ESSENTIAL MATHEMATICS

UNITS 3 & 4

CAMBRIDGE SENIOR MATHEMATICS
FOR QUEENSLAND

NEIL CAPPS | LEANNE BUTLER | DEBORAH BURTON | CASSIE MCKENZIE

INCLUDES INTERACTIVE
TEXTBOOK POWERED BY
CAMBRIDGE HOTMATHS

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Online assessment practice in the Interactive Textbook and Online Teaching Suite

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Online appendices

Printable documents in the interactive textbook and online teaching suite:

A1.1 Glossary of terms

A1.2 Glossary of cognitive verbs

A2.1 Online guide to spreadsheets

A2.2 Online guide to the Desmos graphing calculator

A2.3 Links to online guides to using scientific calculators

A2.4 Guide to problem-solving and modelling, and complex unfamiliar questions

Note: *A printable copy of a formula sheet is available in the Interactive Textbook*

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About the authors

Neil Capps

Neil Capps is the Head of Mathematics at Glasshouse Christian College and has 29 years' experience in teaching. Neil has also taught in the State and Catholic school systems as well as in a variety of Christian schools. Though teaching mainly in Queensland, he taught for four years in Darwin and has also worked as a School Principal in northern New South Wales. Neil is passionate about mathematics education and has enjoyed the implementation of the new curriculum and encouraging both teachers and students to develop their love of mathematics.

Leanne Butler

Leanne Butler is Curriculum Leader of Mathematics at Downlands College in Toowoomba. She has 28 years' teaching experience across all levels from Years 7 to 12 at various schools across south-east Queensland and NSW. She has a keen interest in helping students overcome their apprehension towards mathematics by exploring how students learn mathematics and finding real-life relevance for simple and complex mathematical concepts.

Deborah Burton

Deborah Burton has taught across all age groups from kindergarten, primary, and secondary to adult education, specialising in mathematics. She has a passion for teaching mathematics, with the fruits of her labour being well rewarded with her students performing well academically. She believes that learning is a life-long process for teachers, as well as for students.

Cassie McKenzie

Cassie McKenzie has taught mathematics for 12 years in South Australia and Queensland. Cassie initially struggled with mathematics in high school before the combination of a brilliant teacher and excellent resources meant that she went on to excel in the subject during senior school and university. Cassie is passionate about instilling a love for mathematics by assisting students to gain confidence and find enjoyment through the challenge of mathematics.

Introduction and overview

Cambridge Senior Mathematics for Queensland Essential Mathematics Units 3&4 has been written from the ground up to the QCAA syllabus to be implemented in Year 12 from 2020. Its four components — the print book, downloadable PDF textbook, online Interactive Textbook and Online Teaching Resource* — contain a great range of resources, including worked solutions and revision of Year 10 material, available to schools in a single package at one convenient price.
**The Online Teaching Resource is included with class adoptions, conditions apply.*

► Overview of the print textbook (shown on the page opposite)

1. **Syllabus references** are listed at the beginning of each chapter.
2. **Pre-tests** provide a check of requisite knowledge and skills. Those who used the series in Year 11 will still have access to their Units 1&2 Interactive Textbook to use for revision.
3. **Learning goals** based on the syllabus are given for each section.
4. **What you need to know** boxes list important concepts and principles in concise and accessible format.
5. **Worked examples** detail thinking and the solution in a logical sequence, and are linked to exercises. Video versions are provided in the interactive textbook to encourage independent learning.
6. Exercises are divided into:
   - **Fundamentals –** integrating the Fundamental topic: Calculations throughout the topic, as required by the syllabus.
   - **Applications**: questions in real-life contexts, differentiated into degree-of-difficulty categories as indicated by a bar:

SF: 5–10	CF: 11–15	CU: 16, 17
Simple Familiar	**Complex Familiar**	**Complex Unfamiliar**

   An exercise that covers only complex subject matter has only CF and CU questions; an exercise that covers only simple subject matter has only SF questions. Some exercises cover both simple and complex subject matter, so have questions in all three categories.

Applications questions are also differentiated into **learning-style** and **assessment-style**, the latter category being marked by a star ★. Learning-style questions are scaffolded with steps that guide students to the answer, while assessment-style questions are unscaffolded and are suitable models for examinations or other assessment items. Questions requiring technology other than scientific calculators are learning-style.

7 **Problem-solving and modelling tasks** are provided for every chapter, set out in stages with a flowchart based on the approach to problem-solving and mathematical modelling used by QCAA.

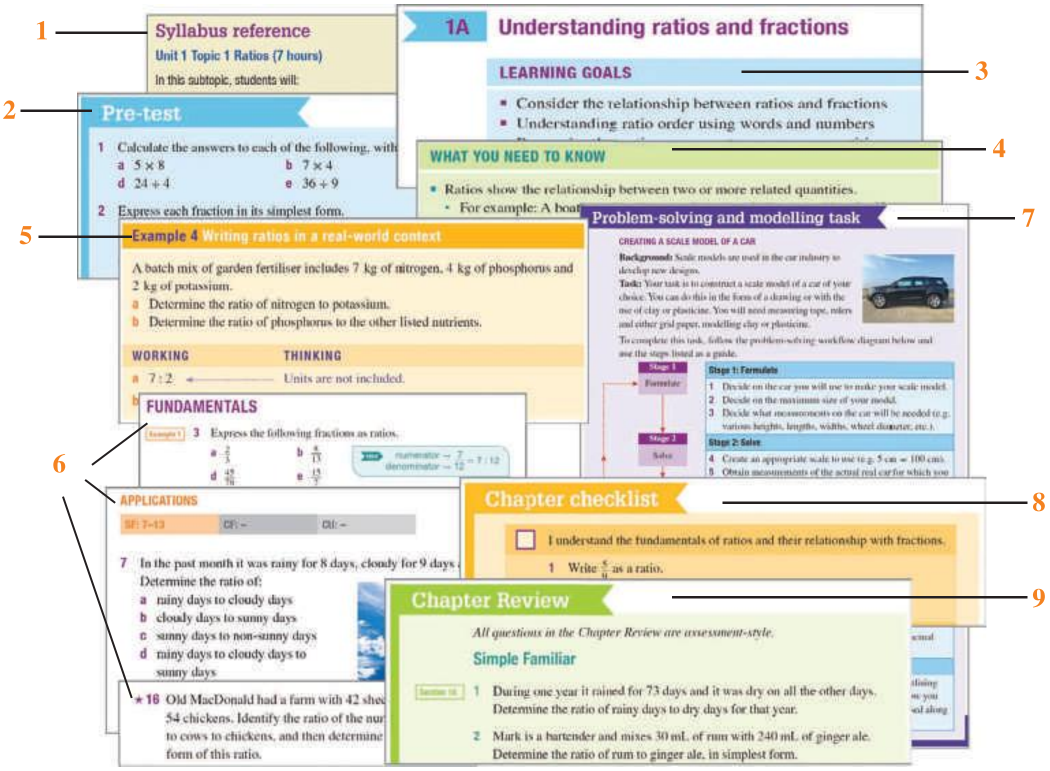
8 **Chapter checklists** comprise short questions assessing achievement of learning goals.

9 **Review** exercises contain only assessment-style questions organised under degree-of-difficulty subheadings.

10 Examples and questions using **spreadsheets** are integrated throughout the text, with accompanying Excel files in the Interactive Textbook.

11 Additional linked resources in the Interactive Textbook and Online Teaching Suite such as practice assessment items are indicated by icons or notes in the text.

Print book features — numbers refer to points above



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► Downloadable PDF textbook

12 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

13 PDF annotation and search features are enabled.

► Overview of the interactive textbook (shown on the page opposite)

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate digital-only product.

14 The material is formatted for on screen use with a convenient and easy-to-use navigation system and links to all resources.

15 **Definitions** pop up for key terms in the text and are also provided in a printable online **glossary**, while the HOTmaths dictionary is also accessible.

16 Examples have **video versions** to encourage independent learning.

17 The **Desmos scientific calculator**, graphics calculator and geometry tool are also available for students to use for their own calculations and exploration.

18 **Spreadsheets** are provided in Excel format.

19 **Quick quizzes** containing automarked multiple-choice questions enable students to check their understanding.

20 Since students can access their previous year's Interactive Textbook, they can use it for revision.

21 The new **Workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing.

22 The new **self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite.

23 **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher.

24 **Practice assessment** items are provided in downloadable PDF and Word files.

25 **Online appendices** provide a glossary of terms and cognitive verbs, guides to spreadsheets and the Desmos graphing calculator, links to scientific calculator guides, and guides to problem-solving, modelling and approaching complex unfamiliar problems.

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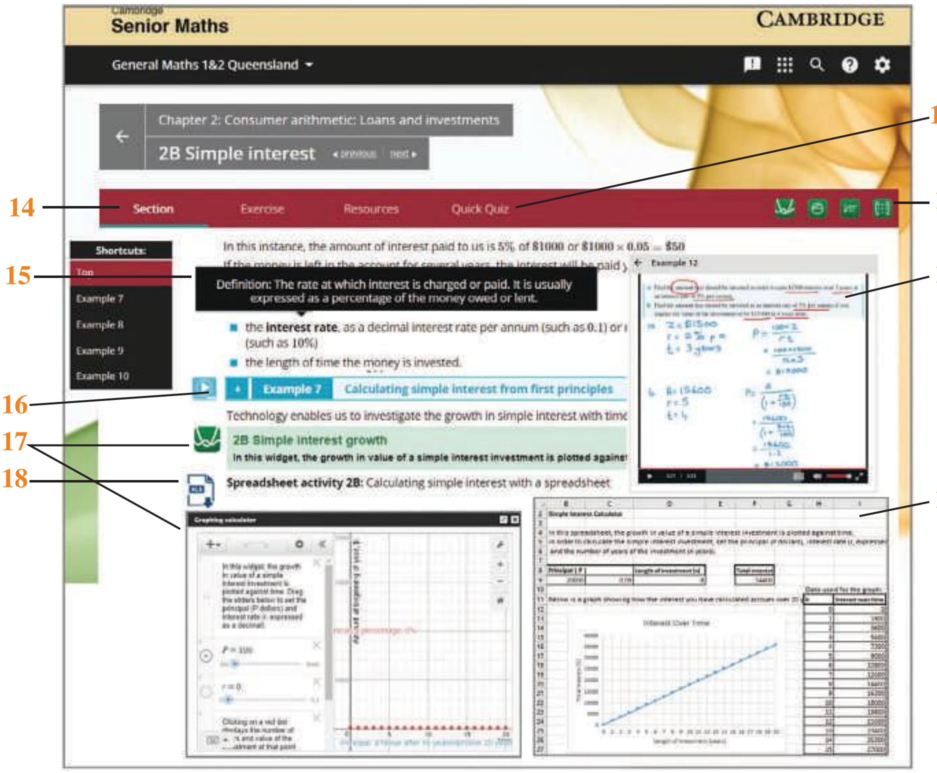
Numbers refer to the descriptions on the opposite page. HOTmaths platform features are updated regularly. Screenshots below show the General Mathematics title in the series — not all the features of Essential Mathematics are shown.



In this instance, the amount of interest paid to us is 5% of \$1000 or \$1000 × 0.05 = \$50

If the money is left in the account for several years, the interest will be paid y

Definition: The rate at which interest is charged or paid. It is usually expressed as a percentage of the money owed or lent.

- the **interest rate**, as a decimal interest rate per annum (such as 0.1) or
(such as 10%)
- the length of time the money is invested.

Example 7 Calculating simple interest from first principles

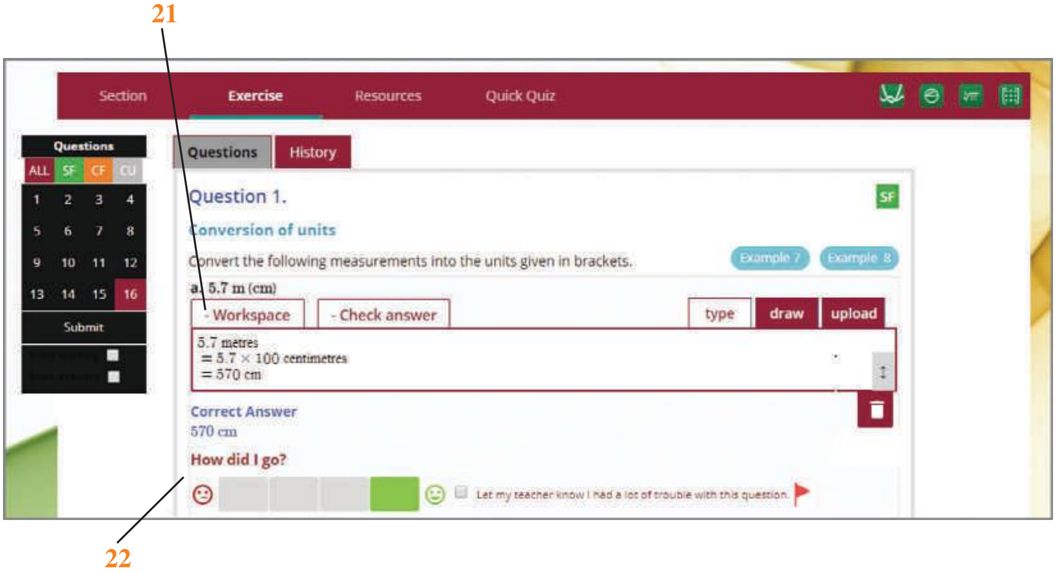
Technology enables us to investigate the growth in simple interest with time

2B Simple interest growth

In this widget, the growth in value of a simple interest investment is plotted against

Spreadsheet activity 2B: Calculating simple interest with a spreadsheet

Workspaces and self-assessment



Question 1.

Conversion of units

Convert the following measurements into the units given in brackets.

a 5.7 m (cm)

5.7 metres
= 5.7 × 100 centimetres
= 570 cm

Correct Answer
570 cm

☐ Let my teacher know I had a lot of trouble with this question.

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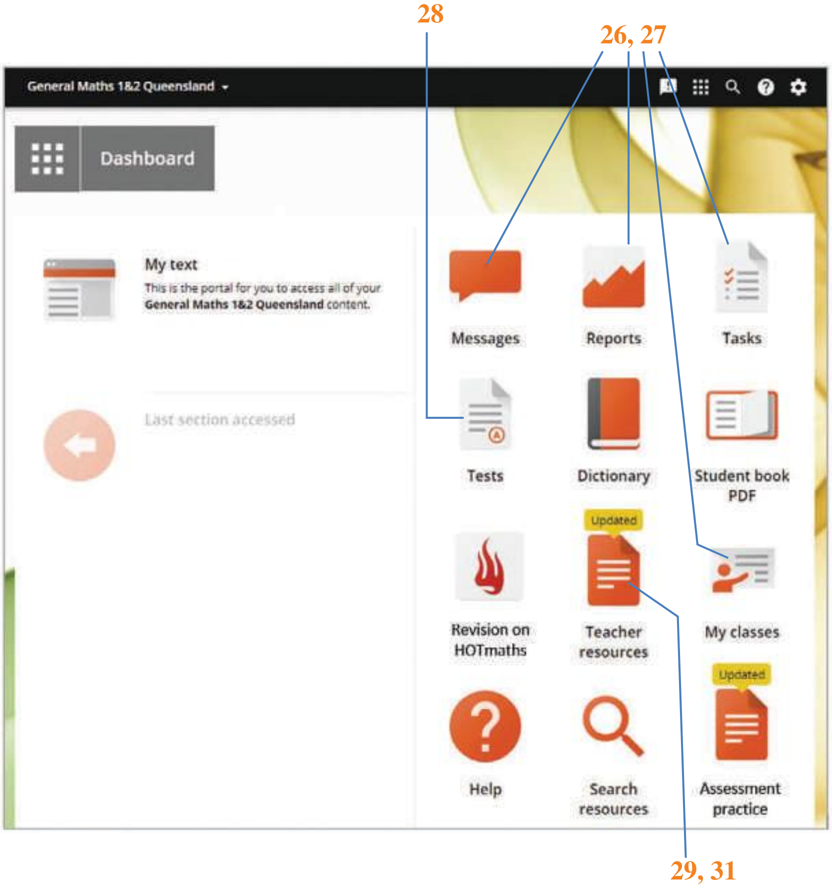
▶ Online teaching suite (shown below)

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the assets and resources are in one place for easy access. The features include:

26 The HOTmaths learning management system with class and student analytics and reports, and communication tools.
27 Teacher's view of a student's working and self-assessment.
28 A HOTmaths-style test generator.
29 Chapter test worksheets as PDFs and editable Word documents.
30 Assessment practice items as PDFs and editable Word documents.
31 Editable curriculum grids and teaching programs.

Online teaching suite powered by the hotmaths platform

Numbers refer to the descriptions on the opposite page. HOTmaths platform features are updated regularly. Screenshots below show the General Mathematics title in the series.



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1

Geometry and linear measure

Maths for an executive assistant: Jenny Marie

Jenny Marie completed a traineeship in Business Administration and started her career as a receptionist, before moving on to become an Executive Assistant in the wine industry.

Tell us a bit about your job. What does a typical day look like?

As an Executive Assistant, my day involves diary and travel management, invoice reconciliation, capital project planning and data analysis. I spend a majority of my day using Excel, not just for data analysis but also for tracking invoices and setting up capital plans for the next five years.

I enjoy the variation of my role, there is always something new to learn. I enjoy using Excel and being able to provide correct information to the General Manager, who is able to use this for business planning decisions.

What maths did you study at school?

I studied Business Maths (equivalent to Essential Maths) at school. I didn't enjoy maths but having realistic applications to life has definitely been beneficial.

How do you use maths in your job?

I use maths on a daily basis in my career. Data analysis in Excel is a big part of my role. I am able to provide the General Manager with our intake plans for the next five years, showing regional and varietal differences, and ensure we have enough capacity to process those volumes by converting the tonnes into litres.

I also analyse our past intake performance to highlight vineyards where we need to create capital plans for redevelopments to meet the best outcomes for the business. These capital projects are also reviewed to ensure they are meeting the required targets.

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In this chapter

Syllabus reference

Unit 3 Topic 1 Measurement

Geometry (3 hours)

In this sub-topic, students will:

- recognise the properties of common two-dimensional geometric shapes, including squares, rectangles and triangles, and three-dimensional solids, including cubes, rectangular-based prisms and triangular-based prisms
- interpret different forms of two-dimensional representations of three-dimensional solids, including nets of cubes, rectangular-based prisms and triangular-based prisms **[complex]**.

Linear measure (5 hours)

In this sub-topic, students will:

- use metric units of length (millimetres, centimetres, metres, kilometres), their abbreviations (mm, cm, m, km), conversions between them, and appropriate levels of accuracy and choice of units
- estimate lengths
- calculate perimeters of familiar shapes, including triangles, squares, rectangles, polygons, circles and arc lengths
- calculate perimeters of familiar composite shapes **[complex]**.

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Pre-test

1 Determine the number of:

a sides in a rectangle
b corners in a rectangle
c edges in a cube
d corners in a cube

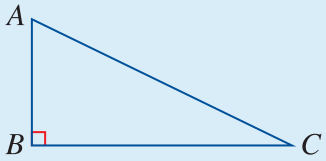
Hint Drawing a diagram may help you to determine the properties or features of a shape or solid.

2 Identify:

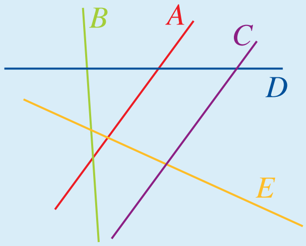
a the unit of measure for angles
b the number of angles in a triangle

3 Identify:

a the right angle in the diagram shown
b the size of a right angle in degrees



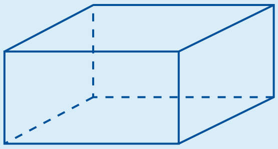
4 Determine which of the following lines are parallel lines.



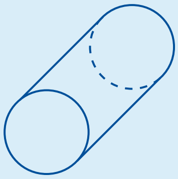
5 For each 3D solid:

i count the number of faces
ii name the 2D shape of each face

a



b



A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

1A Recognising common 2D geometric shapes and 3D solids

LEARNING GOALS

- Identify the names and properties of 2D shapes including:
  - squares
  - rectangles
  - triangles
- Identify the names and properties of 3D solids including:
  - cubes
  - rectangular prisms
  - triangular prisms

Why is it essential to identify the names and properties of various shapes?

Shapes and solids are used and seen everywhere. They can be found in manufactured items such as cardboard and jars. Shapes and solids can also be found in nature such as starfish and tree trunks. It is important to understand the properties of shapes when learning measurement as the properties can be used to describe and create shapes. The properties of shapes can be used in careers such as landscaping, building and engineering.


Modern machines can create extremely complex shapes.

WHAT YOU NEED TO KNOW

- **Two-dimensional (2D)** shapes are flat. They have only two dimensions such as length and width. Examples of 2D shapes include square, rectangle, triangle, trapezium, parallelogram, rhombus, kite and hexagon.
- **Three-dimensional (3D)** solids include a third dimension of depth or height. Examples of 3D solids include sphere, cone, cylinder, cube, rectangular prism and triangular prism.

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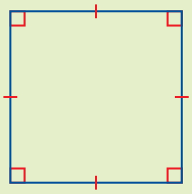
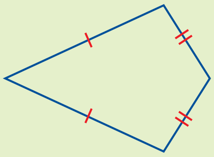
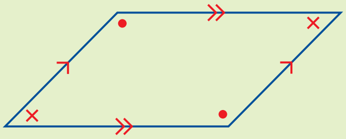
- **Properties** are the description of the shape or solid. The properties of 2D shapes include the number of **sides**, the number of **vertices**, whether the sides are equal, the size of **angles**, whether the angles are equal, and whether the sides are **parallel**. The properties of 3D solids include the number of **faces, edges** and vertices.
- Markings to show properties of 2D shapes
  - Boxed corners represent an angle of 90°. Small ticks on the sides represent equal side lengths.



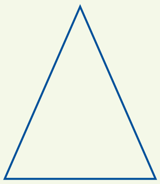
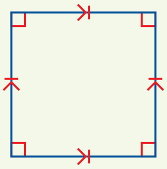
  - Double markings show another set of equal sides. These sides are not the same length as the single marked sides.



  - Arrows represent parallel lines. Double arrows show another set of parallel lines. Small crosses, dots and arcs can be used to show equal angles.



- A **vertex** (plural: vertices) is an angular point on a shape or solid. It is where two or more lines or edges meet.
- 2D shape properties

Shape	**Properties**
Triangle 	3 sides 3 vertices All angles add to 180°
Square 	4 sides 4 vertices All sides are equal All angles are 90° All angles add to 360° Two pairs of parallel sides

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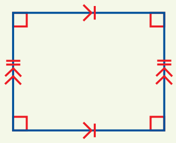
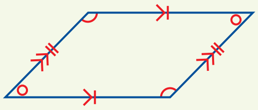
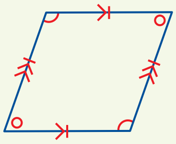
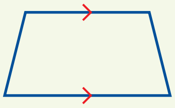
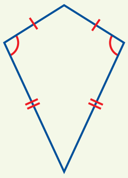
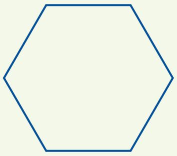
Shape	**Properties**
Rectangle 	4 sides 4 vertices All angles are 90° All angles add to 360° Opposite sides are parallel Opposite sides are equal
Parallelogram 	4 sides 4 vertices All angles add to 360° Opposite sides are parallel Opposite sides are equal Opposite angles are equal
Rhombus 	4 sides 4 vertices All angles add to 360° All sides are equal Opposite angles are equal Opposite sides are parallel
Trapezium 	4 sides 4 vertices All angles add to 360° One pair of parallel sides
Kite 	4 sides 4 vertices All angles add to 360° Adjacent sides are equal One pair of opposite angles are equal
Hexagon 	6 sides 6 vertices All sides equal in a regular hexagon All angles equal in a regular hexagon

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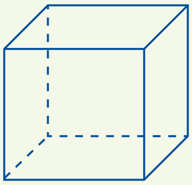
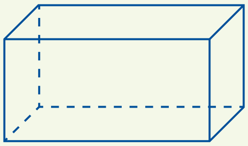
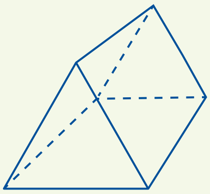
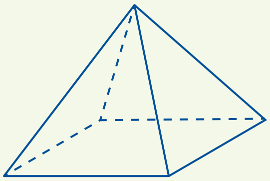
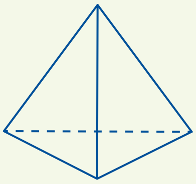
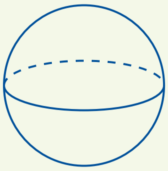
- 3D solid properties

Solid	Properties
Cube 	6 faces 8 vertices 12 edges All faces are squares
Rectangular prism 	6 faces 8 vertices 12 edges All faces are rectangles
Triangular prism 	5 faces 6 vertices 9 edges Three faces are rectangles Two faces are triangles
Square-based and rectangular-based pyramid 	5 faces 8 edges 5 vertices Base is either a square or a rectangle Four faces are triangles
Triangular-based pyramid (tetrahedron) 	4 faces 4 vertices 6 edges Four faces are all triangles
Sphere 	0 faces No faces, vertices nor edges An evenly curved surface Perfectly symmetrical

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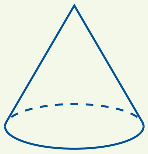
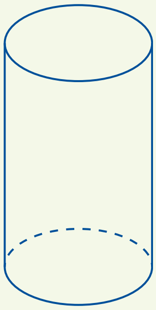
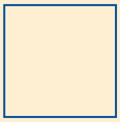
Solid	**Properties**
Cone 	0 faces No vertices nor straight edges One flat circular surface with no straight edge One curved surface around the circle
Cylinder 	0 faces No vertices nor straight edges Two flat, circular surfaces with no straight edges One curved surface that flattens to a rectangle

Example 1 Identifying the properties of 2D shapes

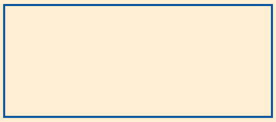
Complete the following for each of the shapes shown.

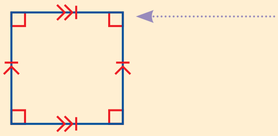
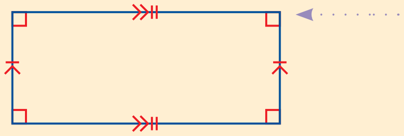
- **i** Identify the name of the shape.
- **ii** Draw the shape.
- **iii** Mark in its properties.

a



b



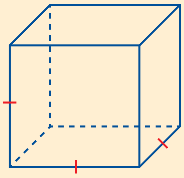
WORKING	**THINKING**
a i Square	
ii, iii 	All angles are 90 degrees. All sides are equal. Two pairs of parallel sides.
b i Rectangle	
ii, iii 	All angles are 90 degrees. Two pairs of parallel sides. Opposite sides are equal.

Example 2 Identifying the properties of 3D solids

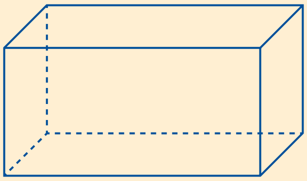
Complete the following for each of the solids shown.

- i Identify the name of the solid.
- ii Identify the number of faces, edges and vertices.

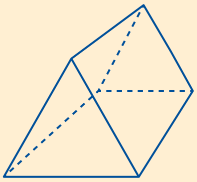
a



b



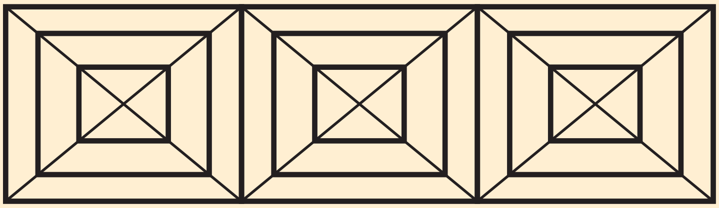
c



WORKING	THINKING
a i Cube	Faces – Count the number of flat sides on each solid.
ii 6 faces, 12 edges and 8 vertices	
b i Rectangular prism	Edges – Count the number of edges (where two faces meet) on each solid.
ii 6 faces, 12 edges and 8 vertices	
c i Triangular prism	Vertices – Count the number of angular points on each solid.
ii 5 faces, 9 edges and 6 vertices	

Example 3 Identifying 2D shapes in a real-world context

Valerie has designed a new fence that she would like to place along the front of her house. Identify the 2D shapes in each panel of the fence shown.



WORKING	THINKING
Triangles Trapeziums Rectangles	Look for shapes that are common 2D shapes. Triangles are in the centre of the fence panels. Each panel is a rectangle and the trapeziums are between the triangles and rectangles in each frame.

Exercise 1A

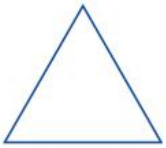
FUNDAMENTALS

1 Identify each 2D shape **a–i** with the correct name from the list **A–I**.

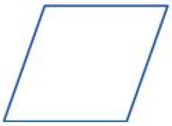
a

b

c



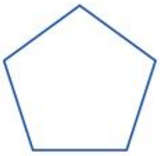
d



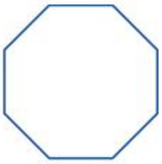
e

f

g



h



i

A parallelogram
D rectangle
G trapezium
B hexagon
E rhombus
H pentagon
C triangle
F octagon
I square

2 Copy the table and tick the boxes to show the correct properties for each of the following shapes.

Property	**Square**	**Rectangle**	**Parallelogram**	**Trapezium**	**Triangle**	**Rhombus**	**Kite**
Four sides							
Three sides							
All sides are equal length							
All angles measure 90°							
Not all angles measure 90°							
Two pairs of parallel sides only							
Two pairs of parallel sides of different lengths							

Example 1

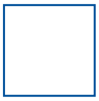
3 Complete the following for each of the shapes shown.

i Identify the name of the shape.

ii Draw the shape and mark in its properties.

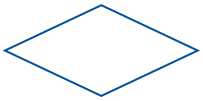
iii Describe the properties of each shape.

a

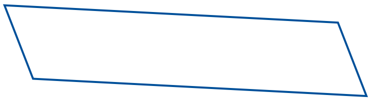


b

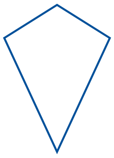
c



d

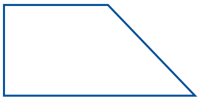


e

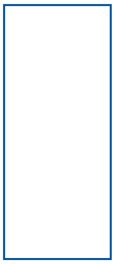


f

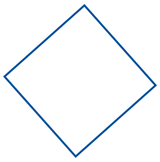
g



h

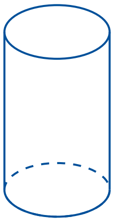


i

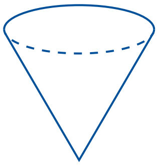


4 Identify each 3D solid **a–g** with the correct name from the list **A–G**.

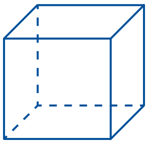
a



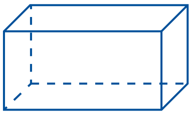
b



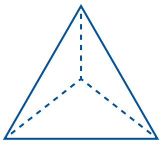
c



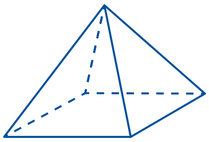
d



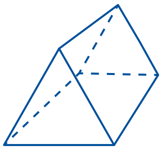
e



f



g



A cube
B triangular prism
C rectangular prism
D triangular-based pyramid (tetrahedron)
E square-based pyramid
F cone
G cylinder

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5 Determine the missing words in the following sentences.

a The flat surface on a 3D solid is called the ________.

b The vertex or vertices describes the ___________ of a 3D solid.

c A side of a 3D solid where two faces meet is called the __________.

Example 2 **6** Using the properties of 3D solids, determine the blank spaces in the following table.

Name of solid	Faces	Edges	Vertices
	6	12	8
Triangular prism			
	5	8	5
Triangular-based pyramid			
Pentagonal prism			
Hexagonal prism			

APPLICATIONS

SF: 7–14 | CF: – | CU: –

Example 3 ★**7** Cubism was a revolutionary style of art that was created by Pablo Picasso and Georges Braque around 1907. It is typically characterised by the use of geometric planes and shapes, similar to the image shown. Identify and create a list of the various 2D shapes within the image.

★**8** Geometric animal art is currently quite popular. It is often used in graphic design as well as in drawings and paintings. This involves using a variety of 2D shapes to create an animal. In the image shown, identify the two main shapes that are used to draw the flamingos.

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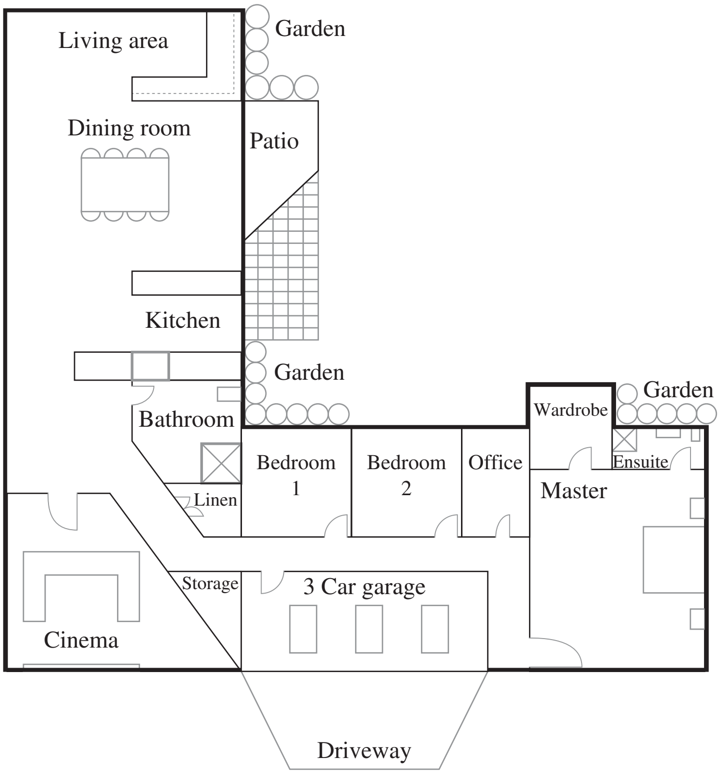
★**9** Sophia has designed the house plan shown. Identify as many 2D shapes as possible and explain what each shape represents in the house.

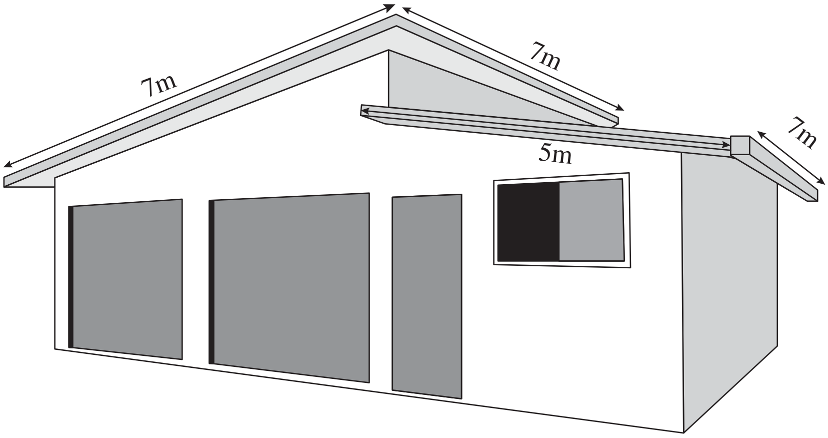


★**10** Ella has just built a new shed for her dance studio.

a Identify the 2D shapes that make up the design of the front of her shed.

b Identify the 2D shapes that make up the design of the sides of her shed.

c Use the roof measurements to identify the 2D shapes that make up the two separate roof designs.



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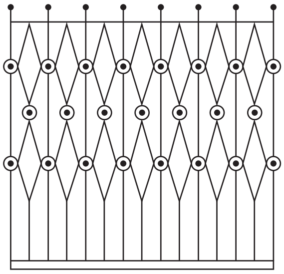
★**11** Lyn has designed a new gate that she would like to have made. Identify the various 2D shapes that she has used.



★**12** Billy and Alana are looking to buy a block of land in a new housing estate. Each block has been numbered in the plan shown below. Identify which blocks are shaped like a:

a rectangle
b parallelogram
c trapezium

Source AV Jennings

★**13** Identify the name of the 3D solid that is most similar to the following real-life objects.

a

b

c

d

e

f

g

h

i

★**14** Identify the name of the 3D solid that is most similar to these landmarks.

Eiffel Tower in Paris

Pyramids in Egypt

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1B Investigating nets of 3D solids COMPLEX

LEARNING GOALS

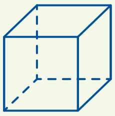
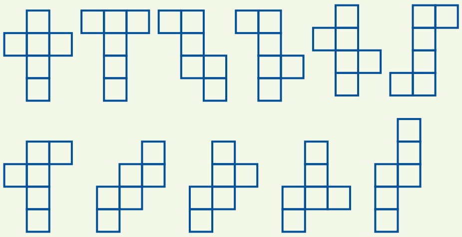
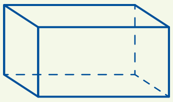
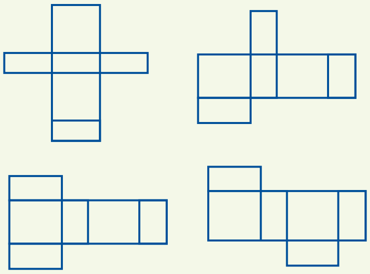
- Interpret various 2D representations of 3D solids in the form of a net, including:
  - cubes
  - rectangular prisms
  - triangular prisms

Why is it essential to interpret 2D representations of 3D solids?

Creating a 2D form of a 3D solid helps us to communicate the exact dimensions and shapes required to create the solid. The concept of designing and creating an object in a solid state requires good communication to the manufacturer, and the best way for this to take place is in the form of a 2D plan. It particularly applies to careers in architecture, manufacturing and design. The 2D design of a 3D solid is known as a net.

WHAT YOU NEED TO KNOW

- A **net** is a flat shape (2D) that can be folded up into a 3D solid. A 3D solid can have more than one type of net.

3D solid	3D image	2D net representations
Cube		
Rectangular prism		

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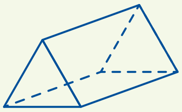
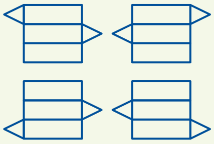
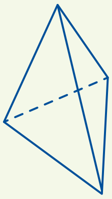
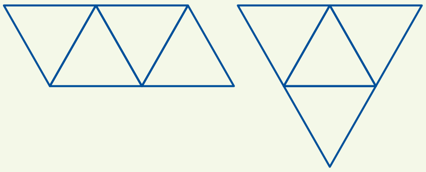
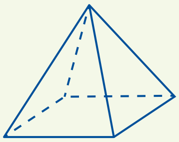
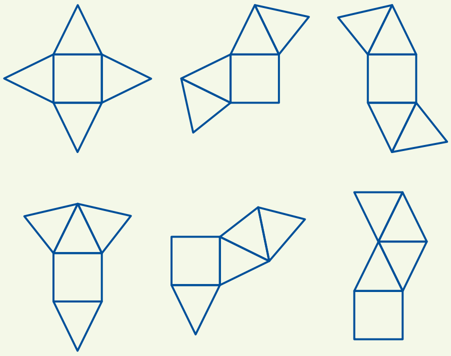
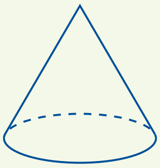
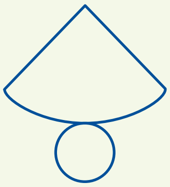
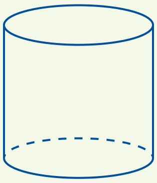
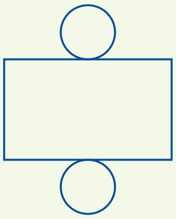
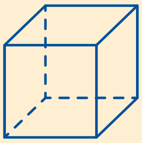
3D solid	3D image	2D net representations
Triangular prism		
Triangular-based pyramid		
Square-based pyramid		
Cone		
Cylinder		

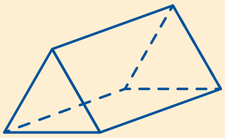
Example 4 Constructing a 2D net of a 3D solid

Construct a net that represents the following 3D solids.

a

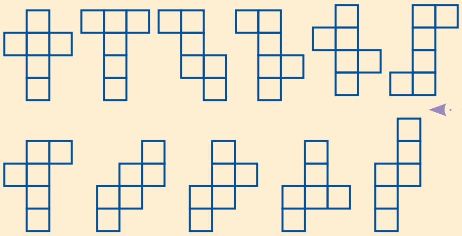


b



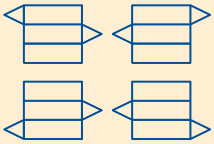
WORKING **THINKING**

a



There are many possible solutions to the design of both nets, all of which are accurate. Check that you have the correct number of faces, edges and vertices to form the solid.

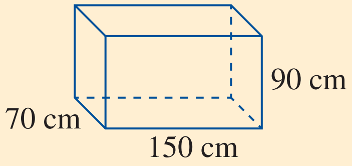
b



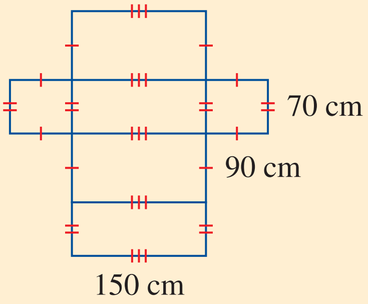
Example 5 Creating a 2D net from a 3D object

Kenji has purchased a rectangular television cabinet that is 90 cm high, 70 cm deep and 150 cm long. Construct the net of this cabinet, showing the measurements.

WORKING **THINKING**



First draw a sketch of the 3D object with the dimensions.

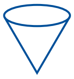
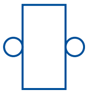
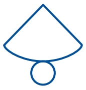
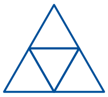


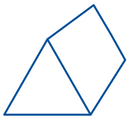
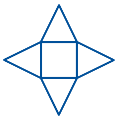
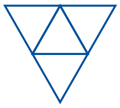
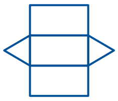
Draw the net of a rectangular-based prism. The net includes 2 rectangles with length 150 cm and width 90 cm. The net includes 2 rectangles with length 150 cm and width 70 cm. The net also includes 2 rectangles with a length of 90 cm and a width of 70 cm.

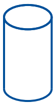
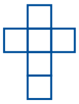
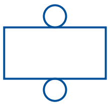
Exercise 1B

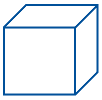
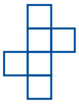
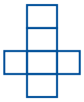
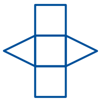
FUNDAMENTALS

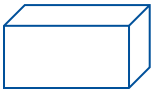
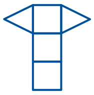
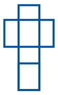
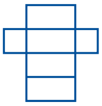
1 Determine the correct net (**A**, **B** or **C**) for each of the following solids.

a

A

B

C


b

A

B

C


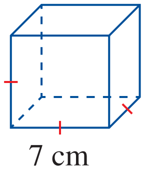
c

A

B

C

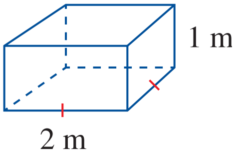

d

A

B

C


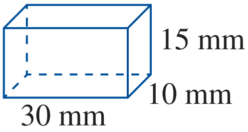
e

A

B

C


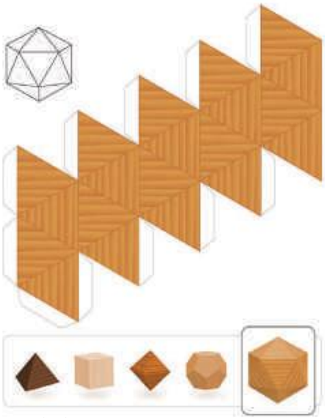
Example 4

2 Draw two different nets for each of the following 3D solids, ensuring that you include the measurements.

a


b


c




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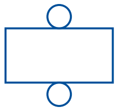
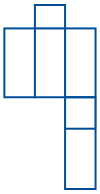
★**3** Match the following nets that are most similar to each of the real-life items in Exercise 1A Q13 (there may be more than one real-life object per net).

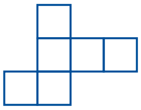
a



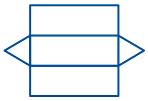
b



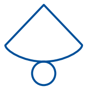
c



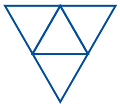
d



e



f



APPLICATIONS

SF: –	CF: 4–5	CU: 6

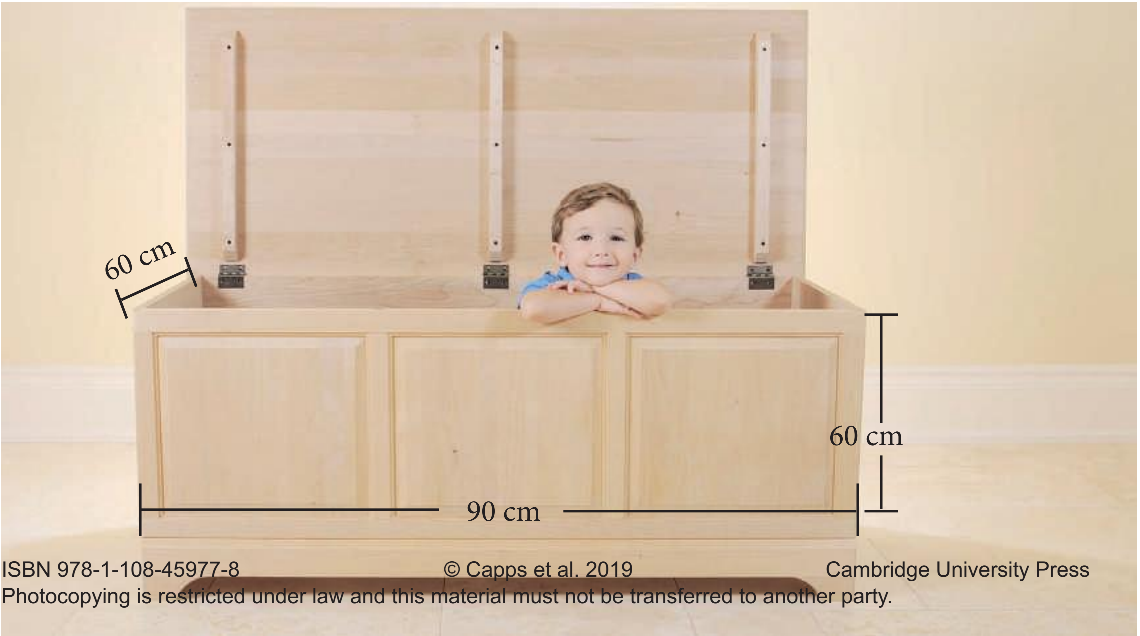
Example 5 ★**4** Sally has purchased a new fridge and it was delivered in a large box. Sally has cut the edges and flattened the box to form a net. Construct a diagram to identify what the net would look like. Ignore the extra flaps that cardboard boxes usually have.

★**5** Ahmed is planning to construct a large cube as part of a sculpture that he is creating as a display for the front of a hotel. He will be making the cube out of sheet metal. In order to make the cube, he must first cut out the net of the cube. He will then fold the sheet metal into a cube and weld it together.

a Design and draw a net of the cube.

b Identify how many folds Ahmed will need to make in order to construct the cube.

★**6** Josh has just purchased a storage box for his son's bedroom. The box has come in a flat pack. The first step on the assembly instructions is to lay out the timber pieces in the form of a net. Draw the net design, including the measurements of the box.



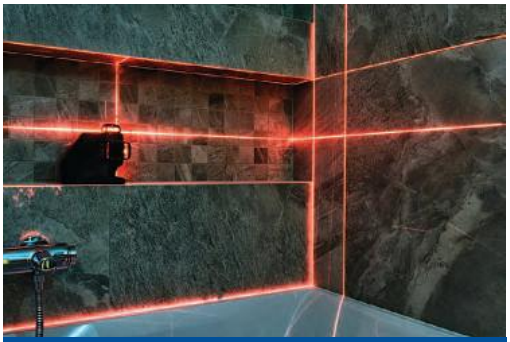
1C Using and converting linear measurements and estimating lengths

LEARNING GOALS

- Use the abbreviations of mm, cm, m, km to represent units of length
- Convert between units of length
- Identify the most appropriate unit of length for measurement
- Estimate the length of objects

Why is it essential to understand units of measure?

Understanding units of measure and the conversion between them is essential as it is not only used in many jobs but also in daily life. The distance people drive or walk per day is often calculated for either working out the best route or for calculating the number of calories burned. Measurements are also used in many industries such as construction, town planning, fashion and sports.



Modern measuring tools include lasers, being used here to check measurements in a bathroom.

WHAT YOU NEED TO KNOW

- **Units of length** include millimetres, centimetres, metres and kilometres, and these can be abbreviated to mm, cm, m and km.
- **Millimetres** are used for small objects of measure, such as an insect or a bolt. They are also used in scale drawings or in construction and manufacturing industries (e.g. to measure lengths of timber and pipes). Anything greater than 2000 mm in these industries is usually measured in metres. For example, a hardware store uses millimetres to identify the width or thickness of timber, pipes, posts and rods, but uses metres to identify and measure the lengths required of each item.
- **Centimetres** are often used for medium-sized objects such as the length of a desk or the height of a child.
- **Metres** are typically used for larger objects such as the length of a room (e.g. the real estate industry use metres but the building industry uses millimetres) or the height of a building.

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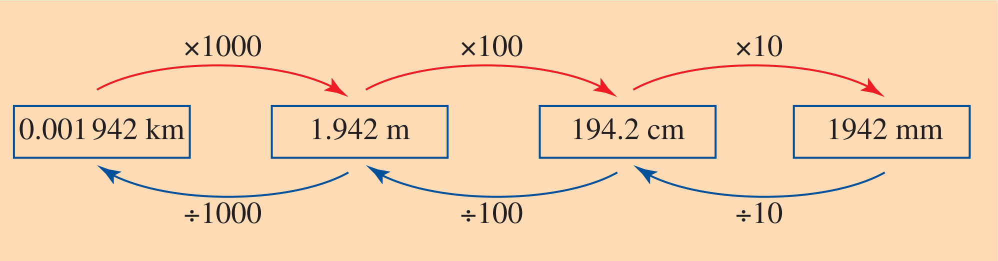
- **Kilometres** are used when calculating long distances (e.g. the distance for a cross-country run or the distance from one city to the next).
- When solving a problem using units of length, ensure that the measurements are in the same units.
- To convert between units:
  - use multiplication when converting from a larger unit to a smaller unit (e.g. converting from km to m)
  - use division when converting from a smaller unit to a larger unit (e.g. converting from cm to m)
- Refer to the following conversion chart when solving problems that require conversion.



- **Estimating** lengths is the application of your knowledge and skills of measuring lengths to approximately guess the length of an object. Estimation mainly takes place through physical, mental and comparative techniques.
  - Physical – knowing the length of a hand span or walking stride can help to estimate the length of an object. For example, if an adult’s stride is around 1 metre, then the length of a room can be estimated based on the number of steps.
  - Mental – recalling the approximate length of an item and comparing this knowledge to a similar item that is being estimated. For example, the recall of the length of a ruler being 30 cm. This knowledge can then help to estimate the length of a desk.
  - Comparative – knowing the height of a particular item and using this measurement to compare the height to another object nearby. For example, the height of a house may be used to estimate the height of a nearby tree.

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Example 6 Converting between units of measurement

Convert the following measurements into the units given in brackets.

a 6.5 km (m)
b 25 mm (cm)
c 0.07 m (mm)

WORKING	THINKING
a $6.5\text{ km} \times 1000 = 6500\text{ m}$	Converting from a larger unit to a smaller unit so multiply. There are 1000 m in each km.
b $25\text{ mm} \div 10 = 2.5\text{ cm}$	Converting from a smaller unit to a larger unit so divide. There are 10 mm in each cm.
c $0.07\text{ m} \times 100 \times 10 = 70\text{ mm}$	Converting from a larger unit to a smaller unit twice so multiply. There are 100 cm in each m and 10 mm in each cm.

Example 7 Applying metric units of length

Blake works as a costume designer and he is making costumes for a dance concert. He has worked out that each dancer will need 82 cm of material. There are 8 dancers in the troupe. Determine how many metres of material Blake will need to purchase.

WORKING	THINKING
$82\text{ cm} \times 8 = 656\text{ cm}$	Calculate how many centimetres of material Blake will require by multiplying the length of material by the number of dancers.
$656\text{ cm} \div 100 = 6.56\text{ m}$	Converting from a smaller unit to a larger unit so divide by 100 as there are 100 cm in each m.
Blake will need to purchase 6.56 m of material.	Communicate your answer in a sentence.

Example 8 Estimating lengths

Estimate the length of the diamond in the ring below, given that the following line is 1.5 cm long and the photo of the ring is actual size.

1.5 cm

WORKING	THINKING
1.5 cm ÷ 2 = 0.75 cm	Compare the length of the line to the diameter of the diamond. The diamond is around half the size of the line so divide the length by 2 to estimate the diameter.
The diamond has a length of approximately 0.75 cm.	Communicate your answer using words.

Exercise 1C

FUNDAMENTALS

Example 6

1 Convert the following measurements into the units given in brackets.

a 5 m (cm)
b 7 cm (mm)
c 20 mm (cm)
d 2 km (m)
e 3.7 m (cm)
f 1.7 km (m)
g 490 cm (m)
h 7 m (mm)
i 1.2 km (mm)
j 3000 cm (km)

Hint If converting from a larger unit to a smaller unit, multiply. If converting from a smaller unit to a larger unit, divide.

2 Calculate the sum of the following measurements. Express your answer in the units given in the brackets.

a 10 mm, 2 cm (cm)
b 3 m, 200 cm (m)
c 1.5 cm, 10 mm (mm)
d 500 m, 2 km (km)
e 150 cm, 3.5 m (cm)
f 200 m, 1.5 km (m)

3 A good way to estimate the length of an object is to compare it to the length of a familiar object. Use a ruler to measure the span of your hand from your thumb to your pinkie (outside finger) in centimetres. Use this measurement to estimate the length of the following items around your classroom.

a length of your desk
b height of your chair
c length of your pen
d width of your book
e length of your eraser
f length of your calculator
g length of your pencil case
h length of the whiteboard

4 Determine the most appropriate unit of length (mm, cm, m or km) for the following measurements.

a length of a car
b width of pencil
c length of a ladybeetle
d length of desk
e flight distance from Brisbane to Cairns
f length of fingernail
g length of arm
h height of a giraffe
i distance around the Earth
j distance from London to Cairo

APPLICATIONS

SF: 5–12	CF: –	CU: –

Example 7 ★**5** Jade is an estimator for a building company. A plan shows a wall in the family room is 5860 mm long. Jade needs the measurement in metres to estimate the cost of the wall panelling. Determine the length of the wall in metres.

★**6** Lachlan is training for a fun run and he runs 4.5 km per day. Determine how many metres Lachlan runs.

★**7** Harrison is looking at house plans to decide which house to build. On one of the house plans, he has measured the length of the master bedroom as 4.2 m. Determine the length of the master bedroom in millimetres.

★**8** At the age of two, Kelly's son is 90 cm. Kelly is told that he will grow to be twice that height. Determine the grown height of Kelly's son in metres.

★**9** Allen is building a garden box for one of his clients. A length of timber is 250 cm long and Allen needs 6 lengths of timber. Determine how many metres of timber in total Allen needs to purchase for the garden box.

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★**10** Choose the correct estimation for each of the measurements shown in the table.

Measurement	**Correct estimation**		
Length of a diamond in a ring	2.5 cm	6 mm	28 mm
Length of a mobile phone	220 mm	0.4 m	12 cm
Height of an old gum tree	32 m	0.8 km	30 000 cm
Height of a 12-storey high skyscraper	0.1 km	36 m	80 m
Distance from Brisbane to Cairns	368 000 m	2500 km	1700 km

Example 8 ★**11** Estimate the length of the following images, given that the blue line is 2 cm long.

a

b

c

★**12** Estimate the height of the palm tree near the front door, given the height of the front rendered wall on the house is 3 m.

1D Calculating perimeters of familiar shapes

LEARNING GOALS

- Calculate the perimeters of familiar shapes including triangles, squares, rectangles and polygons
- Apply the formula to calculate the circumference of a circle
- Apply the formula to calculate the arc length and perimeter of a sector

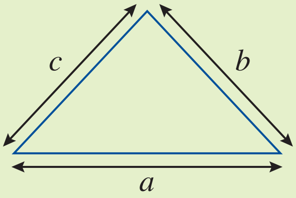
Why is it essential to understand how to calculate perimeters?

Calculating perimeters can be used in industries such as construction and design. In construction, perimeter can be used to calculate the total length around the property when building a fence. Perimeter is used to calculate the amount of timber required to frame a painting. Or it can also be used in fashion to work out the length of trimming required around the hem of a skirt.

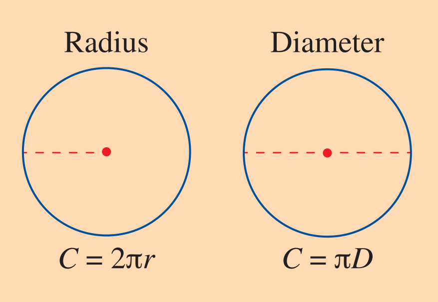
Perimeter is used to calculate the amount of fencing required around a property.

WHAT YOU NEED TO KNOW

- **Perimeter** is the distance around the outside of a shape. For example, the perimeter of the triangle in the image would be the total of $a + b + c$.



- **Circles** – The distance around the outside of a circle is called the **circumference**. The formula to calculate the circumference of a circle is $C = 2\pi r$ or $C = \pi D$, where r stands for radius and D stands for diameter. The radius extends from the centre to the circumference and the diameter extends all the way across the circle through the centre.



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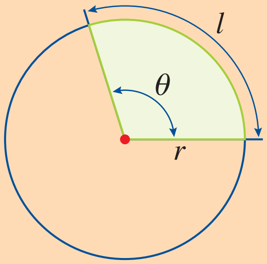
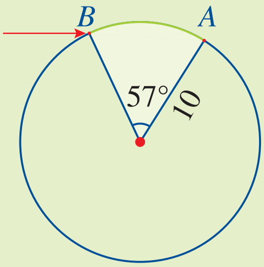
- **Arc length** – An **arc** is a curve that is part of the circumference of a circle. The arc length is the distance along the curved line that makes up a part of a circle.

Formula for arc length:

$$l = \frac{\theta}{360} \times 2\pi r \quad \text{or} \quad l = \frac{\theta}{360} \times \pi D$$



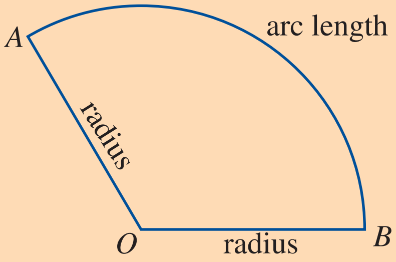
For example: For the arc length A to B in the diagram shown, the arc length is calculated using $l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle at the centre (so $\theta = 57°$) and r is the length of the radius ($r = 10$).



- **Sector** – A sector is a section of the circle formed from two radii. We can identify the perimeter of a sector by calculating the arc length and then adding the two side lengths, which is the radius multiplied by 2.

Formula for perimeter of a sector:

$$P = \frac{\theta}{360} \times 2\pi r + 2r$$



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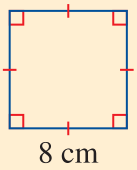
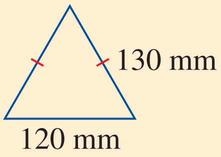
Example 9 Calculating the perimeter of 2D shapes

Calculate the perimeter of the following shapes.

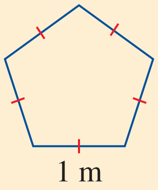
a



b

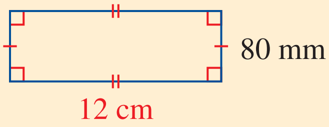


c



For the following question, calculate the perimeter in the units highlighted in red.

d



WORKING	THINKING
a $8 + 8 + 8 + 8 = 32$ cm	Add all the side lengths together.
or	
$4 \times 8 = 32$ cm	Alternatively, as there are 4 equal sides, multiply the side length by 4.
b $130 + 130 + 120 = 380$ mm	Add all the side lengths together.
or	
$2 \times 130 + 120 = 380$ mm	Alternatively, as there are 2 equal sides, multiply the equal side length by 2 and then add the base of the triangle.
c $1 + 1 + 1 + 1 + 1 = 5$ m	Add all the sides together.
or	
$5 \times 1 = 5$ m	Alternatively, as there are 5 equal sides, multiply the side length by 5.
d 80 mm $\div 10 = 8$ cm	Convert mm to cm first. There are 10 mm in every 1 cm so divide 80 by 10.
$8 + 8 + 12 + 12 = 40$ cm	Add all the side lengths together.
or	
$2 \times 8 + 2 \times 12 = 40$ cm	Alternatively, as there are 2 lots of equal side lengths, multiply both side lengths by 2 and add them together.

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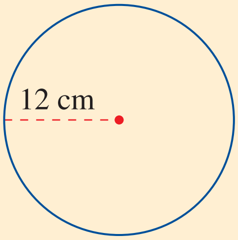
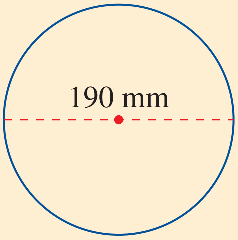
Example 10 Calculating the circumference of a circle

Calculate the circumference of the following circles using the appropriate formula. Round your answers to two decimal places.

a



b



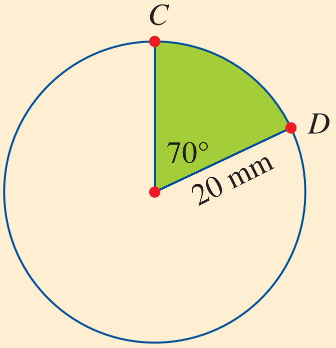
WORKING	THINKING
a $C = 2\pi r$	Given the radius so use $C = 2\pi r$ formula.
$C = 2 \times \pi \times 12$	Substitute $r = 12$ cm.
$C = 75.398223\ldots$ $C \approx 75.40$ cm	Round your answer to two decimal places by looking at the 3rd decimal place. Round up as the 3rd number is greater than 5.
b $C = \pi D$	Given the diameter so use $C = \pi D$ formula.
$C = \pi \times 190$	Substitute $D = 190$ mm.
$C = 596.902604\ldots$ $C \approx 596.90$ mm	Round your answer to two decimal places by looking at the 3rd decimal place. This number is less than 5 so we round down, which means we can keep the 2nd decimal place the same.

Example 11 Calculating the arc length and perimeter of a sector

Use the appropriate formula to calculate:

a the length of the arc *CD*, rounding your answer to two decimal places

b the perimeter of the sector



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WORKING	THINKING
a $l = \frac{\theta}{360} \times 2 \times \pi \times r$	Use the formula $l = \frac{\theta}{360} \times 2\pi r$
$l = \frac{70}{360} \times 2 \times \pi \times 20$	Substitute $\theta = 70^\circ$ and $r = 20$ mm.
$l = 24.434609$ $l = 24.43$ mm	Round your answer to two decimal places by looking at the 3rd decimal place. This number is less than 5 so we round down, which means we can keep the 2nd decimal place the same.
b Arc length = 24.43 mm	Use the arc length found in part **a**.
$P = 24.43 + 2 \times 20$ $P = 64.43$ mm	The sides lengths of the sector are the radius of 20 mm. There are two side lengths so multiply by 2 and add to the length of the arc.

Example 12 Applying perimeter to practical problems

Griffin has 30 m of flexible fencing. He would like to use all of the fencing to make the largest circular pen for his pigs. Calculate the diameter of the largest pen that Griffin can make using all of the fencing. Round your answer to two decimal places.

WORKING	THINKING
$C = \pi \times D$	Use the formula $C = \pi \times D$.
$30 = \pi \times D$	Substitute into the formula the known measurements; that is, the circumference of 30 m.
$D = \frac{30}{\pi}$	To find the diameter, rearrange the equation by dividing the circumference by π.
$D = 9.549296$ $D = 9.55$ m	Round your answer to two decimal places.
The diameter of the pen will be about 9.55 m.	Communicate your answer in a sentence.

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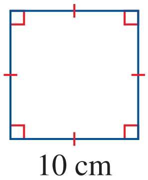
Exercise 1D

FUNDAMENTALS

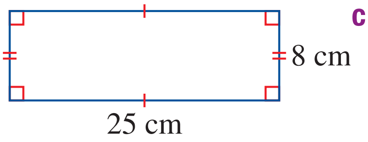
Example 9

1 Calculate the perimeter of the following shapes.

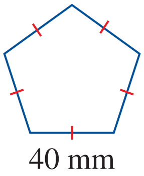
a



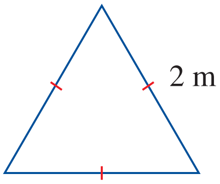
b



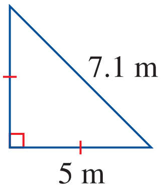
c



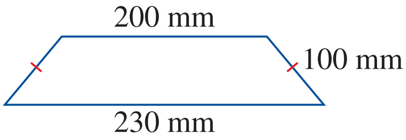
d



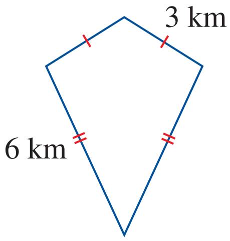
e



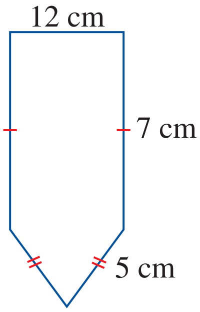
f



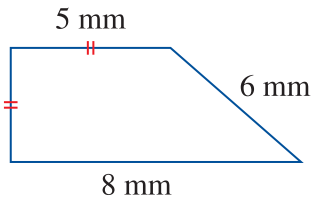
g



h

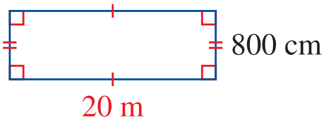


i

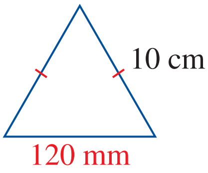


2 Calculate the perimeter for the following shapes using the units indicated in red.

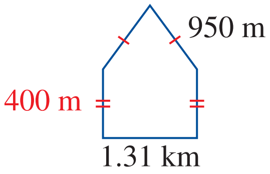
a



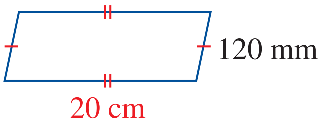
b



c



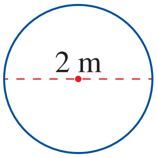
d



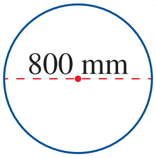
Example 10 **3** Calculate the circumference of the following circles. Round your answer to two decimal places.

Hint Use $C = 2\pi r$ or $C = \pi D$

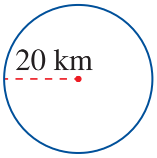
a



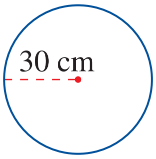
b



c

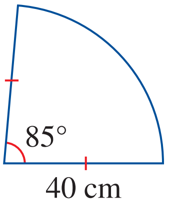


d

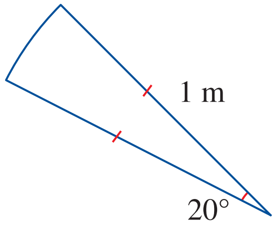


Example 11 **4** Calculate the arc length of the following sectors. Round your answer to two decimal places.

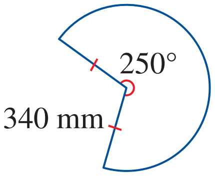
a



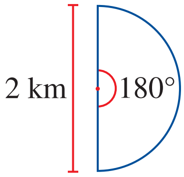
b



c



d



5 Calculate the perimeter for each of the sectors in question **4**.

APPLICATIONS

SF: 6–12	CF: –	CU: –

For the following questions write your answers correct to two decimal places, where appropriate.

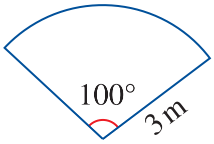
Example 12 ★**6** Quinn is baking her mother a cake for her birthday. Quinn uses a 20 cm square baking dish and plans on decorating the cake by wrapping a ribbon around it. Determine how much ribbon Quinn will require for the cake.

★**7** Lyn lives on a large property and has a new dog that has a reputation for running away. She has decided that she will put in an electric dog fence along the perimeter of her property. The shape of the property is a rectangle. It is 80 metres long and 70 metres wide. Determine how many metres of electric dog fencing Lyn will need to purchase.

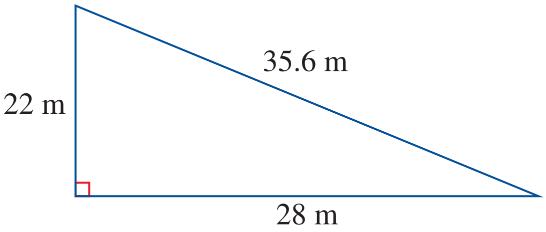
★**8** Bethany is planning on upcycling her old lampshade. It has a circular base that has a 38 cm diameter. She wishes to wrap a length of beading along the edge of the base. Determine the length of beading Bethany should purchase.

★**9** Josh is renovating his bathroom and has purchased a circular mirror. The measurement on the box states that the mirror has a circumference of 4 m. He would like to determine if the mirror will fit on a particular wall in the bathroom. Calculate the diameter of the mirror.

★**10** Stacey is installing a concrete pool in the shape of a sector at her new house. She needs to calculate the perimeter of the pool to work out how many pavers she will need to order. Use the image shown to calculate the perimeter of Stacey's pool.



★**11** Jarrah is building a fence for one of his yards to help keep his stock away from the road. The yard is triangular, as per the image shown. The fence design that he is planning has three railings of timber.



a Calculate the perimeter of the yard.

b Determine how many metres of timber that Jarrah requires to construct the fence.

★**12** Jana has made a circular cake with a radius of 15 cm for her daughter's fifth birthday party. After slicing the cake and serving it to the guests, she notices that there is $\frac{1}{4}$ (90°) of the cake remaining. She divides the cake four ways between her husband, daughter, son and herself. Calculate the arc length of each of these four slices.

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1E Calculating perimeters of familiar composite shapes

COMPLEX

LEARNING GOALS

- Identify composite shapes
- Calculate perimeters of composite shapes

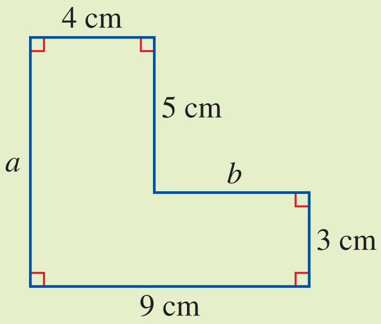
Why is it essential to calculate perimeters of composite shapes?

Often in life, shapes are more intricate in detail and sometimes form a composite shape. They are made by combining two or more familiar shapes. For example, the shape of a house plan can be L-shaped and is made up of two rectangles. The face of a building is another example as it may be a triangle and a rectangle combined.

In a quilt, simple shapes can be sewn together to make more complex composite shapes.

WHAT YOU NEED TO KNOW

- A **composite shape** is a shape that is made up of two or more basic shapes.
- Some composite shapes do not mark the length of every side as it is possible to calculate these lengths using either addition or subtraction.
  - For example:
    The length of the side marked *a* in the diagram is equal to the sum of the lengths of the opposite sides. As $5 + 3 = 8$ cm, $a = 8$ cm.
    The side marked *b* is equal to the side length 9 cm minus the side length 4 cm. In length is equal. As $9 - 4 = 5$ cm, $b = 5$ cm.



The perimeter of the composite shape is $8 + 4 + 5 + 5 + 3 + 9 = 34$ cm.

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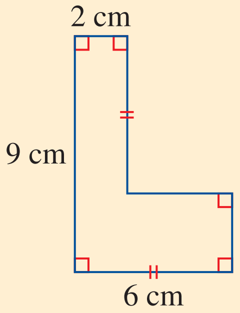
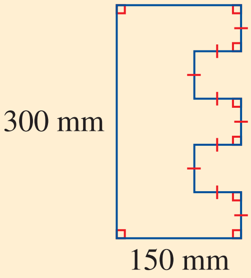
Example 13 Calculating the perimeter of composite shapes

Calculate the perimeter of the following composite shapes.

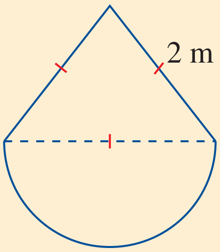
a



b

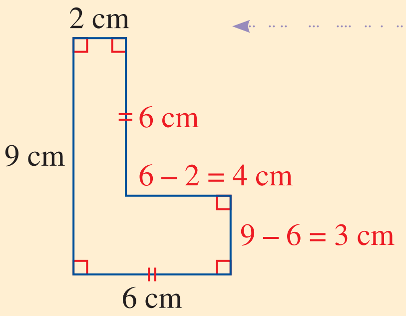


c



WORKING	THINKING

a

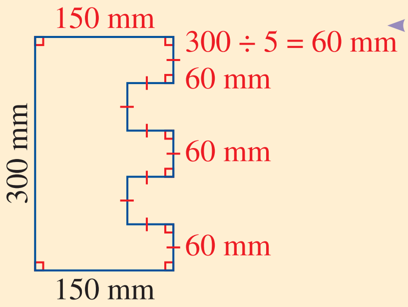


Two side lengths that are unmarked. These lengths are calculated by subtracting the opposite side lengths of the shape.

$$P = 2 + 6 + 4 + 3 + 6 + 9$$
$$P = 30 \text{ cm}$$

Add all the side lengths together to calculate the total perimeter.

b



There are 5 small lengths that equal the total of the opposite side length. Divide 300 by 5 to calculate the length of one of these small lengths. There are 9 of these small lengths in total so multiply the answer by 9.

$$P = (9 \times 60) + (2 \times 150) + 300$$
$$P = 1140 \text{ mm}$$

Add the lengths of the remaining longer sides. Although not marked, we know that there are two side lengths of 150 mm because all the angles are right angled.

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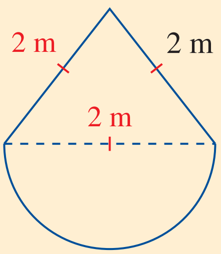
WORKING	**THINKING**

c



$D = 2$

$$\text{Length of arc} = \left(\frac{\pi \times 2}{2}\right)$$
$$\approx 3.14 \text{ m}$$

$P = 3.14 + 2 + 2$

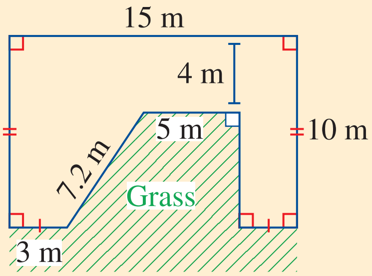
$P = 7.14$ m

NOTE: this shape almost looks like a sector, but it is not because the centre of the semi-circle is on the dotted line not at the point formed by the straight sides.

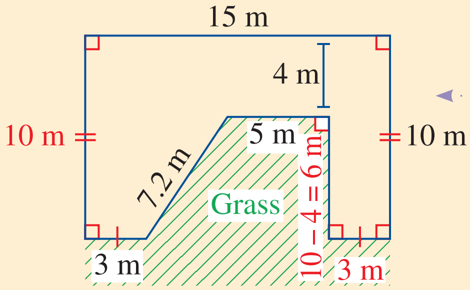
The diameter of the semi-circle is 2 m.
As a semi-circle is half of a circle, calculate the circumference of the circle and then dividing by 2.
Add the remaining two sides of the triangle to calculate the total perimeter for the composite shape.

Example 14 Applying perimeter of composite shapes to practical problems

Noah has designed a garden bed for his new property, as per the image below. He would like to place timber edging along the entire perimeter to hold the soil in place. Calculate how much timber Noah requires for his garden bed.
NOTE: The dimensions are measured around the ‘outside’ of the garden bed after the timber has been added. So you do not have to consider the thickness of the timber.



WORKING	**THINKING**



$P = 15 + 10 + 3 + 6 + 5 + 7.2 + 3 + 10$

$P = 59.2$ m

Noah has used 59.2 metres of timber to build his garden bed.

There are two sides that are marked as 3 cm.
There are two sides that are marked as 10 cm.
There is only one length that is not given. We can calculate this length by subtracting 4 m from the opposite side length of 10 m.
This tells us that the unmarked length is 6 m.
Add all the side lengths together to calculate the total perimeter.
Communicate your solution in a sentence.

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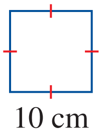
Exercise 1E

FUNDAMENTALS

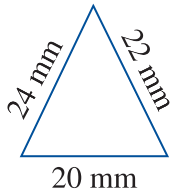
Where appropriate, write your answers to two decimal places.

1 Calculate the perimeter or circumference of the following common shapes.

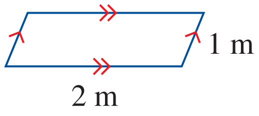
a



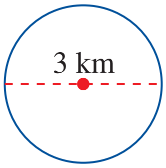
b



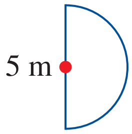
c



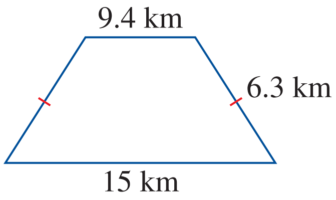
d



e



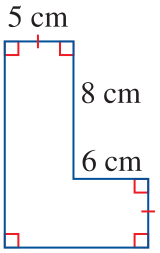
f



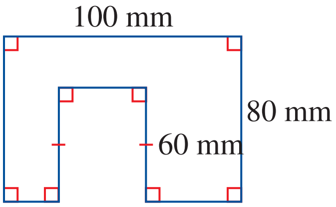
Example 13

2 Calculate the perimeter for the following composite shapes.

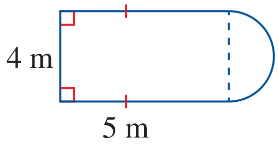
a



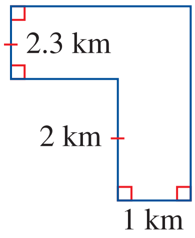
b



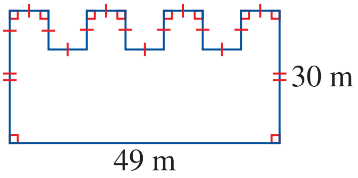
c



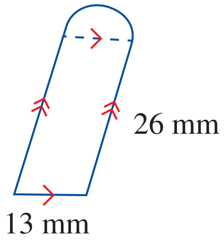
d



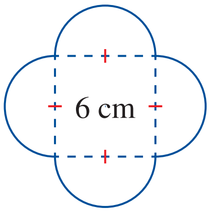
e



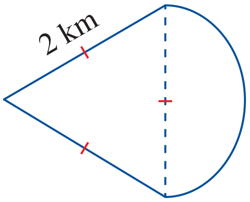
f



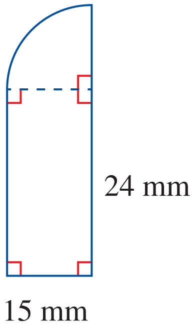
g



h



i



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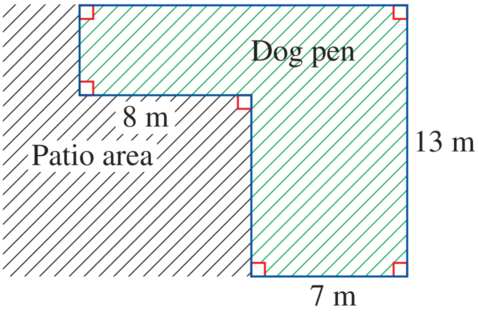
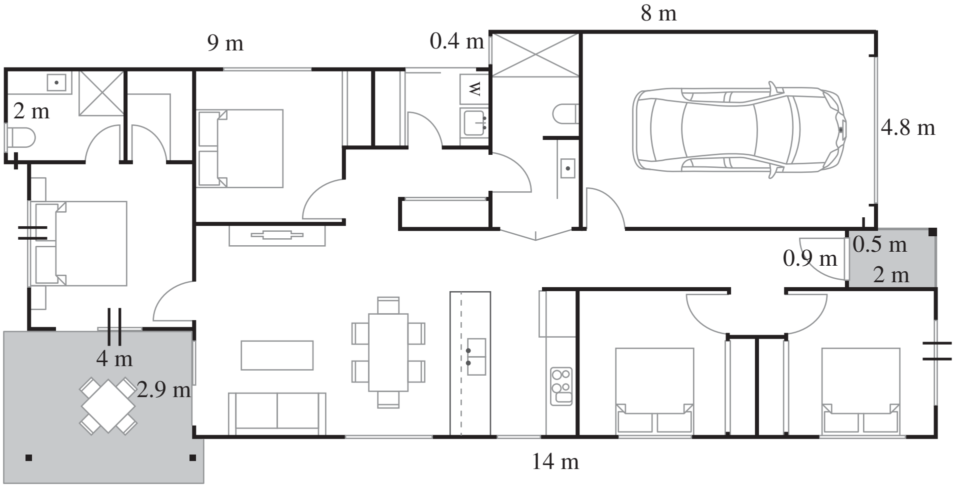
APPLICATIONS

SF: –	CF: 3–7	CU: 8–9

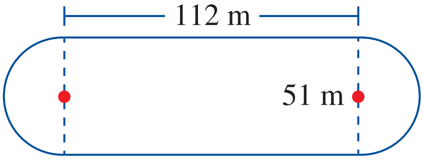
Example 14 ★**3** Russell needs to fence a large patch of grass area next to his patio to create a dog pen with dimensions shown in the diagram. Calculate the total length of fencing that Russell needs to purchase.



★**4** Valerie is renovating her house and she would like to purchase some paint to change the colour of the outside of her house. Her local hardware store has asked her for the perimeter and height of the external walls of her house. She knows the external walls are 3 metres high. Valerie is using her house plan to work out the perimeter. Calculate the perimeter of Valerie's house using the image shown.



★**5** Mason is building a speed skating rink and he is required to place a railing around the outside of the rink for spectators. Use the diagram shown to calculate the length of the railing that will be required.



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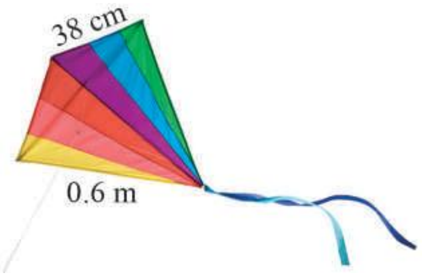
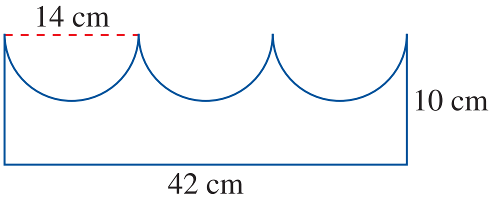
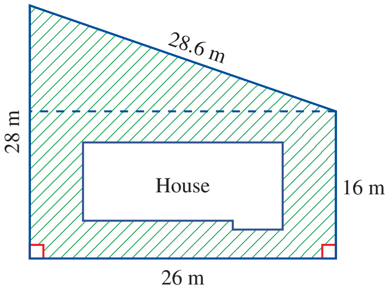
★**6** Mathew has cut and sewn a pattern for a kite in the shape of the image shown. He now needs to buy dowel (wooden rod) to insert along the edges of the kite before he can fly it. Calculate how many metres of dowel Mathew must purchase.



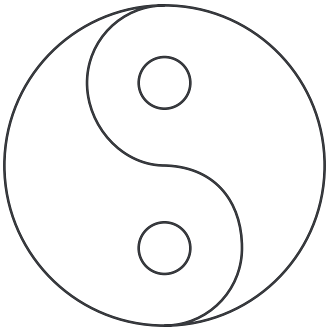
★**7** Anaya is making 8 crowns for her daughter's fifth birthday party. She plans to cut the crowns out of cardboard and stick decorative washi tape on the top and bottom of the crown to provide strength. The image shows an outline for the crown. Calculate how many centimetres of washi tape Anaya will need to purchase to complete the eight crowns.



★**8** Shekila's property contains her house situated in the front rectangular section of the property as shown in the diagram. She wishes to enclose the back triangular section of her yard with fencing. Calculate the length of fencing that Shekila will require.



★**9** Paul has been commissioned to paint a large yin-yang symbol, similar to the image below. The symbol is to have a 2 m diameter and the small dots are to have a 40 cm diameter. Paul needs to purchase black paint for the outline of the symbol. Calculate the perimeter of the black line so that Paul can purchase enough paint to complete the artwork.



Hint The inside swirl line can be calculated using two semi-circles.

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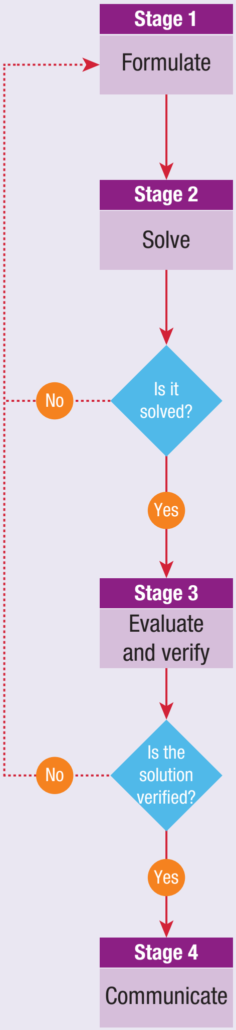
Problem-solving and modelling task

Background: Landscape and garden design involves dealing with composite shapes such as using perimeters to calculate fencing and edging.

Task: Your task is to create a landscape design for a backyard. You must incorporate at least four familiar geometrical shapes, such as squares, triangles, rectangles, to represent aspects of your yard. Use a key to clearly show what they represent. You must incorporate a feature in your landscape design that is a composite shape. Your plans must show measurements. You will then need to calculate the perimeter of your land, the composite-shaped feature and also one other familiar shape.

Approach to problem-solving and modelling task:



Stage 1: Formulate

1 Look at various landscape designs.

2 Research interesting features for a backyard.

3 Research average land sizes.

Stage 2: Solve

4 Sketch the plan of your backyard ensuring that you:
- incorporate at least four familiar shapes
- include measurements
- create a composite-shaped feature
- use a key

5 Calculate the perimeter of the following:
- the backyard
- the composite-shaped feature
- one other 2D familiar shape

Stage 3: Evaluate and verify

6 Check that you have included as many measurements as possible in your design.

7 Check that your composite-shaped feature has at least two familiar shapes.

8 Check that your perimeter calculations are accurate.

Stage 4: Communicate

9 Share your design with a peer. Also describe what your composite-shaped feature represents.

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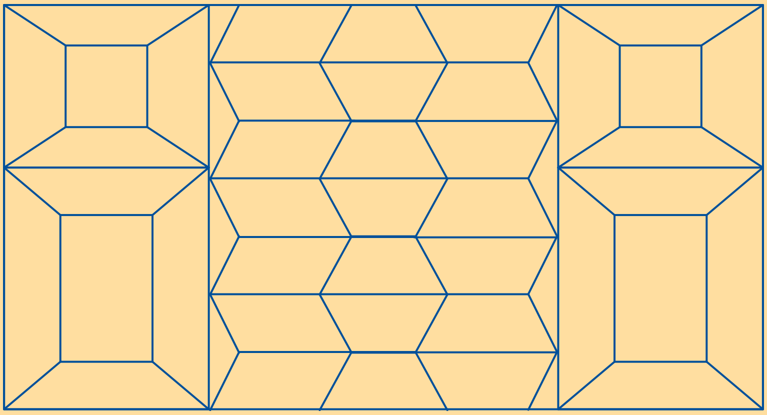
Chapter checklist

☐ I can identify the names and properties of 2D shapes.

1 List the properties for a square, rectangle and triangle.

2 Identify as many 2D shapes as possible in the following image.

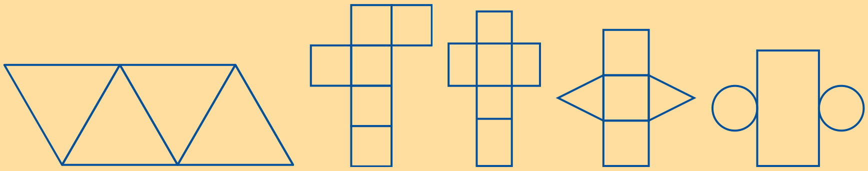


☐ I can identify the names and properties of 3D solids.

3 Construct a table that lists the number of faces, edges and vertices in the following solids: cube, triangular prism, rectangular-based pyramid, triangular-based pyramid, pentagonal prism, hexagonal prism.

☐ I can interpret 2D nets to identify 3D solids. **[complex]**

4 Identify each 3D solid that is formed from the following nets.



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5 Draw nets that best represent each of the following 3D objects.

a

b

c

☐ I can use measurement abbreviations and identify the most appropriate unit of length.

6 Use abbreviations to identify the most appropriate unit of length to measure each of these:
- **a** length of a guinea pig
- **b** height of a large tree
- **c** length of a bolt
- **d** distance between towns

☐ I can estimate lengths of objects.

7 List some techniques that you can use to help you estimate lengths.

8 Choose the best estimate for the following measurements.

a	Thickness of a garden hose	**i** 12 mm	**ii** 100 mm	**iii** 3 cm
b	Length of an adult male foot	**i** 15 cm	**ii** 30 cm	**iii** 50 cm
c	Perimeter of a house	**i** 300 m	**ii** 0.8 km	**iii** 62 m
d	Distance from Gold Coast to Perth	**i** 10 000 m	**ii** 2500 km	**iii** 4000 km

☐ I can convert between units of length.

9 Convert the following measurements into the units given in brackets.

a 4 cm (mm) **b** 30 mm (cm)
c 4.5 km (m) **d** 7.3 m (cm)

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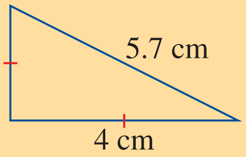
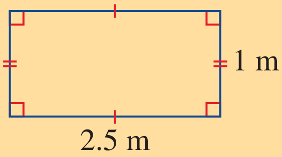
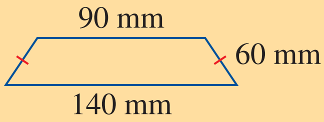
☐ I can calculate the perimeters of familiar shapes.

10 Calculate the perimeter of each of the following familiar 2D shapes.

a


b


c


☐ I can calculate the circumference of a circle using the formula $2 \times \pi \times r$ or $\pi \times D$.

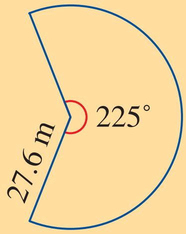
11 Calculate the circumference for the following circles, correct to two decimal places.

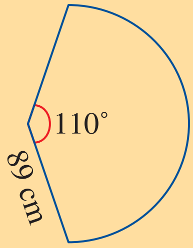
a 3.7 m radius **b** 76 mm diameter

☐ I can calculate arc length and the perimeter of a sector.

12 Correct to two decimal places, calculate the:

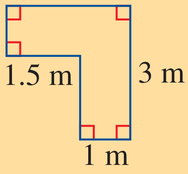
i arc length **ii** perimeter of each sector

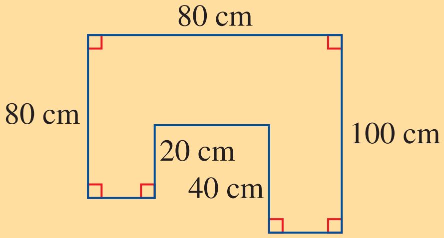
a


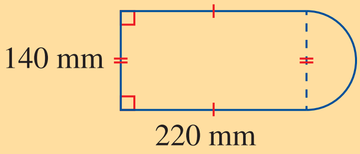
b


☐ I can calculate perimeters of composite shapes. **[complex]**

13 Calculate the perimeter of the following composite shapes. Where appropriate, write your answers correct to two decimal places.

a


b


c


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Chapter review

All questions in the review are assessment style.

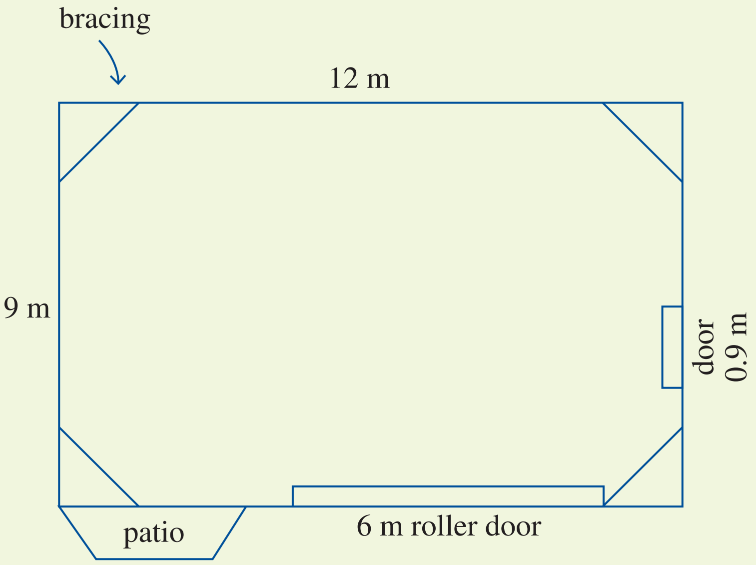
Simple familiar

Where appropriate, write your answers correct to two decimal places.

Section 1A

1 Ashley is a cattle farmer in Toowoomba. He has a plan for a new hay shed, which is shown in the image.

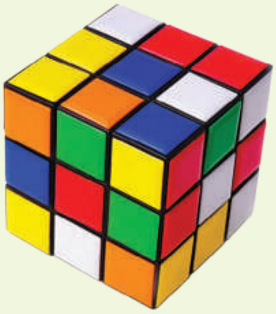
- **a** Identify and list the number of each 2D shape in the hay shed plan.
- **b** List the properties of each of these 2D shapes.



2 Identify which 3D solid is most similar to the following real-life objects.

a

b



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c

d

e

Section 1C

3 A warehouse receives a shipment of widgets, but they have all been labelled with their lengths in the wrong units for their particular uses. Convert the measurements to the units shown in brackets so that they can be correctly labelled.

a 38 m (cm)

b 15 cm (mm)

c 44 mm (cm)

d 1.79 m (cm)

Section 1D

4 Beatrix is planning to sew lace around the perimeter of a 45 cm square cushion that she has just completed. Determine how many metres of lace Beatrix will require.

5 Tim works at a school in maintenance. He has been asked to build a new sandpit that requires a 6 m by 8 m rectangular frame (measured around the outside of the timber frame). Determine how many metres of timber Tim needs to order.

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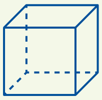
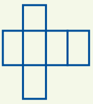
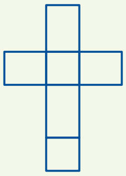
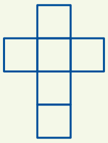
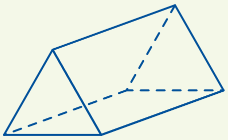
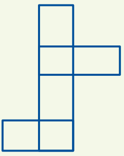
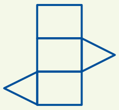
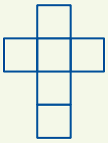
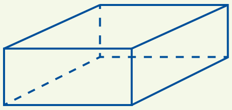
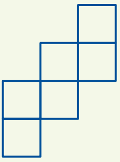
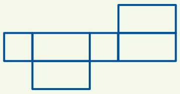
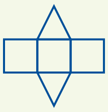
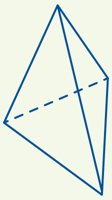
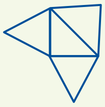
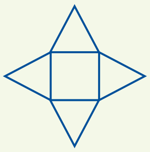
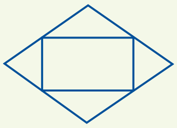
Complex familiar

Section 1B

6 A workshop makes building blocks from metal sheets. Unfortunately, the diagrams of the building blocks and their nets have been mixed up. Decide which net matches with the 3D solid so that the workshop can continue making the building blocks.

3D solid	Net
a 	i  ii  iii 
b 	i  ii  iii 
c 	i  ii  iii 
d 	i  ii  iii 

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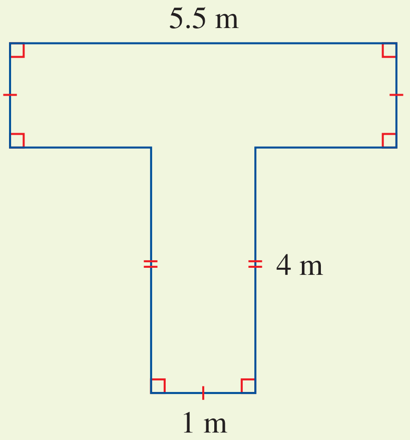
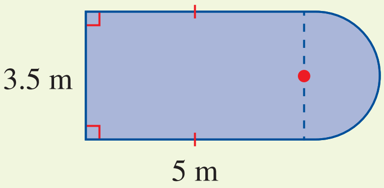
Section 1E **7** Lee-Ann is planning to concrete a path to her front door and underneath her patio. She needs to first construct a timber frame to hold the concrete in place. The diagram at right shows Lee-Ann's plan for the concrete. Calculate how much timber Lee-Ann needs for the timber frame.

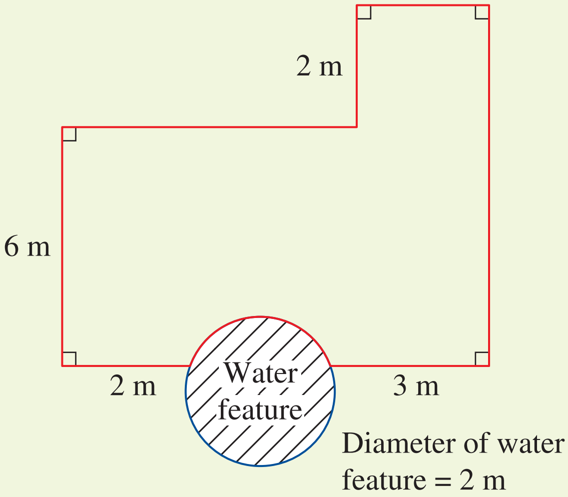


8 Jay is installing some pavers around the edge of his pool, shown in the diagram. Calculate the perimeter of the pool.



Complex unfamiliar

9 Sara is constructing a new garden bed that wraps around her circular water feature. She has drawn a plan with a red line indicating where she will be placing timber as a border for the garden bed. Determine how many metres of timber Sara will need to purchase.



2

Area measure

Maths for an agronomist: Gavin Benjamin

Gavin Benjamin completed a Bachelor of Agriculture to pursue a career in agronomy. Having grown up around vineyards, his interest in the viticulture industry led him to work as a viticulture agronomist at the local rural supply company, and he has since progressed to Operations Manager.

Tell us a bit about your job. What does a typical day look like?

As an Operations Manager, my day involves working with farmers and suppliers to ensure we have the right product on hand to meet their requirements and I advise customers of the best option for their vineyard situation. I also oversee our inventory to ensure we have the right products in store at the right time.

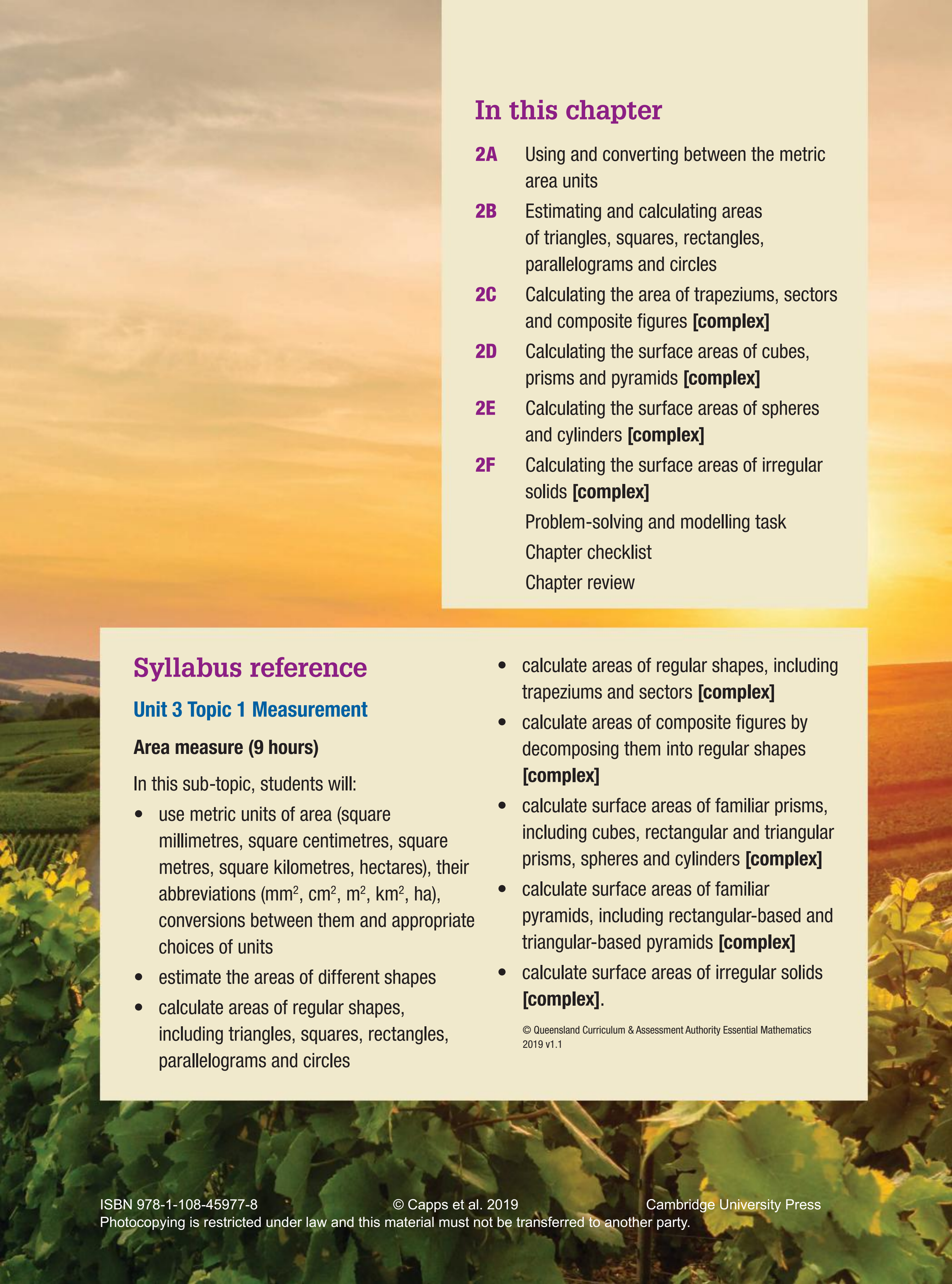
I have a passion for agriculture and enjoy being able to help customers to build their farms and business by providing the best information and solutions.

What maths did you study at school?

I studied Business Maths (equivalent to Essential Maths) in school. I really enjoyed the subject as I was able to apply the learnings to my personal life, and also at work.

How do you use maths in your job?

I use maths on a daily basis in my career. It touches every aspect of my role, including helping customers work out fencing requirements based on property perimeters, fertiliser rates based on planted area and spray water volumes based on vine size. I also analyse business profit margins and forecast stock requirements.



In this chapter

Syllabus reference

Unit 3 Topic 1 Measurement

Area measure (9 hours)

In this sub-topic, students will:

- use metric units of area (square millimetres, square centimetres, square metres, square kilometres, hectares), their abbreviations (mm^2, cm^2, m^2, km^2, ha), conversions between them and appropriate choices of units
- estimate the areas of different shapes
- calculate areas of regular shapes, including triangles, squares, rectangles, parallelograms and circles
- calculate areas of regular shapes, including trapeziums and sectors **[complex]**
- calculate areas of composite figures by decomposing them into regular shapes **[complex]**
- calculate surface areas of familiar prisms, including cubes, rectangular and triangular prisms, spheres and cylinders **[complex]**
- calculate surface areas of familiar pyramids, including rectangular-based and triangular-based pyramids **[complex]**
- calculate surface areas of irregular solids **[complex]**.

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Pre-test

1 Convert 344 cm into millimetres (mm), metres (m) and kilometres (km).

2 Calculate how many squares with a side length of 1 m will fit into a rectangle of length 12 m and width 10 m.

3 Without using a calculator, calculate:

a 7×8 **b** $56 \div 8$

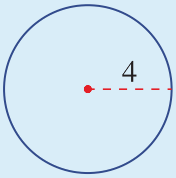
4 Evaluate:

a 1^2 **b** 9^2 **c** 10^2

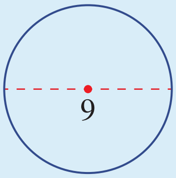
d 100^2 **e** 1000^2 **f** 13^2

5 In the circles below, determine the size of the radius.

a

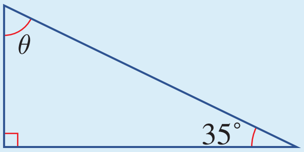


b

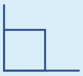


6 State the name of the symbol π and explain what it represents in relation to the lengths of the diameter of a circle and its circumference.

7 In this geometry diagram, identify the feature represented by these symbols:



a



b

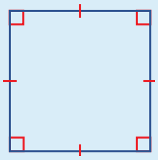
c

8 In these geometry diagrams:

i explain what is meant by the marks on the sides

ii identify the shape

a



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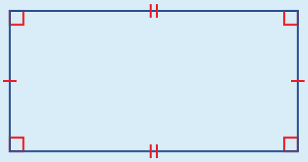

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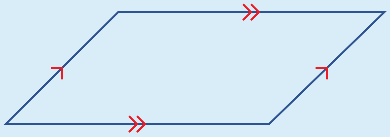

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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2A Using and converting between the metric area units

LEARNING GOALS

- Understand the use and appropriate choice of the metric units of area including square millimetres, square centimetres, square metres, square kilometres and hectares
- Use abbreviations such as mm^2, cm^2, m^2, km^2, ha to represent units of area
- Convert between units of area

Why is it essential to understand units of area and convert between them?

Calculating the area of a shape is used in many careers including landscaping, building, painting, clothes making and interior design. It is important to convert between units of area so that we can compare sizes of objects that may have been measured in different units.

The floor area may have been calculated in square metres, but the tile size is usually given as dimensions in millimetres, so calculating how many tiles are required will involve conversion of units.

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WHAT YOU NEED TO KNOW

- Calculating the **area** of a shape is a measure of the space enclosed by the boundaries of a 2D shape.
- For squares and rectangles, area is length multiplied by its width (also known as breadth).
- If the shape is upright, the term height might be used in place of length or width.
- **Units of area** include square millimetres, square centimetres, square metres, square kilometres and hectares, these can be abbreviated to mm^2, cm^2, m^2, km^2 and ha.
    - **Square millimetres** are typically used in construction or electrical cabling.
    - **Square centimetres** are often used for medium-sized objects such as the surface area of a desk.
    - **Square metres** are typically used for larger objects such as floors and floor coverings, walls or small blocks of land such as a plot for a single house.
    - A **hectare** is an area equal to a square with sides of 100 m. Its area is $100\text{ m} \times 100\text{ m} = 10\,000$ square metres (m^2). This unit of area is typically used for land bigger than an average house plot, such as farm paddocks.
    - **Square kilometres** are typically used for very large areas of land such as the area of a city or region.
- When a 2D shape is referred to as being ‘*l* units square’ (e.g. 60 cm square), it means that it is a square with sides of ‘*l* units’ (e.g. 60 cm). When a 3D solid is referred to as being ‘*l* units cube’ (e.g. 100 cm cube), it means that it is a cube with sides of ‘*l* units’ (e.g. 100 cm).
- When calculating area, always ensure that the area units are in squares of the linear units used to measure the dimensions (i.e. cm and cm^2).
- To convert between units of area:
    - use multiplication when converting from a larger unit to a smaller unit (e.g. from km^2 to m^2)
    - use division when converting from a smaller unit to a larger unit (e.g. from cm^2 to m^2).
- Refer to the following conversion chart when solving problems that require conversion.

Area conversion

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "{\times 1000^2\ (1\,000\,000)}"] & \text{m}^2 \arrow[l, bend left, "{\div 1000^2\ (1\,000\,000)}"] \arrow[r, bend left, "{\times 100^2\ (10\,000)}"] & \text{cm}^2 \arrow[l, bend left, "{\div 100^2\ (10\,000)}"] \arrow[r, bend left, "{\times 10^2\ (100)}"] & \text{mm}^2 \arrow[l, bend left, "{\div 10^2\ (100)}"]
\end{tikzcd}$$

Area conversion with hectares

$$\begin{tikzcd} \mathrm{km}^2 \arrow[r, bend left, "\times 100"] & \mathrm{ha} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10\,000"] & \mathrm{m}^2 \arrow[l, bend left, "\div 10\,000"] \end{tikzcd}$$

Example 1 Converting between units of area

Convert these area measurements to the units given in brackets.

a 0.72 km^2 (m^2)
b 393 mm^2 (cm^2)
c 34.5 km^2 (ha)
d 800 m^2 (ha)

WORKING	THINKING
a $0.72 \times 1000^2 = 720\,000\ \mathrm{m}^2$	As there are 1000^2 square metres in a square kilometre and we are converting from km^2 to a smaller unit of m^2, we need to multiply 0.72 by 1000^2.
b $393 \div 10^2 = 3.93\ \mathrm{cm}^2$	As there are 10^2 square millimetres in a square centimetre and we are converting from mm^2 to a larger unit of cm^2, we need to divide 393 by 10^2.
c $34.5 \times 100 = 3450$ ha	As there are 100 hectares in a square kilometre and we are converting from km^2 to a smaller unit of ha, we need to multiply 34.5 by 100.
d $800 \div 10\,000 = 0.08$ ha	As there are 10 000 square metres in a hectare and we are converting from m^2 to a larger unit of ha, we need to divide 800 by 10 000.

Example 2 Converting between units requiring more than one step

Convert these area measurements to the units given in brackets.

a 5 000 000 cm^2 (km^2)

b 0.07 m^2 (mm^2)

WORKING	THINKING
a $5\,000\,000 \div 10\,000\,000\,000$ $= 0.0005\,\text{km}^2$	As there are 100^2 square centimetres in a square metre and 1000^2 square metres in a square kilometre and we are converting from square centimetres to a larger unit of square kilometres, we need to divide 5 000 000 by $(100^2 \times 1000^2) = 10\,000\,000\,000$.
b $0.07 \times 1\,000\,000$ $= 70\,000\,\text{mm}^2$	As there are 100^2 square centimetres in a square metre and 10^2 square millimetres in a square centimetre and we are converting from square metres to a smaller unit of square millimetres, we need to multiply 0.07 by $(100^2 \times 10^2) = 1\,000\,000$.

Example 3 Applying unit conversion to practical problems

Alana has just purchased some new land that is 1.32 ha. Determine the area of Alana's new property in square metres.

WORKING	THINKING
$1.32 \times 10\,000 = 13\,200\,\text{m}^2$	As there are 10 000 square metres in a hectare and we are converting from ha to a smaller unit of m^2, we need to multiply 1.32 by 10 000.
Alana's new property is 13 200 m^2.	Communicate your answer in a sentence.

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Exercise 2A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Use ______________ when converting from a larger unit of area to a ______________ unit of area.

b Use division when converting from a ______________ unit of area to a ______________ unit of area.

c One hectare is equal to ______________ square metres.

d One square metre is equal to ______________ hectares.

Example 1

2 Convert the following measurements to the units given in brackets.

a 5 m^2 (cm^2)
b 7 cm^2 (mm^2)
c 200 mm^2 (cm^2)
d 2 km^2 (m^2)
e 5 km^2 (ha)
f 3.7 m^2 (cm^2)
g 1.7 km^2 (m^2)
h 490 cm^2 (m^2)
i 527 m^2 (ha)
j 5 mm^2 (cm^2)
k 500 m^2 (km^2)
l 3.75 km^2 (ha)

Example 2

3 Convert the following measurements to the units given in brackets.

a 7 m^2 (mm^2)
b 0.4 km^2 (cm^2)
c 2800 mm^2 (m^2)
d 1.2 km^2 (mm^2)
e 3000 cm^2 (km^2)

Hint If a length unit is 10 times bigger than another length unit, then its square unit for area is 10^2 times bigger (100 times bigger).

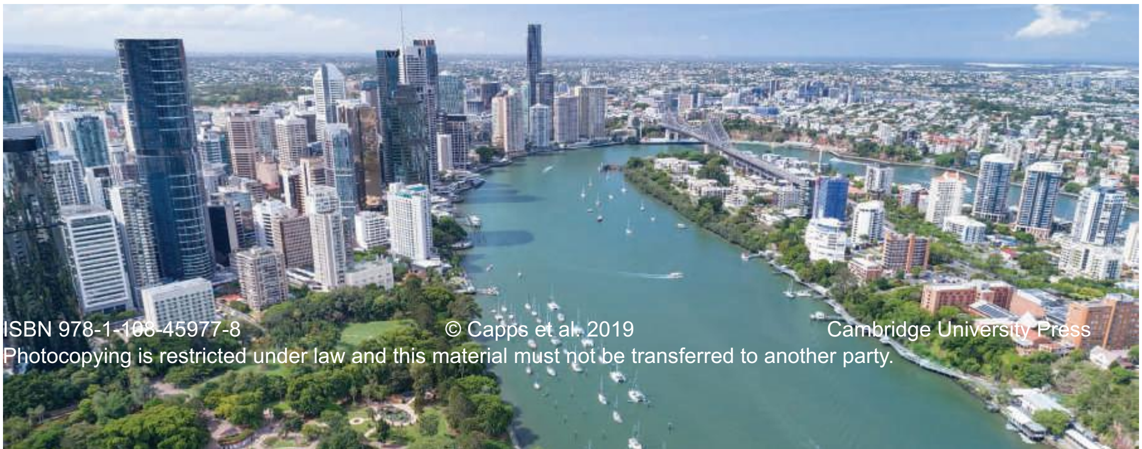
APPLICATIONS

SF: 4–11	CF: –	CU: –

Example 3

★**4** Anne's property is 2.5 ha. Determine the area of land Anne owns in square kilometres.

★**5** Brisbane covers an area of 15 826 km^2. Determine the area of Brisbane in square metres.



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★**6** Molly is making a new dress for her daughter's tiny doll. She has calculated that she needs 19 cm^2 of material. Determine the amount of material that Molly needs in square millimetres.

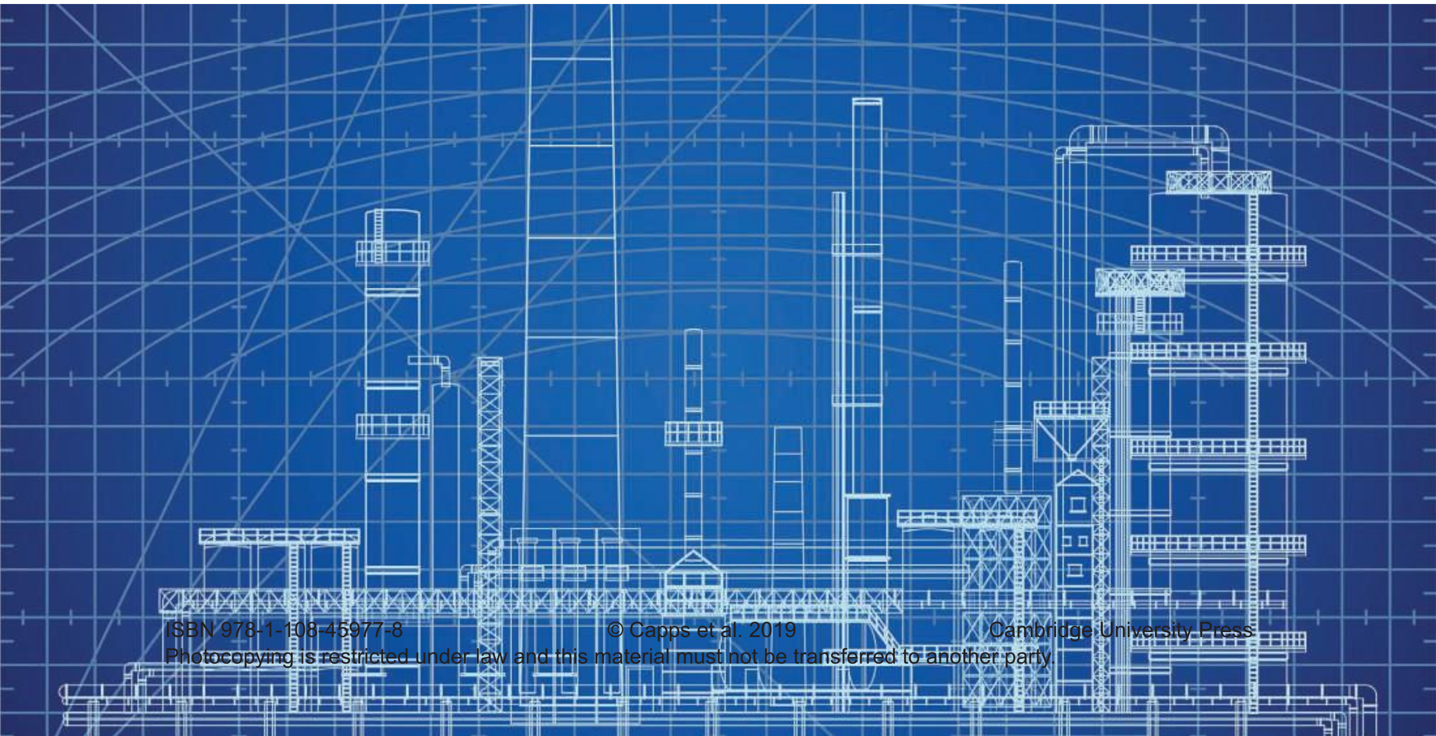
★**7** Ki is replacing the cover on an old chair. He calculates that he requires 1600 cm^2 of material, which is sold in square metres. Determine the amount of material that Ki needs in square metres.

★**8** Harley's horses each have a paddock of 16 180 m^2 to roam. Determine the total area that her two horses have in hectares.

★**9** Kirra works for Surf Life Saving Australia. Part of Kirra's role requires her to wax the rescue boards. She has ten boards in total that have an approximate total surface area of 9200 cm^2 that requires waxing. Determine the total surface area that Kirra will have to wax in square metres.

★**10** Jana is laying a small patch of turf in her backyard. She measures the patch to be 3400 mm long by 2100 mm wide, which she calculates to be an area of 7 140 000 mm^2. The turf is sold in square metres. Determine the amount of turf that Jana requires in square metres.

★**11** Damien is designing a platform to be able to bear a certain weight and is using a computer program to calculate the strength of the beams he will use. The program requires him to input the area of the cross-section of the beam. He calculated the area of the cross-section of the beam to be 0.018 m^2 but he did not realise that the program requires the area to be in mm^2. Determine the area of the cross-section of the beam in mm^2.



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2B Estimating and calculating areas of triangles, squares, rectangles, parallelograms and circles

LEARNING GOALS

- Calculate the area of various familiar shapes including:
  - triangles
  - squares
  - rectangles
  - parallelograms
  - circles
- Estimate the area of familiar shapes

Why is it essential to understand how to calculate the area of a shape?

There are many real-life reasons where you would need to calculate the area of various shapes. This skill is used in careers such as fashion, construction, landscaping and painting. Calculating the area allows a landscaper to find the amount of turf required or a painter to work out how much paint is needed. It is also a good way to compare the sizes of properties when you are looking at buying or leasing (renting) some land.

Real estate agents' signs and ads for small to medium properties for sale or lease usually state the area in square metres – larger blocks such as farms would be in hectares.

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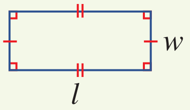
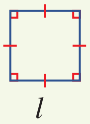
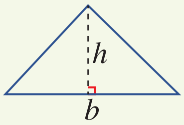
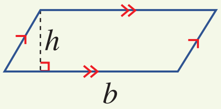
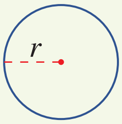
WHAT YOU NEED TO KNOW

- The **formulas** for the areas of some common shapes are given in the table below.

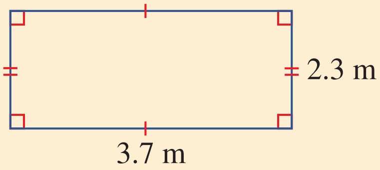
Shape	Area formula
Rectangle 	$A = l \times w$
Square 	$A = l^2$ or $l \times l$
Triangle 	$A = \frac{1}{2}(b \times h)$ or $\frac{b \times h}{2}$
Parallelogram 	$A = b \times h$
Circle 	$A = \pi r^2$ or $\pi \times r \times r$ * Note if given diameter then divide by 2 to calculate the radius.

- Abbreviations: A area, l length, w width, b breadth or base, h height, r radius, π (pi) = 3.142 approximately.
- Units of area include square millimetres, square centimetres, square metres, square kilometres and hectares, these can be abbreviated to mm^2, cm^2, m^2, km^2, ha.

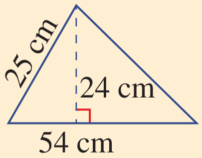
Example 4 Calculating the area of a common 2D shape

Calculate the area of the following common 2D shapes.

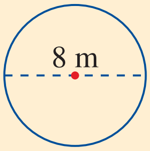
a



b

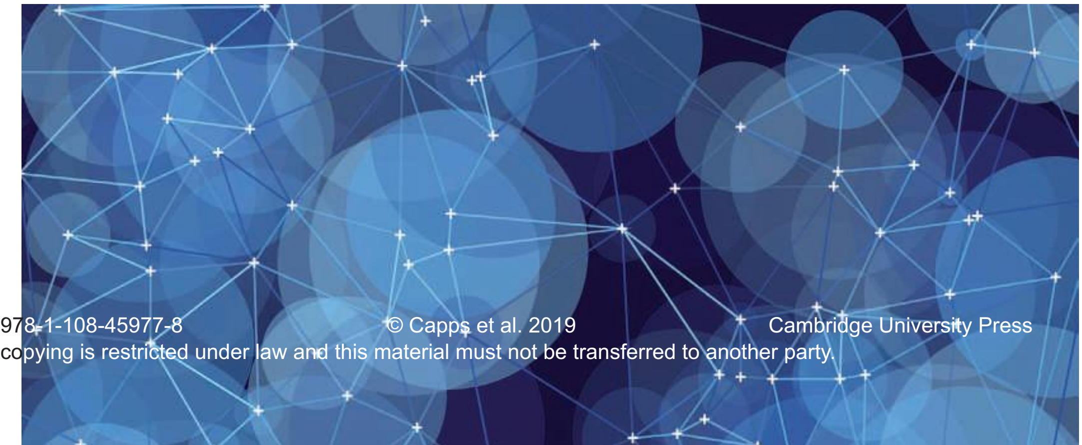


c



Round your answer to 2 decimal places.

WORKING	THINKING
a $A = lw$	Calculate the area of the rectangle using $A = lw$.
$A = l \times w$	
$A = 3.7 \times 2.3$	Substitute the values for l and w.
$A = 8.51 \text{ m}^2$	
b $A = \frac{1}{2}bh$	Calculate the area of the triangle using $A = \frac{1}{2}bh$.
$A = \frac{1}{2}(b \times h)$	
$A = \frac{1}{2}(54 \times 24)$	Substitute the values for b and h.
$A = 648 \text{ cm}^2$	Note: The height must be the *perpendicular* height, meaning it meets the base at a right angle (i.e. 24 is used here)
c $r = 8 \div 2 = 4$	Calculate the radius by dividing the diameter in half.
$A = \pi r^2$	Calculate the area of the circle using $A = \pi r^2$.
$A = \pi \times r^2$	
$A = \pi \times 4^2$	Substitute the value for r.
$A = 50.27 \text{ m}^2$	Round your answer to 2 decimal places.



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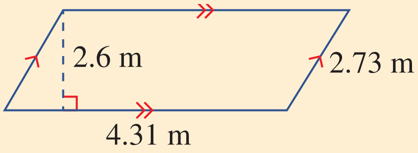
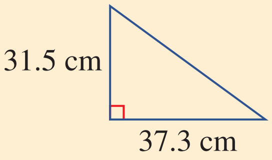
Example 5 Estimating the area of common 2D shapes

Estimate the area of the following common 2D shapes by rounding the dimensions to the nearest whole number.

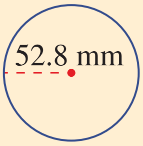
a



b



c



Round your answer to 2 decimal places.

