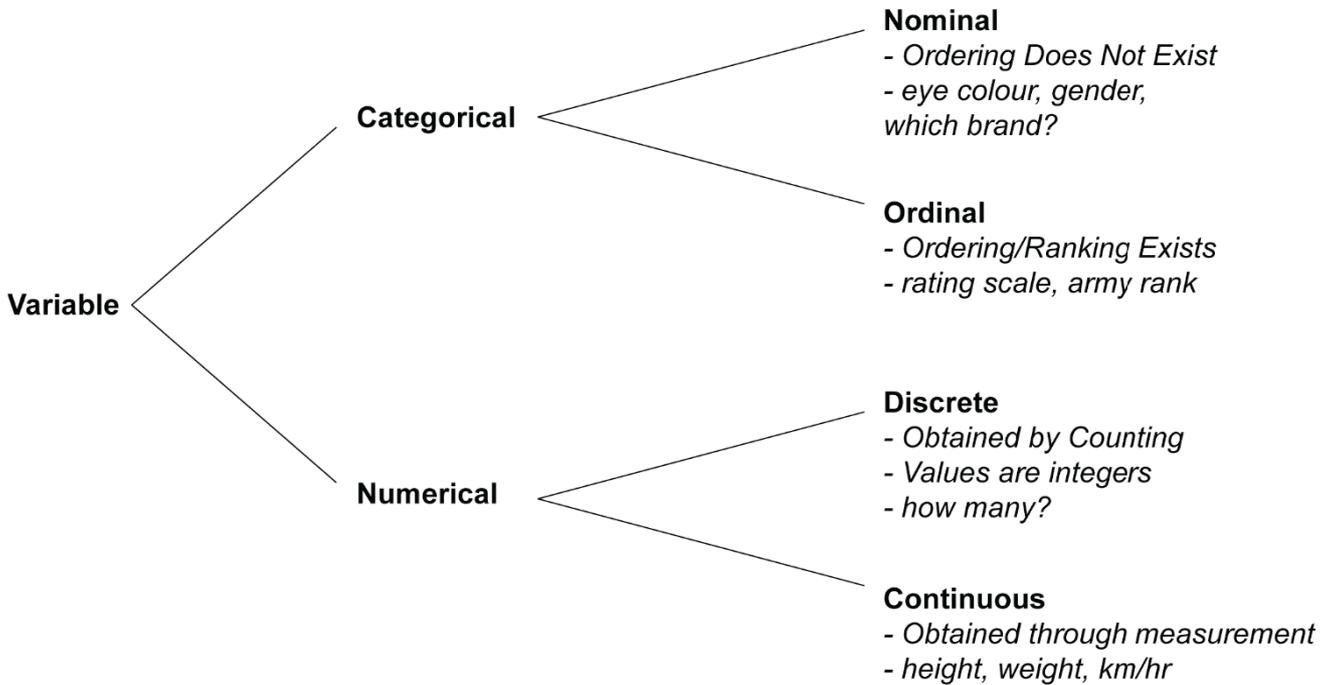
Stem and Leaf Plots

A stem-and-leaf plot resembles a histogram turned on its side. Much like a histogram, the major advantage to organizing the data into a stem-and-leaf plot is that we get a quick visual picture of the shape of the distribution. An added advantage is that a stem plot utilizes the original data, not grouped scores, meaning the identity of each observation is not lost. However, for large data sets, box plots may become too cumbersome.

Symmetrical		Right Skewed		Left Skewed	
Stem	**Leaf**	**Stem**	**Leaf**	**Stem**	**Leaf**
2	1	2	1	2	8
3	0 1	3	0 1	3	7
4	1 2	4	1 2 2 6 9 9	4	6
5	2 6 8	5	2 5 6 8	5	2 5
6	1 2 2 6	6	1 2	6	4
7	0 0 7	7		7	1 2
8	4 8	8	4	8	2 3 3 5 8 9
9	3	9	3	9	2 3 4

Displaying Data and Describing Distributions

A boxplot, or box and whisker diagram, provides a simple graphical summary of a set of data. It shows a measure of central **location** (the median), two measures of **dispersion** (the range and inter-quartile range), the **symmetry** (skewness) (from the orientation of the median relative to the quartiles) and potential outliers (marked individually). Boxplots are especially useful when comparing two or more sets of data.
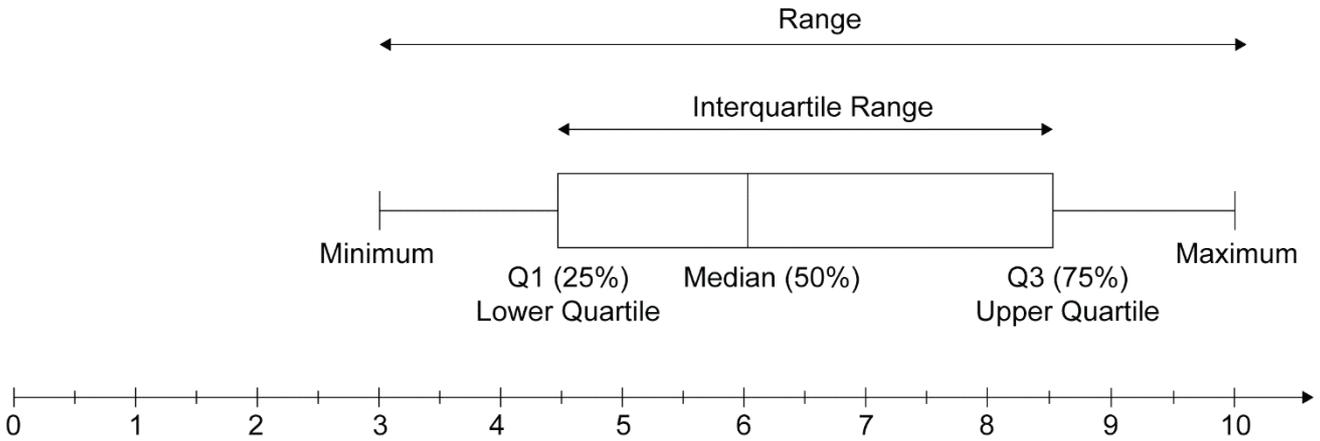
The advantages of a boxplot include

- They graphically display a variable's location at a glance
- Provide some indication of the data's symmetry and skewness
- Quickly providing an indication of the dispersion of the data set
- By using a box plot for each categorical variable side by side on the same axes, data sets can be compared easily

A disadvantage of boxplots is they tend to emphasize the tails of distribution, which are the least certain points in the data set. They also hide many of the details of the distribution.

A frequency histogram can be used to display grouped data. They provide a rough approximation of the frequency distribution of the data. Histograms are similar to bar graphs but, because they represent continuous data, the columns are joined together. Frequency is always displayed on the vertical axis. The midpoint of each class interval is marked on the horizontal axis.
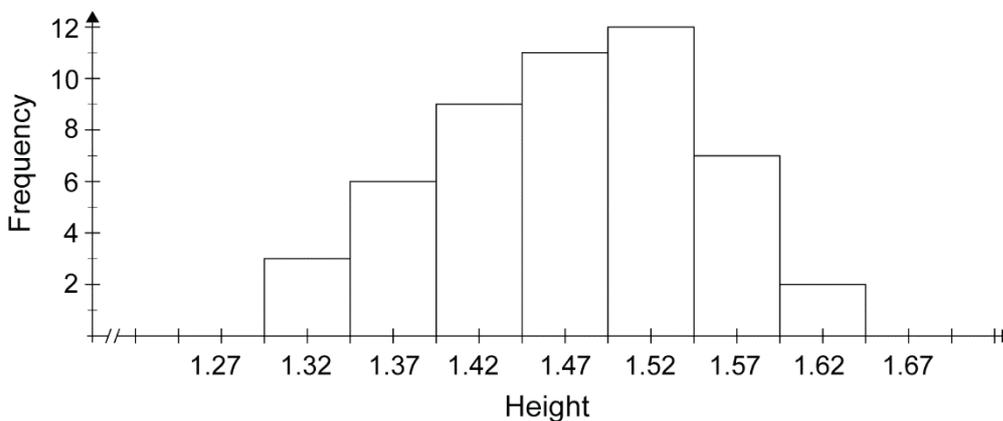
Displaying a histogram in conjunction with a box plot can help to overcome the weaknesses of a box plot by revealing details of the distribution. A histogram provides additional measures of **location** (mean), **dispersion** (standard deviation) and **symmetry** (frequency and modality). It is important to note that if the original scores are unknown, that any statistical measures obtained from a histogram are only approximate, as the scores have been grouped.
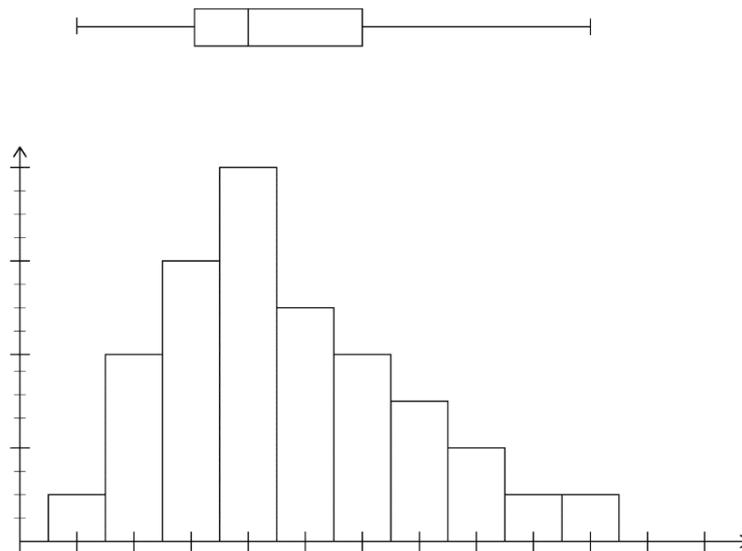
Skewness

Skewness	Description
	If a data set is approximately symmetrical, then the values of the mean and the median will be almost equal. These values will be close to the mode. The box and whisker plot is symmetrical about the median.
	In positively skewed (right skewed) data the mean is usually greater than the mode or the median. This means the median is to the left of the centre of the box, and there is a long whisker of high values to the right.
	In negatively skewed (left skewed) data the mean is likely to be the lowest of the averages. This median is to the right of the centre of the box, and there is a long whisker of low values to the left.

Outliers

An outlier can have a major influence on the results of a statistical analysis. An extreme outlier may impact upon our view of central tendency and variability of the entire distribution and can be a major cause of skewed distributions.

The interquartile range is the difference between the upper and lower quartile. It gives an indication of the spread of the middle 50% of the data and can be used to identify outliers. In this course, an outlier is defined as any score that is more than one-and-a-half interquartile ranges beyond the lower or upper quartiles.

$$ie. \, Q_1 - 1.5 \times IQR > score > Q_3 + 1.5 \times IQR$$

Worked Example

Consider the scores 10, 12, 11, 15, 12, 10 & 6.

Determine any potential outliers in this set.

$Q_1 = 10$ $\qquad\qquad$ $Q_3 = 12$ $\qquad$ $IQR = 2$

$10 - 1.5(2) = 7$ $\qquad$ $12 + 1.5(2) = 15$

Scores that are less than 7, and any scores greater than 15 are considered outliers.

$\therefore 6 \, is \, an \, outlier$

Using the ClassPad to Identify Outliers
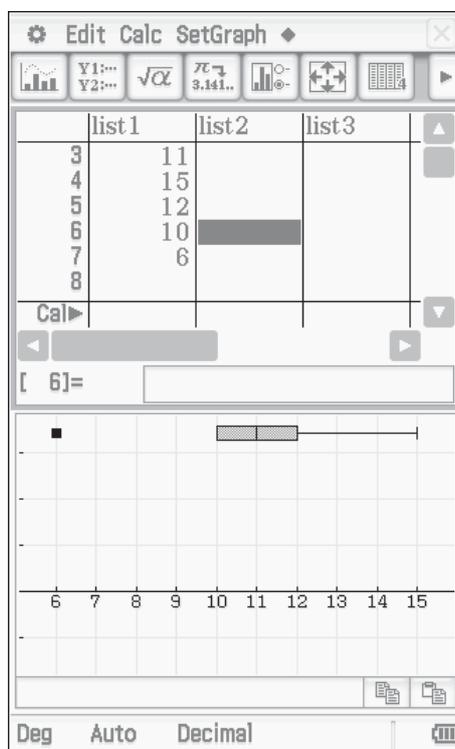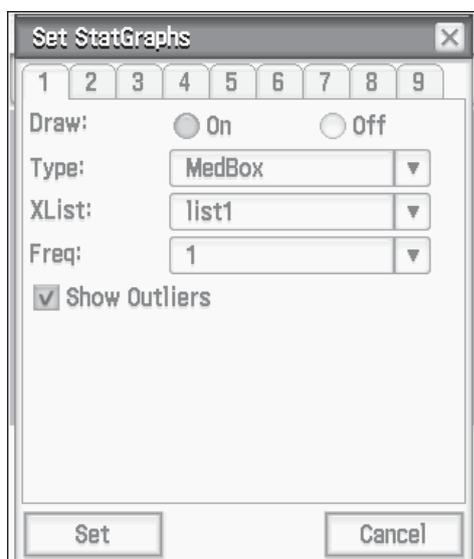
Resource Free Questions

1.   A vending machine dispenses five different types of soft drinks. Explain why the sale of each brand of soft drink is an example of:

   a)   categorical data.
   b)   numerical data.

2.   The following information was collected from patrons working out at the local gymnasium. Classify the data as nominal, ordinal, discrete or continuous.

   a)   Gender
   b)   Brand of shoes worn
   c)   Height
   d)   Occupation
   e)   Number of visits to the gym in the last 7 days.
   f)   Rating of the facilities as excellent, good or poor.

3.   Consider the data shown below.

   32, 51, 63, 17, 44, 65, 69, 68, 73, 8, 74, 72, 35, 63, 52, 26, 56, 72

   a)   Use the data to construct a stem and leaf plot.

   b)   Describe the shape of the scores.

4.   Consider the set of scores shown below.

   2, 3, 3, 4, 4 ,4, 8, 12

   a)   Construct a box-and-whisker plot for the data given.

   b)   Describe the shape of the distribution.

   c)   If an outlier is defined as any score for than 1.5 interquartile ranges above or below the upper or lower quartiles, identify all outliers.

   d)   What effect will the removal of outliers have on the shape of the box-and-whisker plot?

5.   Two classes, A and B, sat an identical test and the results are displayed below. Which class performed better in the test?
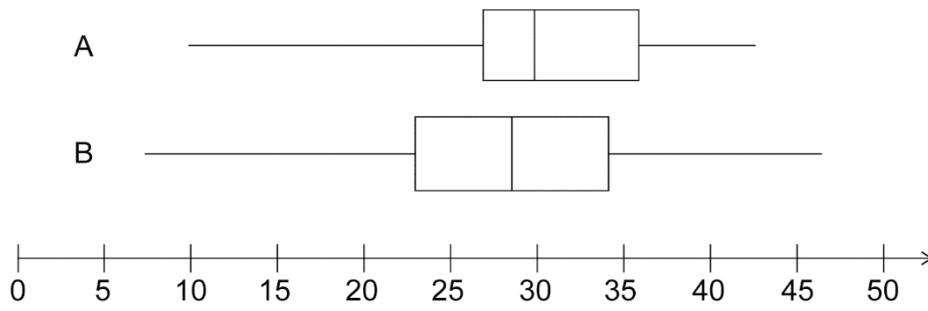
Resource Rich Questions

6.   The boxplot below shows the marks for a group of 180 people in a company safety test.
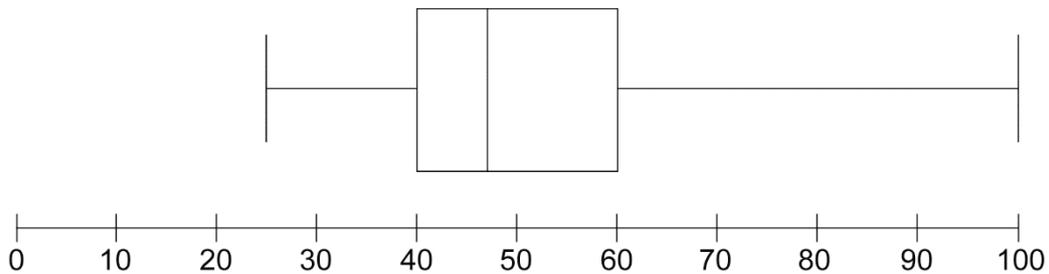
a)   If an employee scored 60%, how can he use the box plot to determine how many of his colleagues scored better than him?

b)   An employee compares the whiskers of the box plot and remarks

"Fewer employees scored between 25% and 40% compared to employees scoring 60% to 100%"

Comment on the validity of this statement.

c)   Which of the following options is most likely? Give reasons for your answer.

i)   mean < median   ii)   mean > median   iii)   mean = median

7.   Under competition rules teams of up to 9 players can be entered into a knockout netball competition. The heights, in cm, of the players from two teams are shown below.

Team A   157, 168, 152, 201, 161, 148, 195, 171, 164
Team B   168, 159, 188, 157, 138, 174, 186, 165, 182

a)   Construct a box-and-whisker plot for both teams. Place each plot on the same set of axes.

b)   Compare and contrast the distribution of heights in the two teams. Which team is taller?

8.   In a cricket team, one batsman must lose his spot when the captain returns from injury.   The last six scores of two batsmen are taken into consideration.

Batsman A: 3, 5, 10, 7, 169, 10
Batsman B: 27, 36, 44, 21, 29, 38

a)   Describe the nature of the data that has been collected.

b)   Calculate the respective means of the two batsmen.

c)   Construct a box and whisker plot for each of the two batsmen.

d)   Compare the distribution of the scores and decide who should lose their place in the team. Give reasons for your answer.

9.   Consider the frequency histogram and its associated boxplot shown below. It shows the tooth length (to the nearest millimetre) in a particular species of fish.
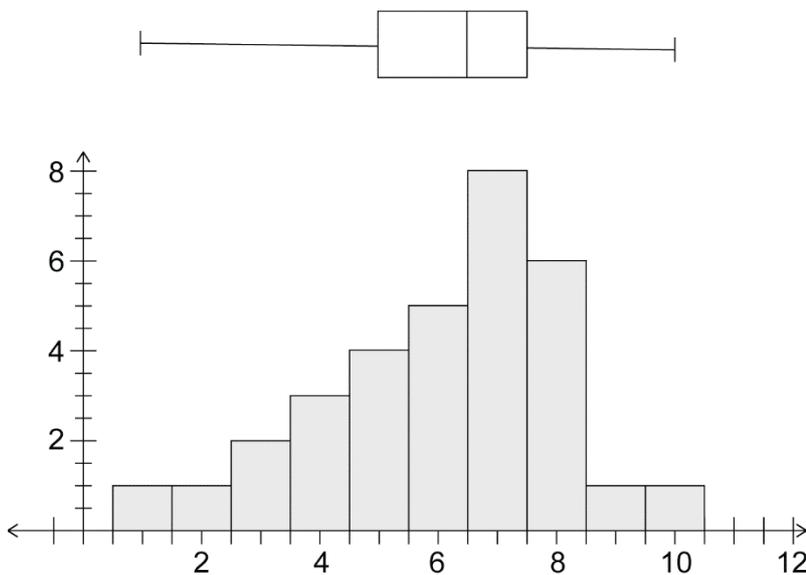
a)   Calculate the mean for the scores shown above.

b)   Explain why your answer to (a) is only an estimate of the mean.

c)   Describe the distribution (location, spread, shape) of data provided.

10. Two new movies have just been released at the theatre. The ages of the patrons have been recorded and the results are displayed in the boxplot shown below.
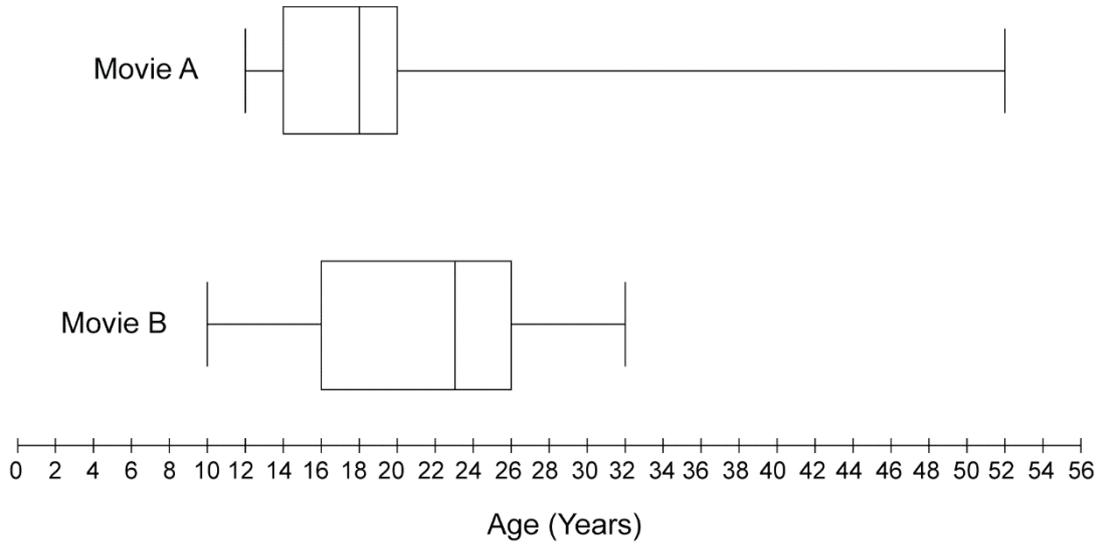
a) Determine the interquartile range for each movie.

b) An observer remarks "Movie A attracts a wider age group than Movie B" Comment on the validity of this statement

c) An observer remarks "Movie B attracts an older audience in comparison with Movie A". Comment on the validity of this statement

Solutions

1. a)   "Name the five brands" – Nominal Categorical Data (no ordering/ranking)
    b)   "How many of each brand was sold?" – Discrete Numerical Data (counting)

2. a)   nominal   b)   nominal   c)   continuous
    d)   nominal   e)   discrete   f)   ordinal

3. a)

Stem	Leaf
0	8
1	7
2	6
3	2 5
4	4
5	1 2 6
6	3 3 5 8 9
7	2 2 3 4

    b)   Negatively skewed

4. a)   n   =   8
          Median   =   4
          IQR   =   3
          Range   =   10
          Min   =   2
          Max   =   12
          UQ   =   6
          LQ   =   3
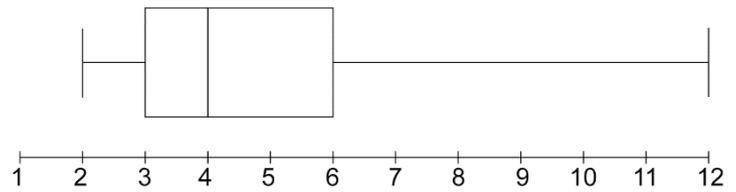
    b)   Positively skewed

    c) $IQR = 6 - 3 = 3$
          $Q_1 - 1.5(IQR) = -1.5$
          $Q_3 + 1.5(IQR) = 10.5$
          $\therefore 12 \text{ is an outlier as } 12 > 10.5$

    d)   The median remains unchanged.
          The interquartile range is reduced from 3 to 1.
          The length of the right whisker is reduced.

5.   Class A's median (29), upper and lower quartiles (36 & 27) are higher than that of
      class B. Overall, Class A performed better in this test than Class B.

6. a) $0.25 \times 180 = 45$
    b)   False. 25% along each whisker
    c)   (ii) Skewed to the right. Mean is likely greater than the median

7.   a)   Team A

Team B

130   140   150   160   170   180   190   200   210
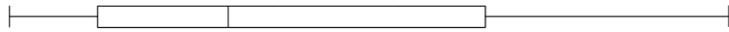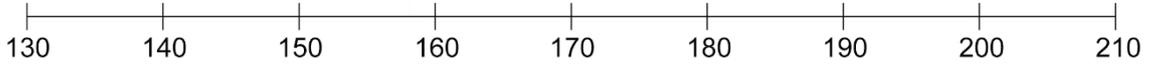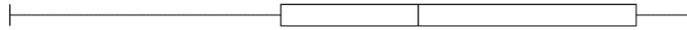
b)   - Means are identical $(168.\dot{5})$.
- Median of Team B (168) is greater than Team A (164)
- Team A is positively skewed, Team B is negatively skewed.
- Lower and Upper Quartiles for Team B are greater than Team A.
- Range and Interquartile ranges for Team A are greater than Team B

Team B is the taller team.

8.   a)   Discrete numerical data
b) $\bar{x}(A) = 34$ $\bar{x}(B) = 32.5$
c)

Batsman A

Batsman B

0   10   20   30   40   50   60   70   80   90   100   110   120   130   140   150   160   170
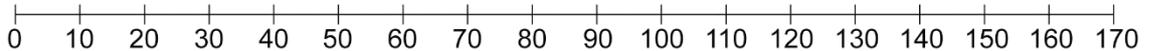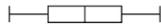
d)   Batsman A should lose his spot.

Although, batsman A has a higher mean, his median score is significantly lower than B.
The boxplot of batsman B is largely skewed, with a large range, indicating that he is an inconsistent scorer.
The boxplot of batsman B is symmetrical with a much lower range, indicating that he is a more consistent batsman.

9.   a) $\bar{x} = 6.06, med = 5.5, mode = 7$

      b)   The original data has been grouped.

      c) $\bar{x} = 6.06, med = 5.5, mode = 7$
            Data is negatively skewed.
            $s = 2.06, \ IQR = 2.5, \ range = 9$

10.   a)   Movie A IQR $= 20 - 14 = 6$
            Movie B IQR $= 26 - 16 = 10$

      b)   Movie A has a maximum of 52.
            $28 + 1.5(15) = 50.5$
            $\therefore$ 52 is an outlier, and has skewed the data. Without the original scores, the
            statement cannot be validated.

      c)   Assuming a sufficient sample size, this statement is valid based upon the median
            and the IQR.

Chapter 13 – Standard Deviation and Making Judgments Between Data Sets

The most common method for comparing data sets is to assess the summary statistics from the sets of data. The **measures of location** such as mean and median are used to compare the typical score in a data set. **Measures of spread** such as range, interquartile range and standard deviation are utilised to assess the overall consistency of scores in the data set.

The standard deviation is a measure of how widely spread the data is from the mean. For a set of data, the greater the spread of the scores from the mean, the larger the value of the standard deviation.

Consider the three following data sets (each contains 10 scores):

Set A   55, 54, 52, 58, 53, 51, 57, 52, 60, 58
Set B   47, 63, 69, 42, 55, 79, 35, 52, 53, 55
Set C   16, 99, 45, 8, 3, 45, 2, 78, 137, 117

	Set A	Set B	Set C
Mean	55	55	55
Standard Deviation	2.93	12.17	47.41

Each set has a mean of 55. Based on mean alone the three sets could be considered the same as each other. The amount of variation within each set is markedly different however as measured by each standard deviation: 2.93 for Set A; 12.17 for Set B and 47.41 for Set C.

Set A is very stable in that all the scores are quite similar whereas Set C has a big difference in the scores from the mean throughout. Although Set A and Set C have the same mean clearly they are not the same. So, in comparing several groups, the means can be used to gauge the variation between the data sets but as you can see from this example, the variation within each group measured by the standard deviation must be considered as well.

Standard Scores

A score can be expressed in terms the number of standard deviations above or below the mean. This is called **standardising** the score.

Worked Example

Consider a set of test scores with a mean of 16 and a standard deviation of 2. To standardise a test score of 21:

$$\text{Standardised score } = \frac{\text{raw score - mean}}{\text{standard deviation}} = \frac{21-16}{2} = \frac{5}{2} = 2.5$$

Standard Deviation Questions

Resource Free

1.   Answer the following questions in relation to the graphs shown below.
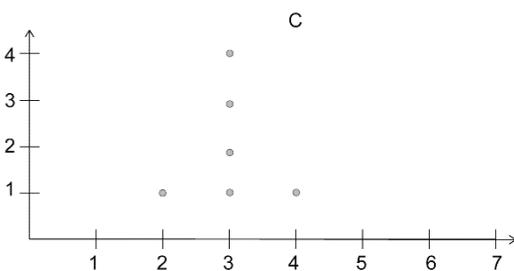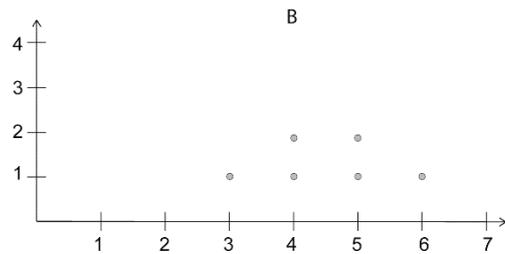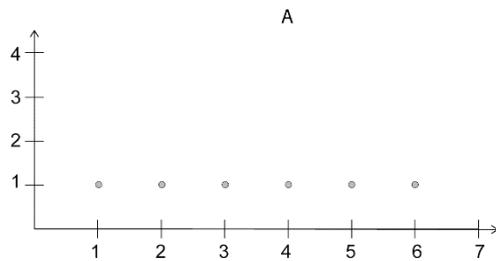
Which graph, A, B or C has the

a)   greatest mean?

b)   lowest mean?

c)   highest standard deviation?

d)   lowest standard deviation?

e)   smallest range?

2.   Consider the following set of scores. 3, 3, 4, 5, 7, 8, 8, 8, 9, 11, 13, 17, 17, 20.

        These scores have a mean of 9.5 and standard deviation of 5.25 (2 d.p.)

        a)   How many of the scores lie within one standard deviation of the mean?

        b)   Standardise a score of 20.

        c)   Interpret your answer to (b).

3.   Compare and contrast the mean and standard deviations of these two sets of scores.

Set A   Set B

Score	Frequency
21	2
22	4
23	3
24	1
25	2
26	3

Score	Frequency
1	2
2	4
3	3
4	1
5	2
6	3

4.   Consider the three stem-and-leaf plots shown below.

	A			**B**			**C**	
Stem	Leaf		Stem	Leaf		Stem	Leaf	
3			3	0 0		3	5	
4	5		4			4		
5	0		5	0		5	0	
6	0 0		6	5		6		
7			7			7	0 0	

a)   Which has the smallest standard deviation?

b)   Which two plots have the same standard deviation?

c)   Which has the lowest mean?

5.   2, 3, 3, 4, 4, 4, 5 & 7

These scores have a standard deviation of 1.51 (2 dp.).

a)   What effect, if any, will the addition of a score of 11 have upon;

i)   the mean?

ii)   the standard deviation?

b)   What effect, if any, will the addition of a score of 4 have upon;

i)   the mean?

ii)   the standard deviation?

Calculator Assumed

6. A set of scores have mean of 22.6 and standard deviation of 3.8.

    a) Standardise a score of 16.9.

    b) How many standard deviations above the mean is a score of 32.86?

7. Consider the frequency table shown below.

Score	Frequency
16	2
17	4
18	3
19	1
22	2
23	3
49	1

    a) Calculate the mean and standard deviation of the above scores.

    b) The score of 49 has been identified as an outlier. Remove the outlier and recalculate the mean and standard deviation.

    c) What effect did this have on the mean and standard deviation and why?

8. Consider the data shown in the stem and leaf plot below.

    Key 2|1 represents 21

    ```
    2 | 1 2 7
    3 | 2 2 5 7
    4 | 1 3 9
    5 | 6
    6 | 3 6
    ```

    a) Calculate the mean and standard deviation for this set of scores.

    b) Add 5 to each of the scores above and recalculate the mean and standard deviation.

    c) What effect did this have on the standard deviation?

    d) Provide an explanation as to why this occurred.

9.   Holly's performances in her Italian and Japanese tests are shown below.

Italian   58, 55, 49, 51, 63, 65, 71, 46

Japanese: 56, 84, 87, 42, 37, 69, 51, 72

a)   In which subject has the student achieved the higher mean?

b)   In which subject was the student more consistent?

10.   A 200 gram packet of "N&N"s contains approximately 210 chocolate candies. A small factory has two machines that fill the bags and seal them for distribution to retailers. 25 packets from each machine were chosen at random and their contents were recorded in the stem and leaf plot below.

Machine 1		Machine 2
1	20_L	4
8 8 8 8 8 8 8 8 8 8	20_H	6 8 8 8 8 8 9 9
3 3 2 2 1 1 0 0 0 0 0	21_L	0 0 0 0 1 1 1 2 2 2 2 3
5 3	21_H	6 6 7
7	22_L	0
	22_H	

a)   Calculate the mean and standard deviation of the number of chocolate candies inserted by each of the machines

b)   Based on the 25 samples;

i)   which of the two machines is more cost effective?

ii)   which of the two machines is more consistent?

Standard Deviation Solutions

1. a) B   b) C   c) A   d) C   e) C

2. a) $\bar{x} \pm s$ gives a lower limit of 4.25 and upper limit of 14.75.
        <u>8 scores lie within this range.</u>

    b) $\frac{20 - 9.5}{5.25} = 2$

    c) A score of 20 is 2 standard deviations above the mean.

3. The mean is higher for set A. The standard deviation for both sets of scores is the same.

4. a) A   b) B&C   c) B

5. a) i)   increase   ii)   increase
    b) i)   same   ii)   decrease

6. a) $\frac{16.9 - 22.6}{3.8} = -1.5$ b) $\frac{32.86 - 22.6}{3.8} = 2.7$

7. a) $\bar{x} = 20.94$ (2 d.p.) $s = 7.68$ (2 d.p.)

    b) $\bar{x} = 19.07$ (2 d.p.) $s = 2.62$ (2 d.p.)

    c) Both the mean and standard deviation decreased. The removal of the outlier caused a decrease in the overall spread of the scores and relieved the upward pressure on the mean.

8. a) $\bar{x} = 40.31$ (2 d.p.) $s = 14.05$ (2 d.p.)

    b) $\bar{x} = 45.31$ (2 d.p.) $s = 14.05$ (2 d.p.)

    c) None. The standard deviation remained the same,

    d) The relationship of the points in the set relative to one another remains the same. i.e. the spread of the scores

9. a) Japanese 62.25   b) Italian, as the standard deviation is lower.

10. a) $\bar{x}_1 = 210.32$ $s_1 = 4.33$ $\bar{x}_2 = 210.84$ $s_2 = 3.49$
    b) i)   Machine 1 as it has a mean closer to 210.
        ii)   Machine 2 as it has the smaller standard deviation.

Chapter 14 – Normal Distribution

There are an infinite number of normal distributions, one for each selection of mean and standard deviation. In this course, we study standard normal distribution, where a normal random variable has a mean of zero and a standard deviation of one.

Normal distributions have a symmetrical, bell shaped curve where the area under the curve represents the probability.
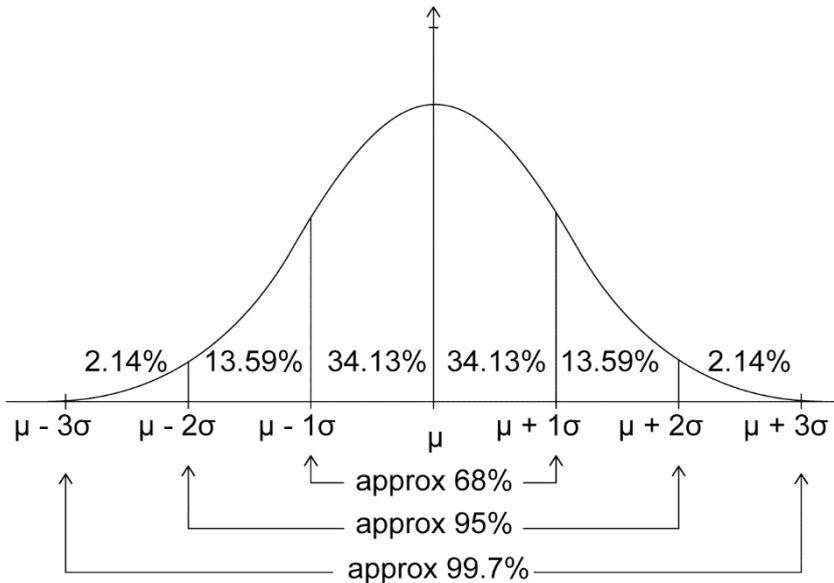
Standardization Notation and Calculating Probability

A standardised score or a '*Z*' score is the normal random variable of a standard normal distribution. When calculating the probability of a normal distribution it must be transformed from the standard normal distribution using the formula;

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

Where   'Z'   is the standardised score
             'X'   is the normal random variable score
             'μ'   is the mean of *X*, and
             'σ' is the standard deviation or $\sqrt{variance}$ of *X*.

The area under the curve represents a probability of 1. This function gives the probability to the left of the value Z. It can be used to calculate;

P(Z > a)   the probability that a standard normal random variable falls between a given value 'a' and plus infinity.

P(Z < a)   the probability that a standard normal random variable falls between minus infinity and a given value 'a'.

P(a < Z < b)   the probability that a standard normal random variable falls between two given values 'a' and 'b'.

Determining Probability Using the Empirical Rule

The empirical rule states, that for a normal distribution:

- Approximately 68% of scores lie within one standard deviation of the mean.

- Approximately 95% of scores lie within two standard deviations of the mean.

- Approximately 99.7% of scores lie within three standard deviations of the mean.

Worked Example 1

If $X \sim N(5, 2^2)$, find $P(X < 7)$.

$\mu = 5$ $\sigma = 2$

If we standardise a score of 7 we get;

$Z = \frac{7-5}{2} = 1$

i.e. a score of 7 is 1 standard deviation above the mean.

$P(X < 7)$ $= P(Z < 1)$

             $= 50\% + 34\%$

             $= \underline{84\% \ approx.}$
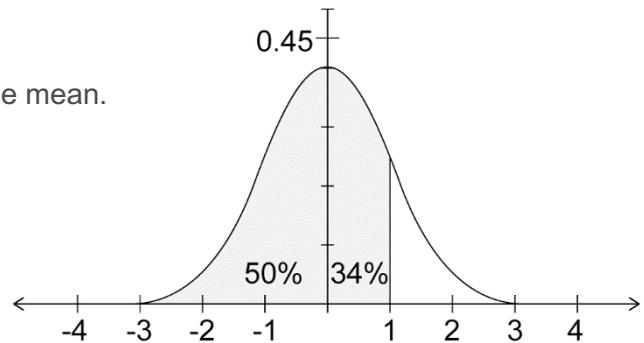
Worked Example 2

If $X \sim N(5, 2^2)$, find $P(3 < X < 9)$.

$\mu = 5$ $\sigma = 2$

If we standardise a score of 3 we get;

$Z = \frac{3-5}{2} = -1$

i.e. a score of 3 is 1 standard deviation below the mean.

If we standardise a score of 9 we get

$Z = \frac{9-5}{2} = 2$

i.e. a score of 9 is 2 standard deviations above the mean.

