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8

ESSENTIAL MATHEMATICS CORE

FOR THE AUSTRALIAN CURRICULUM

SECOND EDITION

David Greenwood
Bryn Humberstone
Justin Robinson
Jenny Goodman
Jennifer Vaughan
Stuart Palmer

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About the Authors

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 30+ years' experience mathematics from Year 7 to 12. He is the lead author for the Cambridge Essential series and has authored more than 80 titles for the Australian Curriculum and for the syllabuses of the states and territories. He specialises in analysing curriculum and the sequencing of course content for school mathematics courses. He also has an interest in the use of technology for the teaching of mathematics.

Bryn Humberstone graduated from the University of Melbourne with an Honours degree in Pure Mathematics, and has 20+ years' experience teaching secondary school mathematics. He was a Head of Mathematics from 2014–2024 at two independent schools in Victoria. Bryn is passionate about applying the science of learning to teaching and curriculum design, to maximise the chances of student success.

Justin Robinson is co-founder of *The Wellbeing Distillery*, an organisation committed to energising, equipping and empowering teachers. He consults with schools globally on the fields of student and educator wellbeing. Prior to this, Justin spent 25 years teaching mathematics, covering all levels of secondary education including teaching VCE, IB and A-Levels. His driving passion was engaging and challenging students within a safe learning environment. Justin is an Honorary Fellow of the University of Melbourne's Graduate School of Education.

Jenny Goodman has taught in schools for over 28 years and is currently teaching at a selective high school in Sydney. Jenny has an interest in the importance of literacy in mathematics education, and in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke Prize for Mathematics. She has written for CambridgeMATHS NSW and was involved in the Spectrum and Spectrum Gold series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

Stuart Palmer was born and educated in NSW. He is a fully qualified high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been Head of Mathematics in two schools. He is very well known by teachers throughout the state for the professional learning workshops he delivers. Stuart also assists thousands of Year 12 students every year as they prepare for their HSC Examinations. At the University of Sydney, Stuart spent more than a decade running tutorials for pre-service mathematics teachers.

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Introduction

The second edition of *Essential Mathematics CORE for the Australian Curriculum* has been significantly revised and updated to suit the teaching and learning of Version 9.0 of the Australian Curriculum. Many of the established features of the series have been retained, but there have been some substantial revisions, improvements and new elements introduced for this edition across the print, digital and teacher resources.

New content and some restructuring

New content has been added at all year levels. In **Year 7**, there is new content on ratios and proportions, volume of triangular prisms, nets of solids and measurement relating to circles. All geometry topics are now contained in a single chapter (Chapter 7). In **Year 8**, there is new content on order of operations, 3D-coordinates, operations with negative fractions, areas of sectors and composite shapes, Pythagoras' theorem, inequalities, similar figures, two-step experiments and tree diagrams. For **Year 9**, there is new content on errors in measurement, inequalities, factorisation, sampling and proportion, quadratics expressions and parabolas. In **Year 10**, there is new content on composite solids, errors in measurement, networks and logarithmic scales.

Version 9.0 places increased emphasis on **investigations** and **modelling**, and this is covered with revised Modelling activities at the end of chapters and downloadable Investigations. There are also many new elaborations covering **First Nations Peoples' perspectives** on mathematics, ranging across all six content strands of the curriculum. These are covered in a suite of specialised investigations provided in the Online Teaching Suite.

Other new features

- **Technology and computational thinking** activities have been added to the end of every chapter to address the curriculum's increased focus on the use of technology and the understanding and application of algorithms.
- **Targeted skillsheets** – downloadable and printable – have been written for every lesson in the series, with the intention of providing additional practice for students who need support at the basic skills covered in the lesson, with questions linked to worked examples in the book.
- **Editable PowerPoint lesson summaries** are also provided for each lesson in the series, with the intention of saving the time of teachers who were previously creating these themselves.

Diagnostic assessment tool

Also new for this edition is a flexible, comprehensive diagnostic assessment tool, available through the Online Teaching Suite. This tool, featuring around 10,000 new questions, allows teachers to set diagnostic tests that are closely aligned with the textbook content, view student performance and growth via a range of reports, set follow-up work with a view to helping students improve, and export data as needed.

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Guide to the working programs

Essential Mathematics CORE for the Australian Curriculum 9.0 contains working programs that are subtly embedded in the exercises. The suggested working programs provide two pathways through the book to allow differentiation for Building and Progressing students.

Each exercise is structured in subsections that match the Australian Curriculum 9.0 proficiency strands (with Problem-solving and Reasoning combined into one section to reduce exercise length), as well as 'Gold star' (★). The questions* suggested for each pathway are listed in two columns at the top of each subsection.

- The left column (lightest shade) shows the questions in the Building working program.
- The right column (darkest shade) shows the questions in the Progressing working program.

Gradients within exercises and proficiency strands

The working programs make use of two gradients that have been carefully integrated into the exercises. A gradient runs through the overall structure of each exercise – where there's an increasing level of sophistication required as a student progresses through the proficiency strands and then on to the 'Gold Star' question(s) – but also within each proficiency strand; the first few questions in Fluency are easier than the last few, for example, and the first few Problem-solving and reasoning questions are easier than the last few.

	Building	Progressing
Understanding	1–3	3
Fluency	4–5(½), 6, 8	4–7(½), 8
Problem-solving and reasoning	9, 10	9(½), 10, 11
★	—	12

The right mix of questions

Questions in the working programs have been selected to give the most appropriate mix of types of questions for each learning pathway. Students going through the Building pathway are given extra practice at the Understanding and basic Fluency questions and only the easiest Problem-solving and reasoning questions. The Progressing pathway, while not challenging, spends a little less time on basic Understanding questions and a little more on Fluency and Problem-solving and reasoning questions. The Progressing pathway also includes the 'Gold star' question(s).

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works best for them. If required, the Warm-up quiz at the start of each chapter can be used as a diagnostic tool. The following are recommended guidelines:

- A student who gets 40% or lower should heavily revise core concepts before doing the Building questions, and may require further assistance.
- A student who gets between 40% and 75% should do the Building questions.
- A student who gets 75% and higher should do the Progressing questions.

For schools that have classes grouped according to ability, teachers may wish to set either the Building or Progressing pathways as the default pathway for an entire class and then make individual alterations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- – : complete none of the questions in this section.

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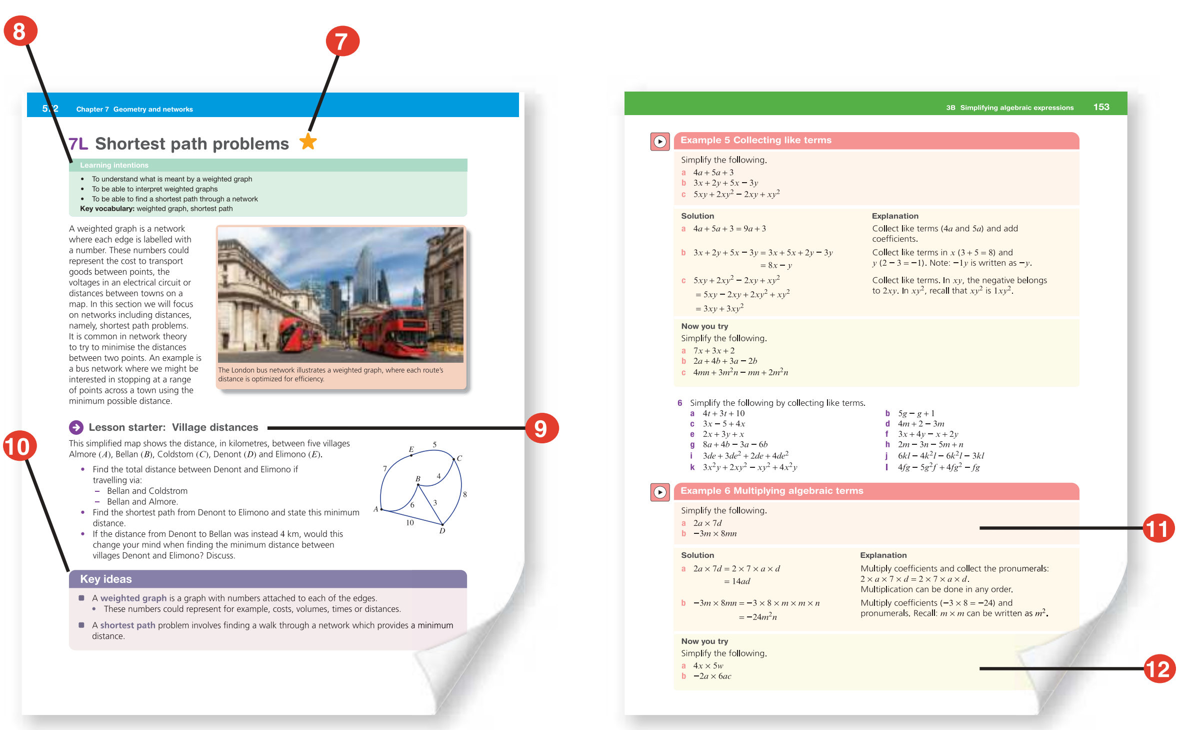
Guide to this resource

PRINT TEXTBOOK FEATURES

1. **NEW** **New lessons:** authoritative coverage of new topics in the Australian Curriculum 9.0 in the form of new, road-tested lessons throughout each book.
2. **Australian Curriculum 9.0:** content strands, sub-strands and content descriptions are listed at the beginning of the chapter (see the teaching program for more detailed curriculum documents)
3. **In this chapter:** an overview of the chapter contents
4. **NEW** **Quick reference:** Multiplication, prime number, fraction wall and divisibility rules tables at the back of the book
5. **Chapter introduction:** sets context for students about how the topic connects with the real world and the history of mathematics
6. **Warm-up quiz:** a quiz for students on the prior knowledge and essential skills required before beginning each chapter
7. **Sections labelled to aid planning:** All non-core sections are labelled as ‘Consolidating’ (indicating a revision section) or with a gold star (indicating a topic that could be considered challenging) to help teachers decide on the most suitable way of approaching the course for their class or for individual students.
8. **Learning intentions:** sets out what a student will be expected to learn in the lesson
9. **Lesson starter:** an activity, which can often be done in groups, to start the lesson
10. **Key ideas:** summarises the knowledge and skills for the section
11. **Worked examples:** solutions and explanations of each line of working, along with a description that clearly describes the mathematics covered by the example. Worked examples are placed within the exercise so they can be referenced quickly, with each example followed by the questions that directly relate to it.
12. **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give students immediate practice



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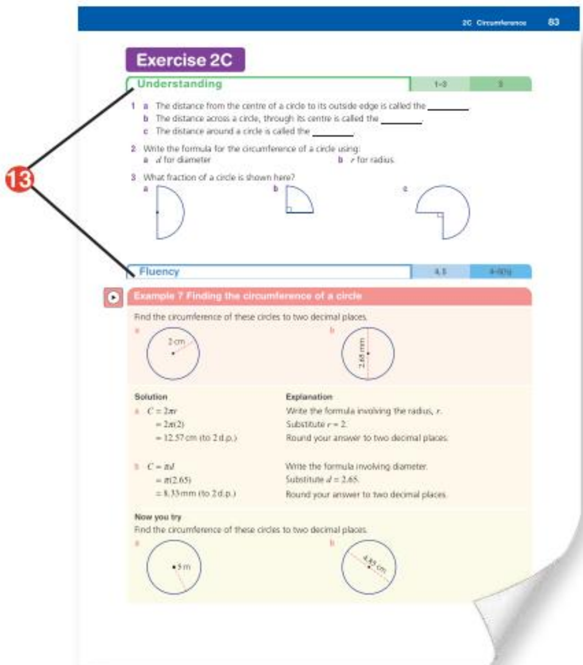
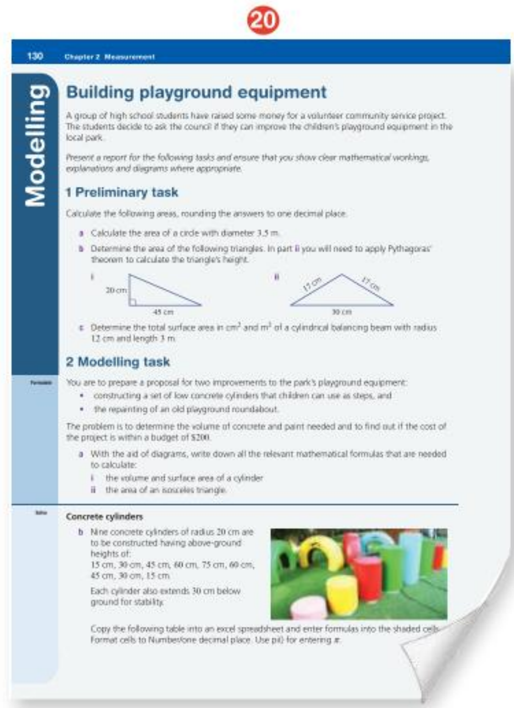
13 **Working programs:** differentiated question sets for two ability levels in exercises

14 **Puzzles and games:** in each chapter provide problem-solving practice in the context of puzzles and games connected with the topic

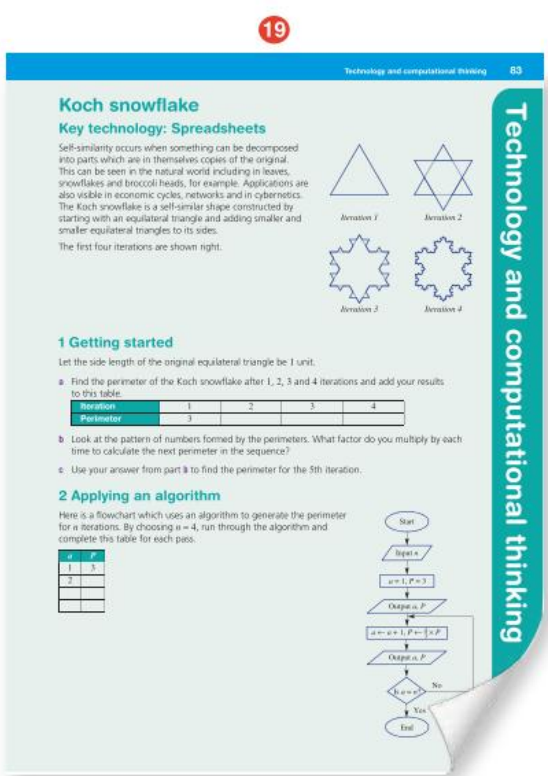
15 **Gentle start to exercises:** the exercise begins at Understanding and then Fluency, with the first question always linked to the first worked example in the lesson

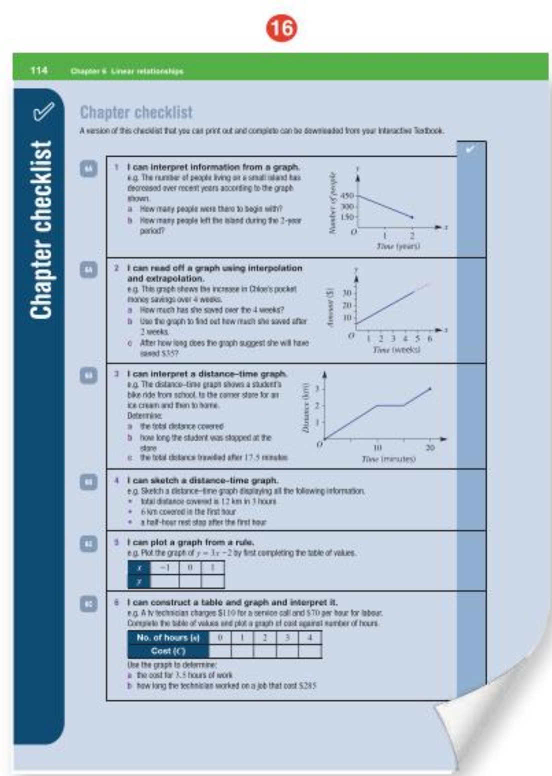




16 **Chapter checklist:** a checklist of the learning intentions for the chapter, with example questions

17 **Chapter reviews:** with short-answer, multiple-choice and extended-response questions; questions that are 'Gold Star' are clearly signposted





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18 **Maths@Work:** a set of extended questions across two pages that give practice at applying the mathematics of the chapter to real-life contexts

19 NEW **Technology and computational thinking** activity in each chapter addresses the curriculum's increased focus on the use of different forms of technology, and the understanding and implementation of algorithms

20 **Modelling activities:** an activity in each chapter gives students the opportunity to learn and apply the mathematical modelling process to solve realistic problems

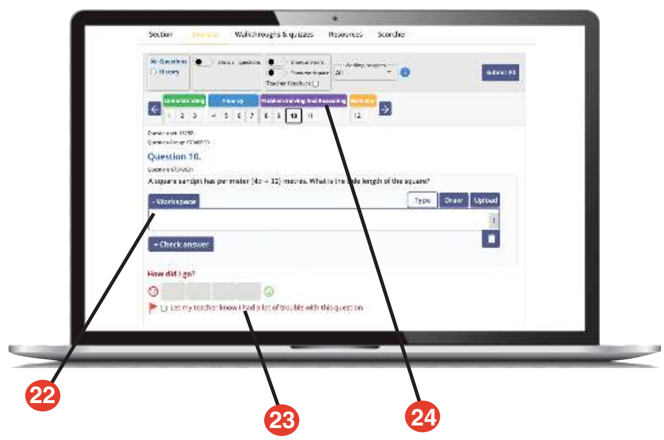
INTERACTIVE TEXTBOOK FEATURES

21 NEW **Targeted Skillsheets**, one for each lesson, focus on a small set of related Fluency-style skills for students who need extra support, with questions linked to worked examples

22 **Workspaces:** almost every textbook question – including all working-out – can be completed inside the Interactive Textbook by using either a stylus, a keyboard and symbol palette, or uploading an image of the work

23 **Self-assessment:** students can then self-assess their own work and send alerts to the teacher. See the Introduction on page x for more information

24 **Interactive question tabs** can be clicked on so that only questions included in that working program are shown on the screen

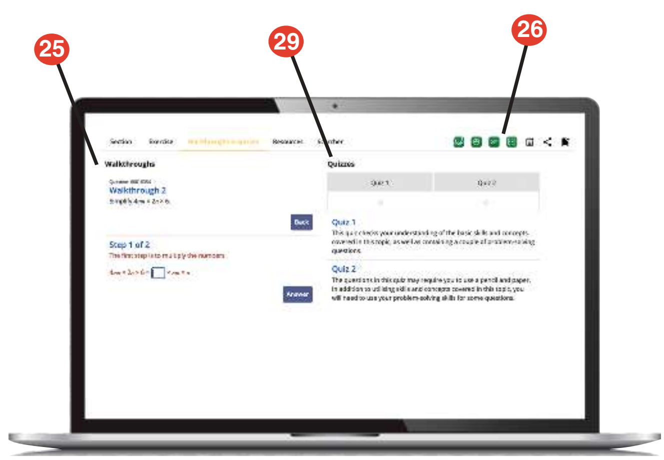


25 **HOTmaths resources:** a huge catered library of widgets, HOTsheets and walkthroughs seamlessly blended with the digital textbook

26 **Desmos graphing calculator**, scientific calculator and geometry tool are always available to open within every lesson

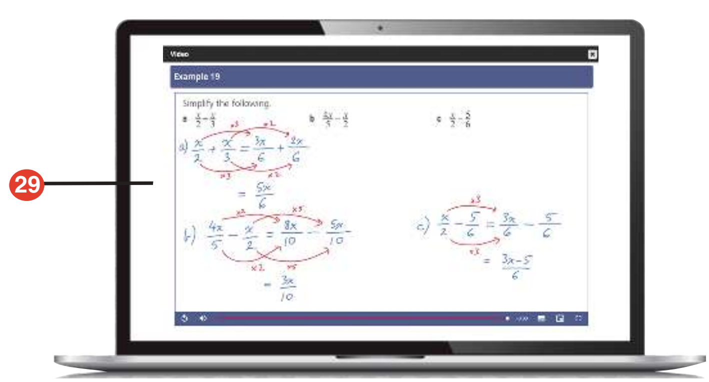
27 **Scorcher:** the popular competitive game

28 **Worked example videos:** every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the flipped classroom



29 A revised set of **differentiated auto-marked practice quizzes** per lesson with saved scores

30 **Auto-marked maths** literacy activities test students on their ability to understand and use the key mathematical language used in the chapter



31 **Auto-marked prior knowledge pre-test** (the 'Warm-up quiz' of the print book) for testing the knowledge that students will need before starting the chapter

32 **Auto-marked progress quizzes and chapter review questions** in the chapter reviews can be completed online

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DOWNLOADABLE PDF TEXTBOOK

33 In addition to the Interactive Textbook, a **PDF version of the textbook** has been retained for times when users cannot go online. PDF search and commenting tools are enabled.

ONLINE TEACHING SUITE

34 NEW **Diagnostic Assessment Tool** included with the Online Teaching Suite allows for flexible diagnostic testing, reporting and recommendations for follow-up work to assist you to help your students to improve

35 NEW **PowerPoint lesson** summaries contain the main elements of each lesson in a form that can be annotated and projected in front of class

36 **Learning Management System** with class and student analytics, including reports and communication tools

37 **Teacher view of students' work and self-assessment** allows the teacher to see their class's workout, how students in the class assessed their own work, and any 'red flags' that the class has submitted to the teacher

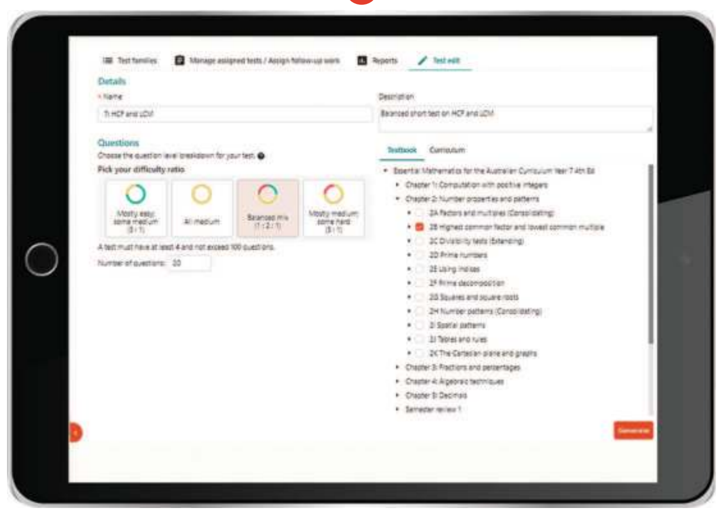
38 **Powerful test generator** with a huge bank of levelled questions as well as ready-made tests

39 **Revamped task manager** allows teachers to incorporate many of the activities and tools listed above into teacher-controlled learning pathways that can be built for individual students, groups of students and whole classes

40 **Worksheets, Skill and drill, maths literacy worksheets**, and **two differentiated chapter tests in every chapter**, provided in editable Word documents

41 **More printable resources:** all Pre-tests and Progress quizzes are provided in printable worksheet versions

34



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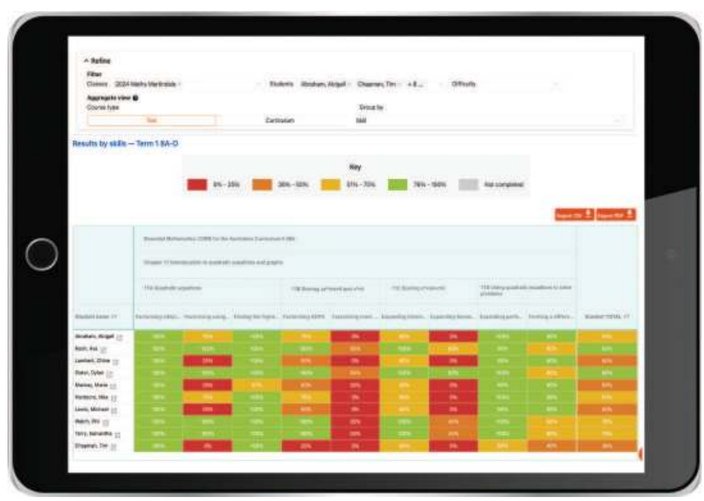

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1 Computation with integers

Essential mathematics: why being able to compute with integers is important

In an emergency, ambulance paramedics often administer intravenous fluids to resuscitate a patient. They use multiplication and division to calculate drip rates, volumes and times.

Electronics engineers and technicians compute squares and square roots when designing audio amplifiers to produce a strong enough musical signal to run loudspeakers.

Boardgame designers use divisibility rules in various ways to make games more engaging and complex. Scoring systems and player progression rules based on whether a player's score is divisible by a certain number, encourage strategic play. Also, distributing skill points evenly across characters ensures equal opportunity for all.

Skiers, snowboarders and mountain climbers select their gear according to temperature forecasts. One winter morning at Australia's Thredbo Top Station, the temperature was −6°C but winds of 72 km/h helped to cause a freezing cold 'feels like' temperature of −21°C.

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In this chapter

Australian Curriculum 9.0

NUMBER

Use the 4 operations with integers and with rational numbers, choosing and using efficient strategies and digital tools where appropriate (AC9M8N04)

Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts; formulate problems, choosing efficient calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8N05)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 Decide if the following expressions relate to **A** addition (+) **S** subtraction (−) **M** multiplication (×) or **D** division (÷).

a Total sum
b Difference
c 10 more than 7
d 10 less than 13
e 5 groups of 3
f How many 3s in 18
g The product
h The quotient
i Increase by 6

2 Complete these additions.

a $12 + 7$
b $50 + 19$
c $42 + 31$
d $146 + 213$
e $15 + 19 + 23$
f $\begin{array}{r} 123 \\ +\ 39 \\ \hline \end{array}$

3 Complete these subtractions.

a $12 - 8$
b $50 - 28$
c $47 - 29$
d $12 - 6 - 6$
e $784 - 163$
f $\begin{array}{r} 336 \\ -289 \\ \hline \end{array}$

4 Complete these multiplications.

a 9×4
b 5×8
c 12×11
d 15×5
e $\begin{array}{r} 121 \\ \times\ \ 9 \\ \hline \end{array}$
f $\begin{array}{r} 338 \\ \times\ 14 \\ \hline \end{array}$

5 Complete these divisions.

a $28 \div 4$
b $99 \div 3$
c $18 \div 6$
d $72 \div 12$
e $3\overline{)453}$
f $7\overline{)364}$

6 **a** List the first 5 multiples of 6.
b List the first 4 multiples of 9.
c What is the lowest common multiple (LCM) of 6 and 9?

7 **a** List all the factors of 12.
b List all the factors of 15.
c What is the highest common factor (HCF) of 12 and 15?

8 Prime numbers have exactly two factors. Of the first 15 positive integers (listed below), list the numbers which are prime. Write your primes in ascending (increasing) order. The first prime is circled.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

9 Answer the following as true (T) or false (F).

a $2 + 3 \times 4 = 2 + 12$
b $10 - 8 \div 2 = 10 - 4$
c $(5 - 2) \times 7 = 3 \times 7$
d $9 \times 3 + 5 = 9 \times 8$
e $9 \times (3 + 5) = 9 \times 8$
f $12 \div 3 \times 4 = 1$

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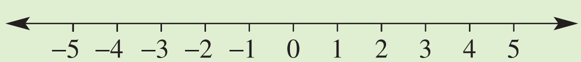
10 State the missing numbers for each part in this table.

a	$2 \times 2 = \square$	$\sqrt{4} = 2$
b	$3 \times 3 = \square$	$\sqrt{9} = \square$
c	$4 \times 4 = \square$	$\sqrt{16} = \square$
d	$6 \times 6 = \square$	$\sqrt{36} = \square$
e	$9 \times 9 = \square$	$\sqrt{\square} = 9$
f	$10 \times 10 = \square$	$\sqrt{\square} = 10$
g	$\square \times \square = 49$	$\sqrt{49} = \square$
h	$\square \times \square = 144$	$\sqrt{144} = \square$

11 What are the next two numbers in each of these patterns?

a $3, 2, 1,$ ___, ___
b $2, 0, -2,$ ___, ___
c $-9, -10, -11,$ ___, ___

12 Use this number line to help find the answer.



a $2 - 5$
b $0 - 3$
c $-4 + 6$
d $-2 + 7$

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1A Adding and subtracting positive integers CONSOLIDATING

Learning intentions

- To understand the commutative law for addition.
- To be able to use the mental strategies of partitioning, compensating and doubling to calculate a sum or difference of whole numbers mentally.
- To be able to use the addition and subtraction algorithms to find the sum and difference of whole numbers.

Key vocabulary: sum, difference, algorithm, commutative law, compensating, doubling, counting on

The number system that we use today is called the Hindu–Arabic or decimal system. It uses the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9.

The value of each digit depends on its place in the number, so, for example, the 4 in 3407 has a place value of 400. Whole numbers include 0 (zero) and the counting (natural) numbers $1, 2, 3, 4, \ldots$ The Counting numbers $1, 2, 3, 4, \ldots$ are called positive integers. We can add or subtract whole numbers to find sums and differences.

Lesson starter: Sum and difference

Use a guess-and-check method to try to find a pair of numbers described by these sentences.

- The sum of two numbers is 41 and their difference is 11.
- The sum of two numbers is 41 and their difference is 1.

Describe the meaning of the words 'sum' and 'difference'. Discuss how you found the pair of numbers in each case.

Key ideas

- You can add in any order.
  e.g. $7 + 5 = 5 + 7$
  $9 + 3 + 1 = 9 + 1 + 3$
  - This is called the **commutative law** for addition.
- You cannot subtract in any order.
  e.g. $7 - 5 \neq 5 - 7$
- If the numbers are large, write numbers in columns and use known **algorithms** to calculate the answer.

$$\begin{array}{r} {}^{1}4\,3\,1 \\ +\,8\,9\,5 \\ \hline 1\,3\,2\,6 \end{array} \qquad \begin{array}{rrr} \cancel{3}^{2} & \cancel{2}^{1} & 4 \\ -\,1 & 7 & 2 \\ \hline 1 & 5 & 2 \end{array}$$

Exercise 1A

Understanding 1–3 3

1 Match each of the questions in the left-hand column (**a**, **b**, **c** and **d**) to the working out in the right-hand column (**I**, **II**, **III**, **IV**).

a The total of 156, 94 and 6 — **I** $\begin{array}{r} 2491 \\ +\ 945 \\ \hline \end{array}$

b Take 856 away from 2491 — **II** $2491 - 856$

c 945 more than 2491 — **III** $156 + 94 + 6$

d 945 less 863 — **IV** $\begin{array}{r} 945 \\ -\ 863 \\ \hline \end{array}$

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2 Write each of the following using an addition (+) or a subtraction (−) sign instead of the words. Do not work out the answer.

a 26 plus 17
b 43 take away 9
c 134 minus 23
d 451 add 50
e The sum of 19 and 29
f The sum of 111 and 236
g The difference between 59 and 43
h The difference between 339 and 298
i 36 more than 8
j 142 more than 421
k 32 less than 49
l 120 less than 251

3 Describe these sums and differences as true (T) or false (F)?

a $15 + 6 = 6 + 15$
b $29 - 6 = 6 - 29$
c $95 + 0 = 95$
d $81 - 81 = 0$
e $15 + 6 + 4 = 15 + 10$
f $41 - 6 + 4 = 41 - 10$

Fluency

4–7(½) 4–7(½)

Example 1 Using mental arithmetic

Evaluate this difference and these sums mentally.

a $347 - 39$
b $125 + 127$
c $28 + 13$

Solution	Explanation	
a $347 - 39 = 308$	$347 - 39 = 347 - 40 + 1$ $= 307 + 1$ $= 308$	This method is called compensating.
b $125 + 127 = 252$	$125 + 127 = 2 \times 125 + 2$ $= 250 + 2$ $= 252$	This method is called doubling.
c $28 + 13 = 41$	$28 + 13 = 28 + 12 + 1$ $= 40 + 1$ $= 41$	This method is called counting on.

Now you try

Evaluate this difference and these sums mentally.

a $194 - 99$
b $220 + 219$
c $53 + 18$

4 Complete these sums.

a $21 + 5$
b $3 + 14$
c $17 + 13$
d $298 + 2$
e $35 + 11$
f $16 + 19$
g $21 + 5$
h $6 + 18$

Hint for Q4 and Q5:
Do these without a calculator or algorithm.

5 Complete these differences.

a $5 - 2$
b $16 - 4$
c $16 - 14$
d $21 - 21$
e $16 - 3$
f $45 - 13$
g $52 - 12$
h $52 - 14$

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1A

6 Evaluate these sums and differences mentally.

a $94-62$	**b** $146+241$	**c** $1494-351$	**d** $36+19$
e $138+25$	**f** $251-35$	**g** $99-20$	**h** $441-50$
i $350+351$	**j** $115+114$	**k** $80-41$	**l** $320-159$

Example 2 Using an algorithm

Use an algorithm to find this sum and difference.

a $\begin{array}{r} 938 \\ +\,217 \\ \hline \end{array}$ **b** $\begin{array}{r} 141 \\ -\;\;86 \\ \hline \end{array}$

Solution

a $\begin{array}{r} 9\,^{1}3\,8 \\ +\,2\;1\;7 \\ \hline 1\;1\;5\;5 \end{array}$

Explanation

$8+7=15$ (carry the 1 to the tens column)
$1+3+1=5$
$9+2=11$

Solution

b $\begin{array}{r} \cancel{1}\,^{13}\cancel{4}\,^{1}1 \\ -\;8\;6 \\ \hline 5\;5 \end{array}$

Explanation

Borrow from the tens column then subtract 6 from 11. Now borrow from the hundreds column and then subtract 8 from 13.

Now you try

Use an algorithm to find this sum and difference.

a $\begin{array}{r} 862 \\ +\,219 \\ \hline \end{array}$ **b** $\begin{array}{r} 362 \\ -\;\;76 \\ \hline \end{array}$

7 Use an algorithm to find these sums and differences.

a $\begin{array}{r} 128 \\ +\;\;46 \\ \hline \end{array}$ **b** $\begin{array}{r} 94 \\ +\,337 \\ \hline \end{array}$ **c** $\begin{array}{r} 9014 \\ +\;\;927 \\ +\;\;421 \\ \hline \end{array}$ **d** $\begin{array}{r} 814 \\ +\,1439 \\ +\;\;326 \\ \hline \end{array}$

e $\begin{array}{r} 94 \\ -\;36 \\ \hline \end{array}$ **f** $\begin{array}{r} 421 \\ -\,204 \\ \hline \end{array}$ **g** $\begin{array}{r} 1726 \\ -\,1699 \\ \hline \end{array}$ **h** $\begin{array}{r} 14\,072 \\ -\;\;\;328 \\ \hline \end{array}$

i $\begin{array}{r} 428 \\ +\,314 \\ +\,107 \\ +\;\;29 \\ \hline \end{array}$ **j** $\begin{array}{r} 1004 \\ +\;\;2407 \\ +\;\;9116 \\ +\,10\,494 \\ \hline \end{array}$ **k** $\begin{array}{r} 3017 \\ -\,2942 \\ \hline \end{array}$ **l** $\begin{array}{r} 10\,024 \\ -\;\;\;936 \\ \hline \end{array}$

Hint for Q7: Carry the 1 for sums larger than 9 and borrow 'ten' for subtraction.

Problem-solving and reasoning

8, 9 | 9–11

8 A racing bike's odometer shows 21 432 km at the start of a race and 22 110 km at the end of the race. How far was the race?

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9 Kristian has \$246 more than Sally. David has \$56 less than Sally. If Sally has \$492, how much do Kristian and David each have?

10 Callum walks 15 km on Monday and 3 km more each day. How many kilometres does Callum walk on Thursday?

11 The sum of two numbers is 39 and their difference is 5. What is the larger number?

Filling the gap and magic triangles — 12

12 **a** Write the digit missing from these sums and differences.

i $\begin{array}{cccc} & 2 & 3 & 7 \\ + & & 4 & \square \\ \hline & 2 & 7 & 9 \end{array}$ **ii** $\begin{array}{ccc} & 4 & 9 \\ + & 3 & 8 \\ \hline & 8 & \square \end{array}$ **iii** $\begin{array}{cccc} & 4 & 9 & 3 \\ + & 2 & 1 & 4 \\ \hline & 7 & \square & 7 \end{array}$ **iv** $\begin{array}{cccc} & 1 & \square & 4 \\ + & 3 & 9 & 2 \\ \hline & 5 & 5 & 6 \end{array}$

v $\begin{array}{ccc} & 3 & 8 \\ - & 1 & 9 \\ \hline & 1 & \square \end{array}$ **vi** $\begin{array}{cccc} & 1 & 2 & 8 \\ - & & 8 & \square \\ \hline & & 3 & 9 \end{array}$ **vii** $\begin{array}{cccc} & 3 & \square & 4 \\ - & 1 & 6 & 2 \\ \hline & 1 & 4 & 2 \end{array}$ **viii** $\begin{array}{cccc} & 2 & 5 & 1 \\ - & 1 & \square & 4 \\ \hline & & 8 & 7 \end{array}$

b Find the missing digits in these sums and differences.

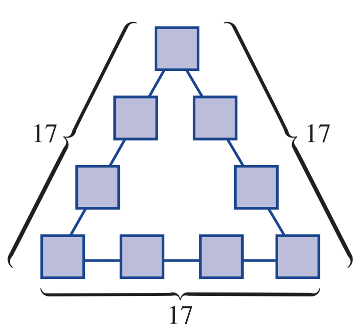
i $\begin{array}{cccc} & 2 & 3 & \square \\ + & \square & 9 & 4 \\ \hline & 6 & \square & 1 \end{array}$ **ii** $\begin{array}{cccc} & \square & 3 & \square \\ + & & \square & 2 \\ \hline & 2 & 1 & 9 \end{array}$ **iii** $\begin{array}{cccc} & \square & 3 & 7 \\ + & 4 & 9 & \square \\ \hline & 7 & \square & 2 \end{array}$

iv $\begin{array}{ccc} & \square & 3 \\ - & 2 & 9 \\ \hline & 6 & \square \end{array}$ **v** $\begin{array}{cccc} & 3 & \square & 2 \\ - & \square & 3 & \square \\ \hline & 1 & 0 & 4 \end{array}$ **vi** $\begin{array}{ccccc} & 2 & \square & \square & 5 \\ - & & 6 & 8 & \square \\ \hline & \square & 3 & 1 & 8 \end{array}$

c The sides of a magic triangle all sum to the same total.

i Show how it is possible to arrange all the digits from 1 to 9 so that each side adds to 17.

ii Show how it is possible to arrange the same digits to a different total. How many different totals can you find?



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1B Multiplying and dividing positive integers CONSOLIDATING

Learning intentions

- To understand the commutative and distributive law for multiplication.
- To be able to use mental strategies to calculate simple products and quotients.
- To be able to use the multiplication and division algorithms to find the product and quotient of whole numbers.

Key vocabulary: product, quotient, remainder, distributive law, commutative law

Multiplying and dividing are two key operations in mathematics and are useful in many practical situations such as finding the cost of 9 tickets at \$109 each or the number of trucks needed to carry 280 tonnes of coal.

Lesson starter: Multiplication or division?

In solving many problems it is important to know whether multiplication or division should be used. Decide if the following situations require the use of multiplication or division. Discuss them in a group or with a partner.

- The number of cookies 4 people can get if a packet of 32 cookies is shared equally between them.
- The cost of paving 30 square metres of courtyard at a cost of \$41 per square metre.
- The number of sheets of paper in 4000 boxes of 5 reams each (1 ream is 500 sheets).
- The number of hours I can afford a plumber at \$75 per hour if I have a fixed budget of \$1650.

Make up your own situation that requires the use of multiplication and another for division.

Key ideas

- A **product** is the result of multiplication.
- Multiplication can be done:
  - mentally
    e.g. $6 \times 5 = 30$
  - using an algorithm
    e.g.
$$\begin{array}{r l} 217 & \\ \times\ \ 26 & \\ \hline 1302 & \leftarrow 217 \times 6 \\ 4340 & \leftarrow 217 \times 20 \\ \hline 5642 & \leftarrow 1302 + 4340 \end{array}$$
- You can multiply numbers in any order.
  e.g. $6 \times 5 = 30$ and $5 \times 6 = 30$
  - This is the **commutative law** for multiplication.
- The **distributive law** is helpful when multiplying.
  e.g.
$$\begin{aligned} 5 \times 34 &= 5 \times (30 + 4) \\ &= 5 \times 30 + 5 \times 4 \\ &= 150 + 20 \\ &= 170 \end{aligned}$$
- Using division results in finding a **quotient** and a **remainder**.

  e.g. $\underset{\text{dividend}}{38} \div \underset{\text{divisor}}{11} = \underset{\text{quotient}}{3}$ and 5 remainder or $38 \div 11 = 3\frac{5}{11}$

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- Division can be done:
  - mentally
    e.g. $56 \div 8 = 7$
  - using an algorithm
    $$\begin{array}{r} 7\ 3\ 2 \\ 7\overline{)5\ 1^{2}2^{1}4} \end{array}$$

Exercise 1B

Understanding 1–3 3

1 Match each of the questions to the working out on the right.

a	The product of 9 and 6	**I**	15×12
b	36 divided by 12	**II**	$15 \div 5$
c	15 lots of 12	**III**	9×6
d	The quotient when 15 is divided by 5	**IV**	$15 \div 12$
e	Divide 12 into 15	**V**	$36 \div 12$

2 Use your knowledge of the multiplication tables to answer the following.

a 5×8	**b** 11×9	**c** 6×7	**d** 9×8
e 11×6	**f** 12×11	**g** 8×4	**h** 7×9
i $100 \div 10$	**j** $88 \div 8$	**k** $121 \div 11$	**l** $144 \div 12$
m $56 \div 7$	**n** $33 \div 3$	**o** $65 \div 5$	**p** $78 \div 6$

Hint for Q2: You should know most of these off by heart.

3 Are these simple equations true (T) or false (F)?

a $4 \times 13 = 13 \times 4$
b $2 \times 7 \times 9 = 7 \times 9 \times 2$
c $6 \div 3 = 3 \div 6$
d $60 \div 20 = 30 \div 10$
e $14 \div 2 \div 7 = 7 \div 2 \div 14$
f $51 \times 7 = (50 \times 7) + (1 \times 7)$
g $79 \times 13 = (80 \times 13) - (1 \times 13)$
h $93 \div 3 = (90 \div 3) + (3 \div 3)$

Fluency 4–7(½) 4–7(½)

Example 3 Using mental strategies for multiplication

Use a mental strategy to evaluate the following.

a 5×160 **b** 7×89 **c** $5 \times 43 \times 2$

Solution	Explanation
a $5 \times 160 = 800$	To multiply by 5 you can multiply by 10 then halve the result. $160 \times 10 = 1600$, $1600 \div 2 = 800$
b $7 \times 89 = 623$	$89 = 90 - 1 \therefore 7 \times 89 = 7 \times 90 - 7 \times 1$ $= 630 - 7 = 623$ (this is the distributive law)
c $5 \times 43 \times 2 = 430$	$5 \times 43 \times 2 = \underbrace{5 \times 2}_{} \times 43$ look for easy pairs $= 10 \times 43$ $= 430$

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1B

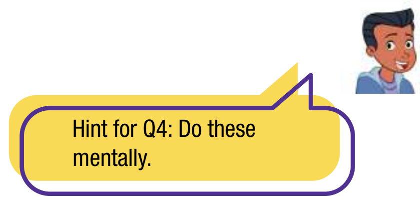
Now you try

Use a mental strategy to evaluate the following.

a 7×110 b 4×51 c $2 \times 36 \times 5$

4 Use a mental strategy to evaluate the following.

a 15×3 b 18×4 c $6 \times 5 \times 2$
d 7×20 e 16×4 f 99×7
g 79×3 h 42×5 i $5 \times 13 \times 2$
j $2 \times 26 \times 5$ k 4×35 l 17×4
m 17×1000 n 136×100 o 59×7
p 119×6 q 9×51 r 6×61
s 4×252 t 998×6



Example 4 Using mental strategies for division

Use a mental strategy to evaluate the following.

a $464 \div 4$ b $480 \div 5 \div 2$

Solution	Explanation
a $464 \div 4 = 116$	To divide by 4 you can divide by 2 twice. $464 \div 4 = 464 \div 2 \div 2$ ($\div$ 2 is the same as halving the number) $= 232 \div 2$ $= 116$
b $480 \div 5 \div 2 = 48$	Dividing by 5 and then by 2 is the same as dividing by 10. $480 \div 10 = 48$

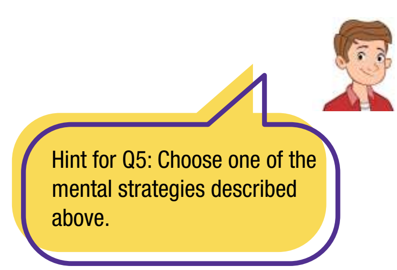
Now you try

Use a mental strategy to evaluate the following.

a $672 \div 8$ b $114 \div 6$

5 Use a mental strategy to evaluate the following.

a $64 \div 2$ b $64 \div 4$
c $640 \div 4$ d $492 \div 4$
e $370 \div 2 \div 5$ f $1980 \div 5 \div 2$
g $128 \div 8$ h $252 \div 4$
i $123 \div 3$ j $508 \div 4$
k $96 \div 6$ l $1016 \div 8$



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Example 5 Using an algorithm for multiplication and division

Use an algorithm to evaluate the following.

a $\begin{array}{r} 412 \\ \times \ \ 25 \\ \hline \end{array}$

b $938 \div 13$

Solution

a $\begin{array}{r} 412 \\ \times \quad 25 \\ \hline 2060 \\ 8240 \\ \hline 10300 \end{array}$

Explanation

$412 \times 5 = 2060$ and $412 \times 20 = 8240$
Add these two products to get the final answer.

Solution

b $\begin{array}{r} 7\ 2\ \text{ rem } 2 \\ 13\overline{)9\ 3\ {}^{2}8} \end{array}$

So $938 \div 13 = 72$ and 2 remainder.
$938 \div 13 = 72\frac{2}{13}$

Explanation

$93 \div 13 = 7$ and 2 remainder
$28 \div 13 = 2$ and 2 remainder
We write remainders as fractions $72\frac{2}{13}$.

Now you try

Use an algorithm to evaluate the following.

a $\begin{array}{r} 137 \\ \times \ \ 12 \\ \hline \end{array}$

b $354 \div 7$

6 Use an algorithm to evaluate the following.

a $\begin{array}{r} 67 \\ \times \ \ 9 \\ \hline \end{array}$

b $\begin{array}{r} 129 \\ \times \quad 4 \\ \hline \end{array}$

c $\begin{array}{r} 294 \\ \times \ \ 13 \\ \hline \end{array}$

d $\begin{array}{r} 1004 \\ \times \quad 90 \\ \hline \end{array}$

e $\begin{array}{r} 690 \\ \times \ \ 14 \\ \hline \end{array}$

f $\begin{array}{r} 96 \\ \times 12 \\ \hline \end{array}$

g $\begin{array}{r} 58 \\ \times 24 \\ \hline \end{array}$

h $\begin{array}{r} 163 \\ \times \ \ 52 \\ \hline \end{array}$

Hint for Q6: Use the setting out described in **Example 5**.

7 Use the short division algorithm to evaluate the following. Write your answer using fractions if there is a remainder.

a $3\overline{)85}$

b $7\overline{)214}$

c $10\overline{)4167}$

d $15\overline{)207}$

e $6\overline{)15\,084}$

f $3\overline{)1236}$

g $12\overline{)2520}$

h $12\overline{)8892}$

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1B

Problem-solving and reasoning

8–10 | 9–12

8 A university student earns $550 for 20 hours of work. What is the student's pay rate per hour?

9 Packets of biscuits are purchased by a supermarket in boxes of 12. The supermarket orders 220 boxes and sells 89 boxes in one day. How many boxes are left? How many packets of biscuits remain in the supermarket?

10 Riley buys a fridge which he can pay for by the following options.
  A 9 payments of $183
  B $1559 up front
  Which option is cheaper and by how much?

11 The shovel of a giant excavator can move 6 tonnes of rock in each load. How many loads are needed to shift 750 tonnes of rock?

12 Tom saves $362 a week. How much will he save in 52 weeks?

Maximum tickets

— | 13

13 A child ticket to a theatre is $7 and an adult ticket is $12.
  a Find the cost of 2 adult and 3 child tickets.
  b Find the cost of 1 adult and 5 child tickets.
  c Gen spends exactly $90 to buy child tickets and adult tickets. Find the maximum number of tickets that Gen could purchase.

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1C Squares, cubes and other powers

Learning intentions

- To understand the meaning of an expression written in the form a^n in terms of repeated multiplication of a.
- To be able to find the square, square root, cube and cube root of certain small whole numbers.

Key vocabulary: base, index, power, index notation, expanded form, product, square, square root, cube, cube root

In mathematics there are many ways to abbreviate expressions.

Using repeated addition, $4+4+4+4+4$ can be written as 5×4 using multiplication.

Using repeated multiplication, $3 \times 3 \times 3 \times 3$ can be written as 3^4 using index notation.

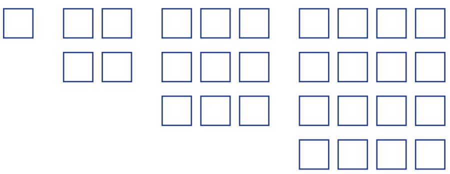
We read 3^4 as "3 to the power of 4".

Lesson starter: Square numbers

Can you explain why we call the numbers $1, 4, 9$ and 16 square numbers?

Draw diagrams for the next two square numbers.

Use centicubes to build the first three cube numbers. Write down the next cube number.



Key ideas

- **Index notation**

  index or power

  expanded form

  $3^4 \qquad 3^4 = 3 \times 3 \times 3 \times 3$

  base

  The **base** of 3 shows the factor that is repeating in multiplication, and the **power** or **index** is the number of times it appears.

- The **square** of a number is written a^2 and it means $a \times a$.
  e.g. 5^2 means 5×5 (we say 5 squared, the square of 5, or 5 to the power of 2)

- The opposite of squaring is finding the **square root** of a number. The symbol $\sqrt{\ }$ means square root.
  e.g. $\sqrt{9} = 3$ as $3^2 = 9$
  - The square root of a number is always positive or zero.

- The **cube** of a number a is $a^3 = a \times a \times a$.
  e.g. $5^3 = 5 \times 5 \times 5$ (we say 5 cubed, or, 5 to the power of 3)

- The opposite of cubing is taking the **cube root** of a number. The symbol for cube root is $\sqrt[3]{\ }$.
  e.g. $\sqrt[3]{8} = 2$ as $2^3 = 2 \times 2 \times 2 = 8$

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Exercise 1C

Understanding

	1–4	3, 4

1 Write each of the following using index notation.

a 2×2 b 4×4 c 5×5

d $5 \times 5 \times 5$ e $6 \times 6 \times 6 \times 6$ f $7 \times 7 \times 7$

2 Match each expression in words to an expression in symbols, given on the right.

Hint for Q2: The cube of 2 is $2^3 = 2 \times 2 \times 2 = 8$.

a The square of 10 I $\sqrt{16}$

b The cube of 1 II $\sqrt[3]{1}$

c The square of 12 III $\sqrt{1}$

d The square root of 1 IV 10^2

e The cube root of 1 V 1^3

f The square root of 16 VI 12^2

3 Copy and complete.

a $3^2 = 3 \times 3 =$ ______ b $7^2 =$ _____ = _______ c $11^2 =$ ______ = ______

4 Copy and complete.

a $2^3 = 2 \times 2 \times 2 =$ ______ b $5^3 =$ ________ = _______ c $10^3 =$ ________ = ________

Fluency

	5–9(½)	6–9(½)

Example 6 Using index notation

Write each product using index notation.

a $8 \times 8 \times 8$ b $7 \times 7 \times 7 \times 7 \times 7 \times 7$

Solution	Explanation
a $8 \times 8 \times 8 = 8^3$	The number 8 appears 3 times. We write 8 to the power of 3.
b $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$	The 7 appears 6 times. We write 7 to the power of 6.

Now you try

Write each product using index notation.

a $6 \times 6 \times 6 \times 6$ b $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

5 Write each of the following products using index notation.

a $7 \times 7 \times 7$ b $10 \times 10 \times 10 \times 10$ c 8×8

d $4 \times 4 \times 4$ e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ f $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

g 12×12 h $5 \times 5 \times 5 \times 5 \times 5 \times 5$ i 6

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Example 7 Using expanded notation

a Write 5^4 in expanded form.

b Find the value of 5^4.

Solution	Explanation
a $5^4 = 5 \times 5 \times 5 \times 5$	The power of 4 tells us that the number 5 appears 4 times. $5^4 = 5 \times 5 \times 5 \times 5$
b $5^4 = 625$	$5^4 = 5 \times 5 \times 5 \times 5$ $= 25 \times 5 \times 5$ $= 125 \times 5$ $= 625$

Now you try

a Write 2^5 in expanded form.

b Find the value of 2^5.

6 Write each index notation in expanded form.

a 8^5
b 3^4
c 9^2
d 4^4
e 2^8
f 11^2

Hint for Q6: $5 \times 5 \times 5$ is the expanded form of 5^3.

7 Find the value of the following by first writing them in expanded form.

a 2^3
b 2^4
c 3^3
d 10^4
e 5^3
f 1^4

Example 8 Finding squares, cubes, square roots and cube roots

Evaluate the following.

a 6^2
b $\sqrt{81}$
c 2^3
d $\sqrt[3]{64}$

Solution	Explanation
a $6^2 = 6 \times 6$ $= 36$	Find the product of 6 with itself.
b $\sqrt{81} = 9$	$9^2 = 9 \times 9 = 81$ so $\sqrt{81} = 9$
c $2^3 = 2 \times 2 \times 2$ $= 8$	In general $x^3 = x \times x \times x$.
d $\sqrt[3]{64} = 4$	$4^3 = 4 \times 4 \times 4 = 64$ so $\sqrt[3]{64} = 4$

Now you try

Evaluate the following.

a 9^2
b $\sqrt{144}$
c 3^3
d $\sqrt[3]{343}$

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1C

8 Evaluate these squares and square roots.

a 4^2	**b** 10^2	**c** 13^2
d 15^2	**e** 100^2	**f** 20^2
g $\sqrt{25}$	**h** $\sqrt{49}$	**i** $\sqrt{121}$
j $\sqrt{900}$	**k** $\sqrt{1600}$	**l** $\sqrt{256}$

Hint for Q8: $3^2 = 9$ and $\sqrt{9} = 3$.

9 Evaluate these cubes and cube roots.

a 2^3	**b** 4^3	**c** 7^3	**d** 5^3
e 6^3	**f** 10^3	**g** $\sqrt[3]{27}$	**h** $\sqrt[3]{8}$
i $\sqrt[3]{125}$	**j** $\sqrt[3]{512}$	**k** $\sqrt[3]{729}$	**l** $\sqrt[3]{1\,000\,000}$

Problem-solving and reasoning

10, 11 | 10–12

10 Decide which of the following is larger.

a 2^3 or 3^2 **b** 2^4 or 3^2 **c** 2^5 or 5^2

11 Copy and complete.

a If $13^2 = 169$, then $\sqrt{169} = \square$

b If $15^2 = 225$, then $\sqrt{225} = \square$

c If $\sqrt{625} = 25$, then $25^2 = \square$

d If $9^3 = 729$, then $\sqrt[3]{729} = \square$

e If $\sqrt[3]{1331} = 11$, then $11^3 = \square$

12 Given $5 \times 5 \times 5 \times 4 \times 4$ is written as $5^3 \times 4^2$ (the different bases of 5 and 4 are kept separate), write each of the following in index form.

a $6 \times 6 \times 7 \times 7 \times 7 \times 7$

b $5 \times 5 \times 5 \times 5 \times 2 \times 2$

c $3 \times 3 \times 8 \times 8$

d $11 \times 9 \times 9 \times 9 \times 9$

e $12 \times 12 \times 4 \times 4 \times 4$

f $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Algebraic indices

— | 13

13 Write each of the following in index form. Remember, different bases are kept separate.

a $m \times m \times m$

b $a \times a \times a \times a \times a$

c $n \times n \times n \times n \times n \times n \times n$

d $p \times p \times p \times p \times p \times p \times p \times p \times p \times p$

e $p \times p \times p \times q \times q$

f $a \times a \times a \times a \times b \times b$

g $a \times a \times b \times b \times b \times b$

h $x \times x \times x \times x \times y$

Hint for Q13:
$$\underbrace{\underbrace{a \times a}_{a^2} \times \underbrace{b \times b \times b}_{b^3}}_{}$$
$$\underbrace{a^2 \times b^3}_{a^2b^3}$$

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1D Number properties

Learning intentions

- To understand that a prime number has exactly two factors and a composite number has more than two factors.
- To be able to find the lowest common multiple (LCM) of two numbers.
- To be able to find the highest common factor (HCF) of two numbers.

Key vocabulary: counting numbers, multiple, factor, lowest common multiple (LCM), highest common factor (HCF), prime numbers, composite numbers

Simple properties of numbers are at the heart of more complex mathematics and associated problems. Prime numbers for example form the basis of our online banking encryption codes as it is very difficult to find the prime factors of large numbers.

Lesson starter: How many in 60 seconds?

In 60 seconds, write down as many numbers as you can that fit each description.

- Multiples of 7
- Factors of 144
- Prime numbers

Compare your lists with the results of the class. What is the largest prime number that the class came up with?

Key ideas

- A **multiple** of a number is obtained by multiplying the number by the **counting numbers** $1, 2, 3, \ldots$
  e.g. Multiples of 9 include $9, 18, 27, 36, 45, \ldots$ (think of your multiplication tables).
- The **lowest common multiple (LCM)** is the smallest multiple of two or more numbers that is common.
  e.g. Multiples of 3 are $3, 6, 9, 12, 15, 18, \ldots$
  Multiples of 5 are $5, 10, 15, 20, 25, \ldots$
  The LCM of 3 and 5 is therefore 15.
- A **factor** of a number has a remainder of zero when divided into the given number.
  e.g. 11 is a factor of 77 since $77 \div 11 = 7$ with 0 remainder.
- The **highest common factor (HCF)** is the largest factor of two or more numbers that is common.
  - Factors of 24 are $1, 2, 3, 4, 6, 8, 12, 24$.
  - Factors of 36 are $1, 2, 3, 4, 6, 9, 12, 18, 36$.
  - The HCF of 24 and 36 is therefore 12.
- **Prime numbers** have only two factors: the number itself and 1.
  - $2, 13$ and 61 are examples of prime numbers.
  - 1 is not considered to be a prime number. (It has only one factor.)
- **Composite numbers** have more than two factors.
  - $6, 20$ and 57 are examples of composite numbers.

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Exercise 1D

Understanding | 1–4 | 3, 4

1 Write down the factors of each number.
a 4 b 6 c 12 d 15 e 20

2 Write down the next term (multiple) in each of these patterns.
a 2, 4, 6, 8, __ b 3, 6, 9, 12, __ c 5, 10, 15, 20, 25, __
d 7, 14, 21, __ e 6, 12, 18, __ f 11, 22, 33, 44, __

3

The factors of 16 are 1, 2, 4, 8, 16.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
The factors of 18 are 1, 2, 3, 6, 9, 18.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30.
The factors of 8 are 1, 2, 4, 8.

Using the information given in the table, write down the HCF of each pair of numbers.
a 16 and 24 b 24 and 30 c 18 and 30 d 16 and 8
e 24 and 18 f 8 and 24 g 16 and 18 h 18 and 8

Hint for Q3: HCF is the Highest Common Factor.

4 Use the first six multiples of the numbers given to find the LCM of each pair of numbers.

Number	Multiples
2	2, 4, 6, 8, 10, 12
4	4, 8, 12, 16, 20, 24
3	3, 6, 9, 12, 15, 18
5	5, 10, 15, 20, 25, 30
6	6, 12, 18, 24, 30, 36

a 2 and 4 b 4 and 3 c 3 and 6
d 4 and 6 e 4 and 5 f 5 and 6

Hint for Q4: LCM is the Lowest Common Multiple.

Fluency | 5, 6, 7–8(½) | 5, 6, 7–8(½)

Example 9 Working with primes and composites

Decide whether each of the following is a prime number or a composite number.
a 29 b 63

Solution	Explanation
a 29 is a prime number	29 has only 2 factors — 1 and 29. It is a prime number.
b 63 is a composite number	63 has factors 1, 3, 7, 9, 21, 63

Now you try

Decide whether each of the following is a prime number or a composite number.
a 39 b 53

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5 Decide whether each of the following numbers is prime or composite.

a 7 **b** 12 **c** 27 **d** 69
e 105 **f** 28 **g** 15 **h** 11
i 31 **j** 37 **k** 49 **l** 99

Hint for Q5: Primes have exactly two factors, composites have more than two factors.

6 Choose the prime numbers from the following list:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

Example 10 Finding the LCM

Find the LCM of 6 and 8.

Solution	Explanation
Multiples of 6 are: 6, 12, 18, **24**, 30, … Multiples of 8 are: 8, 16, **24**, 32, 40, … The LCM is 24.	First, list some multiples of 6 and 8. Continue the lists until there is at least one in common. Choose the smallest number that is common to both lists.

Now you try

Find the LCM of 4 and 10.

7 Find the LCM of these pairs of numbers.

a 2, 3 **b** 5, 9
c 8, 12 **d** 4, 8
e 25, 50 **f** 4, 18
g 8, 60 **h** 12, 20
i 5, 7 **j** 10, 15
k 4, 12 **l** 12, 18

Example 11 Finding the HCF

Find the HCF of 36 and 48.

Solution	Explanation
Factors of 36 are: 1, 2, 3, 4, 6, 9, **12**, 18, 36 Factors of 48 are: 1, 2, 3, 4, 6, 8, **12**, 16, 24, 48 The HCF is 12.	First, list factors of 36 and 48. Choose the largest number that is common to both lists.

Now you try

Find the HCF of 24 and 32.

8 Find the HCF of these pairs of numbers.

a 6, 8 **b** 18, 9 **c** 16, 24
d 24, 30 **e** 7, 13 **f** 19, 31
g 72, 36 **h** 108, 64 **i** 6, 4
j 6, 12 **k** 8, 24 **l** 15, 25

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1D

Problem-solving and reasoning

9–11 | 9, 11, 12

9 Find:

- **a** the LCM of 8, 12 and 6
- **b** the LCM of 7, 3 and 5
- **c** the HCF of 20, 15 and 10
- **d** the HCF of 32, 60 and 48

10 A teacher has 64 students to divide into equal groups of greater than 2 with no remainder. In how many ways can this be done?

11 Below are the numbers 1 to 100.
List all the prime numbers. How many numbers are prime numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

12 Three sets of traffic lights (A, B and C) all turn red at 9:00 am exactly. Light set A turns red every 2 minutes, light set B turns red every 3 minutes and light set C turns red every 5 minutes. How long does it take for all three lights to turn red again at the same time?

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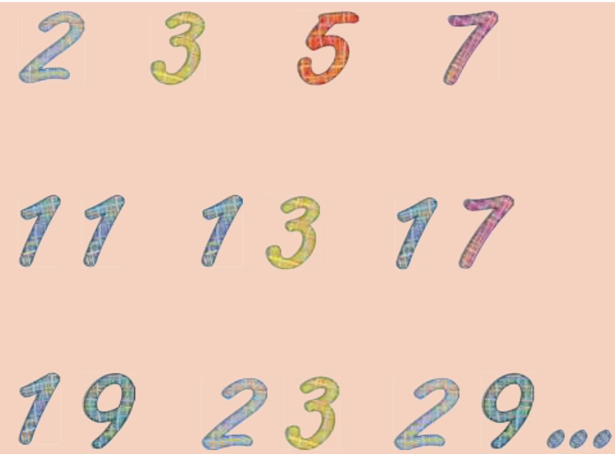
Goldbach's conjecture and twin primes | — | 13, 14

13 *Goldbach's conjecture* is a famous mathematical statement that says that every even number greater than two can be written as the sum of two prime numbers.

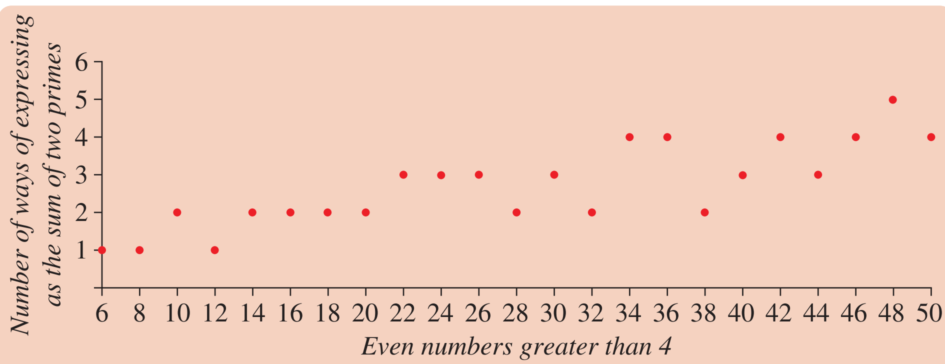
The even numbers 4, 6 and 8 have been written as the sum of two primes.

Show how the even numbers 10 to 30 can be written as the sum of two primes. Some can be done in more than one way.

$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 =$
$12 =$
$14 =$
$16 =$
$18 =$
$20 =$
$22 =$
$24 =$
$26 =$
$28 =$
$30 =$



The first ten prime numbers.



A graph illustrating Goldbach's conjecture up to and including 50 is obtained by plotting the number of ways of expressing even numbers greater than 4 as the sum of two primes.

14 Twin primes are pairs of prime numbers that differ by 2. It has been suggested that there are infinitely many twin primes. Use the table of primes you created in Question 11 of this exercise and list the pairs of twin primes less than 100.

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1E Divisibility and prime factorisation

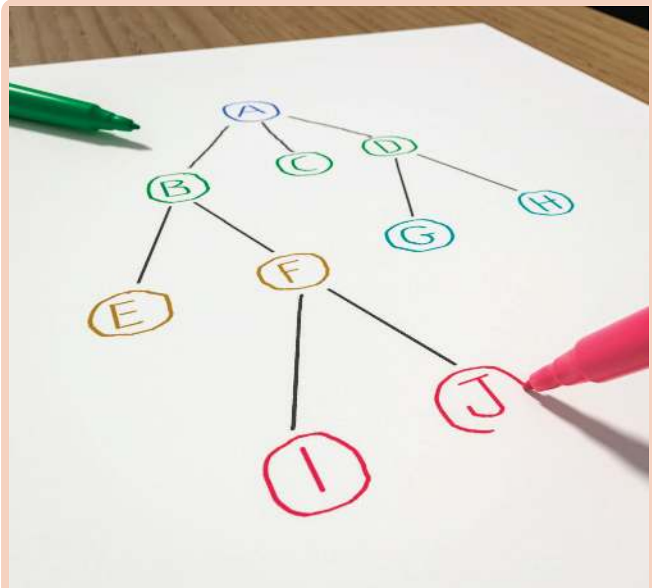
Learning intentions

- To be able to write a number as a product of prime factors.
- To be able to construct a factor tree.
- To be able to use the divisibility tests for single-digit factors other than 7.
- To understand how the lowest common multiple and highest common factor of two numbers can be found using their prime factor form.

Key vocabulary: prime number, factor tree, highest common factor (HCF), lowest common multiple (LCM), divisibility tests, prime factorisation

Every whole number greater than 1 can be written as a product of prime numbers, e.g. $6 = 3 \times 2$ and $20 = 2 \times 2 \times 5$.

Writing numbers as a product of prime numbers can help to simplify expressions and determine other properties of numbers or pairs of numbers.



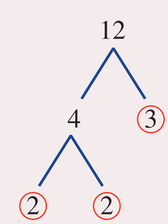
A factor tree can be used to see how a number can be broken down into its prime factors.

Lesson starter: Remembering divisibility tests

To test if a number is divisible by 2, we simply need to see if the number is even or odd. All even numbers are divisible by 2. As a class, can you describe divisibility tests for any of the following?

- Divisible by 3
- Divisible by 4
- Divisible by 5
- Divisible by 6
- Divisible by 8
- Divisible by 9
- Divisible by 10

Key ideas



- A **factor tree** is an illustrated breakdown of a number into its prime factors.
- **Prime factorisation** uses a factor tree, or similar, to write a number as a product of its prime factors.
  e.g. $12 = 2 \times 2 \times 3$ or $2^2 \times 3$ (using indices)
- The **highest common factor (HCF)** can be found using prime factors.
  The HCF = All common primes raised to the smallest power.
  e.g. $12 = 2^2 \times 3 \quad 20 = 2^2 \times 5 \quad \therefore$ HCF $= 2^2$ or 4.
- The **lowest common multiple (LCM)** can be found using prime factors.
  The LCM = All different primes raised to the highest power.
  e.g. $12 = 2^2 \times 3 \quad 20 = 2^2 \times 5 \quad \therefore$ LCM $= 2^2 \times 3 \times 5 = 60$

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- **Divisibility tests**

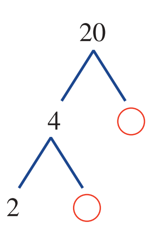
  A number is:
  - divisible by **2** if it is even (ends with the digit 0, 2, 4, 6 or 8), e.g. 24
  - divisible by **3** if the sum of all the digits is divisible by 3
    e.g. 162 where $1 + 6 + 2 = 9$, which is divisible by 3
  - divisible by **4** if the number formed by the last two digits is divisible by 4
    e.g. 148 where 48 is divisible by 4
  - divisible by **5** if the last digit is a 0 or 5
    e.g. 145 or 2090
  - divisible by **6** if it is divisible by both 2 and 3
    e.g. 456 where 6 is even and $4 + 5 + 6 = 15$, which is divisible by 3
  - divisible by **8** if the number formed from the last 3 digits is divisible by 8, or if the last three digits are 000
    e.g. 2112 where 112 is divisible by 8 and 2000 which ends in 000
  - divisible by **9** if the sum of all the digits are divisible by 9
    e.g. 3843 where $3 + 8 + 4 + 3 = 18$ which is divisible by 9
  - divisible by **10** if the last digit is a 0
    e.g. 4230
  - There is no simple test for 7.

Exercise 1E

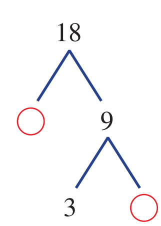
Understanding	1, 2	2

1 Give the missing numbers in these factor trees.

a



b



2 Give the missing word or number. A number is divisible by:

a 5 if the last digit is 0 or ______________.

b 2 if the last digit is ________.

c 10 if the last digit is _________.

d 8 if the number formed by the last _________ digits is divisible by 8.

e 6 if it is divisible by both 2 and _________.

f 3 if the sum of the digits is divisible by _________.

g 9 if the _________ of the digits is divisible by 9.

h 4 if the number formed by the last _________ digits is divisible by 4.

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1E

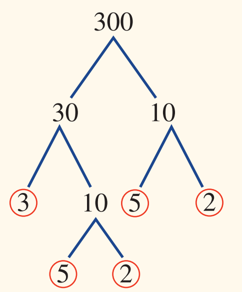
Fluency

3–5(½), 6, 7 | 4–5(½), 6, 7, 8(½)

Example 12 Finding prime factor form

Use a factor tree to write 300 as a product of prime factors.

Solution



$300 = 2 \times 2 \times 3 \times 5 \times 5$
$= 2^2 \times 3 \times 5^2$

Explanation

First, divide 300 into the product of **any** two factors.
Choose the easiest pair.
$300 = 30 \times 10$.

Continue dividing numbers into two factors until the factors are prime.

Circle the prime factors.

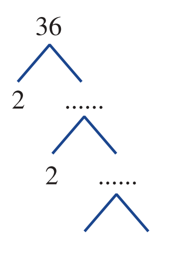
Write the factors in ascending order.

Use index notation (powers) to abbreviate your answer.

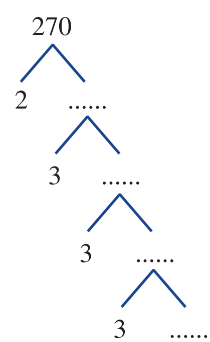
Now you try

Use a factor tree to write 224 as a product of prime factors.

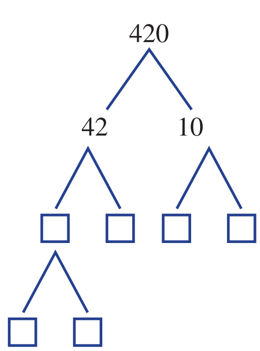
3 Copy and complete these factor trees to help write the prime factor form of the given numbers.

a


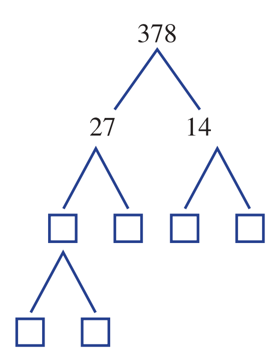
$\therefore 36 = 2^2 \times ......$

b


$\therefore 270 = 2 \times ...... \times ......$

c


420 = ________

d


378 = ________

4 Use a factor tree to find the prime factor form of these numbers.

a 20 b 28 c 40 d 90
e 280 f 196 g 360 h 660

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Example 13 Testing for divisibility

Use divisibility tests to decide if the number 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.

Solution	Explanation
Not divisible by **2** since 7 is odd.	The last digit needs to be even.
Divisible by **3** since $6+2+7=15$ and this is divisible by 3.	The sum of all the digits needs to be divisible by 3.
Not divisible by **4** as 27 is not divisible by 4.	The number formed from the last two digits needs to be divisible by 4.
Not divisible by **5** as the last digit is not a 0 or 5.	The last digit needs to be a 0 or 5.
Not divisible by **6** as it is not divisible by 2.	The number needs to be divisible by both 2 and 3.
Not divisible by **8** as the last 3 digits together are not divisible by 8.	The number formed from the last three digits needs to be divisible by 8.
Not divisible by **9** as $6+2+7=15$ which is not divisible by 9.	The sum of all the digits needs to be divisible by 9.

Now you try

Use divisibility tests to decide if the number 342 is divisible by 2, 3, 4, 5, 6, 8 or 9.

5 Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 51 **b** 126 **c** 248
d 387 **e** 315 **f** 517
g 894 **h** 3107

Hint for Q5: Do the seven tests on each number.

Example 14 Finding the LCM and HCF

Find the LCM and HCF of 105 and 90, using prime factorisation.

Solution	Explanation
$105 = 3 \times 5 \times 7$ $90 = 2 \times 3^2 \times 5$	First, express each number in prime factor form. Note that 3 and 5 are common primes.
$\text{LCM} = 2 \times 3^2 \times 5 \times 7$ $= 630$	For the LCM include all the different primes, raising the common primes to their highest power.
$\text{HCF} = 3 \times 5$ $= 15$	For the HCF include only the common primes raised to the smallest power. 105 and 90 **both** have one 3 and one 5.

Now you try

Find the LCM and HCF of 18 and 42, using prime factorisation.

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6 Copy and complete this table of LCMs and HCFs.

	Number 1	Number 2	LCM	HCF
a	$\mathbf{48} = 2^4 \times 3$	$\mathbf{30} = 2 \times 3 \times 5$		
b	$\mathbf{250} = 2 \times 5^3$	$\mathbf{900} = 2^2 \times 3^2 \times 5^2$		
c	$\mathbf{54} = 2 \times 3^3$	$\mathbf{96} = 2^5 \times 3$		
d	$\mathbf{245} = 5 \times 7^2$	$\mathbf{350} = 2 \times 5^2 \times 7$		
e	$\mathbf{198} = 2 \times 3^2 \times 11$	$\mathbf{693} = 3^2 \times 7 \times 11$		

7 Find the highest common prime factors of these pairs of numbers.
a 10, 45　b 42, 72　c 24, 80　d 539, 525

8 Find the LCM and the HCF of these pairs of numbers, using prime factorisation.
a 10, 12　b 14, 28　c 15, 24　d 12, 15
e 20, 28　f 13, 30　g 42, 9　h 270, 420

Problem-solving and reasoning　9　9, 10

9 What is the smallest number that can be divided, without giving a remainder, by all of the following four numbers?
a 2, 3, 4 and 6　b 2, 6, 8 and 9　c 2, 5, 15 and 6

10 Nana Magoo's two grandchildren love to visit her. Lachlan visits her every 8 days while Bryce visits every 18 days. They both visited her last Monday. How many days will it be from that visit before they both visit her on the same day again?

Hint for Q10: You might like to make a list to help you here!

Find the missing digit　—　11

11 Use the divisibility rules given to you at the start of this section to find the missing digit for each of the following. In some cases there might be more than one digit that works. In these cases, list all the possible answers.

a 2 ☐ 6 if the number is divisible by 3 (remember to list all possible answers).

b 1 ☐ 35 if the number is divisible by 9.

c 4 ☐ 3 if the number is divisible by 3.

d 4 ☐ 3 if the number is divisible by 3 and 9.

e 276 ☐ if the number is divisible by 2.

f 276 ☐ if the number is divisible by 2 and 5.

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Progress quiz

1A **1** Evaluate these sums and these differences mentally.

a $55 + 38$ **b** $215 + 219$ **c** $146 - 25$ **d** $770 - 249$

1A **2** Use an algorithm to find these sums and these differences.

a $\begin{array}{r} 785 \\ +\ 438 \\ \hline \end{array}$ **b** $\begin{array}{r} 68 \\ +215 \\ +187 \\ +\ 11 \\ \hline \end{array}$ **c** $\begin{array}{r} 513 \\ -\ 378 \\ \hline \end{array}$ **d** $\begin{array}{r} 8139 \\ -\ 964 \\ \hline \end{array}$

1B **3** Use a mental strategy to evaluate the following.

a $5 \times 36 \times 2$ **b** 4×79 **c** $342 \div 3$ **d** $600 \div 4$

1B **4** Use an algorithm to evaluate the following.

a $\begin{array}{r} 72 \\ \times\ 31 \\ \hline \end{array}$ **b** $3720 \div 12$

1C **5** Write each product in index notation.

a $7 \times 7 \times 7 \times 7$
b $5 \times 5 \times 2 \times 2 \times 2$
c $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

1C **6** Evaluate the following.

a 5^2 **b** 2^6 **c** $\sqrt{100}$ **d** $\sqrt[3]{27}$

1D **7** Find the LCM of these pairs of numbers.

a $4, 6$ **b** $9, 15$

1D **8** Find the HCF of these pairs of numbers.

a $20, 35$ **b** $11, 17$ **c** $48, 72$

1E **9** Use a factor tree to write 240 as a product of prime factors.

1E **10** Use divisibility tests to decide if 72 is divisible by 2, 3, 4, 5, 6, 8, or 9.

1E **11** Find the LCM and HCF of 40 and 110, using prime factorisation.

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1F Negative integers CONSOLIDATING

Learning intentions

- To understand that integers can be negative, zero or positive.
- To understand how to use a number line to add or subtract positive integers.
- To be able to add a positive integer to a negative integer.
- To be able to subtract a positive integer from a positive or negative integer.

Key vocabulary: integer, positive number, negative number, number line

The Indian mathematician Brahmagupta set out rules for negative numbers in the 7th century.

Today, negative numbers are used in science, engineering and business. They help us describe opposites such as left and right, up and down, profit and loss, and temperatures above and below freezing.

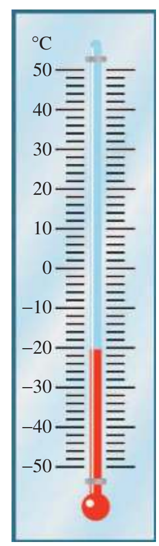
Negative number are used to describe temperatures below freezing (i.e. below $0°$ C).

Lesson starter: A negative world

Describe how to use negative numbers in these situations.

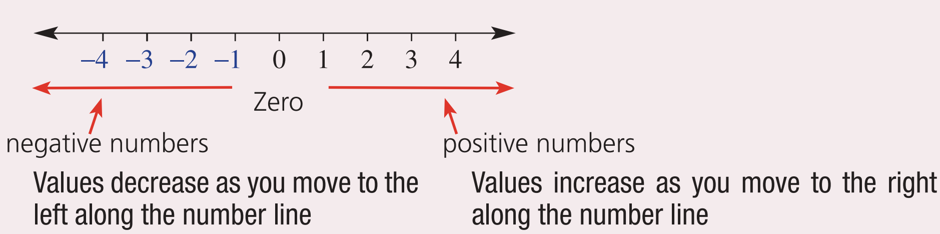
- $6°$C below zero
- A loss of $\$4200$
- 150 m below sea level
- A turn of $90°$ anticlockwise
- The solution to the equation $x + 5 = 3$

Can you describe another situation in which you might make use of negative numbers?



Key ideas

- **Negative numbers** are numbers less than zero.
- The **integers** are $\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots$
  - These include **positive** integers (natural numbers), zero and negative integers.
  - These are illustrated clearly on a **number line**.



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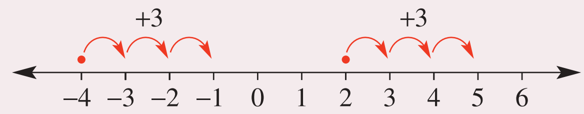
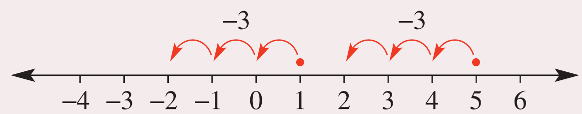
- Adding or subtracting a positive integer can result in a positive or negative number.
  - Adding a positive integer
    e.g. $2 + 3 = 5$
    $-4 + 3 = -1$



  - Subtracting a positive integer
    e.g. $1 - 3 = -2$
    $5 - 3 = 2$



Exercise 1F

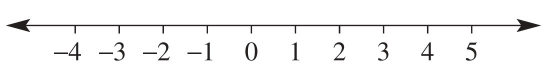
Understanding

1–4 | 3, 4

1 Write down the number suggested by:

a 2 above zero **b** 5 above zero **c** 3 below zero
d 10 below zero **e** 1 below zero

2 Copy the number line below and mark (with a dot) the integers $-3, -1, 1, 3$ and 5.



3 Write the symbol < (less than) or > (greater than) to make these statements true.

a 5 ___ -1 **b** -3 ___ 4 **c** -10 ___ 3 **d** -1 ___ -2
e -20 ___ -24 **f** -62 ___ -51 **g** 2 ___ -99 **h** -61 ___ 62

4 What is the final temperature?

a 10°C is reduced by 12°C **b** 32°C is reduced by 33°C
c −11°C is increased by 2°C **d** −4°C is increased by 7°C

Fluency

5–7(½) | 5–7(½), 8

Example 15 Adding a positive integer

Evaluate the following.

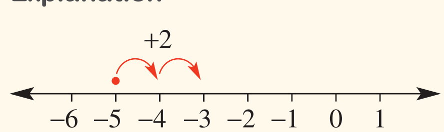
a $-5 + 2$ **b** $-1 + 4$

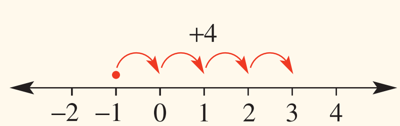
Solution

a $-5 + 2 = -3$

b $-1 + 4 = 3$

Explanation





Now you try

Evaluate the following.

a $-7 + 4$ **b** $-4 + 12$

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1F

5 Evaluate the following.

- **a** $-1 + 2$
- **b** $-3 + 7$
- **c** $-10 + 11$
- **d** $-4 + 12$
- **e** $-20 + 35$
- **f** $-6 + 4$
- **g** $-7 + 2$
- **h** $-15 + 8$
- **i** $-26 + 19$
- **j** $-38 + 24$
- **k** $-10 + 15$
- **l** $-2 + 9$
- **m** $-7 + 3$
- **n** $-7 + 7$
- **o** $-6 + 9$
- **p** $-6 + 1$

Hint for Q5: Start with the left number and move right on the number line.

Example 16 Subtracting a positive integer

Evaluate the following.

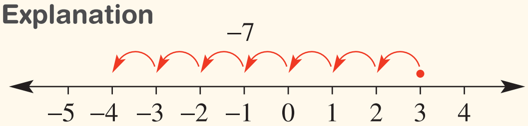
a $3 - 7$ **b** $-2 - 3$

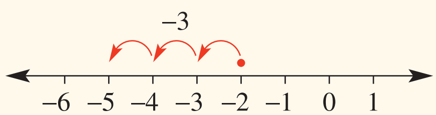
Solution

a $3 - 7 = -4$

b $-2 - 3 = -5$

Explanation





Now you try

Evaluate the following.

a $13 - 20$ **b** $-8 - 17$

6 Evaluate the following.

- **a** $4 - 5$
- **b** $10 - 15$
- **c** $0 - 26$
- **d** $14 - 31$
- **e** $6 - 8$
- **f** $10 - 9$
- **g** $-4 - 7$
- **h** $-11 - 20$
- **i** $-14 - 15$
- **j** $-10 - 100$
- **k** $-11 - 6$
- **l** $0 - 12$
- **m** $-15 - 5$
- **n** $3 - 12$
- **o** $8 - 4$
- **p** $-8 - 4$

Hint for Q6: Start with the left number and move left on the number line.

7 Evaluate the following.

- **a** $-9 + 6$
- **b** $-9 - 6$
- **c** $-12 + 12$
- **d** $-12 - 12$
- **e** $-7 - 7$
- **f** $-7 + 0$
- **g** $15 - 14$
- **h** $15 - 16$
- **i** $-9 - 10$
- **j** $-9 + 10$
- **k** $9 - 15$
- **l** $-20 + 10$
- **m** $100 - 101$
- **n** $-50 - 50$
- **o** $-5 + 25$
- **p** $-9 + 40$

8 Work from left to right to evaluate the following.

- **a** $-3 + 4 - 8 + 6$
- **b** $0 - 10 + 19 - 1$
- **c** $26 - 38 + 14 - 9$
- **d** $9 - 18 + 61 - 53$

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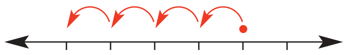
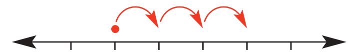
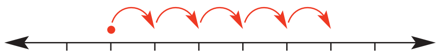
Problem-solving and reasoning

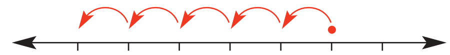
9, 10(½), 11 | 10(½), 11, 12

9 Write the sum (e.g. $-3+4=1$) or difference (e.g. $1-5=-4$) to match these number lines.

a 

b 

c 

d 

10 Write the missing number.

a $-1+\_\_\_=5$ **b** $\_\_\_+30=26$ **c** $\_\_\_+11=-3$ **d** $-32+\_\_\_=-21$

e $5-\_\_\_=-10$ **f** $\_\_\_-17=-12$ **g** $\_\_\_-4=-7$ **h** $-26-\_\_\_=-38$

11 In a high-rise building there are 8 floors above ground level and 6 floors below ground level. A lift starts at the 2nd floor and moves 4 floors up, then 7 floors down before moving down a further 3 floors.

At what floor does the lift finish?

12 On Monday Milly borrows \$35 from a friend. On Tuesday she pays her friend \$40. On Friday she borrows \$42 and pays back \$30 that night. How much does Milly owe her friend then?

Budgets and zero

— | 13, 14

13 **a** Complete Suzanne's account for the week shown. A credit is an addition (+) and a debit is a subtraction (−).

Spending and earning	Credit (+)	Debit (−)	Balance
opening balance			\$500
pays 1 week's rent of \$375		375	
earns \$80 babysitting			
receives \$100 from her parents for her birthday			
buys a pair of jeans for \$90			
buys a top for \$45			
pays her monthly mobile phone bill \$49			
gives \$25 to charity			

b How much would Suzanne need to deposit (credit) into her account so that she can pay the rent for the next week?

14 Find what integer needs to be added or subtracted to each so that the end result is always zero.

a $-6\ \_\_\_\_=0$ **b** $-8\ \_\_\_\_=0$

c $16\ \_\_\_\_=0$ **d** $10-7\ \_\_\_\_=0$

e $-9+7\ \_\_\_\_=0$ **f** $-9-7-2\ \_\_\_\_=0$

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1G Adding and subtracting negative integers

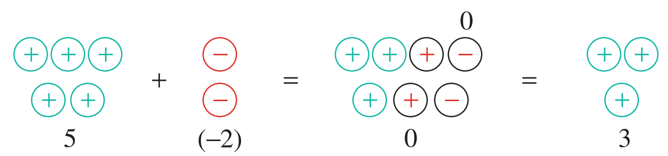
Learning intentions

- To understand that adding a negative number is the same as subtracting its opposite.
- To understand that subtracting a negative number is the same as adding its opposite.
- To be able to add or subtract negative integers.

Key vocabulary: integer, positive number, negative number, opposite

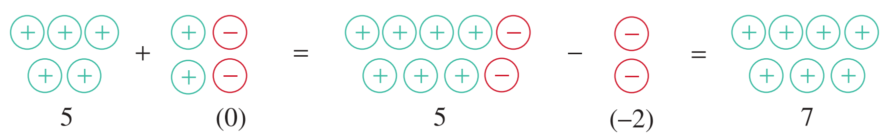
If $\oplus$ represents $+1$ and $\ominus$ represents -1 then $\oplus\ominus$ added together has a value of zero.

Using these symbols, $5 + (-2) = 3$ could be illustrated as the addition of 2 $\ominus$, leaving a balance of 3.



So $5 + (-2)$ is the same as $5 - 2$.

Also $5 - (-2) = 7$ could be illustrated first as 5 $\oplus$ and 2 $\ominus$ together then subtracting the 2 $\ominus$.



So $5 - (-2)$ is the same as $5 + 2$.

When adding or subtracting negative integers we follow the rules set out by the above two illustrations, as well as the patterns below.

Lesson starter: Looking at patterns for adding and subtracting negative numbers

Copy and complete.

A

$6 + 4$	10	
$6 + 3$	9	
$6 + 2$	8	
$6 + 1$		
$6 + 0$		
$6 + (-1)$		$\rightarrow$ same as $6 - 1 = 5$
$6 + (-2)$		$\rightarrow$ same as $6 \;\square\; 2 =$
$6 + (-3)$		$\rightarrow$ same as $6 \;\square\; 3 =$
$6 + (-4)$		$\rightarrow$ same as $6 \;\square\; 4 =$

B

$6 - 4$	2	
$6 - 3$	3	
$6 - 2$	4	
$6 - 1$		
$6 - 0$		
$6 - (-1)$		$\rightarrow$ same as $6 + 1 =$
$6 - (-2)$		$\rightarrow$ same as $\square$
$6 - (-3)$		$\rightarrow$ same as $\square$
$6 - (-4)$		$\rightarrow$ same as $\square$

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Key ideas

- The **opposite** of a number differs by a factor of -1.
  e.g. The opposite of 7 is -7 and the opposite of -12 is 12.
- Adding a **negative number** is the same as subtracting its opposite.
  e.g. $2 + (-3) = 2 - 3 = -1$ two opposite signs give a subtraction/minus
  $-4 + (-7) = -4 - 7 = -11$
- Subtracting a negative number is the same as adding its opposite.
  e.g. $2 - (-5) = 2 + 5 = 7$ two like signs give an addition/plus
  $-6 - (-4) = -6 + 4 = -2$

Exercise 1G

Understanding 1–3 3

1 -3 and 3 are opposites. Write down the opposites of these numbers.
a -6 **b** 10 **c** 38 **d** -46
e -32 **f** 88 **g** 673 **h** -349

2 Write the words 'add' or 'subtract' to suit each sentence.
a To add a negative number, ________________ its opposite.
b To subtract a negative number, ________________ its opposite.

3 Are the following statements true (T) or false (F)?
a $5 + (-2) = 5 + 2$ **b** $3 + (-4) = 3 - 4$ **c** $-6 + (-4) = -6 - 4$
d $-1 + (-3) = 1 - 3$ **e** $8 - (-3) = 8 + 3$ **f** $2 - (-3) = 2 - 3$
g $-3 - (-1) = 3 + 1$ **h** $-7 - (-5) = -7 + 5$ **i** $-6 - (-3) = 6 + 3$

Fluency 4–6(½) 4–6(½)

Example 17 Adding negative numbers

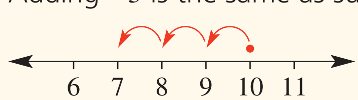
Evaluate the following.
a $10 + (-3)$ **b** $-3 + (-5)$

Solution

a $10 + (-3) = 10 - 3$
$= 7$

Explanation

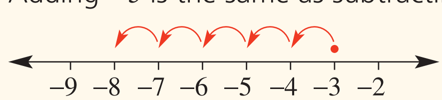
Adding -3 is the same as subtracting 3.



$\boxed{+}$ and $\boxed{-}$ = $\boxed{-}$

b $-3 + (-5) = -3 - 5$
$= -8$

Adding -5 is the same as subtracting 5.



$\boxed{+}$ and $\boxed{-}$ = $\boxed{-}$

Now you try

Evaluate the following.
a $24 + (-7)$ **b** $-13 + (-5)$

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4 Evaluate the following.

a $6+(-2)$	**b** $4+(-1)$	**c** $7+(-12)$	**d** $20+(-5)$
e $2+(-4)$	**f** $26+(-40)$	**g** $-3+(-6)$	**h** $-16+(-5)$
i $-18+(-20)$	**j** $-36+(-50)$	**k** $-83+(-22)$	**l** $-120+(-10)$
m $7+(-8)$	**n** $-9+(-12)$	**o** $6+(-12)$	**p** $-6+(-12)$
q $-8+(-8)$	**r** $5+(-5)$	**s** $-70+(-15)$	**t** $-100+(-6)$

Hint for Q4: To add a negative, subtract its opposite.

Example 18 Subtracting negative numbers

Evaluate the following.

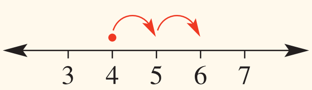
a $4-(-2)$ **b** $-11-(-6)$

Solution

a $4-(-2)=4+2$
$=6$

Explanation

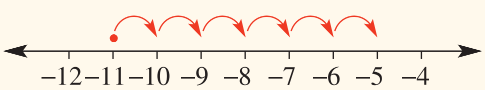
Subtracting -2 is the same as adding 2.



$\boxed{-}$ and $\boxed{-}=\boxed{+}$

b $-11-(-6)=-11+6$
$=-5$

Subtracting -6 is the same as adding 6.



$\boxed{-}$ and $\boxed{-}=\boxed{+}$

Now you try

Evaluate the following.

a $9-(-12)$ **b** $-32-(-4)$

5 Evaluate the following.

a $2-(-3)$	**b** $4-(-4)$	**c** $15-(-6)$
d $24-(-14)$	**e** $59-(-13)$	**f** $147-(-320)$
g $-5-(-3)$	**h** $-8-(-10)$	**i** $-13-(-16)$
j $-10-(-42)$	**k** $-88-(-31)$	**l** $-125-(-5)$
m $60-(-5)$	**n** $-60-(-5)$	**o** $-12-(-12)$
p $-10-(-18)$	**q** $41-(-41)$	**r** $48-(-52)$
s $-46-(-8)$	**t** $-170-(-12)$	

Hint for Q5: To subtract a negative, add its opposite.

6 Evaluate the following mixed problems.

a $46-50$	**b** $46+(-50)$	**c** $9-12$
d $9+(-12)$	**e** $-8+6$	**f** $-8-(-6)$
g $81-15$	**h** $81+(-15)$	**i** $7+(-7)$

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Problem-solving and reasoning

7–10	8–11, 12(½)

7 An ice cube is removed from a freezer at −25°C and placed into a glass of juice at 7°C. What is the difference between the two temperatures?

8 Kelvin owes the bank $\$450\,000$. What must he deposit into his account to only owe $\$270\,000$?

9 State the missing number in each of the following.

a $-6 + \square = 0$ **b** $7 - \square = 0$ **c** $-18 - \square = 0$

10 An addition fact like $2 + 3 = 5$ can be used to generate two subtraction facts: $5 - 2 = 3$ and $5 - 3 = 2$.

a Write two subtraction facts that can be generated from $4 + 6 = 10$.

b Write two subtraction facts that can be generated from $7 + (-2) = 5$.

c Explain why $5 - (-6) = 11$ by using an addition fact involving 5, -6 and 11.

11 If $a = -5$ and $b = -3$, find the value of:

a $a + (-3)$ **b** $a - (-2)$ **c** $b - (-4)$

d $a + b$ **e** $a - b$ **f** $b - a$

Hint for Q11: Replace the pronumeral in the statement with the number it represents.
e.g. $a = -2$
then $a + (-5)$
$= -2 + (-5)$
$= -2 - 5$
$= -7$

12 Write down the missing number.

a $4 + \square = 1$ **b** $6 + \square = 0$

c $-2 + \square = -1$ **d** $\square + (-8) = 2$

e $\square + (-5) = -3$ **f** $\square + (-3) = -17$

g $12 - \square = 14$ **h** $8 - \square = 12$

i $-1 - \square = 29$ **j** $\square - (-7) = 2$

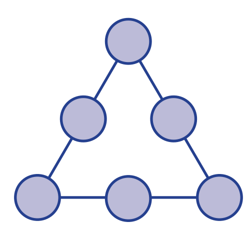
k $\square - (-2) = -4$ **l** $\square - (-436) = 501$

Puzzles with negatives

—	13, 14

13 Place the integers from −3 to 2 in this magic triangle so that each side adds to the given number.

a -3 **b** 0



14 A magic square has each row, column and diagonals adding to the same magic sum. Complete these magic squares.

a

		1
0	−2	−4

b

−12		
	−15	
	−11	−18

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1H Multiplying and dividing negative integers

Learning intentions

- To understand that the product or quotient of two integers will be positive if the two integers have the same sign.
- To understand that the product or quotient of two integers will be negative if the two integers have opposite signs.
- To be able to use order of operations with integers.

Key vocabulary: sign, integer, positive integer, negative integer, product, quotient, order of operations

As a repeated addition, the product $3 \times (-2)$ can be written as $-2 + (-2) + (-2) = -6$. So $3 \times (-2) = -6$ and, since $a \times b = b \times a$ for all numbers a and b, then -2×3 is also equal to -6.

For division we can write the product $3 \times 2 = 6$ as a quotient $6 \div 2 = 3$.

So, if $3 \times (-2) = -6$ then $-6 \div (-2) = 3$.

Also if $-2 \times 3 = -6$ then $-6 \div 3 = -2$.

The quotient of two negative numbers results in a positive number, and the product or quotient of two numbers of opposite sign is a negative number.

Also, $6 \div (-2) = -3$ can also be rearranged to $-3 \times (-2) = 6$. So the product of two negative numbers is a positive number.

Lesson starter: Seeing the pattern

- Write the missing numbers in these tables. You should create a pattern in the third column.

$\square$	$\triangle$	$\square \times \triangle$
3	5	15
2	5	
1	5	
0	5	
−1	5	
−2	5	
−3	5	

$\square$	$\triangle$	$\square \times \triangle$
3	−5	−15
2	−5	−10
1	−5	
0	−5	
−1	−5	
−2	−5	
−3	−5	

- Write the missing numbers in these sentences. Use the tables above to help.

a $3 \times 5 = \_\_\_$ so $15 \div 5 = \_\_\_$

b $-3 \times 5 = \_\_\_$ so $-15 \div 5 = \_\_\_$

c $3 \times (-5) = \_\_\_$ so $-15 \div (-5) = \_\_\_$

d $-3 \times (-5) = \_\_\_$ so $15 \div (-5) = \_\_\_$

Key ideas

- The **product** or **quotient** of two **integers** of the same **sign** is a **positive integer**.
  - Positive × Positive = Positive
  - Positive ÷ Positive = Positive
  - Negative × Negative = Positive
  - Negative ÷ Negative = Positive
- The product or quotient of two integers of opposite signs is a **negative integer**.
  - Positive × Negative = Negative
  - Positive ÷ Negative = Negative
  - Negative × Positive = Negative
  - Negative ÷ Positive = Negative

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Exercise 1H

Understanding

1–3 | 3

1 Choose the correct words to complete each sentence.

a The product (×) or quotient (÷) of two numbers of the _________ sign is a _________ integer.

b The product (×) or quotient (÷) of two numbers of the _________ sign is a _________ integer.

2 Without finding the answer to these products, decide if the answer would be positive or negative.

a 109×4 b -76×5 c $15 \times (-9)$

d $-6 \times (-13)$ e 89×104 f -74×8

g $-94 \times (-5)$ h $80 \times (-7)$ i -37×-3

3 Without finding the answer to these quotients, decide if the answer would be positive or negative.

a $16 \div 2$ b $24 \div (-3)$ c $78 \div (-2)$

d $-56 \div 2$ e $-81 \div 9$ f $-99 \div (-11)$

Fluency

4–6(½) | 4–7(½)

Example 19 Finding products of integers

Evaluate the following.

a $3 \times (-7)$ b $-4 \times (-12)$

Solution	Explanation
a $3 \times (-7) = -21$	The product of two numbers of opposite sign is negative. $\boxed{+} \times \boxed{-} = \boxed{-}$
b $-4 \times (-12) = 48$	-4 and -12 are both negative and so the product will be positive. $\boxed{-} \times \boxed{-} = \boxed{+}$

Now you try

Evaluate the following.

a -4×6 b $-7 \times (-11)$

4 Evaluate the following.

a $4 \times (-5)$ b $6 \times (-9)$ c -4×10 d -11×9

e $-2 \times (-3)$ f -6×7 g -9×8 h $-11 \times (-9)$

i $20 \times (-2)$ j -16×4 k $-5 \times (-7)$ l $8 \times (-4)$

m $-10 \times (-6)$ n $44 \times (-1)$ o $-9 \times (-1)$ p -5×12

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1H

Example 20 Finding quotients of integers

Evaluate the following.

a $-63 \div 7$ b $-121 \div (-11)$

Solution

a $-63 \div 7 = -9$

Explanation

The two numbers are of opposite signs so the answer will be negative.

$\boxed{-} \div \boxed{+} = \boxed{-}$

b $-121 \div (-11) = 11$

-121 and -11 are both negative so the quotient will be positive.

$\boxed{-} \div \boxed{-} = \boxed{+}$

Now you try

Evaluate the following.

a $72 \div (-8)$ b $-45 \div (-9)$

5 Evaluate the following.

a $-10 \div 2$ b $-38 \div 19$ c $-60 \div 15$
d $-120 \div 4$ e $32 \div (-16)$ f $-6 \div 2$
g $6 \div (-2)$ h $-6 \div (-2)$ i $-12 \div 6$
j $-24 \div (-3)$ k $-45 \div 5$ l $-45 \div (-9)$
m $-66 \div (-6)$ n $-5 \div (-5)$ o $-8 \div 1$
p $-8 \div (-1)$

6 Decide if the answer to the following is -2.

a $8 \div (-2)$ b $-1 \times (-2)$
c $-10 \div 5$ d $-16 \div 8$
e -2×1 f -2×0

7 If $(-2)^2 = -2 \times -2 = 4$, find the value of the following.

a $(-5)^2$ b $(-6)^2$
c $(-7)^2$ d $(-8)^2$
e $(-9)^2$ f $(-10)^2$

Problem-solving and reasoning

8, 9 | 9–11

8 Write the missing number.

a $\_\_\_ \times 3 = -9$ b $\_\_\_ \times (-7) = 35$
c $\_\_\_ \times (-4) = -28$ d $-3 \times \_\_\_ = -18$
e $-19 \times \_\_\_ = 57$ f $\_\_\_ \div (-9) = 8$
g $\_\_\_ \div 6 = -42$ h $85 \div \_\_\_ = -17$
i $-150 \div \_\_\_ = 5$

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9 Will $(-2)^3$ give a positive or negative answer? Explain why.

10 Insert × and/or ÷ signs to make these equations true.

a $-2\_\_\_3\_\_\_(-6) = 1$

b $10\_\_\_(-5)\_\_\_(-2) = 25$

c $6\_\_\_(-6)\_\_\_20 = -20$

d $-14\_\_\_(-7)\_\_\_(-2) = -1$

11 The product of two numbers is −24 and their sum is −5. What are the two numbers?

Bracket placements

— | 12(½)

12 Insert brackets in these statements to make them true.

a $-2 + 1 \times 3 = -3$

b $-10 \div 3 - (-2) = -2$

c $-8 \div (-1) + 5 = -2$

d $-1 - 4 \times 2 + (-3) = 5$

e $-4 + (-2) \div 10 + (-7) = -2$

f $20 + 2 - 8 \times (-3) = 38$

g $1 - (-7) \times 3 \times 2 = 44$

h $4 + (-5) \div 5 \times (-2) = -6$

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1I Order of operations and substitution

Learning intentions

- To understand the rules for order of operations
- To be able to evaluate numerical expressions using the order of operations
- To be able to substitute integers for pronumerals in order to evaluate expressions

Key vocabulary: Order of operations, brackets, evaluate, substitute

An expression such as $a + 2 \times b$ can be evaluated if we know the values of a and b. The expression includes the operations addition (listed first) and multiplication; however, by convention, we know that multiplication is done before the addition. In this section we will deal with order of operations, using both positive and negative integers.

Expansion joints prevent bridges from bucking in hot weather. Engineers apply the order of operations after substituting values for the bridge length, Lm, temperatures, t°C to T°C, and a given a value into the expansion length formula: $l = aL(T - t)$.

Lesson starter: Bracket placement

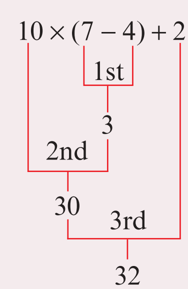
Is it possible, by inserting brackets, to make $3 \times 5 - 2 + 6 = 15$ true?

Insert a pair of brackets to make the equation correct.

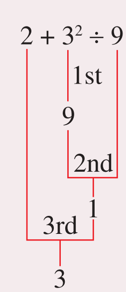
Try making up your own similar problem.

Key ideas

- The rules for order of operations are:
  - Deal with operations inside brackets first.
  - Deal with powers.
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.



- Expressions can be evaluated by substituting numbers for the given pronumerals.
  For example: If $a = -2$ and $b = -3$, then
  $$\begin{aligned} a + 5b &= -2 + 5 \times (-3) \\ &= -2 + (-15) \\ &= -17 \end{aligned}$$
  - Remember, for example, that $5b$ means $5 \times b$ and $\frac{a}{3}$ means $a \div 3$.



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Exercise 1I

Understanding

1–3 | 2, 3

1 By following the order of operations, describe the operation that needs to be done first.
a $2 + 3 \times 9$
b $10 - 2 \div 2$
c $1 \times 3 + 5$
d $6 \times (9 - 6)$
e $(12 + 6) \div 2$

2 Decide if both sides of these simple statements are equal.
a $(2 + 3) - 1 = 2 + 3 - 1$
b $(3 + (-2)) - (-1) = 3 + (-2) - (-1)$
c $5 \times (2 + (-3)) = 5 \times 2 + (-3)$
d $-8 \times 2 - (-1) = -8 \times (2 - (-1))$
e $-10 \div 2 - 4 = -10 \div (2 - 4)$
f $-2 \times 3 + 8 \div (-2) = (-2 \times 3) + (8 \div (-2))$

3 State the missing numbers to complete the working for these substitutions.
a $a + 2b$ $(a = -3, b = 4)$
$a + 2b = -3 + 2 \times 4$
$= \_\_\_\_\_ + \_\_\_\_\_$
$= \_\_\_\_\_$

b $3 \times (a - b)$ $(a = 5, b = -1)$
$3 \times (a - b) = 3 \times (5 - (-1))$
$= 3 \times \_\_\_\_$
$= \_\_\_\_$

Fluency

4–9(½) | 4–10(½)

Example 21 Using order of operations with positive integers

Evaluate the following.
a $10 + 5 \times 3$
b $15 - (7 - 3)$
c $20 \div (2 \times (5 - 3))$

Solution

a $10 + 5 \times 3 = 10 + 15$
$= 25$

Explanation

Multiplication (×) is done BEFORE addition (+). $5 \times 3 = 15$

Solution

b $15 - (7 - 3) = 15 - 4$
$= 11$

Explanation

Brackets need to be done first $(7 - 3) = 4$. Then do the subtraction $15 - 4$.

Solution

c $20 \div (2 \times (5 - 3))$
$= 20 \div (2 \times 2)$
$= 20 \div 4$
$= 5$

Explanation

Start with the innermost brackets $(5 - 3)$.
Finish working with the brackets - we follow the order of operations within the brackets (2×2).
Then the division $20 \div 4$.

Now you try

Evaluate the following.
a $17 - 8 \div 4$
b $10 \times (15 - 9)$
c $27 \div (3 \times (9 - 6))$

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1I

4 Find the answers to each of the following.

a $12 + 5 \times 2$
b $24 - 6 \times 3$
c $10 \times 2 + 6$
d $15 \div 3 - 2$
e $(9 - 2) \times 4$
f $18 - (12 - 8)$
g $28 \div (2 \times 7)$
h $56 - 5 \times 10$
i $120 + 200 \div 5$
j $88 \times 2 \div 8$
k $12 \div (18 \div 6)$
l $16 - 18 \div 9$
m $55 \div 11 \times 5$
n $55 - 25 \div 5$
o $240 \div 10 \times 2$
p $58 + 100 \div 20$
q $100 - 25 \div 5$
r $(24 - 9) \times 3$

5 Evaluate.

a $56 - 4 \times 6$
b $96 \div 4 + 3 \times 6$
c $150 - 7 \times (10 - 3 \times 2)$
d $12 \times (13 - 8) \times (24 - 18)$
e $7 + 30 \div (10 \div (7 - 5))$
f $13 - (6 - (5 - 3)) \times 3 - 1$

Example 22 Using order of operations with integers

Evaluate the following.

a $5 - 6 \times (-2)$
b $-21 \div (5 - (-2))$
c $2 \times 10^2 \div 5$

Solution	Explanation
a $5 - 6 \times (-2) = 5 - (-12)$ $= 17$	Do the multiplication before the addition and remember that $5 - (-12) = 5 + 12$.
b $-21 \div (5 - (-2)) = -21 \div 7$ $= -3$	Deal with brackets first and remember that $5 - (-2) = 5 + 2$.
c $2 \times 10^2 \div 5 = 2 \times 100 \div 5$ $= 200 \div 5$ $= 40$	Deal with powers before other operations. (Note: $2 \times 10^2 \neq 20^2$.)

Now you try

Evaluate the following.

a $12 - 15 \div (-3)$
b $(-3 - 5) \times (7 + (-4))$
c $4 \times 3^2 \div 2$

6 Evaluate the following. Remember to use the normal order of operations.

a $-2 \times 3 \times 5$
b $-6 - 2 \times 3$
c $4 - 8 \times (-1)$
d $-3 \div (-1) + 7 \times (-2)$
e $6 \times (-2) - 10 \div (-5)$
f $4 + 8 \times (-2) \div (-16)$
g $20 - 10 \div (-5) \times 2$
h $0 \times (-3) + 2 \times (-30)$
i $35 - 10 \div (-2) + 0$

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7 Use order of operations to evaluate the following.

a $3 \times (2 - 4)$
b $(7 - (-1)) \times 3$
c $(-8 + (-2)) \div (-5)$
d $40 \div (8 - (-2)) + 3$
e $0 \times (38 - (-4)) \times (-6)$
f $-6 \times (-1 + 3) \div (-4)$
g $((-2) + 1) \times (8 - (-3))$
h $(-6 - 4) \div (50 \div (-10))$
i $-2 \times (8 - 7 \times (-2))$

8 Use order of operations to evaluate the following.

a $5 \times 2^2 \div 10$
b $7 + 3^2 \times 2$
c $(6 - 4^2) \times (-2)$
d $(8 + 1^3) \div (-3)$
e $2^2 - 3^2$
f $3^3 \div 9 + 1$

Hint for Q8: Multiplication and division is calculated before addition and subtraction.

Example 23 Substituting integers

Substitute the given integers to evaluate the expressions.

a $a - 3b$ with $a = -2$ and $b = -4$
b $(a + b) \div (-5)$ with $a = -7$ and $b = 2$
c $a^2 - b^3$ with $a = -2$ and $b = -3$

Solution	Explanation
a $a - 3b = -2 - 3 \times (-4)$ $= -2 - (-12)$ $= -2 + 12$ $= 10$	Substitute $a = -2$ and $b = -4$ and then evaluate, noting that $-2 - (-12) = -2 + 12$.
b $(a + b) \div (-5) = (-7 + 2) \div (-5)$ $= -5 \div (-5)$ $= 1$	Substitute $a = -7$ and $b = 2$ and then deal with the brackets before the division.
c $a^2 - b^3 = (-2)^2 - (-3)^3$ $= 4 - (-27)$ $= 4 + 27$ $= 31$	Use brackets when substituting into expressions with powers. $(-2)^2 = -2 \times (-2) = 4$ $(-3)^3 = -3 \times (-3) \times (-3) = -27$

Now you try

Substitute the given integers to evaluate the expressions.

a $4a + b$ with $a = -3$ and $b = -4$
b $a + (b \div (-2))$ with $a = -10$ and $b = -6$
c $a^3 - b^2$ with $a = -2$ and $b = -3$

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9 Evaluate these expressions using $a = -2$ and $b = 1$.
a $a + b$
b $a - b$
c $2a - b$
d $b - a$
e $a - 4b$
f $3b - 2a$
g $b \times (2 + a)$
h $a(2 - b)$
i $(2b + a) - (b - 2a)$

10 Evaluate these expressions using $a = -3$ and $b = 5$.
a $a + b^2$
b $a^2 - b$
c $b^2 - a$
d $b^3 + a$

Problem-solving and reasoning

11–13 | 12–15

11 True (T) or false (F)?
a $5 + 9 = 5 + 3 \times 3$
b $10 + 2 \times 7 = 12 + 7$
c $18 - 6 + 5 = 12 + 5$
d $3 \times 5 \times 6 = 15 \times 6$
e $120 \div 6 \times 2 = 20 \times 2$
f $(5 + 3) \times 9 = 8 \times 9$

12 Insert operation symbols (+, −, ×, ÷) between the numbers to make each of the following statements true.
a $5 \_\_\_ 4 \_\_\_ 9 = 0$
b $5 \_\_\_ 4 \_\_\_ 9 = 11$
c $5 \_\_\_ 4 \_\_\_ 9 = 41$

13 Insert brackets in these statements to make them true.
a $-2 + 1 \times 3 = -3$
b $-10 \div 3 - (-2) = -2$
c $-8 \div (-1) + 5 = -2$
d $-1 - 4 \times 2 + (-3) = 5$
e $-4 + (-2) \div 10 + (-7) = -2$
f $20 + 2 - 8 \times (-3) = 38$

14 Evaluate the following.
a $3 \times (-2)^2$
b $-2 \times (-2)^3$
c $-16 \div (-2)^3$
d $-4 + \sqrt{25}$
e $7 - \sqrt{16}$
f $-26 + \sqrt[3]{27}$
g $-4 + 2 \times \sqrt[3]{8}$
h $-8 \div \sqrt[3]{-64} + 1$
i $-3 \times (-2)^3 + 4$

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15 Write each of the following situations into mathematical symbols and numbers, and then calculate.

a Murray receives four dollars from his Mum and seven dollars from his Dad as pocket money each week for 12 weeks. How much money does he have at the end of the 12 weeks?

b A raffle prize consists of $\$5000$ cash and 6 shopping vouchers each worth $\$500$. What is the total value of the raffle prize?

c Sally has fifty dollars. She buys four pens at two dollars each and eight exercise books at three dollars each. How much change does Sally get?

Make ten from four

—	16

16 Can you make the first 10 counting numbers ($1, 2, 3, 4, 5, 6, 7, 8, 9$ and 10) using only the four digits $1, 2, 3$ and 4 (once each), brackets and any of the four operations?

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Maths@Work

Retailer of loungeroom furniture

A furniture retailer needs many skills including being a good communicator and a successful salesperson. They need to apply the mathematics of money management to stock orders, delivery costs, insurance and pay rates. It is important that they know their products and have options for clients. A successful retailer relates to customers in a confident, friendly, cheerful and helpful manner.

1 A loungeroom furniture business advertises that all lounges are reduced by $250. What is the sale price on the following lounges currently in stock?

- **a** 2-seater leather lounge marked at $2340.
- **b** 2.5-seater leather lounge with chaise marked at $2599.
- **c** 3-seater + 2-seater sofa set marked at $2099.
- **d** 2-seater recliner lounge in fabric marked at $2249.
- **e** 7-seater corner lounge in fabric marked at $4130.

2 Floor stock is a term describing furniture that has been displayed in the showroom for customers to try out. It is often discounted for a quick sale and is usually available for immediate delivery.

Complete the table below to find the savings on each of these lounges and other available products from the wholesale centre.

Model	Original price	Floor stock price	Savings
Recliner 3-piece	$4499	$2250	
Corner suite	$3299	$1999	
2.5-seater leather chaise	$2295	$1999	
Outdoor sofa	$1120	$895	
Occasional chair	$369	$149	

3 The models on display are available in either leather or fabric.

- **a** Find out the difference in the prices of each lounge suite in leather compared with fabric.
- **b** On average, how much more does the retailer charge for the leather models?

Model	Leather	Fabric	Difference in price
ER 2 + chaise	$2099	$1499	
DW 3450 R	$2350	$1890	
Ebony 3 + 2	$3495	$2599	
Recliner and console 3	$1149	$799	
Victa EL + 1	$3297	$2599	

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4 Jen and Brad decide to buy a grey Boston Chaise lounge suite. They investigate their options from two different outlets to find the best overall price including delivery.
Which is the best buy and by how much?

Option 1: Custom Sofas	Option 2: Leisure Lounges
Leather lounge $1999 Upgrade on colour of leather to grey $160 Delivery $100	Lounge — grey leather $2199 Delivery $70

Using technology

Hint for Q5:
- To increase a price by 25%, multiply it by 1.25.
- To fill formulas down a column, drag the 'fill handle' down.
- Cell G3 formula = C3*D3 + E3*F3

5 A lounge suite business, Luxury Lounges, uses a spreadsheet for orders.

a Copy the following Excel spreadsheet. Format all the number cells to Number with 0 d.p. and all price and cost cells to Currency with 0 d.p.

b If leather furniture costs 25% more than fabric, enter formulas into the 'Price in leather' column to calculate these prices. See the hint box for extra clues.

c Enter formulas in column G to calculate 'Cost of furniture' and the 'Total cost of order'. Costs will be $0 until the numbers of items are entered from part **d**.

	A	B	C	D	E	F	G
1	Luxury Lounges						
2	Catalogue Item number	Furniture description	Price in fabric	Number in fabric	Price in leather	Number in leather	Cost of furniture
3	021-A	Cayman corner lounge with chaise	$2859				
4	021-D	Sophie 3-piece cream lounge with 4 recliners	$1390				
5	021-G	Tuscan red L-shaped sofa set	$2190				
6	054-B	Sapphire blue lounge with 2-seater sofa and ottoman	$3499				
7	054-F	Onyx home theatre entertainment lounge with 4 recliners	$2675				
8	079-L	Mandarin orange recliner chair	$548				
9	079-M	Zara 2-seater sofa	$1099				
10	079-R	Corfu outdoor corner suite	$1563				
11					Total cost of order		

d Use your spreadsheet to calculate the total cost of each of the following orders.

Catalogue item number		021-A	021-D	021-G	054-B	054-F	079-L	079-M	079-R
March order	Fabric	2		1	1			2	1
	Leather		3		1	2	4	1	1
June order	Fabric		1		2		2		3
	Leather	1	2	1	1	1	2	2	

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Modelling

Selling garden gnomes

Wilbur buys garden gnomes from a local supplier and sells them for a profit. There are three sizes of gnomes:

Type	Cost price	Selling price
Small	$5	$8
Medium	$7	$11
Large	$10	$15

Wilbur sets up a balance sheet to keep track of his expenditure and revenue. The following incomplete example shows four transactions starting from an initial balance of $0. Negative numbers are used to indicate money leaving his account, and positive numbers are used for money entering his account.

Transaction	Unit price	Effect on balance	Balance (initially $0)
Purchase 20 small	$5	−$100	−$100
Sell 4 medium	$11	+$44	−$56
Purchase 10 large	$10	−$100	−$156
Sell 6 small	$8		

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

1 Preliminary task

a Explain why the balance after the first transaction on the balance sheet is −$100.

b Explain why the balance after the second transaction on the balance sheet is −$56.

c The table above has two missing numbers.
  i What is the effect on Wilbur's balance when he sells 6 small gnomes?
  ii What is the new balance after he sells these gnomes?

d If Wilbur purchases a further 12 medium garden gnomes from the supplier determine the balance at the end of this transaction.

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2 Modelling task

Formulate

a The problem is to determine sales targets so that Wilbur will be in profit (with a positive balance) at the end of a month. Write down all the relevant information that will help solve this problem.

b Draw up an empty balance sheet using the same headings as the example above. Allow 7 rows for transactions but leave all the rows blank so that a fresh set of transactions can be made. You can assume his initial balance is \$0.

Solve

For part of one particular month Wilbur started with no gnomes of any size and makes the following garden gnome purchases and sales.

- Purchases 30 small
- Purchases 25 medium
- Sells 9 small
- Purchases 15 large
- Sells 15 medium
- Sells 6 small
- Sells 3 large

c Enter these transactions into your balance sheet and calculate the balance after each transaction.

d State the final balance after the above transactions are completed.

e Decide how many gnomes of each type are remaining in Wilbur's stock at the end of the month.

Evaluate and verify

f By considering the balance position from part d above determine one combination of sales using any gnomes in the remaining stock that means that Wilbur will make a profit greater than \$200 in the month. Justify your answer with appropriate calculations.

Communicate

g Summarise your results and describe any key findings.

3 Extension questions

a How much profit does Wilbur make when buying and selling each size of gnome?

b Wilbur wants to make a monthly profit as close to \$200 as possible. Choose a combination of gnomes that he can buy and sell to achieve this. Justify your choice with working.

c Is it possible to achieve a balance equal to \$200 exactly? Justify your answer with working.

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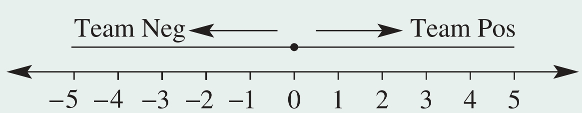
Integer tug of war

Key technology: Programming and spreadsheets

In a game of tug of war, a rope is pulled in opposite directions by two teams. The centre of the rope is marked by a ribbon and starts at position zero. The winning team will move the ribbon a given distance in their direction. In this activity, we will use a number line to record the position of the ribbon where one team is pulling in the positive direction and the other in the negative direction. The winner is the team to move the ribbon past a particular point.

1 Getting started

Let's play a game of tug of war with two teams of equal ability. *Team Pos* tries to pull to the right on this number line and *Team Neg* tries to pull to the left. We will assume that either team can successfully move up to 3 units in their direction on any given 3-second interval.



a Use a random number generator to generate a move up to 3 units left or right: Suggestions:
- Spreadsheet: =RANDBETWEEN $(-3, 3)$
- CAS: RandInt $(-3, 3)$

b Continue to create random moves left or right with your random number generator and add your results into this table. (Your numbers inserted into row 3 onwards will be different from the sample shown here.)

Random number	Position of ribbon
Start	0
−2	−2
1	−1
2	1

c Assume that if the ribbon reaches −5 or 5 then *Team Neg* or *Team Pos* wins. Continue adding to your table until the position of the ribbon is at either −5 or 5. Which team has won your tug of war?

2 Applying an algorithm

a Study this flow chart which describes an algorithm for playing the tug of war game outlined in part **1** above. Describe what you think the variables i, a and b represent.

b Complete this table showing the values of i, a and b through each pass of the loop using your random number generator to calculate the value of a each time. Keep going until b reaches −5 or 5 or $i = 10$.

Pass	i	a	b
Initial	0	–	0
1			
2			
…			

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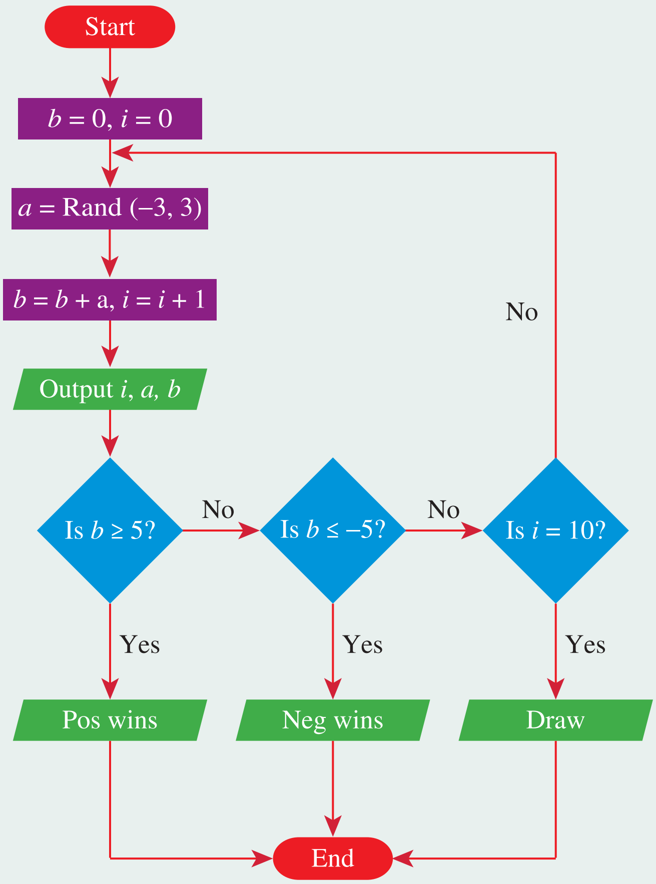

3 Using technology

We will use a spreadsheet to simulate the playing of the above tug of war game but this time the ribbon needs to be pulled through to -10 or $+10$ for one of the teams to win.

a Enter the following into a spreadsheet.

	A	B	C	D	E
1	Integer	tug of war		Result	=IF(C25="Continue","Draw","Win")
2					
3	Pass	Balance point	Status	Pos win?	Neg win?
4					
5	0	0	Start		
6	=A5+1	=B5+RANDBETWEEN(-3,3)	=IF(OR(B6>=10,B6<=-10,C5="Stop"),"Stop","Continue")	=IF(OR(B6>=10,D5="Pos wins"),"Pos wins","")	=IF(OR(B6<=-10,E5="Neg wins"),"Neg wins","")

b Fill down at cells A6, B6, C6, D6 and E6 up to and including row 25. This will mean that 20 passes have taken place.

c Enter the formula in cell E1. What do you think this formula does?

d Press Function F9 to run another game of tug of war. Repeat for a total of 10 games and record how many times Team Pos wins, Team Neg wins or there is a Draw.

Technology and computational thinking

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Puzzles and games

1 Hey, do you know what a wisecracker is?

A $-6-4$	**R** $-8-(-2)$	**I** $-17+10$	**Y** $-17-6$
E $8-10$	**M** $-6-7-4$	**S** $20-7$	**K** $16-(-6)$
O $6-(-4)$	**C** $46+(-6)-8$	**V** $12+(-3)-6$	**T** $-13-7-6+8$

Complete the sums above to unlock the puzzle code.

-10		3	-2	-6	-23	
13	-17	-10	-6	-18		
32	10	10	22	-7	-2	

2 What explosive event was in the year 1000 CE?

Answer the following multiplications and divisions to work out the puzzle code. Write your answer on another sheet of paper.

K -3×4	**N** $8\div -4$
A -1×6	**S** $\dfrac{-36}{-6}$
C $100\div -5$	**U** -9×-7
L -8×-6	**G** $\dfrac{10}{-2}$
W $40\div 8\times -2$	**D** -2×2
H 4×-4	**O** $-12+5$
R $(-10)^2$	**P** $(-4)^2$
E 0×-5	**V** -5×-4
M $-16\div -8$	**F** $(-3)^2$
I $24\div 8$	**T** $-3\times -2\times -4$

-20	-16	3	-2	-6							
-4	0	20	0	48	-7	16	6				
-5	63	-2	16	-7	-10	-4	0	100			
-24	-16	3	6								
48	0	-6	-4	6							
-24	-7										
-24	-16	0									
2	-6	-2	63	9	-6	-20	-24	63	100	0	
-7	9										
9	3	100	0	-10	-7	100	-12	6			

3 Using the symbols $+, -, \times, \div$, make as many sums as you can that have -5 as their answer.

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Chapter summary

Integers
$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Whole numbers
0, 1, 2, 3, ...

Addition and subtraction

$$\begin{array}{r} 2\,{}^{1}4\,7 \\ +\,1\,0\,8 \\ \hline 3\,5\,5 \end{array} \qquad \begin{array}{r} {}^{8}\cancel{9}\,{}^{13}\cancel{4}\,{}^{1}2 \\ -\,3\,6\,8 \\ \hline 5\,7\,4 \end{array}$$

Multiplication and division

$$\begin{array}{rl} 167 & \\ \times\ 15 & \\ \hline 835 & 5 \times 167 \\ 1670 & 10 \times 167 \\ \hline 2505 & 835 + 1670 \end{array} \qquad \begin{array}{r} 5\ 9 \\ 7\overline{)4\ 1\,{}^{6}6} \end{array}$$

$416 \div 7 = 59$ and 3 remainder

Mental strategies

- $156 + 79 = 156 + 80 - 1 = 235$
- $45 + 47 = 45 + 45 + 2 = 92$
- $3 \times 22 = 3 \times 20 + 3 \times 2 = 66$
- $4 \times 88 = 2 \times 176 = 352$
- $164 \div 4 = 82 \div 2 = 41$
- $297 \div 3 = (300 \div 3) - (3 \div 3) = 99$

Negative number operations

$4 - (-3) = 4 + 3$
$4 + (-3) = 4 - 3$

Multiplication and division

$\boxed{+} \times \boxed{+} = \boxed{+}$
$\boxed{-} \times \boxed{-} = \boxed{+}$
$\boxed{+} \times \boxed{-} = \boxed{-}$
$\boxed{-} \times \boxed{+} = \boxed{-}$
$\boxed{+} \div \boxed{+} = \boxed{+}$
$\boxed{-} \div \boxed{-} = \boxed{+}$
$\boxed{+} \div \boxed{-} = \boxed{-}$
$\boxed{-} \div \boxed{+} = \boxed{-}$

Substitution

$a = -2,\ b = 5,\ c = -4$

$$\begin{aligned} 2c - ab &= 2 \times (-4) - (-2) \times 5 \\ &= -8 - (-10) \\ &= 2 \end{aligned}$$

use brackets with negatives

Order of operations

- Brackets, $\times$ and $\div$ then $+$ and $-$

$$\begin{aligned} &10 \times (-3) + 7 \\ &= -30 + 7 \\ &= -23 \end{aligned}$$

Properties

Primes (two factors)

2, 3, 5, 7, 11, 13 ...

Prime factorisation ⭐

$$72 = 2^3 \times 3^2$$

Factor tree: 72 → 2, 36; 36 → 2, 18; 18 → 2, 9; 9 → 3, 3

Squares and cubes

$4^2 = 16,\ \sqrt{16} = 4$
$3^3 = 27,\ \sqrt[3]{27} = 3$

Composite

more than two factors
4, 6, 8, 12

Powers and indices

$9 \times 9 \times 9 \times 9 = 9^4$

Lowest Common Multiple LCM

Multiples of 3: 3, 6, 9, **12** ...
Multiples of 4: 4, 8, **12** ...
$\therefore$ LCM = **12**

Highest Common Factor HCF

Factors of 8: 1, 2, **4**, 8
Factors of 28: 1, 2, **4**, 7, 14, 28
$\therefore$ HCF = **4**

Divisibility ⭐

- 2 Last digit even.
- 3 Sum of digits divisible by 3.
- 4 Number from last 2 digits divisible by 4.
- 5 Last digit 0 or 5.
- 6 Divisible by 2 and 3.
- 8 Number from last 3 digits divisible by 8 or ends in 000.
- 9 Sum of digits divisible by 9.
- 10 Last digit 0.

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
1A	**1 I can use mental addition and subtraction techniques effectively** e.g. Evaluate the following mentally: **a** $347 - 39$ **b** $125 + 127$	
1A	**2 I can use the addition algorithm with whole numbers** e.g. Use an algorithm to find this sum. $\begin{array}{r} 938 \\ +\ 217 \\ \hline \end{array}$	
1A	**3 I can use the subtraction algorithm with whole numbers** e.g. Use an algorithm to find this difference. $\begin{array}{r} 141 \\ -\ 86 \\ \hline \end{array}$	
1B	**4 I can use mental multiplication and division techniques effectively** e.g. Find the following mentally: **a** 5×160 **b** $464 \div 4$	
1B	**5 I can use the multiplication algorithm with whole numbers** e.g. Use an algorithm to evaluate 412×25.	
1B	**6 I can use the division algorithm with whole numbers** e.g. Use an algorithm to evaluate $938 \div 13$.	
1C	**7 I can write products using index notation** e.g. Write $8 \times 8 \times 8$ using index notation.	
1C	**8 I can convert from index notation to expanded notation** e.g. Write 5^4 in expanded form.	
1C	**9 I can find the square and cube of whole numbers** e.g. Find: **a** 6^2 **b** 2^3	
1C	**10 I can find the square root and cube root of certain small whole numbers** e.g. Find: **a** $\sqrt{81}$ **b** $\sqrt[3]{64}$	
1D	**11 I can classify a number as prime or composite (or neither)** e.g. Decide whether each of the following is prime or composite: **a** 29 **b** 63	
1D	**12 I can find the lowest common multiple (LCM) of two whole numbers** e.g. Find the LCM of: 6 and 8	
1D	**13 I can find the highest common factor (HCF) of two whole numbers** e.g. Find the HCF of: 36 and 48	

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		✔
1E	**14 I can write a number as the product of prime factors using a factor tree** e.g. Write 300 as a product of prime factors.	
1E	**15 I can use divisibility tests to determine if a number is divisible by 2, 3, 4, 5, 6, 8 or 9** e.g. Decide whether 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.	
1E	**16 I can find the lowest common multiple (LCM) and highest common factor (HCF) of two whole numbers using prime factorisation** e.g. Find the LCM and HCF of the following using prime factorisation: 105 and 90	
1F	**17 I can add a positive integer to a negative integer** e.g. Evaluate: a $-5+2$ b $-1+4$	
1F	**18 I can subtract a positive integer from another integer** e.g. Evaluate: a $3-7$ b $-2-3$	
1G	**19 I can add negative integers to another integer** e.g. Evaluate: a $10+(-3)$ b $-3+(-5)$	
1G	**20 I can subtract negative integers from another integer** e.g. Evaluate: a $4-(-2)$ b $-11-(-6)$	
1H	**21 I can find the product of integers** e.g. Evaluate: a $3\times(-7)$ b $-4\times(-12)$	
1H	**22 I can find the quotient of integers** e.g. Evaluate: a $-63\div 7$ b $-121\div(-11)$	
1H	**23 I can use order of operations with integers** e.g. Evaluate $-7+6\times(-5)$.	
1I	**24 I can use order of operations to evaluate numerical expressions** e.g. Evaluate the following. a $10+5\times 3$ b $-20\div 2^2-1$	
1I	**25 I can use order of operations to evaluate numerical expressions involving grouping symbols** e.g. Evaluate the following. a $(7+2)\times 5-6$ b $10\div(7-2)-(-1)$	

Chapter checklist

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Chapter review

Short-answer questions

1A

1 Use a mental strategy to evaluate the following.

a $324 + 173$
b $592 - 180$
c $89 + 40$
d $135 - 68$
e $55 + 57$
f $280 - 141$
g $1001 + 998$
h $10\,000 - 4325$

1A

2 Use a mental strategy to find these sums and differences.

a $\begin{array}{r} 392 \\ +\ 147 \\ \hline \end{array}$
b $\begin{array}{r} 1031 \\ +\ 999 \\ \hline \end{array}$
c $\begin{array}{r} 147 \\ -\ 86 \\ \hline \end{array}$
d $\begin{array}{r} 3970 \\ -\ 896 \\ \hline \end{array}$

1B

3 Use a mental strategy for these products and quotients.

a $2 \times 17 \times 5$
b 3×99
c 8×42
d 141×3
e $164 \div 4$
f $357 \div 3$
g $618 \div 6$
h $1005 \div 5$

1B

4 Find these products and quotients using setting out.

a $\begin{array}{r} 139 \\ \times\ 12 \\ \hline \end{array}$
b $\begin{array}{r} 507 \\ \times\ 42 \\ \hline \end{array}$
c $3\overline{)843}$
d $7\overline{)854}$

1B

5 Find the remainder when 673 is divided by these numbers.

a 5
b 3
c 7
d 9

1C

6 Write using powers.

a $6 \times 6 \times 6$
b $8 \times 8 \times 8 \times 8$
c $2 \times 2 \times 5 \times 5 \times 5 \times 5$

1D

7 Evaluate.

a $\sqrt{81}$
b $\sqrt{121}$
c 7^2
d 20^2
e $\sqrt[3]{27}$
f $\sqrt[3]{64}$
g 5^3
h 10^3

1D

8 **a** Find all the factors of 60.
b Find all the multiples of 7 between 110 and 150.
c Find all the prime numbers between 30 and 60.
d Find the LCM of 8 and 6.
e Find the HCF of 24 and 30.

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1E ★ **9** Write these numbers in prime factor form. You may wish to use a factor tree.
a 36 **b** 84 **c** 198

1E ★ **10** Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.
a 84 **b** 155 **c** 124 **d** 621

1E ★ **11** Write the numbers 20 and 38 in prime factor form and then use this to help find the following.
a LCM of 20 and 38
b HCF of 20 and 38

1F **12** Evaluate.
a $-6+9$ **b** $-24+19$
c $5-13$ **d** $-7-24$
e $-62-14$ **f** $-194-136$
g $-111+110$ **h** $-328+426$

1G **13** Evaluate.
a $5+(-3)$ **b** $-2+(-6)$ **c** $-29+(-35)$ **d** $162+(-201)$
e $10-(-6)$ **f** $-20-(-32)$ **g** $-39-(-19)$ **h** $37-(-55)$

1H **14** Evaluate.
a -5×2 **b** $-11\times(-8)$ **c** $9\times(-7)$ **d** $-100\times(-2)$
e $-10\div(-5)$ **f** $48\div(-16)$ **g** $-32\div 8$ **h** $-81\div(-27)$

1H **15** Copy and complete.
a $1^2=$ _____ **b** $(-1)^2=$ _____
c $2^2=$ _____ **d** $(-2)^2=$ _____
e $3^2=$ _____ **f** $(-3)^2=$ _____

1I **16** Evaluate using the order of operations.
a $2+3\times(-2)$
b $-3\div(11+(-8))$
c $-2\times 3+10\div(-5)$
d $-20\div 10-4\times(-7)$

1I **17** Substitute $a=-2$, $b=3$ and $c=-5$ and evaluate these expressions.
a $ab+c$
b a^2-b
c $ac-b$
d $a+b+c$

Chapter review

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Chapter review

Multiple-choice questions

1I **1** $400 \div 5 \times 2$ is the same as:
A $400 \div 10$ **B** 80×2 **C** 16 **D** $400 \div 2 \times 5$ **E** 1600

1A **2** The sum and difference of 97 and 49 are:
A 146 and 58
B 246 and 48
C 136 and 58
D 146 and 48
E 147 and 58

1E **3** 561 is exactly divisible by:
A 5 **B** 2 **C** 3 **D** 9 **E** 10

1B **4** 89×5 is the same as:
A 90×4
B $90 \times 5 - 1 \times 5$
C $89 \times 10 \times 2$
D 178×10
E 450

1C **5** $2 \times 2 \times 2 \times 2 \times 5 \times 5$ is:
A $2^4 \times 5^2$
B $2 \times 4 + 5 \times 2$
C $2^4 + 5^2$
D 10^7
E 1000

1D **6** The LCM of $2^2 \times 3 \times 5$ and 2×7 is:
A 2
B $2^2 \times 3 \times 5 \times 7$
C $2 \times 3 \times 5 \times 7$
D $2^3 \times 3 \times 5 \times 7$
E 7

1F **7** $-15 + 12 - 3$ equals:
A 0 **B** 3 **C** -30 **D** -6 **E** 6

1G **8** $-6 + (-4)$ is the same as:
A $-6 - 4$ **B** $-6 + 4$ **C** $-4 + 6$ **D** $6 + 4$ **E** $6 - 4$

1C **9** If $18^2 = 324$, then $\sqrt{324}$ equals:
A 162 **B** $102\,976$ **C** 18 **D** 9 **E** 324

1F **10** If $a = -2$, $b = 3$ and $c = -1$ then $a^2 + bc$ equals:
A 0 **B** -1 **C** 1 **D** 7 **E** -7

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Extended-response questions

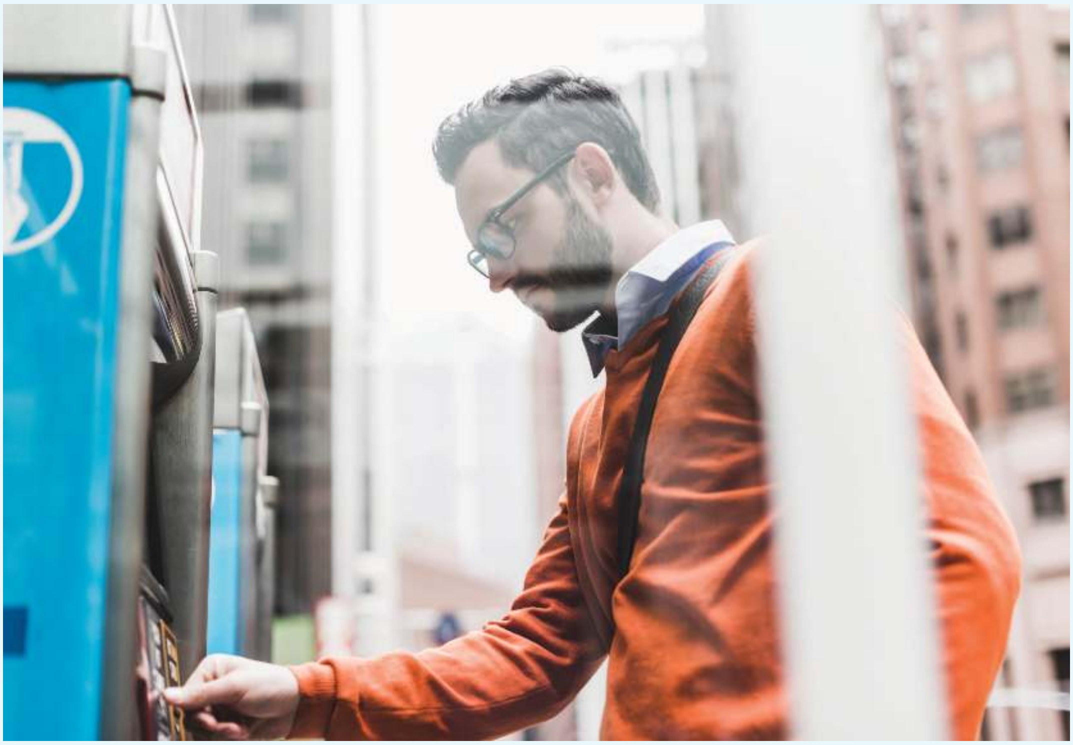
1 A monthly bank account shows deposits as positive numbers and purchases and withdrawals (P + W) as negative numbers.

Details	P + W	Deposits	Balance
Opening balance	–	–	\$250
Water bill	−\$138	–	a
Cash withdrawal	−\$320	–	b
Deposit	–	c	\$115
Supermarket	d	–	−\$160
Deposit	–	\$400	e

a Find the values of a, b, c, d and e.

b If the water bill amount was \$150, what would be the new value for letter e?

c What would the final deposit need to be if the value for e was \$0? Assume the original water bill amount is \$138 as in the table above.



2 Two teams compete at a club games night. Team A has 30 players while team B has 42 players.

a How many players are there in total?

b Write both 30 and 42 in prime factor form.

c Find the LCM and HCF of the number of players representing the two teams.

d Teams are asked to divide into groups with equal numbers of players. What is the largest group size possible if team A and team B must have groups of the same size?

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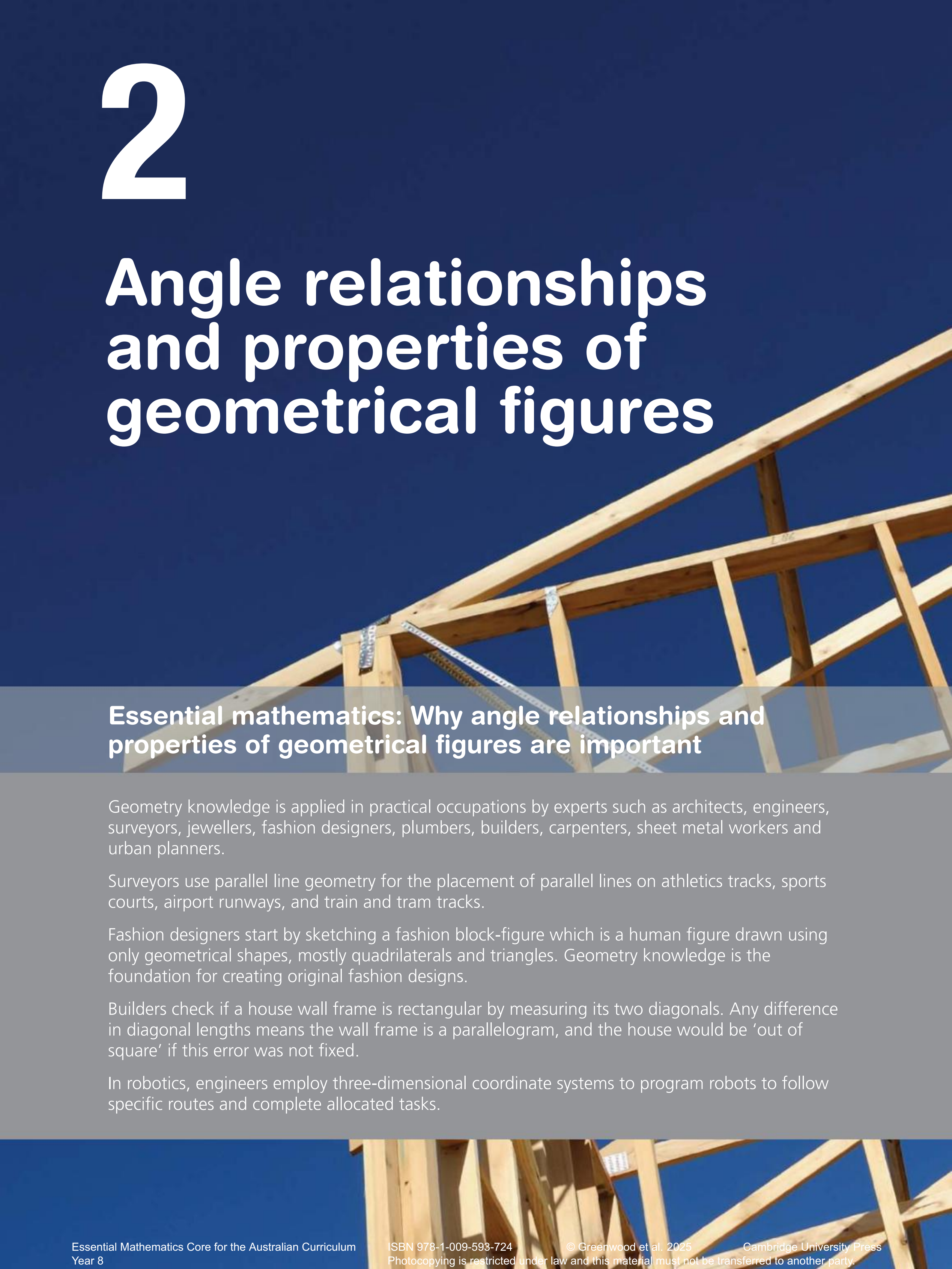

2

Angle relationships and properties of geometrical figures

Essential mathematics: Why angle relationships and properties of geometrical figures are important

Geometry knowledge is applied in practical occupations by experts such as architects, engineers, surveyors, jewellers, fashion designers, plumbers, builders, carpenters, sheet metal workers and urban planners.

Surveyors use parallel line geometry for the placement of parallel lines on athletics tracks, sports courts, airport runways, and train and tram tracks.

Fashion designers start by sketching a fashion block-figure which is a human figure drawn using only geometrical shapes, mostly quadrilaterals and triangles. Geometry knowledge is the foundation for creating original fashion designs.

Builders check if a house wall frame is rectangular by measuring its two diagonals. Any difference in diagonal lengths means the wall frame is a parallelogram, and the house would be 'out of square' if this error was not fixed.

In robotics, engineers employ three-dimensional coordinate systems to program robots to follow specific routes and complete allocated tasks.

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In this chapter

Australian Curriculum 9.0

SPACE

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems explaining reasoning (AC9M8SP02)

Describe the position and location of objects in 3 dimensions in different ways, including using a three-dimensional coordinate system with the use of dynamic geometric software and other digital tools (AC9M8SP03)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

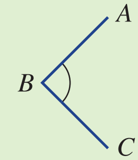
Warm-up quiz

1 Name these objects. Choose from: **A** line AB **B** segment AB **C** point A **D** angle ABC.

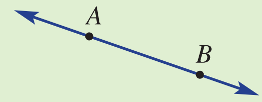
a

b

c

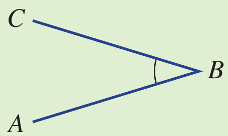


d

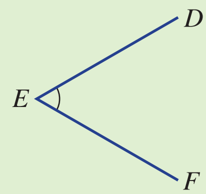


2 Choose a correct angle name from **A** $\angle DEF$ **B** $\angle STU$ **C** $\angle ABC$ for each given diagram.

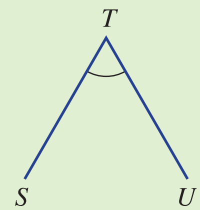
a



b



c



3 Name these angles as **A** *acute* **B** *right* **C** *obtuse* **D** *straight* **E** *reflex* **F** *revolution*.

a $360°$ **b** $90°$ **c** $37°$
d $149°$ **e** $180°$ **f** $301°$

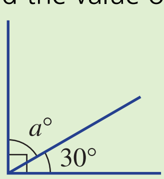
4 Name the triangle that fits the description. Choose from **A** *scalene* **B** *isosceles* **C** *equilateral* **D** *acute* **E** *right* **F** *obtuse*. Draw an example of each triangle to help.

a One obtuse angle **b** 2 equal length sides **c** All angles acute
d 3 different side lengths **e** 3 equal $60°$ angles **f** one right angle

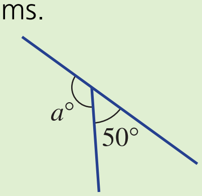
5 Name the six special quadrilaterals with four sides. Choose from **A** *circle* **B** *square* **C** *parallelogram* **D** *line* **E** *triangle* **F** *rectangle* **G** *rhombus* **H** *hexagon* **I** *kite* **J** *trapezium* **K** *tetrahedron*.

6 Find the value of a in these diagrams.

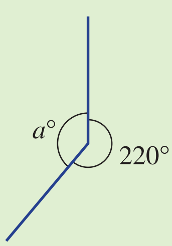
a



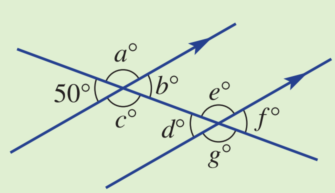
b



c



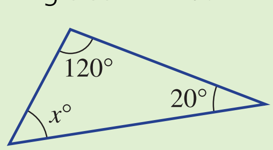
7 This diagram includes a pair of parallel lines and a third line (transversal).



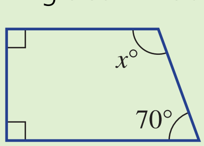
a What is the value of a?
b Which pronumerals (b, c, d, e, f or g) are equal to a? List in alphabetical order.
c Which pronumerals (b, c, d, e, f or g) are equal to 50? List in alphabetical order.

8 Find the value of x in these shapes, using the given angle sum.

a Angle sum $= 180°$



b Angle sum $= 360°$



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2A Angles at a point CONSOLIDATING

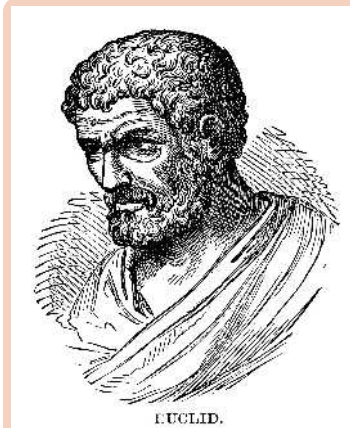
Learning intentions

- To be able to classify angles as acute, right, obtuse, straight, reflex or a revolution.
- To be able to name angles in relation to other angles, for instance, naming the angle vertically opposite to a given angle.
- To be able to determine the angles at a point using angle properties.
- To be able to relate compass bearings to angles.

Key vocabulary: acute, right, obtuse, straight, reflex, revolution, complementary, supplementary, vertically opposite, perpendicular, compass bearing

From three simple objects – point, line and plane – we can develop all the elements of geometry, just as the Greek mathematician Euclid did about 2300 years ago.

We can start by looking at the angles formed when lines meet at a point.

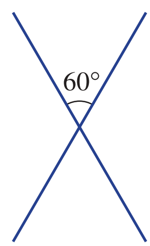


Euclid is considered by many as the 'father of geometry' having written about it around 300 BCE.

Lesson starter: How many angles?

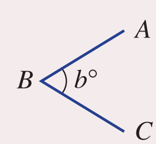
When two lines cross, different angles are formed, like in this example.

- Is there another 60° angle? Why?
- What is the size of one of the obtuse angles? How did you work this out?
- Are there any straight angles in the diagram?
- Are there any reflex angles in the diagram?
- What is a revolution angle?



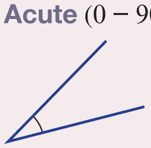
Key ideas

- The angle at right could be named $\angle ABC$, $\angle CBA$, $\angle B$ or $A\hat{B}C$ and has size b°.

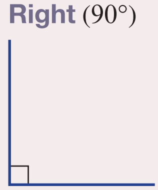


- Types of angles

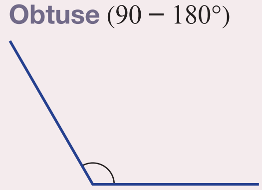
Acute $(0 - 90^\circ)$



Right (90°)

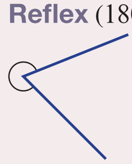


Obtuse $(90 - 180^\circ)$

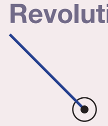


Straight (180°)

Reflex $(180^\circ - 360^\circ)$



Revolution (360°)



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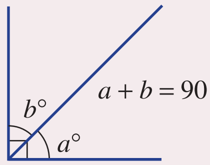
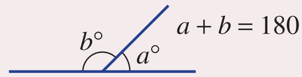
2A

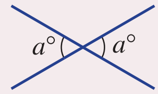
- Special pairs of angles at a point include:
  - **Complementary** angles (add up to 90°)



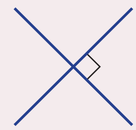
  - **Supplementary** angles (add up to 180°)



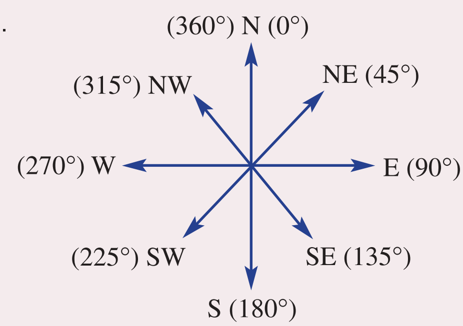
  - **Vertically opposite** angles (equal)



- Angles in a **revolution** add up to 360°.
- Two lines are **perpendicular** if they intersect at right angles (90°).



- Eight point compass bearing
  - Bearings are usually measured clockwise from north.



Exercise 2A

Understanding

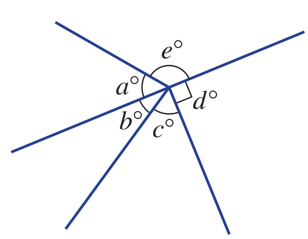
1–4 | 3, 4

1 Write the missing word. Choose from: *equal, supplementary, complementary* and *perpendicular*.
- **a** Angles that add to 90° are called _______ angles.
- **b** Angles that add to 180° are called _______ angles.
- **c** If two lines meet at right angles (90°), then they are said to be _______.
- **d** Vertically opposite angles are _______.

Hint for Q1: Choose from: *acute, right, obtuse, straight, reflex or revolution.*

2 What type of angle are the following?
- **a** 27° **b** 317° **c** 180°
- **d** 90° **e** 360° **f** 139°

3 Complete these sentences for this diagram.
- **a** $b°$ and $c°$ are __________ angles.
- **b** $a°$ and $e°$ are __________ angles.
- **c** $a°, b°, c°, d°$ and $e°$ form a __________.



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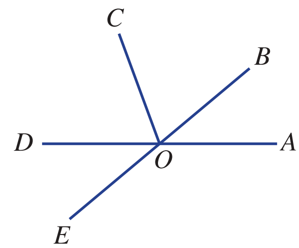
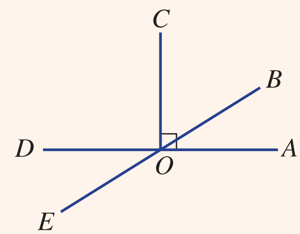
4 Estimate the size of these angles.
  a $\angle AOB$
  b $\angle AOC$
  c Reflex $\angle AOE$



Fluency 5, 6, 7(½), 8 | 5, 7, 8

Example 1 Naming angles

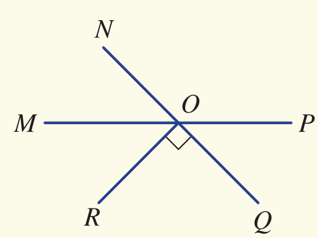
Name an angle which is:
a vertically opposite to $\angle DOE$
b complementary to $\angle COB$
c supplementary to $\angle EOA$



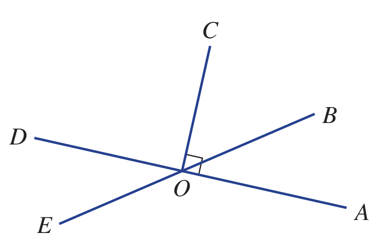
Solution	Explanation
a $\angle AOB$	$\angle DOE$ and $\angle AOB$ are equal and sit opposite each other.
b $\angle BOA$	$\angle COB$ and $\angle BOA$ add to 90°.
c $\angle DOE$ (or $\angle AOB$)	Pairs of angles on a straight line are supplementary (add to 180°).

Now you try

Name an angle which is:
a vertically opposite to $\angle POQ$
b complementary to $\angle MOR$
c supplementary to $\angle MOQ$



5 Name an angle which is:
  a vertically opposite to $\angle DOE$.
  b complementary to $\angle COB$.
  c supplementary to $\angle EOA$.



Hint for Q5: Vertically opposite angles are opposite and equal. Complementary angles add to 90°. Supplementary angles add to 180°.

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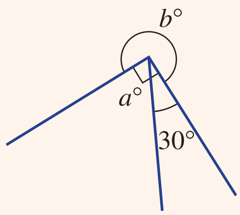
2A

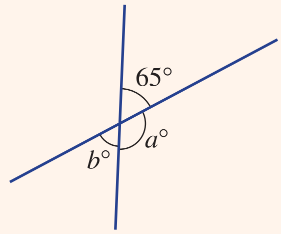
Example 2 Finding angles at a point

Determine the value of the pronumerals in these diagrams.

a



b

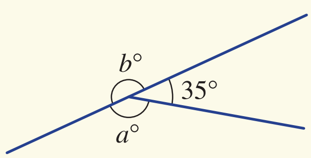


Solution	**Explanation**
a $a + 30 = 90$ $a = 60$ $b + 90 = 360$ $b = 270$	$a°$ and $30°$ make a complementary pair of angles adding to $90°$. Angles in a revolution add to $360°$.
b $a + 65 = 180$ $a = 115$ $b = 65$	$a°$ and $65°$ make a supplementary pair of angles adding to $180°$. $b°$ is vertically opposite the $65°$ angle.

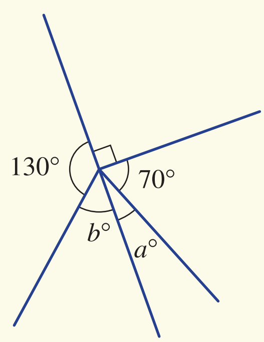
Now you try

Determine the value of the pronumerals in these diagrams.

a

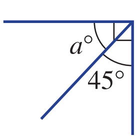


b

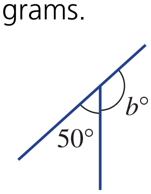


6 State the value of the pronumeral (letter) in these diagrams.

a



b



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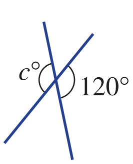
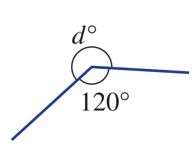
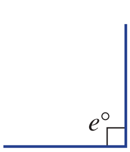


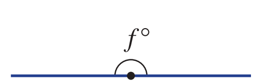
d



e



f



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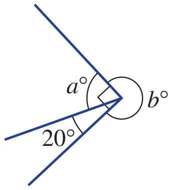
7 Determine the unknown angles marked in these diagrams.

a


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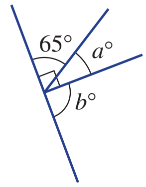
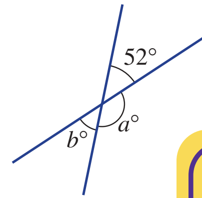
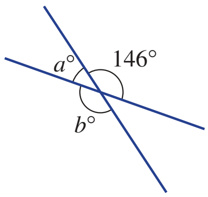


c


d


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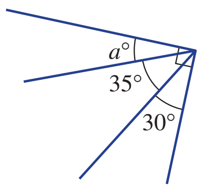

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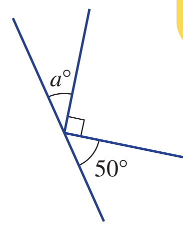
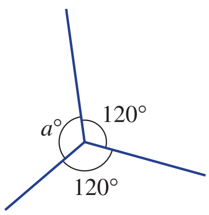
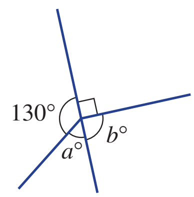
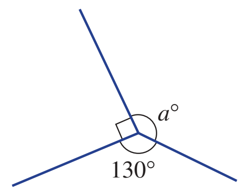


g


h


i


Hint for Q7: Angles in a right angle add to $90°$. Angles on a straight line add to $180°$. Angles in a revolution add to $360°$.

8 Give the compass bearing, in degrees, for these directions.

a West (W) **b** East (E) **c** North (N)
d South (S) **e** NW **f** SE
g SW **h** NE

Hint for Q8: Check the **Key ideas** for help.

Problem-solving and reasoning

9, 10 | 9, 10–11(½), 12

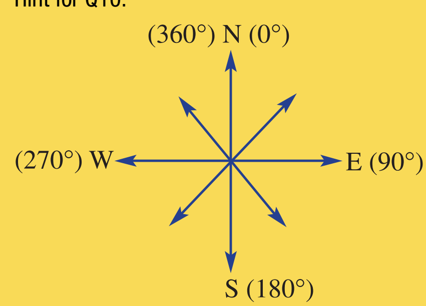
9 A round birthday cake is cut into sectors for nine friends (including Jack) at Jack's birthday party. After the cake is cut there is no cake remaining. What will be the angle at the centre of the cake for Jack's piece if:

a everyone receives an equal share?
b Jack receives twice as much as everyone else? (In parts **b**, **c** and **d** assume his friends have equal shares of the rest.)
c Jack receives four times as much as everyone else?
d Jack receives ten times as much as everyone else?

10 In which direction (e.g. north-east or NE) would you be walking if you were headed on these compass bearings?

a $180°$ **b** $360°$
c $270°$ **d** $90°$
e $45°$ **f** $315°$
g $225°$ **h** $135°$

Hint for Q10:



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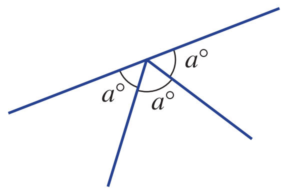
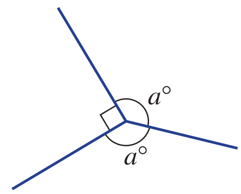
2A

11 Find the value of the pronumerals in these diagrams.

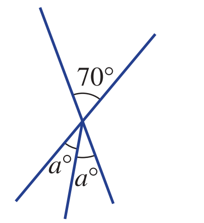
a



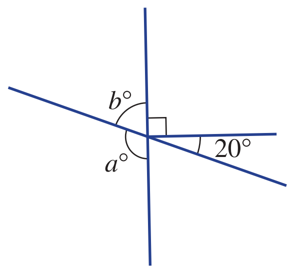
b



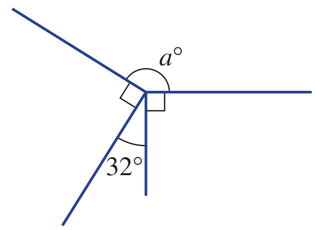
c



d



e



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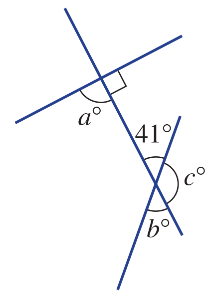
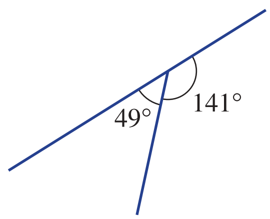


12 Explain, with reasons, what is wrong with these diagrams.

a



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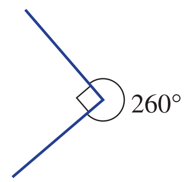

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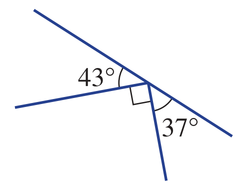

Clock geometry

Clock geometry	—	13, 14

13 Here is a clock face with an hour hand (short arrow) and minute hand (long arrow). The time shown is 8:20.

a How many degrees does the hour hand turn in:
- **i** 6 hours? **ii** 12 hours?
- **iii** 1 hour? **iv** 3 hours?

b How many degrees does the minute hand turn in:
- **i** 1 hour? **ii** 30 minutes?
- **iii** 5 minutes? **iv** 20 minutes?

14 What is the angle between the hour hand and minute hand on a clock at these times?

- **a** 2:30 pm **b** 5:45 am
- **c** 1:40 am **d** 10:20 pm
- **e** 2:35 am **f** 12:05 pm
- **g** 4:48 pm **h** 10:27 am

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2B Parallel lines CONSOLIDATING

Learning intentions

- To understand that parallel lines never intersect and that arrows are used to indicate this on a diagram.
- To be able to use properties of parallel lines to find unknown angles.

Key vocabulary: parallel lines, transversal, corresponding, alternate, cointerior

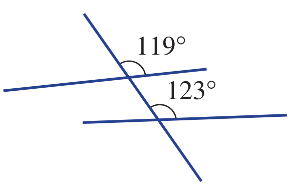
In simple language, Euclid's 5th axiom says that parallel lines do not intersect or meet on a flat plane.

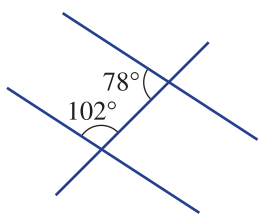
All sorts of shapes and solids both in the theoretical and practical worlds can be constructed using parallel lines. If two lines are parallel and are cut by a third line called a transversal, special pairs of angles are created.

Parallel lines never intersect.

Lesson starter: Are they parallel?

Here are two diagrams that show a pair of lines crossed by a third line called a transversal. Two angles are given.



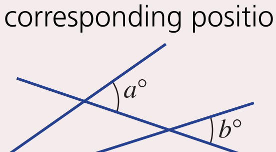


- Do you think that each diagram contains a pair of parallel lines?
- Can you determine all the other angles in the diagrams?
- How many different angles are there in each diagram?

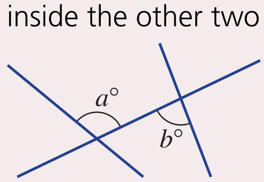
All sorts of shapes and solids can be constructed using parallel lines and transversals.

Key ideas

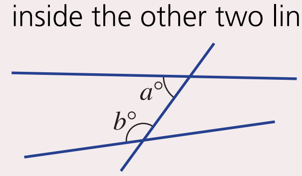
- A **transversal** is a line cutting at least two other lines.
- Pairs of angles formed by transversals can be:
  - **corresponding** (in corresponding positions)
  - **alternate** (on opposite sides of the transversal and inside the other two lines)
  - **cointerior** (on the same side of the transversal and inside the other two lines).



corresponding



alternate



cointerior

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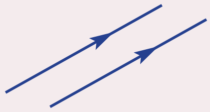
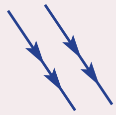
2B

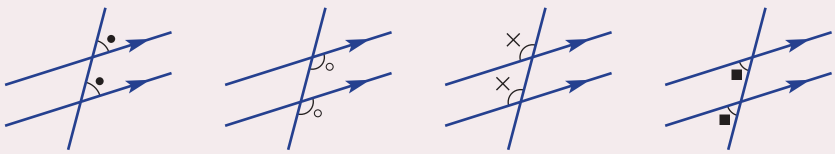
- Lines are **parallel** if they do not intersect.
  - Parallel lines are marked with the same number of arrows.



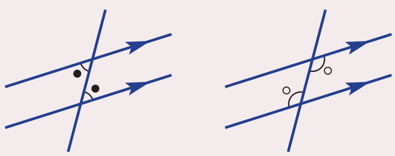
or



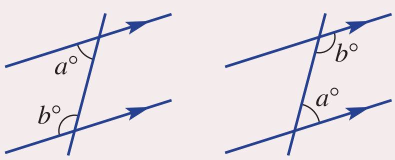
- If two parallel lines are cut by a transversal:
  - the corresponding angles are equal (pairs)



  - the alternate angles are equal (2 pairs)



  - the cointerior angles are supplementary (add up to $180°$) (2 pairs)



$a + b = 180$ $\quad$ $a + b = 180$

Exercise 2B

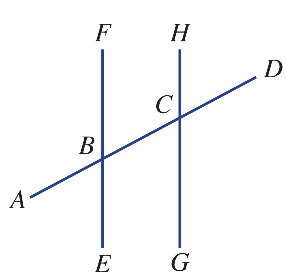
Understanding

1–2 | 2

1 Two parallel lines are cut by a transversal. Write the missing word.
- **a** Corresponding angles are __________.
- **b** Cointerior angles are __________.
- **c** Alternate angles are __________.

Hint for Q1: Choose from *equal* or *supplementary.*

2 Name the angle that is:
- **a** corresponding to $\angle ABF$
- **b** corresponding to $\angle BCG$
- **c** alternate to $\angle FBC$
- **d** alternate to $\angle CBE$
- **e** cointerior to $\angle HCB$
- **f** cointerior to $\angle EBC$
- **g** vertically opposite to $\angle ABE$
- **h** vertically opposite to $\angle HCB$



Hint for Q2: Name angles like this: $\angle ABC$ or $\angle DEF$

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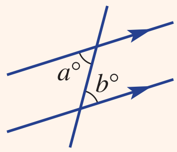
Fluency 3, 4–5(½) 3–5(½)

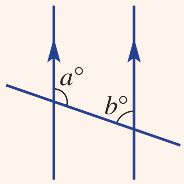
Example 3 Describing related angles in parallel lines

State whether the following marked angles are corresponding, alternate or co-interior. Hence, state whether they are equal or supplementary.

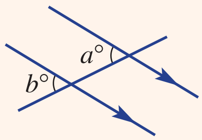
a



b



c

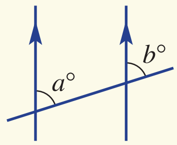


Solution	Explanation
a The marked angles are alternate. Therefore they are equal $(a = b)$.	The two angles are inside the parallel lines and on opposite sides of the transversal, so they are alternate. Alternate angles in parallel lines are equal.
b The marked angles are co-interior. Therefore they are supplementary $(a + b = 180)$.	The two angles are inside the parallel lines and on the same side of the transversal, so they are co-interior. Co-interior angles in parallel lines are supplementary, adding to 180°.
c The marked angles are corresponding. Therefore they are equal $(a = b)$.	The two angles are in corresponding positions (both to the left of the intersection points). Corresponding angles in parallel lines are equal.

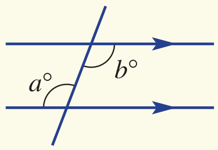
Now you try

State whether the following marked angles are corresponding, alternate or co-interior. Hence state whether they are equal or supplementary.

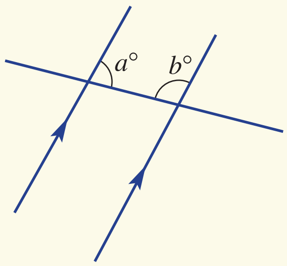
a



b



c



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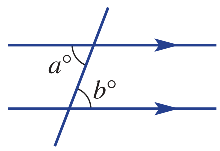
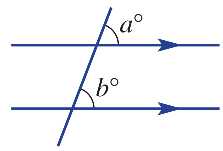
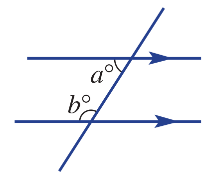
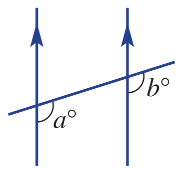
2B

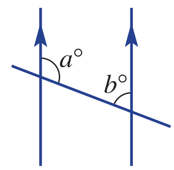
3 State whether the following marked angles are corresponding, alternate or co-interior. Hence, state whether they are equal or supplementary.

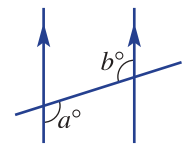
a


b


c


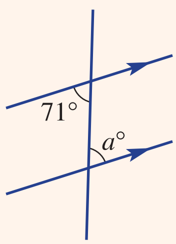
d


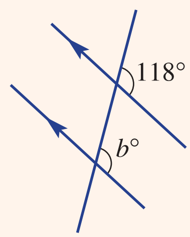
e


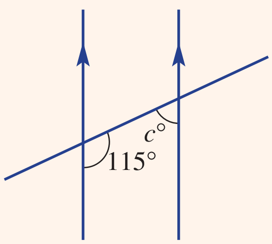
f


Example 4 Working with parallel lines

Find the value of the pronumerals in these diagrams. Give a reason for each answer.

a


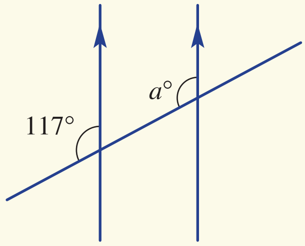
b


c


Solution	Explanation
a $a = 71$, alternate angles in parallel lines.	Alternate angles in parallel lines are equal.
b $b = 118$, corresponding angles in parallel lines.	Corresponding angles in parallel lines are equal.
c $c = 180 - 115 = 65$, cointerior angles in parallel lines.	Cointerior angles in parallel lines add to 180°.

Now you try

Find the value of the pronumerals in these diagrams. Give a reason for each answer.

a


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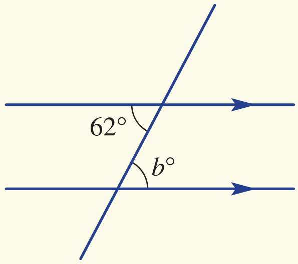

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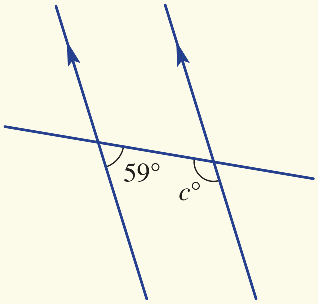

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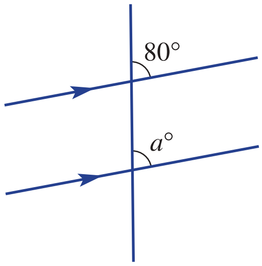
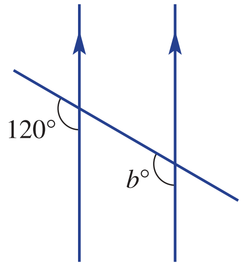
4 Find the value of the pronumerals in these diagrams. Give a reason for each answer.

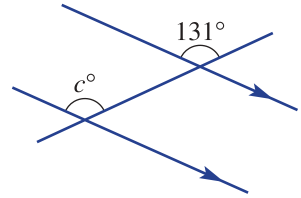
a



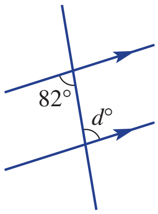
b



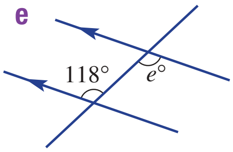
c



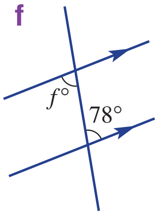
d



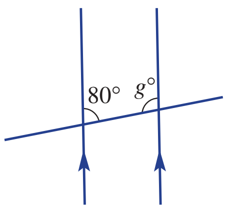
e



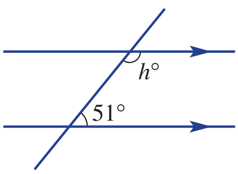
f



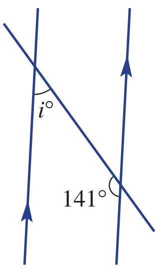
g



h



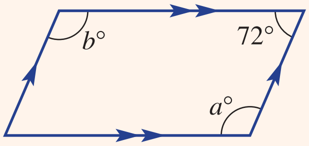
i



Hint for Q4: Corresponding angles are equal in parallel lines. Alternate angles are equal in parallel lines. Cointerior angles in parallel lines are supplementary (add to $180°$).

Example 5 Using parallel lines in shapes

Find the value of the pronumerals in this diagram, stating reasons.

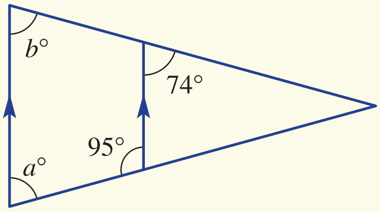


Solution	Explanation
$a + 72 = 180$ $a = 108$	The pairs of angles are cointerior, which are supplementary if the lines are parallel.
$b + 72 = 180$ $b = 108$	This shows that opposite angles in a parallelogram are equal.

Cointerior angles in parallel lines are supplementary.

Now you try

Find the value of the pronumerals in this diagram, stating reasons.



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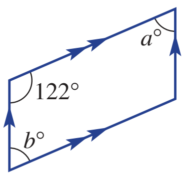
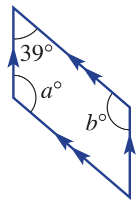
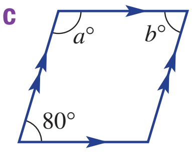
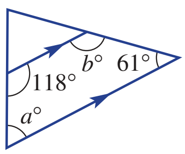
2B

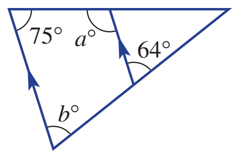
5 Find the value of the pronumerals in these diagrams, stating reasons.

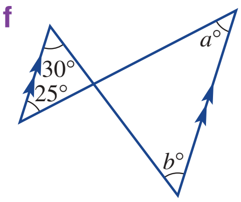
a


b


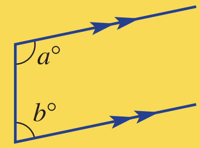
c


d


e


f


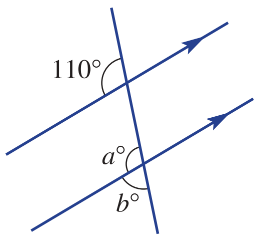
Hint for Q5: Cointerior angles add to 180°.

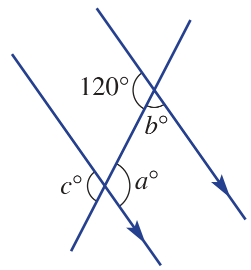


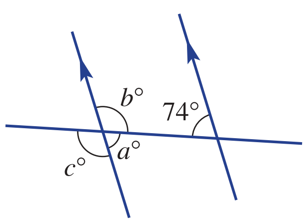
Problem-solving and reasoning

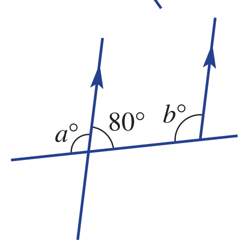
6, 7 | 6(½), 7, 8, 9

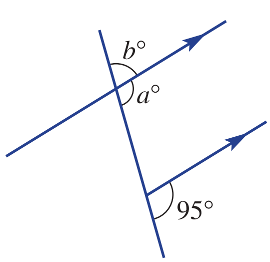
6 Find the value of the pronumerals in these diagrams, stating reasons.

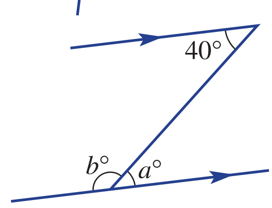
a


b


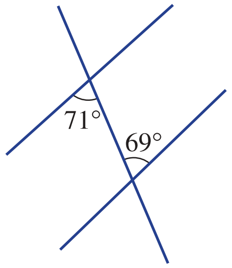
c


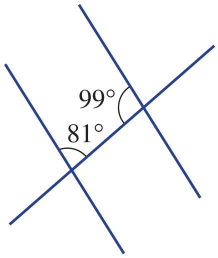
d


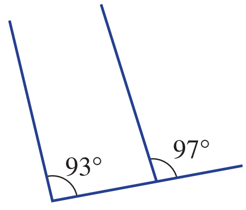
e


f


7 Decide if the following diagrams include a pair of parallel lines. Give a reason for each answer.

a


b


c


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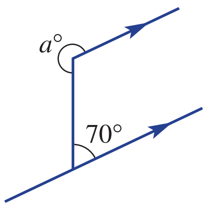
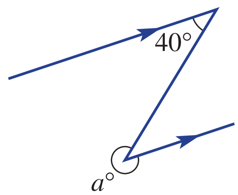
8 Find the value of a in these diagrams.

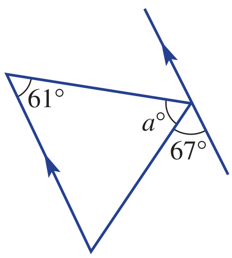
a



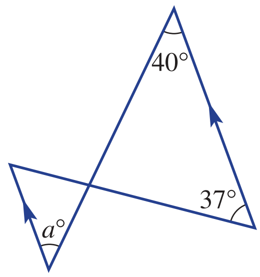
b



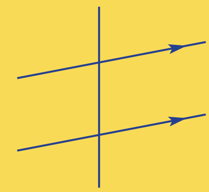
c



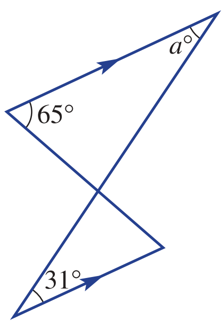
d



Hint for Q8: Extending lines can make it easier to see this type of diagram.



e



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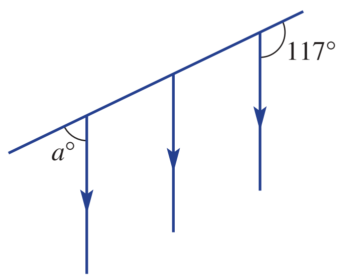
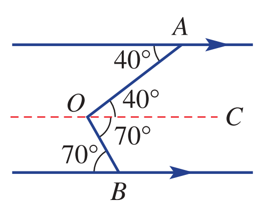
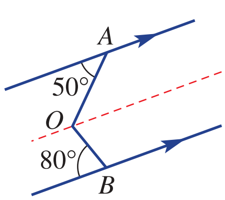


9 Sometimes parallel lines can be added to a diagram to help find an unknown angle. For example, $\angle AOB$ can be found in this diagram by first drawing the dashed line and finding $\angle AOC$ $(40°)$ and $\angle COB$ $(70°)$. So $\angle AOB = 40° + 70° = 110°$.

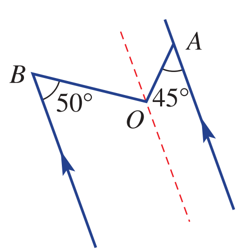


Apply a similar technique to find $\angle AOB$ in these diagrams.

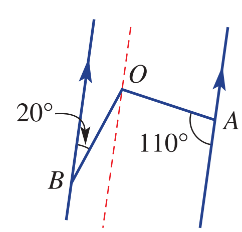
a



b



c



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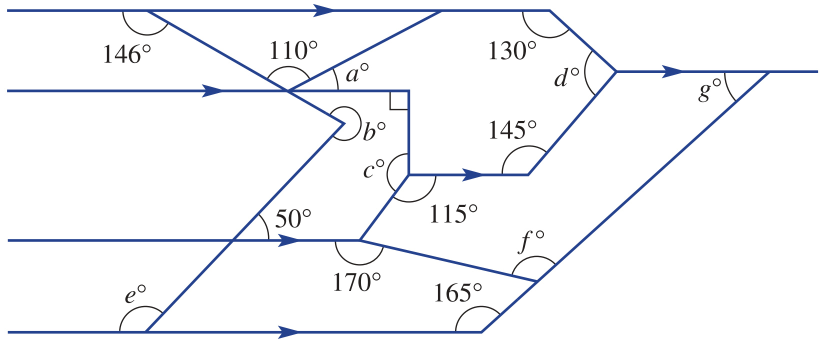
2B

Pipe networks

— 10

10 A plan for a natural gas plant includes many intersecting pipelines, some of which are parallel. Help the designers finish the plans by calculating the size of the angles marked a, b, etc.



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2C Triangles

Learning intentions

- To understand that triangles can be classified by their side lengths as scalene, isosceles or equilateral.
- To understand that triangles can be classified by their interior angles as acute, right or obtuse.
- To know that the angle sum of any triangle is 180°.
- To be able to use the angle sum of a triangle to find unknown angles.
- To be able to use the exterior angle theorem to find unknown angles.

Key vocabulary: scalene, isosceles, apex, base, equilateral, acute, right, obtuse, exterior angle, angle sum

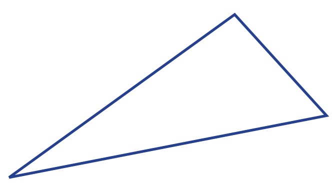
A triangle is a shape with three straight sides. The triangle is a very rigid shape and this leads to its use in the construction of houses and bridges. It is one of the most commonly used shapes in design and construction.

Triangular shapes are often used to striking effect in architecture, as shown by part of the National Gallery of Canada.

Lesson starter: Illustrating the angle sum

You can complete this task using a pencil and ruler or using dynamic geometry software.

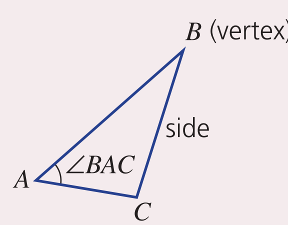
- Draw any triangle and measure each interior angle.
- Add all three angles to find the angle sum of your triangle.
- Compare your angle sum with the results of others. What do you notice?



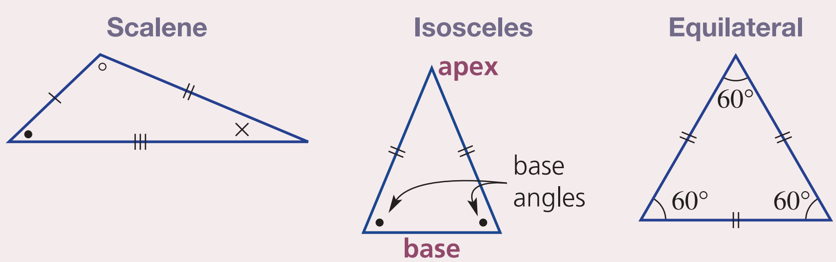
If dynamic geometry is used, drag one of the vertices to alter the interior angles. Now check to see if your conclusions remain the same.

Key ideas

- A **triangle** has:
  - 3 sides
  - 3 vertices (singular: vertex)
  - 3 interior angles.



- Triangles are classified by side lengths:
  - Sides with the same number of dashes are of equal length.



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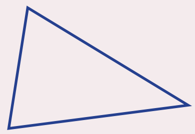
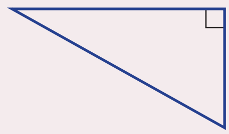
2C

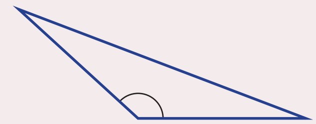
- Triangles are classified by interior angles:
  - **acute** (All angles acute)



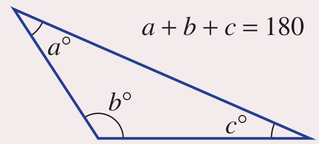
  - **right** (1 right angle)



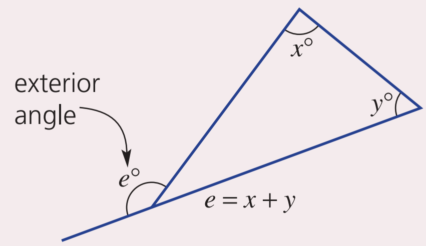
  - **obtuse** (1 obtuse angle).



- The **angle sum** of a triangle is 180°.



- The **exterior angle** theorem:
  The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



Exercise 2C

Understanding

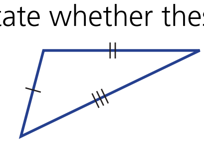
	1–3	2, 3

1 Give the common name of a triangle with these properties. Refer to the **Key ideas** in this section for help.

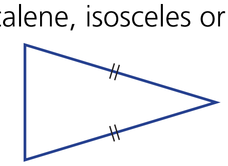
- **a** One right angle
- **b** 2 equal side lengths
- **c** All angles acute
- **d** All angles 60°
- **e** One obtuse angle
- **f** 3 equal side lengths
- **g** 2 equal angles
- **h** 3 different side lengths

2 State whether these triangles are scalene, isosceles or equilateral.

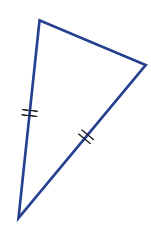
a



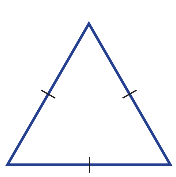
b



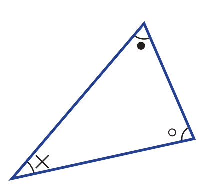
c



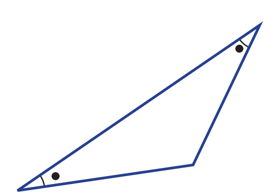
d



e

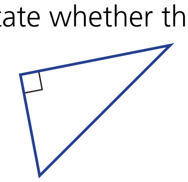


f

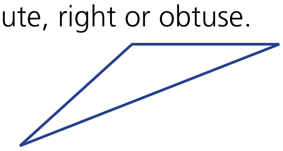


3 State whether these triangles are acute, right or obtuse.

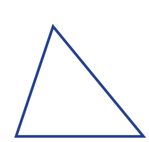
a



b



c



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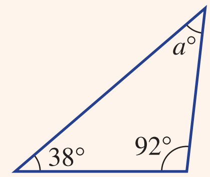
Fluency

4, 5, 6(½) | 4–6(½)

Example 6 Using the angle sum of a triangle

Find the value of a in this triangle.



Solution

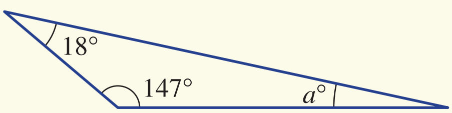
$$a + 38 + 92 = 180$$
$$a + 130 = 180$$
$$a = 50$$

Explanation

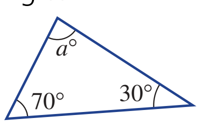
The angle sum of the three interior angles of a triangle is 180°.
Also $38 + 92 = 130$ and $180 - 130 = 50$.

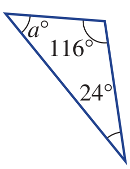
Now you try

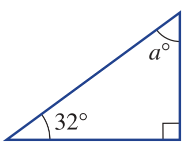
Find the value of a in this triangle.



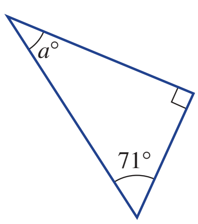
4 Use the angle sum of a triangle to help find the unknown angle in these triangles.

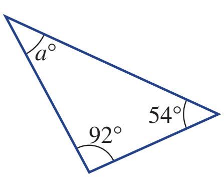
a


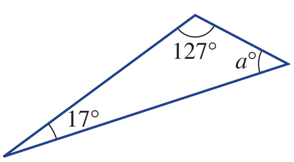
b


c


Hint for Q4: For each one use a mental strategy or start with an equation like $a + 36 + 48 = 180$.

d


e


f


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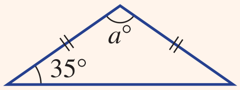
2C

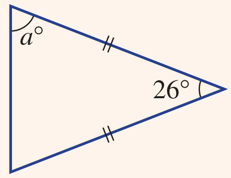
Example 7 Working with isosceles triangles

Find the value of a in these isosceles triangles.

a



b



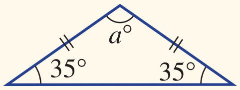
Solution

a

$$a + 35 + 35 = 180$$
$$a + 70 = 180$$
$$a = 110$$

Explanation

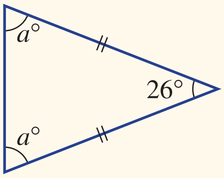
The two base angles in an isosceles triangle are equal.



b

$$2a + 26 = 180$$
$$2a = 154$$
$$a = 77$$

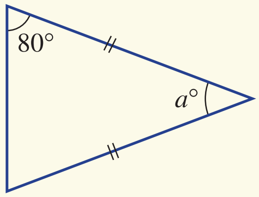
The two base angles in an isosceles triangle are equal.



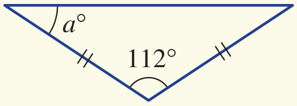
Now you try

Find the value of a in these isosceles triangles.

a

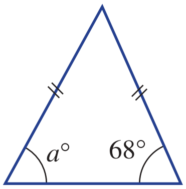


b

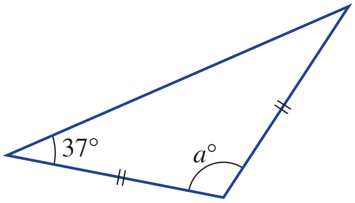


5 These triangles are isosceles. Find the value of a.

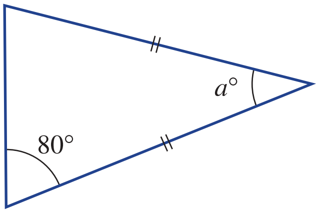
a



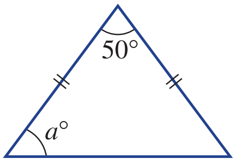
b



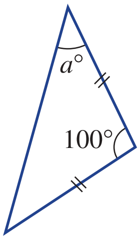
c



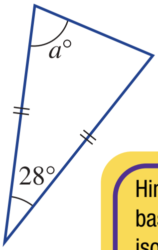
d



e



f



Hint for Q5: The two base angles in an isosceles triangle are equal.

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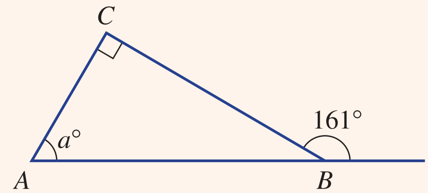
Example 8 Using the exterior angle theorem

Find the size of the unknown angle.



Solution

$a + 90 = 161$

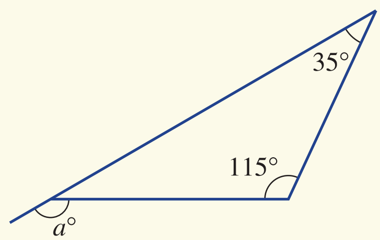
$a = 161 - 90 = 71$

Explanation

Use the exterior angle theorem for a triangle. The exterior angle $(161°)$ is equal to the sum of the two opposite interior angles.

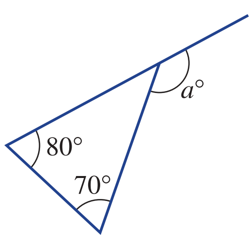
Now you try

Find the size of the unknown angle.

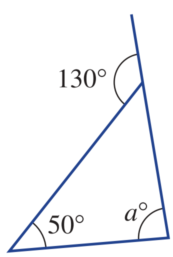


6 Find the value of a.

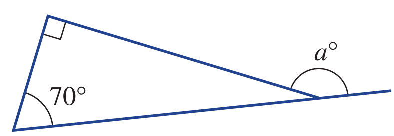
a

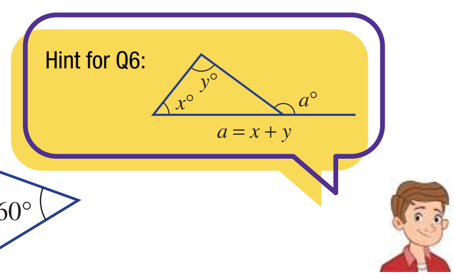


b

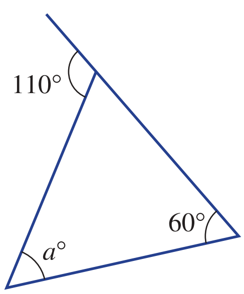


c

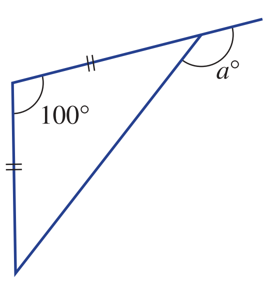




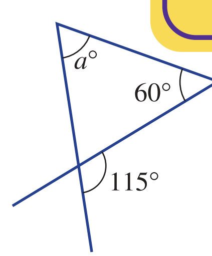
d



e



f



Problem-solving and reasoning 7, 8 7–9

7 Decide if it is possible to draw a triangle with the given description. Draw a diagram to support your answer.

a Right and scalene
b Obtuse and equilateral
c Right and isosceles
d Acute and isosceles
e Acute and equilateral
f Obtuse and isosceles

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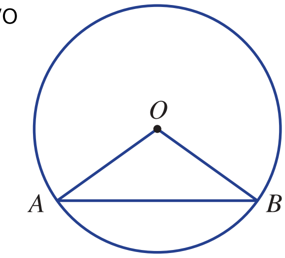
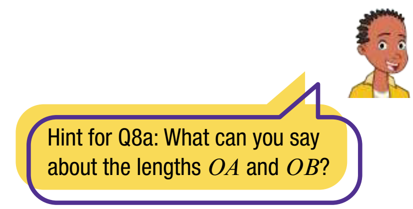
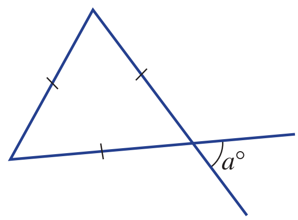
2C

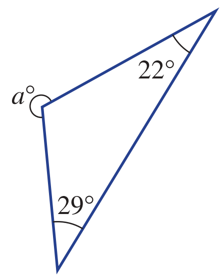
8 A triangle is constructed using a circle and two radius lengths.
  a What type of triangle is ΔAOB and why?
  b Name two angles that are equal.
  c Find $\angle ABO$ if $\angle BAO$ is 30°.
  d Find $\angle AOB$ if $\angle OAB$ is 36°.
  e Find $\angle ABO$ if $\angle AOB$ is 100°.

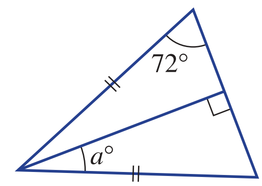


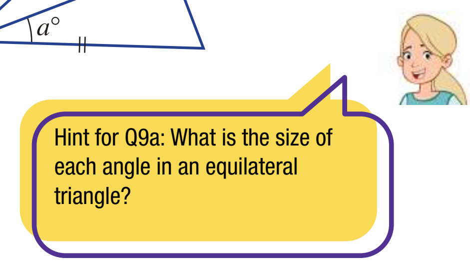


9 Find the value of a in these diagrams.

a


b


c




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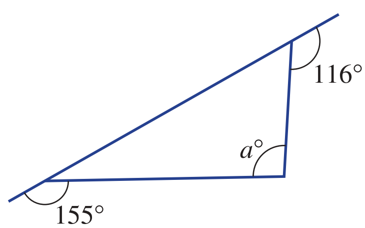
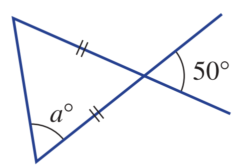


e


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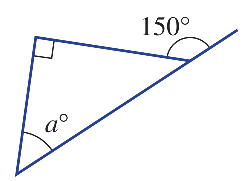
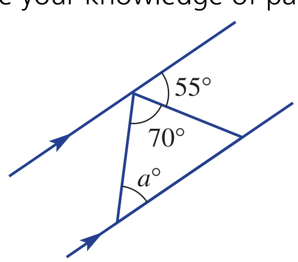
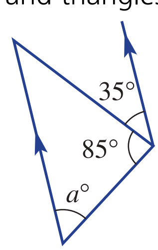
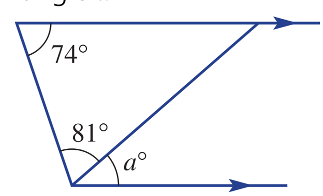


Parallel lines and triangles — 10–11

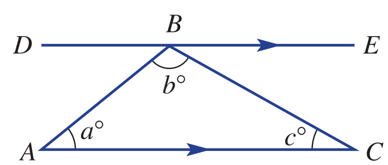
10 Use your knowledge of parallel lines and triangles to find the unknown angle a.

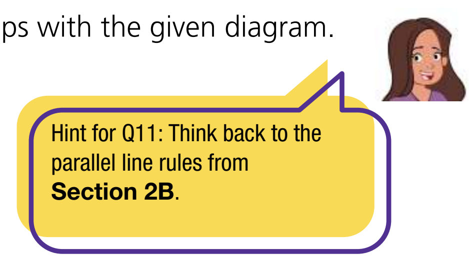
a


b


c


11 To prove that the angle sum of a triangle is 180°, work through these steps with the given diagram.





  a Using the pronumerals a, b or c, give the value of these angles and state a reason.
    i $\angle ABD$
    ii $\angle CBE$
  b What is true about the three angles $\angle ABD$, $\angle ABC$ and $\angle CBE$ and why?
  c What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the angle sum of the triangle ABC?

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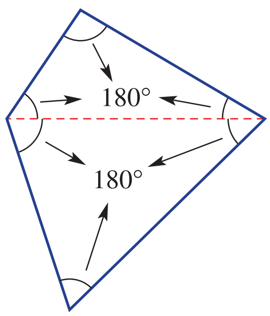
2D Quadrilaterals

Learning intentions

- To be able to classify quadrilaterals as parallelograms, rectangles, rhombuses, squares, kites and/or trapezia.
- To know that the angle sum of any quadrilateral is $360°$.
- To be able to use the angle sum of a quadrilateral to find unknown angles.
- To understand that properties of angles in parallel lines can be used to find unknown angles in trapezia and parallelograms.

Key vocabulary: quadrilateral, parallelogram, square, rectangle, rhombus, kite, trapezium, parallel

Shapes with four sides are called quadrilaterals. All quadrilaterals have the same angle sum. Their other properties depend on such things as pairs of sides of equal length, parallel sides and lengths of diagonals. All quadrilaterals can be divided into two triangles. Since the six angles inside the two triangles make up the four angles of the quadrilateral, the angle sum is $2 \times 180° = 360°$.



Quadrilaterals of many kinds are used by architects.

Lesson starter: Which quadrilaterals suit?

Name all the different quadrilaterals you can think of that have the properties listed below. There may be more than one quadrilateral for each property listed. Draw each quadrilateral to illustrate the shape and its features.

- 4 equal length sides
- 2 pairs of parallel sides
- Equal length diagonals
- 1 pair of parallel sides
- 2 pairs of equal length sides
- 2 pairs of equal opposite angles

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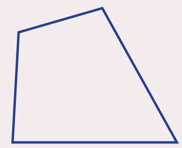
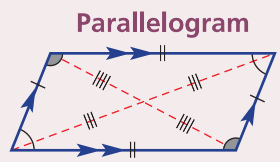
2D

Key ideas

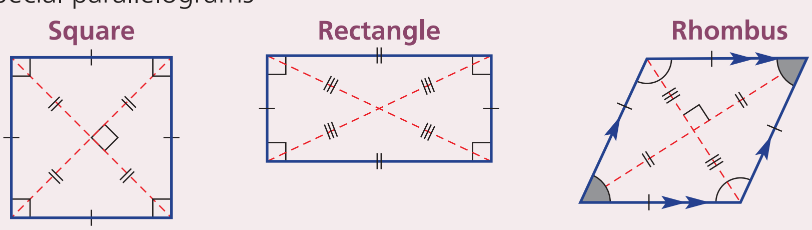
- **Quadrilaterals** are four-sided shapes.



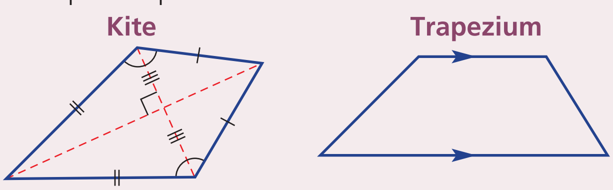
- **Parallelograms** are quadrilaterals with two pairs of **parallel** sides.



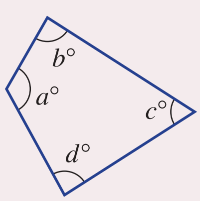
- Special parallelograms



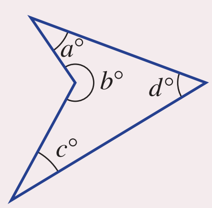
- Other special quadrilaterals



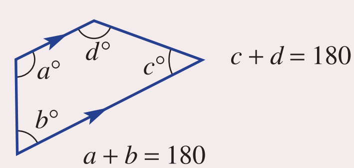
- The angle sum of any quadrilateral is 360°.



$a + b + c + d = 360$



- Quadrilaterals with parallel sides include two pairs of cointerior angles.



Exercise 2D

Understanding

1–3	3

1 What are the six special types of quadrilaterals?

2 Write the missing number or word. Choose from: 90°, *equal*, 360° or *two*.
- **a** The angle sum of a quadrilateral is __________.
- **b** The side lengths of a rhombus are __________ in length.
- **c** A kite has __________ pairs of equal sides.
- **d** The diagonals of squares, rhombuses and kites intersect at __________.

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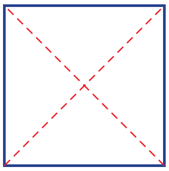
3 Answer true (T) or false (F) to these statements. Refer to the diagrams in the **Key ideas** or accurately draw your own shapes, including the diagonals.

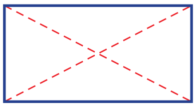
Hint for Q3: The red dashed lines are the diagonals.

a Square



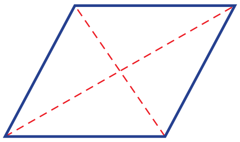
- **i** All sides are of equal length.
- **ii** Diagonals are not equal in length.
- **iii** All sides are parallel to each other.
- **iv** Diagonals intersect at right angles.

b Rectangle



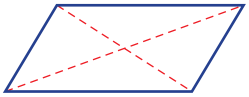
- **i** Diagonals must intersect at right angles.
- **ii** All interior angles are 90°.
- **iii** All sides must be of equal length.
- **iv** There are two pairs of parallel sides.

c Rhombus



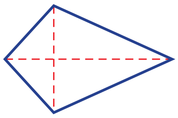
- **i** All interior angles must be equal.
- **ii** All sides are of equal length.
- **iii** Diagonals intersect at right angles.

d Parallelogram



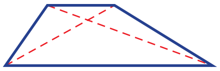
- **i** There are two pairs of parallel sides of equal length.
- **ii** Diagonals must be equal in length.
- **iii** Diagonals must intersect at right angles.

e Kite



- **i** There are two pairs of sides of equal length.
- **ii** There are two pairs of parallel sides.
- **iii** Diagonals intersect at right angles.

f Trapezium



- **i** Diagonals are equal in length.
- **ii** There are two pairs of parallel sides.

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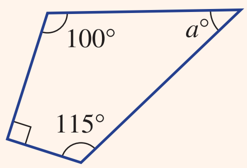
2D

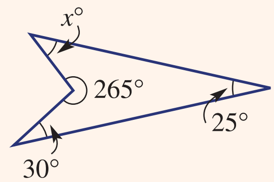
Fluency

4, 5 | 4–5(½)

Example 9 Using the angle sum of a quadrilateral

Find the value of the pronumerals in these quadrilaterals.

a


b


Solution

a $a + 100 + 90 + 115 = 360$
$a + 305 = 360$
$a = 55$

Explanation

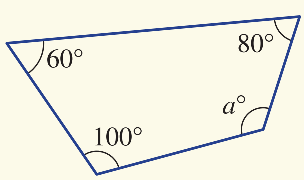
The sum of angles in a quadrilateral is 360°.
Use a mental strategy or solve the equation.

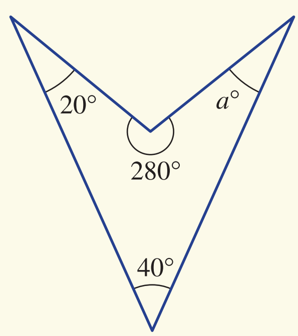
b $x + 265 + 30 + 25 = 360$
$x + 320 = 360$
$x = 40$

Use the angle sum of a quadrilateral.

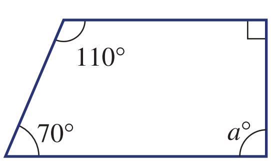
Now you try

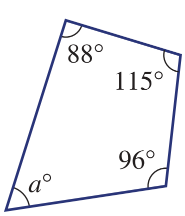
Find the value of the pronumerals in these quadrilaterals.

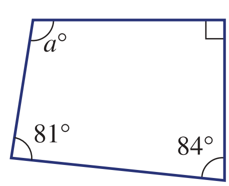
a


b


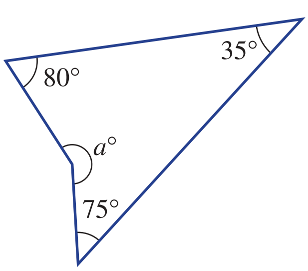
4 Use the quadrilateral angle sum to find the value of a in these quadrilaterals.

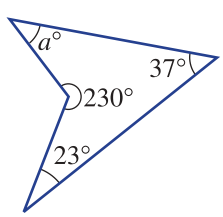
a


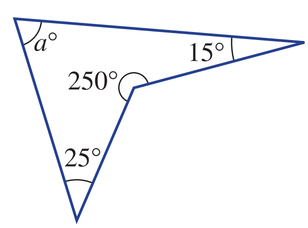
b


c


Hint for Q4: ∟ is a 90° angle. The angle sum of a quadrilateral is 360°.

d


e


f


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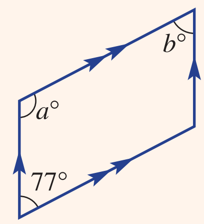
Example 10 Working with parallelograms

Find the value of a and b in this parallelogram.



Solution

$a + 77 = 180$

$a = 103$

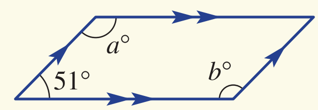
$b = 180 - 103 = 77$

Explanation

Two angles inside parallel lines are cointerior and therefore add up to $180°$.
Note that opposite angles in a parallelogram are equal.

Now you try

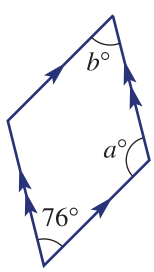
Find the value of a and b in this parallelogram.



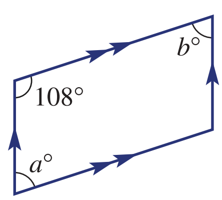
5 Find the value of the pronumerals in these quadrilaterals.

Hint for Q5: Opposite angles in a parallelogram are equal. Other pairs of angles are cointerior (add to $180°$).

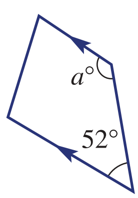
a



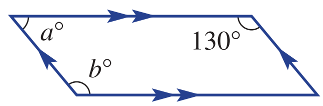
b



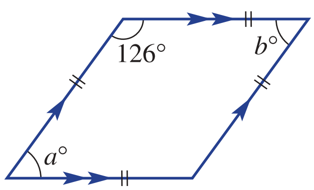
c



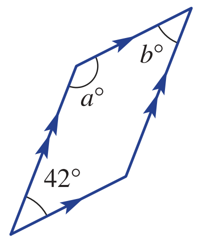
d



e



f



Problem-solving and reasoning

6, 7(½), 8 | 6–9

6 Name the special quadrilaterals which have:

- **a** all sides of equal length
- **b** one pair of parallel lines
- **c** two pairs of equal sides
- **d** diagonals meeting at right angles
- **e** diagonals of equal length.

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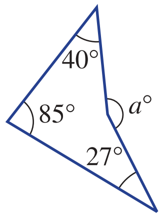
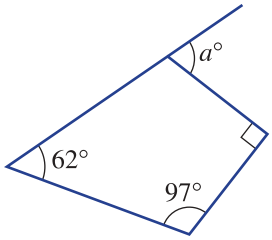
2D

7 Use your knowledge of geometry from the previous sections to find the values of a.

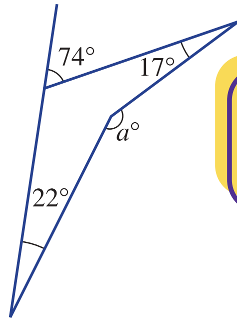
a



b

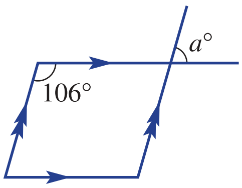


c

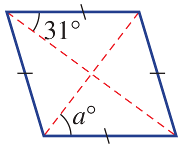


> Hint for Q7: Angles in a revolution add to 360°. Angles on a straight line add to 180°. Vertically opposite angles are equal.

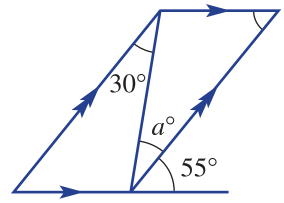
d



e



f

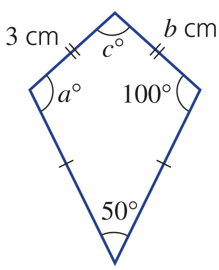


8 Consider the properties of special quadrilaterals. Decide if the following are true (T) or false (F).

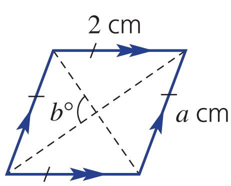
a A square is a type of rectangle.
b A rectangle is a type of square.
c A square is a type of rhombus.
d A rectangle is a type of parallelogram.
e A parallelogram is a type of square.
f A rhombus is a type of parallelogram.

9 Consider the properties of the given quadrilaterals. Give the values of the pronumerals.

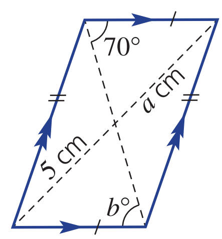
a



b



c

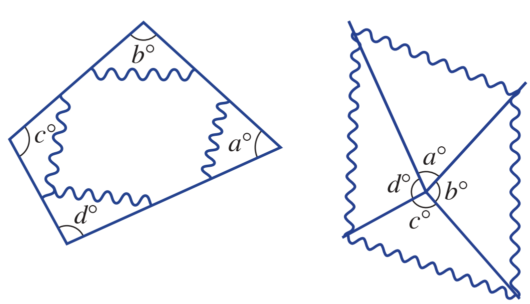


> Hint for Q9:
> **a** A kite
> **b** A rhombus
> **c** A parallelogram

The 'tear off' proof — 10

10 You can confirm that the angle sum of a quadrilateral is 360° by tearing off the corners of any cut-out quadrilateral.

a Use a ruler to draw any quadrilateral and then cut it out.
b Tear off the four corners.
c Arrange the four pieces so the corners (vertices of quadrilateral) all meet at one point as shown. What do you notice?
d What does this tell you about the angle sum of the quadrilaterals?



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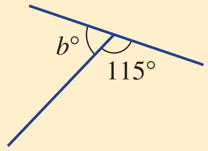
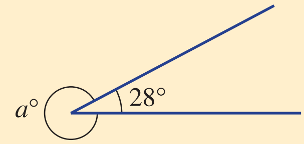
2A **1** Determine the value of the pronumerals in these diagrams.

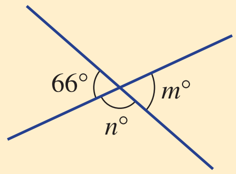
a



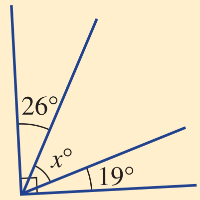
b



c

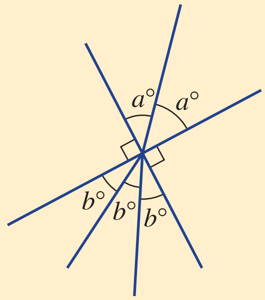


d

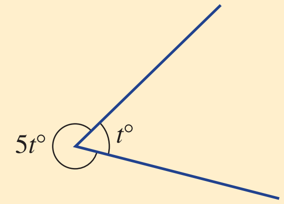


2A **2** Find the value of the pronumerals in these diagrams.

a

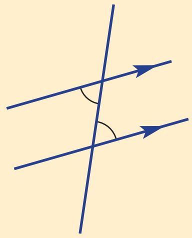


b

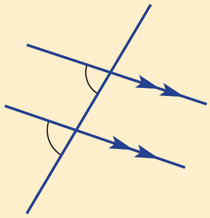


2B **3** Which of the following diagrams have a pair of corresponding angles marked?

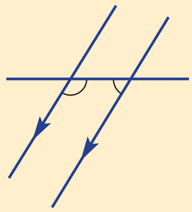
A



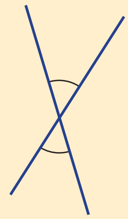
B



C

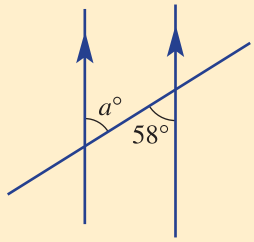


D

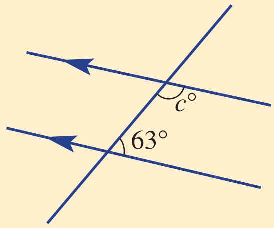


2B **4** Find the value of the pronumerals in these diagrams. Give a reason for each answer.

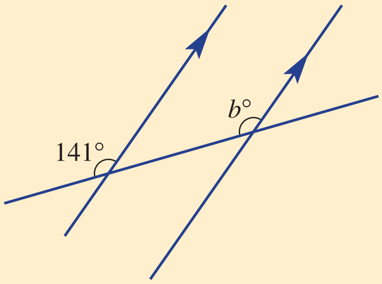
a



b



c



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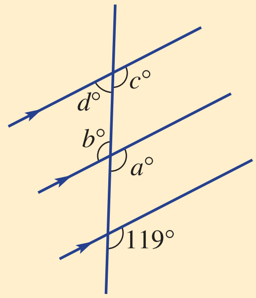
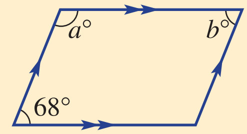
Progress quiz

2B **5** Find the value of the pronumerals in these diagrams, stating reasons.

a

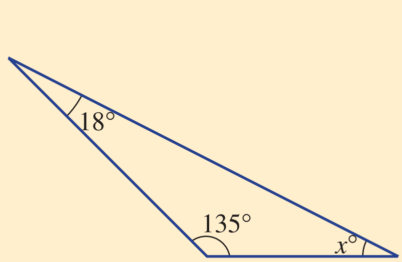


b

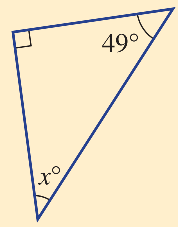


2C **6** Use the angle sum of a triangle to help find the unknown angle in these triangles.

a

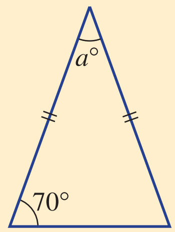


b

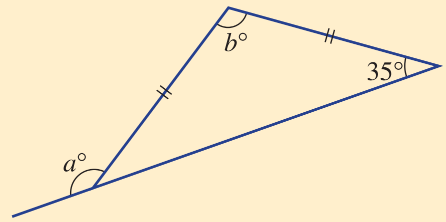


2C **7** Find the size of the unknown angles.

a

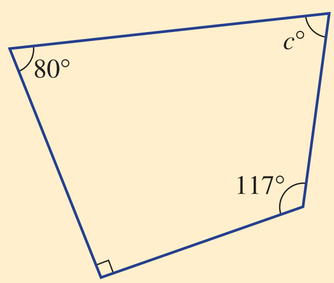


b

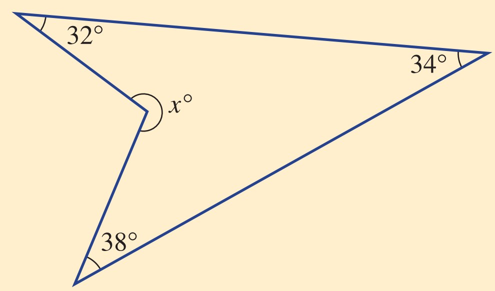


2D **8** Find the value of the pronumerals in these quadrilaterals.

a

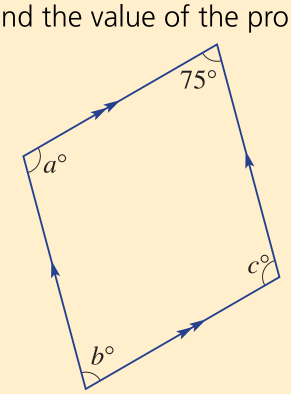


b

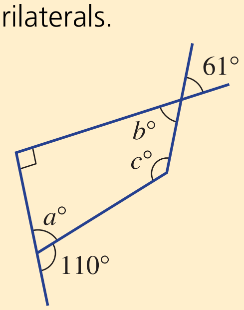


2D **9** Find the value of the pronumerals in these quadrilaterals.

a



b



2D **10** Decide if the following are true (T) or false (F).

a All squares are rectangles.

b All quadrilaterals are rectangles.

c All rectangles are rhombuses.

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2E Polygons ★

Learning intentions

- To know the names of different types of polygons with up to 12 sides.
- To understand what a regular polygon is.
- To be able to find the angle sum of a polygon.
- To be able to use the angle sum of a polygon to find unknown angles.

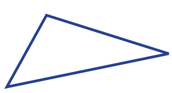
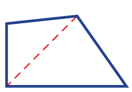
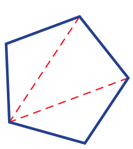
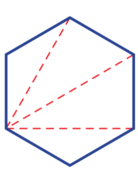
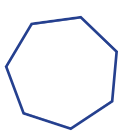
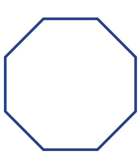
Key vocabulary: polygon, regular polygon, pentagon, hexagon, heptagon, octagon, nonagon, decagon, hendecagon, dodecagon

Triangles and quadrilaterals are both examples of polygons. The word 'polygon' comes from the Greek words *poly*, meaning 'many', and *gonia*, meaning 'angles'. The number of interior angles equals the number of sides. The angle sum of each type of polygon depends on this number. In this section we will explore the rule for the angle sum of a polygon with n sides.

The Pentagon is a famous government office building in Washington DC, USA.

Lesson starter: Developing the rule

The following procedure uses the fact that the angle sum of a triangle is $180°$. Complete the table and try to write the general rule in the final row.

Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180° = 180°$
Quadrilateral 	4	2	$\_\_\_ \times 180° = \_\_\_$
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		$(\_\_\_) \times 180° = \_\_\_$

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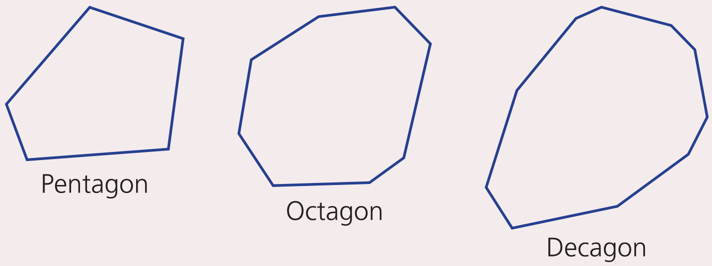
2E

Key ideas

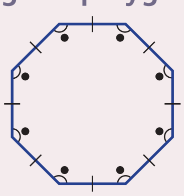
- **Polygons** are shapes with straight sides.



- Polygons are named according to their number of sides.

Number of sides	Name	Angle sum
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1080°
9	Nonagon	1260°
10	Decagon	1440°
11	Hendecagon	1620°
12	Dodecagon	1800°

- The angle sum S of a polygon with n sides is given by the rule: $S = (n - 2) \times 180°$.
- A **regular polygon** has sides of equal length and equal interior angles.



A regular octagon

Exercise 2E

Understanding 1–4 4

1 Name the polygon with the following number of sides.

a 7 **b** 3 **c** 8 **d** 9
e 12 **f** 10 **g** 4 **h** 11

2 State the number of sides on these polygons.

a Hexagon **b** Quadrilateral **c** Decagon
d Heptagon **e** Pentagon **f** Dodecagon

3 Evaluate $(n - 2) \times 180°$ if:

a $n = 6$ **b** $n = 10$ **c** $n = 22$

4 What is the common name given to these polygons?

a Regular quadrilateral
b Regular triangle

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Fluency

5–6(½), 7, 8(½) | 5–6(½), 7, 8

Example 11 Finding the angle sum

Find the angle sum of a heptagon.

Solution

$S = (n - 2) \times 180°$
$= (7 - 2) \times 180°$
$= 5 \times 180°$
$= 900°$

Explanation

A heptagon has 7 sides so $n = 7$. Simplify $(7 - 2)$ before multiplying by $180°$.

Now you try

Find the angle sum of an octagon.

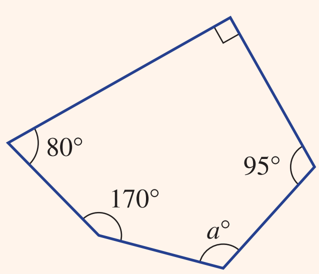
5 Find the angle sum of these polygons.

- **a** Pentagon $(n = 5)$
- **b** Octagon $(n = 8)$
- **c** Decagon $(n = 10)$
- **d** Hexagon
- **e** Nonagon
- **f** Heptagon

Hint for Q5:
Use $S = (n - 2) \times 180°$

Example 12 Finding angles in polygons

Find the value of a in this pentagon by using the given angle sum.



Angle sum = $540°$

Solution

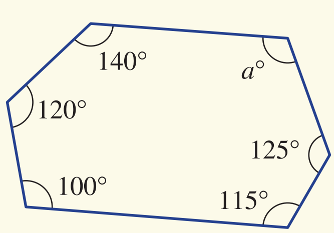
$a + 170 + 80 + 90 + 95 = 540$
$a + 435 = 540$
$a = 105$

Explanation

Add all the angles and set this equal to the angle sum of $540°$. Then simplify and solve for a or use a mental strategy.

Now you try

Find the value of a in this hexagon by using the given angle sum.



Angle sum = $720°$

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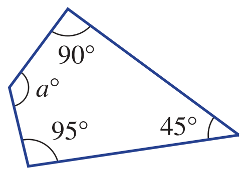
2E

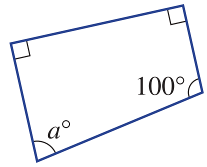
6 Find the value of a in these polygons, by using the given angle sum.

a



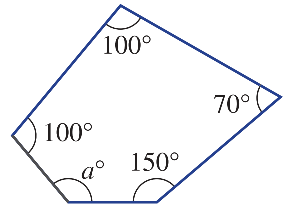
Angle sum = 360°

b



Angle sum = 360°

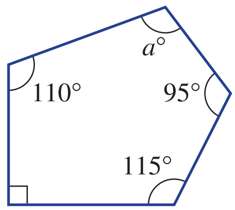
c



Angle sum = 540°

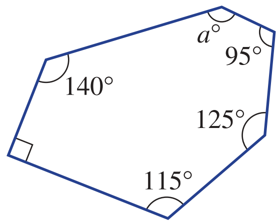
Hint for Q6: Write an equation using the given angle sum, then find the value of a.

d



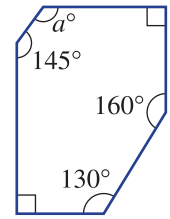
Angle sum = 540°

e



Angle sum = 720°

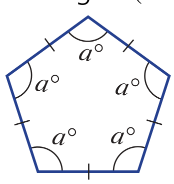
f



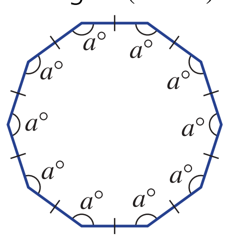
Angle sum = 720°

7 Regular polygons have equal interior angles. Find the size of an interior angle for these regular polygons with the given angle sum.

a Pentagon (540°)

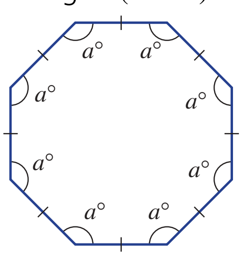


b Decagon (1440°)



Hint for Q7: First find the angle sum then divide by the number of sides.

c Octagon (1080°)



Example 13 Finding interior angles of regular polygons

Find the size of an interior angle in a regular octagon by first finding the angle sum.

Solution	**Explanation**
$S = (n - 2) \times 180°$ $= (8 - 2) \times 180°$ $= 6 \times 180° = 1080°$	First calculate the angle sum of an octagon using $n = 8$ and $S = (n - 2) \times 180°$
Interior angle size $= 1080 \div 8$ $= 135°$	All 8 angles are equal in size so divide the angle sum by 8.

Now you try

Find the size of an interior angle in a regular nonagon by first finding the angle sum.

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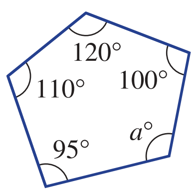
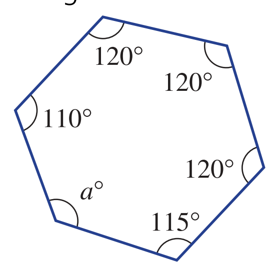
8 Find the size of an interior angle in these regular polygons by firstly finding the angle sum. Round the answer to one decimal place where necessary.
- a Regular pentagon
- b Regular heptagon
- c Regular hexagon
- d Regular decagon
- e Regular octagon
- f Regular hendecagon

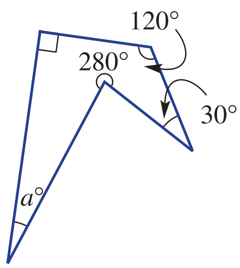
Problem-solving and reasoning

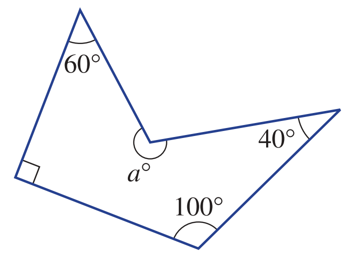
9, 10 | 9(½), 10–12

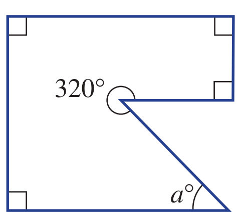
9 Find the value of a in these shapes by firstly finding the angle sum.

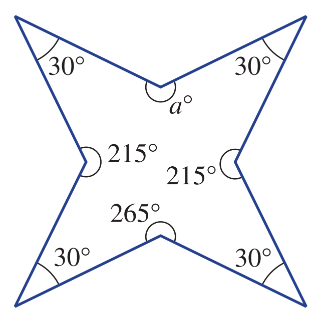
a


b


c


d


e


f


Hint for Q10: The angle sum rule is $S = (n - 2) \times 180°$.

10 Find the number of sides of a polygon with the given angle sums.
- a $1260°$
- b $2340°$
- c $3420°$
- d $29\,700°$

11 Find the number of sides of a regular polygon if each interior angle is:
- a $120°$
- b $162°$
- c $147.272\,727\ldots°$

12 Consider a regular polygon with a very large number of sides (n).
- a What shape does this polygon look like?
- b Is there a limit to the size of a polygon angle sum or does it increase to infinity as n increases?
- c What size does each interior angle approach as n increases?

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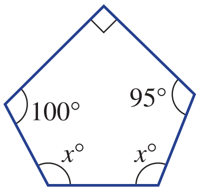
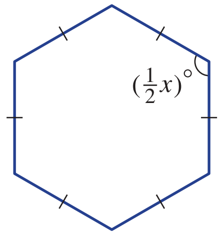
2E

Angle sum challenge

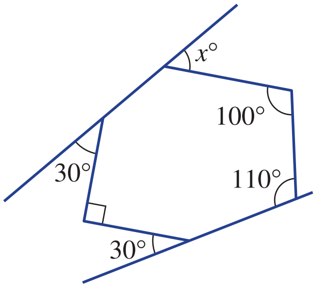
— 13

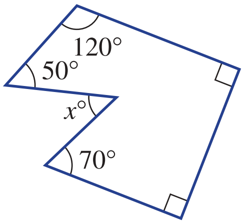
13 Find the value of x in these diagrams.

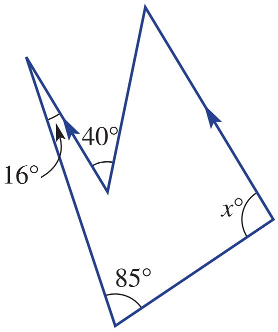
a


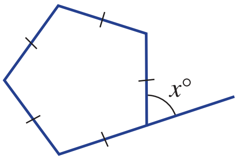
b


Regular hexagon

c


d


e


f


Regular pentagon

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2F Solids ★

Learning intentions

- To be able to name solids using appropriate terminology (e.g. hexagonal prism, square pyramid).
- To be able to count the number of faces, vertices and edges in a solid.

Key vocabulary: polyhedron, prism, pyramid, cross-section, face, vertex (plural: vertices), edge, apex, cylinder, sphere, cone, cube, hexahedron, cuboid, tetrahedron

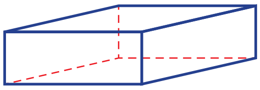
A solid is an object that occupies three-dimensional space. The outside surfaces could be flat or curved. A solid with all flat surfaces is called a polyhedron (plural *polyhedra* or *polyhedrons*). The word 'polyhedron' comes from the Greek words *poly*, meaning 'many', and *hedron*, meaning 'faces'.

The top of this Canary Wharf building in London (left) is a large, complex polyhedron. Polyhedra also occur in nature, particularly in rock or mineral crystals such as quartz (right).

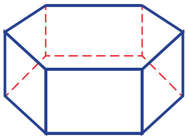
Lesson starter: Naming challenge

Solids can be named by their number of faces. Here are two examples.

Hexahedron (6 faces)



Octahedron (8 faces)



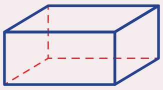
Name and draw a solid with the following number of faces.

- 7 faces
- 5 faces
- 10 faces

Key ideas

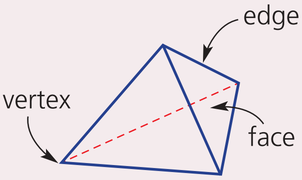
- A **polyhedron** (plural: polyhedra) is a closed solid with flat surfaces (**faces**), **vertices** and **edges**.
  - Polyhedra can be named by their number of faces.

Hexahedron (or rectangular prism or cuboid)



– 6 faces
– 12 edges
– 8 vertices

Tetrahedron (or triangular pyramid)



– 4 vertices
– 4 faces
– 6 edges

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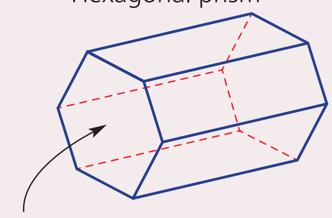
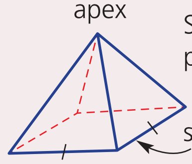
2F

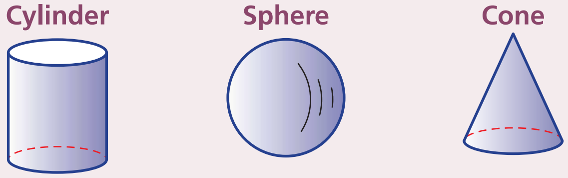
- **Prisms** are polyhedra with two identical (congruent) ends. The congruent ends define the **cross-section** of the prism and also its name. The other faces are parallelograms. If these faces are rectangles, as shown, then the solid is a **right prism**.



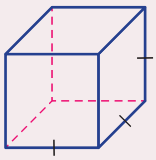
- **Pyramids** are polyhedra with one face that is the base and all other faces meeting at the same vertex point called the **apex**. They are named by the shape of the base.



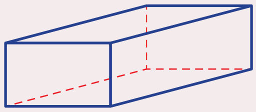
- Some solids have curved surfaces. Common examples include:



- A **cube** is a hexahedron with six square faces.



- Another name for a rectangular prism is **cuboid**. It is also a hexahedron.



Exercise 2F

Understanding

1–3 | 2, 3

1 Write the missing word or number in these sentences. Choose from: *congruent*, *cube*, *six*, *seven*, *vertices*, *octagonal*, *circle*, *seven*.

a A hexahedron has __________ faces.
b The flat face at the base of a cylinder is a __________.
c A hexahedron with six square faces is also called a __________.
d A polyhedron has faces, __________ and edges.
e A heptahedron has _______ faces.
f A prism has two _________ ends.
g A pentagonal prism has _________ faces.
h The base of a pyramid has 8 sides. The pyramid is called an _________ pyramid.

2 Name three solids that have curved surfaces.

3 Which of these solids are polyhedra (i.e. have only flat surfaces)?

A Cube	**B** Pyramid	**C** Cone	**D** Sphere
E Cylinder	**F** Rectangular prism	**G** Tetrahedron	**H** Hexahedron

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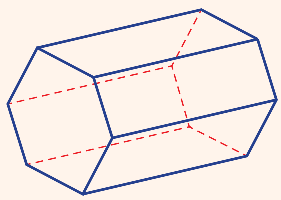
Fluency 4–8 4–8

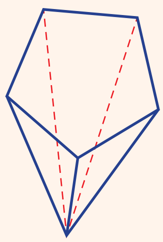
Example 14 Counting faces, vertices and edges

State the number of faces, vertices and edges for these solids.

a Octahedron (or Hexagonal prism)



b Hexahedron (or Pentagonal pyramid)



Solution

a 8 faces
12 vertices
18 edges

Explanation

Faces are the flat surfaces.
Vertices are the corners.
Edges are the lines on the diagram.

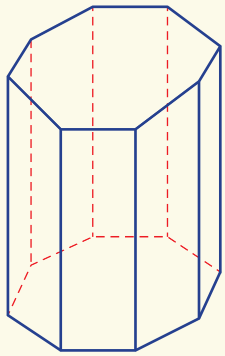
b 6 faces
6 vertices
10 edges

There is one pentagonal face and five triangular faces.
Five vertices are on the base plus one apex.
Five edges are on the base and five meet the apex.