WORKING	THINKING
a 2.6 m ≈ 3 m 4.31 m ≈ 4 m	Begin by rounding your dimensions to the nearest whole number. Look at the first decimal place, if it is 0.5 or more, then round your whole number up, if it is less than 0.5, then drop the decimal place. Note: The perpendicular height of the parallelogram is used (i.e. 2.6 m from this shape)
$A = b \times h$	Calculate the area of the rectangle using $A = b \times h$.
$A = 4 \times 3$ $A = 12\text{ m}^2$	Substitute the values for b and h using the whole numbers to estimate the area of the parallelogram.
b 37.3 ≈ 37 cm 31.5 ≈ 32 cm	Begin by rounding your dimensions to the nearest whole number.
$A = \frac{1}{2}(b \times h)$	Calculate the area of the triangle using $A = \frac{1}{2}(b \times h)$.
$A = \frac{1}{2}(37 \times 32)$ $A = 592\text{ cm}^2$	Substitute the rounded values for b and h.
c 52.8 ≈ 53 mm	Begin by rounding the value to the nearest whole number.
$A = \pi \times r^2$	Calculate the area of the circle using $A = \pi \times r^2$.
$A = \pi \times 53^2$	Substitute the value for r.
$A = 8824.73\text{ mm}^2$	Round your answer to 2 decimal places.

Example 6 Applying the area of a shape to practical problems

Michelle is painting the top surface of a circular stage that has a diameter of 2.4 m. Calculate the area that Michelle will be painting.

WORKING	THINKING
$2.4 \div 2 = 1.2$ m	Divide the diameter by 2 to calculate the radius of the stage.
$A = \pi \times r^2$	Given that the surface is circular, calculate the area using $A = \pi \times r^2$.
$A = \pi \times 1.2^2$	Substitute the value for r.
$A = 4.52\ \text{m}^2$	
Michelle will be painting an area that is $4.52\ \text{m}^2$.	Communicate your answer in a sentence.

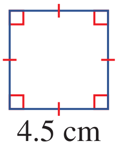
Exercise 2B

FUNDAMENTALS

Example 4

1 Calculate the area of the following common 2D shapes. Round to 2 decimal places where necessary.

a



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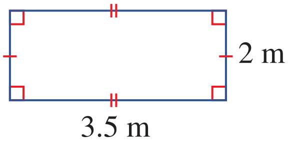
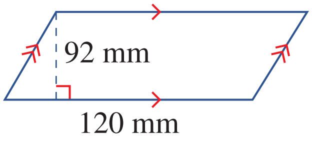


c



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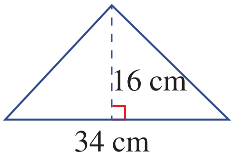
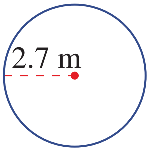


e



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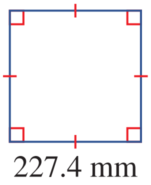

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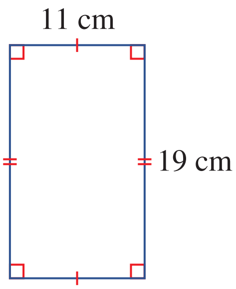
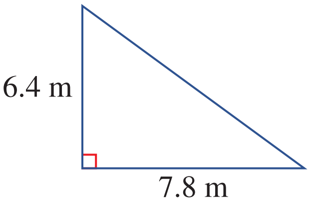


h



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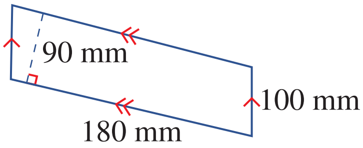

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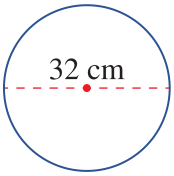
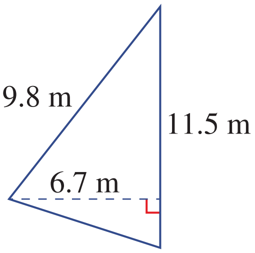
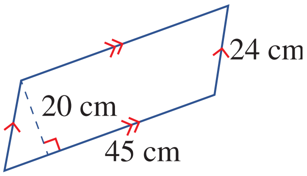
j



k



l

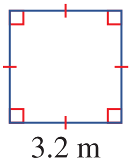


Hint The height must be the perpendicular height for triangles and parallelograms.

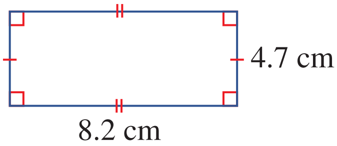
Example 5

2 Estimate the area of the following common 2D shapes by rounding the dimensions to the nearest whole number.

a



b



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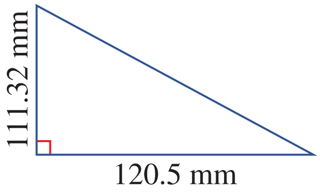

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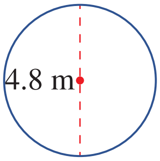
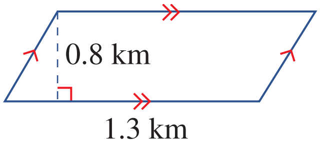
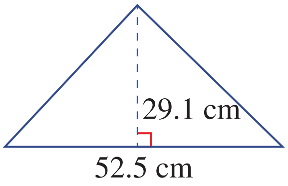


Hint To estimate look at the first decimal place. If it is 0.5 or more, then round your whole number up. If it is less than 0.5, then drop the decimal place.

e



f



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APPLICATIONS

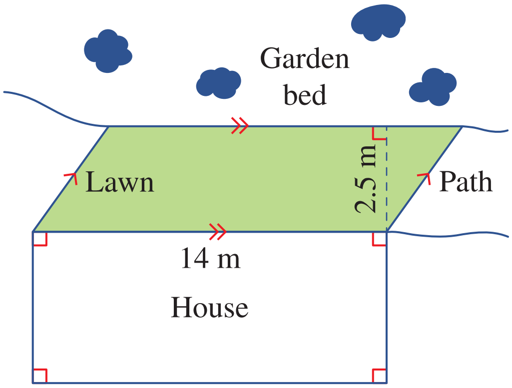
SF: 3–9	CF: –	CU: –

Example 6

3 Jarred is covering a display board with blue paper. The board is a rectangle and it has a length of 3.5 m and height of 1.5 m. Calculate the area that Jarred will be covering in blue paper.

★**4** Haley is tiling the floor of her 2.8 metre square bathroom. Calculate the area that Haley is tiling.

★**5** Ryan is laying an area of lawn that runs parallel between his house and the garden bed. The image below is the outline of the patch that he is turfing. Calculate the area of the new lawn.



★**6** Chelsea is painting the triangular section on the top of her house. The triangle has a base of 8.2 m and a perpendicular height of 3.2 m. Calculate the area that Chelsea is painting so that she can calculate the amount of paint required.

★7 Nathan is constructing a circular sandpit for his children. The sandpit will have a radius of 900 mm. Calculate the area of the sandpit.

★8 Dane has been commissioned to paint a mural on the side of a building. The mural will be rectangular and has a width of 5.7 m and a height of 3 m. Calculate the area of the mural.

9 Graham has just purchased some rectangular shaped land that is 220 m by 100 m. He knows that he can have 10 sheep per hectare, and he would like to have as many sheep as possible on his land.

a Calculate the area of Graham's land in hectares.

b Determine how many sheep Graham can keep on his new property.

Hint 1 hectare is 10 000 m^2.

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2C Calculating the area of trapeziums, sectors and composite figures COMPLEX

LEARNING GOALS

- Apply the formula to calculate the area of trapeziums and sectors
- Apply knowledge of regular shapes to decompose composite shapes
- Calculate the area of composite shapes

Why is it essential to understand how to calculate the area of trapeziums, sectors and composite shapes?

In the previous chapter, there are many real-life reasons where you would need to calculate the area of these shapes. Trapeziums and sectors can be found in construction, painting, town planning, fashion and even cooking. The majority of shapes around the home are actually composite shapes.

Understanding area and composite shapes can help with clothing design and manufacture.

WHAT YOU NEED TO KNOW

- A **trapezium** has 4 sides, two of which are parallel.
- A **sector** is a portion of a circle bounded by two **radii** and a segment of the **circumference**. The two radii form a **vertex** at the centre of a circle. The size of the sector can be specified by the length of the radii and the angle between the two radii at the vertex.

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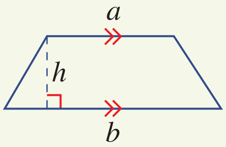
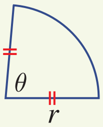
- The formulas for the areas of a sector and a trapezium are in the table below.

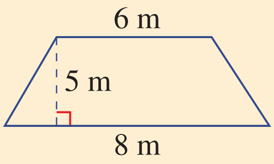
Shape	Area formula
Trapezium 	$A = \frac{1}{2}(a + b) \times h$ or $A = \frac{a + b}{2} \times h$
Sector 	$A = \frac{\theta}{360} \times \pi \times r^2$

- Abbreviations: a and b are lengths of sides, h height, r radius, π (pi) = 3.142 rounded, θ (theta) is unknown angle.
- Units of area include square millimetres, square centimetres, square metres, square kilometres and hectares. These can be abbreviated to mm^2, cm^2, m^2, km^2 and ha.
- A **composite shape** is a shape that is made up of two or more common shapes. Addition and/or subtraction can be used to find the area of composite shapes.
- You will also need to recall the area formulas of the shapes from the previous section.

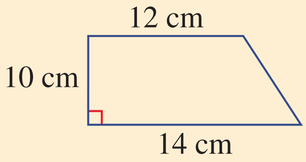
Example 7 Calculating the area of a trapezium

Calculate the area of the following trapeziums.

a



b



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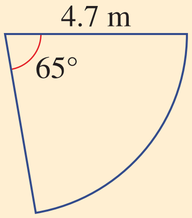
WORKING	THINKING
a $A=\frac{1}{2}\times(a+b)\times h$	Calculate the area of the trapezium using $A=\frac{1}{2}\times(a+b)\times h$.
$A=\frac{1}{2}\times(6+8)\times 5$	Substitute the values for *a*, *b* and *h*.
$A=35\text{ m}^2$	
b $A=\frac{1}{2}\times(a+b)\times h$	Calculate the area of the trapezium using $A=\frac{1}{2}\times(a+b)\times h$.
$A=\frac{1}{2}\times(12+14)\times 10$	Substitute the values for *a*, *b* and *h*.
$A=130\text{ cm}^2$	

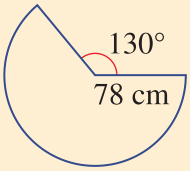
Example 8 Calculating the area of a sector

Calculate the area of the following sectors. Round your answer to 2 decimal places.

a



b

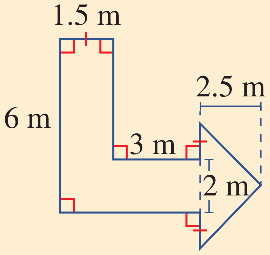


WORKING	THINKING
a $A=\frac{\theta}{360}\times\pi\times r^2$	Calculate the area of the sector using $A=\frac{\theta}{360}\times\pi\times r^2$.
$A=\frac{65}{360}\times\pi\times 4.7^2$	Substitute the values for the internal angle (θ) and the radius (r).
$A=12.53\text{ m}^2$	Round your answer to 2 decimal places.
b $360°-130°=230°$	As the shape is a sector, calculate the internal angle first by subtracting the external angle from 360°.
$A=\frac{\theta}{360}\times\pi\times r^2$	Calculate the area of the sector using $A=\frac{\theta}{360}\times\pi\times r^2$.
$A=\frac{230}{360}\times\pi\times 78^2$	Substitute the values for the internal angle (θ) and the radius (r).
$A=12\,211.37\text{ cm}^2$	Round your answer to 2 decimal places.

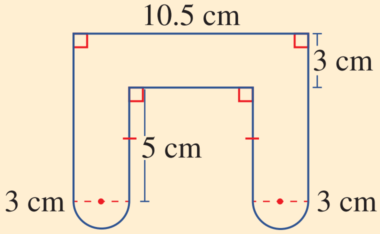
Example 9 Calculating the area of composite shapes with addition

Calculate the area of the following composite shapes, correct to 2 decimal places if needed.

a



b



WORKING	THINKING
a There are 2 rectangles and one triangle.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate area of rectangles by using the formula.
▯ $A = 6 \times 1.5 = 9 \text{ m}^2$	Substitute the values for *l* and *w*.
▭ $A = 3 \times 2 = 6 \text{ m}^2$	
▷ $A = \frac{1}{2} \times b \times h$	Calculate area of triangle using the formula.
$h = 2.5 \text{ m},\ b = 1.5 + 2 + 1.5$ $= 5 \text{ m}$	The unlabelled parts of the base have tick marks indicating they are same as the side labelled 1.5 m.
$A = \frac{1}{2} \times (5 \times 2.5) = 6.25 \text{ m}^2$	Substitute the values for *b* and *h*.
$A = 9 + 6 + 6.25$ $A = 21.25 \text{ m}^2$	Add all the shapes together to find the total area of the composite shape.
b There are 3 rectangles and two semi-circles.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate area of all rectangles using the formula.
▭ $A = 10.5 \times 3 = 31.5 \text{ cm}^2$	Substitute the values for *l* and *w*.
▯ $A = 5 \times 3 = 15 \text{ cm}^2$	
▯ $A = 5 \times 3 = 15 \text{ cm}^2$	
$r = 3 \div 2 = 1.5$	Both semi-circles have a diameter of 3 cm so they can make one whole circle when combined. The radius of the circle is half of the diameter.
$A = \pi \times r^2$	Calculate area of the circle using the formula.
○ $A = \pi \times 1.5^2$	Substitute the value for *r*.
$A \approx 7.07 \text{ cm}^2$	Round your answer to 2 decimal places.
$A = 31.5 + 15 + 15 + 7.07$ $A = 68.57 \text{ cm}^2$	Add all the shapes together to find the total area of the composite shape.

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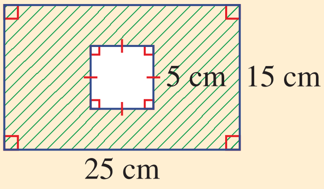
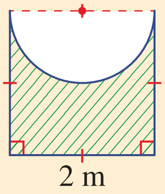
Example 10 Calculating the area of composite shapes with subtraction

For the following composite shapes, calculate the area of the shaded section, correct to 2 decimal places if needed.

a



b



WORKING	THINKING
a There is one rectangle and one square that is cut out from the centre.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate the area of the rectangle using $A = l \times w$.
■ $A = 25 \times 15 = 375\text{ cm}^2$	Substitute the values for l and w.
$A = l \times l$	Calculate the area of the square using $A = l \times l$.
□ $A = 5 \times 5 = 25\text{ cm}^2$	Substitute the value for l.
$A = 375 - 25 = 350\text{ cm}^2$	Subtract the square from the rectangle to find the shaded area.
b There is one square and one semi-circle that is cut out from the top of the square.	Decompose the composite shape by identifying the regular shapes.
$A = l \times l$	Calculate the area of the square using $A = l \times l$.
■ $A = 2 \times 2 = 4\text{ m}^2$	Substitute the value for l.
$r = 2 \div 2 = 1\text{ m}$	Find the radius by halving the diameter (divide by 2).
$A = \pi \times r^2$	Calculate the area of the circle using $A = \pi \times r^2$.
$A = \pi \times 1^2 = 3.14\text{ m}^2$	Substitute the value for r.
◡ Area of semi-circle $A = 3.14 \div 2$ $A = 1.57\text{ m}^2$	Divide the area of the circle by two, as the shape is half of a circle (semi-circle).
$A = 4 - 1.57$ $A = 2.43\text{ m}^2$	Subtract the semi-circle from the square to find the shaded area.

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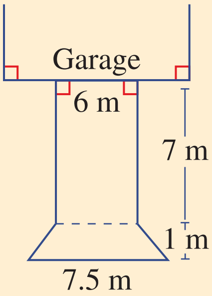
Example 11 Applying the area of composite shapes to practical problems

A driveway, as shown in the diagram, is to be concreted. Calculate the area of the driveway, correct to 2 decimal places.



WORKING	THINKING
There is one rectangle and one trapezium.	Decompose the composite shape by identifying the regular shapes.
$A = l \times w$	Calculate the area of the rectangle using $A = l \times w$.
▯ $A = 7 \times 6 = 42 \text{ m}^2$	Substitute the values l and w.
$A = \frac{1}{2} \times (a + b) \times h$	Calculate the area of the trapezium using $A = \frac{1}{2} \times (a + b) \times h$.
⏢ $A = \frac{1}{2} \times (6 + 7.5) \times 1 = 6.75 \text{ m}^2$	Substitute the values for a, b and h.
$A = 42 + 6.75$ $A = 48.75 \text{ m}^2$	Add the two areas together to calculate the total area of the composite shape.
The area of the driveway is 48.75 m².	Communicate your answer using words.

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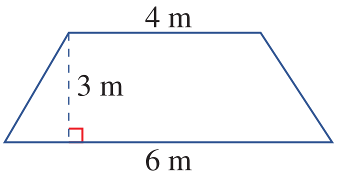
Exercise 2C

FUNDAMENTALS

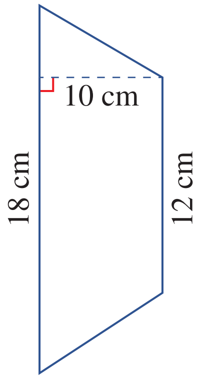
Example 7

1 Calculate the area of the following trapeziums.

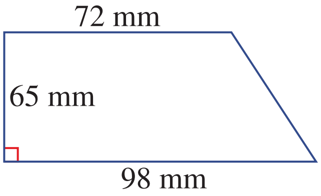
a



b



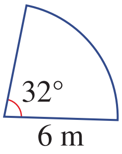
c



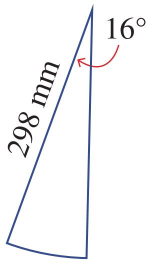
Example 8

2 Calculate the area of the following sectors.

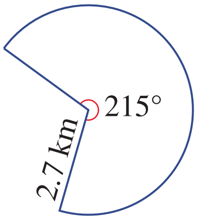
a



b

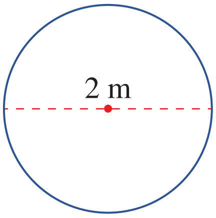


c

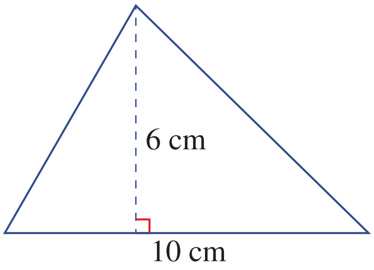


3 Calculate the area of the following common 2D shapes.

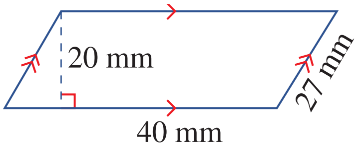
a



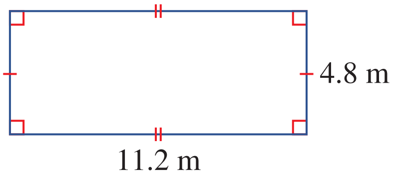
b



c



d



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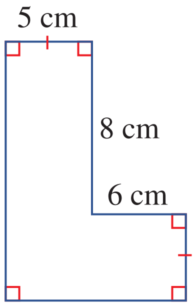
Example 9

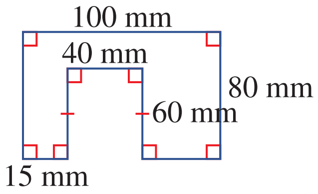
4 Determine the area of the following composite shapes by first decomposing the shapes into regular shapes and then adding the areas of the shapes.

Hint Redraw the shapes into the common shapes to help you to calculate the total area.

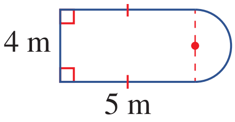
a



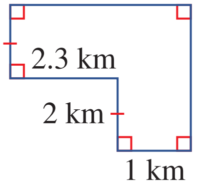
b



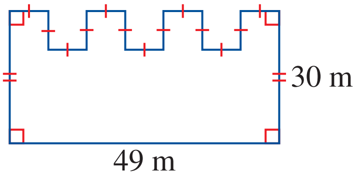
c



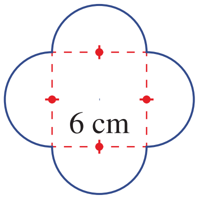
d



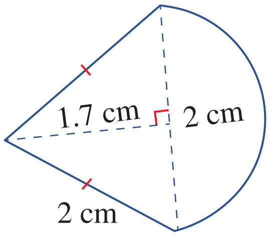
e



f



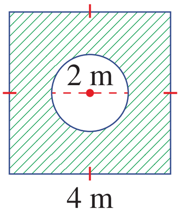
g



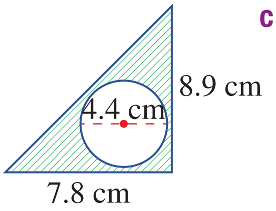
Example 10

5 Determine the area of the shaded section in the following composite shapes by first decomposing the shape into regular shapes and then subtracting the sections that are not shaded. All angles that look like right angles are 90°.

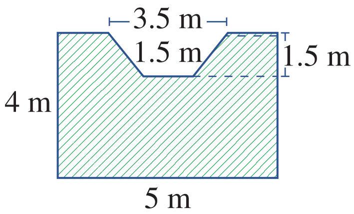
a



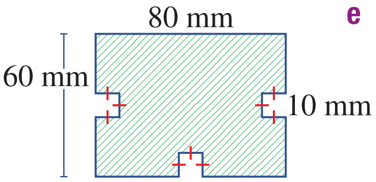
b



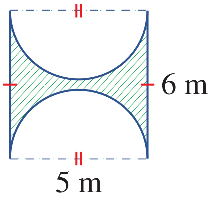
c



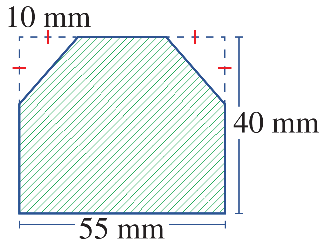
d



e



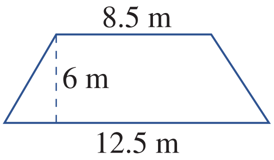
f



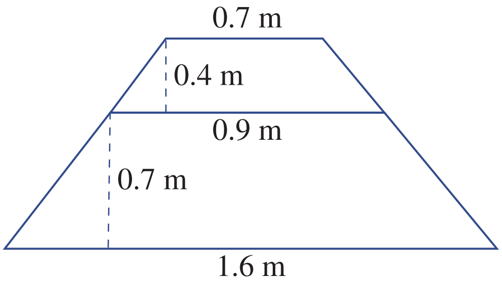
APPLICATIONS

SF: –	CF: 6–9	CU: 10–12

Example 11 ★**6** Ben is planning on making a new shade sail to run from the roof line of his house, across his deck and on to some posts that are near his pool. The design of the sail is shown below. Calculate the area of the material that Ben needs for the sail.



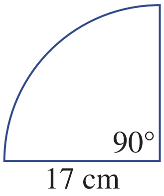
★**7** Hans has been commissioned to re-paint a fading sign of a pizza restaurant. He is calculating how much red paint he will require for the section of the sign shown. He has discovered that the sign makes two trapeziums and has measured the dimensions shown. Calculate the total area that will require red paint.



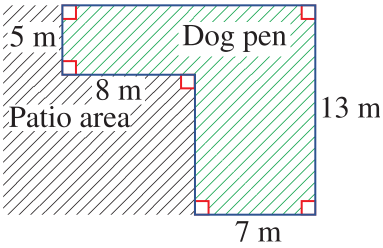
★**8** Leonie has ordered a pizza that has a diameter of 34 cm. She has eaten a quarter of the pizza.

a Use the diagram to calculate the area of pizza that Leonie has consumed.

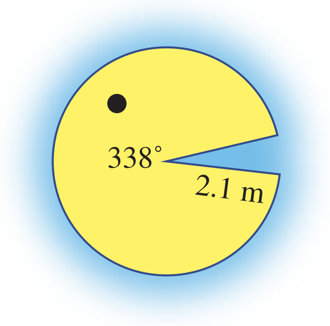
b Determine the area of the pizza that will be left over for Leonie's friends to consume.



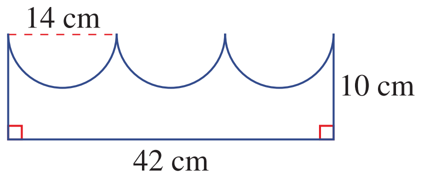
★**9** Ruby is laying a large patch of grass area next to her patio, as shown in the diagram, to create a dog pen. Calculate the total area of turf that Ruby needs to purchase.



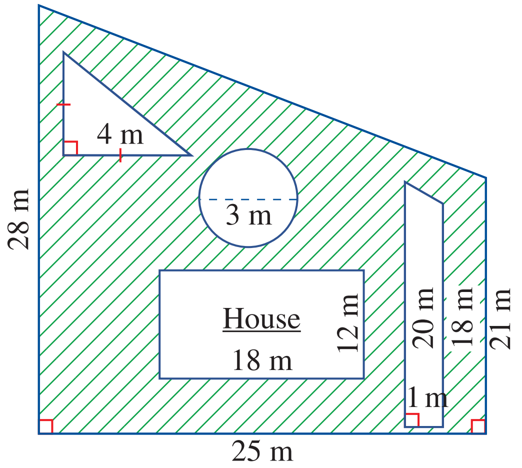
★**10** Randal has been commissioned to paint a large logo on a billboard. He needs to buy the correct amount of yellow paint that covers the area of the logo in the image below. Randal will need to multiply the total area by two as he will need to complete two coats of paint on the logo. If 1 litre of paint covers 15 m^2, determine how many tins of paint Randal will need. Note: Randal will start with yellow paint and will paint the eye of the icon over the top.



★**11** Anaya is making 8 crowns for her daughter's 5th birthday party. She plans on cutting the crowns out of cardboard and covering the outside surface in glitter. The image shown is an outline for the crowns, calculate the total area that Anaya will need to cover.



★**12** Below is an image of Shekila's property. She has outlined her house, water feature and two garden beds. Shekila would like to lay lawn everywhere else on her property. Calculate the total area of Shekila's lawn.



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2D Calculating the surface areas of cubes, prisms and pyramids COMPLEX

LEARNING GOALS

- Draw 2D representations in the form of a net, to show all faces of a 3D solid
- Calculate the surface area of cubes, rectangular and triangular prisms
- Calculate the surface area of triangular-based pyramids
- Calculate the surface area of square-based and rectangular-based pyramids

Why is it essential to calculate the surface area of 3D solids?

Calculating the surface area of 3D objects allows us to calculate how much material is needed to construct the surface of the object or to paint, cover or finish it. Applying yourself to learning this skill will also help you to develop your spatial awareness.

The amount of paint required to cover the surface area of a composite solid can be found by adding the surface areas of the visible regular shapes.

WHAT YOU NEED TO KNOW

- A **prism** is a 3D solid with flat faces, where each **cross-section** in a particular direction is the same shape and size. These cross-sections are shown in the middle column of the table below.
- A **pyramid** has a **polygon** for a base and all the other faces are triangular and meet at one **vertex** (a point). In this course, we only deal with pyramids that have a square, rectangular or triangular base. They are also **right pyramids** which means the vertex is vertically above the centre of the base. Also, the faces (other than the base) are of equal size in triangular and square-based right pyramids. Opposite faces are of equal size in rectangular-based right pyramids.
- When calculating the surface area of a prism or pyramid, start by drawing the **net** of the solid to identify all of its faces.
- A net is a flat shape (2D) made up of the faces that can be folded up into the 3D solid. To identify the net, imagine unfolding all of the faces of the 3D solid

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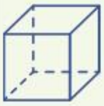
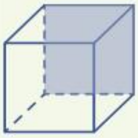
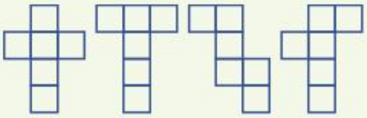
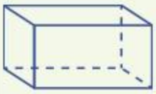
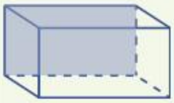
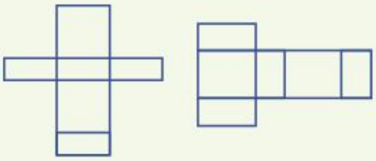
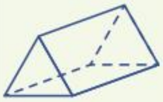
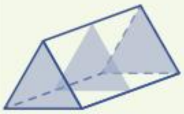
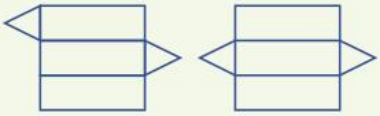
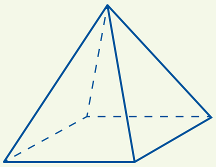
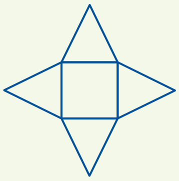
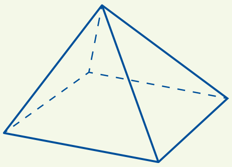
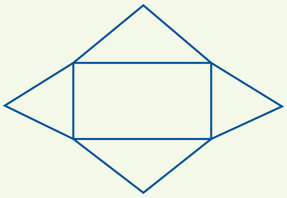
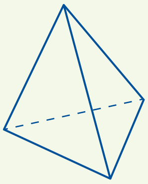
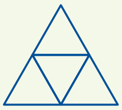
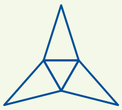
to make a flat shape. A prism will have different nets depending on which faces are left joined and which faces are separated.

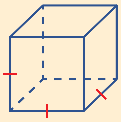
- To calculate the surface area of a 3D solid, calculate the area of all of the **faces** (the 2D shapes) in the net and add them up.
- The formula for the area of a triangular face is $A = \frac{1}{2}(b \times h)$ where A is area, b is the length of the base, and h is the perpendicular height of the triangle.
- The formula for the area of a square face is $A = l^2$.
- The formula for the area of a rectangular face is $A = l \times w$.
- After drawing the net, you may find it helpful to group identical faces together and to draw each type, with dimensions. This is shown in Example 13 and 14.
- There are different types of triangular-based pyramids so be sure to use the correct base and height for each triangle when applying your formula.

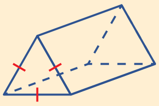
Prism	**Shaded cross-sections**	**Faces**	**2D net representation**
Cube 	A square is the cross-section. 	6 square faces	
Rectangular prism 	A rectangle is the cross-section. 	6 rectangle faces	
Triangular prism 	A triangle is the cross-section. 	2 triangle faces that are the same size and shape 3 rectangle faces	

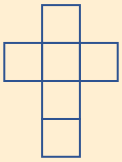
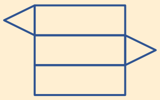
Pyramid	**Faces**	**2D net representation**
Square-based pyramid 	1 square face that is the base 4 triangle faces	
Rectangular-based pyramid 	1 rectangle face that is the base 4 triangle faces	
Triangular-based pyramid 	4 triangle faces	 or 

Example 12 Identifying and drawing the net of a solid

Draw a net to represent the following 3D solids.

a 

b 

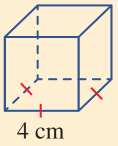
WORKING	THINKING
a 	There are many possible solutions to the design of both nets, all of which are accurate. Check that you have the correct number of faces, edges and vertices to form the solid. This 3D solid is a cube, so all six faces are squares.
b 	This 3D solid is a triangular prism, so there are 2 triangles that are the ends. The other 4 faces are rectangles.

Note: Other answers are possible.

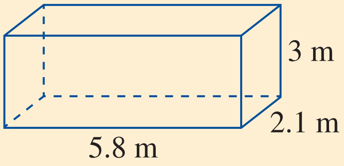
Example 13 Calculating the surface area of a cube, rectangular prism and triangular prism

Calculate the surface area of the following 3D solids.

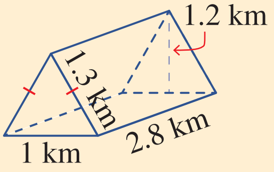
a

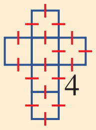
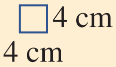
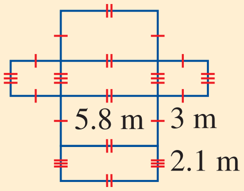
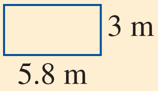
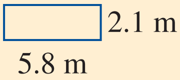
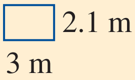


b



c



WORKING	THINKING
a 	Draw a net of the cube, including the dimensions.
	There are 6 square faces in a cube.
$A = l \times l$	Calculate the area of one square using $A = l \times l$.
$A = 4 \times 4 = 16\ \text{cm}^2$	Substitute the value for l.
Total surface area $= 16 \times 6$ $= 96\ \text{cm}^2$	There are 6 identical square faces in the cube so calculate multiply the area of one square by 6.
b 	Draw a net of the rectangular prism, including the dimensions.
	There are 3 pairs of rectangles.
$A1$ 	Two rectangles that are 5.8 m by 3 m.
$A2$ 	Two rectangles that are 5.8 m by 2.1 m.
$A3$ 	Two ends of the prism make two smaller rectangles that are 3 m by 2.1 m.
$A = l \times w$	Calculate the area of each rectangle pair using $A = l \times w$.
$A1 = 5.8 \times 3 = 17.4\ \text{m}^2$ $A2 = 5.8 \times 2.1 = 12.18\ \text{m}^2$ $A3 = 2.1 \times 3 = 6.3\ \text{m}^2$	Substitute the values for l and w in each rectangle pair.

Continued on the next page

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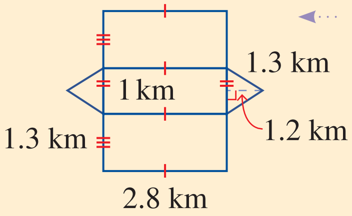
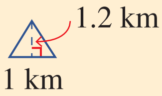
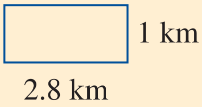
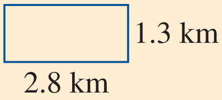
WORKING	THINKING
Total surface area $= (17.4 \times 2) + (12.18 \times 2) + (6.3 \times 2)$ $= 34.8 + 24.36 + 12.6 = 71.76\text{ m}^2$	Double each rectangle area and add all the areas together to find the total surface area of the rectangular prism.
c 	Draw a net of the triangular prism, including the dimensions. There is one pair of triangles, one pair of identical rectangles and one other rectangle.
$A1$ 	Two triangles have a base of 1 km and a height of 1.2 km.
$A2$ 	Two identical rectangles are 2.8 km by 1 km.
$A3$ 	One other rectangle is 2.8 km by 1.3 km.
$A = \frac{1}{2} \times b \times h$	Calculate the area of the triangles using $A = \frac{1}{2} \times b \times h$.
$A1 = \frac{1}{2} \times (1 \times 1.2) = 0.6\text{ km}^2$	Substitute the values for b and h.
$A = l \times w$	Calculate the area of each rectangle pair using $A = l \times w$.
$A2 = 2.8 \times 1.3 = 3.64\text{ km}^2$	Substitute the values for l and w.
$A3 = 2.8 \times 1 = 2.8\text{ km}^2$	
Total surface area $= (0.6 \times 2) + (3.64 \times 2) + 2.8$ $= 1.2 + 7.28 + 2.8 = 11.28\text{ km}^2$	Double rectangle area ($A2$) and the triangle area ($A1$). Add all the areas together to calculate the total surface area of the triangular prism.

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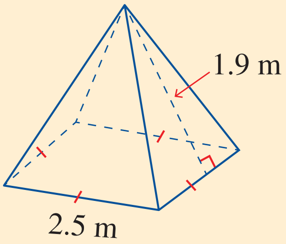
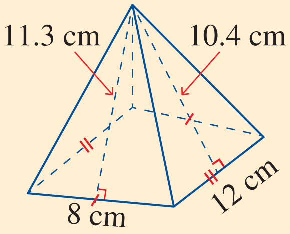
Example 14 Calculating the surface area of pyramids

Calculate the surface area for each of the following pyramids.

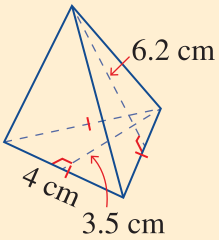
a



b

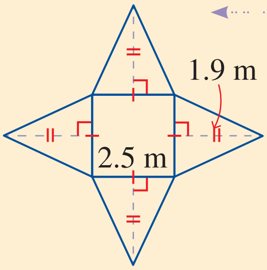


c



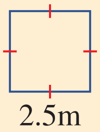
WORKING **THINKING**

a



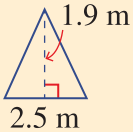
Draw a net of the square-based pyramid, including the dimensions.

$A1$



There is one square with side length of 2.5 m.

$A2$



There are four identical triangles with a base of 2.5 m and height of 1.9 m.

$A = l \times l$

Calculate the area of the square ($A1$) using $A = l \times l$.

$A1 = 2.5 \times 2.5 = 6.25\ \text{m}^2$

Substitute the values for l.

$A = \frac{1}{2}(b \times h)$

Calculate the area of the identical triangles ($A2$) using $A = \frac{1}{2}(b \times h)$.

$A2 = \frac{1}{2}(2.5 \times 1.9) = 2.375\ \text{m}^2$

Substitute the values for b and h.

Total surface area

$= 6.25 + (4 \times 2.375)$

$= 6.25 + 9.5$

$= 15.75\ \text{m}^2$

Multiply the area for the identical triangles by 4. Add the area of the square base to the area of the 4 triangles to calculate the total surface area of the square-based pyramid.

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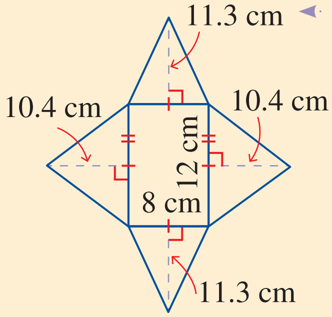
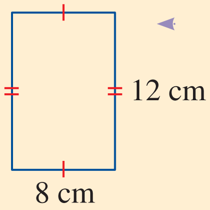
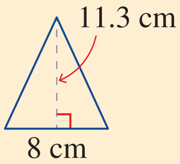
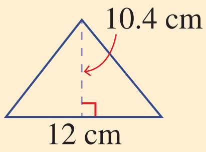
WORKING	THINKING
b 	Draw a net of the rectangular-based pyramid, including the dimensions.
$A1$ 	There is one rectangle with length 12 cm and width 8 cm.
$A2$ 	There are two identical triangles with base 8 cm and height 11.3 cm.
$A3$ 	There are two identical triangles with base 12 cm and height 10.4 cm.
$A = l \times w$	Calculate the area of the rectangle ($A1$) using $A = l + w$.
$A1 = 12 \times 8 = 96 \text{ cm}^2$	Substitute the values for l and w.
$A = \frac{1}{2}(b \times h)$	Calculate the area of the identical triangles ($A2$) using $A = \frac{1}{2}(b \times h)$.
$A2 = \frac{1}{2}(8 \times 11.3) = 45.2 \text{ cm}^2$	Substitute the values for b and h.
$A = \frac{1}{2}(b \times h)$	Calculate the area of the identical triangles ($A3$) using $A = \frac{1}{2}(b \times h)$.
$A3 = \frac{1}{2}(12 \times 10.4) = 62.4 \text{ cm}^2$	Substitute the values for b and h.
Total surface area $= (45.2 \times 2) + (62.4 \times 2) + 96$ $= 90.4 + 124.8 + 96$ $= 311.2 \text{ cm}^2$	Multiply the areas of the triangles by 2 (for $A2$ and $A3$). Add the triangle areas to the rectangle base to calculate the total surface area of the rectangular-based pyramid.

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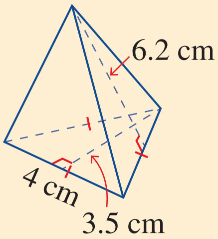
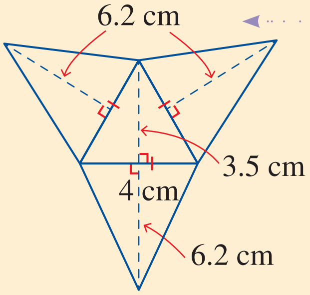
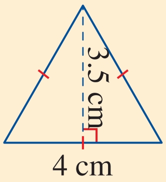
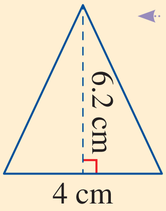
WORKING	THINKING
c 	*A copy of the diagram in the question is repeated here for convenience.*
	Draw a net of the triangular-based pyramid, including the dimensions.
$A1$ 	There is one base triangle with base 4 cm and height 3.5 cm.
$A2$ 	There are three identical triangles with base 4 cm and height 6.2 cm.
$A = \frac{1}{2}(b \times h)$	Calculate the area of the triangle ($A1$) using $A = \frac{1}{2}(b \times h)$.
$A1 = \frac{1}{2}(4 \times 3.5) = 7 \text{ cm}^2$	Substitute the values for b and h.
$A = \frac{1}{2}(b \times h)$	Calculate the area of the identical triangles ($A2$) using $A = \frac{1}{2}(b \times h)$.
$A2 = \frac{1}{2}(4 \times 6.2) = 12.4 \text{ cm}^2$	Substitute the values for b and h.
Total surface area $= (12.4 \times 3) + 7$ $= 37.2 + 7$ $= 44.2 \text{ cm}^2$	Multiply the area of the identical triangle by 3 ($A2$). Add the base triangle area to the other triangle areas to calculate the total surface area of the triangular-based pyramid.

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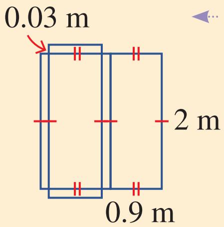
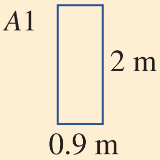
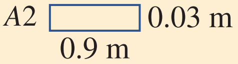
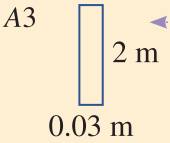
Example 15 Applying the surface area of 3D solids to practical problems

Cailin is painting the inside of her house. She has ten doors that she needs to remove from the hinges and paint. The doors are rectangular prisms in shape, as shown in the image. They are 0.9 m wide, 2 m high and 0.03 m thick. Determine the total surface area of all 10 doors that Cailin needs to paint.

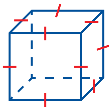
WORKING	THINKING
	Draw a net of the rectangular prism.
	There are three pairs of rectangles. Two rectangles that are 0.9 m by 2 m.
	Two rectangles that are 0.9 m by 0.03 m.
	Two rectangles that are 0.03 m by 2 m.
$A = l \times w$ $A1 = 0.9 \times 2 = 1.8\ \text{m}^2$	Calculate the area of the rectangles using $A = l \times w$.
$1.8 \times 2 = 3.6\ \text{m}^2$ $A2 = 0.9 \times 0.03 = 0.027\ \text{m}^2$ $0.027 \times 2 = 0.054\ \text{m}^2$ $A3 = 0.03 \times 2 = 0.06\ \text{m}^2$ $0.06 \times 2 = 0.12\ \text{m}^2$	Multiply the area of each pair of rectangles by 2.
Total surface area of one door $= 3.6 + 0.054 + 0.12 = 3.774\ \text{m}^2$	Add together the areas of the three pairs of rectangles to find the total surface area of the rectangular prism for one door.
Total surface area of 10 doors $= 3.774 \times 10 = 37.74\ \text{m}^2$	Multiply your answer by 10 to calculate the total surface area for all 10 doors.
Cailin will need to paint a surface area of 37.74 m².	Communicate your solution in a sentence.

Exercise 2D

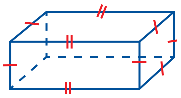
FUNDAMENTALS

Example 12 **1** Draw a net that represents each of the following 3D solids.

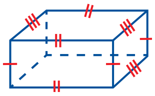
a



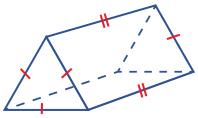
b



c

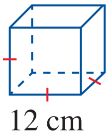


d

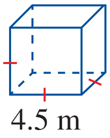


Example 13 **2** Calculate the surface area of the following cubes and rectangular prisms.

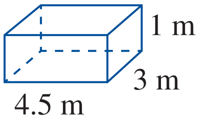
a



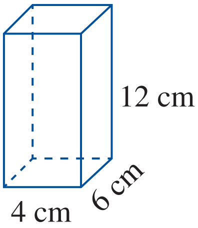
b



c

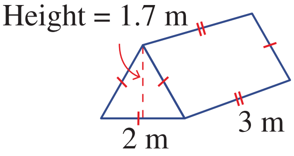


d

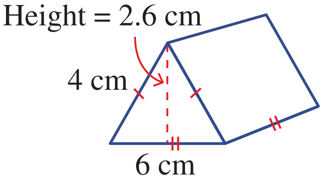


Example 13 **3** Calculate the surface area of the following triangular prisms.

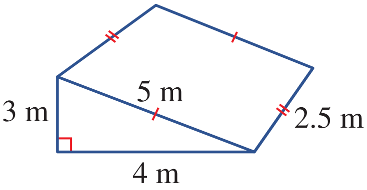
a



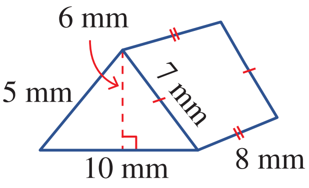
b



c



d

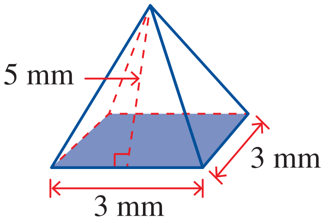


Hint Draw the net of each solid to help you work out the total surface area.

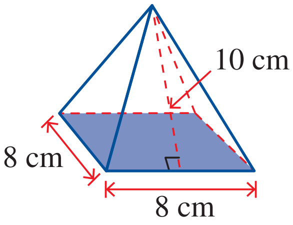
Example 14

4 Calculate the surface area of the following square-based pyramids.

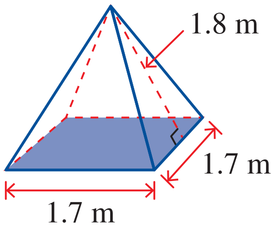
a



b



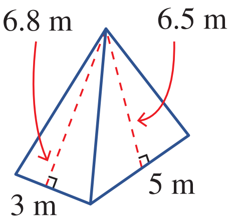
c



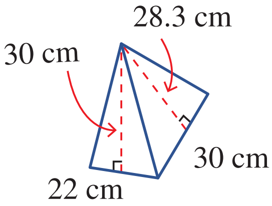
Example 14

5 Calculate the surface area of the following rectangular-based pyramids.

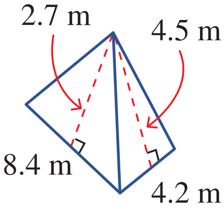
a



b

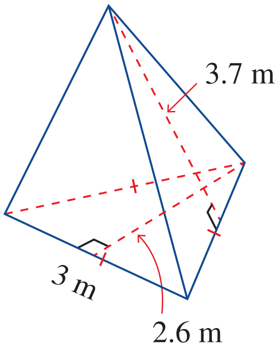


c

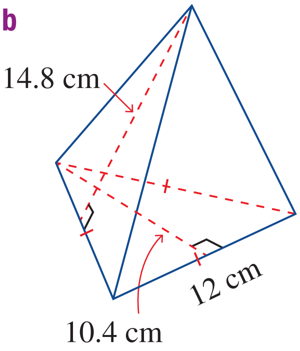


6 Calculate the surface area of the following triangular-based pyramids.

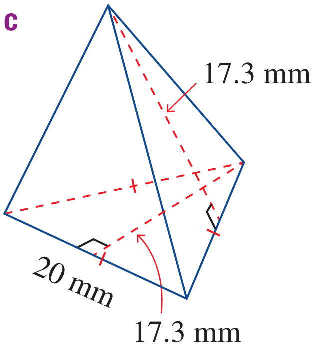
a



b



c



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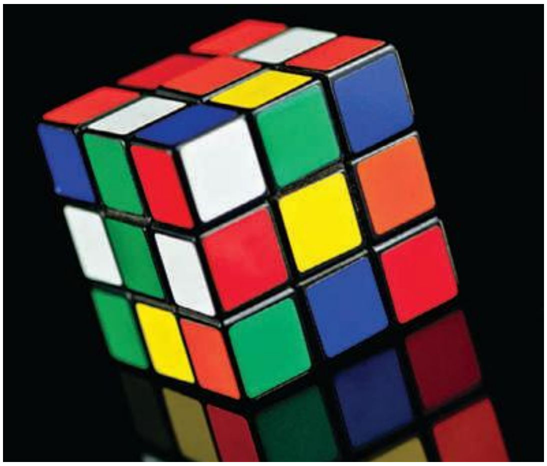
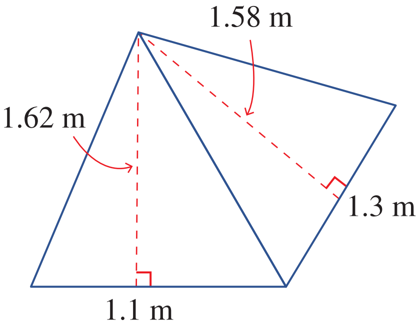
APPLICATIONS

SF: –	CF: 7	CU: 8–12

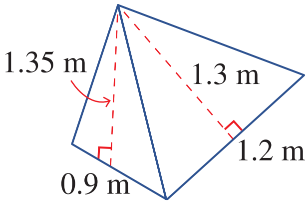
Example 15 ★**7** Scott is sewing together the outer cover of a foot stool that looks like a Rubik's cube. He would like the stool to be 60 cm square. Calculate the total area of material of all colours that Scott will require to cover the outside of the foot stool.



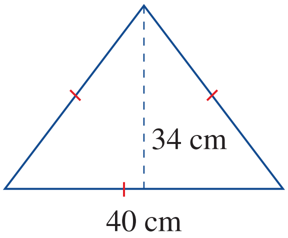
★**8** Michael is making a rectangular-based pyramid out of stainless steel to put on display in his garden. Each triangular face acts like a mirror, reflecting the plants around the pyramid. The rectangular base will be made out of stainless steel for strength. The plan for Michael's pyramid is shown in the diagram. Determine the total area of stainless steel required.



★**9** Anika makes children's play teepees and sells them at the markets. The design of her tepees are rectangular-based pyramids as shown in her sewing plan below. At her last market, she sold 8 teepees. Calculate the total area of material that Anika used to make the teepees that were sold at the market on that day.



★**10** Madison has won a giant Toblerone. She is planning on wrapping it in paper and giving it to her mum for Christmas. The Toblerone is 1.5 metres long and the measurements for the equilateral triangle at either end is shown in the diagram below. Determine the minimum total area of wrapping paper that Madison will require, ignoring the need to overlap the edges of the wrapping paper.



★**11** Renee is designing and sewing a tent for her husband. She plans on making it in the shape of a rectangular prism. It needs to be big enough that her husband can stand in it and it also needs to fit their double air mattress. Her husband is 1.8 metres tall and their air mattress is 1.6 metres wide and 2 metres long. Sketch a plan for Renee's tent and calculate the total area of material that she will require including the base (groundsheet).

★**12** The Eiffel Tower is a monument in Paris, France. It is similar in shape to a square-based pyramid with a base length of 99.9 metres and a height of 304 metres. Suppose a steel square-based pyramid was to be made to transport the Eiffel Tower. Its base length is 100 m and the perpendicular height of the triangles forming the four sides is 308 m. Determine the total surface area of steel required.

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2E Calculating the surface areas of spheres and cylinders COMPLEX

LEARNING GOALS

- Apply a formula to calculate the surface area of spheres
- Apply a formula to calculate the surface area of cylinders

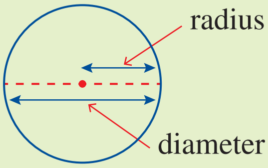
Why is it essential to calculate the surface area of spheres and cylinders?

Spheres and cylinders are used in many industries such as manufacturing, sporting and farming equipment. Being able to calculate the surface area of these shapes allows us to calculate things such as the amount of leather needed for a ball or the amount of steel needed for a silo.

The area of glass needed for this building can be calculated using the methods in this section.

WHAT YOU NEED TO KNOW

- A **sphere** is a perfectly round 3D solid.



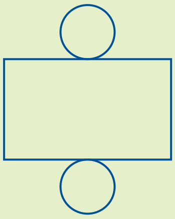
- Note that spheres do not have a net.

Formula for surface area of a sphere: $S = 4 \times \pi \times r^2$, where r stands for radius.

- The **radius** of a sphere is half of the **diameter**. The diameter is the straight-line distance from the surface through the centre of the sphere to the surface on the other side, while the radius is the distance from the centre to the surface.

- A hemisphere is half a sphere, so the area of its curved surface is half that of the sphere (if you need the area to include the flat circular base, add the area of a circle with the same radius as the sphere).
- A **closed cylinder** has two identical parallel faces that are flat circles joined by a rectangle or square curved into a tube.

- An **open cylinder** has either no circles at the ends, like a tube, or a circle at one end, like a container without a lid.
- The net of a closed cylinder, shown in the diagram, is made up of two circles joined by a rectangle or square. The length of the sides touching the circles is the same as the circumference of the circles.

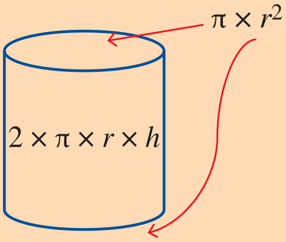


- To calculate the surface area of a closed cylinder, use the net (shown above) as a guide. The curved surface of the cylinder is a rectangle with a length that is the circumference of the circle, $2 \times \pi \times r$, and a width that is the height of the cylinder, h.

  So the area of the curved surface is $A = 2 \times \pi \times r \times h$.

  The ends of the cylinder are circles, so the $A = \pi \times r^2$. We have two circular ends, so we need to double the area of the circle.

$$S = \text{Area of curved rectangle} + 2 \times \text{Area of the circle}$$
$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$



- If the cylinder is closed at one end only, like a tin without a lid, we only need one circular end.

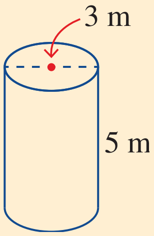
  So, $S = 2 \times \pi \times r \times h + \pi \times r^2$.
- If the cylinder is open at both ends, we do not need the two circular ends.

  So, $S = 2 \times \pi \times r \times h$.

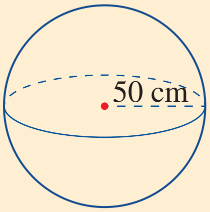
Example 16 Calculating the surface area of spheres and cylinders

Calculate the surface area of the following sphere and cylinder. Round your answers to 2 decimal places.

a



b



WORKING	THINKING
a Radius $= 3 \div 2 = 1.5$ m	Determine the radius first by halving the diameter.
Area of curved rectangular part $A = 2 \times \pi \times r \times h$ $A = 2 \times \pi \times 1.5 \times 5$ $A = 47.12\ \text{m}^2$ Area of 2 circles $A = 2 \times \pi \times r^2$ $A = 2 \times \pi \times 1.5^2$ $A = 14.14\ \text{m}^2$	Calculate the surface area of the cylinder by separating the cylinder into the curved rectangle and two circles. Area of curved rectangular part is $A = 2 \times \pi \times r \times h$ Area of circles is $A = 2 \times \pi \times r^2$
$S = 47.12 + 14.14$ $S = 61.26\ \text{m}^2$	Add together the parts of the surface area.
OR	OR
$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$ $S = 2 \times \pi \times 1.5 \times 5 + 2 \times \pi \times 1.5^2$ $S = 47.12 + 14.14$ $S = 61.26\ \text{m}^2$	By using the formula $S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$ Substitute the values for radius (r) and height (h). Round your answer to 2 decimal places.
b $S = 4 \times \pi \times r^2$ $S = 4 \times \pi \times 50^2$ $S = 31\,415.93\ \text{cm}^2$	Calculate the surface area of the sphere using the formula $S = 4 \times \pi \times r^2$. Substitute the value for the radius (r). Round your answer to 2 decimal places.

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Example 17 Applying the surface area of a cylinder to a practical problem

Lachlan enjoys four-wheel driving and fishing at the beach. He is making a storage container for his fishing rods to attach to his four-wheel drive ute. He has purchased a 5-metre long PVC pipe and two end caps. The caps and pipe have a diameter of 25 cm. He needs to calculate the surface area of the storage container including the end caps, so that he can paint it black to match his vehicle. Calculate the surface area of the storage container.

WORKING	THINKING
Radius $= 25 \div 2 = 12.5$ cm	Determine the radius first by halving the diameter.
$12.5 \div 100 = 0.125$ m	Convert centimetres to metres by dividing by 100. All units are now in metres.
	The storage container creates a cylinder.
$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$	Calculate the surface area of the cylinder using the formula $S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$.
$S = 2 \times \pi \times 0.125 \times 5 + 2 \times \pi \times 0.125^2$	Substitute the values for r and h.
$S = 4.03\ \text{m}^2$	Round your answer to 2 decimal places.
Lachlan needs to paint an area of $4.03\ \text{m}^2$.	Communicate your solution in a sentence.

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Exercise 2E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a The formula to calculate the surface area of a sphere is ________________.

b The formula to calculate the surface area of a cylinder is ________________.

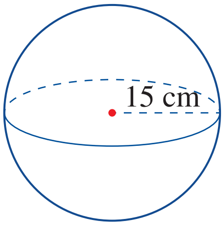
c $2 \times \pi \times r \times h$ calculates the area of the __________ __________ of a cylinder.

d $2 \times \pi \times r^2$ calculates the area of the __________ __________ of a cylinder.

Example 16

2 Calculate the surface area of the following spheres. Round your answer to 2 decimal places where necessary.

a



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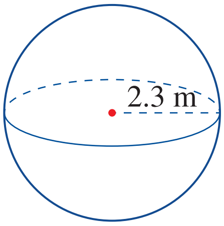

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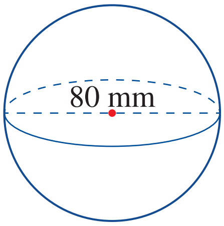

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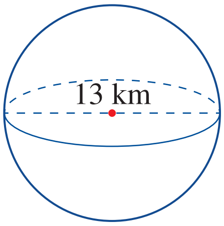
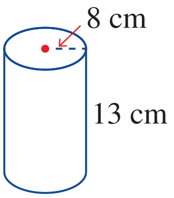


Hint $A = 4 \times \pi \times r^2$

3 Calculate the surface area of the following closed cylinders. Round your answer to 2 decimal places where necessary.

a



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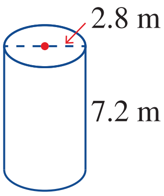

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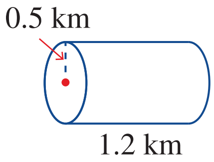
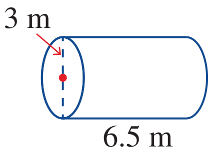
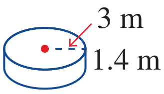


d



e



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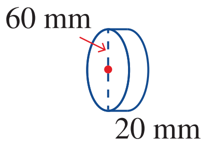

Hint $A = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$

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APPLICATIONS

SF: –	CF: 4–6	CU: 7–9

Example 17 ★**4** Georgie has saved a Pringles container that her mum was going to throw out. It has a height of 30 cm and a diameter of 8 cm. She plans on decorating the container so that she can store some craft items in it. Georgie will be covering the container with contact. Calculate the total surface area of the container.

★**5** Tanya is powder coating a pole that will be holding up a sail. It has a height of 3 m and a radius of 0.08 m. Calculate the total surface area of the pole.

★**6** Asher works in a factory that makes tennis balls. His job is to apply the glue that holds the felt on the ball. An average tennis ball has a diameter of 6.6 cm. Determine the area of glue required for each ball.

★**7** In the heart of Adelaide, South Australia, lies Rundle Mall where a giant sculpture by Bert Flugelman resides called 'On Further Reflection'. It is two stainless steel spheres, one on top of the other, which were gifted to the city in 1977. The spheres have a diameter of 2.15 metres. Calculate the total surface area of both spheres.

★**8** Earth has a diameter of 12 742 km. Determine the total surface area of the Earth assuming it is a sphere.

★**9** Kanku has been given a digeridoo to paint. He will be painting the external curved surface black as a base for his artwork. The digeridoo is a cylinder with a length of 130 cm and a radius of 1.6 cm. Determine the area of the curved surface that Kanku will be painting.

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2F Calculating the surface areas of irregular solids COMPLEX

LEARNING GOALS

- Calculate the surface area of cones
- Calculate the surface area of irregular solids

Why is it essential to calculate the surface area of irregular solids?

Not all objects are formed with the use of familiar solids. Sometimes we need to identify the familiar shapes within an object so that we can calculate the surface area of an irregular solid. This helps in careers such as painting and construction, along with baking.

The main shape of this gingerbread house is made from a triangular prism on top of a rectangular prism.

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WHAT YOU NEED TO KNOW

- For the purposes of this course, an **irregular solid** is either a **composite 3D solid** that consists of two or more of the common solids or it is part of a common solid, such as half a sphere or half a cylinder cut lengthways.
- We can calculate the surface area of irregular solids if their **nets** are formed from the 2D shapes.

Shape	**Formula**
Square	$A = l^2$
Rectangle	$A = l \times w$
Triangle	$A = \frac{1}{2}(b \times h)$
Parallelogram	$A = b \times h$
Trapezium	$A = \frac{1}{2} \times (a + b) \times h$
Circle	$A = \pi \times r^2$
Sector	$A = \frac{\theta}{360} \times \pi \times r^2$

- We may be able to calculate the surface area of irregular solids if they include spheres and cylinders (see the last section), and cones, which are introduced here.

Shape	**Formula**
Sphere	$S = 4 \times \pi \times r^2$
Closed cylinder	$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$
Cone	$S = \pi \times r \times s + \pi \times r^2$

- Surface area of a cone

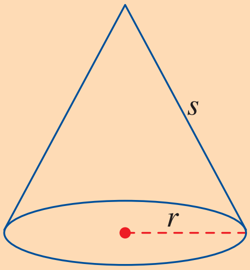
S = curved surface of cone + area of circle

$S = \pi \times r \times s + \pi \times r^2$

S is the total surface area, which includes the area of the circular base, $\pi \times r^2$.

s is the 'slant height' – the shortest distance between the apex and the circumference of the base.

If you only need the curved surface of the cone use $A = \pi \times r \times s$.



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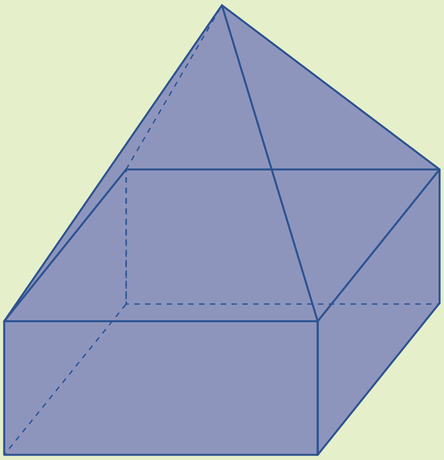
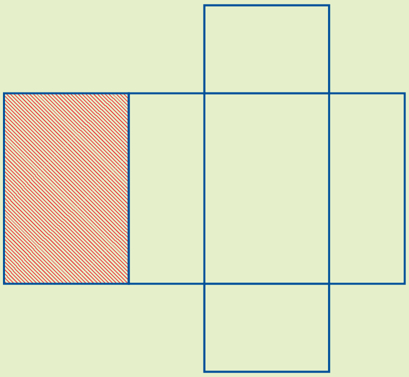
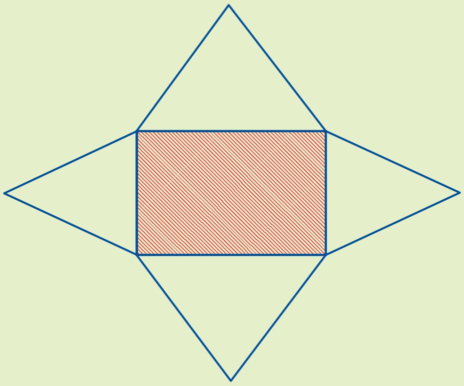
- For an irregular solid that includes a sphere, or a part of a sphere, calculate the surface area of the sphere separately. If the remaining part of the irregular solid has a net, calculate the surface area of the appropriate net. Add all of the surface areas together for the total surface area.
- For an irregular solid that includes a cylinder, or a part of a cylinder, calculate the area of the part of the net that is needed such as:
  - $S = 2 \times \pi \times r \times h + \pi \times r^2$ (a cylinder with only one closed end)
  - $S = 2 \times \pi \times r \times h$ (a cylinder with both ends open)
- For the total surface area of an irregular solid, you need to remember not to include any surfaces that are not visible from the outside of the solid.
  For example, shown is an irregular solid with a rectangular-based pyramid on top of a rectangular prism. Draw a sketch of the nets for each solid. Think about which parts are not visible in the solid:
  - In the rectangular prism, the top side is not included.
  - In the rectangular-based pyramid, the base rectangle is not included.







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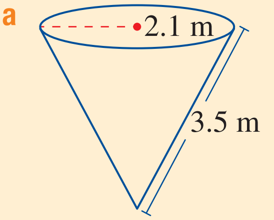
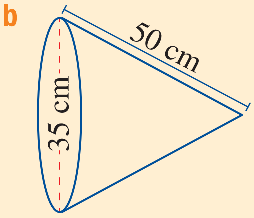
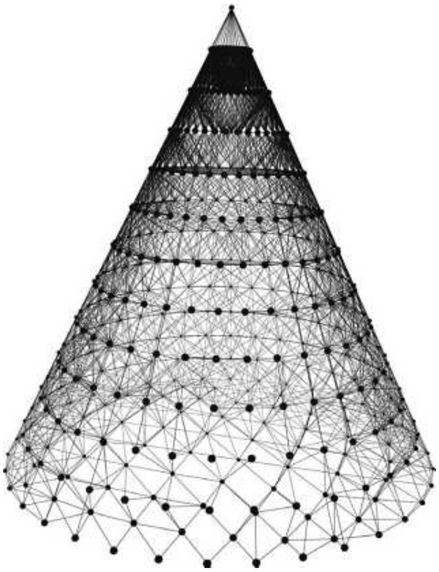
Example 18 Calculating the surface area of cones

Calculate the surface area of the following cones. Round your answer correct to 2 decimal places.





WORKING	THINKING
a $S = \pi \times r \times s + \pi \times r^2$	Calculate the surface area of the cone using $S = \pi \times r \times s + \pi \times r^2$.
$S = \pi \times 2.1 \times 3.5 + \pi \times 2.1^2$	Substitute the values of r and s.
$S = 36.95\text{ m}^2$	Round your answer to 2 decimal places.
b $35 \div 2 = 17.5\text{ cm}$	Determine the radius first by halving the diameter.
$S = \pi \times r \times s + \pi \times r^2$	Calculate the surface area of the cone using $S = \pi \times r \times s + \pi \times r^2$.
$S = \pi \times 17.5 \times 50 + \pi \times 17.5^2$	Substitute the values of r and s.
$S = 3711.01\text{ cm}^2$	Round your answer to 2 decimal places.



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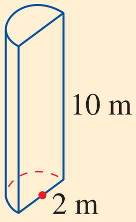
Example 19 Calculating the surface area of irregular solids

Calculate the surface area for each of the following irregular solids.

a



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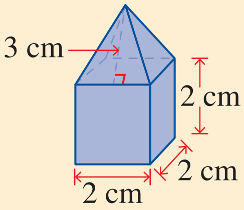

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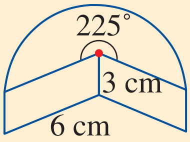
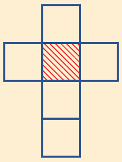


WORKING	THINKING
a Surface area for cylinder $S = 2\times\pi\times r\times h+2\times\pi\times r^2$ $S = 2\times\pi\times1\times10+2\times\pi\times1^2$ $S = 69.12\text{ m}^2$	The formula for finding the surface area of a cylinder is $S = 2\times\pi\times r\times h+2\times\pi\times r^2$ The radius is half the diameter, so radius is 1 m. Substitute $r = 1$ and $h = 10$ into formula.
Surface area for half of cylinder $S1 = 69.12 \div 2$ $S1 = 34.56\text{ m}^2$	This shape is only half a cylinder so divide your answer by 2.
Area of rectangle face $A = lw$ $S2 = 10\times2$ $S2 = 20\text{ m}^2$	Calculate the area of the rectangular face by using $A = lw$. Substitute $l = 10$ and $w = 2$.
Total surface area $= S1 + S2$ $= 34.56 + 20$ $= 54.56\text{ m}^2$	Calculate the total surface area by adding the surface for half the cylinder and the rectangle.
b Area of square base $A = 2\times2 = 4\text{ cm}^2$	Calculate the area of the square base by using $A = lw$.
Surface area of bottom cube $S1 = 4\times5 = 20\text{ cm}^2$ 	The top of the cube is not visible so instead of multiplying by 6, multiply by 5.

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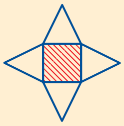
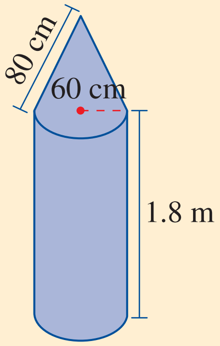
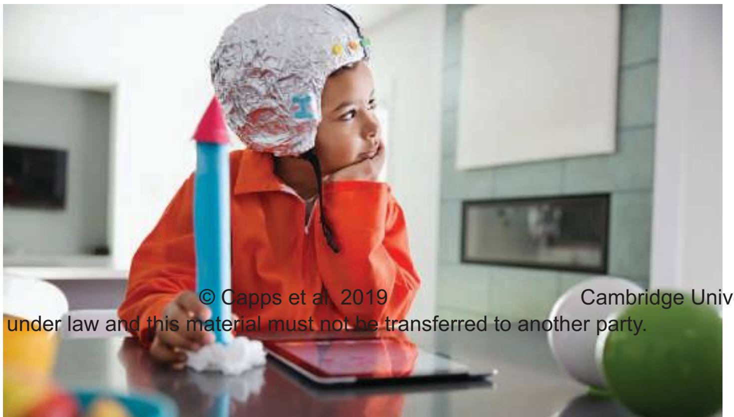
WORKING	THINKING
Surface of the triangle faces $A=\frac{1}{2}(b\times h)$ $A=\frac{1}{2}(2\times 3)=3\ \text{cm}^2$	Calculate the area of each triangle using the formula $A=\frac{1}{2}(b\times h)$.
$S2=3\times 4=12\ \text{cm}^2$ 	There are 4 triangles the same size so multiply this by 4.
Total surface area $=S1+S2$ $=20+12$ $=32\ \text{cm}^2$	Add both sections of the irregular solid to calculate total surface area.
c Area of the sector $A=\frac{\theta}{360}\times\pi\times r^2$ $A=\frac{225}{360}\times\pi\times 6^2=70.69\ \text{cm}^2$	Calculate the area of a sector using the formula $A=\frac{\theta}{360}\times\pi\times r^2$.
Surface area of two sectors $S1=70.69\times 2$ $S1=141.37\ \text{cm}^2$	The top and bottom of the irregular solid is the same sector so multiply this by 2.
Surface area of front two rectangles $A=lw$ $A=6\times 3=18\ \text{cm}^2$ $S2=18\times 2$ $S2=36\ \text{cm}^2$	Calculate the area of the two front panels, which are rectangles so use $A=lw$. As the rectangles are the same size multiply by 2.
Surface area of part of curved rectangle of a cylinder $A=\frac{\theta}{360}\times 2\times\pi\times r\times h$ $S3=\frac{225}{360}\times 2\times\pi\times 6\times 3$ $S3=70.69\ \text{cm}^2$	Calculate the area of the back panel, which is part of the curved rectangle of a cylinder so use $A=\frac{\theta}{360}\times 2\times\pi\times r\times h$.
$S=S1+S2+S3$ $S=141.37+36+70.69$ $S=248.06\ \text{cm}^2$	Add all surface area sections of the irregular solid to calculate total surface area.

Example 20 Applying the surface area of an irregular solid to practical problems

Melfred is painting a cardboard rocket that he has made with his son. Calculate the total surface area of the rocket. The ‘cap’ is a cone and the body is a cylinder. The base of the rocket is closed.



WORKING	THINKING
Radius: $60 \div 100 = 0.6$ m Slant height of cone: $80 \div 100 = 0.8$ m	Convert centimetres to metres by dividing by 100 as all measurements must be in the same units.
Surface area of cone without base $S1 = \pi \times 0.6 \times 0.8 \approx 1.51\text{ m}^2$ rounded to 2 d.p.	Calculate the surface area of the cone without the circular base by using the formula $A = \pi \times r \times s$.
Surface area of the cylinder closed at one end. $S2 = 2 \times \pi \times 0.6 \times 1.8 + \pi \times 0.6^2$ $S2 \approx 7.92\text{ m}^2$	Calculate the area of the cylinder by using the formula for a cylinder closed at one end. $A = 2 \times \pi \times r \times h + \pi \times r^2$
$S = S1 + S2$ $S = 1.51 + 7.92$ $S = 9.43\text{ m}^2$	Add all the areas together to calculate the total surface area of the composite solid (if you use unrounded values you will get 9.42 m^2).
The rocket has a total surface area of 9.43 m^2.	Communicate your solution using words.



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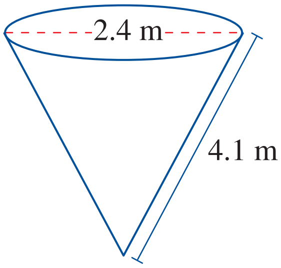
Exercise 2F

FUNDAMENTALS

Example 18 **1** Calculate the surface area of the following cones.

a



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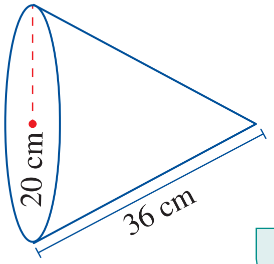

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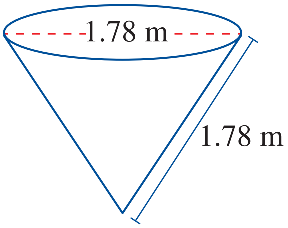
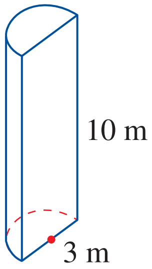


Hint $A = \pi \times r \times s + \pi \times r^2$

Example 19 **2** Calculate the surface area of the following irregular solids.

a



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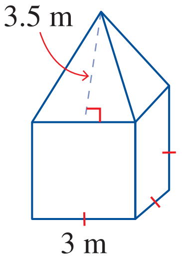

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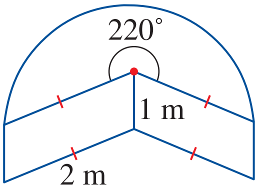

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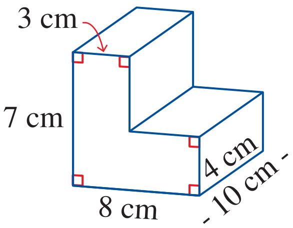

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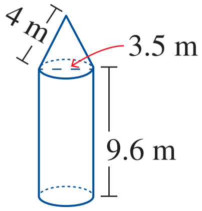

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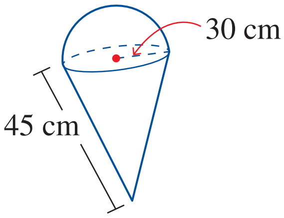
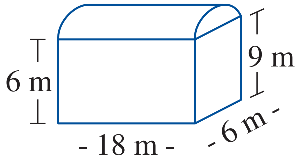
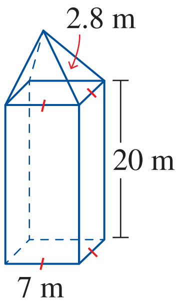


Hint The top is a hemisphere the bottom is a cone.

g



h



Hint draw the shapes that make all of the faces of the solid.

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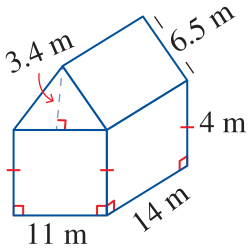
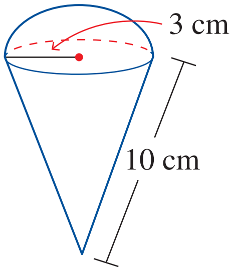
APPLICATIONS

SF: –	CF: 3–5	CU: 6–8

Example 20 ★**3** Millie is planning on painting the outside of her house. Use the diagram shown to calculate the total surface area.

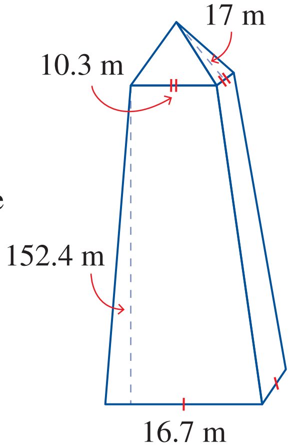


★**4** Tim has carved a wooden ice-cream for his son's play kitchen, as shown in the diagram. He plans on painting it for him. Determine the total area of paint that Tim will need.



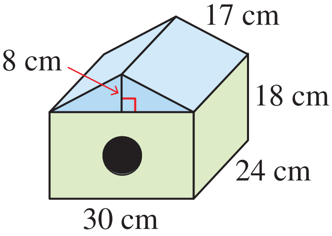
★**5** Christine is making a layered sponge cake for her son's birthday. The rectangular tin she is using has a length of 30 cm, a width of 20 cm and a height of 5 cm. She is planning on stacking 3 cakes on top of each other. She will then ice the cake on all 4 sides and the top. Determine the total surface area for the icing.

★**6** A monument in Washington is 169.2 m tall in total. The main part has four equal-sized faces each in the shape of a trapezium, and there is a square-based pyramid on the top. Calculate the total surface area of the monument, using the approximate measurements in the diagram shown.



★**7** Jean has some camembert cheese in her fridge that has a radius of 7 cm and a height of 3.2 cm. She has eaten a large slice that is approximately 70° of the total cheese. Determine the remaining surface area of the cheese.

★**8** Mason has made a nesting box for his birds. The hole has a diameter of 9 cm. Calculate the external surface area of the nesting box after Mason cuts out the hole for the birds to enter.



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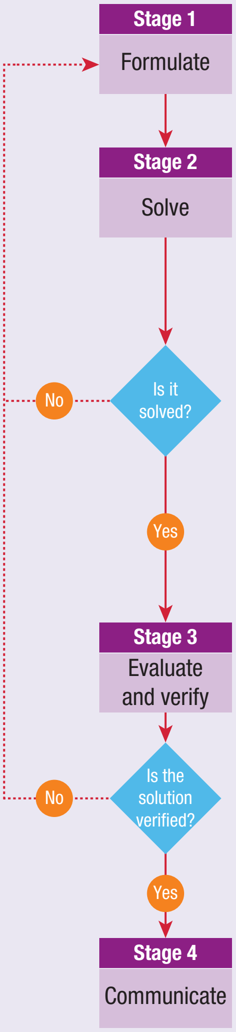
Problem-solving and modelling task

Background: House and land plans provide great applications of area measurement.

Task: Your task is to design a house and land package for a new estate area. You need to show the positioning and outline of the house, along with the measurements for the external walls. You must show the outline of the block of land along with some landscaping ideas including fencing, gardening and other outdoor items (i.e. a pool or water fountain). Your design does not need to be to scale; however, you must show all relevant measurements. You will need to include the area of the land, house, fenced area and the composite shaped backyard feature.

Approach to problem-solving and modelling task:



Stage 1: Formulate

1. Research various house and land designs.
2. Research various features of backyards.

Stage 2: Solve

3. Sketch the bird's eye view of your land, ensuring that you include measurements and consider the functionality of your design. Could this be in the shape of a parallelogram or trapezium?
4. Add the outline of your house, could you make it a composite shape? Include any measurements.
5. Add the fencing design of your house and land package.
6. Add the backyard feature.
7. Calculate the area for the land, house, fenced area and backyard feature.

Stage 3: Evaluate and verify

8. Check that you have included as many measurements as possible in your design.
9. Check that your composite shapes have at least two familiar shapes.
10. Check that you have shown your working for your area calculations.

Stage 4: Communicate

11. Share your design with a peer and have them check your calculations.
12. Write a report to communicate your findings.

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Chapter checklist

☐ I can use units of area and their abbreviations and identify the most appropriate metric units area.

1 Determine the abbreviation for:
- **a** square millimetres
- **b** square kilometres

2 Determine the full unit for:
- **a** cm^2
- **b** m^2
- **c** ha

3 State the most appropriate choice of units to measure the following areas.
- **a** Surface area of a classroom desk
- **b** Suburban block of land
- **c** Horse paddock
- **d** Outback cattle station

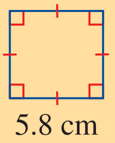
☐ I can convert between units of area.

4 Convert the following measurements into the units given in brackets.
- **a** $6\ m^2\ (cm^2)$
- **b** $9\ cm^2\ (mm^2)$
- **c** $350\ mm^2\ (cm^2)$
- **d** $2.7\ km^2\ (m^2)$
- **e** $6\ km^2$ (ha)
- **f** $7\ m^2\ (mm^2)$

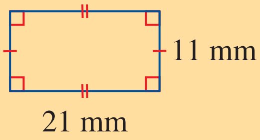
☐ I can calculate and estimate the area of various familiar shapes.

5 Calculate the area of the following common 2D shapes.

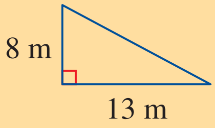
a



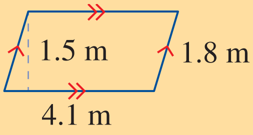
b



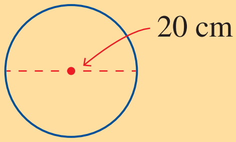
c



d



e



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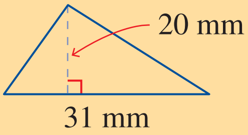

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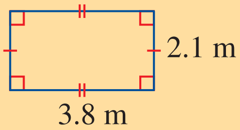
6 Estimate the area of the following common 2D shapes by rounding first to the nearest whole number.

a



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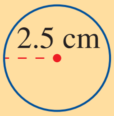

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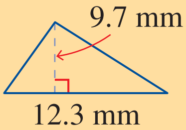
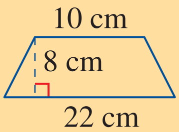


☐ I can calculate the area of trapeziums and sectors.

7 Calculate the area of the following trapeziums.

a



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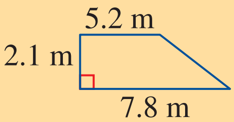
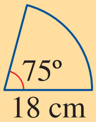


8 Calculate the area of the following sectors. Round your answers to 2 decimal places.

a



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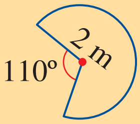
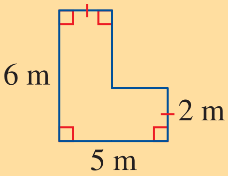


☐ I can calculate the area of composite shapes by decomposing into common shapes.

9 Calculate the area of the following composite shapes.

a



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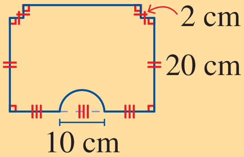
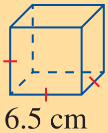


☐ I can calculate the surface area of cubes, rectangular and triangular prisms.

10 Calculate the surface area of the following solids.

a



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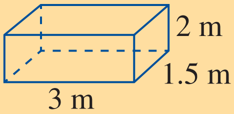
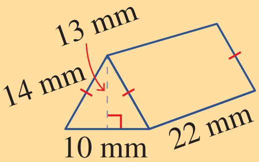


c



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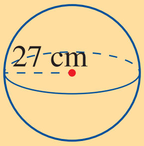
☐ I can calculate the surface area of spheres and cylinders.

11 The formula for the surface area of a sphere is ________.

12 The formula for the surface area of a cylinder is ________.

13 Calculate the area of the following sphere and cylinder.

a



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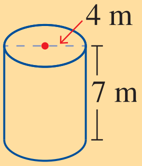
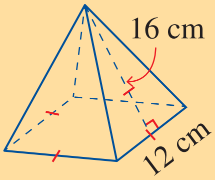


☐ I can calculate the surface area of square-based and rectangular-based pyramids.

14 Calculate the surface area of the following pyramids.

a



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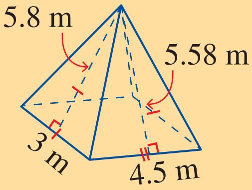

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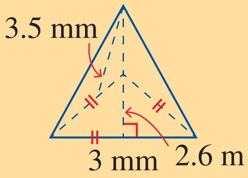
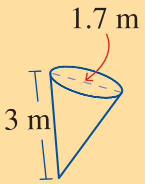


☐ I can calculate the surface area of cones.

15 Calculate the surface area of the following cones.

a



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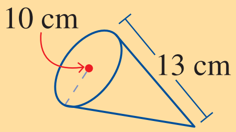
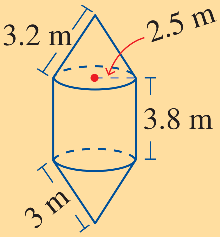


☐ I can calculate the surface area of irregular solids.

16 Calculate the surface area of the following irregular solids.

a



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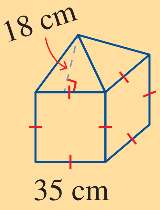

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Chapter review

All questions in the review are assessment style.

Simple familiar

Section 2A

1 Daniel is painting a 19.2 m^2 feature wall in his house. Determine the area of Daniel's wall in square centimetres.

2 Lyla's horse paddock is 27 190 m^2, where her 3 horses roam. Determine the area of the horse paddock in hectares.

3 Ben has calculated that his front door is 1 810 000 mm^2. Determine the area of the front door in square metres.

Section 2B

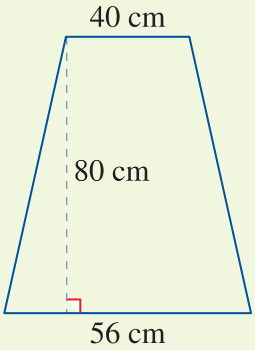
4 Thelma has just purchased a new block of land that is 32.7 metres long by 28 metres wide. Calculate the area of Thelma's land.

5 James has built a new circular training pen for his horses that has a radius of 2.2 metres. Calculate the area of the training pen.

Complex familiar

Section 2C

6 Emily has designed an A-line skirt that she is now planning on sewing. The image below shows the pattern for the front panel of the skirt. Calculate the area of material that Emily will need to make the front of the skirt.



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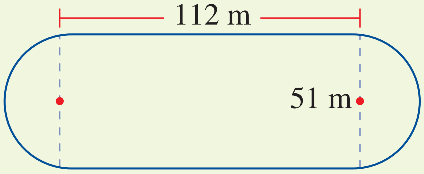
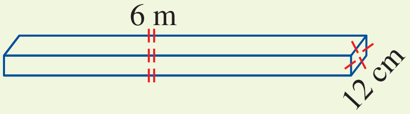
7 Farmer Jo has slashed 310° from one of his circular planted hay crops that has a diameter of 800 metres. Determine the area of land that Jo has slashed for hay.

8 Mason is polishing a speed skating rink, as shown in the diagram. Calculate the area of the rink that Mason is polishing.

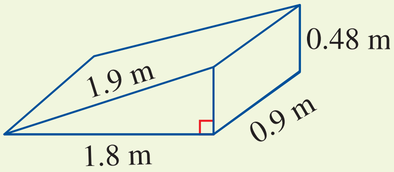


Section 2D **9** Toby's dad has given him a cubed cardboard box that is 70 cm square. Toby is planning on painting the outside of the box red and using it as part of a rocket that he is building. Calculate the total area that Toby will be painting.

10 Bronte is varnishing a timber hand rail that is in the shape of a rectangular prism. Calculate the surface area of the prism in square metres.



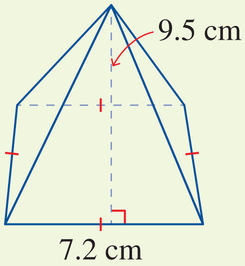
11 Jaz has constructed a triangular prism out of MDF to jump his BMX bike. The dimensions are shown in the diagram. Determine the total surface area of the ramp.



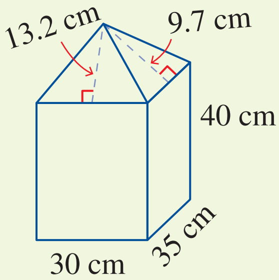
Section 2E **12** Mike is a coach for shot put. So that he can easily locate his equipment at sporting events, he paints each shot put yellow. The shot puts have a radius of 60 mm and he has 10 of them. Determine the total surface area that Mike has painted.

13 Alisha has just finished making a timber table. The table has 4 legs that are shaped like a cylinder and they have a height of 1.2 m and a diameter of 0.2 m. Before she attaches the legs to the table top, she wishes to varnish them first for protection. Determine the total area that Alisha will be varnishing.

14 Haydn has bought his wife a candle in the shape of a square-based pyramid as shown. He has 160 cm^2 of wrapping paper left over at home. Determine if this will be enough wrapping paper to wrap the candle.



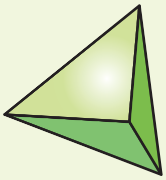
Section 2F **15** James has made a new letter box from steel that he plans on powder coating. Calculate the total surface area of the letter box as shown in the diagram below.



Complex unfamiliar

Section 2C **16** Cooper has ordered a pizza for dinner. There are 8 slices of pizza, so he has calculated that each piece of pizza is 45° of the whole pizza. Cooper eats 4 pieces of pizza and his wife eats 3 pieces. Determine the area of pizza that both Cooper and his wife have consumed, if the pizza has a diameter of 35 cm.

Section 2F **17** A Pyraminx is a puzzle that is similar to the Rubik's cube, but it is the shape of a triangular pyramid with a base that is the same as the sides. (This shape with all four triangular faces the same is called a tetrahedron). The base edge of the Pyraminx is 98 mm and the perpendicular height is 84.9 mm. The company that manufactures the product produces 400 per week and wraps them in plastic. Determine the minimum area of plastic sheet the company would be using each week. Do not include any overlap of the edges of the plastic.



18 Jayden's largest silo on his farm is a cylinder with a cone for the cap. It has a diameter of 7.9 metres and the cylindrical section has a height of 19.7 metres. The slant height of the cone is 5.1 metres. Calculate the surface area of the silo.

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3 Volume, capacity and mass

Maths for a farmer: Lachlan Graeme

Lachlan Graeme works in the agricultural industry as a farmer. His family runs a mixed farm of sheep, cattle and cropping. He has a passion for agriculture, loves being outdoors, achieving new challenges and solving problems.

Tell us a bit about your job. What does a typical day look like?

Being on a farm each day can vary greatly, which keeps it exciting and interesting. Jobs can include feeding and herding sheep and cattle, maintaining machinery and tractors, spraying crops, supervising contractors, checking and monitoring irrigation systems and water troughs, and estimating yields of crops.

I enjoy the variation in each day, and with the changing seasons there is always a new challenge or crop to be grown. Often the more work we put in to the business, the more successful we can be.

What maths did you study at school?

I studied Business Maths (equivalent to Essential Maths) at school.

How do you use maths in your job?

I use maths every day; for example, calculating payments for workers, calibration of spray units, calculating how many kilometres of fencing wire or trellis wire is required, working out how many hectares per hour the grain harvester will be able cover each day and calculating how many litres of diesel per hour the machinery is using. We use maths on a daily basis and without it, our agriculture business would not know if it was making a profit or loss. It is an essential part of the agricultural industry in many ways.

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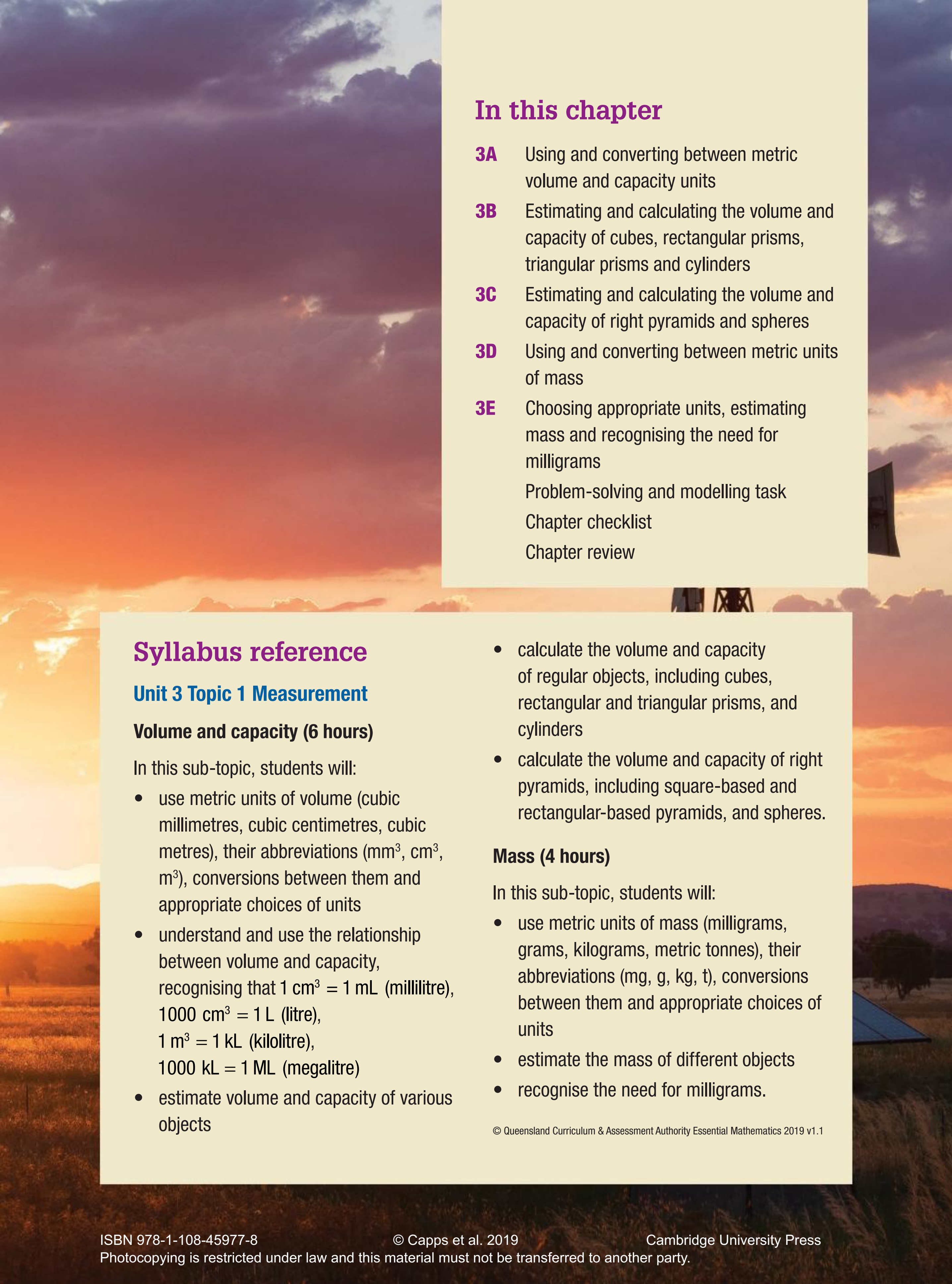


In this chapter

Syllabus reference

Unit 3 Topic 1 Measurement

Volume and capacity (6 hours)

In this sub-topic, students will:

- use metric units of volume (cubic millimetres, cubic centimetres, cubic metres), their abbreviations (mm^3, cm^3, m^3), conversions between them and appropriate choices of units
- understand and use the relationship between volume and capacity, recognising that $1 \text{ cm}^3 = 1 \text{ mL}$ (millilitre),
  $1000 \text{ cm}^3 = 1 \text{ L}$ (litre),
  $1 \text{ m}^3 = 1 \text{ kL}$ (kilolitre),
  $1000 \text{ kL} = 1 \text{ ML}$ (megalitre)
- estimate volume and capacity of various objects
- calculate the volume and capacity of regular objects, including cubes, rectangular and triangular prisms, and cylinders
- calculate the volume and capacity of right pyramids, including square-based and rectangular-based pyramids, and spheres.

Mass (4 hours)

In this sub-topic, students will:

- use metric units of mass (milligrams, grams, kilograms, metric tonnes), their abbreviations (mg, g, kg, t), conversions between them and appropriate choices of units
- estimate the mass of different objects
- recognise the need for milligrams.

Pre-test

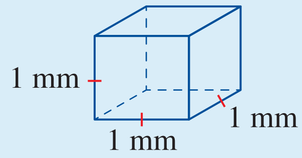
1 Explain the meaning of:

a ‘milli-’ as used in the unit ‘millimetre’

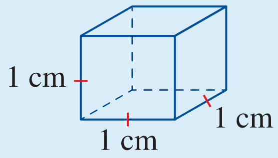
b ‘kilo-’ as used in the unit ‘kilometre’

2 Each cube represents a unit of volume. They are not drawn to the same scale. Write down the full name of each unit and its abbreviation.

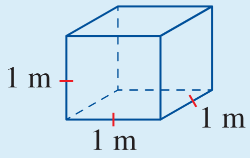
a



b



c



Hint The abbreviation or symbol for these units of volume includes a number as an index (a power) that represents the length, width and depth of the cube (three sides with the same value) being multiplied together.

3 How many millilitres are in a litre?

4 Determine the missing words in the following sentences.

a A rectangular prism has __________ faces. All faces meet at __________ angles.

b A square-based pyramid has _____ side faces (i.e. the base is not included). The faces are in the shape of __________.

c The two bases or ends of a cylinder are always __________ to each other. The shape of each base or end is a __________.

5 (Multiple-choice) ‘Mass’ refers to:

A the capacity of an object.

B the amount of matter in a solid, liquid or gas.

C the amount of space a solid, liquid or gas occupies.

D the pull of gravity.

6 Identify which of these units can be used to measure mass:
litres, grams, square metres, kilograms, hectares, cubic centimetres, tonnes.

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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3A Using and converting between metric volume and capacity units

LEARNING GOALS

- Understand the use and appropriate choice of the metric units of volume including cubic millimetres, cubic centimetres and cubic metres
- Use abbreviations such as mm^3, cm^3 and m^3 to represent units of volume
- Convert between units of volume
- Understand the relationship between volume and capacity including the conversions between millilitres, litres, kilolitres and megalitres

Why is it essential to understand units of volume and capacity?

There are many careers, including pool installation, landscaping, cooking, storage hire or hairdressing, where volume capacity can be used. The production, storage, distribution and use of materials, whether gas, liquid or solid, and their containers, requires measurement and conversion of volume and capacity.

It is an essential skill to be able to convert between the units of volume and capacity so that the capacity (or amount of gas, solid or liquid) can be calculated using the measurements of an object.

Measuring volume is a vital skill in laboratories.

WHAT YOU NEED TO KNOW

- Calculating the **volume** of a solid is a measure of the amount of space taken up by a 3D shape.
- **Units of volume** include cubic millimetres, cubic centimetres, cubic metres and cubic kilometres, which can be abbreviated to mm^3, cm^3, m^3 and km^3.
- To convert between units of volume:
  - use multiplication when converting from a larger unit to a smaller unit (e.g. from m^3 to cm^3)
  - use division when converting from a smaller unit to a larger unit (e.g. from cm^3 to m^3).

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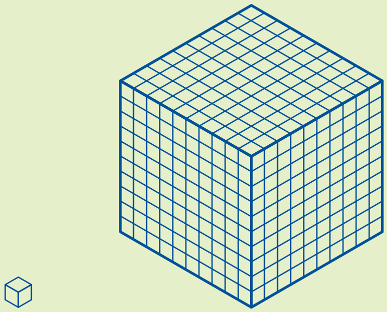
- If one linear unit is 10 times longer than another, then its cubic unit is 10^3 (i.e. 1000 times bigger).



- Refer to the following conversion chart when solving problems that require conversion of units of volume.

$$\begin{tikzcd}[column sep=huge]
\boxed{\text{km}^3} \arrow[r, bend left, "\times 1000^3\ (1\,000\,000\,000)"] & \boxed{\text{m}^3} \arrow[l, bend left, "\div 1000^3\ (1\,000\,000\,000)"] \arrow[r, bend left, "\times 100^3\ (1\,000\,000)"] & \boxed{\text{cm}^3} \arrow[l, bend left, "\div 100^3\ (1\,000\,000)"] \arrow[r, bend left, "\times 10^3\ (1\,000)"] & \boxed{\text{mm}^3} \arrow[l, bend left, "\div 10^3\ (1\,000)"]
\end{tikzcd}$$

- **Capacity** is the same as volume, but in everyday contexts we use capacity for the volume that a container (3D solid) can hold of a substance (solid, liquid or gas).
- Units of capacity include megalitres, kilolitres, litres, millilitres, which can be abbreviated to ML, kL, L and mL.
- Capacity is measured in units based on the litre. Use the following conversion chart for problems that require conversion of units of capacity.

$$\begin{tikzcd}[column sep=huge]
\boxed{\text{ML}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{kL}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{L}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{mL}} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

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- To convert between units of volume and capacity:
  - $1\text{ cm}^3 = 1\text{ mL}$
  - $1000\text{ cm}^3 = 1000\text{ mL} = 1\text{ L}$
  - $1\text{ m}^3 = 1000\text{ L} = 1\text{ kL}$
  - $1000\text{ m}^3 = 1000\text{ kL} = 1\text{ ML}$
  - $1\,000\,000\text{ L} = 1\text{ ML}$

Example 1 Converting between units of volume

Convert these volume measurements into the units given in brackets.

a 0.72 m^3 (mm^3)
b 10.5 cm^3 (mm^3)
c 393 mm^3 (cm^3)
d 800 cm^3 (m^3)

WORKING	THINKING
a $0.72 \times 1\,000\,000\,000$ $= 720\,000\,000\text{ mm}^3$	As there are 1000^3 cubic millimetres in a cubic metre and we are converting from m^3 to a smaller unit of mm^3, we need to multiply 0.72 by 1000^3, which is 1 000 000 000.
b $10.5 \times 10^3 = 10\,500\text{ mm}^3$	As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from cm^3 to a smaller unit of mm^3, we need to multiply 10.5 by 10^3.
c $393 \div 10^3 = 0.393\text{ cm}^3$	As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from mm^3 to a larger unit of cm^3, we need to divide 393 by 10^3.
d $800 \div 100^3 = 0.0008\text{ m}^3$	As there are 100^3 cubic centimetres in a cubic metre and we are converting from cm^3 to a larger unit of cubic metres, we need to divide 800 by 100^3.

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Example 2 Converting between units of capacity

Convert these capacity measurements into the units given in brackets.

a 2125 mL (L)
b 0.5 L (mL)
c 2.8 ML (L)

WORKING	THINKING
a $2125\text{ mL} \div 1000 = 2.125\text{ L}$	To convert millilitres to litres, going from a smaller to a bigger unit, divide by 1000.
b $0.5 \times 1000 = 500\text{ mL}$	To convert litres to millilitres, going from a bigger to a smaller unit, multiply by 1000.
c $2.8 \times 1\,000\,000 = 2\,800\,000\text{ L}$	To convert megalitres to litres, going from a bigger to a smaller unit, multiply by 1 000 000.

Example 3 Converting between units of volume and units of capacity

Convert these volume and capacity measurements into the units given in brackets.

a 325 cm^3 (mL)
b 175 kL (m^3)
c 2 L (cm^3)
d 1825 cm^3 (L)

WORKING	THINKING
a $325\text{ cm}^3 = 325\text{ mL}$	There is 1 millilitre in 1 cubic centimetre.
b $175\text{ kL} = 175\text{ m}^3$	There is 1 cubic metre in 1 kilolitre.
c $2\text{ L} \times 1000 = 2000\text{ mL}$ $= 2000\text{ cm}^3$	As there are 1000 mL in a litre, multiply by 1000 to convert to mL, and then use $1\text{ mL} = 1\text{ cm}^3$.
d $1825\text{ cm}^3 = 1825 \div 1000$ $= 1.825\text{ L}$	As there are 1000 cubic centimetres in a litre, divide by 1000 to convert to litres.

Example 4 Applying unit conversion between volume and capacity

Mackenzie works at a plant nursery and has to fill 640 pots with soil. The pots have a capacity of 750 mL. The soil is measured in cubic metres. Determine the approximate amount of soil in cubic metres Mackenzie needs to fill the pots.

WORKING	THINKING
750 mL = 750 cm^3	Convert 750 mL to cm^3.
750 $\text{cm}^3 \times 640 = 480\,000\ \text{cm}^3$	Multiply this by 640 pots.
$480\,000\ \text{cm}^3 \div 1\,000\,000 = 0.48\ \text{m}^3$	There are 100^3 cubic centimetres in a cubic metre, and we are converting from cm^3 to a larger unit, so divide 480 000 by 100^3, which is 1 000 000.
Mackenzie needs 0.48 m^3 of soil.	Communicate your answer by writing down the result.

Exercise 3A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Use multiplication when converting from a __________ unit to a __________ unit, for example, when converting from km^3 to m^3.
- **b** Use __________ when converting from a smaller unit to a larger unit, for example, when converting from __________ to m^3.
- **c** Capacity is a measure of a 3D shapes ability to __________ a substance, such as a __________, __________ or gas.
- **d** A capacity of 1 __________ is equivalent to 1 cubic centimetre.
- **e** One litre is equivalent to __________ cubic centimetres.
- **f** One cubic metre is equivalent to one __________ (unit of capacity).
- **g** One megalitre is equivalent to 1000 __________ (unit of capacity).

Hint A litre is equivalent to a cube with a side length of 10 cm (100 mm). You can work out the number of cubic centimetres or cubic millimetres in a litre by 'cubing' the side length (raising it to the power of 3).

Hint Ten one-litre cubes (side length 10 cm each) will fit exactly along each side of a one-cubic-metre cube (side length 100 cm).

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Example 1

2 Convert these volume measurements to the units given in brackets.

- **a** 0.86 km^3 (m^3)
- **b** 18.4 cm^3 (mm^3)
- **c** 2.7 m^3 (cm^3)
- **d** 276 mm^3 (cm^3)
- **e** 1.38 km^3 (m^3)
- **f** 68 cm^3 (mm^3)
- **g** 129 cm^3 (m^3)
- **h** 2.7 m^3 (mm^3)

Hint If converting from a larger unit to a smaller unit, multiply. If converting from a smaller unit to a larger unit, divide.

Example 2–3

3 Convert these volume and capacity measurements to the units given in brackets.

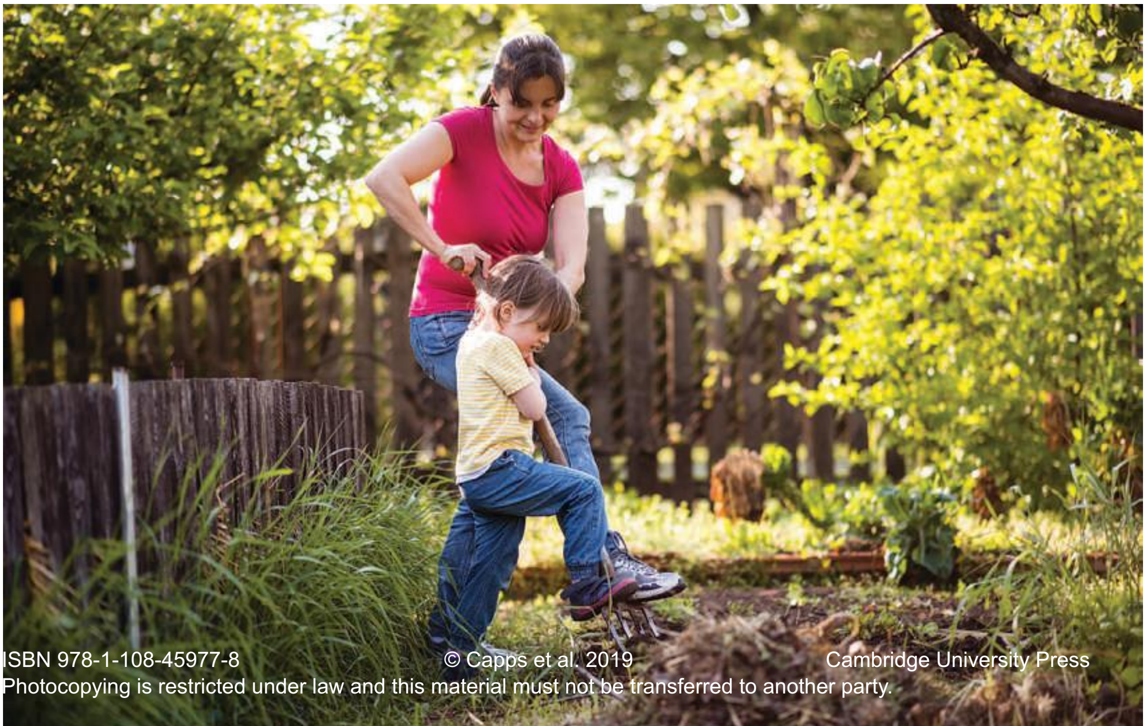
- **a** 265 cm^3 (mL)
- **b** 412 kL (m^3)
- **c** 6 L (cm^3)
- **d** 2128 cm^3 (L)
- **e** 27 mL (cm^3)
- **f** 1247 mL (kL)
- **g** 2.7 ML (kL)
- **h** 27.5 m^3 (kL)
- **i** 267 kL (ML)
- **j** 0.8 L (mL)
- **k** 628 mL (L)
- **l** 3.2 ML (L)

APPLICATIONS

SF: 4–10	CF: –	CU: –

Example 4

★**4** Carlene has calculated that to spread mulch to a depth of 5 cm over a garden bed she needs to buy 800 000 cm^3 of mulch. She finds that mulch is sold in cubic metres. Calculate the number of cubic metres of mulch Carlene should order.



★**5** Tessa has 1089 m^3 of bitumen that she can use to pave the driveway on her farm. To work out how thick the bitumen layer can be she needs to convert this volume to cubic centimetres. Determine how many cm^3 are in 1089 m^3.

★**6** Lee-Ann's pool holds 12 000 L of water. Unfortunately, the pool tiles are damaged so she has decided to fill the pool in with soil and make a big garden instead. Calculate the number of cubic metres of soil Lee-Ann will need in order to fill the pool.

★**7** Jarred works in a factory that makes sugar cubes. A sugar cube with a size of 1 cm^3 is made from 1 mL of liquid sugar. The factory produces 5000 cubes per day. Calculate the number of millilitres of liquid sugar that the factory requires each day.

★**8** An Olympic swimming pool has 2.5 megalitres of water. Calculate the number of litres in an Olympic swimming pool.

★**9** A large fuel truck carries around 10 kilolitres of fuel. Calculate how many litres of fuel a large truck can carry.

★**10** If all the water in Sydney Harbour was poured into a cubic tank with edges of 1 km, it would fill it to a depth of 562 m. Calculate the number of megalitres of water in Sydney Harbour.

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3B Estimating and calculating the volume and capacity of right prisms and cylinders

LEARNING GOALS

- Calculate the volume and capacity of the following regular objects:
  - cubes
  - rectangular prisms
  - triangular prisms
  - cylinders
- Estimate the volume and capacity of the above regular objects

Why is it essential to know how to calculate the volume and capacity?

Among its many uses, calculating volume and capacity is essential for packing, storing and filling containers. For example, when packing a crate into a storage container, we must calculate how much space the crate takes up and how much space is available. Another situation could be when calculating the volume of liquid that can be held by a container or a pool.

This pool is in the shape of a rectangular prism, and you will be able to calculate how much water is required to fill it.

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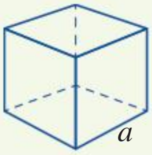
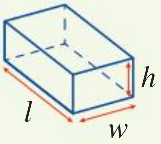
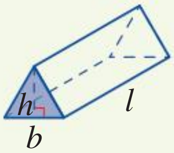
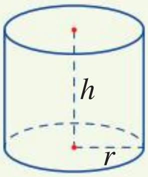
WHAT YOU NEED TO KNOW

- Right prisms have two faces on opposite sides to which the other faces join at right angles, hence the name. A cylinder is not a prism because it includes a curved surface, but it is related, because it has a curved surface joining two circular faces at right angles.
- You will need the rules for area from the previous section.
- The general rule for calculating the volume is the area of the base multiplied by the height. Specific rules for calculating regular solids is found in the table below.

Solid	**Rule**
Cube 	$V = a^3$ (V: volume, a: edge length)
Rectangular prism 	$V = l \times w \times h$ (V: volume, l: length, w: width, h: height)
Triangular prism 	$V = \frac{1}{2}(b \times h) \times l$ (V: volume, b: width of base, h: **perpendicular** height, l: length)
Cylinder 	$V = \pi \times r^2 \times h$ (V: volume, π: 3.142, r: radius of base or end, h: height)

- Linear measurements must be in the same units.
- The volume units will then be cubic units of the linear units used.
- When calculating capacity from linear dimensions, calculate the volume in cubic units and then convert to units of capacity.

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Example 5 Converting volume to capacity units

Convert to the capacity units in the brackets. Round to one decimal place if necessary.

a $1.44\ \text{m}^3$ (kL)
b $1000\ \text{cm}^3$ (L)
c $356\,720\ \text{mm}^3$ (mL)
d $2\,035\,752.04\ \text{cm}^3$ (L)

WORKING	THINKING
a $1.44\ \text{kL}$	Use $1\ \text{m}^3 = 1\ \text{kL}$ to convert to kL.
b $1000\ \text{cm}^3 \div 1000 = 1\ \text{L}$	Given that $1000\ \text{cm}^3 = 1\ \text{L}$, divide by 1000 to convert to litres.
c $356\,702 \div 10^3 = 356.7\ \text{cm}^3$ $= 356.7\ \text{mL}$	Convert the answer to cm^3 by dividing by 10^3. Use $1\ \text{cm}^3 = 1\ \text{mL}$ to convert to mL.
d $2\,035\,752.04 \div 1000 = 2035.8\ \text{L}$	Given that $1000\ \text{cm}^3 = 1\ \text{L}$, divide the answer by 1000 to convert to litres.

The sizes of packaging cartons may be given in units of capacity or volume; both can easily be estimated from the dimensions of the carton.

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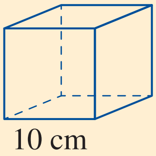
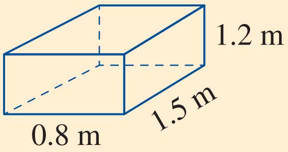
Example 6 Calculating the volume of solids

Calculate the volume of the following solids.

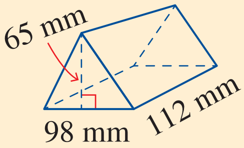
a



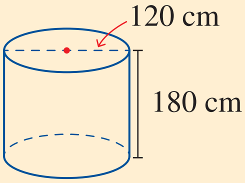
b



c



d



	WORKING	THINKING
a	$V = a^3$ $V = 10^3$ $V = 1000\text{ cm}^3$	As the solid is a cube, apply the formula $V = a^3$. Substitute the value for *a*, which is 10.
b	$V = l \times w \times h$ $V = 0.8 \times 1.5 \times 1.2$ $V = 1.44\text{ m}^3$	As the solid is a rectangular prism, apply the formula $V = l \times w \times h$. Substitute the values for $l = 1.5$, $w = 0.8$ and $h = 1.2$. Note: it does not matter which order you multiply these values in.
c	$V = \frac{1}{2}(b \times h) \times l$ $V = \frac{1}{2}(98 \times 65) \times 112$ $V = 356\ 720\text{ mm}^3$	As the solid is a triangular prism, apply the formula $V = \frac{1}{2}(b \times h) \times l$. Substitute the values for $b = 98$, $h = 65$ and $l = 112$.
d	$r = 120 \div 2 = 60$ $V = \pi \times r^2 \times h$ $V = \pi \times 60^2 \times 180$ $V = 2\ 035752.04\text{ cm}^3$	First determine the radius by dividing the diameter by 2. As the solid is a cylinder, apply the formula $V = \pi \times r^2 \times h$. Then substitute the values for $r = 60$ and $h = 180$.

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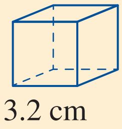
Example 7 Estimating volume and capacity

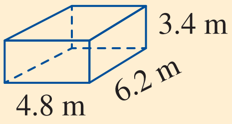
Estimate the volume and capacity of the following shapes by first rounding each measurement to the nearest whole number.

a



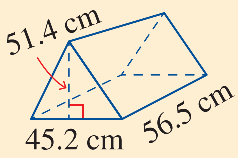
Convert to mL

b



Convert to kL

c



Convert to L

WORKING	THINKING
a 3.2 rounds to 3	Given that the first decimal place is less than 5, round down to the nearest whole number which equals three.
$V = a^3$	The solid is a cube so apply the formula $V = a^3$.
$V = 3^3 = 27\text{ cm}^3$	
The capacity is 27 mL.	Use $1\text{ cm}^3 = 1\text{ mL}$ for capacity.
b 4.8 rounds to 5 6.2 rounds to 6 3.4 rounds to 3	Check the value of the first decimal place, if the value is less than 5 then round down to the nearest whole number. If the first decimal place is 5 or more, round the whole number up by one.
$V = l \times w \times h$	The solid is a rectangular prism so apply the formula $V = l \times w \times h$.
$V = 5 \times 6 \times 3 = 90\text{ m}^3$	
The capacity is 90 kL.	Use $1\text{ m}^3 = 1\text{ kL}$ for capacity.
c 45.2 rounds to 45 51.4 rounds to 51 56.5 rounds to 57	Estimate the values given.
$V = b \times h \times l$	The solid is a triangular prism so apply the formula $V = b \times h \times l$.
$V = \frac{1}{2}(45 \times 51) \times 57$	
$V = 65407.5\text{ cm}^3$	
The capacity is $65407.5 \div 1000 = 65.41\text{ L}$	Use $1000\text{ cm}^3 = 1\text{ L}$ for capacity.

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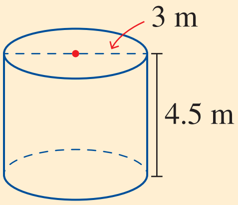
Example 8 Applying volume and capacity to practical problems

Joanne has a rainwater tank on her property that has a diameter of 3 metres and a height of 4.5 metres. Calculate the amount of water in litres that her tank can hold.



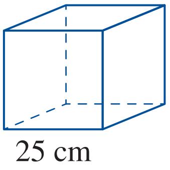
WORKING	THINKING
$r = 3 \div 2 = 1.5$	Determine the radius of the tank by dividing the diameter by 2.
$V = \pi \times r^2 \times h$ $V = \pi \times 1.5^2 \times 4.5$ $V = 31.80852562 \text{ m}^3$	Given that the solid is a cylinder, apply the formula $V = \pi \times r^2 \times h$.
$31.81 \times 100^3 = 31\,808\,525.62 \text{ cm}^3$	Convert the answer to cubic centimetres by multiplying by 100^3.
$31\,808\,525.62 \div 1000 \approx 31\,809 \text{ L}$	Given that $1000 \text{ cm}^3 = 1 \text{ L}$, divide the answer by 1000 to convert to litres, to the nearest litre.
Joanne's tank can hold 31 809 L of water.	Communicate your solution by writing down the result.

Exercise 3B

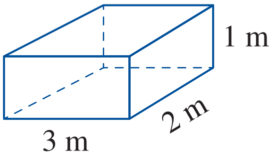
FUNDAMENTALS

1 Calculate the volume of the following solids.

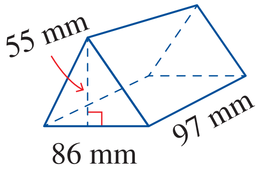
a



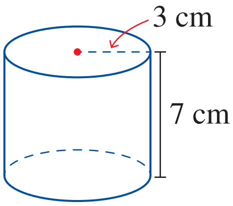
b



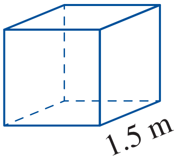
c



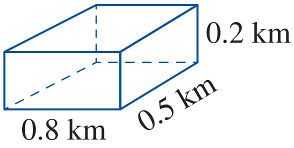
d



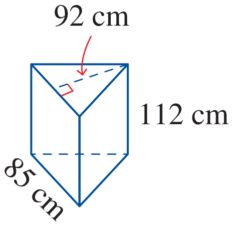
e



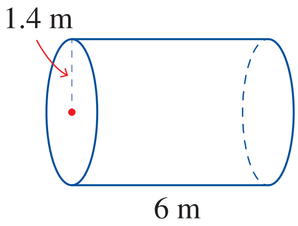
f



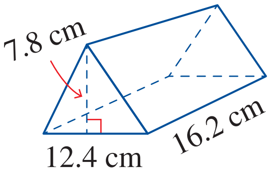
g



h



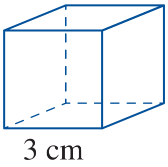
i



Example 5, 6

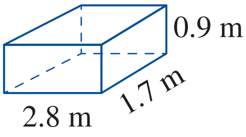
2 Calculate the volume and then the capacity of the following solids.

a



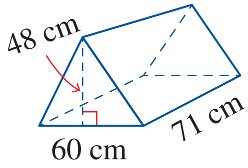
Convert to mL

b



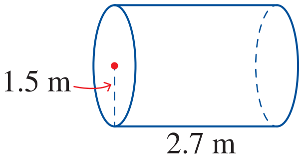
Convert to kL

c



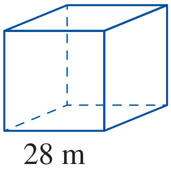
Convert to L

d



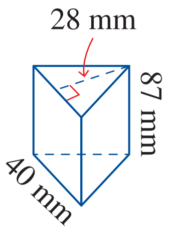
Convert to kL

e



Convert to ML

f



Convert to mL

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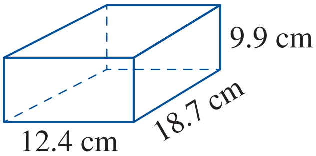
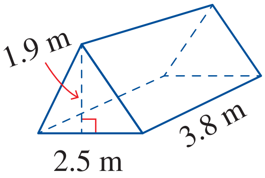
Example 7 **3** Estimate the volume and then the capacity of the following solids.

a



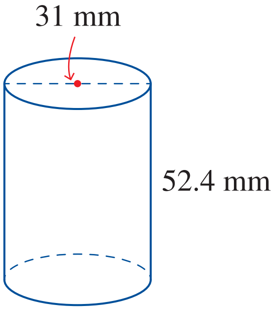
Convert to L

b



Convert to kL

c



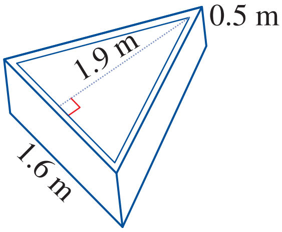
Convert to mL

APPLICATIONS

SF: 4–12	CF: –	CU: –

Example 8 ★**4** Reuben's water tank is a cylinder and has a radius of 1.2 m and a height of 3 m. Calculate the volume of Reuben's water tank.

★**5** Mim has built a triangular garden bed as shown. She plans on filling the garden bed with soil. Determine how many cubic metres of soil Mim will require.



★**6** Lisa has a 45 cm cubic cardboard box. Calculate the volume of the box.

★**7** Michael's can of soft drink is 13 cm tall and has a diameter of 6 cm. Calculate the volume of the can and then the capacity of the can in mL, assuming it is a cylinder.

★**8** A Toblerone box is 30.6 cm long and the base of the triangle is 5.4 cm with a height of 5 cm. Determine the capacity of the box in millilitres.

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★**9** An industrial gas cylinder has a length of 457 cm and a diameter of 124.5 cm. Calculate the capacity of the cylinder in kilolitres, assuming the ends are flat circles, as with a standard geometric cylinder.

★**10** Kyösti's box of cereal is shown. Calculate the capacity of the box in litres.

★**11** Ken is a truck driver and his petrol tanker has a length of 5.94 metres and a diameter of 2.2 metres. Calculate the capacity of the petrol tank in kL. Assume it is a cylinder.

★**12** Rochelle is installing a new pool in her backyard. It will be a flat-bottomed pool with a depth of 1.4 metres. The length of the pool will be 12 metres and the width will be 8 metres. Calculate how much water Rochelle's pool will hold in kilolitres.

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3C Estimating and calculating the volume and capacity of pyramids, cones and spheres

LEARNING GOALS

- Calculate the volume and capacity of the following regular objects:
  - square-based pyramids
  - rectangular-based pyramids
  - spheres
  - cones
- Estimate the volume and capacity of pyramids, cones and spheres

Why is it essential to know how to calculate the volume of pyramids, cones and spheres?

It is important to be able to measure volume and capacity of pyramids, cones and spheres. Spheres are used in manufacturing balls used in sports and industry, and pyramid and cone shapes are used for ornaments, buildings and artwork as well as other applications.

Volume is required to work out how much wax to use to make this candle.