$P(X < 7)$ $= P(-1 < Z < 2)$

             $= 34\% + 47.5\%$

             $= \underline{81.5\% \ approx.}$
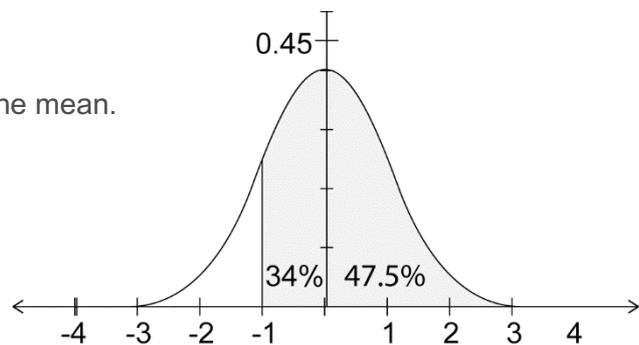
Determining Probability Using Technology

Worked Example 1

The wingspans of a particular type of beetle were normally distributed with a mean of 35 mm and a standard deviation of 5 mm. What proportion of beetles will have wing span less than 42 mm in length?

The first step is to find a standardized score corresponding to $X = 42$ mm.

$$Z = \frac{X - \mu}{\sigma}$$
$$Z = \frac{42 - 35}{5}$$
$$Z = 1.4$$
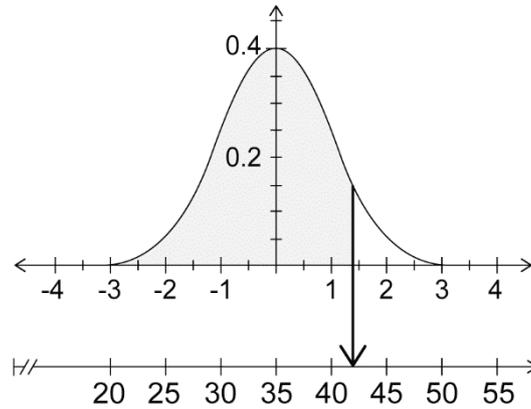
The second step is to find the cumulative probability associated with a Z value less than or equal to 1.4. The shaded (grey) area under the curve is represents the probability.

$$P(Z \leq 1.4) = 0.9192$$

This value can be found by utilising your calculator in one of two ways.

Option 1:   Statistics   Calc > Distribution
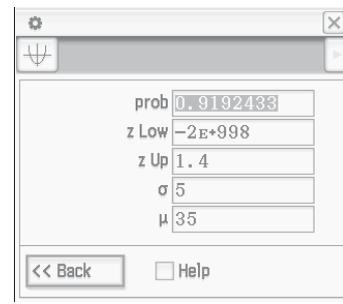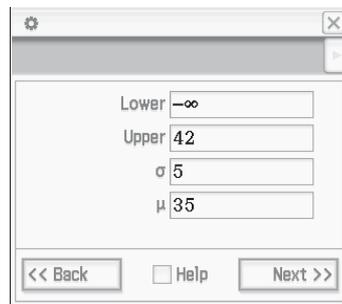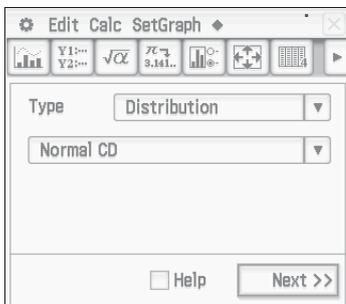
Option 2:   Main   Interactive > Distribution > Continuous > normCDf

$$\text{normCDf}(-\infty, 42, 5, 35)$$
$$0.9192433408$$
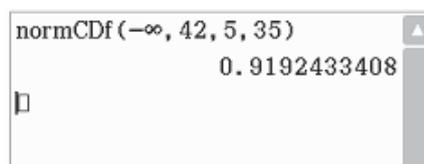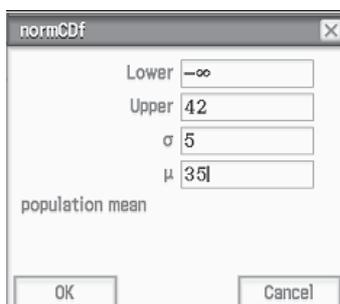
The probability is 0.9192 (4 d.p.)

Worked Example 2

A packet of cooking chocolate contains 500g of chocolate. Suppose the actual weight (X) of these packets is normally distributed with a mean of 512 grams and a standard deviation of 8 grams.

a)   What is the probability of picking a packet between 504 and 520g?

or

$X \sim N$ (512 , 8^2), find the P($504 \leq X \leq 520$)

Type	Distribution	▼
Normal CD		▼

Help   Next >>

Lower 504
Upper 520
σ 8
μ 512

<< Back   Help   Next >>

prob 0.6826895
z Low −1
z Up 1
σ 8
μ 512

<< Back   Help

The probability is 0.6827 (4 d.p.)

b)   What are the minimum and maximum values of the middle 60%?

Type	Inv. Distribution	▼
Inverse Normal CD		▼

Help   Next >>

Tail setting Center ▼
prob 0.6
σ 8
μ 512

<< Back   Help   Next >>

x₁InvN 505.26703
x₂InvN 518.73297
prob 0.6
σ 8
μ 512

<< Back   Help

The minimum and maximum values are 505.5 and 518.5 respectively (to the nearest 0.5)

c)   If $X \sim N$ (512, 64), determine the 23rd percentile.

Type	Inv. Distribution	▼
Inverse Normal CD		▼

Help   Next >>

Tail setting Left ▼
prob 0.23
σ 8
μ 512

<< Back   Help   Next >>

x₁InvN 506.08923
prob 0.23
σ 8
μ 512

<< Back   Help

The 23rd percentile is 506.09.

Resource Free Questions

1. A manufacturing company approximates the length of its matchsticks using a normal distribution model. The matchsticks have a mean of 4.5 cm with a standard deviation of 1 mm.

    For a randomly selected matchstick, use the empirical rule to determine the probability that it measures;

    a)   between 4.4cm and 4.6 cm?

    b)   between 4.3cm and 4.7 cm?

    c)   between 4.2cm and 4.8 cm?

2. 600 scores are normally distributed with a mean of 135 and a standard deviation of 10. Use the empirical rule to determine each of the following.

    a)   Approximately what percentage of scores lie within the interval 125 – 145?

    b)   What interval about the mean includes the middle 95% of the data?

    c)   Approximately what percentage of scores lie within the interval 125 – 155?

    d)   Approximately how many scores lie within the interval 125 – 145?

3. The standby time on a new model of smartphone is normally distributed. The mean standby time is 500 hours with a standard deviation of 18 hours. Use the empirical rule to establish what percentage of this particular model of smart phone has a standby time longer than 536 hours?

4. X is a normally distributed variable with a mean of 56 and a standard deviation of 4. Find, using the empirical rule;

    a) $P(x < 60)$

    b) $P(x > 48)$

    c) $P(52 < x < 64)$

5. The shaded area in the diagram shown opposite is 0.3. If the mean is 50,

    a)   what percentile is 47?

    b)   to which percentile does 53 belong?
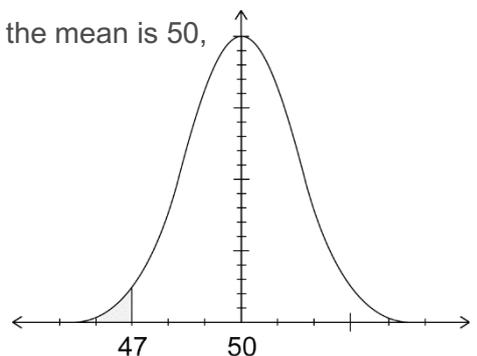
6.   A set of scores is normally distributed with a mean of 14 and a standard deviation of 4.45. If the 25th percentile is 11, determine the interquartile range of the scores.

7.   In the diagram shown on the right, the shaded area is 0.25. Determine the interquartile range of this set of scores.

40   42
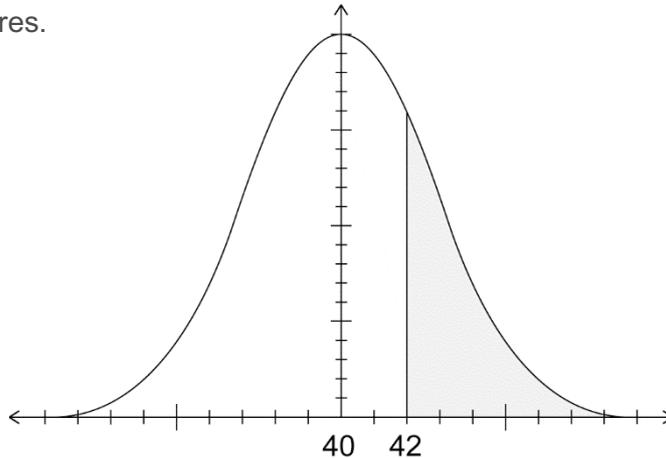
8.   95% of players at a football club weigh between 52 and 80 kg. Assuming that the data is normally distributed, which of the following is most likely to be the mean and standard deviation of scores? Justify your answer.

a)   Mean = 66 kg, Std Dev = 14 kg   b)   Mean = 66 kg, Std Dev = 7 kg

9.   If $X \sim N(12.5, 3^2)$, find, using the empirical rule;

a) $P(X < 15.5)$

b) $P(6.5 < X < 9.5)$

10.   A set of scores is normally distributed with a mean of 80. If the shaded area on the diagram below is approximately 0.475, what is the standard deviation for this set of scores?

80   88
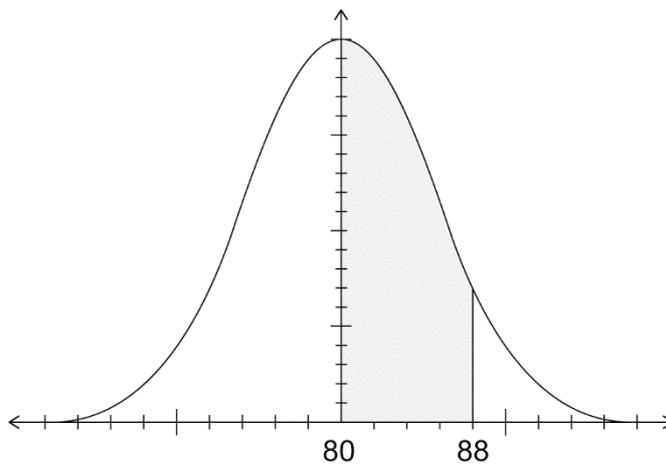
Resource Rich Questions

11.   A set of scores is normally distributed with a mean of 18 and a standard deviation of 2.6. If the 25th percentile is 16.25, determine the interquartile range of the scores.

12.   The contents of a particular brand of tinned soup is normally distributed with a mean of 250 mL and standard deviation of 8 mL. Determine the probability that a randomly selected container tin contains between 250 and 260 mL of soup.

13.   A set of scores is normally distributed with a mean of 85 and a standard deviation of 6. Determine the interquartile range of the scores.

14.   In an international collaboration between universities, 7030 candidates were randomly selected to sit a test. Justin achieved a score of 930. The mean test score was 860 with a standard deviation of 90. Assuming that the test scores are normally distributed, estimate how many students had a higher score than Justin.

15.   The time taken to complete a Public Sector Recruitment Test is normally distributed with a mean of 80 minutes and a standard deviation of 10 minutes.

   a)   What is the probability of a candidate completing the test in one hour of less?

   b)   What is the probability of a candidate taking between 65 and 75 minutes to complete the test.

   c)   Sam completed the test in 55 minutes. What percentage of candidates completed the test faster than Sam?

   d)   What is the interquartile range for completion times?

16.   A coffee machine claims to dispense 150 mL of the selected beverage. The volumes of the drinks dispensed are found to be normally distributed with a mean of 149.5mL and a standard deviation of 1.4mL.

   a)   If the machine dispenses 100 beverages, how many would you expect to contain at least 150 mL?

   b)   What is the least that can be expected in the largest 5% of beverages?

17. The weights of 9 gram sachets of saffron are normally distributed with a mean of 9.12 grams and a standard deviation of 0.15 grams.

    a) What is the probability that a randomly selected sachet of saffron will weigh 9.2 grams or more?

    b) What are the first and ninth deciles for this distribution?

18. A new experimental drug undergoes its first round of clinical trials. The time taken for patients to feel its effects are found to be normally distributed with a mean of 20 minutes and standard deviation of 4 minutes.

    a) What percentage of patients takes longer than 15 minutes to feel the effects of the drug?

    b) What percentage of patients takes between 15 and 25 minutes to feel the effects of the drug?

    The clinical trial found that some patients were "immune" to the experimental drug and experienced no effect after taking the drug. It was determined that if a patient did not react to the drug after 35 minutes, then they are deemed to be "immune" to its effects.

    c) According to the data, how many patients in 100 000 will be immune to the effects of this drug?

19. Julia recently completed her end of year exams. Julia finds out from her language teachers that the Italian exam had a mean of 62% and a standard deviation of 12 and the French exam had a mean of 48% and a standard deviation of 8.

    a) Julia scored 66 in French. Convert this result to a z-score.

    b) Julia scored 71 in Italian. Convert this result to a z-score.

    c) In which subject did Julia attain the "better" result? Justify your answer.

20. Consider the table below displaying Daniel's scores in three different science subjects.

Subject	Daniel's Score	Mean	Std. Dev.
Biology	66	67	7
Chemistry	54	48	5
Physics	61	56	8

    Compare the overall distribution of the scores to determine which of the three subjects Daniel performed best in.

Solutions

1. a)   *approx.* 68%   b)   *approx.* 95%   c)   *approx.* 99.7%

2. a)   *approx.* 68%   b) $115 - 155$

    c) $34 + 47.5 = approx. 81.5\%$ d) $\frac{68}{100} \times 600 = 408$

3. $P(X > 536) = 100 - (50 + 47.5)$

    $\underline{P(X > 536) = 2.5\% \; approx}$

4. a) $50 + 34 = 84\% \; approx$

    b) $50 + 47.5 = 97.5\% \; approx$

    c) $34 + 47.5 = 81.5\% \; approx$

5. a)   30th   b)   70th

6. $25th \; percentile \; is \; 11, then \; 75th \; percentile \; is \; 17$

    $\therefore IQR = 17 - 11 = 6$ $OR \quad (14 - 11) \times 2 = 6$

7. $42 - 38 = 4$

8. Option B. $66 \pm 2(7) = 80 \; and \; 52$

9. a) $50 + 34 = 84\%$

    b) $47.5 - 34 = 13.5\%$

10. $0.475 \times 2 = 0.95 \; or \; 95\%$

    $88 \; is \; two \; standard \; deviations \; above \; the \; mean$

    $\underline{\therefore the \; standard \; deviation \; is \; 4.}$

11. $25th \; percentile \; is \; 16.25, then \; 75th \; percentile \; is \; 19.75$

invNormCDf("L", 0.75, 2.6, 18)
19.75367335

    $OR \quad (18 - 16.25) \times 2 = 3.5$

    $\therefore IQR = 19.75 - 16.25 = 3.5$

12. $0.3944 \; (4 \; dp.)$

normCDf(250, 260, 8, 250)
0.3943502263

13.   Tail setting "centre"

$IQR = 89.05 - 80.95 = 8.09$

x_1InvN	80.953061
x_2InvN	89.046939
prob	0.5
σ	6
μ	85

14.   Approximately 1535 candidates.

```
normCDf(930,∞,90,860)
                      0.2183500154
0.2183500154×7030
                      1535.000608
```

15.   a)   0.0228 (4 dp.)

        b)   0.2417 (4 dp.)

        c)   0.0062 (4 dp.) = 0.62%

        d) $IQR = 13.49$ (2 dp.)

```
☼ Edit Action Interactive                    ⊠

normCDf(-∞,60,10,80)
                      0.02275013195
normCDf(65,75,10,80)
                      0.2417303375
normCDf(-∞,55,10,80)
                      6.209665326E-3
invNormCDf("L",0.25,10,80)
                      73.2551025
invNormCDf("L",0.75,10,80)
                      86.7448975
86.7448975-73.2551025
                      13.489795
```

16.   a)   36 cups   b)   151.80 mL (2 dp.)

```
normCDf(150,∞,1.4,149.5)
                      0.360492431
ans×100
                      36.0492431
invNormCDf("L",0.95,1.4,149.5)
                      151.8027951
□
```

17.   a)   normCDf(9.2, ∞, 0.15, 9.12) = 0.2969

        b)   1st Decile 8.93 (2 dp.)

               9th Decile 9.31 (2 dp.)

```
invNormCDf("L",0.10,0.15,9.12)
                      8.927767265
invNormCDf("L",0.90,0.15,9.12)
                      9.312232735
```

18. a) $0.8943 \Rightarrow 89.43\% \ (2 \ dp.)$

```
normCDf(15,∞,4,20)
                0.8943502263
normCDf(15,25,4,20)
                0.7887004527
normCDf(35,∞,4,20)
                8.84172852E-5
8.84172852E-5×100000
                8.84172852
□
```

     b) $0.7887 \Rightarrow 78.87\% \ (2 \ dp.)$

     c) $8.84 \sim 9$ patients per 100 000

19. a) $Z_{(French)} = \dfrac{66-48}{12}$ b) $Z_{(Italian)} = \dfrac{71-62}{8}$

         $Z_{(French)} = 1.5$ $Z_{(Italian)} = 1.13 \ (2 \ dp.)$

     c) In comparison to the mean of each subject, Julia attained a better result in French, achieving a score 1.5 standard deviations above the mean.

20. $z_{(biology)} = \dfrac{66-67}{7} = -0.1429 \quad z_{(chemistry)} = \dfrac{54-48}{5} = 1.2 \quad z_{(physics)} = \dfrac{61-56}{14} = 0.625$

     Daniel performed best in Chemistry where his score was 1.2 standard deviations above the mean.
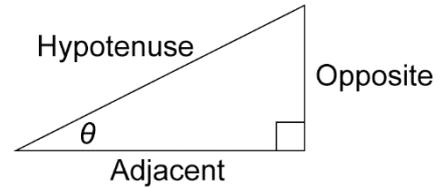
Chapter 15 – Trigonometry

Right Angled Triangles

Pythagorean Theorem $a^2 + b^2 = c^2$ (refer to Chapter 7)

The Trigonometric Ratios

In a right angled triangle, the side opposite to the right angle is called the hypotenuse. If we choose one of the other angles and label it θ, the other sides are often called opposite (the side opposite to θ) and adjacent (the side next to θ).

For a given angle, θ, there is a whole family of right angled triangles that are triangles of different sizes but are the same shape.
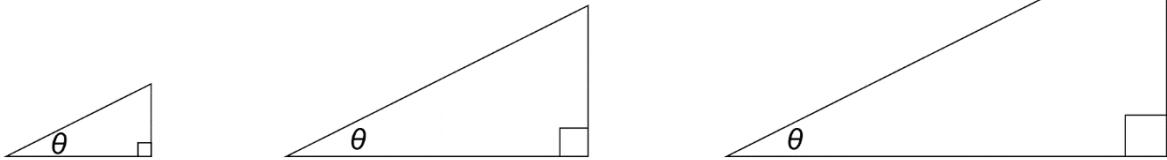
For each of the triangles above, the ratios of corresponding sides (sine, cosine and tangent) have the same values.
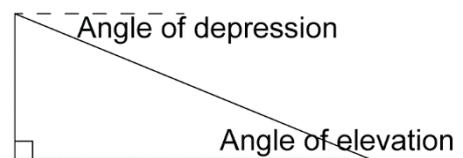
$$sin\,\theta = \frac{opposite}{hypotenuse} \qquad cos\,\theta = \frac{adjacent}{hypotenuse} \qquad tan\,\theta = \frac{opposite}{adjacent}$$

Angles of Elevation and Depression

Elevation is the angle looking upwards and depression is the angle downwards. Both are measured from the horizontal. In the diagram on the right the angle of elevation is equal to the angle of depression.

True Bearings

True bearings are measured in a clockwise direction from North, which is either 0° or 360°. They are expressed using three figures. For example "due east" is expressed as 090°. The difference between two compass bearings gives the angle between them. For example $210^o - 85^0 = 125^0$.
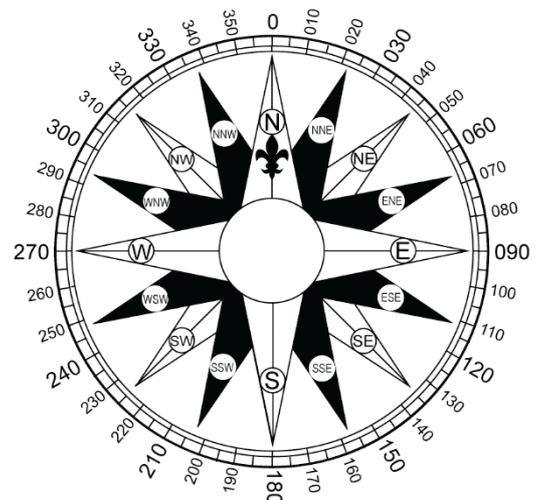
Using the ClassPad to Convert Angles Between Degrees and DMS

*** If you are referring to old textbooks or exam papers you may come across angles expressed in a different form.

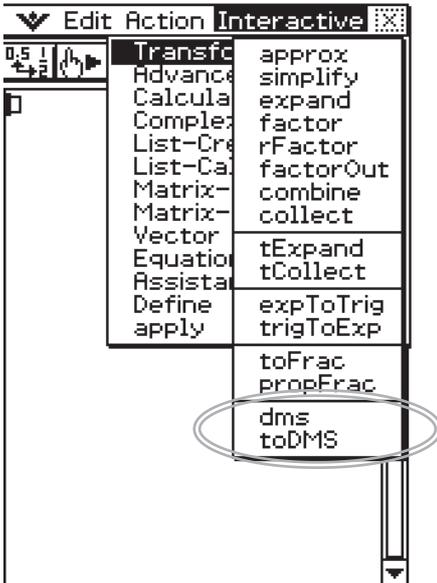18.58^0 is equal to $18^0 34' 48"$. This means 18 degrees, 34 minutes and 48 seconds.

In the main menu screen press the *Interactive* menu, then *Transformation* submenu.

Select **dms** and input 18, 34 and 48 to convert to 18.58.
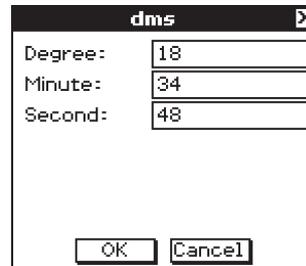
Select **toDMS** and input 18.58 to convert to dms(18,34,48).
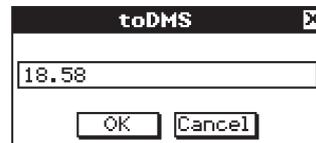
Worked Example

Point A is located 40 meters from a building. If the angle of elevation from point A to the top of the building is 43°, calculate the height of the building.

If x is the height of the building, then

$$\tan \theta = \frac{O}{A}$$

$$\tan 4 3^o = \frac{x}{40}$$

$$x = 40 \times \tan 4 3^o$$

$$x = 37.30 \text{ m (2 d.p.)}$$

Non Right Angled Triangles

The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
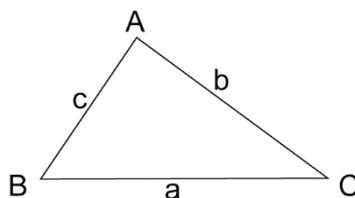
The sine rule relates the sides of any triangle (non right angled or right angled) to the sines of its angles.

Worked Examples

1.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{9}{\sin 58^o} = \frac{x}{\sin 76^o}$$

$$x = \frac{9 \times \sin 76^o}{\sin 58^o}$$
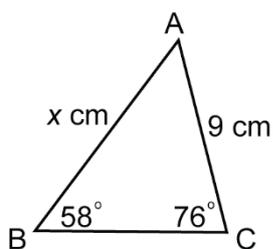
$$x = 10.30 \text{ cm (2 d.p.)}$$

2.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{12}{\sin 75^o} = \frac{8}{\sin x}$$

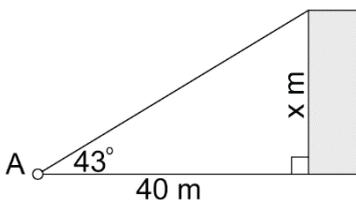$$x = \sin^{-1}(\frac{8 \times \sin 75^o}{12})$$
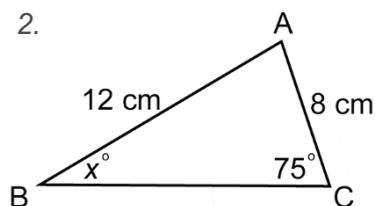
$$x = 40.09^o \text{ (2 d.p.)}$$

Ambiguous Triangles

Consider the following scenario. $\triangle ABC$ has $\overline{AB} = 29cm$, $\angle ABC = 38^o$ and $\overline{AC} = 23cm$. In this situation, the triangle is not uniquely defined by the information given, and therefore, the angle opposite $\overline{AB}$ is not unique.

$$\frac{23}{\sin 3 8^o} = \frac{29}{\sin x}$$

$$x = \sin^{-1}(\frac{29 \times \sin 3 8^o}{23})$$

$$x = \sin^{-1}(0.7762688\dots)$$

$$x = 50.9^o \approx 51^o \text{ or } 180^o - 51^o = 129^o$$
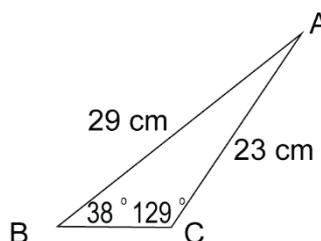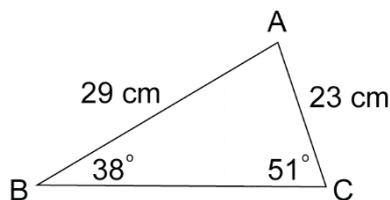
The Cosine Rule

In any triangle, the cosine rule states $a^2 = b^2 + c^2 - 2bc \times \cos A$.

The cosine rule can be used to find the third side when two sides and the included angle are known or the size of any angle when all three sides are known. To find an angle the cosine rule can be rewritten as $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Worked Examples

1.

2.
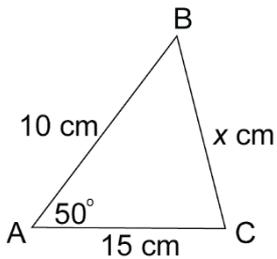
$a^2 = b^2 + c^2 - 2bc \times \cos A.$

$a^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \times \cos 50$

$a^2 = 132.1637171$

$a = 11.50$cm (2 d.p.)
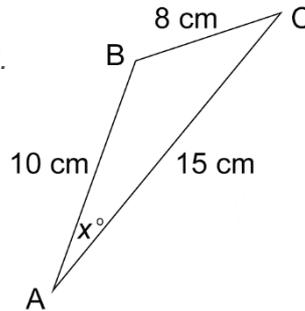
$a^2 = b^2 + c^2 - 2bc \times \cos A.$

$8^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \times \cos A$

$64 = 325 - 300 \cos A$

$A = \cos^{-1} \frac{87}{100}$

$A = 29.54^o$ (2d.p.)

Area of a Triangle

When the perpendicular height of the triangle is not known, the area of a triangle can be found using $Area = \frac{1}{2}ab\sin C$ or $\frac{ab\sin C}{2}$.

Area = 45.38 cm²
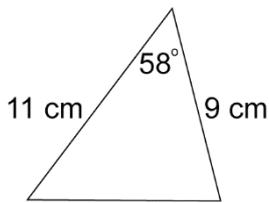
$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2} \times 11 \times 9 \times \sin 58^o$

$A = 41.98cm^2$ (2d.p.)
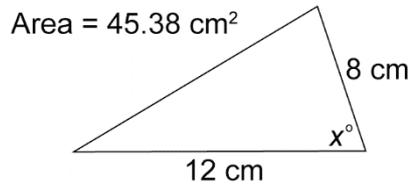
$A = \frac{1}{2}ab\sin C$

$45.38 = 0.5 \times 12 \times 8 \times \sin\theta$

$\theta = \sin^{-1}(0.94541\dot{6})$

$\theta = 70.98 \approx 71^o$

Heron's Formula

If a, b and c are the sides of a triangle, then the semi-perimeter can be calculated using the formula $s = \frac{a+b+c}{2}$. Once the semi-perimeter has been calculated, it can be used to find the area of the triangle using Heron's Formula: $A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$.

Worked Example
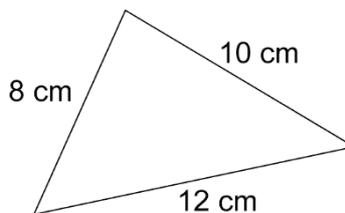
$s = \frac{a+b+c}{2}$

$s = \frac{8+10+12}{2}$

$s = 15$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{15(15-8)(15-10)(15-12)}$

$A = \sqrt{1575}$

$\underline{A = 39.69 \text{ cm (2 d.p.)}}$

Using the ClassPad to Solve Triangles

Select the default triangle and double tap anywhere in white area of the screen. A triangle will appear. Then tap the [▶] in the top right hand corner.

By clicking on one side of the triangle you can set length and by clicking on two sides (one at a time) you can set the angle between the two sides. Once the Classpad has the minimum data required, it automatically calculates all remaining sides and angles, and the area of the triangle.

Consider the following triangle.

$$\angle A = 58^o \qquad\qquad \overline{BC} = 9 \qquad\qquad \angle C = 76^o$$
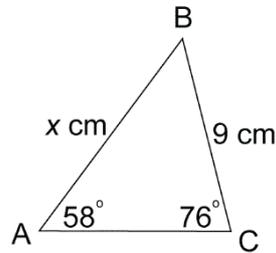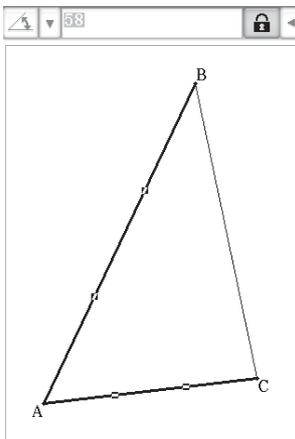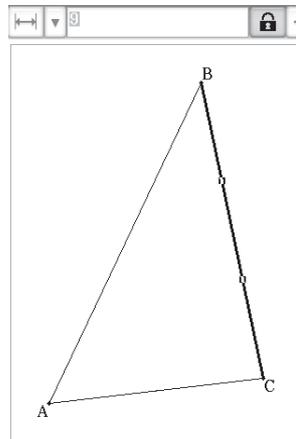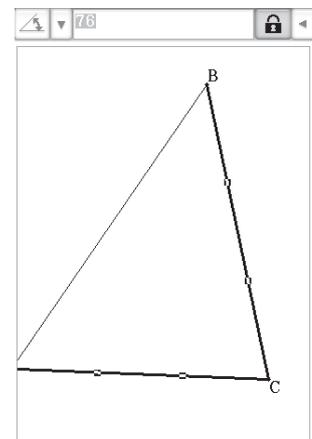
Select side AB to calculate length   Select all three sides to calculate area.
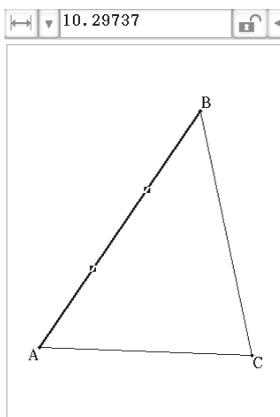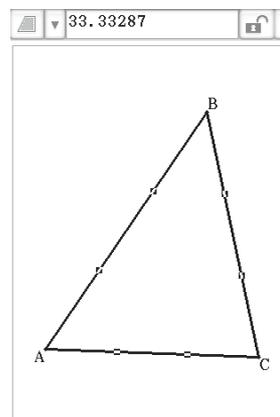
$$\overline{AB} = 10.30 \text{ units} \qquad\qquad \varDelta ABC = 33.33 \text{ units}^2$$

Resource Free Questions

1.   Use the triangle shown below to obtain the fractions for the trigonometric ratios given.
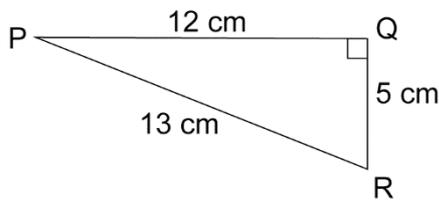
a) $sin P =$
b) $sin R =$
c) $cos R =$
d) $tan P =$

2.   For the triangle shown below, substitute the appropriate pronumerals into the cosine rule to make the statement true.

$r^2 =$
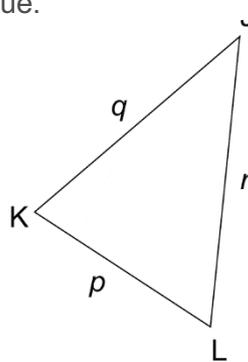
3.   Use Heron's formula to show that the area of this triangle is between 4 cm² and 5 cm².
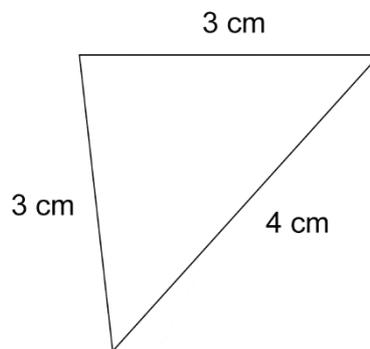
4.   If $sin 30^o = 0.5$, calculate the area of the triangle below.
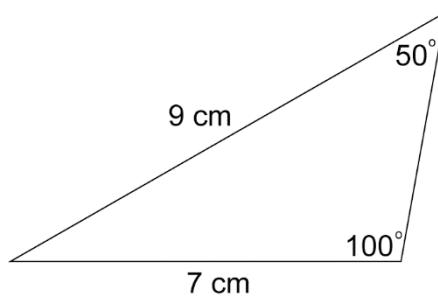
5.   If $\cos 60 = 0.5$, calculate the length of the missing side.

8 cm

60°

5 cm
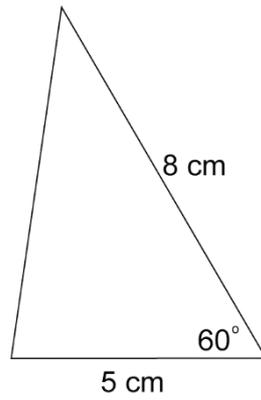
6.   If $\sin 30 = 0.5$, calculate the missing side of the triangle below.

x cm

8 cm

30°
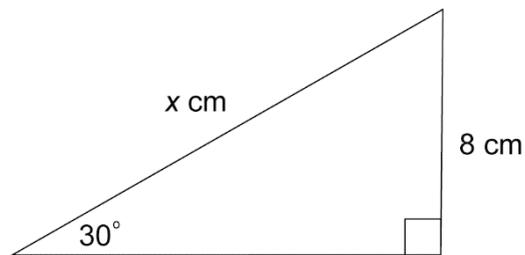
7.   If $\sin 18^o = 0.31$, calculate the missing side of the triangle below.

57 mm

x mm

18°
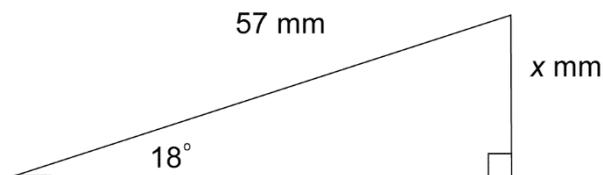
8.   Calculate the bearing of A from B in the following two scenarios.

a)

N

B

θ

N

20°

A

b)

N

110°

A

N

B

θ
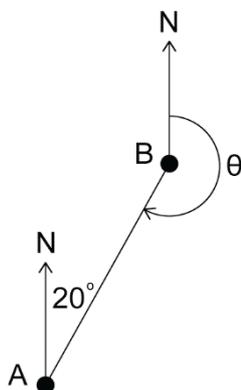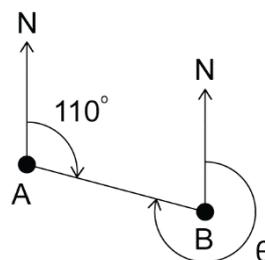
Calculator Assumed Questions

9.   Find, to the nearest degree, two angles whose sine is approximately 0.9205.

10.   Find the value of x in each of the following. (Diagrams are not to scale)

a)

b)

c)

d)
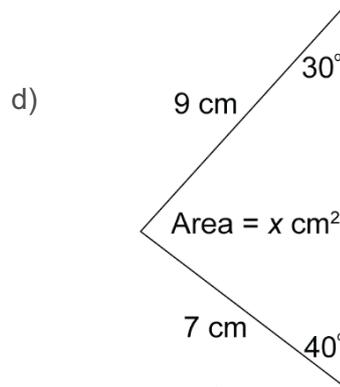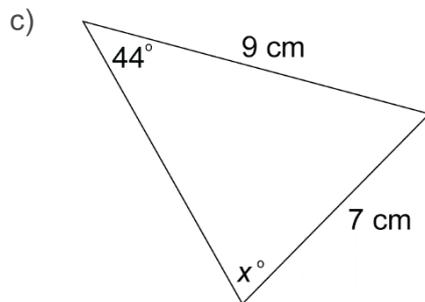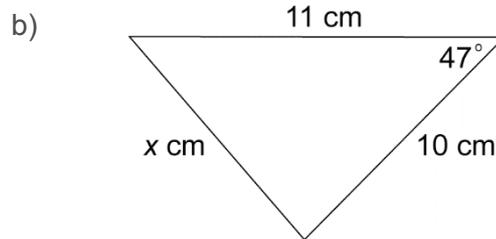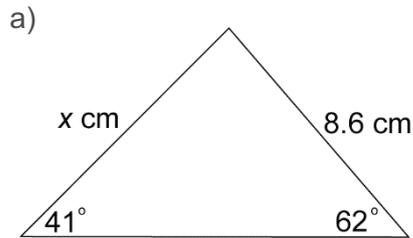
11.   A lake obstructs the direct measurement of a survey line DF, as shown. What is the length of $\overline{DF}$?
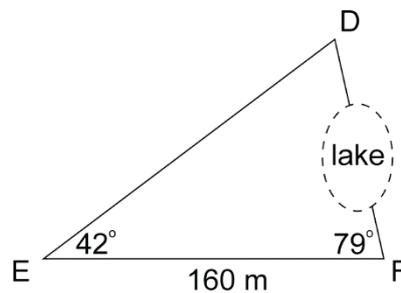
12.   If the area of a triangle is $25.5 cm^2$, and two of the sides measure 8cm and 6.4 cm. What is the angle between the two known sides of this acute triangle?

13.   Triangle PQR is an isosceles triangle with $\angle PQR = \angle PRQ = 35^o$. It is also known that QR = 10 cm.

   a)   Draw a sketch of triangle PQR in the space provided below. Label your diagram indicating the sizes of all angles and the length of the given side.

   b)   What is the perpendicular height of this triangle correct to 2 decimal places.

14.   In ΔABC, AB = 6cm, AC = 10 cm, BC = 14 cm and X is the midpoint of side BC.
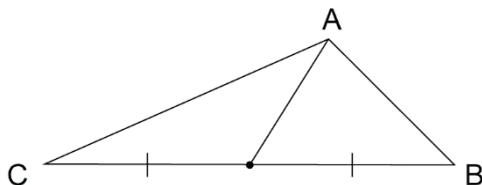
Find correct to one decimal place:
   a) $\angle ACB$
   b)   the length of AX.