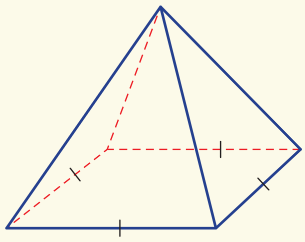
Now you try

State the number of faces, vertices and edges for these solids.

a Decahedron (Octagonal prism)

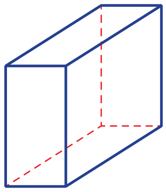


b Pentahedron (Square-based pyramid)

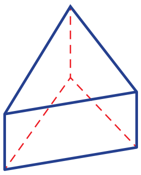


4 Count the number of faces, vertices and edges (in that order) on these polyhedra.

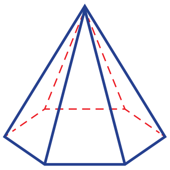
a

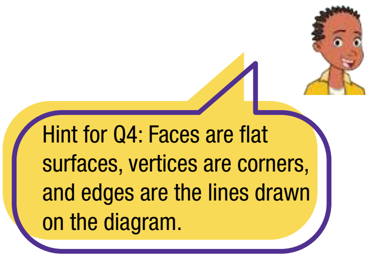


b



c





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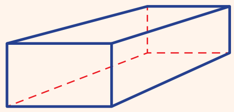
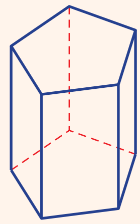
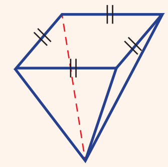
2F

Example 15 Classifying solids using faces

Classify these solids by considering the number of faces.

a 

b 

c 

Solution	Explanation
a Hexahedron	The solid has 6 faces.
b Heptahedron	The solid has 7 faces.
c Pentahedron	The solid has 5 faces.

Now you try

Name the polyhedron that has 8 faces.

5 Name the polyhedron that has the given number of faces.

a 6 b 4 c 5 d 7
e 9 f 10 g 11 h 12

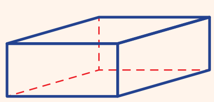
Hint for Q5: Use names such as pentahedron, hexahedron.

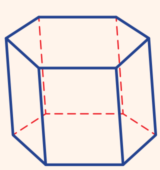
6 How many faces do these polyhedra have?

a Octahedron b Hexahedron c Tetrahedron d Pentahedron
e Heptahedron f Nonahedron g Decahedron h Hendecahedron

Example 16 Naming prisms

Name these solids as a type of prism.

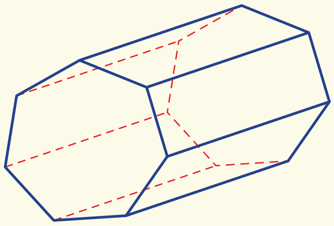
a 

b 

Solution	Explanation
a Rectangular prism	The cross-section is a rectangle.
b Hexagonal prism	The cross-section is a hexagon.

Now you try

Name this solid as a type of prism.



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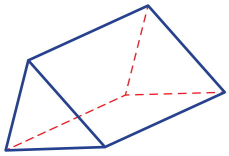
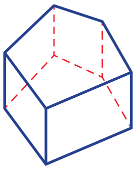
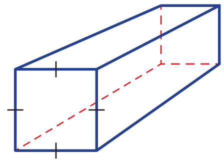
7 Name these prisms.

a


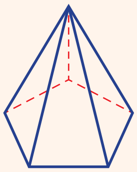
b


c


Hint for Q7: Name using the shape of the cross-section.

Example 17 Naming pyramids

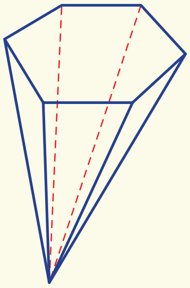
Name this solid as a type of pyramid.



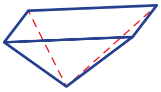
Solution	**Explanation**
Pentagonal pyramid	The base is a pentagon.

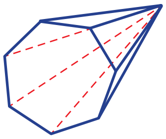
Now you try

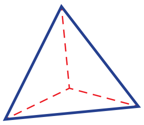
Name this solid as a type of pyramid.



8 Name these pyramids.

a


b


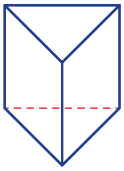
c


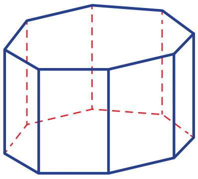
Hint for Q8: Name using the shape of the base.

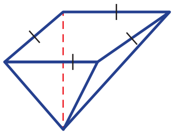
Problem-solving and reasoning

9, 10 | 9–12

9 Name each of these solids in two different ways.

a


b


c


Hint for Q9: As an example: a pentagonal pyramid is also a hexahedron (6 faces).

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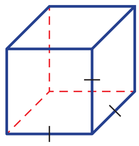
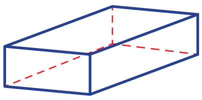
2F

10 Decide if the following statements are true (T) or false (F). Make drawings to help.
- a A tetrahedron is a pyramid.
- b All solids with curved surfaces are cylinders.
- c A cube and a rectangular prism are both hexahedrons.
- d A hexahedron can be a pyramid.
- e There are no solids with 0 vertices.
- f There are no polyhedra with 3 surfaces.
- g All pyramids will have an odd number of faces.

11 Name each of these solids in three different ways.

a


b


12 Investigate if the following statement is true (T) or false (F).
For all pyramids, the number of faces is equal to the number of vertices.

Euler's rule — 13, 14

13 a Copy and complete this table.

Solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)	$F + V$
Cube				
Square pyramid				
Tetrahedron				

b Compare the number of edges (E) with the value $F + V$ for each polyhedron. What do you notice?

14 a A polyhedron has 16 faces and 12 vertices. How many edges does it have?
b A polyhedron has 18 edges and 9 vertices. How many faces does it have?
c A polyhedron has 34 faces and 60 edges. How many vertices does it have?

The National Library of Belarus is a rhombicuboctahedron with 8 triangular faces and 18 square faces.

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2G Three-dimensional coordinate systems

Learning intentions

- To know that the position of objects in three-dimensional (3D) space can be described using a 3D coordinate system
- To be able to describe the position of an object or a point using a 3D coordinate system
- To be able to solve simple geometry problems in 3D space using coordinates

Key vocabulary: Three-dimensional (3D) space, coordinate system

When we find our way around a multi-storey carpark or work with a 3D-printer, we need to consider three-dimensional space. Two dimensions are required to define a space at one level, like zones at ground level in a car park, but a third dimension is required to define spaces above that level.

Three-dimensional coordinate systems are used in many different technologies such as 3D printing.

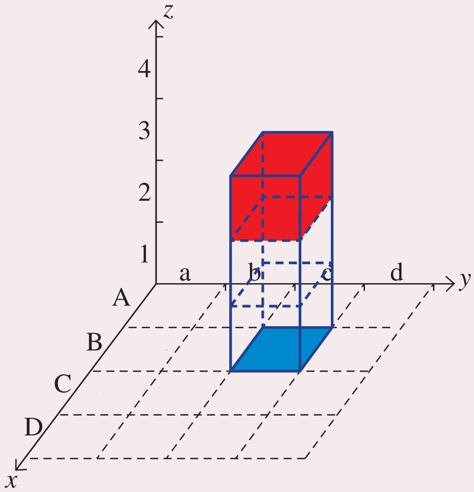
Lesson starter: Where might 3D coordinates be useful?

We know that when trying to describe the position of a zone in a car park you might try to remember three characters. Two characters for the zone on a level like D4 and another character for the level in the building.

- List two or three other examples where a 3D coordinate system might be used to describe position.
- Choose one of your chosen 3D systems and research how the 3D coordinate system is used to describe position. Write down three or four dot points to summarise your findings.

Key ideas

- The position of an object in three-dimensional (3D) space can be described using a coordinate system using three components.
- Three axes: x, y, and z are used to define 3D space.
  - The origin O has coordinates $(0, 0, 0)$.
  - The three axes are all at right angles to each other.
- The position of a cubic unit of space can be described using grid spaces as shown.
  - The red cube is this diagram could be described as $(\mathrm{B}, \mathrm{c}, 3)$.



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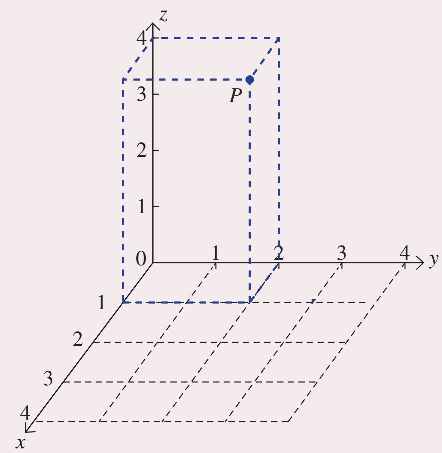
2G

- A point in three-dimensional space is described using (x, y, z) coordinates.
  - The point P is this diagram has coordinates $(1, 2, 4)$.



Exercise 2G

Understanding

1, 2 | 1, 2

1 Consider this set of two blocks sitting in 3D space below.

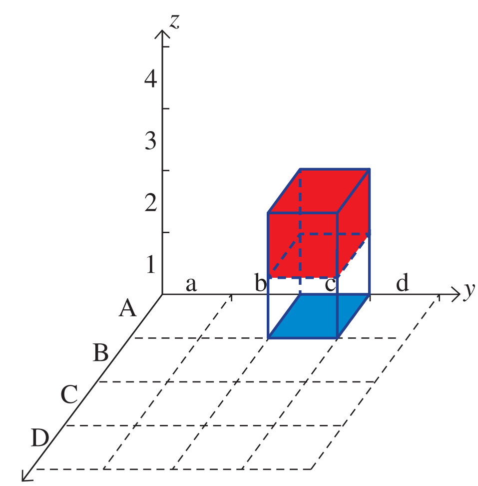
a Which of the following describes the position of the square at the base of the blocks?

A (A, a) **B** (A, c) **C** (B, c)
D (b, A) **E** (C, c)

b Which of the following describes the position of the block which is red?

A (A, a, 2) **B** (B, c, 3) **C** (B, c, 2)
D (A, c, 2) **E** (A, c, 1)

c How many blocks would be required to build a stack where the block at the top is positioned at (B, c, 6)?



2 Consider this 3D coordinate system with point P as shown.

a On the x-y plane, point A has coordinates $(2, 0)$. Which of the following is the coordinates of point A in the 3D system?

A $(2, 0, 1)$ **B** $(2, 0, 3)$
C $(2, 0, 0)$ **D** $(0, 2, 0)$
E $(0, 0, 2)$

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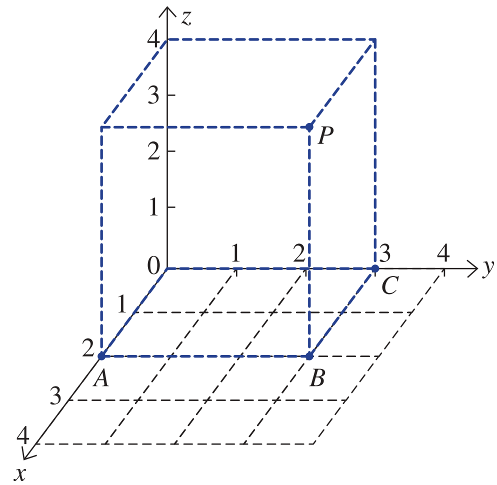
b What are the 3D coordinates of the following points?

i B **ii** C **iii** P

c Pencil the following points onto the given set of axes.

i $A(3, 0, 0)$ **ii** $B(3, 3, 0)$ **iii** $P(3, 3, 3)$

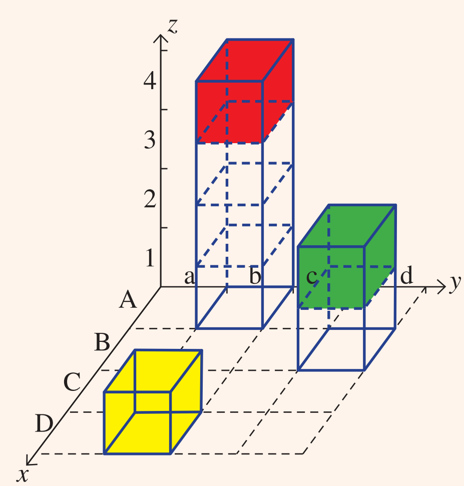


Fluency 3–6 3–7

Example 18 Using grid spaces to describe position in 3D space

Each cube in this 3D space can be described using an $x(\text{A, B, C or D})$, $y(\text{a, b, c or d})$ and $z(1, 2, 3 \text{ or } 4)$ coordinate in that order. Give the coordinates of the cubes with the given colour.

a red
b yellow
c green



Solution

a $(\text{A}, \text{b}, 4)$

Explanation

Using the order x, then y then z, we have the base of the column in the square (A, b) and the red cube is at level 4 on the z-axis.

b $(\text{D}, \text{b}, 1)$

The base of the yellow cube is the square (D, b) and it is at level 1 on the z-axis.

c $(\text{B}, \text{d}, 2)$

The green cube has a base square at (B, d) and is at level 2 on the z-axis.

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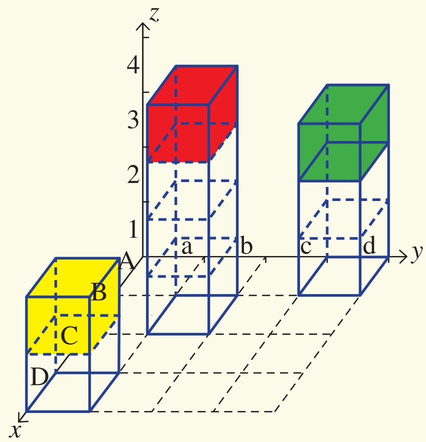
2G

Now you try

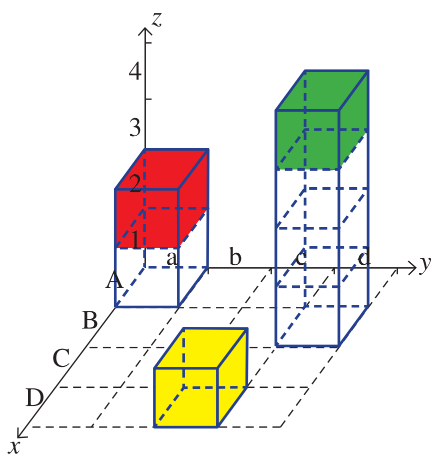
Each cube in this 3D space can be described using an $x(\text{A, B, C or D})$, $y(\text{a, b, c or d})$ and $z(1, 2, 3 \text{ or } 4)$ coordinate in that order. Give the coordinates of the cubes with the given colour.

a red
b yellow
c green



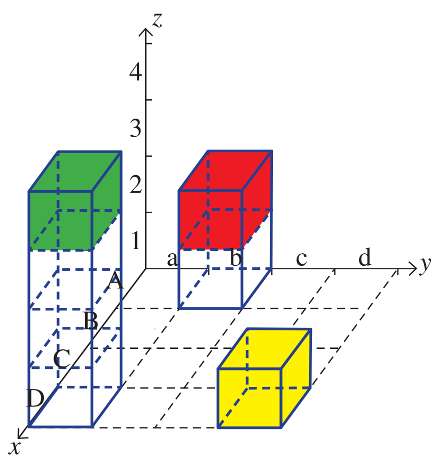
3 Each cube in this 3D space can be described using an $x(\text{A, B, C or D})$, $y(\text{a, b, c or d})$ and $z(1, 2, 3 \text{ or } 4)$ coordinate in that order. Give the coordinates of the cubes with the given colour.

a red
b yellow
c green



4 Each cube in this 3D space can be described using an $x(\text{A, B, C or D})$, $y(\text{a, b, c or d})$ and $z(1, 2, 3 \text{ or } 4)$ coordinate in that order. Give the coordinates of the cubes with the given colour.

a red
b yellow
c green



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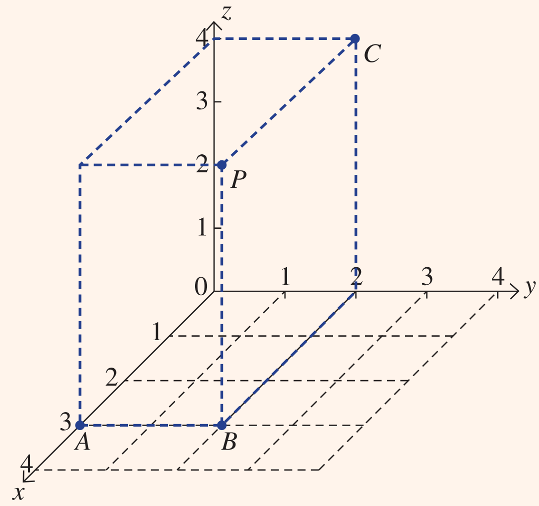
Example 19 Using 3D coordinates to describe a point

Write down the coordinates of the following points shown on this 3D graph.

a A
b B
c C
d P

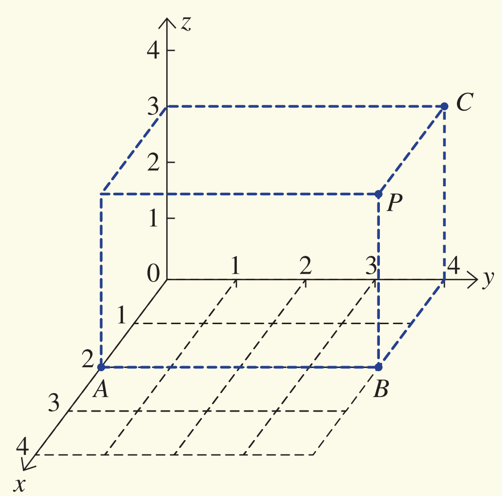


Solution	Explanation
a $(3, 0, 0)$	Point A is positioned at 3 on the x-axis, 0 on the y-axis and 0 on the z-axis.
b $(3, 2, 0)$	Point B is on the x-y plane at point $(3, 2)$ and has a z-coordinate of 0.
c $(0, 2, 4)$	Point C is on the y-z plane and so the x-coordinate is 0.
d $(3, 2, 4)$	On the rectangular prism shown, point P is aligned with the number 3 on the x-axis, 2 on the y-axis and 4 on the z-axis.

Now you try

Write down the coordinates of the following points shown on this 3D graph.

a A
b B
c C
d P



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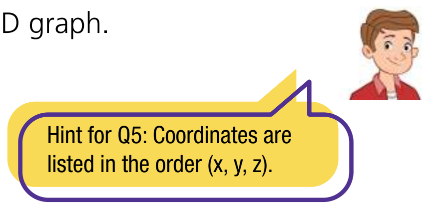
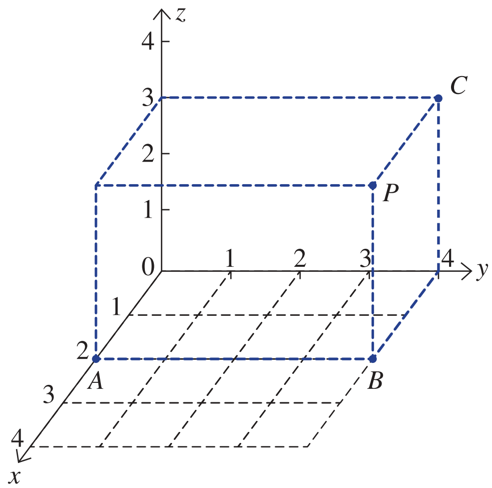
2G

5 Write down the coordinates of the following points shown on this 3D graph.

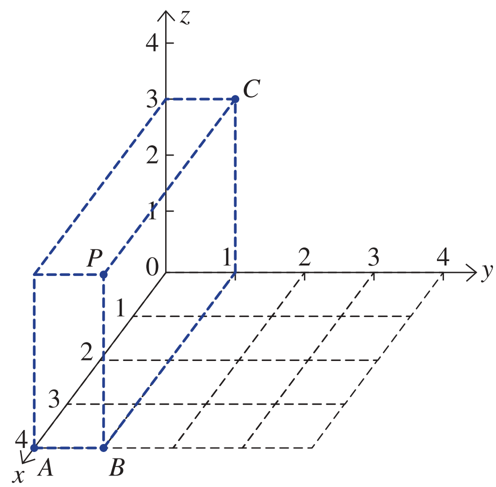
a A
b B
c C
d P





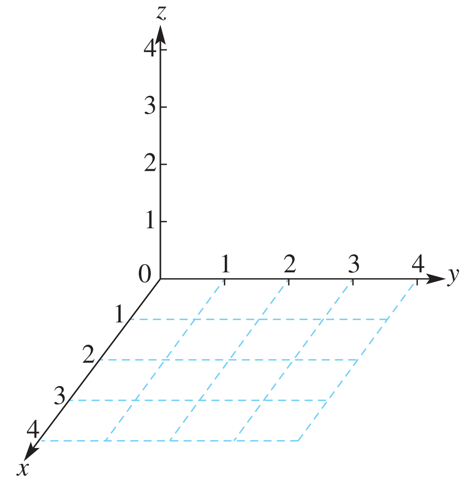
6 Write down the coordinates of the following points shown on this 3D graph.

a A
b B
c C
d P



7 Plot the given points onto this 3D coordinate system.

a $A\ (1, 2, 4)$
b $B\ (3, 1, 2)$
c $C\ (0, 2, 3)$
d $D\ (4, 0, 1)$
e $E\ (3, 3, 0)$
f $F\ (4, 4, 3)$



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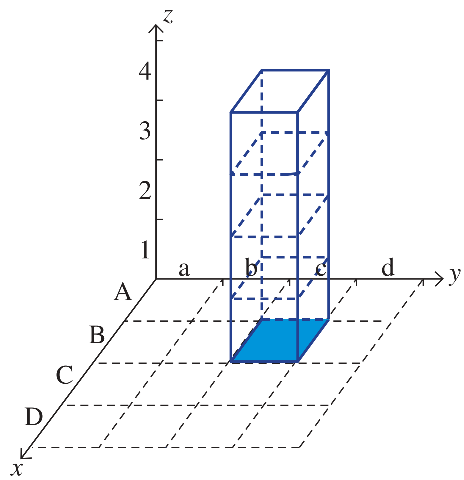
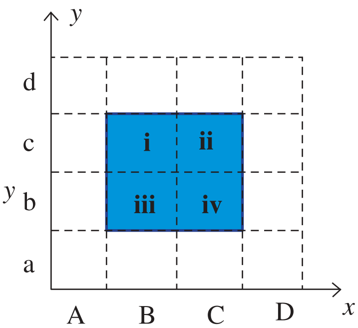
Problem-solving and reasoning

8–10 | 9–12

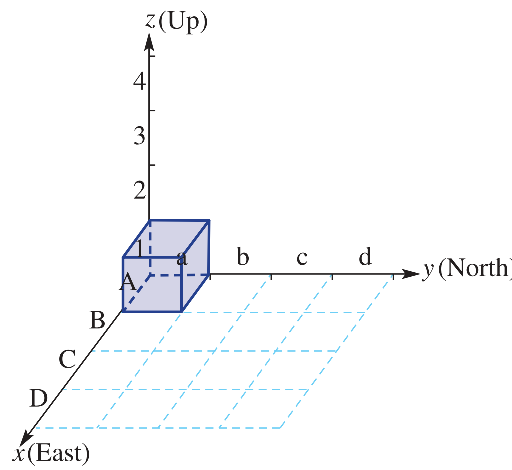
8 Choose the correct square on the 2D diagram (i, ii, iii or iv) that matches the base of the block from the 3D diagram.





9 Imagine the cube at (A, a, 1) that can only move in three directions: East (x), North (y) or Up (z). How many spaces would the cube move in total if it was shifted to the positions with the following coordinates?

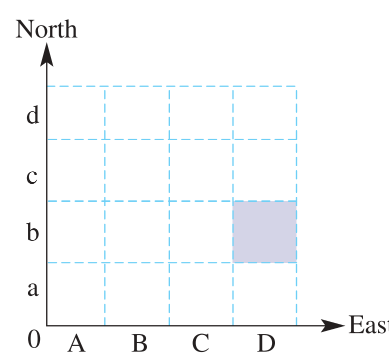
a (C, c, 2)
b (B, a, 3)
c (A, c, 4)
d (D, b, 2)
e (C, d, 4)
f (B, c, 1)



10 A multi-storey carpark has 5 levels and each level is divided into different zones in the same way. The zones for each level are shown in this diagram. A zone which is in the 4th column (East) and 2nd row (North) and on the 3rd level is given the coordinates (D, b, 3).

a Give the coordinates of a zone which is:

i in the 3rd column East, 2nd row North and on the 4th level.
ii in the 2nd column East, 3rd row North and on the 3rd level.



b A car is parked in zone (C, a, 2) and is to move two spaces to the West, three spaces to the North and up three levels. What are the coordinates of the zone where the car is to be moved?

c A car is parked in zone (B, d, 5) and is to move one space to the East, two spaces to the South and down three levels. What are the coordinates of the zone where the car is to be moved?

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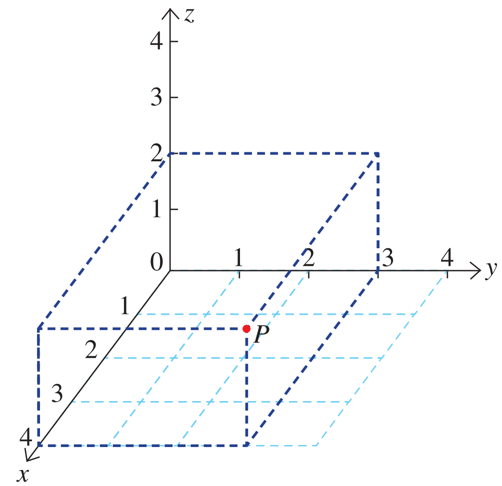
2G

11 Explain why the position of a drone, for example, needs three coordinates rather than just two coordinates.

12 A point P can be combined with the origin to form a rectangular prism with a certain volume. This diagram shows the point P producing a rectangular prism with volume $4 \times 3 \times 2 = 24$ cubic units. Find the volume of the rectangular prisms formed by the following points.

a $P(2, 3, 1)$
b $P(4, 2, 2)$
c $P(5, 3, 2)$
d $P(7, 2, 4)$



3D Drones

— 13

13 A drone is programmed to only move North/South, East/West and up/down in a system using metres as its units.

a Find the minimum distance the drone will travel if it moves from the origin $(0, 0, 0)$ to the following points.
i $(10, 5, 7)$ **ii** $(20, 35, 40)$ **iii** $(12, 29, 18)$

b Find the minimum distance the drone will travel if it moves from the point $(8, 20, 6)$ to the following points.
i $(10, 28, 12)$ **ii** $(2, 15, 26)$ **iii** $(0, 20, 30)$

c Find the minimum distance the drone will travel if it moves from the point $(-2, 16, 21)$ to the following points.
i $(3, 7, 10)$ **ii** $(10, 0, -4)$ **iii** $(-40, 4, -20)$

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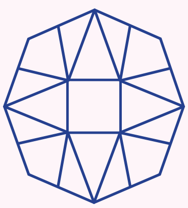
Jewellery designer

A designer of jewellery is a creative person who needs excellent eyesight, a steady hand and good fine-motor skills for their day-to-day work. They work with materials such as gold, silver and gemstones, to create something new and interesting. Designers need to understand the chemistry of their materials and the geometry of design. The angles at which gemstones such as diamonds are cut can account for price variation and can even be the difference between a beautiful, sparkling diamond or one damaged by a chip or crack.

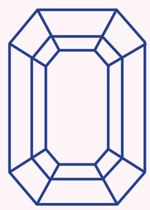
1 The way that gemstones are cut creates many different designs. The cuts form shapes and angles which contribute to the brightness, sparkle and value of the gem or diamond. Look at the following top views and list all the geometrical shapes that you can see in each design.

a



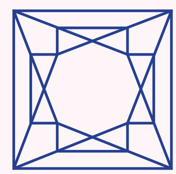
Magna

b



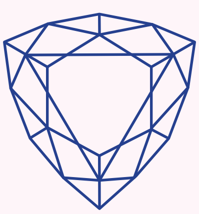
Cabochon

c



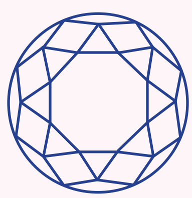
Princess

d



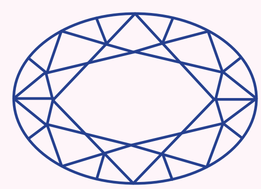
Trilliant

e



Standard round

f

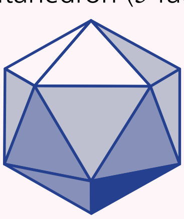


Oval

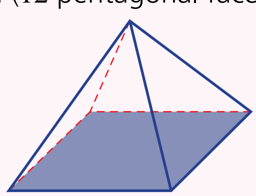
2 Gemstones and diamonds are made from minerals and crystals which naturally form many different geometrical shapes. Choose the correct name for each solid below from the following list:

Tetrahedron (4 faces), Icosahedron (20 triangular faces), Heptahedron (7 faces), Octahedron (8 faces), Pentahedron (5 faces), Dodecahedron (12 pentagonal faces).

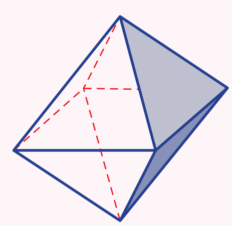
a



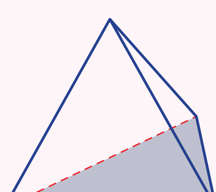
b



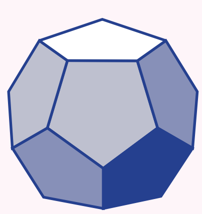
c



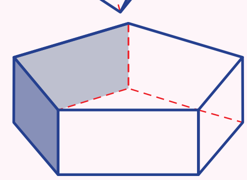
d



e



f



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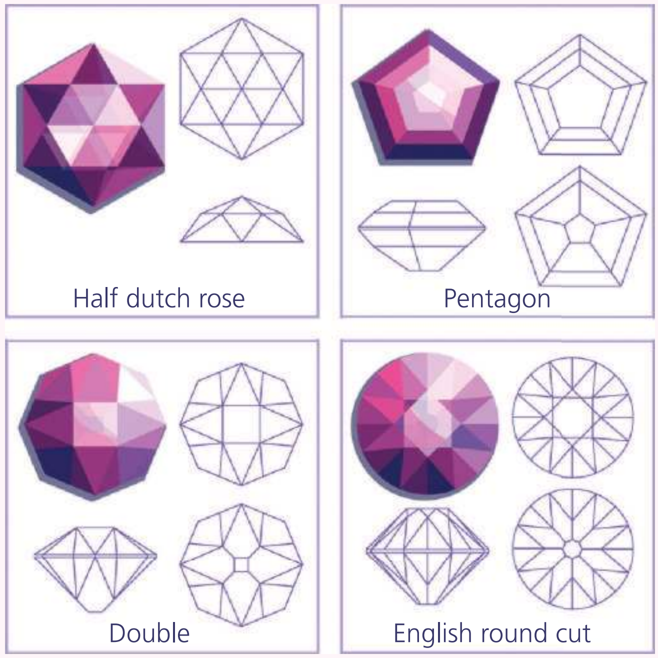
Using technology

3 Use geometry software to digitally draw the top view and side view of a diamond-cut design. You could research diamond-cut designs or choose one from the images below.



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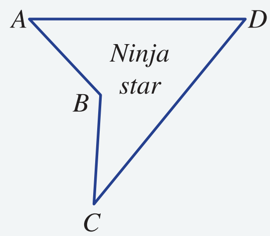
Ninja warrior logo

At his workshop, Shane is cutting out soft plastic ninja stars to give to people to play on his Ninja Warrior course.

Shane makes stars which are quadrilaterals with one interior angle greater than $180°$ like the one shown. He notices that some of the stars are more popular than others and this seems to depend on the angles formed at each vertex.



Shane works only in multiples of 10 degrees because of the limitation of the equipment that he uses.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

1 Preliminary task

a Shane's $40–140$ Ninja star has acute $\angle ADC = 40°$ and obtuse $\angle ABC = 140°$.
  - **i** With a protractor and a straight-edge/ruler, draw an accurate diagram of a star matching this description, and mark in the $40°$ and $140°$ angles.
  - **ii** Find reflex $\angle ABC$.
  - **iii** If $\angle BAD = 60°$ find $\angle BCD$.
  - **iv** If $\angle BCD = 50°$ find $\angle BAD$.

b Shane also has a $50–150$ Ninja star which includes acute $\angle ADC = 50°$ and obtuse $\angle ABC = 150°$. Draw an accurate diagram of the star and mark in all the internal angles if:
  - **i** $\angle BAD = 60°$.
  - **ii** $\angle BCD = 50°$.

c What do you notice about the $40–140$ and $50–150$ stars? Explain why they have some matching angles.

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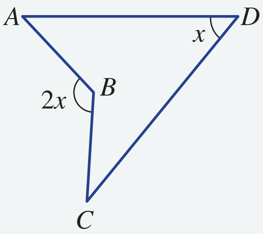
2 Modelling task

Formulate

a The problem is to determine all the angles in a range of popular stars so that Shane knows how to manufacture them. Write and draw all the relevant information that will help solve this problem, including the rule for the angle sum of a quadrilateral.

Solve

b The popular '$2x$' star has the property where obtuse $\angle ABC = 2\angle ADC$, as shown in the diagram below.



- **i** Find all the internal angles if $\angle ADC = 70°$ and $\angle BAD = 60°$. Draw an accurate diagram to illustrate your star, showing the angle at $\angle BCD$.
- **ii** For this type of star Shane knows not to choose $\angle ADC = 40°$. Explain why this would not produce a suitable star. (*Hint*: Try to draw one, marking in all the angles.)

c Another popular type is the '$2.5x$' star, which has the property that obtuse $\angle ABC = 2.5\angle ADC$.

- **i** Find all the internal angles if $\angle ADC = 60°$ and $\angle BAD = 50°$. Draw an accurate diagram to illustrate your star.
- **ii** If Shane only works with multiples of $10°$ and uses $\angle ADC = 60°$, determine the number of possible stars that he could make.

Evaluate and verify

d In the end, Shane decides that the best star is the 'isosceles star', which has the property that $\angle BAD = \angle BCD$. Draw some possible isosceles stars that he could produce, given that it should contain one interior angle greater than $180°$ and all angles are multiples of $10°$.

Communicate

e Summarise your results and describe any key findings.

3 Extension question

a If Shane has the ability to use multiples of $5°$, describe some new $2x$, $2.5x$ and isosceles stars that are now possible to create that were not possible before.

b One type of isosceles star is called the 'perpendicular star'. It has the segment BC perpendicular to AD. If the angles at A and C are $x°$, determine the angles at B and D in terms of x for a perpendicular star.

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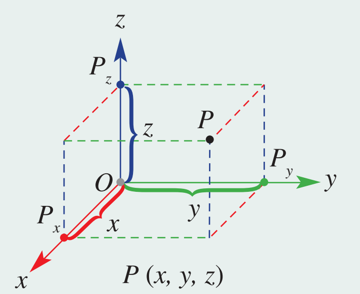
3D city model

Key technology: Dynamic geometry

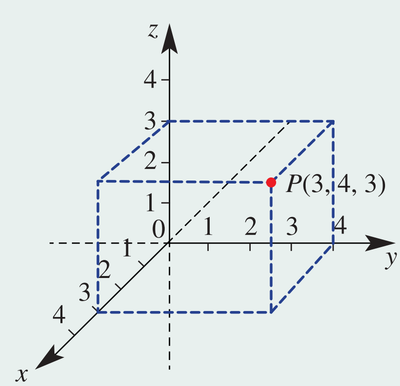
From previous studies, we know that the two-dimensional Cartesian plane allows us to construct and analyse two-dimensional shapes and graph relationships between two variables. When a third dimension is added, we can construct and analyse three-dimensional (3D) solids. This is achieved by using a three-dimensional coordinate system (x, y, z) where all three axes are perpendicular to each other and intersect at the origin $(0, 0, 0)$. Such 3D coordinates systems are used by engineers and architects to explore building designs and build city landscapes.

1 Getting started

The 3D coordinate system uses coordinates with three components (x, y, z). A point *P* in 3D space will therefore have an x-coordinate, a y-coordinate and a z-coordinate.



a For the point $P(3, 4, 3)$ shown in this set of 3D axes, answer the following.
- **i** State the value of the x-coordinate, y-coordinate and z-coordinate.
- **ii** What is the height of the point P above the x-y plane?
- **iii** How far is the point P from the x-z plane?
- **iv** How far is the point P from the y-z plane?



b Use a 3D coordinate system to plot these points.
- **i** $(0, 2, 4)$
- **ii** $(2, 5, 3)$
- **iii** $(4, 1, 0)$

Technology and computational thinking

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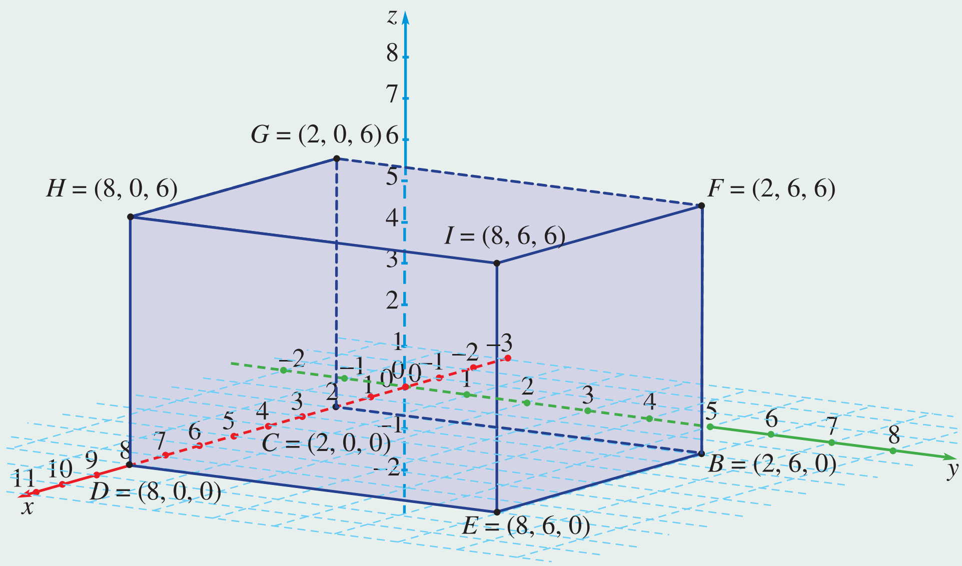
Technology and computational thinking

2 Using technology

a Find a 3D dynamic geometry package, like Geogebra 3D, and spend a few moments becoming familiar with its functions.

b Try constructing a rectangular prism as shown below. This example is shown to be a cube if you consider the coordinates of the vertices and side lengths.

c Label the coordinates of the eight vertices and confirm that they are correct by checking how they align with the scale on the three axes.



3 Applying an algorithm

a We will construct a simple city landscape by adding two or three other rectangular prisms to your 3D graph. Follow this algorithm for each prism.
Step 1: Use the Polygon tool to construct a four-sided polygon on the x-y plane. This is usually achieved by selecting the tool then clicking at four points on the x-y plane and also at the starting point again to finish.
Step 2: Use the Prism or Extrude to prism tool to construct a rectangular prism of a certain altitude of your choice.
Step 3: Adjust the position of your prism (representing buildings) by moving one of the vertices of the polygon base. Also add gaps between prisms to represent city streets.

b List the coordinates of the vertices of each added building (prism).

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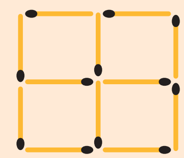
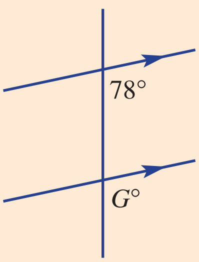
1 This shape includes 12 matchsticks. (To solve these puzzles all matches remaining must connect to other matches at both ends.)
  a Remove 2 matchsticks to form 2 squares.
  b Move 3 matchsticks to form 3 squares.



2 a Use 9 matchsticks to form 5 equilateral triangles.
  b Use 6 matchsticks to form 4 equilateral triangles.

3 Who am I?
I was a female mathematician famous for my work and publications on geometry.
Use your answers to the following to unlock the puzzle code.

a


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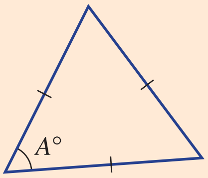

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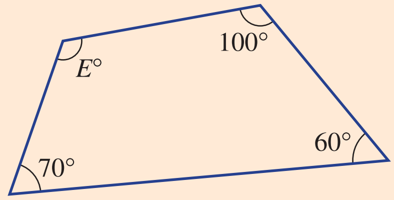

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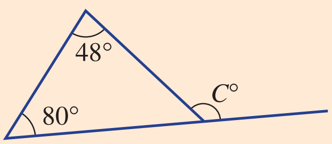

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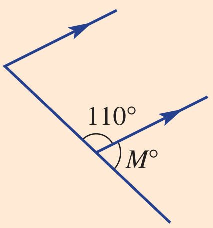

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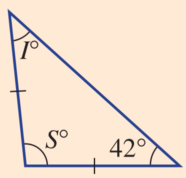

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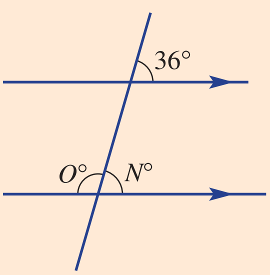

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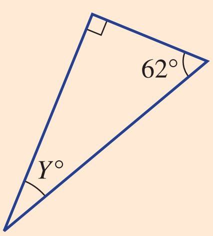

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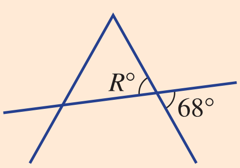

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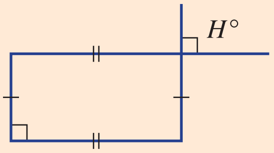

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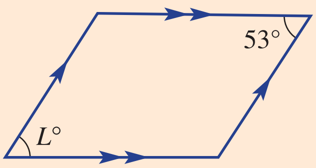
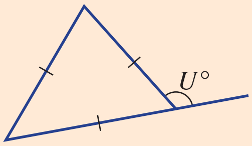


l


$\overline{}$ 78 $\overline{}$ 68 $\overline{}$ 60 $\overline{}$ 128 $\overline{}$ 130 $\overline{}$ 128 $\overline{}$ 90 $\overline{}$ 42 $\overline{}$ 96 $\overline{}$ 90 $\overline{}$ 144 $\overline{}$ 53 $\overline{}$ 70 $\overline{}$ 28 $\overline{}$ 144 $\overline{}$ 120 $\overline{}$ 36 $\overline{}$ 78

4 Find the angle between the hour and minute hands on a clock at these times.
  a 11:30 am
  b 7:45 pm
  c 6:55 pm
  d 2:34 am

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Chapter summary

Angles at a point

Complementary angles
add to 90°

$a + 25 = 90$
$a = 65$

Supplementary angles
add to 180°

$a + 125 = 180$
$a = 55$

Revolution
add to 360°

$a + 240 = 360$
$a = 120$

Vertically opposite angles

$a = 40$

Lines, shapes and solids

Parallel lines
- Corresponding angles are equal ($a = b$)
- Alternate angles are equal ($a = c$)
- Cointerior angles are supplementary ($a + d = 180$)

Triangles
(Scalene, Isosceles, Equilateral, Acute, Right, Obtuse)

Angle sum = 180°

Scalene

$a + 135 = 180$
$a = 45$

Isosceles

$2a + 50 = 180$
$2a = 130$
$a = 65$

Polygons

$S = (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$

Regular pentagon

$a = \frac{540°}{5} = 108°$

Exterior angles

$c = a + b$

$b + 300 = 360$
$b = 60$
$a = 120$

Quadrilaterals
(Square, Rectangle, Rhombus, Parallelogram, Kite, Trapezium)

Angle sum = 360°

Quadrilateral

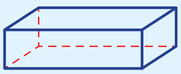
$a + 315 = 360$
$a = 45$

Parallelogram

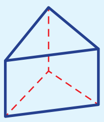
$a + 70 = 180$
$a = 110$

Solids

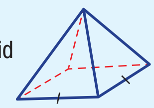
Rectangular prism (or cuboid or hexahedron)



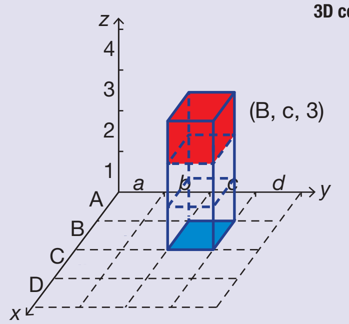
Triangular prism (or pentahedron)

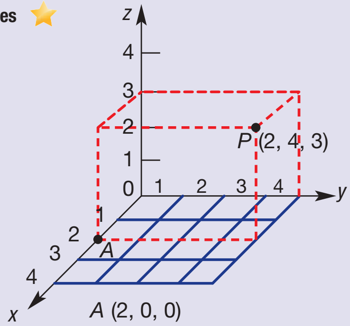


Square pyramid (or pentahedron)



3D coordinates





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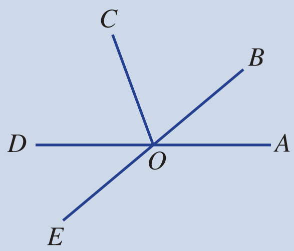
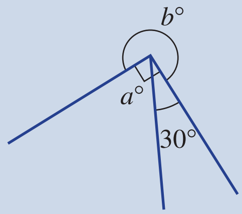
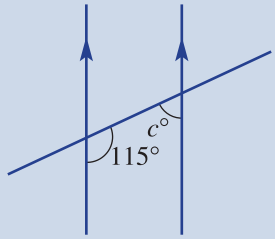
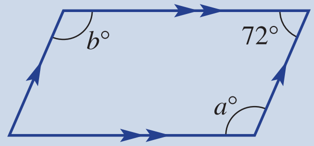
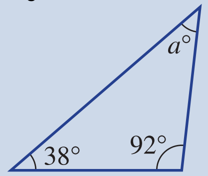
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
2A	**1 I can name angles in relation to other angles** e.g. Name an angle which is (a) vertically opposite to $\angle DOE$, (b) complementary to $\angle COB$, and (c) supplementary to $\angle EOA$.  	
2A	**2 I can find angles at a point using complementary, supplementary or vertically opposite angles** e.g. Determine the value of the pronumerals in this diagram.  	
2B	**3 I can find angles using parallel lines and explain my answers using correct terminology** e.g. Find the value of c in this diagram, giving a reason for your answer.  	
2B	**4 I can use parallel lines to find missing angles in shapes** e.g. Find the value of the pronumerals in this diagram, stating reasons.  	
2C	**5 I can find unknown angles in a triangle using the angle sum** e.g. Find the value of a in this triangle.  	

Chapter checklist

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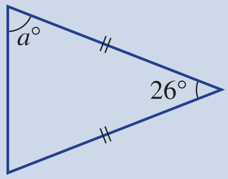
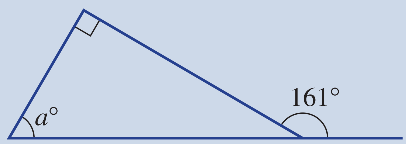
Chapter checklist

✔

2C **6 I can find unknown angles in isosceles triangles**
e.g. Find the value of a in this triangle.

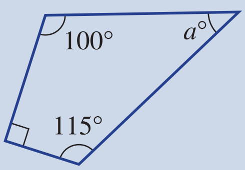


2C **7 I can use the exterior angle theorem**
e.g. Find the value of a in this diagram.

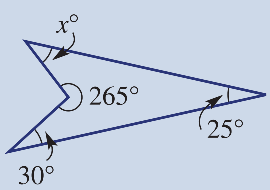


2D **8 I can use the angle sum of a quadrilateral to find unknown angles**
e.g. Find the value of the pronumerals in these quadrilaterals.

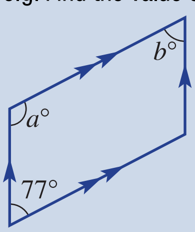
a



b

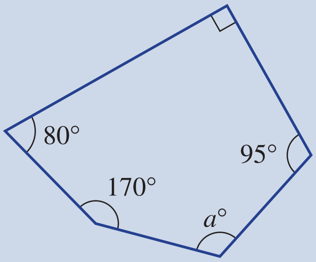


2D **9 I can find unknown angles in quadrilaterals with parallel lines**
e.g. Find the value of a and b in this parallelogram.



2E **10 I can find the angle sum of a polygon**
e.g. Find the angle sum of a heptagon.

2E **11 I can use the angle sum of a polygon to find unknown angles**
e.g. Given that pentagons have an angle sum of $540°$, find the value of a in this diagram.



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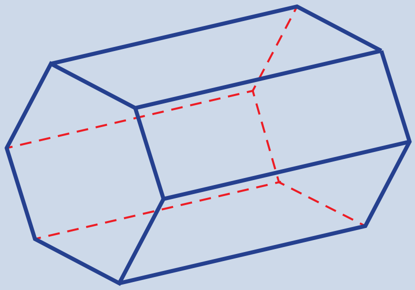
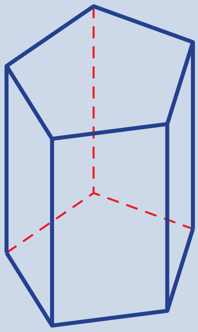
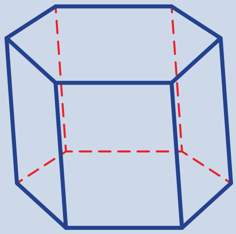
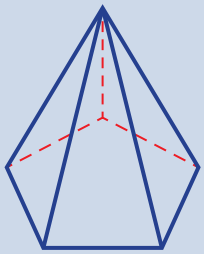
Chapter checklist

		✔
2E	**12 I can find the size of each interior angle in a regular polygon** e.g. Find the size of an interior angle in a regular octagon by first finding the angle sum.	
2F	**13 I can count the faces, vertices and edges of a solid** e.g. State the number of faces, vertices and edges for the solid shown. 	
2F	**14 I can classify solids by their number of faces** e.g. Classify this solid by considering how many faces it has. 	
2F	**15 I can name prisms by considering their cross-section** e.g. Name this solid as a type of prism. 	
2F	**16 I can name pyramids by considering their base** e.g. Name this solid as a type of pyramid. 	

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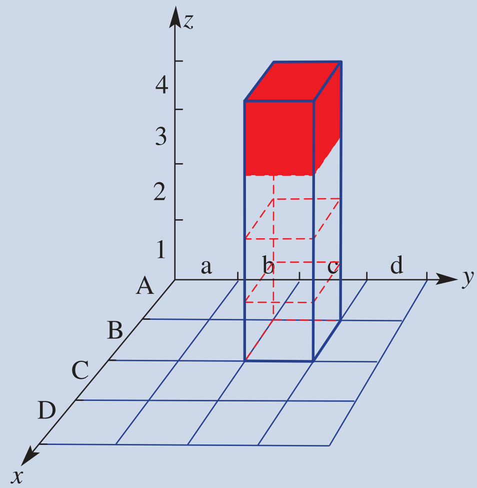
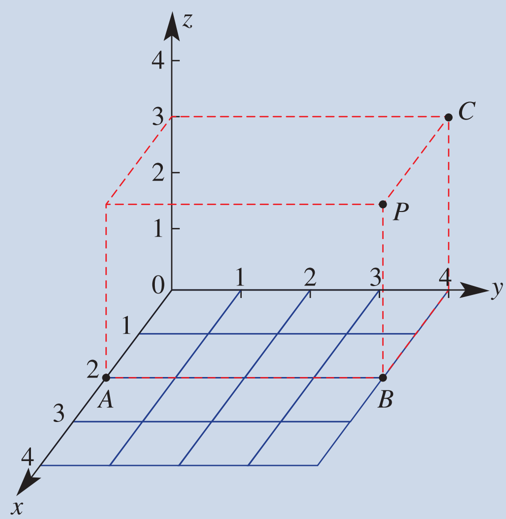
Chapter checklist

	✔
2G **17 I can use grid spaces to describe a position in 3D space.** e.g. Using the given coordinate system, give the coordinates of the red cube. 	
2G **18 I can describe a point in three-dimensional space using an (x, y, z) coordinate system.** e.g. What are the coordinates of the points A, B, C and P? 	

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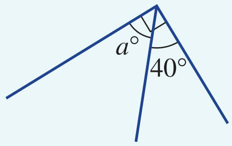
Short-answer questions

2A **1** Find the value of a in these simple diagrams.

a


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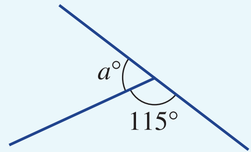

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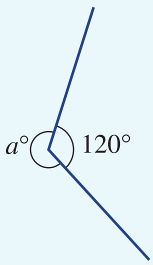

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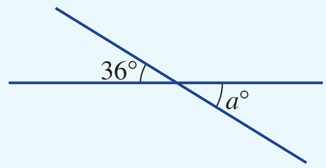

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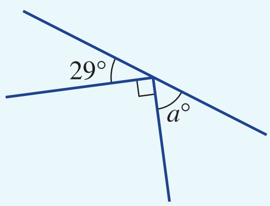

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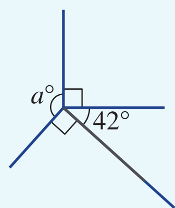
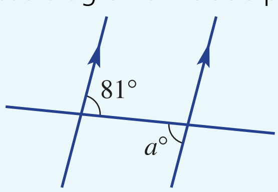


2B **2** These diagrams include parallel lines. Find the value of a.

a


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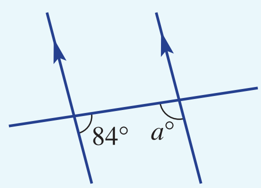

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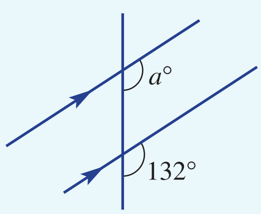

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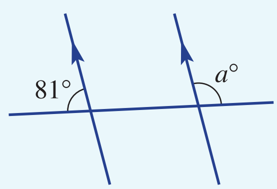

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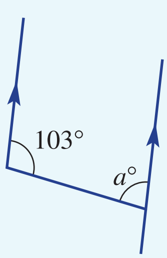

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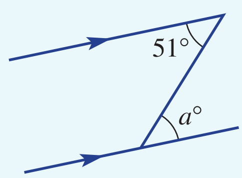
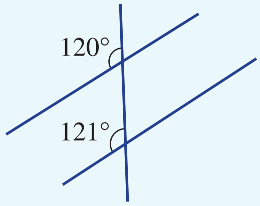


2B **3** Decide if these diagrams include a pair of parallel lines. Give reasons.

a


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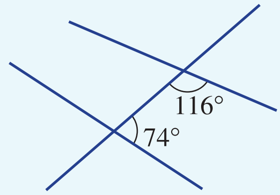

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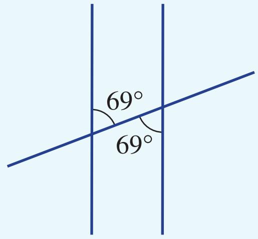

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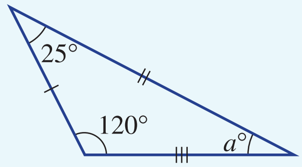
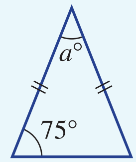
Chapter review

2C **4** Give a name for each triangle and find the value of a.

a



b



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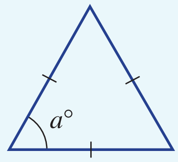

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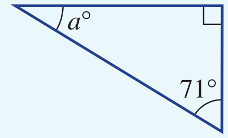

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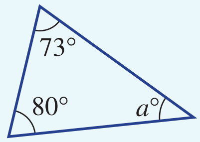

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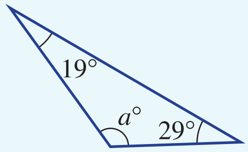
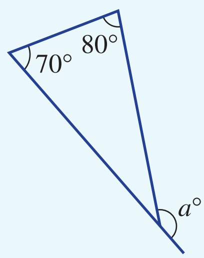


2C **5** These triangles include exterior angles. Find the value of a.

a



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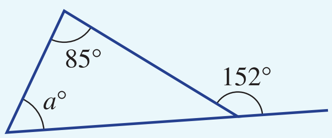

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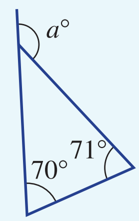
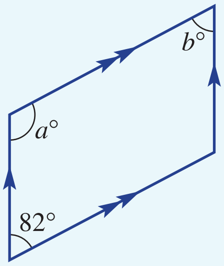


2D **6** Name the quadrilateral(s) which have:

a all sides equal in length
b one pair of parallel lines
c two pairs of equal length sides
d diagonals intersecting at right angles
e equal length diagonals.

2D **7** Find the value of a and b in these quadrilaterals.

a



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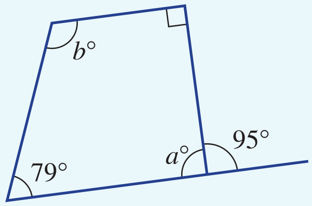

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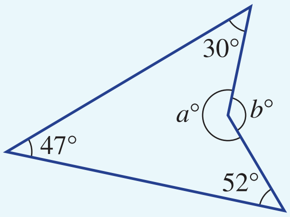

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2E ★ **8** Find the angle sum of these polygons using $S = (n - 2) \times 180°$.

a Heptagon
b Nonagon
c Dodecagon

2E ★ **9** Find the size of an interior angle of these regular polygons by firstly finding the angle sum.

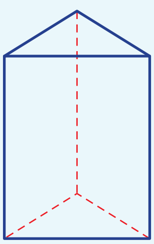
a Regular pentagon
b Regular dodecagon

2F ★ **10** Name the polyhedron that has:

a 6 faces
b 10 faces
c 11 faces

2F ★ **11** What type of prism or pyramid are these solids?

a



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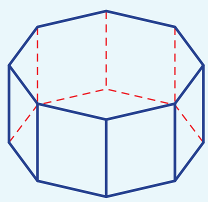

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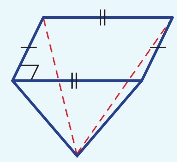
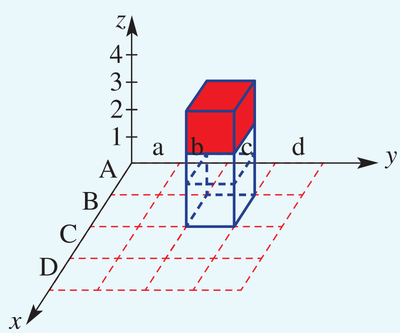
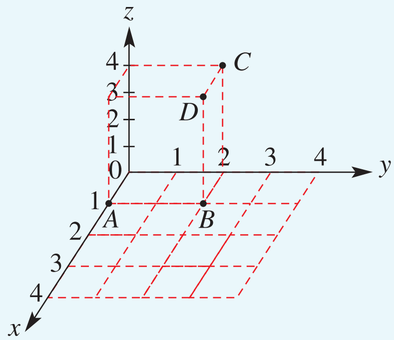


2G ★ **12** Using the given coordinate system, give the coordinate of the red cube.



2G ★ **13** Give the coordinates of the following points.

a A
b B
c C
d D



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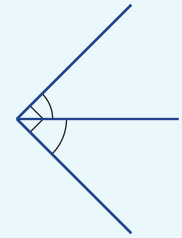
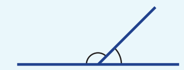
Chapter review

Multiple-choice questions

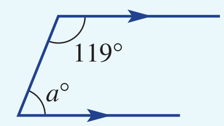
2A **1** What is the name given to two angles that add up to 90°?
- **A** Right
- **B** Supplementary
- **C** Revolutionary
- **D** Complementary
- **E** Vertically opposite



2A **2** Two angles on a straight line add to:
- **A** 180°
- **B** 90°
- **C** 45°
- **D** 270°
- **E** 360°



2B **3** The value of a in this diagram is equal to:
- **A** 45
- **B** 122
- **C** 241
- **D** 119
- **E** 61



2B **4** A pair of alternate angles in parallel lines:
- **A** are not equal
- **B** are vertically opposite
- **C** are equal
- **D** are complementary
- **E** are supplementary

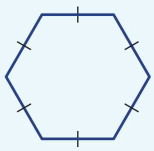
2E **5** The rule for the angle sum S of a polygon with n sides is:
- **A** $S = n \times 180°$
- **B** $S \times n = 180°$
- **C** $S = (n - 1) \times 180°$
- **D** $S = (n - 2) \times 180°$
- **E** $S = (n + 2) \times 180°$

2A **6** The compass bearing for north-east is:
- **A** 90°
- **B** 45°
- **C** 305°
- **D** 60°
- **E** 180°

2E ★ **7** The name given to an eleven-sided polygon is:
- **A** heptagon
- **B** elevenagon
- **C** decagon
- **D** dodecagon
- **E** hendecagon

2E ★ **8** The angle sum (using $S = (n - 2) \times 180°$) of a hexagon is:
- **A** 720°
- **B** 540°
- **C** 900°
- **D** 1080°
- **E** 360°

2E ★ **9** The size of one interior angle of a regular hexagon is:
- **A** 135°
- **B** 180°
- **C** 120°
- **D** 720°
- **E** 108°



2F ★ **10** How many edges does a rectangular prism have?
- **A** 10
- **B** 4
- **C** 6
- **D** 12
- **E** 8

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Extended-response questions

1 This regular polygon has 9 sides.

a Find the angle sum using $S = (n - 2) \times 180°$.

b Find the size of its interior angles correct to the nearest degree.

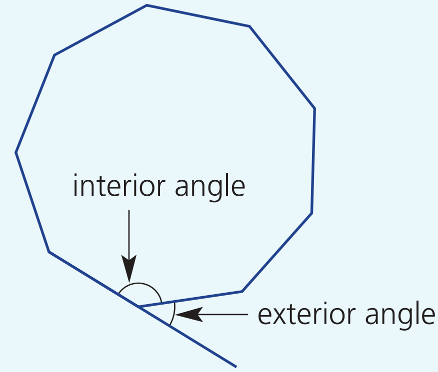
c Find the size of its exterior angles correct to the nearest degree.

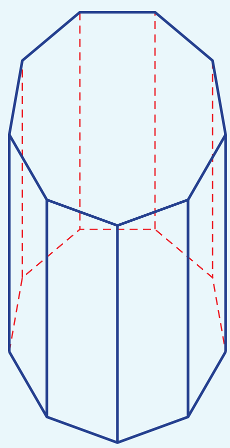
d The polygon is used to form the ends of a prism. For this prism find the number of:

i faces

ii vertices

iii edges.

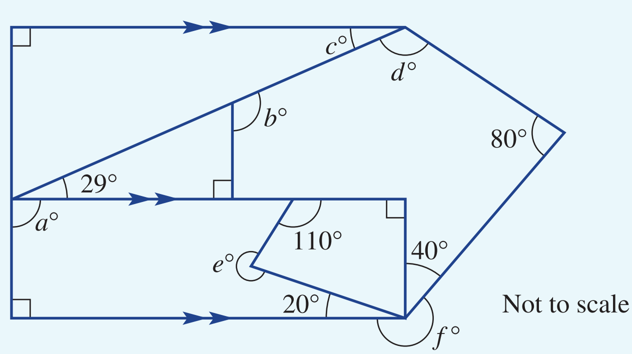




2 A modern house plan is shown here.

a List the names of at least three different polygons that you see.

b Find the values of the pronumerals $a - f$.



Hints:

- a is an angle inside a rectangle
- For b, first find the angle next to b inside the small triangle
- c is alternate to $29°$ inside parallel lines
- d is inside a hexagon, so first use $S = (n - 2) \times 180°$
- e is outside a quadrilateral (angle sum is $360°$) and angles in a revolution add to $360°$
- Try f without a hint!

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3

Fractions, decimals and percentages

Essential mathematics: why skills with fractions, decimals and percentages are important

Machinists build intricate metal parts including for locks, vehicle engine parts and surgical instruments. Digital calipers that measure in mm to 2 or 3 decimal places are used.

Suppose a carpentry apprentice is asked for the drill-bit size in the middle of $\frac{1}{8}$ and $\frac{5}{32}$. The equivalent fractions are $\frac{8}{64}$ and $\frac{10}{64}$, giving the required drill-bit size of $\frac{9}{64}$.

Plumbers bend copper tubing into angles that are often spoken of as fractions of 360°. For example, $\frac{1}{6}$ is 60° and $\frac{3}{4}$ is 270°.

Computers require clock times coded as a decimal. E.g. 11h 25m 18s $= 11 + \frac{25}{60} + \frac{18}{3600}$ $= 11 + 0.41\bar{6} + 0.005 = 11.421\bar{6} = 11.4217$ hours.

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In this chapter

Australian Curriculum 9.0

NUMBER

Recognise irrational numbers in applied contexts, including square roots and π (AC9M8N01)

Recognise terminating and recurring decimals, using digital tools as appropriate (AC9M8N03)

Use the 4 operations with integers and with rational numbers, choosing and using efficient strategies and digital tools where appropriate (AC9M8N04)

Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts; formulate problems, choosing efficient calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8N05)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 Match the following words to the types of fractions:
A whole number **B** improper fraction **C** proper fraction **D** mixed numeral

a $1\frac{2}{5}$ **b** $\frac{3}{7}$ **c** $\frac{10}{2}$ **d** $\frac{7}{4}$

2 How many quarters are in:

a 1 whole? **b** 2 wholes? **c** 5 wholes?

3 Complete the following.

a $1\frac{1}{2} = \frac{\square}{2}$ **b** $2\frac{1}{4} = \frac{\square}{4}$ **c** $1\frac{2}{3} = \frac{5}{\square}$ **d** $1\frac{3}{5} = \frac{\square}{5}$

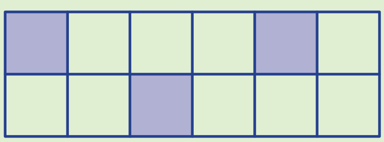
4 Fill in the blanks.

a $\frac{3}{4} = \frac{75}{\square}$ **b** $\frac{3}{6} = \frac{\square}{2}$ **c** $\frac{2}{3} = \frac{\square}{6}$ **d** $\frac{20}{100} = \frac{\square}{5}$

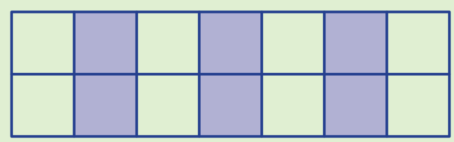
e $\frac{3}{10} = \frac{\square}{100}$ **f** $\frac{3}{5} = \frac{\square}{100}$ **g** $\frac{7}{20} = \frac{35}{\square}$ **h** $\frac{1}{25} = \frac{\square}{100}$

5 What fraction is shaded?

a



b



6 Match the fractions on the left-hand side to their decimal form on the right.

a $\frac{1}{2}$ **A** 3.75

b $\frac{1}{100}$ **B** 0.25

c $\frac{3}{20}$ **C** 0.01

d $3\frac{3}{4}$ **D** 0.5

e $\frac{1}{4}$ **E** 0.15

7 Find:

a $\frac{1}{2} + \frac{1}{4}$ **b** $0.5 + \frac{1}{2}$ **c** $3 - 1\frac{1}{3}$

d $0.3 + 0.2 + 0.1$ **e** $2.4 \div 2$ **f** 0.5×6

8 Write as **i** simple fractions **ii** decimals.

a 10% **b** 25% **c** 50% **d** 75%

9 Find 10% of:

a \$50 **b** \$66 **c** 8 km **d** 6900 m

10 Find:

a 25% of 40 **b** 75% of 24 **c** 90% of \$1

11 Copy and complete the following table.

Fraction	$\frac{3}{4}$			$\frac{2}{5}$				2
Decimal		0.2			0.99		1.6	
Percentage			15%			100%		

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3A Equivalent fractions CONSOLIDATING

Learning intentions

- To understand what equivalent fractions are.
- To understand that fractions have a numerical value, and two distinct fractions can have the same numerical value, like $\frac{3}{5}$ and $\frac{6}{10}$.
- To be able to simplify fractions.

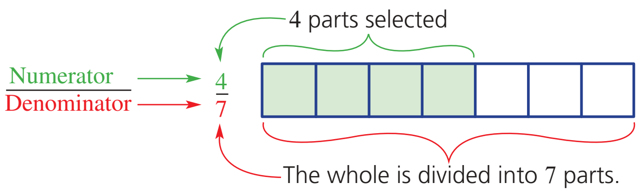
Key vocabulary: fraction, numerator, denominator, equivalent, simplify, simplest form

Fractions are made when whole numbers are broken into parts.

This diagram shows the parts of a fraction.

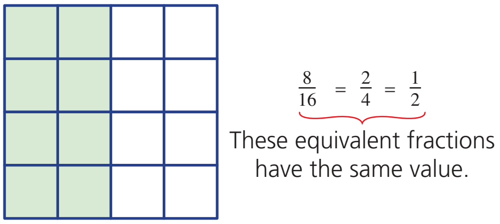
$\frac{4}{7}$ → **n**umerator: parts taken from the whole
→ **d**enominator: number of equal parts the whole is broken into

Think '**u**' for '**u**p the top' and '**d**' for '**d**own the bottom.



There are 7 equal parts in the whole and 4 of them are shaded.

Equivalent fractions are fractions that represent equal portions of a whole amount and so are equal in value. The skill of generating equivalent fractions is needed whenever you add or subtract fractions with different denominators.



Rural properties are broken up into paddocks. The paddocks are a fractional proportion of the property as a whole. Farmers can use this design to diversify their crops.

Fractions are very important whenever we measure or compare. Chefs use them when baking. Builders use them when mixing concrete. A musician can even use fractions when composing music.

Lesson starter: Know your terminology

It is important to know and understand key terms associated with the study of fractions.

As a class give a definition or example of each of the following key terms.

- Numerator
- Improper fraction
- Lowest common multiple (LCM)
- Denominator
- Mixed numeral
- Highest common factor (HCF)
- Equivalent fraction
- Multiple
- Lowest common denominator
- Proper fraction
- Factor

Hint for lesson starter:

$1\frac{1}{2} = \frac{3}{2}$

mixed numeral improper fraction

Hint for lesson starter:

$\frac{7}{3}$ ← numerator ← denominator

This is an improper fraction. Do you know why?

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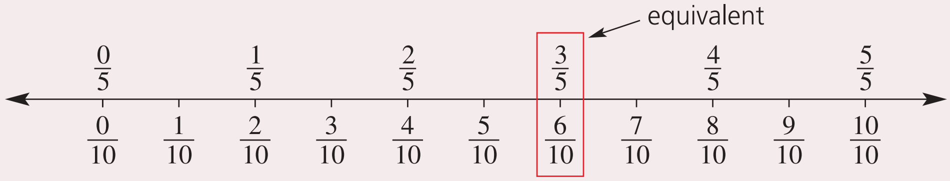
3A

Key ideas

- **Equivalent fractions** are equal in value. They mark the same place on a number line.
  e.g., $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent fractions.



- Equivalent fractions are made by multiplying or dividing the **numerator** *and* **denominator** by the same number.

  e.g. $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$ $\qquad \frac{2}{7} \underset{\times 5}{\overset{\times 5}{=}} \frac{10}{35}$ $\qquad \frac{6}{21} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{7}$

- A fraction can be *cancelled down* (**simplified**) if the top (numerator) *and* bottom (denominator) have a common factor, other than one.

  e.g. $\frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{2}{3}$

  $\frac{12}{18} \underset{\div 6}{\overset{\div 6}{=}} \frac{2}{3}$

Hint for Key ideas: 6 is the HCF of 12 and 18. $\frac{6}{6}$ 'cancels' to 1 because $6 \div 6 = 1$.

- The **simplest form** of a fraction is when the numerator and denominator have no common factors other than one.
  - Two fractions are equivalent if they have the same simplest form.

Exercise 3A

Understanding

	1–4	3, 4

1 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $\overset{\times 3}{\underset{\times 3}{\overbrace{\frac{3}{5} \underset{\times 2}{\overset{\times 2}{=}} \frac{\square}{\square} = \frac{\triangle}{\triangle}}}}$

b $\frac{4}{7} \underset{\times 2}{\overset{\times 2}{=}} \frac{8}{\square} \underset{\times 2}{\overset{\times 2}{=}} \frac{\square}{28} \underset{\times 2}{\overset{\times 2}{=}} \frac{\square}{\square}$

Hint for Q1: Remember always × or ÷ the top and the bottom by the same number!

c $\frac{50}{100} = \frac{25}{\square} = \frac{10}{\square} = \frac{\square}{10} = \frac{1}{\square}$

d $\frac{1}{3} = \frac{2}{\square} = \frac{3}{\square} = \frac{4}{\square}$

2 Which of these two fractions is equivalent to $\frac{2}{3}$: $\frac{10}{15}$ or $\frac{3}{4}$?

3 Which of the following fractions can be cancelled down (simplified)?

A $\frac{3}{7}$ **B** $\frac{10}{12}$ **C** $\frac{8}{6}$ **D** $\frac{5}{9}$ **E** $\frac{3}{9}$

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4 Are the following statements true (T) or false (F)?

a $\frac{1}{2}$ and $\frac{1}{4}$ are equivalent fractions.

b $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions.

c The fraction $\frac{8}{9}$ is written in its simplest form.

d $\frac{14}{21}$ can be simplified to $\frac{2}{3}$.

e $\frac{11}{99}$ and $\frac{1}{9}$ and $\frac{2}{18}$ are all equivalent fractions.

f $\frac{4}{5}$ can be simplified to $\frac{2}{5}$.

Fluency

5–9(½) | 6–9(½)

Example 1 Generating equivalent fractions

Rewrite the following fractions with a denominator of 40.

a $\frac{3}{5}$ **b** $\frac{1}{2}$ **c** $\frac{36}{120}$

Solution	Explanation
a $\frac{3}{5} = \frac{24}{40}$	$\times 8$ $\frac{3}{5} = \frac{\square}{40}$ $\times 8$ Multiply the numerator and denominator by 8.
b $\frac{1}{2} = \frac{20}{40}$	$\times 20$ $\frac{1}{2} = \frac{\square}{40}$ $\times 20$ Multiply both numerator and denominator by 20.
c $\frac{36}{120} = \frac{12}{40}$	$\div 3$ $\frac{36}{120} = \frac{\square}{40}$ $\div 3$ Divide both numerator and denominator by 3.

Now you try

Rewrite the following fractions with a denominator of 32.

a $\frac{3}{8}$ **b** $\frac{1}{4}$ **c** $\frac{8}{64}$

5 Write the missing number in these fractions with a denominator of 24.

a $\frac{1}{3} = \frac{\square}{24}$ **b** $\frac{2}{8} = \frac{\square}{24}$ **c** $\frac{1}{2} = \frac{\square}{24}$ **d** $\frac{5}{12} = \frac{\square}{24}$

e $\frac{5}{6} = \frac{\square}{24}$ **f** $\frac{5}{1} = \frac{\square}{24}$ **g** $\frac{3}{4} = \frac{\square}{24}$ **h** $\frac{7}{8} = \frac{\square}{24}$

Hint for Q5: Multiply or divide top and bottom by the same number.

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3A

6 Write the missing number in these fractions with a denominator of 30.

a $\frac{1}{5}=\frac{\square}{30}$ **b** $\frac{2}{6}=\frac{\square}{30}$ **c** $\frac{5}{10}=\frac{\square}{30}$ **d** $\frac{3}{1}=\frac{\square}{30}$

e $\frac{2}{3}=\frac{\square}{30}$ **f** $\frac{22}{60}=\frac{\square}{30}$ **g** $\frac{5}{2}=\frac{\square}{30}$ **h** $\frac{150}{300}=\frac{\square}{30}$

7 Find the missing value to make equivalent fractions.

a $\frac{1}{5}=\frac{\square}{10}$ **b** $\frac{1}{5}=\frac{\square}{100}$ **c** $\frac{2}{5}=\frac{4}{\square}$ **d** $\frac{3}{4}=\frac{\square}{40}$

e $\frac{2}{3}=\frac{12}{\square}$ **f** $\frac{3}{2}=\frac{6}{\square}$ **g** $\frac{15}{10}=\frac{\square}{2}$ **h** $\frac{90}{100}=\frac{\square}{10}$

i $\frac{2}{5}=\frac{\square}{15}$ **j** $\frac{7}{9}=\frac{14}{\square}$ **k** $\frac{7}{14}=\frac{1}{\square}$ **l** $\frac{21}{30}=\frac{\square}{10}$

m $\frac{4}{3}=\frac{\square}{21}$ **n** $\frac{8}{5}=\frac{80}{\square}$ **o** $\frac{3}{12}=\frac{\square}{60}$ **p** $\frac{7}{11}=\frac{28}{\square}$

Example 2 Converting fractions to simplest form

Write the following fractions in simplest form.

a $\frac{8}{20}$ **b** $\frac{25}{15}$

Solution

a

$$\frac{8}{20} \underset{\div 4}{\overset{\div 4}{=}} \frac{2}{5}$$

b

$$\frac{25}{15} \underset{\div 5}{\overset{\div 5}{=}} \frac{5}{3}$$

Explanation

a The HCF of 8 and 20 is 4.

Both the numerator and the denominator are divided by the HCF of 4.

b The HCF of 25 and 15 is 5.

Both the numerator and the denominator are divided by the HCF of 5.

Now you try

Write the following fractions in simplest form.

a $\frac{10}{24}$ **b** $\frac{40}{25}$

8 Write the following fractions in simplest form.

a $\frac{2}{4}$ **b** $\frac{3}{6}$ **c** $\frac{8}{10}$ **d** $\frac{14}{20}$

e $\frac{3}{9}$ **f** $\frac{4}{8}$ **g** $\frac{10}{12}$ **h** $\frac{15}{18}$

i $\frac{11}{44}$ **j** $\frac{12}{20}$ **k** $\frac{16}{18}$ **l** $\frac{25}{35}$

m $\frac{15}{9}$ **n** $\frac{22}{20}$ **o** $\frac{120}{100}$ **p** $\frac{64}{48}$

Hint for Q8: Divide by the HCF of the numerator and denominator.

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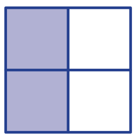
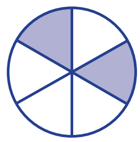
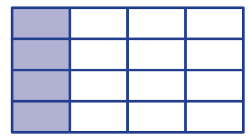
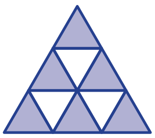
9 Many calculators give the simplified fraction after you press '='. Use a calculator to simplify these fractions.

a $\frac{36}{40}$ b $\frac{16}{12}$ c $\frac{14}{56}$ d $\frac{28}{52}$ e $\frac{32}{48}$ f $\frac{156}{312}$

Problem-solving and reasoning

10, 11 | 11–13

10 Each diagram below shows a fraction of the whole. Write each shaded fraction in more than one way.

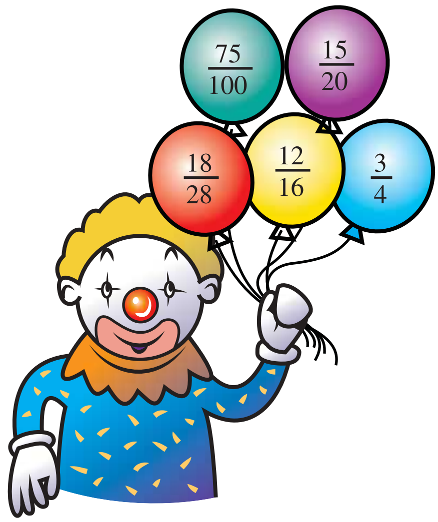
a 
b 
c 
d 

11 a Thomas ate $\frac{1}{4}$ of a 250 gram block of chocolate. Mary ate $\frac{3}{6}$ of her 250 gram block. Who ate the most chocolate?

b A pizza is cut into eight equal pieces. Sian ate 2 slices of pizza, Callum had 4 slices. What fraction of the pizza is left?

12 Write down four fractions that when simplified equal $\frac{1}{5}$.

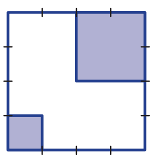
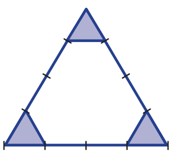
13 Bozzo the Clown, shown below, is holding five balloons with fractions on them. Which balloon is the odd one out?



Tricky shading

— | 14

14 In these diagrams assume that the sides are divided evenly. Find the fraction that is shaded.

a 
b 

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3B Operations with fractions

Learning intentions

- To be able to add and subtract fractions by first finding the lowest common multiple (LCM) of the denominators.
- To be able to multiply and divide fractions.
- To be able to perform the four operations on mixed numerals, converting to improper fractions as required.

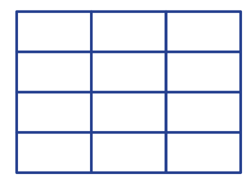
Key vocabulary: lowest common multiple (LCM), lowest common denominator (LCD), reciprocal, numerator, denominator, mixed numeral, improper fraction

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing fractions.

Proper fractions, improper fractions and mixed numerals will be considered for each of the four mathematical operations.

Lesson starter: Shading fractions

In pairs, draw and shade this grid to help work out the following fraction questions.



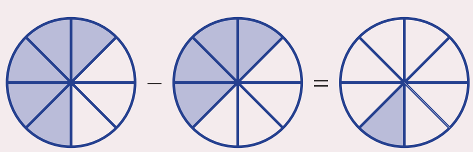
- $\frac{1}{2}+\frac{1}{3}$
- $\frac{7}{12}-\frac{1}{3}$
- $\frac{1}{2}$ of $\frac{1}{2}$

What rules do you know for adding, subtracting, multiplying and dividing fractions?

Key ideas

- Adding and subtracting fractions
  - To add or subtract fractions, convert to the same **denominator**.
    When the denominators are the same, just add or subtract the **numerators**.

    e.g.

    $$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$$

  - The **lowest common multiple (LCM)** of the denominators is used if the denominators are different. This is called the **lowest common denominator (LCD)**.
    e.g. $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$

    $=\frac{5}{6}$

- Multiplying fractions
  - Convert to improper fractions (if needed)
  - Multiply the numerators together
  - Multiply the denominators together
  - Simplify your answer

e.g. $1\frac{1}{2}\times\frac{4}{7}=\frac{3}{2}\times\frac{4}{7}$

$=\frac{3\times 4}{2\times 7}$

$=\frac{12}{14}$

$=\frac{6}{7}$

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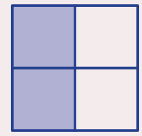
- Dividing fractions
  - To divide by a fraction, remember to flip the fraction after the division sign upside down, then multiply.
    e.g. $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$
    $= 2$



There are 2 quarters in one half.

- To flip a fraction $\frac{a}{b}$ upside down to become $\frac{b}{a}$ is called taking its **reciprocal**.

Exercise 3B

Understanding | 1–4 | 4

1 **a** Which two operations require the denominators to be the same before proceeding?
b Which two operations do not require the denominators to be the same before proceeding?

2 State the lowest common denominator for the following pairs of fractions.
a $\frac{1}{5}+\frac{3}{4}$ **b** $\frac{2}{9}+\frac{5}{3}$ **c** $\frac{11}{25}+\frac{7}{10}$ **d** $\frac{5}{12}+\frac{13}{8}$

3 Rewrite the following equations and fill in the empty boxes.

a $\frac{2}{3}+\frac{1}{4}$
$=\frac{8}{12}+\frac{\square}{12}$
$=\frac{11}{\square}$

b $\frac{7}{8}-\frac{9}{16}$
$=\frac{\square}{16}-\frac{9}{16}$
$=\frac{\square}{16}$

c $1\frac{4}{7}\times\frac{3}{5}$
$=\frac{\square}{7}\times\frac{3}{5}$
$=\frac{\square}{35}$

d $\frac{5}{7}\div\frac{2}{3}$
$=\frac{5}{7}\square\frac{3}{2}$
$=\frac{15}{\square}=\square\frac{\square}{14}$

4 State the reciprocal of the following fractions.
a $\frac{5}{8}$ **b** $\frac{3}{2}$ **c** $3\frac{1}{4}$ **d** $1\frac{1}{11}$

Hint for Q4: Reciprocal means to turn upside down. Convert mixed numerals to improper fractions first!

Fluency | 5–7(½), 8, 9(½) | 5–7(½), 8, 9(½), 10(½)

Example 3 Adding and subtracting fractions

Evaluate.
a $\frac{3}{5}+\frac{4}{5}$ **b** $\frac{5}{3}-\frac{3}{4}$

Solution	Explanation
a $\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$	The denominators are the same so simply add the numerators. Three **fifths** plus four **fifths** equals seven **fifths**.
$=1\frac{2}{5}$	The final answer can be written as a mixed numeral.

Continued on next page

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3B

b $\frac{5}{3} - \frac{3}{4} = \frac{20}{12} - \frac{9}{12}$

$= \frac{11}{12}$

Lowest common multiple of 3 and 4 is 12.
Write equivalent fractions with a LCD of 12.
$\frac{5}{3} = \frac{5 \times 4}{3 \times 4}, \frac{3}{4} = \frac{3 \times 3}{4 \times 3}$
The denominators are now the same, so subtract the numerators.

Now you try

Evaluate.

a $\frac{7}{6} - \frac{5}{6}$

b $\frac{3}{8} + \frac{7}{4}$

5 Evaluate.

a $\frac{1}{3} + \frac{1}{3}$

b $\frac{1}{3} + \frac{1}{6}$

c $\frac{7}{12} - \frac{1}{2}$

d $\frac{11}{10} - \frac{7}{10}$

e $\frac{1}{5} + \frac{2}{5}$

f $\frac{7}{9} - \frac{2}{9}$

g $\frac{5}{8} + \frac{7}{8}$

h $\frac{24}{7} - \frac{11}{7}$

i $\frac{3}{4} + \frac{2}{5}$

j $\frac{3}{10} + \frac{4}{5}$

k $\frac{5}{7} - \frac{2}{3}$

l $\frac{11}{18} - \frac{1}{6}$

Hint for Q5: Convert to a common denominator if required.

Example 4 Adding and subtracting mixed numerals

Evaluate.

a $3\frac{5}{8} + 2\frac{3}{4}$

b $2\frac{1}{2} - 1\frac{5}{6}$

Solution

a $3\frac{5}{8} + 2\frac{3}{4} = \frac{29}{8} + \frac{11}{4}$

$= \frac{29}{8} + \frac{22}{8}$

$= \frac{51}{8} = 6\frac{3}{8}$

Explanation

Convert mixed numerals to improper fractions.
The lowest common multiple of 8 and 4 is 8.
Write equivalent fractions with LCD.
Add numerators together, denominator remains the same. Convert the answer back to a mixed numeral if required.

b $2\frac{1}{2} - 1\frac{5}{6} = \frac{5}{2} - \frac{11}{6}$

$= \frac{15}{6} - \frac{11}{6}$

$= \frac{4}{6} = \frac{2}{3}$

Convert mixed numerals to improper fractions.
The lowest common multiple of 2 and 6 is 6.
Write equivalent fractions with LCD.
Subtract numerators and simplify the answer.

Now you try

Evaluate.

a $2\frac{1}{3} + 1\frac{4}{5}$

b $3\frac{3}{4} - 1\frac{7}{8}$

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6 Evaluate. Write final answers as mixed numerals.

a $3\frac{1}{7}+1\frac{3}{7}$ **b** $7\frac{2}{5}+2\frac{1}{5}$

c $3\frac{5}{8}-1\frac{2}{8}$ **d** $8\frac{5}{11}-7\frac{3}{11}$

e $5\frac{1}{3}+4\frac{1}{6}$ **f** $17\frac{5}{7}+4\frac{1}{2}$

g $6\frac{1}{2}-2\frac{3}{4}$ **h** $4\frac{2}{5}-2\frac{5}{6}$

Hint for Q6: You can add the wholes first if you prefer.

$$3\frac{5}{8}+2\frac{3}{4}=3+2+\frac{5}{8}+\frac{3}{4}$$
$$=5+\frac{5}{8}+\frac{6}{8}$$
$$=5+\frac{11}{8}=6\frac{3}{8}$$

Example 5 Multiplying fractions

Evaluate.

a $\frac{2}{5}\times\frac{3}{7}$ **b** $\frac{8}{5}\times 1\frac{3}{4}$

Solution	Explanation
a $\frac{2}{5}\times\frac{3}{7}=\frac{2\times 3}{5\times 7}$ $=\frac{6}{35}$	Multiply the numerators together. Multiply the denominators together. The answer is in simplest form.
b $\frac{8}{5}\times 1\frac{3}{4}=\frac{\cancel{8}^{2}}{5}\times\frac{7}{\cancel{4}^{1}}$ $=\frac{14}{5}$ $=2\frac{4}{5}$	Use improper fractions, $1\frac{3}{4}=\frac{7}{4}$ Cancel any numerator with any denominator. Multiply the numerators $2\times 7=14$ Multiply the denominators $5\times 1=5$

Now you try

Evaluate.

a $\frac{6}{7}\times\frac{5}{3}$ **b** $\frac{9}{2}\times 3\frac{1}{3}$

7 Evaluate.

a $\frac{3}{5}\times\frac{1}{4}$ **b** $\frac{2}{9}\times\frac{5}{7}$

c $\frac{7}{5}\times\frac{6}{5}$ **d** $\frac{5}{3}\times\frac{8}{9}$

e $\frac{4}{9}\times\frac{3}{8}$ **f** $\frac{12}{10}\times\frac{5}{16}$

g $\frac{12}{9}\times\frac{2}{5}$ **h** $\frac{24}{8}\times\frac{5}{3}$

Hint for Q7: Look to cancel first if possible.

8 Evaluate.

a $2\frac{3}{4}\times 1\frac{1}{3}$ **b** $3\frac{2}{7}\times\frac{1}{3}$

c $4\frac{1}{6}\times 3\frac{3}{5}$ **d** $10\frac{1}{2}\times 3\frac{1}{3}$

Hint for Q8: Convert to improper fractions first.

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3B

Example 6 Dividing fractions

Evaluate.

a $\frac{2}{5} \div \frac{3}{7}$

b $2\frac{1}{4} \div 1\frac{1}{3}$

Solution

a $\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3}$

$= \frac{14}{15}$

Explanation

Change ÷ sign to a × sign and flip the divisor.
Multiply by the reciprocal.
Proceed as for multiplication.
Multiply numerators together and multiply denominators together.

Solution

b $2\frac{1}{4} \div 1\frac{1}{3} = \frac{9}{4} \div \frac{4}{3}$

$= \frac{9}{4} \times \frac{3}{4}$

$= \frac{27}{16} = 1\frac{11}{16}$

Explanation

Convert mixed numerals to improper fractions.
Change ÷ sign to a × sign and flip the divisor.
Multiply by the reciprocal. The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$.
Multiply and simplify.

Now you try

Evaluate.

a $\frac{4}{5} \div \frac{3}{7}$

b $6\frac{2}{5} \div 1\frac{1}{2}$

9 Evaluate the following.

a $\frac{2}{9} \div \frac{3}{5}$
b $\frac{1}{3} \div \frac{2}{5}$
c $\frac{8}{7} \div \frac{11}{2}$
d $\frac{11}{3} \div \frac{5}{2}$
e $\frac{3}{4} \div \frac{6}{7}$
f $\frac{10}{15} \div \frac{1}{3}$
g $\frac{6}{5} \div \frac{9}{10}$
h $\frac{22}{35} \div \frac{11}{63}$

Hint for Q9:
Multiply by the reciprocal.
$\frac{2}{9} \div \frac{3}{5} = \frac{2}{9} \times \frac{5}{3}$

10 Evaluate the following.

a $1\frac{4}{7} \div 1\frac{2}{3}$
b $3\frac{1}{5} \div 8\frac{1}{3}$
c $3\frac{1}{5} \div 2\frac{2}{7}$
d $6\frac{2}{4} \div 2\frac{1}{6}$

Hint for Q10: Convert to improper fractions first.

Problem-solving and reasoning

11	11, 12

11 There are 30 students in Miss Mac's maths class. Find out how many students there are in each of the following groups.

a $\frac{1}{3}$ of the class had brown hair.

b $\frac{1}{2}$ of the class came to school by bus.

c $\frac{5}{6}$ of the class spoke English at home.

d $\frac{1}{10}$ of the class liked maths.

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12 Max and Tanya are painting a large wall together, which is split equally into two halves. Max paints the left half of the wall at the same time as Tanya paints the right half. Max has painted $\frac{3}{7}$ of his half and Tanya has painted $\frac{2}{5}$ of her half.

a What fraction of the overall wall has max painted? Calculate $\frac{3}{7} \times \frac{1}{2}$ to find this.

b What fraction of the overall wall has Tanya painted?

Multiple operations

	—	13

13 Use a calculator to find the answers to the following.

a $\frac{2}{3} \times \frac{1}{4} \div 1\frac{1}{2}$

b $1\frac{2}{3} + 4\frac{4}{5} - \frac{3}{8}$

c $1\frac{1}{4} \div \frac{2}{3} - \frac{5}{7}$

d $\left(1\frac{1}{2} + 2\frac{2}{3}\right) \times \frac{4}{5}$

e What is the lowest common denominator for:

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$?

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3C Operations with negative fractions

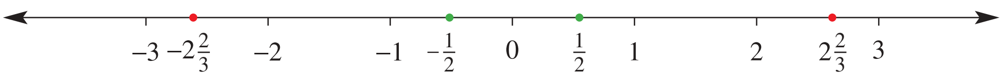
Learning intentions

- To understand that the techniques for adding, subtracting, multiplying and dividing positive fractions also apply to negative fractions
- To understand that the rules for positive and negative integers also apply to fractions
- To be able to add, subtract, multiply and divide negative fractions and mixed numerals

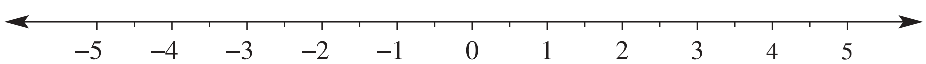
Key vocabulary: negative, LCM, LCD, reciprocal, numerator, denominator, mixed numeral, improper fraction

The English mathematician named John Wallis (1616–1703) invented a number line that displayed numbers extending in both the positive and negative directions.

So, just as we can have negative integers, we can also have negative fractions. In fact, each positive fraction has an opposite (negative) fraction. Two examples are highlighted on the number line below:



Lesson starter: Where do you end up?



You are given a starting point and a set of instructions to follow. You must determine where the finishing point is. The first set of instructions reviews the addition and subtraction of integers. The other two sets involve the addition and subtraction of positive and negative fractions.

- Starting point is 1. Add 3, subtract 5, add -2, subtract -4, subtract 3.
  Finishing point = $\square$
- Starting point is 0. Subtract $\frac{3}{5}$, add $\frac{1}{5}$, add $-\frac{4}{5}$, subtract $\frac{2}{5}$, subtract $-\frac{3}{5}$.
  Finishing point = $\square$
- Starting point is $\frac{1}{2}$. Subtract $\frac{3}{4}$, add $-\frac{1}{3}$, subtract $-\frac{1}{2}$, subtract $\frac{1}{12}$, add $\frac{1}{6}$.
  Finishing point = $\square$

Key ideas

- The techniques for $+, -, \times, \div$ positive fractions also apply to negative fractions.
- The arithmetic rules we observed for integers (**Chapter 1**) also apply to fractions.
- Subtracting a larger positive fraction from a smaller positive fraction will result in a negative fraction.
  For example: $\frac{1}{5} - \frac{2}{3} = \frac{3}{15} - \frac{10}{15} = -\frac{7}{15}$
- Adding a negative fraction is equivalent to subtracting its opposite.
  For example: $\frac{1}{2} + \left(-\frac{1}{3}\right) = \frac{1}{2} - \left(+\frac{1}{3}\right) = \frac{1}{2} - \frac{1}{3}$
- Subtracting a negative fraction is equivalent to adding its opposite.
  For example: $\frac{1}{2} - \left(-\frac{1}{3}\right) = \frac{1}{2} + \left(+\frac{1}{3}\right) = \frac{1}{2} + \frac{1}{3}$

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- The product or quotient of two fractions of the same sign (positive or negative) is a positive fraction.
  - Product: $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$ or $-\frac{1}{3} \times \left(-\frac{2}{5}\right) = \frac{2}{15}$
  - Quotient: $\frac{2}{15} \div \frac{1}{3} = \frac{2}{5}$ or $-\frac{2}{15} \div \left(-\frac{1}{3}\right) = \frac{2}{5}$
- The product or quotient of two fractions of the opposite sign (positive and negative) is a negative fraction.
  - Product: $\frac{1}{2} \times \left(-\frac{1}{4}\right) = -\frac{1}{8}$ or $-\frac{1}{2} \times \frac{1}{4} = -\frac{1}{8}$
  - Quotient: $\frac{1}{8} \div \left(-\frac{1}{2}\right) = -\frac{1}{4}$ or $-\frac{1}{8} \div \frac{1}{2} = -\frac{1}{4}$

Exercise 3C

Understanding

Understanding	1–3	2, 3

1 Using a number line from −4 to 4, indicate the positions of the following negative and positive fractions.

a $-\frac{1}{4}$

b $1\frac{1}{2}$

c $-3\frac{4}{5}$

d $-\frac{7}{3}$

2 State the missing fractions to complete these sentences.

a Adding $\left(-\frac{1}{4}\right)$ is equivalent to subtracting _____.

b Adding $\frac{1}{3}$ is equivalent to subtracting _____.

c Subtracting $\left(-\frac{3}{5}\right)$ is equivalent to adding _____.

d Subtracting $\frac{2}{7}$ is equivalent to adding _____.

3 State whether the answer for the following expressions will be positive or negative. Do not evaluate the expressions.

a $-\frac{3}{5} \times \left(-\frac{1}{3}\right)$

b $-5\frac{1}{5} \times \frac{9}{11}$

c $\frac{5}{3} \div \left(-\frac{3}{5}\right)$

d $-2\frac{1}{7} \div \left(-8\frac{1}{3}\right)$

Dome Argus is in an inland Antarctic weather station. Summer temperatures can reach $-20\frac{3}{4}$°C and a winter temperature of $-82\frac{1}{2}$°C has been recorded. Scientists calculate the difference as: $-82\frac{1}{2} - \left(-20\frac{3}{4}\right) = -61\frac{3}{4}$°C.

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3C

Fluency

4–8(½) 5–8(½)

Example 7 Adding and subtracting negative fractions

Simplify:

a $\frac{2}{7}+\left(-\frac{5}{7}\right)$

b $\frac{2}{3}-\left(-\frac{4}{3}\right)$

c $\frac{1}{5}+\left(-\frac{1}{4}\right)$

d $-\frac{7}{3}-\left(-3\frac{2}{3}\right)$

Solution

Explanation

a $\frac{2}{7}+\left(-\frac{5}{7}\right)=\frac{2}{7}-\frac{5}{7}$
$=-\frac{3}{7}$

Adding $-\frac{5}{7}$ is equivalent to subtracting $\frac{5}{7}$.

b $\frac{2}{3}-\left(-\frac{4}{3}\right)=\frac{2}{3}+\frac{4}{3}$
$=\frac{6}{3}=2$

Subtracting $-\frac{4}{3}$ is equivalent to adding $\frac{4}{3}$.

c $\frac{1}{5}+\left(-\frac{1}{4}\right)=\frac{1}{5}-\frac{1}{4}$
$=\frac{4}{20}-\frac{5}{20}$
$=-\frac{1}{20}$

Adding $-\frac{1}{4}$ is equivalent to subtracting $\frac{1}{4}$.

The LCM of 5 and 4 is 20.

Write equivalent fractions with LCD of 20.

Subtract the numerators.

d $-\frac{7}{3}-\left(-3\frac{2}{3}\right)=-\frac{7}{3}+3\frac{2}{3}$
$=-\frac{7}{3}+\frac{11}{3}$
$=\frac{4}{3}=1\frac{1}{3}$

Subtracting $-3\frac{2}{3}$ is equivalent to adding $3\frac{2}{3}$.

Convert mixed numeral to improper fraction.

Denominators are the same, therefore add numerators $-7+11=4$.

Now you try

Simplify:

a $\frac{5}{11}+\left(-\frac{8}{11}\right)$

b $\frac{4}{5}-\left(-\frac{3}{5}\right)$

c $\frac{2}{3}+\left(-\frac{4}{5}\right)$

d $-\frac{4}{5}-\left(-2\frac{1}{5}\right)$

4 Simplify:

a $\frac{5}{9}+\left(-\frac{1}{9}\right)$

b $\frac{13}{20}+\left(-\frac{4}{20}\right)$

c $-\frac{1}{5}+\frac{4}{5}$

d $-\frac{3}{7}+\frac{5}{7}$

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5 Simplify:

a $-\frac{6}{7}+\frac{2}{7}$ **b** $-\frac{3}{5}+\frac{4}{5}$ **c** $-\frac{5}{9}-\frac{2}{9}$ **d** $-\frac{11}{3}-\frac{5}{3}$

e $\frac{1}{3}+\left(-\frac{2}{3}\right)$ **f** $\frac{1}{5}+\left(-\frac{3}{5}\right)$ **g** $\frac{1}{4}-\left(-\frac{5}{4}\right)$ **h** $\frac{3}{11}-\left(-\frac{4}{11}\right)$

6 Simplify:

a $\frac{1}{4}+\left(-\frac{1}{3}\right)$ **b** $\frac{3}{7}+\left(-\frac{4}{5}\right)$

c $\frac{1}{2}-\left(-\frac{3}{5}\right)$ **d** $\frac{2}{9}-\left(-\frac{2}{3}\right)$

e $-\frac{3}{2}-\left(-\frac{5}{4}\right)$ **f** $-\frac{5}{8}-\left(-\frac{3}{4}\right)$

g $-\frac{7}{5}-\left(-1\frac{1}{4}\right)$ **h** $-\frac{8}{3}-\left(-2\frac{2}{5}\right)$

Hint for Q6: First find a common denominator.

Example 8 Multiplying with negative fractions

Simplify:

a $\frac{2}{3}\times\left(-\frac{4}{5}\right)$ **b** $-\frac{6}{5}\times\left(-\frac{3}{4}\right)$

Solution	Explanation
a $\frac{2}{3}\times\left(-\frac{4}{5}\right)=-\frac{8}{15}$	The two fractions are of opposite sign so the answer is a negative.
b $-\frac{6}{5}\times\left(-\frac{3}{4}\right)=\frac{6}{5}\times\frac{3}{4}$ $=\frac{3}{5}\times\frac{3}{2}$ $=\frac{9}{10}$	The two fractions are of the same sign, so the answer is a positive. Cancel where possible, then multiply numerators and multiply denominators.

Now you try

Simplify:

a $\frac{3}{5}\times\left(-\frac{6}{7}\right)$ **b** $-\frac{3}{4}\times\left(-\frac{2}{9}\right)$

7 Simplify:

a $\frac{3}{5}\times\left(-\frac{4}{7}\right)$ **b** $-\frac{2}{5}\times\frac{8}{11}$

c $-\frac{1}{3}\times\left(-\frac{4}{5}\right)$ **d** $-\frac{5}{9}\times\left(-\frac{3}{2}\right)$

e $-\frac{3}{9}\times\frac{4}{7}$ **f** $\frac{2}{6}\times\left(-\frac{3}{8}\right)$

g $-1\frac{1}{2}\times\left(-\frac{2}{7}\right)$ **h** $-\frac{3}{8}\times 3\frac{1}{5}$

Hint for Q7: First multiply the positive versions of each fraction.

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3C

Example 9 Dividing with negative fractions

Simplify:

a $-\frac{2}{5} \div \left(-\frac{3}{4}\right)$

b $-1\frac{1}{3} \div 3$

Solution

a $-\frac{2}{5} \div \left(-\frac{3}{4}\right) = -\frac{2}{5} \times \left(-\frac{4}{3}\right)$

$= \frac{2}{5} \times \frac{4}{3}$

$= \frac{8}{15}$

Explanation

The reciprocal of $\left(-\frac{3}{4}\right)$ is $\left(-\frac{4}{3}\right)$.

The two fractions are of the same sign so the answer is a positive.

The answer should be in simplest form.

b $-1\frac{1}{3} \div 3 = -\frac{4}{3} \times \frac{1}{3}$

$= -\frac{4}{9}$

The reciprocal of 3 is $\frac{1}{3}$.

The two numbers are of opposite sign, so the answer is a negative.

Now you try

Simplify:

a $-\frac{2}{3} \div \left(-\frac{5}{6}\right)$

b $-2\frac{1}{3} \div 4$

8 Simplify:

a $-\frac{5}{7} \div \frac{3}{4}$

b $-\frac{1}{4} \div \frac{5}{9}$

c $-\frac{2}{3} \div \left(-\frac{5}{4}\right)$

d $-\frac{4}{9} \div \left(-\frac{1}{3}\right)$

e $-\frac{4}{7} \div 2$

f $-\frac{3}{5} \div 4$

g $-1\frac{1}{2} \div (-2)$

h $-5\frac{1}{3} \div \left(-2\frac{2}{9}\right)$

Hint for Q8: Flip the second fraction to find its reciprocal.

Problem-solving and reasoning

9–11 | 10–13

9 Toolapool has an average maximum temperature of $13\frac{1}{2}$°C and an average minimum temperature of $-3\frac{1}{4}$°C. Average temperature range is calculated by subtracting the average minimum temperature from the average maximum temperature. What is the average temperature range for Toolapool?

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10 Arrange these fractions in increasing order.
$\frac{3}{4}, -\frac{1}{2}, -\frac{5}{3}, -\frac{3}{4}, -1\frac{1}{2}, \frac{1}{16}, -\frac{1}{5}, 3\frac{1}{10}$

11 Xaio aims to get, on average, 8 hours of sleep per week night. On Monday night he slept for $6\frac{1}{3}$ hours, on Tuesday night $7\frac{1}{2}$ hours, on Wednesday night $5\frac{3}{4}$ hours and on Thursday night $8\frac{1}{4}$ hours.

a State the difference between the amount of sleep Xaio achieved each night and his goal of 8 hours. Give a negative answer if the amount of sleep is less than 8 hours.
b After four nights, how much is Xaio ahead or behind in terms of his sleep goal?
c If Xaio is to exactly meet his weekly goal, how much sleep must he get on Friday night?

12 Maria's mother wants to make 8 curtains that each require $2\frac{1}{5}$ metres of material in a standard width, but she only has $16\frac{1}{4}$ metres. She asks Maria to buy more material. How much more material must Maria buy?

13 Place an inequality sign (< or >) between the following fraction pairs to make a true statement. (Note: You can use a number line to decide which value is greater.)

a $-\frac{1}{3}\ \square\ -\frac{1}{2}$
b $-3\frac{1}{5}\ \square\ -2\frac{3}{7}$
c $\frac{1}{4}\ \square\ -\frac{1}{2}$
d $-\frac{3}{5}\ \square\ \frac{1}{11}$
e $2\frac{1}{5}\ \square\ -4\frac{3}{5}$
f $0\ \square\ -\frac{1}{100}$
g $\frac{4}{9}\ \square\ \frac{5}{9}$
h $-\frac{4}{9}\ \square\ -\frac{5}{9}$

Interesting fraction products

	—	14, 15

14 It is possible to multiply two negative fractions and obtain a positive integer. For example, $-\frac{4}{5} \times \left(-\frac{5}{2}\right) = 2$. For each of the following, give an example or explain why it is impossible.

a A positive and a negative fraction multiplying to a negative integer
b A positive and a negative fraction adding to a positive integer
c A positive and a negative fraction adding to a negative integer
d Three negative fractions multiplying to a positive integer

15 Do not evaluate the following expressions, just state whether the answer will be positive or negative.

a $-\frac{2}{7} \times \left(-\frac{1}{7}\right) \times \left(-\frac{3}{11}\right)$
b $-4\frac{1}{5} \times \left(-\frac{9}{11}\right)^2$
c $-\frac{5}{6} \div \left(-\frac{2}{7}\right) \times \frac{1}{3} \times \left(-\frac{4}{9}\right)$
d $\left(-\frac{3}{7}\right)^3 \div \left(-4\frac{1}{5}\right)^3$

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3D Understanding decimals CONSOLIDATING

Learning intentions

- To understand place value in a decimal.
- To be able to compare two or more decimals to decide which is largest.
- To be able to convert decimals to fractions.
- To be able to convert fractions to decimals, in cases where the denominator's only prime factors are 2 and/or 5.

Key vocabulary: decimal, decimal point, place value, fraction

Decimals also represent 'parts of a whole'. They represent fractions with denominators of $10,\ 100,\ 1000\ldots$
The decimal point is used to separate the whole number and the fraction part.

$$3\frac{17}{100} = 3.17$$

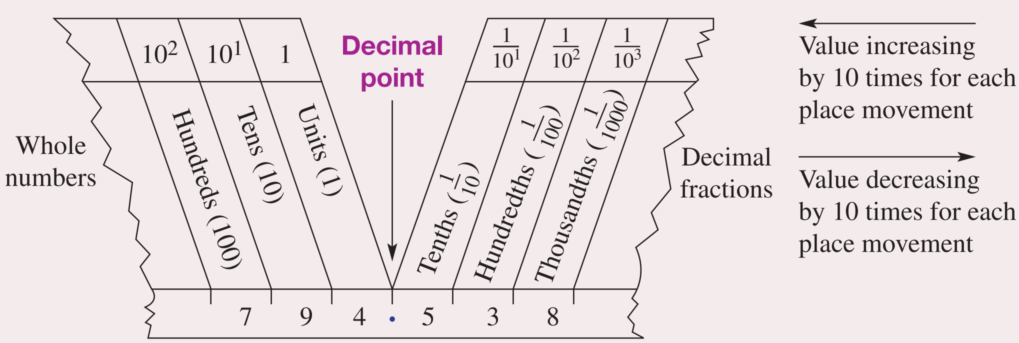
In this section, we review some decimal concepts from Year 7.

Lesson starter: Decimals around us

In pairs, list at least five places where decimals are used and give a specific example for each one.

Key ideas

- The **place value** table is extended for **decimals**.



$$794.538 = 794\frac{538}{1000} = 700 + 90 + 4 + \frac{5}{10} + \frac{3}{100} + \frac{8}{1000}$$

- Comparing and ordering decimals.
  To compare two decimal numbers with digits in the same place-value columns, compare the left-most digits first. Continue comparing digits as you move from left to right until you find two digits that are different.
  e.g. Compare 362.581 and 362.549.

  362.581
  362.549
  the 1st digit that is different: $8 > 4$
  So $362.581 > 362.549$

- Converting decimals to **fractions**
  - Count the number of decimal places used.
  - This is the number of zeroes that you must place in the denominator.
  - Simplify the fraction if required.

  e.g. $0.64 = \frac{64}{100} = \frac{16}{25}$

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- Converting fractions to decimals
  - If the denominator is a power of 10, simply change the fraction directly to a decimal from your knowledge of its place value.

    e.g. $\frac{239}{1000} = 0.239$
  - If the denominator is not a power of 10, try to find an equivalent fraction for which the denominator is a power of 10 and then convert to a decimal.

    e.g. $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15$
  - If the above two methods are not suitable, divide the bottom (denominator) into the top (numerator).

    e.g. $8\overline{)1.{}^{1}0{}^{2}0{}^{4}0}$ with quotient 0.125 $\quad \frac{1}{8} = 0.125$

Exercise 3D

Understanding — 1–4 | 3, 4

1 Which of the following is the mixed numeral equivalent of 8.17?

A $8\frac{1}{7}$ **B** $8\frac{17}{10}$ **C** $8\frac{1}{17}$ **D** $8\frac{17}{1000}$ **E** $8\frac{17}{100}$

2 Which of the following is the mixed numeral equivalent of 5.75?

A $5\frac{75}{10}$ **B** $5\frac{25}{50}$ **C** $5\frac{3}{4}$ **D** $5\frac{15}{25}$ **E** $5\frac{75}{1000}$

3 Which decimal number is equal to $4 + \frac{1}{10} + \frac{5}{100}$?

A 4.015 **B** 40.15 **C** 4.15 **D** 41.5 **E** 4.105

4 Which decimal number is equal to $20 + \frac{3}{10} + \frac{7}{1000}$?

A 20.037 **B** 2.307 **C** 2.37 **D** 20.37 **E** 20.307

Fluency — 5–9(½) | 5–10(½)

Example 10 Comparing decimals

Which is larger?
57.89342 or 57.89631

Solution

57.89631 is larger.

Explanation

Write underneath each other.

57.89 3 42

57.89 6 31

↑

1st digit different from left to right $6 > 3$

$\frac{3}{1000} < \frac{6}{1000}$

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3D

Now you try

Which is larger?

2.14073 or 2.14039

5 Write down the larger decimal in each pair.

a 36.485   37.123

b 21.953   21.864

c 0.0372   0.0375

d 4.21753   4.21809

e 65.4112   64.8774

f 9.5281352   9.5281347

Hint for Q5: Compare digits from left to right.

6 Decide if each of the following statements is true (T) or false (F).

a $\frac{3}{5} = 0.6$

b $\frac{11}{20} = \frac{55}{100} = 5.5$

c $\frac{1}{100} = 0.01$

d $3.6 < 0.36$

e $0.504 > 0.54$

f $0.65 < 0.645$

Example 11 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.725   **b** 5.12

Solution	Explanation
a $\frac{725}{1000} = \frac{29}{40}$	Three decimal places, therefore three zeroes in denominator. 0.725 = 725 thousandths.
b $5\frac{12}{100} = 5\frac{3}{25}$	Two decimal places, therefore two zeroes in denominator. 0.12 = 12 hundredths.

Now you try

Convert the following decimals to fractions in their simplest form.

a 0.58   **b** 12.65

7 Convert the following decimals to fractions in their simplest form.

a 0.31   **b** 0.537

c 0.815   **d** 0.96

e 5.35   **f** 8.22

g 26.8   **h** 8.512

i 0.052   **j** 6.125

k 317.06   **l** 0.424

Hint for Q7: Don't forget to simplify.

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Example 12 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{239}{100}$ **b** $\frac{9}{25}$

Solution

a $\frac{239}{100} = 2\frac{39}{100} = 2.39$

b $\frac{9}{25} = \frac{36}{100} = 0.36$

Explanation

Convert improper fraction to a mixed numeral.
Denominator is a power of 10.

$\frac{9}{25} = \frac{9 \times 4}{25 \times 4} = \frac{36}{100}$

Now you try

Convert the following fractions to decimals.

a $\frac{2341}{1000}$ **b** $\frac{17}{40}$

8 Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $\frac{301}{1000}$ **c** $\frac{45}{100}$

d $\frac{6}{10}$ **e** $\frac{67}{100}$ **f** $\frac{674}{1000}$

g $\frac{15}{100}$ **h** $\frac{79}{100}$ **i** $\frac{7}{10}$

j $\frac{17}{10}$ **k** $\frac{118}{100}$ **l** $\frac{41}{1000}$

9 Convert the following fractions to decimals.

a $\frac{3}{25}$ **b** $\frac{7}{20}$ **c** $\frac{5}{2}$ **d** $\frac{7}{4}$

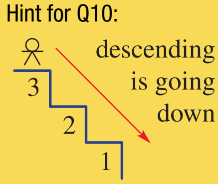
e $\frac{11}{40}$ **f** $\frac{3}{8}$ **g** $\frac{17}{25}$ **h** $\frac{29}{125}$

Hint for Q9: Convert to a denominator of 10, 100 or 1000.

10 Convert the following mixed numerals to decimals and then place them in descending order.

$2\frac{2}{5}, 2\frac{1}{4}, 2\frac{9}{50}, 2\frac{3}{10}$

Hint for Q10:



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3D

Problem-solving and reasoning

11, 12 | 12–14

11 The distances from Nam's locker to his six different classrooms are listed below:

- Locker to room B5 (0.186 km)
- Locker to room A1 (0.119 km)
- Locker to room P9 (0.254 km)
- Locker to gym (0.316 km)
- Locker to room C07 (0.198 km)
- Locker to BW Theatre (0.257 km)

List Nam's six classrooms in order, from the closest classroom to the one furthest away from his locker.

12 The Prime Minister's approval rating is 0.35, while the Opposition Leader's approval rating is $\frac{3}{5}$.
Which leader is ahead in the popularity polls and by how much?

13 Jerome wishes to dig a hole 1.5 metres deep.
Michael has dug a hole $1\frac{3}{4}$ metres deep.

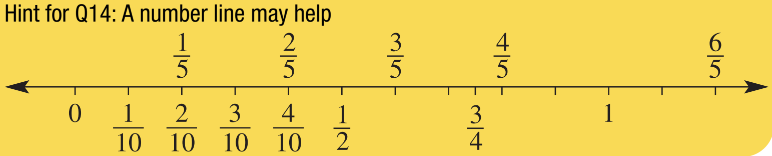
a Whose hole will be the deepest?
b How many centimetres difference is there in the depth of each hole?

Hint for Q13: 1 m = 100 cm

14 Write down a *decimal* that lies between the pairs of fractions.

a $\frac{1}{2}$ and $\frac{9}{10}$
b $\frac{3}{4}$ and 1
c $\frac{1}{4}$ and $\frac{3}{4}$
d $\frac{1}{5}$ and $\frac{7}{10}$

Hint for Q14: A number line may help



Magic squares

— | 15

15 Complete the following magic squares using a mixture of fractions and decimals.

a

2.6		$1\frac{4}{5}$
	$\frac{6}{2}$	
4.2		

b

0.8	1.8		3.2
3.0		2.0	0.6
2.8			
0.2			2.6

Hint for Q15: Each row, column and diagonal add to the same number in each magic square. It's the MAGIC number!

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3E Operations with decimals

Learning intentions

- To be able to add and subtract decimals.
- To be able to multiply decimals.
- To understand that multiplying and dividing by powers of 10 involves moving the digits left or right of the decimal point.
- To be able to divide decimals.

Key vocabulary: decimal point, power of 10, divisor, dividend, quotient

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing decimals.

Precision electronic measuring instruments give a decimal read-out.

Lesson starter: Match the phrases

There are seven different sentence beginnings and seven different sentence endings below. Your task is to match each sentence beginning with its correct ending. When you have done this, write down the seven correct sentences.

Sentence beginnings	Sentence endings
When adding or subtracting decimals	the decimal point moves two places to the right.
When multiplying decimals	the decimal point in the quotient goes directly above the decimal point in the dividend.
When multiplying decimals by 100	make sure you line up the decimal points.
When dividing decimals by decimals	the number of decimal places in the question must equal the number of decimal places in the answer.
When multiplying decimals	the decimal point moves two places to the left.
When dividing by 100	start by ignoring the decimal points.
When dividing decimals by a whole number	we start by changing the question so that the divisor is a whole number.

Key ideas

- Adding and subtracting decimals
  - Ensure digits are correctly aligned in similar place-value columns.
  - Ensure the decimal points are lined up directly under one another.

  $37.56 + 5.231$ $\quad \begin{array}{r} 37.560 \\ +\ 5.231 \\ \hline \end{array}$ ✔ $\quad \begin{array}{l} 37.56 \\ 5.231 \\ \hline \end{array}$ ✘

- Multiplying and dividing decimals by **powers of 10**
  - When multiplying, the decimal point moves to the *right* the same number of places as there are zeroes in the multiplier.

  e.g. $13.753 \times 100 = 1375.3$ $\quad 13.753$

  Multiply by 10 twice.

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3E

- When dividing, the decimal point moves to the *left* the same number of places as there are zeroes in the **divisor**.
  e.g. $586.92 \div 10 = 58.692$

  586.92
  Divide by 10 once.

- Multiplying decimals
  - Initially ignore the decimal points and multiply the wholes.
  - Place the decimal point into the answer using the rule:
    'The number of decimal places in the answer must equal the total number of decimal places in the question.'

e.g. $5.73 \times 8.6 \longrightarrow \begin{array}{r} 573 \\ \times\ \ 86 \\ \hline 49278 \end{array} \longrightarrow 5.\underline{73} \times 8.\underline{6} = 49.\underline{278}$

(3 decimal places in question, 3 decimal places in answer)

- Dividing decimals
  The decimal point in the **quotient** goes directly above the decimal point in the **dividend**.
  e.g. $56.34 \div 3$

  $18.78 \longleftarrow$ Quotient (answer)
  Divisor $\longrightarrow 3\overline{)56.34} \longleftarrow$ Dividend

  We avoid dividing decimals by other decimals. Instead we change the divisor into a whole number. Whatever change we make to the divisor we must also make to the dividend, so it is equivalent to multiplying by 1 and the value of the question is not changed.

  We avoid $\longrightarrow 27.354 \div 0.02$

  $27.354 \div 0.02 = 2735.4 \div 2$

  Preferring to do $\longrightarrow 2735.4 \div 2$

Exercise 3E

Understanding

1–4 | 3, 4

1 Which of the following is the correct set-up for the following addition problem?
$5.386 + 53.86 + 538.6$

A $\begin{array}{r} 5.386 \\ 53.86 \\ +538.6 \\ \hline \end{array}$ **B** $\begin{array}{r} 5.386 \\ 53.860 \\ +538.600 \\ \hline \end{array}$ **C** $\begin{array}{r} 5.386 \\ 53.86 \\ +538.6 \\ \hline \end{array}$ **D** $\begin{array}{r} 538 + 53 + 5 \\ +\ 0.386 + 0.86 + 0.6 \\ \hline \end{array}$

2 The correct answer to the problem $2.731 \div 1000$ is:
A 2731 **B** 27.31 **C** 2.731
D 0.02731 **E** 0.002731

Hint for 3E Understanding: The **Key ideas** section answers all these questions.

3 If $56 \times 37 = 2072$, the correct answer to the problem 5.6×3.7 is:
A 207.2 **B** 2072 **C** 20.72
D 2.072 **E** 0.2072

4 Which of the following divisions would provide the same answer as the division question $62.5314 \div 0.03$?
A $625.314 \div 3$ **B** $6253.14 \div 3$ **C** $0.625314 \div 3$ **D** $625314 \div 3$

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Fluency

5–9(½) | 5–9(½), 10

Example 13 Adding and subtracting decimals

Calculate.

a $23.07 + 9.8$

b $9.7 - 2.86$

Solution

a
$$\begin{array}{r} 23.07 \\ +\ 9.80 \\ \hline 32.87 \end{array}$$

Explanation

Line up the decimal points. Fill in any zeroes then add vertically.

b
$$\begin{array}{r} {}^{8}9.{}^{16}7{}^{1}0 \\ -\ 2.\ \ 8\ \ 6 \\ \hline 6.\ \ 8\ \ 4 \end{array}$$

Align decimal points directly under one another and fill in missing decimal places with zeroes. Carry out subtraction following the same procedure as for subtraction of whole numbers.

Now you try

Calculate.

a $1.05 + 12.96$

b $3.2 - 1.74$

5 Calculate the following.

a $5.6 + 1.2$ **b** $8.4 + 2.1$ **c** $18.6 + 3.3$

d $4.9 + 5.3$ **e** $8.1 + 8.2$ **f** $9.3 + 3.9$

g $23.57 + 39.14$ **h** $64.28 + 213.71$ **i** $5.623 + 18.34$

Hint for Q5: Align digits in matching place-value columns.

6 Calculate the following.

a $5.6 - 1.2$ **b** $8.4 - 2.1$ **c** $18.6 - 3.3$

d $7.9 - 3.8$ **e** $15.6 - 9.5$ **f** $10.4 - 6.4$

g $38.52 - 24.11$ **h** $76.74 - 53.62$ **i** $123.8 - 39.21$

Example 14 Multiplying and dividing by powers of 10

Calculate.

a $9.753 \div 100$

b $27.58 \times 10\,000$

Solution

a $9.753 \div 100 = 0.09753$

Explanation

Dividing by 100 (2 zeroes), therefore the decimal point must move two places to the left. Additional zeroes are inserted as necessary.

$.09.753$

b $27.58 \times 10\,000 = 275\,800$

Multiplying by 10 000 (4 zeroes), therefore the decimal point must move four places to the right. Additional zeroes are inserted as necessary.

$27.5800.$

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3E

Now you try

Calculate.

a $64.3 \div 1000$ b 0.431×100

7 Calculate.

a 9.61×10	b 9.61×100	c 15.463×1000
d $19.4 \div 10$	e $19.4 \div 100$	f $27.4 \div 10$
g 27.4×1000	h 1.6×1000	i 36.5173×100
j 0.08155×1000	k $7.5 \div 10$	l $3.812 \div 100$

Hint for Q7: Move left or right by the number of zeroes.

Example 15 Multiplying decimals

Calculate 25.7×0.3.

Solution

$$\begin{array}{r} {}^{1}2\,{}^{2}5\,7 \\ \times\quad 3 \\ \hline 7\;7\,1 \end{array}$$

$25.7 \times 0.3 = 7.71$

Explanation

Perform multiplication ignoring the decimal point. $(257 \times 3 = 771)$

There are two decimal places in the question, so two decimal places in the answer.

Now you try

Calculate 4.5×1.6.

8 Calculate.

a 0.8×7	b 0.8×0.7	c 15×0.1	d 0.4×0.3
e 15.4×2	f 1.2×0.3	g 0.8×0.4	h 0.8×0.04
i 15×0.2	j 24.5×0.2	k 0.9×9	l 1.2×1.2

Hint for Q8: First ignore the decimal point.

Example 16 Dividing decimals

Calculate.

a $35.756 \div 4$

b $64.137 \div 0.03$

Solution

a 8.939

$$\begin{array}{r} 8.9\,3\,9 \\ 4\,\overline{)\,35.{}^{3}7{}^{1}5{}^{3}6} \end{array}$$

b $64.137 \div 0.03$

$= 6413.7 \div 3$

$= 2137.9$

$$\begin{array}{r} 2\,1\,3\,7.9 \\ 3\,\overline{)\,64{}^{1}1{}^{2}3.{}^{2}7} \end{array}$$

Explanation

Carry out division, remembering that the decimal point in the answer is placed directly above the decimal point in the dividend.

Instead of dividing by 0.03, multiply both the divisor and the dividend by 100, to get a whole number, 3.

(Move **each** decimal point two places to the right.) Carry out the division question $6413.7 \div 3$.

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Now you try

Calculate.

a $1.72 \div 8$

b $34.2 \div 0.6$

9 Calculate.

a $24.54 \div 2$

b $17.64 \div 3$

c $0.0485 \div 5$

d $347.55 \div 7$

e $133.44 \div 12$

f $4912.6 \div 11$

g $2.58124 \div 8$

h $17.31 \div 5$

10 Complete these divisions by filling in the missing numbers.

a $15.6 \div 0.3 = 156 \div 3 = \square$

b $12.4 \div 0.02 = 1240 \div 2 = \square$

c $15.06 \div 0.2 = \square \div 2 = \square$

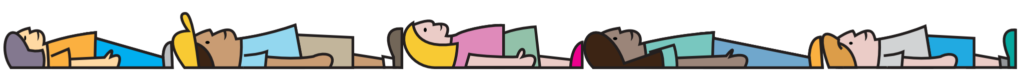
d $45.9 \div 0.03 = 4590 \div \square = \square$

e $0.484 \div 0.4 = \square \div 4 = \square$

Problem-solving and reasoning

11 | 12, 13

11 The heights of Mrs Buchanan's five grandchildren are 1.34 m, 1.92 m, 0.7 m, 1.5 m, and 1.66 m. If the grandchildren laid down in a row, head to toe, how long would the row be?



12 Look closely at the table of canteen prices below.

Canteen prices		
pie \$2.80	chips \$1.70	juice \$3.40
cola \$3.20	chocolate \$2.20	sandwich \$2.60
sauce \$0.60	apple \$0.50	milk \$1.85

a Find the cost of each person's lunch.

Vaughn	Charlotte	Reece
1 pie	1 sandwich	1 pie
1 sauce	1 chocolate	2 colas
1 apple	1 juice	1 sandwich
2 milks		1 pkt chips

b Who had the most change from \$20?

13 An alternative method for multiplying decimals is to convert them to fractions and then multiply them. For example, $0.3 \times 1.2 = \frac{3}{10} \times 1\frac{2}{10} = \frac{3}{10} \times \frac{12}{10} = \frac{36}{100}$, which can be converted into 0.36. Use this method to multiply the following decimals.

a 0.3×0.7

b 0.1×3.07

c 0.2×0.05

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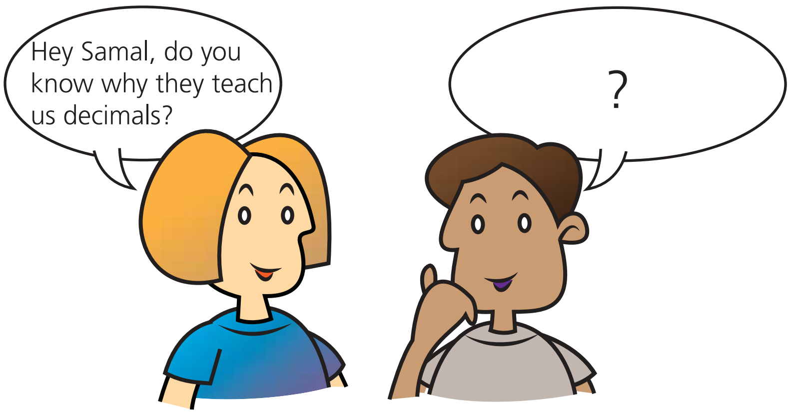
3E

Secret code

—	14

14 Answer each of the 12 questions below to unlock the code and find out how Samal answers Sally's question.



20.7	12.2	4.4			4.4	0.3		
12.2	4.75	14.4	12.2			3.2	160	
24.2	0.3	12.2	4.75			1.32	160	12.2
12.2	4.75	160			17.97	0.3	20.7	0.72	12.2

I $3.2 + 17.5$	O 1.5×0.2	H $47.5 \div 10$	E $96 \div 0.6$
A 1.2×12	T $15.8 - 3.6$	N 0.9×0.8	B $5.9 + 18.3$
W $9.6 \div 3$	P $18.57 - 0.6$	S $9 - 4.6$	G $1.2 + 0.12$

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3F Terminating, recurring and rounding decimals

Learning intentions

- To understand the different notations for recurring decimals (involving dots and dashes)
- To be able to convert a fraction to a terminating decimal using division.
- To be able to convert a fraction to a recurring decimal using division.
- To be able to round decimals to a given number of decimal places by first finding the critical digit.

Key vocabulary: terminating decimal, recurring decimal (or repeating decimal), rounding

Not all fractions convert to the same type of decimal.

For example:

$\frac{1}{2} = 1 \div 2 = 0.5$ (only has one decimal place)

$\frac{1}{3} = 1 \div 3 = 0.33333\ldots$ (keeps going and going)

$\frac{1}{7} = 1 \div 7 = 0.142857\,142857\ldots$ (the pattern repeats)

Decimals that stop (or terminate) are known as terminating decimals, whereas decimals that continue on forever with some form of pattern are known as repeating or recurring decimals.

Lesson starter: Decimal patterns

Use a calculator to perform these divisions. Can you see a pattern?

- $\frac{1}{9} = 1 \div 9 = 0.1111\ldots\ldots$
- $\frac{2}{9}$
- $\frac{3}{9}$
- $\frac{4}{9}$

Without your calculator, write down $\frac{5}{9}$ and $\frac{6}{9}$ as decimals. What do we call these types of decimals?

Key ideas

- A **terminating decimal** has a fixed number of decimal places (i.e. it terminates).

  e.g. $\frac{5}{8} = 5 \div 8 = 0.625$ $\quad 8\overline{)5.{}^{5}0\,{}^{2}0\,{}^{4}0}$ with quotient 0.625 ← Terminating decimal (3 decimal places only)

- A **recurring decimal** (or repeating decimal) keeps going and the decimal places repeat.

  e.g. $\frac{1}{3} = 1 \div 3 = 0.333\ldots$ $\quad 3\overline{)1.{}^{1}0\,{}^{1}0\,{}^{1}0\,{}^{1}0}$ with quotient $0.333\ldots$ ← Recurring decimal

- A convention is to use dots placed above the digits to show the start and finish of a repeating cycle of digits.

  e.g. $0.55555\ldots = 0.\dot{5}$ and $0.3412412412\ldots = 0.3\dot{4}1\dot{2}$

- Another convention is to use a horizontal line placed above the digits to show the repeating cycle of digits.

  e.g. $0.55555\ldots = 0.\overline{5}$ and $0.3412412412\ldots = 0.3\overline{412}$

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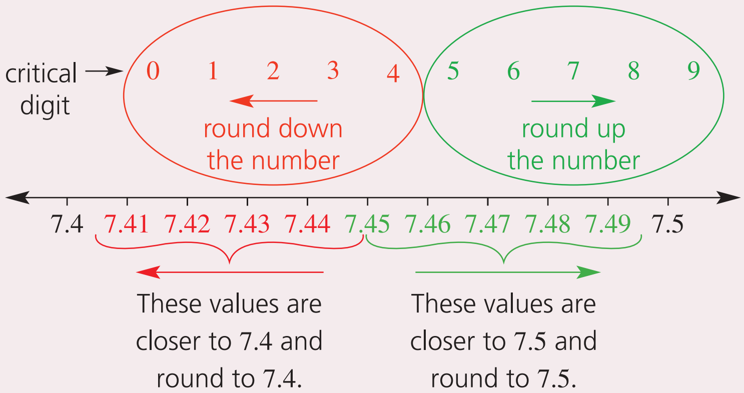
3F

- **Rounding** decimals
  Decimals can be written with fewer decimal places by rounding.
  To round we must look at the digit immediately after the number of places we want. (It's the *critical* digit!)



Exercise 3F

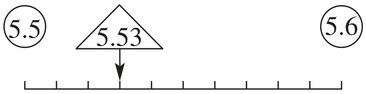
Understanding

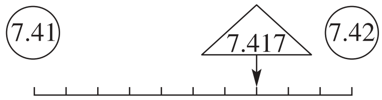
1–4 | 3, 4

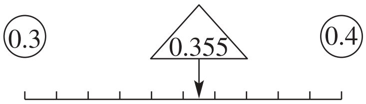
1 State whether the following are terminating decimals (T) or recurring decimals (R).

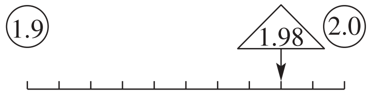
a 5.47 **b** $3.1541\dot{5}$ **c** $8.\dot{6}$ **d** 7.1834
e 0.333 **f** $0.\dot{5}3\dot{4}$ **g** 0.5615 **h** 0.32727…

2 For each line given, which circled decimal is the decimal in the triangle closest to?

a


b


c


d


3 Express the following recurring decimals using the convention of dots or a bar to indicate the start and finish of the repeating cycle.

a 0.33333…
b 6.21212121…
c 8.5764444…
d 2.135635635…
e 11.2857328573…
f 0.003523523…

Hint for Q3: Write 0.7555… as $0.7\dot{5}$.

4 Write down the 'critical' digit (the digit immediately after the rounding digit) for each of the following.

a 3.5724 (rounding to 3 decimal places)
b 15.89154 (rounding to 1 decimal place)
c 0.004571 (rounding to 4 decimal places)
d 5432.726 (rounding to 2 decimal places)

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Fluency

5–8(½) | 5–8(½), 9(½)

Example 17 Writing terminating decimals

Convert the following fractions to decimals.

a $\frac{1}{4}$ b $\frac{7}{8}$

Solution

a $\frac{1}{4} = 0.25$

Explanation

$$\begin{array}{r} 0.\ 2\ 5 \\ 4\overline{)1.{}^{1}0{}^{2}0} \end{array}$$

Write 1 as 1.00.
Divide the bottom (denominator) into the top (numerator).

b $\frac{7}{8} = 0.875$

$$\begin{array}{r} 0.\ 8\ 7\ 5 \\ 8\overline{)7.{}^{7}0{}^{6}0{}^{4}0} \end{array}$$

Write 7 as 7.000.
Divide the bottom (denominator) into the top (numerator).

Now you try

Convert the following fractions to decimals.

a $\frac{4}{5}$ b $\frac{7}{4}$

5 Convert the following fractions to decimals.

a $\frac{3}{5}$ b $\frac{3}{4}$ c $\frac{1}{8}$ d $\frac{11}{20}$

e $\frac{1}{2}$ f $\frac{4}{5}$ g $\frac{1}{25}$ h $\frac{9}{50}$

Hint for Q5: These are all terminating decimals.

Example 18 Writing recurring decimals

Express the following fractions as recurring decimals.

a $\frac{2}{3}$ b $3\frac{5}{7}$

Solution

a $\frac{2}{3} = 0.\dot{6}$

Explanation

$$\begin{array}{r} 0.\ 6\ 6\ldots \\ 3\overline{)2.{}^{2}0{}^{2}0{}^{2}0} \end{array}$$

This pattern continues, it is a repeating decimal.

b $3\frac{5}{7} = 3.\dot{7}1428\dot{5}$ or $3.\overline{714285}$

$$\begin{array}{r} 0.\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ldots \\ 7\overline{)5.{}^{5}0{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}} \end{array}$$

This pattern continues.

Now you try

Express the following fractions as recurring decimals.

a $\frac{7}{15}$ b $1\frac{8}{13}$

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3F

6 Express the following fractions as recurring decimals.

a $\frac{1}{3}$ **b** $\frac{5}{9}$

c $\frac{5}{6}$ **d** $\frac{7}{9}$

e $\frac{3}{7}$ **f** $\frac{1}{6}$

g $\frac{4}{3}$ **h** $1\frac{6}{7}$

Hint for Q6: Remember to use the repeating notation. $0.444\ldots = 0.\dot{4}$.

Example 19 Rounding decimals

a Round 14.258 to 1 decimal place.
b Round 0.671 to 2 decimal places.

Solution	Explanation
a 14.3	14.258 rounded to 1 decimal place — look at next digit (5). Critical digit is 5. Round up $14.258 \approx 14.3$.
b 0.67	0.671 rounded to 2 decimal places — look at the next digit (1). Critical digit is 1. Round down $0.671 \approx 0.67$.

Now you try

a Round 24.9349 to 2 decimal places.
b Round 0.048561 to 3 decimal places.

7 Round each of the following decimals to 1 decimal place.

a 0.57 **b** 0.83 **c** 1.49
d 8.16 **e** 9.47 **f** 8.33
g 1.487 **h** 3.444 **i** 0.333

Hint for Q7: The first decimal place is also called the tenths column.

8 Write each of the following decimals correct to 2 decimal places (the nearest hundredth).

a 0.783 **b** 0.666 **c** 1.478
d 0.893 **e** 15.488 **f** 9.035
g 9.4163 **h** 8.7499 **i** 1.7891

9 **a** Choose the correct answer to each of the following.
- **i** Is 7.9 closer to 7 or 8?
- **ii** Is 7.99 closer to 7.9 or 8.0?
- **iii** Is 4.96 closer to 4.9 or 5.0?

b Round the following to 1 decimal place.
- **i** 4.96 **ii** 8.941 **iii** 5.999

Problem-solving and reasoning 10, 11 10–12

10 Find out how your calculator rounds. Use it to round each of the following decimals to the number of decimal places given in the brackets.

a 0.76581 (3) **b** 9.4582 (1) **c** 6.9701 (1) **d** 21.513426 (4)
e 0.9457 (2) **f** 17.26 (0) **g** 8.5974 (2) **h** 8.10552 (3)

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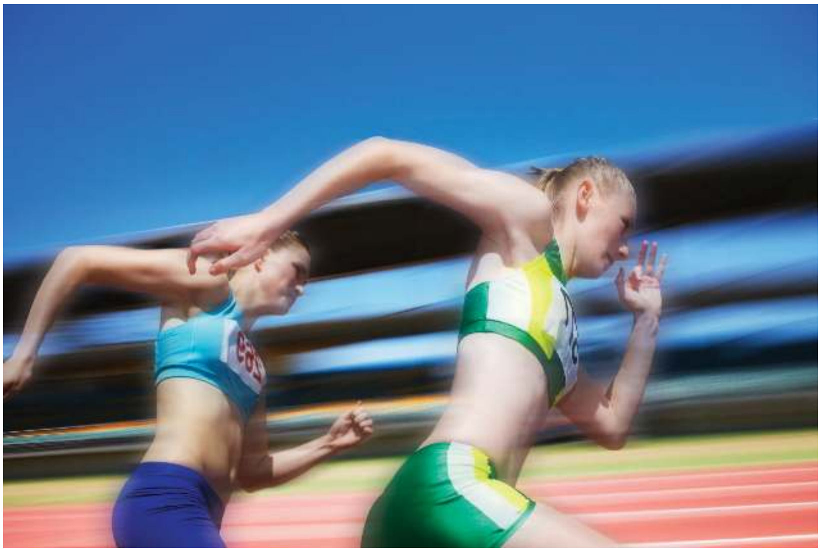
11 Simone and Greer are two very good junior sprinters. Simone ran 100 m in 12.83 seconds, while Greer ran it in 12.77 seconds.

a Who came first, and by how much?

b Round each time to one decimal place. Can you still decide who came first?



12 Petrol is sold at 147.9 cents per litre. Find, correct to the nearest cent, the cost of:

a 1 litre

b 5 litres

c 12 litres

d 39.7 litres

Neither terminating or recurring — 13

13 There are some numbers when written as a decimal neither terminate nor recur. One such set of numbers are called surds, which include a $\sqrt{}$ sign, e.g. $\sqrt{2}$.

a Write $\sqrt{2}$ correct to 7 decimal places using a calculator.

b Find some other surds and write them correct to 3 decimal places.

c Other non-surd numbers which do not terminate or recur include pi and phi. Research these numbers and write a brief report about why they are important.

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Progress quiz

3A **1** Write the following fractions in simplest form.

a $\frac{4}{10}$ b $\frac{6}{9}$ c $\frac{16}{20}$ d $\frac{45}{25}$

3B **2** Evaluate.

a $\frac{5}{11}+\frac{8}{11}$ b $\frac{7}{8}-\frac{3}{4}$ c $2\frac{1}{5}+3\frac{7}{10}$ d $4\frac{2}{3}-2\frac{1}{2}$

3B **3** Evaluate.

a $\frac{2}{3}\times\frac{5}{7}$ b $1\frac{3}{5}\times\frac{3}{4}$ c $\frac{3}{7}\div\frac{5}{11}$ d $1\frac{1}{6}\div\frac{5}{12}$

3C **4** Evaluate.

a $\frac{2}{3}-\left(-\frac{4}{3}\right)$ b $-\frac{1}{2}+\frac{1}{3}$ c $1\frac{1}{2}+\left(-\frac{1}{2}\right)$ d $-2\frac{2}{5}-\frac{7}{2}$

3C **5** Evaluate.

a $\frac{2}{9}\times\left(-\frac{3}{4}\right)$ b $-\frac{6}{7}\times\frac{14}{2}$ c $\frac{8}{3}\div\left(-\frac{4}{9}\right)$ d $-1\frac{1}{2}\div\left(-\frac{1}{4}\right)$

3D **6** Convert the following decimals to fractions in their simplest form.

a 0.35 b 5.25 c 12.8 d 456.14

3D **7** Convert the following fractions to decimals.

a $\frac{7}{10}$ b $\frac{36}{100}$ c $\frac{17}{50}$ d $\frac{9}{4}$

3E **8** Calculate.

a $9.5+12.3$ b $5.78+12.915$ c $35.8-23.6$ d $76.813-56.685$

3E **9** Calculate.

a 6.5734×1000 b $12.754\div10\,000$ c 0.6×0.9 d 45.23×0.5
e $23.845\div5$ f $84.561\div0.03$

3F **10** Convert the following fractions to decimals, expressing your answers as recurring decimals if necessary.

a $\frac{3}{8}$ b $\frac{5}{4}$ c $\frac{5}{3}$ d $\frac{11}{7}$

3F **11** Round each of the following decimals to two decimal places.

a 0.789 b 0.415 c 26.14812 d 379.01099

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3G Converting fractions, decimals and percentages

Learning intentions

- To understand that a percentage (%) is a number out of 100.
- To be able to convert percentages to fractions and decimals.
- To be able to convert fractions and decimals to percentages.

Key vocabulary: percentage, per cent, fraction, decimal

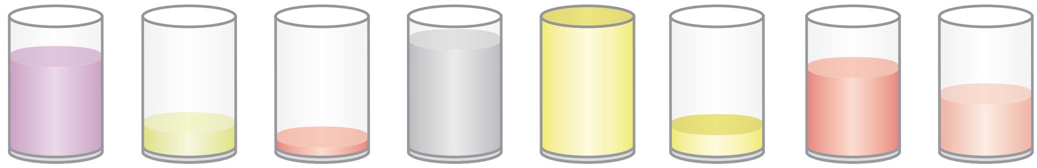
A **percentage** is a particular fraction where the denominator is always 100.

Per cent is Latin for 'out of 100'.

$$\boxed{7\% = 7 \text{ per cent} = 7 \text{ out of } 100 = \frac{7}{100} = 0.07}$$

People use percentages every day in banking, sales and school tests.

Lesson starter: Estimating percentages



- List the drinks in order from the most to the least amount left in the glass.
- Estimate the percentage of drink remaining in each of the glasses shown above.
- Discuss your estimates with a partner.

Key ideas

- A **per cent** sign (%) means 'out of one hundred'.
  $23\% = \frac{23}{100}$
- **Percentages** can be converted to **fractions** and **decimals**.
  $35\% \to \frac{35}{100} = \frac{7}{20}$ (fraction)
  $35\% \to 35 \div 100 = 0.35$ (decimal)
- Fractions and decimals can be converted to percentages.
  $\frac{1}{4}$ or 0.25 as a percentage $\to \frac{1}{4} \times 100 = 25$ and $0.25 \times 100 = 25$
  so $\frac{1}{4} = 0.25 = 25\%$
  Common percentages and their equivalent fractions are shown in the table below.
  It is helpful to know these.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	1
Decimal	0.5	$0.\dot{3}$	0.25	0.2	0.125	$0.\dot{6}$	0.75	1
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	$12\frac{1}{2}\%$	$66\frac{2}{3}\%$	75%	100%

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Exercise 3G

Understanding

1–4 | 4

Hint for Q1: Remember if you see a % sign it means 'out of 100'.

1 The fraction equivalent of 27% is:

A $\frac{2}{7}$ **B** $\frac{27}{100}$ **C** 2700 **D** $\frac{13.5}{10}$

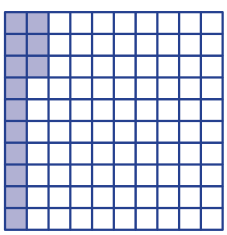
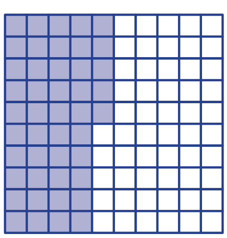
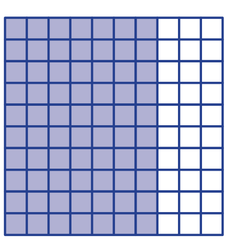
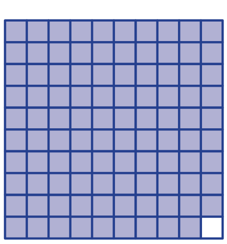
2 The decimal equivalent of 37% is:

A 0.037 **B** 0.37 **C** 3.7 **D** 37.00

3 The percentage equivalent of $\frac{47}{100}$ is:

A 0.47% **B** 4.7% **C** 47% **D** 470%

4 Copy and complete the table or discuss as a group.

		Fraction shaded	Fraction in words	Decimal in figures	Per cent in words	Per cent in figures
a		$\frac{13}{100}$	thirteen hundredths			
b				0.45		
c					seventy per cent	
d						99%

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Fluency 5–8(½) 5–8(½)

Example 20 Converting percentages to fractions and decimals

Write 72% as:

a a simple fraction

b a decimal

Solution	Explanation
a $72\% = \frac{72}{100}$	Change % sign to a denominator of 100.
$= \frac{18 \times 4}{25 \times 4}$	Cancel the highest common factor of 4 to simplify.
$= \frac{18}{25}$	
b $72\% = 72 \div 100$	To change a % to a decimal, divide by 100.
$= 0.72$	

Now you try

Write 65% as:

a a simple fraction

b a decimal

5 Write these percentages as simple fractions.

a 39% b 11%
c 17% d 99%
e 20% f 70%
g 75% h 55%

Hint for Q5: Write each number out of 100.

6 Write these percentages as decimals.

a 39% b 11% c 17% d 99%
e 20% f 70% g 75% h 55%
i 7% j 1% k 10% l 47%

Hint for Q6: Move the decimal point two places to the left.

Example 21 Converting fractions and decimals to percentages

Write the following as percentages.

a $\frac{4}{5}$ b $\frac{3}{8}$ c 0.81

Solution	Explanation
a $\frac{4}{5} \times 100 = 80$	Multiply by 100 to express as a percentage
So $\frac{4}{5} = 80\%$	
b $\frac{3}{8} \times 100 = \frac{300}{8} = 37\frac{1}{2}$	Multiply the fraction by 100. Simplify $300 \div 8 = 37$ remainder 4
So $\frac{3}{8} = 37\frac{1}{2}\%$	

Continued on next page

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c $0.81 \times 100 = 81$

So $0.81 = 81\%$

Multiply the decimal by 100.

$0.81 = 81\%$

Now you try

Write the following as percentages.

a $\frac{23}{50}$ **b** $\frac{5}{16}$ **c** 0.473

7 Write these fractions as percentages.

a $\frac{77}{100}$ **b** $\frac{49}{100}$ **c** $\frac{3}{4}$ **d** $\frac{4}{5}$

e $\frac{7}{25}$ **f** $\frac{9}{20}$ **g** $\frac{11}{20}$ **h** $\frac{19}{50}$

i $\frac{47}{50}$ **j** $\frac{1}{8}$ **k** $\frac{5}{8}$ **l** $\frac{2}{3}$

Hint for Q7: Multiply by 100 and simplify.

8 Write these decimals as percentages.

a 0.16 **b** 0.79 **c** 0.83 **d** 0.97

e 0.03 **f** 0.33 **g** 0.91 **h** 0.09

i 0.56 **j** 0.22 **k** 1.00 **l** 1.01

Problem-solving and reasoning

9, 10 | 9–11

9 **a** If $\frac{1}{5} = 20\%$, what does $\frac{3}{5}$ equal as a percentage?

b If $\frac{1}{8} = 12.5\%$, what does $\frac{7}{8}$ equal as a percentage?

c If $\frac{1}{3} = 33\frac{1}{3}\%$, what does $\frac{2}{3}$ equal as a percentage?

Hint for Q9: What can we multiply each fraction by?

10 Complete the following conversion tables involving common fractions, decimals and percentages.

a

Fraction	Decimal	%
$\frac{1}{4}$		
$\frac{2}{4}$		
$\frac{3}{4}$		
$\frac{4}{4}$		

b

Fraction	Decimal	%
		20%
		40%
		60%
		80%
		100%

c

Fraction		$\frac{3}{10}$		$\frac{1}{3}$
Decimal	0.15			
%			90%	

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11 The Sharks hockey team has won 13 out of 17 games for the season to date. The team still has three games to play. What is the smallest and the largest percentage of games the Sharks could win for the season?

Money and percentages — 12

12 Copy and complete this table. Can you see a connection?

Cents per 100 cents	Cents in the dollar	Percentage
5 cents	\$0.05	
10 cents		
	\$0.09	
		17%
		25%
	\$0.70	
		90%
75 cents		
100 cents		
	\$2	

Hint for Q12:
One dollar equals 100 *cents*.
One *century* is 100 years.

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3H Finding a percentage and expressing as a percentage

Learning intentions

- To be able to express one quantity as a percentage of another.
- To be able to convert units in order to express one quantity as a percentage of another.
- To be able to find a certain percentage of a quantity.

Key vocabulary: percentage, fraction, units

Percentages are useful when comparing discounts, interest rates and even marks in a test.

For example, Huen's report card could be written as marks out of each total or in percentages.

French test $\frac{14}{20}$	French test 70%
German test $\frac{54}{75}$	German test 72%

In this section we look at expressing a number as a percentage of another number as well as finding a percentage of an amount.

Shops often display discounted prices as a percentage of the original price. Calculating the new price helps determine if the discount offers a significant saving.

Lesson starter: What percentage has passed?

Answer the following questions.

- What percentage of your day has passed?
- What percentage of the current month has passed?
- What percentage of the current season has passed?
- What percentage of your school year has passed?
- What percentage of your school education has passed?
- If you live to an average age, what percentage of your life has passed?
- When you turned 5, what percentage of your life was one year?
- When you are 40, what percentage of your life will one year be?

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Key ideas

- To express one quantity as a **percentage** of another:
  1. Write the quantities as a **fraction**. (The 'whole' amount is always the denominator.)
  2. Multiply this fraction by 100.

     e.g. Express a test score of 14 out of 20 as a percentage.

     $\frac{14}{20} \times 100 = \frac{14}{{}_{1}\cancel{20}} \times \frac{{}^{5}\cancel{100}}{1} = 70$

     $\frac{14 \leftarrow \text{part of the whole}}{20 \leftarrow \text{whole amount}}$

     So $\frac{14}{20} = 70\%$

- To find a certain percentage of a quantity:
  1. Express the required percentage as a fraction. (You can also use decimals.)
  2. Change the 'of' to a multiplication sign.
  3. Express the number as a fraction.
  4. Follow the rules for multiplication of fractions.

     e.g. Find 20% of 80.

     $20\% \text{ of } 80 = \frac{20}{100} \times \frac{80}{1} = \frac{{}^{4}\cancel{20}}{{}_{1}\cancel{5}\,\cancel{100}} \times \frac{{}^{4}\cancel{80}}{1} = 16$

Exercise 3H

Understanding — 1–4 | 3, 4

1 The correct working line to express 42 as a percentage of 65 is:

A $\frac{42}{100} \times 65$ **B** $\frac{65}{42} \times 100$ **C** $\frac{100}{42} \times 65$ **D** $\frac{42}{65} \times 100$

2 The correct working line to find 42% of 65 is:

A $\frac{42}{100} \times 65$ **B** $\frac{65}{42} \times 100$

C $\frac{100}{42} \times 65$ **D** $\frac{42}{65} \times 100$

Hint for Q2: The number with the per cent sign is written with the 100 in the denominator.

3 What is the percentage for:

a a score of 20 out of 40?
b a score of 0 out of 10?
c a score of 50 out of 50?

4 Copy and complete the following sentences.

a Finding 1% of a quantity is the same as dividing the quantity by ______.
b Finding 10% of a quantity is the same as dividing the quantity by ______.
c Finding 20% of a quantity is the same as dividing the quantity by ______.
d Finding 50% of a quantity is the same as dividing the quantity by ______.
e Finding 25% of a quantity is the same as dividing the quantity by ______.

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3H

Fluency

5(½), 6, 7, 8(½) 5–8(½)

Example 22 Expressing one quantity as a percentage of another

Express 34 out of 40 as a percentage.

Solution

$$\frac{34}{40}\times\frac{100}{1}=\frac{17}{\cancel{20}^{1}}\times\frac{\cancel{100}^{5}}{1}$$
$$=\frac{17}{1}\times\frac{5}{1}$$
$$=85$$

So $\frac{34}{40}=85\%$

Explanation

Write as a fraction, with the first quantity as the numerator and second quantity as the denominator. Multiply by 100.
Cancel and simplify.

Now you try

Express 13 out of 20 as a percentage.

5 Express each of the following as a percentage.

- **a** 20 out of 25
- **b** 13 out of 20
- **c** 39 out of 50
- **d** 17 out of 25
- **e** 12 out of 20
- **f** 49 out of 50
- **g** 7 out of 10
- **h** 12 out of 30
- **i** 15 out of 20
- **j** 32 out of 40
- **k** 54 out of 90
- **l** 18 out of 24

Hint for Q5: Multiply by 100.

Example 23 Converting units before expressing as a percentage

Express 60 cents as a percentage of \$5.

Solution

$$\frac{60}{500}\times\frac{100}{1}=\frac{60}{5}$$
$$=12$$

So $\frac{60}{500}=12\%$

So 60 cents is 12% of \$5.

Explanation

Units need to be the same.

Convert \$5 to 500 cents.
Write quantities as a fraction and multiply by 100. Cancel and simplify.

Now you try

Express 700 g as a percentage of 4 kg.

6 Express:

- **a** 40c as a percentage of \$8.
- **b** 50c as a percentage of \$2.
- **c** 3 mm as a percentage of 6 cm.
- **d** 400 m as a percentage of 1.6 km.
- **e** 200 g as a percentage of 5 kg.
- **f** 200 m as a percentage of 8 km.

Hint for Q6: Remember:
1 km = 1000 m
1 cm = 10 mm
1 kg = 1000 g
\$1 = 100 cents

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7 Express each quantity as a percentage of the total.
- **a** 28 laps of a 50 lap race completed.
- **b** Saved $450 towards a $600 guitar.
- **c** 172 fans in a train carriage of 200 people.
- **d** Level 7 completed of a 28 level video game.
- **e** 36 students absent out of 90 total.
- **f** 21 km mark of a 42 km marathon.

Example 24 Finding a certain percentage of a quantity

Find 25% of 48.

Solution	Explanation
$25\%\text{ of }48 = \frac{25}{100} \times \frac{48}{1}$	Write the percentage as a fraction over 100. 'Of' means multiply.
$= \frac{1}{4} \times \frac{48}{1} = 12$	Cancel and simplify.

Now you try

Find 7% of 50.

8 Find:
- **a** 50% of 36
- **b** 10% of 80
- **c** 30% of 500
- **d** 9% of 200
- **e** 20% of 40
- **f** 20% of 60
- **g** 75% of 80
- **h** 25% of 88
- **i** 50% of 25
- **j** 5% of 60
- **k** 5% of 6000
- **l** 1% of 720

Hint for Q8: 50% of $36 = \frac{50}{100} \times \frac{36}{1}$

Problem-solving and reasoning

9(½), 10, 11 | 9(½), 11, 12

9 Find:
- **a** 10% of $750
- **b** 5% of 2 km
- **c** 30% of 150 kg
- **d** 20% of 90 minutes
- **e** 10% of 5 litres
- **f** 25% of one hour
- **g** 50% of $6.50
- **h** 2% of $8
- **i** 7% of $\frac{1}{2}$ kg

Hint for Q9: You may like to change the units in the question to make it easier to work with. 3% of 1 km = 3% of 1000 metres.

Hint for Q9: Remember to put the units in your answer. 10% of $50 $= \frac{10}{100} \times \frac{\$50}{1}$ $= \$5$

10 Copy and complete the table of sporting choices.

Sport	Number of students	Fraction of total	Percentage
Tennis	40		
Golf	30		
AFL	70		
NRL	50		
Swimming	10		
Total	200	1	100%

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3H

11 Calculators make working with percentages easier. Use a calculator to answer these questions.
   a Find 8% of $8.40.
   b Find 13% of 2 km.
   c Find $7\frac{1}{4}$% of $500.
   d Find 24% of 1 hour.
   e Find 31.5% of $45 960.
   f 4% of a class of 25 students are away with the flu. How many students are at school?
   g 49.5% of babies born at the local hospital are girls. Of the 200 born in the month, how many were boys?
   h Sean pays 42% of his $86 400 income in tax. How much is left after he pays his tax?

12 Find:
   a $33\frac{1}{3}$% of 15 litres of orange juice.
   b $66\frac{2}{3}$% of 3000 marbles.
   c $12\frac{1}{2}$% of a $64 pair of jeans.
   d 37.5% of 120 donuts.

Hint for Q12: $33\frac{1}{3}\% = \frac{1}{3}$

Percentages and home loans — 13

13 Most banks require a 10% deposit before lending you any money. Ashlee and Matt have 7% of the $450 000 their home costs.
   a How much do Ashlee and Matt have as their deposit?
   b How much do the banks need them to have?
   c How much more do they need to save?
   d If they get a government grant of $14 000, will they have the 10% needed?

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3I Decreasing and increasing by a percentage

Learning intentions

- To be able to find the new value if an amount is increased or decreased by a percentage.
- To understand that percentage mark-ups and discounts correspond to increasing and decreasing a price by a percentage.
- To understand that GST represents a 10% mark-up.

Key vocabulary: reduction, discount, mark-up, profit, loss, selling price, cost price

Percentages are used every day, often when dealing with money. In the world of finance, calculations and percentage increases and decreases are commonplace.

The original amount of something can be thought of as 100%.

Lesson starter: What does it mean?

In pairs, answer the following:

- What does it mean to buy a pair of shoes 'on sale'?
- What does it mean if the sale is '20% off'?
- What does it mean to 'pay the marked price'?
- What does it mean to buy an item on sale?
- Is $\$10$ off better than 10% off? Discuss.
- When would you pay more than the original price?
- What current sales are being advertised in today's paper?

Pie charts use percentages of $360°$ to divide up a circle.

Key ideas

- To increase by a given percentage:
  - find the percentage of the amount
  - add this amount to the original.
- To decrease by a given percentage:
  - find the percentage of the amount
  - subtract this amount from the original.
- Key words:
  - Decrease: reduction, discount, sale, percentage off, loss
  - Increase: mark-up, profit
  - Selling price = cost price + profit *or* cost price − loss
  - GST: Goods and Services Tax (In Australia this is a 10% mark-up.)

Exercise 3I

Understanding 1–3 3

1 Decide if each of these shows an increase or a decrease.
  a Mark's $\$1650$ return airfare to Los Angeles was reduced by 10%.
  b Sonya made 15% profit when she sold her house.
  c The shop discounted all of its computers by 10%.
  d Thomas received a pay rise of 5% on his wage of $\$570$ per week.
  e A tax of 15% is added to the cost of everything in the United Kingdom.

2 Add or subtract these percentages.
  a $100\% + 20\%$ b $100\% + 15\%$ c $100\% - 10\%$ d $100\% - 15\%$

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3I

3 Calculate the new price when:
- **a** an item marked at \$15 is discounted by \$3.
- **b** an item marked at \$25.99 is marked up by \$8.
- **c** an item marked at \$17 is reduced by \$2.50.
- **d** an item marked at \$180 is increased by \$45.

Fluency

4–6(½), 7, 8(½), 9 | 4–5(½), 8(½), 10

Example 25 Increasing a value by a percentage

Find the new value when \$160 is increased by 40%.

Solution	Explanation
$40\% \text{ of } 160 = \frac{40}{100} \times \frac{160}{1} = \$64$	Calculate 40% of \$160. Cancel and simplify.
$\text{New price} = \$160 + \$64$ $= \$224$	New price = original price + increase

Now you try

Find the new value when \$85 is increased by 30%.

4 Find the new value when:
- **a** \$400 is increased by 10%.
- **b** \$240 is increased by 10%.
- **c** \$250 is increased by 10%.
- **d** \$700 is increased by 20%.
- **e** \$500 is increased by 1%.
- **f** \$800 is increased by 25%.
- **g** \$84 is increased by 25%.
- **h** \$90 is increased by 50%.

Hint for Q4: Add the increase to the original amount.

Example 26 Decreasing a value by a percentage

Find the new value when \$63 is decreased by 20%.

Solution	Explanation
$20\% \text{ of } \$63 = \frac{20}{100} \times \frac{63}{1} = \$12.60$	Calculate 20% of \$63. Cancel and simplify.
$\text{New price} = \$63 - \$12.60$ $= \$50.40$	New price = original price − decrease

Now you try

Find the new value when \$120 is decreased by 15%.

5 Find the new value when:
- **a** \$400 is decreased by 10%.
- **b** \$240 is decreased by 10%.
- **c** \$250 is decreased by 10%.
- **d** \$90 is decreased by 20%.
- **e** \$200 is decreased by 15%.
- **f** \$840 is decreased by 25%.
- **g** \$1000 is decreased by 50%.
- **h** \$60 is decreased by 15%.

6
- **a** Find 8% of \$2500.
- **b** Increase \$2500 by 8%.
- **c** Decrease \$2500 by 8%.

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Example 27 Calculating discounts

Find the cost of an $860 television that has been discounted by 25%.

Solution	Explanation
Discount = 25% of \$860 $= \frac{25}{100} \times \frac{860}{1} = \$215$	Calculate 25% discount. Cancel and simplify.
Selling price = \$860 − \$215 = \$645	Selling price = cost price − discount

Now you try

Find the cost of a $450 table that has been discounted by 40%.

7 Find the cost of the following.
- **a** A $600 television that has been discounted by 20%.
- **b** A $150 lipstick that has been reduced by 15%.
- **c** A $52 jumper that has depreciated by 25%.

8 Calculate the selling prices of the following items if they are to be reduced by 25%.
- **a** $16 thongs
- **b** $32 sunhat
- **c** $50 sunglasses
- **d** $85 bathers
- **e** $130 boogie board
- **f** $6.60 surfboard wax

Example 28 Calculating mark-ups

Find the cost of a microwave oven that was originally $250 then marked up by 12%.

Solution	Explanation
Mark-up = 12% of \$250 $= \frac{12}{100} \times \frac{250}{1} = \$30$	Calculate 12% of \$250. Cancel and simplify.
Selling price = \$250 + \$30 = \$280	Selling price = cost price + mark-up

Now you try

Find the cost of a toaster that was originally $64 then marked up by 15%.

9 Find the cost of the following.
- **a** An $80 framed Pink poster that has been marked up by 30%.
- **b** A $14 meal that has been increased by 10%.
- **c** A $420 stereo that has been marked up by 50%.

10 Calculate the selling prices of the following items if they need to have 10% GST added to them.
- **a** $35 T-shirt
- **b** $75 backpack
- **c** $42 massage
- **d** $83 fishing rod
- **e** $52.50 toaster
- **f** $149.99 cricket bat

Hint for Q10: Remember 10% GST adds/increases the price of an item.

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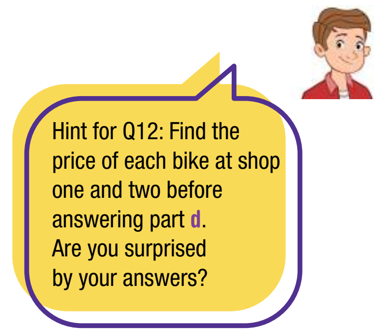
3I

Problem-solving and reasoning 11 11, 12

11 Answer the following problems involving percentages.
- **a** Anne's annual salary was $86 000. Her new salary is 5% more. What is Anne's new salary?
- **b** The state government increases the cost of a $9.60 train trip by 5%. What is the new fare?
- **c** A car worth $47 000 dropped in value by 20% during the year. What is the car now worth?
- **d** The 10% GST needs to be added to the cost of a meal. What does a $74 meal cost once the GST is added in?
- **e** Tax of 40% reduces Saul's wage of $1600. What amount does Saul receive?
- **f** Sally makes a 24% profit on her house. She paid $500 000. What did she sell it for?

12 Two shops advertise the same bike. Both have a recommended retail price of $1800. Shop one offers a 10% discount. Shop two offers $200 off all bikes.
- **a** How much discount does shop one offer on this bike?
- **b** How much do you pay for the bike at each shop?
- **c** Which shop would you recommend and why?
- **d** If the same deal applies to each of the following bikes, would you still buy it from the same shop?
  - **i** $2000 bike
  - **ii** $2200 bike

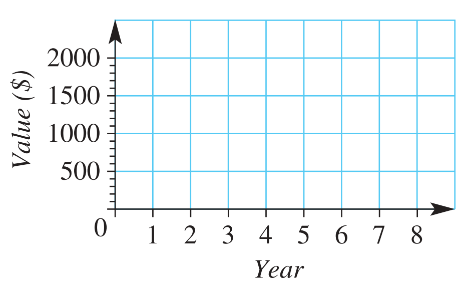


Depreciation — 13

13 The word *depreciation* is used when the value of an item, such as a car, boat or a set of golf clubs, reduces in value each year.
- **a** Rick's set of golf clubs worth $2000 depreciates at a rate of $250 a year.
  - **i** Copy and complete the table showing how the value of the clubs changes over time.

End of year	0	1	2	3	4	5	6	7	8
Value ($)	2000	1750							

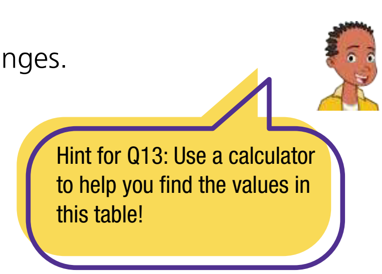
  - **ii** Draw up a set of axes (like those shown below) and graph the values shown in the table.



  - **iii** What shape is your graph?
  - **iv** After how many years is the value of the clubs zero?
- **b** Rick's wife has a set of golf clubs, also valued at $2000, which depreciate at $12\frac{1}{2}\%$ each year.
  - **i** Complete a similar table showing how the value of her clubs changes.

End of year	0	1	2	3	4
Value ($)	2000	1750	1531.25		

  - **ii** Will her clubs ever be worthless?



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3J Calculating percentage change

Learning intentions

- To understand that profit and loss represent the difference between the selling price and cost price of an item.
- To be able to calculate the percentage change (increase or decrease) when prices are increased or decreased.

Key vocabulary: percentage change, percentage profit, percentage loss, profit, loss, selling price, cost price

When selling something, everyone likes to make a profit. This is when you sell it for more than you paid for it.

Unfortunately, people often do the opposite and make a loss.

The percentage change depends on what the item is originally worth. For example:

Car bought for $\$1000$
Car sold for $\$200$

Loss $\$800$, percentage loss 80%

Car bought for $\$16\,000$
Car sold for $\$15\,200$

Loss $\$800$, percentage loss 5%

Lesson starter: Hang on!

Discuss how you could check if the correct price for the jeans had been paid.

Key ideas

- **Profit** = **selling price** − **cost price**
- **Loss** = cost price − selling price
- Calculating a percentage change involves the technique of expressing one quantity as a percentage of another (see **Section 3G**).
  - **Percentage change** $= \dfrac{\text{change}}{\text{original value}} \times 100$
  - **Percentage profit** $= \dfrac{\text{profit}}{\text{original value}} \times 100$
  - **Percentage loss** $= \dfrac{\text{loss}}{\text{original value}} \times 100$

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Exercise 3J

Understanding

1–4 | 4

1 Decide whether each of the following represents a profit or a loss.

a

bought = $250 000
sold = $280 000

b

bought = $795
sold = $210

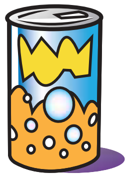
c

bought = $1200
sold = $500

d

bought = $2000
sold = $4500

e



bought = $1.40
sold = $3.20

2 Calculate the profit made in each of the following situations.

a Cost price = $14, Sale price = $21

b Cost price = $75, Sale price = $103

c Cost price = $25.50, Sale price = $28.95

d Cost price = $499, Sale price = $935

Hint for Q2:
Profit = selling price − cost price.

3 Calculate the loss made in each of the following situations.

a Cost price = $22, Sale price = $9

b Cost price = $92, Sale price = $47

c Cost price = $71.10, Sale price = $45.20

d Cost price = $1121, Sale price = $874

4 Which of the following is the correct formula for working out percentage change?

A $\%\text{change} = \dfrac{\text{change}}{\text{original value}}$

B $\%\text{change} = \dfrac{\text{original value}}{\text{change}} \times 100$

C $\%\ \text{change} = \text{change} \times 100\%$

D $\%\text{change} = \dfrac{\text{change}}{\text{original value}} \times 100$

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Fluency

5, 6(½), 7, 8 | 5, 6(½), 8, 9

Example 29 Calculating percentage change: profit

Calculate the percentage profit when \$25 becomes \$32.

Solution

Profit = \$7

$$\%\text{Profit} = \frac{7}{25} \times \frac{100}{1}\%$$
$$= 28\%$$

Explanation

This is percentage **profit** because it was sold for more than the original \$25.

Profit = \$32 − \$25

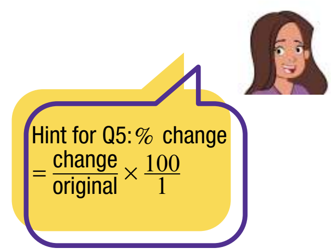
$$\text{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$$

Now you try

Calculate the percentage profit when a second-hand toy purchased for \$36 is sold for \$45.

5 Find the percentage profit when:

- **a** \$20 becomes \$36
- **b** \$10 becomes \$13
- **c** \$40 becomes \$50
- **d** \$25 becomes \$30
- **e** \$12 becomes \$20
- **f** \$8 becomes \$11
- **g** \$10 becomes \$15
- **h** \$6 becomes \$12



Example 30 Calculating percentage change: loss

Calculate the percentage loss when \$60 becomes \$48.

Solution

Loss = \$12

$$\%\text{Loss} = \frac{12}{60} \times \frac{100}{1}$$
$$= 20\%$$

Explanation

This is percentage **loss** because it was sold for less than the original \$60.

Loss = \$60 − \$48

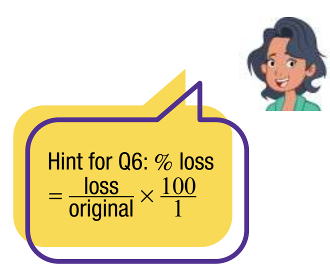
$$\text{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100$$

Now you try

Calculate the percentage loss when a \$140 chair is sold for \$84.

6 Find the percentage loss when:

- **a** \$40 becomes \$30
- **b** \$25 becomes \$21
- **c** \$6 becomes \$3
- **d** \$8 becomes \$2
- **e** \$12 becomes \$8
- **f** \$10 becomes \$9
- **g** \$25 becomes \$20
- **h** \$20 becomes \$18



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3J

Example 31 Solving worded problems

Ross buys a ticket to a concert for $125, but is later unable to go. He sells it to his friend for $75. Calculate the percentage loss Ross made.

Solution

Loss = $125 − $75 = $50

$\% \text{ Loss} = \frac{50}{125} \times \frac{100}{1}$

$= 40\%$

Ross made a 40% loss on the concert ticket.

Explanation

Loss = Cost price − Selling price

$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100$

Now you try

After 5 years an oil painting's value increases from $2000 to $4500. Calculate the percentage increase in its value during this time.

7 Copy and complete the tables below.

a

Cost price ($)	Selling price ($)	Profit ($)	% profit
4	5		
10	12		
24	30		
100	127		

Hint for Q7: $\%\text{profit} = \frac{\text{profit}}{\text{cost price}} \times 100$

b

Cost price ($)	Selling price ($)	Loss ($)	% loss
10	7		
16	12		
50	47		
100	93		

Hint for Q7: $\%\text{loss} = \frac{\text{loss}}{\text{cost price}} \times 100$

8 Find the percentage change (increase or decrease) when:
- **a** 15 kg becomes 18 kg.
- **b** 18 kg becomes 15 kg.
- **c** 4 kg becomes 24 kg.
- **d** 12 kg becomes 30 kg.

9 Find the percentage change in population when:
- **a** a town of 4000 becomes a town of 5000.
- **b** a city of 750 000 becomes a city of 900 000.
- **c** a country of 5 000 000 becomes a country of 12 000 000.

Problem-solving and reasoning

10, 11 | 11–13

10 Gari buys a ticket to a concert for $90, but is unable to go. He sells it to his friend for $72. Calculate the percentage loss Gari made.

11 Xavier purchased materials for $48 and made a dog kennel. He later sold the dog kennel for $84.
- **a** Calculate the profit Xavier made.
- **b** Calculate the percentage profit Xavier made.

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12 Gemma purchased a \$400 foal, which she later sold for \$720.
  a Calculate the profit Gemma made.
  b Calculate the percentage profit Gemma made.

13 Lee-Sen purchased a \$5000 car, which she later sold for \$2800.
  a Calculate the loss Lee-Sen made.
  b Calculate the percentage loss Lee-Sen made.
  c What should Lee-Sen sell the car for to make a 10% profit?

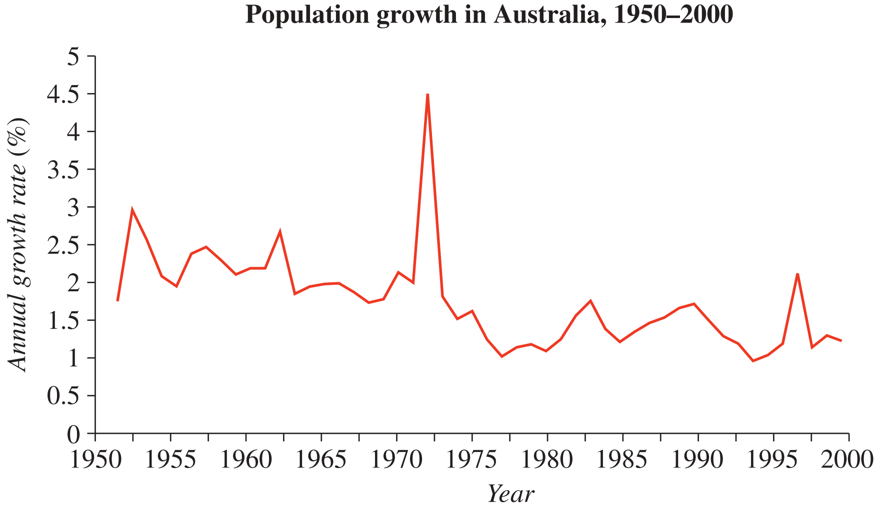
Growth rate for Australia — 14

14 The Australian Bureau of Statistics tracks the population growth of the country and of each individual state and territory.
  a Copy and complete the table below, rounding the % change to one decimal place.

Place	March 2016	Change in the past 12 months	% change
NSW	7 704 300	103 200	
VIC	6 039 100	114 900	
QLD	4 827 000	61 800	
WA	2 613 700	29 800	
SA	1 706 500	9 700	
TAS	518 500	2 200	
ACT	395 200	5 000	
NT	244 000	1 000	
AUSTRALIA	24 048 300	327 600	

Hint for Q14: Use a calculator to help you with this question.

  b Research the current growth rate of Australia and one other country of your choice.



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3K Solving percentage problems using the unitary method

Learning intentions

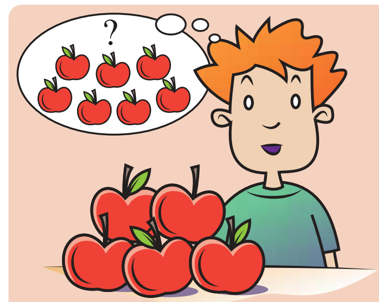
- To understand that the unitary method involves finding the value of 'one unit' as an intermediate step.
- To be able to use the unitary method to find a quantity when only a percentage is known.
- To be able to use the unitary method to find a new percentage when a different percentage is known.
- To be able to apply the unitary method to find the original price when a price has been increased or decreased by a percentage.

Key vocabulary: unitary method, percentage, discount, sale price, original price

You probably did problems like this in primary school: 'If five apples cost $6, what is the cost of 7 apples?'

With questions like this, we find the cost of one apple first.

This is helpful in percentages as well. Once we know what 1% is worth, we can find any percentage amount. This is called the **unitary method**.



If 5 apples cost $6 then one apple costs $1.20.

Lesson starter: Using the unitary method

- Four tickets to a concert cost $100. What does one ticket cost? How much will three tickets cost?
- Ten workers can dig 40 holes in an hour. How many can one worker dig in an hour? How many holes can seven workers dig in an hour?
- Six pizzas cost $54. What does one pizza cost? How much would ten pizzas cost?
- If eight pairs of socks cost $64, how much would 11 pairs of socks cost?
- Five passionfruit cost $2.00. How much will nine passionfruit cost?
- If a worker travels 55 km in 5 trips from home to the worksite, how far will they travel in 7 trips?

Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to answer the question.
- When dealing with **percentages**, finding 'one unit' means finding one per cent (1%).
- Once the value of 1% of an amount is known, it can be multiplied to find the value of any desired percentage.

Exercise 3K

Understanding

1–4 | 3, 4

1 a What do you divide by to go from 8% to 1%?
 b What do you divide by to go from 25% to 1%?
 c What do you multiply by to go from 1% to 100%?
 d What do you multiply by to go from 1% to 50%?

2 If 1% of an amount is $3, what is:
 a 2% of the amount?
 b 10% of the amount?
 c 100% of the amount?

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3 If 1% of an amount is $\$8$, what is:

a 10% of the amount? **b** 100% of the amount?

4 Copy and complete:

If 4% of an amount $= \$16$

then 1% of an amount $= \square$

and 100% of an amount $= \square$

Fluency

5, 6, 7(½) | 5(½), 7(½)

Example 32 Using the unitary method to find the full amount

If 8% of an amount of money is $\$48$, what is the full amount of money?

Solution

$\div 8$ 8% of amount is \$48 $\div 8$

1% of amount is \$6

$\times 100$ 100% of amount is \$600 $\times 100$

Full amount of money is $\$600$.

Explanation

Remember to find 1% first.

Divide by 8 to find the value of 1% $(48 \div 8 = 6)$.

Multiply by 100 to find the value of 100% $(6 \times 100 = 600)$.

Now you try

If 15% of an amount is $\$45$, what is the full amount?

5 Calculate the full amount of money for each of the following.

a 3% of an amount of money is $\$27$.

b 5% of an amount of money is $\$40$.

c 12% of an amount of money is $\$132$.

d 60% of an amount of money is $\$300$.

e 8% of an amount of money is $\$44$.

f 6% of an amount of money is $\$15$.

Hint for Q5: First find the value of 1%.

Example 33 Using the unitary method to find a new percentage

If 11% of the food bill is $\$77$, how much is 25% of the food bill?

Solution

$\div 11$ 11% of food bill is \$77 $\div 11$

1% of food bill is \$7

$\times 25$ 25% of food bill is \$175 $\times 25$

Explanation

Find 1% first.

Divide by 11 to find the value of 1% $(77 \div 11 = 7)$.

Multiply by 25 to find the value of 25% $(7 \times 25 = 175)$.

Now you try

If 20% of a salary bonus is $\$6000$, how much is 75% worth?

6 If 4% of the total bill is $\$12$, how much is 30% of the bill?

Hint for Q6 and Q7: First find the value of 1% of the bill.

7 Calculate:

a 20% of the bill, if 6% of the total bill is $\$36$.

b 80% of the bill, if 15% of the total bill is $\$45$.

c 3% of the bill, if 40% of the total bill is $\$200$.

d 7% of the bill, if 25% of the total bill is $\$75$.

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3K

Problem-solving and reasoning

8, 9(½), 10 | 9(½), 10–12

Example 34 Using the unitary method to find the original price

A pair of shoes has been discounted by 20%. If the sale price was $\$160$, what was the original price of the shoes?

Solution	Explanation
Only paying 80% of original price:	20% discount, so paying $(100 - 20)\%$.
$\therefore 80\%$ of original price is $\$160$	We pay 80% after the 20% discount.
$\therefore 1\%$ of original price is $\$2$	Divide by 80 to find the value of 1% $(160 \div 80 = 2)$.
$\therefore 100\%$ of original price is $\$200$	Multiply by 100 to find the value of 100%
The original price of the shoes was $\$200$.	$(2 \times 100 = 200)$.

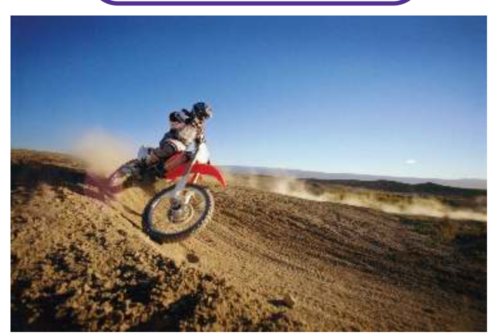
Now you try

A new bed was discounted by 30%. If the sale price was $\$294$, what was the original price of the bed?

8 A necklace in a jewellery store has been discounted by 20%. If the sale price is $\$240$, what was the original price of the necklace?

Hint for Q8: $100\% - 20\%$ $= 80\%$

9 Find the original price of the following items.



- **a** A pair of jeans discounted by 40% has a sale price of $\$30$ (you pay 60%).
- **b** A hockey stick discounted by 30% has a sale price of $\$105$ (you pay 70%).
- **c** A second-hand computer discounted by 85% has a sale price of $\$90$ (you pay 15%).
- **d** A second-hand textbook discounted by 80% has a sale price of $\$6$.
- **e** A standard rose bush discounted by 15% has a sale price of $\$8.50$.
- **f** A motorbike discounted by 25% has a sale price of $\$1500$.

10 Once the GST of 10% is added to a bill, the price is 110%. If the price of a meal at a cafe, including the GST, is $\$55$, how much GST is paid?

Hint for Q10:
$110\% =$
$1\% =$
$10\% =$

11 A pair of jeans, including 10% GST, comes to $\$88$. What is the cost without the GST?

12 If 22% of an amount is $\$8540$, which of the following would give the value of 1% of the amount?

A $\$8540 \times 100$ **B** $\$8540 \div 100$ **C** $\$8540 \times 22$ **D** $\$8540 \div 22$

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Real-life receipts — 13

13 Here are three real-life receipts (with the names of shops changed). GST rate is 10%.
Answer the questions below based on each one.

TAX INVOICE
SUPERBARN
SUPERBARN GYMEA

Description	Total $
O/E PASO TACO KITS 290GM	6.09
TOMATOES LARGE KILO 0.270kg @$4.99/kg	1.35
WATERMELON SEEDLESS WHOLE KILO 1.675kg @$2.99/kg	5.01
LETTUCE ICEBERG EACH	2.49
*PAS M/MALLOWS 250GM	1.89
SubTotal	$16.83
Rounding	$0.02
TOTAL (Inc GST) 5 Items	$16.85
Cash Tendered	$20.00
Change Due	$3.15
GST Amount	$0.17

* Signifies item(s) with GST

Thank you for shopping at Superbarn

GYMEA FRUIT MARKET
HAVE A NICE DAY
DATE 05/07/2011 TUES TIME 11:21

0.090 KG @ $14.99/kg	
BANANA SUGAR	$1.35
SL MUSHROOM	$2.49
PISTACCHIO 11	$6.00
ROUND	$0.01
TOTAL	$9.85
CASH	$9.85
TAX 1	$0.55

XMART
CUSTOMER RECEIPT TAX INVOICE
13/07/11 15:1

*JUNGLE JUMP BALL	6.00
*CR COLOUR SET CARDS	10.00
*CR GLOW STATION	10.00
*STAR OTTOMAN PINK	12.00
*JUNGLE HIDEAWAY	12.00
*LP AIRPORT	29.00
*MY OWN LEAPTOP 2 @ 35.00	70.00
TOTAL	149.00
CASH TENDER	150.00
CHANGE	1.00

* TAXABLE ITEMS

PLEASE RETAIN THIS RECEIPT/TAX INVOICE AS PROOF OF PURCHASE

WE NOW TRADE
24 HOURS A DAY, 7 DAYS A WEEK

Superbarn

a How much was spent at Superbarn?
b How many kilograms of tomatoes were bought?
c Which item included the GST, and how do you tell by looking at the receipt?
d What is the cost of the item if the GST is not included?

Gymea Fruit Market

a What was the cost of bananas per kilogram?
b On what date was the purchase made?
c What does ROUND mean?
d What was the total paid for the items?
e How much tax was included in the bill?
f What percentage of the bill was the tax?

Xmart

a How many toys were purchased?
b What was the cost of the most expensive item?
c Which of the toys attracted GST?
d How much GST was paid in total?
e What percentage of the total bill was the GST?

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Maths@Work

Owner and manager of a fruit and vegetable shop

Owning and running your own business requires hard work and long hours. The skills you need are varied, from being well organised and having good communication skills to being able to manage staff and stock. Perseverance is also a requirement, as many businesses fail within their first three years.

Skill with numbers is important for the day-to-day running of a fruit and vegetable shop to ensure a profit. Stock needs to be ordered and kept fresh. Enough stock needs to be sold to cover wages, store rental, delivery costs and, if possible, also some profit. Prices must be adjusted for seasonal fluctuations as well as unexpected costs.

1 Imagine that you are starting up a fruit and vegetable shop in the town or suburb where you live.
- **a** List some vegetables and fruits that could be supplied from your local markets or farms.
- **b** What costs would your business have, other than buying produce?
- **c** What do you think would be the main challenges to successfully running your shop?

2 When ordering or buying produce from markets, prices per kg or per item are usually stated. Calculate the unit price or cost per kilogram for each of the following.
- **a** 15 kg bag of potatoes costs \$14.55
- **b** 25 kg box of apples costs \$34.50
- **c** Box of lettuce containing 20 heads of lettuce costs \$18
- **d** 8 kg bag of carrots costs \$6.40
- **e** 20 kg box of navel oranges costs \$25.40

3 Fresh produce is generally bought at a wholesale price from suppliers and sold at a higher retail price to cover costs and provide a profit to the store owner. Find the retail price/kg of the items below, given the retail price is 250% of the wholesale price. Round to the nearest cent.
- **a** Fresh asparagus wholesale price is \$1.42 per kg
- **b** Beans wholesale price is \$2.18 per kg
- **c** Broccoli wholesaling at \$2.39 per kg
- **d** Apples 2.5 kg bag wholesaling at \$1.83
- **e** Celery wholesaling at 85 c per kg

4 Find the cost of an individual piece of fruit using the information in this table. Round to the nearest cent.

	Fruit	Cost per kg	Average number of pieces per kg	Cost per piece
a	Gala apples	\$5.00	6	
b	Red delicious apples	\$4.99	5	
c	Bartlett pears	\$3.99	5	
d	Apricots	\$7.95	12	
e	Peaches	\$10.99	9	

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5 Find the cost of the following order for a customer.

$\frac{1}{2}$ kg of pears at \$3.99/kg	2 heads of lettuce at \$2.50 each
1.2 kg of apples at \$5/kg	1 avocado at \$3.50
2 kg bag of potatoes at \$2.99/kg	2.2 kg of tomatoes at \$5.99/kg

6 Compare the cost of 100 g of apples for each option **A** and **B** and state which is the better buy.
A A tray of Pink Lady apples with a weight of 600 grams for \$2.94, or
B 0.75 kg of loose Pink Lady apples for \$3.30.

7 A restaurant in Daintree, north Queensland, ordered the following exotic fruits from a nearby tropical fruit shop. Calculate the total cost of this order including a \$45 packing and delivery fee.

Quantity	Fruit	Price
12	Dragon fruits	\$5.49 each
$2\frac{3}{4}$ kg	Lychees	\$19.90/kg
1	Jackfruit weighing 15.38 kg	\$3.45/kg
16	Star apple fruits	\$2.35 each
4	Custard apples	\$4.55 each
100 g	Soursop dried leaves	\$25 per $\frac{1}{4}$ kg
1900 g	Ice-cream beans	\$2.54 per 100 g
$3\frac{1}{2}$ kg	Chocolate pudding fruits	\$13.99/kg
3.8 kg	Purple Mangosteens	\$10.99/kg


Dragon fruit are very nutritious.

Using technology

8 Many people work overseas and wish to compare living costs. Set up the following Excel spreadsheet to convert American produce prices per lb (pound) to Australian dollars (AUD) per kg. We use the letters USD for American currency.

a Use these conversion factors:
- In cell B2 enter 2.2 as there are 2.2 lb per kg.
- In cell C2 enter 0.75 (i.e. 1 AUD = 0.75 USD) or use the current exchange rate.
- In cell D2 enter the formula = 1/C2. This gives the number of AUD for USD.

b In column C, multiply the prices in USD/lb by 2.2 to give USD/kg. Use \$ signs to anchor the B2 cell value as in the Hint.

c In column D, convert USD to AUD. Multiply by cell D2 (i.e. the number of AUD for 1 USD).

Hint for Q8:
- Formula cell C7 = B7 ∗ \$B\$2
- Formula cell D7 = C7 ∗ \$D\$2
- To fill formulas down a column, drag down the 'fill handle'.

	A	B	C	D
1	Conversion factors	? lb in 1 kg	1 AUD to ? USD	1 USD to ? AUD
2		2.2		
3				
4	Converting USA produce prices to Australian dollars/kg			
5	Produce: fruit or vegetable	USA prices		Australian price
6		USD per lb	USD per kg	AUD per kg
7	Avocado	\$2.20		
8	Bananas	\$0.86		
9	Bok choy	\$2.18		
10	Cauliflower	\$2.49		
11	Granny Smith apples	\$1.95		
12	Mangoes	\$1.71		
13	Oranges - navel	\$1.37		
14	Potatoes	\$1.24		
15	Papaya	\$1.59		

d Select any five of the above items and compare the converted Australian prices/kg to current Australian online supermarket prices/kg. What observation can you make? Give some possible reasons for the difference in prices.

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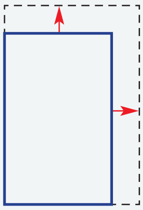
Modelling

Upsized phone screen



The Samsum phone company is considering making a new upsized phone screen compared to one of its smaller models. The smaller model has dimensions 6.3 cm by 11.3 cm.

Market research has indicated that a total increase in screen area of 30% should be enough to meet the demand in the market. To increase the screen size, however, the length and the width need to increase by the same percentage, so the length and width are in the same proportion.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

1 Preliminary task

a Determine the area of the original Samsum phone screen with a length of 11.3 cm and a width of 6.3 cm.

b Find the length and the width of an upsized phone if the dimensions (length and width) are increased by 10%.

c Find the area of an upsized phone screen if the dimensions are increased by 10%.

d What is the change in area between the new and old screens when the dimensions are increased by 10%?

e Find the percentage change in the area of the up-sized phone if the dimensions are increased by 10%.

Remember that: *Percentage change* $= \dfrac{\textit{change}}{\textit{original value}} \times 100\%$

2 Modelling task

Formulate

a The problem is to determine the percentage increase which should be applied to the length and width to achieve a 30% increase in area of the phone screen.

Write down all the relevant information that will help solve this problem.

b Make an accurate drawing of the original Samsum phone screen with dimensions 6.3 cm by 11.3 cm. On your diagram include an illustration of how the dimensions might be increased.

Solve

c Make the following calculations to find the planned, up-sized screen area:

- **i** find the original screen area
- **ii** find 30% of this area
- **iii** increase the original screen area by this change to give the new, up-sized screen area.

d Calculate the dimensions and screen area of an upsized phone if the original Samsum screen's dimensions are increased by the following percentages. Round your answers to three decimal places.

- **i** 5%
- **ii** 15%
- **iii** 25%

e Determine which of the above percentage dimension increases leads to an increase of *more than* 30% in total area. Justify your answer by calculating percentage increases in screen area for each set of new dimensions that you calculated in part **d** above.

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Evaluate and verify

f Examine your results from parts **d** and **e** and use trial and error to determine the required percentage increase in phone dimensions to achieve a 30% increase in area. Answer correct to one decimal place. Remember that the length and the width need to increase by the same percentage.

Communicate

g Summarise your results in a table like the one below and describe any key findings.

Percentage increase of dimensions	Upsized length	Upsized Height	Upsized screen area	Percentage increase of screen area
5%				
15%				
25%				

3 Extension question

One sales executive at Samsum says that to increase the area by 30% you should increase the dimensions by 30%. Demonstrate that the sales executive is wrong.

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Paying for Australia

Key technology: Spreadsheets

As at 2023, Australia's total population is about 25.5 million, its total government expenditure is about $600 billion and the total taxation revenue is about $550 billion. About half of Australia's government revenue comes from personal income tax with most people paying tax based on the following tax table.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

1 Getting started

For this activity, assume the following facts about Australia.

- The current population is 25.5 million, including 20 million adults.
- The total government expenditure is $600 billion.
- The average personal income tax is $12 500 per adult.
- The total revenue from other taxes (not income tax) is $300 billion.

a What is the total revenue to the Australian government from personal income tax paid by adult Australians?

b By combining the total personal income tax and other taxes, does Australia earn enough revenue to pay for its total expenditure? Give reasons.

c Use the tax table above to calculate the total tax payable on the following taxable incomes. (We will ignore the Medicare levy for now.)

  i $30 000
  ii $100 000
  iii $150 000

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2 Using technology

The following spreadsheet is a simple balance sheet for the Australian economy using the information from part 1.

	A	B	C	D	E	F
1	Paying for Australia		Average adult tax paid	12500		
2						
3		Population increase (%)		Other income increase (%)	Expenditure increase (%)	
4		0.5		1	1	
5						
6	Year	Population	Income tax	Other income	Expenditure	Balance
7		million	$billions	$billions	$billions	$billions
8						
9	2023	20	=B9*D$1*1000000/1000000000	300	600	=D9+C9-E9
10	=A9+1	=B9*(1+B$4/100)		=D9*(1+D$4/100)	=E9*(1+E$4/100)	

a Enter all the information into a spreadsheet and answer the following.
- **i** The total revenue from personal income tax is calculated in C9. From which cell does the formula receive the average tax paid by each adult?
- **ii** The population increase year by year is currently set at 0.5%. Explain how the formula in cell B10 uses this information to calculate the correct total figure after the increase.
- **iii** Explain how the formulas in cells D10 and E10 work.

b Fill down at cells A10, B10, C9, D10, E10 and F9 to year 2033. What do you notice about the overall financial balance over this time?

c Change the average amount of income tax paid by Australian adults in cell D1 to $13 000. Does this improve Australia's balance sheet?

d Let's now assume that the average Australian taxable income is $75 000.
- **i** Use the tax table on the previous page to calculate the average tax paid using this taxable income.
- **ii** Use your result from part **d i** in your spreadsheet to update cell D1. Does this improve Australia's balance sheet?

3 Applying an algorithm

a We will systematically apply this algorithm to explore what average taxable income is needed to provide Australia with enough tax revenue. Start with $I = 50\,000$.
- Step 1: Calculate the tax paid by an Australian earning $$I$.
- Step 2: Enter this tax amount into cell D1 of your spreadsheet.
- Step 3: Record Australia's balance at year 2033.
- Step 4: Increase I by 5000.
- Step 5: Return to Step 1.

b To the nearest $5000, what average taxable income provides Australia with a positive balance at year 2033?

c Change the rate of Australia's population growth in cell B4. Explore how this change affects the financial balance at year 2033 compared to the original 0.5% rate.

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Puzzles and games

1 Write down four decimals that when rounded to 2 decimal places give 2.67.

2 Jill has five coins in her pocket: a \$2 coin, \$1 coin, 50 cent piece, 20 cent piece, and one 10 cent coin. If Jill chooses just two coins from her pocket without looking at them, or noticing their size or shape, how many different amounts could she arrive at?

3 Write one half in ten different ways.

4 Complete these magic squares. All rows, columns and the two diagonals add up to the same total.

a

$\frac{4}{3}$		
1	$1\frac{2}{3}$	
$2\frac{2}{3}$		

b

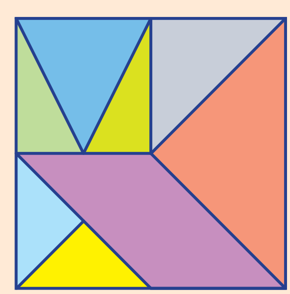
$\frac{5}{3}$		
	$\frac{11}{6}$	$2\frac{1}{6}$
		2

5 A tangram consists of seven geometric shapes (tans) as shown on the right. The tangram puzzle is precisely constructed using vertices, midpoints and straight edges.

a Write each of the separate tan pieces as a percentage, a fraction and a decimal amount of the entire puzzle.

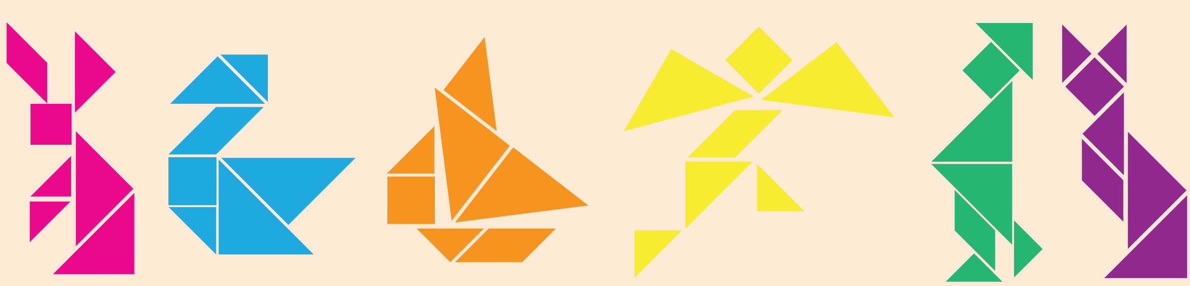
b Check your seven tans add up to a total of 100%.

c Starting with a square, make a new version of a 'modern' tangram puzzle. You must have at least six pieces in your puzzle. An example of a modern puzzle is shown below.



d Write each of the separate pieces of your new puzzle as a percentage, a fraction and a decimal amount of the whole puzzle.

e Separate pieces of tangrams can be arranged to make more than 300 creative shapes and designs, some of which are shown. You may like to research tangrams and attempt to make some of the images.



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Chapter summary

Fractions

What is a fraction?

A fraction is part of a whole. $\frac{1}{4}$

$\frac{1}{4}$ — 1: numerator (parts from the whole); 4: denominator (parts in the whole)

Simplifying fractions

Simplify fractions by dividing the numerator and denominator by their highest common factor..

$\frac{8}{20} = \frac{4 \times 2}{4 \times 5} = \frac{2}{5}$

Equivalent fractions

- represent the same amount, they are the same decimal.

$\frac{2}{4} = \frac{1}{2} = 0.5$

- create equivalent fractions by multiplying or dividing the top (numerator) and bottom (denominator) by the same number.

$\frac{2}{5} \xrightarrow{\times 3} \frac{6}{15}$

Mixed and improper fractions

Mixed $1\frac{1}{3} \xrightarrow{\frac{1 \times 3 + 1}{3}}$ Improper $\frac{4}{3}$

Improper $\frac{4}{3} \xrightarrow{4 \div 3 = 1 \text{ rem } 1}$ Mixed $1\frac{1}{3}$

Adding or subtracting fractions

To add or subtract fractions you should use the lowest common denominator.

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

Negative fractions

$\frac{5}{6} + \left(-\frac{1}{6}\right)$ Subtract opposite $= \frac{5}{6} - \frac{1}{6}$

$\frac{5}{6} - \left(-\frac{1}{6}\right)$ Add opposite $= \frac{5}{6} + \frac{1}{6}$

Reciprocal

To take the reciprocal of a proper or improper fraction is to turn it upside down (flip).

Multiplying fractions

- use proper or improper fractions $\frac{3}{2} \times \frac{1}{4}$
- cancel any numerator with any denominator
- multiply the numerators $\frac{3 \times 1}{2 \times 4}$
- multiply the denominators
- simplify $\frac{3}{8}$

Negative fractions

$\frac{2}{3} \times \left(-\frac{1}{4}\right)$
Different signs → Negative answer
$= -\frac{\cancel{2}^{1}}{\cancel{12}_{6}}$
$= -\frac{1}{6}$

$-\frac{5}{8} \times \left(-\frac{2}{5}\right)$
Same signs → Positive answer
$= \frac{\cancel{10}^{1}}{\cancel{40}_{4}}$
$= \frac{1}{4}$

$-\frac{5}{14} \times 4\frac{1}{5}$
$= -\frac{\cancel{5}^{1}}{\cancel{14}_{2}} \times \frac{\cancel{21}^{3}}{\cancel{5}_{1}}$
$= -\frac{3}{2}$
$= -1\frac{1}{2}$

Note
$\frac{5}{5} = 5 \div 5 = 1$
$\frac{21}{14} = \frac{3}{2}$ ($\div 7$)

Dividing fractions

- change the $\div$ to $\times$
- flip the following fraction (its reciprocal) and multiply.

Negative fractions

$\frac{2}{3} \div \left(-\frac{1}{6}\right)$
$= \frac{2}{3} \times \left(-\frac{6}{1}\right)$
Different signs → Negative answer
$= -\frac{12}{3}$
$= -4$

$-\frac{5}{8} \div \left(-\frac{3}{8}\right)$
$= -\frac{5}{8} \times \left(-\frac{8}{3}\right)$
Same signs → Positive answer
$= \frac{5}{3}$
$= 1\frac{2}{3}$

$-\frac{7}{12} \div \left(-4\frac{2}{3}\right)$
$= -\frac{7}{12} \div \left(-\frac{14}{3}\right)$
$= -\frac{\cancel{7}^{1}}{\cancel{12}_{4}} \times -\frac{\cancel{3}^{1}}{\cancel{14}_{2}}$
$= \frac{1}{8}$

Reciprocal of $-\frac{14}{3}$ is $-\frac{3}{14}$

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Chapter summary

Decimals

Fractions to decimals
(1) Change denominator to 10, 100 or 1000.
or (2) Divide the denominator into the numerator.

Types of decimals
Decimals can terminate (stop).
0.5, 0.75, 0.125
Decimals can repeat/recur.
$0.3333\ldots = 0.\dot{3}$
$0.616161\ldots = 0.\overline{61}$

Multiplying by tens
The decimal point moves the same number of places as the number of zeroes.
$6.413 \times 100 = 641.3$

Adding and subtracting
When adding and subtracting with decimals make sure the decimal points line up.

Dividing by tens
$71.3 \div 100 = 0.713$

Multiplying decimals
The number of decimal places in the question is the same as in the answer.
$0.04 \times 0.3 = 0.012$

Rounding decimals
If the digit after the place you want is 0, 1, 2, 3 or 4, round down; if the digit is 5, 6, 7, 8 or 9, round up.

Dividing decimals
Multiply both numbers by 10, 100, 1000 etc. This is dividing by a whole number.
$$\frac{12.66}{0.3} \underset{\times 10}{\overset{\times 10}{=}} \frac{126.6}{3} = 42.2$$

Percentages and decimals
$0.45 = 45\%$
$20\% = 0.2$

Percentages out of 100

Unitary method ⭐
$4\% =$
$1\% =$ $\div 4$
$100\% =$ $\times 100$

Percentage change ⭐
$$\text{Percentage change} = \frac{\text{change}}{\text{original value}} \times \frac{100}{1}$$

Percentages and fractions
$$\frac{3}{5} = \frac{3}{5} \times 100 = 60\%$$
$$35\% = \frac{35}{100} = \frac{7}{20}$$

Common conversions
$\frac{1}{2} = 0.5 = 50\%$
$\frac{1}{10} = 0.1 = 10\%$
$\frac{1}{100} = 0.01 = 1\%$
$\frac{1}{4} = 0.25 = 25\%$
$\frac{3}{4} = 0.75 = 75\%$
$\frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\%$
$\frac{2}{3} = 0.\dot{6} = 66\frac{2}{3}\%$

Of means ×
7% of 400
$= \frac{7}{100} \times 400$

% of
$7\% \text{ of } 20 = \frac{7}{100} \times \frac{20}{1} = 1.4$

Increase
Find the % increase and add to the original.

Decrease
Find the % decrease and subtract from the original.

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Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
3A	**1 I can generate equivalent fractions** e.g. Rewrite $\frac{3}{6}$ as an equivalent fraction with a denominator of 40.	
3A	**2 I can convert a fraction to simplest form** e.g. Write the fraction $\frac{8}{20}$ in its simplest form.	
3B	**3 I can add and subtract fractions, including mixed numerals** e.g. Simplify: **a** $\frac{5}{3} - \frac{3}{4}$ **b** $3\frac{5}{8} + 2\frac{3}{4}$.	
3B	**4 I can multiply fractions, including mixed numerals** e.g. Simplify: **a** $\frac{2}{5} \times \frac{3}{7}$ **b** $\frac{8}{5} \times 1\frac{3}{4}$.	
3B	**5 I can divide fractions, including mixed numerals** e.g. Simplify: **a** $\frac{2}{5} \div \frac{3}{7}$ **b** $2\frac{1}{4} \div 1\frac{1}{3}$.	
3C	**6 I can add and subtract negative fractions.** e.g. Simplify: $\frac{2}{3} - \left(-\frac{4}{3}\right)$ and $\frac{1}{5} + \left(-\frac{1}{4}\right)$.	
3C	**7 I can multiply and divide negative fractions.** e.g. Simplify: $-\frac{6}{5} \times \left(-\frac{3}{4}\right)$ and $-1\frac{1}{3} \div 3$.	
3D	**8 I can compare decimals** e.g. Compare the following decimals and place the correct inequality sign between them: 57.89342 57.89631	
3D	**9 I can convert decimals to fractions** e.g. Convert 5.12 to a fraction in its simplest form.	
3D	**10 I can convert simple fractions to decimals** e.g. Convert $\frac{9}{25}$ to a decimal.	
3D	**11 I can add and subtract decimals** e.g. Calculate: **a** $9.7 - 2.86$ **b** $2.4 + 4.24$.	
3E	**12 I can multiply and divide decimals by powers of 10** e.g. Calculate: **a** $9.753 \div 100$ **b** $27.58 \times 10\,000$	
3E	**13 I can multiply decimals** e.g. Calculate 25.7×0.3.	

Chapter checklist

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Chapter checklist

		✔
3E	**14 I can divide decimals** e.g. Calculate $64.137 \div 0.03$	
3F	**15 I can convert fractions to terminating decimals** e.g. Write $\frac{7}{8}$ as a terminating decimal.	
3F	**16 I can convert fractions to recurring decimals** e.g. Write $3\frac{5}{7}$ as a recurring decimal.	
3F	**17 I can round terminating decimals** e.g. Round 14.258 to 1 decimal place.	
3F	**18 I can convert percentages to fractions and to decimals** e.g. Write 72% as a simple fraction and as a decimal.	
3G	**19 I can convert fractions and decimals to percentages** e.g. Convert the following to percentages: **a** $\frac{4}{5}$ **b** 0.81	
3H	**20 I can express one quantity as a percentage of another, converting units if required** e.g. Express: **a** 34 out of 40 as a percentage **b** 60 cents as a percentage of $\$5$	
3H	**21 I can find a certain percentage of a quantity** e.g. Find 25% of 48.	
3I	**22 I can find the result when a value is increased by a percentage** e.g. Find the new value when $\$160$ is increased by 40%.	
3I	**23 I can find the result when a value is decreased by a percentage** e.g. Find the new value when $\$63$ is decreased by 20%.	
3I	**24 I can calculate the cost of an item after a discount** e.g. Find the cost of an $\$860$ television that has been discounted by 25%.	
3I	**25 I can calculate the cost of an item after a mark-up** e.g. Find the cost of a $\$250$ microwave oven that has been marked up by 12%.	

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		✔
3J	**26** **I can calculate the percentage profit when prices are increased** e.g. Calculate the percentage profit when $\$25$ becomes $\$32$.	
3J	**27** **I can calculate the percentage loss when prices are decreased** e.g. Calculate the percentage loss when $\$60$ becomes $\$48$.	
3K	**28** **I can use the unitary method to find the full amount** e.g. If 8% of an amount is $\$48$, what is the full amount of money?	
3K	**29** **I can use the unitary method to find a new percentage** e.g. If 11% of the food bill was $\$77$, how much is 25% of the food bill?	
3K	**30** **I can use the unitary method to find the original price** e.g. A pair of shoes has been discounted by 20%. If the sale price was $\$120$, what was the original price of the shoes?	

Chapter checklist

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Chapter review

Short-answer questions

3A **1** Copy and complete.

a $\frac{7}{20}=\frac{\square}{60}$ b $\frac{25}{40}=\frac{5}{\square}$ c $\frac{4}{7}=\frac{\square}{21}$

3A **2** Simplify.

a $\frac{25}{45}$ b $\frac{36}{12}$ c $\frac{16}{12}$

3B **3** Evaluate.

a $\frac{1}{4}+\frac{1}{4}$ b $\frac{5}{6}-\frac{4}{6}$
c $\frac{7}{8}-\frac{4}{8}$ d $\frac{7}{10}+\frac{1}{10}$
e $\frac{7}{8}-\frac{3}{4}$ f $\frac{1}{4}+\frac{1}{2}$
g $\frac{5}{12}+\frac{1}{4}$ h $\frac{3}{5}+\frac{7}{10}$

3B **4** Find:

a $3-1\frac{1}{4}$ b $1\frac{1}{2}+2\frac{1}{2}$ c $10-3\frac{1}{2}$ d $3\frac{4}{5}+1\frac{2}{5}$

3B **5** Find:

a $\frac{2}{3}$ of 6 b $\frac{1}{5}$ of 10 c $\frac{2}{3}\times 12$ d $\frac{3}{5}\times 20$

3B **6** Find:

a $\frac{1}{2}\times\frac{1}{3}$ b $\frac{2}{5}\times\frac{1}{4}$ c $\frac{7}{8}\times\frac{2}{5}$ d $1\frac{1}{2}\times\frac{2}{9}$

3B **7** Calculate these divisions.

a $6\div\frac{1}{2}$ b $\frac{2}{3}\div\frac{1}{3}$ c $\frac{4}{5}\div\frac{1}{2}$ d $1\frac{1}{2}\div\frac{3}{4}$

3C **8** Evaluate each of the following.

a $\frac{1}{5}-\frac{2}{3}$ b $-\frac{3}{4}\times\frac{1}{5}$
c $-\frac{3}{5}\times\left(-\frac{3}{5}\right)$ d $\frac{3}{4}-\left(-\frac{1}{5}\right)$
e $\frac{5}{3}\div\left(-\frac{1}{3}\right)$ f $-6\frac{1}{4}+\left(-1\frac{1}{3}\right)$

3D **9** Convert these fractions to decimals.

a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{3}{5}$ d $\frac{117}{1000}$

3D **10** Write these decimals as simple fractions.

a 0.6 b 0.12 c 0.04 d 0.95

3E **11** Evaluate.

a $12.6+7.4$ b $8.59+5.6$
c $9.4-1.2$ d $10-5.4$
e $9.6+10.1+3.21$ f $12.4-6.22$

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3E **12** Evaluate.

a 3×2
b 0.3×0.2
c 1.2×4
d 0.12×0.4
e 1.5×0.4
f 7.164×100
g 9.6×10
h 0.06×7

3E **13** Find:

a $12 \div 0.3$
b $18.6 \div 3$
c $14.22 \div 0.2$

3F **14** Round these decimals to 3 decimal places.

a $0.666\ldots$
b $3.579\ 64$
c $0.005\ 496\ 31$

3G **15** Copy and complete this table of conversions.

0.1					0.75		
	$\frac{1}{100}$			$\frac{1}{4}$		$\frac{1}{3}$	$\frac{1}{8}$
		5%	50%				

3H **16** Find:

a 10% of \$50
b 25% of \$64
c 5% of 700 g

3H **17** Express each of the following as a percentage.

a \$35 out of \$40
b 6 out of 24
c \$1.50 out of \$2
d 16 cm out of 4 m

3I **18** **a** Increase \$560 by 10%.
b Decrease \$4000 by 15%.

3I **19** If 6% of an amount is \$18, what is the amount?

3J ★ **20** Toni bought a \$194 dress on sale for 20% off.
What did Toni pay for the dress?

3J/K ★ **21** Sally earned \$84 000 last year. This year she got 5% more. What did Sally earn this year?

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Chapter review

Multiple-choice questions

3D **1** 0.36 expressed as a fraction is:

A $\frac{36}{10}$ **B** $\frac{36}{100}$ **C** $\frac{3}{6}$ **D** $\frac{9}{20}$ **E** $\frac{6}{3}$

3B **2** $\frac{1}{8}+\frac{5}{8}$ is equal to:

A $\frac{6}{64}$ **B** $\frac{6}{16}$ **C** $\frac{15}{8}$ **D** $\frac{6}{8}$ **E** 6

3E **3** When 21.63 is multiplied by 13.006, the number of decimal places in the answer is:

A 2 **B** 3 **C** 4 **D** 5 **E** 1

3A **4** $2\frac{1}{3}$ is the same as:

A 7 **B** $\frac{3}{7}$ **C** $\frac{7}{3}$ **D** 2.3 **E** 6

3B **5** The reciprocal of $\frac{3}{4}$ is:

A $\frac{4}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $1\frac{1}{2}$ **E** 4

3D **6** Which decimal has the largest value?

A 6.0061 **B** 6.06 **C** 6.016 **D** 6.0006 **E** 6.007

3E **7** 9.46×1000 is:

A 94 600 000 **B** 9460 **C** 94 600 **D** 0.000 094 6 **E** 0.0946

3H **8** 75% of 84 is the same as:

A $\frac{84}{4} \times 3$

B $\frac{84}{3} \times 4$

C $84 \times 100 \div 75$

D $\frac{(0.75 \times 84)}{100}$

E 75

3K ★ **9** If 1% of the total equals 8, then 5% of the total equals:

A 800 **B** 80 **C** 40 **D** 4 **E** 1.6

3I **10** \$790 increased by 10% gives:

A \$79 **B** \$880 **C** \$771 **D** \$869 **E** 0.79

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Extended-response questions

1 a A \$320 statue has the GST (10%) added to the price. What is the final price?

b The price of a \$670 stove includes GST. What is the price non-inclusive of GST? Round to the nearest cent.

c In another country the GST is 13.5%. If a coat is priced at \$145.28 and this price includes GST, what is the price excluding GST?

2 The following table shows the value of A\$1 (one Australian dollar) in foreign currency. Genevieve is planning an extended holiday to Asia. She plans on visiting India, Singapore, Phuket and Hong Kong.

Currency	A\$ 1
Indian rupee (INR)	42
Singapore dollar (SGD)	1.25
Thai baht (THB)	30
Hong Kong dollar (HKD)	7

a She has decided to change some Australian dollars to each of the above currencies before she flies out. How much of each currency will she receive if she changes A\$500 to each currency?

b If she spent 70% of her Thai baht on hotels, how much Thai baht does she have left to spend?

c After visiting Hong Kong, Genevieve has \$42 HKD left. What does this convert back to in Australian dollars?

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4 Measurement

Essential mathematics: why measurement skills are important

Precise measurements are crucial to the design and assembly of amusement park rides such as Ferris wheels, roller coasters and carousels. Hundreds of components must perfectly fit together to achieve structural strength and stability, providing a safe and comfortable ride.

Sheet metal workers use measurement skills to cut, construct, assemble, and repair metal structures including restaurant kitchens, truck bodies and ducting for HVAC (Heating, Ventilation, and Air Conditioning) installations.

Prism volumes are calculated by builders to determine the volume of concrete in m^3 needed for house foundations and by swimming pool designers to determine a pool's capacity in litres.

Builders and carpenters use Pythagoras' theorem to find the sloping roof rafter length if the horizontal ceiling joist and the vertical height of the roof are known, or, to find the vertical height of the roof when the rafter length and ceiling joist length are known.

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In this chapter

Australian Curriculum 9.0

MEASUREMENT

Solve problems involving the area and perimeter of irregular and composite shapes using appropriate units (AC9M8M01)

Solve problems involving the volume and capacity of right prisms using appropriate units (AC9M8M02)

Solve problems involving the circumference and area of a circle using formulas and appropriate units (AC9M8M03)

Solve problems involving duration, including using 12- and 24-hour time across multiple time zones (AC9M8M04)

Use Pythagoras' theorem to solve problems involving the side lengths of right-angled triangles (AC9M8M06)

NUMBER

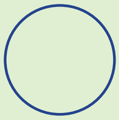
Recognise irrational numbers in applied contexts, including square roots and π (AC9M8N01)

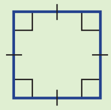
Online resources

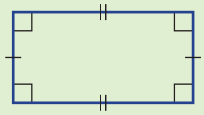
A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

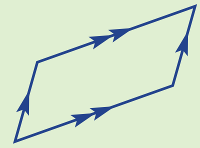
Warm-up quiz

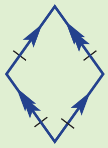
1 For each of the following shapes, choose the most descriptive name from options **A** to **H**.
A *triangle* **B** *rhombus* **C** *square* **D** *parallelogram* **E** *circle* **F** *trapezium* **G** *rectangle* **H** *kite*

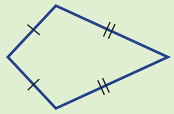
a 

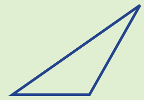
b 

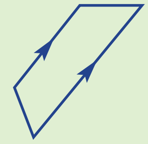
c 

d 

e 

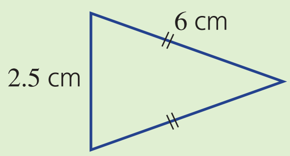
f 

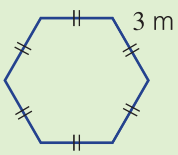
g 

h 

2 Find the perimeter (distance around the outside) of these shapes.

a 

b 

c 

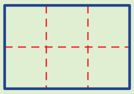
3 Evaluate the following.

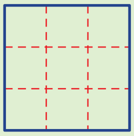
a $\frac{1}{2} \times 5 \times 4$ **b** $\frac{1}{2}(2 + 7) \times 6$ **c** 5^2 **d** 11^2

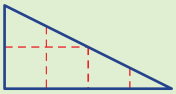
4 Convert these measurements to the units shown in the brackets.

a 3 m (cm) **b** 20 cm (mm) **c** 1.8 km (m)
d 0.25 m (cm) **e** 35 mm (cm) **f** 4200 m (km)
g 500 cm (m) **h** 100 mm (m) **i** 2 minutes (seconds)
j 3 L (mL) **k** 4000 mL (L) **l** 3000 g (kg)

5 Count squares to find the area of these shapes.

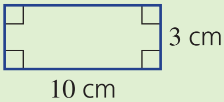
a 

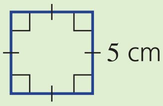
b 

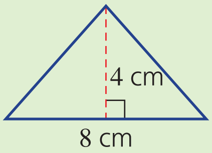
c 

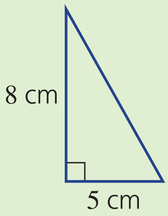
6 Find the area of these rectangles and triangles.

Remember: Area (rectangle) = $l \times w$ and Area (triangle) = $\frac{1}{2}bh$

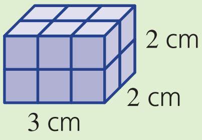
a 

b 

c 

d 

7 Count cubes to find the volume of this solid.



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4A Length and perimeter CONSOLIDATING

Learning intentions

- To understand that perimeter is the distance around a shape and is measured in units such as kilometres, metres, centimetres and millimetres.
- To be able to convert between different metric units of length.
- To be able to find the perimeter of a shape when individual side lengths are known.
- To be able to find an unknown side length of a shape when its perimeter is known.

Key vocabulary: perimeter, length, units, kilometre (km), metre (m), centimetre (cm), millimetre (mm)

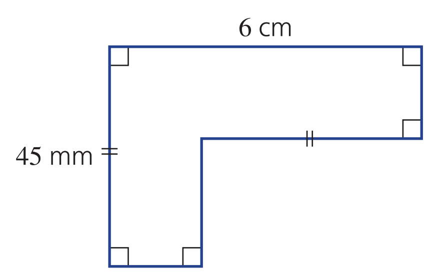
Developed in France in the 1790s, the metric system for measurement includes length units such as millimetre, centimetre, metre and kilometre.

We use such units to describe, for example, the distance between two towns, the perimeter of a block of land, the depth of the ocean or the length of a racetrack.

In F1 racing, the number of laps needed to complete the race changes depending on the length of the circuit.

Lesson starter: Provide the perimeter

In this diagram some of the lengths are given. Three students were asked to find the perimeter.



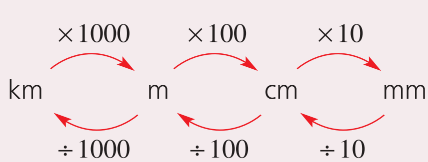
- Will says that you cannot work out some lengths and so the perimeter cannot be found.
- Sally says that there is enough information and the answer is $9 + 12 = 21$ cm.
- Greta says that there is enough information but the answer is $90 + 12 = 102$ cm.

Who is correct?

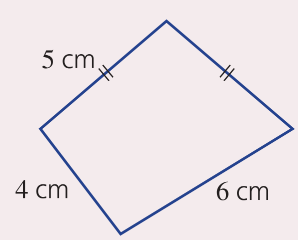
Discuss how each person arrived at their answer.

Key ideas

- The common metric **units** of **length** include:
  - **kilometre** (km) $1 \text{ km} = 1000 \text{ m}$
  - **metre** (m) $1 \text{ m} = 100 \text{ cm}$
  - **centimetre** (cm) $1 \text{ cm} = 10 \text{ mm}$
  - **millimetre** (mm)



- **Perimeter** is the distance around a closed shape.
  - All units must be of the same type when calculating the perimeter.
  - Sides with the same type of markings (dashes) are of equal length.



$P = 2 \times 5 + 4 + 6$
$= 20 \text{ cm}$

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Exercise 4A

Understanding

1–4 | 4

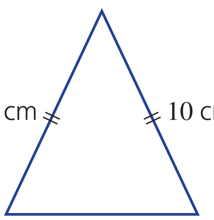
1 Write the missing words.
  a The commonly used measurement system used today is called the _____ system.
  b The common metric units for length include millimetres, ________, _________ and ________.

2 Evaluate the following.
  a 2×100 b 5.2×1000 c 7.8×10
  d $840 \div 100$ e $9610 \div 10$ f $41\,200 \div 1000$

Hint for Q2: Move the decimal point to the right for × and left for ÷.

3 Choose the most appropriate unit of measurement from millimetres (mm), metres (m) and kilometres (km) for the following items being measured.
  a the width of a football ground
  b the length of a small insect
  c the distance for a sprinting race
  d the distance a person drives their car in a week
  e the height of a building
  f the distance between lines on a piece of lined paper

4 Find the value of x in these diagrams.

a


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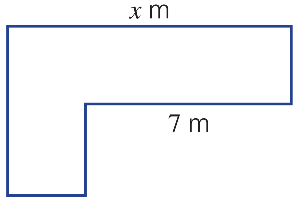

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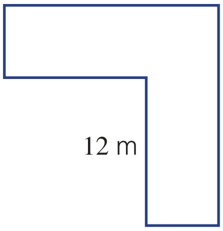

Fluency

5–6(½), 7 | 5–6(½), 7

Example 1 Converting length measurements

Convert these lengths to the units shown in the brackets.

a 5.2 cm (mm) b 2400 m (km)

Solution

a $5.2 \text{ cm} = 5.2 \times 10$
$= 52 \text{ mm}$

Explanation

1 cm = 10 mm, so multiply by 10.

$\text{cm} \xrightarrow{\times 10} \text{mm}$

b $2400 \text{ m} = 2400 \div 1000$
$= 2.4 \text{ km}$

1 km = 1000 m, so divide by 1000.

Now you try

Convert these lengths to the units shown in the brackets.

a 3.61 km (m) b 540 cm (m)

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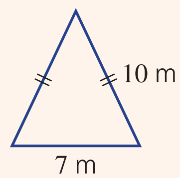
5 Convert these measurements to the units shown in the brackets.

a 3 cm (mm) **b** 6.1 m (cm) **c** 8.93 km (m)
d 3 m (cm) **e** 0.0021 km (m) **f** 320 mm (cm)
g 19 620 m (km) **h** 38 000 cm (m) **i** 48 mm (cm)
j 0.2 cm (mm) **k** 4.2 cm (m) **l** 0.4 m (cm)
m 3700 m (km) **n** 600 m (km) **o** 0.71 km (m)
p 0.02 m (cm)

Hint for Q5:
1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

Example 2 Finding perimeters

Find the perimeter of this triangle.



Solution

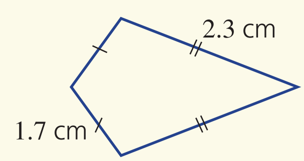
$$P = 2 \times 10 + 7$$
$$= 27 \text{ m}$$

Explanation

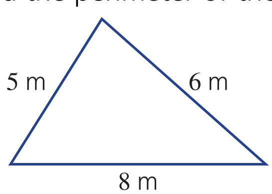
There are two equal 10 m lengths and one 7 m length to add up.

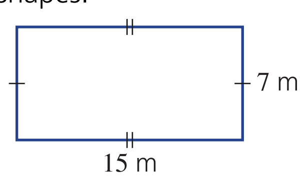
Now you try

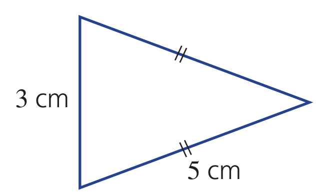
Find the perimeter of this kite.

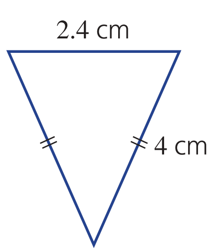


6 Find the perimeter of these shapes.

a


b


c


d


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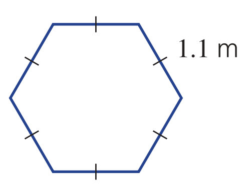
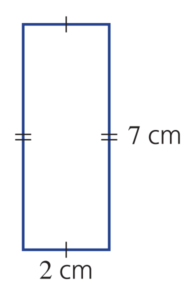
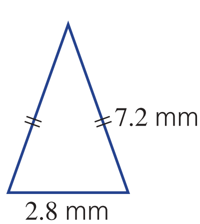


f


Hint for Q6: Sides with the same markings have the same length.

g


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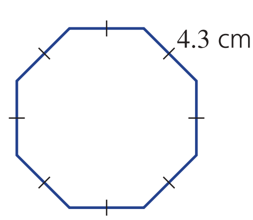

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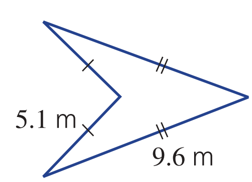

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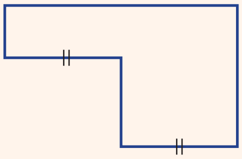
4A

Example 3 Finding perimeters of rectangular shapes

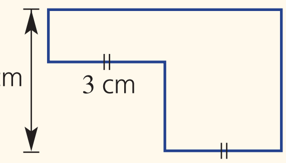
Find the perimeter of this shape.



Solution

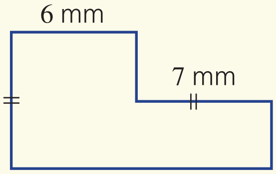
$P = 2 \times (3 + 3) + 2 \times 4$
$= 12 + 8$
$= 20$ cm

Explanation



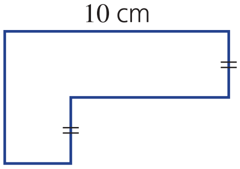
Now you try

Find the perimeter of this shape.

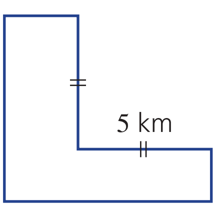


7 Find the perimeter of these shapes.

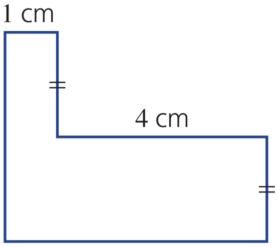
a



b



c



Problem-solving and reasoning

8–9(½), 10 | 8–9(½), 11–13

8 Convert these measurements to the units shown in the brackets.

- **a** 0.0043 m (mm)
- **b** 0.0204 km (cm)
- **c** 23 098 mm (m)
- **d** 342 000 cm (km)
- **e** 194 300 mm (m)
- **f** 10 000 mm (km)
- **g** 0.02403 m (mm)
- **h** 994 000 mm (km)
- **i** 0.00001 km (cm)

Hint for Q8: You will need to multiply or divide by at least two factors. e.g. $\times$ 100 $\times$ 10 or $\div$ 1000 $\div$ 100.

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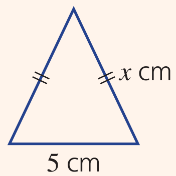
Example 4 Finding an unknown length

Find the unknown value x in this triangle if the perimeter is 19 cm.

$P = 19$ cm



Solution

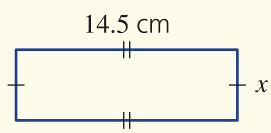
$$2x + 5 = 19$$
$$2x = 14$$
$$x = 7$$

Explanation

$2x + 5$ makes up the perimeter, which is 19.
Solve the equation algebraically or use a guess and check method to find the value of x.

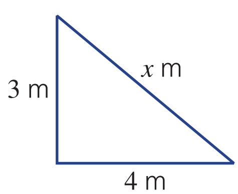
Now you try

Find the unknown value x in this rectangle if the perimeter is 40 cm.



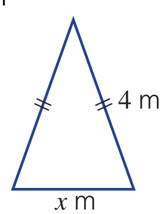
9 Find the unknown value x in these shapes with the given perimeter (P).

a



$P = 12$ m

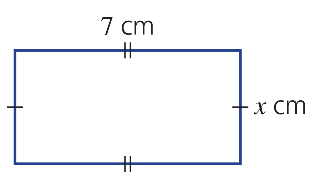
b



$P = 10$ m

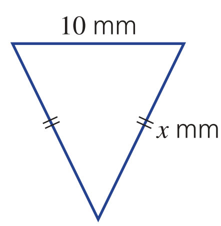
Hint for Q9: Use the given perimeter to find the value of x.

c



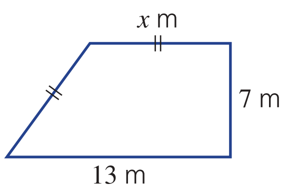
$P = 22$ cm

d



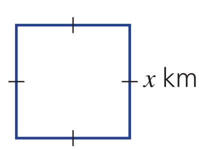
$P = 46$ mm

e



$P = 39$ m

f



$P = 26$ km

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4A

10 Jennifer needs to fence her country house block to keep her dog in. The block is a rectangle with length 50 m and width 42 m. Fencing costs \$13 per metre. What will be the total cost of fencing?

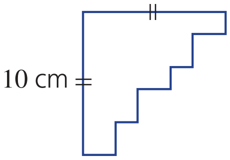
11 Gillian can jog 100 metres in 24 seconds. How long will it take her to jog 2 km? Give your answer in minutes. (There are 60 seconds in one minute.)

12 When a length is measured in practice, the true length of the object might not be exactly the same as the reported measurement. For example, if someone says they are 173 cm tall, they might be anywhere from 172.5 cm to 173.5 cm. Use this principle to give a range for someone whose height is reported as:
a 153 cm
b 178 cm
c 160 cm.

13 A rectangular room is initially estimated to be 4 metres wide and 6 metres long. It is then measured to the nearest centimetre as 403 cm by 608 cm. Finally, it is measured to the nearest millimetre as 4032 mm by 6084 mm.
a Give the perimeters of the room from each of these three sets of measurements.
b What is the difference, in millimetres, between the largest and smallest perimeter?
c Which is the most accurate value to use for the perimeter of the room?

Perimeter challenge — 14

14 Find the perimeter of these shapes. Give your answers in cm.

a


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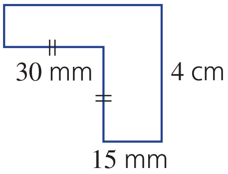

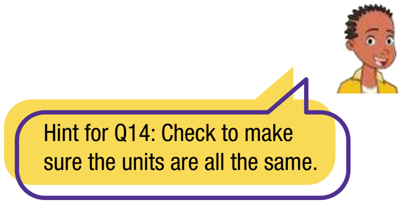

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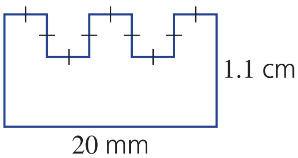

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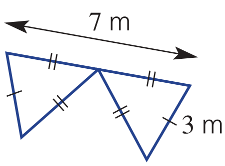
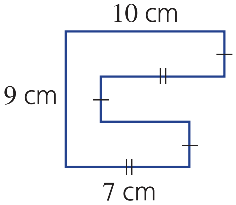


e


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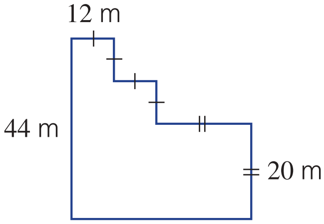

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4B Circumference of a circle

Learning intentions

- To understand that pi (π) is a number that equals the circumference divided by the diameter of any circle.
- To know that pi is approximately 3.14 or 22/7.
- To be able to find the circumference of a circle using a calculator.

Key vocabulary: circle, diameter, radius, circumference, pi (π)

The distance around the outside of a circle, known as the circumference, is connected to the diameter through a special number called pi.

The symbol for pi is π, and as a decimal, $\pi = 3.14159\ldots$. There is no fraction that represents pi exactly, which is why we often use calculators when working with this number.

Pi represents the number of times the diameter of the circle is needed to circumnavigate the circumference of the circle.

Lesson starter: Discovering pi

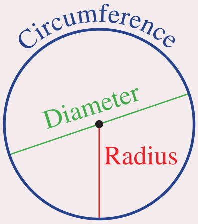
Here are the diameters and circumferences for three circles, correct to two decimal places. Use a calculator to work out the value of Circumference ÷ Diameter and put your results in the third column. Add your own circle measurements by measuring the diameter and circumference of circular objects such as a can or a wheel.

- What do you notice about the numbers for $C \div d$ in the third column?
- Why might the numbers in the third column vary slightly from one set of measurements to another?
- What rule can you write down that links C with d?

Diameter d (mm)	Circumference C (mm)	$C \div d$
2.23	7.01	
5.94	18.66	
20.65	64.87	
Add your own	Add your own	

Key ideas

- Features of a **circle**:
  - **Diameter** (d) is the distance across the centre of a circle.
  - **Radius** (r) is the distance from the centre of a circle to its outside edge. Note $d = 2r$.



- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi r$ or $C = \pi d$
- **Pi** (π) ≈ 3.14159 (correct to five decimal places).
  - Common approximations include 3.14 and $\frac{22}{7}$.
  - A more precise estimate for pi can be found on most calculators or on the internet.

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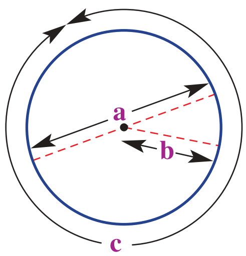
Exercise 4B

Understanding

1–4 | 3, 4

1 Name the features of the circle shown in the diagram.



2 **a** Find the diameter of a circle if its radius is:
i 5 m **ii** 11 cm **iii** 2.3 mm

b Find the radius of a circle if its diameter is:
i 12 cm **ii** 31 mm **iii** 0.42 m

3 Write down the value of π correct to:
a one decimal place
b two decimal places
c three decimal places

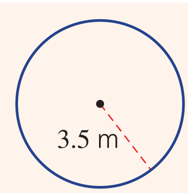
4 Evaluate the following using a calculator and round to two decimal places.
a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

Fluency

5, 6(½) | 5, 6(½)

Example 5 Finding the circumference using the radius

Find the circumference of this circle correct to two decimal places.
Use a calculator for the value of pi.



Solution

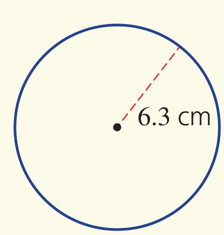
$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 3.5 \\ &= 7\pi \\ &= 21.99 \text{ m (to 2 d.p.)} \end{aligned}$$

Explanation

Since r is given, you can use $C = 2\pi r$.

Now you try

Find the circumference of this circle correct to two decimal places.
Use a calculator for the value of pi.



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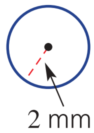
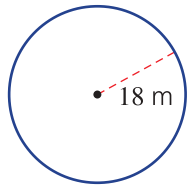
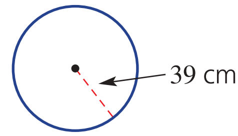
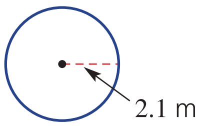
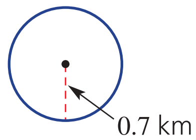
5 Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of pi.

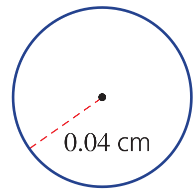
a 

b 

c 

d 

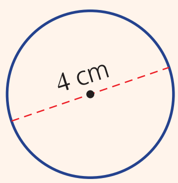
e 

f 

Hint for Q5: Use the rule $C = 2\pi r$ and substitute the value of r.

Example 6 Finding the circumference using the diameter

Find the circumference of this circle, correct to two decimal places.



Solution

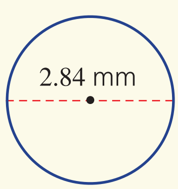
$$\begin{aligned} C &= \pi d \\ &= \pi \times 4 \\ &= 4\pi \\ &= 12.57 \text{ cm (to 2 d.p.)} \end{aligned}$$

Explanation

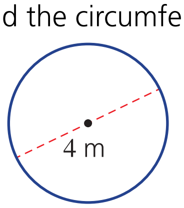
Substitute $d = 4$ into the rule $C = \pi d$ or use $C = 2\pi r$ with $r = 2$.

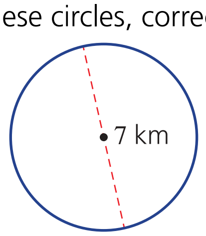
Now you try

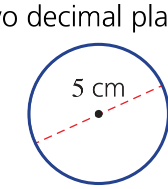
Find the circumference of this circle, correct to two decimal places.