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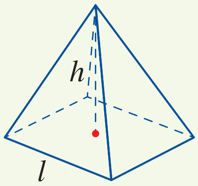
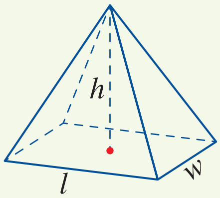
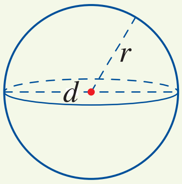
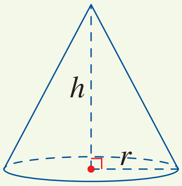
WHAT YOU NEED TO KNOW

- You will need to recall the units and conversions in previous sections.
- Note that the pyramids dealt with are right pyramids, meaning that the point (apex) is above the centre of the base. The same applies to the cones, they are called right cones.
- Formulas for calculating the volume of pyramids and spheres are found in the table below.

Shape	Rule
Square-based pyramid 	$V = \frac{1}{3} \times l^2 \times h$ (V: volume, l: length of side of base, h: height)
Rectangular-based pyramid 	$V = \frac{1}{3} \times l \times w \times h$ (V: volume, l: length of base, w: width of base, h: height)
Spheres 	$V = \frac{4}{3} \times \pi \times r^3$ (V: volume, r: radius, π: 3.142)
Cones 	$V = \frac{1}{3} \times \pi \times r^2 \times h$ (V: volume, r: radius, π: 3.142, h: height)

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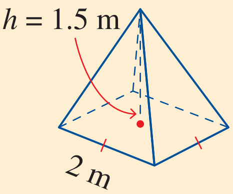
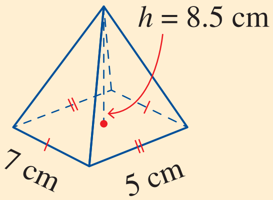
Example 9 Calculating the volume of pyramids, spheres and cones

Calculate the volume of the following solids. Round your answer to 2 decimal places.

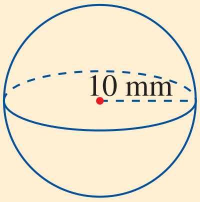
a



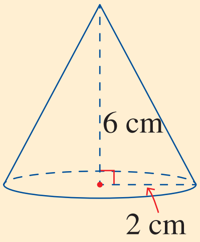
b



c



d



WORKING	THINKING
a $V = \frac{1}{3} \times l^2 \times h$ $V = \frac{1}{3} \times 2^2 \times 1.5$ $V = 2\text{ m}^3$	As the shape is a square-based pyramid, apply the formula $V = \frac{1}{3} \times l^2 \times h$. Substitute the values for l and h.
b $V = \frac{1}{3} \times l \times w \times h$ $V = \frac{1}{3} \times 7 \times 5 \times 8.5$ $V = 99.17\text{ cm}^3$	As the shape is a rectangular-based pyramid, apply the formula $V = \frac{1}{3} \times l \times w \times h$. Substitute the values for l, w and h.
c $V = \frac{4}{3} \times \pi \times r^3$ $V = \frac{4}{3} \times \pi \times 10^3$ $V = 4188.79\text{ mm}^3$	As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$. Substitute the value for r.
d $V = \frac{1}{3} \times \pi \times r^2 \times h$ $V = \frac{1}{3} \times \pi \times 2^2 \times 6$ $V = 25.13\text{ cm}^3$	As the shape is a cone, apply the formula $V = \frac{1}{3} \times \pi \times r^2 \times h$. Substitute the value for r and h.

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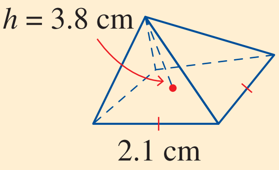
Example 10 Calculating the capacity of solids

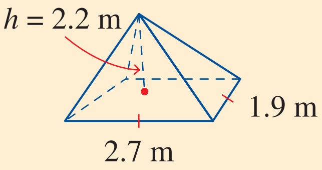
Calculate the capacity of the following solids. Round your answer to 2 decimal places.

a



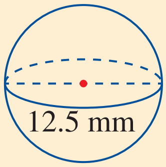
Convert to mL

b



Convert to kL

c



Convert to mL

WORKING	THINKING
a $V = \frac{1}{3} \times l^2 \times h$	As the shape is a square-based pyramid, apply the formula $V = \frac{1}{3} \times l^2 \times h$.
$V = \frac{1}{3} \times 2.1^2 \times 3.8$	Substitute the values for l and h.
$V = 5.59 \text{ cm}^3$	
$V = 5.59 \text{ mL}$	Calculate the capacity using $1 \text{ cm}^3 = 1 \text{ mL}$.
b $V = \frac{1}{3} \times l \times w \times h$	As the shape is a rectangular-based pyramid, apply the formula $V = \frac{1}{3} \times l \times w \times h$.
$V = \frac{1}{3} \times 2.7 \times 1.9 \times 2.2$	Substitute the values for l, w and h.
$V = 3.76 \text{ m}^3$	
$V = 3.76 \text{ kL}$	Calculate the capacity using $1 \text{ m}^3 = 1 \text{ kL}$.
c $r = 12.5 \div 2 = 6.25$	As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$
$V = \frac{4}{3} \times \pi \times r^3$	Substitute the value for r.
$V = \frac{4}{3} \times \pi \times 6.25^3$	
$V = 1022.65 \text{ mm}^3$	
$1022.65 \div 10^3$ $= 1.02 \text{ cm}^3$ $= 1.02 \text{ mL}$	Calculate the capacity by converting to cubic centimetres. As there is 10^3 cubic millimetres in a cubic centimetre, divide by 10^3 to convert to cm^3. Use $1 \text{ cm}^3 = 1 \text{ mL}$.

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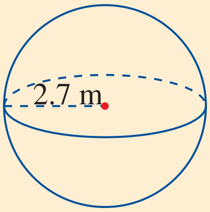
Example 11 Estimating the volume and capacity of solids

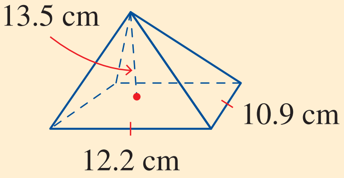
Estimate the volume and capacity of the following solids by first rounding each measurement to the nearest whole number.

a



Convert to ML

b



Convert to L

WORKING	THINKING
a 2.7 rounds to 3	Round the radius to the nearest whole number. As the first decimal place is greater than 5, round the whole number up to 3.
$V = \frac{4}{3} \times \pi \times r^3$ $V = \frac{4}{3} \times \pi \times 3^3$ $V = 113.10\ \text{m}^3$	As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$. Substitute the estimated value for r.
$V = 113.10\,\text{kL}$ $113.10 \div 1000 = 0.1131\,\text{ML}$	Calculate the capacity as $1\ \text{m}^3 = 1\ \text{kL}$. Divide by 1000 to convert to megalitres.
b 13.5 rounds to 14 12.2 rounds to 12 10.9 rounds to 11	Round each value to the nearest whole number. The height of the pyramid has a first decimal place of 5 so round up to 14. The length has a first decimal place of 2 so round down to the whole number 12. The width has a first decimal place of 9 so round the whole number up to 11.
$V = \frac{1}{3} \times l \times w \times h$ $V = \frac{1}{3} \times 12 \times 11 \times 14$ $V = 616\ \text{cm}^3$ $V = 616\ \text{mL}$	As the shape is a rectangular-based pyramid, apply the formula $V = \frac{1}{3} \times l \times w \times h$. Substitute the estimated values for l, w and h.
$616 \div 100 = 0.616\ \text{L}$	Calculate the capacity as $1\ \text{cm}^3 = 1\ \text{mL}$.

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Example 12 Applying volume and capacity to practical problems

Shahida makes large spherical candles. She makes them by pouring melted wax into a spherical mould, which has an internal diameter of 24.3 cm. Calculate the capacity of the mould in litres.

WORKING	THINKING
$V = \frac{4}{3} \times \pi \times r^3$	As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$.
$r = 24.3 \div 2 = 12.15$	Calculate the value of the radius.
$V = \frac{4}{3} \times \pi \times 12.15^3$	Substitute the value for r.
$V = 7513.07 \text{ cm}^3$	
$V = 7513.07 \text{ mL}$	Calculate the capacity as $1 \text{ cm}^3 = 1 \text{ mL}$.
$7513.07 \div 1000 = 7.5 \text{ L}$	Divide by 1000 to answer in litres.
The mould has a capacity of 7.5 litres.	Communicate your solution by writing down the result.

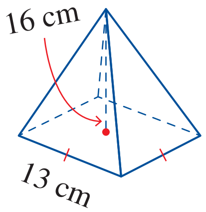
Exercise 3C

FUNDAMENTALS

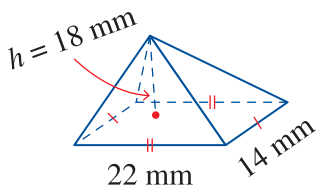
Example 9

1 Calculate the volume of the following solids.

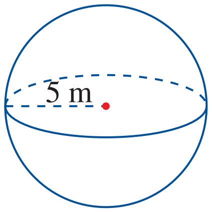
a



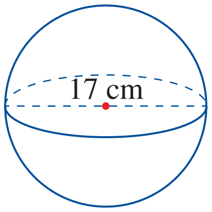
b



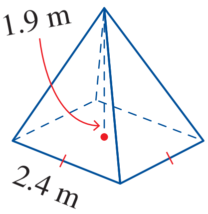
c



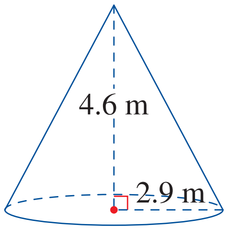
d



e

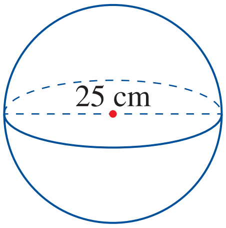


f



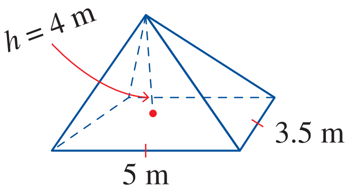
Example 10 **2** Calculate the capacity of the following solids.

a



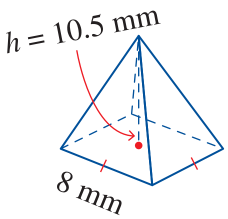
Convert to L

b



Convert to kL

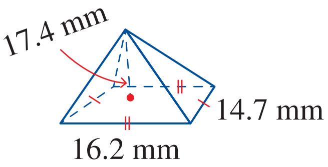
c



Convert to mL

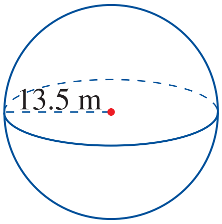
Example 11 **3** Estimate the volume and capacity of the following solids.

a



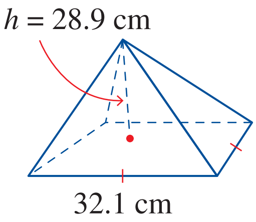
Convert to mL

b



Convert to ML

c



Convert to L

APPLICATIONS

SF: 4–10	CF: –	CU: –

Example 12 ★**4** Bronwyn has designed a rectangular-based pyramid made out of glass that can be used as a paperweight. Calculate the volume if the base has a length of 12 cm and a width of 9 cm, and its height is 8 cm.

★**5** In Richard's barber shop there is a spherical shaped gumball machine. It has an internal diameter of 0.6 metres. Calculate the volume of the gumball machine.

★**6** Phillip makes pyramid shaped snow globes that he then sells as souvenirs at his ski resort. The souvenirs have a 68 mm square base and they are 72 mm high. Calculate the of the volume of the pyramid snow globe.

★**7** Neil runs a factory that makes candles. One of the most popular designs is a square-based pyramid. The pyramids have an 8.4 cm square base and a height of 10.5 cm. Calculate the number of litres of wax that Neil would require if his factory made 150 candles in a day.

★**8** Susan makes freshly squeezed orange juice. She is planning on using 12 oranges that each have a radius of 3.2 cm when peeled and she does not remove the pulp from her juice. Estimate the total number of litres of juice that Susan should get from her oranges.

★**9** In 1483, Leonardo da Vinci designed a parachute that was shaped like an upturned square-based pyramid. He claimed that a piece of material with a square base of 7 metres wide and a height of 7 metres would enable a person to throw himself from any height and not suffer any injuries from landing on the ground. Calculate the capacity of the air in litres in the upturned pyramid of da Vinci's design.

★**10** If Leonardo had used a cone with a base diameter of 7 m and a height of 7 m, calculate the capacity of the air in litres in his parachute from question **9**.

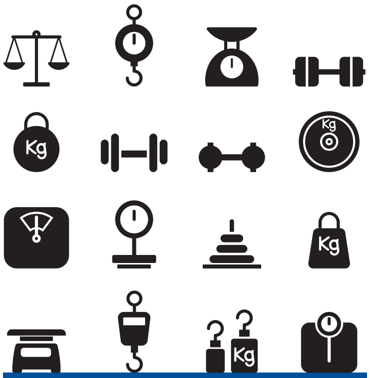
3D Using and converting between metric units of mass

LEARNING GOALS

- Understand the metric units of mass (milligrams, grams, kilograms, metric tonnes)
- Use abbreviations for the metric units of mass (mg, g, kg, t)
- Convert between the metric units of mass
- Recognise appropriate choice of units

Why is it essential to use and understand the metric units of mass?

Understanding how to use and convert mass units and estimate mass is essential in careers but also in many aspects of life. In the transport industry, it is important to know the mass of things being loaded on to trucks and the trucks' maximum load capacity. In the health and medical industry, the application is crucial when administering medication. In the cooking industry, it is important to be able to convert between mass units in order to follow recipes.



A huge variety of scales for measuring mass are used in everyday life.

WHAT YOU NEED TO KNOW

- Mass is the amount of matter in an object. In everyday life we measure mass by its weight, which is the force exerted on it by gravity, and so mass and weight are used interchangeably.
  - 1 tonne = 1000 kilograms
  - 1 kilogram = 1000 grams
  - 1 gram = 1000 milligrams
- Abbreviations for units of mass are metric tonne (t), kilogram (kg), gram (g) and milligram (mg).
- Conversion between units of mass.

$$\begin{tikzcd} \boxed{\text{t}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{kg}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{g}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{mg}} \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$

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Example 13 Converting between units of mass

Convert these mass measurements into the units given in brackets.

a 0.7 kg (g)
b 793 500 mg (g)
c 5 t (kg)
d 2.3 kg (mg)
e 6000 mg (kg)

WORKING	THINKING
a $0.7 \times 1000 = 700$ g	As there are 1000 grams in a kilogram, multiply 0.7 by 1000 to convert to the smaller unit.
b $793500 \div 1000 = 793.5$ g	As there are 1000 milligrams in a gram, divide 793 500 by 1000. to convert to the larger unit.
c $5 \times 1000 = 5000$ kg	As there are 1000 kilograms in a tonne, multiply 5 by 1000 to convert to the smaller unit.
d $2.3 \times 1000 = 2300$ g $2300 \times 1000 = 2300000$ mg	As there are 1000 grams in a kilogram, multiply 2.3 by 1000 to convert to the smaller unit. As there are 1000 milligrams in a gram, multiply 2300 by 1000 to convert to the smaller unit.
e $6000 \div 1000 = 6$ g $6 \div 1000 = 0.006$ kg	As there are 1000 milligrams in a gram, divide 6000 by 1000 to convert to the larger unit. As there are 1000 grams in a kilogram, divide 6 by 1000 to convert to the larger unit.

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Example 14 Applying the conversion of units of mass to practical problems

Doug's new caravan weighs 1.93 tonnes.

a Determine the mass of the caravan in kilograms.

b If Doug added 50 kg of food and water plus 35 kg of linen and clothing to his caravan to go away, determine the weight of the caravan now in tonnes.

WORKING	THINKING
a $1.93 \times 1000 = 1930\,\text{kg}$	As there are 1000 kilograms in a tonne, multiply 1.93 by 1000 to convert to the smaller unit.
Doug's caravan weighs 1930 kilograms.	Communicate your answer in a sentence.
b $1930 + 50 + 35 = 2015$ kg	Use the weight of the caravan in kilograms and add the weight of the food and water and the linen and clothing.
$2015 \div 1000 = 2.015$ t	As there are 1000 kilograms in a tonne, divide by 1000 to convert to the larger unit.
Doug's caravan now weighs 2.015 tonnes.	Communicate your answer in a sentence.

Exercise 3D

FUNDAMENTALS

Example 13

1 Convert these mass measurements into the units given in brackets.

a 0.85 kg (g)
b 973 400 mg (g)
c 2300 g (kg)
d 7.5 t (kg)
e 32 g (mg)
f 3570 kg (t)
g 4.7 kg (mg)
h 0.085 t (g)
i 78 000 mg (kg)
j 3 560 000 g (t)

APPLICATIONS

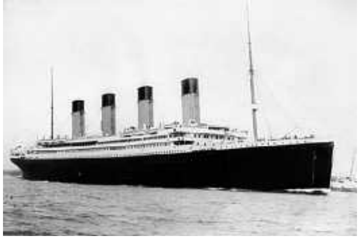
SF: 2–9	CF: –	CU: –

Example 14

★**2** Shae has had a new baby who weighs 3.45 kg. Determine the weight of Shae's baby in grams.

★**3** Jesse works in a food laboratory and needs to know the weight of a steak in milligrams to use in a formula. The supplied steak is labelled as being 450 g. Determine the weight of the steak in milligrams.

★**4** The Titanic ship weighed 52 310 tonnes. Scott needs to know the weight of the ship in kilograms to compare to a table of all the materials used in its construction. Calculate the weight of the Titanic in kilograms.



★**5** Esther has an ant farm that has around 420 worker ants inside and each ant weighs 3 milligrams. Calculate the total weight of the ants in grams.

★**6** Jude's caravan weighs 2.9 tonne.

a Determine the mass of the caravan in kilograms.

b If Jude packs the van with 20 kg of food, 60 kg of water and 40 kg of linen and clothing, determine the total weight of the caravan in kilograms.

★**7** Grace has packed 90 small packets of chips that weigh 45 grams each, ready for a school camp. Determine the total weight of all of the packets of chips in kilograms.

★**8** Isaiah has returned from an overseas holiday. When he departed, his bags weighed 25.2 kg. After a three week holiday in Europe, his bags weighed 28.9 kg. Determine the weight in grams of the items that Isaiah purchased overseas.

★**9** Ivanna is moving house and has hired a shipping container to store her household items. The shipping container weighs 3.7 tonnes and she loads it with 9940 kilograms of furniture and homewares. Calculate, in tonnes, the total weight of the shipping container once it has been packed with Ivanna's items.

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3E Choosing appropriate units, estimating mass and recognising the need for milligrams

LEARNING GOALS

- Recognise appropriate choices of units
- Estimate mass of various objects
- Recognise the need for milligrams

Why is it essential to estimate the mass of an object?

There are many circumstances when you have to be sure that something, such as a boat or trailer, is not overloaded, which may cause it to collapse or sink. For example, when loading a small boat with baggage and people, for safety reasons you must be careful not to overload the boat, bearing in mind its maximum safe carrying capacity. If scales are not available, you will have to estimate the mass of the baggage and people loaded on board.

The boat has a label saying the maximum permitted load is 360 kg, so these people need to estimate how much weight they are putting in the boat.

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WHAT YOU NEED TO KNOW

- There are four main types of unit that are used for mass. Below is a table that provides examples of items that are measured in each of the metric units of mass.

Mass unit	Examples
Milligrams – commonly used for measuring extremely light and small objects	Medicine, vitamins, strand of hair, mosquito or fruit fly
Grams – commonly used for measuring lightweight items	Cotton ball, small business card, insect or small animal
Kilograms – used for measuring heavier items	1 litre of water (= 1 kg), whole pineapple, baseball bat, baby, adult or large animal
Tonnes – used for measuring very heavy items	Car, plane, ship, truck, train, tractor or blue whale

- Why milligrams? Milligrams are necessary as they help with precise measurement of very light objects. They are particularly important in medicine.
- Conversion of units of mass: see page 141 for chart.
- When estimating the mass of an object, it is important to keep in mind some typical weights to which you can compare the object. Below is a common list of items with their average weight.
    - Mosquito 2.5 mg
    - Vitamin C tablet 1000 mg (1 g)
    - Single paperclip 1 g
    - A4 sheet of paper 5 g
    - Mouse 19 g
    - One cup of butter 225 g
    - 1000 paper clips 1 kg
    - 1 litre of water 1 kg
    - Supermarket bag of groceries 5–10 kg
    - Large suitcase packed with clothes and personal items 20 kg
    - Newborn baby 3.5 kg
    - Labrador dog 30 kg
    - Average sized adult human 65–75 kg
    - Small car 1 tonne
    - Full grown polar bear 1 tonne
    - Shipping container 3.5 t
    - Blue whale 150 t
    - Mid-sized airliner 60 t

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Example 15 Choosing the appropriate unit of mass

Decide on the appropriate mass units that would be used when weighing the following.

a

b

c

d

WORKING	THINKING
a grams	Grams is a relatively small unit of mass. A teaspoon of sugar would be quite light.
b kilograms	Large dogs look like they could weigh around half the size of humans. Kilograms would be the most appropriate unit of mass.
c tonnes	Large ships would be extremely heavy therefore tonnes would be the most appropriate unit of mass.
d milligrams	Small butterflies are very lightweight, therefore milligrams would be the best unit of mass.

Example 16 Estimating mass

Select the mass which is the most accurate for the following objects.

a	A bunch of flowers are around …	**i** 800 mg	**ii** 890 g	**iii** 8 kg
b	A car tyre is around …	**i** 750 g	**ii** 7.2 kg	**iii** 1 t
c	A train carriage is around …	**i** 2.8 t	**ii** 45 t	**iii** 980 kg

WORKING	THINKING
a 890 g **b** 7.2 kg **c** 45 t	Think of things that you know the weight of and then use that knowledge to estimate the weight of these items.

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Exercise 3E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a A __________ is a very small mass unit of measure that is for __________ lightweight objects such as __________, vitamins and __________.

b __________ are the units used to measure relatively light objects such as a paperclip or a mouse.

c One litre of __________ has a mass of one __________.

d Extremely large and heavy items can be measured using the unit __________, for example an airplane.

Example 15

2 Decide on the appropriate mass unit (mg, g, kg or t) that would be used when weighing the following objects.

a

b

c

d

Example 16

3 In your surrounds, look for the following items to pick up and estimate their mass. Compare your estimation with a peer, and if scales are available check the actual weight of the items.

a pencil
b classroom desk
c stapler
d chair
e book
f full pencil case
g eraser
h shoe
i an item of jewellery

Hint A millilitre of water weighs 1 g, and a litre of water weighs 1 kg. Sometimes it helps to imagine a volume of water about the same size as the object, and then estimate what that would weigh. Then adjust the estimate up or down taking into account whether it's made from a material that is 'heavier' or 'lighter' than water.

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APPLICATIONS

SF: 4–11	CF: –	CU: –

Example 15 ★**4** Decide on the appropriate mass units that would be used when weighing the following:

a

b

c

d

e

f

g

h

i

j

k

l

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Example 16

5 Select which mass is most likely accurate for the following objects:

a	A watermelon weighs around …	**i** 100 g	**ii** 9 kg	**iii** 1 kg
b	A butterfly weighs around …	**i** 2 g	**ii** 1 kg	**iii** 60 mg
c	An Olympic pool weighs around …	**i** 2400 t	**ii** 980 kg	**iii** 2 t
d	A calculator weighs around …	**i** 0.5 t	**ii** 2 kg	**iii** 65 g
e	An elephant weighs around …	**i** 1000 g	**ii** 1 t	**iii** 100 kg
f	A smart phone weighs around …	**i** 200 g	**ii** 1.5 kg	**iii** 900 g
g	A skateboard weighs around …	**i** 3 kg	**ii** 12 kg	**iii** 600 g
h	A tennis ball weighs around …	**i** 800 g	**ii** 1.3 kg	**iii** 59 g
i	A cricket bat weighs around …	**i** 45 g	**ii** 1 kg	**iii** 3 t
j	A cow weighs around …	**i** 780 kg	**ii** 2 t	**iii** 1230 g

6 Use the images from question **4** and estimate a mass for each object.

Use the following information for questions **7** to **10** and utilise a search engine for your estimations.
Big Ted's Truck Hire have different sized moving trucks that they hire out.
A 500 kg ute, 1-tonne truck, 3-tonne truck and a 10-tonne truck.

7 Graham is moving a washing machine and a dryer. Estimate their weight and hence determine what size vehicle he must hire.

8 Leanne is moving a large concrete water fountain from her front yard. It comes apart for transportation but requires four people to lift each section. Estimate the weight of the fountain and hence determine what size vehicle she will need to hire.

9 Melanie is moving the furniture from her two-bedroom apartment, which includes a fridge, washing machine, two beds, two-piece lounge suite, dining table and chairs, and two chest of drawers and all the contents of the kitchen. Estimate the weight of her furniture and hence determine the size of the truck she will need to hire.

10 Cassie and Ashley are moving house. They have a four-bedroom house and three children with lots of toys including a trampoline, and lots of furniture as they have two living rooms. Estimate the weight of their house contents and hence determine the vehicle they will need to hire.

11 Luan is using a canoe to transport himself, a supermarket bag of groceries and 10 L of drinking water to an island picnic site. The canoe is labelled to take a maximum load of 100 kg. If Luan weighs 75 kg, decide whether he is likely to be under or over the limit.

Problem-solving and modelling task

Background: Prices for storage are often based on volume.

Task: Your task is to calculate the volume required for an average two-person household to store their items in a shipping container while they are overseas. There are three possible options for the shipping container, and the bigger the container, the more expensive it is to hire. The dimensions for each option are as follows:

Small – 3 m long, 2.4 m wide, 2.6 m high

Medium – 6 m long, 2.4 m wide, 2.6 m high

Large – 12 m long, 2.4 m wide, 2.6 m high

You will need to calculate the volume of each shipping container and then the volume of the furniture and household items that you believe an average two-person household will have. To do this you will need to research and estimate the volume of each household item that you believe would need to go into storage such as a couch, television, fridge, washing machine, beds, dining table, and various cardboard boxes. You can research and find websites of storage hire companies that provide calculators to help you estimate the required amount of volume for storage. For example, *https://www.hitechstorage.com.au/self-storage-services/space-estimator/*

Once you have calculated the required volume that you will need stored, you then must decide on which shipping container is the most appropriate size and create a report that shows your working and your findings.

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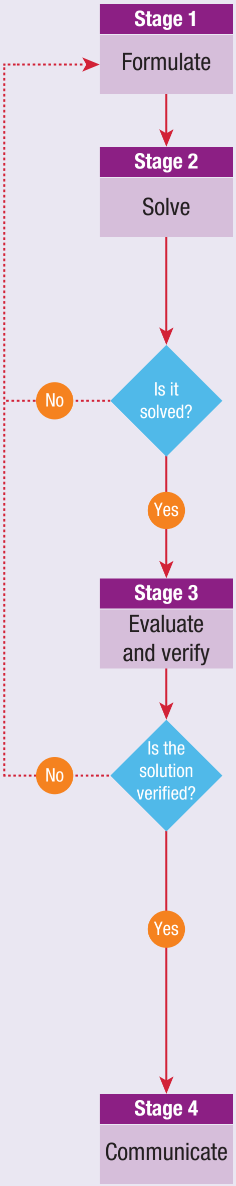
Approach to problem-solving and modelling task



Stage 1: Formulate

1 Research the sizes of various household items and find a storage space estimator website to use.

Stage 2: Solve

2 Make a list of the items you need to be stored.

3 Calculate the volume of each of the shipping containers.

4 Calculate the estimated volume of each of the furniture items.

5 Estimate the number of packing boxes required and calculate the total volume for these.

6 Compare the total volume of the household items to the volume of the shipping containers, and decide which shipping container size would be required.

Stage 3: Evaluate and verify

7 Check that you have included all necessary household items from your original list.

8 Compare your total volume of your household items to an online calculator from a storage item calculator.

9 Check that you have shown all your working for your calculations.

10 Check that you have allowed enough excess volume to reasonably cater for composite-shaped furniture and also for spaces in between (your furniture is unlikely to fit together perfectly like a jigsaw puzzle so you are better to overestimate, of course keeping a budget in mind).

Stage 4: Communicate

11 Present all your working in the form of a report and justify your final decision including your reasoning for your solution.

Chapter checklist

☐ I can use units of volume abbreviations.

1 Express the following metric units of volume to their abbreviated forms.
- **a** Cubic millimetres
- **b** Cubic centimetres
- **c** Cubic metres

☐ I can convert between units of volume.

2 Convert the following measurements into the units given in brackets.
- **a** 1.74 cm^3 (mm^3)
- **b** 2.67 m^3 (cm^3)
- **c** 2340 mm^3 (cm^3)
- **d** 0.0001 km^3 (cm^3)

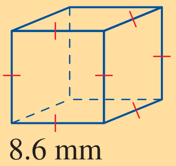
☐ I understand the relationship between volume and capacity including the conversions to millilitres, litres, kilolitres and megalitres.

3 Convert these volume and capacity measurements into the units given in brackets.
- **a** 75 cm^3 (mL)
- **b** 2.5 L (cm^3)
- **c** 6172 cm^3 (L)
- **d** 3.2 ML (kL)

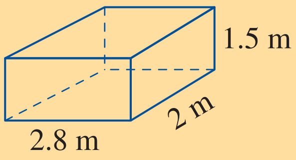
☐ I can calculate the volume and capacity of regular objects.

4 Calculate the volume of the following solids.

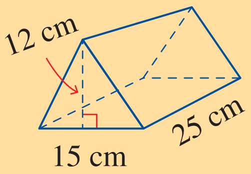
a



b

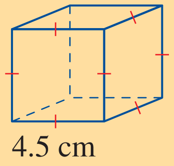


c



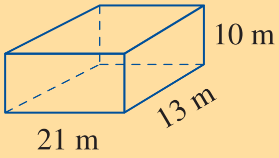
5 Calculate the capacity of the following solids.

a



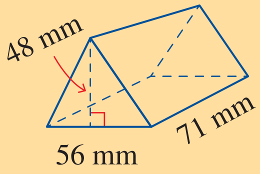
Convert to mL

b



Convert to ML

c



Convert to mL

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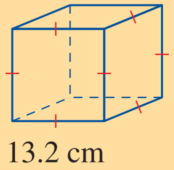
☐ I can estimate the volume and capacity of regular objects.

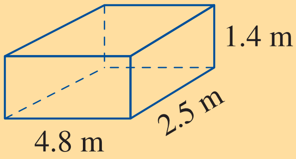
6 Estimate the volume and capacity of the following solids.

a



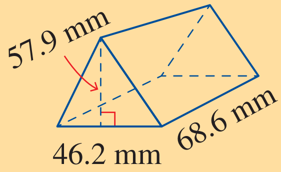
Convert to L

b



Convert to kL

c

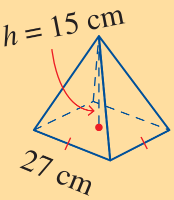


Convert to mL

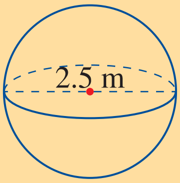
☐ I can calculate the volume and capacity of pyramids, spheres and cones.

7 Calculate the volume of the following solids.

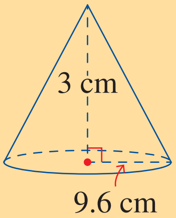
a



b

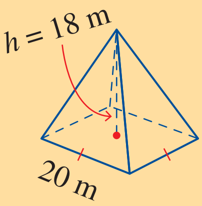


c



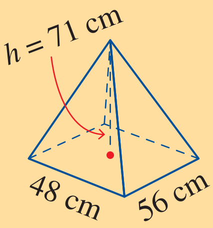
8 Calculate the capacity of the following solids.

a



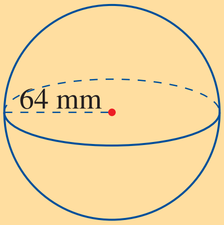
Convert to ML

b



Convert to L

c

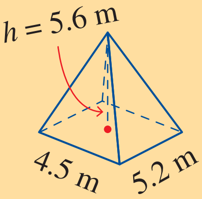


Convert to kL

☐ I can estimate the volume and capacity of pyramids and spheres.

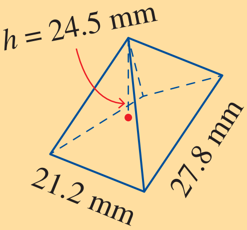
9 Estimate the volume and capacity of the following solids.

a



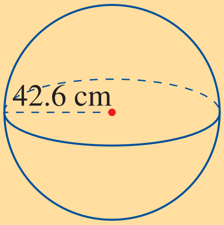
Convert to kL

b



Convert to mL

c



Convert to L

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☐ I can use units of mass abbreviations and convert between units of mass.

10 Convert the following metric units of mass to their abbreviated forms.
- **a** milligrams
- **b** grams
- **c** kilograms
- **d** tonnes

11 Convert these mass measurements into the units given in brackets.
- **a** 0.027 kg (g)
- **b** 973 400 mg (g)
- **c** 2300 g (kg)
- **d** 7.5 t (kg)
- **e** 3570 kg (t)
- **f** 94 000 mg (kg)

☐ I can identify the most appropriate metric units of mass and estimate the mass of various objects.

12 Decide on the appropriate mass units that would be used when weighing the following.

a

b

c

d

13 Use the images above and estimate the mass for each item.

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Chapter review

All questions in the review are assessment style.

Simple familiar

Section 3A

1 Brian has spread 2987 cm^3 of soil. Calculate the volume of soil in cubic metres.

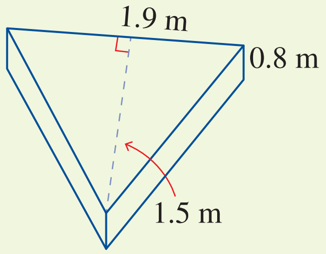
2 Ken's spa holds 1400 L of water. Calculate the capacity of the spa in cubic centimetres.

Section 3B

3 Daniel has a 50 cm cubic fish tank. Calculate the capacity of the tank in litres.

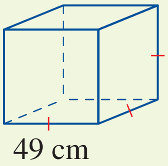
4 Jessica has had a new LPG tank installed in her car. It is a cylinder with a height of 65 cm and a diameter of 27 cm. Calculate the capacity of the tank in litres.

5 William is building a triangular fish pond as shown. Calculate the capacity of the fish pond in kilolitres.

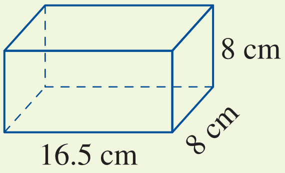


6 Calculate the volume of plastic you would need to make the following solids.

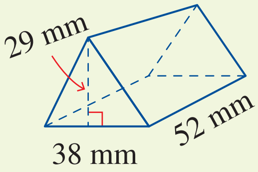
a



b



c



Section 3C

7 Jillian has been given a large beach ball that has a diameter of 900 cm. Calculate the capacity of the beach ball in megalitres.

8 Amara has designed a rectangular-based pyramid made out of plastic that can be used as a paperweight. Calculate the volume of the pyramid if it has a length of 10.5 cm, a width of 7.8 cm and a height of 9 cm.

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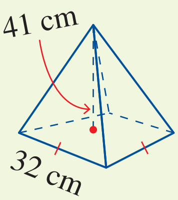
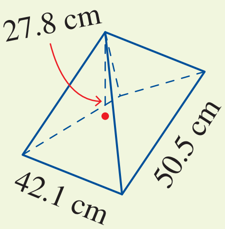
9 Calculate the capacity of melted wax that is needed to make candles of these sizes and shapes, in litres for **a** and **b**, and in millilitres for **c** and **d**.

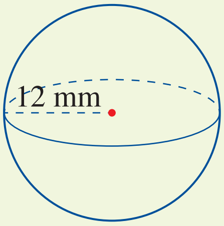
a



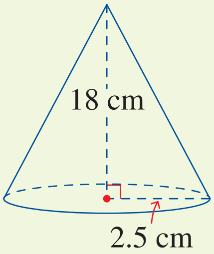
b



c



d



Section 3D

10 Andrew has a box of 80 bolts that weigh 120 g each. Calculate the total weight of the box of bolts in kilograms.

11 Ronslee is packing for an overseas holiday. She has carry-on luggage that weighs 660 g plus luggage that goes in the hold that weighs 22.7 kg. Determine the total weight of Ronslee's luggage in kilograms.

Section 3E

12 Decide on the appropriate units for weighing the following living things.

a

b

c

d

13 Select which mass is most likely accurate for the following objects;

a A banana weighs around … **i** 1 kg **ii** 130 g **iii** 27 mg
b A grasshopper weighs around … **i** 3 g **ii** 1 kg **iii** 60 mg
c A hippopotamus weighs around … **i** 3 t **ii** 692 kg **iii** 999 mg
d A blade of grass weighs … **i** 0.5 t **ii** 100 mg **iii** 65 g

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4 Scale drawings

Maths for an electrician: Justin Sturdee

I am a licenced electrician running my own business as an electrical contractor.

Tell us a bit about your job. What does a typical day look like?

As an electrician you install, repair and maintain electrical installations in buildings and portable devices. This entails working inside a building, climbing into the roofs through manholes as well as working underneath some buildings, basically wherever the cabling goes.

What maths did you study in school?

I did trade and business maths at school, which was an easy practical maths. Unfortunately, when I decided to become an electrician, the maths required was similar to maths B and it took a lot of work to catch up, as it was a lot harder than the maths I did at school.

How do you use mathematics in your job?

I usually work out the load on the circuit if I want to install an air conditioner, so I need to use maths to work out how to use the wattage. I use some basic algebra to work out what sort of cable I need to use. On new builds, I use the builder's plans to set up and install the cabling throughout the new build.

In this chapter

Syllabus reference

Unit 3 Topic 2 Scales, plans and models

Interpret scale drawings (6 hours)

In this sub-topic, students will:

- interpret commonly used symbols and abbreviations in scale drawings
- find actual measurements from scale drawings, including lengths, perimeters and areas
- estimate and compare quantities, materials and costs using actual measurements from scale drawings **[complex]**.

Creating scale drawings (4 hours)

In this sub-topic, students will:

- understand and apply drawing conventions of scale drawings, including scales in ratio, clear indications of dimensions and clear labelling **[complex]**
- construct scale drawings by hand and by using software packages **[complex]**.

Pre-test

1 Determine the following ratios in simplest form.

a 12 : 144

b 25 : 200

c 50 : 750

d 14 : 280

e 11 : 330

Hint Find a common factor of the two numbers in the ratio
Divide both numbers by the common factor

2 Convert the following measurements in millimetres to metres by dividing by 1000.

a 2400 mm

b 1800 mm

c 14 000 mm

d 36 000 mm

3 Convert the following measurements in metres to millimetres by multiplying by 1000.

a 2.25 m

b 1.4 m

c 3.84 m

d 0.279 m

4 Calculate the perimeters of rectangles with the following widths and lengths.

a 10 m, 15 m

b 8 cm, 20 cm

c 7 m, 28 m

d 120 mm, 200 mm

Hint Recall the definition of a perimeter

5 Determine the cost of laying carpet in the following rooms, given the carpet costs $62 per square metre laid.

a 10 m^2

b 24 m^2

c 42 m^2

d 60 m^2

Hint 'm^2' is the symbol for a square metre

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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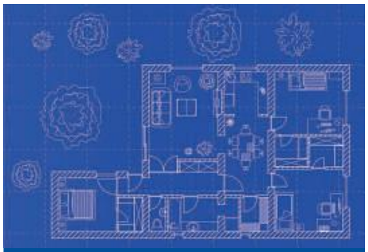
4A Reviewing scales and interpreting scale symbols and abbreviations

LEARNING GOALS

- Review the concept of a scale
- Convert between units of measure
- Simplify scales
- Identify and interpret common symbols and abbreviations in scale diagrams
- Use the internet to research common symbols and abbreviations

Why it is essential that we use symbols and abbreviations on a scale diagram?

Symbols and abbreviations are added to scale diagrams to help users interpret the information in the diagram and identify the use of each space or detail. Without symbols and abbreviations a detailed plan would be difficult to read as there would be too much information written on the plan.



House plans use abbreviations and symbols to represent different parts of the house.

WHAT YOU NEED TO KNOW

- A **scale** is a comparison of like quantities usually expressed as a **ratio** such as 1 : 100. The first number is the size on the diagram or plan, the second number is the actual size in real life. Remember this as Diagram : Actual.
- **Simplify** a scale by changing both the diagram and actual measurements to the same units, and then simplifying the ratio using common factors.
- Scales are used in construction and manufacturing industries.
- Scale drawings, scale diagrams, and plans are the same thing – the terms are used interchangeably
- Plans are drawn to scale by designers, planners and architects prior to construction.
- Builders and manufacturers use **scale drawings** to build at full size.
- To convert between units of length use these calculations:

$$\begin{tikzcd}\boxed{\text{km}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{m}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \boxed{\text{cm}} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \boxed{\text{mm}} \arrow[l, bend left, "\div 10"]\end{tikzcd}$$

- **Symbols** and **abbreviations** are used on plans to avoid covering detail with text labels.
- Some special or unfamiliar terms such as **void** may be used on house plans.

Example 1 Simplifying scales in ratio

Simplify the following scales.

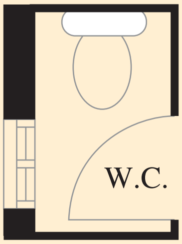
a 1 cm : 2 m

b 5 mm : 2 m

WORKING	THINKING
a 1 cm : 2 m	Convert both sides of the scale to the same units.
1 cm : 200 cm	There are 100 cm in 1 m so multiply by 100. $2 \times 100 = 200$ cm
1 : 200	As both units are the same, remove the cm symbol. Since the scale has 1 as the first number, it doesn't need to be simplified any more.
b 5 mm : 2 m	Convert both sides of the scale to the same units.
5 mm : 2000 mm	There are 1000 mm in 1 m so multiply by 1000. $2 \text{ m} \times 1000 = 2000$ mm
1 mm : 400 mm	Divide both sides by 5 to simplify.
1 : 400	As both units are the same, remove the mm symbol.

Example 2 Interpreting abbreviations used on scale drawings

Interpret the meaning of the abbreviation W.C. from the diagram.



WORKING	THINKING
The abbreviation W.C. stands for Water Closet, which is a toilet.	Type 'W.C.' abbreviation into an internet search engine to discover its meaning.

Activity 4A Symbols, abbreviations and special terms used on house plans: See the Interactive Textbook for this activity to research and list abbreviations, symbols and special terms used on house plans.

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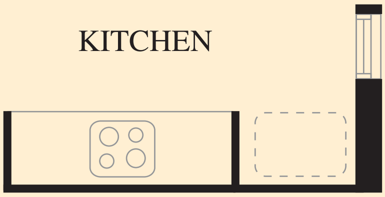
Example 3 Interpreting symbols used on scale drawings

Interpret the meaning of the symbol  from the diagram.



WORKING	THINKING
It is a cooktop.	As it is in the kitchen, we can use our general knowledge of kitchens to identify that a cooktop would look like this symbol.

Exercise 4A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** A scale is a comparison of __________ quantities.
- **b** __________ are used to represent objects on a scale diagram.
- **c** We use __________ to simplify words on a scale diagram.
- **d** To convert metres to kilometres we must __________ by 1000.
- **e** To convert metres to centimetres we must __________ by 100.

2 Convert the following measurements into the unit indicated in brackets.

- **a** 5 m (cm)
- **b** 28 cm (mm)
- **c** 3.73 m (cm)
- **d** 2.75 cm (mm)
- **e** 4 m (mm)
- **f** 6.75 m (mm)
- **g** 250 cm (m)
- **h** 1200 mm (m)

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Example 1

3 Simplify the following scales.

a 1 cm : 4 m
b 1 mm : 20 cm
c 1 mm : 2 m
d 1 mm : 3.75 m
e 5 cm : 3 m
f 7.5 cm : 3.6 m
g 12 mm : 7.2 m
h 15 mm : 4.5 m

Hint The scales must have the same units for both numbers
Divide both numbers by a common factor

Example 2

4 Interpret the meaning of each of the following abbreviations by first identifying its position in the scale diagram of the house.

a L'DRY
b SRD
c REF
d ROBE
e ASD
f ASW

Hint Activity 4A will help set up to answer these questions

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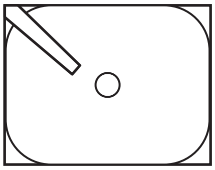
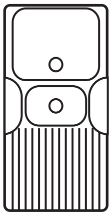
Example 3

5 Interpret the meaning of each of the following symbols by first identifying its position in the scale diagram of the house in question **4**.

a

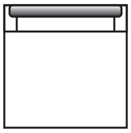


b

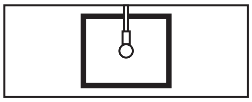


c

d



e



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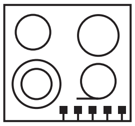
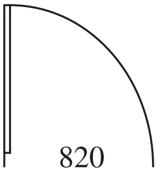
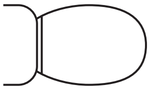


g



h



6 This plan is the top floor of a two-storey house. Interpret the meaning of each of the following by first identifying its position in the scale diagram of the house.

a W.I.R

b PDR

c W.I.L.

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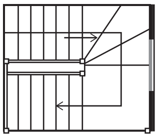
7 Interpret the meaning of each of the following symbols by first identifying its position in the scale diagram of the house in question **6**.

a



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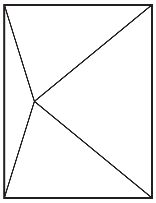
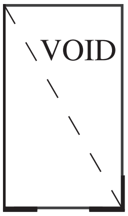


c



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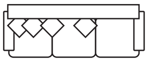
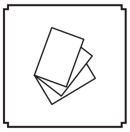


e



f

APPLICATIONS

SF: 8–10	CF: –	CU: –

★ **8** Luther was looking over the internal plans for his new house from the builder and discovered some abbreviations that he did not understand. These were AS, U/G and ENS. Determine what the abbreviations represent on the plan by researching their meaning.

★ **9** Lillian saw the abbreviations of FW, DP, HWS on her house plans and did not understand what they represented. Determine what the abbreviations represent on the plan by researching their meaning.

★ **10** Lana wants to create a symbol to represent her grand piano on her new house plans. Create a symbol for her to use.

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4B Identifying and calculating measurements of length, perimeter and area from scale diagrams

LEARNING GOALS

- Take precise measurements from drawings to create a scale
- Determine a scale from a plan
- Simplify scales
- Revise calculating perimeters and areas of shapes
- Use a scale to calculate actual measurements and perimeters from plans
- Determine costs in building by using perimeters

Why is it essential that we know how to read and interpret scale plans and diagrams?

Many professions and occupations use scale diagrams and plans to complete projects such as designing a new car or constructing a building. These professions and occupations include planners, architects, landscapers, builders, designers and manufacturers. By knowing how to read and interpret scale plans and diagrams, a builder can accurately build the owner's/designer's house to the correct measurements and design.

Understanding the symbols, abbreviations and scales of a house plan are important when interpreting the designer's details

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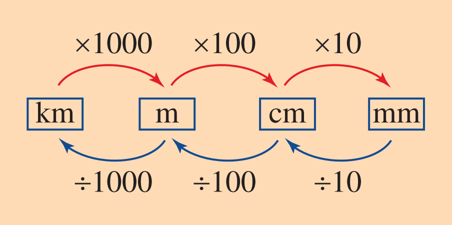
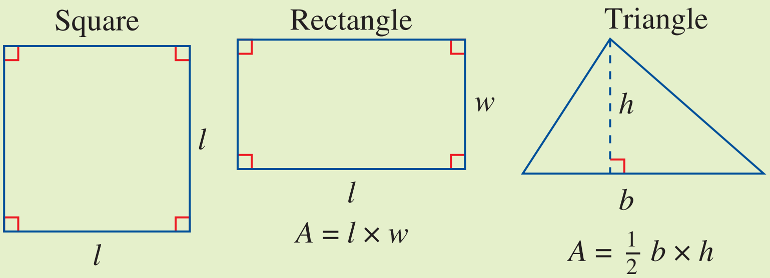
WHAT YOU NEED TO KNOW

- Use a ruler to measure to the nearest millimetre or tenth of a centimetre and ensure that you start at zero.
- Length is measured in metres or units based on the metre.
- Convert measurements of length by using these calculations:



- A scale is the ratio Diagram : Actual for a dimension.
- The **scale factor** is the second number in a scale ratio that starts with 1.
- To convert a diagram length to actual length, multiply by the scale factor.
- To convert an actual length to the diagram length, divide by the scale factor.
- **Simplify** a scale by changing both the scale and actual measurements to the same units, and then simplifying the ratio using common factors.
- A perimeter is the outside distance of a 2D shape calculated by adding up the distances of all the sides.
- The area is the number of unit squares in a 2D shape.
- Substitute values into formulas to calculate areas.
- Common area formulas are:



$A = l^2$

$A = l \times w$

$A = \frac{1}{2} b \times h$

A	area
l	length
w	width
h	height
b	breadth

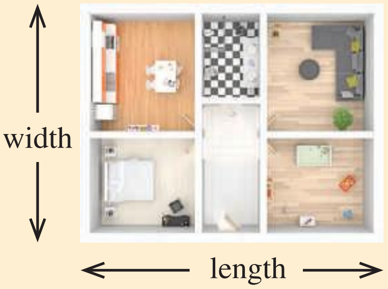
- A **composite shape** is a shape that is made up of two or more common shapes. Addition and/or subtraction can be used to find the area of composite shapes.
- You may also need to recall the area formulas of other shapes from sections 2B and 2C.

Example 4 Determining a scale from a plan

The actual length of the front of the house is 10 m.

a Determine the scale of the plan.

b Calculate the perimeter of the actual house.



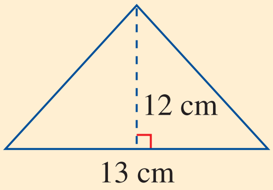
WORKING	THINKING
a The length of the front of the house on the diagram is 4 cm.	Measure the external length of the front of the house on the diagram.
Diagram : Actual 4 cm : 10 m	Write the scale as the ratio of the length from the diagram to the actual length of the house. There are 100 cm in 1 m.
4 cm : 1000 cm	Convert the actual length to cm by multiplying by 100.
1 : 250	Remove the units and simplify the scale by dividing both sides by 4.
b Perimeter = length + width + length + width	We need the length and width to calculate the perimeter of the house. We know the length, now we need to calculate the width.
Width = 31 mm	Measure the external width of the house on the scale diagram. (This time we have used millimetres)
$= 31 \text{ mm} \times 250$ $= 7750 \text{ mm}$	Calculate its actual measurement by using the scale 1 : 250, which means multiply the diagram length by the scale factor which is 250.
$= 7.75 \text{ m}$	Convert the measurement to metres. There are 1000 mm in 1 m, so we divide by 1000.
Perimeter $= 10 \text{ m} + 7.75 \text{ m} + 10 \text{ m} + 7.75 \text{ m}$ $= 35.5 \text{ m}$	Calculate the perimeter.

Note that in the example above we could have calculated the scale from the width, and used it to then calculate length. Any dimension can be used to calculate scales.

Example 5 Calculating the areas of 2D shapes

Calculate the area of the following shapes.

a



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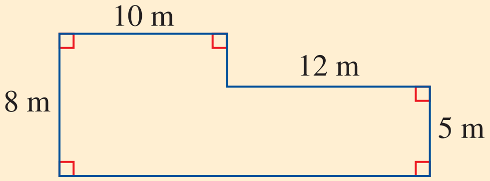
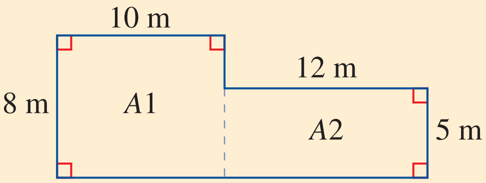


WORKING	THINKING
a $A = \frac{1}{2} \times b \times h$	Write the area formula for a triangle.
$A = \frac{1}{2} \times 13 \times 12$	Substitute the measurements into the formula.
$A = 78 \text{ cm}^2$	Calculate the area of the triangle. Include the units.
b 	Break the shape into two familiar shapes, which are rectangles. Call them $A1$ and $A2$.
$A = l \times w$ $A1 = 10 \times 8$	Write the area formula for a rectangle. For the first rectangle, substitute the measurements into the formula.
$A1 = 80 \text{ m}^2$	Calculate the area of the first rectangle. Include the units.
$A = l \times w$ $A2 = 12 \times 5$ $A2 = 60 \text{ m}^2$	For the second rectangle, substitute the measurements into the formula. Calculate the area of the second rectangle. Include the units.
$A = 80 + 60$ $A = 140 \text{ m}^2$	Add the areas together to get the total area, A.

Example 6 Calculating the actual measurements from a scale diagram

The actual width of the toilet cubicle is 1 m.

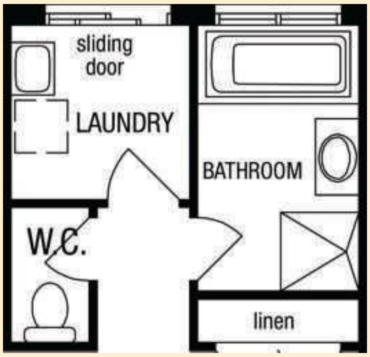
a Determine the scale for the diagram shown.

b Use the scale to determine the dimensions (in metres) of:

- **i** the laundry
- **ii** the bathroom

c Calculate the perimeter (in metres) of:

- **i** the laundry
- **ii** the bathroom



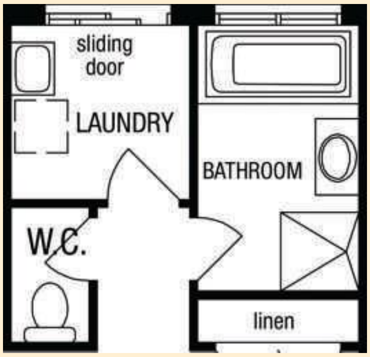
WORKING	THINKING
a The width = 1 cm	The toilet cubicle is labelled W.C.
Diagram : Actual	Measure the internal width of the W.C.
1 cm : 1 m	Write as a scale of the W.C. width on the diagram to the actual W.C. width.
1 cm : 100 cm	Convert the actual width to cm by multiplying by 100.
1 : 100	Remove the units.
b i Diagram dimensions are 2.2 cm by 2.2 cm	Measure the internal length and internal width of the laundry.
Actual dimensions are 220 cm by 220 cm	Multiply both measurements by the scale factor of 100.
= 2.2 m by 2.2 m	Convert to metres by dividing by 100.
ii Diagram dimensions are 2.1 cm by 3.5 cm	Measure the internal length and internal width of the bathroom.
Actual dimensions are 210 cm by 350 cm	Multiply both measurements by the scale factor of 100.
= 2.1 m by 3.5 m	Convert to metres by dividing by 100.
c i $P = l + w + l + w$ $P = 2.2 + 2.2 + 2.2 + 2.2$ $P = 8.8$ m	Use the length and width to calculate the perimeter of the laundry.
ii $P = l + w + l + w$ $P = 2.1 + 3.5 + 2.1 + 3.5$ $P = 11.2$ m	Use the length and width to calculate the perimeter of the bathroom.

Example 7 Calculating areas using measurements from a scale diagram

The actual width of the toilet cubicle is 1 m.

a Determine the scale for the diagram below.

b Use the scale to calculate the area of the bathroom, in metres.



WORKING	THINKING
a The width = 1 cm	Measure the width of the toilet.
Scale : Actual	Write as a scale to the actual toilet width.
1 cm : 1 m	
1 cm : 100 cm	Convert the actual width to cm by multiplying by 100.
1 : 100	Remove the units.
b Diagram dimensions are 2.1 cm by 3.5 cm	Measure the length and width of the bathroom.
Actual dimensions are 210 cm by 350 cm	Multiply by the scale factor of 100. Convert to metres by dividing by 100.
= 2.1 m by 3.5 m	
$A = l \times w$	Write down the area formula for a rectangle.
$A = 2.1 \times 3.5$	Substitute the values to calculate the area of the bathroom and write the units.
$A = 7.35 \text{ m}^2$	

Technology activity 4B: Using the measuring tool in Adobe Acrobat Reader to measure dimensions on a scale drawing in a PDF file. See the Interactive Textbook for this activity.

Exercise 4B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

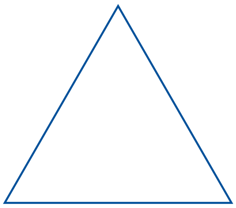
- **a** A ______________ is the outside distance on a 2D shape.
- **b** The ______________ is the number of unit squares in a 2D shape.
- **c** We ______________ values into ______________ to calculate areas.
- **d** Area of a rectangle = ______________ × ______________
- **e** Area of a triangle = $\frac{1}{2}$ × ______________ × ______________

Example 4

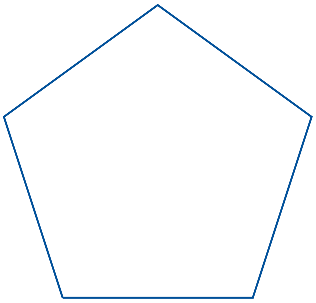
2 The actual length of the side of each shape is given in brackets. For each shape, measure the side lengths to:

- **i** determine the scale
- **ii** calculate the actual perimeter

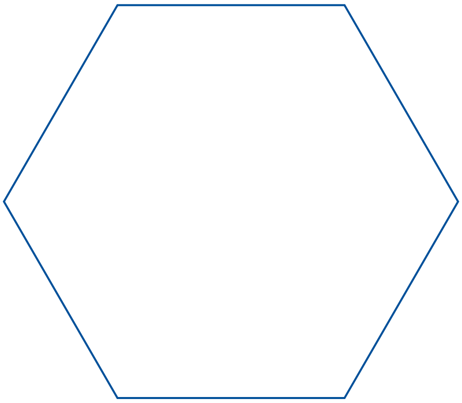
a (6 m)



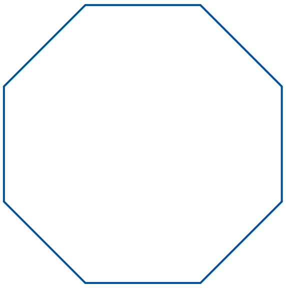
b (2 m)



c (1.5 m)



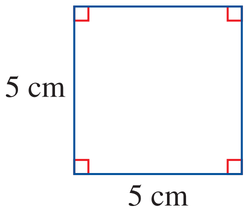
d (7.5 m)



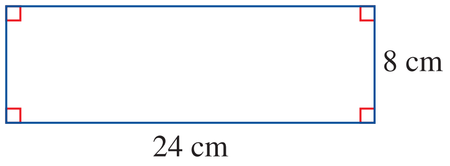
Hint A scale is a ratio of size in diagram to actual size

Example 5 **3** Calculate the area of the following shapes to 1 decimal place where necessary.

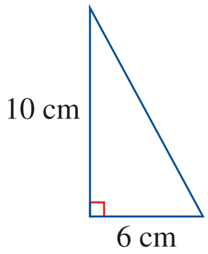
a



b



c



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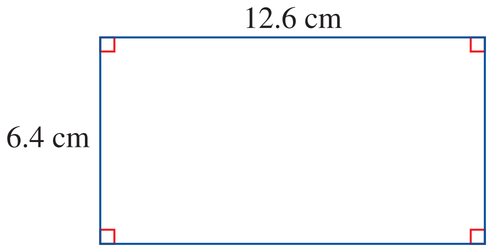
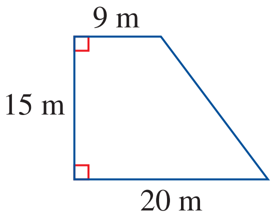


e



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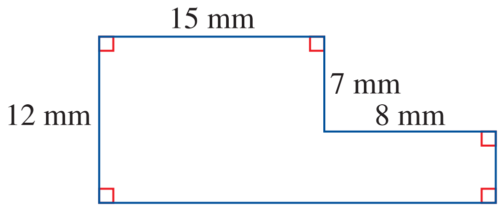
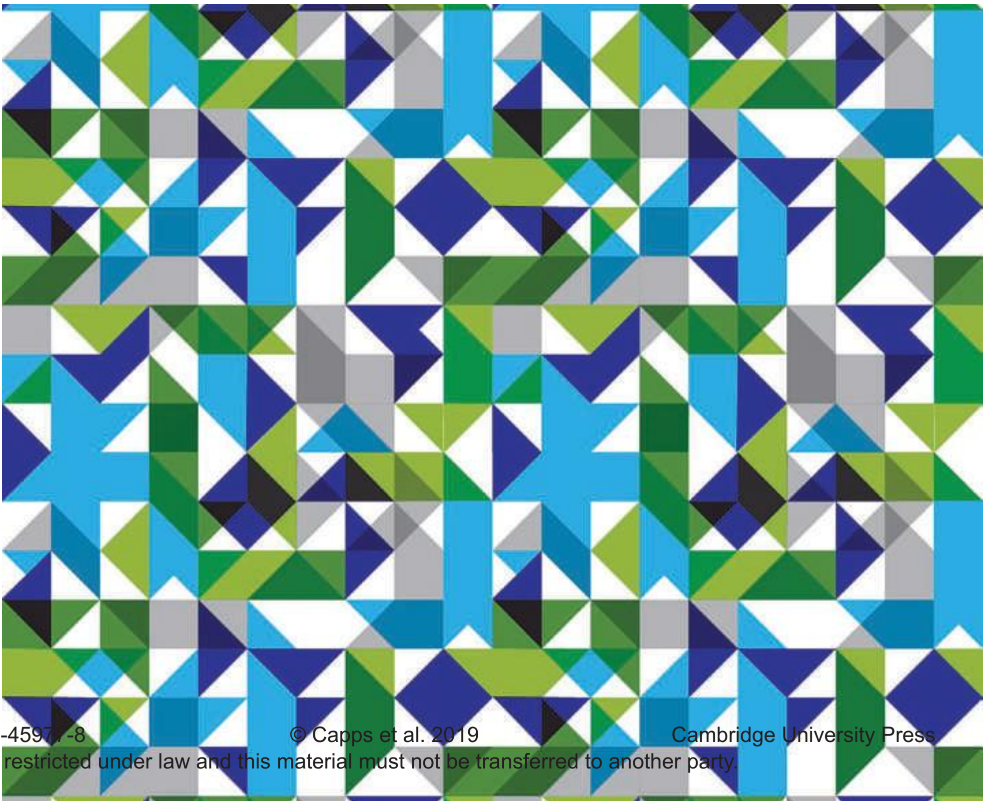


Hint These last two are composite shapes that should be divided up.



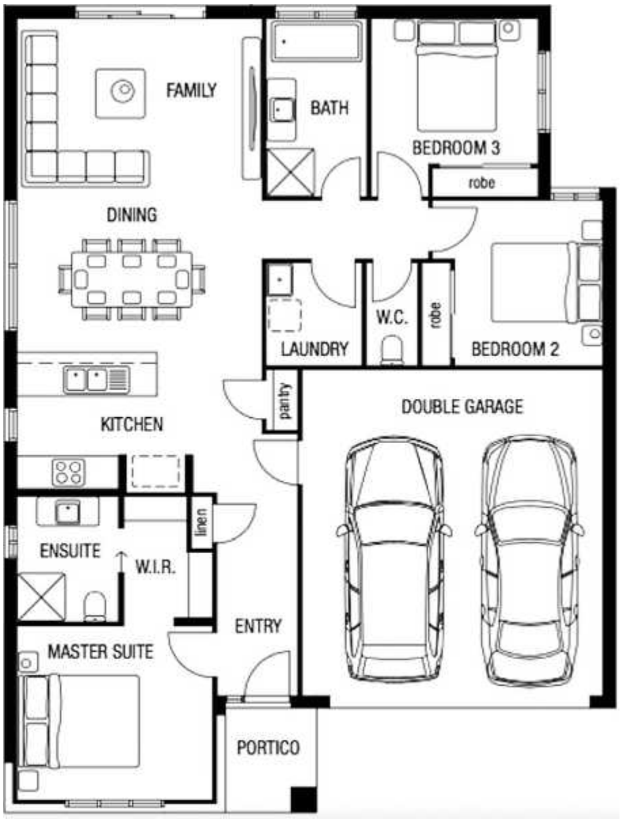
APPLICATIONS

SF: 4–10	CF: –	CU: –

Example 6

4 Refer to the plan below. The actual width of the garage is 6 m.

- **a** Determine the scale of the plan.
- **b** Use the scale to determine the actual dimensions (in millimetres) of the:
  - **i** bedroom 3 (including robe)
  - **ii** W.C.
  - **iii** laundry
  - **iv** family/dining area
  - **v** ensuite
  - **vi** garage
- **c** Calculate the actual perimeter (in metres) of each part of the house in metres from part **b**.



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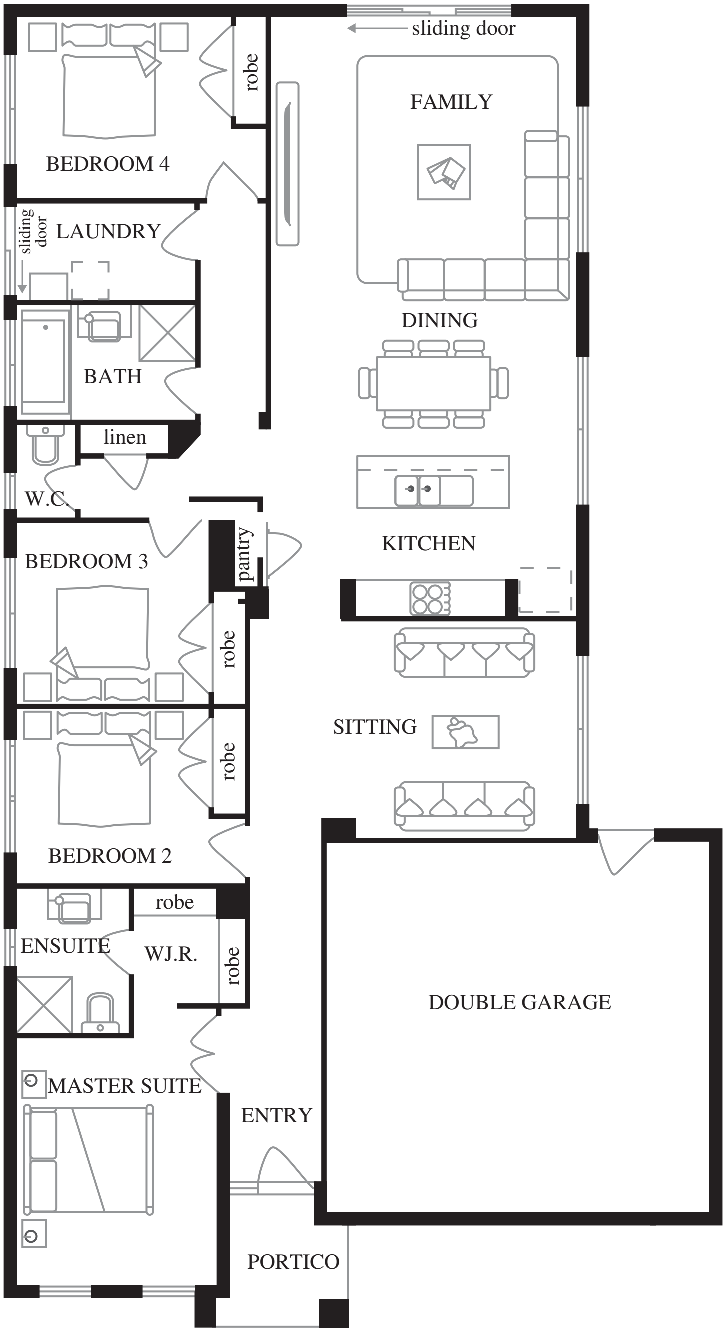
Example 7

5 Refer to the plan below. The actual width of the garage is 5 m.

- **a** Determine the scale for the plan.
- **b** Use the scale to calculate the dimensions (in millimetres) of the:
  - **i** master suite (not including the ensuite and W.I.R.)
  - **ii** garage
  - **iii** bathroom
  - **iv** ensuite
- **c** Calculate the area of the rooms listed in part **b** in square metres.



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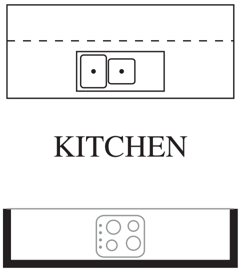
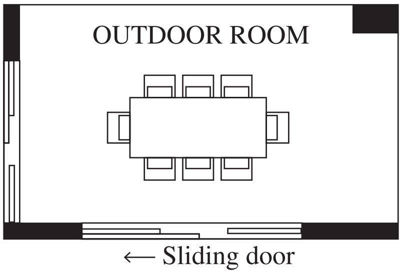
★6 Monica measured the actual length of the island bench to be 3.6 m. Determine the scale of the plan.



★7 Wayne measured the actual internal length of the outdoor room to be 4 m. Determine the scale of the plan.



★8 James measured the land frontage to the road of his new block (Lot 37) to be 20 m wide (This means the side of the plot next to the road.). He wants to determine the area of the block so that he can choose a house to fit on the block.

a Determine the scale for the plan of the block of land.

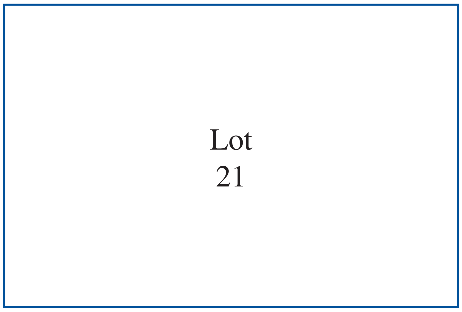
b Use the scale to calculate the area of Lot 37.

Assume the block of land is a rectangle.

★9 Wilma has bought a block of land named Lot 21. The actual length of the shortest side is 20 metres.

a Determine the scale for this diagram of the block of land.

b Calculate the perimeter of her block of land.



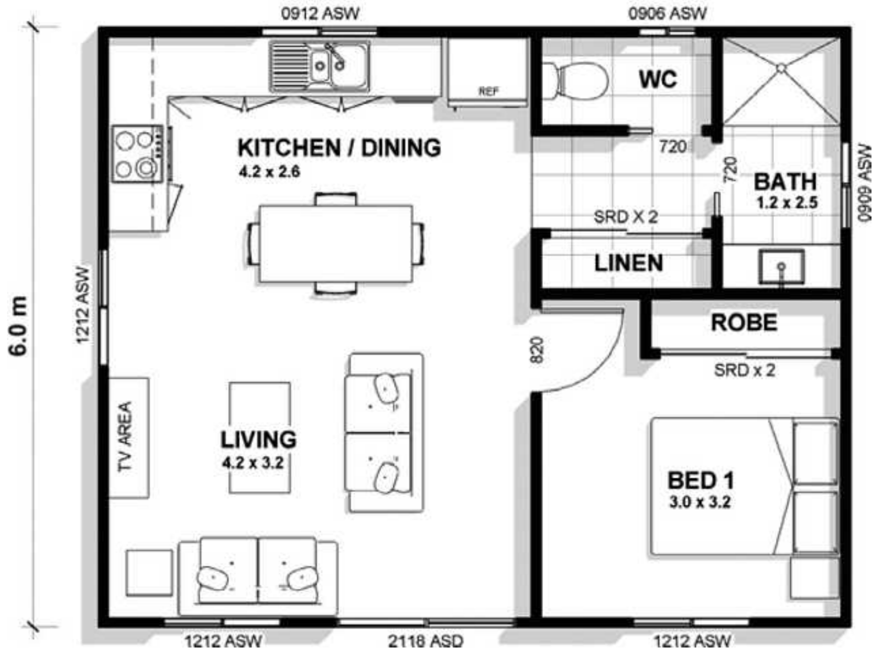
★10 Henry is building a granny flat in his backyard for his mother. The council allows a maximum area for a granny flat of 60 m². One side length of the plan for the granny flat is marked as 6 m.

a Determine the scale for the plan of the granny flat.

b Calculate the actual area of the Henry's granny flat.

c Consider whether the granny flat meets council requirements.

Hint Measure on the diagram the side length that is labelled as 6 m, which will give you the scale



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4C Estimating and comparing quantities, materials and costs from scale diagrams COMPLEX

LEARNING GOALS

- Determine costs from dimensions and areas calculated from scale diagrams
- Estimate quantities of materials needed
- Compare costs of materials to determine best values

Why is it essential to be able to calculate dimensions and areas from plans?

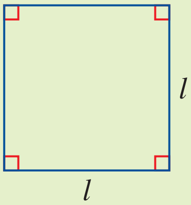
Homeowners need to budget for home renovations, building homes and landscaping. Many commonly-used materials are sold by length or area. Skirting boards are sold by length. Stores sell floor coverings such as carpet and tiles by the square metre. Knowing how to calculate areas in square metres from a plan helps people to determine what tiles and carpet they can afford.

Tilers are priced in dollars per m^2, and the tiler charges for labour in the same units.

WHAT YOU NEED TO KNOW

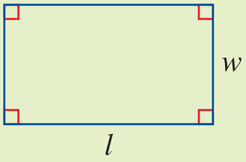
- Use a ruler to measure to the nearest millimetre or tenth of a centimetre, and ensure that you start at zero.
- Simplify a scale by changing both the scale and actual measurements to the same units, and then simplifying the ratio using common factors.
- The area is the number of unit squares in a 2D shape.
- Substitute values into formulas to calculate areas.
- Area formulas for some familiar 2D shapes.

Square



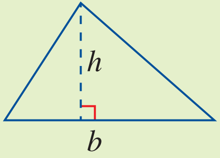
$A = l^2$

Rectangle



$A = l \times w$

Triangle



$A = \frac{1}{2} b \times h$

- A ‘linear’ or ‘lineal’ metre is just a metre of length, the term is used to emphasise it is not a square metre.
- Cost is usually calculated as number of units multiplied by price in dollars per the same unit. E.g. the cost of 3 m of material sold by length and priced at \$5.00 per metre is \$15.

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Example 8 Determining costs using calculated areas

Ryan is tiling his home theatre, which is a rectangle with dimensions 5.5 m by 4 m. The tiles that Ryan has chosen cost $29.95 per square metre and the tiler charges $45 per square metre to lay the tiles.

a Calculate the area of Ryan's home theatre.

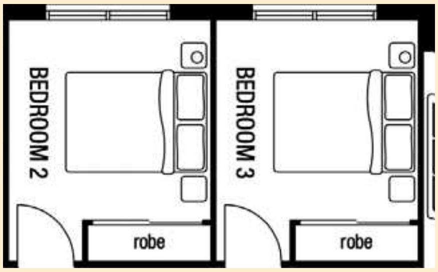
b Determine how much it will cost Ryan to tile his home theatre.

c After tiling the tiler offers to install a decorative skirting board around the perimeter of the room at a cost of $3.60 per metre. Calculate the extra cost, ignoring the doorway.

WORKING	THINKING
a $A = l \times w$ $A = 5.5 \times 4$ $A = 22\text{ m}^2$	Write the formula for area of a rectangle. Substitute the values of length and width into the formula. Calculate the area.
b Add cost of tiles and tiler: $\$29.95 + \$45 = \$74.95\text{ per m}^2$	Add up the cost per m^2 of the tiles and the labour of the tiler.
$\text{Total cost} = 22\text{ m}^2 \times \$74.95$ $= \$1648.90$	Calculate the total cost by multiplying the area by the cost per m^2.
Perimeter = **c** length + width + length + width $= 5.5 + 4 + 5.5 + 4$ $= 19\text{ m}$	The perimeter is the distance around the edge of the room. Add up the sides.
$\text{Cost of skirting} = 19\text{ m} \times \$3.60/\text{m}$ $= \$68.40$	Calculate the cost by multiplying the perimeter by the cost per m.

Example 9 Determining costs using areas calculated from a scale diagram

Erin is replacing the carpet in two of her bedrooms as shown in the plan. The carpet that Erin has chosen costs $43.75 per square metre fully laid. The actual robe length is 2 m.



a Determine the scale for the plan of the bedrooms.

b Determine the combined area of the bedrooms, including the robes, ignoring the joining wall and robe walls.

c Calculate the cost of the carpet.

WORKING	THINKING
a 16 mm	Measure the length of the robe on the drawing.
16 mm : 2000 mm	Write the scale as a ratio of length in the drawing to the actual robe length in the same units.
1 : 125	Simplify the scale.
b The rooms are both 54 mm by 26 mm	Measure the length and width of the 2 rooms together, ignore the joining wall and robe walls in your calculations.
54 mm $\times$ 125 by 26 mm $\times$ 125 6750 mm by 3250 mm	Determine the actual dimensions of the rooms by multiplying by the scale factor of 125.
6.75 m by 3.25 m	Convert to metres.
$A = l \times w$	Use the area of rectangle formula.
$A = 6.75 \times 3.25$ $A = 21.94 \text{ m}^2$	Calculate the area of the rooms.
c Cost = 21.94 $\text{m}^2 \times \$43.75$ = \$959.88	Calculate the cost by multiplying the area by the cost per m^2.

Exercise 4C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** We ______________ a scale by dividing both parts by the same number.
- **b** A _______ _______ is the second number in the scale ratio when the first number is 1.
- **c** We calculate cost by multiplying the ______________ per unit by the number of ______________ .

Example 8

2 Bella has just completed her new rumpus room that she built underneath her house. She wants to buy skirting boards to place around the edge of the rumpus room. Bella's rumpus room is 6 m by 4 m and the skirting boards cost \$20 per metre.

- **a** Calculate the perimeter of the rumpus room.
- **b** Calculate the cost of the skirting boards by multiplying the cost per metre by the perimeter.

3 Tom wants to carpet his bedroom. His bedroom is 3 m by 3.5 m in dimensions and the cost of carpet is $52 fully laid per square metre.

a Calculate the area of the bedroom.

b Calculate the cost to carpet the room.

Hint Total cost is the price per unit times the number of units.

Example 9

4 Tamika wants to tile her patio. The patio is 6 m by 5.5 m and the cost of tiling is $65 fully laid per square metre.

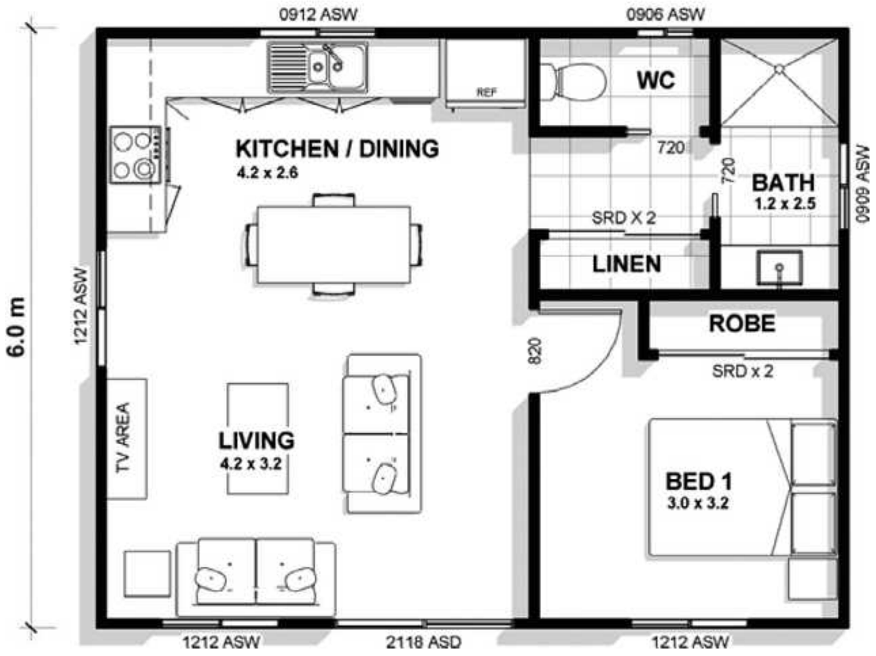
a Calculate the area of the patio.

b Calculate the cost to tile the patio.

APPLICATIONS

SF: –	CF: 5–14	CU: –

Henry is working on some extra features for the granny flat for his mother. Use the following scale drawing to answer questions **5** to **7**.



5 Henry has decided that he wants to have a concrete mowing strip poured around the edge of the granny flat in his backyard.

a Determine the scale of the granny flat plan.

b Use the scale to calculate the length of the granny flat.

c Calculate the perimeter using the actual length and width of the granny flat. The concreting will cost $20 per metre of the perimeter of the flat.

d Calculate the total cost of the mowing strip by multiplying the cost per metre by the perimeter.

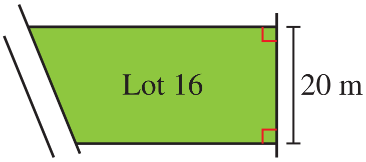
6 Henry's mum Dorethy has decided that she wants a special cornice (decorative strip to border the ceiling and the walls) in her bedroom. The cost of the cornice is \$4.75 per linear metre.

- **a** Measure the length and width of the granny flat bedroom on the diagram, ignoring the robe and door.
- **b** Using the scale you have found in the previous question, determine the actual length and width of the bedroom.
- **c** Calculate the perimeter of the bedroom.
- **d** Calculate the cost of the cornice by multiplying the cost per linear metre by the perimeter.

7 Henry wants to lay wooden floorboards in the kitchen, dining and living space. The cost of the floorboards including laying is \$62 per square metre.

- **a** Measure the length and width of the kitchen, dining and living area ignoring the cabinets and furniture.
- **b** Using the scale you have found in the previous question, determine the actual length and width of the kitchen, dining and living area.
- **c** Calculate the area of the kitchen, dining and living area.
- **d** Calculate the cost of laying the floorboards in the granny flat by multiplying the cost per square metre by the area.

8 Alaskah is buying a block of land as shown in the scale drawing. If she is fencing all 4 sides at a cost of \$47.50 a metre, calculate the total cost of fencing Alaskah's property.



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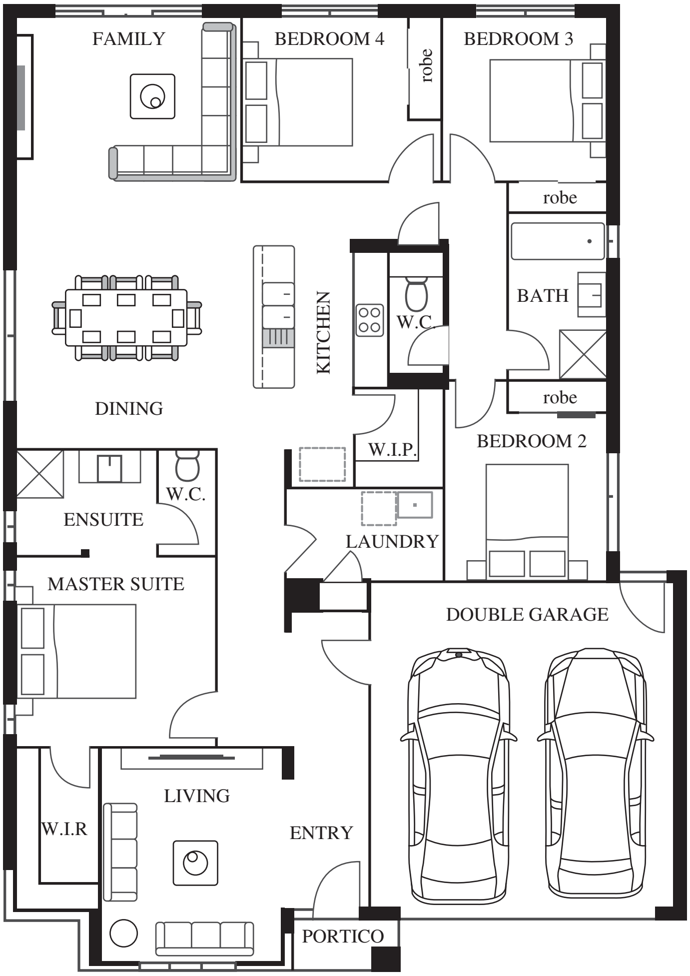
For questions **9** to **13** use the following house plan where the actual width of the garage is 6 metres.



★ **9** Rae is a tiler who is quoting to tile the laundry floor. If Rae charges $70 per square metre to lay the tiles and the tiles cost $34 per square metre, determine the cost to tile the laundry floor.

★ **10** Julie wants to carpet the living room, which is beside the entry.

- **a** If the carpet costs $53.75 per square metre, calculate the cost to carpet the living room.
- **b** Julie sees an advertisement for carpeting a lounge area for $600, determine whether this would be a cheaper way for Julie to carpet her lounge area.

★ **11** Aaston wants to buy paving paint to paint the floor of his garage. One litre of paint covers 8 m^2 and costs $54.

- **a** Calculate how many tins of paving paint are needed to paint the floor of the garage.
- **b** Determine the total cost for the paint.

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★ **12** Jang has discovered that the garage does not have any ceiling insulation and she has decided to purchase the insulation and install it herself.

- **a** If it costs $11.50 per square metre, calculate the total cost for Jang to insulate the ceiling herself.
- **b** If the builder offers to insulate the garage for $450, determine which will be the cheapest way for Jang to insulate her garage.

★ **13** Jii built this house on a 30 metre and 20 metre block and he is going to lay turf at a cost of $7.70 per square metre. He estimates the area of the house to the nearest square metre by measuring the greatest length by the greatest width. Determine the approximate cost to turf the block.

Hint The turf is laid on the area of the block minus the area of the house.

★ **14** Gerties Guttering is quoting for supplying gutters for the roof of the building shown. Unfortunately, the plan does not have a scale but the owner knows that the garage is 6.4 m wide. The guttering costs $12 per metre.

- **a** Determine the scale.
- **b** Calculate the perimeter of the building.
- **c** Calculate the overall cost of the guttering assuming that the perimeter of the roof is the same as the perimeter of the building.

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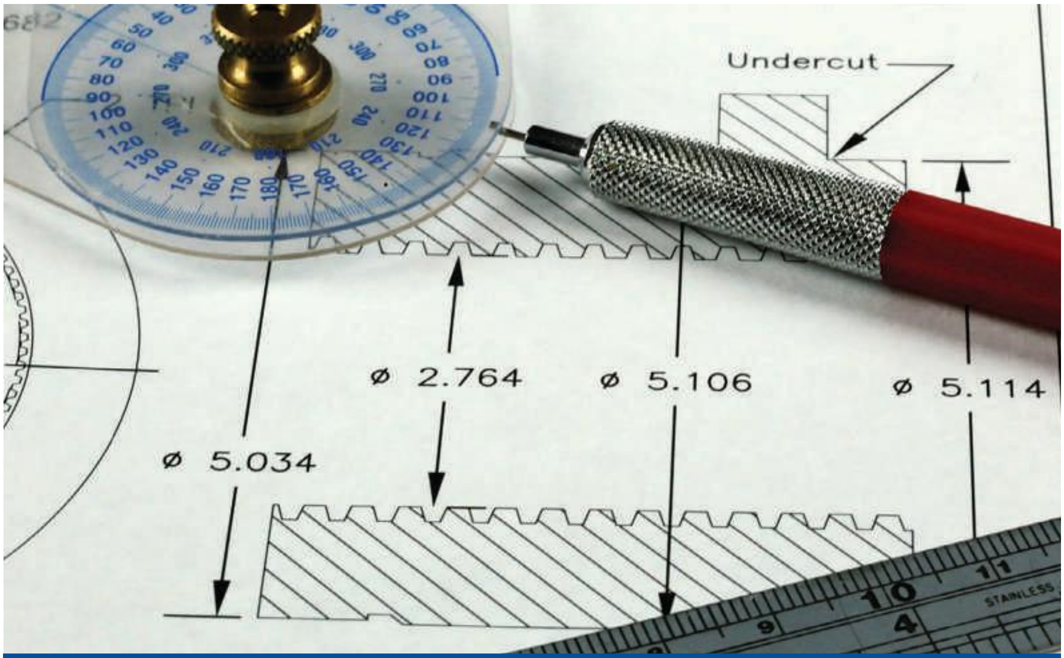
4D Understanding and applying drawing conventions of scale drawings COMPLEX

LEARNING GOALS

- Identify drawing conventions for scale diagrams
- Identify labelling techniques of ratio in scale diagrams
- Read measurements from scale diagrams
- Understand and apply drawing conventions of scale drawings

Why are drawing conventions essential?

In order for plans to be used by multiple people in the process of any construction, a universally agreed convention must be used so that there is an understanding of what the plans say. Some people create plans and other people need to read the plans to build or construct so drawing conventions are vital.



Using drawing conventions to draw scale plans helps others interpret the plans for building or manufacturing.

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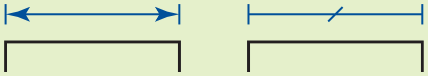
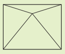
WHAT YOU NEED TO KNOW

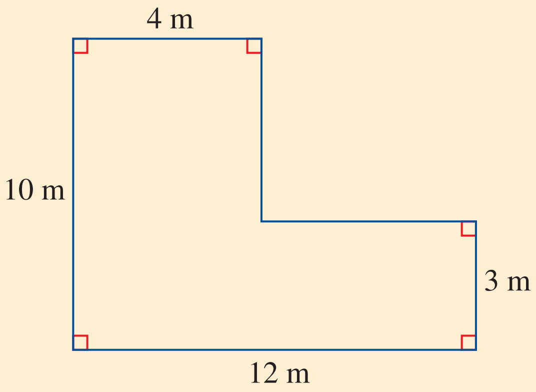
- Depending on what types of plans you are reading, the measurements are marked in slightly different ways, either with arrows or with lines marked on a parallel line.



- A very common scale is 1 : 100 for building plans meaning that 1 centimetre on the plan indicates 1 metre in the actual building.
- An engineering scale where a small part is drawn enlarged on a plan would be 10 : 1 meaning that 10 millimetres (1 centimetre) on the plan would be 1 millimetre in reality on the part.
- Common symbols, as discussed in section 4A, are used in plans creating a uniformity such as the symbol for a shower. 
- In order not to crowd the plans, not all measurements are named; however, all the measurements can be found from other markings on the plans.

Example 10 Reading measurements from indicated dimensions

Determine the missing side measurements on the following diagram.



WORKING	THINKING
Vertical measurement = 10 m – 3 m = 7 m	Determine what is left when the 3 m side is subtracted from the 10 m side.
Horizontal measurement = 12 m – 4 m = 8 m	Determine what is left when the 4 m side is subtracted from the 12 m side.

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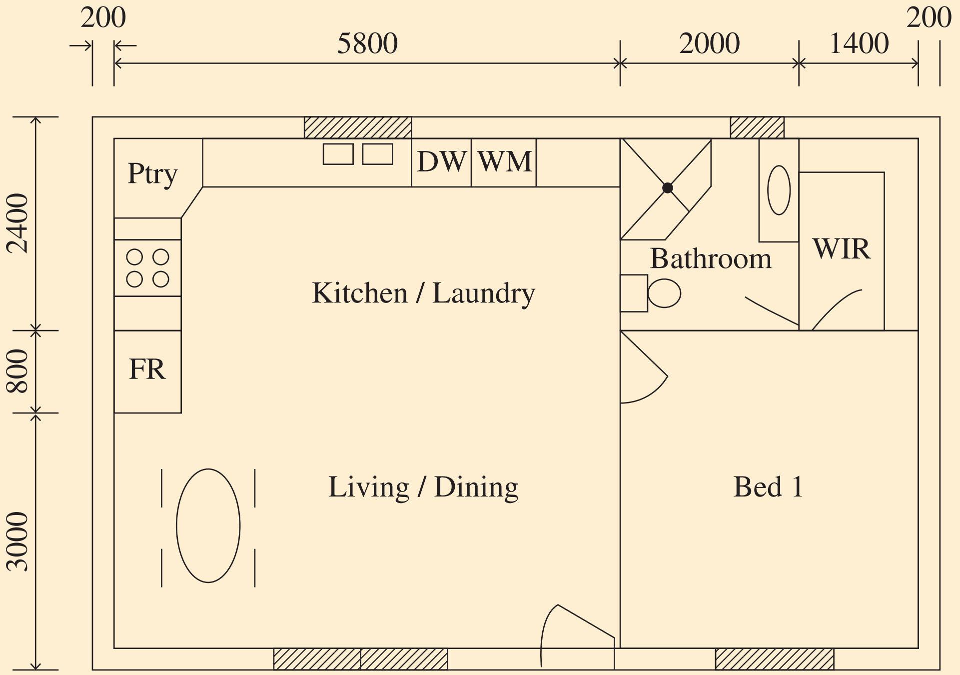
Example 11 Understanding and applying drawing conventions of scale drawings

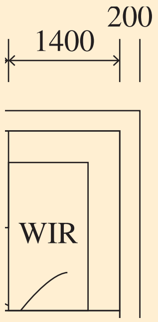


Use the plan shown to answer the following questions. Measurements are in mm.

- **a** Identify the width of the external walls.
- **b** Identify the internal dimensions of the kitchen/laundry/living/dining area.
- **c** Determine what the measurement of 2000 mm found on the top side of the plan measures.
- **d** Determine the internal measurements of bedroom 1.
- **e** The council requires a granny flat to be less than 60 m^2. Calculate the area of this plan to determine if it will fit council requirements.

WORKING **THINKING**

a 200 mm ← The width of the external wall is shown as the first measurement along the length.



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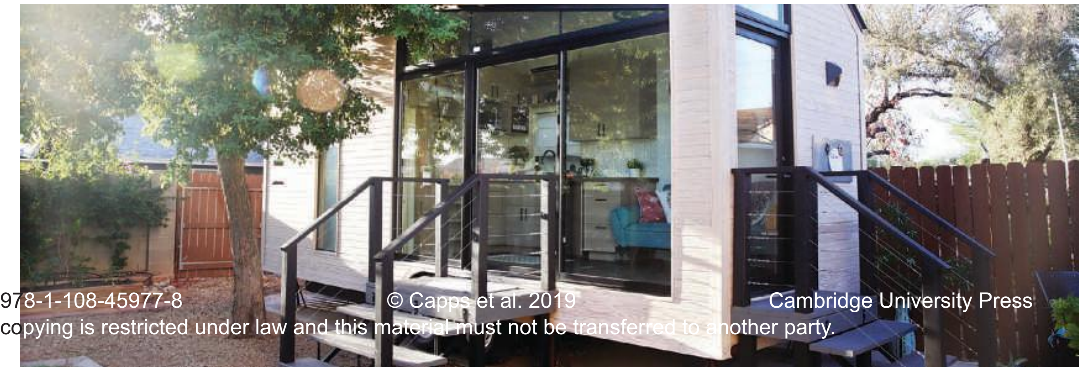
WORKING	THINKING
b	Read the dimensions of the kitchen/laundry/living/dining area.
$\text{Length} = 5800\text{ mm}$ $\text{Width} = 3000 + 800 + 2400 - 200 - 200$ $= 5800\text{ mm}$	$\text{Length} = 5800$ $\text{Width} = 3000 + 800 + 2400 - 200 - 200$ (subtracting the external wall from each end)
Dimensions are 5800 mm by 5800 mm.	
c The width of the bathroom.	Follow the lines on the measurement to identify the measured section.
d $\text{Width} = 2000 + 1400 = 3400\text{ mm}$ $\text{Length} = 3000 + 800 - 200$ $= 3600\text{ mm}$	Reading the dimensions from the plan. $\text{Width} = 2000 + 1400$ $\text{Length} = 3000 + 800 - 200$ (subtract the width of the outer wall)
Dimensions are 3600 mm by 3400 mm.	
e $\text{Length} = 200 + 5800 + 2000 + 1400 + 200$ $= 9600\text{ mm}$ $\text{Width} = 2400 + 800 + 3000$ $= 6200\text{ mm}$	Read the outside dimensions of the building.
$9600\text{ mm} = 9.6\text{ m}$ $6200\text{ mm} = 6.2\text{ m}$	Convert the measurements to metres.
$A = 9.6\text{ m} \times 6.2\text{ m}$ $A = 59.52\text{ m}^2$	Calculate the area by multiplying the length by the width.
The area will fit within the council requirements as it less than 60 m^2.	Identify whether or not the granny flat fits the council requirements.



Exercise 4D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a A common scale used for ____________ is ______: 100.

b Common ____________ create uniformity in plans.

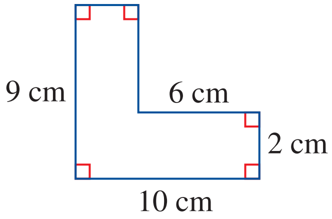
c Not all ________________ are always marked on a ________________, however all ________________ can be calculated from a plan.

d Dimensions in plans are not always marked the same, but they usually have ________________ or lines on parallel lines.

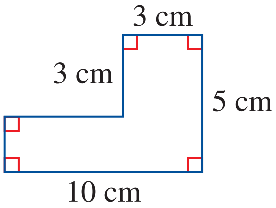
Example 10

2 Determine the missing side measurements in the following diagrams.

a

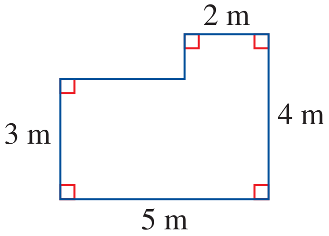


b

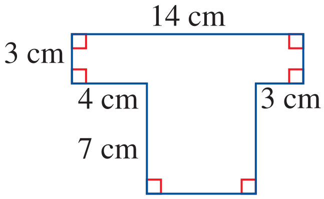


Hint The unknown side lengths are the difference between two or more known side lengths.

c



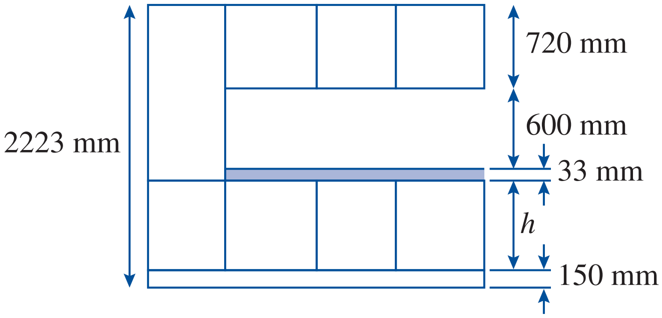
d



APPLICATIONS

SF: –	CF: 3–6	CU: –

★ **3** Janice wants to work out the actual height of the base cabinets from this kitchen plan shown.



She realises that 33 mm is the thickness of the benchtop and 150 mm is the height of the kickboards under the bench. Use the measurements from the plan to calculate *h*, the height of the bottom cabinet doors for Janice.

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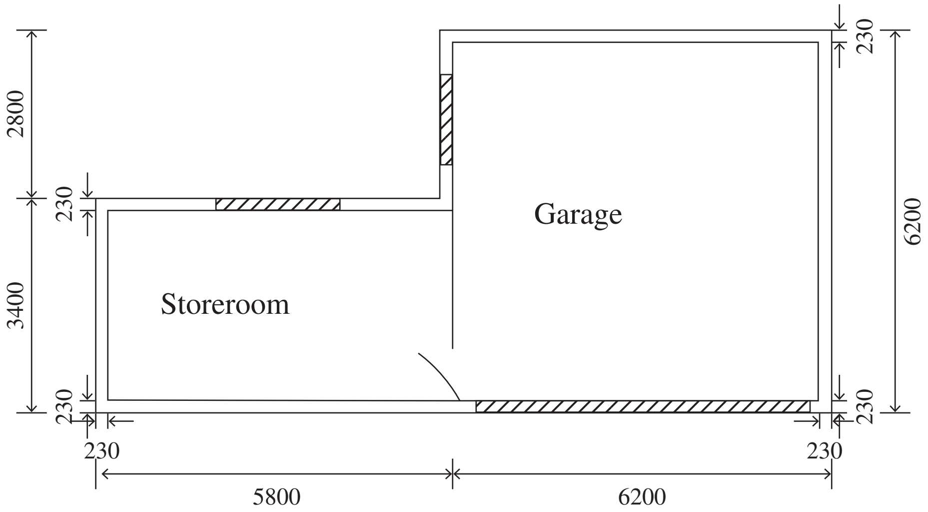
Example 11 ★ **4** Use the plan shown to answer the following questions. The measurements are in millimetres.



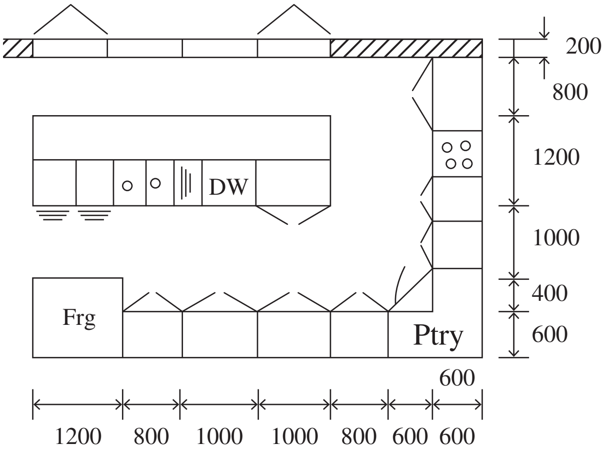
a Determine the actual internal dimensions of the garage, in metres.

b Determine the actual external dimensions of the garage, in metres.

c Determine the actual width of the external walls.

d Identify the internal dimensions of the storeroom.

★ **5** Use the plan shown to answer the following questions. The measurements are in millimetres.



a Identify the window width in the kitchen plan.

b Identify the dimensions of the island bench in the kitchen.

c Determine the walk space distance between the island bench and the external wall/window.

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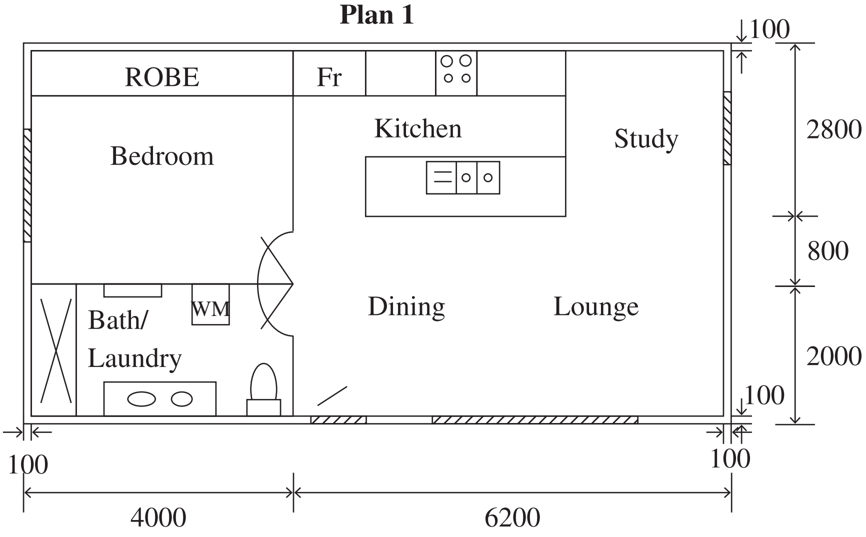
★ **6** Use plan 1 and plan 2 shown to answer the following questions. The measurements are in millimetres.

Plan 1



a Identify the width of the external walls in both plans.

b Identify the internal dimensions of the:

- **i** dining/lounge area in plan 1
- **ii** bathroom/laundry in plan 2.

c Determine what these measurements indicate:

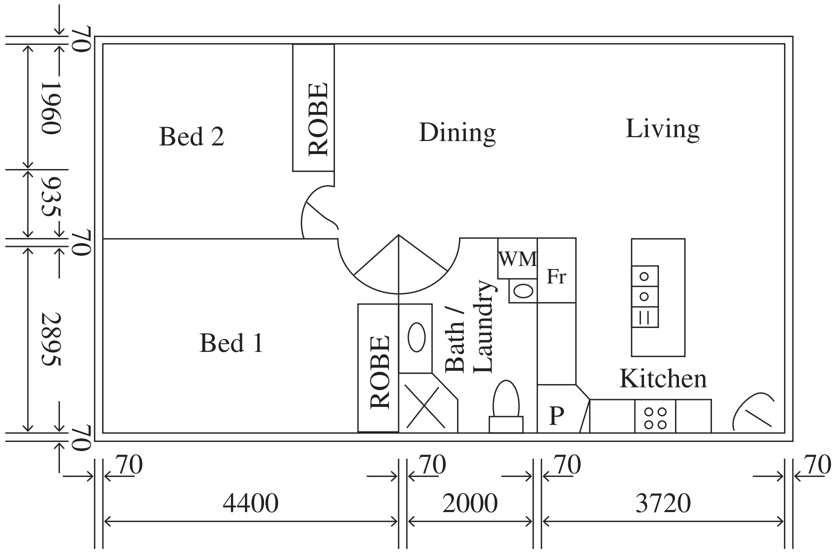
- **i** 2000 on the right-hand side of plan 1
- **ii** 1960 on the left-hand side of plan 2.

d Determine the internal measurements including the robes of:

- **i** the bedroom in plan 1
- **ii** bedroom 1 in plan 2.

e The council requires a granny flat to be less than 60 m^2. Calculate the area of both plans to determine if they will fit council requirements.

Plan 2



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4E Constructing scale diagrams COMPLEX

LEARNING GOALS

- Construct enlargement scale diagrams by hand
- Construct reduction scale diagrams by hand
- Construct scale diagrams using technology

Why is it essential to be able to construct scale diagrams?

Although there are many jobs that construct scale diagrams such as landscaper, builder, architect and cabinet makers, it is a useful skill for everyday people. It allows us to create an image to share our vision for renovating around our own homes and any landscaping we might do over the years. It is a useful process allowing us to share ideas, visions and inventions with other to help our dreams become a reality.

Scale drawings are necessary to renovate homes and kitchens.

WHAT YOU NEED TO KNOW

- Recall the use of a scale to determine measurements in diagrams and how to indicate it with a ratio.
- Enlargement makes something bigger and reduction makes something smaller.
- Scales used so far have a small number first for a dimension in the diagram and a larger number second for the actual dimension in real life, such as 1 : 10. This indicates the diagram is a reduction of a large thing such as a building, a machine or a piece of furniture.
- Small objects such as machine parts are usually drawn with an enlargement, which is indicated by a scale where the first number is larger than the second, such as 10 : 1. This means 10 mm on the diagram is 1 mm of the actual object.
- To calculate by how many times the object is enlarged or reduced in the diagram, express the scale as a fraction with the first number on top and the second number on the bottom, and simplify if necessary. So 1 : 10 indicates the object in the diagram is $\frac{1}{10}$ of its actual size, and 10 : 1 indicates it is 10 times larger in the diagram.
- Grid paper is useful for drawing lines and shapes accurately with the help of a ruler.

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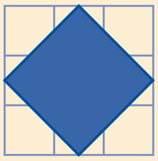
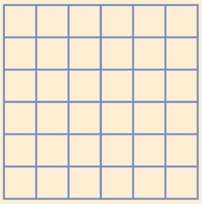
Example 12 Constructing a scale diagram by hand

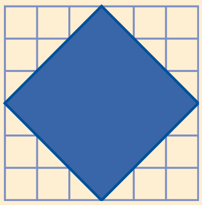
Use an enlargement scale of 2 : 1 to construct a scale diagram of the following shape.



WORKING

The diagram has to be $\frac{2}{1} = 2$ times bigger than the original shape.





2 : 1

THINKING

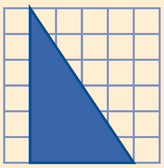
The scale of 2 : 1 has the first number larger than the second which confirms it means an enlargement. Express it as a fraction with the first number on top and the second number on the bottom; this is how much bigger the diagram has to be.

The original shape is on a grid of 3 by 3 squares. $3 \times 2 = 6$ so to draw it two times bigger, construct a grid of 6 by 6 squares of the same size.

Draw the same shape on the new grid, and label it with the scale (2 : 1).

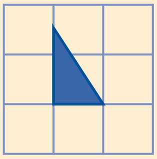
Example 13 Constructing a scale diagram by hand

Use a reduction scale of 1 : 4 to construct a scale diagram of the following shape.



WORKING

The diagram has to be $\frac{1}{4}$ the size of the original shape.



THINKING

Express the reduction scale as a fraction with the first number on top and the second number on the bottom; this is the fraction of the original shape that the diagram has to be.

The triangle has a height of 6 squares and a base of 4 squares, dividing by 4 gives a height of 1.5 squares and a base of 1 square. A grid of 3×3 squares of the same size will be enough for the diagram. Draw the reduced shape on the gird with the new dimensions.

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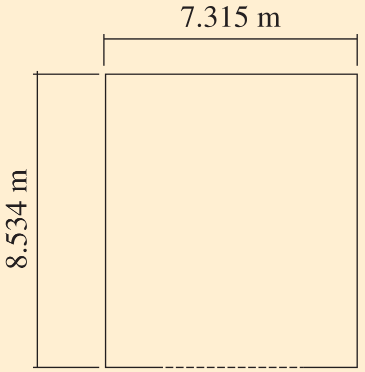
Example 14 Constructing a scale diagram using grid paper

Use the following steps to construct a scale diagram of the garage.
Use a scale of 1 : 100.

a Convert the measurements to mm, given the measurements are in metres.

b Apply the scale of 1 : 100 to convert the actual measurements.

c Draw a rectangle using the scale measurements on the grid paper.



WORKING | **THINKING**

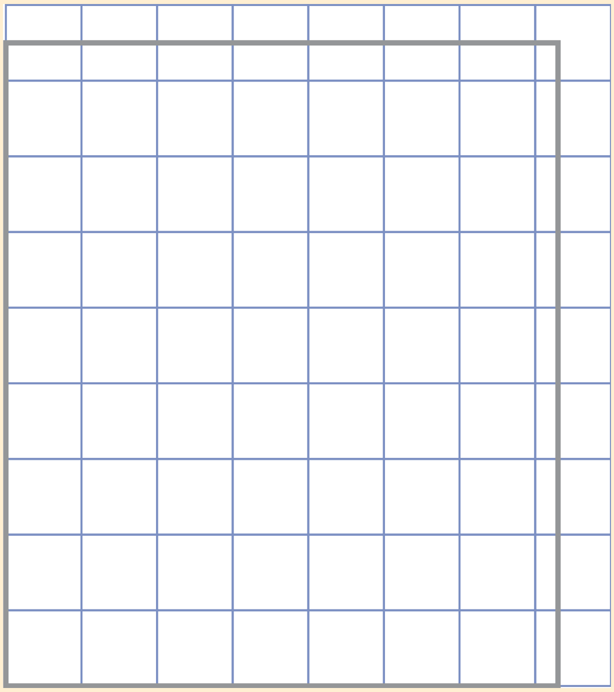
a 7.315 m × 1000 = 7315 mm
8.534 m × 1000 = 8534 mm

Multiple the measurements by 1000 to convert metres to millimetres.

b 7315 mm ÷ 100 = 73.15 mm
8534 m ÷ 100 = 85.34 mm

Apply the scale by dividing the measurements by the scale factor, 100.

c 1 : 100



Draw a rectangle using the measurements on the grid paper. The longer side is about 8.5 squares and the shorter one is about 7.3 squares. To make it more accurate you could use grid paper with 1 mm squares, or use a ruler.

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Exercise 4E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

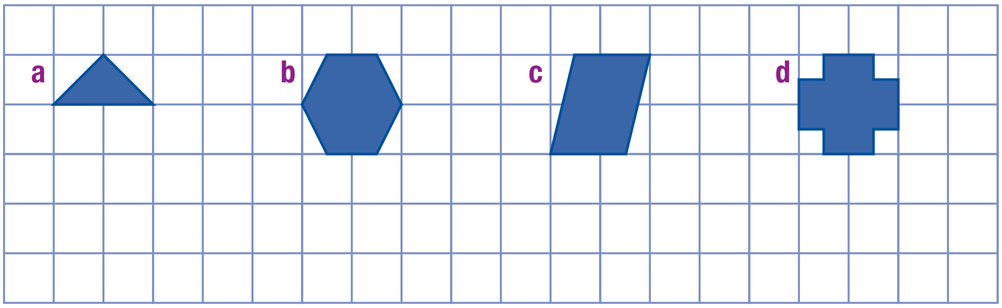
a __________ of scale drawings enables people to __________ ideas.

b We can create scale diagrams by __________ and using __________.

c An __________ constructs scale __________ of a house and a __________ will build the house.

Example 12

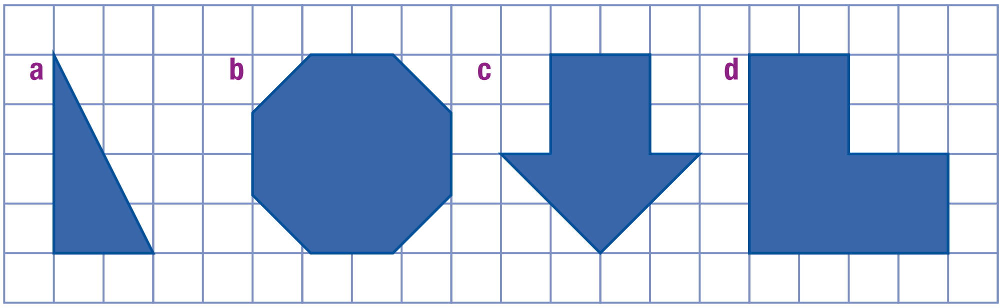
2 Use an enlargement scale of 3 : 1 to construct a scale diagram of the following shapes.



Hint Work out how many times bigger the scale diagram is to the original shape

Example 13

3 Use a reduction scale of 1 : 4 to construct a scale diagram of the following shapes.



Hint Work out from the scale ratio what fraction of the original shape the scale diagram dimensions need to be

APPLICATIONS

SF: –	CF: 4–8	CU: 9–11

★ **4** Construct a scale enlargement diagram using grid paper and a scale of 5 : 1 for an outline of the bolt shown. It has an overall length of 18 mm, a head with dimensions 10 mm by 4 mm and the threaded portion has a diameter of 4 mm. Do not show the threads, draw the outline of that part with straight not zig-zag lines. Indicate the scale on the diagram.

★ **5** Construct a scale enlargement diagram using grid paper and a scale of 4 : 1 to show the outline of this drill bit. It has a length of 20 mm and a diameter of 2 mm. Do not show the cutting edges (called the *flutes*), use a straight line for the outline. Indicate the scale on the diagram.

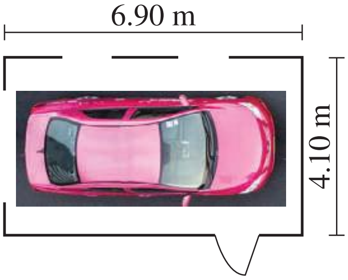
Example 14

6 Use the following steps to construct a scaled diagram of the garage. Use a scale of 1 : 100.

a Convert the actual measurements to mm, given the measurements are in metres.

b Apply the scale of 1 : 100 to convert the actual measurements to diagram measurements.

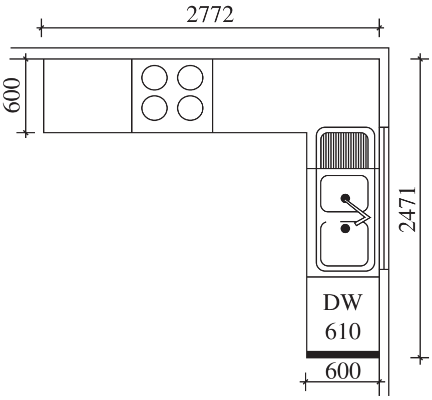
c Draw a rectangle using the scale measurements on the grid paper.



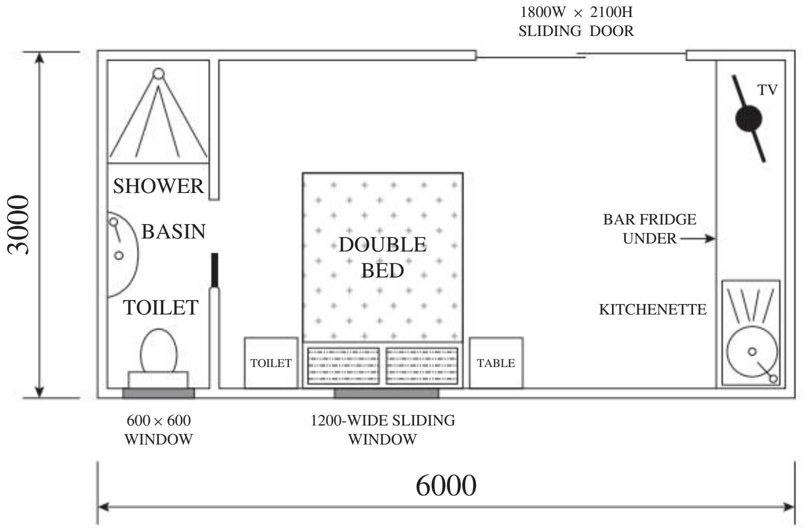
7 Use the following steps to construct a scale diagram of the outline of these kitchen cabinets using grid paper and a scale of 1 : 50.

a Apply the scale to the actual measurements to convert to diagram measurements.

b Draw the outline using the scale measurements on the grid paper. You do not have to draw the individual kitchen units, just the outline.



★ **8** Construct a scale diagram of the exterior dimensions of this garage conversion using grid paper and a scale of 1 : 75.



★ **9** Construct a 1 : 100 scale drawing showing the boundary of a rectangular block of land and the exterior walls of a rectangular shed placed in the centre of it. The block of land is 26 m × 16.5 m, and the shed is 12.5 m by 9 m. The longer side of the shed runs in the same direction as the longer side of the block. The yard is of equal width on opposite sides of the shed. Label the dimensions of the block of land in metres and of the shed in millimetres. Add the scale to the drawing.

10 Use technology to create a kitchen of your own design. Online kitchen programs are available through the internet and most schools will have a CAD (Computer Aided Design) program that can be useful.

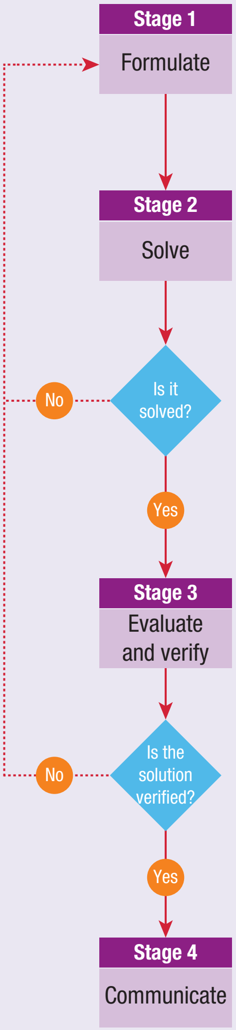
11 Use technology to create a bathroom of your own design. Online bathroom programs are available through the internet and most school will have a CAD (Computer Aided Design) program that can be useful.

Problem-solving and modelling task

Background: Floor plans of dwellings are the most common kind of scale drawing that most people will experience in their lives.

Task: The council have rezoned the rules for the construction of granny flats in the Sunshine Coast area. They are now allowing an area of 90 m^2. Your task is to design a granny flat that meets the council requirements. You can either use technology or draw by hand the granny flat plan. Your client wants to have a separate laundry and two toilets in the granny flat and a spare bedroom for guests and family to stay.

Approach to problem-solving and modelling tasks:



Stage 1: Formulate

1 Determine the elements you want to include in the granny flat.

2 Research minimum bedroom and bathroom sizing.

Stage 2: Solve

3 Sketch the outline of your granny flat ensuring it is less than or equal to 90 m^2.

4 Sketch the rooms into the outline of the granny flat.

5 Draw symbols onto the plan to identify fixtures and features.

6 Use abbreviations to label the smaller rooms on the plan.

Stage 3: Evaluate and verify

7 Show someone else your work for comment and check your area.

Stage 4: Communicate

8 Submit your plan along with a paragraph identifying the positive traits of your design and the mathematical expression showing that it will meet council requirements.

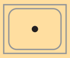
Chapter checklist

☐ I understand and interpret scale symbols and abbreviations.

1 Convert 3.54 m into mm.

2 Simplify the scale 8 mm : 48 cm.

3 Identify the meaning of the abbreviation ROBE.

4 Identify the  symbol.

☐ I can identify and calculate measurements of length, perimeter and area from scale diagrams.

5 Calculate the area of a rectangle with a length of 6.4 m and a width of 5.3 m.

6 Jayden measured the actual internal length (wall above the word laundry) of the laundry to be 2.7 m long, determine the scale of the plan.

☐ I can estimate and compare quantities, materials and costs from scale diagrams. **[complex]**

7 Tutu wants to carpet his bedroom. His bedroom has dimensions of 4 m by 4.2 m and the cost of carpet is $48 fully laid per square metre.

a Calculate the area of the bedroom.

b Calculate the cost to carpet the room.

8 The internal length of the garage from the plan shown is 6 m. Jonah wants to tile the ensuite. If the tiles and tiling costs $62 per square metre, calculate the cost to tile the ensuite.

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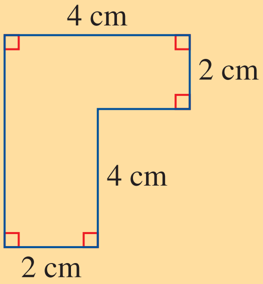
☐ I can understand and apply drawing conventions of scale drawings. **[complex]**

9 Determine the missing side measurements from the following diagram.



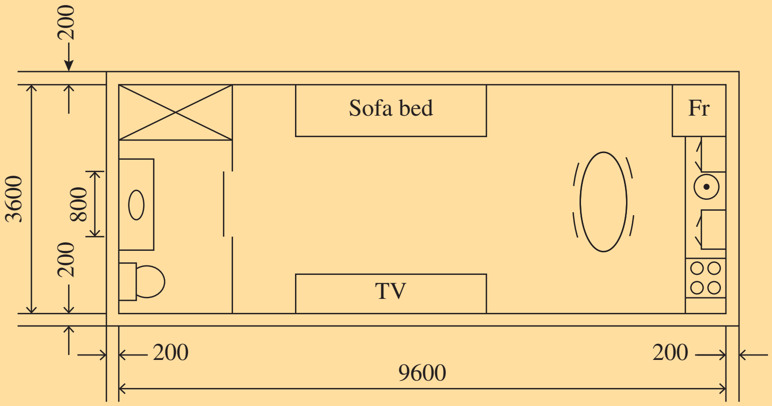
10 Use the following plan to complete the following.

a Identify the width of the external walls.

b Identify the internal dimensions of the entire apartment.

c Determine what the measurement of 800 mm found on the left-hand side of the plan measures.

d Determine the overall area of the building including external walls.



☐ I can construct a scale diagram. **[complex]**

11 Construct a scale enlargement diagram using grid paper and a scale of 4 : 1 for a bolt. It has a length of 15 mm, a head with dimensions 12.5 mm by 5 mm and the threaded portion has a diameter of 5 mm. Indicate the scale on the diagram.

12 Construct a scale diagram of a garage with dimensions of 5.8 m by 6 m using grid paper and a scale of 1 : 100.

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Chapter review

All questions in the review are assessment style.

Simple familiar

Refer to this diagram for Questions **1** to **5**.

Section 4A

1 Simone is reading the plans of the house she would like to have built on her block of land. She does not know what the abbreviations W.C. and W.I.R. mean. Interpret these abbreviations for Simone.

Section 4B

2 Amanda knows that the internal width of the garage is 5.75 m.

- **a** Determine the scale for the house plan.
- **b** Calculate the area of the sitting room.
- **c** Calculate the area the master suite (not including the W.I.R. and ensuite).

Complex familiar

Section 4C

3 Aaron wants to retile the ensuite in his home. The costs of laying the tiles is \$35 per square metre and the tiles cost \$42.25 per square metre. Determine the area of the ensuite and calculate the total cost for Aaron to have it retiled.

4 Gina wants to paint the floor of the garage with paving paint. If the paint covers 6 m^2 per litre and she needs to do 2 coats of paving paint, calculate the number of 4-litre tins Gina will need to purchase.

5 Dan is going to carpet bedroom 3 excluding the walk-in robe. Calculate the cost if the carpeting costs \$34 per square metre fully laid.

Section 4D

6 Use the plan shown below to answer the following questions, given that the measurements are in millimetres.

- **a** Identify the width of the external walls.
- **b** Identify the dimensions of the bedroom including the robe.
- **c** Determine what the measurement of 4000 mm found at the top side of the plan measures.

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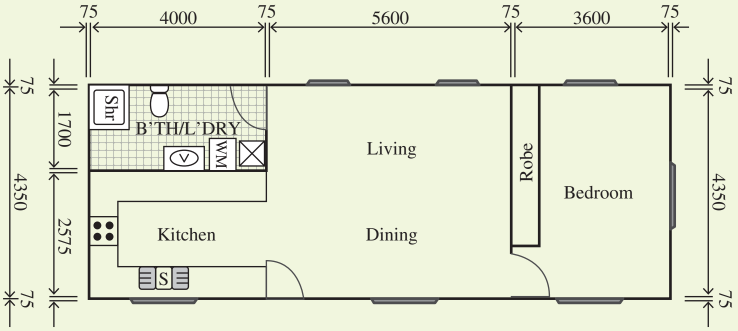
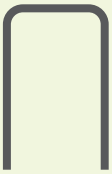
d Determine the internal measurements of the living/dining area.

e The council requires a granny flat to be less than 60 m^2. Calculate the area of this plan to determine if it will fit council requirements.



Section 4E

7 Construct a scale enlargement diagram using grid paper and a scale of 3 : 1 for the U-bolt shown. It has a height of 30 mm, a width of 15 mm and a thickness of 1.5 mm. Indicate the scale on the diagram and label its actual dimensions.