15.   Consider the triangle below.

14 cm

19 cm

24 cm
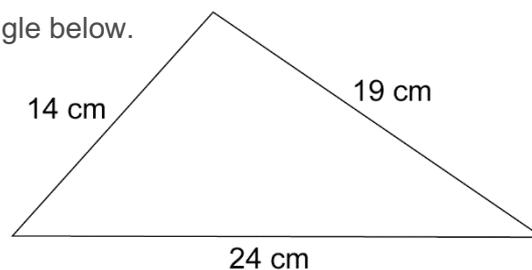
   a)   Use Heron's formula to calculate the area of this triangle.

   b)   Hence, determine the volume of the prism below.

19 cm

14 cm

11 cm

24 cm
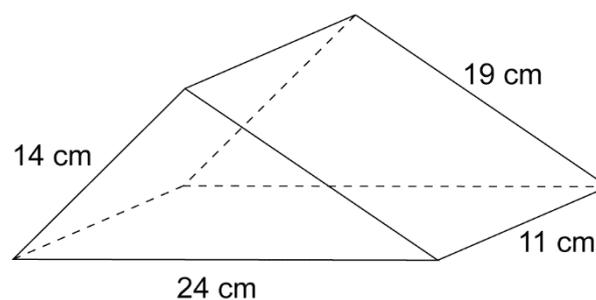
16.   A 53 metre wire attached to the top of a vertical pole attaches to fixed point on the ground where the angle of elevation is 24°. What is the height of the pole?

17.   John leaves camp and hikes for 3.2 km on a bearing of 167°. How far south of the camp has he travelled?

18.   From a 9.5 metre observation tower, a lifesaver sees a shark. He measures the angle of depression of his line of sight as 2°. Two minutes later, the angle of depression is 4° and his line of sight is in the same direction. How far has the shark swum in two minutes? Give your answer to the nearest metre.

19.   From the top of a cliff, an eagle (E) spots a mouse (M) and a rat (R). The angle of elevation from the mouse to the eagle is 55° and the angle of elevation from the rat to the eagle is 27°. The mouse and the rat are 425 metres apart.
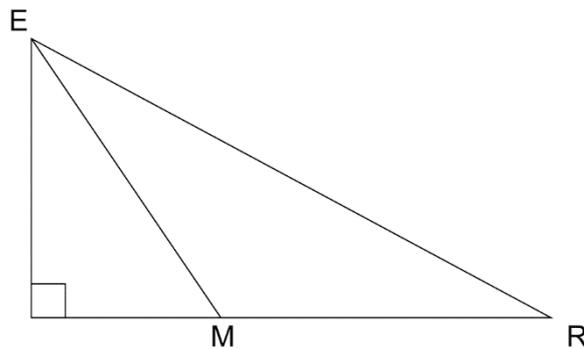
a)   What is the direct distance that the eagle must fly to catch the stationary mouse ie. (EM)?

b)   What is the direct distance that the eagle must fly to catch the stationary rat ie. (ER)?

c)   How high above the ground is the eagle perched?

20.   A ship sails from point A on a bearing of 042° for 200 km to Point B.
        From Point B, it then sails on a bearing of 155° degrees for 345 km.

a)   Calculate the size of angle θ.

b)   How far is the boat from point A?
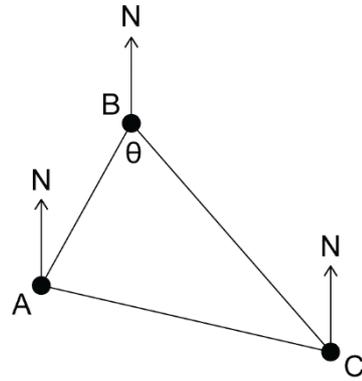
c)   A piece of cargo falls off the ship during the journey and is thought to be in
        the area marked grey. Calculate the size of the search area.

Trigonometry Solutions

1. a) $\sin P = \frac{5}{13}$ b) $\sin R = \frac{12}{13}$ c) $\cos R = \frac{5}{13}$ d) $\tan P = \frac{5}{12}$

2. $r^2 = p^2 + q^2 - 2pq.\cos K$

3. $s = \frac{3+3+4}{2}$ $A = \sqrt{5(5-3)(5-3)(5-4)}$

    $s = 5$ $A = \sqrt{20}$

    $\sqrt{16} = 4, \sqrt{25} = 5 \ \therefore \ \sqrt{20}$ is between 4 and 5.

4. $Area = 0.5 \ \times a \times b \times \sin C$

    $Area = 0.5 \times 9 \times 7 \times \sin 30°$

    $\underline{Area \ = 0.5 \times 9 \times 7 \times 0.5 = 15.75 \ cm^2}$

5. $a^2 = b^2 + c^2 - 2bc \times \cos A$

    $a^2 = 5^2 + 8^2 - 2(5)(8) \times \cos 60$

    $a^2 = 89 - 80 \times 0.5$

    $a^2 = 49$

    $\underline{a = 7 \ cm}$

6. $\sin 30° = \frac{8}{x}$

    $x = \frac{8}{0.5}$

    $\underline{x = 16 \ cm}$

7. $\sin 18° = \frac{x}{57}$

    $x = 0.31 \times 57$

    $\underline{x = 17.67 \ cm}$

8. a)   Bearing of A from B is 200°   b)   Bearing of A from B is 290°

9. $\sin^{-1}(0.9205) \approx 67°$

    $180 - 67° = 113°$ <u>The two angles are 67° and 113° to the nearest degree.</u>

10. a) $\frac{x}{\sin 62°} = \frac{8.6}{\sin 41°}$ b) $x^2 = 11^2 + 10^2 - 2 \times 11 \times 10 \times \cos 47°$

    $\underline{x = 11.57 \ cm \ (2 \ d.p.)}$ $x \ = \ 8.42 \ cm \ (2d.p.)$

c) $\dfrac{9}{\sin x} = \dfrac{7}{\sin 44^o}$

$\underline{x = 63.27^o \ (2 \ dp)}$

*** the diagram, although not to scale,

shows x^o to be acute. If a diagram

was not provided a solution of $\underline{116.73^o}$

would also be valid.

d) $x = \dfrac{9 \times 7 \times \sin 110^o}{2}$

$\underline{x = 29.60 \ cm^2 \ (2 \ dp)}$

11. $\angle D = 180 - 79 - 42^o$ $\dfrac{\overline{DF}}{\sin 42^o} = \dfrac{160}{\sin 59^o}$

$\angle D = 59^0$ $\overline{DF} = \underline{124.90 \ m \ (2d.p.)}$

12. $25.5 = \dfrac{6.4 \times 8 \times \sin x}{2}$

$x = 84.93^o$ (acute) or $\underline{95.07^o}$ (obtuse)

13. a)

P

Q 35°   35° R
       10 cm
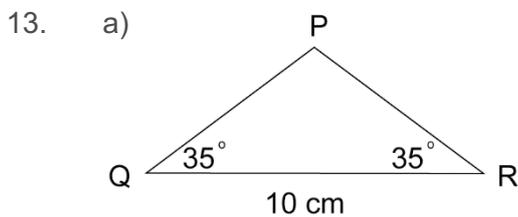
b) Let 'S' be the midpoint of $\overline{QR}$

$\tan 35^o = \dfrac{\overline{PS}}{5cm}$

$\underline{O = 3.50 \ cm \ (2 \ dp.)}$

14. a) $6^2 = 10^2 + 14^2 - 2 \times 10 \times 14 \times \cos C$
         $\underline{\angle C = 21.79^o}$ (2 d.p.)

     b) $a^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 21.79$
         $\underline{\overline{AX} = 4.36cm}$ (2 d.p.)
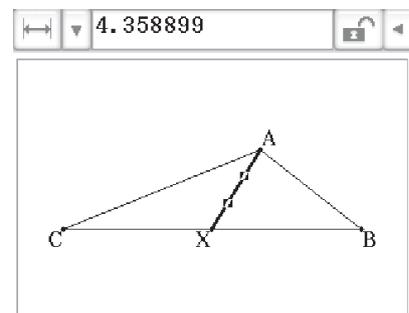
15. a) $s = \dfrac{(14+19+24)}{2}$
         $s = 28.5$

         $A = \sqrt{(28.5(28.5 - 14)(28.5 - 19)(28.5 - 24))}$
         $\underline{A = 132.92 \ cm^2 (to \ 2 \ d.p.)}$

     b) $V = Area \ of \ the \ front \ face \times Depth$
         $V = 132.91515 \times 11$
         $\underline{V = 1462.07 \ cm^3}$

16. $\sin 24° = \frac{x}{53}$

        $x = 53 \times \sin 24°$

        $\underline{x = 21.56 \, m \, (2 \, dp.)}$

17. $\cos 13° = \frac{x}{3.2}$

        $3.2 \times \cos 13°$

        $\underline{x = 3.12 \, km \, (2dp.)}$

18. $Distance = Initial \, Postion - Final \, Position$

        $D = \frac{9.5}{\tan 2^0} - \frac{9.5}{\tan 4^0}$ OR $D = 9.5 \times \tan(88) - 9.5 \times \tan(86)$

        $\underline{D = 136 \, m \, (nearest \, metre)}$ $\underline{D = 136 \, m \, (nearest \, metre)}$

19.   a) $\angle EMR = 180 - 55 = 125^o$ b) $\frac{425}{\sin(28)} = \frac{x}{\sin(125)}$

             $\therefore \angle MER = 180 - 125 - 27 = 28^o$ $\underline{x = 741.56 \, m \, (2 \, dp.)}$

             $\frac{425}{\sin(28)} = \frac{x}{\sin(27)}$

             $\underline{x = 410.98 \, m \, (2 \, dp.)}$ c) $\sin(55) = \frac{x}{410.98}$

                                                                    $\underline{x = 336.66 \, m \, (2 \, dp.)}$

        a)   b)   c)
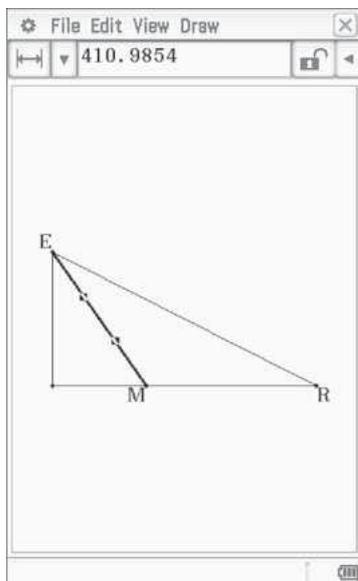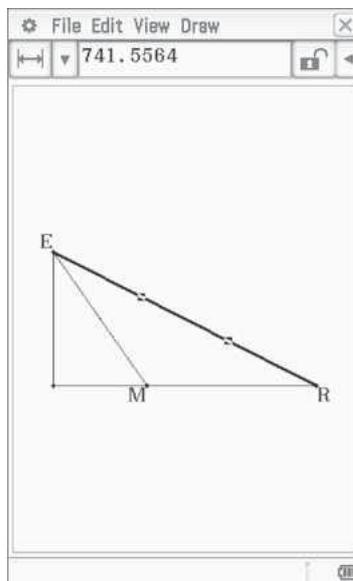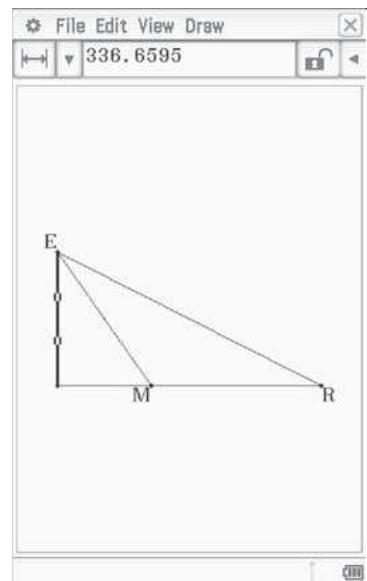
20.   a) $42 + (180 - 155) = 67^o$

       b)   324.20 km (2 dp.)   c)   31 757.42 km²
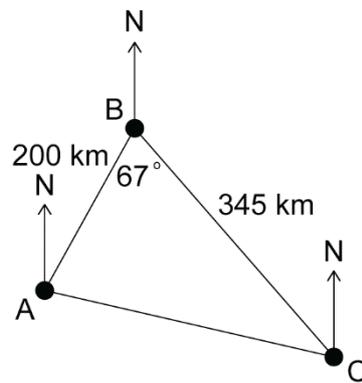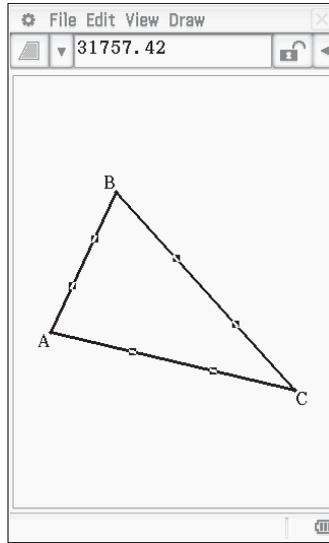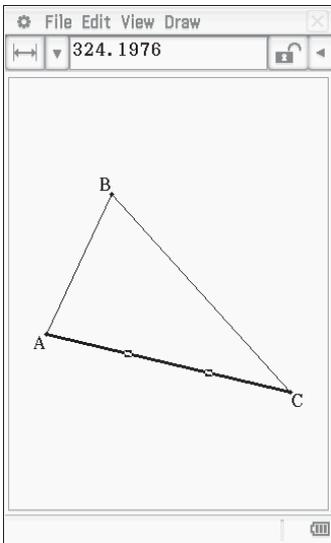
Chapter 16 – Solving Linear Equations

Worked Example 1

$3x - 5 = -23$

$3x = -23 + 5$

$3x = -18$

$x = \dfrac{-18}{3}$

$\underline{x = -6}$

Worked Example2

$3(x - 2) = 15$

$3x - 6 = 15$ [expand the brackets]

$3x = 15 + 6$

$3x = 21$

$x = \dfrac{21}{3}$

$\underline{x = 7}$

Worked Example 3 (Unknown on both sides of the equation)

$5x - 4 = -2x - 18$

$5x + 2x = -18 + 4$

$7x = -14$

$x = \dfrac{-14}{7}$

$\underline{x = -2}$

Worked Example 4 (Equations with Fractions)

$\dfrac{x-7}{4} = \dfrac{x+3}{6}$

$\dfrac{3(x-7)}{3(4)} = \dfrac{2(x+3)}{2(6)}$ [LCD =12]

$\dfrac{3x-21}{12} = \dfrac{2x+6}{12}$

$3x - 21 = 2x + 6$ [Multiplying both sides by 12, removes the denominator]

$3x - 2x = 6 + 21$

$\underline{x = 27}$

Solving More Complex Linear Equations

Worked Example 5

Solve $\quad \frac{19-x}{4} = 11 + x$

$\qquad 19 - x = 4(11 + x)$

$\qquad 19 - x = 44 + 4x$

$\qquad -x - 4x = 44 - 19$

$\qquad -5x = 25$

$\qquad x = \frac{25}{-5}$

$\qquad \underline{x = -5}$

```
 ⚙ Edit Action Interactive                    ✕
 0.5 1  👆▶  ∫dx⊐  Simp  ∫dx  ▼  ⨀  ▼  ▶
 ↳2           ∫dx↵

 solve( 19−x =11+x, x )
         ────
          4
                                  {x=−5}

 ▯
```

Worked Example 6

$\qquad \frac{2x+1}{3} + 2 = \frac{3x-1}{4}$

$\qquad \frac{4(2x+1)}{4(3)} + \frac{24}{12} = \frac{3(3x-1)}{3(4)} \quad$ (LCD = 12 $\therefore$ 2 = $\frac{24}{12}$)

$\qquad \frac{8x+4}{12} + \frac{24}{12} = \frac{9x-3}{12}$

$\qquad 8x + 4 + 24 = 9x - 3$

$\qquad 8x - 9x = -3 - 4 - 24$

$\qquad -x = -31$

$\qquad \underline{x = 31}$

```
 ⚙ Edit Action Interactive                    ✕
 0.5 1  👆▶  ∫dx⊐  Simp  ∫dx  ▼  ⨀  ▼  ▶
 ↳2           ∫dx↵

 solve( 2•x+1 +2= 3•x−1 , x )
         ────      ────
          3         4
                                  {x=31}

 ▯
```

Solving Worded Problems

1.   Carefully read the situation and choose a letter to represent the unknown(s) in the problem. In situations involving one unknown, it can be useful to look for comparisons.

2.   Translate the problem into an equation.

3.   Solve the equation.

4.   Answers the question in the context of the situation.

Worked Example 1

Kira, Terry and Dave all collect bottle caps. Terry has 5 more than Dave and Kira has 17 less than Dave. Altogether, they have 237 bottle caps. How many bottle caps does each person have?

1.   Terry and Kira's amounts have been compared to Dave. Let 'x' represent the number of bottle caps Dave owns.

2.   Dave x Terry $x + 5$ Kira $x - 17$

$x + x + 5 + x - 17 = 237$

$3x - 12 = 237$

3.   Solve $3x - 12 = 237$

$3x = 237 + 12$

$3x = 249$

$x = \frac{249}{3}$

$x = 83$

4.   Dave has 83 bottle caps; Kira has 66 and Terry has 88.

Worked Example 2

Dana is 25 years younger than her father. In 10 years' time, her father will be twice Dana's age. What are their current ages?

1.   Let the father's current age be represented by x

      Let Dana's current age be represented by $x - 25$

2. $x + 10 = 2(x - 25 + 10)$

      $x + 10 = 2(x - 15)$

3.   Solve $x + 10 = 2(x - 15)$

      $x + 10 = 2x - 30$

      $x - 2x = -30 - 10$

      $-x = 40$

      $x = 40$

Dana is currently $40 - 25 = 15$ years old and her father is 40 years old.

Resource Free Questions

1.   Solve the following equations

   a) $5x - 3 = 32$ b) $-27 - 2x = -15$ c) $\frac{5}{x} = 20$

   d) $1.5 - 3x = -6$ e) $-6(2x - 3) = -30$ f) $\frac{x}{5} - \frac{x}{2} = \frac{3}{10}$

2.   Evaluate x in each of the following equations.

   a) $4x - 3 = 2x - 11$ b) $\frac{(x-5)}{3} = (2x + 5)$ c) $\frac{5}{(x+3)} = \frac{2}{7}$

   d) $28x - 6(3x - 5) = 40$ e) $\frac{3x-7}{4} = \frac{2x-5}{3}$

3.   I think of a number, double it, reduce it by 7 and divide it by 3. The result is 17. What is the number.

4.   The sum of 3 consecutive integers is 144. Determine each of the integers.

5.   Gemma is 3 years younger than Robert and Ron is twice the age of Robert. The sum of their ages is 121. How old is each person?

6.   For each of the following triangles, write an equation to show the relationship between the variable and the given quantities, then solve the equation to find x.
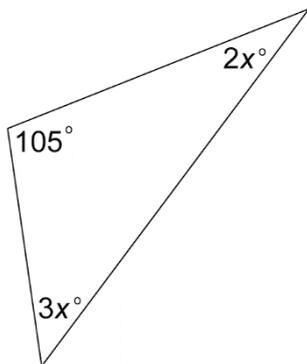
   a)   b)
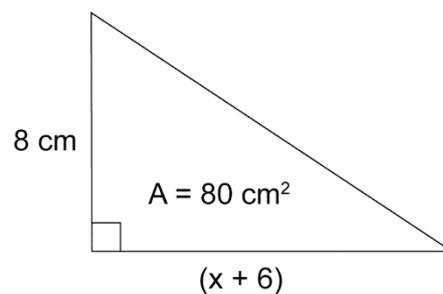
7.   Determine the dimensions of the rectangle shown below.

```
              _____//_____
             |                        |
             |                        |
     <  Perimeter = 80 m     >  2x - 4
             |                        |
             |_____//_____|
                     5x + 2
```
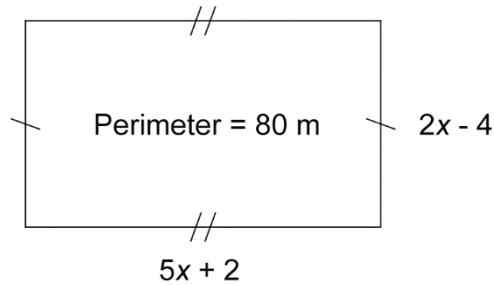
8.   I think of a number increase it by 3 and multiply the result by 12. I get the same value if I take the original number, double it, and add 16. What is the number I first thought of?

9.   A jar contains three colours of jelly beans. The number of red jelly beans is one more than double the amount of green jelly beans. The number of purple jelly beans is three less than four times the number of green ones. If there are 250 jelly beans in the jar, how many of each colour can be found?

10.   Lisa is 31 years younger than her father. In 14 years time, her father will be twice Lisa's age. What are their current ages?

Resource Rich Questions

11.   Solve the following equations using your CAS capabilities of your calculator.

a) $\frac{3x-7}{4} = \frac{2x-5}{3} + 2$ b) $x + \frac{1}{2} = \frac{x}{2} - \frac{2}{3}$ c) $6 + 0.5x = \frac{x}{4} - 5 + \frac{x}{6}$

12.   One seventh of a number increased by two, is equivalent to one less than an eleventh of the same number. What is the original number?

13.   Consider the rectangle shown below.
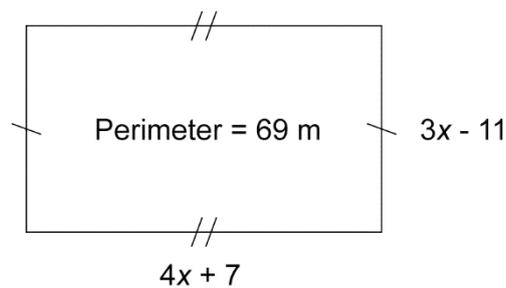
a)   Determine the value of x.

b)   Calculate the area of the rectangle.

14.   Consider the quadrilateral shown below.

a)   Calculate the size of $\angle ADC$.

b)   Calculate the distance from A to C.
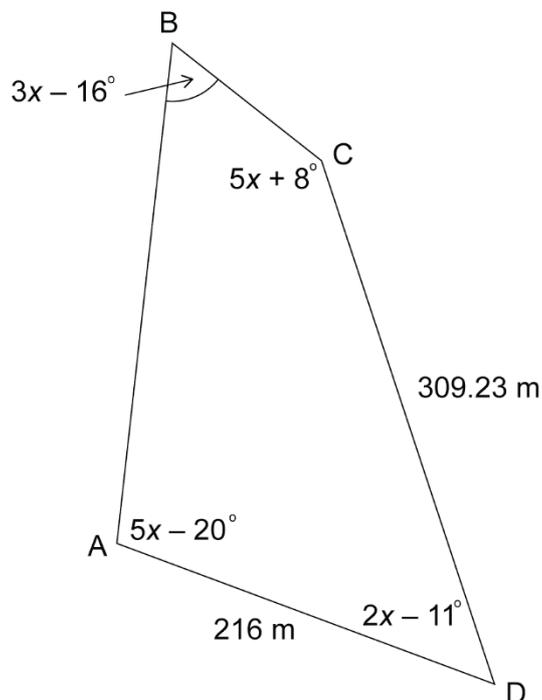
15.   The price of a dining chair is reduced by \$32 from its recommended retail price
       (RRP). To clear the remaining stock, the manager further reduces the price by
       12.5%. David buys 6 chairs of these chairs at a cost of \$708.75.

       Write an equation to show the price that David paid and solve the equation to find the
       RRP of a single chair (x).

Solutions

1.

a) $5x - 3 = 32$

$5x = 35$

$\underline{x = 7}$

b) $-27 - 2x = -15$

$-2x = 12$

$\underline{x = -6}$

c) $\frac{5}{x} = 20$

$x = \frac{5}{20}$

$\underline{x = 0.25}$

d) $1.5 - 3x = -6$

$-3x = -7.5$

$\underline{x = 2.5}$

e) $-6(2x - 3) = -30$

$-12x + 18 = -30$

$-12x = -48$

$\underline{x = 4}$

f) $\frac{x}{5} - \frac{x}{2} = \frac{3}{10}$

$\frac{2x}{10} - \frac{5x}{10} = \frac{3}{10}$

$-3x = 3$

$\underline{x = -1}$

2.   a) $4x - 3 = 2x - 11$

$2x = -8$

$\underline{x = -4}$

b) $\frac{(x-5)}{3} = (2x + 5)$

$x - 5 = 3(2x + 5)$

$x - 5 = 6x + 15$

$-5x = 20$

$\underline{x = -4}$

c) $\frac{5}{(x+3)} = \frac{2}{7}$

$5 = \frac{2(x+3)}{7}$

$35 = 2x + 6$

$29 = 2x$

$\underline{x = 14.5}$

d) $28x - 6(3x - 5) = 40$

$28x - 18x + 30 = 40$

$10x = 10$

$\underline{x = 1}$

e) $\frac{3x-7}{4} = \frac{2x-5}{3}$

$3(3x - 7) = 4(2x - 5)$

$9x - 21 = 8x - 20$

$\underline{x = 1}$

3. $\frac{2x-7}{3} = 17$

$2x - 7 = 51$

$2x = 58$

$\underline{x = 29}$

4. $x + (x + 1) + (x + 2) = 144$

       $3x + 3 = 144$

       $3x = 141$

       $x = 47$

       <u>The three numbers are 47, 48 & 49.</u>

5.   *Let x represent Robert's age*

       $x + (x - 3) + 2x = 121$

       $4x - 3 = 121$

       $4x = 124$

       $x = 31$

       <u>*Robert is 31, Gemma is 28 and Ron is 62.*</u>

6.   a) $3x + 2x + 105 = 180$ b) $\frac{8}{2}(x + 6) = 80$

              $5x = 75$ $4x + 24 = 80$

              <u>$x = 15$</u> $4x = 56$

                                                     <u>$x = 14$</u>

7. $5x + 2 + 2x - 4 + 5x + 2 + 2x - 4 = 80$

       $14x - 4 = 80$

       $14x = 84$

       $x = 6$

       <u>The dimensions of the rectangle are 32 m by 8 m.</u>

8. $12(x + 3) = 2x + 16$

       $12x + 36 = 2x + 16$

       $10x = -20$

       <u>$x = -2$</u>

9.   *Let x represent the number of green jelly beans*

$x + (2x + 1) + (4x - 3) = 250$

$7x - 2 = 250$

$7x = 252$

$x = 36$

If x = 36, there are 36 green, 73 red and 141 purple jelly beans.

10.   Solve $x + 14 = 2(x - 31 + 14)$

$x + 14 = 2x - 34$

$x - 2x = -34 - 14$

$x = 48$

∴ Lisa is 17 and her father is 48

11.   a) $x = 25$

        b) $x = -\dfrac{7}{3}$

        c) $x = -132$

$$\text{solve}\left(\frac{3 \cdot x - 7}{4} = \frac{2 \cdot x - 5}{3} + 2, x\right)$$
$$\{x=25\}$$
$$\text{solve}\left(x + \frac{1}{2} = \frac{x}{2} - \frac{2}{3}, x\right)$$
$$\left\{x = -\frac{7}{3}\right\}$$
$$\text{solve}\left(6 + 0.5 \cdot x = \frac{x}{4} - 5 + \frac{x}{6}, x\right)$$
$$\{x=-132\}$$

12.   *Solve* $\dfrac{x}{7} + 2 = \dfrac{x}{11} - 1$

$x = -57.75$

13.   a) $14x - 8 = 69$ b) $A = 29 \times 5.5$

                $x = 5.5$ $A = 159.5 \, m^2$

14.   a) $3x - 16 + 5x + 8 + 5x - 20 + 2x - 11 = 360^o$

                $15x - 39 = 360^o$

                $15x = 399^o$

                $x = 26.6^o$

                $\therefore \angle ADC = 2(26.6) - 11$

                $\underline{\therefore \angle ADC = 42.2^o}$

    b) $\overline{AC}^2 = 216^2 + 309.23^2 - 2(216)(309.23) \times \cos 42.2$

                $\overline{AC} = \sqrt{43317.06261}$

                $\underline{\overline{AC} = 208.13 \; m \; (2 \; dp.)}$
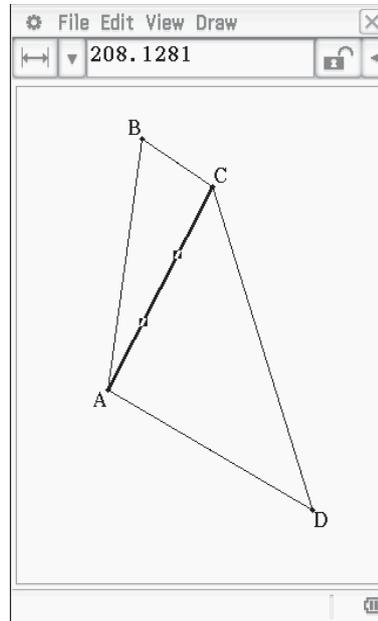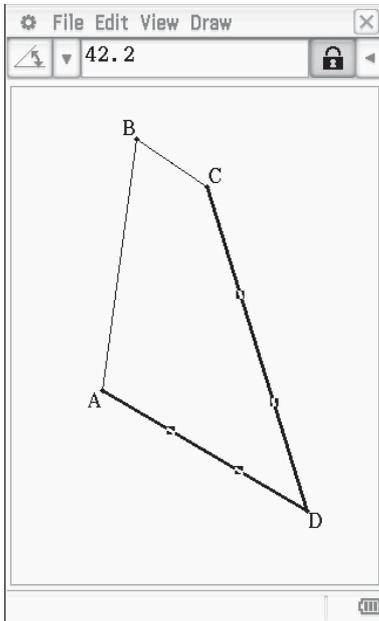
15. $\frac{87.5}{100}(x - 32) \times 6 = 708.75$

        $\underline{x = \$167.00}$

Chapter 17 – Graphs of Linear Equations

Linear equations commonly take the form $y = mx + c$ where:

- 'x' and 'y' are the coordinates of the points that satisfy the function and so lie on the straight line graph,

- 'm' is the gradient of the straight line graph, and

- 'c' is the 'y-intercept' of the straight line graph.

The **gradient** refers to the slope of a line and is defined to be the gradient of any interval within the line. The y**-intercept** is the point at which the line cuts the y-axis. The value of x is always zero at a y-intercept.

Positive Gradient	Negative Gradient	Zero Gradient	Undefined Gradient
$y = mx \pm c$	$y = -mx \pm c$	$y = \pm k$	$x = \pm k$

$$* \ 'k' is \ a \ constant$$

Consider the equation $y = 3x - 2$. It has a positive gradient of 3, that is, as x increases by 1, y increases by 3. The y-intercept is (-2) and is located at (0 , -2)

To graph the function;

- Plot the y-intercept (0 , -2).

- From the y-intercept, go 1→ and 3↑.

- Join the points and label graph.

$y = 3x - 2$
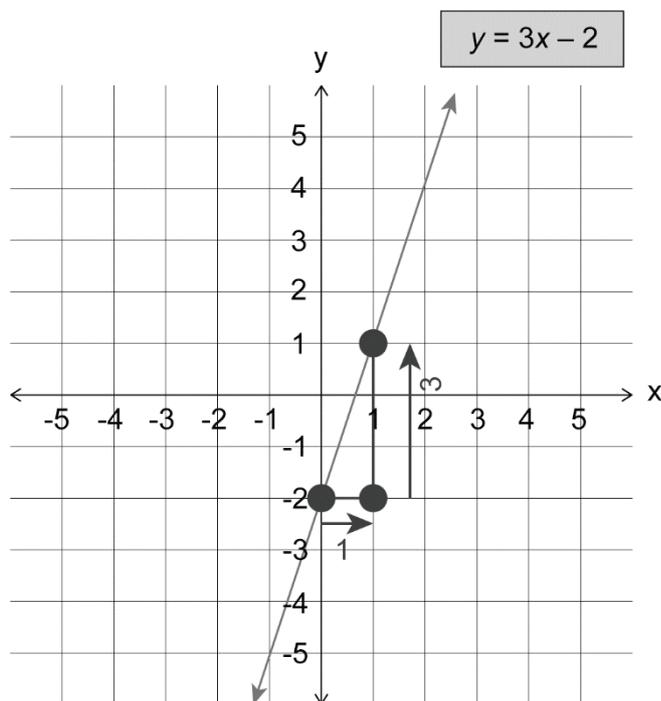
Determining the Equation From a Table of Values

A table of values shows the how the y variable changes in relation to the x variable.

x	0	1	2	3	4
y	1	3	5	7	9

y-intercept
(x = 0)

+2   +2   +2   +2

Constant 1st Order Difference Pattern (+2)
Gradient = 2

The equation of the line is $y = 2x + 1$. It has a positive gradient of 2, that is, as x increases by 1, y increases by 2. The y-intercept is (1) and is located at the coordinate (0 , 1)

Using The ClassPad To Determine the Equation From A Table Of Value

Statistics

Enter the x scores from the table into List 1 and the y scores into List 2.

Calc > Regression > Linear Reg > OK
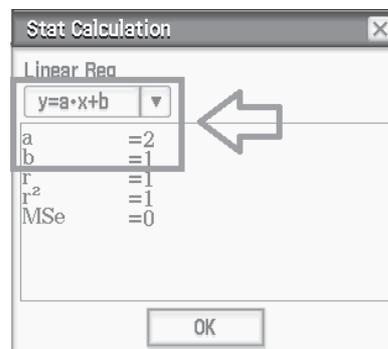
*** Note that the ClassPad uses the $y = ax + b$ in the place of $y = mx + c$.

Hence, the equation is $y = 2x + 1$.

Graphing Linear Equations Not in the Form $y = mx + c$

Consider the equation $4x - 2y = 8$. An alternative approach to graphing this linear relationship is to utilise the x & y intercept method. The y-intercept can be found by substituting $x = 0$ into the equation and likewise, the x-intercept can be found by substituting $y = 0$ into the equation.

y-intercept ($x = 0$)	x-intercept ($y = 0$)
$4x - 2y = 8$	$4x - 2y = 8$
$4(0) - 2y = 8$	$4x - 2(0) = 8$
$-2y = 8$	$4x = 8$
$y = -4$	$x = 2$
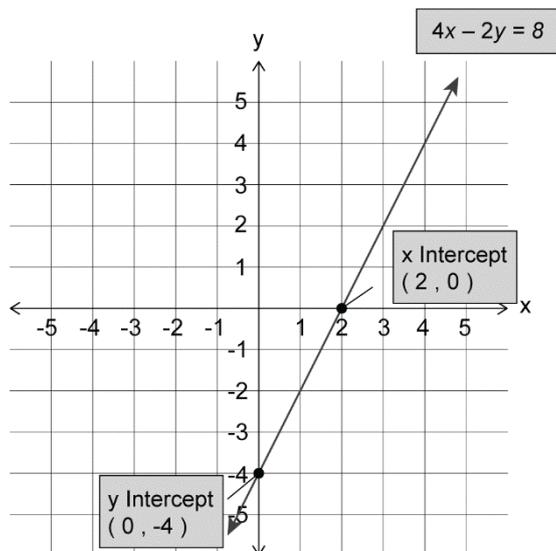
Using the ClassPad to Determine the Equation of the Line Not in the Form $y = mx + c$

Consider the equation $4x - 2y = 8$

- In the main application, type in the equation $4x - 2y = 8$.
- Click the graphing application to activate the split screen, highlight, and drag the equation into the xy-axis that appears.
- Analysis > Trace
- In the top right-hand corner of the graphing window the equation will appear in $y = mx + c$ form ($y = 2x - 4$)
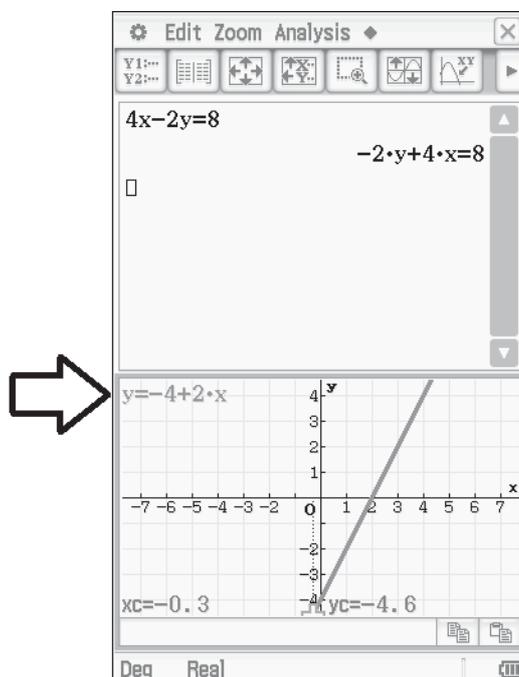
Resource Free Questions

1.   Consider the graphs of linear equations shown below.
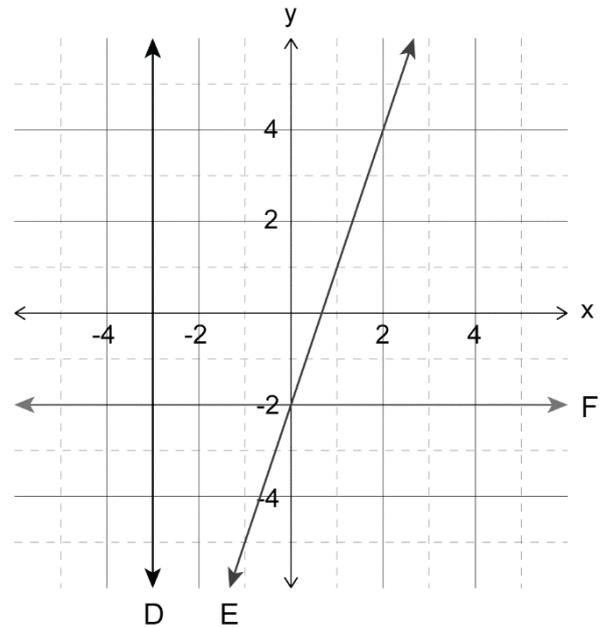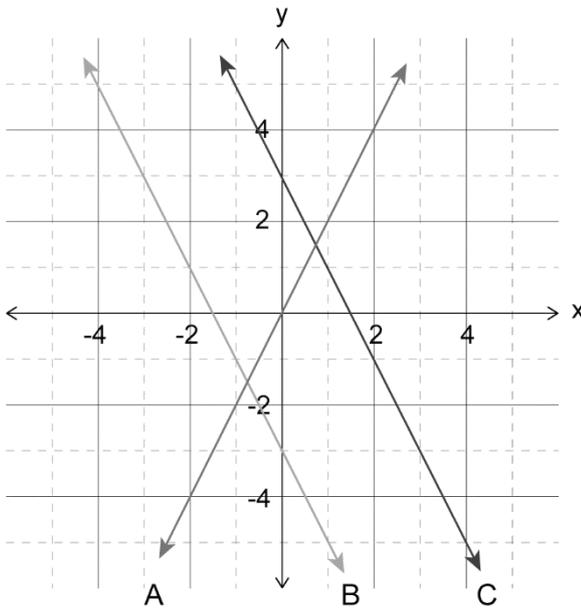
Match the graphs above to the description in the table below.