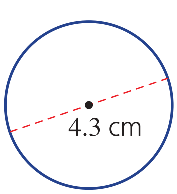


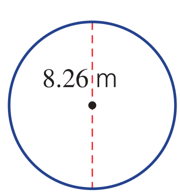
6 Find the circumference of these circles, correct to two decimal places.

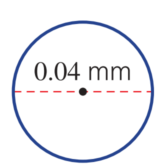
a 

b 

c 

d 

e 

f 

Hint for Q6: Use the rule $C = \pi d$ and substitute the value of d.

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4B

Problem-solving and reasoning

7–10	9–13

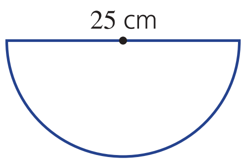
7 The diameter of the circular face of a metal drum is 80 cm. Find its circumference, correct to the nearest whole centimetre.

8 A water tank has a diameter of 3.5 m. Find its circumference, correct to one decimal place.

9 A wheel of radius 28 cm rolls one full turn. Find how far it rolls, correct to the nearest centimetre.

10 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

11 These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to two decimal places.

a


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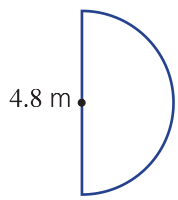
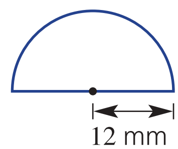


c


Hint for Q11: The perimeter is half of the circumference of a full circle plus the diameter.

12 Draw an accurate number line showing the integers 0, 1, 2, 3 and 4.

a Label the points 3.14, $\frac{22}{7}$ and π at their locations on the line.

b Sort the numbers 3.14, $\frac{22}{7}$, π in ascending order.

c When finding the circumference of a circle with a known diameter, you could use 3.14 or $\frac{22}{7}$ as an approximate value for π. Which of these values will give you an approximate value that is bigger then the true circumference? Explain your answer.

13 Explain why the rule $C = 2\pi r$ is equivalent to (i.e. the same as) $C = \pi d$.

Memorising pi

—	14

14 The box shows π correct to 100 decimal places. In 2020 the unofficial world record for the most number of digits of π recited from memory was held by Akira Haraguchi from Japan. He recited 100 000 digits non-stop over a $16\frac{1}{2}$-hour period.

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679

Challenge your friends to see who can remember the most number of digits in the decimal representation of π.

Number of digits memorised	Report
10+	A good show
20+	Great effort
35+	Superb
50+	Amazing memory

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4C Area of basic shapes

Learning intentions

- To understand what the area of a two-dimensional shape refers to.
- To be able to convert between different metric units of area, including hectares.
- To be able to find the area of rectangles, parallelograms, triangles and squares.

Key vocabulary: area, perpendicular, rectangle, square, parallelogram, triangle, hectares (ha)

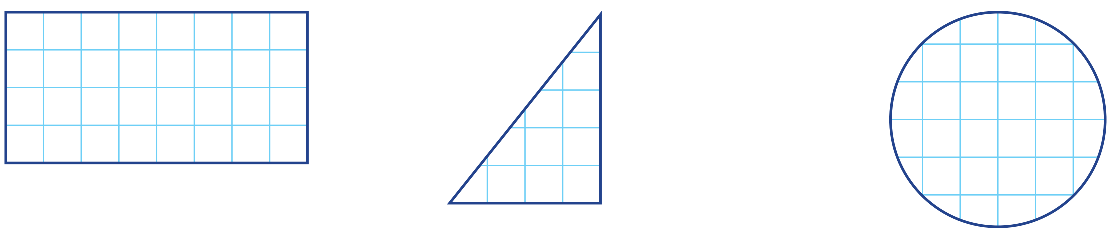
The amount of space on a surface is called area. Area is measured in square units and the common metric units are square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), square kilometres (km^2) and hectares (ha).

The hectare is often used to describe areas of land, since the square kilometre for such areas is considered to be too large a unit and the square metre too small. A school football oval might be about 1 hectare, for example, and a small forest might be about 100 hectares.


A typical athletics field, including the running track, is around 1.2 hectares in size.

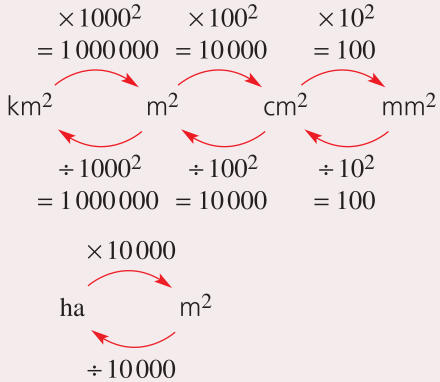
Lesson starter: Estimating area

By counting squares, or by using an estimate, you can find the area of a shape. For the following shapes, find or estimate their area. Explain your method for each one.



Key ideas

- The common metric units for **area** include:
  - square millimetres (mm^2)
  - square centimetres (cm^2)
  - square metres (m^2)
  - square kilometres (km^2)
  - **hectares** (ha)

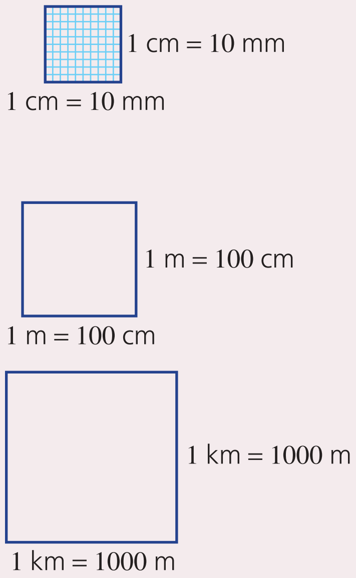


$1\ cm^2$
$= 10\ mm \times 10\ mm$
$= 100\ mm^2$

$1\ m^2$
$= 100\ cm \times 100\ cm$
$= 10\,000\ cm^2$

$1\ km^2$
$= 1000\ m \times 1000\ m$
$= 1\,000\,000\ m^2$

$1\ ha$
$= 100\ m \times 100\ m$
$= 10\,000\ m^2$



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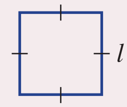
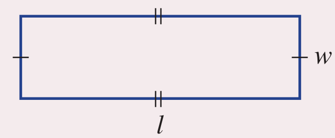
4C

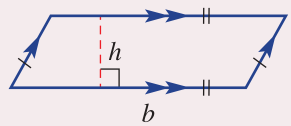
- Area of **squares**, **rectangles**, **parallelograms** and **triangles**
  - Square $A = l \times l = l^2$



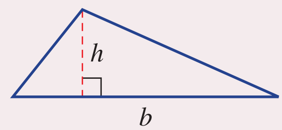
  - Rectangle $A = l \times w = lw$



  - Parallelogram $A = b \times h = bh$



  - Triangle $A = \frac{1}{2} \times b \times h = \frac{1}{2}bh$



  - The dashed line which gives the height is **perpendicular** (at right angles) to the base.

Exercise 4C

Understanding

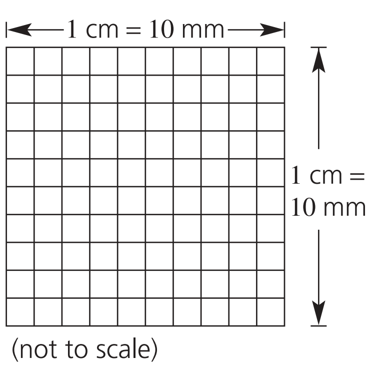
Understanding	1–3	1, 3

1 Convert these area measurements to the units shown in the brackets.

a $2\ \text{m}^2$ (cm^2) **b** $5\ \text{cm}^2$ (mm^2) **c** $400\ \text{mm}^2$ (cm^2)

d $30\,000\ \text{cm}^2$ (m^2) **e** $4\ \text{km}^2$ (m^2) **f** $8\,000\,000\ \text{m}^2$ (km^2)

2 By considering the given diagrams, answer the questions.



a **i** How many mm^2 in $1\ \text{cm}^2$?

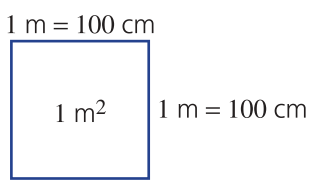
ii How many mm^2 in $4\ \text{cm}^2$?

iii How many cm^2 in $300\ \text{mm}^2$?

b **i** How many cm^2 in $1\ \text{m}^2$?

ii How many cm^2 in $7\ \text{m}^2$?

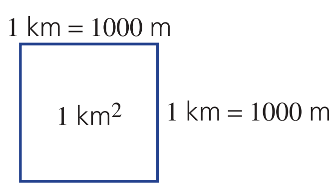
iii How many m^2 in $40\,000\ \text{cm}^2$?



c **i** How many m^2 in $1\ \text{km}^2$?

ii How many m^2 in $5\ \text{km}^2$?

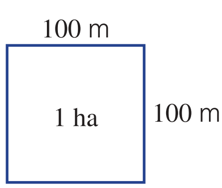
iii How many km^2 in $2\,500\,000\ \text{m}^2$?



d **i** How many m^2 in 1 ha?

ii How many m^2 in 3 ha?

iii How many ha in $75\,000\ \text{m}^2$?



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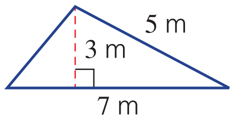
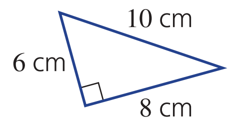
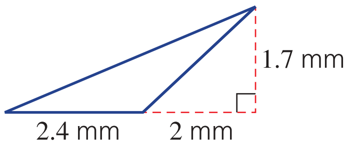
3 Which length measurements would be used for the *base* and the *height* (in that order) to find the area of these triangles?

a


b


c


Hint for Q3: Recall that the base and height are perpendicular (at 90°).

Fluency

4–7(½) 4–7(½)

Example 7 Converting units of area

Convert these area measurements to the units shown in the brackets.

a $0.248\ \text{m}^2$ (cm^2)
b $3100\ \text{mm}^2$ (cm^2)

Solution

a $0.248\ \text{m}^2 = 0.248 \times 10\,000$
$= 2480\ \text{cm}^2$

b $3100\ \text{mm}^2 = 3100 \div 100$
$= 31\ \text{cm}^2$

Explanation

$1\ \text{m}^2 = 100^2\ \text{cm}^2 = 10\,000\ \text{cm}^2$
Multiply since you are changing to a smaller unit.

$\text{m}^2 \xrightarrow{\times 100^2} \text{cm}^2$

$1\ \text{cm}^2 = 10^2\ \text{mm}^2 = 100\ \text{mm}^2$
Divide since you are changing to a larger unit.

$\text{cm}^2 \xleftarrow{\div 10^2} \text{mm}^2$

Now you try

Convert these area measurements to the units shown in the brackets.

a $0.43\ \text{cm}^2$ (mm^2)
b $52\,500\ \text{cm}^2$ (m^2)

4 Convert these area measurements to the units shown in the brackets.

a $2\ \text{cm}^2$ (mm^2)
b $7\ \text{m}^2$ (cm^2)
c $0.5\ \text{km}^2$(m^2)
d 3 ha (m^2)
e $0.34\ \text{cm}^2$ (mm^2)
f $700\ \text{cm}^2$ (m^2)
g $3090\ \text{mm}^2$ (m^2)
h $0.004\ \text{km}^2$ (m^2)
i $2000\ \text{cm}^2$ (m^2)
j $450\,000\ \text{m}^2$ (km^2)
k $4000\ \text{m}^2$ (ha)
l $3210\ \text{mm}^2$ (cm^2)
m $320\,000\ \text{m}^2$ (ha)
n $0.0051\ \text{m}^2$ (cm^2)
o $0.043\ \text{cm}^2$ (mm^2)
p $4802\ \text{cm}^2$ (m^2)
q $19\,040\ \text{m}^2$ (ha)
r $2933\ \text{m}^2$ (ha)
s 0.0049 ha (m^2)
t 0.77 ha (m^2)

Hint for Q4:
$1\ \text{cm}^2 = 10 \times 10$
$= 100\ \text{mm}^2$
$1\ \text{m}^2 = 100 \times 100$
$= 10\,000\ \text{cm}^2$
$1\ \text{km}^2 = 1000 \times 1000$
$= 1\,000\,000\ \text{m}^2$
$1\ \text{ha} = 100 \times 100$
$= 10\,000\ \text{m}^2$

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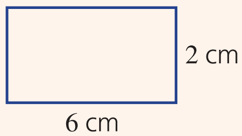
4C

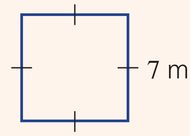
Example 8 Finding the area of rectangles and squares

Find the area of this rectangle and square.

a



b



Solution

a $A = lw$

$= 6 \times 2$

$= 12\ \text{cm}^2$

Explanation

Write the formula for the area of a rectangle and substitute $l = 6$ and $w = 2$.

b $A = l^2$

$= 7^2$

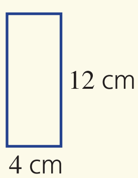
$= 49\ \text{m}^2$

For a square, multiply the length of a side by itself to get the area.

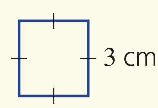
Now you try

Find the area of this rectangle and square.

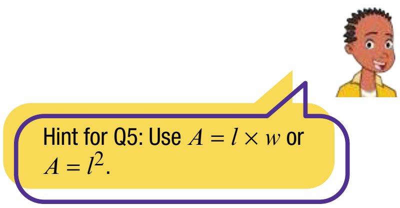
a



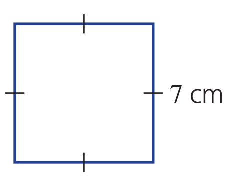
b



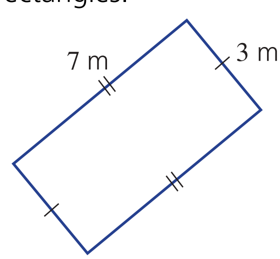
5 Find the areas of these squares and rectangles.



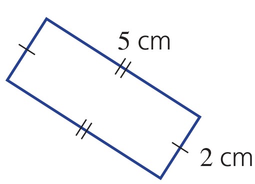
a



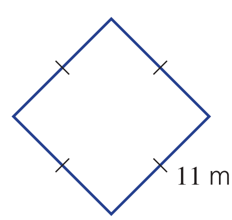
b



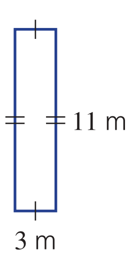
c



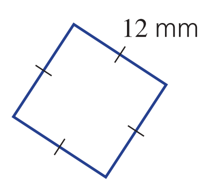
d



e



f



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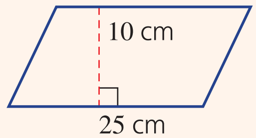
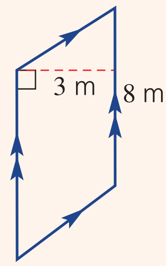
Example 9 Finding the area of parallelograms

Find the area of these parallelograms.

a



b



Solution

a $A = bh$

$= 25 \times 10$

$= 250\ \text{cm}^2$

b $A = bh$

$= 8 \times 3$

$= 24\ \text{m}^2$

Explanation

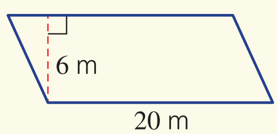
Use $A = bh$ with $b = 25$ and $h = 10$

The height is measured at right angles to the base.

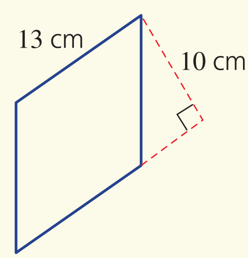
Now you try

Find the area of these parallelograms.

a

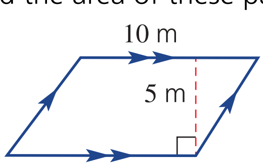


b

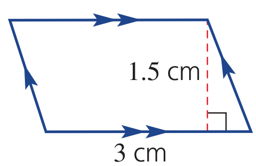


6 Find the area of these parallelograms.

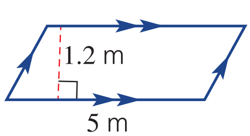
a



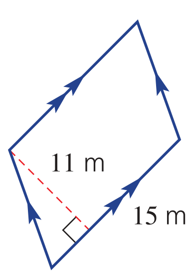
b



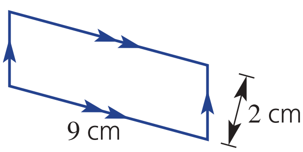
c



d



e



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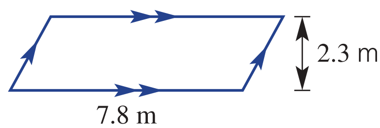

Hint for Q6: Use $A = bh$ and choose your base and perpendicular height.

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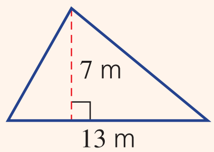
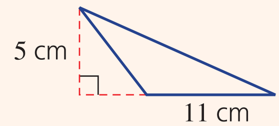
Example 10 Finding the area of triangles

Find the area of these triangles.

a



b



Solution

a $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 7$

$= 45.5 \text{ m}^2$

b $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 11 \times 5$

$= 27.5 \text{ cm}^2$

Explanation

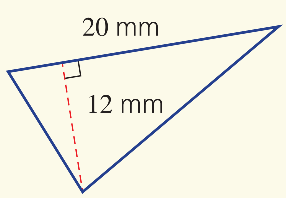
Remember that the height is measured using a line that is perpendicular to the base.

The base is 11 cm and the height is 5 cm so use $b = 11$ and $h = 5$.

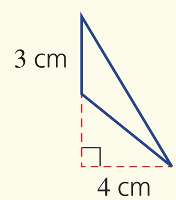
Now you try

Find the area of these triangles.

a

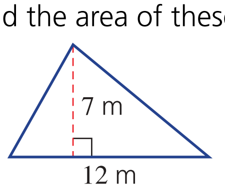


b



7 Find the area of these triangles.

a



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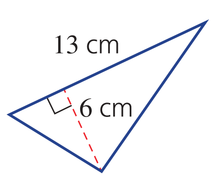

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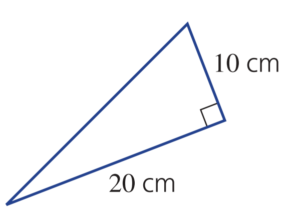
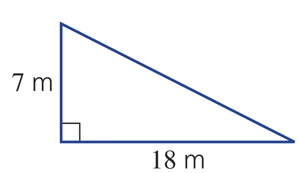
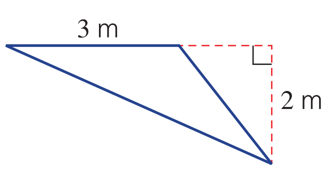


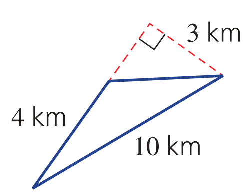
d



e



f



Hint for Q7: Use $A = \frac{1}{2}bh$ and choose the base and height so they are perpendicular (at 90°).

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Problem-solving and reasoning

8–12 | 10–14

8 **a** A rectangular park has a length of 100 m and an area of 5000 m^2. What is its width?
b A parallelogram has an area of 26 m^2 and its base length is 13 m. What is its perpendicular height?
c A triangle has an area of 20 cm^2 and a base of 4 cm. Find its height.

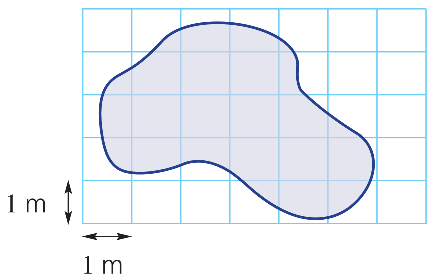
9 Find the side length of a square if its area is:
a 36 m^2
b 2.25 cm^2

10 **a** Find the area of a square if its perimeter is 20 m.
b Find the area of a square if its perimeter is 16 cm.
c Find the perimeter of a square if its area is 49 cm^2.
d Find the perimeter of a square if its area is 169 m^2.

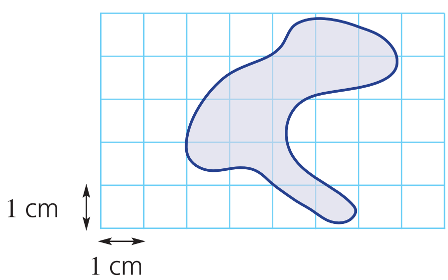
Hint for Q10: First find the side length of the square.

11 Paint costs $12 per litre and can only be purchased in a full number of litres. One litre of paint covers an area of 10 m^2. A rectangular wall is 6.5 m long and 3 m high and needs two coats of paint. What will be the cost of paint for the wall?

12 **a** Estimate the area of the following irregular shapes using the grid provided.
i



ii



b Explain how the accuracy of your answer would change if the grids above used more squares with a smaller area (e.g. using cm^2 for (**i**) or mm^2 for (**ii**)).

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13 An inaccurate measurement can make a big difference when seeking the answer in an area calculation. A square room could be measured as having side length 4 metres, then remeasured in centimetres as 396 cm, then finally measured in millimetres as 3957 mm.

a Find the three areas based on each measurement.

b Calculate the difference in mm^2 between the largest and smallest area.

14 Use your knowledge of area units to change these measurements to the units shown in the brackets.

a $0.2\ m^2$ (mm^2)

b $0.000043\ km^2$ (cm^2)

c $374\,000\ cm^2$ (km^2)

d $10\,920\ mm^2$ (m^2)

e 0.0000002 ha (cm^2)

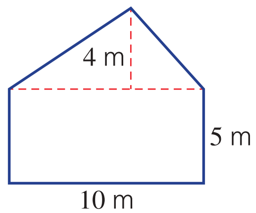
f $1\,000\,000\,000\ mm^2$ (ha)

Composite shapes — 15

15 Find the area of these composite shapes by using addition or subtraction.

Hint for Q15: Divide into two or more shapes then add or subtract.

a



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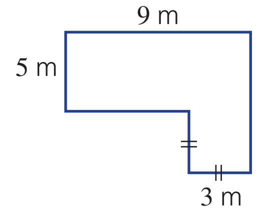

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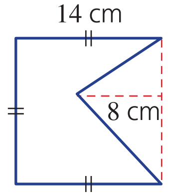

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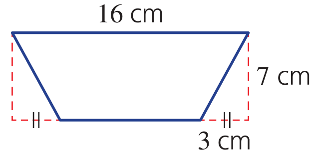

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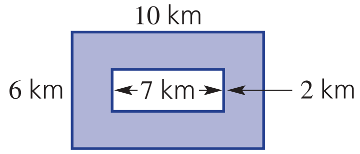
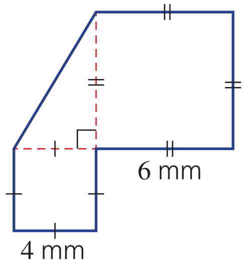


(Find the area of the shaded region.)

f



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4D Area of kites, rhombuses and trapeziums

Learning intentions

- To understand that the area for special quadrilaterals can be determined from the formulas for the area of rectangles and triangles.
- To be able to find the area of rhombuses, kites and trapezia.

Key vocabulary: area, rhombus, kite, trapezium, diagonals (of a quadrilateral)

We have used formulas to work out the area of rectangles ($A = lw$), squares ($A = l^2$), parallelograms ($A = bh$) and triangles ($A = \frac{1}{2}bh$).

In this section, we will develop and use formulas for a set of special quadrilaterals including the rhombus, kite and trapezium.

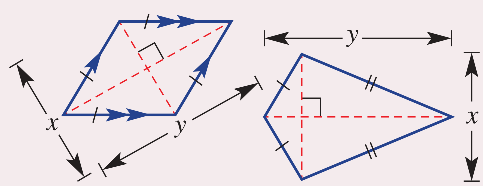
Lesson starter: Develop the rule $A = \frac{1}{2}xy$

The intersecting diagonals of a rhombus form four identical right-angled triangles.

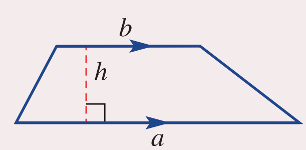
- By cutting along the diagonals and rearranging the triangles, what other shapes can be formed?
- Using the pronumerals x and y as the length of the diagonals, what are the side lengths of your newly formed shape?
- How does this help to explain the rule $A = \frac{1}{2}xy$ for the area of a rhombus?
- Can you apply this thinking to the area rule for a kite?

Key ideas

- **Area** of a **rhombus** and **kite**
  Area $= \frac{1}{2} \times$ **diagonal** $x \times$ diagonal y
  or $A = \frac{1}{2}xy$



- Area of a **trapezium**
  Area $= \frac{1}{2} \times$ sum of parallel sides $\times$ perpendicular height
  or $A = \frac{1}{2}(a+b)h$ or $A = \frac{h}{2}(a+b)$



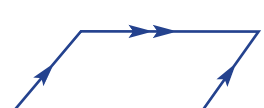
Exercise 4D

Understanding

1–3 | 3

1 Match each formula with a shape.

a $A = lw$ **b** $A = \frac{1}{2}xy$ **c** $A = bh$ **d** $A = \frac{1}{2}(a+b)h$

A


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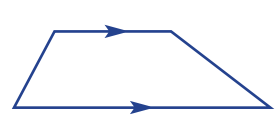

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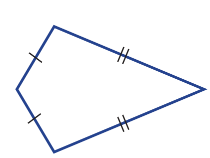

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4D

2 Find the value of A using these formulas and given values. Substitute the given values into the formulas.

a $A = bh\ (b = 2,\ h = 3)$

b $A = \frac{1}{2}xy\ (x = 5,\ y = 12)$

c $A = \frac{1}{2}(a + b)h\ (a = 2,\ b = 7,\ h = 3)$

d $A = \frac{1}{2}(a + b)h\ (a = 7,\ b = 4,\ h = 6)$

Hint for Q3: Choose from: *height, 90, parallel, kite, rhombus, perpendicular.*

3 Complete these sentences.

a Lines that are perpendicular meet at _______ degrees.

b The two diagonals in a kite or a rhombus are ______________.

c To find the area of a trapezium you multiply $\frac{1}{2}$ by the sum of the two ______________ sides and then multiply by the ______________.

d The two special quadrilaterals that have the same area formula using diagonal lengths x and y are the ______________ and the ______________.

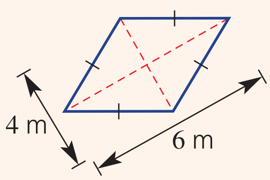
Fluency

4, 5(½) | 4, 5(½)

Example 11 Finding the area of rhombuses and kites

Find the area of the rhombus and kite.

a



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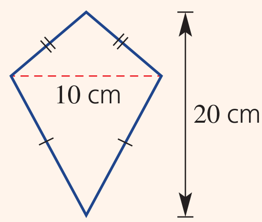
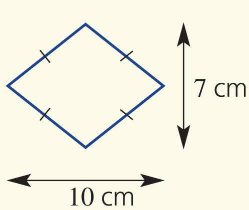


Solution	Explanation
a $A = \frac{1}{2}xy$ $= \frac{1}{2} \times 6 \times 4$ $= 12\ \text{m}^2$	Use $A = \frac{1}{2}xy$ when the diagonals are given with $x = 6$ and $y = 4$ (or vice versa).
b $A = \frac{1}{2}xy$ $= \frac{1}{2} \times 10 \times 20$ $= 100\ \text{cm}^2$	Use the formula $A = \frac{1}{2}xy$ since both diagonals are given. This formula can also be used for a rhombus.

Now you try

Find the area of the rhombus and kite.

a



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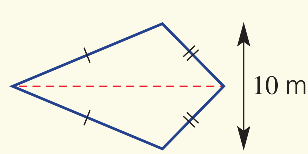

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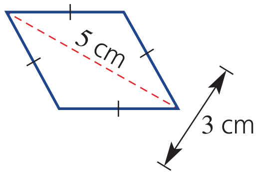
4 Find the area of these rhombuses and kites.

a



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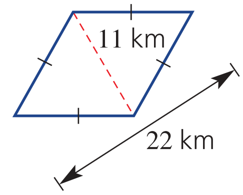

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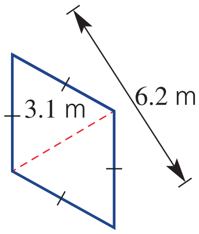
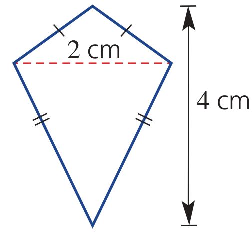


d



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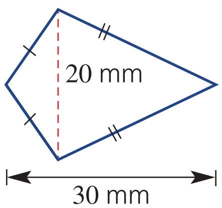

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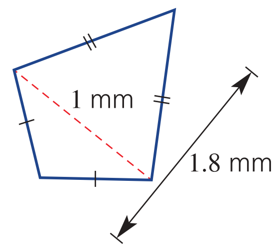
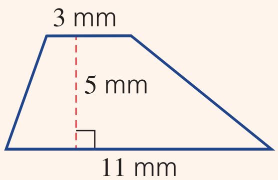


Hint for Q4: Recall that $A = \frac{1}{2}xy$ for both rhombuses and kites with x and y as the diagonals.

Example 12 Finding the area of trapeziums

Find the area of this trapezium.



Solution

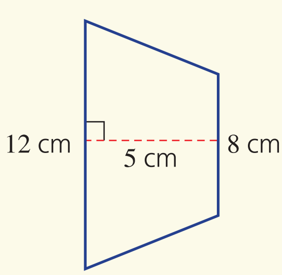
$$A = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2} \times (11 + 3) \times 5$$
$$= \frac{1}{2} \times 14 \times 5$$
$$= 35\ \text{mm}^2$$

Explanation

The two parallel sides are 11 mm and 3 mm in length. The perpendicular height is 5 mm.

Now you try

Find the area of this trapezium.



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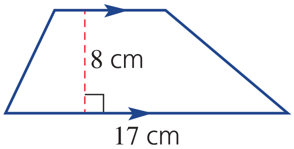
4D

5 Find the area of these trapeziums.

a


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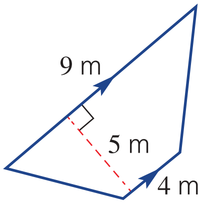

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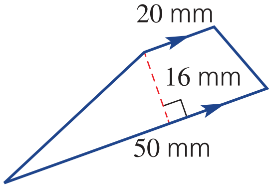

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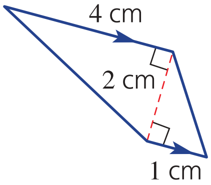
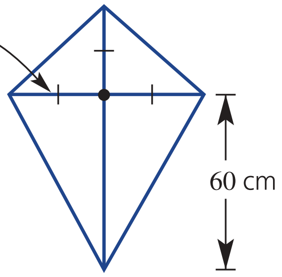


Hint for Q5: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides.

Problem-solving and reasoning

6–8 | 6–9

6 A flying kite is made from four centre rods all connected near the middle of the kite as shown. Three of the rods are 30 cm in length and one is 60 cm as shown. What area of plastic, in square metres, is needed to cover the kite?



7 A landscape gardener charges \$20 per square metre of lawn. A lawn area is in the shape of a rhombus and its diagonals are 8 m and 14.5 m. What would be the cost of laying this lawn?

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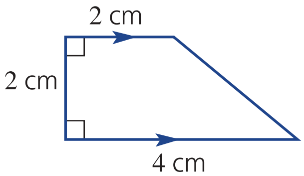
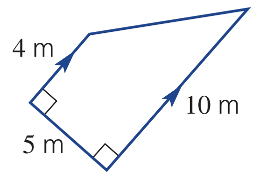
8 These trapeziums have one side at right angles to the two parallel sides. Find the area of each.

a



b



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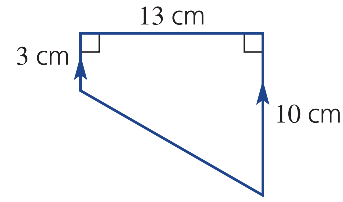
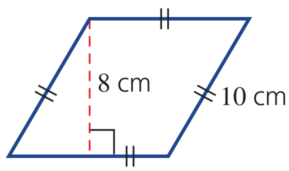


9 Would you use the formula $A = \frac{1}{2}xy$ to find the area of this rhombus? Explain.



Proving formulas — 10

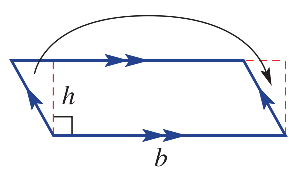
10 Copy and complete these proofs to give the formula for the area of a parallelogram, a rhombus and a trapezium.

a Parallelogram

$A = \text{length} \times \text{width}$

$= \_\_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$



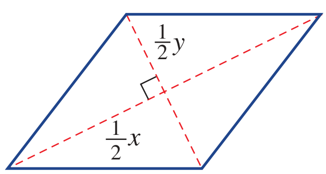
b Rhombus

$A = 4 \text{ triangle areas}$

$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 4 \times \frac{1}{2} \times \_\_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$



c Trapezium

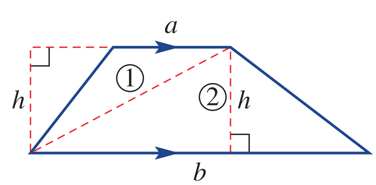
$A = \text{Area (triangle 1)} + \text{Area (triangle 2)}$

$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$

$= \frac{1}{2} \times \_\_\_\_\_\_ \times \_\_\_\_\_\_ + \frac{1}{2} \times \_\_\_\_\_\_ \times \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$



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4E Area of a circle

Learning intentions

- To be able to find the area of a circle given its radius or diameter.
- To understand how to find the area of a semicircle or quadrant by multiplying a circle's area by $\frac{1}{2}$ or $\frac{1}{4}$.

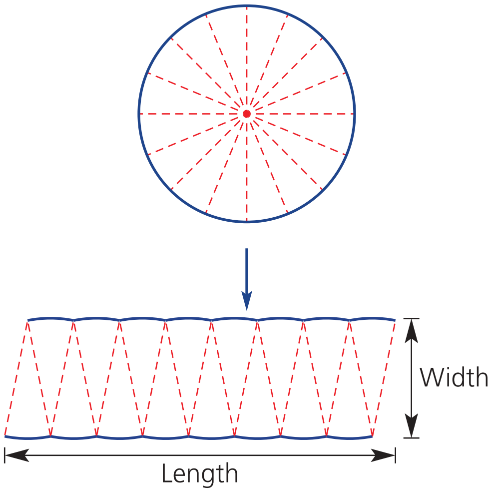
Key vocabulary: circle, pi π, semicircle, quadrant

Like the circumference of a circle, the area of a circle is linked to the number pi (π).

One way to consider the area of a circle is to divide it into sectors, then arrange them into a rectangular shape. If very thin sectors are used, then the arrangement will be close to a rectangle with a length that is half the circumference of the circle, or $\frac{1}{2} \times 2\pi r = \pi r$ and width r.

This leads to the area formula: $A = \text{length} \times \text{width}$

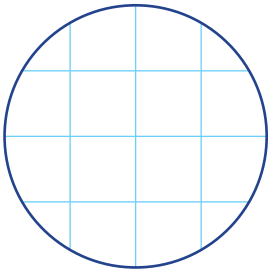
$$= \pi r \times r$$
$$= \pi r^2$$



Lesson starter: Just count squares

To find an estimate for the area of a circle you can count the number of squares.

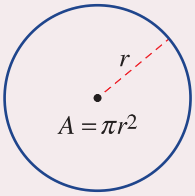
- Count squares to estimate the area of this circle in cm^2.
- Ask your teacher to give you an accurate measure of its area. Who was the closest?



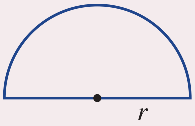
Key ideas

- The area of a **circle** is given by the formula $A = \pi r^2$.
  - The diameter is twice the radius: $d = 2r$
  - Substitute the radius into the formula to find the area.
    e.g. If $r = 2$

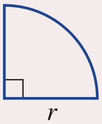
$$A = \pi \times 2^2$$
$$= \pi \times 4$$
$$= 12.57 \text{ (to 2 d.p.)}$$



- A half circle is called a **semicircle**.
  $A = \frac{1}{2}\pi r^2$



- A quarter circle is called a **quadrant**.
  $A = \frac{1}{4}\pi r^2$



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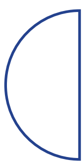
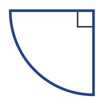
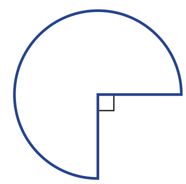
Exercise 4E

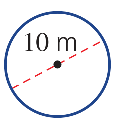
Understanding

Understanding	1–4	4

1 Write the rule for:
a the circumference of a circle.
b the area of a circle.

2 Use a calculator to evaluate these, correct to two decimal places.
a $\pi \times 5^2$
b $\pi \times 13^2$
c $\pi \times 3.1^2$
d $\pi \times 9.8^2$

3 What fraction of a full circle is shown here?
a

b

c


4 What is the length of the radius in these shapes?
a


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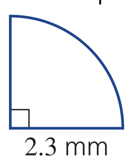

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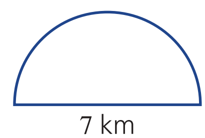


Fluency

Fluency	5, 6(½)	5, 6(½), 7

Example 13 Finding circle areas using a radius

Find the area of this circle, correct to two decimal places.

Solution

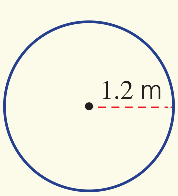
$A = \pi r^2$
$= \pi \times 4^2$
$= 50.27\ \text{cm}^2$ (to 2 d.p.)

Explanation

Use the π button on the calculator and enter $\pi \times 4^2$ or $\pi \times 16$.

Now you try

Find the area of this circle, correct to two decimal places.



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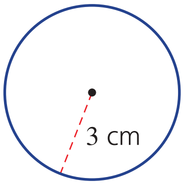
4E

5 Find the area of these circles, correct to two decimal places.

a



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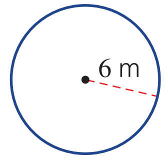
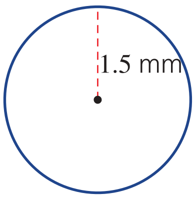


Hint for Q5: Substitute the radius for r in $A = \pi r^2$.

c



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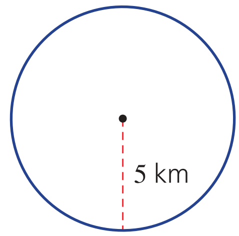
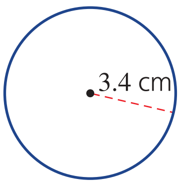


e



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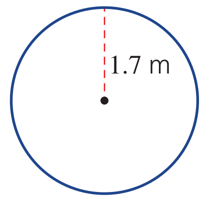
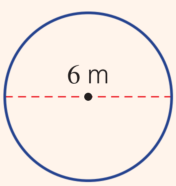


Example 14 Finding circle areas using a diameter

Find the area of this circle, correct to two decimal places.



Solution

$r = d \div 2$
$= 6 \div 2$
$= 3$

$A = \pi r^2$
$= \pi \times 3^2$
$= 28.27 \text{ m}^2$ (to 2 d.p.)

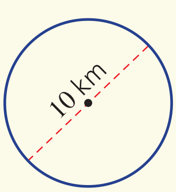
Explanation

First work out the radius as half of the diameter.

Substitute $r = 3$ into the rule, then round to two decimal places.

Now you try

Find the area of this circle, correct to two decimal places.



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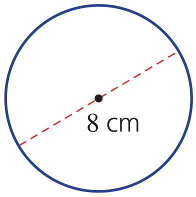
6 Find the area of these circles, correct to two decimal places.

Hint for Q6: First work out the radius.

a



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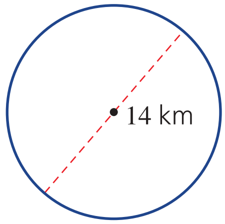
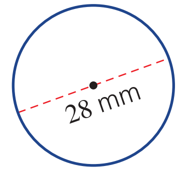
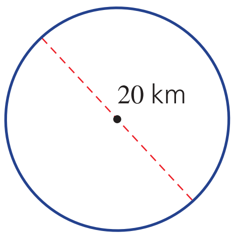


c



d



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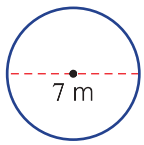

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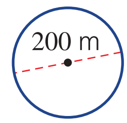
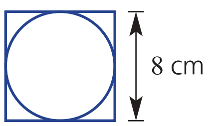


7 Find the area of the circle inside these shapes. Round to two decimal places.

a



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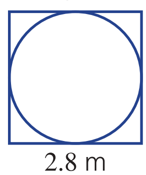

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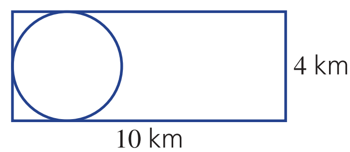

Problem-solving and reasoning

8–10 | 10–13

8 A circular pizza tray has a diameter of 30 cm. Calculate its area to the nearest whole number of cm^2.

9 A tree trunk is cut to show a circular cross-section of radius 60 cm. Is the area of the cross-section more than 1 m^2? If so, by how much? Round your answer to the nearest whole number of cm^2.

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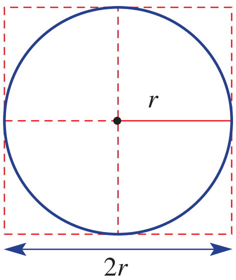
4E

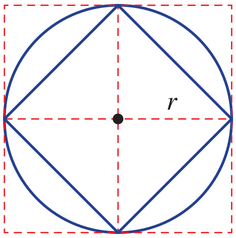
10 In this question, you will be finding an upper bound and lower bound for π.

a Use the diagram to explain why the area of a circle of radius r must be less than $4r^2$.



b Use the diagram to explain why the area of a circle of radius r must be more than $2r^2$.

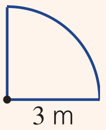
c Is the true area of a circle greater than, less than or equal to the average of the two squares' areas?



Example 15 Finding the area of quadrants and semicircles

Find the area of this quadrant and semicircle, correct to two decimal places.

a



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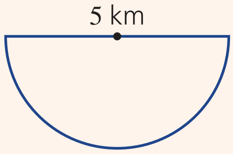


Solution

a $A = \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \pi \times 3^2$

$= 7.07 \text{ m}^2$ (to 2 d.p.)

Explanation

The area of a quadrant is $\frac{1}{4}$ the area of a circle with the same radius.

b $r = \frac{5}{2} = 2.5$

$A = \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 2.5^2$

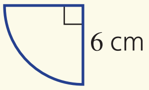
$= 9.82 \text{ km}^2$ (to 2 d.p.)

The radius is half the diameter.
The area of a semicircle is $\frac{1}{2}$ the area of a circle with the same radius.

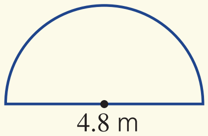
Now you try

Find the area of this quadrant and semicircle, correct to two decimal places.

a



b



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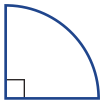
11 Find the area of these quadrants and semicircles, correct to two decimal places.

a



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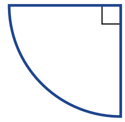

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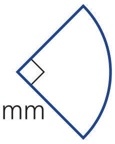
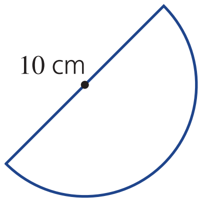


Hint for Q11: The radius is half the diameter.

d



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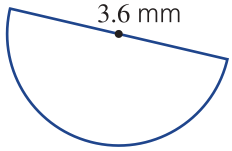

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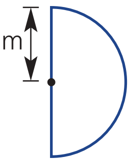

12 Two circular plates have radii 12 cm and 13 cm. Find the difference in their area, correct to two decimal places.

13 A square of side length 10 cm has a hole in the middle. The diameter of the hole is 5 cm. What is the area remaining? Round the answer to the nearest whole number.

$A = \pi r^2$ in reverse

	—	14

14 Reverse the rule $A = \pi r^2$ to find the radius in these problems.

a If $A = 10$, use your calculator to show that $r \approx 1.78$.

b Find the radius of circles with these areas. Round the answer to two decimal places.

i 17 m^2 **ii** 4.5 km^2 **iii** 320 mm^2

c Can you write a rule for r in terms of A? Check that it works for the circles defined in part **b**.

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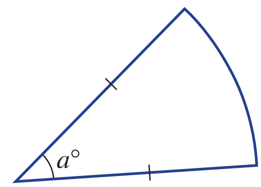
4F Area of sectors and composite shapes

Learning intentions

- To know what a sector is
- To understand that a sector's area can be found by taking a fraction of the area of a circle with the same radius
- To be able to find the area of a sector given its radius and the angle at the centre
- To be able to find the area of composite shapes involving sectors

Key vocabulary: sector, composite shape, radii

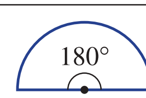
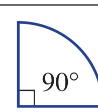
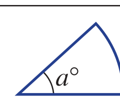
A slice of pizza or a portion of a round cake cut from the centre forms a shape called a sector. The area cleaned by a windscreen wiper could also be thought of as a difference of two sectors with the same angle but different radii. Clearly the area of a sector depends on its radius, but it also depends on the angle between the two straight edges.



A computer hard drive disk stores data on concentric circular tracks that are divided into track sectors. Each track sector stores 512 bytes of data and its area is the difference in area between two geometric sectors. The engineers of IBM at the San Jose California Laboratory are the ones who invented the hard drive in the year 1953.

Lesson starter: The sector area formula

Complete this table to develop the rule for finding the area of a sector.

Angle	Fraction of area	Area rule	Diagram
180°	$\frac{180}{360} = \frac{1}{2}$	$A = \frac{1}{2} \times \pi r^2$	
90°	$\frac{90}{360} =$ ________	$A =$ ___ $\times \pi r^2$	
45°	$\frac{45}{360} =$ ________		
30°			
a°		$A =$ ___ $\times \pi r^2$	

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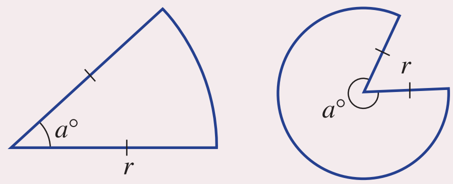
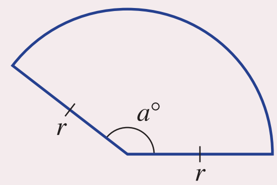
Key ideas

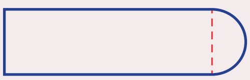
- A **sector** is formed by dividing a circle with two radii.



- A sector's area is determined by calculating a fraction of the area of a circle with the same radius.
  - Fraction is $\frac{a}{360}$
  - Sector area $= \frac{a}{360} \times \pi r^2$



- The area of a **composite shape** can be found by adding or subtracting the areas of more basic shapes.



$A = lw + \frac{1}{2}\pi r^2$

Exercise 4F

Understanding

1–3 | 2, 3

1 Simplify these fractions.

a $\frac{180}{360}$

b $\frac{90}{360}$

c $\frac{60}{360}$

d $\frac{45}{360}$

2 Evaluate the following using a calculator. Give your answer correct to two decimal places.

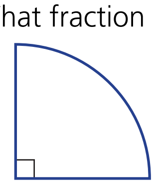
a $\frac{80}{360} \times \pi \times 2^2$

b $\frac{20}{360} \times \pi \times 7^2$

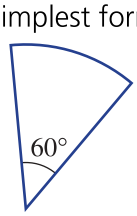
c $\frac{210}{360} \times \pi \times 2.3^2$

3 What fraction of a circle in simplest form is shown by these sectors?

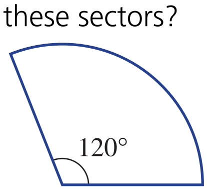
a

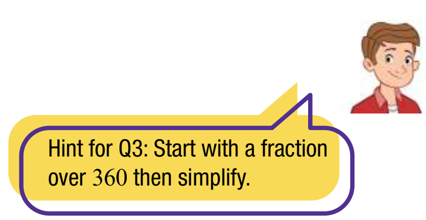


b



c





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4F

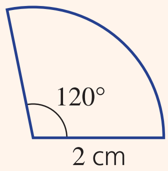
Fluency

4, 5–6(½) 5–6(½)

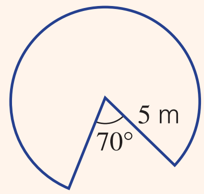
Example 16 Finding areas of sectors

Find the area of these sectors correct to two decimal places.

a



b



Solution

a

$$A = \frac{a}{360} \times \pi r^2$$
$$= \frac{120}{360} \times \pi \times 2^2$$
$$= \frac{1}{3} \times \pi \times 4$$
$$= 4.19\,\text{cm}^2 \ \ (\text{to 2 d.p.})$$

Explanation

First, write the rule for the area of a sector.

Substitute $a = 120$ and $r = 2$.

Note that $\frac{120}{360}$ simplifies to $\frac{1}{3}$.

b

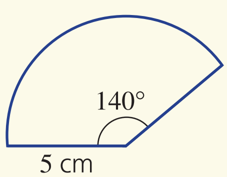
$$a = 360 - 70 = 290$$
$$A = \frac{a}{360} \times \pi r^2$$
$$= \frac{290}{360} \times \pi \times 5^2$$
$$= 63.27\,\text{m}^2 \ \ (\text{to 2 d.p.})$$

First, calculate the angle inside the sector and remember that a revolution is 360°. Then substitute $a = 290$ and $r = 5$.

Now you try

Find the area of these sectors, correct to two decimal places.

a



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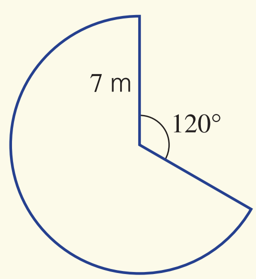
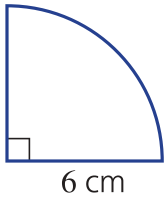


4 Find the area of these sectors, correct to one decimal place.

a



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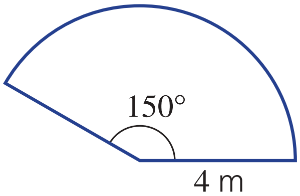

Hint for Q4: Start with $\frac{a}{360} \times \pi r^2$.

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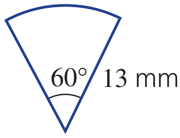
5 Find the area of these sectors, correct to two decimal places.

a


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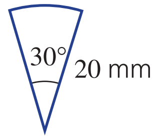

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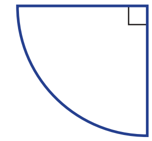

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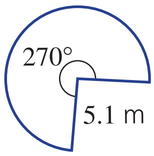

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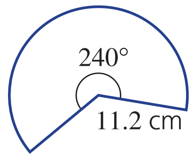

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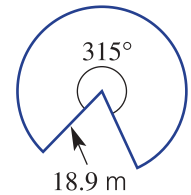
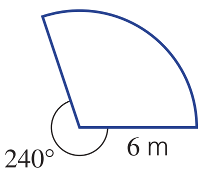


6 Find the area of these sectors, correct to two decimal places.

a


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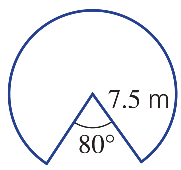

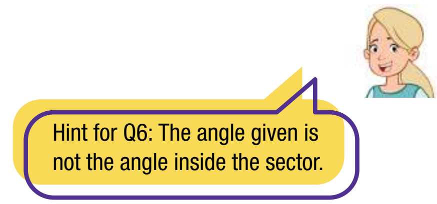

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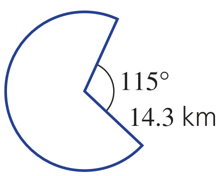
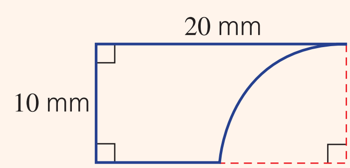


Example 17 Finding areas of composite shapes

Find the area of this composite shape, correct to the nearest whole number of mm^2.



Solution

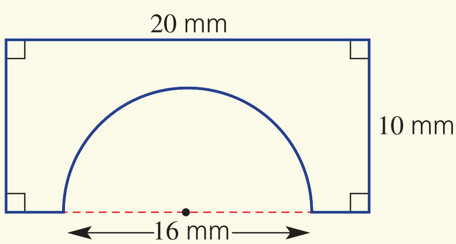
$$A = lw - \frac{1}{4}\pi r^2$$
$$= 20 \times 10 - \frac{1}{4} \times \pi \times 10^2$$
$$= 200 - 25\pi$$
$= 121\ mm^2$ (to nearest whole number)

Explanation

The area can be found by subtracting the area of a quadrant from the area of a rectangle.

Now you try

Find the area of this composite shape, correct to two decimal places.



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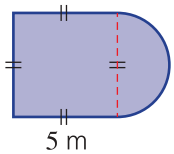
4F

7 Find the areas of these composite shapes using addition or subtraction. Round the answer to two decimal places.

a



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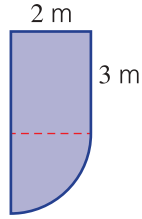

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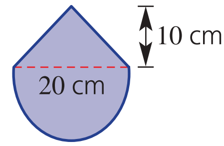

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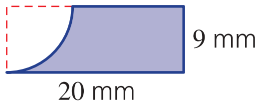

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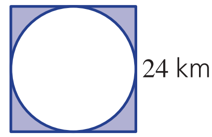

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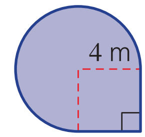

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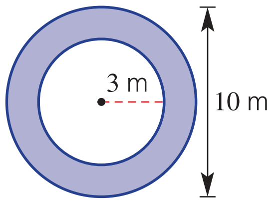

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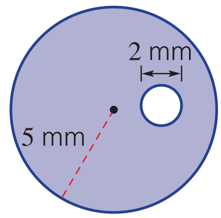

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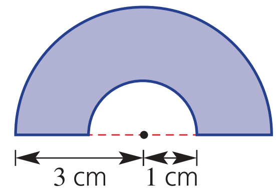
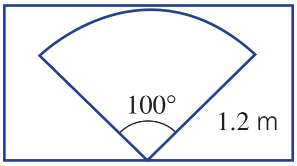


Problem-solving and reasoning

8–10 | 9–11

8 A simple bus wiper blade wipes an area over 100° as shown. Find the area wiped by the blade, correct to two decimal places.



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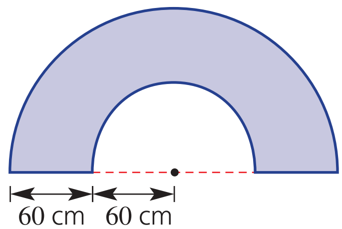
9 At Buy-by-the-sector Pizza they offer a sector of a 15 cm radius pizza with an angle of 45° or a sector of a 13 cm radius pizza with an angle of 60°. Which piece gives the bigger area and by how much? Round the answer to two decimal places.

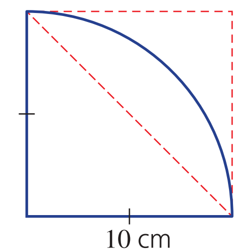
10 An archway is made up of an inside and outside semicircle as shown. Find the area of the arch, correct to the nearest whole cm^2.



11 Consider the sector shown.

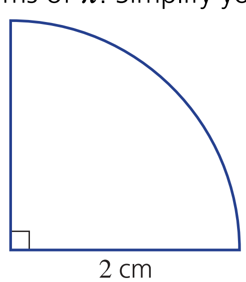
a Use the dashed lines to explain why the sector's area is between 50 cm^2 and 100 cm^2.

b The average of 50 and 100 is 75. How close is the sector's area to 75 cm^2? Answer correct to one decimal place.



Exact areas — 12

12 An exact area measure in terms of π might look like $\pi \times 2^2 = 4\pi$. Find the exact area of these shapes in terms of π. Simplify your answer.

a


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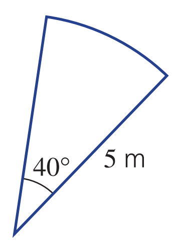
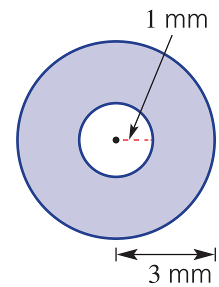


c Find the shaded area.


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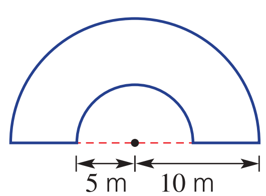

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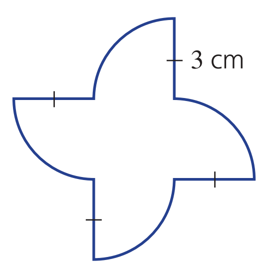

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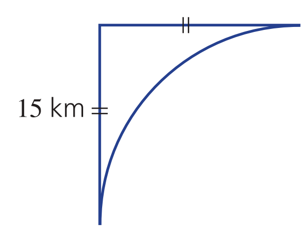

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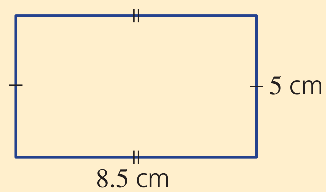
Progress quiz

4A **1** Convert these measurements to the units shown in the brackets.

a 12 cm (m)
b 585 mm (cm)
c 6.2 m (mm)
d 2.57 km (m)

4A **2** Find the perimeter of these shapes.

a



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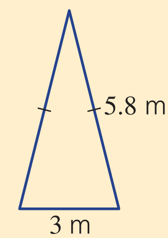

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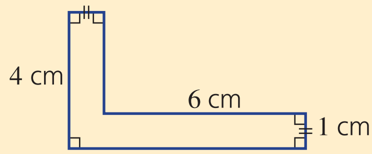

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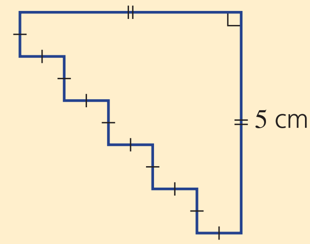
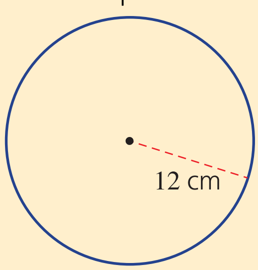


4B **3** Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of pi.

a



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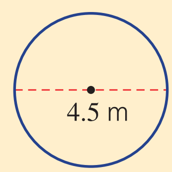


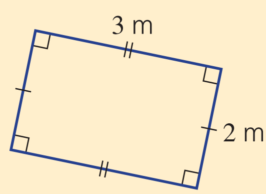
4B **4** Find the perimeter of a mathematics protractor, which is the shape of a semicircle, with a base of 14 cm. Round the answer to two decimal places.

4B **5** Convert these area measurements to the units shown in the brackets.

a 7 cm^2 (mm^2)
b 4500 mm^2 (cm^2)
c 0.0034 m^2 (mm^2)
d 30 km^2 (ha)

4C **6** Find the area of the following shapes.

a



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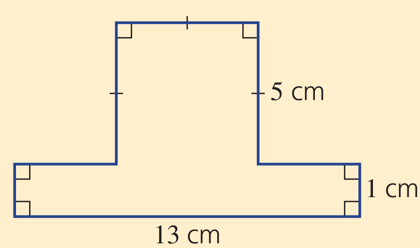

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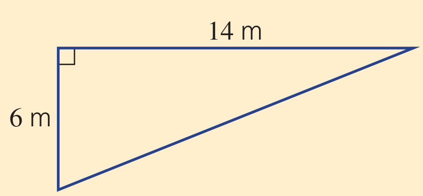

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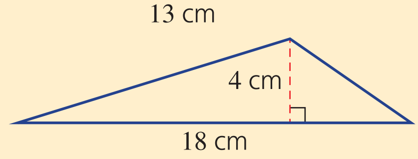

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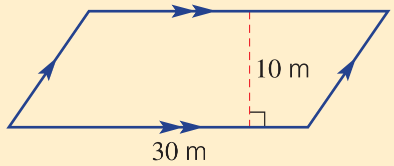
4D **7** Find the area of the following special quadrilaterals.

a



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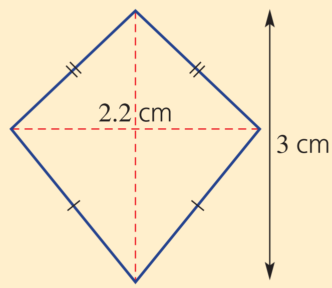

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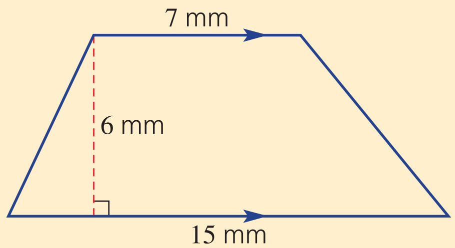

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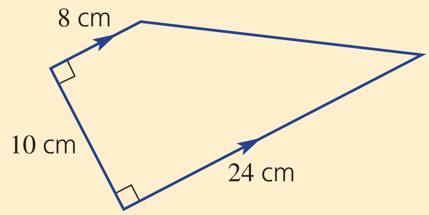
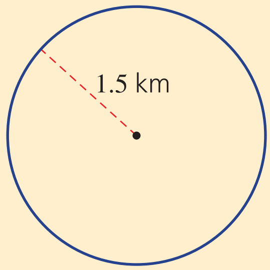


4D **8** A parallelogram has an area of $40\ \text{m}^2$ and its perpendicular height is 8 m. What is the length of its base?

4E **9** Find the area of these circles, correct to two decimal places.

a



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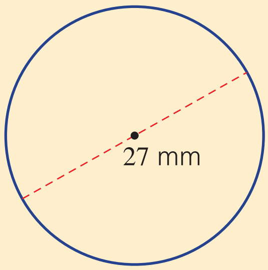
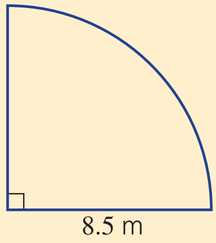


4E **10** Find the area of this quadrant and semicircle, correct to two decimal places.

a



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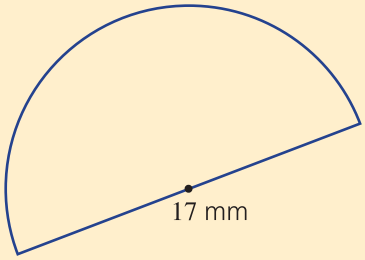
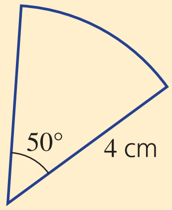


4F **11** Find the area of these composite shapes, correct to two decimal places.

a



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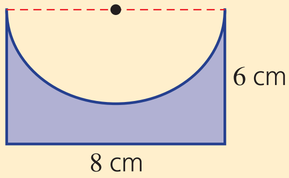

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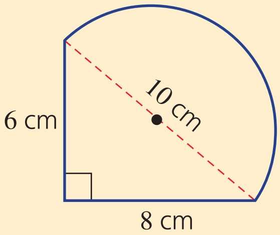

Progress quiz

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4G Volume and capacity

Learning intentions

- To understand that volume is the space occupied by a three-dimensional object.
- To understand that capacity is the volume of fluid or gas that a container can hold.
- To be able to convert between units for volume and capacity.
- To be able to find the volume of rectangular prisms, including cubes.

Key vocabulary: volume, capacity, rectangular prism, cube

Volume is a measure of the space occupied by a three-dimensional object. It is measured in cubic units. Common metric units for volume given in abbreviated form include mm^3, cm^3, m^3 and km^3. We also use mL, L, kL and ML to describe volumes of fluids or gas. The volume of space occupied by a room in a house, for example, might be calculated in cubic metres (m^3), and the capacity of a fuel tanker might be measured in litres (L) or kilolitres (kL).

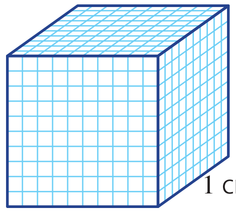
The capacity of a fuel tanker could be measured in litres (L) or kilolitres (kL).

Lesson starter: Why are there $1000\ mm^3$ in $1\ cm^3$?

Shown here is a 1-cm cube (not to scale) that is also divided up into cubes.

- How many $1\ mm^3$ blocks sit along one edge?
- How many $1\ mm^3$ blocks sit on one layer?
- How many layers of $1\ mm^3$ blocks make up the full $1\ cm^3$?
- Now try to explain why there are $1000\ mm^3$ in $1\ cm^3$.
- How many cm^3 are in $1\ m^3$? How many m^3 are there in $1\ km^3$?

Give reasons.



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Key ideas

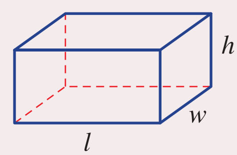
- **Volume** is measured in cubic units. Common metric units are:
  - cubic millimetres (mm^3)
  - cubic centimetres (cm^3)
  - cubic metres (m^3)
  - cubic kilometres (km^3)

$$\text{km}^3 \underset{\div 1000^3}{\overset{\times 1000^3}{\rightleftarrows}} \text{m}^3 \underset{\div 100^3}{\overset{\times 100^3}{\rightleftarrows}} \text{cm}^3 \underset{\div 10^3}{\overset{\times 10^3}{\rightleftarrows}} \text{mm}^3$$

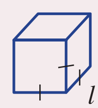
- **Capacity** is the volume of fluid or gas that a container can hold. Common metric units are:
  - millilitre (mL)
  - litre (L)
  - kilolitre (kL)
  - megalitre (ML)

$$\text{ML} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \text{kL} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \text{L} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \text{mL}$$

- Some common conversions are:
  - $1\text{ mL} = 1\text{ cm}^3$
  - $1\text{ L} = 1000\text{ mL} = 1000\text{ cm}^3$
  - $1\text{ kL} = 1000\text{ L} = 1\text{ m}^3$
- Volume of a **rectangular prism**
  - Volume = length × width × height $V = lwh$



- Volume of a **cube** $V = l^3$



Exercise 4G

Understanding

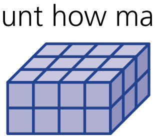
1–3 | 3

1 State if the following are units for length, area or volume.

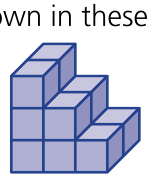
a cm **b** cm^2 **c** cm^3 **d** mm^2 **e** m^3
f mm **g** km^2 **h** mm^3 **i** m^2 **j** km

2 Count how many cubic units are shown in these cube stacks.

a



b



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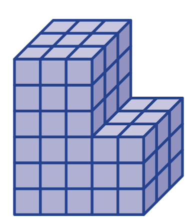

3 Write the missing number in the following unit conversions.

a $1\text{ L} = \_\_\_\text{ mL}$ **b** $\_\_\_\text{kL} = 1000\text{ L}$ **c** $1000\text{ kL} = \_\_\_\text{ ML}$
d $1\text{ mL} = \_\_\_\text{ cm}^3$ **e** $1000\text{ cm}^3 = \_\_\_\text{ L}$ **f** $1\text{ cm}^3 = \_\_\_\text{ mm}^3$

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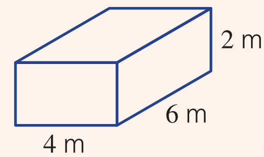
4G

Fluency

4–6(½) 4–6(½)

Example 18 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



Solution

$V = lwh$

$= 6 \times 4 \times 2$

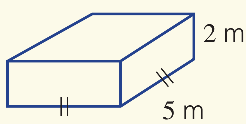
$= 48\ \text{m}^3$

Explanation

First write the rule and then substitute for the length, width and height. Any order will do since $6 \times 4 \times 2 = 4 \times 6 \times 2 = 2 \times 4 \times 6$ etc.

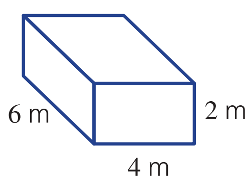
Now you try

Find the volume of this rectangular prism.



4 Find the volume of these rectangular prisms.

a



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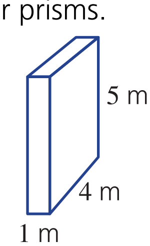
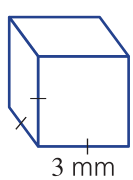


Hint for Q4: Use $V = lwh$ or use $l^3 = l \times l \times l$ for cubes.

c



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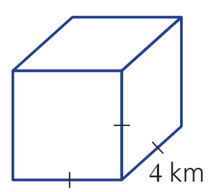

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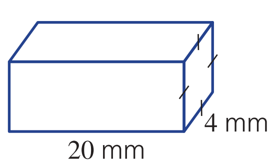

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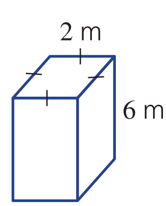

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Example 19 Converting units

Convert these measurements to the units in the brackets.

a 0.5 L (mL)
b 6400 kL (ML)
c 3500 cm^3 (L)

Solution	Explanation
a $0.5 \text{ L} = 0.5 \times 1000$ $= 500 \text{ mL}$	There are 1000 mL in 1 L so multiply by 1000.
b $6400 \text{ kL} = 6400 \div 1000$ $= 6.4 \text{ ML}$	1 ML = 1000 kL and ML is the larger unit so divide by 1000.
c $3500 \text{ cm}^3 = 3500 \div 1000$ $= 3.5 \text{ L}$	1 L = 1000 mL and 1 mL = 1 cm^3, so 1 L = 1000 cm^3.

Now you try

Convert these measurements to the units in the brackets.

a 750 mL (L)
b 0.04 kL (L)
c 0.37 L (cm^3)

5 Convert the measurements to the units shown in the brackets.

a 2 L (mL)
b 5 kL (L)
c 0.5 ML (kL)
d 3000 mL (L)
e 4 mL (cm^3)
f 50 cm^3 (mL)
g 2500 cm^3 (L)
h 5.1 L (cm^3)
i 1 m^3 (L)

Hint for Q5:
1 L = 1000 mL = 1000 cm^3
1 kL = 1000 L
1 ML = 1000 kL

Example 20 Finding capacity

Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.

Solution	Explanation
$V = lwh$ $= 20 \times 10 \times 15$ $= 3000 \text{ cm}^3$ $3000 \div 1000 = 3 \text{ L}$	First calculate the volume of the container in cm^3. Then convert to litres using 1 L = 1000 cm^3.

Now you try

A fish tank is 50 cm long, 40 cm wide and 30 cm high. Find the capacity in litres.

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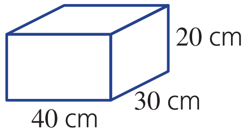
4G

6 Find the capacity of these containers, converting your answer to litres.

a


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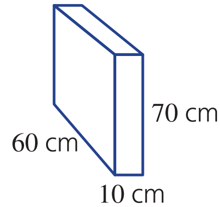

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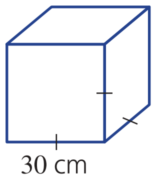

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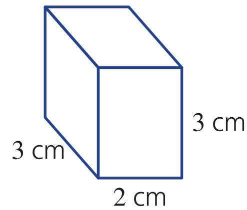

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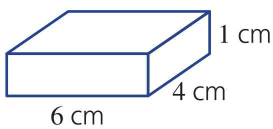

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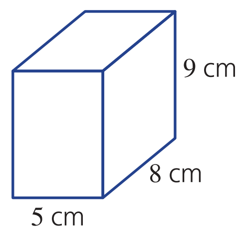
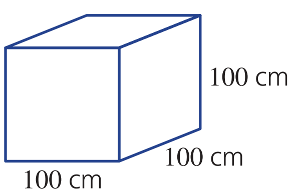


Hint for Q6: First find the volume in cm^3 using $V = lwh$, then divide by 1000 to convert to litres.

Problem-solving and reasoning

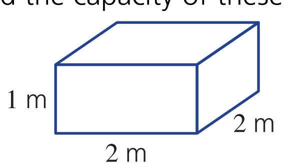
7–9 | 8(½), 9–12

7 Here is a 1 m cube with each edge 100 cm.



a Find its volume in cm^3.
b Complete this statement: $1\ m^3 =$ __________ cm^3
c How many cm^3 make 1 L?
d Complete this statement: $1\ m^3 =$ __________ L

8 Find the capacity of these rectangular prisms in litres.

a


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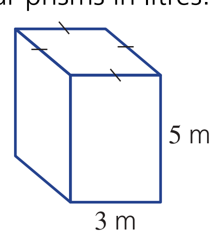

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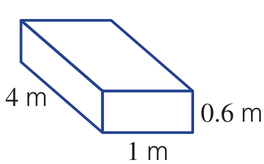

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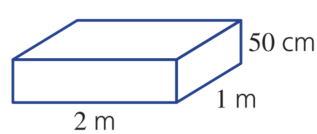

Hint for Q8: First find the volume in m^3, then use $1\ m^3 = 1000$ L.

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9 An oil tanker has a capacity of $60\,000\ \text{m}^3$.

a What is the ship's capacity in:

i litres?

ii kilolitres?

iii megalitres?

b If the ship leaks oil at a rate of 300 000 litres per day, how long will it take for all the oil to leak out?

Hint for Q9: $1\ \text{m}^3 = 1000$ L.

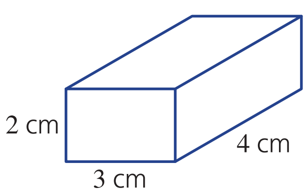
10 If 1 kg is the mass of 1 L of water, what is the mass of water in a full container that is a cube with side length 2 m?

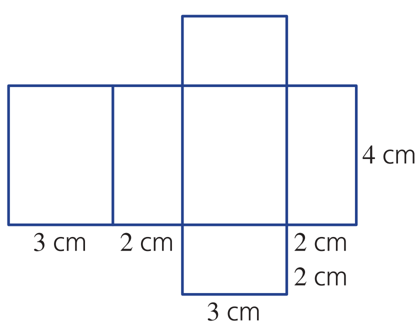
11 Water is being poured into a fish tank at a rate of 2 L every 10 seconds. The tank is 1.2 m long by 1 m wide by 80 cm high. How long will it take to fill the tank? Give the answer in minutes.

12 How many cubic containers (with side lengths that are a whole number of centimetres) have a capacity of less than 1 litre?

Surface area of rectangular prisms — 13

13 You can find the total surface area of solids by adding all the areas of each outside surface. Here is an example.





Total surface area

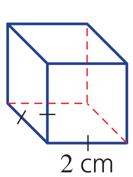
$= 2 \times (3 \times 4) + 2 \times (3 \times 2) + 2 \times (2 \times 4)$

$= 24 + 12 + 16$

$= 52\ \text{cm}^2$

Find the surface area of these rectangular prisms.

a



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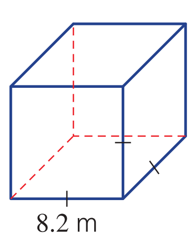

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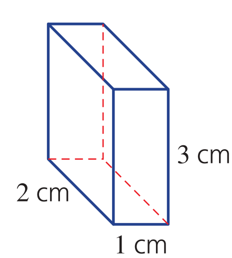

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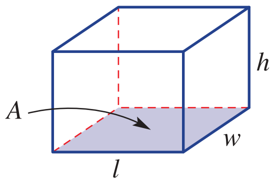
4H Volume of prisms

Learning intentions

- To understand what a cross-section of a prism is.
- To be able to find the volume of a prism given the area and height of its cross-section.
- To be able to find the volume of a prism by first calculating the area of the cross-section.

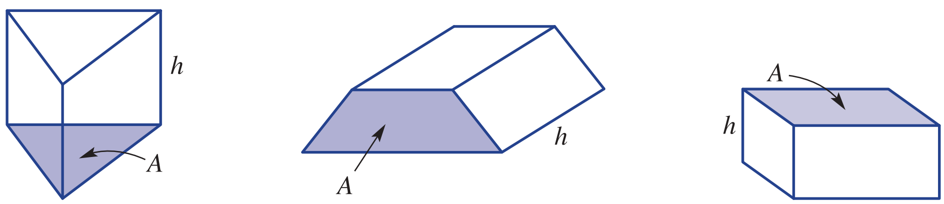
Key vocabulary: volume, prism, right prism, cross-section, perpendicular height

We know that for a rectangular prism, its volume V is given by the rule $V = lwh$. Length × width (lw) gives the area of the base A. So $V = lwh$ could also be written as $V = Ah$.



The rule $V = Ah$ can also be applied to prisms that have different shapes as their bases. One condition, however, is that the area of the base must represent the area of the cross-section of the solid. The height h is measured perpendicular to the cross-section.

Here are some examples of prisms with A and h marked.



The height of a building is measured perpendicular to the ground. The hight of the Gold Coast's Q1 tower is 322.5 meters. The Q1 building is currently the tallest skyscraper in Australia and the 6th tallest in the world.

Lesson starter: Drawing prisms

Try to draw prisms that have the following shapes as their cross-sections.

- Rectangle
- Trapezium
- Parallelogram
- Triangle
- Pentagon
- Kite

The cross-section of a prism should be the same size and shape along the entire length of the prism. Check this property on your drawings.

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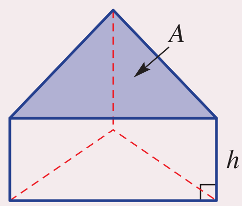
Key ideas

- A **prism** is a solid with a constant (uniform) **cross-section**.
  - Its sides between the two congruent ends are parallelograms.
  - A **right prism** has rectangular sides between the congruent ends.
- **Volume** of a prism = Area of **cross-section** × **perpendicular height** or $V = Ah$.



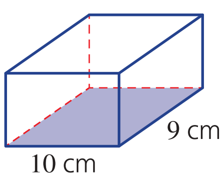
$V = Ah$

Exercise 4H

Understanding 1–3 | 3

1 What is the name of the shape of the cross-section in these prisms (shaded)?

a



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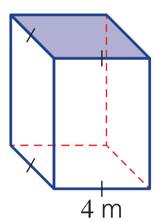

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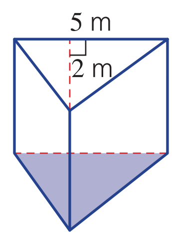


2 What is the area of the shaded cross-sections in Question **1**? You will need the formulas: $A = lw$, $A = l^2$ and $A = \frac{1}{2}bh$.

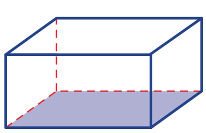
3 For these solids below:

i state whether or not it is a prism

ii if it is a prism, state the shape of its cross-section.

Hint for Q3: Prisms must have constant cross-sections.

a



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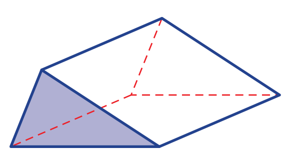

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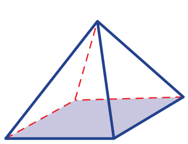

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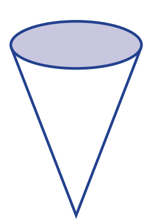

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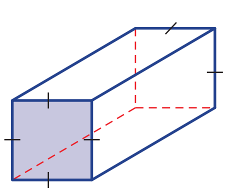

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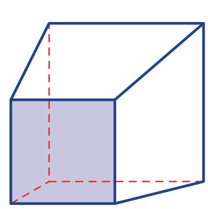

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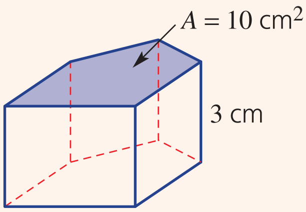
4H

Fluency

4(½), 5 | 4, 5(½), 6

Example 21 Finding the volumes of prisms given the cross-section

Find the volume of this prism using $V = Ah$.



Solution

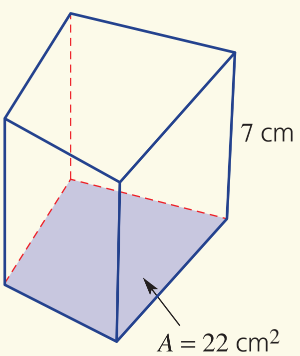
$$\begin{aligned} V &= Ah \\ &= 10 \times 3 \\ &= 30\text{ cm}^3 \end{aligned}$$

Explanation

Write the rule and substitute the given values of A and h, where A is the area of the cross-section.

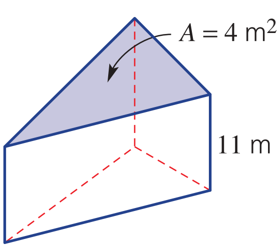
Now you try

Find the volume of this prism using $V = Ah$.

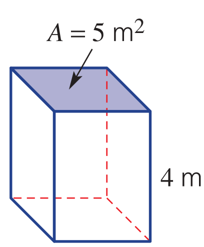


4 Find the volume of these solids using $V = Ah$.

a



b



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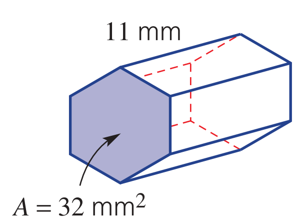

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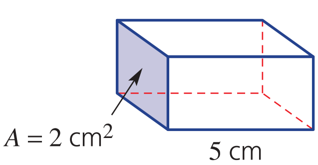

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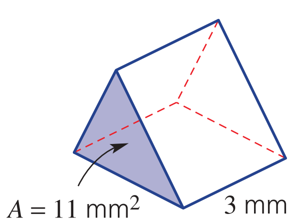

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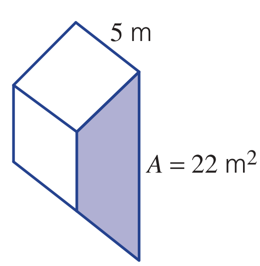

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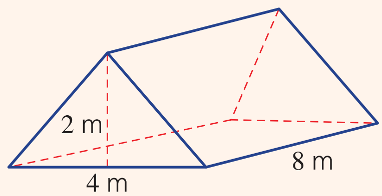
Example 22 Finding the volume of prisms

Find the volume of this prism.



Solution

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 4 \times 2$$
$$= 4 \text{ m}^2$$

$$V = Ah$$
$$= 4 \times 8$$
$$= 32 \text{ m}^3$$

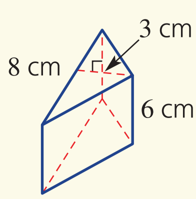
Explanation

The cross-section is a triangle, so use $A = \frac{1}{2}bh$ with base 4 m and height 2 m.

Then multiply by 8 using $V = Ah$, with $h = 8$.

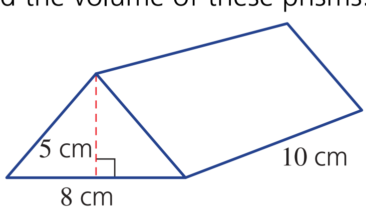
Now you try

Find the volume of this prism.



5 Find the volume of these prisms.

a



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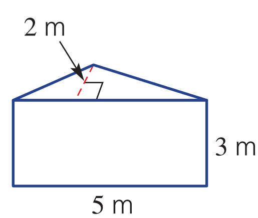
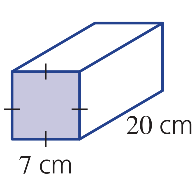


Hint for Q5: First find the area of the cross-section then multiply by h.

c



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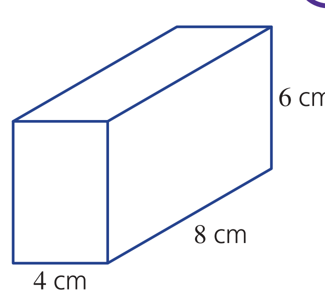

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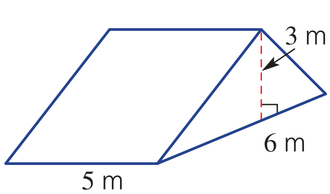

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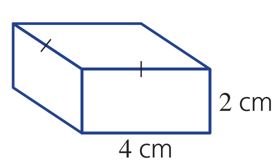

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6 A rectangular drain pipe has a cross-sectional area of 4 m^2 and is 10 m long. Find its volume.

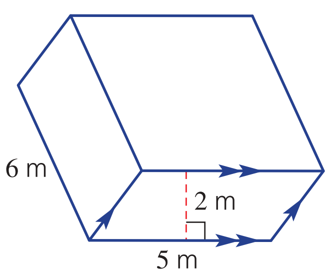
A capsule hotel has rows of small rectangular prism shaped bedrooms.

Problem-solving and reasoning

Problem-solving and reasoning	7	7(½), 8

7 These solids have cross-sections which are parallelograms, trapeziums, rhombuses or kites. Find their volume.

a



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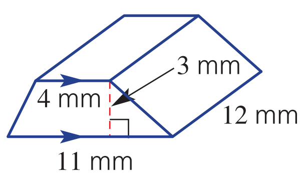
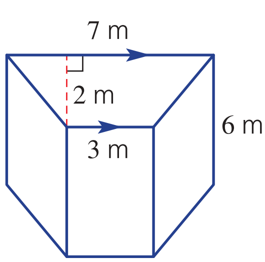


Hint for Q7: First find the area of the cross-section using $A = bh$

$A = \frac{1}{2}(a + b)h$ or

$A = \frac{1}{2}xy$

c



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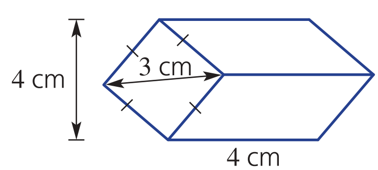

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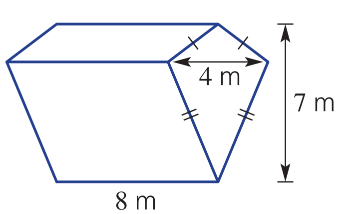

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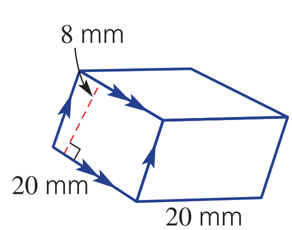

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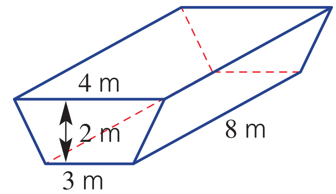
8 A swimming pool is a prism with a cross-section that is a trapezium. The pool is being filled at a rate of 1000 litres per hour.

a Find the capacity of the pool in litres.

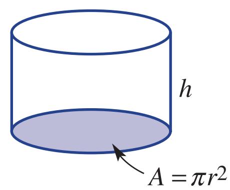
b How long will it take to fill the pool?



Volume of a cylinder

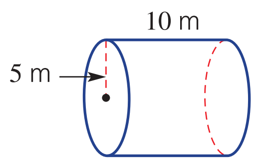
Volume of a cylinder	—	9

9 Although a cylinder is not a prism, because it has curved sides, the volume of a cylinder can be calculated using $V = Ah$ where $A = \pi r^2$, so $V = \pi r^2 h$.



Find the volume of these cylinders. Round your answers to two decimal places.

a



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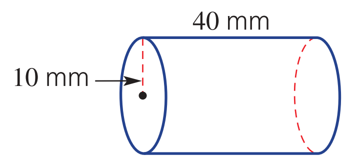

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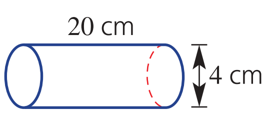

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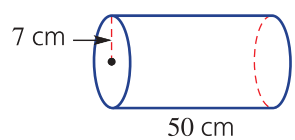

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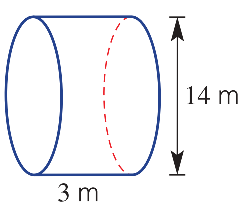

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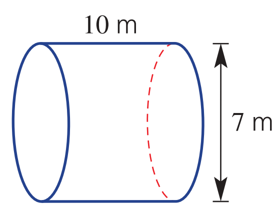

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4I Time

Learning intentions

- To be able to convert between different units of time.
- To be able to convert between times in 24-hour time and am/pm.
- To be able to use a world time zone map to relate times in different locations around the world.

Key vocabulary: time zone, duration, am, pm, longitude

The origin of seconds and minutes dates back to the ancient Babylonians, who used a base 60 number system. The 24-hour day dates back to the ancient Egyptians, who described the day as 12 hours of day and 12 hours of night. Today, we use am (*ante meridiem*, which is Latin for 'before noon') and pm (*post meridiem*, which is Latin for 'after noon') to represent the hours before and after noon (midday).

During the rule of Julius Caesar, the ancient Romans introduced the Julian calendar, which recognised that the Earth takes about $365\frac{1}{4}$ days to orbit the Sun. This gave rise to the leap year, which includes one extra day (on the 29th of February) every 4 years.

Julius Caesar was born in 100BC and died 15 March 44 BC.

Lesson starter: Time quiz

In less than five seconds per question, see if you can write the answers to the following:

- How many seconds in a minute?
- How many hours in two days?
- How many months in a year?
- How many seconds in an hour?
- Which months have 31 days?
- What do BCE (or BC) and CE (or AD) mean on time scales?

Key ideas

- The standard unit of time is the second.
- Units of time include:
  - 1 minute (min) = 60 seconds (s)
  - 1 hour (h) = 60 minutes (min)
  - 1 day = 24 hours (h)
  - 1 week = 7 days
  - 1 year = 12 months

$$\begin{tikzcd} \text{days} \arrow[r, bend left, "\times 24"] & \text{hours} \arrow[l, bend left, "\div 24"] \arrow[r, bend left, "\times 60"] & \text{minutes} \arrow[l, bend left, "\div 60"] \arrow[r, bend left, "\times 60"] & \text{seconds} \arrow[l, bend left, "\div 60"] \end{tikzcd}$$

- We use **am** or **pm** to describe the 12 hours before and after noon (midday).
- 24-hour time shows the number of hours and minutes after midnight.
  - 0330 is 3:30 am.
  - 1121 is 11:21 am.
  - 1530 is 3:30 pm.
  - 2247 is 10:47 pm.

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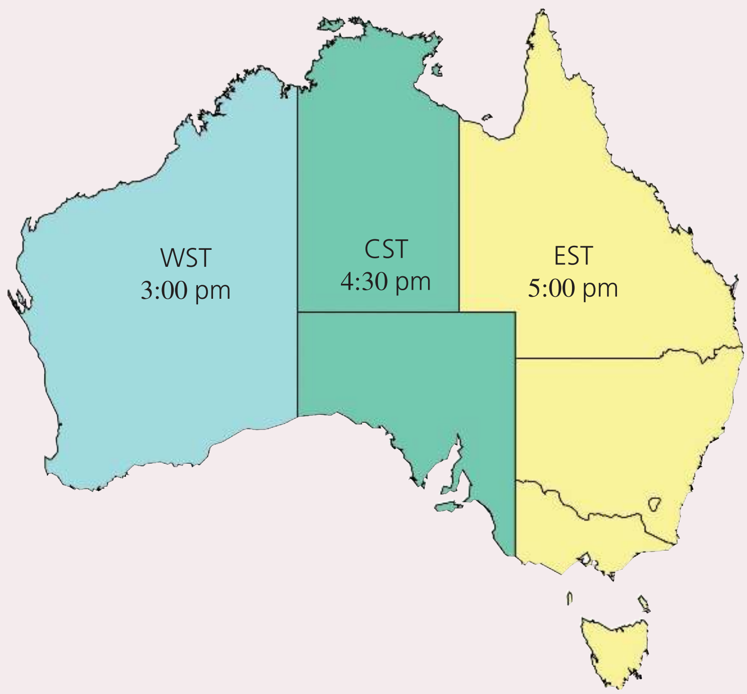
- The Earth is divided into 24 major **time zones** (one for each hour) and several minor time zones.
  - Twenty-four 15° lines of **longitude** divide the Earth into its time zones. Time zones also depend on a country's borders and how close it is to other countries. (See the world time zone map on pages 260–261 for details.)
  - Time is based on the time in Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
  - Places east of Greenwich are ahead in time.
  - Places west of Greenwich are behind in time.
- Australia has three time zones:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.



Exercise 4I

Understanding

1–3 | 3

1 Write the missing number.

- **a** 1 minute = _____ seconds
- **b** _____ days = 1 week
- **c** _____ hours = 1 day
- **d** 2 hours = _____ minutes
- **e** 240 seconds = _____ minutes
- **f** March has ___ days

2 Find the number of:

- **a** seconds in 2 minutes
- **b** minutes in 180 seconds
- **c** hours in 120 minutes
- **d** minutes in 4 hours
- **e** hours in 3 days
- **f** days in 48 hours
- **g** weeks in 35 days
- **h** days in 40 weeks

3 What is the time difference between these times?

- **a** 12:00 noon to 6:30 pm
- **b** 12:00 midnight to 10:45 am
- **c** 12:00 midnight to 4:20 pm
- **d** 11:00 am to 3:30 pm

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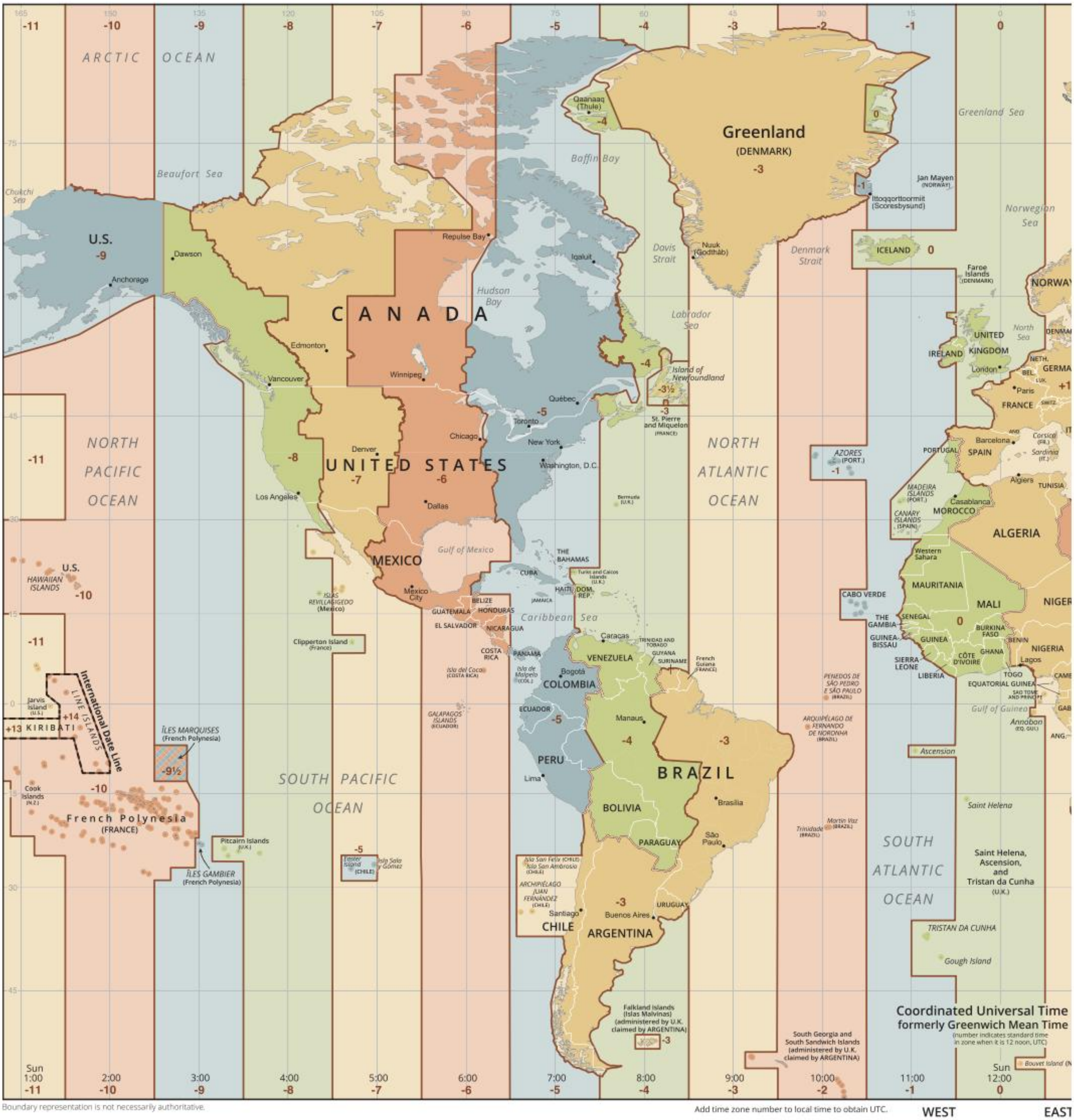

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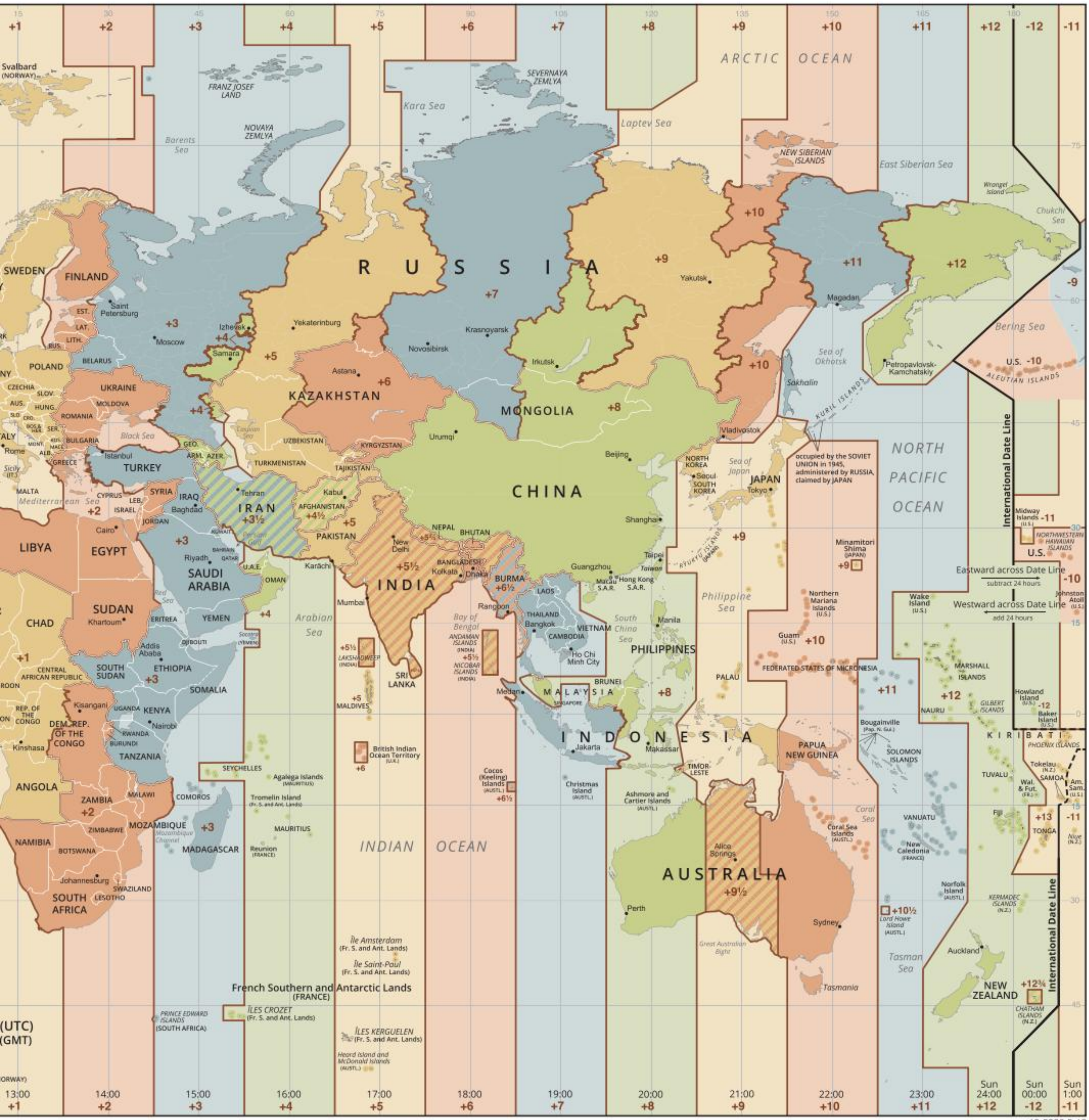

Subtract time zone number from local time to obtain UTC.

18-5386 5-18

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4I

Fluency

4(½), 5, 6(½), 7, 8 | 4–6(½), 7, 8–9(½)

Example 23 Converting units of time

Convert these times to the units shown in brackets.

a 3 days (minutes)

b 30 months (years)

Solution	Explanation
a 3 days $= 3 \times 24$ hours $= 3 \times 24 \times 60$ min $= 4320$ min	1 day = 24 hours 1 hour = 60 minutes
b 30 months $= 30 \div 12$ years $= 2\frac{1}{2}$ years	There are 12 months in 1 year.

Now you try

Convert these times to the units shown in brackets.

a 6.5 min (seconds)

b 750 min (hours)

4 Convert these times to the units shown in brackets.

a 2 min (s)
b 48 h (days)
c 21 days (weeks)
d 3 h (min)
e 10.5 min (s)
f 240 s (min)
g 90 min (h)
h 6 days (h)
i 72 h (days)
j 1 week (h)
k 1 day (min)
l $3\frac{1}{2}$ h (min)

Hint for Q4:
1 min = 60 sec
1 hr = 60 min
1 day = 24 hrs
1 week = 7 days

5 Write the time for these descriptions.

a 4 hours after 2:30 pm
b 10 hours before 7:00 pm
c $3\frac{1}{2}$ hours before 10:00 pm
d $7\frac{1}{2}$ hours after 9:00 am
e $6\frac{1}{4}$ hours after 11:15 am
f $1\frac{3}{4}$ hours before 1:25 pm

Example 24 Using 24-hour time

Write these times using the system given in brackets.

a 4:30 pm (24-hour time)

b 1945 hours (am, pm)

Solution	Explanation
a 4:30 pm $= 1200 + 0430$ $= 1630$ hours	Since the time is pm, add 12 hours to 0430 hours.
b 1945 hours = 7:45 pm	Since the time is after 1200 hours, subtract 12 hours.

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Now you try

Write these times using the system given in brackets.

a 10:25 am (24-hour time) b 2236 (am, pm)

6 Write these times using the system shown in brackets.

a 1:30 pm (24-hour) b 8:15 pm (24-hour)
c 10:23 am (24-hour) d 11:59 pm (24-hour)
e 0630 hours (am/pm) f 1300 hours (am/pm)
g 1429 hours (am/pm) h 1938 hours (am/pm)
i 2351 hours (am/pm) j 0426 hours (am/pm)
k 6:47 pm (24-hour) l 4:32 am (24-hour)

Hint for Q6: 6:00 am is 0600 hours. 12:00 noon is 1200 hours. 6:00 pm is 1800 hours.

7 Round these times to the nearest hour.

a 1:32 pm b 5:28 am
c 1219 hours d 1749 hours

Example 25 Using time zones

Use the world time zone map (on pages 260–261) to answer the following.

a When it is 2:00 pm EST (Eastern Standard Time), find the time in these places.
i Adelaide ii Perth iii Queensland iv Phillipines

b When it is 9:35 am in Western Australia, find the time in these places.
i Alice Springs ii Tasmania iii Brisbane iv China

Solution

a i 1:30 pm
ii 12:00 noon
iii 2:00 pm
iv 12:00 noon

Explanation

Adelaide is in the Central Standard Time zone, which is half an hour behind Eastern Standard Time.
Perth is in the WST zone, 2 hours behind EST.
Queensland is in the EST zone.
Phillipines is in the same zone as Western Australia.

Solution

b i 11:05 am
ii 11:35 am
iii 11:35 am
iv 9:35 am

Explanation

Alice Springs uses Central Standard Time, which is $1\frac{1}{2}$ hours ahead of Western Standard Time.
Tasmania uses Eastern Standard Time, which is 2 hours ahead of Western Standard Time.
Brisbane is in the EST zone, 2 hours ahead of WST.
China is in the same zone as Western Australia.

Now you try

Use the world time zone map (on pages 260–261) to answer the following.

a When it is 8:30 am in Western Australia, find the time in these places.
i Sydney ii Northern Territory iii Victoria iv Japan

b When it is 2:40 pm EST, find the time in these places.
i Perth ii New Zealand iii Madagascar iv France

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4I

8 Use the world time zone map on pages 260–261 to find the time in the following places when it is 10:00 am EST (Eastern Standard Time).

- **a** Melbourne
- **b** Darwin
- **c** Adelaide
- **d** Perth
- **e** Sydney
- **f** Tasmania
- **g** China
- **h** Papua New Guinea

Hint for Q8: CST is $\frac{1}{2}$ an hour behind EST, WST is 2 hours behind EST.

9 Use the world time zone map on pages 260–261 to find the time in these places when it is 3:30 pm in Perth.

- **a** Melbourne
- **b** Phillipines
- **c** Sydney
- **d** China
- **e** Hobart
- **f** Queensland
- **g** Alice Springs
- **h** New Zealand
- **i** Japan

Problem-solving and reasoning

10–13 | 13–16

10 From options **A** to **F**, match up the time units with the most appropriate description.

- **a** Single heartbeat
- **b** 40 hours of work
- **c** Duration of a university lecture
- **d** Bank term-deposit
- **e** 200-m run
- **f** Flight from Australia to the UK

- **A** 1 hour
- **B** 1 minute
- **C** 1 day
- **D** 1 week
- **E** 1 year
- **F** 1 second

11 What is the time difference between these time periods?

- **a** 10:30 am and 1:20 pm
- **b** 9:10 am and 3:30 pm
- **c** 2:37 pm and 5:21 pm
- **d** 10:42 pm and 7:32 am
- **e** 1451 and 2310 hours
- **f** 1940 and 0629 hours

12 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

13 An international company wishes to schedule a virtual meeting between workers in Hobart, London and New York.

- **a** If the meeting is scheduled for 9 a.m. in Hobart, what time is it in the other two cities?
- **b** If the meeting needs to be between 8 a.m. and 8 p.m. in both London and New York (but not necessarily Hobart), in what range of times would it occur for the Hobart workers?
- **c** Explain why it might be challenging for this virtual meeting to take place.

14 On a flight to Europe, Janelle spends 8 hours and 36 minutes on the flight from Melbourne to Kuala Lumpur, Malaysia; 2 hours and 20 minutes at the airport at Kuala Lumpur; and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

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15 A pre-paid phone plan charges 11 cents per 30 seconds. The 11 cents are added to the bill at the beginning of every 30-second block of time.

- **a** What is the cost of a 70-second call?
- **b** What is the cost of a call that lasts 6 minutes and 20 seconds?

16 A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

- **a** per day
- **b** per hour
- **c** per minute
- **d** per second (in cents)

World time zones

	—	17–19

17 Use the world time zone map to find the time in the following places if it is 3:30 pm in Victoria.

- **a** United Kingdom
- **b** Libya
- **c** Sweden
- **d** Perth
- **e** Japan
- **f** Central Greenland
- **g** Alice Springs
- **h** New Zealand

18 Use the world time zone map to find the time in the following places if it is 10:00 am UTC in England.

- **a** Spain
- **b** Turkey
- **c** Tasmania
- **d** Darwin
- **e** Argentina
- **f** Peru
- **g** Alaska
- **h** Portugal

19 **a** Explain why you gain time when you travel from Australia to Europe.

- **b** Explain why you lose time when you travel from Germany to Australia.
- **c** Explain what happens to the date when you fly from Australia to Canada across the International Date Line.

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4J Introducing Pythagoras' theorem

Learning intentions

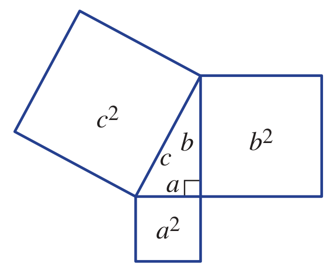
- To be able to identify the hypotenuse in a right-angled triangle
- To be able to determine if three numbers form a Pythagorean triple
- To be able to use Pythagoras' theorem to determine if a triangle has a right angle based on its side lengths

Key vocabulary: right-angled triangle, Pythagoras' theorem, Pythagorean triple, hypotenuse

Pythagoras was a philosopher in ancient Greece who lived in the 6th century BCE. Pythagoras was believed to provide a proof for the theorem that bears his name, and methods to find Pythagorean triples, which are sets of three whole numbers that make up the sides of right-angled triangles.

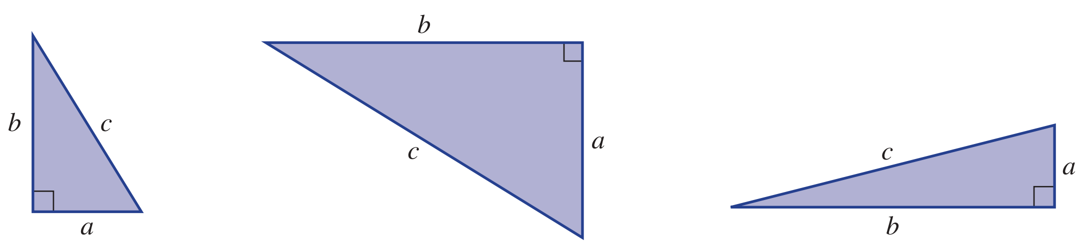
The ancient Babylonians, 1000 years before Pythagoras' time, and the Egyptians also knew of this relationship between the sides of a right-angled triangle.

Pythagoras' theorem states that the square of the hypotenuse (longest side) of a right-angled triangle is equal to the sum of the squares of the other two sides. An illustration of the theorem includes squares drawn on the sides of the right-angled triangle. The area of the larger square (c^2) is equal to the sum of the two smaller squares $(a^2 + b^2)$.



Lesson starter: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest mm. Then complete the table.

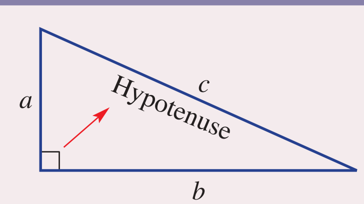


	a	b	c	a^2	b^2	c^2
Triangle 1						
Triangle 2						
Triangle 3						

- Can you see any relationship between the numbers in the columns for a^2 and b^2 and the number in the column for c^2?
- Can you write down this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it was unknown.

Key ideas

- The **hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.



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- **Pythagoras' theorem**
  - The square of the length of the hypotenuse is the sum of the squares of the lengths of the other two shorter sides.
  - $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$
- A **Pythagorean triple** (or triad) is a set of three whole numbers which satisfy Pythagoras' theorem. e.g. 3, 4, 5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$
- A triangle can be classified based on its side lengths a, b, c (in increasing order) as
  - right-angled if $c^2 = a^2 + b^2$
  - acute if $c^2 < a^2 + b^2$
  - obtuse if $c^2 > a^2 + b^2$

Exercise 4J

Understanding

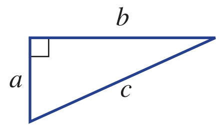
1–4 | 2–4

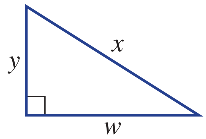
1 Calculate these squares and sums of squares.
a 3^2 **b** 1.5^2 **c** $2^2 + 4^2$ **d** $3^2 + 7^2$

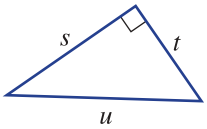
2 Decide if these equations are true or false.
a $2^2 + 3^2 = 4^2$ **b** $6^2 + 8^2 = 10^2$ **c** $6^2 - 3^2 = 2^2$

3 State the missing words in this sentence.
The __________ is the longest side of a right-angled __________.

4 Which letter represents the length of the hypotenuse in these triangles?

a


b


c


Fluency

5, 6–8(½) | 6–8(½)

Example 26 Checking Pythagorean triples

Decide if the following are Pythagorean triples.
a 6, 8, 10 **b** 4, 5, 9

Solution

a
$$\begin{aligned} a^2 + b^2 &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100\ (= 10^2) \end{aligned}$$
$\therefore$ 6, 8, 10 is a Pythagorean triple.

Explanation

Let $a = 6$, $b = 8$ and $c = 10$ and check that $a^2 + b^2 = c^2$.

b
$$\begin{aligned} a^2 + b^2 &= 4^2 + 5^2 \\ &= 16 + 25 \\ &= 41\ (\neq 9^2) \end{aligned}$$
$\therefore$ 4, 5, 9 is not a Pythagorean triple.

Explanation

$a^2 + b^2 = 41$ and
$9^2 = 81$ so
$a^2 + b^2 \neq c^2$

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4J

Now you try

Decide if the following are Pythagorean triples.

a $4, 6, 8$

b $3, 4, 5$

5 Decide if the following are Pythagorean triples.

a $3, 4, 6$

b $4, 2, 5$

c $5, 12, 13$

Hint for Q5: Check if $a^2 + b^2 = c^2$.

6 Decide if the following are Pythagorean triples.

a $9, 12, 15$

b $8, 15, 17$

c $2, 5, 6$

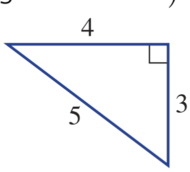
d $9, 40, 41$

e $10, 12, 20$

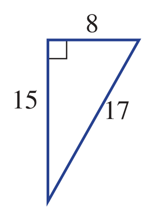
f $4, 9, 12$

7 Check that $a^2 + b^2 = c^2$ for all these right-angled triangles. Write out the statement $(\text{e.g.}\, 3^2 + 4^2 = 5^2)$ and check that the two sides are equal.

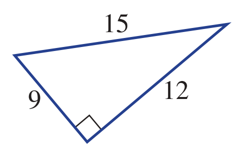
a



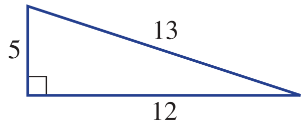
b



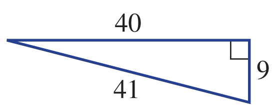
c



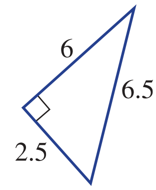
d



e



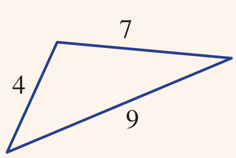
f



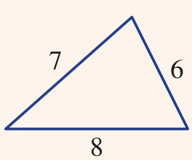
Example 27 Classifying a triangle using Pythagoras' theorem

Classify the following triangles as right-angled, acute or obtuse based on their side lengths.

a



b



Solution	Explanation
a $a = 4, b = 7, c = 9$	List the side lengths in ascending order so c is largest.
$c^2 = 9^2$ $= 81$	Calculate c^2.
$a^2 + b^2 = 4^2 + 7^2$ $= 16 + 49$ $= 65$	Calculate $a^2 + b^2$.
$81 > 65$ $\therefore$ This is an obtuse triangle.	If $c^2 > a^2 + b^2$, then the triangle has an obtuse angle.

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b $a = 6, b = 7, c = 8$ — List the side lengths in ascending order.

$c^2 = 8^2$ — Calculate c^2.
$= 64$

$a^2 + b^2 = 6^2 + 7^2$ — Calculate $a^2 + b^2$.
$= 36 + 49$
$= 85$

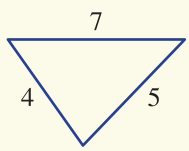
$64 < 85$ — If $c^2 < a^2 + b^2$, then the triangle has only acute angles.

$\therefore$ This is an acute triangle.

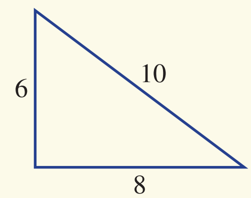
Now you try

Classify the following triangles as right-angled, acute or obtuse based on their side lengths.

a



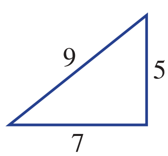
b



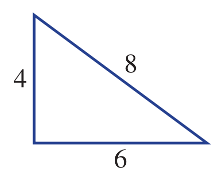
8 Classify the following triangles as right-angled, acute or obtuse based on their side lengths. Note that the triangles are not drawn to scale.

Hint for Q8: Compare c^2 to $a^2 + b^2$, e.g. $9^2 = 81$ and $5^2 + 7^2 = 74$.

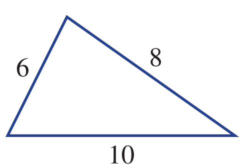
a



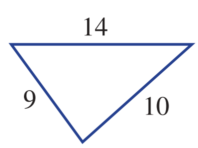
b



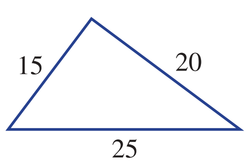
c



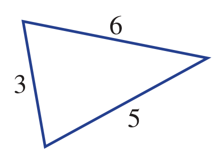
d

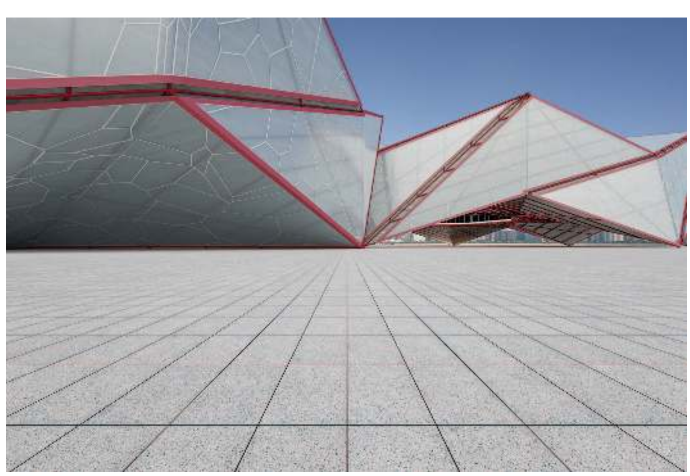


e



f





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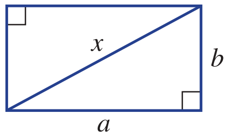
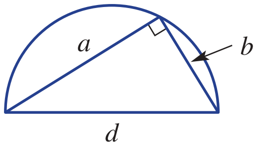
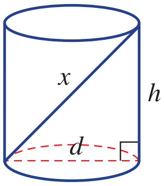
4J

Problem-solving and reasoning 9–11 10–12

9 Write down an equation using the pronumerals given in these diagrams.

a


b


c


10 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

a Using $c = 40$, decide if $a^2 + b^2 = c^2$. (*Hint:* Draw a diagram of the situation first.)

b Do you think the triangle formed by the mast and the cable is right angled? Give a reason.

11 If the side lengths of a triangle are all multiplied by the same positive number, the angles stay the same. Use this to explain why there are infinitely many Pythagorean triples.

12 If $a^2 + b^2 = c^2$ is true, complete these statements.

a $c^2 - b^2 =$ ____________

b $c^2 - a^2 =$ ____________

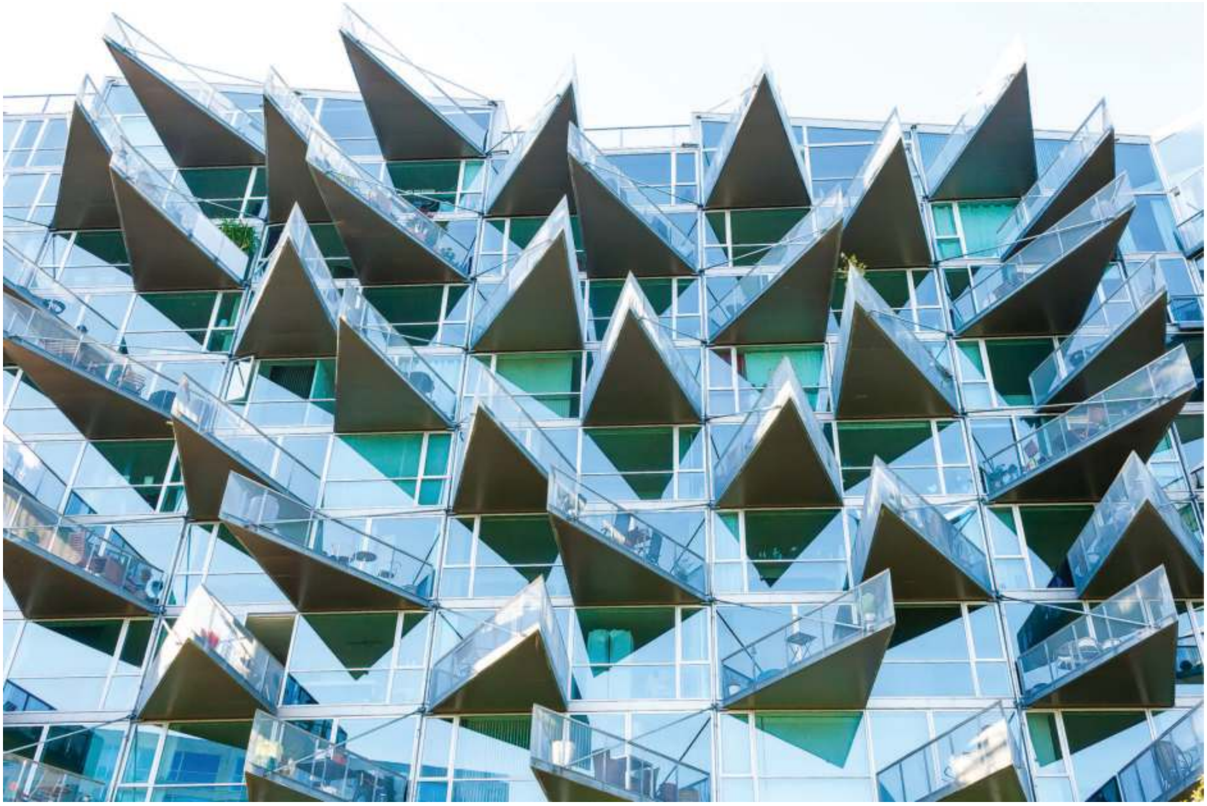
c $c =$ ______________

How many triples can you find? — 13

13 $(3, 4, 5)$ and $(5, 12, 13)$ are Pythagorean triples since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$.

a Find 10 more Pythagorean triples using whole numbers all less than 100.

b Find the total number of Pythagorean triples with whole numbers all less than 100.



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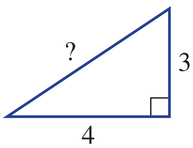
4K Using Pythagoras' theorem

Learning intentions

- To be able to use Pythagoras' theorem to find the hypotenuse of a right-angled triangle
- To understand what a surd is
- To be able to apply Pythagoras' theorem to simple worded problems involving an unknown hypotenuse or diagonal

Key vocabulary: surd, hypotenuse

From our understanding of algebra, we know that equations can be solved to find the value of an unknown. This is also the case for equations derived from Pythagoras' theorem, where, if two of the side lengths of a right-angled triangle are known, then the third can be found.



So if $c^2 = 3^2 + 4^2$ then $c^2 = 25$ and $c = 5$.

We also notice that if $c^2 = 25$ then $c = \sqrt{25} = 5$ (if $c > 0$).

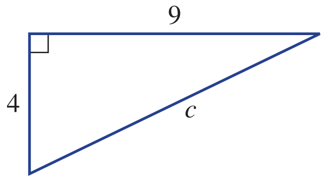
This use of Pythagoras' theorem has a wide range of applications wherever right-angled triangles can be drawn.

Note that a number using a $\sqrt{}$ sign may not always result in a whole number. For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers and neither can be written as a fraction, so they are irrational numbers. These types of numbers are called surds and they can be approximated using rounded decimals.

Marine engineers and builders use Pythagoras' theorem to calculate the length of a sloping boat ramp that will be above water at both low and high tides. Sloping boat ramps are used by fishermen and by people driving onto car ferries.

Lesson starter: Correct layout

Three students who are trying to find the value of c in this triangle using Pythagoras' theorem write their solutions on a board. There are only very minor differences between each solution and the answer is written rounded to two decimal places. Which student has all the steps written correctly? Give reasons why the other two solutions are not laid out correctly.



Student 1	Student 2	Student 3
$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$	$c = a^2 + b^2$
$= 4^2 + 9^2$	$= 4^2 + 9^2$	$= 4^2 + 9^2$
$= 97$	$= 97$	$= 97$
$= \sqrt{97}$	$\therefore c = \sqrt{97}$	$= \sqrt{97}$
$= 9.85$	$= 9.85$	$= 9.85$

Key ideas

- Using Pythagoras' theorem
  - If $c^2 = a^2 + b^2$, then $c = \sqrt{a^2 + b^2}$.
  - The final answer may not always result in a whole number. For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers.

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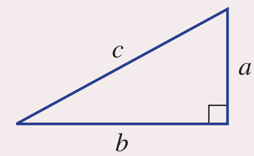
4K

- **Surds** are numbers that have a $\sqrt{\ \ }$ sign when written in simplest form.
  - They are not whole numbers and cannot be written as a fraction, so they are irrational numbers.
  - Written as a decimal, the decimal places would continue forever with no repeated pattern (just like the number π). Surds are therefore classified as irrational numbers.
  - $\sqrt{2}, \sqrt{5}, 2\sqrt{3}$ and $\sqrt{90}$ are all examples of surds.



- Note:
  - $\sqrt{a^2+b^2} \neq a+b$, for example, $\sqrt{3^2+4^2} \neq 3+4$
  - If $c^2 = k$, then $c = \sqrt{k}$ if $c > 0$.

Exercise 4K

Understanding

1–3 | 1, 3

1 Decide if these numbers written with a $\sqrt{\ \ }$ simplify to a whole number. Answer Yes or No.

a $\sqrt{9}$ **b** $\sqrt{11}$ **c** $\sqrt{20}$ **d** $\sqrt{121}$

2 Round these surds correct to two decimal places using a calculator.

a $\sqrt{10}$ **b** $\sqrt{26}$ **c** $\sqrt{65}$

3 State the missing parts to complete this working out.

a
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= \_\_\_ \\ \therefore c &= \sqrt{\_\_\_\_\_} \\ &= \_\_\_ \end{aligned}$$

b
$$\begin{aligned} c^2 &= \_\_\_\_\_ \\ &= 9^2 + 40^2 \\ &= \_\_\_ \\ \therefore c &= \sqrt{\_\_\_\_\_} \\ &= \_\_\_ \end{aligned}$$

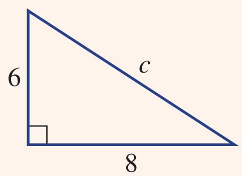
Fluency

4, 5–6(½) | 5–6(½)

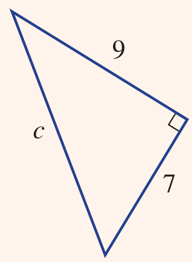
Example 28 Finding the length of the hypotenuse

Find the length of the hypotenuse for these right-angled triangles. Round the answer for part **b** to two decimal places.

a



b



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Solution

a

$$c^2 = a^2 + b^2$$
$$= 6^2 + 8^2$$
$$= 100$$
$$\therefore c = \sqrt{100}$$
$$= 10$$

Explanation

Write the equation for Pythagoras' theorem and substitute the values for the shorter sides.

Find c by taking the square root.

b

$$c^2 = a^2 + b^2$$
$$= 7^2 + 9^2$$
$$= 130$$
$$\therefore c = \sqrt{130}$$
$$= 11.40 \text{ (to 2 d.p.)}$$

First calculate the value of $7^2 + 9^2$.

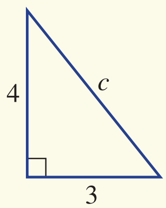
$\sqrt{130}$ is a surd, so round the answer as required.

A calculator can be used to find this answer.

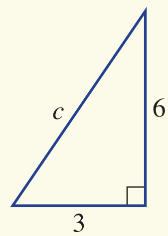
Now you try

Find the length of the hypotenuse for these right-angled triangles. Round the answer for part **b** to two decimal places.

a

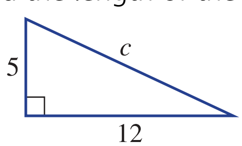


b



4 Find the length of the hypotenuse for these right-angled triangles.

a



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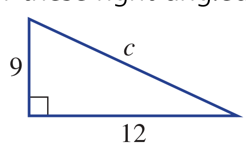
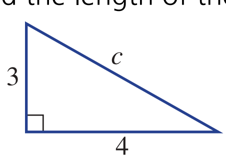


Hint for Q4: Start by writing $c^2 = a^2 + b^2$ and then put in the values for a and b.

5 Find the length of the hypotenuse of these right-angled triangles.

a



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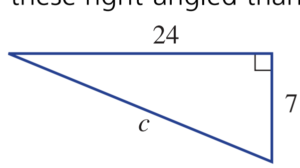
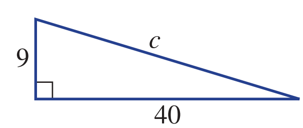
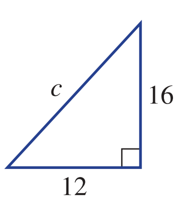


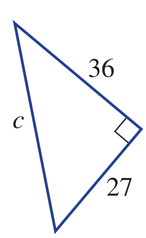
c



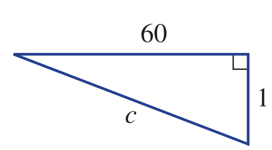
d



e



f



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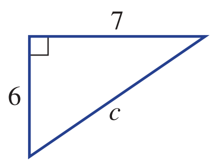
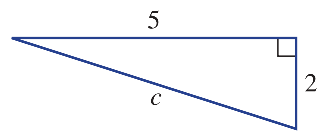
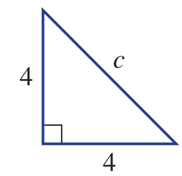
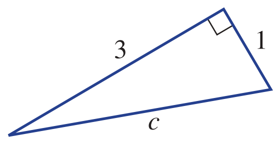
4K

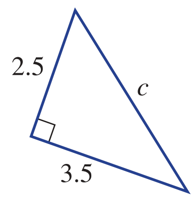
6 Find the length of the hypotenuse of these right-angled triangles correct to two decimal places.

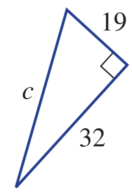
a


b


c


d


e


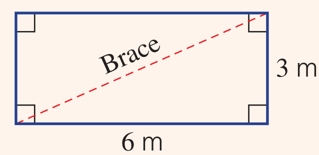
f


Problem-solving and reasoning

7–9 8–11

Example 29 Applying Pythagoras' theorem to find the hypotenuse

A rectangular wall is to be strengthened by a diagonal brace. The wall is 6 m wide and 3 m high. Find the length of brace required correct to the nearest cm.

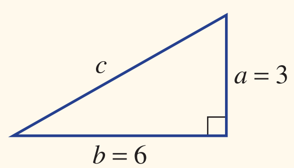


Solution

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 3^2 + 6^2 \\ &= 45 \\ \therefore c &= \sqrt{45} \\ &= 6.71 \text{ m or } 671 \text{ cm (to nearest cm)} \end{aligned}$$

The length of the brace is 6.71 metres.

Explanation

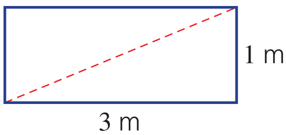


Write your answer in a sentence.

Now you try

A rectangular wall is 5 m wide and 4 m high. Find the length of a diagonal brace correct to the nearest cm.

7 A rectangular board is to be cut along one of its diagonals. The board is 1 m wide and 3 m high. What will be the length of the cut, correct to the nearest cm?



Hint for Q7: There is a right-angle triangle with $a = 3$ and $b = 1$.

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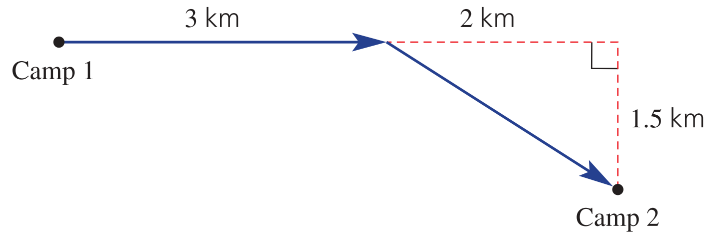
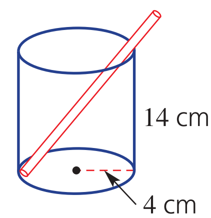
8 The size of a television screen is determined by its diagonal length. Find the size of a television screen that is 1.2 m wide and 70 cm high. Round the answer to the nearest cm.

9 Here is a diagram showing the path of a bushwalker from Camp 1 to Camp 2. Find the total distance rounded to one decimal place.



10 A 20 cm straw sits in a cylindrical glass as shown. What length of straw sticks above the top of the glass? Round the answer to two decimal places.



11 Explain the error in each set of working.

a
$$\begin{aligned} c^2 &= 2^2 + 3^2 \\ \therefore c &= 2 + 3 \\ &= 5 \end{aligned}$$

b
$$\begin{aligned} c^2 &= 3^2 + 4^2 \\ &= 7^2 \\ &= 49 \\ \therefore c &= 7 \end{aligned}$$

c
$$\begin{aligned} c^2 &= 2^2 + 5^2 \\ &= 4 + 25 \\ &= 29 \\ &= \sqrt{29} \end{aligned}$$

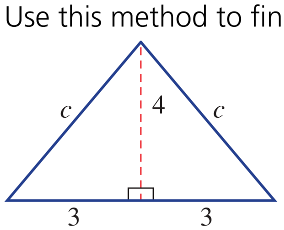
Hint for Q11: Look at the examples to see the correct layout.

Working with isosceles triangles

—	12

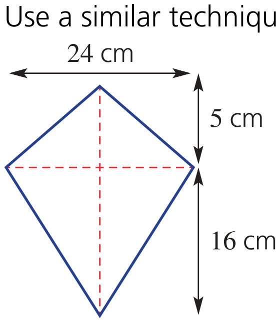
12 An isosceles triangle can be split into two right-angled triangles. Pythagoras' theorem can be used to find side lengths.

a Use this method to find c in the triangle shown, with base 6 and height 4.



b Hence, find the perimeter of this isosceles triangle.

c Use a similar technique to find the perimeter of the kite shown.



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4L Calculating the length of a shorter side

Learning intentions

- To be able to use Pythagoras' theorem to find the length of a shorter side in a right-angled triangle
- To be able to apply Pythagoras' theorem to simple worded problems involving an unknown shorter side

Key vocabulary: hypotenuse

We know that if we are given the two shorter sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the hypotenuse. Generalising further, we can say that if given *any* two sides of a right-angled triangle, we can use Pythagoras' theorem to find the length of the third side.

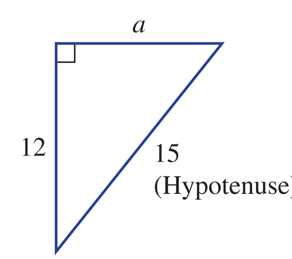
Firefighters can use Pythagoras' theorem to find the vertical height that a ladder can reach up a wall, from knowing the ladder's length and its distance to the base of the wall.

Lesson starter: What's the setting out?

The triangle shown has a hypotenuse length of 15 and one of the shorter sides is of length 12. Here is the setting out to find the length of the unknown side a.

Fill in the missing gaps and explain what is happening at each step.

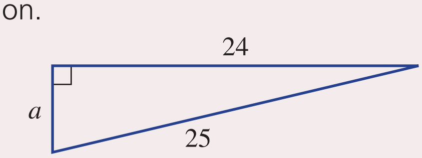
$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + \underline{\quad}^2 &= \underline{\quad}^2 \\
a^2 + \underline{\quad} &= \underline{\quad} \\
a^2 &= \underline{\quad} \text{ (Subtract } \underline{\quad} \text{ from both sides)} \\
\therefore a &= \sqrt{\underline{\quad}} \\
&= \underline{\quad}
\end{aligned}
$$



Key ideas

- Pythagoras' theorem can be used to find the length of the shorter sides of a right-angled triangle if the length of the hypotenuse and another side are known.
- Use subtraction to make the unknown the subject of the equation.
For example:

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + 24^2 &= 25^2 \\
a^2 + 576 &= 625 \\
a^2 &= 49 \text{ (Subtract 576 from both sides)} \\
\therefore a &= \sqrt{49} \\
&= 7
\end{aligned}
$$



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Exercise 4L

Understanding

1, 2 | 1(½), 2

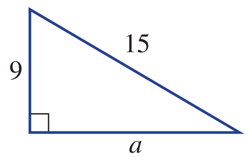
1 Find the value of a in these equations. (Assume a is a positive number.)

a $a^2 = 16$ **b** $a^2 + 16 = 25$ **c** $a^2 + 36 = 100$

d $a^2 + 441 = 841$ **e** $10 + a^2 = 19$ **f** $6 + a^2 = 31$

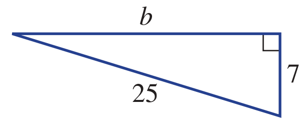
2 State the missing numbers to complete the following working.

a



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 9^2 &= \_\_\_ \\ a^2 + \_\_\_ &= 225 \\ a^2 &= \_\_\_ \\ \therefore a &= \sqrt{\_\_\_} \\ &= \_\_\_ \end{aligned}$$

b



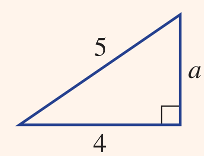
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 7^2 + 9^2 &= \_\_\_ \\ \_\_\_ + b^2 &= \_\_\_ \\ b^2 &= 576 \\ \therefore b &= \sqrt{\_\_\_} \\ &= \_\_\_ \end{aligned}$$

Fluency

3, 4–5(½) | 4–5(½)

Example 30 Finding the length of a shorter side

Find the length of the unknown side in this right-angled triangle.



Solution

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 4^2 &= 5^2 \\ a^2 + 16 &= 25 \\ a^2 &= 9 \\ \therefore a &= \sqrt{9} \\ &= 3 \end{aligned}$$

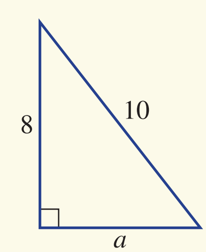
Explanation

Write the equation using Pythagoras' theorem and substitute the known values.

Subtract 16 from both sides. Find a by taking the square root.

Now you try

Find the length of the unknown side in this right-angled triangle.



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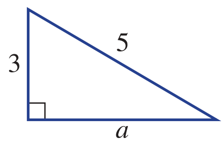
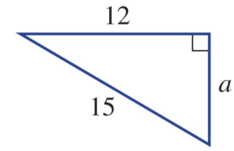
4L

3 Find the length of the unknown side in these right-angled triangles.

a



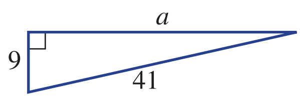
b



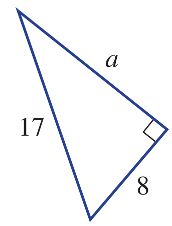
Hint for Q4: For the first question, start with $a^2 + 9^2 = 41^2$ and then use a calculator to help find a.

4 Find the length of the unknown side in these right-angled triangles.

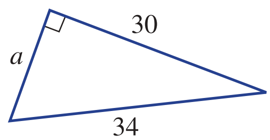
a



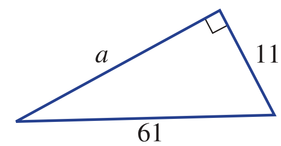
b



c

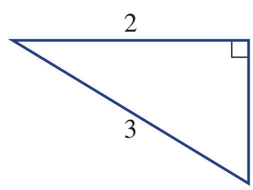


d

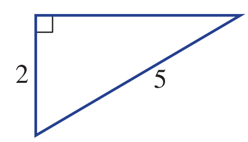


5 Find the length of the unknown side in these right-angled triangles, giving the answer correct to two decimal places.

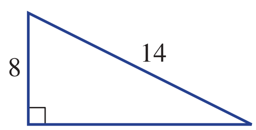
a



b



c



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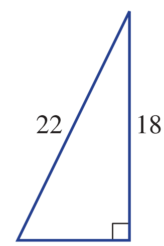
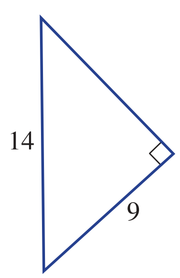
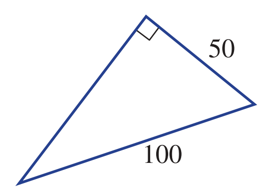


e



f

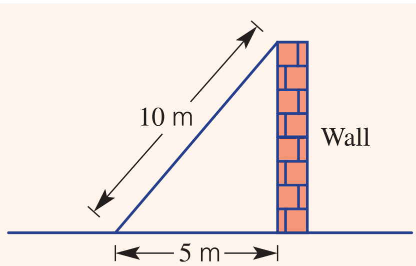


Problem-solving and reasoning

6–8 | 7–10

Example 31 Applying Pythagoras' theorem to find a shorter side

A 10 m steel brace holds up a concrete wall. The bottom of the brace is 5 m from the base of the wall. Find the height of the concrete wall correct to two decimal places.



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Solution

Let a metres be the height of the wall.

$a^2 + b^2 = c^2$
$a^2 + 5^2 = 10^2$
$a^2 + 25 = 100$
$a^2 = 75$
$\therefore a = \sqrt{75}$
$= 8.66$ (to 2 d.p.)

The height of the wall is 8.66 metres.

Explanation

Choose a letter (pronumeral) for the unknown height.

Substitute into Pythagoras' theorem.

Subtract 25 from both sides.

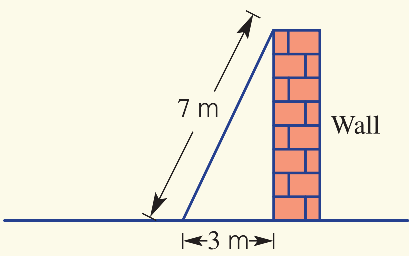
$\sqrt{75}$ is the exact answer.

Round as required.

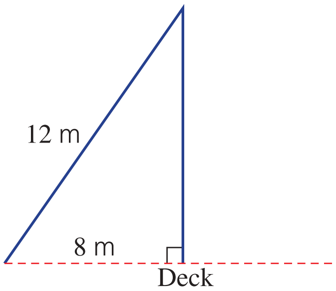
Answer a worded problem using a full sentence.

Now you try

A 7 m ladder is placed 3 m from the base of a wall as shown. Find the height of the wall correct to two decimal places.



6 A yacht's mast is supported by a 12 m cable attached to its top. On the deck of the yacht, the cable is 8 m from the base of the mast. How tall is the mast? Round the answer to two decimal places.



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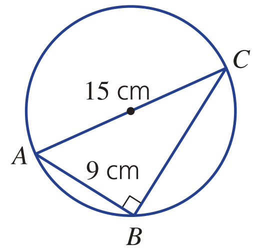
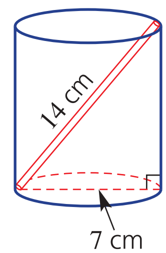
4L

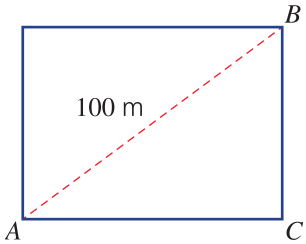
7 A circle's diameter AC is 15 cm and the chord AB is 9 cm. Angle ABC is $90°$. Find the length of the chord BC.

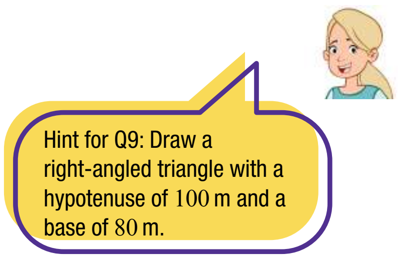


8 A 14 cm drinking straw just fits into a can as shown. The diameter of the can is 7 cm. Find the height of the can correct to two decimal places.



9 To cut directly through a rectangular field from A to B, the distance is 100 metres. The path from A to C is 80 metres. Find how much further a person travels by walking $A \to C \to B$ compared to directly $A \to B$.





10 Describe what is wrong with the second line of working in each step.

a $a^2 + 10 = 24$
$a^2 = 34$

b $a^2 = 25$
$= 5$

c $a^2 + 25 = 36$
$a + 5 = 6$

Pythagorean families

—	11

11 Recall that $(3, 4, 5)$ is called a Pythagorean triple because the numbers 3, 4 and 5 satisfy Pythagoras' theorem $\left(3^2 + 4^2 = 5^2\right)$.

a Explain why $(6, 8, 10)$ is also a Pythagorean triple.
b Explain why $(6, 8, 10)$ is considered to be in the same family as $(3, 4, 5)$.
c List three other Pythagorean triples in the same family as $(3, 4, 5)$ and $(6, 8, 10)$.
d Find another triple not in the same family as $(3, 4, 5)$, but has all three numbers less than 20.
e List five triples that are each the smallest triple of five different families.

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Hairdresser

Hairdressing is a demanding job that involves long hours on your feet and high levels of concentration. Good communication skills are needed. Being good at maths is also important. Colour combinations need to be weighed and mixed in correct volumes, and temperatures maintained, as well as being able to manage times. Clients do not like to be kept waiting or to have their experience rushed due to scheduling issues.

1 Tubes of colour are kept in each salon and their volume is usually quoted in cc.

> cc means cubic centimetres
> $1 \text{ mL} = 1 \text{ cc} = 1 \text{ cm}^3$

Convert the following from cubic centimetres (cc) into millilitres (mL).

a 10 cc **b** 20 cc **c** 45 cc **d** 100 cc
e 1000 cc

2 Before it is applied, hair colour is mixed with a chemical called developer. The volume of developer is twice the volume of the colour. Determine the volume of developer, in cc, that is needed for:

a 10 cc of colour **b** 30 cc of colour **c** 50 cc of colour

3 A client, Chloe, has thick, long hair. Her salon records show:

> Chloe's July appointment:
> - mixed and used 3 amounts of colour
> - standard (10 cc) + standard + half of a standard volume

On Chloe's next visit the hairdresser decided to mix enough colour all in one batch.

a What volume of colour should be used?
b What volume of developer should be used?
c What is the total volume of mixture that is applied to Chloe's hair?
d Chloe books her appointments every 10 weeks on a Friday. Her last appointment was on Friday the 26th of August. What is the date of her next appointment?
e It takes 15 minutes for the consultation, 45 minutes for applying the colour, 20 minutes for the wash and 45 minutes for the cut and blow dry. If Chloe's appointment was for 9.15 am what time would she expect to finish?
f If the same stylist consults and cuts Chloe's hair, while another hairdresser applies the colour and does the wash, how much time between the consultation and the cut is available for the stylist to work on another client?

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4 Happy Hair is a busy salon where Amelia and Layla are the hairdressers. Answer the following questions about this day's appointments.

<table>
<tr><td colspan="3">Happy Hair appointment book</td></tr>
<tr><td colspan="3">Tuesday, March 18</td></tr>
<tr><td>Start times</td><td>Amelia</td><td>Layla</td></tr>
<tr><td>9:00 am</td><td rowspan="3">Jessica: Long hair, colour, cut, wash, head massage, blow-dry and style</td><td>Liam: Men's haircut</td></tr>
<tr><td>9:30 am</td><td rowspan="2">Mrs White: short hair, cut and style</td></tr>
<tr><td>10:00 am</td></tr>
<tr><td>10:30 am</td><td>Ella: Girls' haircut</td><td rowspan="3">Mrs Davis: Short hair, colour, cut, wash, eye-brows, blow-dry and style</td></tr>
<tr><td>11:00 am</td><td>Jessica: continued</td></tr>
<tr><td>11:30 am</td><td rowspan="3">Mrs Williams: Short hair, cut, wash and style</td></tr>
<tr><td>12:00 noon</td><td>Lunch</td></tr>
<tr><td>12:30 pm</td><td>Mrs Davis: continued</td></tr>
<tr><td>1:00 pm</td><td>Max: Men's haircut</td><td>Joel: Boys' haircut</td></tr>
<tr><td>1:30 pm</td><td>Lunch</td><td rowspan="2">Chelsea: Short haircut and style</td></tr>
<tr><td>2:00 pm</td><td rowspan="6">Mrs Babb: Short hair, colour, cut, wash, eye-brows, head massage, blow-dry and style</td></tr>
<tr><td>2:30 pm</td><td rowspan="2">Zoe: Braiding</td></tr>
<tr><td>3:00 pm</td></tr>
<tr><td>3:30 pm</td><td rowspan="2">Ruby: Deep conditioning and wash</td></tr>
<tr><td>4:00 pm</td></tr>
<tr><td>4:30 pm</td><td rowspan="3">Holly: Half-head of foils and wash</td></tr>
<tr><td>5:00 pm</td><td>Luke: Boys' haircut</td></tr>
<tr><td>5:30 pm</td><td></td></tr>
</table>

Amelia

a How many customers does Amelia have on this Tuesday?
b How long is Jessica's appointment?
c At what times does Jessica start and finish her appointment?
d What could Jessica be doing while Amelia cuts Ella's hair?
e If Max was running 10 minutes late, could Amelia still fit him in?
f If Gary rang in the morning and wanted a hair trim that same day, at what times could Amelia fit him in?

Hint for part c: The times are start times, e.g. Ella's haircut starts at 10:30 am and finishes by 11 am.

Layla

g How many customers does Layla have on this Tuesday?
h At what times does Mrs Davis start and finish her appointment?
i How much time is allocated for Zoe's braiding?
j If Holly's foils went overtime by 20 minutes, at what time would Layla finish her appointments?

Using technology

5 For this task, use digital technology such as: a table in Word, an Excel spreadsheet or a day diary from a digital calendar. Imagine that you are a hairdresser in a hairdressing salon.

a You are to fill in appointment times for your customers, as listed below, who are all coming on one day. Include the salon's name, a day and date, your name and the name of each customer.
- Two 2.5-hour appointments for a person with long hair: colour, cut, wash, head massage, eye-brows, blow-dry and style.
- Four 30-minute appointments for haircuts for two men, a boy and a girl.
- A 90-minute appointment for foils, wash and blow-dry.

b Some digital diaries have the option of attaching extra notes to an entry, e.g. in Excel, right-click/insert comment. Suggest what other information could be included in a hairdresser's digital appointment diary.

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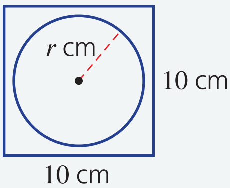
Carving table legs



Kosta is carving cylindrical table legs out of square 10 cm by 10 cm wooden poles, each of length 1.2 metres. The cross-section of the pole is shown in this diagram. He uses a wood lathe to remove the timber outside the circle leaving a timber cylinder of radius r cm and length 1.2 metres.

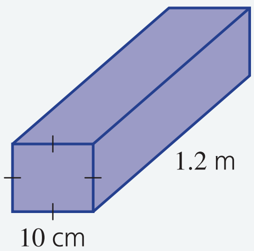
Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Round measurements to two decimal places.

1 Preliminary task

a Write down the formulas required to calculate the volume of
 i a cube **ii** a rectangular prism **iii** a cylinder.

b Convert 1.2 m into cm.

c Find the volume of the uncarved 10 cm by 10 cm pole of length 1.2 m, as shown in the diagram below. Give your answer in cubic centimetres, ensuring you first convert all dimensions to the same unit.



d If the radius of the circular cross-sectional area of the carved pole is 3 cm, find:
 i the cross-sectional area of the carved pole
 ii the volume of the carved pole
 iii the volume of wood wasted in the process
 iv the percentage of wood wasted in the process.

2 Modelling task

Formulate

a The problem is to determine the radius of the carved pole so that no more than 25% of the original timber pole is wasted. Write down all the relevant information that will help solve this problem with the aid of one or more diagrams.

b By first calculating areas, determine the volume of timber wasted if the carved pole is created using the following radii:
 i 2 cm
 ii 3 cm
 iii 4 cm.

Modelling

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Solve

c By calculating the percentage of timber wasted, decide if any of the three radii listed in part **b** satisfy the requirement that no more than 25% of the timber can be wasted.

d If the largest cylinder possible is created, determine the percentage of timber wasted.

Evaluate and verify

Kosta likes to waste slightly more than the absolute minimum amount of timber because there is a better chance of producing a smoother finish. He therefore aims for a figure closer to 25% timber wastage.

e Explain why you only need to consider the cross-sectional area of the pole rather than looking at the entire volume to solve this problem.

f Use trial and error to determine the radius that Kosta should aim for to achieve a 25% timber wastage correct to as many decimal places as possible.

Communicate

g Summarise your results in a table like the one below.

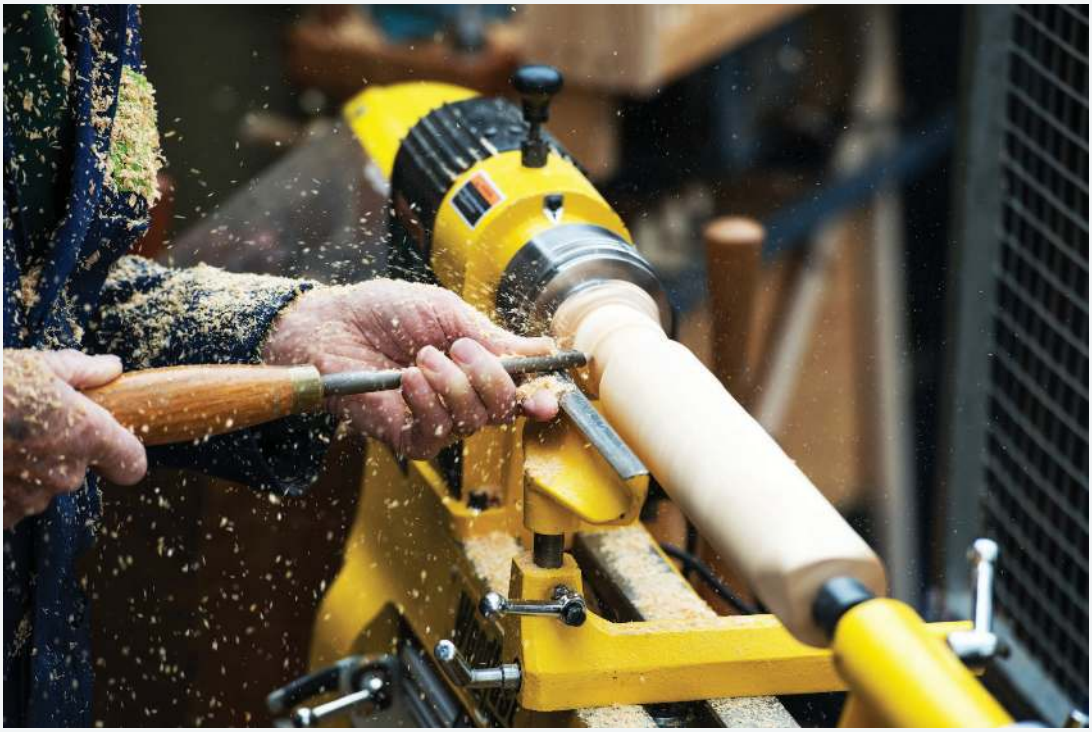
Trial	Radius chosen	Volume of the cylindrical leg	Volume of timber wasted	Percentage of timber wasted
1				
2				
3				

3 Extension questions

a Write an expression for the percentage of timber wasted if the radius of the pole is r cm.

b By using your expression from part **a**, outline a direct method for finding the radius of the pole that delivers exactly 25% timber wastage.

c Find, correct to three decimal places, the value of r that would result in 50% of the wood being wasted.



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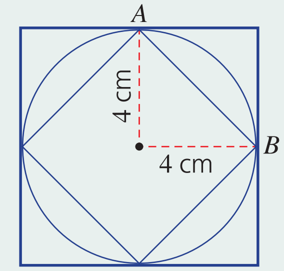
Approximating a circle with 2 squares

Key technology: Dynamic geometry and spreadsheets

We know that to find the area of a circle precisely we use the irrational number π, which is an infinitely non-recurring decimal and part of the area formula $A = \pi r^2$. The area of a circle can, however, be approximated using squares and Pythagoras' theorem using the circle's radius only.



1 Getting started

Consider this circle of radius 4 cm and the two squares constructed as shown.

a State the side length of the larger square.

b Use Pythagoras' theorem to find the side length of the smaller square AB.

c Find the areas of both squares.

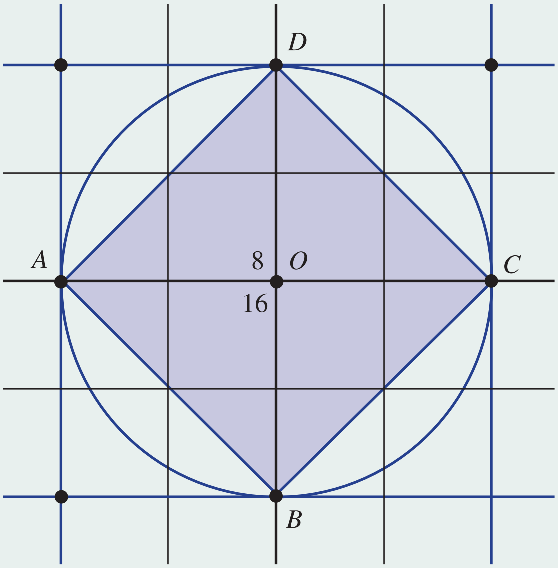
d Find the average of the areas of the two squares. This is your approximation to the area of the circle.

e Use the formula for the area of a circle, $A = \pi r^2$, and compare this to your approximation found in part **d** above.

f Repeat parts **b** to **e** above using a radius of 5 cm.

2 Using technology

a Construct a dynamic circle similar to above using a dynamic geometry package like Desmos Geometry. Follow these steps.
- Step 1: Construct a segment AC and centre O as the midpoint.
- Step 2: Construct the circle with centre O and radius OC.
- Step 3: Construct the perpendicular line BD and the points B and D.
- Step 4: Construct the perpendicular lines to form the larger square.
- Step 5: Construct the smaller and larger squares using the Polygon tool.
- Step 6: Find the areas of the smaller and larger squares.



b Find the average of the two areas calculated in your construction above. This is your approximation to the area of the circle.

c Drag one of your starting points A or C to alter the size of the circle and change the areas of the squares. Recalculate the average of the areas of the two squares.

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3 Applying an algorithm

We note that if the radius of the circle is r, then the side length of the area of the larger square l_1 is given by $l_1 = 2r$ and the smaller square l_2 is given by $l_2 = \sqrt{r^2 + r^2} = \sqrt{2r^2} = \sqrt{2}r$.

a Use these above formulas to set up a spreadsheet to approximate the area of a circle as follows.

	A	B	C
1	**Approximating**	**circles with squares**	
2			
3	**Circle radius**	**Approximation**	**Exact**
4	1	=(4*A4^2+(SQRT(2)*A4)^2)/2	=PI()*A4^2
5	=A4+1		

b Fill down at cells A5, B4 and C4. Compare your approximation for the area of the circle with the more exact values in column C, which uses the $A = \pi r^2$ formula.
You can see that your approximation is always less than the exact area. Follow this algorithm to search for a particular percentage increase that is required to bring the approximation closer to the exact.

c In cell D3, enter a chosen percentage increase and in cell D4, calculate a new approximation using the given formula. Fill down at cell D4.

d Follow this algorithm to find an accurate percentage increase that gives a precise approximation to the area of a circle.
- Step 1: Change the percentage increase value in cell D3 to a different value.
- Step 2: Compare the approximate area values in column D to the exact values in columns C.
- Step 3: Decrease the percentage increase value in cell D3 if the approximation is too large or increase the percentage increase value in cell D3 if the approximation is too small.
- Step 4: Repeat from Step 2 until you have found a percentage increase that makes your area approximation accurate to two decimal places.

D
% increase
5
=B4*(1+D$3/100)

e Persist with your algorithm above until you have found a percentage increase that makes your area approximation accurate to 3 or 4 decimal places. Compare your result with a friend.

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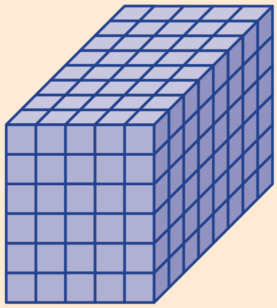
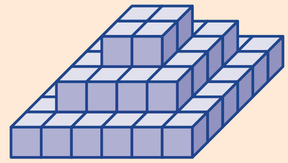
1 How many cubes are in each solid stack?

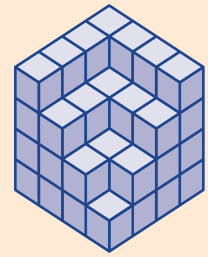
a



b

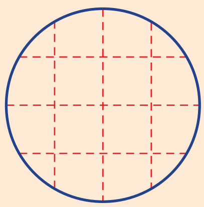


c

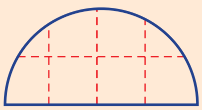


2 Estimate the area of these shapes by counting squares.

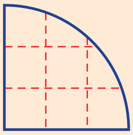
a



b



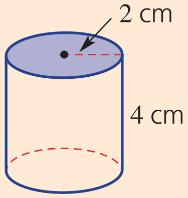
c



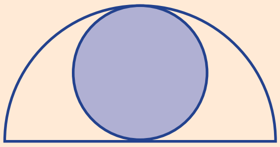
3 A cube has a capacity of 1 L. What are its dimensions in cm?

4 A fish tank is 60 cm long, 30 cm wide, 40 cm high and contains 70 L of water. Rocks with a volume of 3000 cm^3 are placed into the tank. Will the tank overflow?

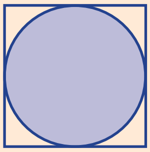
5 Find the total surface area of this cylinder.



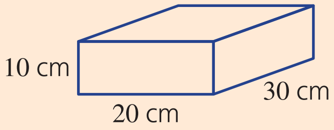
6 What proportion (fraction or percentage) of the semicircle does the full circle occupy?



7 A circle just fits inside a square. What percentage of the square is occupied by the circle?



8 1.8 L of water is poured into this container. What will be the depth of the water?



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Chapter summary

Measurement

Length

Perimeter

4 cm, 10 cm

$P = 2 \times 10 + 2 \times 4$
$= 28$ cm

Units

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

Circumference

$C = 2\pi r$ or πd
$= 2 \times \pi \times 3$
$= 18.85$ m^2

3 m

Area

Triangle

3 cm, 6 cm

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 6 \times 3$
$= 9$ cm^2

Units

$\times 1000^2$ $\times 100^2$ $\times 10^2$
km^2 m^2 cm^2 mm^2
$\div 1000^2$ $\div 100^2$ $\div 10^2$

1 ha = 10 000 m^2

Quadrilaterals

- Square $A = l^2$
- Rectangle $A = lw$
- Parallelogram $A = bh$
- Rhombus $A = \frac{1}{2}xy$
- Kite $A = \frac{1}{2}xy$
- Trapezium $A = \frac{1}{2}(a + b)h$

Circle

$A = \pi r^2$
$= \pi \times 7^2$
$= 153.94$ cm^2

7 cm

Sectors

$A = \frac{\theta}{360} \times \pi r^2$
$= \frac{280}{360} \times \pi \times 2^2$
$= 9.77$ m^2

2 m, 280°

Time

1 min = 60 s
1 h = 60 min
0311 is 03:11 am
2049 is 08:49 pm

Volume

Units

km^3, m^3, cm^3, mm^3

$\times 1000$ $\times 1000$ $\times 1000$
ML kL L mL
$\div 1000$ $\div 1000$ $\div 1000$

1 mL = 1 cm^3
1 m^3 = 1000 L

Rectangular prism

$V = lwh$
$= 10 \times 20 \times 30$
$= 6000$ cm^3
$= 6$ L

30 cm, 20 cm, 10 cm

Triangular prism

$V = Ah$
$= \frac{1}{2} \times 3 \times 1 \times 2$
$= 3$ m^3

1 m, 3 m, 2 m

Pythagoras' theorem

Theorem

a, b, c

$a^2 + b^2 = c^2$

Finding c

$c^2 = 5^2 + 7^2$
$= 74$
$\therefore c = \sqrt{74}$

7, 5, c

Finding a shorter side

a, 1, 2

$a^2 + 1^2 = 2^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \sqrt{3}$

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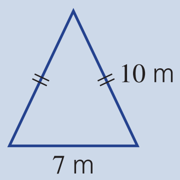
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

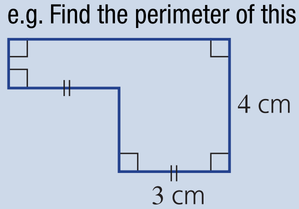
✔

4A **1 I can convert length measurements**
e.g. Convert:
a 5.2 cm to mm **b** 2400 m to km.

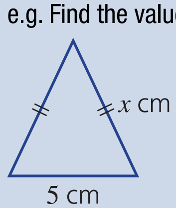
4A **2 I can find the perimeter of simple shapes**
e.g. Find the perimeter of this triangle.



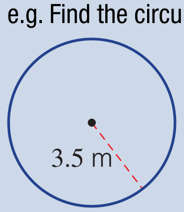
4A **3 I can find the perimeter of rectangular shapes**
e.g. Find the perimeter of this shape.



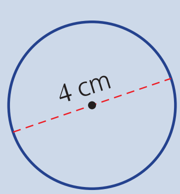
4A **4 I can find unknown side lengths in a shape, given the perimeter**
e.g. Find the value of x given that this triangle's perimeter is 19 cm.



4B **5 I can find the circumference of a circle using the radius**
e.g. Find the circumference, correct to two decimal places, using a calculator for the value of π.



4B **6 I can find the circumference of a circle using the diameter**
e.g. Find the circumference of this circle, correct to two decimal places.



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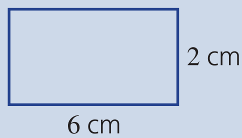
Chapter checklist

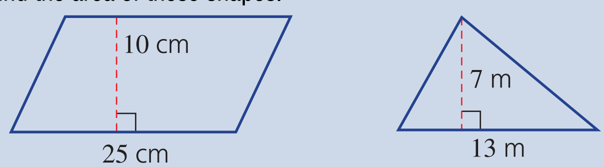
✔

4C **7 I can convert units of area**
e.g. Convert:
a $0.248\ \text{m}^2$ to cm^2 **b** $3100\ \text{mm}^2$ to cm^2.

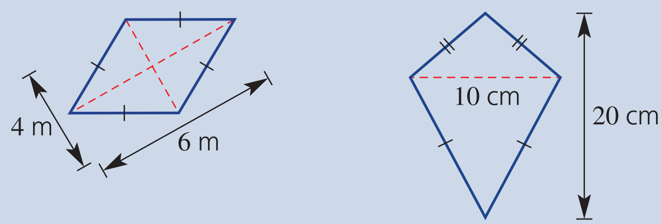
4C **8 I can find the area of rectangles and squares**
e.g. Find the area of this shape.



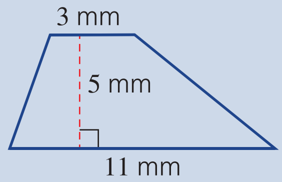
4C **9 I can find the area of parallelograms and triangles**
e.g. Find the area of these shapes.



4D **10 I can find the area of rhombuses and kites**
e.g. Find the area of this rhombus and kite.

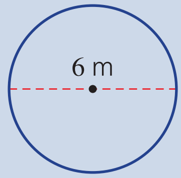


4D **11 I can find the area of trapeziums**
e.g. Find the area of this shape.



4E **12 I can find circle areas using a radius**
e.g. Find the area of a circle that has a radius of 4 cm, correct to two decimal places.

4E **13 I can find circle areas using a diameter**
e.g. Find the area of this circle, correct to two decimal places.



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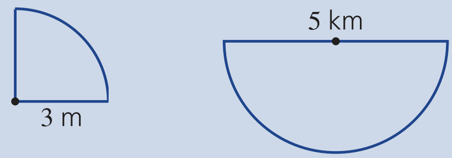
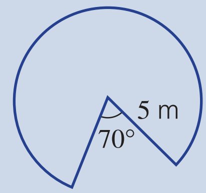
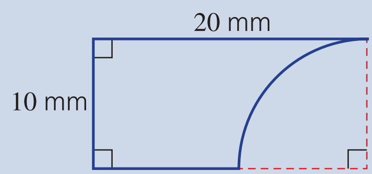
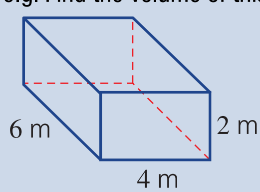
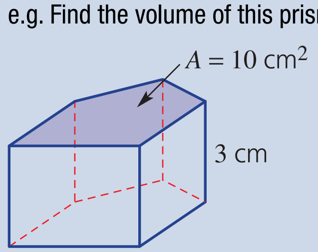
		✔
4E	**14 I can find the area of semicircles and quadrants** e.g. Find the area of this quadrant and semicircle, correct to two decimal places.	
4F	**15 I can find the area of sectors.** e.g. Find the area of this sector, correct to two decimal places.	
4F	**16 I can find the area of composite shapes involving sectors.** e.g. Find the area of this composite shape, correct to the nearest whole number of mm^2.	
4G	**17 I can find the volume of a rectangular prism** e.g. Find the volume of this rectangular prism.	
4G	**18 I can convert between units of volume or capacity** e.g. Convert: **a** 0.5 L to millilitres **b** 3500 cm^3 to litres.	
4G	**19 I can find the capacity of a rectangular prism** e.g. Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.	
4H	**20 I can find the volume of a prism using its cross-sectional area** e.g. Find the volume of this prism.	











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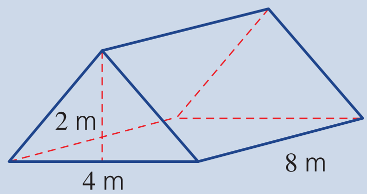
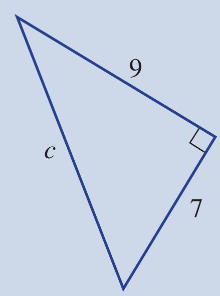
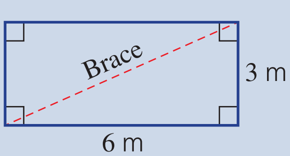
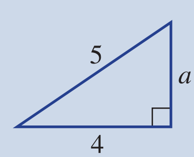
Chapter checklist

Section			✔
4H	21	**I can find the volume of a prism by first calculating its cross-sectional area** e.g. Find the volume of this prism. 	
4I	22	**I can convert between different units of time** e.g. Convert 3 days to minutes.	
4I	23	**I can convert between 24-hour time and am/pm** e.g. Write: **a** 4:30 pm in 24-hour time **b** 1945 hours in am/pm.	
4I	24	**I can use time zones** e.g. Use a world time zone map to find the time in China when it is 9:35 am in New South Wales, Australia.	
4J	25	**I can decide if three numbers form a Pythagorean triple.** e.g. Decide if 6, 8, 10 is a Pythagorean triple.	
4J	26	**I can classify a triangle as right-angled, acute or obtuse.** e.g. A triangle has side lengths 4 m, 7 m and 9 m. Decide if it is right-angled, acute or obtuse.	
4K	27	**I can find the length of the hypotenuse of a right-angled triangle.** e.g. Find the length of c correct to two decimal places. 	
4K	28	**I can identify the diagonal of a rectangle as the hypotenuse of a right-angled triangle and find its length.** e.g. Find the length of this diagonal brace, to the nearest centimetre. 	
4L	29	**I can find the length of a shorter side in a right-angled triangle.** e.g. Find the value of a. 	

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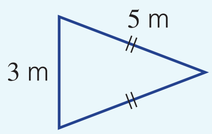
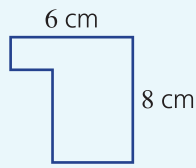
Short-answer questions

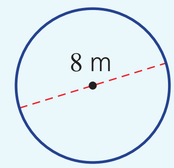
4A/C/G **1** Convert these measurements to the units given in the brackets.

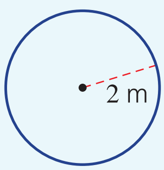
- **a** 2 m (mm)
- **b** 50 000 cm (m)
- **c** 320 m (km)
- **d** 0.04 km (m)
- **e** 3 cm^2 (mm^2)
- **f** 4000 cm^2 (m^2)
- **g** 0.01 km^2 (m^2)
- **h** 350 mm^2 (cm^2)
- **i** 4000 mL (L)
- **j** 3 cm^3 (mm^3)
- **k** 400 cm^3 (L)
- **l** 4300 kL (ML)

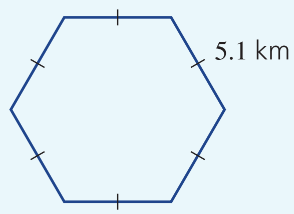
4B/C **2** Find the perimeter/circumference of these shapes. Round the answer to two decimal places where necessary.

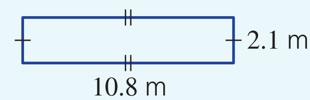
a


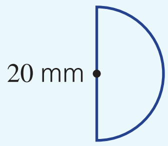
b


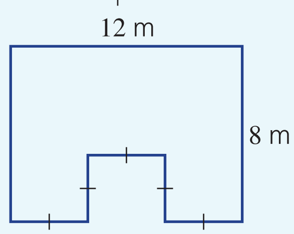
c


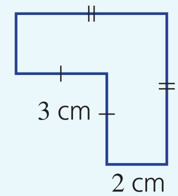
d


e


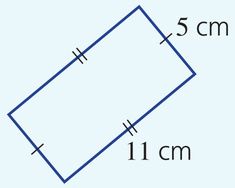
f


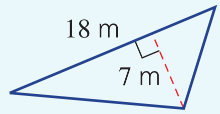
g


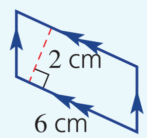
h


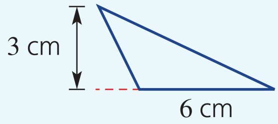
i


4B/D/E **3** Find the area of these shapes. Round the answer to two decimal places where necessary.

a


b


c


d


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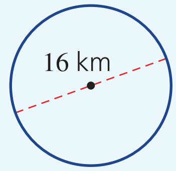

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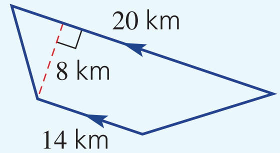
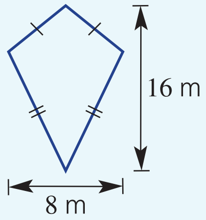


g


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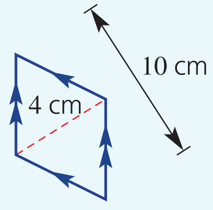

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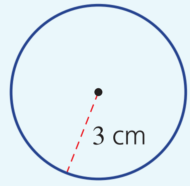

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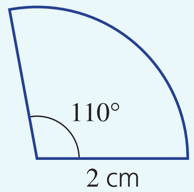
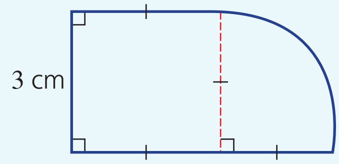
Chapter review

4F **4** Find the area of these shapes. Round to two decimal places where necessary.

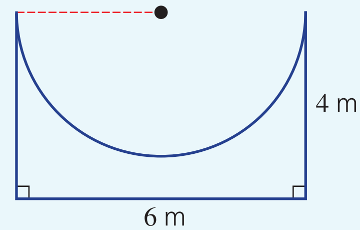
a



b

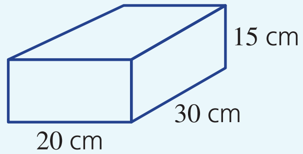


c



4G **5** Find the capacity of these rectangular prisms in litres. Recall $1\text{ L} = 1000\text{ cm}^3$.

a



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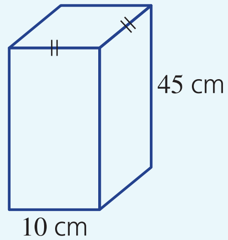
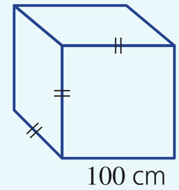
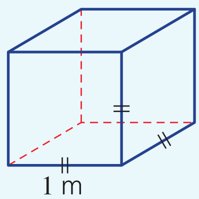


c



4H **6** Find the volume of each prism.

a



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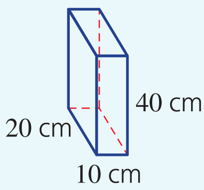

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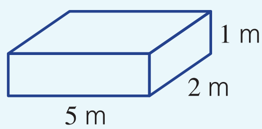

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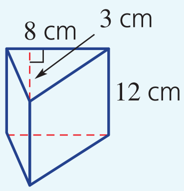

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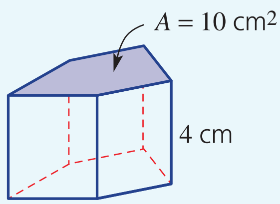

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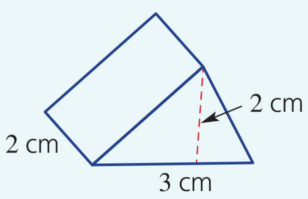

4I **7** An oven is heated from 23°C to 310°C in 18 minutes and 37 seconds. It then cools by 239°C in 1 hour, 20 minutes and 41 seconds.

a Give the temperature:

i increase

ii decrease.

b What is the total time taken to heat and cool the oven?

c How much longer does it take for the oven to cool down than to heat up?

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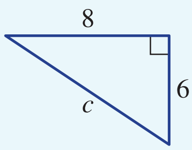
4I **8** **a** What is the time difference between 4:20 am and 2:37 pm?
b Write 2145 hours in am/pm time.
c Write 11:31 pm in 24-hour time.

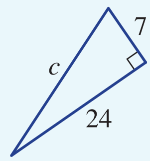
4I **9** When it is 4:30 pm in Western Australia, state the time in each of these places.
a New South Wales
b Adelaide
c Darwin
d China
e Perth
f Phillipines
g New Zealand
h Tasmania
i Queensland

4K **10** Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round the answer to two decimal places in part **c**.

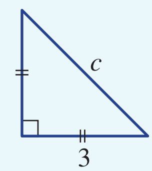
a



b

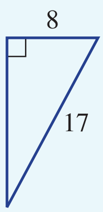


c

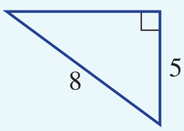


4L **11** Use Pythagoras' theorem to find the unknown length in these right-angled triangles. Round the answer to two decimal places in parts **b** and **c**.

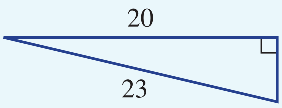
a



b

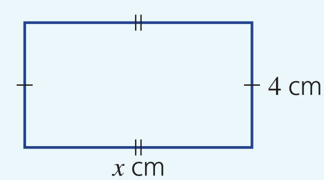


c



Multiple-choice questions

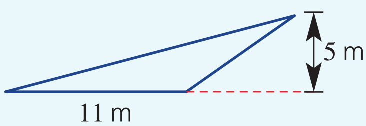
4A **1** The perimeter of this rectangle is 20 cm. The unknown value x is:



A 4 **B** 16 **C** 5
D 10 **E** 6

4B/E **2** A wheel has a diameter of 2 m. Its circumference and area (in that order) are given by:
A $\pi,\ \pi^2$
B $2\pi,\ \pi$
C $4\pi,\ 4\pi$
D $2,\ 1$
E $4,\ 4$

4C **3** The area of this triangle is:



A $27.5\ \text{m}^2$
B 55 m
C $55\ \text{m}^2$
D 110 m
E $16\ \text{m}^2$

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4E **4** Using $\pi = 3.14$, the area of a circular oil slick with radius 100 m is:

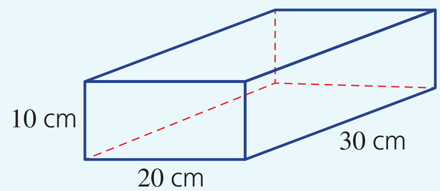
A 7850 m^2
B 314 m^2
C 31 400 m^2
D 78.5 m^2
E 628 m^2

4G **5** 2.5 L is the same as:

A 250 mL
B 2500 cm^3
C 1 ML
D 0.025 kL
E 25 000 mL

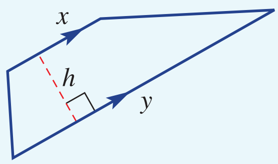
4G **6** The volume of this rectangular prism is:

A 60 L **B** 60 cm
C 6 m^3 **D** 600 cm^3
E 6000 cm^3



4D **7** The rule for the area of the trapezium shown is:

A $\frac{1}{2}xh$ **B** $\frac{1}{2}(x+y)$
C $\frac{1}{2}xy$ **D** πxy^2
E $\frac{1}{2}(x+y)h$

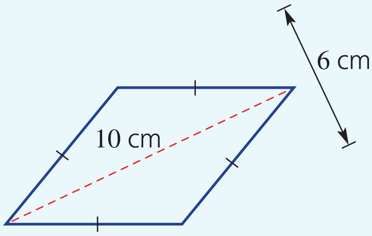


4G **8** The volume of a rectangular prism is 48 cm^3. If its width is 4 cm and height 3 cm, its length would be:

A 3 cm **B** 4 cm **C** 2 cm **D** 12 cm **E** 96 cm

4D **9** The diagonals of a rhombus measure 10 cm and 6 cm. Its area is:

A 120 cm^2 **B** 16 cm^2
C 15 cm^2 **D** 30 cm^2
E 60 cm^2



4D **10** A square has area 49 m^2. Its side length is:

A 5 m
B 8 m
C 49 m
D 7 m
E 4 m

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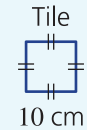
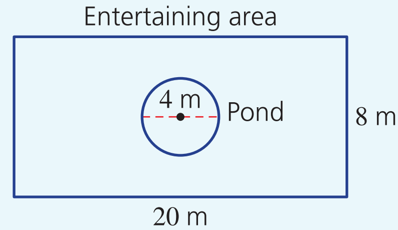
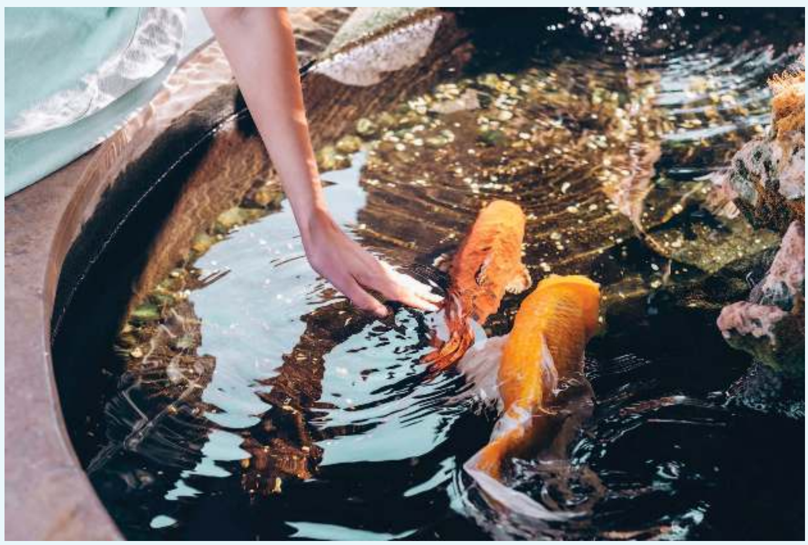
Extended-response questions

1 A rectangular entertaining area is to be tiled.
The tiles are 10 cm square and the entertaining area is 20 m by 8 m.
A circular pond of diameter 4 m is to be built in the centre.



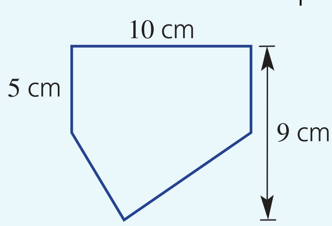


- **a** Find the total area of the entertaining area in m^2.
- **b** Find the perimeter of the entertaining area.
- **c** Find the area of the pond, correct to two decimal places.
- **d** Find the area to be tiled (not including the pond area), correct to two decimal places.
- **e** Find the area of one tile in:
  - **i** cm^2
  - **ii** m^2
- **f** Find the minimum number of tiles required for the job.
- **g** Why might a tiler use more tiles than the minimum number?

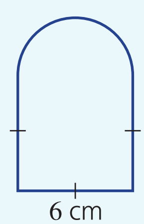


2 Find the area of these composite shapes.

a



b



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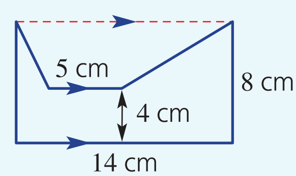

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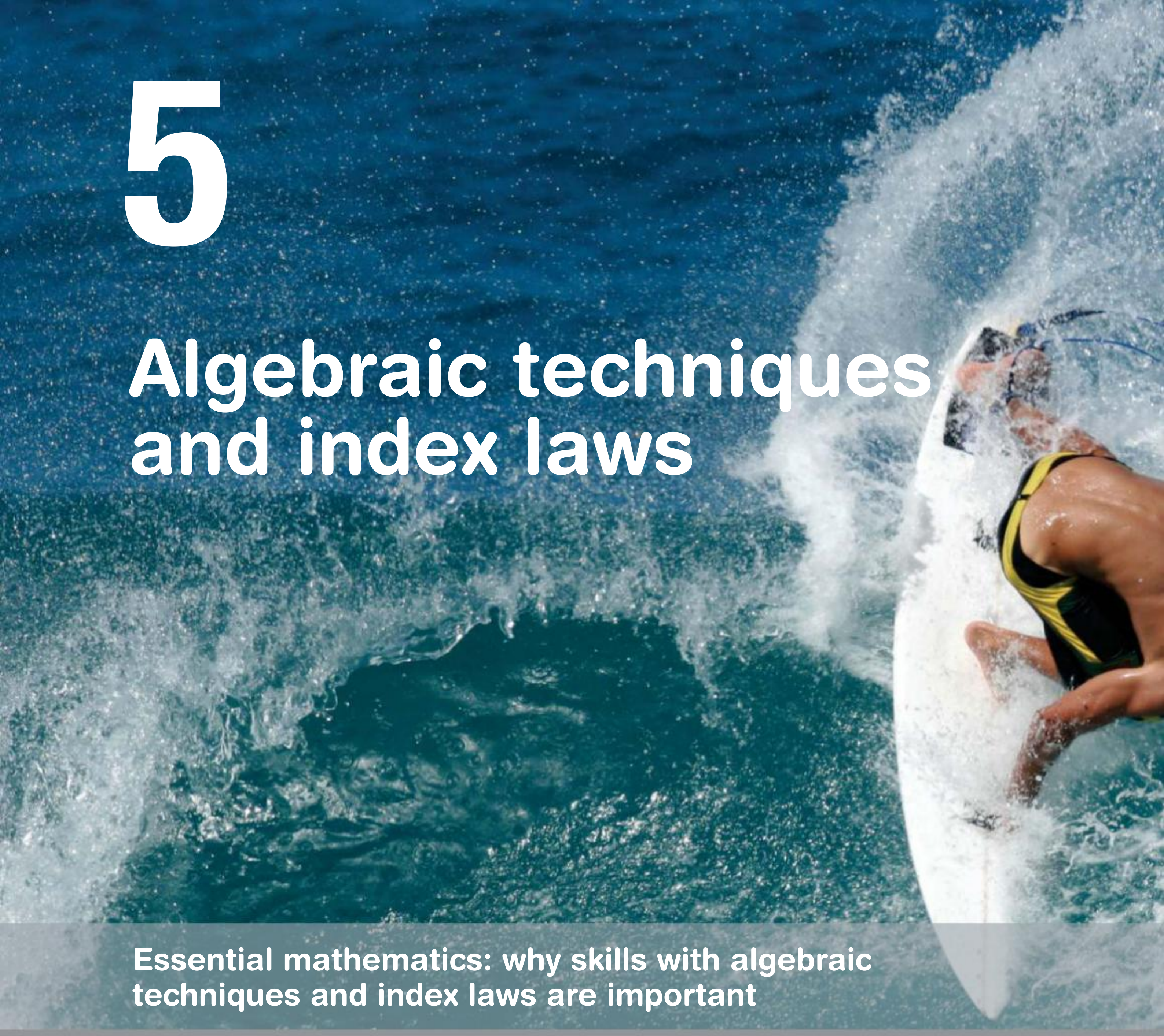

5

Algebraic techniques and index laws

Essential mathematics: why skills with algebraic techniques and index laws are important

Algebra skills are essential when applying formulas, solving problems and coding algorithms.

Algebra formulas are coded into the computer models that run virtual sports activities to test potential design changes in sporting equipment, e.g. Formula One racing cars, kayaks, yachts, surfboards and snowboards.

Financial advisors substitute different values into relevant financial equations, predicting possible future returns on investments.

HVAC technicians apply algebra techniques to calculate air flow rates, heat generated from people and appliances, heat absorbed by a cooling process, and energy consumption for heating and cooling systems.

The index laws are widely applied in science, economics, computer technology and medicine. For example, when calculating the decay time of a radioactive tracer used for medical scans.

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In this chapter

Australian Curriculum 9.0

ALGEBRA

Create, expand, factorise, rearrange and simplify linear expressions, applying the associative, commutative, identity, distributive and inverse properties (AC9M8A01)

NUMBER

Establish and apply the exponent laws with positive integer exponents and the zero exponent, using exponent notation with numbers (AC9M8N02)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 Evaluate.
a $8 + 4 \times 6$
b $4 \times 5 - 2 \times 3$
c $12 - (6 + 2) + 8$
d $3(6 + 4)$

2 Evaluate.
a The sum of 7 and 10
b The product of 2 and 6
c The sum of 12, 10 and 8
d Half of 24

3 If $\square = 10$, write the value of:
a $\square + 2$
b $\square \times 7$
c $\square - 3$
d $\square + \square$

4 Find the value of $\square \times \square$ if:
a $\square = 4$
b $\square = 2$
c $\square = 11$
d $\square = 100$

5 Write an expression for:
a 5 more than x
b 7 less than m
c the product of x and y
d half of w

6 If $y = 2x + 5$, find the value of y when $x = 10$.

7 Complete the tables using the given rules.
a $M = 2A + 3$

A	0	3	7	10
M				

b $y = x + 12$

x	1	3	11	0
y				

8 Substitute $x = 6$ and $y = 2$ into each expression and then evaluate.
a $x + y$
b xy
c $3x - y$
d $2x + 3y$

9 Write down the HCF (highest common factor) of:
a 24 and 36
b 15 and 36
c 48 and 96

10 Evaluate the following
a $3 \times 3 \times 3$
b 2^4
c $(-1)^2$
d 5×2^2

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5A The language of algebra CONSOLIDATING

Learning intentions

- To know the basic terminology of algebra.
- To know how to identify coefficients, terms and constant terms within expressions, including in situations where coefficients are zero or negative.
- To know how to write expressions from worded descriptions.

Key vocabulary: pronumeral, variable, expression, coefficient, term, constant term, sum, difference, product, quotient

A pronumeral (or variable) is a letter that can represent any number. For instance, x could represent the number of goals a particular football player scored last year.

The expression $6x + y$ gives the total number of points in an AFL game, if x is the number of goals and y is the number of points.

Lesson starter: Algebra sort

Consider the four expressions $x + 2$, $x \times 2$, $x - 2$ and $x \div 2$.

- If you know that x is 10, can you sort the four expressions from lowest to highest?
- Give an example of a value of x that would make $x \times 2$ less than $x + 2$.

Key ideas

- In algebra, letters can be used to represent numbers. These letters are called **variables** or **pronumerals**.
- $a \times b$ is written ab and $a \div b$ is written $\frac{a}{b}$.
- An **expression** combines numbers and pronumerals with mathematical operations, e.g. $3x + 2yz$ and $8 \div (3a - 2b) + 41$ are expressions.
- A **term** is part of an expression with only pronumerals, numbers, multiplication and division, e.g. $9a$, $10cd$ and $\frac{3x}{5}$ are all terms.
- A term that does not contain any pronumerals is called a **constant term**.
- A **coefficient** is the number in front of a pronumeral. If the term is being subtracted, the coefficient is a negative number, and if there is no number in front, the coefficient is 1.
  e.g. For the expression $3x + y - 7z$, the coefficient of x is 3, the coefficient of y is 1 and the coefficient of z is -7. There are three terms in the expression.
- Mathematical operations.

Words	Symbols
sum	$+$
difference	$-$
product	$\times$
quotient	$\div$

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Exercise 5A

Understanding — 1–4 | 3, 4

1 Find the following values.
  a The sum of 3 and 5
  b The product of 2 and 6
  c The difference between 10 and 7
  d Double the value of 5

2 The expression $3a + 2b + 5c$ has three terms.
  a List the terms.
  b State the coefficient of:
    i a
    ii b
    iii c
  c Write another expression with three terms.

Hint for Q3: A constant term has no pronumerals.

3 The expression $5a + 7b + c - 3ab + 6$ has five terms.
  a State the constant term.
  b State the coefficient of:
    i a
    ii b
    iii c
  c Write another expression that has five terms.

Hint for Q4: Coefficients are negative if the term is subtracted.

4 For each of the following expressions, state the coefficient of b.
  a $3a + 2b + c$
  b $3a + b + 2c$
  c $4a + 9b + 2c + d$
  d $3a - 2b + f$
  e $5a - 6b + c$
  f $7a - b + c$

Fluency — 5–7 | 5, 6(½), 7, 8

Example 1 Creating expressions from a description

Write an expression for each of the following.
  a The sum of 3 and k
  b The product of m and 7
  c 5 is added to one half of k
  d The sum of a and b is doubled

Solution	Explanation
a $3 + k$	The word 'sum' means +.
b $m \times 7$ or $7m$	The word 'product' means ×.
c $\frac{1}{2}k + 5$ or $\frac{k}{2} + 5$	One half of k can be written $\frac{1}{2} \times k$ (because 'of' means ×), or $\frac{k}{2}$ because k is being divided by two.
d $(a + b) \times 2$ or $2(a + b)$	The values of a and b are being added and the result is multiplied by 2. Brackets are required to multiply the whole result by two and not just the value of b.

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Now you try

Write an expression for each of the following.

a 5 more than x
b The product of 6 and m
c 4 less than twice y
d The sum of a and b is tripled

5 Match each of the following worded statements with the correct mathematical expression.

a The sum of x and 7
b 3 less than x
c x is divided by 2
d x is tripled
e x is subtracted from 3
f x is divided by 3

A $3 - x$
B $\frac{x}{3}$
C $x - 3$
D $3x$
E $\frac{x}{2}$
F $x + 7$

6 Write an expression for each of the following.

a 7 more than y
b 3 less than x
c The sum of a and b
d The product of 4 and p
e Half of q is subtracted from 4
f One third of r is added to 10
g The sum of b and c multiplied by 2
h The sum of b and twice the value of c

7 Describe each of the following expressions in words.

a $3 + x$
b $a + b$
c $2 \times k$
d $\frac{m}{2}$

8 Describe each of the following expressions in words.

a $4 \times b \times c$
b $2a + b$
c $(4 - b) \times 2$
d $4 - 2b$

Problem-solving and reasoning

9, 10 | 10, 11

9 Write an expression for:

a the total cost of buying 10 litres of petrol at $\$x$ per litre
b the time spent shopping if you spend A minutes in the supermarket and B minutes in the department store
c the difference in age between Oliver, who is 22 years old, and his younger cousin Ben, who is k years old
d the volume of water left in a 50-litre vat after x litres are removed.

Hint for Q9: If petrol is \$2 per litre, then the cost for 10 litres is \$20.

10 Marcela buys 7 plants from the local nursery.

a If the cost is \$10 for each plant, what is the total cost?
b If the cost is $\$x$ for each plant, write an expression for the total cost in dollars.
c If the cost of each plant is decreased by \$3 during a sale, write an expression for:
  i the new cost per plant in dollars
  ii the new total cost in dollars of the 7 plants.

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5A

11 Francine earns $\$p$ per week for her job. She works for 48 weeks each year. Write an expression for the amount she earns:
- a in a fortnight
- b in one year (of 48 weeks)
- c in one year if her wage is increased by $20 per week after she has already worked 30 weeks in the year.

DVD Dilemma

— 12

12 Tom would like to purchase some DVDs of two television shows.
- a Write an expression for the total cost of:
  - i 4 seasons of Numbers
  - ii 7 seasons of Proof by Induction
  - iii 5 seasons of both shows
  - iv all 7 seasons of both shows if the final price is halved in a sale.
- b If a is 20 and b is 30, what is the maximum number of seasons he could buy with $200 without getting duplicates?

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5B Substitution and equivalence

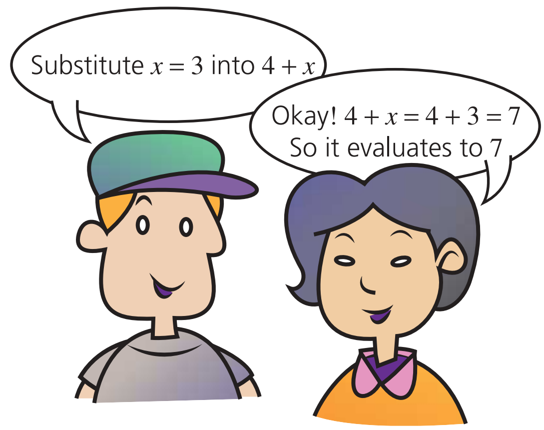
Learning intentions

- To be able to substitute in values to evaluate algebraic expressions.
- To understand what it means for two expressions to be equivalent.
- To understand how the commutative and associative laws for arithmetic can be used to determine equivalence.
- To be able to show that two expressions are not equivalent using substitution.

Key vocabulary: evaluate, substitute, equivalent

Replacing pronumerals with numbers is called substitution. We can evaluate (find the value of) an expression once we substitute in numbers.

If two expressions always evaluate to the same number, they are called equivalent. For instance, $4 + x$ and $x + 4$ are equivalent.



Lesson starter: AFL algebra

In Australian Rules football, the final team score is given by $6x + y$, where x is the number of goals and y is the number of behinds scored.

- State the score if $x = 3$ and $y = 4$.
- If the score is 29, what are the values of x and y? Try to list all the possibilities.
- If $y = 9$ and the score is a 2-digit number, what are the possible values of x?

Key ideas

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value.
  e.g. If $a = 3$, then we can evaluate the expression $7a + 13$:
  $$\begin{aligned} 7a + 13 &= 7 \times 3 + 13 \\ &= 21 + 13 \\ &= 34 \end{aligned}$$
- Two expressions are **equivalent** if they have equal values regardless of the number that is substituted for each pronumeral.

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Exercise 5B

Understanding

1–4 | 4

1 State the value of:
a $5 + 3 \times 2$
b $5 \times 3 + 2$
c $17 - 2 \times 4$
d $20 \div 5 + 3$

Hint for Q1: Brackets first, then division and multiplication, then addition and subtraction.

2 If $\square = 6$, determine the value of each expression.
a $\square + 5$
b $\square \times 2$
c $\square - 3$
d $\square \div 2$

3 Find the value of $\square + 11$ if:
a $\square = 5$
b $\square = 10$
c $\square = 100$
d $\square = 59$

4 Fill in the missing words.
Two expressions that are always equal are called ________________.

Fluency

5–8, 9(½), 10, 11 | 6, 8, 9(½), 10, 11

Example 2 Substituting for a pronumeral

Substitute $x = 3$ to evaluate $5x$.

Solution

$$5x = 5 \times 3$$
$$= 15$$

Explanation

Substitute 3 for x and note that $5x$ means $5 \times x$.

Now you try

Substitute $b = 5$ to evaluate $7 - b$.

5 a What number is obtained when $x = 5$ is substituted into the expression $3 \times x$?
b What is the result of evaluating $20 - b$ if b is equal to 12?
c What is the value of $2b$ if b is equal to 10?

Hint for Q5: $2b$ means $2 \times b$.

6 a State the value of $4 + 2x$ if $x = 5$.
b State the value of $40 - 2x$ if $x = 5$.
c Are $4 + 2x$ and $40 - 2x$ equivalent expressions?

7 Substitute the following values of x into the expression $7x + 2$.
a 4
b 5
c 2
d 8

8 If $y = 4$, find the value of:
a $y + 3$
b $9 - y$
c $3y - 2$
d $5y + 3$

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Example 3 Substituting for multiple pronumerals

Substitute $x = 3$ and $y = 6$ to evaluate $3x + 2y$.

Solution

$$\begin{aligned} 3x + 2y &= 3 \times 3 + 2 \times 6 \\ &= 9 + 12 \\ &= 21 \end{aligned}$$

Explanation

Replace all the pronumerals with their values and remember the order in which to evaluate (multiplication before addition).

Now you try

Substitute $a = 2$ and $b = 9$ to evaluate $12a - 2b$.

9 If $a = 4$ and $b = 7$, evaluate:

a $3a + 2$ **b** $2b - 1$ **c** $a + b$ **d** $6 + ab$
e $3a + b$ **f** $2a + 3b$ **g** $b - a$ **h** $3b - a$

10 Evaluate the expression $2x - 3y$ when:

a $x = 10$ and $y = 4$
b $x = 4$ and $y = 2$

Example 4 Deciding if expressions are equivalent

a Are $x - 3$ and $3 - x$ equivalent expressions?
b Are $a + b$ and $b + a$ equivalent expressions?

Solution

a No

Explanation

The two expressions are equal if $x = 3$ (both equal zero).
But if $x = 7$ then $x - 3 = 4$ and $3 - x = -4$.
Because they are not equal for every single value of x, they are not equivalent.

Solution

b Yes

Explanation

Regardless of the values of a and b substituted, the two expressions are equal. This is because it does not matter the order in which numbers are added.

Now you try

a Are $2a + b$ and $2b + a$ equivalent expressions?
b Are $4x + 2 - x$ and $3x + 2$ equivalent expressions?

11 For the following, state whether they are equivalent (E) or not (N).

a $x + y$ and $y + x$
b $3 \times x$ and $x \times 3$
c $4a + b$ and $4b + a$
d $4 + 2x$ and $2 + 4x$
e $\frac{1}{2} \times a$ and $\frac{a}{2}$
f $3 + 6y$ and $3(2y + 1)$

Hint for Q11: Try different values to see if the expressions are always equal.

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5B

Problem-solving and reasoning | 12 | 12, 13

12 a A number is substituted for k in the expression $7k$ and the result is 56. What is the value of k?
b The variable m is chosen so that $4m$ is a two-digit number and $4+m$ is a single-digit number. List the possible values of m.

13 The expressions ab and $a+b$ are not equivalent.
a Explain why they are not equivalent.
b If $a=0$ and $b=0$, the two expressions are equal. Give an example of another pair of values that make them equal.
c Explain why $a+2$ and $a-2$ are not equivalent.
d Will $a+2$ and $a-2$ ever evaluate to the same number? Why/why not?

Hint for Q13: Find values for a and b where ab and $a+b$ are not equal.

Substituting with negatives | — | 14

14 Copy and complete the following table.

x	3				4	2
y	8	7		-3		
$x+y$		12	5			
$x-2y$			-4		8	
xy				0		12

A vet substitutes values into algebraic formulas to find correct vaccination volumes.

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5C Adding and subtracting terms

Learning intentions

- To understand that 'like terms' contain exactly the same pronumerals, possibly in a different order.
- To be able to decide if two terms are like terms.
- To be able to combine like terms to simplify expressions.

Key vocabulary: term, pronumeral, like terms, sign, simplify

Two terms with the same pronumerals are called like terms, and they can be collected and combined. For example, $2a + 6a$ can be simplified to $8a$ because $2a$ and $6a$ are like terms.

The order of the pronumerals does not matter, so $3ab$ and $5ba$ are like terms because they both include a and b.

Lesson starter: Like terms

The terms $2abc$ and $5cab$ are like terms, and $2abc + 5cab = 7abc$.

In a short amount of time, see how many ways you can fill in the boxes: $\square + \square = 7abc$.

Can you explain why abc and cab are equivalent?

Key ideas

- **Like terms** contain exactly the same **pronumerals** with the same powers; the pronumerals do not need to be in the same order, e.g. $4ab$ and $7ba$ are like terms.
- Like terms can be combined when they are added or subtracted to **simplify** an expression, e.g. $3xy + 5xy = 8xy$.

  – sign stays with following term

  $3x + 7y \boxed{-2x} + 3y + x \boxed{-4y}$
  $= 3x - 2x + x + 7y + 3y - 4y$
  $= 2x + 6y$
- A subtraction **sign** stays in front of a **term** even when it is moved.

Exercise 5C

Understanding

1–4 | 3, 4

1 Fill in the blanks.

a Two terms with exactly the same pronumerals are called _______ _______.

b If two expressions are always equal when evaluated, they are called _______ expressions.

2 **a** If $x = 3$, evaluate $5x + 2x$.

b If $x = 3$, evaluate $7x$.

c $5x + 2x$ is equivalent to $7x$. True or false?

3 **a** If $x = 3$ and $y = 4$, evaluate $5x + 2y$.

b If $x = 3$ and $y = 4$, evaluate $7xy$.

c $5x + 2y$ is equivalent to $7xy$. True or false?

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5C

4 a List the pronumerals that occur in $3abc$.
b List the pronumerals that occur in $7bca$.
c Are $3abc$ and $7bca$ like terms?

Hint for Q4: Like terms have the same pronumerals, possibly in a different order.

Fluency

5–7, 8(½) | 5–8(½), 9

Example 5 Identifying like terms with a single pronumeral

Classify the following pairs as like terms (L) or not like terms (N).

a $3x$ and $12x$
b $5y$ and $7z$

Solution	Explanation
a L	Both $3x$ and $12x$ have the same pronumeral (x) so they are like terms.
b N	$5y$ and $7z$ have different pronumerals so they are not like terms.

Now you try

Classify the following pairs as like terms (L) or not like terms (N).

a $9a$ and $4b$
b $6y$ and $15y$

5 Classify the following pairs as like terms (L) or not like terms (N).
a $5x$ and $2x$
b $5x$ and $2y$
c $3k$ and $4k$
d $2q$ and $7x$

Example 6 Identifying like terms with multiple pronumerals

Classify the following pairs as like terms (L) or not like terms (N).

a $2ab$ and $3ba$
b $4x$ and $2xy$

Solution	Explanation
a L	They have the same pronumerals (order does not matter).
b N	$4x$ has the pronumeral x. $2xy$ has the pronumerals x and y. Since the terms have different pronumerals they are not like terms.

Now you try

Classify the following pairs as like terms (L) or not like terms (N).

a $7ab$ and $4a$
b $14xy$ and $3yx$

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6 Classify the following pairs as like terms (L) or not like terms (N).

a $4pq$ and $3pq$
b $2ab$ and $5bc$
c $7rs$ and $12sr$
d $5ab$ and $6a$
e $7abc$ and $2cba$
f $8x$ and $8xy$
g $12ab$ and $14ba$
h $8xyz$ and $9yzx$

Example 7 Simplifying by combining like terms

Simplify the following by combining like terms.

a $7t + 2t - 3t$ **b** $4x + 3y + 2x + 7y$ **c** $8b + 7ac - 5b + 2ca$

Solution	Explanation
a $7t + 2t - 3t = 6t$	These are like terms, so they can be combined: $7 + 2 - 3 = 6$.
b $4x + 3y + 2x + 7y$ $= 4x + 2x + 3y + 7y$ $= 6x + 10y$	Move the like terms next to each other. Combine the pairs of like terms.
c $8b + 7ac - 5b + 2ca$ $= 8b - 5b + 7ac + 2ca$ $= 3b + 9ac$	Move like terms together. The subtraction sign stays in front of $5b$ when it is moved. $8 - 5 = 3$ and $7 + 2 = 9$

Now you try

Simplify the following by combining like terms.

a $13y - 9y$ **b** $16a + 2b - 5a - b$ **c** $7ab - b - 6ba + 6b$

7 Simplify the following by combining like terms.

a $3x + 2x$
b $7a + 12a$
c $15x - 6x$
d $9y - 2y$
e $4xy + 3xy$
f $16uv - 3uv$
g $10ab + 4ba$
h $3pq + 12pq$

8 Simplify the following by combining like terms.

a $7f + 2f + 8 + 4$
b $10x + 3x + 5y + 3y$
c $2a + 5a + 13b - 2b$
d $10a + 5b + 3a + 4b$
e $10 + 5x + 2 + 7x$
f $10a + 3 + 4b - 2a - b$
g $10x + 31y - y + 4x$
h $11a + 4 - 2a + 12a$
i $2b + 4c + 3b + 5c$
j $3a - b + 4b - a$
k $2qr + 3q + 4qr + 6rq$
l $12xy - 5yx + 3x + 6x$
m $10ab - 4b - 6ba + 11b$
n $20kl + 10kl - 7lk + 2l$

Hint for Q8: Pair up the like terms.
Note: $ab = ba$.

9 For each expression, choose an equivalent expression from the options listed.

a $7x + 2x$
b $12y + 3x - 2y$
c $3x + 3y$
d $8y - 2x + 6y - x$
e $4xy + 5yx$

A $10y + 3x$
B $9xy$
C $9x$
D $3y + 3x$
E $14y - 3x$

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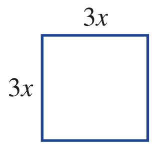
5C

Problem-solving and reasoning | 10, 11 | 10–12

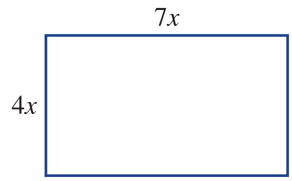
10 Write expressions for the perimeters of the following shapes in simplest form.

Hint for Q10: Perimeter = total distance around a shape.

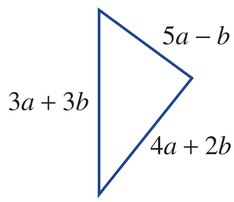
a



b



c



11 Towels cost $\$c$ each at a shop.

a John buys 3 towels, Mary buys 6 towels and Naomi buys 4 towels. Write a fully simplified expression for the total amount spent on towels.

b On another occasion, Chris buys n towels, David buys twice as many as Chris, and Edward buys 3 times as many as David. Write a simplified expression for the total amount they spent on towels.

12 **a** Make a substitution to prove that $4a + 3b$ is not equivalent to $7ab$.

b Is $4a+3b$ ever equal to $7ab$? Try to find some values of a and b to make $4a+3b = 7ab$ a true equation.

c Is $4a + 3a$ ever not equal to $7a$? Explain your answer.

Filling the blanks | — | 13

13 The expression $4a + 7b + 6a$ is equivalent to $10a + 7b$.

a Give another way to fill in the blanks to make this statement true:

$\square\, a + \square\, b + \square\, a = 10a + 7b$

b Assuming the blanks above must be filled by positive integers, how many ways could they be filled to make a true statement?

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5D Multiplying and dividing terms

Learning intentions

- To understand that the order in which pronumerals are multiplied is not important.
- To understand the meaning of x^2.
- To be able to multiply terms and simplify the result.
- To be able to divide terms and simplify the result.

Key vocabulary: term, pronumeral, common factor, simplify

Recall that $4ab$ is shorthand for $4 \times a \times b$. Observing this helps us to see how we can multiply terms.

$$\begin{aligned} 4ab \times 3c &= 4 \times a \times b \times 3 \times c \\ &= 4 \times 3 \times a \times b \times c \\ &= 12abc \end{aligned}$$

Division is written as a fraction so $\frac{12ab}{9ad}$ means $(12ab) \div (9ad)$. To simplify a division we look for common factors.

$$\frac{{}^{4}\cancel{12} \times \cancel{a} \times b}{{}^{3}\cancel{9} \times \cancel{a} \times d} = \frac{4b}{3d}$$ $a \div a = 1$ for any value of a except 0, so $\frac{a}{a}$ cancels to 1.

Lesson starter: Multiple ways

Multiplying $4a \times 6b$ gives you $24ab$.

- In how many ways can positive integers fill the blanks in $\square\, a \times \square\, b = 24ab$?
- Can you explain why there are more ways to fill in the blanks for $\square\, a \times \square\, b = 24ab$ than for $\square\, a \times \square\, b = 25ab$?

Key ideas

- $12abc$ means $12 \times a \times b \times c$.
- When multiplying, the order is not important: $2 \times a \times 4 \times b = 2 \times 4 \times a \times b$.
- x^2 means $x \times x$.
- When dividing, cancel any **common factors**.
  For example: $\frac{{}^{3}\cancel{15}\, x\cancel{y}}{{}^{4}\cancel{20}\, \cancel{y}z} = \frac{3x}{4z}$

Exercise 5D

Understanding

	1–4	4

1 Are the following true (T) or false (F)?
- **a** $3 \times a$ can be written as $3a$.
- **b** $k \times 5$ can be written as $5k$.
- **c** $2x$ is short for $2 + x$.
- **d** $4ab$ could also be written as $4a \div b$.
- **e** $q \times q$ can be written as q^2.

2 Which is the correct way to write $3 \times a \times b \times b$?
- **a** $3ab$
- **b** $3ab^2$
- **c** ab^3
- **d** $3a^2b$

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5D

3 Simplify these fractions.

a $\frac{12}{20}$ **b** $\frac{5}{15}$ **c** $\frac{12}{8}$ **d** $\frac{15}{25}$

4 Write these without multiplication signs.

a $3 \times x \times y$ **b** $5 \times a \times b \times c$ **c** $12 \times a \times b \times b$ **d** $4 \times a \times c \times c \times c$

Fluency

5–7(½), 8 5–8(½)

Example 8 Multiplying terms

Simplify $7a \times 2bc \times 3d$.

Solution

$$\begin{aligned} 7a \times 2bc \times 3d &= 7 \times a \times 2 \times b \times c \times 3 \times d \\ &= 7 \times 2 \times 3 \times a \times b \times c \times d \\ &= 42abcd \end{aligned}$$

Explanation

Write the expression with multiplication signs and bring the numbers to the front.
Simplify: $7 \times 2 \times 3 = 42$ and $a \times b \times c \times d = abcd$

Now you try

Simplify $3x \times 7yz$.

5 Simplify the following.

a $7d \times 9$ **b** $5a \times 2$ **c** $3 \times 12x$
d $4k \times 6$ **e** $3 \times 2q$ **f** $3x \times 10y$
g $4a \times 2b \times cd$ **h** $3a \times 10bc \times 2d$ **i** $4a \times 6de \times 2b$

Example 9 Multiplying terms with repeated pronumerals

Simplify $3xy \times 5xz$.

Solution

$$\begin{aligned} 3xy \times 5xz &= 3 \times x \times y \times 5 \times x \times z \\ &= 3 \times 5 \times x \times x \times y \times z \\ &= 15x^2yz \end{aligned}$$

Explanation

Write the expression with multiplication signs and bring the numbers to the front.
Simplify, remembering that $x \times x = x^2$.

Now you try

Simplify $4a \times 7ab$.

6 Simplify the following.

a $x \times x$ **b** $a \times a$ **c** $3d \times d$ **d** $5d \times 2d \times e$
e $7x \times 2y \times x$ **f** $5xy \times 2x$ **g** $4xy \times 2xz$ **h** $4abc \times 2abd$
i $12xy \times 4x$ **j** $9ab \times 2a$ **k** $3xy \times 2x \times 4y$ **l** $2ab \times 4a \times 3b$

7 Write each expression without a division sign.

a $k \div 4$ **b** $x \div 5$
c $2q \div 5$ **d** $3k \div 10$
e $5 \div a$ **f** $a \div b$
g $x \div y$ **h** $12 \div g$

Hint for Q7: $\frac{k}{4}$ is the same as $k \div 4$.

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Example 10 Dividing terms

Simplify $\frac{10ab}{15bc}$.

Solution

$$\frac{10ab}{15bc} = \frac{{}^{2}\cancel{10} \times a \times \cancel{b}}{{}^{3}\cancel{15} \times \cancel{b} \times c}$$

$$= \frac{2a}{3c}$$

Explanation

Write the numerator and denominator in full, with multiplication signs. Cancel any common factors and remove the multiplication signs.

Now you try

Simplify $\frac{16x}{8xy}$.

8 Simplify the following divisions by cancelling any common factors.

a $\frac{5a}{10a}$ **b** $\frac{7x}{14y}$ **c** $\frac{10xy}{12y}$ **d** $\frac{ab}{4b}$

e $\frac{7xyz}{21yz}$ **f** $\frac{2}{12x}$ **g** $\frac{4xy}{7x}$ **h** $\frac{3abc}{6b}$

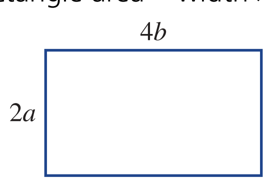
Hint for Q8: Cancel numbers and pronumerals where possible.

Problem-solving and reasoning

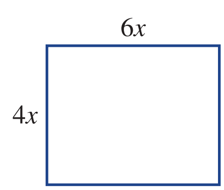
9, 10 | 9–12

9 Write a simplified expression for the area of the following shapes. Recall that rectangle area = width × length.

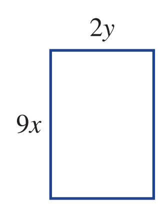
a



b



c



Hint for Q10: You can combine any like terms.

10 Simplify the following completely.

a $2a \times 3b + 5ab$ **b** $6q \times 2r + 4q \times 3r$

c $10x \times 2y - 3y \times 6x$

11 Fill in the missing terms to make the following equivalences true.

a $3x \times \square \times z = 6xyz$ **b** $4a \times \square = 12ab$ **c** $\frac{\square}{4r} = 7s$ **d** $\frac{\square}{2ab} = 4b$

12 Joanne claims that the following three expressions are equivalent: $\frac{2a}{5}, \frac{2}{5} \times a, \frac{2}{5a}$.

a Is she right? Try different values of a.

b Which two expressions are equivalent?

c There are two values of a that make all three expressions equal. State one of them.

Missing coefficients

— | 13

13 **a** Simplify $2a \times 3b + 5b \times 2a$ to a single term.

b State another way to fill in the blanks to make the simplification correct:

$\square a \times \square b + \square b \times \square a = 16ab$

c Give an example of an even longer expression that is equivalent to $16ab$.

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5E Expanding brackets

Learning intentions

- To understand that the distributive law can be used to expand brackets.
- To be able to relate the distributive law to the area of rectangles.
- To be able to expand brackets using the distributive law.
- To be able to use expansion together with combining like terms to simplify expressions.

Key vocabulary: distributive law, expand, brackets

Two expressions can look different and still be equivalent, like $x + x$ and $2x$.

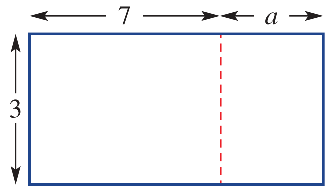
Note that $3 \times (7 + a) = 3(7 + a)$, which is equivalent to 3 groups of $7 + a$, so:

$$\begin{aligned} 3(7 + a) &= 7 + a + 7 + a + 7 + a \\ &= 21 + 3a \end{aligned}$$

This means that $3(7 + a)$ and $21 + 3a$ are equivalent.

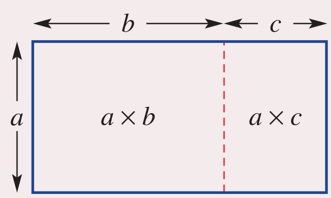
Lesson starter: Equivalent areas

What is the total area of the rectangle shown below? Try to write two expressions: one with brackets and the other without brackets.



Key ideas

- **Expanding brackets** involves writing an equivalent expression without brackets:
  $2(a + b) = a + b + a + b$ or $2(a + b) = 2 \times a + 2 \times b$
  $= 2a + 2b$ $= 2a + 2b$
- To expand brackets, you can use the **distributive law**, which states that:
  - $a(b + c) = ab + ac$
    $a(b - c) = ab - ac$
- The distributive law can be demonstrated by considering rectangle areas: $a(b + c) = ab + ac$



$$\left.\begin{aligned} \text{Area} &= a(b + c) \\ \text{Area} &= ab + ac \end{aligned}\right\} a(b + c) = ab + ac$$

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Exercise 5E

Understanding

1–3 | 3

1 Copy and complete.

a $a(b+c) = ab + \_\_\_$

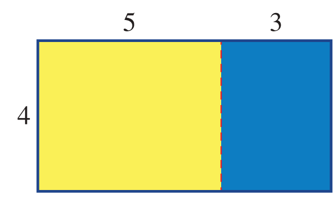
b $a(b-c) = \_\_\_ - \_\_\_$

2 The rectangle shown has a width of 4 and a length of $5 + 3 = 8$.

a What is the area of the yellow rectangle?

b What is the area of the blue rectangle?

c What is the total combined area?



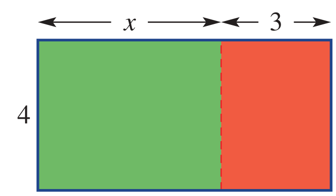
3 The area of the rectangle shown can be written as $4(x + 3)$.

a What is the area of the green rectangle?

b What is the area of the red rectangle?

c Write the total area as an expression without using brackets.

d Fill in the blank: The expressions $4(x + 3)$ and $4x + 12$ are __________ expressions.

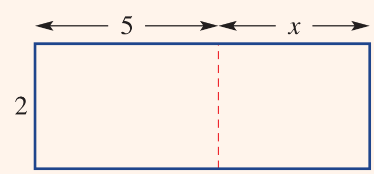


Fluency

4–6, 7(½) | 4–7(½), 8

Example 11 Expanding brackets using rectangle areas

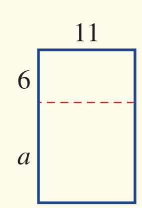
Write two equivalent expressions for the total area of the rectangle shown: one with brackets and the other without brackets.



Solution	Explanation
Using brackets: $2(5 + x)$	The whole rectangle has a width of 2 and a length of $5 + x$.
Without brackets: $10 + 2x$	The smaller rectangles have area $2 \times 5 = 10$ and $2 \times x = 2x$, which are added.

Now you try

Write two equivalent expressions for the total area of the rectangle shown: one with brackets and the other without brackets.



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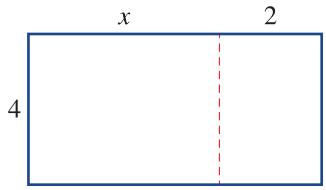
5E

4 For each of the following rectangles, write two equivalent expressions for the total area.

a


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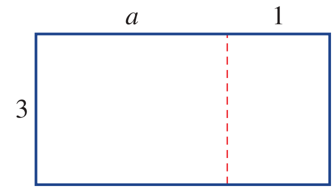
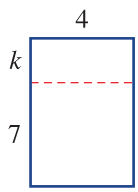
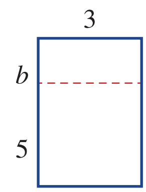


c


d


Hint for Q4: One of the expressions should have brackets.

Example 12 Expanding using the distributive law

Expand the following expressions.

a $5(x+3)$ b $3(a-4)$ c $2(3p-7q)$

Solution

a $5(x+3) = 5x + 5 \times 3$
$= 5x + 15$

Explanation

Using the distributive law

$5(x+3) = 5 \times x + 5 \times 3$

Simplify the result.

Alternative layout:

	x	$+3$
5	$5x$	$+15$

b $3(a-4) = 3a - 3 \times 4$
$= 3a - 12$

Using the distributive law

$3(a-4) = 3 \times a - 3 \times 4$

Simplify the result.

Alternative layout:

	a	-4
3	$3a$	-12

c $2(3p-7q) = 2 \times 3p - 2 \times 7q$
$= 6p - 14q$

Using the distributive law
$2(3p-7q) = 2 \times 3p - 2 \times 7q$
Simplify the result, remembering $2 \times 3p = 6p$ and $2 \times 7q = 14q$.

Alternative layout:

	$3p$	$-7q$
2	$6p$	$-14q$

Now you try

Expand the following expressions.

a $4(x+9)$ b $2(a-7)$ c $12(4m-3q)$

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5 Use the distributive law to expand the following.
a $6(y+8)$ **b** $7(l+4)$ **c** $9(a+7)$ **d** $2(t+6)$

6 Use the distributive law to expand the following.
a $2(m-10)$ **b** $8(y-3)$ **c** $3(e-7)$ **d** $7(e-3)$

7 Use the distributive law to expand the following.
a $10(6g-7)$ **b** $5(3e-8)$ **c** $5(7w+10)$ **d** $5(2u+5)$
e $7(8x-2)$ **f** $3(9v-4)$ **g** $7(2q-4)$ **h** $4(5c-v)$
i $4(2+5x)$ **j** $3(7+2y)$ **k** $8(9-3x)$ **l** $11(2-4k)$

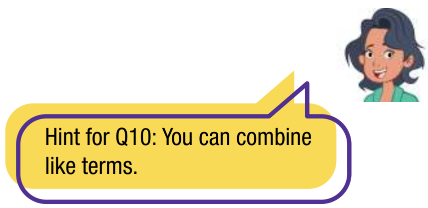
8 Fill in the missing number in the following expansions.
a $4(x+5)=4x+\square$
b $3(x+2)=3x+\square$
c $5(3a+2)=15a+\square$
d $7(4x-2)=28x-\square$

Problem-solving and reasoning

9, 10 | 10–12

9 The perimeter of a rectangle is given by the expression $2(l+w)$ where l is the length and w is the width. What is an equivalent expression for this?

10 Expand the brackets in the following and then simplify the result.
a $3(x+2)+4x$
b $4(a+3)-2a$
c $5(3b-2)+10$
d $6(2c+4)-2c$



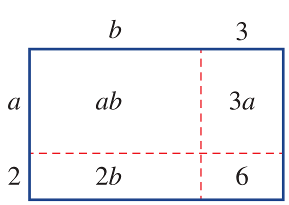
11 Write an expression for each of the following and then expand it.
a A number x has 3 added to it and the result is multiplied by 5.
b A number b has 6 added to it and the result is doubled.
c A number z has 4 subtracted from it and the result is multiplied by 3.
d A number y is subtracted from 10 and the result is multiplied by 7.

12 When expanded, $4(2a+6b)$ gives $8a+24b$. Find two other expressions that expand to $8a+24b$.

Bigger expansions

— | 13

13 The diagram below helps to demonstrate that $(a+2)(b+3)=ab+2b+3a+6$.



Use a diagram like the one above to expand the following expressions.
a $(a+4)(b+2)$
b $(x+3)(y+5)$
c $(2a+5)(3c+2)$
d $(4a+1)(5b+3)$

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Progress quiz

5A **1** For each of the following expressions, state the coefficient of b.

a $8a + 5b - 2c$
b $12 - 4b$
c $15a + 7c - 11b$
d $a + b - 3d$

5A **2** Match each of the following worded statements with the correct mathematical expression.

a The sum of x and y — A $x - 4$
b 4 is subtracted from x — B $\frac{x}{3} + y$
c x is quadrupled — C $4 - x$
d x is divided by 3 and y is added — D $x + y$
e x is subtracted from 4 — E $\frac{x}{2} + 6$
f x is halved and 6 is added — F $4x$

5B **3** If $a = 5$, find the value of:

a $11a$
b $24 - 3a$
c $a + 2a + 3a$
d $100 - a^2 + 2a$

5B **4** Evaluate the expression $4x - 3y$ when:

a $x = 8$ and $y = 5$
b $x = 2$ and $y = 3$
c $x = 11$ and $y = 0$
d $x = 100$ and $y = 1$

5C **5** Classify the following pairs as like terms (L) or not like terms (N).

a $5p$ and 5
b $12p$ and $17pq$
c $40x$ and $5x$
d $21ft$ and $2tf$

5C **6** Simplify the following by combining like terms.

a $5h + 8h - 3h$
b $12t + 7r - 3t$
c $4x + 4xy - 5y + 3xy$
d $9kt - 5k + 6k - 4tk$

5D **7** Simplify the following.

a $5w \times 3$
b $6y \times 3z$
c $2a \times 3b \times 4c$
d $5ef \times 11 \times 2m$

5D **8** Simplify the following.

a $y \times y$
b $4t \times 3t$
c $5h \times 3jh$
d $6g \times 3f \times 2f \times g$

5D **9** Simplify the following.

a $\frac{3f}{12}$
b $\frac{15xy}{5y}$
c $\frac{3ac}{9bcd}$
d $\frac{14xy}{21x}$

5E **10** Expand the following expressions.

a $4(x + 6)$
b $2(5y - 7)$
c $5(4m - 3n)$
d $x(8 - 3x)$

5E **11** Expand the brackets in the following and then simplify the result.

a $6(x + 3) - 2x$
b $2(5 - 3x) + 7$
c $4(3x - 2y) - 8x$
d $x(x + 3) + 7x$

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5F Factorising expressions

Learning intentions

- To understand that factorising is the reverse of expanding.
- To be able to find the highest common factor (HCF) of two terms.
- To be able to factorise expressions.

Key vocabulary: factorise, expand, highest common factor (HCF)

Factorising is the opposite procedure to expanding. Because $3(2x+5)$ expands to $6x+15$, this means that a factorised form of $6x+15$ is $3(2x+5)$.

Lesson starter: Expanding gaps

Try to fill in the gaps to make the following equivalence true: $\square(\square+\square) = 12x+24$.

- In how many ways can this be done? Try to find as many ways as possible.
- If the aim is to make the term outside the brackets as large as possible, what is the best possible solution to the puzzle?

Key ideas

- The **highest common factor (HCF)** of two terms is the largest factor that divides into each term.
  e.g. HCF of $15x$ and $21y$ is 3.
  HCF of $10a$ and $20c$ is 10.
  HCF of $12x$ and $18xy$ is $6x$.
- To **factorise** an expression, first take the HCF of the terms outside the brackets, and divide each term by it, leaving the result in brackets.
  e.g. $10x+15y$
  HCF = 5

  Result $5(2x+3y)$

  HCF $\quad 10x \div 5 \quad 15y \div 5$

Exercise 5F

Understanding

	1–3	3

1 Find the highest common factor of the following pairs of numbers.
a 12 and 18 **b** 15 and 25 **c** 40 and 60 **d** 24 and 10

2 Fill in the blanks.
a $5x \times \square = 15x$ **b** $7 \times \square a = 28a$ **c** $3 \times \square = 6b$ **d** $2 \times \square = 14x$

3 Fill in the blanks to make these expansions correct.
a $3(4x+1) = \square x + 3$
b $5(7-2x) = \square - 10x$
c $6(2+5y) = \square + \square$
d $7(2a-3b) = \square - \square$
e $3(2a+\square) = 6a+21$
f $4(\square - 2y) = 12 - 8y$
g $7(\square + \square) = 14 + 7q$
h $\square(2x+3y) = 8x+12y$

Hint for Q3: $a(b+c) = ab+ac$

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5F

Fluency

4, 5, 6(½) | 4–7(½)

Example 13 Finding the highest common factor (HCF) of terms

Find the highest common factor (HCF) of:

a $12k$ and 20

b $18x$ and $24xy$

Solution	Explanation
a 4	There are no pronumerals in common so choose the HCF of 12 and 20.
b $6x$	6 is the largest number that divides into 18 and 24, and x is in both terms.

Now you try

Find the highest common factor (HCF) of:

a 16 and $30x$

b $12ab$ and $8b$

4 Find the highest common factor (HCF) of the following pairs of terms.

a 15 and $10x$

b $20a$ and 12

c 27 and $9b$

d $7y$ and $14x$

e $3a$ and $6b$

f $12x$ and $18y$

5 Find the HCF of the following pairs of terms.

a $12x$ and $18xy$

b $8a$ and $16ab$

c $9bc$ and $12b$

d $36xy$ and $24y$

e $10q$ and $12qr$

f $8p$ and $20pq$

Hint for Q5: The HCF can include pronumerals.

Example 14 Factorising expressions

Factorise the following expressions.

a $6x + 15$

b $12a + 18ab$

c $21x - 14y$

Solution	Explanation
a $6x + 15 = 3(2x + 5)$	HCF of $6x$ and 15 is 3. $6x \div 3 = 2x$ and $15 \div 3 = 5$
b $12a + 18ab = 6a(2 + 3b)$	HCF of $12a$ and $18ab$ is $6a$. $12a \div 6a = 2$ and $18ab \div 6a = 3b$
c $21x - 14y = 7(3x - 2y)$	HCF of $21x$ and $14y$ is 7. $21x \div 7 = 3x$ and $14y \div 7 = 2y$

Now you try

Factorise the following expressions.

a $4x - 14$

b $14b + 35ab$

c $15ab - 10a$

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6 Factorise the following by first finding the HCF.

a $3x+6$ b $8v+40$ c $15x+35$ d $10z+25$
e $40+4w$ f $5j-20$ g $9b-15$ h $12-16f$
i $5d-30$ j $10x+5$ k $6k-12$ l $18p+20$

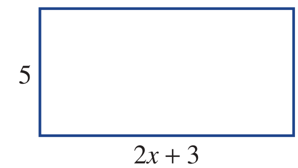
7 Factorise the following.

a $10cn+12n$ b $24y+8ry$ c $14jn+10n$ d $24g+20gj$
e $10h+4z$ f $30u-20n$ g $21p-6c$ h $12a+15b$

Problem-solving and reasoning

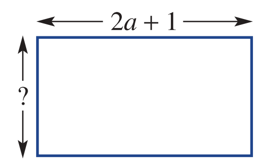
8, 9 | 9, 10

8 The rectangle shown to the right has an area of $10x+15$. Draw a rectangle that would have an area of $12x+16$.

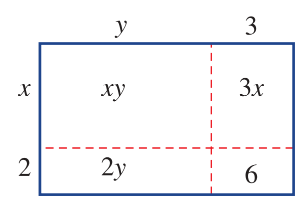


9 The area of the rectangle shown to the right is $10a+5$. One side's measurement is unknown.

a What is the value of the unknown measurement?
b Write an expression for the perimeter of the rectangle.



10 Consider the diagram shown to the right. What is the factorised form of $xy+3x+2y+6$?



The factorising photographer

— | 11

11 A group of students lines up for a photo. They are in 6 rows with x students in each row. Another 18 students join the photo.

a Write an expression for the total number of students in the photo.
b Factorise the expression above.
c How many students would be in each of the 6 rows now? Write an expression.
d If the photographer wanted just 3 rows, how many students would be in each row? Write an expression.
e If the photographer wanted just 2 rows, how many students would be in each row? Write an expression.

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5G Applying algebra ★

Learning intentions

- To be able to model simple situations using algebra.
- To be able to write expressions from descriptions.
- To understand that applying a model requires defining what the variables stand for.

Key vocabulary: modelling, expression, variable, units

The skills of algebra can be applied to many situations involving unknown or varying quantities.

Lesson starter: Carnival conundrum

Alwin, Bryson and Calvin have each been offered special deals for the local carnival.

- Alwin can pay \$50 to go on all the rides all day.
- Bryson can pay \$20 to enter the carnival and then pay \$2 per ride.
- Calvin can enter the carnival at no cost and then pay \$5 per ride.

- Which of them do you think has the best deal?
- In the end, they each went on 12 rides. Who paid the most? Who paid the least?

Algebra can be applied to both the engineering of a carnival ride and the price of tickets.

Key ideas

- Different situations can be **modelled** with algebraic **expressions**.
- To apply a rule, the **variables** should first be clearly defined.
  e.g. total cost is $2 \times n + 3 \times d$
  n = number of minutes d = distance in km

Exercise 5G

Understanding

Understanding	1–4	4

1 The cost of a newspaper is \$2 and the cost of an ice-cream is \$3. Find the cost of:
 a 5 newspapers
 b 4 ice-creams
 c 10 newspapers and 2 ice-creams

2 An episode of Joshua's favourite show lasts 30 minutes.
 a How long would it take him (in minutes) to watch:
  i 2 episodes? ii 5 episodes? iii 10 episodes?
 b Which of the following expressions gives the total time to watch n episodes?
  A $n + 30$ B $30n$ C $n \div 30$ D $30 - n$

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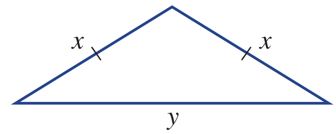
3 Evaluate the expression $3d+5$ when:

a $d=10$ **b** $d=12$ **c** $d=0$

4 Consider the isosceles triangle shown.

a Write an expression for the perimeter of the triangle.

b Find the perimeter when $x=3$ and $y=2$.



Fluency

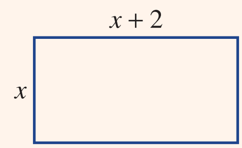
Fluency	5–7	5–8

Example 15 Writing expressions from descriptions

Write an expression for the following situations.

a The total cost of k bottles if each bottle costs \$4.

b The perimeter of a rectangle if its length is 2 cm more than its width, and its width is x cm



c The total cost of hiring a plumber for n hours if he charges a \$40 call-out fee and \$70 per hour.

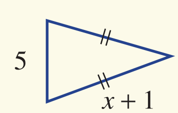
Solution	Explanation
a $4\times k=4k$	Each bottle costs \$4 so the total cost is \$4 multiplied by the number of bottles purchased.
b $x+x+2+x+x+2=4x+4$	Width $=x$, so length $=x+2$. The perimeter is width + length + width + length.
c $40+70n$	\$70 per hour means that the cost to hire the plumber would be $70\times n$. Additionally, \$40 is added for the call-out fee, which is charged regardless of how long the plumber stays.

Now you try

Write expressions for the following situations.

a The amount received by each person if \$100 is divided equally between n people.

b The perimeter of this isosceles triangle.

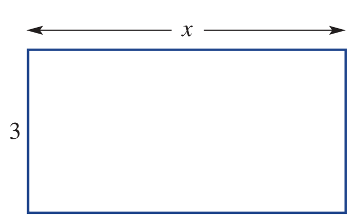


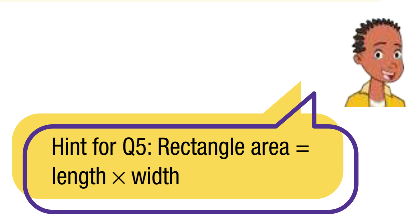
c The cost of hiring a car for n hours if it costs \$100 up-front plus \$20 per hour.

5 **a** Write an expression for the total perimeter of the shape shown.

b If $x=9$, what is the perimeter?

c Write an expression for the area.





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6 Pens cost $3 each.

a How much would 10 pens cost?

b Write an expression for the total cost of n pens.

c If $n = 12$, find the total cost.

7 An electrician charges a call-out fee of $30 and $90 per hour.

a How much does a 2-hour visit cost?

b Which of the following represents the total cost for a visit of x hours?

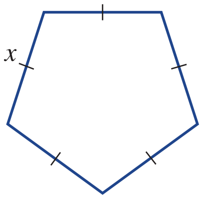
A $x(30+90)$ B $30x+90$

C $30+90x$ D $120x$

8 a Give an expression for the perimeter of this regular pentagon.

b If each side length were doubled, what would the perimeter be?

c If each side length has 3 added to it, write a new expression for the perimeter.



Problem-solving and reasoning

9, 10 | 10–12

9 An indoor soccer pitch costs $40 per hour to hire plus a $30 booking fee.

a Write an expression for the cost of hiring the pitch for x hours.

b What is the cost of hiring the pitch for an 8-hour tournament?

10 A plumber says that the cost in dollars to hire her for x hours is $50 + 60x$.

a What is her call-out fee?

b How much does she charge per hour?

c How much does a 3-hour visit cost?

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11 A repairer says the cost in dollars to hire his services for x hours is $20(3+4x)$.

a How much would it cost to hire him for 1 hour?

b Expand the expression he has given you.

c What is:

i his call-out fee

ii the amount he charges per hour?

Hint for Q11: $20(3+4x)$

12 Tamir notes that whenever he hires an electrician, they charge a call-out fee of $\$F$ and an hourly rate of $\$H$ per hour.

a Write an expression for the cost of hiring an electrician for one hour.

b Write an expression for the cost of hiring an electrician for two hours.

c Write an expression for the cost of hiring an electrician for 30 minutes.

Hint for Q12: Your expressions should involve F and H.

★ Ticket sales — 13

13 Three deals are available at a fair.
Deal 1: Pay \$10, rides cost \$4/each.
Deal 2: Pay \$20, rides cost \$1/each.
Deal 3: Pay \$30, all rides are free.

a Write an expression for the total cost of n rides using deal 1. (The total cost includes the entry fee of \$10.)

b Write an expression for the total cost of n rides using deal 2.

c Write an expression for the total cost of n rides using deal 3.

d Which of the three deals is best for someone going on just two rides?

e Which of the three deals is best for someone going on 20 rides?

f Fill in the gaps:

i Deal 1 is best for people wanting up to ____ rides.

ii Deal 2 is best for people wanting between ____ and ____ rides.

iii Deal 3 is best for people wanting more than ____ rides.