Complex unfamiliar

Section 4C

8 Construct a 1 : 100 scale diagram of a granny flat on grid paper with the following rectangular dimensions. Draw the first dimension in each pair horizontally on the plan, and the second dimension vertically on the plan.

- External dimensions 8950 mm by 5500 mm, with the longer side horizontal in the plan
- Bedroom 1 located at the top left of the plan, internal dimensions 3000 mm by 3760 mm
- Bathroom located at the top centre of plan, next to bedroom 1, internal dimensions 2400 mm by 2800
- Study located at bottom left of plan, next to bedroom 1, internal dimensions 3000 mm by 1340 mm
- The kitchen/dining and living areas make up the space left over, with the living area at the top right of the plan, and kitchen/dining in the bottom right of the plan.

Show the front door at the top of the plan, and doors to the enclosed rooms. Don't show windows or fixtures and features. Label the plan with the external dimensions, the names of the rooms and indicate their internal dimensions in a suitable way. Indicate the scale.

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5

Right-angled triangles

Maths for a builder/painter: John Vreeling

I am a qualified painter and have also completed courses as an owner builder and have built three houses. I have built three homes from scratch, one in Darwin, one in Grantham where our house was destroyed by the floods and our current home in Peachester.

Tell us a bit about your job. What does a typical day look like?

When working as a builder, I make sure I have all my tools and keep them maintained. I use my phone to coordinate other trades to schedule them and being a builder requires good organisation to keep the project running smoothly.

What maths did you study in school?

I did two levels of maths at school, I did 3 unit maths but unfortunately I bombed out of that subject and did general maths predominately.

How do you use mathematics in your job?

We constantly use Pythagoras' theorem in construction to make sure the groundwork has been set out correctly and is square. It is vital in construction to have correct foundations or your project will end up a disaster.

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In this chapter

Syllabus reference

Unit 3 Topic 2 Scales, plans and models

Right-angled triangles (5 hours)

In this sub-topic, students will:

- apply Pythagoras’ theorem to solve problems for all side lengths
- apply the tangent, sine and cosine ratios to find unknown angles and sides **[complex]**
- use the concepts of angle of elevation and angle of depression to solve practical problems **[complex]**.

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Pre-test

1 Calculate the squares of the following numbers using your calculator.

a 6
b 8
c 11
d 24
e 33

2 Calculate the square roots of the following numbers using your calculator. Round to 2 decimal places where necessary.

a 169
b 289
c 320
d 3600
e 4336

Hint On most calculators, type in the number followed by the key with the square root sign.

3 Determine the value of the unknowns in the following problems using your calculator. Round to 2 decimal places where necessary.

a $c = \sqrt{12^2 + 16^2}$
b $c = \sqrt{10^2 + 24^2}$
c $c = \sqrt{21^2 + 43^2}$
d $a = \sqrt{39^2 - 15^2}$
e $b = \sqrt{23^2 - 17^2}$

Hint Follow 'order of operations'. Remember also that the root sign with a bar over the top of an expression means to treat the expression as if it is in brackets.

4 Determine the value the following problems using your calculator. Round to 2 decimal places where necessary.

a $15 \times \cos 20°$
b $\sin 62° \times 13$
c $\tan 45° \div 21$
d $78 \div \cos 12°$

Hint If your calculator has a mode for radians ('Rad'), make sure it is in degree mode ('Deg').

5 Construct a diagram using a right-angled triangle to show a firefighter who is looking to the top of a 60 m wall of a building and standing on the ground 100 m horizontally from the base of the wall of the building. Draw a line to show the firefighter's line of sight to the top of the wall.

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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5A Calculating the hypotenuse by applying Pythagoras' theorem

LEARNING GOALS

- Understand Pythagoras' theorem
- Identify the sides of a right-angle triangle
- Identify the hypotenuse of a right-angled triangle
- Calculate the hypotenuse given the other two sides
- Calculate the hypotenuse in a real-world context
- Verify square angles in construction by using Pythagoras'

Why is it essential to understand the use of Pythagoras' theorem?

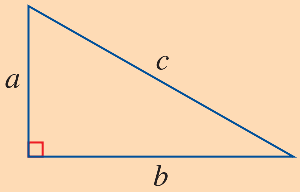
Pythagoras' theorem is a very practical way to help builders verify right angles on building sites as it is vital on most constructions to have a perfect right angle on edges. This is a method of checking the edges of the walls are square. It is one of the most common mathematical theorems used in many occupations such as farming, landscaping, metalworking, and other manufacturing and construction industries.


Pythagoras's theorem can be used to check that the corner of this soccer field is a right angle.

WHAT YOU NEED TO KNOW

- A **right angle** is a 90° angle, the same as is found at the corners of squares and rectangles, and it is also known as a square angle.
- The **hypotenuse** is the longest side of a triangle, which is found opposite the right-angle.

The formula is known as $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.



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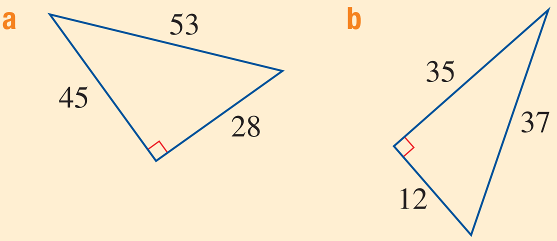
- **Pythagoras' theorem** states that the square of the longest side is equal to the sum of the other sides squared in a right-angled triangle.
- To find the value of c rearrange the formula to $c = \sqrt{a^2 + b^2}$.
- Rounding a decimal to a specific number of places.

Example 1 Identifying the hypotenuse on a right-angled triangle

Identify the length of the hypotenuse in each of the following triangles.



WORKING | **THINKING**

a The hypotenuse is 53. ◄ The hypotenuse is the longest side and is also opposite the right angle.

b The hypotenuse is 37. ◄ The hypotenuse is the longest side and is also opposite the right angle.

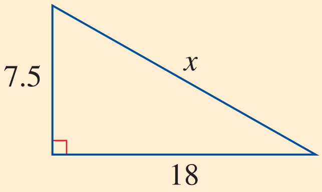
Example 2 Determining the value of the hypotenuse given the other two sides

Complete the following steps to determine the value of the hypotenuse for the triangle shown.

a Identify the values of a, b and c.

b Substitute the values into the formula, $c^2 = a^2 + b^2$.

c Calculate the value of the hypotenuse using your calculator. Round to 1 decimal place when necessary.



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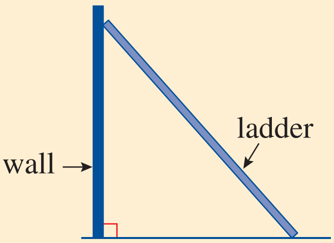
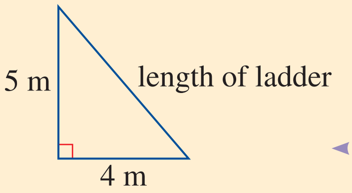
WORKING	THINKING
a $c = x, a = 7.5, b = 18$	Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are *a* and *b* in any order.
b $c^2 = a^2 + b^2$	Write the formula.
$x^2 = 7.5^2 + 18^2$	Substitute the values for *a, b, c* into the formula.
c $x = \sqrt{7.5^2 + 18^2}$	Calculate the value of the hypotenuse.
$x = \sqrt{380.25}$	
$x = 19.5$	Round the final answer to 1 decimal place.

Example 3 Determining the value of the hypotenuse in a real-world context

A ladder is leaning against a vertical wall at a height of 5 metres up the wall. If the base of the ladder is 4 metres from the base of the wall, determine the length of the ladder, correct to 1 decimal place.



WORKING	THINKING
$a = 5 \text{ m}, \; b = 4 \text{ m}, \; c = \text{length of ladder}$	Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are *a* and *b* in any order.
	Draw a diagram with the known information.
$c^2 = a^2 + b^2$	Write the formula.
$c^2 = 5^2 + 4^2$	Substitute the values of *a, b, c* into the formula.
$c = \sqrt{4^2 + 5^2}$	Calculate the value of the hypotenuse.
$c = \sqrt{41}$	
$c = 6.403124237$	
$c = 6.4 \text{ m}$	Round the answer to 1 decimal place.
The length of the ladder is 6.4 m.	Communicate your answer in a sentence.

Example 4 Determining if an angle is a right angle

Ziza is building a shed in her backyard. She wants to verify that the concrete slab she is having poured has been set out correct and is at right angles. She measured the two adjoining sides as 5 m and 6 m and the diagonal as 7.8 m. Verify the concrete slab will be square (within 0.2 m difference is allowable).

WORKING	THINKING
$a = 5\text{ m}, b = 6\text{ m}, c = 7.8\text{ m}$	Identify the hypotenuse and the other two sides.
$c^2 = a^2 + b^2$	Write the formula.
$c^2 = 7.8^2$	Compare both sides of Pythagoras' theorem.
$c^2 = 60.84$	
$a^2 + b^2 = 5^2 + 6^2$	
$= 25 + 36$	
$= 61$	
Each side of the equation is the same within 0.2 m.	Communicate your answer in a sentence.

Exercise 5A

FUNDAMENTALS

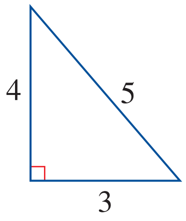
1 Determine the missing words in the following sentences.

- **a** The ____________ theorem is the ____________ squared equals the ________ of the other two sides squared.
- **b** The ____________ theorem can only be applied to ________________ triangles.
- **c** The formula for Pythagoras' Theorem is $\underline{\qquad\quad}^2 = \underline{\qquad\quad}^2 + \underline{\qquad\quad}^2$

Example 1

2 Identify which side is the hypotenuse in these right-angled triangles.

a



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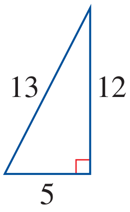

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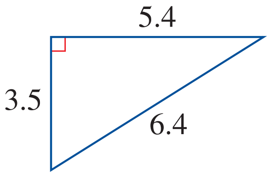

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3 Calculate the value of the following using your calculator. Round to 1 decimal place where necessary.

a $\sqrt{3^2 + 4^2}$

b $\sqrt{5^2 + 12^2}$

c $\sqrt{7^2 + 11^2}$

d $\sqrt{13^2 + 24^2}$

Example 2

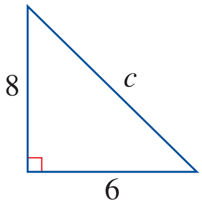
4 Complete the following for each of the triangles shown.

i Identify the values of a, b and c.

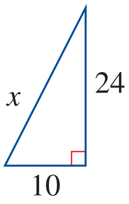
ii Substitute the values of a, b and c into the formula $c^2 = a^2 + b^2$.

iii Calculate the value of the unknown side by using your calculator. Round your answer to 1 decimal place when necessary.

a



b



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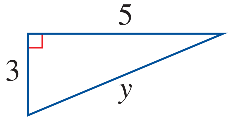
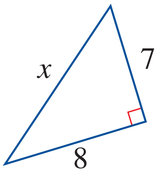
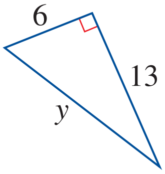


d



e



Hint The value of *c* is important to identify; *a* and *b* are just the other 2 short sides.

Example 4

5 Verify that the triangles with the following side lengths are right-angled triangles. An allowance of 0.1 difference may be applied due to rounding.

a 8, 15, 17

b 7, 24, 25

c 33, 56, 65

d 4.5, 6.3, 7.74

e 12.7, 18.6, 22.52

Hint The hypotenuse is the longest side.

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APPLICATIONS

SF: 6–14	CF: –	CU: –

Example 3

6 The ladder rests against a tree 10 m above the ground. The base of the ladder is 3 m from the base of the tree.

a Substitute the values of a and b into the formula $c^2 = a^2 + b^2$.

b Determine the length of the ladder using your calculator. Round to 1 decimal place.

Hint The diagrams all involve right-angled triangles.

7 Khia is flying a kite that is at a vertical height of 80 metres and is 40 metres away from her measured horizontally along the ground. The string of the kite forms a straight line.

a Draw and label a diagram to show the situation.

b Identify a right-angled triangle in your diagram. Label the horizontal and vertical sides of the triangle with the values given. Label the hypotenuse as the unknown side length.

c Write the formula to find the hypotenuse. Substitute the values for a and b.

d Calculate the length Khia's kite string, correct to 1 decimal place.

8 Chen is on top of a vertical cliff with a height of 120 m. He is about to ride a flying fox that lands at a horizontal distance of 175 m from the base of the cliff.

a Draw and label a diagram to show the situation.

b Label the horizontal and vertical side with the values given.

c Write the formula to find the hypotenuse. Substitute the values for a and b.

d Calculate the length the flying fox ride, correct to 1 decimal place.

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★**9** Arthur is participating in a re-enactment and he needs to organise a siege on a castle that has a 10-metre wide moat around the castle. If the castle wall is 8 metres high, calculate the minimum ladder length needed to reach over the moat to the top of the castle wall. Round your answer correct to 1 decimal place.

★**10** Anmarie is designing a cable-stayed bridge. She needs to calculate the length of a cable that will attach to the bridge pylon (the column) with a vertical distance of 40 metres above the bridge deck. The other end of the cable will attach to the bridge deck at a horizontal distance of 20 metres from the middle of the bridge pylon. Calculate the required length of the cable, correct to 1 decimal place.

★**11** Holly is building a gate in the shape of a rectangle and she needs to add a diagonal brace (a piece of wood running along the diagonal of the rectangular face of the gate). If the gate is 1.2 metres high and 2.6 metres wide, calculate the length of the brace needed. Round your answer correct to 1 decimal place.

★**12** Hayden has designed a television that is 140 cm wide and 90 cm high. He is told that to market his television he needs to advertise the length of the screen diagonal, which is the measure commonly used for screen size. Calculate the length of the television screen diagonal, correct to 1 decimal place.

Example 4

★**13** Bitta the builder is checking the right angle between two walls in a rectangular room by measuring the lengths of the sides of the rectangle and its diagonal. She measures the sides of the room as 8 metres and 6 metres, and the diagonal as 10 metres. Verify that the walls are built with a right angle between the two walls.

★**14** Lance is wanting to renovate his kitchen. Before he orders new cabinets, he wants to confirm that the room is actually square. If one wall is 3 m and the adjoining wall is 3.6 m and the diagonal is approx. 4.69 m, verify the kitchen is square (within 0.1 m difference is allowable).

5B Calculating a short side by applying Pythagoras' theorem

LEARNING GOALS

- Understand Pythagoras' theorem
- Identify the hypotenuse on a right-angled triangle
- Calculate a short side given the hypotenuse and the other short side

Why is calculating side lengths in a right-angled triangle essential?

Right angles are everywhere in manufacturing and construction. Roof trusses are a good example. But they also occur in surveying, landscaping, sewing, graphic design, and other arts and crafts.

The importance of calculating side lengths is well illustrated by the image of the timber roof truss below. Its base length is determined by the width of the building, and its height is determined to give a practical and attractive slope to the roof. The length of every other piece of timber from which the truss is made can then be calculated using right-angled triangles and Pythagoras' theorem.

This roof truss is in the shape of two right-angled triangles joined along their shortest sides.

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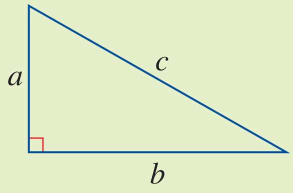
WHAT YOU NEED TO KNOW

- It does not matter which side is labelled *a* or *b*, as long as they are not the hypotenuse.
- Pythagoras' theorem states that the unknown side squared in a right-angled triangle is equal to square of the longest side minus the square of the other side.
- The formula is known as $c^2 = a^2 + b^2$, where *c* is the length of the hypotenuse.
- To find the value of *a* rearrange the formula to $a^2 = c^2 - b^2$.
- Rounding to a specific number of decimal places.



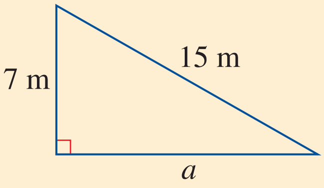
Example 5 Determining the unknown side given the hypotenuse and the other side

Complete the following steps to determine the unknown side for the triangle shown.

a Identify the values of *a, b* and *c*.

b Substitute the values of *a, b* and *c* into the formula $a^2 = c^2 - b^2$

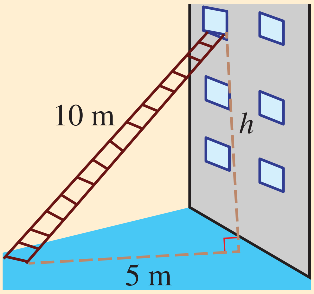
c Determine the value of the short side using your calculator. Round your answer to 1 decimal place if necessary.



WORKING	THINKING
a $c = 15$ m, $a = ?$, $b = 7$ m	Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are *a* and *b* in any order.
b $a^2 = c^2 - b^2$	Write the formula.
$a^2 = 15^2 - 7^2$	Substitute the values for *a, b, c* into the formula.
c $a = \sqrt{15^2 - 7^2}$	Calculate the value of the short side.
$a = \sqrt{176}$	
$a = 13.26649916$	
$a = 13.3$	Round the final answer to 1 decimal place.

Example 6 Calculating the unknown side given the hypotenuse and the other side in a real-world context

A 10-metre ladder is leaning against a vertical wall at a height of *h* metres. If the base of the ladder is 5 metres from the base of the wall, calculate the height that the ladder reaches up the wall, correct to 1 decimal place.



WORKING	THINKING
$a = h$ m, $b = 5$ m, $c = 10$ m	Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are *a* and *b* in any order.
$a^2 = c^2 - b^2$	Write the formula.
$h^2 = 10^2 - 5^2$ $h^2 = 100 - 25$ $h^2 = 75$	Substitute the values of *a, b, c* into the formula.
$h = \sqrt{75}$ $h = 8.66$ m	Calculate the value of the height of the ladder up the wall.
$h = 8.7$ m	Round the answer to 1 decimal place.
The ladder reaches a height of 8.7 m.	Communicate your answer in a sentence.

Exercise 5B

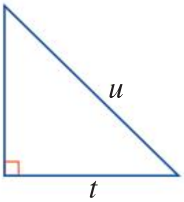
FUNDAMENTALS

1 Determine the missing words in the following sentences.

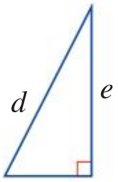
- **a** The ______________ theorem is the ______________ side squared equals the ______________ squared minus the other ______________ squared.
- **b** A ______________ is used to calculate the ______________.
- **c** The formula to calculate a short side on a right-angled triangle is ________^{2} = ________^{2} – ________^{2}.

2 The unknown side is 'a' in these right-angled triangles shown. Write the rule to find the value of a using the other values on the triangles.

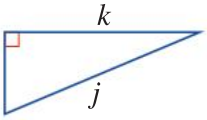
a



b



c



3 Calculate the value of the following using your calculator. Round to 1 decimal place where necessary.

a $\sqrt{13^2 - 12^2}$ **b** $\sqrt{25^2 - 20^2}$ **c** $\sqrt{17^2 - 14^2}$

Example 5

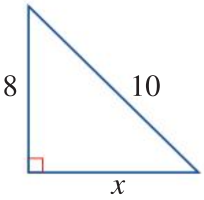
4 Complete the following for each of the triangles shown.

i Identify the values of a, b and c.

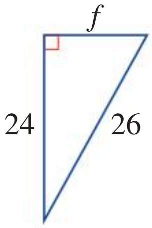
ii Substitute the values of a, b and c into the formula $a^2 = c^2 - b^2$.

iii Calculate the value of the unknown side by using your calculator. Round your answer to 1 decimal place when necessary.

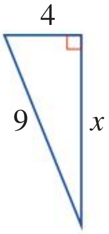
a



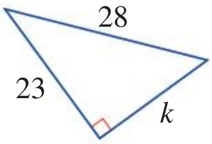
b



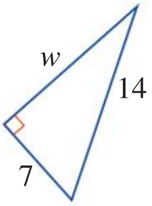
c



d



e



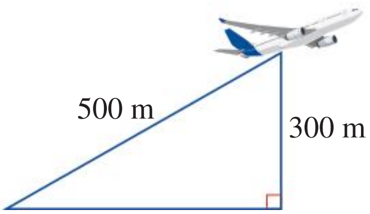
APPLICATIONS

SF: 5–12	CF: –	CU: –

5 A plane has just taken off and has travelled 500 m in a straight line sloping upwards. It is 300 m vertically above the ground.

a Substitute the values of c and b into the equation $a^2 = c^2 - b^2$.

b Determine the horizontal distance the plane has travelled, using your calculator. Round your answer to 1 decimal place.



6 Paul has been flying a kite but it is now stuck in the top of a tree. He had the full 50 metres of line out and it is pulled tight. The line forms a straight line from where he is standing, which is 29 metres horizontally from the base of the tree.

a Draw and label a diagram to show the situation.
b Label the known sides with values given.
c Write the formula to find a short side. Substitute the known values into the formula.
d Calculate the height of the tree, correct to 1 decimal place.

7 Barbie has climbed up the ladder of a radio mast. She is level with the attachment point of a cable. The other end of the cable is attached to a point that is 40 metres horizontally from the base of the mast. The cable forms a straight line and is known to be 234 metres long.

a Draw and label a diagram to show the situation.
b Label the known sides with values given.
c Write the formula to find a short side. Substitute the known values into the formula.
d Calculate the height that Barbie is above the ground, correct to 1 decimal place.

Example 6 ★**8** Boyd has 5 metres of a ladder to climb onto a roof 3 metres high. Calculate how far from the base of the wall that base of the ladder should be placed.

★**9** Julie wants to design a slippery slide to fit in her townhouse courtyard. She only wants it to take up 3 metres across the ground and has 3.5 metres of material for the actual slide. Determine the vertical height of the slide from the ground.

★**10** Jonah has a television with a 150 cm screen, which is measured as the length of the diagonal. If the screen is 75 cm high, calculate the width of the screen.

★**11** Melanie has bought an odd-shaped block of land in the shape of a right-angled triangle. If the two longest sides are 50 metres and 40 metres respectively, calculate the length of the third side of the triangle.

★**12** Sally wants to know what width of tarp material she needs to buy to make a two person tent. She has a 1.5 metre pole for the middle of the tent and wants to allow 2.5 metres across the bottom of the tent for two people to fit inside the tent. Determine the width of the tarp that Sally needs to buy, rounded to 2 decimal places.

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5C Determining unknown side lengths in problems by applying trigonometric rules COMPLEX

LEARNING GOALS

- Understand the concept of a trigonometric ratio
- Determine the adjacent and opposite sides to an angle in a right-angled triangle
- Learn how to use the rules for a trigonometric ratio
- Use the rules to calculate the unknown sides of a right-angled triangle
- Calculate unknown sides in right-angled triangles in a real-world context

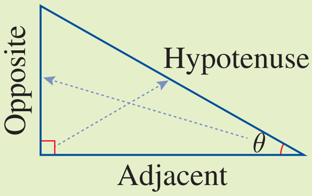
Why is it essential to understand other applications of calculating sides lengths of a right-angled triangle?

Several questions in the chapter so far have asked for calculations of vertical height and horizontal distance. These are examples of the use of right-angled triangles in calculating location and position, which is used in surveying, map-making, navigation and positioning systems.

Right-angled triangles have applications in navigation.

WHAT YOU NEED TO KNOW

- For a right-angled triangle with an angle labelled θ, the three sides should be named adjacent (next to the angle), opposite (not touching the angle) and the hypotenuse (the longest side opposite the right angle).



- We use trigonometric ratios to form a relationship between two sides and an angle in a right-angled triangle. The abbreviation SOH CAH TOA is used to help remember the trigonometric formulas shown in the diagram.

SOH CAH TOA

$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

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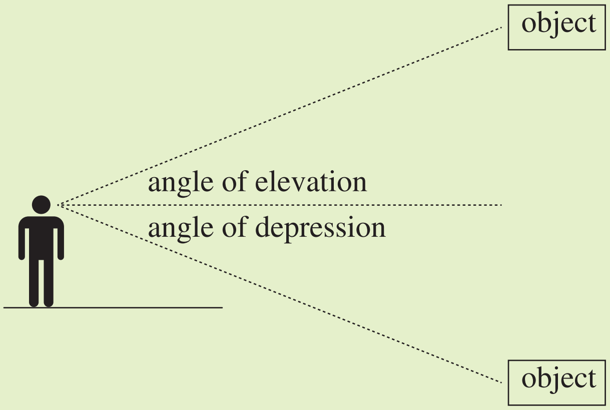
- Our calculators must be set to degree mode often seen by deg or D on the top of the calculator screen. Sin, cos and tan buttons can be found on all scientific calculators. To calculate the angle, shift or 2nd function sin, cos or tan is used.
- The angle of **elevation** is the angle from the horizontal going up. The angle of **depression** is the angle from the horizontal going down.



Example 7 Calculating the unknown side given the angle and one other side

Solve for the unknown value in the following. Round the answer to 2 decimal places.

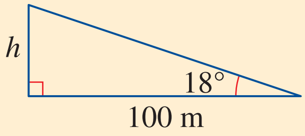
a $\sin 48 = \frac{x}{15}$

b $\cos 22 = \frac{19}{y}$

WORKING	THINKING
a $\sin 48 = \frac{x}{15}$	Rewrite the equation.
$15 \times \sin 48 = x$	Move divided by 15 to the other side as multiplied 15. (If the unknown value is on the top, then multiply.)
$x = 11.147$	Determine the unknown value using your calculator.
$x = 11.15$	As the third decimal place is a 5 or greater, round your answer up to 2 decimal places.
b $\cos 22 = \frac{19}{y}$	Rewrite the equation.
$y = 19 \div \cos 22$	Swap the cos 22 with the y (If the unknown is on the bottom, then swap the values.)
$y = 20.492$	Determine the unknown value using your calculator.
$y = 20.49$	As the third decimal place is below 5, your answer stays the same at 2 decimal places.

Example 8 Calculating the unknown side given the angle and one other side, with the unknown as a numerator

Complete the following steps to calculate the unknown side in the triangle shown.



a Identify the known sides on the triangle.
b Determine which trigonometric ratio is required.
c Substitute the values into the ratio.
d Solve the ratio for the unknown value. Round answer to 1 decimal place.

WORKING	**THINKING**
a The angle is 18°. 100 m is the adjacent side.	Identify the known sides in relation to the named angle in the triangle. The 100 m is next to the angle but is not the hypotenuse so it is the adjacent side.
h is the opposite side.	The *h* is opposite the named angle is the opposite side.
b $\tan\theta = \frac{\text{opp}}{\text{adj}}$	The ratio that has the adjacent (A) and opposite (O) side named is tangent. (TOA)
c $\tan 18 = \frac{h}{100}$	Substitute the values into the tangent ratio with $\theta = 18°$, $\text{O} = h$, $\text{A} = 100\ \text{m}$.
d $100 \times \tan 18 = h$	Multiply tan 18 by 100 to solve for *h*.
$h = 32.491$	Calculate the answer using your calculator.
$h = 32.5\ \text{m}$	Round your answer to 1 decimal place.

Land surveying makes extensive use of trigonometry.

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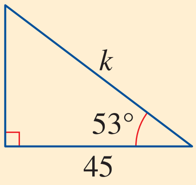
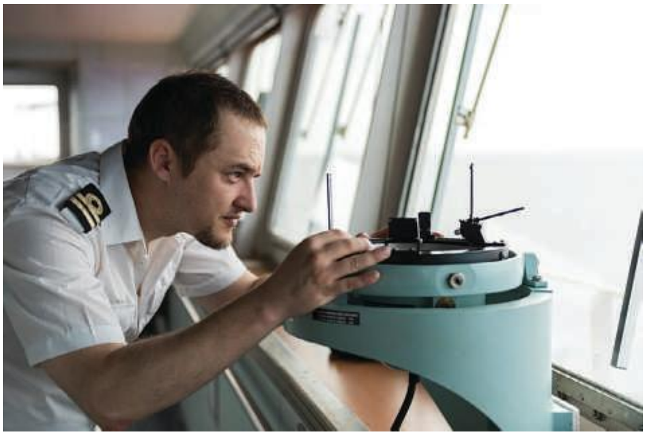
Example 9 Calculating the unknown side given the angle and one other side with the unknown as a denominator

Complete the following steps to calculate the unknown side in the triangle shown.



- **a** Identify the known sides on the triangle.
- **b** Determine which trigonometric ratio is required.
- **c** Substitute the values into the ratio.
- **d** Solve the ratio for the unknown value. Round your answer to 1 decimal place.

WORKING	THINKING
a The angle is 53°.	Identify the known sides in relation to the named angle in the triangle.
45 is the adjacent side.	The 45 is next to the angle but is not the hypotenuse so it is the adjacent side.
k is the hypotenuse.	The k is opposite the right angle so is the hypotenuse.
b $\cos\theta = \frac{\text{adj}}{\text{hyp}}$	The ratio that has the adjacent (A) and hypotenuse (H) side is cosine. (CAH)
c $\cos 53 = \frac{45}{k}$	Substitute the values into the cosine ratio with $\theta = 53°$, H $= k$, A $= 45$.
d $k = \frac{45}{\cos 53}$	Swap the position of the 45 and the k within the ratio.
$k = 74.77$	Calculate the answer using your calculator.
$k = 74.8$	Round the answer to 1 decimal place.



Navigation is based on trigonometry.

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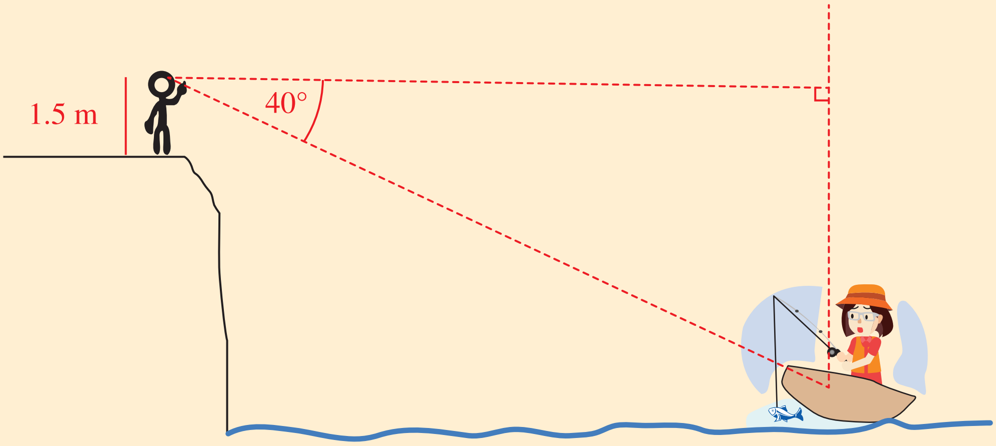
Example 10 Calculating the unknown side given the angle and one other side with the unknown in a real-world context

Rex is fishing 100 metres from the base of a cliff. His wife Adel waves to him to come home for dinner and looks down to him with an angle of 40 degrees. Determine the height of the cliff given that Adel's eyeline is 1.5 metres above the cliff.



WORKING	THINKING
$\theta = 40°$ $\text{adj} = 100\,\text{m}$ $\text{opp} = x$	Identify the values given for the right-angled triangle.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	Identify the correct ratio required from SOH CAH TOA. The tangent ratio uses O and A.
$\tan 40° = \frac{x}{100}$	Substitute the values into the tangent ratio.
$100 \times \tan 40° = x$ $x = 83.9$	Rearrange the ratio to determine the unknown value.
The cliff is $83.9 - 1.5 = 82.4$ m	Calculate the height of the cliff by subtracting Adel's height from x.
The cliff is 82.4 metres high.	Communicate your answer in a sentence.

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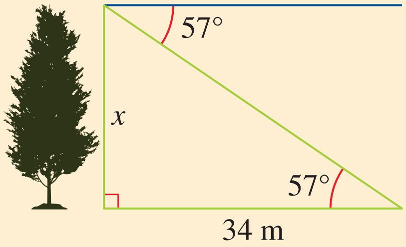
Example 11 Calculating the unknown side given the angle and one other side in a real-world context

Jane climbs to the top of a tree. She sees her friend standing 34 metres from the base of the tree with an angle of depression of 57 degrees. Determine the height of the tree that Jane has climbed.

WORKING	THINKING
	Draw a diagram to represent the information given. Label the information on the diagram noting the angle of depression from Jane is equal to the angle of elevation from her friend 34 m away.
x = opposite 34 m = adjacent $\tan\theta = \frac{\text{opp}}{\text{adj}}$	Identify the known sides in the diagram. Identify the correct ratio required from SOH CAH TOA. The tangent ratio uses O and A.
$\tan 57 = \frac{x}{34}$ $34 \times \tan 57 = x$ $x = 52.35$ $x = 52.4$	Substitute the known values into the tangent ratio. Rearrange the ratio to determine the unknown value.
The height of the tree is 52.4 metres.	Communicate your answer in a sentence.

Exercise 5C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a The tangent ratio equals the ______________ side ÷ the ______________ side.

b The sine ratio equals the ______________ side ÷ the ______________ side.

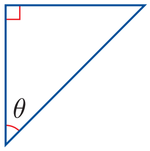
c The cosine ratio equals the ______________ side ÷ the ______________ side.

d Drawing a ______________ and labelling it will help solve trigonometric ______________.

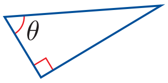
2 Label the sides of the triangles below with A (adjacent), O (opposite) and H (hypotenuse).

Hint The names relate to the angle θ.

a



b



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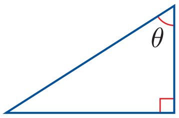


3 Calculate the value of the following, rounding the answer rounded to 1 decimal place where necessary.

a $24 \times \cos 50°$ **b** $33 \times \sin 37°$ **c** $48 \div \tan 82°$ **d** $67 \div \cos 12°$

Example 7

4 Solve for the unknown value in the following, rounding to 2 decimal places.

a $\sin 23 = \frac{x}{17}$ **b** $\cos 47 = \frac{y}{86}$ **c** $\tan 12 = \frac{a}{36}$

d $\tan 68 = \frac{23}{x}$ **e** $\sin 41 = \frac{16}{g}$ **f** $\cos 62 = \frac{118}{b}$

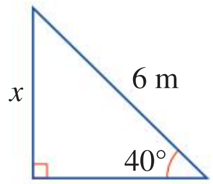
Estimating the height of tall objects uses angles of elevation, and trigonometry.

Example 8 & 9

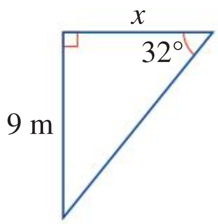
5 Complete the following steps to calculate the unknown side in the triangle shown.

i Identify the known sides of the triangle.
ii Determine which trigonometric ratio is required.
iii Substitute the values into the ratio.
iv Solve the ratio for the unknown value. Round your answer to 1 decimal place

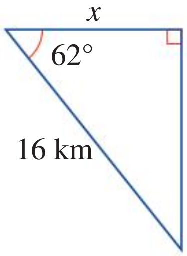
a



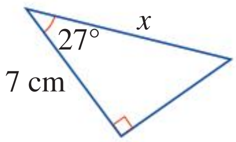
b



c



d



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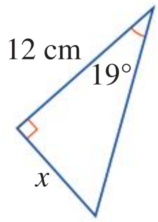
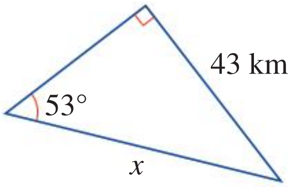


f

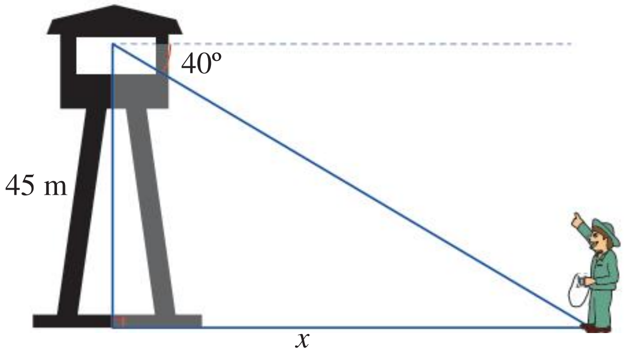


APPLICATIONS

SF: –	CF: 6–15	CU: 16

Example 10

6 Bridget is in the tower at 45 m above the ground, and she can see her Aunty Maree on the ground at an angle of 40°. Complete the following steps to calculate how far her Aunty Maree is from the base of the tower.



a Determine which sides are the known sides (A, O, H).
b Determine the correct ratio to use from SOH CAH TOA.
c Substitute the values into the appropriate ratio.
d Calculate the correct value for the distance Aunty Maree is from the base of the tower. Round your answer to 1 decimal place.

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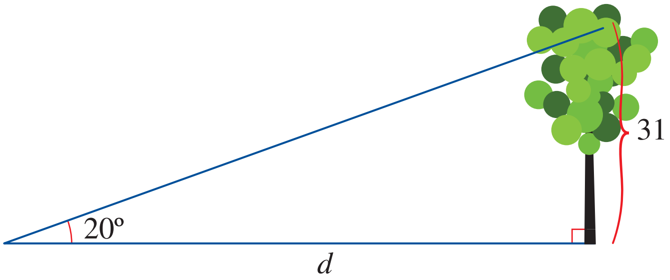
7 Rove can see the top of a tree in the distance at an angle of 20°. The height of the tree is 31 metres. Complete the following steps to calculate how far he is standing from the base of the tree.



a Determine which sides are the known sides (A, O, H).

b Determine the correct ratio to use from SOH CAH TOA.

c Substitute the values into the appropriate ratio.

d Calculate the correct value for the distance Rove is standing from the base of the tree. Round your answer to 1 decimal place.

8 Karen is looking up to the top of a building at an angle of 60 degrees. The base of the building is 36 metres from her on the ground. Complete the following steps to calculate the height of the building.

a Draw and label a diagram with the known sides and angle.

b Determine which ratio is needed to calculate the unknown side.

c Substitute the values into the appropriate ratio.

d Calculate the height of the building.

Hint The diagram involves a right-angled triangle.

Example 11 **9** Heidi is flying her rescue helicopter 1200 m above the ocean and sees a person in trouble at an angle of depression of 38 degrees. Complete the following steps to calculate the direct distance that the helicopter is from the person in the water.

a Draw and label a diagram with the known sides and angle.

b Determine which ratio is needed to calculate the unknown side.

c Substitute the values into the appropriate ratio.

d Calculate the direct distance that the helicopter is from the person in the water.

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10 Roland is sailing his boat and sees the top of the 120 m cliff at an angle of elevation of 15 degrees from the boat to the top of the cliff. Complete the following steps to calculate the distance to the boat is to the base of the cliff.

- **a** Draw and label a diagram with the known sides and angle.
- **b** Determine which ratio is needed to calculate the unknown side.
- **c** Substitute the values into the appropriate ratio.
- **d** Calculate the distance the boat is to the base of the cliff.

★**11** A cable is to be attached to the top of a large antenna that is 32 metres tall. If the cable is to have an angle from the ground to the top of the antenna of 65 degrees, calculate the length of the cable.

★**12** Pebbles is flying a kite and has let out the entire 55 metres of string. If the angle of the kite string is 44 degrees from the ground, calculate the horizontal distance to the kite.

★**13** Billy is running down a 322-metre slope on the hill. If he found the angle of depression of the hill to be 21 degrees, calculate how high the hill is above ground level.

★**14** A straight waterslide has an angle of elevation of 38 degrees and is 12 metres high. Calculate the actual length of the waterslide.

★**15** The school flagpole is casting a 4.5-metre shadow. If the angle of depression of the sunlight is 36 degrees, calculate the actual height of the flagpole.

★**16** Margret has seen a lighthouse 1350 metres horizontally from her boat. She has measured the angle of elevation from her boat to the top of the lighthouse as 52 degrees. Margret knows that the lighthouse is 50 metres tall on the top of the cliff. Determine the height of the cliff.

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5D Determining unknown angles in problems by applying trigonometric rules COMPLEX

LEARNING GOALS

- Determine the named sides in relation to the unknown angle in a right-angled triangle
- Use the rules to calculate the unknown angle of a right-angled triangle
- Identify which trigonometric rules are required to solve real-world context questions, and hence calculate the unknown angle

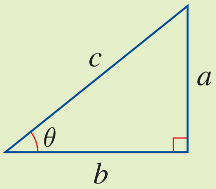
Why is it essential to calculate angles in right-angled triangles in a real-world context?

Wooden or metal supports and also cabling use angles within a specific range in order to maintain strength within a structure. Building inspectors use length and width measurements of a roof to check that the roof pitch meets building codes.

Many buildings use right-angle triangles in their construction.

WHAT YOU NEED TO KNOW

- Inverse sine ($\sin^{-1}$), inverse cosine ($\cos^{-1}$) and inverse tangent ($\tan^{-1}$) can be used to find angles in right-angled triangles.
  - $\sin\theta = \frac{a}{c}$ means $\theta = \sin^{-1}\left(\frac{a}{c}\right)$
  - $\cos\theta = \frac{b}{c}$ means $\theta = \cos^{-1}\left(\frac{b}{c}\right)$
  - $\tan\theta = \frac{a}{b}$ means $\theta = \tan^{-1}\left(\frac{a}{b}\right)$
- Note that $\sin^{-1} x$ does *not* mean $\frac{1}{\sin x}$.



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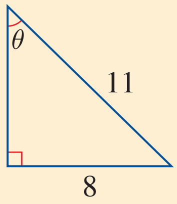
Example 12 Calculating the unknown angle given two other sides of the right-angled triangle

Complete the following steps to calculate the unknown side in the triangle shown.



a Identify the known sides of the triangle in relation to the angle you want to find.

b Determine which ratio uses the two sides given from SOH CAH TOA.

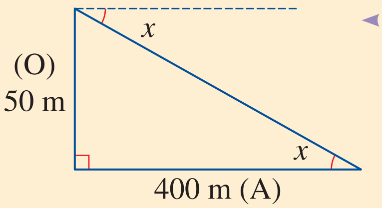
c Substitute the values into the ratio.

d Solve for the unknown angle in the ratio, rounding to 1 decimal place.

WORKING	THINKING
a 8 = opposite side 11 = hypotenuse	The 8 is opposite the named angle and 11 is the hypotenuse.
b $\sin\theta = \frac{\text{opp}}{\text{hyp}}$	Identify the correct ratio required from SOH CAH TOA. The sine ratio uses O and H.
c $\sin\theta = \frac{8}{11}$	Substitute the values into sine ratio with $\theta°$, O = 8 and H = 11.
d $\theta = \sin^{-1}\frac{8}{11}$ $\theta = 46.65°$	Move sin to the other side of the equals sign as $\sin^{-1}$. Calculate the angle size using your calculator.
$\theta = 46.7°$	Round the answer to 1 decimal place.

Example 13 Calculating the unknown angle given two other sides of the right-angled triangle in a real-world context

Ranger Yogi wants to build a flying fox (cable glider) to go from the top of the lookout to the ranger station on the ground. The lookout is 50 metres high and the base of the lookout is 400 metres from the ranger station. Determine the angle of depression of the cable connecting the top of the lookout to the ranger station.

WORKING	**THINKING**
	Draw a diagram. Identify that the angle of depression equals the angle of elevation at the opposite side of the right-angled triangle. Label the information on the diagram indicating A, O, H.
50 m = opposite 400 m = adjacent $\tan\theta = \frac{\text{opp}}{\text{adj}}$	Identify the correct ratio required from SOH CAH TOA. The tangent ratio uses O and A.
$\tan x = \frac{50}{400}$	Substitute the values into the tangent ratio with $O = 50$ and $A = 400$.
$x = \tan^{-1}\frac{50}{400}$	Move tan to the other side of the equal sign as $\tan^{-1}$.
$x = 7.12°$	Calculate the angle size using your calculator.
$x = 7.1°$	Round your answer to 1 decimal place.

Exercise 5D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** The __________ ratio equals the ____________ side ÷ the adjacent side.
- **b** The __________ ratio equals the adjacent side ÷ the _______________.
- **c** The sine ________ equals the ___________ side ÷ the hypotenuse.
- **d** To solve a word problem, we should always _________ a diagram and ____________ it.

2 Calculate the following, rounding to 2 decimal places where necessary.

- **a** $\cos^{-1}\frac{12}{13}$
- **b** $\sin^{-1}\frac{37}{61}$
- **c** $\tan^{-1}\frac{7}{38}$
- **d** $\tan^{-1}\frac{18}{16}$

Hint Remember that a fraction can be reduced to a single decimal number by dividing the top number by the bottom one on a calculator.

3 Solve for the unknown value in the following, rounding to 2 decimal places.

a $\sin\theta = \frac{12}{13}$

b $\cos\theta = \frac{5}{13}$

c $\tan\theta = \frac{14}{17}$

d $\sin\theta = \frac{29}{83}$

Hint You need the inverse trigonometric ratio buttons ($\sin^{-1}$, etc.) on your calculator to solve these equations. Such buttons are usually 'second function' buttons meaning you have to press a button labelled '2nd' before using or accessing them.

Example 12 **4** Complete the following steps to calculate the unknown side in the triangle shown.

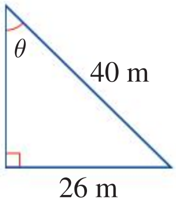
i Identify the known sides of the triangle.

ii Determine which trigonometric ratio is required.

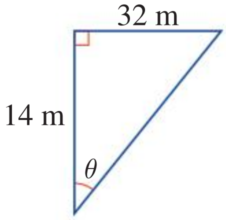
iii Substitute the values into the ratio.

iv Solve the ratio for the unknown value. Round your answer to 1 decimal place.

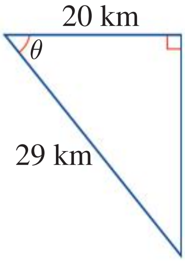
a



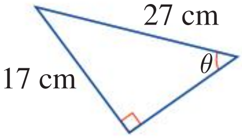
b



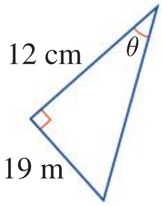
c



d



e



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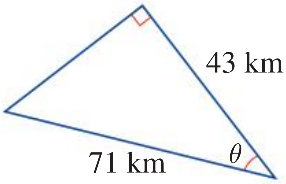
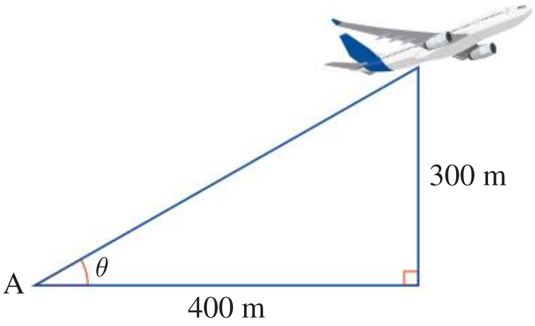


APPLICATIONS

SF: –	CF: 5–12	CU: 13

Example 13 **5** Peta the pilot has just taken off from the runway from a point A as shown in the diagram. Complete the following steps to calculate the angle of take-off from the ground.



a Determine which sides are the known sides using A, O, H.

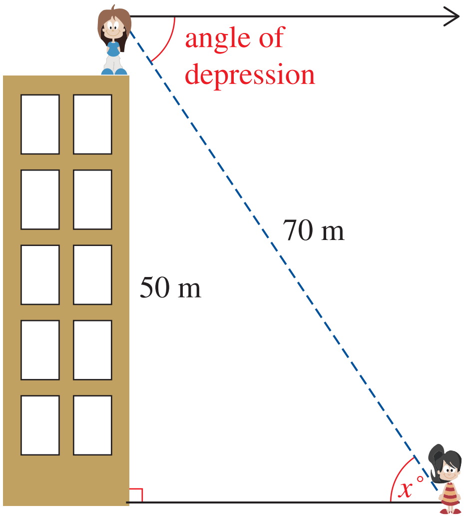
b Determine the correct ratio to use from SOH CAH TOA.

c Substitute the values into the appropriate ratio.

d Calculate the angle of take-off from the ground, correct to 1 decimal place.

6 Charlie is on top of a building that is 50 metres above the ground and she can see her friend Georgie outside the building on the ground. The direct distance between Charlie and Georgie is 70 metres. Complete the following steps to calculate the angle of depression from Charlie to Georgie.

- a Determine which sides are the known sides using A, O, H.
- b Determine the correct ratio to use from SOH CAH TOA.
- c Substitute the values into the appropriate ratio.
- d Calculate the angle of depression from Charlie to Georgie, correct to 1 decimal place.



7 Douglass has climbed a big tree which is 20 metres high. His father leans a 30-metre ladder against the tree so that he can climb up the tree and help Douglass down to the ground. Complete the following steps to calculate the angle that the ladder makes with the tree.

- a Draw and label a diagram with the known sides.
- b Determine which of the known sides are A, O, H.
- c Determine the correct ratio to use from SOH CAH TOA.
- d Substitute the values into the ratio and calculate the angle that the ladder makes with the tree, correct to 1 decimal place.

★8 Holly has climbed to the top of Mount Coolum, which is 208 metres high above sea level. She saw the ocean, which is 1100 metres horizontally from Holly’s current position. Determine the angle that Holly is looking down towards the ocean.

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★**9** A ski lift travels 324 metres to the top of a hill that is 212 metres high. Calculate the angle of elevation the ski lift is from the start of the ski lift.

★**10** A castle has a 20-metre wall and a 40-metre wide moat. If an archer wanted to fire an arrow from the top of the wall at someone on the edge of the moat, determine the angle of depression that the arrow would need to be fired. (Assume the arrow will travel straight.)

★**11** A right-angled triangular roof is being built on a house that is 20 metres wide. The roof has a slope length of 28 metres. Calculate the angle of elevation of the roof.

★**12** A straight slippery slide has a height at the top of 2.5 metres and the slide itself is 5 metres long. Determine the angle of depression from the top of the slide.

★**13** Henry is 1.80 metres tall and sees a 24-metre flagpole while he is standing 20 metres away from the base of the flagpole. Determine the angle of elevation that Henry must raise his eyes to see the top of the flagpole. (Assume his eyes are 1.8 metres above the ground.)

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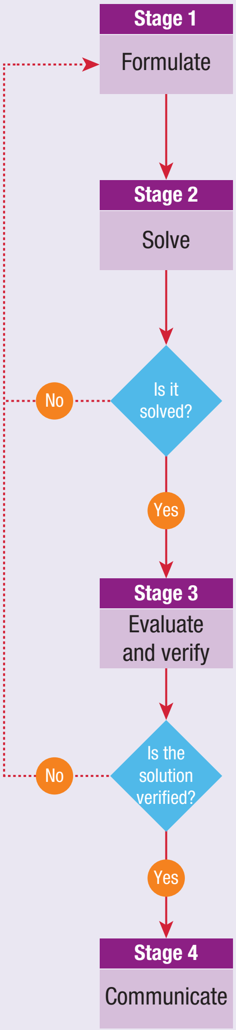
Problem-solving and modelling task

Background: If you don't have an electronic range-finder, how do you find the height of an object that you can't climb or reach the top of with a long tape measure? A common solution is to use shadow lengths. The sun's rays are for practical purposes parallel, which means the length of shadows of objects measured at the same time of day are proportional to their height.

Task: In a group, you are to use shadows to determine the heights of 3 different objects. An example may be the height of the football posts, the flagpole and a building. You are to use your knowledge of trigonometry and a ruler to determine the angle of depression of the sun by measuring a person's height and the length of their shadow. You are to then measure the length of the shadows on the various objects and use trigonometry to calculate the heights of the objects given your calculation of the angle of depression.

Approach to problem-solving and modelling tasks:



Stage 1: Formulate

1 Decide which objects to determine their heights.
2 Allocate jobs to each person in the team.

Stage 2: Solve

3 Measure the height of a member of the team.
4 Measure the length of that person's shadow.
5 Use trigonometry to calculate the angle of elevation of the sun.
6 Measure the length of the shadows of the chosen objects.
7 Use trigonometry to calculate the heights of the chosen objects using the angle of depression of the sun and the length of the object's shadow.

Stage 3: Evaluate and verify

8 Estimate the heights of the chosen objects.
9 Compare the calculated height with your estimations for validity.

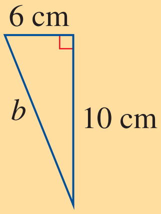
Stage 4: Communicate

10 Communicate your results by writing a report showing all your calculations and results.

Chapter checklist

☐ I can calculate the hypotenuse using Pythagoras' theorem.

1 Calculate the value of b in the triangle shown, rounding to 1 decimal place.

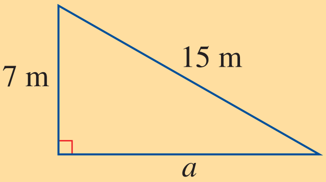


2 Calculate the hypotenuse of a right-angled triangle given its base is 8 m and its height is 18 m. Round your answer to 1 decimal place.

3 Kristy is making a gate that is 2600 mm by 1800 mm and she needs to build a diagonal brace. Determine the length of the brace that Kristy will need to cut, rounding to the nearest whole number.

☐ I can calculate unknown short sides using Pythagoras' theorem.

4 Solve for a in the diagram shown, rounding to 1 decimal place.



5 Calculate the base of a right-angled triangle given its hypotenuse is 26 metres and its height is 11 metres. Round your answer to 1 decimal place.

6 Brad ran 2.5 km due west and then ran due north for some distance. If he then ran 4.2 km back to his starting point, determine how far north Brad had run.

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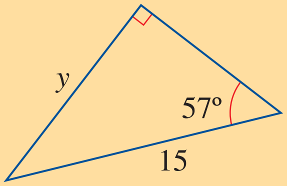
☐ I can apply the tangent, sine and cosine rule to determine unknown sides lengths. **[complex]**

7 Calculate the value of y in the triangle shown, rounding to 1 decimal place.



8 A right-angled triangle has an angle of 30 degrees and an adjacent side of 25 metres.

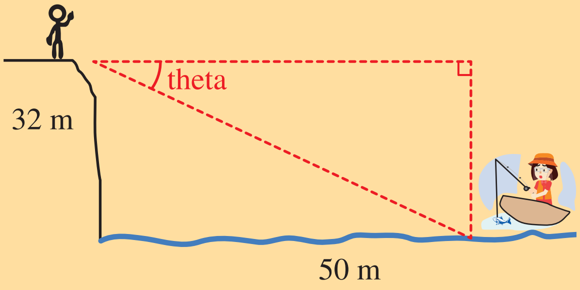
a Calculate the length of the opposite side.

b Calculate the length of the hypotenuse.

9 Terri is walking her puppy with a 2.5 m lead. The puppy is walking ahead and pulling the lead tight and it makes an angle of elevation from the dog to Terri's hand of 21°. Determine how much higher Terri's hand is than the puppies' collar.

☐ I can apply the tangent, sine and cosine rule to determine unknown angles. **[complex]**

10 Freda is standing on the top of a cliff and can see a person in a boat fishing in the ocean. The person in the boat is 50 m from the base of the 32 m-high cliff. Ignoring the height of Freda, calculate the angle of depression from Freda to the person in the boat.



11 A stunt ramp is 14 metres long and 5 metres high. Determine the angle of elevation of the ramp from the ground.

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Chapter review

All questions in the review are assessment style.

Simple familiar

Section 5A

1 A castle drawbridge is 20 metres long and has chains connecting the end of the drawbridge to the top of the castle wall which is 25 metres high. Determine the length of the chains to 1 decimal place.

2 An old tree needs a steel cable to help hold it upright. If the cable is attached 2 metres from the base of the tree and 5 metres high on the tree, determine the length of cable between the tree and the ground to the nearest whole number.

3 Esmae walked due north on a 4 km hike and then turned 90 degrees and walked a further 2.5 km due west. Determine how far Esmae is from her starting point to 1 decimal place.

Section 5B

4 Tarek has a 3.5-metre ladder and leans it against a wall that is 2.5 metres high. Calculate the distance the base of the ladder is from the base of the wall, rounding to 1 decimal place.

5 A tent pole that is 2.1 metres high has a rope attached that is 2.6 metres long. Determine how far the peg needs to be hammered into the ground from the base of the pole so that the rope will be tight.

Complex familiar

Section 5C

6 River is standing 40 metres from the base of a waterfall. If she measured the angle to the top of the waterfall as 52 degrees, calculate the height of the waterfall.

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7 Zek is flying a plane and approaching a runway at 1000 metres above the ground. He notices that the beginning of the runway is at an angle of 28 degrees from the plane. Calculate the distance the plane still needs to travel before reaching the beginning of the runway.

Section 5D

8 Nell slides down a straight slippery slide that is 6.2 metres long and 2.5 metres high. Calculate the angle of the slippery slide from the ground.

9 James hang glides from the top of a 368-metre cliff to land 823 metres away from the base of the cliff. Assuming James travelled in a straight line, determine the angle of depression that he flew.

10 The lighthouse keeper looks due east from the top of her lighthouse 182 metres above sea level and sees a ship at an angle of depression of 12 degrees. She sees another ship in the same direction at an angle of depression of 18 degrees. Calculate the distance between the two ships.

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6

Summarising and interpreting data

Maths for a police officer: Brendan Harding

Brendan Harding joined the police straight out of high school and initially went to the police academy in Brisbane. He was then transferred to Innisfail where he worked in several stations within the Innisfail District. He also had the opportunity to go to Kowanyama for a short period of time. In 2010 Brendan was promoted to Sergeant in the Toowoomba Road Policing Unit.

Tell us a bit about your job. What does a typical day look like?

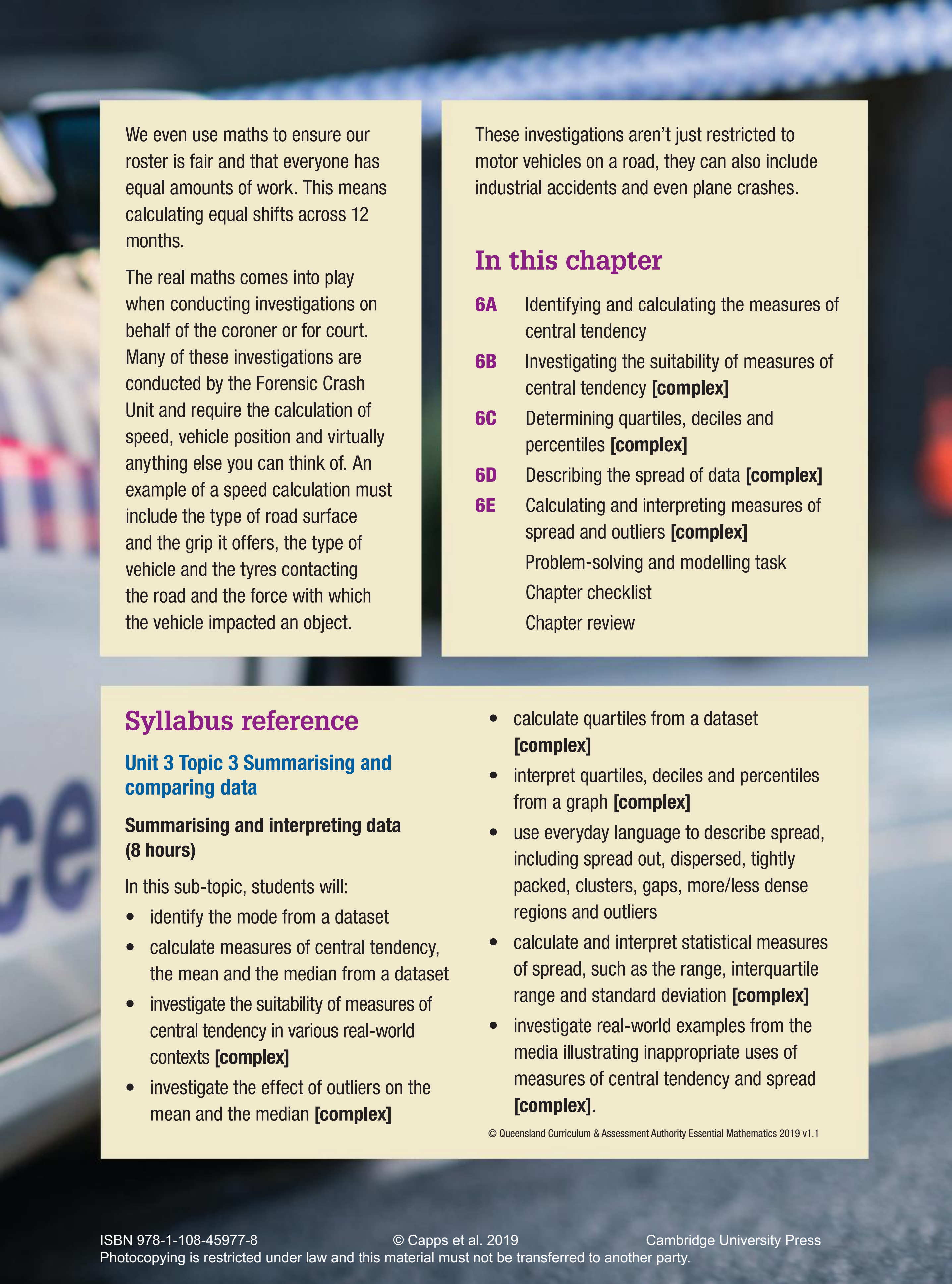
It's hard to describe a typical day as no two days are the same. One day, I might be in the office checking correspondence such as court briefs for the prosecutor and risk management paperwork. A day on the road could involve school patrols making sure the kids are safe coming to, and going from, school. Another day could be a major traffic operation where my team and I conduct high visibility static interceptions of motor vehicles looking for drink drivers, drug drivers, unlicensed drivers, unregistered and defective vehicles. The days on the road are the best as you get out of the office and get to talk to lots of different people. Today, I'm sitting on a cordon waiting for a stolen car to come past that has just done a fuel drive off.

What maths did you study at school?

I wasn't necessarily the smartest kid at school and, if honest, was pretty average. I really didn't like study but did it when I had to. When I think about it, I should have asked my teacher more questions at school but didn't want to appear stupid in front of the class.

How do you use maths in your job?

A lot of what I do comes down to statistics and dictates where we conduct our enforcement activities. Many times, I will have to collect raw data such as traffic count data from Main Roads and match that data with times, days and locations of traffic crashes to identify the right time to be at the right place for specific enforcement.



We even use maths to ensure our roster is fair and that everyone has equal amounts of work. This means calculating equal shifts across 12 months.

The real maths comes into play when conducting investigations on behalf of the coroner or for court. Many of these investigations are conducted by the Forensic Crash Unit and require the calculation of speed, vehicle position and virtually anything else you can think of. An example of a speed calculation must include the type of road surface and the grip it offers, the type of vehicle and the tyres contacting the road and the force with which the vehicle impacted an object.

These investigations aren't just restricted to motor vehicles on a road, they can also include industrial accidents and even plane crashes.

In this chapter

Syllabus reference

Unit 3 Topic 3 Summarising and comparing data

Summarising and interpreting data (8 hours)

In this sub-topic, students will:

- identify the mode from a dataset
- calculate measures of central tendency, the mean and the median from a dataset
- investigate the suitability of measures of central tendency in various real-world contexts **[complex]**
- investigate the effect of outliers on the mean and the median **[complex]**
- calculate quartiles from a dataset **[complex]**
- interpret quartiles, deciles and percentiles from a graph **[complex]**
- use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions and outliers
- calculate and interpret statistical measures of spread, such as the range, interquartile range and standard deviation **[complex]**
- investigate real-world examples from the media illustrating inappropriate uses of measures of central tendency and spread **[complex]**.

Pre-test

1 Evaluate the following correct to 2 decimal places.

a $x = \frac{145}{7}$ **b** $y = \frac{3}{6}$ **c** $x = \frac{1}{3}$

2 Arrange the following numbers into order from lowest to highest and identify the median (middle number).

11, 13, 7, 9, 4, 1, 17, 19, 21, 6, 5, 10, 13

3 Evaluate the following.

a $\frac{22 + 31}{2}$ **b** $\frac{43 + 45}{2}$ **c** $\frac{56 + 71}{2}$

4 Evaluate x, leaving your answer as a percentage.

a $x = \frac{4}{16}$ **b** $x = \frac{7}{16}$

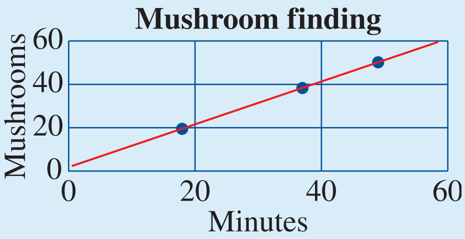
5 Determine the value of x that satisfies each percentage.

a $\frac{x}{8} = 12.5\%$ **b** $\frac{x}{36} = 25\%$

6 Evaluate the following squares.

a 2^2 **b** 7^2 **c** 0.5^2

7 From this graph, estimate how many minutes it took to collect 24 mushrooms.



8 Identify the smallest, largest and most common number in this stem-and-leaf plot.

Stem-and-leaf plot

4	1
5	2 7 8
6	5 6
7	0 5 8 8 8
8	0 0
9	5

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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6A Identifying and calculating the measures of central tendency

LEARNING GOALS

- Identify the mode from a dataset
- Calculate the mean from a dataset
- Calculate the median from a dataset

Why is it essential to understand how to determine the measures of central tendency?

There are three main measures of central tendency: the mode, the mean and the median. The aim of each measure is to pinpoint the location of the centre of a data distribution with numerical data.

The measures can be used as a summary of the data, and any conclusion drawn from the data should refer to them. Careers that might use measures of central tendency are car salespeople, real estate salespeople, scientists, business administrators and any vocation that requires the use of statistics.

Measures of central tendency help us to make sense of a large data distribution.

WHAT YOU NEED TO KNOW

- A **measure of central tendency** describes a set of data by identifying the central position within that dataset. There are three main measures of central tendency: the mode, the mean and the median.
- The **mode** reveals the most frequent value in the dataset, i.e. the value that occurs most often.
  - A dataset can also have two modes (it is **bimodal**), or no mode at all.
  - An example for using the mode is when a dress shop wants to know the most popular size when ordering stock.

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- The **mean** is the average; it is equal to the sum of all the values in the data set divided by the number of values in the data set.
  - The symbol for mean in mathematics is $\bar{x}$.
  - The formula for identifying the mean is $\bar{x} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$.
  - An example for using the mean is when a teacher wants to calculate the average marks for a class, as this value summarises how well the class is going as a whole.
- The **median** is the middle value of a dataset, when the dataset is sorted in order from the smallest value to the largest value.
  - The median is the number that falls exactly in the middle of the data.
  - The median is the $\frac{n+1}{2}$th value when ranked in order from smallest to largest where n is the number of pieces of data. So if a dataset has 21 values, then the median will be the 11th value because $\dfrac{21+1}{2} = 11$.
  - If the dataset has an even number of values, then the median is the **average** of the two middle values.

Odd number of values	Even number of values
4, 6, 7, 9, 11	4, 6, 7, 9, 11, 14
Median = 7	$\text{Median} = \dfrac{7+9}{2}$ $= 8$

  - An example for using the median is when a real estate agent uses the median for house prices in a particular suburb, especially if the suburb includes either some very expensive or very cheap properties that might have a big effect on the mean.

Example 1 Identifying the mode from a dataset

Emily is a dressmaker and needs to make some more dresses for her pop-up shop. She recalls the sizes of the dresses she sold over the past three months:

Size 6 – 15 dresses
Size 8 – 5 dresses
Size 10 – 6 dresses
Size 12 – 12 dresses
Size 14 – 2 dresses
Size 16 – 22 dresses

Identify the mode for the dataset.

WORKING	THINKING
The mode is size 16 at 22 dresses.	The mode is the most common value. The dresses purchased the most are size 16 at 22.

Example 2 Calculating the mean from a dataset

The following are the term grades for a student's tests.

67%, 78%, 65%, 72%, 64%, 76%, 78%, 80%, 82%, 85%

Calculate the mean for the student's grades.

WORKING	THINKING
$\bar{x} = \frac{67 + 78 + 65 + 72 + 64 + 76 + 78 + 80 + 82 + 85}{10}$ $\bar{x} = 74.7$	Mean $= \bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$
The mean of the student's tests for the term is 74.7%.	Add all the values and divide by the number of values (with this dataset, 10).

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Example 3 Calculating the median from a dataset

The following are the prices of some houses in a particular area.
$345 000, $300 000, $450 000, $290 000, $390 000, $670 000, $345 000, $410 000
Calculate the median of the house sales.

WORKING	THINKING
290 000, 300 000, 345 000, 345 000, 390 000, 410 000, 450 000, 670 000	The data must first be ordered from smallest to largest. The median is the middle value.
The number of scores is 8. So, $n = 8$ The median score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score. The median will lie between the 4th and 5th scores. The 4th score = 345 000 The 5th score = 390 000	Determine the number of scores. This is an even number of scores, so we need to average the middle two scores.
$\text{Median} = (345000 + 390000) \div 2$ $= 367500$	Add the scores and divide by 2.
The median for the house prices is $367 500.	Communicate your answer in a sentence.

Exercise 6A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** The aim of measures of central tendency is to pinpoint the location of the __________ of data distribution.
- **b** There are three main measures of central tendency: the __________, the __________ and the __________.
- **c** __________ is the most frequent value in the data.
- **d** A dataset can have __________ modes (which means it is __________), or it can have no mode at all.
- **e** The mean is the __________; it is equal to the __________ of all the values in the dataset __________ by the number of values in the data set.
- **f** The formula for identifying the mean is ______________.

g The median is the ________ value of a dataset, when the dataset is sorted in _________ order.

h If the dataset has an _______ number of values, then the median is the average of the _______ middle values.

Example 1–3

2 For each dataset, identify:

i the mean

ii the median

iii the mode

a 6, 4, 3, 5, 6, 2, 7, 6, 5, 9, 5, 4

b 85, 85, 95, 55, 75, 85, 75, 85, 55

c 6.7, 8.5, 8.6, 9.2, 7.4, 7.5, 7.9, 8.0

d 12, 18, 19, 11, 23, 24, 21, 18, 35

e 1, 3, 2, 2, 3, 2, 2, 1, 3, 3, 0, 3, 3, 2

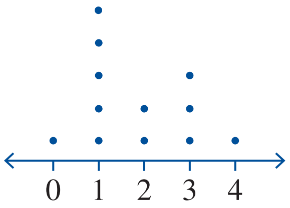
Hint The mean is the average.
$\bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$

Hint The median is the middle score. Place all of your data in order from smallest to largest before finding the median.

3 Identify the mode for the following datasets.

Hint The mode is the most frequent.

a **Number of children**

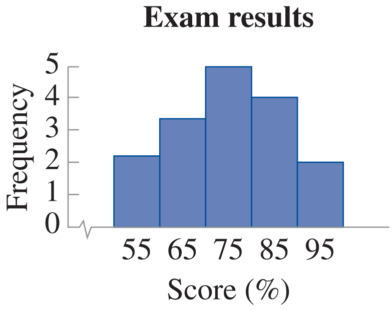


b **Number of goals**

0	9
1	2 2 5
2	3 5
3	0

2|3 = 23

c



APPLICATIONS

SF: 4–8	CF: –	CU: –

4 A doctor is researching anaemia and he has recorded the following systolic pressure values from a group of patients' blood pressure readings.

115, 115, 107, 128, 122, 113, 108, 130, 115, 170, 120, 106

- **a** Calculate the mean number of systolic pressure values.
- **b** Identify the median number of systolic pressure values.
- **c** Determine the mode.

Hint Mean is average; median is the middle; mode is the most frequent.

5 Markus wants to purchase a new laptop. The following are the amounts recorded from a selection of options.

\$700, \$900, \$200, \$1200, \$400, \$300, \$900, \$1000, \$700, \$850, \$2600

- **a** Calculate the mean price for a laptop.
- **b** Identify the median price for a laptop.
- **c** Determine the mode.

6 Micaela is a hockey coach and she is preparing for a new season. The following dataset is her team's scores for the previous season.

0, 4, 1, 1, 2, 1, 2, 1, 5, 4, 3, 5, 2, 1, 0, 2, 1, 0, 3, 2, 3, 3, 2, 1, 0, 2, 1, 2, 4, 0

- **a** Calculate the mean score.
- **b** Identify the median score.
- **c** Determine the mode.

7 For the data in the following stem-and-leaf plots, identify:

i the mean

ii the mode

iii the median

Hint A stem-and-leaf plot splits each value into a stem (the first digit/s) and a leaf (usually the last digit). For example, 22|5 is 225.

a

0	8 9
1	2 2 3 5 7
2	4 6 9

2|4 = 24

b

20	0 0 1 3
21	2 3 4 5 5
22	4 5 5 5 5 8 9
23	6 8
24	0 0 3 4

22|5 = 225

c

4	0 1 3 8
5	2 2 2 3 7 9
6	6 7
7	0 1 3 5

5|2 = 52

d

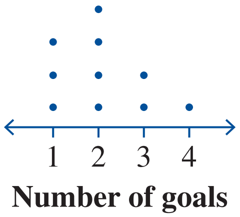
10	0 0 0 0 1 7
11	1 2 2 3 4 5 9
12	1 1 2 2 4 6 8
13	0 0 7

13|7 = 137

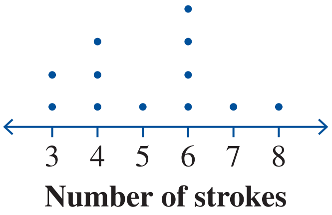
8 For the data in the following dot plots, identify:

i the mean **ii** the mode **iii** the median

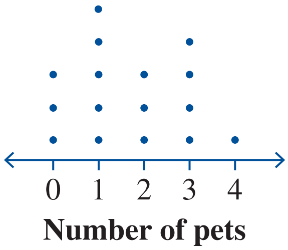
a



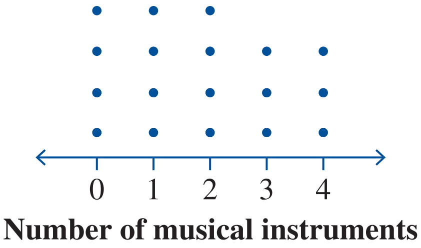
b



c



d



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6B Investigating the suitability of measures of central tendency COMPLEX

LEARNING GOALS

- Investigate the suitability of measures of central tendency in various real-world contexts
- Investigate real-world examples from the media illustrating inappropriate uses of measures of central tendency

Why is it essential to be able to determine the suitability of the measures of central tendency?

Deciding which measure of central tendency to use in real-world contexts is important when trying to understand the distribution of a dataset. Measures of central tendency are the most typical representation of the collected data and have their own purposes.

Deciding which measure of central tendency to use can be important when interpreting datasets.

WHAT YOU NEED TO KNOW

- The mode reveals the most frequent value in the dataset.
  - Advantages of using the mode:
    - Simple to understand (the value that occurs most often).
    - Not affected by extreme values (**outliers**).
  - Disadvantages of using the mode:
    - Not based on all the values in a dataset.
    - Sometimes the data has more than one mode, and sometimes there is no mode at all.
- The mean is the average of the dataset.
  - Advantages of using the mean:
    - All the data is taken into account.
    - Easy to understand (the value you would have if all the data points were equal) and calculate.
  - Disadvantages of using the mean:
    - Outliers (extreme values) can distort the results.

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        - If the data is in the form of percentages or ratios, it could be challenging to calculate the mean.
- The median is the middle value of a dataset, when the dataset is sorted in order from the smallest value to the largest value.
    - Advantages of using the median:
        - Simple to understand (the data point in the middle, with an equal number of greater and lesser values) and easy to calculate.
        - Not affected by outliers.
    - Disadvantages of using the median:
        - The median is based only on the middle value of an ordered dataset and does not include values from the other data points at all.
        - Need to remember that if there is an even number of data points to take the average of the middle two.
    - For example: For the dataset 1, 2, 1, 2, 20 (outlier):

The mean including the outlier	The mean not including the outlier
$\frac{1+2+1+2+20}{5} = 5.2$	$\frac{1+2+1+2}{4} = 1.5$
The mean with the outlier is 5.2.	The mean without the outlier is 1.5.

An outlier does not affect the median so much:

The median including the outlier:	The median not including the outlier:
$1\ 1\ \boxed{2}\ 2\ 20$	$1\ \boxed{1\ 2}\ 2$
The median with the outlier is 2.	The median without the outlier is 1.5.

- The **misuse** of measures of central tendency may arise from the following issues:
    - The mean may be misleading when there are outliers or extreme values.
    - The median does not consider the exact value of each observation and is capable of misleading when all information is required.
    - The mode should not be used if the data is continuous, such as the heights of people in a basketball team, as it is not likely to have any one value that is more frequent than any other, or if the most frequent value is far away from the rest of the data.
- To sum up, measures of central tendency can be very useful for understanding a dataset, but each measure has the potential to give a misleading representation of the data if used in the wrong context. That's why it is important to consider the characteristics of the data before choosing an appropriate measure.
    - Is the data continuous? If so, be wary of using the mode.
    - Does the data have extremely large or small values compared to the rest of the data? If so, be wary of using the mean.
    - Is it possible to logically order the data? If not, don't use the median.

Example 4 Investigating the suitability of measures of central tendency in various real-world contexts

Henry's goal is to achieve 50% correct on his weekly maths test across 10 weeks. He has recorded his grades out of 20 questions for the past 8 weeks. He has two weeks to go.

4, 6, 9, 11, 8, 10, 12, 6

- **a** Calculate the mean grade.
- **b** Identify the median grade.
- **c** Determine the mode grade.
- **d** Which is a better measure to assist with Henry's preparation to reach his goal? Give a reason.

WORKING	THINKING
a $\bar{x} = \frac{4+6+9+11+8+10+12+6}{8}$	Mean $= \bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$
$\bar{x} = 8.25$ The mean of grades is 8.25.	Add all the values and divide by the number of values (with this dataset, 8).
b 4, 6, 6, ⑧, ⑨, 10, 11, 12	The data must first be ordered from smallest to largest. The median is the middle value.
The number of scores is 8. So $n = 8$	Determine the number of scores. This is an even number of scores, so we need to average the middle two scores.
The median score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score The median will lie between the 4th and 5th scores. The 4th score = 8 The 5th score = 9	
Median $= (8+9) \div 2$ $= 8.5$	Add the scores and divide by 2.
The median for the grades is 8.5.	Communicate your answer in a sentence.
c The mode for the grade is 6.	The mode is the most common value.

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WORKING	THINKING
d The better measure would be mean, as it is the average score of all grades. The median score does not take into account all grades recorded. The mode has nothing to do with how well the other grades have contributed to his results.	← Consider the pros and cons of each measure of tendency and determine the purpose of the real-world central measure.

Worksheet 6A Investigating a real-world example from the media illustrating inappropriate uses of measures of central tendency: see the Interactive Textbook for this activity

Exercise 6B

FUNDAMENTALS

Example 4

1 Describe the advantages and disadvantages of each measure of central tendency.

Measure of central tendency	Advantages	Disadvantages
Mean		
Median		
Mode		

APPLICATIONS

SF: – CF: 2–6 CU: 7

2 Verity is a shoe designer and she is preparing for a new pop-up shop. Verity has recorded the size of shoes purchased over the past week.

12, 7, 7, 7, 7, 8, 6, 7, 7, 10, 10, 5, 9, 9, 10, 9, 11

a Calculate the mean shoe size.

b Identify the median shoe size.

c Determine the mode shoe size.

d Which is a better measure to assist with Verity's preparation? Give a reason.

Hint Mean is average; median is the middle; mode is the most frequent.

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3 Noah is trying to improve his test scores and has been recording his results out of 40 questions in preparation for passing the term.

26, 13, 8, 2, 30, 17

- **a** Calculate the mean result.
- **b** Identify the median result.
- **c** Determine the mode result.
- **d** Clarify which measure would best assist Noah with his preparation to improve his test scores. Give a reason.

Hint Give examples from your findings in your reasons.

4 The following back-to-back stem-and-leaf plot displays the homework results over the term of two students.

- **a** For each student, identify:
  - **i** the mean
  - **ii** the mode
  - **iii** the median
- **b** Compare the performance of the two students using the measures of central tendency.

Hint Include examples of central tendency in your comparison.

Homework scores (%)

Jamie		Scarlett
8 5	6	7
6 4	7	5 5
5 5 2	8	8 9 9
8 6 5	9	5 5 5 9

7 | 5 = 75%

5 Mitchell is a town planner and he needed to determine the number of children in families from two high schools. He chose 300 students from the two local high schools and asked how many siblings each student had. Determine the median number of siblings for all the students surveyed.

Hint Think of a way you could do this without writing out each number.

Number of siblings	Town High School	Rangeville High School
0	118	142
1	82	108
2	59	31
3	31	9
4	10	10

6 Ben and his father were trying to work out a fair way to determine how much per month Ben should have for pocket money. Ben surveyed 13 of his friends (one being the daughter of an extremely wealthy family) and wrote out a list of how much pocket money they received each month.

Friend	Monthly pocket money $
1	140
2	50
3	40
4	70
5	2350
6	140
7	120
8	50
9	100
10	50
11	75
12	110
13	50

Ben's father says he should go by the median, Ben's younger sister says he should go by the mode, Ben says the mean is the fairest solution.

a Calculate the mean.

b Identify the median.

c Determine the mode.

d Why do you think each person suggested their particular measure of central tendency? Give a reason to which is the fairest in this situation.

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★7 Kenzi, a maths teacher, needs to choose who to give the maths award to in her class. She records all the results for her top three students.

Student A	33	45	23	24	47	48	46	25	45
Student B	30	43	20	26	50	50	49	30	47
Student C	30	42	20	23	48	50	48	32	43

Determine which student should win the maths award. Justify your answer, using the mean median and modes for the students and explaining which of these is the fairest for her class.

6C Determining quartiles, deciles and percentiles COMPLEX

LEARNING GOALS

- Calculate quartiles from a dataset
- Interpret quartiles from a graph
- Interpret deciles from a graph
- Interpret percentiles from a graph

Why is understanding quartiles, deciles and percentiles essential?

The mean and median both describe the 'centre' of a distribution; however, there are other parts of the distribution that could be of more interest. To identify these, data can be divided into smaller equal parts, such as quartiles, deciles and percentiles. These can be used to break down the data and describe the information contained in the data more clearly.

When data is put into specific parts, such as quartiles, deciles and percentiles, the information begins to make more sense and you can see the overall 'picture' more clearly.

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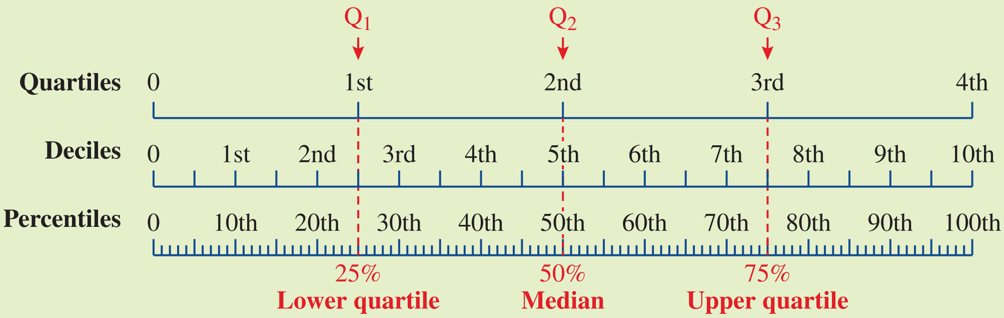
WHAT YOU NEED TO KNOW

- Parts or groups in an ordered dataset are divided by values that collectively are known as **quantiles**. ('Ordered' means the dataset is sorted in order from the smallest value to the largest value.) Quantiles are not groups; they are the boundaries between the groups.
  - Quantiles that divide ordered data into four equal parts are called **quartiles** (like 'quarters'). There are three quartiles, Q_1, Q_2, Q_3, shown on the diagram on the next page.
  - Quantiles that divide ordered data into 10 equal parts are called **deciles** (think of 'decimal', which relates to tenths). There are nine deciles, as shown on the diagram on the next page. (They can be called D_1, D_2, D_3 etc.)
  - Quantiles that divide ordered data into 100 equal parts are called **percentiles** (think of 'percentage', which relate to hundredths). There are 99 percentiles, as shown on the diagram below. (They can be called P_1, P_2, P_3 etc.)
- Quartiles, deciles and percentiles can be compared and converted with this diagram. The first quartile lies between the 2nd and 3rd decile, and it is the same as the 25th percentile. The first decile is the same as the 10th percentile, and so on.



- There are the same number of data points between the quartiles. So, from 0 to Q_1 there are the same number of data points as there are between Q_1 and Q_2, and so on. Remember that quantiles divide the number of data points (not their values) into equal groups. The same principles apply to the deciles and percentiles: there is an equal number of data points between each one.
- Q_2 is the **median** of the whole dataset, the data point that splits the ordered data into two equally sized groups. It is the same as the 5th decile and the 50th percentile. Then Q_1 is the median of the upper half of the data, and Q_3 is the median of the lower half of the data. Q_1 is also called the lower quartile, and Q_3 is also called the upper quartile. Q_1 is the same as the 25th percentile, and Q_3 is the same as the 75th percentile.

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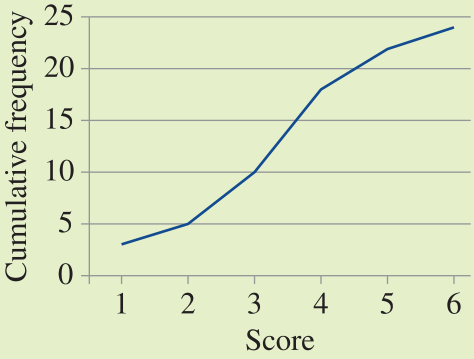
- Cumulative frequency graphs are often used with quartiles, deciles, and percentiles. **Cumulative frequency** is the running total of the **frequency distribution** of the dataset, which is shown in a **frequency table**. The frequency table for a dataset of scores of 1 to 6 is shown below left. (These scores could be any kind of data, such as throws of a single die in a board game). Next to the frequency table is a table showing how we could add up the running totals of the frequencies in a cumulative frequency column.

Score	Frequency
1	3
2	2
3	5
4	8
5	4
6	2
Total	24

Cumulative frequency	Cumulative frequency %
3	$= 3/24 \times 100 = 12.5\%$
$3 + 2 = 5$	20.8%
$5 + 5 = 10$	41.7%
$10 + 8 = 18$	75.0%
$18 + 4 = 22$	91.7%
$22 + 2 = 24$	100.0%
Total	100%

  - The cumulative frequencies could also be calculated as a **cumulative frequency percentage** of the final total (which is 24 in this case). This is shown in the fourth column.
  - The frequency table shows that the dataset contains 24 data points (the total of the frequencies, and the last value of the cumulative frequency). As this is an even number, the median is the average of the two middle values: the 12th and 13th.
- Graphs of cumulative frequency and cumulative frequency percentage:
  - The cumulative frequency graph is represented by the cumulative frequency on the vertical axis and scores on the horizontal axis (shown in graph on left).
  - If you plot the cumulative frequency percentage against the score, you get the same graph but now the vertical axis is in percentages (shown in graph on right).

Cumulative frequency graph



Cumulative frequency percentage graph

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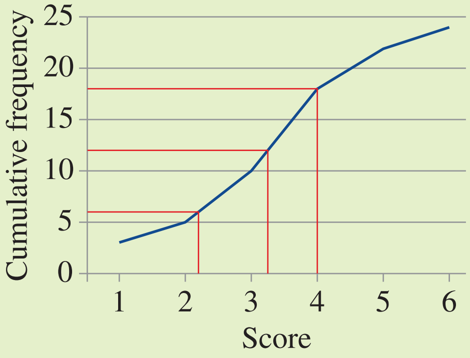
- A very useful characteristic of a cumulative frequency graph is that the quartiles can be easily marked on both axes, as shown below.



Cumulative frequency graph

For the cumulative frequency graph, the maximum value is 24. On the vertical axis, Q_1 is a quarter of the maximum value (6), Q_2 is half the maximum value (12) and Q_3 is three-quarters the maximum value (18).

Cumulative frequency percentage graph

It is even easier to mark Q_1, Q_2 and Q_3 on the cumulative frequency percentage graph. On the vertical axis, Q_1 is at 25% on the vertical axis, Q_2 is at 50% and Q_3 is at 75%.

  - Vertical lines can now be drawn from the intersection of each quartile with the graph down to the horizontal axis for the scores. This gives the Q_1, Q_2 and Q_3 of the scores, i.e. $Q_1 \approx 2$, $Q_2 \approx 3$, $Q_3 = 4$ on both graphs.
  - The same method can be applied to finding deciles and percentiles of the scores.
- Interpretation of quartiles, deciles and percentiles mainly involves determining where a particular value lies in relation to them, and the percentage of scores that are above or below a particular quartile, decile or percentile. For example:
  - 75% of scores are above Q_1 and 25% are below it.
  - 40% of scores are above the sixth decile and 60% are below it.
  - 70% of scores are above the thirtieth percentile and 30% are below it.
- Further interpretation depends on the subject of the data, and whether higher or lower scores are ‘better’. For example, if the subject is marks in a test, it is better to be in the 80th percentile than the 20th, but if the data is the amount of time it takes to download 1 gigabyte, it is better to be in the 20th percentile than the 80th.

Example 5 Calculating quartiles from a dataset

Consider this dataset.

9, 10, 7, 7, 8, 6, 12, 28, 6

a Identify the median (2nd quartile: Q_2).

b Determine the lower quartile (1st quartile: Q_1).

c Determine the upper quartile (3rd quartile: Q_3).

d Interpret what the quartiles mean in relation to the fraction and percentage of scores that lie above and below each one.

WORKING	THINKING
a 6, 6, 7, 7, 8, 9, 10, 12, 28	
The number of scores is 9.	The data must first be ordered from smallest to largest.
The middle value is the 5th, which is 8.	
The median (Q_2) is 8.	The median is the middle value.
b (6, 6, 7, 7,) 8, 9, 10, 12, 28 Average of 6 and 7 = $\frac{6 + 7}{2} = 6.5$	The lower quartile is the middle number of the lower half of the data. Place brackets around the lower half of the dataset, excluding the median, and identify its middle data point or points (ringed). There is an even number of scores, so the middle value is the average of the two middle scores (ringed).
The lower quartile Q_1 is 6.5.	This score is Q_1.
c 6, 6, 7, 7, 8, (9, 10, 12, 28) Average of 10 and 12 = $\frac{10 + 12}{2} = 11$	The upper quartile is the middle number of the upper half. Place brackets around the upper half of the dataset, excluding the median, and identify its middle data point or points (ringed). There is an even number of scores, so the middle value is the average of the two middle scores (ringed).
The upper quartile Q_3 is 11.	This score is Q_3.

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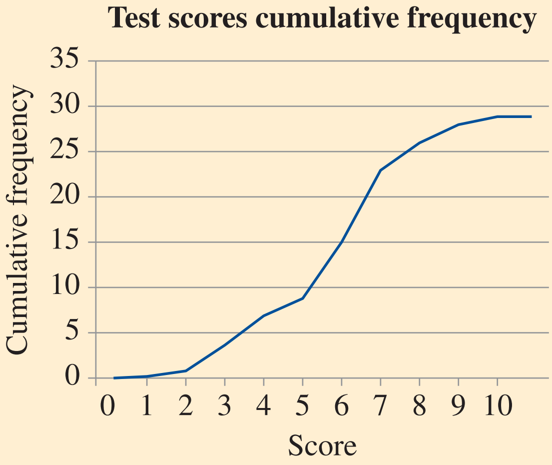
WORKING	THINKING
d A quarter or 25% of the data has a value that is less than or equal to 6.5, and three-quarters or 75% of the data has a value that is more than or equal to 6.5.	One quarter is 25%, a half is 50%, three-quarters is 75%. 25% of the data has a value that is less than or equal to Q_1, and 75% of the data has a value that is more than or equal to Q_1.
Half or 50% of the data has a value that is less than or equal to 8, and half or 50% of the data has a value that is more than or equal to 8.	50% of the data has a value that is less than or equal to Q_2, and 50% of the data has a value that is more than or equal to Q_2.
Three-quarters or 75% of the data has a value that is less than or equal to 11, and a quarter or 25% of the data has a value that is more than or equal to 11.	75% of the data has a value that is less than or equal to Q_3, and 25% of the data has a value that is more than or equal to Q_3.

Example 6 Determining the quartiles from a cumulative frequency graph

The graph below shows the cumulative frequency of test scores (out of ten) in a class's test results. Determine the lower quartile Q_1, median Q_2 and upper quartile Q_3 of the test scores.



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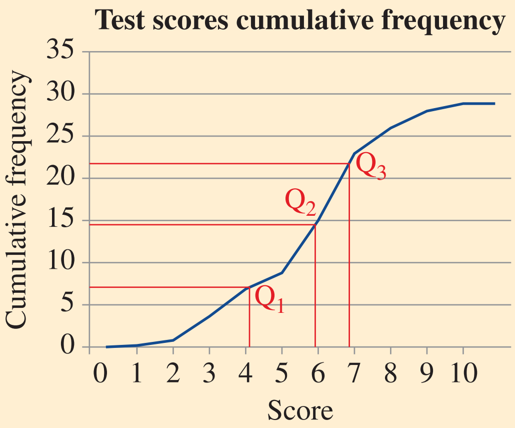
WORKING	THINKING
The cumulative frequency total is 29.	Write down the cumulative frequency total.
$Q_1 = 7.25$ $Q_2 = 14.5$ $Q_3 = 21.75$	Q_1 (one quarter of the total) Q_2 (half of the total) Q_3 (three quarters of the total)
	Draw horizontal lines at 7.25, 14.5, and 21.75 on the cumulative frequency axis and label them Q_1, Q_2 and Q_3. Draw vertical lines from where these horizontal lines meet the graph to the horizontal axis.
Q_1 score = 4 Q_2 score = 6 Q_3 score = 7	Read off the values of the vertical lines Q_1, Q_2 and Q_3 on the horizontal axis and round to the nearest whole number.
The lower quartile of the test scores is 4. The median of the test scores is 6 The upper quartile of the test scores is 7.	Communicate your answers in sentences.

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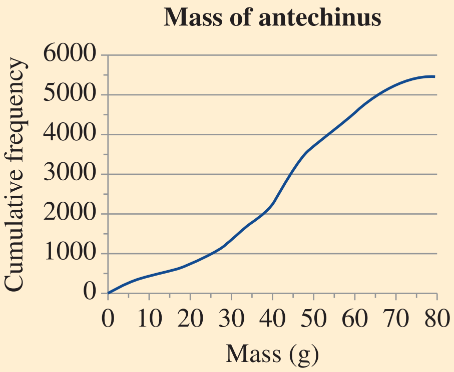
Example 7 Interpreting the deciles from a cumulative frequency

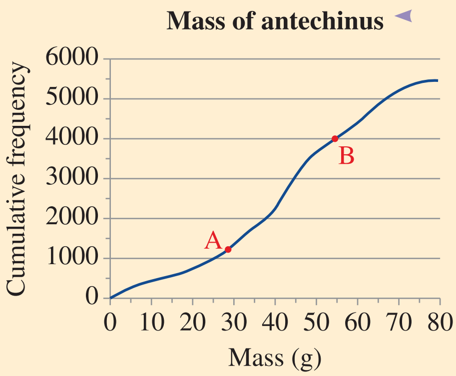
The cumulative frequency graph shows the results of a survey of the mass of marsupial mice (antechinus) in a population.

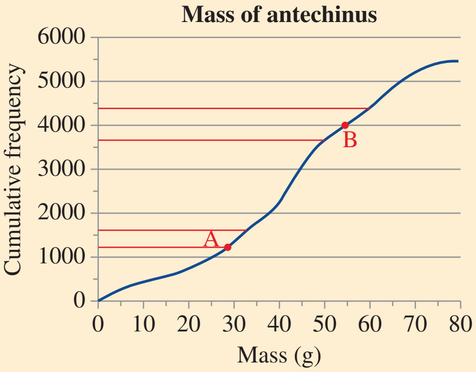
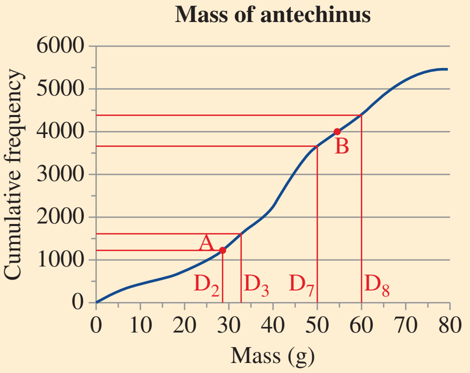
Two more antechinus specimens have been weighed, with mass of (A) 30 g and B (55 g).



a Determine the deciles of mass between which these two specimens lie.

b Interpret what this means in terms of the proportion of the population that are lighter or heavier than the two new specimens.

WORKING	THINKING
a 	Mark the position on the graph for A (30 g) and B (55 g).
Maximum cumulative frequency is 5300.	Read the maximum cumulative frequency on the graph.
Position of A on vertical axis is 1250. 1250 ÷ 5300 = 0.24	Determine the position of A on the vertical axis.
A lies between the 2–3 deciles.	Decide between what deciles A lies.
Position of B on the vertical axis is 4000. 4000 ÷ 5300 = 0.75	Determine the position of B on the vertical axis.
B lies between the 7–8 deciles.	Decide between what deciles B lies.

WORKING	**THINKING**
So we should mark D_2, D_3, D_7 and D_8. $D_2 = 2/10 \times 5300 = 1060$ $D_3 = 3/10 \times 5300 = 1590$ $D_7 = 7/10 \times 5300 = 3710$ $D_8 = 8/10 \times 5300 = 4240$	Calculate the values for D_2, D_3, D_7 and D_8 on the vertical axis.
	Draw the position of D_2, D_3, D_7 and D_8 as horizontal lines against the vertical axis.
	Where the horizontal lines meet the graph, draw vertical lines down to the horizontal axis, and label as D_2, D_3, D_7 and D_8.
Specimen A lies between the second and third deciles, and specimen B lies between the seventh and eighth deciles.	Communicate your solution in words.
b Specimen A is heavier than two-tenths of the population and is lighter than seven-tenths of the population. Specimen B is heavier than seven-tenths of the population and is lighter than two-tenths of the population.	Two-tenths of the data lies below the second decile, and seven-tenths lies above the third decile $(10 - 3 = 7)$. Data lower than a decile are lighter and data above a decile are heavier.

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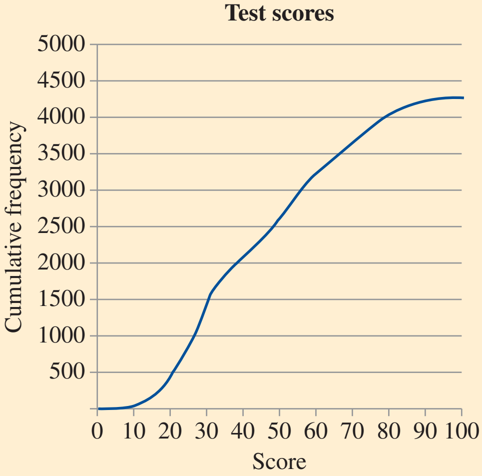
Example 8 Interpreting the percentiles from a cumulative frequency

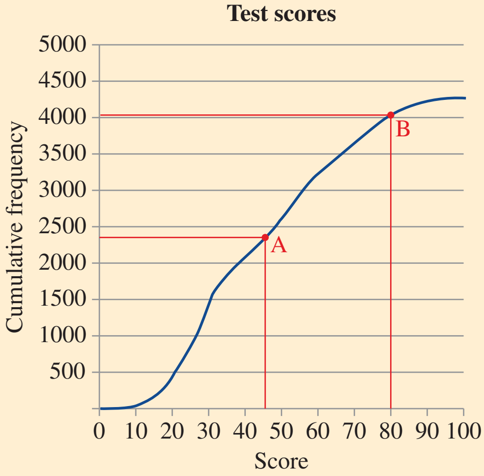
The cumulative frequency graph below shows the results of scores in a test. Student A had a score of 44 and Student B had a score of 80.

a Determine which percentiles of the scores between which these two scores lie.

b Interpret what this means in terms of the percentage of the students who did better or worse on the test than students A and B.



WORKING	THINKING
a	Mark the position on the graph for A at a score of 44. Draw a vertical line from A to the horizontal axis. Draw a horizontal line from A to the vertical axis. Mark the position on the graph for B at a score of 80. Draw a vertical line from B to the horizontal axis. Draw a horizontal line from B to the vertical axis.



WORKING	THINKING
Maximum cumulative frequency is 4250.	Read the maximum cumulative frequency on the graph.
A is about 2350 on the vertical axis. B is about 4000 on the vertical axis.	Read the position of A and B on the vertical axis.
$A = \frac{2350}{4250} \times 100\% = 55.3\%$	Calculate the vertical axis reading of A as a percentage of the maximum cumulative frequency.
$B = \frac{4000}{4250} \times 100\% = 94.1\%$	Calculate the vertical axis reading of B as a percentage of the maximum cumulative frequency.

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WORKING	THINKING
Student A's score of 44 lies between the 55th and 56th percentiles. Student B's score of 80 lies between the 94th and 95th percentiles.	Percentiles are equivalent to percentage points, so 55% is the 55th percentile. 55.3% lies between 55th and 56th percentiles, and 94.1% lies between the 94th and 95th percentiles.
b Student A scored better than 55% of students and worse than 44% of students. Student B scored better than 94% of students and worse than 5% of students.	55% of scores lie below the 55th percentile and 44% of scores lie above the 56th percentile (100 – 56 = 44). Scores lower than a percentile are worse and scores above a percentile are better.

Exercise 6C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Datasets can be divided into parts or groups by values that collectively are known as __________.
- **b** Cumulative frequency is the __________ __________ of the frequency distribution of the dataset.
- **c** Quartiles are __________ values that split the data into __________ equally sized groups.
- **d** A quartile is a __________, not a group of numbers.
- **e** Deciles of a distribution are the __________ values that split the data into __________ equally sized groups.
- **f** Percentiles of a distribution are the __________ values that split the dataset into a __________ equal parts.

2 Copy and complete this table to show cumulative frequency, cumulative frequency percentage and the totals.

Score	Frequency	Cumulative frequency	Cumulative frequency %
1	1		
2	6		
3	25		
4	19		
5	31		
6	12		
Total			

3 Draw a graph of cumulative frequency against score for the data in question **2**.

4 On the graph in question **3**, draw horizontal and vertical lines to show Q_1, Q_2 and Q_3, and determine the approximate value of the score for each of them.

APPLICATIONS

SF: – CF: 5–8 CU: 9–12

Example 5

5 The dataset below is the number of matches won by teams in a league.

12, 10, 2, 4, 6, 7, 6, 9, 9, 8, 5

a Identify the median (2nd Quartile → Q_2).

b Determine the lower quartile (1st Quartile → Q_1).

c Determine the upper quartile (3rd Quartile → Q_3).

d Interpret what the quartiles mean in relation to the fraction of matches won that lie above and below each one.

Hint Determine the median, and then the medians for the upper and lower halves of the data.

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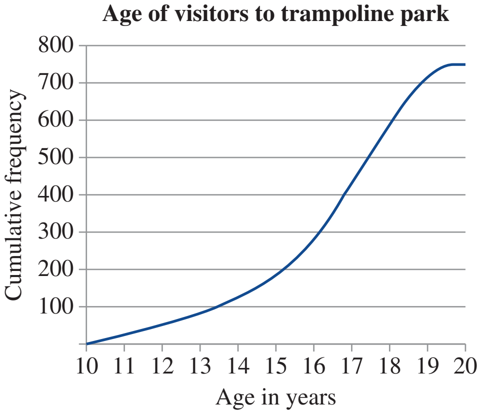
Example 6

6 Caroline is a trampoline park operator. She has recorded the ages of the visitors to the park to plan new equipment for the following year. Caroline has made a cumulative frequency graph of the results.



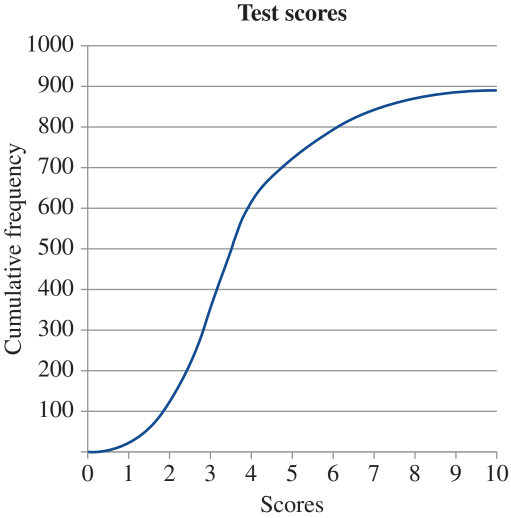
a Determine the median of the ages (2nd Quartile → Q_2).

b Determine the lower quartile of the ages (1st Quartile → Q_1).

c Determine the upper quartile of the ages (3rd Quartile → Q_3).

Example 7

7 This cumulative frequency graph shows the scores of students sitting a science test.



Andrew scored $4\frac{1}{2}$ and Brad scored 9.

a Determine the deciles of the scores between which these students' scores lie.

b Interpret what this means in terms of the proportion of the students that did better or worse than Andrew and Brad.

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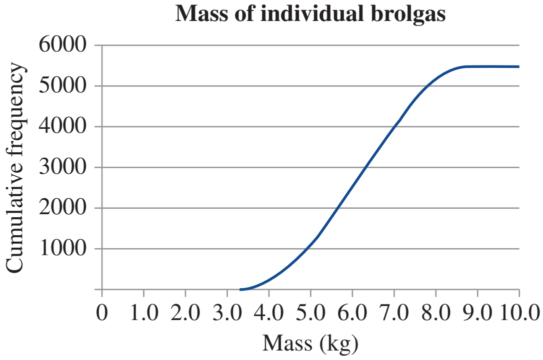
Example 8 **8** The mass of individual brolgas arriving at a site in North Queensland has been recorded and the results are shown in the cumulative frequency graph. Two new brolgas visit, brolga A weighs 4.2 kg and brolga B weighs 7.5 kg.



a Determine the percentiles of the masses between which the masses of A and B lie.

b Interpret what this means in terms of the percentage of the brolgas that are heavier or lighter than A and B.

★**9** Using the quartile information from the trampoline park graph in question **6**.

a 75% of the visitors are younger than Virat. Determine his age.

b Zara is younger than three-quarters of the visitors to the trampoline park. Determine how old she is.

c What percentage of the visitors to the park are between the ages of Zara and Virat?

★**10** When receiving results for an exam, is it better to receive results with a high or low percentile? Explain your answer.

★**11** Troy is currently in his doctor's waiting rooms. He has been there for 32 minutes, which is the 85th percentile of waiting times. Is this good or bad? Explain your answer.

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★**12** Katrina and Elliot are looking at purchasing a house. Their real estate agent has told them that the most expensive house they can afford is in the 25th percentile. Their research has shown that the 25th percentile of houses in the area that they're looking at is currently \$350 000. Does that mean they can afford 25% of the houses in their area or 75% of the houses?

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6D Describing the spread of data COMPLEX

LEARNING GOAL

- Use everyday language to describe the spread of the data, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions and outliers.

Why is essential to be able to use everyday language to describe spread in data?

One very useful way to describe the graphs of datasets is talk about the **spread** of the data. The spread describes the relationship with measures of central tendency, and explains how the data is distributed. We can use the language of spread to describe how well the mean, for example, represents the data. Being able to describe the spread of data also allows us to compare and contrast sets of related data to each other.

Datasets are easier to understand when you are able to identify and describe the spread.

WHAT YOU NEED TO KNOW

- Data needs to be displayed to show spread. A good kind of graph to show **spread** is a histogram or a column graph.
- Two distributions differ in spread if the values of the data in one distribution are more spread out or dispersed than the values of the data in the other distribution.

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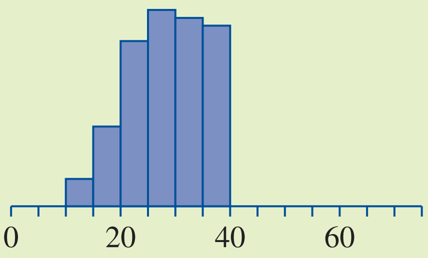
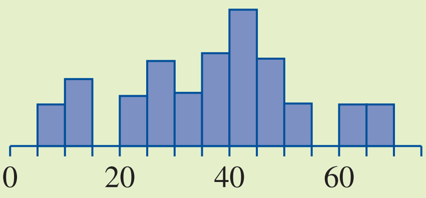
Histogram A

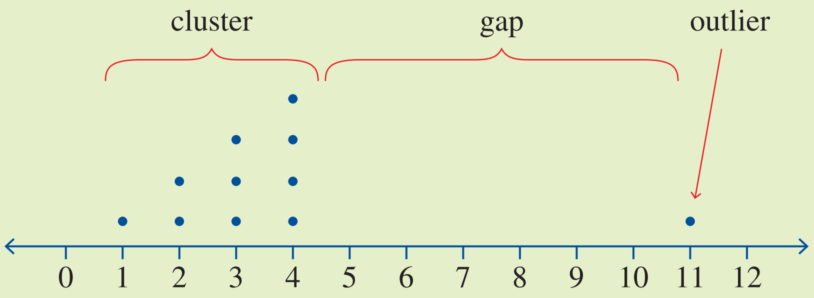


Histogram B



Words describing a distribution that is not spread out	*Words describing a distribution that is spread out*
not spread out	spread out
tightly packed	loosely packed
clustered	dispersed
more dense	less dense
narrowly (or tightly) distributed	widely distributed
narrowly dispersed	widely dispersed

- The spread also describes how a dataset is distributed around the mean or median. The location of the median can be estimated, and when the data is displayed using histograms, the total area of the columns either side of the median will be equal. Looking at the histograms above, we would say ‘Histogram A shows data that is tightly packed around the median, whereas histogram B shows data that is loosely packed around the median’.
- A **cluster** is produced when several data points lie in a group.
- A **gap** is a section that contains no data.
- An outlier has a value that is much greater than or much less than other data in the set.

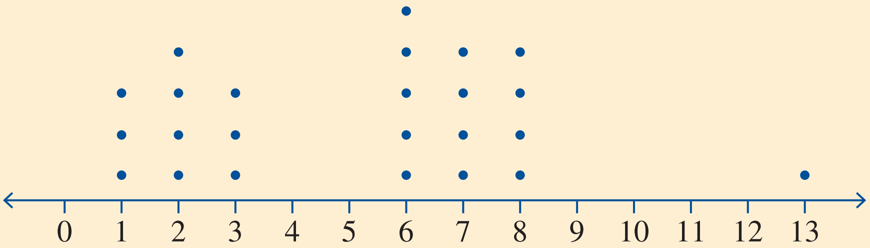


- A large dataset may have a distribution that is mixed, with some parts or regions being tightly packed or clustered and other parts being loosely packed.

Example 9 Describing the spread in a dataset

An organic store is conducting research into the shelf life of a variety of berries.

Shelf time for a variety of berries



Expand on each description below in as much detail as you can.

a The distribution has an outlier.

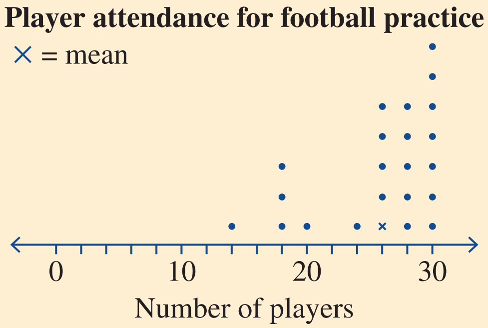
b The distribution has gaps.

c The distribution has clusters.

WORKING	THINKING
a The distribution has an outlier at day 13, which is much greater than the other observations.	Does the distribution have an outlier? Is there a data point situated away from the other data points?
b There are two gaps in the dot plot, between 3 and 6, as well as between 8 and 13.	Does the distribution have a gap? Is there a section that contains no data?
c The distribution has clusters between 1 to 3 days and 6 to 8 days.	Is there a cluster of points?

Example 10 Describing the spread from a dot plot

A coach is recording how many players are attending football practice.



Describe the distribution, making use of the terms below where possible.

- spread out
- tightly packed
- loosely packed
- dispersed
- more/less dense regions
- clusters
- gap
- outliers.

WORKING	THINKING
The data is tightly packed around the mean. The dot plot has a small dispersion. There are two clusters. One between 18 and 20, and the other between 24 and 30. The data has two gaps, one between 14 and 18, the other between 20 and 24. 14 could be considered an outlier.	Go through each term and see if they correspond to the distribution. Where possible include examples from the dataset.

Exercise 6D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

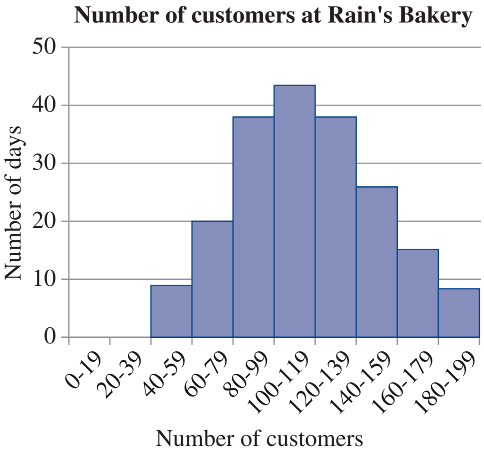
- **a** Data needs to be ________ to show the spread.
- **b** The spread is used to describe the ________ between datasets.
- **c** The best kinds of graphs to show spread is a ________ or a ________ graph.
- **d** The spread can be described by how it is distributed around the ________.
- **e** A ________ is produced when several data points lie in a group.
- **f** A ________ is a section that contains no data.

APPLICATIONS

SF: –	CF: 2-6	CU: –

Example 9

2 Rain has just opened her new bakery and has recorded the number of customers who purchased cakes from her shop.



Expand on each description below in as much detail as you can.

- **a** The distribution has an outlier.
- **b** The distribution has a gap.
- **c** The data is tightly packed around the mean.

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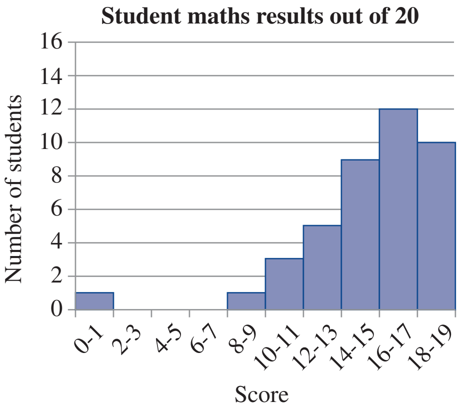
Example 10

3 A teacher is recording her students' test scores (out of 20). Describe the distribution, making use of the terms below where possible.

- spread out
- widely scattered
- dispersed
- tightly packed
- clusters
- gaps
- more/less dense regions
- outliers.



Hint A gap is a section that contains no data.

★**4** An apiarist was doing some research on his honey farm. He counted how many bees were seen in January and February and compiled the data in a stem-and-leaf plot. Describe and compare the spread for both months.

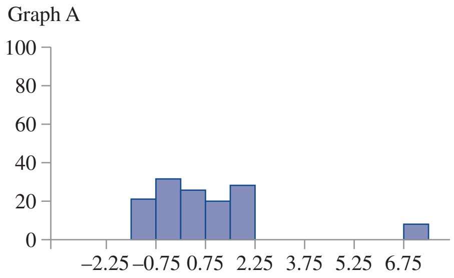
Bees seen in a day

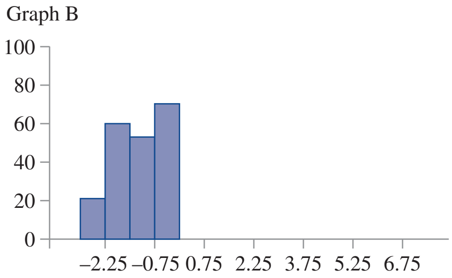
Jan		**Feb**
1100	10	01
	11	01
53321000	12	0134
988732000	13	1789
986542110	14	1233
	15	
	16	135556
9	17	022334

17 | 3 = 173

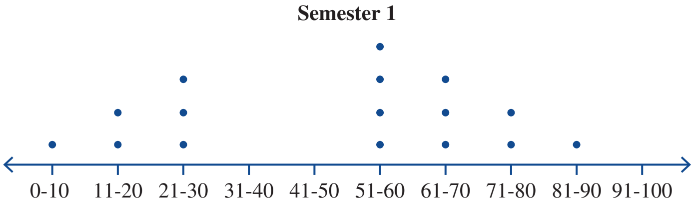
Hint A stem-and-leaf plot is like a column graph turned on its side. This is a back-to-back stem-and-leaf plot, so it's like a double column graph, with one dataset on the left and one on the right.

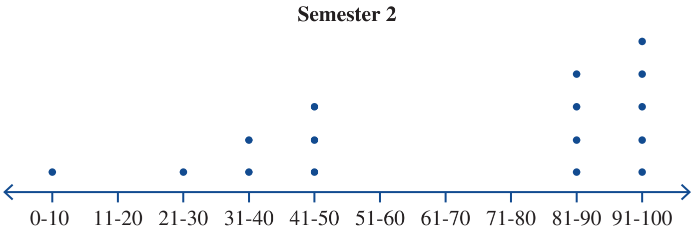
★**5** Two graphs were used in research. Describe and compare the spread for both graphs.





★**6** A teacher uses dot plots to display how her students are progressing after each semester. Describe and compare the spread for both semesters.





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6E Calculating and interpreting measures of spread and outliers COMPLEX

LEARNING GOALS

- Calculate and interpret statistical measures of spread using the range
- Calculate and interpret statistical measures of spread using interquartile range
- Calculate and interpret statistical measures of spread using standard deviation
- Investigate real-world examples from the media illustrating inappropriate uses of measures of spread
- Identify an outlier using a formula
- Investigate the effect of outliers on the mean
- Investigate the effect of outliers on the median

Why is it essential to understand how to calculate and interpret statistical measures of spread and outliers?

Being able to describe the spread using everyday language goes some way to understanding a data distribution, but the next step towards working mathematically is to measure the spread. When the spread of values in a dataset is large, then the mean is not as effective a representation of the data as when the spread of data is small. Understanding how to calculate and interpret the spread using range, interquartile range and standard deviation allows us to analyse data with a greater degree of precision.

The same goes for outliers. Intuitively, an outlier is a score or observation that lies beyond the ‘obvious edge’ of a dataset but in this section we will learn to determine outliers (and whether there are any in the first place) using a precise formula.

An outlier lies beyond the other observations.

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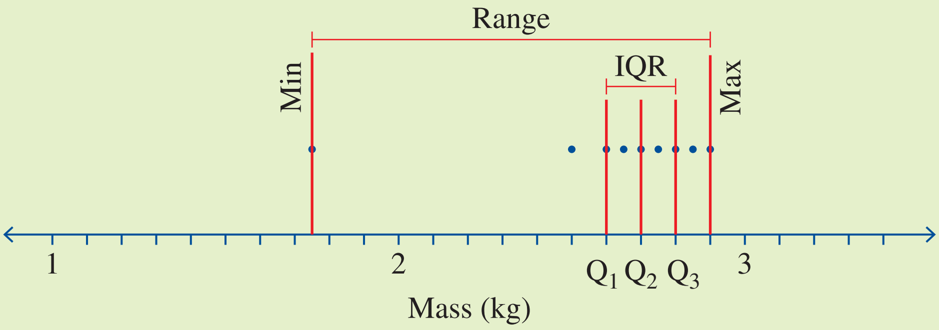
WHAT YOU NEED TO KNOW

- The range, interquartile range and standard deviation are useful measures of spread to compare datasets.
- The simplest measurement of spread is the **range**.
  - The range represents the limits of a dataset and only gives basic detail about the spread.
  - It explains how wide or narrow the spread is, especially when compared to another dataset.
  - The range is found by subtracting the minimum value in a dataset from the maximum value.
  - The range can be distorted by extreme values (outliers), which are values that are very large or very small compared to the rest of the dataset.
- The **interquartile range** (IQR) is the range of the central half of the data, either side of the median. It is calculated by subtracting the lower quartile (Q_1) from the upper quartile (Q_3): $\text{IQR} = Q_3 - Q_1$.
  - If the IQR is small compared to the range, the dataset likely contains outliers. In such cases the IQR is often a better measure of spread as it excludes outliers.
  - If the IQR is half or more of the value of the range, the data is likely to be evenly spread or tightly packed.
  - There may be good reasons for examining only the data within the IQR. For example, if you are studying how to improve the shelf life of pawpaws and customers do not want fruit that is too small or too large, you include only pawpaws from the IQR for size in the results of your study.

Pawpaw size



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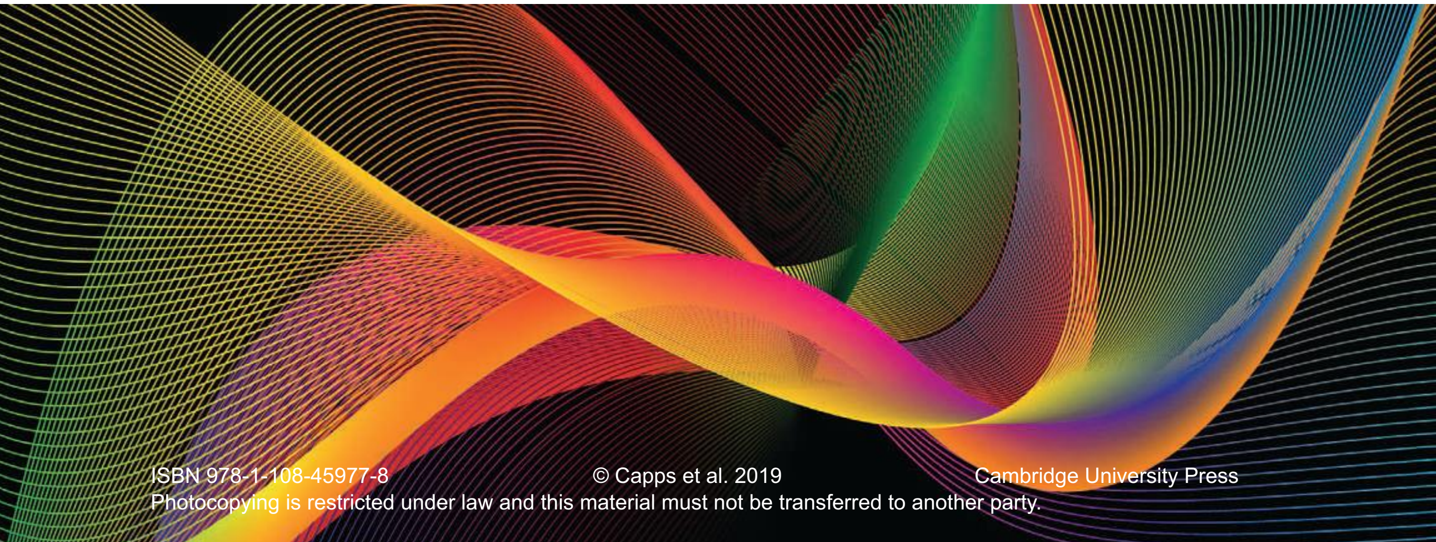
- The most important purpose of **standard deviation** from the mean is to understand how spread out a data set is. It is a measure of the average distance of data away from the mean.
  - A low standard deviation means that most of the numbers are tightly packed around the mean.
  - A high standard deviation means that the numbers are spread out from the mean.
- The **variability** of a dataset is the amount by which data points differ from the mean and from each other, similar to the spread, and which can also be measured by the range, IQR and standard deviation.
- As we have learned previously, an outlier is an observation that appears to be inconsistent with the remainder of that set of data.
- Outliers can be determined mathematically using the following formula:

A data point is an outlier if it is either:
less than $(Q_1 - 1.5 \times \text{IQR})$ or
greater than $(Q_3 + 1.5 \times \text{IQR})$.

  - The mean can be affected by outliers; therefore, it should not be used when outliers are present.
  - An outlier may be due to an inconsistency in the measurement or it may indicate experimental error. They are therefore sometimes removed from the dataset, but this needs to be justified.
  - The mean is usually affected more by the outlier than the median, especially for small datasets.
- Misuse of measures of spread may arise in the following cases.
  - Standard deviation may be misleading when there are extremes in the data (extremely high or low values) as the average is skewed.
  - The range may be misleading to measure spread when there are outliers or extreme values.



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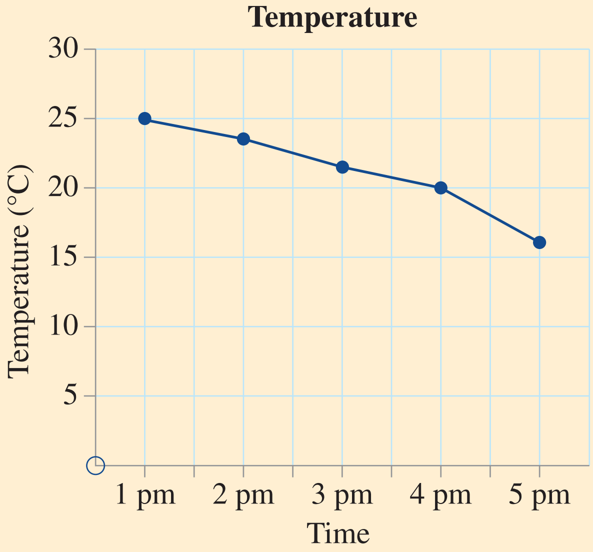
Example 11 Calculating and interpreting statistical measures of spread using the range

Kari has measured some of today's temperature.



a Identify the maximum and minimum temperatures recorded in the dataset.
b Calculate the range.
c Interpret the range of the dataset.