Criteria	Graph(s)
Has a positive gradient	
Has a gradient of zero	
Has a positive gradient and a negative y-intercept	
Has a negative gradient and a negative y-intercept	
Has an infinite gradient	
Has a y-intercept of zero	

2.   Determine the equation of the straight lines with the following properties.

a)   Has a gradient of -3 and y-intercept of 4

b)   Has gradient of 5 and a y-intercept of 0.

c)   Has a gradient of -1 and a y-intercept of -1.

3.   Determine the equation of the straight line that;

a)   has a gradient of 3 and passes through the point (0 , -2)

b)   has a gradient of -2 and passes through the point (0 , 5)

c)   has gradient of -3 and passes through the point (0 , 0)

d)   has a gradient of 0 and passes through the point (0 , 4)

4.   Determine the gradient for each of these lines.

   a) $y = 3x - 2$ b) $y = 12 - 2x$ c) $y = 9 - x$

5.   Determine the y-intercept for each of these lines.

   a) $y = -3x - 7$ b) $y = 12 + 4x$ c) $y = 3x$

6.   For each of the following lines, state whether the slope of the line is positive or negative.

   a) $y = -3x + 2$ b) $y = -12 + 5x$ c) $y = -3 - 2x$

7.   Determine the relationship between x and y in each of the table of values.

a)

x	0	1	2	3	4
y	1	4	7	10	13

b)

x	0	1	2	3	4
y	-2	1	4	7	10

c)

x	0	1	2	3	4
y	-2	-1	0	1	2

d)

x	0	1	2	3	4
y	2	2	2	2	2

8.   Determine equations of the straight lines shown below.

a)

b)

c)

d)
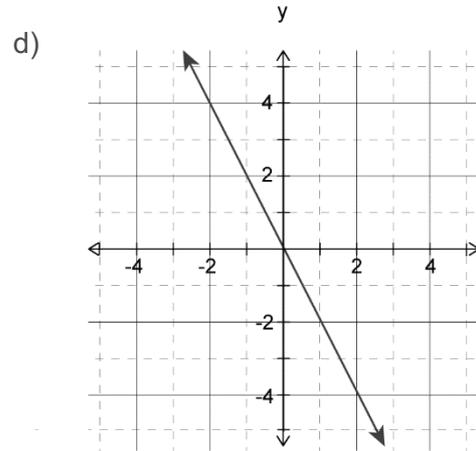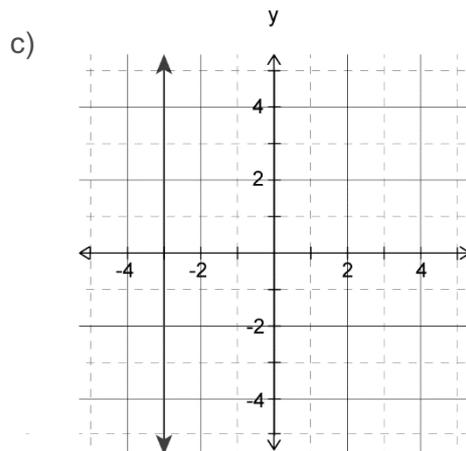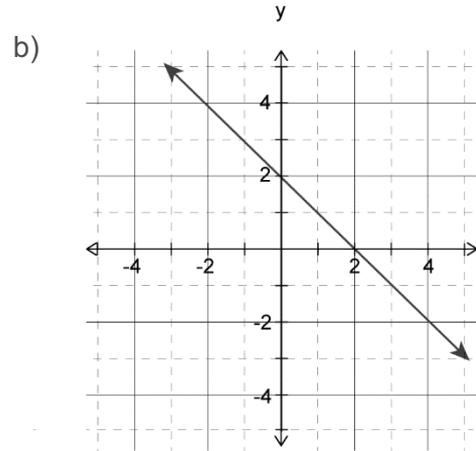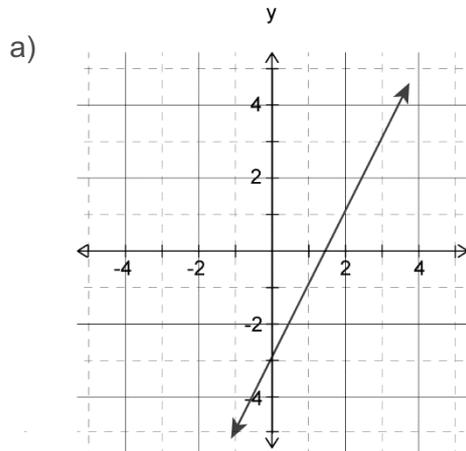
9.   Graph the following functions
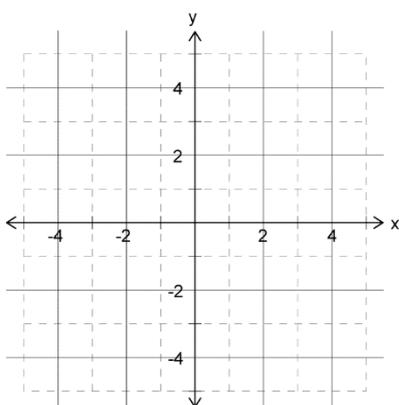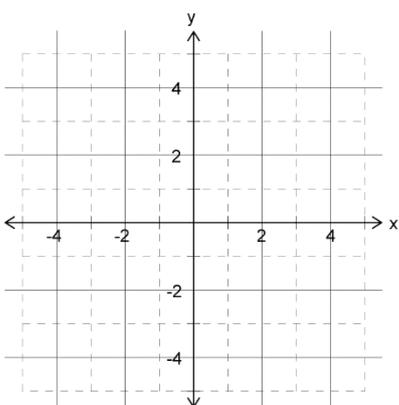
a) $y = 3x - 4$ b) $y = \frac{1}{4}x$ c) $y = -3$
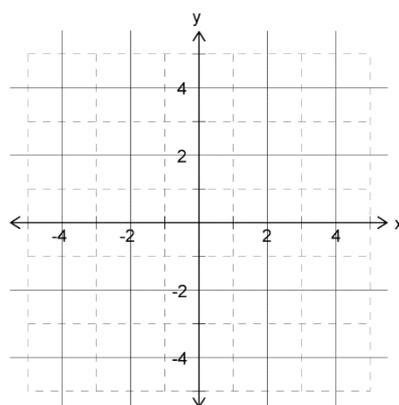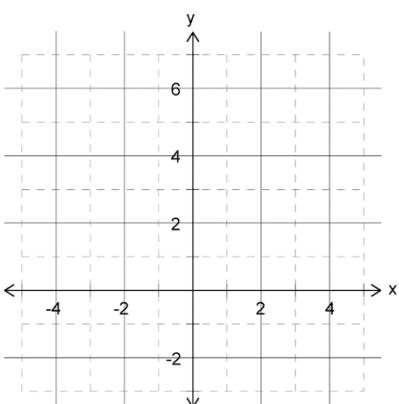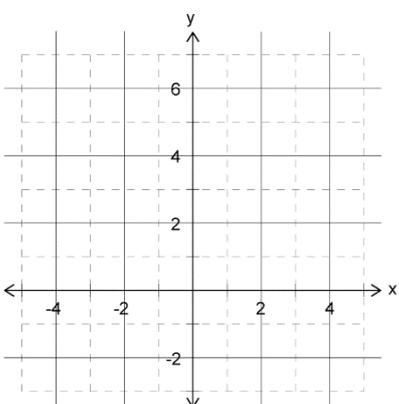
d) $x = -4$ e) $y = 3 - 2x$ f) $2x + y = 6$
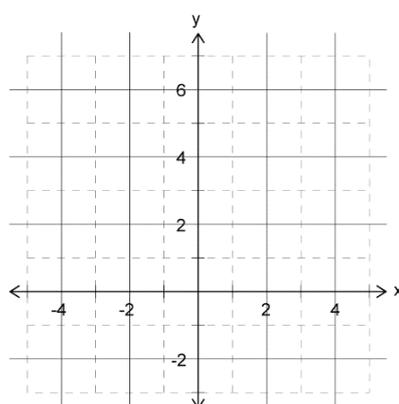
10.   Determine the equation of the straight lines that;

a)   passes through the points (0 , -2) and (1 , 1).

b)   passes through the point (0 , 5) and (1 , 7).

c)   passes through the point (0 , -3) and (1 , -5).

Resource Rich Questions

11. Utilise the statistics application on your calculator to determine the equation of the linear relationship between x and y.

a)

x	2	3	4	5	6
y	4.5	5	5.5	6	6.5

b)

x	4	5	6	7	8
y	1	-1	-3	-5	-7

12. Determine the coordinates of where $y - 9 = -1.2x$ cuts the x-axis

(i.e. the x-intercept)

13. Use your calculator to generate a table of values for the equation

$y = 0.75x - 3.2$ for the domain $0 \leq x \leq 4$.

x	0	1	2	3	4
y					

14. Use the calculator to rearrange the equation $6x = 4 - 2y$ and determine the gradient of the line and the coordinates of the y-intercept.

15. Use the calculator to rearrange the equation $x - 4y = 12$ and determine the gradient of the line and the coordinates of the y-intercept.

Solutions

1.

Criteria	Graph(s)
Has a positive gradient	A & E
Has a gradient of zero	F
Has a positive gradient and a negative y-intercept	E
Has a negative gradient and a negative y-intercept	B
Has an infinite gradient	D
Has a y-intercept of zero	A

2.   a) $y = -3x + 4$ b) $y = 5x$ c) $y = -x - 1$

3.   a) $y = 3x - 2$ b) $y = -2x + 5$ c) $y = -3x$ d) $y = 4$

4.   a) $m = 3$ b) $m = -2$ c) $m = -1$

5   a) $y - int = (0, -7)$ b) $y - int = (0, 12)$ c) $y - int = (0, 0)$

6.   a)   *Negative*   b)   *Positive*   c)   *Negative*

7.   a) $y = 3x + 1$ b) $y = 3x - 2$ c) $y = x - 2$ d) $y = 2$

8   a) $y = 2x - 3$ b) $y = -x + 2$ c) $x = -3$ d) $y = -2x$

9.
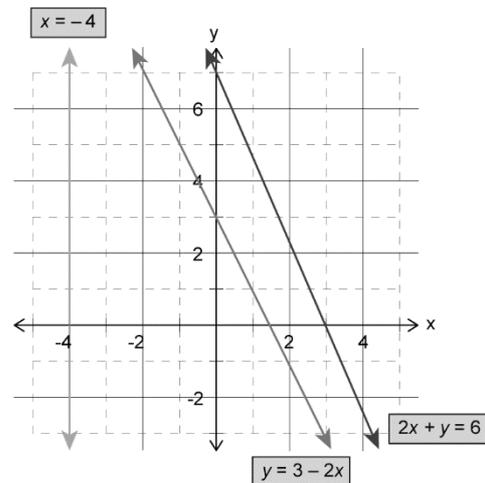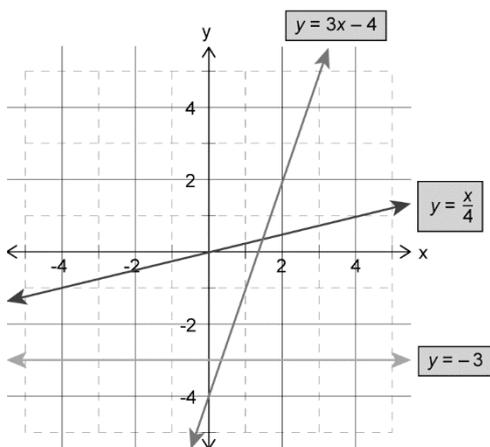
10.   a) $m = \dfrac{1-(-2)}{1-0}$ b) $m = \dfrac{7-5}{1-0}$ c) $m = \dfrac{-5-(-3)}{1-0}$

            $m = 3$ $m = 2$ $m = 2$

            $y - int (-2)$ $y - int (5)$ $y - int (-3)$

            $\underline{y = 3x - 2}$ $\underline{y = 2x + 5}$ $\underline{y = -2x - 3}$

11. a) $y = 0.5x + 3.5$ b) $y = -2x + 9$

12. Analysis > GSolv > Root   (7.5 , 0)

13.

x	0	1	2	3	4
y	-3.2	-2.45	-1.7	-0.95	-0.2
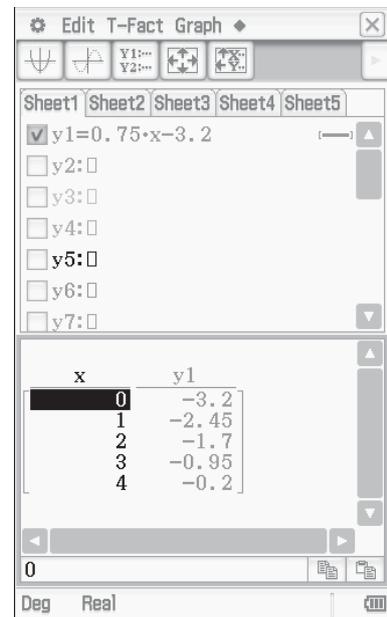
14. $y = -3x + 2$ $m = -3 \ \& \ y - int = (0 , 2)$

15. $y = \frac{x}{4} - 3$ $m = \frac{1}{4} \ \& \ y - int = (0 , -3)$
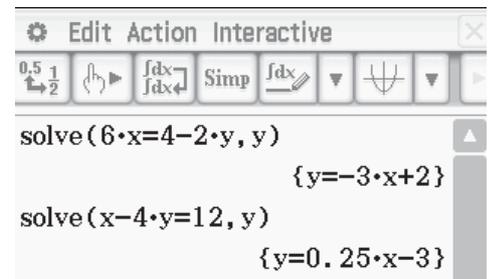
Chapter 18 – Applications of Linear Equations

Constant Rate Problems

If the rate does not alter over time, it is said to be a **constant rate**. The gradient represents the rate of change in the dependent variable per unit of change in the independent variable. As such, straight line graphs can be used to solve problems involving constant rates.

The cost of hiring a celebrity guest to a fundraising dinner is given by the function

$$C = 500 + 100H$$

where 'C' represents the total cost of hiring the celebrity, and
'H' represents the number of hours the celebrity is at the event.

In this function, the y-intercept of 500 represents a fixed cost of $500, while the gradient of 100 represents a constant rate, i.e. $100 per hour.
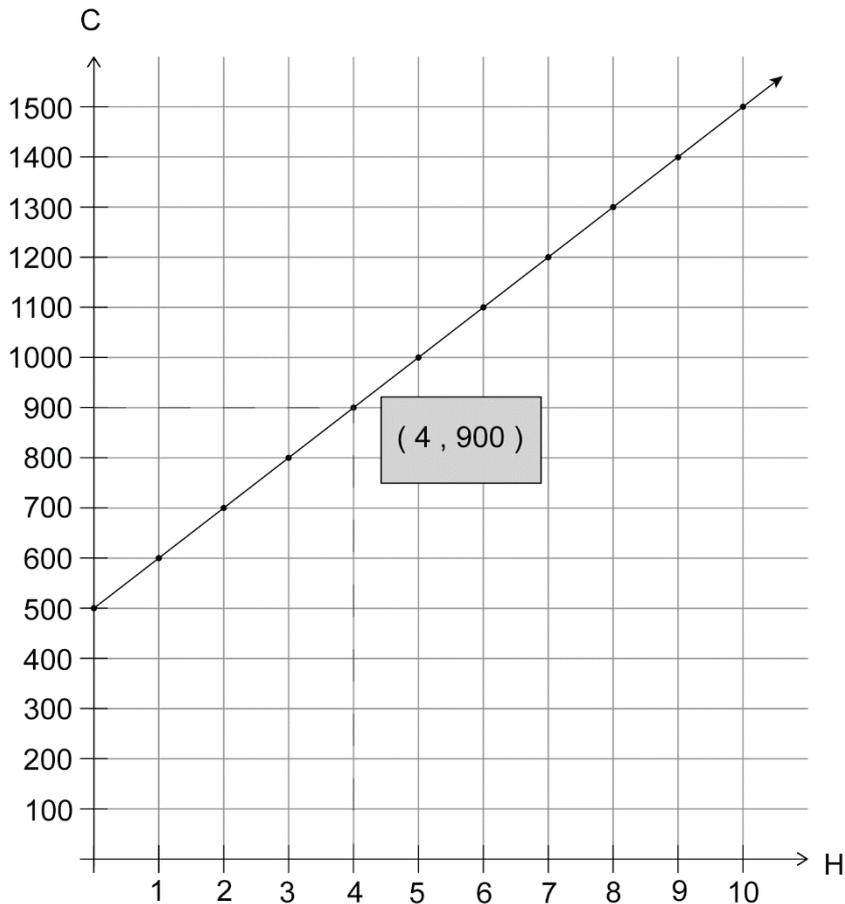
The graph can be utilised to ascertain the cost of hiring the celebrity for four hours. The coordinate (4 , 900) indicates that the celebrity will charge $900 for a four hour appearance at this event.

Resource Fee Questions

1.   A wedding singer charges $200 plus $60 an hour for her services. Construct a linear
      graph to show the cost (C) of the singer for (h) hours.
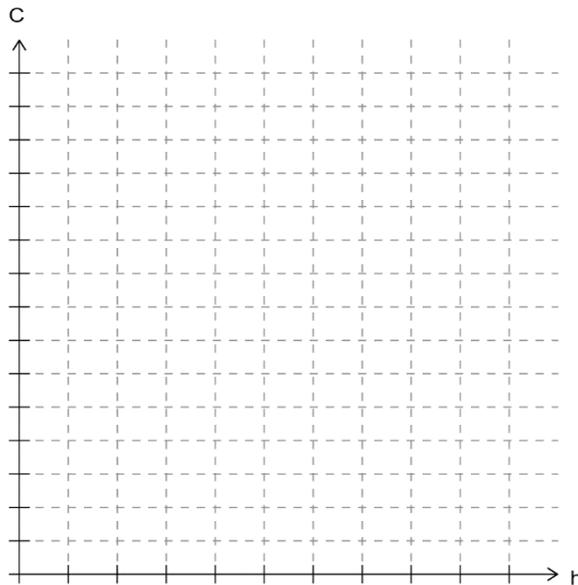
2.   Consider the chart below showing the cost, in dollars, of an international phone call
      from Australia to Belgium.
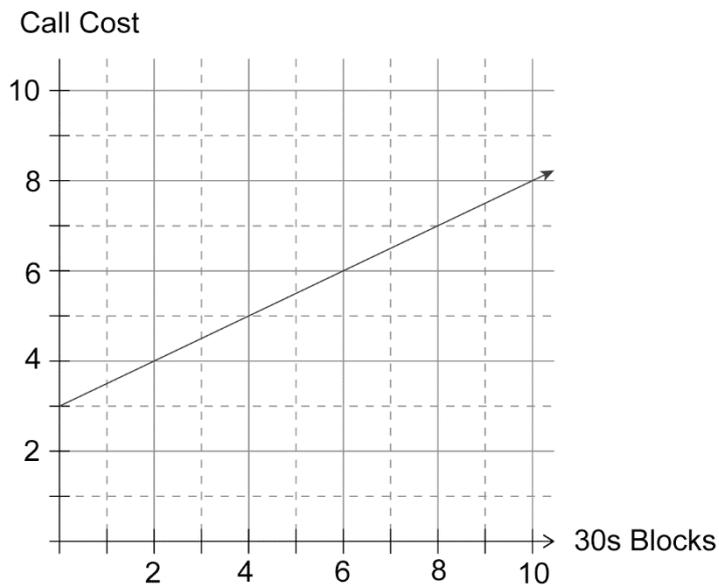
      a)   What is the gradient of the line?

      b)   What is the y-intercept of the line?

      c)   Describe the cost of a phone call to Belgium using your answers from
            (a) & (b)

      d)   Determine the cost of a phone call to Belgium lasting exactly 5 minutes.

3.   In a radio competition, $1 000 dollars is on offer to any contestant who can answer
        10 general knowledge questions correctly. For every question the contestant gets
        wrong, $100 is deducted from the total.

        a)   Construct a graph to show the decline in prize money (P) as the number of
                incorrect answers (a) increases.
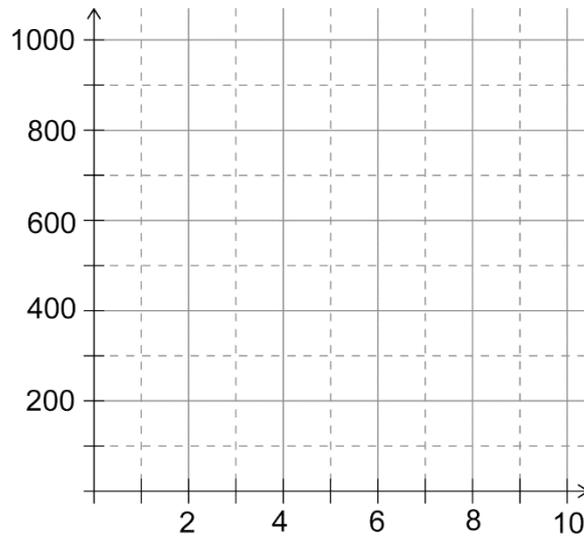
        b)   What is the gradient of the line?

4.   At a dog grooming show, the organisers create individual pens for all the dogs in the
        competition. The first pen requires four fencing panels. Each subsequent pen only
        requires three additional fencing panels when attached to another pen.

                    1 Pen   2 Pens   3 Pens
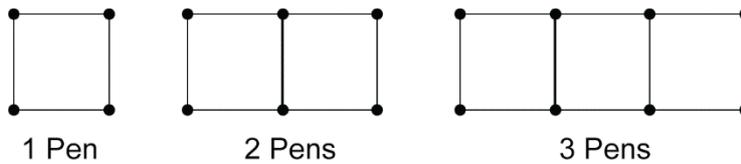
        a)   Complete the table below to show the number of panels required for the
                growing number of pens.

Number of Pens	1	2	3	4	5
Number of Fencing Panels					

        b)   Write a linear equation to model the relationship between the number of pens
                (P) and the number of the fencing panels (f) required.

        c)   How many fencing panels will be required if 32 pens are needed?

        d)   How many pens can be constructed from 76 fencing panels?

5.   Consider the conversion chart shown below.
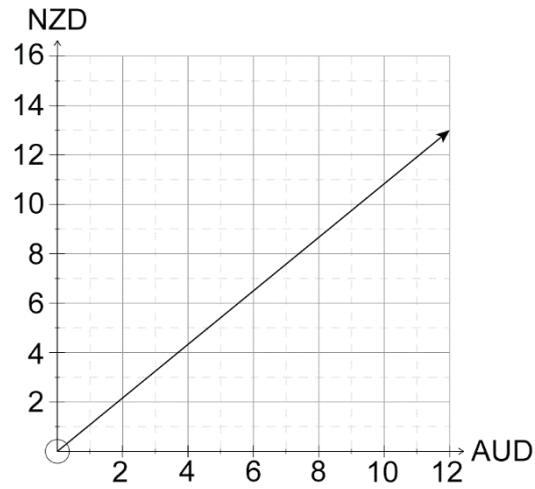
a)   Use the chart to convert 10 Australian Dollars to New Zealand Dollars (NZD).

b)   What is the gradient of the line above?

c)   Using the exchange rate established in the conversion chart, convert 23AUD to NZD.

Resource Rich Questions

6.   Kane runs out of fuel on his way home from work. He walks down to the petrol station
        when he purchases a fuel container for $13 and fills it with unleaded fuel costing
        $1.43 a litre.

        a)   How much will it cost Kane to fill his car with 6 litres of fuel?

        b)   If Kane only has $25 in his wallet, how much fuel will he be able to purchase
                and take back to his car?

7.   A full storage tank is punctured and begins to slowly leak water. The volume of the
        tank (V) in litres, t hours after it is punctured is given by the formula,
        $$V = 600 - 12t$$

        a)   What is the initial volume of the storage tank?

        b)   What is the tank's volume after 6 hours?

        c)   How long will it take for the tank to lose half its initial volume?

        d)   What is the total time taken for the tank to completely empty?

8.   Jack works at a call centre for an insurance company selling insurance policies. On
        any given day, if he signs up 20 new customers, he receives a $300 bonus from his
        manager, plus $18 for every subsequent customer. If any fewer than 20 new policies
        are signed, he receives no bonus at all.

        a)   Write a linear expression to show the bonus payment *(B)* that Jack receives
                for signing up 20 or more customers *(c)* on any given day.

        b)   On a very successful day, Jack receives a $606 bonus from his manager.
                How many new customers did Jack successfully attract on that day?

9.   For a science experiment, a student is asked to analyse the growth of a bamboo
        plant. Initially, the plant was 6.8 cm high. In the first hour it grew by 28 mm.

        If the bamboo plant continues to grow by the same amount every hour:

        a)   Write a linear model to represent the height *(H)* of the plant after *'t'* hours.

        b)   How long will it take to reach a height of 0.6 metres?

10.   Phoebe receives an annual base salary of $32 000 per annum plus a commission of 3% on her total sales for the year.

    a)   Write linear equation to show Phoebes total income (I), based on her total sales of 'c' dollars.

    b)   If Phoebe has annual sales of a quarter of million dollars, what will be her total income for the year?

    c)   If Phoebe wants to earn $65 000 in a year, what must her total sales be for the year?

Solutions

1.
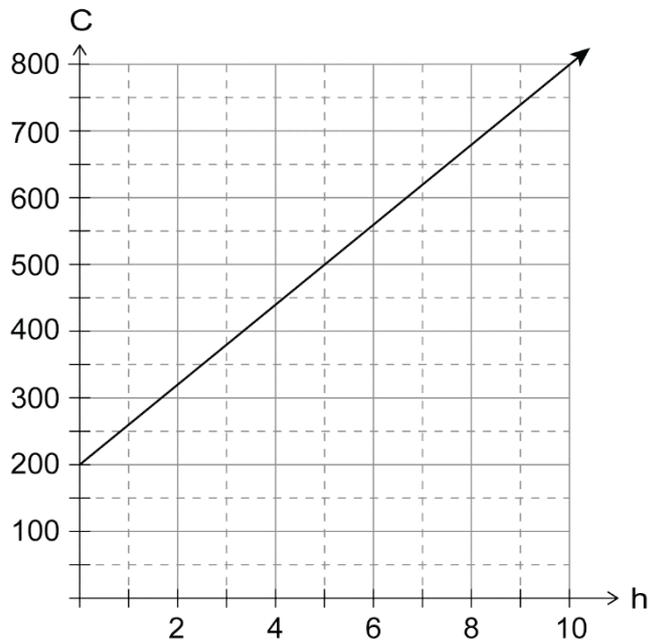
2.   a)   0.5   b)   (0 , 3)

      c)   A phone call has a connection (fixed) fee of $3.00 and is charged at a fixed
            rate of $0.50 per 30 second block.

      d) $8.00

3.   a)
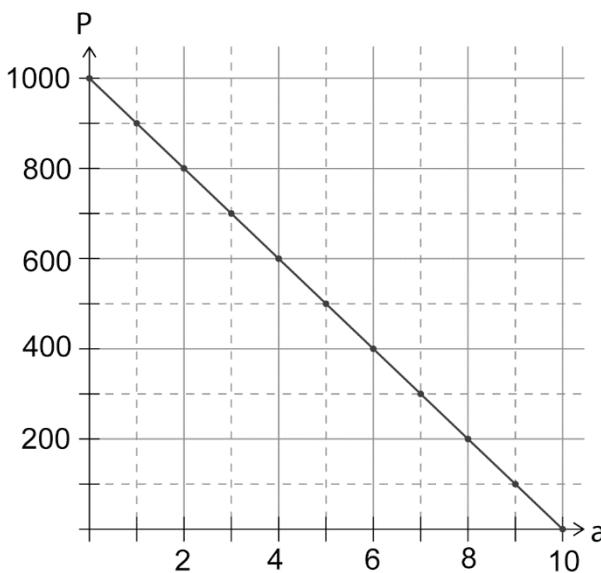
b) -100

4.   a)

Number of Pens	1	2	3	4	5
Number of Fencing Panels	4	7	10	13	16

      b) $f = 3P + 1$ c) $f = 3(32) + 1$ d) $76 = 3P + 1$

                                         $\underline{f = 97}$ $\underline{P = 25}$

5.   a) $10AUD = 11NZD$ b)   1.1   c) $1AUD = 1.1NZD$

$$\frac{11}{10} = \frac{x}{23}$$

$$x = \frac{(23 \times 11)}{10}$$

$$\therefore 23AUD = 25.30NZD$$

6.   a) $C = 13 + 1.43(6)$ b) $25 = 13 + 1.43L$

$\underline{C = \$21.58}$ $\underline{L = 8.39 \quad i.e.\,8.39\ litres\ of\ fuel.}$

7.   a) $600\ litres\ (y - intercept)$ b) $V = 600 - 12(6)$
                                                                    $\underline{V = 528\ L}$

     c) $300 = 600 - 12t$ d) $0 = 600 - 12t$
          $\underline{t = 25\ hours}$ $\underline{t = 50\ hours}$

8.   a)   Let $'c'$ represent the number of customers exceeding 20.

          $B = \$300 + 18c$

                              OR

          Let $'c'$ represent the number of customers signed per day.

          $B = \$300 + 18(c - 20)$

     b) $606 = 300 + 18c$ $606 = 300 + 18(20 - c)$

          $306 = 18c$ OR $306 = 18(20 - c)$

          $c = 17$ $17 = 20 - c$

          $\underline{Jack\ successfully\ attracted}$ $c = 37$

          $\underline{17 + 20 = 37\ new\ customers}$ $\underline{Jack\ successfully\ attracted\ 37}$

                                                     $\underline{new\ customers}$

9.   a) $H = 6.8 + 2.8t$ b) $60 = 6.8 + 2.8t$

                                         $\underline{t = 19\ hours}$

10. a) $I = 32\,000 + 0.03c$

     b) $I = 32\,000 + 0.03c$

          $I = 32\,000 + 0.03(250\,000)$

          $\underline{I = \$39\,500}$

     c) $65\,000 = 32000 + 0.03x$

          $x = \frac{(65\,000 - 32\,000)}{0.03}$

          $\underline{x = \$1\,100\,000}$

Chapter 19 – Simultaneous Equations

Solving Simultaneous Equations

Graphical Method

Simultaneous equations can also be solved by using graphs.

The diagram on the right shows the graphs of;

$2x + y = 10$

$y = x + 1$

$2x + y = 10$ can be graphed using x and y intercepts

y-intercept $(x = 0)$ *x*-intercept $(y = 0)$

$2x + y = 10$ $2x + (0) = 10$
$2(0) + y = 10$ $2x = 10$
$y = 10$ $x = 5$
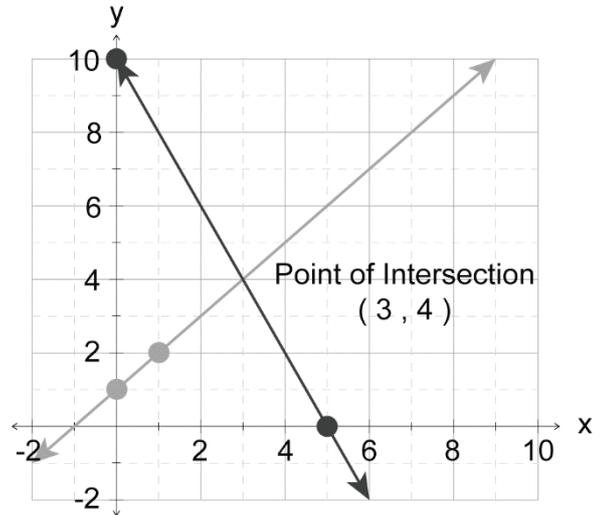
$y = x + 1$ can be graphed using gradient and *y* intercept. Start at (0 , 1). Move $1 \rightarrow$ and $1\uparrow$.

Substitution Method

In some cases, simultaneous equations can be solved more simply by substitution.

$2x + y = 10$

$y = x + 1$

Substitute $x + 1$ for *y*.   Substitute $x = 3$ into either equation

$2x + x + 1 = 10$

$3x + 1 = 10$ $y = x + 1$ $2x + y = 10$

$3x = 10 - 1$ $y = 3 + 1$ or $2(3) + y = 10$

$3x + 9$ $\underline{y = 4}$ $6 + y = 10$

$\underline{x = 3}$ $\underline{y = 4}$

The point of intersection is (3 , 4)

Elimination Method

If one of the equations is not in the form '$x =$' or '$y =$', it may be easier to use the elimination method. One of the unknowns, x or y is eliminated when the equations are added or subtracted.

$3x + 5y = -3$

$+ \qquad x - 5y = 19$ $\qquad\qquad$ (add the equations as $5y + (-5y) = 0$

$= \qquad 4x = 16$

$\qquad x = \frac{16}{4}$

$\qquad \underline{x = 4}$

Substitute $x = 4$ into either equation.

$3x + 5y = -3$		$x - 5y = 19$
$3(4) + 5y = -3$		$4 - 5y = 19$
$12 + 5y = -3$	or	$-5y = 19 - 4$
$5y = -15$		$-5y = 15$
$y = -3$		$y = -3$

Break Even Analysis

Profit = Revenue – Costs

The **break-even point** (equilibrium) of supply and demand is where costs equal revenue i.e. there is no profit or loss

Worked Example

A small company builds it its own game controllers.

The cost (C) of manufacturing game controllers
is represented by the equation $C = 12x + 52\,400$.

The sale price (S) for each controller is $52
ie. $S = 52x$.

How many units must the company sell to cover
the manufacturing costs?

Answer: 1310 units must be sold to "break even"
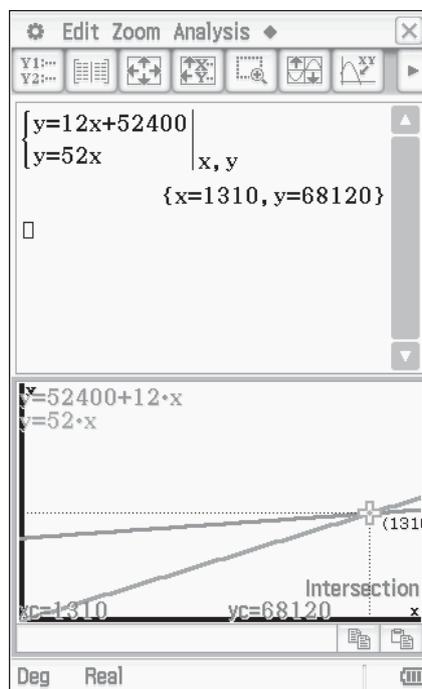
Resource Free Questions

1. Answer "yes" or "no" to the following statements.

    a) Is (2, 0) a solution to the system $\left\{ \begin{array}{l} 3x + y = 6 \\ x - y = 2 \end{array} \right\}$?

    b) Is (5 , 7) a solution to the system $\left\{ \begin{array}{l} x + y = 12 \\ y = x + 2 \end{array} \right\}$?

    c) Is (4 , -2) a solution to the system $\left\{ \begin{array}{l} 3x - y = 10 \\ y = x - 6 \end{array} \right\}$?

2. What is the point of intersection for the following pairs of graphs?
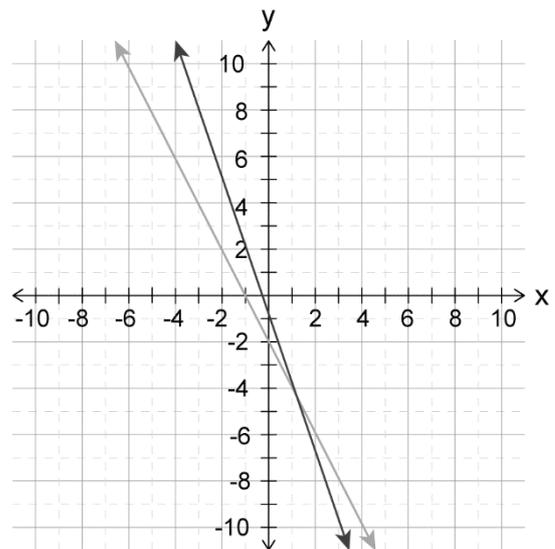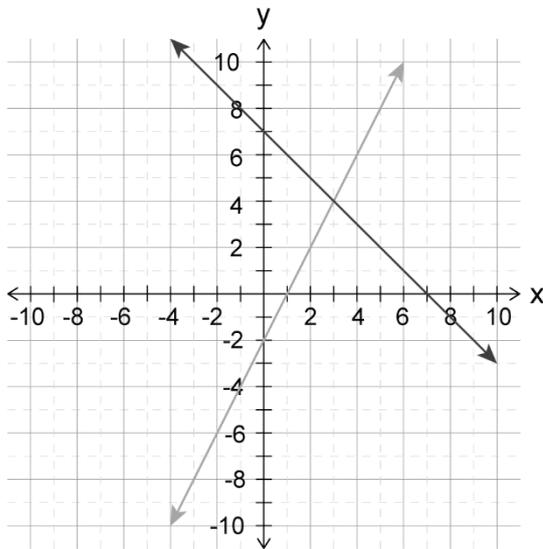
3. Use the set of axes below to show why there are no solutions to the following pair of equations $y = 2x - 3$ and $y = 2x + 5$
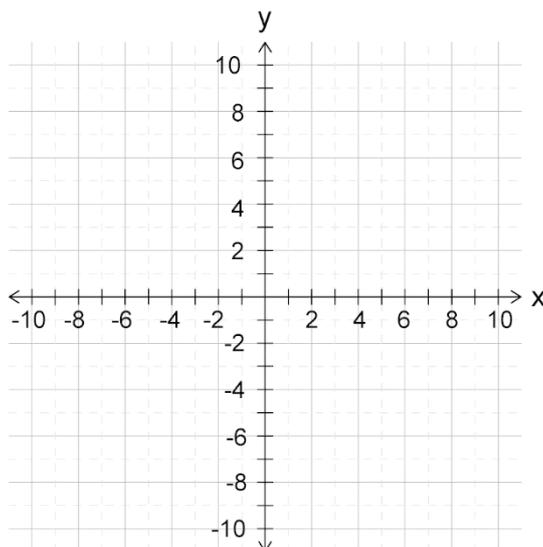
4. Graph the following pairs of equations using the axes provided and identify the point of intersection.

a) $y = 4x - 2$
    $y = 3x - 3$
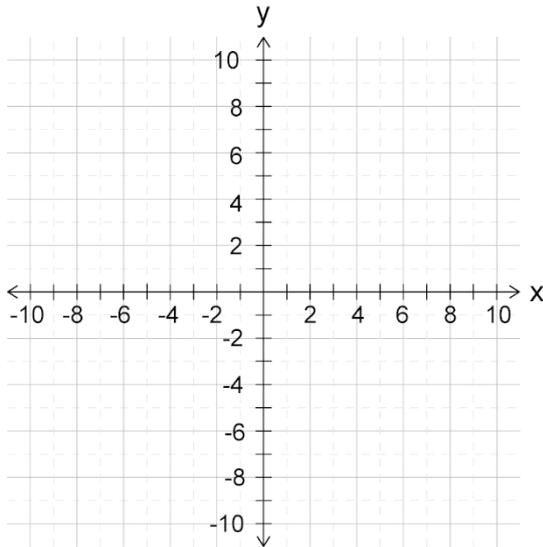
b) $y = x + 1$
    $y = -x + 5$
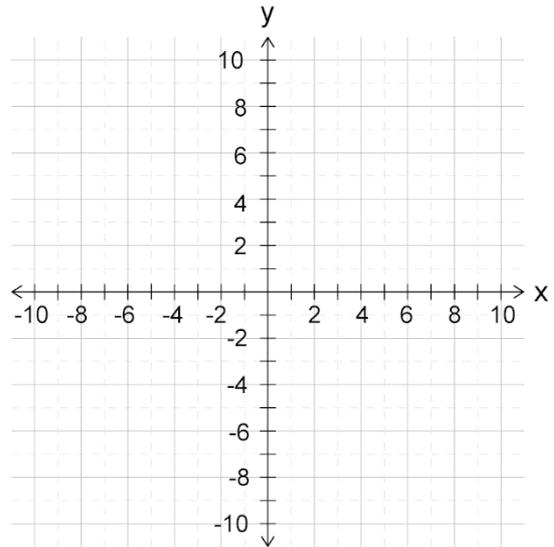
c) $y = x - 5$
    $x + y = 3$
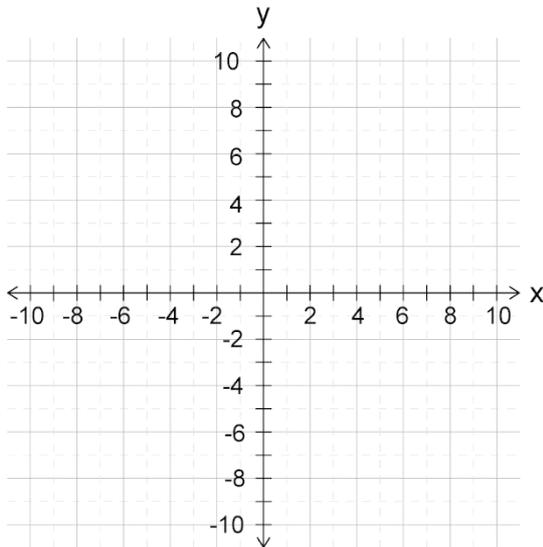
d) $y = 2x + 3$
    $x + 2y = -4$
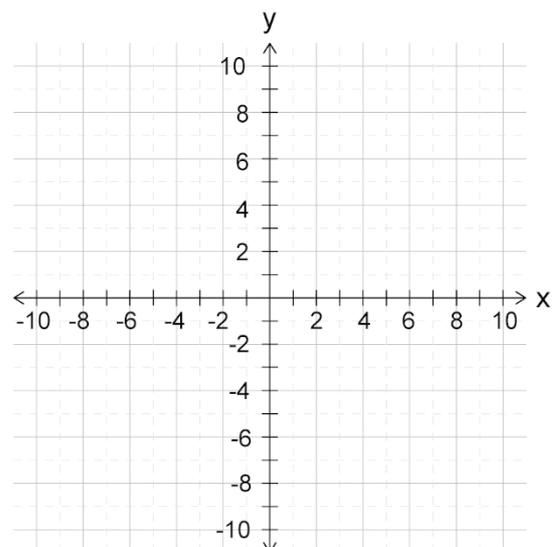
5. Solve these pairs of linear equations using the substitution method.

a) $2x + y = 14$
    $y = 3x + 4$

b) $y = 3 - x$
    $3x - 2y = -1$

c) $x = y - 3$
    $3x + 2y = 11$

6. Solve using the elimination method.

a) $4x - 3y = 3$
    $2x + 3y = 15$

b) $3x - 2y = 7$
    $4x - 3y = 5$

7. James sells watches that he imports from overseas for $90 each. Each watch costs him $15, and the cost of his retail space is $600 per week. How many watches must he sell each week to reach the break-even point? Show algebraically, how you obtained your answer.