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5H The index laws

Learning intentions

- To be able to apply the index law for multiplying numbers written in index notation terms with the same base.
- To be able to apply the index law for dividing numbers written in index notation terms with the same base.
- To be able to apply the index law for a number in index notation is raised to another power.
- To be able to simplify expressions in which the index is zero.

Key vocabulary: index notation, base, index, index laws, simplify

When working with one or more numbers written using index form there are a number of rules that help to simplify expressions.

These rules are called the index laws.

Lesson starter: Investigating the first two index laws

Write out 3^7 in expanded notation, that is, as seven copies of 3 being multiplied.

Now write out 3^4 in expanded notation.

What do you get when 3^7 is multiplied by 3^4? How many times does the base of 3 appear in this product?

What do you get when 3^7 is divided by 3^4? How many times does the base of 3 appear in this quotient, when you cancel any common factors?

Key ideas

- **Index law for multiplying powers: $a^m \times a^n = a^{m+n}$**
  Use when multiplying numbers written in **index notation**. If the **base** is the same, you keep the base and add the powers together.
  - e.g. $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$
    $= 2^5$ (here the base of 2 appears 5 times $(3 + 2)$)
- **Index law for dividing powers: $a^m \div a^n = a^{m-n}$**
  Use when dividing numbers written in index notation. If the base is the same, you keep the base and subtract the powers.
  - e.g. $2^6 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2)$
    $= \dfrac{2 \times 2 \times 2 \times 2 \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2}}$
    $= 2^4$ (here the base of 2 appears 4 times $(6 - 2)$)
- **Index law for raising powers: $(a^m)^n = a^{m \times n}$**
  Use when a number written in index notation is raised to another power.
  The base remains the same and the two powers (indices) are multiplied together.
  - e.g. $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$
    $= 2^{3+3+3+3}$
    $= 2^{12}$ (here the base of 2 appears in total 12 times (3×4))
- **The zero power: $a^0 = 1$**
  Any non-zero number raised to the power of zero gives an answer of one.
  - e.g. $2^0 = 1$
  - e.g. $2^3 \div 2^3 = 2^{3-3} = 2^0$ (but $2^3 \div 2^3 = 1$ so we observe that $2^0 = 1$)

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Exercise 5H

Understanding

1–4 | 3, 4

1 Which of the following is the same as $4^3 \times 4^4$?

A $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
B $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$
C 16^7
D 16^{12}

2 Which of the following is equal to $3^6 \div 3^2$?

A $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
B $3 \times 3 \times 3$
C $3 \times 3 \times 3 \times 3$
D 1^4

3 Write the missing word or number.

a When multiplying numbers with the same base you ____________ the powers.
b When dividing numbers with the same base you ____________ the powers.
c When a power is raised to another power you ____________ the powers.
d Any number (except zero) raised to the power of zero is equal to ____________.

4 Which of the following is the same as $(2^2)^3$?

A 2^5
B 4^5
C $(2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$
D 2^{23}

Fluency

5–10(½) | 6–7(½), 9–10(½)

Example 16 Using the index laws for multiplication and division

Simplify each of these, leaving your answer in index form.

a $6^4 \times 6^7$
b $5^7 \div 5^4$

Solution

a $6^4 \times 6^7 = 6^{11}$

Explanation

Use index law: $a^m \times a^n = a^{m+n}$
(keep the base and add the powers)
$6^4 \times 6^7$ (the base of 6 appears 4 times in the first term and 7 times in the next term)
The base of 6 appears 11 times in the product.

b $5^7 \div 5^4 = 5^3$

Use index law: $a^m \div a^n = a^{m-n}$

$$5^7 \div 5^4 = 5^{7-4} = 5^3$$

Now you try

Simplify each of these, leaving your answer in index form.

a $3^6 \times 3^4$
b $11^9 \div 11^4$

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5H

5 Copy and complete the following.

a $7^4 \times 7^2 = 7^{\square}$ **b** $8^2 \times 8^1 = 8^{\square}$

c $9^6 \times 9^3 = 9^{\square}$ **d** $5^4 \times 5^3 = 5^{\square}$

e $2^{10} \times 2^3 = 2^{\square}$ **f** $2^{\square} \times 2^9 = 2^{15}$

g $5^8 \div 5^2 = 5^{\square}$ **h** $6^4 \div 6^1 = 6^{\square}$

i $2^{12} \div 2^8 = 2^{\square}$ **j** $1^{16} \div 1^{13} = 1^{\square}$

k $8^{\square} \div 8^4 = 8^2$ **l** $10^7 \div 10^{\square} = 10^2$

Hint for Q5: $a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$

6 Simplify each of the following using the index law for multiplication.

a $3^4 \times 3^2$ **b** $2^2 \times 2^3$

c $10^3 \times 10^1$ **d** $9^6 \times 9^4$

e $4^4 \times 4$ **f** $2^3 \times 2^9$

g $8^7 \times 8^3$ **h** $12^9 \times 12$

i $16^5 \times 16^3$

Hint for Q6: Index law is $a^m \times a^n = a^{m+n}$.

7 Simplify each of the following using the index law for division.

a $3^4 \div 3^2$ **b** $2^7 \div 2^5$

c $9^6 \div 9^2$ **d** $4^5 \div 4^2$

e $17^{26} \div 17^{20}$ **f** $11^9 \div 11^3$

Hint for Q7: Index law is $a^m \div a^n = a^{m-n}$.

Example 17 Using the index law for raising powers

Simplify $(4^5)^2$.

Solution	Explanation
$(4^5)^2 = 4^{10}$	Use index law: $(a^m)^n = a^{m \times n}$ $(4^5)^2 = 4^{5\times2}$ The base of 4 stays the same and the powers are multiplied together.

Now you try

Simplify $(5^3)^4$.

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8 Copy and complete.

a $(2^3)^4 = 2^{\square}$ **b** $(3^2)^5 = 3^{\square}$

c $(5^2)^2 = 5^{\square}$ **d** $(2^4)^3 = 2^{\square}$

e $(7^3)^2 = 7^{\square}$ **f** $(8^4)^5 = 8^{\square}$

Hint for Q8: $(a^m)^n = a^{m \times n}$

9 Simplify the following.

a $(7^2)^2$ **b** $(2^5)^4$

c $(3^7)^2$ **d** $(8^4)^2$

e $(3^4)^2$ **f** $(10^6)^5$

g $(9^2)^7$ **h** $(5^5)^3$

Example 18 Using the power of zero

Simplify.

a 9^0 **b** $(3 \times 2)^0$ **c** 4×5^0

Solution	Explanation
a $9^0 = 1$	A number (except zero) raised to the power of zero equals one.
b $(3 \times 2)^0 = 6^0$ $= 1$	As the overall power of the brackets is zero – the expression equals one.
c $4 \times 5^0 = 4 \times 1$ $= 4$	$5^0 = 1$ so the product of 4 and 5^0 is the same as 4×1.

Now you try

Simplify.

a 7^0 **b** $(6 \times 3)^0$ **c** 6×3^0

10 Simplify the following.

a 5^0 **b** 6^0

c 19^0 **d** 15^0

e $(27 \times 25)^0$ **f** $5^0 + 7$

g $8 - 3^0$ **h** 10×2^0

i $5^0 \times 6^0$ **j** $5^0 + 6^0$

k $6^0 + 5$ **l** $12^0 \times 3$

Hint for Q10: $a^0 = 1$

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5H

Problem-solving and reasoning | 11 | 11, 12

11 Complete the following.

a Given $4 = 2^2$, write the product $2^7 \times 4$ as $2^{\square}$.

b Write $5^4 \times 25$ as $5^{\square}$.

c Write down the numerical value of $6^{14} \div 6^{12}$.

d What do you notice about $(3^4)^2$ and $(3^2)^4$?

e Write down the numerical value of $4^2 \times 3^2$. Is it the same as 7^2 or 12^2?

12 Simplify the following.

a $2^7 \times 2^4 \div 2^3$

b $(2^3)^3 \times 2^4$

c $10^7 \div 10^2 \div 10^2$

d $7^9 \times 7^3 \times 7^2$

e $6^4 \times 6^5 \div 6^8$

f $3^7 \times 3 \times 3$

Hint for Q12: Combine the index laws where required.

Index laws with pronumerals | — | 13–14(½)

13 Use the index laws to complete these index law questions involving pronumeral bases.

a $a^7 \times a^4$

b $m^4 \times m^3$

c $a^5 \times a^4$

d $x^5 \times x^8$

e $n^7 \times n^4$

f $m^6 \times m^7 \times m$

g $n^9 \div n^3$

h $a^{10} \div a^7$

i $m^6 \div m^4$

j $a^7 \times a^2 \times a^3$

k $w^{12} \div w^3$

l $p^8 \times p^2 \div p^6$

Hint for Q13: Remember, the base stays the same.

$m^{20} \times m^4$
$= m^{20+4}$
$= m^{24}$

14 Simplify these using the given hint.

a $5m^4 \times m^3$

b $6m^2 \times 4m^6$

c $8m^6 \times 2m^4$

d $3a^2 \times 4a^7$

e $7x^3 \times 3x^4$

f $5x^9 \times 4x^3$

Hint for Q14: $5x^7 \times 3x^2$
$= 5 \times 3 \times x^7 \times x^2$
$= 15 \times x^{7+2}$
$= 15x^9$

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Pharmacist

Pharmacists complete a university degree, enjoy science and maths, and want to work in the medical industry. They need to understand and accurately fill prescriptions, and know multiple safe drug combinations and side effects so that patients' lives are not at risk.

Pharmacists talk with customers about their health and lifestyle and can suggest vitamin supplements and healthy products, measure blood pressure and blood sugar levels and, if needed, advise people to see their doctor for further treatment.

Compounding pharmacists have the skills and apparatus to mix their own products using various formulas, rather than just supplying pre-packaged medications. For example, a doctor may prescribe an iron supplement for a baby and the pharmacist will prepare the correct dose in a flavour the child will accept.

1 Pharmacists can use the Body Mass Index (BMI) as an indication of a client's health. BMI for adults is calculated by dividing a person's weight (in kg) by the square of their height (in m). A healthy BMI is considered between 20 and 25 kg/m^2.
Find the nearest whole number BMI for these people and state if it is within the healthy range.

$$BMI = \frac{weight(kg)}{[height\ (m)]^2}$$

- **a** Sally, weight 56 kg, 1.54 m tall.
- **b** Ahmed, weight 67 kg, 1.73 m tall.
- **c** Ainslie, weight 55 kg, 1.6 m tall.
- **d** Blake, weight 72 kg, 1.75 m tall.
- **e** Dominic, weight 105 kg, 1.9 m tall.

2 A pharmacist needs to precisely dilute various solutions. The concentration, C, (grams per litre, g/L) decreases as the volume, V, (litres, L) increases. The formula in the box below is used for the following procedures:

$$C_1 \times V_1 = C_2 \times V_2$$

Original concentration and volume	Final concentration and volume

- **a** Hari has a solution with an original concentration of $C_1 = 4$ g/L.
  He dilutes it to create 8 litres of a 2g/L solution.
  Using the formula above, calculate the original volume, V_1, of solution.

Hint for Q2: Write rule: $C_1 \times V_1 = C_2 \times V_2$
Substitute: $4 \times V_1 = 2 \times 8$
Find unknown amount: $4 \times V_1 = 16$
$4 \times ? = 16$
$V_1 = ?$ *litres*

- **b** Mal has a solution with a concentration of 5 g/L. He dilutes it to create 10 Litres of a 2 g/L solution. What original volume, V_1, of solution did Mal use?
- **c** Kelsey has $\frac{1}{2}$ L of a 4 g/L solution and she adds $1\frac{1}{2}$ litres of distilled water. What is the volume, V_2, and concentration, C_2, of the final solution?

3 A baby is given an antibiotic in oral form. The prescribed amount is 2.5 mL three times a day for 5 days. How many mL does the pharmacist need to make up for the parents to ensure the doctor's instructions are fulfilled?

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Using technology

4 Medications come in certain 'stock' strengths and a pharmacist needs to calculate the number of 'stock' doses per day that a patient has been prescribed.

Set up the Excel spreadsheet shown below and enter formulas to calculate the prescribed stock dosage numbers.

	A	B	C	D	E	F
1			**Prescribed amounts for the patient**			
2	**Medical condition**	**Stock strength in mg**	**Prescribed dose in mg/day**	**Number of stock doses per day**	**Number of days**	**Total number of stock doses**
3	Arthritis pain	200	400		5	
4	Asthma	300	600		10	
5	Stomach reflux	20	40		7	
6	Antibiotic	500	1000		5	
7	Alzheimer's disease	5	5		28	
8	Type 1 diabetes	20	40		30	
9	Urinary tract infections	250	500		5	
10	Heart disease	0.05	0.1		30	
11	Type 2 diabetes	500	1000		25	

5 A person's body surface area (BSA) is used for calculating medication dose amounts. BSA is calculated from weight (in kg) and height (in cm) using this algebra rule:

$$BSA = \sqrt{\frac{weight(kg) \times height(cm)}{3600}}$$

a Set up the Excel spreadsheet shown below for calculating medication dose amounts.

b In column D, enter formulas to calculate the BSAs to 4 decimal places.

	A	B	C	D	E	F
14	**Child medication dose calculations**					
15	**Child's name (Age)**	**Weight in kg**	**Height in cm**	**BSA in m²**	**Adult dose in mg /day**	**Child dose in mg /day**
16	Amelia (2)	9	82			
17	Georgia (14)	58	160			
18	Dylan (8)	18.5	130			
19	Hunter (15)	61	168			
20	Ella (10)	25	145			
21	Chelsea (6)	13	112			
22	Benjamin (11)	63	135			

Hint for Q5: Use SQRT for $\sqrt{}$
Formula cell D16
= SQRT(B16*C16/3600)

c Children have smaller bodies than adults, so they need smaller doses of medication. The average adult body surface area is 1.7 m^2. This formula calculates child dose amounts:

$$Child\ dose = adult\ dose \times \frac{child\ BSA}{1.7}$$

Enter this formula in column F, e.g. cell F16 formula = E16*D16/1.7. Format cells to 0 d. p. Column F will have answers of zero until you have completed the next question.

6 Use your Excel spreadsheet from question 5 to find the following prescribed dose amounts. Enter values into column E and the spreadsheet will calculate the answers you need.

a An antibiotic, Keflex, has an adult dose of 500 mg/day. What is the Keflex dose that sisters Amelia and Georgia are prescribed?

b Dylan and Hunter are prescribed Amoxil for a chest infection. If the adult dose is 250 mg, state their prescribed dosages. Why is Hunter's dose almost the same as the adult dose?

c Ella is 4 years older than her sister Chelsea. They are both prescribed a Nuelin for asthma which has 200 mg/day adult dose. How much more does Ella take per dose than Chelsea?

d A blood pressure medication called Tenormin has an adult dose of 50 mg/day. What is the dose that Benjamin is prescribed? What percentage is this of an adult's dose?

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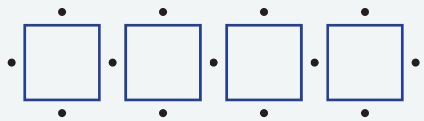
Tiling spacers

When tiling a wall, plastic spacers are used to ensure that equal width gaps remain between the tiles while the glue is drying. Tommy is working on a set of square tiles and uses one spacer on each side of every square tile. This diagram shows an example with 4 tiles laid in a single row.



Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

1 Preliminary task

a If Tommy completes a single row of square tiles, how many plastic spacers are needed for the following number of tiles used?

i 1 **ii** 2 **iii** 5

b Complete this table of values showing the number of plastic spacers (S) for a given number of square tiles (n).

Square tiles (n)	1	2	3	4
Spacers (S)				

c Describe any patterns you see in your table of values.

d Write an expression for the number of spacers required for n square tiles.

e How many spacers would be required for a single row of 20 square tiles?

2 Modelling task

Formulate

a The problem is to determine the total number of spacers for tiling a square array of square tiles. Write down all the relevant information that will help solve this problem.

b Draw a diagram for a 3 by 3 square array of square tiles using 3 rows and 3 columns.

c Using dots, show the spacers that are needed for this array of tiles.

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Solve

d If Tommy completes a square array of tiles with 3 rows and 3 columns, how many plastic spacers are needed?

e Complete this table of values showing the number of plastic spacers (S) for a square array of tiles with n rows and n columns of square tiles. Construct drawings to support your results.

Rows and columns (n)	1	2	3	4
Spacers (S)				

f Describe any patterns you see in your table of values.

g Write an expression in terms of n for the number of spacers required for a square array with n by n square tiles.

h How many spacers would be required for a square array of tiles with 20 rows and 20 columns?

Evaluate and verify

i Compare your answer to part **g** with others in your class. Is there more than one way that you can write your expression? Provide an explanation.

Communicate

j Summarise your results and describe any key findings.

3 Extension questions

When Tommy lays large square tiles, he uses 2 spacers on *each* side of each tile. Tommy now tiles a floor using large square tiles but with a rectangular array.

a Draw a diagram showing 3 rows and 4 columns of square tiles and use dots to mark 2 spacers on each side of each tile.

b If Tommy completes this rectangular array of tiles with 3 rows and 4 columns, how many plastic spacers are needed?

c Tommy tiles a wall with m rows and n columns. Determine an expression for the number of spacers required in terms of m and n.

d Use your expression to find the number of spacers required for a wall with 20 rows and 15 columns.

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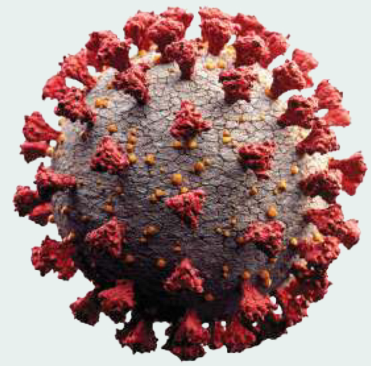
Technology and computational thinking

Shortcuts to large sums

Key technology: Spreadsheets and programming

Whenever we see patterns in the real world, we can try to describe the pattern using numbers and pronumerals. For example, the cost of hiring a car for n days will reveal a number pattern that will depend on the pronumeral n; or the total population of people infected by a virus where the number of new infections is increasing by 20% per week will also form a number pattern. Such number patterns can be described using algebraic expressions and it is these expressions which help us make calculations regarding the value of a particular term in the pattern or the total sum of the terms in a pattern up to a particular point.



1 Getting started

We will start by considering the set of even numbers as shown in this table where:

- n represents the even number
- t_n is the nth even number
- S_n represents the sum of all the even numbers up to and including the nth even number.

n	1	2	3	4	5	6	7	8
t_n	2	4	6	8	10			
S_n	2	6	12					

a The second even number t_2 is 4. State:
  i the fourth even number, t_4
  ii the seventh even number, t_7.

b The sum of the first two even numbers S_2 is 6. State:
  i the sum of the first three even numbers, S_3
  ii the sum of the first five even numbers, S_5.

c Complete the table above and describe any patterns that you see.

d Write an expression for the nth even number, t_n using the pronumeral n. Use your expression to find the 8th even number and check this against your completed table.

e Evaluate the expression $n(n+1)$ for the following values of n. What do you notice about your answers and the numbers produced in the table above?
  i 3 **ii** 6 **iii** 8

f Use the expression given in part **e** to find the sum of the first 20 even numbers, S_{20}.

2 Applying an algorithm

We will now use an algorithm as shown to explore the sum of the odd numbers.

- m is how many odd numbers you wish to sum.
- n represents the odd number.
- t_n is the nth odd number.
- S_n represents the sum of all the odd numbers up to and including the nth odd number.

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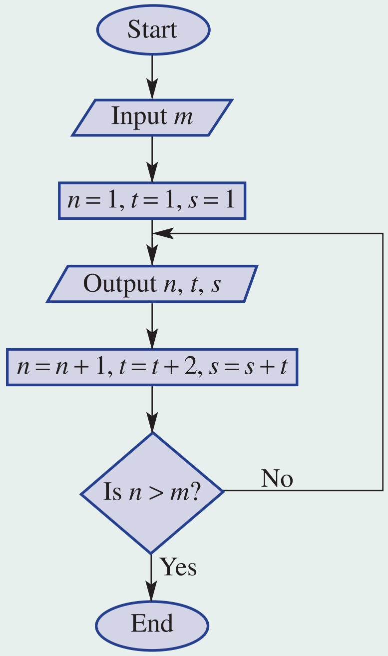

a Run through the algorithm for $m = 5$. State the output using this table.

n	1	2	3	4	5
t_n	1	3			
S_n	1				

b What do you notice about the values of t_n in relation to the value of n? Write an expression that gives the value of t_n using the n value.

c What do you notice about the values of S_n in relation to the value of n? Write an expression that gives the value of S_n using the n value.

d Use your expression in part **c** to find the sum of the first 10 odd numbers, S_{10}.

3 Using technology

A spreadsheet will be used to apply the above algorithm and generate the sum of odd numbers that were studied above in part **2**.

	A	B	C
1	Generating patterns		
2			
3	***n***	***t***	***s***
4			
5	1	1	=B5+C4
6	=A5+1	=B5+2	

a Construct the following spreadsheet that generates the sequence of odd numbers and the total sum.

b Fill down at cells A6, B6 and C5 for at least 10 rows. Check that your results agree with the results generated in part **2** above.

c Now generate a similar spreadsheet but this time use your expressions from part **2b** and **2c** above.

d Fill down at cells E6, F5 and G5 for at least 10 rows. Check that your results agree with the results generated in part **b** above.

E	F	G
Patterns with	expressions	
n	***t***	***s***
1	=2*E5-1	=E5^2
=E5+1		

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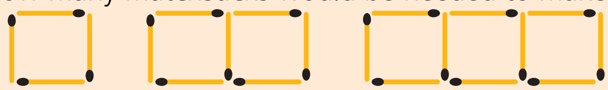
Puzzles and games

1 How many matchsticks would be needed to make 1000 squares?



$n = 1$	$n = 2$	$n = 3$	… $n = 1000$
4 sticks	7 sticks	10 sticks	? sticks

2 Find the values of A, B and C so that the rows and columns add up correctly.

A	B	C	Sum = 14
A	C	B	Sum = 14
A	C	B	Sum = 14
Sum = 15	Sum = 16	Sum = 11	

3 Fill in the missing expressions to make the six equivalences true.

$3x$	+		=	$7x + 3y + 1$
+		+		+
	+		=	
=		=		=
$2y + 3x$	+		=	$7x + 6y + 1$

4 Think of a number, n.
Double it and add 4.
Triple the result and subtract 12.
You now have 6 times the original number. Use algebra to see if this was just a coincidence.

Design a puzzle like this and try it on your friends.

5 Find the largest value for each of the following.

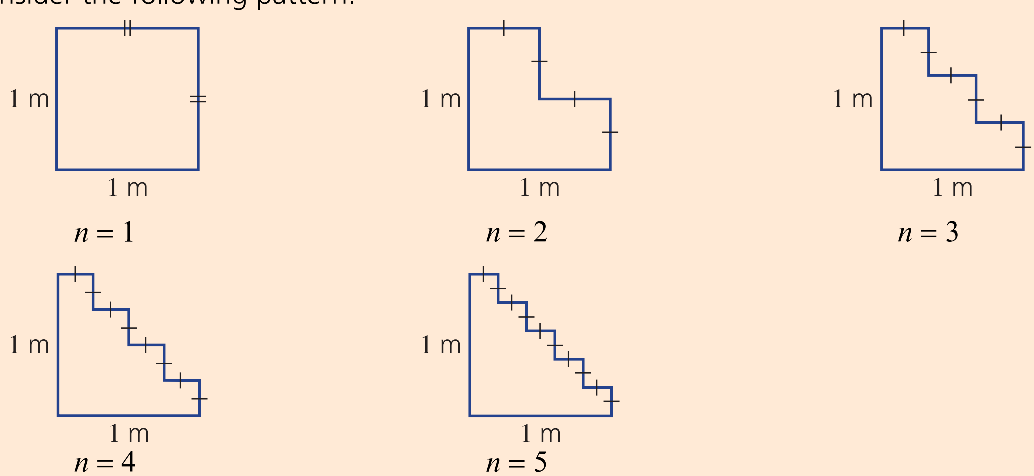
a If b can be any number, what is the largest value of $b \times (10 - b)$?

b If m can be any number, what is the largest value that $10 - m(m + 5)$ could have?

c If $x + y$ evaluates to 15, what is the largest value that $x \times y$ could have?

d If a and b are chosen so that $a^2 + b^2$ is equal to $(a + b)^2$, what is the largest value of $a \times b$?

6 Consider the following pattern.



The perimeter for the shape when $n = 1$ is given by the expression 4 m and the area is 1 m^2.

a Calculate the perimeter and area of the other shapes shown above and try to find a pattern.

b If $n = 1000$, state the perimeter and give the approximate area.

Hint for Q6: Think about what the shape looks like.

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Chapter summary

Algebra

Pronumeral: a letter that stands for a numerical value

Language

+	−	×	÷
sum more than added increased	difference less than minus decreased	product times double (2×) twice (2×) triple (3×)	quotient divide

× ÷ not used in algebraic expressions

an expression

terms

$5x - 7xy - 8$

5 is the coefficient of x

−7 is the coefficient of xy

constant term is −8

Like terms

Have the same pronumerals

$6x$ and $5x$ ✓

$3x$ and $7y$ ✗

can be in different order

$6ab$ and $12ba$ ✓

Adding and subtracting like terms

$\boxed{6a}$ $\boxed{+12b}$ $\boxed{+3a}$ $\boxed{-7b}$

$= 6a + 3a + 12b - 7b$

$= 9a + 5b$

Substitution

'evaluate' 'substitute'

Replace pronumerals with numbers and calculate answer

If $a = 3$ then

$$\begin{aligned} 5a + 7 &= 5 \times 3 + 7 \\ &= 15 + 7 \\ &= 22 \end{aligned}$$

Equivalent expressions

Always evaluate to the same number

e.g. $2x$ and $x + x$

Expanding

$3(2x + 5y) = 6x + 15y$

$2a(5 - 7b) = 10a - 14ab$

Index laws

$4^2 \times 4^3 = 4^5$ $\quad 4^9 \div 4^3 = 4^6$

$(2^4)^3 = 2^{12}$ $\quad 5^0 = 1$

Multiplying and dividing terms

$$\begin{aligned} 3a \times 2b &= 3 \times a \times 2 \times b \\ &= 6 \times a \times b \\ &= 6ab \end{aligned}$$

$$\frac{\overset{2}{\cancel{12}}a}{\underset{3}{\cancel{18}}b} = \frac{2a}{3b}$$

Factorising ⭐

$12x + 6$ (HCF = 6)

$= 6 \times 2x + 6 \times 1$

$= 6(2x + 1)$

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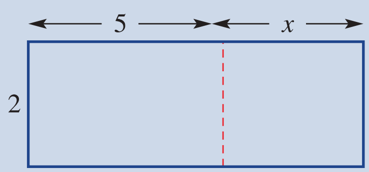
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

			✔
5A	1	**I can state coefficients of pronumerals** e.g. In the expression $4a + b - 12c + 5$, state the coefficients of a, b, and c.	
5A	2	**I can create expressions from descriptions** e.g. Write an expression for "The sum of a and b is doubled."	
5B	3	**I can substitute values into expressions** e.g. Substitute $x = 3$ to evaluate $5x$.	
5B	4	**I can substitute for multiple pronumerals** e.g. Substitute $x = 3$ and $y = 6$ to evaluate $3x + 2y$.	
5B	5	**I can decide if expressions are equivalent** e.g. Decide if $x - 3$ and $3 - x$ are equivalent.	
5C	6	**I can decide if two terms are like terms** e.g. Decide whether $2ab$ and $3ba$ are like terms or not.	
5C	7	**I can simplify expressions by combining like terms** e.g. Simplify $4x + 3y + 2x + 7y$.	
5D	8	**I can multiply terms and simplify the result** e.g. Simplify: **a** $7a \times 2bc \times 3d$ **b** $3xy \times 5xz$.	
5D	9	**I can divide terms and simplify the result** e.g. Simplify $\frac{10ab}{15bc}$.	
5E	10	**I can expand brackets using rectangle areas** e.g. Write two equivalent expressions for the total area of the rectangle shown: one with brackets and the other without brackets. 	
5E	11	**I can expand brackets using the distributive law** e.g. Expand: **a** $5(x + 3)$ **b** $2(3p - 7q)$.	
5F	12	**I can find the highest common factor (HCF) of algebraic terms** e.g. Find the HCF of: $18x$ and $24xy$.	

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Chapter checklist

		✔
5F	**13 I can factorise expressions by taking out the highest common factor** e.g. Factorise: **a** $12a + 18ab$ **b** $21x - 14y$.	
5G	**14 I can write an expression to model a practical situation** e.g. Write an expression for the total cost of hiring a plumber for n hours if they charge a \$40 call-out fee and \$70 per hour.	
5H	**15 I can multiply powers and use an index law to simplify** e.g. Simplify $6^4 \times 6^7$.	
5H	**16 I can divide powers and use an index law to simplify** e.g. Simplify $5^7 \div 5^4$.	
5H	**17 I can simplify powers of powers using an index law** e.g. Simplify $(4^5)^2$, giving your answer in index notation.	
5H	**18 I can simplify expressions in which the index is zero** e.g. Simplify 4×5^0.	

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Short-answer questions

5A **1** State whether each of the following is true (T) or false (F).
- **a** The constant term in the expression $5x + 7$ is 5.
- **b** $16xy$ and $5yx$ are like terms.
- **c** The coefficient of d in the expression $6de + 7d + 8abd + 3$ is 7.
- **d** The highest common factor of $12abc$ and $16c$ is $2c$.
- **e** The coefficient of x in $5y - 3x$ is -3.

5A **2** For the expression $6xy + 2x - 4y + 3$, state:
- **a** the coefficient of x
- **b** the constant term
- **c** the number of terms
- **d** the coefficient of xy

5B **3** Substitute the following values of a to evaluate the expression $12 - 2a$.
- **a** 1
- **b** 2
- **c** 4
- **d** 6

5B **4** Substitute $A = 2$ and $B = 5$ into the following expressions.
- **a** $10A$
- **b** $A + B$
- **c** $B - A$
- **d** $3A + 2B$

5A **5** Substitute $x = 2$ and $y = 3$ into each of the following.
- **a** $2y + 3$
- **b** $3x + y$
- **c** $xy + y$
- **d** $4x - 2y$

5C **6** Simplify each of these expressions by collecting like terms.
- **a** $7m + 9m$
- **b** $3a + 5b - a$
- **c** $3y - x + y + 1$
- **d** $5x + 3y + 2x + 4y$
- **e** $7x - 4xy + 5xy + 2x$
- **f** $7m - 2n + 3m - 4n$

5D **7** Simplify.
- **a** $9a \times 4b$
- **b** $30 \times x \times y$
- **c** $2x \times 5y \times 3z$

5D **8** Simplify.
- **a** $\dfrac{10x}{5}$
- **b** $\dfrac{12ab}{4b}$
- **c** $\dfrac{4xz}{20xy}$

5E **9** Expand.
- **a** $3(x - 4)$
- **b** $2(5 + x)$
- **c** $3(2y + 4)$
- **d** $10(2x + 7)$
- **e** $3(x - 5)$
- **f** $11(z - 2)$
- **g** $4(3a - 11)$
- **h** $2(6b - 3)$

5F **10** Find:
- **a** the HCF of $12x$ and 16
- **b** the HCF of $14ab$ and $21a$

5F **11** Factorise fully.
- **a** $2x + 6$
- **b** $24 - 16g$
- **c** $12x + 3xy$
- **d** $7a + 14ab$

5G **12** If apricots cost $\$a$ each and pears cost $\$p$ each, write an expression for:
- **a** the cost of 5 apricots
- **b** the cost of 3 pears
- **c** the cost of 5 apricots and 3 pears

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Chapter review

5G **13** Greg runs 10 km each day.
a How far (in km) does he run in one week (7 days)?
b Write an expression for how far he runs in n days.

5H **14** Simplify these powers.
a $4^9 \times 4^2$ **b** $3^4 \div 3^2$ **c** 5^0 **d** $(3^4)^5$

Multiple-choice questions

5A **1** The sum of x and y can be written as:
A $2x$ **B** $2xy$ **C** $x + y$ **D** $x - y$ **E** xy

5A **2** Consider the expression $5a - 3b + 8$. Which one of the following statements is true?
A The coefficient of a is 5.
B It has 5 terms.
C The constant term is -8.
D The coefficient of b is 3.
E The coefficient of a is 10.

5A **3** If n is a number, which of the following represents one third of n?
A $\frac{3}{n}$ **B** $0.3n$ **C** $3n$ **D** $\frac{n}{3}$ **E** $n - 3$

5B **4** If $a = 2$, then $17 + 2a$ is:
A 3 **B** -3 **C** 21 **D** 11 **E** 13

5D **5** $3 \times x \times y$ is equivalent to:
A $3x + y$ **B** xy **C** $3 + x + y$ **D** $3x + 3y$ **E** $3xy$

5D **6** $\frac{12ab}{24a}$ can be simplified to:
A $2ab$ **B** $\frac{2a}{b}$ **C** $\frac{b}{2a}$ **D** $\frac{ab}{2}$ **E** $\frac{b}{2}$

5E **7** The expanded form of $2(3 + 5y)$ is:
A $6x + 5y$ **B** $3x + 5y$ **C** $6x + 5xy$ **D** $6 + 10y$ **E** $6x + 10xy$

5D **8** Simplifying $3a \div 6b$ gives:
A 2 **B** $\frac{a}{b}$ **C** $\frac{2a}{b}$ **D** $\frac{ab}{2}$ **E** $\frac{a}{2b}$

5C **9** When like terms are combined, $3a + 4b + 2a - 2b$ simplifies to:
A $5a + 6b$ **B** $7ab$ **C** $11ab$ **D** $5a + 2b$ **E** $a + 6b$

5F **10** The factorised form of $3a - 6ab$ is:
A $3a(1 - 2b)$ **B** $3a(a - 2b)$ **C** $3a(a - b)$ **D** $6a(a - b)$ **E** $3(a - 2ab)$

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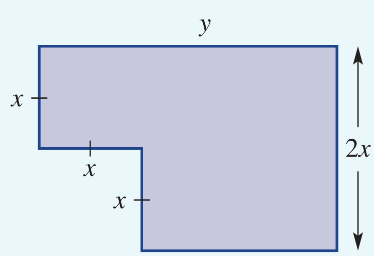
Extended-response questions

1 Two bus companies have different pricing structures. Each charge a call-out fee to cover business running costs. The price per hour pays for the drivers' wages.

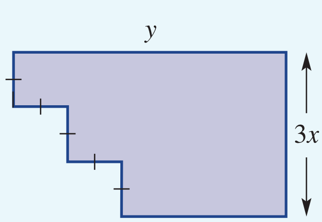
Company A	Company B
$\$120$ call-out fee, plus $\$80$ per hour	$\$80$ call-out fee, plus $\$100$ per hour

a Write an expression for the total cost of travelling for n hours with company A.
b Write an expression for the total cost of travelling for n hours with company B.
c What is the cost of travelling for 3 hours with each company?
d For how long would you need to hire a bus to make company A the cheaper option?
e If a school hired one bus from each company for n hours, what would the total cost be?

2 Consider the floor plan shown.



a Write an expression for the floor's area in terms of x and y.
b Using that expression, find the floor's area if $x = 3$ metres and $y = 7$ metres.
c Write an expression for the floor's perimeter in terms of x and y.
d Using that expression, find the floor's perimeter if $x = 3$ metres and $y = 7$ metres.
e Another floor plan is shown below. Write an expression for the floor's area and an expression for its perimeter.



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Computation with integers

Short-answer questions

1 Evaluate, without using a calculator.

a $4973 + 196$ b $1506 - 156$ c -96×3
d 139×5 e 14×99 f $14 \times 99 + 14 \times 101$
g 9^2 h 4^3 i $-9 - 7 - 3$

2 Evaluate.

a $10 - 6 \times 4$ b $15 \times 4 \div 2$ c $24 \div 2 \times 6$
d $-3 + (-10 - (-6))$ e $-81 \div (-3) \times 2$ f $73 - 72 - 7$

3 Find the HCF of:

a 24 and 42
b 35 and 42
c 100 and 60
d 15 and 45

4 Write down the LCM of:

a 24 and 42 b 8 and 9 c 100 and 60 d 15 and 45

5 Write using powers.

a $7 \times 7 \times 7$ b 8×8 c $3 \times 3 \times 3 \times 3 \times 3$

6 Write down the value of:

a 9^2 b $\sqrt{49}$
c 5^3 d $\sqrt{27}$

7 If $a = 5$ and $b = -7$, what is the value of:

a $a + b$ b $a - b$ c $a \times b$
d $15 - b$ e a^2 f b^2

Multiple-choice questions

1 $156 \div 4$ is the same as:

A $156 \div 2 \times 2$ B $156 \div 2 \div 2$ C $312 \div 2$ D $156 \times 2 \div 2$

2 $-24 + 6 \times (-3)$ is equal to:

A 6 B 42 C −42 D −6

3 What is the smallest number that can be added to 1923 to make the answer divisible by 9?

A 1 B 2 C 3 D 4

4 $(-15)^2$ equals:

A 225 B 30 C −30 D −225

5 Two numbers have a sum of −10 and a product of −56. The larger of the two numbers is:

A −4 B 4 C 14 D −14

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Extended-response questions

1 The weather for a November day is given for different cities around the world.

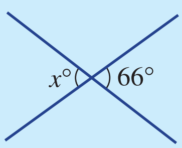
	Minimum (°C)	Maximum (°C)
Amsterdam	3	12
Auckland	11	18
LA	8	14
Hong Kong	16	28
Moscow	6	8
Beijing	3	0
New York	8	10
Paris	6	13
Tel Aviv	16	23
Wollongong	18	22

a Which city recorded the highest temperature on the day shown in the table?
b Which two cities only had a 2° difference in temperature between minimum and maximum temperature?
c Which city had the largest difference in temperature on this November day?
d What is the difference in the minimum temperatures of Beijing and Auckland?

Angle relationships and properties of geometrical figures

Short-answer questions

1 Find the value of x.

a


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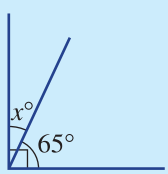

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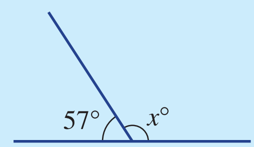

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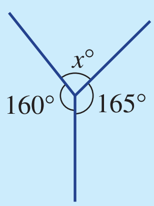

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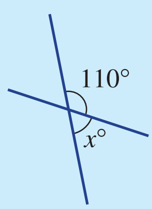

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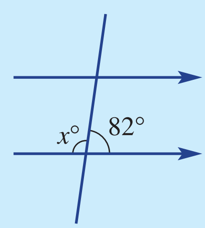
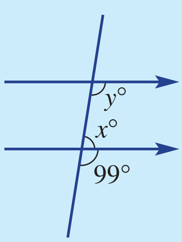


2 Find the value of each pronumeral.

a


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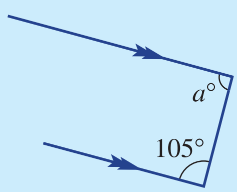

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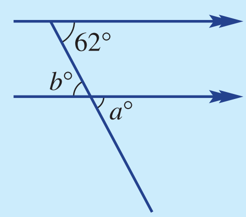

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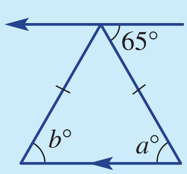

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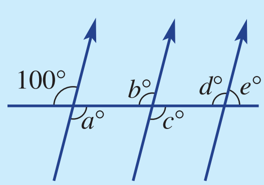

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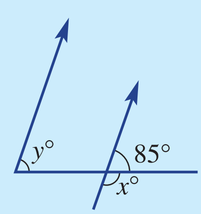

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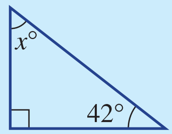
3 Find the value of the pronumeral in these triangles.

a


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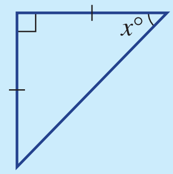

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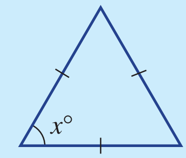

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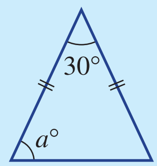

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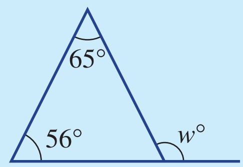

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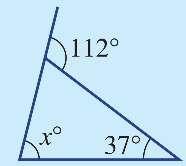
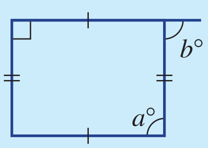


4 Find the value of a and b in these quadrilaterals.

a


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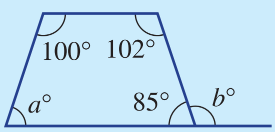

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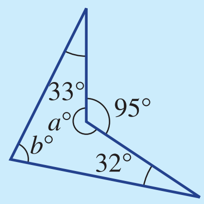


5 Find the internal angle sum of:

a a pentagon **b** an octagon

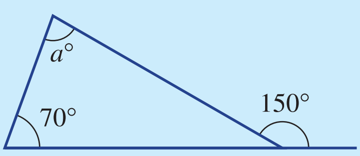
Multiple-choice questions

1 The supplementary angle to 80° is:

A 10° **B** 100° **C** 280° **D** 20°

2 In this diagram a equals:

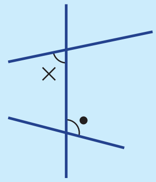
A 150° **B** 220°
C 70° **D** 80°

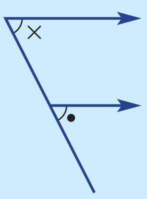


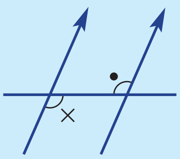
3 The angle sum of a regular pentagon is:

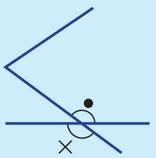
A 72° **B** 108° **C** 540° **D** 120°

4 Which diagram shows equal alternate angles?

A


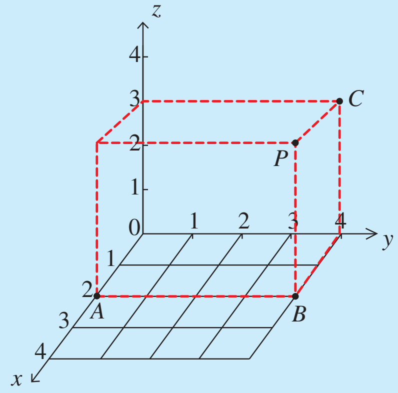
B


C


D


5 Give the coordinates of the point P shown in this three-dimensional diagram.

A (2, 0, 0) **B** (2, 4, 0) **C** (0, 4, 3)
D (2, 4, 3) **E** (4, 2, 3)



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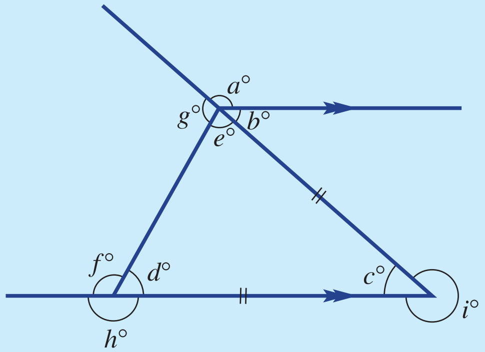
Extended-response questions

1 **a** If $a = 115$, find the size of each angle marked. Give a reason for each answer. Write your answers in the order you found them.



b Is the order the same for everybody in the class? Discuss any differences and the reasons associated with each.

Fractions, decimals and percentages

Short-answer questions

1 Copy and complete these equivalent fractions.

a $\frac{3}{5} = \frac{\square}{30}$

b $\frac{\square}{11} = \frac{5}{55}$

c $1\frac{4}{6} = \frac{\square}{3}$

2 Evaluate each of the following.

a $\frac{3}{4} - \frac{1}{2}$

b $\frac{4}{5} + \frac{3}{5}$

c $1\frac{1}{2} + 1\frac{3}{4}$

d $\frac{4}{7} - \frac{2}{3}$

e $\frac{4}{9} \times \frac{3}{4}$

f $1\frac{1}{2} \times \frac{3}{5}$

3 Write the reciprocal of:

a $\frac{2}{5}$

b 8

c $4\frac{1}{5}$

4 Evaluate.

a $2\frac{1}{2} \times 1\frac{4}{5}$

b $1\frac{1}{2} \div 2$

c $3 - 2\frac{1}{3}$

5 Calculate each of the following.

a $3.84 + 3.09$

b $10.85 - 3.27$

c $12.09 \div 3$

d $6.59 - 0.08$

e 96.37×40

f $15.84 \div 0.02$

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6 Evaluate.

a 5.3×100

b 9.6×1000

c $61.4 \div 100$

7 Copy and complete this table of decimals, fractions and percentages.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$				
Decimal								0.99	0.005
Percentage						80%	95%		

8 Find:

a 10% of 56

b 12% of 98

c 15% of 570 m

d 99% of \$2

e 25% of \$840

f 50% of 8500 g

9 a Increase \$560 by 25%

b Decrease \$980 by 12%

10 A \$348 Charlie Brown dress was bought for \$261.

a How much money did the buyer save?

b What percentage off does this equal?

Multiple-choice questions

1 $\frac{150}{350}$ simplifies to:

A $\frac{6}{14}$ B $\frac{3}{70}$ C $\frac{15}{35}$ D $\frac{3}{7}$

2 Sienna spends $\frac{3}{7}$ of her \$280 income on clothes and saves the rest. She saves:

A \$470 B \$120 C \$160 D \$2613

3 0.008×0.07 is equal to:

A 0.056 B 0.0056 C 0.00056 D 56

4 0.24 expressed as a fraction is:

A $\frac{1}{24}$ B $\frac{6}{25}$ C $\frac{12}{5}$ D $\frac{24}{10}$

5 If 5% of x is 8, then 10% of x equals:

A 4 B 16 C 64 D 80

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Extended-response questions

1 A laptop decreases in value by 20% a year.

- **a** Find the value of a \$2000 laptop at the end of:
  - **i** 1 year
  - **ii** 2 years
  - **iii** 3 years
- **b** After how many years is the laptop worth less than \$800?
- **c** Is the laptop ever going to have a value of zero dollars? Explain.

Measurement

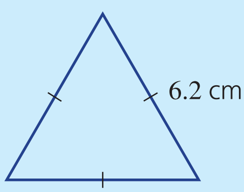
Short-answer questions

1 Complete these conversions.

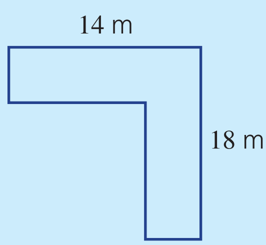
- **a** 5 m = ______ cm
- **b** 1.8 m = ______ cm
- **c** 9 m^2 = ______ cm^2
- **d** 1800 mm = ______ cm
- **e** 4 L = ______ cm^3
- **f** 0.01 km^2 = ______ m^2

2 Find the perimeter of these shapes.

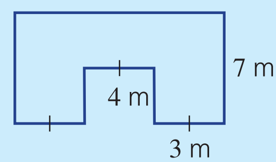
a



b



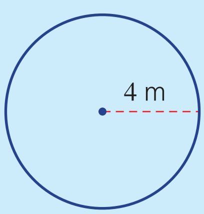
c



3 Find, correct to two decimal places:

- **i** the circumference
- **ii** the area

a



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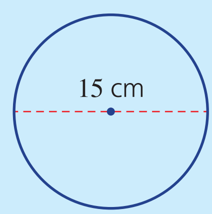
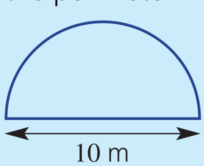


4 Find, correct to two decimal places:

- **i** the perimeter
- **ii** the area

a



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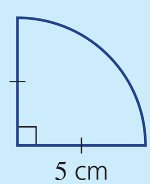

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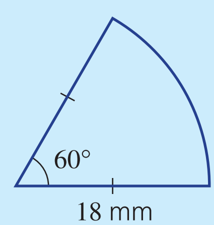
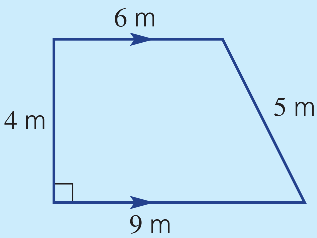


5 Find the area of these shapes.

a



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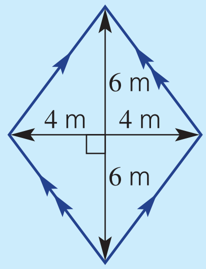

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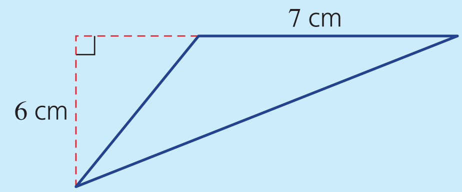

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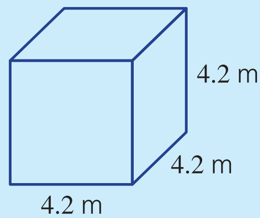
6 Find the volume of these solids.

a



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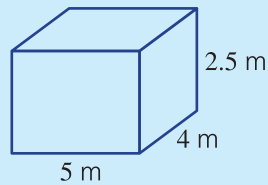
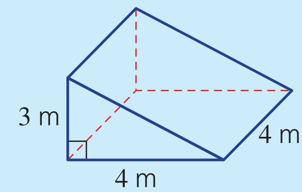


c

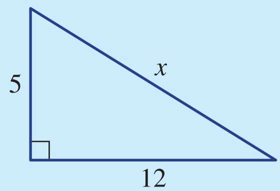


7 Write these times using 24-hour time.

a 3:30 pm **b** 7:35 am

8 Find the value of x in these triangles. Round to two decimal places for part **b**.

a



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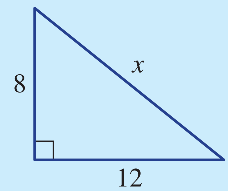


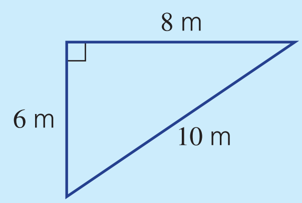
Multiple-choice questions

1 A cube has a volume of 8 cubic metres. The side length of the cube is:

A 8 m **B** 4 m **C** 2 m **D** 16 m

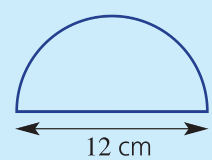
2 The area of this triangle is:

A 48 m^2
B 24 m^2
C 30 m^2
D 40 m^2



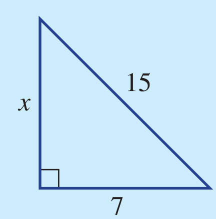
3 The perimeter of this semicircle is closest to:

A 38 cm
B 30 cm
C 19 cm
D 31 cm



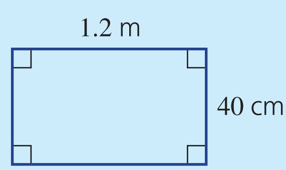
4 The value of x in this triangle is closest to:

A 176
B 13
C 274
D 17



5 The area of this rectangle is:

A 48 m^2
B 48 000 cm^2
C 480 cm^2
D 0.48 m^2



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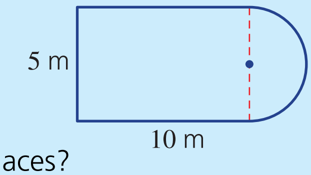
Extended-response questions

1. A paved area is in the shape of a rectangle with a semicircular end as shown.



   a What is the radius of the semicircle?
   b What is the area of the semicircle, correct to two decimal places?
   c What is the total area of the paved area, correct to two decimal places?
   d A special brick border is to go around the perimeter of the area. Find this length, correct to the nearest metre.

Algebraic techniques and index laws

Short-answer questions

1. Write an expression for:
   a the sum of p and q
   b the product of p and 3
   c half the square of m
   d the sum of x and y, divided by 2

2. Find the value of $7k - 2$ if:
   a $k = 3$ **b** $k = 10$ **c** $k = 5$ **d** $k = 100$

3. If $a = 6$, $b = 4$ and $c = 1$, evaluate:
   a $a + b + c$ **b** $ab - c$
   c $a(b - c)$ **d** $3a + 2b$
   e abc **f** $a - (-2b) + 3c$

4. Simplify each algebraic expression.
   a $4 \times 6k$ **b** $a + a + a$
   c $a \times a \times a$ **d** $7p \div 14$
   e $3ab + 2 + 4ab$ **f** $7x + 9 - (-6x) - (-10)$
   g $18xy \div 9x$ **h** $m + n - (-3m) + n$

5. Simplify.
   a $\frac{5xy}{5}$ **b** $\frac{30x}{21y}$ **c** $\frac{2w}{10}$ **d** $\frac{17abc}{5bc}$

6. Expand, and simplify where necessary.
   a $2(x + 5)$
   b $6(2m - 3)$
   c $10 + 2(m - 3)$

7. State the HCF of:
   a $12x$ and $6y$
   b $15k$ and $20kl$
   c $120ab$ and $100bc$

8. Factorise.
   a $18a - 12$
   b $6mn + 12m$
   c $8x + 12$

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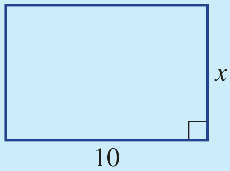
9 Write an expression for the rectangle's:
   a perimeter
   b area



10 If pens cost \$2 each and notepads cost \$3 each, write expressions for:
   a the cost of x pens
   b the cost of y notepads
   c the total cost of x pens and y notepads.

11 Simplify using the index laws.
   a $3^4 \times 3^5$
   b $4^7 \div 4^5$
   c 6^0
   d $(5^3)^4$

Multiple-choice questions

1 If $x = 3$, then $7x + 2$ equals:
   A 21
   B 75
   C 73
   D 23

2 $4x + 5 + 3x$ simplifies to:
   A $7x + 5$
   B $12x$
   C $12 + x^2$
   D $2x + 12$

3 $2(6x + 5)$ expands to:
   A $12x + 5$
   B $12x + 10$
   C $6x + 10$
   D $60x$

4 $12m + 18$ factorises to:
   A $2(6m - 9)$
   B $-6(2m - 3)$
   C $6(3 - 2m)$
   D $6(2m + 3)$

5 The coefficient of x in $12 - 3x + 4y$ is:
   A 12
   B 3
   C -3
   D 4

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Extended-response questions

1 A repairer charges a $\$60$ call-out fee plus $\$80$ per hour.
  a Find the cost of a 2-hour visit.
  b Write an expression for the cost of an n-hour visit.
  c Another repairer charges no call-out fee but $\$100$ per hour.
    i Write an expression for this repairer's total cost.
    ii For how many hours were they hired if the total cost was the same for both repairers?

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6

Ratios and rates

Essential mathematics: why skills with ratios and rates are important

Ratio skills are very useful everywhere as the following examples illustrate.

Builders mix concrete, e.g. cement to sand = $1 : 3$.

Gardeners and fishermen mix petrol to oil = $50 : 1$ for lawn mowers and outboard motors.

Jewellers mix gold with alloys in a given ratio.

Farmers mix fertilizer in a ratio, e.g. nitrogen : phosphorus : potassium = $3 : 1 : 2$.

Hairdressers use ratios to mix hair dyes, e.g. hair colour : developer = $1 : 2$

Music producers mix vocals to background music, e.g. in the ratio $2 : 1$.

A **map scale** is the ratio of map distance : ground distance, e.g. $1 : 10\,000$.

The Ferrari F1 can reach a speed of 340 km/h using its top gear ratio of $0.92 : 1$. This ratio means the engine crankshaft completes 0.92 of a revolution for one full rotation of the wheels.

Rates help us to compare quantities of different materials. For example, fuel prices (cents/L), fuel economy (litres/100 km), pay rates ($/h), food energy (kJ/100g), and data download rates (Mb/s).

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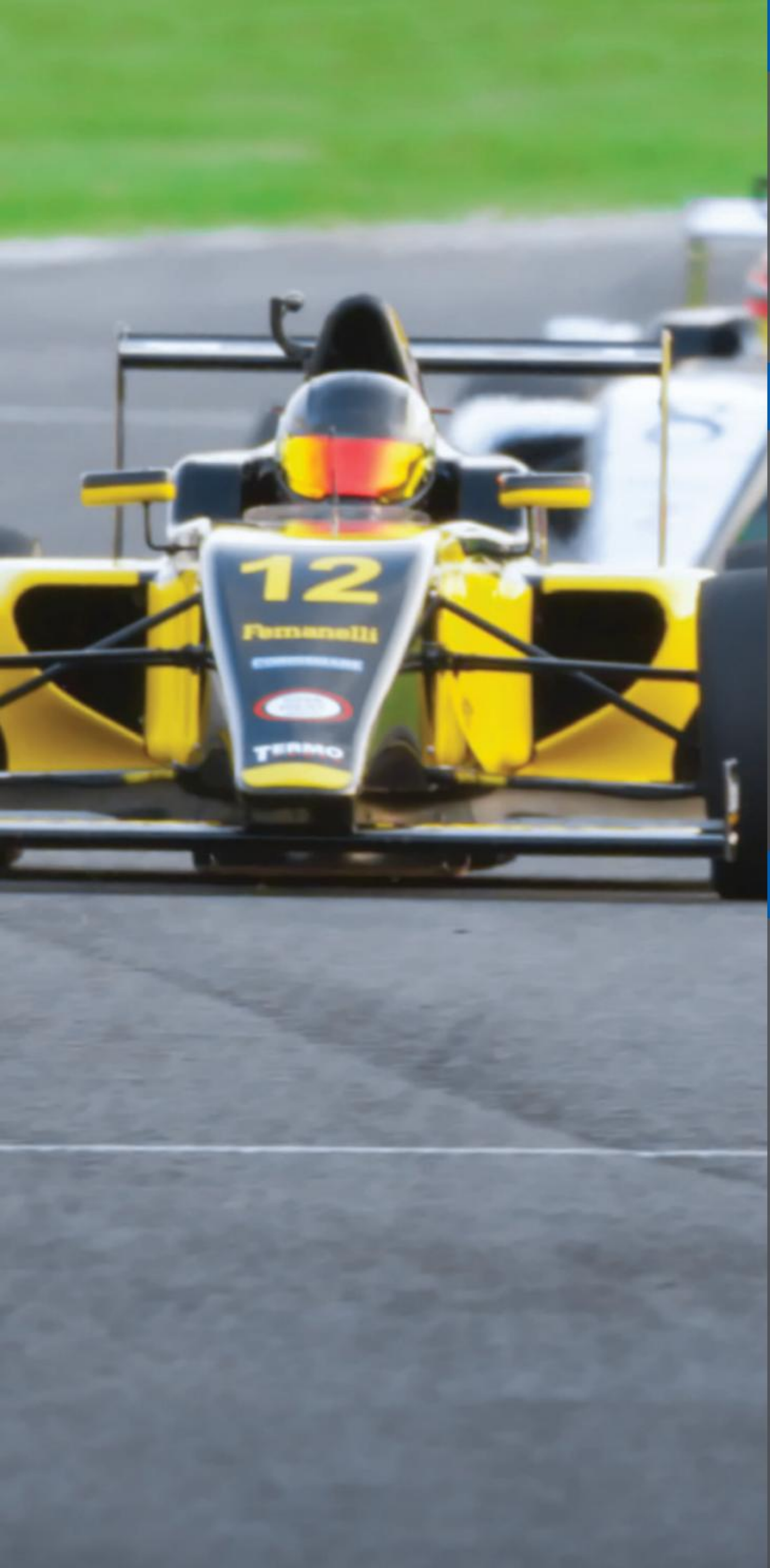


In this chapter

Australian Curriculum 9.0

MEASUREMENT

Recognise and use rates to solve problems involving the comparison of 2 related quantities of different units of measure (AC9M8M05)

Use mathematical modelling to solve practical problems involving ratios and rates, including financial contexts; formulate problems; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8M07)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 State the missing number.

a $\div 2$
$$\frac{2}{4} = \frac{1}{\square}$$
$\div 2$

b $\div 5$
$$\frac{15}{20} = \frac{\square}{4}$$
$\div 5$

c $\div ?$
$$\frac{12}{15} = \frac{\square}{5}$$
$\div ?$

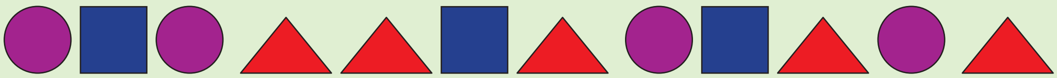
2 State the missing number.

a $\times 2$ $2 : 5$ $\times 2$
$4 : \square$

b $\div 4$ $20 : 28$ $\div 4$
$\square : 7$

c $\times ?$ $3 : 2$ $\times ?$
$12 : \square$

3



Write the ratio of:

a squares to circles
b circles to triangles
c triangles to total shapes

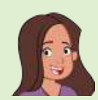
4 Convert:

a 5 m = ____ cm
b 6 km = ____ m
c 500 cm = ____ m
d 80 mm = ____ cm
e 120 cm = ____ m
f 15 000 m = ____ km

Hint for Q4:
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

5 Write these ratios in the same units and simplify.

a 3 cm : 15 mm
b 45 cm : 1 m
c 45 minutes : 1 hour
d 10 minutes : $\frac{1}{2}$ hour
e 2 km : 500 m
f 40 m : $\frac{1}{2}$ km



6 If 4 mangoes cost $6:

a how much would one mango cost?
b how much would 12 mangoes cost?

Hint for Q6:
÷4 4 mangoes cost $6 ÷4
1 mango costs $?

7 The cost of 1 kg of bananas is $4.99. Find the cost of:

a 2 kg
b 5 kg
c 10 kg
d $\frac{1}{2}$ kg

8 Kevin walks 3 km in one hour. How far did he walk in 30 minutes?

9 Tao earns $240 for working 8 hours. How much did Tao earn each hour?

10 A car travels at an average speed of 60 km per hour.

a How far does it travel in the following times?
  i 2 hours
  ii 5 hours
  iii $\frac{1}{2}$ hour

b How long would it take to travel the following distances? (Answer in fractions of hours.)
  i 180 km
  ii 90 km
  iii 20 km

c If the car's speed was 70 km per hour, how many minutes would it take to travel 7 km?

Hint for Q10:
÷10 70 km in 60 minutes ÷10
7 km in ? minutes

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6A Introducing ratios

Learning intentions

- To understand that ratios show a relationship between quantities.
- To understand that the order in which values are written in a ratio is important.
- To be able to write a ratio from a situation.
- To be able to write equivalent ratios to a given ratio by multiplying or dividing each quantity by the same number.

Key vocabulary: ratio, equivalent, colon (:)

Ratios are regularly used in everyday life. They are used to show the relationship between two (or more) related quantities.

Here are five common uses of ratios:

- Ingredients – the ratio of different ingredients in a recipe (cooking, medicines, industrial)
- Maps – most maps include a scale which is written as a ratio
- Sporting success – showing a team's win to loss ratio, or the ratio of kicking goals to points
- Comparing size – the ratio of length, area or volume of different shapes
- Legal requirements – minimum standards of supervision, staff to student ratios.

When dealing with ratios, the order in which the ratio is written is very important. For example, a team's win : loss ratio of $5 : 2$ is very different to a team's win : loss ratio of $2 : 5$.

Ratios compare quantities of the same type with the same unit. Therefore, a ratio is not generally written with a unit of measurement.

When we say a ratio, we use the word 'to' or 'is to' for the colon (:). So $3 : 5$ is spoken "3 to 5" or "3 is to 5".

Using the scale of a map is important for trip planning as it helps accurately estimate distances, travel times, and the best routes, ensuring a smooth and well-organised journey.

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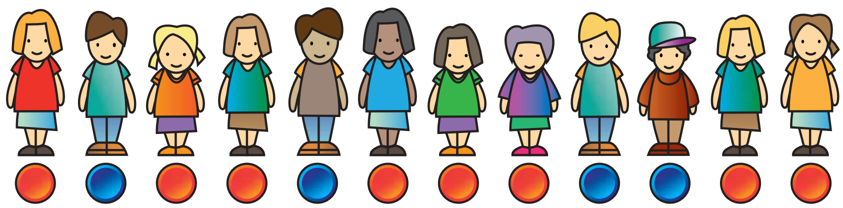
6A

Lesson starter: Student teams

In pairs, work through this activity using different coloured counters for girls and boys. Discuss the answers to these questions.



Ms D's class of 12 students has 8 girls and 4 boys.

- Imagine the students lined up with the girls on the left and the boys on the right. What is the ratio of girls to boys?
- Now consider boys on the left and girls on the right. What is the ratio of boys to girls?
- Divide Ms D's class into halves, making two equal teams so each team has the same number of girls and boys.

  ? girls ? boys | ? girls ? boys

  For one team, what are the ratios of *boys to girls* = $\square : \square$ and *girls to boys* = $\square : \square$?
- Divide Ms D's class into quarters by arranging the 8 girls and 4 boys into four equal teams so each team has the same number of girls and boys.

  ? girls ? boys | ? girls ? boys | ? girls ? boys | ? girls ? boys

  For one team, what are the ratios of *boys to girls* = $\square : \square$ and *girls to boys* = $\square : \square$?
- List these ratios for girls : boys from the groups above.
  Ms D's whole class = $\square : \square$
  one team (half class) = $\square : \square$
  one team (quarter class) = $\square : \square$

These are equivalent ratios because the proportion of girls and boys is the same for each group.

Key ideas

- A **ratio** shows the relationship between two (or more) amounts of the same type.
- Each quantity must first be written with the same units and then the ratio is written without units.
  e.g. 23 minutes to 1 hour = 23 minutes : 60 minutes = 23 : 60
- The **colon (:)** is the mathematical symbol used to represent ratios.
- The written ratio of a : b is read as the ratio of 'a to b' or 'a is to b'.
- The order in which the quantities are written in a ratio is important.
  e.g. cars : bikes = 11 : 2 means 11 cars for every 2 bikes.
- If each number in a ratio is multiplied or divided by the same amount an **equivalent** ratio is formed. e.g. $\times 2$ $\begin{matrix} 1:3 \\ 2:6 \end{matrix}$ $\times 2$ are equivalent ratios.

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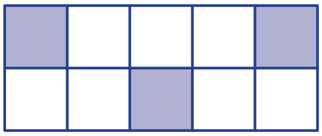
Exercise 6A

Understanding

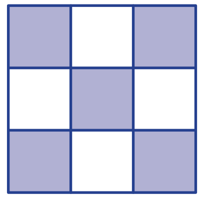
1–3 | 3

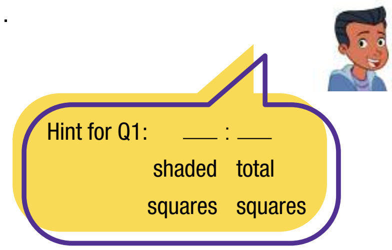
1 Write down the ratio of shaded parts to unshaded parts for each grid.

a

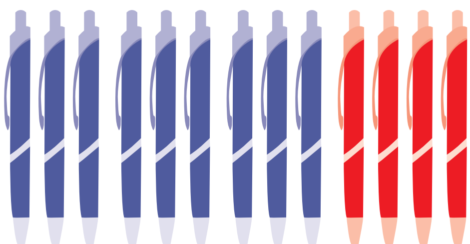


b

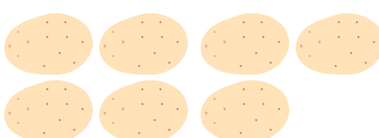


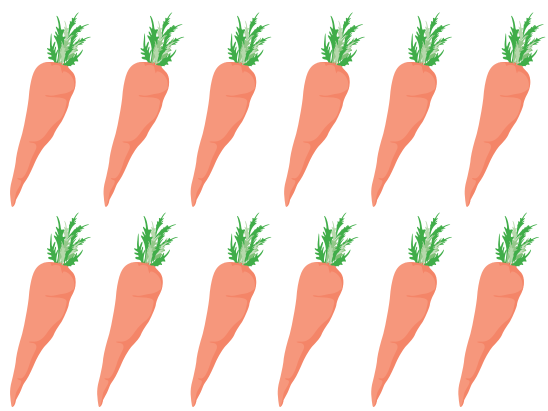


2 **a** What is the ratio of blue pens to red pens?



b What is the ratio of potatoes to carrots?

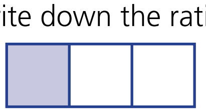




c What is the ratio of cordial to water?

3 Write down the ratio of shaded parts to total parts for each grid.

a



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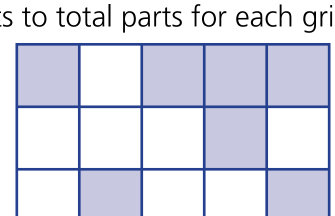

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6A

Fluency

4, 6, 7(½) | 5, 6, 7(½)

Example 1 Writing ratios

A sample of mixed nuts contains 5 cashews and 12 peanuts.
Write down:

a the ratio of cashews to peanuts
b the total number of nuts
c the ratio of cashews to the total number of nuts

Solution	Explanation
a $5 : 12$	cashews : peanuts
b 17	$5 + 12 = 17$
c $5 : 17$	5 cashews, total nuts 17

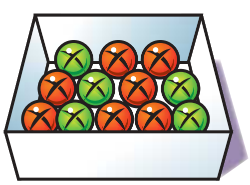
Now you try

A container has 6 chocolate and 5 plain biscuits. Write down:

a the ratio of chocolate biscuits to plain biscuits
b the total number of biscuits
c the ratio of chocolate biscuits to the total number of biscuits

4 A box contains 5 green and 7 red marbles.
 a Write the ratio of green marbles to red marbles.
 b What is the total number of marbles?
 c Write the ratio of green marbles to the total number of marbles.

Hint for Q4: Remember the order of numbers is important in a ratio.



5 Over the past fortnight, it has rained on eight days and it has snowed on three days.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
fine	fine	rain	rain	snow	snow	snow
rain	rain	rain	rain	rain	rain	fine

Write down the ratio of:
 a rainy days to snowy days
 b snowy days to total days
 c fine days to rainy and snowy days
 d rainy days to non-rainy days

6 In a box of 40 flavoured icy poles there were 13 green, 9 lemonade, 11 raspberry and 7 orange icy poles.
Write down the ratio of:
 a green icy poles to orange icy poles
 b raspberry icy poles to lemonade icy poles
 c the four different flavours of icy poles; green : lemonade : raspberry : orange
 d green and orange icy poles to raspberry and lemonade icy poles

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Example 2 Producing equivalent ratios

Complete each pair of equivalent ratios.

a $4 : 9 = 16 : \square$ **b** $30 : 15 = \square : 5$ **c** $2 : 4 : 7 = \square : 12 : \square$

Solution	Explanation
a $\times 4$ $4 : 9 = 16 : 36$ $\times 4$	$4 \times 4 = 16$ $9 \times 4 = 36$
b $\div 3$ $30 : 15 = 10 : 5$ $\div 3$	$15 \div 3 = 5$ $30 \div 3 = 10$
c $\times 3$ $2 : 4 : 7 = 6 : 12 : 21$ $\times 3$	$4 \times 3 = 12$ so multiply each number by 3.

Now you try

Complete each pair of equivalent ratios.

a $3 : 2 = \square : 10$ **b** $36 : 48 = \square : 4$ **c** $1 : 5 : 3 = \square : 20 : \square$

7 Copy and complete each pair of equivalent ratios.

a $1 : 3 = 4 : \square$
b $1 : 7 = 2 : \square$
c $2 : 5 = \square : 10$
d $3 : 7 = \square : 21$
e $5 : 10 = 1 : \square$
f $12 : 16 = 3 : \square$
g $12 : 18 = \square : 3$
h $20 : 50 = \square : 25$
i $2 : 3 : 5 = 4 : \square : \square$
j $4 : 12 : 16 = \square : 6 : \square$
k $0.5 : 3 = 1 : \square$
l $0.1 : 100 = \square : 1000$

Problem-solving and reasoning

8, 9 | 9–11

8 Write three equivalent ratios for each of the following ratios.

a $1 : 2$
b $2 : 5$
c $8 : 6$
d $9 : 3$

Hint for Q8: Multiply or divide both parts of the ratio by the same number.

9 Sort the following ratios into three pairs of equivalent ratios.

$2 : 5, \quad 6 : 12, \quad 7 : 4, \quad 1 : 2, \quad 4 : 10, \quad 70 : 40$

10 Write the ratio of vowels to consonants for each of the following words.

a Queensland
b Canberra
c Wagga Wagga
d Australia

Hint for Q10: The vowels are a, e, i, o, u.

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6A

11 There are four groups of students with 12 students in each group. From the ratios given below, work out the number of boys and the number of girls in each group.

a **Group A**
boys : girls = 2 : 1

b **Group B**
girls : boys = 2 : 1

c **Group C**
girls : boys = 1 : 3

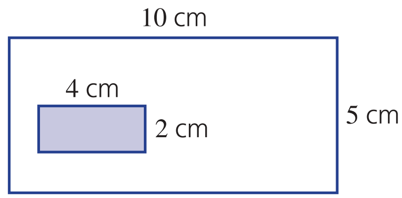
d **Group D**
boys : girls = 1 : 5

Hint for Q11: For group A, find how many lots of 2 boys and 1 girl are needed to make a group of 12 children.

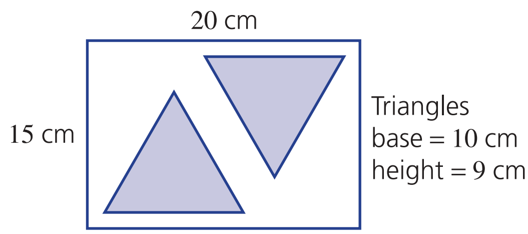
Area ratios — 12, 13

12 Using the dimensions provided, find the ratio of the shaded area to the unshaded area for each of the following diagrams.

a



b



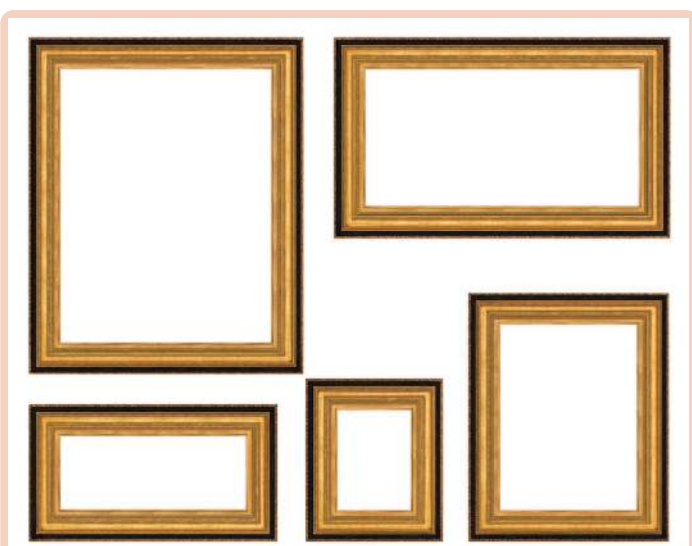
Hint for Q12: Area of triangle $= \dfrac{\text{base} \times \text{height}}{2}$

13 Use your ruler to measure the length and width of some rectangular objects that are on your desk. Measure in mm and round each answer to the nearest 5 mm.

For example: calculator, pencil case, exercise book, textbook and desk.

a For each object, find these ratios and simplify.
 i Length : Width
 ii Length : Area

b What do you notice about your answers to part ii when each ratio is simplified?



Rectangular picture frames have various length to width ratios.

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6B Simplifying ratios

Learning intentions

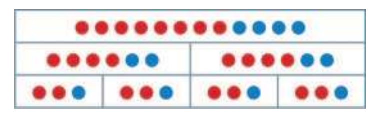
- To understand that simplifying involves finding an equivalent ratio with no common factors.
- To be able to simplify a ratio involving whole numbers by dividing by the highest common factor.
- To be able to write simplified ratios involving quantities by converting units if necessary.

Key vocabulary: ratio, simplify, highest common factor (HCF)

In a similar way to fractions, ratios are simplified by dividing each term by a common factor.

A ratio is said to be in its simplest form when it contains whole numbers only and the highest common factor (HCF) between the terms in the ratio is 1.

Ms D's class had 8 girls and 4 boys. Equal teams (same number of boys and girls in each team) were made by dividing up the class, first into halves and then into quarters.



Ratio of girls : boys

Ms D's whole class $= 8 : 4$

one team (half of class) $= 4 : 2$

one team (quarter of class) $= 2 : 1$

So in Ms D's class the ratio of girls : boys $= 8 : 4 = 4 : 2 = 2 : 1$.

The smallest ratio $2 : 1$ is called the simplest form of these equivalent ratios. This simplest form ratio shows that the proportion of girls to boys in Ms D's class is 2 girls for every 1 boy.

Lesson starter: Class ratios

Look around your classroom and write down the following ratios.

a Ratio of girls to boys
b Ratio of teachers to students
c Ratio of left-handed to right-handed students
d Ratio of white socks to black socks
e Ratio of textbooks open to textbooks closed
f Ratio of not having a pencil case to having a pencil case
g Ratio of blonde hair to brown hair to black hair
h Ratio of blue eyes to brown eyes to other colour eyes

Design your own ratio question for your class or classroom. Can any of your ratio answers be simplified?

Key ideas

- Before **ratios** are **simplified** the quantities must be expressed in the same unit.
- A ratio is simplified by dividing both numbers in the ratio by their **highest common factor (HCF)**.
  e.g. the ratio $15 : 25$ can be simplified to $3 : 5$.

$$\begin{array}{ccc} & 15 : 25 & \\ \div 5 \downarrow & & \downarrow \div 5 \\ & 3 : 5 & \end{array}$$

- Ratios are usually written in their simplest form.
- Ratios in simplest form use whole numbers only.
- If a ratio is expressed with fractions, it is simplified by converting the quantities to whole numbers. This is generally done by multiplying by the lowest common denominator (LCD).

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Exercise 6B

Understanding | 1–3 | 3

1 Copy and complete writing these ratios in simplest form.

a $\div 5$ $5 : 10 = \square : \square$ $\div 5$

b ? $12 : 20 = \square : \square$?

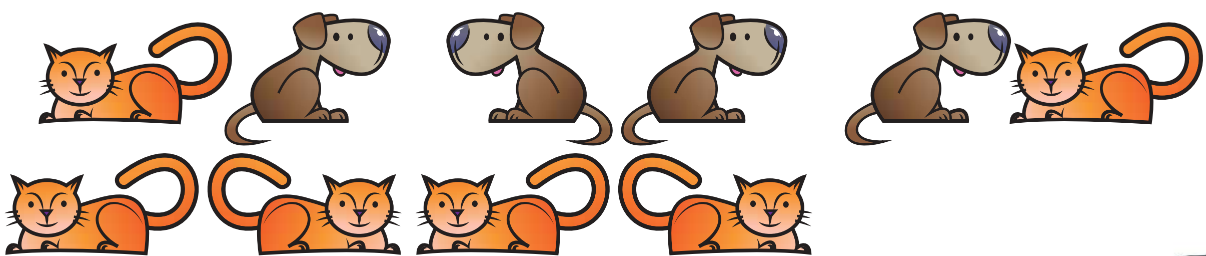
c ? $6 : 18 = \square : \square$?

d ? $15 : 35 = \square : \square$?

e ? $80 : 50 = \square : \square$?

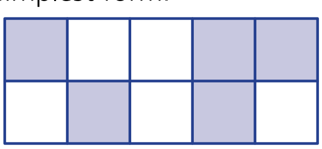
f ? $72 : 60 = \square : \square$?

2 Write the ratio of cats to dogs in simplest form.

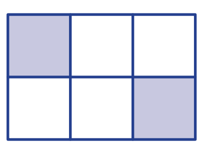


3 Write down the ratio of shaded parts to unshaded parts for each of the following in simplest form.

a



b



Hint for Q3:
purple squares : white squares
$\square : \square$

Fluency | 4–5(½) | 4–6(½)

Example 3 Simplifying ratios

Simplify the following ratios.

a $7 : 21$ **b** $450 : 200$

Solution	Explanation
a $\div 7$ $7 : 21 = 1 : 3$ $\div 7$	Highest common factor (HCF) of 7 and 21 is 7. Divide both numbers by 7.
b $\div 50$ $450 : 200 = 9 : 4$ $\div 50$	HCF of 450 and 200 is 50. Divide both numbers by 50. Alternatively, divide both numbers by 10 first and then by 5.

Now you try

Simplify the following ratios.

a $9 : 3$ **b** $125 : 475$

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4 Simplify the following ratios.

a 2 : 8	**b** 10 : 50	**c** 4 : 24
d 6 : 18	**e** 8 : 10	**f** 25 : 40
g 21 : 28	**h** 24 : 80	**i** 18 : 14
j 26 : 13	**k** 45 : 35	**l** 81 : 27
m 51 : 17	**n** 20 : 180	**o** 300 : 550
p 150 : 75	**q** 1200 : 100	**r** 70 : 420
s 200 : 125	**t** 90 : 75	

Hint for Q4: Divide both numbers by the highest common factor.

5 Simplify the following ratios.

a 2 : 4 : 6	**b** 12 : 21 : 33	**c** 42 : 60 : 12
d 85 : 35 : 15	**e** 12 : 24 : 36	**f** 100 : 300 : 250
g 270 : 420 : 60	**h** 24 : 48 : 84	

Hint for Q5: Divide all three numbers by the HCF.

Example 4 Simplifying ratios that have different units

First, change the quantities to the same unit by changing the larger unit to the smaller unit. Then express each pair of quantities as a ratio in simplest form.

a 4 mm to 2 cm
b 25 minutes to 2 hours

Solution	Explanation
a 4 mm to 2 cm = 4 mm to 20 mm = 4 : 20 = 1 : 5	2 cm = 20 mm Once in the same unit, write as a ratio. Simplify ratio by dividing by HCF of 4.
b 25 minutes to 2 hours = 25 minutes to 120 minutes = 25 : 120 = 5 : 24	2 hours = 120 minutes Once in the same unit, write as a ratio. Simplify ratio by dividing by HCF of 5.

Now you try

First, change the quantities to the same unit by changing the larger unit to the smaller unit. Then express each pair of quantities as a ratio in simplest form.

a 2.5 tonnes to 500 kg
b 250 seconds to 5 minutes

6 First, change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 12 mm to 3 cm	**b** 7 cm to 5 mm
c 120 m to 1 km	**d** 60 mm to 2.1 m
e 3 kg to 450 g	**f** 200 g to 2.5 kg
g 2 tonnes to 440 kg	**h** 1.25 L to 250 mL
i 400 mL to 1 L	**j** 20 minutes to 2 hours
k 3 hours to 15 minutes	**l** 3 days to 8 hours
m 180 minutes to 2 days	**n** 8 months to 3 years
o 4 days to 4 weeks	**p** 8 weeks to 12 days
q 50 cents to $4	**r** $7.50 to 25 cents

Hint for Q6: Change the larger unit to the smaller unit.

Hint for Q6:
1 tonne = 1000 kg
1000 g = 1 kg
1 L = 1000 mL
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

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6B

Problem-solving and reasoning

7–10 9–12

7 To express the ratio 4 : 16 in simplest form, you would:

A multiply both quantities by 2
B subtract 4 from both quantities
C divide both quantities by 2
D divide both quantities by 4

8 Decide which of the following ratios is not written in simplest form.

A 1 : 5
B 3 : 9
C 2 : 5
D 11 : 17

Hint for Q8: Find the ratio that has a common factor.

9 Decide which of the following ratios is written in simplest form.

A 2 : 28
B 15 : 75
C 14 : 45
D 13 : 39

Hint for Q9: Find the ratio that does not have a common factor.

10 When Lisa makes fruit salad for her family, she uses 5 bananas, 5 apples, 2 passionfruit, 4 oranges, 3 pears, 1 lemon (for juice) and 20 strawberries.

a Write the ratio of the fruits in Lisa's fruit salad (in the same order as given in the question).

b Lisa wanted to make four times the amount of fruit salad to take to a party. Write an equivalent ratio that shows how many of each fruit Lisa would need.

c Write these ratios in simplest form.

i bananas to strawberries
ii strawberries to other fruits

Hint for Q10: The ratio in part **a** will have 7 numbers.

11 Andrew incorrectly simplified 12 cm to 3 mm as a ratio of 4 : 1. What was Andrew's mistake and what is the correct simplified ratio?

Hint for Q11: First write 12 cm : 3 mm with the same units.

12 **a** Write two quantities of time, in different units, which have a ratio of 2 : 5.

b Write two quantities of distance, in different units, which have a ratio of 4 : 3.

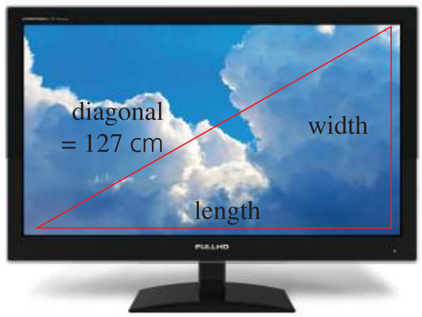
Hint for Q12: 2 hours : 5 hours
= 2 hours : $\square$ minutes

Essential Mathematics Core for the Australian Curriculum Year 8 ISBN 978-1-009-593-724

Aspect ratios

— 13

13 Aspect ratio is the relationship between the width and height of an image displayed on a screen. The aspect ratio of a rectangle is the ratio of the length to the width.



Size of TV = diagonal length of image
Aspect ratio = length : width

Investigate aspect ratios and create a poster or PowerPoint showing:

a how to calculate the aspect ratio for a rectangular image
b examples of aspect ratios.

Research these examples of aspect ratios.

1. Use the internet or a newspaper to find an advertisement for the various enlargements available from a local photo print shop. State the aspect ratio for each of these enlargements.
2. What is the difference between the size of a television (e.g.127 cm) and the aspect ratio of the television?
3. Find out the aspect ratio of:
   - analog televisions
   - high-definition digital televisions
   - old cinema movies
   - modern cinema movies
   - widescreen movies shown on television
4. Calculate the aspect ratio for different-sized newspapers.

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6C Dividing a quantity in a given ratio ★

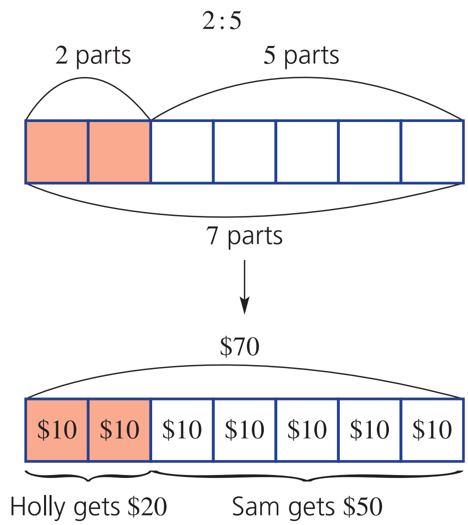
Learning intentions

- To understand that a quantity can be divided in a ratio.
- To be able to divide a quantity in a particular ratio (with two or three terms).
- To be able to find the total quantity given a ratio and the actual size of one component.

Key vocabulary: ratio, parts, unitary method, equivalent ratios

Some ways that ratios can be used are to:

- share money between people
- divide any quantity into certain proportions
- find the correct amount of each portion in a mixture.

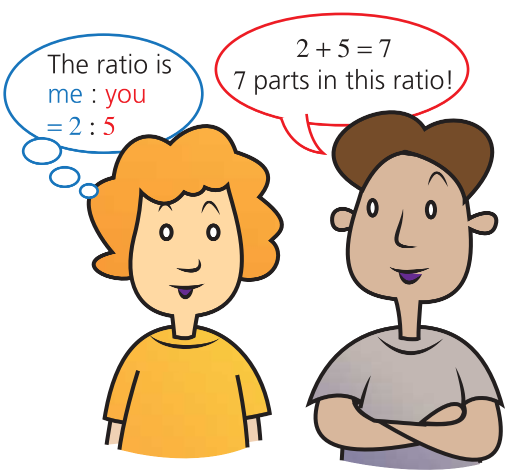


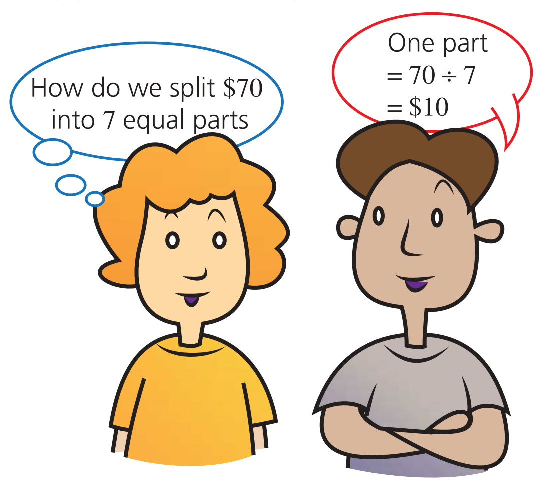
For example:

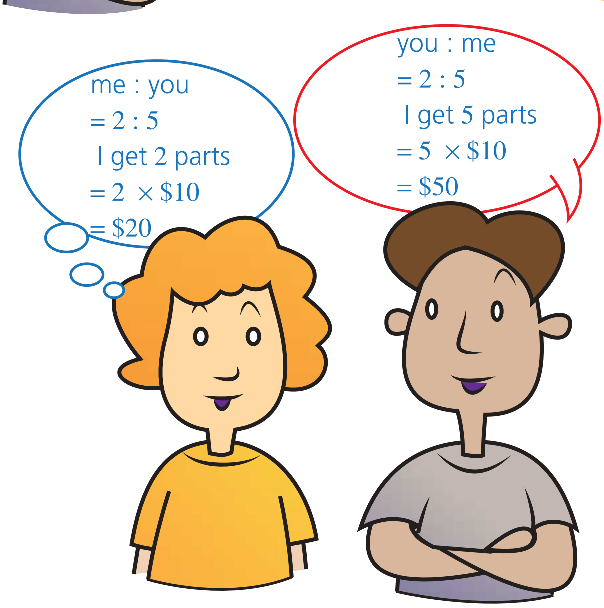
Each weekend Holly and Sam deliver brochures to letterboxes.

Holly works for 2 hours and Sam works for 5 hours and altogether they earn $70.

They worked out how to divide up $70 in the ratio 2 : 5 using a diagram like the one shown on the right.







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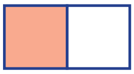
Lesson starter: Sharing money

With a partner, work out how to share these amounts using the given ratios.

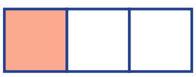
- How can you check that your answers are correct?

\$60 is shared between Lucy and Bronte in the ratio $1 : 1$.



What is the total number of parts?
What is the value of one part?
How much does Lucy get?
How much does Bronte get?

\$120 is shared between Andrew and Matt in the ratio $1 : 2$.



What is the total number of parts?
What is the value of one part?
How much does Andrew get?
How much does Matt get?

\$120 is shared between Christine, Prue and Karol in the ratio $3 : 2 : 5$.

What is the total number of parts?
What is the value of one part?
How much does Christine get?
How much does Prue get?
How much does Karol get?

Key ideas

- Think of a **ratio** in terms of '**parts**'.
  A ratio of $2 : 3$ has 2 parts of one quantity for every 3 parts of another quantity, and a total of 5 parts.
- Use the **unitary method** to divide a quantity in a given ratio:
  - find the total number of parts in the ratio
  - find the value of one part
  - find the value of the number of parts required in the ratio.

  e.g. Share \$20 in the ratio of $2 : 3$.
  Think of sharing \$20 into 2 parts and 3 parts.
  Total number of parts $= 2 + 3 = 5$.
  Value of one part $= \$20 \div 5 = \$4$.
  Therefore 2 parts $= \$8$, and 3 parts $= \$12$.

$$\begin{array}{rcl} & \$20 = 5 \text{ parts} & \\ \div 5 & & \div 5 \\ & \$4 = 1 \text{ part} & \\ \times 2 & & \times 2 \\ & \$8 = 2 \text{ parts} & \end{array}$$

- To find a total quantity from a given ratio:
  - Find the value of one part and then the value of the total parts can be calculated.

  Or
  - Use **equivalent ratios** to find the value of each quantity in the total ratio, then add the numbers together to find the total.

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Exercise 6C

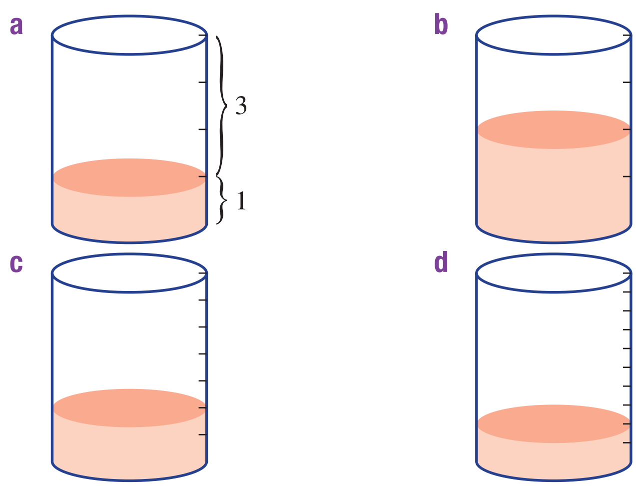
Understanding 1–3 3

1 Find the total number of parts in the following ratios.
- **a** 3 : 7
- **b** 1 : 5
- **c** 11 : 3
- **d** 2 : 3 : 4

Hint for Q1: Add the numbers in the ratio to find the total parts.

2 Marta and Joshua earned \$25 between them. They want to share it in the ratio Marta : Joshua = 3 : 2. Copy and complete these steps.
- **a** In the ratio 3 : 2, the total parts = __ + __ = __
- **b** __ parts = \$25, so 1 part = ________
- **c** Marta gets 3 parts, so Marta gets 3 × \$________ = \$________
- **d** Joshua gets 2 parts, so Joshua gets 2 × \$________ = \$________
- **e** Total amount = \$________ + ________ = \$________

3 The diagram shows four glasses that contain different amounts of cordial. Water is then added to fill each glass right to the top. For each drink shown, what is the ratio of cordial to water?



Hint for Q3: Cordial : water
$\square$: 3

Hint for Q3: Write ratios in simplest form.

Fluency 4(½), 5, 6(½) 4(½), 5, 6(½), 7

Example 5 Dividing a quantity in a particular ratio

Divide 54 metres in a ratio of 4 : 5.

Solution

Total number of parts = 9

÷9 9 parts = 54 m ÷9
1 part = 6 m
×4 4 parts = 24 m ×4

×5 1 part = 6 m ×5
5 parts = 30 m

The amounts are 24 m and 30 m.

Explanation

Total number of parts = 4 + 5 = 9
Value of 1 part = 54 m ÷ 9 = 6 m
Check numbers add to total: 24 + 30 = 54.
Write the answers with units.

Now you try

Divide \$80 in a ratio of 5 : 3.

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4 Divide:

a $60 in the ratio of 2 : 3
b $110 in the ratio of 7 : 4
c $1000 in the ratio of 3 : 17
d 48 kg in the ratio of 1 : 5
e 14 kg in the ratio of 4 : 3
f 360 kg in the ratio of 5 : 7
g 72 m in the ratio of 1 : 2
h 40 m in the ratio of 3 : 5
i 155 m in the ratio of 4 : 1

5 Share $400 in the ratio:

a 1 : 3
b 2 : 3
c 3 : 5
d 9 : 11

Hint for Q5: Start by finding the:
- total parts
- value of one part.

Example 6 Dividing a quantity in a ratio with three numbers

Divide $300 in the ratio of 2 : 1 : 3.

Solution

Total number of parts = 6

$$\begin{aligned} 6 \text{ parts} &= \$300 \\ \div 6 \quad 1 \text{ part} &= \$50 \quad \div 6 \\ \times 2 \quad 2 \text{ parts} &= \$100 \quad \times 2 \end{aligned} \qquad \begin{aligned} 1 \text{ part} &= \$50 \\ \times 3 \quad 3 \text{ parts} &= \$150 \quad \times 3 \end{aligned}$$

The three amounts are $100, $50 and $150.

Explanation

Total number of parts = 2 + 1 + 3 = 6.
Value of 1 part = $300 ÷ 6 = $50
Check numbers add to total:
$100 + $50 + $150 = $300.

Now you try

Divide 60 kg in the ratio 7 : 2 : 3.

6 Divide:

a $200 in the ratio of 1 : 2 : 2
b $400 in the ratio of 1 : 3 : 4
c 12 kg in the ratio of 1 : 2 : 3
d 88 kg in the ratio of 2 : 1 : 5
e 320 kg in the ratio of 12 : 13 : 15
f $50 000 in the ratio of 1 : 2 : 3 : 4

Hint for Q6: What is the total number of parts?

7 Share 600 lollies in the ratio:

a 1 : 9
b 2 : 1 : 3
c 2 : 5 : 5
d 12 : 7 : 8 : 3

Hint for Q7: Write units in the answers.

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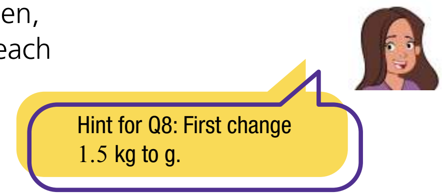
6C

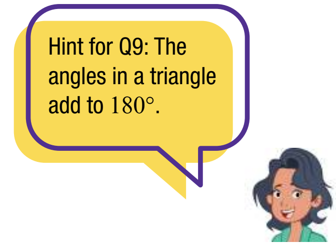
Problem-solving and reasoning

8, 9	9–11

8 Evergreen Fertiliser is made up of the three vital nutrients — nitrogen, potassium and phosphorus — in a ratio of $4:5:3$. How much of each nutrient is in a 1.5 kg bag?

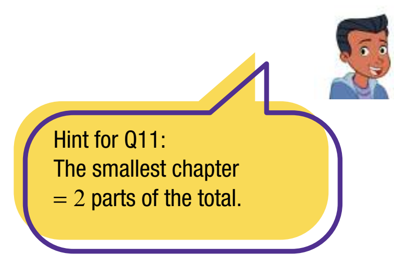


9 The angles of a triangle are in the ratio of $2:3:4$. Find the size of each angle.



10 Three friends, Cam, Molly and Seb, share a prize of $\$750$ in a ratio of $3:4:8$. How much more money does Seb receive than Cam?

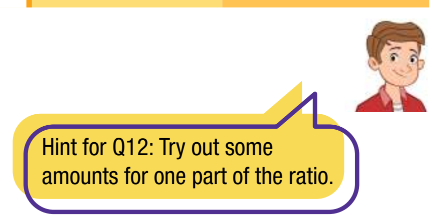
11 A textbook has three chapters and the ratio of pages in these chapters is $3:2:5$. If there are 24 pages in the smallest chapter, how many pages are in the textbook?



Changing ratios

—	12, 13

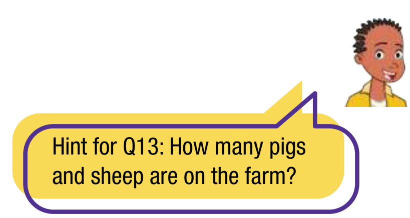
12 The ratio of the cost of a shirt to the cost of a jacket is $2:5$. If the jacket cost $\$240$ more than the shirt, find the cost of the shirt and the cost of the jacket.



13 On a farm with 24 pigs and sheep the ratio of sheep to pigs is $1:2$.

a On one day, the ratio of sheep to pigs was $3:7$. How many pigs and how many sheep were missing?

b If 4 more sheep and 4 more pigs joined the farm, what would be the new ratio of sheep : pigs?



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6D Scale drawings

Learning intentions

- To understand that scale drawings can be used to depict large or small objects.
- To be able to convert from a distance on a map or diagram to the actual distance in real life.
- To be able to convert from the actual distance in real life to a distance on a map or diagram.
- To be able to determine the scale factor given a distance on a diagram and the distance in real life.
- To be able to convert between different units of length.

Key vocabulary: scale, scale factor, ratio

A scale drawing is used when the actual object has measurements too large to fit on a page.

For example, scale drawings are used for:

- house plans
- maps
- drawings of large objects, like a car or plane.

Scale drawings allow very large objects to be drawn on a single page, essential for design work.

A scaled-up image of an insect's eyes, highlights intricate details and structures not visible to the naked eye.

A scale drawing is also used for very small objects so we can see the details clearly. For example, scale drawings could show a 'close-up' of a:

- flea
- strand of hair
- computer chip.

Let's say this picture of a dragonfly is 5 times larger than a real dragonfly.

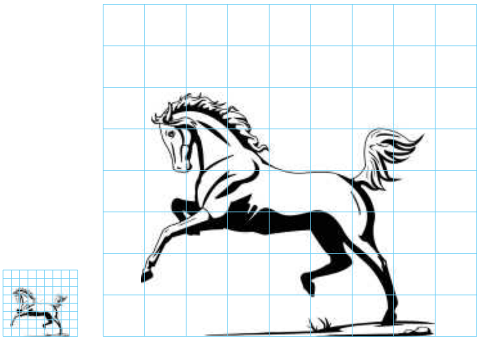
The real dragonfly is $\frac{1}{5}$ of the size of this picture.

How wide is the real dragonfly's head?

Lesson starter: Enlarging pictures

The larger picture of a horse is an enlargement of the smaller picture.

- Discuss what is meant by the scale factor between the two drawings.
- What method would you use to calculate the scale factor.
- What is the approximate scale factor between the two horse drawings.



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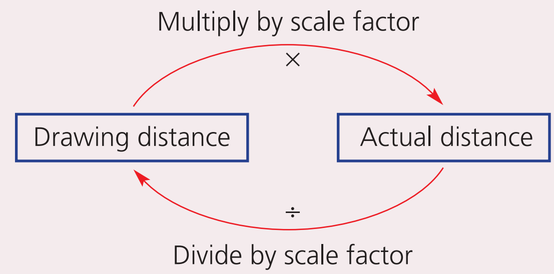
6D

Key ideas

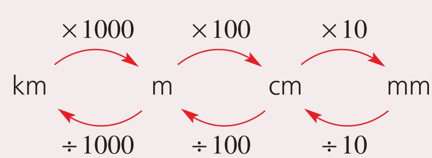
- A scale drawing has exactly the same shape as the original object, but it is a different size.
- The **scale** on a drawing is written as a scale **ratio** of the drawing length : actual (real) length.
  e.g. a scale ratio of 1 : 100 means that the real object is 100 times larger than the drawing.
- Scales should begin with the number 1. The second number in the ratio is called the **scale factor**.
  e.g.

$$\div 5 \;\; \begin{array}{c} 5 : 25\,000 \\ 1 : 5000 \end{array} \;\; \div 5$$

  The scale factor is 5000.
  The real object is 5000 times larger than the drawing.



- To change a drawing distance to an actual (real) distance, multiply by the scale factor.
  - Multiplying by a scale factor does not change the units.
    e.g. On a house plan with scale 1 : 200 a room is 2 cm wide.
    The real room will be 2 cm × 200 = 400 cm wide
    = 400 ÷ 100 m
    = 4 m wide
- To change an actual (real) distance to a drawing distance, you divide by the scale factor.
  - Dividing by a scale factor does not change the units.
    e.g. A real house is 12 m wide and the house plan has a scale 1 : 200.
    The house plan will be 12 m ÷ 200 = 0.06 m wide
    = 0.06 × 100 cm
    = 6 cm wide
- It is important to remember how to correctly convert measurements of length when dealing with scales.



Exercise 6D

Understanding

1–3 | 3

1 **a** Convert 10 000 cm to:
- **i** mm
- **ii** m
- **iii** km

b Convert 560 m to:
- **i** km
- **ii** cm
- **iii** mm

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2 Here are pictures of a real classic racing car and a model racing car.
  a Write the model car length in cm and real car length in cm.
  b How many times bigger is the real car compared to the model?
  c What is the scale ratio for the model car : real car?


Classic racing car length: 5.7 m


Model racing car length: 57 mm

3 A model ship is 60 cm long and the real ship is 300 m long.
  a Write the model length in cm and the real ship length in cm.
  b How many times bigger is the real ship compared to the model?
  c What is the scale ratio for the model ship : real ship?

Fluency 4–6, 7(½) 4–7(½)

Example 7 Converting scale distance to actual distance

A map has a scale of $1 : 20\,000$.
Find the actual distance in m for each scaled distance (map distance).

a 2 cm b 5 mm

Solution

a Actual distance $= 2\text{ cm} \times 20\,000$
$= 40\,000\text{ cm}$
$= 400\text{ m}$

Explanation

Scale factor $= 20\,000$
Multiply scaled distance by scale factor.
Then $\div 100$ to convert cm to m.

b Actual distance $= 5\text{ mm} \times 20\,000$
$= 100\,000\text{ mm}$
$= 100\text{ m}$

Multiply scaled distance by scale factor.
5 mm times scale factor gives answer in mm.
Then $\div 10 \div 100$ to convert mm to cm to m.

Now you try

A map has a scale of $1 : 600$.

Find the actual distance in m for each scaled distance (map distance).

a 4 cm b 3 mm

4 Find the actual distance for each of the following scaled distances.
Give your answer in the unit that is in brackets after each question.
  a Scale $1 : 2$
    i 310 cm (cm) ii 2.5 mm (mm)
  b Scale $1 : 10\,000$
    i 2 cm (m) ii 4 mm (m)
  c Scale $1 : 400$
    i 16 mm (m) ii 72 cm (m)
  d Scale $1 : 0.01$
    i 3 cm (mm) ii 0.815 m (mm)

Hint for Q4:
- Multiply by the scale factor.
- Keep the units the same as the question.
- Then convert to the required units.

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6D

Example 8 Converting actual distance to scaled distance

A model boat has a scale of 1 : 500.
Find the scaled length in mm for these actual lengths.

a 50 m **b** 4550 mm

Solution

a Scaled distance = 50 m ÷ 500
= 0.1 m
= 10 cm
= 100 mm

b Scaled distance = 4550 mm ÷ 500
= 45.5 mm ÷ 5
= 9.1 mm

Explanation

a Divide by the scale factor, 500.
50 m ÷ 500 gives the answer in m.
× 100 to convert m to cm.
× 10 to convert cm to mm.

b Divide actual distance by scale factor.
Shortcut: ÷ 100, then ÷ 5 (or vice versa)
The answer is in mm.

$\boxed{\text{Scaled length}} \xleftarrow{\div} \boxed{\text{Actual length}}$

Divide by scale factor

Now you try

A print of a painting has a scale of 1 : 25.
Find the scaled length in cm for these actual lengths.

a 3 m **b** 220 cm

5 Find the scaled length for each of these actual lengths. Give your final answer in the unit that is in brackets after each question.

a Scale 1 : 200
  i 200 m (m) **ii** 4 km (m)

b Scale 1 : 500
  i 10 000 m (m) **ii** 1 km (m)

c Scale 1 : 10 000
  i 1350 m (cm) **ii** 736.5 m (cm)

d Scale 1 : 0.05
  i 7.5 cm (m) **ii** 8.2 mm (m)

Hint for Q5:
- Divide by the scale factor.
- Keep the units the same as the question.
- Then convert to the required units.

6 Change the two measurements provided in each scale into the same unit, and then write the scale as a ratio of two numbers in simplest form.

a 2 cm : 200 m
b 5 mm : 500 cm
c 12 mm : 360 cm
d 4 mm : 600 m
e 4 cm : 5 m
f 1 cm : 2 km
g 28 mm : 2800 m
h 3 cm : 0.6 mm
i 1.1 m : 0.11 mm

Hint for Q6:
- Convert larger unit to smaller unit.
- Divide by HCF.

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Example 9 Determining the scale factor

State the scale factor in the following situations.

a 4 mm on a scale drawing represents an actual distance of 50 cm.
b An actual length of 0.1 mm is represented by 3 cm on a scaled drawing.

Solution	Explanation
a Scale ratio = 4 mm : 50 cm	Write the ratio drawing length : actual length.
= 4 mm : 500 mm	Convert to 'like' units.
Scale ratio = 4 : 500	Write the scale ratio without units.
= 1 : 125	Divide both numbers by 4. (HCF = 4)
Scale factor = 125	Ratio is now in the form 1 : scale factor. The actual size is 125 times larger than the scaled drawing.
b Scale ratio = 3 cm : 0.1 mm	Write the ratio drawing length : actual length.
= 30 mm : 0.1 mm	Convert to 'like' units.
Scale ratio = 30 : 0.1	Write the scale ratio without units.
= 300 : 1	Multiply both numbers by 10.
$= 1 : \frac{1}{300}$	Divide both numbers by 300. Ratio is now in the form 1 : scale factor.
Scale factor = $\frac{1}{300}$	The actual size is 300 times smaller than the scaled drawing.

Now you try

Find the scale factor in the following situations.

a 2.5 cm on a scale drawing represents an actual distance of 750 m.
b An actual length of 0.02 cm is represented by 4 cm on a drawing.

7 Find the scale ratio and the scale factor for each of the following.
  a 2 mm on a scale drawing represents an actual distance of 50 cm.
  b 4 cm on a scale drawing represents an actual distance of 2 km.
  c 1.2 cm on a scale drawing represents an actual distance of 0.6 km.
  d 5 cm on a scale drawing represents an actual distance of 900 m.
  e An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.
  f An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

Hint for Q7:
- Same units.
- Scale ratios start with 1.
- Write scale factors as whole numbers or fractions.

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6D

Problem-solving and reasoning

8, 9 | 9–11

8 A model city has a scale ratio of 1 : 1000.
 a Find the actual height in m of a skyscraper that has a scaled height of 8 cm.
 b Find the scaled length in cm of a train platform that is 45 m long in real life.

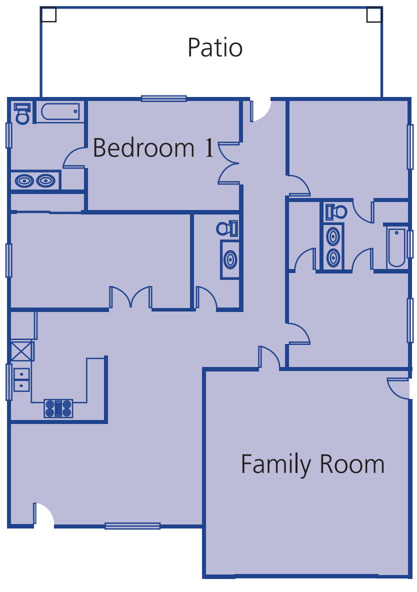
Hint for Q8: Scaled height means model height.

9 Blackbottle and Toowoola are 17 cm apart on a map with a scale of 1 : 50 000. How many km apart are the towns in real life?

10 This house plan has a scale of 1 : 150. For each room listed below, do these two steps.
- Use a ruler to measure its length and width in mm.
- Use the scale to calculate the real dimensions in m to one decimal place.

 a Bedroom 1 b Family room c Patio

Hint for Q10: Length in mm $\times$ scale factor = real length in mm

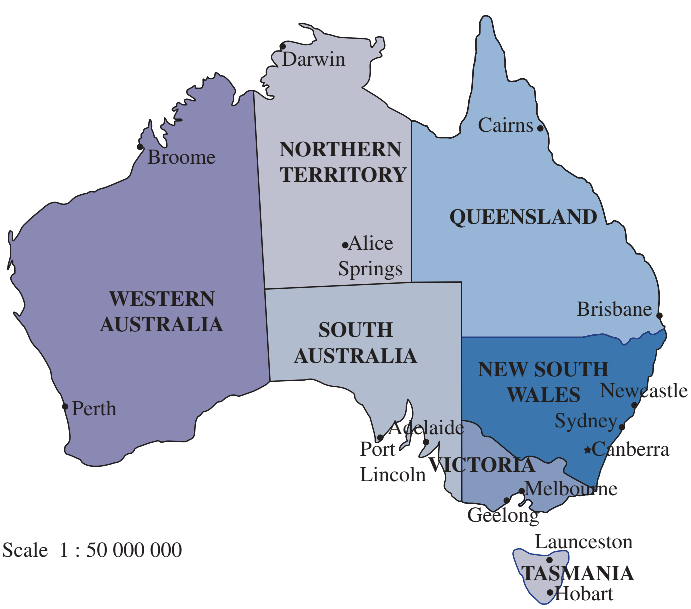


2 m

Scale 1 : 150

11 For each question below, do these two steps using the map.
- Use a ruler to measure the straight-line map distance in cm to one decimal place.
- Use the scale to calculate the real distance and give each answer to the nearest 100 km.

 a Hobart to Cairns
 b Perth to Sydney
 c Darwin to Adelaide
 d Brisbane to Melbourne
 e Australia's furthest west point to furthest east point.



Scale 1 : 50 000 000

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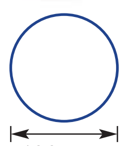
Design a bedroom

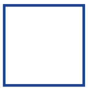
—	12

12 For this activity you will design and draw the bedroom of a house plan. Builders use millimetres for units so keep all units in millimetres for this activity.
This bedroom has length = 3000 mm and width = 4000 mm.

Hint for Q12: How many times larger is the bedroom length than your page length? Choose a whole number for the scale factor.
scale = 1: *scale factor*

The furniture in the bedroom is illustrated below. These pictures are **not** shown to scale. The real dimensions are given for the 'top view'. The 'top view' is how it is seen looking down from above.





You are to design a scaled drawing of a bedroom including this furniture.

a Find a scale that will allow the drawing of this bedroom to fit on one page.
b Use your scale to change all the real dimensions to scaled lengths and widths in mm.
c Draw a scaled rectangle for the bedroom.
d Choose where each piece of furniture will be placed in the bedroom and draw the scaled top view of each.
e Choose where a window and a door will be placed in the bedroom and draw the scaled top view of each.
f Label dimensions with the real measurements in mm.
g Write the scale next to your bedroom plan.

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Progress quiz

6A **1** A group of mixed balls contains 7 tennis balls and 5 basketballs. Write down:
- **a** the ratio of tennis balls to basketballs.
- **b** the total number of balls.
- **c** the ratio of basketballs to the total number of balls.

6A **2** Complete each pair of equivalent ratios.
- **a** $12 : 15 : 24 = 24 : \square : 48$
- **b** $35 : 25 = \square : 5$
- **c** $3 : 5 : 8 = 12 : \square : \square$

6B **3** Simplify the following ratios.
- **a** $6 : 27$
- **b** $100 : 25$
- **c** $42 : 28 : 7$

6B **4** First change the quantities to the same unit, by changing the larger unit to the smaller unit. Then express each pair of quantities as a ratio in simplest form.
- **a** 15 minutes to $1\frac{1}{2}$ hours
- **b** 8 mm to 1.2 cm
- **c** \$6.00 to 40 cents

6B **5** Decide which of the following ratios is not written in simplest form.

A $13 : 15$ **B** $24 : 41$ **C** $34 : 15$ **D** $51 : 17$

6C ★ **6** Divide:
- **a** 50 kg in the ratio of $2 : 3$
- **b** 18 m in the ratio of $5 : 1$
- **c** \$4000 in the ratio of $7 : 13$
- **d** 4 hours in the ratio of $1 : 5$

6C ★ **7** Divide \$1000 in the ratio:
- **a** $3 : 7$
- **b** $19 : 1$
- **c** $65 : 35$
- **d** $23 : 27$

6C ★ **8** The ratio of goals to behinds scored by the U15 Newtown Eagles is $3 : 5$. If the team scored 248 times for the season, how many goals did the Eagles score for the season?

6D **9** A map has a scale ratio of $1 : 50\,000$.
- **a** What is the scale factor?
- **b** What actual real distance in m would 3 cm on the map represent?
- **c** What actual real distance in km would 8 cm on the map represent?

6D **10** A model car has a scale of $1 : 200$. Find the scaled length in mm for each of these actual lengths.
- **a** 8 m
- **b** 4600 mm

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6E Introducing rates

Learning intentions

- To understand that rates compare two quantities measured in different units.
- To be able to simplify rates.
- To be able to find average rates.
- To understand that a rate like $12/h means $12 for 1 hour.

Key vocabulary: rate, simplified rate, average rate, per (/)

If you monitored what you said each day, you might find that you speak about rates many times!

A ratio shows the relationship between the same type of quantities with the same units, but a rate shows the relationship between two different types of quantities with different units.

The following are all examples of rates:

- Cost of petrol was $1.45 per litre.
- Rump steak was on special for $18/kg.
- Dad drove to school at an average speed of 52 km/h.
- After the match, your heart rate was 140 beats/minute.

A ratio compares two amounts of the same type and the same units, so a ratio does not include units.

- For example: the ratio of girls to boys in a group was 4 : 5.

A rate compares different types of quantities so both units must be shown.

- For example: the average rate of growth of a teenage boy is 6 cm/year.

In a year's time, this boy could expect to grow by 6 cm, based on the average growth rate for boys.

Lesson starter: State the rate

For each of the following statements, write down a corresponding rate.

- The Lodges travelled 400 km in 5 hours. What is their speed in km/h?
- Gary was paid $98 for a 4-hour shift at work. What is the rate of pay in $/h?
- Felicity spent $600 on a two-day shopping spree. What is Felicity's spending rate in $/day?
- Max had grown 9 cm in the last three months. What is Max's growth rate in cm/month?
- Vuong paid $37 for half a cubic metre of crushed rock. What is the cost in $/cubic metre?
- Paul cycled a total distance of 350 km for the week. At what rate did Paul cycle in km/day?

What was the rate (in questions/minute) at which you answered these questions?

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6E

Key ideas

- **Rates** compare quantities measured in different units.
- All rates must include two different units.
- The two different units are separated by a slash '/', which is the mathematical symbol for '**per**'.
  e.g. 20 km/h = 20 km per hour = 20 km for each hour.
- It is usual to write rates in their simplest form. This involves writing the rate for only one unit of the second quantity.
  e.g. Avi earned \$45 in 5 hours = \$45 in 3 hours ← non-simplified rate
  ÷ 3 ÷ 3
  = \$15 in 1 hour
  = \$15/h ← **simplified rate**
- The **average rate** is calculated by dividing the total change in the first quantity by the total change in the second quantity.
  e.g. reading a 400-page book in 4 days

  Average reading rate = 400 pages in 4 days
  ÷ 4 ÷ 4
  = 100 pages in 1 day
  Average reading rate = 100 pages/day

Exercise 6E

Understanding 1–3 3

1 Which of the following are examples of rates?

A \$5.50 **B** 180 mL/min
C \$60/h **D** $\frac{5}{23}$
E 4.2 runs/over **F** 0.6 g/L
G 200 cm^2 **H** 84 c/L

Hint for Q1: Remember that rates have two different units.

2 Match each rate in the first column with its most likely rate in the second column.

a	Employee's wage	90 people/day
b	Speed of a car	\$2100/m^2
c	Cost of building a new home	68 km/h
d	Population growth	64 beats/min
e	Resting heart rate	\$15/h

3 Select from this list the most typical units for each of the following rates.
\$/L mg/tablet \$/kg kJ/serve runs/over words/minute goals/shots (on goal) L/minute

a Price of sausages **b** Petrol costs
c Typing speed **d** Goal conversion rate
e Energy nutrition information **f** Water usage in the shower
g Pain relief medication **h** Cricket team's run rate

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Fluency

4(½), 5, 6 | 4–5(½), 6

Example 10 Writing simplified rates

Express each of the following as a simplified rate.

a 12 students for two teachers

b $28 for 4 kilograms

Solution	Explanation
a 12 students/2 teachers = 6 students/teacher	Divide both quantities by the second amount. $12 \div 2 = 6$ students for 1 teacher. Include both units separated by /.
b $28/4 kg = $7/kg	$28 \div 4 = \$7$ for 1 kg When writing a cost rate, the $ sign is written before the number.

Now you try

Express each of the following as a simplified rate.

a 40 sandwiches for 20 people

b $68 per 4 kg

4 Write each of the following as a simplified rate.

a 12 days in 4 years

b 15 goals in 3 games

c $180 in 6 hours

d $17.50 for 5 kilograms

e $126 000 to purchase 9 acres

f 36 000 cans in 8 hours

g 12 000 revolutions in 10 minutes

h 80 mm rainfall in 5 days

i 60 minutes to run 15 kilometres

j 15 kilometres run in 60 minutes

Hint for Q4: Divide both amounts by the second number. The answer includes both units separated by (/).

Example 11 Finding average rates

Find the average rate of change in each situation.

a 15 000 revolutions in 5 minutes

b 30 minutes to run 6 km

Solution	Explanation
a Average rate = 15 000 revs/5 mins = 3000 revs/min	Divide both quantities by the second amount. $15\,000 \div 5 = 3000$ revs for 1 minute on average.
b Average rate = 30 minutes/6 km = 5 minutes/km	$30 \div 6 = 5$ minutes for 1 km on average. Include both units separated by a /.

Now you try

Find the average rate of change in each situation.

a 180 km covered in 3 hours

b 240 pages read in 8 days

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5 Find the average rate of change for each situation.

- a Relma drove 6000 kilometres in 20 days.
- b Holly saved \$420 over 3 years.
- c A cricket team scored 78 runs in 12 overs.
- d Saskia grew 120 centimetres in 16 years.
- e Russell gained 6 kilograms in 4 years.
- f The temperature dropped 5°C in 2 hours.

Hint for Q5: In your answer, write the units in the same order as the question.

Example 12 Finding average rates over a period

Tom was 120 cm tall when he turned 10 years old. He was 185 cm tall when he turned 20 years old. Find Tom's average rate of growth per year between 10 and 20 years of age.

Solution	Explanation
Average rate = 65 cm/10 years = 6.5 cm/year	Growth = 185 − 120 = 65 cm Divide both numbers by 10.

Now you try

At 1 year old, Pam the cat was 2 kg and at 9 years old, Pam was 4 kg. Find Pam's average weight gain over this period of time.

6
- a Liam was 150 cm tall at 10 years old and 188 cm tall when 20 years old. Find Liam's average rate of growth per year between 10 and 20 years of age.
- b Brittany was 140 cm tall at 10 years old and 164 cm at 18 years old. Find Brittany's average rate of growth per year between 10 and 18 years of age.

Problem-solving and reasoning 7–9 9–12

7 A dripping tap filled a 9 litre bucket in 3 hours.

- a What was the dripping rate of the tap in litres/hour?
- b How long would it take the tap to fill a 21 litre bucket?

Hint for Q7:
? litres in 1 hour
×? 21 litres in ? hours ×?

8 Martine grew at an average rate of 6 cm/year for the first 18 years of her life. If Martine was 50 cm long when she was born, how tall was Martine when she turned 18?

Hint for Q8:
6 cm in 1 year
×? ? cm in 18 years ×?

9 If 30 salad rolls were bought to feed 20 people at a picnic and the total cost was \$120, find the following rates.

- a Salad rolls/person
- b Cost/person
- c Cost/roll

10 Harvey finished a 10 kilometre race in 37 minutes and 30 seconds.
Jacques finished a 16 kilometre race in 53 minutes and 20 seconds.
Calculate the running rate of each runner in min/km.
Which runner had a faster running pace?

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11 The Tungamah Football Club had 12 000 members. After five successful years they now have 18 000 members.

a What has been the average rate of membership growth per year for the past 5 years?

b If this membership growth rate continues, how many more years will it take for the club to have 32 400 members?

12 **a** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:

i km/L

ii L/km (give answer as a fraction)

b How can you convert km/L to L/km?

Hint for Q12: Start with: 216 km uses 24 litres.

Target 155 — 13

13 In Victoria, due to repeated drought experiences, the state government has urged all residents to save water. The goal was set for each person to use no more than 155 litres of water per day.

a How many people live in your household?

b According to the Victorian government, how many litres of water can your household use per day?

If you live in a different state of Australia, find the target volume of water use per person for your state and determine how many litres of water your household can use each day.
Use the following rates of water flow for the questions below.

Shower rate (10 L/min)	Washing machine (100 L/load)
Hose (24 L/min)	Toilet (4.5 L/half flush)
Running tap (16 L/min)	Drinking water (3 L/day)
Dishwasher (20 L/wash)	Water for food preparation (15 L/day)

Hint for Q13b: Draw a table.

c Estimate the average daily rate of water usage for your household.

d Ask your parents for a recent water bill and find out what your family household water usage rate was for the past three months.

e What is the rate at which your family is charged for its water?

Before the water saving plan, Victorians were using an average of 164 litres/day/person. Twelve months later, Victorians were using 151 litres/day/person.

f How much water per year for the state of Victoria does this saving of 13 litres/day/person represent?

Hint for Q13f: The population of Victoria is about 6 million.

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6F Speed and applications of other rates

Learning intentions

- To understand that rates can be used to model many situations.
- To be able to solve problems involving rates.
- To understand that speed is a rate relating distance and time.
- To be able to find an average speed (given a distance and the time taken).
- To be able to find the distance travelled (given an average speed and the time taken).
- To be able to find the time taken (given an average speed and the distance).

Key vocabulary: rate, speed, constant speed, average speed, distance, time

We are interested in how things change over a period of time. A rate that we come across almost every day is speed. Speed is the rate of distance travelled per unit of time.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

A snail can move at 1 m/hour, an Olympic sprint runner can run at a speed of 10 m/second and the Earth travels around the Sun at a speed of around 30 km/second.

- How far does the Earth travel in an hour?
- Can you estimate the speed of a passenger jet in m/second?

Lesson starter: Racing rates

Work with a partner and help each other to calculate these rates.

One day while at a school camp, students completed an adventure race. Students kayaked down a river, cycled along a country road and finally jogged back to camp.

Piper kayaked 16 km in 2 hours.

- On average, at what speed was Piper kayaking in km/h?

In a 2 minute period, Piper counted 80 paddle strokes.

- What rate was Piper paddling in paddle strokes/minute?

Summer cycled 36 km in 2 hours.

- On average, what speed was Summer cycling in km/h?

Cycling uphill, Summer counted 150 pedal turns in 3 minutes.

- What was Summer's pedalling rate in pedal turns/minute?
- At this rate, how many pedal turns would Summer make in 7 minutes of uphill cycling?

Luca jogged the final section of the race at a speed of 200 m/minute.

- At this rate, how long did it take Luca to run 1000 m?
- At this rate, how far would Luca run in 60 minutes?
- What speed did Luca jog at in km/h?
- Running at this speed, how long did it take Luca to complete the 3 km run back to the camp?

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Key ideas

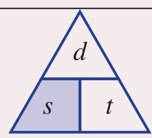
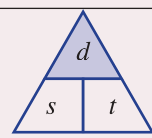
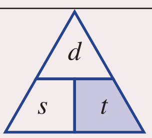
- When a **rate** is provided, a change in one quantity implies that an equivalent change must occur in the other quantity.
  e.g. Patrick earns \$20/hour. How much will he earn in 6 hours?

$$\times 6 \left(\begin{array}{c}\$20 \text{ for 1 hour}\\ \$120 \text{ for 6 hours}\end{array}\right) \times 6$$

  e.g. Patrick earns \$20/hour. How long will it take him to earn \$60?

$$\times 3 \left(\begin{array}{c}\$20 \text{ in 1 hour}\\ \$60 \text{ in 3 hours}\end{array}\right) \times 3$$

- **Speed** is a measure of how fast an object is travelling.
  - If the speed of an object does not change over **time**, the object is travelling at a **constant speed**. 'Cruise control' helps a car travel at a constant speed.
  - When speed is not constant, due to acceleration or deceleration, we are often interested to know the **average speed** of the object.
  - Average speed is calculated by the formula:
    $$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \quad \text{or} \quad s = \frac{d}{t}$$
  - Depending on the unknown value, the above formula can be rearranged to make d or t the subject. The three formulas involving s, d, and t are:

$s = \dfrac{d}{t}$	$d = s \times t$	$t = \dfrac{d}{s}$
		

  - Care must be taken with units for speed, and on occasions units will need to be converted. The most common units of speed are m/s and km/h.

Exercise 6F

Understanding

1–4 | 3, 4

1 Fill in the gaps.

a $$\times 3 \left(\begin{array}{c}\text{60 km in 1 hour}\\ \text{180 km in } \underline{\quad} \text{ hours}\end{array}\right) \times 3$$

b $$\times 5 \left(\begin{array}{c}\$25 \text{ in 1 hour}\\ \$125 \text{ in } \underline{\quad} \text{ hours}\end{array}\right) \underline{\quad}$$

c $$\underline{\quad} \left(\begin{array}{c}\text{7 questions in 3 minutes}\\ \text{70 questions in } \underline{\quad} \text{ minutes}\end{array}\right) \underline{\quad}$$

d $$\underline{\quad} \left(\begin{array}{c}\text{120 litres in 1 minute}\\ \underline{\quad} \text{ litres in 6 minutes}\end{array}\right) \underline{\quad}$$

2 Fill in the gaps.

a $$\begin{array}{c} \div 3 \left(\begin{array}{c}\$36 \text{ for 3 hours}\\ \underline{\quad} \text{ for 1 hour}\end{array}\right) \div 3 \\ \times 5 \left(\begin{array}{c}\underline{\quad} \text{ for 1 hour}\\ \underline{\quad} \text{ for 5 hours}\end{array}\right) \underline{\quad} \end{array}$$

b $$\begin{array}{c} \underline{\quad} \left(\begin{array}{c}\text{150 rotations in 5 minutes}\\ \underline{\qquad} \text{ in 1 minute}\end{array}\right) \underline{\quad} \\ \underline{\quad} \left(\begin{array}{c}\underline{\qquad} \text{ in 1 minute}\\ \underline{\qquad} \text{ in 7 minutes}\end{array}\right) \underline{\quad} \end{array}$$

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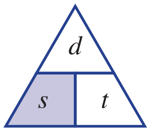
6F

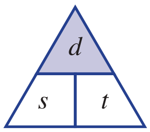
3 Copy and complete by writing in the missing words.

a



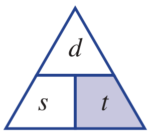
speed = $\dfrac{?}{\text{time}}$

b



distance = ? × time

c



time = $\dfrac{?}{?}$

Hint for Q3: Use the triangles to help work out each rule.

4 Which of the following is not a unit of speed?

A m/s **B** km/h **C** cm/h
D L/kg **E** m/min

Hint for Q4: Units of speed have a length unit and a time unit.

Fluency

5, 6, 7–9(½) | 6, 7–9(½)

Example 13 Solving rate problems

a Rachael can touch type at 74 words/minute. How many words can she type in 15 minutes?
b Leanne works in a donut van and sells on average 60 donuts every 15 minutes. How long is it likely to take her to sell 800 donuts?

Solution	Explanation
a ×15 (74 words in 1 minute / <u>1110 words</u> in 15 minutes) ×15	Calculate $74 \times 15 = 1110$
Rachael can type 1110 words in 15 minutes.	
b ÷15 (60 donuts in 15 minutes / 4 donuts in 1 minute) ÷15 ×200 (4 donuts in 1 minute / 800 donuts in <u>200 minutes</u>) ×200	Selling rate = 60 donuts/15 minutes. Divide both quantities by 15. Multiply both quantities by 200. Convert answer to hours and minutes.
Leanne is likely to take 3 hours and 20 minutes to sell 800 donuts.	

Now you try

a A car factory produces 8 cars per day. How many cars can it produce in a 5-day working week?
b On average, Leo can run 100 m every 10 seconds. How far can Leo run in 5 minutes?

5 **a** Lewis can touch type at 80 words/minute. How many words can he type in 20 minutes?
b Robbie works at a bakery and, on average, he sells 4 loaves of bread every 10 minutes. How long will it take him to sell 20 loaves of bread?

6 A factory produces 40 plastic bottles/minute.
a How many bottles can the factory produce in 60 minutes?
b How many bottles can the factory produce in an 8 hour day of operation?

Hint for Q6: $\square$ bottles in 1 hour. $\square$ bottles in 8 hours.

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Example 14 Finding average speed

Find the average speed in km/h of:

a a cyclist who travels 140 km in 5 hours.
b a runner who travels 3 km in 15 minutes.

Solution	Explanation
a $s = \dfrac{d}{t}$	The unknown value is speed. Write the formula for speed.
$= \dfrac{140\text{ km}}{5\text{ h}}$	Distance travelled = 140 km. Time taken = 5 h. Calculate 140 ÷ 5.
$= 28\text{ km/h}$	Speed unit is km/h.
Alternative unitary method $\begin{array}{ccc} & 140\text{ km in 5 hours} & \\ \div 5 & & \div 5 \\ & 28\text{ km in 1 hour} & \end{array}$ Average speed = 28 km/h	Write down the rate provided in the question. Divide both quantities by 5.
b $s = \dfrac{d}{t}$ $= \dfrac{3\text{ km}}{15\text{ min}}$ $= \dfrac{1}{5}\text{ km/min} \quad = 12\text{ km/h}$	Distance travelled = 3 km divided by the time taken of 15 minutes. $\begin{array}{ccc} & \frac{1}{5}\text{ km in 1 minute} & \\ \times 60 & & \times 60 \\ & 12\text{ km in 60 minutes} & \end{array}$
Alternative unitary method $\begin{array}{ccc} & 3\text{ km in 15 minutes} & \\ \times 4 & & \times 4 \\ & 12\text{ km in 60 minutes} & \end{array}$ Average speed = 12 km/h	Write down the rate provided in the question. $15 \times 4 = 60$ minutes = 1 hour. Multiply both quantities by 4.

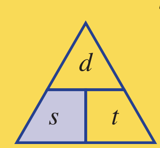
Now you try

Find the average speed in m/s of:

a a walker who travels 7.2 m in 6 seconds.
b a cyclist who travels 300 m in 1 minute.

7 Find the average speed of:
a a sprinter running 200 m in 20 seconds (in m/s)
b a skateboarder travelling 840 m in 120 seconds (in m/s)
c a car travelling 180 km in 3 hours (in km/h)
d a truck travelling 400 km in 8 hours (in km/h)
e a train travelling 60 km in 30 minutes (in km/min and km/h)
f a tram travelling 15 km in 20 minutes (in km/min and km/h)

Hint for Q7: $s = \dfrac{d}{t}$



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6F

Example 15 Finding the distance travelled

Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.

Solution

$d = s \times t$

$= 95 \text{ km/h} \times 15 \text{ h}$

$= 1425 \text{ km}$

Explanation

The unknown value is distance.
Write the formula for distance.
Distance unit is km.

Alternative unitary method

$$\times 15 \left(\begin{array}{c} 95 \text{ km in } 1 \text{ hour} \\ 1425 \text{ km in } 15 \text{ hours} \end{array} \right) \times 15$$

Truck travels 1425 km in 15 hours.

Write the rate provided in the question.
Multiply both quantities by 15.

Now you try

Find the distance travelled by a car travelling for 6 hours at an average speed of 90 km/h.

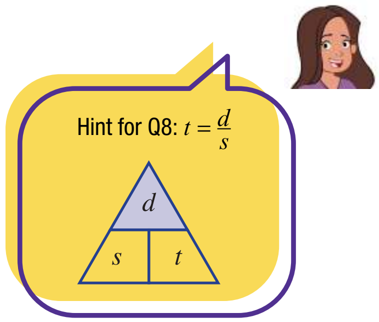
8 Find the distance travelled by:

a a cyclist travelling at 12 m/s for 90 seconds

b an ant travelling at 2.5 cm/s for 3 minutes

c a bushwalker who has walked for 8 hours at an average speed of 4.5 km/h

d a tractor ploughing fields for 2.5 hours at an average speed of 20 km/h



Example 16 Finding the time taken

Find the time taken for a hiker walking at 4 km/h to travel 15 km.

Solution

$t = \dfrac{d}{s}$

$= \dfrac{15 \text{ km}}{4 \text{ km/h}}$

$= 3.75 \text{ h}$

$= 3 \text{ h } 45 \text{ min}$

Explanation

The unknown value is time.
Write the formula with t as the subject.
The time unit is h. Leave answer as a decimal or convert to hours and minutes.
$0.75 \text{ h} = 0.75 \times 60 = 45 \text{ min}$

Alternative unitary method

$$\begin{array}{rcl} & 4 \text{ km in } 1 \text{ hour} & \\ \div 4 & & \div 4 \\ & 1 \text{ km in } \frac{1}{4} \text{ hour} & \\ \times 15 & & \times 15 \\ & 15 \text{ km in } \frac{15}{4} \text{ hours} & \end{array}$$

It takes 3 h 45 min to travel 15 km.

Express the rate as provided in the question.
Divide both quantities by 4.
Multiply both quantities by 15.

Now you try

Find the time taken for a jogger to travel 10 km at 8 km/h.

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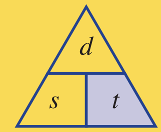
9 Find the time taken by:
- **a** a sports car to travel 1200 km at an average speed of 150 km/h
- **b** a bus to travel 14 km at an average speed of 28 km/h
- **c** a plane to fly 6900 km at a constant speed of 600 km/h
- **d** a ball moving through the air at a speed of 12 m/s to travel 84 m

Hint for Q9: $t = \frac{d}{s}$



Problem-solving and reasoning

10, 11	11–13

10 Putra is an elite rower. When training, his goal is a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.
- **a** How many times does Putra's heart beat during a 30 minute workout?
- **b** How many times does Putra's heart beat during 30 minutes of 'rest'?
- **c** If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

Hint for Q10: Putra is training when he has a workout.

11 A plane is flying at a cruising speed of 900 km/h. How far will the plane travel from 11:15 am to 1:30 pm on the same day?

12 The wheels on Charlie's bike have a circumference of 1.5 m. When Charlie is riding fastest, the wheels turn at a speed of five turns per second.
- **a** What is the fastest speed Charlie can ride his bike, in km/h?
- **b** How far would Charlie travel in 5 minutes at his fastest speed?

13 The Ghan train is an Australian icon. You can board the Ghan in Adelaide and 2979 km later, after travelling via Alice Springs, you arrive in Darwin. For these questions, round the answers correct to one decimal place.
- **a** If you board the Ghan in Adelaide on Sunday at 2:20 pm and arrive in Darwin on Tuesday at 5:30 pm, what is the average speed of the train journey?
- **b** There are two major rest breaks. The train stops for $4\frac{1}{4}$ hours at Alice Springs and 4 hours at Katherine.

  Taking these breaks into account, what is the average speed of the train when it is moving?

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6F

Speed research — 14

14 Carry out research to find answers to the following questions.

Light and sound

a What is the speed of sound in m/s?

b What is the speed of light in m/s?

c How long would it take sound to travel 100 m?

d How long would it take light to travel 100 km?

e How many times quicker is the speed of light than the speed of sound?

f What is a Mach number?

Spacecraft

g What is the escape velocity needed by a spacecraft to 'break free' of Earth's gravitational pull? Give this answer in km/h and also km/s.

h What is the orbital speed of planet Earth around the Sun? Give your answer in km/h and km/s.

i What is the average speed of a space shuttle on a journey from Earth to the International Space Station?

Knots

Wind speed and boat speed are often given in terms of knots (kt).

j What does a knot stand for?

k What is the link between nautical miles and a system of locating positions on Earth?

l How do you convert a speed in knots to a speed in km/h?

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Development officer for a fragrance company

Many small businesses are taking advantage of the increasing popularity of the wellbeing industry and the desire for natural products. A group of interested people can form a business producing and distributing fragrance products such as perfumes, candles, soaps, body and incense oils and various lotions. The fragrances can be mixed at home and the products sold at community markets and online. Such businesses are taking an increasing share of profits in the global economy.

Understanding and applying the science and mathematics of mixing oils and fragrances is vital to the success of such businesses. To be a qualified development officer in this field you need to have a good sense of smell, a detailed knowledge of fragrances and oils and the skills to safely mix chemicals in the correct ratios.

Note to the Teacher: This activity is intended to get students thinking about ratios and rates in a practical context, but is not intended as a practical activity. However, these are real recipes and the creation of these perfumes could be done in class in a safe laboratory environment.

Maths@Work

1 To make a perfume for a summer candle, 1 part coconut is mixed with 3 parts vanilla. How much vanilla is needed for:
- **a** 10 mL of coconut?
- **b** 15 mL of coconut?
- **c** 240 mL of mixture?

2 To make Apple Jack essence, mix 1 drop of apple fragrance with 1 drop of cinnamon fragrance to 2 drops of grapefruit fragrance.
- **a** How many drops of grapefruit fragrance are needed for:
  - **i** 2 drops of apple?
  - **ii** 5 drops of cinnamon?
- **b** If 10 mL of apple fragrance is used, how much cinnamon is needed?

3 To make a Charleston candle fragrance, 10 parts sandalwood fragrance are mixed with 2 parts cinnamon fragrance.
- **a** Write this as a simplified ratio.
- **b** How much cinnamon is needed for:
  - **i** 10 drops of sandalwood
  - **ii** 25 drops of sandalwood
  - **iii** 120 drops of scent.

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4 A 'Quiet time' candle uses a perfume of chamomile and spearmint mixed in the ratio of 1 to 3.
  a How many grams of chamomile are needed for:
    i 6 grams of spearmint
    ii 12 grams of spearmint
    iii 30 grams of spearmint
    iv total 240 grams of fragrance
  b How many grams of spearmint are needed for:
    i 20 grams of chamomile
    ii 100 grams of chamomile
    iii total 360 grams of fragrance

5 In a certain perfume, 3 drops of cedarwood are mixed with 10 drops of lavender to 5 drops of bergamot. These are then mixed with alcohol to form the final product.
  a How many drops of lavender are needed for:
    i 15 drops of bergamot?
    ii 12 drops of cedarwood?
    iii 15 drops of cedarwood?
  b How many drops of cedarwood are needed for 40 drops of lavender?
  c If one drop is approximately 0.065 mL, how many millilitres are in:
    i 10 drops of lavender?
    ii 20 drops of cedarwood?
    iii 24 drops of bergamot?
  d 360 drops of fragrance are used in total. How many drops of each of the three individual scents are used?

6 A seaweed face mask lotion can be made from 3 parts seaweed powder, 6 parts sweet almond oil (or jojoba oil), 1 part aloe vera gel and 1 part honey.
  a Given that 1 tablespoon is equal in volume to 3 teaspoons, rewrite the recipe above using spoon measurements.
  b Josie grinds 2 sheets of dried seaweed and that amount fills 2 tablespoons. State the spoon quantities of the remaining ingredients that Josie needs to make her face mask if she uses jojoba oil.

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Using technology

An Essential Oils Development Officer would instruct pupils about safety. For example, undiluted essential oils or fragrances must never be put on skin or near eyes, never swallowed and always kept away from children. An important mathematical method to teach is the procedure for mixing Essential oils with Carrier oils to achieve the recommended 1% or 2% dilution.

7 When diluting Essential oils or fragrances with Carrier oils the following rates apply:
- For 1% dilution of Essential oil use 1 drop/5 mL of Carrier oil
- For 2% dilution of Essential oil use 2 drops/5 mL of Carrier oil

a Set up the Excel spreadsheet shown below and enter formulas to calculate the number of drops needed for these dilutions.

	A	B	C	D	E
1	**Dilution of Essential oils**				
2	**Carrier oil**	**Volume of Carrier oil in mL**	**Dilution %**	**Essential oil**	**Drops of Essential oil**
3	Sweet Almond oil	10	1	Chamomile	
4	Sweet Almond oil	10	2	Lemon	
5	Coconut oil	10	1	Rosewood	
6	Jojoba oil	5	1	Rosemary	
7	Jojoba oil	25	2	Lavender	
8	Grapeseed oil	20	2	Rosemary	

b Use more rows in your spreadsheet to find the number of drops of Essential oil needed to make:
- 1% dilution of Peppermint Essential oil with 25 mL of Sweet Almond oil
- 2% dilution of Geranium Essential oil with 20 mL of Coconut oil
- 1% dilution of Lavender Essential oil with 10 mL of Jojoba oil
- 2% dilution of Lemon Grass Essential oil with 15 mL of Sunflower oil

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Modelling

Ethanol fuel mix

Abbey is planning to make a 2000 km trip from Brisbane to Melbourne. A local fuel retailer advises her that it might be cheaper to buy one of their fuel mixes that contain both petrol and ethanol. The currently available types with their ratios, costs and projected fuel economy for Abbey's car are shown below.

Type	Petrol-ethanol ratio	Fuel economy	Price
E20	4 : 1	9 L/100 km	\$1.30/L
E10	9 : 1	8 L/100 km	\$1.45/L
Petrol	N/A (100% petrol)	7.5 L/100 km	\$1.60/L

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

1 Preliminary task

a How much does Abbey spend if she buys:
  i 60 litres of petrol? **ii** 65 litres of E10? **iii** 67.5 litres of E20?

b The fuel economy for petrol is 7.5 L/100 km. How far can Abbey travel using 60 litres of petrol?

c How far can Abbey travel if she purchases fuel according the different options from part **a**?

d The E10 fuel has a petrol-ethanol ratio of 9 : 1. Divide 65 litres in this ratio to find the amount of ethanol in this mix.

e Determine the amount of ethanol purchased if Abbey buys 67.5 litres of the E20 mix.

2 Modelling task

Formulate

a The problem is to determine the minimum cost to spend on fuel for her 2000 km trip from Brisbane to Melbourne by considering the different fuel options. Write down all the relevant information that will help solve this problem.

Solve

b Determine the total amount of fuel Abbey needs to purchase for the trip if she uses:
  i petrol fuel **ii** E10 fuel **iii** E20 fuel.

c Determine the total cost of purchasing the following fuel for the entire trip.
  i petrol fuel **ii** E10 fuel **iii** E20 fuel.

d Determine the total saving if Abbey purchases:
  i E20 instead of petrol fuel **ii** E10 instead of petrol fuel.

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e Abbey thinks that she can buy petrol for the trip at an average price of \$1.55/L. Will this mean that petrol is the cheapest option? Justify your response.

f By hunting around Abbey can find a better price for E20 for the 2000 km trip. At what price should Abbey purchase E20 to make the overall cost less than the overall cost of purchasing E10?

Evaluate and verify

g Summarise your results and describe any key findings.

Communicate

3 Extension questions

A friend of Abbey's warned her against ethanol-type fuels and said that for each litre of ethanol consumed by the car, it would add a wear and tear cost of 50 cents.

a Determine the amount of ethanol consumed by Abbey's car for the 2000 km trip if E10 is used and also if E20 is used.

b Does this extra "wear and tear" cost make the petrol option the cheapest for the 2000 km trip?

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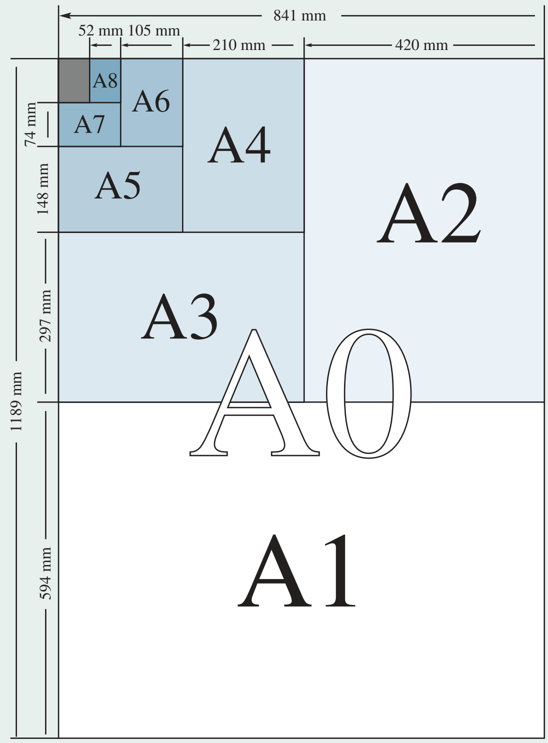
Technology and computational thinking

Paper size ratios

Key technology: Spreadsheets and programming

The standard international paper size is called ISO 216 and consists of 11 sizes from A0 through to A10. Most of the time we use A4 in our printers and exercise books. The ratio of the length of any standard size paper to its width is constant regardless of the size of paper. A0 to A8 is shown here with measurements rounded to the nearest mm.



1 Getting started

First consider these facts for standard paper.

1. The total area of A0 is $1\,\text{m}^2$.
2. The length of A0 is $\sqrt{\sqrt{2}}$ m.
3. The width of A0 is $\dfrac{1}{\sqrt{\sqrt{2}}}$ m.
4. The width of all paper sizes is the length divided by $\sqrt{2}$.

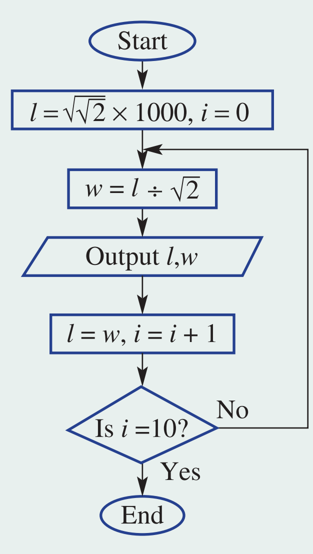
   a Use the above facts to show the following, rounded to the nearest mm.
   - **i** The length of A0 is 1189 mm.
   - **ii** The width of A0 is 841 mm.

   b Note for example that the width of A3 is equal to the length of A4. Use this fact and fact 4 above to find the lengths and widths of the following paper sizes. Check that your results agree with the figures in the diagram above.
   - **i** A1
   - **ii** A2
   - **iii** A3
   - **iv** A4

   c Use fact 4 to state the ratio *length* : *width*.

2 Applying an algorithm

This flow chart is designed to determine the length and width of the standard paper sizes A0 through A10 in millimetres.



a Run through the algorithm for at least five passes and enter the output in the following table. Round to the nearest mm.

i	0	1	2	3	4
l					
w					

b As we have stated, the width of one piece of paper becomes the length of the paper on the next size down. Which line in the algorithm updates the next length with the old width?

c Explain why the flow chart contains the line $w = l \div \sqrt{2}$.

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3 Using technology

A spreadsheet can be used to execute the algorithm on the previous page.

	A	B	C	D
1	**Paper ratios**			
2				
3	**A size**	**Length (mm)**	**Width (mm)**	**Area (sq. metres)**
4				
5	0	=SQRT(SQRT(2))*1000	=1/(SQRT(SQRT(2)))*1000	=B5*C5/1000000
6	=A5+1	=C5	=B6/SQRT(2)	
7				

a Create a spreadsheet to generate the lengths and widths of standard paper sizes by entering the given information.

b Fill down at cells A6, B6, C6 and D5 to produce the information for paper sizes A0 through to A10. Check your results with parts **1** and **2** on the previous page.

c The total area of A0 is $1\ \text{m}^2$. Use the formula =SUM(D6:D15) to find the total sum of A1 through to A10. Can you explain why the sum is not equal to 1 when we know that the total area of A0 is $1\ \text{m}^2$? (*Hint:* Look at the original diagram showing how an A0 page is divided to form all the other sizes. What do you notice about the remainder each time the next size down is created?)

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Puzzles and games

1 Write these ratios in simplest form to solve the riddles below.

A 4 : 8	**C** 4 : 16	**E** 6 : 10	**F** 4 : 12	**H** 8 : 12
I 20 : 16	**K** 10 : 4	**L** 12 : 3	**O** 9 : 6	**P** 15 : 5
R 25 : 15	**S** 20 : 10	**T** 35 : 25	**V** 2 : 12	

a What do termites eat for dessert?

7 : 5	3 : 2	3 : 2	7 : 5	2 : 3	3 : 1	5 : 4	1 : 4	5 : 2	2 : 1

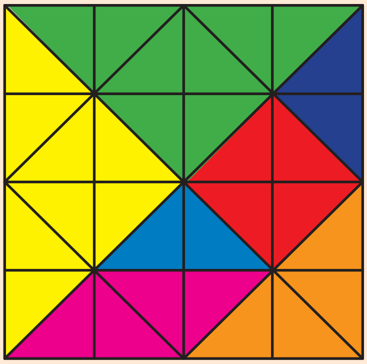
b Where do geologists go to have a good time?

7 : 5	3 : 2

5 : 3	3 : 2	1 : 4	5 : 2

1 : 3	3 : 5	2 : 1	7 : 5	5 : 4	1 : 6	1 : 2	4 : 1	2 : 1

2 The ancient Chinese puzzle known as a tangram consists of 7 geometric shapes (tans) as shown.

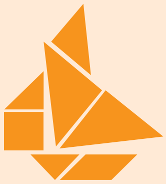


a Write the ratio of the areas of the seven shapes in this tangram. Write each of the ratios in simplest form in ascending order.

b The pieces (tans) of a tangram can be arranged to make many creative shapes and designs. For the shapes shown here, find:

i the ratio of the yacht's sails to the boat hull.

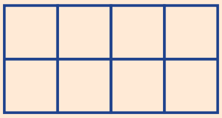
ii the ratio of the cat's head to the rest of the body.



3 Hannah is 14 years old and her brother Blake is 9 years old. Find their ages when the ratio of Hannah's age to Blake's age is:

a 3 : 2 **b** 5 : 4 **c** 11 : 10

4 This diagram is made up of 8 equal-sized squares.



How many squares need to be shaded if the ratio of shaded squares to unshaded squares is:

a 1 : 3 **b** 2 : 3 **c** 1 : 2

Give each answer as a mixed fraction if necessary.

5 Bottle A has 1 L of cordial drink with a cordial to water ratio of 3 : 7. Bottle B has 1 L of cordial drink with a cordial to water ratio of 1 : 4. The drink from both bottles is combined to form a 2 L drink. What is the new cordial to water ratio?

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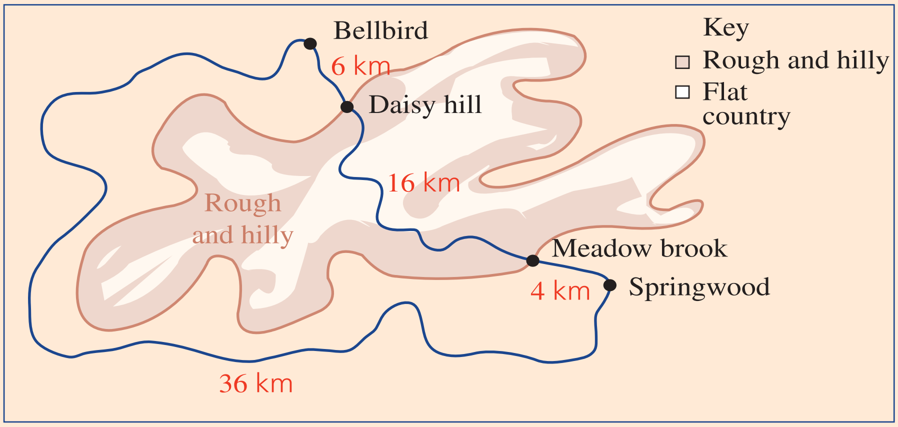
6 A group of cyclists decide to have a race from Springwood to Bellbird. The towns and distances are shown on the sketch map below. Over flat country a cyclist averages 20 km/h but through rough and hilly country the average is 12 km/h. Which route would be fastest and by how much?



7 Brothers Marco and Matthew start riding from home into town, which is 30 km away. Marco rode at 10 km/h and Matthew took 20 minutes longer to complete the trip. Assuming that they both rode at a constant speed, how fast was Matthew riding?

8 Solve the questions below to find the answer to the riddle:
Why did the monkey put a steak under himself?

5 m 4 m 8 m 2 m 1 : 8000 1 : 4000 4 m 70 cm 4 m

1 : 3000 70 cm 1 m 1 : 8000 90 cm 70 cm 1 : 3000

70 cm 4 m 1 : 80 2 m 1 : 4000 2 m

90 cm 1 : 500 10 cm 25 cm 25 cm 4 m 1 : 500

1 : 3	3 : 5	2 : 1	7 : 5	5 : 4	1 : 6	1 : 2	4 : 1	2 : 1

If the scale is 1 : 100, find the real length in metres shown by:
a 2 cm **b** 5 cm **c** 8 cm **d** 6 cm **e** 4 cm

If the scale is 1 : 10, find the real length shown by:
f 3 cm **g** 9 cm **h** 7 cm **i** 1 cm **j** 15 cm

If the scale is 1 : 5, find the real length shown by:
k 3 cm **l** 5 cm **m** 10 cm **n** 30 cm **o** 20 cm

Write each scale in the simplest ratio form:
p 1 cm to 2 m **q** 1 cm to 10 m **r** 1 cm to 5 m **s** 1 m to 4 km **t** 1 m to 3 km
u 1 m to 8 km **v** 1 mm to 3 cm **w** 1 mm to 8 cm **x** 1 mm to 15 cm **y** 1 mm to 6 cm
z 1 mm to 2 cm

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Chapter summary

RATIOS

Ratios are written in
- simplest form
- with whole numbers

Simplest form

Eliminate decimals
$0.8 : 1.2$
$\times 10$ $8 : 12$ $\times 10$

HCF 4
$\div 4$ $2 : 3$ $\div 4$

Same units
25 minutes : $1\frac{1}{4}$ hours
$\div 25$ $25 : 75$ $\div 25$
$1 : 3$

Equivalent ratios
$2 : 3$
$\times 4$ $8 : 12$ $\times 4$
$\div 2$ $4 : 6$ $\div 2$

$a : b$
a is to b
a to b

Comparison of two quantities of the
- same type and
- same unit

Dividing a quantity in a given ratio

Divide \$180 in the ratio 4 : 5

Unitary method

Divide \$180 in the ratio 4 : 5

Total number of parts	$4 + 5 = 9$
One part	$\div 9$: 9 parts = \$180, 1 part = \$20 ($\div 9$); $\times 4$: 4 parts = \$80 ($\times 4$); $\times 5$: 1 part = \$20, 5 parts = \$100 ($\times 5$)
Sentence answer	\$180 divided in the ratio 4 : 5 is \$80 and \$100

Finding a quantity from a ratio

Cows to horses in ratio 5 : 2
There are 8 horses, how many cows?

Unitary method
cows : horses
5 : 2
2 parts = 8 horses
1 part = 4 horses
5 parts = 20 cows

Equivalent ratio method
C : H = 5 : 2
C : H = $\times 4$ 5 : 2 $\times 4$
= 20 : 8
There are 20 cows.

Scale ratios

Drawing length : Actual length
= 1 : scale factor

Actual size from model
Model car 17 cm long
Scale 1 : 25
Scale factor = 25
Actual car = 25 × 17 cm
= 425 cm ← same units
= 4.25 m

Length units
km
$\times 1000$ $\div 1000$
m
$\times 100$ $\div 100$
cm
$\times 10$ $\div 10$
mm

RATES

Average rates
720 km driven in 10 hours
Average speed = $\frac{720}{10}$
= 72 km/h

s : speed
d : distance
t : time

$s = \frac{d}{t}$

$d = s \times t$

$t = \frac{d}{s}$

Simplified rates
- 25 km/h 25 km per one hour
- \$12/kg \$12 per one kg

Comparing two quantities with different units

Unitary method with rates
$\div 5$ \$60 for 5 hours $\div 5$
\$12 for 1 hour
$\times 40$ \$480 for 40 hours $\times 40$

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Chapter checklist

Chapter checklist: Success criteria

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
6A	**1 I can write a ratio from a description** e.g. A sample of mixed nuts contains 5 cashews and 12 peanuts. Write down the ratio of: **a** cashews to peanuts **b** cashews to the total number of nuts.	
6A	**2 I can produce a ratio that is equivalent to a given ratio** e.g. State the missing number in the equivalence $30 : 15 = ? : 5$.	
6B	**3 I can simplify ratios involving whole numbers** e.g. Simplify $450 : 200$.	
6B	**4 I can write simplified ratios involving quantities by first converting units** e.g. Write the relationship "25 minutes to 2 hours" as a ratio by first changing the quantities to the same unit.	
6C	**5 I can divide a quantity in a ratio with two components** e.g. Divide 54 m in a ratio of $4 : 5$.	
6C	**6 I can divide a quantity in a ratio with three components** e.g. Divide $\$300$ in the ratio of $2 : 1 : 3$.	
6C	**7 I can find a total quantity from a given ratio and the actual size of one component** e.g. The ratio of boys to girls at Birdsville College is $2 : 3$. If there are 246 boys at the school, how many students attend Birdsville College?	
6D	**8 I can convert from scale distance to actual distance using a scale** e.g. A map has a scale of $1 : 20\,000$. Find the actual distance for a scale distance of 5 mm. Answer in metres.	
6D	**9 I can convert from actual distance to scale distance using a scale** e.g. A model boat has a scale of $1 : 500$. Find the scaled length for an actual length of 50 m. Answer in millimetres.	
6D	**10 I can determine the scale factor** e.g. Determine the scale factor if 4 mm on a scale drawing represents an actual distance of 50 cm.	
6E	**11 I can write simplified rates** e.g. Express $\$28$ for 4 kilograms as a simplified rate.	
6E	**12 I can find average rates** e.g. Find the average rate of change for $15\,000$ revolutions in 5 minutes.	

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Chapter checklist

		✔
6E	**13** **I can find average rates in harder problems** e.g. Tom was 120 cm tall when he turned 10 years old, and 185 cm when he turned 20 years old. Find Tom's average rate of growth per year over this period.	
6F	**14** **I can solve rate problems** e.g. Rachael can type at 74 words/minute. How many words can she type in 15 minutes?	
6F	**15** **I can find an average speed** e.g. Find the average speed in km/h of a runner who travels 3 km in 15 minutes.	
6F	**16** **I can find the distance travelled** e.g. Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.	
6F	**17** **I can find the time taken** e.g. Find the time taken for a hiker walking at 4 km/h to travel 15 km.	

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Short-answer questions

6A **1** In Lao's pencil case there are 6 coloured pencils, 2 black pens, 1 red pen and 3 lead pencils. Find the ratio of:

- **a** lead pencils to coloured pencils
- **b** black pens to red pens
- **c** all pens to all pencils

6B **2** True (T) or false (F)?

- **a** $1 : 4 = 3 : 6$.
- **b** The ratio $2 : 3$ is the same as $3 : 2$.
- **c** The ratio $3 : 5$ is written in simplest form.
- **d** 40 cm : 1 m is written as $40 : 1$ in simplest form.

6B **3** Copy and complete.

- **a** $4 : 50 = 2 : \square$
- **b** $3 : 7 = \square : 21$
- **c** $\square : 12 = 8 : 3$
- **d** $1 : \square : 5 = 5 : 15 : 25$

6B **4** Simplify the following ratios.

- **a** $10 : 40$
- **b** $36 : 24$
- **c** $75 : 100$
- **d** $8 : 64$
- **e** $27 : 9$
- **f** $5 : 25$
- **g** $6 : 4$
- **h** $52 : 26$
- **i** $6 : 9$
- **j** $8 : 4 : 20$

6B **5** Simplify the following ratios by first changing to the same units.

- **a** 2 cm : 8 mm
- **b** 5 mm : 1.5 cm
- **c** 3 L : 7500 mL
- **d** 30 min : 1 h
- **e** 400 kg : 2 tonnes
- **f** 6 h : 1 day
- **g** 120 m : 1 km
- **h** 45 min : $2\frac{1}{2}$ h

6C **6** Divide:

- **a** \$80 in the ratio $7 : 9$
- **b** 200 kg in the ratio $4 : 1$
- **c** 40 m in the ratio $6 : 2$
- **d** \$1445 in the ratio $4 : 7 : 6$
- **e** \$100 in the ratio $3 : 1 : 1$

6C **7** Orange juice, pineapple juice and guava juice are mixed in the ratio $4 : 3 : 2$. If 250 mL of guava juice is used, how many litres of drink does this make?

6D **8** A map has a scale of $1 : 20\,000$. Find the real distance for each of these scaled distances.

- **a** 3 cm (answer in m)
- **b** 12 cm (answer in km)

6D **9** For each of these situations, find the scale ratio and also state the scale factor.

- **a** 5 mm on a scale drawing represents a real length of 1 m.
- **b** 4 cm on a map represents an actual length of 10 km.

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6D **10** Two towns are 5 km apart. How many millimetres apart are they on a map that has a scale of 1 : 100 000?

6E **11** Express each rate in simplest form.

a 10 km in 2 hours (? km/h)
b $650 for 13 hours ($?/h)
c 2800 km in 20 days (? km/day)

6E **12** Copy and complete.

a
×? 7 km uses 1 L of fuel ×?
280 km uses ? L of fuel

b
×? 60 words typed in 1 minute ×?
? words typed in 10 minutes

6F **13** **a** A truck uses 12 litres of petrol to travel 84 km. How far will it travel on:

i 1 L of petrol?
ii 42 L of petrol?

b Samira earns $67.20 for a 12-hour shift. How much will she earn for:

i 1 hour?
ii 7 hours?

6F **14** **a** Sandra drives to her mother's house. It takes 2 hours. Calculate Sandra's average speed in km/h if her mother lives 150 km away.

b How long does it take Ari to drive 180 km along the freeway to work if he manages to average 100 km/h for the trip? Give your answer in hours.

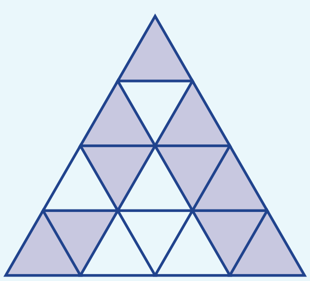
c How far does Siri ride his bike if he rides at 12 km/h for 45 minutes?

Multiple-choice questions

6A/B **1** A school has 315 primary students, 378 secondary students and 63 teachers. The ratio of students to teachers is:

A 11 : 1 **B** 1 : 11 **C** 5 : 6 **D** 6 : 5 **E** 1 : 10

6A/B **2** Find the ratio of the shaded area to the unshaded area in this triangle.



A 3 : 5 **B** 8 : 5 **C** 5 : 3 **D** 5 : 8 **E** 1 : 2

6B **3** The ratio 500 mm to 20 cm is the same as:

A 50 : 2 **B** 2500 : 1 **C** 2 : 5 **D** 5 : 2 **E** 10 : 1

6B **4** The ratio 1 hour : 30 minutes simplifies to:

A 2 : 1 **B** 1 : 2 **C** 1 : 30 **D** 4 : 3 **E** 1 : 3

6C **5** $750 is divided in the ratio 1 : 3 : 2. The smallest share is:

A $250 **B** $125 **C** $375 **D** $750 **E** $150

6C **6** The ratio of the areas of two triangles is 5 : 2. The area of the larger triangle is 60 cm^2. What is the area of the smaller triangle?

A 12 cm^2 **B** 24 cm^2 **C** 30 cm^2 **D** 17 cm^2 **E** 36 cm^2

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6E/F **7** Callum fills his car with 28 litres of petrol at 142.7 cents per litre. His change from $50 cash is:

A $10 **B** $39.95 **C** $10.05 **D** $40 **E** $12.50

6F **8** Madison cycled 20 km in 1.25 hours. Her average speed was:

A 25 km/h **B** 20 km/h **C** 16 km/h **D** 18.75 km/h **E** 30 km/h

6D **9** A house plan has a scale of 1 : 200. On the plan, the lounge room is 25 mm in length. The real length of the lounge room would be:

A 50 m **B** 5 m **C** 50 cm **D** 8 m **E** 80 cm

6D **10** On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?

A 1 : 5 **B** 1 : 5000 **C** 1 : 50 000 **D** 1 : 5 000 000 **E** 1 : 10 000

Extended-response question

1 From Canberra, ACT, to Melbourne, Victoria, it is 660 km. Two families, the Harrisons and the Nguygens, both leave Canberra at 8 am to drive to Melbourne.

The Harrison family's trip

- The Harrison's 17-year-old son drives for the first 2 hours at an average speed of 80 km/h.
- Then they stop for a rest of 1.5 hours.
- Mr Harrison drives the rest of the way to Melbourne with no more stops.

a How far did the Harrison's son drive?
b How far did Mr Harrison drive?
c At what time did the Harrison family finish their morning rest break?
d If the Harrisons arrive in Melbourne at 4:30 pm, for how long did Mr Harrison drive?
e What was Mr Harrison's average speed?

The Nguygen family's trip

- The Nguygen family drove to Melbourne with one 30-minute break.
- It took them $8\frac{1}{4}$ hours in total.

f At what time did the Nguygen family arrive in Melbourne?
g Calculate the average speed that the Nguygen family drove at, not counting the break.

Comparing the cost of each trip

h Using the information below, calculate the cost of each car's fuel for the trip.
Petrol costs 152.7 cents/L.
The Harrison family's car uses 8 L/100 km.
The Nguygen family's car uses 11 L/100 km.

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7

Equations and inequalities

Essential mathematics: why skills for solving equations and inequalities are important

Solutions to algebraic equations solve problems in a vast number of occupations, including in agriculture, business, financial services, food production, healthcare, and the trades.

Small business owners such as pet groomers, event photographers and hairdressers can solve equations to find the number of clients needed to make a certain weekly or monthly profit.

Horse trainers and assistants measure a horse's pace by timing it between distance markers and then calculating its speed using $S = \frac{d}{t}$, where d is distance covered, and t is time taken. Knowing the various paces of a horse helps trainers to adjust workout programs. A horse that learns how to maintain pace develops a rhythm and balance that maximises its efficiency, comfort and performance in races.

Nurses record a patient's daily pain medication of n mg. The inequation $10 \le n \le 25$ can show 10 mg/day is the minimum effective dose and 25 mg/day is the maximum safe dose.

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In this chapter

Australian Curriculum 9.0

ALGEBRA

Create, expand, factorise, rearrange and simplify linear expressions, applying the associative, commutative, identity, distributive and inverse properties (AC9M8A01)

Graph linear relations on the Cartesian plane using digital tools where appropriate; solve linear equations and one-variable inequalities using graphical and algebraic techniques; verify solutions by substitution (AC9M8A02)

Use mathematical modelling to solve applied problems involving linear relations, including financial contexts; formulate problems with linear functions, choosing a representation; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8A03)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 Fill in the missing number in these equations.
a $5 + 7 = \square$ **b** $3 \times 9 = \square$ **c** $12 \div 4 = \square$ **d** $5 \times 2 = \square$

2 Find the value of $\square$ to make these equations true.
a $4 + \square = 12$ **b** $6 \times \square = 12$ **c** $\square + 14 = 19$ **d** $\square - 4 = 11$

3 If $x = 6$, find the value of:
a $x + 2$ **b** $x \times 7$ **c** $x - 2$ **d** $8 - x$

4 Simplify these algebraic expressions.
a $9m + 2m$
b $4a - 3a$
c $7n + 3n - n$
d $8a + 2a - 10$
e $4x + 2 + 7x$
f $5b + 4 + 3b$

5 Expand these algebraic expressions using the distributive law.
a $3(m + 4)$ **b** $2(a + 6)$ **c** $3(x + 7)$ **d** $4(k - 6)$

6 I think of a number, double it, and then add three to get 27. What is the number?

7 If $x = 5$, are the following equations true (T) or false (F)?
a $x + 2 = 7$ **b** $3x = 35$ **c** $x - 1 = 6$ **d** $2x = 10$

8 Solve each of the following equations by inspection or using guess and check.
a $x + 8 = 12$ **b** $4x = 32$ **c** $m - 6 = -2$ **d** $3m = 18$

9 State the opposite operation of each of the following.
Choose from: **A** $+3$, **B** -2, **C** $\div 5$ or **D** $\times 3$
a $\times 5$ **b** $+2$ **c** $\div 3$ **d** -3

10 The sum of k and 3 is written as $k + 3$. Write expressions for the following.
Choose from: **A** $q - 6$, **B** $2z$, **C** $p + 10$ or **D** $4x$
a The sum of p and 10
b The product of 4 and x
c Double z
d 6 less than q

11 Copy and complete.

a

x	-2	-1	0	1	2	3
$3x - 1$	-7					

b

x	-2	-1	0	1	2	3
$2(x + 3)$			6			

12 True (T) or False (F).
a $<$ is the symbol for 'less than'
b $>$ is the symbol for 'less than'
c $\geq$ is the symbol for 'greater than or equal to'
d $\leq$ is the symbol for 'greater than or equal to'

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7A Equations review CONSOLIDATING

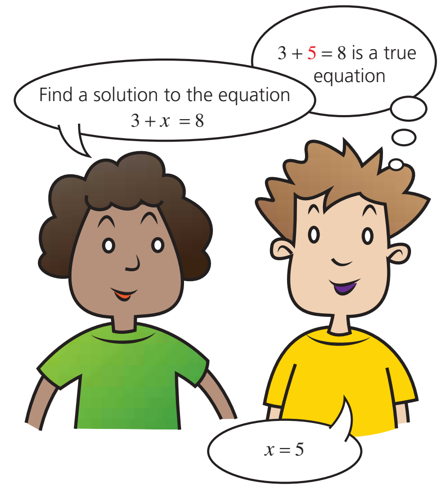
Learning intentions

- To understand that an equation is a mathematical statement that can be true or false.
- To understand that a solution is a value for the unknown that makes an equation true.
- To be able to find a solution to simple equations by inspection.
- To be able to write equations from worded descriptions.

Key vocabulary: equation, expression, solution, solving, LHS, RHS

Equations are mathematical statements saying that two things are equal. For example, $2 + 2 = 4$ is an equation.

If there is a pronumeral involved, then a solution is a value for that pronumeral that makes the equation true.



Lesson starter: What's missing?

Rory has erased a number in each of the equations below.

- If the equations were originally true, find the missing values:
  $10 + \square = 57$ $\quad \square - 31 = 40$ $\quad 2 \times \square + 5 = 19$
- In one equation he erased two numbers to get $\square \times 2 = \square$.
- Is it possible to find the missing values? Why or why not?

Key ideas

- An **equation** is a mathematical statement that two **expressions** are equal, such as $3 \times 5 = 15$ (which is true) or $2 + 2 = 100$ (which is false) or $2x + 1 = 9$ (which is true if $x = 4$).
- The parts of an equation are:

$$3 + 7 = 2 \times 5$$

left-hand side (**LHS**) — equals sign — right-hand side (**RHS**)

- A **solution** to an equation is a value for a pronumeral that makes an equation true. The process of finding a solution is called **solving**.

Exercise 7A

Understanding

	1–4	3, 4

1 Classify these equations as true (T) or false (F).
a $5 \times 3 = 15$ **b** $7 + 2 = 12$ **c** $5 + 3 = 16 \div 2$
d $8 - 6 = 6$ **e** $4 \times 3 = 12 \times 1$ **f** $2 = 8 - 3 - 3$

2 Find the value of $A + 5$ if:
a $A = 3$ **b** $A = 7$ **c** $A = 10$ **d** $A = 40$

3 If the value of x is 3, what is the value of the following?
a $10 + x$ **b** $3x$ **c** $5 - x$ **d** $6 \div x$

Hint for Q3: $3x$ means $3 \times x$.

4 State the value of the missing number to make the following equations true.
a $5 + \square = 12$ **b** $10 \times \square = 90$ **c** $\square - 3 = 12$ **d** $3 + 5 = \square$

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7A

Fluency

5–8(½) 6–8(½)

Example 1 Classifying equations involving pronumerals as true or false

If $x = 10$, is the equation $x + 20 = 3 \times x$ true or false?

Solution

True

Explanation

LHS $= x + 20 = 10 + 20 = 30$.
RHS $= 3 \times x = 3 \times 10 = 30$.
LHS equals RHS, so the equation is true.

Now you try

If $x = 7$, is the equation $4x = 34 - x$ true or false?

5 If $x = 2$, state whether the following equations are true (T) or false (F).

a $x + 4 = 6$
b $10x = 5$
c $8 = 10 - x$
d $7x = 8 + 3x$
e $10 - x = 4x$
f $3x = 5 - x$

6 If $a = 3$, state whether the following equations are true (T) or false (F).

a $7 + a = 10$
b $2a + 4 = 12$
c $8 - a = 5$
d $4a - 3 = 9$
e $7a + 2 = 8a$
f $a = 6 - a$

7 For each equation below, choose the correct solution from the options on the right.

a $x + 12 = 20$
b $10x + 5 = 35$
c $12 = x + 5$
d $10 + x = 3x + 2$
e $3 + 2x = 5$

Hint for Q7: $x = 1$ $x = 7$ $x = 3$ $x = 4$ $x = 8$

Example 2 Stating a solution to an equation

State a solution to each of the following equations.

a $4 + x = 25$
b $5y = 45$

Solution

a $x = 21$

b $y = 9$

Explanation

a We need to find a value of x that makes the equation true. If $4 + 21 = 25$ is a true equation, $x = 21$ is a solution.

b If $y = 9$ then $5y = 5 \times 9 = 45$, so the equation is true.

Now you try

State a solution to each of the following equations.

a $7 = x - 12$
b $10x = 90$

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8 State a solution to each of the following equations.

a $5 + x = 12$ b $3 = x - 10$ c $4u = 28$

d $17 = p - 2$ e $10x = 20$ f $77 = 7k$

Problem-solving and reasoning 9, 10 10–12

Example 3 Writing equations from a description

Write equations for the following.

a The number k is doubled, then three is added and the result is 52.

b Akira works n hours, earning \$12 per hour. The total she earned was \$156.

Solution	Explanation
a $2k + 3 = 52$	The number k is doubled, giving $k \times 2$. This is the same as $2k$. If 3 is added, the left-hand side is $2k + 3$, which must be equal to 52 according to the description.
b $12n = 156$	If Akira works n hours at \$12 per hour, the total amount earned is $12 \times n$, or $12n$.

Now you try

Write equations for the following.

a Four is subtracted from double m and the result is 20.

b Apples cost \$$a$ each and bananas cost \$1 each. Seven apples and 6 bananas cost \$9.50.

9 'A number x is tripled and the result is 12.' Which of the following equations describes this?

A $x + 3 = 12$ B $12x = 3$ C $3x = 12$ D $12 - x = 3$

10 Write equations to describe the following problems. You do not need to solve the equations.

a The number k is increased by 4 and the result is 20.

b A number x is doubled and then 7 is added. The result is 10.

c The sum of x and half of x is 12.

d Fel's height is h cm and her brother Pat is 30 cm taller. Pat's height is 147 cm.

e Coffee costs \$$c$ per cup and tea costs \$3. Four cups of coffee and two cups of tea cost a total of \$22.

f Chairs cost \$$c$ each. To purchase 8 chairs and a \$2000 table costs a total of \$3600.

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11 Find the value of the number for the following problems.
- **a** A number is tripled to obtain the result 21.
- **b** Half of a number is 21.
- **c** Six less than a number is 7.
- **d** A number is doubled and the result is 52.

12 Berkeley buys x kg of oranges at \$3.20 per kg. He spends a total of \$9.60.
- **a** Write an equation involving x to describe this situation.
- **b** State a solution to this equation.

More than one unknown — 13

13 **a** There are six equations in the square below. Find the values of a, b, c, d and e to make all six equations true.

$$\begin{array}{ccccc}
\boxed{a} & + & \boxed{12} & = & \boxed{22} \\
\times & & \div & & - \\
\boxed{2} & \times & \boxed{b} & = & \boxed{c} \\
= & & = & & = \\
\boxed{d} & \div & \boxed{e} & = & \boxed{10}
\end{array}$$

b If the four numbers above $(2, 10, 12, 22)$ are doubled, what would the values of a, b, c, d and e become?

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7B Solving equations using backtracking

Learning intentions

- To understand that expressions can be built from a single pronumeral by performing operations.
- To understand that an equation can be solved by performing opposite operations in reverse.
- To be able to use backtracking to solve simple equations.

Key vocabulary: backtracking, pronumeral, expression, opposite operation, solution

Backtracking using flowcharts is one way to solve simple equations. An expression can be built up from a single pronumeral.

Performing operations to a number

$$\boxed{3} \xrightarrow{\times 2} \boxed{6} \xrightarrow{+5} \boxed{11}$$

Performing operations to a pronumeral

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x+5}$$

The arrows can also be reversed to break down the expression but the opposite operation is used.

Reversing operations on a number

$$\boxed{3} \xleftarrow{\div 2} \boxed{6} \xleftarrow{-5} \boxed{11}$$

Reversing operations on a pronumeral

$$\boxed{x} \xleftarrow{\div 2} \boxed{2x} \xleftarrow{-5} \boxed{2x+5}$$

Lesson starter: One percenter

Starting with the number 100, you can get to the number 1 in many different ways (two are shown).

- How could you get from 100 to 1 in just one step?
- Describe how you could do it using 10 steps. Draw a flowchart.

$$\boxed{100} \xrightarrow{\div 10} \boxed{10} \xrightarrow{-9} \boxed{1}$$

$$\boxed{100} \xrightarrow{-80} \boxed{20} \xrightarrow{\times 0.05} \boxed{1}$$

Key ideas

- **Expressions** like $2x + 5$ can be built from a single **pronumeral**, like x.

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x+5}$$

Equations can be solved by reversing the arrows and using the **opposite operation**. This process is called **backtracking**.

e.g. Equation
$2x + 5 = 11$

building →
$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x+5}$$
$$\boxed{3} \xleftarrow{\div 2} \boxed{6} \xleftarrow{-5} \boxed{11}$$
backtracking →

Solution $x = 3$

Operation	Opposite
$+3$	-3
$\times 5$	$\div 5$
-10	$+10$
$\div 7$	$\times 7$

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Exercise 7B

Understanding

1–3 | 3

1 Fill in the gaps with the appropriate word.
- **a** The opposite of adding 6 is __________ 6.
- **b** The opposite of multiplying by 3 is __________ by 3.
- **c** The opposite of subtracting 20 is __________ 20.
- **d** The opposite of dividing by 12 is __________ by 12.

2 Copy and complete the flowcharts below.
- **a** $\boxed{3} \xrightarrow{\times 5} \boxed{} \xrightarrow{-2} \boxed{}$
- **b** $\boxed{7} \xrightarrow{-2} \boxed{} \xrightarrow{\times 3} \boxed{}$
- **c** $\boxed{x} \xrightarrow{+2} \boxed{}$
- **d** $\boxed{a} \xrightarrow{\times 3} \boxed{}$
- **e** $\boxed{f} \xrightarrow{\times 2} \boxed{} \xrightarrow{+3} \boxed{}$
- **f** $\boxed{p} \xrightarrow{\times 5} \boxed{} \xrightarrow{-4} \boxed{}$

3 State the operation used (e.g. $\times$ 4) on the arrow for the flowcharts below.
- **a** $\boxed{k} \xrightarrow{?} \boxed{4k}$
- **b** $\boxed{x} \xrightarrow{?} \boxed{x - 3}$
- **c** $\boxed{q} \xrightarrow{?} \boxed{q + 12}$

Fluency

4–10 | 6–9, 10–11(½)

Example 4 Using backtracking to solve simple equations

Use backtracking to solve the equation $4k = 12$.

Solution

$$\begin{array}{ccc} & \times 4 & \\ \boxed{k} & \longrightarrow & \boxed{4k} \\ \boxed{} & \longleftarrow & \boxed{} \\ & \div 4 & \end{array}$$

Explanation

First set up a flowchart for $4k$, showing the opposite operation on the bottom arrow.

$$\begin{array}{ccc} & \times 4 & \\ \boxed{k} & \longrightarrow & \boxed{4k} \\ \boxed{3} & \longleftarrow & \boxed{12} \\ & \div 4 & \end{array}$$

Put the number 12 in below $4k$ and follow the arrow back to find k.

Solution: $k = 3$

Now you try

Use backtracking to solve the equation $x - 3 = 16$.

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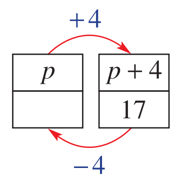
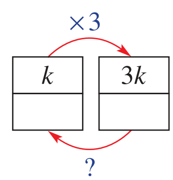
4 **a** Copy and complete the flowchart shown for the equation $p + 4 = 17$.

b What is the solution to the equation $p + 4 = 17$?



5 **a** Copy and complete the flowchart below for the equation $3k = 18$.



Hint for Q5: The number 18 goes below $3k$.

b What is the solution to $3k = 18$?

6 Solve the following equations by first making a flowchart.

a $3k = 30$ **b** $p + 4 = 30$ **c** $r - 12 = 30$

d $5x = 40$ **e** $w \times 12 = 132$ **f** $s \div 3 = 10$

Example 5 Using backtracking to solve two-step equations

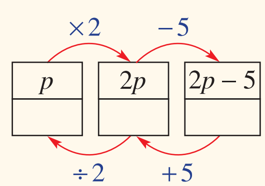
Solve the equation $2p - 5 = 15$ using backtracking.

Solution

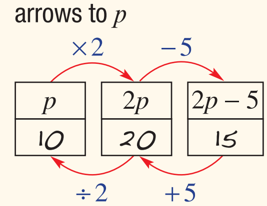
Solution: $p = 10$.

Explanation

Step 1
Make a flowchart for $2p - 5$



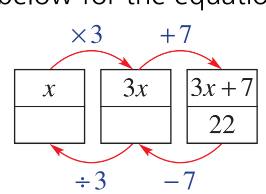
Step 2
Put 15 in and follow the arrows to p



Now you try

Solve the equation $2a + 5 = 33$ using backtracking.

7 **a** Copy and complete the flowchart below for the equation $3x + 7 = 22$.



b What is the solution to $3x + 7 = 22$?

8 Solve the following equations using backtracking.

a $2p - 5 = 25$ **b** $10x + 3 = 43$ **c** $3q + 7 = 25$

d $5r - 11 = 24$ **e** $6 + 10u = 26$ **f** $3 + 2p = 45$

Hint for Q8: Draw a flowchart for each one.

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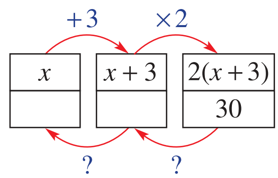
7B

9 **a** Copy and complete the flowchart below for the equation $2(x+3)=30$.



b What is the solution to $2(x+3)=30$?

10 Solve the following equations using backtracking.

a $2(x+5)=16$ **b** $4(q+3)=20$ **c** $3r+7=22$

d $7t-10=39$ **e** $10(s-20)=60$ **f** $10s-20=60$

11 The following equations involve negative numbers. Use backtracking to find the solutions.

a $3x=-15$ **b** $p+10=4$ **c** $5x+12=-13$

d $4(r-3)=-20$ **e** $3(n+40)=30$ **f** $7u-10=-31$

Problem-solving and reasoning

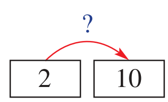
12, 13 | 13, 14

12 Oliver doubles a number and then adds 7. The result is 59.

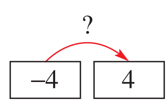
a If x is the number that he started with, draw a flowchart to describe this situation.

b Use backtracking to find the value of x.

13 **a** Give two separate operations that could be used to fill in the question mark below.



b If one of the above operations can be used for the flowchart below, what is the operation?



14 **a** Draw flowcharts to solve the equations $2x+4=10$ and $2(x+4)=10$.

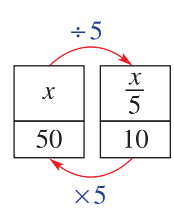
b Describe how the flowcharts differ from each other.

Fractional flowcharts

— | 15

15 Remember that $\frac{x}{5}$ means $x \div 5$. This can be used to solve the equation $\frac{x}{5}=10$

Use flowcharts to solve the equations using fractions.



a $\frac{x}{5}=7$ **b** $\frac{r}{2}=9$

c $\frac{x}{10}+4=11$ **d** $\frac{y}{3}-2=7$

e $2\left(\frac{x}{5}\right)+3=7$ **f** $4\left(\frac{m}{3}\right)-1=7$

g $\frac{x+4}{5}=10$ **h** $\frac{r-3}{2}=4$

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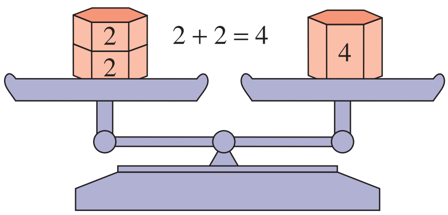
7C Solving equations using the balancing method

Learning intentions

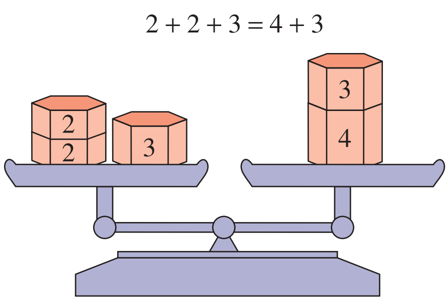
- To understand what it means for two equations to be equivalent.
- To be able to find equivalent equations by applying an operation to both sides.
- To be able to solve one-step and two-step equations algebraically by finding equivalent equations.

Key vocabulary: equivalent, balancing method, substitute

Sometimes it is helpful to think of an equation as two weights balancing on scales.

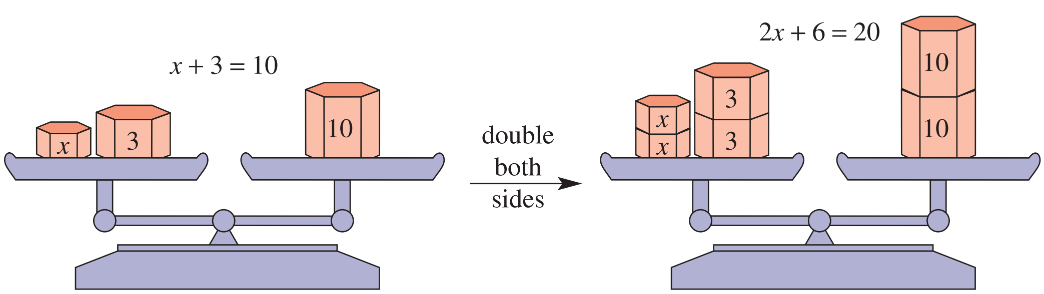


If the same weight is added to both sides, the scales still balance.



Three medium stones plus two small stones balance with one large stone.

We can also subtract a value from both sides, or multiply/divide both sides by the same value, and the scales will still balance.



Equations are called equivalent if you can get from one to the other by performing the same operations on both sides.

The operations are written next to arrows, like this:

$$x + 3 = 10 \xrightarrow{\times 2} 2x + 6 = 20$$

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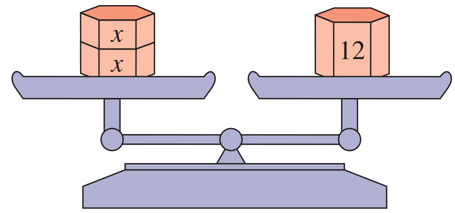
7C

Lesson starter: Equivalent equations



- In 60 seconds, write as many equations as you can that are equivalent to $2x = 12$.
- For one equation that you wrote down, show it as a pair of scales like this diagram.
- Show one of them with arrows like this diagram.

$$\begin{array}{ccc} & 2x = 12 & \\ ? & & ? \\ & \_\_ = \_\_ & \end{array}$$

- What is the simplest (smallest) equation that is equivalent to $2x = 12$?

Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side and right-hand sides of the equation
- To solve an equation using the **balancing method**, you should repeatedly find an equivalent equation that is simpler. For example:

$$\begin{array}{ccc} & 5x + 2 = 32 & \\ -2 & & -2 \\ & 5x = 30 & \\ \div 5 & & \div 5 \\ & x = 6 & \end{array}$$

Check: LHS $= 5 \times 6 + 2$ RHS $= 32$
$= 32$

- Check that your solution is correct by **substituting** into the original equation to see if LHS = RHS.

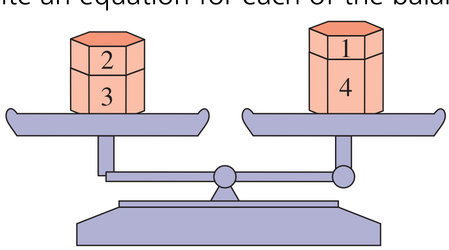
Exercise 7C

Understanding

1–4 | 3, 4

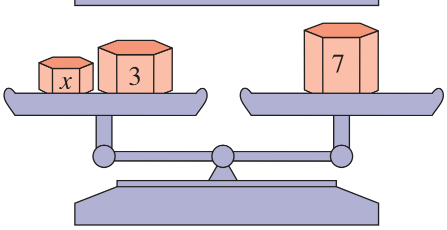
1 Write an equation for each of the balancing scales below.

a

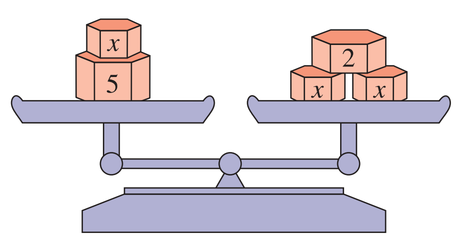


Hint for Q1: An example could be $5 + 2 = 6 + 1$ or $3x + 1 = x + 4$.

b



c



2 Write the equivalent equations by filling in the blanks.

a
$$\begin{array}{ccc} & 2x = 12 & \\ \div 2 & & \div 2 \\ & \_\_ = \_\_ & \end{array}$$

b
$$\begin{array}{ccc} & x - 3 = 5 & \\ +3 & & +3 \\ & \_\_ = \_\_ & \end{array}$$

c
$$\begin{array}{ccc} & 3q + 4 = 16 & \\ -4 & & -4 \\ & \_\_ = \_\_ & \end{array}$$

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3 Consider the equation $4x = 32$.

a Copy and complete the following working.

$$\begin{array}{ccc} & 4x = 32 & \\ \div 4 & & \div 4 \\ & x = \_\_ & \end{array}$$

b What is the solution to the equation $4x = 32$?

Hint for Q3:
A solution is a value of x that makes the equation true.

4 To solve the equation $10x + 5 = 45$, which of the following operations would you first apply to both sides?

A Divide by 5 **B** Subtract 5 **C** Divide by 10 **D** Subtract 45

Fluency 5–8(½) 5–9(½)

Example 6 Finding equivalent equations

Show the result of applying the given operation to both sides of these equations.

a $8y = 40$ [$\div$ 8] **b** $10 + 2x = 36$ [-10] **c** $5a - 3 = 12$ [+3]

Solution

a
$$\begin{array}{ccc} & 8y = 40 & \\ \div 8 & & \div 8 \\ & y = 5 & \end{array}$$

Explanation

Write the equation out and then divide both sides by 8.
$40 \div 8$ is 5 and $8y \div 8$ is y.

b
$$\begin{array}{ccc} & 10 + 2x = 36 & \\ -10 & & -10 \\ & 2x = 26 & \end{array}$$

Write the equation out and then subtract 10 from both sides.
$36 - 10$ is 26
$10 + 2x - 10$ is $2x$

c
$$\begin{array}{ccc} & 5a - 3 = 12 & \\ +3 & & +3 \\ & 5a = 15 & \end{array}$$

Write the equation out and then add 3 to both sides.
$12 + 3$ is 15
$5a - 3 + 3$ is $5a$

Now you try

Show the result of applying the given operation to both sides of these equations.

a $x - 7 = 12$ [+7] **b** $6 + 2x = 44$ [-6] **c** $7y - 4 = 45$ [+4]

5 For each equation, show the result of applying the given operation to both sides.

a $10 + 2x = 30$ [-10] **b** $4 + q = 12$ [-2] **c** $13 = 12 - q$ [+5]
d $4x = 8$ [$\times$ 3] **e** $7p = 2p + 4$ [+6] **f** $3q + 1 = 2q + 1$ [-1]

6 Copy and complete the following to solve the given equations using the balancing method.

a
$$\begin{array}{ccc} & 10x = 30 & \\ \div 10 & & \div 10 \\ & x = \_\_ & \end{array}$$

b
$$\begin{array}{ccc} & q + 5 = 12 & \\ -5 & & -5 \\ & \_\_ = \_\_ & \end{array}$$

c
$$\begin{array}{ccc} & k - 3 = 8 & \\ +3 & & +3 \\ & \_\_ = \_\_ & \end{array}$$

d
$$\begin{array}{ccc} & 4x + 2 = 22 & \\ -2 & & -2 \\ & 4x = \_\_ & \\ \div 4 & & \div 4 \\ & \_\_ = \_\_ & \end{array}$$

e
$$\begin{array}{ccc} & 7p + 2 = 30 & \\ -2 & & -2 \\ & \_\_ = \_\_ & \\ \square & & \square \\ & \_\_ = \_\_ & \end{array}$$

f
$$\begin{array}{ccc} & 10x - 4 = 26 & \\ +4 & & +4 \\ & \_\_ = \_\_ & \\ \square & & \square \\ & \_\_ = \_\_ & \end{array}$$

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7C

Example 7 Solving equations using the balancing method

Solve the following equations and check your solution using substitution.

a $x - 4 = 16$ b $2u + 7 = 17$ c $10 = 3k - 11$

Solution	Explanation
a $\begin{array}{ccc} & x - 4 = 16 & \\ +4 & & +4 \\ & x = 20 & \end{array}$	By adding 4 to both sides of the equation, we get an equivalent equation. So the solution is $x = 20$.
Check: LHS $= 20 - 4$ RHS $= 16$ $= 16$	Check your solution by substituting $x = 20$ into the LHS.
b $\begin{array}{ccc} & 2u + 7 = 17 & \\ -7 & & -7 \\ & 2u = 10 & \\ \div 2 & & \div 2 \\ & u = 5 & \end{array}$	To get rid of the +7, we subtract 7 from both sides. Finally, we divide by 2 to reverse the $2u$. Remember that $2u$ means $2 \times u$. So the solution is $u = 5$.
Check: LHS $= 2 \times 5 + 7$ RHS $= 17$ $= 17$	Check your solution using substitution.
c $\begin{array}{ccc} & 10 = 3k - 11 & \\ +11 & & +11 \\ & 21 = 3k & \\ \div 3 & & \div 3 \\ & 7 = k & \end{array}$	First add 11 to 'undo' the -11 Then divide by 3 since $3k$ means $k \times 3$. Swap the $7 = k$ to put the pronumeral first in our solution. So the solution is $k = 7$.
Check: LHS $= 10$ RHS $= 3 \times 7 - 11$ $= 10$	Check that both sides of $10 = 3k - 11$ are equal using $k = 7$.

Now you try

Solve the following equations and check your solution using substitution.

a $y + 2 = 29$ b $3m - 4 = 26$ c $17 = 9 + 4k$

7 Solve the following equations and check your solution using substitution.

a $a + 5 = 8$ b $t \times 2 = 14$ c $q - 2 = 7$ d $k + 2 = 11$
e $x + 9 = 19$ f $3h = 30$ g $9l = 36$ h $g \div 3 = 3$

8 Solve the following equations and check your solution using substitution.

a $9h + 5 = 32$ b $9u - 6 = 30$ c $5s - 2 = 13$ d $3w - 6 = 18$
e $8 + 5x = 28$ f $6 + 10w = 56$ g $8a - 8 = 8$ h $4y - 8 = 40$

9 Solve the following equations and check your solution using substitution.

a $10 = 5x$ b $12 = k + 7$ c $30 = x - 12$ d $5 = x \div 4$
e $32 = 4k + 4$ f $50 = 2x - 10$ g $12 = 3y - 6$ h $14 = x \div 2 + 4$

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Problem-solving and reasoning

10, 11 | 10–13

10 The solutions to the following equations are negative numbers. Solve the equations to find them.

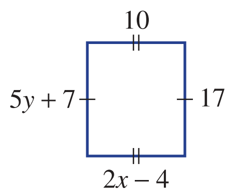
a $x + 10 = 4$ **b** $7a = -21$ **c** $3x + 4 = -26$
d $2k + 20 = 10$ **e** $7 = 2k + 15$ **f** $1 = 7p + 8$
g $-2 = p \div 8$ **h** $-3 = 2x + 7$

11 For each of the following, write an equation and solve.

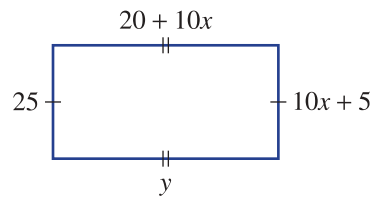
a The sum of p and 8 is 15.
b The product of q and 3 is 12.
c 4 is subtracted from double the value of k and the result is 18.
d When r is tripled and 4 is added the result is 34.

12 The following shapes are rectangles. By solving equations, find the value of the variables.

a

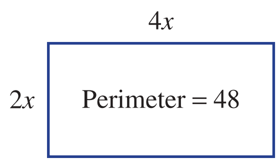


b

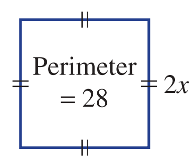


Hint for Q12: Find the value of x first.

c



d



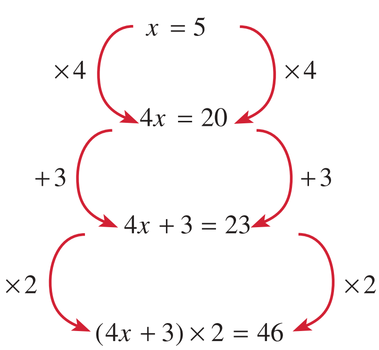
13 Solve the following equations. More than two steps are involved.

a $14 \times (4x + 2) = 140$ **b** $8 = (10x - 4) \div 2$ **c** $3 + (2x + 1) \times 4 = 47$

From solutions to equations

— | 14

14 A student has taken the equation $x = 5$ and performed some operations to both sides:



a Solve $(4x + 3) \times 2 = 46$.
b Describe how the steps you used in your solution compare with the steps the student used.
c Give an example of another equation that has $x = 5$ as its solution.

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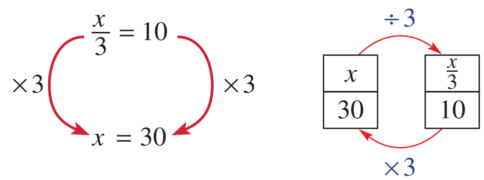
7D Equations with fractions

Learning intentions

- To understand that fractions are used in algebra to indicate division.
- To be able to solve equations involving algebraic fractions.

Key vocabulary: fraction, denominator, backtracking, equivalent equations

Recall from algebra that a fraction such as $\frac{x}{3}$ represents $x \div 3$. This means that to solve an equation with $\frac{x}{3}$ on one side, we should first multiply both sides by 3. For example:



Lesson starter: Practising with fractions

- If $x = 10$, find out what each of these expressions would equal:

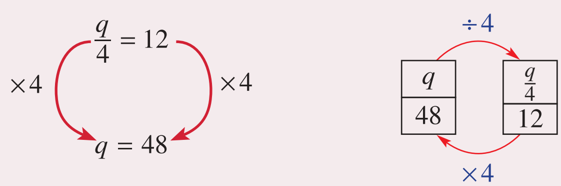
  $\frac{2x+1}{2}$ $\qquad 2\left(\frac{x}{2}+1\right)$ $\qquad \frac{2}{x+1}$ $\qquad \frac{2+2x}{2}$ $\qquad 2\left(x+\frac{1}{2}\right)$

- Which of the above expressions are equal if $x = 0$?

Key ideas

- $\frac{a}{b}$ means $a \div b$.
- To solve an equation with a **fraction** on one side, multiply both sides by the **denominator**.

  Using **equivalent equations** Using **backtracking** Solution: $q = 48$



Exercise 7D

Understanding

1–4 | 3, 4

1 Which of the following expressions represents 'x divided by 5'?

A $x+5$ **B** $\frac{x}{5}$ **C** $\frac{5}{x}$ **D** $5x$

2 If $x = 20$, state whether the following equations are true (T) or false (F).

a $\frac{x}{4} = 5$ **b** $\frac{x}{2} = 40$ **c** $\frac{x}{5} = 5$ **d** $\frac{x}{10} = 2$

3 **a** If $x = 4$, find the value of $\frac{x}{2} + 6$.

b If $x = 4$, find the value of $\frac{x+6}{2}$.

c Are $\frac{x}{2} + 6$ and $\frac{x+6}{2}$ equivalent expressions?

Hint for Q3: Expressions are equivalent if they are always equal.

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4 Fill in the missing steps to solve these equations.

a $\dfrac{x}{3} = 10$ $\xrightarrow{\times 3}$ $x = \_\_$

b $\dfrac{m}{5} = 2$ $\xrightarrow{\times 5}$ $m = \_\_$

c $11 = \dfrac{q}{2}$ $\xrightarrow{\boxed{}}$ $\_\_ = q$

d $\dfrac{p}{10} = 7$ $\xrightarrow{\boxed{}}$ $p = \_\_$

Fluency

5–8(½), 9 | 6–8(½), 9, 10(½)

Example 8 Solving equations with fractions

Solve the following equations.

a $\dfrac{k}{10} = 4$

b $\dfrac{4x}{3} = 8$

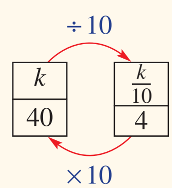
Solution

a

$$\frac{k}{10} = 4$$
$$\downarrow \times 10 \quad \times 10 \downarrow$$
$$k = 40$$

Explanation

Multiplying both sides by 10 removes the denominator of 10. Alternatively, backtracking can be used.



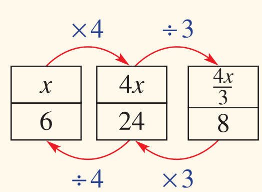
The solution is $k = 40$.

b

$$\frac{4x}{3} = 8$$
$$\downarrow \times 3 \quad \times 3 \downarrow$$
$$4x = 24$$
$$\downarrow \div 4 \quad \div 4 \downarrow$$
$$x = 6$$

Multiplying both sides by 3 removes the denominator of 3

Both sides are divided by 4 to solve the equation. Alternatively, backtracking can be used.



The solution is $x = 6$.

Now you try

Solve the following equations.

a $\dfrac{x}{3} = 7$

b $\dfrac{9x}{7} = 9$

5 Solve the following equations.

a $\dfrac{b}{5} = 4$ **b** $\dfrac{g}{10} = 2$ **c** $\dfrac{a}{5} = 3$ **d** $\dfrac{k}{6} = 3$

6 Solve the following equations.

a $\dfrac{2l}{5} = 8$ **b** $\dfrac{7w}{10} = -7$ **c** $\dfrac{3s}{2} = -9$ **d** $\dfrac{5v}{4} = 15$

e $\dfrac{3m}{7} = 6$ **f** $\dfrac{3n}{7} = 6$ **g** $\dfrac{-6j}{5} = 6$ **h** $\dfrac{-6f}{5} = -24$

Hint for Q6: Multiply both sides by a chosen number.

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7D

Example 9 Solving more complex equations with fractions

Solve the equation: $\dfrac{4y+15}{9}=3$.

Solution

$$\frac{4y+15}{9}=3$$

$\times 9$ $\qquad$ $\times 9$

$$4y+15=27$$

-15 $\qquad$ -15

$$4y=12$$

$\div 4$ $\qquad$ $\div 4$

$$y=3$$

Explanation

Multiplying both sides by 9 removes the denominator of 9.

The equation $4y+15=27$ is solved in the usual fashion (subtract 15, divide by 4).
Alternative solution using backtracking:

$\times 4$ $\quad$ $+15$ $\quad$ $\div 9$

y	$4y$	$4y+15$	$\frac{4y+15}{9}$
3	12	27	3

$\div 4$ $\quad$ -15 $\quad$ $\times 9$

The solution is $y=3$.

Now you try

Solve the equation: $\dfrac{2m-11}{5}=1$.

7 Solve the following equations.

a $\dfrac{t-8}{2}=10$ **b** $\dfrac{h+10}{3}=4$ **c** $\dfrac{a+12}{5}=5$

d $\dfrac{c-7}{2}=5$ **e** $\dfrac{s-2}{8}=1$ **f** $\dfrac{5j+6}{8}=2$

Hint for Q7: First multiply.

Example 10 Solving more equations with fractions

Solve the equation: $4+\dfrac{5x}{2}=29$.

Solution

$$4+\frac{5x}{2}=29$$

-4 $\qquad$ -4

$$\frac{5x}{2}=25$$

$\times 2$ $\qquad$ $\times 2$

$$5x=50$$

$\div 5$ $\qquad$ $\div 5$

$$x=10$$

Explanation

We must subtract 4 first because we do not have a fraction by itself on the left-hand side.
Once there is a fraction by itself, multiply by the denominator (2).
Alternative solution using backtracking:

$\times 5$ $\quad$ $\div 2$ $\quad$ $+4$

x	$5x$	$\frac{5x}{2}$	$4+\frac{5x}{2}$
10	50	25	29

$\div 5$ $\quad$ $\times 2$ $\quad$ -4

The solution is $x=10$.

Now you try

Solve the equation: $\dfrac{3y}{4}-7=2$.

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8 Solve the following equations.

a $\frac{y}{10}+3=5$ **b** $2+\frac{x}{4}=7$ **c** $\frac{y}{2}-6=1$

d $\frac{2x}{5}+6=10$ **e** $\frac{6p}{7}-4=2$ **f** $9+\frac{3k}{2}=18$

9 Match each of these equations with the correct first step to solve it.

a $\frac{x}{4}=7$ **b** $\frac{x-4}{2}=5$ **c** $\frac{x}{2}-4=7$ **d** $\frac{x}{4}+4=3$

A Multiply both sides by 2.
B Add 4 to both sides.
C Multiply both sides by 4.
D Subtract 4 from both sides.

10 Solve the following equations.

a $\frac{g-3}{5}=1$ **b** $\frac{2x}{7}=4$ **c** $\frac{k}{3}+1=6$ **d** $\frac{x}{4}=9$ **e** $3=\frac{q}{2}-2$

f $15=\frac{3+x}{2}$ **g** $2=\frac{5p}{15}$ **h** $\frac{2x+7}{3}=3$ **i** $9=\frac{2r}{4}-1$

Problem-solving and reasoning

11, 12 | 11–13

11 For the following puzzles, write an equation and solve it to find the unknown number.

a A number x is divided by 5 and the result is 7.
b Half of y is 12.
c A number p is doubled and then divided by 7. The result is 4.
d Four is added to x. This is halved to get a result of 10.
e x is halved and then 4 is added to get a result of 10.
f A number k is doubled and then 6 is added. This result is halved to obtain 14.

12 The average of two numbers can be found by adding them and then dividing the result by 2.

a Find the average of 9 and 5.
b If the average of x and 5 is 12, what is x? Solve the equation $\frac{x+5}{2}=12$ to find out.
c The average of 7 and p is 5. Find p by writing and solving an equation.
d The average of a number and double that number is 18. What is that number?
e The average of $4x$ and 6 is 19. What is the average of $6x$ and 4?

Hint for Q12e: Find x first.

13 A restaurant bill is to be paid. Blake puts in \$40 which is one-third of the amount in his wallet.

a Write an equation to describe this situation, if b represents the amount in Blake's wallet before he pays.
b Solve the equation to find out how much money Blake has in his wallet.

Variable denominators

— | 14

14 To solve an equation with a variable in the denominator we can first multiply both sides by that variable. Use this method to solve the equations.

a $\frac{12}{x}=2$ **b** $\frac{15}{x}=5$ **c** $\frac{20}{x}=4$

d $4+\frac{20}{x}=14$ **e** $\frac{16}{x}+1=3$ **f** $\frac{12}{x}=1$

$$\begin{aligned} \frac{30}{x} &= 10 \\ &\downarrow \times x \\ 30 &= 10x \\ &\downarrow \div 10 \\ 3 &= x \end{aligned}$$

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7E Equations with brackets ★

Learning intentions

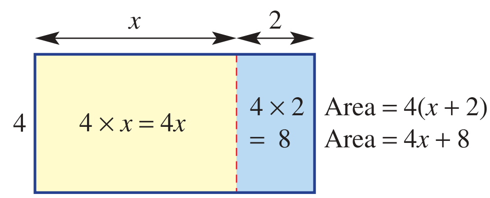
- To understand that the distributive law can be used to expand brackets within equations.
- To be able to solve equations by expanding brackets.

Key vocabulary: expand, distributive law, like terms, simplify

In **Chapter 5** it was noted that expressions with brackets could be expanded by considering rectangle areas.

We can see from the demonstration on the right that $4(x+2) = 4x + 8$.

Expansion can also be used to help solve equations with brackets.



Lesson starter: T-shirts and shorts

Harrison buys two sporting outfits at a shop where shorts cost \$5 more than T-shirts.

- If each pair of shorts is \$10, how much does one outfit cost?
- If the two outfits cost \$60 in total, can you give the cost of each item?
- Try to find an expression for the total cost of the outfits.

Key ideas

- To **expand** brackets, use the **distributive law** which states that:
  - $a(b+c) = ab + ac$. e.g. $3(x+4) = 3x + 12$.
  - $a(b-c) = ab - ac$. e.g. $4(b-2) = 4b - 8$.
- **Like terms** are terms that contain exactly the same pronumerals and can be collected to **simplify** expressions. For example, $5x + 10 + 7x$ can be simplified to $12x + 10$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms. e.g. $2(x-3) = 10$ becomes $2x - 6 = 10$, which can then be solved using the balancing method or backtracking.

Exercise 7E

Understanding

	1–4	3, 4

1 Fill in the missing numbers.

a $4(y+3) = 4y + \square$

b $7(2p-5) = \square\, p - 35$

c $2(4x+5) = \square\, x + \square$

d $10(5+3q) = \square + \square\, q$

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2 Match each expression (**a**–**d**) with its expanded form (**A**–**D**).

a $2(x+4)$	**A** $4x+8$
b $4(x+2)$	**B** $2x+4$
c $2(2x+1)$	**C** $2x+8$
d $2(x+2)$	**D** $4x+2$

3 If $x = 5$, state whether the following equations are true (T) or false (F).

a $3(x+1) = 18$ **b** $4(x-2) = 16$
c $2(2x+1) = 22$ **d** $5(x-1) = 20$

4 $3(x-7) = 12$ is the same equation as:

A $3x - 7 = 12$ **B** $3x - 12 = 7$
C $3x - 21 = 12$ **D** $7x - 3 = 12$

Fluency 5–6(½) 5–6(½)

Example 11 Solving equations with brackets

Solve the following equations by first expanding any brackets.

a $3(p+4) = 18$
b $4(2x-5) + 3x = 57$

Solution

a

$$\begin{aligned} 3(p+4) &= 18 \\ 3p + 12 &= 18 && -12 \\ 3p &= 6 && \div 3 \\ p &= 2 \end{aligned}$$

Explanation

Use the distributive law to expand the brackets.
Alternative solution using backtracking:

$p \xrightarrow{\times 3} 3p \xrightarrow{+12} 3p+12$

p	$3p$	$3p+12$
2	6	18

$2 \xleftarrow{\div 3} 6 \xleftarrow{-12} 18$

The solution is $p = 2$

b

$$\begin{aligned} 4(2x-5) + 3x &= 57 \\ 8x - 20 + 3x &= 57 \\ 11x - 20 &= 57 && +20 \\ 11x &= 77 && \div 11 \\ x &= 7 \end{aligned}$$

Use the distributive law to expand the brackets.
Combine the like terms: $8x + 3x = 11x$.
Alternative solution using backtracking:

$x \xrightarrow{\times 11} 11x \xrightarrow{-20} 11x-20$

x	$11x$	$11x-20$
7	77	57

$7 \xleftarrow{\div 11} 77 \xleftarrow{+20} 57$

The solution is $x = 7$

Now you try

Solve the following equations by first expanding any brackets.

a $12(x-2) = 36$
b $3(2+5x) - 4x = 28$

5 Solve the following equations by first expanding the brackets.

a $4(x+1) = 24$ **b** $3(k+5) = 18$ **c** $2(r-7) = 20$
d $2(4u+2) = 52$ **e** $3(3j-4) = 15$ **f** $5(2p-4) = 40$
g $15 = 5(2m-5)$ **h** $2(5n+5) = 60$ **i** $26 = 2(3a+4)$

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7E

6 Solve the following equations by expanding and combining like terms.

a $2(x+3)+x=30$ **b** $3(x-1)+2x=47$
c $5(r-2)+r=50$ **d** $4(3y+2)+2y=50$
e $5(2l-5)+3l=1$ **f** $4(5+3w)+5=49$
g $49=5(3c+5)-3c$ **h** $28=4(3d+3)-4d$
i $58=4(2w-5)+5w$ **j** $23=4(2p-3)+3$
k $44=5(3k+2)+2k$ **l** $49=3(2c-5)+4$

Hint for Q6: First expand then solve using backtracking or using the balancing method.

Problem-solving and reasoning

7, 8 | 8–10

7 A number is increased by 5 and then the result is doubled.

a If the number is n, write an expression for the final result.

b If the final result equals 40, which of the following equations describes this?

A $n+5\times 2=40$ **B** $2(n+5)=40$ **C** $2n+5=40$ **D** $40(n+2)=5$

c What was the original number?

8 Desmond notes that in 4 years' time his age when doubled will give the number 50. Desmond's current age is d.

a Write an expression for Desmond's age in 4 years' time.

b Write an expression for double his age in 4 years' time.

c Write an equation to describe the situation described above.

d Solve the equation to find his current age.

9 Amos buys 3 shirts and 2 pairs of trousers for a total of \$225. Each pair of trousers costs \$20 more than a shirt.

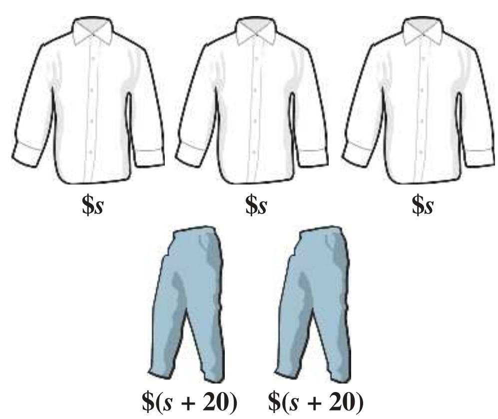
a Explain why the total cost is $3s+2(s+20)$ if $\$s$ is the cost of one shirt.

b Solve the equation $3s+2(s+20)=225$.

c How much does one shirt cost?

d How much does one pair of trousers cost?

e What would the total cost be for 5 shirts and 3 pairs of trousers?



10 Rahda's usual hourly wage is $\$w$. She works for 5 hours at this wage and then 3 more hours at an increased wage of $\$(w+4)$.

a Write an expression for the total amount Rahda earns for the 8 hours.

b Rahda earns \$104 for the 8 hours. Write and solve an equation to find her usual hourly wage.

Negative brackets

— | 11

11 The following equations involve negative numbers. Use the methods from the previous page to solve them.

a $2(x+1)=-10$ **b** $3(p-2)=-18$
c $10(q+9)=-100$ **d** $-2(r+1)=-10$
e $-5(r+6)=-40$ **f** $2(x+5)=-12$
g $3(k+1)+k=-37$ **h** $-10(s-5)=50$

Hint for Q11:

$-10(s-5)=-10s+50$

(-10×-5)

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Progress quiz

7A **1** If $x = 8$, is the equation $2x + 22 = 5x - 4$ true (T) or false (F)?

7A **2** State a solution to each of the following equations.
- **a** $18 - x = 13$
- **b** $7x = 56$
- **c** $x + 6 = 40$

7A **3** Write equations for the following.
- **a** The number t is tripled, then four is added and the result is 19.
- **b** Jack sleeps for n hours on week nights and 8 hours on weekend nights. The total he sleeps in a week is 61 hours.

7B **4** Solve the following equations using backtracking.
- **a** $4f + 7 = 27$
- **b** $5k + 19 = 34$
- **c** $5 + 3p = 65$
- **d** $7d - 12 = 37$

7B **5** Jennifer quadruples a number and then adds 5. The result is 49.
- **a** If x is the number that she started with, draw a flowchart to describe this situation.
- **b** Use backtracking to find the value of x.

7C **6** Solve the following equations.
- **a** $m - 12 = 3$
- **b** $h + 21 = 40$
- **c** $3b - 6 = 39$
- **d** $99 = 11g - 22$

7C **7** The solutions to the following equations are negative numbers. Solve the equations to find them.
- **a** $p + 14 = 9$
- **b** $j + 100 = 40$
- **c** $4b - 4 = -12$
- **d** $-8 = 3y + 7$

7D **8** Solve the following equations.
- **a** $\frac{x}{5} = 20$
- **b** $\frac{g}{11} = 8$
- **c** $\frac{3f}{5} = 12$
- **d** $\frac{2g}{-5} = -4$

7D **9** Solve the following equations.
- **a** $\frac{p + 11}{3} = 5$
- **b** $\frac{3p - 4}{2} = 7$
- **c** $\frac{p}{5} - 3 = 10$
- **d** $7 + \frac{2p}{5} = 19$

7E ★ **10** Expand the brackets for:
- **a** $3(6y - 4)$
- **b** $11(y - 7)$
- **c** $5(8 + 3y)$

7E ★ **11** Solve the following equations by first expanding the brackets.
- **a** $5(x - 3) = 10$
- **b** $3(3n + 12) = 63$
- **c** $71 = 8(2d - 6) + 7$

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7F Formulas and relationships

Learning intentions

- To be able to substitute values into equations containing two or more variables.
- To be able to apply a formula to find an unknown value.

Key vocabulary: formula, variable, rule, subject, substitute

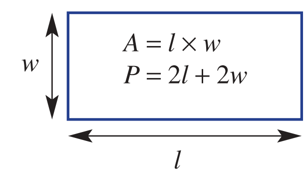
Formulas occur in many areas of maths and science.

The famous formula $E = mc^2$ relates to energy (*E*), mass (*m*) and the speed of light (*c*).

Formulas are a special type of equation that relate to two or more variables.

Lesson starter: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = l \times w$ and $P = 2l + 2w$.



- If $l = 10$ and $w = 7$ find the perimeter and the area.
- If $l = 8$ and $w = 2$ find the perimeter and the area.
- Notice that sometimes the number for the area is bigger than the number for the perimeter and sometimes the number for the area is less than the number for the perimeter. If $l = 10$, is it possible to make the numbers for the area and the perimeter equal?
- If $l = 2$ can you make the numbers for the area and the perimeter equal? Discuss.

Key ideas

- The **subject** of an equation is a pronumeral (or **variable**) that occurs by itself on the left-hand side, e.g. V is the subject of $V = 3x + 2y$.
- A **formula** or **rule** is an equation containing two or more variables, one of which is the subject of the equation.
- To use a formula, **substitute** all the known values into the equation and then solve the equation to find the unknown value.

Exercise 7F

Understanding	1–4	3, 4

1 Fill in the blanks: Choose from: *area*, *formula* or *subject*.
  a A _______ or rule is an equation relating two or more variables.
  b A variable by itself on the left-hand side of an equation is called the _______.
  c The formula $A = l \times w$ is used to find the _______ of a rectangle.

2 If you substitute $l = 5$ and $w = 3$ into the formula $A = l \times w$, which of the following equations would you get?
  A $A = 5 + 3$
  B $A = 53$
  C $A = 5 \times 3$
  D $A = 5 - 3$

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3 If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?
A $10 = 6 + x$
B $10 = 3m + 2$
C $2 = 3m + 10$
D $P = 30 + 2$

4 If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?
A $12 = 40 + Q$
B $L = 40 + 12$
C $12 = 410 + Q$
D $10 = 48 + Q$

Fluency 5–8 6–9

Example 12 Applying a formula

Apply the formula for a rectangle's perimeter, $P = 2l + 2w$, to find:
a P when $l = 4$ and $w = 7$
b l when $P = 40$ and $w = 3$

Solution	Explanation
a $P = 2l + 2w$	Write the formula.
$P = 2 \times 4 + 2 \times 7$	Substitute in the values for l and w.
$P = 22$	Simplify the result.
b $P = 2l + 2w$	Write the formula.
$40 = 2l + 2 \times 3$	Substitute in the values for P and w to obtain an equation.
$40 = 2l + 6$ -6 -6 $34 = 2l$ $\div 2$ $\div 2$ $17 = l$ $\therefore\ l = 17$	Solve the equation to obtain the value of l.

Now you try

Apply the formula for the perimeter of an isosceles triangle, $P = 2a + b$, to find:
a P when $a = 3$ and $b = 4$
b a when $b = 6$ and $P = 20$

5 Look at the rule $A = 4p + 7$.
a Find A if $p = 3$.
b Find A if $p = 11$.
c Find A if $p = 0$.
d Find A if $p = 100$.

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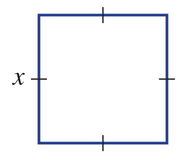
6 The perimeter of a square is given by $P = 4x$, where x is the width.



a Find the value of P if x is:

i 10

ii 3

iii 7.5

b Solve the equation $44 = 4x$.

c If $P = 44$, what is the width of the square?

7 Look at the rule $U = 8a + 4$.

a Find the value of a if $U = 20$. Set up and solve an equation.

b Find a if $U = 44$. Set up and solve an equation.

c Find a if $U = 92$. Set up and solve an equation.

8 Look at the relationship $y = 2x + 4$.

a Find y if $x = 3$.

b By solving an appropriate equation, find the value of x that makes $y = 16$.

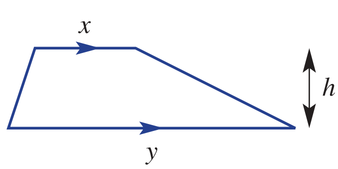
c Find the value of x if $y = 0$.

Hint for Q8c: Your answer will be a negative number.

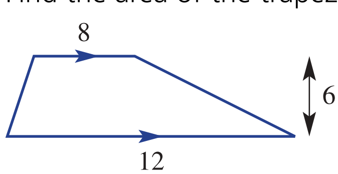
9 Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

Problem-solving and reasoning | 10, 11 | 10–12

10 The formula for the area of a trapezium is $A = \dfrac{h(x + y)}{2}$.

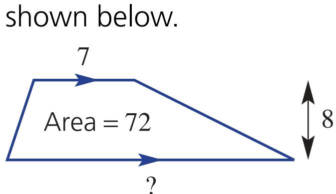


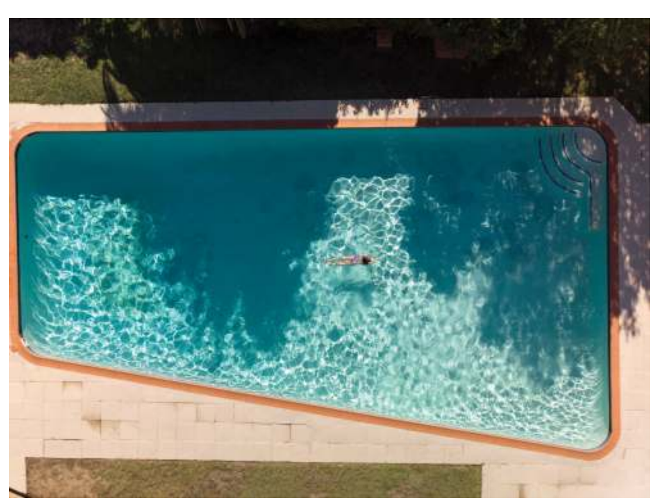
a Find the area of the trapezium shown below.



b Find the value of h if $A = 20$, $x = 3$ and $y = 7$.

c Find the missing value in the trapezium shown below.





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11 The cost, $\$C$, to hire a taxi for a trip of length d km is $C = 3 + 2d$.

a Find the cost of a 10 km trip (i.e. for $d = 10$).

b A trip has a total cost of $\$161$.

  i Set up an equation by substituting $C = 161$.

  ii Solve the equation algebraically.

  iii How far did the taxi travel? (Give your answer in km.)

12 Look at the rule $G = 120 - 4p$.

a If p is between 7 and 11, what is the largest value of G?

b Is it possible to make G equal to zero? What would p equal?

Mobile phone plans

— 13

13 Two companies have pre-paid mobile phone plans where the cost of a call depends on how much time (t minutes) you talk for.

Company A's cost in dollars: $A = 0.1 + 0.05t$

Company B's cost in dollars: $B = 0.06t$

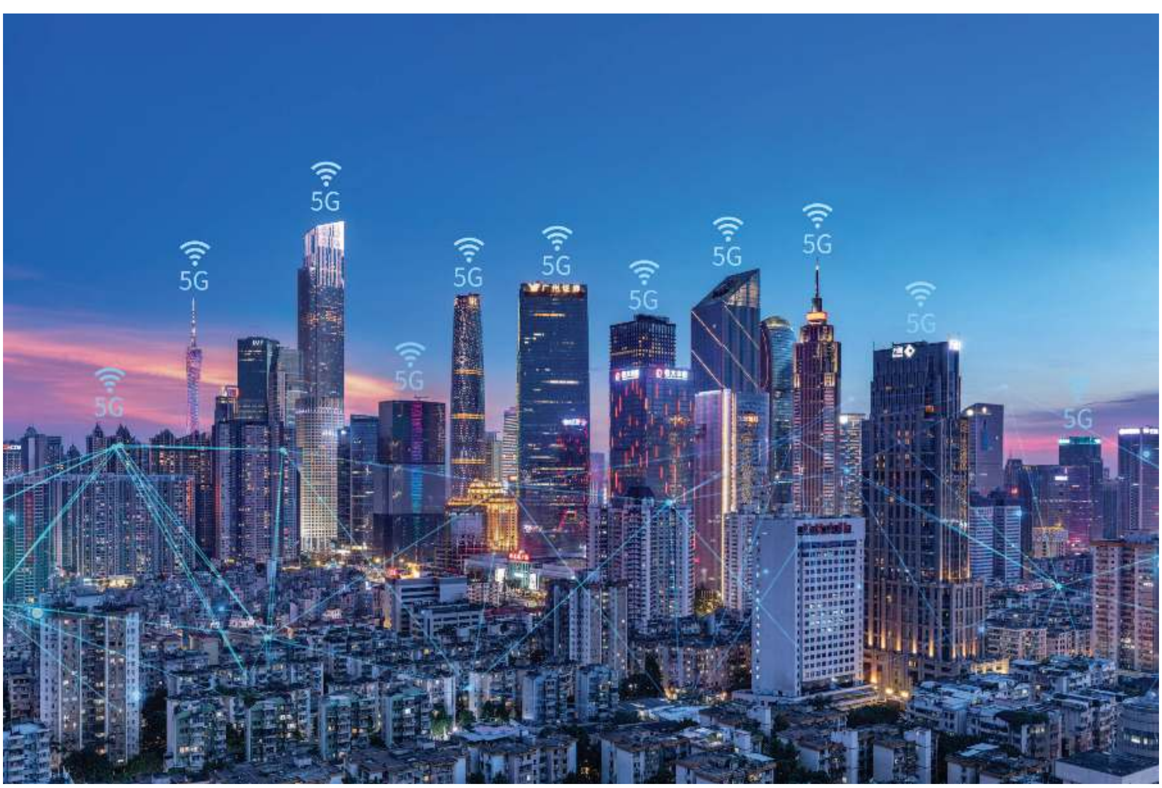
Hint for Q13: Use trial and error to solve this equation.

a Find the cost of a 20-minute call with each company.

b If company A charged $\$0.30$ for a call, how long did it take?

c If company B charged $\$0.30$ for a call, how long did it take?

d How long would a call have to be if the cost for company A and company B is the same?



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7G Applications

Learning intentions

- To understand that equations can be applied to real-world situations.
- To be able to solve problems using equations.

Key vocabulary: equation, model, variable, unknown

Most problems in science, engineering, finance and other fields can be solved by setting up and solving equations. One critical element of the process is to define the unknowns and then set up an appropriate equation.

Problems involving two people's ages can be expressed as an equation.

Lesson starter: Sibling sum

John and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 32, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if a is used for John's age?
- How would the equation change if the *product* of their ages is 32?

Key ideas

- An **equation** can be used to describe any situation in which two values are equal.
- To solve a problem, follow these steps.

1 Define **variables** to stand for **unknown** numbers.	**1** Let a = John's current age (from above example).
2 Write an equation to link the facts to the question.	**2** $a + a + 4 = 32$
3 Solve the equation if possible.	**3** $\begin{aligned} 2a + 4 &= 32 \quad (-4) \\ 2a &= 28 \quad (\div 2) \\ a &= 14 \end{aligned}$
4 Make sure you answer the original question and include the correct units (e.g. dollars, years, cm).	**4** John is 14 years old.

Exercise 7G

Understanding | 1–3 | 3

1 Match each of these worded descriptions with an appropriate expression.

a The sum of x and 3	**A** $2x$
b The cost of 2 apples if they cost $\$x$ each	**B** $x + 1$
c The cost of x oranges if they cost $\$1.50$ each	**C** $3x$
d Triple the value of x	**D** $x + 3$
e One more than x	**E** $1.5x$

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2 For the following problems, choose the equation to describe them.

a The sum of x and 5 is 11.

A $5x = 11$ **B** $x + 5 = 11$ **C** $x - 5 = 11$ **D** $11 - 5$

b The cost of 4 pens is \$12. Each pen costs \$$p$.

A $4 = p$ **B** $12p$ **C** $4p = 12$ **D** $12p = 4$

c Josh's age next year is 10. His current age is j.

A $j + 1 = 10$ **B** $j = 10$ **C** 9 **D** $j - 1 = 10$

d The cost of n pencils is \$10. Each pencil costs \$2.

A $n \div 10 = 2$ **B** 5 **C** $10n = 2$ **D** $2n = 10$.

3 For each of the following, choose the best variable definition to start solving the problem.

a Frank grew by 10 cm and is now 107 cm. How tall was Frank last year?

A Let f = Frank. **B** Let f = Frank's height this year.

C Let f = Frank's age. **D** Let f = Frank's height last year.

b Waleed worked for 20 hours and earned \$300. How much does he earn per hour?

A Let w = Waleed's height. **B** Let $w = 300$.

C Let w = Waleed's hourly wage. **D** Let $w = 20$.

c Louise spent \$400 on 12 identical calculators for her class. How much does a calculator cost?

A Let c = cost of one calculator. **B** Let c = number of calculators.

C Let l = Louise. **D** Let l = Louise's income.

Fluency

4–7 5–8

Example 13 Solving a problem using equations

The weight of 6 identical books is 1.2 kg. What is the weight of one book?

Solution

Let b = weight of one book.

$6b = 1.2$

$$\div 6 \quad 6b = 1.2 \quad \div 6$$
$$b = 0.2$$

The books weigh 0.2 kg each, or 200 g each.

Explanation

1 Define a variable to stand for the unknown number.

2 Write an equation to link the facts in the question.

3 Solve the equation.

4 Answer the original question. It is not enough to give the final answer as 0.2 — this is not the weight of a book, it is just a number.

Now you try

A mobile mechanic charges a call-out fee of \$90 plus \$80 per hour. The total cost of one visit is \$330. How long did the mechanic stay for?

4 Jerry buys 4 cups of coffee for \$14.

a Choose a variable to stand for the cost of one cup of coffee.

b Write an equation to link the facts to the question.

c Solve the equation.

d What is the cost of one cup of coffee?

Hint for Q4d: Remember to include the \$ sign in your answer.

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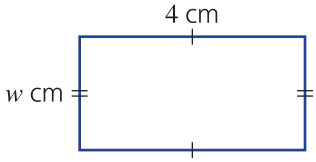
7G

5 A plumber charges a $\$70$ call-out fee and $\$80$ per hour. The total cost of a particular visit was $\$310$.
  a Define a variable to stand for the length of the visit in hours.
  b Write an equation to link the facts to the question.
  c Solve the equation.
  d What is the length of the plumber's visit?

6 When 6 chairs are bought, a "bulk buy" discount reduces the final price by $\$200$. The total becomes $\$1300$.
  a Define a variable for the cost of one chair.
  b Write an equation to link the facts to the question.
  c Solve the equation.
  d What is the cost of one chair?

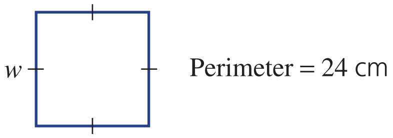
7 The combined age of twin girls is 26. Let a = the age of one girl.
  a Solve the equation $a + a = 26$.
  b How old is each girl?

8 The perimeter of this rectangle is 72 cm. Let w cm be the width.
  a Write an equation using the given diagram.
  b Solve the equation.
  c What is the width of the rectangle?



Problem-solving and reasoning | 9, 10 | 10, 11

9 A square has a perimeter of 24 cm.
  a Solve an equation to find its width.
  b What is the area of the square?



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Example 14 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

Solution

Let l = Luke's age.

$l + 2l = 60$

$$3l = 60 \quad (\div 3)$$
$$l = 20$$

Luke is 20 years old and Jane is 40 years old.

Explanation

1. Define a variable for the unknown. Once Luke's age is found, we can double it to find Jane's age.
2. Write an equation to link the facts in the question. Note that Jane's age is $2l$ because she is twice as old as Luke.
3. Solve the equation by first combining like terms.
4. Answer the original question. Include units and write a sentence answer.

Now you try

A rectangular paddock has a length which is 20 m longer than its width. Use a diagram and equation the find the paddock's width and area if the perimeter is 400 m.

10 Alison and Flynn's combined age is 40. Flynn is 4 years older than Alison.
- **a** Define a variable, write an equation and solve it to find Alison's age.
- **b** How old is Flynn?

Hint for Q10b: Include the units in your final answer.

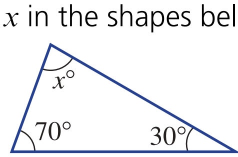
11 The length of a rectangular pool is 5 metres longer than the width.
The perimeter of the pool is 58 metres.
- **a** Draw a diagram of this situation.
- **b** Use an equation to find the pool's width.
- **c** What is the area of the pool?

Equational geometry — 12

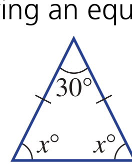
12 The sum of angles in a triangle is 180° and the sum of angles in a quadrilateral is 360°. Find the value of x in the shapes below by first solving an equation.

Hint for Q12a: $x + 70 + 30 = 180$

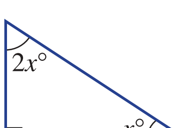
a



b



c



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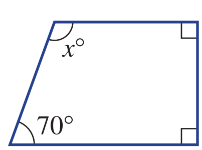
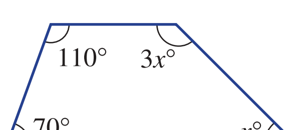


e



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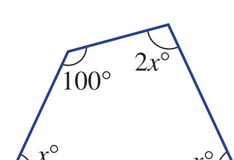

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7H Inequalities

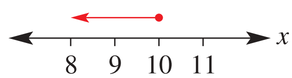
Learning intentions

- To understand that an inequality is a mathematical statement that one value is larger than (or as large as) another value
- To be able to represent inequalities on a number line using open or closed circles and/or arrows
- To be able to describe real-life situations using inequalities

Key vocabulary: inequality, number line

An inequality is like an equation but, instead of indicating that two expressions are equal, it indicates which of the two has a greater value. For example, $2+4<7$, $3\times5\geqslant15$ and $x\leqslant10$ are all inequalities. The first two are true, and the last one could be true or false depending on the value of x. For instance, the numbers 9.8, 8.45, 7 and -120 all make this inequality true.

We could represent all the values of x that make $x\leqslant10$ a true statement.



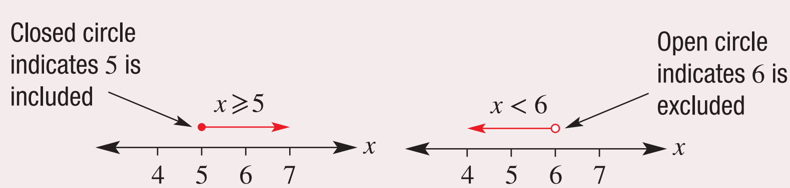
Lesson starter: Small sums

Two positive whole numbers are chosen: x and y. You are told that $x+y\leq5$.

- How many possible pairs of numbers make this true? For example, $x=2$ and $y=1$ is one pair and it is different from $x=1$ and $y=2$.
- If $x+y\leqslant10$, how many pairs are possible? Try to find a pattern rather than listing them all.
- If all you know about x and y is that $x+y>10$, how many pairs of numbers could there be? Explain.

Key ideas

- An **inequality** is a statement of the form:
  - LHS > RHS (greater than). For example: $5>2$
  - LHS ⩾ RHS (greater than or equal). For example: $7\geqslant7$ or $10\geqslant7$
  - LHS < RHS (less than). For example: $2<10$
  - LHS ⩽ RHS (less than or equal). For example: $5\leqslant5$ or $2\leqslant5$
- Inequalities can be reversed: $3<x$ and $x>3$ are equivalent.
- Inequalities can be represented on a number line, using closed circles at the end points if the value is included, or open circles if it is excluded.



- A range can be represented as a segment on the number line using appropriate closed and open end points.