WORKING	THINKING
a The maximum temperature is 25°C. The minimum temperature is 16°C.	Identify the maximum (the highest) and minimum (the lowest) temperatures.
b 25° – 16° = 9° The range of the data is 9°C.	Range = Highest value – Lowest value
c The range of the data represents the limits of the dataset. Given the data was collected for four hours during the afternoon, it may not give an accurate measure of spread of the day's temperature range. However it might be useful if compared with measurements taken during the same part of the day across several days, weeks or months.	Refer to the definition of range.

Example 12 Calculating and interpreting statistical measures of spread using interquartile range

The test results of ten students are 5, 7, 10, 5, 6, 7, 9, 4, 6, 9.

a Arrange the data in order from the smallest value to the largest value.
b Identify the median (Q_2), lower quartile (Q_1) and upper quartile (Q_3).
c Calculate the interquartile range.
d Interpret the interquartile range.

WORKING	THINKING
a 4 5 5 6 6 7 7 9 9 10	Put values in order from lowest to highest.
b 4 5 5 6 6 7 7 9 9 10	To determine the median of a distribution:
$Q_2 = \frac{6+7}{2}$	• The number of scores is 10, so $n = 10$.
$Q_2 = \frac{13}{2}$	• The median score will be $\frac{10+1}{2} = \frac{11}{2} = 5.5\text{th}$ score.
$Q_2 = 6.5$	• The median will lie between the 5th and 6th scores.
(4 5 5 6 6) 7 7 9 9 10	The lower quartile is the middle number of the lower half. Place brackets around the lower half of the dataset.
$Q_1 = 5$	Identify the median of the lower half.
4 5 5 6 6 (7 7 9 9 10)	The upper quartile is the middle number of the upper half. Place brackets around the upper half of the dataset.
$Q_3 = 9$	Identify the median of the upper half.
c $\text{IQR} = Q_3 - Q_1$ $\text{IQR} = 9 - 5$ $\text{IQR} = 4$	The interquartile range is the difference between Q_3 and Q_1.
d The IQR is the range of the central half of the data. the value of 4 compared to the range of 6 indicates that there are no outliers in the data and it is fairly evenly spread across the range.	Refer to the definition and usefulness of interquartile range.

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Example 13 Calculating and interpreting statistical measures of spread using standard deviation without using technology

Note: In the assessment for this course you will not be asked to calculate standard deviation without using technology. It is done here to help you learn what the standard deviation is. It is suggested that you work through part a in order to see what the technology does when it calculates standard deviation.

A dog breeder records the number of pups in each of their dogs' litters (2, 4, 4, 4, 5, 5, 7, 9).

a Calculate the standard deviation, without using technology.

b Interpret the standard deviation of the dataset.

WORKING	THINKING
a Step one: The mean for the eight litters is $\bar{x} = \frac{2+4+4+4+5+5+7+9}{8}$ $\bar{x} = 5$	*Step one*: Calculate the mean.
Step two: $2 - 5 = -3$ $5 - 5 = 0$ $4 - 5 = -1$ $5 - 5 = 0$ $4 - 5 = -1$ $7 - 5 = 2$ $4 - 5 = -1$ $9 - 5 = 4$	*Step two*: Find the difference of each number from the mean.

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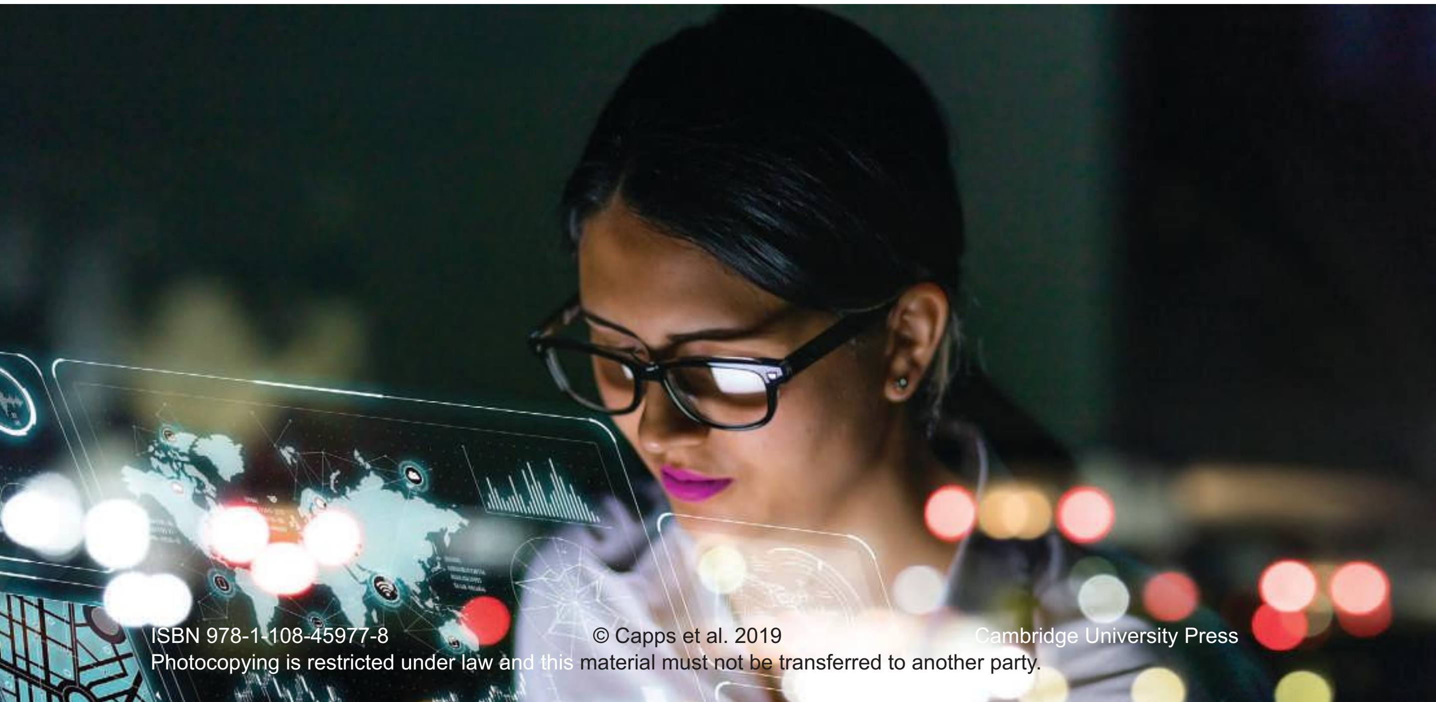
WORKING	THINKING
Step three: $(-3)^2 = 9 \quad 0^2 = 0$ $(-1)^2 = 1 \quad 0^2 = 0$ $(-1)^2 = 1 \quad 2^2 = 4$ $(-1)^2 = 1 \quad 4^2 = 16$	*Step three*: Square the difference of each number from the mean. This makes all of them positive, so they don't cancel each other out. It also magnifies larger differences and minimises smaller differences.
$\frac{9+1+1+1+0+0+4+16}{8} = 4$	*Step four*: Calculate the mean of the squared differences.
$\sqrt{4} = 2$ The standard deviation from the mean for the litter of puppies is 2.	*Step five*: Finally, calculate the square root of the answer. This counteracts the squaring from step three and allows the standard deviation to be expressed in the original units.
b The standard deviation is 2 from the mean of 5. this indicates neither a very narrow nor a very wide dispersion of the data. The standard deviation would be most useful for comparing to another dataset.	Refer to the definition of standard deviation.



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Example 14 Calculating and interpreting statistical measures of spread with technology

Sally recorded the heights of her friends in centimetres:
160, 171, 158, 167, 163
Calculate the mean, standard deviation, range and IQR using a calculator.
The instructions given below are for a Casio fx.

WORKING	THINKING
The mean is 163.8. Standard deviation for the heights of Sally's friends is 4.71 from the mean. Range = 171 – 158 = 13 IQR = 169 – 159 = 10	Clear any data already in the calculator. Press Mode [setup] > $\boxed{2}$ > $\boxed{1}$. A table appears. Type individual data values followed by the = key to enter the data, then click AC[off]. For these statistics press these keys: Mean, $\bar{x}$: Shift > $\boxed{1}$ > $\boxed{4}$ > $\boxed{2}$ > enter Standard deviation, SX: Shift > $\boxed{1}$ > $\boxed{4}$ > $\boxed{4}$ > enter Minimum value, minX: Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{1}$ > enter Maximum value, maxX: Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{2}$ > enter Median, med: Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{4}$ > enter Quartile 1, Q1: Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{3}$ > enter Quartile 3, Q3: Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{5}$ > enter For the range, subtract the minimum value from the maximum value. For the IQR, subtract Q3 from Q1.

Calculator activity 6E: Calculating statistical measures of spread with scientific calculators.

Spreadsheet activity 6E: Calculating statistical measures of spread using a spreadsheet:
These technology activities are in the Interactive Textbook.

Example 15 Identifying an outlier using a formula and demonstrating the effect on the mean and median

The following is a dataset representing the number of wedge-tail eagles spotted each day on a property.

5, 1, 2, 17, 3, 1, 4, 5, 3, 4, 3

$(Q_1 = 2; Q_2 = 3; Q_3 = 5; \text{IQR} = 3)$

a Determine whether any numbers are outliers using the given formula.

A data point is an outlier if it is either:
less than $Q_1 - 1.5 \times \text{IQR}$ or
greater than $Q_3 + 1.5 \times \text{IQR}$

b Give a possible reason for the outlier.

c Calculate the mean:
  i with the outlier
  ii without the outlier

d Compare your answers to **c i** and **ii** and consider if the outlier should be included or removed.

e Calculate the median:
  i with the outlier
  ii without the outlier

f Compare your answers to **e i** and **ii** and consider if the outlier should be included or removed.

WORKING	THINKING
a $Q_1 - 1.5 \times \text{IQR} = 2 - 1.5 \times 3$ $= 2 - 4.5$ $= -2.5$ $Q_3 + 1.5 \times \text{IQR} = 5 + 1.5 \times 3$ $= 5 + 4.5$ $= 9.5$ 17 is an outlier.	A data point is an outlier if it is either: • less than $Q_1 - 1.5 \times \text{IQR}$ or • greater than $Q_3 + 1.5 \times \text{IQR}$ There is no number in the dataset less than –2.5, but 17 is greater than 9.5.
b There may have been an abundance of prey on the property or a large dead animal.	*What possible reason could there be for this larger amount of wedge-tail eagles?*

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WORKING	THINKING
c **i** with the outlier $\frac{5+1+2+17+3+1+4+5+3+4+3}{11}$ The mean, with the outlier $= 4.46$	Calculate the mean including 17.
ii without the outlier $\frac{5+1+2+3+1+4+5+3+4+3}{10}$ The mean, without the outlier $= 3.1$	Calculate the mean, removing 17 from the dataset.
d $4.46 - 3.1 = 1.36$ With the difference between the two means being larger than some of the actual observations, it is reasonable to determine that the outlier greatly affects the data and should be removed.	Compare the answers with/without the outlier and determine whether the data is affected.
e **i** with the outlier 1, 1, 2, 3, 3, 3, 4, 4, 5, 5, 17 The median, with the outlier = 3	Calculate the median including 17.
ii without the outlier 1, 1, 2, 3, 3, 3, 4, 4, 5, 5 The median, without the outlier = 3	Calculate the median, removing 17 from the dataset.
f Both datasets have a median of 3. Therefore, the outlier does not affect the median and should be included in the dataset.	Compare the answers with/without the outlier and determine whether the data is affected.

PDF **Worksheet 6D** Investigating a real-world example from the media illustrating inappropriate uses of measures of spread: see the interactive textbook for this activity

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Exercise 6E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** The simplest measurement of spread is the ________.
- **b** The range is the ________ between the ________ value and the ________ value.
- **c** The ________ ________ (IQR) is calculated using the central section of data, either side of the median.
- **d** Standard deviation is a ________ of the ________ distance of data away from the mean.
- **e** An outlier is a score that lies ________ the obvious edge of a dataset.
- **f** When a small set of data has an outlier, the ________ is usually affected more by the outlier than the ________.

2 For the dataset shown, calculate the range, IQR and standard deviation.
2, 11, 5, 15, 17, 12, 7, 2, 11, 3

3 Complete the following for the following dataset.
15, 17, 14, 22, 0, 25, 13, 19, 16, 20, 15, 14, 17, 11, 20

- **a** Calculate Q_1, Q_3 and the IQR.
- **b** Calculate the value of the formula $Q_1 - 1.5 \times \text{IQR}$ and determine if any data value in the dataset is less than this.
- **c** Calculate the value of the formula $Q_3 + 1.5 \times \text{IQR}$ and determine if any data value in the dataset is greater than this.
- **d** State whether there are any outliers in the data.

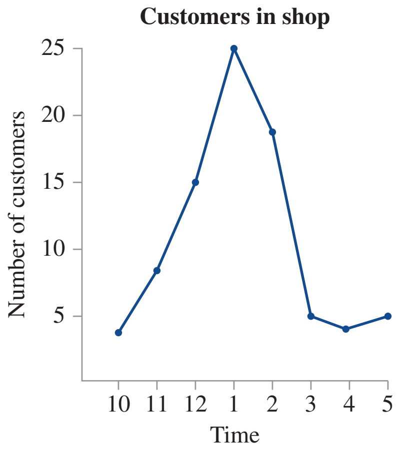
APPLICATIONS

SF: –	CF: 4–12	CU: –

Example 11

4 Ken is deciding whether he needs to hire an assistant and is recording the number of customers in his shop on an hourly basis.

- **a** Identify the maximum and minimum values.
- **b** Calculate the range of the distribution.



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Example 12 **5** A maths teacher has recorded some of her students' results.

Results	%
Amy	85
Fari	40
Grace	37
Sarah	80
Blake	75
Craig	100
Skye	20

a Arrange the data in order from lowest to highest.

b Identify the median (Q_2), lower quartile (Q_1) and upper quartile (Q_3).

c Calculate the interquartile range of the spread.

Example 13 **6** Rose was trying to explain to her friends that they need to eat more vegetables. She recorded how many different types of vegetables were in each of her friends' refrigerators.

2 3 5 6 9

Without the use of technology:

a calculate the mean

b calculate the standard deviation

Example 14 **7** Carrie and her friends are currently trying to lose weight. They have recorded their current weights (kg).

90 120 72 78 85

Using a calculator:

a calculate the mean

b calculate the standard deviation

c identify the minimum, Q_1, median, Q_3 and the maximum values

d calculate the range

e calculate the IQR

8 Sheree has recorded the heights (cm) of some children in her class.
150 140 130 102 105 163 110 152 145 143 147 139 140
Using a calculator:
- **a** calculate the mean
- **b** calculate the standard deviation
- **c** identify the minimum, Q_1, median, Q_3 and the maximum values
- **d** calculate the range
- **e** calculate the IQR

9 Rodney records the weekly weather temperatures for the local newspaper. Calculate the mean and standard deviation using a spreadsheet.
25°C 27°C 32°C 31°C 29°C 32°C 30°C 29°C

10 A couple are recording home loan rates in preparation to buying a home. Calculate the mean and standard deviation using a spreadsheet.
2.5% 3% 4.5% 3.5% 2.7% 3.4% 2.7% 4% 3.7% 2.8%

Example 15

11 Cooper has been practising shooting baskets every day in preparation for his basketball tournament. He recorded the number of shots he made each day from 20 shots.
13, 20, 17, 2, 17, 20, 15
- **a** Find the mean of the basketball shots.
- **b** Identify any outliers using the formula.
- **c** Explain whether the outliers affect the mean.
- **d** Determine whether outliers need to be removed.

12 Faith is researching the purchase of a new car. Her top seven car prices are:
\$45 000, \$34 000, \$40 000, \$120 000, \$52 000, \$27 000, \$30 000
- **a** Find the median of the car prices.
- **b** Identify any outliers using the formula.
- **c** Explain whether the outliers affect the median.
- **d** Determine whether outliers need to be removed.

Problem-solving and modelling task

Background: Being able to organise and analyse data to present statistics and describe the data is a highly regarded skill in the workplace and for many interests that people have outside of work.

Task: Your task is to use statistics to report on a topic or area that is of interest to you. You will need to gather one or two datasets (we suggest they should contain 20 to 50 values) relevant to your interest. You will need to:

1. organise the data into either a stem-and-leaf plot or a graph of your choice
2. determine the mean, median and mode
3. determine the range, interquartile range and standard deviation
4. describe the centre and spread.

With this information, create a report including your calculations to communicate and perhaps promote your interest, demonstrating that statistics promote expertise. Some ideas for data that you could collect:

- sports or digital game results or scores that you or your team have achieved
- results or scores of a sportsperson, team or gamer that you support
- data from things that you or your family create or collect as a hobby, pastime or business
- data collected from reliable internet sources to do with places, people, animals or plants that interest you.

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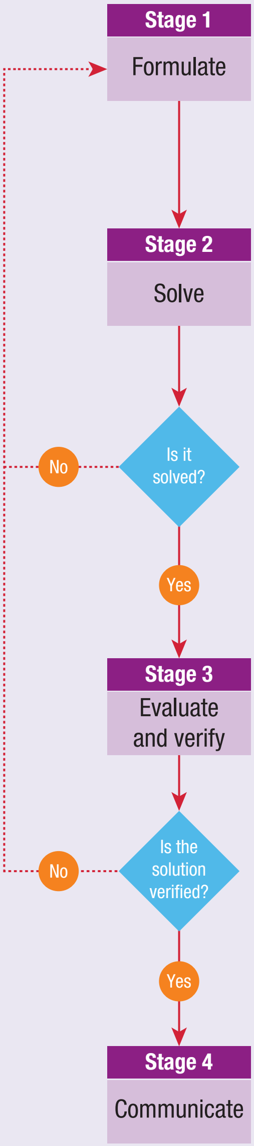
Approach to problem-solving and modelling tasks:



Stage 1: Formulate

1 What is of particular interest?
2 What data do you need?
3 Do you already have some data?
4 How can you find the data?

Stage 2: Solve

5 Collect the required amount of data.
6 Produce the required plots and/or graphs.
7 Complete the calculations and descriptions required.

Stage 3: Evaluate and verify

Once the data has been collected and compiled, consider:

8 Are they reasonable?
9 Have you presented them in the most suitable way?
10 Are all statistics measured correctly?

Stage 4: Communicate

11 Communicate your findings in a short report that justifies your findings. Include:
Introduction (What is your particular interest?)
Body (include evidence in the form of statements, graphs and statistics)
Conclusion (How has statistics enabled you to be more of an expert?)

Chapter checklist

☐ I can identify the mode, mean and median from a dataset.

1 Identify the mode from this dataset.
5 4 8 7 3 6 7 5 8 7 5 9 4 9 7 7 8 9 10 2

☐ I can calculate the mean and median from a dataset.

2 Calculate the mean and median from the dataset in question **1**.

☐ I can investigate the suitability of measures of central tendency in various real-world contexts, and their inappropriate use in the media.

3 Determine the best measure of central tendency for this house price data.
$145 000, $360 000, $1 700 000, $650 000, $170 000, $300 000, $390 000

4 **a** List the measures of central tendency that are not much affected by extreme values.

b Explain which measure of central tendency is best to use when you want the value of all the data points in the dataset to be included in its calculation.

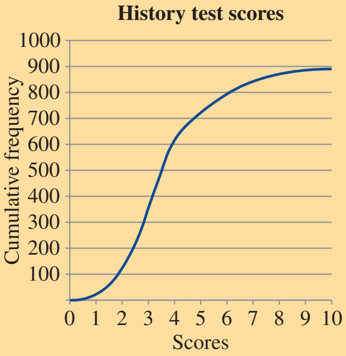
5 List the features of datasets that could make the mean, median and mode misleading as measures of spread.

☐ I can calculate quartiles from a dataset.

6 Calculate the 1st, 2nd and 3rd quartiles for this dataset:
5 4 8 7 3 6 7 5 8 7 5 9 4 9 7 7 8 9 10 2

☐ I can interpret quartiles from a graph.

7 From this graph of history test scores, determine the lowest score needed to be in the top quarter of the history class.



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☐ I can interpret deciles from a graph.

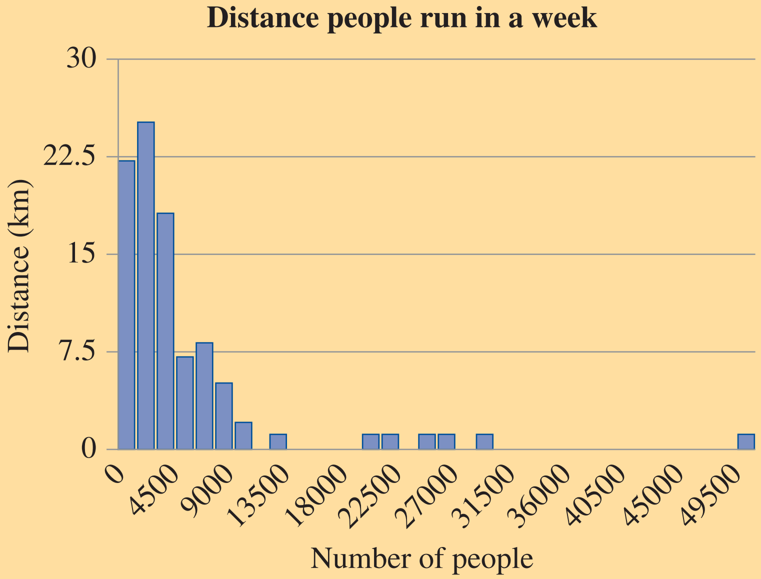
8 On the history test score graph from question **7**, my score is better than 3 tenths of the class. Determine the value of my score.

☐ I can interpret percentiles from a graph.

9 On the history test score graph in question **7**, determine the percentile of a score of 8, and explain what percentage of the class did better than this score.

☐ I can describe the spread of data including the terms spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions and outliers.

10 Use as many of the terms listed to describe spread of data to describe the spread of the data in this graph.



☐ I can calculate statistical measures of spread using the range, interquartile range and standard deviation. **[complex]**

11 Calculate the range, interquartile range and standard deviation of the data.
6 7 5 24 25 27 3 0 0 1 45

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☐ I can interpret statistical measures of spread using the range, interquartile range and standard deviation.

12 Explain what the range, interquartile range and standard deviation of the dataset in question **11** tells us about the spread of the data.

☐ I can identify outliers and investigate their effect on the mean and the median.

13 The following is a dataset recording the number of crows found in a field each day.
50 49 40 50 107 45 37 35 20 10 6 18
Identify any outliers and then calculate the mean and median with and without the outlier.

☐ I can investigate real-world examples from the media illustrating inappropriate uses of measures of spread.

14 Give examples of features of datasets that could make the standard deviation and range misleading as measures of spread.

Chapter review

All questions in the review are assessment style.

Simple familiar

Section 6A

1 A netball coach was preparing for a new season. The following dataset is her team's scores for the previous season.

56 76 45 78 67 48 65 45 65 60 65 59 79

Use the dataset to:

a identify the mode

b calculate the mean

c calculate the median

2 Explain which measure, or measures, of central tendency are the best to use in the following situations.

a You don't want the measure of central tendency to be distorted by extreme values.

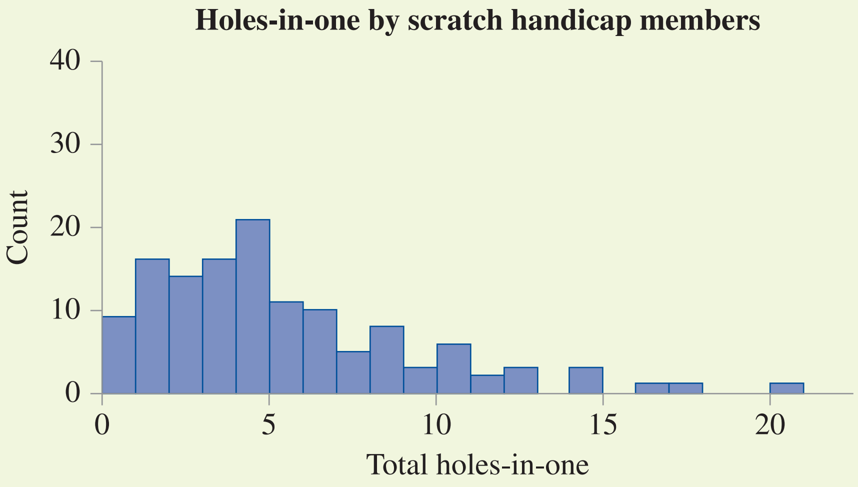
b You want to include the value of all the points in the measure.

c You want a measure that has an equal number of data points above it as below it.

d You want a measure that tells you which value occurs most often in the dataset.

Section 6D

3 A golf club recorded how many holes-in-one its scratch handicap members achieved during their memberships at the club.



Use everyday language to describe the measure of spread.

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Complex familiar

Section 6A **4** Abel’s goal is to achieve a 50% grade on his mathematics tests. He has recorded his grades out of 20 for the past 8 weeks. He has two weeks to go.

5, 8, 9, 3, 2, 4, 12, 15

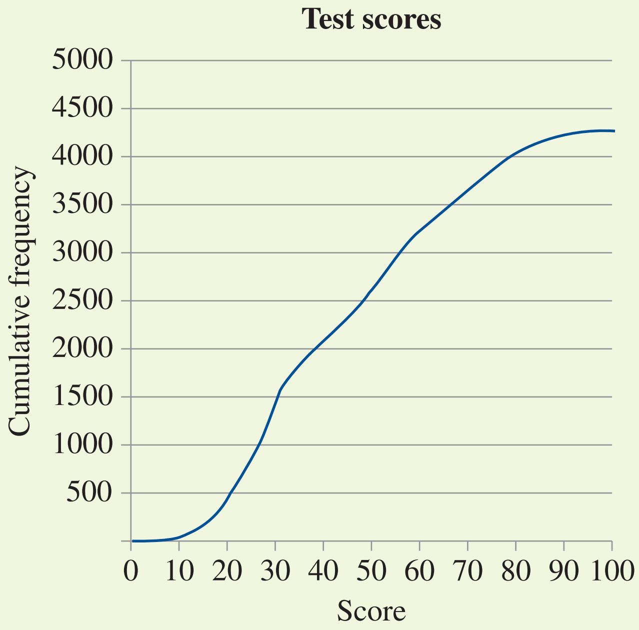
a Calculate the mean grade.

b Determine the median grade

c Determine the mode grade.

d Which is a better measure to assist with his preparation to reach his goal? Give a reason.

Section 6C **5** The test scores for a class are displayed in the cumulative frequency graph.



a Calculate the median of the test scores (2nd Quartile → Q_2).

b Determine the lower quartile of the test scores (1st Quartile → Q_1).

c Determine the upper quartile of the test scores (3rd Quartile → Q_3).

d Determine below which test score will a quarter of the test scores lie.

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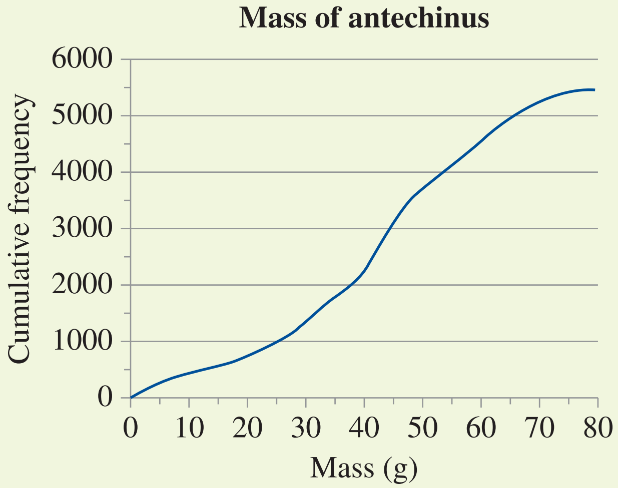
6 The cumulative frequency graph below shows the results of a survey of the mass of marsupial mice (antechinus) in a population.



a Estimate the 90th percentile of the mass.

b An individual antechinus weighs 25 g. What percentage of the populations are heavier than the antechinus?

Section 6E

7 Tom is recording the following rainfall (mm) for the past number of days. Determine the range; interquartile range and standard deviation.

3 7 5 2 25 2 3 0 0 1 45

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8 Tom has recorded the following maximum temperature (in °C) for the past number of days. Using a calculator, determine the range, interquartile range, mean, five-number summary (the minimum value, Q_1, median, Q_3, the maximum value) and standard deviation.

18 18 16 22 22 25 23 27 20 25 25 27

9 Tom has recorded the following minimum temperatures (in °C) for the past number of days. Using technology determine the range, mean and standard deviation.

4 7 4 7 8 5 5 7 8 7 5 7

10 The following is a dataset recording the number of dolphins seen from a cruiseship each day.

35 49 36 50 107 43 37 34 20 10 6 7

a For the dataset, determine:

- **i** the median (Q_2)
- **ii** the upper quartile (Q_3)
- **iii** the lower quartile (Q_1)
- **iv** IQR

b Determine whether any numbers are outliers. Remember that:

A data point is an outlier if it is either:
less than $Q_1 - 1.5 \times \text{IQR}$ or
greater than $Q_3 + 1.5 \times \text{IQR}$

c Give a possible reason for the outlier.

d Determine whether outliers need to be removed.

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Complex unfamiliar

Section 6A **11** Mitchell works five days a week, Tuesday to Saturday, in sales. He receives a set wage as well as commission from his sales. Mitchell is currently saving to buy a house and has a set budget requiring his commission to be an average of $50 per working day. He has recorded his commission for the week so far.

Day	**Commission ($)**
Tuesday	80
Wednesday	20
Thursday	30
Friday	40
Saturday	

Calculate the minimum commission that Mitchell needs to earn on Saturday to meet his weekly budget.

Section 6C **12** Jackie works in administration and is concerned with her current salary. She surveys some colleagues working in administration in other companies and finds that her salary is in the 78th percentile. Should she be concerned? Explain your answer.

Section 6E **13** Consider the three datasets A, B, C

$A = \{8, 10, 13, 7, 12\}$

$B = \{10, 10, 10, 10, 10\}$

$C = \{1, 1, 10, 19, 20\}$

a Calculate the standard deviation of each dataset.

b Determine which dataset has the largest standard deviation.

c Is it possible to answer question **b** without completing question **a**?

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7 Comparing datasets

Maths for a store owner: Kristy Catania

Kristy Catania and her husband own a supermarket.

Tell us a bit about your job. What does a typical day look like?

I oversee the daily operations of the business, serve customers, and manage staff focusing on good customer relations. I also buy and order all stock; organise all of the advertising and local market research; as well as recruitment and rostering of our staff.

What maths did you study at school?

Actually, I did my schooling in Tasmania and only did maths to Year 10. I didn't think it would be useful in my job, so I didn't do any maths in Years 11 and 12. However, I now realise in my job that I need to use mathematics every day and it would have been very helpful to have studied more at school.

How do you use maths in your job?

Mostly with regards to sales analysis and profit margins. Maths is especially needed when ordering stock and calculating prices. My staff and I use maths when serving on the till, as we need to know if we have made a mistake when entering the prices. I need maths skills when I create the roster, as I have a wage budget and need to calculate which staff members I can employ staying under the planned budget. I also use maths in summarising and comparing data; as well as graphs, change and loans.

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In this chapter

Syllabus reference

Unit 3 Topic 3 Summarising and comparing data

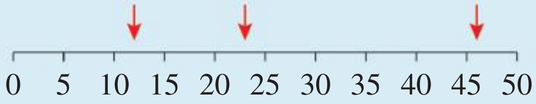
Comparing datasets (5 hours)

In this sub-topic, students will:

- complete a five-number summary for different datasets
- construct box plots using a five-number summary
- compare parallel box plots and back-to-back stem-and-leaf plots for different datasets **[complex]**
- compare the characteristics of the shape of histograms using symmetry, skewness and bimodality, where applicable **[complex]**

Pre-test

1 Identify the numbers marked by the arrows on this number line.



2 Given the following group of numbers, determine the range (largest number in the data minus smallest number) of the data and show this on the number line provided.
17, 34, 19, 41, 29

3 Organise the following numbers into a stem-and-leaf plot.
41, 52, 57, 58, 65, 66, 70, 75, 78, 78, 78, 80, 80, 95

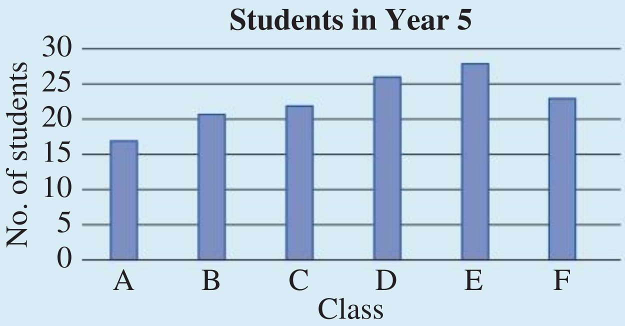
4 The following numbers represent the number of students in every year for a school.

Year	7	8	9	10	11	12
Students	212	225	210	223	195	165

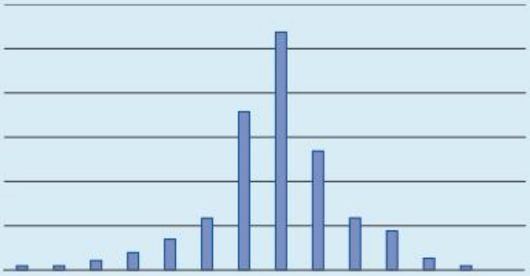
The most useful scale that should be used to plot the data is:

A increments of 1
B increments of 0.5
C increments of 10
D increments of 100

5 The following graph shows the number of Year 5 students in every class. Identify which class has the most students and which has the least.



6 Estimate the position of the median on this column graph.



A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

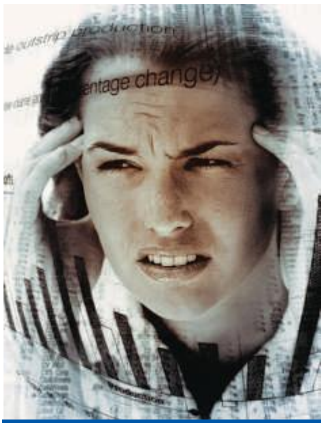
7A Completing a five-number summary

LEARNING GOALS

- Explore the structure of a five-number summary
- Create a five-number summary without using technology
- Create a five-number summary using technology

Why are five-number summaries essential?

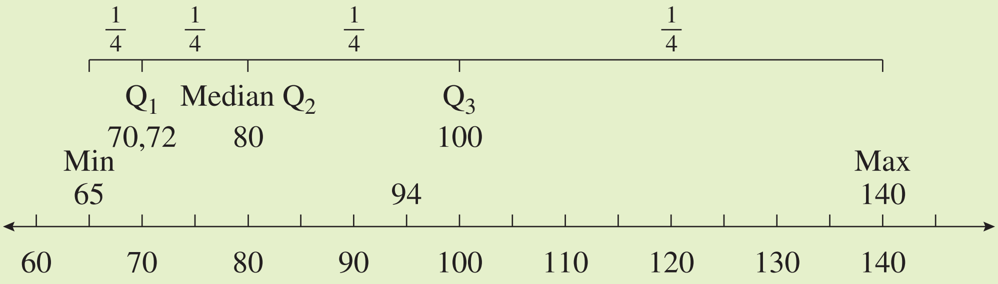
Sometimes it is overwhelming looking at so much data and too many numbers. The five-number summary contains the five most important numbers of the data. In our world, we are surrounded by sets of data for a wide range of things. It is essential that you are able to recognise the structure and create five-number summaries so that you can reduce the overwhelming sets of numbers to just five important ones.



Too many numbers are overwhelming. It is essential to be able to recognise the structure and create five-number summaries.

WHAT YOU NEED TO KNOW

- To find the **five-number summary**, the dataset must first be placed in ascending order.
- The median is the $\frac{n+1}{2}$th score when ranked in order from smallest to largest where n represents the number of data pieces.
- The quartiles of a ranked (ordered) set of data values are the three points that divide the dataset into four equal groups.



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- A five-number summary is made up of the:
  1. minimum score (the smallest number in the set of data)
  2. lower quartile, Q_1 (the median of the lower half of the data)
  3. median, Q_2 or M (the number that falls exactly in the middle)
  4. upper quartile, Q_3 (the median of the upper half of the data)
  5. maximum score (the largest number in the set of data).
- The interquartile range (IQR) is another useful measure of spread: $\text{IQR} = Q_3 - Q_1$
- When identifying the upper and lower halves of a dataset with an odd number of data points, the upper and lower halves do not include the median. With an even number of data points, they do include the median.

Example 1 Exploring the structure of a five-number summary

Sally runs two health classes and has asked for everyone's weight. The lists of weights are shown below.

Green class: 72 kg, 65 kg, 80 kg, 140 kg, 70 kg, 100 kg, 94 kg

Blue class: 65 kg, 80 kg, 100 kg, 105 kg, 96 kg, 79 kg, 90 kg, 110 kg

Sally requires an idea of the dataset distribution.

a Sort the data in order from smallest to largest for each class.

b Determine the minimum and maximum values for each class.

c Determine the median for each class.

d Determine the lower quartile, Q_1 for each class.

e Determine the upper quartile, Q_3 for each class.

f State the five-number summary for each class.

WORKING	THINKING
a 65, 70, 72, 80, 94, 100, 140 65, 79, 80, 90, 96, 100, 105, 110	Put the numbers in each set in order from smallest to largest.
b $\boxed{65}$, 70, 72, 80, 94, 100, $\boxed{140}$ Minimum = 65 Maximum = 140 $\boxed{65}$, 79, 80, 90, 96, 100, 105 $\boxed{110}$ Minimum = 65 Maximum = 110	Find the minimum and maximum (smallest number and largest number) data values.

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WORKING	THINKING
c $65, 70, 72, \boxed{80}, 94, 100, 140$ Median = 80	The median is the middle number. To determine the median of an odd distribution: • The number of scores is 7. So $n = 7$. • The median score will be $\frac{7+1}{2} = \frac{8}{2} = 4$th score.
$65, 79, 80, \boxed{90, 96}, 100, 105, 110$ Median $= (90 + 96) \div 2$ $= 186 \div 2$ Median $= 93$	To determine the median of an even distribution: • The number of scores is 8. So $n = 8$. • The median score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score. • The median will lie between the 4th and 5th scores.
d $(65, 70, 72,)\boxed{80}, 94, 100, 140$ $(65, \boxed{70}, 72,)\boxed{80}, 94, 100, 140$ $Q_1 = 70$	Put brackets around the numbers below the median (for an odd number do not include the median). This helps in finding Q_1. Find the median of the lower half.
$(65, 79, 80, 90,)\boxed{96}, 100, 105, 110$ $(65, 79, 80 , 90,)\boxed{96}, 100, 105, 110$ $Q_1 = (79 + 80) \div 2$ $Q_1 = 159 \div 2$ $Q_1 = 79.5$	Put brackets around the numbers below the median (for an even number include the median). This helps in finding Q_1. Find the median of the lower half.
e $65, 70, 72, \boxed{80}, (94, 100, 140)$ $65, 70, 72, \boxed{80}, (94, \boxed{100}, 140)$ $Q_3 = 100$	Put brackets around the numbers above the median (for an odd number do not include the median). This helps in finding Q_3. Find the median of the upper half.
$65, 79, 80, \boxed{90}, (96, 100, 105, 110)$ $65, 79, 80, \boxed{90}, (96, 100, 105, 110)$	Put brackets around the numbers above the median (for an even number include the median). This helps in finding Q_3.
$Q_3 = (100 + 105) \div 2$ $Q_3 = 205 \div 2$ $Q_3 = 102.5$	Find the median of the upper half.

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WORKING	THINKING
f The five-number summary for the Green class is: Minimum = 65 $Q_1 = 70$ Median = 80 $Q_3 = 100$ Maximum = 140 The five-number summary for the Blue class is: Minimum = 65 $Q_1 = 79.5$ Median = 93 $Q_3 = 102.5$ Maximum = 110	State the five-number summary.

Example 2 Creating a five-number summary from a stem-and-leaf plot

A charity has collected the following amounts of donations in the first hour of an event. Their collector placed the amounts in a stem-and-leaf plot.

First hour takings

0	5 5 5 6 8 9
1	0 0 5 5
2	0 5 5 7
3	0 5

a Decide if the data is in order.

b Determine the minimum and maximum scores.

c Determine the median.

d Determine lower quartile, Q_1.

e Determine the upper quartile, Q_3.

f State the five-number summary.

WORKING	THINKING
a Yes, the stem-and-leaf plot has the data in order.	Does this stem-and-leaf plot display the data in order?

b **First hour takings**

0	⑤ 5 5 6 8 9
1	0 0 5 5
2	0 5 5 7
3	0 ⑤

Minimum = 5
Maximum = 35

Find the minimum and maximum (first number and last number) data value.

c **First hour takings**

0	5 5 5 6 8 9
1	0 ⓪ ⑤ 5
2	0 5 5 7
3	0 5

The 8th score = 10
The 9th score = 15
Median = (10 + 15) ÷ 2
Median = 12.5

Find the median. The median is the middle number.

- There are 16 terms, so $n = 16$.
- The median score will be $\frac{16+1}{2} = \frac{17}{2} = 8.5$th score.
- The median will lie between the 8th and 9th scores.

d **First hour takings**

0	5 5 5 ⑥ ⑧ 9
1	0 0 5 5
2	0 5 5 7
3	0 5

The 4th score = 6
The 5th score = 8
$Q_1 = (6 + 8) \div 2$
$Q_1 = 7$

Find lower quartile, Q_1.

- There are 8 terms in the bottom half, so $n = 8$.
- The lower quartile score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score.
- The lower quartile will lie between the 4th and 5th scores.

Find the median of the lower half.

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WORKING	THINKING
e **First hour takings** 0 | 5 5 5 6 8 9 1 | 0 0 | 5 5 2 | 0 5 5 7 3 | 0 5	Find upper quartile, Q_3. - There are also 8 terms in the top half so $n = 8$ - The upper quartile score will be $\frac{8+1}{2} = \frac{9}{2} = 4.5$th score. - The upper quartile will lie between the 4th and 5th scores after the median.
The 4th score after the median = 25 The 5th score after the median = 25 $Q_3 = (25 + 25) \div 2$ $Q_3 = 25$	Find the median of the upper half.
f The five-number summary for the data is: Minimum = 5 $Q_1 = 7$ Median = 12.5 $Q_3 = 25$ Maximum = 35	State the five-number summary.

Example 3 Creating a five-number summary using a calculator

Tom recorded the height of his friends in centimetres.

160, 171, 158, 167, 163

Create the five-number summary, using a calculator.

Note: this example is based on a Casio fx-82. For examples using a TI-30XB and Sharp EL531TH see the interactive textbook (a link is below this example).

WORKING	THINKING
Reset All Press [AC] key	Reset your calculator to remove all past data records. To clear, press:
Clear? 1: Setup 2: Memory 3: All	SHIFT $\overset{\text{CLR}}{\boxed{9}}$ $\boxed{3}$ $\boxed{=}$
X 1 160 2 171 3 158	Press Mode [setup] > $\boxed{2}$ > $\boxed{1}$. A table appears. Type individual data values followed by the = key to enter the data. After entering the five data points click AC[off].
minX 158	Press Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{1}$ > enter for the minimum value.
Q1 159	Press shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{3}$ > enter for quartile 1.
med 163	Press shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{4}$ > enter for the median value.
Q3 169	Press shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{5}$ > enter for quartile 3.
maxX 171	Press Shift > $\boxed{1}$ > $\boxed{5}$ > $\boxed{2}$ > enter for the maximum value.
The five-number summary is: Minimum = 158 Q_1 = 159 Median = 163 Q_3 = 169 Maximum = 171	State the five-number summary.

Calculator activity 7A for TI and Sharp calculators: see the interactive textbook for this activity on using a TI-30XB and Sharp EL531TH to create a five-number summary.

Spreadsheet activity 7A: see the interactive textbook for this activity on using a spreadsheet to create a five-number summary

Exercise 7A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** To find the five-number summary, the dataset must first be placed in order from ____________ to ____________.
- **b** The smallest number in the set of data is also called the ____________.
- **c** The lower and upper quartiles are represented by the symbols ____________ and ____________.
- **d** The ____________ can also be represented by the symbol Q_2. It is the number that falls exactly in the ____________, when all the numbers have been placed from smallest to largest.
- **e** The largest number in the set of data is also called the ____________.

2 Write each dataset in order from smallest to largest.

- **a** 2, 1, 0, 5, 2, 2, 0, 7, 4, 2, 9, 1, 0, 2, 3, 3
- **b** 8.9, 8.7, 9, 7.7, 8.6, 9.6, 8.7, 8.5, 7.9, 9.2
- **c** 45, 65, 46, 43, 42, 48, 46, 42, 49, 41, 47, 45
- **d** \$45.90, \$34.70, \$35.80, \$36.50, \$36.00, \$36.30

3 Determine the minimum and maximum scores in each dataset.

- **a** 7, 5, 9, 4, 6, 8, 5, 4, 7, 8, 6, 9, 6, 4, 7, 3, 7, 8, 5
- **b** 8.9, 8.0, 8.7, 8.6, 8.6, 8.8, 8.6, 8.9, 8.5, 8.2
- **c** 23, 26, 26, 21, 26, 27, 23, 21, 15, 28, 26, 24, 21
- **d** 0.87, 0.76, 0.56, 0.88, 0.76, 0.65, 0.55, 0.76, 0.5

4 Determine $\frac{n+1}{2}$ when n is:

a 9 **b** 7 **c** 6

d 30 **e** 27 **f** 120

Hint *n* is the number. Then add 1 and divide the answer by 2.

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5 Write out the data represented by this stem-and-leaf plot in order from smallest to largest.

Stem	Leaf
0	6 7 4 7
1	3 6 2 6 3 7
2	9 7
3	
4	8 6 5
5	4 0
6	7
7	2
8	4 6 3

8|6 = 86

APPLICATIONS

SF: 6–17 **CF: –** **CU: –**

Example 1

6 A group of students was asked to record the number of pets each student owned. The data below shows the results.

0, 7, 4, 2, 0, 1, 0, 2, 3, 3, 0, 2, 1, 0, 3, 2, 2

a Sort the data into order.

b Determine the minimum and maximum values.

c Determine the median.

d Determine the lower quartile, Q_1.

e Determine the upper quartile, Q_3.

f State the five-number summary.

Hint Minimum (smallest number)
Q_1 (median of lower half)
Q_2 (middle value)
Q_3 (median of upper half)
Maximum (largest number)

7 Tony collects data on wedge-tail eagles in Highfields. Below are the number of eagles he spotted over the past 12 days.

6, 4, 3, 5, 6, 2, 7, 6, 5, 9, 5, 4

a Sort the data into order.

b Determine the minimum and maximum values.

c Determine the median.

d Determine the lower quartile, Q_1.

e Determine the upper quartile, Q_3.

f State the five-number summary.

8 The following number of bikes were recorded over 20 days on a street where the residents were asking for a bike way.

12, 9, 15, 19, 12, 21, 8, 12, 11, 10, 29, 12, 17, 28, 10, 15, 16, 34, 12, 18

a Sort the data into order.
b Determine the minimum and maximum values.
c Determine the median.
d Determine the lower quartile, Q_1.
e Determine the upper quartile, Q_3.
f State the five-number summary.

Example 2

9 An apple farmer recorded the number of apples picked, per hour, by a new group of pickers.

New pickers

9	8
10	1 3 5 7 9
11	2 4 6 8 9
12	7 7 7 8
13	4 5 5 7 8 9
14	5 5 6 7 8 9
15	0 0 0 5 5 5 7

11|4 = 114

a Sort the data into order.
b Determine the minimum and maximum values.
c Determine the median.
d Determine the lower quartile, Q_1.
e Determine the upper quartile, Q_3.
f State the five-number summary.

Example 3

10 The following datasets are the measurements of the leg lengths of animals in centimetres. State the five-number summary for each of the following datasets, using technology.

a 24, 67, 54, 87, 56, 32, 76, 45, 31, 53
b 78, 65, 98, 68, 98, 65, 105, 45, 32, 48, 27, 41
c 84, 99, 48, 34, 93, 27, 12, 36, 73, 112, 117, 38, 96
d 4.7, 8.98, 9.56, 4.83, 2.76, 3.95, 2.98, 2.74, 1.05, 4, 3, 6.5, 6.74, 2.84

11 A doctor is researching diabetes and she has recorded the following blood pressure numbers from a group of patients. Create a five-number summary, without the use of technology, to assist with her research.

128, 122, 113, 108, 115, 115, 107, 130, 115, 107, 120, 106

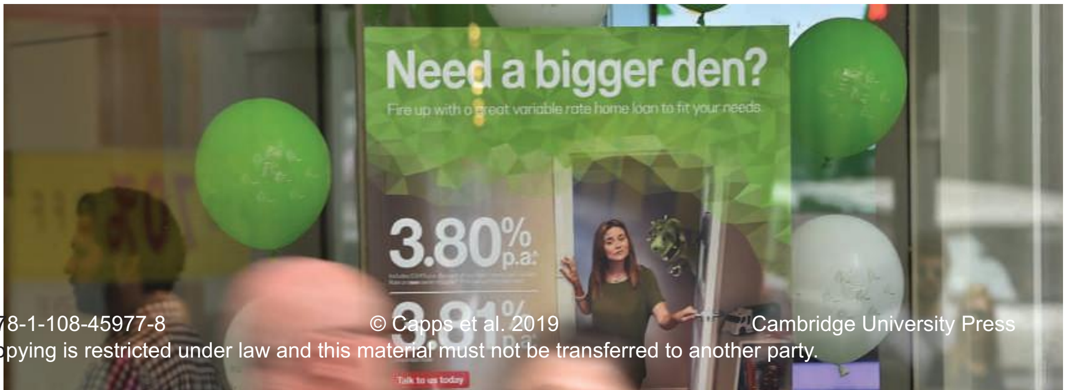
12 A student was wanting to purchase a smartphone. The following dataset is the amounts recorded from a selection of options. Create a five-number summary, without the use of technology, to assist with the student's decision.
\$200, \$1200, \$400, \$300, \$900, \$700, \$900, \$1000, \$700, \$850

13 A soccer coach was preparing for a new season. The following dataset is his team's scores for the previous season. Create a five-number summary, without the use of technology, to assist with his preparation.
1, 5, 4, 3, 5, 2, 1, 0, 2, 1, 0, 3, 2, 0, 4, 1, 1, 2, 1, 2, 3, 3, 2, 1, 0, 2, 1, 2, 4, 0

14 A couple were deciding on a bank for a loan. The following dataset is the various interest rates on offer by different banks.
3.5%, 6.4%, 4.6%, 3.7%, 3.7%, 5.8%, 7.3%, 5.3%, 7%, 5.4%, 4%, 3.2%, 6%, 6.5%, 6.4%, 3.6%, 7%, 8.4%, 6%, 7%, 4.5%, 5.6%, 7%, 5.4%

a Organise the data into a stem-and-leaf plot.

b Create a five-number summary, with the use of technology, to assist with their decision.



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15 The following datasets are the scores from quizzes in a mathematics class. Determine the dataset that the following five-number summary corresponds to.

Minimum = 15

Q_1 = 20

Median = 22

Q_3 = 27

Maximum = 32

A 15, 20, 21, 32, 27, 16, 30

B 28, 15, 21, 32, 26, 22, 19

C 22, 27, 20, 23, 24, 32, 15

D 32, 20, 15, 22, 20, 22, 27

16 The following datasets are the scores from quizzes in a science class. Determine the dataset that the following five-number summary corresponds to.

Minimum = 23

Q_1 = 31

Median = 36

Q_3 = 45

Maximum = 51

A 22, 21, 45, 47, 52, 45, 36, 23, 51, 40, 45, 36, 45, 30, 46, 46, 23

B 36, 37, 40, 42, 33, 33, 32, 45, 32, 23, 23, 51, 45, 24, 30, 46, 50

C 51, 50, 42, 34, 22, 27, 20, 23, 24, 32, 15, 50, 51, 51, 37, 45, 23

D 36, 23, 32, 36, 45, 32, 20, 15, 22, 20, 22, 27, 51, 50, 50, 51, 36

17 Will is recording the number of cars passing per 20 minutes.

20, 20, 19, 27, 45, 30, 34, 45, 36, 44, 28, 26, 45, 45, 40, 46, 25, 28

a Arrange the data into a stem-and-leaf plot.

b State the five-number summary, with the use of technology.

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7B Constructing box plots

LEARNING GOAL

- Construct box plots using a five-number summary

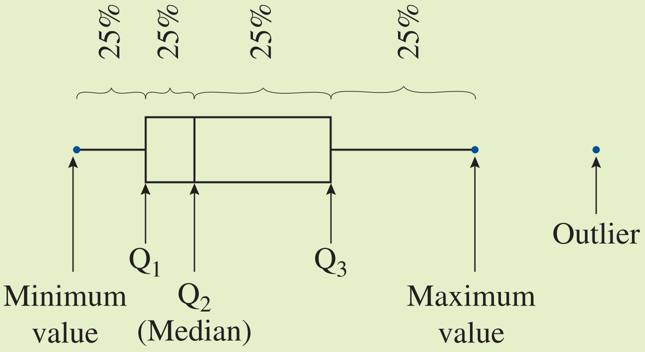
Why are box plots essential?

Understanding five-number summaries is just the beginning of coping with increasing data in our ever-growing world. Box plots are the visual representation of the five-number summary of a dataset. They are particularly useful for presenting the information in a large dataset in a simplified way. Being able to construct box plots is essential in understanding how data can be visually represented.

Understanding how to construct a box plot will lessen the feeling of being overwhelmed with large datasets.

WHAT YOU NEED TO KNOW

- **Box plots** are the visual representation of the five-number summary of a dataset.
- They are drawn against a scale.
- Box plots can be drawn vertically or horizontally.
- They are divided into four sections with a quarter (25%) of the data in each section.



Note: *Compare this diagram to the one of quartiles at the bottom of the first page in section 7A, and note the similarities.*

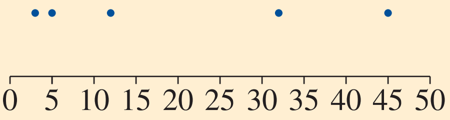
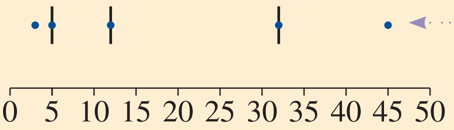
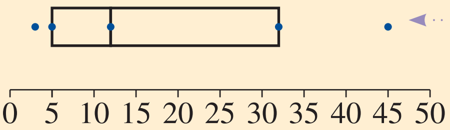
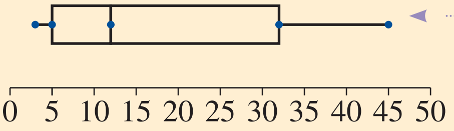
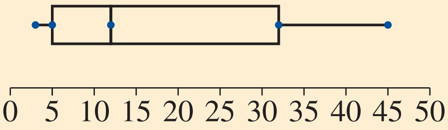
- Box plots are also known as box-and-whisker plots, with a box surrounding the Q_1, median and Q_3, and whiskers extending to the minimum and maximum scores.
- Outliers are also used in box plots. They are an observation that appears to be inconsistent with the remainder of that set of data; a surprising observation.

Example 4 Constructing box plots using a five-number summary

Greg records the number of runners from each team who complete a marathon.
3, 6, 8, 4, 12, 20, 45, 36, 28

a State the five-number summary for the dataset.

b Construct a box plot using the five-number summary, without the use of technology.

WORKING	THINKING
a $(\textcircled{3}, 4, \overset{5}{\uparrow} 6, 8)\ \boxed{12}\ (20, 28, \overset{32}{\uparrow} 36, \textcircled{45})$	Write the dataset in ascending order.
Minimum = 3 $Q_1 = 5$ Median = 12 $Q_3 = 32$ Maximum = 45	Find the minimum and maximum values. Find the median. Put the remaining numbers either side in brackets. Find the Q_1 and Q_3.
b $(\textcircled{3}, 4, \overset{5}{\uparrow} 6, 8)\ \boxed{12}\ (20, 28, \overset{32}{\uparrow} 36, \textcircled{45})$ 	Choose an appropriate scale to represent the data (e.g. go up only by 1s, 2s or 5s) Draw a number line representing the data. Place a dot above each of the five numbers.
	Extend the dots to small lines for the median, Q_1 and Q_3.
	Create the ‘box’.
	Draw lines (whiskers) out to the minimum and maximum values.
Number of members from each team 	Give your box plot a title.

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Exercise 7B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Box plots are the ______________ representation of the five-number summary of a ______________.
- **b** Box plots are drawn against a ______________.
- **c** Box plots are also known as box-and-______________ plots.
- **d** The upper 25% of the dataset is above ______________.
- **e** Outliers are an observation that appears to be ______________ with the remainder of that set of data; a ______________ observation.

2 Choose an appropriate scale for the following data.

- **a** 0, 5, 3, 1, 5, 4, 2, 4, 0, 1, 3, 3, 4, 7, 3, 4, 2
- **b** 14, 14, 21, 16, 15, 17, 14, 18, 14, 23, 19, 16, 16, 17, 18, 14
- **c** 29, 12, 17, 28, 10, 15, 16, 34, 12, 18, 34, 43, 32, 46, 34, 32, 39, 34, 32, 19
- **d** 0, 52, 4, 11, 0, 0, 7, 8, 0, 2, 18, 0, 0, 4, 0, 0, 5, 13, 2, 13, 1, 1, 14, 1, 12

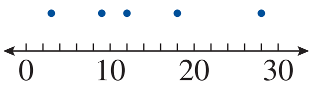
Hint Do increments of 2, 5, 10 suit the data best?

3 Draw a number line for each dataset. Represent each data value using a dot above the line.

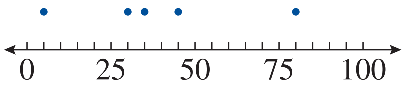
- **a** 5, 8, 7, 9, 6
- **b** 0.7, 0.4, 0.8, 0.2, 0.6
- **c** 95, 56, 32, 67, 75

4 Record the numbers represented on the following number lines.

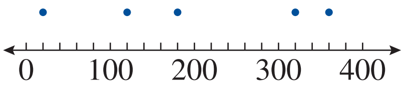
a



b



c



Hint To work out the value of the interval between the tick marks, divide the difference between two labelled values by the number of tick mark intervals between them.

APPLICATIONS

SF: 5–17	CF: –	CU: –

Example 4

5 A group of students were asked to record the number of hours of homework they completed each week. The data below shows the results.

0, 5, 3, 1, 5, 4, 2, 4, 0, 1, 3, 3, 4, 7, 3, 4, 2

- **a** Create a five-number summary.
- **b** Construct a box plot.

Hint Remember to put the data in order from smallest to largest.

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6 A takeaway store recorded the ages of their staff.

14, 14, 21, 16, 15, 17, 14, 18, 14, 23, 19, 16, 16, 17, 18, 14

a Create a five-number summary.

b Construct a box plot.

Hint Organise your scale and number line first.

7 The following number of cars were recorded on a residential street over 20 days where the residents were requesting a traffic calming speed hump.

29, 12, 17, 28, 10, 15, 16, 34, 12, 18, 34, 43, 32, 46, 34, 32, 39, 34, 32, 19

a Create a five-number summary.

b Construct a box plot.

8 A librarian is interested in the number of books people borrow from a library. She selected a sample of 25 people and recorded the number of books each person had borrowed in the previous year. Here are her results:

0, 52, 4, 11, 0, 0, 7, 8, 0, 2, 18, 0, 0, 4, 0, 0, 5, 13, 2, 13, 1, 1, 14, 1, 12

a Identify any possible outliers and write down their values.

b Construct a box plot of the data, showing outliers.

Hint Outliers are observations that appear to be inconsistent with the remainder of that set of data.

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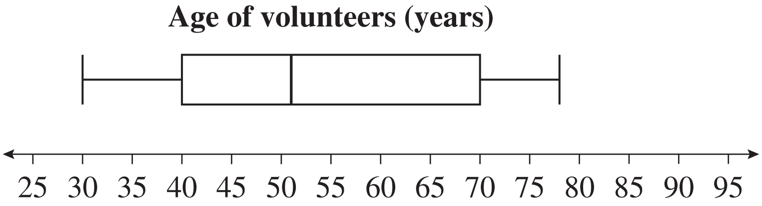
9 The time taken, in minutes, for a group of students to complete a maths problem is:
10, 8, 60, 6, 6, 14, 15, 6, 7, 6, 5, 7, 8, 6, 18, 9, 7, 10, 5, 8, 6, 14, 11, 5

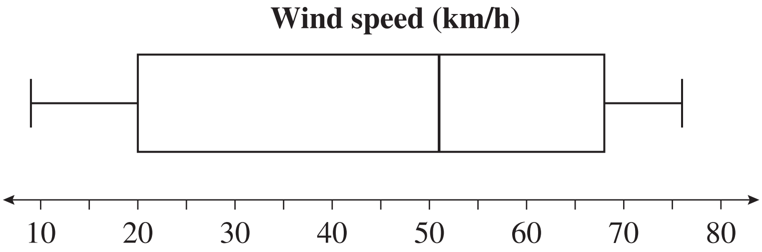
a Identify any possible outliers and write down their values.

b Construct a box plot of the data, showing outliers.

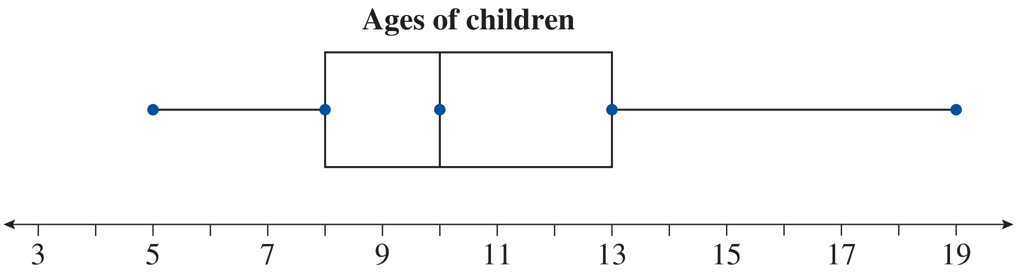
10 The box plot below shows the age of people volunteering for a particular charity. Create a five-number summary.



11 A group of drone enthusiasts meet at a park every day. To monitor wind speed, they have recorded the wind speed (in km/h) over a month. Create a five-number summary of their recorded data.

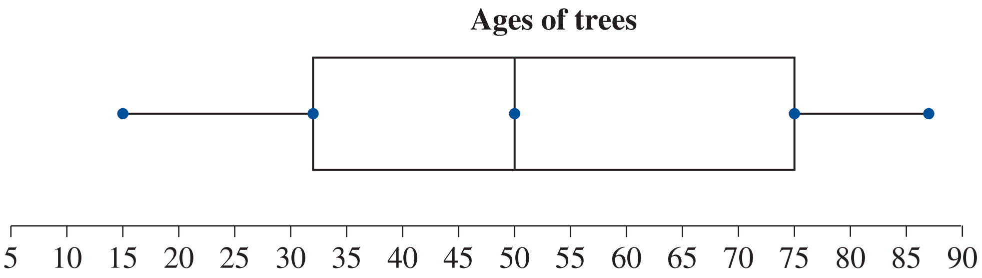


12 The following box plot represents the children's ages in a large family.


Ages of children

a What age are 75% of the children older than?
b Is it possible to determine how many children there are from this data?
c What percentage of the children are between 13 and 19?
d How old is the youngest child?
e The family are going to the agricultural show. Child tickets are for children up to 13 years old. What percentage of the children qualify for a child ticket?

13 A local arborist determined the age of 100 trees in his suburb.


Ages of trees

a What age are 75% of the trees older than?
b What is the range of the data?
c What percentage of the trees are between 15 and 32?
d How old is the oldest tree?

14 The stem-and-leaf plot represents a sample of 50 scores in a golf tournament.

Golf scores

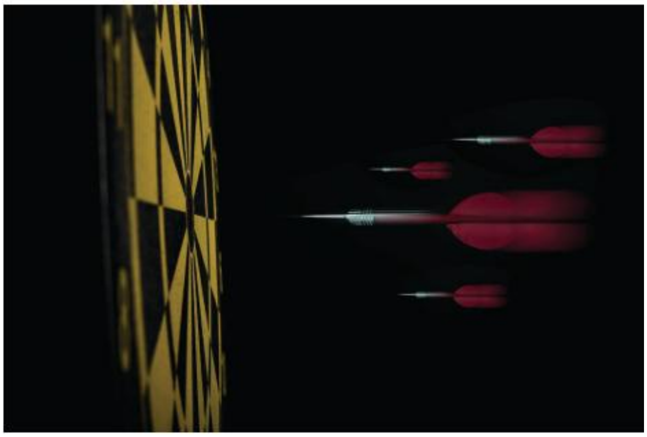
5	8 9 9 9
6	2 3 3 4 4 7 7 8 9 9 9 9
7	0 0 0 0 1 2 2 2 5 7 8 8 8 8
8	0 1 1 1 3 4 6 6 7 8 9 9 9
9	2 3 3 4 8 8 9

7|0 represents a score of 70

a Create a five-number summary of the dataset.
b Construct a box plot.

15 A group of men throw one dart each at a dart board and their scores are recorded.

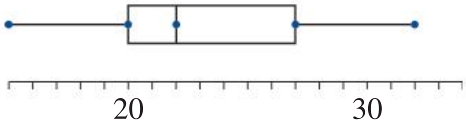
Name	Scores
Tony	8
James	22
Steve	28
Jon	14
Ted	30
Tom	13
Nick	20
Greg	24
David	18
Brian	15
Mitch	7



a Create a five-number summary.

b Construct a box plot.

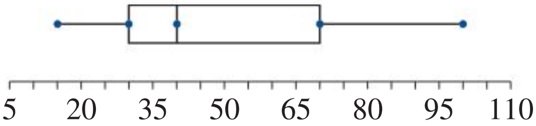
16 The following datasets are the scores from quizzes in a science class. Determine the dataset that the following box plot corresponds to.



A 22, 27, 20, 23, 24, 32, 15 **B** 15, 20, 21, 32, 27, 16, 30

C 32, 20, 15, 22, 20, 22, 27 **D** 15, 32, 26, 28, 20, 20, 22

17 The following datasets are the scores from quizzes in a history class. Determine the dataset that the following box plot corresponds to.



A 95, 63, 15, 76, 45, 57, 31 **B** 20, 43, 67, 32, 69, 110, 100

C 71, 45, 50, 100, 34, 17, 36 **D** 58, 53, 80, 100, 43, 41, 79

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7C Comparing datasets COMPLEX

LEARNING GOALS

- Compare parallel box plots for different datasets
- Compare back-to-back stem-and-leaf plots for different datasets

Why is comparing datasets essential?

Understanding data summaries and being able to visually represent information is essential to living in our current overwhelming world of numbers and statistics. Being able to compare data is also essential, as it provides the information to make decisions effectively. It's impossible to make sound judgements and decisions without comparing data.

Comparing data is an essential skill to help us make better decisions.

WHAT YOU NEED TO KNOW

- Parallel box plots can be used to compare two or more groups.
- When comparing box plots, they must always be placed against the same axis. This enables the median, spread and possible outliers of the data to be easily identified and compared.
- The range can be used as a measure of spread in a dataset, but it is extremely sensitive to the presence of outliers and should only be used with care.
- The IQR should be compared and contrasted to the range.
- The variability of a dataset is the amount by which data points differ from the mean and from each other, similar to the spread, and which can also be measured by the range, IQR and standard deviation.

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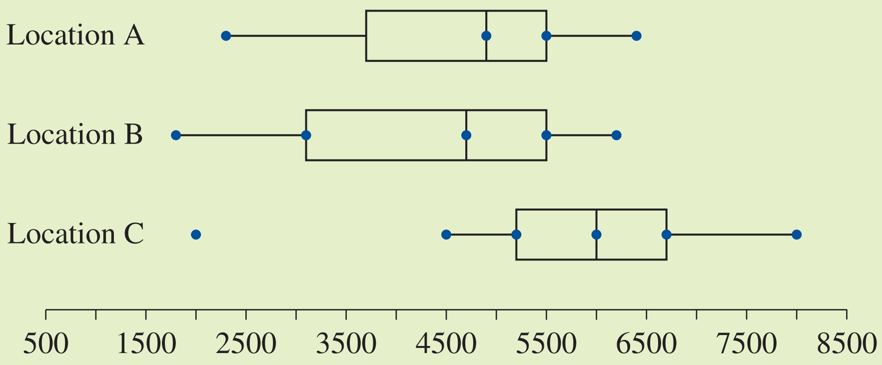
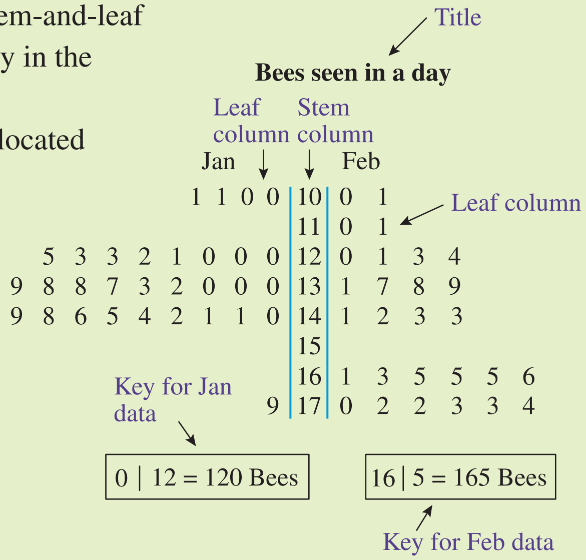
- Outliers are identified by a mark located beyond the ‘whiskers’ of the box plot. An outlier is an observation that is numerically distant from the rest of the data.

Comparison of the prices paid at auction at three different locations



- **Back-to-back stem-and-leaf plots** have a single stem with two sets of leaves. Each set of leaves is separated for the two groups being compared. The leaves for one set of data is on one side and the leaves for the second set of data is on the other side.
- When comparing back-to-back stem-and-leaf plots, the leaves must always be placed against the corresponding stem. This enables the shape, median, spread and possible outliers of the data to be easily identified and compared.
  - The shape of a back-to-back stem-and-leaf plot can be seen by looking at the shape and length of the leaves. This shows whether a dataset is symmetric (roughly the same on each side when cut down the middle) or skewed (lopsided).
  - A symmetric back-to-back stem-and-leaf plot shows the median roughly in the middle of the spread.
- Outliers are identified by a leaf located beyond the main cluster of data. An outlier is an observation that is numerically distant from the rest of the data.



Bees seen in a day

Jan		Feb
1 1 0 0	10	0 1
	11	0 1
5 3 3 2 1 0 0 0	12	0 1 3 4
9 8 8 7 3 2 0 0 0	13	1 7 8 9
9 8 6 5 4 2 1 1 0	14	1 2 3 3
	15	
	16	1 3 5 5 5 6
9	17	0 2 2 3 3 4

0 | 12 = 120 Bees

16 | 5 = 165 Bees

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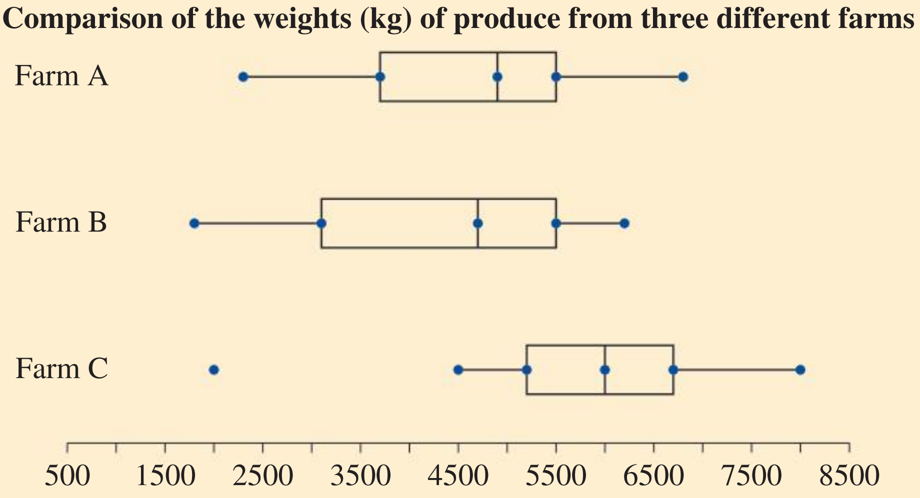
Example 5 Comparing parallel box plots for different datasets

The parallel box plots represent the weights (kg) of produce from three different farms.



a Compare the medians of all datasets.
b Compare the range of all datasets.
c Compare the spread using the IQR, which can be read directly from the box plot (the width of the boxes).
d Locate any outliers.
e Write a paragraph comparing the farms.

WORKING	THINKING
a Median for Farm A = 4900 Median for Farm B = 4700 Median for Farm C = 6000 The farm with the highest median is Farm C with 6000 kg of produce.	Locate the median (middle value) for each dataset by comparing the vertical line in the middle of the boxes.

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WORKING	THINKING
b Farm A $6800 - 2300 = 4500$ Farm B $6200 - 1800 = 4400$ Farm C $8000 - 4500 = 3500$ Farm A has the largest range with 4500 kg of produce; and Farm C has the smallest range of 3500 kg of produce.	To calculate the range, find the difference between the minimum and maximum of each dataset.
c Farm A $5500 - 3700 = 1800$ Farm B $5500 - 3100 = 2400$ Farm C $6700 - 5200 = 1500$ The farm with the largest spread between the interquartile range is Farm B, whereas the smallest spread is Farm C with 1500 kg of produce.	Compare the spread by reading the width of the boxes. $\text{IQR} = Q_3 - Q_1$
d There are no outliers for Farms A and B; however, Farm C has an outlier at 2000 kg of produce.	Record any marks beyond the ‘whiskers’.
e The farm with the lightest weight is Farm B (1800 kg). The farm with the heaviest weight is Farm C (8000 kg). Farm A has the largest range with 4500 kg of produce. The smallest range is Farm C with 3500 kg of produce. The farm with the highest median is Farm C with 6000 kg of produce. Farm C is the only farm to have an outlier at 2000 kg of produce.	Compare and contrast the median, spread and outliers of each farm.

Example 6 Comparing back-to-back stem-and-leaf plots for different datasets

The back-to-back stem-and-leaf plot compares the female life expectancy across 15 countries for 1960 and 2015.

Female life expectancy
(in years)

1960		2015
9 9	4	8
9 9 5 3	5	9
5 5 2 1 1 1 0	6	8 9
5 5	7	2 3 4 5 5 9
	8	2 3 3 8
	9	0

7|2 = 72 years

a Compare the median of both datasets.
b Compare the range of values of both datasets.
c Compare the spread between the IQR (interquartile range) of both datasets.
d Write a paragraph comparing both datasets.

WORKING **THINKING**

a **Female life expectancy**
(in years)

Locate the median (middle value) for each dataset using $(n+1)/2$.

1960		2015
9 9	4	8
9 9 5 3	5	9
5 5 2 1 1 ① 0	6	8 9
5 5	7	2 3 4 ⑤ 5 9
	8	2 3 3 8
	9	0

7|2 = 72 years

1960
$(15 \text{ numbers} + 1)/2 = 8\text{th number}$
Median for 1960 = 61 years
2015
$(15 \text{ numbers} + 1)/2 = 8\text{th number}$
Median for 2015 = 75 years
The median life expectancy of females in 2015 was 14 years higher than in 1960.

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WORKING **THINKING**

b 1960

$75 - 49 = 26$

2015

$90 - 48 = 42$

2015 has the wider range with 42 years compared to 26 years.

To calculate the range, find the difference between the minimum and maximum of each dataset.

c **Female life expectancy**
(in years)

1960		2015
9 9	4	8
9 9 5 3	5	9
5 5 2 1 1 1 0	6	8 9
5 5	7	2 3 4 5 5 9
	8	2 3 3 8
	9	0

7|2 = 72 years

1960

$65 - 55 = 10$

2015

$83 - 69 = 14$

The spread of life expectances of females in 1960 (IQR = 10 years) is smaller than the spread in 2015 (IQR = 14 years).

To compare the spread, find the quartiles.
Quartiles are found by finding the median of the lower half of the data; and the median of the upper half of the data.

Then find IQR : $\text{IQR} = Q_3 - Q_1$

d The year with the lowest life expectancy was 2015; however, this was only by one year. The year with the highest life expectancy was 2015 by fifteen years. The median life expectancy for the particular countries has increased by fourteen years. The variability in life expectancy has increased, as demonstrated by the larger IQR and range in 2015.

Compare and contrast the median, range and spread summaries of each year.

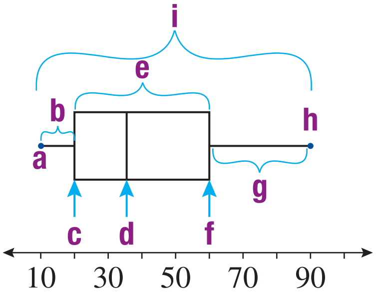
Exercise 7C

FUNDAMENTALS

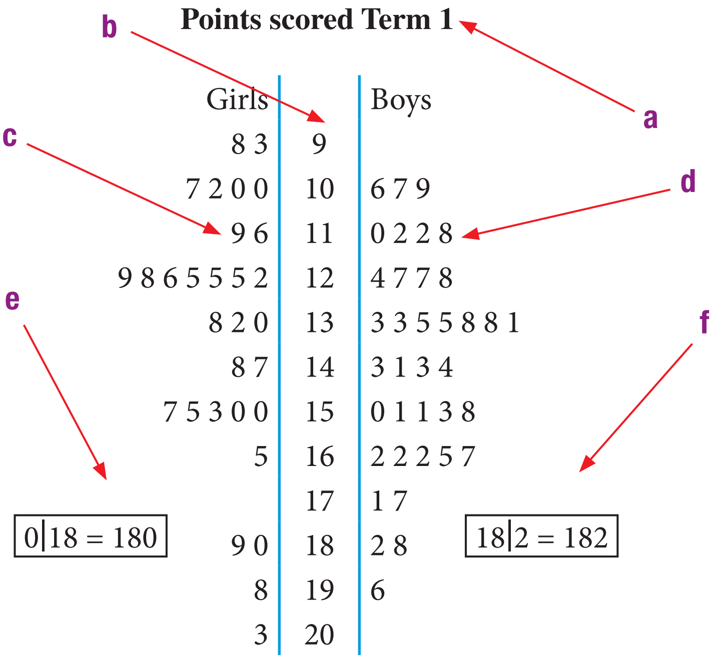
1 Determine the missing words in the following sentences.

- **a** Parallel box plots can be used to compare ____________ or more groups.
- **b** When comparing box plots, they must always be placed against the ____________ axis.
- **c** The range can be used as a ____________ of spread in a dataset, but it is extremely ____________ to the presence of ____________ and should only be used with care.
- **d** An ____________ is an observation that is numerically distant from the rest of the data.
- **e** Back-to-back stem-and-leaf plots have a ____________ stem with ____________ sets of leaves.
- **f** Each set of leaves is ____________ for the two groups being ____________.

2 Draw the box plot and label the parts.



3 Label the parts of this back-to-back stem-and-leaf plot.



Points scored Term 1

Girls		Boys
8 3	9	
7 2 0 0	10	6 7 9
9 6	11	0 2 2 8
9 8 6 5 5 5 2	12	4 7 7 8
8 2 0	13	3 3 5 5 8 8 1
8 7	14	3 1 3 4
7 5 3 0 0	15	0 1 1 3 8
5	16	2 2 2 5 7
	17	1 7
9 0	18	2 8
8	19	6
3	20	

0|18 = 180

18|2 = 182

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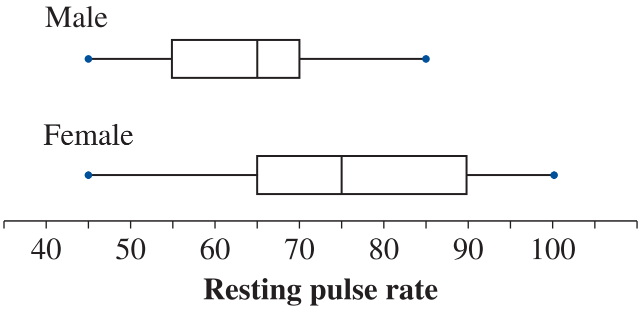
APPLICATIONS

SF: –	CF: 4–7	CU: 8–11

Example 5

4 A group of 50 male and 50 female students were asked to record their resting pulse rates. The datasets below show the results.



a Compare the medians of both datasets.

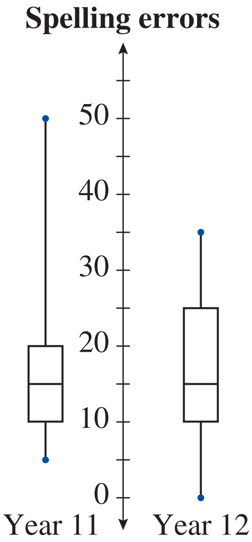
b Compare the range of both datasets.

c Compare the spread using the interquartile range.

d Write a statement comparing both datasets.

Hint Q_1 is the median of the lower half of the data; Q_3 is the median of the upper half of the data.

5 One hundred Year 11 students and one hundred Year 12 students sit the same writing task with their spelling errors being recorded.
The datasets below show the results.



a Compare the medians of both datasets.

b Compare the range of both datasets.

c Compare the spread using the IQR.

d Write a statement comparing both datasets.

Hint $IQR = Q_3 - Q_1$

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Example 6

6 A sample of Year 7 and Year 12 students were asked to record the amount of homework they completed in hours each week. The datasets below show the results.

Home work time (hours per week)

3|0 = 3 hours per week

Year 7		Year 12
9 8 6 6 5 4 3 3	0	5 5 7 9
5 5 5 4 4 0 0	1	0 0 2 4 7 8 9
	2	0 1 1 1

1|0 = 10 hours per week

a Compare the medians of both datasets.

b Compare the range of both datasets.

c Compare the spread using the interquartile range.

d Write a statement comparing both datasets.

7 The parallel stem-and-leaf plot displays the number of burpees a group of Year 12 students could do in a minute at the beginning of a netball season and at the end of the season.

Burpees per minute

0|4 = 40 burpees per min

Beginning of Netball season		End of Netball season
	0	
9 8	1	
8 3 1 0 0	2	0 5 5 7
6 5 5 3 2 2	3	0 1 3 7 9 9 9
5 0	4	9 9
	5	0 0

5|0 = 50 burpees per min

a Compare the medians of both datasets.

b Compare the range of both datasets.

c Compare the spread of the two datasets using the interquartile range.

d Write a statement comparing the two datasets.

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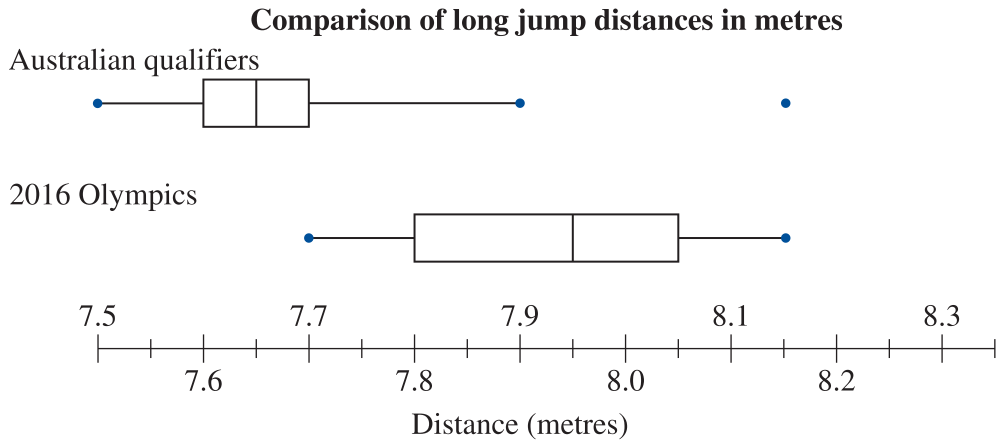
8 The following parallel box plots show the long jump distances of the Australian qualifiers compared to the distances achieved at the 2016 Olympic games.



Explain how the following conclusions can be made from this data.

a The Australian qualifiers' long jumps were generally shorter than the Olympic long jumps.

b The Olympic jumps are spread out more than the Australians.

c Most of the Olympic long jumps were longer than all of the Australians' jumps.

9 A popular café is currently concerned by a new café in town stating that they have the fastest service around. The following datasets are the delivery times (in minutes) for both cafés. Compare the range and measures of central tendency of the datasets for the two cafés.

Comparison of delivery times
(in minutes)

Popular café		New café
9 8	0	
9 8 6 5 4 3 3 2	1	0 0 1 2 4 5 6 6 8 9
8 3 3 1	2	0 1 2 5
0	3	0

0|3 = 30 min

2|5 = 25 min

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10 Toowoomba High School had their annual 10-pin bowling championships. The back-to-back stem-and-leaf plot shows the highest score of each player, by gender. The mean of the boys' highest scores was 143.3 and the mean of the girls' highest scores was 137.4. Compare the range and measures of central tendency of the datasets for the boys and the girls.

Toowoomba U18 bowling scores

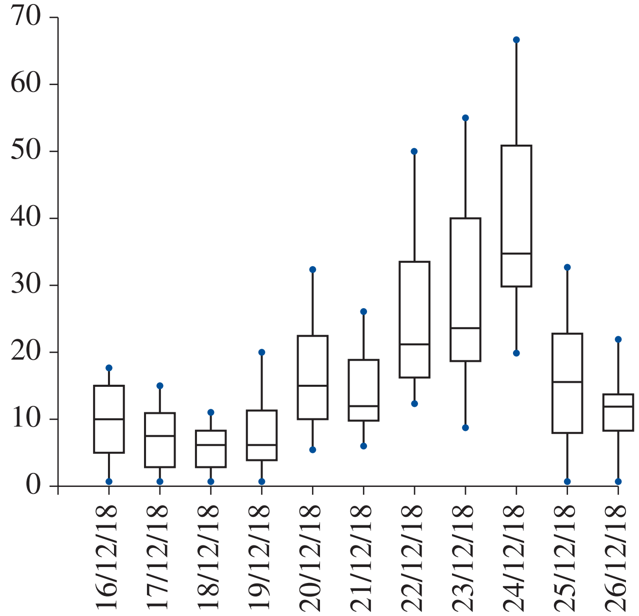
Girls		Boys
8 3	9	
7 2 0 0	10	6 7 9
9 6	11	0 2 2 8
9 8 6 5 5 5 2	12	4 7 7 8
8 2 0	13	3 3 5 5 8 8
8 7	14	3 3 4
7 5 3 0 0 0	15	0 1 1 3 8
5	16	2 2 2 5 7
	17	1 7
9 0	18	2 8
8	19	6
3	20	

3|20 = 203 score

19|6 = 196 score

11 The following set of box plots were used to compare the delay times of plane flights (in minutes) during the 2018 Christmas holidays. Compare the delay times for each dataset.

Plane delays in minutes



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7D Comparing the characteristics of histograms COMPLEX

LEARNING GOALS

- Compare the characteristics of the shape of histograms using symmetry
- Compare the characteristics of the shape of histograms using skewness
- Compare the characteristics of the shape of histograms using bimodality

Why is comparing the characteristics of the shape of histograms essential?

A histogram is a graphical method of displaying data using columns that reflect the distribution and frequency of each data group. As a histogram gives information in a compact and organised manner, it provides quick communication for informed decisions. Understanding how to quickly compare the characteristics of the shape of histograms using symmetry, skewness and bimodality are essential skills used to interpret data.

Comparing the shape of histograms is an essential skill to help us easily interpret data.

WHAT YOU NEED TO KNOW

- A **histogram** is a special type of column graph:
  - The data scores are shown on the horizontal x-axis and are organised into groups, each specified by a range.
  - The y-axis is the frequency (it may also be labelled as the number of things being measured or scored as this is equivalent to frequency).
  - There are no gaps between the columns.
  - Usually a histogram has vertical columns, but they may be horizontal.
- Spread is the measure of the range of the distribution.
- The distribution is also characterised by the shape of the columns. The main ways to describe shape are symmetry, skewedness, number and height of peaks and whether it is uniform.

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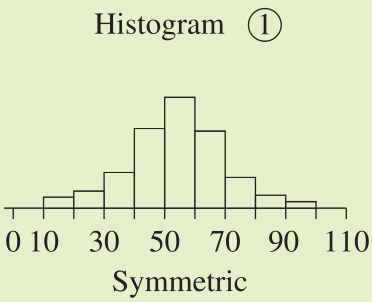
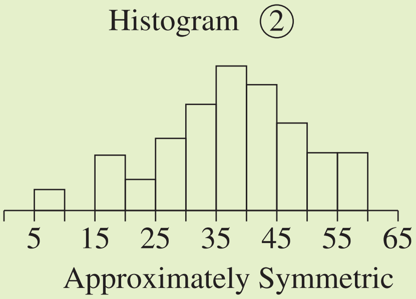
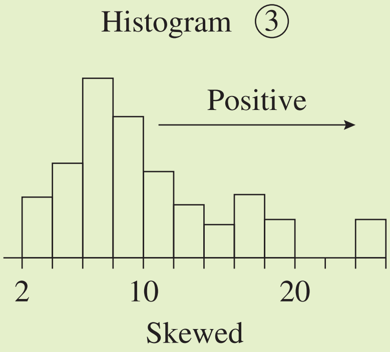
- **Symmetric** (mathematical term for symmetrical) shape is when the columns form a mirror image. (see Histogram 1)
  - The **axis** is the line through the distribution showing each half.
  - The **centre**, or half-way mark, of the shape is found by eye, where approximately half of the data is on one side and the rest on the other.



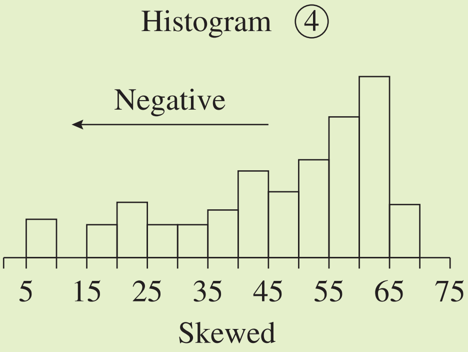
- **Approximately symmetric** is identified where the shape is approximately the same on both sides. (see Histogram 2)
- The mode of a distribution is the observation that appears most often. It is the value that is most likely to be recorded.



- **Positively skewed** is when the distribution's peak is higher on the left, with the tail stretching to the right. This is because the shorter columns are towards the positive section of the distribution. (see Histogram 3)



- **Negatively skewed** is when the distribution's peak is higher on the right, with the tail stretching to the left. This is because the shorter columns are towards the negative section of the distribution. (see Histogram 4)



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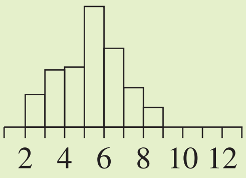
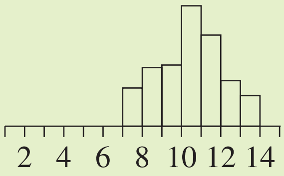
- **Comparing location**: Histogram 5 and 6 are similar in shape; however, they are in different locations on the axis.

Histogram ⑤

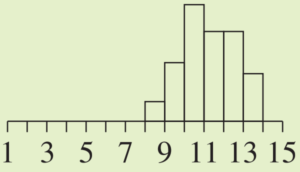


Histogram ⑥

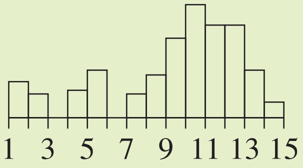


- **Comparing spread**: Histogram 7 and 8 are both centred in a similar location; however, Histogram 8 is more spread out than Histogram 7.

Histogram ⑦

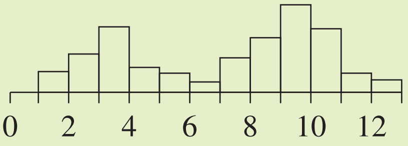


Histogram ⑧



- **Bimodal**: is having two modes, which means there is not one data value that occurs with the highest frequency, but two data values having high frequencies. (see Histogram 9)

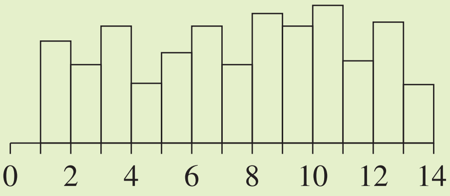
Histogram ⑨



Bimodal

- **Uniform**: Histogram 10 also known as 'multimodal distribution' has a fairly constant or even frequency distribution.

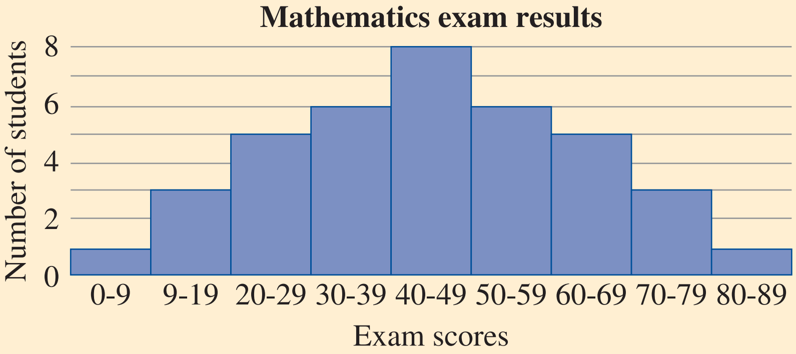
Histogram ⑩

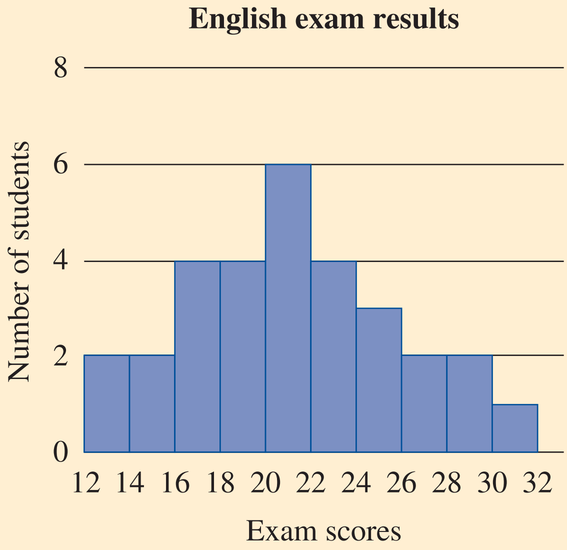


Uniform

Example 7 Comparing the characteristics of the shape of histograms using symmetry

The histograms display the results from a class of their Maths (/89) and English (/32) exams.





a Compare the shape of the histograms.

b Compare the centre of both histograms.

c Compare the spread of both histograms.

WORKING | **THINKING**

a The Maths exam histogram is a symmetric shape, whereas the English exam histogram is approximately symmetric. ◂ Which shape best describes each histogram?

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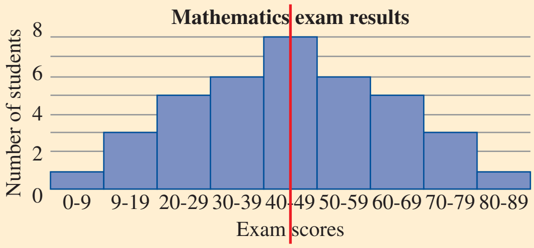
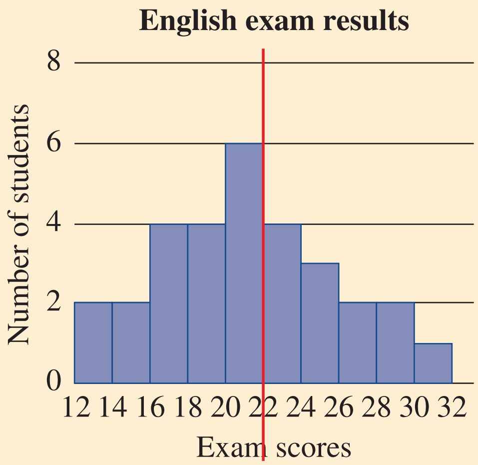
WORKING	THINKING
b The Maths exam histogram has a centre of 44.5 out of 89.	Where is the centre column?
$\frac{44.5}{89} \times 100 = 50\%$	Use your eye to find the centre. 
The English exam histogram has a centre of 22 out of 32.	
$\frac{22}{32} \times 100 = 68.75\%$ Therefore, the English exam has a higher centre.	To be able to compare fairly, convert both histograms' centres to a percentage.
c The spread for the Maths exam histogram is $89 - 0 = 89$ The spread for the English exam histogram $32 - 12 = 20$ Therefore, the spread for the Maths exam histogram is greater than for the English exam histogram.	How are the columns spread across the *x*-axis? Spread is measured by the range of the distribution. Find the range of the distributions. Maximum value – minimum value

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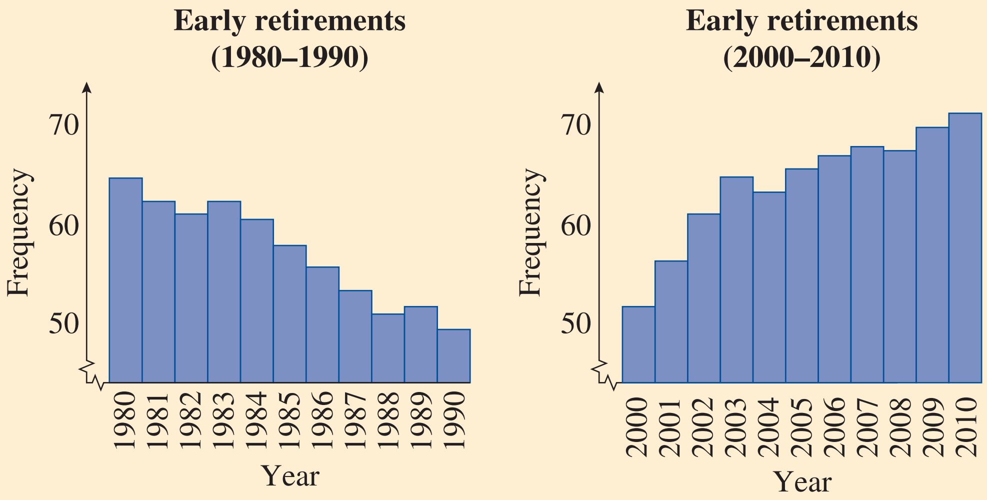
Example 8 Comparing the characteristics of the shape of histograms using skewness

The two histograms are comparing the frequency of retirements from a company between 1980–1990 and 2000–2010.



a Compare the shape of the histograms.
b Compare the centre of both histograms.
c Compare the spread of both histograms.

WORKING	THINKING
a The 1980–1990 histogram is positively skewed, as the distribution's peak is higher on the left, with the tail stretching to the right. The 2000–2010 histogram is negatively skewed, as the distribution's peak is higher on the right, with the tail stretching to the left. This is because the shorter columns are towards the left of the distribution.	Which shape best describes each histogram?

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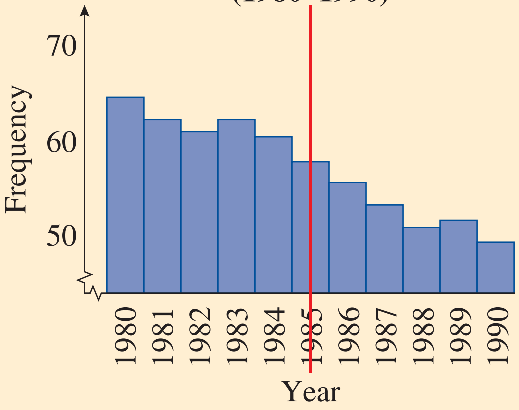
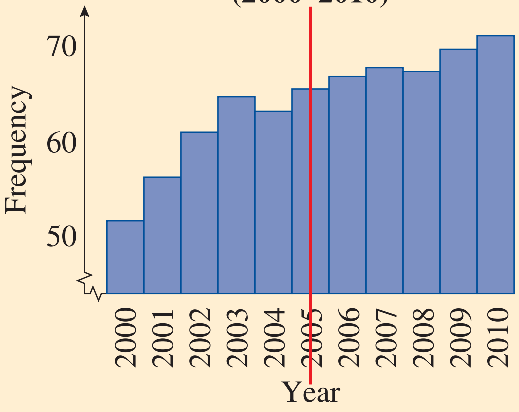
WORKING	THINKING
b The 1980–1990 histogram has a centre of 1985. The 2000–2010 histogram has a centre of 2005. The centre for both histograms is the 6th year of 11 years. Therefore, the centre is the same for both histograms.	Where is the centre column? Use your eye to find halfway along the horizontal axis. **Early retirements (1980–1990)**  **Early retirements (2000–2010)** 
c The spread for the 1980–1990 histogram is across the whole axis (11 years). The spread for the 2000–2010 histogram is across the whole axis (11 years). Even though the histograms cover different years, the range is the same size – 11 years. Therefore, the spread of both histograms is the same.	How are the columns spread across the *x*-axis? Spread is the measure of the range of the distribution. Find the range of the distributions.

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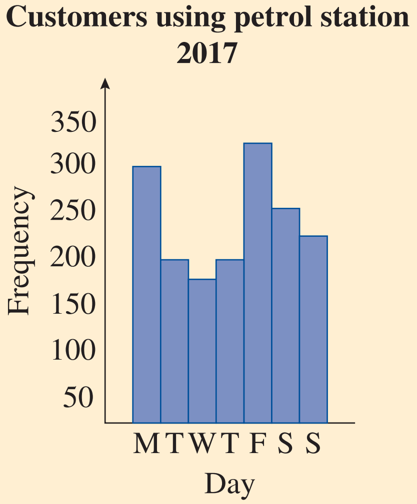
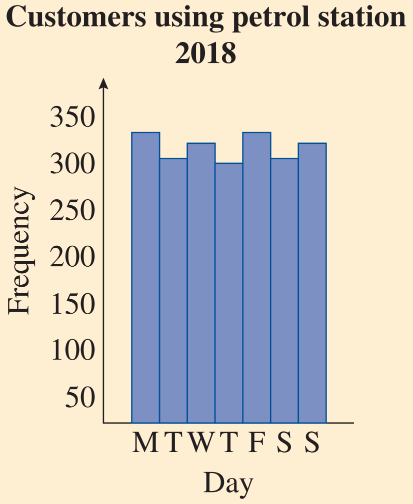
Example 9 Comparing the characteristics of the shape of histograms using bimodality

The two histograms are comparing the frequency of people using a petrol station in 2017 and 2018.


Customers using petrol station 2017


Customers using petrol station 2018

a Compare the shape of the histograms.
b Compare the centre of both histograms.
c Compare the spread of both histograms.

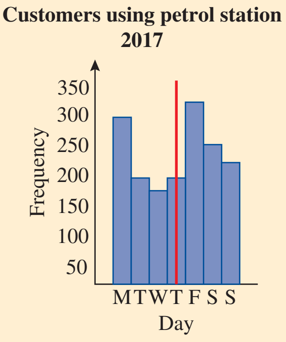
WORKING **THINKING**

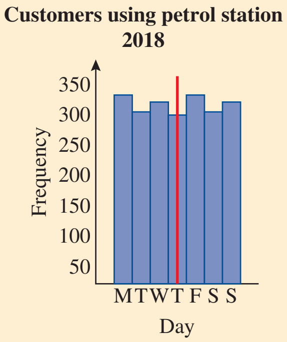
a The 2017 histogram's shape is bimodal as it is double-peaked at Monday and Friday. The 2018 histogram's shape is uniform distribution.

Which shape best describes each histogram?

b The 2017 histogram has a centre of Thursday.
The 2018 histogram has a centre of Thursday.
Therefore, both histograms have the same centre.

Where is the centre column?
Use your eye to find halfway along the horizontal axis.


Customers using petrol station 2017


Customers using petrol station 2018

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WORKING	THINKING
c The spread for the 2017 histogram is across the whole axis (1 week).	How are the columns spread across the *x*-axis?
The spread for the 2018 histogram is across the whole axis (1 week).	Spread is the measure of the range of the distribution.
Therefore, the spread of both histograms is the same.	Find the range of the distributions.

Exercise 7D

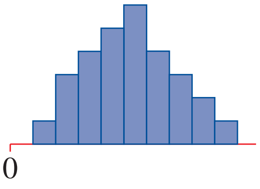
FUNDAMENTALS

1 Determine the missing words in the following sentences.

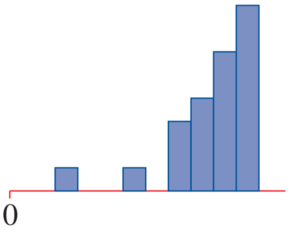
- **a** Symmetric shape is when the columns form a ____________ image.
- **b** Approximately symmetric is identified where the shape is ____________ the same on both sides.
- **c** Positively skewed is when the distribution's peak is higher on the ____________, with the tail stretching to the ____________.
- **d** ____________ skewed is when the distribution's peak is higher on the right, with the tail stretching to the ____________.
- **e** ____________ is having two modes, which means there is not one data value that occurs with the highest frequency, but ____________ data values having high frequencies.

2 Describe the shape of each of the following histograms.

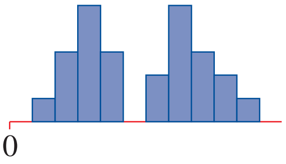
a



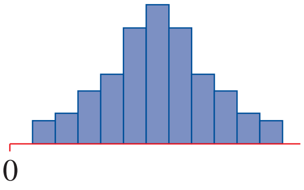
b



c



d



3 Draw a histogram that is:

- **a** symmetric
- **b** approximately symmetric
- **c** positively skewed
- **d** negatively skewed
- **e** bimodal
- **f** uniform

APPLICATIONS

SF: –	CF: 4–7	CU: 8–9

Example 7

4 The scores of the first 11 throws and the second 11 throws of a die are shown in the histograms.

Hint When finding the centre, use your eye to find halfway along the horizontal axis.

- **a** Compare the shape of the histograms.
- **b** Compare the centre and spread of both histograms.

Example 8

5 A wind farm compared their wind speed over two days. The histograms on the next page show the results.

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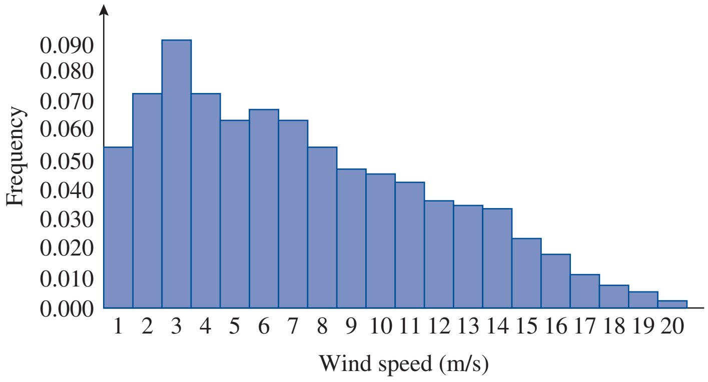
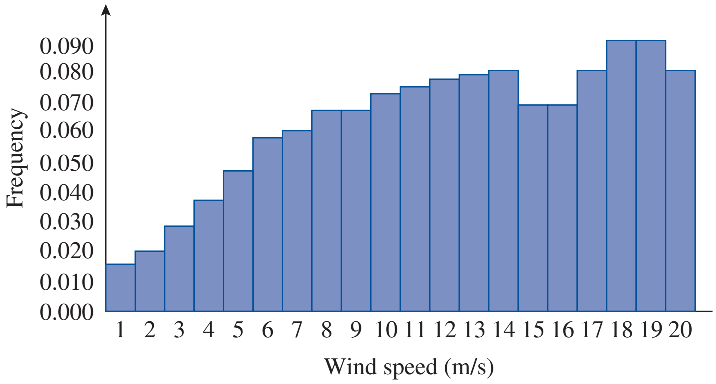
Wind speed distributions Monday



Wind speed distributions Tuesday



a Compare the shape of the histograms.

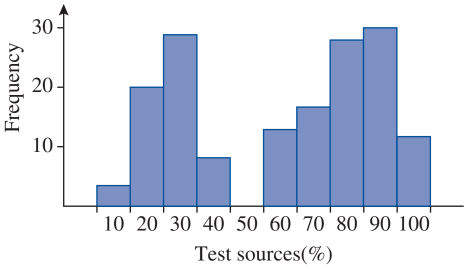
b Compare the centre and spread of both histograms.

Hint The spread describes the distribution.

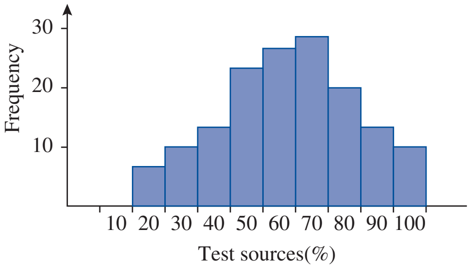
Example 9

6 A teacher uses histograms to display the results of her students' mathematics test scores. The histograms below show the results.

Term one test scores



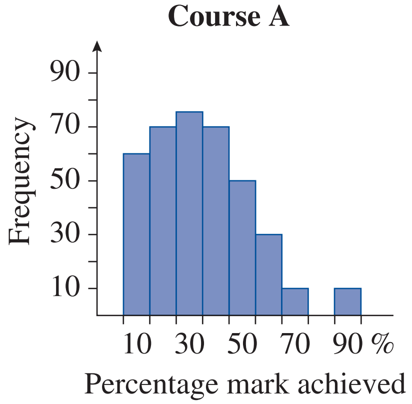
Term two test scores

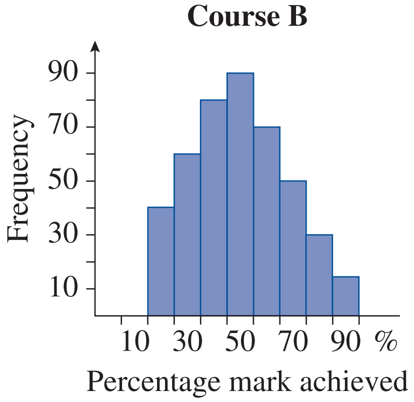


a Compare the shape of the histograms.

b Compare the centre and spread of both histograms.

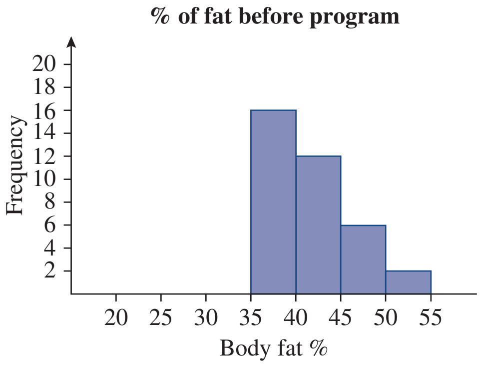
7 A Year 12 student is trying to choose between two TAFE courses. She is using the following histograms to help her decision.

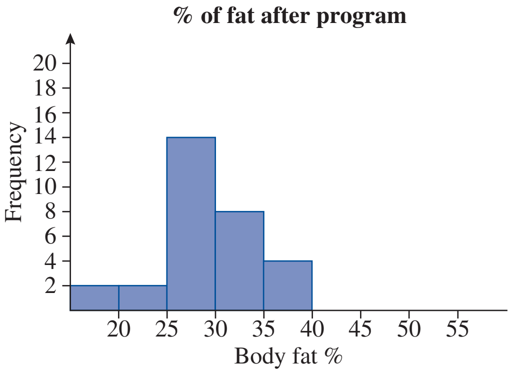




- **a** Write a statement comparing the shape of the histograms.
- **b** Explain which course would give the student a greater chance of passing. Justify your answer.

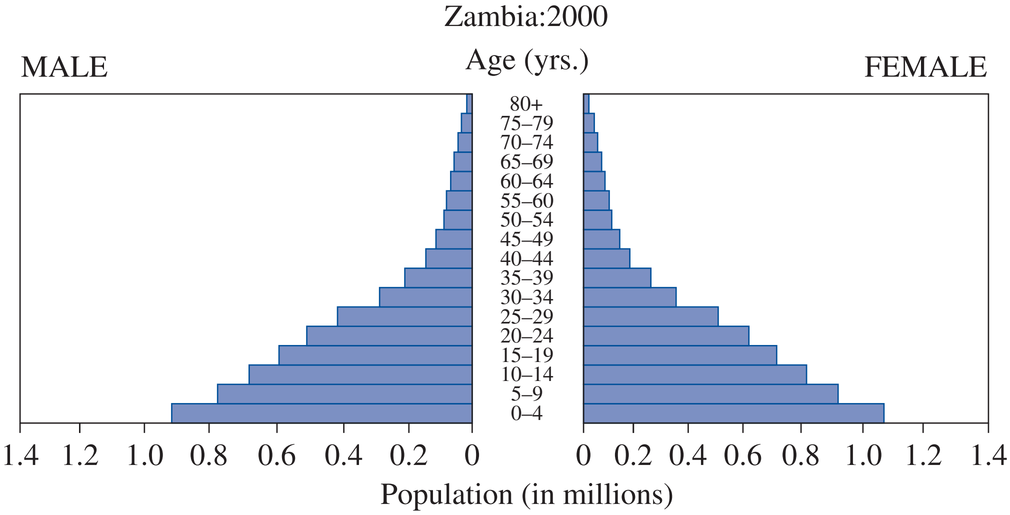
8 A personal trainer is advertising their new program using the following histograms. Compare the shape of the histograms.





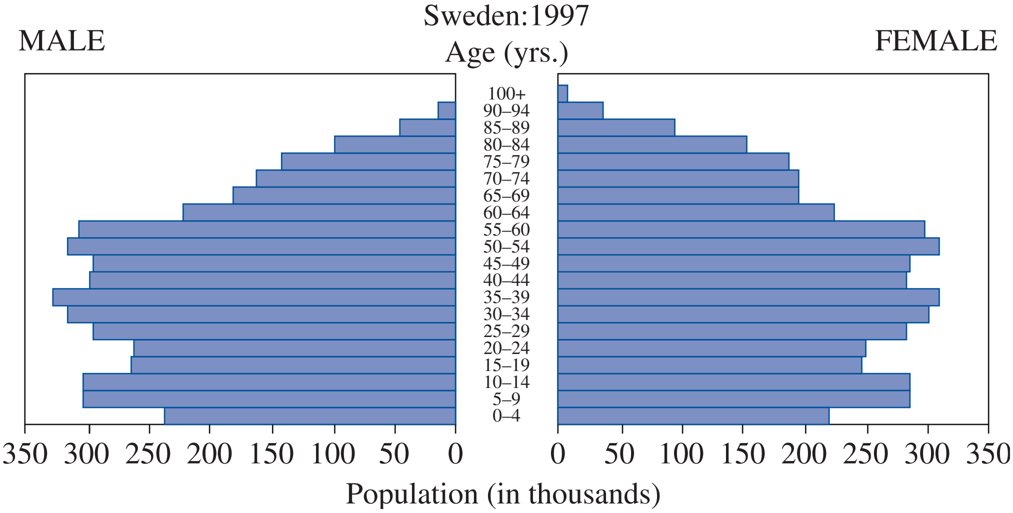
9 The two population distribution histograms display the percentage of a population by age and gender in Zambia and Sweden.

Population Distribution of Zambia by Age and Sex, 2000



Source: U.S. Census Bureau [Internet]. Washington, DC: IDB Population Pyramids [cited 2004 Sep 10]. Available from http://www.census.gov/ipc/www/idb/.

Population Distribution of Sweden by Age and Sex, 1997



Source: U.S. Census Bureau [Internet]. Washington, DC: IDB Population Pyramids [cited 2004 Sep 10]. Available from http://www.census.gov/ipc/www/idbpyr.html.

a Compare the shape of the male and female histograms for Zambia.

b Compare the shape of the male and female histograms for Sweden.

c What conclusions can be made about the difference in population distribution by age and sex in Zambia and Sweden?

Problem-solving and modelling task

Background: Various articles have been published about the difference in total earning between males and females in Australia.

Task: Your task is to find and compare datasets for male and female income in Australia.
Here are possible sources:
https://www.businessinsider.com.au/australia-gender-pay-gap-2018–8
http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/6302.0May%20 2018?OpenDocument
http://www.abs.gov.au/AUSSTATS/abs@.nsf/Lookup/6302.0Explanatory%20 Notes1May%202018?OpenDocument

From such sources you need to compile the datasets, display them in an appropriate way, and write a short report comparing the datasets. Write a conclusion that addresses whether there is any difference between male and female incomes in Australia.

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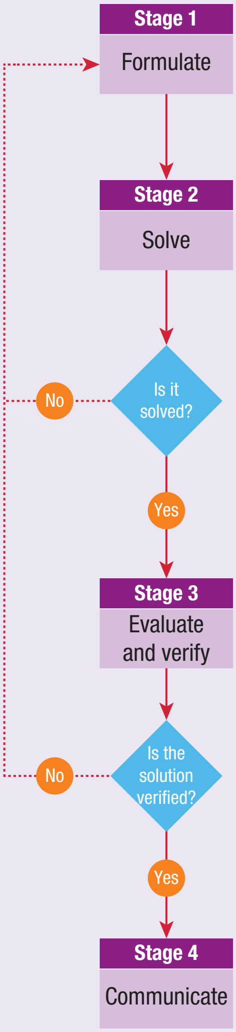
Approach to problem-solving and modelling tasks:



Stage 1: Formulate

1 Research the data sources available.
2 Research the best way to display the data such as parallel box plots or a back-to-back stem-and-leaf plot.

Stage 2: Solve

3 Examine the datasets and the information that comes with them thoroughly to ensure understanding of each dataset.
4 Organise the two datasets (male total earnings and female total earnings).
5 Construct displays of the two datasets.
6 Decide what comparisons of the two datsets are relevant.

Stage 3: Evaluate and verify

7 Check how reasonable they are.
8 Have you presented them in the most suitable way?
9 Is there enough information to contrast?

Stage 4: Communicate

10 Communicate your findings in a short report that introduces the datasets, the displays and the comparison of the datasets.
11 Write a conclusion about any difference between male and female total earnings.

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Chapter checklist

☐ I can recognise the structure of a five-number summary.

1 List the names of the numbers in a five-number summary in order from smallest to largest.

2 Write a definition of each of the five numbers.

☐ I am able to create a five-number summary.

3 Create a five-number summary of the following data.
4, 8, 7, 4, 9, 13, 9, 8, 7

☐ I can create a five-number summary with the use of technology.

4 Create a five-number summary of the data from question **3**, using technology.

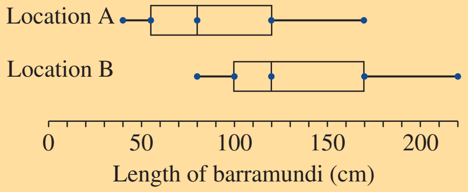
☐ I can construct box plots using a five-number summary.

5 Construct a box plot for the following dataset.
3, 4, 2, 5, 3, 2, 1, 6, 5

☐ I can compare datasets using parallel box plots. **[complex]**

6 Compare the length of barramundi found in location A and B from these parallel box plots.

Length of Barramundi taken from two locations



☐ I can compare datasets using back-to-back stem-and-leaf plots. **[complex]**

7 Compare the number of days to ripening of tomato batches A and B in the back-to-back stem-and-leaf plots.

Number of days to tomatoes ripening

Batch A		Batch B
9	0	
9 8 2 1 0 0	1	1 2 3 3 7 8
	2	0
	3	

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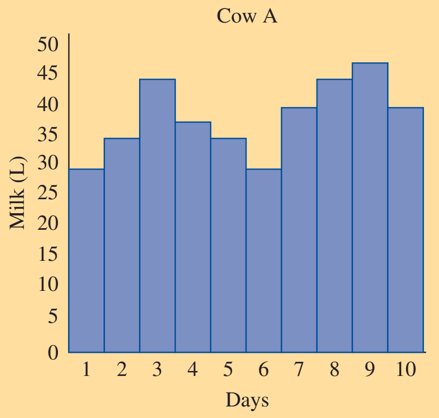
☐ I can compare the characteristics of the shape of histograms. **[complex]**.

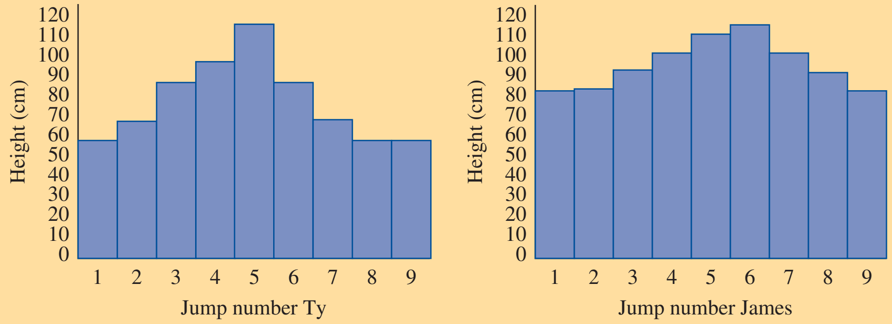
8 Compare the shape of the following histograms using symmetry, skewness or bimodality as appropriate.

a



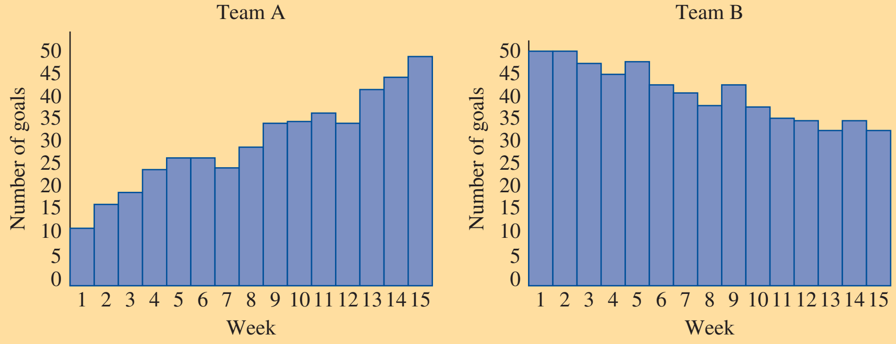
Litres of milk per day

b



High jumps

c



Goals scored for Team A + Team B

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Chapter review

All questions in the review are assessment style.

Simple familiar

1 Christine owns a dog shelter, she has recorded the current weights of the dogs in her care. Determine the minimum, Q_1, median, Q_3 and maximum of her current dataset below.

7 kg, 5 kg, 8 kg, 14 kg, 7 kg, 10 kg, 4 kg, 5 kg, 3 kg, 4 kg, 7 kg, 5 kg, 4 kg

Section 7A

2 A soccer coach was preparing for a new season. The following dataset is her team's scores for the previous season. Create a five-number summary, without technology, to assist with their preparation.

2, 1, 0, 3, 2, 0, 4, 1, 1, 2, 1, 2, 3

3 Fajalla is a diabetic. Her blood glucose levels are recorded in mmol/L.

4, 8, 7, 4, 9, 13, 9, 8, 7

Create a five-number summary of her latest results using technology.

Section 7B

4 A group of Year 12 students were asked to record the number of hours of homework they completed each week. The data below shows the results.

8, 9, 11, 4, 2, 4, 1, 3, 3, 6, 10, 10

a Create a five-number summary using technology.

b Construct a box plot.

5 Sam records the number of players from each football team who are selected to play in the Queensland representative team. Construct a box plot from Sam's dataset.

3, 4, 2, 5, 3, 2, 1, 6, 5

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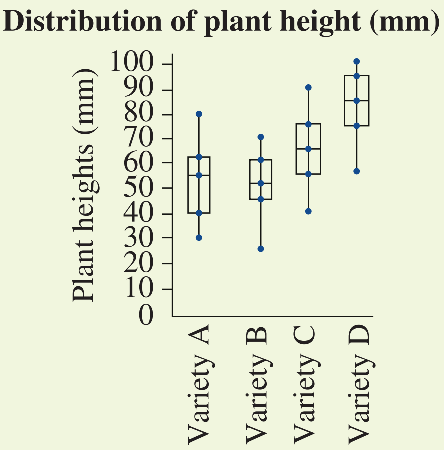
Complex familiar

Section 7C **6** The following set of box plots were used to compare the heights of different varieties of a plant.



Compare the distributions of the plant varieties' heights, concluding which is tallest and commenting on the variability.

7 The following back-to-back stem-and-leaf plot represents the distributions of life expectancies for males in 15 countries in 1988 and 2018.