8. I am 13 years older than my brother. The sum of our ages is 47. What are our ages?

9. For her birthday, Tiffany received a total of $146 from her siblings. The amount of money she received from her brother was $38 more than what she received from her sister.

    a) Write a pair of linear equations to represent the information shown above.

    b) Solve algebraically to show the amount of money received from each family member.

10. Two identical tanks are standing side by side. Each has a capacity of 3000 litres.

    a) One tank is full and is leaking water at a rate of 60 litres an hour.
        Display this graphically, using the axes below.

    b) The second tank is empty and is being filled at a rate of 40 litres an hour.
        Display this graphically, using the axes below.
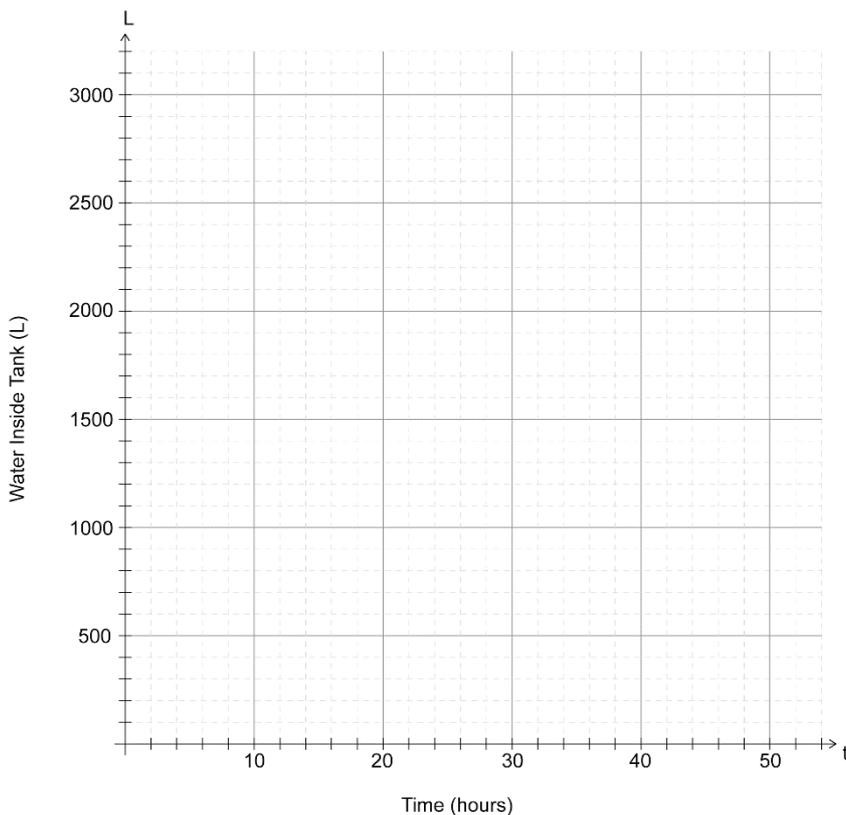
    c) After how many hours (t) will the water contents (L) of the two tanks be identical? Use your graphs to obtain your answer.

Resource Rich Questions

11. a) Solve each of the following pairs of simultaneous equations using the graphing capabilities of your ClassPad.

        i) $4x + 5y = 12$ ii) $2x + 7y = 15$
                  $3x - 4y = 9$ $9x - 6y = 3$

     b) Solve each of the following pairs of simultaneous equations using the solver capabilities of your ClassPad.

        i) $3x - 4y = 9$ ii) $4x + 5y = 19.4$
                  $4x - 2y = -5$ $5x - 2y = -30.2$

12. James and Sarah are both electricians. James charges a flat rate of $82.00 per hour and Sarah charges a call-out-fee of $60 and charges $70.00 per hour. For what length of time will the cost of labour for the two electricians be the same amount?

13. In five years time, Sammy's grandmother will be eight times as old as her. Currently, the sum of their ages is 71. How old are they now?

14. A solution containing 13% of methanol is mixed with a solution containing 3% methanol to make 18 L of a solution containing 7.5% methanol. How much of each solution is required to obtain the desired 7.5% methanol concentration?

15. The sum of the numerator and denominator of a certain fraction is 18. If the denominator is increased by seven, then the fraction will be a quarter. Find the value of the numerator and the denominator.

16. a) On Monday, Steve buys 20 lollypops and 30 icy poles for $40 from the school canteen. On Tuesday he buys 30 lollypops and 10 icy poles from the school canteen for $25. Find individual prices of lollypops and icy poles at the school canteen.

     b) Five bottles of juice and three bottles of water cost $17.30. Bottles of water are 90 cents cheaper than a bottle of juice. Calculate the costs of each type of drink.

17. Two fitness centres, Alpha Fitness and Omega Training both offer memberships packages. Alpha fitness charges a once-off $104 joining fee and an ongoing charge of $10 per week. Omega Fitness has no joining fee and charges $12 per week.

     a) Use linear equations to model the relationship between cost (C) and the number of weeks (w) and find the point of intersection between the two lines.

    b)   Which fitness centre is more economical if a client continues their
           memberships for;

         i)   6 months?

         ii)   1 year?

         iii)   18 months?

18.   Determine the value of each of the variables in the shapes below.

a

$5x - 2y$

$x + y$

5 cm

11 cm

b)

$2x + 5y$

$x + 3y$

5 cm

9 cm

c)

$20°$

$4x + 2y°$ $5y°$

d)

$3x - 4y$

13

$x + 3y$
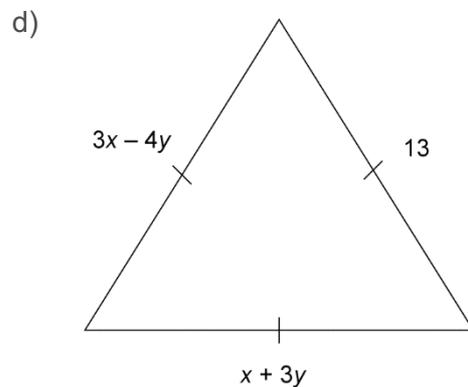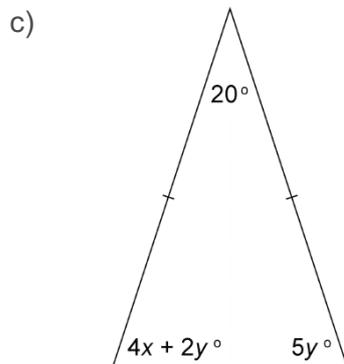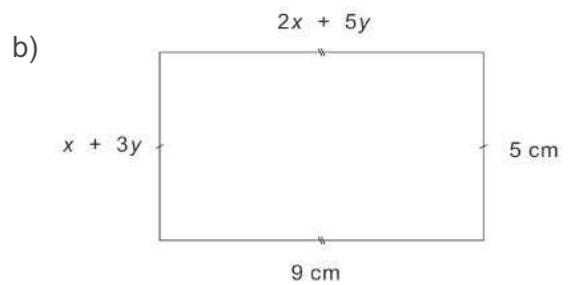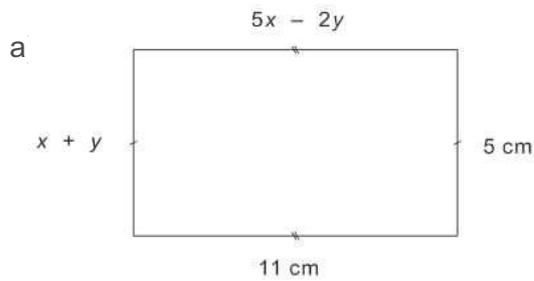
19.   Two car companies offer different rates on the hire of identical cars.

      Company A charges \$15 per day plus \$1.60 per kilometre travelled.
      Company B charges \$40 a day and 0.60c per kilometre travelled.

    a)   If I plan to travel 30 kilometres in a day which, company would give me the
           better deal?

    b)   For what distance travelled will the price of the hiring a car, be identical for
           both companies?

20.   Kate owns a small business. The cost of running the store (rent, electricity, phone
      etc) is \$2500 per fortnight.

      She imports shoes from overseas at a cost of \$6.50 per pair and she sells them in
      her store for \$20.00.

a)   What is the percentage profit Kate makes on the sale of each pair of shoes?
      Give your answer to the nearest whole.

b)   If 'C' represents Kate's running costs and 'p' represents the number of pairs of
      shoes sold;

      i)   Write an equation to show her fortnightly costs.

      ii)   Write an equation to show the revenue generated from the sale of 'p'
              shoes.

c)   Graph both equations from part (b) using the axes below.
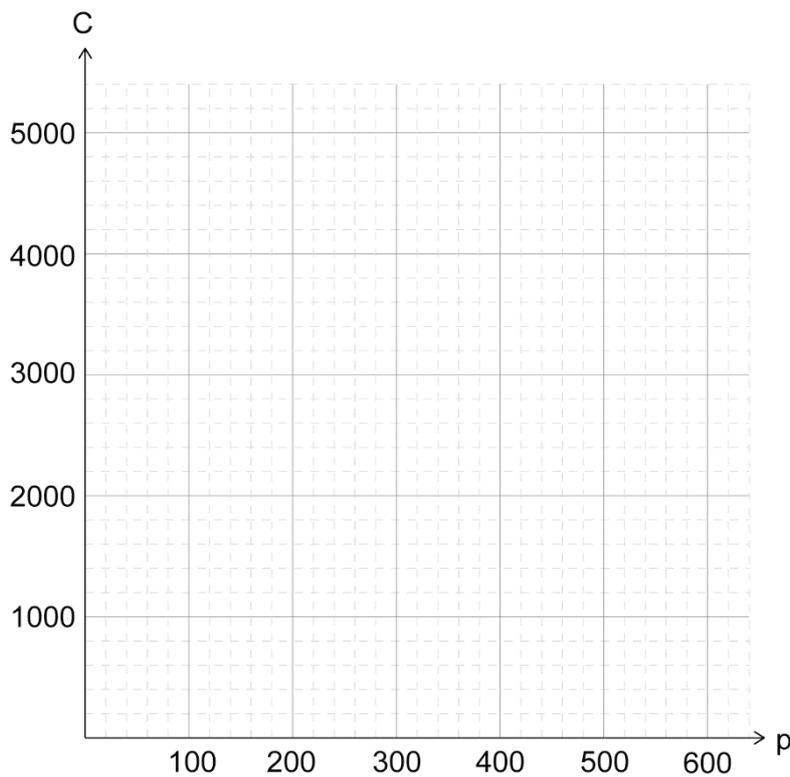
d)   How many pairs of shoes must be sold each week to "break even" i.e. cover
      costs only?

Solutions

1.   a)   Substitute (2,0) into x & y in both equations to check if the point lies on both
             of the lines

$$\begin{Bmatrix} 3(2) + 0 = 6 \\ 2 - 0 = 2 \end{Bmatrix}$$ ∴ Yes (2,0) is a solution

      b) $$\begin{Bmatrix} 5 + 7 = 12 \\ 7 = 5 + 2 \end{Bmatrix}$$ ∴ Yes (5,7) is a solution

      c) $$\begin{Bmatrix} 3(4) - (-2) \neq 12 \\ -2 = 4 - 6 \end{Bmatrix}$$ ∴ No (4,-2) is a not a solution.

2.   a)   (3 , 4)   b)   (1 , -4)

3.
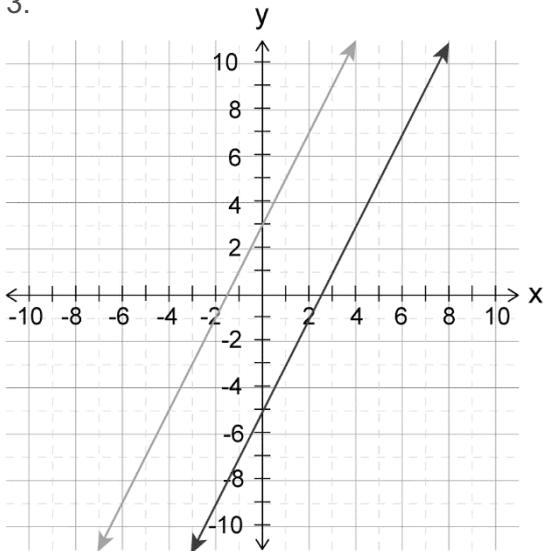
There is no solution as both lines have the
same gradient (parallel) and will never intersect
(cross) each other.

4.

      a)   b)

Point of intersection
(-1 , -6)

Point of intersection
(2 , 3)

Solution is (-1,-6)   Solution is (2,3)

c)

Solution is (4,-1)

d)
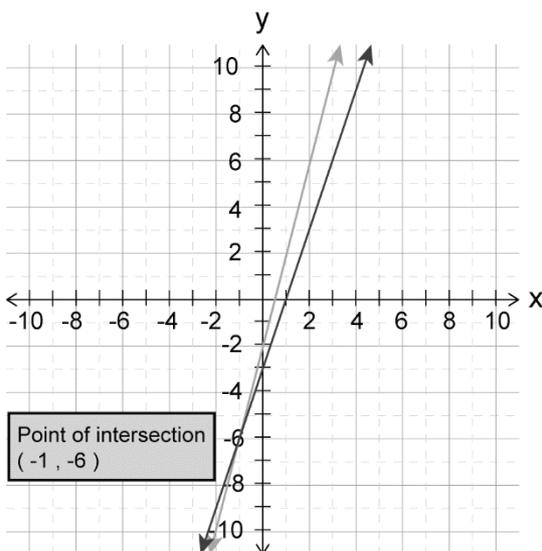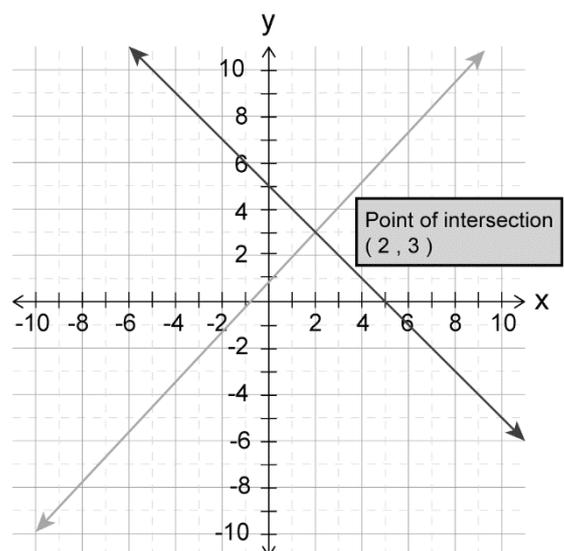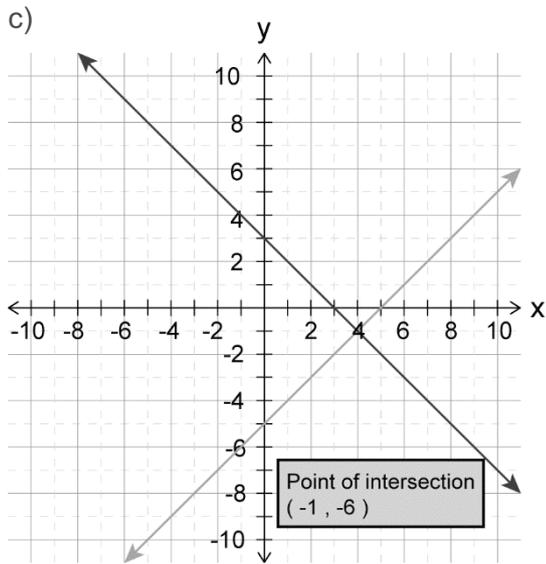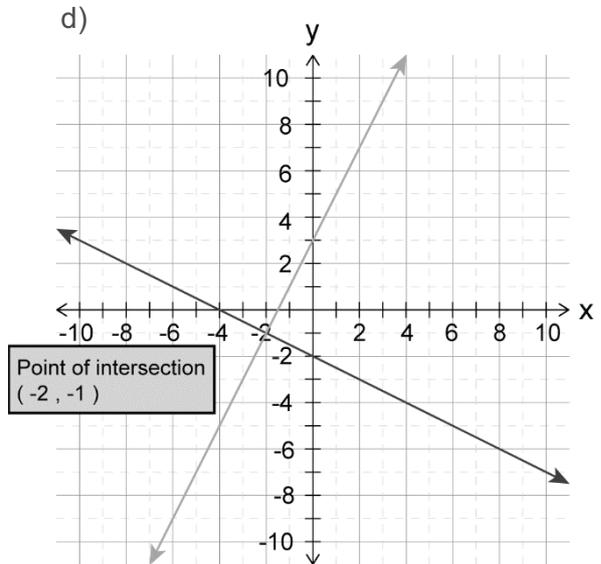
Solution is (-2,-1)

5.   a)   Substitute equation 2 into equation 1 $2x + (3x + 4) = 14$

$$5x = 14 - 4$$

$$5x = 10$$

$$x = 2$$

Substitute $x = 2$ into equation 2 $y = 3(2) + 4 = 10$

Solutions are $x = 2, y = 10$

b)   Substitute equation 1 into equation 2 $3x - 2(3 - x) = -1$

$$3x - 6 + 2x = -1$$

$$5x = -1 + 6$$

$$5x = 5$$

$$x = 1$$

Substitute $x = 2$ into equation 1 $y = 3 - 1 = 2$

Solutions are $x = 1, y = 2$

c)   Substitute equation 1 into equation 2 $3(y - 3) + 2y = 11$

$$3y - 9 + 2y = 11$$

$$5y = 11 + 9$$

$$5y = 20$$

$$y = 4$$

Substitute $y = 4$ into equation 1 $x = 4 - 3 = 1$

Solutions are $x = 1, y = 4$

6. a) [1] + [2]   b) [1] × 4 − [2] × 3

        $6x = 18$ $\underline{y = 13}$ Sub $y = 13$ into [1]

        $\underline{x = 3}$ Sub $x = 3$ into [2] $3x - 2(13) = 7$

        $2(3) + 3y = 15$ $\underline{x = 11}$

        $\underline{y = 3}$

7. $y = 90x$

    $y = 600 + 15x$

    $90x = 600 + 15x$

    $75x = 600$

    $x = 8$

    $\underline{8\ watches\ must\ be\ sold\ to\ break\ even.}$

8. $Brother = x\ and\ Me = y$ ∴ $y - x = 13 \ldots \ldots [1]$

                                          $x + y = 47 \ldots \ldots [2]$

                                          [1] + [2]

                                          $2y = 60$

                                          $y = 30$

    $\underline{I\ am\ 30\ years\ of\ age\ and\ my\ brother\ is\ 17\ years\ of\ age.}$

9. a) Let x represent the amount of money received from her brother.

        Let y represent the amount of money received from her sister.

        $x + y = 146\ (\ rearrange\ to\ \boldsymbol{x = 146 - y}\ )$

        $x - y = 38$

    b) $(\boldsymbol{146 - y}) - y = 38$

        $146 - 2y = 38$

        $-2y = -108$

        $y = 54$ $Substitute\ y = 54\ into\ x = 146 - y$ to get $x = 92$

        $\underline{∴\ Tiffany\ received\ \$92\ from\ her\ brother\ and\ \$54\ from\ her\ sister}$

10.
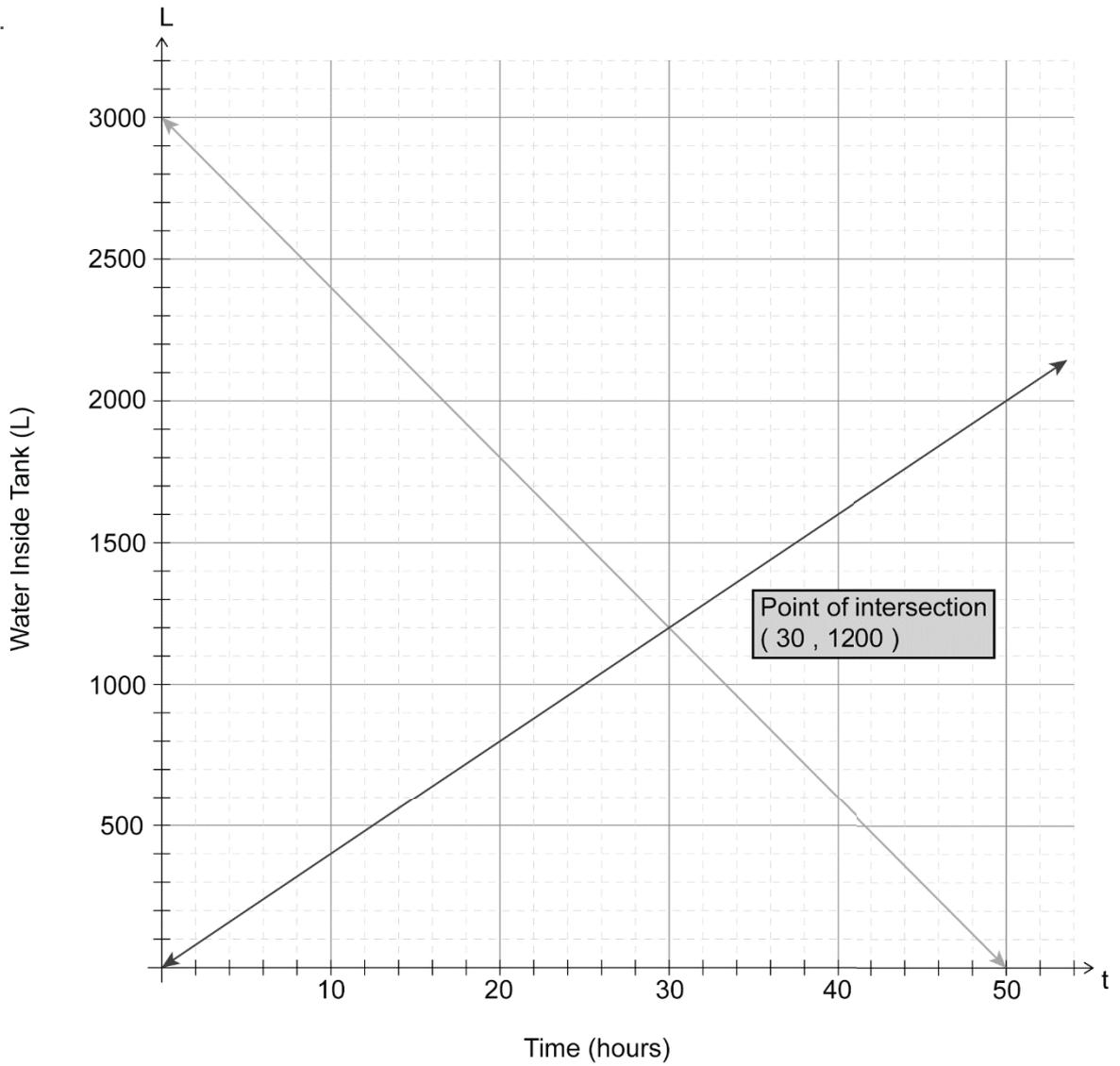
c)   The volume of the two tanks will be identical after 30 hours.

11.   a)   i) $x = 3, y = 0$ ii) $x = 1.48, y = 1.72$
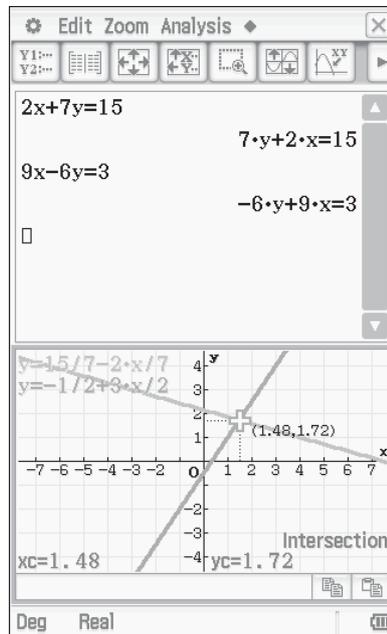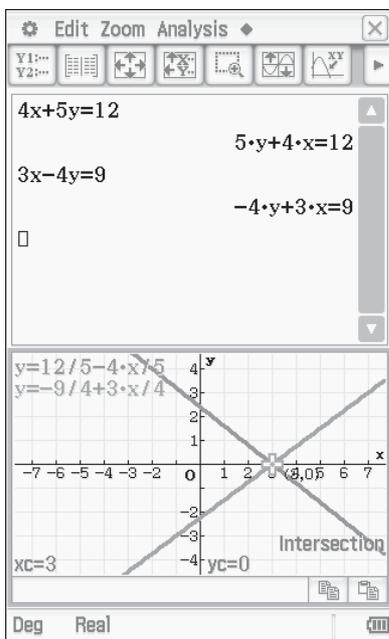
b)   i) $x = -3.8, y = -5.1$

       ii) $x = -3.4, y = 6.6$

$\begin{cases} 3x-4y=9 \\ 4x-2y=-5 \end{cases}\Big|_{x,\,y}$

$\{x=-3.8, y=-5.1\}$

$\begin{cases} 4x+5y=19.4 \\ 5x-2y=-30.2 \end{cases}\Big|_{x,\,y}$

$\{x=-3.4, y=6.6\}$

☐

12.   James $C = 82h$

       Sarah $C = 60 + 70h$

$\begin{cases} C=82h \\ C=60+70h \end{cases}\Big|_{C,\,h}$

$\{C=410, h=5\}$

<u>James and Sarah will charge the same in labour for a 5-hour job.</u>

13. $x + y = 71$
       $8(x + 5) = y + 5$

$\begin{cases} x+y=71 \\ 8(x+5)=y+5 \end{cases}\Big|_{x,\,y}$

$\{x=4, y=67\}$

Currently, Sammy is 4 and his grandmother is 67.

14. $\frac{7.5}{100} \times 18 = 1.35$

       $0.13x + 0.03y = 1.35$

       $x + y = 18$

       $x = 8.1, \quad y = 9.9$

       $\therefore$ <u>8.1 L of 13% solution and 9.9 L of 3% solution</u>

15. $x + y = 18$

       $\frac{x}{(y+7)} = 0.25$

       $x = 5, y = 13$

16.   a)   Let $x = lollypops$ & $y = icy\ poles$

              $20x + 30y = 40$

              $30x + +10y = 25$

              $x = \$0.50$ & $y = \$1.00$

       b)   Let $x = Juice$ & $y = Water$

              $5x + 3y = 17.30$

              $y = x - 0.90$

              $x = \$2.50$ & $y = \$1.60$

17.   a)   Alpha $C = 104 + 10w$

              Omega $C = 12w$

$\begin{cases} C=104+10w \\ C=12w \end{cases}\Big|_{C,\,w}$

$\{C=624, w=52\}$

              <u>Same charge for 52 weeks ($624.00)</u>

       b)   i) $C = 104 + 10(26) = \$364.00$

                      $C = 12(26) = \$312.00$

                      $\therefore$ <u>Omega Training</u>

              ii)   Same charge for 52 weeks (1 year)

                      $\therefore$ <u>Both represent same value for money</u>

iii) $C = 104 + 10(78) = \$884.00$
        $C = 12(78) = \$936.00$
        $\therefore$ <u>Alpha Fitness</u>

18.   a) $x + y = 5$ b) $2x + 5y = 9$
             $5x + 2y = 11$ $x + 3y = 5$
             $x = 3$, $y = 2$ $x = 2$, $y = 1$

       c) $4x + 2y = 80$ d) $3x - 4y = 13$
             $5y = 80$ $x + 3y = 13$
             $x = 12$, $y = 16$ $x = 7$, $y = 2$

```
⌈x+y=5
⌊5x−2y=11│x, y
                        {x=3, y=2}
⌈2x+5y=9
⌊x+3y=5 │x, y
                        {x=2, y=1}
⌈4x+2y=80
⌊5y=80    │x, y
                        {x=12, y=16}
⌈3x−4y=13
⌊x+3y=13 │x, y
                        {x=7, y=2}
□
```

19.   a)   For 30 km, Company B is $5 cheaper

```
|15+1.6*30
                        63
40+0.6*30
                        58
⌈y=15+1.60x
⌊y=40+0.60x│x, y
                        {x=25, y=55}
```

       b)   For 25 km, the price is identical ($55)

20.   a) $\dfrac{13.50}{6.50} \times 100 = 208\%$

       b)   i) $C = 6.5p + 2500$

             ii) $Revenue\ generated = 20p$

       c)
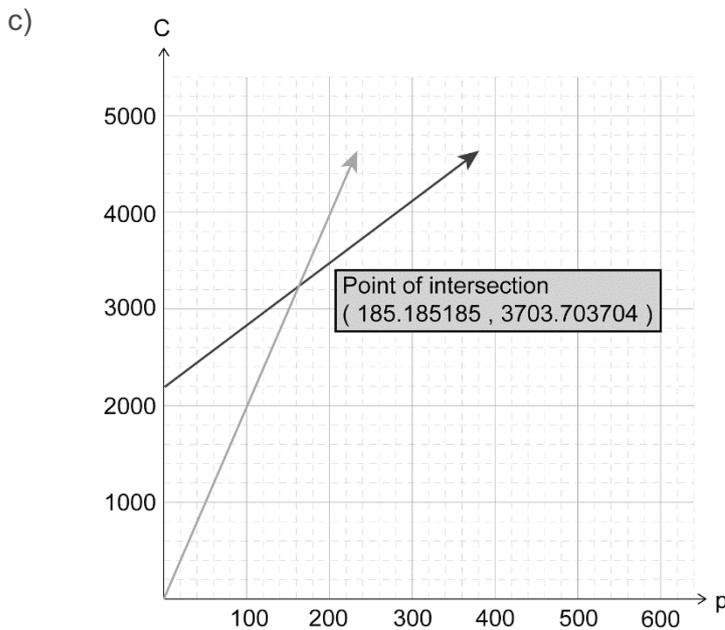
       d)   <u>$p = 186\ (nearest\ whole)$</u>

             <u>Kate has to sell 186 pairs of shoes to break even</u>

Chapter 20 – Piecewise Functions

Inequalities on a Number Line

$-2 \leq x < 3$

$x < -2$

$x \geq 3$
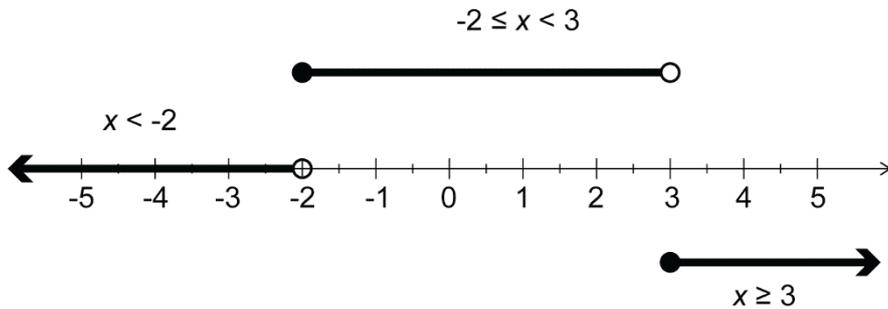
Piecewise Functions and Step Graphs

A piecewise function is a function defined by two or more rules over two or more intervals. The equation for a piecewise function varies as each rule corresponds to different part of the domain (range of x values.).

For example, the cost of peppercorns from a wholesaler varies according to the amount purchased. As such, cost (C) will differ for each interval of the domain (weight - w).

Black Peppercorns

Pricing (up to 10 kg)

$16.00 per kilo for the first 2 kg,

$13.50 per kilo for the next 3 kg,

$16.00 per kilo for the next 5 kg.

C ($)

w (kg)
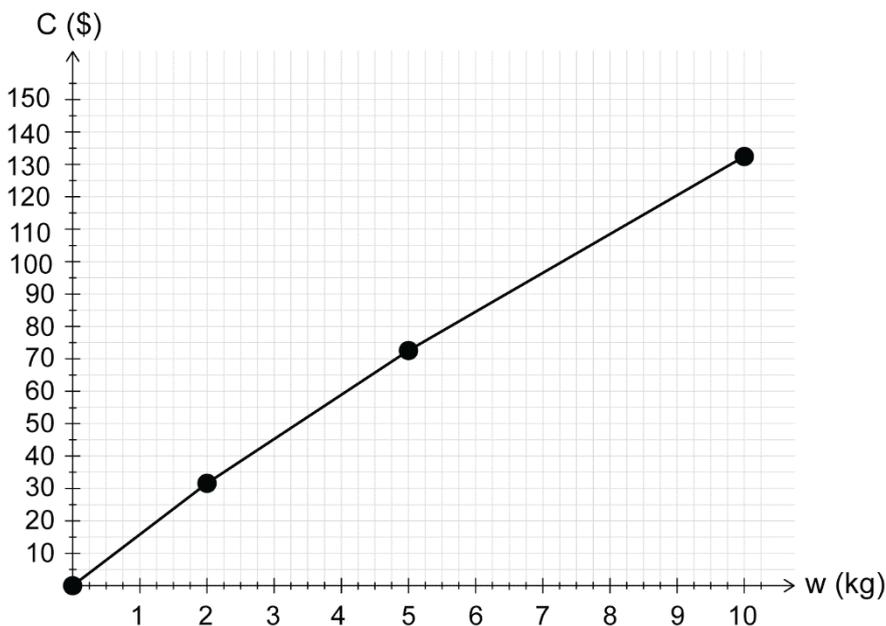
The piecewise function shown can be defined as:

For $0 \leq x \leq 2$ $y = 16x$

For $2 < x \leq 5$ $y = 13.5x + 5$

For $5 < x \leq 10$ $y = 12x + 12.50$

A piecewise function that is constant for each interval of its domain is known as a **step function**. Consider parking charged as a flat fee rather than an hourly or half hourly rate.

Fee ($)

Parking Fees

$10.00 for 0 - 12 hours

$18.00 for 12 - 24 hours
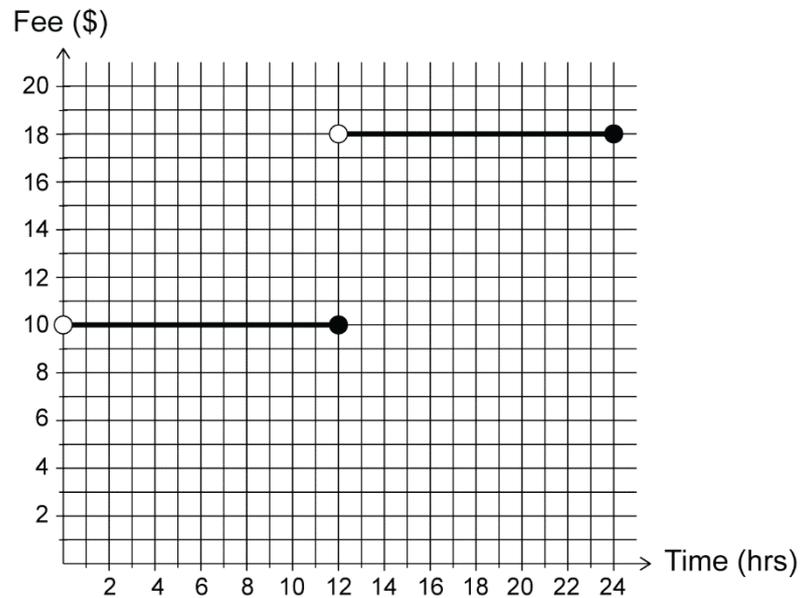
Under this arrangement, the price of parking remains constant for the interval 0 – 12 hours then again for 12 – 24 hour interval.