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Exercise 7H

Understanding

1–4 | 2–4

1 Classify the following statements as true or false.
a $5 > 3$ b $7 < 5$ c $13 < 13$ d $13 \leqslant 13$

2 Match each of these inequalities with the appropriate description.
a $x > 5$ b $x < 5$ c $x \geqslant 3$ d $x \leqslant 3$
A The number x is less than 5.
B The number x is greater than or equal to 3.
C The number x is less than or equal to 3.
D The number x is greater than 5.

3 For each of the following, state whether they make the inequality $x > 4$ true or false.
a $x = 5$ b $x = -2$ c $x = 4$ d $x = 27$

4 If $x = 12$, classify the following inequalities as true or false.
a $x > 2$ b $x < 11$ c $x \geqslant 13$ d $x \leqslant 12$

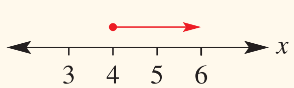
Fluency

5, 6(½), 7, 8(½) | 6(½), 7, 8(½)

Example 15 Representing inequalities on a number line

Represent the following inequalities on a number line.
a $x \geqslant 4$ b $x < 6$ c $1 < x \leqslant 5$

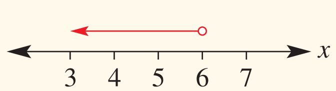
Solution

a


Explanation

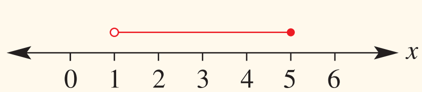
A circle is placed at 4 and then the arrow points to the right, towards all numbers greater than 4.

The circle is filled (closed) because 4 is included in the set.

b


A circle is placed at 6 and then the arrow points to the left, towards all numbers less than 6.

The circle is hollow (open) because 6 is not included in the set.

c


Circles are placed at 1 and 5, and a line goes between them to indicate that all numbers in between are included. The circle at 1 is open because the inequality is $<$ not $\leqslant$.

Now you try

Represent the following inequalities on a number line.
a $x \geqslant 5$ b $x < 9$ c $3 \leqslant x < 6$

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7H

5 Draw a number line from 0 to 8 and show these inequalities.

a $x \geqslant 3$ **b** $x < 7$ **c** $x \leqslant 2$

6 Represent the following inequalities on separate number lines.

a $x > 3$ **b** $x < 10$ **c** $x \geqslant 2$ **d** $x < -5$
e $x < -9$ **f** $x < -6$ **g** $x \geqslant -3$ **h** $x \leqslant 5$
i $10 > x$ **j** $2 < x$ **k** $5 \geqslant x$ **l** $-3 \leqslant x$

Hint for Q5–8: Use a filled circle for $\leqslant$ or $\geqslant$. Use a hollow circle for $<$ or $>$.

7 **a** List which of the following numbers make the inequality $2 \leqslant x < 7$ true.
8, 1, 3, 4, 6, 4.5, 5, 2.1, 7, 6.8, 2

b Represent the inequality $2 \leqslant x < 7$ on a number line.

8 Represent the following inequalities on separate number lines.

a $1 \leqslant x \leqslant 6$ **b** $4 \leqslant x < 11$ **c** $-2 < x \leqslant 6$ **d** $-8 \leqslant x \leqslant 3$
e $2 < x \leqslant 5$ **f** $-8 < x < -1$ **g** $7 < x \leqslant 8$ **h** $0 < x < 1$

Problem-solving and reasoning

9, 10 9–11

Example 16 Using inequalities to describe real-life situations

Describe the following situations as an inequality, using x to stand for the unknown quantity.

a Fred is shorter than 160 cm.

b John is at least as old as Maria, who is 10.

c Rose's test score is between 40 and 50 inclusive.

Solution	Explanation
a $x < 160$	Using x to stand for Fred's height in cm, x must be less than 160.
b $x \geqslant 10$	John is at least 10, so his age is greater than or equal to 10.
c $40 \leqslant x \leqslant 50$	x is between 40 and 50. The word 'inclusive' tells us that 40 and 50 are both included, so $\leqslant$ is used (rather than $<$ if the word 'exclusive' is used).

Now you try

Describe the following situations as an inequality, using x to stand for the unknown quantity.

a Michelle is taller than 170 cm.

b Peter's salary is at least \$100 000 per annum.

c The average rating for this café is between 3 and 4 exclusive.

9 For each of the following descriptions, choose an appropriate inequality from **A–H** below.

a John is more than 12 years old.

b Marika is shorter than 150 cm.

c Matthew is at least 5 years old but he is younger than 10.

d The temperature outside is between −12°C and 10°C inclusive.

A $x < 150$ **B** $x < 12$ **C** $x > 12$ **D** $x \leqslant 150$
E $10 \leqslant x \leqslant -12$ **F** $-12 \leqslant x \leqslant 10$ **G** $5 \leqslant x < 10$ **H** $5 < x \leqslant 10$

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10 It is known that Tim's age is between 20 and 25 inclusive, and Nick's age is between 23 and 27 inclusive.
  a If t = Tim's age and n = Nick's age, write two inequalities to represent these facts.
  b Represent both inequalities on the same number line.
  c Nick and Tim are twins. What is the possible range of their ages? Represent this on a number line.

11 An inequality statement can be reversed and have the opposite symbol used. For example, $2 < 5$ is equivalent to $5 > 2$, and $7 \geqslant 3$ is equivalent to $3 \leqslant 7$.
Write an equivalent statement for each of these inequalities.
  a $2 \leqslant 8$ b $6 > 4$ c $3 < x$ d $8 \geqslant y$

School grading system — 12

12 At a certain school the following grades are awarded for different scores.

Score	$x \geqslant 80$	$60 \leqslant x < 80$	$40 \leqslant x < 60$	$20 \leqslant x < 40$	$x < 20$
Grade	A	B	C	D	E

  a Convert the following scores into grades.
    i 15 ii 79 iii 80 iv 60 v 30
  b Emma got a B on one test, but her sister Rebecca got an A with just 7 more marks. What is the possible range for Emma's score?
  c Hugh's mark earned him a C. If he had scored half this mark, what grade would he have earned?
  d Alfred and Reuben earned a D and a C respectively. If their scores were added together, what grade or grades could they earn?
  e Michael earned a D and was told that if he doubled his mark he would have a B. What grade or grades could he earn if he got an extra 10 marks?

Bacteria multiply in temperatures $5°C \leqslant T \leqslant 60°C$; high-risk foods include meat, seafood, eggs and cooked rice. Bacteria hibernate in a freezer, $T \leqslant -18°C$, or in a fridge, $T < 5°C$. When food is cooked at $T \geqslant 75°C$, bacteria are killed.

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7I Solving inequalities

Learning intentions

- To understand that inequalities can be solved using equivalent inequalities
- To understand that the sign of an inequality is reversed if both sides are multiplied or divided by a negative number
- To understand that the sign of an inequality is reversed if the two sides are switched
- To be able to solve inequalities algebraically

Key vocabulary: inequality, equivalent, solve

Sometimes a problem arises in which an inequality is more complicated than something such as $x > 5$ or $y \leqslant 40$. For instance, you could have the inequality $2x + 4 > 100$. To **solve** an inequality means to find all the values that make it true. For the inequality above, $x = 50$, $x = 90$ and $x = 10\,000$ are all part of the solution, but the solution is best described as $x > 48$, because any number greater than 48 will make this inequality true and any other number makes it false.

The rules for solving inequalities are very similar to those for equations: perform the same operation to both sides. The one exception occurs when multiplying or dividing by a negative number. We can do this, but we must flip the sign because of the following observation.

Manufacturing companies employ financial analysts to determine the number of sales, n, to make a minimum profit. If an orange juice company makes a profit of $1.25/bottle, then for $1000/week minimum profit, solving: $1.25n \geqslant 1000$ gives $n \geqslant 800$ bottles/week to be sold.

$$5 > 2 \xrightarrow{\times(-1)} -5 > -2$$

Incorrect method

$$5 > 2 \xrightarrow{\times(-1)} -5 < -2$$

Correct method

Lesson starter: Limousine costing

A limousine is hired for a wedding. The charge is a $50 hiring fee plus $200 per hour.

- If the total hire time was more than 3 hours, what can you say about the total cost?
- If the total cost is less than $850 but more than $450, what can you say about the total time the limousine was hired?

Key ideas

- Given an inequality, an **equivalent** inequality can be obtained by:
  - adding or subtracting an expression from both sides
  - multiplying or dividing both sides by any positive number
  - multiplying or dividing both sides by a negative number and reversing the inequality symbol
  - swapping sides and reversing the inequality symbol.

  For example:

$$2x + 4 < 10 \xrightarrow{-4} 2x < 6 \xrightarrow{\div 2} x < 3$$

$$-4x + 2 < 6 \xrightarrow{-2} -4x < 4 \xrightarrow{\div(-4)} x > -1$$

Sign is reversed multiplying or dividing by a negative

$$2 > x + 1 \xrightarrow{-1} 1 > x \rightarrow x < 1$$

Sign is reversed when swapping sides

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Exercise 7I

Understanding 1–4 2–4

1 If $x = 3$, classify the following inequalities as true or false.
- **a** $x + 4 > 2$
- **b** $5x \geqslant 10$
- **c** $10 - x < 5$
- **d** $5x + 1 < 16$

2 State whether the following choices of x make the inequality $2x + 4 \geqslant 10$ true or false.
- **a** $x = 5$
- **b** $x = 1$
- **c** $x = -5$
- **d** $x = 3$

3 **a** State the missing number.

$$\begin{aligned} 2x &< 8 \\ &\downarrow \div 2 \\ x &< \_\_ \end{aligned}$$

b What is the solution to the inequality $2x < 8$?

4 **a** State the missing numbers.

$$\begin{aligned} 2x + 4 &\geqslant 10 \\ &\downarrow -4 \\ 2x &\geqslant \_\_ \\ &\downarrow \div 2 \\ x &\geqslant \_\_ \end{aligned}$$

b What is the solution to the inequality $2x + 4 \geqslant 10$?

c If $x = 7.1328$, is $2x + 4 \geqslant 10$ true or false?

Fluency 5–8(½) 6–8(½)

Example 17 Solving inequalities

Solve the following inequalities.

a $5x + 2 < 47$

b $\dfrac{3 + 4x}{9} \geqslant 3$

Solution

a

$$\begin{aligned} 5x + 2 &< 47 \\ &\downarrow -2 \\ 5x &< 45 \\ &\downarrow \div 5 \\ x &< 9 \end{aligned}$$

Explanation

The inequality is solved in the same way as an equation is solved: 2 is subtracted from each side and then both sides are divided by 5. The sign does not change throughout.

Continued on next page

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b

$$\frac{3+4x}{9} \geqslant 3$$
$$3+4x \geqslant 27$$
$$4x \geqslant 24$$
$$x \geqslant 6$$

$\times 9$ $\times 9$ -3 -3 $\div 4$ $\div 4$

The inequality is solved in the same way as an equation is solved. Both sides are multiplied by 9 first to eliminate 9 from the denominator.

Now you try

Solve the following inequalities.

a $3x+6<21$ **b** $\dfrac{4+2x}{3} \leqslant 2$

5 Solve the following inequalities.

a $x+3<7$ **b** $x-2>9$ **c** $x+4 \geqslant 6$ **d** $x-5 \leqslant 12$

6 Solve the following inequalities.

a $x+9>12$ **b** $4l+9 \geqslant 21$ **c** $8g-3>37$ **d** $2r-8 \leqslant 6$
e $9k+3>21$ **f** $8s-8<32$ **g** $8a-9>23$ **h** $2+n \geqslant 7$
i $9+2d \geqslant 23$ **j** $8+6h<38$ **k** $10+7r \leqslant 24$ **l** $6+5y<26$

7 Solve the following inequalities involving fractions.

a $\dfrac{d-9}{2}>10$ **b** $\dfrac{y+4}{2} \leqslant 7$ **c** $\dfrac{x-3}{4}>2$ **d** $\dfrac{q+4}{2} \leqslant 11$
e $\dfrac{2x+4}{3}>6$ **f** $\dfrac{7+3h}{2}<5$ **g** $\dfrac{4+6p}{4} \geqslant 4$ **h** $\dfrac{8j+2}{7}<6$

Example 18 Reversing the inequality

Solve the inequality $15-2x>1$

Solution

$$15-2x>1$$
$$-2x>-14$$
$$x<7$$

-15 -15 $\div(-2)$ $\div(-2)$

Explanation

15 is subtracted from each side.

Both sides are divided by -2. Because this is a negative number, the inequality is reversed from $>$ to $<$.

Now you try

Solve the inequality $20-3x<8$.

8 Solve the following inequalities involving negative numbers.

a $6-2x<4$ **b** $24-6s \geqslant 12$ **c** $43-4n>23$
d $34-2j<14$ **e** $2-9v \leqslant 20$ **f** $2-7j \leqslant 37$
g $48-8c \geqslant 32$ **h** $42-8h \leqslant 42$ **i** $7-8s>31$
j $6-8v>22$ **k** $10-4v \geqslant 18$ **l** $4-5v<29$

Hint for Q8: Remember to reverse the inequality whenever you divide by a negative number.

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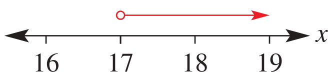
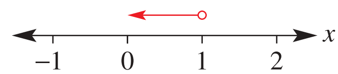
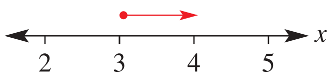
Problem-solving and reasoning | 9–11 | 9, 11–13

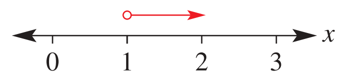
9 Match the following inequalities with their solutions depicted on a number line.

a $5x + 2 \geqslant 17$ **b** $\dfrac{x+1}{6} > 3$ **c** $9(x + 4) < 45$ **d** $5 - 2x < 3$

A


B


C


D


10 Kartik buys 4 cartons of milk and a \$20 phone card. The total cost of his shopping was greater than \$25.

a If c is the cost of a carton of milk, write an inequality to describe the situation above.

b Solve the inequality to find the possible values of c.

c If the milk's cost is a multiple of 5 cents, what is the minimum price it could be?

11 In AFL football the score is given by $6g + b$ where g is the number of goals and b is the number of behinds. A team scored 4 behinds and their score was less than or equal to 36.

a Write an inequality to describe this situation.

b Solve the inequality.

c Given that the number of goals must be a whole number, what is the maximum number of goals that they could have scored?

12 A puzzle is given below with four clues.
Clue A: $3x > 12$ Clue B: $5 - x \leqslant 4$ Clue C: $4x + 2 \leqslant 42$ Clue D: $3x + 5 < 36$

a Two of the clues are unnecessary. State which two clues are not needed.

b Given that x is a whole number divisible by 4, what is the solution to the puzzle?

13 Multiplying or dividing by a negative number can be avoided by adding the variable to the other side of the equation. For example:

$$\begin{array}{ccc} & -4x + 2 < 6 & \\ +4x & \downarrow & +4x \\ & 2 < 6 + 4x & \\ -6 & \downarrow & -6 \\ & -4 < 4x & \\ \div 4 & \downarrow & \div 4 \\ & -1 < x & \end{array}$$

This can be rearranged to $x > -1$, which is the same as the answer obtained using the method shown in the **Key ideas**. Use this method to solve the following inequalities.

a $-5x + 20 < 10$ **b** $12 - 2a \geqslant 16$ **c** $10 - 5b > 25$ **d** $12 < -3c$

Temperature inequality | — | 14

14 Recall that to convert temperature in degrees Celsius (C) to degrees Fahrenheit (F) the rule is $F = 1.8C + 32$. Pippa informs you that the temperature is between 59° and 68° Fahrenheit inclusive.

a Solve $1.8C + 32 \geqslant 59$.

b Solve $1.8C + 32 \leqslant 68$.

c Hence state the solution to $59 \leqslant 1.8C + 32 \leqslant 68$, giving your answer as a single inequality.

d Pippa later realised that the temperatures she gave you should have been doubled – the range was actually 118° to 136° Fahrenheit. State the range of temperatures in degrees Celsius, giving your answer as an inequality.

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Maths@Work

Financial officers at a local council

The financial officers who are employed by local councils need to work as part of a team, understand accounts, use Excel spreadsheets and pay attention to detail. They require mathematical skills to work with the equations and formulas that councils use to calculate rates.

Councils supply rate-payers with yellow lid recycling bins.

All Australian property owners pay rates to their local council for services such as waste management, public transport, roads and bridges, parks, libraries and future planning. The rates (a \$ amount) that councils charge vary with the value and category of each property. Each local council determines the rates due by using an equation that calculates the revenue (total money collected) needed to cover its expenditure (the money spent by a council).

1 'Rate in the dollar' is the rate that councils charge and it is the number of cents charged per dollar for the value, in dollars, of the property (cents/\$). For the following property values, calculate the council rates per year using 'Rate in the dollar' = 0.46 cents/\$.

a $\$350\,000$ **b** $\$425\,000$ **c** $\$784\,000$

Hint for Q1: Rates calculation
$= \$\text{property} \times c/\$ \div 100$
$= 350\,000 \times 0.46 \div 100$
$= \$1610$

2 Local councils can use CIV for the property values when calculating the annual rates payable. CIV stands for the Capital Improved Value and is the total market value of the land and buildings and any improvements.

One council uses the following equation to calculate its rates:

RATES (\$) = CIV × Rate in the dollar

RATES (\$) = CIV × 0.5739 cents/\$

a Use this equation to find the council rates on properties with CIV of:

i $\$500\,000$ **ii** $\$560\,000$ **iii** $\$675\,000$
iv $\$750\,000$ **v** $\$1\,000\,000$

b Write an equation to calculate quarterly rates, Q, payable from the annual rates, R.

3 Another council calculates its rates using:

- 'Rate in the dollar' = 0.482075 cents/\$, charged on land value, LV.
- plus, a fixed charge of $\$456.30$ for domestic waste services.

a Write an equation for the rates, R, payable on a property with land value, LV.

b Find the annual rates payable on a property with land value of $\$345\,000$.

c Thomas pays $\$428.70$/quarter. Using your formula, find the current land value, to the nearest \$, for Thomas's property.

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4 A different system of rate calculation uses the following equation:

Annual Rates(\$) $= AUV \times P + \$765$

AUV in \$ Average Unimproved Value of land	*P* Percentage rating factor
$1 - 150\,000$	0.2746%
$151\,000 - 300\,000$	0.39%
$300\,001 - 450\,000$	0.48%
$450\,001 - 600\,000$	0.54%
$600\,001$ and over	0.575%

Hint for Q4a: Annual Rates
$= AUV \times P + 765$
$= 260\,000 \times 0.0039 + 765$
$= \$1779$

a Calculate the annual rates payable to council on a property with
  i $AUV = \$295\,000$ **ii** $AUV = \$547\,000$

b Write an equation for annual rates payable for a property with $AUV = \$x$, where x lies between $\$450\,001$ and $\$600\,000$.

c If a property has $AUV = \$500\,000$, use your equation from part **b** to find the annual rates payable on this property.

d Calculate the quarterly rates on a property with $AUV = \$700\,000$.

e Write an equation that can be used to find the quarterly rates payable on a property with $AUV = \$y$ where y is over $\$600\,001$.

Using technology

5 Each local council in Australia determines, in advance, its total expenditure for the next financial year. The council must collect enough revenue from rates to pay for all the expenses.

a Set up the following Excel spreadsheet. Format columns B and C to Number (2 decimal places) and column D to Number (4 decimal places).

b Enter formulas to calculate Rate in the dollar in cents/\$. It equals the Expenditure divided by the Total of property values.

	A	B	C	D
1	Council Rate in the dollar calculations			
2	Council	Expenditure in millions	Total property value in billions	Rate in the dollar in cents/\$
3	A Large city	245.00	42.58	
4	B Medium city	117.50	24.45	
5	C Coastal city	97.50	19.85	
6	D Coastal town	14.50	3.05	
7	E Country town	8.95	2.54	

Hint for Q5b: In the column D formulas: Use brackets and multiply millions by $1\,000\,000$, billions by $1\,000\,000\,000$, and multiply by 100 to give the final answer in cents/\$.

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Modelling

Wedding marquee

Natasha and Mark wish to hire marquees for their wedding reception. They need to hire the marquees for a number of days to allow for preparation, the reception itself and pack up. The local supplier charges a fixed amount per marquee that covers the setting up and packing up of the marquee, as well as a cost per day to hire. The rates are shown in the table below.

Type	Total set up and pack up fee	Fee per day
Small	\$200	\$600
Large	\$620	\$1140

Natasha and Mark do not think that hiring one of the small marquees will provide enough space to house all the guests coming to the reception, so their options are to hire either two small marquees or one large marquee.

The marquee company only accepts the hiring of marquees for a whole number of days.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

1 Preliminary task

a Determine the cost of hiring one large marquee for:
 i 2 days **ii** 5 days.

b Determine the cost of hiring two small marquees (the setting up and packing up fee of \$200 must be paid on both marquees) for:
 i 2 days **ii** 5 days.

2 Modelling task

Formulate

The problem is to determine the cheapest marquee option for Natasha and Mark's wedding depending on the number of days that are required.

a Write down all the relevant information that will help solve this problem.

b Let C = the total hiring cost and n = the number of days hired. Construct a formula for the cost C, of hiring the following for n days:
 i 1 large marquee
 ii 2 small marquees.

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Solve

c Apply the cost formulas, from part **b**, to find the cost for 3 days hire of one large marquee or two small marquees. Which is the cheaper option?

d Mark and Natasha spend \$5180 hiring 1 large marquee. Complete the following:
 i Write the formula for the hiring cost C, of one large marquee for n days.
 ii Set up an equation by substituting $C = 5180$. Solve this equation algebraically for n.
 iii For how many days was the large marquee hired?

e Mark and Natasha spend \$6400 hiring 2 small marquees. Complete the following:
 i Write the formula for the hiring cost C, of 2 small marquees for n days.
 ii Set up an equation by substituting $C = 5200$. Solve this equation algebraically for n.
 iii For how many days were the 2 small marquee hired?

Evaluate and verify

f Show possible hiring costs for various numbers of days of hire. You could use a table like this one.

Cost for hiring marquees				
	2 days	3 days	4 days	5 days
2 small marquees				
1 large marquee				

g State the smallest number of days that Mark and Natasha can hire for so that the single large marquee is the cheaper option.

Communicate

h Summarise your results and describe any key findings.

3 Extension questions

The marquee company is considering changing the per day cost of the single large marquee but retaining the \$620 set up/pack up fee.
Let C = the total hiring cost and p = new price per day for the large marquee.

a Write a formula for the cost C of hiring a large marquee for 4 days, using a \$620 set up/pack up fee and p dollars per day.

b The cost of hiring this large marquee is to be \$5100 for the 4 days. Set up and solve an equation to find the new price per day, p.

c Using this new per day price, find the smallest number of days that they can hire for so that the single large marquee is the cheaper option.

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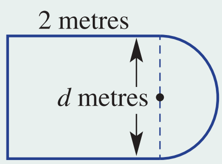
Technology and computational thinking

The 6 metre benchtop

Key technology: Spreadsheets

A kitchen company produces special stone benchtops including a rectangular island with a semicircular end.

One of their island benchtops includes a rectangle of length 2 metres and perimeter 6 metres as shown.



1 Getting started

a We know that the circumference of a circle is given by $C = \pi d$. Use this information to find the perimeter of a benchtop if the diameter, d metres, is the following. Round your answers correct to one decimal place.

i 1 metre **ii** 2 metres **iii** 0.7 metres

b Knowing that the perimeter of the benchtop for this kitchen company is 6 metres, use trial and error to find the value of d correct to one decimal place.

c Explain why the equation that relates to what we are finding in part **b** is $d + 4 + \frac{1}{2}\pi d = 6$.

2 Using technology

The equation above can be solved with a high degree of accuracy using a spreadsheet.

a Enter the given information into a spreadsheet.

b Fill down at cells A6, B5 and C5 until the perimeter is beyond 6 metres.

c Which value of d gives the perimeter which is closest to 6 metres? What is the percentage error shown in this case?

	A	B	C
1	**Stone benchtops**		
2			
3	***d***	**Perimeter**	**Percentage error**
4			
5	0.1	=A5+4+0.5*PI()*A5	=ABS(6-B5)/6*100
6	=A5+0.1		
7			

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3 Applying an algorithm

To obtain a more accurate solution for the value of d that gives a 6-metre perimeter, we will apply this algorithm.

- Step 1: Change the starting number in cell A5 to be closer to the solution for d.
- Step 2: Change the increment in cell A6 to a smaller number. Suggest 0.01, 0.001 etc.
- Step 3: Fill down until the perimeter is beyond 6 metres.
- Step 4: Choose the value of d that gives a perimeter closest to 6 metres and has the smallest percentage error.
- Step 5: Repeat from Step 1.

a Apply this algorithm until you are satisfied that you have found the solution for d, correct to two decimal places.

b Persist with your algorithm to now find the solution for d, correct to three or even four decimal places.

	A	B	C
1	**Stone benchtops**		
2			
3	***d***	**Perimeter**	**Percentage error**
4			
5	0.77	5.979513172	0.341447139
6	0.771	5.982083968	0.298600534
7	0.772	5.984654764	0.255753929
8	0.773	5.987225561	0.212907323
9	0.774	5.989796357	0.170060718
10	0.775	5.992367153	0.127214112
11	0.776	5.99493795	0.084367507
12	0.777	5.997508746	0.041520901
13	0.778	6.000079542	0.001325704
14	0.779	6.002650339	0.04417231
15	0.78	6.005221135	0.087018915
16	0.781	6.007791931	0.12986552

Technology and computational thinking

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Puzzles and games

1 Find the value of the square, triangle and circle using the following clues.
- $\square \times \triangle = 24$
- $\bigcirc + \bigcirc + \bigcirc + \bigcirc = 36$
- $\square - \triangle = \bigcirc + 1$
- $\square + \triangle + \triangle + \triangle = \bigcirc + \bigcirc$

2 Find the unknown value in the following puzzles.

a A number is halved, then halved again, then halved again. The result is 11.

b A number is tripled, then it is added to its original value. The result is 24.

c A number is increased by 2, then doubled, then increased by 3 and then tripled. The result is 99.

d The price of a shirt is increased by 10% for GST and then decreased by 10% on a sale. The new price is \$44. What was the original price?

e The average of a number and double that number is 30.

3 Consider the following solution that appears to show that $0 = 1$.

$$\begin{aligned} 2x + 5 &= 3x + 5 \\ -5 \downarrow \quad &\quad \downarrow -5 \\ 2x &= 3x \\ \div x \downarrow \quad &\quad \downarrow \div x \\ 2 &= 3 \\ -2 \downarrow \quad &\quad \downarrow -2 \\ 0 &= 1 \end{aligned}$$

a Which step caused the problem in this working? (Hint: Consider the actual solution to the equation.)

b Prove that $0 = 1$ is equivalent to the equation $22 = 50$ by adding, subtracting, multiplying and dividing both sides.

4 Consider the expressions below.

$4x + 2 \qquad 2(x + 4) \qquad 2x + 4 \qquad 4(x + 2) \qquad 4\left(x + \frac{1}{2}\right)$

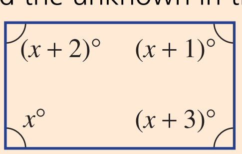
a If $x = 0$, which pairs are equal?

b Use two of the expressions above to form an equation that is always true.

c Use two of the expressions to form an equation that is never true.

5 Find the unknown in these geometric figures.

a



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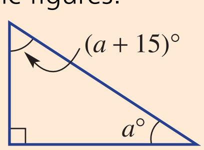

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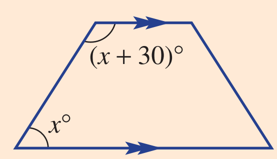

6 A certain pair of scales only registers weights between 100 kg and 150 kg, but it allows more than one person to get on at a time.

a If three people weigh themselves in pairs and the first pair weighs 117 kg, the second pair weighs 120 kg and the third pair weighs 127 kg, what are their individual weights?

b If another three people weigh themselves in pairs and get weights of 108 kg, 118 kg and 130 kg, what are their individual weights?

c A group of four children who all weigh less than 50 kg, weigh themselves in groups of three, getting the weights 122 kg, 128 kg, 125 kg and 135 kg. How much do they each weigh?

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Chapter summary

Solving equations

- Choose variables to make equation true

e.g. $x + 5 = 12$
Solution: $x = 7$

Backtracking

$3x = 12$

$\times 3$	
x	$3x$
4	12
$\div 3$	

Solution: $x = 4$

$4k + 6 = 42$

$\times 4$	$+6$	
k	$4k$	$4k + 6$
9	36	42
$\div 4$	-6	

Solution: $k = 9$

Checking solutions

Put variables in to see if equation is true.
Is $x = 3$ a solution to
$4x + 2 = 14$?
$4 \times 3 + 2 = 14 \longrightarrow$ true

Balancing

- Same operation to both sides

$$\begin{aligned} 3x &= 12 \quad (\div 3) \\ x &= 4 \end{aligned}$$

$$\begin{aligned} 4k + 6 &= 42 \quad (-6) \\ 4k &= 36 \quad (\div 4) \\ k &= 9 \end{aligned}$$

Equations

- A statement that two values are equal

e.g. $2 + 2 = 4$

LHS, Equal sign, RHS

Formulas ⭐

- Equations with 2 or more variables, one as the subject on the LHS.

e.g. $F = ma$
$S = 2x + 3$

- Substitute known values to find unknowns

Equations with fractions

- Multiply by denominator

$$\begin{aligned} \frac{x}{4} &= 10 \quad (\times 4) \\ x &= 40 \end{aligned}$$

$$\begin{aligned} \frac{k+3}{7} &= 9 \quad (\times 7) \\ k + 3 &= 63 \quad (-3) \\ k &= 60 \end{aligned}$$

Equations with brackets ⭐

Expand using distributive law

$$\begin{aligned} 2(x + 4) &= 14 \\ 2x + 8 &= 14 \quad (-8) \\ 2x &= 6 \quad (\div 2) \\ x &= 3 \end{aligned}$$

Combine like terms after expanding.

$4(x + 3) + 2x$ becomes
$4x + 12 + 2x \Rightarrow 6x + 12$

Applications

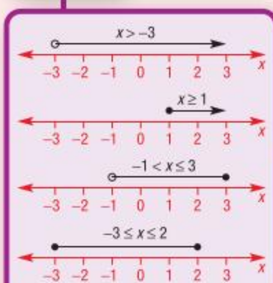
Whenever two things are equal

1. Define variables — Let \$$c$ = car cost
2. Write equation — $2c = 60000$
3. Solve equation — $c = 30000$
4. Answer question — A car costs \$30 000.

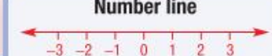
Inequalities

- $>$ greater than
- $\geq$ greater than or equal to
- $<$ less than
- $\leq$ less than or equal to

○ exclude
● include



Number line



$$\begin{aligned} 3 &> -1 \quad \div(-1) \\ -3 &< 1 \end{aligned}$$

Reverse the inequality sign

- Multiplying or dividing by a negative number
- Swapping sides

$$\begin{aligned} 6 - 3x &> 12 \quad (-6) \\ -3x &> 6 \quad \div(-3) \\ x &< -2 \end{aligned}$$

Inequality sign unchanged

- Add a number to both sides.
- Multiply or divide both sides by a positive number.
- Subtract a number from both sides.

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

			✔
7A	1	**I can classify equations as true or false** e.g. If $x = 10$, is the equation $x + 20 = 3 \times x$ true or false?	
7A	2	**I can state a solution to a simple equation** e.g. State a solution to the equation $4 + x = 25$.	
7A	3	**I can write an equation from a description** e.g. Write an equation for the following scenario: The number k is doubled, then three is added and the result is 52.	
7B	4	**I can solve simple equations using backtracking** e.g. Use backtracking to solve the equation $4k = 12$.	
7B	5	**I can solve two-step equations using backtracking** e.g. Solve the equation $2p - 5 = 15$ using backtracking.	
7C	6	**I can find equivalent equations** e.g. Show the result of adding 3 to both sides of the equation $5a - 3 = 12$.	
7C	7	**I can solve simple equations using the balancing method and check my solution** e.g. Solve $2u + 7 = 17$ using the balancing method and check the solution by substitution.	
7D	8	**I can solve simple equations involving fractions** e.g. Solve $\frac{k}{10} = 4$.	
7D	9	**I can solve equations involving fractions** e.g. Solve: **a** $\frac{4y + 15}{9} = 3$ **b** $4 + \frac{5x}{2} = 29$	
7E	10	**I can expand brackets** e.g. Expand the brackets for $2(5x + 3)$.	
7E	11	**I can solve equations with brackets by expanding and collecting like terms** e.g. Solve $4(2x - 5) + 3x = 57$ by first expanding any brackets.	
7F	12	**I can apply a formula to find unknown values by substituting** e.g. Apply the formula for a rectangle's perimeter, $P = 2l + 2w$, to find the value of P when $l = 10$ and $w = 7$.	

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		✔
7F	**13 I can apply a formula to find unknown values by solving** e.g. Apply the formula for a rectangle's perimeter, $P = 2l + 2w$, to find the value of l when $P = 40$ and $w = 3$.	
7G	**14 I can solve problems using equations** e.g. The weight of 6 identical books is 1.2 kg. Set up and solve an equation to find the weight of one book.	
7G	**15 I can use equations to solve problems involving two related unknowns** e.g. Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.	
7H	**16 I can represent an inequality on a number line** e.g. Represent $1 < x \leq 5$ on a number line.	
7H	**17 I can use inequalities to describe real-life situations** e.g. Fred is shorter than 160 cm. Describe this as an inequality, using x to stand for Fred's height in cm.	
7I	**18 I can solve inequalities algebraically** e.g. Solve $15 - 2x > 1$ algebraically.	

Chapter checklist

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Chapter review

Short-answer questions

7A

1 Are the following equations true (T) or false (F)?

- **a** If $x = 3$, then $3x = 6$.
- **b** If $a = 21$, then $a - 14 = \frac{a}{3}$.
- **c** $5 \times 4 = 10 + 10$

7A

2 State the solutions to these equations.

- **a** $4m = 16$
- **b** $m + 5 = 11$
- **c** $20 = 4q$
- **d** $z - 10 = 40$

7A

3 Write an equation to represent each of the following statements. You do not need to solve the equations.

- **a** Double m plus 3 equals 27.
- **b** The sum of n and four is tripled; the answer is 18.
- **c** The sum of two consecutive numbers, the first being x, is 7.

7B

4 Use backtracking to solve the following equations.

- **a** $3x + 2 = 14$
- **b** $4u + 5 = 21$
- **c** $3d - 5 = 13$
- **d** $2b - 1 = 13$
- **e** $6f - 2 = 16$
- **f** $12k + 3 = 27$

7C

5 Copy and complete the following equivalent equations.

- **a** $3x + 2 = 14$, -2 on both sides: $\_\_ = \_\_$
- **b** $2b - 1 = 13$, $+1$ on both sides: $\_\_ = \_\_$
- **c** $4x = 20$, $\div 4$ on both sides: $\_\_ = \_\_$

7C

6 For each equation below, state the first operation you would apply to both sides.

- **a** $15 + 2x = 45$
- **b** $\frac{x}{2} - 5 = 6$
- **c** $\frac{3a + 1}{2} = 11$

7C

7 Solve the following equations (using the balancing method).

- **a** $7a + 3 = 38$
- **b** $4b - 10 = 14$
- **c** $2n + 9 = 41$
- **d** $12 = 4c + 4$
- **e** $12 = 3 + x$
- **f** $10 = 8x - 6$

7D

8 Solve the following equations.

- **a** $\frac{m}{3} = 2$
- **b** $\frac{5x}{2} = 20$
- **c** $5 = \frac{k}{6}$
- **d** $\frac{2y}{3} = 12$
- **e** $\frac{k + 3}{11} = 5$
- **f** $10 = \frac{x - 2}{3}$

7E

9 Expand the brackets then simplify the following expressions.

- **a** $2(x + 5)$
- **b** $3(q - 10)$
- **c** $4(2r + 3)$
- **d** $5(x + 3) + 2x$
- **e** $4(z + 2) + 10$
- **f** $3(q - 5) + 2q$

7E

10 Solve the following equations by first expanding the brackets.

- **a** $2(x + 5) = 16$
- **b** $3(x + 1) = 9$
- **c** $5(p + 2) + p = 46$
- **d** $18 = 2(2x - 1)$
- **e** $3(2x + 1) + 4 = 67$
- **f** $5(k - 2) + 2k = 74$

7F

11 Look at the formula $F = ma$, relating force, mass and acceleration.

- **a** Find F, if $m = 10$ and $a = 3$.
- **b** Find m, if $F = 20$ and $a = 5$.
- **c** If $F = 100$ and $a = 100$, what is the value of m?

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7F **12** **a** If $P = 2(I + b)$, find I when $P = 48$ and $b = 3$.

b If $M = \dfrac{f}{f - d}$ find M when $f = 12$ and $d = 8$.

c If $F = \dfrac{5c}{2} + 20$, find c when $F = 30$.

7G **13** Hugo buys 4 mangoes and a \$20 gift voucher from the supermarket, giving a total cost of \$26.

a Let m = the cost of a mango. Which of the following equations describes this situation?

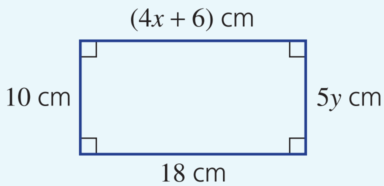
A $m = 20$ **B** $20m + 4 = 26$ **C** $4m = 20$

D $4m + 20 = 26$ **E** $4m + 26 = 20$

b Solve the equation chosen in part a.

c What is the cost of a mango?

7G **14** **a** Find the value of x and y for this rectangle.

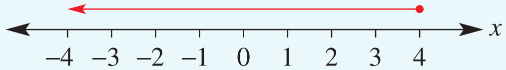


b The sum of three consecutive numbers is 39. First write an equation and then find the value of the smallest number.

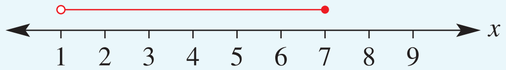
c The difference between a number and three times that number is 17. What is the number?

7H **15** Write the inequality represented by each number line below.

a



b



7H **16** Represent the following inequalities on separate number lines.

a $x \leq -1$ **b** $x > 2$ **c** $x < -1$

d $x \geq -2$ **e** $1 > x$ **f** $-3 \leq x$

g $x < 7$ **h** $-2 < x \leq 4$ **i** $-1 \leq x \leq 1$

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Chapter review

7H **17** Write an inequality to represent these situations, where x is the unknown value.

a The profit of a company is at least \$100 000.

b The cost of a new car cannot exceed \$6700.

c To ride on the roller-coaster, a person's height must be between 1.54 m and 1.9 m inclusive.

7I **18** Solve the following inequalities.

a $x + 3 > 5$ **b** $x - 2 < 6$ **c** $x - 2 < -6$

d $6x \geq 12$ **e** $4x < -8$ **f** $\frac{x}{4} \geq 2$

Multiple-choice questions

7A **1** If $x = 3$, then the value of $2x + 5$ is:

A 28 **B** 11 **C** 7 **D** 25 **E** 1

7A **2** If $a = 10$, which one of the following equations is true?

A $a + 5 = 10$ **B** $10 - a = 20$ **C** $a + a = 20$

D $3 = a - 5$ **E** $10 = a + 10$

7A **3** Which one of the following equations does not have the solution $x = 9$?

A $4x = 36$ **B** $x + 7 = 16$ **C** $\frac{x}{3} = 3$

D $x + 9 = 0$ **E** $14 - x = 5$

7B **4** The solution to the equation $6 = 2x$ is:

A $x = 12$ **B** $x = 3$ **C** $x = 6$ **D** $x = 4$ **E** $x = 8$

7C **5** The solution to the equation $3a + 8 = 29$ is:

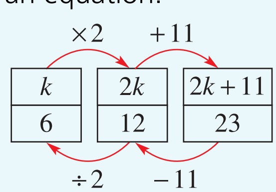
A $a = 21$ **B** $a = 12\frac{1}{3}$ **C** $a = 7$ **D** $a = 18$ **E** $a = 3$

7D **6** 'Three less than half a number is 4' can be expressed as an equation by:

A $\frac{x}{2} - 3 = 4$ **B** $\frac{(x-3)}{2} = 4$ **C** $2x - 3 = 4$

D $\frac{x}{2} + 3 = 4$ **E** $\frac{x+3}{2} = 4$

7B **7** A flowchart is used to solve an equation.



What equation is being solved?

A $k = 6$ **B** $2k + 11 = 6$ **C** $2k = 6$ **D** $2k + 11 = 23$ **E** $2k = 12$

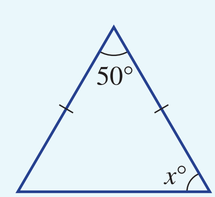
7E **8** The solution to the equation $3(m + 4) + m = 24$ is:

A $m = 7$ **B** $m = 8$ **C** $m = 4$ **D** $m = 1$ **E** $m = 3$

7F **9** Using the formula $F = 3k + b$, if $b = 7$ and $F = 34$ then k equals:

A 27 **B** 3 **C** 9 **D** 14 **E** 13

7G **10** An equation that could be used to find x in this isosceles triangle is:



A $50 + x = 180$ **B** $50 + 2x = 180$ **C** $2x = 180$

B $x = x$ **E** $50 = x$

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Extended-response questions

1 At a theme park, customers pay $\$10$ entry fee and then $\$5$ for each ride.

a Write an expression for the total cost to go on n rides.

b Inga spent a total of $\$55$ one afternoon at the theme park.

i Write an equation to link the cost of n rides with how much Inga spent.

ii Solve the equation.

iii How many rides did Inga go on?

A parent and three children visit the park together. The parent does not go on any rides so a formula for the total cost is:

$$T = 3(\underbrace{5n + 10}_{\text{ride plus entry cost for each child}}) + 10 \leftarrow \text{parent entry}$$

c If the children go on 4 rides together $(n = 4)$, what is the total cost?

d If the total cost was $\$145$, how many rides did the children go on?

2 To upload an advertisement to the www.searches.com.au website costs $\$20$ and then 12 cents whenever someone clicks on it.

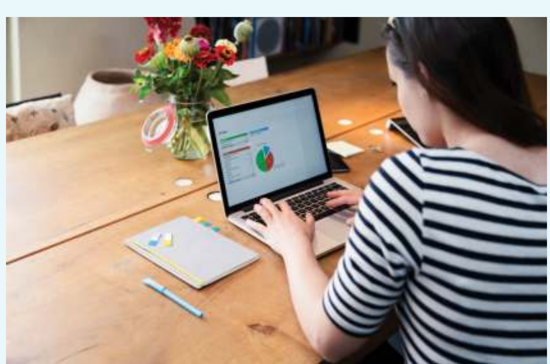
a Write a formula relating the total cost ($\$S$) and the number of clicks (n) on the advertisement.

b If the total cost is $\$23.60$, write and solve an equation to find out how many times the advertisement has been clicked on.

c To upload to the www.yousearch.com.au website costs $\$15$ initially and then 20 cents for every click. Write a formula for the total cost $\$Y$ when the advertisement has been clicked n times.

d If a person has at most $\$20$ to spend, what is the maximum number of clicks they can afford on their advertisement at yousearch.com.au?

e Use trial and error to find the minimum number of clicks for which the total cost of posting an advertisement to searches.com.au is less than the cost of posting to yousearch.com.au.



Chapter review

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8

Statistics and probability

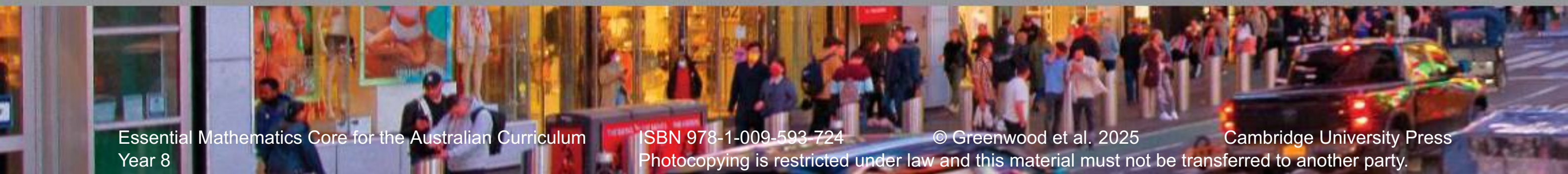
Essential mathematics: why skills in statistics and probability are important

Market research analysts conduct surveys for businesses such as retail stores, restaurants, game developers and movie studios to determine customer attitudes about products and service.

To make sense of all this data and to discover which items customers want to buy, the data is organised into tables and graphs and statistical measures like the mean, median and range are calculated. Statistical analysis can influence decisions about future promotions.

Selling techniques include TV ads; branded items in online games and movies; companies sponsoring sports competitions; using billboards; and running social media campaigns. Sales results are then statistically analysed to check the success of the various techniques.

Advertising agencies also conduct surveys to determine which age groups are more likely to respond to specific advertisements. Venn diagrams and two-way tables are effective tools for finding these probabilities.



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In this chapter

Australian Curriculum 9.0

STATISTICS

Investigate techniques for data collection including census, sampling, experiment and observation, and explain the practicalities and implications of obtaining data through these techniques (AC9M8ST01)

Analyse and report on the distribution of data from primary and secondary sources using random and non-random sampling techniques to select and study samples (AC9M8ST02)

Compare variations in distributions and proportions obtained from random samples of the same size drawn from a population and recognise the effect of sample size on this variation (AC9M8ST03)

Plan and conduct statistical investigations involving samples of a population; use ethical and fair methods to make inferences about the population and report findings, acknowledging uncertainty (AC9M8ST04)

PROBABILITY

Recognise that complementary events have a combined probability of one; use this relationship to calculate probabilities in applied contexts (AC9M8P01)

Determine all possible combinations for 2 events, using two-way tables, tree diagrams and Venn diagrams, and use these to determine probabilities of specific outcomes in practical situations (AC9M8P02)

Conduct repeated chance experiments and simulations, using digital tools to determine probabilities for compound events, and describe results (AC9M8P03)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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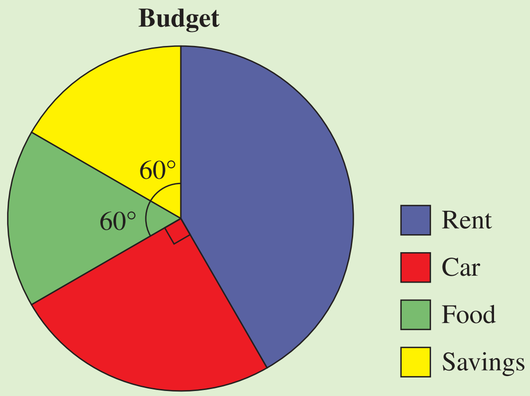
Warm-up quiz

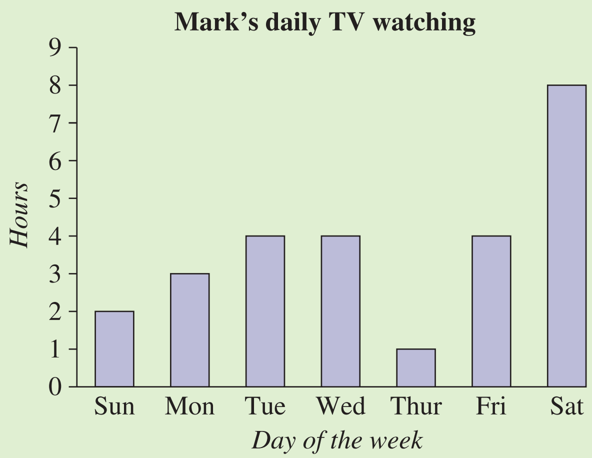
1 Arrange the following in ascending order.
a 2, 4, 10, 7, 1, 0, 6, 14, 9
b 101, 20, 30.6, 204, 36, 100
c 1.2, 1.9, 2.7, 1.7, 3.5, 3.2

2 Write down the total and the average (mean) as a decimal for each of the sets below.
a 4, 6, 8, 10 and 12
b 15, 17, 19, 19 and 24
c 0.6, 0.6, 0.6, 0.7 and 0.8

3 Use the information in the sector graph to answer the following questions.
a What fraction of the income was spent on food?
b What is the size of the angle for the rent sector?
c If $420 is saved each month, find how much is spent on:
i food?
ii the car?



4 **a** How many hours of television were watched on Wednesday?
b How many hours of television were watched on Monday?
c On which day was the most TV watched?
d How many hours of TV were watched over the week shown?
e What fraction of Saturday was spent watching TV?



5 If a fair die is rolled, state the probability as a fraction that:
a the number 2 is rolled
b an even number is rolled
c a number less than 5 is rolled
d the number 8 is rolled.

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8A Interpreting graphs and tables

CONSOLIDATING

Learning intentions

- To be able to interpret column graphs, line graphs and pie charts.
- To be able to interpret data presented in a table.

Key vocabulary: column graph, pie chart, sector graph, line graph, divided bar graph

Statistics give us a way to understand information about our world, from weather patterns to outcomes of scientific experiments. Graphs and tables are the most commonly used ways to represent and display data that has been collected.

From microscope to graph: visualising data from scientific experiments in column graphs, line graphs, and pie charts helps us understand the world.

Lesson starter: Movement graphs

This is a whole class activity.

Two volunteers are needed: the *walker* who completes a journey, and the *grapher* who graphs the journey on the whiteboard. All other students in the class are *support graphers* and draw their own graph of the journey in their books.

A graph can show the distance walked relative to the time taken.

1 To set up your graph, draw two axes: the vertical axis labelled 'Distance from the front of the room' and horizontal labelled 'Time'. No numbers are required.
2 The walker starts from the front of the room, walks steadily to the back of the classroom, stops for a few seconds and then walks steadily back to the front of the room.
3 As the walker moves, all graphers draw a line on their graphs to show the walker's distance from the front of the room versus time. No numbers are required.
4 At the end of the journey, discuss the graph that has been drawn on the whiteboard.
   - Do you agree with this graph?
   - How does the graph show that the walker stopped for a short time?
   - The total distance walked is increasing so why does the graph start and end with zero distance?
5 Make up some other journeys and repeat this activity.
   Some ideas you could try are: walking slowly then quickly, stopping and then reversing direction for a few steps, starting the 'journey' in the middle of the room or at the back of the room.
6 Now complete this activity in reverse. A graph is drawn on the whiteboard and a volunteer walks to match the graph. The class checks to make sure the 'walker' is following the graph correctly.

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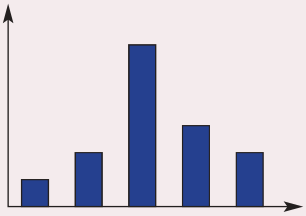
8A

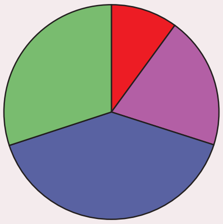
Key ideas

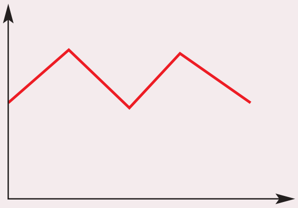
- Data can be represented as a graph or a table.
  The numbers in a table link the row and column headings.
  For example:

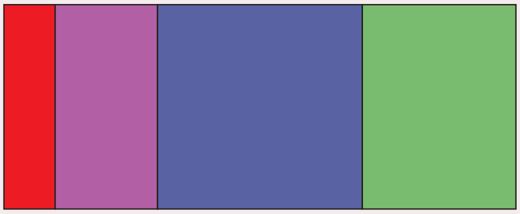
	Boys	Girls
Year 8	53 (Year 8 boys)	50 (Year 8 girls)
Year 9	49 (Year 9 boys)	52 (Year 9 girls)

- Common types of graphs include:


Column graphs


Pie charts (also called sector graphs)


Line graphs


Divided bar graphs

Exercise 8A

Understanding 1, 2 2

1 Name four different types of graphs used to illustrate data.

2 The following table shows the population of some small towns over a 10-year period.

Year	Expton	Calcville	Statsland
2010	400	200	300
2015	320	240	310
2020	180	270	290

a What was the population of Expton in 2015?
b What was the population of Calcville in 2020?
c What was the population of Statsland in 2010?
d Which town's population kept decreasing over time?
e Which town's population kept increasing over time?

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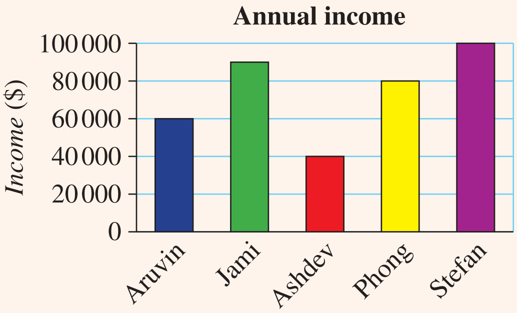
Fluency

3–7 | 4–8

Example 1 Interpreting column graphs

This column graph represents the annual income of five different people.

a What is Aruvin's annual income?
b What is the difference between Jami's income and Ashdev's income?
c Who earns the most?

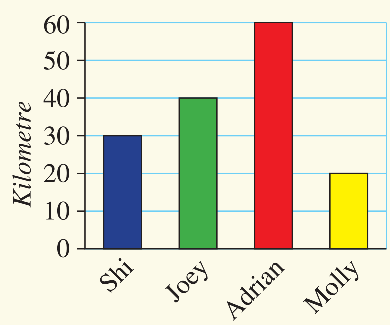


Solution	Explanation
a $60 000	Reading off the graph, Aruvin earns $60 000.
b 90 000 − 40 000 = $50 000	Jami earns $90 000 and Ashdev earns $40 000, so the difference is $50 000.
c Stefan earns the most.	With the highest column, Stefan earns the most ($100 000).

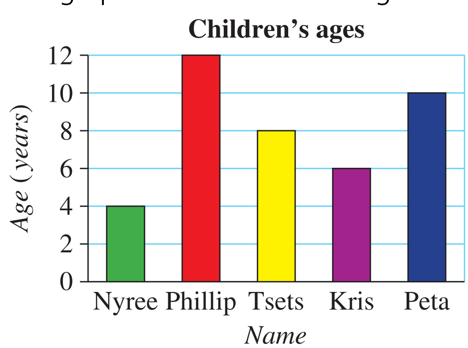
Now you try

This column graph represents the number of kilometres run by four people during a week.

a How far did Joey run?
b How much further did Adrian run compared to Shi?
c Who ran the least?



3 The graph below shows the ages of 5 children.



a How old is Peta?
b How old is Kris?
c Who is the oldest of the five children?
d Who is the youngest of the five children?
e What is the difference in age between Tsets and Nyree?

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4 Six Year 8 classes are asked to vote for which sport they would like to do next in Physical Education. Their results are shown in the table.

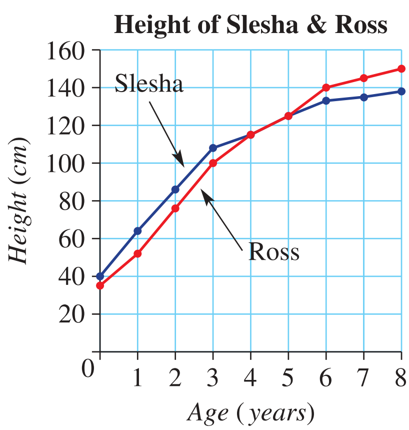
Sport	8A	8B	8C	8D	8E	8F
Badminton	3	5	7	0	8	12
Water polo	9	9	8	14	11	9
Handball	12	10	11	11	7	3

Hint for Q4: For **a**, look in the 8B column.

Hint for Q4: For **d**, add the numbers in the rows.

a How many students in 8B want to do water polo?
b How many students in 8A want to do handball?
c What is the most popular sport in 8F?
d Find the total number of students that chose each sport.
e Which sport had the most votes in total?

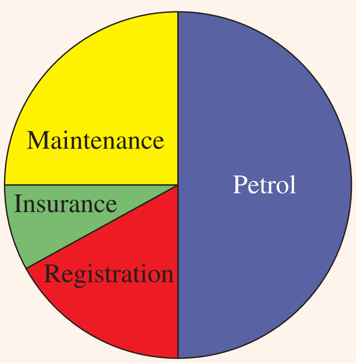
5 The line graph shows the height of Slesha and her twin brother Ross from the time they were born.
a Which of the children was taller on their first birthday?
b Which of the children was taller on their eighth birthday?
c On which birthdays were the twins the same height?



Example 2 Interpreting sector graphs

A car owner graphs the amount of money spent per year on car-related expenses.

a What is the largest expense each year?
b What percentage of the car's expenses is devoted to maintenance?
c If the car owner spends \$3000 per year on petrol, what is the total amount spent on the car each year?



Solution	Explanation
a Petrol	Since petrol occupies the largest area of the graph, it is the largest expense.
b 25%	Maintenance occupies $\frac{1}{4}$ of the graph's area, which equals 25%.
c 50% of expenses $= \$3000$ $\times 2 \downarrow \quad \downarrow \times 2$ 100% of expenses $= \$6000$ The car owner spends \$6000 each year.	Petrol occupies half the graph's area, which is 50%. This is doubled to find the total amount spent.

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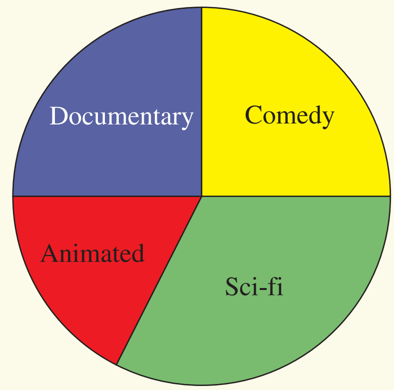
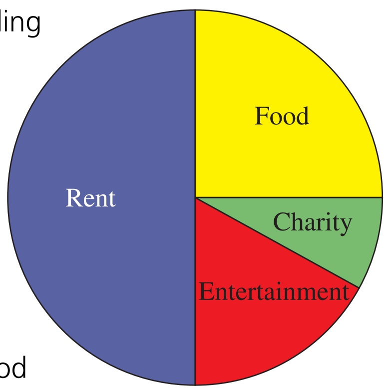
Now you try

This pie chart shows the amount of time that Freddie watches different types of TV shows.

a What percentage of time is devoted to comedy?
b If Freddie watches 20 hours of TV in a week, how many hours does he spend watching documentaries?



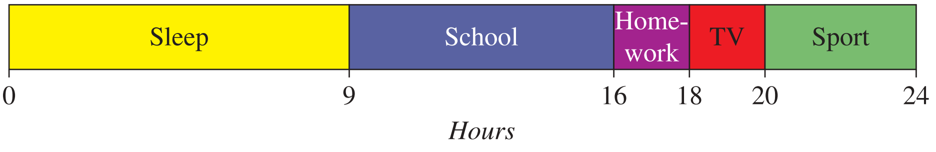
6 This sector graph shows one person's spending in a month.
a What is the largest expense in that month?
b What is the smallest expense in that month?
c What percentage of the month's spending was on rent?
d What percentage was spent on food?
e If the person spent a total of $600 on food in the month, what was their total spending?



Hint for Q6e:

$$\begin{aligned} 25\% &= \$600 \\ 100\% &= \,? \end{aligned} \quad \times ?$$

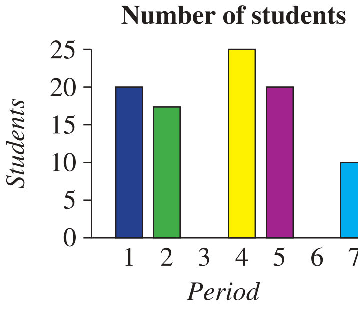
7 A student has recorded how she spent her time on one day, shown in the divided bar graph below.



a How much time did she spend doing homework on that day?
b How much time was spent at school during that day?
c What did she spend the most time doing?
d What fraction of her day was spent playing sport?

Hint for Q7: Use the scale to find the width of each section.

8 A teacher records the number of students in her room during a 7-period day in a column graph.
a How many students were in the room during period 1?
b During which periods was the classroom empty?
c During which period was the smallest class in the room?
d One class used the room twice on that day. In which periods was that class in the room?



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8A

Problem-solving and reasoning

9, 10 | 10–12

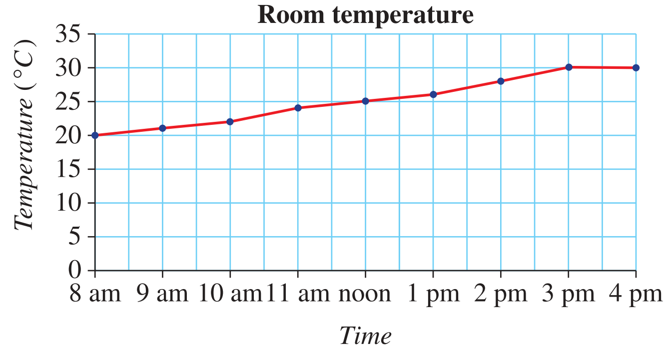
9 The temperature in a classroom is graphed over an eight-hour period.

a What was the temperature at 8 am?

b By how much did the temperature increase in the eight-hour period from 8 am to 4 pm?

c Students complain that it is uncomfortably hot when the temperature is 25°C or greater. At what time does it become uncomfortably hot?

Hint for Q9b: First state the temperature at 8 am and at 4 pm.



10 Two families, the Red family driving a red car and the Blue family driving a blue car, leave Coffs Harbour together at 8 am and drive to the Gold Coast.

a Why does a flat part of the graph show the car is stopped?

b When did the Red family stop for a morning tea break?

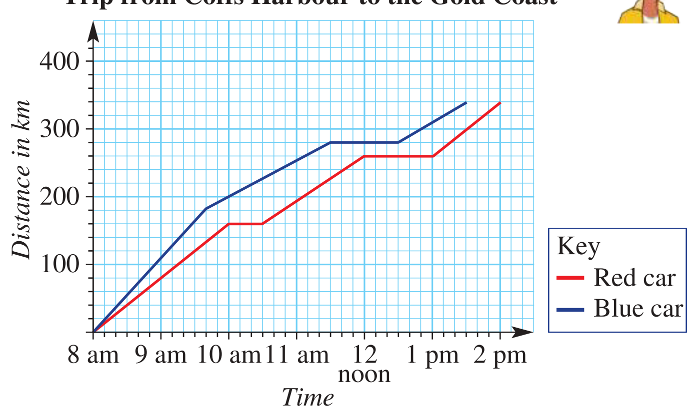
c How far had the blue car travelled when the Blue family stopped for lunch?

d How long did the Blue family stop for lunch?

e What distance had each family travelled at 10 am?

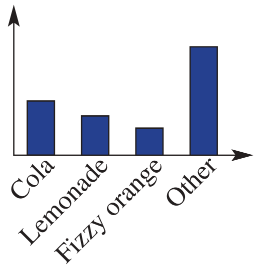
f What was the total driving time for each family (exclude stops)?

Hint for Q10a: Does the distance or time change along the flat part of the graph?

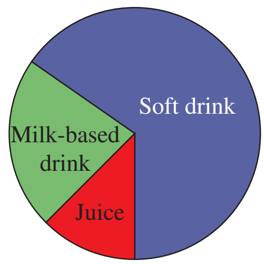


11 Three different surveys are conducted to establish whether soft drinks should be sold in the school canteen.

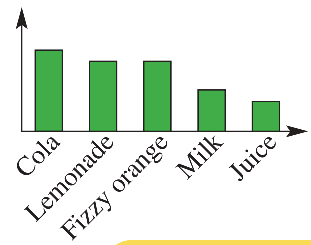
Survey 1 : Favourite drink



Survey 2 : Favourite type of drink



Survey 3 : Sugar content per drink

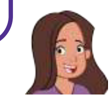


a Which graph could be used to show the financial benefit to the canteen of selling soft drinks?

b Which graph could be used to show there was not much desire for Fizzy orange?

c Which graph could be used to show how unhealthy soft drink is?

Hint for Q11a: Financial benefit comes from a lot of sales.



Hint for Q11c: Unhealthy drinks have a high sugar content.

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12 The population of three nearby towns is shown over a 10-year period.

	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Town A	1414	1277	1204	1118	1026	1083	1171	1254	1317	1417
Town B	1062	1137	1188	1285	1371	1447	1502	1571	1665	1728
Town C	1042	1100	1174	1250	1312	1176	1075	992	895	783

a Match these statements with the correct town.
  1 The population increased then decreased.
  2 The population decreased then increased.
  3 The population kept increasing.

b Find the average population of the three towns in 2020. Round your answer to the nearest whole number.

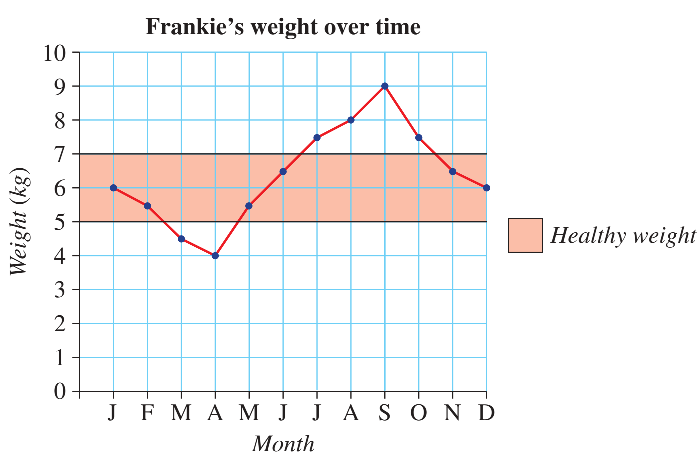
c Find the average population for town C over the 10-year period. Round your answer to the nearest whole number.

Hint for Q12a: Observe how the numbers change along the rows.

Hint for Q12b and c:
Average = $\dfrac{\text{total}}{\text{number of values}}$

Weight over time — 13

13 The owner of a dog called Frankie has graphed his weight over the year.



a If a healthy weight for Frankie is between 5 kg and 7 kg, fill out the following table.

	Underweight	Healthy weight	Overweight	Total
Number of months				12
Fraction of 12				
Angle				

b Represent the results of the table as a pie chart.

c What is the advantage of a line graph over a pie chart?

d What is the advantage of a pie chart over a line graph?

e Draw a line graph showing another dog's weight over 12 months, given that the dog is underweight for 2 months, overweight for 3 months and the healthy weight for 7 months.

Hint for Q13b:
Sector angle = Fraction $\times 360°$.

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8B Frequency tables and tallies

Learning intentions

- To understand that a tally can be used for counting data as it comes in.
- To be able to interpret tallies.
- To be able to construct a tally and frequency table from a set of data.

Key vocabulary: tally, tally marks, frequency table, data

Often the actual values in a set of data are not required – just knowing how many numbers fall into different categories is all the information that is needed. A frequency table allows us to do this by listing how often the different values occur.

Frequency tables can be used for listing particular values or ranges of values.

Number of cars	Frequency
0	10
1	12
2	5
3	3

Age	Frequency
0–4	7
5–9	12
10–14	10
15–19	11

Lesson starter: Subject preferences

- Survey a group of peers to find their favourite school subject out of Maths, English, Science, Music and Sport.
- Represent your results in a table like the one below.

	Maths	English	Science	Music	Sport
Tally	$\cancel{\|\|\|\|}$	$\cancel{\|\|\|\|}\ \|$	$\cancel{\|\|\|\|}\ \|\|\|$	$\|\|\|\|$	$\|\|$
Frequency	5	6	8	4	2

- How would you expect the results to differ for different classes at your school, or for different schools?

Key ideas

- A **tally** is a tool used for counting as results are gathered. Numbers are written as vertical lines called **tally marks** with every 5th number having a cross through a group of lines.
  e.g. 4 is $||||$ and 7 is $\cancel{||||}\ ||$.
- A **frequency table** has a column for the items (values or categories) and another column for the frequency of each item.

  The frequency shows how often each item occurs. A tallying column is also often used as **data** is gathered.
- The items can be individual values or intervals of values.
  - Individual scores

Hours worked	4	5	6	7	8	9
Tally	$\|\|$	$\|\|\|\|$	$\cancel{\|\|\|\|}$	$\cancel{\|\|\|\|}\ \|$	$\cancel{\|\|\|\|}\ \|\|$	$\|\|\|$
Frequency	2	4	5	6	7	3

  - Scores in intervals or groups

Hours worked	4–6	7–9
Tally	$\cancel{\|\|\|\|}\ \cancel{\|\|\|\|}\ \|$	$\cancel{\|\|\|\|}\ \cancel{\|\|\|\|}\ \cancel{\|\|\|\|}\ \|$
Frequency	11	16

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Exercise 8B

Understanding | 1–3 | 3

1 The table below shows survey results for students' favourite colours.

Colour	Frequency
Red	5
Green	2
Orange	7
Blue	3

Are the following true (T) or false (F)?

a 5 people chose red as their favourite colour.
b 9 people chose orange as their favourite colour.
c Blue is the favourite colour of 3 people.
d More people chose green than orange as their favourite colour.

2 Fill in the blanks.

a The tally |||| represents the number ______ .
b The tally ~~||||~~ || represents the number ______.
c The tally ______ represents the number 2.
d The tally ______ represents the number 11.

Hint for Q2: ~~||||~~ = 5

3 This is a list of some students' handspans measured in cm.
19, 18, 20, 17, 22, 19, 22, 20, 24, 18, 20, 19
Copy and complete each of these frequency tables.

a

Handspans	Frequency
17	
18	
19	
20	
21	
22	
23	
24	

b

Handspans	Frequency
17–19	
20–22	
23–25	

Hint for Q3: Frequency for 17–19: Count how many values were 17, 18 or 19.

Fluency | 4–6 | 4, 5, 7

Example 3 Interpreting tallies

The different car colours along a quiet road are noted.

White	Black	Blue	Red	Yellow
|||	~~||||~~ ~~||||~~ |||	~~||||~~ ~~||||~~ ~~||||~~ ||	~~||||~~ |	~~||||~~ ||||

a Convert the following tally into a frequency table.
b How many red cars were seen?
c What was the total number of cars seen?

Continued on next page

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Solution

a

Colour	White	Black	Blue	Red	Yellow
Frequency	3	13	17	6	9

b 6 red cars were seen.

c 48 cars seen in total.

Explanation

Each tally is converted into a frequency. For example, black is two groups of 5 plus 3, giving $10 + 3 = 13$.

This can be read directly from the table.

$3 + 13 + 17 + 6 + 9 = 48$
Add the frequencies to find the total.

Now you try

The heights of some people were recorded rounded to the nearest cm as shown.

Height (cm)	150–159	160–169	170–179	180–189
Tally	||	𝍸 |	𝍸 𝍸 ||	||||

a Convert the table into a frequency table.
b How many people were recorded as 170–179 cm tall?
c How many people were recorded in this experiment?

4 A basketball player's performance in one game is recorded in the following table.

	Passes	Shots at goal	Shots that go in	Steals
Tally	|||	𝍸 𝍸 ||	𝍸 |||	||
Frequency				

a Copy and complete the table, filling in the frequency row.
b How many shots did the player have at goal?
c How many shots went in?
d How many steals did the player have during the game?

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Example 4 Constructing tables from data

Put the following data into a frequency table: 1, 4, 1, 4, 1, 2, 3, 4, 6, 1, 5, 1, 2, 1.

Solution

Number	1	2	3	4	5	6
Tally	卌 |	||	|	|||	|	|
Frequency	6	2	1	3	1	1

Explanation

Construct the tally as you read through the list. Then go back and convert the tally to frequencies.

Now you try

Put the following data into the given frequency table:

15, 9, 7, 19, 24, 42, 16, 14, 3, 26, 37, 30, 21.

Number	0–9	10–19	20–29	30–39	40–49
Tally					
Frequency					

5 A student surveys her class to ask how many people are in their family. The results are:
6, 3, 3, 2, 4, 5, 4, 5, 8, 5, 4, 8, 6, 7, 6, 5, 8, 4, 7, 6

- **a** Construct a frequency table. Include a row for family size, a tally row and a frequency row.
- **b** How many students have exactly 5 people in their family?
- **c** How many students have at least 6 people in their family?

Hint for Q5: Check that the number of scores in the list equals the total of the frequencies in the table.

6 Braxton surveys a group of people to find out how much time they spend watching television each week. They give their answers rounded to the nearest hour.

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–168
Tally	卌	|||	卌 卌 ||	卌 卌 卌	卌 ||||	||||	||

- **a** Draw a frequency table of his results, converting the tallies to frequencies.
- **b** How many people altogether did Braxton survey?
- **c** How many people spend 15–19 hours per week watching television?
- **d** How many people watch television for less than 5 hours per week?
- **e** How many people watch television 2 hours *per day* or less on average?

Hint for Q6b: Add the frequencies to find the total number surveyed.

Hint for Q6d: Less than 5 hours doesn't include 5 hours. So 0–1 and 2–4.

7 The heights of a group of 21 people are shown below, given to the nearest cm.
174 179 161 132 191 196 138 165 151 178 189
147 145 145 139 157 193 146 169 191 145

- **a** Copy and complete the frequency table below.

Height (cm)	130–139	140–149	150–159	160–169	170–179	180–189	190+
Tally							
Frequency							

- **b** How many people are in the range 150–159 cm?
- **c** How many people are 180 cm or taller?
- **d** How many people are between 140 cm and 169 cm tall?

Hint for Q7: 180 cm or taller means 180–189 and 190+.

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8B

Problem-solving and reasoning

8, 9 | 9–11

8 A tennis player records the number of double faults they serve per match during one month.

Double faults	0	1	2	3	4	5
Frequency	4	2	1	0	2	1

a How many matches did they play in total during the month?
b How many times did they serve exactly 1 double fault?
c In how many matches did they serve no double faults?
d How many double faults did they serve in total during the month?

Hint for Q8: Add the frequencies to find the total number of matches played.

9 Match each of these data sets with the correct column (A, B, C or D) in the frequency table shown below.
a $1, 1, 2, 3, 3$
b $1, 2, 2, 2, 3$
c $1, 1, 1, 2, 3$
d $1, 2, 3, 3, 3$

Hint for Q9: Column A starts with 3 lots of 1.

Number	A Frequency	B Frequency	C Frequency	D Frequency
1	3	2	1	1
2	1	1	1	3
3	1	2	3	1

10 Five different classes are in the same building in different rooms at the same time. The ages of students in each room are recorded in the frequency table below.

Age	Room A Frequency	Room B Frequency	Room C Frequency	Room D Frequency	Room E Frequency
12	3	2	0	0	0
13	20	18	1	0	0
14	2	4	3	0	10
15	0	0	12	10	11
16	0	0	12	10	11
17	0	0	0	1	0

a How many students are in room C?
b How many students are in the building?
c How many 14-year-olds are in the building?
d What is the average (mean) age of students in room B? Answer to one decimal place.
e Make a frequency table showing age and the number of each age group in the building.

Hint for Q10d: Average $= \frac{\text{sum}}{n}$

11 Some exam results are presented in the frequency table below.

0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–100
0	0	3	1	2	5	8	12	10	2

a Redraw the table so that the intervals are of width 20 rather than 10 (i.e. so the first column is 0–19, the second is 20–39, and so on).
b Redraw the table with the intervals 0–29, 30–59, 60–89, 90–100.

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Homework puzzle — 12

12 Priscilla records the numbers of hours of homework she completes each evening from Monday to Thursday. Her results are shown in this frequency table.

Number of hours	Frequency
1	1
2	1
3	2

a On how many nights did Priscilla do 3 hours of homework?

b One possibility is that she worked 3 hours on Monday, 2 hours on Tuesday, 3 hours on Wednesday and 1 hour on Thursday. Copy and complete this table to show other ways her time could have been allocated for the four nights.

Monday	Tuesday	Wednesday	Thursday
☐ hours	☐ hours	☐ hours	☐ hours
☐ hours	☐ hours	☐ hours	☐ hours
☐ hours	☐ hours	☐ hours	☐ hours

c Priscilla's brother Joey did homework on all five nights. On two nights he worked for 1 hour, on two nights he worked for 2 hours and on one night he worked for 3 hours. Show three ways that the table below could be filled in to match his description.

Monday	Tuesday	Wednesday	Thursday	Friday
☐ hours	☐ hours	☐ hours	☐ hours	☐ hours

d Calculate the average hours of homework per night for Priscilla and Joey.

e How many hours more homework per week would Joey have to do over 5 nights to make his average per night equal to Priscilla's average?

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8C Graphs of frequency tables

Learning intentions

- To understand that a frequency table can be represented as a graph.
- To be able to construct a graph from a frequency table.
- To understand that dot plots can be used for small collections of discrete numerical data.
- To be able to construct and analyse a dot plot.

Key vocabulary: frequency table, graph, vertical axis

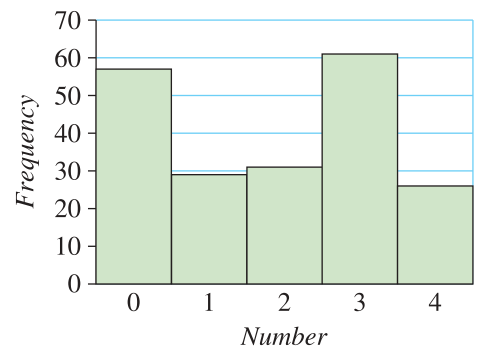
A graphical representation of a frequency table can be constructed so that patterns can be observed more easily.

For example, the data below is represented as a frequency table and as a frequency graph.

a As a table

Number	Frequency
0	57
1	29
2	31
3	61
4	26

b As a graph



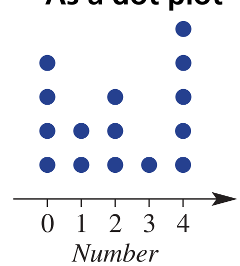
At a glance you can see from the graph that 0 and 3 are about twice as common as the other values. This is harder to read straight from the table. A graph makes comparisons of frequency easier.

An alternative way to show a frequency table graphically is with a dot plot, where each data point is shown as a single dot.

As a table

Number	Frequency
0	4
1	2
2	3
3	1
4	5

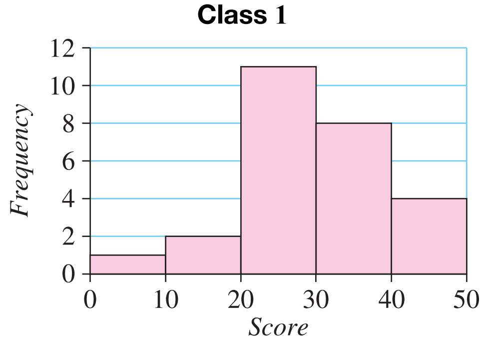
As a dot plot



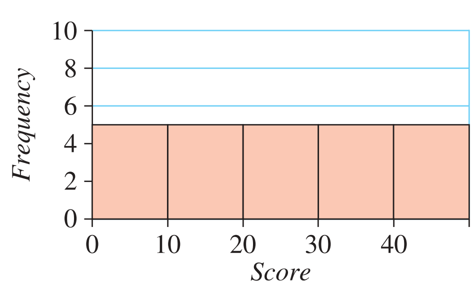
Lesson starter: Test analysis

The results for some end-of-year tests are shown for four different classes in four different graphs.

Class 1



Class 2



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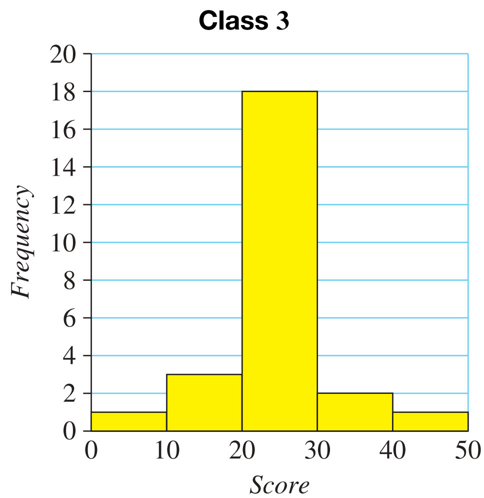
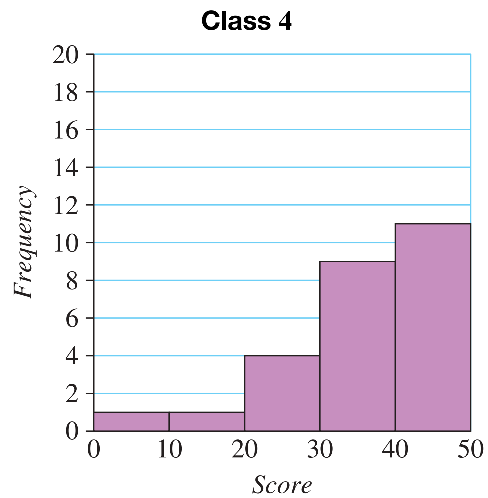


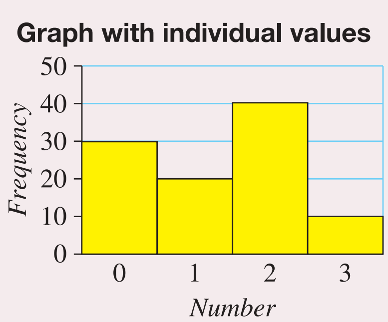


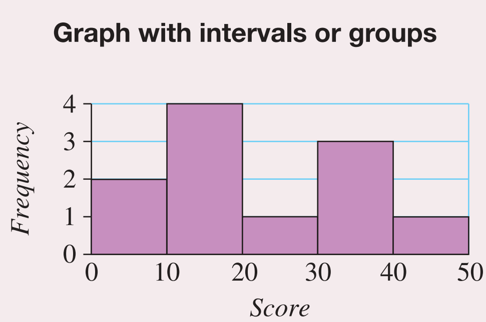
Work with a classmate and discuss the answers to these questions.

1 Choose which class has results that can be described as:
   a a few low scores, a few high scores and a lot of scores around the middle.
   b equal numbers of students getting low, middle and high scores.
   c more students getting high scores than low scores.
   d more students getting middle scores than either high or low scores.
2 Which class has the highest average score?
3 Which class has the highest overall score?
4 Which class would be the easiest to teach and which would be the hardest, do you think?

Key ideas

- **Frequency tables** can be represented using a **graph**.
- The **vertical axis** (y-axis) is used to represent the frequency of each item.
- Sometimes values are grouped (e.g. 0–9, 10–19, 20–29) before a graph is drawn.





- A half column-width space is sometimes placed between the vertical axis and the first column of the graph if the first vertical bar does not start at zero.
- A dot plot is an alternative graph of frequency tables for discrete data with a small number of values.
- An outlier is a value noticeably distinct from the main cluster of points in a dot plot.

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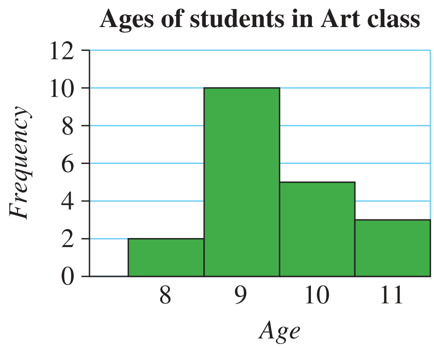
Exercise 8C

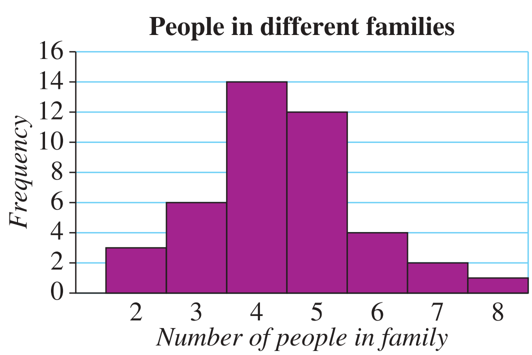
Understanding

1–3 | 3

1 This graph below shows the ages of people in an Art class.
  a How many 8-year-olds are in this class?
  b What is the most common age for students in this class?
  c What is the age of the oldest person in the class?



2 A survey is conducted of the number of people in different families. The results are shown in this graph.
  a What is the most likely number of people in a family, on the basis of this survey?
  b How many people responding to the survey said they had a family of 6?
  c What is the least likely number (from 2 to 8) of people in a family, on the basis of this survey?



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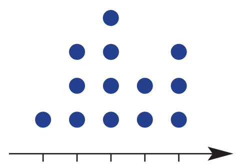
3 The graph on the right shows the number of computers owned by a group of surveyed households.
   a How many households were surveyed?
   b How many households had 4 computers?
   c What was the most common number of computers owned?



Fluency

4, 5, 6–8(½) | 5, 6–8(½)

Example 5 Constructing and interpreting dot plots

a Construct a dot plot for the frequency table shown of the number of books read in the past month.

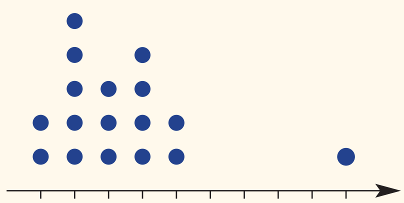
b Use your dot plot to find the most common number of books read.

c Use your dot plot to identify any outliers.

Number of books	Frequency
0	2
1	5
2	3
3	4
4	2
9	1

Solution

a



Explanation

The scale is chosen to fit all the values from the minimum (0) to the maximum.

Each dot represents one value, so there are 5 dots above 1, representing the frequency of 5.

b 1

The highest point is at 1, with a frequency of 5.

c 9 books is an outlier.

The dot at 9 is separated from the main cluster.

Now you try

a Construct a dot plot for the frequency table shown.

b Use your dot plot to find the most common number of games.

c Use your dot plot to identify any outliers.

Games	Frequency
1	3
2	1
3	4
4	2
5	1
10	1

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8C

4 Construct a dot plot for the following frequency tables.

a

Number	Frequency
0	3
1	5
2	1
3	2

b

Number	Frequency
1	4
2	2
3	1
4	3
5	2

5 **a** Construct a dot plot for the frequency table showing the number of aces served by a tennis player.
b Use your dot plot to find the most common number of aces served.
c Use your dot plot to identify any outliers.

Aces	Frequency
0	1
1	3
2	4
3	2
4	6
5	1
10	1

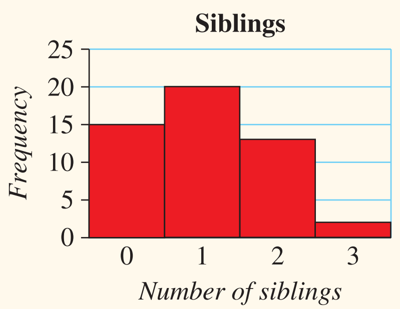
Hint for Q5: Your horizontal axis should go from 0 to 10.

Example 6 Constructing graphs from frequency tables with individual labels

Represent this frequency table as a graph.

Number of siblings	Frequency
0	15
1	20
2	13
3	2

Solution



Explanation

The scale 0–25 is chosen to fit the highest frequency (20).

Each different number of siblings in the frequency table is given a column in the graph.

Now you try

Represent this frequency table as a graph.

Number of goals	Frequency
0	6
1	4
2	10
3	2

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6 Represent the following frequency tables as graphs.

a

Number of pets	Frequency
0	5
1	3
2	5
3	2
4	4

b

Number of bikes	Frequency
0	3
1	9
2	3
3	10
4	7

c

Age	Frequency
12	15
13	10
14	25
15	20
16	28

d

Number of cars	Frequency
0	4
1	5
2	4
3	2

Hint for Q6: Remember to rule up even scales.

7 For the following sets of data:

i create a frequency table

ii draw a graph from the frequency table.

a 1, 2, 5, 5, 3, 4, 4, 4, 5, 5, 5, 1, 3, 4, 1

b 5, 1, 1, 2, 3, 2, 2, 3, 3, 4, 3, 3, 1, 1, 3

c 4, 3, 8, 9, 7, 1, 6, 3, 1, 1, 4, 6, 2, 9, 7, 2, 10, 5, 5, 4

d 60, 52, 60, 59, 56, 57, 54, 53, 58, 56, 58, 60, 51, 52, 59, 59, 52, 60, 50, 52

Hint for Q7: Frequency table

number	tally	frequency

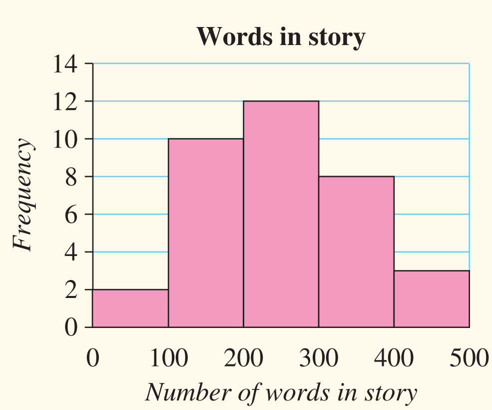
Hint for Q7: Frequency shows how many of each number.

Example 7 Constructing graphs from frequency tables using intervals

Draw the frequency table below as a graph.

Number of words in story	Frequency
0–99	2
100–199	10
200–299	12
300–399	8
400–500	3

Solution



Explanation

The scale 0–14 is chosen to fit the highest frequency (12).
The different intervals (0–99 words, 100–199 words etc.) are displayed on the horizontal axis.

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8C

Now you try

Draw the frequency table below as a graph.

Number of characters in a book	Frequency
0–9	4
10–19	18
20–29	12
30–39	6
40–49	2

8 Represent the following frequency tables as graphs.

a

Score	Frequency
0–19	1
20–39	4
40–59	10
60–79	12
80–100	5

b

Age	Frequency
0–5	5
6–10	12
11–15	14
16–20	11
21–25	5
26–30	8
31–35	2
36–40	1

Hint for Q8: Frequency is shown on the vertical axis.

Hint for Q8: Mark even scales.

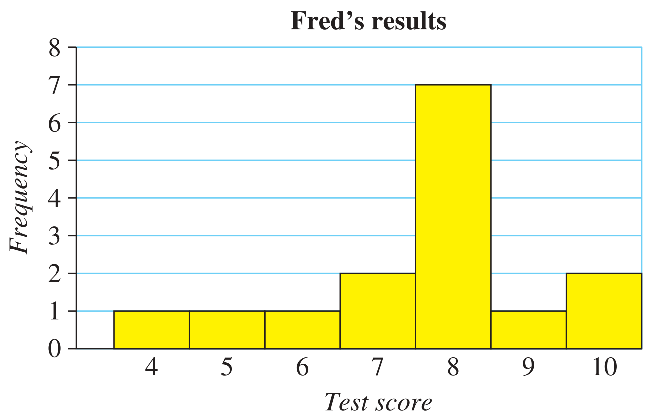
Problem-solving and reasoning

9–11 | 10–13

9 Edwin records the results for his spelling tests out of 10.
They are 3, 9, 3, 2, 7, 2, 9, 1, 5, 7, 10, 6, 2, 6, 4.

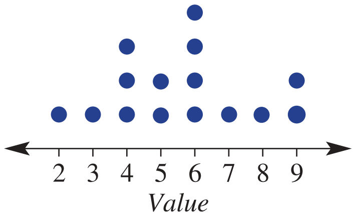
a Draw a graph for his results.

b Fred's results are given by the graph shown below. Is Edwin a better or a worse speller generally than Fred? Give a reason for your answer.



10 **a** Draw a frequency table to match the dot plot shown.

b If one of the data points is removed, the dot plot and frequency table will both be reduced in size. What is the point?



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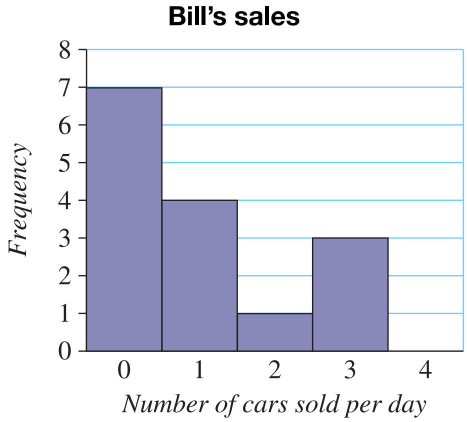
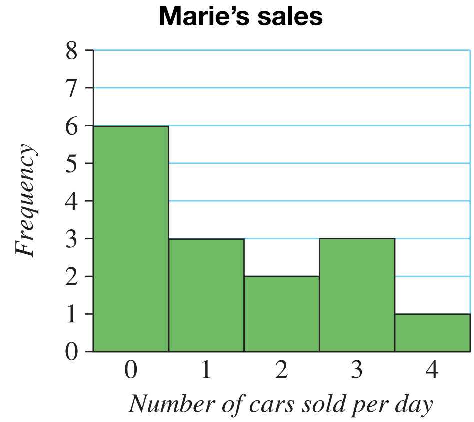
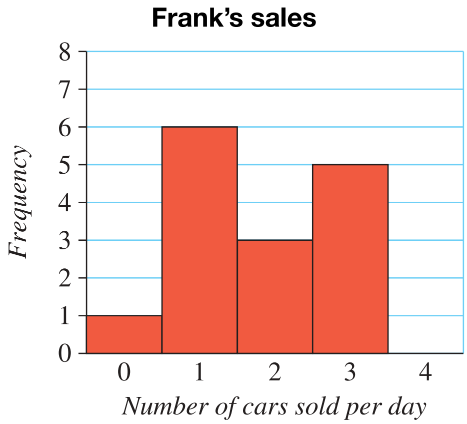
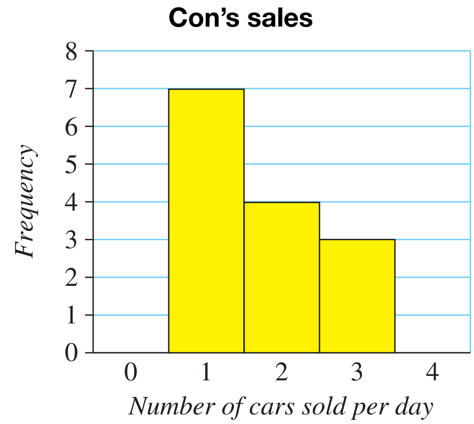
11 A car dealership records the number of sales each salesperson makes per day over three weeks.









- **a** On how many days did Bill not make any sales?
- **b** For how many days did Bill sell one car per day?
- **c** What is the record for the greatest number of sales in one day and who holds this record?
- **d** Which salesperson made at least one sale every day?
- **e** Over the whole period, which salesperson made the most sales in total? How many cars did they sell?
- **f** Over the whole period, which salesperson made the fewest sales in total? How many cars did they sell?

12 This graph shows the ages of a group of people in a room.

- **a** Which part of this graph would change if a graph is drawn for the ages of the same group of people in exactly 12 years' time?
- **b** How would this graph look if it showed the ages of the same group of people exactly 12 years ago?

Hint for Q12: The frequency shows the number of people of each age.

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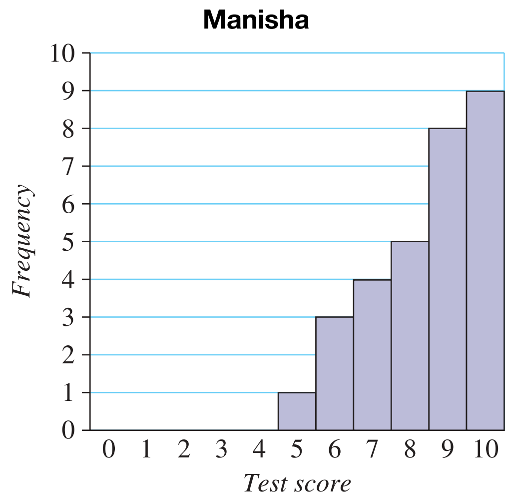
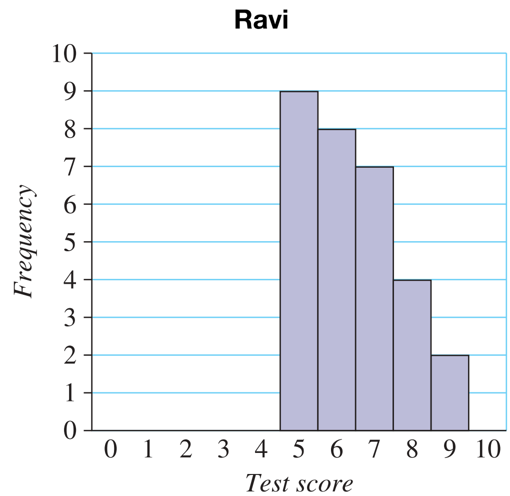
8C

13 Two students have each drawn a graph that shows their results for a number of spelling tests. Each test is out of 10 and there has been one test per week for 30 weeks.



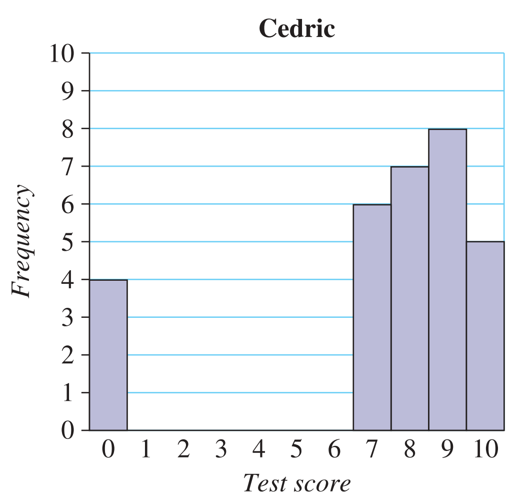


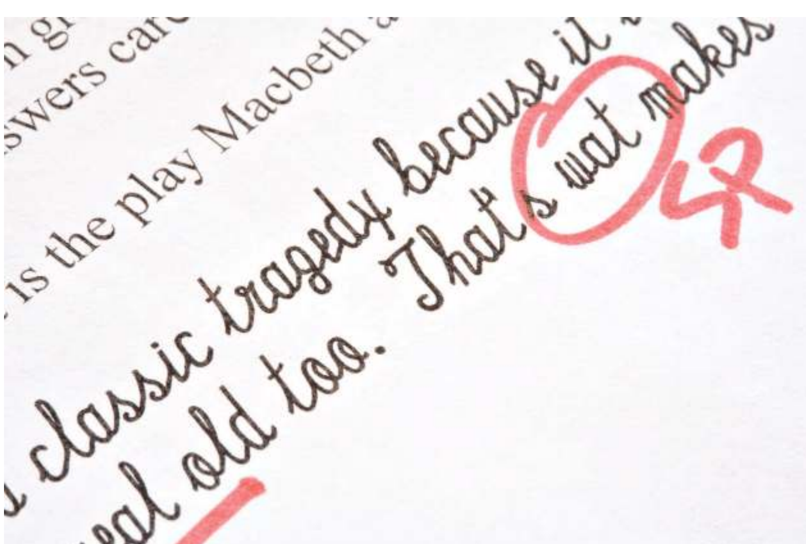
a Manisha's scores started very high but have got worse during the year.
Give an example of a list of scores that Manisha might have received over the 30 weeks.

b Ravi's spelling has actually improved consistently over the course of the year.
Give an example of a list of the scores he might have received for the 30 weeks.

c A third student, Cedric, has the following results.
What is a likely explanation for the '0' results?

Hint for Q13: The frequency shows the number of tests for each result.





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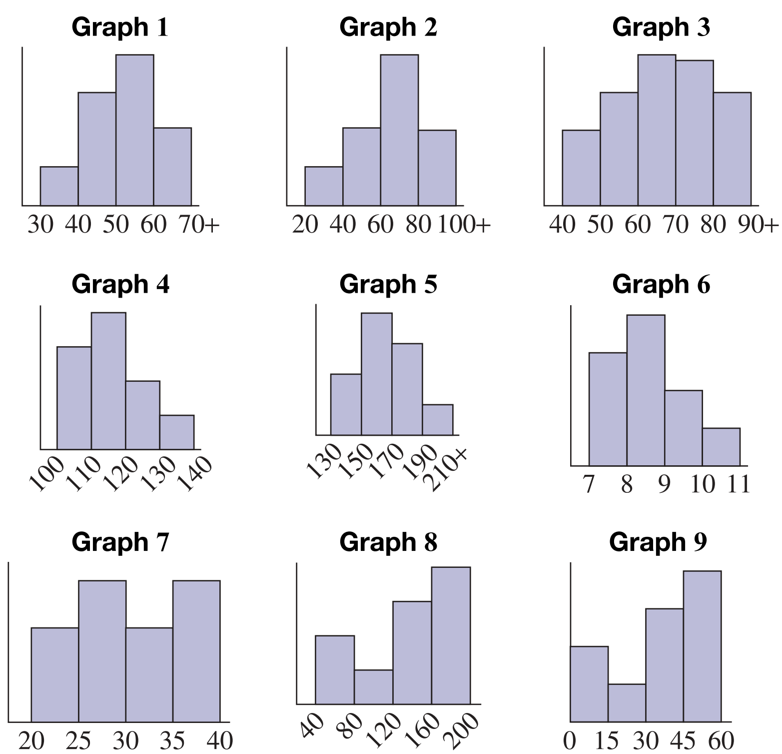
Heights, weights and ages mix-up — 14

14 Three students survey different groups of people to find out their heights, weights and ages. Unfortunately they have mixed up all the graphs they obtained.

a Copy and complete the table below, stating which graph corresponds to which set of data.

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4		
Shopping centre			
Teachers' common room			



b Show with rough sketches how the age graphs would look for:

i people in a retirement village
ii students at a secondary school
iii guests at a 30-year high school reunion.

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8D Range and measures of centre

Learning intentions

- To understand that the mean of a set of data can be affected significantly by an outlier, whereas the median and mode are not affected.
- To be able to calculate the mean, median and mode for a set of numerical data.
- To be able to calculate the range of a set of numerical data.

Key vocabulary: mean, median, mode, modal category, outlier, range

It is sometimes useful to summarise a large group of data as a single value. The concept of 'average' is familiar to most people, but more precise mathematical terms to use are 'mean', 'median' and 'mode'.

Lesson starter: Family heights

Each New Year, the Green family measure and record their heights. This year their height measurements are:

Georgia 78 cm, Emily 130 cm, Amy 130 cm, Ethan 188 cm, Mrs Green 165 cm, Mr Green 182 cm.

Work with a classmate to help each other to complete these activities.

Range

1 Who is the shortest and who is the tallest person in the Green family?
2 What is the range (the difference) between the shortest and tallest heights in the Green family?
3 The shortest person in the world is 55 cm and the tallest person in the world is 251 cm. What is the current range of heights for all adult humans?
4 The Green family had a snow-skiing holiday. One morning it was −8°C and that afternoon it was 5°C. What was the range of temperature that day?

Median

1 List the heights of the Green family in ascending (increasing) order.
2 What are the two middle heights?
3 Find the median (middle of these two central heights).
4 If the tallest man in the world, height 251 cm, is added into the Green family heights list, what is the median (middle) height now?
5 By how much has the median height changed by adding the tallest man into the list?
6 Does the median value always have to be one of the scores in the list?

Mean

1 Add up the total of all the heights of the Green family.
2 Now find the mean height. (mean = total of heights divided by the number of heights)
3 If the tallest man in the world is included with the Green family heights, what is the mean height now?
4 By how much has the mean height changed by including the tallest man into the list?
5 Does the mean value always have to be one of the scores in the list?

Mode

1 The Green family has twins. Who are they and what is their height?
2 What is the mode (most common) of the Green family heights?
3 The Pink family has heights: 125 cm, 142 cm, 142 cm, 142 cm, 160 cm and 178 cm. The Pink family have a set of triplets. What is the height of the Pink triplets?
4 What is the mode of the Pink family heights?
5 Does the mode value always have to be one of the scores in the list?

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Key ideas

- The **range** of a set of data is given by:
  Range = highest number − lowest number.
- The **mean** (sometimes called the average) of a set of numbers is given by:
  Mean = (sum of all the values) ÷ (total number of values).
  e.g. $7 + 8 + 1 + 10 + 2 + 1 + 6 = 35$
  Mean = $35 \div 7 = 5$
- The **median** is the middle value if the values are in order (ascending or descending). If there are two middle values then the average of them is taken, by adding them together and dividing by 2.
  e.g. $1 \quad 1 \quad 2 \quad 6 \quad 7 \quad 8 \quad 10$
  Middle ⇒ Median = 6
  e.g. $2 \quad 3 \quad 5 \quad 9 \quad 10 \quad 12$
  $\dfrac{5+9}{2} \Rightarrow$ Median = 7
- The **mode** is the most common value, i.e. the score with the highest frequency. There can be more than one mode.
  e.g. $1 \quad 1 \quad 2 \quad 6 \quad 7 \quad 8 \quad 10$
  Mode = 1
  For data sorted into categories, the most common category is called the **modal category**.
  e.g. bananas 25, oranges 20, mangoes 30
  The modal category is mangoes.
- An **outlier** is a score that is much larger or smaller than the rest of the data.
  - The median and mode are generally unaffected by outliers whereas the mean can be affected significantly by an outlier.

Exercise 8D

Understanding | 1–5 | 2–5

1 Fill in the blanks. Choose from: *range, outlier, mode, mean or median.*
  a The most common value in a set of data is called the _________.
  b The sum of all values, divided by the number of values is called the ________.
  c The ________ can be calculated by finding the middle value(s) of the numbers placed in ascending order.
  d The difference between the highest and lowest values is called the _______.
  e A value that is much larger or smaller than the other values is called an _______.

Hint for Q2:
Mean = $\dfrac{\text{sum of scores}}{\text{number of scores}}$

2 Use the set of numbers $1, 7, 1, 2, 4$.
  a Find the sum of these numbers.
  b How many numbers are listed?
  c Hence find the mean.

Hint for Q3: The median is the middle value when listed in ascending order.

3 Use the values $5, 2, 1, 7, 9, 4, 6$.
  a Sort these numbers from smallest to largest.
  b What is the middle value in your sorted list?
  c What is the median of this set?

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8D

4 Use the set $1, 5, 7, 9, 10, 13$.
   a State the two middle values.
   b Find the sum of the two middle values.
   c Divide your answer by 2 to find the median of the set.

5 Use the set of numbers $1, 3, 2, 8, 5, 6$.
   a State the largest number.
   b State the smallest number.
   c Hence state the range, by finding the difference of these two values.

Fluency

6–8(½), 9 | 6–8(½), 10

Example 8 Finding the range

Find the range of the following sets of data.

a $1, 5, 2, 3, 8, 12, 4$
b $-6, -20, 7, 12, -24, 19$

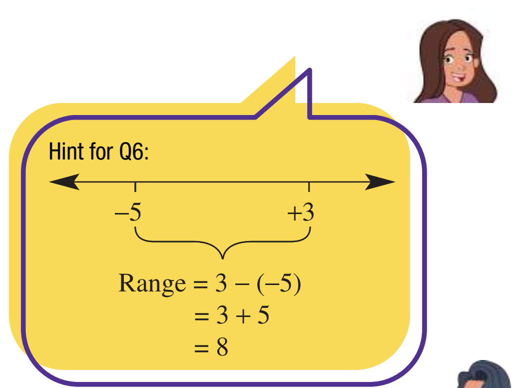
Solution	Explanation
a Range $= 12 - 1$ $= 11$	Maximum: 12, minimum: 1 Range = maximum − minimum
b Range $= 19 - (-24)$ $= 43$	Maximum: 19, minimum: −24 Range $= 19 - (-24) = 19 + 24 = 43$

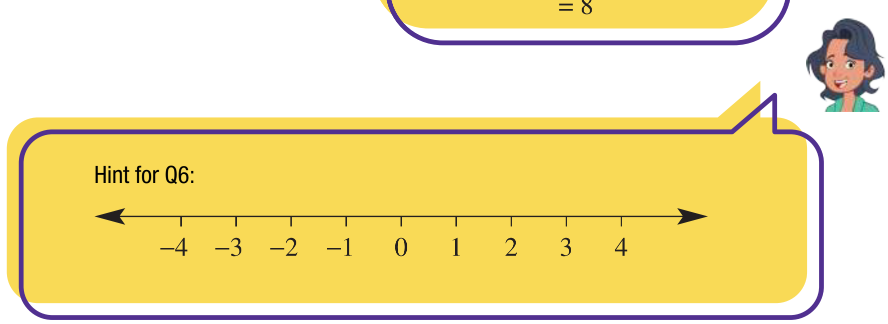
Now you try

Find the range of the following sets of data.

a $9, 17, 21, 11, 32, 26, 5, 22, 14$
b $0, -3, -14, -4, -8, -6$

6 Find the range of the following sets.
   a $5, 1, 7, 9, 10, 3, 10, 6$
   b $9, 3, 9, 3, 10, 5, 0, 2$
   c $4, 13, 16, 9, 1, 6, 5, 8, 11, 10$
   d $16, 7, 17, 13, 3, 12, 6, 6, 3, 6$
   e $-7, 4, 12, -5, -18, -16, 7, 9$
   f $16, -3, -5, -6, 18, -4, 3, -9$
   g $3.5, 6.9, -9.8, -10.0, 6.2, 0.8$
   h $-4.6, 2.6, -6.1, 2.6, 0.8, -5.4$





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Example 9 Finding the mean and the mode

For the set of numbers $10, 2, 15, 1, 15, 5, 11, 19, 4, 8$ find:

a the mean
b the mode

Solution	Explanation
a $10 + 2 + 15 + 1 + 15 + 5 + 11 + 19 + 4 + 8 = 90$	The numbers are added to find the total.
Mean $= 90 \div 10 = 9$	The mean is found by dividing the total by the number of items (in this case 10).
b Mode $= 15$	The most common value is 15, so this is the mode.

Now you try

For the set of numbers $6, -1, 3, 4, 0, 2, -2, 4$ find:

a the mean **b** the mode

7 For each of the following sets find:

i the mean **ii** the mode

a $5, 6, 3, 4, 4, 8$
b $2, 2, 1, 2, 1, 4, 2$
c $4, 3, 3, 10, 10, 2, 3$
d $-10, -4, 0, 0, -2, 0, -5$
e $3, 4, 5, -9, 6, -9$
f $3, -6, 7, -4, -3, 3$
g $13, 15, 7, 7, 20, 9, 15, 15, 11, 17$
h $20, 12, 15, 11, 20, 3, 18, 2, 14, 16$
i $18, 12, 12, 14, 12, 3, 3, 16, 5, 16$
j $18, 5, 14, 5, 19, 12, 13, 5, 10, 3$
k $-15, -6, -6, 16, 6, 13, 3, 2, 19, -8$
l $-13, -6, -6, -13, -6, 10, -15, 6, 7, 2$

Hint for Q7i:
Mean $= \dfrac{\text{sum of scores}}{\text{number of scores}}$

Hint for Q7ii:
The mode is the most common score.

Example 10 Finding the median

Find the median of:

a $16, 18, 1, 13, 14, 2, 11$ **b** $7, 9, 12, 3, 15, 10, 19, 3, 19, 1$

Solution	Explanation
a Sorted: $\boxed{1,2,11}$, 13, $\boxed{14,16,18}$	Sort the numbers from smallest to largest. Split the list into two equal halves.
Median $= 13$	The middle value is 13.
b Sorted: $\boxed{1,3,3,7}$, 9, 10, $\boxed{12,15,19,19}$	Sort the numbers from smallest to largest. Split the list into two equal halves.
Median $= \dfrac{9 + 10}{2} = 9.5$	There are two middle values (9 and 10) so we add them and divide by 2.

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8D

Now you try

Find the median of:

a 2, 8, 4, 6, 5, 10, 1

b 12, 15, 11, 19, 26, 14

8 For each of the following sets, calculate the median.

- a 3, 5, 6, 8, 10
- b 3, 4, 4, 6, 7
- c 1, 2, 4, 8, 10, 13, 13
- d 2, 5, 5, 5, 8, 12, 14
- e 14, 15, 7, 1, 11, 2, 8, 7, 15
- f 4, 14, 5, 7, 12, 1, 12, 6, 11
- g 2, 2, 4, 6, 7, 9
- h 1, 1, 2, 9, 9, 10
- i 1, 3, 5, 7, 8, 10, 13, 14
- j 0, 1, 9, 13, 1, 10, 7, 12, 9, 2
- k 12, 17, 7, 10, 2, 17, −2, 15, 11, −8
- l −2, −1, −3, 15, 13, 11, 14, 17, 1, 14

Hint for Q8: List the score from smallest to largest.

Hint for Q8: The median is the middle score.

9 Bernie writes down how many hours he works each day for one week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of hours	8	10	8	7	9

- a What is the mean number of hours Bernie worked each day?
- b What is the median number of hours Bernie worked each day?
- c What is the mode number of hours Bernie worked each day?

10 State the modal category for the following frequency tables.

a Colours of cars are noted as they drive past.

Colour	Red	Blue	Orange	White	Green	Black
Frequency	21	14	3	42	7	25

Hint for Q10: The modal category has the highest frequency.

b Pizza preferences are noted within a group of teenagers.

Hawaiian	Meat-lovers	Vegetarian	Cheese
5	7	4	2

c The favourite day of the week of a group of people.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Frequency	4	12	41	16	28

d The number of gymnasts in different states.

New South Wales	Queensland	South Australia	Tasmania	Victoria	Western Australia
152	135	193	86	144	159

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Problem-solving and reasoning

11, 12 | 12–14

11 Federica is in a dancing competition and each week she is rated out of 10. Her results for one term are shown in the frequency table below.

Score	7	8	9	10
Frequency	3	0	3	4

a In how many weeks did she get 7 out of 10?
b What score did she receive the most often?
c List out all the scores.
d What is her mean dancing score for the 10 weeks?
e What is her median dancing score for the 10 weeks?
f What is the range of Federica's dancing scores?

12 Business A pays wages of $\$42\,000$, $\$48\,000$, $\$50\,000$, $\$50\,000$ and the boss gets $\$70\,000$. Business B pays wages of $\$42\,000$, $\$48\,000$, $\$50\,000$, $\$50\,000$ and the boss gets $\$200\,000$.
a Which group of wages includes an outlier? What is its value?
b Find the mean wage of each business.
c How much larger is the mean wage of Business B than the mean wage of Business A?
d State the median wage of each business.
e Has the outlier affected the median wage?
f Which measure better shows how much the workers are paid in each business, the mean or median? Give a reason for your answer.

Hint for Q12: An outlier is a value much larger (or smaller) than the other values.

13 Gary and Sarah compare the number of runs they score in cricket over a number of weeks.

Gary: $17, 19, 17, 8, 11, 20, 5, 13, 15, 15$
Sarah: $39, 4, 26, 28, 23, 18, 37, 18, 16, 20$

a Calculate Gary's range.
b Calculate Sarah's range.
c Who has the greater range?
d Which cricketer is more consistent on the basis of their ranges only?

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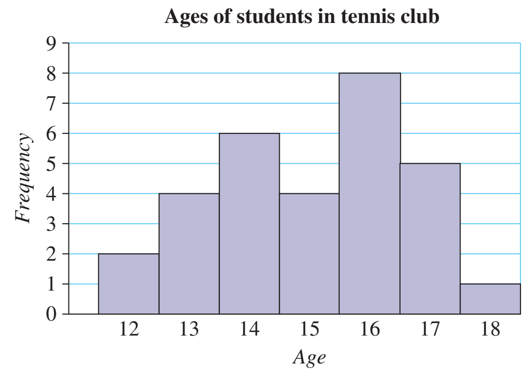
8D

14 The graph at right shows the ages of all students in a school's tennis club.



- **a** List all the ages from smallest to largest.
- **b** What is the range of the ages in the tennis club?
- **c** What is the most common age?
- **d** Calculate the mean age correct to two decimal places.
- **e** Calculate the median age.
- **f** Now include the teacher's age of 52 in the list of ages.
  - **i** Find the new mean age.
  - **ii** Find the new median age.
  - **iii** Which measure has changed the most, the mean or the median?

House for sale — 15

15 The prices of all the houses in School Court are recorded: \$520 000, \$470 000, \$630 000, \$580 000, \$790 000, \$540 000, \$710 000, \$8.4 million, \$660 000.

- **a** What is the mean house price in School Court, correct to the nearest dollar?
- **b** What is the median house price in School Court?
- **c** What effect does having a single \$8.4 million mansion in School Court have on the mean?
- **d** What effect does having a single \$8.4 million mansion in School Court have on the median?
- **e** Why might 'median house price' be a more useful measure than 'mean house price' when people are looking at living in a particular area?

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8E Surveying and sampling

Learning intentions

- To understand that a sample needs to be representative of a larger group in order for the conclusions to be meaningful.
- To be able to interpret results from a survey.
- To be able to decide whether a bias is introduced by different methods of collecting data.

Key vocabulary: population, sample, survey, census, symmetrical data, skewed data

To find information about a large number of people it is generally not possible to ask everybody to complete a survey, so instead a sample of the population is chosen and surveyed. It is hoped that the information given by this smaller group is representative of the larger group of people. Choosing the right sample size and obtaining a representative sample are harder than many people realise.

Lesson starter: Average word length

To decide how hard the language is in a book, you could try to calculate the average length of the words in it. Because books are generally too large to record the length of every word, instead you can choose a smaller sample. For this exercise, you must decide or be assigned to the 'small sample', 'medium sample' or 'large sample' group. Then:

1. Pick a page from the book at random.
2. Find the average (mean) length of any words on this page, choosing the first 10 words if you are in the 'small sample' group, the first 30 words for the 'medium sample' group, and the first 50 words for the 'large sample' group.

Discuss as a class:

- Which of the groups would have the best estimate for the average word length in the book?
- What are the advantages and disadvantages of choosing a large sample?
- Does this sample help to determine the average length of words in the English language?
- How could the results of a whole class be combined to get the best possible estimate for average word length in the book?
- If all students are allowed to choose the page on which to count words, rather than choosing one at random, how could this bias the results?

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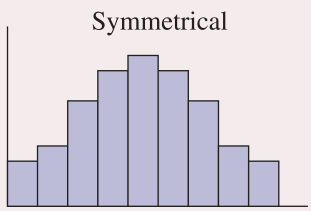
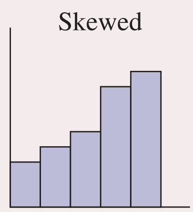
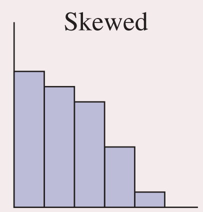
8E

Key ideas

- A **population** is the entire group that we are interested in. For example, if we want to find the average height of 14-year-old girls in Australia, the population is all the 14-year-old girls in Australia.
- A **sample** is a small group randomly selected out of a population. For example, a sample could be 100 randomly chosen 14-year-old Australian girls.
  - A **simple random sample** is found by randomly selecting from the population.
  - When a population has distinct groups within it (e.g. different year levels in a school), a **stratified sample** is found by randomly selecting separately from each group (e.g. randomly selecting 10 people from each year level, rather than randomly selecting 60 people from the school).
  - A **convenience sample** is found by selecting easily available data (e.g. surveying the people in your class) and is likely not to be representative.
- A **survey** is a set of questions used in a sample to obtain information about a larger group.
- The accuracy of the survey's conclusion can be affected by:
  - the sample size (number of participants or items considered)
  - whether the sample is representative of the larger group, or biased
  - whether there were any measurement errors, which could lead to outliers – values that are noticeably different from the other values.
- Data represented as a graph can be seen as **symmetrical data** or **skewed data**.







- If a data distribution is symmetrical, the mean and the median are approximately equal.

Exercise 8E

Understanding

1–3 | 2, 3

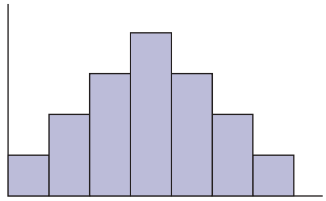
1 Write down the missing word from each statement. Choose from: *sample, symmetrical, skewed, survey* or *biased.*

a A ________ is a set of questions.

b A small group out of a population is called a ________.

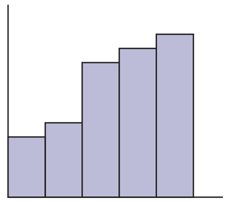
c A ________ sample doesn't represent the population.

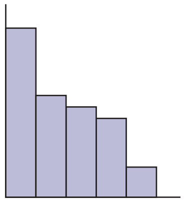
d



This graph has a ________ shape.

e





These graphs have a ________ shape.

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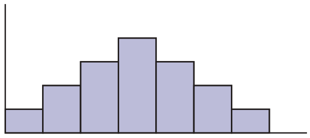
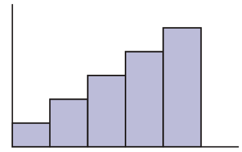
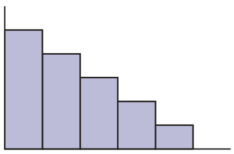
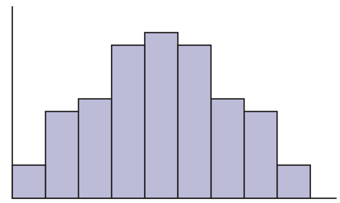
2 Marieko wishes to know the average age of drivers in her city. She could survey 10 of her friends, or survey 1000 randomly selected drivers.
  a Which of these options would give a more accurate result?
  b Which would be easier for Marieko to perform?

3 Classify the following distributions as symmetrical or skewed.

a


b


c


d


Fluency

	4–7	4, 5, 7, 8

Example 11 Calculating population numbers from random sample data

Out of a random sample of 10 Tasmanian devils, there are 7 that have a facial tumour.
  a What proportion of this population has facial tumours?
  b If there are 200 Tasmanian devils in this region, on the basis of this sample, how many would you expect to have facial tumours?
  c If there are 100 Tasmanian devils in this region, how many would you expect not to have a facial tumour?

Solution	Explanation
a $\frac{7}{10}$	7 have tumours out of a total of 10.
b $\frac{7}{10} \times 200 = 140$	The sample proportion $\times$ 200.
c $\frac{3}{10} \times 100 = 30$	3 out of 10 don't have a tumour.

Now you try

Out of a random sample of 20 books in a library, 4 had been printed in colour.
  a What proportion of books had colour?
  b If there were 1000 books in the library, based on this sample how many would you expect to have colour?
  c If there were 5200 books in the library, based on this sample how many would you expect not to have colour?

4 Ajith looks at a random sample of penguins and notes that of the 50 he sees, 20 of them have spots on their bodies.
  a What proportion of the population has spots?
  b If there are 5000 penguins in a region, on the basis of this sample how many would you expect to have spots on their bodies?
  c If there are 500 penguins in a region, how many would you expect to not have spots on their bodies?

Hint for Q4:
A proportion can be written as a fraction.

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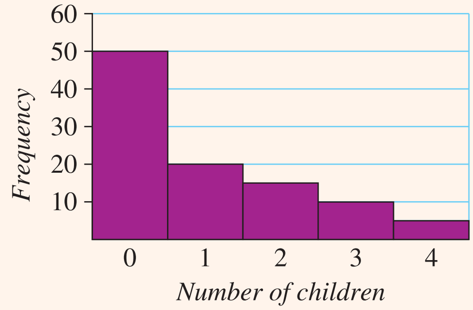
8E

Example 12 Interpreting survey results

A survey is conducted asking 100 randomly selected adults how many children they have. You can assume that this sample is representative of the adult population. The results are shown in this graph:

a Is this distribution symmetrical or skewed?
b What proportion of the adult population has two or more children?
c In a group of 9000 adults, how many would you expect to have 4 children?
d Which of the following methods of conducting the survey could lead to bias? Give a reason why.



Method 1 Asking people waiting outside a childcare centre.
Method 2 Randomly selecting people at a night club.
Method 3 Choosing 100 adults at random from the national census and noting how many children they claimed to have.

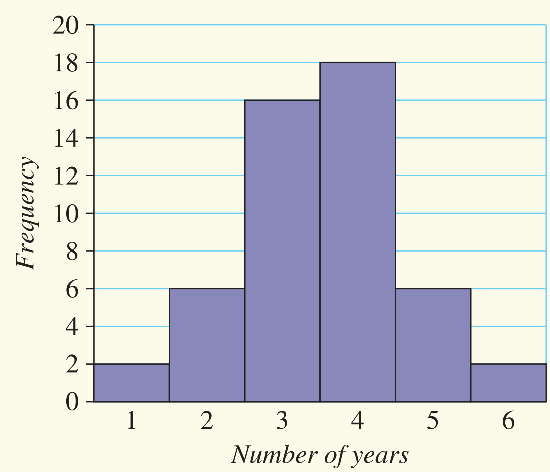
Solution	Explanation
a Skewed	Many more people have 0 children, so the distribution is not symmetrical.
b $\frac{3}{10}$	$15 + 10 + 5 = 30$ adults have two or more children. Proportion $= \frac{30}{100} = \frac{3}{10}$
c $\frac{1}{20} \times 9000 = 450$	In the survey, $\frac{5}{100} = \frac{1}{20}$ of the population have four children.
d Method 1 could lead to bias. If someone is waiting outside a childcare centre they are more likely to have at least one child. Method 2 could lead to bias. If someone is at a night club they are likely to be a younger adult, and so less likely to have a child.	

Now you try

In a survey of 50 university graduates, they were asked how many years they had spent at university. The results are shown in this graph.

a Is the distribution symmetrical or skewed?
b What proportion of graduates spent at least 4 years at university?
c In a group of 800 graduates, how many would you expect to have spent less then 3 years at university?
d Which of the following methods of conducting a survey could lead to bias? Give a reason why.

Method A Randomly selecting graduates from a university list which contained all the graduates from one year.
Method B Randomly selecting graduates who had completed a medical degree.



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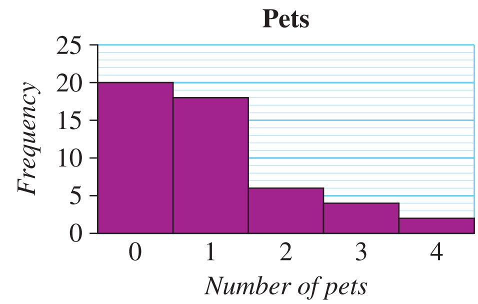
5 A survey is conducted asking 50 people how many pets they own. You can assume it is a representative sample of the population. The results are shown in the graph on the right.

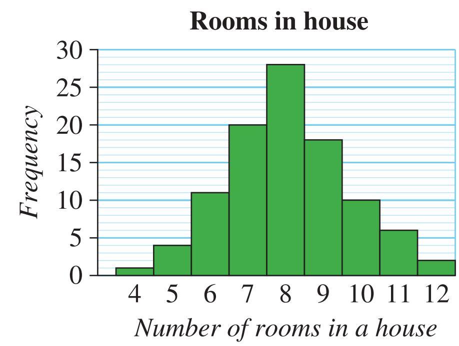
Hint for Q5: Expected number = proportion × total.

- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people had no pets?
- **c** Of a group of 1000 people, how many of them would you expect to have no pets?
- **d** What proportion of people had 2 or more pets?
- **e** Of a group of 5000 people, how many of them would you expect to have 2 or more pets?
- **f** Why would conducting this survey outside a veterinary clinic cause a bias in the results?



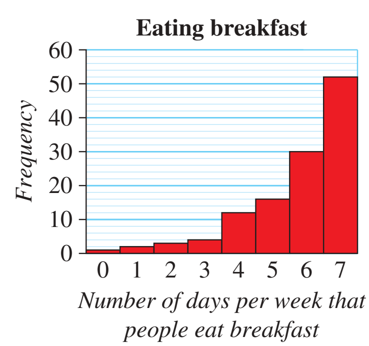
6 A survey was conducted of 100 randomly selected people who live in a house. The survey asked how many rooms were in their house. You can assume that it is a representative sample of the population. The results are shown in this graph.

- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people live in an 8-room house?
- **c** In a group of 1500 people, how many would you expect to live in an 8-room house?
- **d** What proportion of people live in a house with 5, 6 or 7 rooms?
- **e** In a group of 3000 people, how many would you expect to live in a house with 5, 6 or 7 rooms?
- **f** Why would conducting this survey in a wealthy suburb cause bias in the results?



7 A survey of 120 randomly selected people asked how many days per week each person ate breakfast. You can assume that it is a representative sample of the population. The results are shown in this graph.

- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people eat breakfast 7 days a week?
- **c** In a group of 36 000 people, how many would you expect to eat breakfast 7 days a week?
- **d** What proportion of people eat breakfast 4 or 5 days a week?
- **e** In a group of 4800 people, how many would you expect to eat breakfast 4 or 5 days a week?
- **f** Why would conducting this survey on a 6 am suburban train to the city cause bias in the results?



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8 In a factory producing chocolate bars, a sample of bars is taken and automatically weighed to check whether they are between 50 and 55 grams. The results are shown in a frequency table.

Weight (g)	49	50	51	52	53	54	55	108
Frequency	2	5	10	30	42	27	11	1

Hint for Q8: Add up the frequencies to find the total number of chocolate bars in the sample.

a Which weight value is an outlier?
b If you leave out the 108 gram result, is this distribution skewed or symmetrical?
c What proportion of chocolate bars are 53 g, 54 g or 55 g?
d In a batch of 800 chocolate bars, how many would be expected to be 53 g, 54 g or 55 g?
e What proportion of chocolate bars are less than 52 g?
f In a batch of 2048 chocolate bars, how many would be expected to be less than 52 g?

Problem-solving and reasoning 9, 10 10–12

9 Zeke attempts to find a relationship between people's ages and their incomes. He is considering some questions to put in his survey. For each question, decide whether it should be included in the survey, giving a brief explanation.
a What is your current age in years?
b Are you rich?
c Are you old?
d How much money do you have?
e What is your name?
f How much money did you earn in the past year?
g How much money did you receive today?

10 For each of the following survey questions, give an example of an unsuitable location and time to conduct the survey if you wish to avoid a bias.
a A survey to find the average number of children in a car.
b A survey to find how many people are happy with the current prime minister.
c A survey to find the proportion of Australians who are vegetarians.
d A survey to find the average cost of supermarket groceries.

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11 A survey is being conducted to decide how many adults use Mathematics later in life.
- **a** If someone wanted to make it seem that most adults do not use Mathematics, where and when could they conduct the survey?
- **b** If someone wanted to make it seem that most adults use Mathematics a lot, where and when could they conduct the survey?
- **c** How could the survey be conducted to provide less biased results?

12 Imagine you wanted to know the average number of subjects taken by a student in your high school. You cannot ask everyone in the school but want a sample of 20–30 students.
- **a** For the following sampling methods, classify them as 'simple random', 'convenience' or 'stratified'.
  - **i** Method 1: Ask the first 24 students you see
  - **ii** Method 2: Randomly select 4 students from each year level (Year 7, Year 8, up to Year 12).
  - **iii** Method 3: Randomly select 24 students from a list of all students in the school.
- **b** Compare the three methods above in terms of reliability of any conclusions.

13 In a population of 100 people, a census was conducted to find the number of bedrooms in their main residence.

The results are shown below.

Number of bedrooms	1	2	3	4	5	6
Frequency	5	14	30	38	11	2

The mean number of bedrooms for the population is 3.42. Unfortunately, the complete set of results shown in the table above was not published so a researcher chooses a sample of ten to estimate the number of bedrooms.
- **a** Explain how it could be possible for the researcher to find that the mean number of bedrooms is 4. (State a sample of ten people where the mean is 4.)
- **b** Explain how it is possible to choose a sample with a mean of 1.5.
- **c** What is the largest mean value the sample could have?
- **d** Assuming the researcher is already using a random sample, how could they reduce the amount of variation in sample means?

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8E

Design a survey and graph sample results

—	14

14 **Task 1**

Design survey questions to find out the following information.

- The mean number of siblings of the students in your class.
- The mean number of car trips made to school each week by families in your class.
- The mean number of computers owned by families in your class.

Task 2

Write down how you will choose an unbiased sample of students for your survey.
Run the survey on your chosen sample students. Keep a record of all results.

Task 3

In an Excel spreadsheet, record your results as tables showing the frequency of each answer.
Use Excel to draw a graph for each table. Comment on whether each set of data is symmetrical or skewed.

Task 4

In each table add a column for the proportions and enter the proportion that each frequency is of the total.
Multiply these proportions by the total number of students in Year 8 at your school to find the expected numbers from your year level.

Task 5

For each set of data, use an Excel spreadsheet to help you to find the expected mean for the students in your year level. Write your conclusions in sentences.

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8F Probability

Learning intentions

- To understand that a probability is a number between 0 and 1, representing the likelihood of an event.
- To be able to calculate the probability of simple events.
- To understand that higher probabilities correspond to more likely events.

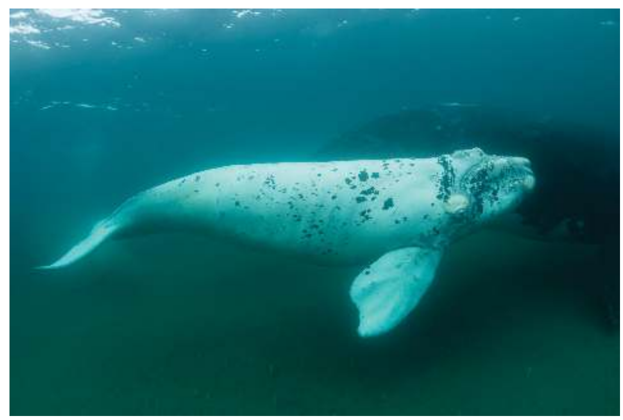
Key vocabulary: experiment, trial, outcome, event, sample space, complement

Most people would agree that being hit by lightning and getting rained upon are both possible when going outside, but that rain is more likely. Probability gives us a way to describe how much more likely one event is than another. A probability is a number between 0 and 1, where 0 means 'impossible' and 1 means 'certain'.

If the outcomes are equally likely, we find the probability of an event by counting the ways it can happen and dividing by the total number of outcomes.

Lesson starter: Estimating probabilities

Try to estimate the probability of the following events, giving a number between 0 and 1. Compare your answers with other students in the class and discuss any differences.



1. Flipping a 'tail' on a 50-cent coin.
2. An albino whale being born.
3. Rolling three 6s in a row on a fair die.
4. Correctly guessing a number between 1 and 10.
5. Tomorrow being a rainy day.
6. Seeing a wombat in the Australian bush.

Are there some events for which there is more than one correct answer?

Key ideas

- An **experiment** is a situation involving chance which leads to a set of results.
- A **trial** is a process which can be repeated to produce results. Examples could be flipping a coin, rolling a die or spinning a spinner.
- An **outcome** is a possible result from an experiment; for example, 'rolling a 3' or 'flipping tails on the coin'.
- An **event** is a single outcome (e.g. rolling a 3) or a collection of outcomes (e.g. rolling a 3, 4 or 5).
- The **probability** of an event is a number between 0 and 1 that represents the chance that the event occurs. If all the outcomes are equally likely:

$$\Pr(\text{Event}) = \frac{\text{number of outcomes where the event occurs}}{\text{total number of outcomes}}$$

- Probabilities are often written as fractions, but can also be written as decimals or percentages.

More likely →

Probability: → 0 $\quad \frac{1}{2}$ $\quad 1$

Description: → Impossible $\quad$ Even chance $\quad$ Certain

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8F

- The **sample space** is the set of possible outcomes of a trial or event. For example, the sample space for the roll of a die is $1, 2, 3, 4, 5, 6$.
- The **complement** of some event E is written E' (or not E). E' is the event that E does not occur. For example, the complement of 'rolling the number 3' is 'rolling a number other than 3'. Note: $\Pr(E) + \Pr(E') = 1$.
- The following language is also commonly used in probability.
  - 'at least', for example, 'at least 3' means $3, 4, 5, \ldots$.
  - 'at most', for example, 'at most 7' means $\ldots, 5, 6, 7$.
  - 'or', for example, 'rolling an even number or a 5' means rolling $2, 4, 5$ or 6.
  - 'and', for example, 'rolling an even number and a prime number' means rolling a 2.

Exercise 8F

Understanding

1–4 | 3, 4

1 Write the missing word from each statement. Choose from: *outcomes*, A', *complement*, *trial* or *sample space*.
- **a** An example of a _________ is flipping a coin.
- **b** After rolling a die, the possible _________ are $1, 2, 3, 4, 5$ and 6.
- **c** The set of all possible outcomes from a trial is called the _________.
- **d** The _________ of an event is the opposite of that event.
- **e** If an event is called A then the complement is written as _________.

2 Match each experiment (**a**-**d**) with the set of possible outcomes (**A**-**D**).
- **a** Flipping a coin
- **b** Choosing a number between 1 and 5
- **c** Choosing a letter of the word MATHS
- **d** Rolling a die

A $1, 2, 3, 4, 5, 6$ **B** Heads, Tails **C** $1, 2, 3, 4, 5$ **D** M, A, T, H, S

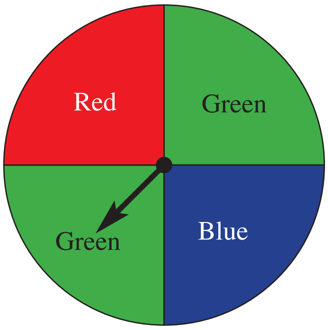
3 The following events are shown with their probabilities.

Event A: 0 Event B: 0.9 Event C: 1 Event D: 0.5

- **a** Which of the four events is most likely to occur?
- **b** Which of the four events is sure not to occur?
- **c** Which is more likely – event B or event D?
- **d** Which event is sure to occur?

Hint for Q3: Impossible events are sure not to occur.

4 The spinner is spun and could land with the pointer on any of the four sections. Answer true (T) or false (F):
- **a** Red and blue are equally likely outcomes.
- **b** Green is less likely to occur than blue.
- **c** The probability of the spinner landing on orange is 0.
- **d** Red is less likely to occur than green.



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Fluency

5–7 | 5, 6, 8

Example 13 Working with probabilities

The letters of the word PRINCE are written onto 6 equally sized cards and one is chosen at random.

a State the sample space.
b Find Pr(the letter N is chosen).
c List the outcomes of the event V = choosing a vowel.
d Find $\Pr(V)$.
e List the outcomes of the complement of choosing a vowel, written V'.
f Find $\Pr(V')$.

Solution	Explanation
a P, R, I, N, C, E	The sample space is all the possible outcomes when a single card is chosen. In this case, each of the letters in the word.
b $\Pr(N) = \frac{1}{6}$	There are 6 equally likely cards and 1 of them has the letter N.
c I, E	The outcomes of V include all the vowels in the word PRINCE.
d $\Pr(V) = \frac{2}{6}$ $= \frac{1}{3}$	There are 2 cards with vowels, so probability $= 2 \div 6$.
e V' includes P, R, N, C	The complement of V (V') is all the outcomes that are not in V, i.e. all the letters that are not vowels.
f $\Pr(V') = \frac{4}{6}$ $= \frac{2}{3}$	There are 4 cards that do not have vowels, so $\Pr(V') = 4 \div 6$.

Now you try

The numbers $1, 2, 3, \ldots, 10$ are written onto 10 equally sized cards and one is chosen at random.

a State the sample space.
b Find Pr(the number 7 is chosen).
c List the outcomes of the event M = choosing a multiple of 3.
d Find $\Pr(M)$.
e Find $\Pr(M')$.

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8F

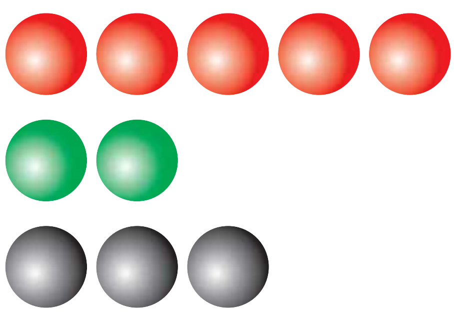
5 The letters of the word PIANO are written on 5 cards and then one card is drawn from a hat at random.
   a List the sample space.
   b Find Pr(the letter A is chosen).
   c Find Pr(a vowel is chosen).
   d Find Pr(a consonant is drawn).
   e Find Pr(the letter chosen is not an N).
   f List the outcomes of the complement of choosing a vowel, written V'.
   g Find $\mathrm{Pr}(V')$.

Hint for Q5: Pr means probability.

Hint for Q5: Write probability answers as fractions.

6 A fair die is rolled.
   a List the sample space.
   b Find Pr(5). That is, find the probability that a 5 is rolled.
   c Find Pr(even number).
   d List the outcomes of the complement of 'rolling a 5'.
   e State the probability that a 5 is not rolled.
   f What is the probability of rolling a 14?

7 There are five red marbles, two green marbles and three black marbles. The 10 marbles are placed into a hat and one is picked out.
   a What is Pr(red)? That is, what is the probability that the picked marble is red?
   b Find Pr(green).
   c Find Pr(black).
   d Find Pr(a black or a red marble is drawn).
   e Find Pr(red'), that is, find the probability of the complement of choosing a red marble.
   f Find Pr(black').
   g Give an example of an event that has a probability of 0.



8 The numbers 1 to 10 are written on cards. A card is chosen at random.
   a List the sample space.
   b Find the probability of choosing a 5.
   c Find Pr(7 or 9).
   d Find Pr(a multiple of 3 is chosen).
   e Find Pr(prime number).
   f Find Pr(a factor of 24).

Hint for Q8: A factor of 24 divides into 24 with no remainder. A prime has 2 factors. 1 is not prime.

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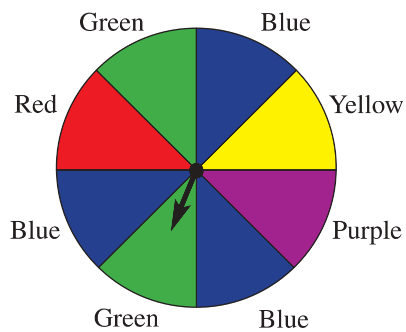
Problem-solving and reasoning

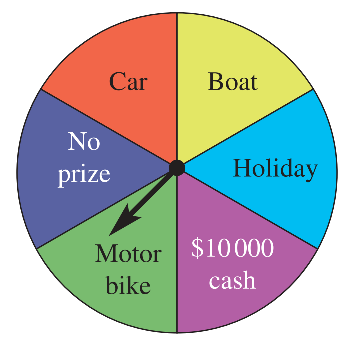
9–11 | 10–13

9 A spinner has the arrangement of colours as shown.
- a List the sample space when this spinner is spun.
- b Find Pr(red).
- c State Pr(green).
- d Find Pr(blue).
- e List the outcomes of the complement of 'spinner landing on blue'.
- f What is Pr(not blue)?
- g Find Pr(red or green or blue).
- h What is an event that is equally likely to 'spinning red'?
- i Give an example of an event that has a probability of 0.

Hint for Q8: List the colour as many times as it is on the spinner.



10 On a game show, a wheel is spun for a prize with the options as shown.
- a Joan wants to go on a $\$10\,000$ holiday so she is happy with the cash or the holiday. What is the probability she will get what she wants?
- b What is the probability of getting a prize that is not the cash?
- c What is Pr(car or motorbike)?
- d What is the probability of winning a prize?



11 Each of the numbers 1 to 10 are written on 10 cards and one card is chosen at random. Find the following probabilities.
- a Pr(even).
- b Pr(3 or even).
- c Pr(3 and even).
- d Pr(at least 6).
- e Pr(at most 5).
- f Pr(prime or even).

12 Six counters coloured red, purple or orange are placed in a pocket. You are told that Pr(red or orange) $= \frac{1}{2}$ and Pr(red or purple) $= \frac{2}{3}$.
- a How many counters of each colour are there?
- b State Pr(red).
- c Find Pr(purple).
- d Find Pr(orange′).

Hint for Q12: Change the probabilities to have a common denominator.

13 Draw a spinner that has Pr(red) $= \frac{1}{8}$, Pr(blue) $= \frac{5}{8}$ and Pr(green) $= \frac{1}{4}$.

Hint for Q13: First divide a circle into 8 equal sectors.

Changing probabilities

— | 14

14 In a large bucket there are 2 red balls and 8 blue balls.
- a State Pr(red).
- b One of each colour is added. What is the new Pr(red)?
- c The procedure of adding a red ball and a blue ball is repeated several times. How many balls are in the bucket when Pr(red) $= \frac{1}{3}$?
- d Imagine the procedure is repeated many times. What value does Pr(red) eventually approach as more balls are added? It might be helpful to imagine 1000 balls of each colour are added and use decimals.

Hint for Q14: Make a table.

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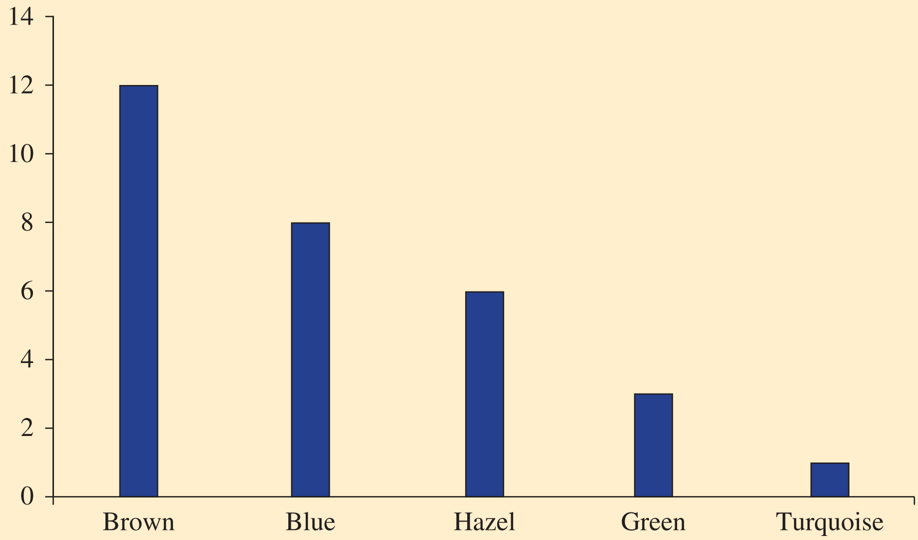
Progress quiz

8A **1** A class of Year 8 students was surveyed for their eye colour and the results are shown in the column graph below.

a How many students are in the class?
b How many students have blue or hazel eyes?
c How many more students have brown eyes than blue eyes?



8B **2** Put the following data into a frequency table.
2, 5, 3, 3, 3, 5, 1, 4, 2, 5, 2, 1, 1, 3, 2, 2, 2, 5, 2, 4

8C **3** Represent this frequency table below as a graph.

Number of broken bones	Frequency
0	12
1	7
2	2
3	4
4	1

8D **4** Find the range of the following sets of data.
a 2, 11, 3, 6, 7, 15, 3, 4, 8
b 12, 7, −10, −6, 29, 32, 3, 0, −11, −3, 1, 16, 18

8D **5** For each of the following sets, find: **i** the mean **ii** the mode.
a 1, 2, 5, 4, 4, 3, 2, 2, 2, 5
b −5, −4, −8, −1, 0, 0, −3, −4, −2, −7, −4, 2

8D **6** For each of the following sets, calculate the median.
a 1, 4, 6, 12, 15, 17, 23
b 53, 56, 57, 57, 61, 67, 68, 85
c 3, 11, 6, 4, 5
d −5, −8, 1, 15, −2, −4

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8D **7** Paschmal cares for 8 hens that generally lay an egg each day. Paschmal has recorded, in the frequency table below, the daily number of eggs he has collected over the past 4 weeks.

Daily number of eggs	5	6	7	8
Frequency	11	7	9	1

a In how many days did each hen lay an egg?
b What was the most common number of eggs Paschmal collected?
c List out all the number of eggs Paschmal collected over the 28 days.
d What is the mean number of eggs collected over the 28 days?
e What is the median number of eggs collected over the 28 days?
f What is the range of the number of eggs Paschmal collected over the 28 days?

8E **8** Marcia spends an hour bird watching and notes that of the 25 birds she sees, 15 of them are a type of wren.
a From this sample, what proportion of the population of birds in this area do we think are wrens?
b If there are 1000 birds in this area, on the basis of this sample how many would be wrens?
c If there are 30 000 birds in this area, on the basis of this sample how many birds that are not wrens are there?

8F **9** The letters of the word RESILIENCE are written on 10 cards and placed in a hat. One card is then drawn from the hat at random.
a List the sample space.
b Find Pr(the letter E is chosen).
c Find Pr(a consonant is chosen).
d Find Pr(the letter I is not chosen).
e State the probability that a letter with some curved lines is not drawn.

8F **10** Matty has a bowl full of 50 coloured lollies: 20 red, 5 yellow, 6 orange, 10 green and the remainder purple.
a How many purple lollies are in the bowl?
b Matty eats two lollies of each of the different colours. How many lollies are left in the bowl?
c Matty then gives his friend 5 of his favourite red lollies. How many red lollies are now left in the bowl?
d Matty now decides to close his eyes and pick a lolly at random from the bowl. What is the probability Matty chooses a purple one?

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8G Two-step experiments

Learning intentions

- To understand that a table can be used to list the sample space of a two-step experiment
- To be able to calculate the probability of events in two-step experiments

Key vocabulary: two-step experiment, independent steps

Sometimes an experiment consists of two independent steps, such as when a coin is tossed and then a die is rolled. Or perhaps a card is pulled from a hat and then a spinner is spun. We can use tables to list the sample space.

Consider the following example in which a coin is flipped and then a die is rolled.

		Die					
		1	2	3	4	5	6
Coin	Heads	H1	H2	H3	H4	H5	H6
	Tails	T1	T2	T3	T4	T5	T6

There are 12 outcomes listed in the table. So the probability of getting a 'tail' combined with the number 5 is $\frac{1}{12}$.

Psychologists record observations of people's social interactions and thinking processes. Statistical methods are used to analyse and interpret the data, providing psychologists with a better understanding of another person's experiences.

Lesson starter: Monopoly mystery

In a board game, two dice are rolled and the player moves forward by their sum.

- What are the possible values that the sum could have?
- Are some values more likely than others? Discuss.
- How likely is it that the numbers showing on the two dice will add to 5?

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Key ideas

- If an experiment has two independent steps, the outcomes can be listed as a table.
- The probability is still given by:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

Exercise 8G

Understanding

1, 2 | 1, 2

1 A coin is flipped and then a spinner is spun. The possible outcomes are listed in the table below.

	1	2	3	4	5
H	H1	H2	H3	H4	H5
T	T1	T2	T3	T4	T5

a How many outcomes are possible?
b List the four outcomes in which an even number is displayed on the spinner.
c Hence, state the probability that an even number is displayed.
d List the outcomes for which tails is flipped and an odd number is on the spinner.
e What is $\text{Pr}(\text{T, odd number})$?

2 Two coins are flipped and the four possible outcomes are shown below.

		20-cent coin	
		H	T
50-cent coin	H	HH	HT
	T	TH	TT

a What is the probability that the 50-cent coin will be heads and the 20-cent coin will be tails?
b For which outcomes are the two coins displaying the same face?
c What is the probability of the two coins displaying the same face?

The sample space from rolling two dice can be listed in a table.

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8G

Fluency 3–5 4–6

Example 14 Using a table for two-step experiments

A spinner with the numbers $1, 2$ and 3 is spun, and then a card is chosen at random from the letters ATHS written on four cards.

a Draw a table to list the sample space of this experiment.

b How many outcomes does the experiment have?

c Find the probability of the combination 2S.

d Find the probability of an odd number being spun and the letter H being chosen.

Solution

a

	A	T	H	S
1	1A	1T	1H	1S
2	2A	2T	2H	2S
3	3A	3T	3H	3S

Explanation

The sample space of the spinner $(1, 2, 3)$ is put into the left column.

The sample space of the cards (A, T, H, S) is put into the top row.

b There are 12 outcomes.

The table has $4 \times 3 = 12$ items in it.

c $\Pr(2S) = \frac{1}{12}$

All 12 outcomes are equally likely. Spinning 2 and choosing an S is one of the 12 outcomes.

d $\Pr(\text{odd, H}) = \frac{2}{12} = \frac{1}{6}$

Possible outcomes are 1H and 3H, so probability $= 2 \div 12$.

Now you try

A spinner with the numbers $1, 2, 3$ and 4 is spun and then a card is chosen at random from the letters PIE written on three cards.

a Draw a table to list the sample space of this experiment.

b How many outcomes does the experiment have?

c Find the probability of the combination 3P.

d Find the probability of an even number being chosen together with a vowel.

3 A coin is flipped and then a die is rolled.

a Copy and complete the table shown to list the sample space of this experiment.

	1	2	3	4	5	6
H	H1					
T						T6

b How many possible outcomes are there?

c Find the probability of the pair H3.

d Find the probability of 'heads' on the coin with an odd number on the die.

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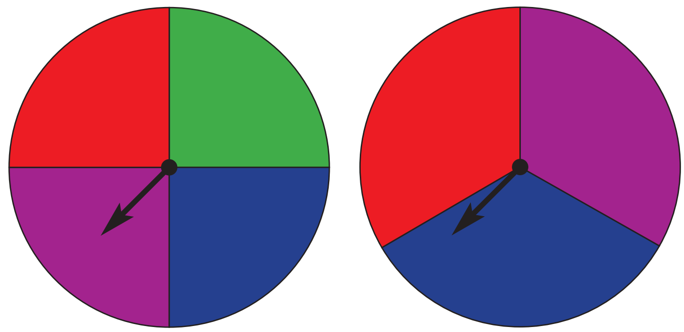
4 A letter is chosen from the word LINE and another is chosen from the word RIDE.
- **a** Draw a table to list the sample space.
- **b** How many possible outcomes are there?
- **c** Find Pr(NR), i.e. the probability that N is chosen from LINE and R is chosen from RIDE.
- **d** Find Pr(LD).
- **e** Find the probability that two vowels are chosen.
- **f** Find the probability that two consonants are chosen.
- **g** Find the probability that the two letters chosen are the same.

Hint for Q4: Your table row headings should be L, I, N, E and column headings should be R, I, D, E.

5 The spinners shown below are each spun.



Spinner 1 **Spinner 2**

- **a** Draw a table to list the sample space. Use R for red, P for purple and so on.
- **b** Find the probability that spinner 1 will display red and spinner 2 will display blue.
- **c** Find the probability that both spinners will display red.
- **d** What is the probability that spinner 1 displays red and spinner 2 displays purple?
- **e** What is the probability that one of the spinners displays red and the other displays blue?
- **f** What is the probability that both spinners display the same colour?
- **g** What is the probability that the spinners display a different colour (that is, the complement of displaying the same colour)?

6 A letter from the word EGG is chosen at random and then a letter from ROLL is chosen at random. The sample space is shown below.

	R	O	L	L
E	ER	EO	EL	EL
G	GR	GO	GL	GL
G	GR	GO	GL	GL

- **a** Find Pr(ER).
- **b** Find Pr(GO).
- **c** Find Pr(both letters are vowels).
- **d** Find Pr(both letters are consonants).

Problem-solving and reasoning 7, 8 8–10

7 Two dice are rolled for a board game. The numbers showing are then added together to get a number between 2 and 12.
- **a** Draw a table to describe the sample space.
- **b** Find the probability that the two dice add to 5.
- **c** Find the probability that the two dice do not add to 5. Recall that complementary events add to 1.
- **d** What is the most likely sum to occur?
- **e** What are the two least likely sums to occur between 2 and 12?

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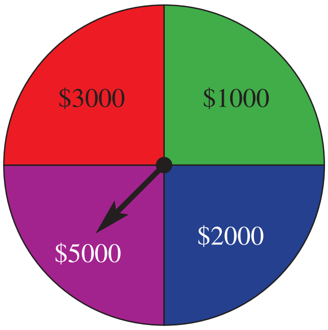
8 In Rosemary's left pocket she has two orange marbles and one white marble. In her right pocket she has a yellow marble, a white marble and 3 blue marbles. She chooses a marble at random from each pocket.

- **a** Draw a table to describe the sample space.
- **b** Find the probability that she will choose an orange marble and a yellow marble.
- **c** What is the probability that she chooses a white marble and a yellow marble?
- **d** What is the probability that she chooses a white marble and an orange marble?
- **e** Find the probability that a white and a blue marble are selected.
- **f** What is the probability that the two marbles selected are the same colour?

Hint for Q8: The left-pocket outcomes are W, O, O.

9 In a game show, a wheel is spun to determine the prize money and then a die is rolled. The prize money shown is multiplied by the number on the die to give the total winnings.

- **a** What is the probability that a contestant will win $\$6000$?
- **b** What is the probability that they win more than $\$11\,000$?
- **c** What is the probability they will win $\$11\,000$ or less?



10 Two separate experiments are conducted simultaneously. The first has 7 possible outcomes and the second has 9 outcomes. How many outcomes are there in the combined experiment?

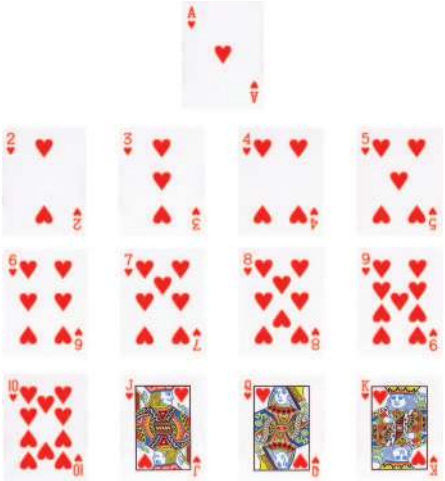
The deck of cards

—	11

11 In a deck of cards there are four suits (♥, ♦, ♣, ♠) and 13 cards in each suit (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K). ♥, ♦ are red suits.

- **a** If a card is chosen at random, what is Pr(3♦)?
- **b** What is Pr(red king)?
- **c** If two cards are chosen at random from separate decks, what is the probability that they are both diamonds?
- **d** If two cards are chosen at random from separate decks, what is the probability that they are both red cards?
- **e** What is the probability that 3♦ is chosen from both decks?
- **f** Why is it important that the two cards are chosen from separate decks? How would your answers to parts **c**–**e** change if the two cards were drawn from the same deck?

Hint for Q11: Do not draw a 52×52 table.



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8H Tree diagrams

Learning intentions

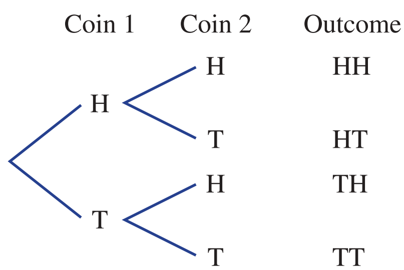
- To understand that a tree can be used to list the outcomes of experiments involving two or more steps
- To be able to use a tree diagram to determine the probability of events in multi-step events

Key vocabulary: tree diagram, multistep experiment

When two coins are flipped, we can draw a table to list the sample space. But if three coins are flipped, then we would need a three-dimensional table to list all outcomes. Imagine trying to find probabilities when five coins are flipped!

Another tool that mathematicians use for probability is the tree diagram. This tree diagram describes the four outcomes when two coins are flipped.

It is important to be able to read a tree diagram correctly. The first row (HH) represents the outcome where the first coin flipped was heads and the second coin flipped was heads. The third row (TH) represents the outcome where the first coin was tails and the second was heads.



Viral marketing occurs when social media users advertise a product to friends. A calculation based on a tree diagram shows that if each person shares with 5 others, then, after 9 stages of sharing, over 2 million people have received this advertising.

Lesson starter: Coin puzzle

- If two coins are flipped, rank these outcomes from most likely to least likely.
  - Exactly two heads are flipped.
  - Exactly one head and exactly one tail are flipped.
  - At least one coin shows tails.
  - Three tails are shown.
- How might the order change if three coins are flipped?
  Compare your answers with other students.

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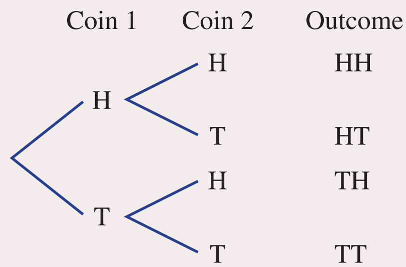
8H

Key ideas

- A **tree diagram** can be used to list the outcomes of experiments that involve two or more steps.

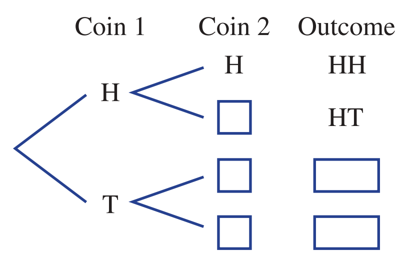


- At this stage, we will only consider tree diagrams for which each branch corresponds to an equally likely outcome.

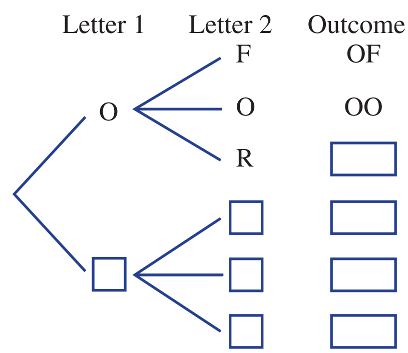
Exercise 8H

Understanding 1, 2 1, 2

1 Two coins are flipped.
  a State the missing parts to complete the tree diagram on the right.
  b How many equally likely outcomes are possible?



2 A letter from the word ON is chosen and then a letter from the word FOR is chosen.
  a State the missing parts to complete the tree diagram.
  b State the missing outcomes. The sample space is OF, OO, ______, ______, ______, ______.
  c How many equally likely outcomes are there in total?
  d How many outcomes have two consonants?



Fluency 3–5 3–5

Example 15 Using tree diagrams

Three fair coins are flipped.
  a List the sample space using a tree diagram.
  b How many possible outcomes are there?
  c Find the probability that the first coin is heads and the next two are tails.
  d Find the probability that exactly two of the coins show heads.

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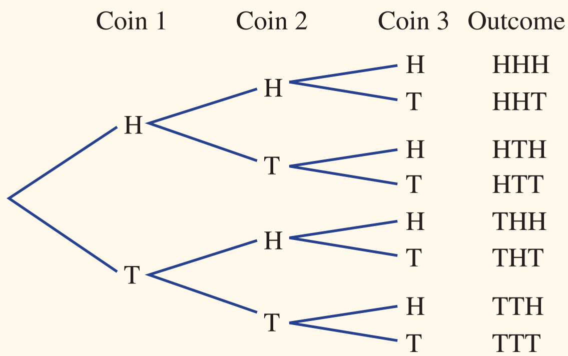
Solution

a



Explanation

Each coin has two outcomes: heads (H) and tails (T).

After each coin is flipped, the next coin has two outcomes, so the tree branches out.

b There are 8 possible outcomes.

They are listed: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

c $\text{Pr(HTT)} = \frac{1}{8}$

This is just one of the eight equally likely outcomes.

d Outcomes: HHT, HTH, THH

$\text{Pr(exactly 2 heads)} = \frac{3}{8}$

List the outcomes with exactly two heads. There are 3 of them so the probability is $\frac{3}{8}$.

Now you try

A spinner consists of two equally sized regions: red and blue. It is spun three times.

a List the sample space using a tree diagram.
b How many possible outcomes are there?
c Find the probability that the first spin is blue and the next two spins show red.
d Find the probability that all spins show the same colour.

3 A letter from the word CAT is chosen and then a letter from the word GO is chosen.
  a List the sample space using a tree diagram.
  b How many outcomes are possible?
  c Find Pr(C then G).
  d Find Pr(T then O).
  e Find Pr(2 consonants).

Hint for Q3: The first three branches are C, A and T. Then have G and O as options for each of these three branches

4 A spinner with numbers 1, 2 and 3 is spun twice.
  a Show the sample space in a tree diagram.
  b Find Pr(1 then 1).
  c Find Pr(1 then 2).
  d Find Pr(1 and 2 spun in either order).
  e Find Pr(both show the same number).
  f Find Pr(numbers add to 4).

5 A coin is tossed three times.
  a Draw a tree diagram to represent the sample space.
  b Find Pr(3 tails).
  c Find Pr(at least one head).
  d Find Pr(2 tails then 1 head).
  e Find Pr(2 tails and 1 head, in any order).
  f Which is more likely: getting exactly 3 tails or getting exactly 2 tails?
  g Find the probability of getting at least 2 tails.

Hint for Q5: “At least one head” is the complement of “3 tails”.

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8H

Problem-solving and reasoning | 6–8 | 7–9

6 Two letters are chosen from the word CAR. Once a letter is chosen it cannot be chosen again.
  a Draw a tree diagram of the six possible outcomes.
  b What is the probability that A and C will be chosen?
  c Find $\text{Pr}(2 \text{ consonants})$.
  d Find $\text{Pr}(2 \text{ vowels})$.
  e What is the probability that the letters chosen will be different?

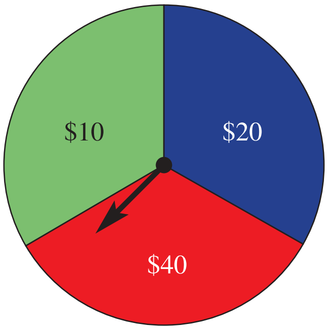
7 The letters of the word PIPE are placed on four cards. Two of the cards are chosen.
  a Draw a tree diagram showing all 12 outcomes.
  b Find $\text{Pr}(2 \text{ vowels})$.
  c Find $\text{Pr}(\text{the same letter is on the 2 cards})$.
  d What is $\text{Pr}(\text{at least one letter is a P})$?

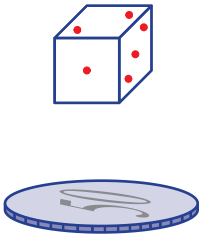
8 If 2 coins are tossed there are 4 outcomes. If 3 coins are tossed there are 8 outcomes. How many outcomes are there if 5 coins are tossed?

9 a If a coin is flipped 4 times, what is the probability that it will display heads four times?
  b If a coin is flipped 4 times, what is the probability that it will display H, T, T, H in that order?
  c If a coin is flipped 5 times, which is more likely: the result HHHHH or the result HTHHT?
  d If a coin is flipped 5 times, which is more likely: 5 heads or 3 heads?
  e Explain why your answers to parts **c** and **d** are different.

The wheel, die and coin | — | 10

10 In a game a prize wheel is spun, then a die is rolled and finally a coin is flipped. If the coin displays heads, you win the prize multiplied by the amount on the die. If the coin displays tails, you get nothing.
  a What amount do you win if you spin \$20 then roll a 5 and then flip heads?
  b Draw a tree diagram showing the 36 possible outcomes.
  c What is the probability that you win \$80?
  d Find $\text{Pr}(\text{win } \$100 \text{ or more})$.
  e Find $\text{Pr}(\text{receive less than } \$15)$. Include the possibility that you get nothing.





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8I Venn diagrams

Learning intentions

- To understand that Venn diagrams can be used to view the number of possible outcomes when two different events are considered.
- To understand that 'or' can mean 'inclusive or' or 'exclusive or' depending on the context.
- To be able to construct a Venn diagram from a worded situation.

Key vocabulary: Venn diagram, categories, probability, outcome

When two events are being considered a Venn diagram gives another way to view the probabilities. They are especially useful when survey results are being considered and converted into probabilities.

Lesson starter: Free dress day

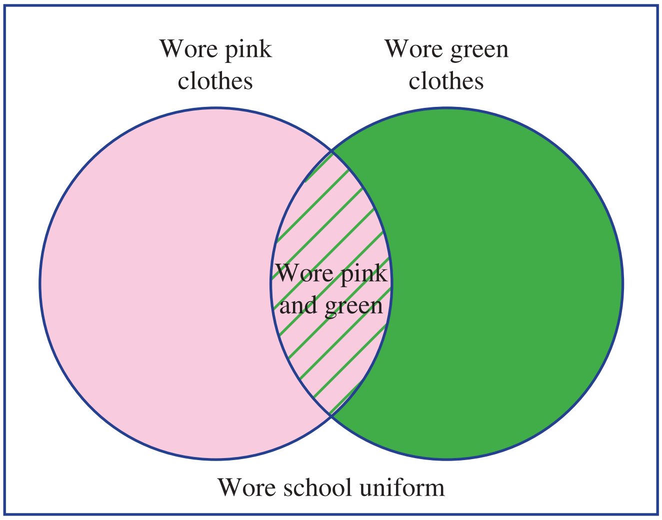
Work with a classmate and help each other to answer the questions in each activity.

On a free dress day a Year 8 class decided to wear either pink only or green only or both pink and green. A few students came in their school uniform.

This is how the students dressed:

- 9 wore pink only
- 5 wore green only
- 8 wore both pink and green
- 4 wore school uniform.

In Maths class that day, the students drew a Venn diagram showing the colours that the students dressed in on free dress day.



1. Copy this Venn diagram.
2. Write the number of students in each area that matches the colours worn.
3. How many students in total wore pink casual clothes?
4. How many students in total wore green casual clothes?
5. How many students in total wore green casual clothes or pink casual clothes or both?
6. How many students altogether are in this class?

Key ideas

- A **Venn diagram** is a pictorial representation using overlapping circles showing the number of objects in two or more **categories**.
- The words 'and' and 'or' usually have the following meanings in the area of **probability**. For two events A and B then:
  - an **outcome** belongs to 'A and B' if it belongs to both.
  - an outcome belongs to 'A or B' if it belongs to A or B or both.

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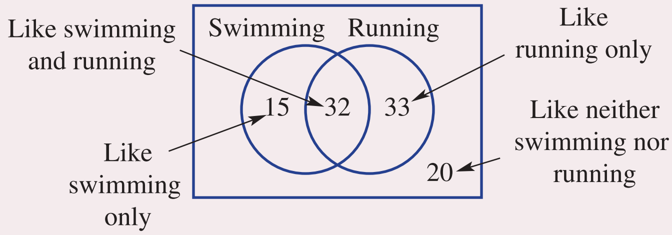
- Venn diagrams can be used to find probabilities.



$\text{Pr(swimming)} = \dfrac{47}{100}$

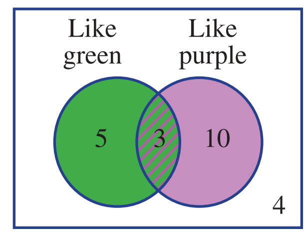
$\text{Pr(swimming and running)} = \dfrac{32}{100}$

$\text{Pr(swimming or running)} = \dfrac{80}{100}$

Exercise 8I

Understanding 1–3 3

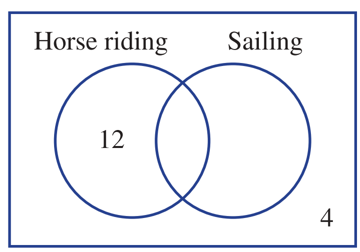
1 Here is a Venn diagram showing how many students like green or purple.



a How many students like green?
b How many students like purple?
c How many students like both green and purple?
d How many students like either green or purple or both?
e How many students don't like either green or purple?
f How many students were in this survey?

2 A class of Year 8 students were asked who liked sailing and who liked horse riding. Some of the results of this survey are in this Venn diagram.

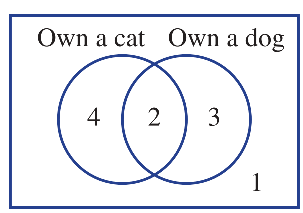
a Copy this Venn diagram and complete it by writing these numbers in the correct parts.
i 6 students liked both horse riding and sailing.
ii 8 students liked sailing but didn't like horse riding.



Hint for Q2: The overlap of the circles shows students who like both horse riding and sailing.

b How many students in total were in this class?

3 Look at the Venn diagram representing cat and dog ownership. State the missing number (1, 2, 3 or 4) to make the following statements true.



Hint for Q3: The number outside of the circles shows the people who don't own either a cat or a dog.

a The number of people who own both a cat and a dog is _____.
b The number of people who own a cat but do not own a dog is _____.
c The number of people who own neither a cat nor a dog is _____.
d The number of people who own a dog but do not own a cat is _____.

Fluency 4–7 5–8

4 A survey asked students if they liked oranges or bananas. Draw and label a Venn diagram showing the results of this survey as listed here.

15 students liked only oranges.
12 students liked both oranges and bananas.
8 students liked only bananas.
6 students prefer other fruit.

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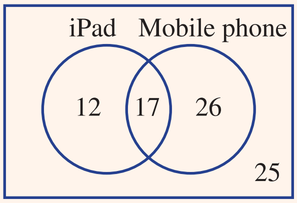
5 In a group of 30 students it is found that 10 play both cricket and soccer, 5 play only cricket and 7 play only soccer.
a How many people do not play cricket or soccer?
b Represent the survey findings in a Venn diagram.
c How many of the people surveyed play cricket?
d How many of the people surveyed play either cricket or soccer or both?

Example 16 Using Venn diagrams to find probabilities

This Venn diagram shows the results of asking some Year 8 students whether they owned an iPad or a mobile phone.



a How many students in total were surveyed?
b What proportion of students owned an iPad only?
c What proportion of students owned both an iPad and a mobile phone?
d What is the probability of choosing a student who owns a mobile phone?
e What is the probability of choosing a student who owns an iPad or a mobile phone or both?

Solution

a Total $= 80$

b $\frac{12}{80} = \frac{3}{20}$

c $\frac{17}{80}$

d $\Pr(\text{mobile phone}) = \frac{43}{80}$

e $\Pr(\text{mobile phone or iPad or both}) = \frac{55}{80}$
$= \frac{11}{16}$

Explanation

a Add all the numbers so $12 + 17 + 26 + 25 = 80$

b $\frac{\text{iPad only}}{\text{total}}$
$= \frac{\text{number in the iPad circle but not in the overlap}}{\text{total}}$
$= \frac{12}{80} = \frac{3}{20}$

c $\frac{\text{iPad and mobile}}{\text{total}}$
$= \frac{\text{number in the overlap of the circles}}{\text{total}}$
$= \frac{17}{80}$

d $\frac{\text{total number in the mobile phone circle}}{\text{total}} = \frac{43}{80}$

e $\frac{\text{total of mobile phone circle and iPad circle and overlap}}{\text{total}}$
$= \frac{12 + 17 + 26}{80} = \frac{55}{80} = \frac{11}{16}$

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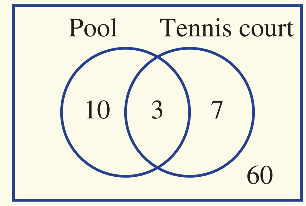
Now you try

This Venn diagram shows the results of asking home owners whether they owned a pool or a tennis count.



a How many homeowners were surveyed?

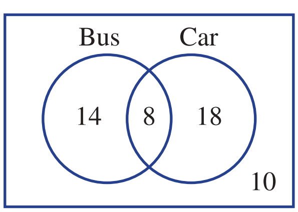
b What proportion of homeowners owned a pool only?

c What is the probability of choosing a homeowner who:
- i owns a pool
- ii owns a pool and a tennis court
- iii owns a pool or a tennis court?

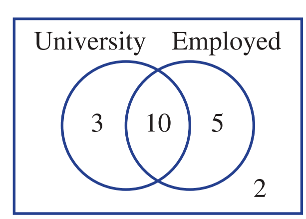
6 The Venn diagram below shows the results of surveying some Year 8 students about whether they travel to school by bus or car.
- a How many students in total were surveyed?
- b What proportion of students travelled by bus only?
- c What proportion of students travelled by both car and bus?
- d What is the probability of randomly choosing a student who travels by car?
- e What is the probability of randomly choosing a student who travels either by car or by bus or both?

Hint for Q6: Add all the numbers in the rectangle to find the total number of people surveyed.

Hint for Q6: All proportions and probabilities are fractions out of the total number surveyed.



7 Look at this Venn diagram showing the number of people who have a university degree and the number who are now employed.



- a What is the total number of people surveyed?
- b What is the total number of people in the survey who are employed?
- c What proportion of people in the survey are employed?
- d What is the number of people who have a university degree and are also employed?
- e What is the probability of randomly choosing a person who has a university degree and is also employed?

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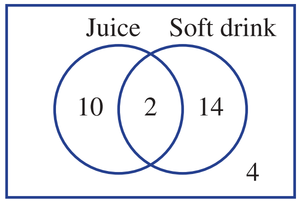
8 The Venn diagram shows the number of people who like juice and/or soft drinks.



Hint for Q8: Add the number outside of the 'like juice' circle to find how many do not like juice.

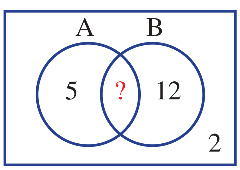
a What is the total number of people who like neither juice nor soft drink?
b What is the probability that a randomly selected person likes neither juice nor soft drink?
c What is the probability that a randomly selected person likes either juice or soft drink or both?
d What is the probability that a randomly selected person does not like juice?

Problem-solving and reasoning

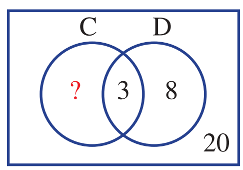
9, 10 | 9, 11, 12

9 Find all of the missing numbers (?) in each of these Venn diagrams.

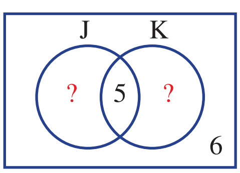
a Overall Total = 20



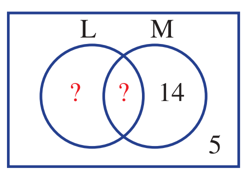
b Overall Total = 40



c Total of J = 18
Total of K = 21



d Total of L = 26
Total of M = 21

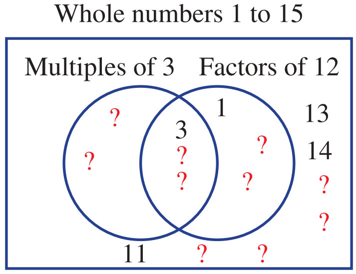


Hint for Q9: Remember that all the numbers in the Venn diagram add to the overall total.

Hint for Q9: The total in a circle is the sum of the two numbers in that circle.

10 **a** Copy and complete this Venn diagram by writing in the numbers missing from the whole numbers 1 to 15.

Whole numbers 1 to 15



Hint for Q10: Multiples of 3 are found when 3 is multiplied by whole numbers. So 3, 6…

Hint for Q10: Factors of 12 are numbers that divide into 12 with no remainder.

b How many numbers that are multiples of 3 are also factors of 12?
c How many factors of 12 are not multiples of 3?
d Out of the numbers 1 to 15, what proportion (fraction) are multiples of 3?
e What is the probability of choosing a factor of 12 out of the numbers 1 to 15?

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8I

11 For this question you will select from the numbers from 1 to 30.

a List the even numbers.

b List the factors of 30.

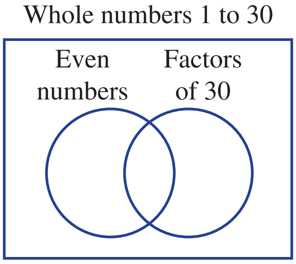
c Make a large copy of this Venn diagram and copy the *values* from your lists into the correct parts of the diagram.

d How many even numbers are also factors of 30?

e What is the probability of choosing an even number that is also a factor of 30 out of the numbers 1 to 30?

f How many even numbers are there in the Venn diagram?

g What is the probability of choosing a factor of 30 out of the even numbers?



Hint for Q11g: $\dfrac{?\ \text{(even factors of 30)}}{?\ \text{(even numbers)}}$

12 In Year 8 at a school there are 40 girls, half of whom are in the racing club. Of the 100 students in Year 8, 35 are in the racing club.

a Copy and complete the Venn diagram to describe the situation.

b What is the probability, as a percentage, that a randomly selected person in Year 8:

i is a girl in the racing club?

ii is a boy in the racing club?

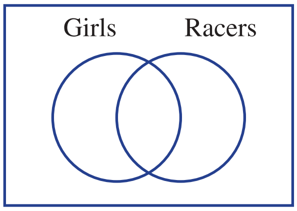
iii is not a girl?

iv is not in the racing club?

c What proportion of Year 8 racers are boys?

d What proportion of Year 8 racers are girls?

e What is the probability of randomly choosing a Year 8 racer out of the Year 8 girls?



Hint for Q12: Write proportions as fractions in simplest form.

Triple Venn diagrams — 13

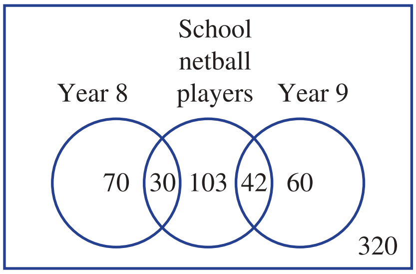
13 This Venn diagram shows the numbers in Years 8 and 9 and the netball players in a school.

a How many students in total are at this school?

b What is the probability of randomly choosing a student in this school who is:

i a Year 8 netball player?

ii a Year 9 netball player?



c How many netball players in total are there at this school?

d Out of the netball players only, what is the probability of choosing a:

i Year 8 netball player?

ii Year 9 netball player?

e If a student is randomly chosen out of Year 8, what is the probability that the student is a netball player?

f In Year 10, there are 105 students in total and 32 play netball. Copy and complete the Venn diagram above including a new circle for Year 10s. Be careful to enter the numbers correctly.

g If a student is randomly chosen out of the netball students, what is the probability it is a Year 10 student?

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8J Two-way tables

Learning intentions

- To understand that two-way tables can be used to view the number of possible outcomes when two different events are considered.
- To understand that 'or' can mean 'inclusive or' or 'exclusive or' depending on the context.
- To be able to construct a two-way table from a worded situation.

Key vocabulary: two-way table, row, column, outcome, Venn diagram, probability

When two events are being considered, two-way tables give another way to view events and calculate the probabilities. They can be used alongside or instead of a Venn diagram.

Lesson starter: Are English and Mathematics enemies?

Conduct a poll among students in the class, asking whether they like English and whether they like Maths. Use a tally like the one shown.

	Like Maths	Do not like Maths
Like English	卌 |	卌 ||||
Do not like English	卌 卌 |	卌 ||

Use your survey results to debate these questions:

- Are the students who like English more or less likely to enjoy Maths?
- If you like Maths does that increase the probability that you will like English?
- Which is the more popular subject within your class?

Key ideas

A **two-way table** lists the number of **outcomes** or people in different categories, with the final **row** and **column** being the total of the other entries in that row or column. For example:

	Like Maths	Do not like Maths	Total
Like English	28	33	61
Do not like English	5	34	39
Total	33	67	100

- A two-way table can be used to find **probabilities**:
  e.g. $\Pr(\text{like Maths}) = \frac{33}{100}$, $\Pr(\text{like Maths and not English}) = \frac{5}{100}$.

Exercise 8J

Understanding 1–4 3, 4

1 a Copy and complete the two-way table by writing in the missing totals.

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	
Total		35	75

b How many people like both apples and bananas?
c How many people dislike apples and dislike bananas?
d How many people were surveyed?

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2 Copy and complete the two-way table below using this information:
- 23 students like Anzac biscuits and also like lamingtons.
- 14 students like Anzac biscuits but dislike lamingtons.
- 12 students like lamingtons but dislike Anzac biscuits.
- 3 students dislike lamingtons and also dislike Anzac biscuits.

	Like lamingtons	Dislike lamingtons	Total
Like Anzac biscuits			
Dislike Anzac biscuits			
Total			

3 Here is a two-way table showing the results of surveying some teenagers about exercise. Use the numbers in this table to answer the questions below.

	Like jogging	Dislike jogging	Total
Like cycling	15	10	25
Dislike cycling	12	3	15
Total	27	13	40

Hint for Q3: The number of teenagers who 'like jogging' is equal to the total of the column 'like jogging'.

a How many teenagers like jogging?
b How many teenagers like both jogging and cycling?
c How many teenagers like jogging but dislike cycling?
d How many teenagers like cycling?
e How many teenagers like cycling but dislike jogging?
f How many teenagers were surveyed?

4 Answer the questions below about this two-way table:

	Like skateboards	Dislike skateboards	Total
Like BMX bikes	24	12	36
Dislike BMX bikes	15	9	24
Total	39	21	60

If a student is chosen randomly from this group, find these probabilities and simplify your answers.
a Pr(like skateboards but dislike BMX bikes).
b Pr(like BMX bikes and like skateboards).
c Pr(dislike skateboards but like BMX bikes).
d Pr(dislike BMX bikes and also dislike skateboards).

Hint for Q4: Write probabilities as a fraction out of the total (60) and then simplify this fraction.

Fluency

5–7(½) 5–8(½)

Example 17 Completing a two-way table

Copy and complete this two-way table.

	Like Macs	Dislike Macs	Total
Like PCs			
Dislike PCs		3	13
Total	35	40	75

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Solution

	Like Macs	Dislike Macs	Total
Like PCs	25	37	62
Dislike PCs	10	3	13
Total	35	40	75

Explanation

Start with a row or column that has only one number missing.
'dislike PCs' row, $10 + 3 = 13$

'dislike Macs' column, $37 + 3 = 40$

Now complete 'like Macs' column, $25 + 10 = 35$

The total in the 'like PCs' row is $25 + 37 = 62$

Now you try

Copy and complete this two-way table.

	Like running	Dislike running	Total
Like swimming	9		
Dislike swimming			17
Total	15	25	

5 Copy and complete this two-way table.

	Like surfing	Dislike surfing	Total
Like hiking			
Dislike hiking		5	30
Total	70	15	85

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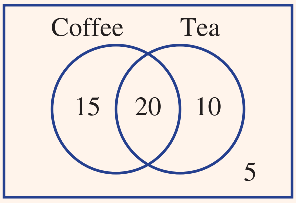
Example 18 Constructing two-way tables from Venn diagrams

Consider this Venn diagram showing the number of people who like coffee and who like tea.



a Represent the survey findings in a two-way table.
b How many people like neither tea nor coffee?
c How many people surveyed like tea?
d How many people like both coffee and tea?
e How many people like coffee or tea (or both)?

Solution

a

	Like coffee	Dislike coffee	Total
Like tea	20	10	30
Dislike tea	15	5	20
Total	35	15	50

b 5 do not like either tea or coffee.

c $20 + 10 = 30$ like tea.

d 20 like both coffee and tea.

e 45 like tea or coffee or both.

Explanation

a The two-way table has the four numbers from the Venn diagram and also a 'total' column (e.g. $20 + 10 = 30$, $15 + 5 = 20$) and a 'total' row. Note that 50 in the bottom corner is both $30 + 20$ and $35 + 15$.

b $50 - 20 - 15 - 10 = 5$ people who do not like either.

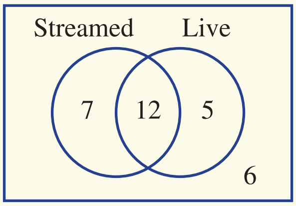
c 10 people like tea but not coffee, but 20 people like both. In total 30 people like tea.

d 20 out of 50 people like both coffee and tea.

e $15 + 20 + 10 = 45$ people like either coffee or tea or both.

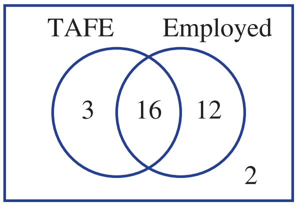
Now you try

Consider this Venn diagram showing the number of people who like streamed or live TV.



a Represent the survey findings in a two-way table.
b How many people liked neither streamed nor live TV?
c How many people liked live TV?
d How many people liked live and streamed TV?
e How many people liked live or streamed TV?

6 Look at this Venn diagram showing the number of people who have a TAFE degree and the number who are now employed.



a Copy and complete the two-way table shown below.

	Employed	Unemployed	Total
TAFE degree			
No TAFE degree			
Total			

b How many people were unemployed with no TAFE degree?
c How many people surveyed were employed?
d How many people had a TAFE degree and were also employed?
e How many people had a TAFE degree or were employed (or both)?

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Example 19 Using two-way tables to calculate probabilities

Consider the two-way table below of the eating and sleeping preferences of zoo animals.

	Eats meat	No meat	Total
Sleeps during day	20	12	32
Only sleeps at night	40	28	68
Total	60	40	100

For a randomly selected animal find:

a Pr(sleeps only at night)

b Pr(eats meat or sleeps during day)

Solution

a Pr(sleeps only at night)

$= \frac{68}{100}$

$= \frac{17}{25}$

Explanation

The total of animals that sleep at night is 68.

So $\frac{68}{100} = \frac{17}{25}$.

b Pr(eats meat or sleeps during day)

$= \frac{72}{100}$

$= \frac{18}{25}$

$20 + 12 + 40 = 72$ animals eat meat or sleep during the day (or both). $\frac{72}{100} = \frac{18}{25}$.

Now you try

Consider the two-way table below showing the number of students allergic to nuts and eggs.

	Nut allergy	No nut allergy	Total
Egg allergy	8	4	12
No egg allergy	3	35	38
Total	11	39	50

For a randomly selected student find:

a Pr(only a nut allergy)

b Pr(nut or egg allergy)

7 The two-way table below shows the results of a poll conducted of a group of boys and girls who own mobile phones to see who pays their own bills.

	Boys	Girls	Total
Pay own bill	4	7	11
Do not pay own bill	8	7	15
Total	12	14	26

a How many people participated in this poll?

b How many boys were surveyed?

c How many of the people surveyed pay their own bill?

d Find the probability that a randomly selected person:

i is a boy who pays his own bill.

ii is a girl who pays her own bill.

iii is a girl.

iv does not pay their own bill.

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8 The two-way table shows the results of a survey on car and home ownership at a local supermarket.

	Own car	Do not own car	Total
Own home	8	2	10
Do not own home	17	13	30
Total	25	15	40

Hint for Q8: Pr means probability.

a Find Pr(randomly selected person owns a car and a home).
b Find Pr(randomly selected person owns a car but not a home).
c What is the probability that a randomly selected person owns their own home?
d What is the probability that a randomly selected person does not own a car?

Problem-solving and reasoning

9, 10 | 10–12

9 Copy and complete the following two-way tables.

a

	B	Not B	Total
A	20		70
Not A			
Total		60	100

b

	B	Not B	Total
A		5	
Not A		3	7
Total	10		

10 A car salesman notes that among his 40 cars there are 15 automatic cars and 10 sports cars. Only two of the sports cars are automatic.
a Create a two-way table of this situation.
b What is the probability that a randomly selected car will be a sports car that is not automatic?
c What is the probability that a randomly selected car will be an automatic car that is not a sports car?

11 A car hire firm has 60 cars for hire. There are 33 four-wheel drive cars (4WD) and 26 automatic cars. There are 10 cars that are neither automatic nor four-wheel drive.
a Copy and complete this two-way table.

	Automatic car	Not automatic	Total
4WD			
Not 4WD			
Total			

b If a customer randomly selects a car for hire, find these probabilities:
i Pr(automatic and 4WD)
ii Pr(4WD)
iii Pr(automatic)
iv Pr(not 4WD)
v Pr(not automatic)
vi Pr(neither automatic nor 4WD)

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Example 20 Using two-way tables in worded problems

A total of 50 students were asked whether they liked or disliked skiing and snowboarding. Of these students, 25 liked both skiing and snowboarding and 12 disliked skiing but liked snowboarding. The total number of students who disliked skiing was 17.

a Complete a two-way table showing this information.
b If a student is randomly chosen from this group, what is the probability that they dislike skiing but like snowboarding?
c How many students like skiing?
d What proportion of students who like skiing also like snowboarding?
e If a student is selected out of those that like snowboarding, what is the probability that they dislike skiing?

Solution | **Explanation**

a

	Like skiing	Dislike skiing	Total
Like snowboarding	25	12	37
Dislike snowboarding	8	5	13
Total	33	17	50

First fill in the given numbers.
Use the totals to find any missing values.

b $\Pr$(dislike skiing but like snowboarding)

$= \frac{12}{50} = \frac{6}{25}$

12 dislike skiing but like snowboarding out of a total of 50. Use $\Pr()$ for all probability answers.

c 33 students like skiing

33 is the total in the like skiing column.

d $\frac{25}{33}$

Out of the 33 who like skiing there are 25 who like snowboarding.

e $\Pr$(dislike skiing given that like snowboarding) $= \frac{12}{37}$

Out of the 37 who like snowboarding there are 12 who dislike skiing.

Now you try

Out of a total of 40 outdoor enthusiasts, 25 liked hiking and 4 liked both hiking and climbing. The number who disliked climbing was 28.

a Complete a two-way table showing this information.
b One of the enthusiasts was selected at random. What is the probability that they dislike hiking?
c How many enthusiasts liked hiking or climbing?
d What proportion of enthusiasts liked only hiking.
e What is the probability that one of the enthusiasts likes neither hiking nor climbing?

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12 A total of 33 students were asked whether they liked or disliked volleyball and tennis. Of these students, 12 liked both volleyball and tennis and 6 disliked volleyball but liked tennis. The total number of students who liked volleyball was 23.

a Copy and complete a two-way table.

	Like volleyball	Dislike volleyball	Total
Like tennis			
Dislike tennis			
Total			

b If a student is randomly chosen from this group, what is the probability that they dislike tennis but like volleyball?

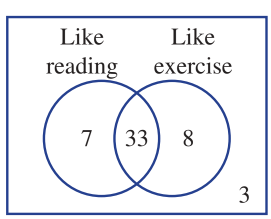
c How many students like tennis?

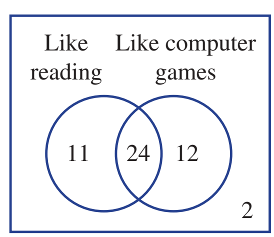
d What proportion of students who like tennis also like volleyball?

e If a student is selected out of those who like tennis, what is the probability that they dislike volleyball?

Two-way table from two Venn diagrams — 13

13 Two surveys of two different groups of people showed how many students liked reading compared to how they like exercise and computer games. The results are shown in these Venn diagrams.





a Copy and complete this two-way table showing this information.

	Like reading	Don't like reading	Total
Like exercise			
Don't like exercise			
Like computer games			
Don't like computer games			
Total			

b If a student is selected out of those who like exercise, what is the probability that they like reading also?

c If a student is selected out of those who don't like exercise, what is the probability that they don't like reading also?

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8K Experimental probability

Learning intentions

- To understand that the theoretical probability of an event can be estimated by running an experiment, and that running more trials generally gives a better estimate.
- To be able to calculate the experimental probability of an event.
- To be able to calculate the expected number of occurrences given a probability and a number of trials.
- To understand that a simulation using random devices can be used to generate experimental probabilities.

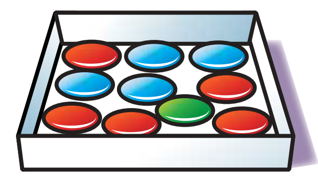
Key vocabulary: experimental probability, expected number, simulation, random number generator

Sometimes the probability of an event is unknown or cannot be determined using the techniques learnt earlier. An experiment or survey results can be used to estimate an event's probability and this estimate is called an experimental probability.

Lesson starter: A horse race

Work in pairs, small groups or as a class for this activity.

Equipment: Container with 5 red counters, 4 blue counters and 1 green counter.



1 Use the colour of each counter in the name of your horses, e.g. Red Racer, Blue Beauty, Green Lightning.
2 Copy the table below to track the progress of each horse.
3 Randomly select a counter. The horse with that colour in its name moves forward 100 m. Shade in a cell in the table to show that the horse has moved forward 100 m.
4 Return the counter to the container.
5 Continue selecting a counter, moving that horse colour forward, and replacing the counter.
6 The winning horse is the first to reach the finish at 1000 m.
7 Stop playing when a horse has won.

Horse	100 m	200 m	300 m	400 m	500 m	600 m	700 m	800 m	900 m	1000 m
Red Racer										
Blue Beauty										
Green Lightning										

Discussion questions

Discuss these questions with your classmate and write down the answers.

1 Which 'horse' colour won your race?
2 How many counters were selected in total until this horse won?
3 What proportion of the total number of selected counters did the winning colour have? Name this the experimental probability.
4 Use this experimental probability to calculate how many times you would expect to select your winning colour if you selected a counter and replaced it 800 times.
5 What is the actual probability of selecting the colour of your winning horse?
6 Use the actual probability to find out how many times you would expect to select each colour if you selected a counter and replaced it 800 times.
7 Why do you think there can be different expected numbers depending on whether you use experimental probability or actual probability? Give one reason.

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Key ideas

- The **experimental probability** of an event based on a particular experiment is defined as $\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$.
- The **expected number** of occurrences = probability × number of trials.
- Complex events can be simulated. A **simulation** is conducted using random devices such as coins, dice, spinners or **random number generators**.

Exercise 8K

Understanding

1, 2	2

Hint for Q1: The experimental probability of green is the proportion (fraction) of times that green has occurred.

1 A spinner is spun 10 times and the colour shown is recorded.
Blue, blue, green, red, blue, green, blue, red, blue, blue.
  a How many times was green shown?
  b What is the experimental probability of green being spun?
  c What is the experimental probability of blue being spun?

2 A coin is tossed 10 times and the result shown is recorded.
Head, tail, tail, head, head, tail, head, tail, head, head.
  a How many times did heads appear?
  b What is the experimental probability of a head appearing?
  c What is the experimental probability of a tail appearing?

Fluency

3–8	4, 5, 7–9

Example 21 Working with experimental probability

A coin is tossed 20 times and the results are shown in this table.

Outcome	Head	Tail
Frequency	8	12

a What is the experimental probability of tossing a head?
b Using this experimental probability, calculate the expected number of heads you would get if you tossed this coin 500 times.
c What is the actual probability of a head? (Assume the coin is as equally likely to fall on heads as tails.)
d Using the actual probability, calculate the expected number of heads you would get if you tossed this coin 500 times.

Solution	Explanation
a $\frac{8}{20} = \frac{2}{5}$	A head came up 8 times out of a total of 20 throws.
b $\frac{8}{20} \times 500 = 200$	Expected number = experimental probability times number of trials

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c $\Pr(\text{Heads}) = \frac{1}{2}$

The coin is equally likely to fall on heads as tails so a 1 in 2 chance for heads.

d $\frac{1}{2} \times 500 = 250$

Expected number = actual probability times number of trials.

Now you try

A 6-sided die is tossed 40 times and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	7	4	7	9	8	5

a What is the experimental probability of a 6?

b Using the experimental probability, calculate the expected number of 6s if the die is rolled 240 times.

c Using the theoretical probability, calculate the expected number of 6s if the die is rolled 240 times.

3 A coin is tossed 50 times and the results are shown in this table:

Outcome	Head	Tail
Frequency	27	23

a What is the experimental probability of a head?

b Using this experimental probability, calculate the expected number of heads you would get if you tossed this coin 700 times.

c What is the actual probability of a head? (Assume the coin is as equally likely to fall on heads as tails.)

d Using the actual probability, calculate the expected number of heads you would get if you tossed this coin 700 times.

4 A die is tossed 100 times and the results are shown in this table:

Outcome	Even number	Odd number
Frequency	55	45

a What is the experimental probability of an even number?

b Using this experimental probability, calculate how many times you would expect to get an even number if you tossed this die 1000 times.

c What is the actual probability of obtaining an even number when a die is tossed?

d Using the actual probability, calculate how many times you would expect to get an even number if you tossed this die 1000 times.

Example 22 Working with probabilities and expected numbers

A number of red, white and orange marbles are placed in a jar. Repeatedly, a marble is taken out, its colour is noted and the marble is replaced in the jar. The results are tallied in the table.

Colour	Red	White	Orange
Tally	卌 |||	卌 卌 ||	卌 卌
Frequency	8	12	10

a What is the experimental probability of a red marble being chosen next?

b What is the experimental probability of a red or a white marble being chosen?

c If the experiment is done 600 times, what is the expected number of times that an orange marble is selected?

Continued on next page

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Solution

a $\Pr(\text{red}) = \frac{8}{30} = \frac{4}{15}$

Explanation

Experimental probability

$= \frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$.

b $\Pr(\text{red or white}) = \frac{20}{30} = \frac{2}{3}$

Red or white marbles were selected 20 times out of the 30 trials.

c Expected number $= \frac{1}{3} \times 600 = 200$

$\Pr(\text{orange}) = \frac{10}{30} = \frac{1}{3}$

Expected number = probability × number of trials.

Now you try

A number of white, milk and dark chocolates are selected from a box and replaced after each selection. The results are tallied in the table.

Type	White	Milk	Dark
Tally	||||	卌 卌 ||	卌 |||

a What is the experimental probability of a white chocolate being chosen next?
b What is the experimental probability of a milk or dark chocolate being chosen next?
c If the experiment is done 900 times, what is the expected number of times that a milk chocolate is selected?

5 A spinner is spun 50 times and the results are shown in the frequency table below.

Colour	Red	Blue	White	Purple
Tally	卌 卌 卌 卌 卌 卌	卌	||	卌 卌 |||
Frequency	30	5	2	13

Hint for Q5d: Expected number = probability × number of trials.

a What is the experimental probability of red?
b What is the experimental probability of blue?
c What is the experimental probability of red or purple?
d If the spinner were spun 1000 times, what is the expected number of times that white would be spun?

6 A group of households are surveyed on how many cars they own. The results are shown.

0 cars	1 car	2 cars	3 cars	4 cars
卌 卌 ||	卌 卌 卌 卌 卌 卌 卌 ||	卌 卌 卌 卌 卌 卌 卌 卌 |	卌 |||	||

a Write the tallied results as a frequency table, with headings 'Number of cars' and 'Frequency'.
b How many households in total were surveyed?
c What is the experimental probability that a randomly chosen household owns no cars?
d What is the experimental probability that a randomly chosen household owns 2 or more cars?

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7 A die is painted so that 3 faces are blue, 2 faces are red and 1 face is green.
  a What is the actual probability that it will display red on one roll?
  b Using the actual probability, how many times would you expect it to display red on 600 rolls?
  c Using the actual probability, how many times would you expect it to display blue on 600 rolls?

8 A spinner displays the numbers 1, 2, 3 and 4 on four sectors of different sizes.
It is spun 20 times and the results are 1, 3, 1, 2, 2, 4, 1, 1, 3, 1, 2, 4, 4, 2, 4, 3, 1, 1, 3, 2.
  a Give the experimental probability that the spinner will land on:
    i 1 ii 2 iii 3 iv 4
  b On the basis of this experiment, what is the expected number of times in 1000 trials that the spinner will land on 3?

Hint for Q8: Write these probabilities as decimals.

9 A fair die is rolled 100 times and the number 5 occurs 19 times.
  a What is the experimental probability of a 5 being rolled? Give a decimal answer.
  b What is the actual probability of a 5 being rolled on a fair die? Round the answer to two decimal places.
  c For this experiment, which is greater: the experimental probability or the actual probability?

Problem-solving and reasoning

10, 11 | 11–13

10 A basketball player has a 1 in 2 chance of getting a shot in from the free-throw line. To simulate this, use a coin: heads represents the shot going in, tails represents missing.
  a Flip a coin 20 times and write down the results.
  b Based on your experiment, what is the experimental probability that a shot will go in?
  c Based on the actual probability of $\frac{1}{2}$, how many of 20 throws are expected to go in?
  d Is it possible that this basketball player could have 20 shots from the free-throw line and 18 go in? Could all 20 go in?

11 A number of marbles are placed in a bag – some are red and some are green.
A marble is selected from the bag and then replaced after its colour is noted.
The results are shown in the table.

Red	Green
28	72

Based on the experiment, give the most likely answer to the following questions.
  a If there are 10 marbles in the bag, how many are red?
  b If there are 6 marbles in the bag, how many are red?
  c If there are 50 marbles, how many are red?
  d If there are 4 marbles, how many are green?
  e If there are 14 green marbles in the bag, how many marbles are there in total?
  f If there are 3 red marbles, how many green marbles are there?

12 Four coins are tossed together and the number of tails is noted. This is repeated 11 times.

Number of tails	0	1	2	3	4
Frequency	1	3	5	2	0

  a Based on this experiment, what is the experimental probability of obtaining 4 tails?
  b Based on this experiment, what is the experimental probability of obtaining 3 heads?
  c True (T) or False (F)? If the experimental probability is 0 then the actual probability is 0.
  d If 4 coins are tossed at the same time, what is the actual probability of obtaining 5 tails?
  e True (T) or False (F)? If the actual probability is 0 then the experimental probability is 0.

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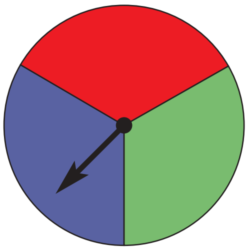
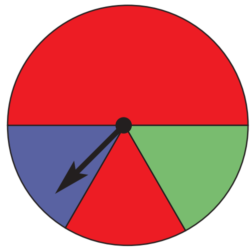
8K

13 In a probability simulation the probability of the outcomes must stay the same. For example, it is possible to simulate (model) a coin using a die by using the numbers $1-3$ to stand for tails $\left(\text{probability} = \frac{1}{2}\right)$ and $4-6$ to stand for heads $\left(\text{probability} = \frac{1}{2}\right)$. Which of the following spinners could be simulated using a single roll of a die? Explain your answer.

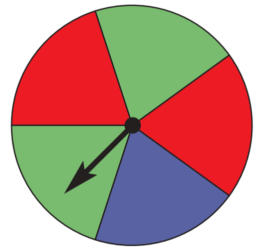
a



b



c



Hint for Q13: Explain which numbers on the die correspond to the colours on the spinner.

Running a simulation

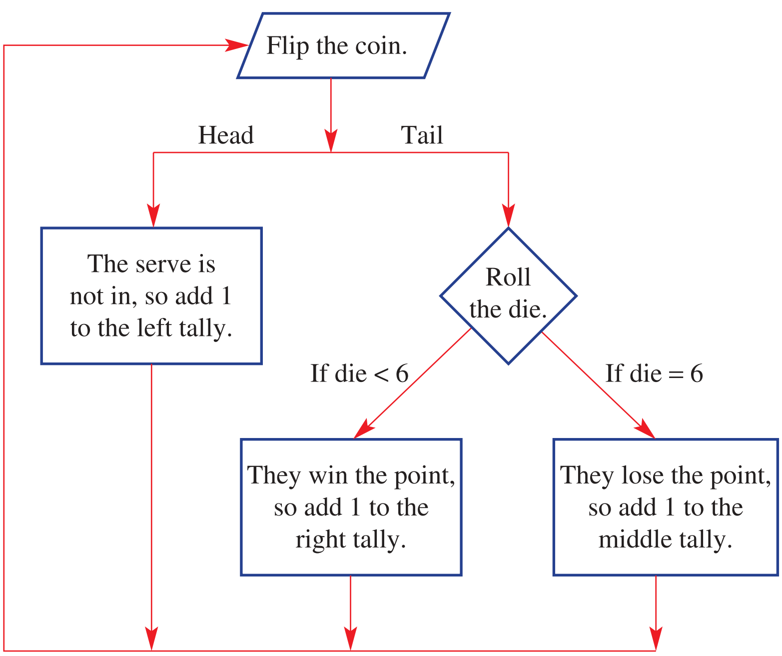
— 14

14 A tennis player has a 1 in 2 chance of getting their first serve in. When the serve goes in, they have a 5 in 6 chance of winning the point and a 1 in 6 chance of losing the point.

a Copy this table and keep a record of results from your 'tennis' simulation.

	Serve not in	Serve in but lose point	Serve in and win point
Tally			
Frequency			

b Using a coin and a die, run the simulation outlined below at least 20 times.



c From your results, state your experimental probabilities for:
- **i** serve not in.
- **ii** serve in but lose a point.
- **iii** serve in and win a point.

d Using these experimental probabilities, find the expected number of points won in a match for a player who makes 120 serves.

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Student Representative Council (SRC) coordinator

Statistics is an area of mathematics used in many occupations and by individuals on a daily basis. Statistics refers to the collection and analysis of numerical data. The Australian government uses statistics about population growth, people's health, economic performance and the state of our environment, etc. Federal, state and local governments all use statistics to determine which issues need addressing with new policies. Counting election votes and deciding final results all fall under the banner of statistics.

At a school level, SRC elections are held each year and the coordinating teacher uses statistics to determine which students are elected.

SRC students often use statistics to find out which issues to focus on within their school.

1 The Year 8 SRC needs 5 students. Fourteen students were nominated and every Year 8 student voted for 5 SRC students. The results of this vote are in the table below.

Student	Votes	Student	Votes
Charlie K	83	Sara H	34
Tahlia P	120	William B	45
Hicham J	94	Vivaan P	51
Aanya N	72	Braydon B	29
Eden V	102	Austin C	37
Scarlett K	19	Monique H	11
Josiah G	45	Elizabeth S	8

a How many students are in the year group?

b What was the highest number of votes that a nominated student received?

c List the names of the 5 SRC members for the year.

d What percentage of the total vote did each SRC member receive?

e Explain why this method of voting is not used on a larger scale, such as in federal government elections.

2 The results from the Year 12 SRC election are shown below as a percentage of the total votes cast. A total of 750 votes were cast.

Student	Percentage
Alex P	44%
Jia Hao N	22%
Kelly Y	14%
Samantha W	12%
Nelson C	8%

a How many votes did each of the final five SRC members receive?

b Which of the SRC elected students had more than 150 votes?

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3 The Year 8 SRC decided to run a whole-school survey to determine the usage of the school's computer system in the library at lunch time. The survey question asked: 'How many times a week do you use the library computers at lunch time?'. The results are recorded below:

Usage of library computers	
Number of times per week	Frequency
0	80
1	138
2	234
3	245
4	123
5	65

a Given that every student in the school participated in the survey, how many students are in the school?
b What percentage of the students never used the library computers? Round to the nearest whole percent.
c What is the mean number of times that any student used the library computers in a typical week? State the answer to 2 d.p.
d What percentage of the student body used the computers more than twice a week? Round the answer to a whole number.
e Based on the results of the survey, should the SRC run a lunch time computer competition in the library?
f What other survey questions could the SRC ask to help them accurately determine if the computer competition would be a success?
g Carry out a similar survey in your own class/school.

Using technology

4 The Year 8 SRC students came up with the suggestions shown in the table below. They then conducted a survey where all Year 8 students selected their two highest preferences. The results are shown below.
a Copy this table into an Excel spreadsheet.

	A	B	C	D	E	F	G	H
1	SRC suggestions							
2	Ideas	More recycling bins	Technology games challenge	Dance, music and art competition	Disco or dance evening	Lunch-time clubs and activities	Hand ball courts	Self-esteem and 'stress-less' posters
3	Votes of Year 8 students	25	32	62	78	52	14	29

b Choose one or more of the charts below and display the SRC results. You need to first select rows 2 and 3 of the table, choose 'Insert', then follow the selected instructions.

Column charts
i Click 'Insert Column or Bar Chart' and select either the vertical '2D (or 3D) Clustered Column' or the horizontal '2D (or 3D) Clustered Bar'.
ii Select the chart, choose Design and select the colours and style that you want.

A treemap and divided bar graph
i Click 'Insert Hierarchy Chart', select 'Treemap'.
ii The 'key' is optional as names are within each rectangular area.
iii Title the Treemap and choose your colours and its overall rectangular shape.
iv One long rectangle makes it into a divided bar graph.

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Seven free chocolate bars

Sasha notices that a chocolate company claims that one in six chocolate bars has a message that entitles you to a free chocolate bar. He plans to purchase one bar each day for 10 days in the hope of winning at least 3 free bars.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

1 Preliminary task

Use a 6-sided die to simulate buying a chocolate bar. If the number 6 is rolled, this represents finding the 'free chocolate bar' message inside the wrapper.

- **a** Roll the die once and see what number comes up. Did you receive a free chocolate bar?
- **b** Repeat part **a** for a total of 10 trials. How many 6s did you obtain?
- **c** Using your result from part **b**, decide how many free chocolates Sasha received when he bought 10 chocolate bars. How does this compare to other students in your class?

2 Modelling task

Modelling

Formulate

- **a** The problem is to determine a good estimate for the probability that Sasha will receive at least 3 free chocolate bars after 10 purchases. Write down all the relevant information that will help solve this problem.
- **b** Describe how a 6-sided die can be used to simulate the purchase of a chocolate bar and decide whether or not you win a free one.

Solve

- **c** Repeat the simulation including 10 trials and count the number of times a 6 (free chocolate bar) is obtained.
- **d** Continue to repeat part **c** for a total of 12 simulations. Record your results in a table similar to the following using a tally.

Simulation	1	2	3	4	5	6	7	8	9	10	11	12
Number of 6s tally (out of 10)												
Number of 6s (frequency)												

- **e** Out of the 12 simulations, how many indicate that at least 3 free chocolate bars will be obtained?

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Evaluate and verify

f By considering your results from the 12 simulations, determine the experimental probability that Sasha will obtain at least 3 free chocolate bars after 10 purchases.

g Compare your result from part **f** with others in your class.

h Explain how you might alter your experiment so that your experimental probability might be closer to the theoretical probability.

Communicate

i Summarise your results and describe any key findings.

3 Extension questions

a Find the average experimental probability that Sasha will obtain at least 3 chocolate bars after 10 purchases using the data collected from the entire class.

b Compare your result from part **a** with the theoretical value of 0.225, correct to three decimal places.

c Explore how random number generators and technology could be used to repeat this experiment for a large number of trials.

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Cutting the hay

Key technology: Spreadsheets and programming

The Williams family own a farm and cut hay each year to store for the winter or sell on the market. The timing is quite tight as the grass needs to be cut and baled over a 3-week period. This work can only be done on Saturday as the members of the Williams family hold other work positions on the weekdays and are otherwise busy on Sunday. They therefore have three Saturdays available of which at least two out of the three need to be dry in order to complete the work. History shows that there is a 3 in 5 chance that any particular Saturday at that time of year is dry.

1 Getting started

We will use a random number generator to simulate the number of dry days on the three Saturdays available. By generating a random integer between 1 and 5 inclusive we will allocate the outcomes as follows.

- Dry weather: $\{1, 2, 3\}$
- Wet weather: $\{4, 5\}$

a Search for 'Random number generator' on the internet and set the Min as 1 and the Max as 5. Then generate a random integer.

b Decide if your given random number from part **a** represents dry weather or wet weather.

c Now generate three random integers between 1 and 5 inclusive (representing three Saturdays) and count how many times dry weather is obtained.

d Based on your result from part **c**, will the Williams family be able to complete the work required to make the hay? Remember that at least two out of the three need to be dry in order to complete the work making hay.

2 Using technology

We will use a spreadsheet to complete the three-day simulation described above.

a Create a spreadsheet using the given information and fill down at cells A5 and B5 for three rows.

b Explain why the formula in cell B5 allocates the word 'Dry' for the integers $1, 2$ or 3.

c Press Function F9 to rerun the simulation for a total of 10 times. For each simulation, note the total number of dry days and enter your results into this table. For example, if your simulation results in 2 dry days, then add a dash to the tally in the number 2 column.

	A	B
1	**Making hay**	
2		
3	**Integer**	**Dry/Wet?**
4		
5	=RANDBETWEEN(1,5)	=IF(A5<=3,"Dry","Wet")
6	=RANDBETWEEN(1,5)	=IF(A6<=3,"Dry","Wet")
7	=RANDBETWEEN(1,5)	=IF(A7<=3,"Dry","Wet")
8		
9	**Total Dry**	=COUNTIF(B5:B7,"Dry")

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d By counting the dashes made in each of the four tally cells, fill in the cells for Frequency. How many of the 10 simulations resulted in at least two dry days?

Total dry	0	1	2	3
Tally				
Frequency				

e What fraction of the 10 simulations resulted in at least two dry days?

f Your answer to part **e** is your experimental probability that the Williams family will be able to complete the work making hay. Compare your answer with your fellow students. Are your experimental probabilities reasonably close together or spread out?

3 Applying an algorithm

We will now try to improve on your experimental probability result above by increasing the number of simulations completed.

a Use your spreadsheet to apply this algorithm.
- Step 1: Use Function F9 to rerun the simulation.
- Step 2: Note the total number of dry days.
- Step 3: Record a dash in the corresponding tally.
- Step 4: Repeat from Step 1 until the simulation as been repeated 50 times.

b Calculate the frequencies in your table and count the total number of simulations that resulted in at least two dry days.

c Calculate the experimental probability that the Williams family will be able to complete the work making hay.

d Compare your result with your fellow students. Are the class results closer together or more spread out compared to the result obtained in part **2f** above? Can you provide a reason for this observation?

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1 John Venn was an English mathematician who invented Venn diagrams to make sorting data and probability calculations easier. He was also a fan of cricket. Solve the questions below to find the answer to this question:
What did John Venn invent that, in 1909, clean bowled (i.e. bowled out) one of our best Australian cricket batsmen four times?

20

7	30	6	E'	23	F	12

5	20	15	14	23	F	T

A Find the mean of these scores: 26, 25, 13, 24, 12 and 20.

B What is the median of these scores: 2, 4, 4, 7, 8, 9, 10?

C What is the range of these scores: 13, 18, 5, 7, 16, 3?

E True (T) or False (F): A pie chart is in the shape of a circle.

G What frequency does this tally represent? ~~||||~~ ~~||||~~ ||

H If 8 students have a cat only, and 6 have both a cat and a dog, and 9 a dog only, how many have a cat?

I If 8 students have a cat only, and 6 have both a cat and a dog, and 9 a dog only, how many have a cat or a dog or both?

L How is the complement of the event E written?

M What is the mode of these scores: 5, 6, 7, 5, 4, 5, 3, 5, 2

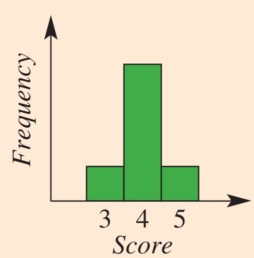
N True (T) or False (F): A skewed graph has its highest frequency in the middle.

O How many times would we expect to throw either a 4 or a 5 if a die was tossed 90 times?

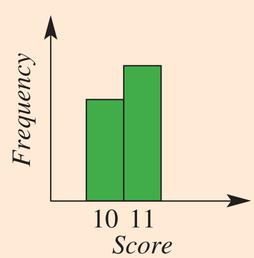
W How many of these ages are in the interval 12 − 15 years? 12, 13, 16, 11, 12, 15, 19, 19, 14, 16, 14.

2 The following graphs are drawn to scale but the frequency scale has been omitted. Determine the median for each one.

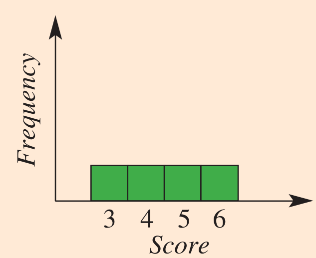
a



b



c



3 At the local sports academy, everybody plays netball or tennis or both. What is the probability that a randomly chosen person at the academy plays both netball and tennis, given that:
- 10 people play both netball and tennis,
- half the tennis players also play netball,
- one-third of the netballers also play tennis.

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4 Choose a word from this list that matches each of the following descriptions. FLUFF, DING, OFF, PERSONALITY, TEN, PROBABILITY, MOON, STUMBLE, TOY, TRY.

a $\Pr(\text{vowel}) = \frac{1}{2}$

b $\Pr(\text{F}) = \frac{2}{3}$

c $\Pr(\text{vowel}) = \frac{1}{4}$ and $\Pr(\text{D}) = \frac{1}{4}$

d $\Pr(\text{I}) = \frac{2}{11}$ and $\Pr(\text{consonant}) = \frac{7}{11}$.

e $\Pr(\text{M}) = \frac{1}{7}$ and $\Pr(\text{T}) = \frac{1}{7}$ and $\Pr(\text{S}) = \frac{1}{7}$.

f $\Pr(\text{vowel}) = 0$ and $\Pr(\text{T}) = \frac{1}{3}$

5 In a certain town of 100 adults there are 22 women who can cook and 18 men who cannot cook. Given that half the town is male and 54% of the town can cook, how many men in the town can cook?

6 **Monopoly mystery**

In a board game, two dice are rolled and the player moves forward by the sum of the numbers on each die. E.g. a throw of a 4 and a 6 means the player would move by 10 places.

Are you as likely to obtain a 9 as the sum of two dice as any other number for the total? Explain with an example.

7 Jayden is an AFL player and notes the number of points he scores in his 22-week season. His results are shown in this table.

Jayden said he scored a different number of points for each of the 22 games. Is he correct? Use an example to explain your answer.

Points	Frequency
0–4	3
5–9	11
10–14	6
15–19	1
20–25	1

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Chapter summary

Representing Data

Graphs

- Column graph

- Line graph

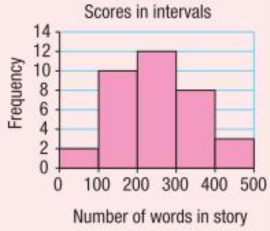
- Sector graph

- Divided bar graph

Graphs from frequency tables

Individual scores

Scores in intervals



Tables

Frequency tables

Individual scores

Number of children Score	Number of families Tally	Frequency
0	III	3
1	𝍸	5
2	𝍸 𝍸 I	11
3	𝍸	5
4	𝍸 I	6
		$n = 30$

Scores in intervals

Age	Frequency
0–4	3
5–9	7
10–14	5
	$n = 15$

Measures of centre

numerical order

0, 1, 4, 4, 5, 6, 9, 30

Median (middle) $= \frac{4+5}{2} = 4.5$

Mode (most frequent) = 4

Mean $= \frac{59}{8} = 7.4$

Measure of range

1 5 7 9 11 15

Range = 15 – 1 = 14

Probability

Theoretical probability (actual probability)

Trial: Roll a fair die
Sample space (possible outcomes)

{ 1, 2, 3, 4, 5, 6 }

Pr(odd number) $= \frac{3}{6} = \frac{1}{2}$

Trial: Select a playing card and note its suit.
Sample space: spade, diamond, club, heart

Theoretical probabilities

Pr(black) $= \frac{26}{52} = \frac{1}{2}$

Pr(heart) $= \frac{13}{52} = \frac{1}{4}$

Pr(not spade) $= \frac{39}{52} = \frac{3}{4}$

Pr(either red or a spade) $= \frac{39}{52} = \frac{3}{4}$

Pr(red ace) $= \frac{2}{52} = \frac{1}{26}$

Probabilities are written as

- fractions
- percentages
- decimals

100%	1	Certain
		Likely
0.5 50%	$\frac{1}{2}$	Even chance (50–50)
		Unlikely
0%	0	Impossible

Experimental probability

Playing card selected and replaced 20 times, and its suit noted.

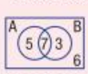
Outcome	Frequency	Experimental probability
Heart	4	$\frac{4}{20}$
Diamond	5	$\frac{5}{20}$
Club	4	$\frac{4}{20}$
Spade	7	$\frac{7}{20}$
	$n = 20$	

Expected number of outcomes

Outcome	Theoretical probability	Expected number in 20 trials
Heart	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Diamond	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Club	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Spade	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$

Venn diagrams and two-way tables

These organise data from two or more categories.



Venn diagram

	A	not A	
B	7	3	10
not B	5	6	11
	12	9	21

6 in neither category,
7 in both categories.

Pr(A) $= \frac{12}{21} = \frac{4}{7}$

Pr(B only) $= \frac{3}{21} = \frac{1}{7}$

Playing cards

Spades
Diamonds
Clubs
Hearts

Sample space from experiments with two or more steps

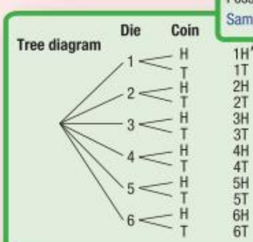
Table

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

(inside the table)

Possible outcomes
Sample space



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Chapter checklist

Chapter checklist

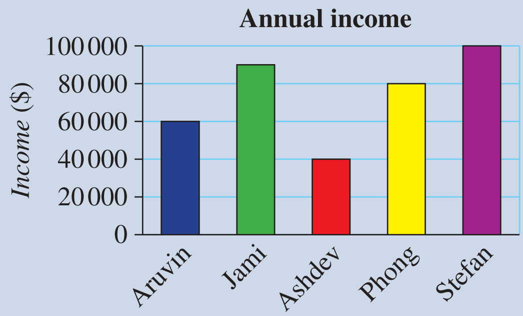
A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

8A

1 I can interpret data presented in graphical form

e.g. State the difference between Jami's income and Ashdev's income based on this column graph.



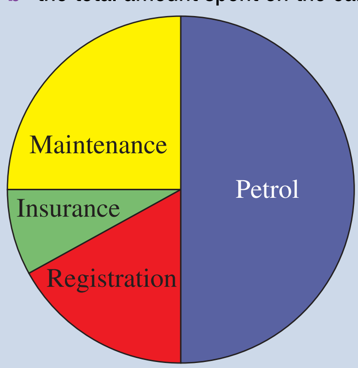
8A

2 I can interpret sector graphs

e.g. Use this sector graph to determine:

a the amount of the car's expenses that is devoted to maintenance.

b the total amount spent on the car each year if the owner spends $3000 per year on petrol.



8B

3 I can interpret tallies

e.g. Write the frequency for each car colour based on the tally below.

White	Black	Blue	Red	Yellow
𝍷𝍷𝍷	𝍸 𝍸 𝍷𝍷	𝍸 𝍸 𝍸 𝍷𝍷	𝍸 𝍷	𝍸 𝍷𝍷𝍷𝍷

8B

4 I can construct a tally and frequency table from a set of data

e.g. Put the following data into a frequency table: $1, 4, 1, 4, 1, 2, 3, 4, 6, 1, 5, 1, 2, 1$.

8C

5 I can construct a graph from a frequency table.

e.g. Represent the table below as a frequency graph and as a dot plot.

Number of siblings	Frequency
0	3
1	4
2	2
3	1

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Chapter checklist

		✔

8C **6 I can construct a graph from a frequency table using intervals**
e.g. Represent the following table as a graph.

Number of words in story	Frequency
0–99	2
100–199	10
200–299	12
300–399	8
400–500	3

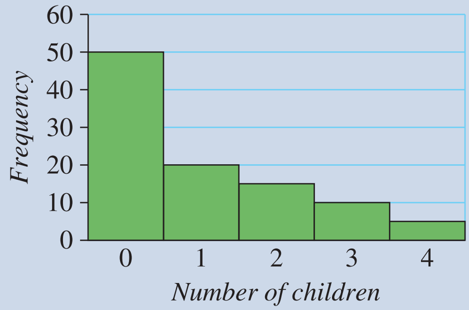
8D **7 I can find the range of a set of numerical data**
e.g. Find the range of $1, 5, 2, 3, 8, 12, 4$.

8D **8 I can find the mean and mode for a set of numerical data**
e.g. Find the mean and mode for the set of numbers: $10, 2, 15, 1, 15, 5, 11, 19, 4, 8$.

8D **9 I can find the median for a set of numerical data with an odd or even number of values**
e.g. Find the median of the following sets:
a $16, 18, 1, 13, 14, 2, 11$ **b** $7, 9, 12, 3, 15, 10, 19, 3, 19, 1$

8E **10 I can calculate population estimates using random sample data**
e.g. Out of a random sample of 10 Tasmanian devils, there are 7 that have a facial tumour. If there are 200 devils in the region, based on this sample how many would you expect to have facial tumours?

8E **11 I can interpret survey results**
e.g. A survey is conducted and the results are shown. Assuming it is a representative sample, what proportion of the population has 2 or more children?



8E **12 I can decide whether a method of data collection is likely to lead to biased samples**
e.g. In conducting a survey to determine how many children a person generally has, explain why randomly selecting people outside a childcare centre is likely to lead to bias.

8F **13 I can find the probability of a simple event**
e.g. The letters of the word PRINCE are written out on cards and one is chosen at random. Find the probability that a vowel will be chosen.

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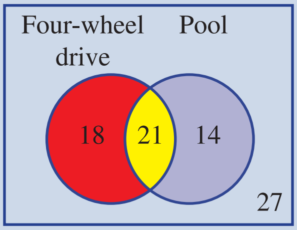
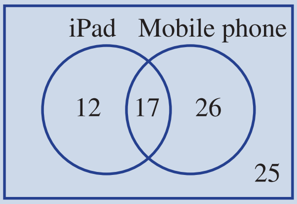
8G **14** **I can use a table to find probabilities in two-step experiments.**
e.g. A spinner with the numbers 1, 2, and 3 is spun, and then a card is chosen at random from the letters ATHS written on four cards. Find the probability of an odd number being spun and the letter H being chosen.

8H **15** **I can use a tree diagram to find probabilities in multi-step experiments.**
e.g. Three fair coins are flipped. Use a tree diagram to find the probability that exactly two of the coins show heads.

8I **16** **I can interpret a Venn diagram**
Use this Venn diagram to find how many families own a pool or a four-wheel drive car or both.



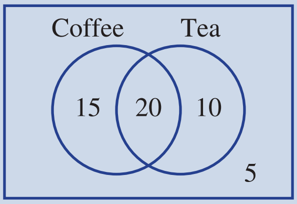
8I **17** **I can construct a Venn diagram from a situation**
e.g. Based on the survey results shown in the Venn diagram, what is the probability of choosing a student who owns a mobile phone?



8J **18** **I can complete a two-way table with missing numbers**
e.g. Fill in the missing numbers in this two-way table.

	Like Macs	Dislike Macs	Total
Like PCs			
Dislike PCs		3	13
Total	35	40	75

8J **19** **I can construct a two-way table from a Venn diagram**
e.g. Represent this Venn diagram as a two-way table and then state how many people surveyed like tea.



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		✓
8J	**20 I can use a two-way table to calculate probabilities** e.g. The eating and sleeping preferences of zoo animals are shown below. Find the probability that an animal only sleeps at night.	
8K	**21 I can find the expected number of times an event will occur** e.g. A weighted coin is tossed 20 times and it lands heads 8 times. Find the experimental probability of a head and use this to calculate the expected number of heads you would get if you tossed this coin 500 times.	
8K	**22 I can find the experimental probability of an event** e.g. Coloured marbles are in a jar. Repeatedly, a marble is taken out, its colour noted and then it is placed back in the jar. Use the results tallied below to find the experimental probability that the next marble chosen will be red.	

Table for item 20:

	Eats meat	No meat	Total
Sleeps during day	20	12	32
Only sleeps at night	40	28	68
Total	60	40	100

Table for item 22:

Colour	Red	White	Orange
Tally	𝍸 |||	𝍸 𝍸 ||	𝍸 𝍸
Frequency	8	12	10

Chapter checklist

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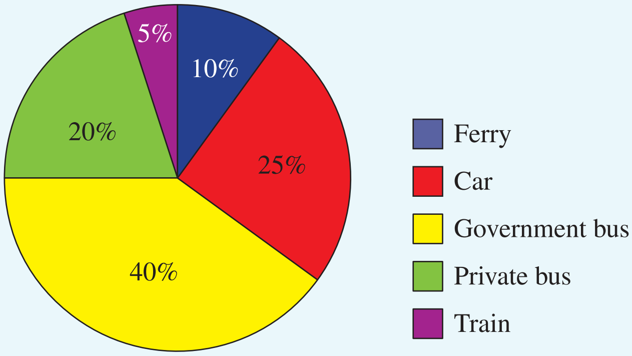
Short-answer questions

8A **1** The sector graph shows the type of transport office workers use to get to work every day.



a Which type of transport is the most popular?
b Which type of transport is the least popular?
c What percentage of office workers did not travel by car?
d If 20 000 workers were surveyed, how many people travelled to work each day by train?
e The year after this survey was taken, it was found that the number of people using government buses had decreased. Give a reason why this could have occurred.

8B **2** Some students were asked how many hours of study they did before their half-yearly Maths exam. Their responses are represented in a tally.

<table>
<tr><th>0 hours</th><th>1 hour</th><th>2 hours</th><th>3 hours</th><th>4 hours</th></tr>
<tr><td>||</td><td>卌 |</td><td>|||</td><td>|||</td><td>卌 |||</td></tr>
</table>

a How many students are in the class?
b Convert the tally above into a frequency table.
c Draw a graph to represent the results of the survey.
d What proportion of the class did no study for the exam?
e Find the total number of hours of study done by this group.
f Calculate the mean number of hours per student in the class that were spent studying for the exam, giving the answer correct to one decimal place.

8D **3** **a** Rewrite the following data in ascending order:
56 52 61 63 43 44 44 72 70 38 55
60 62 59 68 69 74 84 66 53 71 64
b What is the mode?
c What is the median for these scores?
d Calculate the range.

8D **4** The ages of students in an after-school athletics squad are shown in the table below.

Age	Frequency
10	2
11	3
12	4
13	8
14	10

a State the total number of students in the squad.
b List the ages of all these students in ascending order.
c Calculate the mean age of the squad, correct to two decimal places.
d What is the median age of the students in the squad?

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8C **5** A group of teenagers were weighed and their weights recorded to the nearest kilogram. The results are as follows:

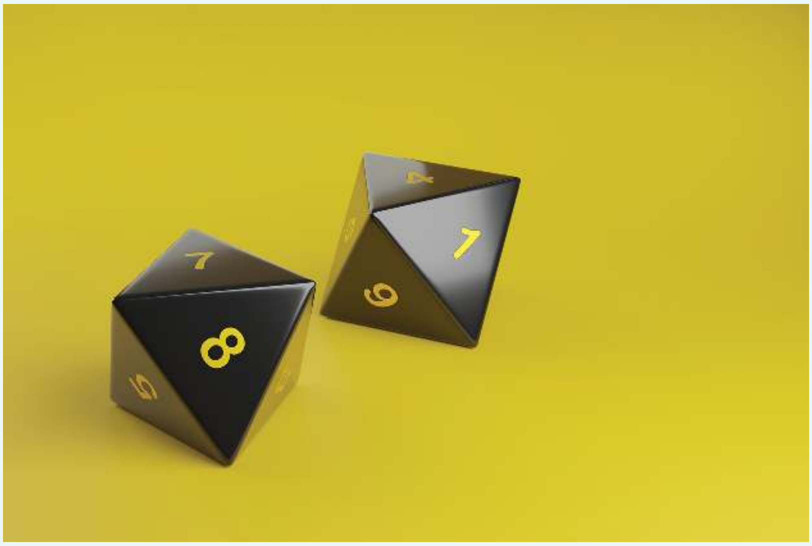
56 64 72 81 84 51 69 69 63 57 59 68 72 73 72 80 78 61 61 70
57 53 54 65 61 80 73 52 64 66 66 56 50 64 60 51 59 69 70 85

a Find the highest and lowest weights and the range.
b Create a grouped frequency distribution table using the groups (intervals) of 50–54, 55–59, 60–64 etc.
c Find the modal group.
d Why is this sample not representative of the whole human population?

8D **6** **a** Use the data $5, 1, 7, 9, 1, 6, 4, 10, 12, 14, 6, 3$ to find:
i the mean **ii** the median **iii** the range.
b An extra score of 52 is added into the list in part **a**. Calculate the new median and mean, and state which measure has changed the most.
c What is the name for a score that is much larger than all the other values in a list?

8E **7** In an attempt to find the average number of hours of homework that a Year 8 student completes, Samantha asks 10 of her friends in Year 8 how much homework they do.
a Explain two ways in which Samantha's sampling is inadequate for representing the population of Year 8s in her school.
b If Samantha wished to convey to her parents that she did more than the average, how could she choose 10 people to bias the results in this way?

8F **8** An eight-sided die has the numbers $1, 2, 3, 4, 5, 6, 7, 8$ on its faces.
a Find the probability that the number 4 is rolled.
b What is the probability that the number rolled is odd?
c What is the probability that the number rolled is both even and greater than 5?
d If P is the event that a prime number is rolled, state the sample space of P', the complement of P.
e What is $\Pr(P')$?



8F **9** The letters of the word MATHEMATICIAN are written on 13 cards. The letters are placed in a bag and one card is drawn at random.
a State the sample space.
b Find the probability of choosing the letter M.
c Find the probability of a vowel being drawn.
d What is the probability of a consonant being drawn?
e What is the probability that the letter chosen will be a letter in the word THEMATIC?

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Chapter review

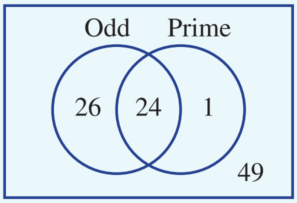
8G **10** A die is rolled and then a coin is flipped.

a Draw a table to list the sample space of this experiment.
b Find the probability that the die shows an even number and the coin shows tails.
c Find the probability of obtaining the pair $(3$, heads$)$.

8H **11** A two-digit number is to be made from the digits $3, 4$ and 5.

a Draw a tree diagram to show all outcomes. The digits are chosen randomly and can be used more than once (e.g. 44 is possible).
b What is the probability of creating an even number?
c Find the probability that the number is divisible by 3.
d What is the probability that the sum of the two digits is greater than 8?
e Find the probability that the number starts with 3 or 5.
f If the numbers cannot be used more than once, what is the probability of creating an even number?

8I **12** The Venn diagram below shows how many whole numbers between 1 and 100 are odd and how many are prime.



Consider the whole numbers between 1 and 100.

a How many are odd?
b How many prime numbers are there?

For parts **c**, **d**, **e**, **f** give answers both as a percentage and a simplified fraction.

c What is the probability that a randomly selected number will be odd and prime?
d What is the probability that a randomly selected number will be prime but not odd?
e What is the probability that a prime number is chosen out of all the odd numbers?
f What is the probability that an odd number is chosen out of all the prime numbers?

8J **13** Of 20 workshops in a town, 12 fit tyres (T), 10 fit mufflers (M), and 8 fit both tyres and mufflers.

a Represent this information in a two-way table.

	Mufflers	No mufflers	Total
Tyres			
No tyres			
Total			

b How many workshops fit neither tyres nor mufflers?
c Find the probability that a randomly selected workshop fits:

i tyres
ii tyres and mufflers
iii tyres or mufflers
iv mufflers only
v neither tyres nor mufflers

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8K **14** Of 50 people surveyed, 20 said that they intend to send their children to a private school.

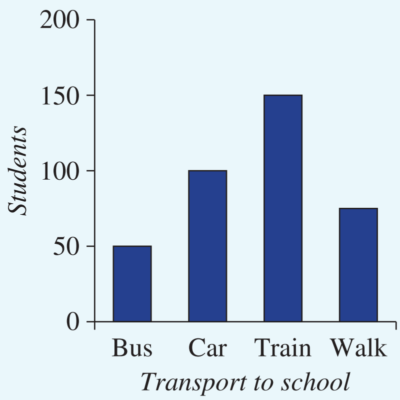
a Find the experimental probability that a person intends to send their children to private school.

b 200 people are selected from the population. What is the expected number who intend to send their children to private school?

Multiple-choice questions

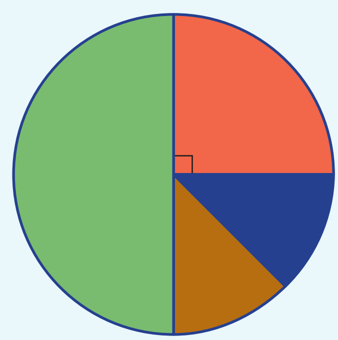
8A **1** Using the information in the column graph, how many students do not walk to school?

A 75 **B** 150 **C** 300 **D** 375 **E** 100



8A **2** The chocolates in a bag are grouped by colour and the proportions shown in the sector graph. If there are equal numbers of blue and brown chocolates, how many are blue, given that the bag contains 28 green ones?

A 112 **B** 7 **C** 56
D 14 **E** 28



8B **3** The table below shows the number of goals scored by a soccer team over a season.

Goals	0	1	2	3	4
Tally for number of games	卌 ||	卌	|||	|||	||

The total number of games played by the soccer team is:

A 28 **B** 10 **C** 20 **D** 13 **E** 5

8B **4** For the soccer results in the table in question **3**, the total number of goals scored in the season is:

A 28 **B** 10 **C** 4 **D** 20 **E** 13

8D **5** Which is the best description of the mode in a set of test scores?

A The average of the scores
B The score in the middle
C The score with the highest frequency
D The difference between the highest and lowest score
E The lowest score

Chapter review

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Chapter review

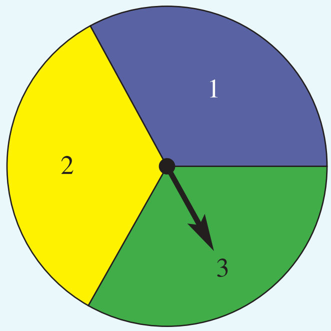
8D **6** For the set of data $1, 5, 10, 12, 14, 20$ the range is:

A 1 **B** 19 **C** 4
D 11 **E** 6

8F **7** The letters of the word STATISTICS are placed on 10 different cards and placed into a hat. If a card is drawn at random, the probability that it will show a vowel is:

A 0.2 **B** 0.3 **C** 0.4
D 0.5 **E** 0.7

8F **8** The spinner shown is spun. The probability that the spinner will display an odd number is:



A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{2}{3}$ **D** $\frac{1}{2}$ **E** 1 and 3

8K **9** A coin is tossed 30 times. The expected number of tails is:

A 29
B 30
C $\frac{1}{2}$
D 15
E 25

8K **10** An experiment is conducted in which a die is rolled 300 times. The results are shown in this frequency table.