Male life expectancy (in years)

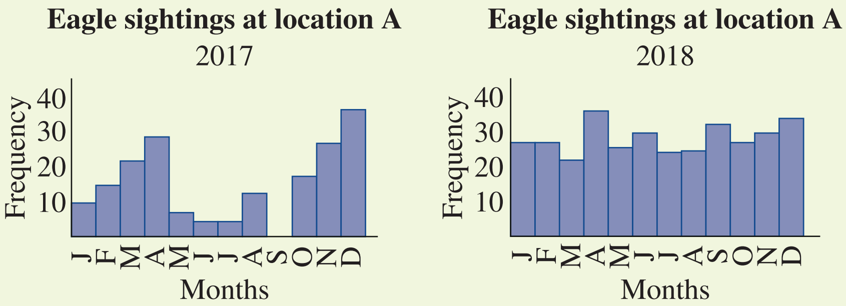
1988		2018
9	4	
8 6 5	5	5 5
9 9 7 5 0	6	0 0 5 5
7 5 5	7	5 5 5 9 9
5 5 0	8	5 5 5
	9	0

5|5 = 55 years

a Compare these distributions in terms of median, range and IQR.

b Write a paragraph concluding whether male life expectancy in these countries changed between 1988 and 2018.

Section 7D **8** An avid bird watcher has been recording wedge-tail eagle sightings at a particular location in each month in 2017 and 2018. Compare the shape of the two histograms.



Complex unfamiliar

9 Complete the following for each of the tables shown.

i Draw the histogram.

ii Determine the shape the histogram would produce.

a

Digital article sizes	
Bytes per article	Frequency
500	1500
1000	3000
1500	5000
2000	5500
2500	6000
3000	7000
3500	7500
4000	7500
4500	9000
5000	8500

b

Drug dispensing times	
Times (minutes)	Frequency
0–10	3
11–20	4
21–30	5.5
31–40	7.5
41–50	5.5
51–60	4

c

Petal lengths	
Petal length (cm)	Frequency
1	7.5
2	20
3	10
4	12.5
5	27.5
6	12.5

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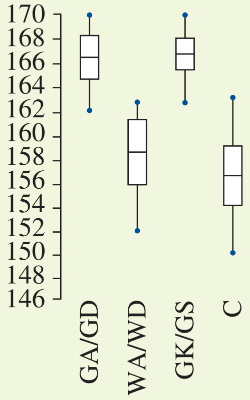
10 The following set of box plots show the heights (in cm) of 100 netball players grouped according to their positions on their teams. Compare the heights for the grouped positions, and their variability. Relate the height of the players and the variability to positions that involve goal and those that do not.

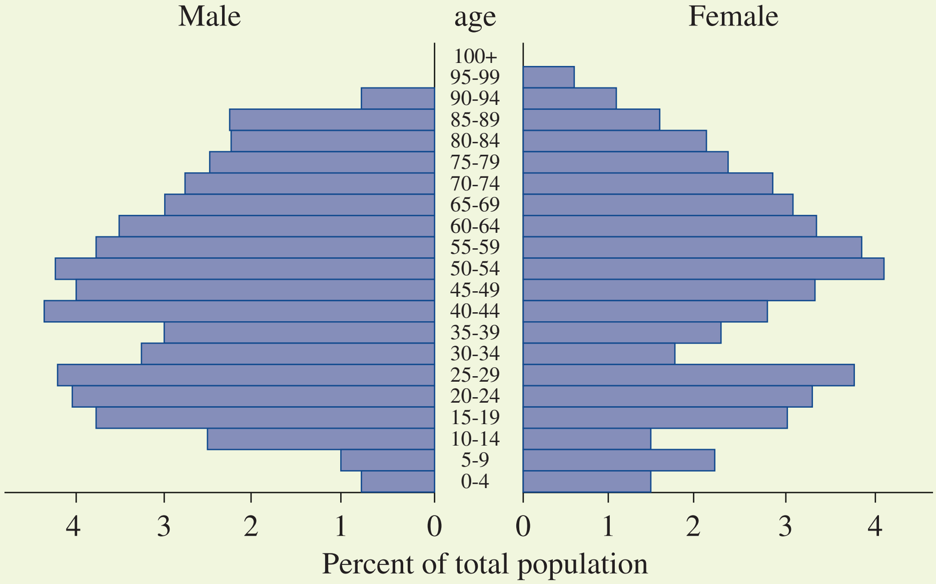
Height (in cm) by position for netball teams



Abbreviations
GA: Goal attack
GD: Goal defence
GS: Goal shooter
GK: Goal keeper
WA: Wing attack
WD: Wing defence
C: Centre

11 Histograms were used to display the distribution of age by sex from the data supplied by a recent census. Compare the shape of the histograms.

Population distribution by age and sex



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8

The Cartesian plane and bivariate scatterplots

Maths for a homemaker: Melanie Capps

I am a qualified primary teacher who is now the primary homemaker at my home after having my boys. I still work casually as a relief teacher as well as a volunteer at a local church and the school bands.

Tell us a bit about your job. What does a typical day look like?

My day is different and can change due to a phone call to work replacing a teacher who has called in sick. My main job in the morning is getting my boys to eat their breakfast and get ready for school. After they leave, I will often be on the phone seeking the best deals for my family as I run our household budget. I do some cleaning but my family including my husband chip in with the chores, which spreads the load. I also research for purchases of bigger household items and planning family trips.

What maths did you study in school?

I did Maths A at school and did quite well. I would like to have pushed myself a bit harder as I do actually like maths.

How do you use mathematics in your job?

When we need to have some work done around the house, I will research using the internet different tradies' costs, making sure I compare their call out fees plus the amount of time expected to do the job. If they are quick workers but have a bigger call out fee, they may be the best for the job. I use maths constantly for our household budgets as well as the church kitchen budget where I volunteer and purchase bulk amounts for my volunteering position.

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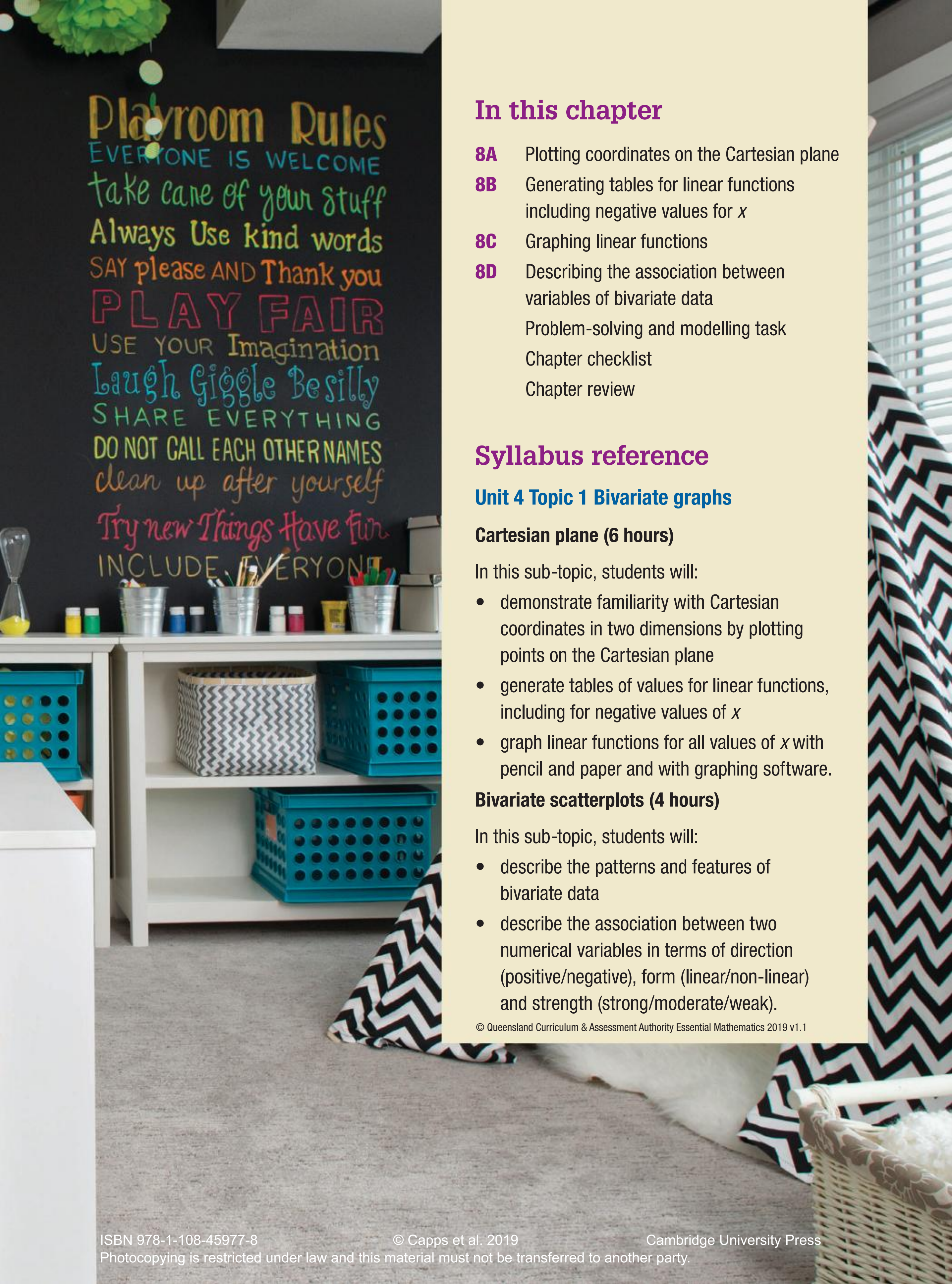

In this chapter

Syllabus reference

Unit 4 Topic 1 Bivariate graphs

Cartesian plane (6 hours)

In this sub-topic, students will:

- demonstrate familiarity with Cartesian coordinates in two dimensions by plotting points on the Cartesian plane
- generate tables of values for linear functions, including for negative values of x
- graph linear functions for all values of x with pencil and paper and with graphing software.

Bivariate scatterplots (4 hours)

In this sub-topic, students will:

- describe the patterns and features of bivariate data
- describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak).

Pre-test

1 Identify the x-coordinate in the following points.

- **a** (3, 6)
- **b** (12, –10)
- **c** (–5, 3)
- **d** (0, –2)

2 Identify the y-coordinate in the following points.

- **a** (7, 2)
- **b** (–5, 13)
- **c** (–3, –6)
- **d** (3, 0)

3 Carlos earns $22 per hour, calculate how much he will earn for the following job times.

- **a** 3 hours
- **b** 8 hours
- **c** 2.5 hours
- **d** 6 hours 30 minutes
- **e** 4.75 hours

4 Margareta earns $40 per hour plus a $50 call out fee, calculate how much she will earn for the following job times.

- **a** 1 hour
- **b** 3 hours
- **c** 2.5 hours
- **d** 4 hours 30 minutes
- **e** 1.75 hours

5 Determine the next number in the following number patterns.

- **a** 2, 4, 6, …
- **b** 3, 6, 9, …
- **c** 4, 7, 10, …
- **d** 2, 9, 16, …

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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8A Plotting coordinates on the Cartesian plane

LEARNING GOALS

- Understand what a Cartesian plane is and the standard convention for naming coordinates
- Determine the x and y values of a coordinate from a graph
- Graph coordinates on a Cartesian plane
- Plot coordinates on a graph based on a Cartesian plane in a real-world context

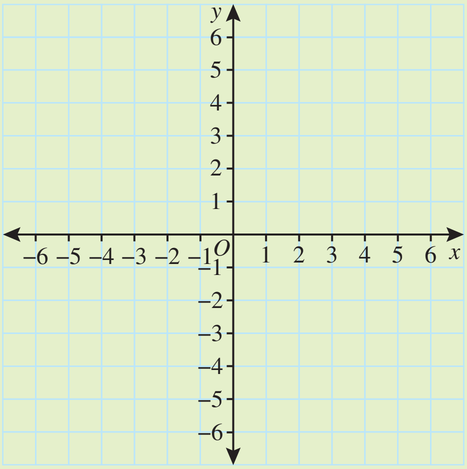
Why is it essential to be able to plot coordinates?

The ability to plot coordinates is an essential skill to being able to graph linear functions and bivariate data, especially scatterplots. And in business and finance, line graphs and scatterplots are the most widely used way of displaying data.

Linear graphs are used in a variety of areas particularly in finance and science fields.

WHAT YOU NEED TO KNOW

- A **Cartesian plane** is a graph with axes x and y labelled with whole number values including negative values. In its basic form it is arranged in a square with O at the centre.
- The ***x*-coordinate** is the horizontal distance on the x-axis.
- The ***y*-coordinate** is the vertical distance on the y-axis.
- A coordinate is written by conventions as the x-coordinate first followed by the y-coordinate, in brackets: (x, y).



- Line graphs and scatterplots in real-world contexts are usually based on a Cartesian plane displaying only the positive values of x and y.

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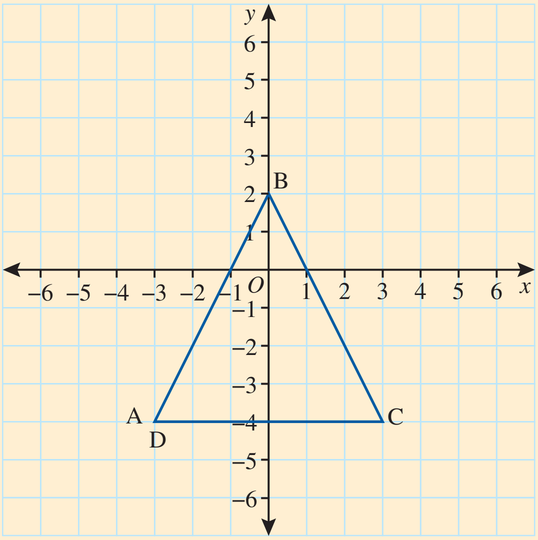
Example 1 Plotting coordinates on a Cartesian plane

Plot the following set of points connecting them in order and name the shape it makes.
A (–3, –4), B (0, 2), C (3, –4), D (–3, –4)

WORKING



The coordinate points form a triangle.

THINKING

To plot point A, go to –3 on the horizontal axis and then down 4 places to –4 on the vertical axis and mark point A.

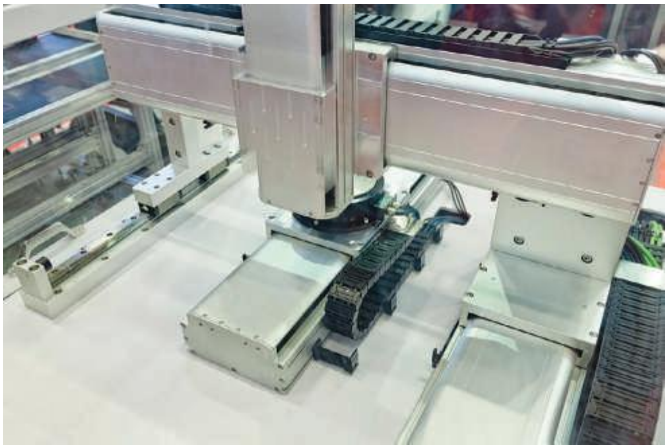
To plot point B, go to 0 on the horizontal axis and then up 2 places to 2 on the vertical axis and mark point B.

To plot point C, go to 3 on the horizontal axis and then down 4 places to –4 on the vertical axis and mark point C.

To plot point D, go to –3 on the horizontal axis and then down 4 places to –4 on the vertical axis and mark point D.

Connect points in order A to B to C to D with 3 straight lines.

Identify the shape formed.



This industrial robot positions itself using Cartesian coordinates to carry out a variety of functions.

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Example 2 Plotting coordinates on a Cartesian plane in a real-world context

Ziah starts a gardening business to make some extra cash. In the first month he earned $15, in the second month he earned $45, in the third month he earned $30 and in the fourth month he earned $40.

a Complete the table using the values identified in the question.

Month	1	2	3	4
Earnings $				

b Plot the coordinates (month, earning) on a graph.

c Join the coordinates in order of time.

d Determine the month that Ziah earned the most money.

e Calculate the total Ziah earned in the 4 months.

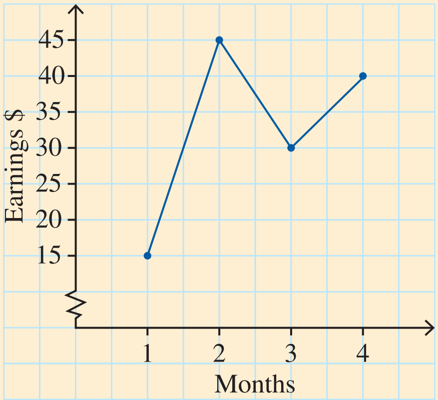
WORKING	THINKING

a

Month	1	2	3	4
Earnings $	15	45	30	40

Complete the table by entering the corresponding amount to the month.

b
c



Identify that the graph needs to go to at least $45 on the vertical axis and 4 on the horizontal axis.
Draw the graph axes and label accordingly.
Plot the points as per the table using (month, earning)
Join the points.

d Ziah earned the most in the 2nd month.

Find the highest point in the graph.

e $\$15 + \$45 + \$30 + \$40 = \$130$

Calculate the total earnings by adding the monthly earnings.

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Exercise 8A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

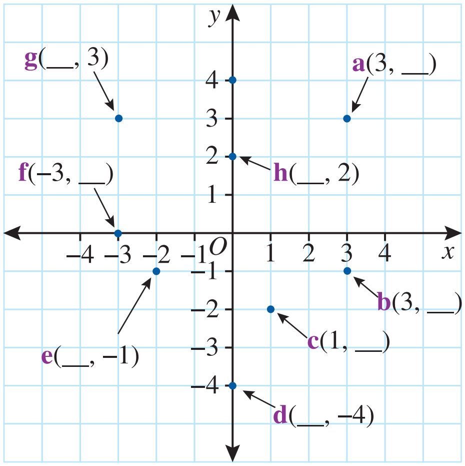
a The *x*- ______________ is the distance along the ______________ axis.

b The *y*-coordinate is the ______________ along the ______________ axis.

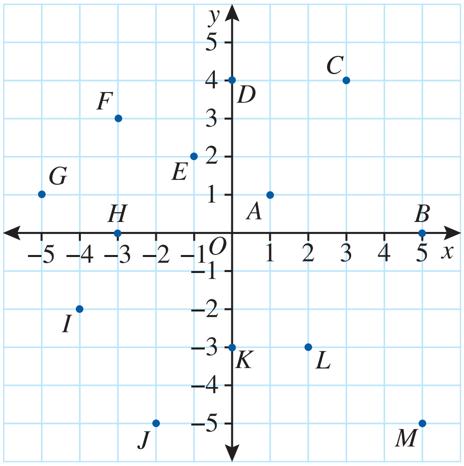
c The ______________ plane contains both ______________ and ______________ numbers.

2 Determine the missing coordinates labelled **a** to **h** below, left.

Question 2



Question 3



3 Identify and write the coordinates of the points labelled *A* to *M* above, right.

Example 1

4 Plot the following sets of points connecting them in order and name the shape it makes.

a A (0, 2), B (2, 0), C (0, –4), D (–2, 0), E (0, 2)

b A (–3, 1), B (–1, 2), C (1, 1), D (0, –1), E (–2, –1), F (–3, 1)

c A (–4, –3), B (–4, 2), C (1, 2), D (1, –3), E (–4, –3)

d A (–3, –1), B (–2, 1), C (3, 1), D (2, –1), E (–3, –1)

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APPLICATIONS

SF: 5–9	CF: –	CU: –

Example 2

5 Dylan starts a pet walking business to make some extra cash. In the first month he earned \$28, in the second month he earned \$35, in the third month he earned \$49 and in the fourth month he earned \$42.

a Complete the table using the values identified in the question.

Month	1	2	3	4
Earning \$				

b Plot the coordinates (month, earning) on a graph.
c Join the coordinates in order of time.
d Determine the month that Dylan earned the most money.
e Calculate the total Dylan earned in the 4 months.

6 Darlene starts a mowing business to make some extra cash. In the first month she earned \$40, in the second month she earned \$60, in the third month she earned \$50 and in the fourth month she earned \$80.

a Complete the table using the values identified in the question.

Month	1	2	3	4
Earning \$				

b Plot the coordinates (month, earning) on a graph.
c Join the coordinates in order of time.
d Determine the month that Darlene earned the most money.
e Calculate the total Darlene earned in the 4 months.

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★7 Irene is making wraps for premature babies to help them stay warm. She made 10 in 2015, 14 in 2016, 18 in 2017 and 20 in 2018.

a Create a table of values for this data.
b Create a graph to display this data.
c Determine the year that Irene made the most wraps.
d Calculate the total number of wraps that Irene knitted in the 4 years.

★8 James has some stock and wants to buy a car by selling his stock to get money for a car. Given each share of the stock was worth \$26.50 in 2016, \$27.50 in 2017, \$27.00 in 2018 and \$26.50 in 2019. If James has 1000 shares, complete a table of values and then draw a graph to determine when he should have sold his stock to make the most money and how much he would receive.

★9 Jayde has some stock and needs to buy her first car by selling her stock. Given each share of the stock was worth \$3.50 in 2016, \$4.75 in 2017, \$4.25 in 2018 and \$4.00 in 2019. If Jayde has 2000 shares, draw a graph and determine when she should have sold her stock to make the most money.

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8B Generating tables for linear functions including negative values for *x*

LEARNING GOALS

- Substitute values into equations to calculate solutions
- Work with negative numbers
- Complete a table of values

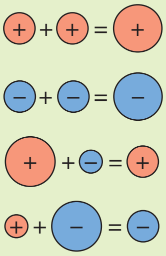
Why is it essential to be able to use a table of values?

Tables of values are the starting point for graphing linear functions. As will be seen in the next chapter, these graphs can be used to make predictions for the future. They are relevant to many occupations in business, finance, science, technology, health, economics and social studies.

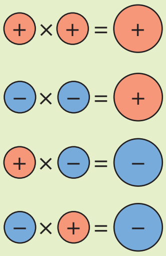
Linear functions can be used in a variety of fields to predict future values.

WHAT YOU NEED TO KNOW

- A **linear function** is an equation of the form $y = mx + c$ where m and c are constants.
- Addition rules of **positive** and **negative** numbers.



- Multiplication rules of **positive** and **negative** numbers.



The size of the circles indicates the size of the number.

- Solve an equation by substituting values for the variables into the equation. For example: For $y = 2x + 3$, if $x = 4$ then $y = 2 \times 4 + 3 = 11$ (Calculate using BIDMAS)
- Complete a table of values for a linear function by substituting values for the variables into the equation.
  For example: For $y = 2x + 3$, complete the table below by first substituting $x = -3$ and calculating the value of y:
  $y = (2 \times -3) + 3 = -6 + 3 = -3$

x	-3	-2	-1	0	1	2	3
y	-3						

In the column under −3, enter −3 in the row for y.
Continue for the remaining values of x.

Example 3 Generating tables of values for linear functions including negative values for *x*

Complete the table of values for the linear equation $y = -5x + 2$.

x	-2	0	2
y			

WORKING	THINKING
When $x = -2, y = -5 \times -2 + 2$ $y = 10 + 2 = 12$	Substitute the value of −2 into the equation. Use order BIDMAS to simplify.
When $x = 0, y = -5 \times 0 + 2$ $y = 0 + 2 = 2$	Substitute the value of 0 into the equation. Use order BIDMAS to simplify.
When $x = 0, y = -5 \times 2 + 2$ $y = -10 + 2 = -8$	Substitute the value of 2 into the equation. Use order BIDMAS to simplify.

x	-2	0	2
y	12	2	-8

Complete the table using the values for y found above.

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Example 4 Generating tables for linear functions in a real-world context

Erin is an electrician and she charges a \$60 call out fee and \$45 per hour. This can be mapped by the equation $y = 45x + 60$, where x is the number of hours worked and y is the cost for the client.

a Complete the table of values for Erin's television repair costs.

x	1	2	3
y			

b Calculate the cost of hiring Erin for $1\frac{1}{2}$ hours.

WORKING | **THINKING**

a $y = 45 \times 1 + 60 = 105$

$y = 45 \times 2 + 60 = 150$

$y = 45 \times 3 + 60 = 195$

x	1	2	3
y	105	150	195

Substitute the value of 1 into the equation. Use order BIDMAS to simplify.

Substitute the value of 2 into the equation. Use order BIDMAS to simplify.

Substitute the value of 3 into the equation. Use order BIDMAS to simplify.

Use the values calculated to complete the table of values.

b $y = 45 \times 1\frac{1}{2} + 60 = \$127.50$

Substitute $x = 1\frac{1}{2}$ into the equation to calculate Erin's pay.

Exercise 8B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a To complete the ______________ of values we must ______________ values into the equation.

b A ______________ number multiplied by a negative number equals a positive ______________.

c A ______________ number multiplied by a negative number ______________ a negative number.

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2 Substitute $x = 2$ into the following equations to calculate the value of y.

a $y = 3x$ **b** $y = -5x$
c $y = x + 3$ **d** $y = x - 4$
e $y = 2x - 4$ **f** $y = 6 - 5x$

3 Substitute $x = -3$ into the following equations to calculate the value of y.

a $y = 4x$
b $y = -2x$
c $y = x + 4$
d $y = x - 3$
e $y = 2x + 3$
f $y = 3 - 4x$

Hint Use the addition and multiplication rules for positive and negative numbers.

Example 3

4 Complete the table of values for the following linear equations.

a $y = 3x$

x	-2	0	2
y			

b $y = -2x$

x	-3	0	3
y			

c $y = x + 3$

x	-2	0	2
y			

d $y = x - 2$

x	-3	0	3
y			

e $y = 4x + 1$

x	-2	0	2
y			

f $y = -2x - 2$

x	-3	0	3
y			

APPLICATIONS

SF: 5–9	CF: –	CU: –

Example 4

★**5** Lucinda is a television repair person and she charges a \$50 call out fee and \$60 per hour. This can be mapped by the equation $y = 60x + 50$, where x is the number of hours worked and y is the cost for the client.

a Complete the table of values for Lucinda's television repair costs.

x	1	2	3
y			

b Calculate the cost of hiring Lucinda for $2\frac{1}{2}$ hours.

★**6** Mario is a plumber and he charges a $60 call out fee and $45 per hour. This can be mapped by the equation $y = 45x + 60$, where x is the number of hours worked and y is the cost for the client.

a Complete the table of values for Mario's plumbing cost.

x	2	4	6
y			

b Calculate the cost of hiring Mario for $4\frac{1}{2}$ hours.

★**7** Francis has bought 15 kg of strawberries and unfortunately 2 kg of the strawberries go rotten each week. This can be mapped by the equation $y = 15 - 2x$, where x is the number of weeks and y is the total amount of non-rotten strawberries. Complete the table of values for Francis' strawberries.

x	0	2	4
y			

★**8** Lisa won $1000 and spends $150 per week from her winnings. This can be mapped by the equation $y = 1000 - 150x$, where x is the number of weeks and y is the amount Lisa has left. Complete the table of values for Lisa.

x	1	3	5
y			

9 In questions 7 and 8, if the table was extended for more weeks there would come a point where all the strawberries would be rotten, and all the money would be gone. Determine approximately how many weeks it would take for:

a all Francis's strawberries to go rotten

b all Lisa's winnings to be gone.

10 Trev's Truck hire costs an initial $120 plus $50 per hour that the truck is rented. Create an equation to map this relationship and then create a table of values to show the cost to hire his truck for the first 5 hours.

Hint In the equation, the initial cost of $120 stays the same, while separately, the time in hours has to be multiplied by the cost per hour.

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8C Graphing linear functions

LEARNING GOALS

- Create graphs based on linear functions by hand
- Create graphs based on linear functions using technology
- Read values from the graphs created

Why is it essential to graph linear functions?

The ability to graph linear functions is an essential skill for understanding the relationships between two variables contained in bivariate data. This skill enables people in finance, business and many other fields to use graphs to illustrate trends from data. In the next chapter this will lead to the ability to make predictions from the graphs. This is the most valuable application of graphs in everyday life.

The money markets and financial companies use linear graphs to illustrate trends in the stock market.

WHAT YOU NEED TO KNOW

- A linear function is an equation of the form $y = mx + c$ where m and c are constants.
- Knowledge of Cartesian planes, substituting values into an equation and creating a table of values from these values are required for this section.
- The y-axis is the vertical axis and the x-axis is the horizontal axis.
- A scale for the x- and y-axis is created by reading the range of the x values and y values in the data to be graphed.
- A plotted graph line is simply a series of points joined together.
- A straight line can be drawn using two coordinates, but we generally use three coordinates to ensure no mistakes have been made in determining these coordinates.
- Linear graphs can be plotted by hand and technology, such as spreadsheets, and online graphing calculators, such as Desmos, can also be used to create straight line graphs.

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Example 5 Graphing linear equations by hand

Use the equation $y = 2x - 3$ to complete the following.

a Complete a table of values for $x = 0, 2, 4$.

b Determine the range of x and y values needed on the graph.

c Draw the graph axes and labels on graph paper.

d Plot the coordinates and draw a line through the points.

WORKING **THINKING**

a

x	0	2	4
y	−3	1	5

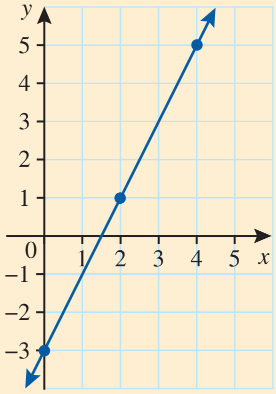
Substitute the values of 0, 2 and 4 into the equation. Use order BIDMAS to simplify.

b x is from 0 to 4, y is from −3 to 5.

Identify the range of x and y required to be graphed.

c

d

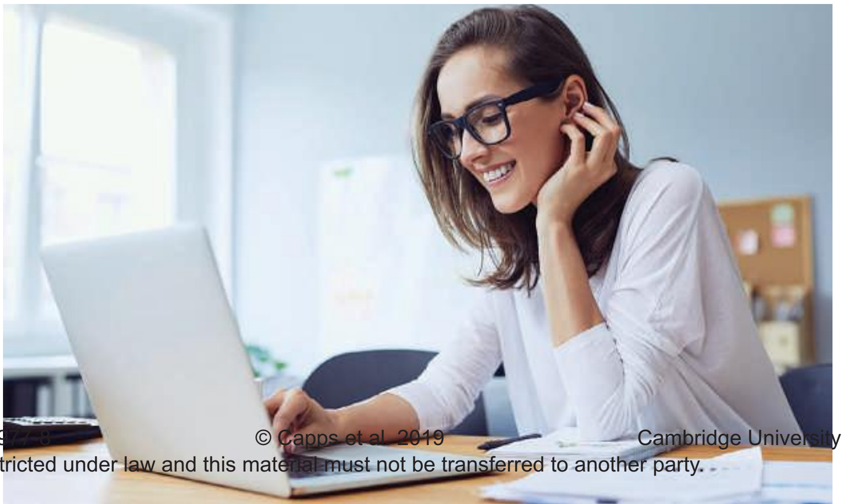


Use the range of x and y to draw a graph large enough to display all the coordinates.

Plot the coordinates on the graph and draw a line passing through all three points.

Desmos activity 8C See the interactive textbook for this activity on how to graph a linear function using the Desmos online graphing calculator.

Spreadsheet activity 8C See the interactive textbook for this activity on how to graph a linear function using a spreadsheet.



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Example 6 Graphing linear equations in a real-world context

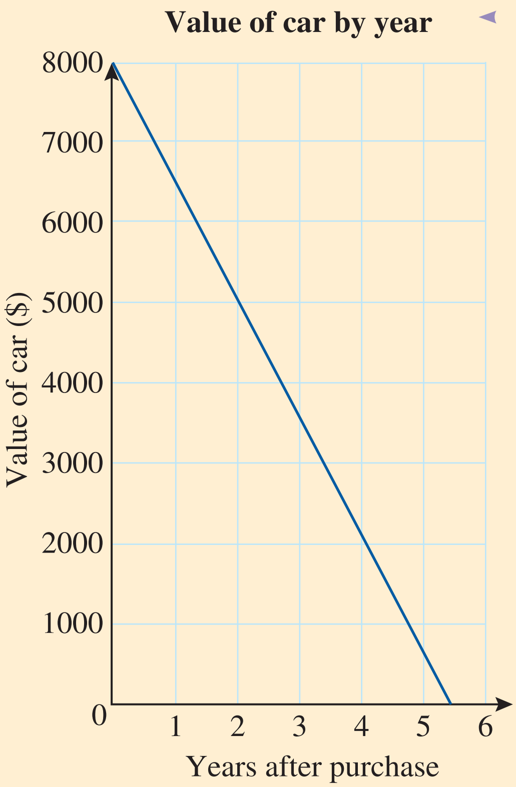
Tamara bought a used car worth \$8000. The car depreciates (goes down in value) by \$1500 per year. The car's value can be described as $y = 8000 - 1500x$, where y is the car's value and x is the number of years.

a Graph the equation of the value of her car by hand or with technology.

b Determine how many years it will take for the car to be worth under \$1000.

WORKING **THINKING**

a



The range of y values was from 0 to 8000, so to show the car's value decrease with a step of 1000.
The range of x values was from 0 to 6 with a step of 1 as the car is worth nothing by the 6th year.

b After 5 years the car will be less than \$1000.

Once the line goes below the 1000 mark, the car is worth less than \$1000.

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Exercise 8C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a A ______________ can be used to make future ______________.

b To ______________ a graph we need to find the ______________ of values for the x- and y-axis.

c ______________ programs allow us to create graphs ______________ and ______________.

2 For the following set of points determine the vertical and horizontal range required for the graph axes. For example: The table shown has x values from 0 to 4 and y values from −2 to 6.

x	0	2	4
y	−2	2	6

a

x	1	2	3
y	−3	2	7

b

x	−2	0	2
y	−5	−1	3

c

x	−3	0	3
y	−12	2	16

d

x	0	3	6
y	0	13	26

3 Graph the following tables of values on the same graph.

i Plot the coordinates on the graph paper.

ii Connect the three coordinates by drawing a straight line through them.

a

x	−2	0	2
y	−1	3	7

b

x	0	2	4
y	4	1	−2

Example 5

4 For each of the following equations, complete the following.

i Complete a table of values for $x = 0, 2, 4$.

ii Determine the range of x and y values needed on the graph.

iii Draw the graph axes and labels on graph paper.

iv Plot the coordinates and draw the lines.

a $y = x + 3$

b $y = 2x$

c $y = 3x - 3$

d $y = -2x + 1$

5 Use technology to assist you to complete the following.

a Graph the following equations.

i $y = x$ **ii** $y = 2x$

iii $y = \frac{1}{2}x$ **iv** $y = -2x$

b Identify what the number in front of the x does to each graph in part **a**.

c Graph the following equations.

i $y = x + 1$ **ii** $y = x - 3$

iii $y = x + 4$ **iv** $y = x$

d Identify what the number at the end (not next to the x) does to each graph in part **b**.

APPLICATIONS

SF: 6–12	CF: –	CU: –

Example 6

★**6** Davina is a salesperson selling high quality saucepans. She receives $100 per week and $50 per saucepan set that she sells. Her income can be described as $y = 50x + 100$, where y is her income and x is the number of saucepans she sells.

a Graph the equation of Davina's income by hand or with technology.

b Determine the total income Davina that will receive if she sells 7 saucepans.

★**7** Charlie is a salesperson who sells jetskis. He receives $225 per week and $250 per jetski he sells. His income can be described as $y = 250x + 225$, where y is his income and x is the number of jetskis he sells.

a Graph the equation of his income by hand or with technology.

b Determine the total income Charlie will receive if he sells 3 jetskis.

★**8** Bobbie is a farmer who has bought a tractor worth $25 000. The tractor depreciates (goes down in value) by $4500 per year and the value of the tractor can be described as $y = 25\,000 - 4500x$, where y is the value of the tractor and x is the number of years.

a Graph the equation of the value of the tractor by hand or with technology.

b Determine how many years it will take for the value of the tractor to be under $5000.

★9 Tom is a fisherman who has bought a boat worth \$38 000. The boat depreciates (goes down in value) by \$6000 per year and the value of the boat can be described as $y = 38\,000 - 6000x$, where y is the value of the boat and x is the number of years.

a Graph the equation of the value of the boat by hand or with technology.

b Determine how many years it will take for the value of the boat to be worth under \$15 000.

★10 Mandeep is a taxi driver who charges a \$4.50 fee when picking up clients and then charges \$0.80 per km travelled. The taxi fare can be described as $y = 0.8x + 4.5$, where y is the total taxi fare and x is the number of km travelled. If Mandeep picks up a client at Brisbane airport and takes them to the city which is 17 km away, create a graph to determine how much the total fare will be.

★11 Taj is a limousine driver who charges a \$24 fee when picking up clients and then charges \$1.20 per km travelled. The taxi fare can be described as $y = 1.2x + 24$, where y is the total limousine fare and x is the number of km travelled. If Taj picks up a client at Cairns airport and takes them to Palm Cove which is 26 km away, create a graph to determine how much the total fare will be.

12 Kirra has a surfboard shop and it costs her \$800 a day in wages and rent to stay open. If she makes \$210 per surfboard she sells, write this as an equation and graph the equation to determine the amount of surfboards required to be sold to make a profit each day.

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8D Describing the association between variables of bivariate data

LEARNING GOALS

- Understand definition of bivariate data
- Identify patterns and trends from bivariate data
- Determine positive/negative data correlations
- Determine linear/non-linear data correlations
- Determine strong/moderate/weak/no data correlations

Why is it essential to know about bivariate data and to identify associations between two variables?

The investigation of whether one thing affects another is at the heart of research and development across science, technology, health, social studies, economics and business. Data gathered when one thing affects another is bivariate data – there are two variables, and ‘bi’ means ‘two’ and ‘variate’ means varying. An important element of statistics is determining if any relationship exists between the variables in a set of bivariate data. These relationships can be used to make conclusions and even predictions.

Statistics can help determine whether there is a relationship between the diameter of the weather balloon and the weight of instruments it can lift.

WHAT YOU NEED TO KNOW

- **Bivariate data** is data for two **variables** that may have an **association**. An example is hours of study in maths and maths test scores.

Hours of study	1	2	3	4	5
Test scores out of 20	3	6	11	13	19

- The association between them is that the hours of study affects the test scores. An association is also called a relationship or **correlation**.

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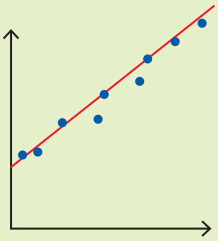
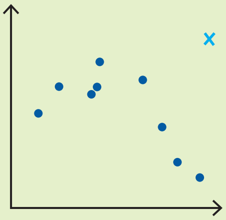
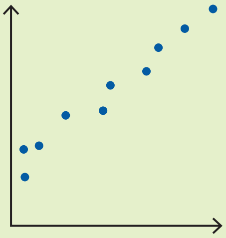
- A **scatterplot** is the most usual form of a graph of bivariate data, where the data are represented by individual dots on the graph.
- A **linear correlation** is when a graph of the data appears similar to a straight line with a slope, as shown here.



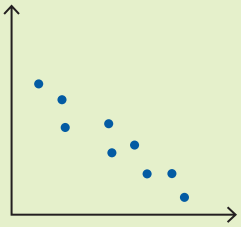
- A **non-linear correlation** is when a graph of the data produces a curve.
- An outlier is a data point that does not fit with the general pattern, like the data point marked 'x' in this scatterplot.



- A **positive correlation** is as one data variable increases so does the other data variable, so the graph slopes up to the right (e.g. comparing height to shoe size – the taller the person the larger the shoe size).

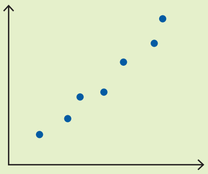


- A **negative correlation** is as one data variable increases the other data variable decreases, so the graph slopes down to the right (e.g. the larger number of days with sunny weather will mean the less water left in the dam due to evaporation).

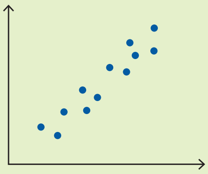


- The correlations can be described as strong, moderate, weak or none depending on the scatter and how close to a straight line the data lies.

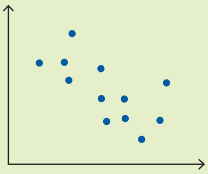
Strong correlation



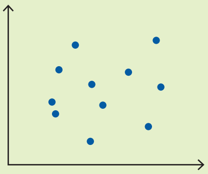
Moderate correlation



Weak correlation



No correlation



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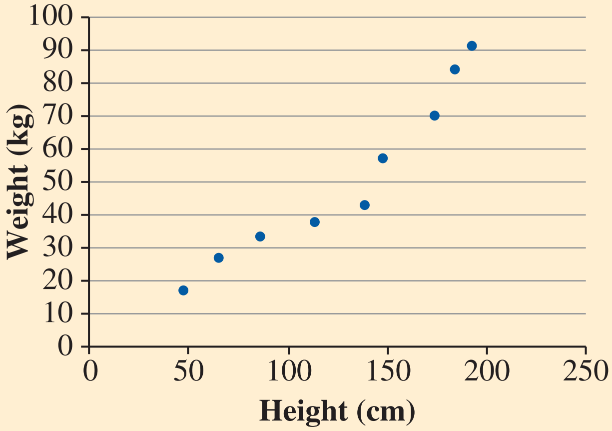
Example 7 Describing the association of variables of bivariate data from a scatterplot

The scatterplot shown maps the height and the weight of nine people.

a Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.

b Determine what relationship if any exists between a person's height and weight.



WORKING	THINKING
a The scatterplot has a positive, linear and strong correlation.	As the scatterplot goes up from left to right it is positive. The data is close to a straight line and is therefore linear, and as it is close to a line it is a strong correlation.
b As the person's height increases, the person's weight will also increase.	As there is a definite correlation, we can conclude a statement describing the relationship between a person's height and weight.

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Example 8 Creating a scatterplot and describing the association of variables of bivariate data

The lifesaver association want to show that the number of lifesavers patrolling a beach affect the number of people who are saved each year.

a Using the data in the table, create a scatterplot to determine any correlation between the number of lifesavers and the number of people saved that exists.

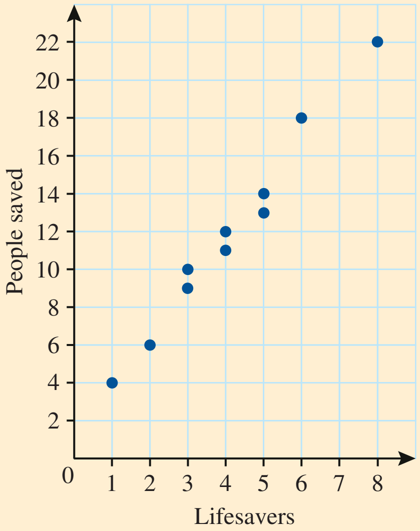
b Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.

c Determine if the number of lifesavers have an effect on the number of people saved each year.

Number of lifesavers	4	6	2	1	5	8	4	3	6	5	3
Number of people saved	12	18	6	4	13	22	11	10	19	14	9

WORKING **THINKING**

a



Identify the range of 'Number of lifesavers' and 'Number of people saved' required to be graphed. Use the range of both to draw a graph large enough to display all the coordinates. Plot the coordinates on the graph from the table of data points to create a scatterplot.

b The correlation is positive, linear and strong.

As the scatterplot goes up from left to right it is positive. The data is close to a straight line and is therefore linear, and as it is close to a line it is a strong correlation.

c It appears that the more lifesavers on patrol will result in more people being saved.

As there is a definite correlation, we can conclude with a statement describing the relationship between the number of lifesavers and the number of people saved.

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Exercise 8D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a A ______________ correlation displays data increasing from ______________ to ______________.

b A strong correlation describes a ______________ that forms a pattern similar to a ______________ line.

c Data with no ______________ is said to have no correlation.

d A ______________ correlation shows data that has some pattern but is not very strong.

2 Decide if the data variables have a relationship.

a weight of cars and fuel consumption

b temperature and the cost of a textbook

c number of flowers and number of bees

d height of a door and size of door handles

e amount of rain and the size of vegetables in the garden

f length of student's hands and the length of student's feet

Hint If two variables are related, one of these should be true:
- when one of them changes, it will produce a change in the other; but not necessarily the other way around;
- or something else, when it changes, causes both of them to change.

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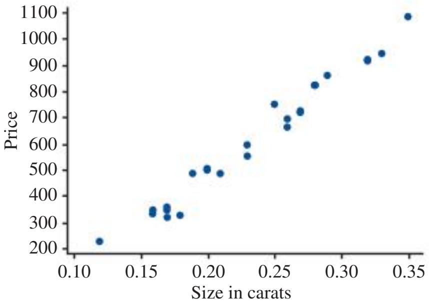
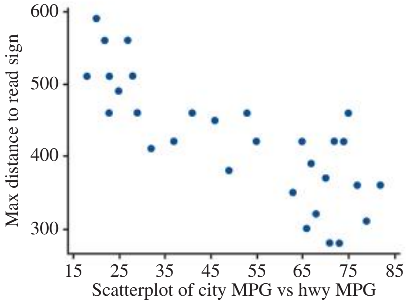
3 Describe the correlations between the variables in these graphs in terms of:

i direction (positive or negative)
ii shape (linear or non-linear)
iii strength (strong, moderate, weak or none)

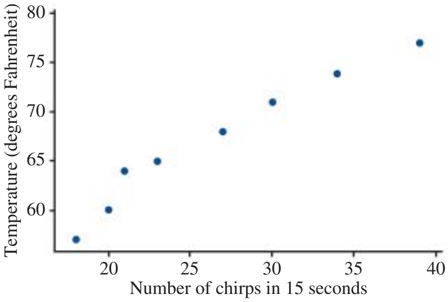
a



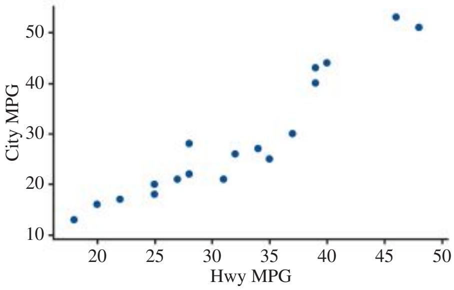
b



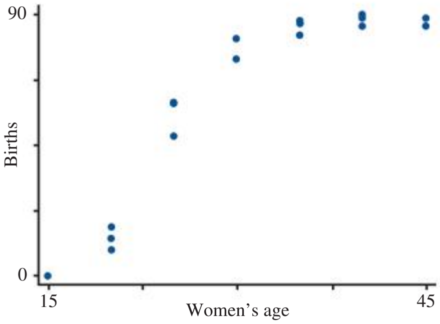
c



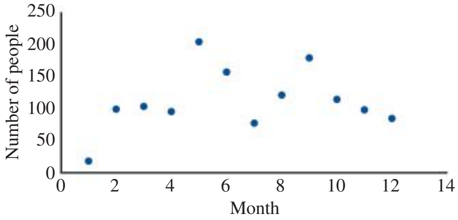
d



e



f



4 Construct a scatterplot using technology (or by hand on graph paper) using the data below. From this scatterplot determine the correlation between the data variables that exists in terms of:

a direction (positive, negative, none)
b shape (linear or non-linear)
c strength (strong, moderate, weak or none)

Hint An outlier can be disregarded when identifying a pattern.

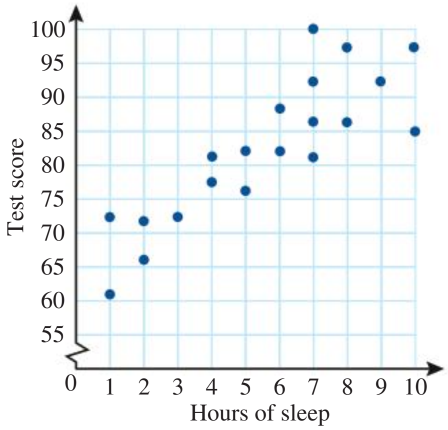
Hours studied	9	1	5	4	3	5	0	1	2
Test score	90	86	84	92	91	100	76	82	85

APPLICATIONS

SF: 5–10	CF: –	CU: –

Example 7 ★**5** The scatterplot shown below, left, maps student's test scores and the average hours of sleep per night. Describe any correlations in the terms of direction, shape and strength, and determine what effect the hours of sleep have on student's test scores, if any.

Question 5



Question 6

★**6** The scatterplot shown above, right is mapping the age of a driver against their reading distance of street signs. Describe any correlations in the terms of direction, shape and strength, and determine what effect the age of the driver makes to the driver's reading distance of street signs, if any.

Example 8 ★**7** The police union are wanting to prove that the number of police serving in an area will lower the incidence of crime. Using the data in the table, create a scatterplot by hand or with technology to determine any correlation between the number of police and the incidence of crime that exists. Describe any correlations in terms of direction, shape and strength, and determine if the police union's belief is correct.

Number of police	15	21	8	14	19	31	17	12	18	9	12	14
Incidence of crime	28	16	36	24	21	19	21	26	22	31	24	26

★8 The CSIRO are examining the effects of fertiliser on the yield per crop. Using the data in the table, create a scatterplot by hand or with technology to determine any correlations that exist between the amount of fertiliser used and the crop yield. Describe any correlations in terms of direction, shape and strength, and determine if the amount of fertiliser has an effect on the crop yield.

Fertiliser (kilograms)	100	125	180	80	250	140	276	112	211
Crop yield (tonnes)	7.2	7.7	8.4	6.4	12.4	8.0	13.0	7.3	10.9

★9 It has been thought that the number of hours of playing video games per day by teenagers negatively affects their overall number of hours of sleep. Using the data in the table, create a scatterplot by hand or with technology to determine any correlations that exist between the hours per day of playing video games and the amount of sleep. Describe any correlations in the terms of direction, shape and strength, and determine if the number of hours playing video games affects the teenagers' sleep.

Number of hours playing video games per day	5	0	3	2.5	6	4	3.5	8	2	5	3	1.5
Number of hours of sleep per day	7	10	8	8.5	6	8	7.5	5	8.5	7.5	8.5	9

★10 A survey was conducted with the school basketball team asking players how many hours per week they practised and the average points per game they scored. Using the data in the table, create a scatterplot by hand or with technology to determine any correlations that exist between the hours per week practising and the average points per week. Describe any correlations in terms of direction, shape and strength and determine if the practice amount affects the points scored.

Hours practising per week	4	10	2	6	3.5	7	9	1	5.5	8
Average points per game	12	18	8	5	9	8	7	14	6	9

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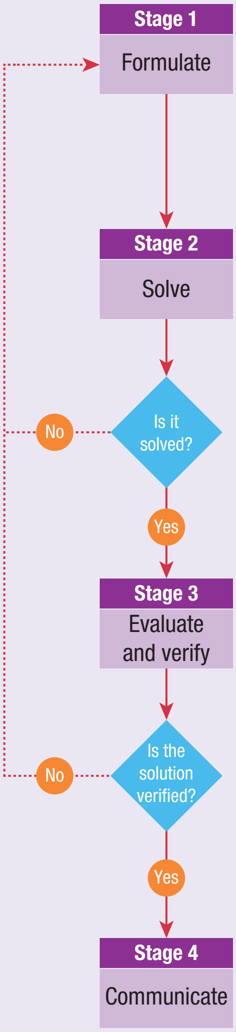
Problem-solving and modelling task

Background: For most people, their arm span is about equal to their height.

Task: To test this idea you are to measure the heights of your classmates and also measure their arm span. Plot this data on a scatterplot to determine if the student's height does actually correlate with their arm span. A method of measuring a person's arm span is to have each student hold a piece of string from left fingertip to right fingertip with their arms outstretched. You can then measure the piece of string with a ruler or measuring tape.

Approach to problem-solving and modelling tasks:



Stage 1: Formulate

1 Decide whose heights and arm spans you will measure.
2 Decide how you will record the data.
3 Formulate the method you will use to obtain your data.

Stage 2: Solve

4 Take your measurements of each student's height and arm span.
5 Record those measurements.
6 Enter the values into technology and create (or draw by hand) a scatterplot.

Stage 3: Evaluate and verify

7 Determine what correlation if any has occurred by observation of the scatterplot.
8 Record the types of correlations that have occurred.

Stage 4: Communicate

9 Create a report that shows the raw data, a scatterplot and a conclusion based on observations made from the scatterplot.

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Chapter checklist

☐ I can plot coordinates on the Cartesian plane in two dimensions.

1 Plot the set of points (2, –2), (1, 1), (–1, 1), (–2, –2), (2, –2) and connect them in order. Name the shape that is made.

2 Luke starts a car cleaning business to make some extra cash. In the first month he earned \$80, in the second month he earned \$55, in the third month he earned \$100, and in the fourth month he earned \$90.

a Complete the table using the values identified in the question.

Month	1	2	3	4
Earning \$				

b Plot the coordinates (month, earning) on a graph.

c Join the coordinates in order of time.

d Determine the month that Luke earned the most money.

e Calculate the total Luke earned in the 4 months.

☐ I can generate tables for linear functions including negative values of x.

3 Complete the table of values for the linear equation $y = 3x - 2$.

x	–2	0	2
y			

4 Delta is a fridge mechanic and she charges a \$40 call out fee and \$70 per hour. This can be mapped by the equation $y = 70x + 40$, where x is the number of hours worked and y is the cost for the client.

a Complete the table of values for Delta's fridge repair costs.

x	1	2	3
y			

b Calculate the cost of hiring Delta for $2\frac{1}{2}$ hours.

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☐ I can graph linear functions by hand and using technology.

5 Use the equation $y = 2x + 3$ to complete the following.

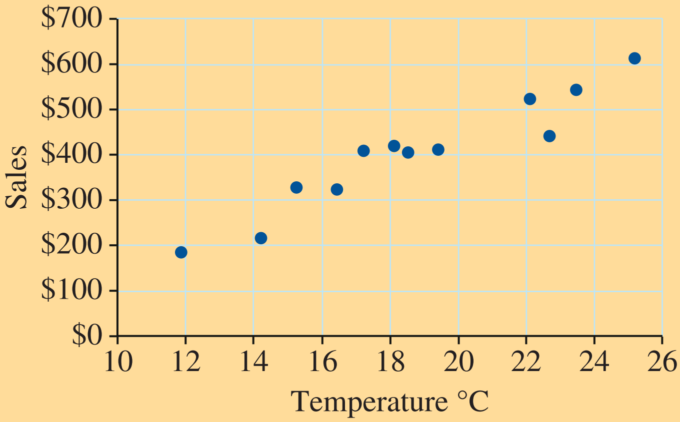
- **a** Complete a table of values for $x = -2, 0, 2$.
- **b** Determine the range of x and y values needed on the graph.
- **c** Draw the graph axes and labels on graph paper.
- **d** Plot the coordinates and draw a line through the points.

6 Graph the equation, $y = 2 - 3x$ using technology.

7 John is a helicopter pilot who charges a $120 fee when picking up clients and then charges $5 per km travelled. The helicopter costs can be described as $y = 5x + 120$, where y is the total helicopter cost and x is the number of kilometres travelled. If John picks up a client at Brisbane airport and takes them to Toowoomba, which is 138 km away, create a graph to determine how much the total fare will be.

☐ I can identify patterns in bivariate data in direction, form and strength.

8 Describe the graph shown in terms of direction, form and strength.



9 Cassie is a maths teacher and she wants to test her idea that there is a relationship between the length of a person's forearm and their height. Cassie has collected measurements from the class as shown in the table. Plot the data from the table to determine if there is a correlation between a person's forearm length and their height. Comment on your observations from the graph.

Length of forearm (cm)	40	35	32	44	46	28	41	38	52	36	47
Height of student (cm)	172	168	165	176	180	162	175	170	183	166	182

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Chapter review

All questions in the review are assessment-style.

Simple familiar

Section 8A

1 Glen has started busking with his guitar to make some extra cash. In the first month he earned $48, in the second month he earned $41, in the third month he earned $63, and in the fourth month he earned $54.

a Complete the table using the values identified in the question.

Month	1	2	3	4
Earning				

b Plot the coordinates (month, earning) on a graph.

c Join the coordinates in order of time.

d Determine the month that Glen earned the most money.

e Calculate the total Glen earned in the 4 months.

2 Charlette sells jewellery at the local markets each week. In week 1 she made $112, in week 2 she made $146, in week 3 she made $134, and in week 4 she made $121. Enter Charlette's income into a table of values and draw the graph. Calculate the total Charlette made in the 4 weeks.

Section 8B

3 Jack is a pool cleaner and he charges a $20 call out fee and $38 per hour for cleaning the pool. This can be mapped by the equation $y = 38x + 20$, where x is the number of hours and y is Jack's total fee.

a Complete the table of values for Jack's fee.

x	0	1	2	3
y				

b Determine after how many hours Jack has earned more than $100.

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4 Fen won \$2000 and spends \$300 per week from her winnings until she has no money left. This can be mapped by the equation $y = 2000 - 300x$, where x is the number of weeks and y is the amount of money Fen has remaining. Complete the table of values for Fen's remaining winnings.

x	1	3	5
y			

Section 8C

5 Ian has purchased a mobile crane for \$90 000. The crane depreciates (goes down in value) by \$12 500 per year and its value can be described as $y = 90\,000 - 12\,500x$, where y is the crane's value and x is the number of years.

a Graph the equation of his crane's value.

b Determine after how many years the crane will be worth under \$16 000.

6 Zac is a taxi driver who charges a \$6.50 fee when picking up clients and then charges \$0.90 per km travelled. The taxi fare can be described as $y = 0.9x + 6.50$, where y is the total taxi fare and x is the number of kilometres travelled. If Zac picks up a client at Cairns airport and takes them to the city, which is 8 km away, create a graph and determine how much the total fare will be.

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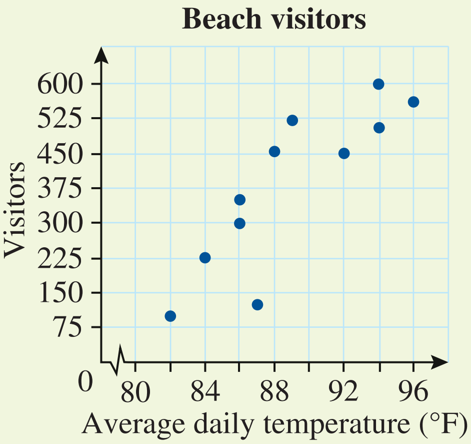
Section 8D

7 The scatterplot shown maps the daily temperature at the beach and the number of visitors to the beach. Determine if any correlation exists in terms of direction, shape and strength. Comment on whether a relationship exists between the number of visitors attending the beach and the daily temperature.



8 Climate scientists have been mapping the temperatures at Antarctica, and they have recorded the average yearly temperature and the area of the ice. Determine if any correlation exists in terms of direction, shape and strength. Comment on whether a relationship exists between the average yearly temperature and the area of the ice.

Area of Antarctic ice (‘000 000 km²)	14	13.8	13.3	13.5	13.2	12.9	13.1	12.7	12.5
Average yearly temperature (°C)	8.3	8.2	8	7.8	7.9	7.5	8	7.8	7.3

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9 Line of best fit

Maths for a firefighter: David Southey

David Southey plays a crucial role in the front-line response to emergency and disaster situations, defending people, property and the environment from harm as a firefighter.

Tell us a bit about your job. What does a typical day look like?

No day is the same. Beyond fighting fires, we work at heights and enclosed spaces, as we negotiate a variety of emergency and non-urgent situations.

As a firefighter, I value and embody integrity, respect, courage, trust and loyalty as I work in a team to help people in their time of need, and I have a desire to serve the community.

Every day is different we never know what job is coming or when. Our station is open 24 hours 7 days a week 365 days a year. As part of our job we maintain our station (cleaning, mowing etc.), our equipment (servicing and operational readiness) and ourselves (skills training and physical fitness). At any time, we could respond to a wide variety of jobs.

What maths did you study at school?

At school I studied Maths B.

How do you use maths in your job?

Maths is used a lot in our work and in many different jobs. We use maths to work out how much air we have in our cylinders on our breathing apparatus. Also, maths is used to determine the Safe Working Load SWL, which is how much load we can put on our equipment.

We use mechanical advantage and maths to assist us in hauling people to safety. We also use maths to work out loads on anchors (things we attach our rescue lines to) and our lines or slings. We also use maths to calculate how we use our pumps to pump water to the fire.

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In this chapter

Syllabus reference

Unit 4 Topic 1 Bivariate graphs

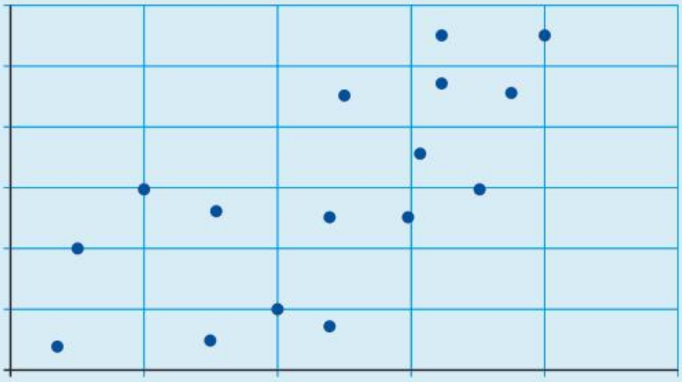
Line of best fit (10 hours)

In this sub-topic, students will:

- identify the dependent and independent variable
- determine the line of best fit by eye
- use technology to determine the line of best fit **[complex]**
- interpret relationships in terms of the variables **[complex]**
- use technology to calculate the correlation coefficient (an indicator of the strength of linear association) **[complex]**
- use the line of best fit to make predictions, both by interpolation and extrapolation **[complex]**
- recognise the dangers of extrapolation **[complex]**
- distinguish between causality and correlation through examples **[complex]**.

Pre-test

1 Draw a line of best fit for the following distribution of points so that there are roughly as many points above the line and below.

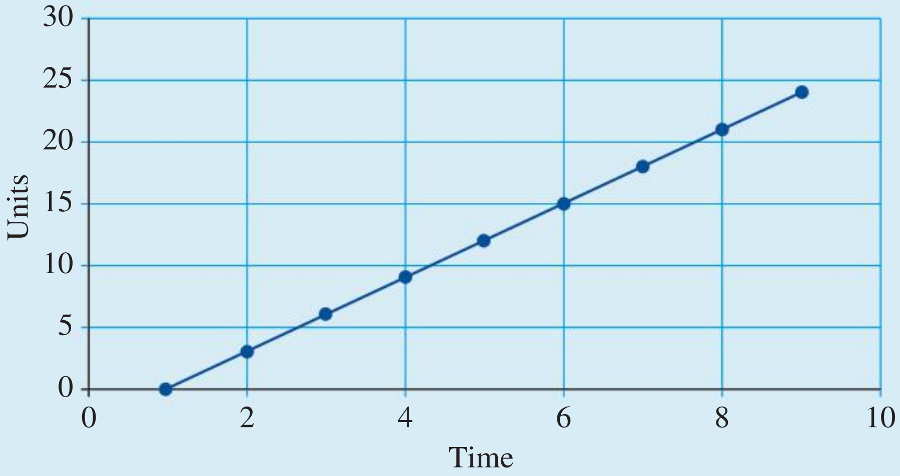


Hint Use a clear ruler so you can see when there are approximately the same number of dots either side of the line.

2 Create a scatterplot using the following dataset observing different students in a class rolling 2 dice.

Student	1	2	3	4	5	6	7	8	9	10
Roll 1	5	2	3	2	4	1	3	6	4	2
Roll 2	1	2	6	4	1	6	3	4	3	5

3 This is a graph of a quantity against time.



a Determine the number of units at time 6.

b For every increase in 1 unit of time, determine the increase in the number of units.

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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9A Identifying the dependent and independent variables

LEARNING GOAL

- Identify the dependent and independent variable

Why is understanding the independent and dependent variable essential?

The previous chapter introduced bivariate data and the idea of an association between two variables and that they could be correlated. To understand this further we need to consider a vital difference between the variables, that a change in one variable (the independent variable) may cause a change in the other (the dependent variable). This is very important for research and development in science, health and technology, as well as in education, economics and social science.

WHAT YOU NEED TO KNOW

- A **variable** is a measure that may change.
- An **independent variable** is the variable in an experiment that we change or select in order to see what effect it has, or which changes naturally, like time or the weather. It is independent of what we are going to measure.
    - 'Independent' here does not mean independent of the researcher, because it is the thing that the researcher changes or allows to change.
    - An independent variable that changes naturally may need to be measured, e.g. the temperature of the environment or amount of rainfall. Often it is a characteristic of a location that we select, such as sea temperature. The fact that a variable is measured is therefore not the only thing that determines whether it is independent or dependent.
- A **dependent variable** 'depends' on the independent variable.
    - It is what we measure because we think it is changed by the independent variable. It could be something that is affected in an experiment or discovered in a survey, e.g. the number of organisms in locations that we select because those locations have differences in the independent variable.
    - The dependent variable reacts to the independent variable.
- The independent variable is the cause and the dependent variable is the effect.
- When graphing, the independent variable is placed on the horizontal (x-axis) and the dependent variable is on the vertical (y-axis).
- A line of best fit could be straight (the relationship is **linear**) or curved (the relationship is **non-linear**). In this course we will deal mainly with linear relationships.

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A scientist applying a treatment (the independent variable) will then measure the effect on plant growth (the dependent variable).

Example 1 Identifying the independent and dependent variable

Sigrid is interested in how different types of stress affects a patient's heart rates.

a What is the independent variable?

b What is the dependent variable?

WORKING	THINKING
a The independent variable is the type of stress, as this can be changed by the researcher.	What can be changed by the researcher?
b The dependent variable is the heart rate, as it is dependent on the type of stress.	What will be affected? What will be responding? What will be changed by the independent variable?

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Exercise 9A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** A variable is a measure that may ___________.
- **b** When graphing, the independent variable is placed on the ___________ and the dependent variable is on the ___________.
- **c** The ___________ ___________ is actually being measured and what is affected during the research.
- **d** An ___________ ___________ is the variable that is changed or allowed to change by the researcher.
- **e** The values of both variables may ___________; however, the difference is that the value of the independent variable is ___________ by the researcher, while the value of the dependent variable only changes in ___________ to the independent variable.

Example 1

2 Define using everyday language:

- **a** variable
- **b** independent variable
- **c** dependent variable

APPLICATIONS

SF: 3–8	CF: –	CU: –

★**3** Alix is researching a new medication for asthma. She needs to alter the amount of medication given to see if the breathing rate changes.

- **a** What could be the independent variable? Give a reason.
- **b** What could be the dependent variable? Give a reason.

★**4** Michael is wanting to see how much fertiliser is required to helps his plants produce more beans.

- **a** What could be the independent variable? Give a reason.
- **b** What could be the dependent variable? Give a reason.

Hint The independent variable is the variable that is not affected by the other variable in the experiment.

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★**5** Sonia is studying how rising ocean temperatures are impacting on the amount of coral in the Great Barrier Reef.

a What could be the independent variable? Give a reason.

b What could be the dependent variable? Give a reason.

★**6** Noah is researching how classical music can increase children's reading ability. He sets a variety of students to listen to classical music for different times and then collects their reading results.

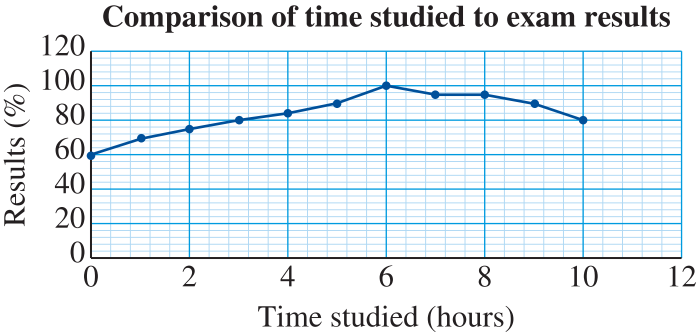
a What could be the independent variable? Give a reason.

b What could be the dependent variable? Give a reason.

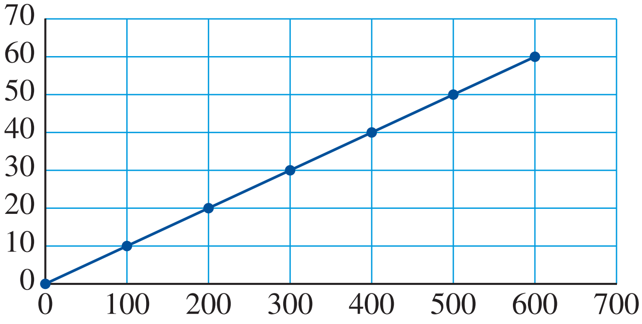
★**7** The graph shows how the number of hours a group of students studied affected their exam results.

a What could be the independent variable? Give a reason.

b What could be the dependent variable? Give a reason.



★**8** Kaity has been delivering pamphlets to gain some extra pay. She graphed the number of pamphlets delivered and her pay. However, she has lost her titles and labels from her graph and cannot remember which is the dependent variable and which is the independent variable.



a What could be the independent variable? Give a label.

b What could be the dependent variable? Give a label.

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9B Determining the line of best fit SIMPLE/COMPLEX

LEARNING GOALS

Note: this section contains both simple and complex subject matter

- Determine the line of best fit by eye
- Use technology to determine and create the line of best fit [complex]

Why is it essential to be able to determine the line of best fit by eye?

The line of best fit is used to examine the relationship between two variables. It is a straight line drawn so there is an approximately equal number of points above and below the line on a scatterplot. The line of best fit will then tell us if the relationship between the variables is proportional; that is, if a change in the independent variable causes a proportional change in the dependent variable.

Learning how to find the line of best fit by eye, with the help only of a ruler, gives you an understanding of what a line of best fit is and what it means. The most accurate way to find the line of best fit is to enter the data into your calculator or computer and let the software do the work.

WHAT YOU NEED TO KNOW

- In surveys and experiments, most bivariate data collected results in a **scatterplot** rather than a neat line graph. This is because in the real world it can be hard to get accurate measurements and hard to prevent other factors from affecting your measurements.
- When graphing, the **independent** variable is placed on the x-axis and the **dependent** variable is on the y-axis.
- A **line of best fit** (or trend line or regression line) is a straight line that best represents the data on a scatterplot.
  - When drawing by eye, a straight line is drawn through the mean, balancing an approximately equal number of points above and below the line. It may pass through some of the points or none of the points on the graph.
  - The line of best fit shows the general direction of the change in the dependent variable in responding to change in the independent variable. It is a way of removing the influence of all those other factors or random effects that also influence each measurement of the dependent variable.
  - When drawing a line of best fit by eye, start by placing your ruler on its edge in the position you think the line should be, so you can easily see the dots on either side. Mark the position of both ends of the line, then place the ruler flat to draw the complete line so that it is straight.

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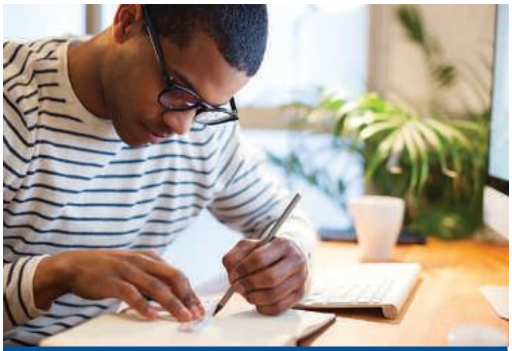

Drawing a line of best fit by eye helps you understand the relationship between two variables.

Example 2 Drawing the line of best fit by eye

Liv is working in a café and has suspected that there was a relationship between the outside temperature and her sales of hot chocolate. She has recorded the data in a table.

Outside temperature °C	20	18	10	11	13	10	11	8	9	4	2	1
Number of hot chocolates sold	0	1	8	6	7	7	10	13	14	16	18	20

a Determine the independent and dependent variables.

b Draw a scatterplot by hand.

c **i** Calculate the mean ($\bar{x}$) of the x-axis.

  ii Calculate the mean ($\bar{y}$) of the y-axis.

d Draw the line of best fit by eye and draw in the line.

WORKING **THINKING**

a Independent variable is Outside temperature, as it is what is changing naturally in the environment and is not affected by sales of hot chocolate.
Dependent variable is Number of hot chocolates sold, as that is what is expected to change when the temperature changes.

Determine the independent (thing that is controlled by researcher or environment) and dependent variables (thing that is predicted to change based on the independent variable).

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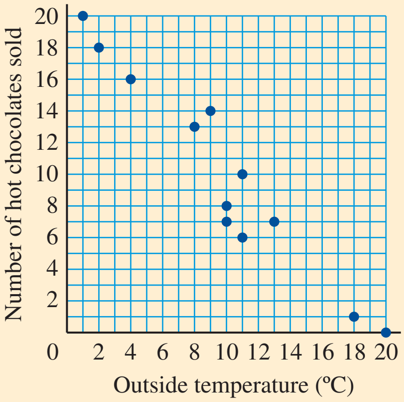
WORKING **THINKING**

b



Use grid paper. Place the independent variable on the *x*-axis. Place the dependent variable on the *y*-axis. Plot the points.

c

i $$\bar{x} = \frac{20 + 18 + 10 + 11 + 13 + 10 + 11 + 8 + 9 + 4 + 2 + 1}{12}$$

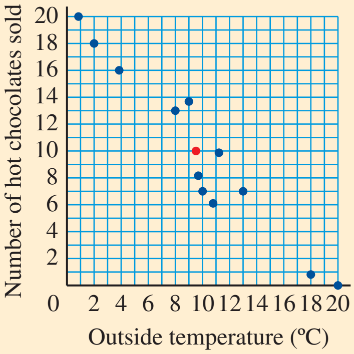
$$\bar{x} = 9.75$$

Find the mean ($\bar{x}$) of the *x*-axis (Outside temperature).

ii $$\bar{y} = \frac{0 + 1 + 8 + 6 + 7 + 7 + 10 + 13 + 14 + 16 + 18 + 20}{12}$$

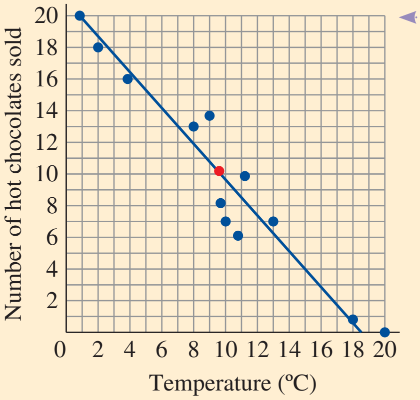
$$\bar{y} = 10$$

Find the mean ($\bar{y}$) of the *y*-axis (Number of hot chocolate).



Plot the mean (9.75, 10).

d



Using your ruler, draw a line through the mean in the same directions as most of the data points. Ensure that as many data points are above the line as below it.

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Example 3 Using a spreadsheet to create the line of best fit [complex]

Nate has noticed the relationship between the age of a person and the amount of emails they receive per day.

Age (years)	50	64	23	25	24	72	26	47	60	30	70	28
Number of emails per day	14	10	28	27	24	5	27	15	7	28	7	26

Use a spreadsheet to:

a graph the data in a scatterplot

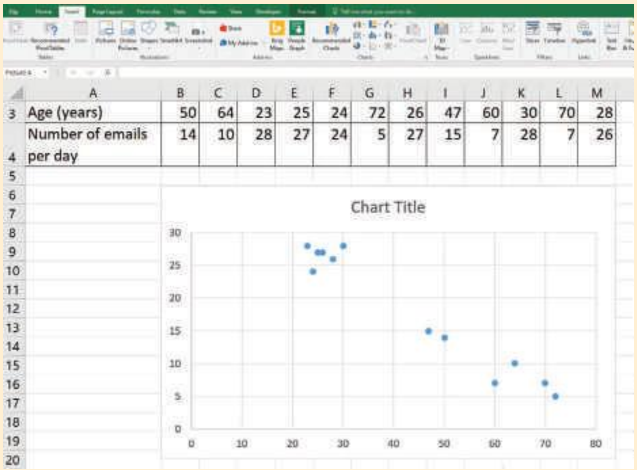
b create the line of best fit.

WORKING **THINKING**

a

	A	B	C	D	E	F	G	H	I	J	K	L	M
2													
3	Age (years)	50	64	23	25	24	72	26	47	60	30	70	28
4	Number of emails per day	14	10	28	27	24	5	27	15	7	28	7	26
5													

Type data into spreadsheet cells.



	A	B	C	D	E	F	G	H	I	J	K	L	M
3	Age (years)	50	64	23	25	24	72	26	47	60	30	70	28
4	Number of emails per day	14	10	28	27	24	5	27	15	7	28	7	26

Select the data.
Click on the Insert tab > Charts > Scatter.

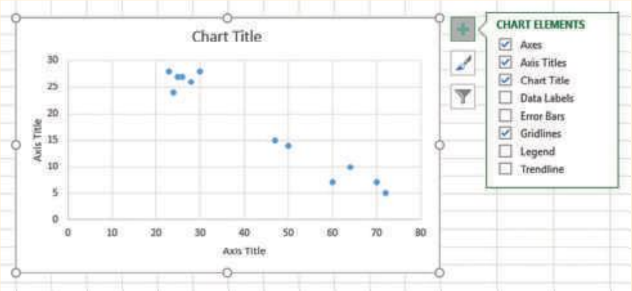
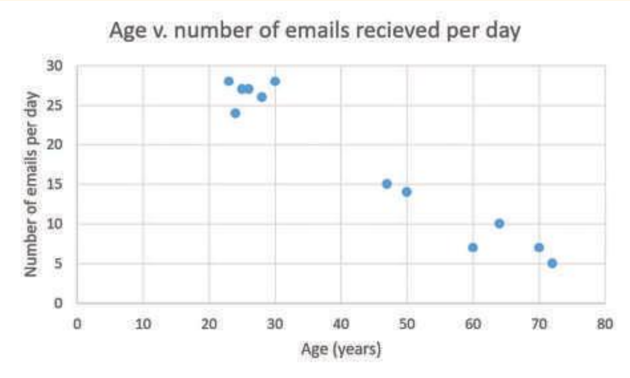


CHART ELEMENTS
☑ Axes
☑ Axis Titles
☑ Chart Title
☐ Data Labels
☐ Error Bars
☑ Gridlines
☐ Legend
☐ Trendline

Click on chart to bring up the + icon.
Click Axis Titles.

WORKING **THINKING**



Insert titles.

b

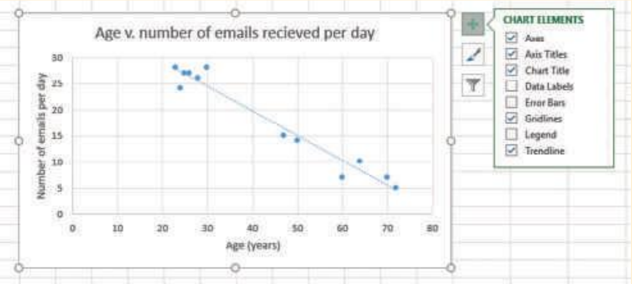
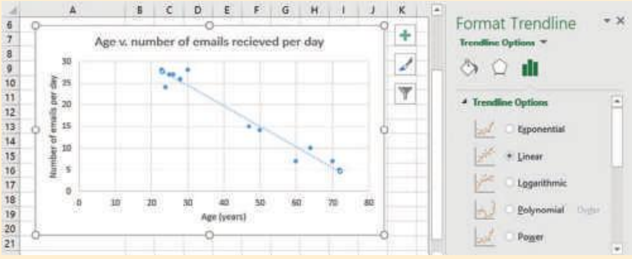


CHART ELEMENTS
☑ Axes
☑ Axis Titles
☑ Chart Title
☐ Data Labels
☐ Error Bars
☑ Gridlines
☐ Legend
☑ Trendline

To find and create the line of best fit, click on chart to bring up the $\boxed{+}$ icon. Click Trendline.



Format Trendline
Trendline Options
Trendline Options
○ Exponential
● Linear
○ Logarithmic
○ Polynomial Order
○ Power

Right click the trendline Format Trendline. Adjust the settings in the Format Trendline pane to suit your requirements. Copy and Paste your graph to your report.

Desmos Activity: See the interactive textbook for an example and activity based on using the Desmos graphing calculator to fit a line to a scatterplot.

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Exercise 9B

Note: *It is suggested in questions 3 and 4 that the line of best fit be done by eye. The rest of the questions in the exercise could be done by eye, with a spreadsheet, with Desmos, or with another technology. Check with your teacher.*

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** The line of best fit is used to examine the ________ between two variables.
- **b** A line of best fit is a _____ line that best represents the data on a _____ plot.
- **c** The line of best fit is also called the ________ line or the ________ line.
- **d** The line of best fit is intended to ________ the ________ of the data.
- **e** When graphing, the ________ variable is placed on the x-axis and the ________ variable is on the y-axis.
- **f** The line of best fit is intended to highlight the ________ between the ________ and represent the ________ of the data.

2 Using everyday language for each of the following.

- **a** Explain the process of drawing a scatterplot by hand.
- **b** Explain the process of finding the line of best fit by eye.

APPLICATIONS

SF: 3–5	CF: 6–10	CU: –

Example 2

3 Felicity is studying a type of soft coral that is often found in cooler waters, and she has collected the following data in an attempt to find out if the coral is more plentiful in cooler southern seas compared to tropical waters.

Ocean temp (°C)	19	18	20	27	30	17	27	31	24
Number of corals per 100 m^2	140	160	122	40	12	154	19	2	80

- **a** Determine the independent and dependent variables.
- **b** Create a scatterplot by hand.
- **c** Calculate the mean of the ocean temperature and the mean number of soft corals and plot the coordinates on the graph.
- **d** Draw the line of best fit by eye.

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4 Sonia is training for a marathon and is recording her latest times.

Time (min)	25	40	35	37	90	40	115	35	95
Distance (km)	6	10	15	10	20	15	25	10	20

a Determine the independent and dependent variables.
b Create a scatterplot by hand.
c Calculate the mean of time taken and the mean distance run and plot the coordinates on the graph.
d Draw the line of best fit by eye.

XLS **Spreadsheet Ex9BQ5–7** The data is available in a spreadsheet in the Interactive Textbook.

See the note at the start of this exercise about which method to use for these questions. The data is available in a spreadsheet in the Interactive Textbook.

5 David is wanting to see how much fertiliser helps his plants produce the most beans.

Amount of fertiliser (mL)	10	10	15	30	5	50	2	55	60
Number of beans per plant	6	8	9	18	5	28	3	32	40

a Determine the independent and dependent variables.
b Create a scatterplot of the data.
c Calculate the mean of the amount of fertiliser and the mean of the number of beans and plot the coordinates.
d Draw the line of best fit.

Hint Remember to have approximately half of the points above and half of the points below the line of best fit.

Example 3 ★**6** Deb is wanting to see if there is a relationship between a group of people's annual salary and how many years of work experience they had. She has collected the following data.

Years of experience	15	45	35	25	5	30	30	40	45	20	3	3	5	30	25
Annual salary ('000)	25	80	100	30	80	55	85	110	90	80	25	20	30	100	75

a Create a scatterplot of the data.
b Create the line of best fit.

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★7 Kay has been a nurse at a health centre for over 20 years and has noticed a relationship between children's height at 2 years and at 18 years. She has collected the following data.

a Create a scatterplot of the data.

b Create the line of best fit.

2 yo height (cm)	80	83	87	81	83	88	84	85	86	83	85	80	88
18 yo height (cm)	157	167	171	160	162	172	167	170	170	164	165	159	171

8 Char volunteers as a surf life saver and has noticed a relationship between the average temperature and the amount of people on the beach. She collects the data below with the thought that it may help in planning how many lifesavers to have on duty according to the temperature forecast for each day.

a Create a scatterplot of the data.

b Create the line of best fit.

Average temperature (°C)	23	28	25	34	29	35	27	37	26	29	28	30	36
Number of people on the beach	70	85	74	87	87	105	80	112	78	88	84	92	110

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9 Mel is a Personal Trainer and has noticed a relationship between the hours of exercise people do and their resting heart rate (BPM).

a Create a scatterplot of the data.

b Create the line of best fit.

Exercise (hours per week)	0.2	0.4	2.3	9	0.5	0.7	6	8	4	1	1.2	1.5	2	8.5	5	3
Resting heart rate (BPM)	81	80	69	45	81	78	55	47	64	75	71	70	72	45	61	66

10 Leanne's writing a report about the relationship between a city's height (m) above sea level and their average yearly rainfall (mm).

a Create a scatterplot of the data.

b Create the line of best fit.

City	**Average yearly rainfall (mm)**	**Height above sea level (m)**
Armidale	1000	80
Orange	800	230
Bathurst	180	670
Goulburn	170	640
Toowoomba	275	600
Canberra	280	580
Alice Springs	300	580
Ballarat	690	450
Tamworth	680	400
Kalgoorlie	265	380

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9C Interpreting relationships between variables COMPLEX

LEARNING GOAL

- Interpret relationships in terms of the variables

Why is being able to interpret relationships between variables essential?

Graphing bivariate data in scatterplot and fitting a line to it is just a display. Saying what it means, in other words interpreting the relationship between variables, is an integral part. This is how you draw conclusions from a bivariate dataset. Without an understanding of this, you will not be able to understand your research and could present the incorrect results from your data.

Interpreting the relationship between variables ensures that you present the correct results.

WHAT YOU NEED TO KNOW

- **Regression** analysis is the overall term for the process of identifying the relationships between an independent variable and a dependent variable using a scatterplot and line of best fit.
  - Regression analysis helps us understand how the typical value of the dependent variable changes when the independent variable is altered.

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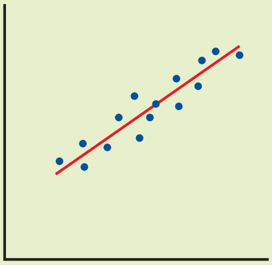
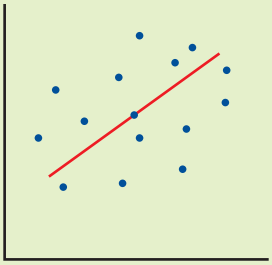
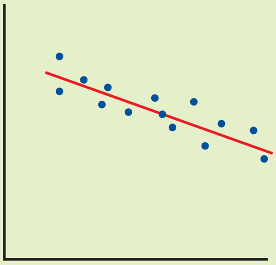
- A **correlation** is a measure of the strength of the linear relationship between two variables.
  - The correlations can be described as strong, moderate, weak or none depending on how close the points are to the line of best fit.
  - A positive correlation is a relationship between two variables that move either positively or negatively together.
    - ○ As one variable increases, the other variable increases.
    - ○ As one variable decreases, the other variable decreases.
- When looking for a relationship, focus on:
  - the closer the points cluster to the line of best fit, the stronger the relationship that exists between the two variables, and there is most likely a correlation (i.e. **strong correlation**)

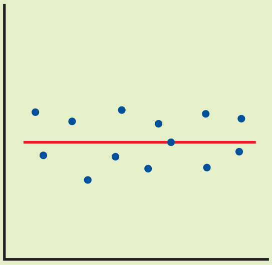


  - the further the points are from the line of best fit, the weaker the relationship that exists between the two variables, and therefore the less likely it is that there is a correlation (i.e. **weak correlation**).



- When the points rise from lower left to upper right, as in the graphs above, there is a **positive correlation** (if x increases, y increases).
- When the points fall from upper left to lower right, there is a **negative correlation** (if x increases, y decreases).
- When the line of best fit is horizontal, there is **no correlation**.





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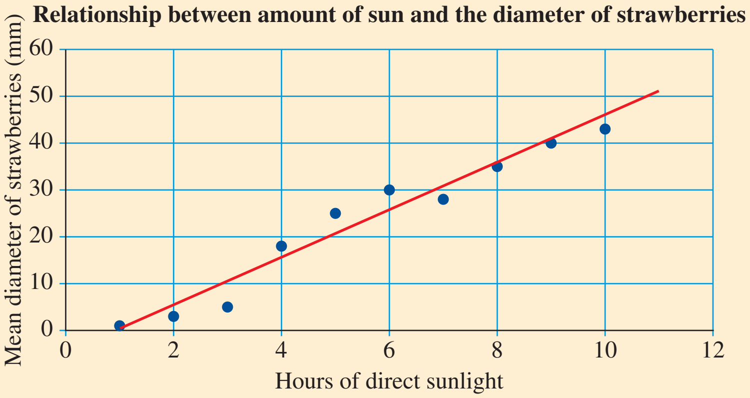
Example 4 Interpreting relationships in terms of the variables [complex]

Erica owns a strawberry farm. She is collecting data to see if there is a relationship between the amount of direct sun and the diameter of her strawberries. She has exposed different plants to different hours of direct sunlight. All plants are then placed in the same shade to make up to a total of 12 hours of daylight. She gathered the following data after measuring the strawberries and taking the mean diameter.

Amount of direct sunlight per day (hours)	1	2	3	4	5	6	7	8	9	10
Mean diameter of strawberries (mm)	1	3	5	18	25	30	28	35	40	43

She has created this scatterplot and line of best fit for the data.



a Determine which is the independent variable and which is the dependent variable.

b Describe any correlations in terms of positive or negative, linear or non-linear and strong, moderate, weak or none.

c Interpret the relationship between the amount of sunlight and the diameter of the strawberries and explain what it means for the strawberry farm.

	WORKING	THINKING
a	The independent variable is the amount of direct sunlight, as Erica changes it for the experiment. The dependent variable is the diameter of the strawberries, as the amount of sunlight can change the growth of the strawberries.	Which variable possibly causes a change in the other variable?
b	The correlation is positive, linear and strong.	The line of best fit goes up from left to right, therefore it is positive. The points follow a definite line, therefore it is linear. The points are close to the line of best fit, therefore there is a strong correlation.
c	There is a strong relationship between the amount of sunlight and the diameter of the strawberries. As the amount of sunlight increases, the diameter of the strawberries increases. More sunlight causes larger strawberries to grow. If Erica wants larger strawberries, she should provide more sunlight to her produce.	As there is a strong correlation there is a relationship between the variables, which can be interpreted as direct sunlight causing larger strawberries to grow.

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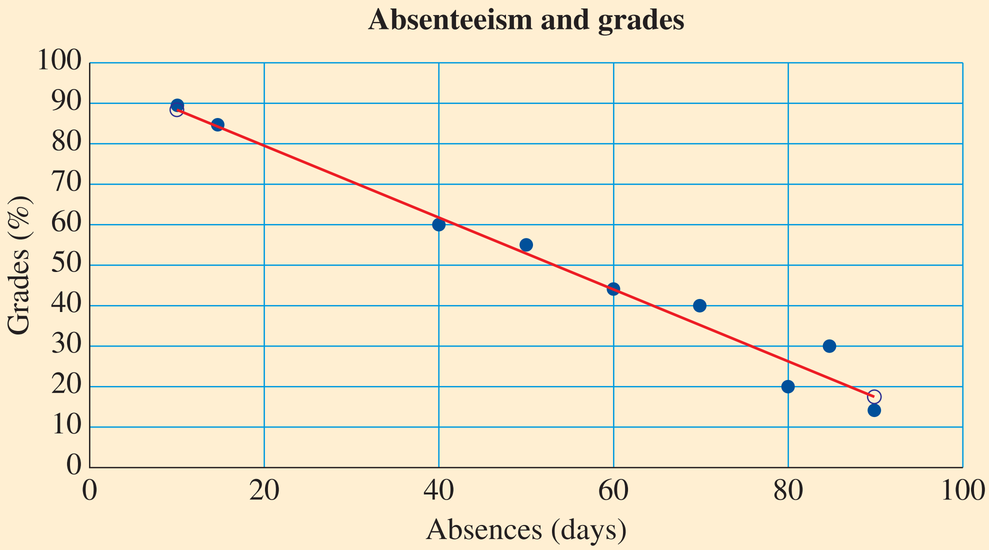
Example 5 Interpreting relationships in terms of the variables [complex]

Ty is a deputy of a school and is noticing a relationship between student absences and their grades. He has created the following scatterplot and line of best fit from the data.



a Determine which is the independent variable and which is the dependent variable.

b Describe any correlations in terms of positive or negative, linear or non-linear and strong, moderate, weak or none.

c Interpret the relationship between the number of days absent and the grades achieved.

d Explain how the deputy might use this information.

WORKING	THINKING
a The independent variable is the number of days absent, as it is not affected by the grades.	What is the cause? See the x-axis as a hint.
The dependent variable is the grades, as it is reasonable to expect that decreasing the time spent learning through absences will lead to a decrease in grades.	What is the effect? See the y-axis as a hint.

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WORKING	**THINKING**
b The correlation is negative, linear and strong.	The line of best fit goes down from left to right, therefore it is negative. The points follow a definite line and is therefore linear. The points are close to the line of best fit, therefore there is a strong correlation.
c There is a strong relationship between the amount of days absent and the students' grades. The correlation is negative, as when days absent increase, the grades decrease.	As there is a strong correlation there is a relationship between the variables.
d The deputy principal could use the information to show students and parents that absenteeism strongly affects grades.	How can this information assist the deputy principal?

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Exercise 9C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** __________ analysis is the process for identifying the __________ between a dependent variable and an independent variable.
- **b** A correlation is a measure of the __________ of the linear relationship between two variables.
- **c** A positive correlation is a relationship between two variables that move either __________ or __________ __________.
- **d** A negative correlation is a relationship between two variables where one variable __________ as the other __________.

APPLICATIONS

SF: –	CF: 2–8	CU: 9

Example 4, 5

2 Lee organises a rodeo once a year and is wondering if there is a relationship between the age of the rodeo riders and the amount of injuries at the rodeo.

Age of riders (years)	35	30	25	15	20	23	32
Number of injuries	18	14	13	8	12	14	16

- **a** Determine which is the dependent variable and which is the independent variable.
- **b** Create a scatterplot.
- **c** Determine the line of best fit.
- **d** Describe any correlations in terms of positive or negative, linear or non-linear and strong, moderate, weak or none.
- **e** Interpret relationships between the age of the rider and the injuries sustained.
- **f** Explain how a rodeo rider might use this data.

Hint When the points rise in a line going from lower left to upper right, there is a positive correlation (if *x* increases, *y* increases).

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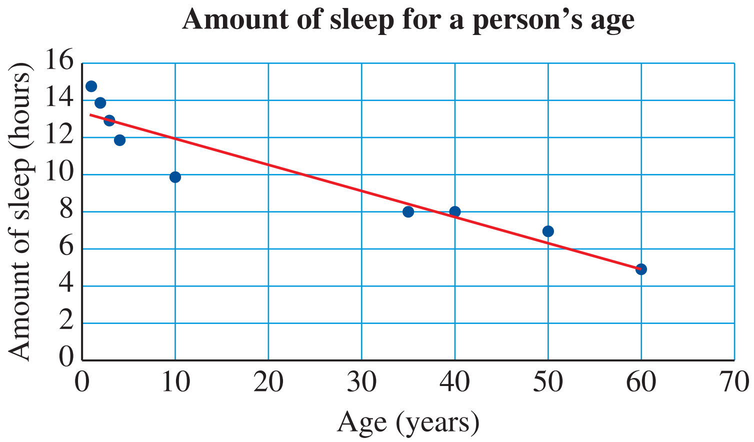
3 Eleanora has collected data to see if there is a relationship between a person's age and the number of hours of sleep they have each day. She has created this scatterplot and line of best fit from the data.



a Determine which is the dependent variable and which is the independent variable.

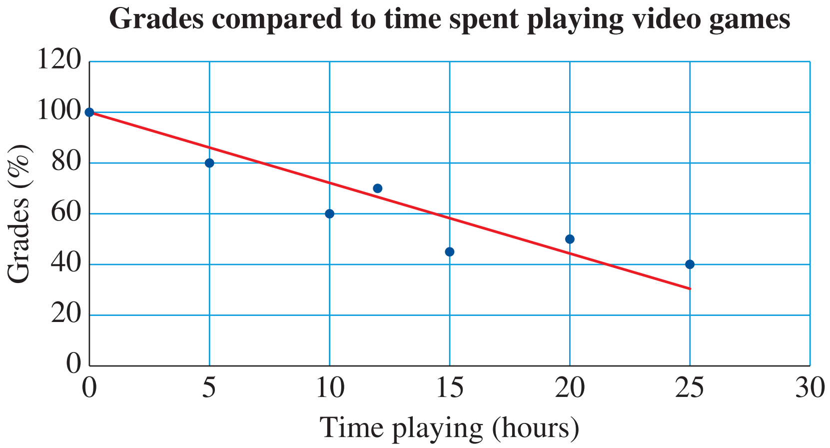
b Describe any correlations in terms of positive or negative, linear or non-linear and strong, moderate, weak or none.

c Interpret the relationship between the age and amount of sleep.

d Explain how this information might be used.

Hint When the points fall in a line from upper left to lower right, there is a negative correlation (if *x* increases, *y* decreases).

4 A teacher is trying to prove to his students that there is a relationship between hours spent playing video games and their grades. She has created this scatterplot and line of best fit from the data.



a Determine which is the dependent variable and which is the independent variable.

b Describe any correlations in terms of positive or negative, linear or non-linear and strong, moderate, weak or none.

c Interpret relationships between time spent playing games and the grades achieved.

d Explain how the teacher could use this information.

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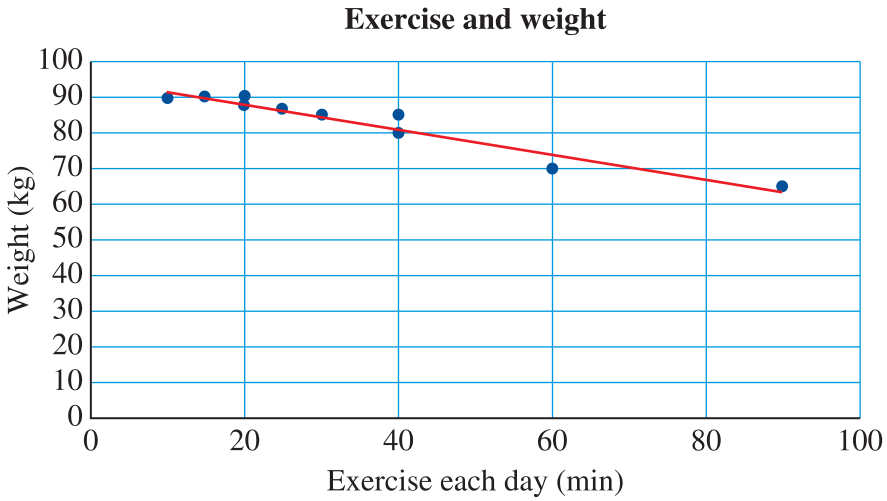
5 A personal trainer is trying to persuade their clients to exercise more. He has created this scatterplot and line of best fit from the data he has gathered.



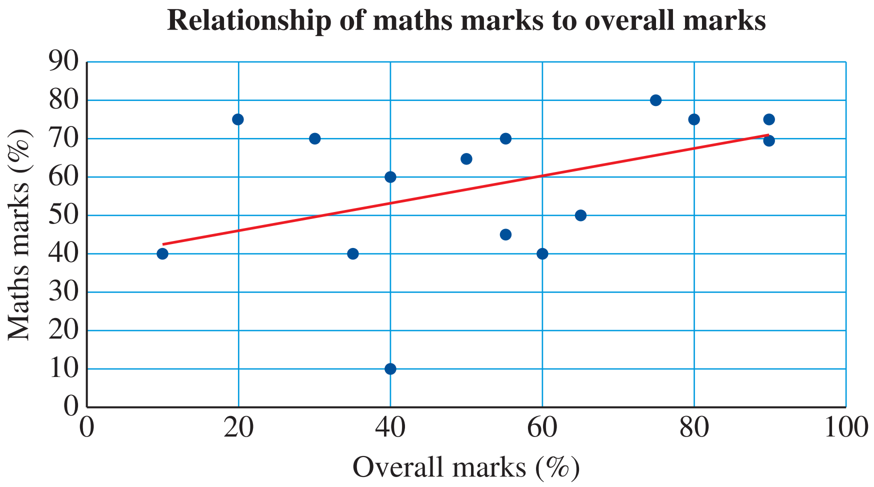
a Determine which is the dependent variable and which is the independent variable.

b Describe any correlations in terms of positive or negative, linear or non-linear and strong, moderate, weak or none.

c Interpret the relationship between the hours spent exercising and weight.

d Explain how the personal trainer could use this information.

★**6** A maths teacher believes that their students' end-of-year marks in maths also relate to their overall marks in all subjects. He has created this scatterplot and line of best fit from data collected from his students.



a Interpret the relationship between the overall marks and maths marks.

b Explain how the teacher could use this information.

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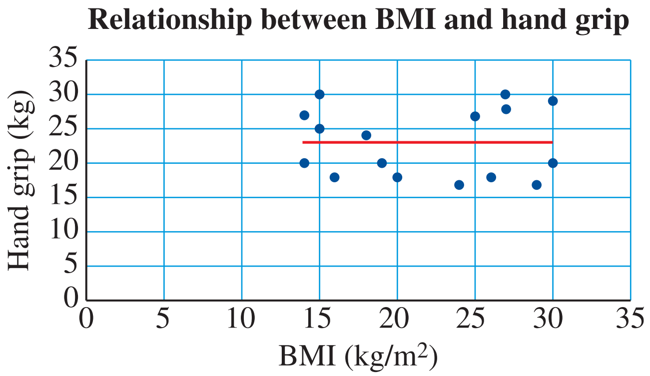
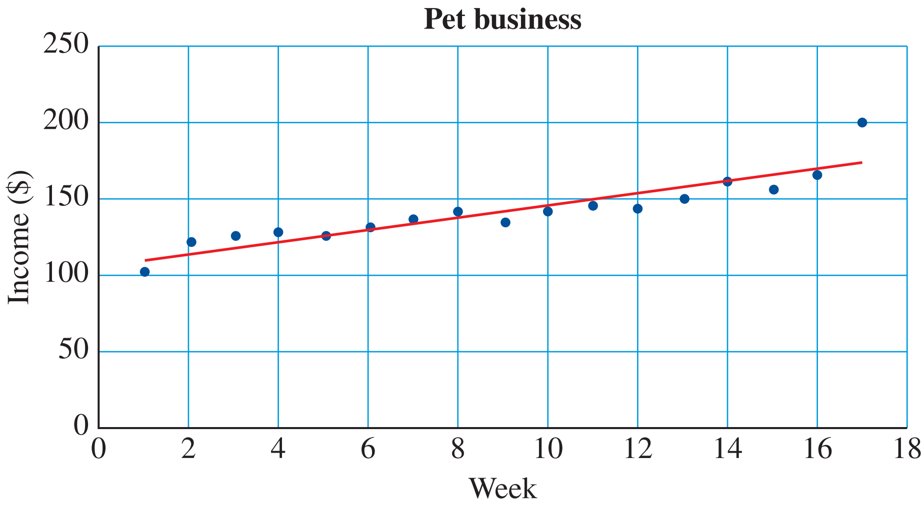
★7 A group of people were measured by a researcher to see if there is a relationship between hand grip strength and BMI. She has created this scatterplot and line of best fit from the data.



a Interpret relationships between the BMI and hand grip.

b Explain how this information could be used.

★8 Will has started his own business, walking people's dogs. He is hoping to see strong positive relationship between each week of his business and his weekly income. He has created this scatterplot and line of best fit from data he has collected.



a Interpret relationships between each week and income.

b Explain how Will could use this information.

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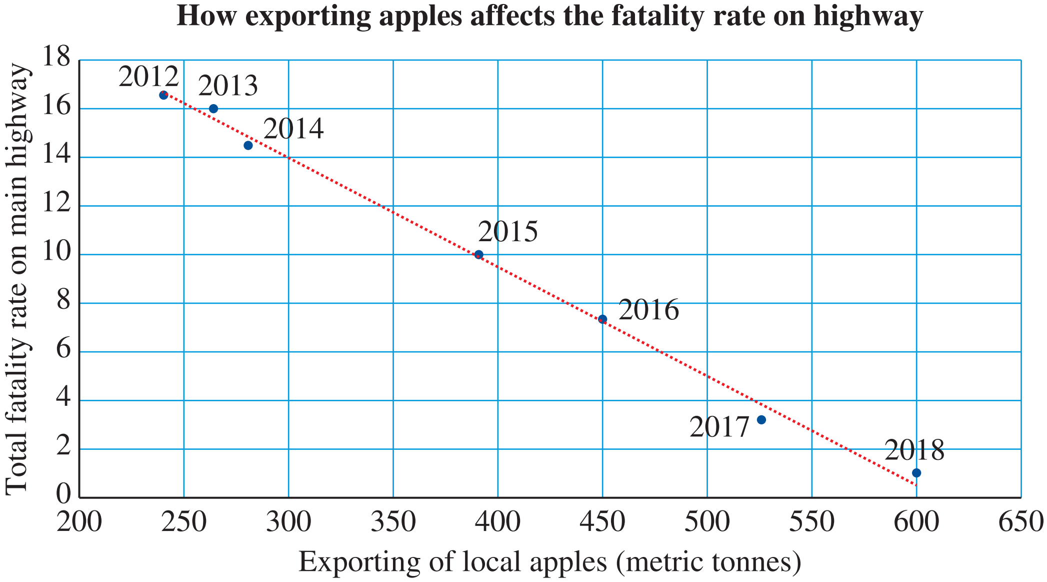
★**9** A Queensland farmer has informed his local newspaper that he has discovered a relationship between exporting local apples and the fatality rate on their main highway each year. He has collected the following data to illustrate his theory.



a Interpret relationships in terms of the two variables.

b Explain what this means and justify whether or not you think the interpretation is valid.

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9D Calculating the correlation coefficient using technology

COMPLEX

LEARNING GOAL

- Use technology to find the correlation coefficient (an indicator of the strength of linear association)

Why is being able to find the correlation coefficient essential?

So far, we have determined whether two variables are correlated by the appearance of the line of best fit. Working mathematically and scientifically however requires working with numbers – in this case, a measurement of the size of the correlation. This is essential in research and development, in all subjects and fields.

The correlation coefficient shows the degree to which points on a scatterplot lie on a line of best fit.

WHAT YOU NEED TO KNOW

- Correlation tells us about the strength of the linear relationship between two variables.
  - To measure correlation and the strength of the relationship we calculate the correlation coefficient.

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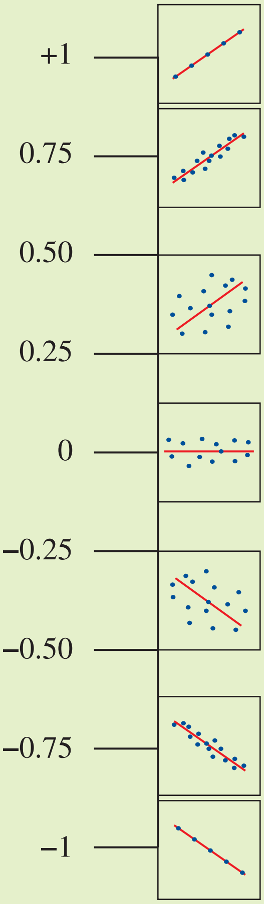
- The **correlation coefficient** is based on the degree to which points on a scatterplot lie on a line of best fit.
  - The correlation coefficient has the symbol (r), which measures the strength and direction of a linear relationship between variables.
  - The value of r is always between +1 and –1.
  - To interpret the correlation coefficient, use the following as a guide:

Value of correlation coefficient **Typical scatterplot** **Interpretation**



1: perfect positive correlation

0.75 to 0.99: strong positive correlation

0.5 to 0.74: moderate positive correlation

0.25 to 0.49: moderate positive correlation

Above 0 up to 0.24: weak or no positive correlation

0: no correlation

Below 0 down to –0.24: weak or no negative correlation

–0.25 to –0.49: moderate negative correlation

–0.5 to –0.74: moderate negative correlation

–0.75 to –0.99: strong negative correlation

–1: perfect negative correlation

- To calculate the correlation coefficient, use a scientific calculator, a spreadsheet or other technology.

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Example 6 Using technology (scientific calculator) to find the correlation coefficient [complex]

Dan is a used car salesman and needs to understand if there is a relationship between the value of cars and their age. He has collected the following data.

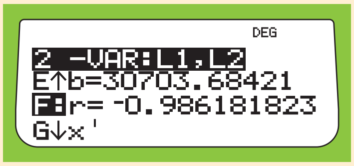
Age (years)	6	3	4	1	8	5	7	2	5
Value	13 000	21 000	18 000	29 800	9 000	15 700	11 000	25 000	15 300

a Use a scientific calculator to determine the correlation coefficient for the data showing the value of cars with respect to their age in years.

b Interpret the meaning of this value of the correlation coefficient (*r*).

WORKING	**THINKING**

a



```
DEG
2 -VAR:L1,L2
E↑b=30703.68421
F:r= -0.986181823
G↓x'
```

The correlation coefficient for this data is $r = -0.986$.

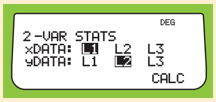
Press stat data to get to the tables to enter data.

Press stat data 1 to clear away any old data.

Type age of car in L1.

Type value of car in L2.

Press 2nd stat data 2 to get into 2-Variable Statistics:



```
DEG
2 -VAR STATS
xDATA: L1 L2 L3
yDATA: L1 L2 L3
CALC
```

Arrow down to CALC and press enter

Arrow down to find r (correlation).

b $r = -0.986$

The correlation between the age of cars and their value has a very strong, negative relationship. This means that as the age of a car increases the value of a car decreases.

Is the correlation positive/negative?

Is the correlation strong/moderate/weak/none?

The correlation is very strong, as it is very close to being −1.

Calculator activity 9D Using a scientific calculator to find the correlation coefficient. See the interactive textbook for this activity.

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Example 7 Using technology (spreadsheet) to determine the correlation coefficient [complex]

Sue is a nutritionist researching whether there is a relationship between eating chocolate and weight gain. The people in her trial ate the same basic healthy diet and followed the same exercise regime. They were given identical 200 g bars of chocolate to eat and their weight gain was recorded. She has collected the following data and calculated the mean of the weight gain.

No. of bars eaten per week	1	20	3	10	15	5	7	18	4
Mean weight gained per week (kg)	0.1	1.2	0.2	0.6	0.8	0.4	0.5	1	0.3

a Use a spreadsheet to create a scatterplot of the data, including a line of best fit.

b Use a spreadsheet to determine the correlation coefficient for the data showing relationship between the number of chocolate bars to weight gain.

c Interpret the meaning of this value of the correlation coefficient (r).

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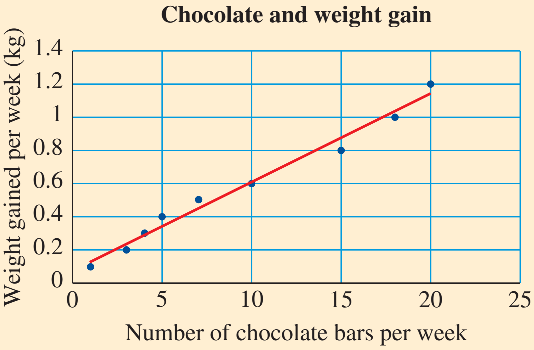
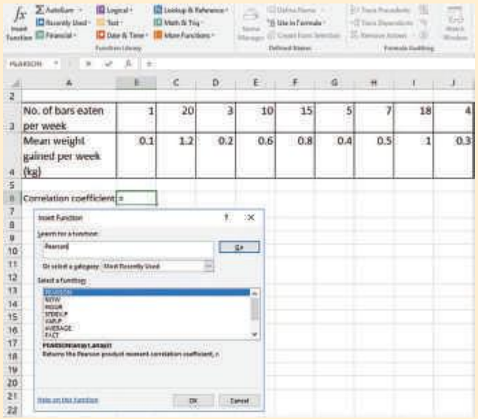
WORKING **THINKING**

a

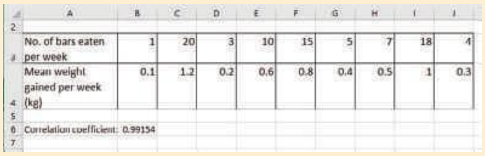


Draw a scatterplot to the data given in the table.
Insert a line of best fit to the scatterplot.

b



	A	B	C	D	E	F	G	H	I	J
2										
3	No. of bars eaten per week	1	20	3	10	15	5	7	18	4
4	Mean weight gained per week (kg)	0.1	1.2	0.2	0.6	0.8	0.4	0.5	1	0.3
5										
6	Correlation coefficient	=								



	A	B	C	D	E	F	G	H	I	J
2										
3	No. of bars eaten per week	1	20	3	10	15	5	7	18	4
4	Mean weight gained per week (kg)	0.1	1.2	0.2	0.6	0.8	0.4	0.5	1	0.3
5										
6	Correlation coefficient:	0.99154								
7										

The correlation coefficient is 0.99.

To find the correlation coefficient, select another cell.
Select Formulas
Insert Function
Write Pearson
(The correlation coefficient was originally called the Pearson Product Moment Correlation Coefficient, which is why we choose the formula PEARSON rather than correlation coefficient.)
Click OK.
Select the independent values into Array One.
Select the dependent values into Array Two.

c $r = +0.99$

The correlation between the amount of chocolate bars eaten each week and weekly weight gain has a very strong, positive relationship. This means that as the number of chocolate bars increase, the amount of weight gain also increases.

Is the correlation positive/negative?
Is the correlation strong/moderate/weak/none?
The correlation is very strong, as it is very close to being +1.

Spreadsheet activity 9D Using a spreadsheet to find the correlation coefficient.
See the Interactive Textbook for this activity.

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Exercise 9D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Understanding how to find the correlation coefficient is essential as it enables the ___________ in a relationship to be easily ___________.

b To measure the ___________ of the relationship, we calculate the correlation ___________.

c The correlation coefficient has the symbol ___________, which measures the ___________ and ___________ of a linear relationship between variables.

d The correlation coefficient can have values from ___________ to ___________ :

i +1 = ____________________

ii ____________________ = strong positive relationship

iii ____________________ = moderate positive relationship

iv 0.25 to 0.50 = ____________________

v ____________________ = no linear relationship

vi ____________________ = weak negative relationship

vii –0.50 to – 0.75 = ____________________

viii ____________________ = strong negative relationship

ix ____________________ = perfect negative relationship

2 Define using everyday language:

a correlation

b correlation coefficient

APPLICATIONS

SF: –	CF: 3–6	CU: 7

Example 6 ★**3** Charlie is wondering if there is a relationship between time spent studying for assessment and the amount of errors she makes in her assessments. She has collected the following data.

Hint A lot of people are confused by the word *negative*, (e.g. strong negative relationship) it just means that as one variable increases, the other decreases. It is not negative, as in a bad thing.

Assessment study time (min)	52	24	26	45	39	28	60
Assessment errors	3	16	15	7	10	14	1

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a Use a scientific calculator to determine the correlation coefficient for the data to 3 decimal places.

b Interpret the meaning of this value of the correlation coefficient (r).

★4 A tailor has been told that there is a relationship between the height and weight of men. He has collected the following data.

Hint Remember to always clear away any old data.

Men's height (m)	1.5	1.2	1.7	2	1.4	1.8	1.3	1.7	1.6
Men's weight (kg)	77	90	82	95	92	106	105	94	68

a Use a scientific calculator to determine the correlation coefficient for the data.

b Interpret the meaning of this value of the correlation coefficient (r).

Example 7

5 A doctor has collected data on the age of her patients with a certain condition and number of sick days they have been forced to take in the past year.

Age of patients (years)	25	60	38	24	45	58	30	40	65	40
Number of days sick	20	5	15	20	10	7	5	12	8	18

a Use a spreadsheet to create a scatterplot of the data, including a line of best fit.

b Use a spreadsheet to determine the correlation coefficient for the data.

c Interpret the meaning of this value of the correlation coefficient (r).

Example 7

6 Deb has noticed that whenever she goes in the sun, the following day she has a better sense of wellbeing. She has collected the following data to see if there is a relationship. Deb measured wellbeing by a scale of 1, with 10 being 'feeling excellent' and 1 being 'feeling low'.

Feeling of wellbeing (1–10)	10	2	5	7	4	3	9	8	10	9
Time in the sun each day (min)	20	5	7	15	2	5	18	16	20	18

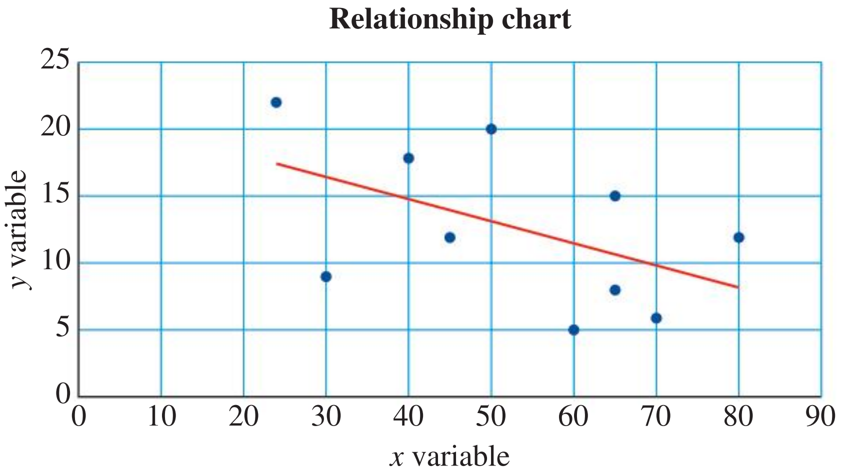
a Use a spreadsheet to create a scatterplot of the data, including a line of best fit.

b Use a spreadsheet to determine the correlation coefficient for the data showing whether there is a relationship between being in the sun and wellbeing.

c Interpret the meaning of this value of the correlation coefficient (r).

Hint Select the independent values into Array One.
Select the dependent values into Array Two.

7 The following scatterplot and table has a correlation coefficient for the variables of –0.50.



***x* variable**	50	60	65	24	45	70	30	80	65	40
***y* variable**	20	5	15	22	12	6	9	12	8	18

Using a spreadsheet, change some of the x and y variables to produce a set of data where the correlation coefficient is:

a strong positive correlation

b weak negative correlation

c moderate positive correlation

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9E Making predictions COMPLEX

LEARNING GOALS

- Use the line of best fit to make predictions, both by interpolation and extrapolation
- Recognise the dangers of extrapolation

Why is understanding how to use the line of best fit to make predictions essential?

Being able to interpret the line of best fit and measure the correlation between two variables is only the starting point in research and development. The main purpose of the line is to determine its equation and to use it to predict values that are not in the dataset. It enables prediction of future values within the range of our data and outside the range. This is the real purpose of the topic covered in this chapter.

The purpose of creating a line of best fit for data and finding its equation is to be able to make predictions from it.

WHAT YOU NEED TO KNOW

- A **prediction** in this context is a forecast of the value of the dependent variable based on a value of the independent variable, even though neither value was in the original dataset.
- A **line of best fit** enables a prediction about the data to be formed.
- The **correlation coefficient** is a measurement of the relationship between two variables, as well as the strength of that relationship. A coefficient of either +1 or –1 indicates the strongest relationship, and 0 indicates no relationship.
- If the line of best fit is a straight line, its equation has the form $y = mx + c$.
  - x represents the independent variable and y represents the dependent variable
  - m represents the gradient (slope of the line)
  - Gradient $= \frac{\text{rise}}{\text{run}} = \frac{\text{amount the line goes up}}{\text{amount the line goes across}}$
  - c represents the y-intercept (the point where the line crosses the y-axis).

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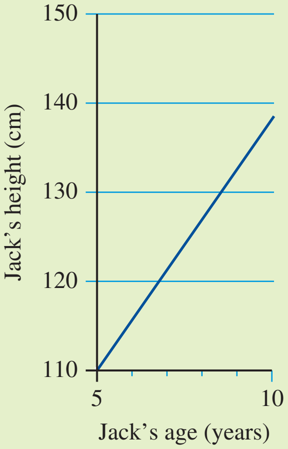
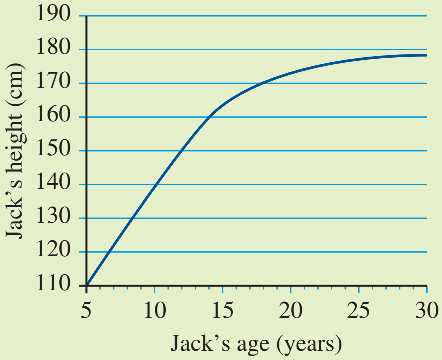
- **Interpolation** is an estimation of a value on the line of best fit. The accuracy of prediction by interpolation generally depends on the strength of the correlation: stronger correlations mean more accurate predictions.
- **Extrapolation** is an estimation of a value based on extending the line of best fit.
  - To extrapolate is to infer something that is not within the range of the particular dataset used to determine the line of best fit.
  - The accuracy of a prediction made by extrapolation is also affected by the strength of the correlation, as with interpolation. In addition, the danger of extrapolation is that there may be natural reasons why the straight line of best fit is not valid beyond the range of the dataset. Consider this graph of Jack's height between the ages of 5 and 10 years old. Jack grew 30 cm in five years so if that is extrapolated to age 30, he would be 260 cm tall!



  - The second graph shows the shape of Jack's age/height chart for ages 5 to 30 (not to the same scale), which explains why you cannot extrapolate from his growth between the ages of 5 and 10 years to predict his height at age 30. The line of best fit is only linear for part of the range.



- Where the independent variable is time, for example the years when something had a particular monetary value, the main danger of extrapolating is that unknown future events could affect the future value.

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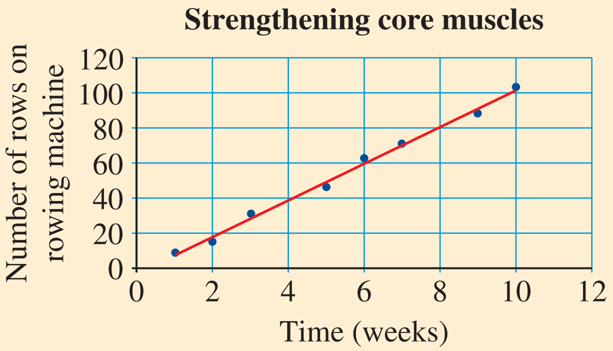
Example 8 Using the line of best fit to make predictions, both by interpolation and extrapolation [complex]

Bec has injured her core muscles. To gain strength she has been using a rowing machine. The line of best fit shows her progress after 10 weeks.



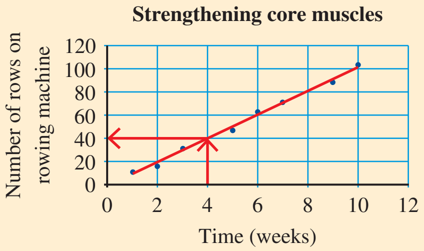
a Bec has lost her record of how many rows she completed at 4 weeks. Using interpolation, estimate how many rows she could have completed.

b Bec has also forgotten which week she was able to row 80 times. Using interpolation, estimate the week she rowed 80 times.

c Determine the equation of the line of best fit in the form $y = mx + c$.

d Using extrapolation with the equation of the line of best fit, predict how many rows Bec will be able to achieve by week 15.

WORKING	THINKING
a	◂ Find 4 weeks on the x-axis, go up to the line of best fit, and across to the y-axis.



Bec was able to row approximately 40 times in the fourth week.

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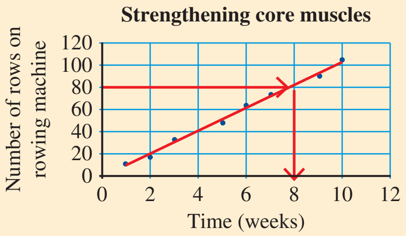
WORKING **THINKING**

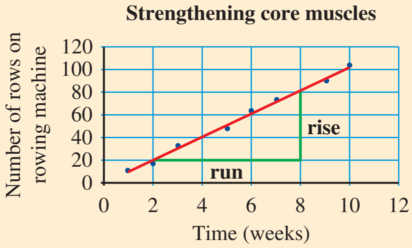
b



Bec was able to row 80 times around week 8.

Find 80 on the y-axis, go across to the line of best fit, and down to the x-axis.

c



The equation for the line of best fit is $y = mx + c$, where m is the gradient and c is the y-intercept.

Gradient:

rise $= 80 - 20$

$= 60$

run $= 8 - 2$

$= 6$

$m = \frac{60}{6}$

$= 10$

$y = 10x + c$

Calculate the gradient:

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{amount the line goes up}}{\text{amount the line goes across}}$$

Choose two sections on the line of best fit that have whole number coordinates. Draw a right-angled triangle to measure the *rise* and the *run*.

Substitute 10 for m in the equation.

y-intercept:

$80 = 10 \times 8 + c$

$80 = 80 + c$

$c = 80 - 80$

$c = 0$

As we are unable to see where the line crosses the y-axis, we will need to substitute values of x, y and m to find the value of c. Then rearrange to make c the subject of the equation and evaluate for c.

Therefore, the equation for this line of best fit is:

$y = 10x + 0$

which can be shortened to $y = 10x$

d $y = 10x$

$y = 10 \times 15$

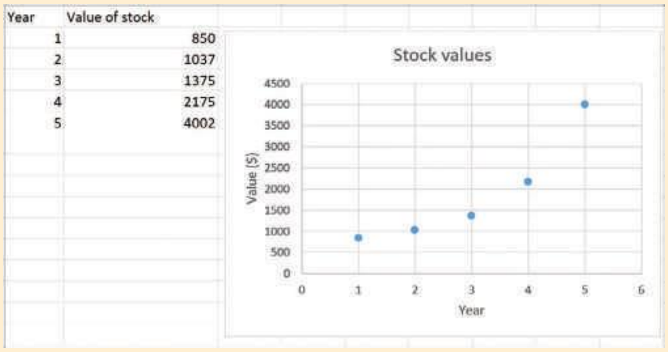
$y = 150$

Bec will be able to row 150 times by week 15.