Resource Free Questions

1.   The entry fee to the annual Agricultural Show is charged according the patron's age.

Ticket Prices	
Age (Years)	Cost
0 – 2	Free
3 – 11	
12 – 17	$10.00
18 – 54	$14.00
55 plus	

Cost ($)
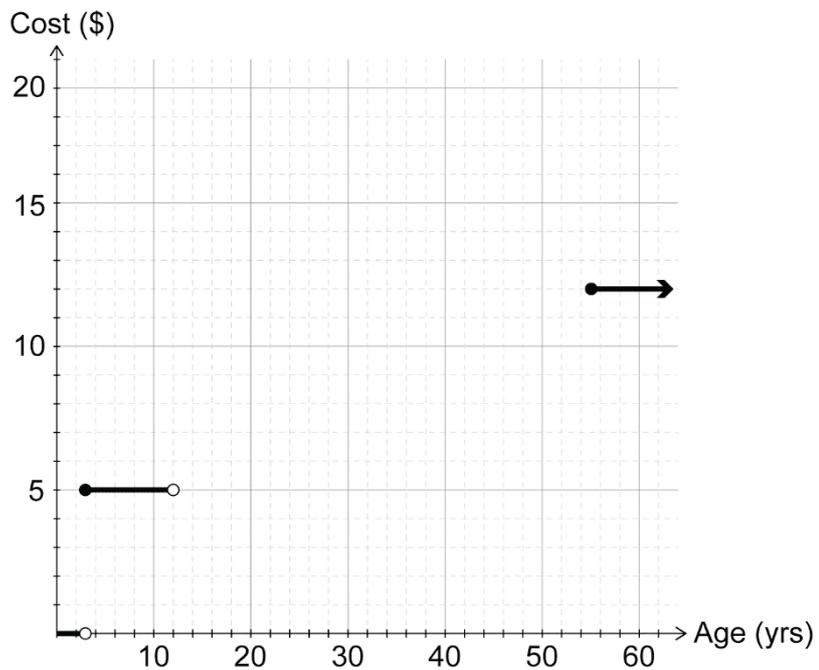
Use the information provided to complete the table and the step graph shown above.

2.   A children's entertainer charges a flat fee for his services plus an hourly rate
      dependent on the length of time

Equipment and Setup Fee	$300
0 to 3 hours	$75 per hour
3 – 6 hours (max)	$50 per hour

a)   What is the cost of hiring the entertainer for 3 hours?

b)   What is the cost hiring the entertainer for 6 hours?

c)   Display this information in the form of piecewise graph using the axes below.

Cost ($)

No. Hours
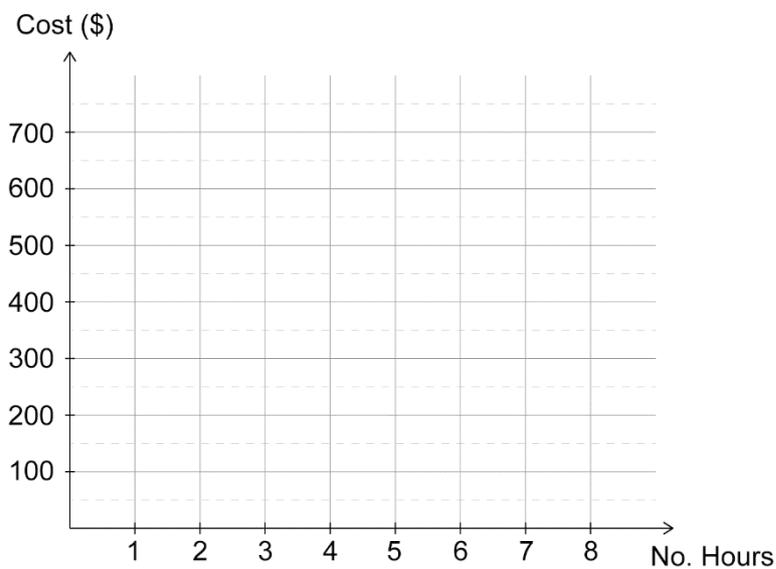
3.   Tom is paid $20 per hour for a standard forty-hour week. He is paid time-and-a-half for his first 5 hours of overtime and double-time for any additional hours. Last week, Tom worked 49 hours. Draw a piecewise graph to show Tom's pay.

Income ($)

```
1200 ┤
     │
1000 ┤
     │
 800 ┤
     │
 600 ┤
     │
 400 ┤
     │
 200 ┤
     │
     └────┬────┬────┬────┬────┬──────→ No. Hours
        10   20   30   40   50
```
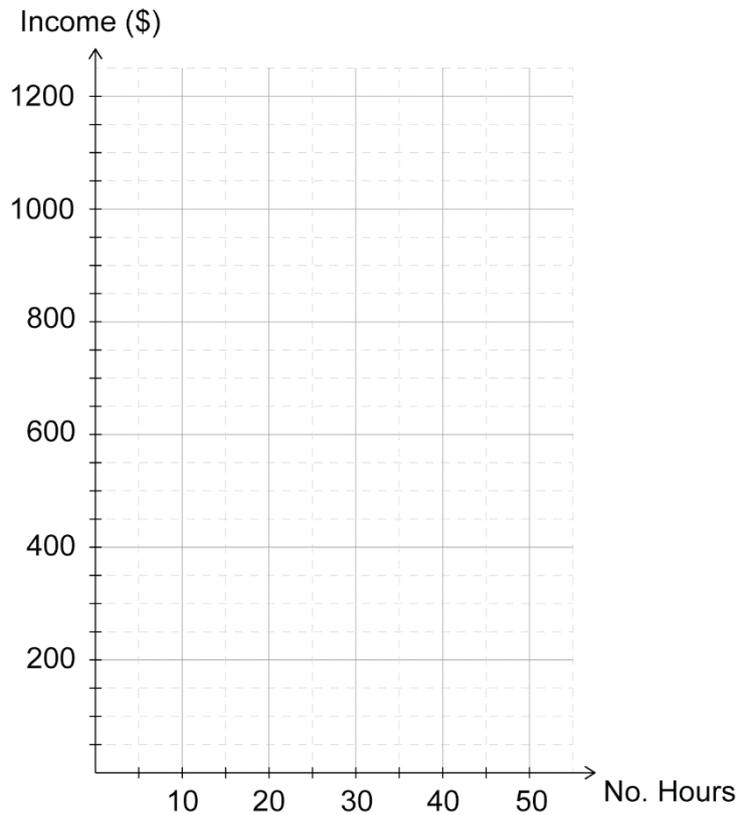
4.   The postal service offers non-denominational concession stamps for senior citizens. The stamps can only be purchased in booklets of five for $3.00 per booklet. The number of stamps for large letters, up to B4 size, is dependent on the weight of the letter.

No. Stamps

```
10 ┤
 9 ┤
 8 ┤                        ○────────●
 7 ┤
 6 ┤
 5 ┤            ○──────●
 4 ┤
 3 ┤    ○────●
 2 ┤●───●
 1 ┤
   └──┬────┬────┬────┬────┬────┬────┬───→ Weight (g)
    100  200  300  400  500  600  700
```
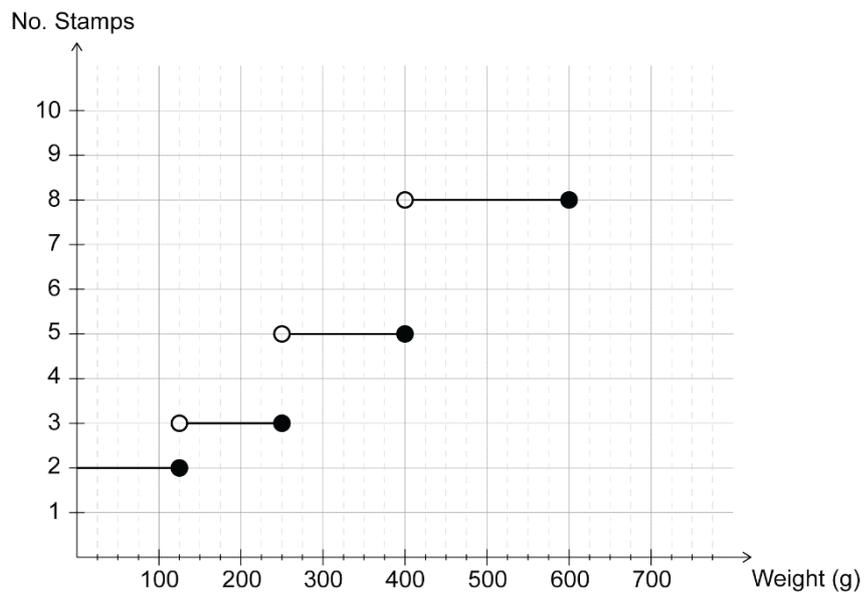
a)   How many stamps are required for a large letter weighing 140 grams?

b)   If Beryl wishes to send a 350 gram letter and a 476 gram letter, how many booklets will she need to buy?

5.   A doctor's surgery charges patients according to the length of the consultation. The step graph below shows the cost of consultations up to $120.

Fee ($)

Time (min)
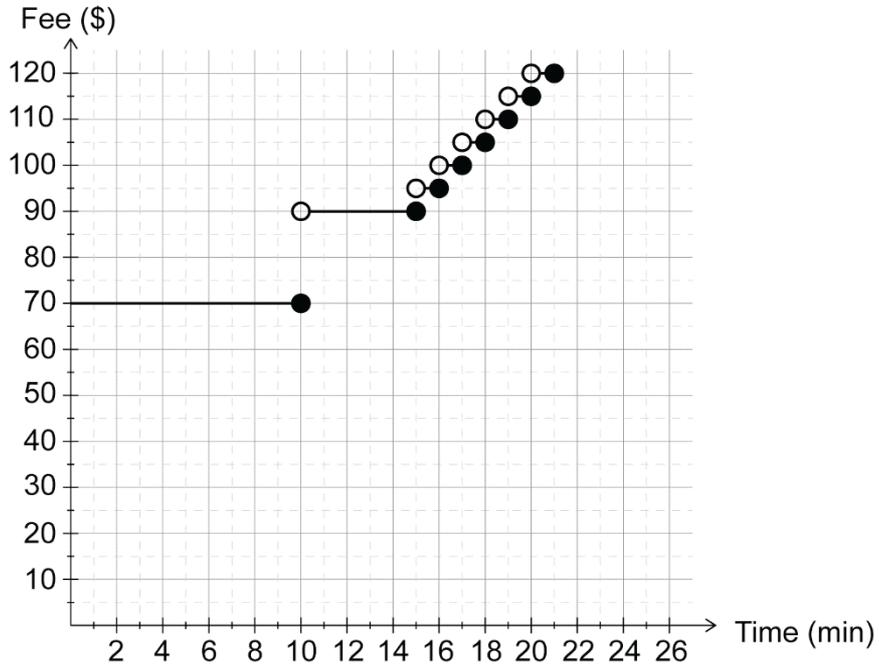
Use the step graph to complete the statements below.

- A consultation of up to 10 minutes cost $(a).

- A consultation over (b) minutes and up to (c) minutes cost $90.

- Over 15 minutes costs $(d) plus $(e) per minute above 15 minutes.

Resource Rich Questions

6.   A travel company has small minibuses that groups of up to 10 people can hire to see sites around WA. The costs, as outlined below, are based on the distance the location is from Perth, not upon the total distance travelled. The maximum distance is 400 km to allow enough time to return to Perth before the end of the day.

Destination Location	Charge
Less than 100 km	$180.00
100 km to less than 280 km	$350.00
280 km to a maximum of 400 km	$580.00

a)   Represent the data above as a step graph using the axes below.
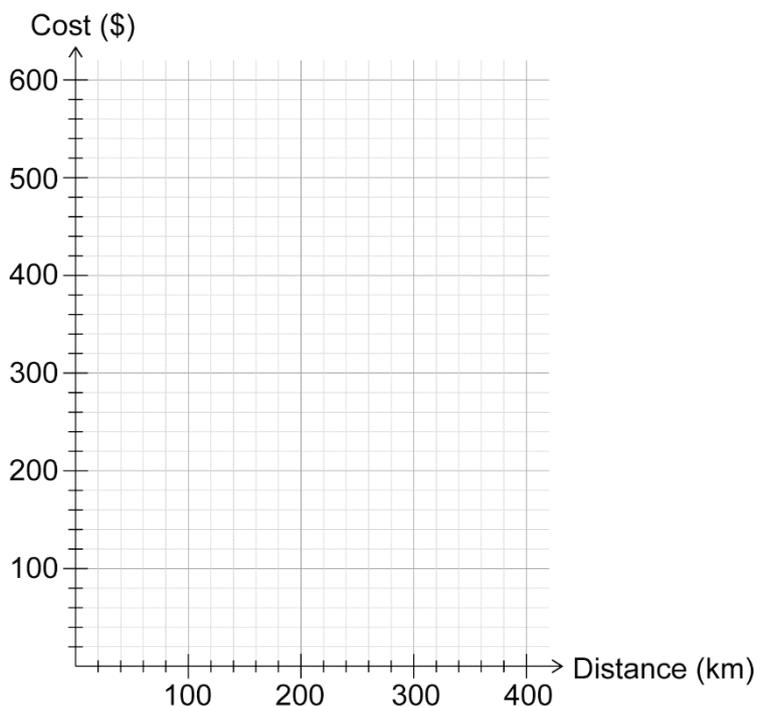
A group of 8 tourists want to visit the tin horses in the regional town of Kulin. It is located 280 km from Perth.

b)   What is the cost of hiring the minibus?

c)   If the costs are split evenly, what will be the cost per passenger?

A rival company charges their fee on the total distance travelled. The cost of hiring a 10-seater bus is charged according to the following formula

$C = \$200 + 0.7d$, where d is the distance of the return trip in km.

The town of Kulin is exactly 280 km by road from Perth.

d)   Which company has the better price for a trip to Kulin? Explain

7.   A discount airline allows up to 25 kg of luggage on its international flights. The baggage cost is charged per kilogram and increases with increasing weights.
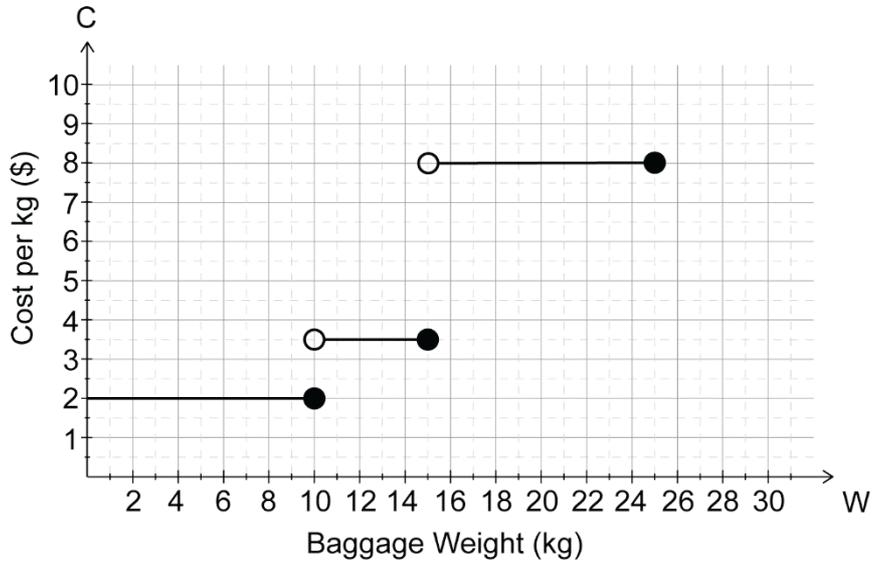
   a)   Determine the total baggage charge of a suitcase weighing 12 kg

   b)   A man travelling with his son has 18 kg of luggage in his suitcase and his son has 2 kg in his suitcase. The man decides to place 3 kg of his luggage into his son's suitcase to save on baggage costs. How much does the man save by doing this?

8.   Consider the graph below. Its shows the distance a cyclist travelled in the first two 2 hours of his training session.
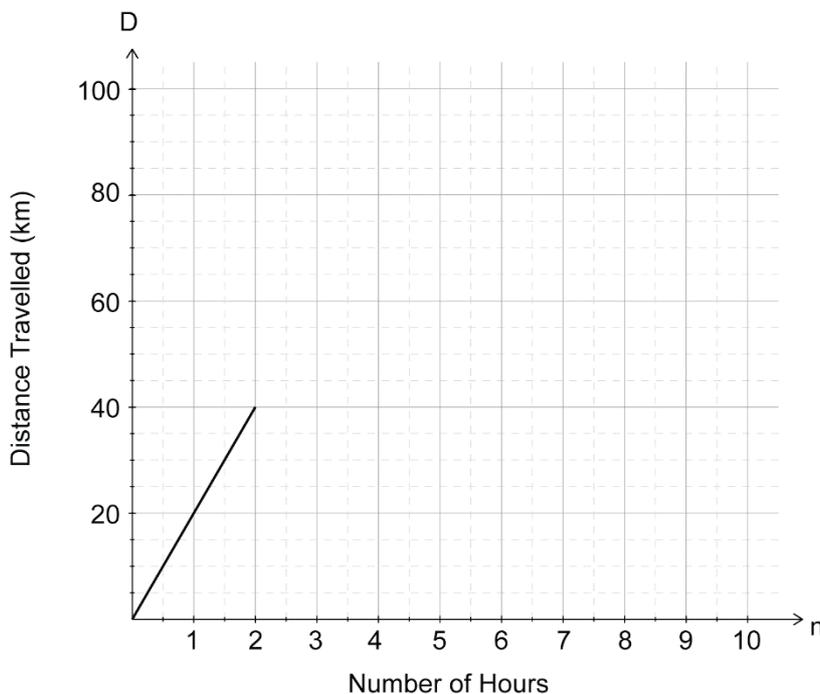
   a)   What is the cyclist's average speed in km/hr in the first two hours of his journey?

   b)   The cyclist then travelled at 15km/hr for the next 3 hours, then 9 km an hour for the next 90 minutes. Show this information on the graph.

9.   The cost, C dollars, charged to supply and deliver x m³ of river stones is given by the equations

$$C = 50 + 40x \quad (0 \le x < 3) \qquad\qquad C = 80 + 30x \quad (3 \le x \le 8)$$

a)   Use an appropriate equation to determine the cost to supply and deliver 2.5 m³ of river stones.

b)   Use the equations to construct a piecewise linear graph for $0 \le x \le 8$
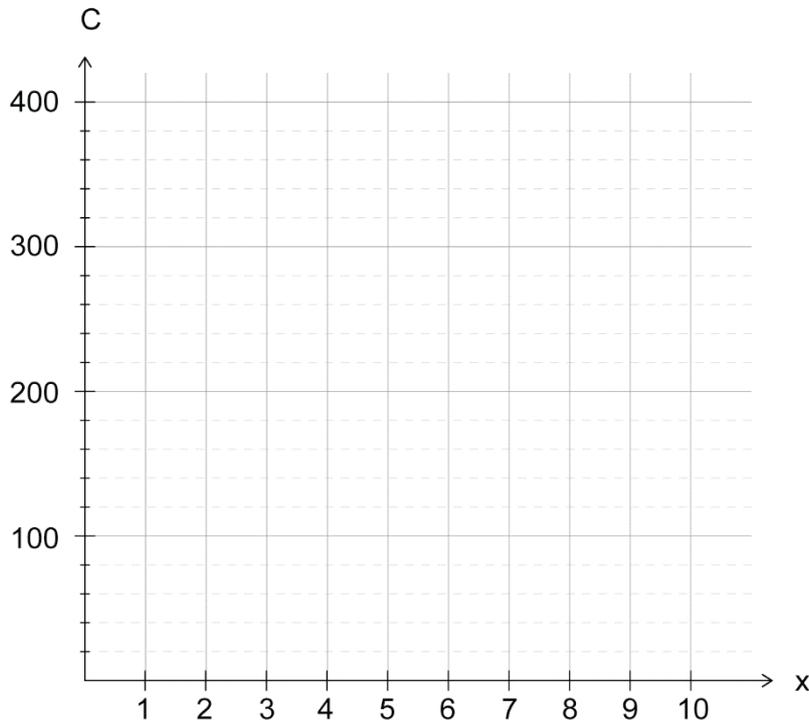
10.   Consider the income tax information shown in the table below.

Taxable income	Tax On This Income
0 – $6, 500	Nil
$6, 501 – $35,000	15% on each $1 over $6,500
$35,001 – $80,000	$3,500 plus 30% on each $1 over $35,000
$80,001 – $180,000	$17,500 plus 35% on each $1 over $80,000
$180,001 and over	$55,000 plus 45% on each $1 over $180,000

a)   Calculate the tax payable on an income of $35 000.

b)   Calculate the tax payable on an income of $80 000.

c)   Calculate the tax payable on an income of $ 180 000.

d)   Draw a piecewise graph to show the tax payable on incomes up to $180 000. (Axes have been provided on the following page.)

Tax ($1000s)
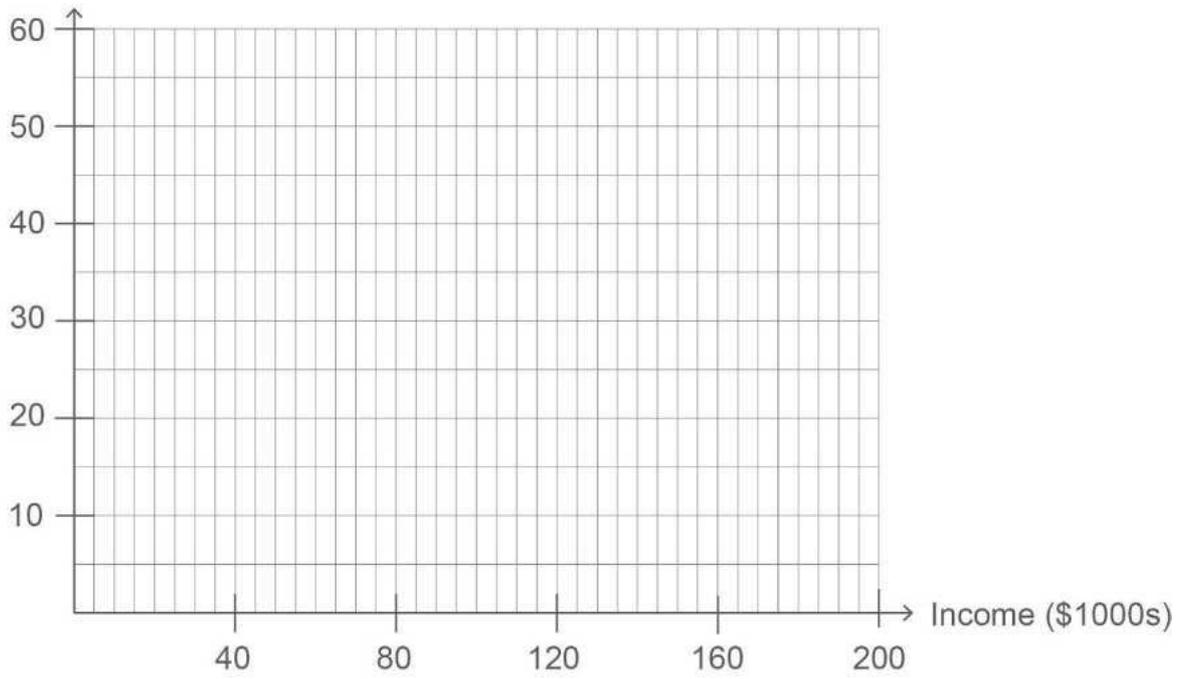
Solutions

1.

Ticket Prices	
Age (Years)	Fee
0 – 2	Free
3 – 11	$5.00
12 – 17	$10.00
18 – 54	$14.00
55 plus	$12.00

2.   a) $525   b) $675

   c)

3.
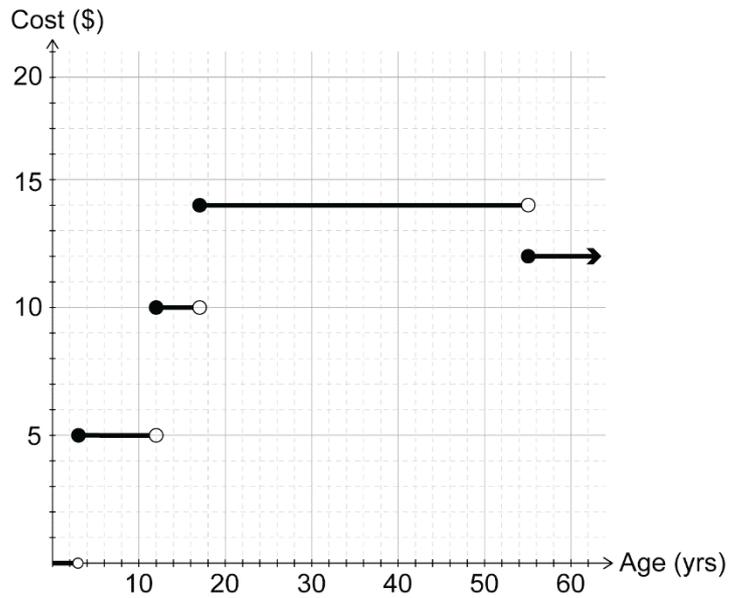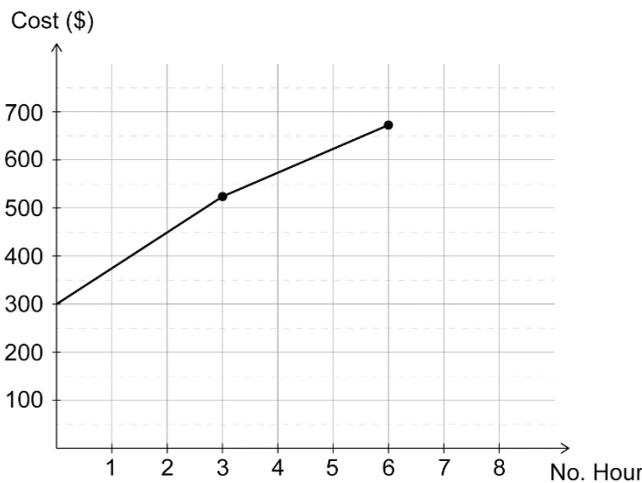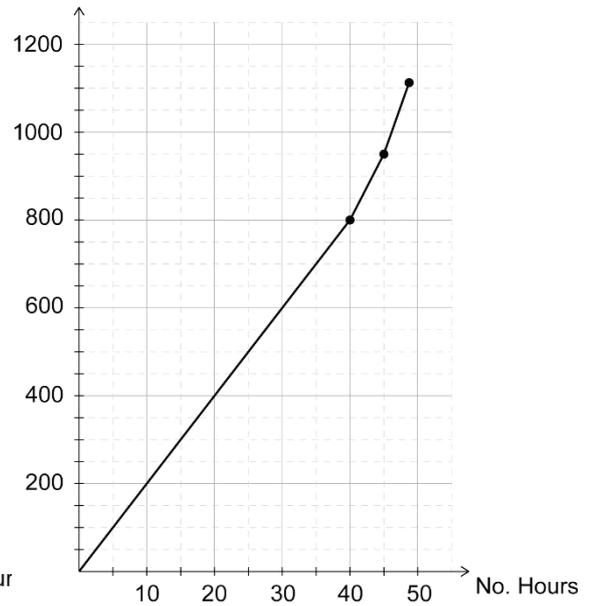
4.   a)   3   b) $5 + 8 = 13$ stamps, which requires 3 booklets of 5 stamps.

5.   a) $70   b)   10   c)   15   d)   90   e)   5

6.   a)   Cost ($)
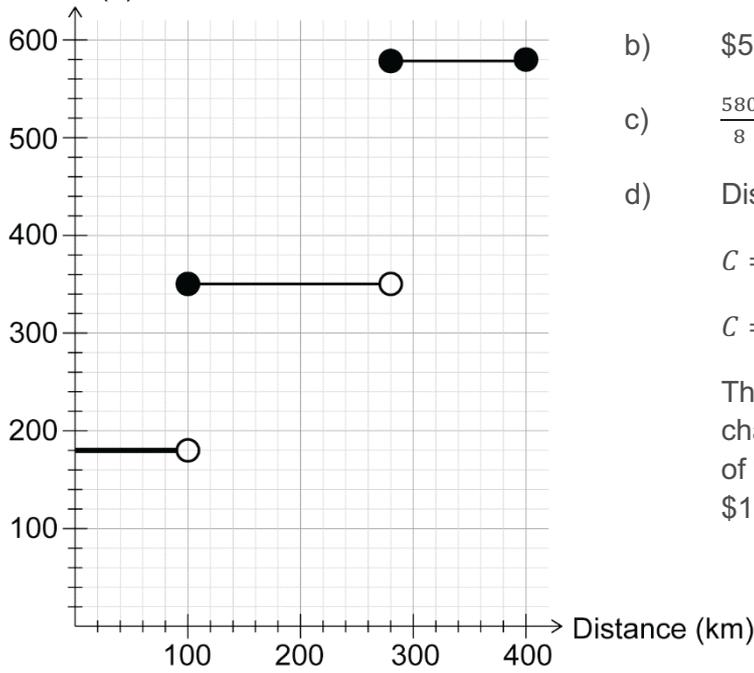
b) $580.00

c) $\frac{580}{8} = 72.50 per person

d)   Distance is $280 \times 2 = 560$ km

$C = 200 + 0.7(560)$

$C = 592.00

The company that
charges the flat fee
of $580.00 is cheaper by
$12.00.

7.   a) $12 \times 3.50 = 42 b) $12 \times 3.5 = 42

$(18 \times 8) + (2 \times 2) = 148.00

$(15 \times 3.5) + (5 \times 2) = 62.50

Saving $85.50

8.   a)   *40 km in 2 hours is equivalent to an average of 20 km/hr*

b)
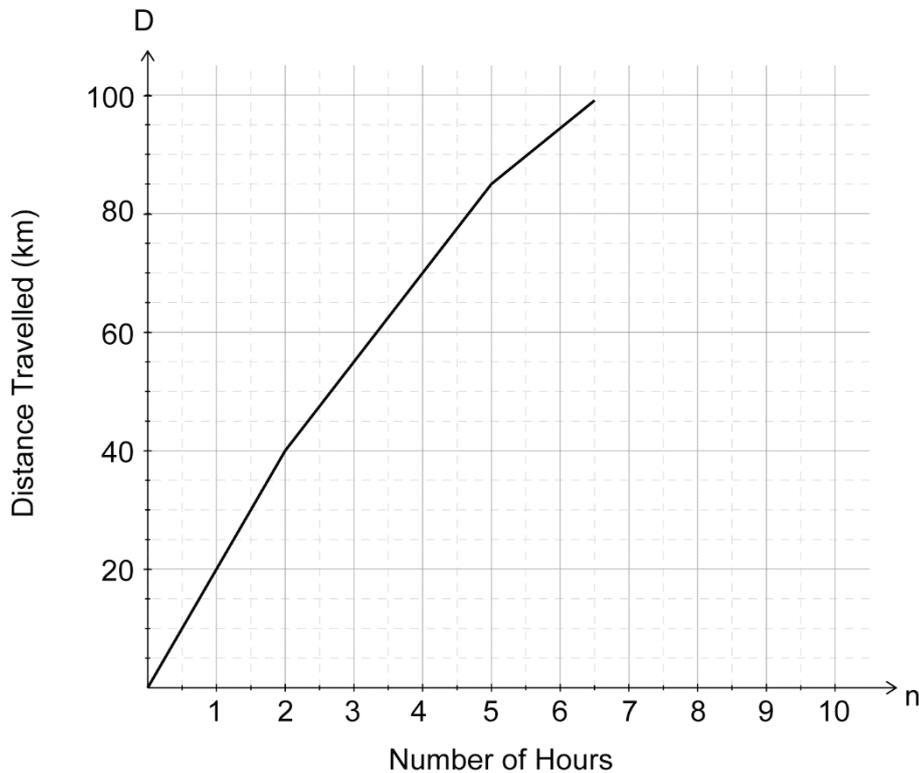
9.   a) $C = 50 + 40(2.5)$ b)   C

      $C = \$150.00$
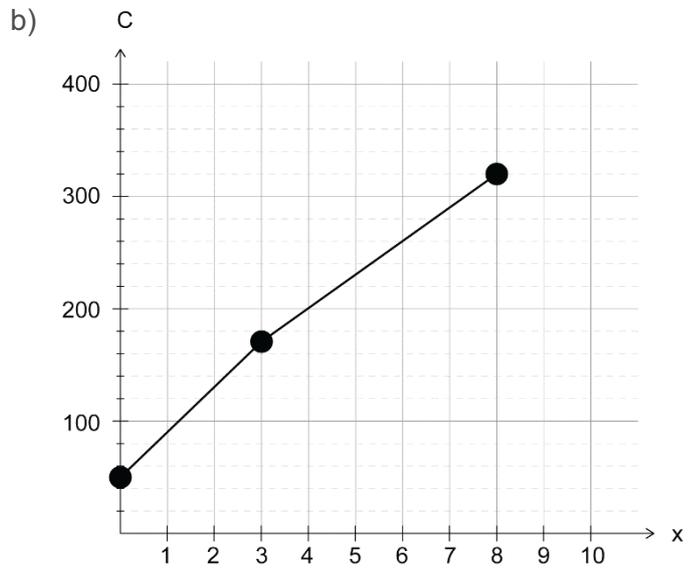
10.   a) $4275   b) $17 000   c) $52 500

       d)

Tax ($1000s)
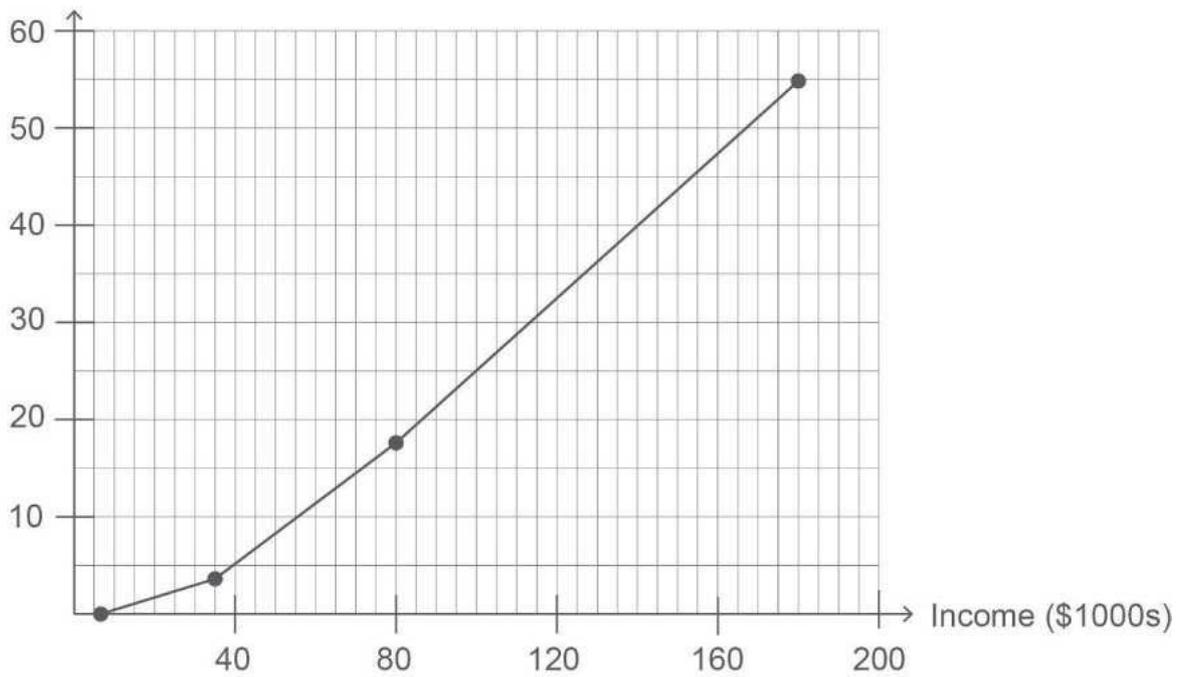
Practice Exam Papers

MATHEMATICS APPLICATIONS

Unit 1

Calculator Free Paper

TIME ALLOWED FOR THIS PAPER

Reading time before commencing work: Five minutes
Working time for paper: Fifty minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
This Question/Answer Booklet
Formula Sheet

TO BE PROVIDED BY THE CANDIDATE
Standard Items: Pens, pencils, eraser or correction fluid, ruler.

IMPORTANT NOTE FOR CANDIDATES:
The standard items listed above and the Calculator–free examination paper are the only items permitted on your table during the Calculator –free examination.

INSTRUCTIONS TO CANDIDATES

This section contains **9** questions.
You are required to attempt **ALL** questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.
Spare pages are available at the back of this booklet. If these extra pages are used, label your answers carefully.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

The examiners recommend that candidates **do not use pencil**, except in diagrams.

STRUCTURE OF THE PAPER

QUESTION	MARKS AVAILABLE
1	6
2	6
3	5
4	4
5	4
6	7
7	6
8	5
9	9

TOTAL MARKS = 52

1. **[6 marks: 2, 2, 2]**

If $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 6 \\ 2 & 4 \end{bmatrix}$

Calculate;

a) $A + B$ b) AB

c) $3A - B$

2. **[6 marks: 4, 2]**

a)   Show that $\Delta ABC \sim \Delta CDE$
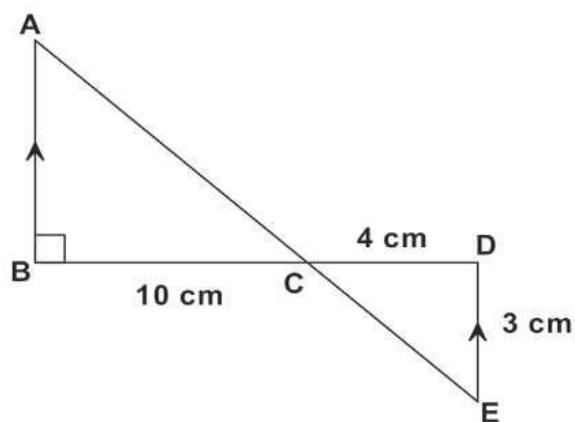
b)   Hence, find the length of AB.

3.   **[5 marks: 1, 2, 2]**

Four employees work in a small call centre for a marketing company. The performance data for each employee for the first week of March is shown below.

> Employee A: 142 calls, 19 sales
> Employee B: 125 calls, 25 sales
> Employee C: 150 calls, 27 sales
> Employee D: 121 calls, 20 sales

a)   Express this data in a 4×2 matrix.

b)   For employee B, express the number of sales as a percentage of the number of phone calls made.

The following matrix shows the performance data of the same four employees for the second week of March.

$$\begin{matrix} & A & B & C & D \\ Calls & \begin{bmatrix} 130 & 106 & 162 & 101 \\ Sales & 14 & 24 & 23 & 17 \end{bmatrix} \end{matrix}$$

This data undergoes the following multiplication;

$$\begin{bmatrix} 130 & 106 & 162 & 101 \\ 14 & 24 & 23 & 17 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

c)   i)   What will be the dimensions of the resulting matrix?

       ii)   What will be the meaning of the values contained in the resulting matrix?