Outcome	1	2	3	4	5	6
Frequency	48	53	44	55	51	49

Based on this, the experimental probability of obtaining an even number is:

A $\frac{48}{300}$
B $\frac{55}{300}$
C $\frac{50}{300}$
D $\frac{150}{300}$
E $\frac{157}{300}$

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Extended-response questions

1 The two-way table below shows the results of a survey on car ownership and public transport usage. You can assume the sample is representative of the population.

	Uses public transport	Does not use public transport	Total
Own a car	20	80	
Do not own a car	65	35	
Total			

- **a** Copy and complete the table.
- **b** How many people were surveyed in total?
- **c** What is the probability that a randomly selected person will have a car?
- **d** What is the probability that someone does not use public transport?
- **e** What is the probability that someone will use public transport and also own a car?
- **f** Out of the people who own cars, what is the probability that someone will use public transport?
- **g** In what ways could the survey produce biased results if it had been conducted:
  - **i** outside a train station?
  - **ii** in regional New South Wales?

2 A spinner is made using the numbers $1, 3, 5$ and 10 in four sectors. The spinner is spun 80 times, and the results obtained are shown in the table.

Number on spinner	Frequency
1	30
3	18
5	11
10	21
	80

- **a** Display the data as a frequency graph.
- **b** Which sector on the spinner occupies the largest area? Explain.
- **c** Two sectors of the spinner have the same area. Which two numbers do you think have equal areas, and why?
- **d** What is the experimental probability of obtaining a 1 on the next spin?
- **e** Draw an example of what you think the spinner might look like, in terms of the area covered by each of the four numbers.

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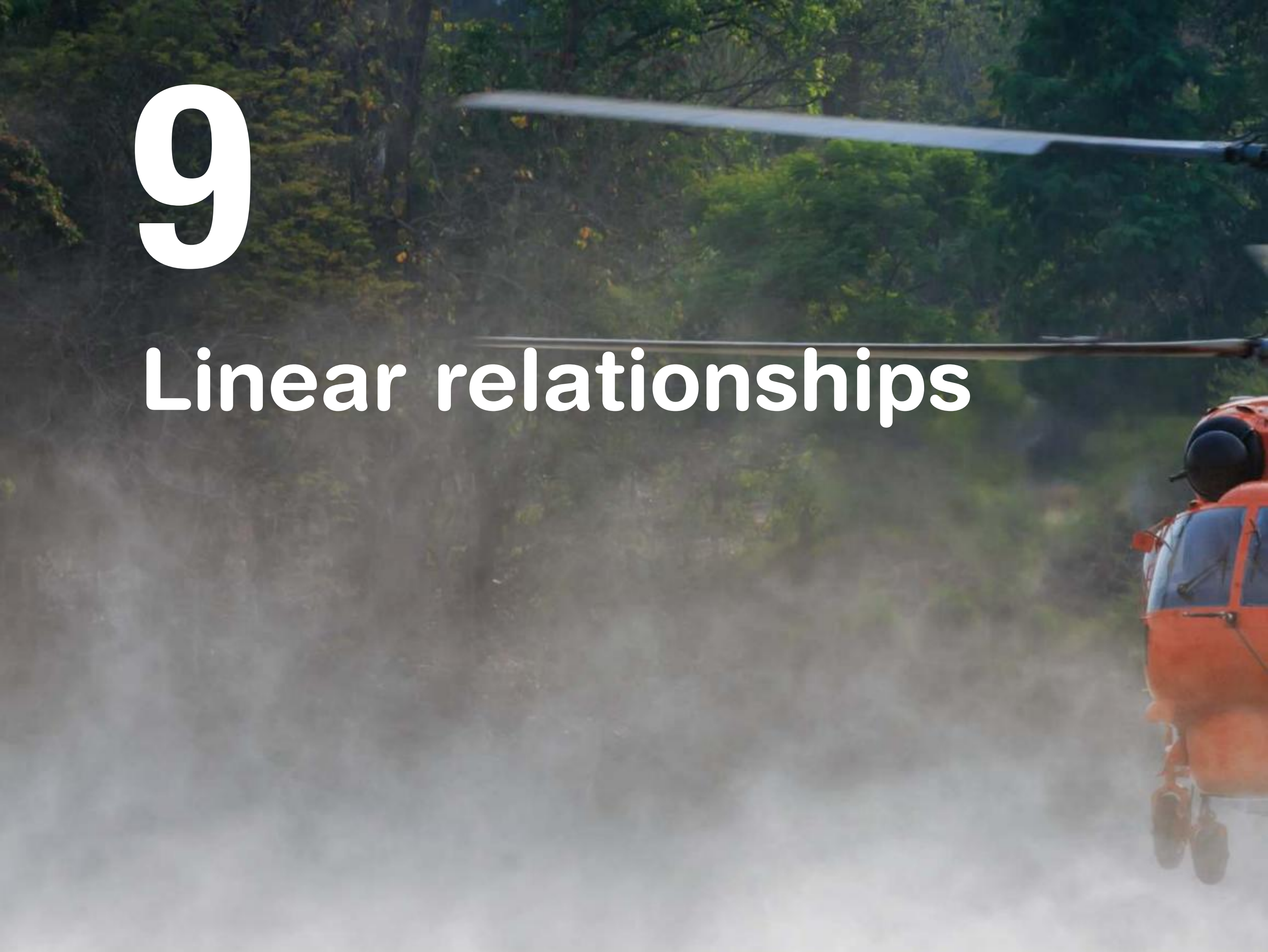

9 Linear relationships

Essential mathematics: why understanding linear relationships is important

Linear relationship skills using straight line graphs, tables and rules are essential for solving many problems in the trades and professions, and in science, industry and business.

Linear relationships arise in many contexts, such as,

- Farmers who calculate fertilizer weight = kg/acre × number of acres,
- Nurses who calculate medical doses = amount/kg × patient's weight in kg,
- Accountants who calculate wages = pay \$/hour × hours worked.

When renting a helicopter there is a linear relationship between the cost of hiring it and the number of hours flown. For example, renting a Skycrane heavy-lift firefighting helicopter can have an operational cost of $\$10\,000$/hour plus an insurance fee of $\$2000$. The linear rule for the hire cost C for n hours is $C = 10\,000n + 2000$.

The cable on a suspension bridge forms a curve called a parabola. Engineers model parabola shaped curves using rules that include an x^2 term.

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In this chapter

Australian Curriculum 9.0

ALGEBRA

Graph linear relations on the Cartesian plane using digital tools where appropriate; solve linear equations and one-variable inequalities using graphical and algebraic techniques; verify solutions by substitution (AC9M8A02)

Use mathematical modelling to solve applied problems involving linear relations, including financial contexts; formulate problems with linear functions, choosing a representation; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8A03)

Experiment with linear functions and relations using digital tools, making and testing conjectures and generalising emerging patterns (AC9M8A04)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

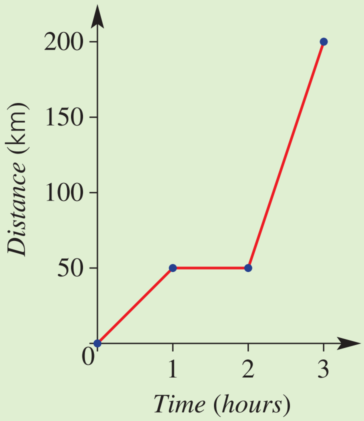
1 This graph relates distance and time for a journey in a train.

a How far did the train travel:
  i in the first hour?
  ii in the second hour?
  iii in the third hour?

b What was the total distance travelled?

c During which hour was the train at rest?

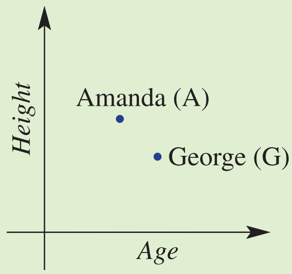
d During which hour was the train travelling the fastest?



2 This graph shows the relationship between age and height of two people, Amanda (A) and George (G).

a Who is older? (A or G)

b Who is taller? (A or G)



3 Write the missing numbers.

a $-3, -2, \_\_, 0, 1, \_\_, \_\_$

b $-7, \_\_, -3, -1, 1, \_\_, \_\_$

c $12, 7, \_\_, -3, \_\_, \_\_, -18$

d $-31, \_\_, -13, -4, \_\_, \_\_$

4 The x-coordinate in $(2, -3)$ is 2. Write the x-coordinate for these points.

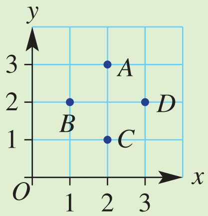
a $(1, 2)$ **b** $(1, 5)$ **c** $(0, -1)$ **d** $(-3, 0)$

5 The y-coordinate in $(2, -3)$ is -3. Write the y-coordinate for these points.

a $(1, 6)$ **b** $(-4, -1)$ **c** $(-3, 0)$ **d** $(-4, 2)$

6 The coordinates of the point A on the graph below are $(2, 3)$. What are the coordinates of these points?

a B **b** C **c** D



7 If $\square = \Delta + 4$, find the value of $\square$ when:

a $\Delta = 3$ **b** $\Delta = 0$ **c** $\Delta = -2$

8 If $\square = 2 \times \Delta - 5$, find the value of $\square$ when:

a $\Delta = 2$ **b** $\Delta = 0$ **c** $\Delta = -3$

9 Complete the tables for the given rules.

a $\square = 2 \times \Delta$

Δ	-2	-1	0	1	2
$\square$					

b $\square = 3 \times \Delta - 4$

Δ	-2	-1	0	1	2
$\square$					

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9A The Cartesian plane CONSOLIDATING

Learning intentions

- To understand that coordinates can be used to describe locations in two-dimensional space on a number plane.
- To be able to state the coordinates of points shown on a number plane.
- To be able to plot points at given coordinates.
- To know the location of the four quadrants.

Key vocabulary: number plane, Cartesian plane, coordinates, x-axis, y-axis, origin, quadrant

To describe a relationship between two variables, like water volume and time, we might use a graph. The graph would include a pair of axes and a set of points. The points can be joined to form a line or curve.

In mathematics, a pair of axes defines a number plane. The number plane is also called the Cartesian plane after its inventor, Rene Descartes, who lived in France in the 17th century. The horizontal axis (x) and vertical axis (y) of a number plane can be extended to include negative numbers. The point where these axes cross over is called the origin, and it provides a reference point for all other points on the plane.

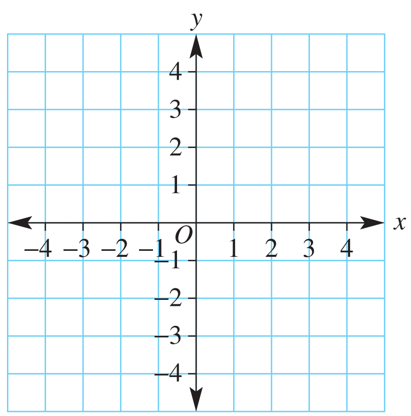
Consider plotting a graph to visualise the change in water volume in a leaking bucket over time.

Lesson starter: Make the shape

In groups or as a class, see if you can remember how to plot points on a number plane. Then decide what type of shape is formed by each set.

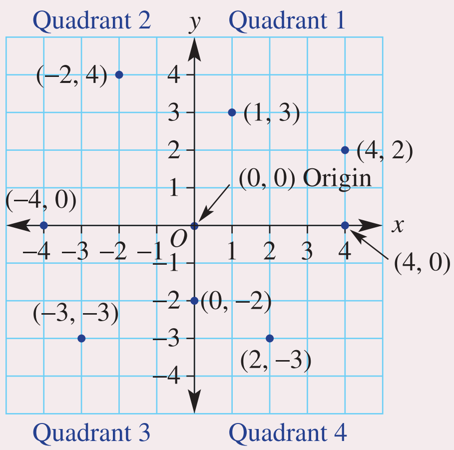
- $A(0, 4)$, $B(3, -3)$, $C(-3, -3)$
- $A(3, 3)$, $B(0, -4)$, $C(-3, 3)$
- $A(2, 4)$, $B(2, -4)$, $C(-2, -4)$, $D(-2, 4)$

Discuss the basic rules for plotting points on a number plane.



Key ideas

- A **number plane** (or **Cartesian plane**) includes a vertical **y-axis** and a horizontal **x-axis** intersecting at right angles.
  - There are 4 **quadrants** labelled as shown.
- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first followed by the y-coordinate.
- The point $(0, 0)$ is called the **origin**, and is often labelled O.



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Exercise 9A

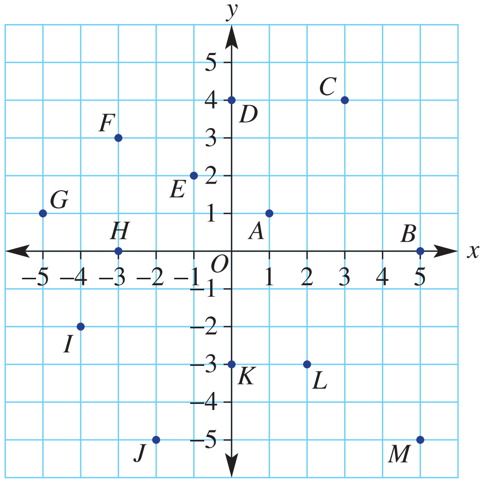
Understanding

1–3 | 3

1 Complete these sentences.

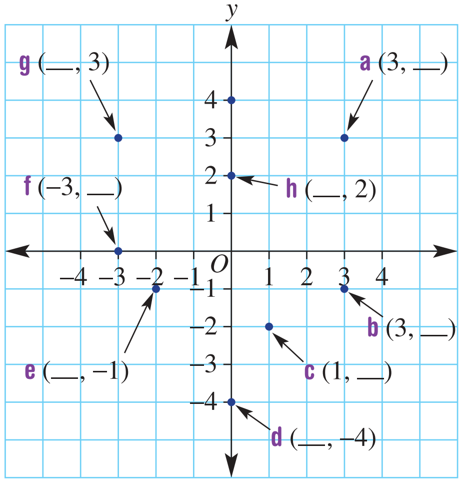
- **a** The x-coordinate in $(3, -4)$ is ______.
- **b** The x-coordinate in $(-4, 7)$ is ______.
- **c** The y-coordinate in $(2, 5)$ is ______.
- **d** The y-coordinate in $(-4, -8)$ is ______.
- **e** The coordinates of the origin are _____.
- **f** The vertical axis is called the ___-axis.

2 Write the coordinates of the points labelled A to M.



Hint for Q2: Write a pair of coordinates (x, y). x is positive on the right and y is positive on the upper side.

3 Write the missing number for the coordinates of the points **a**–**h**.



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Fluency

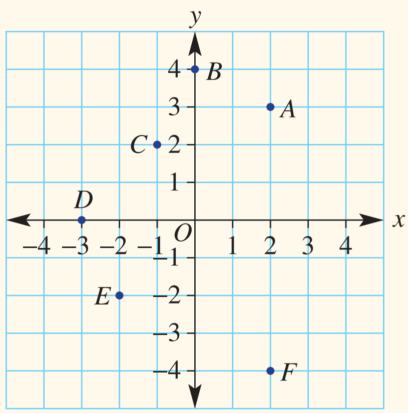
4, 5 | 4, 5

Example 1 Plotting points

Draw a number plane extending from −4 to 4 on both axes then plot and label these points.

a $A(2, 3)$ **b** $B(0, 4)$ **c** $C(-1, 2)$ **d** $D(-3, 0)$ **e** $E(-2, -2)$ **f** $F(2, -4)$

Solution



Explanation

The x-coordinate is listed first followed by the y-coordinate.

For each point, start at the origin $(0, 0)$ and move left or right or up and down to suit both x- and y-coordinates. For point $C(-1, 2)$, for example, move 1 to the left and 2 up.

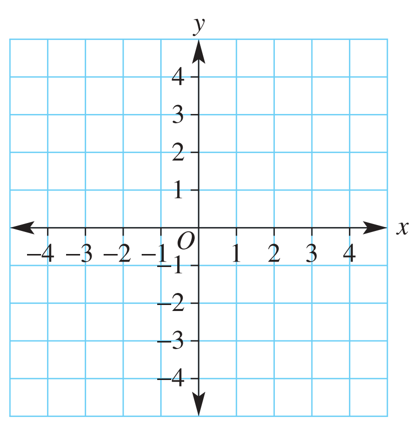
Now you try

Draw a number plane extending from −4 to 4 on both axes then plot and label these points.

a $A(1, 4)$ **b** $B(0, -2)$ **c** $C(-2, 3)$
d $D(-3, -4)$ **e** $E(0, 1)$ **f** $F(2, -2)$

4 Draw a number plane extending from −4 to 4 on both axes and then plot and label these points.

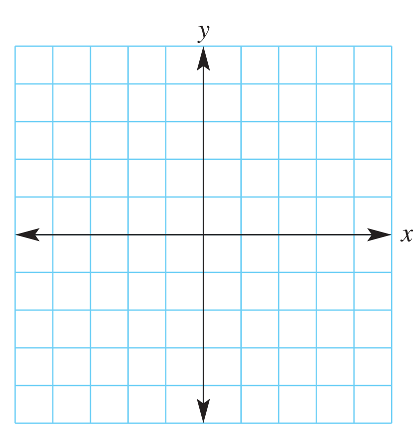
a $A(4, 1)$ **b** $B(2, 3)$
c $C(0, 1)$ **d** $D(-1, 3)$
e $E(-3, 3)$ **f** $F(-2, 0)$
g $G(-3, -1)$ **h** $H(-1, -4)$
i $I(0, -2)$ **j** $J(0, 0)$
k $K(3, -1)$ **l** $L(1, -4)$



Hint for Q4: First move horizontally for x then vertically for y.

5 Plot each set of points. What do you notice about each graph?

a $(-4, -4), (-3, -3), (-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)$
b $(-2, 4), (-1, 2), (0, 0), (1, -2), (2, -4)$
c $(-4, 2), (-2, 1), (0, 0), (2, 1), (4, 2)$



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9A

Problem-solving and reasoning

6, 7 | 6, 8, 9

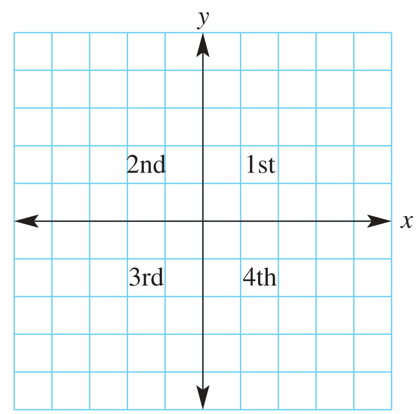
6 Complete these sentences.

- **a** The point $(2, 4)$ is in the ______ quadrant.
- **b** The point $(1, -5)$ is in the ______ quadrant.
- **c** The point $(-3, 6)$ is in the ______ quadrant.
- **d** The point $(-7, -20)$ is in the ______ quadrant.
- **e** The quadrant that has positive coordinates for both x and y is the ______ quadrant.
- **f** The quadrant that has negative coordinates for both x and y is the ______ quadrant.

Hint for Q6: Answer as first, second, third or fourth.

7 One point in each set below is not 'in line' with the other points. Plot the points then name the point not in line with the others in the same set.

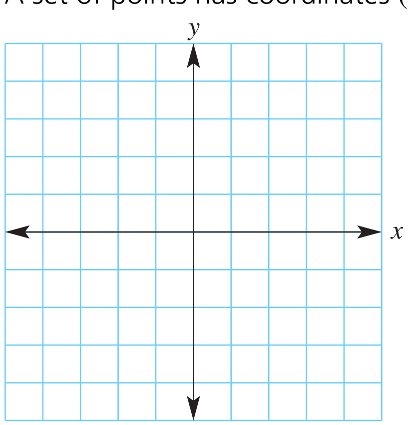
- **a** $A(1, 2)$, $B(2, 4)$, $C(3, 4)$, $D(4, 5)$, $E(5, 6)$
- **b** $A(-5, 3)$, $B(-4, 1)$, $C(-3, 0)$, $D(-2, -3)$, $E(-1, -5)$
- **c** $A(-4, -3)$, $B(-2, -2)$, $C(0, -1)$, $D(2, 0)$, $E(3, 1)$
- **d** $A(6, -4)$, $B(0, -1)$, $C(4, -3)$, $D(3, -2)$, $E(-2, 0)$



8 Each set of points forms a basic shape. Describe the shape without drawing a graph if you can.

- **a** $A(-2, 4)$, $B(-1, -1)$, $C(3, 0)$
- **b** $A(-3, 1)$, $B(2, 1)$, $C(2, -6)$, $D(-3, -6)$
- **c** $A(-4, 2)$, $B(3, 2)$, $C(4, 0)$, $D(-3, 0)$
- **d** $A(-1, 0)$, $B(1, 3)$, $C(3, 0)$, $D(1, -9)$

9 A set of points has coordinates $(0, y)$, where y is any number. What does this set of points represent?



Plotting pictures

— | 10

10 Using a scale extending from -5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

- **a** $(-2, -2)$, $(2, -2)$, $(2, 2)$, $(1, 3)$, $(1, 4)$, $\left(\frac{1}{2}, 4\right)$, $\left(\frac{1}{2}, 3\frac{1}{2}\right)$, $(0, 4)$, $(-2, 2)$, $(-2, -2)$
- **b** $(2, 1)$, $(0, 3)$, $(-1, 3)$, $(-3, 1)$, $(-4, 1)$, $(-5, 2)$, $(-5, -2)$, $(-4, -1)$, $(-3, -1)$, $(-1, -3)$, $(0, -3)$, $(2, -1)$, $(1, 0)$, $(2, 1)$

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9B Using rules and tables to explore linear relationships

Learning intentions

- To understand that a table can be used to show values connected by a rule.
- To be able to construct a table of values for a rule.

Key vocabulary: rule, equation, table of values, variable, substitute

When two variables like *number of kilograms of bananas* and *cost* are related, we can use a rule to describe the relationship. For example, if bananas are \$5 per kg, then a rule connecting the number of kilograms (n) and the cost (\$$C$) would be $C = 5n$.

This rule can then be used to create a table of values, and this can then be used to create a graph which will be studied in the next section.

Lesson starter: Weigand's emu eggs

Weigand has a small emu farm and sells emu eggs for \$50 each. It costs him \$5000 per year to run the farm, so the rule connecting his profit (\$$P$) and number of emu eggs sold (n) is:

$$P = 50n - 5000$$

- Find Weigand's profit if he sells 120 eggs.
- Find Weigand's profit if he sells 300 eggs.
- Find Weigand's profit if he sells 50 eggs. What do you notice, and what does this mean?
- What values are missing from this table?

n	0	50	100	150	
P	-5000			2500	5000

- What is Weigand's 'break even' point?

Key ideas

- A **rule** is an **equation** that connects two or more **variables**.
  Examples: $d = 50t$, $y = 3x - 2$, $V = 100 - 10t$, $C = 100n - 1000$.
- A rule can be used to construct a **table of values**.
  - **Substitute** the values of one variable into the rule to find the value of the other variables.
    Rule: $C = 2n + 3$

n	-1	0	1	2	3
C	1	3	5	7	9

If $n = 3$, $C = 2 \times 3 + 3 = 9$
If $n = -1$, $C = 2 \times -1 + 3 = 1$

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Exercise 9B

Understanding

1–3 | 3

1 Write the missing values in each table for the given rules.

a $\square = \Delta + 2$

Δ	0	1	2	3
$\square$	2		4	

b $\square = 2 \times \Delta$

Δ	0	1	2	3
$\square$		2	4	

c $\square = \Delta - 7$

Δ	4	5	6	7
$\square$		-2		0

d $\square = 2 \times \Delta + 4$

Δ	0	2	4	6
$\square$	4		12	

e $\square = 3 \times \Delta - 8$

Δ	0	1	2	3
$\square$	-8			1

f $\square = \Delta \div 2 - 1$

Δ	0	2	4	6
$\square$	-1			2

2 If $y = 2x - 5$, find the value of y for these values of x.

a $x = 4$ **b** $x = 3$
c $x = 0$ **d** $x = -2$

Hint for Q2: A positive times a negative equals a negative. e.g. $2 \times (-1) = -2$.

3 If $y = -3x + 1$, find the value of y for these values of x.

a $x = 0$ **b** $x = 4$
c $x = -1$ **d** $x = -3$

Hint for Q3: A negative times a negative equals a positive. e.g. $-3 \times (-1) = 3$.

Fluency

4, 6(½) | 5, 6(½)

Example 2 Constructing tables using positive numbers

The rule connecting the distance travelled (d km) and time (t hours) is $d = 60t$.

a Construct a table of values using the following values of t : 0, 1, 2, 3, 4.
b What distance is travelled after 3 hours?
c How long does it take to travel 90 km?

Solution

a

t	0	1	2	3	4
d	0	60	120	180	240

Explanation

Substitute each value of t into the rule $d = 60t$.
For example, when $t = 3$, $d = 60 \times 3 = 180$.

b 180 km

The value of d at $t = 3$ is 180.

c 1.5 hours

60 km is travelled every hour, so 1.5 hours is required for 90 km.

Now you try

The rule connecting the volume of gas (V litres) and time (t hours) is $V = 10 + 2t$.

a Construct a table of values using the following values of t : 0, 1, 2, 3, 4.
b What is the volume after 4 hours?
c How long does it take to get to a volume of 15 L?

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4 The rule connecting the distance travelled (d km) and time (t hours) is given by $d = 40t$.

- **a** Construct a table of values using the following values of $t: 0, 1, 2, 3, 4$.
- **b** What distance is travelled after 3 hours?
- **c** How long does it take to travel 80 km?

Hint for Q4: 40 km is travelled every hour.

5 The rule connecting the volume of water in a tank (V litres) after t minutes is given by $V = 20t + 1000$.

- **a** Construct a table of values using the following values of $t: 0, 1, 2, 3, 4, 5$.
- **b** What is the volume after 4 minutes?
- **c** How long does it take for the volume to reach 1100 litres?

Example 3 Constructing tables using negative numbers

For the given rules, construct a table of values for x from -2 to 2.

a $y = 2x - 3$ **b** $y = -x + 4$

Solution

a

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

Explanation

If $y = 2x - 3$ then when $x = -2$,
$y = 2 \times (-2) - 3 = -4 - 3 = -7$
Repeat for other values of x.

b

x	-2	-1	0	1	2
y	6	5	4	3	2

If $y = -x + 4$ then when $x = -2$,
$y = -(-2) + 4 = 2 + 4 = 6$
Repeat for other values of x.

Now you try

For the given rules, construct a table of values for x from -2 to 2.

a $y = 3x - 7$ **b** $y = -2x + 5$

6 For the given rules, complete the given tables.

a $y = 3x$

x	-2	-1	0	1	2
y					

b $y = x - 2$

x	-2	-1	0	1	2
y					

c $y = 2x + 1$

x	-2	-1	0	1	2
y					

d $y = 2x - 3$

x	-2	-1	0	1	2
y					

e $y = -x + 2$

x	-2	-1	0	1	2
y					

f $y = -x - 1$

x	-2	-1	0	1	2
y					

Hint for Q6: A positive times a negative is a negative.

Hint for Q6: A negative times a negative is a positive.

g $y = -2x - 1$

x	-2	-1	0	1	2
y					

h $y = -4x + 2$

x	-2	-1	0	1	2
y					

i $y = -6x - 11$

x	-2	-1	0	1	2
y					

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9B

Problem-solving and reasoning

7, 8 | 7–9

7 To hire a car costs \$70 per day so the rule for the cost (\$$C$) for n days is $C = 70n$.

a What is the cost for:

i $n = 2$?

ii $n = 10$?

b What does it cost to hire the car for 2 weeks?

c How long can you hire the car if you have \$280 to spend?

8 A rule linking x and y is given by $y = 3x - 4$.

a What is the value of y when:

i $x = 2$? ii $x = 1$?

b What x-value gives a y-value of:

i 5? ii -4?

c What is the smallest whole number for x which makes y positive?

d What is the largest whole number for x which makes y negative?

9 The profit equation for a small company producing watches is given by $P = 15n - 15\,000$, where \$$P$ is the profit and n is the number of watches sold.

a What is the profit if:

i $n = 2000$? ii $n = 1000$?

b How many watches need to be sold to make a profit of:

i \$30 000?

ii \$45 000?

iii \$75 000?

c Explain what happens to the profit if $n < 1000$. i.e. less than 1000 watches are sold.

d What is the loss if only 200 watches are sold?

The sum shortcut

— | 10

10 To add the first 3 consecutive positive whole numbers would mean to calculate $1 + 2 + 3$, which equals 6. Note also that $(3 \times 4) \div 2 = 6$. In a similar way, $1 + 2 + 3 + 4 + 5$ can be calculated as $(5 \times 6) \div 2 = 15$. In general, the rule is: Sum $= \frac{n \times (n + 1)}{2}$ so for $n = 5$, Sum $= \frac{5 \times 6}{2} = 15$.

a Use the rule: Sum $= \frac{n \times (n + 1)}{2}$ for:

i $n = 4$ ii $n = 8$ iii $n = 10$

b Use the rule to find the sum of:

i the first six positive whole numbers ($n = 6$)

ii the first twelve positive whole numbers.

c Use the rule to calculate:

i $1 + 2 + 3 + \ldots + 7 \quad (n = 7)$ ii $1 + 2 + 3 + \ldots + 20$ iii $1 + 2 + 3 + \ldots + 100$

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9C Plotting straight line graphs

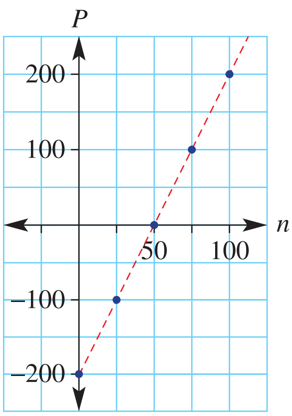
Learning intentions

- To understand that a relationship between variables x and y can be shown as a graph.
- To be able to plot a graph from a table of values.
- To be able to plot a graph from a rule.

Key vocabulary: rule, table of values, plot, coordinates, graph, linear

The next step in illustrating the relationship between two variables is to plot points to form a graph. The points are taken from a table of values and plotted on a number plane. If the points form a single straight line, the relationship is said to be linear.

For example, if the profit ($\$P$) from producing and selling n kg of raspberries is given by $P = 4n - 200$, then the graph would look like the following:



Farmers can use a linear relationship between kilograms of raspberries sold and profit to find how many need to be sold to make a positive profit.

Lesson starter: Which set of points form a straight line?

Consider the set of points (x, y) from these three tables of values.

A $y = 1 \div x$

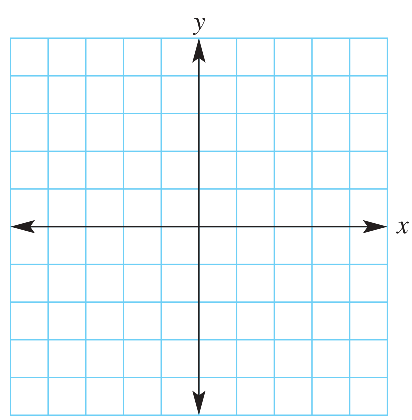
x	-2	-1	0	1	2
y	-0.5	-1		1	0.5

B $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

C $y = 2x$

x	-2	-1	0	1	2
y	-4	-2	0	2	4



- If each set is plotted on a number plane, which set would form a single straight line?
- What do you notice about the values in the table that gives a straight line graph?

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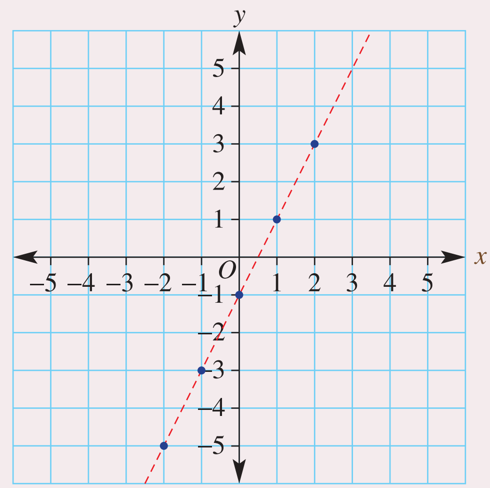
9C

Key ideas

- A **linear** relationship gives a straight line **graph**.
  For example, $y = 2x - 1$

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



- To draw a linear graph using a **rule**:
  - Construct a **table of values**, finding a y-coordinate for each given x-coordinate. Substitute each x-coordinate into the rule.
  - **Plot** the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

Exercise 9C

Understanding

1–3 | 3

1 For the rule $y = 2x + 3$, find the y-coordinate for these x-coordinates.

a 1 **b** 2 **c** 0 **d** -1
e -5 **f** -7 **g** 11 **h** -12

2 Write the missing number in these tables for the given rules.

a $y = 2x$

x	0	1	2	3
y	0		4	6

b $y = x - 3$

x	-1	0	1	2
y	-4	-3		-1

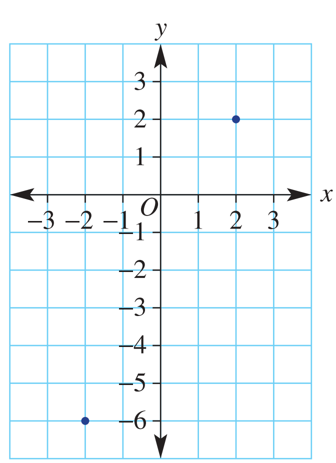
c $y = 5x + 2$

x	-3	-2	-1	0
y		-8	-3	2

3 Complete the graph to form a straight line from the given rule and table. Two points have been plotted for you.

$y = 2x - 2$

x	-2	-1	0	1	2
y	-6	-4	-2	0	2



Hint for Q3: For $(-1, -4)$ move 1 left and 4 down.
For $(0, -2)$ just move 2 down from the origin $(0, 0)$.

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Fluency

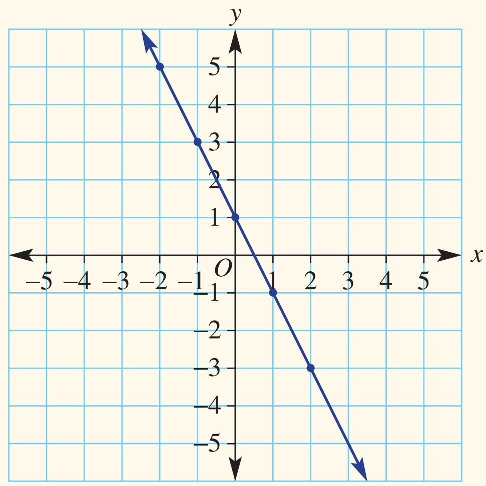
4, 5(½) | 4–5(½)

Example 4 Plotting a graph from a table

Plot a graph from this table of values.

x	-2	-1	0	1	2
y	5	3	1	-1	-3

Solution



Explanation

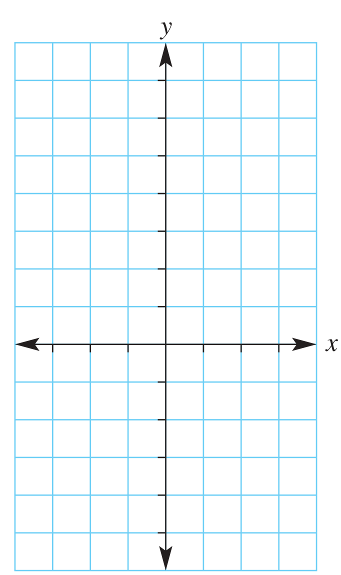
Plot the five points $(-2, 5)$, $(-1, 3)$, $(0, 1)$, $(1, -1)$ and $(2, -3)$. Then join to form a straight line.

Now you try

Plot a graph from this table of values.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

4 Plot a graph from these tables of values.



a

x	-2	-1	0	1	2
y	2	1	0	-1	-2

b

x	-2	-1	0	1	2
y	-3	-2	-1	0	1

c

x	-3	-2	-1	0	1	2	3
y	-3	-2	-1	0	1	2	3

d

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

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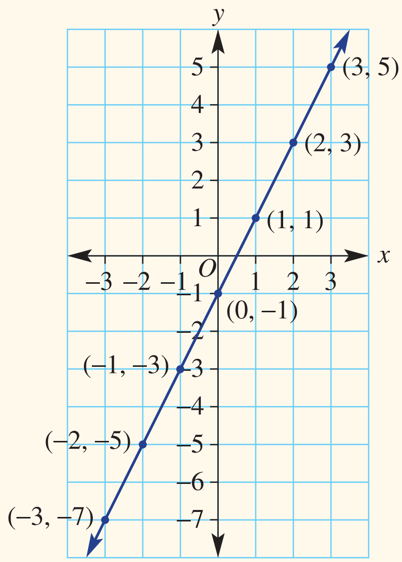
9C

Example 5 Plotting a graph from a rule

For the rule $y = 2x - 1$, construct a table and draw a graph.

Solution

x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5



Explanation

Substitute each x-coordinate in the table into the rule to find the y-coordinate.

Plot each point $(-3, -7), (-2, -5)\ldots$ and join them to form the straight line graph.

Now you try

For the rule $y = -x - 1$, construct a table and draw a graph.

5 For each rule, construct a table then plot and draw a graph. Use a table like the one shown here for each rule.

x	-3	-2	-1	0	1	2	3
y							

a $y = x + 1$ **b** $y = x - 2$
c $y = 2x - 3$ **d** $y = 2x + 1$
e $y = -2x + 3$ **f** $y = -3x - 1$
g $y = -x$ **h** $y = -x + 4$

Hint for Q5: Substitute each value of x into the rule to find the value of y. Then plot this pair on a graph.

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Problem-solving and reasoning | 6, 7 | 6–9

6 Decide if the following tables of values would give a straight line graph.

a

x	0	1	2	3	4
y	0	2	4	6	8

b

x	0	1	2	3	4
y	0	3	6	9	12

c

x	0	1	2	3	4
y	2	0	1	5	2

d

x	0	1	2	3	4
y	7	0	4	1	6

e

x	−1	0	1	2	3
y	6	−2	4	1	8

f

x	−2	1	0	1	2	3
y	2	1	0	−1	−2	−3

7 The distance a car travelled (d km) is given by the rule $d = 80t$ where t is in hours.

a Complete this table of values.

t	0	1	2	3
d	0			

b Draw a graph which illustrates the rule and table. Use t for time on the horizontal axis.

c How far does the car travel after 4 hours?

d How long would it take for the car to travel 400 km?

8 **a** What is the minimum number of points needed to draw a graph of a straight line?

b Draw the graph of these rules by plotting only two points. Use $x = 0$ and $x = 1$.

i $y = \frac{1}{2}x$ **ii** $y = 2x - 1$

9 The graphs of $y = x$, $y = 3x$ and $y = -2x$ all pass through the origin $(0, 0)$. Explain why.

The intersection point | — | 10

10 It is possible to find the intersection point of two graphs by plotting each graph and observing the intersection. For each pair of rules, follow these steps.

i Construct a table of values for each rule.

ii Plot to form two straight line graphs.

iii Give the coordinates of the intersection point (if any).

a $y = x + 2$
$y = -x + 2$

b $y = 2x - 3$
$y = -x + 3$

c $y = 2x - 2$
$y = 2x + 1$

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9D Finding the rule using a table of values

Learning intentions

- To understand that a rule can be determined from a table of values.
- To be able to find the rule for a table of values.
- To understand that a linear rule is of the form $y = \triangle \times x + \square$ where $\square$ is the value of y when $x = 0$.

Key vocabulary: rule, coordinates, linear, increase, decrease

A mathematical rule is a convenient way of describing a relationship between two variables. While a table and a graph are limited by the number of points they show, a rule can be used to find any value of y for any given x-value quickly. Finding such a rule from a collection of points on a graph or table is an important step in the development and application of mathematics.

For example, if a straight fence requires two upright posts per metre, then the rule for the number of posts (P) is $P = 2n + 1$, where n is the number of metres. An extra post is required for the ends.

A simple rule gives the number of fence posts in every metre of fence.

Lesson starter: What's my rule?

Each of the tables describes a linear relationship between y and x.

x	y
0	4
1	5
2	6
3	7
4	8

x	y
−3	−5
−2	−3
−1	−1
0	1
1	3

x	y
−2	5
−1	4
0	3
1	2
2	1

Rules to choose from:
$y = 2x + 1$
$y = -x + 3$
$y = x + 4$

- For each table, find the correct rule.
- Discuss your strategy for finding the three different rules. What patterns did you notice and how did these patterns help determine the rule?

Key ideas

- A **rule** must satisfy every pair of **coordinates** (x, y) in a table.
  - Most **linear** rules can be written in the form $y = \triangle x + \square$

    insert a number here (for $\triangle$)   insert a number here (for $\square$)

- Consider a linear rule of the form $y = \triangle \times x + \square$.
  - The value of $\triangle$ will be the **increase** in y as x increases by 1. If there is a **decrease** in y then $\triangle$ will be negative.

x	−2	−1	0	1	2
y	−1	1	3	5	7

+2 +2 +2 +2

$y = 2x + 3$

x	−2	−1	0	1	2
y	1	0	−1	−2	−3

−1 −1 −1 −1

$y = -x - 1$

  - The value of $\square$ will be the value of y when $x = 0$.

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Exercise 9D

Understanding 1–3 2, 3

1 Match the rules **A**, **B** and **C** with the tables **a**, **b** and **c**.

a

x	-1	0	1	2	3
y	-5	-4	-3	-2	-1

b

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

c

x	-2	-1	0	1	2
y	2	1	0	-1	-2

A $y = 2x - 3$ **B** $y = -x$ **C** $y = x - 4$

2 By how much does y increase for each increase by 1 in x? If y is decreasing, give a negative answer.

a

x	-2	-1	0	1	2
y	-1	1	3	5	7

b

x	-3	-2	-1	0	1
y	4	3	2	1	0

c

x	-4	-3	-2	-1	0
y	4	2	0	-2	-4

d

x	-3	-2	-1	0	1
y	-6	-3	0	3	6

3 For each of the tables in Question **2**, state the value of y when $x = 0$.

Fluency 4–5(½) 4–5(½)

Example 6 Finding rules from tables

Find the rule for these tables of values.

a

x	-2	-1	0	1	2
y	0	1	2	3	4

b

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

Solution

a $\triangle = 1$, $\square = 2$

$y = x + 2$

b $\triangle = 3$, $\square = -2$

$y = 3x - 2$

Explanation

x	-2	-1	0	1	2
y	0	1	2	3	4

$+1$ $+1$ $+1$ $+1$

$y = \triangle x + \square$

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

$+3$ $+3$ $+3$ $+3$

$y = \triangle x + \square$

Now you try

Find the rule for these tables of values.

a

x	-2	-1	0	1	2
y	-3	-1	1	3	5

b

x	-2	-1	0	1	2
y	-12	-8	-4	0	4

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9D

4 Find the rule for these tables of values.

a

x	−1	0	1	2	3
y	0	1	2	3	4

b

x	−2	−1	0	1	2
y	−4	−2	0	2	4

c

x	−2	−1	0	1	2
y	0	2	4	6	8

d

x	−2	−1	0	1	2
y	−7	−4	−1	2	5

e

x	−2	−1	0	1	2
y	−8	−4	0	4	8

f

x	−2	−1	0	1	2
y	−3	0	3	6	9

Hint for Q4: In $y = \triangle x + \square$
$\triangle$ = increase in y
$\square$ = y value at $x = 0$.

Example 7 Finding rules when y is decreasing

Find the rule for these tables of values.

a

x	−1	0	1	2	3
y	1	0	−1	−2	−3

b

x	−2	−1	0	1	2
y	5	3	1	−1	−3

Solution

a $\triangle = -1$ and $\square = 0$

$y = -x$

Explanation

x	−1	0	1	2	3
y	1	0	−1	−2	−3

$\triangle = -1$

$\square = 0$

−1 −1 −1

$y = \triangle x + \square$

b $\triangle = -2$ and $\square = 1$

$y = -2x + 1$

x	−2	−1	0	1	2
y	5	3	1	−1	−3

$\triangle = -2$

$\square = 1$

−2 −2 −2 −2

$y = \triangle x + \square$

Now you try

Find the rule for these tables of values.

a

x	−2	−1	0	1	2
y	3	2	1	0	−1

b

x	−3	−2	−1	0	1
y	8	5	2	−1	−4

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5 Find the rule for these tables of values.

a

x	-2	-1	0	1	2
y	2	1	0	-1	-2

b

x	-2	-1	0	1	2
y	1	0	-1	-2	-3

c

x	-3	-2	-1	0	1
y	4	3	2	1	0

d

x	-1	0	1	2	3
y	8	6	4	2	0

e

x	-2	-1	0	1	2
y	4	2	0	-2	-4

f

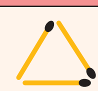
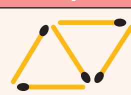
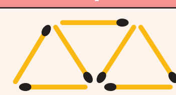
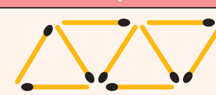
x	-3	-2	-1	0	1
y	10	7	4	1	-2

Hint for Q5: Since y is decreasing Δ will be negative.

Problem-solving and reasoning 6, 7 6–8

Example 8 Finding rules for patterns

If x = number of triangles and y = number of matchsticks, use a table to help find a rule for this pattern.

Shape 1	Shape 2	Shape 3	Shape 4
			

Solution

x	1	2	3	4
y	3	5	7	9

$y = 2x + 1$

Explanation

x is the number of triangles and y is the number of matchsticks.

x	1	2	3	4
y	3	5	7	9

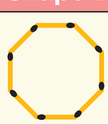
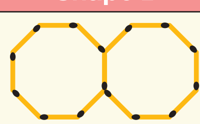
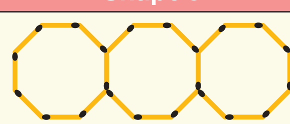
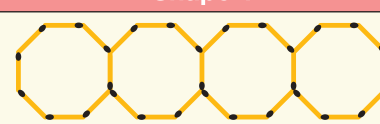
$+2$ $+2$ $+2$

$y = 2x + \square$

$\square = 1$ since $2 \times 1 + 1 = 3$

Now you try

If x = number of octagons and y = number of matchsticks, use a table to help find a rule for this pattern.

Shape 1	Shape 2	Shape 3	Shape 4
			

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9D

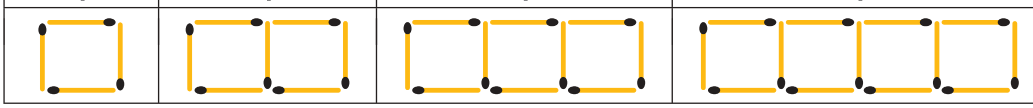
6 Write a rule for these matchstick patterns.

a x = number of squares and y = number of matchsticks

Hint for Q6: First draw a table and fill it in.

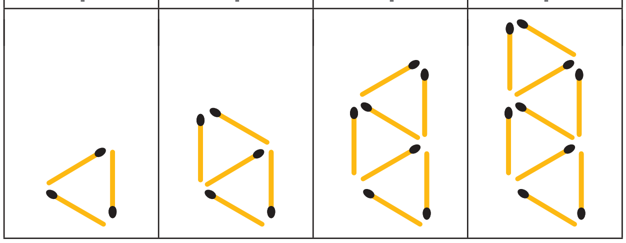
x	1	2	3	4
y	4	7		

Shape 1	Shape 2	Shape 3	Shape 4



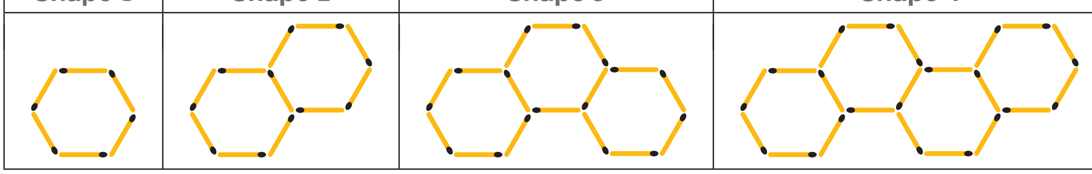
b x = number of triangles and y = number of matchsticks

Shape 1	Shape 2	Shape 3	Shape 4



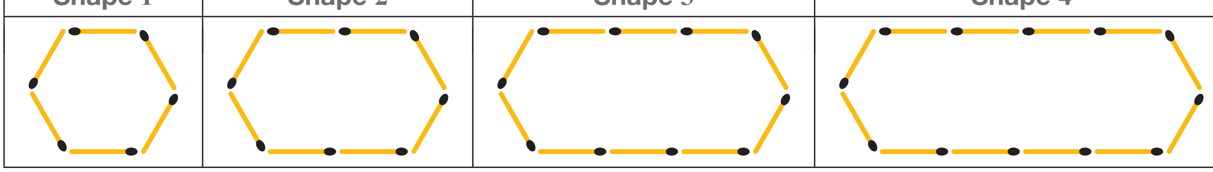
c x = number of hexagons and y = number of matchsticks

Shape 1	Shape 2	Shape 3	Shape 4



d x = number of matchsticks on top row and y = number of matchsticks

Shape 1	Shape 2	Shape 3	Shape 4



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7 A rule is of the form $y = 3x + \square$. Find the value of $\square$, if the values of (x, y) are:
a $(1, 4)$ **b** $(-1, 0)$ **c** $(-2, 1)$ **d** $(0, 0)$

8 Look at this table of values.

x	-2	0	2	4
y	-4	-2	0	2

a The increase in y for each unit increase in x is not 2. Explain why.
b If the pattern is linear, state the increase in y for each increase by 1 in x.
c Write the rule for the relationship.
d Find the rule for these tables.

i

x	-4	-2	0	2	4
y	-5	-1	3	7	11

ii

x	-6	-3	0	3	6
y	15	9	3	-3	-9

iii

x	-3	-1	1	3	5
y	-10	-4	2	8	14

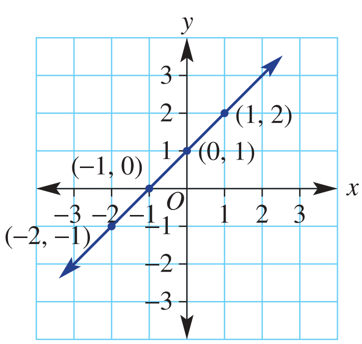
iv

x	-10	-8	-6	-4	-2
y	20	12	4	-4	-12

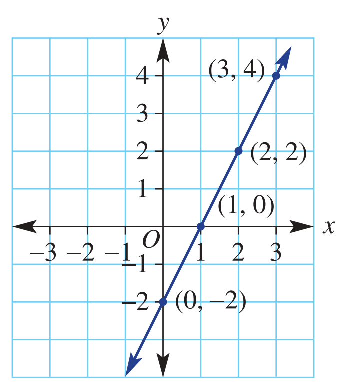
Rules from graphs — 9

9 Find the rule for these graphs by first constructing a table of (x, y) values.

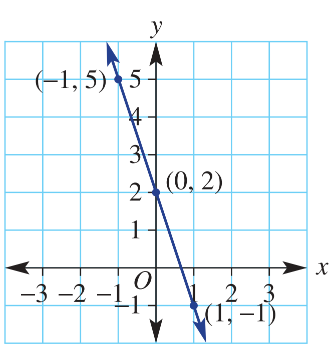
a



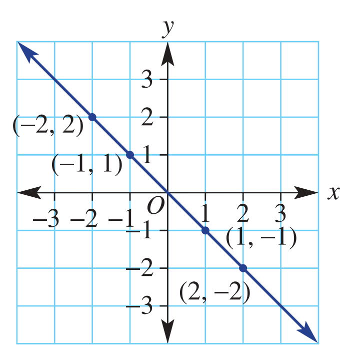
b



c



d



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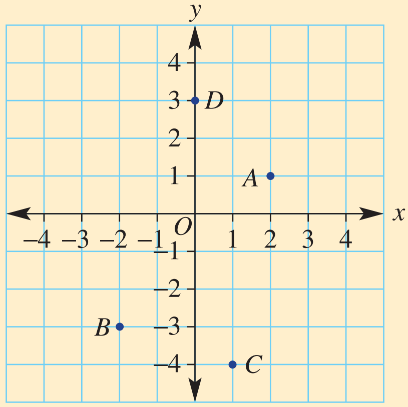
9A **1** Write down the coordinates of the points A to D on this number plane.



9A **2** The following set of points forms a basic shape. Describe the shape, with or without drawing a graph.
$A(-3, 4), B(-1, 4), C(-1, -3), D(-3, -3)$

9B **3** If $y = 3x + 4$, find the y value for these values of x.
a $x = 4$ **b** $x = 1$ **c** $x = -3$ **d** $x = 11$

9B **4** For the given rules, complete the given tables.
a $y = 4x$

x	-2	-1	0	1	2
y					

b $y = -2x + 5$

x	-2	-1	0	1	2
y					

9B **5** To rent a surfboard costs \$12 per hour, so the rule for the cost (\$$C$) for t hours is $C = 12t$.
a What is the cost for:
i $t = 3$? **ii** $t = 7$?
b What would it cost to hire the surfboard for 4 hours?
c A secondhand surfboard is available to purchase for \$230. How many hours of surfing would you need to do for it to be better value to purchase the secondhand surfboard rather than keep renting a board?

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9C **6** Plot a graph from this table of values and then join the dots to form a straight line.

x	-2	-1	0	1	2
y	-1	1	3	5	7

9C **7** For the rule $y = 2x - 2$, construct a table and draw a graph.

9C **8** For the following tables of values, fill in the missing numbers that would give a straight line graph.

a

x	-2	-1	0	1	2
y	5	3		-1	

b

x	-2	-1	0	1	2
y		0		6	9

9D **9** Find the rules for these tables of values.

a

x	-2	-1	0	1	2
y	-3	-2	-1	0	1

b

x	-2	-1	0	1	2
y	8	6	4	2	0

c

x	-2	-1	0	1	2
y	-4	-1	2	5	8

9D **10** A rule is of the form $y = 2x - \square$. Find the value of $\square$, if the values of (x, y) are:

a $(2, 3)$
b $(1, -3)$
c $(7, 14)$

Progress quiz

ISBN 978-1-009-593-724

9E Using graphs to solve linear equations

Learning intentions

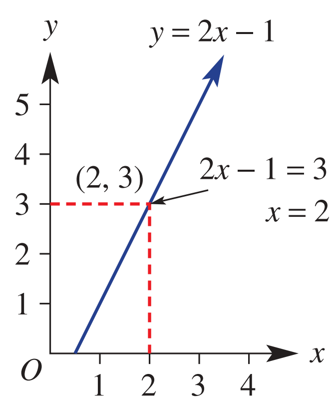
- To understand that each point on a graph represents a solution to an equation relating x and y.
- To understand that the point of intersection of two straight lines is the only solution that satisfies both equations.
- To be able to solve a linear equation using a graph.
- To be able to solve an equation with pronumerals on both sides using the intersection point of two linear graphs.

Key vocabulary: equation, solution, intersection, substitute, x-coordinate, y-coordinate

The rule for a straight line shows the connection between the x-coordinate and y-coordinate of each point on the line. We can substitute a given x-coordinate into the rule to calculate the y-coordinate. When we substitute a y-coordinate into the rule, it makes an equation that can be solved to give the x-coordinate. So, for every point on a straight line, the value of the x-coordinate is the solution to a particular equation.

The point of intersection of two straight lines is the shared point where the lines cross over each other. This is the only point with coordinates that satisfy both equations; that is, makes both equations true (LHS = RHS).

For example, in the graph below, the point $(2, 3)$ on the line $y = 2x - 1$ shows us that when $2x - 1 = 3$ the solution is $x = 2$.



Lesson starter: Matching equations and solutions

When a value is substituted into an equation and it makes the equation true (LHS = RHS), then that value is a solution to that equation.

- From the lists below, match each equation with a solution. Some equations have more than one solution.

Equations	
$2x - 4 = 8$	$y = x + 4$
$3x + 2 = 11$	$y = 2x - 5$
$y = 10 - 3x$	$5x - 3 = 2$

Possible solutions			
$x = 1$	$(1, 5)$	$x = 2$	$(3, 1)$
$x = -1$	$x = 6$	$(2, -1)$	$(2, 6)$
$(-2, -9)$	$(-2, 16)$	$x = 3$	$(2, 4)$

- Which two equations share the same solution and what is this solution?
- List the equations that have only one solution. What is a common feature of these equations?
- List the equations that have more than one solution. What is a common feature of these equations?

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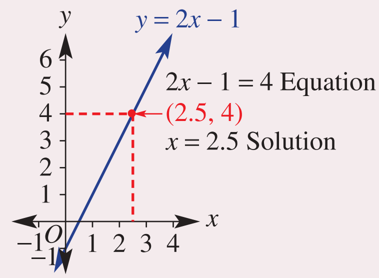
Key ideas

- The **x-coordinate** of each point on the graph of a straight line is a **solution** to a particular linear **equation**.
  - A particular linear equation is formed by **substituting** a chosen **y-coordinate** into a linear relationship.

  For example, if $y = 2x - 1$ and $y = 4$, then the linear equation is $2x - 1 = 4$.
  - The solution to this equation is the x-coordinate of the point with the chosen y-coordinate.

  For example, the point $(2.5, 4)$ shows that $x = 2.5$ is the solution to $2x - 1 = 4$.

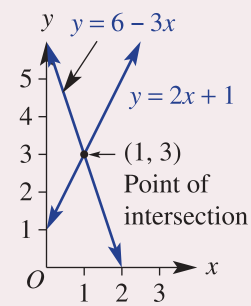


- A point (x, y) is a solution to the equation for a line if its coordinates make the equation true.
  - An equation is true when LHS = RHS after the coordinates are substituted.
  - Every point on a straight line is a solution to the equation for that line.
  - Every point that is not on the straight line is not a solution to the equation for that line.
- The point of **intersection** of two straight lines is the only solution that satisfies both equations.
  - The point of intersection is the shared point where two straight lines cross each other.
  - This is the only point with coordinates that make both equations true.

  For example, $(1, 3)$ is the only point that makes both $y = 6 - 3x$ and $y = 2x + 1$ true.

  Substituting $(1, 3)$

$y = 6 - 3x$	$y = 2x + 1$
$3 = 6 - 3 \times 1$	$3 = 2 \times 1 + 1$
$3 = 3$ (True)	$3 = 3$ (True)



Exercise 9E

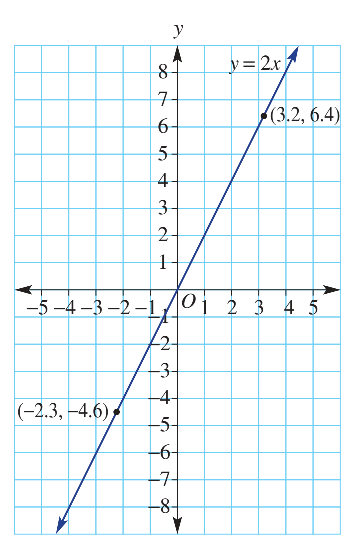
Understanding 1–3 3

1 Use the given rule to complete this table and then plot and join the points to form a straight line. $y = 2x - 1$.

x	-2	-1	0	1	2	3
y						

2 State the coordinates (x, y) of the point on this graph of $y = 2x$ where:

- **a** $2x = 4$ (i.e. $y = 4$)
- **b** $2x = 6.4$ (i.e. $y = 6.4$)
- **c** $2x = -4.6$
- **d** $2x = 7$
- **e** $2x = -14$
- **f** $2x = 2000$



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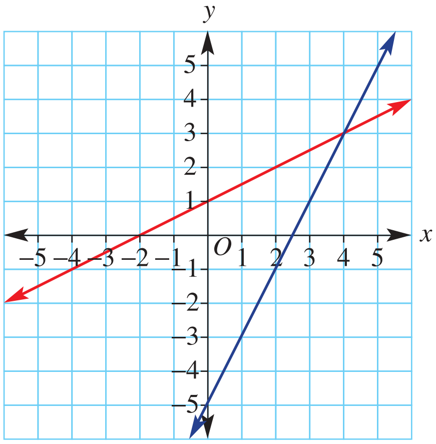
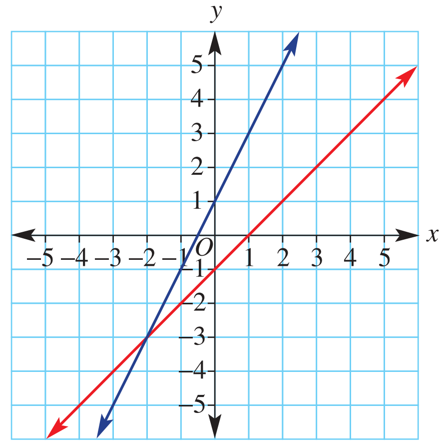
9E

3 For each of these graphs, write down the coordinates of the point of intersection (i.e. the point where the lines cross over each other).

a



b

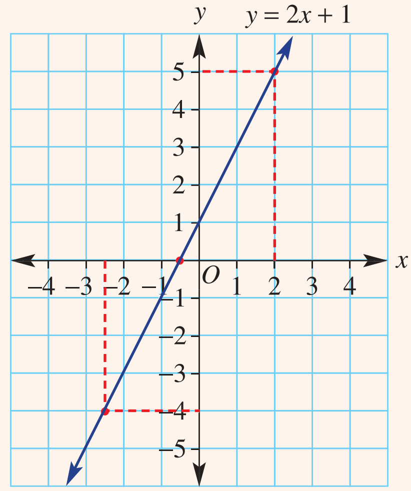


Fluency 4–5 4, 6, 7

Example 9 Using a linear graph to solve an equation

Use the graph of $y = 2x + 1$, shown here, to solve each of the following equations.

a $2x + 1 = 5$
b $2x + 1 = 0$
c $2x + 1 = -4$



Solution	Explanation
a $x = 2$	Locate the point on the line with y-coordinate 5. The x-coordinate of this point is 2 so $x = 2$ is the solution to $2x + 1 = 5$.
b $x = -0.5$	Locate the point on the line with y-coordinate 0. The x-coordinate of this point is -0.5 so $x = -0.5$ is the solution to $2x + 1 = 0$.
c $x = -2.5$	Locate the point on the line with y-coordinate -4. The x-coordinate of this point is -2.5 so $x = -2.5$ is the solution to $2x + 1 = -4$.

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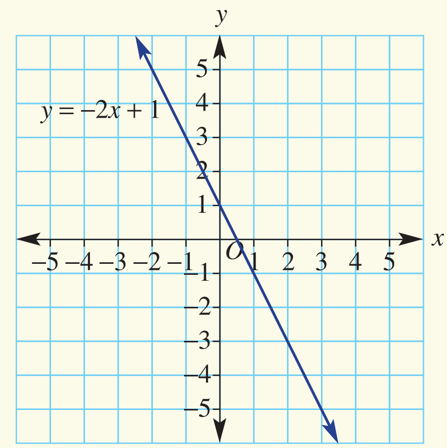
Now you try

Use the graph of $y=-2x+1$, shown here, to solve each of the following equations.

a $-2x+1=-3$

b $-2x+1=0$

c $-2x+1=4$



4 Use the graph of $y=2x-1$, shown here, to find the solution to each of these equations.

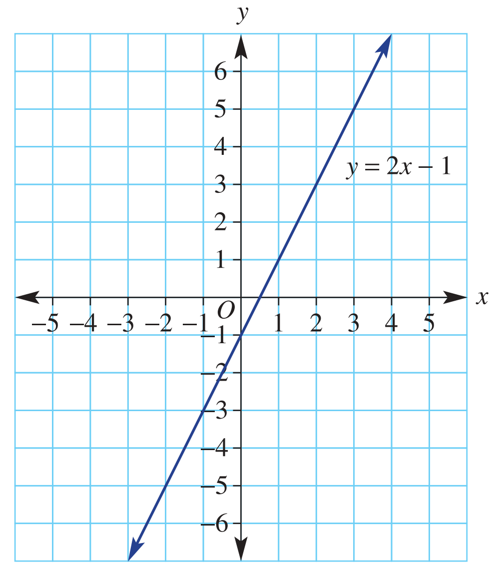
a $2x-1=3$

b $2x-1=0$

c $2x-1=5$

d $2x-1=-6$

e $2x-1=-4$



5 Use the graph of $y=3-x$, shown here, to solve each of the following equations.

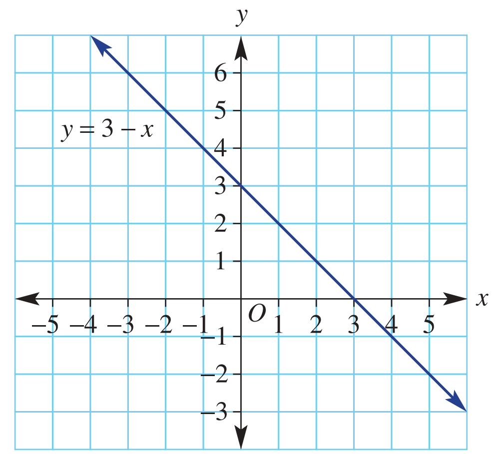
a $3-x=5.5$

b $3-x=0$

c $3-x=3.5$

d $3-x=-1$

e $3-x=-2$



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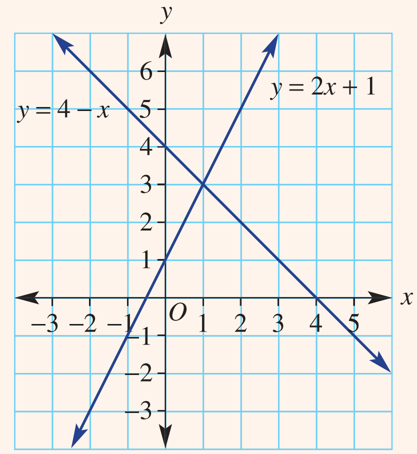
9E

Example 10 Using the point of intersection of two lines to solve an equation

Use the graph of $y = 4 - x$ and $y = 2x + 1$, shown here, to answer these questions.

a Write four solutions (x, y) for the line with equation $y = 4 - x$.
b Write four solutions (x, y) for the line with equation $y = 2x + 1$.
c Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.
d Solve the equation $4 - x = 2x + 1$.



Solution

a $(-2, 6), (-1, 5), (1, 3), (4, 0)$

Explanation

Many correct answers. Each point on the line $y = 4 - x$ is a solution to the equation for that line.

b $(-2, -3), (0, 1), (1, 3), (2, 5)$

Many correct answers. Each point on the line $y = 2x + 1$ is a solution to the equation for that line.

c

$(1, 3)$	$(1, 3)$
$y = 4 - x$	$y = 2x + 1$
$3 = 4 - 1$	$3 = 2 \times 1 + 1$
$3 = 3$ (True)	$3 = 3$ (True)

The point of intersection $(1, 3)$ is the solution that satisfies both equations. Substitute $(1, 3)$ into each equation and show that it makes a true equation (LHS = RHS).

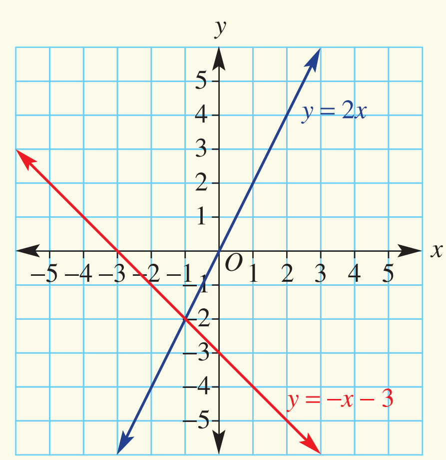
d $x = 1$

The solution to $4 - x = 2x + 1$ is the x-coordinate at the point of intersection. The value of both rules is equal for this x-coordinate.

Now you try

Use the graph of $y = -x - 3$ and $y = 2x$, shown here, to answer these questions.

a Write four solutions (x, y) for the line with equation $y = -x - 3$.
b Write four solutions (x, y) for the line with equation $y = 2x$.
c Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.
d Solve the equation $-x - 3 = 2x$



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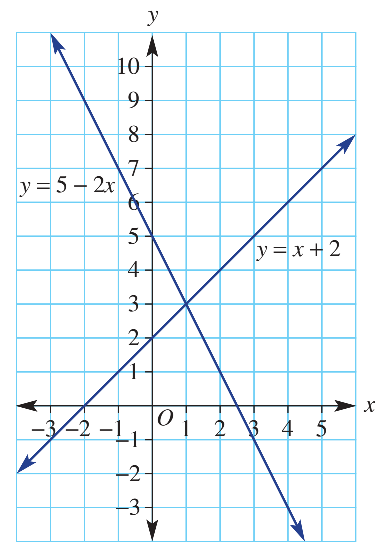
6 Use the graph of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.

a Write four solutions (x, y) for the equation $y = 5 - 2x$.

b Write four solutions (x, y) for the equation $y = x + 2$.

c Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.

d Solve the equation $5 - 2x = x + 2$ from the graph.



7 Graph each pair of lines on the same set of axes and read off the point of intersection.

a $y = 2x - 1$

x	-2	-1	0	1	2	3
y						

$y = -x$

x	-2	-1	0	1	2	3
y						

b $y = x + 1$

x	-2	-1	0	1	2	3
y						

$y = x + 2$

x	-2	-1	0	1	2	3
y						

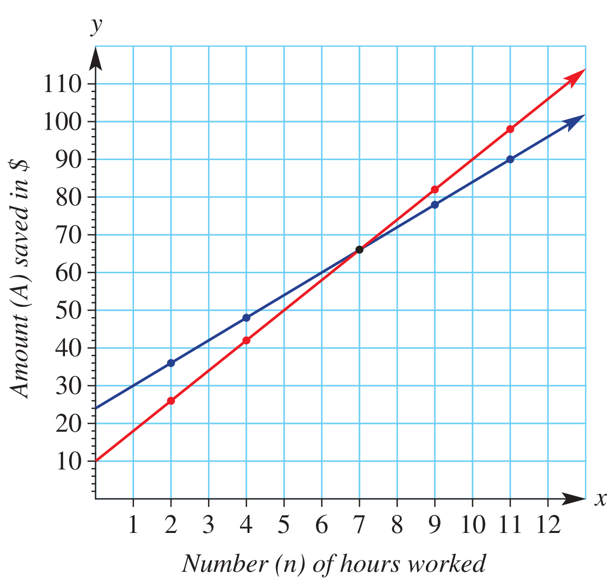
Problem-solving and reasoning

8, 10 | 8–10

8 Jayden and Ruby are saving all their money for the school ski trip.

- Jayden has saved \$24 and earns \$6 per hour mowing lawns.
- Ruby has saved \$10 and earns \$8 per hour babysitting.

This graph shows the total Amount (A) in dollars of their savings for the number (n) of hours worked.



a Here are rules for calculating the Amount (A) saved for working for n hours.

$A = 10 + 8n$ and $A = 24 + 6n$

Which rule applies to Ruby and which to Jayden? Explain why.

b Use the appropriate line on the above graph to find the solution to the following equations.

i $10 + 8n = 42$ ii $24 + 6n = 48$ iii $10 + 8n = 66$

iv $24 + 6n = 66$ v $10 + 8n = 98$ vi $24 + 6n = 90$

c From the graph, write three solutions (n, A) that satisfy $A = 10 + 8n$.

d From the graph, write three solutions (n, A) that satisfy $A = 24 + 6n$.

e Write the solution (n, A) that is true for both Ruby's and Jayden's equations and show that it satisfies both equations.

f From the graph, find the solution to the equation: $10 + 8n = 24 + 6n$ (i.e. find the value of n that makes Ruby's and Jayden's savings equal to each other).

g Explain how many hours have been worked and what their savings are at the point of intersection of the two lines.

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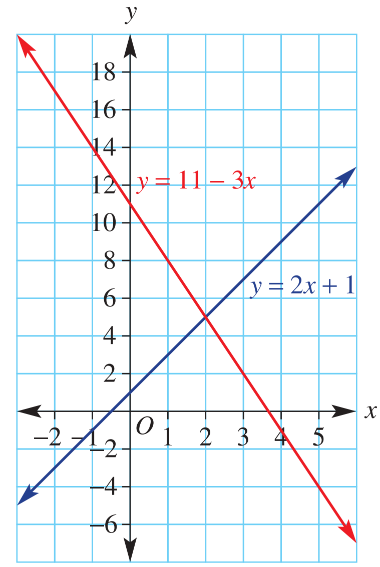
9E

9 This graph shows two lines with equations $y = 11 - 3x$ and $y = 2x + 1$.



a Copy and complete the coordinates of each point that is a solution for the given linear equation.

i $y = 11 - 3x$

$(-2, ?), (-1, ?), (0, ?), (1, ?), (2, ?), (3, ?), (4, ?), (5, ?)$

ii $y = 2x + 1$

$(-2, ?), (-1, ?), (0, ?), (1, ?), (2, ?), (3, ?), (4, ?), (5, ?)$

b State the coordinates of the point of intersection.

c Explain why the point of intersection is the only solution that satisfies both equations.

10 Use digital technology to sketch a graph of $y = 1.5x - 2.5$ for x and y values between −7 and 7. Use the graph to solve each of the following equations. Round answers to two decimal places.

a $1.5x - 2.5 = 3$

b $1.5x - 2.5 = -4.8$

c $1.5x - 2.5 = 5.446$

The Max and Jessica race — 11

11 Jessica and Max have a 10 second running race.

- Max runs at 6 m/second.
- Jessica is given a 10 m head-start and runs at 4 m/second.

a Copy and complete this table showing the distance run by each athlete.

Time (t) in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance (d) in metres	0										
Jessica's distance (d) in metres	10										

b Plot these points on a distance-time graph and join the points to form two straight lines, labelling them 'Jessica' and 'Max'.

c Find the rule linking distance (d) and time (t) for Max.

d Using the rule for Max's race, write an equation that has the solution:

i $t = 3$

ii $t = 5$

iii $t = 8$

e Find the rule linking distance (d) and time (t) for Jessica.

f Using the rule for Jessica's race, write an equation that has the solution:

i $t = 3$

ii $t = 5$

iii $t = 8$

g Write the solution (t, d) that is true for both distance equations and show that it satisfies both equations.

h Explain what is happening in the race at the point of intersection, and for each athlete state the distance from the starting line and time taken.

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9F Using graphs to solve linear inequalities

Learning intentions

- To understand that graphs can be used to solve inequalities.
- To be able to sketch horizontal and vertical lines given their equation.
- To be able to graph inequalities using horizontal and vertical lines as regions in the plane.
- To be able to solve simple inequalities using graphs.

Key vocabulary: inequality, horizontal, vertical, region in the plane

We have already been introduced to inequalities, which are statements containing an inequality symbol $<, \leqslant, >$ and $\geqslant$. For example: $3x > 6$ or $2x - 1 \leqslant 7$. Such a statement might arise from a situation, for example, where you want to find out the possible number of hours you can hire a sports car if your budget is at most \$500 and the cost is \$80 up-front plus \$60 per hour. The inequality could be written as $80 + 60t \leqslant 500$, where t is the number of hours.

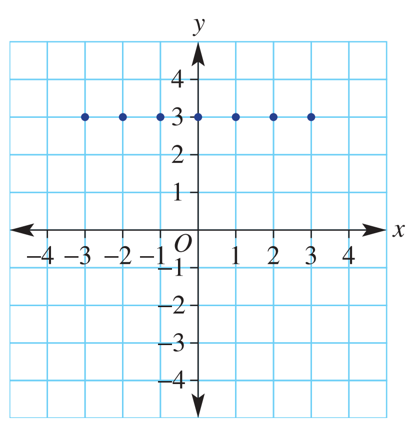
While it is possible to solve inequalities using algebraic techniques, it is also possible to solve them using graphs. To achieve this, we can use the graphs of horizontal and vertical lines.

Lesson starter: Why are the equations of horizontal and vertical lines so simple?

The equation $y = 3$ might not look like the equation of a straight line in two-dimensions. However, it does describe an infinite set of points which can be graphed on a Cartesian place as a straight line.

Look at this Cartesian plane including the seven given points.

- Write down the coordinates of all seven points.
- What do the coordinates of all the points have in common?
  Now imagine a line passing through all seven points.



- Which of the following points would also sit on this line?
  A $(4, 3)$ **B** $(-1.5, 3)$
  C $(2, 2)$ **D** $(-3, 5)$
- Give reasons why $y = 3$ is the equation of the line described above and why the equation does not depend on the value of x. Now think about a set of points which are aligned vertically, all with $x = 2$ as its x-coordinate.
- Give reasons why $x = 2$ is the equation of the line joining the points and why the equation does not depend on the value of y.

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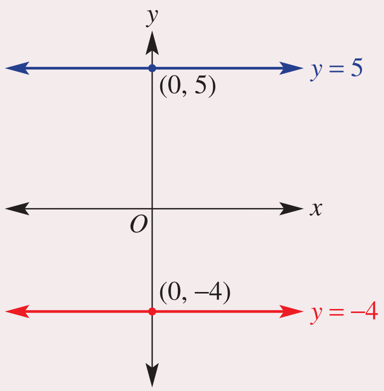
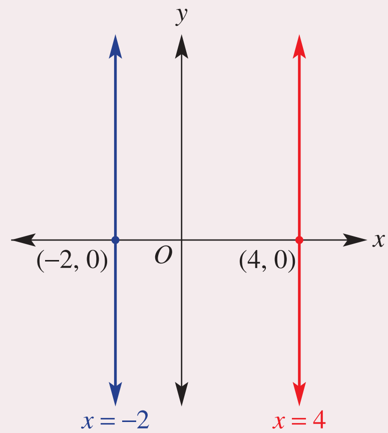
9F

Key ideas

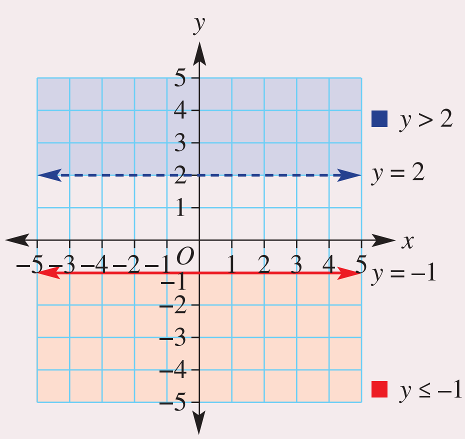
- A horizontal line (parallel to the x-axis) has a rule in the form $y = c$, where c is any number.

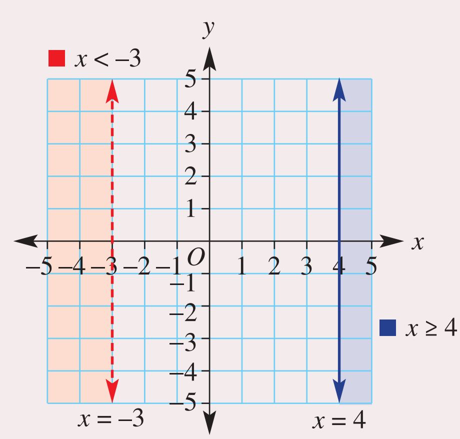


- A vertical line (parallel to the y-axis) has a rule in the form $x = k$, where k is any number.

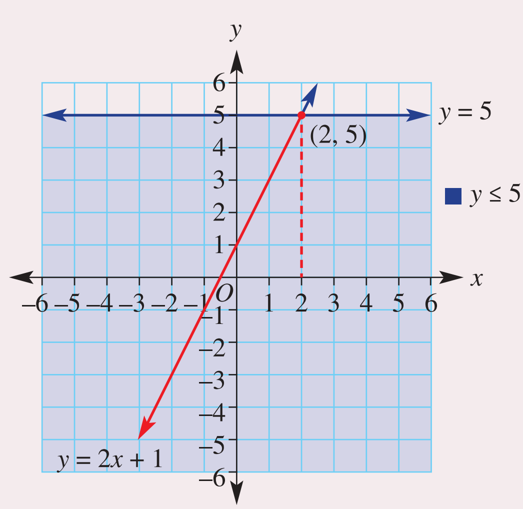


- A region in the plane is described using an inequality.
  - All the points that satisfy the inequality form the shaded region.
  - A dashed line is used to show that the points on the line are not included in the region. A dashed line is used when the < or > symbols are given.
  - A full line is used to show that the points on the line are included in the region. A full line is used when the $\leqslant$ or $\geqslant$ symbols are given.





- Graphs can be used to solve inequalities. To solve $2x + 1 \leqslant 5$ for example:
  - Consider the graph of $y = 2x + 1$ and the region $y \leqslant 5$.
  - The solution to $2x + 1 \leqslant 5$ will be all the x values of the points that are on the line $y = 2x + 1$ and also in the region $y \leqslant 5$, shown in red. The solution is therefore $x \leqslant 2$.
  - Alternatively, the solution to $2x + 1 > 5$ will be $x > 2$ where the line $y = 5$ would be drawn as a dashed line.



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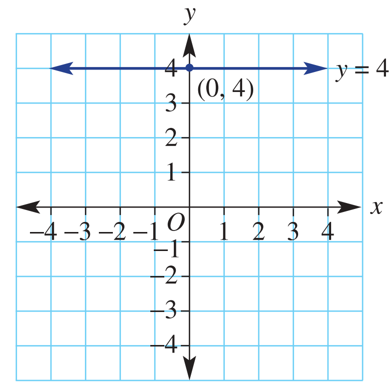
Exercise 9F

Understanding

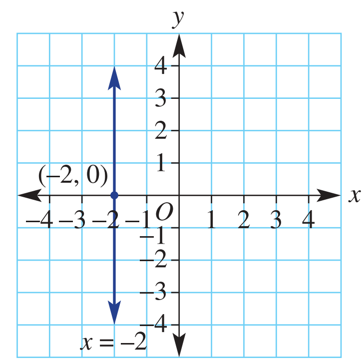
	1–3	1–3

1 This horizontal line has equation $y = 4$. Answer true or false to the following questions.



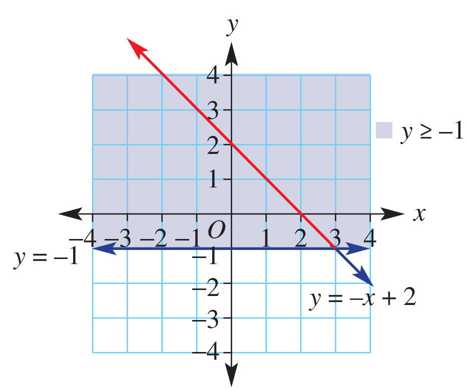
- **a** All the points on the line have a y-coordinate of 4.
- **b** All the points on the line have an x-coordinate of 4.
- **c** The point $(1, 3)$ is on the line.
- **d** The point $(2, 4)$ is on the line.
- **e** When graphing the region $y \leqslant 4$, you would shade above the line.
- **f** When graphing the region $y \leqslant 4$, you would shade below the line.
- **g** When graphing the region $y < 4$ or $y > 4$, a dashed line should be used.

2 This vertical line has equation $x = -2$. Answer true or false to the following questions.



- **a** All the points on the line have an x-coordinate of -2.
- **b** All the points on the line have a y-coordinate of -2.
- **c** The point $(-2, 5)$ is on the line.
- **d** The point $(-1, 3)$ is on the line.
- **e** When graphing the region $x \leqslant -2$, you would shade to the left of the line.
- **f** When graphing the region $x \geqslant -2$, you would shade to the right of the line.
- **g** When graphing the region $x < -2$ or $x > -2$, a full line should be used.

3 This graph shows the graph of the equation $y = -x + 2$ and the region $y \geqslant -1$.



- **a** True or false? The position of all the points that are on both the line $y = -x + 2$ and in the region $x \geqslant -1$ are shown in red.
- **b** Describe all the possible x coordinates of the points outlined in part **a**.
- **c** The values of x outlined in part **b** are the solution to the inequality $-x + 2 \geqslant -1$. Write this solution using an inequality symbol.

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9F

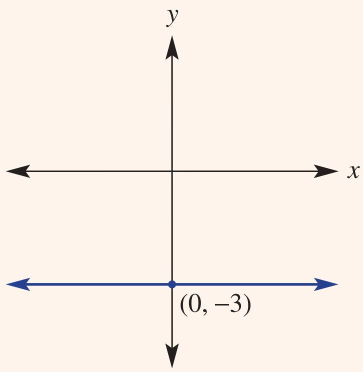
Fluency

4, 5–8(½) | 4–9(½)

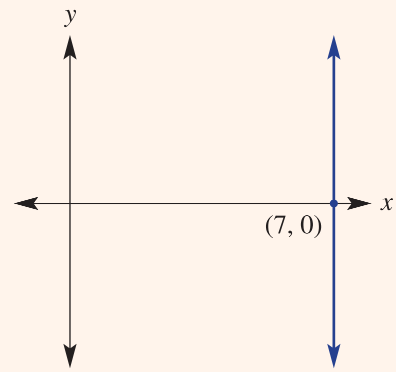
Example 11 Finding rules for horizontal and vertical lines

Write the rule for these horizontal and vertical lines.

a



b



Solution

a $y = -3$

b $x = 7$

Explanation

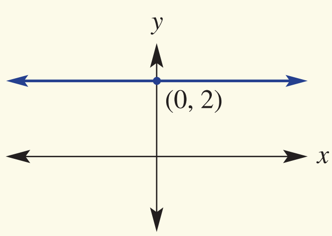
All points on the line have a y-value of -3.

Vertical lines take the form $x = k$. Every point on the line has an x-value of 7.

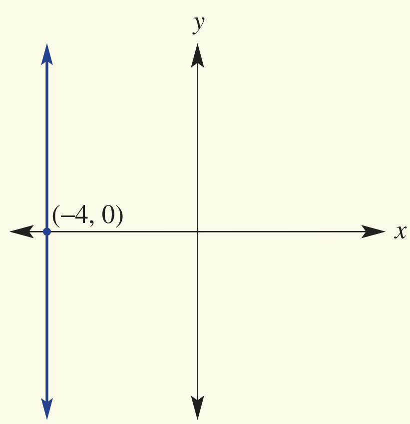
Now you try

Write the rule for these horizontal and vertical lines.

a

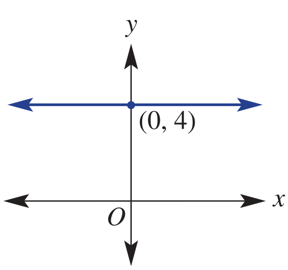


b

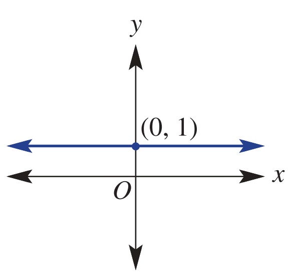


4 Write the rule for these horizontal and vertical lines.

a



b



Hint for Q4: Horizontal lines have rules in the form $y = c$.

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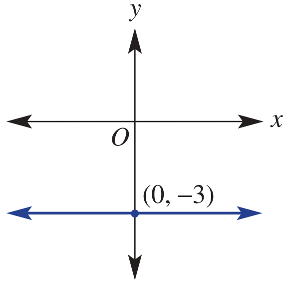
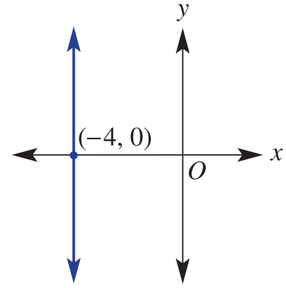
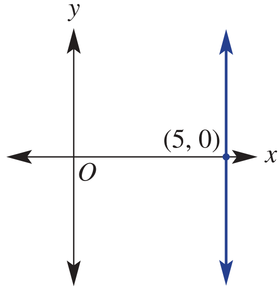
c



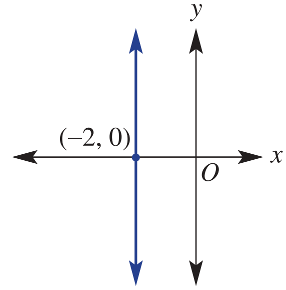
d



e



f



5 Sketch horizontal or vertical graphs for these rules.

a $y = 2$ **b** $y = -1$ **c** $y = -4$ **d** $y = 5$

e $x = -3$ **f** $x = 4$ **g** $x = 1$ **h** $x = -1$

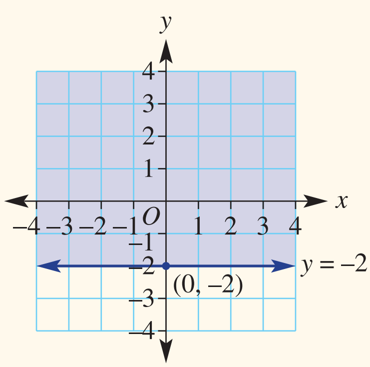
Example 12 Sketching regions in the plane

Sketch the following regions.

a $y \geqslant -2$ **b** $x < 5$

Solution

a

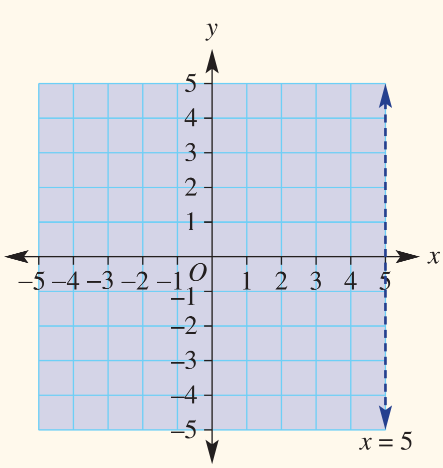


Explanation

First, sketch $y = -2$ using a full line and shade above the line since the $\geqslant$ (greater than or equal to) symbol is used.

All the points on or above the line $y = -2$ satisfy $y \geqslant -2$.

b



First, sketch $x = 5$ using a dashed line and shade to the left of the line since the $<$ (less than) symbol is used.

All the points left of the line $x = 5$ satisfy $x < 5$.

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9F

Now you try

Sketch the following regions.

a $y < 1$

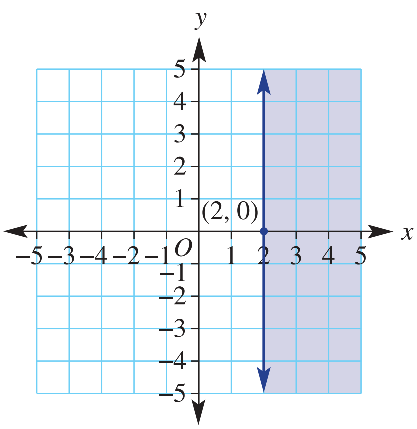
b $x \geqslant -4$

6 Sketch the following regions.

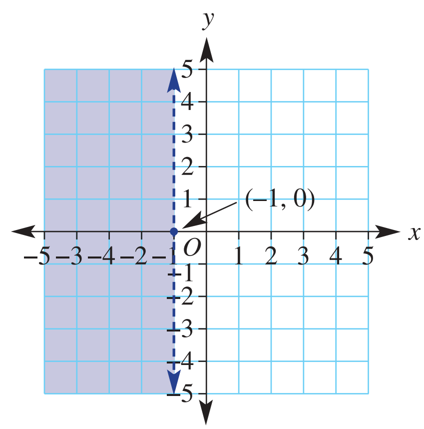
a $y \geqslant 1$
b $y > -2$
c $y < 3$
d $y \leqslant -4$
e $x < 2$
f $x \leqslant -4$
g $x \geqslant 2$
h $x > -1$

7 Write the inequalities matching these regions.

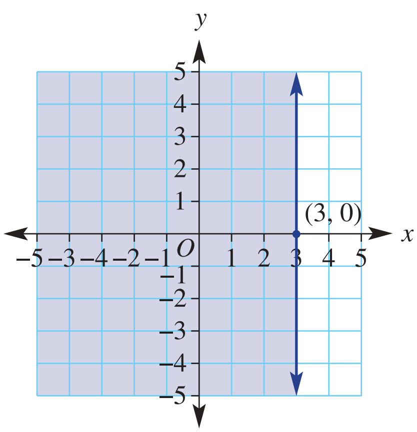
a



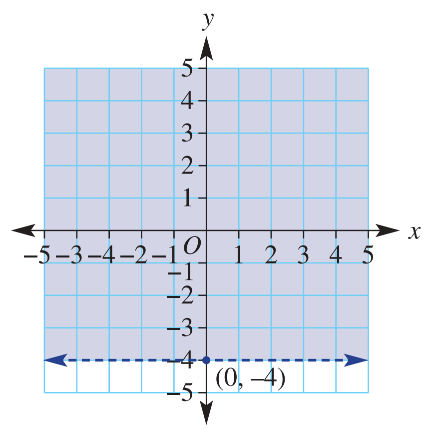
b



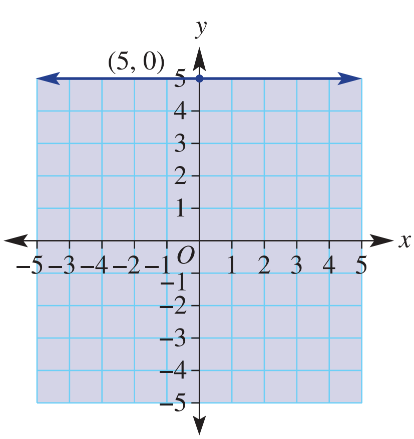
c



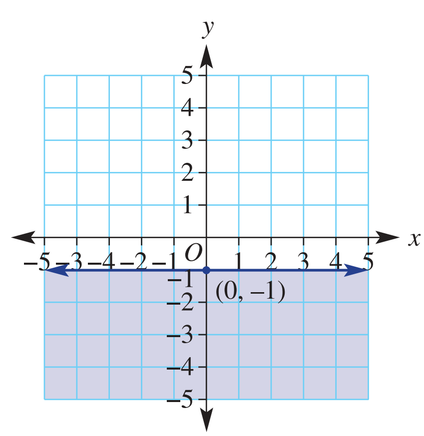
d



e



f



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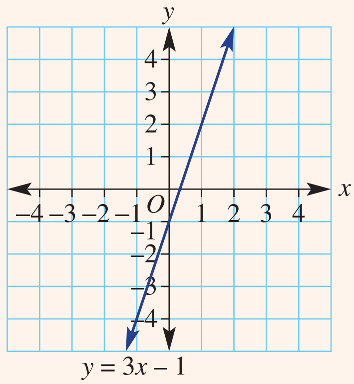
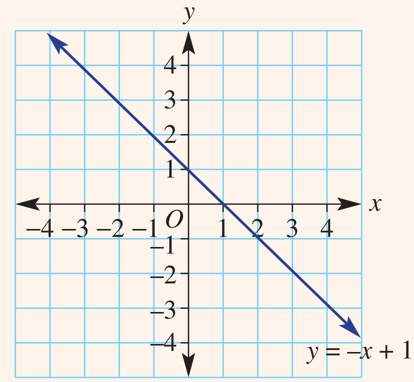
Example 13 Solving inequalities using graphs

Solve the following inequalities using the given graphs.

a $3x - 1 \leq 2$

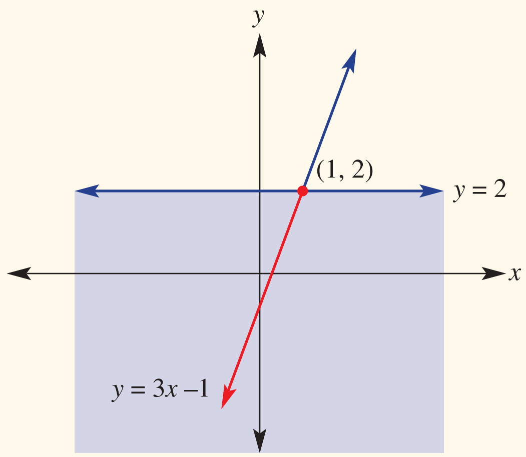


b $-x + 1 > 3$



Solution

a



The solution is $x \leqslant 1$.

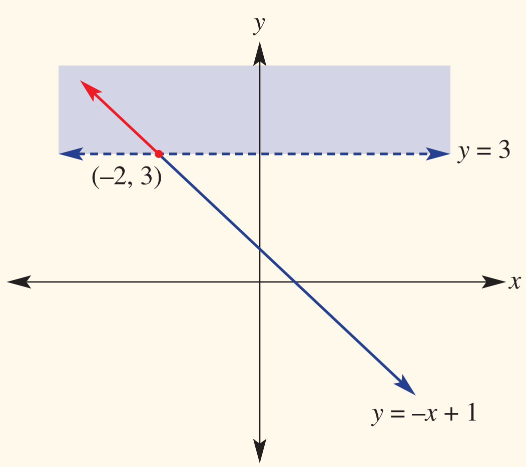
Explanation

Sketch the region $y \leqslant 2$ on the graph.

The solution to $3x - 1 \leqslant 2$ will be all the x values of the points that are both on the line $y = 3x - 1$ and in the region $y \leqslant 2$.

These points are highlighted in red where $x \leqslant 1$.

b



The solution is $x < -2$.

Sketch the region $y > 3$ on the graph.

The solution to $-x + 1 > 3$ will be all the x values of the points that are both on the line $y = -x + 1$ and in the region $y > 3$.

These points are highlighted in red where $x < -2$.

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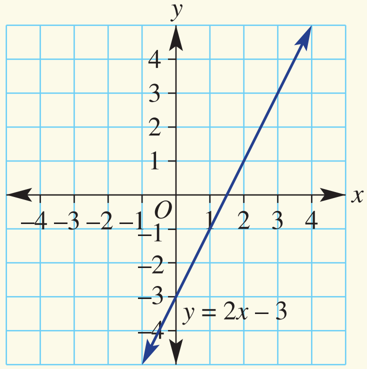
9F

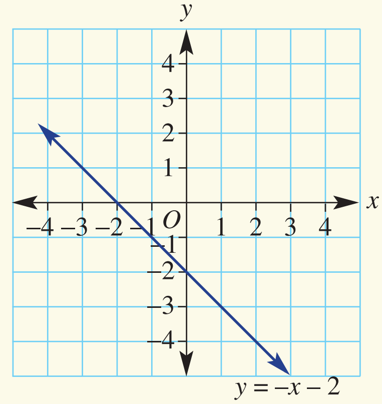
Now you try

Solve the following inequalities using the given graphs.

a $2x - 3 \geqslant 1$

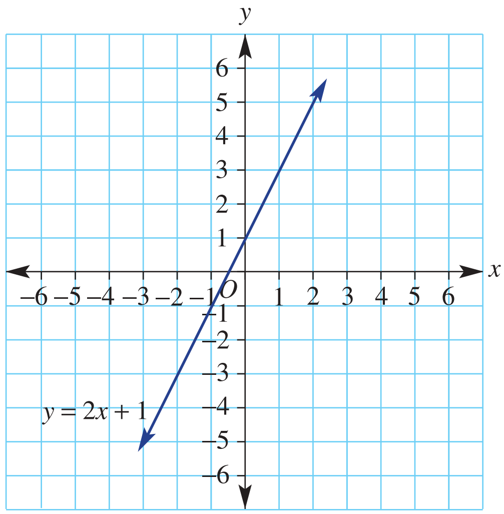


b $-x - 2 < 2$

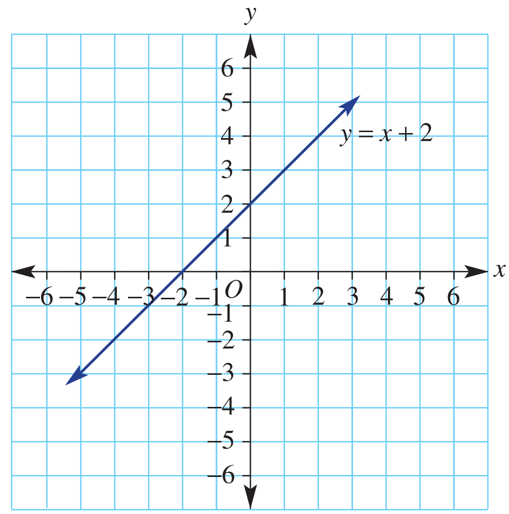


8 Solve the following inequalities using the given graphs.

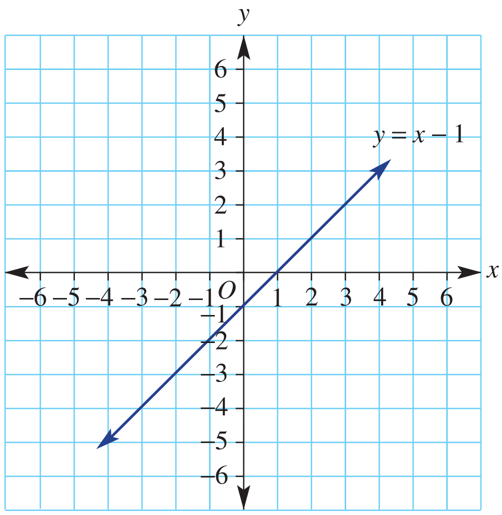
a $2x + 1 \leq 5$



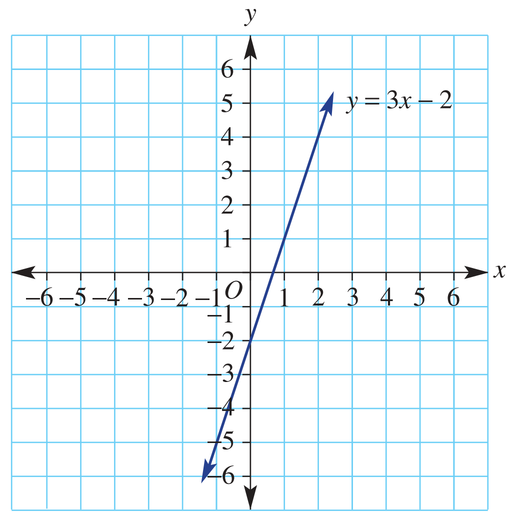
b $x + 2 \leq 3$



c $x - 1 \geq 4$



d $3x - 2 > 4$



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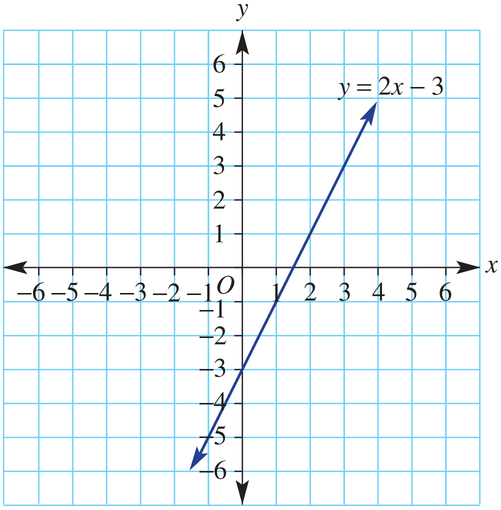
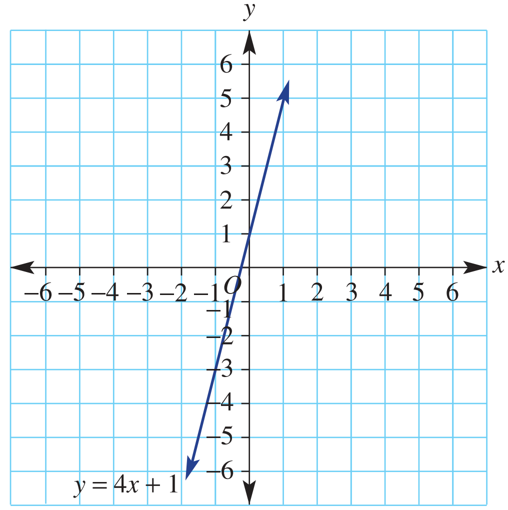
e $2x - 3 < -1$

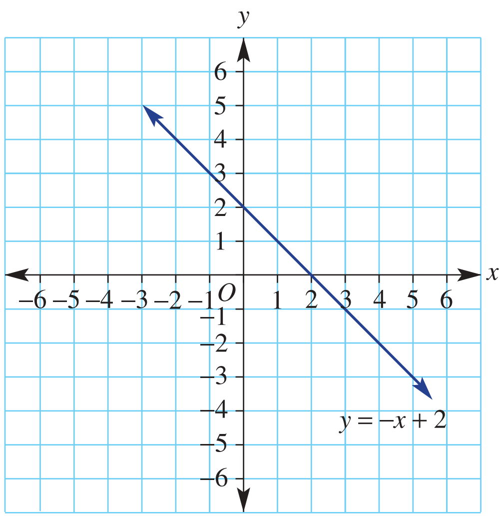


f $4x + 1 > -3$

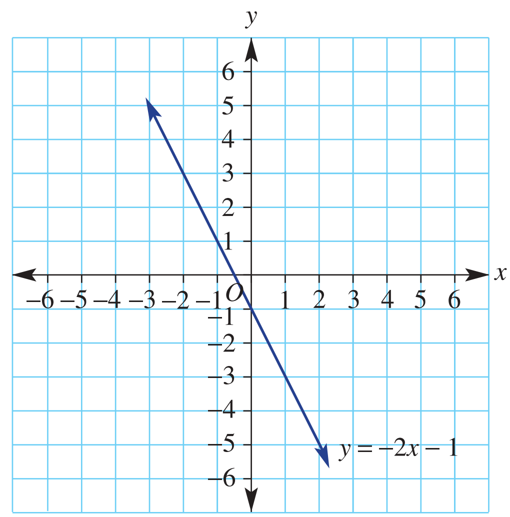


9 Solve the following inequalities using the given graphs.

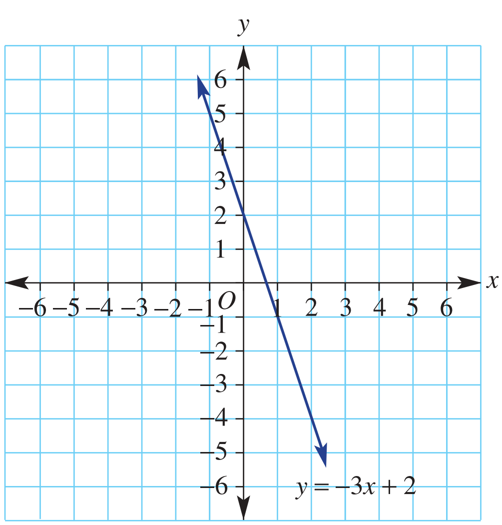
a $-x + 2 \geq 1$



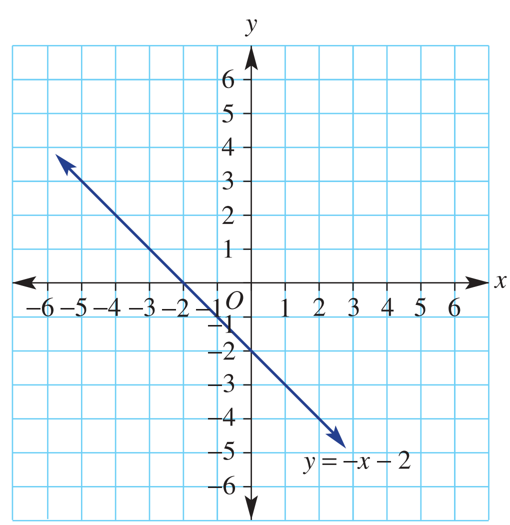
b $-2x - 1 \leq 3$



c $-3x + 2 \geq 5$



d $-x - 2 > -3$



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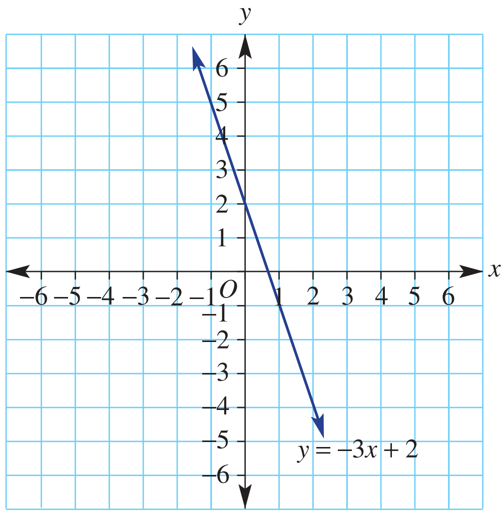
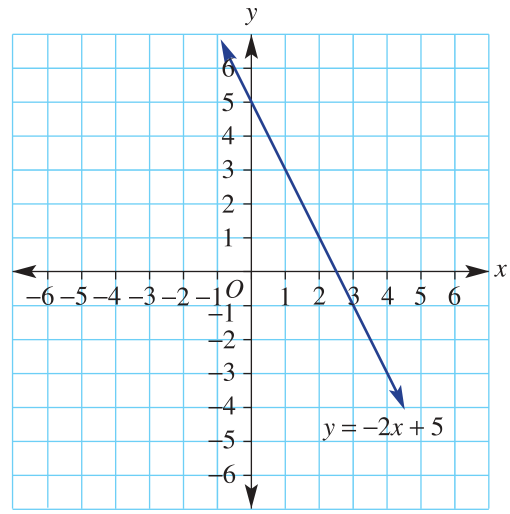
e $-3x + 2 < -4$



f $-2x + 5 < 1$



Problem-solving and reasoning

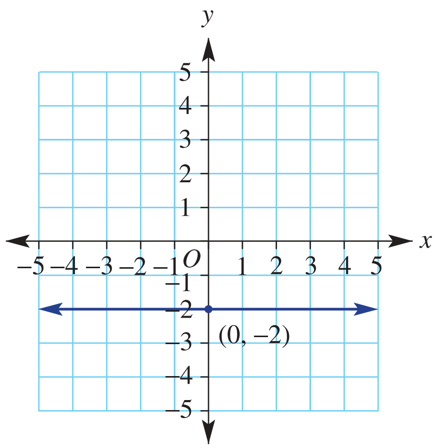
10, 11–12 (½) | 10, 11–12 (½), 13

10 Match the equations **a**–**f** with the graphs **A**–**F**.

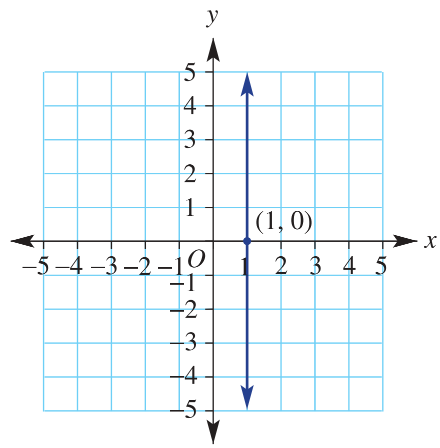
a $x > 4$
b $y \le 3$
c $x = 1$
d $x = -5$
e $y = -2$
f $y > -2$

Hint for Q10: Inequalities are shown with a shaded region, but equations are shown with a line.

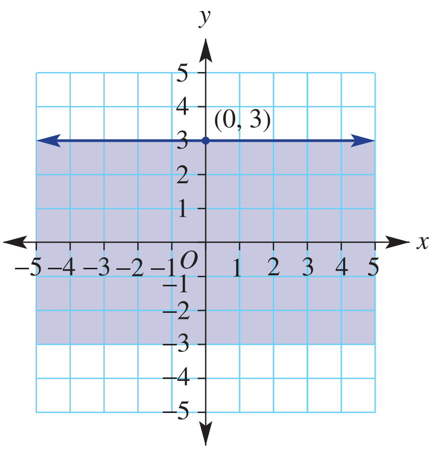
A



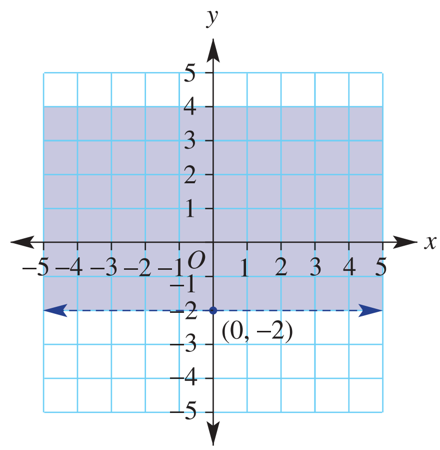
B



C



D



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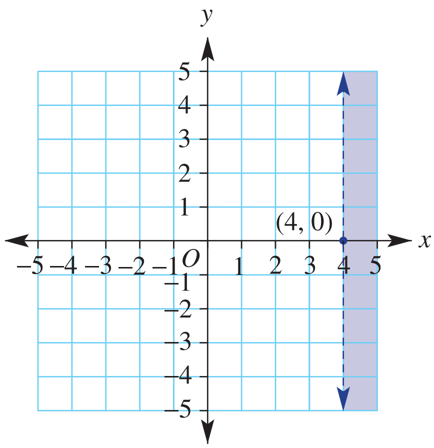
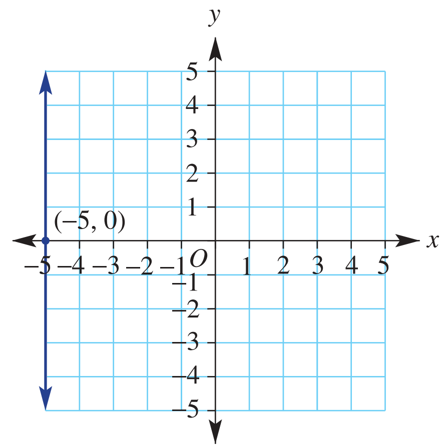
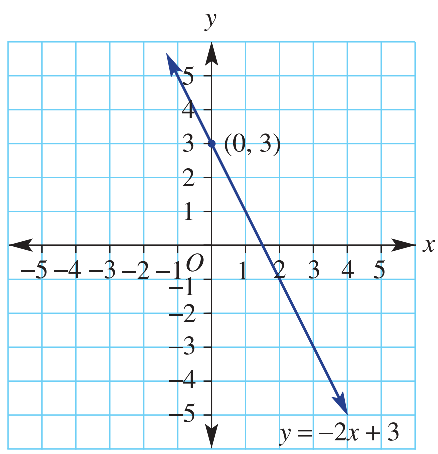
E



F



11 This diagram shows the graph of $y = -2x + 3$.



Use the graph to solve the following equations and inequalities.

a $-2x + 3 = 5$
b $-2x + 3 = -1$
c $-2x + 3 = 0$
d $-2x + 3 < 0$
e $-2x + 3 \geq 0$
f $-2x + 3 \geq 1$

12 Decide if the point $(1, 4)$ is on the given line or within the given region.

a $y = 4$
b $y > 4$
c $x = -1$
d $x = 1$
e $x = 4$
f $x \leq 1$
g $x > 1$
h $y \leq 4$
i $y < 1$
j $y < 6$
k $x > -2$
l $y \geq -7$

Hint for Q12: Draw a Cartesian plane and plot the point $(1, 4)$ on it first.

13 **a** Explain why the rule for the x-axis is given by $y = 0$.
b Explain why the rule for the y-axis is given by $x = 0$.

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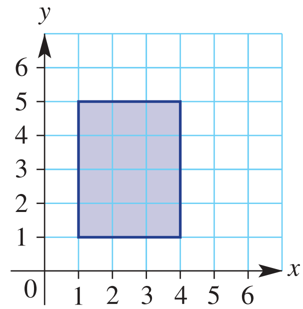
9F

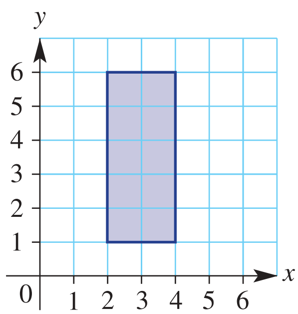
Making rectangles with inequalities — 11

14 The given rectangular region could be described as all the points in common with the four inequalities: $x \geqslant 1$, $x \leqslant 4$, $y \geqslant 1$ and $y \leqslant 5$.

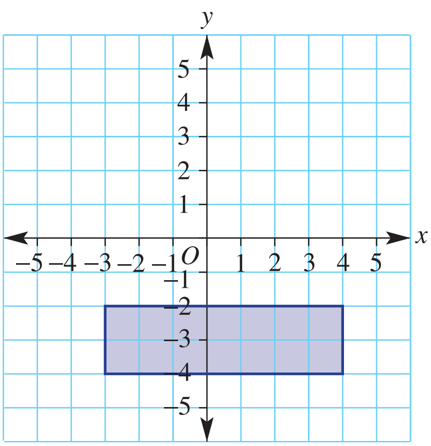


Give the four inequalities that describe the following rectangles.

a



b



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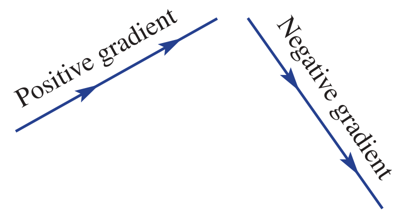
9G Gradient ★

Learning intentions

- To understand that gradient is a number measuring the slope of a line.
- To understand that gradient can be positive, negative, zero or undefined.
- To be able to find the gradient of a straight line.

Key vocabulary: gradient, slope, rise, run, positive number, negative number, horizontal, vertical, undefined

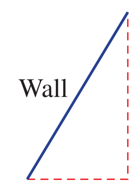
When we think about how two variables (like population and time) are related, we are often concerned about how quickly one variable changes with respect to the other.



Graphically, we use the concept of gradient to describe this change. This is illustrated by the slope of a line. The steepness or slope of a line depends on how far it rises or falls over a given horizontal distance. This is why the gradient is calculated by dividing the vertical rise by the horizontal run between two points. Lines that rise (from left to right) have a positive gradient and lines that fall (from left to right) have a negative gradient.

Lesson starter: Which is the steepest?

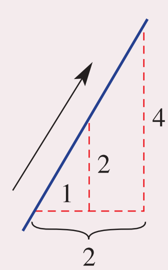
At a children's indoor climbing centre there are three types of sloping walls to climb. The blue wall rises 2 metres for each metre across. The red wall rises 3 metres for every 2 metres across and the yellow wall rises 7 metres for every 3 metres across.



- Draw a diagram showing the slope of each wall.
- Label your diagrams with the information given above.
- Discuss which wall might be the steepest, giving reasons.
- Discuss how it might be possible to accurately compare the slope of each wall.

Key ideas

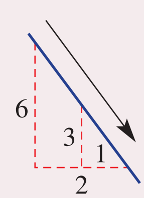
- The **gradient** is a measure of **slope**.
- Gradient $= \dfrac{\text{rise}}{\text{run}}$
  - **Rise** = change in y.
  - **Run** = change in x.
  - The run is always considered to be **positive** when moving from left to right.



Positive gradient

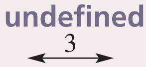
Gradient $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{4}{2}$ $= 2$ or Gradient $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{2}{1}$ $= 2$

- A gradient is **negative** if y decreases as x increases. The rise is considered to be negative.
- The gradient of a **horizontal** line is 0.
- The gradient of a **vertical** line is **undefined**.

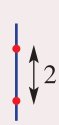


Negative gradient

Gradient $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{-6}{2}$ $= -3$ or Gradient $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{-3}{1}$ $= -3$



Gradient $= \dfrac{0}{3}$ $= 0$



Gradient $= \dfrac{2}{0}$ Which is undefined

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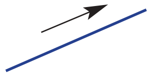
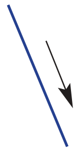
Exercise 9G

Understanding

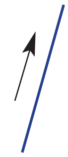
1–3 3(½)

1 Decide if the gradients of these lines are positive or negative.

a 

b 

c 

d 

2 This graph shows four points A, B, C and D.

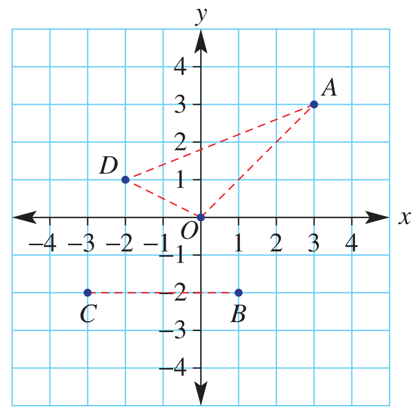
a Write down the coordinates of the four points A, B, C and D.

b What is the rise between these pairs of points?

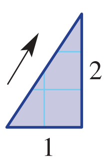
- **i** From O to A **ii** From C to B
- **iii** From D to O **iv** From D to A

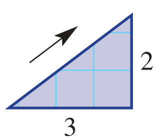
c What is the run between these pairs of points?

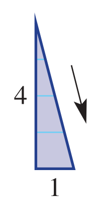
- **i** From O to A **ii** From C to B
- **iii** From D to O **iv** From D to A



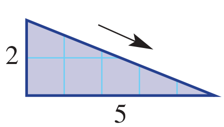
3 Write down the gradient, using $\frac{\text{rise}}{\text{run}}$ for each of these slopes.

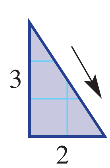
a 

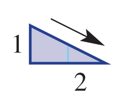
b 

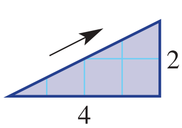
c 

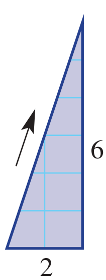
Hint for Q3: Remember that the rise is negative when the arrow is pointing downwards.

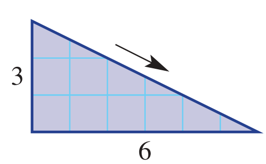
d 

e 

f 

g 

h 

i 

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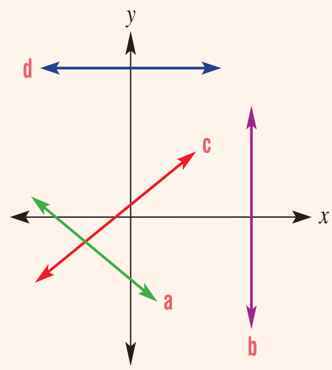
Fluency

4–6 | 4, 5–6(½)

Example 14 Deciding a type of gradient

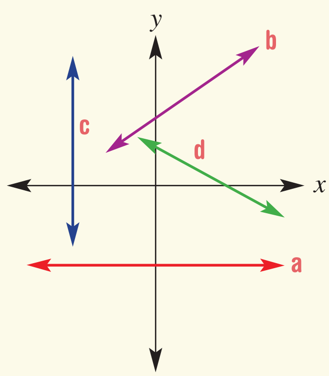
Decide if the lines labelled a, b, c and d on this graph have a positive, negative, zero or undefined gradient.



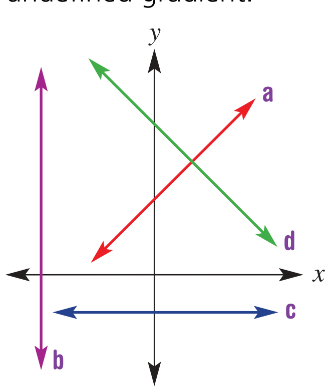
Solution	Explanation
a Negative gradient	As x increases, y decreases.
b Undefined gradient	The line is vertical.
c Positive gradient	y increases as x increases.
d Zero gradient	There is no increase or decrease in y as x increases.

Now you try

Decide if the lines labelled a, b, c and d on this graph have a positive, negative, zero or undefined gradient.



4 Decide if these lines labelled a, b, c and d on this graph have a positive, negative, zero or undefined gradient.



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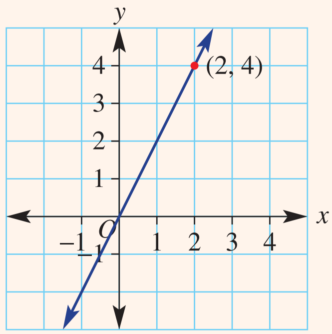
9G

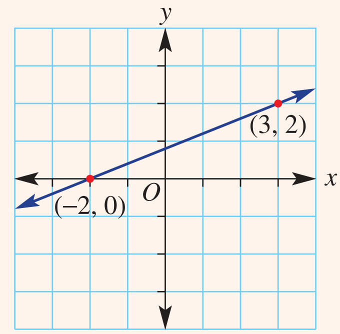
Example 15 Finding a positive gradient from a graph

Find the gradient of these lines.

a



b



Solution

a Gradient $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{4}{2} = 2$

Explanation

The rise is 4 for every 2 across to the right.
So rise = 4 and run = 2.

b Gradient $= \dfrac{\text{rise}}{\text{run}}$

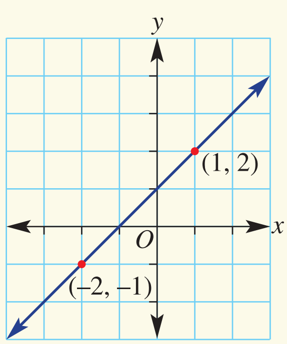
$= \dfrac{2}{5}$

From $(-2, 0)$ to $(3, 2)$ the rise is 2 and the run is 5. Leave your answer as a fraction.

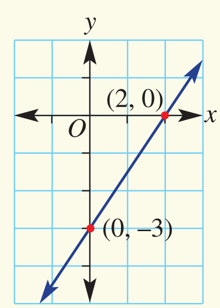
Now you try

Find the gradient of these lines.

a



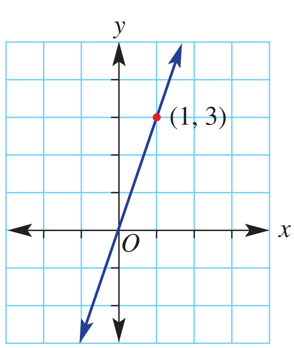
b



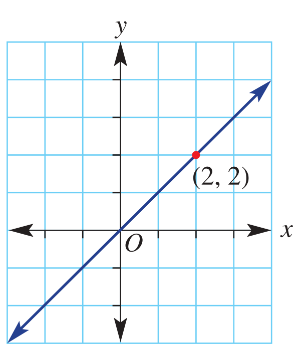
Hint for Q5: The run is always positive.

5 Find the gradient of these lines. Use gradient $= \dfrac{\text{rise}}{\text{run}}$.

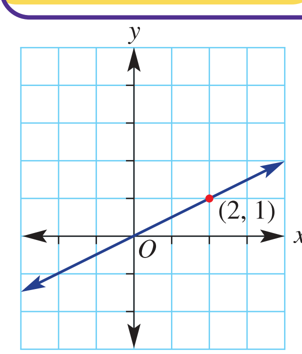
a



b



c



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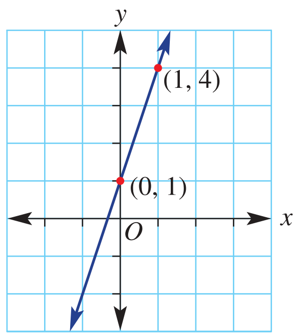
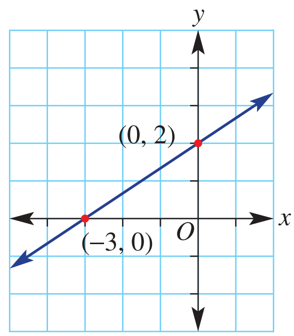
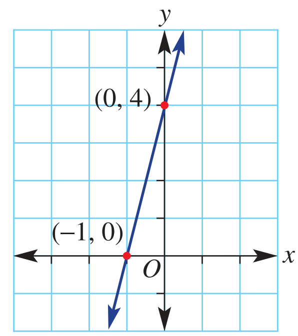
d



e



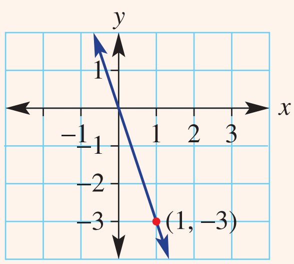
f



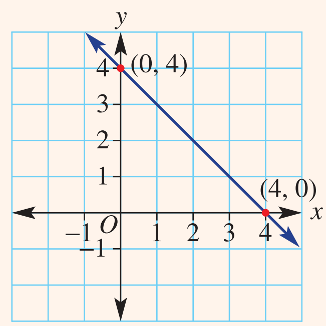
Example 16 Finding a negative gradient

Find the gradient of these lines.

a



b



Solution

a Gradient $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-3}{1}$

$= -3$

b Gradient $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-4}{4}$

$= -1$

Explanation

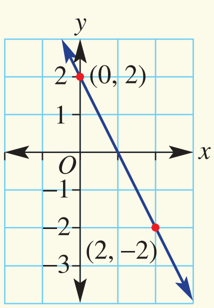
Between $(0, 0)$ and $(1, -3)$ the rise is -3 and the run is 1.

The y value falls 4 units while the x value increases by 4.

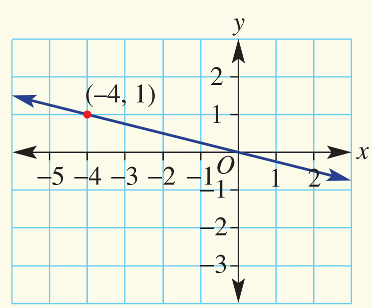
Now you try

Find the gradient of these lines.

a



b



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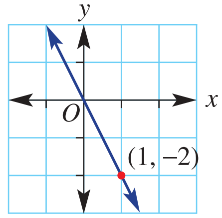
9G

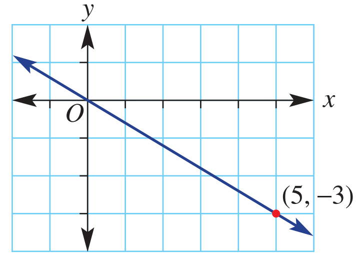
6 Find the gradient of these lines. Use Gradient = $\frac{\text{rise}}{\text{run}}$.

Hint for Q6: The run is positive and the rise will be negative for these lines.

a



b



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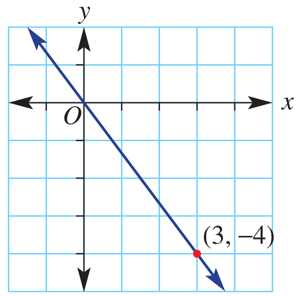

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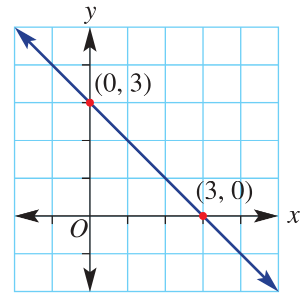

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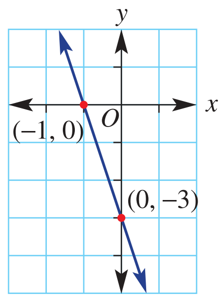

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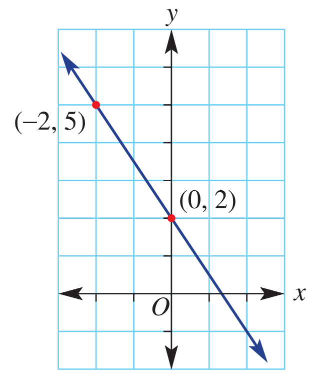
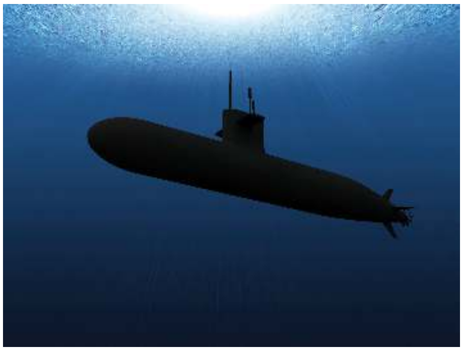


Problem-solving and reasoning

7, 8 | 7–10

7 Abdullah climbs a rocky slope that rises 12 m for each 6 metres across. His friend Jonathan climbs a nearby grassy slope that rises 25 m for each 12 m across. Which slope is steeper?

8 A submarine falls 200 m for each 40 m across and a torpedo falls 420 m for each 80 m across in pursuit of the submarine. Which has the steeper gradient, the submarine or torpedo?



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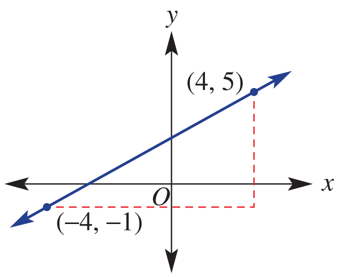
9 **a** Explain why the rise between the two points $(-1, 1)$ and $(3, 7)$ is 6.
b Explain why the rise between the two points $(-4, 2)$ and $(3, -5)$ is -7.
c Explain why the run between the two points $(-1, 1)$ and $(3, 7)$ is 4.
d Explain why the run between the two points $(-4, 2)$ and $(3, -5)$ is 7.

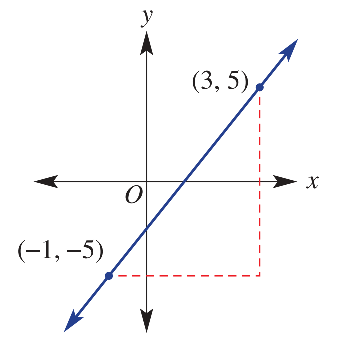
10 A line joins the point $(0, 0)$ to the point (a, b) with a gradient of 2.
a If $a = 1$, find b. **b** If $a = 5$, find b.

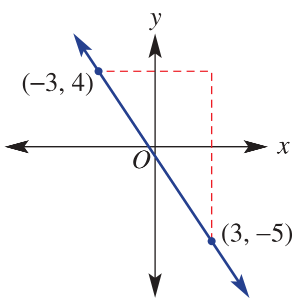
Gradients without grids

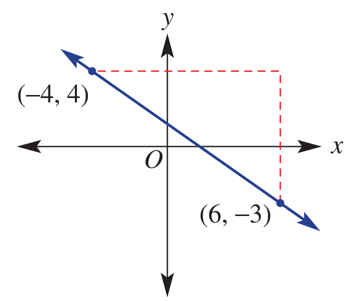
—	11, 12

11 Find the gradient of these lines. You will need to first calculate the rise and the run.

a


b


c


d


12 Find the gradient of the line joining these pairs of points.
a $(0, 2)$ and $(2, 7)$ **b** $(0, -1)$ and $(3, 4)$
c $(-3, 7)$ and $(0, -1)$ **d** $(-5, 6)$ and $(1, 2)$
e $(-2, -5)$ and $(1, 3)$ **f** $(-5, 2)$ and $(5, -1)$

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9H Applications of linear graphs ★

Learning intentions

- To understand that straight line graphs can be applied to situations where there is a constant rate of change.
- To be able to apply straight line graphs to model and solve problems arising in real-world situations.

Key vocabulary: linear, straight line, rate of change, variables

Rules, tables and graphs can be applied to many practical situations where a relationship exists between two variables. It is also often the case that this relationship is linear, meaning that the graph is a straight line. For example, if a pile of coal being emptied out of a pit increases at a rate of 12 tonnes per hour, then the graph of the mass of dirt over time would be a straight line. For every hour, the mass of dirt increases by 12 tonnes.

A constant rate of excavation creates a linear relationship and a straight line graph.

Lesson starter: Mining for coal

A coal mining pit produces 12 tonnes of coal per hour.

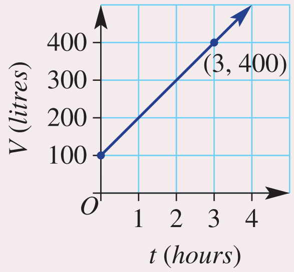
- Describe the two related variables in this situation.
- Discuss whether or not the relationship between the two variables is linear. Give reasons.
- Use a table and a graph to illustrate the relationship.
- Find a rule that links the two variables and discuss how your rule might be used to find the mass of coal at a given time.

Key ideas

- If the **rate of change** of one **variable** with respect to another is constant, then the relationship between the two variables is **linear**.
- When applying **straight line** graphs, choose letters to replace x and y to suit the variables.
  For example, V for volume and t for time.

t	0	1	2	3
V	100	200	300	400

$V = 100t + 100$



Exercise 9H

Understanding — 1–4 — 4

1 Find a rule linking d and t in these tables.

a

t	0	1	2	3
d	0	3	6	9

b

t	0	1	2	3
d	2	3	4	5

c

t	0	1	2	3
d	4	3	2	1

Hint for Q1: Put d on the left by itself as in $d = 2t$ or $d = -t + 3$.

2 A rule linking distance (d) and time (t) is given by $d = 10t + 5$. Use this rule to find the value of d for the given values of t.

a $t = 1$ **b** $t = 4$ **c** $t = 0$ **d** $t = 12$

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3 The height (in cm) of fluid in a flask increases at a rate of 30 cm every minute starting at 0 cm. Find the height of fluid in the flask at these times.
a 2 minutes b 5 minutes c 11 minutes

4 The volume of gas in a tank decreases from 30 L by 2 L every second. Find the volume of gas in the tank at these times.
a 1 second b 3 seconds c 10 seconds

Fluency 5, 7 6, 7

Example 17 Linking distance with time

A hiker walks at a constant rate of 4 kilometres per hour for 4 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 4.
b Draw a graph by plotting the points given in the table in part a.
c Write a rule linking d with t.
d Use your rule to find the distance travelled for 2.5 hours of walking.
e Use your rule to find the time taken to travel 8 km.

Solution

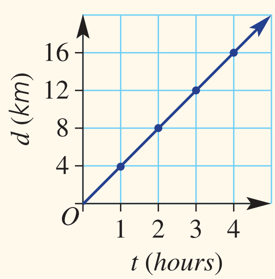
a

t	0	1	2	3	4
d	0	4	8	12	16

Explanation

d increases by 4 for every increase in t by 1.

b



Plot the points on a graph using a scale that matches the numbers in the table.

c

t	0	1	2	3	4
d	0	4	8	12	16

+4 +4 +4 +4

$d = 4t$

For $d = \Delta t + \square$, $\Delta = 4$ and $\square = 0$.
The rule is: $d = 4t$.

d $d = 4t$
$= 4 \times 2.5$
$= 10$
The distance is 10 km after 2.5 hours of walking.

Substitute $t = 2.5$ into your rule and find the value for d.

e $d = 4t$
$8 = 4t$
$2 = t$
It takes 2 hours to travel 8 km.

Substitute $d = 8$ into your rule then divide both sides by 4. Check by looking at your graph from part b.

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Now you try

A canoeist paddled downstream at 4 km/h for 7 hours.

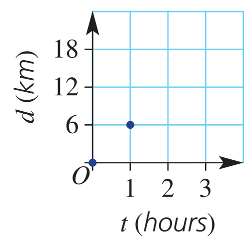
- **a** Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 7.
- **b** Draw a graph by plotting the points given in the table in part **a**.
- **c** Write a rule linking d with t.
- **d** Use your rule to find the distance travelled in 3.5 hours.
- **e** Use your rule to find the time taken to travel 26 km.

Hint for Q5: Add 6 km for every hour.

5 A jogger runs at a rate of 6 kilometres per hour for 3 hours.

- **a** Complete this table of values using t for time in hours and d for distance in kilometres.
- **b** Complete this graph by plotting the points given in the table in part **a**.
- **c** Write a rule linking d with t.
- **d** Use your rule to find the distance travelled for 1.5 hours of jogging.
- **e** Use your rule to find how long it takes to travel 12 km. Check by looking at your graph from part **b**.

t	0	1	2	3
d	0	6		



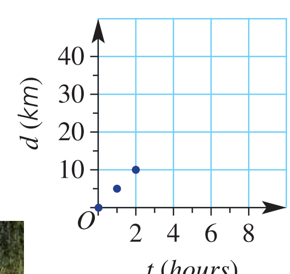
Hint for Q6: Add 5 km for every hour.

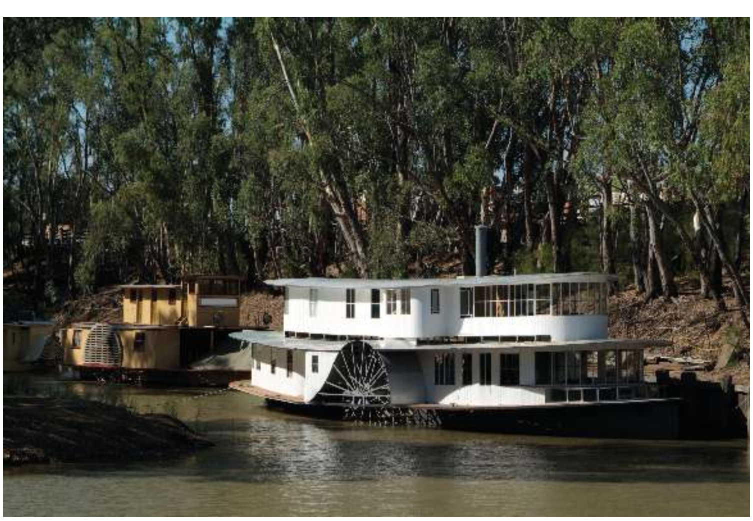
6 A paddle steamer moves up the Murray River at a constant rate of 5 kilometres per hour for 8 hours.

- **a** Complete this table of values using t for time in hours and d for distance in kilometres.

t	0	1	2	3	4	5	6	7	8
d	0	5	10						

- **b** Complete this graph by plotting the points given in the table in part **a**.
- **c** Write a rule linking d with t.
- **d** Use your rule to find the distance travelled after 4.5 hours.
- **e** Use your rule to find how long it takes to travel 20 km. Check by looking at your graph from part **b**.





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Example 18 Applying graphs when the rate is negative

The initial volume of water in a dish in the sun is 300 mL. The water evaporates and the volume decreases by 50 mL per hour for 6 hours.

- **a** Draw a table of values using t for time in hours and V for volume in millilitres.
- **b** Draw a graph by plotting the points given in the table in part **a**.
- **c** Write a rule linking V with t.
- **d** Use your rule to find the volume of water in the dish after 4.2 hours in the sun.
- **e** Use your rule to find the time taken for the volume to reach 75 mL.

Solution

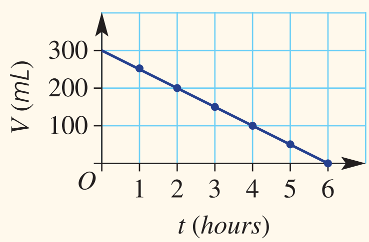
a

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

Explanation

The volume starts at 300 millilitres and decreases by 50 millilitres every hour.

b



Use numbers from 0 to 300 on the V-axis and 0 to 6 on the t-axis to accommodate all the numbers in the table.

c $V = -50t + 300$

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

−50 −50 −50 −50 −50

$V = \Delta t + \square$,
$\Delta = -50$ and $\square = 300$.

d

$$\begin{aligned} V &= -50t + 300 \\ &= -50 \times 4.2 + 300 \\ &= 90 \end{aligned}$$

The volume of water in the dish is 90 millilitres after 4.2 hours.

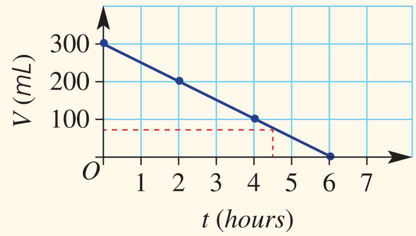
Substitute $t = 4.2$ into your rule to find V.

e

$$\begin{aligned} V &= -50t + 300 \\ 75 &= -50t + 300 \\ -225 &= -50t \\ 4.5 &= t \end{aligned}$$

It takes 4.5 hours for the volume to reach 75 mL. Check by looking at your graph from part **b**.

Substitute $V = 75$ into your rule.
Subtract 300 from both sides.
Divide both sides by −50.
Check by looking at your graph from part **b**.



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Now you try

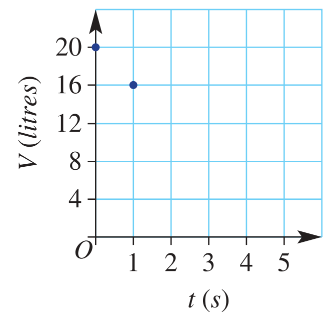
The initial volume of gas in a cylinder is 120 L. It leaks at a rate of 10 litres per hour.

a Draw a table of values using t for time in hours and V for volume in litres.
b Draw a graph by plotting the points given in the table in part a.
c Write a rule linking V with t.
d Use your rule to find the volume of gas after 7 hours.
e Use your rule to find the time taken for the cylinder to be empty (0 litres).

7 The volume of water in a sink is 20 L. The plug is pulled out and the volume decreases by 4 L per second for 5 seconds.

a Complete this table of values using t for time in seconds and V for volume in litres.

t	0	1	2	3	4	5
V	20	16				



b Complete this graph by plotting the points given in the table in part a.
c Write a rule linking V with t.
d Use your rule to find the volume of water in the sink 2.2 seconds after the plug is pulled.
e Use your rule to find how long it takes for the volume to fall to 8 L. Check by looking at your graph.

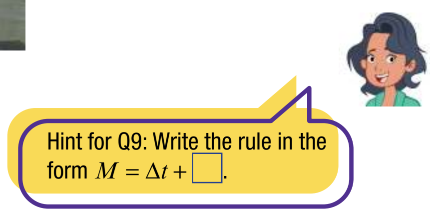
Problem-solving and reasoning

8, 9 | 9, 10

8 A weather balloon at a height of 500 m starts to descend at a rate of 125 m per minute for 4 minutes.

a Draw a table of values using t for time in minutes and h for height in metres.
b Draw a graph by plotting the points given in the table in part a.
c Write a rule linking h with t.
d Use your rule to find the height of the balloon after 1.8 seconds.
e Use your rule to find how long it takes for the balloon to fall to a height of 125 m. Check by looking at your graph from part b.

9 A BBQ gas bottle starts with 3.5 kg of gas. Gas is used at a rate of 0.5 kg per hour for a long lunch.



a Write a rule for the mass of gas (M) in terms of time (t).
b How long will it take for the gas bottle to empty?
c How long will it take for the mass of the gas in the bottle to reduce to 1.25 kg?

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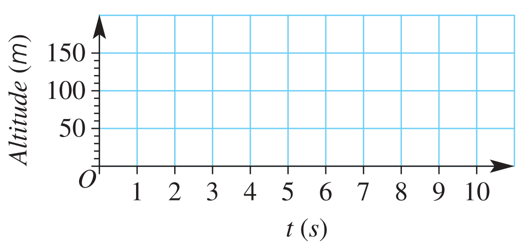
10 A cyclist races 50 km at an average speed of 15 km per hour.
- **a** Write a rule for the distance travelled (d) in terms of time (t).
- **b** How long will it take the cyclist to travel 45 km?
- **c** How long will the cyclist take to complete the 50-km race? Give your answer in hours and minutes.

Danger zone — 11

11 Two small planes take off and land at the same airfield. One plane takes off from the runway and gains altitude at a rate of 15 metres per second. At the same time, the second plane flies near the runway and reduces its altitude from 100 metres at a rate of 10 metres per second.
- **a** Complete this table of values using t between 0 and 10 seconds and h for height in metres of both planes.

$t(s)$	0	1	2	3	4	5	6	7	8	9	10
$h_1(m)$	0	15									
$h_2(m)$	100	90									

- **b** On the same set of axes draw a graph of the height of each plane during the 10-second period.



- **c** How long does it take for the second plane to touch the ground?
- **d** Write a rule for the height of each plane.
- **e** At what time are the planes at the same height?
- **f** At what time is the first plane at a height of 37.5 m?
- **g** At what time is the second plane at a height of 65 m?

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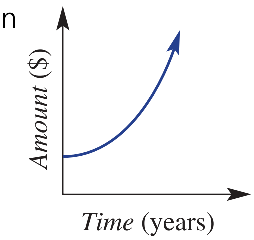
9I Non-linear graphs

Learning intentions

- To understand that a rule relating x and y can result in a graph where the points do not lie on a straight line.
- To be able to plot a non-linear relationship by creating a table of values.

Key vocabulary: non-linear, parabola

Not all relationships between two variables are linear. The amount of money invested in a compound interest account, for example, will not increase at a constant rate. Over time, the account balance will increase more rapidly, meaning that the graph of the relationship between *Amount* and *Time* will be a curve and not a straight line.



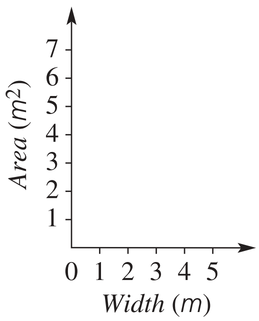
Lesson starter: The fixed perimeter play-pen

Imagine you have 10 metres of fencing material to make a rectangular play-pen.

- List some possible dimensions of your rectangle.
- What is the area of the play-pen for some of your listed dimensions?
- Complete this table showing all the positive integer dimensions of the play-pen.

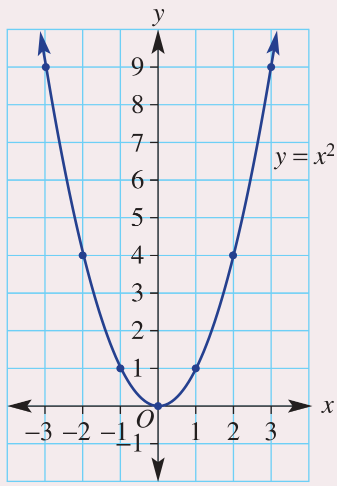
Width (m)	1	2	3	4
Length (m)		3		
Area (m^2)		6		

- Plot the *Area* against *Width* to form a graph.
- Discuss the shape of your graph.
- Discuss the situation and graphical points when the width is 1 m or 4 m.
- What dimensions would deliver a maximum area?
  Explain how your graph helps determine this.



Key ideas

- To plot **non-linear** curves given their rule, follow these steps.
  - Construct a table of values using the rule.
  - Plot the points on a set of axes.
  - Join the plotted points to form a smooth curve.
- The graph of $y = x^2$ is an example of a non-linear graph called a **parabola**.



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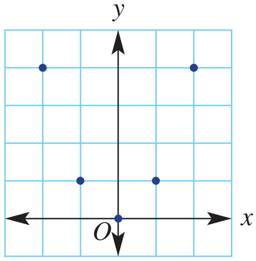
Exercise 9I

Understanding

1–3 1–3

1 Copy these graphs then join the points to form smooth curves.

a



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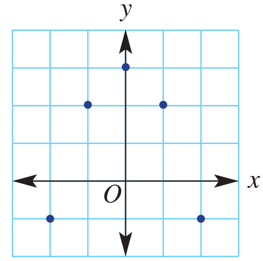

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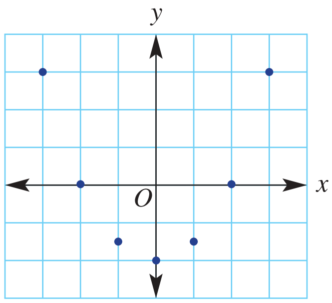
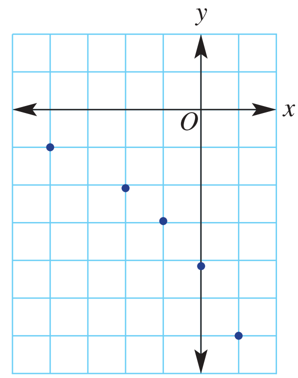
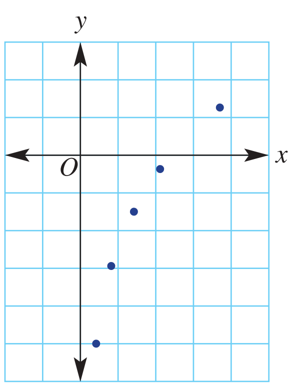


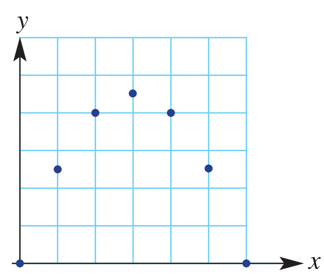
d



e



f



2 If $y = x^2 - 1$, find the value of y for these x values.

a $x = 0$

b $x = 3$

c $x = 2$

d $x = -4$

3 Decide if the following rules would give a non-linear graph.

a $y = x^2$

b $y = x$

c $y = 2x$

d $y = x^2 - 1$

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9I

Fluency

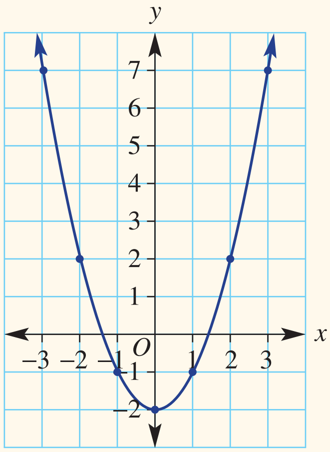
4, 5 | 4, 5

Example 19 Plotting a non-linear relationship

Plot points to draw the graph of $y = x^2 - 2$ using a table.

Solution

x	-3	-2	-1	0	1	2	3
y	7	2	-1	-2	-1	2	7



Explanation

Find the value of y by substituting each value of x into the rule. Plot the points and join with a smooth curve. The curve is called a parabola.

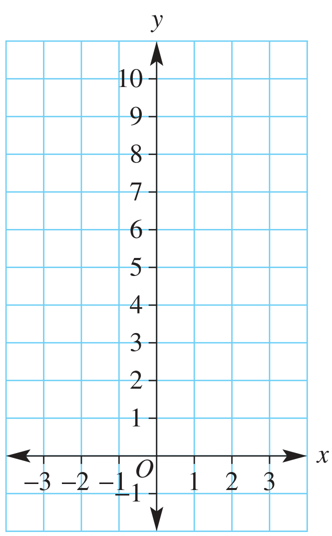
Now you try

Plot points to draw the graph of $y = 4 - x^2$ using a table.

4 Plot points to draw the graph of each of the given rules. Use the table and set of axes as a guide.

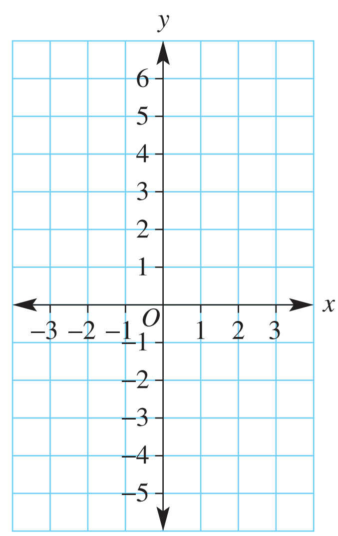
a $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9						



b $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	5						



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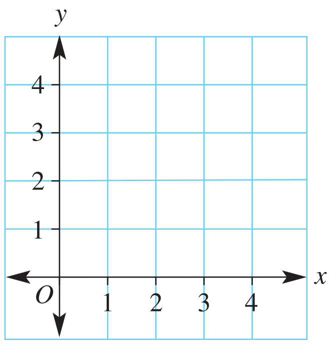
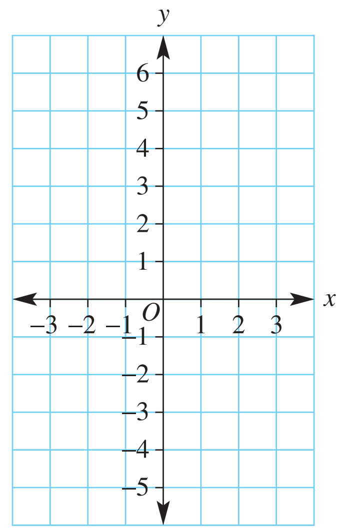
c $y = x(4 - x)$

x	0	1	2	3	4
y					



d $y = 5 - x^2$

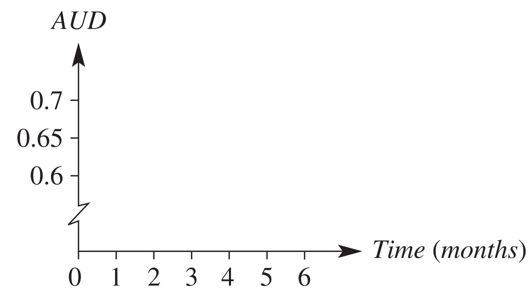
x	−3	−2	−1	0	1	2	3
y		1					



5 The behaviour of the Australian dollar against the British pound over a 6-month period is summarised by the data in this table.

Time (months)	0	1	2	3	4	5	6
AUD	0.69	0.64	0.61	0.6	0.61	0.64	0.69

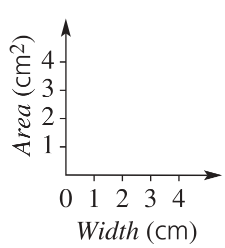
a Plot the data on the given graph and join the points to form a smooth curve.
b Describe the shape of your graph.
c By how much has the Australian dollar:
  i decreased in the first month?
  ii increased in the fifth month?
d Estimate the value of the Australian dollar after 7 months.



Problem-solving and reasoning 6, 7 6–8

6 James has 8 cm of string to form a rectangular space.
a For the given width values, complete this table of values.
b Plot the *Area* against the *Width* to form a graph.
c Describe the shape of your graph.
d What rectangle dimensions appear to provide the maximum area?

Width (cm)	0	1	2	3	4
Length (cm)			2		
Area (cm^2)			4		



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9I

7 Explain why the graph of the rule $y = x^2$ is curved and not straight.

8 By choosing and plotting a selection of points, decide if the following rules would deliver linear or non-linear curves.

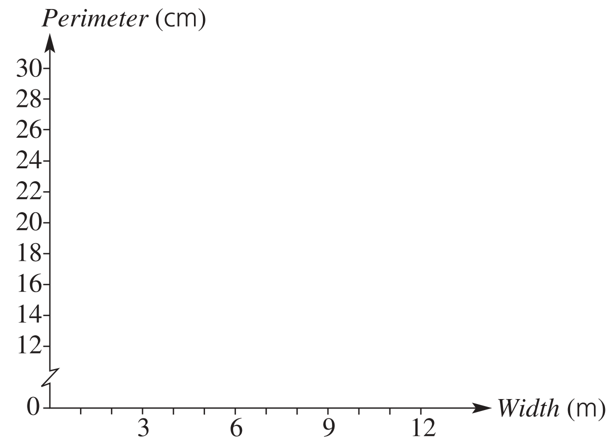
a $y = 5x$ b $y = 1 - x$ c $y = x^2 + 2$

d $y = \frac{1}{x}$ e $y = \frac{2}{x}$ f $y = x \times (x + 1)$

Lara's toy paint — 9

9 Lara has enough paint to cover $12\ \text{cm}^2$ of space. She intends to paint a rectangular area.

Width (cm)	1	2	3		6	
Length (cm)				3		
Perimeter (cm)						26



a For the given values, complete the table above.

b Plot the *Perimeter* against *Width* to form a graph.

c Would you describe the curve to be linear or non-linear?

d Look at the point where there is a minimum perimeter.

i Estimate the width of the rectangle at this point.

ii Estimate the perimeter at this point.

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Maths@Work

Economists and household expenditure

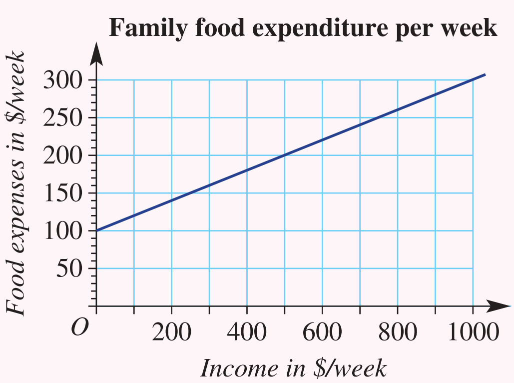
Economists study how the wealth of a nation is created and shared between governments, businesses, people and households. Economists usually have a university degree and have studied mathematics.

Economists use many equations and formulae in tables, spreadsheets and graphs. They study relationships such as supply and demand, profit and loss, and household income and expenditure. Economists work in teams to write reports interpreting data and advising of future trends for governments and the boards of large corporations.

1 Positive gradients represent growth and negative gradients show a decline over time. For each of the following, state whether it has a positive or negative gradient.

- **a** The petrol price went from $\$1.34$/litre to $\$1.37$/litre in a week.
- **b** An electricity bill was $\$789$ last quarter and is $\$684.35$ this quarter.
- **c** The Australian milk price per litre has been steadily decreasing over the last decade.
- **d** Each year more tourists visit the Great Barrier Reef than in the previous year.
- **e** Over the last year, Australia's unemployment rate has increased.

2 This straight line graph shows the relationship between how much money ($\$y$/week) a family spends on food, and family income ($\$x$/week).



Hint for Q2f: Rule $y = \Delta x + \square$
Δ = the gradient
$\square = y$ value when $x = 0$

Answer the questions below about this graph.

- **a** Food is essential and can be bought using savings or a credit card. How much is spent weekly on food even if there is zero weekly income?
- **b** How much is spent weekly on food by a family with a $\$1000$/week income?
- **c** State the change in food expenditure for each $\$100$ increase in family income?
- **d** Is the gradient of this line positive or negative?
- **e** What is the gradient of the line?
- **f** Write a rule for this straight line.

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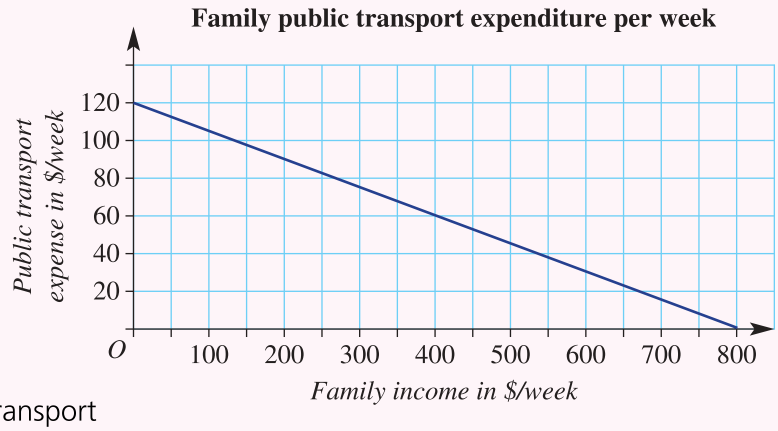
3 This straight line graph shows a simple relationship between a family's public transport expenses ($\$y$/week) and a family's income ($\$x$/week).
Answer the questions below about this graph.



- **a** How much is spent on public transport for families with:
  - **i** zero weekly income?
  - **ii** \$400 weekly income?
- **b** State the change in public transport expenditure in \$ per \$100 increase in family income.
- **c** Is the gradient of this graph positive or negative?
- **d** Find the gradient of this line.
- **e** Write the rule for this straight line.
- **f** At what family income is the public transport expense zero?
- **g** Do you think this graph is realistic? Can you think of why families with higher incomes might use less public transport, or more?

4 Economists use the terms below for types of 'goods' (goods are objects or products).
- Normal goods: an *increasing* income (x) results in *increasing* expenditure (y), e.g. food.
- Inferior goods: an *increasing* income (x) results in *decreasing* expenditure (y), e.g. cheap mass-produced clothing.

Which of these equations represent Normal goods, and which represent Inferior goods?
- **A** $y = 150 + 0.2x$
- **B** $y = 200 - 0.5x$
- **C** $y = 250 + 0.05x$
- **D** $y = -0.02x + 60$

Using technology

Hint for Q5: Format cells: Currency/0 d.p. $y = 0.25x - 120$. In cell B3, this rule is $= 0.25 \times$ B2 $- 120$.

5 The rule $y = 0.25x - 120$ calculates family restaurant spending ($\$y$/week) from family income ($\$x$/week).
- **a** Set up the following Excel spreadsheet and enter the given data.

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Restaurant expenditure vs Income**											
2	**Income \$/week**	\$0	\$200	\$400	\$600	\$800	\$1,000	\$1,200	\$1,400	\$1,600	\$1,800	\$2,000
3	**Restaurant expenditure \$/week**											

- **b** Enter formulas to calculate restaurant expenditure from income.
- **c** Follow these instructions for inserting a graph:
  - **i** Select rows 2 and 3 of the table.
  - **ii** Choose Insert/Scatter graph/icon for 'Scatter with Straight Lines'.
  - **iii** Right-click on each graph axes and select 'Add minor gridlines'.
  - **iv** Choose Design and select one of the 'Quick layout' formats and add a title and label both axes. (A key is not required and can be deleted.)
- **d** Economists call Restaurant spending a 'luxury good' as only families with higher weekly incomes consume restaurant meals. Above what weekly income does a family start spending money on restaurant meals?
- **e** What is the weekly income of a family that spends \$200/week on restaurant meals?

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Fidget spinners

Manoj thinks that there is money to be made by importing and selling fidget spinners to children and their parents. He considers 3 models of spinners with prices shown in this table.

Model	Cost price ($)	Selling price ($)
Simple	6.50	10.00
Super	8.00	12.00
Luxury	11.00	16.00

His other fixed costs for running the business each month total to $200.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

1 Preliminary task

a If Manoj only buys and sells the Simple fidget spinners, find the total cost and the total revenue from buying and selling the following numbers of spinners over one month. Include the fixed costs of $200.

i 20 **ii** 50 **iii** 80

b Explain why the cost ($\$C$) of Manoj buying n Simple spinners in one month is given by the rule $C = 6.5n + 200$.

c Explain why the revenue ($\$R$) for Manoj from selling n Simple spinners in one month is given by the rule $R = 10n$.

d For the Simple spinners and using n ranging from 0 to 80:

i calculate the cost and revenue for three values of n

ii sketch straight-line graphs of the cost and revenue on the same set of axes.

e Use your graph to estimate the number of Simple spinners that need to be bought and sold so that the cost of buying n spinners in one month equals the revenue from selling n spinners.

Modelling

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2 Modelling task

Formulate

a The problem is to determine the number of spinners that Manoj should purchase and sell so that a profit is made. Within a month he only buys and sells one type of spinner (e.g. he just buys and sells Super fidget spinners).

Write down all the relevant information that will help solve this problem.

b Construct rules for the monthly cost ($\$C$) and revenue ($\$R$) in terms of n for the Super fidget spinners, including the fixed $\$200$ cost he pays each month.

c Construct rules for the monthly cost ($\$C$) and revenue ($\$R$) in terms of n for the Luxury fidget spinners, including the fixed $\$200$ cost he pays each month.

Solve

d For the Super fidget spinners and using n ranging from 0 to 80:

- **i** calculate the cost and revenue for three values of n
- **ii** sketch straight-line graphs of the cost and revenue on the same set of axes
- **iii** use the graph to estimate the number of Super fidget spinners that need to be bought and sold in one month so that Manoj starts to make a profit.

e Repeat the process above, using two graphs on the same set of axes, to estimate the number of Luxury fidget spinners that need to be bought and sold in one month to begin making a profit.

Evaluate and verify

f Decide which type of spinner delivers a profit for the least number of spinners bought and sold in a month. Justify your response.

Communicate

g Summarise your results and describe any key findings.

3 Extension questions

a For each spinner, solve an equation to determine algebraically the number of that type required to make a profit.

b Compare the answers you found graphically with the answers you found algebraically. Describe how similar they were.

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Sausages and drinks

Key technology: Graphing

Marissa has a certain amount of money to spend on sausages and drinks at a community fair for her son's friends. Each drink costs $\$2$ and each sausage costs $\$3$.

We will use the following variables.

- x is the number of drinks purchased.
- y is the number of sausages purchased.

1 Getting started

We will first consider the case where Marissa has $\$48$ to spend.

a Find the total cost of purchasing the following.
  i 7 drinks and 6 sausages
  ii 10 drinks and 8 sausages

b Will Marissa be able to afford 9 drinks and 11 sausages? Give a reason.

c If Marissa aims to spend exactly $\$48$, write an equation using the variables x and y.

d Using your equation in part **c**, find:
  i the value of x if $y = 0$
  ii the value of y if $x = 0$.

e Sketch a graph of your rule in part **c** using x values between 0 and 24 inclusive.

2 Using technology

a Use graphing software like Desmos to draw a graph of your equation found in part **1c** above. Add a restriction to the range of x values so that both x and y are greater than or equal to zero.

b Add the points representing the different purchases in parts **1a** and **b** above. What do you notice about the positions of these points in relation to the line?

Technology and computational thinking

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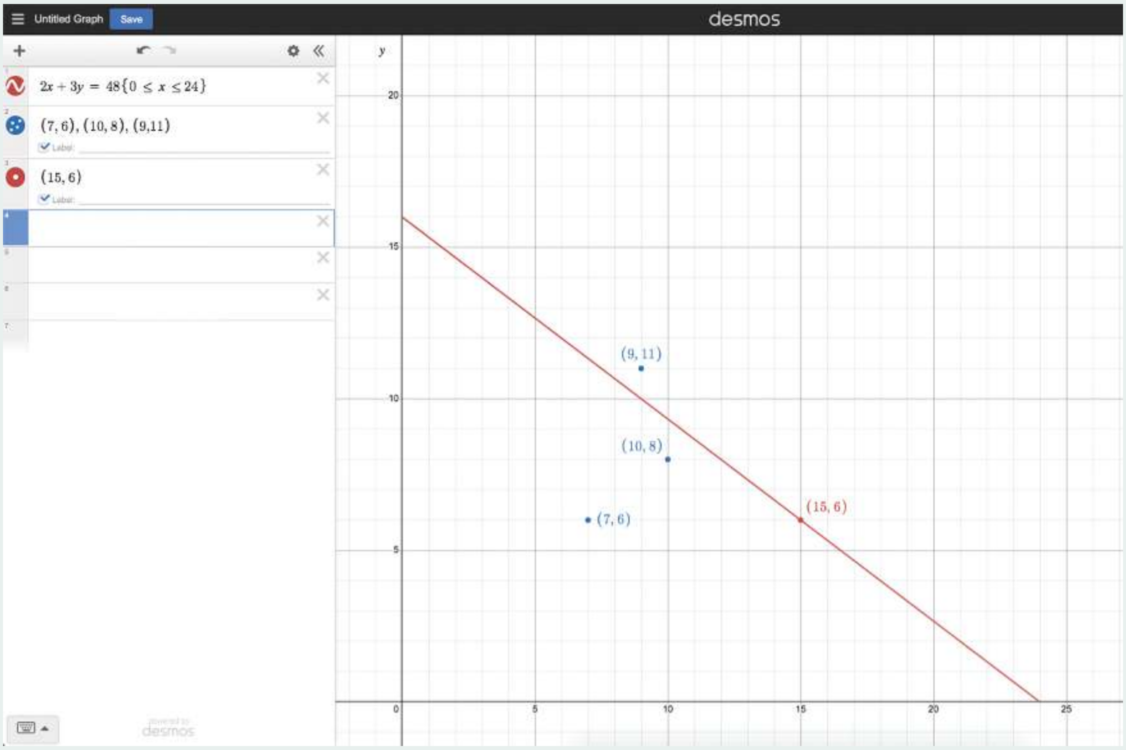
c Plot and label the point $(15, 6)$. What does this point represent in terms of the number of drinks and sausages purchased and the total amount spent?



d Find two other points using integer values of x and y which result in Marissa spending exactly $\$48$.

e Which region on the graph represents the points where Marissa spends less than or equal to $\$48$? Write an inequality that describes this region.

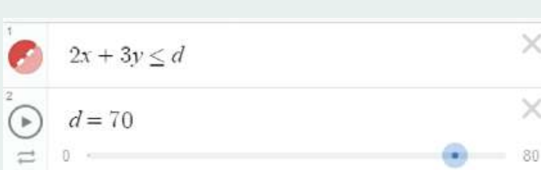
f Which region on the graph represents the points where Marissa spends more than $\$48$? Write an inequality that describes this region.

g If Marissa must buy 10 sausages, what is the maximum number of drinks that she can buy?

3 Applying an algorithm

Marissa realises that to feed all her son's friends she will need to spend more than $\$48$. She also needs to buy 15 sausages and at least 14 drinks.

a Use your graphing package to sketch the region given by $2x + 3y \leqslant d$, where d represents Marissa's maximum spend. Add a slider for d and change the range of possible values of d from 0 to 80 as shown.



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b Investigate the possible number of drinks that Marissa can buy if her maximum spend is between \$50 and \$80 by following this algorithm.

- Step 1: Adjust the d value to 50.
- Step 2: Find a point that describes the situation where Marissa buys 15 sausages and a maximum number of drinks with the remaining change.
- Step 3: Add your result into this table including the actual amount spent for your chosen values of x. The first column is completed for you.

d (Maximum spend)	\$50	\$55	\$60	\$65	\$70	\$75	\$80
Number of drinks x	2						
Number of sausages y	15	15					
\$Actual total spent	\$49						

- Step 4: Increase the d value by 5 and repeat from Step 2 until you reach $d = 80$.

c What was the smallest value of d listed in your table above where Marissa can buy 15 sausages and 14 drinks?

d Now adjust the value of d using your graphing package to find the smallest value of d such that Marissa buys 15 sausages and at least 14 drinks.

Technology and computational thinking

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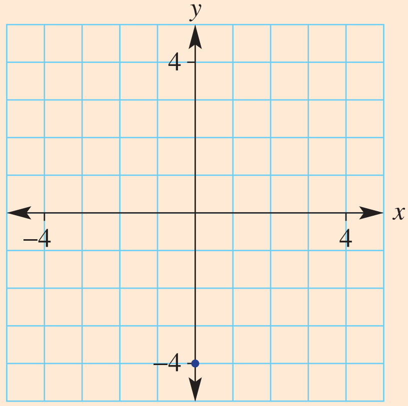
Puzzles and games

1 Plot and join the set of points in order to form the picture. What is the picture of?
(4, 1), (2, 1), (1, −1), (0, 1), (−3, 1), (−3, 3), (−2, 2), (3, 2), (4, 1)



2 Find the rule linking y and x.

a

x	−2	−1	0	1	2
y	−15	−11	−7	−3	1

b

x	1	2	3	4	5
y	10	9	8	7	6

c

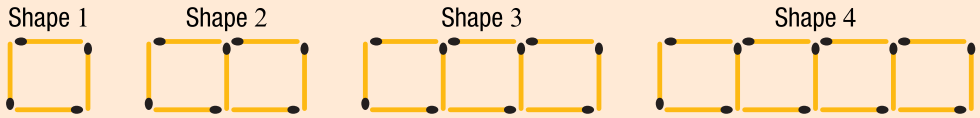
x	−10	−9	−8	−7	−6
y	−100	−95	−90	−85	−80

d

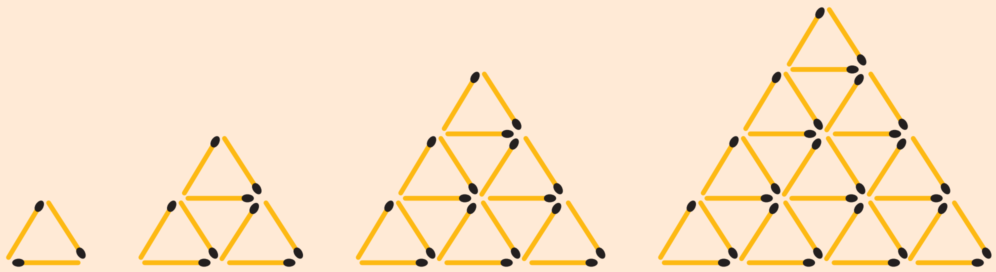
x	0	3	6	9	12
y	−10	−7	−4	−1	2

3 How many matchsticks would be needed for the 10th shape in each pattern?

a



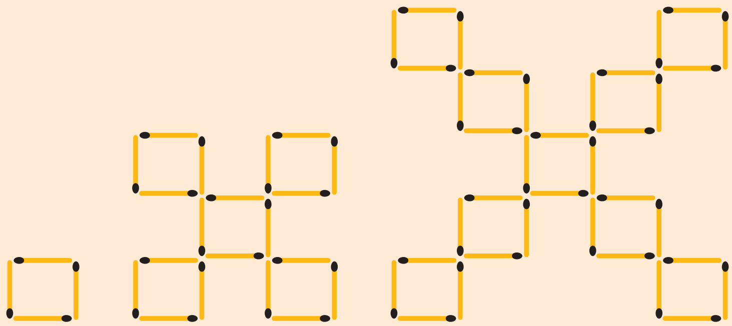
b



4 A trekker hikes along a track at 3 km per hour. Two hours later, a second trekker sets off on the same track at 5 km per hour. How long is it before the second trekker catches up with the first?

5 Two cars travel toward each other on a 100 km stretch of road. One car travels at 80 km per hour and the other at 70 km per hour. If they set off at the same time, how long will it be before the cars meet?

6 Find the number of matchsticks needed in the 100th diagram in the pattern given below. The first three diagrams in the pattern are given.



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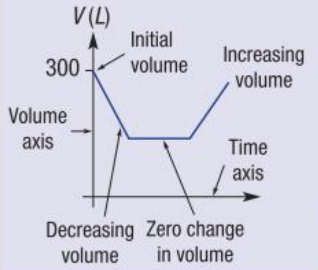
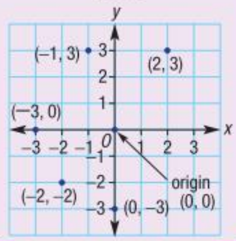
Chapter summary

Linear relationships

Graphs



Number/Cartesian plane



Rules and tables

$y = -2x + 3$

x	–2	–1	0	1	2
y	7	5	3	1	–1

If $x = -2$

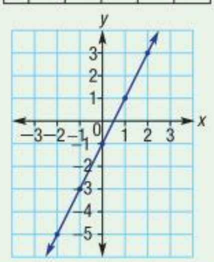
$y = -2 \times (-2) + 3$
$= 4 + 3$
$= 7$

Plotting straight line graphs

Rule $y = 2x - 1$

Table

x	–2	–1	0	1	2
y	–5	–3	–1	1	3



Finding the rule using tables

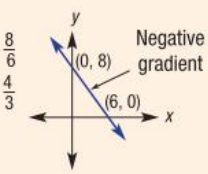
x	–2	–1	0	1	2
y	–8	–5	–2	1	4

+3 +3 +3 +3

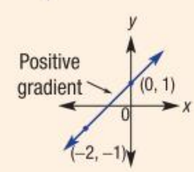
$y = \triangle \times x + \square$
$= 3x + (-2)$
$= 3x - 2$

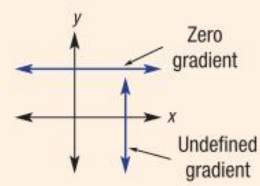
Gradient ⭐

$\frac{\text{rise}}{\text{run}} = -\frac{8}{6}$
$= -\frac{4}{3}$



$\frac{\text{rise}}{\text{run}} = \frac{2}{2}$
$= 1$





Applications ⭐

- Distance increases by 20 km per hour.

t	0	1	2	3
d	0	20	40	60

$d = 20t$

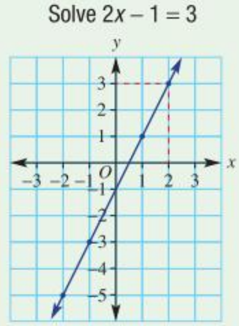
- Volume decreases from 1000 L by 200 L per minute.

t	0	1	2	3	4	5
V	1000	800	600	400	200	0

$V = -200t + 1000$

Using graphs to solve linear equations

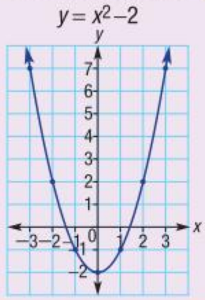
Solve $2x - 1 = 3$



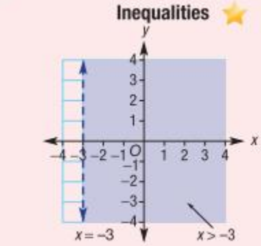
The solution is $x = 2$

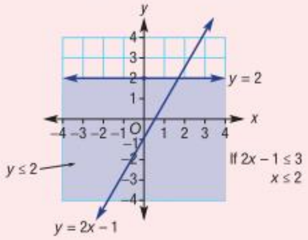
Non-linear graphs ⭐

$y = x^2 - 2$



Inequalities ⭐





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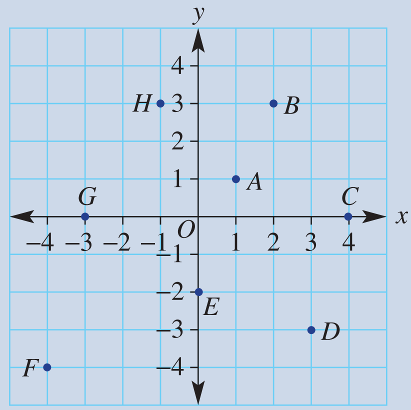
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
9A	**1 I can state the coordinates of points shown on a number plane** e.g. Write down the coordinates of the points A to H on this number plane.	



9A	**2 I can plot points at a given location** e.g. Draw a number plane extending from −4 to 4 on both axes then plot and label the points $A(2, 3)$, $C(1, 2)$ and $F(2, 4)$.	
9B	**3 I can construct and interpret a table of values for a practical situation** e.g. The rule connecting the distance (d km) and time (t hours) is $d = 60t$. Construct a table using values of t from 0 to 4, and use this to state how long it takes to travel 90 km.	
9B	**4 I can create a table of values for a rule including negative numbers** e.g. For the rule $y = 2x - 3$ construct a table using values of x between −2 and 2.	
9B	**5 I can plot a graph from a table of values** e.g. Plot a graph from this table of values.	

x	−2	−1	0	1	2
y	5	3	1	−1	−3

9C	**6 I can plot a graph from a rule** e.g. For the rule $y = 2x - 1$, construct a table and draw a graph (using values of x between −3 and 3).	
9D	**7 I can find the rule from a table of values** e.g. Find the rule for these tables.	

a

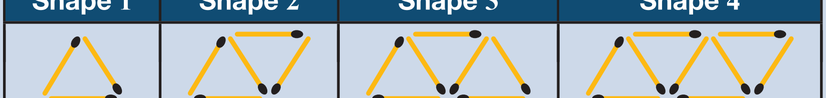
x	−2	−1	0	1	2
y	−8	−5	−2	1	4

b

x	−2	−1	0	1	2
y	5	3	1	−1	−3

9D	**8 I can find the rule for a spatial pattern** e.g. If x = number of triangles and y = number of matchsticks, use a table to find a rule for this pattern.	

Shape 1	Shape 2	Shape 3	Shape 4



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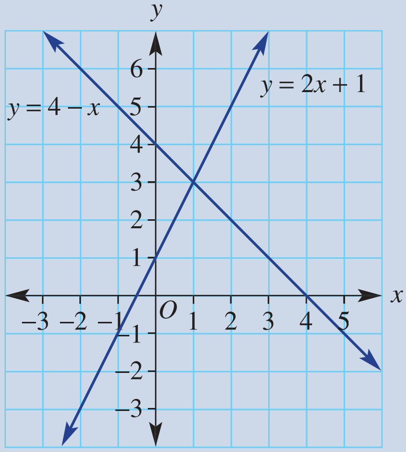
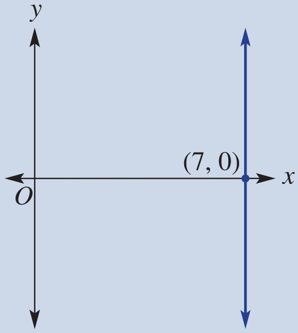
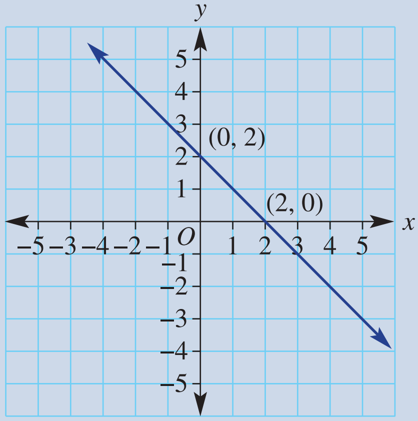
Chapter checklist

		✔
9E	**9 I can use a linear graph to solve an equation** e.g. Use a graph of $y = 2x + 1$ to solve the equation $2x + 1 = 5$.	
9E	**10 I can use the point of intersection of two lines to solve an equation** e.g. Use the graphs of $y = 4 - x$ and $y = 2x + 1$ to solve the equation $4 - x = 2x + 1$. 	
9F	**11 I can find the rule for horizontal and vertical lines** e.g. Find the rule for this vertical line. 	
9F	**12 I can sketch a region using horizontal or vertical lines** e.g. Sketch the region $y < 4$.	
9F	**13 I can use graphs to solve a linear inequality** e.g. Use the given graph to solve $-x + 2 > -1$. 	

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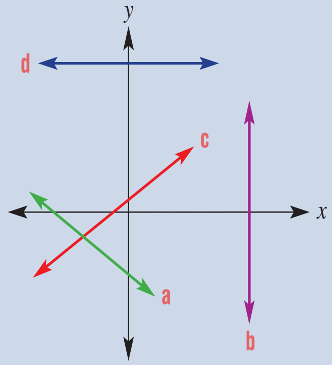
Chapter checklist

	✔

9G

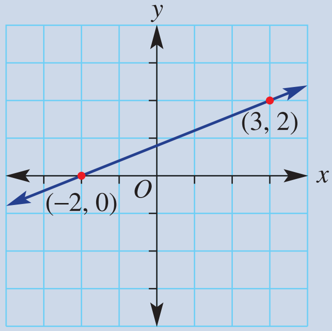
14 I can decide if a gradient is positive, negative, zero or undefined
e.g. Decide for each line whether the gradient is positive, negative, zero or undefined.



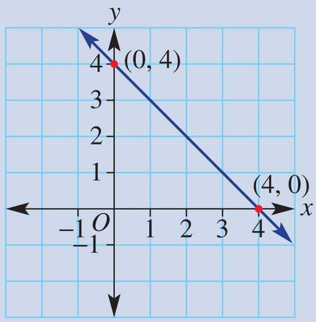
9G

15 I can calculate the gradient of a line
e.g. Find the gradient of these lines.

a



b



9H

16 I can apply graphs where distance and time are related
e.g. A hiker walks at a constant rate of 4 km/h for 4 hours. Draw a graph showing the distance at each time and find the distance travelled in 2.5 hours.

9H

17 I can apply graphs to model real-world situations
e.g. The volume of water in a dish is 300 mL initially and decreases by 50 mL per hour for 6 hours.

a Draw a table of values using t for time in hours and V for volume in millilitres.
b Draw a graph of the relationship between V and t.
c Write a rule linking V and t.
d Find the time taken for the volume to reach 75 mL.

9I

18 I can plot a non-linear relationship
e.g. Plot points to draw the graph of $y = x^2 - 2$ using a table.

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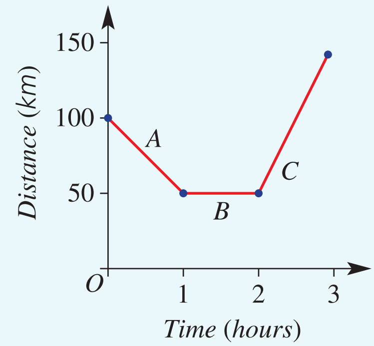
Short-answer questions

9A

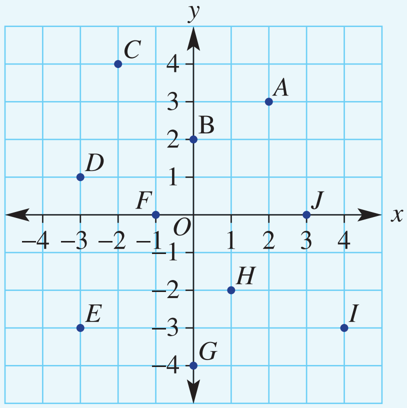
1 The graph shows how far a bird is from its nest.

a How far was the bird from its nest initially (at the start)?
b For how long did the bird rest?
c How far did the bird travel in:
  i section A? **ii** section C?
d During which section did the bird fly the fastest?



9A

2 Write the coordinates of all the points $A - J$ in the graph below.



9B

3 Use the given rules to find the missing values in the tables.

a $y = x - 1$

x	-2	-1	0	1	2
y	-3				

b $y = 2x$

x	-2	-1	0	1	2
y		-2			

c $y = 3x + 1$

x	-2	-1	0	1	2
y		-2			

d $y = -x + 1$

x	-2	-1	0	1	2
y		2			

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9B **4** For each rule, create a table using x values from -3 to 3 and plot to draw a straight line graph.

a $y = 2x$ **b** $y = 3x - 1$ **c** $y = 2x + 2$

d $y = -x + 1$ **e** $y = -2x + 3$ **f** $y = 3 - x$

x	-3	-2	-1	0	1	2	3
y							

9D **5** Write the rule for these tables of values.

a

x	-2	-1	0	1	2
y	-3	-1	1	3	5

b

x	-2	-1	0	1	2
y	-4	-1	2	5	8

c

x	3	4	5	6	7
y	6	7	8	9	10

d

x	-3	-2	-1	0	1
y	4	3	2	1	0

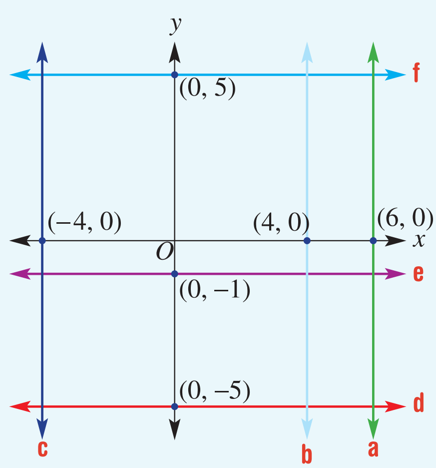
e

x	-1	0	1	2	3
y	3	-1	-5	-9	-13

f

x	0	1	2	3	4
y	8	7	6	5	4

9F **6** Write the rule for each of these horizontal and vertical lines.

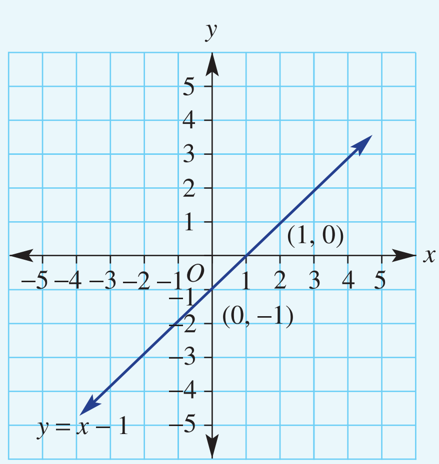


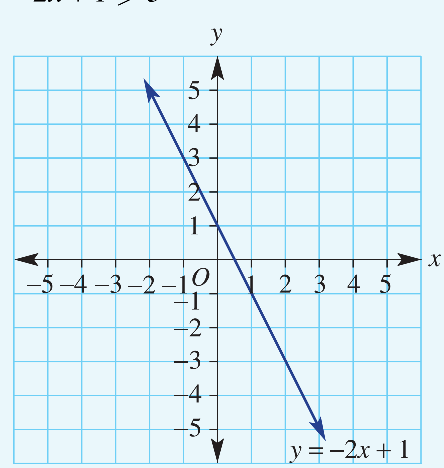
9F **7** Sketch the following regions.

a $x < 2$ **b** $y \geqslant -3$

9F **8** Use the given graphs to solve the following inequalities.

a $x - 1 < 1$ **b** $-2x + 1 \geqslant 3$





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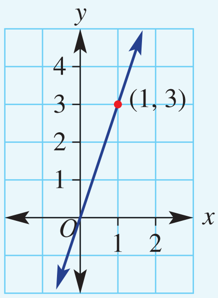
9G ★ **9** Find the gradient of each of these lines.

a



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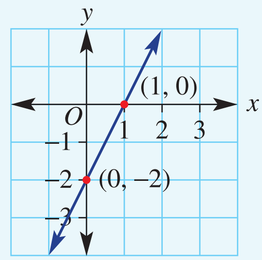
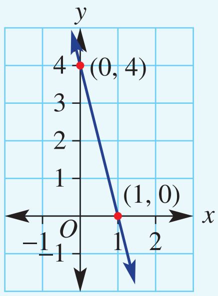
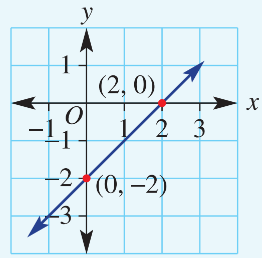


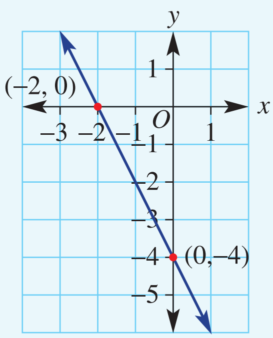
c



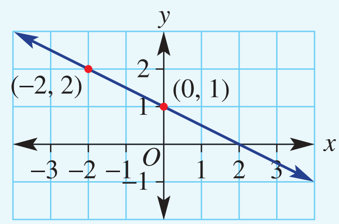
d



e



f



9E/H ★ **10** A bicycle hire company has the following cost structure: \$40 upfront plus \$20 per hour after that.

a Copy and complete this graph showing the relationship between total cost (\$$C$) and time ($t$ hours) for 4 hours.

b Use your graph to find the cost of hiring a bike for 2.5 hours.

c Use your graph to find how long you can hire a bike using \$110.

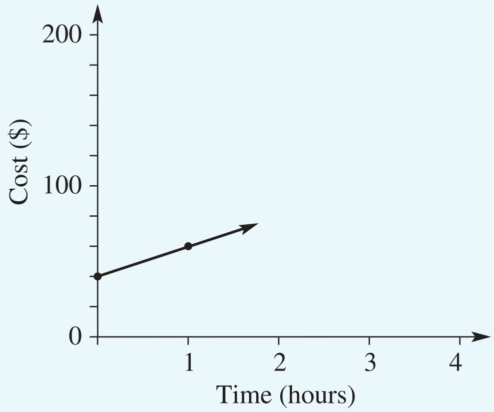
d Write a rule connecting C and t.

e Use your rule to confirm your answers to parts **b** and **c** above.

f Use your graph to solve:

i $80 = 20t + 40$

ii $70 = 20t + 40$



Another company offers hire at \$30 per hour and no upfront fee.

g Sketch a graph on the same set of axes from part **a** to illustrate this company's cost structure.

h Use your graph to determine the number of hours of hire for which the cost for both companies is equal.

9I **11** Plot a graph of the rule $y = 2 - x^2$ for x values from -2 to 2.

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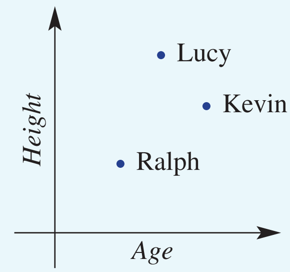
Multiple-choice questions

9A **1** This graph shows the relationship between the height and age of three people. Who is the tallest person?



A Ralph
B Lucy
C Kevin
D Lucy and Ralph together
E Kevin and Lucy together

9A **2** The name of the point $(0, 0)$ on a number (Cartesian) plane is the:
A y-intercept **B** gradient **C** origin
D axis **E** x-intercept

9A **3** Which point is not in line with the other points?
$A(-2, 3), B(-1, 2), C(0, 0), D(1, 0), E(2, -1)$
A A **B** B **C** C **D** D **E** E

9B **4** If $d = 10t$, then the value of d when $t = 6$ is:
A 600 **B** 6 **C** 10 **D** 60 **E** 100

9B **5** The rule for this table of values is:

x	-2	-1	0	1	2
y	-1	0	1	2	3

A $y = x$ **B** $y = x + 1$ **C** $y = -x$ **D** $y = -x - 1$ **E** $y = 1$

9G **6** The gradient of a line joining the two points $(0, 0)$ and $(1, -6)$ is:
A 3 **B** 6 **C** 1 **D** -6 **E** -3

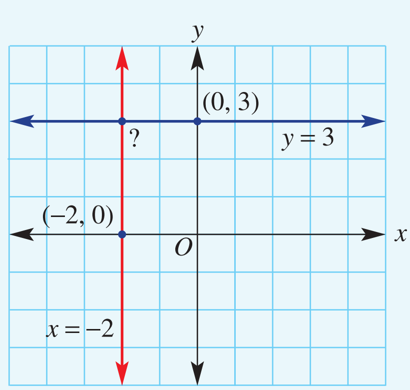
9D **7** The rule for this table of values is:

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5

A $y = -x - 3$ **B** $y = x - 3$ **C** $y = -x + 3$ **D** $y = x + 3$ **E** $y = -(x - 1)$

9G **8** A vertical line has what type of gradient?
A Fraction **B** Positive **C** Negative **D** Zero **E** Undefined

9E **9** Use the graph and find the coordinates of the point of intersection of the graphs of $y = 3$ and $x = -2$.
A $(0, 3)$ **B** $(2, -3)$ **C** $(-2, -3)$
D $(-2, 3)$ **E** $(2, 3)$



9H **10** The water level in a dam starts at 300 cm deep and decreases by 5 cm every day for 10 days. The water level after 7 days would be:
A 35 cm **B** 275 cm **C** 230 cm **D** 160 cm **E** 265 cm

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Extended-response questions

1 A seed sprouts and the plant grows 3 millimetres per day in height for 6 days.

a Complete this table of values using t for time in days and h for height in millimetres.

t	0	1	2	3	4	5	6
h	0						

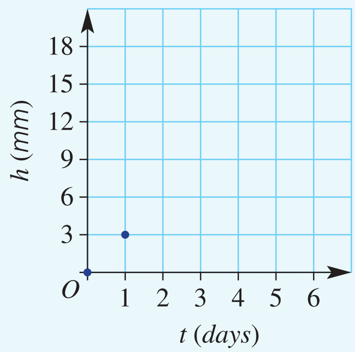
b Complete this graph using the points from your table.

c Find a rule linking h with t.

d Use your rule to find the height of the plant after 3.5 days.

e If the linear pattern continued, what would be the height of the plant after 10 days?

f How long will it be before the plant grows to 15 mm in height?



2 A speed boat at sea is initially 12 km from a distant rock. The boat travels towards the rock at a rate of 2 km per minute. The distance between the boat and the rock will therefore decrease over time.

a Complete this table showing t for time in minutes and d for distance to the rock in kilometres.

t	0	1	2	3	4	5	6
d	12	10					

b Draw a graph using the points from your table. Use t on the horizontal axis.

c How long does it take the speed boat to reach the rock?

d What is the gradient of the line drawn in part **b**?

e Find a rule linking d with t.

f Use your rule to find the distance from the rock at the 2.5 minute mark.

g How long does it take for the distance to reduce to 3.5 km?

Chapter review

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10 Transformations and congruence

Essential mathematics: why skills with transformation and congruence are important

Geometric transformation and congruence principles are applied in graphic design, digital gaming and robotics, fashion design, architecture, engineering, surveying and urban planning.

The magnificent Baha'i temple, New Delhi, India, has architectural features imitating a lotus flower. Geometric rotational symmetry can be seen in the layout of its 27 lotus-like 'petals' arranged in three circles of 9 'petals' each, radiating symmetrically around its central axis.

Animators and digital game creators write algorithms applying geometrical translation, reflection and rotation to move characters around obstacles and build changing digital scenery.

Engineers create truss supports for bridges using a series of congruent triangles. These triangles provide stability and strength, symmetrically distributing the bridge's weight.

Tessellations form intriguing mathematical art seen in designs on tiles and flooring, fabric textile patterns for clothing and furnishings, and some exterior faces of modern architecture.

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In this chapter

Australian Curriculum 9.0

SPACE

Identify the conditions for congruence and similarity of triangles and explain the conditions for other sets of common shapes to be congruent or similar, including those formed by transformations (AC9M8SP01)

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems explaining reasoning (AC9M8SP02)

Design, create and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works (AC9M8SP04)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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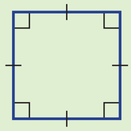
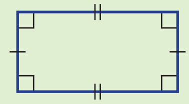
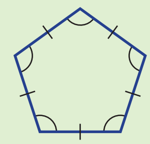
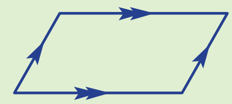
Warm-up quiz

1 How many lines of symmetry are there in these shapes?

a 
square

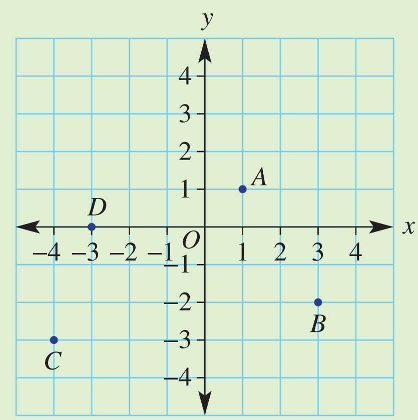
b 
rectangle

c 
regular pentagon

d 
parallelogram

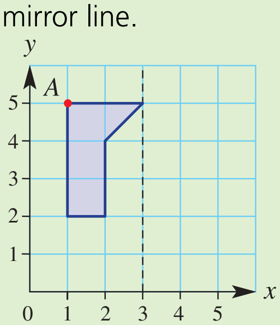
2 What is the order of rotational symmetry for the shapes in Question **1**?

3 This number plane shows four points A, B, C, D.
- **a** State the coordinates of the points A, B, C, D.
- **b** What would be the coordinates of point A if:
  - **i** it were shifted left by 1 unit?
  - **ii** it were shifted right by 2 units and 1 unit down?
  - **iii** it were shifted left by 5 units and 3 units down?
- **c** What would be the coordinates of point C if it were reflected in the x-axis?

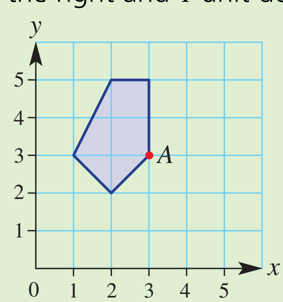


4 Complete the simple transformations of the given shapes as instructed then state the coordinates of the image of point A after the given transformation.

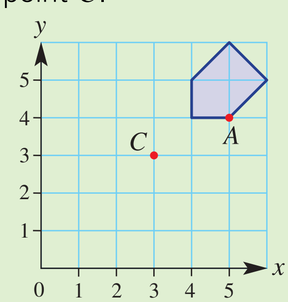
a Reflect this shape over the mirror line.



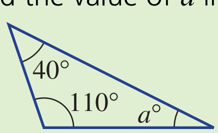
b Shift this shape 2 units to the right and 1 unit down.

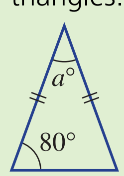


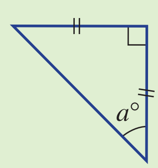
c Rotate this shape 180° around point C.

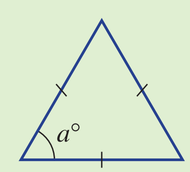


5 Find the value of a in these triangles.

a 

b 

c 

d 

6 Which of the special quadrilaterals (**A**–**F**) fit the descriptions (**a**–**d**)?

A Square
B Rectangle
C Rhombus
D Parallelogram
E Kite
F Trapezium

- **a** Opposite sides are of equal length.
- **b** It has at least one pair of equal opposite angles.
- **c** Diagonals are of equal length.
- **d** Diagonals intersect at right angles.

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10A Reflection

Learning intentions

- To understand that an object can be reflected over a line.
- To be able to draw the image of a point or shape that is reflected in a mirror line.
- To understand that lines of symmetry are the mirror lines that reflect a shape directly onto itself.

Key vocabulary: transformation, reflection, image, mirror line, line of symmetry

When an object is shifted from one position to another, rotated about a point, reflected over a line or enlarged by a scale factor, we say the object has been transformed. The names of these types of transformations are reflection, translation, rotation and enlargement.

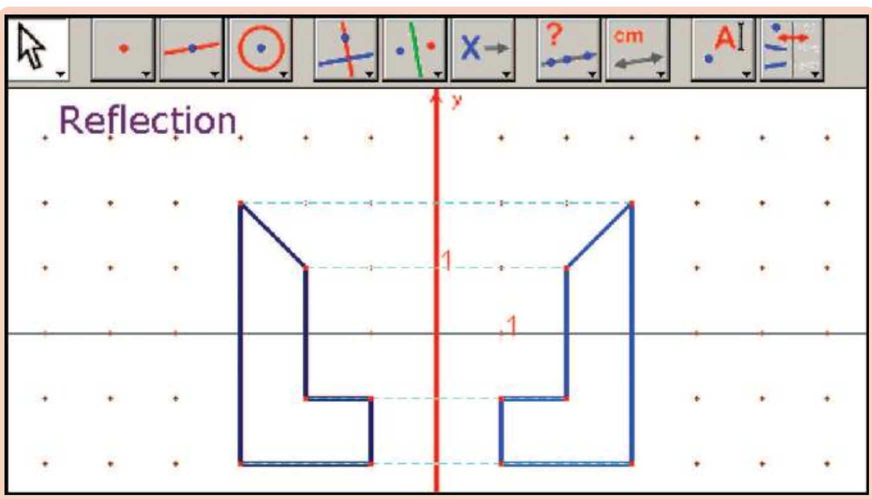
The first three of these transformations studied in this chapter are called isometric transformations because the object's geometric properties are unchanged and the transformed object will be congruent to the original object. This means that the size and shape of the object are not altered. The word 'isometric' comes from the Greek words *isos* meaning 'equal' and *metron* meaning 'measure'.

Reflection creates an image reversed as in a mirror or in water.

Lesson starter: Visualising the image

This activity could be done by hand on a page, in a group using a whiteboard or using dynamic geometry projected onto a whiteboard.

- Draw any shape with straight sides.
- Draw a vertical or horizontal mirror line outside the shape.
- Try to draw the reflected image of the shape in the mirror line.
- If dynamic geometry is used, reveal the precise image (the answer) using the Reflection tool to check your result.
- For a further challenge, redraw or drag the mirror line so it is not horizontal or vertical. Then try to draw the image.



Dynamic geometry software provides a reflection tool.

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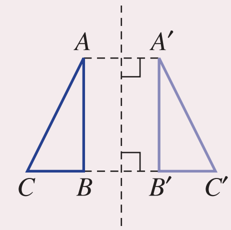
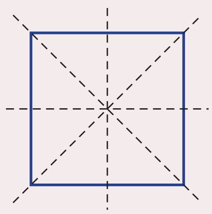
10A

Key ideas

- A **transformation** is a process which can change the size and/or position of an object. The four geometric transformations include: translation, reflection, enlargement and rotation.
- **Reflection** is a transformation in which the size and shape of the object is unchanged.
  - Reflection is the result of flipping a geometrical figure across a line.
- The **image** is the result of a transformation.
  - The image of a point A is denoted A'.
- A **mirror line** is a line over which a figure is reflected.
  - Each point is reflected at right angles to the mirror line.
  - The distance from a point A to the mirror line is equal to the distance from the image point A' to the mirror line.



- **Lines of symmetry** are mirror lines that result in an image being reflected onto itself.
  - A square has four lines of symmetry.



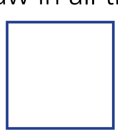
Exercise 10A

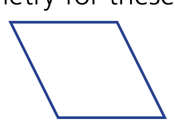
Understanding

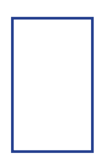
1–3 | 3

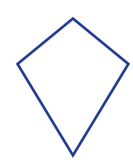
1 Give the missing words or symbols.
- **a** Reflection is one of the four geometric __________.
- **b** A _______ _______ is a line over which a figure is reflected.
- **c** The image of the point A is denoted _____.
- **d** Lines of _________ are mirror lines that result in an image being reflected onto itself.

2 Draw in all the lines of symmetry for these shapes.

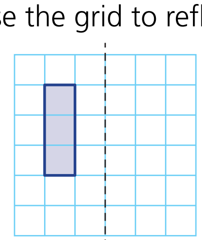
a

square

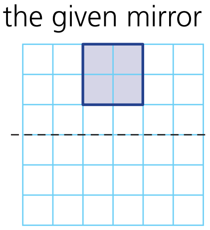
b

rhombus

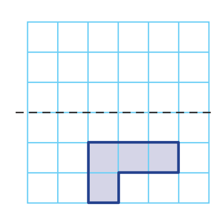
c

rectangle

d

kite

3 Use the grid to reflect each shape in the given mirror line.

a


b


c


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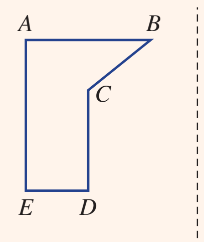
Fluency

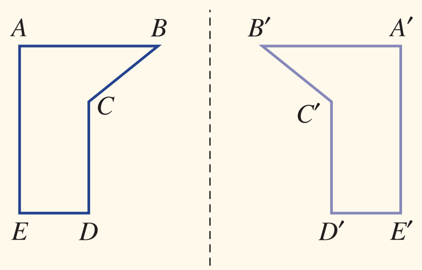
4, 5–7(½), 8 | 5–7(½), 8, 9

Example 1 Drawing simple reflected images

Copy the diagram and draw the reflected image over the given mirror line.



Solution

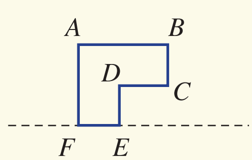


Explanation

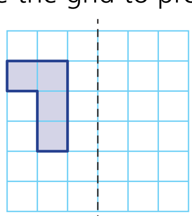
Reflect each vertex point at right angles to the mirror line. Join the image points to form the final image.
Use A' as the image point of A.

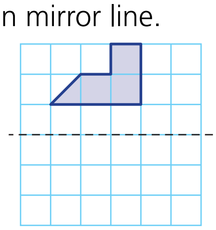
Now you try

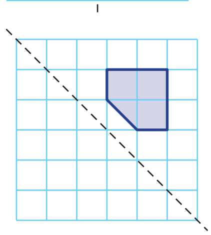
Copy the diagram and draw the reflected image over the given mirror line.

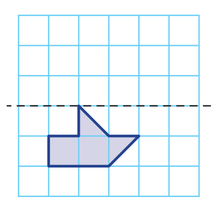


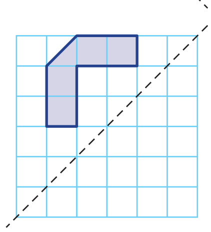
4 Use the grid to precisely reflect each shape in the given mirror line.

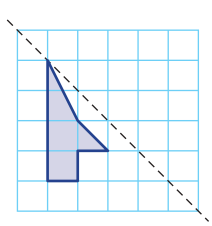
a


b


c


d


e


f


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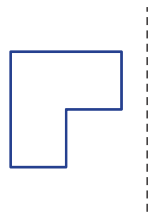
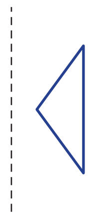
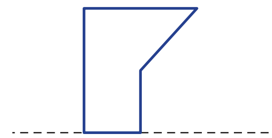
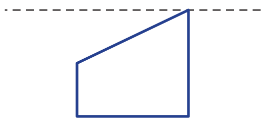
10A

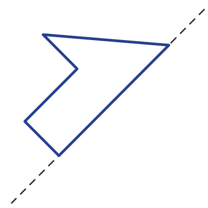
5 Copy the diagram and draw the reflected image over the given mirror line.

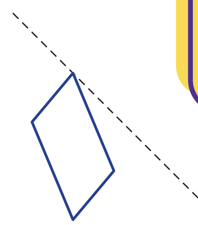
a 

b 

c 

d 

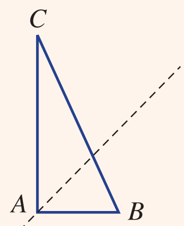
e 

f 

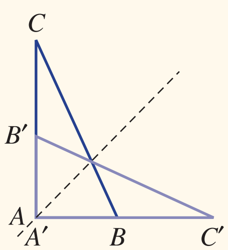
Hint for Q5: Start by reflecting each vertex point at 90° across the mirror line. Then join these points to form the shape.

Example 2 Drawing more complex reflected images

Copy and reflect over the mirror line.



Solution

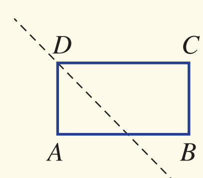


Explanation

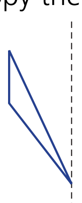
Reflect points A, B and C at right angles to the mirror line to form A', B' and C'. Note that A' is in the same position as A as it is on the mirror line. Join the image points to form the image triangle.

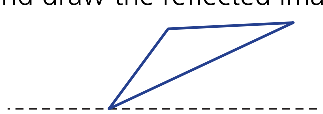
Now you try

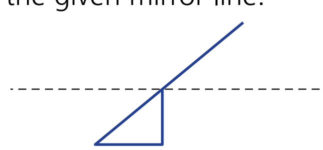
Copy and reflect over the mirror line.

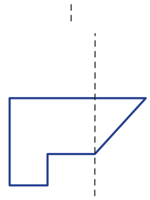


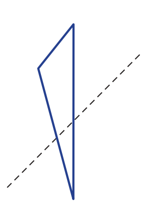
6 Copy the diagram and draw the reflected image over the given mirror line.

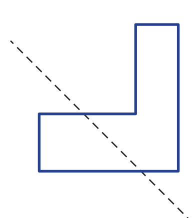
a 

b 

c 

d 

e 

f 

Hint for Q6: Reflect the vertex points first. Then join the points to finish.

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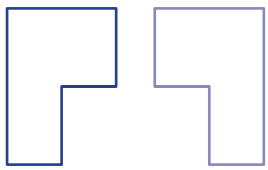
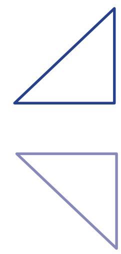
7 Copy the diagram and accurately locate and draw the mirror line.

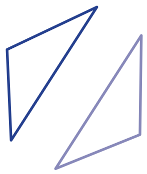
a



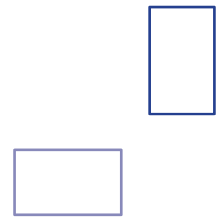
b



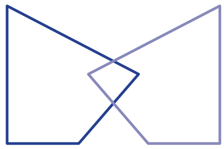
c



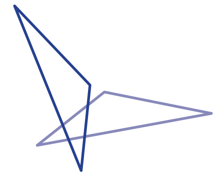
d



e



f

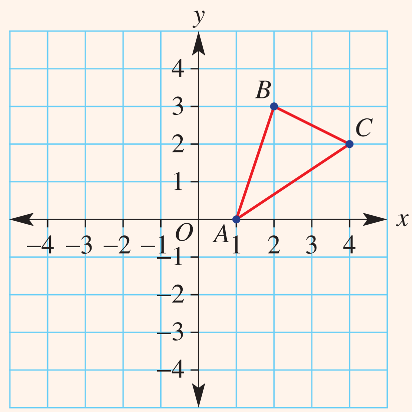


Example 3 Using coordinates in reflection

State the coordinates of the vertices A', B' and C' after this triangle is reflected in the given axes.

a x-axis

b y-axis



Solution

a $A' = (1, 0)$

$B' = (2, -3)$

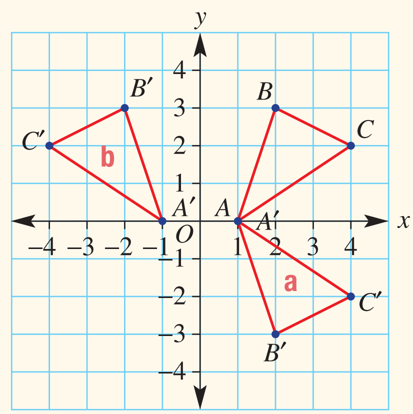
$C' = (4, -2)$

b $A' = (-1, 0)$

$B' = (-2, 3)$

$C' = (-4, 2)$

Explanation



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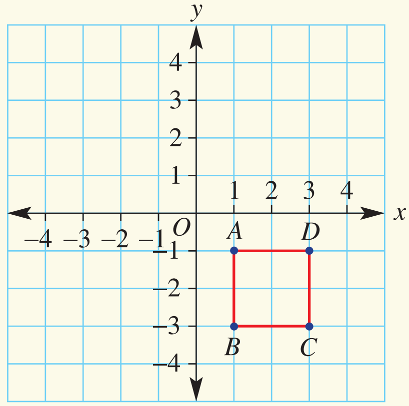
Now you try

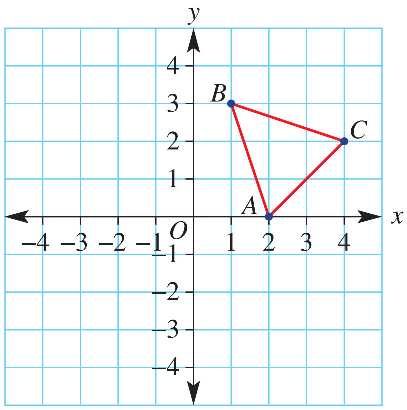
State the coordinates of the vertices A', B', C' and D' after this square is reflected in the given axes.

a x-axis
b y-axis



8 State the coordinates of the vertices x and y after the triangle (below) is reflected in the given axes.

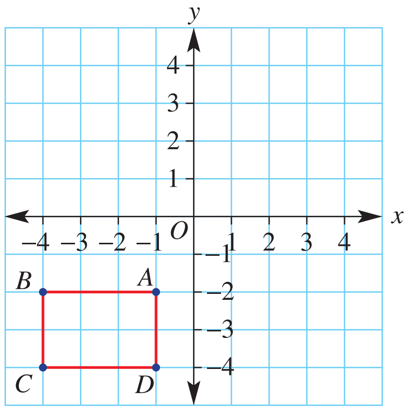
a x-axis
b y-axis



Hint for Q8: Pencil in the reflection then look at the position of the image points. The x-axis is the horizontal axis and the y-axis is the vertical axis.

9 State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

a x-axis
b y-axis



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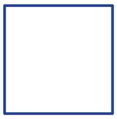
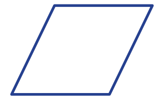
Problem-solving and reasoning 10–12 11–14

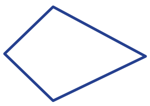
Hint for Q10: Be careful: not all diagonals are lines of symmetry.

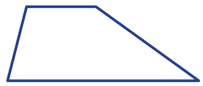
10 How many lines of symmetry do these shapes have?

a 
Square

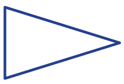
b 
Rectangle

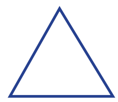
c 
Rhombus

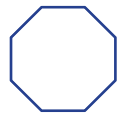
d 
Kite

e 
Trapezium

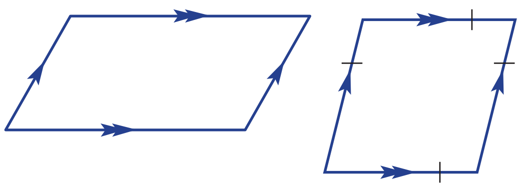
f 
Parallelogram

g 
Isosceles triangle

h 
Equilateral triangle

i 
Regular octagon

11 Explain why a parallelogram in general has no lines of symmetry but a rhombus has two lines of symmetry.



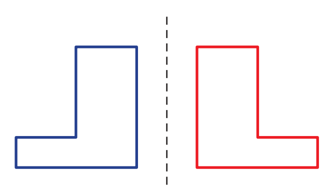
12 A shape with area $10\ \text{m}^2$ is reflected in a line. What is the area of the image shape? Give a reason for your answer.

13 How many lines of symmetry does a regular polygon with n sides have? Write an expression.

14 A point is reflected in the x-axis then in the y-axis and finally in the x-axis again.
What single reflection could replace all three reflections?

Computer reflection — 15

15 Use computer geometry to construct a shape and a mirror line.



- **a** Reflect your shape in the mirror line.
- **b** Drag the mirror line. What do you notice?
- **c** Drag your original shape. What do you notice?
- **d** Drag the mirror line across the middle of your original shape. What do you notice?

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10B Translation

Learning intentions

- To understand that an object can be translated up, down, left or right using a vector.
- To be able to determine the vector that moves a given point to its image.
- To be able to draw the image of an object after it has been translated.

Key vocabulary: transformation, translation, image, vector

Translation is a shift of every point on an object in a given direction and by the same distance. The direction and distance is best described by the use of a translation vector. This vector describes the overall direction using a horizontal component (for shifts left and right) and a vertical component (for shifts up and down). Negative numbers are used for translations to the left and down.

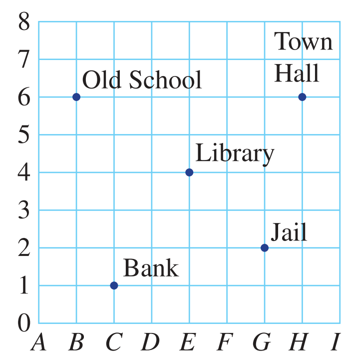
Designers of animated movies translate images in many of their scenes. Computer software is used and translation vectors help to define the specific movement of the objects and characters on the screen.

Animated characters move through a series of translations.

Lesson starter: City walking tour

The position of places in a city square is given by this grid. For example, the Town Hall is H6.

On a walking tour, vectors are used to describe the walk between two places. For example, the vector $(-3, -2)$ takes you from the Town Hall (H6) to the Library (E4). The $(-3, -2)$ vector means move 3 blocks left and 2 blocks down.

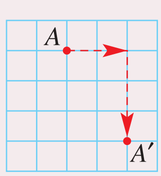


- What is the translation vector that takes you from:
  - the Town Hall to the Jail?
  - the Town Hall to the Old School?
  - the Bank to the Library?
  - the Old School to the Jail?
- Write down, in order, the places you would visit if you started at the Town Hall and followed these vectors: $(-3, -2)$, $(-2, -3)$, $(4, 1)$ and $(-5, 4)$
- Write down the vector that takes you from:
  - B1 to F3
  - H4 to D7
  - D3 to A8.

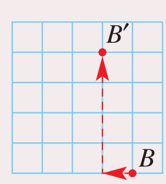
Key ideas

- **Translation** is a **transformation** that involves a shift by a given distance in a given direction.
- A **vector** (x, y) is used to describe the distance and direction of a translation.

Vector $(2, -3)$
Right 2 Down 3



Vector $(-1, 4)$
Left 1 Up 4



- If x is positive you shift to the right.
- If x is negative you shift to the left.
- If y is positive you shift up.
- If y is negative you shift down.

- The **image** of a point A is denoted A'.

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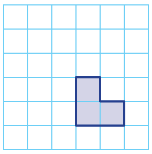
Exercise 10B

Understanding

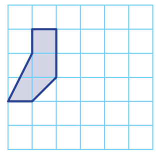
	1–3	3

1 Copy the diagrams then draw in the image of these shapes for the given translations.

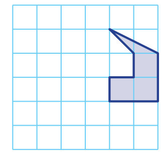
a Shift 2 left and 1 up
vector $(-2, 1)$



b Shift 3 right and 2 down
vector $(3, -2)$



c Shift 3 left and 1 down
vector $(-3, -1)$



2 Use the words left, right, up or down, to complete these sentences.

a The vector $(2, 4)$ means to move 2 units to the ____ and 4 units ____.

b The vector $(-5, 6)$ means to move 5 units to the ____ and 6 units ____.

c The vector $(3, -1)$ means to move 3 units to the ____ and 1 unit ____.

d The vector $(-10, -12)$ means to move 10 units to the ____ and 12 units ____.

Hint for Q2: As an example the vector $(2, -1)$ means to move 2 right and 1 down.

3 Write the vector (x, y) that describes these transformations.

a 5 units to the right and 2 units down

b 2 units to the left and 6 units down

c 7 units to the left and 4 units up

d 9 units to the right and 17 units up

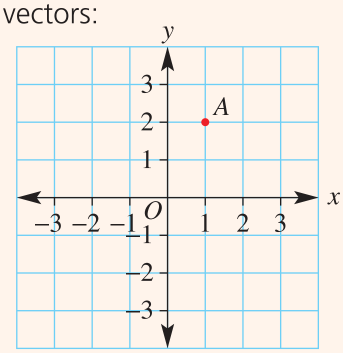
Fluency

	4, 5(½), 6	4, 5–6(½)

Example 4 Translating points

Give the coordinates of the image of the point A if it is translated by these vectors:

a vector $(2, -3)$ b vector $(-3, -2)$

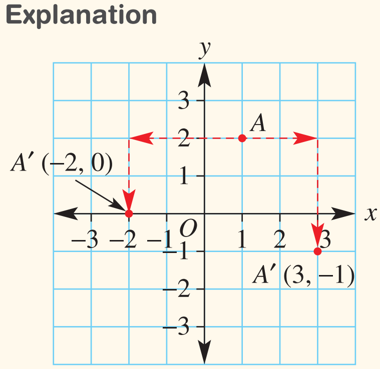


Solution

a $(3, -1)$

b $(-2, 0)$

Explanation



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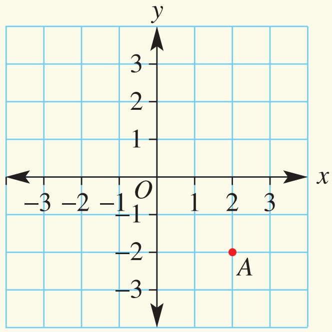
Now you try

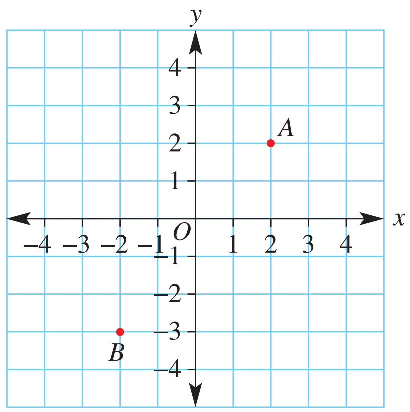
Give the coordinates of the image of the point A if it is translated by these vectors:

a vector $(-1, 4)$
b vector $(-3, 2)$



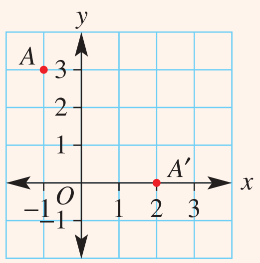
4 **a** Give the coordinates of the image of the point A if it is translated by these vectors:
- **i** vector $(1, 2)$
- **ii** vector $(-3, 1)$
- **iii** vector $(-4, -2)$
- **iv** vector $(-5, -3)$

b Give the coordinates of the image of the point B if it is translated by these vectors.
- **i** vector $(1, 4)$
- **ii** vector $(5, 3)$
- **iii** vector $(-1, 4)$
- **iv** vector $(0, 6)$



Example 5 Finding the translation vector

State the translation vector that moves the point $A(-1, 3)$ to $A'(2, 0)$.



Solution	Explanation
vector $(3, -3)$	To shift A to A' move 3 units to the right and 3 units down.

Now you try

State the translation vector that moves the point $A(-3, -1)$ to $A'(4, -2)$.

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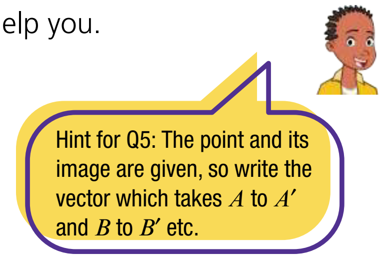
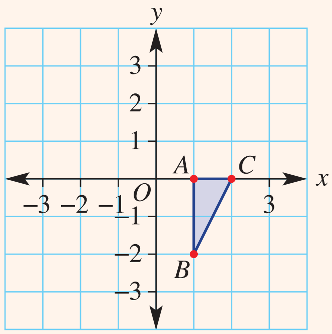
5 Write the vector that takes each point to its image. Use a grid to help you.

a $A(2, 3)$ to $A'(3, 2)$
b $B(1, 4)$ to $B'(4, 3)$
c $C(-2, 4)$ to $C'(0, 2)$
d $D(-3, 1)$ to $D'(-1, -3)$
e $E(-2, -4)$ to $E'(1, 3)$
f $F(1, -3)$ to $F'(-2, 2)$
g $G(0, 3)$ to $G'(2, 0)$
h $H(-3, 5)$ to $H'(0, 0)$
i $I(5, 2)$ to $I'(-15, 10)$
j $J(-3, -4)$ to $J'(-12, -29)$

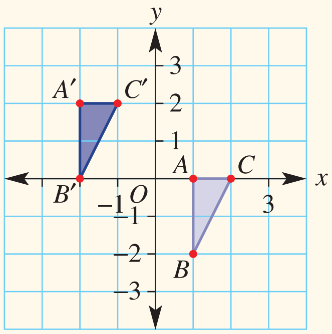


Example 6 Drawing images using translation

Draw the image of the triangle ABC after a translation by the vector $(-3, 2)$.



Solution

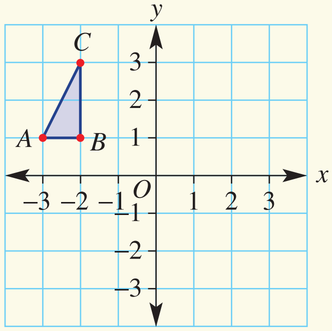


Explanation

First translate each vertex, A, B and C, 3 spaces to the left, and then 2 spaces up. Then join the image vertices A', B' and C'.

Now you try

Draw the image of the triangle ABC after a translation by the vector $(4, -3)$.



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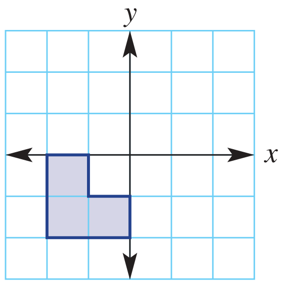
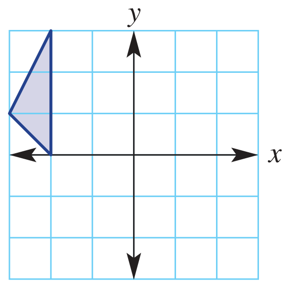
10B

6 Copy the diagrams and draw the image of the shapes translated by the given vectors:

a vector $(2, 3)$

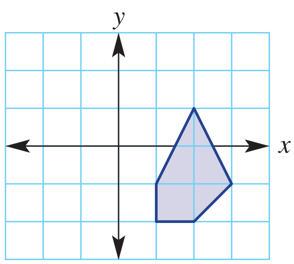


b vector $(4, -2)$

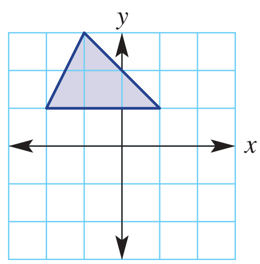


Hint for Q6: First move each vertex (corner) then join to form the image shape.

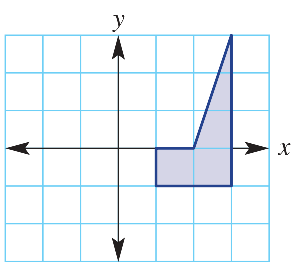
c vector $(-3, 1)$



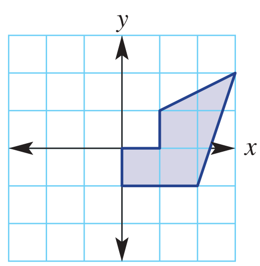
d vector $(0, -3)$



e vector $(-4, -1)$



f vector $(-3, 0)$



Problem-solving and reasoning

7, 8 | 7–9

7 Decide if these vectors describe vertical or horizontal translation.

a $(2, 0)$ **b** $(0, 7)$ **c** $(0, -4)$ **d** $(-6, 0)$

8 Write the coordinates of the image of the point $A(13, -1)$ after a translation by the given vectors.

a $(2, 3)$ **b** $(8, 0)$ **c** $(0, 7)$
d $(-4, 3)$ **e** $(-2, -1)$ **f** $(-10, 5)$
g $(-2, -8)$ **h** $(6, -9)$

Hint for Q8: Always start at $(13, -1)$ then move to the image point using the given vector.

9 A reverse vector takes a point in the reverse direction by the same distance.
For example, the reverse vector of $(2, -3)$ is $(-2, 3)$. Write the reverse vectors of these vectors.

a $(3, -2)$ **b** $(-5, 0)$ **c** (x, y) **d** $(-x, -y)$

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City street vectors

	—	10

10 A car makes its way around a city street grid. A vector $(2, 3)$ represents travelling 200 m east and 300 m north.

a What vector would be used to describe travelling:

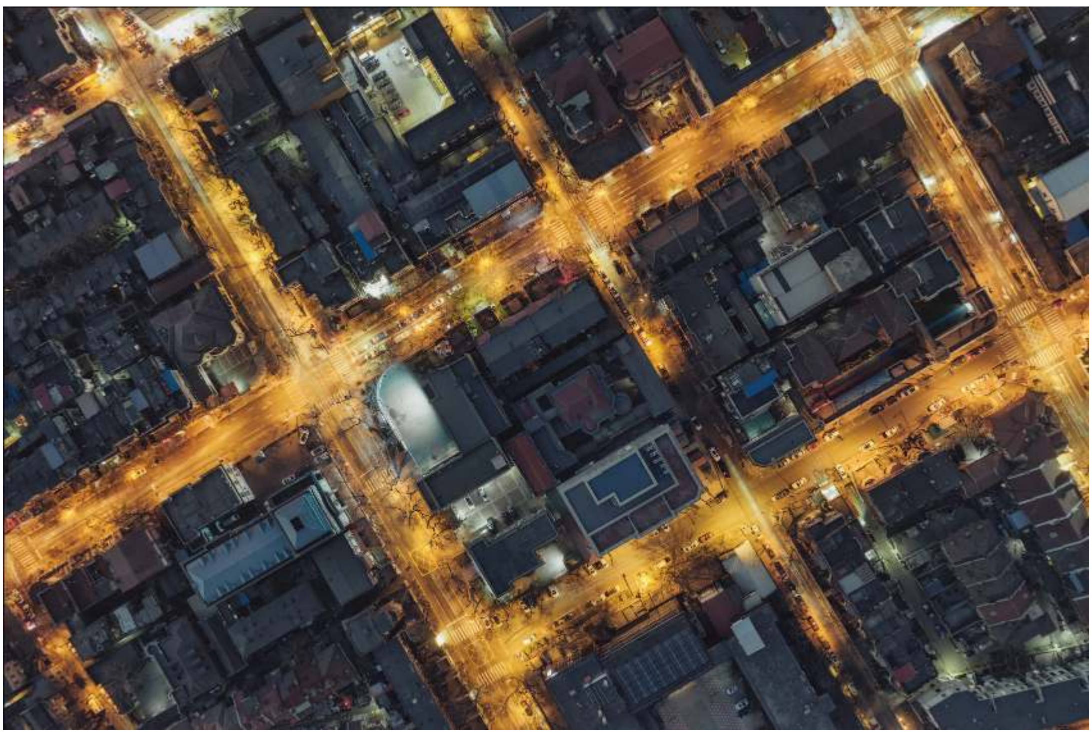
i 100 m east and 200 m south?

ii 300 m west and 400 m north?

iii 300 m south only?

b Find how far the car travels in total if it follows these vectors in order. $(2, 3)$, $(-5, 1)$, $(3, -3)$ and $(-2, -4)$.

c What vector takes the car back to the origin $(0, 0)$, assuming it started at the origin and used the travel vectors in part **b**?



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10C Rotation

Learning intentions

- To understand that an object can be rotated about a given centre point by an angle either clockwise or anticlockwise.
- To understand the order of rotation is the number of times that the shape's image will be an exact copy of the shape in a 360° rotation.
- To be able to find the order of rotational symmetry of a given shape.
- To be able to draw the result of a rotation.

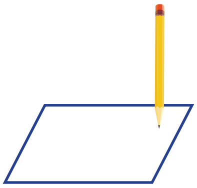
Key vocabulary: transformation, rotation, centre of rotation, rotational symmetry, order of rotational symmetry

When the arm of a crane moves left, right, up or down, it undergoes a rotation about a fixed point. This type of movement is a transformation called a rotation. The pivot point on a crane would be called the centre of rotation and all other points on the crane's arm move around this point by the same angle in a circular arc.

When the arm of a crane moves, it is undergoing a transformation known as a rotation, where any point on the arm moves by the same angle around a central pivot point.

Lesson starter: Parallelogram centre of rotation

This activity will need a pencil, paper, ruler and scissors.



- Accurately draw a large parallelogram on the piece of paper and cut it out.
- Place the tip of a pencil at any point on the parallelogram and spin the shape around the pencil.
- At what position do you put the pencil tip to produce the largest circular arc?
- At what position do you put the pencil tip to produce the smallest circular arc?
- Can you rotate the shape by an angle of less than 360° so that it exactly covers the area of the shape in its original position? Where would you put the pencil to achieve this?

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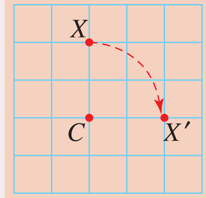
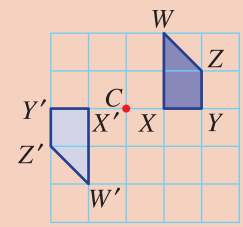
Key ideas

- **Rotation** is a **transformation** about a centre point and by a given angle.
- An object can be rotated clockwise ↻ or anticlockwise ↺.
- Each point is rotated on a circular arc about the **centre of rotation** C.

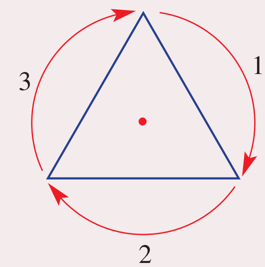


x is rotated 90° clockwise about C



Shape $XYZW$ is rotated 180° about C

- A shape has **rotational symmetry** if it can be rotated about a centre point to produce an exact copy covering the entire area of the original shape.
  - The number of times the shape can make an exact copy in a 360° rotation is called the **order of rotation**. If the order of rotation is 1, then it is said that the shape has no rotational symmetry.
  - This equilateral triangle has rotational symmetry of order 3.



Exercise 10C

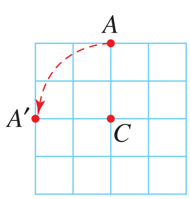
Understanding

1–2 | 2

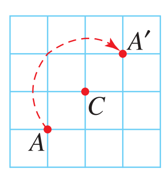
1 Match each description **a**, **b** and **c** with each diagram **A**, **B** and **C**.

a Rotation 90° clockwise **b** Rotation 90° anticlockwise **c** Rotation 180° clockwise

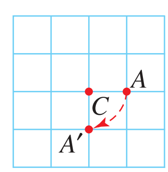
A



B

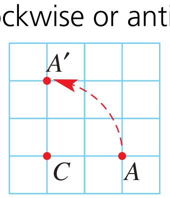


C

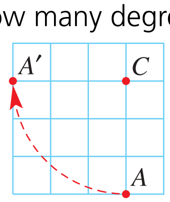


2 Point A has been rotated to its image point A'. For each part, state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

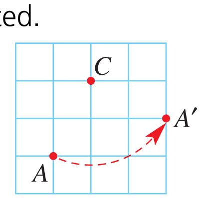
a



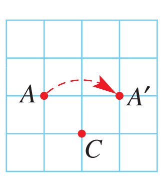
b



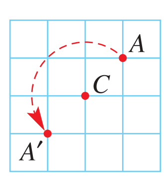
c



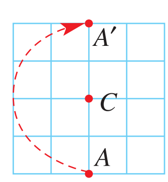
d



e



f



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10C

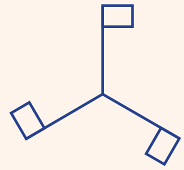
Fluency

3–5 | 3–6(½)

Example 7 Finding the order of rotational symmetry

Find the order of rotational symmetry for these shapes.

a 

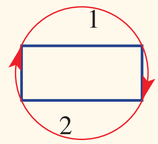
b 

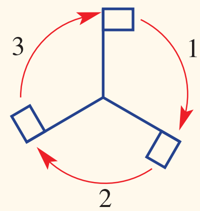
Solution

a Order of rotational symmetry = 2

b Order of rotational symmetry = 3

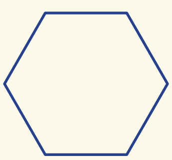
Explanation

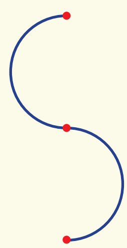




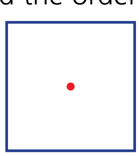
Now you try

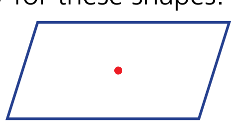
Find the order of rotational symmetry for these shapes.

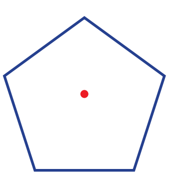
a 

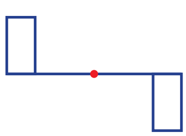
b 

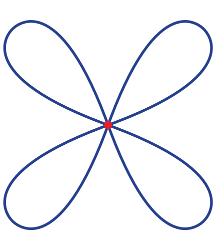
3 Find the order of rotational symmetry for these shapes.

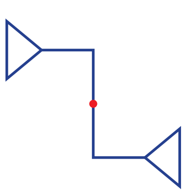
a 

b 

c 

d 

e 

f 

Hint for Q3: How many times can you make an exact copy of the original in a 360° turn?