Substitute 15 for x in the equation from part c.

Example 9 Recognising the dangers of extrapolation [complex]

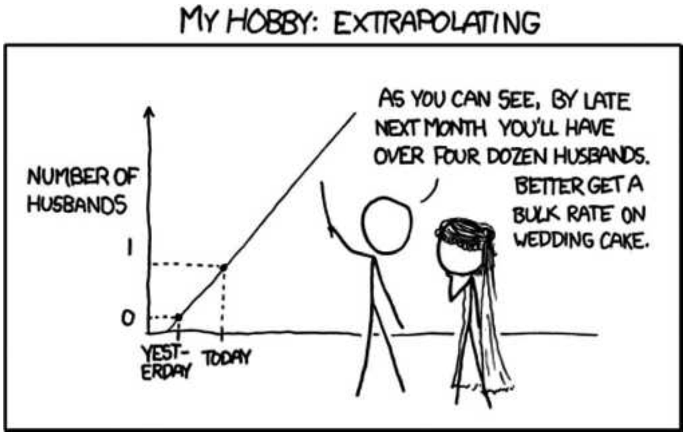
A stock broker was watching the increase of a particular company's stock. He put the past five year's values in a table and also created a graph.



Year	Value of stock
1	850
2	1037
3	1375
4	2175
5	4002

He then predicted the stock values in the next two years. Determine the dangers of the predictions.

WORKING	THINKING
There is no information to tell us why the stock was increasing in value over the past five years, so there is no way of knowing if the conditions for the increase will continue into the future. The danger with this prediction is that the stock market is not stable and future events are likely to be different from past history. Stock prices cannot be relied upon to continually increase.	Is the stock market stable, i.e. could unknown future events affect the value of the stock in the future? **Note:** The actual stock that this example was taken from did fall. Their 8th year value was $2354; and their 9th year was $1840.



Source: stackexchange.com

Exercise 9E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** The equation for the line of best fit enables ______________ to be made both ______________ the plot and ______________ of the plot.
- **b** The equation of a straight line is ______________.
- **c** m represents the ______________ (______________ of the line).
- **d** c represents the ______________(the point where the line crosses the________).
- **e** ______________ is an estimation of a value within two given points on the line of best fit.
- **f** ___________ is an estimation of a value based on extending the line of best fit

2 Define using everyday language:

- **a** interpolation
- **b** extrapolation

3 Determine which dataset A, B or C would be the most reliable based on the correlations stated. Explain your response.

- **A** $r = -0.94$
- **B** moderate positive
- **C** $r = 0.12$

4 Determine which dataset A, B or C would be the least reliable based on the correlations stated. Explain your response.

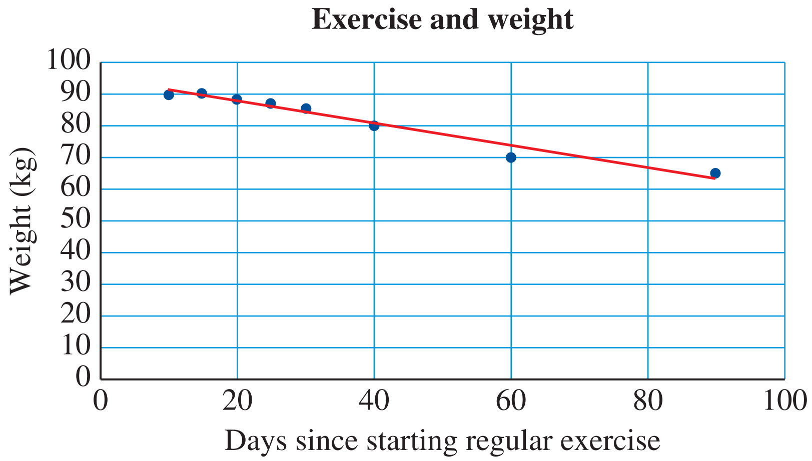
- **A** weak linear negative
- **B** $r = 0.75$
- **C** strong positive

APPLICATIONS

SF: –	CF: 5–8	CU: 9–10

Example 8, 9

5 Mindy is a 20-year-old who has been measuring her weight and the days since she started exercising regularly. She has recorded most of her measurements, but she has lost a couple of entries.



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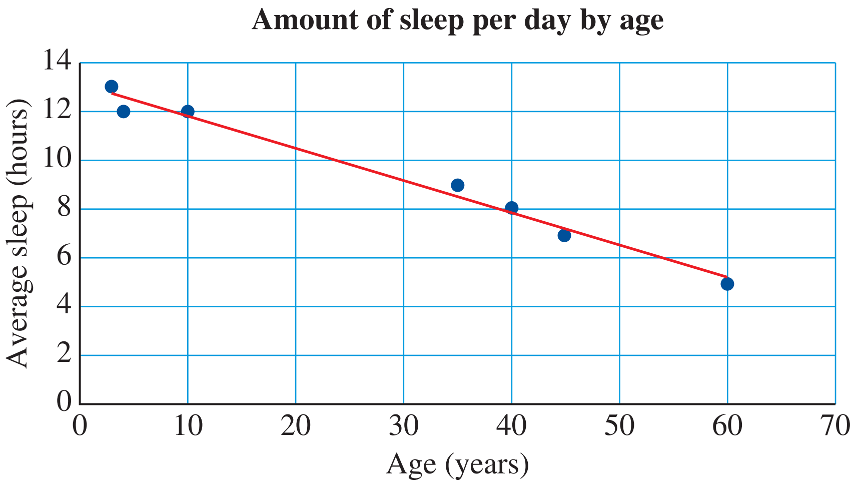
Hint Draw a right-angled triangle to measure the rise and the run.

Gradient $= \frac{\textit{rise}}{\textit{run}} = \frac{\textit{amount the line goes up}}{\textit{amount the line goes across}}$

- **a** Using interpolation, estimate what her weight was when she had been exercising for 50 days.
- **b** Using interpolation, estimate how many days she had been exercising when she had reached 70 kg.
- **c** Without technology, determine the equation of the line of best fit using $y = mx + c$.
- **d** Use the equation of the line of best fit to predict what Mindy's weight will be after she has exercised for 120 days.
- **e** Explain how reliable is the prediction for **d**.

6 Kent is researching the relationship between sleep and age, however he has lost a section of his data.



- **a** Using interpolation, estimate the average amount of sleep required for someone who is 25 years old.
- **b** Using interpolation, estimate on average the age of people who have 6 hours sleep per day.
- **c** Without technology, determine the equation of the line of best fit using $y = mx + c$.
- **d** Use the equation of the line of best fit to predict what how much sleep on average an eighty-year-old has.
- **e** Explain how reliable is the prediction for **d**.

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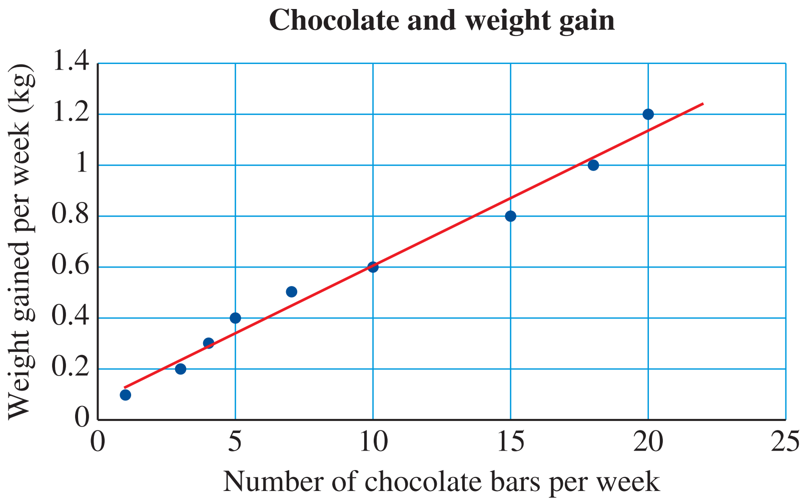
7 In an experiment, a group of people have had the same diet and exercise but have been given different numbers of chocolate bars per week to eat. Their weight gain per week has been recorded and graphed against the number of chocolate bars eaten per week.



a Using interpolation, estimate what the weight gain would be for someone eating 12 bars of chocolate per week.

b Using interpolation, estimate how many chocolates would be eaten per week to gain 1.1 kg per week.

c Using technology, determine the equation of the line of best fit.

Hint When using a calculator, the equation is $y = ax + b$.

d Using a calculator, predict how many chocolates are required per week to gain 5 kg per week.

e Explain how reliable is the prediction for **d**.

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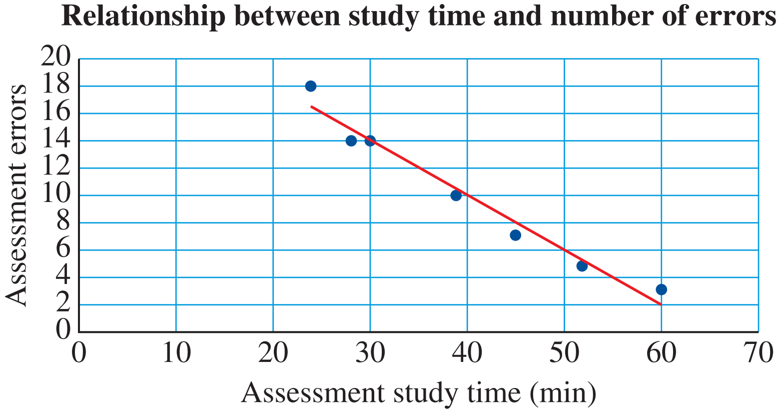
8 A teacher has collected data and produced this graph to try to convince his students that there is a relationship between the study time for an assessment and the number of errors they make in the assessment.



a Using interpolation, estimate how many errors there will be after 35 minutes of study.

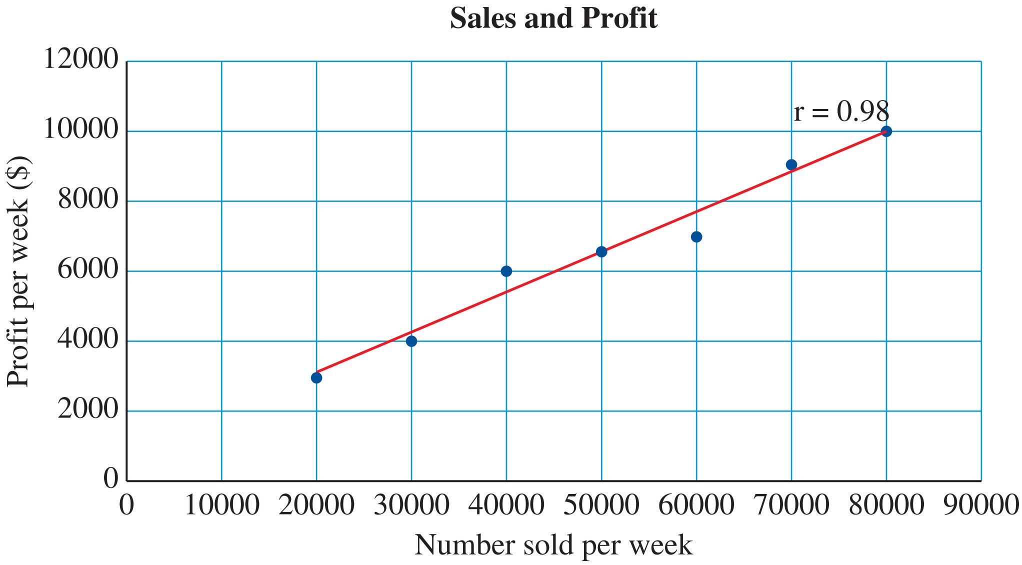
b Using interpolation, estimate how much study time is required for only 4 errors.

c Using technology, determine the equation of the line of best fit.

d Using a calculator, predict how much study time results in there being 30 errors.

e Explain how reliable is the prediction for **d**.

9 A business owner is wanting to sell a young couple part of his business and is using extrapolation above 80 000 sales per week from the following chart to enhance his sales pitch. Explain how extrapolation in this situation could be dangerous.



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9F Distinguishing between causality and correlation COMPLEX

LEARNING GOAL

- Distinguish between causality and correlation through examples

Why is it essential to understand the difference between causality and correlation?

Two variables may have a relationship; however, it does not necessarily mean that one causes the other. Correlation is a specific number that describes the size and direction of a relationship between variables. Causation, however, reveals that the value of one variable is caused by the value of the other variable.

Typical reasons why correlation is not due to causation include:

- coincidence
- the value of both variables is the result of a third variable.

An association between two things does not necessarily mean one caused the other.

WHAT YOU NEED TO KNOW

- **Causation**, or causality or a causal relationship is the ability of one variable to impact another.
  - The first variable could bring the second variable into being or may cause the value of the second variable to change.
  - Only when the change in one variable causes the change in another parameter is there a causal relationship.
- **Correlation** is a measure of the strength of the relationship between two variables.

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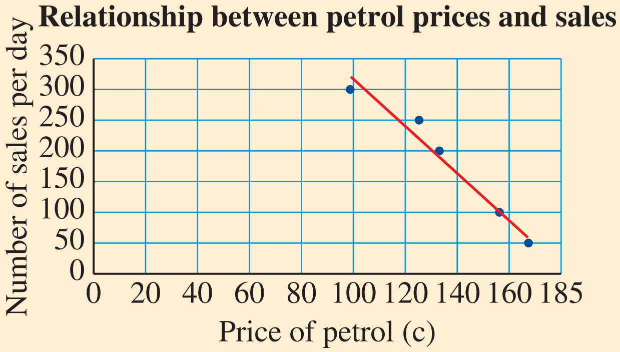
- Correlation does not imply causation.
  - Correlation does not explain why and how there is a relationship; it just reveals that a relationship may exist.
  - Causation states that any change in one variable will cause a change in another variable. This is also known as cause and effect. Distinguishing between a causal relationship and an association without causation is often a matter of common sense, but sometimes you cannot be certain of the answer and the data just shows that more research is needed.

Example 10 Distinguishing between causality and correlation (causality) [complex]

Guy works at a petrol station and has noticed a relationship between the price of petrol and the number of sales in a day.



a Determine if there is a correlation between the two variables. Explain.
b Determine if it is likely that there is causality between the two variables. Explain.

WORKING	THINKING
a There is a strong negative correlation between the price of petrol and the number of sales per day. As the price of petrol increases, the number of sales decreases.	Does the graph show a correlation?
b It is likely that there is causality, as people will find another supplier or wait for the petrol prices to go down before they purchase petrol. The price affects the number of sales.	Is there a cause and effect between the two variables?

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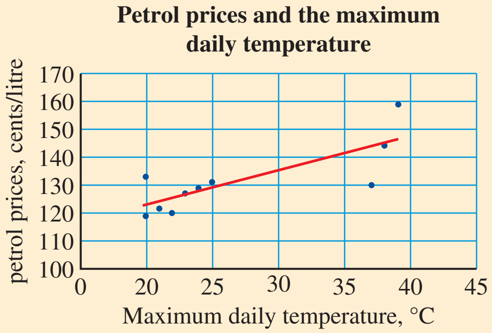
Example 11 Distinguishing between causality and correlation (no causality)

Kate is wondering if there is a relationship between petrol prices and the maximum daily temperature.



a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

WORKING	THINKING
a The line of best fit has a slight positive slope, which means there is a relationship between petrol prices and the maximum daily temperature.	Does the graph show a correlation?
b There does not appear to be any causality. There isn't a mechanism whereby a change in maximum daily temperature can cause the prices to change. There are many other variables that determine the change in petrol prices, for example supply and demand, location, and exchange rates. The correlation is likely to be a coincidence, and if Kate surveys prices and temperature over a longer period of time she may find there is no correlation.	Is there a cause and effect between the two variables?

Exercise 9F

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Two variables may have a ____________; however, it does not necessarily mean that one ____________ the other.

b Causation is the ability of one variable to ____________ another.

c Correlation is a measure of the ____________ of the linear relationship between two ____________.

d Correlation does not ____________ causation.

2 Define using everyday language:

a correlation

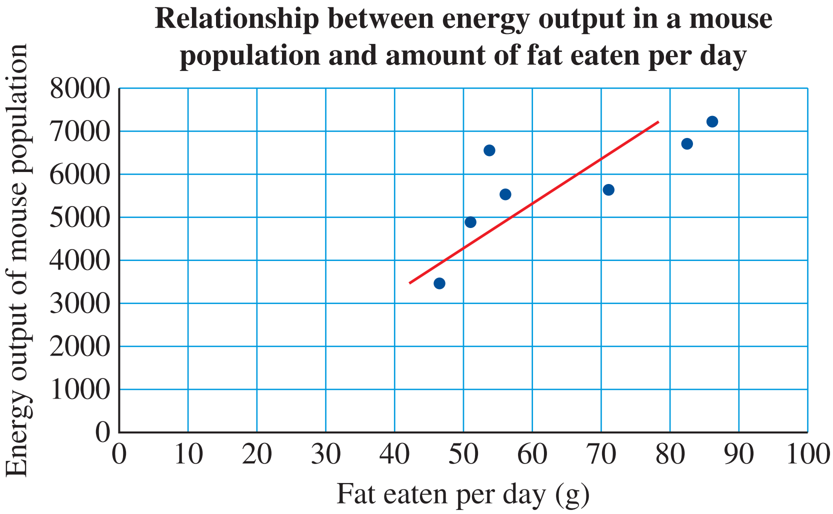
b causation

APPLICATIONS

SF: –	CF: 3–4	CU: 5–7

Example 10, 11

3 Ella is researching the relationship between fat in the diet of laboratory mice and their energy output. Using laboratory equipment, she has obtained the data shown in the graph.



a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

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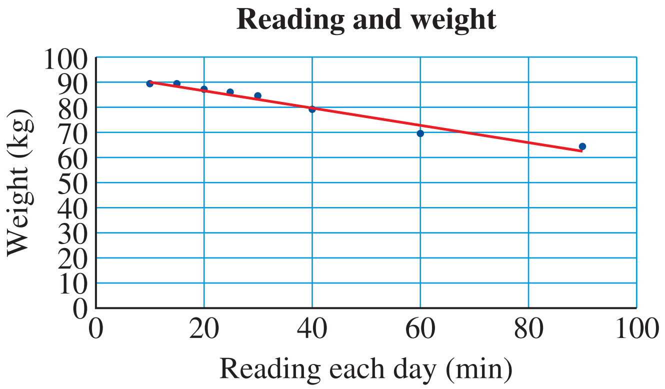
4 Ebony is trying to lose weight. She thinks that perhaps when she reads books, she loses weight.

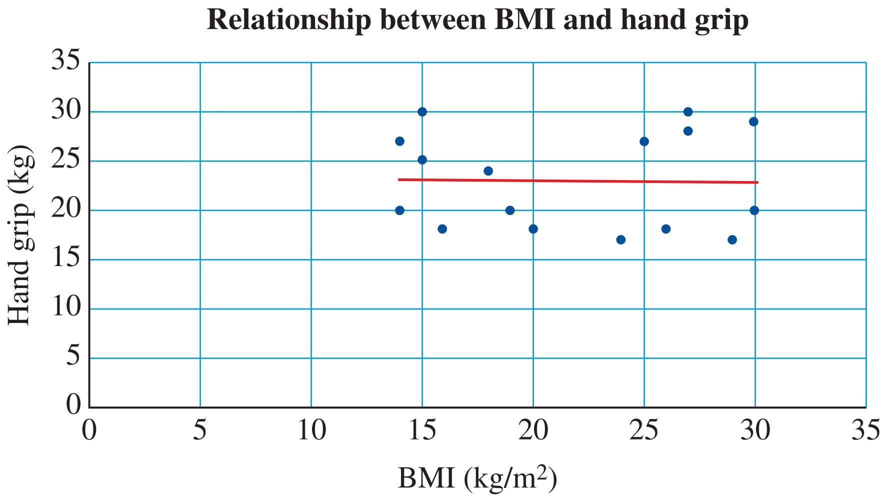


a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

Hint Is there a cause and effect between the two variables?

5 Finn is researching to see if there is a relationship between people's BMI and the strength of their hand grip. Her data (which was used in a previous question, in Exercise 9C) is shown below.



a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

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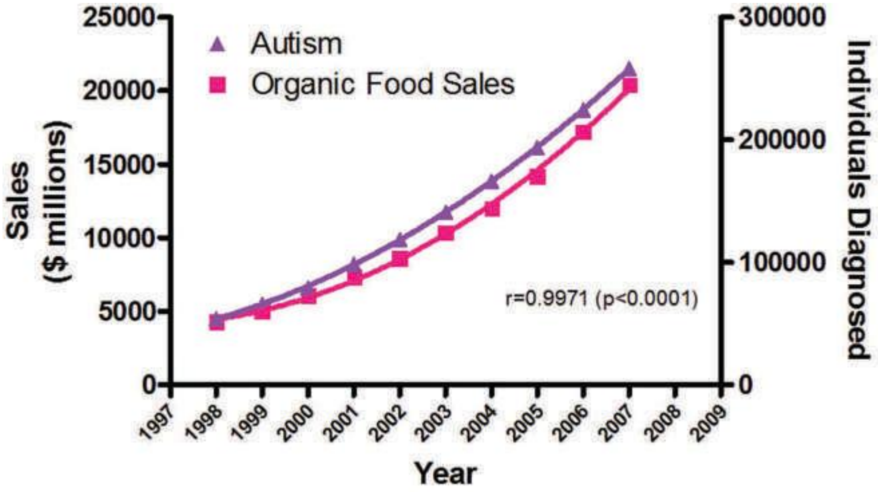
6 A researcher is studying whether there is a relationship between organic food sales and autism in children. The following graph was presented as evidence.

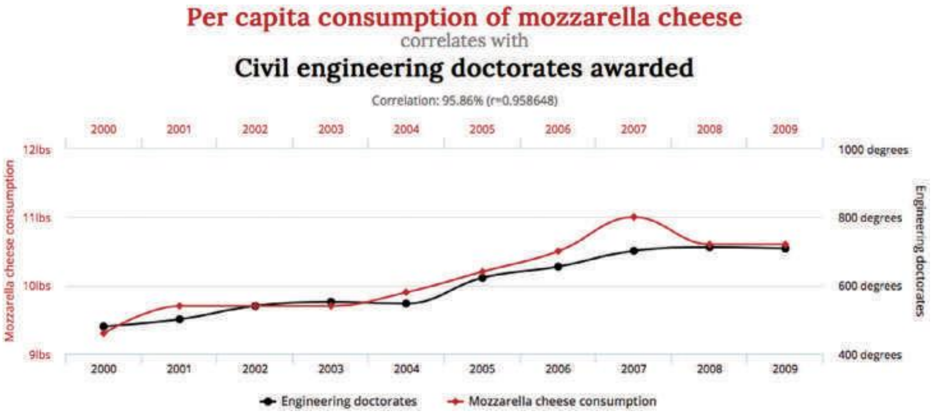


Sources: Organic Trade Association, 2011 Organic Industry Survey; U.S. Department of Education, Office of Special Education Programs, Data Analysis System (DANS), OMB# 1820-0043: "Children with Disabilities Receiving Special Education Under Part B of the Individuals with Disabilities Education Act

a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

7 A scientist is studying whether there is a relationship between civil engineering doctorates awarded and the consumption of mozzarella cheese. The following graph has been submitted as evidence.



a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

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Problem-solving and modelling task

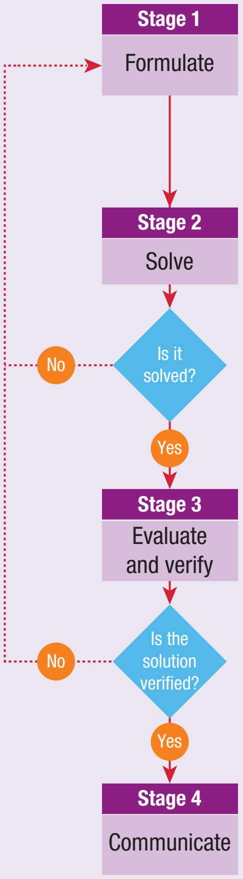
Background: $C = \pi D$

You have always been told that to find the circumference of a circle, you simply multiply its diameter by pi.

Task: Your task is to prove this formula by:

1. Finding at least ten different circular objects (with different diameters).
2. Create a table with their diameter (cm) being the x and their circumference (cm) being the y.
3. From the table create a scatterplot and find the line of best fit.

Approach to problem-solving and modelling tasks:



Stage 1: Formulate

1 What does pi stand for?
2 What part of the circle is the circumference?
3 How do you find the diameter?
4 How can you find the data?

Stage 2: Solve

5 Collect the required amount of data.
6 Produce the table and scatterplot.
7 Complete the calculations to find the line of best fit.
8 Find the measurement of the slope.

Stage 3: Evaluate and verify

9 Do you have enough variety in your measurements?
10 Have you presented them in the most suitable way?
11 What are the relationships between diameter and circumference?
12 What is the gradient similar to? Why?

Stage 4: Communicate

13 Communicate your findings in a short report that justifies your findings.
14 Include:
Introduction (What is the dependent and independent variable? Why?)
Body (Include statements about the line of best fit and the relationship between variables)
Conclusion (Distinguish between causality and correlation through your examples).

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Chapter checklist

☐ I can identify the dependent and independent variable.

1 Identify both the dependent and independent variable from this data. Give reasons.

Number of cups of chai tea sold per hour in the winter recorded against outside air temperature.

Temperature °C	11	8	9	4	20	18	10	11	13	10	2	1
Number of chai teas sold	10	13	14	16	0	1	8	6	7	7	18	20

☐ I can determine the line of best fit by eye.

2 Draw a scatterplot from the previous data, then determine the line of best fit by eye.

☐ I can use technology to determine the line of best fit. **[complex]**

3 Determine the line of best fit from the previous data using a spreadsheet.

☐ I can interpret relationships in terms of the variables. **[complex]**

4 Interpret and explain the relationships in terms of the two variables from the previous data.

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☐ I can use technology to calculate the correlation coefficient (an indicator of the strength of linear association).

5 Use a scientific calculator to determine the correlation coefficient from the previous data.

☐ I can use the line of best fit to make predictions, both by interpolation and extrapolation. **[complex]**

6 From the previous data make both an interpolation and an extrapolation prediction.

☐ I can recognise the dangers of extrapolation.

7 In relation to previous extrapolation prediction, state whether there are any dangers.

☐ I can distinguish between causality and correlation through examples. **[complex]**

8 In relation to previous data distinguish between causality and correlation.

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Chapter review

All questions in the review are assessment-style.

Simple familiar

Section 9A

1 Jon is studying how higher summer temperatures are impacting on the growth of his crops.

a What could be the dependent variable? Give a reason.

b What could be the independent variable? Give a reason.

Section 9B

2 Leanne is training for a bike ride and is recording her latest times. Draw a scatterplot and determine the line of best fit by hand.

Time (min)	25	40	35	37	90	40	115	35	95
Distance (km)	6	10	15	10	20	15	25	10	20

Complex familiar

3 Zek is researching growth in snow peas. Identify the dependent variable and list three possible independent variables.

4 Lilly has noticed a relationship between the average temperature and the amount of sales of ice cones. Determine the line of best fit using technology.

Average temperature (°C)	23	28	25	34	29	35	27	37	26	29	28	30	36
Number of ice cones sold	70	85	74	87	87	105	80	112	78	88	84	92	110

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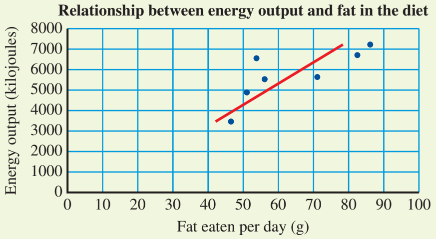
5 Glenn has been recording the value of his stock over the past five years. Determine the line of best fit using technology.

Year	Value of stock $
2014	850
2015	1037
2016	1375
2017	2175
2018	4002

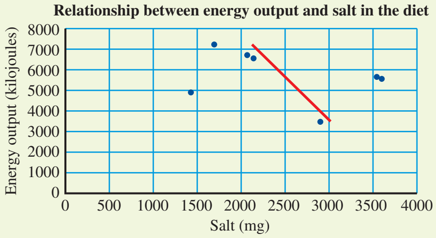
Section 9C

6 Interpret the relationship in terms of the variables of the following graphs.

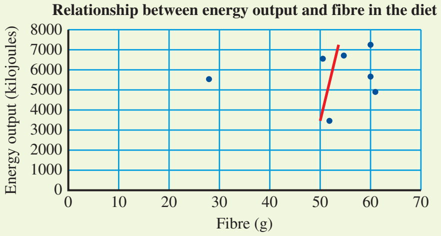
a



b



c



Section 9D

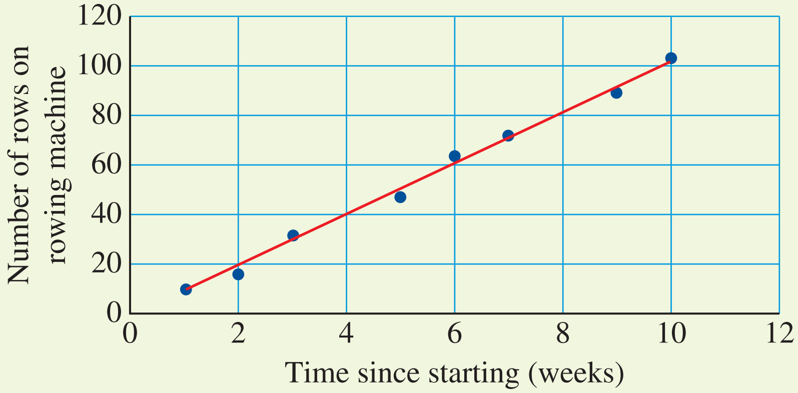
7 Roxy is wondering if there is a relationship between time spent studying for an assessment and her grades for the assessment. Use a scientific calculator to determine the correlation coefficient for the data.

Assessment study time (min)	52	24	26	45	39	28	60
Assessment grades	87	23	25	42	40	27	90

Section 9E

8 Rachel is currently strengthening her core muscles by using a rowing machine. The data shows the number of rows (rowing strokes) she can do each week after she started.

Strengthening core muscles



a Using interpolation, estimate the week could do 40 rows.

b Without technology, use the equation of the line of best fit in the form $y = mx + c$, to predict how many rows Rachel will be able to achieve by week 16.

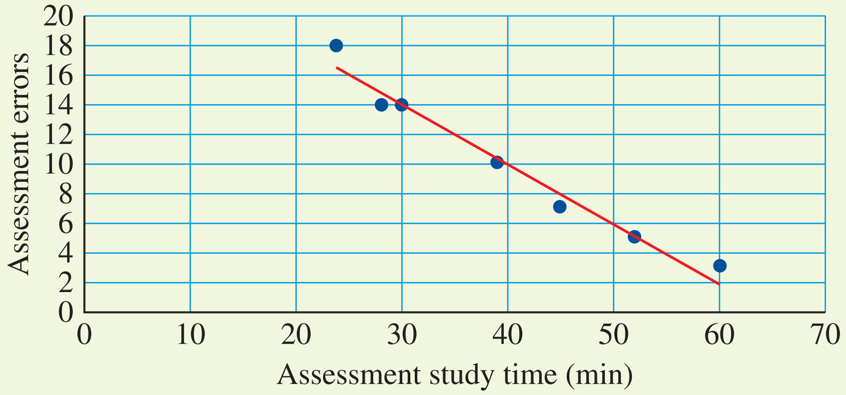
c Use a calculator to predict how many rows Rachel will be able to achieve by week 20.

d Explain the dangers of extrapolation with regards to this graph.

Section 9F

9 Scarlett has noticed that when she studies more for an assessment she makes fewer errors in the assessment.

Relationship between study time and number of errors



a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

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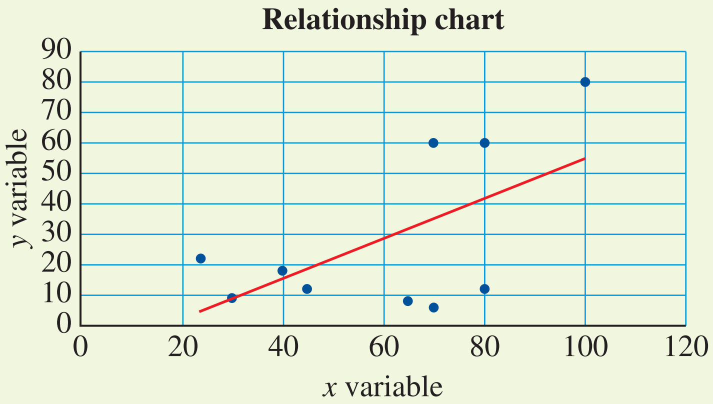
Complex unfamiliar

10 Your teacher wants to show examples of what the data and graphs of different correlation relationships look like, and has produced the following scatterplot and table with a correlation coefficient for the variables of 0.60.



x **variable**	80	70	100	24	45	70	30	80	65	40
y **variable**	60	60	80	22	12	6	9	12	8	18

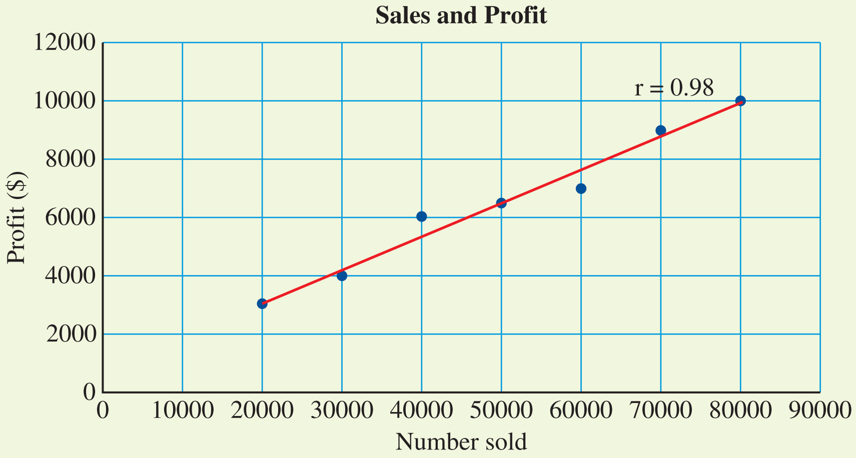
Using a spreadsheet, change some of the *x* and *y* variables to produce a set of data where the correlation coefficient is:

a weak positive correlation

b strong negative correlation

c moderate negative correlation

11 Use the following graph to devise an interpolation statement and an extrapolation statement. Explain the dangers of extrapolation.



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12 Use the following graph to determine the correlation and causality statements.

Per capita conpumption of sour cream

correlates with

Motorcycle riders killed in noncollision transport accident

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Per capita conpumption of sour cream Half-pints (USDA)	6.1	6.5	6.7	7.5	7.9	8.3	7.9	8.2	7.9	7.8
Motorcycle riders killed in noncollision transport accident Deaths (US) (CDC)	35	34	33	47	54	63	44	56	55	51

Correlation: 0.916391

Permalink • Mark as interesting (201) • Not interesting (152)

Source: Tylervigen.com

a Determine if there is a correlation between the two variables. Explain.

b Determine if there is causality between the two variables. Explain.

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10

Simple probabilities and simulations

Maths for a retail salesperson: Mitchell Burton

Mitchell Burton works for a large electronics retailer.

Tell us a bit about your job. What does a typical day look like?

I work for a large electronics retailer. I've worked there since 2010 in various roles from sales management to managing the counter and the tills. My current role is counter coordinator. This involves being responsible for putting all cash, EFT and finance sales through the till after a salesperson has assisted the customer with finding the right solution. We also take and direct multiple phone calls every hour.

A typical day involves opening and counting the tills to make sure the float (cash in the register) is correct. We then have a team meeting for about fifteen minutes to discuss the daily and weekly budget and any relevant sales information and then we open the doors. On average we serve around 400 customers per day. At the end of the day, the tills are counted again and then we close the shop.

What maths did you study at school?

I studied Maths B in Year 11 and then went to Maths A in Year 12.

How do you use maths in your job?

Retail is primarily a maths-based environment. Not only are we handling money and processing various transactions through the day, but we're also constantly thinking about sales budgets versus wages budgets. We also need to monitor our stock levels and measure the supply of our stock versus the demand of our customers to make sure we're not overstocking or understocking particular items. I personally still like to do a lot of the maths involved with my job in my head, (i.e. calculating percentage discounts and cash change) as I find it's healthy to keep my mind active and I am also able to pick up on pricing mistakes more easily.

In this chapter

Syllabus reference

Unit 4 Topic 2 Probability and relative frequencies

Simulations (9 hours)

In this sub-topic, students will:

- perform simulations of probability experiments using technology
- recognise that the repetition of chance events is likely to produce different results
- identify relative frequency as probability
- identify factors that could complicate the simulation of real-world events **[complex]**.

Simple probabilities (6 hours)

In this sub-topic, students will:

- construct a sample space for an experiment
- use a sample space to determine the probability of outcomes for an experiment
- use arrays or tree diagrams to determine the outcomes and the probabilities for experiments.

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Pre-test

1 Round each of the following decimals correct to 2 decimal places.

a 0.454 **b** 0.438 **c** 0.595

2 Convert the following fractions to decimals correct to 2 decimal places.

a $\frac{17}{98}$ **b** $\frac{16}{52}$ **c** $\frac{2}{43}$

Hint Use division.

3 Express each fraction in its simplest form.

a $\frac{10}{15}$ **b** $\frac{6}{8}$ **c** $\frac{12}{14}$

4 Convert the following fractions to percentages correct to 2 decimal places.

a $\frac{16}{81}$ **b** $\frac{5}{36}$ **c** $\frac{4}{18}$

5 Calculate the following correct to 2 decimal places.

a $\frac{6}{8} \times 65$ **b** $\frac{12}{14} \times 50$ **c** $\frac{7}{12} \times 38$

6 Calculate the following.

a 10% of 45 **b** 18% of 460 **c** 1.5% of 120

7 A die is rolled and the number on the upper most face is noted.

a Identify the possible outcomes.

b Calculate the probability that the upper most face is:

i a 6 **ii** an even number

iii more than 2 **iv** less than 5

8 A list of random numbers are generated on a calculator to simulate the gender of 12 babies born in a week at a hospital. If the last digit is odd = boy, even = girl.

0.278635 0.731297 0.573965 0.776794 0.86989 0.868027

0.66248 0.289817 0.801859 0.716876 0.648063 0.280808

a Organise the results of the simulation into a frequency table.

b Calculate the relative frequency of boys based on the simulation.

c Calculate how many of the 12 babies you would expect to be boys.

d Reflect on any similarities or differences between your answers to **b** and **c**.

Outcome	Tally	Frequency
Odd/Boy		
Even/Girl		
Total		

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

10A Performing probability experiments using technology

LEARNING GOALS

- Perform probability experiments
- Perform simulations of probability experiments using technology

Why is simulation of real-life scenarios essential?

We all have said ‘what will happen if … ’ many times in our lives. When faced with a decision, having some idea of what outcome to expect can help with the process. Simulations are quick, cheap and effective ways to answer the ‘what will happen if … ’ question without the need to wait and see or running a probability experiment which can be time-consuming and expensive.

Engineers run simulations of traffic flow using computer software when designing new freeways to predict bottlenecks and find ways to eliminate them.

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WHAT YOU NEED TO KNOW

- An **experiment** is a procedure undertaken to make a discovery. We can conduct probability experiments to simulate real-life problems (e.g. the probability of having a baby boy versus a baby girl)
    - Each repeat of an experiment is called a **trial**.
    - The results of the probability experiment need to be organised into a **frequency** table. Frequency is a count of how often an outcome appears.
- Physical techniques that can be used for simulation include:
    - Tossing a coin. For example: if using simulation to predict the gender of a baby before it is born, heads = girl, tails = boy
    - Rolling a die. For example: even result = girl, odd result = boy
    - Spinning a spinner. For example: even result = girl, odd result = boy
    - Drawing a selection ‘from a hat’.
    - Drawing cards from a standard pack of playing cards which has four suits and two Jokers. Red suits: Hearts♥ and Diamonds♦; Black suits: Spades♠ and Clubs♣. Each suit has 13 cards: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

- Simple technology that can be used for simulation includes:
    - The random number feature on a scientific calculator returns a random decimal number between 0 and 1. For example: if the last digit is even = girl, odd = boy.

      rand .23640104

      Note: Most smart phones also have a scientific calculator app with a RAND function that works in the same way. On an iPhone, turn it sideways with the calculator app open to access the scientific calculator.
    - RandomBetween function in Excel. For example: entering =RANDBETWEEN(1,2) in a cell returns a random whole number between 1 and 2 that could be allocated to your selection. For instance, 1 = girl, 2 = boy. Pressing F9 in an Excel sheet with a probability simulation will repeat the probability experiment. The outcomes are never exactly the same.
    - Online calculators are useful when lots of trials are required. We will use online calculators in section 10B.

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Example 1 Analysing physical probability experiments

Toss a coin to simulate the gender of 20 babies born at a hospital during a week. Use heads = girl, tails = boy.

a Organise the result of 20 coin tosses in a frequency table.

b Calculate the percentage of girls from your simulation.

c Decide if the simulation results varied from what you expected. Explain how they may differ.

WORKING	THINKING
a Result of 20 coin tosses: T H H H T H H T T H H H H H H H T T H H	You can use tally marks and record as you go or just record each result of the coin toss and count when done.

Result	Frequency
heads/girls	14
tails/boys	6

WORKING	THINKING
b Percentage girls $= \frac{14}{20} \times 100 = 70\%$	Percentage $= \frac{\textit{amount}}{\textit{total}} \times 100$
c The simulation did not produce the expected result. The percentage of girls in the simulation was 70%, you would expect it to be around 50%.	Simulations do not always give the results you would expect. Lots of trials need to be done to get a result that is close to what the expected value might be. **Note:** In real life, the probability of a boy or a girl is not actually equal. 51.2% of babies born in industrialised countries are boys. For the purposes of this chapter, we will assume that it is equal.

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Example 2 Conducting a simulation of a probability experiment using technology

Conduct a simulation to estimate how many students need to be chosen at random for each of the four school houses to be represented.

a Use Excel to generate data for which school house a student is from. Use the numbers 1, 2, 3, 4 to represent the four school houses.

b Copy and complete this frequency table to record which school house the simulation shows the students are from. Stop when all four school houses have been selected at least once.

House	1	2	3	4
Tally				
Frequency				

c Determine how many simulations it took to represent all four school houses.

d Identify which school house was represented more than other houses in the simulation.

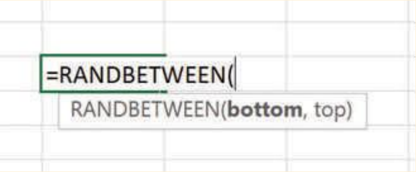
e Explain a way the simulation experiment could be conducted using a physical technique.

WORKING	THINKING

a Data: houses of 12 students:

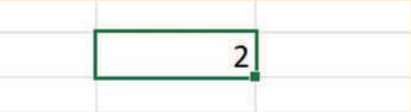
4 2 2 1
1 3 3 1
1 1 4 3

=RANDBETWEEN(1,4) will randomly select a whole number between 1 and 4 to represent the four school houses.



=RANDBETWEEN(
RANDBETWEEN(**bottom**, top)

Tip: Grabbing the little box in the bottom right corner and dragging down or across will give you more random numbers within the same range.



2

WORKING **THINKING**

b $\cancel{4}$ $\cancel{2}$ $\cancel{2}$ $\cancel{1}$
$\cancel{1}$ $\cancel{3}$ 3 1
1 1 4 3

Cross off each simulated school house and record in the table. Stop once all the houses have been represented.

Frequency table:

House	1	2	3	4
Tally	||	||	|	|
Frequency	2	2	1	1

c It took 6 simulations for all four school houses to be represented.

Count the number of houses crossed off or total up the frequencies.

d The first two school houses were both represented twice before the final school house, 3, was selected.

Look for the highest frequency.

e Playing cards could be used where each suit represented a school house. The card would need to be replaced after each selection is noted.

Any technique that has four possible choices would be suitable.
Replacing the card each time keeps the chance of each being selected remaining the same.

Exercise 10A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Physical techniques that can be used for _______ include tossing a _______, rolling a _______ and spinning a _______.

b Simple _______ that can be used for simulation include the _______ number feature on our _______ that returns a random _______ number between 0 and 1. The RANDBETWEEN function in _______ returns a random whole number _______ two values that could be allocated to your selection.

Spreadsheet activity 10A: See the interactive textbook for this activity using the random number and counting functions in Excel to conduct a simulation for this exercise.

APPLICATIONS

SF: 2–10	CF: –	CU: –

Example 1

2 Complete the following questions based on this simulation.

a Toss two coins twenty times to simulate the gender of children in twenty 2-child families. Use Heads = Girl, Tails = Boy. Copy and complete the following frequency table to record your results.

2-child combination	Tally	Frequency
boy + boy		
boy + girl		
girl + girl		

b Calculate the percentage of 2-child families with a boy and a girl combination in your simulation.

c Decide if the results of your simulation differ from what you might have expected. Explain how they may differ.

3 Complete the following questions based on this simulation.

a Toss three coins twenty times to simulate the gender of children in twenty 3-child families. Use Heads = Girl, Tails = Boy. Record your results in a frequency table as follows:

3-child combination	Tally	Frequency
3 × boys		
2 × boys + 1 × girl		
1 × boy + 2 × girls		
3 × girls		

b Calculate the percentage of 3-child families with a boy/girl combination in your simulation.

c Decide if the results of your simulation differ from what you might have expected. Explain how they may differ.

4 A supermarket runs a promotion where a free collectable toy is given away with each $30 spent in the store. There are 20 different toys to collect. Ashleigh runs a simulation using a spreadsheet to estimate how much money she would need to spend to collect all 20 toys. The simulation at the top of the next page shows which toy she receives after spending $30.

9	7	5	6	2	17	9	11	9	5
6	13	7	5	19	12	19	6	15	15
15	7	11	13	18	11	4	13	4	14
20	18	4	17	2	9	9	10	7	13
19	4	5	10	15	7	14	18	3	8
19	2	12	20	15	18	3	17	11	9
7	18	19	16	9	8	14	9	1	9

a Copy and complete this frequency table to record which toys the simulation shows she will receive. Stop when all 20 toys have been collected at least once.

Toy number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Tally																				
Frequency																				

b Determine how many simulations it took to collect all 20 toys.

c Calculate how much money she will need to spend to collect all 20 toys using the results of the simulation.

Hint Look for the highest frequency.

d Determine which toy was collected more than others in the simulation.

e Calculate the average number of each toy she collected so that she collected all 20 toys in the simulation.

Hint Average = $\frac{\text{total of all data values}}{\text{number of data values}}$

5 Ian has 7 different ties that he keeps in a box on his desk. He randomly selects a tie from the box each morning when he arrives at work. His workmate, David, uses a spreadsheet to conduct a simulation of which tie Ian will wear each day for the next month. The results are as follows:

Week	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
One	5	7	1	1	4
Two	1	3	7	3	7
Three	1	5	7	5	7
Four	7	5	5	3	2

a Copy and complete this frequency table for how many times each tie is worn over the four-week simulation.

Tie number	1	2	3	4	5	6	7
Tally							
Frequency							

b Determine if any tie is worn more, or less, often than another.

c Calculate how often the simulation shows that Ian wears the same tie more than once in a week.

d Identify how often the simulation shows that Ian wears the same tie on two consecutive days.

e Calculate the average number of times you would expect each tie to be worn over the 4-week period. Determine if any of the results from the simulation differ to what you would expect.

Hint Expected number = trials ÷ number of possible outcomes

Example 2 ★**6** Ben wants to run a simulation to estimate how many people he would need to ask before he finds two with the same birth month.

a Explain why each of the following simulation techniques would or would not be suitable to use:

- **i** Using a standard pack of cards and removing the Kings. The card drawn at random represents the month of birth. i.e. A = Jan, 2 = Feb, 3 = Mar …. J = Nov, Q = Dec. The card is not replaced after being drawn.
- **ii** Using a deck of cards, as in part **i**, but the card is replaced after the selection is noted.
- **iii** Rolling two standard die and adding the two faces to simulate the month of birth.
- **iv** Using Excel and entering the formula =RANDBETWEEN(1,12).

b Ben decides to use the deck of cards without the Kings and replace the card after each selection. He has the following results after 40 trials:

A	9	10	8	6	Q	J	5
3	5	5	5	3	8	A	7
7	A	A	2	2	3	8	4
J	8	7	3	Q	8	4	10
3	7	8	6	5	A	2	10

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Organise the results of the simulation into a frequency table like the one following. STOP when one of the months has been selected twice:

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally												
Frequency												

c Determine how many trials Ben needed to conduct to find two people with the same birth month.

★**7** From question **6b**, Ben notices that not all the months seem to be represented evenly in his trials. He decides to see how big the differences are.

a Organise the results of all 40 trials into a frequency table like the one following.

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally												
Frequency												

b Decide if Ben was correct: were all the months represented the same amount of times in the 40 trials? Comment on the results.

Example 2 ★**8** From question **6b**, repeat Ben's experiment for 40 trials using Excel and the formula =RANDBETWEEN(1,12).

a Determine how many trials it takes to find two people with the same birth month.

b Decide if all the months were represented evenly over the 40 trials.

★**9** Faith wants to conduct a simulation to determine the average number of children a couple would need to have to ensure they have a combination of sons/daughters in their family. She decides to use the random number feature on her calculator. The last digit will determine the gender of a baby where even = girl, odd = boy. She starts recording her simulations in a table as follows.

Trial number	Outcome of trial	Family size
1	9(B), 4(G)	2
2	3(B), 9(B), 7(B), 8(G)	4
3	…	

a Conduct 20 trials of Faith's simulation. Organise your results in a table like the one above.

b Calculate the average size family based on the results of your simulation.

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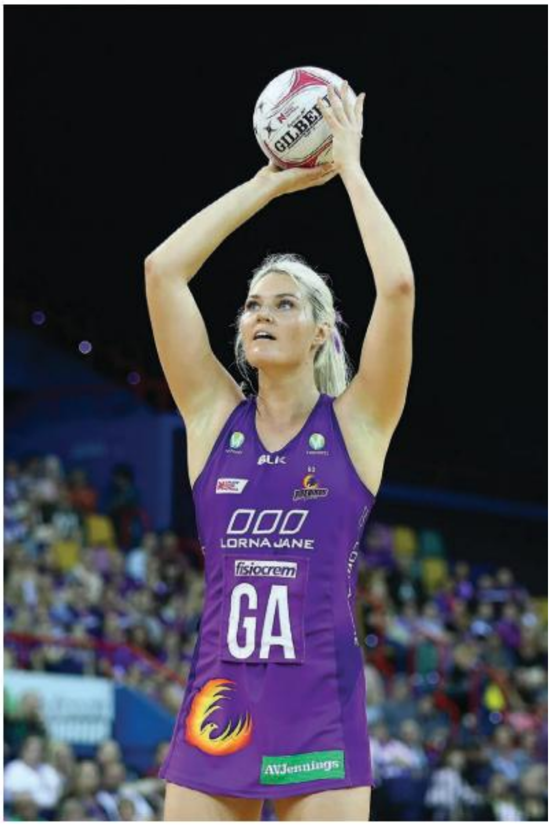
★**10** Gretel Tippett is the Goal Attack for the Qld Firebirds Netball team. In the 2018 season she scored 283 goals from 315 attempts.

a Show that her success rate for shooting a goal is 90%.

b Use the random number feature on your calculator to simulate 30 attempts at goal. If the last two digits are between 01–90, she successfully shoots the goal, if the last two digits are between 91–00, she misses the goal. Copy and complete the following frequency table to record your results.



Attempt at goal	Tally	Frequency
Successful		
Miss		
Total		

c Calculate the percentage success rate for the 30 trials.

d With a 90% success rate, determine how many of the 30 goals you would expect Gretel to successfully shoot.

e Compare your answer in part **d** to the number of successful attempts found in the simulation in part **b**. Comment on differences or similarities between the two results.

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10B Recognising the repetition of chance events

LEARNING GOALS

- Use technology to perform simulations
- Recognise how repetition will affect results

Why is an understanding of repetition of chance essential?

Gambling is a popular pastime in Australia with many adults participating in a ‘flutter’ on the pokies, scratchies, sports results or racing. Unfortunately, gambling addiction is also a significant public health issue, with Australian Gambling Statistics reporting a total gambling expenditure of $23.694 billion in 2017. This is an average of $1251.39 per Australian adult. Problem gambling can cause emotional and financial distress. A common misconception that many people have in games of chance is that past results can affect future ones; this is not true and is a contributor to problem gambling.

In games of chance, like Lotteries, some numbers come out more often than others. This does not improve the chance of them appearing in the next draw.

Source: Queensland Government Statistician’s Office, Queensland Treasury, *Australian Gambling Statistics 35th*, edition.

WHAT YOU NEED TO KNOW

- The **outcome** of previous trials has no impact on the chance of the outcome occurring in successive trials. As an example, say you tossed five heads in a row with a single coin. When lining up to toss the coin for a 6th time, even though tossing 6 heads in a row is very unlikely (around 1.56%), the chance of tossing a head remains at one in two, or 50%, for the 6th toss because chance has no memory.
- When conducting **trials** where all outcomes have an equal chance, we would expect the different outcomes to appear approximately the same number of times.
  - Expected number of occurrences of an outcome = trials ÷ number of different outcomes
- When conducting **simulations**, we can get unusual results when only running a small number of trials. Increasing the number of trials can improve the reliability of our predictions.

Example 3 Conducting an experiment with repetition of trials

Using a single coin, conduct an experiment where the coin is tossed twice for ten trials.

a Organise the results into a table.

Trial	1	2	3	4	5	6	7	8	9	10
First toss										
Second toss										

b Determine how often the outcome in the second toss was the same as in the first toss.

c Calculate how many heads were recorded in the first toss.

d Calculate how many heads were recorded in the second toss.

e Calculate how many heads you would expect over ten trials.

f Determine the average number of heads observed per toss over the ten trials.

g Decide if the results were as expected.

WORKING THINKING

a

Trial	1	2	3	4	5	6	7	8	9	10
First toss	T	T	H	T	T	H	T	T	T	H
Second toss	T	H	H	T	T	H	H	H	H	H

Insert the results into a table.

b 6 out of the 10 trials had the same result in both tosses.

Since the chance of a head or a tail is equally likely, we would expect about half the time to toss the same result as the previous toss. While we did not get exactly one half, it is very close.

c 3 heads were recorded in the first toss

d 7 heads were recorded in the second toss

e Expected number of heads $= 10 \div 2$

$= 5$

Therefore, we would expect to toss 5 heads from the ten trials.

The two outcomes should appear the same amount of times.

f Average number of heads $= \dfrac{3 + 7}{2} = 5$

$\text{Average} = \dfrac{\text{Sum}}{\text{How many}}$

g While the individual trials did not achieve the expected number of heads (i.e. 5), the average over the two trials was what was expected.

They were expected because we only did two trials. Repeating trials gives a better result.

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Exercise 10B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** The outcome of ______ trials has no impact on the ______ of the outcome occurring in successive trials.
- **b** When conducting trials where all outcomes have an ______ chance, we would expect the ______ outcomes to appear approximately the ______ amount of times.
- **c** When conducting simulations, we can get ______ results when only running a limited number of ______.
- **d** Increasing the number of trials can improve the ______ of our predictions.

APPLICATIONS

SF: 2–7	CF: –	CU: –

Example 3 ★**2** A coin is tossed 20 times. The results are shown below.

H	T	H	T	H	H	T	H	T	T
T	T	H	H	T	T	T	T	H	H

- **a** Determine how many different outcomes there are when a coin is tossed.
- **b** Calculate how many times you would expect to get the same result as the previous toss in the 20 trials.
- **c** Determine how many times the same result appears in consecutive tosses. Compare this answer with what you expected from part **b**.
- **d** Calculate how many times you would expect each outcome to appear in the 20 rolls.
- **e** Decide if any results appear more/less often than expected.

Regardless of previous results, each time a coin is tossed the chance remains the same for tossing a head.

⋆**3** A standard die is rolled 30 times. The results are shown below.

4	3	4	5	2	1	5	6	5	6
1	3	1	2	3	5	5	1	4	4
2	4	1	1	4	3	4	4	4	4

a Determine how many different outcomes there are when a die is rolled.
b Calculate how many times you would expect to get the same result as the previous roll in the 30 trials.
c Determine how many times the same result appears in consecutive rolls over the 30 trials. Compare this answer with what you expected from part **b**.
d Calculate how many times you would expect each outcome to appear in the 30 rolls.
e Decide if any results appear more/less often than expected.

⋆**4** A standard die is rolled 20 times and the results are recorded. The results of 10 trials are displayed.

	Dice Outcome Frequency						
Trial	1	2	3	4	5	6	Total
1	1	2	4	5	5	3	20
2	4	1	4	4	4	3	20
3	4	2	3	4	3	4	20
4	5	3	2	2	5	3	20
5	5	3	3	3	2	4	20
6	5	6	2	4	0	3	20
7	5	2	1	2	6	4	20
8	1	5	3	2	5	4	20
9	4	5	3	4	3	1	20
10	3	3	5	3	3	3	20
Total							

a Calculate how many times you would expect each face of the die to appear in 20 rolls.
b Determine if any outcome appeared more than another in:
  i the first trial **ii** the sixth trial
c Calculate and complete the totals for each outcome in the bottom row of the table.
d Calculate how many times you would expect each face of the die to appear in 200 rolls, which is the grand total of rolls of the die from the last column of the table.
e Decide if any results from the 200 rolls differ from what you would expect.

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5 A lottery website displays data of numbers drawn over time as a way of offering strategy tips for picking numbers to encourage people to participate in the lottery.

HOT NUMBERS statistics show these numbers have been drawn more often	6, 30, 37, 40, 36, 9
COLD NUMBERS statistics show these numbers have been drawn less often	16, 29, 4, 27, 12, 4
MOST PROFITABLE NUMBERS statistics show these numbers have resulted in bigger payouts	36, 10, 39, 25, 24, 21
LEAST PROFITABLE NUMBERS statistics show these numbers have resulted in smaller payouts	28, 37, 25, 6, 9, 15

a Decide if the 'Hot Numbers' are more likely to come up in the next draw than the 'Cold Numbers'. Explain why or why not.

b Determine how many of the 'Hot Numbers' also appear in the:

- **i** 'Most Profitable Numbers'
- **ii** 'Least Profitable Number'

c Reflect on the results from part **b**. Explain why might this happen.

d The results from the last draw are shown on the right:
Determine how many of the numbers drawn were:

- **i** Hot Numbers
- **ii** Cold Numbers
- **iii** Most Profitable Numbers
- **iv** Least Profitable Numbers

e To win a division 6 prize of $13.50, you need to have a minimum of 1 blue number and the two white numbers. If you played the numbers from the website, determine if any of the Hot, Cold, Most Profitable, Least Profitable number sets would have won a prize.

6 Use an online simulator to investigate tossing a coin with a very large number of repetitions. A suggested site is *http://www.shodor.org/interactivate/activities/Coin/* Determine what happens to the distribution of Heads/Tails as the number of trials is increased.

7 Use an online simulator to investigate rolling a die with a very large number of repetitions. A suggested site is *https://academo.org/demos/dice-roll-statistics/* Determine what happens to the distribution of the outcomes as the number of trials is increased for:

- **a** one die
- **b** two dice

Spreadsheet activity 10B: See the interactive textbook for this activity using Excel to simulate the results of rolling a die 30 times with many repetitions.

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10C Identifying and calculating relative frequency

LEARNING GOALS

- Calculate probability
- Identify relative frequency as a probability

Why is calculating relative frequency essential when determining probability?

In many situations, it is difficult to determine the chance of an event occurring, for example, the chance of a football team winning their next match. We need to look at past events and records to estimate the chance of the team winning.

Past performance is used to estimate the chance of a team winning a match.

WHAT YOU NEED TO KNOW

- **Probability** is a numerical indicator of the chance of an event occurring.
  - Probability of an event $= \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
  - Probability ranges in value from 0 (will not happen) to 1 (will happen).
- **Relative frequency** can be used to estimate the probability from a simulation or survey.
  - Relative frequency of an event $= \dfrac{\text{number of favourable outcomes observed}}{\text{total number of trials}}$
  - Relative frequency ranges from 0 (did not happen) to 1 (happened in every trial).
  - Probability of a coin coming down heads: favourable outcomes 1 (heads) total number of outcomes 2 (heads and tails) probability 0.5.
  - In six coin tosses where 4 came down heads, relative frequency of heads is the number of favourable outcomes observed (4), the total number of trials is 6, the relative frequency of heads is 0.67.
  - When increasing the amount of trials to a very large amount, the relative frequency approaches the expected (calculated) probability.

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Example 4 Calculating probabilities using relative frequency

A survey asked students how many times they had visited the dentist in the past year. The table below shows the findings:

Number of visits	0	1	2	3	4	5
Number of students	2	6	4	2	0	1

Determine the following based on selecting a student at random.

a Calculate the probability that the student has visited the dentist twice in the past year. Round your answer to 2 decimal places.

b Calculate the probability that the student has been to the dentist more than once. Round your answer to 2 decimal places.

WORKING	THINKING
a Total number of students $= 2 + 6 + 4 + 2 + 0 + 1$ $= 15$ Probability of 2 visits $= \frac{4}{15}$ $= 4 \div 15 = 0.27$	Relative frequency of an event = $\frac{\text{number of favourable outcomes observed}}{\text{total number of trials}}$
b Probability of more than 1 visit $= \frac{4+2+0+1}{15}$ $= \frac{7}{15}$ $= 7 \div 15 = 0.47$	More than once will mean they have visited 2, 3, 4 or more times.

Example 5 Using relative frequency to make estimations

A deputy principal wants to look at ways of improving traffic congestion around the school by encouraging students to either walk, ride a bike, or catch a bus to school. He does a survey of a group of students and finds the following results:

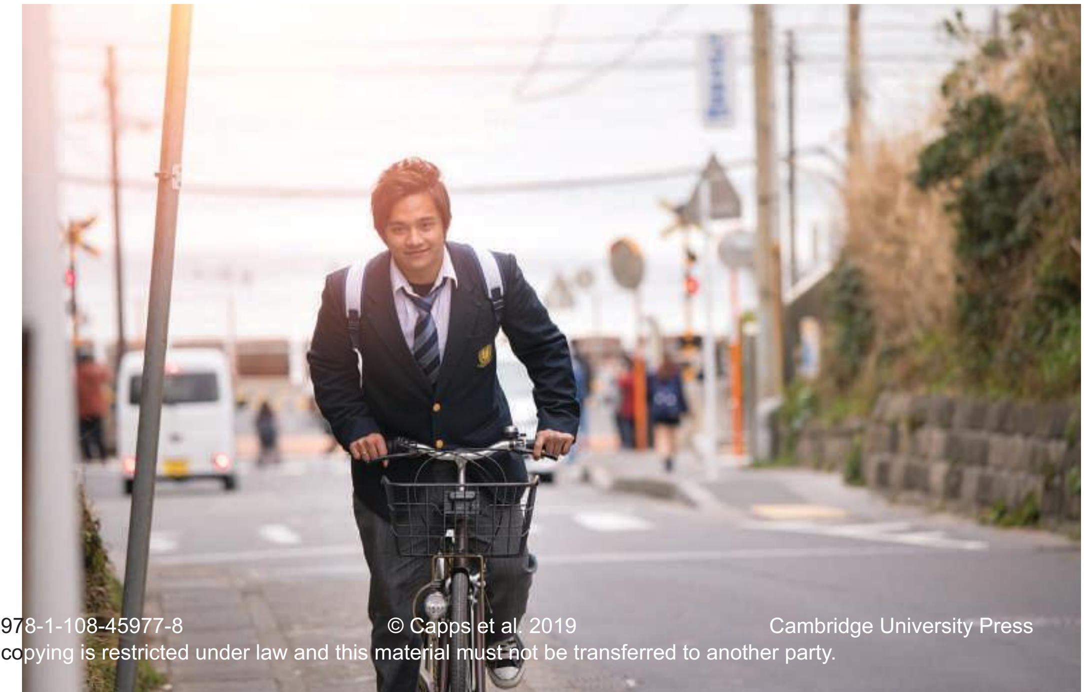
Transport method	Car	Walk	Bike	Bus
Number of students	56	27	12	43

a Determine how many students were surveyed.

b Calculate the relative frequency of students who travel by car.

c The school has a total of 827 students. Use the relative frequency to estimate the number of students in the school who travel to school in a car.

WORKING	THINKING
a Number of students surveyed $= 56 + 27 + 12 + 43 = 138$	Total the number of students.
b Relative frequency of car travel $= \frac{56}{138}$ ≈ 0.41	Relative frequency of an event = $\frac{\text{number of favourable outcomes observed}}{\text{total number of trials}}$
c Estimate of students in the school who travel by car $= \frac{56}{138} \times 827$ $= 336$	It is better to use the exact value and not a rounded value in subsequent calculations so use the fraction form of the relative frequency.



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Exercise 10C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Probability is a numerical indicator of the ________ of an event occurring.

b $\text{Probability of an event} = \dfrac{\text{number of }\underline{\hspace{2.5cm}}\text{ outcomes}}{\text{total number of }\underline{\hspace{2.5cm}}}$

c Probability ________ in value from 0 (will _____ happen) to 1 (_____ happen).

d Relative frequency can be used to _____ the probability from a simulation or _____.

e $\text{Relative}\underline{\hspace{2.5cm}}\text{of an event} = \dfrac{\text{number of favourable outcomes }\underline{\hspace{2cm}}}{\text{total number of trials}}$

2 Convert the following fractions to decimals. Round your answer to 2 decimal places.

a $\frac{14}{25}$

b $\frac{125}{267}$

c $\frac{54}{87}$

d $\frac{17}{693}$

Hint top ÷ bottom

3 Calculate the following. Round your answer to the nearest whole number.

a 0.28×568

b 0.79×682

c $\frac{24}{52} \times 467$

d $\frac{39}{586} \times 1276$

APPLICATIONS

SF: 4–8 | CF: – | CU: –

Example 4 ★**4** A class is surveyed about how many siblings each student has. The results are:

Number of siblings	0	1	2	3	4	5	6
Number of students	3	6	10	4	2	0	1

a Determine how many students were surveyed.

b Calculate the relative frequency of a student having 2 siblings. Express your answer as a decimal correct to 2 decimal places.

c Calculate the relative frequency of a student having more than 2 siblings.

d Calculate the relative frequency of a student having at least 2 siblings.

e Calculate the relative frequency of a student having fewer than 2 siblings.

5 The form guide for a horse race displays information about past performance of the horses. The numbers in the Career column show:

- how many races the horse has participated in
- how many times the horse has finished first
- how many times the horse has finished second
- how many times the horse has finished third

So, a Career of 10 : 2–1–5 would mean the horse has participated in 10 races, won two races, came second once and third on five occasions.

This table shows the field for a 1200 m race:

# Number, Horse (Barrier)	Career
1. Frequency Follie (6)	5 : 2–0–1
2. Statistics Sally (3)	4 : 1–0–0
3. Tally Marks (4)	4 : 2–0–1
4. Sum Total (2)	4 : 1–0–1
5. Decimal Diva (1)	6 : 1–1–1
6. Essential Ellie (7)	3 : 1–1–0
7. Relative Rita (5)	3 : 1–0–0

a For each horse in the race, calculate the relative frequency of them finishing in first place. Express your answer as a decimal correct to 2 decimal places.

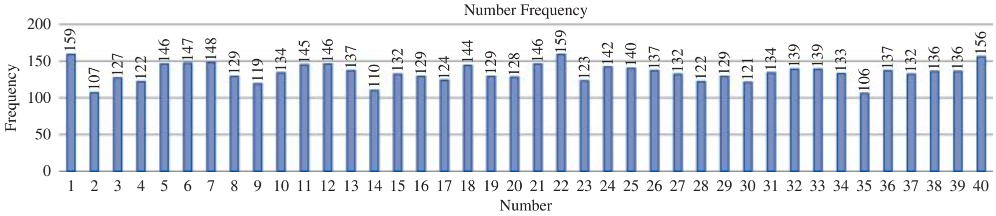
b For each horse in the race, calculate the relative frequency of them finishing in either first, second or third place. Express your answer as a decimal correct to 2 decimal places.

c Based on your results from **a** and **b**, determine which horse has the best chance of:

  i winning this race

  ii placing in this race

6 The following graph shows how many times each number has be drawn in a lottery over a period of time.



a Determine how many numbers have been drawn over the period of time.
b Calculate the relative frequency of the number drawn being 13. Express your answer as a decimal correct to 2 decimal places.
c Determine the relative frequency of the number drawn being a birthdate. Express your answer as a decimal correct to 2 decimal places.
d Based on the past numbers being drawn, explain which numbers are more likely to be selected in future draws.

★**7** The age of migrants to Queensland over a ten-year period is shown below:

Age (years)	0–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65 and over
Number	31 002	13 945	11 577	21 032	28 107	27 936	23 144	16 306	8392	4209	2640	4292

a Calculate how many migrants arrived in Queensland over the period of study.
b Determine the relative frequency of migrant children who are aged 0–14 years.
c Calculate the relative frequency of migrants who are aged under 20 years.
d Calculate the relative frequency of migrants who are aged 60 years or older.

Example 5 ★**8** Brittany wants to decorate a birthday cake with green chocolate coated candy and wants to know how many bags of mixed colour candy to buy. She knows from an internet search that there are approximately 105 pieces of candy per 100 grams and that a standard bag is 180 grams. She counted out a handful of candy and found that of the 38 pieces of candy, 7 were green. Brittany estimates that she will need 50 green candies for her project.

a Calculate how many pieces of candy would be in 20 grams.
b Determine how many pieces of candy would be in a standard 180-gram bag.
c Calculate the relative frequency of the green candy in her sample.
d Use your answers from **b** and **c** to estimate how many pieces of green candy will be in one 180-gram packet.
e Determine how many bags of candy Brittany will need to buy for her decorated cake project.

Technology Activity 10C: See the Interactive Textbook for this activity on using Excel to simulate rolling 2 dice and calculating relative frequency.

10D Identifying complication factors with real-life simulations COMPLEX

LEARNING GOAL

- Identify factors that cause unreliable results when conducting a simulation using techniques other than technology

Why is identifying complication factors in simulations essential?

When conducting a simulation, the outcomes need to be completely random in their selection. Poor techniques, such as not returning cards to a deck after selection or always tossing a coin with heads facing up, could alter the results achieved.

Poor techniques could complicate the results obtained from a simulation.

WHAT YOU NEED TO KNOW

- Poor techniques could complicate the results obtained from a simulation.
- If selecting from a hat or pack of cards:
  - always return the item to the hat/deck
  - mix/shuffle thoroughly before making the next selection
  - ensure there are no cards missing, but check there are no Jokers if the simulation doesn't involve them.
- When rolling a die, tossing a coin or spinning a spinner:
  - ensure that it is well tossed/spun so that the same selection does not continue to come up
  - do not always start with the same side up or at the same place
  - check the spinner rotates freely without sticking (e.g. that is does not stop in the same place each time).

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Example 6 Identifying complicating factors with real-life simulations

Identify potential problems that could occur with the following simulation experiments.

a When simulating the gender of a baby, tossing a coin and always starting on heads.

b When simulating the month of birth for a person, rolling two dice and adding the faces together.

c When simulating the meal choice of chicken or beef for wedding guests, selecting a card from the top of a new unshuffled deck of cards, noting the colour of the card and then not returning the card to the deck before making the next selection.

WORKING	THINKING
a Tossing a coin and always starting on heads has the potential problem that depending on how high the coin is tossed and with how much spin, always starting on heads each time could bias the results towards the same result.	Always start at a different position.
b Simulating month of birth by rolling two dice has the potential problem that minimum sum is 2 so January would not be able to be achieved. February needs both dice to be 1, and December needs both dice to be 6, but June could come from either 3 + 3, 2 + 4 or 1 + 5.	The simulation must be able to produce all results, and each outcome needs to be equally likely to occur.
c A new, unshuffled deck of cards is likely to have the cards grouped by suit, and so the first 13 drawn from the top will be the same colour – the deck should be shuffled, or the card drawn at random. If the card is not returned, then the number of options reduces, so if the first card is red, and not returned, then there are now more black cards than red cards to choose from. A deck has two Jokers, which would not indicate either meal choice if drawn.	Each outcome needs to be equally likely to occur.

Exercise 10D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Poor ______ could complicate the results obtained from a __________.
- **b** If ______ from a hat or pack of cards, always ______ the item to the hat/deck and mix/shuffle thoroughly before making the __________ selection.
- **c** When rolling a die, tossing a coin or spinning a spinner, ensure that it is well tossed/spun so that the same ______ does not continue to come up and do not always start with the ______ side up or at the same place.

APPLICATIONS

SF: – | CF: 2–4 | CU: 5

Example 6

2 Use a set of standard playing cards to simulate which sport house each member of a group of 20 students is in, from a choice of four houses. Remove both Jokers from the deck and use these variations on simulation technique to compare results.

- **i** Return the selected card to the deck after each selection and shuffle the deck between selections
- **ii** Return the selected card to the deck after each selection but do not shuffle the deck between selections
- **iii** Do not return the selected card to the deck after each selection but thoroughly shuffle between selections
- **iv** Do not return the selected card to the deck after each selection and do not shuffle between selections
- **a** Copy and complete this frequency table to collate your results.

	Frequency			
Complicating factor	**Hearts**	**Diamonds**	**Clubs**	**Spades**
return/ shuffle				
return/ no shuffle				
no return/ shuffle				
no return/ no shuffle				

- **b** Evaluate which of the techniques could have possibly caused a problem with the results collected. Explain how.

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3 Use a coin to simulate the gender of 20 babies born in a hospital in one week. Use these variations on simulation technique to compare results.

- **i** Always start with heads
- **ii** Always start with the previous result up. For instance, if a head is tossed, start with a head facing up next turn
- **iii** High toss
- **iv** Low toss
- **v** Catch the coin off the toss and reveal the result on the back of your hand
- **vi** Allow the coin to fall to the ground from the toss
- **a** Copy and complete this frequency table to collate your results.

	Frequency	
Complicating factor	**Heads**	**Tail**
heads up		
previous result up		
high toss		
low toss		
catch and reveal		
fall to the ground		

- **b** Evaluate which of the techniques could have possibly caused a problem with the results collected. Explain how.

4 Use a die to sample the following variations on simulation techniques over 20 trials and compare the results collected.

- **i** Use a cup to shake the die before rolling
- **ii** Pick up the die and flick straight out of your hand without shaking
- **iii** Shake the die in your hands before rolling
- **a** Copy and complete this frequency table to collate your results.

	Frequency					
Complicating factor	**1**	**2**	**3**	**4**	**5**	**6**
use cup to shake						
no shaking of die						
shake in hands						

- **b** Decide which of the techniques could have possibly caused a problem with the results collected. Explain how.

5 A club manager runs a ‘Happy Hour’ promotion each Sunday afternoon. At the start of ‘Happy Hour’, the bartender spins a wheel to determine the discount applied to all drinks sold during the hour. A discount of 50 cents, \$1, \$1.50 or ‘no discount’ is applied for the hour.

- **a** Use a simulation technique (cards, spinner etc.) to estimate the relative frequency of each outcome over one year (trials = 52).
- **b** Determine the average discount offered during ‘Happy Hour’ over the year.
- **c** The manager notices that the bar staff re-spin the wheel if a ‘No discount’ comes up to keep the members happy.
  - **i** In what way does this complicate the simulation that was conducted?
  - **ii** Based on your results from part **a**, how often will the wheel need to be re-spun?
  - **iii** Use a simulation technique to re-spin the wheel for the ‘No discount’ outcomes. Calculate the new relative frequency for each of the discounts.
  - **iv** Calculate the new average discount offered during Happy Hour over the year.
- **d** Whenever the ‘No discount’ comes up, the bar staff always re-spin the wheel. Suggest a better simulation method.

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10E Constructing a sample space

LEARNING GOALS

- Determine the size of a sample space
- Use a table to construct a sample space
- Use a systematic list to construct a sample space

Why is the use of sample spaces essential?

A sample space is a list of all possible outcomes in a probability experiment, such as the six possible outcomes of rolling one die. Being able to construct a list of all possible outcomes of a probability experiment allows us to calculate the probability of an event occurring.

A sample space is a list of all possible outcomes. The image shows the sample space for throwing one die.

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WHAT YOU NEED TO KNOW

- A **sample space** is a list of all possible outcomes. The list can be constructed by either using a table or a systematic list.
- A **table** is used when there are two stages in a probability experiment. The outcomes of one stage are listed across the top and the other stage is listed along the side. The table is then infilled with the combinations of the two trials.
- For example, when tossing a coin and rolling a die:

Die outcomes

		1	2	3	4	5	6
Coin outcomes	Head	Head, 1	Head, 2	Head, 3	Head, 4	Head, 5	Head, 6
	Tail	Tails, 1	Tails, 2	Tails, 3	Tails, 4	Tails, 5	Tails, 6

- The **size of the sample space** is determined by the product of the number of outcomes in each stage.
  - For example: A die has 6 outcomes $\{1, 2, 3, 4, 5, 6\}$ and a coin has two outcomes $\{H, T\}$. Therefore, size of sample space $= 6 \times 2 = 12$.
- A **systematic list** is constructed by systematically listing all the outcomes of the first stage with each of the outcomes of the second stage in turn.
  - For example: A coin is tossed and a spinner is spun with the outcomes A, B, C on each selection. The sample space would be:

Head, A   Head, B   Head, C

Tail, A   Tail, B   Tail, C

- If selecting two items from a list, repeats cannot happen.
  - For example: Colby, Sharlah, Bridget and Tom nominate for the school tennis team. Only two positions are available. The possible team combinations would be:

First person selected

		Colby	Sharlah	Bridget	Tom
Second person selected	Colby	X	S, C	B, C	T, C
	Sharlah	C, S	X	B, S	T, S
	Bridget	C, B	S, B	X	T, B
	Tom	C, T	S, T	B, T	X

Example 7 Using a table to construct a sample space

Nicole is packing for an overseas holiday. She plans on taking:

- 3 shirts that are blue, white and black
- 2 pairs of trousers that are tan and black

a If each combination of shirt/trousers is an outcome, calculate the size of the sample space.

b Use a table to construct a sample space of the different outfit combinations.

WORKING

THINKING

a Sample space size = $3 \times 2 = 6$

The product of the number of outcomes in each stage determines the size of the sample space.

b Sample space:

Construct a table showing all outcomes.

		Shirts		
		blue	white	black
Trousers	tan	blue, tan	white, tan	black, tan
	black	blue, black	white, black	black, black

Example 8 Using a systematic list to construct a sample space

A pizza restaurant offers two types of base (thin or thick) and two types of sauce (BBQ or tomato). These can be ordered in any combination.

a Calculate the size of the sample space (the number of pizza combinations).

b Make a systematic list of the pizza combinations.

WORKING

THINKING

a Sample space size = $2 \times 2 = 4$

Sample space size = product of number of outcomes in each stage

b List of combinations:
Thin, BBQ; Thin, Tomato Thick, BBQ;
Thick, Tomato

Start with the first stage (bases) and add each of the elements from the second stage (sauces).

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Example 9 Using a table without repetition to determine a sample space

Patrick is in the gym and wants to add weights to the bench press to increase the load. He can choose from 2 kg, 5 kg, 10 kg or 20 kg weights. There is only enough space to add two weights to the machine and only one of each of the weights to choose from.

a Calculate the size of the sample space.

b Use a table to construct a sample space of the different weight combinations.

WORKING	THINKING
a Sample space size $= 4 \times 3 = 12$	Once a weight has been selected, it cannot be selected again so the second selection only has 3 possible outcomes.
b Sample space:	Construct a table without repetition of the outcomes.

Weight No. 1

Weight No. 2	2	5	10	20
2	X	5,2	10,2	20,2
5	2,5	X	10,5	20,5
10	2,10	5,10	X	20,10
20	2,20	5,20	10,20	X

Exercise 10E

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** A table is used when there are _____ stages in a probability experiment. The outcomes of one stage are listed across the top and the other stage is listed along the side. The table is then infilled with the _____ of the two trials.
- **b** The size of the sample _____ is determined by the _____ of the number of outcomes in each stage.
- **c** A systematic list is constructed by _____ listing all the outcomes of the first stage with each of the outcomes of the _____ stage in turn.
- **d** If selecting two items from a list, repeats _____ happen.

2 For each of the following probability experiments:

i Rolling a die

ii Tossing a coin

iii Selecting a card from a standard deck and noting the colour

iv Selecting a card from a standard deck and noting the suit

a Determine how many elements are in the sample space.

b List the sample space.

APPLICATIONS

SF: 3–8	CF: –	CU: –

Example 7 ★**3** When buying a car, Zara had the choice of body type (hatch, sedan, SUV) and colour (white, red, grey, blue).

a Calculate the size of the sample space to determine how many different combinations of car are possible.

b Use a table to determine the sample space for car choices.

★**4** A couple is expecting twin babies.

a Calculate the size of the sample space to determine how many different combinations of Boy/Girl are possible.

b Use a table to determine the sample space for the gender of the two babies.

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Example 8 ★5 Ryan is ordering a takeaway meal from the local Thai Restaurant. He has a choice of:

Meat: Chicken, Beef, Prawns

Accompaniment: Steamed Rice, Fried Rice, Noodles

a Calculate the size of the sample space to determine how many different meal combinations are possible.

b Use a systematic list to determine the sample space for meal choice combinations.

★6 Grace is organising her 18th birthday celebrations at a local tavern. The menu has the following options for the set menu:

Entrée: Salt & Pepper Calamari, Garlic Prawns, Chicken Satay

Main: Fillet Steak, Chicken Schnitzel, Barramundi, Chicken Parmigiana, Pasta Carbonara

a Calculate the size of the sample space to determine how many different combinations of entrée and main are possible.

b Use a systematic list to determine the sample space for meal choice combinations.

Example 9 ★7 Greer has a messy sock drawer with two striped, two dotty (spotted), one orange and one black sock all thrown in together. She randomly selects two socks without checking if they are a matching pair.

a Calculate the sample size to determine how many different sock combinations are possible.

b Use a table to determine the sample space for sock choices.

★8 Mike, Daniel, Brooke, Abby and Holly all nominate to be the Student Representative for their class. Only two positions are available on the SRC.

a Calculate the sample size to determine how many different SRC combinations are possible.

b Use a table to determine the sample space for SRC combinations.

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10F Determining probabilities for an experiment

LEARNING GOAL

Use a sample space to determine the probability of outcomes for an experiment

Why is the ability to calculate probabilities essential?

In the same way that relative frequency can be calculated based on simulations, theoretical probability can be determined by using a sample space; without the need to conduct probability experiments. There are many examples of fields where the ability to calculate probabilities is vital, such as weather and environmental forecasting, insurance (predicting the rate of losses and accidents), engineering (the chance of components failing), and business and finance (chance of future events). In science every time that results are reported it is necessary to calculate the chances that a particular score or measurement could have arisen by chance.

Scientists have to calculate the probability that their results arose by chance.

WHAT YOU NEED TO KNOW

- **Probability** is a numerical indicator of the chance of an event occurring.
  - $\text{Probability of an event} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
- The total number of **outcomes** is the size of the **sample space**.
- **Favourable outcomes** are the elements of the sample space that we are interested in, or which suit our needs.

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Example 10 Calculating probabilities from a simple sample space

A standard die is rolled.

a Determine how many elements are in the sample space.
b List the sample space.
c Calculate the probability that a 4 is rolled.
d Calculate the probability that more than a 4 is rolled.
e Calculate the probability that at least 4 is rolled.
f Calculate the probability that less than 4 is rolled.
g Calculate the probability of rolling an odd number.

WORKING	THINKING
a Sample space size = 6	There are 6 faces on a standard die.
b Sample space = 1, 2, 3, 4, 5, 6	Include all results possible.
c $1, 2, 3, \boxed{4,} 5, 6$ Number of favourable outcomes = 1 Probability of 4 = $\frac{1}{6}$	Highlighting, or underlining, favourable outcomes in your sample space is a great exam technique.
d $1, 2, 3, 4, \boxed{5, 6}$ Number of favourable outcomes = 2 Probability of more than 4 = $\frac{2}{6}$ or $\frac{1}{3}$	'More than 4' does not include 4.
e $1, 2, 3, \boxed{4, 5, 6}$ Number of favourable outcomes = 3 Probability of at least 4 = $\frac{3}{6}$ or $\frac{1}{2}$	'At least 4' includes 4.
f $\boxed{1, 2, 3,} 4, 5, 6$ Number of favourable outcomes = 3 Probability of less than 4 = $\frac{3}{6}$ or $\frac{1}{2}$	'Less than 4' does not include 4.
g $\boxed{1,} 2, \boxed{3,} 4, \boxed{5,} 6$ Number of favourable outcomes = 3 Probability of odd = $\frac{3}{6} = \frac{1}{2}$	Odd numbers are 1, 3, 5, …

Example 11 Calculating probabilities from a sample space in a table

Two standard dice are rolled, and the two faces are added together.

a Determine how many elements are in the sample space.
b Use a table to determine the sample space.
c Calculate the probability that a total of 4 is rolled.
d Calculate the probability that a total of more than a 4 is rolled.
e Calculate the probability that a total of at least 4 is rolled.
f Calculate the probability that a total less than 4 is rolled.
g Calculate the probability of rolling an even total.

WORKING	THINKING
a Sample size $= 6 \times 6 = 36$	Sample size is the product of the number of elements in each stage of the experiment.
b Sample space:	Construct a table to show all outcomes.

Die 1 (columns), **Die 2** (rows)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

WORKING	THINKING
c Number of favourable outcomes = 3 Probability of a total of 4 $= \frac{3}{36} = \frac{1}{12}$	Highlighting, or underlining, favourable outcomes in your sample space is a great exam technique.
d Number of favourable outcomes = 30 Probability of total more than 4 $= \frac{30}{36}$ or $\frac{5}{6}$	‘More than 4’ does not include 4.
e Number of favourable outcomes = 33 Probability of total of at least 4 $= \frac{33}{36}$ or $\frac{11}{12}$	‘At least 4’ includes 4.

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WORKING | **THINKING**

Die 1

	+	1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The sample space is copied here for convenience.

f Number of favourable outcomes = 3

‘Less than 4’ does not include 4.

Probability of total of less than $4 = \frac{3}{36}$ or $\frac{1}{12}$

g Number of favourable outcomes = 18

Even numbers are 2, 4, 6, …

Probability of an even total = $\frac{18}{36}$ or $\frac{1}{2}$

Exercise 10F

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Probability is a numerical ______ of the chance of an ______ occurring.

b Probability of an event = $\dfrac{\text{\_\_\_\_\_\_ of favourable outcomes}}{\text{total number of \_\_\_\_\_\_\_\_\_\_}}$

c The ______ number of outcomes is the ______ of the sample space.

d ______ outcomes are the elements of the sample space that suit our______.

Example 10

2 A card is selected from a standard deck of 52 cards.

a If the colour of the card is noted:

i determine the size of the sample space

ii list the sample space

iii calculate the probability of selecting a Red card

b If the suit of the card is noted:

i determine the size of the sample space

ii list the sample space

iii calculate the probability of selecting a Heart

c If the face of the card is noted:

i determine the size of the sample space

ii list the sample space

iii calculate the probability of selecting an Ace

Hint A standard pack of cards has 4 suits.
Red suits: Hearts♥ and Diamonds♦
Black suits: Spades♠ and Clubs♣
Each suit has 13 cards: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

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APPLICATIONS

SF: 3–8	CF: –	CU: –

Example 11 ★**3** Two coins are tossed and the result noted.

- **a** Determine the size of the sample space.
- **b** Use a table to list the sample space.
- **c** Calculate the probability of tossing no tails.
- **d** Calculate the probability of tossing one tail.
- **e** Calculate the probability of tossing two tails.
- **f** Calculate the probability of tossing at least one tail.

★**4** An ice-cream shop offers two types of cones (Waffle and Sugar) and four different flavours of ice-cream (Chocolate, Vanilla, Strawberry and Salted Caramel).

- **a** Determine the size of the sample space.
- **b** Use a table to list the sample space.
- **c** Calculate the probability of a person ordering Salted Caramel in a Waffle cone.
- **d** Calculate the probability of a person ordering Salted Caramel ice-cream.
- **e** Calculate the probability of a person ordering ice-cream in a Waffle cone.
- **f** Calculate the probability of a person ordering Chocolate ice-cream or a Waffle cone.

5 A card is selected from a standard deck of 52 cards. Complete the following if the suit and face of the card is noted.

- **a** Determine the size of the sample space.
- **b** Use a table to list the sample space.
- **c** Calculate the probability of selecting the Ace of Hearts.
- **d** Calculate the probability of selecting a Picture Card (J, Q, K).
- **e** Calculate the probability of selecting an Ace or a Queen.
- **f** Calculate the probability of selecting a Spade or a Queen.
- **g** Calculate the probability of selecting a Picture Card or a Club.

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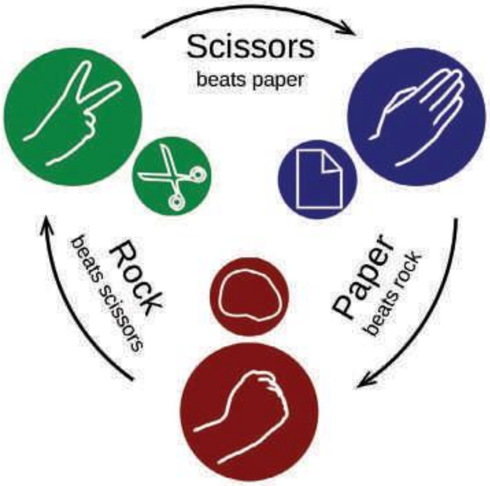
★**6** Two four-sided dice are rolled, and the two faces are added together.

a Determine the size of the sample space.
b Use a table to list the sample space.
c Calculate the probability of a total of 5 being rolled.
d Calculate the probability of a total of at least 5 being rolled.
e Calculate the probability of a total that is more than 5 being rolled.
f Calculate the probability of a total that is less than 5 being rolled.

★**7** A student guesses the answer to two multiple choice questions in a test. Each question has 4 options, with only one being correct.

a Determine the size of the sample space.
b Use a table to list the sample space.
c Calculate the probability of getting both questions correct.
d Calculate the probability of getting one question correct.
e Calculate the probability of getting at least one question correct.
f Calculate the probability of getting no questions correct.

★**8** Hayley and Ben play ‘Rock-Paper-Scissors’ to decide who will pay for ice-cream. Each player randomly selects to be ‘rock’ or ‘paper’ or ‘scissors’. The rules are:



a Determine the size of the sample space.
b Use a table to list the sample space.
c Calculate the probability of Ben playing ‘Scissors’.
d Calculate the probability of both players playing ‘Rock’.
e Calculate the probability of both players playing the same move.
f Calculate the probability of Hayley winning by playing ‘Paper’.

Hint If two players play the same move, it is a draw and the game is played again.

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10G Using tree diagrams to determine probabilities

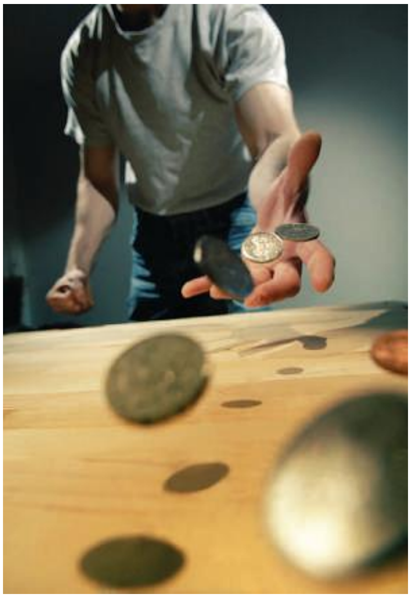
LEARNING GOALS

- Construct a tree diagram for two or more trials
- Determine a sample space from a tree diagram
- Determine probabilities for experiments using a tree diagram

Why are tree diagrams essential?

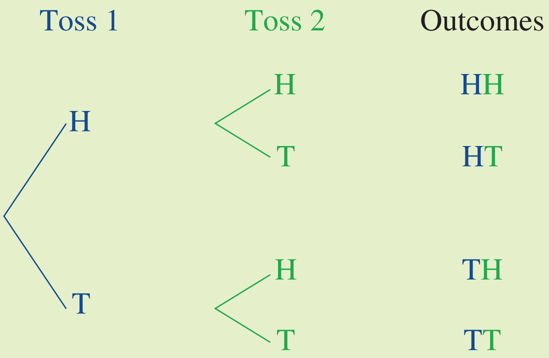
Tables and systematic lists are useful for determining sample spaces for probability experiments with up to two trials. If there are more than two trials, these methods are no longer useful, and we use a tree diagram instead.

Tree diagrams help us determine sample spaces and probabilities for experiments with two or more trials.



WHAT YOU NEED TO KNOW

- **Outcomes** for each **trial** are listed in columns. The sample space is a list of all possible outcomes written in a column at the end of the branches.
  - For example, if a coin is tossed two times, the **tree diagram** would be:



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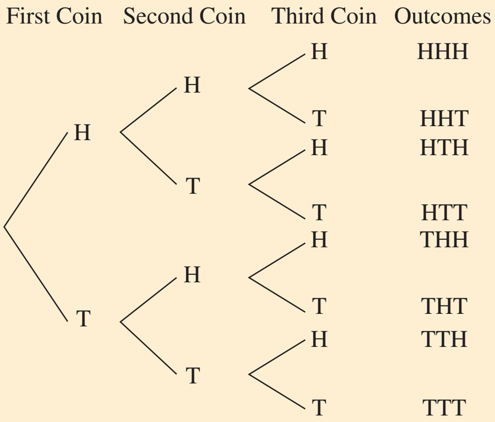
Example 12 Using a tree diagram to determine probabilities

Lilly tosses three coins for a probability experiment, noting the result from each coin as a Head (H) or a Tail (T).

a Construct a tree diagram showing all possible outcomes of the experiment.
b Determine the number of outcomes.
c Use the tree diagram to list all possible outcomes of the experiment.
d Calculate the probability of tossing three heads.
e Calculate the probability of tossing two heads.
f Calculate the probability of tossing one or two heads.
g Calculate the probability of tossing at least one head.

WORKING	THINKING
a 	Construct a tree diagram ensuring that only the possible outcomes at each stage are included.
b There are 8 outcomes	Count how many outcomes are in the final column.
c The outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, TTT	Read across each branch set to create a list of the outcomes.
d Number of favourable outcomes = 1 Probability of HHH $= \frac{1}{8}$	There is only one outcome of HHH.
e Number of favourable outcomes = 3 Probability of two heads $= \frac{3}{8}$	HHT, HTH, THH are all favourable outcomes.
f Number of favourable outcomes = 6 Probability of one or two heads $= \frac{6}{8} = \frac{3}{4}$	HTT, THT, TTH, HHT, HTH, THH are all favourable outcomes.

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WORKING | **THINKING**

g Number of favourable outcomes = 7

Probability of at least one head = $\frac{7}{8}$

Except for TTT, all other outcomes are favourable.

Example 13 Using a tree diagram without replacement to determine probabilities

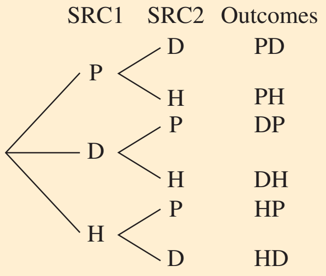
Preston, Doug, and Hannah all nominate for the two positions as class SRC representative.

a Construct a tree diagram showing all possible outcomes.
b Determine the number of outcomes.
c List all possible outcomes of the experiment from the tree diagram.
d Calculate the probability that Doug and Hannah are the SRC representatives.
e Calculate the probability that Preston is an SRC representative.

WORKING | **THINKING**

a All possible outcomes:

Each person can only be selected once. If selected in the first round, they cannot be selected in the second round.



b There are 6 outcomes.

Count how many outcomes are in the final column.

c The outcomes are
PD, PH, DP, DH, HP, HD

Read across each branch set to create a list of the outcomes.

d Number of favourable outcomes = 2,

Probability that Doug and Hannah are the representatives = $\frac{2}{6}$ or $\frac{1}{3}$

DH and HD are both favourable outcomes.

e Number of favourable outcomes = 4,

Probability that Preston is the representative = $\frac{4}{6}$ or $\frac{2}{3}$

Count the outcomes with P.

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Exercise 10G

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Tables and systematic lists are useful for ______ sample spaces for probability experiments with up to ______ trials.

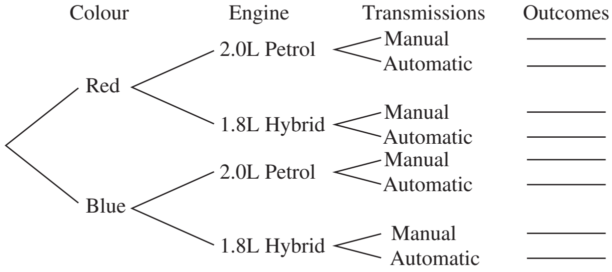
b Tree diagrams help us determine sample spaces and ______ for experiments with 2 or ______ trials.

c Outcomes for each trial are listed in______. The sample space is a list of all possible ______ written in a column at the end of the branches.

APPLICATIONS

SF: 2–8	CF: –	CU: –

2 Tenille is looking to buy herself a car. The car she has decided on comes with a choice of colour, engine type and transmission. A tree diagram of the possible choices is shown.



a Complete the Outcomes column.

b Determine how many car combinations are possible.

c Calculate the probability that Tenille selects a Red Hybrid Automatic car.

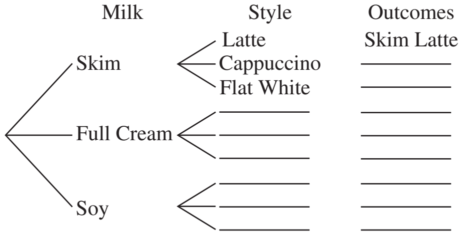
d Calculate the probability that Tenille selects a Red Automatic car.

e Calculate the probability that Tenille selects a an Automatic car.

3 When ordering a coffee, Jayde has a choice of:

- Milk: Skim, Full Cream, Soy
- Style: Latte, Cappuccino, Flat White

a Complete the tree diagram for her coffee order possible combinations:



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b Determine how many order combinations are possible.

c List all the possible coffee combinations.

d Calculate the probability Jayde orders a Skim Milk Latte.

e Calculate the probability Jayde orders a Latte.

f Calculate the probability Jayde does not order a coffee made with Soy.

Example 12 ★**4** Felix conducts a probability experiment by selecting a card at random from a standard deck of cards and noting the colour as Red (R) or Black (B). He conducts three trials of the experiment and returns the card to the deck after each selection.

a Construct a tree diagram showing all possible outcomes of the experiment.

b Determine the number of outcomes.

c Use your tree diagram to list all possible outcomes of the experiment.

d Calculate probability of Felix selecting exactly three red cards.

e Calculate probability of Felix selecting exactly two red cards.

f Calculate probability of Felix selecting less than two red cards.

★**5** Lochie is enjoying a meal at Murphy's Grill. The menu is shown:

Murphy's Grill

200G EYE FILLET	34
250G RIB FILLET	35
350G RUMP	32

OUR GRILLS ARE SERVED WITH
CHIPS OR MASH
VEGETABLES OR SALAD

a Construct a tree diagram showing all possible steak meal combinations of:
- Steak: Eye (E), Rib (R) or Rump (P)
- Potato: Chips (C) or Mash (M)
- Side: Vegetables (V) or Salad (S)

b Determine the number of meal combinations.

c List all the meal combinations.

d Calculate the probability that Lochie has the Rump steak served with chips and salad.

e Calculate the probability that Lochie has the Rib Fillet steak served with chips.

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★**6** Will has noticed that the school bus is equally likely to be either on Time (T) or Late (L) each morning. He conducts a probability experiment by rolling a die to determine the likelihood of being late for the next three mornings. If the result on the die is even then the bus is on Time (T); if the result is odd then the bus is Late (L).

a Construct a tree diagram showing all possible outcomes of the experiment.

b Determine the number of outcomes.

c List all the possible outcomes.

d Calculate the probability that Will is on time for school all three days.

e Calculate the probability that Will is late for school on one of the three days.

f Calculate the probability that Will is late for school at least once over the three days.

Example 13 ★**7** Chloe, Gabby, Libby and Michelle all nominate for the two positions in the school tennis team.

a Construct a tree diagram showing all possible outcomes.

b Determine the number of outcomes.

c List all the possible outcomes.

d Calculate the probability that Libby and Gabby are in the tennis team.

e Calculate the probability that Chloe is in the tennis team.

★**8** Cortay wants to have an ice-cream. He can have his choice of Waffle (W) or Sugar Cone (S) and likes to have two scoops of ice-cream with *different* flavours. The flavour choices are: Chocolate (C), Wild Berry (B) and Vanilla (V).

a Construct a tree diagram showing all possible ice-cream outcomes.

b Determine the number of outcomes.

c List all the possible outcomes.

d Calculate the probability that Cortay has a Waffle cone with Wild Berry and Chocolate ice-cream.

e Calculate the probability that Cortay has no Chocolate ice-cream.

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Problem-solving and modelling task

Background: During periods of extended drought, farmers reduce the size of their breeding flock due to a lack of available food.

Once the drought finally breaks, it takes several breeding seasons to return the herd to its original size through the breeding program.

Task: Assume a sheep farmer reduces his herd of 2400 female breeders to just 500 female breeders during a drought. He knows from previous breeding seasons that the chance of a female ewe having twins is $\frac{1}{6}$. You are to perform a simulation of several breeding seasons and estimate how many breeding seasons it will take for the farmer to return his female breeding herd to the original size.

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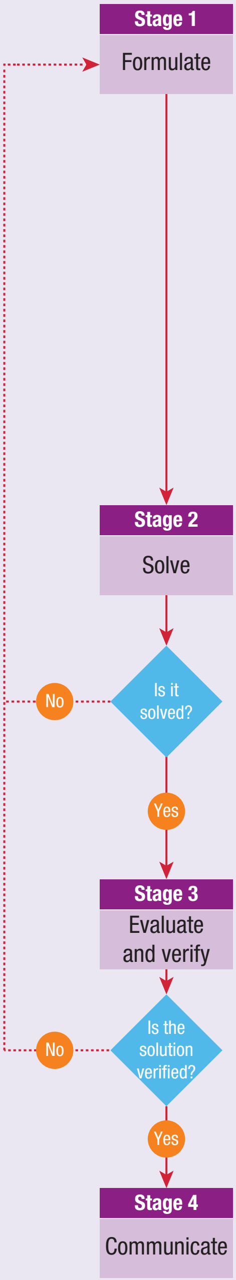
Approach to problem-solving and modelling task:



Stage 1: Formulate

1 What are you required to do?
2 What information do you have?
3 What other information do you need? E.g. For how many years is a breeding ewe able to produce lambs?
4 What assumptions will you make? E.g. What about the male lambs?
5 How could you conduct the simulation? E.g. Will you start with a smaller size sample?
6 How will you record the results of your simulation?

Stage 2: Solve

7 Gather information by conducting a simulation.
8 Use mathematics to calculate information required to solve the problem.
9 Produce graphs/tables required to solve the problem.
10 Have you been able to determine a time line for restoring the breeding flock to full capacity following the drought?

Stage 3: Evaluate and verify

11 Have you answered the question?
12 Is the answer you have reasonable?

Stage 4: Communicate

Write a report with an introduction, body and conclusion.

13 State your main point.
14 Include the evidence in the form of statements, graphs and tables.
15 Explain the evidence. Use a sentence starter like ‘This means … ’
16 In the conclusion, link back to your main point.

Chapter checklist

☐ I can perform simulations of probability experiments using technology.

1 Generate 20 random numbers with a calculator or spreadsheet.

☐ I can recognise that repetition of chance events is likely to produce different results.

2 If a die is rolled 300 times, calculate how many times you would expect each side to appear.

☐ I can identify relative frequency as probability.

3 A survey asked students how many sports they currently participate in. The table below shows the findings.

Number of sports	0	1	2	3
Number of students	42	57	24	2

If a student is selected at random:

a calculate the probability that the student plays one sport. Round your answer to 2 decimal places.

b calculate the probability that the student plays fewer than two sports. Round your answer to 2 decimal places.

☐ I can identify factors that could complicate the simulation of real-world events [complex].

4 Explain the potential problems of doing a simulation by drawing cards from an unshuffled pack of cards.

5 In a probability simulation, explain if the coin should always be held the same way up before tossing.

☐ I can construct a sample space for an experiment.

6 A coin is tossed twice. Determine the sample space by using:

a table

b a systematic list

c a tree diagram

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☐ I can use a sample space to determine the probability of outcomes of an experiment.

7 Use your sample space from the previous question to determine the probability of:

- **a** two tails
- **b** one tail
- **c** at least one tail

☐ I can use a tree diagram to determine the outcomes and probabilities for experiments.

8 Sam has three shirts: they are blue, red and white. He is packing to go away for the weekend and needs to pack two shirts.

- **a** Construct a tree diagram to show the possible combinations of the shirts he packs.
- **b** Determine the probability that he packs the blue and red shirts.
- **c** Determine the probability that he packs the white shirt.

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Chapter review

All questions in the review are assessment-style.

Simple familiar

Section 10A

1 Sam tosses a coin to simulate the gender of 30 job applicants at a fast food store during a week. Heads = girl, tails = boy. The results of the simulation are:

T	T	T	H	T	T	H	H	T	H
T	H	H	H	T	T	H	T	T	T
H	T	T	T	T	T	H	H	T	T

a Record the results of the simulation in a frequency table.

Result	**Frequency**
heads/girls	
tails/boys	

b Determine the percentage of boys from your simulation.

c Calculate how many boys would you expect to find in the sample.

d Decide if the simulation varied from what you expected. Explain.

2 Celeste tosses two coins to simulate the gender of the children in twenty families with two children. Heads = girl, tails = boy. The results of the simulation are:

T H	H H	H T	T H	H T
T T	H T	H H	T T	T H
T T	H T	H H	T H	T H
H T	T T	T T	H H	H T

a Copy and complete the following frequency table to record your results.

2-child combination	**Tally**	**Frequency**
boy + boy		
boy + girl		
girl + girl		

b Calculate the relative frequency of 2-child families with a boy and a girl combination in your simulation.

Section 10B

3 In a probability experiment, Ayuen selects a card from a standard deck of cards and notes the suit. He repeats the experiment 20 times. The results are:

H D C S D D H C C D
S D S H D C H D S H

a Identify how many different outcomes there are when a card is selected at random from a deck and the suit noted.

b Determine how many times you would expect to get the same result as the previous selection in the 20 trials.

c Calculate how many times the same result appears in consecutive selections. Compare this answer with what you expected from part **b**.

d Determine how many times you would expect each outcome to appear in the 20 selections.

Section 10C

4 Ayuen thinks that some results appear more than others in his probability experiment.

a Using the data from question **3**, complete a frequency table for the experiment.

b Calculate the relative frequency for each outcome.

c Determine if any results appear more/less often than expected.

d Describe how Ayuen could change the experiment so that each outcome appears the same amount of times.

5 A survey asked shoppers how many re-usable bags they had with them when entering a supermarket. The table below shows the findings.

Number of bags	0	1	2	3	4	5
Number of shoppers	15	24	18	35	18	12

If a shopper is selected at random:

a calculate the probability that the shopper has 2 bags with them. Round your answer to 2 decimal places.

b determine the probability that the shopper has fewer than 4 bags with them. Round your answer to 2 decimal places.

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Section 10D **6** Identify potential problems that could occur with the following simulation experiments.

- **a** Not returning a selected card to the deck after each selection but thoroughly shuffling between selections.
- **b** Always selecting the top card from a deck of cards.
- **c** Always starting with heads when tossing a coin.
- **d** Allowing the coin to fall to the ground from the toss.
- **e** Picking up a die and flicking it straight out of your hand without shaking first.
- **f** Using a cup to shake a die before rolling.

Section 10E **7** Two regular dice are rolled, and the two faces are added together.

- **a** Determine the size of the sample space.
- **b** Use a table to list the sample space.

8 Two coins are tossed, and the result noted.

- **a** Determine the size of the sample space.
- **b** Use a systematic list to determine the sample space.

Section 10F **9** Two regular dice are rolled and the two faces are added together. Use the sample space from question **7b** to:

- **a** Calculate the probability that a total of 8 is rolled.
- **b** Calculate the probability that a total of at least 8 is rolled.
- **c** Calculate the probability that a total more than 8 is rolled.
- **d** Calculate the probability that a total less than 8 is rolled.

10 Two coins are tossed, and the result noted. Use the sample space from question **8b** to:

- **a** Calculate the probability of tossing two heads.
- **b** Calculate the probability of tossing one head.
- **c** Calculate the probability of tossing no heads.
- **d** Calculate the probability of tossing at least one head.

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Section 10G

11 A die is rolled three times and the outcome is noted as Odd (O) or Even (E)

- **a** Draw a tree diagram showing all possible outcomes of the experiment.
- **b** Determine the number of outcomes.
- **c** From your tree diagram, list all possible outcomes of the experiment.
- **d** Calculate the probability of rolling two even numbers.
- **e** Calculate the probability of rolling 3 even numbers.
- **f** Calculate the probability of rolling less than 2 even numbers.

Complex familiar

12 Connor estimates that his school bus is late once a week.

- **a** Show that P(bus is late) is 0.20.
- **b** Use the Random number feature on your calculator to simulate if the bus is late (0.001–0.200) or on time (0.201–0.999) for the next 4 school weeks.
  - **i** Organise your results in a table.

  - **ii** Determine many times is the bus ‘late’ in the simulation.
- **c** Connor estimated that the bus was late 20% of the time.
  - **i** In 4 weeks, determine how many times would you expect the bus to be late.
  - **ii** Describe any differences or similarities between the simulation in part **b** and the expected answer found in **c i**.

13 Tanya wants to order Chinese takeaway for her family. They have four favourite dishes that she usually orders from: Lemon Chicken (C), Mongolian Lamb (L), Sweet and Sour Pork (P) and Satay Beef (B). Tanya needs to only order two *different* meals tonight.

- **a** Determine the size of the sample space.
- **b** Use a table to list the sample space.
- **c** Calculate the probability that Tanya orders Sweet and Sour Pork.
- **d** Calculate the probability that Tanya orders Lemon Chicken and Satay Beef.

e Calculate the probability that Tanya orders Lemon Chicken and Satay Beef or Mongolian Lamb.

f Calculate the probability that Tanya orders Lemon Chicken or Satay Beef.

Complex unfamiliar

14 As part of a History investigation into the one-child policy once adopted in rural communities in China, Anna wants to conduct a probability experiment to determine the average number of children a family would need to have in order to have at least one son. She decides to toss a coin, and say that Head = Girl, Tail = Boy. The results of her tosses are below. She has indicated the number of children required for the first three families to have a boy child by grouping results together until a Tail is tossed.

H	T	H	T	H	H	T	H	H	T
H	T	H	H	T	T	T	T	T	T
H	H	H	H	T	T	T	H	H	H
T	T	T	T	H	H	T	T	T	H

a Continue to identify the size of twenty families in Anna's simulation.

b Organise the results into a frequency table as follows.

Number of children	Tally	Frequency
1		
2		
3		
4		
5		
Total		

c Calculate the average family size based on Anna's simulation.

d Calculate the relative frequency for:

- **i** 2 children
- **ii** 1 child

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11 Simple and compound interest

Maths for a painting contractor: Jordan Rubie

Jordan Rubie runs his own painting business.

Tell us a bit about your job. What does a typical day look like?

I am a painting contractor, specialising in large commercial work. A typical day consists of an early start, usually around 6 a.m. My job consists of preparing surfaces, and applying paint and protective coatings by either brush, roller or spray. I will consult with a project manager during the day to organise upcoming work and materials required.

What maths did you study at school?

I completed Maths B until Year 11 and switched to Maths A for Year 12.

How do you use maths in your job?

I use maths on a daily basis. Figuring out quantities of paint needed requires me to figure out surface area by the square metre and calculating the spread rate of different paint products.

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In this chapter

Syllabus reference

Unit 4 Topic 3 Loans and compound interest

Compound interest (12 hours)

Note: Only investments will be dealt with in this chapter. Reducing balance loans are covered in Chapter 12.

In this sub-topic, students will:

- review the principles of simple interest through substitution of given values for other pronumerals into a mathematical formula to find the value of the subject of the formula
- understand the concept of compound interest as a recurrence relation
- consider similar problems involving compounding **[complex]**
- use technology (online calculator) to calculate the future value of a compound interest (loan or) investment and the total interest (paid or) earned
- use technology (spreadsheet) to calculate the future value of a compound interest (loan or) investment and the total interest (paid or) earned **[complex]**
- use technology (online calculator) to compare, numerically and graphically, the growth of simple interest and compound interest (loans and) investments
- use technology (spreadsheet) to compare, numerically and graphically, the growth of simple interest and compound interest (loans and) investments **[complex]**
- use technology (online calculator) to investigate the effect of the interest rate and the number of compounding periods on the future value of a (loan or) investment
- use technology (spreadsheet) to investigate the effect of the interest rate and the number of compounding periods on the future value of a (loan or) investment **[complex]**.

Pre-test

1 Convert the following percentages to fractions.
- **a** 15%
- **b** 6%
- **c** 2.4%
- **d** 11.25%

2 Calculate the following amounts correct to 2 decimal places.
- **a** 12% of \$45
- **b** 8% of \$1590
- **c** 1.8% of \$24 900
- **d** 3.1% of \$800

3 Change the following amounts by the given percentage.
- **a** Increase \$5750 by 12%
- **b** Increase \$600 by 3%
- **c** Decrease \$2120 by 4%
- **d** Decrease \$30 000 by 15%

4 Convert the following time periods.
- **a** 5 years to months
- **b** 3 years to weeks
- **c** 1 year to days
- **d** 7 years to fortnights

5 Use a calculator to the following. Express your answer correct to 2 decimal places.
- **a** 2.01^5
- **b** 1.025^3
- **c** 0.85^7
- **d** 0.55^3

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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11A Understanding and calculating simple interest

LEARNING GOALS

- Understand contexts where simple interest is appropriate
- Use the simple interest formula to calculate interest
- Apply simple interest to real life applications
- Use a spreadsheet for simple interest calculations

Why is simple interest essential?

Interest is the cost of borrowing money, or the reward for investing money. It is quoted using percentages. When investigating loans, or investments, it is important to use the potential costs of interest on a loan, or the interest earned on an investment, to help make good decisions.

Interest is the reward for investing, or the cost of borrowing money.

WHAT YOU NEED TO KNOW

- **Simple interest** is calculated using an interest rate on the same amount of money each **time period**.
- The formula to calculate simple interest is $I = Pin \left(\text{or } I = P \times i \times n\right)$
  - I = interest calculated
  - P = **Principal** (the money borrowed or invested)
  - i = interest rate as a decimal per a time period, for example, 0.04 p.a. (p.a. means '**per annum**' which is 'per year'). However we are usually given the interest rate as a percentage, r %, so it needs to be written as a fraction, $\frac{r}{100}$ when substituting into the formula.
  - n = number of time periods for which interest is calculated.
  - The time units of the time period and the interest rate MUST be the same, e.g. $i = \frac{4}{100}$ p.a and $n = 3$ years; or $i = \frac{4}{100}$ per month and $n = 6$ months.
- Loans using simple interest require the borrowed amount and the interest to be repaid.
  - Total to repay = amount borrowed + interest
  - Regular repayment = total to repay ÷ number of repayments

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Example 1 Calculating simple interest for yearly time periods

Maxine deposits \$600 into her bank account. The account offers an interest rate of 3.2% p.a. simple interest. She leaves the money in the bank for four years.

a Calculate the total interest she would earn over four years.

b Determine how much money Maxine will have after the four years.

WORKING	WORKING
a $P = \$600$ $i = \frac{3.2}{100}$ $n = 4$	Identify the values that need to be substituted into the formula $I = Pin$.
$I = Pin$ $I = 600 \times \frac{3.2}{100} \times 4$ $I = \$76.80$	Substitute the values into the formula. Keystrokes on scientific calculator: 6 0 0 × 3 . 2 ÷ 1 0 0 × 4 enter Money always has two decimal places because of the cents.
b Balance $= \$600 + \$76.80$ $= \$676.80$	Add the interest to the original amount.

Example 2 Calculating simple interest for time periods other than a year

Maxine deposits \$600 into her bank account. The account offers an interest rate of 3.2% p.a. simple interest. Calculate the total interest she would earn if the money is invested for:

a 10 months

b 15 weeks

c 3 years and 6 months

d 60 days

WORKING	THINKING
a $P = \$600$ $i = \frac{3.2}{100}$ $n = \frac{10}{12}$	Identify the values that need to be substituted into the formula $I = Pin$. To change months to years, divide *number of months* by 12.

WORKING	THINKING
$I = Pin$	Substitute the values into the formula.
$I = 600 \times \frac{3.2}{100} \times \frac{10}{12}$ $I = \$16.00$	Keystrokes on scientific calculator: 6 0 0 × 3 . 2 ÷ 1 0 0 × 1 0 ÷ 1 2 enter
b $P = \$600$ $i = \frac{3.2}{100}$	Identify the values that need to be substituted into the formula $I = Pin$.
$n = \frac{15}{52}$	To change weeks to years, divide *number of weeks* by 52.
$I = 600 \times \frac{3.2}{100} \times \frac{15}{52}$ $I = \$5.54$	Substitute the values into the formula.
c $P = \$600$ $i = \frac{3.2}{100}$	Identify the values that need to be substituted into the formula $I = Pin$.
$n = 3 + \frac{6}{12} = 3\frac{6}{12}$	To change months to years, divide *number of months* by 12.
$I = 600 \times \frac{3.2}{100} \times 3\frac{6}{12}$ $I = \$67.20$	Substitute the values into the formula. Keystrokes on scientific calculator: 6 0 0 × 3 . 2 ÷ 1 0 0 × 3 2nd $\frac{n}{d}$ 6 $\frac{n}{d}$ 1 2 enter
d $P = \$600$ $i = \frac{3.2}{100}$	Identify the values to substitute into $I = Pin$.
$n = \frac{60}{365}$	To change days to years, divide *number of days* by 365.
$I = 600 \times \frac{3.2}{100} \times \frac{60}{365}$ $I = \$3.16$	Substitute the values into the formula.

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Example 3 Calculating simple interest loan amounts

John borrows \$1600 with a simple interest rate of 8.7% p.a over 9 months to buy a new laptop. He agrees to repay the loan in equal monthly repayments.

a Calculate the total amount of interest owed after 9 months.

b Calculate the total amount he will need to repay after 9 months.

c Calculate the monthly repayment.

WORKING	THINKING
a $P = \$1600$ $i = \frac{8.7}{100}$ $n = \frac{9}{12}$	Identify the values that need to be substituted into the formula $I = Pin$. To change months to years, divide *number of months* by 12.
$I = Pin$ $I = 1600 \times \frac{8.7}{100} \times \frac{9}{12}$ $I = \$104.40$	Substitute the values into the formula.
b Total to repay = amount borrowed + interest $= 1600 + 104.40$ $= \$1704.40$	Add the amount borrowed to the total interest.
c Regular repayment = total to repay ÷ number of repayments $= 1704.40 \div 9$ $= \$189.38$	Divide the total to repay by the number of repayments. Money always has two decimal places because of the cents.

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Exercise 11A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Interest is the cost of ______________ money, or the reward for ______________ money. It is calculated using ______________.

b Simple interest is calculated using a percentage ______________ rate on the ______________ amount of money each time period.

c Loans using simple interest require the ______________ amount and the ______________ to be repaid.

2 Express the following time periods as a fraction of a year.

a 7 months = _____ year

b 31 weeks = _____ year

c 25 days = _____ year

d 18 months = _____ year

e 270 days = _____ year

f 9 weeks = _____ year

Hint

1 Year = 12 months
= 52 weeks
= 26 fortnights
= 365 days

3 Calculate the value of the following percentages.

a 30% of $23.70

b 10% of $80.90

c 5% of $126.45

d 3.45% of $456.21

e 1.05% of $364.85

f 0.65% of $63.90

Hint First divide the percentage by 100 to make it a decimal.

APPLICATIONS

SF: 4–10	CF: 11	CU: –

Example 1

4 Calculate the simple interest earned on the following investments.

a $780 is invested at 4.57% p.a. for 4 years

b $4260 is invested at 6.32% p.a. for 3 years

c $2030 is invested at 9.05% p.a. for 5 years

d $625 is invested at 5.9% p.a. for 2 years

e $1650 is invested at 4.2% p.a. for 5 years

f $5050 is invested at 3.9% p.a. for 7 years

g $8880 is invested at 5.79% p.a. for 6 years

h $7280 is invested at 5.36% p.a. for 2 years

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★**5** Angus invested \$10 000 in a fixed-term account for 4 years paying a simple interest rate of 3.65% p.a.

a Calculate the total amount of interest earned over the 4 years.
b Calculate the total value of the investment after 4 years.

Example 2 **6** Calculate the simple interest earned on the following investments.

a \$6200 is invested at 6.8% p.a. for 4 months
b \$590 is invested at 5.11% p.a. for 20 weeks
c \$8290 is invested at 4.48% p.a. for 15 days
d \$6700 is invested at 2.26% p.a. for 2 weeks
e \$7740 is invested at 5.35% p.a. for 14 days
f \$858 is invested at 7.07% p.a. for 7 months
g \$3700 is invested at 6.1% p.a. for 9 years and 6 months
h \$6160 is invested at 5.3% p.a. for 8 years and 3 months

★**7** A company invested \$2 000 000 in the short-term money market at 8% p.a. simple interest for 30 days.

a Calculate the total amount of interest earned over the 30 days.
b Calculate the total value of the investment after 30 days.

Hint: The short-term money market is used for borrowing and investing money for up to one year.

Example 3 ★**8** Caleb borrowed \$3000 with a simple interest rate of 9.7% p.a. over 18 months to buy a new lounge.

a Calculate the total amount of interest owed after 18 months.
b Calculate the total amount Caleb will need to repay after 18 months.
c Calculate the monthly repayment.

Hint Total to repay = amount borrowed + interest
Regular repayment = total to repay ÷ number of repayments

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★9 Olivia borrowed $5000 with a simple interest rate of 11.2% p.a. over 4 years to buy a car.

a Calculate the total amount of interest owed after 4 years.

b Calculate the total amount Olivia will need to repay after 4 years.

c Calculate the monthly repayment.

★10 A bank offers the following simple interest rates on fixed-term deposits.

Term	Interest Rate for $10 000 to $49 999	Interest Rate for $50 000 to $1 999 999
60 months	2.65% p.a.	2.75% p.a.
24–33 months	2.60% p.a.	2.70% p.a.
12 months	2.20% p.a.	2.30% p.a.
6 months	2.05% p.a.	2.05% p.a.
3 months	2.00% p.a.	2.00% p.a.

a Ezikiel wins $15 000 in a lottery. He decides to invest it for two years to use as a home loan deposit.

  i Determine what interest rate the bank will offer.

  ii Calculate how much interest he will earn in two years.

  iii Determine how much his investment will be worth after the two years.

b Georgia inherits $65 000. She decides to invest it for one year while she considers what to do with the money.

  i Determine what interest rate the bank will offer.

  ii Calculate how much interest will she earn in one year.

  iii Determine how much her investment will be worth after one year.

★ c Jason has saved $11 400 from his part-time job. He decides to invest it for six months while he hunts for a car to buy.

  i Determine what interest rate the bank will offer.

  ii Calculate how much interest he will earn.

  iii Determine how much his investment will be worth after six months.

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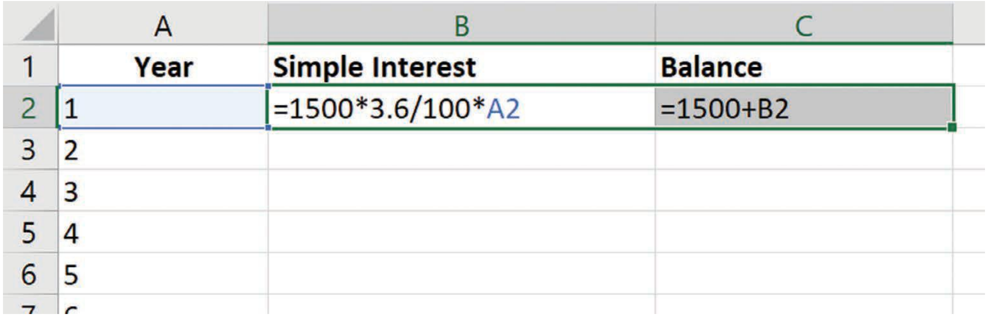
11 Cortay invests $1500 into a bank account that pays 3.6% p.a.

a Set up a spreadsheet to model this investment over 10 years. A sample set-up is shown:



	A	B	C
1	Year	Simple Interest	Balance
2	1	=1500*3.6/100*A2	=1500+B2
3	2		
4	3		
5	4		
6	5		

Hint Only complete the first row. Highlight cells, grab the little square in bottom right corner and drag down to auto fill the formulas.