4.   **[4 marks: 2, 2]**

If $\mathbf{V} = \frac{RT}{P}$, calculate the value of;

a) V, when $R = 6, T = 12 \ \& \ P = 8$

b) P, when $V = -2, R = -6 \ \& \ T = 3$

5.   **[4 marks: 1, 1, 2]**

Consider the shape as shown on the grid below.

a)   Draw an enlargement of shape A, scale factor ½ and label it B.

b)   Draw an enlargement of shape A, scale factor 2 and label it C.
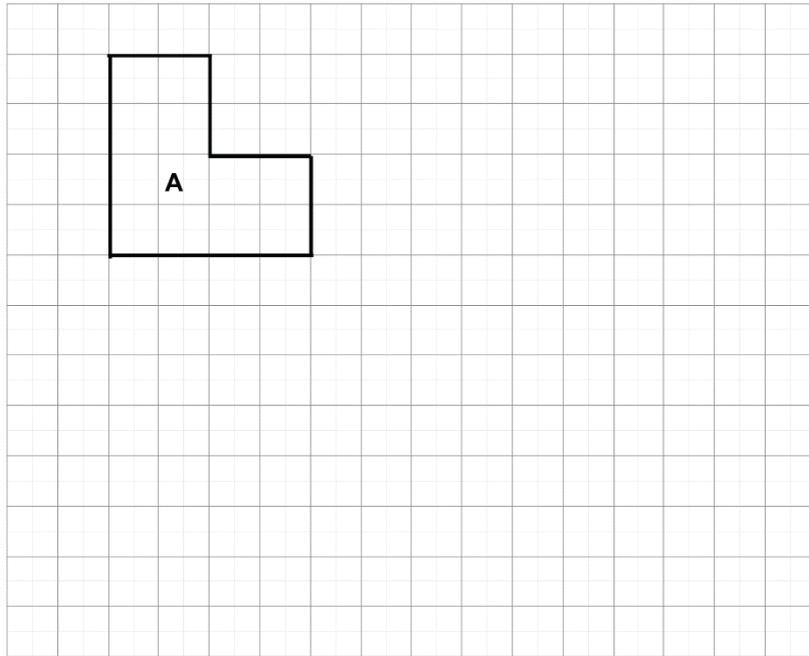
c)   Compare the area of C to the area of B.

6. [7 marks: 2, 5]

The graph below shows the annual interest earned, in dollars, for an amount invested over a 5 year period using both simple interest and compound interest calculations. For both interest methods, interest is calculated annually.
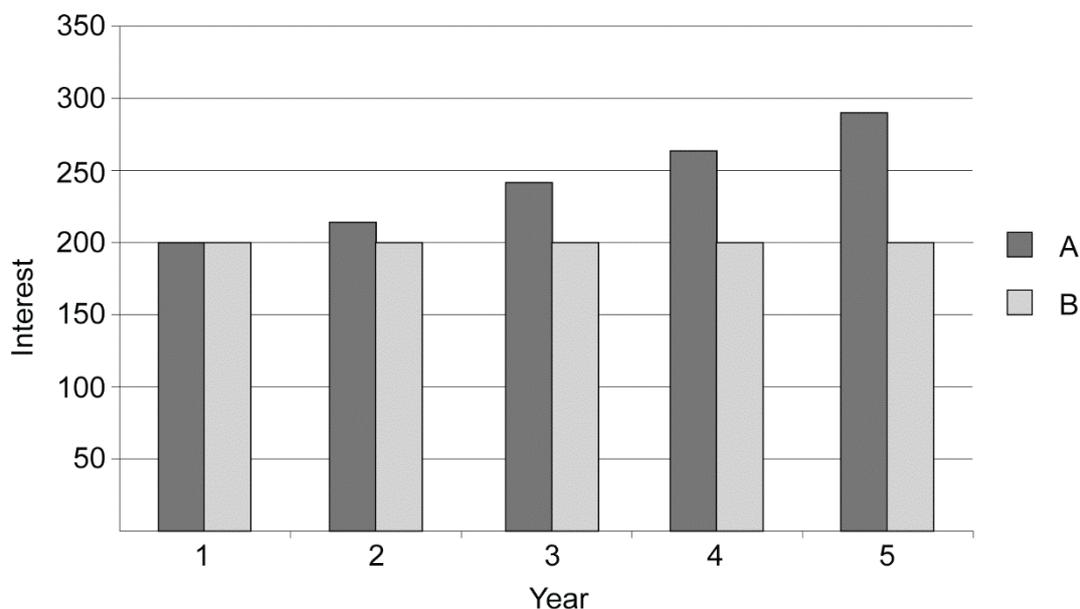
a) Which of the columns, A or B, demonstrate interest earned by the simple interest method? Justify your answer.

b) If the initial investment was $2000, use the graph to determine;

    i) the value of the investment after 3 years, using the simple interest method.

    ii) the approximate value of the investment after 2 years using the compound interest method.

    iii) the annual rate of interest.

7.   [6 marks: 1, 1, 2, 2]

Lizzy obtains a tourist map of the city. She notices that the scale utilised on the map shows that 2 cm represents 300 metres.

a)   If the distance between two tourist attractions is 7 cm on the map, what is the actual distance?

b)   If the church is 1.8 km from the museum, how far apart will these two landmarks be on the map?

Lizzy knows that a week ago, she walked 6 km in 90 minutes.

c)   If she walks at the same pace, how long will it take her to walk 5 km?

d)   Express her average pace in km/hr.

8.   [5 marks: 3, 2]

a)   Solve for the following:

        i)   Find 20% of $350.

        ii)   Increase $350 by 15%.

b)   Belinda buys a medicine ball for $80 and sells it for $56. Express this as a percentage loss.

9.   [9 marks: 2, 2, 3, 2]

a)   A triangle has a perpendicular height of 10 cm and an area of 20 cm². What is
        the length of its base?

b)   Show that the missing length of the triangle below is between 8 and 9 cm.
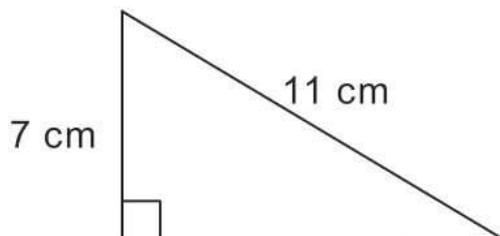
c)   Show that the surface area of the object below is 128π
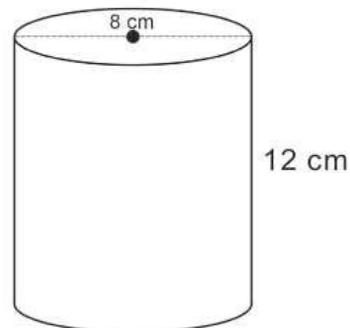
d)   A student comments that "If you double the dimensions of an object, the
        surface area of the object will also double". Is the student correct? Give
        evidence to support or refute this statement.

MATHEMATICS APPLICATIONS UNIT 1

Calculator Assumed Paper

TIME ALLOWED FOR THIS PAPER

Reading time before commencing work: Ten minutes
Working time for paper: One hundred minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
This Question/Answer Booklet.

TO BE PROVIDED BY THE CANDIDATE
Standard Items: Pens, pencils, eraser or correction fluid, ruler.

Special Items: Please refer to the SCSA guidelines in relation to items such as:

- Drawing instruments
- Approved Calculators
- Notes
- Formula Sheets

INSTRUCTIONS TO CANDIDATES

This section contains **11** questions.
You are required to attempt **ALL** questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.
Spare pages are available at the back of this booklet. If these extra pages are used, label your answers carefully.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

The examiners recommend that candidates **do not use pencil**, except in diagrams.

STRUCTURE OF THE PAPER

QUESTION	MARKS AVAILABLE
10	7
11	8
12	12
13	9
14	8
15	9
16	14
17	9
18	8
19	8
20	6

TOTAL MARKS = 98

10.   [7 marks: 1, 1, 2, 3]

On July 1st, Jane decided to invest the $20 000 inheritance left to her by her grandmother.
Jane purchased 1200 shares for a total of $10 368.
The remaining amount was placed into an investment account earning 3.8% per
annum compounded monthly.

a)   What is the total price per share?

b)   In the previous twelve months, the earnings were $1.08 per share.
        Determine the P/E ratio.

c)   One year later, each share was now worth $10.80 a unit. Express this increase as a
        percentage of Jane's purchase price.

d)   How much interest will Jane earn from placing her money into the investment
        account for one year?

11.   [8 marks:3, 2, 3]

Consider the square-based pyramid shown below. It has a base length of 7 feet (ft) and each triangular face has a perpendicular height of 6.5 feet.
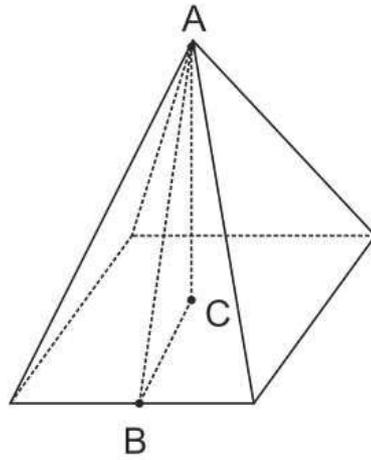
a)   Use triangle ACB to find the height of the pyramid to three decimal place accuracy.

b)   Use your answer to part (a) to calculate the volume of the pyramid in cubic feet.

c)   If 12ft = 365.76 cm, convert your answer to part (b) into cm³.

12.   [12 marks: 3, 2, 2, 4, 1]

Consider the triangle shown below.
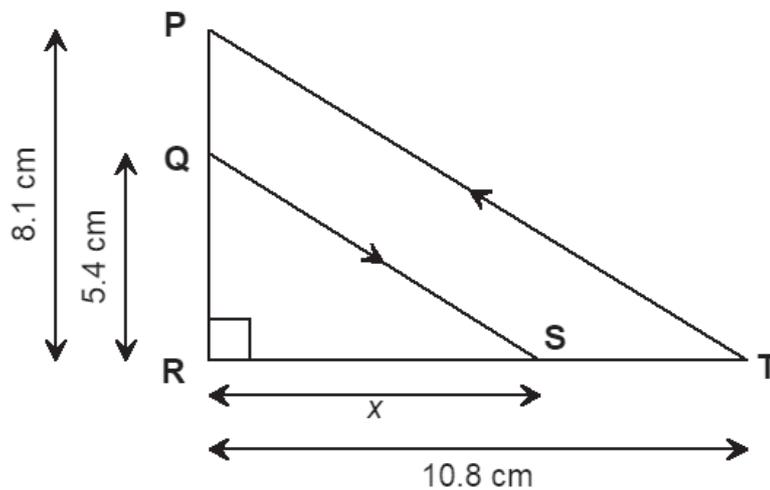
a)   Show that ΔPRT and ΔQRS are similar triangles.

b)   Calculate the value of x.

c)   Calculate the area of the trapezium PTSQ.

Question 12 continued

d)   What is the difference in between the length of the two parallel lines QS and PT?

e)   The area of a trapezium can be determined using the formula $A = \frac{h}{2}(a + b)$; where a and b are lengths of the two parallel sides and h is the perpendicular height of the trapezium. Use this formula to determine the perpendicular height of the trapezium $PTSQ$.

13.   [9 marks:1, 1, 3, 3, 1]

The volume of the prism ABCDEFGH below is 67.2 cm³. The front face, ADCB, is a parallelogram and is also shown below.
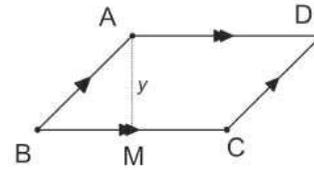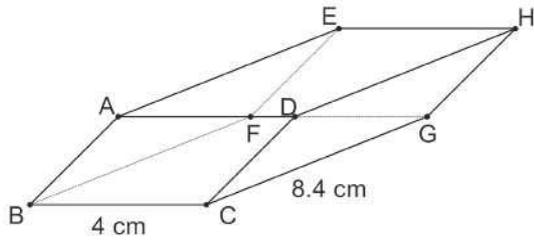
a)   Determine the area of the parallelogram ADCB.

b)   Determine value of y, the perpendicular height of the parallelogram.

Point M is a midpoint, i.e. it is exactly halfway between B and C.

c)   i)   Determine the length of BM.

        ii)   Determine the length of side AB. Give your answer to two decimal place accuracy.

d)   Determine the surface area of the prism ABCDEFGH.

e)   The prism shown below is an enlargement of the prism shown above. Determine the scale factor.
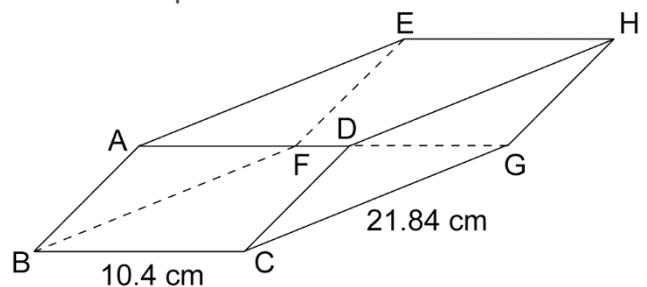
14.   **[8 marks:2, 2, 1, 3]**

An open-ended cylinder is formed when a rectangular piece of paper is joined at its <u>two shorter sides</u> with no overlap. If the piece of paper is 44 cm by 18 cm;
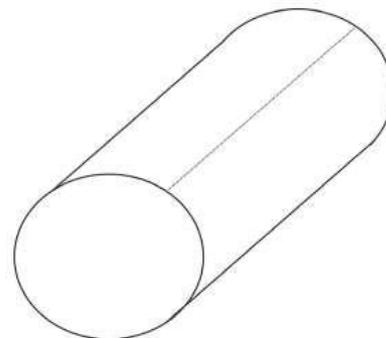
a)   Show that length of the radius is approximately 7 cm.

b)   Calculate the surface area of this open-ended cylinder

c)   Give your answer to (b) in mm^2

d)   Michael has a box that is deep enough to hold the cylinders standing on their circular ends with just enough room to close the lid and not damage them. The box has a length of 50 centimetres and a width of 30 centimetres.

He determines the area of the rectangular opening and divides it by the area of the circular end of one cylinder.

i)   According to this calculation, how many cylinders can the box hold?

ii)   Explain why this is not a valid way to determine the number of cylinders that can fit into the box.

15.   [9 marks: 2, 2, 3, 2]

Kevin earns $32 an hour, working for a plumbing company. His normal working hours are forty hours per week. He occasionally works weekends where he earns double time.

Last financial year, Kevin worked for 48 weeks at regular pay and put in 188 hours of work over the weekends. For each of his four weeks of holidays, he was paid the equivalent of a normal working week.

a)   Calculate Kevin's gross pay for the financial year.

Consider the tax table shown below.

Taxable income	Tax On This Income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

b)   Calculate the tax payable on Kevin's gross income.

c)   This finanacial year, Kevin's wife paid $29 572 in tax. What is her taxable income?

Question 15 continued

Kevin estimates his grocery bills to be $650 per fortnight and fuel costs of $55 per week. His utility bills average $410 per quarter.

d)   Express his estimated living costs as a percentage of his annual net (after tax) income.

.

Question 15 continued

Kevin estimates his grocery bills to be $650 per fortnight and fuel costs of $55 per week. His utility bills average $410 per quarter.

16.   **[14 marks: 2, 2, 3, 3, 2, 2]**

For the half time show at the AFL grand final, event organisers have come up with a design for a stage to be wheeled out to the centre of the field. To make transport easier, the stage has been broken up into 4 sections that fit together to form the stage. The design is shown below.
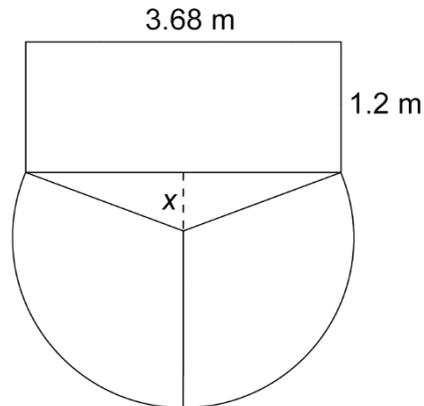
The front section consists of two segments each with internal angles of 113° and a radius of 2 m.

a)   Calculate the area of one of these segments.

b)   Show that the perpendicular height of the triangular section, x, is 0.78 m to two decimal place accuaracy.

c)   What is the area of the entire stage?

d)   If the stage is 1.5 metres high, what is the surface area of the stage when all the sections are connected excluding the underside of the stage?

Question 16 continued

e)   If a designer quotes $26.50 per square metre to paint and decorate all visible surfaces of the stage area, what is the total cost of decorating the stage?

f)   When quoting her fees, the designer forgot to add GST. What is the total cost of the designer's services plus GST?

17.   [9 marks: 2, 1, 3, 1, 2]

At the community fair, there are three attractions in sideshow alley. The clowns cost $7, the bumper cars cost $11 and the petting zoo costs $6. On Saturday, 94 kids played the clowns, 85 rode the bumper cars and 114 visited the petting zoo. On Sunday, 117 played the clowns, 92 rode the bumper cars and 156 visited the petting zoo.

a)   Give the breakdown of people visiting each attraction as a 2×3 matrix. Label this matrix A.

b)   Write the costs in the form of a 3×1 cost matrix. Label this matrix B.

c)   Calculate AB. Interpret the meaning of the values in AB.

d)   What is the total income for the 3 attractions in sideshow alley over the weekend period?

e)   Compare the total incomes over the two day period. Calculate the percentage increase from Saturday to Sunday.

18.   [8 marks: 1, 2, 2, 3]

A person's Body Mass Index, BMI, is determined using the formula

$$B = \frac{m}{h^2}$$

Where; B is the patient's BMI,
               m is the mass of the person in kilograms, and
               h is the patient's height in metres.

a)   Calculate the BMI for a patient who is 74 kg and 1.75 metres tall

b)   Calculate the BMI for a patient who is 53 kg and 152 centimetres tall.

A patient is considered to be healthy if their BMI falls between 21.5 and 24.5.

c)   If a patient has a BMI of 23.8 and is 1.83 metres tall, what is the patient's
       weight?

d)   If the same patient's body weight increases by 6%, will the patient's BMI still
       fall within the range considered healthy? Give evidence for your answer.

19.   [8 marks: 4, 1, 1, 2]

Three friends each from Australia, are holidaying in Hong Kong.

- Anna exchanged 250 Australian Dollars (AUD) for 1825 Hong Kong Dollars (HKD)

- Belinda exchanged 560AUD for 4060HKD.

- Carrie exchanged 400AUD for 2912HKD.

a)   Which of the three friends received the best exchange rate? Give evidence to support your answer.

b)   The friend who received the best rate exchanged more of her Australian Dollars the following day. She received a total of 2336HKG. Assuming she received the same rate as the previous day, what was the total in AUD that she exchanged?

c)   After two weeks, the three friends fly home with a 14 hour stopover in Singapore where the currency is known as Singaporean Dollars (SGD) Belinda still has 460 HKD remaining. If 1SGD buys 6.25HKD, calculate the equivalent value in Singaporean Dollars.

d)   Belinda notices that the exchange rate of 1SGD buys 6.25HKG offered by one company is 1.2% higher than that offered by a rival company. If someone exchanged 125SGD, what is the difference in HKD between the two currency exchange companies? Give your answer to the nearest cent.

20.   [6 marks: 2, 2, 2]

Consider the situation below.
A large rectangular block has a "pill shaped" concreted area in the middle of the
block. Local residents have nicknamed it "The Pill" and many like to use the edge of
the concreted area as running track.
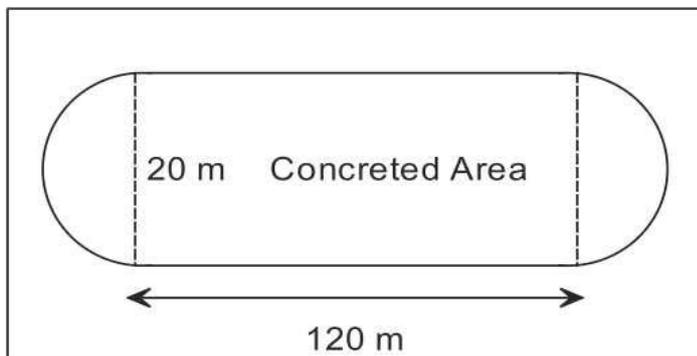
a)   If a person were to run around the edge of the concreted area, what would be
        the total distance covered by one lap?

Over time, the concreted area has become stained and unsightly. The council
considers painting or replacing the concrete pad.

b)   The local council decided to paint the surface of the "The Pill". What is the
        total area of concrete that will need to be painted?

c)   If the concrete pad is to be 20 cm thick, what is the total volume of concrete
        required?

Resource Free Exam Solutions

1. a) $A + B = \begin{bmatrix} 2+1 & 4+6 \\ 3+2 & 5+4 \end{bmatrix}$ b) $AB = \begin{bmatrix} 2 \times 1 + 4 \times 2 & 2 \times 6 + 4 \times 4 \\ 3 \times 1 + 5 \times 2 & 3 \times 6 + 5 \times 4 \end{bmatrix}$

    $A + B = \begin{bmatrix} 3 & 10 \\ 5 & 9 \end{bmatrix}$ ✓✓ $AB = \begin{bmatrix} 10 & 28 \\ 13 & 38 \end{bmatrix}$ ✓✓

    c) i) 2×1 ✓

        ii)   *Total calls and total sales for all employees*   ✓

2. a) $\angle ACB = \angle ECD$ *(vertically opposite angles)*   ✓
        $\angle ABC = \angle EDC$ *(alternate angles)*   ✓
        $\angle BAC = \angle DEC$ *(alternate angles)*   ✓

        $\therefore \Delta ABC \sim \Delta CDE\ (AAA)$ ✓

    b) $\dfrac{BC}{CD} = \dfrac{10}{4} = 2.5$ ✓
        $AB = 2.5 \times 3$
        $AB = 7.5cm$ ✓

3. a) $\begin{bmatrix} 142 & 19 \\ 125 & 25 \\ 150 & 27 \\ 121 & 20 \end{bmatrix}$ ✓   b) $\dfrac{25}{125} \times 100 = 20\%$ ✓✓

    c) $\dfrac{17-20}{20} \times 100 = -15\ \%$ ✓

        15% *decrease in sales from Week 1 to Week 2* ✓

4. a) $V = \dfrac{6 \times 12}{8} = 9$ ✓   b) $-2 = \dfrac{-6 \times 3}{P}$ ✓

        $V = 9$ ✓ $P = \dfrac{-18}{-2} = 9$ ✓

5. a) & b)
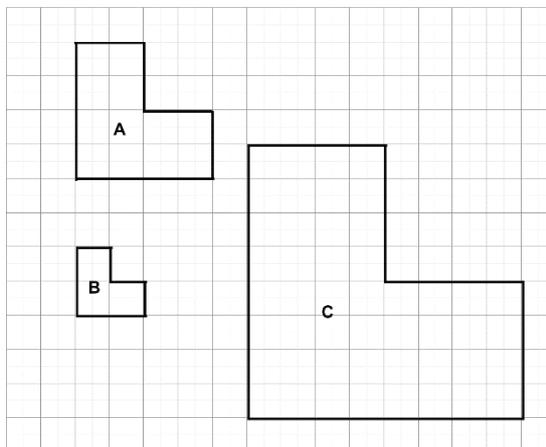
    c) The area of BC is $2^2 \div 0.5^2 = 16$ times the area of C. ✓✓

6. a)   <u>Column B</u> demonstrates the simple interest earned.   ✓
         The amount earned remaings constant at $200 each year. ✓

    b)   i) $2000 + 3 \times 200 = \$2600$ ✓
         ii) $2000 + 200 + 220 = \$2420$ ✓✓
         iii) $\frac{200}{2000} \times 100 = 10\%$ ✓

7. a) $\frac{300}{2} = \frac{x}{7}$ $\therefore$ $\frac{7 \times 300}{2} = \frac{x}{1}$ $\therefore$ $x = 1050m$ ✓

    b) $\frac{300}{2} = \frac{1800}{x}$ $\therefore$ $\frac{x}{1} = \frac{1800 \times 2}{300}$ $\therefore$ $x = 12cm$ ✓

    c) $\frac{6}{90} = \frac{5}{x}$ ✓

         $\therefore$ $\frac{x}{1} = \frac{5 \times 90}{6}$ $\therefore$ $x = 75mins$ ✓

    d) $speed = \frac{distance\ (km)}{time(hrs)} = \frac{6}{1.5} = 4$ km per hour ✓✓

8. a)   i) $20\%\ of\ 350 = 2(10\%) = 2(35) = \$70.00$ ✓

         ii) $\$350 + \$35 + \$17.50 = \$402.50$ ✓✓

    b) $80 - 56 = loss\ of\ 24$ ✓ $\therefore$ $\frac{24}{80} = \frac{3}{10} = 30\%\ loss$ ✓

9. a) $Area = \frac{Base \times p.height}{2}$ $\therefore$ $20 = \frac{b \times 10}{2}$ $\therefore b = \frac{20 \times 2}{10} = 4cm$ ✓✓

    b) $11^2 = 7^2 + x^2$
         $x^2 = 11^2 - 7^2$
         $x = \sqrt{72}cm$ ✓

         $\sqrt{64} = 8\ \&\ \sqrt{81} = 9 \therefore \sqrt{72}\ cm\ is\ between\ 8\ \&\ 9$

    c) $Surface\ Area = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h = 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 12$
         $Surface\ Area = 2 \times 16 \times \pi + 8 \times 12 \times \pi = 32\pi + 96\pi$ ✓✓

    d)   If we take a rectangle with dimensions of 3cm x 4cm it's area = 12cm²
         If we double its dimensions, ie. 6cm x 8cm shape it's area = 48cm²
         <u>$48 \div 12 = 4$ $\therefore$ the area is 4 times larger so student's claim is false</u>   ✓

Calculator Assumed Solutions

10. a) $\frac{10368}{1200} = \$8.64$ ✓   b) $\frac{8.64}{1.08} = 8$ ✓

c) $\frac{10.8}{8.64} \times 100 = 125$ ✓   d) $A = 9632\left(1 + \frac{0.038}{12}\right)^{12}$ ✓

∴ 125%   ✓ $A = \$10004.46$ ✓

$I = \$10004.46 - 9632$

$I = \$372.46$ ✓

11.

a) $h = \sqrt{6.5^2 - 3.5^2}$ ✓✓

$h = 5.477 \, ft \, (3 \, dp.)$ ✓

b) $Volume = \frac{1}{3} \times area \, of \, base \times p.\,height$

$Volume = \frac{1}{3} \times 7^2 \times 5.477$ ✓

$Volume = 89.458 \, ft^3$ ✓

c) $1ft = \frac{365.76}{12} = 30.48 \, cm$ ✓

$Volume = 89.458 \times 30.48^3$ ✓

$Volume = 2\,533\,168.462 \, cm^3$ ✓

12. a) $\angle QRS = \angle PRT$ common angle   ✓

$\angle QSR = \angle PTR$ corresponding angles   ✓

$\angle RQS = \angle RPT$ corresponding angles   ✓

∴ $\Delta PRT \sim \Delta QRS \, (AAA)$

b) $\frac{5.4}{8.1} \times 10.8 = 7.2 \, cm$ ✓✓

c) $A(trapezium) = A(triangle \, PRT) - A(triangle \, QRT)$

$A(trapezium) = 0.5 \times 10.8 \times 8.1 - 0.5 \times 7.2 \times 5.4$

$\underline{A(trapezium) = 24.3 \, cm^2}$ ✓✓

d) $QS = \sqrt{5.4^2 + 7.2^2}$ ✓ $PT = 1.5 \times 9$

$QS = 9 \, cm$ ✓ $PT = 13.5 \, cm$ ✓

$13.5 - 9 = 4.5$ ✓

e) $24.3 = \frac{h}{2}(9 + 13.5)$

$\underline{h = 2.16 \, cm}$ ✓

13. a) $\frac{67.2}{8.4} = 8 \, cm^2$ ✓   b) $\frac{8}{4} = 2 \, cm$ ✓

c) i) $BM = \frac{4}{2} = 2$ cm   ✓   ii) $AB = \sqrt{2^2 + 2^2}$ ✓

$AB = 2.83\ cm\ (2dp.)$ ✓

d) $TSA = 2 \times 8cm^2 + 2 \times 4 \times 8.4\ cm + 2 \times 2.83 \times 8.4cm$ ✓✓

$TSA = 130.74\ cm^2$ ✓

e) $\frac{10.4}{4} = 2.6$ ✓

14. a) $2 \times \pi \times r = 44$ ✓

$\therefore\ r = \frac{44}{2 \times \pi} = 7.00$cm (2 d. p.)   ✓

b) $Area = 18 \times 44 = 792cm^2$ or $2 \times \pi \times 7.0028 \times 18 = 792.00\ cm^2$ ✓

$\underline{792 \times 2 = 1584\ cm^2}$ ✓

c) $\underline{1\ cm^2 = 100\ mm^2\ \ \therefore\ \ 792\ cm^2\ = 79200\ mm^2}$ ✓

d) i) $\frac{30 \times 50}{\pi \times 7^2} = 9.744180191 \approx 9$ cylinders   ✓

ii) This method does not take into account that the shape of the cylinders
cannot be altered to fill the gaps.   ✓
The box can only hold 6 cylinders (refer to the scale drawing   ✓
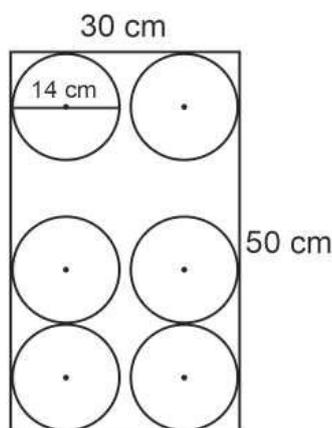
15. a) $Gross\ Pay = 32 \times 40 \times 52 + 188 \times 32 \times 2$ ✓
$Gross\ Pay = \$78\ 592$ ✓

b) $Tax\ payable = 3572 + (0.325 \times (78592 - 37000))$
$Tax\ payable = \$17\ 089.40$ ✓✓

c) $0.37(x - 80000) + 17547 = 29\ 572$ ✓
$0.37(x - 80000) = 12025$
$x = \frac{12025}{0.37} + 80000$

$x = \$112\ 500$ ✓

d) Expenses(per year) $= 650 \times 26 + 55 \times 52 + 410 \times 4 = \$21400$

Expenses(per year) $= \frac{21400}{(78592 - 17089.40)} \times 100$ ✓

Expenses(per year) $= 34.80\%$ (2 dp.) ✓

16. a) $A = \frac{113}{360} \times \pi \times 2^2$ ✓

$\underline{A = 3.94\ m^2 (2dp.)}$ ✓

113°
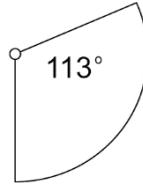
b) $\frac{3.68}{2} = 1.84$

$x = \sqrt{2^2 - 1.84^2}$ ✓

$\underline{x = 0.78\ m\ (2\ dp.)}$ ✓

c) $A = 2 \times 3.94 + 0.5 \times 3.68 \times 0.78 + 3.68 \times 1.2$ ✓✓

$\underline{A = 13.73\ m^2\ (2dp.)}$ ✓

d) $SA = 13.73 + \frac{226}{360} \times 2 \times \pi \times 2 \times 1.5 + 2 \times 1.2 \times 1.5 + 3.68 \times 1.5$ ✓

$SA = 13.73 + 20.95333233$ ✓

$\underline{SA = 34.68\ m^2\ (2\ dp.)}$ ✓

e) $C = 34.68 \times 26.5$ ✓   f) $C = \$919.02 \times 1.1$ ✓

$C = \$919.02$ ✓ $\underline{C = \$1010.92}$ ✓

17. a) $A = \begin{bmatrix} 94 & 85 & 114 \\ 117 & 92 & 156 \end{bmatrix}$ ✓✓   b) $B = \begin{bmatrix} 7 \\ 11 \\ 6 \end{bmatrix}$ ✓

c) $A \times B = \begin{bmatrix} 94 & 85 & 114 \\ 117 & 92 & 156 \end{bmatrix} \times \begin{bmatrix} 7 \\ 11 \\ 6 \end{bmatrix} = \begin{bmatrix} 2277 \\ 2767 \end{bmatrix}$ ✓✓

$\$2277\ total\ revenue\ on\ Saturday\ and\ \$2767\ on\ Sunday.$ ✓

d) $2277 + 2767 = \$5044$ ✓

e) $\frac{(2767 - 2277)}{2767} \times 100$ ✓

$= 21.52\%\ increase$ ✓

18.   a) $\frac{74}{1.75^2} = 24.16 \ (2 \ dp.)$ ✓

      b) $\frac{53}{1.52^2} = 22.94 \ (2 \ dp.)$ ✓✓

      c) $23.8 = \frac{x}{1.83^2}$ ✓

           $\underline{x = 79.70 \ kg \ (2 \ dp.)}$ ✓

      d) $\frac{(79.70 \times 1.06)}{1.83^2}$ ✓

           $= 25.23 \ (2 \ dp.)$ ✓

```
74/1.75^2
                  24.16326531
53/1.52^2
                  22.93975069
solve(23.8= x/1.83^2 , x)
                  {x=79.70382}
(79.70×1.06)/1.83^2
                  25.22679089
```

           The patient's BMI would fall outside the "healthy" range with a 6% increase in body weight.   ✓

19.   a) $\frac{1825}{250} = 7.3$ $\frac{4060}{560} = 7.25$ $\frac{2912}{400} \ 7.28$ ✓✓✓

           Anna got the best rate $1AUD = 7.3HKD$ ✓

      b) $\frac{2336}{7.3} = \$320AUD$ ✓

      c) $\frac{460}{6.25} = 73.6SGD$ ✓

      d) $125 \times 6.25 = 781.25 \ HKD$ ✓

           $\frac{1.2}{100} \times 781.25 = 9.38HKD (nearest \ cent)$ ✓

```
1825/250
                      7.3
4060/560
                     7.25
2912/400
                     7.28
2336/7.3
                      320
460/6.25
                     73.6
125×0.988-125
                     -1.5
1.5×6.25
                    9.375
□
```

20.   a) $Distance = 120 + 120 + 2 \times \pi \times 10 = 302.83m (to \ 2 \ d.p.)$ ✓✓

      b) $A = \pi r^2 + l \times w$

           $A = \pi \times 10^2 + 120 \times 20$

           $\underline{A = 2714.16 \ m^2 \ (2 \ dp.)}$ ✓✓

      c) $V = 2714.16 \times 0.2$ ✓

           $\underline{V = 542.83 \ m^3 \ (2 \ dp.)}$ ✓

Practice Exam Paper 2 – Units 1 & 2

MATHEMATICS APPLICATIONS

Units 1 & 2

Calculator Free Paper

TIME ALLOWED FOR THIS PAPER

Reading time before commencing work: Five minutes
Working time for paper: Fifty minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
This Question/Answer Booklet
Formula Sheet

TO BE PROVIDED BY THE CANDIDATE
Standard Items: Pens, pencils, eraser or correction fluid, ruler.

IMPORTANT NOTE FOR CANDIDATES:
The standard items listed above and the Calculator–free examination paper are the only items permitted on your table during the Calculator –free examination.

INSTRUCTIONS TO CANDIDATES

This section contains 9 questions.
You are required to attempt **ALL** questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.
Spare pages are available at the back of this booklet. If these extra pages are used, label your answers carefully.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

The examiners recommend that candidates **do not use pencil**, except in diagrams.

STRUCTURE OF THE PAPER

QUESTION	MARKS AVAILABLE
1	5
2	6
3	6
4	7
5	3
6	6
7	9
8	4
9	4

TOTAL MARKS = 50

1.   [5 marks: 3, 1, 1]

Consider the linear graphs on the axes below.
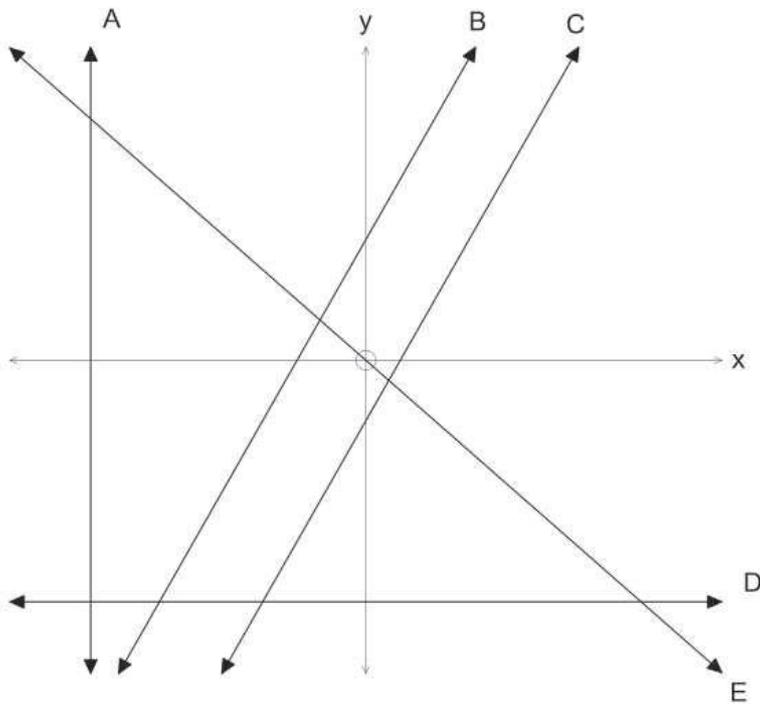
a)   Match the graphs shown to the descriptions provided.

Description	Line
Positive gradient and positive y-intercept	
$y = -4$	
Has a gradient of zero	

b)   If line E passes through the points (0 , 0) and (1 , -1), determine the equation of the line.

c)   If line C has a gradient of 2 and a y-intercept of 1, determine the equation of
the   line.

d)   If two graphs have the equations $3x - y = 16$ and $5x + 2y = 1$ find the point of intersection.

2.   [6 marks: 1, 3, 2]

Consider the set of scores listed below. It shows the weekly sales of couches at a furniture store over a 12 week period.

$$w, 53, 55, 56, 57, 57, x, y, z, 68, 69, 70$$

a)   Classify the type of data that has been collected.

b)   These scores are listed in ascending order and possess the following properties.

$Q_1 = 55.5$ $median = 57.5$ $range = 17$ $IQR = 10.5$ $\bar{x} = 60$

The sum of the 8 known scores is 485.

Determine the values of $w, x, y \ and \ z$.

c)   In the 13th week, 62 couches were sold. What effect, if any, will the addition of this score have upon the mean, median and standard deviation?

3.   **[6 marks: 1, 1, 1, 1, 2]**

If 5000 scores $X \sim N(15, 2^2)$;

a)   what is the mean?

b)   what is the standard deviation?

c)   determine 12 as a z score.

      Use the empirical rule to determine;

d)   approximately what percentage of scores lie between 13 and 19?

e)   approximately how many of the scores will fall between 13 and 17?

4.   [7 marks: 5, 2]

$A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 4 \\ 3 & -2 \end{bmatrix} \quad \& \quad C = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

a)   Determine,

i) $A - B$ ii) AB iii) $-2B$

b)   A student gives the answer to A^2 as $\begin{bmatrix} 4 & 9 \\ 16 & 1 \end{bmatrix}$. Is the student correct? Give reasons for your answer.