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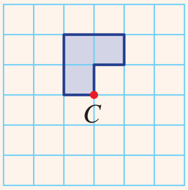
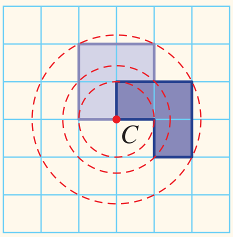
Example 8 Rotating a shape by 90°

Rotate this shape about C clockwise by 90°.



Solution



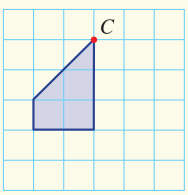
Explanation

Take each vertex point and rotate about C by 90°, but it may be easier to visualise a rotation of some of the sides first.

Horizontal sides will rotate to vertical sides in the image and vertical sides will rotate to horizontal sides in the image.

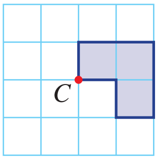
Now you try

Rotate this shape about C anticlockwise by 90°.

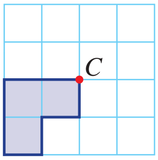


4 Rotate these shapes about the point C by 90° in the given direction.

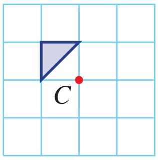
a Clockwise



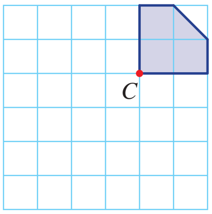
b Anticlockwise



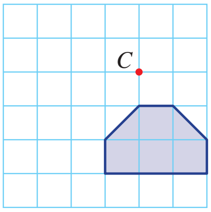
c Anticlockwise



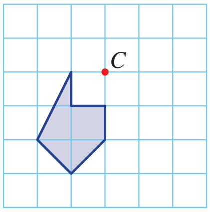
d Clockwise

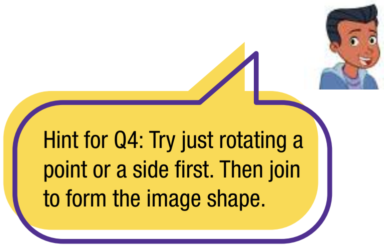


e Clockwise



f Anticlockwise





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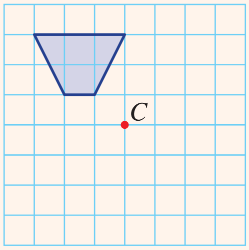
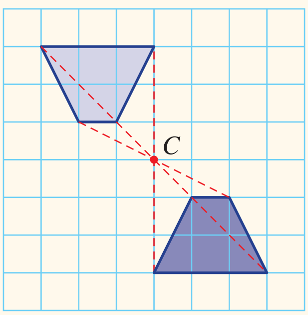
10C

Example 9 Rotating a shape by 180°

Rotate this shape about point C by 180°.



Solution

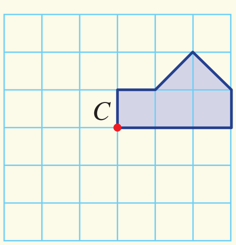


Explanation

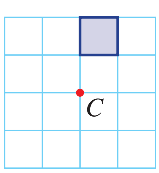
You can draw a dashed line from each vertex through C to a point at an equal distance on the opposite side.

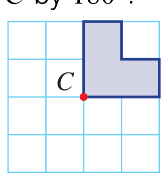
Now you try

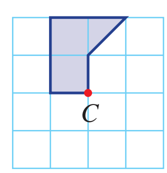
Rotate this shape about point C by 180°.

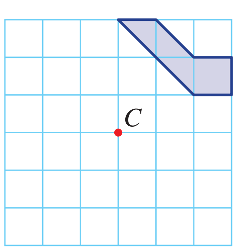


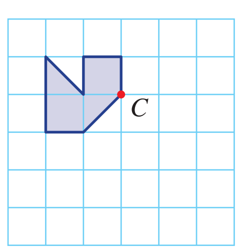
5 Rotate these shapes about the point C by 180°.

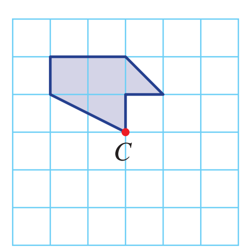
a


b


c


d


e


f


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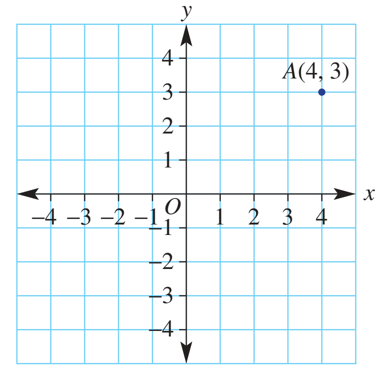
6 The point $A(4, 3)$ is rotated about the origin $O(0, 0)$ by the given angle and direction. Give the coordinates of A'.
  - a $180°$ clockwise
  - b $180°$ anticlockwise
  - c $90°$ clockwise
  - d $90°$ anticlockwise
  - e $270°$ clockwise
  - f $270°$ anticlockwise
  - g $360°$ clockwise

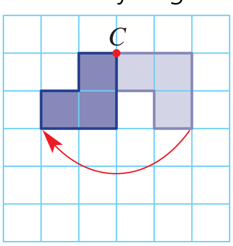


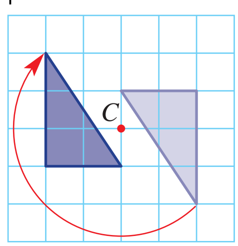
Problem-solving and reasoning

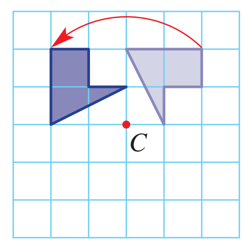
	7, 8	8–10

7 Complete these sentences.
  - a A rotation clockwise by $90°$ is the same as a rotation anticlockwise by _____.
  - b A rotation anticlockwise by $180°$ is the same as a rotation clockwise by _____.
  - c A rotation anticlockwise by _____ is the same as a rotation clockwise by $58°$.
  - d A rotation clockwise by _____ is the same as a rotation anticlockwise by $296°$.

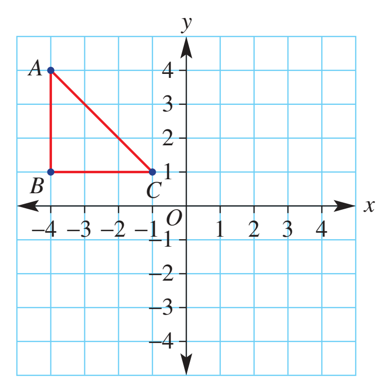
8 By how many degrees have these shapes been rotated?

a


b


c


9 The triangle shown below is rotated about $(0, 0)$ by the given angle and direction. Give the coordinates of the image points A', B' and C'.
  - a $180°$ clockwise
  - b $90°$ clockwise
  - c $90°$ anticlockwise



10 Which capital letters of the alphabet (A,B,C…Z) have rotational symmetry of order 2 or more?

Combining line and rotational symmetry

	—	11

11 Draw an example of a shape that has these properties:
  - a Rotational symmetry of order 4 with 4 lines of symmetry.
  - b Rotational symmetry of order 2 with no line symmetry.
  - c Rotational symmetry of order 6 with 6 lines of symmetry.
  - d Rotational symmetry of order 4 with no line symmetry.
  - e No rotational symmetry with 1 line of symmetry.

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10D Congruent figures

Learning intentions

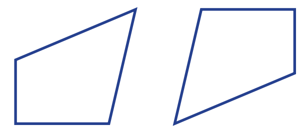
- To understand that two figures are congruent (have the same size and shape) if one can be transformed to the other using any combination of reflections, translations and rotations.
- To be able to name corresponding pairs of vertices, sides and angles in congruent shapes.

Key vocabulary: figure, congruent figures, corresponding, vertex (plural vertices), side, angle, reflection, translation, rotation

If two objects are identical and have the same size and shape, we say they are congruent. The images on the front cover of your Year 8 maths textbook, for example, would be congruent to the images on the front of another Year 8 maths textbook, assuming it's the same type of book. Even if one text was flipped over, shifted or rotated you would still say the images on the books were congruent.

Lesson starter: Are they congruent?

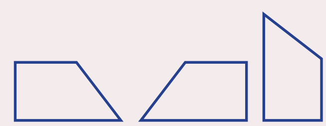
Here are two shapes. To be congruent they need to be exactly the same shape and size.



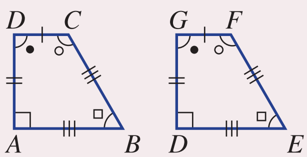
- Do you think they look congruent? Give reasons.
- What measurements could be taken to help establish whether or not they are congruent?
- Can you just measure angles or do you need to measure lengths as well? Discuss.

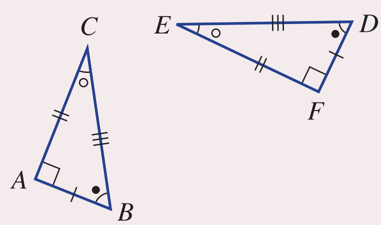
Key ideas

- A **figure** is a shape, diagram or illustration.
- **Congruent figures** have the same size and shape.
- The image of a figure that is **reflected**, **translated** or **rotated** is congruent to the original figure.

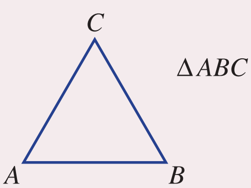


- **Corresponding** (matching) parts of a figure have the same geometric properties. For example:
  - **Vertex** B corresponds to vertex E.
  - **Side** CD corresponds to side FG.
  - **Angle** $\angle C$ corresponds to $\angle F$.
  - Vertex C corresponds to vertex E.
  - Side AB corresponds to side FD.
  - Angle $\angle B$ corresponds to $\angle D$.





- The symbol for 'triangle' is Δ.



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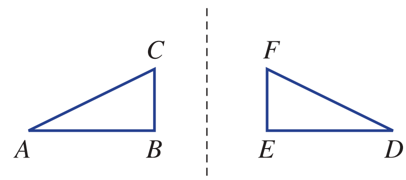
Exercise 10D

Understanding

1–4 | 4

1 Answer true (T) or false (F).
 a Congruent shapes can be of different size.
 b Congruent shapes have equal matching sides.
 c Congruent shapes have equal matching angles.
 d The image of a shape after reflection is congruent to the original shape.
 e The image of a shape after rotation is congruent to the original shape.
 f The image of a shape after translation is congruent to the original shape.

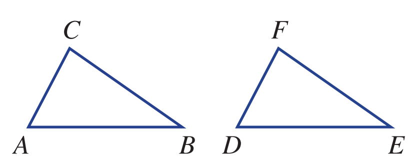
2 In this diagram ΔABC has been reflected to give the image triangle ΔDEF.



Hint for Q2: Choose the matching vertex (corner), side or angle on the opposite triangle.

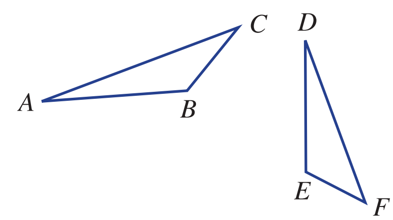
 a Is ΔDEF congruent to ΔABC?
 b Name the vertex on ΔDEF which corresponds to:
  i vertex A ii vertex B iii vertex C
 c Name the side on ΔDEF which corresponds to:
  i side AB ii side BC iii side AC
 d Name the angle in ΔDEF which corresponds to:
  i $\angle B$ ii $\angle C$ iii $\angle A$

3 In this diagram ΔABC has been translated (shifted) to give the image triangle ΔDEF.



 a Is ΔDEF congruent to ΔABC?
 b Name the vertex on ΔDEF which corresponds to:
  i vertex A ii vertex B iii vertex C
 c Name the side on ΔDEF which corresponds to:
  i side AB ii side BC iii side AC
 d Name the angle in ΔDEF which corresponds to:
  i $\angle B$ ii $\angle C$ iii $\angle A$

4 In this diagram ΔABC has been rotated to give the image triangle ΔDEF.
 a Is ΔDEF congruent to ΔABC?
 b Name the vertex on ΔDEF which corresponds to:
  i vertex A ii vertex B iii vertex C
 c Name the side on ΔDEF which corresponds to:
  i side AB ii side BC iii side AC
 d Name the angle in ΔDEF which corresponds to:
  i $\angle B$ ii $\angle C$ iii $\angle A$



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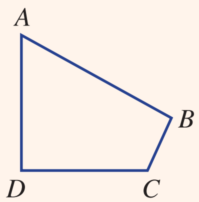
10D

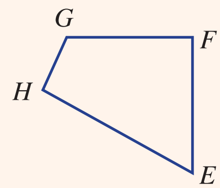
Fluency | 5 | 5, 6

Example 10 Naming corresponding pairs

These two quadrilaterals are congruent. Name the objects in quadrilateral $EFGH$ that correspond to these objects in quadrilateral $ABCD$.

a Vertex C b Side AB c $\angle C$



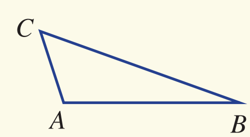


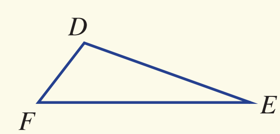
Solution	Explanation
a Vertex G	C sits opposite A and $\angle A$ is the smallest angle. G sits opposite E and $\angle E$ is also the smallest angle.
b Side EH	Sides AB and EH are both the longest sides of their respective shapes. A corresponds to E and B corresponds to H.
c $\angle G$	$\angle C$ and $\angle G$ are both the largest angle in their corresponding quadrilateral.

Now you try

These two triangles are congruent. Name the objects in ΔDEF that correspond to these objects in ΔABC.

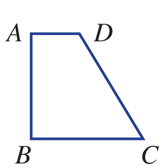
a Vertex B
b Side AC
c $\angle A$

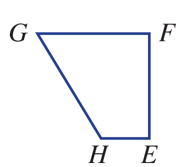




5 These two quadrilaterals are congruent. Name the object in quadrilateral $EFGH$ which corresponds to these objects in quadrilateral $ABCD$.

a i Vertex A ii Vertex D
b i Side AD ii Side CD
c i $\angle C$ ii $\angle A$

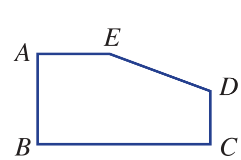


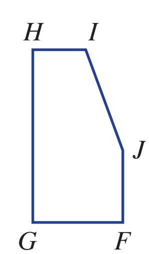


Hint for Q5: Corresponding angles will be equal and corresponding sides are the same length.

6 These two pentagons are congruent. Name the object in pentagon $FGHIJ$ which corresponds to these objects in pentagon $ABCDE$.

a i Vertex A ii Vertex D
b i Side AE ii Side CD
c i $\angle C$ ii $\angle E$





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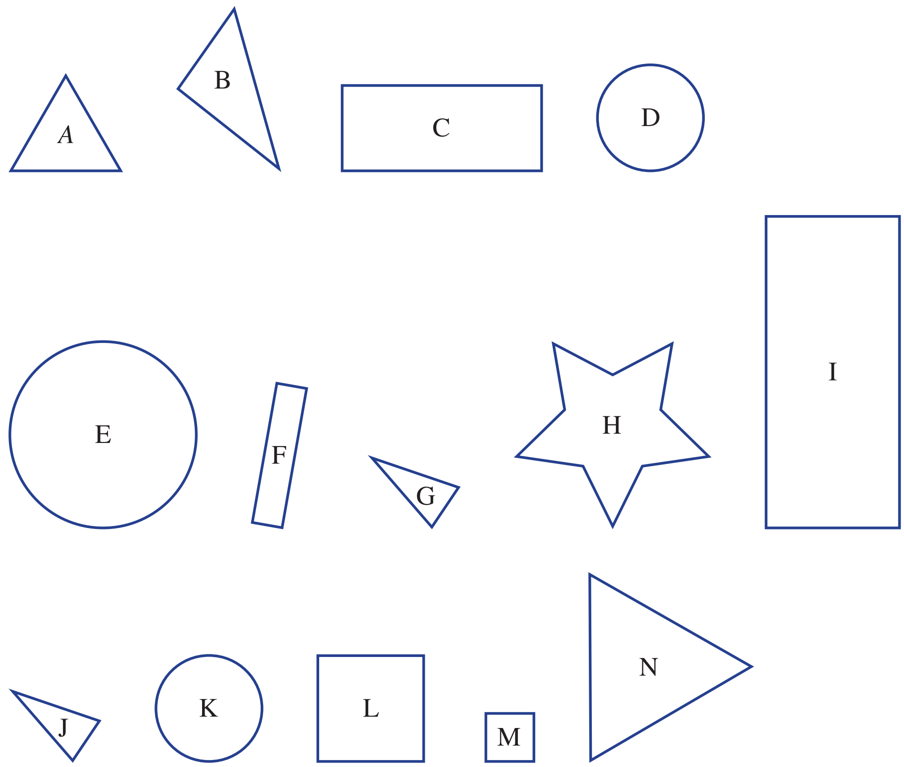
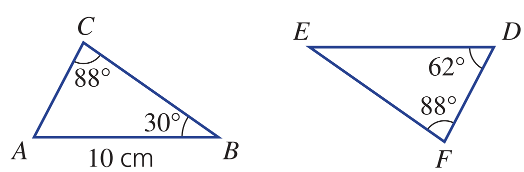
Problem-solving and reasoning

7–9 8–11

7 From all the shapes shown here, find 2 pairs that look congruent.

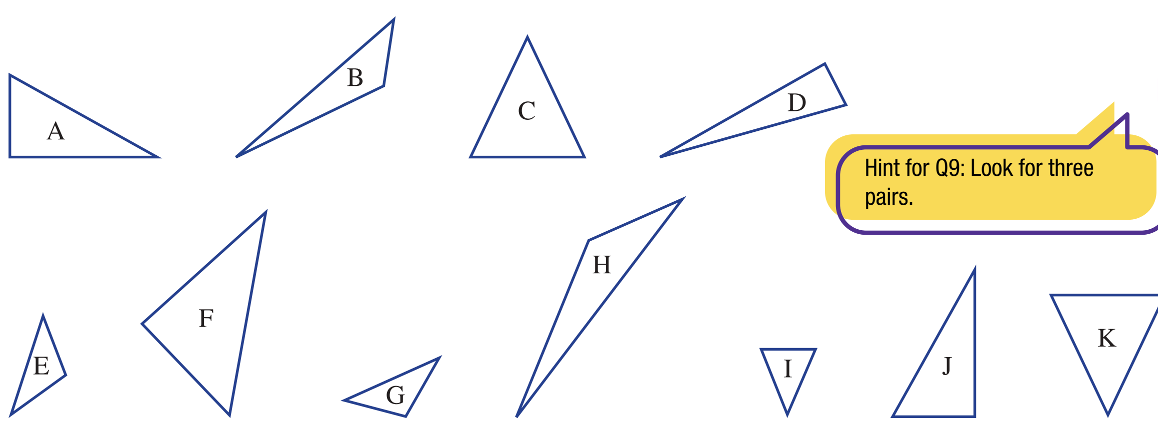


8 These triangles are congruent.



a Which side on ΔDEF corresponds to side AB?
b Which angle on $\angle ABC$ corresponds to $\angle E$?
c What is the length DE?
d What is the size of $\angle A$?
e What is the size of $\angle E$?

9 List the pairs of the triangles below that look congruent.



Hint for Q9: Look for three pairs.

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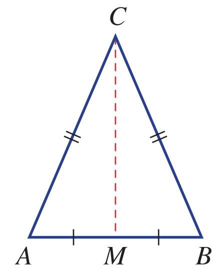
10 An isosceles triangle is cut as shown, using the midpoint of AB.

a Name the two triangles formed.

b Will the two triangles be congruent? Give reasons.



11 If a parallelogram is cut by either diagonal, will the two triangles be congruent?

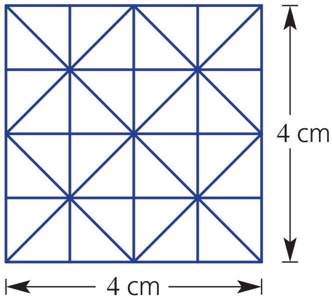
Counting triangles

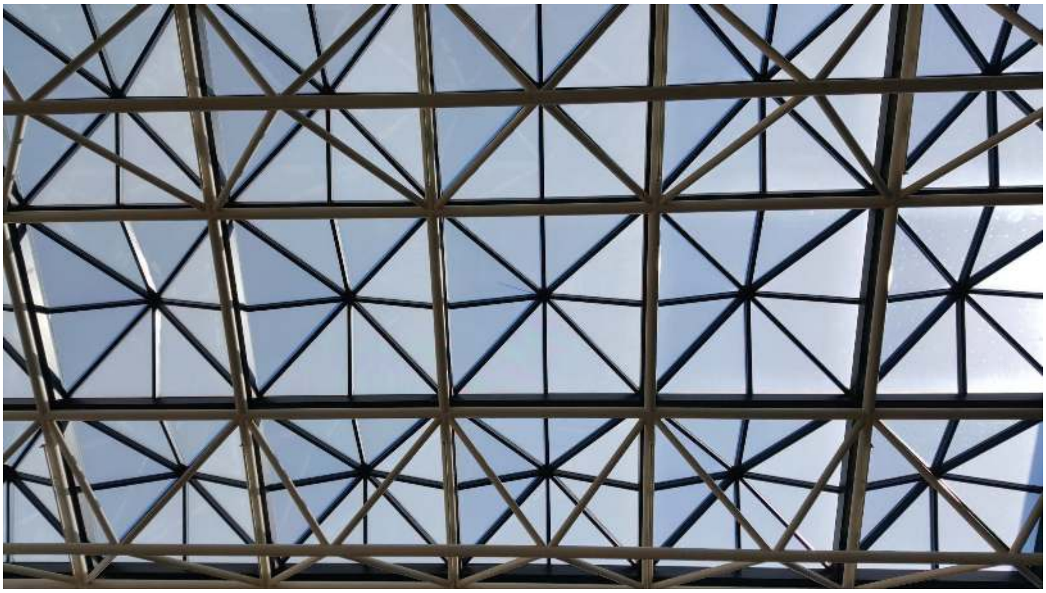
— 12

12 How many congruent triangles are there in this diagram with:

a area $\frac{1}{2}$ cm^2 **b** area 1 cm^2? **c** area 2 cm^2?

d area 4 cm^2? **e** area 8 cm^2?





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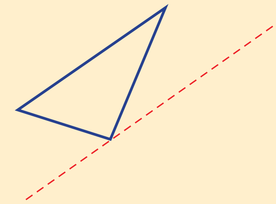
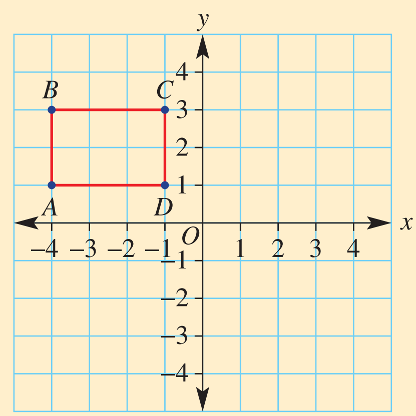
Progress quiz

10A **1** Copy the diagram and draw the reflected image over the given mirror line.

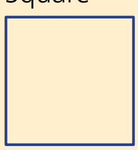


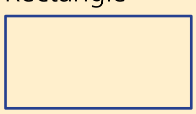
10A **2** State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

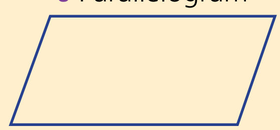
a x-axis
b y-axis



10A **3** How many lines of symmetry do these quadrilaterals have?

a Square


b Rectangle


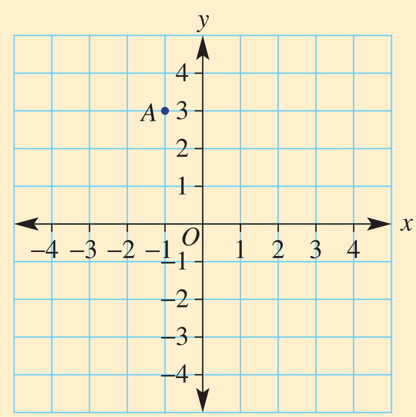
c Parallelogram


10B **4** Write the vector (x, y) that describes the following transformations.

a 4 units to the left and 3 units up
b 6 units to the right and 2 units down

10B **5** Give the coordinates of the image of the point A if it is translated by these vectors.

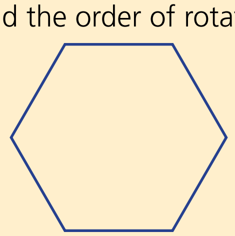
a $(2, 3)$ **b** $(-3, 1)$ **c** $(5, 0)$

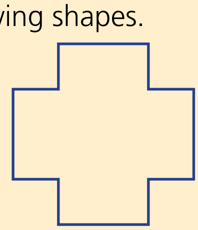


10B **6** Write the vector that takes each point to its image. Use a grid to help you.

a $A(3, 4)$ to $A'(2, -1)$ **b** $A(-1, 0)$ to $A'(-5, 3)$

10C **7** Find the order of rotational symmetry for the following shapes.

a


b


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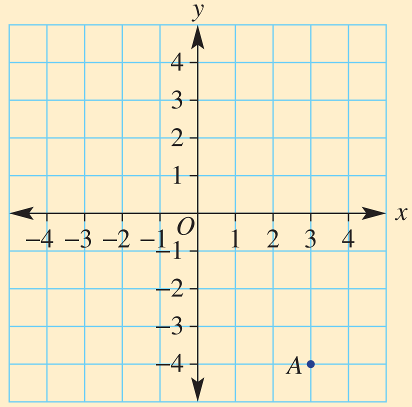
Progress quiz

10C

8 The point $A(3, -4)$ is rotated about the origin $O(0, 0)$ by the given angle and direction. Give the coordinates of A':

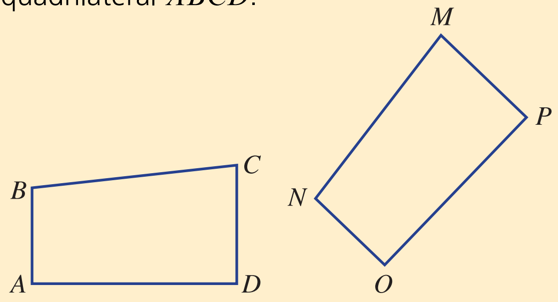
- **a** 180° clockwise
- **b** 180° anticlockwise
- **c** 90° clockwise
- **d** 90° anticlockwise
- **e** 360° clockwise
- **f** 270° anticlockwise



10D

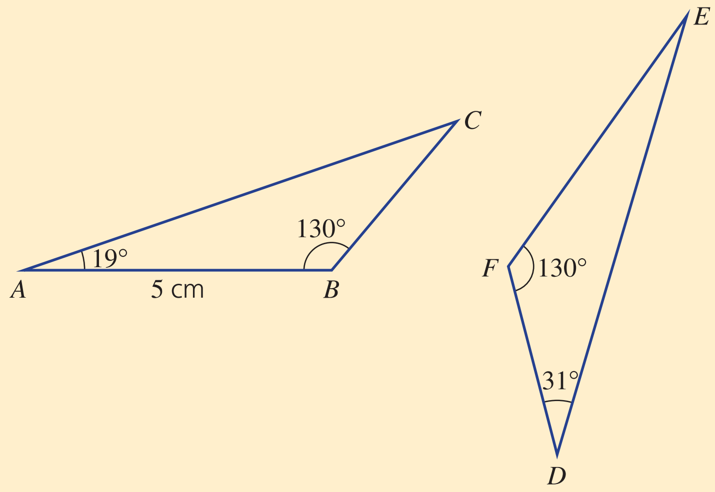
9 These two quadrilaterals are congruent. Name the object in quadrilateral $MNOP$ which corresponds to these objects in quadrilateral $ABCD$.

- **a** Vertex A
- **b** Vertex C
- **c** Side AB
- **d** Side BC
- **e** $\angle D$
- **f** $\angle B$



10D

10 These triangles are congruent.



- **a** Which side on ΔDEF corresponds to side BC?
- **b** Which angle on ΔABC corresponds to $\angle D$?
- **c** What is the length of EF?
- **d** What is the size of $\angle C$?

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10E Congruent triangles

Learning intentions

- To understand that determining whether triangles are congruent can be done using the congruence tests SSS, SAS, AAS and RHS.
- To be able to determine which congruence test should be used to determine if two triangles are congruent.

Key vocabulary: congruent, corresponding, included angle, hypotenuse, congruence statement

Imagine the sorts of design and engineering problems we would face if we could not guarantee that two objects such as window panes or roof truss frames were not the same size or shape. Also, it might not be possible to measure every length and angle to test for congruence. In the case of triangles, it is possible to consider only a number of pairs of sides or angles to test whether or not they are congruent. This leads to a special set of minimum conditions (tests) which can be used to establish that two triangles are congruent.

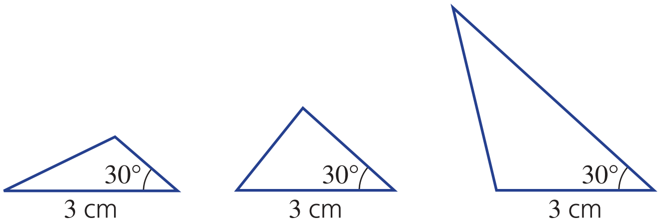
The principles of triangle congruence are applied to measure roof trusses, ensuring precise calculation of angles and lengths. This accuracy guarantees the correct placement and alignment of each truss, providing structural stability and uniformity to the roof.

Lesson starter: How much information is enough?

Given one corresponding angle (say 30°) and one corresponding equal side length (say 3 cm), it is clearly not enough information to say two triangles are congruent. This is because more than one triangle can be drawn with the given information.

Does the following information allow you to draw only one kind of triangle? If you can draw two non-identical triangles then there is not enough information. You could use a ruler and a protractor or simply try this by hand labelling vertices, sides and angles as you go.

- ΔABC with $AC = 4$ cm and $\angle C = 40°$
- ΔABC with $AB = 5$ cm and $AC = 4$ cm
- ΔABC with $AB = 5$ cm, $AC = 4$ cm and $\angle A = 45°$
- ΔABC with $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm
- ΔABC with $AB = 4$ cm, $\angle A = 40°$ and $\angle B = 60°$



Knowing one corresponding side and one corresponding angle is not enough to say that two triangles will be congruent.

This wall of a building at Federation Square in Melbourne includes many congruent triangles.

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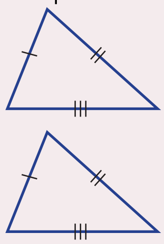
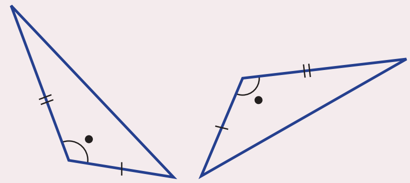
Key ideas

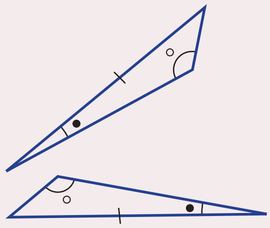
- Two triangles are **congruent** if one of these four sets of tests is satisfied.
  - **SSS**
    3 equal **corresponding** sides



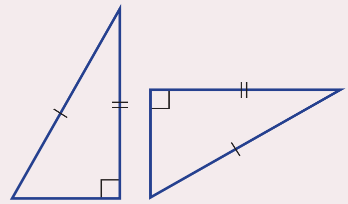
  - **SAS**
    2 equal corresponding sides and 1 equal corresponding angle between them. This angle is called the **included angle**.



  - **AAS**
    2 equal corresponding angles and 1 equal corresponding side. Any order is accepted AAS, ASA, SAA.



  - **RHS**
    2 right-angled triangles with equal **hypotenuse** lengths and 1 other pair of equal corresponding sides.



- If triangle ABC is congruent to triangle DEF, we write $\Delta ABC \equiv \Delta DEF$.
  - This is called a **congruence statement**.
  - Letters are usually written in matching order.

Exercise 10E

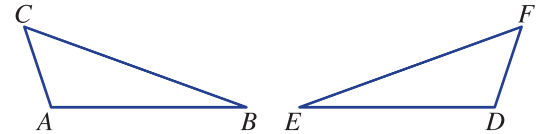
Understanding

	1–2	2

1 Which of the following are tests for congruent triangles?

A SSS **B** SAS **C** AAA
D AAS **E** RHS **F** SSA

2 Look at this pair of congruent triangles.



a Which vertex on ΔDEF corresponds to (matches) these vertices on ΔABC?
- **i** Vertex C
- **ii** Vertex A
- **iii** Vertex B

b Which angle on ΔABC corresponds to these angles on ΔDEF?

i $\angle D$ **ii** $\angle F$ **iii** $\angle E$

c Which side on ΔDEF corresponds to these sides on ΔABC?

i AB **ii** CA **iii** BC

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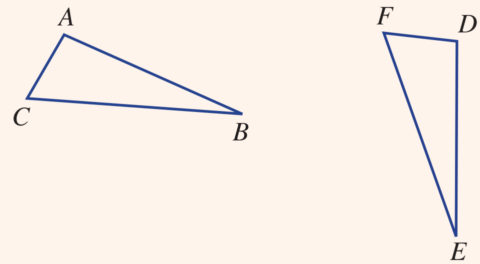
Fluency

3, 4(½) | 3–5(½)

Example 11 Writing a congruence statement

Write a congruence statement for this pair of congruent triangles.



Solution

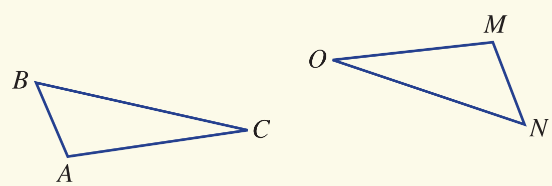
$\Delta ABC \equiv \Delta DEF$

Explanation

Given the size of the angles and the side lengths, it appears that A matches D, B matches E and C matches F.

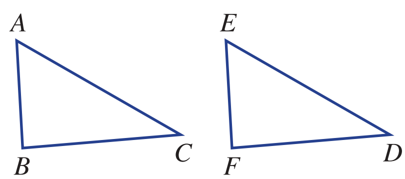
Now you try

Write a congruence statement for this pair of congruent triangles.

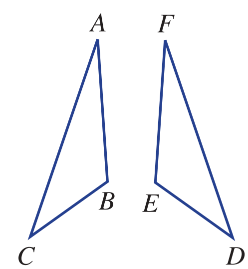


3 Write a congruence statement (e.g. $\Delta ABC \equiv \Delta DEF$) for these pairs of congruent triangles. Try to match vertices.

a

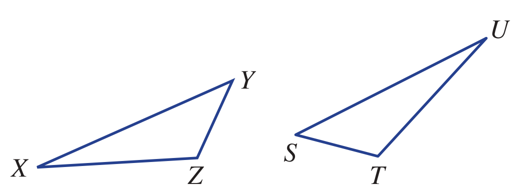


b

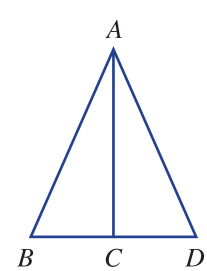


Hint for Q3: Match vertices which have the same matching angles.

c



d



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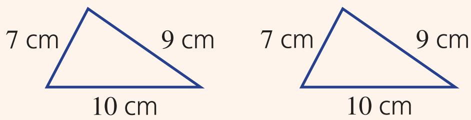
10E

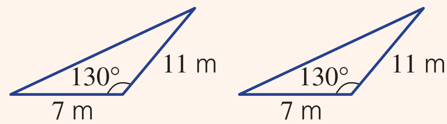
Example 12 Deciding on a test for congruence

Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these pairs of triangles?

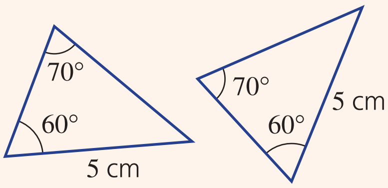
a



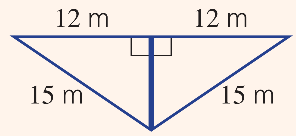
b



c



d

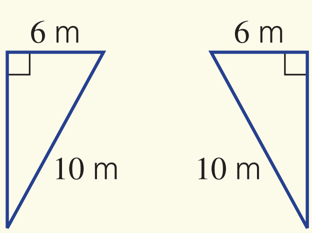


Solution	Explanation
a SSS	There are 3 equal corresponding pairs of sides.
b SAS	There are 2 equal corresponding pairs of sides and 1 equal angle between them.
c AAS	There are two equal angles and 1 pair of equal corresponding sides. The side that is 5 cm is adjacent to the 60° angle on both triangles.
d RHS	There are a pair of right angles with hypotenuses of equal lengths. A second pair of corresponding sides are also of equal length.

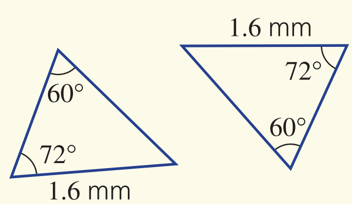
Now you try

Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these pairs of triangles?

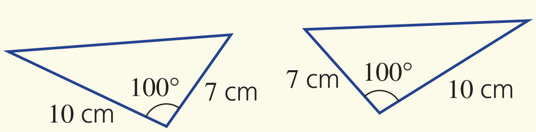
a



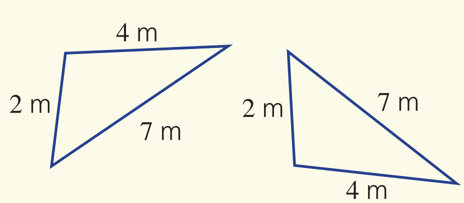
b



c



d



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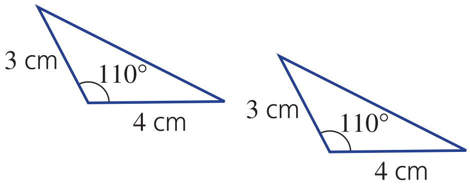
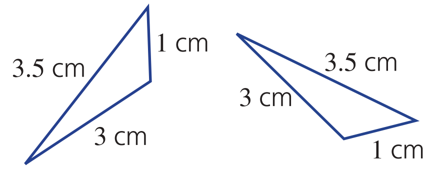
4 Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these triangles?

a



b



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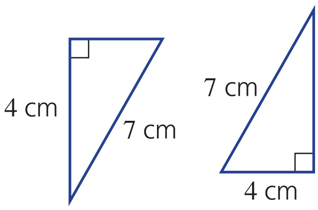
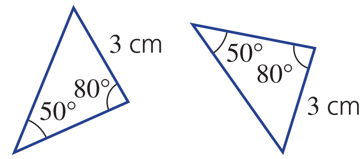
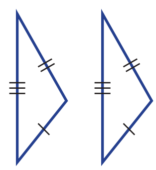


d



5 Pick the congruence test (SSS, SAS, AAS or RHS) that matches each pair of congruent triangles.

a



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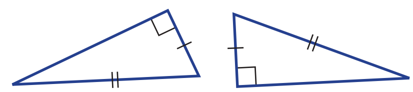

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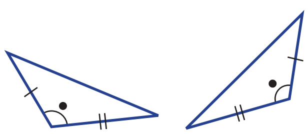

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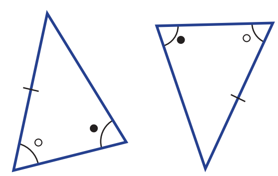
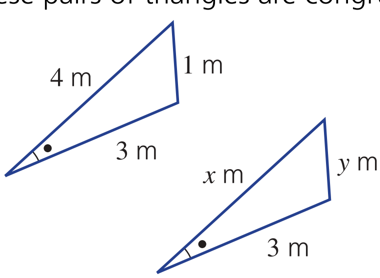


Problem-solving and reasoning

6, 7 | 6–9

6 These pairs of triangles are congruent. Find the values of the pronumerals.

a



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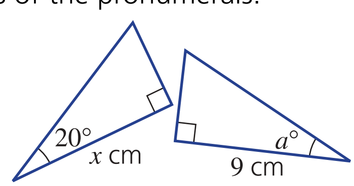
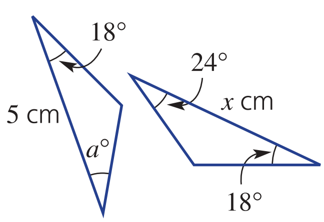


Hint for Q6: Matching sides will be equal and matching angles will be equal.

c



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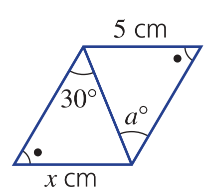
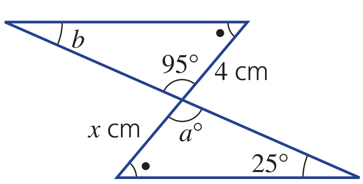
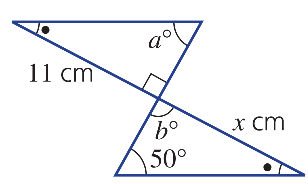


e



f



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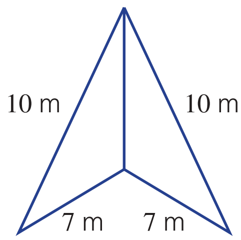
10E

7 Which of SSS, SAS, AAS or RHS would you choose to say that each pair of triangles is congruent?

a



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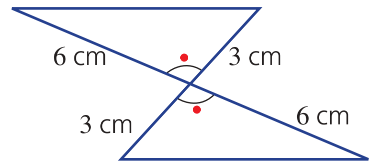
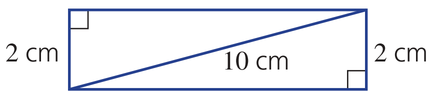


Hint for Q7: Use the information given in the diagram.

c



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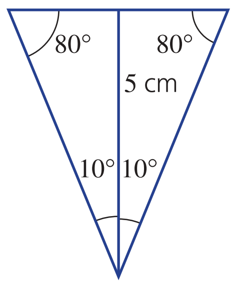
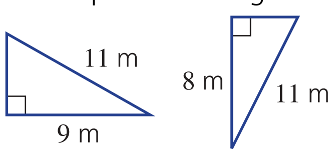


8 Are these pairs of triangles congruent? If they are, give a reason.

a



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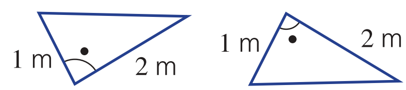

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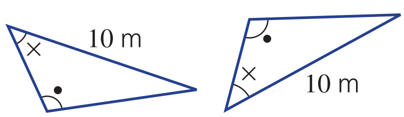

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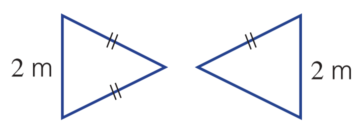
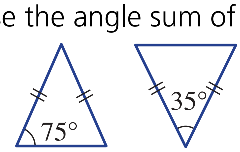


9 Explain why AAA is not sufficient to prove that two triangles are congruent. Draw diagrams to show your reasoning.

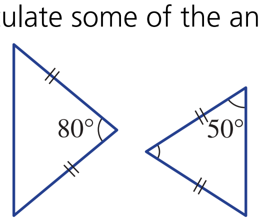
Using the angle sum of a triangle — 10

10 Decide if each pair of triangles is congruent. You may first need to use the angle sum of a triangle to help calculate some of the angles.

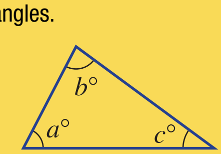
a



b

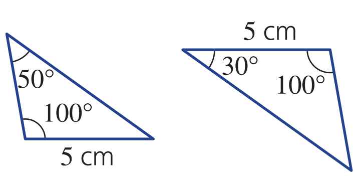


Hint for Q10: First work out all the missing angles in the triangles.

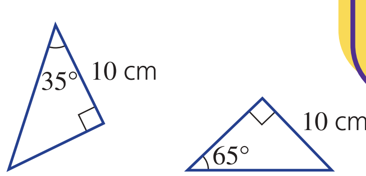


$a + b + c = 180$

c



d



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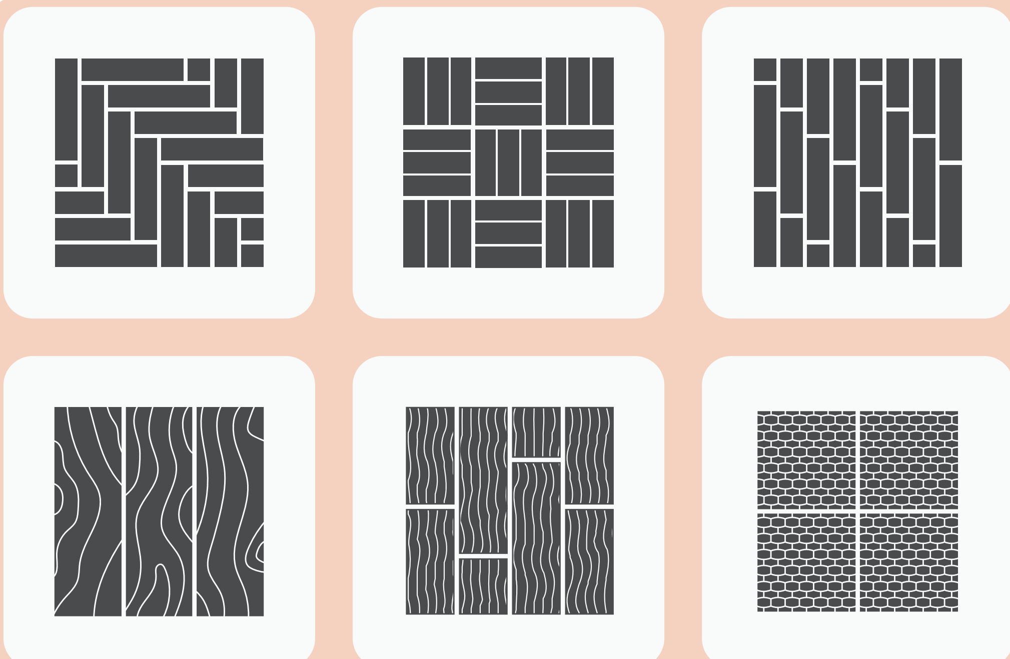
10F Tessellations

Learning intentions

- To be able to tessellate a basic shape.
- To be able to name a regular or semi-regular tessellation based on a picture.

Key vocabulary: tessellation, regular tessellation, semi-regular tessellation, reflection, translation, rotation

Architects, builders and interior designers have great interest in arranging basic congruent shapes to create interesting patterns within a new home. These patterns are often formed using tiles or pavers and can be found on bathroom walls, interior floors or exterior courtyards.

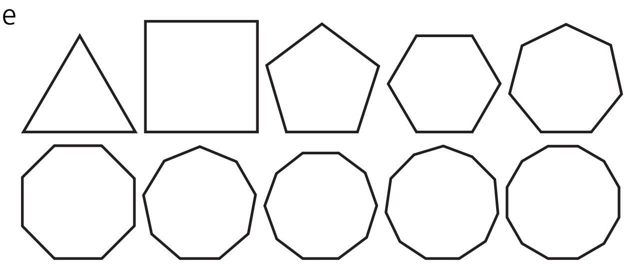


A common range of paving patterns that show how a rectangle can be arranged without leaving any gaps or holes. Which is your preferred pattern if you were wanting to pave an outdoor entertaining area?

The words *tessellate* and *tessellation* originate from the Latin noun, *tessera*, referring to a small tile used in the construction of a mosaic. Tessellated tile designs are commonly used throughout history in the fields of Art and Design and continue to be extensively employed today. It is most likely that various tessellations exist within your home and your school.

Lesson starter: To tessellate or not to tessellate?

In **Section 2F** you were introduced to the concept of *regular polygons* as shapes with sides of equal length and equal interior angles. Can you remember the name given to the first ten polygons?



Johann Kepler, back in 1619, was the first mathematician to prove that there are only three polygons that will tessellate by themselves. Working with a partner, can you determine which polygons these are?

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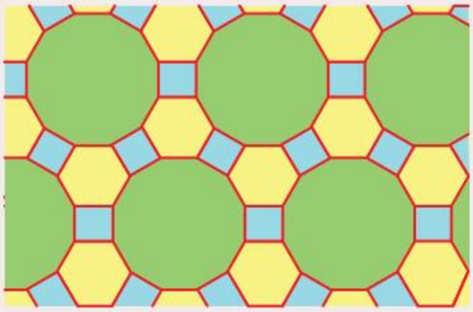
10F

Key ideas

- A **tessellation** is a pattern made up of shapes that fit together without any gaps and without any overlaps.
- Transformations, such as **reflections, translations** and **rotations**, are used with appropriate shapes to produce tessellated patterns.
- **Regular tessellations** are formed by arranging multiple copies of one regular polygon. There are only three regular polygons that tessellate by themselves: triangle, square and hexagon.
- **Semi-regular tessellations** are formed by arranging multiple copies of two or more regular polygons. There are eight distinct semi-regular tessellations.



- Regular and semi-regular tessellations can be named by counting the number of sides each regular polygon has at any of the identical vertices.
  For example: the following semi-regular tessellation consists of squares, hexagons and dodecagons. It can be named as a 4.6.12 tessellation.
- Other tessellated patterns can be formed by any combination of shapes (regular, irregular, composite). Curved shapes and images can also be used to form tessellated patterns, like the one shown on the right.

Exercise 10F

Understanding 1–3 3

When asked to construct tessellations you can do these by hand using grid or dot paper; alternatively, you may prefer to use geometry software.

1 Which of the following phrases best describes what a *tessellation* is?
   A A group of shapes joined together.
   B A group of shapes all stacked on top of one another.
   C A group of shapes arranged together without any overlaps or any gaps.
   D A group of shapes positioned in such a way to form an attractive pattern.

2 Which of the following words best matches the mathematical term *congruence*?
   A Parallel **B** Similar **C** Related **D** Identical

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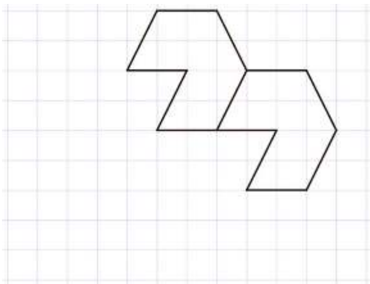
3 The following image shows the start of a tessellation, with two identical shapes joined together. Continue the tessellation by adding on another six identical shapes.



Fluency

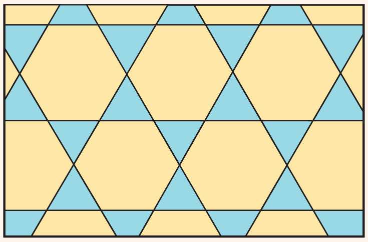
4–6, 8 5–8

4 Using the following trapezium shape  draw ten identical shapes to show that a trapezium will tessellate. You can translate, rotate or reflect this shape to form your tessellation.

5 Draw regular tessellations using only:
a triangles
b squares
c hexagons

Example 13 Naming tessellations

By considering any vertex, name the following semi-regular tessellation.

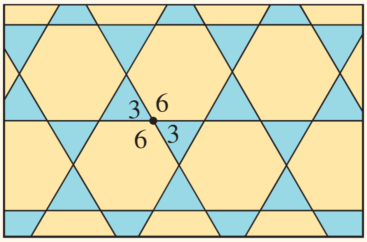


Solution

(3.6.3.6)

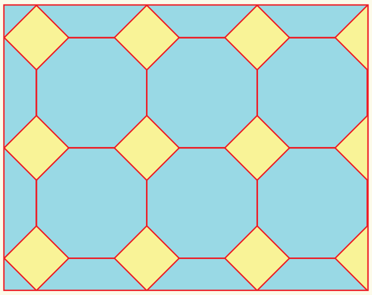
Explanation

Select any vertex and as you go around the vertex, count the number of sides each polygon has.



Now you try

By considering any vertex, name the following semi-regular tessellation.



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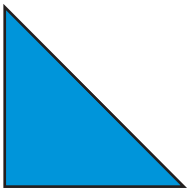
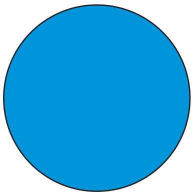
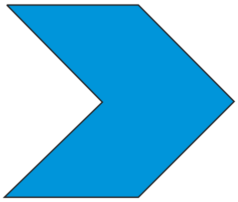
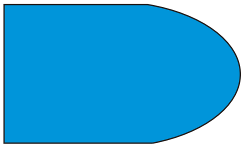
6 By looking at vertices, label each of the tessellations drawn in Question 5.

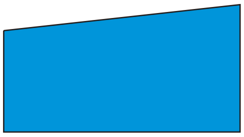
7 Which of the following shapes tessellate by themselves? Reflections, rotations and translations of the original shape can be used.

a 

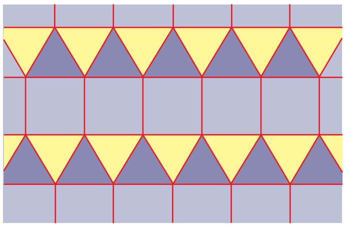
b 

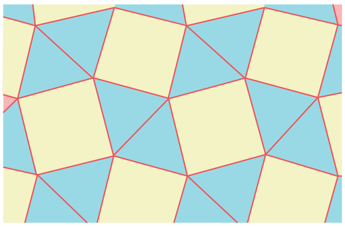
c 

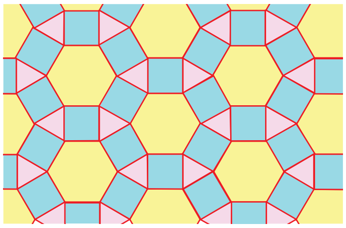
d 

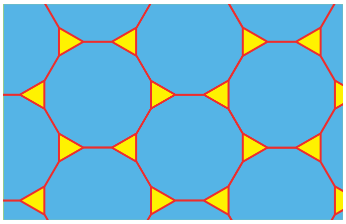
e 

8 By considering any vertex, name the following semi-regular tessellations.

a 

b 

c 

d 

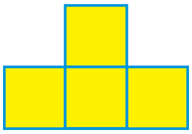
Problem-solving and reasoning

9, 10 | 10, 11

9 Design a tessellation using the following. Use rotations, reflections and translations if needed.

a only the following shape

b only the following shape



c any combination of the above two shapes

10 Produce a tessellation using only regular octagons and squares.

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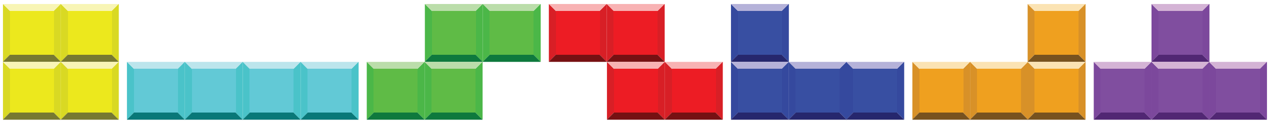
Cambridge University Press

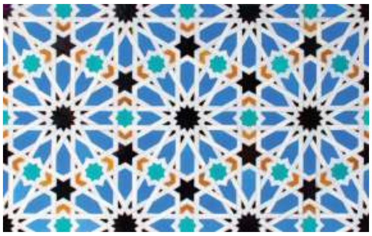
11 The object of the game *Tetris* is to produce rows with no gaps, or in other words to produce a tessellation with the tiles as they appear. Using 1 cm grid paper, draw a large rectangle of width = 10 cm and height = 20 cm.
The following image shows the seven different *Tetris* pieces, with each small cube representing a 1 cm × 1 cm square.



a How many *Tetris* pieces will be needed to completely fill the 10 cm × 20 cm rectangle?
b Using at least three of each piece, design a tessellated pattern to fill the 10 cm × 20 cm rectangle.

Ancient and modern tessellations — 12, 13

12 Ancient tessellations
During the Middle Ages the Moorish people, particularly of Spain, were well known for their distinctive and elaborate tile designs. Several images are shown below.



a Carry out research on Moorish tile designs and print two of your favourite tessellations.
b Using grid paper, design your own intricate 10 × 10 tile, consisting of a range of simple coloured shapes which tessellate and completely cover the tile.
c Either by hand or using appropriate geometry software, repeatedly draw your intricate tile to show how it tessellates and see how effective it looks as a design that could go in a modern home.

13 Modern tessellations
The Dutch artist M.C. Escher (1898–1972) is famous for making irregular tessellations involving repeated images which gradually change form. An example of Escher's work is shown below.
a Carry out research on M.C. Escher and print two of your favourite Escher designs.
b Either by hand or using appropriate geometry software, design your own irregular tessellation consisting of the one repeated image.

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10G Congruence and quadrilaterals ★

Learning intentions

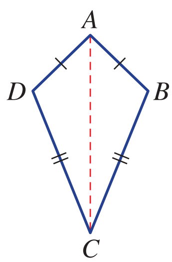
- To understand that properties of special quadrilaterals (e.g. kites, parallelograms) can be proved using congruent triangles.
- To be able to prove properties of special quadrilaterals using facts about congruent triangles.

Key vocabulary: congruence, quadrilateral, bisect, parallelogram, rhombus, rectangle, square, trapezium, kite

The properties of special quadrilaterals, including the parallelogram, rhombus, rectangle, square, trapezium and kite, can be examined more closely using congruence. By drawing the diagonals and using the tests for the congruence of triangles we can prove many of the properties of these special quadrilaterals.

Lesson starter: Why are a pair of opposite angles in a kite equal?

A kite with two pairs of equal length sides can be divided into two triangles, as shown.



- Are these two triangles congruent?
- Which congruent triangle test (SSS, SAS, AAS, RHS) would be used to confirm this?
- What does this say about $\angle B$ and $\angle D$?

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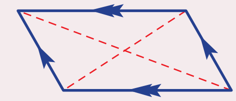
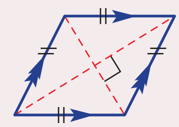
Key ideas

This is a summary of the properties of the special quadrilaterals. Many of the proofs of these properties will be considered in the following exercise.

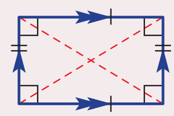
- **Parallelogram** – A **quadrilateral** with two pairs of parallel sides.
  - Opposite sides are equal.
  - Opposite angles are equal.
  - Diagonals **bisect** each other. (*Bisect* means to cut in half.)



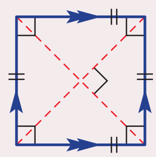
- **Rhombus** – A parallelogram with all sides equal.
  - Opposite angles are equal.
  - Diagonals bisect each other at 90°.
  - Diagonals bisect the interior angles.



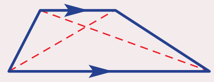
- **Rectangle** – A parallelogram with all interior angles 90°.
  - Opposite sides are equal.
  - Diagonals bisect each other.
  - Diagonals are equal in length.



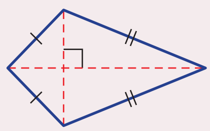
- **Square** – A rectangle with all sides equal.
  - Diagonals bisect each other at 90°.
  - Diagonals are equal in length.



- **Trapezium** – A quadrilateral with one pair of parallel sides.



- **Kite** – A quadrilateral with two pairs of equal sides.
  - One pair of opposite angles are equal.
  - Diagonals bisect each other at right angles.
  - One of the diagonals is bisected by the other.



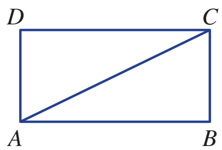
Exercise 10G

Understanding — 1–4 — 4

1 SSS is one test for congruence of triangles. Write down the other three.

2 Name the side (e.g. AB) that is common to both triangles in each diagram.

a



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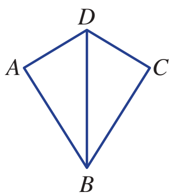

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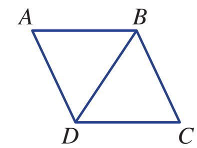

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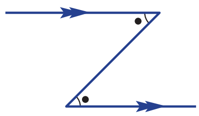
3 Give the reason why the two marked angles are equal.

a


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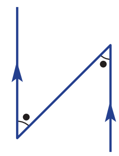

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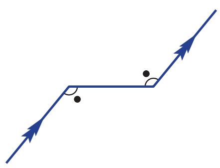
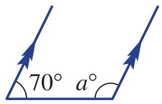


Hint for Q3:
Recall that in parallel lines:
- corresponding angles are equal
- alternate angles are equal
- cointerior angles add to $180°$

4 Give the reason why the two marked angles add to $180°$ and then state the value of a.

a


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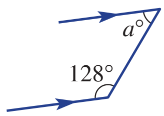


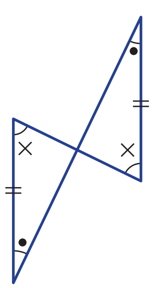
Fluency | 5, 6 | 5, 6

5 Answer true (T) or false (F).

a Opposite sides of a parallelogram are parallel.
b Opposite sides of a kite are equal.
c A trapezium has two pairs of parallel sides.
d The diagonals of a rectangle are equal.
e The diagonals of a kite are equal.
f The diagonals of a parallelogram are equal.
g The diagonals of a trapezium are equal.
h The diagonals of a rhombus are equal.
i The diagonals of a square are equal.
j All angles inside a square are $90°$.
k Opposite angles in a kite are equal.
l The diagonals of a parallelogram intersect at right angles.
m The diagonals of a rhombus intersect at right angles.
n The diagonals of a kite intersect at right angles.
o The diagonals of a rhombus bisect each other.
p The diagonals of a parallelogram bisect each other.
q The diagonals of a rectangle bisect each other.

Hint for Q5: Use the information in the **Key ideas** to help.

6 Which of the four tests for congruence of triangles would be used to prove that each pair of triangles is congruent? Angles and sides with the same markings are equal.

a


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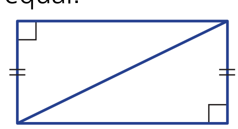

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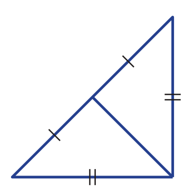

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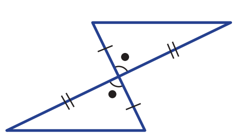

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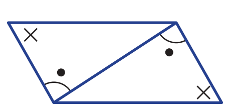
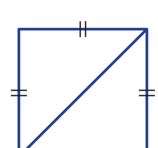


f


Hint for Q5: *Bisect* means to cut in half.

Hint for Q6: Two are SSS, one is SAS, two are AAS and one is RHS.

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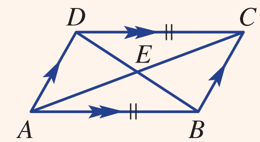
Problem-solving and reasoning 7–9 9–12

Example 14 Exploring the diagonals of a parallelogram

Answer these questions regarding the diagonals of this parallelogram.



a Are the diagonals equal in length?
b What can be said about $\angle BAE$ and $\angle DCE$?
c What can be said about $\angle ABE$ and $\angle CDE$?
d Does $AB = DC$?
e Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABE \equiv \Delta CDE$?
f Why do parallelogram diagonals bisect each other?

Solution

a No.

b They are equal.

c They are equal.

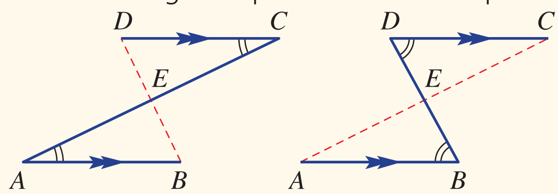
d Yes.

e AAS.

f $\Delta ABE \equiv \Delta CDE$ so $BE = DE$ and $AE = CE$

Explanation

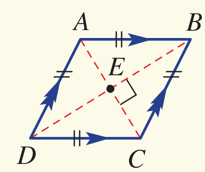
Alternate angles in parallel lines are equal.



The two triangles are congruent using the AAS test. Since ΔABE and ΔCDE are congruent the corresponding sides are equal.

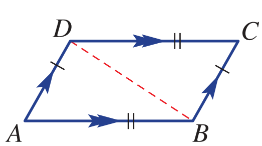
Now you try

Answer these questions regarding the diagonals of this rhombus.



a Are the diagonals equal in length?
b What can be said about $\angle AEB$ and $\angle CED$?
c What can be said about $\angle BAE$ and $\angle DCE$?
d Does $AB = DC$?
e Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABE \equiv \Delta CDE$?
f Why does BD bisect (cut in half) AC?

7 Answer these questions about angles in this parallelogram.



a List two triangles formed by the diagonal.
b What can be said about sides AB and DC?
c What can be said about sides AD and BC?
d Which side is common to both triangles?
e Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABD \equiv \Delta CDB$?
f Why is $\angle A = \angle C$?

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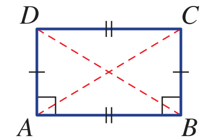
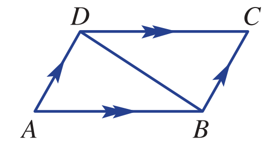
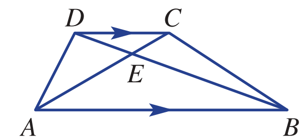
8 Answer these questions about diagonals in this rectangle.
  a Locate ΔABD and ΔBAC. Is $\angle A = \angle B$?
  b Is $AD = BC$?
  c Is $AB = CD$?
  d Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABC \equiv \Delta BAD$?
  e Why is $AC = BD$?



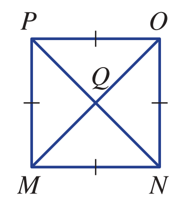
9 A parallelogram $ABCD$ has two pairs of parallel sides.
  a What can be said about $\angle ABD$ and $\angle CDB$ and give a reason?
  b What can be said about $\angle BDA$ and $\angle DBC$ and give a reason?
  c Which side is common to both ΔABD and ΔCDB?
  d Which congruence test would be used to show that $\Delta ABD \equiv \Delta CDB$?
  e If $\Delta ABD \equiv \Delta CDB$, what can be said about the opposite sides of a parallelogram?



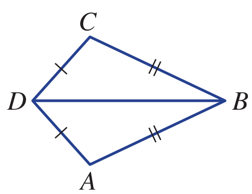
10 A trapezium $ABCD$ has one pair of parallel sides.
  a Which angle is equal to $\angle BAE$?
  b Which angle is equal to $\angle ABE$?
  c Explain why ΔABE is not congruent to ΔCDE.



11 For this square assume that $MQ = QO$ and $NQ = PQ$.
  a Give reasons why $\Delta MNQ \equiv \Delta ONQ$.
  b Give reasons why $\angle MQN = \angle OQN = 90°$.
  c Give reasons why $\angle QMN = 45°$.

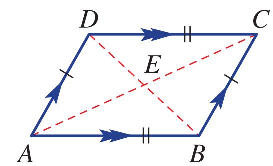


12 Use the information in this kite to prove these results.
  a $\Delta ABD \equiv \Delta CBD$
  b $\angle DAB = \angle DCB$
  c $\angle ADB = \angle CDB$



Writing a formal proof — 13

13 Prove by giving reasons that the diagonals in a parallelogram bisect each other. Opposite sides are equal, so use $AB = CD$. Complete the proof by following these steps.
Step 1. List the pairs of equal angles in ΔABE and ΔCDE giving reasons why they are equal.
Step 2. List the pairs of equal sides in ΔABE and ΔCDE giving reasons why they are equal.
Step 3. Write $\Delta ABE \equiv \Delta CDE$ and give the reason SSS, SAS, AAS or RHS.
Step 4. State that $BE = DE$ and $AE = CE$ and give a reason.



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10H Similar figures

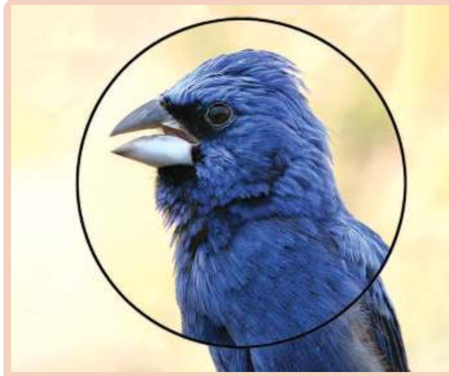
Learning intentions

- To understand that similar figures have the same shape (angles, ratios of sides) but can be of a different size
- To be able to identify corresponding features of a pair of similar figures
- To be able to find the scale factor in a pair of similar figures
- To be able to decide if shapes are similar by considering angles and side ratios

Key vocabulary: similar figures, ratio, scale factor

When you look at an object through a telescope or a pair of binoculars, you expect the image to be much larger than the one you would see with the naked eye. Both images would be the same in every way except for their size. Such images in mathematics are called similar figures.

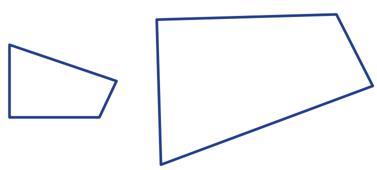
Images that have resulted in a reduction in size (rather than an enlargement in size) are also considered to be similar figures.



An item through a telescope is 'similar' to how it appears to the naked eye.

Lesson starter: Are they similar?

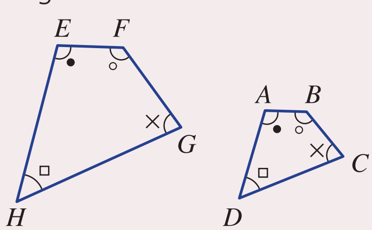
Here are two quadrilaterals.



- Do they look similar in shape? Why?
- How could you use a protractor to help decide if they are similar in shape? What would you measure and check? Try it.
- How could you use a ruler to help decide if they are similar in shape? What would you measure and check? Try it.
- Describe the geometrical properties that similar shapes have. Are these types of properties present in all pairs of shapes that are 'similar'?

Key ideas

- **Similar figures** have the same shape but can be of different size.
- For similar figures, all corresponding (matching) angles are equal.
- For similar figures, all corresponding (matching) sides are in the same ratio.
- The ratio of sides is often written as a fraction, shown here. It is often called the **scale factor**. Unless specified, the scale factor will be from the smaller shape to the larger shape, so will be larger than 1.



$$\text{Scale factor} = \frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD} = \frac{HE}{DA}$$

- If the scale factor is 1, the figures are congruent.

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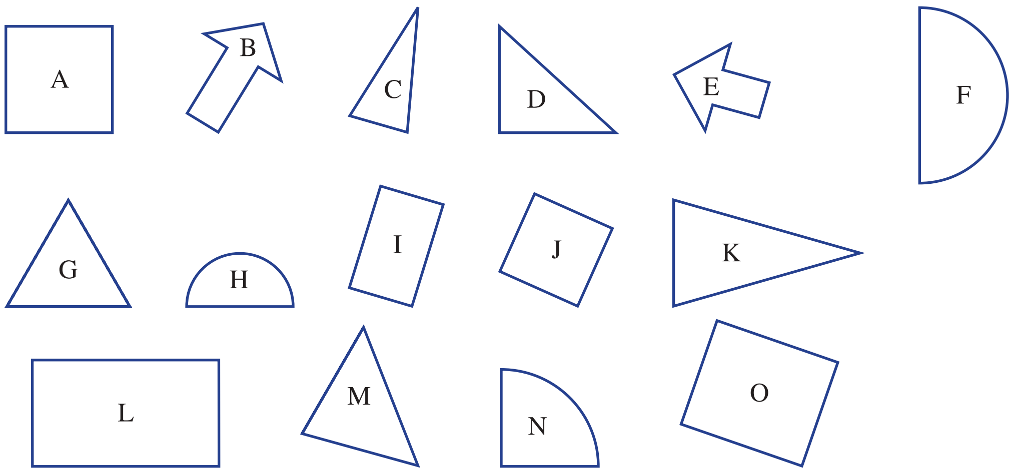
Exercise 10H

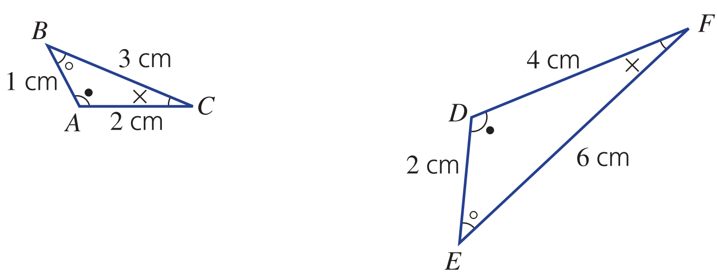
Understanding

	1, 2	1–2

1 State any pairs of shapes that look similar (i.e. same shape but different size).



2 Consider these two triangles.



a Name the angle on the larger triangle which corresponds to:

i $\angle A$ **ii** $\angle B$ **iii** $\angle C$.

b Name the side on the smaller triangle which corresponds to:

i DE **ii** EF **iii** FD.

c Find these scale factors.

i $\dfrac{DE}{AB}$ **ii** $\dfrac{EF}{BC}$ **iii** $\dfrac{FD}{CA}$

d Would you say that the two triangles are similar? Why or why not?

Fluency

	3–5	3–5

Example 15 Identifying corresponding features

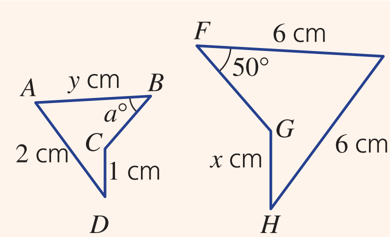
For the pair of similar figures shown on the right, complete these tasks.

a List the pairs of corresponding sides.

b List the pairs of corresponding angles.

c Find the scale factor.

d Find the values of the pronumerals.



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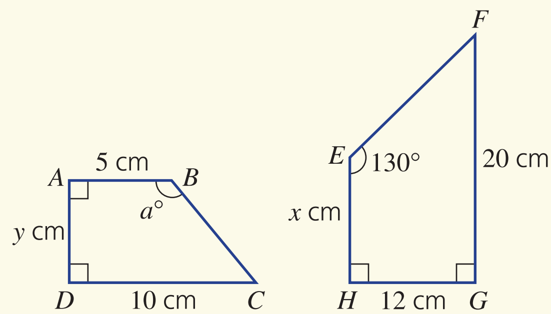
Solution	Explanation
a $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$	Pair up each vertex, noticing that G corresponds with C, H corresponds with D and so on.
b $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$	$\angle G$ and $\angle C$ are clearly the largest angles in their respective shapes. Match the other angles in reference to these angles.
c $\frac{HE}{DA} = \frac{6}{2} = 3$	HE and DA are corresponding sides both with given measurements. Divide the larger by the smaller.
d $a = 50$	$\angle B$ and $\angle F$ are equal corresponding angles.
$x = 3 \times 1 = 3$ cm	The scale factor $\frac{HE}{DA} = 3$ so $\frac{GH}{CD}$ should also equal 3.
$y = 6 \div 3 = 2$ cm	Alternatively, say that GH is 3 times the length CD. Similarly, EF should be 3 times the length AB (y cm).

Now you try

For the following pair of figures complete these tasks.

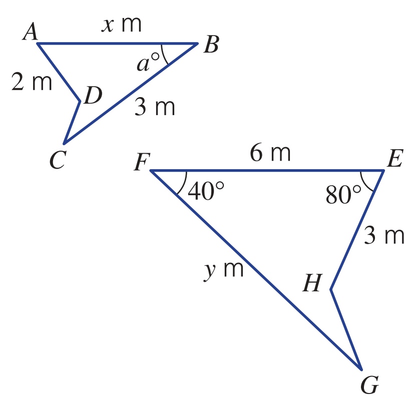
a List the pairs of corresponding sides.
b List the pairs of corresponding angles.
c Find the scale factor.
d Find the values of the pronumerals.



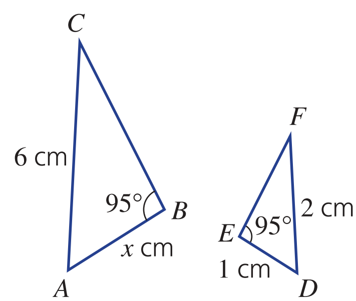
3 For the following pairs of similar figures, complete these tasks.

i List the pairs of corresponding sides.
ii List the pairs of corresponding angles.
iii Find the scale factor.
iv Find the values of the pronumerals.

a



b



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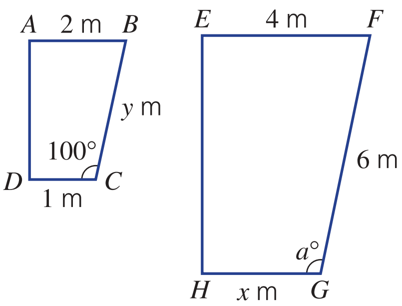
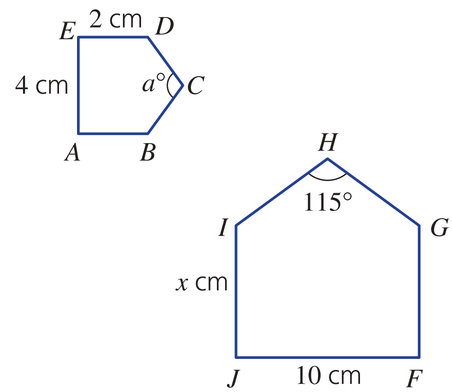
4 For the following pairs of similar figures, complete these tasks.

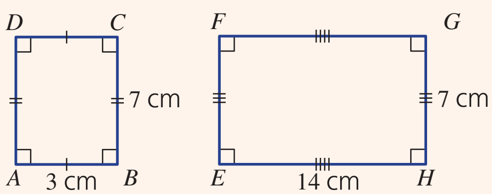
  i List the pairs of corresponding sides.
  ii List the pairs of corresponding angles.
  iii Find the scale factor.
  iv Find the values of the pronumerals.

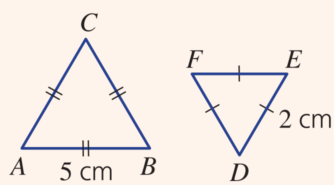
a 

b 

Example 16 Deciding if shapes are similar

Decide if these shapes are similar by considering corresponding angles and the ratio of sides.

a 

b 

Solution

a All corresponding angles are equal.

$$\frac{EH}{BC} = \frac{14}{7} = 2$$

$$\frac{GH}{AB} = \frac{7}{3} = 2.\dot{3}$$

Scale factors are not equal.

Shapes are not similar.

Explanation

All angles are 90° and so corresponding angles are equal.

Match pairs of sides to find scale factors.

The scale factor needs to be equal if the shapes are to be similar.

Solution

b All angles are 60°

$$\frac{AB}{DE} = \frac{5}{2} = 2.5$$

$$\frac{BC}{EF} = \frac{5}{2} = 2.5$$

$$\frac{CA}{FD} = \frac{5}{2} = 2.5$$

Triangles are similar.

Explanation

All angles in an equilateral triangle are 60°.

All scale factors are equal so the triangles are similar.

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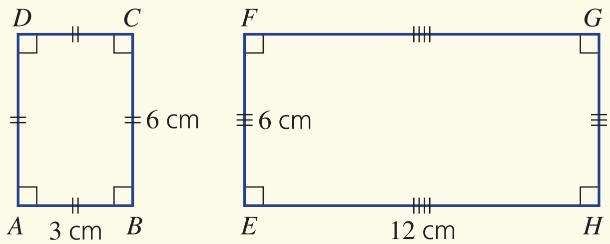
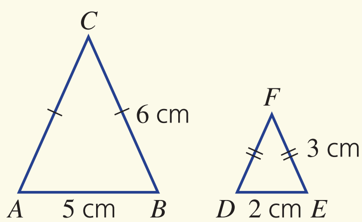
Now you try

Decide if these shapes are similar by considering corresponding angles and the ratio of sides.

a

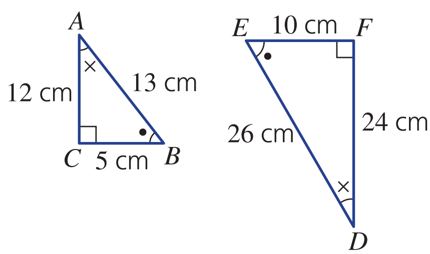


b



5 Decide if these shapes are similar by considering corresponding angles and the ratios of sides.

a



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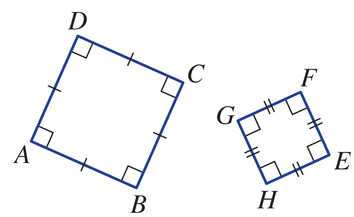

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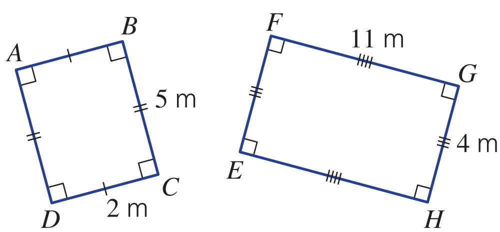

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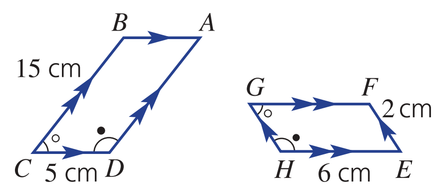
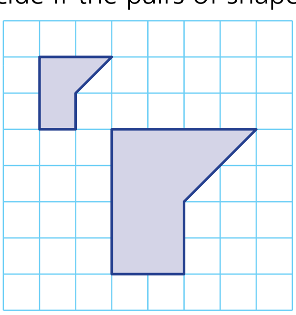


Problem-solving and reasoning

6–8 | 6, 8, 9

6 Decide if the pairs of shapes on these grids are similar. If so, state the scale factor.

a



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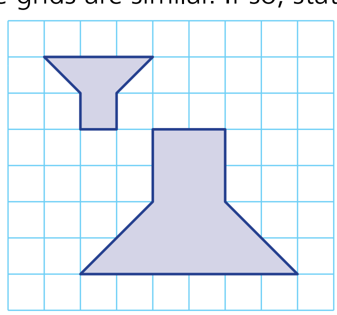
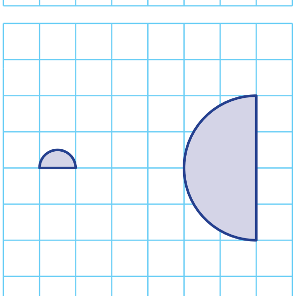


Hint for Q6: To find the scale factor, divide the lengths of corresponding sides.

c



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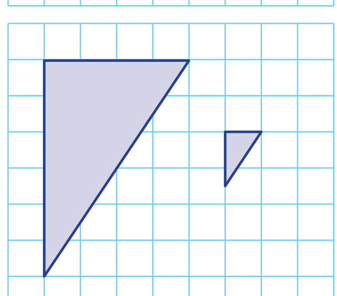

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10H

7 Two rectangular picture frames are similar in shape. A corresponding pair of sides are of length 50 cm and 75 cm. The other side length on the smaller frame is 70 cm. Find the perimeter of the larger frame.

8 Two similar triangles have a scale factor of $\frac{7}{3}$. If the larger triangle has a side length of 35 cm, find the length of the corresponding side on the smaller triangle.

9 A square photo of area 100 cm^2 is enlarged to an area of 900 cm^2. Find the scale factor of the side lengths of the two photos.

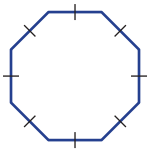
What's true about these shapes?

	—	10, 11

10 Are the following statements true or false? Give reasons for your answers.

a All squares are similar.
b All rectangles are similar.
c All equilateral triangles are similar.
d All isosceles triangles are similar.
e All rhombuses are similar.
f All parallelograms are similar.
g All kites are similar.
h All trapeziums are similar.
i All circles are similar.

11 **a** If a regular polygon such as this regular octagon is enlarged, do the interior angles change?



b Are all polygons with the same number of sides similar? Give reasons.

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10I Similar triangles

Learning intentions

- To understand that determining whether triangles are similar can be done using the similarity tests AAA, SSS, SAS and RHS
- To be able to decide if two triangles are similar
- To be able to find missing lengths using a pair of similar triangles

Key vocabulary: similarity test, similar triangles

Finding the approximate height of a tree or the width of a gorge using only simple equipment is possible without actually measuring the distance directly. Similar triangles can be used to calculate distances without the need to climb the tree or cross the gorge. It is important, however, to ensure that if the mathematics of similar triangles is going to be used, then the two triangles are in fact similar. We learned earlier that there were four tests that help to determine if two triangles are congruent. Similarly, there are four tests that help establish whether or not triangles are similar. Not all side lengths or angles are required to prove that two triangles are similar.

Before building the Landwasser Viaduct, Switzerland, surveyors could use similar triangles to calculate the width of this 70 m deep gorge. Then, using algebra, geometry and trigonometry, engineers designed a suitable bridge.

Lesson starter: How much information is enough?

Given a certain amount of information, it may be possible to draw two triangles that are guaranteed to be similar in shape.

Decide if the information given is enough to guarantee that the two triangles will always be similar. If you can draw two triangles (ΔABC and ΔDEF) that are not similar, then there is not enough information provided.

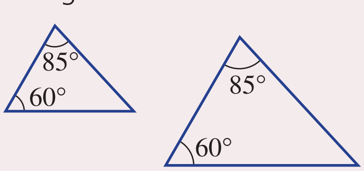
- $\angle A = 30°$ and $\angle D = 30°$
- $\angle A = 30°$, $\angle B = 80°$ and $\angle D = 30°$, $\angle E = 80°$
- $AB = 3$ cm, $BC = 4$ cm and $DE = 6$ cm, $EF = 8$ cm
- $AB = 3$ cm, $BC = 4$ cm, $AC = 5$ cm and $DE = 6$ cm, $EF = 8$ cm, $DF = 10$ cm
- $AB = 3$ cm, $\angle A = 30°$ and $DE = 3$ cm, $\angle D = 30°$
- $AB = 3$ cm, $AC = 5$ cm, $\angle A = 30°$ and $DE = 6$ cm, $DF = 10$ cm, $\angle D = 30°$

Key ideas

- Two triangles are similar if:

  AAA

  - Three pairs of corresponding angles are equal. If two pairs are known, then the angle sum of a triangle can be used to show that the third pair of angles are also equal.



$$\begin{aligned}\text{Missing angle} &= 180° - (60° + 85°)\\ &= 35°\end{aligned}$$

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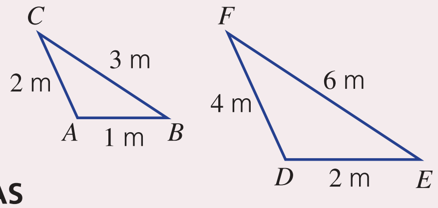
10I

SSS

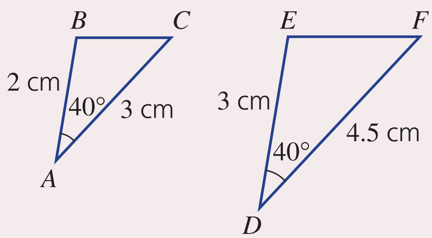
- All pairs of corresponding sides are in the same ratio.



$$\frac{DE}{AB}=\frac{EF}{BC}=\frac{DF}{AC}=2$$

SAS

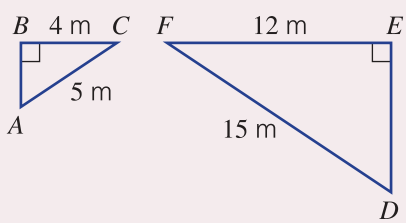
- Two pairs of corresponding sides are in the same ratio and the included angles are equal.



$$\frac{DE}{AB}=\frac{DF}{AC}=1.5$$

RHS

- A pair of right angles, the pair of hypotenuse lengths and another pair of corresponding sides are in the same ratio.



$$\frac{DF}{AC}=\frac{EF}{BC}=3$$

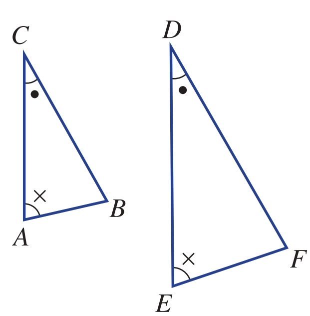
- If two triangles ΔABC and ΔDEF are similar, we write $\Delta ABC \sim \Delta DEF$ or $\Delta ABC \,|||\, \Delta DEF$.
  - This is called a similarity statement.
  - Letters are usually written in matching order.

Exercise 10I

Understanding	1, 2	1, 2

1 Give a similarity statement for these pairs of similar triangles, e.g. $\Delta ABC \sim \Delta FED$ or $\Delta ABC \,|||\, \Delta DEF$. Be careful with the order of letters and make sure each letter matches the corresponding vertex.

a



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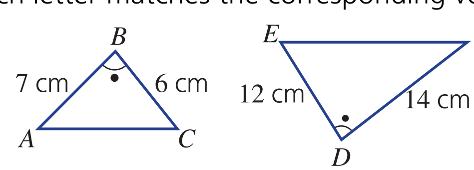

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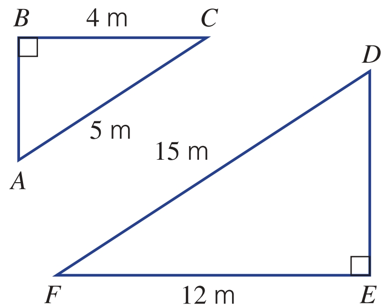
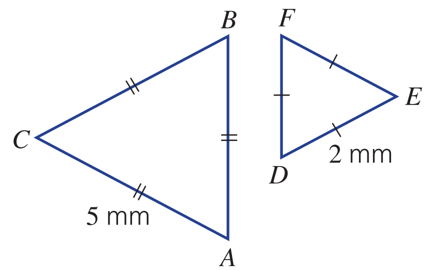
c



d



2 For the pairs of triangles in Question **1**, which of the four similar triangle tests (AAA, SSS, SAS or RHS) would be used to show their similarity? See the **Key ideas** for a description of each test. There is one pair for each test.

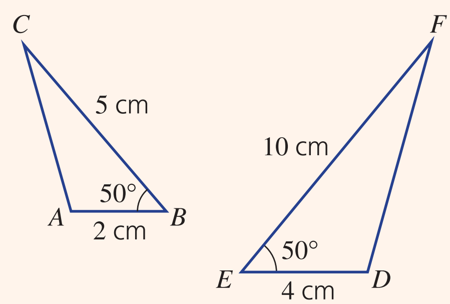
Fluency

	3–5	4, 5

Example 17 Explaining why two triangles are similar

Explain, with reasons, why these pairs of triangles are similar.

a



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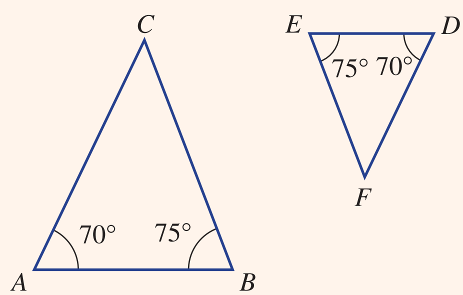

Solution

a $\dfrac{ED}{AB} = \dfrac{4}{2} = 2$

$\angle B = \angle E = 50°$

$\dfrac{EF}{BC} = \dfrac{10}{5} = 2$

$\therefore \Delta ABC$ is similar to ΔDEF

Using SAS

Explanation

Work out the ratio of the two pairs of corresponding sides to see if they are equal. Note that the angle given is the included angle between the two given pairs of corresponding sides.

b $\angle A = \angle D$

$\angle B = \angle E$

$\therefore \Delta ABC$ is similar to ΔDEF

Using AAA

Two pairs of equal angles are given. This implies that the third pair of angles are also equal and that the triangles are similar.

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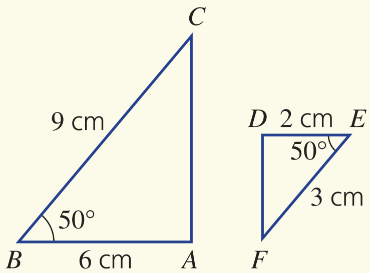
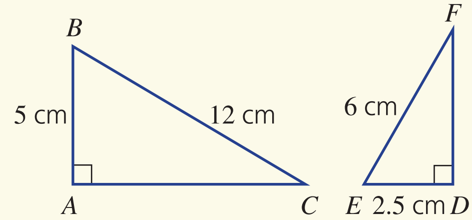
Now you try

Explain, with reasons, why these pairs of triangles are similar.

a



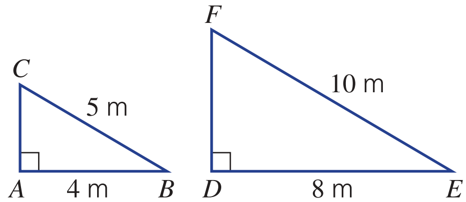
b



3 Explain, with reasons, why these pairs of triangles are similar.

Hint for Q3: The **Key ideas** section lists the tests for similarity.

a



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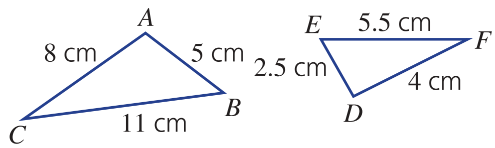
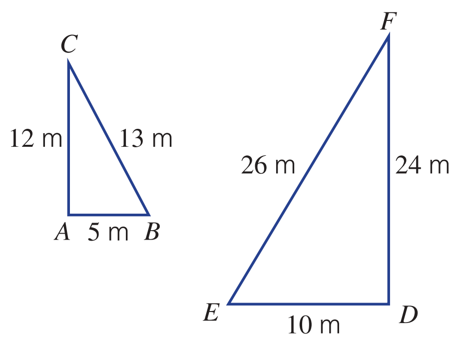


4 Explain, with reasons, why these pairs of triangles are similar. State which of the four tests (AAA, SSS, SAS or RHS) applies.

a



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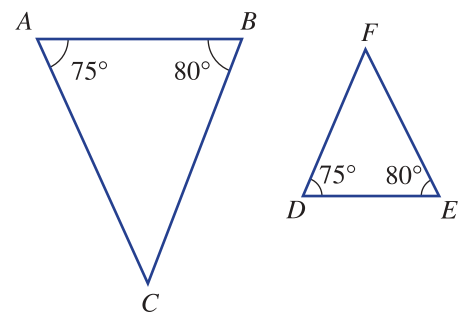

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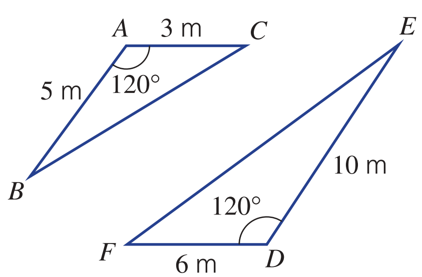

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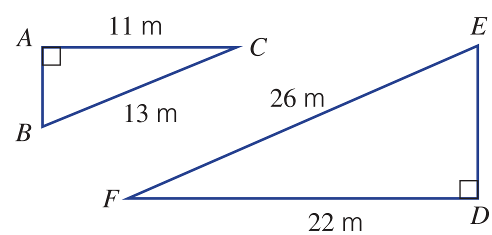

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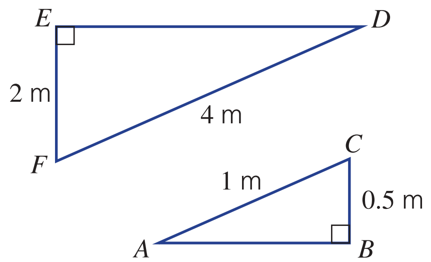

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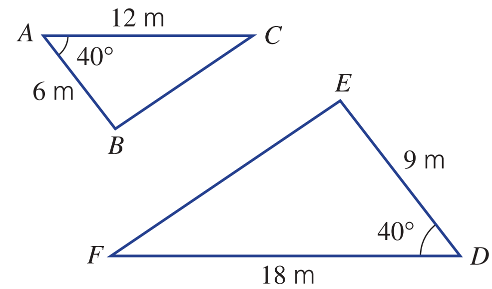

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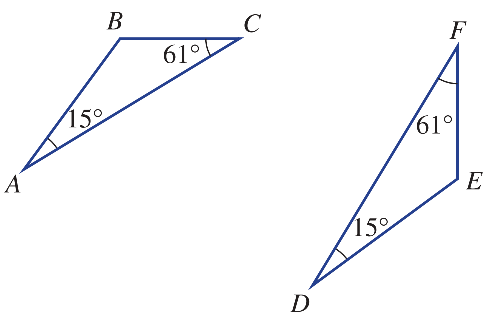

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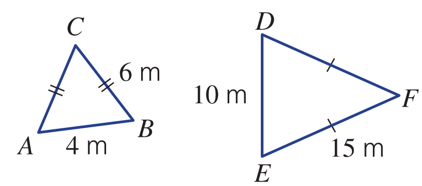
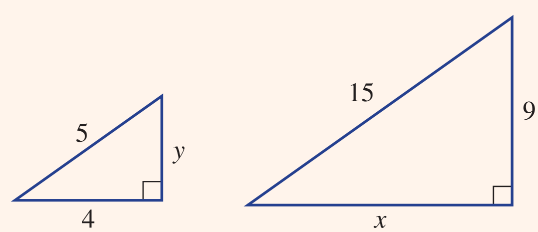


Example 18 Finding a missing length

Given that these pairs of triangles are similar, find the value of the pronumerals.



Solution

Scale factor $= \frac{15}{5} = 3$

$4 \xrightarrow{\times 3} x \quad \therefore \quad x = 12$

$y \xrightarrow{\times 3} 9 \quad \therefore \quad y = 3$

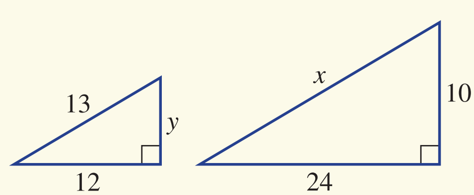
Explanation

First find the scale factor.

Multiply or divide by the scale factor to find the values of the pronumerals.

Now you try

Given that this pair of triangles are similar, find the value of the pronumerals.



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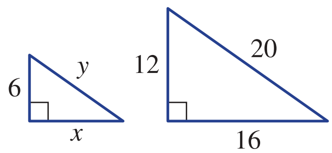
5 Given that these pairs of triangles are similar, find the value of the pronumerals.

a



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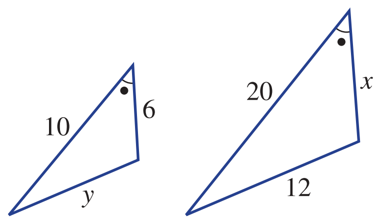

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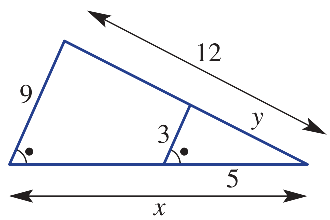

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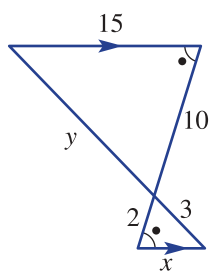
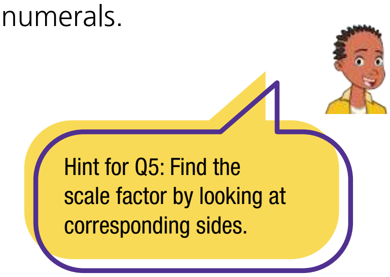




Problem-solving and reasoning

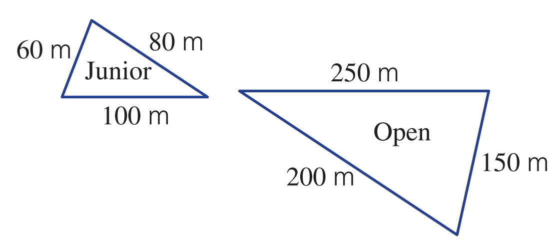
6–8 7–9

6 Two triangular tracks are to be used for the junior and open divisions for a school event.

a Work out the scale factor for each of the three corresponding pairs of sides.

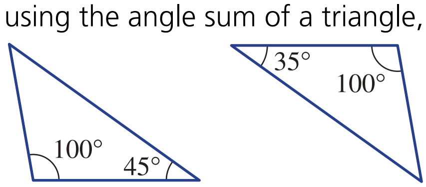
b Are the two tracks similar in shape? Give a reason.

c How many times would a student need to run around the junior track to cover the same distance as running one lap of the senior track?



7 By using the angle sum of a triangle, decide if the given pairs of triangles are similar.

a



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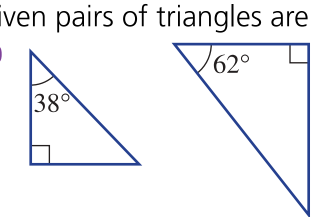

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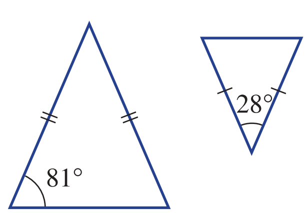

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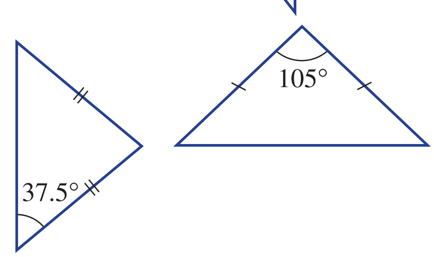

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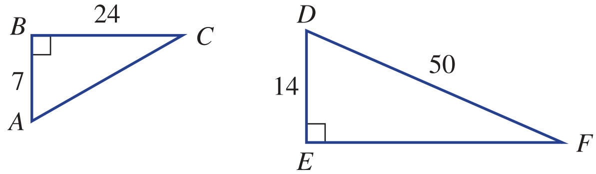
8 Explain why only two pairs of angles are required to prove that two triangles are similar.

9 Show how Pythagoras' theorem can be used to prove that these two triangles are similar.

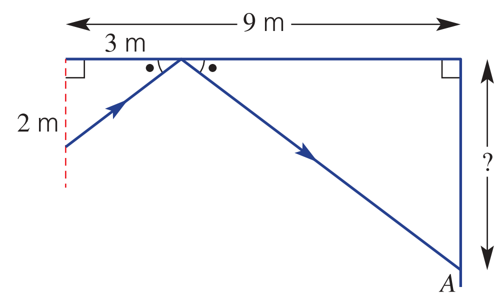


Hint for Q9: Find the missing side length in each triangle first.

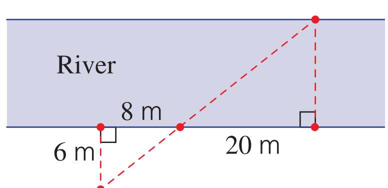
Rivers and minigolf

Rivers and minigolf	—	10, 11

10 In a game of minigolf, a ball bounces off a wall as shown. The ball proceeds and hits another wall at point A. How far down the right side wall does the ball hit?



11 Trees on a river bank are to be used as markers to form two triangular shapes. Each tree is marked by a red dot as shown in the diagram.



- **a** Are the two triangles similar? Explain why.
- **b** Calculate the scale factor.
- **c** Calculate how far it is across the river.

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Maths@Work

Fashion designer

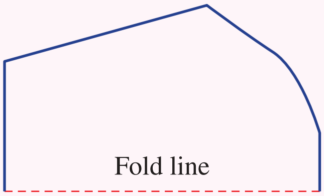
The fashion industry is a glamorous but extremely competitive market where many talented designers fail to survive. A degree in Fashion Design provides up-to-date technical skills and the opportunity to meet industry professionals.

Aspiring designers should study fashion trends, take art classes, be inventive with colour and learn to sew a variety of fabrics. Designers need to budget money to survive without income at first. A good working relationship with a manufacturer is vital to success, and it is also important to be resilient since designers often have their work critically assessed.

Mathematics is an essential tool in fashion, both in the business aspect and especially in the technical package development. Garment pattern and fabric placement, cutting angles, accurate measurements and design geometry, are all examples of how mathematics is embedded in fashion.

1 Many patterns use symmetry in their designs. Copy (e.g. trace) and complete each design below, using each dashed line as the axis of symmetry.

a



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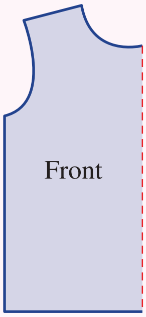

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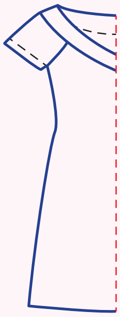

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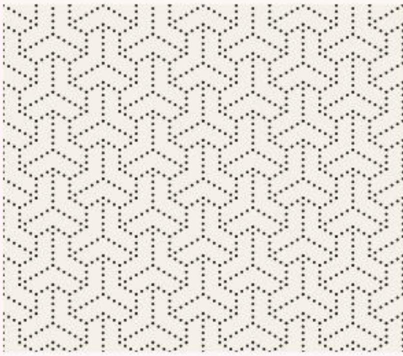
2 Designers often use transformations when designing their own fabrics. Describe the transformations used in each of the following fabric designs.

a


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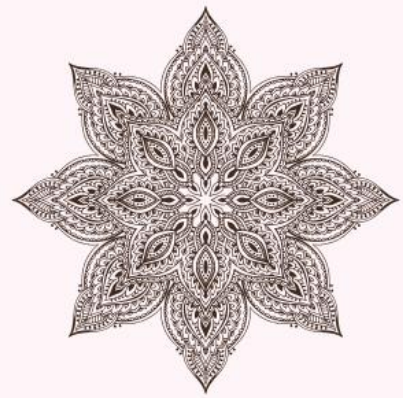
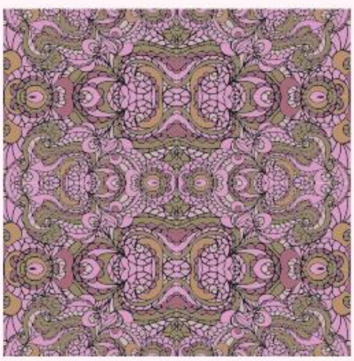


c


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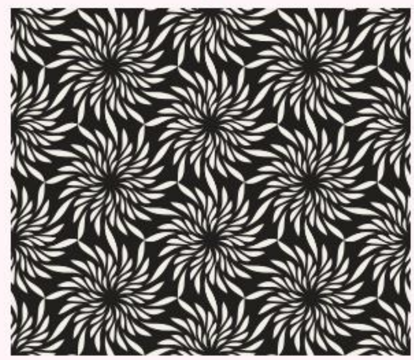
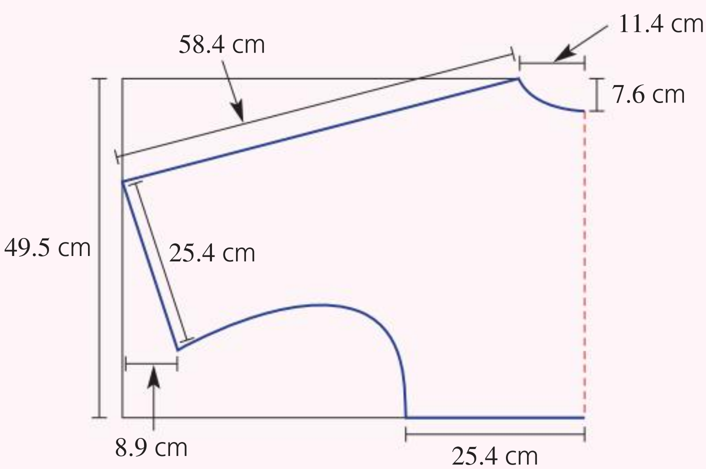


3 Garment pattern makers develop a designer's sketch for the manufacturer. It is a very technical and detailed process resulting in a pattern that can be successfully sewn by numerous workers in a large factory. Garments need to be manufactured in identical (i.e. congruent) sets of each size as well as scaled copies (i.e. similar sets) that are the various sizes.

Reproduce an exact copy of the scale drawing below of a garment pattern. First make accurate measurements of lengths and angles.



Using technology

4 Use geometry software to digitally design a fabric pattern using geometrical transformations. Colour the fabric in three different combinations for presentation to a commercial fabric manufacturer.

Explain the transformations you have used in your design.

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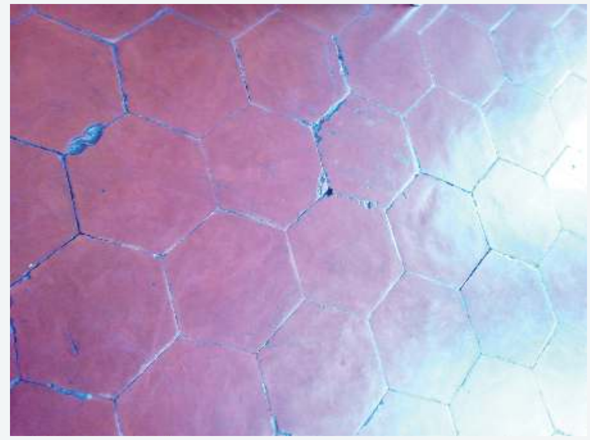
Modelling

Restoring old tiles

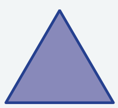
Isaac is in the business of cutting new tiles to replace broken tiles in old houses. The old tiles are mostly regular polygons, so he focuses on these types of shapes for his new cuts.

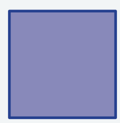
Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

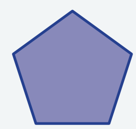


1 Preliminary task

a Name each of the following regular polygons.

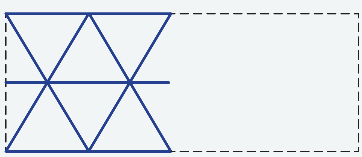
i 

ii 

iii 

b Isaac cuts a number of equilateral triangles of equal size for a tiling job.

i Copy and complete the drawing below to show how such tiles can join together without gaps (tessellate).



ii At one vertex point inside your tessellation, determine all the angles surrounding that point.

c Repeat part **b** if square tiles are used.

2 Modelling task

Formulate

a The problem is to determine the types of shapes that Isaac can use to form tessellations for the purposes of tiling. Write down all the relevant information that will help solve this problem.

b Describe what it means for a shape to tessellate, illustrating your description with one or more diagrams.

Solve

c Apart from an equilateral triangle and a square, there is only one other regular polygon that Isaac can use that tessellates by itself. State the shape and illustrate how it tessellates.

d Try to construct a tessellation using only octagons of equal size. Explain why Isaac cannot use only octagons for a tessellating tile pattern. Justify your response using a diagram.

e Isaac decides to use two different regular polygon shapes to make a tile pattern.

i If he uses an octagon as one of the shapes, determine what other shape is required to form the tessellation. Justify using a drawing.

ii If he uses only equilateral triangles and squares, determine how a tessellation can be formed. Justify using a drawing.

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Evaluate and verify

f Isaac's favourite three regular polygon tiles are the hexagon, square and equilateral triangle. Explore if it is possible for Isaac to combine all three shapes to form a tile tessellation. Illustrate your solution using a drawing and also determine the angles at one of the vertices inside the tessellation.

Communicate

g Summarise your results and describe any key findings.

3 Extension questions

a We know that there are only three regular polygons that tessellate by themselves. If two or more different regular polygons tessellate together, these are called semi-regular tessellations. Draw some examples of how semi-regular tessellations could be used to tile a region.

b Find out how many possible tessellations exist if:

i two regular polygons are used

ii any number of regular polygons can be used.

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Flowchart geometric proofs

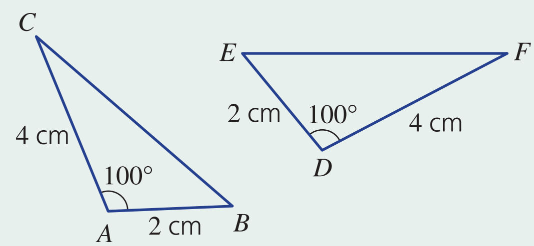
Key technology: Dynamic geometry

A proof in mathematics involves a sequence of ordered and logical steps which show that a given statement is true. An example of this in geometry is an argument that two triangles are congruent where observations about pairs of angles and side lengths are listed and connected to one of the tests for congruence, namely: SSS, SAS, AAS or RHS.

1 Getting started

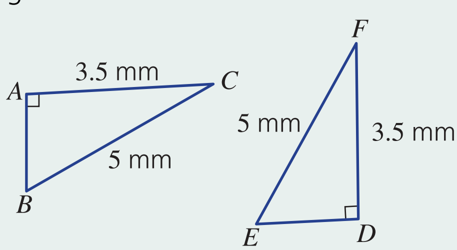
a Consider this pair of triangles.

- **i** List the corresponding pairs of equal length sides.
- **ii** List any pairs of corresponding equal angles.
- **iii** Which of the reasons (SSS, SAS, AAS or RHS) would you choose to show that the pair of triangles are equal?



b By following the steps in part **a**, complete a proof showing that the following pairs of triangles are congruent.

i



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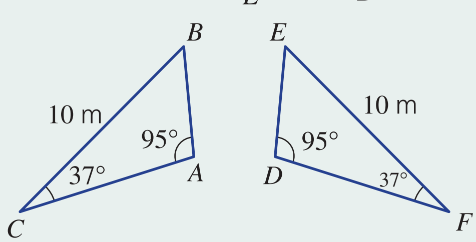

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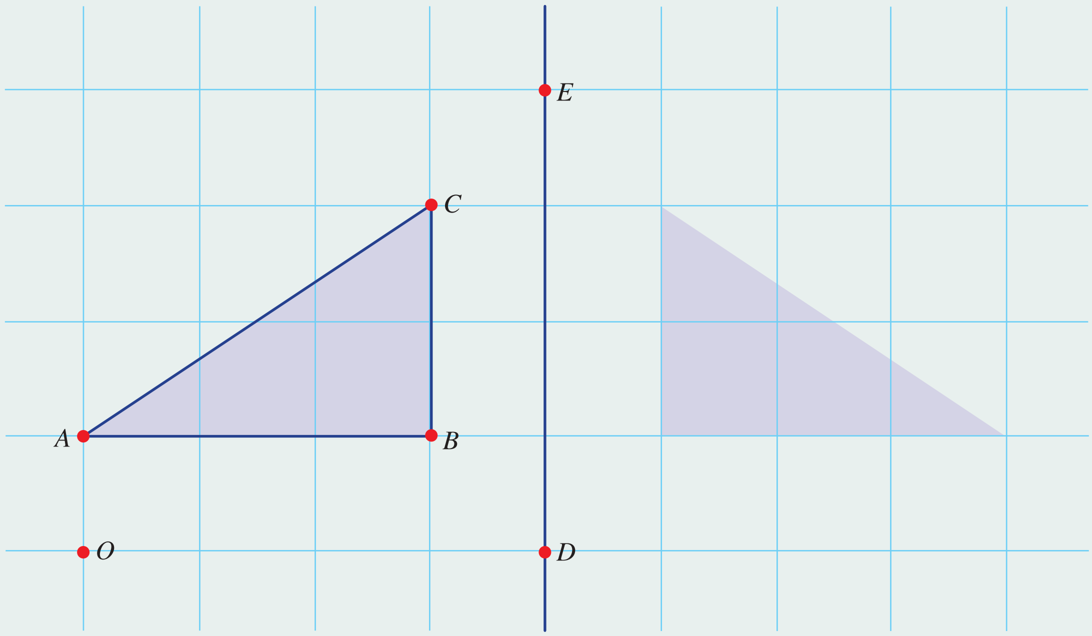
2 Using technology

We will explore congruent shapes using transformations and a dynamic geometry package like Desmos Geometry, or Geogebra.



a Construct a triangle using the Polygon tool. Remember to finish the construction by selecting the first point again. Move each vertex so that it snaps onto the grid. Label your triangle ABC.

b Construct the line DE and point O as shown.

c Reflect triangle ABC in the mirror line DE by using the Reflection tool in the Transform and Define Transformation menu.

d Decide if the image created in part **c** is congruent to triangle ABC. Give reasons.

e Drag one of the points A, B, C, D or E and describe what you notice. Is the image always congruent to triangle ABC?

f Now rotate triangle ABC about the point O. Use the Rotation tool in the Transform and Define Transformation menu. Experiment by changing the angle of rotation or by altering the positions of points A, B, C or O. Is the image always congruent to triangle ABC? Give reasons.

g Now translate triangle ABC using two points F and G. Use the Translation tool in the Transform and Define Transformation menu. Experiment by changing the positions of points F and G or by altering the positions of points A, B or C. Is the image always congruent to triangle ABC? Give reasons.

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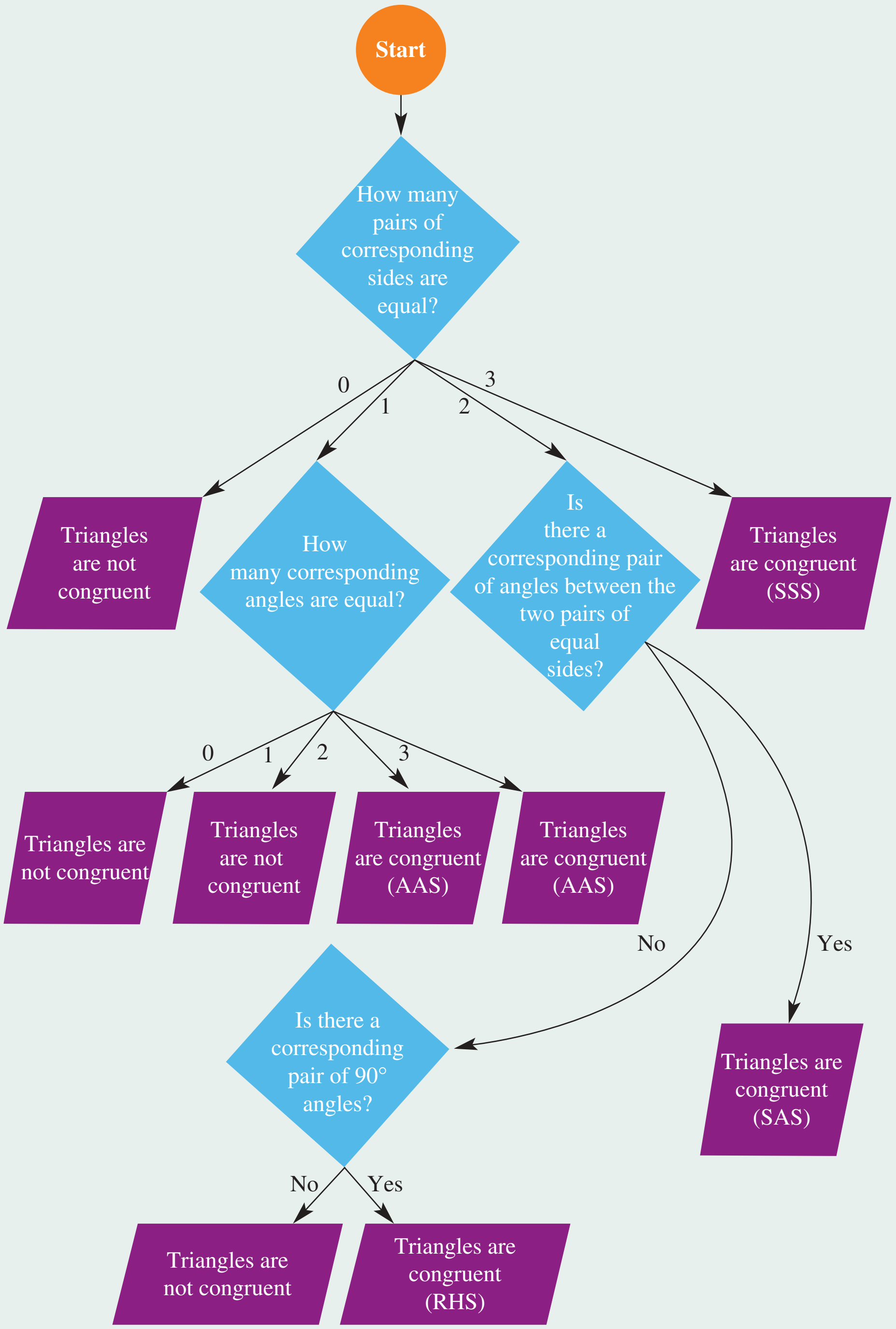
3 Applying an algorithm

Here is a flow chart that decides if two triangles are congruent and gives the abbreviated reason.



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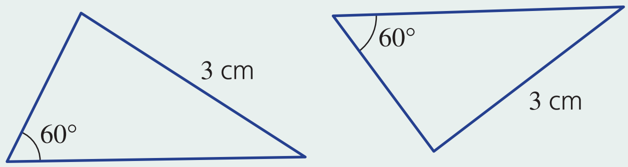
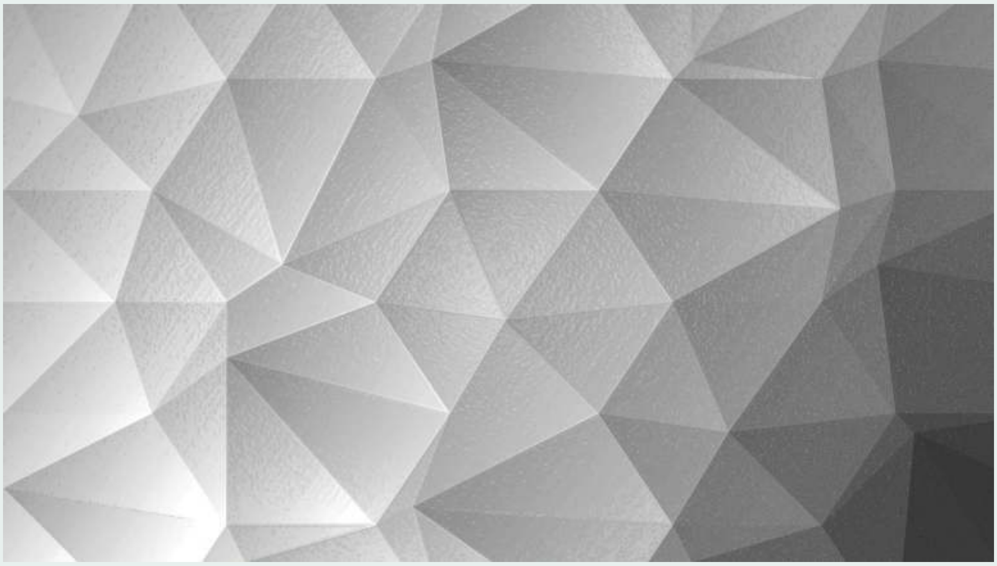
a Run through the flow chart for the three pairs of triangles outlined in parts **1a** and **b**. Does the result agree with the outcome from part **1**?

b Run through the algorithm in the flow chart for this pair of triangles. Decide if the flow chart delivers the correct result.



c One of the reasons out of SSS, SAS, AAS and RHS was not used on part **1** earlier. Decide which one was not used and draw a set of triangles which would use this remaining reason to prove that they are congruent. Check your drawings and conclusions by running through the flow chart.



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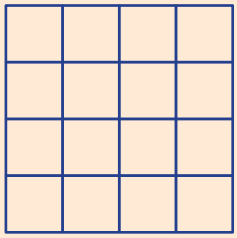
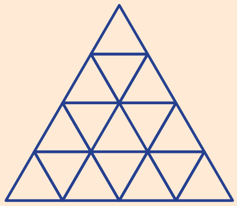
Puzzles and games

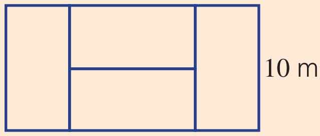
1 How many squares are there in this diagram?



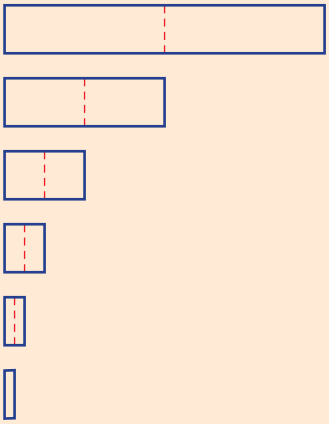
2 How many triangles are there in this diagram?



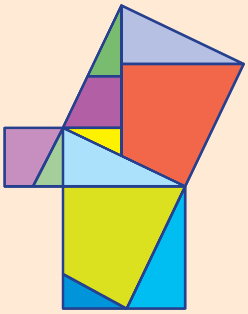
3 The four rectangles inside this diagram are congruent. What is the perimeter of each rectangle?



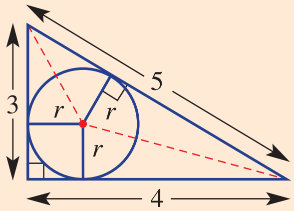
4 A strip of paper is folded 5 times in one direction only. How many creases will there be in the original strip when it is folded out?



5 Can you fit the shapes in the two smaller squares into the largest square? Try drawing or constructing the design and then use scissors to cut out each shape.



6 Use congruent triangles to find the radius (r) in this diagram.



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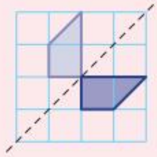
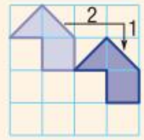
Chapter summary

Transformations and congruence

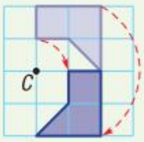
Reflection



Translation vector (2, –1)

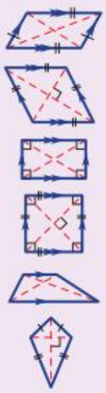


Rotation clockwise by 90°

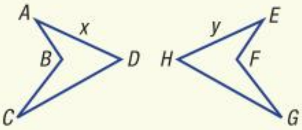


Quadrilaterals and congruence

- Parallelogram
- Rhombus
- Rectangle
- Square
- Trapezium
- Kite



Congruent figures: same size and shape

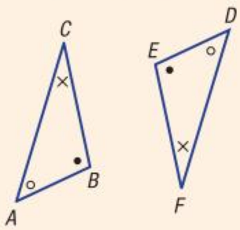


$ABCD \equiv EFGH$ $\quad x = y$

$AB = EF$

$\angle C = \angle G$

Similar triangles



$\Delta ABC \,|||\, \Delta DEF$

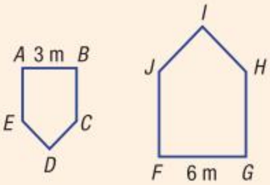
or $\Delta ABC \sim \Delta DEF$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Tests: AAA, SSS, SAS or RHS (see **Key ideas**)

Similar figures



Ratio $= \frac{FG}{AB} = \frac{6}{3} = 2$

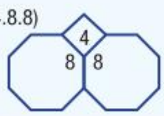
$\angle A = \angle F$ etc.

$GH = AE \times 2$

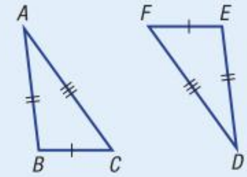
Tessellations

- Patterns including shapes with no gaps.
- Named by counting the number of sides of each polygon at a vertex.

Name: (4.8.8)



Congruent triangles



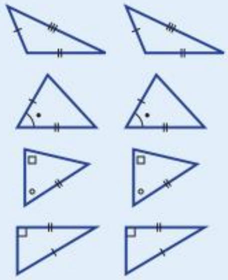
$\Delta ABC \equiv \Delta DEF$

$AB = DE$, $BC = EF$, $AC = DF$

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Tests for congruent triangles

- SSS
- SAS
- AAS
- RHS



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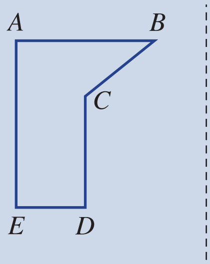
Chapter checklist

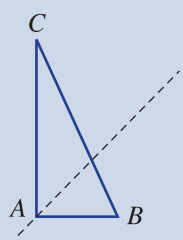
A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

10A **1 I can draw reflected images**
e.g. Copy the diagram and draw the reflected image over the given mirror line.



10A **2 I can draw reflected images when the mirror line cuts through the shape**
e.g. Copy the diagram and draw the reflected image over the given mirror line.

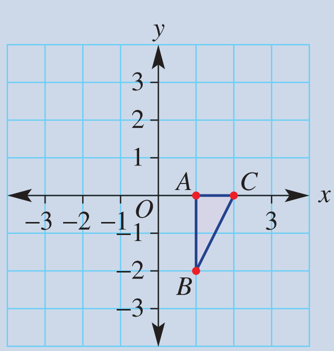


10A **3 I can state the coordinates of an image point that has been reflected in a horizontal or vertical line**
e.g. Consider the point $B(2, 3)$. State the image (B') after it is reflected in the x-axis.

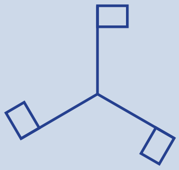
10B **4 I can translate points**
e.g. The point $A(1, 2)$ is translated by vector $(2, -3)$. Give the coordinates of the image A'.

10B **5 I can find a translation vector given a source and image point**
e.g. State the translation vector that moves the point $A(-1, 3)$ to $A'(2, 0)$.

10B **6 I can draw the result of a translation**
e.g. Draw the image of the triangle ABC after a translation by the vector $(-3, 2)$.



10C **7 I can find the order of rotational symmetry of a shape**
e.g. Find the order of rotational symmetry for this shape.



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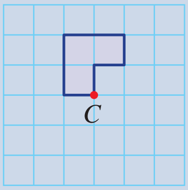
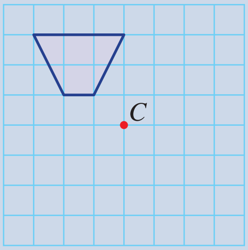
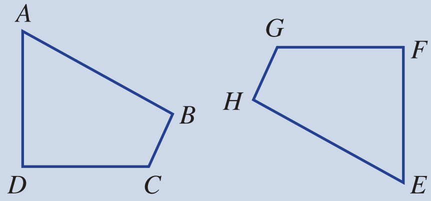
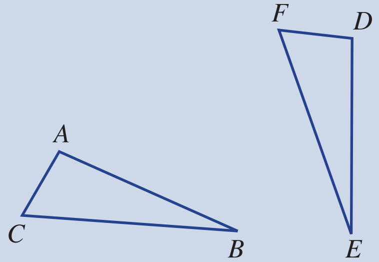
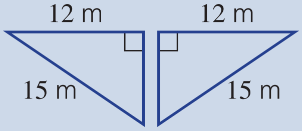
Chapter checklist

		✔
10C	**8 I can rotate a shape by $90°$** e.g. Rotate this shape $90°$ clockwise about the point C. 	
10C	**9 I can rotate a shape by $180°$** e.g. Rotate this shape about point C by $180°$. 	
10D	**10 I can name corresponding pairs in congruent figures** e.g. These two quadrilaterals are congruent. Name the objects in quadrilateral $EFGH$ that correspond to vertex C and side AB. 	
10E	**11 I can write a congruence statement** e.g. Write a congruence statement for this pair of congruent triangles. 	
10E	**12 I can decide on an appropriate test for congruence of two triangles** e.g. Which test (SSS, SAS, AAS or RHS) could be used to test the congruence of this pair of triangles? 	

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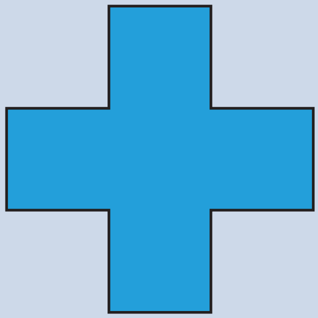
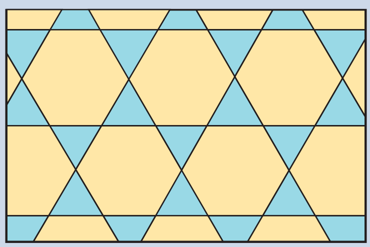
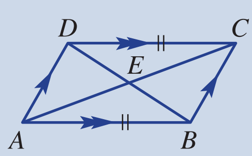
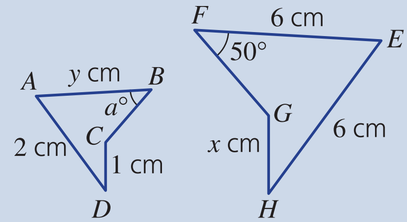
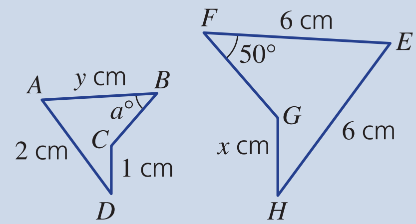
Chapter checklist

		✔
10F	**13 I can tessellate a basic shape** e.g. Use the 'plus sign' shape below to draw ten identical plus signs to show that this shape will tessellate. 	
10F	**14 I can name a tessellation** e.g. By considering any vertex, name the following semi-regular tessellation. 	
10G	**15 I can prove facts about quadrilaterals using congruent triangles** e.g. A parallelogram $ABCD$ is shown below. Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABE \equiv \$\Delta CDE$? Explain why this means the diagonals of a parallelogram bisect each other (that is, cut each other into two equal length segments). 	
10H	**16 I can identify corresponding features in similar figures.** e.g. These figures are similar. List the pairs of corresponding sides and the pairs of corresponding angles. 	
10H	**17 I can find the scale factors and unknown sides and angles in similar figures.** e.g. These figures are similar. State the scale factor and hence find the values of the pronumerals. 	

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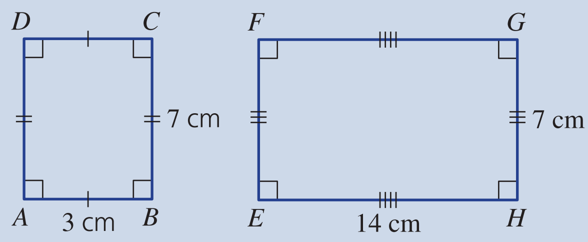
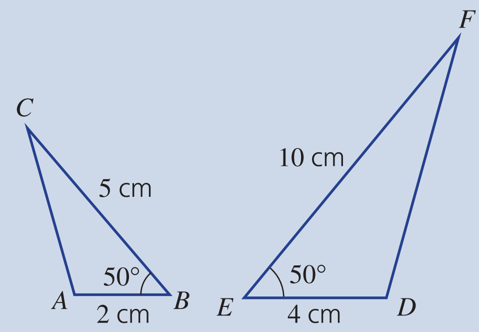
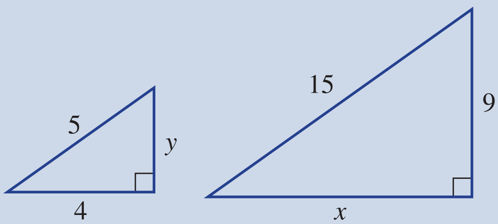
		✔
10H	**18 I can decide if shapes are similar.** e.g. Decide if these shapes are similar by considering corresponding angles and the ratio of sides. 	
10I	**19 I can decide if triangles are similar using a similarity test.** e.g. Decide with reasons, whether these triangles are similar. 	
10I	**20 I can find missing lengths in similar triangles.** e.g. Given that these triangles are similar, find the value of the pronumerals. 	

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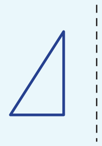
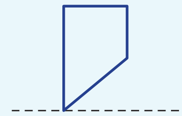
Short-answer questions

10A **1** Copy these shapes and draw the reflected image over the mirror line.

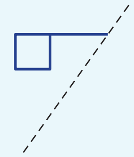
a



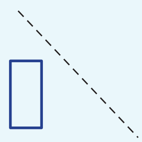
b



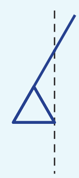
c



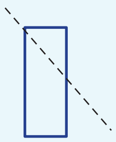
d



e

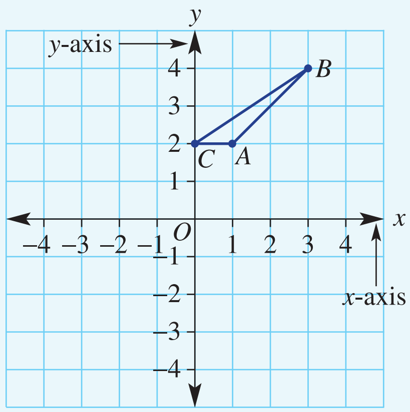


f



10A **2** The triangle $A(1, 2)$, $B(3, 4)$, $C(0, 2)$ is reflected in the given axis. State the coordinates of the image points A', B' and C'.

a x-axis
b y-axis

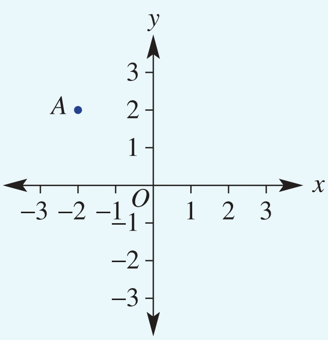


10A **3** How many lines of symmetry do these shapes have?

a Square
b Isosceles triangle
c Rectangle
d Kite
e Regular hexagon
f Parallelogram

10B **4** Give the coordinates of the image of the point A if it is translated by these vectors.

a $(2, 1)$
b $(4, -3)$
c $(-1, -1)$
d $(-1, -4)$



10B **5** Write the vectors that translate each point A to its image A'.

a $A(2, 5)$ to $A'(3, 9)$
b $A(-1, 4)$ to $A'(2, -2)$
c $A(0, 7)$ to $A'(-3, 0)$
d $A(-4, -6)$ to $A'(0, 0)$

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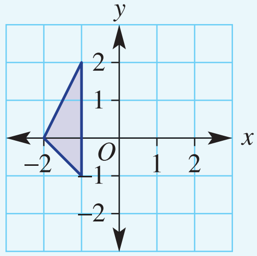
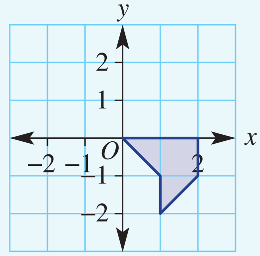
10B **6** Copy these shapes and draw the translated image using the given translation vector.

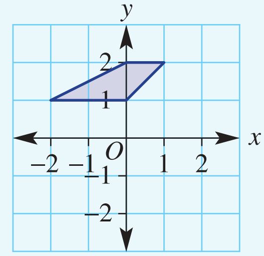
A vector $(3, -1)$



B vector $(-2, 2)$



C vector $(0, -3)$

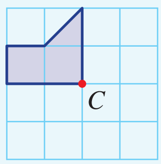


10C **7** What is the order of rotational symmetry of these shapes?

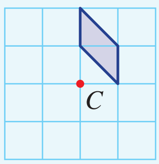
a Equilateral triangle **b** Parallelogram **c** Isosceles triangle

10C **8** Rotate these shapes about the point C by the given angle.

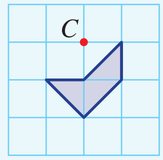
a Clockwise $90°$



b Clockwise $180°$



c Anticlockwise $270°$

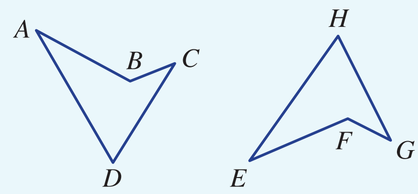


10D **9** For these congruent quadrilaterals, name the object in quadrilateral $EFGH$ that corresponds to the given object in quadrilateral $ABCD$.

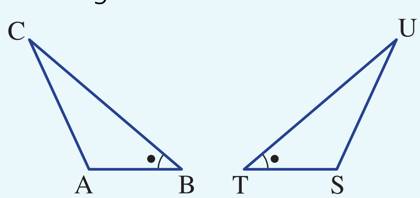
a **i** Vertex B **ii** Vertex C

b **i** Side AD **ii** Side BC

c **i** $\angle C$ **ii** $\angle A$

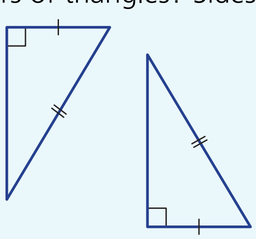


10E **10** Write a congruence statement for these congruent triangles.

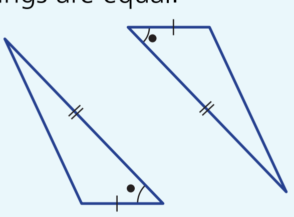


10E **11** Which of the tests SSS, SAS, AAS or RHS would you choose to explain the congruence of these pairs of triangles? Sides or angles with the same markings are equal.

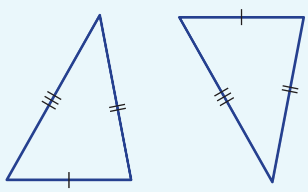
a



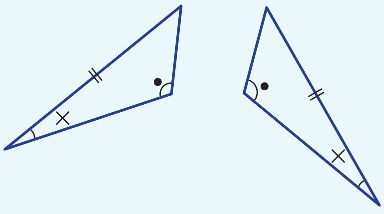
b



c



d



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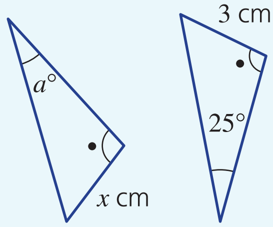
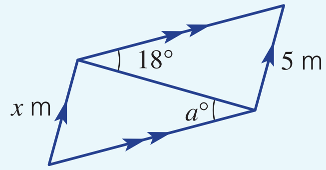
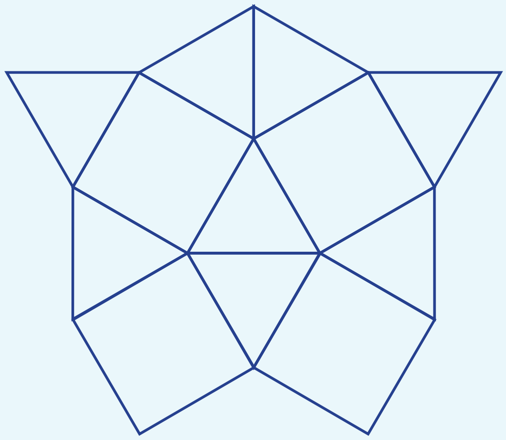
10E **12** Find the values of the pronumerals for these congruent triangles.

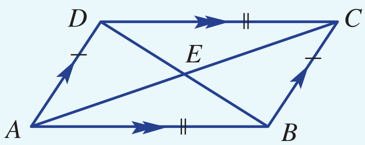
a 

b 

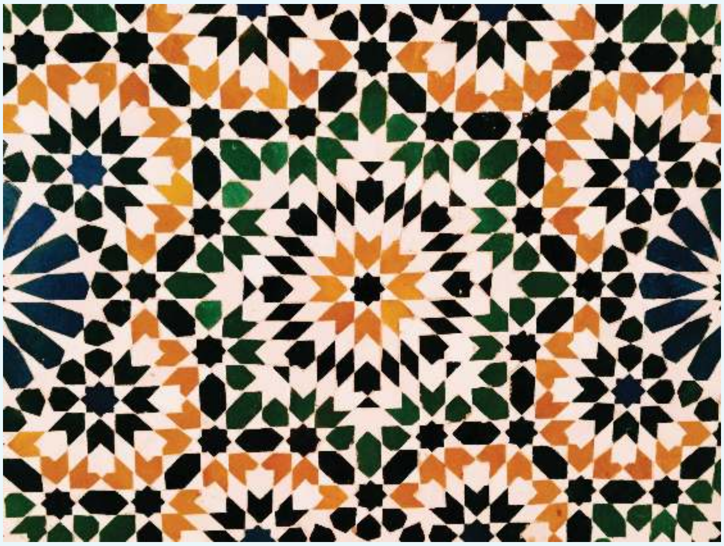
10F **13** Name this semi-regular tessellation.



10G **14** This quadrilateral is a parallelogram with 2 pairs of parallel sides. You can assume that $AB = DC$ as shown.



a Is $\angle BAE = \angle DCE$? Give a reason.
b Is $\angle ABE = \angle CDE$? Give a reason.
c Is $AB = DC$?
d Which reason (SSS, SAS, AAS, RHS) would be used to explain that $\Delta ABE \equiv \Delta CDE$?
e Explain why BD and AC bisect each other.



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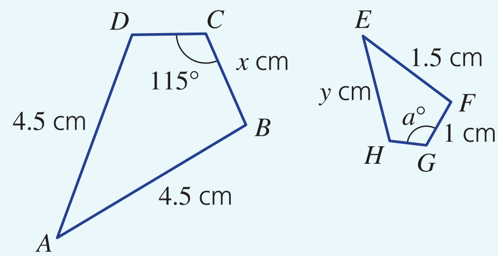
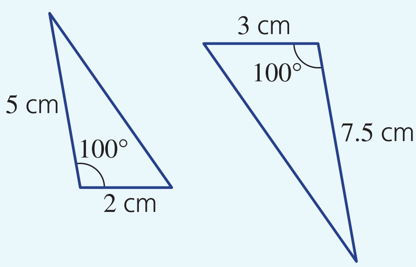
10H **15** For these similar shapes, complete the following tasks.

a List the pairs of corresponding sides.
b List the pairs of corresponding angles.
c Find the scale factor.
d Find the value of the pronumerals.

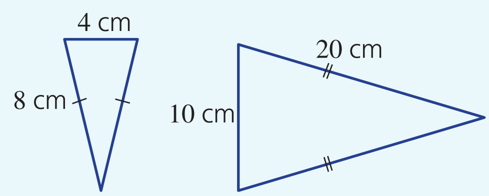


10I **16** Decide, with reasons, if these pairs of triangles are similar. State which tests (AAA, SSS, SAS or RHS) is used.

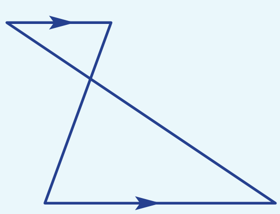
a



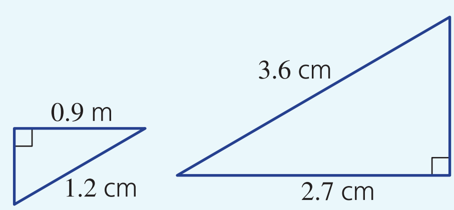
b



c

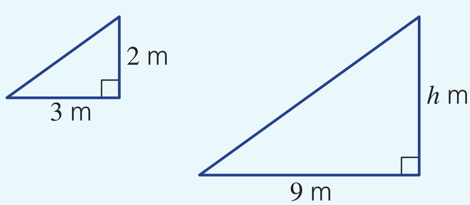


d



10I **17** Given that the triangles are similar, find the values of the pronumerals.

a



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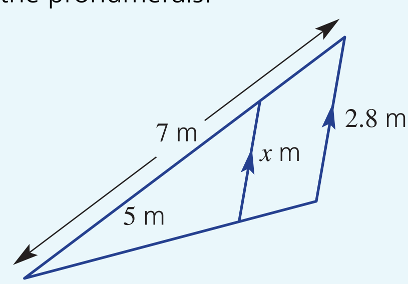

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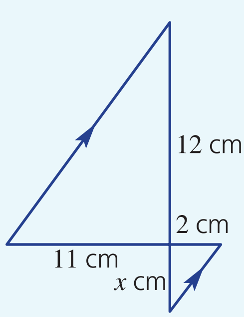

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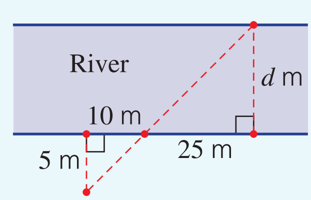

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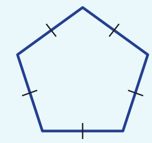
Multiple-choice questions

10A **1** The number of lines of symmetry in a regular pentagon is:



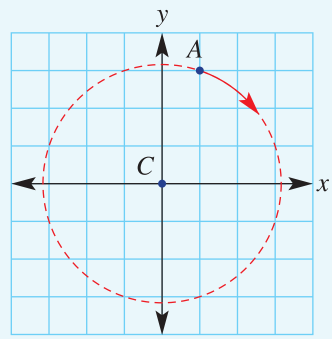
A 10 **B** 5 **C** 2
D 1 **E** 0

10B **2** Which vector describes a translation of 5 units to the left and 3 units up?
A $(3, -5)$ **B** $(-3, 5)$ **C** $(5, -3)$ **D** $(-5, 3)$ **E** $(-5, -3)$

10B **3** The point $A(-3, 4)$ is translated to the point A' by the vector $(6, -4)$. The coordinates of point A' are:
A $(3, 8)$ **B** $(-9, 8)$ **C** $(3, 0)$ **D** $(0, 3)$ **E** $(-9, 0)$

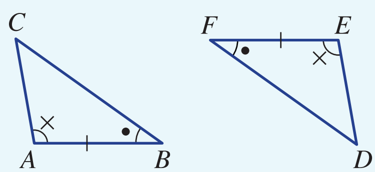
10C **4** An anticlockwise rotation of $125°$ about a point is the same as a clockwise rotation about the same point of:
A $235°$ **B** $65°$ **C** $55°$ **D** $135°$ **E** $245°$

10C **5** Point $A(1, 3)$ is rotated clockwise about C by $90°$ to A'. The coordinates of A' are:



A $(3, 0)$ **B** $(3, -1)$ **C** $(-3, 1)$
D $(-1, 3)$ **E** $(3, 1)$

Questions **6**, **7**, and **8** relate to this pair of congruent triangles.

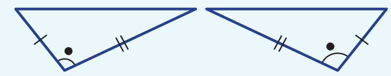


10D **6** The angle on ΔDEF that corresponds to $\angle A$ is:
A $\angle C$ **B** $\angle B$ **C** $\angle F$ **D** $\angle D$ **E** $\angle E$

10D **7** If $AC = 5$ cm then ED is equal to:
A 5 cm **B** 10 cm **C** 2.5 cm **D** 15 cm **E** 1 cm

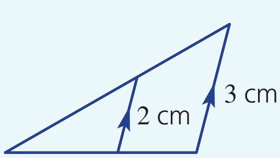
10E **8** A congruence statement with ordered vertices for the triangles is:
A $\Delta ABC \equiv \Delta FED$ **B** $\Delta ABC \equiv \Delta EDF$ **C** $\Delta ABC \equiv \Delta DFE$
D $\Delta ABC \equiv \Delta DEF$ **E** $\Delta ABC \equiv \Delta EFD$

10E **9** Which of the four tests (SSS, SAS, AAS, RHS) would be chosen to show that these two triangles are congruent?



A AAS **B** RHS **C** AAA
D SAS **E** SSS

10I **10** Which of the four tests (see **Section 10I**) would be chosen to show that these two triangles are similar?



A AAS **B** RHS **C** SAS
D AAA **E** SSS

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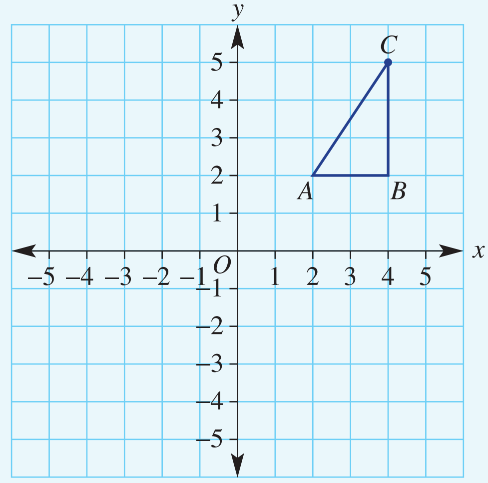
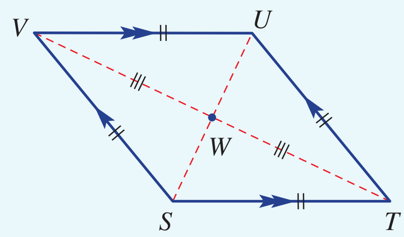
Extended-response questions

1 The shape on this set of axes is to be transformed by a succession of transformations. The image of the first transformation is used to start the next transformation. For each set of transformations write down the coordinates of the vertices A', B' and C' of the final image.
Parts **a** and **b** are to be treated as separate questions.

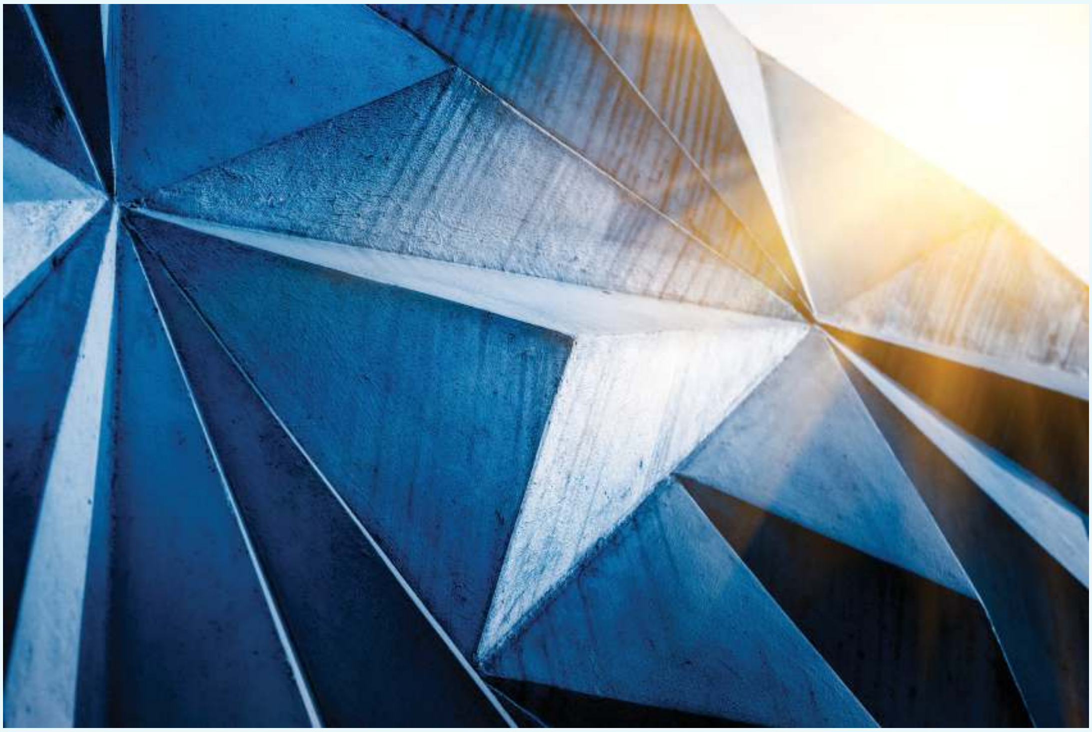


- **a** Set 1
  - **i** Reflection in the x-axis.
  - **ii** Translation by the vector $(-2, 1)$.
  - **iii** Rotation about $(0, 0)$ by $180°$.
- **b** Set 2
  - **i** Rotation about $(0, 0)$ clockwise by $90°$.
  - **ii** Reflection in the y-axis.
  - **iii** Translation by the vector $(5, 3)$.

2 For this rhombus note that $VW = WT$.



- **a** Give reasons why $\Delta VWU \equiv \Delta TWU$.
- **b** Give reasons why $\angle VWU = \angle TWU = 90°$.



Chapter review

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Ratios and rates

Short-answer questions

1 Simplify these ratios.
- **a** 24 to 36
- **b** $15 : 30 : 45$
- **c** 0.6 m to 70 cm
- **d** 15 cents to $\$2$
- **e** 2 kg to 400 g
- **f** 30 seconds to $1\frac{2}{3}$ minutes

2 **a** Divide 960 cm in the ratio of $3 : 2$. **b** Divide $\$4000$ in the ratio of $3 : 5$.
c Divide $\$8$ in the ratio of $2 : 5 : 3$.

3 A business has a ratio of profit to costs of $5 : 8$. If the costs were $\$12\,400$, how much profit was made?

4 A map has a scale $1 : 10\,000$. Find the real distance in cm and also in m between two towns that are 6 cm apart on the map.

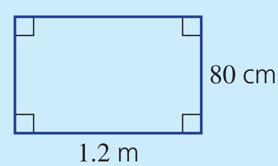
5 Write each of the following as a simplified rate.
- **a** 84 mm rainfall in 7 days
- **b** 18 goals in 6 games
- **c** $\$15$ for 750 g of meat

6 A shop sells $1\frac{1}{2}$ kg bags of apples for $\$3.40$. Find the cost of one kilogram at this rate.

7 A family travels the 1070-km road from Rockhampton to Cairns in 12.5 hours. Calculate their average speed.

Multiple-choice questions

1 The ratio of the length to the width of this rectangle is:



A $12 : 80$ **B** $3 : 20$
C $3 : 2$ **D** $20 : 3$

2 Which of the following ratios is not written in simplest form?
A $2 : 3$ **B** $5 : 10$ **C** $11 : 3$ **D** $3 : 7$

3 $\$18$ is divided in the ratio $2 : 3$. The larger part is:
A $\$3.60$ **B** $\$7.20$ **C** $\$10.80$ **D** $\$12$

4 Calvin spent $\$3$ on his mobile phone plan for every $\$4$ he spent on his internet connection. Calvin spent $\$420$ on his phone last year. How much did he spend for his internet connection the same year?
A $\$140$ **B** $\$315$ **C** $\$560$ **D** $\$240$

5 A boat sailed 30 kilometres in 90 minutes. What was the average speed of the boat?
A 15 km/h **B** 45 km/h **C** 3 km/h **D** 20 km/h

Extended-response questions

1 A small car uses 30 litres of petrol to travel 495 km.
- **a** What is the average distance travelled per litre?
- **b** At this rate, what is the maximum distance a small car can travel on 45 litres of petrol?
- **c** Find the number of litres used to travel 100 km, correct to one decimal place.
- **d** Petrol costs 117.9 cents/litre. Find the cost of petrol for the 495-km trip.
- **e** A larger car uses 42 litres of petrol to travel 378 km. The smaller car holds 36 litres of petrol while the larger car holds 68 litres. How much further can the larger car travel on a full tank of petrol?

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Equations and inequalities

Short-answer questions

1 Solve each of these equations.

a $3w = 27$ **b** $12 = m + 5$ **c** $2x - 1 = 9$

d $4a + 2 = 10$ **e** $2w + 6 = 32$ **f** $4 = 6x - 2$

2 Solve each of these equations.

a $\frac{x}{3} = 10$ **b** $\frac{2q}{5} = 4$ **c** $3 = \frac{p}{5}$

d $\frac{x+2}{4} = 3$ **e** $\frac{r-3}{12} = 1$ **f** $2 = \frac{3a-4}{10}$

3 Solve the following equations.

a $2(x + 3) = 16$

b $4(2k + 1) = 84$

c $3(r + 2) + r = 6$

d $10(z - 4) + 2z = 80$

4 Double a number less three is the same as 9. What is the number?

5 The formula $S = 6g + b$ relates an AFL score (S) to the number of goals (g) and behinds (b).

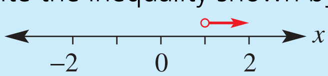
a Find S if $g = 3$ and $b = 2$.

b Find b if $S = 62$ and $g = 10$.

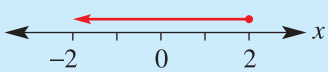
c Find g if $S = 50$ and $b = 8$.

6 Write the inequality shown by each number line below.

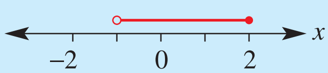
a



b



c



7 Solve each of these inequalities.

a $2x > -16$ **b** $3x + 8 \le 17$ **c** $\frac{x}{5} - 6 \le 0$

Multiple-choice questions

1 If $x = 5$, which one of these equations is true?

A $x + 3 = 2$ **B** $7x = 75$ **C** $7 - x = 2$ **D** $2x = 20$

2 The sum of a number and three is doubled. The result is 12. This can be written as:

A $x + 3 \times 2 = 12$ **B** $2(x + 3) = 12$ **C** $2x + 3 = 12$ **D** $x + 3 = 24$

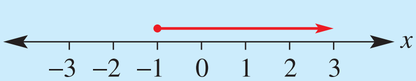
3 The solution to the equation $2m - 4 = 48$ is:

A $m = 8$ **B** $m = 22$ **C** $m = 20$ **D** $m = 26$

4 The solution to $\frac{x}{4} - 3 = 4$ is:

A $x = 20$ **B** $x = 28$ **C** $x = 4$ **D** $x = 24$

5 Which of the following inequalities is represented by the number line below?



A $x \le -1$ **B** $x \ge -1$ **C** $x > -1$ **D** $x < -1$

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Extended-response questions

1 EM Publishing has fixed costs of \$1500 and production costs of \$5 per book.

a Write an expression for the cost of producing n books.
b The total costs for one year were \$2000. Use an equation to find how many books were produced.
c Write an expression for the money made by selling n books, if they sell each book for \$20.
d If the total revenue is \$1000, find the number of books sold.
e Given that the profit is given by the formula, $P = 15n - 1500$, find:
  i the profit if 200 books are sold
  ii the profit if 1000 books are sold
  iii the number of books sold if the profit is \$0.
f If $n = 50$, the profit is $-\$750$. Explain what this means for EM Publishing.

Statistics and probability

Short-answer questions

1 Find: i the mean, ii the median and iii the range of these data sets.
a 10, 15, 11, 14, 14, 16, 18, 12
b 1, 8, 7, 29, 36, 57
c 1.5, 6, 17.2, 16.4, 8.5, 10.4

2 Draw a graph for this frequency table.

Score	Frequency
10	2
11	3
12	5
13	1

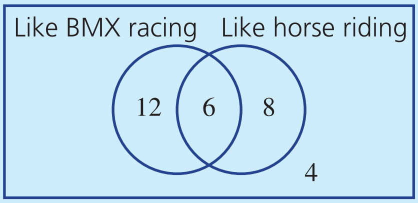
3 A bag contains 16 balls of equal size and shape. Of these balls, 7 are yellow, 1 is blue and the rest are black. If one ball is chosen from the bag at random, find the probability that it is:
a yellow b blue c not blue d black e pink

4 The ages of 50 people at a party are shown in the table below.

Ages	0–9	10–19	20–29	30–39	40–49	50–59	Over 60
Frequency	3	7	1	28	6	2	3

If one person is chosen at random to prepare a speech, find the probability that the person is aged:
a $0 - 9$ b 30 or older c in their twenties d not in their fifties

5 The Venn diagram to the right shows the results of a survey asking students if they like BMX racing and if they like horse riding.



a How many students like BMX racing?
b How many students like both BMX racing and horse riding?
c How many students like horse riding but not BMX racing?
d How many students don't like horse riding and also don't like BMX racing?
e How many students don't like horse riding?

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6 Answer the questions below about this two-way table:

	Likes rugby	Dislikes rugby	Total
Likes soccer	20	4	24
Dislikes soccer	9	3	12
Total	29	7	36

If a student is chosen randomly from this group, find these probabilities and simplify your answers.

a Pr(likes soccer but dislikes rugby)
b Pr(likes soccer and likes rugby)
c Pr(dislikes soccer but likes rugby)
d Pr(dislikes soccer and also dislikes rugby).

Multiple-choice questions

1 For the set of numbers 3, 2, 1, 3, 5, 1, 3, 9, 3, 5 the mode is:

A 3 B 3.5 C 8 D 35

2 Look at the set of numbers 8, 9, 10, 10, 16, 19, 20, 20. Which of the following statements is true?

A Median = 13 B Mean = 13 C Mode = 13 D Range = 13

3 In a bag there are 5 green marbles, 6 blue marbles, 2 red marbles and 3 purple marbles. The probability of randomly selecting a blue marble is:

A $\frac{6}{10}$ B $\frac{2}{16}$ C $\frac{6}{10}$ D $\frac{3}{8}$

4 A coin and a six-sided die are tossed together. The number of elements in the sample space is:

A 2 B 6 C 12 D 8

5 A die is rolled 60 times. The number 4 appears exactly 24 times. The experimental probability of obtaining the number 4 is:

A 0 B $\frac{2}{5}$ C $\frac{2}{3}$ D $\frac{1}{6}$

Extended-response questions

1 Two groups of students have their pulse rates recorded as beats per minute. The results are listed here:

Group A: 65, 70, 82, 81, 67, 74, $\square$, 81, 88, 84, 72, 65, 66, 81, 72, 68, 86, 86
Group B: 83, 88, 78, 60, 81, 89, 91, 76, 78, 72, 86, 80, 64, 77, 62, 74, 87, 78

a How many students are in group B?
b If the median pulse rate for group A is 76, what number belongs in the $\square$?
c What is the median pulse rate for group B?
d Which group has the largest range?

Semester review 2

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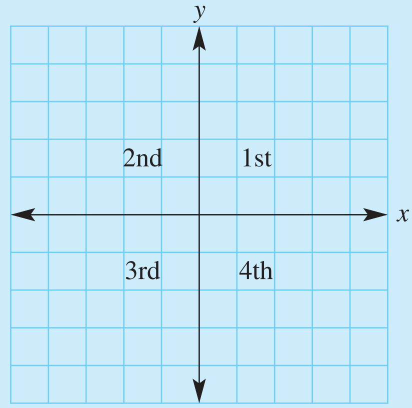
Linear relationships

Short-answer questions

1 In which quadrant does each point lie?

a $(5, 1)$
b $(-3, 4)$
c $(-5, -1)$
d $(8, -3)$



2 a Complete these tables of values.

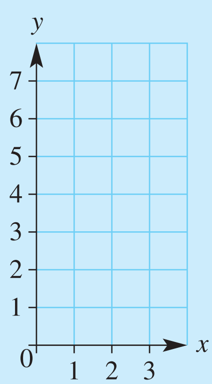
i $y = 2x + 1$

x	0	1	2	3
y				

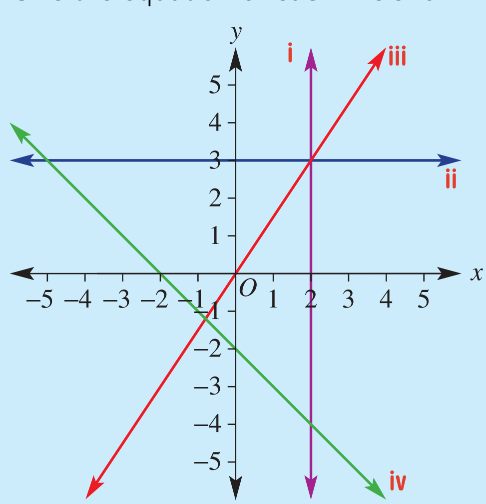
ii $y = 4 - x$

x	0	1	2	3
y				

b Plot the points from both tables and join to form two graphs.



3 a Give the equation of each line shown on this grid.



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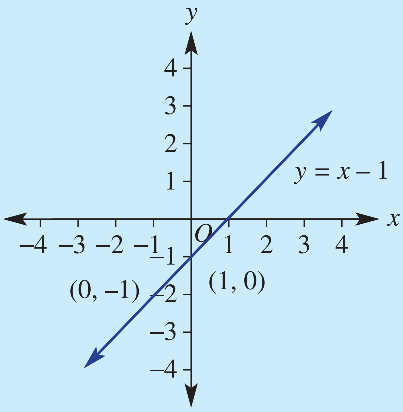
b Solve the inequality $x - 1 > 2$ using this graph.



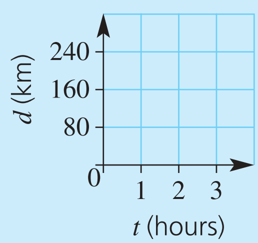
4 Decide if the gradient for each graph in Question **2** is zero, positive, negative or undefined.

5 The distance a car travels (d km) over t hours is given by $d = 80t$.

a Complete this table of values.

t	0	1	2	3
d				

b Plot a graph using your table.
c How far does the car travel after 3 hours?
d How long would it take for the car to travel 320 km?



6 For each rule, complete a table of values like the one shown and plot to form a graph.

a $y = 2x - 2$ **b** $y = -x + 1$

x	−3	−2	−1	0	1	2	3
y							

Multiple-choice questions

1 The value of y in the rule $y = 2x - 1$ when $x = -1$ is:

A 3 **B** 1 **C** -1 **D** -3

2 The coordinates of the point 3 units directly above the origin is:

A $(0, 0)$ **B** $(0, 3)$ **C** $(0, -3)$ **D** $(3, 0)$

3 The rule for the table of values shown is:

A $y = 2x$ **B** $y = 2x + 2$
C $y = 2(x + 2)$ **D** $y = x + 4$

x	0	2	4
y	4	8	12

4 The gradient of the line through $A(4, 7)$ and $B(8, -1)$ is:

A $-\frac{1}{2}$ **B** 2 **C** $\frac{1}{2}$ **D** -2

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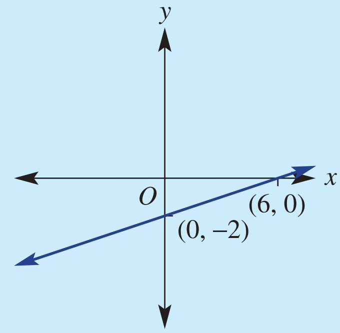
5 Which equation suits the given graph?

A $y = 6x - 2$
B $y = 3x - 2$
C $y = x - 2$
D $3y = x - 6$



Extended-response questions

1 The cost ($\$C$) of running a coffee shop is given by the rule $C = 400 + 5n$, where n is the number of customers on any given day. The revenue (income) is $\$R$ and is given by $R = 13n$.

a Complete this table.

n	0	10	20	30	40	50	60
C							
R							

b Plot a graph for both C and R on the same set of axes.
c What is the 'break even' point for the coffee shop i.e. where does the cost = revenue?
d If they are particularly busy on a Saturday and serve 100 people, calculate the shop's profit (profit = revenue − cost).

Transformations and congruence

Short-answer questions

1 How many lines of symmetry does each of these shapes have?

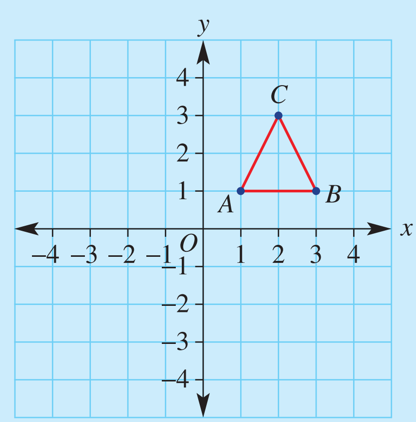
a Scalene triangle **b** Rhombus **c** Rectangle **d** Semicircle

2 Write the vectors that translate each point P to its image P'.

a $P(1, 1)$ to $P'(3, 3)$ **b** $P(-1, 4)$ to $P'(-2, 2)$

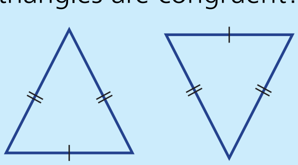
3 Triangle ABC is on a Cartesian plane as shown. List the coordinates of the image points A', B' and C' after:

a a reflection in the x-axis
b translation by the vector $(-4, -2)$
c a rotation $90°$ clockwise about $(0, 0)$
d a rotation $180°$ about $(0, 0)$.

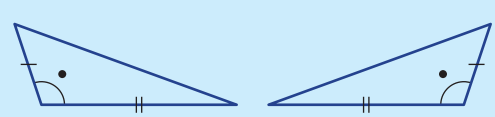


4 Which congruency test (SSS, SAS, AAS or RHS) would be used to prove the following pairs of triangles are congruent?

a



b



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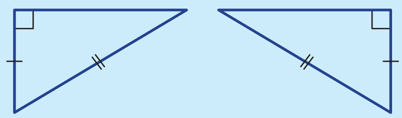
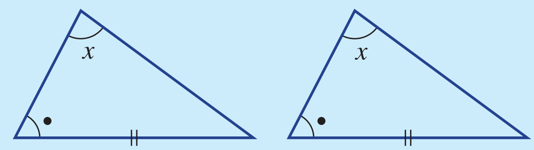
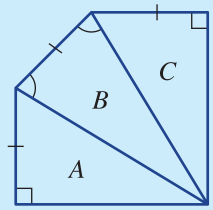
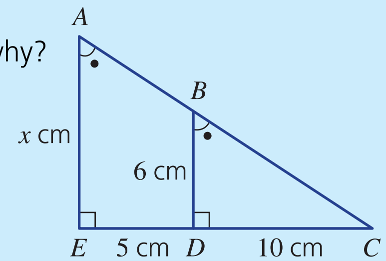
c


d


5 Which two triangles are congruent?



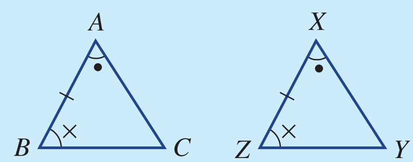
6 a Which two shapes in the diagram below are similar, and why?
b Which vertex in ΔAEC corresponds to vertex C in ΔBDC?
c Find the value of x.



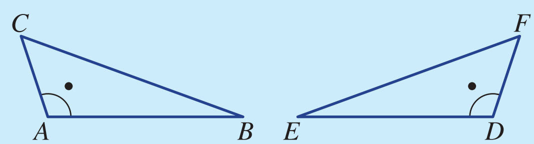
Multiple-choice questions

1 The number of lines of symmetry in a square is/are:
A 0 B 2 C 4 D 6

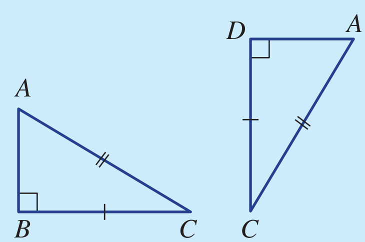
2 The side AC corresponds to:
A XZ B XY
C ZY D BC



3 A congruence statement for these triangles is:
A $\Delta ABC \equiv \Delta DFE$ B $\Delta ABC \equiv \Delta EFD$
C $\Delta ABC \equiv \Delta EDF$ D $\Delta ABC \equiv \Delta DEF$



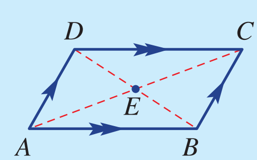
4 Which test is used to show triangle ABC is congruent to triangle ADC?
A SSS B SAS C AAS D RHS



5 Which of the following codes is not enough to prove congruency for triangles?
A SSS B AAS C AAA D SAS

Extended-response questions

1 For this parallelogram with $AB = DC$, answer the following.
a Why is $\angle BAE = \angle DCE$?
b Why is $\angle ABE = \angle CDE$?
c Give the reason (SSS, SAS, AAS, RHS) why $ABE \equiv \Delta CDE$.
d Explain why the diagonals bisect each other.



Semester review 2

 ISBN 978-1-009-593-724

A

Acute angle An angle between 0 and 90 degrees

Algorithm A procedure involving a number of steps to find the answer of a problem

Alternate angles Two angles that lie between two lines on either side of a transversal

am Ante meridiem, before midday

Angle The amount of turn between two lines around their vertex

Angle sum The value of all the angles added together for a particular shape

Apex The top or highest point (vertex), usually referred to as the vertex opposite the base

Area The amount of surface a shape covers

Average rate The overall representative rate of two related quantities

Average speed A representative speed that is calculated by dividing the distance travelled by the time taken

B

Backtracking A method of solving equations systematically by applying opposite operations

Balance method The process of solving an equation by doing the same thing to both sides of the equation

Base (Measurement) One side of a shape, at right angles to the height

Base (Number) The number or pronumeral that is being raised to a power or index

Bisect To divide a line, angle, or shape into two equal parts

Braces A pair of symbols used to group things together (same as Brackets)

Brackets A pair of symbols used to group things together

C

Capacity How much liquid or gas a container can hold

Cartesian plane A plane on which every point is related to a pair of numbers called coordinates (same as Number plane)

Categories Different types of groups used to sort or label data

Census Collection of data from the whole population

Centimetre (cm) A metric unit for length, equal to 10 millimetres

Centre of rotation Fixed point about which a figure rotates

Circle A closed (2D) curved shape where all points are equal distance from the centre point

Circumference The distance around a circle

Coefficient A numeral placed before a pronumeral to indicate that the pronumeral is to be multiplied by that factor

Cointerior angles A pair of angles lying between two lines on the same side of a transversal

Colon (:) A punctuation mark used in ratios representing the phrase ‘is to’

Column A vertical line of cells within a table

Column graph A graph where the height of each column represents a value

Common factor Factors which are common across a given pair or set of numbers

Commutative law When adding and multiplying, the order in which two numbers are combined does not matter

Compass bearing A measurement of direction, numbered clockwise with north as $0°$

Compensating A mental strategy where you round a number and then add or subtract a smaller amount

Complement The complement of some event E is written E' (or not E). E' is the event that E does not occur

Complementary Having a sum of $90°$

Composite number A whole number greater than 1 that has at least three factors (i.e. it has a factor other than itself and 1)

Composite shape A shape made up of two or more basic shapes

Cone A solid with a circular base and a slanting curved surface that tapers to a point called the apex

Congruence statement A statement that says that two shapes are congruent

Congruent Having the same shape and size

Congruent figures Figures that are exactly the same size and shape

Constant speed A speed which does not vary or change

Constant term The part of an expression without any pronumerals

Coordinates Numbers or letters used to give a location or position, often an ordered pair written in the form (x, y)

Coordinate system A combination of axes (e.g. 2D or 3D) used to define location

Corresponding To be in similar positions and equal in value

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Corresponding angles Pairs of angles in the same position, formed by two lines cut by a transversal

Cost price The price for which an item is purchased

Counting numbers The set of whole numbers starting at 1

Counting on A mental strategy where you add or subtract part of a number to arrive at a round number and then add or subtract the remaining part of the number

Cross-section The shape formed when a solid is cut through parallel to its base

Cube (Geometry) A solid (3D shape) with six square faces that are congruent (the same size and shape)

Cube (Operation) To multiply a number by itself three times

Cube root The opposite operation of cubing

Cuboid A box-shaped solid object, also known as a rectangular prism

Cylinder A solid with two circular faces joined by a curved surface

D

Data Information (often numerical) gathered by observation, survey or measurement

Decagon A ten sided polygon

Decimal A number containing a decimal point

Decimal point Symbol that separates the whole part of a number from its fractional part

Decreasing Becoming smaller or fewer

Denominator The bottom part of a fraction

Diagonals (of a quadrilateral) A line connecting two non adjacent vertices of a quadrilateral

Diameter A line interval from one point on a circle through the centre to another point on the circle

Difference A number that is the result of subtraction

Discount Deducting an amount from the normal price

Distance The length of the space between two points

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

Divided bar graph A graph where the size of each bar represents a value

Dividend The number being divided

Divisibility test A way to work out whether a whole number is divisible by another whole number, without actually doing the division

Divisible When divided by a certain number gives a whole number answer

Divisor The number you are dividing by

Dodecagon A twelve sided polygon

Doubling Multiplying a number by two

Duration The amount of time during which something continues or lasts

E

Edge A line segment where two faces meet on a solid shape

Equation A mathematical statement that two expressions (numeric or algebraic) have the same value

Equilateral triangle A triangle with three equal sides and three equal angles

Equivalent Equal in value

Equivalent equations Equations that are equal in value and are formed by doing the same operation to both sides of the equation

Equivalent fractions Fractions that represent the same amount. They can be reduced to the same basic fraction

Equivalent ratios Ratios that represent the same comparison of quantities

Evaluate To find the numerical value of an expression

Event Either a single outcome (e.g. rolling a 3) or a collection of outcomes (e.g. rolling a 3, 4 or 5)

Expand To remove grouping symbols (brackets)

Expanded form A way of writing out in full a number written in index form

Expected number The number of occurrences you would expect to happen

Experiment A series of repeated probability trials that lead to outcomes

Experimental probability Probability based on recording the outcomes of trials of an experiment

Expression A group of mathematical terms that does not contain an equals sign

Exterior angle The angle between any side of a shape and a line extended from the next side of the shape

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F

Face Each flat surface of a solid shape

Factor A whole number that will divide into another number exactly

Factor tree A diagram showing the breakdown of a number into its prime factors

Factorise To write an expression as a product of factors

Figure A shape, diagram or illustration

Formula An equation that shows the relationship between variables

Fraction Part of a whole

Frequency table A table summarising data by showing all possible scores from lowest to highest in one column, and the frequency of each score in another column

G

Gradient A measure of slope

Graph A pictorial representation or a diagram that represents data in an organised manner

Grouping symbols Parentheses ( ), brackets [ ] and braces { } used to collect terms and operations together

GST Goods and Services Tax

H

HCF Abbreviation of 'highest common factor'

Hectare (ha) A unit of area equal to $10\,000$ square metres

Hendecagon An eleven-sided polygon

Heptagon A seven sided polygon

Hexagon A six sided polygon

Hexahedron A polyhedron with six faces

Highest common factor (HCF) The largest number that is a factor of all the given factors

Histogram A special type of column graph for quantitative data with no gaps between the columns

Horizontal A flat line or plane

Hypotenuse The longest side of a right-angled triangle

I

Image The result of transforming a figure

Improper fraction A fraction where the numerator is greater than or equal to the denominator

Included angle The angle between two given sides of a shape

Increasing Becoming bigger or larger

Independent steps Steps in an experiment that do not impact on each other

Index (plural: indices) The number of times a factor is repeated under multiplication

Index form A method of writing numbers that are multiplied by themselves

Index laws Specific rules that help when working with one or more numbers written using index form

Index notation A method of writing numbers that are multiplied by themselves

Inequality An equation involving an inequality symbol $<, \leq, >$ or $\geq$

Integers The set of positive and negative whole numbers and zero

Intersection The point where two or more lines meet

Isosceles triangle A triangle with two equal sides and two equal angles

K

Kilometre (Km) A metric unit for length, equal to 1000 metres

Kite A quadrilateral with exactly two pairs of equal adjacent sides

L

LCM Abbreviation of 'lowest common multiple'

Left-hand side (LHS) (Equations) The expression on the left side of the equals sign

Length The distance between two points

LHS See Left-hand side

Like terms Terms with the same pronumerals and same powers

Line graph A graph where the height of each dot represents a value, and the dots are joined together by lines

Line of symmetry The line (axis) along which a figure could be folded to produce identical halves

Linear A linear graph in two dimensions is a straight line

Longitude The angular distance of a place east or west of the Greenwich meridian, usually expressed in degrees and minutes

Loss The amount of money lost by selling for less than the cost

Lowest common denominator (LCD) The smallest common multiple of the denominators of two or more fractions

Lowest common multiple (LCM) The smallest number that two or more numbers divide into evenly

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M

Mark-up Adding an amount to the cost price

Mean An average value calculated by dividing the total of a set of numbers by the number of values

Median The middle score when all the numbers in a set are arranged in order

Metre (m) The standard metric unit for length, equal to 100 centimetres

Millimetre (mm) A metric unit for length, equal to one tenth of a centimetre

Mirror line A line over which a figure is reflected or a line which can be drawn onto a shape to show that both sides are identical.

Mixed number A number with a whole number part and a fraction part

Modal category The most frequently occurring category or group in a set of data

Mode The most frequently occurring value in a set of data

Model A mathematical representation of a system

Modelling A mathematical representation of a situation

Multiple A multiple of a number is the product of that number and any other whole number

Multi-step experiment An experiment involving more than one step

N

Negative integer A whole number less than zero

Negative number A number less than zero

Nonagon A nine sided polygon

Non-linear An expression or graph that does not result in a straight line

Number line A line on which numbers are represented by points

Number plane A plane on which every point is related to a pair of numbers called coordinates (same as Cartesian plane)

Numerator The top part of a fraction

O

Obtuse angle An angle between 90 and 180 degrees

Octagon An eight sided polygon

Opposite (Angles) The angles on either side of a pair of intersecting lines

Opposite (Number) A number that has the same size, but has a different sign (i.e. the opposite number differs by a factor of -1)

Opposite operation A mathematical process that undoes what was done by the previous operation

Order of operations A particular order that needs to be followed when working with more than one operation

Order of rotational symmetry The number of times a figure matches its original position during rotation of $360°$

Origin The point with coordinates $(0, 0)$ on the number plane

Original price The price of an item prior to the mark-up being added or the discount being subtracted

Outcome One of the possible results of a chance experiment

Outlier Any value that is much larger or much smaller than the rest of the data in a set

P

Parabola An example of a non-linear graph, which is symmetrical and approximately U-shaped

Parallel lines Lines in the same plane that are a fixed distance apart and never intersect

Parallelogram A quadrilateral with two pairs of opposite sides parallel

Parentheses A pair of symbols used to group things together (same as Brackets)

Parts A particular amount of a quantity

Pentagon A five sided polgyon

per (/) For each

Per cent (%) Per hundred, out of a hundred

Percentage A way of writing a fraction with a denominator of 100

Percentage change Expressing the change in value as a percentage of the original value

Percentage loss Expressing the loss in value as a percentage of the original value

Percentage profit Expressing the profit as a percentage of the original value

Perimeter The total distance (length) around the outside of a figure

Perpendicular At right angles to another line or surface

Perpendicular height The distance between the base and the perpendicular apex, or top, of a shape

pi (π) The 16th letter of the greek alphabet, which is used in mathematics to represent the ratio of the circumference of a circle to its diameter

Pie chart A graph or chart where the size of each segment represents a value (same as Sector graph)

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Place value The value of where a digit is within a particular number

Plot To draw on a graph

pm Post meridiem, after midday

Polygon A two-dimensional shape where three or more straight lines are joined together to form a closed figure

Polyhedron A three-dimensional figure made by joining polygons at their edges

Population The entire group we are interested in

Positive integer A whole number greater than zero

Positive number A number greater than zero

Power (Index) The number that a base is being raised to

Power of 10 Numbers which correspond to a base number of ten and an associated index number (e.g. $100 = 10^2$)

Prime factorisation Writing a number as a product of its prime factors

Prime number A whole number with only two factors, itself and 1.

Prism A solid where each cross-section in a particular direction is exactly the same and all faces are polygons

Probability The likelihood that an event will occur, measured on a scale between 0 and 1

Product A number that is the result of multiplication

Profit The amount of money made by selling for more than the cost

Pronumeral A letter or a symbol used to represent a number

Pythagoras' theorem A theorem which states that for a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Pythagorean triple Three positive integers which satisfy Pythagoras' theorem

Pyramid A solid where the base is a polygon and the other faces are formed by triangles with a common vertex

Q

Quadrant A sector which is one quarter of a circle

Quadrilateral A four-sided plane (2D) shape with straight sides

Quotient A number that is the result of division

R

Radius (plural: Radii) A line interval from the centre of a circle to its circumference (boundary), or the length of that interval

Random number generator A device that generates a sequence of random numbers

Range The difference between the highest and lowest numbers in a set

Rate A comparison of two related quantities

Rate of change A rate that describes how one quantity changes in relation to another. For linear graphs, rate of change equals the change in y values divided by the change in x values

Ratio A comparison of quantities usually written as a fraction or in the form a : b

Reciprocal A fraction in which the numerator and denominator have changed places

Rectangle A quadrilateral with both pairs of opposite sides equal and parallel, and with four right angles

Rectangular prism A box-shaped solid object, also known as a cuboid

Recurring decimal A decimal number with a digit (or group of digits) that repeats forever

Reduction Making something smaller or less

Reflection Flipping a geometrical figure across a line

Reflex angle An angle between 180 and 270 degrees

Region in the plane A portion or area of the Cartesian plane defined by lines or curves

Regular polygon A polygon with all sides equal and all angles equal

Regular tessellation A tessellation formed by arranging multiple copies of only one type of regular polygon

Remainder Leftover amount after one number has been divided by another

Revolution A full turn or circle (360°)

Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal

RHS See Right-hand side

Right angle An angle of 90 degrees

Right prism A prism with rectangular side faces

Right-hand side (RHS) (Equations) The expression on the right side of the equals sign

Rise The change in y values between two points on a line

Rotation A turn around a centre point or axis

Rotational symmetry When a figure rotated less than 360° matches its original position

Rounding Approximating a number to a specified number of places

Row A horizontal line of cells within a table

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Rule An equation that describes the relationship between two or more variables or amounts

Run The change in x values between two points on a line

S

Sale price The price for which an item is sold (same as Selling price)

Sample Collection of data from a smaller subset of the whole population

Sample space The list of all the possible outcomes of a trial

Scale A ratio that compares a drawing or a model to the real object

Scale factor The number you multiply each side length by to enlarge or reduce a shape

Scalene triangle A triangle with no equal sides or angles

Sector A portion of a circle with two sides as radii.

Sector graph A graph or chart where the size of each segment represents a value (same as Pie chart)

Selling price The price for which an item is sold (same as Sale price)

Semicircle Half a circle

Semi-regular tessellation A tessellation formed by arranging multiple copies of two or more types of regular polygons

Side A line segment that joins two vertices in a shape

Sign The sign of a number refers to if the number is positive or negative

Similar figures Shapes that have the same overall shape but can be of different size

Similarity test A set of minimum conditions which prove that two triangles are similar (AAA, SSS, SAS, RHS)

Similar triangles Triangles that are the same shape but can be of different size

Simplest form Writing an expression or fraction as simply as possible. For fractions, this is when the numerator and the denominator have no common factors other than one

Simplified rate A comparison of two related quantities expressed in simplest form

Simplify To make something as simple as possible

Simplifying Finding the simplest possible expression

Simulation A way to model random events

Skewed data Data that is unevenly distributed either side of the mean or median

Slope A rising or falling surface

Solution The value/s that give a true statement when substituted for the unknown in an equation

Solving Finding the value of an unknown variable

Speed A measure of how fast an object is moving

Sphere A solid (3D shape) where every point on the surface is the same distance from the centre, also known as a ball

Square (Geometry) A quadrilateral with all sides equal in length and four right angles

Square (Operation) To multiply a number by itself

Square root The opposite operation of squaring

Straight angle An angle of 180 degrees

Straight line A linear graph in two dimensions

Subject A pronumeral (or variable) that occurs by itself on one side of an equation

Substitution Replacing pronumerals (letters) with values (numbers)

Sum A number that is the result of addition

Supplementary Having a sum of 180°

Surd A number involving a $\sqrt{}$ that cannot be simplified to an integer or fraction

Survey A set of questions that are asked of the people in a sample

Symmetrical data Data that is balanced on either side of the mean and median

T

Table of values A list of numbers shown for one or more variables to show the relationship between the variables

Tally A tool used for counting as results are gathered

Tally marks Line strokes used to record data, made in groups of 5

Term One of the numbers in a sequence

Terminating decimal A decimal that contains a fixed number of digits

Tessellation A pattern made up of shapes that fit together without any gaps and without any overlaps

Tetrahedron A polyhedron with four faces

Three dimensional (3D) space A space defined by three dimensions e.g. an x, y, z axes system

Time The duration of an event

Time zone Any geographic region of the world in which the same standard time is kept

Transformation Changing a figure's position, size or shape through a mathematical process

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Translation Moving a shape a certain distance in a given direction

Transversal A line that cuts two or more lines

Trapezium A quadrilateral with exactly one pair of parallel sides

Tree diagram A diagram which maps out all the outcomes in a multistep experiment

Trial One run of a chance experiment

Triangle A plane (2D) shape with three straight sides and three angles

Two step experiment An experiment involving two steps like tossing a six-sided die twice

Two-way table A way of listing the number of outcomes in different categories

U

Undefined An expression in mathematics which does not have any meaning

Unit A type of measurement (e.g. centimetres or litres)

Unitary method Calculating the value of one unit of an item and then using this to calculate the value of a number of items

Unknown A pronumeral with a value that is yet to be found to make the equation true

V

Variable Something that is measurable and observable, which is expected to change over time or between each observation

Vector A pair of numbers used to describe a translation

Venn diagram A diagram using overlapping circles to show the relationships between two or more categories

Vertex (plural: vertices) A point where two straight lines meet to make an angle

Vertical A line or plane at right angles to a horizontal line or plane

Vertical axis A vertical reference line drawn on a graph

Vertically opposite Opposite each other across a common vertex

Volume The amount of three-dimensional space in (or occupied by) an object

X

x-axis Horizontal axis of the number plane

x-coordinate The first number in an ordered pair of coordinates

Y

y-axis Vertical axis of the number plane

y-coordinate The second number in an ordered pair of coordinates

Z

z-axis A third axis perpendicular to the x and y-axes used in 3D graphing

z-coordinate The third number in an ordered triplet of coordinates in a 3D graph

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Answers

Chapter 1

Warm-up quiz

1 **a** A **b** S **c** A **d** S **e** M
f D **g** M **h** D **i** A
2 **a** 19 **b** 69 **c** 73
d 359 **e** 57 **f** 162
3 **a** 4 **b** 22 **c** 18
d 0 **e** 621 **f** 47
4 **a** 36 **b** 40 **c** 132
d 75 **e** 1089 **f** 4732
5 **a** 7 **b** 33 **c** 3 **d** 6
e 151 **f** 52
6 **a** 6, 12, 18, 24, 30 **b** 9, 18, 27, 36 **c** 18
7 **a** 1, 2, 3, 4, 6, 12 **b** 1, 3, 5, 15 **c** 3
8 2, 3, 5, 7, 11, 13
9 **a** T **b** T **c** T
d F **e** T **f** F
10 **a** 4 **b** 9, 3 **c** 16, 4 **d** 36, 6
e 81, 81 **f** 100, 100 **g** 7, 7, 7 **h** 12, 12, 12
11 **a** 0, −1 **b** −4, −6 **c** −12, −13
12 **a** −3 **b** −3 **c** 2 **d** 5

1A

Now you try

Example 1

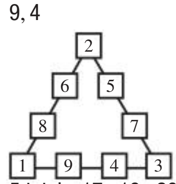
a 95 **b** 439 **c** 71

Example 2

a 1081 **b** 286

Exercise 1A

1 **a** III **b** II **c** I **d** IV
2 **a** 26 + 17 **b** 43 − 9 **c** 134 − 23
d 451 + 50 **e** 19 + 29 **f** 111 + 236
g 59 − 43 **h** 339 − 298 **i** 8 + 36
j 421 + 142 **k** 49 − 32 **l** 251 − 120
3 **a** T **b** F **c** T
d T **e** T **f** F
4 **a** 26 **b** 17 **c** 30 **d** 300
e 46 **f** 35 **g** 26 **h** 24
5 **a** 3 **b** 12 **c** 2 **d** 0
e 13 **f** 32 **g** 40 **h** 38
6 **a** 32 **b** 387 **c** 1143 **d** 55
e 163 **f** 216 **g** 79 **h** 391
i 701 **j** 229 **k** 39 **l** 161
7 **a** 174 **b** 431 **c** 10 362 **d** 2579
e 58 **f** 217 **g** 27 **h** 13 744
i 878 **j** 23 021 **k** 75 **l** 9088
8 678 km
9 David has $436 and Kristian has $738
10 24 km
11 22
12 **a** **i** 2 **ii** 7 **iii** 0 **iv** 6
v 9 **vi** 9 **vii** 0 **viii** 6
b Answers given from top row down and from left to right.
i 7, 3, 3 **ii** 1, 7, 8 **iii** 2, 5, 3
iv 9, 4 **v** 4, 2, 8 **vi** 0, 0, 7, 1
c **i**



ii 5 totals, 17, 19, 20, 21 and 23

1B

Now you try

Example 3

a 770 **b** 204 **c** 360

Example 4

a 84 **b** 19

Example 5

a 1644 **b** $50\frac{4}{7}$

Exercise 1B

1 **a** III **b** V **c** I **d** II **e** IV
2 **a** 40 **b** 99 **c** 42 **d** 72 **e** 66 **f** 132
g 32 **h** 63 **i** 10 **j** 11 **k** 11 **l** 12
m 8 **n** 11 **o** 13 **p** 13
3 **a** T **b** T **c** F **d** T
e F **f** T **g** T **h** T
4 **a** 45 **b** 72 **c** 60 **d** 140
e 64 **f** 693 **g** 237 **h** 210
i 130 **j** 260 **k** 140 **l** 68
m 17 000 **n** 13 600 **o** 413 **p** 714
q 459 **r** 366 **s** 1008 **t** 5988
5 **a** 32 **b** 16 **c** 160 **d** 123
e 37 **f** 198 **g** 16 **h** 63
i 41 **j** 127 **k** 16 **l** 127
6 **a** 603 **b** 516 **c** 3822 **d** 90 360
e 9660 **f** 1152 **g** 1392 **h** 8476
7 **a** 28 rem 1 or $28\frac{1}{3}$ **b** 30 rem 4 or $30\frac{4}{7}$
c 416 rem 7 or $416\frac{7}{10}$ **d** 13 rem 12 or $13\frac{12}{15} = 13\frac{4}{5}$
e 2514 **f** 412 **g** 210 **h** 741
8 $27.50 an hour
9 131 boxes; 1572 packets
10 Option B by $88
11 125 loads
12 $18 824
13 **a** $45 **b** $47 **c** 4 adults and 6 kids = 10 tickets

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1C

Now you try

Example 6

a 6^4 b 2^7

Example 7

a $2 \times 2 \times 2 \times 2 \times 2$ b 32

Example 8

a 81 b 12 c 27 d 7

Exercise 1C

1 a 2^2 b 4^2 c 5^2
d 5^3 e 6^4 f 7^3

2 a IV b V c VI
d III e II f I

3 a 9 b $7 \times 7 = 49$ c $11 \times 11 = 121$

4 a 8 b $5 \times 5 \times 5 = 125$ c $10 \times 10 \times 10 = 1000$

5 a 7^3 b 10^4 c 8^2
d 4^3 e 2^7 f 6^7
g 12^2 h 5^6 i 6^1

6 a $8 \times 8 \times 8 \times 8 \times 8$ b $3 \times 3 \times 3 \times 3$
c 9×9 d $4 \times 4 \times 4 \times 4$
e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ f 11×11

7 a 8 b 16 c 27
d 10 000 e 125 f 1

8 a 16 b 100 c 169 d 225
e 10 000 f 400 g 5 h 7
i 11 j 30 k 40 l 16

9 a 8 b 64 c 343 d 125
e 216 f 1000 g 3 h 2
i 5 j 8 k 9 l 100

10 a 3^2 b 2^4 c 2^5

11 a 13 b 15 c 625 d 9 e 1331

12 a $6^2 \times 7^4$ b $2^2 \times 5^4$
c $3^2 \times 8^2$ d $9^4 \times 11$
e $4^3 \times 12^2$ f $2^6 \times 3^3$

13 a m^3 b a^5 c n^7 d p^{10}
e p^3q^2 f a^4b^2 g a^2b^4 h x^4y

1D

Now you try

Example 9

a Composite b Prime

Example 10

20

Example 11

8

Exercise 1D

1 a 1, 2, 4 b 1, 2, 3, 6 c 1, 2, 3, 4, 6, 12
d 1, 3, 5, 15 e 1, 2, 4, 5, 10, 20

2 a 10 b 15 c 30
d 28 e 24 f 55

3 a 8 b 6 c 6 d 8
e 6 f 8 g 2 h 2

4 a 4 b 12 c 6
d 12 e 20 f 30

5 a Prime (P) b Composite (C) c C
d C e C f C
g C h P i P
j P k C l C

6 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

7 a 6 b 45 c 24 d 8
e 50 f 36 g 120 h 60
i 35 j 30 k 12 l 36

8 a 2 b 9 c 8 d 6
e 1 f 1 g 36 h 4
i 2 j 6 k 8 l 5

9 a 24 b 105 c 5 d 4

10 4 ways excluding factor of 64

11 25: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

12 30 minutes

13 $10 = 5 + 5$ $12 = 5 + 7$ $14 = 7 + 7$
$16 = 5 + 11$ $18 = 5 + 13$ $20 = 3 + 17$
$22 = 5 + 17$ $24 = 5 + 19$ $26 = 7 + 19$
$28 = 5 + 23$ $30 = 7 + 23$

14 (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)

1E

Now you try

Example 12

$2^5 \times 7$

Example 13

Divisible by 2, 3, 6, 9

Example 14

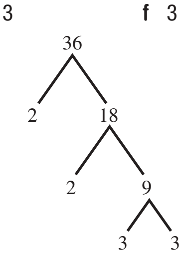
LCM = 126, HCF = 6

Exercise 1E

1 a 2, 5 b 2, 3

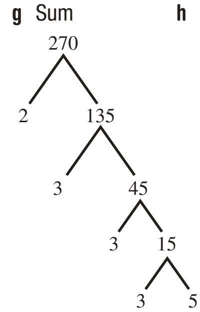
2 a 5 b Even c 0 d Three
e 3 f 3 g Sum h Two

3 a



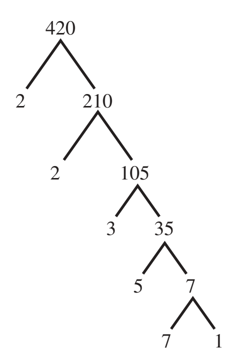
$\therefore 36 = 2^2 \times 3^2$

b



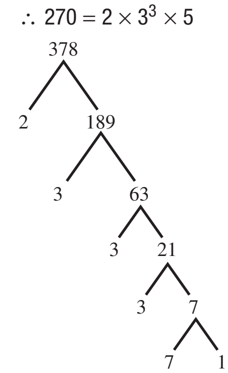
$\therefore 270 = 2 \times 3^3 \times 5$

c



$420 = 2^2 \times 3 \times 5 \times 7$

d



$378 = 2 \times 3^3 \times 7$

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4 a $2^2 \times 5$ **b** $2^2 \times 7$ **c** $2^3 \times 5$
d $2 \times 3^2 \times 5$ **e** $2^3 \times 5 \times 7$ **f** $2^2 \times 7^2$
g $2^3 \times 3^2 \times 5$ **h** $2^2 \times 3 \times 5 \times 11$
5 a Divisible by 3 **b** Divisible by 2, 3, 6, 9
c Divisible by 2, 4, 8 **d** Divisible by 3, 9
e Divide by 3, 5, 9 **f** None
g 2, 3, 6 **h** None
6 a $\text{LCM} = 2^4 \times 3 \times 5$ $\text{HCF} = 2 \times 3 = 6$
b $\text{LCM} = 2^2 \times 3^2 \times 5^3$ $\text{HCF} = 2 \times 5^2 = 50$
c $\text{LCM} = 2^5 \times 3^3$ $\text{HCF} = 2 \times 3 = 6$
d $\text{LCM} = 2 \times 5^2 \times 7^2$ $\text{HCF} = 5 \times 7 = 35$
e $\text{LCM} = 2 \times 3^2 \times 7 \times 11$ $\text{HCF} = 3^2 \times 11 = 99$
7 a 5 **b** 3 **c** 2 **d** 7
8 a 60, 2 **b** 28, 14 **c** 120, 3 **d** 60, 3
e 140, 4 **f** 390, 1 **g** 126, 3 **h** 3780, 30
9 a 12 **b** 72 **c** 30
10 72 days
11 a 1, 4, 7 **b** 0, 9 **c** 2, 5, 8
d 2 **e** 0, 2, 4, 6, 8 **f** 0

Progress Quiz

1 a 93 **b** 434 **c** 121 **d** 521
2 a 1223 **b** 481 **c** 135 **d** 7175
3 a 360 **b** 316 **c** 114 **d** 150
4 a 2232 **b** 310
5 a 7^4 **b** $5^2 \times 2^3$ **c** 1^8
6 a 25 **b** 64 **c** 10 **d** 3
7 a 12 **b** 45
8 a 5 **b** 1 **c** 24
9 $2^4 \times 3 \times 5$
10 Divisible by 2, 3, 4, 6, 8 and 9 but not 5
11 LCM = 440, HCF = 10

1F

Now you try

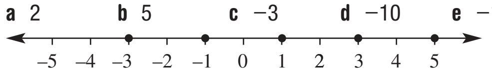
Example 15

a −3 **b** 8

Example 16

a −7 **b** −25

Exercise 1F

1 a 2 **b** 5 **c** −3 **d** −10 **e** −1
2


3 a > **b** < **c** < **d** > **e** >
f < **g** > **h** <
4 a −2°C **b** −1°C **c** −9°C **d** 3°C
5 a 1 **b** 4 **c** 1 **d** 8
e 15 **f** −2 **g** −5 **h** −7
i −7 **j** −14 **k** 5 **l** 7
m −4 **n** 0 **o** 3 **p** −5
6 a −1 **b** −5 **c** −26 **d** −17
e −2 **f** 1 **g** −11 **h** −31
i −29 **j** −110 **k** −17 **l** −12
m −20 **n** −9 **o** 4 **p** −12
7 a −3 **b** −15 **c** 0 **d** −24
e −14 **f** −7 **g** 1 **h** −1
i −19 **j** 1 **k** −6 **l** −10
m −1 **n** −100 **o** 20 **p** 31
8 a −1 **b** 8 **c** −7 **d** −1
9 a $1 - 4 = -3$ **b** $-9 + 3 = -6$
c $-1 + 5 = 4$ **d** $-15 - 5 = -20$
10 a 6 **b** −4 **c** −14 **d** 11
e 15 **f** 5 **g** −3 **h** 12
11 4 floors below ground level (−4)
12 \$7
13 a

+	−	Balance
		500
	375	125
80		205
100		305
	90	215
	45	170
	49	121
	25	96

b \$279
14 a +6 **b** +8 **c** −16
d −3 **e** +2 **f** +18

1G

Now you try

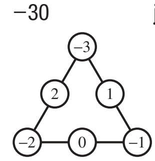
Example 17

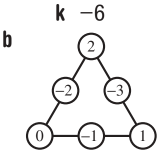
a 17 **b** −18

Example 18

a 21 **b** −28

Exercise 1G

1 a 6 **b** −10 **c** −38 **d** 46
e 32 **f** −88 **g** −673 **h** 349
2 a Subtract **b** Add
3 a F **b** T **c** T **d** F **e** T
f F **g** F **h** T **i** F
4 a 4 **b** 3 **c** −5 **d** 15
e −2 **f** −14 **g** −9 **h** −21
i −38 **j** −86 **k** −105 **l** −130
m −1 **n** −21 **o** −6 **p** −18
q −16 **r** 0 **s** −85 **t** −106
5 a 5 **b** 8 **c** 21 **d** 38
e 72 **f** 467 **g** −2 **h** 2
i 3 **j** 32 **k** −57 **l** −120
m 65 **n** −55 **o** 0 **p** 8
q 82 **r** 100 **s** −38 **t** −158
6 a −4 **b** −4 **c** −3 **d** −3 **e** −2
f −2 **g** 66 **h** 66 **i** 0
7 32°
8 \$180 000
9 a 6 **b** 7 **c** −18
10 a $10 - 6 = 4, 10 - 4 = 6$ **b** $5 - 7 = -2, 5 - (-2) = 7$
c Because $11 + (-6) = 5$, which can be rewritten as $5 - (-6) = 11$.
11 a −8 **b** −3 **c** 1
d −8 **e** −2 **f** 2
12 a −3 **b** −6 **c** 1 **d** 10
e 2 **f** −14 **g** −2 **h** −4
i −30 **j** −5 **k** −6 **l** 65
13 a


b


14 a

−1	−6	1
0	−2	−4
−5	2	−3

b

−12	−19	−14
−17	−15	−13
−16	−11	−18

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1H

Now you try

Example 19

a −24 **b** 77

Example 20

a −9 **b** 5

Exercise 1H

1 a Same, positive **b** Opposite, negative
2 a + **b** − **c** − **d** + **e** +
f − **g** + **h** − **i** +
3 a + **b** − **c** −
d − **e** − **f** +
4 a −20 **b** −54 **c** −40 **d** −99
e 6 **f** −42 **g** −72 **h** 99
i −40 **j** −64 **k** 35 **l** −32
m 60 **n** −44 **o** 9 **p** −60
5 a −5 **b** −2 **c** −4 **d** −30
e −2 **f** −3 **g** −3 **h** 3
i −2 **j** 8 **k** −9 **l** 5
m 11 **n** 1 **o** −8 **p** 8
6 a No **b** No **c** Yes
d Yes **e** Yes **f** No
7 a 25 **b** 36 **c** 49
d 64 **e** 81 **f** 100
8 a −3 **b** −5 **c** 7 **d** 6 **e** −3
f −72 **g** −252 **h** −5 **i** −30
9 Negative, because Neg × Neg is Pos, so Neg × Neg × Neg is Pos × Neg, Which is Neg.
10 a ×, ÷ **b** ×, ÷ **c** ÷, × **d** ÷, ÷
11 −8 and 3
12 a $(-2+1)\times 3=-3$
b $-10\div(3-(-2))=-2$
c $-8\div(-1+5)=-2$
d $(-1-4)\times(2+(-3))=5$
e $(-4+-2)\div(10+(-7))=-2$
f $20+(2-8)\times(-3)=38$
g $(1-(-7)\times 3)\times 2=44$
h $(4+-5\div 5)\times(-2)=-6$

1I

Now you try

Example 21

a 15 **b** 60 **c** 3

Example 22

a 17 **b** −24 **c** 18

Example 23

a −16 **b** −7 **c** −17

Exercise 1I

1 a Multiplication **b** Division **c** Multiplication
d Subtraction **e** Addition
2 a Equal **b** Equal
c Not Equal **d** Not Equal
e Not Equal **f** Equal
3 Missing numbers are:
a −3, 8, 5 **b** 6, 18
4 a 22 **b** 6 **c** 26 **d** 3
e 28 **f** 14 **g** 2 **h** 6
i 160 **j** 22 **k** 4 **l** 14
m 25 **n** 50 **o** 48 **p** 63
q 95 **r** 45
5 a 32 **b** 42 **c** 122
d 360 **e** 13 **f** 0
6 a −30 **b** −12 **c** 12
d −11 **e** −10 **f** 5
g 24 **h** −60 **i** 40
7 a −6 **b** 24 **c** 2
d 7 **e** 0 **f** 3
g −11 **h** 2 **i** −44
8 a 2 **b** 25 **c** 20 **d** −3
e −5 **f** 4 **g** −30 **h** −7
9 a −1 **b** −3 **c** −5
d 3 **e** −6 **f** 7
g 0 **h** −2 **i** −5
10 a 22 **b** 4 **c** 28 **d** 122
11 a T **b** F **c** T
d T **e** T **f** T
12 a $5+4-9$ **b** $5\times 4-9$ **c** $5+4\times 9$
13 a $(-2+1)\times 3=-3$
b $-10\div(3-(-2))=-2$
c $-8\div(-1+5)=-2$
d $(-1-4)\times(2+(-3))=5$
e $(-4+-2)\div(10+(-7))=-2$
f $20+((2-8)\times(-3))=38$
14 a 12 **b** 16 **c** 2 **d** 1
e 3 **f** −23 **g** 0 **h** 3
i 28 **j** 26 **k** 0 **l** −22
15 a $(4+7)\times 12=\$132$
b $5000+6\times 500=\$8000$
c $50-(4\times 2+8\times 3)=\$18$
16 Some suggestions include:

1	$(3+2-4)\div 1$
2	$1+2+3-4$
3	$(1+2\times 3)-4$
4	$(1+2)\div 3\times 4$
5	$(1+2)\div 3+4$
6	Change of order gives $4\times 3\div 2\times 1$
7	$(4+3)\times(2-1)$
8	$(1+3)\times 4\div 2$
9	$1\times 2+3+4$
10	$1+2+3+4$

Maths@Work: Retailer of loungeroom furniture

1 a \$2090 **b** \$2349 **c** \$1849
d \$1999 **e** \$3880

2

Model	Savings
Recliner 3 piece	\$2249
Corner suite	\$1300
2.5 seat leather chaise	\$296
Outdoor sofa	\$225
Occasional chair	\$220

3 a

Model	Difference in price
ER 2+ chaise	\$600
DW 3450 R	\$460
Ebony 3 + 2	\$896
Recliner and console 3	\$350
Victa EL +1	\$698

b \$600.80
4 Custom Sofas is cheaper by \$10.

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5 a–d

March order		
Catalogue item number	**Price in leather**	**Cost of furniture**
021-A	\$3574	\$5718
021-D	\$1738	\$5213
021-G	\$2738	\$2190
054-B	\$4374	\$7873
054-F	\$3344	\$6688
079-L	\$685	\$2740
079-M	\$1374	\$3572
079-R	\$1954	\$3517
Total cost of order		\$37 509

June order		
Catalogue item number	**Price in leather**	**Cost of furniture**
021-A	\$3574	\$3574
021-D	\$1738	\$4865
021-G	\$2738	\$2738
054-B	\$4374	\$11 372
054-F	\$3344	\$3344
079-L	\$685	\$2466
079-M	\$1374	\$2748
079-R	\$1954	\$4689
Total cost of order		\$35 794

Puzzles and games

1 A very smart cookie.
2 China develops gunpowder this leads to the manufacture of fireworks.
3 Answers will vary.

Checklist Answers

1 a 308 b 252
2 a 1155
3 55
4 a 800 b 116
5 10 300
6 $72\frac{10}{13}$
7 8^3
8 $5 \times 5 \times 5 \times 5$
9 a $6^2 = 36$ b $2^3 = 8$
10 a 9 b 4
11 a 29 is prime b 63 is composite
12 LCM(6 and 8) = 24
13 HCF(36 and 48) = 12
14 $300 = 3 \times 5^2 \times 2^2$
15 3
16 HCF = 15 LCM = $2 \times 3^2 \times 5 \times 7$
17 a −3 b 3
18 a −4 b −5
19 a 7 b −8
20 a 6 b −5
21 a −21 b 48
22 a −9 b 11
23 −37
24 a 25 b −6
25 a 39 b 3

Short-answer questions

1 a 497 b 412 c 129 d 67
e 112 f 139 g 1999 h 5675
2 a 539 b 2030 c 61 d 3074
3 a 170 b 297 c 336 d 423
e 41 f 119 g 103 h 201
4 a 1668 b 21 294 c 281 d 122
5 a 3 b 1 c 1 d 7
6 a 6^3 b 8^4 c $2^2 \times 5^4$
7 a 9 b 11 c 49 d 400
e 3 f 4 g 125 h 1000
8 a 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
b 112, 119, 126, 133, 140, 147
c 31, 37, 41, 43, 47, 53, 59 d 24 e 6
9 a $2^2 \times 3^2$ b $2^2 \times 3 \times 7$ c $2 \times 3^2 \times 11$
10 a Divisible by 2, 3, 4, 6 b Divisible by 5
c Divisible by 2, 4 d Divisible by 3, 9
11 a 380 b 2
12 a 3 b −5 c −8 d −31
e −76 f −330 g −1 h 98
13 a 2 b −8 c −64 d −39
e 16 f 12 g −20 h 92
14 a −10 b 88 c −63 d 200
e 2 f −3 g −4 h 3
15 a 1 b 1 c 4
d 4 e 9 f 9
16 a −4 b −1 c −8 d 26
17 a −11 b 1 c 7 d −4

Multiple-choice questions

1 B 2 D 3 C 4 B 5 A
6 B 7 D 8 A 9 C 10 C

Extended-response questions

1 a $a = \$112,\ b = -\$208,\ c = \$323,\ d = -\$275,\ e = \$240$
b \$228 c \$160
2 a 72 b $30 = 2 \times 3 \times 5, 42 = 2 \times 3 \times 7$
c LCM = 210, HCF = 6 d 6

Chapter 2

Warm-up quiz

1 a B: segment AB b C: point A
c D: angle ABC d A: line AB
2 a C: $\angle ABC$ b A: $\angle DEF$ c B: $\angle STU$
3 a F: revolution b B: right c A: acute
d C: obtuse e D: straight f E: reflex
4 a F: obtuse b B: isosceles
c D: acute d A: scalene
e C: equilateral f E: right
5 B, C, F, G, I, J
6 a 60 b 130 c 140
7 a 130 b $(c,\ e,\ g)$ c $(b,\ d,\ f)$
8 a 40 b 110

2A

Now you try

Example 1

a $\angle MON$ b $\angle MON$ c $\angle POQ$ or $\angle MON$

Example 2

a $a = 145,\ b = 180$ b $a = 20,\ b = 50$

Exercise 2A

1 a Complementary b Supplementary
c Perpendicular d Equal
2 a Acute b Reflex c Straight
d Right e Revolution f Obtuse
3 a Complementary b Supplementary c Revolution

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4 a 40° b 110° c 220°
5 a $\angle AOB$ b $\angle BOA$ (or $\angle DOE$)
c $\angle AOB$ (or $\angle EOD$)
6 a 45 b 130 c 120
d 240 e 90 f 180
7 a $a = 70,\ b = 270$ b $a = 25,\ b = 90$
c $a = 128,\ b = 52$ d $a = 34,\ b = 146$
e $a = 25$ f $a = 40$ g $a = 120$
h $a = 50,\ b = 90$ i $a = 140$
8 a 270° b 90° c 0° (or 360°) d 180°
e 315° f 135° g 225° h 45°
9 a 40° b 72° c 120° d 200°
10 a S b N c W d E
e NE f NW g SW h SE
11 a 60 b 135 c 35 d $a = 110,\ b = 70$
e $a = 148$ f $a = 90,\ b = 41,\ c = 139$
12 a Angles on a straight line should add to 180°.
b Angles in a revolution should add to 360°.
c Angles on straight line should add to 180°.
13 a i 180° ii 360° iii 30° iv 90°
b i 360° ii 180° iii 30° iv 120°
14 a 105° b 97.5° c 170° d 170°
e 132.5° f 27.5° g 144° h 151.5°

2B

Now you try

Example 3

a Corresponding, equal ($a = b$)
b Alternate, equal ($a = b$)
c Co-interior, supplementary ($a + b = 180$)

Example 4

a $a = 117$, corresponding angles in parallel lines
b $b = 62$, alternate angles in parallel lines
c $c = 121$, cointerior angles in parallel lines

Example 5

a $a = 85$, cointerior with the 95° angle.
$b = 74°$, corresponding with the 74° angle.

Exercise 2B

1 a equal b supplementary c equal
2 a $\angle BCH$ b $\angle ABE$ c $\angle GCB$ d $\angle BCH$
e $\angle FBC$ f $\angle GCB$ g $\angle FBC$ h $\angle DCG$
3 a Alternate, equal b Corresponding, equal
c Co-interior, supplementary d Corresponding, equal
e Co-interior, supplementary f Alternate, equal
4 a 80 (corresponding) b 120 (corresponding)
c 131 (corresponding) d 82 (alternate)
e 118 (alternate) f 78 (alternate)
g 100 (cointerior) h 129 (cointerior)
i 39 (cointerior)
5 a $a = 58,\ b = 58$ (both cointerior to 122°)
b $a = 141,\ b = 141$ (both cointerior to 39°)
c $a = 100$ (cointerior to 80°), $b = 80$ (cointerior to $a°$)
d $a = 62$ (cointerior to 118°), $b = 119$ (cointerior to 61°)
e $a = 105$ (cointerior to 75°), $b = 64$ (corresponding to 64°)
f $a = 25$ (alternate to 25°), $b = 30$ (alternate to 30°)
6 All reasons assume that lines are parallel.
a $a = 110$ (corresponding to 110°), $b = 70$ (supplementary to $a°$)
b $a = 120$ (alternate to 120°), $b = 60$ (cointerior to $a°$),
$c = 120$ (corresponding to 120°)
c $a = 74$ (alternate to 74°), $b = 106$ (cointerior to 74°),
$c = 106$ (supplementary to $a°$)
d $a = 100$ (supplementary to 80°), $b = 100$ (cointerior to 80°)
e $a = 95$ (corresponding to 95°), $b = 85$ (supplementary to $a°$)
f $a = 40$ (alternate to 40°), $b = 140$ (cointerior to 40°)
7 a No, the alternate angles are not equal.
b Yes, the cointerior angles are supplementary.
c No, the corresponding angles are not equal.
8 a 250 b 320 c 52
d 40 e 31 f 63
9 a 130° b 95° c 90°
10 $a = 36,\ b = 276,\ c = 155,\ d = 85,\ e = 130,\ f = 155,\ g = 15$

2C

Now you try

Example 6

$a = 15$

Example 7

a $a = 20$ b $a = 34$

Example 8

$a = 150$

Exercise 2C

1 a Right-angled triangle b Isosceles triangle
c Acute-angled triangle d Equilateral triangle
e Obtuse-angled triangle f Equilateral triangle
g Isosceles triangle h Scalene triangle
2 a Scalene b Isosceles
c Isosceles d Equilateral
e Scalene f Isosceles
3 a Right b Obtuse c Acute
4 a 80 b 40 c 58
d 19 e 34 f 36
5 a 68 b 106 c 20
d 65 e 40 f 76
6 a 150 b 80 c 160
d 50 e 140 f 55
7 a Yes b No c Yes
d Yes e Yes f Yes
8 a Isosceles, the two radii are of equal length.
b $\angle OAB,\ \angle OBA$ c 30° d 108°
e 40°
9 a 60 b 231 c 18
d 91 e 65 f 60
10 a 55 b 60 c 25
11 a i a, alternate angles in parallel lines
ii c, alternate angles in parallel lines
b They add to 180°, they are on a straight line.
c $a + b + c = 180$, angles in a triangle add to 180°.

2D

Now you try

Example 9

a $a = 120$ b $a = 20$

Example 10

$a = 129$

$b = 129$

Exercise 2D

1 Square, rectangle, parallelogram, rhombus, kite, trapezium
2 a 360° b equal c 2 d 90°

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3 a i T ii F iii F iv T
b i F ii T iii F iv T
c i F ii T iii T
d i T ii F iii F
e i T ii F iii T
f i F ii F
4 a 90 b 61 c 105
d 170 e 70 f 70
5 a $a = 104,\ b = 76$ b $a = 72,\ b = 72$
c $a = 128$ d $a = 50,\ b = 130$
e $a = 54,\ b = 54$ f $a = 138,\ b = 42$
6 a Square, rhombus b Trapezium
c Rectangle, parallelogram, kite
d Square, rhombus, kite e Square, rectangle
7 a 152 b 69 c 145
d 74 e 59 f 30
8 a T b F c T
d T e F f T
9 a $a = 100,\ b = 3,\ c = 110$
b $a = 2,\ b = 90$
c $a = 5,\ b = 70$
10 a–c Answers will vary. d Angle sum = 360°

Progress Quiz

1 a 65 b 332
c $m = 66,\ n = 114$ d $x = 45$
2 a $a = 45,\ b = 30$ b $t = 60$
3 B
4 a $a = 58$ (alternate) b $c = 117$ (cointerior)
c $b = 141$ (corresponding)
5 a $a = 119$ (corresponding), $b = 119$ (vertically opposite), $c = 119$ (corresponding to a, alternate to b), $d = 61$ (cointerior to b, or supplementary to c)
b $a = 112$ (cointerior), $b = 68$ (cointerior to a)
6 a $x = 27$ b $x = 41$
7 a $a = 40$ b $a = 145,\ b = 110$
8 a $c = 73$ b $x = 256$
9 a $a = 105,\ b = 75,\ c = 105$
b $a = 70,\ b = 61,\ c = 139$
10 a T b F c F

2E

Now you try

Example 11

1080°

Example 12

$a = 120$

Example 13

$S = 1260°,\ a = 140$

Exercise 2E

1 a Heptagon b Triangle c Octagon
d Nonagon e Dodecagon f Decagon
g Quadrilateral h Undecagon
2 a 6 b 4 c 10 d 7
e 5 f 12
3 a 720° b 1440° c 3600°
4 a Square b Equilateral triangle
5 a 540° b 1080° c 1440°
d 720° e 1260° f 900°
6 a 130 b 80 c 120
d 130 e 155 f 105
7 a 108° b 144° c 135°
8 a 108° b 128.6° c 120°
d 144° e 135° f 147.3°
9 a 115 b 135 c 20
d 250 e 40 f 265
10 a 9 b 15 c 21 d 167
11 a 6 b 20 c 11
12 a Circle b Increases to infinity c 180°
13 a 127.5 b 240 c 60
d 60 e 79 f 72

2F

Now you try

Example 14

a 10 faces, 16 vertices, 24 edges
b 5 faces, 5 vertices, 8 edges

Example 15

Octahedron

Example 16

Heptagonal prism

Example 17

Hexagonal pyramid

Exercise 2F

1 a Six b Circle c Cube
d Vertices e Seven f Congruent
g Seven h Octagonal
2 Cylinder, sphere, cone
3 A, cube; B, pyramid; F, rectangular prism; G, tetrahedron; H, hexahedron
4 a 6, 8, 12 b 5, 6, 9 c 7, 7, 12
5 a Hexahedron b Tetrahedron
c Pentahedron d Heptahedron
e Nonahedron f Decahedron
g Undecahedron h Dodecahedron
6 a 8 b 6 c 4 d 5
e 7 f 9 g 10 h 11
7 a Triangular prism b Pentagonal prism
c Square prism
8 a Rectangular pyramid b Heptagonal pyramid
c Triangular pyramid
9 a Pentahedron, triangular prism
b Octagonal prism, decahedron
c Square pyramid, pentahedron
10 a T b F c T d T
e F (sphere) f T g F
11 a Cube, square prism, hexahedron
b Cuboid, rectangular prism, hexahedron
12 T
13 a

Solid	Faces (F)	Vertices (V)	Edges (E)	$F + V$
Cube	6	8	12	14
Square pyramid	5	5	8	10
Tetrahedron	4	4	6	8

b $F + V$ is 2 more than E.
14 a 26 b 11 c 28

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2G

Now you try

Example 18

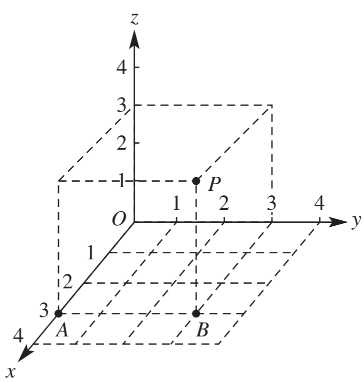
a (B, b, 4)
b (D, a, 2)
c (A, d, 3)

Example 19

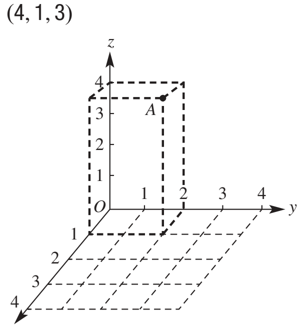
a (2, 0, 0)
b (2, 4, 0)
c (0, 4, 3)
d (2, 4, 3)

Exercise 2G

1 a B
b D
c 6
2 a C
b i (2, 3, 0)
ii (0, 3, 0)
iii (2, 3, 4)
c



3 a (A, a, 2)
b (D, c, 1)
c (B, d, 4)
4 a (A, b, 2)
b (D, d, 1)
c (D, a, 4)
5 a (2, 0, 0)
b (2, 4, 0)
c (0, 4, 3)
d (2, 4, 3)
6 a (4, 0, 0)
b (4, 1, 0)
c (0, 1, 3)
d (4, 1, 3)
7 a



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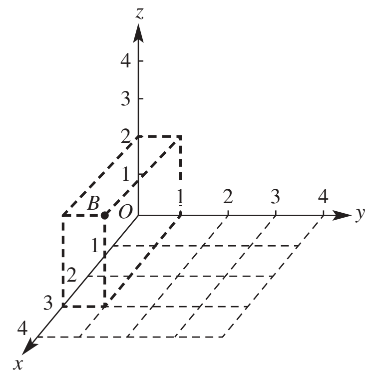
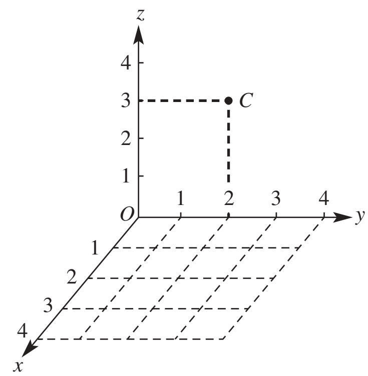


c



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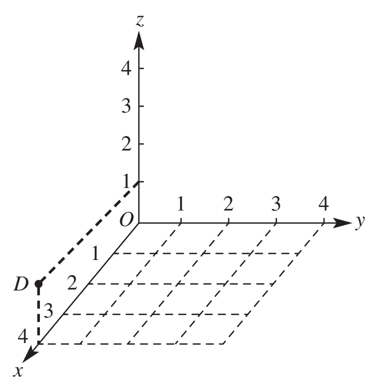
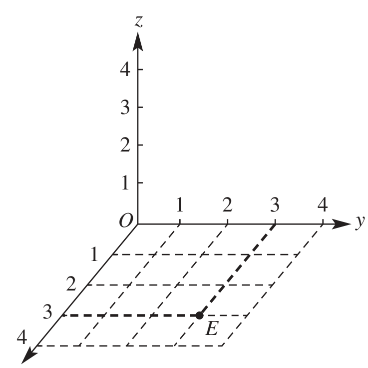


e



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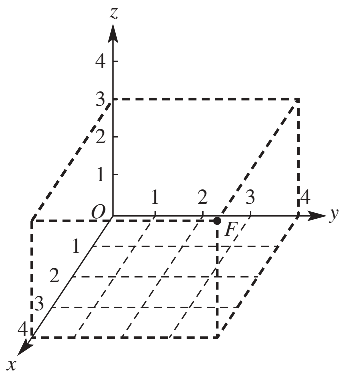
f



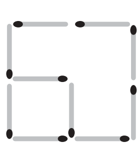
8 Square i
9 a 5
b 3
c 5
d 5
e 8
f 3
10 a i (C, b, 4) **ii** (B, c, 3)
b (A, d, 5)
c (C, b, 2)
11 A drone can travel in three-dimensions, two for the horizontal plane and one for the altitude.
12 a 6
b 16
c 30
d 56
13 a i 22 m
ii 95 m
iii 59 m
b i 16 m
ii 31 m
iii 32 m
c i 25 m
ii 53 m
iii 91 m

Maths@Work: Jewellery designer

1 a Magna: octagon, square, isosceles triangle, scalene triangle, kite.
b Cabochon: octagon, trapezium.
c Princess: Square, rhombus, octagon, isosceles triangle, kite, trapezium.
d Trilliant: equilateral triangle, isosceles triangle, right angled triangle, quadrilateral, nonagon.
e Standard round: octagon, circle, isosceles triangle, parallelogram or rhombus.
f Oval: square, octagon, right angled triangle, kite, isosceles triangle.
2 a Icosahedron **b** Pentahedron
c Octahedron **d** Tetrahedron
e Dodecahedron **f** Heptahedron
3 Answers will vary.

Puzzles and games

1 a



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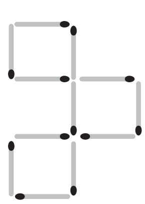
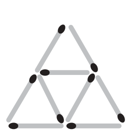
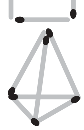


2 a



b



3 GRACE CHISHOLM YOUNG
4 a 165° **b** 37.5° **c** 122.5° **d** 127°

Checklist Answers

1 a $\angle BOA$ **b** $\angle BOA$
c $\angle BOA$ or $\angle DOE$
2 $a = 60, b = 270$
3 $c = 65$
4 $a = 108$ (co-interior angles supplementary in parallel lines)
$b = 108$ (co-interior angles supplementary in parallel lines)
5 $a = 50$
6 $a = 77$
7 $a = 71$
8 $a = 55, b = 40$
9 $a = 103, b = 77$
10 900°
11 $a = 105$
12 Angle sum of an octagon is 1080° interior angle is 135°
13 $V = 12$ $E = 18$ $F = 8$
14 Heptahedron (also a pentagonal prism)
15 Hexagonal prism
16 Pentagonal pyramid
17 $(B, C, 4)$
18 $A(2, 0, 0)B(2, 4, 0)C(0, 4, 3)P(2, 4, 3)$

Short-answer questions

1 a 50 **b** 65 **c** 240 **d** 36 **e** 61
f 138
2 a 81 **b** 96 **c** 132 **d** 99 **e** 77
f 51
3 a No – corresponding angles are not equal.
b No – cointerior angles do not add to 180°.
c Yes – alternate angles are equal.
4 a Scalene or obtuse, 35 **b** Isosceles or acute, 30
c Equilateral or acute, 60 **d** Right angle or scalene, 19
e Acute or scalene, 27 **f** Obtuse or scalene, 132
5 a 150 **b** 67 **c** 141
6 a Square, rhombus **b** Trapezium
c Rectangle, kite, parallelogram
d Square, rhombus, kite **e** Square, rectangle
7 a $a = 98,\ b = 82$ **b** $a = 85,\ b = 106$
c $a = 231,\ b = 129$
8 a 900° **b** 1260° **c** 1800°
9 a 108° **b** 150°
10 a Hexahedron **b** Decahedron
c Undecahedron, or hendecahedron
11 a Triangular prism **b** Octagonal prism
c Rectangular pyramid
12 $(B, c, 3)$
13 a (1, 0, 0) **b** (1, 2, 0)
c (0, 2, 4) **d** (1, 2, 4)

Multiple-choice questions

1 D	**2** A	**3** E	**4** C	**5** D
6 B	**7** E	**8** A	**9** C	**10** D

Extended-response questions

1 a 1260° **b** 140° **c** 40°
b i 11 **ii** 18 **iii** 27
2 a Triangle, quadrilateral, pentagon, hexagon
b $a = 90,\ b = 119,\ c = 29,\ d = 121,\ e = 270,\ f = 230$

Chapter 3

Warm-up quiz

1 a D: mixed numeral **b** C: proper
c A: whole number **d** B: improper

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2 a 4 b 8 c 20
3 a 3 b 9 c 3 d 8
4 a 100 b 1 c 4 d 1 e 30
f 60 g 100 h 4
5 a $\frac{1}{4}$ b $\frac{3}{7}$
6 a D b C c E d A e B
7 a $\frac{3}{4}$ b 1 c $1\frac{2}{3}$ d 0.6
e 1.2 f 3
8 a i $\frac{1}{10}$ ii 0.1
b i $\frac{1}{4}$ ii 0.25
c i $\frac{1}{2}$ ii 0.5
d i $\frac{3}{4}$ ii 0.75
9 a \$5 b \$6.60 c 0.8 km d 690 m
10 a 10 b 18 c 90 cents
11

Fraction	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{2}{5}$	$\frac{99}{100}$	1	$\frac{8}{5}$	2
Decimal	0.75	0.2	0.15	0.4	0.99	1.0	1.6	2.0
Percentage	75%	20%	15%	40%	99%	100%	160%	200%

3A

Now you try

Example 1

a $\frac{12}{32}$ b $\frac{8}{32}$ c $\frac{4}{32}$

Example 2

a $\frac{5}{12}$ b $\frac{8}{5}$

Exercise 3A

1 a $\frac{6}{10}, \frac{9}{15}$ b $14, 16, \frac{32}{56}$ c 50, 20, 5, 2 d 6, 9, 12
2 $\frac{10}{15}$
3 B, C and E
4 a F b T c T d T
e T f F
5 a 8 b 6 c 12 d 10
e 20 f 120 g 18 h 21
6 a 6 b 10 c 15 d 90
e 20 f 11 g 75 h 15
7 a 2 b 20 c 10 d 30
e 18 f 4 g 3 h 9
i 6 j 18 k 2 l 7
m 28 n 50 o 15 p 44
8 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{4}{5}$ d $\frac{7}{10}$
e $\frac{1}{3}$ f $\frac{1}{2}$ g $\frac{5}{6}$ h $\frac{5}{6}$
i $\frac{1}{4}$ j $\frac{3}{5}$ k $\frac{8}{9}$ l $\frac{5}{7}$
m $\frac{5}{3}$ n $\frac{11}{10}$ o $\frac{6}{5}$ p $\frac{4}{3}$
9 a $\frac{9}{10}$ b $1\frac{1}{3}$ c $\frac{1}{4}$ d $\frac{7}{13}$
e $\frac{2}{3}$ f $\frac{1}{2}$
10 a $\frac{1}{2}$ or $\frac{2}{4}$ b $\frac{1}{3}$ or $\frac{2}{6}$ c $\frac{1}{4}, \frac{4}{16}$ or $\frac{2}{8}$ d $\frac{2}{3}$ or $\frac{6}{9}$
11 a Mary ate the most (125 grams) b $\frac{1}{4}$
12 Answers vary, some include: $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{10}{50}, \frac{20}{100}$
13 $\frac{18}{28}$ as it does not simplify to $\frac{3}{4}$
14 a $\frac{5}{16}$ b $\frac{3}{16}$

3B

Now you try

Example 3

a $\frac{1}{3}$ b $\frac{17}{8}$ or $2\frac{1}{8}$

Example 4

a $\frac{62}{15}$ or $4\frac{2}{15}$ b $\frac{15}{8}$ or $1\frac{7}{8}$

Example 5

a $\frac{10}{7}$ or $1\frac{3}{7}$ b 15

Example 6

a $\frac{28}{15}$ or $1\frac{13}{15}$ b $\frac{64}{15}$ or $4\frac{4}{15}$

Exercise 3B

1 a +, − b ×, ÷
2 a 20 b 9 c 50 d 24
3 a 3, 12 b 14, 5 c 11, 33 d ×, 14, 1, 1
4 a $\frac{8}{5}$ b $\frac{2}{3}$ c $\frac{4}{13}$ d $\frac{11}{12}$
5 a $\frac{2}{3}$ b $\frac{1}{2}$ c $\frac{1}{12}$ d $\frac{2}{5}$
e $\frac{3}{5}$ f $\frac{5}{9}$ g $1\frac{1}{2}$ h $1\frac{6}{7}$
i $1\frac{3}{20}$ j $1\frac{1}{10}$ k $\frac{1}{21}$ l $\frac{4}{9}$
6 a $4\frac{4}{7}$ b $9\frac{3}{5}$ c $2\frac{3}{8}$ d $1\frac{2}{11}$
e $9\frac{1}{2}$ f $22\frac{3}{14}$ g $3\frac{3}{4}$ h $1\frac{17}{30}$
7 a $\frac{3}{20}$ b $\frac{10}{63}$ c $1\frac{17}{25}$ d $1\frac{13}{27}$
e $\frac{1}{6}$ f $\frac{3}{8}$ g $\frac{8}{15}$ h 5
8 a $3\frac{2}{3}$ b $1\frac{2}{21}$ c 15 d 35
9 a $\frac{10}{27}$ b $\frac{5}{6}$ c $\frac{16}{77}$ d $1\frac{7}{15}$
e $\frac{7}{8}$ f 2 g $1\frac{1}{3}$ h $3\frac{3}{5}$
10 a $\frac{33}{35}$ b $\frac{48}{125}$ c $1\frac{2}{5}$ d 3
11 a 10 b 15 c 25 d 3
12 a $\frac{3}{14}$ b $\frac{1}{5}$
13 a $\frac{1}{9}$ b $6\frac{11}{120}$ c $1\frac{9}{56}$ d $3\frac{1}{3}$
e 60

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3C

Now you try

Example 7

a $-\frac{3}{11}$ **b** $1\frac{2}{5}$ **c** $-\frac{2}{15}$ **d** $1\frac{2}{5}$

Example 8

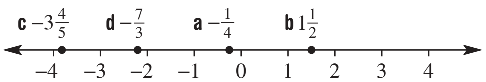
a $-\frac{18}{35}$ **b** $\frac{1}{6}$

Example 9

a $\frac{4}{5}$ **b** $-\frac{7}{12}$

Exercise 3C

1



2 **a** $\frac{1}{4}$ **b** $-\frac{1}{3}$ **c** $\frac{3}{5}$ **d** $-\frac{2}{7}$

3 **a** Positive **b** Negative
c Negative **d** Positive

4 **a** $\frac{4}{9}$ **b** $\frac{9}{20}$ **c** $\frac{3}{5}$ **d** $\frac{2}{7}$

5 **a** $-\frac{4}{7}$ **b** $\frac{1}{5}$ **c** $-\frac{7}{9}$ **d** $-5\frac{1}{3}$
e $-\frac{1}{3}$ **f** $-\frac{2}{5}$ **g** $\frac{3}{2}$ **h** $\frac{7}{11}$

6 **a** $-\frac{1}{12}$ **b** $-\frac{13}{35}$ **c** $1\frac{1}{10}$ **d** $\frac{8}{9}$
e $-\frac{1}{4}$ **f** $\frac{1}{8}$ **g** $-\frac{3}{20}$ **h** $-\frac{4}{15}$

7 **a** $-\frac{12}{35}$ **b** $-\frac{16}{55}$ **c** $\frac{4}{15}$ **d** $\frac{5}{6}$
e $-\frac{4}{21}$ **f** $-\frac{1}{8}$ **g** $\frac{3}{7}$ **h** $-1\frac{1}{5}$

8 **a** $-\frac{20}{21}$ **b** $-\frac{9}{20}$ **c** $\frac{8}{15}$ **d** $1\frac{1}{3}$
e $-\frac{2}{7}$ **f** $-\frac{3}{20}$ **g** $\frac{3}{4}$ **h** $2\frac{2}{5}$

9 $16\frac{3}{4}$°C

10 $-\frac{5}{3}, -1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{5}, \frac{1}{16}, \frac{3}{4}, 3\frac{1}{10}$

11 **a** Mon $= -1\frac{2}{3}$, Tue $= -\frac{1}{2}$, Wed $= -2\frac{1}{4}$, Thur $= \frac{1}{4}$
b $-4\frac{1}{6}$ **c** $12\frac{1}{6}$ hours

12 $1\frac{7}{20}$ metres

13 **a** $>$ **b** $<$ **c** $>$ **d** $<$
e $>$ **f** $>$ **g** $<$ **h** $>$

14 **a** $\frac{4}{5} \times \frac{-5}{2} = -2$ (Other answers possible)
b $-\frac{1}{5} + \frac{6}{5} = 1$ (Other answers possible)
c $\frac{1}{5} + \left(-\frac{6}{5}\right) = -1$ (Other answers possible)
d Not possible, neg $\times$ neg = pos, so neg $\times$ neg $\times$ neg = pos $\times$ neg = neg, so the product must be negative.