5.   [3 marks: 1, 1, 1]

Place $>, <$ or $=$ in the boxes below to make the statement true.

a) $\overline{x}_A$ [ ] $\overline{x}_B$ b)   Range A [ ] Range B

c) $S_x A$ [ ] $S_x B$

6.   [6 marks: 1, 3, 2]

A family hires a car while on holidays in Los Angeles. The cost of parking is determined by the following rates charged in US Dollars (USD).

$1.00 per hour for the first two hours.
$2.00 for the third hour
$0.50 for each 30 minutes thereafter.
$6.00 maximum.

a)   What will the cost be to park for 4 hours and 45 minutes?

b)   Construct a step graph to display the information in the table.
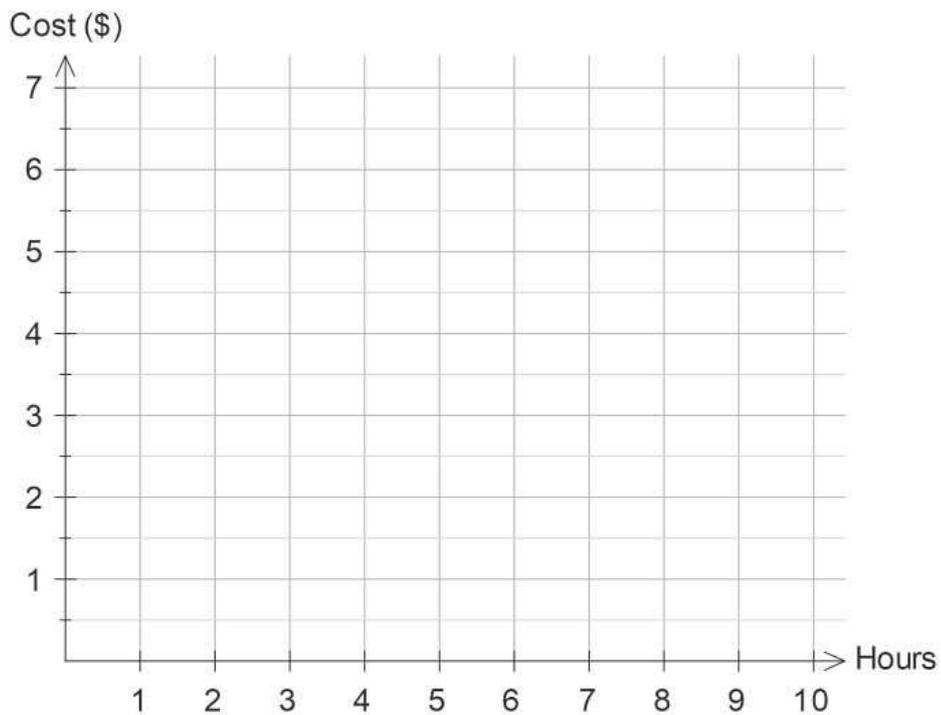
Cost ($)

c)   If 1 Australian dollar (AUD) is equivalent to 0.80 USD, what will be the cost of parking for 8 hours in AUD?

7.   **[9 marks: 2, 3, 2, 2]**

Consider the triangle shown below.
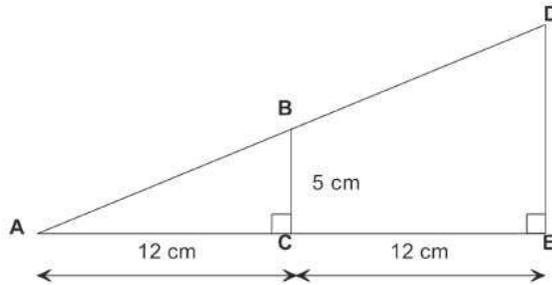
a)   Show that the length of AB is 13 cm.

b)   If $\overline{AD} = 26\ cm$, prove that triangles ABC and ADE are similar.

c)   Calculate the area of the quadrilateral BDEC.

d)   Express the area of quadrilateral BDEC as a percentage of triangle ADE .

8.   **[4 marks: 1, 1, 1, 1]**

ABCD is a square with sides of 4 cm. BG divides the square into a triangle BCG and a trapezium ABGD. The distance from D to G is 1 cm.
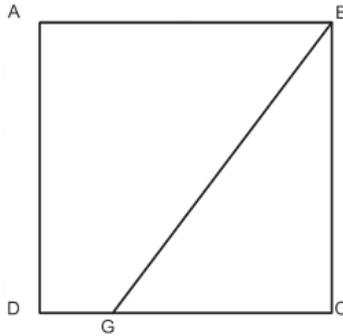
a)   Calculate the length of BG.

b)   Calculate the area of triangle BCG.

c)   Calculate the area of the trapezium ABDG.

d)   Express the area of the trapezium as a percentage of the square's area.

9.   [4 marks: 1, 1, 2]

Paediatricians use Fried's Rule, to estimate the medicine dosage (D) for a child. It is given by the formula;

$$D = \frac{mA}{150}$$

where; D is the dosage,
              m is the age of the infant in months, and
              A is the adult dosage

a)   Calculate the child's dose for

      i)   a ten month old baby if the adult dosage is 45 mL.

      ii)   a two year old child if the adult dosage is 62.5 mL

b)   If 8 mL of a particular type of medicine was given to a 6 month old infant, what is the adult dosage?

MATHEMATICS APPLICATIONS

Units 1 & 2

Calculator Assumed Paper

TIME ALLOWED FOR THIS PAPER

Reading time before commencing work: Ten minutes
Working time for paper: One hundred minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
This Question/Answer Booklet.

TO BE PROVIDED BY THE CANDIDATE
Standard Items: Pens, pencils, eraser or correction fluid, ruler.

Special Items: Please refer to the SCSA guidelines in relation to items such as:

- Drawing instruments
- Approved Calculators
- Notes
- Formula Sheets

INSTRUCTIONS TO CANDIDATES

This section contains **12** questions.
You are required to attempt **ALL** questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.
Spare pages are available at the back of this booklet. If these extra pages are used, label your answers carefully.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

The examiners recommend that candidates **do not use pencil**, except in diagrams.

STRUCTURE OF THE PAPER

QUESTION	MARKS AVAILABLE
10	11
11	6
12	7
13	12
14	4
15	6
16	8
17	8
18	6
19	8
20	10
21	14

TOTAL MARKS = 100

10.   **[11 marks: 1, 2, 2, 1, 2, 2, 1]**

At the world championships, the winner of the high jump recorded a height of 2.43 metres. The best jumps of the top three places are recorded in the table below.

Place	Best Jump
1st	2.43 m
2nd	2.25 m
3rd	2.15 m

a)   Classify the nature of the data collected.

b)   Express the 1st place jump as a percentage of the 2nd place jump and interpret your answer.

The best jumps of the 12 other competitors are shown in the table below.

2.11	2.03	2.08	2.01	2.16	1.99	1.97	2.13	2.07	2.08	2.11	2.12

c)   Construct a stem-and-leaf plot using 5 cm class intervals to show the best jump of each competitor.

d)   Describe the shape of the distribution.

e)   Calculate the mean and standard deviation of the best jumps for the 15 competitors.

f)   How many of the scores lie within one standard deviation of the mean?

g)   Standardise the score of 15th placed competitor.

11.   [6 marks: 2, 2, 2]

The Leoning M-8 was an American fighter monoplane with a wingspan of 35 feet.

a)   If 7 feet is equivalent to 2.1336 metres, calculate the wingspan of the M8 in
        metres to two decimal place accuracy.

b)   Write a formula for converting feet (f) to metres (m).

c)   A plane enthusiast wishes to construct a model of the M8 using a 1:25 scale.
        Calculate the wingspan of the model plane in centimetres to two decimal
        place accuracy.

12.   [7 marks: 3, 1, 1, 2]

A shopper has three options when purchasing a particular brand of adhesive.

- Pack of four, 1litre tins for $29.15
- One, 1 litre tin for $7.30
- Pack of 3, 220 mL tins for $4.80

a)   Which of the following represents the best buy?

b)   Determine the cost price of the 1 litre tin if it is 250% of its original price.

c)   The pack of four 1 L tins includes GST. Calculate the pre-GST price.

d)   If a shopper buys five, three-pack 220 mL cartons, the overall price is
         discounted by 7.5%. Determine the cost of buying five of this product.

13.   [12 marks: 3, 2, 2, 2, 3]

Consider the quadrilateral SRQP shown below. It is composed of two triangles.
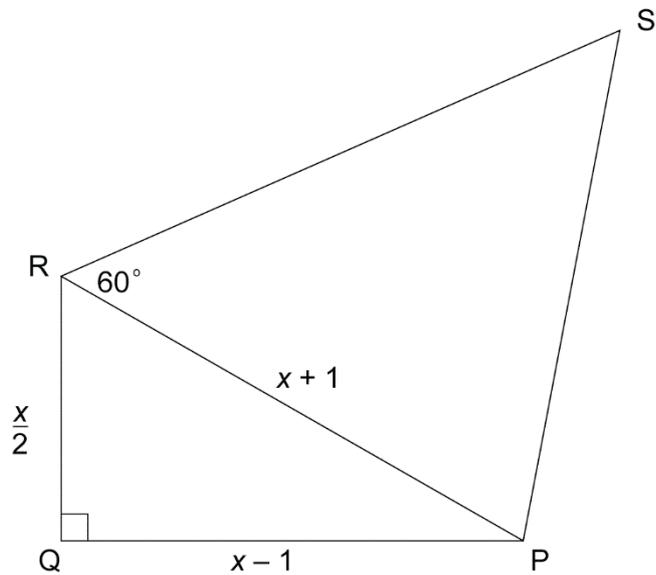
a)   If the perimeter of $\triangle RPQ$ is 40 cm, calculate the value of 'x'.

b)   Use the value of x to calculate $\angle RPQ$.

c)   If the area of $\triangle SRP$ is $132.50\,cm^2$, calculate the length of $\overline{RS}$ to the nearest centimetre.

Question 13 continued

d)   If $\overline{PS} = 17.52$ cm, calculate the size of $\angle RPS$ to one decimal place.

e)   Calculate the straight line distance from Q to S.

14.   [5 marks]

Which of the following represents the best investment option? Comment on your findings.

Option A $5000 invested at 16% pa. compounded quarterly, for 9 months?

Option B $5000 invested at 5% pa. earning simple interest for 30 months?

Option C $5000 invested at 4% pa. compounded monthly, for 3 years?

15.   [6 marks: 2, 2, 2]

Maxine is 17, has no children, lives at home, and attends TAFE. Maxine spends $58 a week on transport and has a mobile phone bill of $59.95 per month. She gives her mum $75 a fortnight in board to help with food and bills.

a)   What is the total weekly cost of these expenditures?

b)   Maxine receives a youth allowance payment of $226.80 per fortnight. Will these payments be enough to cover her living expenses? Explain.

The total cost of her TAFE course fees, text books and stationary is $920.70. Maxine decides she should take up an opportunity to work part time at the local deli, earning $18.60 per hour.

c)   How many hours will she need to work to earn the equivalent amount of her TAFE fees?

16.   [8 marks: 1, 2, 4, 1]

Frank is paid on commission. The graph below shows Frank's monthly income based on his total sales.

C ($)

Commission (1000s)

Sales (1000s)

S ($)
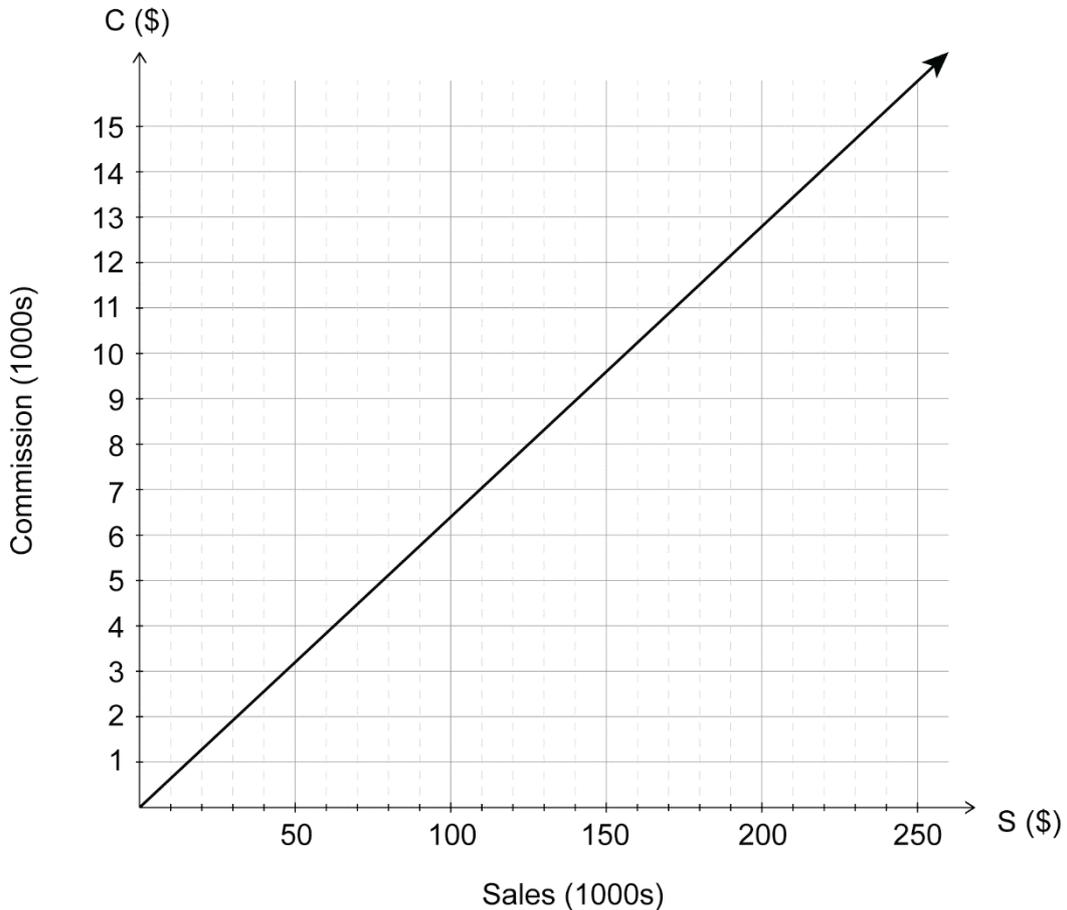
a)   Use the graph to determine the commission Frank will earn on $200 000 of sales.

b)   Express Frank's commission as a percentage of total sales.

Daniel is paid a retainer plus a commission on total sales.

c)   If Daniel receives a monthly base salary of $2000 plus 2% of total monthly sales (s);

    i)   use a linear model to express Daniel's monthly pay (P).

    ii)   construct a graph of this linear model using the axes above.

d)   For what total of sales will Daniel and Frank receive the same monthly pay?

17. **[8 marks: 2, 3, 2, 1]**

Consider the pipe section shown below. It is 20 m long and has a diameter of 4 m
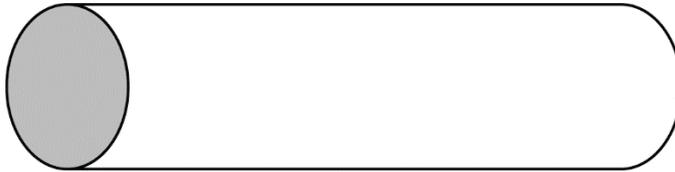
a)   If it was closed at both ends what is the volume of the cylinder?

Imagine the cylinder is open at one end.

b)   What would be the total surface area of the cylinder?

c)   If it costs $1320 to powder coat every surface of the pipe, what is the cost per square metre?

d)   Express your answer to (c) as a cost per square kilometre.

18.   **[6 marks: 2, 2, 2]**

Consider the triangle below.

68 cm

85 cm
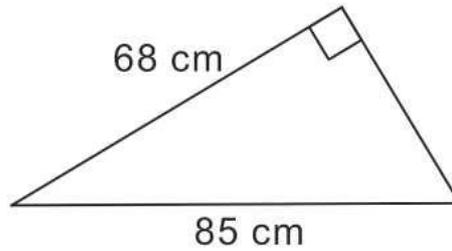
a)   Calculate the length of the missing side.

b)   If three of the triangles were placed together as shown below, what would the perimeter of the resulting shape be?
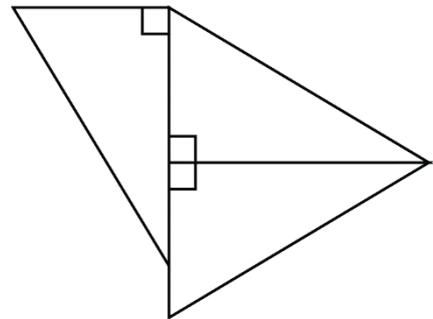
c)   If a single triangle was dilated by a scale factor of 0.6, what would it's area be?

19.   [8 marks: 2, 4, 2]

The operating life of two types of engines (Type R and Type S) were factory tested. The results are displayed below.

Distance (1000s) km
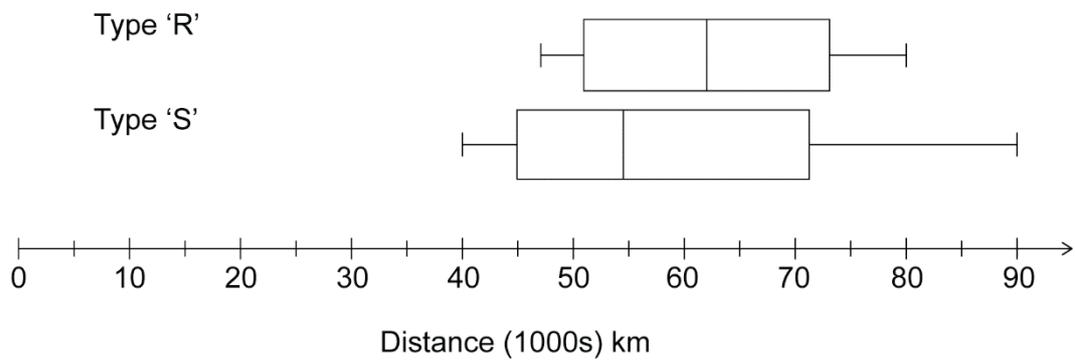
a)   On average, which motor type has the longest life? Justify your answer.

b)   Contrast the two engine types in terms of location, dispersion and shape.

c)   Which of the two engine types "performs best"? Give reasons for your answer.

20.   [10 marks: 2, 2, 3, 3]

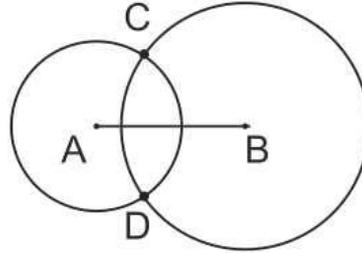Two circles overlap, and intersect at points C and D. The smaller circle has a radius of 10 cm, and the larger has a radius of 12 cm. The centres of the two circles (A and B respectively) are exactly 16 cm apart.

a)   Calculate $\angle DAB$

b)   Point X is located 6 cm to the left of point B. What is the shortest distance from X to D along the edge of the smaller circle?
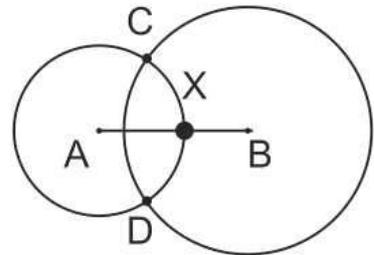
c)   Determine the distance from CD.

d)   Use Heron's formula to determine the area of triangle CDB.

21.   [14 marks: 1, 1, 1, 1, 2, 1, 1, 2, 2, 2]

Julie owns a small business. She imports vases from overseas which she sells for $51 each including GST.

a)   Write an equation for the revenue, R, that Julie receives for the sale of n, vases.

b)   What is the pre-GST price for each vase?

The cost (C) of importing the vases is given by the equation $C = 3600 + 26n$.

c)   What does the figure of 3600 represent?

d)   What does the figure of 26 represent?

e)   Use algebraic methods to determine the "break-even" point for Julie.

Julie makes the decision to import 5000 vases.

f)   What is the cost of importing 5000 vases?

Question 21 continued

g)   What is the profit made on the sale of the vases?

h)   Express the profit made on the sale of 5000 vases, as a percentage of the cost of importing 5000 vases.

Julie has a taxable income of $68 540. In this tax bracket, the tax owing is $3,572 plus 32.5c for each $1 over $37,000

i)   How much income tax is Julie liable for?

j)   After tax, what is Julie's average fortnightly income?

Resource Free Exam Solutions

1.   a)

Description	Line
Positive gradient and positive y-intercept	B ✓
$y = -4$	D ✓
Has a gradient of zero	D ✓

b) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-0}{1-0} = -1$

Substitute $m = -1$ and $(0,0)$ into $y = mx + c$ to get c
$0 = -1 \times 0 + c$
$c = 0$ Equation of the line is $y = -x$ ✓

c)   Equation of the line is $y = 2x + 1$ ✓

d)   (3, -7)

2.   a)   Numerical, Discrete data. ✓

b)   53, 53, 55, 56, 57, 57, 58, 60, 64, 68, 69, 70

$w = 70 - 17$ $\frac{x+57}{2} = 57.5$ $\frac{z+68}{2} = 55.5 + 10.5$

$w = 53$✓ $x = 58$ ✓ $z = 64$ ✓✓

Sum of all scores $= \bar{x} \times n$
$= 60 \times 12 = 720$
$y = 720 - 485 - 53 - 58 - 64$
$y = 720 - 660 = 60$ ✓✓

c)   Mean would increase. ✓
Median would increase to 58 (the 7th score) ✓
Standard deviation would be lowered. ✓

3.   a)   15✓   b)   2✓   c) $Z = \frac{12-15}{2} = -1.5$✓

d) $34 + 34 + 13.5 = 81.5\%$✓   e) $34\% + 34\% = 68\%$
$0.68 \times 5000 = 3400\ scores$✓✓

4   a)   i) $A - B = \begin{bmatrix} -1 & -7 \\ 1 & 3 \end{bmatrix}$✓   ii) $A \times B = \begin{bmatrix} -3 & 14 \\ 15 & 14 \end{bmatrix}$ ✓✓✓
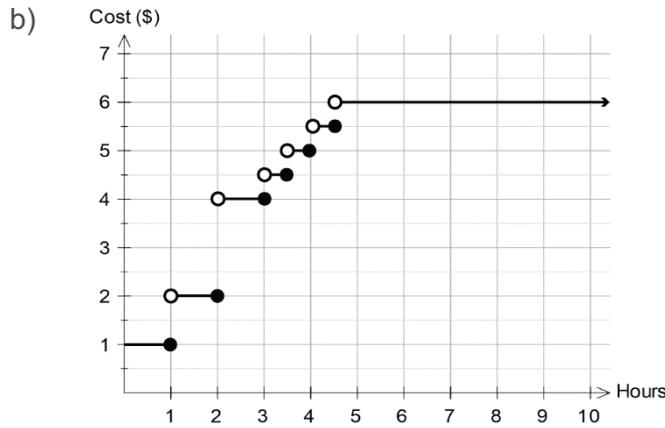
iii) $-2B = \begin{bmatrix} -6 & -8 \\ -6 & 4 \end{bmatrix}$ ✓

b) $A^2 = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -8 & -9 \\ 12 & -11 \end{bmatrix}$✓the student is incorrect✓

5. a) $\overline{x}_A$ ☐ $= \checkmark$ $\overline{x}_B$ b) Range A ☐ $= \checkmark$ Range B

    c) S_x ☐ $> \checkmark$ $S_x B$

6. a) $6.00 ✓

    b) Cost ($)

    ✓✓✓

    c) $\frac{6}{0.8} = \frac{6 \times 5}{4} = \underline{7.50\ AUD}$ ✓✓

7. a) $AB = \sqrt{12^2 + 5^2} = 13cm$ ✓

    b) $\angle BAC = \angle DAE$ $(common\ angle)$ ✓

        $\frac{AD}{AB} = \frac{AE}{AC}$ $(pair\ of\ corresponding\ sides\ in\ proportion)$ ✓

        $\therefore \Delta ABC\ ||| \Delta ADE$ (SAS) ✓

                            or

        $\angle BAC = \angle DAE$ $(common\ angle)$ ✓

        $\angle ACB = \angle AED$ $(given\ as\ 90^o)$ ✓

        $\angle CBA = \angle EDA$ $(sum\ of\ angles\ in\ a\ triangle)$

        $\therefore \Delta ABC\ ||| \Delta ADE$ (AAA) ✓

    c) $DE = \sqrt{26^2 + 24^2} = 10cm$
        $A = (0.5 \times 24 \times 10) - (0.5 \times 12 \times 5)$ or $A = \frac{12}{2}(5 + 10)$
        $A = 120 - 30\ cm^2$ $A = 6(15)cm$
        $\underline{A = 90\ cm^2}$ $\underline{A = 90\ cm^2}$ ✓

8. a) $BG = \sqrt{3^2 + 4^2} = 5cm$✓

    b) $Area = 0.5 \times base \times p.h. = 0.5 \times 3 \times 4 = 6cm^2$✓

    c) $Area(trapezium) = 16 - 6 = 10\ cm^2$✓

    d) $\frac{10}{16} = \frac{5}{8} = 5 \times 0.125 = 0.625 = 62.5\%$ ✓

9.   a)   i) $D = \frac{10 \times 45}{150} = \frac{450}{150}$ ii) $D = \frac{24 \times 62.5}{150} = \frac{250}{25}$

$\underline{D = 3 \; mL}$ ✓ $\underline{D = 10 \; mL}$ ✓

b) $8 = \frac{6A}{150}$ ✓

$1200 = 6A$

$\underline{A = 200}$ ✓ <u>The adult dosage is 200ml</u>

Resource Rich Exam Solutions

10. a)   Continuous, numerical data. ✓

b) $\frac{2.43}{2.25} = 108\%$ ✓

1st placed jump was 8% higher than 2nd placed.   ✓

c)

190_H	7 9
200_L	1 3
200_H	7 8 8
210_L	1 1 2 3
210_H	5 6
220_L	
220_H	5
230_L	
230_H	
240_L	3

✓✓

d)   positively skewed   ✓

e) $\bar{x} = 2.11 (2\ dp.)$ ✓
     $s = 0.11\ (2\ dp.)$ ✓✓

```
x̄        =2.1126667
Σx       =31.69
Σx²      =67.1307
σx       =0.1096337
Sx       =0.1134817
n        =15
minX     =1.97
Q₁       =2.03
Med      =2.11
         =2.15
```

f) $(\bar{x} + 1\sigma) \geq x \geq (\bar{x} - 1\sigma)$

$(2.112 + 0.109) \geq x \geq (2.112 - 0.109) = 11$ scores ✓✓

g) $\frac{1.97 - 2.11}{0.11} = -1.27\ (2\ dp.)$ ✓

11. a) $1ft = \frac{2.1336}{7} = 0.3048\ m$
          $35ft \times 0.3048 = 10.67\ m$ ✓✓

b) $m = 0.3048 \times f$ ✓✓

c) $\frac{10.67}{25} = 0.4268\ m$ ✓
     $0.4268m = 42.68\ cm$ ✓

12. a)   Find cents per ml for each quantity and compare them:

Pack of four $= \frac{2915}{4000} = 0.72875$ ✓

$One\ Tin = \frac{730}{1000} = 0.730$ ✓

$Pack\ of\ three \times 220ml = \frac{480}{660} = 0.7272$ Pack of three is the best buy ✓

b) $7.30 \div 2.5 = \$2.92$ ✓

c) $\$29.15 = 1.1 \times x$
$29.15 \div 1.1 = x$
$x = \$26.50\ Pre - G.S.T.price$ ✓

d) $5 \times 4.8 = 24 \times (1.00 - 0.0725) = \$22.20$ ✓

13. a) $0.5x + x - 1 + x + 1 = 40$
$2.5x = 40$
$\underline{x = 16}$

solve$(0.5 \cdot x + x - 1 + x + 1 = 40, x)$
$\{x=16\}$

b) $\cos p^o = \frac{15}{17} \quad \therefore p = 28.07^o$ ✓✓

c) $Area = \frac{1}{2} \times a \times b \times sinC \quad \therefore \quad 131.51 = 0.5 \times 17 \times \overline{RS} \times \sin 60$

$132.51 = 7.3612 \times \overline{RS} \quad \therefore \quad \overline{RS} = 18cm$ ✓✓

d) $\frac{18}{\sin(\angle RPS)} = \frac{17.52}{\sin 60^o}$

$\sin(\angle RPS) = \frac{18 \times \sin 60^o}{17.52}$ ✓

$\angle RPS = 62.8^o$ ✓

e) $x^2 = 15^2 + 17.52^2 - 2(15)(17.52) \times \cos(62.8 + 28.07)$ ✓✓

$x = 23.24\ cm\,(to\ 2\ d.p.)$ ✓

14. a) $5000 - 5000(1 + \frac{0.16}{4})^3 = \$624.32$ ✓

b) $5000 \times 0.05 \times \frac{30}{12} = \$625$ ✓

c) $5000 - 5000(1 + \frac{0.04}{12})^{36} = \$636.36$ ✓

Option C offers the best return by $11.36 over Option B and $12.04 over Option A. However, it requires the longest period of time ✓

Option A offers a short term investment with quick returns. ✓

15.   a)   *weekly cost* $= 58 + (59.95 \div 4) + 37.50 = \$110.49$✓

       b) $(58 \times 52) + (59.95 \times 12) + (75 \times 26) = 5685.40$
             Youth Allowance $= 226.80 \times 26 = 5896.80$
             This would easily cover her costs ✓

       c) $920.70 \div 18.60 = 49.5$
             Maxine would need to work $49\frac{1}{2}$ *hours to cover her total TAFE fees* ✓✓

16   a) $12000 ✓   b) $\frac{12000}{200000} \times 100 = 6\%$ ✓✓
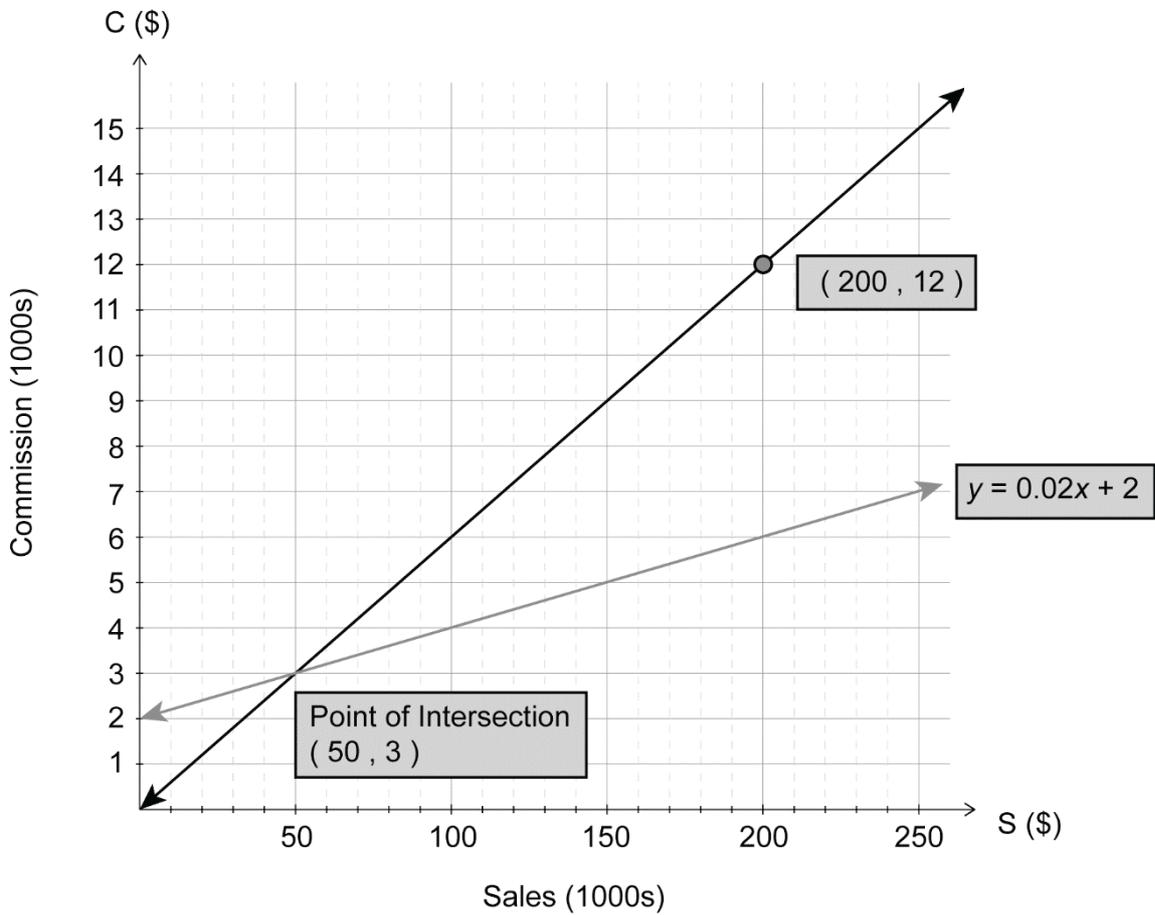
       c)   i) $P = 0.02 \times s + 2000$ ✓✓ ii)   on axis ✓

       d) $50000 worth of sales✓

17. a) $Area = \pi \times r^2 = \pi \times 2^2 = 12.566 m^2$
$Volume = 12.566 \times 20 = 251.33 m^3 ✓$

b) $TSA = inner\ circle + outer\ circle + inner\ rectangle + outer\ rectangle$

$TSA = 2\pi r^2 + 2(2\pi rh)$
$TSA = 2 \times \pi \times 2^2 + 2(2 \times \pi \times 2 \times 20)$
$TSA = 527.79\ m^2\ (2\ dp.)\ ✓✓$

c) $\frac{1320}{527.79} = \$2.50\ per\ m^2\ ✓✓$

d) $2.5 \times 1000^2 = \$2\ 500\ 000\ per\ sq.\ km ✓$

18. a) $missing\ side = \sqrt{85^2 - 68^2} = 51 cm✓✓$
b) $85 + 85 + 85 + 51 + (51 + 51 - 68) = 340 cm\ ✓✓$
c) $Area\ (after\ dilation) = 0.5 \times (0.6 \times 51) \times (0.6 \times 68) = 624.24 cm^2✓✓$

19. a) Type S has an average of 55 in comparison to Type R which has a
Higher average of 63 so Tyre R has a longer life on average by 8000km. ✓✓

b) *Type R engines have a higher average life and have a less variable lifetime✓*
Type R has a symmetrical distribution, centred around a median value of
62 000 km.✓
Type S is positively skewed, with a median value of 54 000 km.✓
Type R has a smaller range and IQR.✓

c) *Type R performed better because it operated for a higher average distance*
and overall has a spread meaning it is more consistent.✓✓

20. a) $BD^2 = AD^2 + AB^2 - 2 \times AD \times AB \times cos\angle DAB$
$12^2 = 10^2 + 16^2 - 2 \times 10 \times 16 \times cos\angle DAB\ ✓$
$\angle DAB = 48.51°\ ✓✓$

b) $XD^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times cos\angle 48.51°\quad ✓$
$XD = 8.22 cm(to\ 2\ d.p.)\qquad\qquad ✓$

c) $CD^2 = AD^2 + AC^2 - 2 \times AD \times AC \times cos\angle DAC$
$BD^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times cos\ (2 \times 48.51)\ ✓$
$BD^2 = 224.44$
$BD = 14.98 cm(to\ 2\ d.p.)\ ✓✓$

d) $s = \frac{(12+12+14.98)}{2} = 19.49\quad ✓$

$A = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}.$

$A = \sqrt{19.49 \times (19.49 - 12) \times (19.49 - 12) \times (19.49 - 14.98)}\ ✓$

$A = 70.22\ cm(to\ 2\ d.p.)\quad ✓$

21.   a) $R = 51 \times n$✓   b) $51 \div 1.1 = \$46.36$✓

       c) $3600\ represents\ her\ fixed\ running\ costs$✓

       d) $26\ represents\ the\ cost\ of\ each\ vase\ from\ her\ overseas\ supplier.$ ✓

       e) $51n = 3600 + 26n$
              $51n - 26n = 3600$
              $n = \dfrac{3600}{25}$
              $n = 144\ vases\ to\ break\ even$ ✓✓

       f) $Cost = 3600 + 5000 \times 26 = \$133\ 600$✓

       g) $Profit = 51 \times 5000 = \$255\ 000 - \$133\ 600 = \$121\ 400\ in\ profit\ made$✓

       h) $\dfrac{121400}{133600} \times 100 = 90.86\%$✓✓

       i) $3572 + 0.325(68540 - 37000) = \$13\ 822.50$ ✓✓

       j) $\dfrac{(68540 - 13822.5)}{26}$ ✓
              $= \$2\ 104.52\ (2\ dp.)\ per\ fortnight$ ✓

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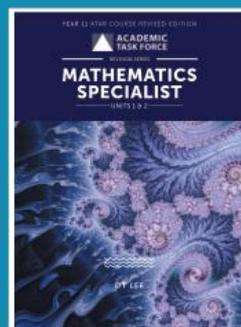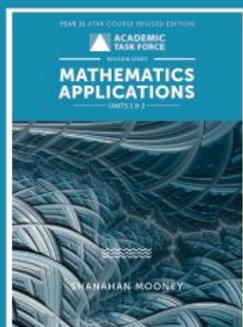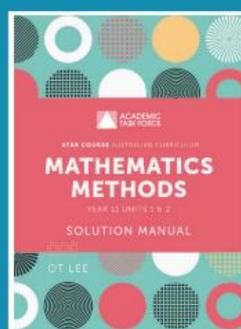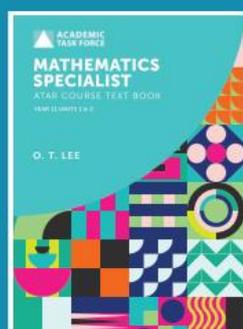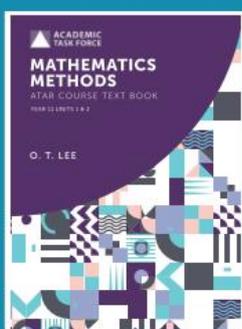Academic Task Force and Academic Associates are part of the Academic Group

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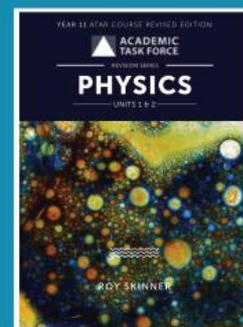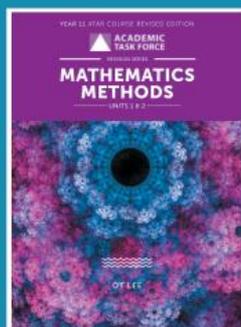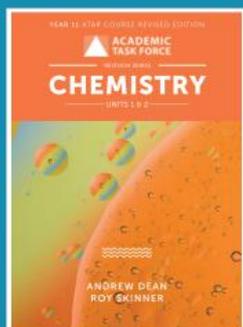
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