15 **a** Negative **b** Negative
c Negative **d** Positive

3D

Now you try

Example 10

2.14073

Example 11

a $\frac{29}{50}$ **b** $12\frac{13}{20}$

Example 12

a 2.341 **b** 0.425

Exercise 3D

1 E
2 C
3 C
4 E
5 **a** 37.123 **b** 21.953 **c** 0.0375 **d** 4.218 09
e 65.4112 **f** 9.528 135 2
6 **a** T **b** F **c** T **d** F **e** F **f** F
7 **a** $\frac{31}{100}$ **b** $\frac{537}{1000}$ **c** $\frac{163}{200}$ **d** $\frac{24}{25}$
e $5\frac{7}{20}$ **f** $8\frac{11}{50}$ **g** $26\frac{4}{5}$ **h** $8\frac{64}{125}$
i $\frac{13}{250}$ **j** $6\frac{1}{8}$ **k** $317\frac{3}{50}$ **l** $\frac{53}{125}$
8 **a** 0.17 **b** 0.301 **c** 0.45 **d** 0.6
e 0.67 **f** 0.674 **g** 0.15 **h** 0.79
i 0.7 **j** 1.7 **k** 1.18 **l** 0.041
9 **a** 0.12 **b** 0.35 **c** 2.5 **d** 1.75
e 0.275 **f** 0.375 **g** 0.68 **h** 0.232
10 2.18, 2.25, 2.3, 2.4
11 A1, B5, C07, P9, BW Theatre, gym
12 Opposition leader by 0.25.
13 **a** Michael's by 0.25 m **b** 25 cm
14 Answers vary, one possible is given for each:
a 0.7 **b** 0.8 **c** 0.5 **d** 0.6

15 **a**

2.6	4.6	$1\frac{4}{5}$
2.2	$\frac{6}{2}$	3.8
4.2	1.4	$3\frac{2}{5}$

b

0.8	1.8	1.0	3.2
3.0	1.2	2.0	0.6
2.8	1.4	2.2	0.4
0.2	2.4	1.6	2.6

3E

Now you try

Example 13

a 14.01 **b** 1.46

Example 14

a 0.0643 **b** 43.1

Example 15

7.2

Example 16

a 0.215 **b** 57

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Exercise 3E

1 B
2 E
3 C
4 B
5 **a** 6.8 **b** 10.5 **c** 21.9 **d** 10.2
e 16.3 **f** 13.2 **g** 62.71 **h** 277.99
i 23.963
6 **a** 4.4 **b** 6.3 **c** 15.3 **d** 4.1
e 6.1 **f** 4 **g** 14.41 **h** 23.12
i 84.59
7 **a** 96.1 **b** 961 **c** 15 463 **d** 1.94
e 0.194 **f** 2.74 **g** 27 400 **h** 1600
i 3651.73 **j** 81.55 **k** 0.75 **l** 0.038 12
8 **a** 5.6 **b** 0.56 **c** 1.5 **d** 0.12
e 30.8 **f** 0.36 **g** 0.32 **h** 0.032
i 3 **j** 4.9 **k** 8.1 **l** 1.44
9 **a** 12.27 **b** 5.88 **c** 0.0097 **d** 49.65
e 11.12 **f** 446.6 **g** 0.322 655 **h** 3.462
10 **a** 52 **b** 620 **c** 150.6; 75.3
d 3; 1530 **e** 4.84; 1.21
11 7.12 m
12 **a**

Vaughn	Charlotte	Reece
cost \$7.60	cost \$8.20	cost \$13.50
change = \$12.40	change = \$11.80	change = \$6.50

b Vaughn had the most change from \$20
13 **a** $\frac{21}{100} = 0.21$ **b** $\frac{307}{1000} = 0.307$
c $\frac{1}{100} = 0.01$
14 Answer comes from the puzzle – ask your teacher if your answer does not make sense.

3F

Now you try

Example 17

a 0.8 **b** 1.75

Example 18

a $0.4\dot{6}$ **b** $1.\dot{6}1538\dot{4}$ or $1.\overline{615384}$

Example 19

a 24.93 **b** 0.049

Exercise 3F

1 **a** T **b** R **c** R **d** T
e T **f** R **g** T **h** R
2 **a** 5.5 **b** 7.42 **c** 0.4 **d** 2.0
3 **a** $0.\dot{3}$ **b** $6.\dot{2}\dot{1}$ or $6.\overline{21}$ **c** $8.576\dot{4}$
d $2.1\dot{3}5\dot{6}$ or $2.1\overline{356}$ **e** $11.\dot{2}857\dot{3}$ or $11.\overline{28573}$
f $0.00\dot{3}5\dot{2}$ or $0.00\overline{352}$
4 **a** 4 **b** 9 **c** 7 **d** 6
5 **a** 0.6 **b** 0.75 **c** 0.125 **d** 0.55
e 0.5 **f** 0.8 **g** 0.04 **h** 0.18
6 **a** $0.\dot{3}$ **b** $0.\dot{5}$ **c** $0.8\dot{3}$ **d** $0.\dot{7}$
e $0.\overline{428571}$ **f** $0.1\dot{6}$ **g** $1.\dot{3}$ **h** $1.\dot{8}5714\dot{2}$
7 **a** 0.6 **b** 0.8 **c** 1.5 **d** 8.2
e 9.5 **f** 8.3 **g** 1.5 **h** 3.4
i 0.3
8 **a** 0.78 **b** 0.67 **c** 1.48 **d** 0.89
e 15.49 **f** 9.04 **g** 9.42 **h** 8.75
i 1.79
9 **a** **i** 8 **ii** 8.0 **iii** 5.0
b **i** 5.0 **ii** 8.9 **iii** 6.0
10 **a** 0.766 **b** 9.5 **c** 7.0 **d** 21.5134
e 0.95 **f** 17 **g** 8.60 **h** 8.106
11 **a** Greer by 0.06 of a second
b 12.8 for both, as they are the same to one decimal place you can't tell who came first.
12 **a** \$1.48 **b** \$7.40 **c** \$17.75 **d** \$58.72
13 **a** 1.4142136 **b** Answers will vary
c Answers will vary

Progress Quiz

1 **a** $\frac{2}{5}$ **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $\frac{9}{5}$ or $1\frac{4}{5}$
2 **a** $1\frac{2}{11}$ **b** $\frac{1}{8}$ **c** $5\frac{9}{10}$ **d** $2\frac{1}{6}$
3 **a** $\frac{10}{21}$ **b** $1\frac{1}{5}$ **c** $\frac{33}{35}$ **d** $2\frac{4}{5}$
4 **a** 2 **b** $-\frac{1}{6}$ **c** 1 **d** $-\frac{49}{10}$
5 **a** $-\frac{1}{6}$ **b** 6 **c** −6 **d** 6
6 **a** $\frac{7}{20}$ **b** $5\frac{1}{4}$ **c** $12\frac{4}{5}$ **d** $456\frac{7}{50}$
7 **a** 0.7 **b** 0.36 **c** 0.34 **d** 2.25
8 **a** 21.8 **b** 18.695 **c** 12.2 **d** 20.128
9 **a** 6573.4 **b** 0.0012754 **c** 0.54
d 22.615 **e** 4.769 **f** 2818.7
10 **a** 0.375 **b** 1.25 **c** $1.\dot{6}$ **d** $1.\dot{5}7142\dot{8}$
11 **a** 0.79 **b** 0.42 **c** 26.15 **d** 379.01

3G

Now you try

Example 20

a $\frac{13}{20}$ **b** 0.65

Example 21

a 46% **b** 31.25% **c** 47.3%

Exercise 3G

1 B
2 B
3 C
4

	Fraction	Decimal in words	Decimal in figures	Per cent in words	Per cent in figures
a	$\frac{13}{100}$	thirteen hundredths	0.13	thirteen per cent	13%
b	$\frac{45}{100}$	forty five hundredths	0.45	forty five per cent	45%
c	$\frac{70}{100}$	seven tenths	0.7	seventy per cent	70%
d	$\frac{99}{100}$	ninety nine hundredths	0.99	ninety nine per cent	99%

5 **a** $\frac{39}{100}$ **b** $\frac{11}{100}$ **c** $\frac{17}{100}$ **d** $\frac{99}{100}$
e $\frac{1}{5}$ **f** $\frac{7}{10}$ **g** $\frac{3}{4}$ **h** $\frac{11}{20}$
6 **a** 0.39 **b** 0.11 **c** 0.17 **d** 0.99
e 0.2 **f** 0.7 **g** 0.75 **h** 0.55
i 0.07 **j** 0.01 **k** 0.1 **l** 0.47
7 **a** 77% **b** 49% **c** 75% **d** 80%
e 28% **f** 45% **g** 55% **h** 38%
i 94% **j** 12.5% **k** 62.5% **l** $66\frac{2}{3}\%$

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8 a 16% b 79% c 83% d 97%
e 3% f 33% g 91% h 9%
i 56% j 22% k 100% l 101%

9 a $3 \times 20\% = 60\%$ b $7 \times 12.5\% = 87.5\%$
c $66\frac{2}{3}\%$

10 a

Fraction	Decimal	%
$\frac{1}{4}$	0.25	25%
$\frac{2}{4}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{4}{4}$	1	100%

b

Fraction	Decimal	%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{5}{5}$	1	100%

c

$\frac{3}{20}$	$\frac{3}{10}$	$\frac{9}{10}$	$\frac{1}{3}$
0.15	0.3	0.9	$0.\dot{3}$
15%	30%	90%	$33\frac{1}{3}\%$

11 65%, 80%

12

Cent per 100 cents	Cents in the dollar	Percentage
5c	\$0.05	5%
10c	\$0.10	10%
9c	\$0.09	9%
17c	\$0.17	17%
25c	\$0.25	25%
70c	\$0.70	70%
90c	\$0.90	90%
75c	\$0.75	75%
100c	\$1	100%
200c	\$2	200%

3H

Now you try

Example 22

65%

Example 23

17.5%

Example 24

3.5

Exercise 3H

1 D
2 A
3 a Half the test correct, 50%
b No answers correct, 0%
c Every answer correct, 100%

4 a 100 b 10 c 5 d 2 e 4

5 a 80% b 65% c 78% d 68%
e 60% f 98% g 70% h 40%
i 75% j 80% k 60% l 75%

6 a 5% b 25% c 5% d 25%
e 4% f 2.5%

7 a 56% b 75% c 86% d 25%
e 40% f 50%

8 a 18 b 8 c 150 d 18 e 8
f 12 g 60 h 22 i 12.5 j 3
k 300 l 7.2

9 a \$75 b 100 m c 45 kg d 18 minutes
e 500 mL f 15 minutes g \$3.25 h 16 cents
i 35 g

10

$\frac{1}{5}$	20%
$\frac{3}{20}$	15%
$\frac{7}{20}$	35%
$\frac{1}{4}$	25%
$\frac{1}{20}$	5%

11 a 67 c b 260 m c \$36.25
d 14 min 24 s (14.4 min) e \$14 477.40
f 24 g 101 h \$50 112

12 a 5 L b 2000 marbles c \$8
d 45 donuts

13 a \$31 500 b \$45 000
c \$13 500 d Yes, \$500 more

3I

Now you try

Example 25

\$110.50

Example 26

\$102

Example 27

\$270

Example 28

\$73.60

Exercise 3I

1 a Decrease b Increase c Decrease
d Increase e Increase

2 a 120% b 115% c 90% d 85%

3 a \$12 b \$33.99 c \$14.50 d \$225

4 a \$440 b \$264 c \$275 d \$840
e \$505 f \$1000 g \$105 h \$135

5 a \$360 b \$216 c \$225 d \$72
e \$170 f \$630 g \$500 h \$51

6 a \$200 b \$2700 c \$2300

7 a \$480 b \$127.50 c \$39

8 a \$12 b \$24 c \$37.50 d \$63.75
e \$97.50 f \$4.95

9 a \$104 b \$15.40 c \$630

10 a \$38.50 b \$82.50 c \$46.20 d \$91.30
e \$57.75 f \$164.99

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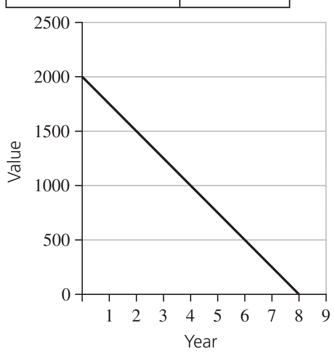
11 a \$90 300 **b** \$10.08 **c** \$37 600
d \$81.40 **e** \$960 **f** \$620 000

12 a \$180
b Shop 1 = \$1620, shop 2 = \$1600
c 2 as the bike is cheaper
d i Bikes are the same price so either shop is recommended
ii Shop 1 is now cheaper; \$1980 versus \$2000

13 a i

End of year	Value
0	2000
1	1750
2	1500
3	1250
4	1000
5	750
6	500
7	250
8	0

ii



iii Straight line

iv After 8 years

b i

3	\$1339.84
4	\$1172.36

ii Never

3J

Now you try

Example 29

25%

Example 30

40%

Example 31

125%

Exercise 3J

1 a Profit **b** Loss **c** Loss **d** Profit
e Profit

2 a \$7 **b** \$28 **c** \$3.45 **d** \$436

3 a \$13 **b** \$45 **c** \$25.90 **d** \$247

4 D

5 a 80% **b** 30% **c** 25% **d** 20%
e $66\frac{2}{3}$% **f** $37\frac{1}{2}$% **g** 50% **h** 100%

6 a 25% **b** 16% **c** 50% **d** 75%
e $33\frac{1}{3}$% **f** 10% **g** 20% **h** 10%

7 a

Cost price (\$)	Selling price (\$)	Profit (\$)	% Profit
4	5	1	25%
10	12	2	20%
24	30	6	25%
100	127	27	27%

b

Cost price (\$)	Selling price (\$)	Loss (\$)	% Loss
10	7	3	30%
16	12	4	25%
50	47	3	6%
100	93	7	7%

8 a 20% increase **b** $16\frac{2}{3}$% decrease
c 500% increase **d** 150% increase

9 a 25% increase **b** 20% increase
c 140% increase

10 20% loss

11 a \$36 **b** 75% profit

12 a \$320 **b** 80%

13 a \$2200 **b** 44% loss **c** \$5500

14 a

Place	March 2016	Change in the past 12 months	% Change
NSW	7704300	103200	1.3
VIC	6039100	114900	1.9
QLD	4827000	61800	1.3
WA	2613700	29800	1.1
SA	1706500	9700	0.6
TAS	518500	2200	0.4
ACT	395200	5000	1.3
NT	244000	1000	0.4
AUSTRALIA	24051400	327600	1.4

b Answers will vary.

3K

Now you try

Example 32

\$300

Example 33

\$22 500

Example 34

\$420

Exercise 3K

1 a 8 **b** 25 **c** 100 **d** 50

2 a \$6 **b** \$30 **c** \$300

3 a \$80 **b** \$800

4 \$4, \$400

5 a \$900 **b** \$800 **c** \$1100 **d** \$500
e \$550 **f** \$250

6 \$90

7 a \$120 **b** \$240 **c** \$15 **d** \$21

8 \$300

9 a \$50 **b** \$150 **c** \$600 **d** \$30
e \$10 **f** \$2000

10 \$5

11 \$80

12 D

13

Superbarn	Gymea fruit market	Xmart
a \$16.85 **b** 0.27 kg **c** Marshmallows – marked with * **d** 1.89 – 0.17 = \$1.72	**a** \$14.99 per kg **b** 5th July 2011 **c** Taking the cash amount to the nearest 5 cents **d** \$9.85 **e** 55 cents **f** 5.6%(1 dec pl)	**a** 8 **b** \$35 **c** All toys **d** \$13.55 **e** 9.09%

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Maths@Work: Owner and manager of a fruit and vegetable shop

1 a Answers will vary.
  b Answers could include: rent; wages; shop fit-out e.g. shelves and fridges; a vehicle; home delivery service; and advertising.
  c Keeping produce fresh and also including a big variety despite seasonal availability.
2 a \$0.97/kg b \$1.38/kg c \$0.90/head
  d \$0.80/kg e \$1.27/kg
3 a \$3.55 b \$5.45 c \$5.98
  d \$1.83 e \$2.13
4 a \$0.83/apple b \$1.00/apple
  c \$0.80/pear d \$0.66/apricot
  e \$1.22/peach
5 \$35.65
6 Option A: \$0.49/100 g; Option B: \$0.44/100 g, the best buy.
7 \$423.45
8 **a**, **b** and **c** using 1 AUD = \$0.75 US

Produce: fruit or vegetable	USA prices		Australian price
	per lb	per kg	per kg
Avocado	\$2.20	\$4.84	\$6.45
Bananas	\$0.86	\$1.89	\$2.52
Bok choy	\$2.18	\$4.80	\$6.39
Cauliflower	\$2.49	\$5.48	\$7.30
Granny Smith apples	\$1.95	\$4.29	\$5.72
Mangoes	\$1.71	\$3.76	\$5.02
Oranges - navel	\$1.37	\$3.01	\$4.02
Potatoes	\$1.24	\$2.73	\$3.64
Papaya	\$1.59	\$3.50	\$4.66

  a Australian fresh produce prices are generally dearer than USA prices. Possible reasons include Australia's smaller population and larger distances for transporting produce from farms to shops.

Puzzles and games

1 Answers vary, some include: 2.6701, 2.666, 2.668, 2.6712 . . .
2 10
3 $\frac{1}{2}$, 50%, 0.5, $\frac{2}{4}$, $\frac{10}{20}$ etc.
4 a

$\frac{4}{3}$	3	$\frac{2}{3}$
1	$1\frac{2}{3}$	$\frac{7}{3}$
$2\frac{2}{3}$	$\frac{1}{3}$	2

  b

$\frac{5}{3}$	$\frac{5}{2}$	$\frac{4}{3}$
$\frac{3}{2}$	$\frac{11}{6}$	$2\frac{1}{6}$
$\frac{7}{3}$	$\frac{7}{6}$	2

5 See teacher if your answer to the puzzle does not make sense.

Checklist Answers

1 $\frac{20}{40}$
2 $\frac{2}{5}$
3 a $\frac{11}{12}$ b $6\frac{3}{8}$
4 a $\frac{6}{35}$ b $2\frac{4}{5}$
5 a $\frac{14}{15}$ b $\frac{27}{16} = 1\frac{11}{16}$
6 $\frac{2}{3} - -\frac{4}{3} = 2, \frac{1}{5} + \frac{-1}{4} = -\frac{1}{20}$
7 $-\frac{6}{5} \times -\frac{3}{4} = \frac{9}{10}, -\frac{4}{3} \div 3 = -\frac{4}{9}$
8 57.89342 < 57.89631
9 $5\frac{3}{25}$
10 0.36
11 a 6.84 b 6.34
12 a 0.09753 b 275800
13 7.71
14 2137.9
15 0.875
16 $3.\dot{7}1428\dot{5}$
17 14.3
18 $\frac{18}{25} = 0.72$
19 a 80% b 81%
20 a 85% b 12%
21 12
22 \$224
23 \$50.40
24 \$645
25 \$280
26 28%
27 20%
28 \$600
29 \$175
30 \$150

Short-answer questions

1 a 21 b 8 c 12
2 a $\frac{5}{9}$ b 3 c $1\frac{1}{3}$
3 a $\frac{1}{2}$ b $\frac{1}{6}$ c $\frac{3}{8}$ d $\frac{4}{5}$
  e $\frac{1}{8}$ f $\frac{3}{4}$ g $\frac{2}{3}$ h $1\frac{3}{10}$
4 a $1\frac{3}{4}$ b 4 c $6\frac{1}{2}$ d $5\frac{1}{5}$
5 a 4 b 2 c 8 d 12
6 a $\frac{1}{6}$ b $\frac{1}{10}$ c $\frac{7}{20}$ d $\frac{1}{3}$
7 a 12 b 2 c $1\frac{3}{5}$ d 2
8 a $\frac{-7}{15}$ b $\frac{-3}{20}$ c $\frac{9}{25}$ d $\frac{19}{20}$
  e −5 f $-7\frac{7}{12}$
9 a 0.5 b 0.25 c 0.6 d 0.117
10 a $\frac{3}{5}$ b $\frac{3}{25}$ c $\frac{1}{25}$ d $\frac{19}{20}$
11 a 20 b 14.19 c 8.2 d 4.6
  e 22.91 f 6.18
12 a 6 b 0.06 c 4.8 d 0.048
  e 0.6 f 716.4 g 96 h 0.42
13 a 40 b 6.2 c 71.1
14 a 0.667 b 3.580 c 0.005
15

0.1	0.01	0.05	0.5	0.25	0.75	$0.\dot{3}$	0.125
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{8}$
10%	1%	5%	50%	25%	75%	$33\frac{1}{3}$%	12.5%

16 a \$5 b \$16 c 35 g
17 a 87.5% b 25% c 75% d 4%
18 a \$616 b \$3400
19 \$300
20 \$155.20
21 \$88 200

Answers
CH3
M@W

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Multiple-choice questions

1 B 2 D 3 D 4 C 5 A
6 B 7 B 8 A 9 C 10 D

Extended-response question

1 a $352 b $609.09 c $128
2 a Indian = 21 000 INR, SINGAPORE = $625 SGD Thai = 15 000 THB, Hong Kong = $3500 HKD
b 4500 THB c $6 Australian

Chapter 4

Warm-up quiz

1 a E b C c G d D
e B f H g A h F
2 a 30 m b 14.5 cm c 18 m
3 a 10 b 27 c 25 d 121
4 a 300 cm b 200 mm c 1800 m d 25 cm
e 3.5 cm f 4.2 km g 5 m h 0.1 m
i 120 s j 3000 mL k 4 L l 3 kg
5 a 6 b 9 c 4
6 a 30 cm^2 b 25 cm^2 c 16 cm^2 d 20 cm^2
7 12 cm^3

4A

Now you try

Example 1

a 3610 m b 5.4 m

Example 2

8 cm

Example 3

40 mm

Example 4

$x = 5.5$

Exercise 4A

1 a Metric b Centimetres, metres, kilometres
2 a 200 b 5200 c 78 d 8.4
e 961 f 41.2
3 a m b mm c m d km
e m f mm
4 a 10 b 10 c 2
5 a 30 mm b 610 cm c 8930 m d 300 cm
e 2.1 m f 32 cm g 19.62 km h 380 m
i 4.8 cm j 2 mm k 0.042 m l 40 cm
m 3.7 km n 0.6 km o 710 m p 2 cm
6 a 19 m b 44 m c 13 cm d 10.4 cm
e 6.6 m f 18 cm g 17.2 mm h 34.4 cm
i 29.4 m
7 a 32 cm b 28 km c 18 cm
8 a 4.3 mm b 2040 cm
c 23.098 m d 3.42 km
e 194.3 m f 0.01 km
g 24.03 mm h 0.994 km
i 1 cm
9 a 5 b 2 c 4 d 18
e 9.5 f 6.5
10 $2392
11 8 min
12 a 152.5 cm to 153.5 cm b 177.5 cm to 178.5 cm
c 159.5 cm to 160.5 cm
13 a 20 m, 2022 cm, 20 232 mm b 232 mm
c 20 232 mm is most accurate.
14 a 40 cm b 17 cm c 7.8 cm d 2000 cm
e 46 cm f 17 600 cm

4B

Now you try

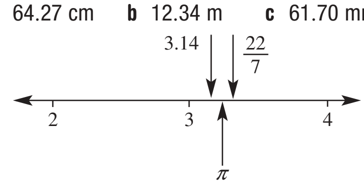
Example 5

39.58 cm

Example 6

8.92 mm

Exercise 4B

1 a Diameter b Radius c Circumference
2 a i 10 m ii 22 cm iii 4.6 mm
b i 6 cm ii 15.5 mm iii 0.21 m
3 a 3.1 b 3.14 c 3.142
4 a 15.71 b 40.84 c 18.85 d 232.48
5 a 12.57 mm b 113.10 m
c 245.04 cm d 13.19 m
e 4.40 km f 0.25 cm
6 a 12.57 m b 21.99 km
c 15.71 cm d 13.51 cm
e 25.95 m f 0.13 mm
7 251 cm
8 11.0 m
9 176 cm
10 12 566 m
11 a 64.27 cm b 12.34 m c 61.70 mm
12 a


b 3.14, π, $\frac{22}{7}$
c $\frac{22}{7}$ will give a larger value, because $\frac{22}{7} = 3.1428\ldots$ which is larger than $\pi = 3.14159\ldots$
13 $d = 2r$, so $2\pi r$ is the same as πd.
14 Answers will vary.

4C

Now you try

Example 7

a 43 mm^2 b 5.25 m^2

Example 8

a 48 cm^2 b 9 cm^2

Example 9

a 120 m^2 b 130 cm^2

Example 10

a 120 m^2 b 6 cm^2

Exercise 4C

1 a 20 000 cm^2 b 500 mm^2 c 4 cm^2
d 3 m^2 e 4 000 000 m^2 f 8 km^2

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2 a i 100 ii 400 iii 3
b i 10 000 ii 70 000 iii 4
c i 1 000 000 ii 5 000 000 iii 2.5
d i 10 000 ii 30 000 iii 7.5
3 a 7 m, 3 m b 8 cm, 6 cm (or other way around)
c 2.4 mm, 1.7 mm
4 a 200 mm^2 b 70 000 cm^2 c 500 000 m^2
d 30 000 m^2 e 34 mm^2 f 0.07 m^2
g 0.00309 m^2 h 4000 m^2 i 0.2 m^2
j 0.45 km^2 k 0.4 ha l 32.1 cm^2
m 32 ha n 51 cm^2 o 4.3 mm^2
p 0.4802 m^2 q 1.904 ha r 0.2933 ha
s 49 m^2 t 7700 m^2
5 a 49 cm^2 b 21 m^2 c 10 cm^2 d 121 m^2
e 33 m^2 f 144 mm^2
6 a 50 m^2 b 4.5 cm^2 c 6 m^2 d 165 m^2
e 18 cm^2 f 17.94 m^2
7 a 42 m^2 b 39 cm^2 c 100 cm^2
d 63 m^2 e 3 m^2 f 6 km^2
8 a 50 m b 2 m c 10 cm
9 a 6 m b 1.5 cm
10 a 25 m^2 b 16 cm^2 c 28 cm d 52 m
11 $48
12 a i Approx. 15 m^2 (Answers may vary)
ii Approx. 10 cm^2 (Answers may vary)
b This would allow a more accurate estimate as you could count the shaded squares with higher precision.
13 a 16 m^2, 156 816 m^2, 15 657 849 mm^2 b 342 151 mm^2
14 a 200 000 mm^2 b 430 000 cm^2
c 0.000 037 4 km^2 d 0.010 92 m^2
e 20 cm^2 f 0.1 ha
15 a 70 m^2 b 54 m^2 c 140 cm^2 d 91 cm^2
e 46 km^2 f 64 mm^2

4D

Now you try

Example 11

a 35 cm^2 b 80 m^2

Example 12

50 cm^2

Exercise 4D

1 a B b D c A d C
2 a 6 b 30 c 13.5 d 33
3 a 90° b Perpendicular
c Parallel, perpendicular d Rhombus, kite
4 a 7.5 cm^2 b 121 km^2 c 9.61 m^2 d 4 cm^2
e 300 mm^2 f 0.9 mm^2
5 a 96 cm^2 b 32.5 m^2 c 560 mm^2 d 5 cm^2
6 0.27 m^2
7 $1160
8 a 6 cm^2 b 35 m^2 c 84.5 cm^2
9 No, use formula for parallelogram $A = bh$, as we already know these lengths.
10 a $A = \text{length} \times \text{width}$
$= b \times h$
$= bh$
b $A = 4 \text{ triangle areas}$
$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$
$= 4 \times \frac{1}{2} \times \frac{1}{2} x \times \frac{1}{2} y$
$= \frac{1}{2} xy$
c $A = \text{Area (triangle 1)} + \text{Area (triangle 2)}$
$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$
$= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$
$= \frac{1}{2} ah + \frac{1}{2} bh$
$= \frac{1}{2}(a+b)h$

4E

Now you try

Example 13

4.52 m^2

Example 14

78.54 km^2

Example 15

a 28.27 cm^2 b 9.05 m^2

Exercise 4E

1 a $C = 2\pi r$ or $C = \pi d$ b $A = \pi r^2$
2 a 78.54 b 530.93 c 30.19 d 301.72
3 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{3}{4}$
4 a 5 m b 2.3 mm c 3.5 km
5 a 28.27 cm^2 b 113.10 m^2 c 7.07 mm^2
d 78.54 km^2 e 36.32 cm^2 f 9.08 m^2
6 a 50.27 cm^2 b 153.94 km^2 c 615.75 mm^2
d 314.16 km^2 e 38.48 m^2 f 31 415.93 m^2
7 a 50.27 cm^2 b 6.16 m^2 c 12.57 km^2
8 707 cm^2
9 Yes, by 1310 cm^2
10 a Outer square contains 4 small squares of area r^2 each; circle fits entirely within outer square.
b Inner square contains 4 small triangles of area $\frac{1}{2} r^2$ each (so $4 \times \frac{1}{2} r^2 = 2r^2$) and fits entirely within the circle.
c Actual area is bigger, since $\pi > 3$, so $\pi r^2 > 3r^2$, which is the average of $2r^2$ and $4r^2$.
11 a 3.14 cm^2 b 201.06 cm^2 c 226.98 mm^2
d 39.27 cm^2 e 5.09 mm^2 f 100.53 m^2
12 78.54 cm^2
13 80 cm^2
14 a True
b i 2.33 m ii 1.20 km iii 10.09 mm
c $r = \sqrt{\frac{A}{\pi}}$

4F

Now you try

Example 16

a 30.54 cm^2 b 102.63 m^2

Example 17

99.47 mm^2

Answers 4D

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Exercise 4F

1 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{1}{6}$ d $\frac{1}{8}$
2 a 2.79 b 8.55 c 9.69
3 a $\frac{1}{4}$ b $\frac{1}{6}$ c $\frac{1}{3}$
4 a 28.3 cm^2 b 20.9 m^2
5 a 88.49 mm^2 b 104.72 mm^2
c 4.91 cm^2 d 61.28 m^2
e 262.72 cm^2 f 981.93 m^2
6 a 37.70 m^2 b 137.44 m^2
c 437.21 km^2
7 a 34.82 m^2 b 9.14 m^2
c 257.08 cm^2 d 116.38 mm^2
e 123.61 km^2 f 53.70 m^2
g 50.27 m^2 h 75.40 mm^2
i 12.57 cm^2
8 1.26 m^2
9 13 cm radius pizza by 0.13 cm^2
10 16 965 cm^2
11 a Triangle area is $\frac{1}{2} \times 10 \times 10 = 50$ cm^2, and square's area is 100 cm^2. The sector lies between the triangle and the square.
b Actual area of $\frac{1}{4} \times \pi \times 10^2 \approx 78.5$ cm^2 is approximately 3.5 cm^2 bigger than 75 cm^2.
12 a π cm^2 b $\frac{25\pi}{9}$ m^2
c 8π mm^2 d $\frac{75\pi}{2}$ m^2
e $(9\pi + 9)$ cm^2 f $\left(225 - \frac{225\pi}{4}\right)$ km^2

Progress Quiz

1 a 0.12 m b 58.5 cm c 6200 mm d 2570 m
2 a 27 cm b 14.6 m c 22 cm d 20 cm
3 a 75.40 cm b 14.14 m
4 35.99 cm
5 a 700 mm^2 b 45 cm^2 c 3400 mm^2
d 3000 ha
6 a 6 m^2 b 38 cm^2 c 42 m^2 d 36 cm^2
7 a 300 m^2 b 3.3 cm^2 c 66 mm^2 d 160 cm^2
8 5 m
9 a 7.07 km^2 b 572.56 mm^2
10 a 56.75 m^2 b 113.49 mm^2
11 a 6.98 cm^2
b 22.87 cm^2
c 63.27 cm^2

4G

Now you try

Example 18

50 m^3

Example 19

a 0.75 L b 40 L c 370 cm^3

Example 20

60 L

Exercise 4G

1 a Length b Area c Volume d Area
e Volume f Length g Area h Volume
i Area j Length
2 a 24 b 12 c 72
3 a 1000 b 1 c 1 d 1
e 1 f 1000
4 a 48 m^3 b 20 m^3 c 27 mm^3
d 64 km^3 e 320 mm^3 f 24 m^3
5 a 2000 mL b 5000 L c 500 kL d 3 L e 4 cm^3
f 50 mL g 2.5 L h 5100 cm^3 i 1000 L
6 a 24 L b 42 L c 27 L d 0.018 L
e 0.024 L f 0.36 L
7 a 1 000 000 cm^3 b 1 000 000 c 1000 d 1000
8 a 4000 L b 45000 L c 2400 L d 1000 L
9 a i 60 000 000 L ii 60 000 kL iii 60 ML
b 200 days
10 8000 kg
11 80 minutes
12 9
13 a 24 cm^2 b 403.44 m^2 c 22 cm^2

4H

Now you try

Example 21

154 cm^3

Example 22

72 cm^3

Exercise 4H

1 a Rectangle b Square c Triangle
2 a 90 cm^2 b 16 m^2 c 5 m^2
3 a i Prism ii Rectangle
b i Prism ii Triangle
c i Not a prism (pyramid)
d i Not a prism (cone)
e i Prism ii Square
f i Not a prism (truncated pyramid)
4 a 44 m^3 b 20 m^3 c 352 mm^3 d 10 cm^3
e 33 mm^3 f 110 m^3
5 a 200 cm^3 b 15 m^3 c 980 cm^3 d 192 cm^3
e 45 m^3 f 32 cm^3
6 40 m^3
7 a 60 m^3 b 270 mm^3 c 60 m^3 d 24 cm^3
e 112 m^3 f 3200 mm^3
8 a 56 000 L b 56 hours
9 a 785.40 m^3 b 12 566.37 mm^3 c 251.33 cm^3
d 7696.90 cm^3 e 461.81 m^3 f 384.85 m^3

4I

Now you try

Example 23

a 390 s b 12.5 min

Example 24

a 1025 hours b 10:36 pm

Example 25

a i 10:30 am ii 10:00 am iii 10:30 am
iv 9:30 am
b i 12:40 pm ii 4:40 pm iii 7:40 am
iv 5:40 am

Exercise 4I

1 a 60 b 7 c 24
d 120 e 4 f 31

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2 a 120 s b 3 min c 2 h d 240 min
e 72 h f 2 days g 5 weeks h 280 days
3 a 6 h 30 min b 10 h 45 min
c 16 h 20 min d 4 h 30 min
4 a 120 s b 2 days c 3 weeks d 180 min
e 630 s f 4 min g 1.5 h h 144 h
i 3 days j 168 h k 1440 min l 210 min
5 a 6:30 pm b 9:00 am c 6:30 pm
d 4:30 pm e 5:30 pm f 11:40 am
6 a 1330 h b 2015 h c 1023 h
d 2359 h e 6:30 am f 1:00 pm
g 2:29 pm h 7:38 pm i 11:51 pm
j 4:26 am k 1847 h l 0432 h
7 a 2:00 pm b 5:00 am c 1200 hours
d 1800 hours
8 a 10:00 am b 9:30 am c 9:30 am
d 8:00 am e 10:00 am f 10:00 am
g 8:00 am h 10:00 am
9 a 5:30 pm b 3:30 pm c 5:30 pm
d 3:30 pm e 5:30 pm f 5:30 pm
g 5:00 pm h 7:30 pm i 4:30 pm
10 a F b D c A d E e B f C
11 a 2 h 50 min b 6 h 20 min c 2 h 44 min
d 8 h 50 min e 8 h 19 min f 10 h 49 min
12 17 min 28 s
13 a London 11 p.m., New York 6 p.m.
b Between 11 p.m. and 6 a.m. in Hobart
c There are no times where all three cities are between 8 a.m. and 8 p.m., so it cannot be conducted in normal business hours for everyone.
14 23 h 15 min
15 a 33c b 143c or $1.43
16 a $900 b $90 c $1.50 d 2.5c
17 a 5:30 am b 6:30 am c 6:30 am
d 1:30 pm e 2:30 pm f 2:30 am
g 3:00 pm h 5:30 pm
18 a 11:00 am b 12 noon c 8:00 pm
d 7:30 pm e 7:00 am f 5:00 am
g 1:00 am h 10:00 am
19 a You have to turn your clock back.
b You have to turn your clock forward.
c You adjust the date back one day.

4J

Now you try

Example 26

a Not a Pythagorean triple
b This is a Pythagorean triple.

Example 27

a Obtuse because $7^2 > 4^2 + 5^2$
b Right-angled because $10^2 = 6^2 + 8^2$

Exercise 4J

1 a 9 b 2.25 c 20 d 58
2 a False b True c False
3 Hypotenuse, triangle
4 a c b x c u
5 a No b No c Yes
6 a Yes b Yes c No
d Yes e No f No
7 a $3^2 + 4^2 = 5^2$ b $8^2 + 15^2 = 17^2$
c $9^2 + 12^2 = 15^2$ d $5^2 + 12^2 = 13^2$
e $9^2 + 40^2 = 41^2$ f $2.5^2 + 6^2 = 6.5^2$
8 a Obtuse b Obtuse c Right-angled
d Obtuse e Right-angled f Obtuse
9 a $a^2 + b^2 = x^2$ b $a^2 + b^2 = d^2$ c $d^2 + h^2 = x^2$
10 a No
b No, $a^2 + b^2 = c^2$ must be true for a right-angled triangle.
11 Given any one Pythagorean triple, e.g. (3, 4, 5), you can multiply it by any positive number other than 1 to generate a different Pythagorean triple. Since there are infinitely many positive numbers other than 1, there must be infinitely many Pythagorean triples.
12 a a^2 b b^2 c $\sqrt{a^2 + b^2}$
13 a Answers may vary. See answer to part **b** for the list of possible answers.
b $\{(6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (21, 28, 35), (24, 32, 40), (27, 36, 45), (30, 40, 50), (33, 44, 55), (36, 48, 60), (39, 52, 65), (42, 56, 70), (45, 60, 75), (48, 64, 80), (51, 68, 85), (54, 72, 90), (57, 76, 95)\}$, $\{(5, 12, 13), (10, 24, 26), (15, 36, 39), (20, 48, 52), (25, 60, 65), (30, 72, 78), (35, 84, 91)\}$, $\{(7, 24, 25), (14, 48, 50), (21, 72, 75)\}$, $\{(8, 15, 17), (16, 30, 34), (24, 45, 51), (32, 60, 68), (40, 75, 85)\}$, $\{(9, 40, 41), (18, 80, 82)\}$, $\{(11, 60, 61)\}$, $\{(20, 21, 29), (40, 42, 58), (60, 63, 87)\}$, $\{(12, 35, 37), (24, 70, 74)\}$, $\{(28, 45, 53)\}$, $\{(33, 56, 65)\}$, $\{(16, 63, 65)\}$, $\{(48, 55, 73)\}$, $\{(13, 84, 85)\}$, $\{(36, 77, 85)\}$, $\{(39, 80, 89)\}$, $\{(65, 72, 97)\}$

4K

Now you try

Example 28

a $c = 5$ b $c = 6.71$ (to 2 d.p.)

Example 29

The length of the brace is 6.40 m or 640 cm.

Exercise 4K

1 a Yes b No c No d Yes
2 a 3.16 b 5.10 c 8.06
3 a
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= 169 \\ \therefore c &= \sqrt{169} \\ &= 13 \end{aligned}$$
b
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 9^2 + 40^2 \\ &= 1681 \\ \therefore c &= \sqrt{1681} \\ &= 41 \end{aligned}$$
4 a 13 b 15
5 a 5 b 25 c 41
d 20 e 45 f 61
6 a 9.22 b 5.39 c 5.66
d 3.16 e 4.30 f 37.22
7 3.16 m or 316 cm
8 139 cm
9 5.5 km
10 3.88 cm
11 a 2nd line is incorrect, cannot take the square root of each term.
b 2nd line is incorrect, cannot add $3^2 + 4^2$ to get 7^2.
c Last line should say $\therefore c = \sqrt{29}$.
12 a 5 b 16 c 66 cm

4L

Now you try

Example 30

$a = 6$

Example 31

The height of the wall is 6.32 m.

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Exercise 4L

1 a 4 b 3 c 8
d 20 e 3 f 5
2 a 15^2, 81, 144, 144, 12 b 25^2, 49, 625, 576, 24
3 a 4 b 9
4 a 40 b 15 c 16 d 60
5 a 2.24 b 4.58 c 11.49 d 12.65
e 10.72 f 86.60
6 8.94 m
7 12 cm
8 12.12 cm
9 40 metres
10 a Should subtract not add 10.
b Should say $a = 5$.
c Can't take the square root of each term.
11 a $6^2 + 8^2 = 10^2$
b It is a multiple of $(3, 4, 5)$.
c $(9, 12, 15)$, $(12, 16, 20)$, $(15, 20, 25)$
d $(8, 15, 17)$
e $(3, 4, 5)$, $(5, 12, 13)$, $(8, 15, 17)$, $(7, 24, 25)$, $(9, 40, 41)$, etc.

Maths@Work: Hairdresser

1 a 10 mL b 20 mL c 45 mL d 100 mL
e 1000 mL
2 a 20 cc of developer b 60 cc of developer
c 100 cc of developer
3 a 25 cc b 50 cc c 75 cc
d Friday, November 4 e 11:20 am f 65 minutes
4 a 6 customers
b 2.5 hours c Start 9 am, finish 11:30 am
d Under the hair dryer.
e Yes, she could work into her lunch time.
f Around 3:30 or 4 pm while Mrs Babb's hair dried or 5:30 pm
g 8 customers
h Start 10:30 am, finish 1 pm i 1 hour
j 6:20 pm
5 a Various answer similar to this page of appointments.

Salon appointment book	
Day and date	
Start Time	Hairdresser's name
9:00 am	Name: long hair, colour, cut, wash, head massage, eye-brows, blow-dry and style.
9:30 am	
10:00 am	
10:30 am	Name: men's hair cut
11:00 am	First customer, cont'd.
11:30 am	Name: foils wash and blow-dry
12:00noon	
12:30 pm	
1:00 pm	Name: men's hair cut
1:30 pm	Lunch
2:00 pm	Name: long hair, colour, cut, wash, head massage, eye-brows, blow-dry and style
2:30 pm	
3:00 pm	
3:30 pm	Name: boy's hair cut
4:00 pm	After lunch customer, cont'd.
4:30 pm	Name: girl's hair cut

b Customers phone numbers, colour mixing records and price charged.

Puzzles and games

1 a 240 b 56 c 50
2 a 12.5 b 6.3 c 7
3 10 cm each side
4 Yes, 1 L will overflow.
5 75.4 cm^2
6 $\frac{1}{2}$
7 78.5%
8 3 cm

Checklist Answers

1 a 52 mm b 2.4 km
2 27 m
3 20 cm
4 $x = 7$
5 21.99 m
6 12.57 cm
7 a 2480 m^2 b 31 cm^2
8 12 cm^2
9 a 250 cm^2 b 45.5 m^2
10 12 m^2, 100 cm^2
11 35 mm^2
12 50.27 cm^2
13 28.27 m^2
14 7.07 m^2, 9.82 km^2
15 63.27 m^2
16 121 mm^2
17 48 m^2
18 a 500 mL b 3.5 L
19 3 L
20 30 cm^3
21 32 m^3
22 4320 minutes
23 a 1630 hours b 7:45 pm
24 7:35 am
25 6, 8, 10 is a Pythagorean triple (triad) as $6^2 + 8^2 = 10^2$
26 $9^2 > 4^2 + 7^2$ making the triangle obtuse
27 $c = 11.40$
28 6.71 m or 671 cm
29 $a = 3$

Short-answer questions

1 a 2000 mm b 500 m c 0.32 km d 40 m
e 300 mm^2 f 0.4 m^2 g 10 000 m^2 h 3.5 cm^2
i 4 L j 3000 mm^3 k 0.4 L l 4.3 ML
2 a 13 m b 28 cm c 25.13 m d 12.57 m
e 30.6 km f 25.8 m g 51.42 mm h 48 m
i 20 cm
3 a 55 cm^2 b 63 m^2 c 12 cm^2 d 9 cm^2
e 201.06 km^2 f 136 km^2 g 64 m^2 h 20 cm^2
i 28.27 cm^2
4 a 3.84 cm^2 b 12.14 cm^2
c 9.86 m^2
5 a 9 L b 4.5 L c 1000 L
6 a 1 m^3 b 8000 cm^3 c 10 m^3
d 144 cm^3 e 40 cm^3 f 6 cm^3
7 a i 287°C ii 239°C
b 1 h 39 min 18 s
c 1 h 2 min 4 s
8 a 10 h 17 min b 9:45 pm c 2331 hours
9 a 6:30 pm b 6:00 pm c 6:00 pm
d 4:30 pm e 4:30 pm f 4:30 pm
g 8:30 pm h 6:30 pm i 6:30 pm
10 a 10 b 25 c 4.24
11 a 15 b 6.24 c 11.36

Multiple-choice questions

1 E 2 B 3 A 4 C 5 B
6 E 7 E 8 B 9 D 10 D

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Extended-response questions

1 a $160\ m^2$
b 56 m
c $12.57\ m^2$
d $147.43\ m^2$
e i $100\ cm^2$ ii $0.01\ m^2$
f 14 744 tiles
g Some will break and more are needed to go around the pond.
2 a $70\ cm^2$ b $50.14\ cm^2$ c $74\ cm^2$

Chapter 5

Warm-up quiz

1 a 32 b 14 c 12 d 30
2 a 17 b 12 c 30 d 12
3 a 12 b 70 c 7 d 20
4 a 16 b 4 c 121 d 10 000
5 a $x+5$ b $m-7$ c xy d $\frac{w}{2}$
6 25
7 a

A	0	3	7	10
M	3	9	17	23

b

X	1	3	11	0
Y	13	15	23	12

8 a 8 b 12 c 16 d 18
9 a 12 b 3 c 48
10 a 27 b 16 c 1 d 20

5A

Now you try

Example 1

a $x+5$ b $6m$ c $2y-4$ d $3(a+b)$

Exercise 5A

1 a 8 b 12 c 3 d 10
2 a $3a, 2b, 5c$
b i a: 3 ii b: 2 iii c: 5
c $2x+5y+8z$. Other answers are possible
3 a 6
b i a: 5 ii b: 7 iii c: 1
c $x+2y+3z+4w+91k$. Other answers are possible.
4 a 2 b 1 c 9 d −2
e −6 f −1
5 a F b C c E d D
e A f B
6 a $y+7$ b $x-3$ c $a+b$ d $4p$
e $4-\frac{q}{2}$ f $10+\frac{r}{3}$ g $2(b+c)$ h $b+2c$
7 a The sum of 3 and x b The sum of a and b
c Double k d Half m
8 a The product of 4, b and c
b Double a is added to b
c b is subtracted from 4 and the result is doubled.
d b is doubled and the result is subtracted from 4.
9 a $10x$ b $A+B$ c $22-k$ d $50-k$
10 a \$70 b $7x$ c i $x-3$ ii $7(x-3)$
11 a $2p$ b $48p$ c $30p+18(p+20)$
12 a i $4a$
ii $7b$
iii $5a+5b$
iv $\frac{7a+7b}{2}$
b 7 numbers, 2 Proof by induction $\Rightarrow$ total = 9 DVDs

5B

Now you try

Example 2

2

Example 3

6

Example 4

a No b Yes

Exercise 5B

1 a 11 b 17 c 9 d 7
2 a 11 b 12 c 3 d 3
3 a 16 b 21 c 111 d 70
4 Equivalent expressions
5 a 15 b 8 c 20
6 a 14 b 30 c No
7 a 30 b 37 c 16 d 58
8 a 7 b 5 c 10 d 23
9 a 14 b 13 c 11 d 34
e 19 f 29 g 3 h 17
10 a 8 b 2
11 a E b E c N d N
e E f E
12 a 8 b 3, 4, 5
13 a If $a=3$ and $b=4$ $3+4=7, 3\times 4=12$
b $a=2$ and $b=2$
c Not equal if $a=10$ $(12 \neq 8)$
d No, always 4 apart.
14

x	3	5	2	0	4	2
y	8	7	3	−3	−2	6
$x+y$	11	12	5	−3	2	8
$x-2y$	−13	−9	−4	6	8	−10
xy	24	35	6	0	−8	12

5C

Now you try

Example 5

a N b L

Example 6

a N b L

Example 7

a $4y$ b $11a+b$ c $7ab-b-6ba+6b$

Exercise 5C

1 a Like terms b Equivalent
2 a 21 b 21 c True
3 a 23 b 84 c False
4 a a, b, c b a, b, c c Yes
5 a L b N c L d N
6 a L b N c L d N
e L f N g L h L
7 a $5x$ b $19a$ c $9x$ d $7y$
e $7xy$ f $13uv$ g $14ab$ h $15pq$
8 a $9f+12$ b $13x+8y$ c $7a+11b$
d $13a+9b$ e $12+12x$ f $8a+3b+3$
g $14x+30y$ h $21a+4$ i $5b+9c$

Answers CH5

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j $2a + 3b$ **k** $12qr + 3q$ **l** $7xy + 9x$
m $4ab + 7b$ **n** $23lk + 2l$
9 a C **b** A **c** D **d** E **e** B
10 a $12x$ **b** $22x$ **c** $12a + 4b$
11 a $13c$ **b** $9nc$
12 a If $a = 1$, $b = 2$: $4a + 3b = 10$, $7ab = 14$. Other answers are possible.
b Yes, for example if $a = 0$ and $b = 0$.
c No, they are equivalent.
13 a $5a + 7b + 5a$. Other answers are possible.
b 9 ways

5D

Now you try

Example 8

$21xyz$

Example 9

$28a^2b$

Example 10

$\frac{2}{y}$

Exercise 5D

1 a T **b** T **c** F **d** F **e** T
2 B
3 a $\frac{3}{5}$ **b** $\frac{1}{3}$ **c** $\frac{3}{2}$ **d** $\frac{3}{5}$
4 a $3xy$ **b** $5abc$ **c** $12ab^2$ **d** $4ac^3$
5 a $63d$ **b** $10a$ **c** $36x$ **d** $24k$
e $6q$ **f** $30xy$ **g** $8abcd$ **h** $60abcd$
i $48abde$
6 a x^2 **b** a^2 **c** $3d^2$ **d** $10d^2e$
e $14x^2y$ **f** $10x^2y$ **g** $8x^2yz$ **h** $8a^2b^2cd$
i $48x^2y$ **j** $18a^2b$ **k** $24x^2y^2$ **l** $24a^2b^2$
7 a $\frac{k}{4}$ **b** $\frac{x}{5}$ **c** $\frac{2q}{5}$ **d** $\frac{3k}{10}$
e $\frac{5}{a}$ **f** $\frac{a}{b}$ **g** $\frac{x}{y}$ **h** $\frac{12}{g}$
8 a $\frac{1}{2}$ **b** $\frac{x}{2y}$ **c** $\frac{5x}{6}$ **d** $\frac{a}{4}$
e $\frac{x}{3}$ **f** $\frac{1}{6x}$ **g** $\frac{4y}{7}$ **h** $\frac{ac}{2}$
9 a $8ab$ **b** $24x^2$ **c** $18xy$
10 a $11ab$ **b** $24qr$ **c** $2xy$
11 a $2y$ **b** $3b$ **c** $28rs$ **d** $8ab^2$
12 a No **b** $\frac{2a}{5}$ and $\frac{2}{5} \times a$ **c** $a = 1$ or $a = -1$
13 a $16ab$ **b** 2, 5, 6, 1 others possible
c $2a \times 3b + 3a \times 2b + 4a \times b$. Others possible.

5E

Now you try

Example 11

$11(a + 6)$, $11a + 66$

Example 12

a $4x + 36$
b $2a - 14$
c $48m - 36q$

Exercise 5E

1 a $ab + ac$ **b** $ab - ac$
2 a 20 **b** 12 **c** 32
3 a $4x$ **b** 12 **c** $4x + 12$ **d** Equivalent
4 a $4(x + 2) = 4x + 8$ **b** $3(a + 1) = 3a + 3$
c $4(k + 7) = 4k + 28$ **d** $3(b + 5) = 3b + 15$
5 a $6y + 48$ **b** $7l + 28$ **c** $9a + 63$ **d** $2t + 12$
6 a $2m - 20$ **b** $8y - 24$ **c** $3e - 21$ **d** $7e - 21$
7 a $60g - 70$ **b** $15e - 40$ **c** $35w + 50$
d $10u + 25$ **e** $56x - 14$ **f** $27v - 12$
g $14q - 28$ **h** $20c - 4v$ **i** $8 + 20x$
j $21 + 6y$ **k** $72 - 24x$ **l** $22 - 44k$
8 a 20 **b** 6 **c** 10 **d** 14
9 $2l + 2w$
10 a $7x + 6$ **b** $2a + 12$ **c** $15b$ **d** $10c + 24$
11 a $5(x + 3) = 5x + 15$ **b** $2(b + 6) = 2b + 12$
c $3(z - 4) = 3z - 12$ **d** $7(10 - y) = 70 - 7y$
12 $2(4a + 12b)$ and $8(a + 3b)$. Others possible.
13 a $ab + 4b + 2a + 8$ **b** $xy + 3y + 5x + 15$
c $6ac + 15c + 4a + 10$ **d** $20ab + 5b + 12a + 3$

Progress Quiz

1 a 5 **b** −4 **c** −11 **d** 1
2 a D **b** A **c** F **d** B
e C **f** E
3 a 55 **b** 9 **c** 30 **d** 85
4 a 17 **b** −1 **c** 44 **d** 397
5 a N **b** N **c** L **d** L
6 a $10h$ **b** $9t + 7r$ **c** $4x - 5y + 7xy$
d $5kt + k$
7 a $15w$ **b** $18yz$ **c** $24abc$ **d** $110efm$
8 a y^2 **b** $12t^2$ **c** $15jh^2$ **d** $36f^2g^2$
9 a $\frac{f}{4}$ **b** $3x$ **c** $\frac{a}{3bd}$ **d** $\frac{2y}{3}$
10 a $4x + 24$ **b** $10y - 14$ **c** $20m - 15n$
d $8x - 3x^2$
11 a $4x + 18$ **b** $17 - 6x$ **c** $4x - 8y$
d $x^2 + 10x$

5F

Now you try

Example 13

a 2 **b** $4b$

Example 14

a $2(2x - 7)$
b $7b(2 + 5a)$
c $5a(3b - 2)$

Exercise 5F

1 a 6 **b** 5 **c** 20 **d** 2
2 a 3 **b** 4 **c** $2b$ **d** $7x$
3 a 12 **b** 35 **c** 12, $30y$ **d** $14a$, $21b$
e 7 **f** 3 **g** 2, q **h** 4
4 a 5 **b** 4 **c** 9 **d** 7 **e** 3
f 6
5 a $6x$ **b** $8a$ **c** $3b$ **d** $12y$ **e** $2q$
f $4p$
6 a $3(x + 2)$ **b** $8(v + 5)$ **c** $5(3x + 7)$
d $5(2z + 5)$ **e** $4(10 + w)$ **f** $5(j - 4)$
g $3(3b - 5)$ **h** $4(3 - 4f)$ **i** $5(d - 6)$
j $5(2x + 1)$ **k** $6(k - 2)$ **l** $2(9p + 10)$
7 a $2n(5c + 6)$ **b** $8y(3 + r)$ **c** $2n(7j + 5)$
d $4g(6 + 5j)$ **e** $2(5h + 2z)$ **f** $10(3u - 2n)$
g $3(7p - 2c)$ **h** $3(4a + 5b)$

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8 For example: length = 2, width = $6x + 8$. Other answers are possible.
9 a 5 b $4a + 12$
10 $(x + 2)(y + 3)$
11 a $6x + 18$ b $6(x + 3)$ c $x + 3$ d $2x + 6$
e $3x + 9$

5G

Now you try

Example 15

a $\frac{100}{n}$ b $2x + 7$ c $20n + 100$

Exercise 5G

1 a \$10 b \$12 c \$26
2 a i 60 mins ii 150 mins iii 300 mins
b B
3 a 35 b 41 c 5
4 a $2x + y$ b 8
5 a $2x + 6$ b 24 c $3x$
6 a \$30 b $3n$ c \$36
7 a \$210 b C
8 a $5x$ b $10x$ c $5(x + 3)$ or $5x + 15$
9 a $30 + 40x$ b \$350
10 a \$50 b \$60 c \$230
11 a \$140 b $60 + 80x$
c i \$60 ii \$80
12 a $F + H$ b $F + 2H$ c $F + \frac{H}{2}$
13 a $10 + 4n$ b $20 + n$ c 30 d Deal 1
e Deal 3
f i 3 ii 4, 10 iii 9

5H

Now you try

Example 16

a 3^{10} b 11^5

Example 17

5^{12}

Example 18

a 1 b 1 c 6

Exercise 5H

1 A
2 C
3 a add b subtract c multiply
d one
4 C
5 a 6 b 3 c 9 d 7 e 13
f 6 g 6 h 3 i 4 j 3
k 6 l 5
6 a 3^6 b 2^5 c 10^4 d 9^{10} e 4^5
f 2^{12} g 8^{10} h 12^{10} i 16^8
7 a 3^2 b 2^2 c 9^4 d 4^3 e 17^6
f 11^6
8 a 12 b 10 c 4 d 12 e 6
f 20
9 a 7^4 b 2^{20} c 3^{14} d 8^8 e 3^8
f 10^{30} g 9^{14} h 5^{15}
10 a 1 b 1 c 1 d 1 e 1
f 8 g 7 h 10 i 1 j 2
k 6 l 3
11 a 2^9 b 5^6 c $6^2 = 36$
d The same e $144 = 12^2$
12 a 2^8 b 2^{13} c 10^3 d 7^{14}
e $6^1 = 6$ f 3^9
13 a a^{11} b m^7 c a^9 d x^{13}
e n^{11} f m^{14} g n^6 h a^3
i m^2 j a^{12} k w^9 l p^4
14 a $5m^7$ b $24m^8$ c $16m^{10}$ d $12a^9$
e $21x^7$ f $20x^{12}$

Maths@Work: Pharmacist

1 a 24, healthy b 22, healthy c 21, healthy
d 24, healthy e 29, unhealthy
2 a $V_1 = 4$ litres b $V_1 = 4$ litres
c $V_2 = 2$ litres, $C_2 = 1$ g/L
3 37.5 mL
4

Prescribed amounts for the patient					
Medica condition	Stock strength in mg	Prescribed dose in mg/day	Number of stock doses per day	Number of days	Total number of stock doses
Arthritis pain	200	400	2	5	10
Asthma	300	600	2	10	20
Stomach reflux	20	40	2	7	14
Antibiotic	500	1000	2	5	10
Alzheimer's disease	5	5	1	28	28
Type 1 diabetes	20	40	2	30	60
Urinary tract infections	250	500	2	5	10
Heart disease	0.05	0.1	2	30	60
Type 2 diabetes	500	1000	2	25	50

5 a–c

Child medication dose calculations					
Child's name (Age)	Weight in kg	Height in cm	BSA in m^2	Adult dose in mg/day	Child dose in mg/day
Amelia (2)	9	82	0.4528	500	133
Georgia (14)	58	160	1.6055	500	472
Dylan (8)	18.5	130	0.8173	250	120
Hunter (15)	61	168	1.6872	250	248
Ella (10)	25	145	1.0035	200	118
Chelsea (6)	13	112	0.6360	200	75
Benjamin (11)	63	135	1.5370	50	45

6 a Amelia 133 mg/day; Georgia 472 mg/day
b Dylan 120 mg/day; Hunter 248 mg/day; Hunter's BSA = 1.6872 which is almost the same as the adult BSA of 1.7.
c Ella takes 43 mg/day more than Chelsea
d Benjamin 45 mg/day; 90%

Answers

5G

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Puzzles and games

1 3001 sticks
2 $A = 5,\ B = 2,\ C = 7$
3
$$\begin{array}{ccccc} \boxed{3x} & + & \boxed{4x+3y+1} & = & \boxed{7x+3y+1} \\ + & & + & & + \\ \boxed{2y} & + & \boxed{y} & = & \boxed{3y} \\ = & & = & & = \\ \boxed{2y+3x} & + & \boxed{4x+4y+1} & = & \boxed{7x+6y+1} \end{array}$$
4 $3(2n+4) - 12$ simplifies to $6n \rightarrow$ Not a coincidence
5 **a** 25 **b** 16.25 **c** 56.25 **d** 0
6 **a** All perimeters = 4 m
Areas: 1 m^2, $\frac{3}{4}m^2$, $\frac{6}{9}m^2$, $\frac{10}{16}m^2$, $\frac{15}{25}m^2$
b Perimeter = 4 m
Area approximately $\frac{1}{2}m^2$.

Checklist Answers

1 a is 4 b is 1 and $c = -12$
2 $2(a+b)$
3 15
4 21
5 Not equivalent
6 $2ab$ & $3ba$ are like terms
7 $6x + 10y$
8 **a** $42abcd$ **b** $15x^2yz$
9 $\frac{2a}{3c}$
10 $2(x+5) = 2x + 10$
11 **a** $5x + 15$ **b** $6p - 14q$
12 $6x$
13 **a** $6a(2+3b)$ **b** $7(3x - 2y)$
14 $C = 40 + 70h$
15 6^{11}
16 5^3
17 4^{10}
18 4

Short-answer questions

1 **a** F **b** T **c** T **d** F **e** T
2 **a** 2 **b** 3 **c** 4 **d** 6
3 **a** 10 **b** 8 **c** 4 **d** 0
4 **a** 20 **b** 7 **c** 3 **d** 16
5 **a** 9 **b** 9 **c** 9 **d** 2
6 **a** $16m$ **b** $2a + 5b$ **c** $4y - x + 1$ **d** $7x + 7y$
e $9x + xy$ **f** $10m - 6n$
7 **a** $36ab$ **b** $30xy$ **c** $30xyz$
8 **a** $2x$ **b** $3a$ **c** $\frac{z}{5y}$
9 **a** $3x - 12$ **b** $10 + 2x$ **c** $6y + 12$
d $20x + 70$ **e** $3x - 15$ **f** $11z - 22$
g $12a - 44$ **h** $12b - 6$
10 **a** 4 **b** $7a$
11 **a** $2(x+3)$ **b** $8(3 - 2g)$ **c** $3x(4+y)$
d $7a(1+2b)$
12 **a** $5a$ **b** $3p$ **c** $5a + 3p$
13 **a** 70 km **b** $10n$
14 **a** 4^{11} **b** 3^2 **c** 1 **d** 3^{20}

Multiple-choice questions

1 C 2 A 3 D 4 C 5 E
6 E 7 D 8 E 9 D 10 A

Extended-response questions

1 **a** $120 + 80n$ **b** $80 + 100n$
c A costs $360, B costs $380.
d Any more than two hours
e $200 + 180n$
2 **a** $2xy - x^2$ **b** 33 m^2 **c** $4x + 2y$ **d** 26 m
e Area = $3xy - 3x^2$; Perimeter = $6x + 2y$

Semester review 1

Computation with integers

Short-answer questions

1 **a** 5169 **b** 1350 **c** −288 **d** 695
e 1386 **f** 2800 **g** 81 **h** 64
i −19
2 **a** −14 **b** 30 **c** 72 **d** −7
e 54 **f** −6
3 **a** 6 **b** 7 **c** 20 **d** 15
4 **a** 168 **b** 72 **c** 300 **d** 45
5 **a** 7^3 **b** 8^2 **c** 3^5
6 **a** 81 **b** 7 **c** 125 **d** 3
7 **a** −2 **b** 12 **c** −35 **d** 22
e 25 **f** 49

Multiple-choice questions

1 B 2 C 3 C 4 A 5 B

Extended-response questions

1 **a** Hong Kong **b** Moscow, New York
c Hong Kong **d** 14°

Angle relationships and properties of geometrical figures

Short-answer questions

1 **a** 66 **b** 25 **c** 123 **d** 35 **e** 70
f 98
2 **a** $x = 81,\ y = 99$ **b** $a = 75$ **c** $a = 62,\ b = 62$
d $a = 65,\ b = 65$ **e** $a = b = c = d = 100,\ e = 80$
f $x = 95,\ y = 85$
3 **a** 48 **b** 45 **c** 60 **d** 75
e 121 **f** 75
4 **a** $a = b = 90$ **b** $a = 73,\ b = 95$
c $a = 265,\ b = 30$
5 **a** 540° **b** 1080°

Multiple-choice questions

1 B 2 D 3 C 4 C 5 D

Extended-response questions

1 **a** $b = 65$ (supplementary to a)
$c = 65$ (alternate to b)
$d = e = 57.5$ (isosceles triangle)
$f = 122.5$ (supplementary to d)
$g = 122.5$ (revolution angle = 360)
$h = 180$ (straight angle)
$i = 295$ (revolution) **b** Answers will vary.

Fractions, decimals and percentages

Short-answer questions

1 **a** 18 **b** 1 **c** 5
2 **a** $\frac{1}{4}$ **b** $\frac{7}{5}$ **c** $3\frac{1}{4}$
d $-\frac{2}{21}$ **e** $\frac{1}{3}$ **f** $\frac{9}{10}$

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3 a $\frac{5}{2}$ b $\frac{1}{8}$ c $\frac{5}{21}$
4 a $\frac{9}{2}$ b $\frac{3}{4}$ c $\frac{2}{3}$
5 a 6.93 b 7.58 c 4.03 d 6.51
e 3854.8 f 792
6 a 530 b 9600 c 0.614
7 *See table at bottom of page*
8 a 5.6 b 11.76 c 85.5 m d \$1.98
e \$210 f 4250 g
9 a \$700 b \$862.40
10 a \$87 b 25%

Multiple-choice questions

1 D 2 C 3 C 4 B 5 B

Extended-response questions

1 a i \$1600 ii \$1280 iii \$1024
b 5 years
c No, there will always be 80% of the previous value.

Measurement

Short-answer questions

1 a 500 cm b 180 cm c 90 000 cm^2
d 180 cm e 4000 cm^3 f 10 000 m^2
2 a 18.6 cm b 64 m c 40 m
3 a i 25.13 m ii 50.27 m^2
b i 47.12 cm ii 176.71 cm^2
4 a i 25.71 m ii 39.27 m^2
b i 17.85 cm ii 19.63 cm^2
c i 54.85 mm ii 169.65 mm^2
5 a 30 m^2 b 48 m^2 c 21 cm^2
6 a 74.088 m^3 b 50 m^3 c 24 m^3
7 a 1530 b 0735
8 a 13 b 14.42

Multiple-choice questions

1 C 2 B 3 D 4 B 5 D

Extended-response questions

1 a 2.5 m b 9.82 m^2 c 59.82 m^2
d 32.85 m

Algebraic techniques and index laws

Short-answer questions

1 a $p+q$ b $3p$ c $\frac{m^2}{2}$ d $\frac{x+y}{2}$
2 a 19 b 68 c 33 d 698
3 a 11 b 23 c 18 d 26
e 24 f 17
4 a $24k$ b $3a$ c a^3 d $\frac{p}{2}$
e $7ab+2$ f $x-1$ g $2y$ h $2n-2m$
5 a xy b $\frac{10x}{7y}$ c $\frac{w}{5}$ d $\frac{17a}{5}$
6 a $2x+10$ b $12m-18$ c $4+2m$
7 a 6 b $5k$ c $20b$
8 a $6(3a-2)$ b $6m(n+2)$ c $4(2x+3)$
9 a $2x+20$ b $10x$
10 a $2x$ b $3y$ c $2x+3y$
11 a 3^9 b 4^2 c 1 d 5^{12}

Multiple-choice questions

1 D 2 A 3 B 4 B 5 C

Extended-response questions

1 a \$220
b $60+80n$
c i $100n$ ii 3 hours

Chapter 6

Warm-up quiz

1 a 2 b 3 c 4
2 a 10 b 5 c 8
3 a 3 : 4 b 4 : 5 c 5 : 12
4 a 500 b 6000 c 5 d 8
e 1.2 f 15
5 a 2 : 1 b 9 : 20 c 3 : 4 d 1 : 3
e 4 : 1 f 2 : 25
6 a \$1.50 b \$18.00
7 a \$9.98 b \$24.95 c \$49.90 d \$2.50
8 1.5 km
9 \$30/h
10 a i 120 km ii 300 km iii 30 km
b i 3 hours ii $1\frac{1}{2}$ hours iii $\frac{1}{3}$ hour
c 6 minutes

6A

Now you try

Example 1

a 6 : 5 b 11 c 6 : 11

Example 2

a 15 b 3 c 4, 12

Exercise 6A

1 a 3 : 7 b 5 : 4
2 a 9 : 4 b 7 : 12 c 10 : 75 (or 2 : 15)
3 a 1 : 3 b 7 : 15
4 a 5 : 7 b 12 c 5 : 12
5 a 8 : 3 b 3 : 14 c 3 : 11 d 8 : 6 or 4 : 3
6 a 13 : 7 b 11 : 9 c 13 : 9 : 11 : 7
d 20 : 20 or 1 : 1
7 a 12 b 14 c 4 d 9
e 2 f 4 g 2 h 10
i 4 : 6 : 10 j 2 : 6 : 8 k 6 l 1

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{19}{20}$	$\frac{99}{100}$	$\frac{1}{200}$
Decimal	0.25	0.5	0.2	$0.\dot{3}$	$0.\dot{6}$	0.8	0.95	0.99	0.005
Percentage	25%	50%	20%	$33\frac{1}{3}$%	$66\frac{2}{3}$%	80%	95%	99%	0.5%

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8 Answers may vary, some include:
a $2:4, 3:6, 5:10$ b $4:10, 20:50, 200:500$
c $4:3, 16:12, 40:30$ d $3:1, 6:2, 18:6$
9 $2:5$ and $4:10$, $6:12$ and $1:2$, $7:4$ and $70:40$
10 a $4:6$ or $2:3$ b $3:5$ c $4:6$ or $2:3$ d $5:4$
11 a 8 boys, 4 girls b 4 boys, 8 girls
c 9 boys, 3 girls d 2 boys, 10 girls
12 a $8:42$ or $4:21$ b $90:210$ or $3:7$
13 a i and ii Answers will vary.
b Each ratio length : area simplifies to 1 : width.

6B

Now you try

Example 3

a $3:1$ b $5:19$

Example 4

a $5:1$ b $5:6$

Exercise 6B

1 a $1:2$ b $3:5$ c $1:3$
d $3:7$ e $8:5$ f $6:5$
2 $3:2$
3 a $1:1$ b $1:2$
4 a $1:4$ b $1:5$ c $1:6$ d $1:3$
e $4:5$ f $5:8$ g $3:4$ h $3:10$
i $9:7$ j $2:1$ k $9:7$ l $3:1$
m $3:1$ n $1:9$ o $6:11$ p $2:1$
q $12:1$ r $1:6$ s $8:5$ t $6:5$
5 a $1:2:3$ b $4:7:11$ c $7:10:2$ d $17:7:3$
e $1:2:3$ f $2:6:5$ g $9:14:2$ h $2:4:7$
6 a $2:5$ b $14:1$ c $3:25$ d $1:35$
e $20:3$ f $2:25$ g $50:11$ h $5:1$
i $2:5$ j $1:6$ k $12:1$ l $9:1$
m $1:16$ n $2:9$ o $1:7$ p $14:3$
q $1:8$ r $30:1$
7 D
8 B
9 C
10 a $5:5:2:4:3:1:20$
b $20:20:8:16:12:4:80$
c i $1:4$ ii $1:1$
11 Andrew did not convert amounts to the same units. Correct ratio is $40:1$.
12 Answers may vary, some include:
a 2 hours to 300 minutes, 24 minutes to 1 hour
b 4 kilometres to 3000 metres, 2 kilometres to 1500 metres
13 Answers will vary.

6C

Now you try

Example 5

\$50 and \$30

Example 6

35 kg, 10 kg and 15 kg

Exercise 6C

1 a 10 b 6 c 14 d 9
2 a In the ratio $3:2$ the total parts $= 2 + 3 = 5$
b 5 parts = \$25, so 1 part = \$5
c Marta gets 3 parts, so Marta gets $3 \times \$5 = \$15$
d Joshua gets 2 parts, Joshua gets $2 \times \$5 = \$10$
e Total amount $= \$15 + \$10 = \$25$
3 a $1:3$ b $1:1$ c $2:5$ d $1:4$
4 a \$24 and \$36 b \$70 and \$40
c \$150 and \$850 d 8 kg and 40 kg
e 8 kg and 6 kg f 150 kg and 210 kg
g 24 m and 48 m h 15 m and 25 m
i 124 m and 31 m
5 a \$100 and \$300 b \$160 and \$240
c \$150 and \$250 d \$180 and \$220
6 a \$40, \$80, \$80 b \$50, \$150, \$200
c 2 kg, 4 kg, 6 kg d 22 kg, 11 kg, 55 kg
e 96 kg, 104 kg, 120 kg
f \$5000, \$10 000, \$15 000, \$20 000
7 a 60, 540 b 200, 100, 300
c 100, 250, 250 d 240, 140, 160, 60
8 Nitrogen: 500 g, potassium: 625 g, phosphorus: 375 g
9 $40°, 60°, 80°$
10 \$250
11 120 pages
12 Shirt \$160, jacket \$400
13 a 2 pigs and 2 sheep were missing or 5 sheep and 9 pigs.
b $3:5$

6D

Now you try

Example 7

a 24 m b 1.8 m

Example 8

a 12 cm b 8.8 cm

Example 9

a 30 000 b $\frac{1}{200}$

Exercise 6D

1 a i 100 000 mm ii 100 m iii 0.1 km
b i 0.56 km ii 56 000 cm iii 560 000 mm
2 a 5.7 cm, 570 cm b Real car 100 × bigger
c $1:100$
3 a 60 cm, 30 000 cm b Real ship 500 × bigger
c $1:500$
4 a i 620 cm ii 5 mm
b i 200 m ii 40 m
c i 6.4 m ii 288 m
d i 0.3 mm ii 8.15 mm
5 a i 1 m ii 20 m
b i 20 m ii 2 m
c i 13.5 cm ii 7.365 cm
d i 1.5 m ii 0.164 m
6 a $1:10\,000$ b $1:1000$ c $1:300$
d $1:150\,000$ e $1:125$ f $1:200\,000$
g $1:100\,000$ h $50:1$ i $10\,000:1$
7 a $1:250$, 250 b $1:50\,000$, 50 000
c $1:50\,000$, 50 000 d $1:18\,000$, 18 000
e $1:\frac{1}{7}, \frac{1}{7}$ f $1:\frac{1}{600}, \frac{1}{600}$
8 a 80 m b 4.5 cm
9 8.5 km
10 a 3.8 m × 2.7 m b 5 m × 5 m c 8.3 m × 2.1 m
11 a 2800 km b 3300 km c 2500 km
d 1300 km e 3900 km

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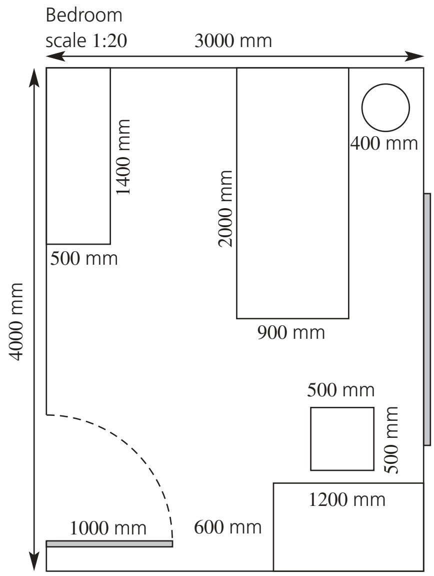
12 a–g Note: Different furniture arrangements also correct.



Progress Quiz

1 a 7 : 5 **b** 12 **c** 5 : 12
2 a 30 **b** 7 **c** 20, 32
3 a 2 : 9 **b** 4 : 1 **c** 6 : 4 : 1
4 a 1 : 6 **b** 2 : 3 **c** 15 : 1
5 D
6 a 20 kg : 30 kg **b** 15 m : 3 m
c \$1400 : \$2600 **d** 40 min : 200 min
7 a \$300 : \$700 **b** \$950 : \$50
c \$650 : \$350 **d** \$460 : \$540
8 93 goals
9 a 50 000 **b** 1500 m **c** 4 km
10 a 40 mm **b** 23 mm

6E

Now you try

Example 10

a 2 sandwiches/person **b** \$17/kg

Example 11

a 60 km/h **b** 30 pages/day

Example 12

0.25 kg/year

Exercise 6E

1 B, C, E, F, H
2 a Employee's wage: \$15/h
b Speed of a car: 68 km/h
c Cost of building new home: \$2100/m^2
d Population growth: 90 people/day
e Resting heart rate: 64 beats/min
3 a \$/kg **b** \$/L **c** Words per minute
d Goals/shots on goal **e** kJ/serve
f L/min **g** mg/tablet **h** runs/over
4 a 3 days/year **b** 5 goals/game **c** \$30/h
d \$3.50/kg **e** \$14 000/acre **f** 4500 cans/hour
g 1200 revs/min **h** 16 mm/day **i** 4 min/km
j 0.25 km/min or 250 m/min
5 a 300 km/day **b** \$140/year
c 6.5 runs/over **d** 7.5 cm/year
e 1.5 kg/year **f** Dropped 2.5°C/h or −2.5°C/h
6 a 3.8 cm/year **b** 3 cm/year
7 a 3 L/h **b** 7 hours
8 158 cm
9 a 1.5 rolls/person **b** \$6/person **c** \$4/roll
10 Harvey: 3.75 min/km, Jacques: 3.33 min/km; Jacques
11 a 1200 members/year **b** 12 years
12 a i 9 km/L **ii** $\frac{1}{9}$ L/km
b Find the reciprocal.
13 Answers will vary.

6F

Now you try

Example 13

a 40 cars **b** 3000 m

Example 14

a 1.2 m/s **b** 5 m/s

Example 15

540 km

Example 16

1.25 hours

Exercise 6F

1 a 3 hours **b** 5 hours, × 5
c × 10, 30 minutes, × 10 **d** × 6, 720 litres, × 6
2 a \$12, \$60, × 5
b ÷ 5, 30 rotations, ÷ 5, × 7, 210 rotations, × 7
3 a speed $= \frac{\text{distance}}{\text{time}}$ **b** distance = speed × time
c time $= \frac{\text{distance}}{\text{speed}}$
4 D
5 a 1600 words **b** 50 minutes
6 a 2400 bottles **b** 19 200 bottles
7 a 10 m/s **b** 7 m/s **c** 60 km/h
d 50 km/h **e** 2 km/min, 120 km/h
f 0.75 km/min, 45 km/h
8 a 1080 m **b** 4.5 m **c** 36 km **d** 50 km
9 a 8 hours **b** $\frac{1}{2}$ hour or 30 minutes
c 11.5 hours **d** 7 seconds
10 a 3750 beats **b** 1380 beats **c** 80 minutes
11 2025 km
12 a 27 km/h **b** $2\frac{1}{4}$ km
13 a 58.2 km/h **b** 69.4 km/h
14 a 343 m/s
b 299 792 458 m/s
c 0.29 s
d 0.0003 s
e 874 030
f How many times the speed of sound (mach 1 = speed of sound)
g 40 320 km/h, 11.2 km/s
h 107 208 km/h, 29.78 km/s
i 7.7 km/s
j–l Answers vary.

Maths@Work: Development officer for a body and fragrance company

1 a 30 mL **b** 45 mL **c** 180 mL
2 a i 4 drops **ii** 10 drops
b 10 mL

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3 **a** 5 : 1
b **i** 2 drops **ii** 5 drops **iii** 20 drops
4 **a** **i** 2 g **ii** 4 g **iii** 10 g **iv** 60 g
b **i** 60 g **ii** 300 g **iii** 270 g
5 **a** **i** 30 drops **ii** 40 drops **iii** 50 drops
b 12 drops
c **i** 0.65 mL **ii** 1.3 mL **iii** 1.56 mL
d 60 drops of cedarwood; 200 drops of lavender; 100 drops of bergamot.
6 **a** 1 tablespoon of seaweed powder
2 tablespoons of sweet almond oil (or jojoba)
1 teaspoon of aloe vera gel
1 teaspoon of honey.
b 2 tablespoons of seaweed powder
4 tablespoons of jojoba oil
2 teaspoons of aloe vera gel
2 teaspoons of honey.
7 **a**

Dilution of Essential oils				
Carrier oil	**Volume of Carrier oil in mL**	**Dilution %**	**Essential oil**	**Drops of Essential oil**
Sweet Almond Oil	10	1	Chamomile	2
Sweet Almond Oil	10	2	Lemon	4
Coconut oil	10	1	Rosewood	2
Jojoba oil	5	1	Rosemary	1
Jojoba oil	25	2	Lavender	10
Grapeseed oil	20	2	Rosemary	8

b

Dilution of Essential oils				
Carrier oil	**Volume of Carrier oil in mL**	**Dilution %**	**Essential oil**	**Drops of Essential oil**
Sweet Almond oil	25	1	Peppermint	5
Coconut oil	20	2	Geranium	8
Jojoba oil	10	1	Lavender	2
Sunflower oil	15	2	Lemon Grass	6

Puzzles and games

1 **a** TOOTHPICKS **b** TO ROCK FESTIVALS
2 **a** 1 : 1 : 2 : 2 : 4 : 4
b **i** 13 : 3 **ii** 1 : 3
3 **a** Hannah 15, Blake 10 **b** Hannah 25, Blake 20
c Hannah 55, Blake 50
4 **a** 2 **b** $3\frac{1}{5}$ **c** $2\frac{2}{3}$
5 1 : 3
6 A flat route (1 h 48 min) faster by 2 minutes.
7 9 km/h
8 Because he thought he was a griller.

Checklist Answers

1 **a** 5 : 12 **b** 5 : 17
2 10
3 9 : 4
4 25 : 120 = 5 : 24
5 25 m to 30 m
6 \$100, \$50 and \$150
7 615 students
8 100 m
9 100 mm
10 1 : 125, scale factor = 125
11 \$7/kg
12 3000 rev/min or 50 rev/s
13 6.5 cm/yr
14 1110 words
15 12 km/h
16 1425 km
17 3.75 hours or 3 hours 45 minutes

Short-answer questions

1 **a** 1 : 2 **b** 2 : 1 **c** 1 : 3
2 **a** F **b** F **c** T **d** F
3 **a** 25 **b** 9 **c** 32 **d** 3
4 **a** 1 : 4 **b** 3 : 2 **c** 3 : 4 **d** 1 : 8 **e** 3 : 1
f 1 : 5 **g** 3 : 2 **h** 2 : 1 **i** 2 : 3 **j** 2 : 1 : 5
5 **a** 5 : 2 **b** 1 : 3 **c** 2 : 5 **d** 1 : 2
e 1 : 5 **f** 1 : 4 **g** 3 : 25 **h** 3 : 10
6 **a** \$35, \$45 **b** 160 kg, 40 kg
c 30 m, 10 m **d** \$340, \$595, \$510
e \$60, \$20, \$20
7 1.125 L or $1\frac{1}{8}$ L
8 **a** 600 m **b** 2.4 km
9 **a** Scale ratio = 1 : 200 **b** Scale ratio = 1 : 250 000
Scale factor = 200 Scale factor = 250 000
10 50 mm
11 **a** 5 km/h **b** \$50/h **c** 140 km/day
12 **a**
7 km uses 1L
×40 ×40
280 km uses 40 L
b
60 words typed in 1 minute
×10 ×10
600 words typed in 10 minutes
13 **a** **i** 7 km **ii** 294 km
b **i** \$5.60 **ii** \$39.20
14 **a** 75 km/h **b** 1.8 hours **c** 9 km

Multiple-choice questions

1 A **2** C **3** D **4** A **5** B
6 B **7** C **8** C **9** B **10** D

Extended-response questions

1 **a** 160 km **b** 500 km
c 11 : 30 am **d** 5 hours
e 100 km/h **f** 4 : 15 pm
g 85.2 km/h **h** Harrison's cost \$80.63
Nguyen's cost \$110.86

Chapter 7

Warm-up quiz

1 **a** 12 **b** 27 **c** 3 **d** 10
2 **a** 8 **b** 2 **c** 5 **d** 15
3 **a** 8 **b** 42 **c** 4 **d** 2
4 **a** $11m$ **b** a **c** $9n$
d $10a - 10$ **e** $11x + 2$ **f** $8b + 4$
5 **a** $3m + 12$ **b** $2a + 12$
c $3x + 21$ **d** $4k - 24$
6 12
7 **a** T **b** F **c** F **d** T
8 **a** $x = 4$ **b** $x = 8$ **c** $m = 4$ **d** $m = 6$
9 **a** ÷ 5 (C) **b** −2 (B) **c** × 3 (D) **d** +3 (A)
10 **a** $p + 10$ (C) **b** $4x$ (D) **c** $2z$ (B) **d** $q - 6$ (A)

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11 a

x	-2	-1	0	1	2	3
$3x-1$	-7	-4	-1	2	5	8

b

x	-2	-1	0	1	2	3
$2(x+3)$	2	4	6	8	10	12

12 a T **b** F **c** T **d** F

7A

Now you try

Example 1

False

Example 2

a 19 **b** 9

Example 3

a $2m - 40 = 20$ **b** $7a + 6 = 9.5$

Exercise 7A

1 a T **b** F **c** T **d** F
e T **f** T
2 a 8 **b** 12 **c** 15 **d** 45
3 a 13 **b** 9 **c** 2 **d** 2
4 a 7 **b** 9 **c** 15 **d** 8
5 a T **b** F **c** T **d** T **e** T **f** F
6 a T **b** F **c** T **d** T **e** F **f** T
7 a $x = 8$ **b** $x = 3$ **c** $x = 7$ **d** $x = 4$
e $x = 1$
8 a $x = 7$ **b** $x = 13$ **c** $u = 7$ **d** $p = 19$
e $x = 2$ **f** $x = 11$
9 C
10 a $k + 4 = 20$ **b** $2x + 7 = 10$
c $x + \dfrac{x}{2} = 12$ **d** $h + 30 = 147$
e $4c + 6 = 22$ **f** $8c + 2000 = 3600$
11 a 7 **b** 42 **c** 13 **d** 26
12 a $3.2x = 9.6$ **b** $x = 3$
13 a $a = 10,\ b = 6,\ c = 12,\ d = 20,\ e = 2$
b $a = 20,\ b = 6,\ c = 24,\ d = 80,\ e = 4$

7B

Now you try

Example 4

$x = 19$

Example 5

$a = 14$

Exercise 7B

1 a Subtracting **b** Dividing **c** Adding
d Multiplying
2 a $\boxed{3} \xrightarrow{\times 5} \boxed{15} \xrightarrow{-2} \boxed{13}$ **b** $\boxed{7} \xrightarrow{-2} \boxed{5} \xrightarrow{\times 3} \boxed{15}$
c $\boxed{x} \xrightarrow{+2} \boxed{x+2}$ **d** $\boxed{a} \xrightarrow{\times 3} \boxed{3a}$
e $\boxed{f} \xrightarrow{\times 2} \boxed{2f} \xrightarrow{+3} \boxed{2f+3}$ **f** $\boxed{p} \xrightarrow{\times 5} \boxed{5p} \xrightarrow{-4} \boxed{5p-4}$
3 a $\times 4$ **b** -3 **c** $+12$
4 a $\boxed{p} \xrightarrow{+4} \boxed{p+4}$, $\boxed{13} \xleftarrow{-4} \boxed{17}$ **b** $p = 13$
5 a $\boxed{k} \xrightarrow{\times 3} \boxed{3k}$, $\boxed{6} \xleftarrow{?} \boxed{18}$ **b** $k = 6$
6 a $k = 10$ **b** $p = 26$ **c** $r = 42$ **d** $x = 8$
e $w = 11$ **f** $s = 30$
7 a $\boxed{\dfrac{x}{5}} \underset{\div 3}{\overset{\times 3}{\rightleftarrows}} \boxed{\dfrac{3x}{15}} \underset{-7}{\overset{+7}{\rightleftarrows}} \boxed{\dfrac{3x+7}{22}}$ **b** $x = 5$
8 a $p = 15$ **b** $x = 4$ **c** $q = 6$ **d** $r = 7$
e $u = 2$ **f** $p = 21$
9 a $\boxed{\dfrac{x}{12}} \underset{-3}{\overset{+3}{\rightleftarrows}} \boxed{\dfrac{x+3}{15}} \underset{\div 2}{\overset{\times 2}{\rightleftarrows}} \boxed{\dfrac{2(x+3)}{30}}$ **b** $x = 12$
10 a $x = 3$ **b** $q = 2$ **c** $r = 5$ **d** $t = 7$
e $s = 26$ **f** $s = 8$
11 a $x = -5$ **b** $p = -6$ **c** $x = -5$ **d** $r = -2$
e $n = -30$ **f** $u = -3$
12 a $\boxed{x} \underset{\div 2}{\overset{\times 2}{\rightleftarrows}} \boxed{2x} \underset{-7}{\overset{+7}{\rightleftarrows}} \boxed{\dfrac{2x+7}{59}}$ **b** $x = 26$
13 a $\times 5, +8$ **b** $+8$
14 a $x = 3,\ x = 1$ **b** Order of $\times 2 + 4$ is reversed
15 a $x = 35$ **b** $r = 18$ **c** $x = 70$ **d** $y = 27$
e $x = 10$ **f** $m = 6$ **g** $x = 46$ **h** $r = 11$

7C

Now you try

Example 6

a $x = 19$ **b** $2x = 38$ **c** $7y = 49$

Example 7

a $y = 27$ **b** $m = 10$ **c** $k = 2$

Exercise 7C

1 a $2 + 3 = 1 + 4$ **b** $x + 3 = 7$ **c** $x + 5 = 2x + 2$
2 a $x = 6$ **b** $x = 8$ **c** $3q = 12$
3 a 8 **b** $x = 8$
4 B
5 a $2x = 20$
b $2 + q = 10$
c $18 = 17 - q$
d $12x = 24$
e $7p + 6 = 2p + 10$
f $3q = 2q$
6 a $x = 3$
b $q = 7$
c $k = 11$
d $4x = 20,\ x = 5$
e $7p = 28,\ p = 4$ (missing operation $\div 7$)
f $10x = 30,\ x = 3$ (missing operation $\div 10$)
7 a $a = 3$ **b** $t = 7$ **c** $q = 9$ **d** $k = 9$
e $x = 10$ **f** $h = 10$ **g** $l = 4$ **h** $g = 9$
8 a $h = 3$ **b** $u = 4$ **c** $s = 3$ **d** $w = 8$
e $x = 4$ **f** $w = 5$ **g** $a = 2$ **h** $y = 12$
9 a $x = 2$ **b** $k = 5$ **c** $x = 42$ **d** $x = 20$
e $k = 7$ **f** $x = 30$ **g** $y = 6$ **h** $x = 20$
10 a $x = -6$ **b** $a = -3$ **c** $x = -10$ **d** $k = -5$
e $k = -4$ **f** $p = -1$ **g** $p = -16$ **h** $x = -5$

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11 a $p+8=15, p=7$ **b** $3q=12, q=4$
c $2k-4=18,\ k=11$ **d** $3r+4=34,\ r=10$
12 a $x=7,\ y=2$ **b** $x=2,\ y=40$
c $x=4$ **d** $x=3.5$
13 a $x=2$ **b** $x=2$ **c** $x=5$
14 a $x=5$ **b** Opposite operations from bottom to top.
c For example, $7-3x=-8$

7D

Now you try

Example 8

a $x=21$ **b** $x=7$

Example 9

$m=8$

Example 10

$y=12$

Exercise 7D

1 B
2 a T **b** F **c** F **d** T
3 a 8 **b** 5 **c** No
4 a 30 **b** 10 **c** $\times 2, 22$ **d** $\times 10, 70$
5 a $b=20$ **b** $g=20$ **c** $a=15$ **d** $k=18$
6 a $l=20$ **b** $w=-10$ **c** $s=-6$ **d** $v=12$
e $m=14$ **f** $n=14$ **g** $j=-5$ **h** $f=20$
7 a $t=28$ **b** $h=2$ **c** $a=13$ **d** $c=17$
e $s=10$ **f** $j=2$
8 a $v=20$ **b** $x=20$ **c** $y=14$ **d** $x=10$
e $p=7$ **f** $k=6$
9 a C **b** A **c** B **d** D
10 a $g=8$ **b** $x=14$ **c** $k=15$ **d** $x=36$
e $q=10$ **f** $x=27$ **g** $p=6$ **h** $x=1$
i $r=20$
11 a $x=35$ **b** $y=24$ **c** $p=14$ **d** $x=16$
e $x=12$ **f** $k=11$
12 a 7 **b** 19 **c** 3 **d** 12 **e** 26
13 a $\dfrac{b}{3}=40$ **b** \$120
14 a $x=6$ **b** $x=3$ **c** $x=5$ **d** $x=2$
e $x=8$ **f** $x=12$

7E

Now you try

Example 11

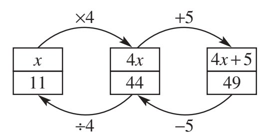
a $x=5$ **b** $x=2$

Exercise 7E

1 a 12 **b** 14 **c** 8, 10 **d** 50, 30
2 a C **b** A **c** D **d** B
3 a T **b** F **c** T **d** T
4 C
5 a $x=5$ **b** $k=1$ **c** $r=17$ **d** $u=6$
e $j=3$ **f** $p=6$ **g** $m=4$ **h** $n=5$
i $a=3$
6 a $x=8$ **b** $x=10$ **c** $r=10$ **d** $y=3$
e $l=2$ **f** $w=2$ **g** $c=2$ **h** $d=2$
i $w=6$ **j** $p=4$ **k** $k=2$ **l** $c=10$
7 a $2(n+5)$ **b** B **c** 15
8 a $d+4$ **b** $2(d+4)$ **c** $2(d+4)=50$ **d** 21
9 a $3 \times$ cost of shirt $+ 2 \times$ cost of trouser **b** $s=37$
c \$37 **d** \$57 **e** \$356
10 a $5w+3(w+4)$ **b** \$11.50
11 a $x=-6$ **b** $p=-4$ **c** $q=-19$ **d** $r=4$
e $r=2$ **f** $x=-11$ **g** $k=-10$ **h** $s=0$

Progress Quiz

1 F
2 a 5 **b** 8 **c** 34
3 a $3t+4=19$ **b** $5n+16=61$
4 a $f=5$ **b** $k=3$ **c** $p=20$ **d** $d=7$
5 a $4x+5=49$ **b** $x=11$



6 a $m=15$ **b** $h=19$ **c** $b=15$ **d** $g=11$
7 a $p=-5$ **b** $j=-60$ **c** $b=-2$ **d** $y=-5$
8 a 100 **b** 88 **c** 20 **d** 10
9 a 4 **b** 6 **c** 65 **d** 30
10 a $18y-12$ **b** $11y-77$ **c** $40+15y$
11 a $x=5$ **b** $n=3$ **c** $d=7$

7F

Now you try

Example 12

a $P=10$ **b** $a=7$

Exercise 7F

1 a Formula **b** Subject **c** Area
2 C
3 B
4 A
5 a $A=19$ **b** $A=51$ **c** $A=7$ **d** $A=407$
6 a i 40 **ii** 12 **iii** 30
b $x=11$
c 11 units
7 a $a=2$ **b** $a=5$ **c** $a=11$
8 a $y=10$ **b** $x=6$ **c** $x=-2$
9 $m=5.5$
10 a $A=60$ **b** $h=4$ **c** 11
11 a \$23
b i $161=3+2d$ **ii** $d=79$ **iii** 79 km
12 a 92 **b** Yes, if $p=30$
13 a A: \$1.10, B: \$1.20 **b** 4 minutes
c 5 minutes **d** 10 minutes

7G

Now you try

Example 13

3 hours

Example 14

Width = 90 m, area = 9900 m^2

Exercise 7G

1 a D **b** A **c** E **d** C **e** B
2 a B **b** C **c** A **d** D
3 a D **b** C **c** A
4 a Let \$$c$ = cost of one cup. **b** $4c=14$
c $c=3.5$ **d** \$3.50
5 a Let t = time spent (hours) **b** $70+80t=310$
c $t=3$ **d** 3 hours
6 a Let \$$c$ = cost of one chair. **b** $6c-200=1300$
c $c=250$ **d** \$250

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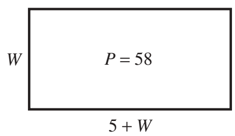
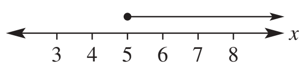
7 a $a = 13$ **b** 13 years old
8 a $2(4 + w) = 72$ or $8 + 2w = 72$ **b** $w = 32$
c 32 cm
9 a $4w = 24$, $w = 6$, so width = 6 cm. **b** 36 cm^2
10 a $a + a + 4 = 40$, $a = 18$, so Alison is 18 years old.
b 22 years old
11 a


b 12 m **c** 204 m^2
12 a $x = 80$ **b** $x = 75$ **c** $x = 30$ **d** $x = 110$
e $x = 45$ **f** $x = 65$

7H

Now you try

Example 15

a

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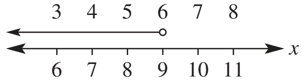
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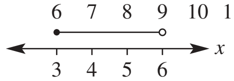
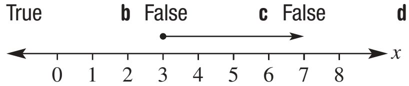


Example 16

a $x > 170$ **b** $x \geqslant 100000$
c $3 < x < 4$

Exercise 7H

1 a True **b** False **c** False **d** True
2 a D **b** A **c** B **d** C
3 a True **b** False **c** False **d** True
4 a True **b** False **c** False **d** True
5 a

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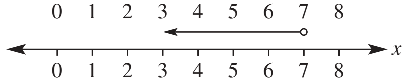
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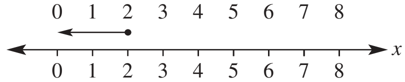
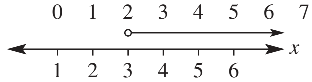

6 a

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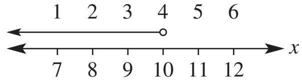
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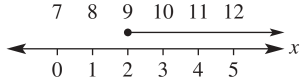
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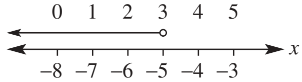
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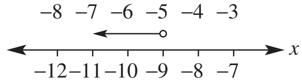
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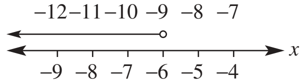
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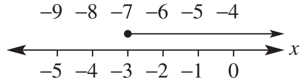
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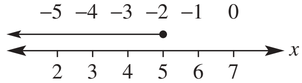
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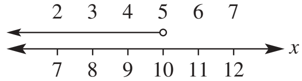
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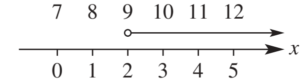
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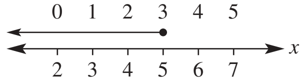
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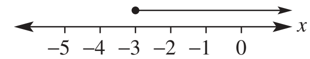
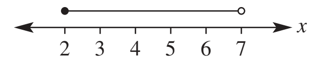

7 a 3, 4, 6, 4.5, 5, 2.1, 6.8, 2
b

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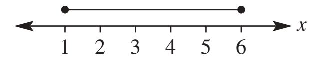
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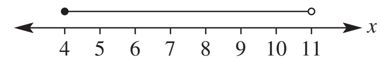
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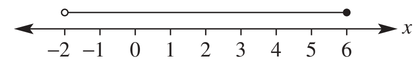
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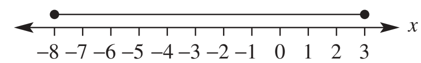
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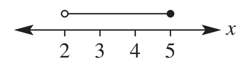
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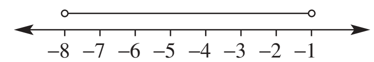
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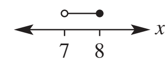
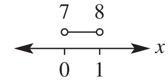
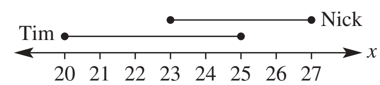
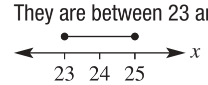

h

9 a C **b** A **c** G **d** F
10 a $20 \leqslant t \leqslant 25$ and $23 \leqslant n \leqslant 27$
b


c They are between 23 and 25 years old.

11 a $8 \geqslant 2$ **b** $4 < 6$ **c** $x > 3$ **d** $y \leqslant 8$
12 a i E **ii** B **iii** A **iv** B
v D
b $73 \leqslant x < 80$
c D
d B or A
e C only

7I

Now you try

Example 17

a $x < 5$ **b** $x \leqslant 1$

Example 18

$x > 4$

Exercise 7I

1 a True **b** True **c** False **d** False
2 a True **b** False **c** False **d** True
3 a 4 **b** $x < 4$
4 a 6, 3 **b** $x \geqslant 3$ **c** True
5 a $x < 4$ **b** $x > 11$ **c** $x \geqslant 2$ **d** $x \leqslant 17$
6 a $x > 3$ **b** $l \geqslant 3$ **c** $g > 5$ **d** $r \leqslant 7$
e $k > 2$ **f** $s < 5$ **g** $a > 4$ **h** $n \geqslant 5$
i $d \geqslant 7$ **j** $h < 5$ **k** $r \leqslant 2$ **l** $y < 4$
7 a $d > 29$ **b** $y \leqslant 10$ **c** $x > 11$ **d** $q \leqslant 18$
e $x > 7$ **f** $h < 1$ **g** $p \geqslant 2$ **h** $j < 5$
8 a $x > 1$ **b** $s \leqslant 2$ **c** $n < 5$ **d** $j > 10$
e $v \geqslant -2$ **f** $j \geqslant -5$ **g** $c \leqslant 2$
h $h \geqslant 0$ **i** $s < -3$ **j** $v < -2$ **k** $v \leqslant -2$
l $v > -5$
9 a C **b** A **c** B **d** D
10 a $4c + 20 > 25$ **b** $c > 1.25$ **c** \$1.30

Answers

7H

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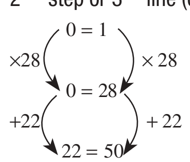
11 a $6g + 4 \leqslant 36$ **b** $g \leqslant \frac{16}{3}$ **c** 5 goals
12 a Clues B and D **b** $x = 8$
13 a $x > 2$ **b** $a \leqslant -2$ **c** $b < -3$ **d** $c < -4$
14 a $C \geqslant 15$ **b** $C \leqslant 20$
c $15 \leqslant C \leqslant 20$ **d** $47\dot{7} \leqslant C \leqslant 57\dot{7}$

Maths@Work: Financial officers at a local council

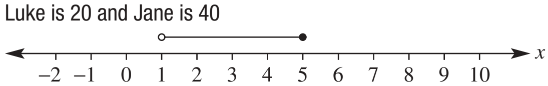
1 a \$1610 **b** \$1955 **c** \$3606.40
2 a i \$2869.50 **ii** \$3213.84
iii \$3873.83 **iv** \$4304.25
v \$5739.00
b $Q = \frac{R}{4}$
3 a $R = 0.00482075 \times LV + 456.3$ **b** \$2119.46
c \$261 058.96
4 a i \$1915.50 **ii** \$3718.80
b $R = 0.0054x + 765$
c \$3465.00
d \$1197.50
e $R = \frac{(0.00575y + 765)}{4}$
5 b 0.575, 0.481, 0.491, 0.475, 0.352

Puzzles and games

1 □ = 12, △ = 2, ◯ = 9
2 a 88 **b** 6 **c** 13 **d** \$44.44 **e** 20
3 a 2[nd] step or 3[rd] line (can't divide by 0)
b


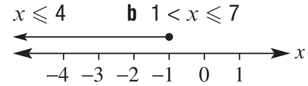
4 a $4x + 2$ and $4\left(x + \frac{1}{2}\right)$, $2(x + 4)$ and $4(x + 2)$
b $4x + 2 = 4\left(x + \frac{1}{2}\right)$ **c** $2(x + 4) = 2x + 4$
5 a $x = 88.5$ **b** $a = 37.5$ **c** $x = 75$
6 a 65 kg, 62 kg, 55 kg **b** 70 kg, 60 kg, 48 kg
c 35 kg, 42 kg, 45 kg, 48 kg

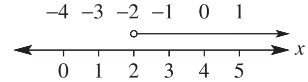
Checklist Answers

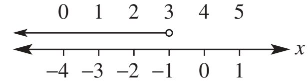
1 True $(10 + 20 = 3 \times 10)$
2 $x = 21$
3 $2k + 3 = 52$
4 $k = 3$
5 $p = 10$
6 $5a = 15$
7 $u = 5$
8 $k = 40$
9 a $y = 3$ **b** $x = 10$
10 $10x + 6$
11 $x = 7$
12 $P = 34$
13 $l = 17$
14 $6b = 1.2$, $b = 0.2$ one book is 0.2 kg or 200 g
15 Luke is 20 and Jane is 40
16


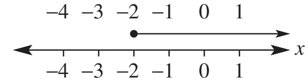
17 $x < 160$
18 $x < 7$

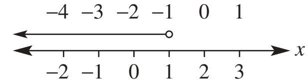
Short-answer questions

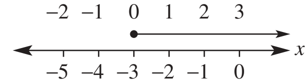
1 a F **b** T **c** T
2 a $m = 4$ **b** $m = 6$ **c** $q = 5$ **d** $z = 50$
3 a $2m + 3 = 27$ **b** $3(n + 4) = 18$
c $x + x + 1 = 7$ (or $2x + 1 = 7$)
4 a $x = 4$ **b** $u = 4$ **c** $d = 6$ **d** $b = 7$
e $f = 3$ **f** $k = 2$
5 a $3x = 12$ **b** $2b = 14$ **c** $x = 5$
6 a Subtract 15 **b** Add 5 **c** Multiply by 2
7 a $a = 5$ **b** $b = 6$ **c** $n = 16$ **d** $c = 2$
e $x = 9$ **f** $x = 2$
8 a $m = 6$ **b** $x = 8$ **c** $k = 30$ **d** $y = 18$
e $k = 52$ **f** $x = 32$
9 a $2x + 10$ **b** $3q - 30$ **c** $8r + 12$
d $7x + 15$ **e** $4z + 18$ **f** $5q - 15$
10 a $x = 3$ **b** $x = 2$ **c** $p = 6$ **d** $x = 5$
e $x = 10$ **f** $k = 12$
11 a $F = 30$ **b** $m = 4$ **c** $m = 1$
12 a $I = 21$ **b** $M = 3$ **c** $c = 4$
13 a D **b** $m = 1.5$ **c** \$1.50
14 a $x = 3, y = 2$ **b** $x + x + 1 + x + 2 = 39, x = 12$
c 8.5
15 a $x \leqslant 4$ **b** $1 < x \leqslant 7$
16 a


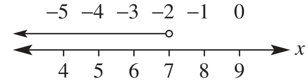
b


c


d


e


f


g


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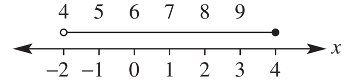
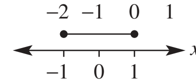


i


17 a $x \geqslant 100\,000$ **b** $x \leqslant 6700$
c $1.54 \leqslant x \leqslant 1.9$
18 a $x > 2$ **b** $x < 8$ **c** $x < -4$ **d** $x \geqslant 2$
e $x < -2$ **f** $x \geqslant 8$

Multiple-choice questions

1 B **2** C **3** D **4** B **5** C
6 A **7** D **8** E **9** C **10** B

Extended-response questions

1 a $10 + 5n$
b i $10 + 5n = 55$ **ii** $n = 9$ **iii** 9 rides
c \$100
d 7 rides
2 a $S = 20 + 0.12n$ **b** 30 times
c $Y = 15 + 0.2n$ **d** 25
e 63 is the minimum number.

Chapter 8

Warm-up quiz

1 a 0, 1, 2, 4, 6, 7, 9, 10, 14 **b** 20, 30.6, 36, 100, 101, 204
c 1.2, 1.7, 1.9, 2.7, 3.2, 3.5
2 a Total = 40, average = 8
b Total = 94, average = 18.8
c Total = 3.3, average = 0.66
3 a $\frac{1}{6}$ **b** 150°
c i \$420 **ii** \$630

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4 **a** 4 **b** 3 **c** Saturday **d** 26 **e** $\frac{1}{3}$

5 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** 0

8A

Now you try

Example 1

a 40 km **b** 30 km **c** Molly

Example 2

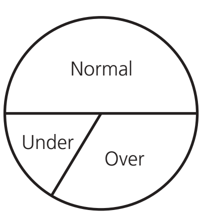
a 25% **b** 5 hours

Exercise 8A

1 Column, line, pie chart, divided bar chart. Answers may vary.
2 **a** 320 **b** 270 **c** 300
d Expton **e** Calcville
3 **a** 10 **b** 6 **c** Phillip
d Nyree **e** 4 years
4 **a** 9 **b** 12 **c** Badminton
d Badminton 35, Water polo 60, Hand ball 54
e Water polo
5 **a** Slesha **b** Ross **c** 4th and 5th
6 **a** Rent **b** Charity **c** 50%
d 25% **e** $2400
7 **a** 2 hours **b** 7 hours **c** Sleeping **d** $\frac{1}{6}$
8 **a** 20 **b** Periods 3 and 6 **c** Period 7
d Periods 1 and 5
9 **a** 20°C **b** 10°C **c** Midday or noon
10 **a** The distance doesn't change for a period of time.
b 10 am to 10 : 30 am **c** 280 km
d 1 hour **e** Red 160 km, Blue 200 km
f Red $4\frac{1}{2}$ hours, Blue $4\frac{1}{2}$ hours
11 **a** Survey 2 **b** Survey 1 **c** Survey 3
12 **a** 1 Town C, 2 Town A, 3 Town B **b** 1235 people/town
c 1080 people
13 **a**

	Under weight	Normal weight	Over weight	Total
Number of months	2	6	4	12
Fraction of 12	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	1
Angle	60°	180°	120°	360°

b



c Can see how weight changes over time.
d Can see how much of the year the dog was underweight, overweight and normal weight.
e Answers will vary.

8B

Now you try

Example 3

a

Height (cm)	150–159	160–169	170–179	180–189
Frequency	2	6	12	4

b 12 **c** 24

Example 4

Number	0–9	10–19	20–29	30–39	40–49
Tally	|||	||||	|||	||	|
Frequency	3	4	3	2	1

Exercise 8B

1 **a** T **b** F **c** T **d** F
2 **a** 4 **b** 7 **c** || **d** ~~||||~~ ~~||||~~ |
3 **a**

Handspan	Frequency
17	1
18	2
19	3
20	3
21	0
22	2
23	0
24	1

Handspan	Frequency
17–19	6
20–22	5
23–25	1

4 **a**

	Passes	Shots at goal	Shots that go in	Steals
Frequency	3	12	8	2

b 12 **c** 8 **d** 2

5 **a**

People in family	2	3	4	5	6	7	8
Tally	|	||	||||	||||	||||	||	|||
Frequency	1	2	4	4	4	2	3

b 4
c 9

6 **a**

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–166
Frequency	5	3	12	15	9	4	2

b 50 **c** 9 **d** 8 **e** 35

7 **a**

Height (cm)	Tally	Frequency
130–139	|||	3
140–149	~~||||~~	5
150–159	||	2
160–169	|||	3
170–179	|||	3
180–189	|	1
190+	||||	4

b 2 **c** 5 **d** 10
8 **a** 10 **b** 2 **c** 4 **d** 17
9 **a** B **b** D **c** A **d** C
10 **a** 28 **b** 130 **c** 19 **d** 13.1 years old
e

Age	12	13	14	15	16	17
Frequency	5	39	19	33	33	1

11 **a**

Score	0–19	20–39	40–59	60–79	80–100
Frequency	0	4	7	20	12

b

Score	0–29	30–59	60–89	90–100
Frequency	3	8	30	2

12 **a** 2
b All arrangement of 3, 3, 2, 1 will be correct.
c All arrangements of 3, 2, 2, 1, 1 will be correct.
d Priscilla = 2.25 hrs/night, Joey 1.8 hrs/night
e 2.25 hours more homework.

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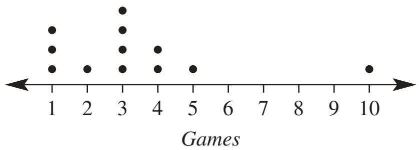
8C

Now you try

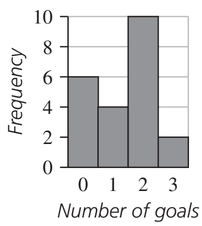
Example 5

a

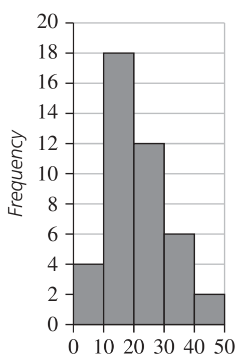


b 3
c 10

Example 6

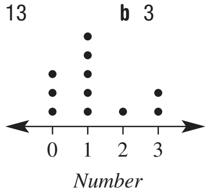


Example 7

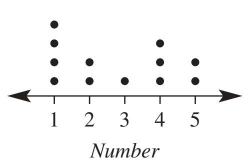


Exercise 8C

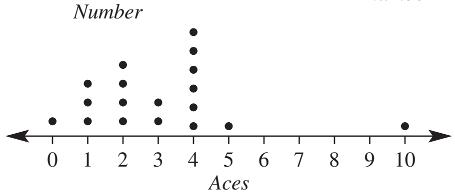
1 **a** 2 **b** 9 **c** 11 years old
2 **a** 4 **b** 4 **c** 8
3 **a** 13 **b** 3
4 **a**



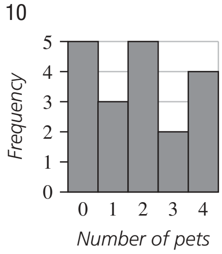
b



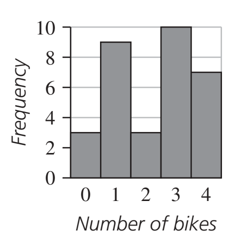
5 **a**



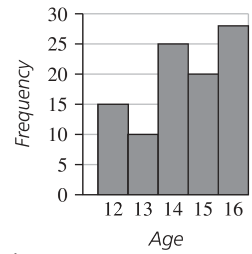
b 4
c 10
6 **a**



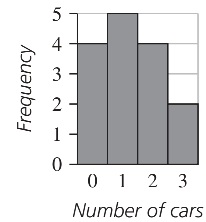
b



c



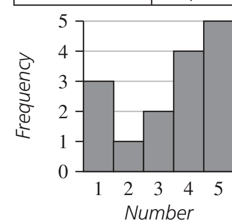
d



7 **a** **i**

Number	Tally	Frequency
1	|||	3
2	|	1
3	||	2
4	||||	4
5	~~||||~~	5

ii

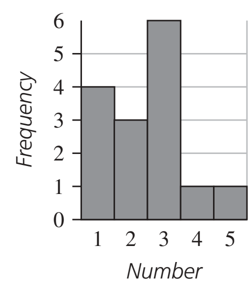


b

i

Number	Tally	Frequency
1	||||	4
2	|||	3
3	~~||||~~ |	6
4	|	1
5	|	1

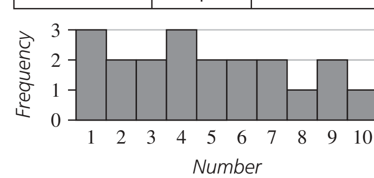
ii



c **i**

Number	Tally	Frequency
1	|||	3
2	||	2
3	||	2
4	|||	3
5	||	2
6	||	2
7	||	2
8	|	1
9	||	2
10	|	1

ii



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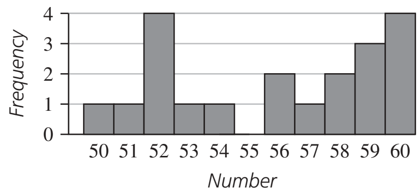
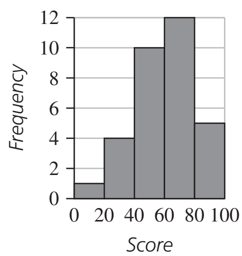
d i

Number	Tally	Frequency
50	|	1
51	|	1
52	||||	4
53	|	1
54	|	1
55		0
56	||	2
57	|	1
58	||	2
59	|||	3
60	||||	4

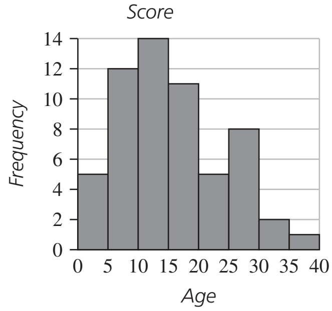
ii



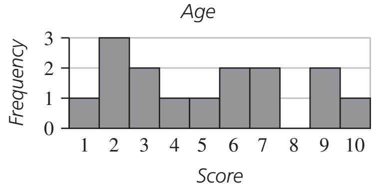
8 a



b



9 a



b Edwin is worse than Fred as most of Fred's scores are 8 or higher.

10 a

Value	Frequency
2	1
3	1
4	3
5	2
6	4
7	1
8	1
9	2

b The value 2 (one row less in the frequency table, and the dot plot can then start at 3).

11 a 7 days
b 4 days
c 4 cars in one day sold by Marie
d Con
e Frank who sold 27 cars
f Bill who sold 15 cars

12 a It would look identical but the age labels would start at 22 and go to 26.
b It would look just like the right half (12, 13, 14) but with the age axis labelled 0, 1, 2.

13 a 9 weeks of 10, 8 weeks of 9, 5 weeks of 8, 4 weeks of 7, 3 weeks of 6, 1 week of 5 (any list with the higher scores coming first and the lowest scores last is correct).
b 9 weeks of 5, then 8 weeks of 6, then 7 weeks of 7, then 4 weeks of 8, then 2 weeks of 9 out of 10
c They were absent from the tests perhaps.

14 a

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4	Graph 7	Graph 6
Shopping centre	Graph 8	Graph 2	Graph 9
Teachers common room	Graph 5	Graph 3	Graph 1

b i–iii Answers will vary.

8D

Now you try

Example 8

a 27 **b** 14

Example 9

a 2 **b** 4

Example 10

a 5 **b** 14.5

Exercise 8D

1 a Mode **b** Mean **c** Median
d Range **e** Outlier
2 a 15 **b** 5 **c** 3
3 a 1, 2, 4, 5, 6, 7, 9 **b** 5 **c** 5
4 a 7 and 9 **b** 16 **c** 8
5 a 8 **b** 1 **c** 7
6 a 9 **b** 10 **c** 15 **d** 14
e 30 **f** 27 **g** 16.9 **h** 8.7
7 a i 5 **ii** 4
b i 2 **ii** 2
c i 5 **ii** 3
d i −3 **ii** 0
e i 0 **ii** −9
f i 0 **ii** 3
g i 12.9 **ii** 15
h i 13.1 **ii** 20
i i 11.1 **ii** 12
j i 10.4 **ii** 5
k i 2.4 **ii** −6
l i −3.4 **ii** −6
8 a 6 **b** 4 **c** 8 **d** 5 **e** 8
f 7 **g** 5 **h** 5.5 **i** 7.5 **j** 8
k 10.5 **l** 12
9 a 8.4 **b** 8 **c** 8
10 a White **b** Meat-lovers **c** Wednesday
d South Australia
11 a 3 **b** 10 **c** 7, 7, 7, 9, 9, 9, 10, 10, 10, 10
d 8.8 **e** 9 **f** 3
12 a Business B, \$200 000
b Mean A = \$52 000, mean B = \$78 000
c \$26 000 larger
d \$50 000 for both A and B
e No
f The median, \$50 000 as it is not affected by the outlier.

Answers

8D

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13 a 15 **b** 35 **c** Sarah **d** Gary
14 a 12, 12, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 15, 15, 15, 15, 16, 16, 16, 16, 16, 16, 16, 16, 17, 17, 17, 17, 17, 18
b 6 years **c** 16 years old **d** 15.03 years old
e 15
f **i** 16.23 years
ii 15 years
iii The mean has increased the most. The median is unchanged.
15 a \$1 477 778 **b** \$630 000
c A strong effect – it makes the mean significantly higher.
d Only increases the median by a small amount.
e Median is not easily distorted by a few very large values.

8E

Now you try

Example 11

a $\frac{1}{5}$ or 0.2 **b** 200 **c** 4160

Example 12

a Symmetrical **b** 52% or $\frac{13}{25}$ **c** 128
d B, you would expect most medical graduates to have spent at least 5 or 6 years at university.

Exercise 8E

1 a Survey **b** Sample **c** Biased **d** Symmetrical
e Skewed
2 a Surveying 1000 randomly selected people
b Surveying 10 friends
3 a Symmetrical **b** Skewed
c Skewed **d** Symmetrical
4 a $\frac{2}{5}$ **b** 2000 **c** 300
5 a Skewed **b** $\frac{2}{5}$ **c** 400 **d** $\frac{6}{25}$
e 1200
f More likely that people will have pets if near a vet clinic.
6 a Symmetrical **b** $\frac{28}{100}=\frac{7}{25}$ **c** 420
d $\frac{35}{100}=\frac{7}{20}$ **e** 1050
f In a wealthy suburb the houses are more likely to be larger.
7 a Skewed **b** $\frac{52}{120}=\frac{13}{30}$
c 15 600 **d** $\frac{28}{120}=\frac{7}{30}$ **e** 1120
f The people on this train probably start work early and are less likely to eat breakfast.
8 a 108 g **b** Symmetrical **c** $\frac{5}{8}$
d 500 **e** $\frac{17}{128}$ **f** 272
9 a Yes, it is required information.
b No, it is too vague or personal.
c No, it is too vague or personal.
d No, it addresses wealth but not income.
e No, it is irrelevant.
f Yes, it can be used to decide income.
g No, if it is not a pay day then results will be distorted.
10 a During school hours.
b Outside a political party office.
c In a butcher's shop.
d At 11 pm, when people will buy just a few items.
11 a At a professional dance studio in the afternoon.
b In a Bank.
c Choose a large random sample.
12 a **i** Convenience
ii Stratified
iii Simple random
13 a The sample could be just 10 people who all live in 4–bedroom houses.
b 1, 1, 1, 1, 1, 2, 2, 2, 2, 2 has a mean of 1.5.
c 5.2 is the largest, if the sample was 6, 6, 5, 5, 5, 5, 5, 5, 5, 5
d Increasing the sample size

8F

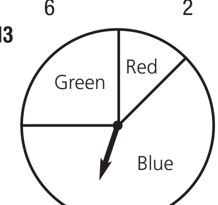
Now you try

Example 13

a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **b** $\frac{1}{10}$
c 3, 6, 9 **d** $\frac{3}{10}$ **e** $\frac{7}{10}$

Exercise 8F

1 a Trial **b** Outcomes **c** Sample space
d Complement **e** A'
2 a B **b** C **c** D **d** A
3 a Event C **b** Event A **c** Event B **d** Event C
4 a T **b** F **c** T **d** T
5 a P, I, A, N, O **b** $\frac{1}{5}$ **c** $\frac{3}{5}$ **d** $\frac{2}{5}$ **e** $\frac{4}{5}$
f $V'=P,N$ **g** $\Pr(V')=\frac{2}{5}$
6 a 1, 2, 3, 4, 5, 6 **b** $\frac{1}{6}$ **c** $\frac{1}{2}$
d 1, 2, 3, 4, 6 **e** $\frac{5}{6}$ **f** 0
7 a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{3}{10}$ **d** $\frac{4}{5}$ **e** $\frac{1}{2}$
f $\frac{7}{10}$ **g** Choosing a purple marble
8 a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **b** $\frac{1}{10}$ **c** $\frac{1}{5}$ **d** $\frac{3}{10}$
e $\frac{2}{5}$ **f** $\frac{3}{5}$
9 a Red, green, blue, yellow, purple
b $\frac{1}{8}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$
e Green, red, yellow, purple **f** $\frac{5}{8}$ **g** $\frac{3}{4}$
h Spinning purple (or spinning yellow)
i Spinning orange
10 a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$ **d** $\frac{5}{6}$
11 a $\frac{1}{2}$ **b** $\frac{3}{5}$ **c** 0 **d** $\frac{1}{2}$
e $\frac{1}{2}$ **f** $\frac{4}{5}$
12 a 1 red, 2 orange, 3 purple
b $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$
13



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14 a $\frac{1}{5}$ b $\frac{1}{4}$ c 18

d It approaches $\frac{1}{2}$ or 0.5.

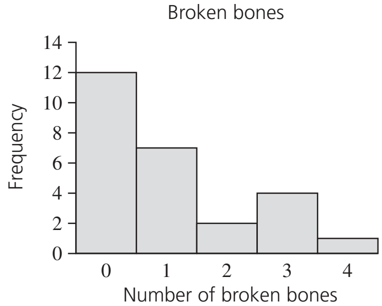
Progress Quiz

1 a 30 b 14 c 4

2

Number	1	2	3	4	5
Frequency	3	7	4	2	4

3



4 a 13 b 43

5 a i 3 ii 2
b i −3 ii −4

6 a 12 b 59 c 5 d −3

7 a 1 b 5
c 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 8
d 6 e 6 f 3

8 a $\frac{3}{5}$ b 600 c 12 000

9 a R, E, S, I, L, I, E, N, C, E b $\frac{3}{10}$ c $\frac{1}{2}$ d $\frac{4}{5}$

e $\frac{7}{10}$

10 a 9 b 40 c 13 d $\frac{1}{5}$

8G

Now you try

Example 14

a

	P	I	E
1	1P	1I	1E
2	2P	2I	2E
3	3P	3I	3E
4	4P	4I	4E

b 12

c $\Pr(3P) = \frac{1}{12}$

d $\Pr(\text{even, vowel}) = \frac{1}{3}$

Exercise 8G

1 a 10 b H2, H4, T2, T4 c $\frac{2}{5}$

d T1, T3, T5 e $\frac{3}{10}$

2 a $\frac{1}{4}$ b HH, TT c $\frac{1}{2}$

3 a

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

b 12
c $\frac{1}{12}$
d $\frac{1}{4}$

4 a

	R	I	D	E
L	LR	LI	LD	LE
I	IR	II	ID	IE
N	NR	NI	ND	NE
E	ER	EI	ED	EE

b 16
c $\frac{1}{16}$
d $\frac{1}{16}$
e $\frac{1}{4}$
f $\frac{1}{4}$
g $\frac{1}{8}$

5 a

	R	P	B
R	RR	RP	RB
P	PR	PP	PB
G	GR	GP	GB
B	BR	BP	BB

b $\frac{1}{12}$
c $\frac{1}{12}$
d $\frac{1}{12}$
e $\frac{1}{6}$
f $\frac{1}{4}$
g $\frac{3}{4}$

6 a $\frac{1}{12}$ b $\frac{1}{6}$ c $\frac{1}{12}$ d $\frac{1}{2}$

7 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{9}$
c $\frac{8}{9}$
d 7
e 2 and 12

8 a

	Y	W	B	B	B
W	WY	WW	WB	WB	WB
O	OY	OW	OB	OB	OB
O	OY	OW	OB	OB	OB

b $\frac{2}{15}$
c $\frac{1}{15}$
d $\frac{2}{15}$
e $\frac{1}{5}$

Answers

8G

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f $\frac{1}{15}$

9 a $\frac{1}{8}$ b $\frac{1}{3}$ c $\frac{2}{3}$

10 63

11 a $\frac{1}{52}$

b $\frac{1}{26}$

c $\frac{1}{16}$

d $\frac{1}{4}$

e $\frac{1}{2704}$

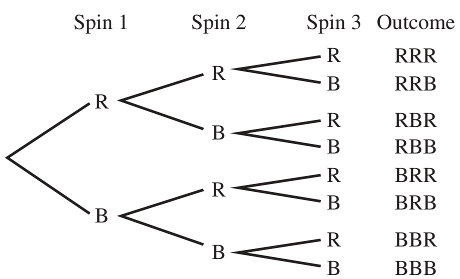
f The second card would then depend on the first card (e.g. if a red card was selected, it would be a little less likely that the next card would be red).

8H

Now you try

Example 15

a



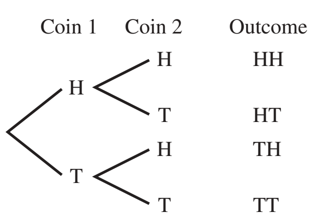
b 8

c $\Pr(\text{BRR}) = \frac{1}{8}$

d $\Pr(\text{RRR or BBB}) = \frac{1}{4}$

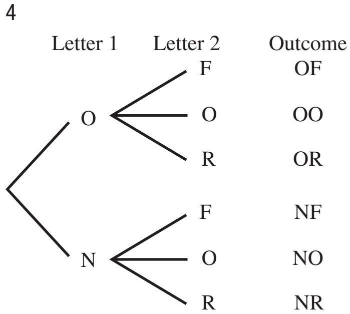
Exercise 8H

1 a



b 4

2 a

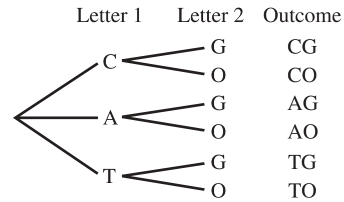


b OR, NF, NO, NR

c 6

d 2

3 a



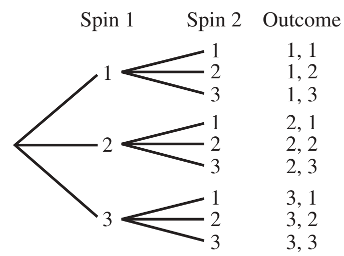
b 6

c $\frac{1}{6}$

d $\frac{1}{6}$

e $\frac{1}{3}$

4 a



b $\frac{1}{9}$

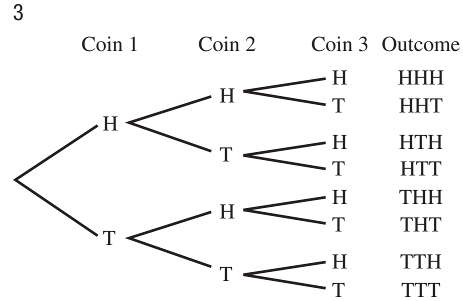
c $\frac{1}{9}$

d $\frac{2}{9}$

e $\frac{1}{3}$

f $\frac{1}{3}$

5 a



b $\frac{1}{8}$

c $\frac{7}{8}$

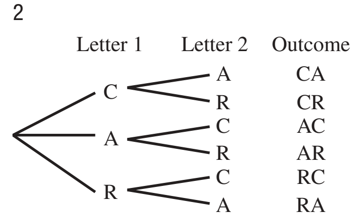
d $\frac{1}{8}$

e $\frac{3}{8}$

f Exactly 2 tails

g $\frac{1}{2}$

6 a



b $\frac{1}{3}$

c $\frac{1}{3}$

d 0

e 1

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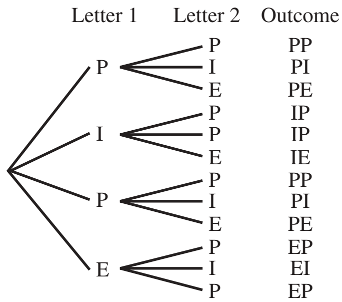
7 a



b $\frac{1}{6}$

c $\frac{1}{6}$

d $\frac{5}{6}$

8 32

9 a $\frac{1}{16}$

b $\frac{1}{16}$

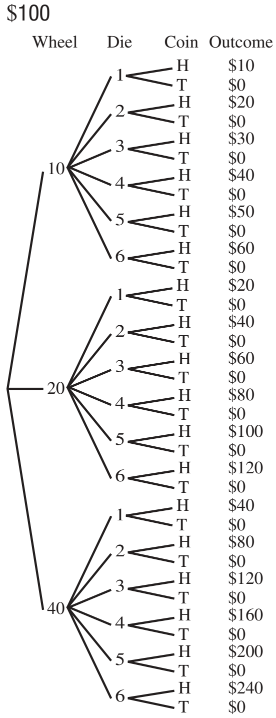
c They are equally likely.

d 3 heads

e 3 heads could be HTHHT or HTTHH or THTHH etc. but heads must be HHHHH $\left(\frac{1}{32}\right)$.

10 a \$100

b



c $\frac{1}{18}$

d $\frac{1}{6}$

e $\frac{19}{36}$

8I

Now you try

Example 16

a 80

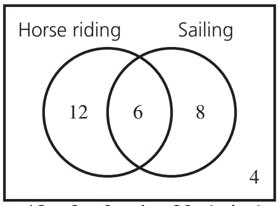
b $\frac{1}{8}$

c i $\frac{13}{80}$ **ii** $\frac{3}{80}$ **iii** $\frac{1}{4}$

Exercise 8I

1 a 8 **b** 13 **c** 3 **d** 18 **e** 4 **f** 22

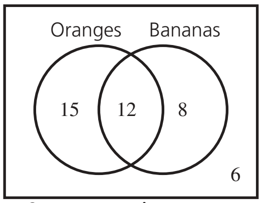
2 a i–ii



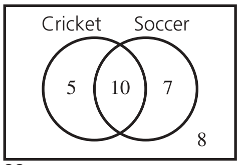
b Total $= 12 + 6 + 8 + 4 = 30$ students

3 a 2 **b** 4 **c** 1 **d** 3

4



5 a 8 **b**



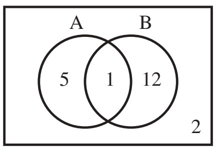
c 15 **d** 22

6 a $14 + 8 + 18 + 10 = 50$ **b** $\frac{14}{50} = \frac{7}{25}$ **c** $\frac{8}{50} = \frac{4}{25}$ **d** $\frac{26}{50} = \frac{13}{25}$ **e** $\frac{40}{50} = \frac{4}{5}$

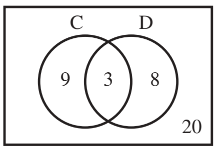
7 a 20 **b** 15 **c** $\frac{15}{20} = \frac{3}{4}$ **d** 10 **e** $\frac{10}{20} = \frac{1}{2}$

8 a 4 **b** $\frac{2}{15}$ **c** $\frac{13}{15}$ **d** $\frac{3}{5}$

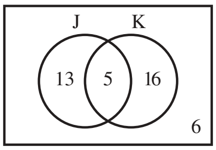
9 a



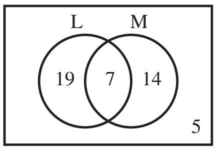
b



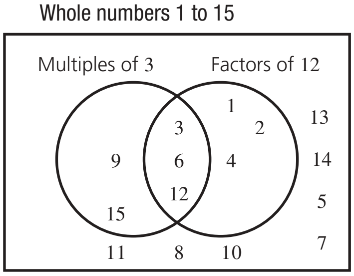
c



d



10 a Whole numbers 1 to 15



b 3 **c** 3 **d** $\frac{5}{15} = \frac{1}{3}$ **e** $\frac{6}{15} = \frac{2}{5}$

11 a 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

b 1, 2, 3, 5, 6, 10, 15, 30

Answers

8I

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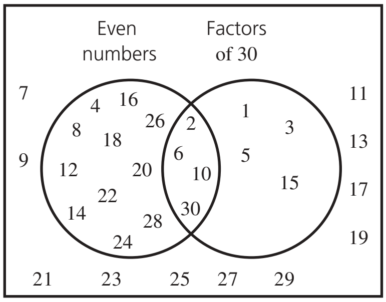
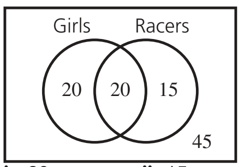
c



d 4 **e** $\frac{4}{30} = \frac{2}{15}$ **f** 15 **g** $\frac{4}{15}$

12 a



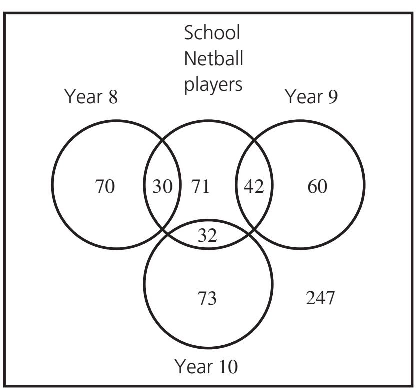
b i 20% **ii** 15% **iii** 60% **iv** 65%

c $\frac{3}{7}$ **d** $\frac{4}{7}$ **e** $\frac{1}{2}$

13 a 625 **b i** $\frac{6}{125}$ **ii** $\frac{42}{625}$

c 175 **d i** $\frac{6}{35}$ **ii** $\frac{6}{25}$ **e** $\frac{3}{10}$

f



g $\frac{32}{175}$

8J

Now you try

Example 17

	Like running	Dislike running	Total
Like swimming	9	14	23
Dislike swimming	6	11	17
Total	15	25	40

Example 18

a

	Like streamed	Dislike streamed	Total
Like Live	12	5	17
Dislike Live	7	6	13
Total	19	11	30

b 6 **c** 17 **d** 12 **e** 24

Example 19

a $\frac{3}{50}$ **b** $\frac{3}{10}$

Example 20

a

	Like climbing	Dislike climbing	Total
Like hiking	4	21	25
Dislike hiking	8	7	15
Total	12	28	40

b $\frac{3}{8}$ **c** 33 **d** $\frac{21}{40}$ **e** $\frac{7}{40}$

Exercise 8J

1 a

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	30
Total	40	35	75

b 30 **c** 20 **d** 75

2

	Like Lamingtons	Like Lamingtons	Total
Like Anzacs	23	14	37
Dislike Anzacs	12	3	15
Total	35	17	52

3 a 27 **b** 15 **c** 12 **d** 25 **e** 10
f 40

4 a $\frac{15}{60} = \frac{1}{4}$ **b** $\frac{24}{60} = \frac{2}{5}$ **c** $\frac{12}{60} = \frac{1}{5}$ **d** $\frac{9}{60} = \frac{3}{20}$

5

	Like surfing	Dislike surfing	Total
Like hiking	45	10	55
Dislike hiking	25	5	30
Total	70	15	85

6 a

	Employed	Unemployed	Total
TAFE degree	16	3	19
No TAFE degree	12	2	14
Total	28	5	33

b 2 **c** 28 **d** 16 **e** 31

7 a 26 **b** 12 **c** 11

d i $\frac{2}{13}$ **ii** $\frac{7}{26}$ **iii** $\frac{7}{13}$ **iv** $\frac{15}{26}$

8 a $\frac{1}{5}$ **b** $\frac{17}{40}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$

9 a

	B	Not B	Total
A	20	50	70
Not A	20	10	30
Total	40	60	100

b

	B	Not B	Total
A	6	5	11
Not A	4	3	7
Total	10	8	18

10 a

	Sports	Not Sports	Total
Automatic	2	13	15
Not Automatic	8	17	25
Total	10	30	40

b $\frac{1}{5}$ **c** $\frac{13}{40}$

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11 a

	Automatic car	Not automatic	Total
4 WD	9	24	33
Not 4WD	17	10	27
Total	26	34	60

b i Pr(Automatic and 4 WD) $= \frac{9}{60} = \frac{3}{20}$

ii Pr(4 WD) $= \frac{33}{60} = \frac{11}{20}$

iii Pr(automatic) $= \frac{26}{60} = \frac{13}{30}$

iv Pr(not 4 WD) $= \frac{27}{60} = \frac{9}{20}$

i Pr(not automatic) $= \frac{34}{60} = \frac{17}{30}$

vi Pr (neither automatic nor 4WD) $= \frac{10}{60} = \frac{1}{6}$

12 a

	Like volleyball	Dislike volleyball	Total
Like tennis	12	6	18
Dislike tennis	11	4	15
Total	23	10	33

b $\frac{11}{33} = \frac{1}{3}$
c 18
d $\frac{12}{18} = \frac{2}{3}$
e $\frac{6}{18} = \frac{1}{3}$

13 a

	Like reading	Don't like reading	Total
Like exercise	33	8	41
Don't like exercise	7	3	10
Like computer games	24	12	36
Don't like computer games	11	2	13
Total	75	25	100

b $\frac{33}{41}$ **c** $\frac{3}{10}$

8K

Now you try

Example 21

a $\frac{1}{8}$ **b** 30 **c** 40

Example 22

a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** 450

Exercise 8K

1 a 2 **b** $\frac{1}{5}$ **c** $\frac{3}{5}$

2 a 6 **b** $\frac{6}{10} = \frac{3}{5}$ **c** $\frac{4}{10} = \frac{2}{5}$

3 a $\frac{27}{50}$ **b** 378 **c** $\frac{1}{2}$ **d** 350

4 a $\frac{55}{100} = \frac{11}{20}$ **b** 550 **c** $\frac{1}{2}$ **d** 500

5 a $\frac{3}{5}$ **b** $\frac{1}{10}$ **c** $\frac{43}{50}$ **d** 40

6 a

No. of cars	0	1	2	3	4
Frequency	12	37	41	8	2

b 100 **c** $\frac{3}{25}$ **d** $\frac{51}{100}$

7 a $\frac{1}{3}$ **b** 200 **c** 300

8 a i $\frac{7}{20}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{5}$ **iv** $\frac{1}{5}$
b 200

9 a $\frac{19}{100}$ **b** 0.17 **c** Experimental

10 a Answers will vary. **b** Answers will vary.
c 10 **d** Possible but unlikely.

11 a 3 **b** 2 **c** 14 **d** 3
e 19 **f** 8

12 a 0 **b** $\frac{3}{11}$ **c** F **d** 0 **e** T

13 a Could be (Red: 1, 2, Blue: 3, 4, Green: 5, 6)
b Could be (Red: 1, 2, 3, 4, Blue: 5, Green: 6)
c Could not be; probability of $\frac{1}{5}$ cannot be achieved with single die roll.

14 a–d Answers will vary.

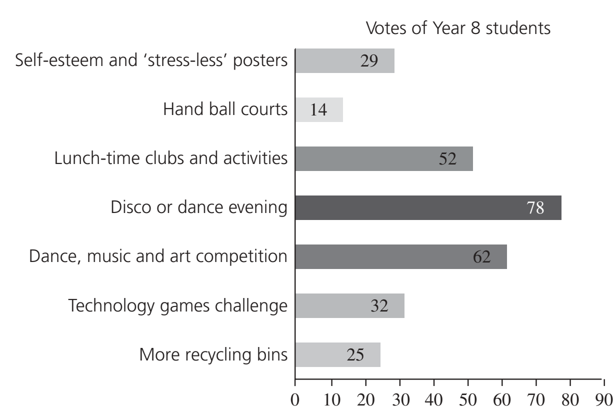
Maths@Work: Student Representative Council (SRC) coordinator

1 a 150 **b** 120
c Tahlia P, Eden V, Hicham J, Charlie K, Aanya N
d Tahlia P 16%, Eden V 14%, Hicham J 13%, Charlie K 11%, Aanya N 10% **e** Answers will vary.

2 a Alex P 330, Jia Hao N 165, Kelly Y 105, Samantha W 90, Nelson C 60
b Alex P and Jia Hao N

3 a 885 students **b** 9%
c 2.44 times per week **d** 49%
e From the survey results it appears that there are students working on library computers every lunchtime. Whether a competition could be run depends on how many students will be in the competition and how many library computers are spare.
f Answers will vary. Possible questions could include:
'Which days do you regularly work on the library computers at lunchtime?'
'Would you be interested being part of a lunch-time computer competition in the library?'
g Answers will vary.

4 Horizontal Column chart



Answers

8K

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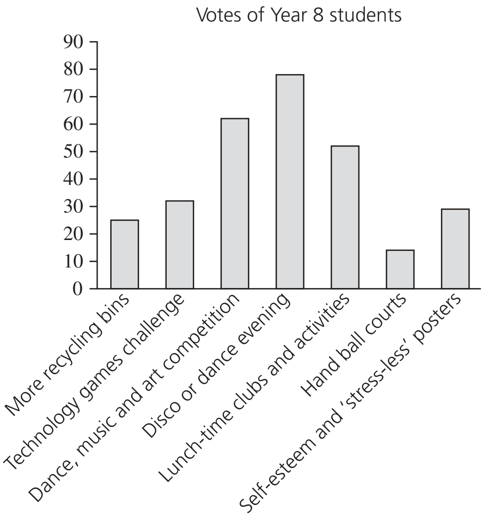
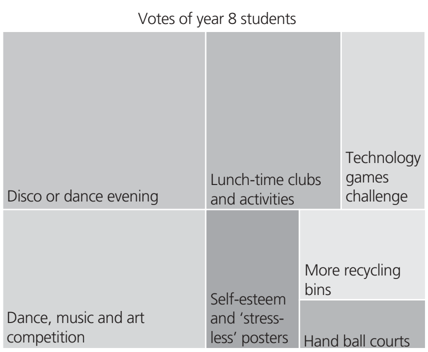
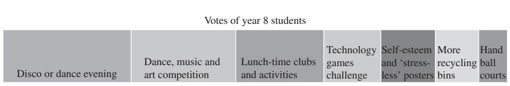
Vertical Column chart



Treemap



Divided bar chart



Puzzles and games

1 A BOWLING MACHINE
2 **a** 4 **b** 11 **c** 4.5
3 0.25
4 **a** MOON **b** OFF **c** DING **d** PROBABILITY
e STUMBLE **f** TRY
5 32
6 No, many reasons e.g. sum of 12 only from 6, 6 but sum of 9 from 4, 5; 5, 4; 6, 3 or 3, 6 so sum of 9 more likely than sum of 12.
7 No, must have repeated points in $5-9$ and $10-14$.

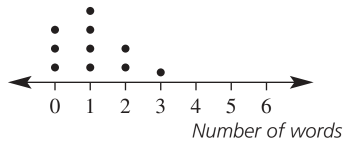
Checklist Answers

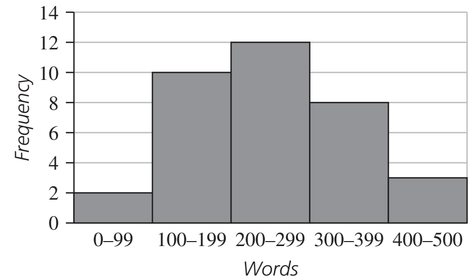
1 \$50 000
2 **a** 25% **b** \$6000
3

Colour	white	black	blue	red	yellow
Frequency	3	12	17	6	9

4

Score	1	2	3	4	5	6
Frequency	6	2	1	3	1	1

5


6


7 Range $= 12-1=11$
8 Mean $= 9$, mode $= 15$
9 **a** 13
b 9.5
10 140
11 $\frac{30}{100}=3$ in $10(30\%)$
12 As people outside a childcare centre are likely to have been dropping off or picking up their own children or grandchildren they are likely to have kids and produce data that is bias.
13 $\frac{2}{6}=\frac{1}{3}$
14 $\frac{1}{6}$
15 $\frac{3}{8}$
16 53
17 $\frac{43}{80}$
18

	Like Mac	Doesn't like Mac	TOTALS
Like PC	25	37	62
Don't like PC	10	3	13
TOTALS	35	40	75

19

	Like Coffee	Dislike coffee	TOTALS
Like Tea	20	10	30
Dislike Tea	15	5	20
TOTALS	35	15	50

20 $\frac{68}{100}=\frac{17}{25}$
21 200
22 $\frac{8}{30}=\frac{4}{15}$

Short-answer questions

1 **a** Government bus **b** Train **c** 75%
d 1000 **e** Example: Prices went up for government buses.
2 **a** 22
b

Hours	0	1	2	3	4
Frequency	2	6	3	3	8

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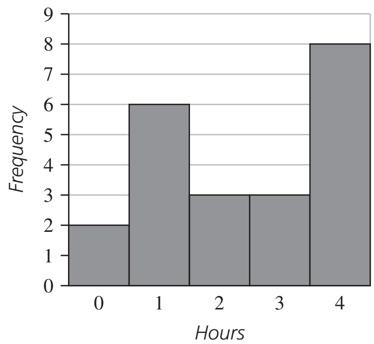
c



d $\frac{1}{11}$ **e** 53 hours **f** 2.4

3 a 38, 43, 44, 44, 52, 53, 55, 56, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 74, 84

b 44 **c** 61.5 **d** 46

4 a 27

b 10, 10, 11, 11, 11, 12, 12, 12, 12, 13, 13, 13, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14

c 12.78 years **d** 13 years

5 a Lowest: 50 kg, highest: 85 kg, range = 35 kg

b

Weight	Frequency
50 – 54	6
55 – 59	6
60 – 64	8
65 – 69	7
70 – 74	7
75 – 79	1
80 – 85	5

c 60–64 kg

d Only teenagers were chosen, not including children or adults.

6 a i 6.5 **ii** 6 **iii** 13

b Median = 6
Mean = 10
The mean changes the most.

c Outlier.

7 a Not enough people, and her friends might work harder (or less hard) than other students.

b She could choose 10 people who worked less hard than her.

8 a $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** 1, 4, 6, 8

e $\frac{1}{2}$

9 a M, A, T, H, E, M, A, T, I, C, I, A, N **b** $\frac{2}{13}$

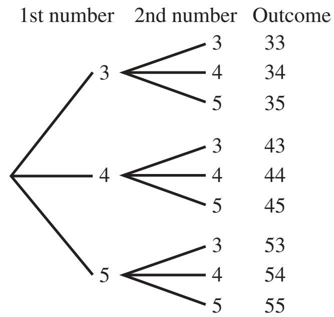
c $\frac{6}{13}$ **d** $\frac{7}{13}$ **e** $\frac{12}{13}$

10 a

	H	T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

b $\frac{1}{4}$ **c** $\frac{1}{12}$

11 a



b $\frac{1}{3}$

c $\frac{1}{3}$

d $\frac{1}{3}$

e $\frac{2}{3}$

f $\frac{1}{3}$

12 a 50 **b** 25 **c** $24\% = \frac{6}{25}$

d $1\% = \frac{1}{100}$ **e** $\frac{12}{25} = 48\%$

f $\frac{24}{25} = 96\%$

13 a

	Mufflers	No mufflers	Total
Tyres	8	4	12
No tyres	2	6	8
Total	10	10	20

b 6

c i $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{7}{10}$ **iv** $\frac{1}{10}$

v $\frac{3}{10}$

14 a $\frac{2}{5}$ **b** 80

Multiple-choice questions

1 C **2** B **3** C **4** A **5** C
6 B **7** B **8** C **9** D **10** E

Extended-response questions

1 a

	Uses public transport	Does not use public transport	Total
Own a car	20	80	100
Do not own a car	65	35	100
Total	85	115	200

b 200 **c** $\frac{1}{2}$ **d** $\frac{23}{40}$ **e** $\frac{1}{10}$ **f** $\frac{1}{5}$

g i More public transport users expected.
ii People less likely to use public transport in regional area.

Answers

CH8
P&G

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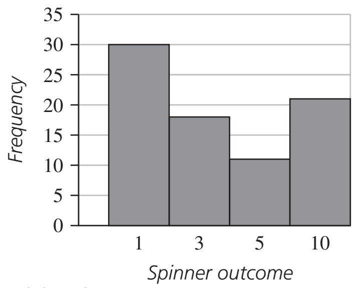
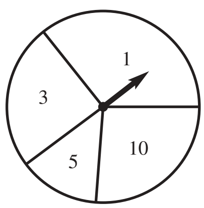
2 a



b 1, it has the most occurrences.
c 3 and 10, as they occur a similar number of times.
d $\frac{3}{8}$
e



Chapter 9

Warm-up quiz

1 a i 50 km **ii** 0 km **iii** 150 km
b 200 km **c** Second hour
d Third hour
2 a George **b** Amanda
3 a −1, 2, 3 **b** −5, 3, 5
c 2, −8, −13 **d** −22, 5, 14
4 a 1 **b** 1 **c** 0 **d** −3
5 a 6 **b** −1 **c** 0 **d** 2
6 a (1, 2) **b** (2, 1) **c** (3, 2)
7 a 7 **b** 4 **c** 2
8 a −1 **b** −5 **c** −11
9 a

△	−2	−1	0	1	2
□	−4	−2	0	2	4

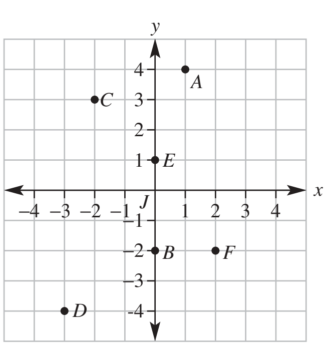
b

△	−2	−1	0	1	2
□	−10	−7	−4	−1	2

9A

Now you try

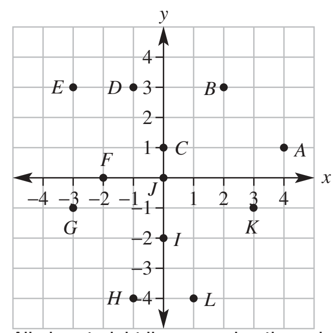
Example 1



Exercise 9A

1 a 3 **b** −4 **c** 5 **d** −8
e (0, 0) **f** y

2 $A(1, 1)$, $B(5, 0)$, $C(3, 4)$, $D(0, 4)$, $E(-1, 2)$, $F(-3, 3)$, $G(-5, 1)$, $H(-3, 0)$, $I(-4, -2)$, $J(-2, -5)$, $K(0, -3)$, $L(2, -3)$, $M(5, -5)$
3 a 3 **b** −1 **c** −2 **d** 0
e −2 **f** 0 **g** −3 **h** 0
4



5 All give straight lines, passing through origin.
6 a First **b** Fourth **c** Second **d** Third
e First **f** Third
7 a B **b** C **c** E **d** D
8 a Triangle **b** Rectangle
c Parallelogram **d** Kite
9 A line on the y-axis
10 a House **b** Fish

9B

Now you try

Example 2

a

t	0	1	2	3	4
V	10	12	14	16	18

b 18 L **c** 2.5 hours

Example 3

a

x	−2	−1	0	1	2
y	−13	−10	−7	−4	−1

b

x	−2	−1	0	1	2
y	9	7	5	3	1

Exercise 9B

1 a 3, 5 **b** 0, 6 **c** −3, −1 **d** 8, 16
e −5, −2 **f** 0, 1
2 a 3 **b** 1 **c** −5 **d** −9
3 a 1 **b** −11 **c** 4 **d** 10
4 a

t	0	1	2	3	4
d	0	40	80	120	160

b 120 km **c** 2 hours
5 a

t	0	1	2	3	4	5
v	1000	1020	1040	1060	1080	1100

b 1080 L **c** 5 minutes
6 a

x	−2	−1	0	1	2
y	−6	−3	0	3	6

b

x	−2	−1	0	1	2
y	−4	−3	−2	−1	0

c

x	−2	−1	0	1	2
y	−3	−1	1	3	5

d

x	−2	−1	0	1	2
y	−7	−5	−3	−1	1

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e

x	−2	−1	0	1	2
y	4	3	2	1	0

f

x	−2	−1	0	1	2
y	1	0	−1	−2	−3

g

x	−2	−1	0	1	2
y	3	1	−1	−3	−5

h

x	−2	−1	0	1	2
y	10	6	2	−2	−6

i

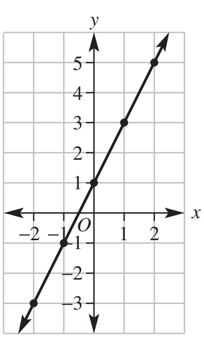
x	−2	−1	0	1	2
y	1	−5	−11	−17	−23

7 a i \$140 **ii** \$700
b \$980 **c** 4 days
8 a i 2 **ii** −1
b i 3 **ii** 0
c 2 **d** 1
9 a i \$15 000 **ii** \$0
b i 3000 **ii** 4000 **iii** 6000
c A loss is made **d** \$12 000
10 a i 10 **ii** 36 **iii** 55
b i 21 **ii** 78
c i 28 **ii** 210 **iii** 5050

9C

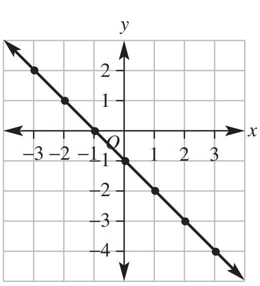
Now you try

Example 4



Example 5

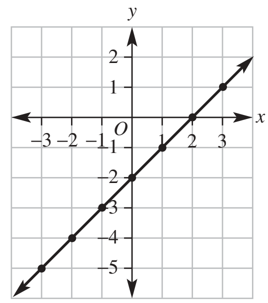
x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4



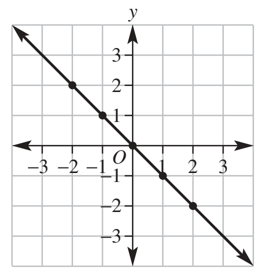
Exercise 9C

1 a 5 **b** 7 **c** 3 **d** 1
e −7 **f** −11 **g** 25 **h** −21
2 a 2 **b** −2 **c** −13

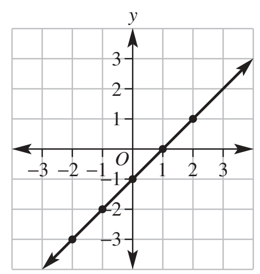
3



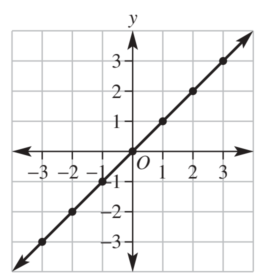
4 a



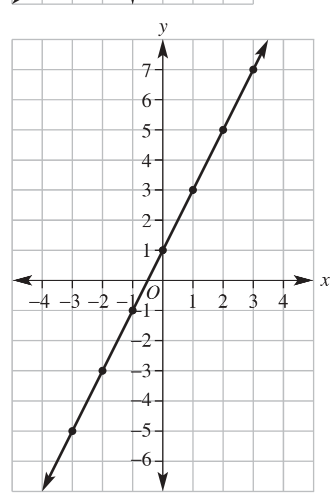
b



c



d



5 a

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

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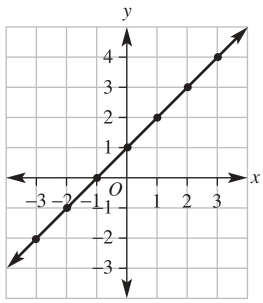
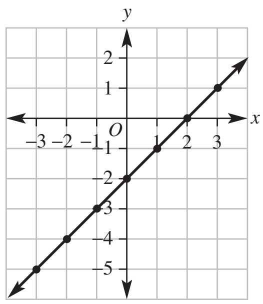


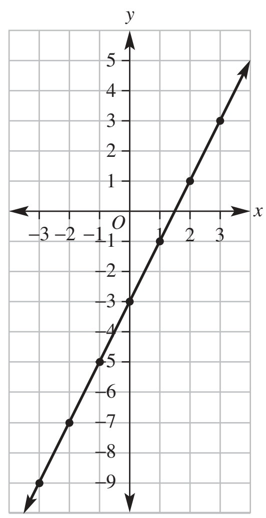
b

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1



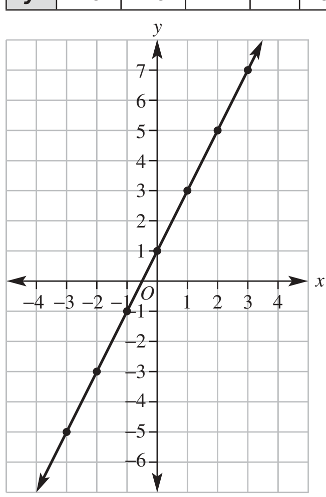
c

x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3



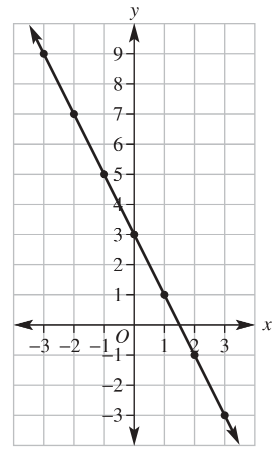
d

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7



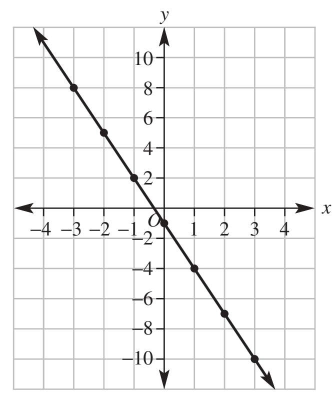
e

x	−3	−2	−1	0	1	2	3
y	9	7	5	3	1	−1	−3



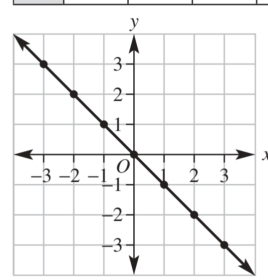
f

x	−3	−2	−1	0	1	2	3
y	8	5	2	−1	−4	−7	−10



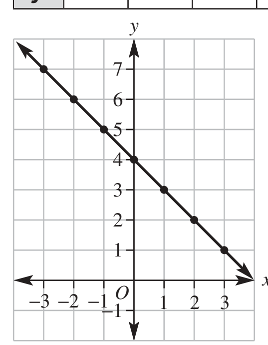
g

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3



h

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1



6 **a** Yes **b** Yes **c** No **d** No
e No **f** Yes

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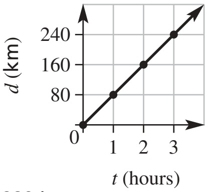
7 a

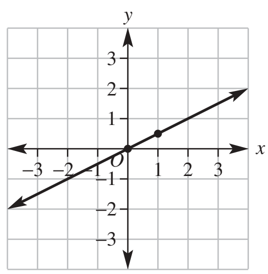
t	0	1	2	3
d	0	80	160	240

b

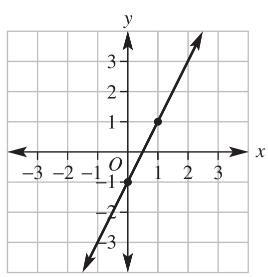


c 320 km
d 5 hours

8 a 2
b i



ii



9 When $x = 0$, $y = 0$ for each.
10 Intersection points are:
a $(0, 0)$ **b** $(2, 1)$ **c** No intersection

9D

Now you try

Example 6

a $y = 2x + 1$ **b** $y = 4x - 4$

Example 7

a $y = -x + 1$ **b** $y = -3x - 1$

Example 8

x	1	2	3	4
y	8	15	22	29

$y = 7x + 1$

Exercise 9D

1 a C **b** A **c** B
2 a 2 **b** −1 **c** −2 **d** 3
3 a 3 **b** 1 **c** −4 **d** 3
4 a $y = x + 1$ **b** $y = 2x$ **c** $y = 2x + 4$
d $y = 3x - 1$ **e** $y = 4x$ **f** $y = 3x + 3$
5 a $y = -x$ **b** $y = -x - 1$
c $y = -x + 1$ **d** $y = -2x + 6$
e $y = -2x$ **f** $y = -3x + 1$
6 a $y = 3x + 1$ **b** $y = 2x + 1$
c $y = 5x + 1$ **d** $y = 2x + 4$
7 a 1 **b** 3 **c** 7 **d** 0
8 a x is not increasing by 1. **b** 1 **c** $y = x - 2$
d i $y = 2x + 3$ **ii** $y = -2x + 3$
iii $y = 3x - 1$ **iv** $y = -4x - 20$
9 a $y = x + 1$ **b** $y = 2x - 2$
c $y = -3x + 2$ **d** $y = -x$

Progress Quiz

1 $A(2, 1), B(-2, -3), C(1, -4), D(0, 3)$
2 Rectangle
3 a 16 **b** 7 **c** −5 **d** 37
4 a

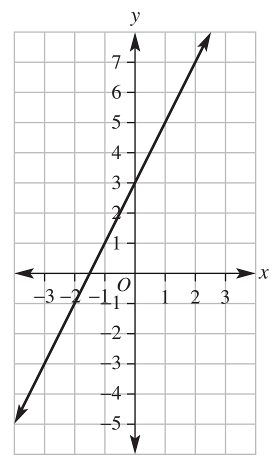
x	−2	−1	0	1	2
y	−8	−4	0	4	8

b

x	−2	−1	0	1	2
y	9	7	5	3	1

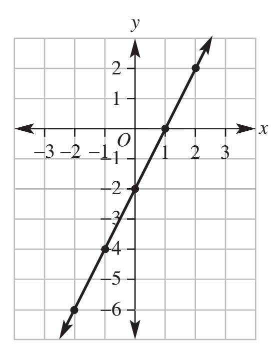
5 a i \$36 **ii** \$84
b \$48 **c** 20

6



7

x	−2	−1	0	1	2
y	−6	−4	−2	0	2



8 a

x	−2	−1	0	1	2
y	5	3	1	−1	−3

b

x	−2	−1	0	1	2
y	−3	0	3	6	9

9 a $y = x - 1$ **b** $y = -2x + 4$
c $y = 3x + 2$
10 a 1 **b** 5 **c** 0

9E

Now you try

Example 9

a $x = 2$ **b** $x = 0.5$ **c** $x = -1.5$

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Example 10

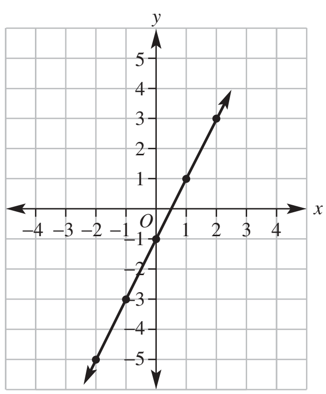
a $(-2, -1), (-1, -2), (0, -3), (1, -4)$, others possible
b $(-1, -2), (0, 0), (1, 2), (2, 4)$, others possible
c $(-1, -2)$

$y = -x - 3$	$y = 2x$
$-2 = -(-1) - 3$	$-2 = 2(-1)$
$-2 = -2$	$-2 = -2$
True	True

d $x = -1$

Exercise 9E

1



x	−2	−1	0	1	2
y	−5	−3	−1	1	3

2 **a** $(2, 4)$ **b** $(3.2, 6.4)$ **c** $(-2.3, -4.6)$
d $(3.5, 7)$ **e** $(-7, -14)$ **f** $(1000, 2000)$
3 **a** $(4, 3)$ **b** $(-2, -3)$
4 **a** $x = 2$ **b** $x = 0.5$ **c** $x = 3$
d $x = -2.5$ **e** $x = -1.5$
5 **a** $x = -2.5$ **b** $x = 3$ **c** $x = -0.5$
d $x = 4$ **e** $x = 5$
6 **a** Any point that lies on the line is correct, e.g. $(-2, 9), (0, 5), (1, 3), (2, 1)$
b Any point that lies on the line is correct, e.g. $(-2, 0), (0, 2), (1, 3), (3, 5)$
c $(1, 3)$

$y = x + 2$	$y = 5 - 2x$
$3 = 1 + 2$	$3 = 5 - 2 \times 1$
$3 = 3$ True	$3 = 3$ True

d $x = 1$
7 **a** $(2, 3)$ **b** $(-1, 1)$
8 **a** $A = 10 + 8n$ applies to Ruby as she has \$10 to start with and adds to her savings by \$8 times the number (n) of hours worked.
$A = 24 + 6n$ applies to Jayden as he has \$24 to start with and increases his savings by \$6 times the number (n) of hours worked.
b **i** $n = 4$ **ii** $n = 4$ **iii** $n = 7$ **iv** $n = 7$
v $n = 11$ **vi** $n = 11$
c Many solutions e.g. $(2, 26), (4, 42), (7, 66), (9, 82), (11, 98)$
d Many solutions e.g. $(2, 36), (4, 48), (7, 66), (9, 78), (11, 90)$
e $(7, 66), (7, 66)$

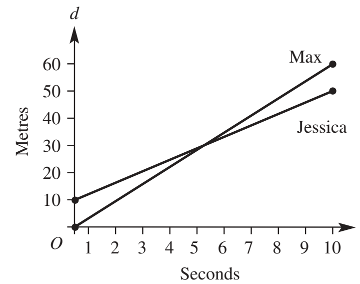
$A = 10 + 8n$	$A = 24 + 6n$
$66 = 10 + 8 \times 7$	$66 = 24 + 6 \times 7$
$66 = 10 + 56$	$66 = 24 + 42$
$66 = 66$ True	$66 = 66$ True

f $n = 7$
g Ruby and Jayden have both worked 7 hours and both have \$66 saved.
9 **a** **i** $(-2, 17), (-1, 14), (0, 11), (1, 8), (2, 5), (3, 2), (4, -1), (5, -4)$
ii $(-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5), (3, 7), (4, 9), (5, 11)$
b $(2, 5)$
c It is the only shared point.
10 **a** $x = 3.67$ **b** $x = -1.53$
c $x = 5.30$

11 **a**

Time in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance in metres	0	6	12	18	24	30	36	42	48	54	60
Jessica's distance in metres	10	14	18	22	26	30	34	38	42	46	50

b



c $d = 6t$
d **i** $6t = 18$ **ii** $6t = 30$
iii $6t = 48$
e $d = 10 + 4t$
f **i** $10 + 4t = 22$ **ii** $10 + 4t = 30$
iii $10 + 4t = 42$
g $(5, 30)$ $(5, 30)$

$d = 6t$	$d = 10 + 4t$
$30 = 6 \times 5$	$30 = 10 + 4 \times 5$
$30 = 30$ True	$30 = 30$ True

h Max catches up to Jessica. They are both 30 m from the starting line and have each run for 5 seconds.

9F

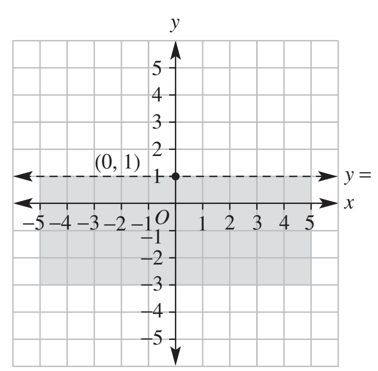
Now you try

Example 11

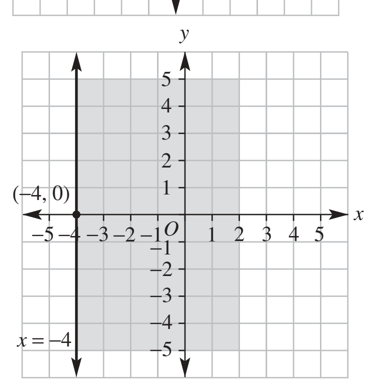
1 $y = 2$ **2** $x = -4$

Example 12

1



2



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Example 13

a $x \geq 2$ **b** $x > -4$

Exercise 9F

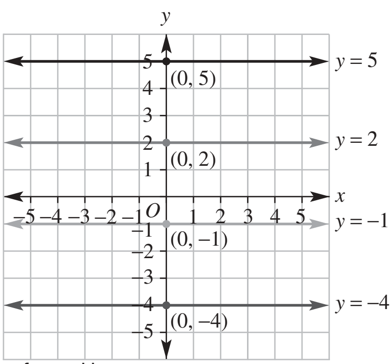
1 **a** True **b** False **c** False **d** True
e False **f** True **g** True

2 **a** True **b** False **c** True **d** False
e True **f** True **g** False

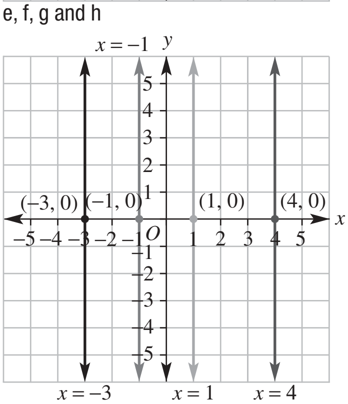
3 **a** True
b All less than or equal to 3
c $x \leq 3$

4 **a** $y = 4$ **b** $y = 1$ **c** $y = -3$ **d** $x = -4$
e $x = 5$ **f** $x = -2$

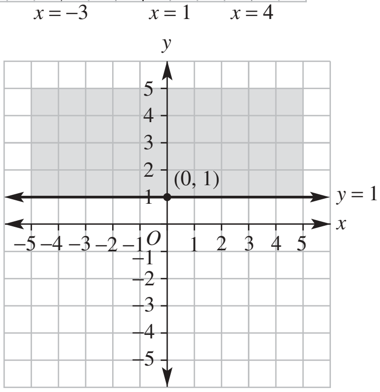
5 a, b, c and d



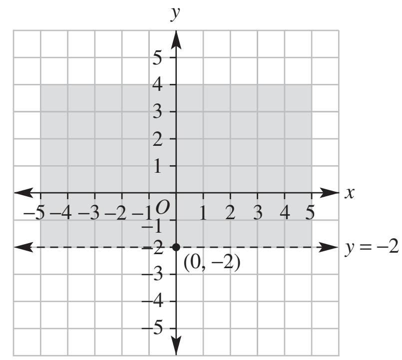
e, f, g and h



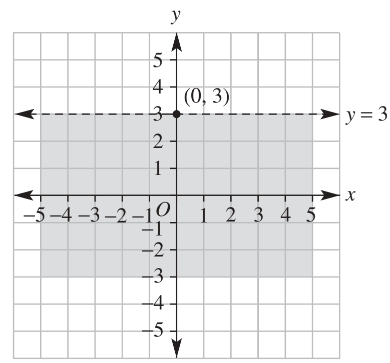
6 **a**



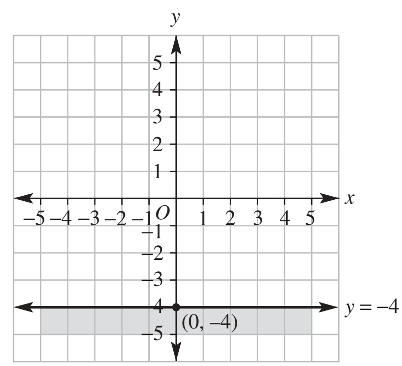
b



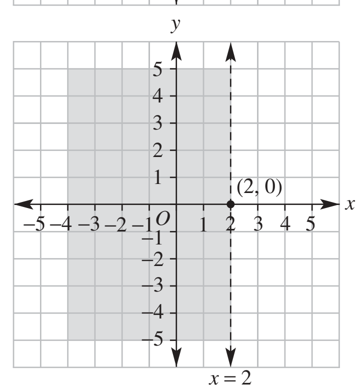
c



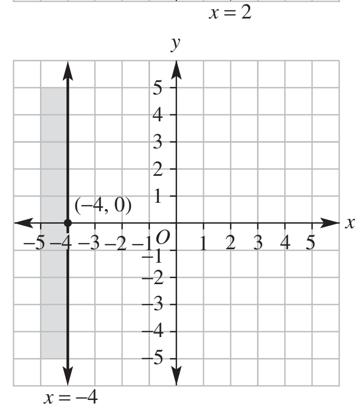
d



e



f



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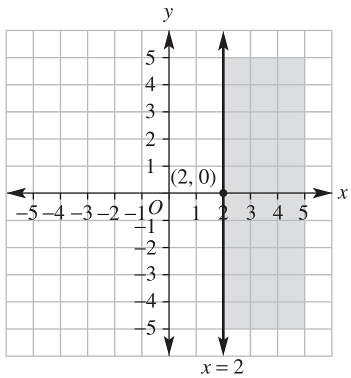
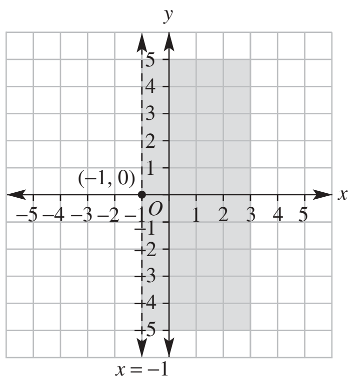
g



h



7 **a** $x \geq 2$ **b** $x < -1$ **c** $x \leq 3$
d $y > -4$ **e** $y \leq 5$ **f** $y \leq -1$

8 **a** $x \leq 2$ **b** $x \leq 1$ **c** $x \geq 5$
d $x > 2$ **e** $x < 1$ **f** $x > -1$

9 **a** $x \leq 1$ **b** $x \geq -2$ **c** $x \leq -1$
d $x < 1$ **e** $x > 3$ **f** $x > 2$

10 **a** E **b** C **c** B
d F **e** A **f** D

11 **a** $x = -1$ **b** $x = 2$ **c** $x = 1.5$
d $x > 1.5$ **e** $x \leq 1.5$ **f** $x \leq 1$

12 **a** Yes **b** No **c** No **d** Yes
e No **f** Yes **g** No **h** Yes
i No **j** Yes **k** Yes **l** Yes

13 **a** Each point on the x-axis has coordinate with $y = 0$ and any x value.
b Each point on the y-axis has coordinate with $x = 0$ and any y value.

14 **a** $x \geq 2, x \leq 4, y \geq 1, y \leq 6$
b $x \geq -3, x \leq 4, y \geq -4, y \leq -2$

9G

Now you try

Example 14

a Zero gradient **b** Positive gradient
c Undefined gradient **d** Negative gradient

Example 15

a 1 **b** $\frac{3}{2}$

Example 16

a −2 **b** $\frac{-1}{4}$

Exercise 9G

1 **a** Positive **b** Negative
c Negative **d** Positive

2 **a** $A(3, 3),\ B(1, -2),\ C(-3, -2),\ D(-2, 1)$
b **i** 3 **ii** 0 **iii** −1 **iv** 2
c **i** 3 **ii** 4 **iii** 2 **iv** 5

3 **a** 2 **b** $\frac{2}{3}$ **c** −4 **d** $\frac{-2}{5}$ **e** $\frac{-3}{2}$
f $\frac{-1}{2}$ **g** $\frac{1}{2}$ **h** 3 **i** $\frac{-1}{2}$

4 **a** Positive **b** Undefined **c** Zero **d** Negative

5 **a** 3 **b** 1 **c** $\frac{1}{2}$
d 3 **e** $\frac{2}{3}$ **f** 4

6 **a** −2 **b** $\frac{-3}{5}$ **c** $\frac{-4}{3}$
d −1 **e** −3 **f** $\frac{-3}{2}$

7 Grassy slope

8 Torpedo

9 **a** The y-value rises from 1 to 7.
b The y-value falls from 2 to −5.
c The x-value increases from −1 to 3.
d The x-value increases from −4 to 3.

10 **a** 2 **b** 10

11 **a** $\frac{3}{4}$ **b** $\frac{5}{2}$ **c** $\frac{-3}{2}$ **d** $\frac{-7}{10}$

12 **a** $\frac{5}{2}$ **b** $\frac{5}{3}$ **c** $\frac{-8}{3}$
d $\frac{-2}{3}$ **e** $\frac{8}{3}$ **f** $\frac{-3}{10}$

9H

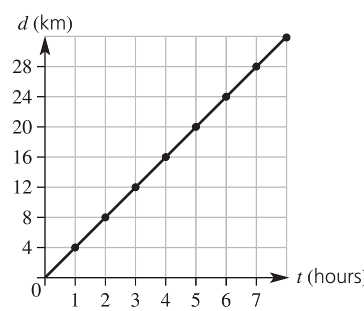
Now you try

Example 17

a

t	0	1	2	3	4	5	6	7
d	0	4	8	12	16	20	24	28

b



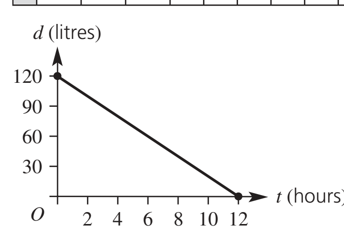
c $d = 4t$
d 14 km
e 6.5 hours

Example 18

a

t	0	1	2	3	4	5	6	7	8	9	10	11	12
v	120	110	100	90	80	70	60	50	40	30	20	10	0

b



c $V = -10t + 120$
d 50 litres
e 12 hours

Exercise 9H

1 **a** $d = 3t$ **b** $d = t + 2$ **c** $d = -t + 4$

2 **a** 15 **b** 45 **c** 5 **d** 125

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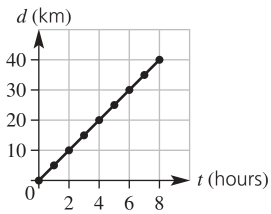
3 a 60 cm b 150 cm c 330 cm

4 a 28 L b 24 L c 10 L

5 a

t	0	1	2	3
d	0	6	12	18

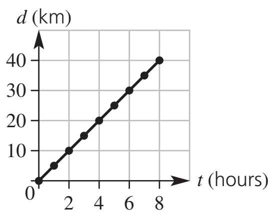
b


c $d = 6t$

d 9 km e 2 hours

6 a

t	0	1	2	3	4	5	6	7	8
d	0	5	10	15	20	25	30	35	40

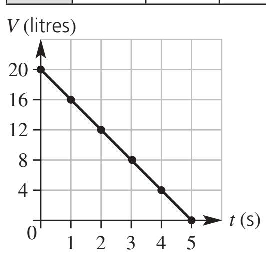
b


c $d = 5t$

d 22.5 km e 4 hours

7 a

t	0	1	2	3	4	5
d	20	16	12	8	4	0

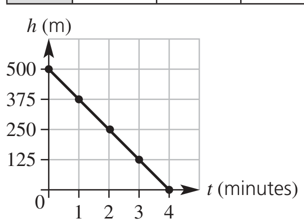
b


c $V = -4t + 20$

d 11.2 L e 3 seconds

8 a

t	0	1	2	3	4
h	500	375	250	125	0

b


c $h = -125t + 500$

d 275 m e 3 minutes

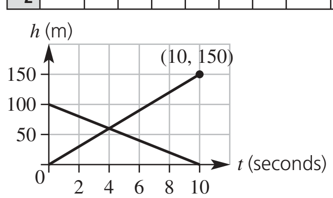
9 a $M = -0.5t + 3.5$ b 7 hours c 4.5 hours

10 a $d = 15t$ b 3 hours

c 3 hours 20 minutes

11 a

t	0	1	2	3	4	5	6	7	8	9	10
h_1	0	15	30	45	60	75	90	105	120	135	150
h_2	100	90	80	70	60	50	40	30	20	10	0

b


c 10 seconds

d $h = 15t,\ h = -10t + 100$ e At 4 seconds

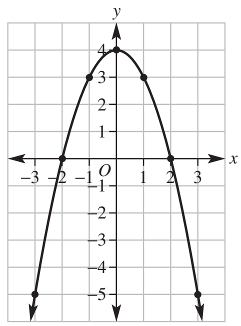
f At 2.5 seconds g At 3.5 seconds

9I

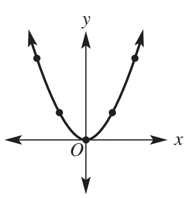
Now you try

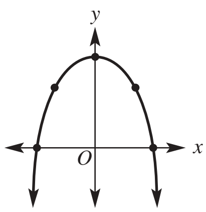
Example 19

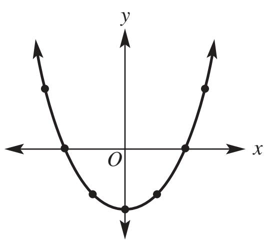
x	−3	−2	−1	0	1	2	3
y	−5	0	3	4	3	0	−5

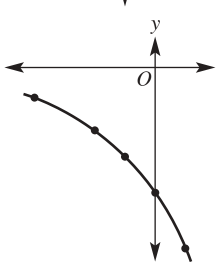


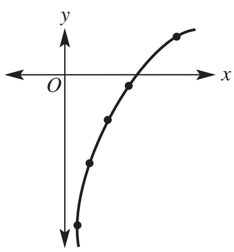
Exercise 9I

1 a


b


c


d


e


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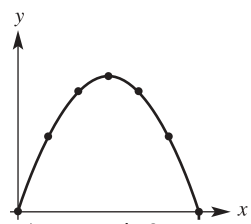
f

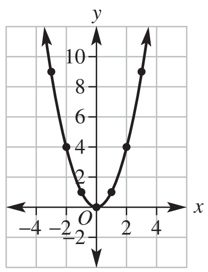


2 **a** −1 **b** 8 **c** 3 **d** 15

3 **a** Yes **b** No **c** No **d** Yes

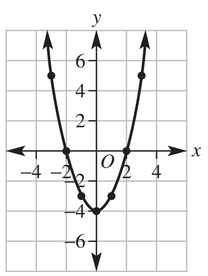
4 **a** $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



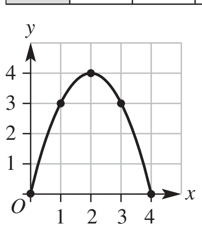
b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5



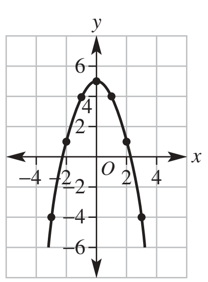
c $y = x(4 - x)$

x	0	1	2	3	4
y	0	3	4	3	0

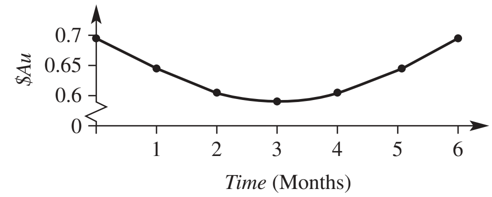


d $y = 5 - x^2$

x	−3	−2	−1	0	1	2	3
y	−4	1	4	5	4	1	−4



5 **a**



b Non-linear (parabolic)

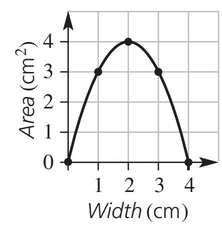
c **i** \$0.05 **ii** \$0.03

d ≈ 0.76

6 **a**

Width (cm)	0	1	2	3	4
Length (cm)	4	3	2	1	0
Area (cm^2)	0	3	4	3	0

b



c Non-linear (parabolic) **d** 2 cm by 2 cm

7 For each unit change in x there are variable changes in y.

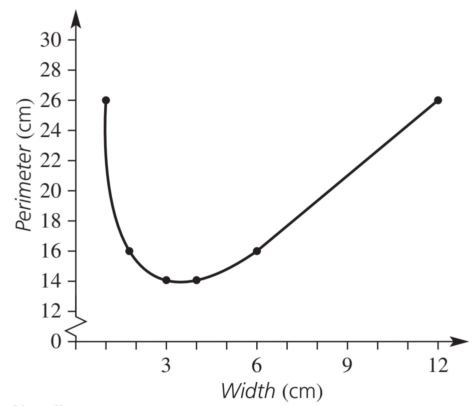
8 **a** Linear **b** Linear **c** Non-linear

d Non-linear **e** Non-linear **f** Non-linear

9 **a**

Width (cm)	1	2	3	4	6	12
Length (cm)	12	6	4	3	2	1
Perimeter (cm)	26	16	14	14	16	26

b



c Non-linear

d **i** ≈ 3.5 cm **ii** ≈ 13.9 cm

Maths@Work: Economists and household expenditure

1 **a** Positive **b** Negative **c** Negative

d Positive **e** Positive

2 **a** \$100/week **b** \$300/week **c** \$20 increase

d Positive **e** 0.2 **f** $y = 0.2x + 100$

3 **a** **i** \$120/week **ii** \$60/week

b \$15 less or −\$15 **c** Negative

d −0.15 **e** $y = -0.15x + 120$ **f** \$800/week

g Answers will vary.

4 **A** Normal **B** Inferior **C** Normal **D** Inferior

5 **a** **a** and **b** *See table at bottom of next page*

c **i–iv** *See figure at bottom of next page*

d \$480 **e** \$1280

Puzzles and games

1 Aeroplane

2 **a** $y = 4x - 7$ **b** $y = -x + 11$

c $y = 5x - 50$ **d** $y = x - 10$

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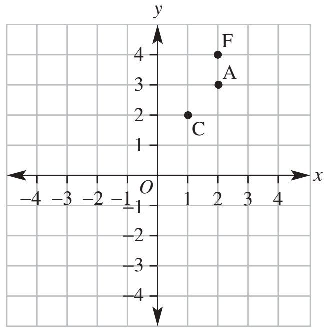
3 **a** 31 **b** 165

4 3 hours **5** 40 min **6** 1588

Checklist Answers

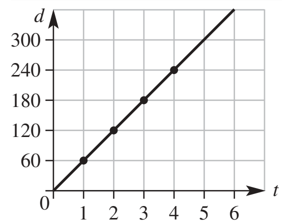
1 $A(1,1)$ $B(2,3)$ $C(4,0)$ $D(3,-3)$ $E(0,-2)$ $F(-4,-4)$ $G(-3,0)$ $H(-1,3)$

2



3

t	0	1	2	3	4
d	0	60	120	180	240

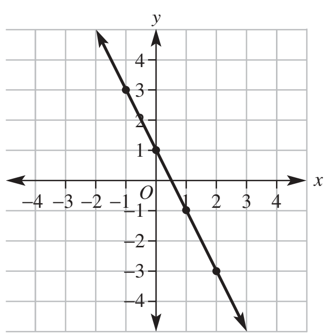


1.5 hours to travel 90 km

4

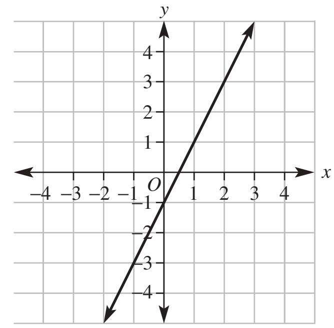
x	−2	−1	0	1	2
y	−7	−5	−3	−1	1

5 $y = 1 - 2x$



6 $y = 2x - 1$

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5



7 **a** $y = 3x - 2$

b $y = 1 - 2x$

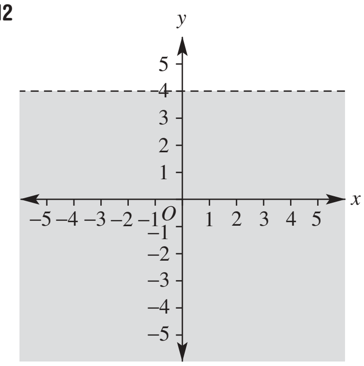
8 $y = 2x + 1$

9 $x = 2$

10 $x = 1$

11 $x = 7$

12



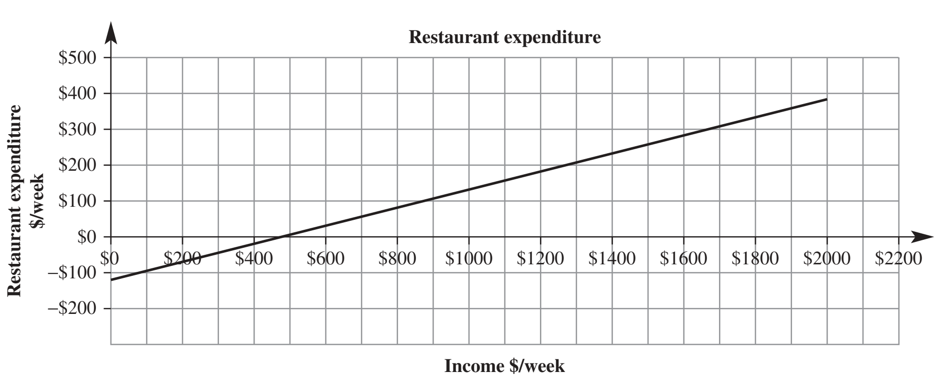
13 $x < 3$

14 **a** Negative **b** Undefined

c Positive **d** Zero

15 **a** $\frac{2}{5}$ **b** −1

Weekly Income	$0	$200	$400	$600	$800	$1000	$1200	$1400	$1600	$1800	$2000
Restaurant Expenditure	−$120	−$70	−$20	$30	$80	$130	$180	$230	$280	$330	$380



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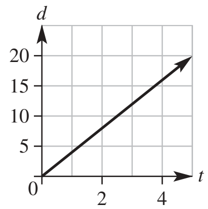
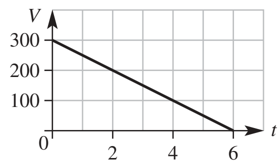
16 $d = 4t$ 10 km in 2.5 hours



17 a

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

b

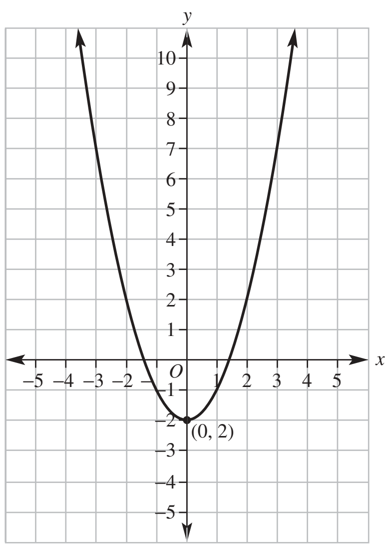


c $V = 300 - 50t$

d $4\frac{1}{2}$ *hours*

18

x	−2	−1	0	1	2
y	2	−1	−2	−1	2



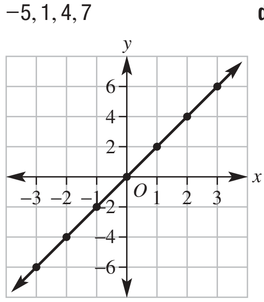
Short-answer questions

1 **a** 100 km **b** 1 hour
c **i** 50 km **ii** 100 km
d Section C

2 $A(2, 3), B(0, 2), C(-2, 4), D(-3, 1), E(-3, -3),$
$F(-1, 0), G(0, -4), H(1, -2), I(4, -3), J(3, 0)$

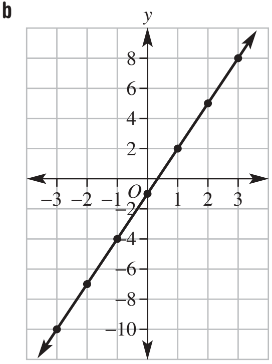
3 **a** $-2, -1, 0, 1$ **b** $-4, 0, 2, 4$
c $-5, 1, 4, 7$ **d** $3, 1, 0, -1$

4 **a**



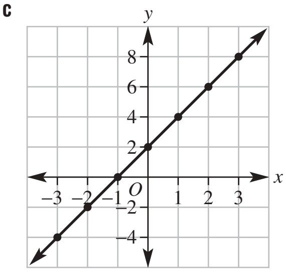
x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

b



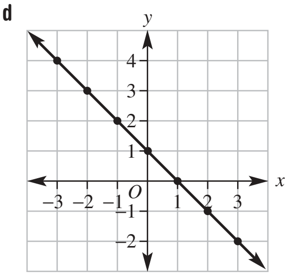
x	−3	−2	−1	0	1	2	3
y	−10	−7	−4	−1	2	5	8

c



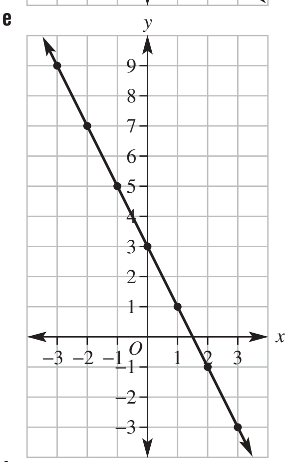
x	−3	−2	−1	0	1	2	3
y	−4	−2	0	2	4	6	8

d



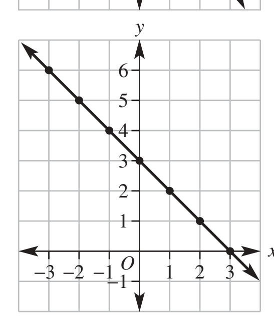
x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	−1	−2

e



x	−3	−2	−1	0	1	2	3
y	9	7	5	3	1	−1	−3

f



x	−3	−2	−1	0	1	2	3
y	6	5	4	3	2	1	0

5 **a** $y = 2x + 1$ **b** $y = 3x + 2$
c $y = x + 3$ **d** $y = -x + 1$
e $y = -4x - 1$ **f** $y = -x + 8$

6 **a** $x = 6$ **b** $x = 4$ **c** $x = -4$ **d** $x = -5$
e $x = -1$ **f** $y = 5$

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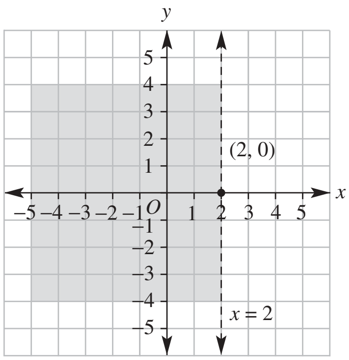
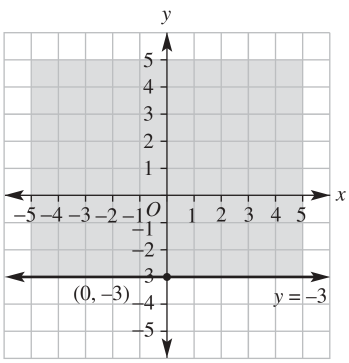
7 a



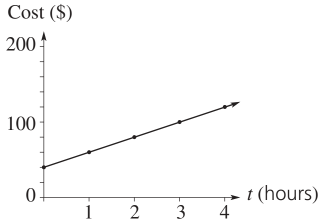
b



8 a $x < 6$ **b** $x \leqslant -1$

9 a 3 **b** 2 **c** −4 **d** 1
e −2 **f** $\frac{-1}{2}$

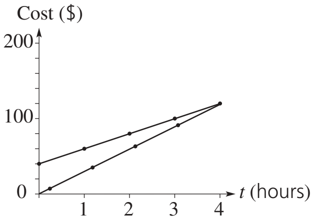
10 a



b $90

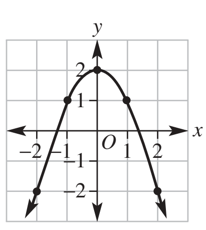
c 3.5 hours **d** $C = 20t + 40$
e $90 and 3.5 hours
f i $t = 2$ **ii** $t = 1.5$

g



h 4 hours

11



Multiple-choice questions

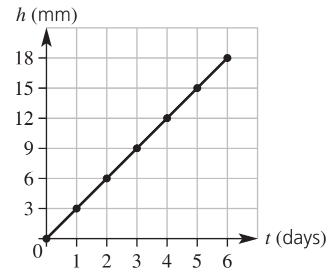
1 B **2** C **3** C **4** D **5** B
6 D **7** A **8** E **9** D **10** E

Extended-response questions

1 a

t	0	1	2	3	4	5	6
h	0	3	6	9	12	15	18

b

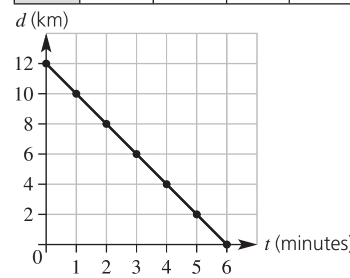


c $h = 3t$
d 10.5 mm
e 30 mm
f 5 days

2 a

t	0	1	2	3	4	5	6
h	12	10	8	6	4	2	0

b



c 6 minutes **d** −2 **e** $h = -2t + 12$
f 7 km **g** 4 minutes 15 seconds

Chapter 10

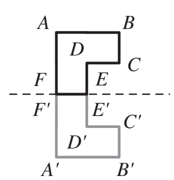
Warm-up quiz

1 a 4 **b** 2 **c** 5 **d** 0
2 a 4 **b** 2 **c** 5 **d** 2
3 a $A(1, 1)$, $B(3, -2)$, $C(-4, -3)$, $D(-3, 0)$
b i (0, 1) **ii** (3, 0) **iii** (−4, −2)
c (−4, 3)
4 a (5, 5) **b** (5, 2) **c** (1, 2)
5 a $a = 30$ **b** $a = 20$ **c** $a = 45$ **d** $a = 60$
6 a A, B, C, D
b A, B, C, D, E
c A, B
d A, C, E

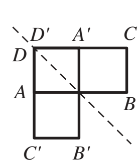
10A

Now you try

Example 1



Example 2



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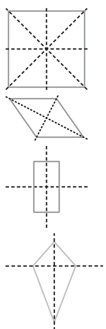
Example 3

a $A'(1, 1), B'(1, 3), C'(3, 3), D'(3, 1)$
b $A'(-1, -1), B'(-1, -3), C'(-3, -3), D'(-3, -1)$

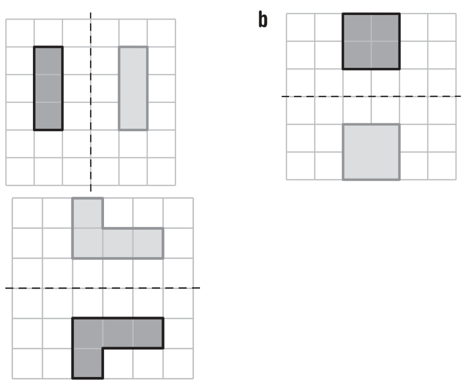
Exercise 10A

1 **a** Transformations **b** Mirror line
c A' **d** Symmetry

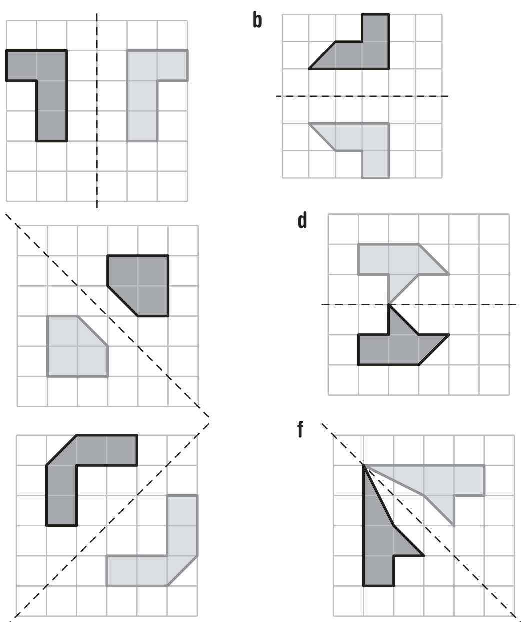
2 **a** **b** **c** **d**



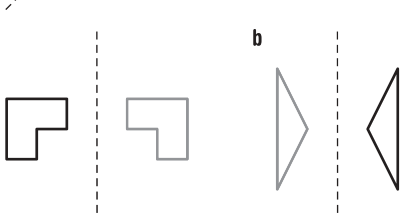
3 **a** **b** **c**



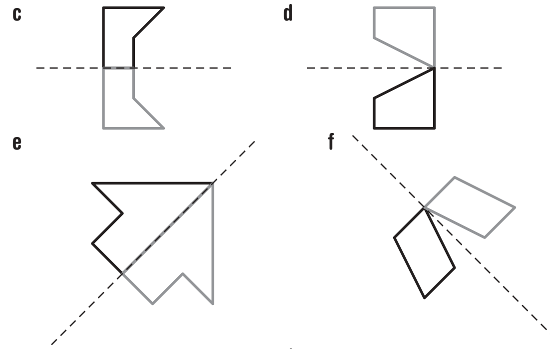
4 **a** **b** **c** **d** **e** **f**



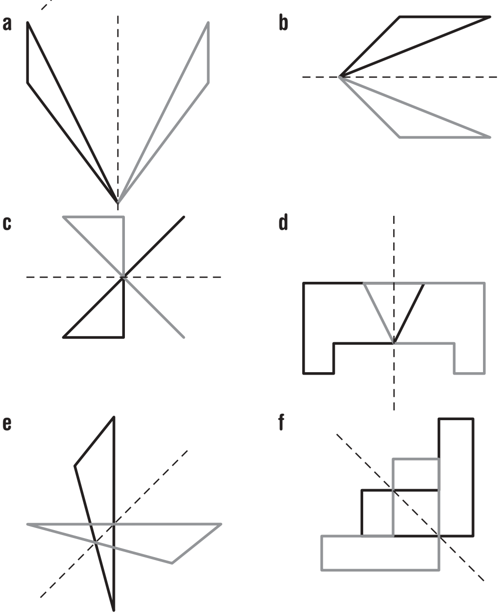
5 **a** **b**



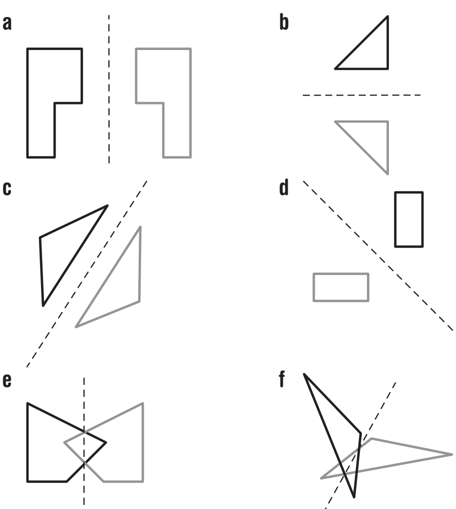
c **d** **e** **f**



6 **a** **b** **c** **d** **e** **f**



7 **a** **b** **c** **d** **e** **f**



8 **a** $A'(2, 0),\ B'(1, -3),\ C'(4, -2)$
b $A'(-2, 0),\ B'(-1, 3),\ C'(-4, 2)$

9 **a** $A'(-1, 2),\ B'(-4, 2),\ C'(-4, 4),\ D'(-1, 4)$
b $A'(1, -2),\ B'(4, -2),\ C'(4, -4),\ D'(1, -4)$

10 **a** 4 **b** 2 **c** 2 **d** 1 **e** 0
f 0 **g** 1 **h** 3 **i** 8

11 A parallelogram cannot be reflected onto itself over a mirror line but a rhombus can do this in two ways.

12 10 m^2, the area is unchanged after reflection.

13 n

14 Reflection in the y-axis.

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15 a–d Computer geometry required, answers will vary.

10B

Now you try

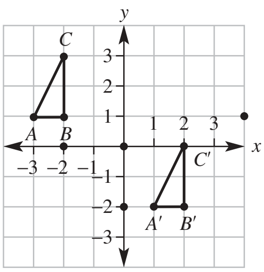
Example 4

a (1, 2) **b** (−1, 0)

Example 5

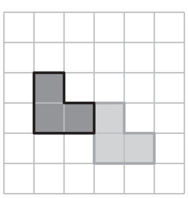
(7, −1)

Example 6

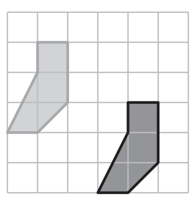


Exercise 10B

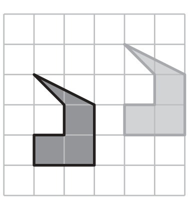
1 a



b



c



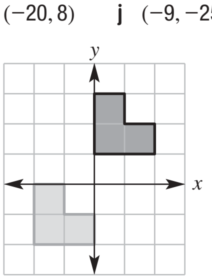
2 a Right, up **b** Left, up
c Right, down **d** Left, down

3 a (5, −2) **b** (−2, −6)
c (−7, 4) **d** (9, 17)

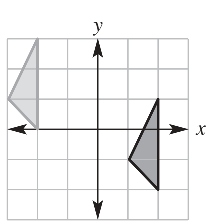
4 a i (3, 4) **ii** (−1, 3) **iii** (−2, 0) **iv** (−3, −1)
b i (−1, 1) **ii** (3, 0) **iii** (−3, 1) **iv** (−2, 3)

5 a (1, −1) **b** (3, −1) **c** (2, −2) **d** (2, −4)
e (3, 7) **f** (−3, 5) **g** (2, −3) **h** (3, −5)
i (−20, 8) **j** (−9, −25)

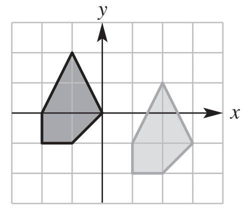
6 a



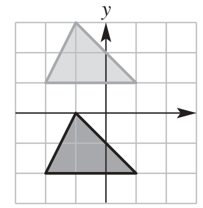
b



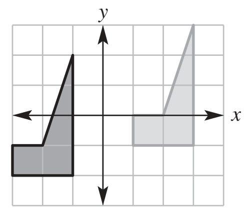
c



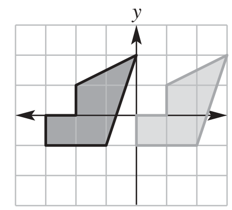
d



e



f



7 a Horizontal **b** Vertical
c Vertical **d** Horizontal

8 a (15, 2) **b** (21, −1) **c** (13, 6) **d** (9, 2)
e (11, −2) **f** (3, 4) **g** (11, −9) **h** (19, −10)

9 a (−3, 2) **b** (5, 0) **c** $(-x, -y)$ **d** (x, y)

10 a i (1, −2) **ii** (−3, 4) **iii** (0, −3)
b 1742 m **c** (2, 3)

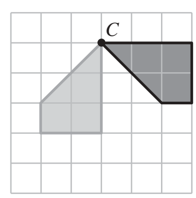
10C

Now you try

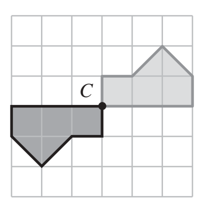
Example 7

a 6
b 2

Example 8



Example 9



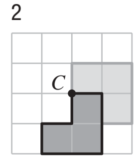
Exercise 10C

1 a C **b** A **c** B

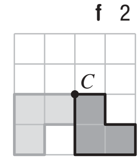
2 a Anticlockwise, 90° **b** Clockwise, 90°
c Anticlockwise, 90° **d** Clockwise, 90°
e Anticlockwise, 180° **f** Clockwise, 180°

3 a 4 **b** 2 **c** 5
d 2 **e** 4 **f** 2

4 a



b



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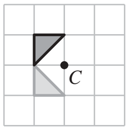
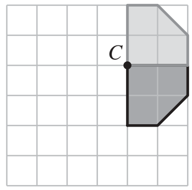
c


d


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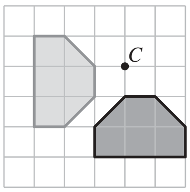

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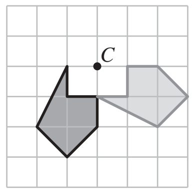
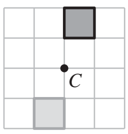


5 a


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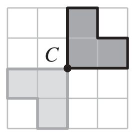

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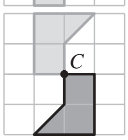

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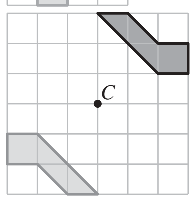

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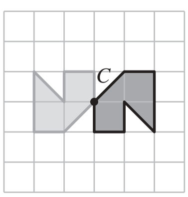
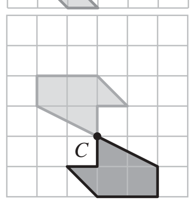
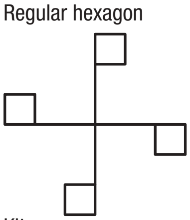


f


6 a (−4, −3) b (−4, −3) c (3, −4) d (−3, 4)
e (−3, 4) f (3, −4) g (4, 3)
7 a 270° b 180° c 302° d 64°
8 a 90° b 180° c 90°
9 a $A'(4,-4)$, $B'(4,-1)$, $C'(1,-1)$
b $A'(4,4)$, $B'(1,4)$, $C'(1,1)$
c $A'(-4,-4)$, $B'(-1,-4)$, $C'(-1,-1)$
10 H, I, N, O, S, X and Z
11 Answers may vary. Examples are:
a Square
b Parallelogram
c Regular hexagon
d

e Kite

10D

Now you try

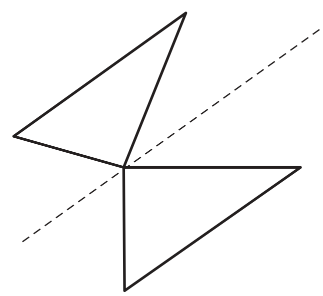
Example 10

a Vertex E b Side DF c $\angle D$

Exercise 10D

1 a F b T c T
d T e T f T
2 a Yes
b i D ii E iii F
c i DE ii EF iii DF
d i $\angle E$ ii $\angle F$ iii $\angle D$
3 a Yes
b i D ii E iii F
c i DE ii EF iii DF
d i $\angle E$ ii $\angle F$ iii $\angle D$
4 a Yes
b i D ii E iii F
c i DE ii EF iii DF
d i $\angle E$ ii $\angle F$ iii $\angle D$
5 a i E ii H
b i EH ii GH
c i $\angle G$ ii $\angle E$
6 a i F ii I
b i FJ ii HI
c i $\angle H$ ii $\angle J$
7 (J, G), (D, K)
8 a DE b $\angle B$ c 10 cm
d 62° e 30°
9 (A, J), (C, K), (E, G)
10 a $\Delta AMC, \Delta BMC$
b Yes, all corresponding sides and angles will be equal.
11 Yes
12 a 32 b 24 c 20 d 8 e 4

Progress Quiz

1


2 a $A'(-4,-1)$, $B'(-4,-3)$, $C'(-1,-3)$, $D'(-1,-1)$
b $A'(4,1)$, $B'(4,3)$, $C'(1,3)$, $D'(1,1)$
3 a 4 b 2 c 0
4 a (−4, 3) b (6, −2)
5 a (1, 6) b (−4, 4) c (4, 3)
6 a (−1, −5) b (−4, 3)
7 a 6 b 4
8 a (−3, 4) b (−3, 4) c (−4, −3)
d (4, 3) e (3, −4) f (−4, −3)
9 a Vertex O b Vertex M
c Side ON d Side NM
e $\angle P$ f $\angle N$
10 a FD b $\angle C$ c 5 cm d 31°

10E

Now you try

Example 11

$\Delta ABC \equiv \Delta MNO$

Example 12

a RHS b AAS c SAS d SSS

Exercise 10E

1 a Yes b Yes c No d Yes
e Yes f No
2 a i F ii D iii E
b i $\angle A$ ii $\angle C$ iii $\angle B$
c i DE ii FD iii EF
3 a $\Delta ABC \equiv \Delta EFD$ b $\Delta ABC \equiv \Delta FED$
c $\Delta XYZ \equiv \Delta UST$ d $\Delta ABC \equiv \Delta ADC$

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4 **a** SAS **b** SSS **c** RHS **d** AAS
5 **a** SSS **b** RHS **c** SAS **d** AAS
6 **a** $x = 4,\ y = 1$ **b** $x = 9,\ a = 20$
c $x = 5,\ a = 24$ **d** $x = 5,\ a = 30$
e $x = 4,\ a = 95,\ b = 25$ **f** $x = 11,\ a = 50,\ b = 90$
7 **a** SSS **b** SAS **c** RHS **d** AAS
8 **a** No **b** Yes, SAS **c** Yes, AAS **d** No
9 You can draw an infinite number of triangles with the same shape but of different size.
10 **a** No **b** Yes **c** Yes **d** No

10F

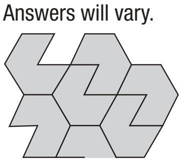
Now you try

Example 13

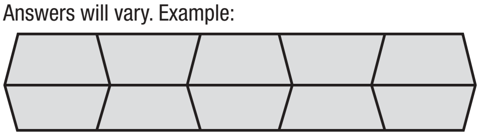
(4.8.8)

Exercise 10F

1 C
2 D
3 Answers will vary.

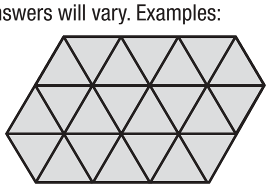


4 Answers will vary. Example:

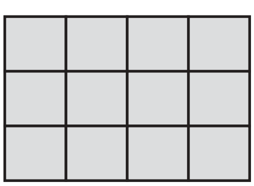


5 Answers will vary. Examples:

a



b



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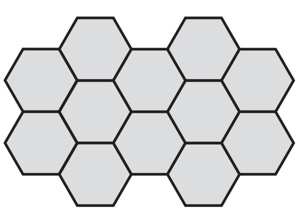
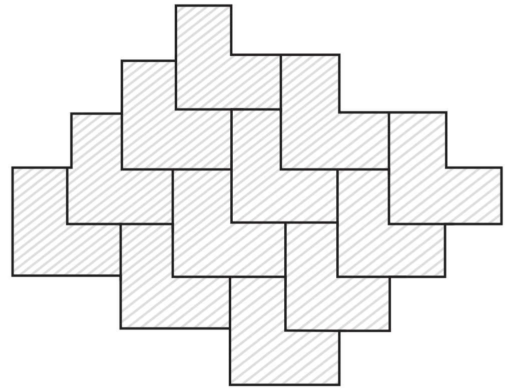


6 **a** 3.3.3.3.3.3 **b** 4.4.4.4 **c** 6.6.6
7 **a** Yes
b No
c Yes
d No
e Yes
8 **a** 3.3.3.4.4
b 3.3.4.3.4
c 3.4.6.4
d 3.12.12
9 Answers will vary. Examples:

a



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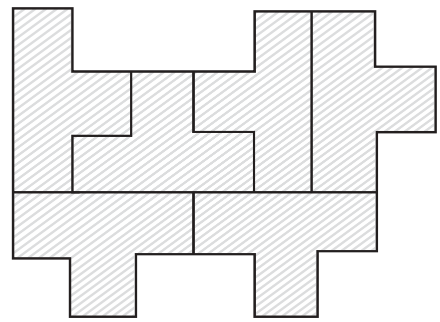
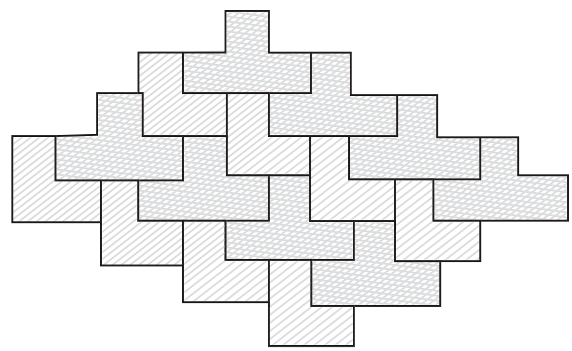
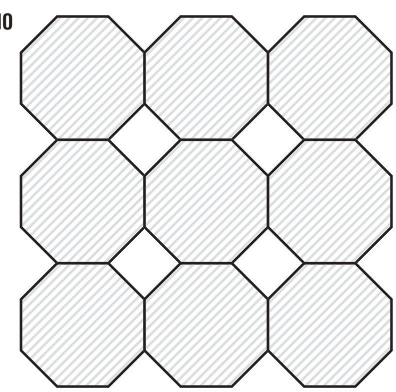


c



10



11 **a** 50 **b** Answers will vary.
12 **a–c** Answers will vary.
13 **a** and **b** Answers will vary.

10G

Now you try

Example 14

a No **b** Equal **c** Equal **d** Yes **e** AAS
f Yes since $AE = CE$

Exercise 10G

1 SAS, AAS and RHS
2 **a** AC **b** BD **c** DB

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3 **a** Alternate angles in parallel lines
b Alternate angles in parallel lines
c Alternate angles in parallel lines
4 **a** Co-interior angles in parallel lines, $a = 110$
b Co-interior angles in parallel lines, $a = 52$
5 **a** T **b** F **c** F **d** T **e** F
f F **g** F **h** F **i** T **j** T
k F **l** F **m** T **n** T **o** T
p T **q** T
6 **a** AAS **b** RHS **c** SSS **d** SAS
e AAS **f** SSS
7 **a** ΔABD, ΔCDB **b** Equal **c** Equal
d BD **e** SSS
f Corresponding angles in congruent triangles.
8 **a** Yes (90°) **b** Yes **c** Yes
d Sas **e** Corresponding sides in congruent triangles.
9 **a** Equal (alternate angles in parallel lines)
b Equal (alternate angles in parallel lines)
c BD
d AAS
e They must be equal.
10 **a** $\angle DCE$
b $\angle CDE$
c There are no pairs of equal sides.
11 **a** SSS (3 equal sides)
b They are equal and add to 180° so each must be 90°.
c Since ΔQMN is isosceles and $\angle MQN$ is 90° then $\angle QMN = 45$.
12 **a** $AB = CB$, $AD = CD$ and BD is common, so $\Delta ABD \equiv \Delta CBD$ by SSS.
b $\Delta ABD \equiv \Delta CBD$ so $\angle DAB \equiv \angle DCB$
c $\Delta ABD \equiv \Delta CBD$ so $\angle ADB \equiv \angle CDB$
13 $\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$\angle BAE = \angle DCE$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\angle ABE \equiv \angle CDE$ (AAS)
$BE = DE$ and $AE = CE$ because corresponding sides on congruent triangles are equal.

10H

Now you try

Example 15

a (AB, HE), (BC, EF), (CD, FG), (DA, GH)
b $(\angle A, \angle H)$, $(\angle B\ \angle E)$, $(\angle C\ \angle F)$, $(\angle D\ \angle G)$
c 2
d $a = 130, x = 10, y = 6$

Example 16

a Similar (scale factor is 2) b Not similar

Exercise 10H

1 (A, J), (C, K), (F, H), (I, L)
2 **a** **i** $\angle D$ **ii** $\angle E$ **iii** $\angle F$
b **i** AB **ii** BC **iii** CA
c **i** 2 **ii** 2 **iii** 2
d Yes, all side ratios are equal and all interior angles are equal.
3 **a** **i** (AB, EF), (BC, FG), (CD, GH), (DA, HE)
ii $(\angle A, \angle E)$, $(\angle B, \angle F)$, $(\angle C, \angle G)$, $(\angle D, \angle H)$
iii 1.5
iv $a = 40, x = 4, y = 4.5$
b **i** (AB, DE), (BC, EF), (CA, FD)
ii $(\angle A, \angle D)$, $(\angle B, \angle E)$, $(\angle C, \angle F)$
iii 3
iv $x = 3$
4 **a** **i** (AB, EF), (BC, FG), (CD, GH), (DA, HE)
ii $(\angle A, \angle E)$, $(\angle B, \angle F)$, $(\angle C, G)$, $(\angle D, \angle H)$
iii 2
iv $a = 100, x = 2, y = 3$
b **i** (AB, FG), (BC, GH), (CD, HI), (DE, IJ), (EA, JF)
ii $(\angle A, \angle F)$, $(\angle B, \angle G)$, $(\angle C, \angle H)$, $(\angle D, \angle I)$, $(\angle E, \angle J)$
iii 2.5
iv $a = 115, x = 5$
5 **a** Yes, all ratios are 2.
b Yes, squares of different sizes.
c No, ratios are not equal.
d Yes, ratios are both 2.5 with equal angles.
6 **a** Yes, 2
b Yes, 2
c Yes, 4
d Yes, 4
7 360 cm
8 15 cm
9 3
10 **a** True, angles and side ratios will be equal.
b False, side ratios may be different.
c True, angles and side ratios will be equal.
d False, side ratios may be different.
e False, angles and side ratios may be different.
f False, side ratios and angles may be different.
g False, side ratios and angles may be different.
h False, side ratios and angles may be different.
i True, shape is always the same.
11 **a** No
b No, they can have different shapes.

10I

Now you try

Example 17

1 $\dfrac{AB}{DE} = \dfrac{6}{2} = 3$
$\angle B = \angle E$
$\dfrac{BC}{EF} = \dfrac{9}{3} = 3$
$\therefore \Delta ABC$ is similar to ΔDEF using SAS
2 $\angle A = \angle D = 90°$
$\dfrac{BC}{EF} = \dfrac{12}{6} = 2$
$\dfrac{AB}{DE} = \dfrac{5}{2.5} = 2$

$\therefore \Delta ABC$ is similar to ΔDEF using RHS.

Example 18

$x = 26, y = 5$

Exercise 10I

1 **a** $\Delta ABC \mid\mid\mid \Delta EFD$ **b** $\Delta ABC \mid\mid\mid \Delta FDE$
c $\Delta ABC \mid\mid\mid \Delta DEF$
d $\Delta ABC \mid\mid\mid \Delta DEF$ (order does not matter)
2 **a** AAA **b** SAS **c** RHS **d** SSS
3 **a** RHS
$\angle A = \angle D = 90°$
$\dfrac{EF}{BC} = \dfrac{10}{5} = 2$
$\dfrac{DE}{AB} = \dfrac{8}{4} = 2$
$\therefore \Delta ABC \sim \Delta DEF$
b SSS
$\dfrac{AB}{DE} = \dfrac{5}{2.5} = 2$
$\dfrac{BC}{EF} = \dfrac{11}{5.5} = 2$
$\dfrac{AC}{DF} = \dfrac{8}{4} = 2$
$\therefore \Delta ABC \sim \Delta DEF$

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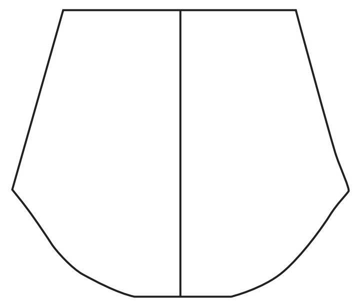
4 a SSS
$\frac{DE}{AB} = \frac{10}{5} = 2$
$\frac{DF}{AC} = \frac{24}{12} = 2$
$\frac{EF}{BC} = \frac{26}{13} = 2$
$\therefore \Delta ABC \sim \Delta DEF$
b AAA
$\angle A = \angle D$
$\angle B = \angle E$
$\therefore \Delta ABC \sim \Delta DEF$
c SAS
$\frac{DE}{AB} = \frac{10}{5} = 2$
$\angle D = \angle A$
$\frac{DF}{AC} = \frac{6}{3} = 2$
$\therefore \Delta ABC \sim \Delta DEF$
d RHS
$\angle D = \angle A = 90°$
$\frac{EF}{BC} = \frac{26}{13} = 2$
$\frac{DF}{AC} = \frac{22}{11} = 2$
$\therefore \Delta ABC \sim \Delta DEF$
e RHS
$\angle E = \angle B = 90°$
$\frac{DF}{AC} = \frac{4}{1} = 4$
$\frac{EF}{BC} = \frac{2}{0.5} = 4$
$\therefore \Delta ABC \sim \Delta DEF$
f SAS
$\frac{DE}{AB} = \frac{9}{6} = 1.5$
$\angle D = \angle A$
$\frac{DF}{AC} = \frac{18}{12} = 1.5$
$\therefore \Delta ABC \sim \Delta DEF$
g AAA
$\angle A = \angle D$
$\angle C = \angle F$
$\therefore \Delta ABC \sim \Delta DEF$
h SSS
$\frac{DE}{AB} = \frac{10}{4} = 2.5$
$\frac{DF}{AC} = \frac{15}{6} = 2.5$
$\frac{EF}{BC} = \frac{15}{6} = 2.5$
$\therefore \Delta ABC \sim \Delta DEF$
5 a $x = 8, y = 10$
b $x = 12, y = 6$
c $x = 15, y = 4$
d $x = 3, y = 15$
6 a 2.5
b Yes (SSS)
c 2.5
7 a Yes
b No
c No
d Yes
8 If two angles are known, then the third is automatically known using the angle sum of a triangle.
9 Using Pythagoras' theorem, $AC = 25$.
$\frac{DF}{AC} = \frac{50}{25} = 2$
$\frac{ED}{AB} = \frac{14}{7} = 2$
$\therefore \Delta ABC \sim \Delta DEF$ (RHS)
10 4 m
11 a Yes (AAA) b 2.5 c 15 m

Maths@Work: Fashion designer

1 a

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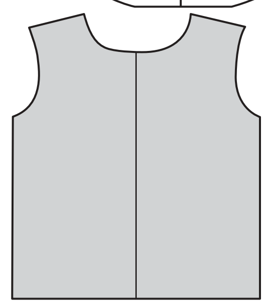
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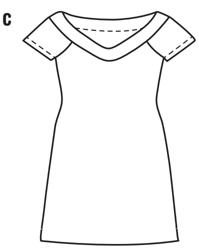
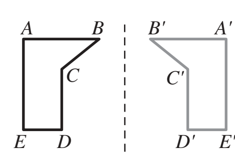


2 a Rotation by 120°, translation
b Rotation by 45°, reflection in the 8 axes of symmetry
c Reflection in the vertical and horizontal axes of symmetry
d Rotation by 30°, translation
3 A drawing congruent to the image in the question.
4 Answers will vary.

Puzzles and games

1 30
2 27
3 30 m
4 31
5 Yes, illustrates Pythagoras' theorem using areas.
6 $(3 - r) + (4 - r) = 5$, so $r = 1$

Checklist Answers

1


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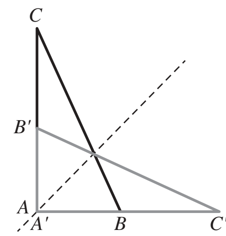

3 $(2, -3)$
4 $(3, -1)$
5 $(3, -3)$

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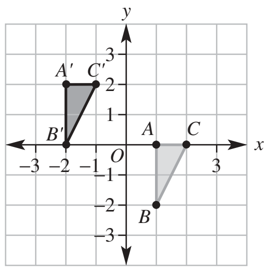
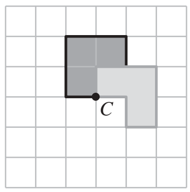
6


7 3

8


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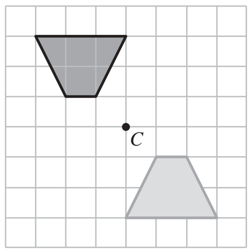
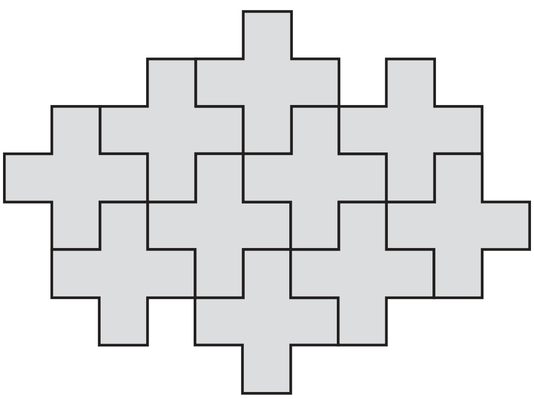
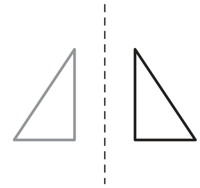


10 EFGH ≡ ABCD
11 ΔABC ≡ ΔDEF
12 RHS
13 Answers will vary Example:


14 (3.6.3.6)
15 DC = BA (given)
∠BDC = ∠DBA (alternative angles DC // BA)
∠DCA = ∠CAB (alternative angles DC // BA)
ΔDCE ≡ ΔBEA (AAS)
DE = EB & CE = AE (corresponding sides in congruent triangles)
∴ Diagonals of a parallelogram bisect each other.

16 (AB, EF), (BC, FG), (CD, GH), (DA, HE); (∠A, ∠E), (∠B, ∠F), (∠C, ∠G), (∠D, ∠H)
17 3; a = 50; x = 3; y = 2
18 Scale factors are not equal. Shapes are not similar.
19 $\frac{DE}{AB} = \frac{4}{2} = 2$ ∠B = ∠E = 50° $\frac{EF}{BC} = \frac{10}{5} = 2$
ΔABC is similar to ΔDEF (similarity SAS)
20 $x = 12$; $y = 3$

Short-answer questions

1 a

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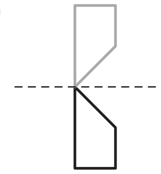
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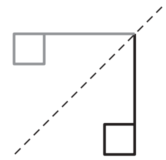
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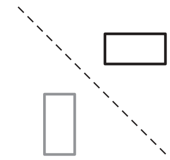
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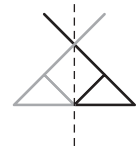
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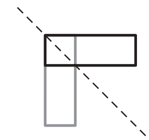
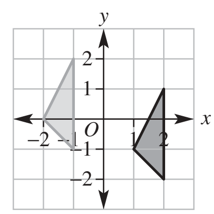


2 a $A'(1, -2)$, $B'(3, -4)$, $C'(0, -2)$
b $A'(-1, 2)$, $B'(-3, 4)$, $C'(0, 2)$
3 a 4 b 1 c 2 d 1 e 6
f 0
4 a (0, 3) b (2, −1) c (−3, 1) d (−3, −2)
5 a (1, 4) b (3, −6) c (−3, −7)
d (4, 6)
6 a


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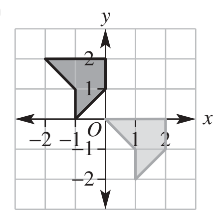

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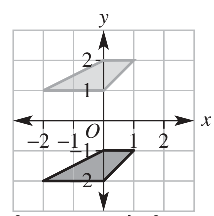
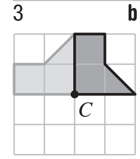


7 a 3 b 2 c No rotational symmetry
8 a


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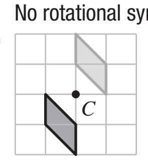

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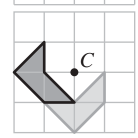

9 a i F ii G
b i EH ii FG
c i $\angle G$ ii $\angle E$
10 $\Delta ABC \equiv \Delta STU$
11 a RHS b SAS c SSS d AAS
12 a $x = 3,\ a = 25$ b $x = 5,\ a = 18$
13 (3.3.4.3.4)
14 a Yes – alternate angles in parallel lines
b Yes – alternate angles in parallel lines
c Yes – given
d AAS
e $AE = CE$ (matching sides of congruent triangles), $BE = DE$ (matching sides of congruent triangles), therefore AC and BD bisect each other.

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15 a $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$
b $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$
c 3
d $y = 1.5, a = 115, x = 3$

16 a Yes, SAS
b Yes, SSS
c Yes, AAA
d Yes, RHS

17 a $h = 6$
b $x = 2$
c $x = \frac{24}{11}$
d $d = 12.5$

Multiple-choice questions

1 B 2 D 3 C 4 A 5 B
6 E 7 A 8 E 9 D 10 D

Extended-response questions

1 a $A'(0, 1),\ B'(-2, 1),\ C'(-2, 4)$
b $A'(3, 1),\ B'(3, -1),\ C'(0, -1)$

2 a SSS
b $\angle VWU = \angle TWU$ and $\angle VWU + \angle TWU = 180°$ so $\angle VWU = \angle TWU = 90°$

Semester review 2

Ratios and rates

Short-answer questions

1 a 2 : 3 b 1 : 2 : 3 c 6 : 7
d 3 : 40 e 5 : 1 f 3 : 10

2 a 576 cm, 384 cm b \$1500, \$2500
c \$1.60, \$4, \$2.40

3 \$7750

4 60 000 cm, 600 m

5 a 12 mm/day b 3 goals/game
c 2 cents/g or \$0.02 /g or \$20/kg

6 \$2.27

7 85.6 km/h

Multiple-choice questions

1 C 2 B 3 C 4 C 5 D

Extended-response questions

1 a 16.5 km
b 742.5 km
c 6.1 L
d \$35.37
e 18 km

Equations and inequalities

Short-answer questions

1 a $w = 9$ b $m = 7$ c $x = 5$ d $a = 2$
e $w = 13$ f $x = 1$

2 a $x = 30$ b $q = 10$ c $p = 15$ d $x = 10$
e $r = 15$ f $a = 8$

3 a $x = 5$ b $k = 10$ c $r = 0$ d $z = 10$

4 6

5 a 20 b 2 c 7

6 a $x > 1$ b $x \le 2$ c $-1 < x \le 2$

7 a $x > -8$ b $x \le 3$ c $x \le 30$

Multiple-choice questions

1 C 2 B 3 D 4 C 5 B

Extended-response questions

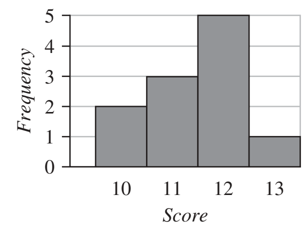
1 a $1500 + 5n$ b 100 books
c $20n$ d 50 books
e i \$1500 ii \$13 500 iii 100 books
f They make a \$750 loss

Probability and statistics

Short-answer questions

1 a i 13.5 ii 14 iii 8
b i 23 ii 18.5 iii 56
c i 10 ii 9.45 iii 15.7

2



3 a $\frac{7}{16}$ b $\frac{1}{16}$ c $\frac{15}{16}$ d $\frac{1}{2}$ e 0

4 a $\frac{3}{50}$ b $\frac{39}{50}$ c $\frac{1}{50}$ d $\frac{24}{25}$

5 a 18 b 6 c 8 d 4 e 16

6 a $\frac{1}{9}$ b $\frac{5}{9}$ c $\frac{1}{4}$ d $\frac{1}{12}$

Multiple-choice questions

1 A 2 A 3 D 4 C 5 B

Extended-response questions

1 a 18 b 78 c 78 d Group B

Linear relationships

Short-answer questions

1 a 1st b 2nd c 3rd d 4th

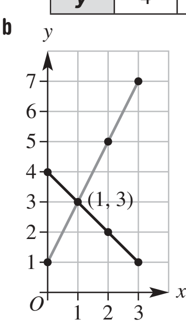
2 a i

x	0	1	2	3
y	1	3	5	7

ii

x	0	1	2	3
y	4	3	2	1

b



3 a i $x = 2$ ii $y = 3$ iii $y = \frac{3}{2}x$ iv $y = -x - 2$
b $x > 3$

4 i Positive ii Negative

5 a

t	0	1	2	3
d	0	80	160	240

Answers

SR2

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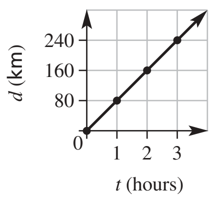
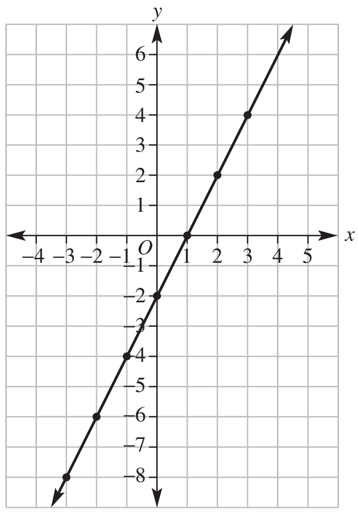
b



c 240 km

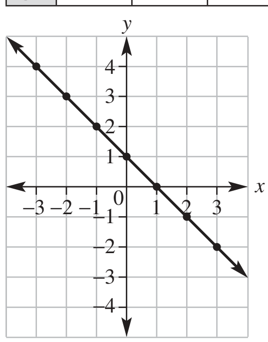
d 4 hours

6 a



x	−3	−2	−1	0	1	2	3
y	−8	−6	−4	−2	0	2	4

b



x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	−1	−2

Multiple-choice questions

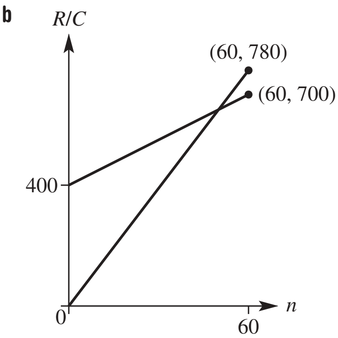
1 D **2** B **3** C **4** D **5** D

Extended-response questions

1 a

n	0	10	20	30	40	50	60
C	400	450	500	550	600	650	700
R	0	130	260	390	520	650	780

b



c (50, 650) **d** \$400

Transformation and congruence

Short-answer questions

1 a 0 **b** 2 **c** 2 **d** 1

2 a (2, 2) **b** (−1, −2)

3 a $A'(1, -1),\ B'(3, -1),\ C'(2, -3)$

b $A'(-3, -1),\ B'(-1, -1),\ C'(-2, 1)$

c $A'(1, -1),\ B'(1, -3),\ C'(3, -2)$

d $A'(-1, -1),\ B'(-3, -1),\ C'(-2, -3)$

4 a SSS **b** SAS **c** RHS **d** AAS

5 A, C

6 a Triangle AEC and triangle BDC (AAA) **b** C **c** 9

Multiple-choice questions

1 C **2** B **3** D **4** D **5** C

Extended-response questions

1 a Alternate angles in parallel lines.

b Alternate angles in parallel lines.

c AAS

d $\Delta ABE \equiv \Delta CDE$ so $AE = CE$ and $BE = DE$.

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