Hint Highlight first two columns>Insert> Recommended Charts. Select a line graph.

Hint Highlight first and third columns while holding down Ctrl> Insert>Recommended Charts. Select a line graph.

b Use the spreadsheet table to determine the interest earned after:

- **i** 5 years
- **ii** 10 years.

c Use a table of values to determine the value of the investment after:

- **i** 5 years
- **ii** 10 years.

d Graph the Year vs Simple Interest using Excel.

e Graph the Year vs Balance using Excel.

Spreadheet activity 11A: See the interactive textbook for this activity to set up a spreadsheet that will allow you to change the Principal and Interest Rate to investigate how this changes the interest earned.

11B Understanding and calculating compound interest

LEARNING GOALS

- Understand that compound interest is repeated interest calculations
- Apply the compound interest formula to annual compounding periods
- Apply the compound interest formula to various compounding periods
- Use a spreadsheet to calculate recurring interest calculations

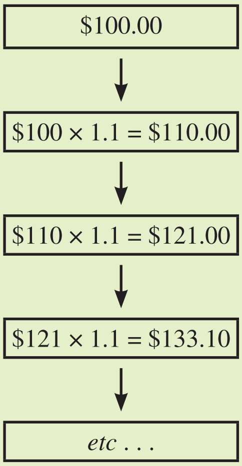
Why is compound interest essential?

We investigated simple interest calculations in the previous section. More commonly though, interest is added to the account at the end of each time period. Future interest calculations are then made on the new balance, that also includes any previous interest payments. Interest then compounds, so we call this method of interest calculation *compound interest*.

With compound interest, interest is earned on the amount including the interest already earned and so the investment will grow more quickly.

WHAT YOU NEED TO KNOW

- At the end of each time period, the same interest calculation rule is applied to the amount of money in the account. The calculation 'recurs' (is repeated) each time. If no money is withdrawn or paid in, and the interest rate remains the same, this produces a sequence where each amount depends only on the previous amount. For example: \$100 is increased by 10% (which means it is multiplied by 1.1) and the result is multiplied by 1.1, and the result of that is multiplied by 1.1, and so on, as shown in the diagram.



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- This is called a **recurrence relation**, of which compound interest is just one type. In this course, we won't use the mathematical form of a recurrence relation, we will use a formula which gives us the amounts in the sequence, which we will show as successive lines in a table or spreadsheet.
- Consider an investment of $2000 placed in an account paying 4% p.a. for 3 years with interest added to the account at the end of each year. We can use a table to find the balance at the end of every year as follows:

Year	Principal	Interest	Balance
1	\$2000.00	$\$2000.00 \times \frac{4}{100} = \$80.00$	\$2000.00 + \$80.00 = \$2080.00
2	\$2080.00	$\$2080.00 \times \frac{4}{100} = \$83.20$	\$2080.00 + \$83.20 = \$2163.20
3	\$2163.20	$\$2163.20 \times \frac{4}{100} = \$86.53$	\$2163.20 + \$86.53 = \$2249.73

- Notice how the balance becomes the principal in the next line, and that the principal is multiplied by the same amount each time ($\frac{4}{100}$ in this case). This is what is meant by a recurrence relation.
- Another method is to consider that by adding 4% interest, at the end of the year we would now have 104% of the starting principal.

  This can be expressed as $(100\% + 4\%) = \left(\frac{100}{100} + \frac{4}{100}\right) = \left(1 + \frac{4}{100}\right)$

$$\text{After 1 year, Balance} = 2000 \times \left(1 + \frac{4}{100}\right) = \$2080$$

$$\text{After 2 years, Balance} = 2000 \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{4}{100}\right) = \$2163.20$$

$$\text{After 3 years, Balance} = 2000 \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{4}{100}\right) = \$2249.73$$

Here we are multiplying the starting principal by (1 + interest) and repeating the multiplication according to the number of time periods. For 3 years we multiply three times.

- We can use indices to write repeated, or recurring, multiplication. Hence after three years:

$$\text{Balance} = 2000 \times \left(1 + \frac{4}{100}\right)^3 = \$2249.73$$

- A formula for **compound interest** can now be developed.

  $\boldsymbol{A = P(1 + i)^n}$, where:
  - A = the *accumulated* amount at the end of the investment
  - P = the *principal* amount that you started with
  - i = the *interest* rate as a decimal per a time period. Using the example with 4% above, $\frac{4}{100} = 0.04$, so $i = 0.04$ p.a. However we are usually given the

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interest rate, as a percentage r % p.a., it needs to be written as $\frac{r}{100}$ when put into the formula.

- n = the *number* of time periods (compounding periods).
- The time units of the compounding period and the interest rate MUST be the same, e.g. $i = \frac{4}{100}$ p.a and $n = 3$ years; or $i = \frac{4}{100}$ per month and $n = 6$ months.

Example 4 Calculating compound interest using a table

Titan invests \$900 into an account paying 3% p.a. interest for four years.

a Complete the following table to calculate Titan's interest and balance at the end of each year.

Year	Principal	Interest	Balance
1	\$900		
2			
3			
4			

b Determine how much interest he earned in four years.

WORKING **THINKING**

a

Year	Principal	Interest	Balance
1	\$900.00	$900.00 \times \frac{3}{100}$ $= 27.00$	$900.00 + 27.00$ $= \$927.00$
2	\$927.00	$927.00 \times \frac{3}{100}$ $= 27.81$	$927.00 + 27.81$ $= \$954.81$
3	\$954.81	$954.81 \times \frac{3}{100}$ $= 28.64$	$954.81 + 28.64$ $= \$983.45$
4	\$983.45	$\$983.45 \times \frac{3}{100}$ $= 29.50$	$983.45 + 29.50$ $= \$1012.95$

The interest is added to the principal at the end of each year.

The balance becomes the principal in the next year.

Money should always have two decimal places because of the cents.

b Interest $= 1012.95 - 900.00 = \$112.95$

Subtract the starting principal from the final balance to calculate the interest earned.

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Example 5 Calculating compound interest with yearly compounding periods using the formula

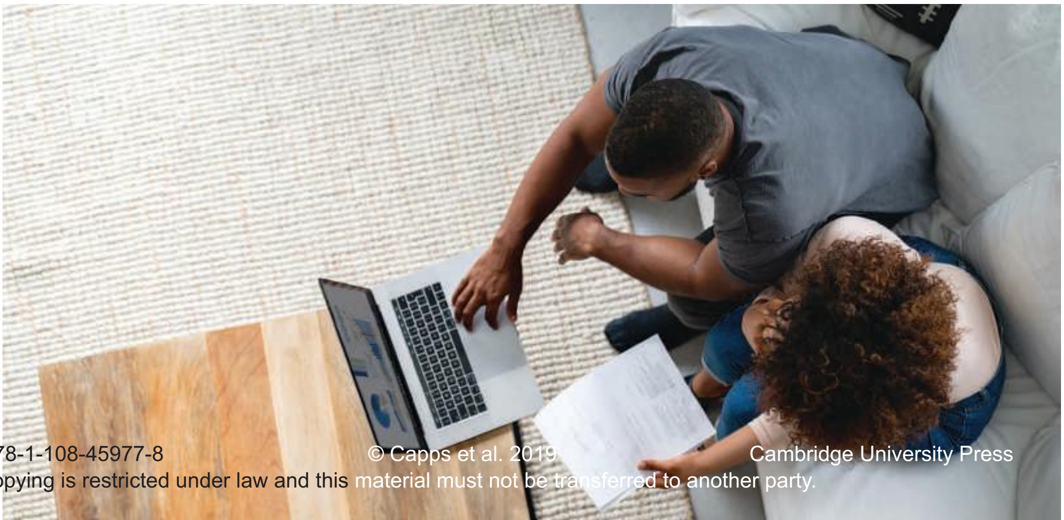
Riley invests \$2800 into an account paying 2.5% p.a. interest that compounds annually.

a Use the compound interest formula to find what Riley's investment will be worth after five years.

b Determine how much interest he earned in five years.

WORKING	THINKING
a $P = \$2800$ $i = \frac{2.5}{100}$ $n = 5$	Identify the values that need to be substituted into the formula $A = P(1+i)^n$.
$A = P(1+i)^n$ $A = 2800\left(1+\frac{2.5}{100}\right)^5$ $A = \$3167.94$	Substitute the values into the formula. To enter the power, use the $\boxed{\wedge}$ or $\boxed{x^{\sqrt{}}}$ button on your calculator. Keystrokes on scientific calculator:
b Interest $= 3167.94 - 2800$ $= \$367.94$	Subtract the starting principal from the final balance to calculate the interest earned.

2	8	0	0	×	(	1	+
2	.	5	÷	1	0	0	
)	^	5	enter				



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Example 6 Calculating compound interest with various compounding periods using the formula

Joshua invests \$1600 into an account paying 2.5% p.a. interest for three years. Use the compound interest formula to find what his investment will be worth after three years if interest compounds:

a yearly	**b** half-yearly	**c** monthly
d weekly	**e** daily	

WORKING	**THINKING**
a $P = \$1600$ $i = \frac{2.5}{100}$ $n = 3$	Identify the values that need to be substituted into the formula $A = P(1+i)^n$.
$A = P(1+i)^n$ $A = 1600(1+\frac{2.5}{100})^3$ $A = \$1723.03$	Substitute into the formula and evaluate with a calculator. Money should always have two decimal places because of the cents.
b $P = \$1600$ $i = \frac{2.5 \div 2}{100} = \frac{2.5}{200}$ per half-year $n = 3 \times 2 = 6$ half-years	The interest rate will need to be divided by 2 (and 100) since there are 2 half-years in a year. Leaving as a fraction will give a more exact answer. If interest compounds twice per year, then it will compound 6 times in three years.
$A = P(1+i)^n$ $A = 1600\left(1+\frac{2.5}{200}\right)^6$ $A = \$1723.81$	Substitute into the formula and evaluate with a calculator.

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WORKING	THINKING
c $P = \$1600$ $i = \frac{2.5 \div 12}{100} = \frac{2.5}{1200}$ per month $n = 3 \times 12 = 36$ months	The interest rate will need to be divided by 12 (and 100) since there are 12 months in a year. Leaving as a fraction will give a more exact answer. If interest compounds every month, then it will compound 36 times in three years.
$A = P(1+i)^n$ $A = 1600\left(1+\frac{2.5}{1200}\right)^{36}$ $A = \$1724.48$	Substitute into the formula and evaluate with a calculator.
d $P = \$1600$ $i = \frac{2.5 \div 52}{100} = \frac{2.5}{5200}$ per week $n = 3 \times 52 = 156$ weeks	The interest rate will need to be divided by 52 (and 100) since there are 52 weeks in a year. Leaving as a fraction will give a more exact answer. If interest compounds every week, then it will compound 156 times in three years.
$A = P(1+i)^n$ $A = 1600\,(1+\frac{2.5}{5200})^{156}$ $A = \$1724.58$	Substitute into the formula and evaluate with a calculator.
e $P = \$1600$ $i = \frac{2.5 \div 365}{100} = \frac{2.5}{36\,500}$ per day $n = 3 \times 365 = 1095$ days	The interest rate will need to be divided by 365 (and 100) since there are 365 days in a year. Leaving as a fraction will give a more exact answer. If interest compounds every day, then it will compound 1095 times in three years.
$A = P(1+i)^n$ $A = 1600\,(1+\frac{2.5}{36\,500})^{1095}$ $A = \$1724.61$	Substitute into the formula and evaluate with a calculator.

Exercise 11B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a With compound interest, ____________ is earned on the ____________ already earned.

b The formula for interest is $A = P(1 + i)^n$, where:

i A = the ____________ amount at the end of the investment.

ii P = the ____________ amount that you started with.

iii i = the ____________ rate, as a percentage.

iv n = the ____________ of compounding periods.

c i and n _____ be in the same time units as the compounding ____________ (per month, per year etc).

2 Express the following percentages as fractions.

a 15% **b** 6% **c** 3.4% **d** 2.03%

3 Convert one year to:

a months **b** weeks **c** half-years

d days **e** quarters **f** fortnights

4 A bank advertises interest rates of 2.4% p.a. Express this as a:

a half-yearly rate **b** quarterly rate **c** monthly rate

d fortnightly rate **e** weekly rate **f** daily rate

APPLICATIONS

SF: 5–12 **CF: 13** **CU: –**

Example 4 ★**5** Kate invests $2480 into an account that pays 3.2% p.a. compound interest for three years. Interest is compounded yearly.

a Copy and complete the following table to calculate her interest and balance at the end of each year.

Year	Principal	Interest	Balance
1	$2480		
2			
3			

b Determine how much interest she earned in three years.

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★**6** Jasper invests $3850 into an account that pays 2.10% p.a. compound interest for four years. Interest is compounded yearly.

a Copy and complete the following table to calculate his interest and balance at the end of each year.

Year	Principal	Interest	Balance
1			
2			
3			
4			

b Determine how much interest he earned in four years.

Example 5 ★**7** Abby has saved $9600 from her part-time job. She invests it for two years at 3.20% p.a. until she gets her P's and can buy a car. Interest is compounded yearly.

a Use the compound interest formula to calculate the balance of her account after two years.

b Determine how much interest she earned over the two years.

Hint To enter the power, use the ^ or x^n button on your calculator.

★**8** Ben invests the $4500 he earned from the sale of cattle for four years at 2.20% p.a. to save for a trip at the end of school. Interest is compounded yearly.

a Use the compound interest formula to calculate the balance of his account after four years.

b Determine how much interest he earned over the four years.

9 Calculate the final balance of the following investments.

a $680 invested at 5% p.a. compounded annually for three years.

b $275 invested at 3% p.a. compounded annually for six years.

c $682 invested at 4% p.a. compounded annually for five years.

d $1240 invested at 2.2% p.a. compounded annually for four years.

e $5760 invested at 1.8% p.a. compounded annually for seven years.

f $4030 invested at 2.15% p.a. compounded annually for five years.

Example 6 ★**10** Natalie invests $5000 in an account that earns interest at the rate of 3.6% p.a. compounding monthly. Use the compound interest formula to calculate how much is in her account after five years.

★**11** Georgia invests $3800 in an account that earns interest at the rate of 2.4% p.a. compounding weekly. Use the compound interest formula to calculate how much is in her account after two years.

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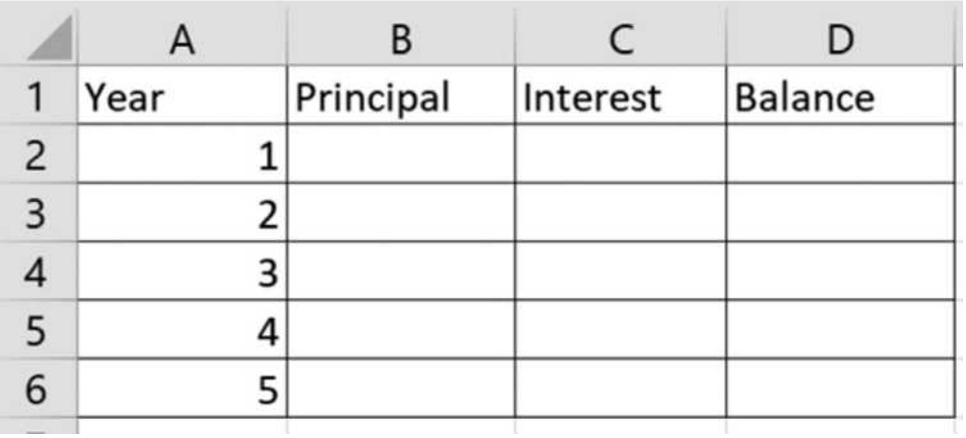
12 Calculate the final balance of the following investments.

- **a** \$1370 invested at 8.85% p.a. compounded monthly for three years.
- **b** \$54 600 invested at 1.20% p.a. compounded half-yearly for eight years.
- **c** \$4300 invested at 5.07% p.a. compounded weekly for two years.
- **d** \$3350 invested at 6.1% p.a. compounded monthly for three years.
- **e** \$7960 invested at 4.92% p.a. compounded fortnightly for two years.
- **f** \$4220 invested at 4.66% p.a. compounded quarterly for four years.

13 Charlie invests \$3500 in an account that pays 4% p.a. interest compounded every year.

★ **a** Use a spreadsheet to calculate the interest and balance each year for five years. A sample set is shown:

Hint All formulas begin with =
Multiplication is *
Divide is /



	A	B	C	D
1	Year	Principal	Interest	Balance
2	1			
3	2			
4	3			
5	4			
6	5			

- **i** Identify which cell you will need to enter the \$3500 principal.
- **ii** Determine the formula will you need to enter in cell C2.
- **iii** Determine the formula will you need to enter in cell D2.
- **iv** Determine the formula will you need to enter in cell B3.

b **i** Fill down to complete the spreadsheet.
- **ii** Use your spreadsheet to calculate the value of Charlie’s investment after five years.

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11C Applying compounding to practical problems COMPLEX

LEARNING GOALS

- Apply the principles of compounding to real-life growth contexts such as inflation, population growth and investments
- Apply the principles of compounding to real-life decay contexts such as population decline and depreciation

Why are growth applications essential?

The compound interest formula can be applied to many real-life situations other than banking. Any situation where there is growth that can be expressed as a recurrence relation can be modelled using the compound interest formula. While the percentage change used in the recurring calculations stays the same, the amount of increase gets bigger with each time period. Common examples are inflation, population growth, and appreciation in investments.

In the case of growth that can be modelled using the compound interest formula, the amount of growth increases with each time period.

If the percentage change is a decrease (i.e. a negative percentage) rather than an increase, then the formula can model such things as depreciation on equipment and vehicles.

WHAT YOU NEED TO KNOW

- Real-life applications, where there is a *percentage increase or decrease* over time, can be modelled using the compound interest formula, $A = P(1 + i)^n$. We use compound rather than simple interest because the increase or decrease is applied to previously increased or decreased values in a **recurrence relation**. Common applications include:
  - **Inflation**: a term that describes the increasing cost of items we need to buy. It is measured using the annual percentage change in the Consumer Price Index (CPI). We can use average percentage inflation over recent past time periods and the compound interest formula to predict the cost, or value, of items we need to purchase in the future. When we do this we assume that the inflation rate in the recent past will continue into the future, but the actual inflation rate varies somewhat randomly, as the graph at the top of the next page shows.

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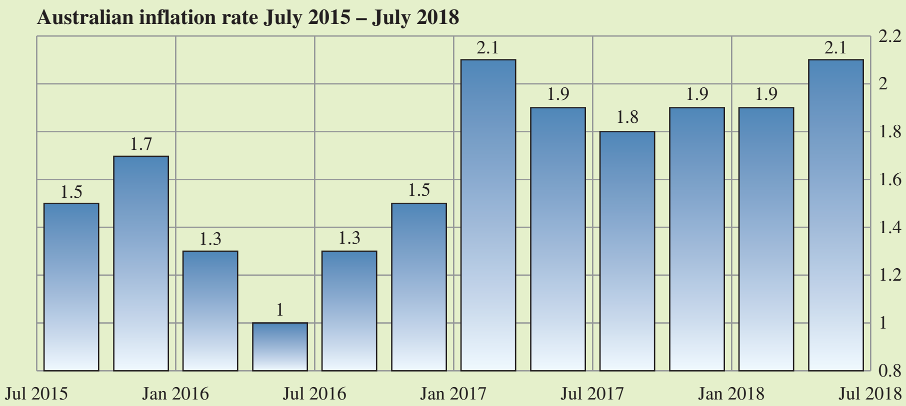
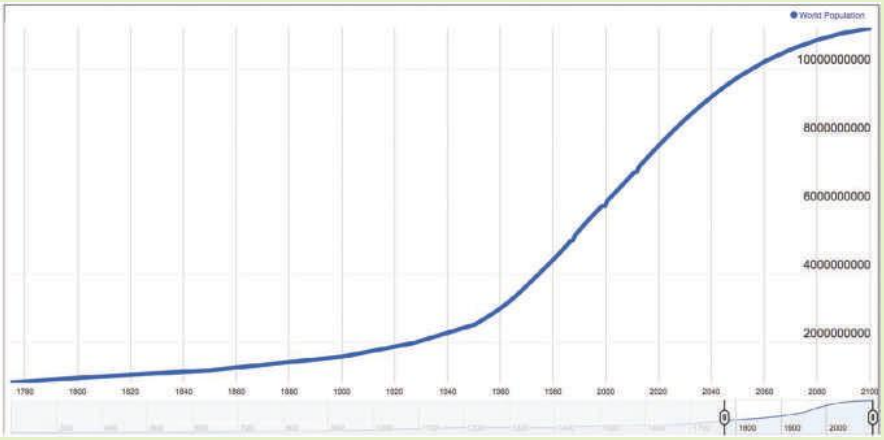


- **Population growth** is measured using percentage change on previous years. Australia's population growth in September 2018 had been 1.6% p.a. over the previous 12 months, reaching 25.1 million people. We can use population growth rates and the compound interest formula to predict the future size of populations.
- This graph shows the growth in the world population since the arrival First Fleet in Australia in 1788. The shape of the graph indicates a growth rate that follows the rules of the compound interest formula with a positive percentage rate.



Source: http://www.worldometers.info/world-population/

- **Appreciation** is a measure of the increasing value of an investment over time. (The term is generally used for investments other than bank accounts.) Investments in real estate are common and when looking for a suburb to invest in, researching the annual growth of housing values in an area and applying the compound interest formula can give an indication of the future value of properties. For example:

Show insights for All houses in East Toowoomba, QLD 4350

Loan Repayments Calculator
Calculate repayments for this suburb

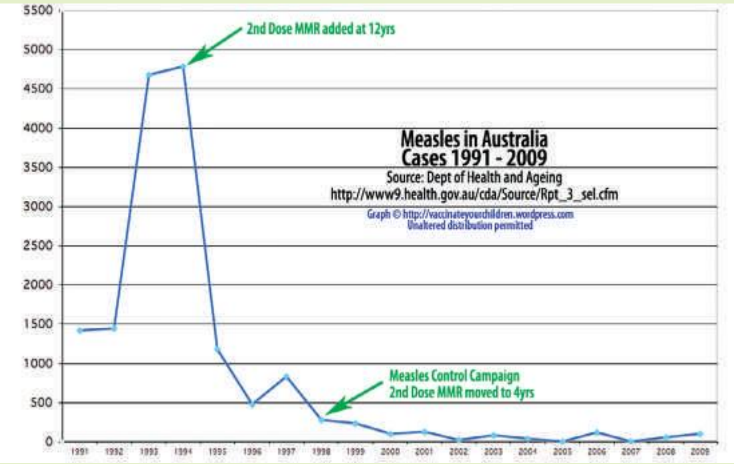
East Toowoomba

Investment property data for all houses

Annual growth	Median price	Median rent	Rental yield	Rental demand	Est. cash flow
5.4%	$495K	$350/wk	3.7%	Average 747 visits per house	-$149/mo

Source: realestate.com.au

- Real-life applications where there is a *percentage decrease* over time can be modelled using the compound interest formula by using a negative percentage interest rate. That is, $A = P(1 - i)^n$. Common applications include:
  - **Depreciation** is the reducing value of items like cars, computers or machinery. For example, a new car generally loses 20% of its value in the first year and 15% of its value every year after that. We can use the compound interest formula, with a negative interest rate, to calculate the value of a car after several years.
  - **Population decline** can be applied to endangered species or instances of disease. The compound interest formula can be applied to predict the population after several years. For example, the graph below shows that the number of cases of the measles in Australia has declined since the introduction of a vaccine. The shape of the graph indicates a depreciation rate that between about 1993 and 2000 roughly followed the rules of the compound interest formula with a negative percentage rate.



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Example 7 Applying compound interest to a growth (increasing) problem

A 3 L bottle of milk is \$3.30 at a supermarket in 2018. If the average rate of inflation is 2.1% p.a., estimate how much a 3 L bottle of milk will cost in:

a five years

b 2030

WORKING	THINKING
a $P = \$3.30$ $i = \frac{2.1}{100}$ $n = 5$	Identify the values that need to be substituted into the formula.
$A = P(1+i)^n$ $A = 3.30\left(1+\frac{2.1}{100}\right)^5$ $A = \$3.66$	Substitute into the formula and evaluate with a calculator. Money should always have two decimal places because of the cents.
b $P = \$3.30$ $i = \frac{2.1}{100}$ $n = 2030 - 2018 = 12$	Identify the values that need to be substituted into the formula.
$A = P(1+i)^n$ $A = 3.30\left(1+\frac{2.1}{100}\right)^{12}$ $A = \$4.23$	Substitute into the formula and evaluate with a calculator. Money should always have two decimal places because of the cents.

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Example 8 Applying compound interest to a decreasing problem

A new car is worth \$29 990. If new cars depreciate at 15% p.a., estimate the value of the car in 6 years.

WORKING	THINKING
$P = \$29990$ $i = \frac{15}{100}$ $n = 6$	Identify the values that need to be substituted into the formula. Decreasing value, so a negative interest rate.
$A = P(1 - i)^n$ $A = 29990\left(1 - \frac{15}{100}\right)^6$ $A = \$11\,310.71$	Substitute into the formula and evaluate with a calculator. Money should always have two decimal places because of the cents.

Exercise 11C

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Real-life applications, where there is a ____________ increase over time, can be modelled using the ____________ interest formula, $A = P(1 + i)^n$.
- **b** Inflation is a term that describes the ____________ cost of items we need to buy.
- **c** Population growth can be ____________ using percentage change on previous years.
- **d** Appreciation is a measure of the increasing ____________ of an investment over time.
- **e** Real-life applications where there is a percentage ____________ over time can be modelled using the compound interest formula by using a negative ____________ rate.
- **f** Depreciation is the ____________ value of items like cars, computers or machinery.

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2 Decide if the following real-life situations have an increasing or decreasing value.
  - **a** The cost of a loaf of bread in the future.
  - **b** The value of a car a few years after purchase.
  - **c** The population of an endangered animal in the future.
  - **d** The value of an investment property in the future.
  - **e** The population of Australia in the future.

APPLICATIONS

SF: –	CF: 3–7	CU: 8–10

Example 7

★3 The average rate of inflation in Australia is 2.1% p.a.
  - **a** A burger combo meal costs \$10.65 at a fast food store. If the price increases with inflation, use the compound interest formula to estimate how much will it cost in 4 years.
  - **b** A new car sells for \$20 490. If the price increases with inflation, use the compound interest formula to estimate how much will it cost in 20 years.

★4 Bec receives a salary of \$93 000. Each year, her salary increases to match the rate of inflation. Calculate her salary in the next year if the rate of inflation is:
  - **a** 1.5% p.a.
  - **b** 1.9% p.a.

★5 The cost of petrol today is 145.9 cents per litre.
  - **a** Calculate the price of a litre of petrol in 20 years' time if inflation averages 1.9% p.a. over that time. Express your answer to 1 decimal place.
  - **b** Calculate the price of a litre of petrol in 10 years' time if inflation averages 2.1% p.a. over that time. Express your answer to 1 decimal place.

★6 The population of Australia was 24.8 million in 2018 with an annual growth rate of 1.32%. If the population continues to grow at the same rate, use the compound interest formula to estimate the population:
  - **a** 10 years after 2018
  - **b** in 2100

★**7** Henry buys a home for \$447 500. If the annual growth rate in the suburb is 3.1% p.a., use the compound interest formula to estimate the value of the house in:

- **a** 15 years
- **b** 20 years

Example 8 ★**8** Rodney buys a new truck for his transport business for \$320 000. If the truck depreciates in value by 15% p.a., use the compound interest formula to estimate the value of the truck in:

- **a** 5 years
- **b** 10 years
- **c** 15 years

★**9** Following the introduction of a vaccine for a disease, the number of people contracting the disease decreased by 20% each year.

- **a** If 6500 people suffer from the disease when the vaccine was first introduced, use the compound interest formula to estimate how many people have the disease after:
  - **i** three years
  - **ii** five years
- **b** Discuss why the disease is not fully eradicated after five years of 20% reduction each year.

★**10** Computers lose 45% of their value each year.

- **a** Harrison buys a new laptop computer for \$1549.00. Determine the value of the laptop in two years' time.
- **b** Lexi spends \$2099 on a new laptop computer for her daughter at the start of Year 10. Determine the value of the laptop when her daughter finishes Year 12.

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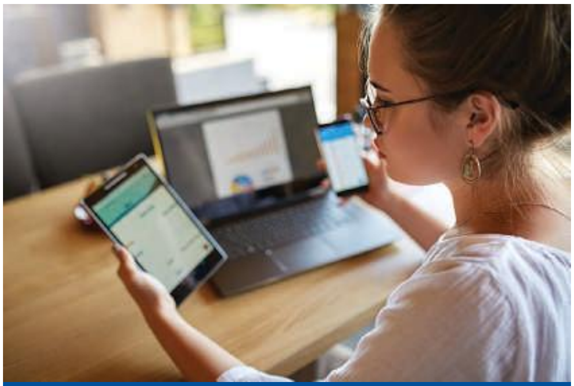
11D Using technology with investment problems

LEARNING GOALS

- Calculate the future value of a compound interest investment and the total interest earned using:
  - an online calculator
  - a spreadsheet [complex]
- Compare the growth of simple interest and compound interest investments using:
  - an online calculator
  - a spreadsheet [complex]

Why is using technology essential when investigating investments?

We work hard to earn our money and we want to be able to save for things like holidays, cars, homes and our future. Having a plan for your savings and understanding the impact of your decisions will help you to be more successful in achieving your financial goals.



Understanding the impact of your decisions will help you to be more successful in achieving your financial goals.

WHAT YOU NEED TO KNOW

- There are many places online where you can go to calculate future value of investments. It is the core business of all the banks, so they all have calculators on their websites. The Australian Securities and Investment Commission (ASIC) Moneysmart website is free from bias, is a useful place to find information about savings and loans, and has lots of calculators to help you. (*https://www.moneysmart.gov.au/*)

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- For example: With the Moneysmart compound interest calculator, you can easily find the total interest and savings if you save $100 every month for 5 years at 4% p.a.

Compound interest calculator

Your strategy

Initial deposit: $0

Regular deposit: $100

Deposit frequency: Monthly

Compound frequency: Monthly

Number of years: (max 50) 5 years

Interest rate: (max 20%) 4.00%

Effective interest rate: 4.07%

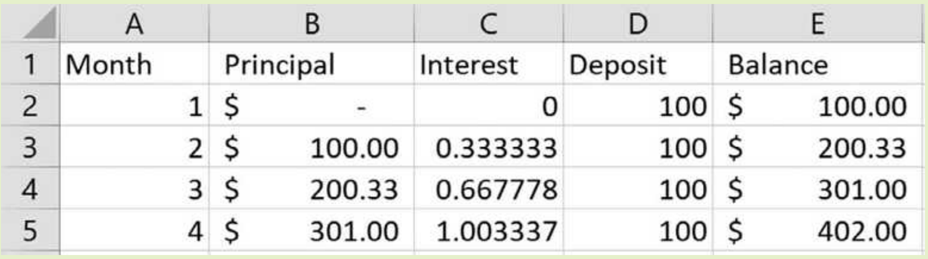
Results

Your strategy:

Initial deposit:	$0
Regular deposits:	$6,000
Total interest:	$630
Total savings:	$6,630

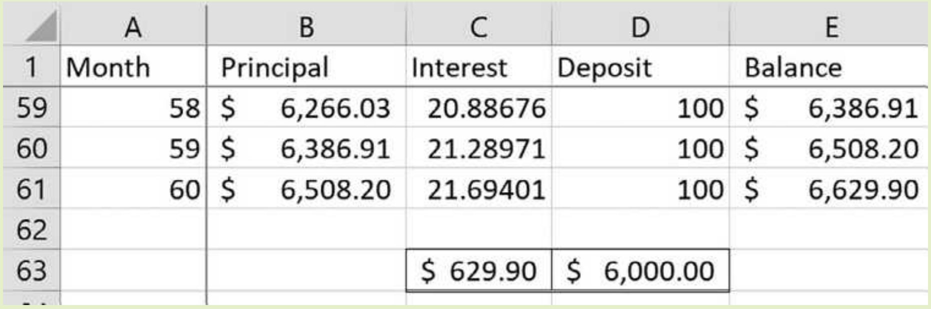
Alternative strategy:

- [Complex] A spreadsheet can also be used to calculate this information:



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2	1	$ -	0	100	$ 100.00
3	2	$ 100.00	0.333333	100	$ 200.33
4	3	$ 200.33	0.667778	100	$ 301.00
5	4	$ 301.00	1.003337	100	$ 402.00

Scrolling down to the 60th month (5 years), we can see that the balance, total interest and regular deposits match the figures from the Moneysmart website:



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
59	58	$ 6,266.03	20.88676	100	$ 6,386.91
60	59	$ 6,386.91	21.28971	100	$ 6,508.20
61	60	$ 6,508.20	21.69401	100	$ 6,629.90
62					
63			$ 629.90	$ 6,000.00	

We will develop this spreadsheet in Example 10.

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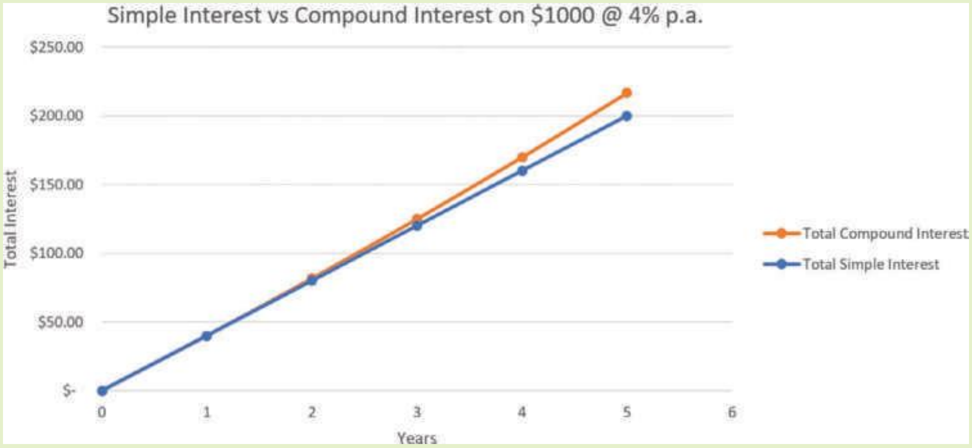
- Simple interest investments earn the same amount of interest every time period. Compound interest investments have the interest added after each time period and the amount of interest earned increases over time.
  - For example: $1000 is invested at 4% p.a. for 5 years.

Year	Simple interest	Total simple interest
1	\$ 40.00	\$ 40.00
2	\$ 40.00	\$ 80.00
3	\$ 40.00	\$ 120.00
4	\$ 40.00	\$ 160.00
5	\$ 40.00	\$ 200.00

Year	Compound interest	Total compound interest
1	\$ 40.00	\$ 40.00
2	\$ 41.60	\$ 81.60
3	\$ 43.26	\$ 124.86
4	\$ 44.99	\$ 169.86
5	\$ 46.79	\$ 216.65

Graphically, the difference between total interest earned is:



Notice how the simple interest generates a straight line and the compound interest generates a curved line. We will develop this table and graph as a technology activity in the interactive textbook for Exercise 11D.

Example 9 Using an online calculator to find future value and interest for a compound interest investment

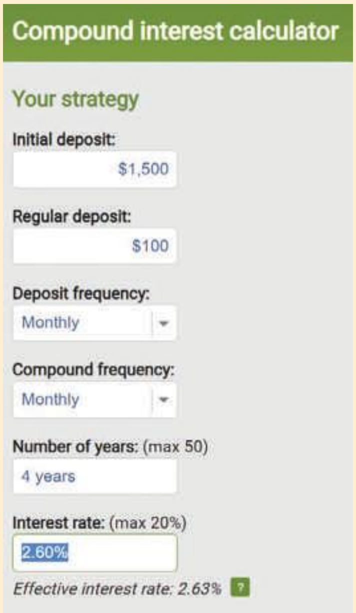
Chris has saved $1500 from his part-time job. He deposits it into an account that pays 2.6% p.a., compounding monthly. He continues to deposit $100 into the account every month for the next 4 years. Use an online calculator to find:

a the future value of Chris's account after 4 years

b the total interest Chris earned with his account

WORKING **THINKING**

a



Compound interest calculator

Your strategy

Initial deposit: $1,500

Regular deposit: $100

Deposit frequency: Monthly

Compound frequency: Monthly

Number of years: (max 50) 4 years

Interest rate: (max 20%) 2.60%

Effective interest rate: 2.63% ?

Identify the values to enter in the Moneysmart compound interest calculator *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator*

Your strategy:

■ Initial deposit:	$1,500
■ Regular deposits:	$4,800
■ Total interest:	$417
Total savings:	$6,717

Read the total savings from the online calculator.

Chris will have $6717 in his account after 4 years.

Communicate your answer in a sentence.

b Chris will have earned $417 interest over the 4 years.

Read the total interest from the online calculator.

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Example 10 Using a spreadsheet to find future value and interest for a compound interest investment [complex]

Emma has already saved $2500 from her online business selling handbags. She deposits it into an account that pays 2.15% p.a., compounding monthly. She continues to deposit $200 into the account every month for the next 2 years.

Use a spreadsheet to find:

a the future value of Emma's account after 2 years

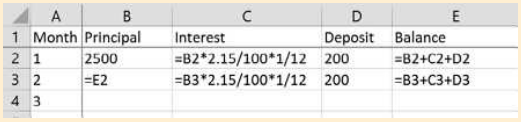
b the total interest Emma earned with her account

WORKING | **THINKING**

a Set up the spreadsheet with the following headings and formulas:

Interest for one month = Pin

$$= P \times \frac{2.15}{100} \times \frac{1}{12}$$



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2	1	2500	=B2*2.15/100*1/12	200	=B2+C2+D2
3	2	=E2	=B3*2.15/100*1/12	200	=B3+C3+D3
4	3				

The time period, n, needs to be in years to match the interest rate.

Balance = principal + interest + deposit

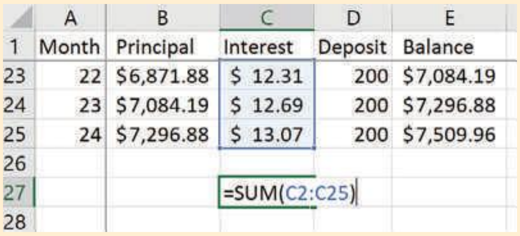
Principal next month = balance of last month

Fill down for 2 years (24 months).

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
22	21	$6,659.95	$ 11.93	200	$6,871.88
23	22	$6,871.88	$ 12.31	200	$7,084.19
24	23	$7,084.19	$ 12.69	200	$7,296.88
25	24	$7,296.88	$ 13.07	200	$7,509.96
26					

Make sure the deposit stays as 200 for every month.

Tip: Freeze the top row so the headings are still visible when you scroll down. See Excel Help.



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
23	22	$6,871.88	$ 12.31	200	$7,084.19
24	23	$7,084.19	$ 12.69	200	$7,296.88
25	24	$7,296.88	$ 13.07	200	$7,509.96
26					
27			=SUM(C2:C25)		
28					

The formula =sum(cell range) will find the total interest and deposits made.

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
23	22	$6,871.88	$ 12.31	200	$7,084.19
24	23	$7,084.19	$ 12.69	200	$7,296.88
25	24	$7,296.88	$ 13.07	200	$7,509.96
26					
27			$ 209.96	$4,800.00	

Read the value in the final Balance cell.

Emma will have $7509.96 after the two years.

Communicate your answer in a sentence.

b Emma will have earned $209.96 interest over the two years.

Read the sum from the interest column.

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Exercise 11D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Having a ____________ for your savings and understanding the impact of your decisions will help you to be more ____________ in achieving your financial goals.
- **b** There are many places online that you can go to ____________ future value of investments.
- **c** A spreadsheet can be used to calculate the future ____________ of investments and ____________ earned.
- **d** Simple interest investments earn the ____________ amount of interest ____________ time period.
- **e** Compound interest investments have the interest ____________ after each time period and the amount of interest earned ____________ over time.

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APPLICATIONS

SF: 2–6	CF: 7–8	CU: –

Use the Moneysmart compound interest calculator to answer the following questions. *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator*

Example 9

2 Greg saves $250 each fortnight into his superannuation account for twenty years. It earns 6% p.a. interest compounded monthly.

- **a** Select the values that will need to be entered for the:
  - **i** initial deposit
  - **ii** regular deposit
  - **iii** deposit frequency
  - **iv** compound frequency
  - **v** number of years
  - **vi** interest rate
- **b** Use the Moneysmart compound interest calculator to determine how much he will have in his superannuation account after 20 years.
- **c** Use the Moneysmart compound interest calculator to determine how much he will have earned in interest.

3 Jeff wants to retire with a million dollars in his superannuation fund. He is currently 18 and has just finished school. He dreams of retiring when he is 55. His superannuation fund pays 5.6% p.a. compounded yearly and he plans on making a deposit every fortnight into the fund.

- **a** Select the values that will need to be entered for the:
  - **i** initial deposit
  - **ii** deposit frequency
  - **iii** compound frequency
  - **iv** number of years
  - **v** interest rate
- **b** Use the Moneysmart compound interest calculator to determine how much he will need to deposit each fortnight to reach his millionaire retirement dream.
- **c** Use the Moneysmart compound interest calculator to determine how much he will need to earn in interest.

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4 Shirley is president of her local sports club. The committee have decided that they need to plan for refurbishments of the club house that they think will need to be done in two years' time. They already have $12 000 in the bank; they can save $500 each month and the bank offers them 3.2% p.a. interest compounding monthly.

- a Select the values that will need to be entered for the:
  - i initial deposit
  - ii regular deposit
  - iii deposit frequency
  - iv compound frequency
  - v number of years
  - vi interest rate
- b Use the Moneysmart compound interest calculator to determine how much the committee will have in their account after two years.
- c Use the Moneysmart compound interest calculator to determine how much they will have earned in interest.
- d The committee estimate that they will need to have at least $30 000 for the refurbishments. Use the online calculator to find how long it will take to reach their goal.
- e The committee do not think that they can wait more than two years for the refurbishments. Use the Moneysmart compound interest calculator to determine how much they would need to save each month to reach the $30 000 goal in two years.

5 Nicole is saving for a deposit on a house. She saves $150 from her pay every fortnight into an account that pays 3.5% p.a. with interest compounding monthly.

- a Use the Moneysmart compound interest calculator to calculate how much Nicole will have in her house deposit account after five years.
- b Use the Moneysmart compound interest calculator to calculate how much Nicole will have in her house deposit account after 10 years.

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6 Phill is Nicole's friend. After learning about how much Nicole had saved in five years, he decides he had better do the same thing to buy his own house. Since he is starting five years later, he doubles his regular payment, to catch up to Nicole. He also earns 3.5% p.a. interest that compounds monthly.

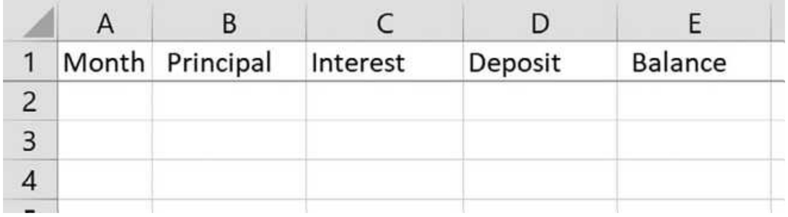
- **a** Determine the size of Phill's regular fortnightly deposit.
- **b** Use the Moneysmart compound interest calculator to calculate how much Phill will have in his house deposit account after five years.
- **c** Decide whether Phill will be able to catch up to Nicole by doubling the size of his regular deposit. Explain why or why not.

Use a spreadsheet to answer the following questions.

Example 10

7 Allyson is planning an overseas holiday. She has just received her tax return which was $1256 and she will use this towards her holiday. She budgets to save $210 each month and can earn 3.15% p.a. compound interest that compounds each month.

- **a** Set up a spreadsheet with the following headings:



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2					
3					
4					

- **b** Decide what will need to be entered in the following cells:
  - **i** A2
  - **ii** B2
  - **iii** C2
  - **iv** D2
  - **v** E2
  - **vi** A3
- **c** Fill down to determine how much she has saved after 12 months.
- **d** Sum the values in the interest column by using =SUM(C2:C13) in any cell to the right of the table. Determine how much interest Allyson earned in the 12 months.
- **e** Allyson believes she will need to save $5000 for her holiday. Continue to fill down your table to determine how many months it will take to reach her goal.

8 Bev is saving to buy a car. She already has $3460 saved from her part-time job and can save $500 per month. Her bank account pays 2.8% p.a. interest compounding monthly.

a Set up a spreadsheet with the following headings:

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2					
3					
4					

b Decide what will need to be entered in the following cells:

i A2
ii B2
iii C2
iv D2
v E2
vi A3

c Fill down to determine how much she has saved after 24 months.

d Sum the values in the interest column by using =SUM(C2:C25) in any cell to the right of the table. Determine how much interest Bev earned in the 24 months.

e Bev believes she will need to save $20 000 for her car. Continue to fill down your table to determine how many months it will take to reach her goal.

Spreadsheet activity 11D: See the interactive textbook for this activity using a spreadsheet to compare the growth of simple interest and compound interest investments.

Desmos activity 11D: See the interactive textbook for this activity using the Desmos graphing calculator to compare the growth of simple interest and compound interest investments.

11E Investigating the effects of changing interest rates and compounding periods using technology

LEARNING GOALS

- Investigate the effect of the interest rate and the number of compounding periods on the future value of an investment using:
  - an online calculator
  - a spreadsheet [complex]

Why is understanding the impact of interest rates and number of compounding periods essential?

We work hard for our money and knowing how to invest it wisely to earn the maximum possible amount of interest is essential to ensuring a comfortable financial future. Understanding the effect that interest rates and compounding periods have on the growth of your investments is essential.

The dollar amount of interest paid or earned can be affected by changes to interest rates and frequency of compounds.

WHAT YOU NEED TO KNOW

- Making changes to the interest rate and frequency of compounds will have an impact on the amount of interest earned.
- Compound interest was covered in Section 11B. The formula for compound interest is $A = P(1 + i)^n$.

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Example 11 Determining the impact of interest rate changes using an online calculator

a Use Desmos activity 11D from Exercise 11D in the interactive textbook to calculate the value of $6000 after 15 years when it is invested at:

  i 10% p.a. compounded yearly

  ii 15% p.a. compounding yearly

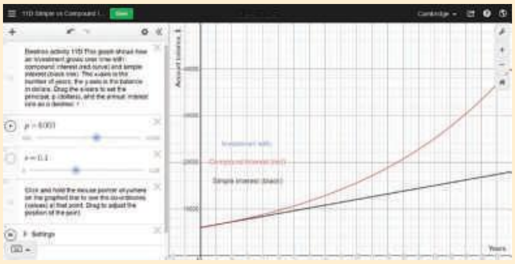
b Calculate the difference in the amount of interest earned.

c Describe the changes to the shape of the compound interest curve when the interest rate increased.

WORKING

THINKING

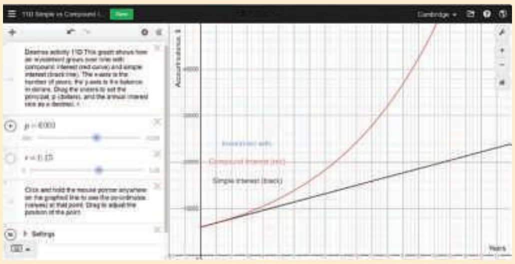
a



Use the sliders for p and r to change to required value.

Track along the red line with the mouse, click and hold to find the value of the investment after 15 years.

i After 15 years, investment = $25 063.49.



ii After 15 years, investment = $48 822.37.

b Difference = 48 822.37 – 25 063.49
= $23 758.88

The amount of interest earned almost doubled over the same time period when the interest rate was increased by half.

c The curve for the compound interest investment (red line) became a lot steeper.

View the graph to describe the change when the interest increased.

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Example 12 Using an online calculator to determine the impact of frequency of compounding periods

Baxter has $15 000 to invest for two years. The bank offers a 2.5% p.a. interest rate.

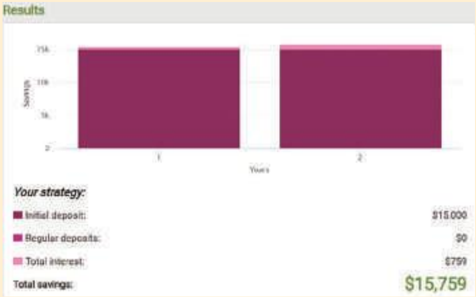
a Use the Moneysmart compound interest calculator *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator* to calculate the interest earned if interest compounds:

  i annually **ii** monthly

b Describe the impact of increasing the frequency of the compounds on the amount of interest earned.

WORKING | **THINKING**

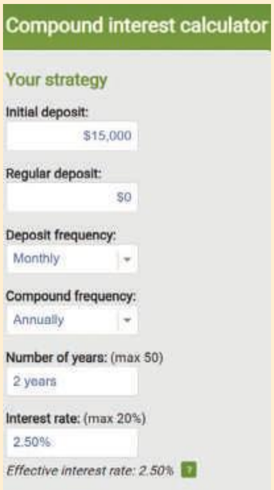
a i



Your strategy:
- ■ Initial deposit: $15 000
- ■ Regular deposits: $0
- ■ Total interest: $759

Total savings: $15,759

Total interest is $759 when compounded annually.



Compound interest calculator

Your strategy

Initial deposit: $15,000

Regular deposit: $0

Deposit frequency: Monthly

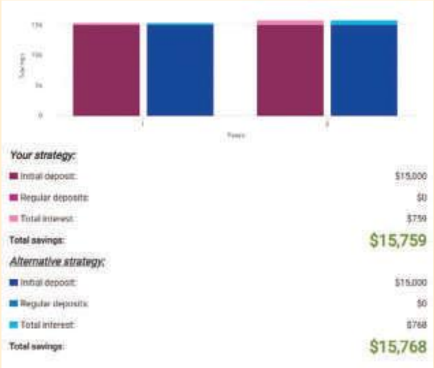
Compound frequency: Annually

Number of years: (max 50) 2 years

Interest rate: (max 20%) 2.50%

Effective interest rate: 2.50%

ii



Your strategy:
- ■ Initial deposit: $15,000
- ■ Regular deposits: $0
- ■ Total interest: $759

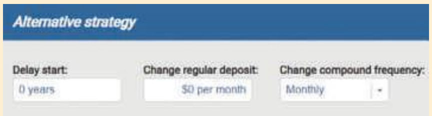
Total savings: $15,759

Alternative strategy:
- ■ Initial deposit: $15,000
- ■ Regular deposits: $0
- ■ Total interest: $768

Total savings: $15,768

Total interest is $768 when compounded monthly.

Use the *Alternate strategy* tab to change compounding frequency.



Alternative strategy

Delay start: 0 years

Change regular deposit: $0 per month

Change compound frequency: Monthly

b Extra interest was earned (768 – 759 = $9.00) over the same time period with the same interest rate.

Subtract the total interest from each time period.

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Example 13 Using a spreadsheet to calculate impact of changes to interest on investments [complex]

Set up a spreadsheet that will allow you to enter the variables for principal, interest rate, years and compounds per year, and then calculate the final amount when compounding at different time periods. A sample set up is:

	A	B
1	Principal, P	
2	Interest %p.a.	
3	Years	
4	Compounds per year	
5	Amount $A	

a The formula for compound interest is $A = P(1+i)^n$. Determine which cell has the value for:

i P **ii** i

b **i** Determine what is in cell B4.

ii Determine how to find the value of n required for the formula $A = P(1+i)^n$.

iii Explain what will need to happen to the interest rate in the formula $A = P(1+i)^n$ when compounding at intervals other than a year.

c Use the information above to construct a formula in cell B5 that will calculate the amount, \$$A$, of the investment.

d Use the spreadsheet to calculate the final amount in an account for \$5000 deposited at 7% p.a. for four years compounding:

i weekly **ii** monthly **iii** quarterly

e Describe the impact of increasing the number of compounding periods over the same time period.

WORKING	THINKING
a **i** Principal = B1 **ii** Interest rate = B2	Cell referencing uses the column letter followed by the row number.
b **i** B4 is how many compounds occur per year.	
ii n=B4*B3	Number of compounds = number of compounds per year × years
iii i/B4/100	Divide interest rate by number of compounds per year, and 100 because it is a percentage.

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WORKING **THINKING**

c $A = P(1 + i)^n$

P is in cell B1.

i is in cell B2 and needs to be divided by B4 and 100 (i.e. B2/B4/100).

n will be the product of B4 and B3.

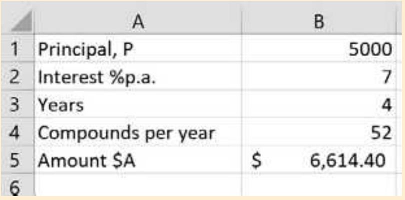
So, the formula for $A = P(1 + i)^n$ in cell B5 is

=B1*(1+B2/B4/100)^(B3*B4)

Number of compounds = number of compounds per year × years

The interest rate will need to be divided by the number of compounds per year and 100 because it is a percentage. Note that powers are entered in a spreadsheet using ^.

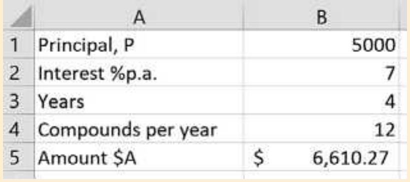
d **i**



	A	B
1	Principal, P	5000
2	Interest %p.a.	7
3	Years	4
4	Compounds per year	52
5	Amount $A	$ 6,614.40
6		

There are 52 weeks in a year.

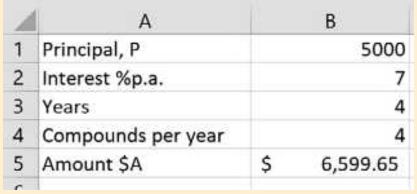
ii



	A	B
1	Principal, P	5000
2	Interest %p.a.	7
3	Years	4
4	Compounds per year	12
5	Amount $A	$ 6,610.27

There are 12 months in a year.

iii



	A	B
1	Principal, P	5000
2	Interest %p.a.	7
3	Years	4
4	Compounds per year	4
5	Amount $A	$ 6,599.65

There are 4 quarters in a year.

e The more compounds the greater the amount of interest earned over the same time period.

The highest final amount was with 52 compounds per year.

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Exercise 11E

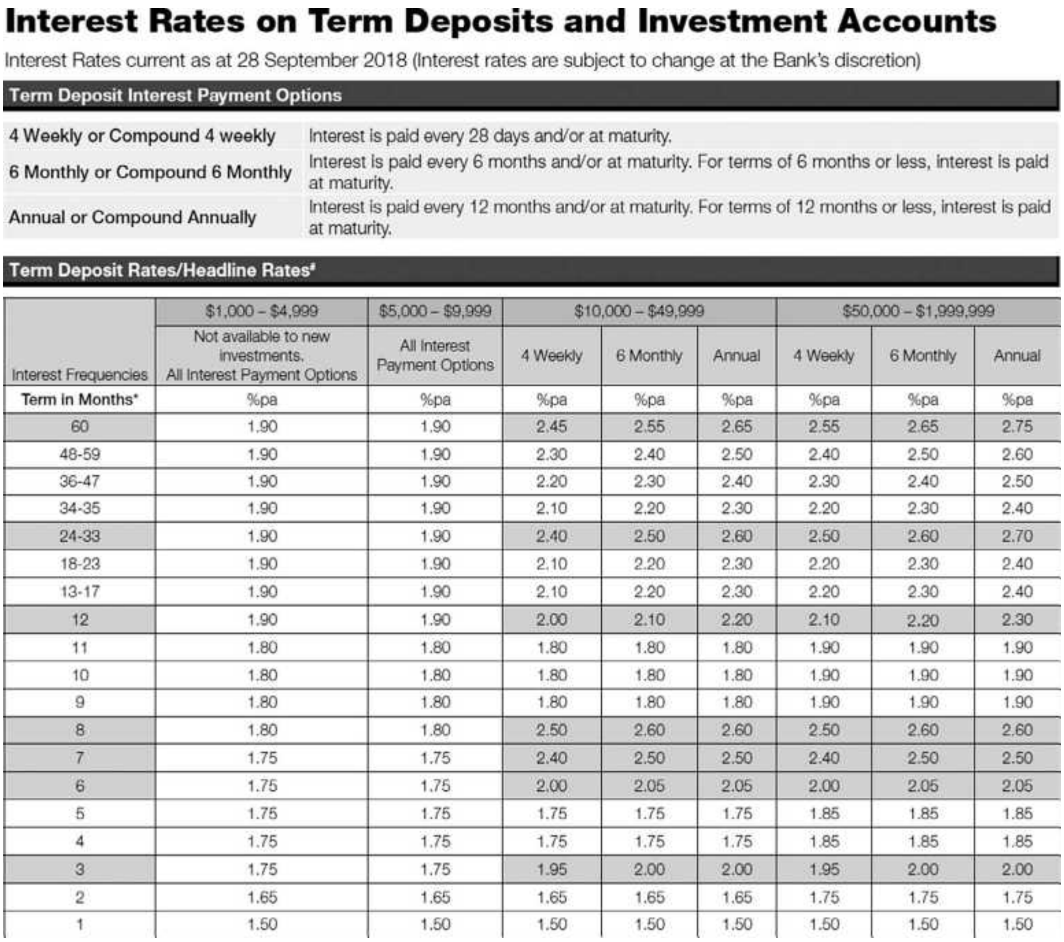
FUNDAMENTALS

1 Determine the missing words in the following sentences.
Making ____________ to the interest rate and frequency of compounds will have an ____________ on the amount of ____________ earned.

2 Complete the following conversions.

a 1 year = _______ months
b 1 year = _______ weeks
c 1 year = _______ days
d 1 year = _______ fortnights
e 1 year = _______ 6 months
f 1 year = _______ 4 weeks
g 1 year = _______ quarters

Use this table to answer questions in the exercise that follow on the next page.



Interest Rates on Term Deposits and Investment Accounts

Interest Rates current as at 28 September 2018 (Interest rates are subject to change at the Bank's discretion)

Term Deposit Interest Payment Options

4 Weekly or Compound 4 weekly	Interest is paid every 28 days and/or at maturity.
6 Monthly or Compound 6 Monthly	Interest is paid every 6 months and/or at maturity. For terms of 6 months or less, interest is paid at maturity.
Annual or Compound Annually	Interest is paid every 12 months and/or at maturity. For terms of 12 months or less, interest is paid at maturity.

Term Deposit Rates/Headline Rates#

	$1,000 – $4,999	$5,000 – $9,999	$10,000 – $49,999			$50,000 – $1,999,999		
Interest Frequencies	Not available to new investments. All Interest Payment Options	All Interest Payment Options	4 Weekly	6 Monthly	Annual	4 Weekly	6 Monthly	Annual
Term in Months*	%pa	%pa	%pa	%pa	%pa	%pa	%pa	%pa
60	1.90	1.90	2.45	2.55	2.65	2.55	2.65	2.75
48-59	1.90	1.90	2.30	2.40	2.50	2.40	2.50	2.60
36-47	1.90	1.90	2.20	2.30	2.40	2.30	2.40	2.50
34-35	1.90	1.90	2.10	2.20	2.30	2.20	2.30	2.40
24-33	1.90	1.90	2.40	2.50	2.60	2.50	2.60	2.70
18-23	1.90	1.90	2.10	2.20	2.30	2.20	2.30	2.40
13-17	1.90	1.90	2.10	2.20	2.30	2.20	2.30	2.40
12	1.90	1.90	2.00	2.10	2.20	2.10	2.20	2.30
11	1.80	1.80	1.80	1.80	1.80	1.90	1.90	1.90
10	1.80	1.80	1.80	1.80	1.80	1.90	1.90	1.90
9	1.80	1.80	1.80	1.80	1.80	1.90	1.90	1.90
8	1.80	1.80	2.50	2.60	2.60	2.50	2.60	2.60
7	1.75	1.75	2.40	2.50	2.50	2.40	2.50	2.50
6	1.75	1.75	2.00	2.05	2.05	2.00	2.05	2.05
5	1.75	1.75	1.75	1.75	1.75	1.85	1.85	1.85
4	1.75	1.75	1.75	1.75	1.75	1.85	1.85	1.85
3	1.75	1.75	1.95	2.00	2.00	1.95	2.00	2.00
2	1.65	1.65	1.65	1.65	1.65	1.75	1.75	1.75
1	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50

Source: www.commbank.com.au

★**3** Harry has $20 000 to invest for two years.

a Determine the number of months in two years.

b Use the table on page 554 to determine the interest rate if interest is compounded:

i annually **ii** 6 monthly **iii** 4 weekly

c Determine how many times the interest will compound in a year if it compounds:

i annually **ii** 6 monthly **iii** 4 weekly

d Use the compound interest formula to calculate the value of the investment after two years if the interest is compounded:

i annually **ii** 6 monthly **iii** 4 weekly

e Decide which investment option is best for the $20 000 over two years.

APPLICATIONS

SF: 4–7	CF: 8–10	CU: –

Example 11

4 Use the Desmos activity 11D in the interactive textbook to answer the following questions.

a Calculate the value of $8500 after 15 years when it is invested at:

i 5% p.a. compounded yearly

ii 10% p.a. compounding yearly

b Calculate the difference in the amount of interest earned.

c Comment on the difference in the amount of interest earned between the two rates.

d Describe the changes to the shape of the compound interest curve when the interest rate increased.

5 Use the Desmos activity from Exercise 11D in the interactive textbook to answer the following questions

a Calculate the value of $10 000 after 20 years when it is invested at:

i 9% p.a. compounded yearly

ii 12% p.a. compounding yearly

b Calculate the difference in the amount of interest earned.

c Comment on the difference in the amount of interest earned between the two rates.

d Describe the changes to the shape of the compound interest curve when the interest rate increased.

Use the compound interest calculator on the Moneysmart website to answer the following questions. *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator*

Example 12

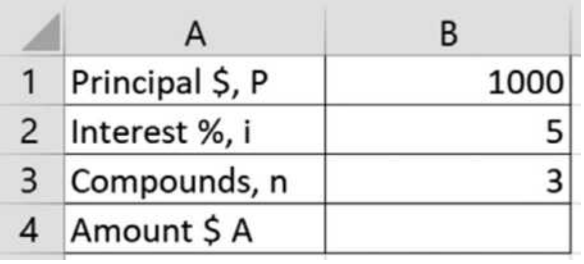
6 Baxter has \$8000 to invest for four years. The bank offers a 2.7% p.a. interest rate.

- **a** Use the Moneysmart compound interest calculator to find the interest earned if interest compounds:
  - **i** annually
  - **ii** monthly
- **b** Describe the impact of increasing the frequency of the compounds on the amount of interest earned.

7 Cherokee has \$5000 to invest for three years. The bank is offering two different types of accounts:

- **i** 2.4% p.a. compounding annually
- **ii** 2.2% p.a. compounding monthly
- **a** Use the Moneysmart compound interest calculator to determine how much she will have in her account if she goes with the first option.
- **b** Use the Moneysmart compound interest calculator to determine how much she will have in her account if she goes with the second option.
- **c** Decide which account option Cherokee should go with.

8 Use a spreadsheet to answer the following question.



	A	B
1	Principal \$, P	1000
2	Interest %, i	5
3	Compounds, n	3
4	Amount \$ A	

- **a** Set up a spreadsheet that will allow you to enter the variables for principal, interest rate and number of compounds and calculate the final amount when compounding yearly, as shown above.
- **b** The formula for compound interest is $A = P(1+i)^n$. Select the cell that has the value for:
  - **i** P
  - **ii** i
  - **iii** n

Hint Powers are entered using ^

- ★**c** Construct the formula that will need to be entered in cell B4 to find the accumulated amount.
- **d** Use the spreadsheet to calculate the amount in an account after \$1000 is deposited for three years at:
  - **i** 5% p.a. compounding annually
  - **ii** 10% p.a. compounding annually
  - **iii** 2% p.a. compounding annually
- **e** Use the spreadsheet to calculate the amount in an account after \$65 000 is deposited for six years at:
  - **i** 3.25% p.a. compounding annually
  - **ii** 6.15% p.a. compounding annually
  - **iii** 2.01% p.a. compounding annually
- **f** Describe the impact that changing the interest rate has on the amount of interest earned.

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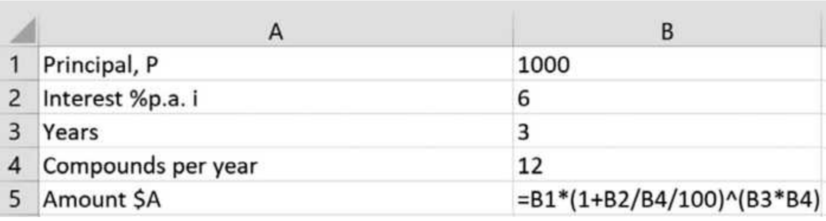
Example 13

9 Use a spreadsheet to answer the following question.

a Set up a spreadsheet that will allow you to enter the variables for principal, interest rate and compounds per year and then calculate the final amount when compounding at different time periods. A sample set is shown for \$1000 deposited at 6% p.a. for three years compounding monthly:



	A	B
1	Principal, P	1000
2	Interest %p.a. i	6
3	Years	3
4	Compounds per year	12
5	Amount $A	=B1*(1+B2/B4/100)^(B3*B4)

★ **b** The formula for compound interest is $A = P(1+i)^n$. Select the cell that has the value for:

i P **ii** i

★ **c** Use the spreadsheet to answer the following questions.

i Identify what is shown in cell B4.

ii Explain why the power needs to be (B3*B4) for the formula to work in cell B5.

iii Explain why B2 is divided by B4 and 100 in cell B5.

d Use the spreadsheet to calculate the amount in an account after \$1000 is deposited at 6% p.a. for three years compounding:

i weekly **ii** monthly **iii** quarterly

e Use the spreadsheet to calculate the amount in an account after \$65 000 is deposited at 3.25% p.a. for six years compounding:

i weekly **ii** monthly **iii** quarterly

f Describe the impact of increasing how often interest is compounded on the amount of interest earned.

10 William is treasurer of his local sports club. The club has \$120 000 that they want to invest for two years.

★**a** Use the table on page 554 to select the interest rate if interest is compounded:

i 4 weekly **ii** 6 monthly **iii** annually

b Use the spreadsheet from question **9** to calculate the amount in an account if interest is compounded:

i 4 weekly **ii** 6 monthly **iii** annually

c The bank offered different interest rates over different time periods. Describe what impact this may have on any advantage that is usually gained from more frequent compounds.

Problem-solving and modelling task

Background: Buying your own home is a goal for most young people, but coming up with a deposit for a home loan is a challenge for many.

Task: You are to design a budget and savings plan for yourself with the goal of saving enough money for a deposit on a home loan. You will need to make assumptions about your income, living expenses and potential regular savings that can be made. Research savings options and interest rates with a bank that you could use for a potential future loan. Clearly identify and justify the size of the deposit required for your future home and the time you will take to achieve your goal.

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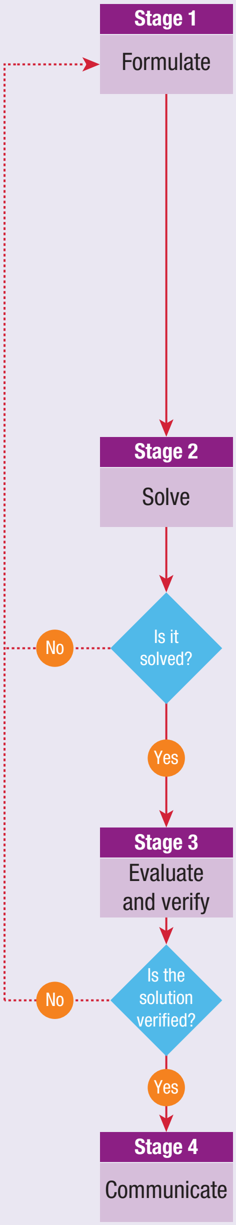
Approach to problem-solving and modelling task:



Stage 1: Formulate

1 What are you required to do?

2 What information do you have?

3 What other information do you need? What is the minimum percentage deposit required? You will also need to save for additional costs like stamp duty, insurance, legal fees and loan application fees.

4 How could you gather more information?

5 What assumptions will you make?

Stage 2: Solve

6 Gather more information.

7 Use mathematics to calculate information required to solve the problem.

8 Produce calculations, graphs or tables required to solve the problem.

9 Have you been able to develop a suitable savings plan to achieve your financial goal?

Stage 3: Evaluate and verify

10 Have you answered the question?

11 Is the answer you have reasonable based on potential increases in the value of your chosen home due to inflation and market changes?

Stage 4: Communicate

12 State your main point.

13 Include the evidence in the form of statements, calculations, graphs and tables.

14 Explain the evidence. Use a sentence starter like ‘This means … ’

15 In the conclusion, link back to your main point.

Chapter checklist

☐ I can calculate simple interest.

1 \$2682 invested at 4.2% p.a. for three years.

a Calculate the total interest earned over three years.

b Determine how much money will be in the account after the three years.

☐ I can calculate compound interest.

2 \$3540 invested at 2.7% p.a. compounded annually for four years.

a Determine how much money will be in the account after the four years.

b Calculate the total interest earned over four years.

☐ I can solve real-life problems involving compounding.

3 The population of Queensland is 5.04 million people in 2018. The annual growth rate averages 1.6% p.a. Calculate the expected population of Queensland in:

a 15 years

b 2050

☐ I can use the Moneysmart compound interest calculator to find the future value of an investment.

4 Anson is saving to buy a car. He already has \$8760 saved from his part-time job and can save \$400 per month. His bank account pays 2.7% p.a. interest compounding monthly. Use the Moneysmart compound interest calculator to calculate how much he has in his account after 2 years.

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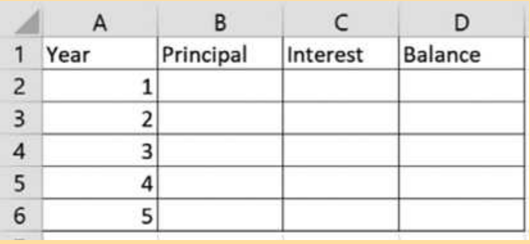
☐ I can use a spreadsheet to show growth of a compound interest investment. **[complex]**

5 Cody invests $500 in an account that pays 4% p.a. interest compounded every year. A spreadsheet is used to calculate the interest and balance each year.



	A	B	C	D
1	Year	Principal	Interest	Balance
2	1			
3	2			
4	3			
5	4			
6	5			

- **a** Identify the value that will be entered in cell B2.
- **b** Decide which formula you will need to enter in cell C2.
- **c** Determine the formula you will need to enter in cell D2.
- **d** Determine the formula will you need to enter in cell B3.

☐ I can identify the effect of interest rate changes on investments.

6 $7500 is deposited in an account for five years.

- **a** Calculate the amount in the account after the five years if the interest rate is:
  - **i** 3.75% p.a. compounding annually
  - **ii** 5.15% p.a. compounding annually
- **b** Describe the effect of the interest rate change.

☐ I can identify the effect of the number of compounding periods on an investment.

7 $7500 is deposited in an account for five years at 5% p.a. compounding interest.

- **a** Calculate the amount in the account after five years if the interest is compounding:
  - **i** annually
  - **ii** fortnightly
- **b** Describe the effect of the change in compounding frequency.

Chapter review

All questions in the review are assessment style.

Simple familiar

Section 11A

1 Riley invested \$60 000 in a fixed term account for 3 years paying a simple interest rate of 2.65% p.a.

a Calculate the total amount of interest earned over the 3 years.

b Calculate the total value of the investment after 3 years.

2 A company invested \$5 000 000 in the short-term money market at 6.7% p.a. simple interest for 90 days.

a Calculate the total amount of interest earned over the 90 days.

b Calculate the total value of the investment after 90 days.

Section 11B

3 Electra invests \$1400 into an account that pays 3.8% p.a. compound interest for three years. Interest is compounded yearly.

a Copy and complete the following table to calculate her interest and balance at the end of each year.

Year	Principal	Interest	Balance
1	\$1400		
2			
3			

b Determine how much interest she earned in three years.

4 Shae-Leah invests \$2000 in an account that earns interest at the rate of 4.6% p.a. compounding monthly. Use the compound interest formula to determine how much is in her account after four years.

5 Smithy has saved \$5200 from his part-time job. He invests it for two years at 3.20% p.a. so he can buy a car when he turns 18. Interest is compounded yearly.

a Use the compound interest formula to calculate the balance of his account after two years.

b Calculate how much interest he earned over the two years.

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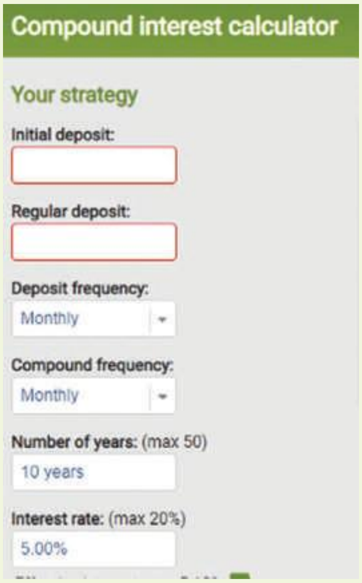
Section 11D **6** Tom is saving for a deposit on a house. He saves \$450 from his pay every fortnight into an account that pays 3.6% p.a. with interest compounding monthly. He uses the Moneysmart compound interest calculator to determine how much he will have saved after 5 years.

a Decide what he will need to enter on the online calculator at:
  - **i** initial deposit
  - **ii** regular deposit
  - **iii** deposit frequency
  - **iv** compound frequency
  - **v** number of years
  - **vi** interest rate



Compound interest calculator

Your strategy

Initial deposit:

Regular deposit:

Deposit frequency: Monthly

Compound frequency: Monthly

Number of years: (max 50) 10 years

Interest rate: (max 20%) 5.00%

b The calculator returns the following values:

Your strategy:

Initial deposit:	\$0
Regular deposits:	\$58,500
Total interest:	\$5,491
Total savings:	\$63,991

  - **i** Determine how much he will have in his account after 5 years.
  - **ii** Identify how much interest he has earned.

Section 11E **7** \$3500 is deposited for three years in an account.

a Calculate the amount in the account after three years at:
  - **i** 4.75% p.a. compounding annually
  - **ii** 5.25% p.a. compounding annually

b Describe the effect of the interest rate change.

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8 $3500 is deposited for five years in an account at 4.5% p.a. compounding interest.

- **a** Calculate the amount in the account after five years if the interest compounds:
  - **i** every 6 months
  - **ii** weekly
- **b** Describe the effect of the change in compounding frequency.

Complex familiar

Section 11C

9 The population of Brisbane was 2.4 million in 2018 with an annual growth rate of 3.42% p.a. If the population continues to grow at the same rate, use the compound interest formula to estimate the population:

- **a** 5 years after 2018
- **b** in 2100

10 The average rate of inflation in Australia is 2.1% p.a. If a 3 L bottle of milk costs $3.30 in 2018, use the compound interest formula to estimate how much will it cost:

- **a** 15 years after 2018
- **b** in 2050

Section 11D

11 The following table shows the interest and balance earned on a $5000 investment earning 5% p.a. for five years when interest is calculated using simple interest vs compound interest.

	Simple interest			Compound interest		
Year	**Principal**	**Interest**	**Balance**	**Principal**	**Interest**	**Balance**
1	$5000.00	$250.00	$5250.00	$5000.00	$250.00	$5250.00
2	$5000.00	$250.00	$5500.00	$5250.00	$262.50	$5512.50
3	$5000.00	$250.00	$5750.00	$5512.50	$275.63	$5788.13
4	$5000.00	$250.00	$6000.00	$5788.13	$289.41	$6077.53
5	$5000.00	$250.00	$6250.00	$6077.53	$303.88	$6381.41

- **a** Calculate the total interest earned in the:
  - **i** simple interest account
  - **ii** compound interest account
- **b** Explain why the balance is the same at the end of the first year.
- **c** Explain the key difference in how simple interest and compound interest is calculated.

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Complex unfamiliar

Section 11C

12 Computers lose 45% of their value each year. Biar buys a new laptop computer for $1899. Use the compound interest formula to estimate how much the laptop will be worth in two years' time.

Section 11D

13 Amy is planning an overseas holiday. She has just inherited $3400 which she will use towards her holiday. She budgets to save $360 each month and can earn 4.25% p.a. compound interest that compounds each month. She sets up a spreadsheet as follows.

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2					
3					
4					

Determine what needs to be entered in the following cells.

a A2 **b** B2

c C2 **d** D2

e E2 **f** B3

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12

Reducing balance loans

Maths for a dental assistant: Annie Wauchope

Annie Wauchope is a dental assistant.

Tell us a bit about your job. What does a typical day look like?

I am a dental assistant. My job is to welcome patients into our reception area, comfort patients in the treatment rooms and assist the dentist with all procedures.

A typical day for me: I check that all appointments have been confirmed, and set my surgery room up for the day. We then start seeing patients for treatment. Throughout the day, I sterilise instruments and take care of patients while assisting the dentist. At the end of the day, I pack up my surgery room and all the assistants help each other to clean and close the practice.

What maths did you study at school?

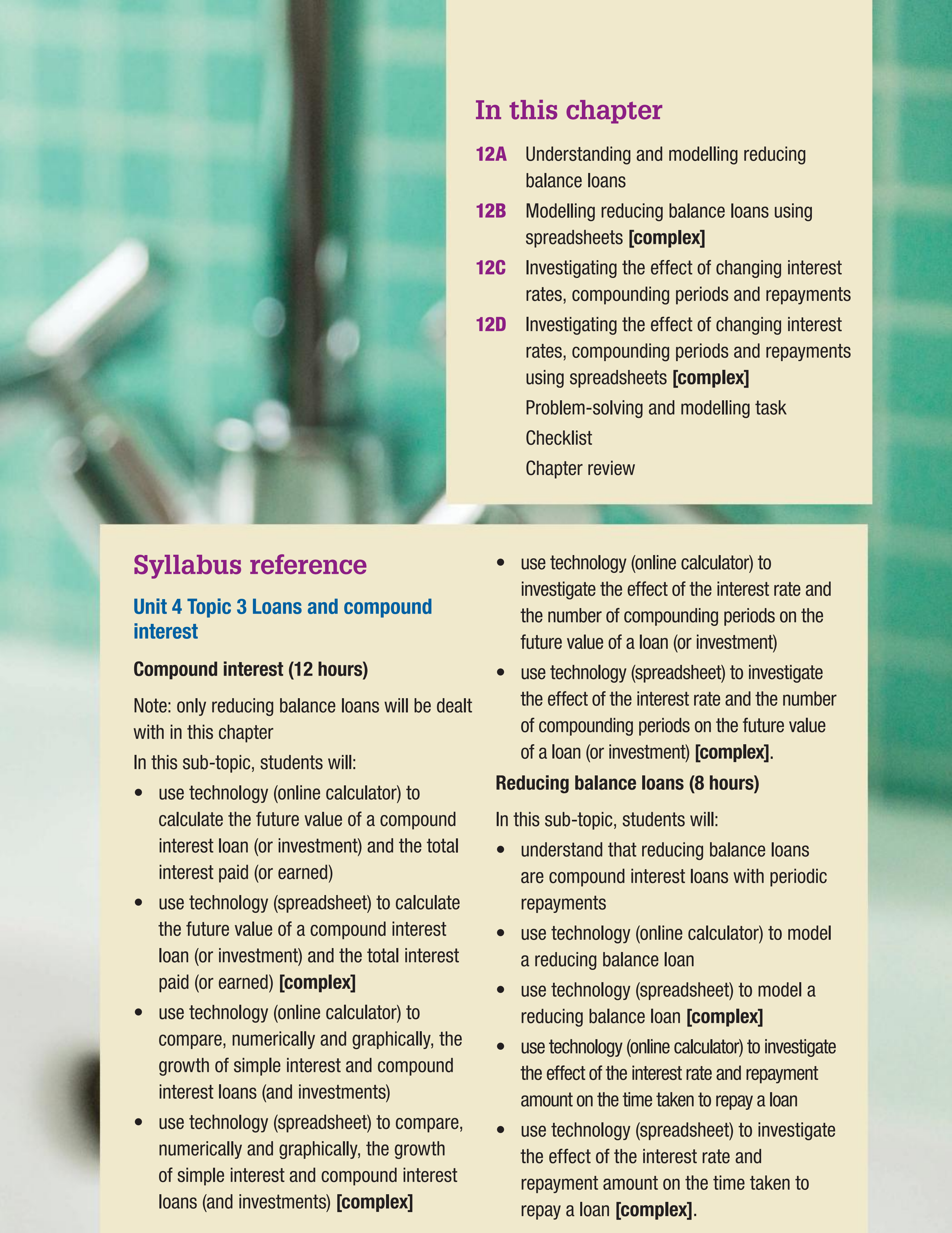
In school, I studied Maths A throughout grades 11 and 12.

How do you use maths in your job?

I have to count instruments, we count teeth, I use maths when doing the banking and taking payments.

I need maths when making patient’s appointments, because I need to work out how much time the dentist needs to complete all treatment in the appointment, and then I need to fit the appointment into a day when the dentist has time. I also try to ensure every day is booked with appointments back to back so there’s not any wasted time throughout the day, and our patients do not have to wait weeks to be seen.

I also use maths when taking payments to calculate the gap payment from a patients’ health fund and when giving a patient a discount. I need to be able to count cash and give patients the correct amount of change. Maths is very important in my job.



In this chapter

Syllabus reference

Unit 4 Topic 3 Loans and compound interest

Compound interest (12 hours)

Note: only reducing balance loans will be dealt with in this chapter

In this sub-topic, students will:

- use technology (online calculator) to calculate the future value of a compound interest loan (or investment) and the total interest paid (or earned)
- use technology (spreadsheet) to calculate the future value of a compound interest loan (or investment) and the total interest paid (or earned) **[complex]**
- use technology (online calculator) to compare, numerically and graphically, the growth of simple interest and compound interest loans (and investments)
- use technology (spreadsheet) to compare, numerically and graphically, the growth of simple interest and compound interest loans (and investments) **[complex]**
- use technology (online calculator) to investigate the effect of the interest rate and the number of compounding periods on the future value of a loan (or investment)
- use technology (spreadsheet) to investigate the effect of the interest rate and the number of compounding periods on the future value of a loan (or investment) **[complex]**.

Reducing balance loans (8 hours)

In this sub-topic, students will:

- understand that reducing balance loans are compound interest loans with periodic repayments
- use technology (online calculator) to model a reducing balance loan
- use technology (spreadsheet) to model a reducing balance loan **[complex]**
- use technology (online calculator) to investigate the effect of the interest rate and repayment amount on the time taken to repay a loan
- use technology (spreadsheet) to investigate the effect of the interest rate and repayment amount on the time taken to repay a loan **[complex]**.

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Pre-test

1 Calculate the simple interest on the following investments.
- **a** $450 at 6% p.a. for one month
- **b** $980 at 5% p.a. for one week
- **c** $387 at 4% p.a. for one fortnight
- **d** $1345 at 5% p.a. for one quarter
- **e** $350 at 6% p.a. for 6 months

Hint 'p.a.' means per annum, i.e. per year.

2 $4000 is borrowed at 6% p.a. simple interest for 2 years.
- **a** Calculate the interest for the two years.
- **b** Calculate the total amount repaid over the two years.
- **c** Calculate the size of the monthly repayment.

3 Convert the following time periods to years and months.
- **a** 48 months
- **b** 260 weeks
- **c** 78 fortnights
- **d** 42 months
- **e** 338 weeks
- **f** 91 fortnights
- **g** 200 months
- **h** 200 weeks
- **i** 200 fortnights

A link to a HOTmaths lesson is provided in the Interactive Textbook to revise this topic.

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12A Understanding and modelling reducing balance loans

LEARNING GOALS

- Understand that reducing balance loans are compound interest loans with periodic repayments
- Model a reducing balance loan by using technology (online calculator)
- Calculate the future value of a compound interest loan and the total interest paid by using technology (online calculator)
- Compare the growth of simple interest and compound interest loans by using technology (online calculator)

Why are reducing balance loans essential?

We all need to borrow money at some stage of our lives. It may be to purchase a car or your first home. Most loans have interest calculated on the remaining balance. Understanding how the loan works and the savings to be made by paying more frequently, or just a little bit more, will help you to achieve your financial goals.

A home loan is the biggest expense you will ever have, so it is wise to look for ways to make savings.

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WHAT YOU NEED TO KNOW

Simple interest loans were dealt with in Exercise 11A. They are not as common as reducing balance loans but are easy to calculate the interest and size of the repayments. **Reducing balance loans** have the interest calculated on the remaining balance each interest period.

- The mortgage calculator on the Moneysmart website allows you to enter how much you need to borrow and different variables such as the interest rate and the length of time you will take to repay the loan. The online calculator then generates for you the size of the regular repayments required for that loan.

  Moneysmart Mortgage Calculator: *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-much-will-my-repayments-be*

  - For example: If you borrow \$200 000 over 25 years with an interest rate of 5.6% p.a. calculated on the reducing balance, the following calculations will be shown.

Mortgage calculator

How much will my mortgage repayments be?

Mortgage details

Amount borrowed: \$200,000

Interest rate: 5.60%

Repayment frequency: Monthly

Length of loan: 25 years

Fees: \$0 Monthly

Your repayments will be: \$1,240 per month

Source https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-much-will-my-repayments-be

The monthly repayments would be \$1240.

The total amount repaid = 1240 × 12 × 25 = \$372 000

$$\begin{aligned}\text{The total interest} &= \text{total repaid} - \text{amount borrowed}\\ &= 372\,000 - 200\,000 = \$172\,000\end{aligned}$$

Note: The total interest is represented by the pink part of the bar graph, sitting on top of the principal.

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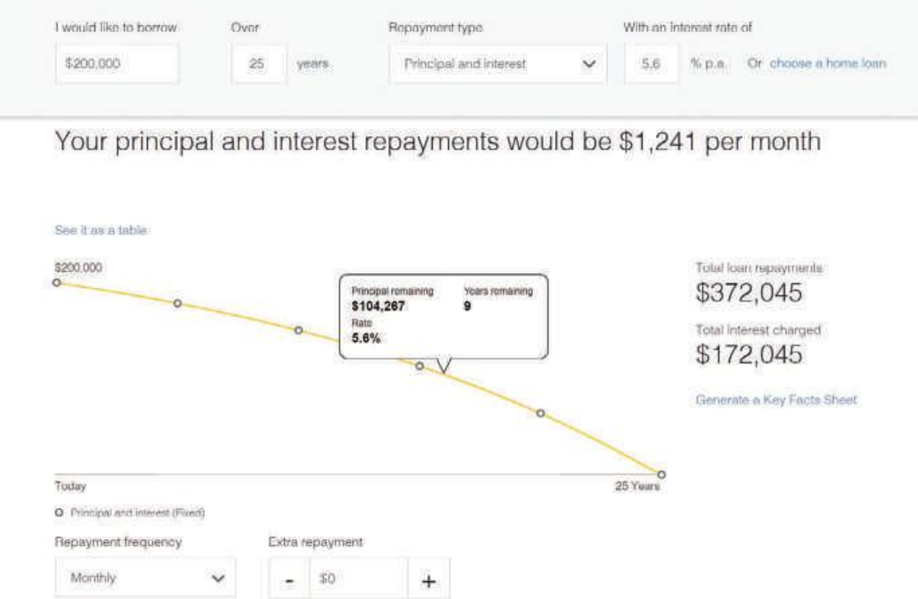
- All banks provide loan calculators on their websites to help you with understanding the costs of a loan. We will use the Commonwealth Bank home loan calculator: *https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments*
  - For example: If $200 000 was borrowed at 5.6% p.a. reducible interest over 25 years, the debt remaining would be modelled as:



I would like to borrow: $200,000

Over: 25 years

Repayment type: Principal and interest

With an interest rate of: 5.6 % p.a.

Your principal and interest repayments would be $1,241 per month

Total loan repayments: $372,045

Total interest charged: $172,045

Repayment frequency: Monthly

Extra repayment: $0

Source https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments

Note: the difference in repayment amounts between the two online calculators is because one website has rounded down and the other has rounded up.

  - By floating your cursor above the graph, we see that it takes more than 16 years to pay the first half of the debt, and less than 9 years to pay the second half of the debt. We could also see this information in a table by clicking on the link at the top left of the graph.
    The shape of the graph shows that the debt remaining on the loan is slow to decrease at the start and decreases faster towards the end. This is because the *interest is calculated on the reducing balance of the loan* so will reduce over time; while the repayment remains the same. Repayments made towards the end of the loan will have more impact on the balance remaining.

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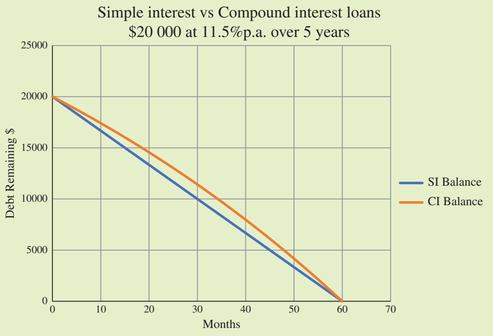
- We can see this by looking at the first few months of this loan in a table:

Month	Principal	Interest	Repay	Balance
1	200 000.00	$I = 200\,000.00 \times \frac{5.6}{100} \times \frac{1}{12}$ $= 933.33$	1 241	$200\,000.00 + 933.33$ $-1241 = 199\,692.33$
2	199 692.33	$I = 199\,692.33 \times \frac{5.6}{100} \times \frac{1}{12}$ $= 931.90$	1 241	$199\,692.33 + 931.90$ $-1241 = 199\,383.23$
3	199 383.23	$I = 199\,383.23 \times \frac{5.6}{100} \times \frac{1}{12}$ $= 930.46$	1 241	$199\,383.23 + 930.46$ $-1241 = 199\,072.69$

  - Interest for each month is calculated using the formula $I = Pin$.
  - The balance at the end of each month is found by adding the interest and subtracting the repayment from the principal at the start of the month.
  - The balance from the previous month becomes the principal in the next month.
  - The interest reduces each month as it is calculated on the reducing balance.
- When comparing the graphs of debt remaining on simple interest and compound interest loans, their shape is different, as they were in Section 11D for investments. The simple interest graph is a straight line as the interest remains the same during the entire loan. The compound interest graph is curved as the interest decreases as the loan is paid off.



  - The interest costs across the two loans is also significantly different:

    $\text{Total interest (simple interest)} = \$11500$

    $\text{Total interest (compound interest)} = \$6390.97$

  Note: This graph will be developed as a technology (spreadsheet) activity in Exercise 12B.

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Example 1 Calculating a reducing balance loan

Tait has borrowed $235 000 to buy a unit. The terms of his loan are 6.55% p.a. reducible interest over 20 years and his monthly repayments are $1759 per month.

a Complete this table to determine how much Tait will owe on his loan after the first three months.

Month	Principal	Interest	Repayment	Balance
1				
2				
3				

b Determine the total of Tait's monthly repayments over the first three months.

c Calculate the total of the interest charges over the first three months.

d Calculate the difference between the amount repaid and the interest charges over the first three months.

e Determine how much Tait reduced his debt by in the first three months.

WORKING

THINKING

a Month 1

Interest is calculated using the simple interest formula $I = Pin$.

Principal = 235 000

$$\text{Interest} = 235\,000 \times \frac{6.55}{100} \times \frac{1}{12}$$

$$= 1282.71$$

Put the percentage interest rate over 100 to convert to a decimal.

$n = 1$ month $= \frac{1}{12}$ year

Repayment = $1759

$$\text{Balance} = 235\,000 + 1282.71 - 1759$$

$$= \$234\,523.71$$

Balance = starting principal + interest − repayment made

Month 2

Previous month's balance becomes this month's principal.

Principal = $234 523.71

$$\text{Interest} = \$234\,523.71 \times \frac{6.55}{100} \times \frac{1}{12}$$

Interest formula $I = Pin$

$$= 1280.11$$

Repayment = $1759

$$\text{Balance} = \$234\,523.71 + 1280.11 - 1759$$

$$= \$234\,044.82$$

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WORKING	THINKING
Month 3 Principal = $234 044.82	Previous month's balance becomes this month's principal.
$\text{Interest} = \$234\,044.82 \times \frac{6.55}{100} \times \frac{1}{12}$ $= 1277.49$ Repayment = $1759 Balance = $234 044.82 + 1277.49 − 1759 = $233 563.31	Interest formula $I = Pin$

Complete the values in the table.

Month	Principal	Interest	Repayment	Balance
1	$235 000.00	$1282.71	$1759.00	$234 523.71
2	$234 523.71	$1280.11	$1759.00	$234 044.82
3	$234 044.82	$1277.49	$1759.00	$233 563.31

b Total repayments = 1759 × 3 — Three equal repayments were made.
= $5277

c Total interest — The interest is reducing each month.
= 1282.71 + 1280.11 + 1277.49
= $3840.31

d Difference = 5277 − 3840.31 — Difference = total repaid − total interest
= $1436.69

e Difference = 235 000 − 233 563.31 — Difference = original loan − debt remaining
= $1436.69

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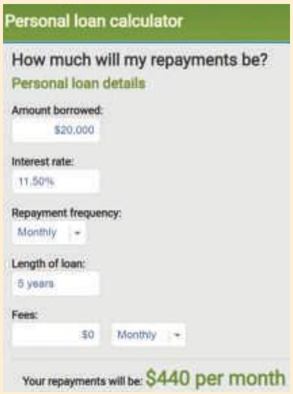
Example 2 Using an online calculator for a compound interest loan

Alexander has borrowed $20 000 over 5 years at 11.5% p.a. with reducible (compound) interest to buy a car. Use the Moneysmart personal loan calculator to determine:

a the monthly repayment required
b the total repaid
c the total interest

WORKING **THINKING**

a



Personal loan calculator

How much will my repayments be?

Personal loan details

Amount borrowed: $20,000

Interest rate: 11.50%

Repayment frequency: Monthly

Length of loan: 5 years

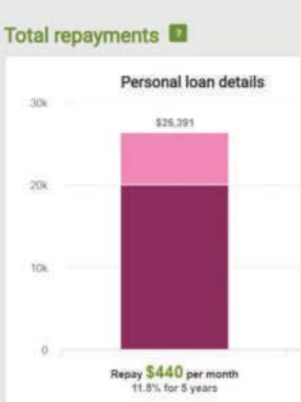
Fees: $0 Monthly

Your repayments will be: $440 per month

https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/personal-loan-calculator#!how-much-will-my-repayments-be
Enter values into an online calculator.

The monthly repayment will be $440.

b



This is different to the value on the graph ($26 391) because the monthly repayment has been rounded to the nearest dollar.

$$\begin{aligned}\text{Total repaid} &= 440 \times 12 \times 5\\ &= \$26\,400\end{aligned}$$

c

$$\begin{aligned}\text{Total interest} &= 26\,400 - 20\,000\\ &= \$6400\end{aligned}$$

Total interest = total repaid − amount borrowed
You can also hover over the pink section of the graph to find the interest.

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Example 3 Using an online calculator to model a reducing balance loan

Ella has borrowed $315 000 to buy a house. She will repay the loan monthly over 25 years with 5.37% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to answer the following.

The Commonwealth Bank home loan calculator: *https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments*

a Determine the size of the monthly repayments.
b Determine the total loan repayments.
c Determine the total interest charges.
d Determine the amount left owing after 20 years.
e Determine how many years it takes to pay off half the loan.

WORKING **THINKING**

Your principal and interest repayments would be $1,910 per month

Total loan repayments
$573,000

Total interest charged
$258,000

Input variables into online calculator. Read off required information.

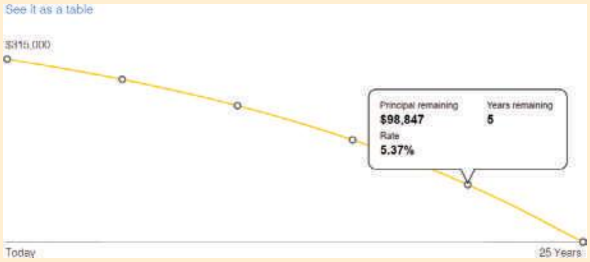
a Monthly repayments = $1910

b Total loan repayments = $573 000

c Total interest charges = $258 000

d 20 years: 25 – 20 = 5 years remaining

Hover over graph to find required value.



Principal remaining $98,847
Years remaining 5
Rate 5.37%

There was $98 847 owing after 20 years.

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WORKING **THINKING**

e Half the loan = 315 000 ÷ 2

= 157 500

Need to find the first time the principal remaining is less than $157 500.

With 8 years remaining, the principal falls below $157 500.

Time taken = 25 – 8 = 17 years

It takes 17 years to pay off the first half of the loan.

Exercise 12A

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** All loans require you to repay what you ________ plus ________.
- **b** Reducing balance loans have the interest calculated on the ________ balance each ________ period.
- **c** Interest for each month is calculated using the ________ $I = Pin$.
- **d** The balance at the end of each month is found by ________ the interest and ________ the repayment from the principal at the start of the month.
- **e** The balance from the previous month becomes the ________ in the next month.
- **f** The interest ________ each month as it is calculated on the reducing balance.
- **g** Repayments made towards the ________ of the loan will have more impact on the ________ remaining.
- **h** Online ________ help you find the amount of interest you pay and the ________ cost of the loan.

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APPLICATIONS

SF: 2–12	CF: –	CU: –

Example 1

2 Lana has borrowed $2300 to purchase a new bed. She has agreed to pay off the loan in 12 months with reducible interest calculated at 12.65% p.a. Her monthly repayments are $205. Lana is given a table of her loan schedule as follows.

Month	Principal	Interest	Repayment	Balance
1	$ 2300.00	$ 24.25	$ 205.00	$ 2119.25
2	$ 2119.25	$ 22.34	$ 205.00	$ 1936.59
3	$ 1936.59	$ 20.41	$ 205.00	$ 1752.00
4	$ 1752.00	$ 18.47	$ 205.00	$ 1565.47
5	$ 1565.47	$ 16.50	$ 205.00	$ 1376.97
6	$ 1376.97	$ 14.52	$ 205.00	$ 1186.49
7	$ 1186.49	$ 12.51	$ 205.00	$ 994.00
8	$ 994.00	$ 10.48	$ 205.00	$ 799.47
9	$ 799.47	$ 8.43	$ 205.00	$ 602.90
10	$ 602.90	$ 6.36	$ 205.00	$ 404.26
11	$ 404.26	$ 4.26	$ 205.00	$ 203.52
12	$ 203.52	$ 2.15	$ 205.00	$ 0.66

a Decide if Lana will completely pay off the loan in 12 months. Why will this happen?

Hint Consider the final balance.

b Explain how much Lana should pay in the last month to completely pay off the loan.

c Determine how much Lana will still owe on her loan after 6 months.

d Describe what happens to the interest each month.

e Calculate the total interest Lana pays on the loan.

f Calculate 12.65% of $2300. Explain why this is different to your answer in **e**.

★**3** Todd repays $373 each month for four years on his $15 000 loan to buy a motorbike.

a Determine how many repayments will he make.

b Calculate how much he will repay in total.

c Calculate how much he will pay in interest.

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Example 1

4 Lochy has borrowed $14 700 to buy a car. The terms of his loan are 11.45% p.a. reducible interest over four years and his monthly repayments are $383 per month.

a Copy and complete this table to find how much Lochy will owe on his loan after the first three months.

Month	Principal	Interest	Repay	Balance
1	14 700	$147\,00 \times \frac{11.45}{100} \times \frac{1}{12}$ $= 140.26$	383	$14\,700 + 140.26 - 383$ $= 14\,457.26$
2	14 457.26			
3				

b Calculate the total of Lochy's monthly repayments over the first three months.

c Calculate the total of the interest charges over the first three months.

d Determine the difference between the amount repaid and the interest charges over the first three months.

e Determine how much Lochy will reduce his debt by in the first three months.

★**5** Stephanie has borrowed $5600 to fund a trip overseas. The terms of her loan are 12.10% p.a. reducible interest over two years and her monthly repayments are $264 per month.

a Copy and complete this table to find how much Stephanie will owe on her loan after the first four months.

Month	Principal	Interest	Repay	Balance
1				
2				
3				
4				

b Determine how much interest Stephanie will pay in the first month.

c Calculate how much Stephanie will reduce her debt by in the first month.

d Determine how much interest Stephanie will pay in the fourth month.

e Calculate how much Stephanie will reduce her debt by in the fourth month.

f Interpret the difference between your answers to parts **c** and **e**.

★6 Gus has borrowed $28 000 to start up a business. He has agreed to repay the loan in two years with 4.35% p.a. reducible interest. He will repay the loan with quarterly payments of $3673.

- **a** Determine how many repayments Gus will make.
- **b** Construct a loan schedule for Gus with the headings: Quarter, Principal, Interest, Repayment, Balance.
- **c** **i** Determine how much will be left owing after eight quarters.
  - **ii** Explain why this will happen.
  - **iii** Describe how can Gus fix this issue.
- **d** Calculate the total interest if this had been a simple interest loan.
- **e** Calculate how much interest Gus will pay on his reducible interest loan.

Use the calculators on the Moneysmart website *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/loan-credit-and-debt-calculators* to answer the following questions:

Example 2

7 Lakeisha has borrowed $18 000 to buy a car. Interest is calculated at 9.85% p.a. on the reducing balance (compound interest) and she will repay the loan over four years. Use the Moneysmart personal loan calculator to determine:

- **a** the monthly repayment required
- **b** the total repaid
- **c** the total interest

8 Tilly has borrowed $12 000 to pay for an overseas holiday. Interest is calculated at 10.85% p.a. on the reducing balance (compound interest) and she will repay the loan over three years. Use the Moneysmart personal loan calculator to determine:

- **a** the monthly repayment required
- **b** the total repaid
- **c** the total interest

9 Charles has borrowed $250 000 to buy a home. Interest is calculated at 5.85% p.a. on the reducing balance (compound interest) and he will repay the loan over 25 years.

- **a** Use the Moneysmart Mortgage Calculator to determine:
  - **i** the monthly repayment required
  - **ii** the total repaid
  - **iii** the total interest

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b Charles thinks he could afford to repay $1700 each month.
  i Use the 'How can I repay my home loan sooner?' tab to determine how long it would now take Charles to repay the loan.
  ii Determine how much Charles will now repay in total.
  iii Determine how much Charles will now pay in interest.
  iv Calculate the saving in interest by paying more each month.
  v Determine how much extra Charles will repay each month.
  vi Explain what recommendations you would make about paying extra in repayments.

Use the Commonwealth Bank home loan calculator to answer the following questions: *https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments*

Example 3

10 Kayla has borrowed $260 000 to buy a home. She will repay the loan monthly over 25 years with 4.59% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to determine the:

a size of the monthly repayments
b total loan repayments
c total interest charges
d amount left owing after 20 years
e number of years it will take to pay off half the loan

11 Declan has borrowed $22 000 to buy a car. He will repay the loan monthly over 5 years with 10% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to determine the:
a size of the monthly repayments
b total loan repayments
c total interest charges
d amount left owing after 3 years
e number of years it takes to pay off half the loan

12 A school has borrowed $5 million to build a new sports centre. They will repay the loan in equal monthly payments over 5 years with 4.89% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to determine the:
a size of the monthly repayments
b total loan repayments
c total interest charges
d amount left owing after 4 years
e number of years it takes to pay off half the loan

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12B Modelling reducing balance loans using spreadsheets COMPLEX

LEARNING GOALS

- Model a reducing balance loan by using technology (spreadsheet)
- Calculate the future value of a compound interest loan and the total interest paid by using a spreadsheet
- Compare the growth of simple interest and compound interest loans by using a spreadsheet

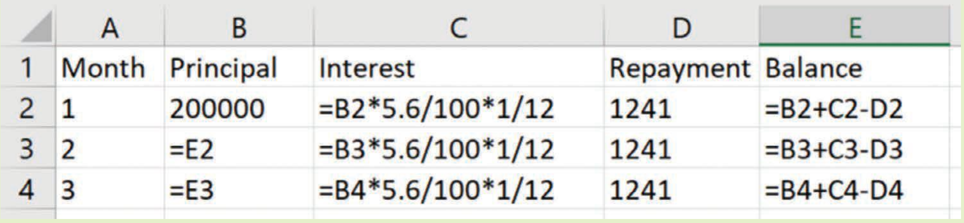
Why is it essential to use technology for reducing balance loans?

We saw in the last section that the calculations required for a reducing balance loan schedule are repetitive and time consuming. Using a spreadsheet can make the process a lot quicker with less potential to make careless errors.

Using computer spreadsheets to do reducing balance loan calculations saves time.

WHAT YOU NEED TO KNOW

- We can use a spreadsheet to perform the table calculations we did in the last section.
  - For example: If \$200 000 was borrowed at 5.6% p.a. reducible interest over 25 years, a suitable set up for the debt remaining would be:



	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
2	1	200000	=B2*5.6/100*1/12	1241	=B2+C2-D2
3	2	=E2	=B3*5.6/100*1/12	1241	=B3+C3-D3
4	3	=E3	=B4*5.6/100*1/12	1241	=B4+C4-D4

For a 25-year home loan, we would need to fill down $(25 \times 12) = 300$ rows to fully pay off the loan.

- Excel can calculate the size of a loan repayment using the formula:

  =PMT(rate, nper, pv, [fv], [type])

  where:

rate	The interest rate for the loan as a fraction or decimal
nper	The total number of payments to pay off the loan
pv	The present value of the principal; the amount of money borrowed
fv	The future value of the loan. As we will pay off our loans, this will be 0.
type	0 means the payment is made at the end of the period, 1 means the payment is made at the beginning of the period. It is usual to pay at the end of the time period, so we will enter 0 for type.

Example 4 Using a spreadsheet for a compound interest loan

Note: the spreadsheet in this example can be accessed in the Interactive Textbook by clicking on the icon at left.

Georgina has borrowed $335 000 over 20 years at 6.2% p.a. with reducible (compound) interest to buy a house. Use a spreadsheet to determine:

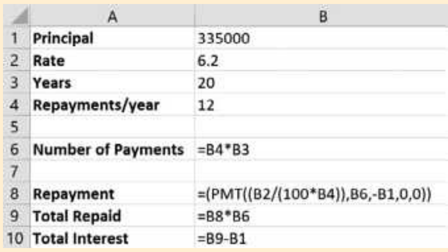
a the monthly repayment required

b the total repaid

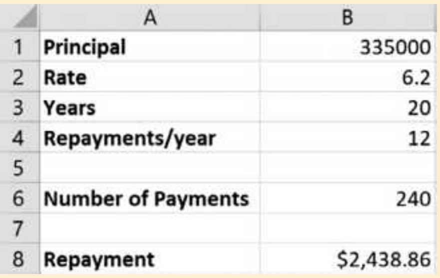
c the total interest

WORKING **THINKING**

a



	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=(PMT((B2/(100*B4)),B6,-B1,0,0))
9	Total Repaid	=B8*B6
10	Total Interest	=B9-B1



	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/year	12
5		
6	Number of Payments	240
7		
8	Repayment	$2,438.86

The monthly repayment is $2439.

The Excel formula needs to know rate, number of payments and principal.

=PMT(rate, nper, pv, [fv], [type])

The rate will need to be divided by number of payments per year and 100 to make it a decimal.

Number of payments = payments per year × number of years

As a formula this is =**B4*B3**

The principal is entered as a negative value in the formula since it is a debt. For [fv] and [type] we need to enter 0. Set up spreadsheet as shown.

Round answer to nearest dollar amount.

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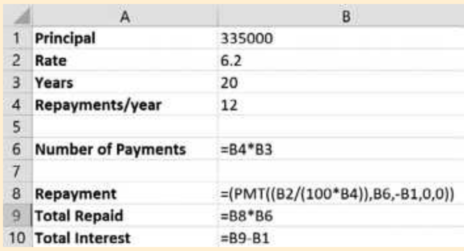
WORKING **THINKING**

b For the total amount repaid:

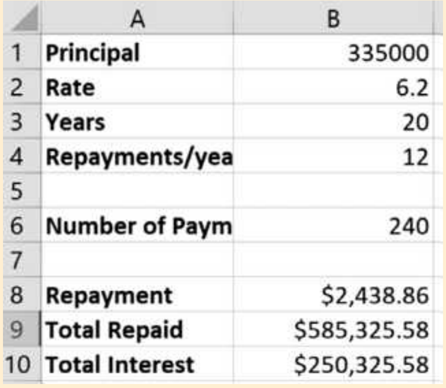


	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=(PMT((B2/(100*B4)),B6,-B1,0,0))
9	Total Repaid	=B8*B6
10	Total Interest	=B9-B1

Total repaid = $585325.58

Total repaid = repayment × number of payments
As a formula this is =**B8*B6**

c For the total interest:



	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/yea	12
5		
6	Number of Paym	240
7		
8	Repayment	$2,438.86
9	Total Repaid	$585,325.58
10	Total Interest	$250,325.58

Total interest = $250325.58

Total interest = Total repaid – amount borrowed
As a formula this is =**B9 –B1**

Example 5 Using a spreadsheet to model a reducing balance loan

Note: the spreadsheet in this example can be accessed in the Interactive Textbook by clicking on the icon at left.

Josh has borrowed $25 000 to buy a car. He will repay the loan with monthly repayments over 4 years with 13.9% p.a. reducible interest.

a Use a spreadsheet to calculate the size of the monthly repayment to the nearest dollar.

b Set up a spreadsheet to model the loan schedule over 4 years.

c Determine the amount left owing after 48 months.

d Calculate the size of the last repayment to fix the overpayment.

e Determine how many months it takes to pay off half the loan.

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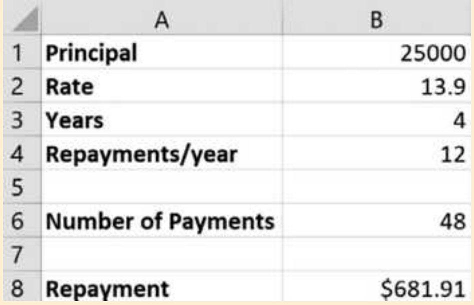
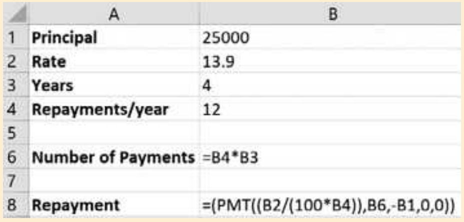
WORKING	**THINKING**

a Monthly repayment = $682



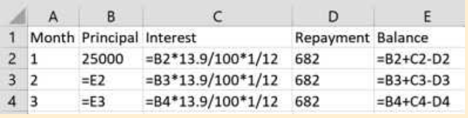
	A	B
1	**Principal**	25000
2	**Rate**	13.9
3	**Years**	4
4	**Repayments/year**	12
5		
6	**Number of Payments**	48
7		
8	**Repayment**	$681.91

Use the spreadsheet from Example 4.



	A	B
1	**Principal**	25000
2	**Rate**	13.9
3	**Years**	4
4	**Repayments/year**	12
5		
6	**Number of Payments**	=B4*B3
7		
8	**Repayment**	=(PMT((B2/(100*B4)),B6,-B1,0,0))

b

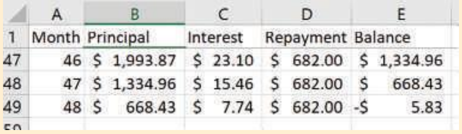


	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
2	1	25000	=B2*13.9/100*1/12	682	=B2+C2-D2
3	2	=E2	=B3*13.9/100*1/12	682	=B3+C3-D3
4	3	=E3	=B4*13.9/100*1/12	682	=B4+C4-D4

All formulas need to start with =.
Use the same method as used in Exercise 12A to calculate values in cells.
Fill down for 48 months.

c –$5.83 is left owing after 48 months. The loan has been overpaid by $5.83.

Scroll to the bottom to find answer.

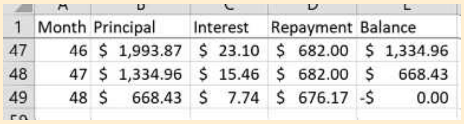


	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
47	46	$ 1,993.87	$ 23.10	$ 682.00	$ 1,334.96
48	47	$ 1,334.96	$ 15.46	$ 682.00	$ 668.43
49	48	$ 668.43	$ 7.74	$ 682.00	-$ 5.83

Tip: freeze the top row so that the headings stay visible when scrolling.

d Final payment = 682 – 5.83
= $676.17

Reduce final payment to finish with a $0 balance.

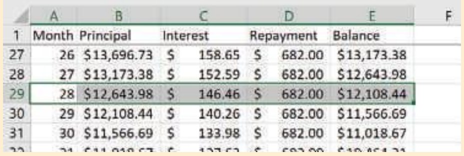


1	Month	Principal	Interest	Repayment	Balance
47	46	$ 1,993.87	$ 23.10	$ 682.00	$ 1,334.96
48	47	$ 1,334.96	$ 15.46	$ 682.00	$ 668.43
49	48	$ 668.43	$ 7.74	$ 676.17	-$ 0.00

e Half the loan = 25 000 ÷ 2
= $12 500

It takes 28 months to repay half the loan.

Scroll until you first find a balance that is less than $12 500.



	A	B	C	D	E	F
1	Month	Principal	Interest	Repayment	Balance	
27	26	$13,696.73	$ 158.65	$ 682.00	$13,173.38	
28	27	$13,173.38	$ 152.59	$ 682.00	$12,643.98	
29	28	$12,643.98	$ 146.46	$ 682.00	$12,108.44	
30	29	$12,108.44	$ 140.26	$ 682.00	$11,566.69	
31	30	$11,566.69	$ 133.98	$ 682.00	$11,018.67	

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Exercise 12B

FUNDAMENTALS

1 Determine the missing words in the following sentences.

a Using __________ calculators and computer __________ to do reducing balance loan calculations saves time.

b A spreadsheet can help you understand the total __________ of a loan.

c A spreadsheet can do the repetitive table __________ when we enter in the formula.

APPLICATIONS

SF: –	CF: 2–8	CU: –

Spreadsheet Example 4 applies to question 2.

Example 4

2 Ayla has borrowed \$17 000 to buy a car. She will repay the loan with monthly repayments over 3 years with 6.9% p.a. reducible interest.

a Use a spreadsheet to calculate the size of each repayment to the nearest dollar.

b Calculate the total loan repayments.

c Calculate the total interest charges.

Spreadsheet Example 5 applies to questions 3-8

Example 5

3 Use the information in question **2** to complete the following.

a Set up the spreadsheet to model the loan schedule over 3 years.

b Determine how much is left owing after 36 months.

c Calculate the size of the last repayment to fix the underpayment.

d Determine how many months it will take to pay off half the loan.

4 Clayton has borrowed \$6000 to fund an overseas holiday. He will repay the loan with fortnightly repayments over 2 years with 9.73% p.a. reducible interest.

a Use a spreadsheet to calculate the size of the repayment to the nearest dollar.

b Determine the total number of payments.

c Calculate the total loan repayments.

d Calculate the total interest charges.

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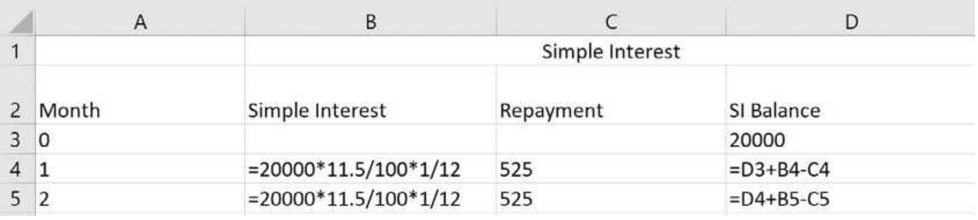
5 Use the information in question **4** to complete the following.

- **a** Set up a spreadsheet to model the loan schedule over 2 years.
- **b** Calculate the interest for the first fortnight.
- **c** Determine how much is left owing after 2 years.
- **d** Calculate the size of the last repayment to fix the underpayment.
- **e** Determine how many fortnights it will take to pay off half the loan.

Note: a spreadsheet for questions **6–8** can be accessed in the interactive textbook by clicking on the icon at left.

6 Lexi has borrowed $20 000 to buy a car. Simple interest is charged at 11.5% p.a. and she will repay the loan over five years in equal monthly repayments.

- ★**a** Calculate:
  - **i** the total interest.
  - **ii** the total to repay
  - **iii** the number of repayments
  - **iv** how much she will need to repay each month
- ★**b** Use a spreadsheet to show the amount still owing over the life of the loan. Use the following headings and formulas and fill down for 60 months.



	A	B	C	D
1			Simple Interest	
2	Month	Simple Interest	Repayment	SI Balance
3	0			20000
4	1	=20000*11.5/100*1/12	525	=D3+B4-C4
5	2	=20000*11.5/100*1/12	525	=D4+B5-C5

  - ★**i** Explain the formula used in cell B4.
  - ★**ii** Explain the formula in cell D4.
  - ★**iii** Explain why the formula in B5 the same as B4.
- **c** In cell B65 enter the formula =SUM(B4:B63). Decide if the answer matches your answer from part **a i**.
- **d** In cell C65 enter the formula =SUM(C4:C63). Decide if the answer matches your answer from part **a ii**.
- **e** Describe the values in the Simple Interest column (Column B).

Hint Adding month 0 (start of the loan) will assist later when we want to graph the loans schedules.

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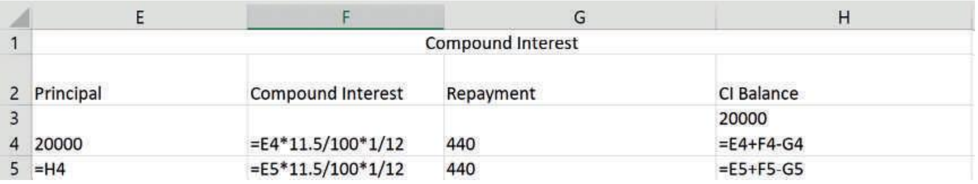
7 Daniel is a friend of Lexi. He also has borrowed \$20 000 to buy a car. Reducing balance (compound) interest is charged at 11.5% p.a. and he also decides he will repay the loan over five years in equal monthly repayments.

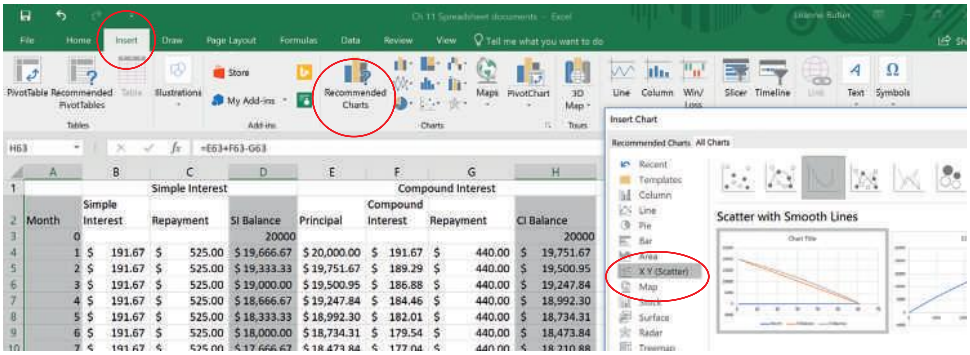
- **a** Use a spreadsheet to calculate:
  - **i** the monthly repayment to the nearest dollar.
  - **ii** the total to repay.
  - **iii** the total interest.
- ★**b** Use the same spreadsheet as in question **6**, and add the following headings and formulas and fill down for 60 months.



	E	F	G	H
1		Compound Interest		
2	Principal	Compound Interest	Repayment	CI Balance
3				20000
4	20000	=E4*11.5/100*1/12	440	=E4+F4-G4
5	=H4	=E5*11.5/100*1/12	440	=E5+F5-G5

  - **i** Explain the formula used in cell F4.
  - **ii** Explain the formula in cell H4.
  - **iii** Explain the formula in cell E5.
- **c** Describe what is happening to the values in the Compound Interest column (Column F).
- **d** In cell F65 enter the formula =SUM(F4:F63).
  - **i** Decide if the answer matches your answer from part **a iii**.
  - **ii** Calculate the difference in the interest paid in the compound interest loan to the interest paid in the simple interest loan in Question **6**.
- **e** In cell G65 enter the formula =SUM(G4:G63).
  - **i** Decide if the answer matches your answer from part **a ii**.
  - **ii** Calculate the difference in the total paid in the compound interest loan to the total paid in the simple interest loan in Question **6**.

8 **a** Produce a graph to compare the balance owing on the loans from questions **6** and **7** over the five years. Hold down the Ctrl key and highlight the month, SI Balance and CI Balance columns. From the Insert Tab>Recommended Charts>All Charts>XY Scatter>Scatter with Smooth Lines as below.



H63 =E63+F63-G63

	A	B	C	D	E	F	G	H
1			Simple Interest			Compound Interest		
2	Month	Simple Interest	Repayment	SI Balance	Principal	Compound Interest	Repayment	CI Balance
3	0			20000				20000
4	1	\$ 191.67	\$ 525.00	\$ 19,666.67	\$ 20,000.00	\$ 191.67	\$ 440.00	\$ 19,751.67
5	2	\$ 191.67	\$ 525.00	\$ 19,333.33	\$ 19,751.67	\$ 189.29	\$ 440.00	\$ 19,500.95
6	3	\$ 191.67	\$ 525.00	\$ 19,000.00	\$ 19,500.95	\$ 186.88	\$ 440.00	\$ 19,247.84
7	4	\$ 191.67	\$ 525.00	\$ 18,666.67	\$ 19,247.84	\$ 184.46	\$ 440.00	\$ 18,992.30
8	5	\$ 191.67	\$ 525.00	\$ 18,333.33	\$ 18,992.30	\$ 182.01	\$ 440.00	\$ 18,734.31
9	6	\$ 191.67	\$ 525.00	\$ 18,000.00	\$ 18,734.31	\$ 179.54	\$ 440.00	\$ 18,473.84
10	7	\$ 191.67	\$ 525.00	\$ 17,666.67	\$ 18,473.84	\$ 177.04	\$ 440.00	\$ 18,210.88

- **b** Add axis labels and a title to your graph.
- **c** Comment on the difference between the graphs of the two loans.

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12C Investigating the effect of changing interest rates, compounding periods and repayments

LEARNING GOALS

- Investigate the effect of the interest rate and the number of compounding periods on the future value of a loan using technology (online calculator)
- Investigate the effect of the interest rate and repayment amount on the time taken to repay a loan using technology (online calculator)

Why is understanding interest rate changes essential?

The dollar amount of interest paid or earned can be affected by changes to rates and frequency of compounds.

The Board of the Reserve Bank of Australia meets on the first Tuesday of every month to check on the state of the economy and set interest rates. At 2:30 p.m., they release a statement about their interest rate decision for that month. Their decisions have an impact on people around the country with variable interest rate home loans because banks can pass on the rates change to their customers. If rates go up, their monthly repayments will also go up. On the other hand, if rates go down, so will their repayments.

WHAT YOU NEED TO KNOW

- Changes to the size of repayments, frequency of repayments and interest rate changes all have an impact on the time taken to repay a loan and the amount of interest that will be paid. We will use the Commonwealth Bank home loan calculator and the Moneysmart Mortgage Calculator to further explore these effects:
  - *https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments?ei=tools_repayments*
  - *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-can-i-repay-my-loan-sooner*

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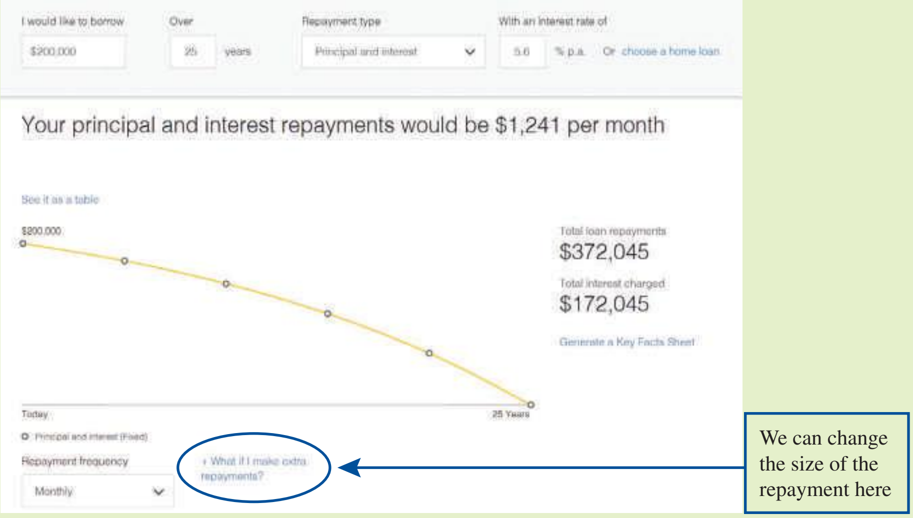
- For example: A $200 000 loan over 25 years at 5.6% p.a. reducible interest generated the following results for monthly repayments on the Commonwealth Bank online calculator.



I would like to borrow $200,000

Over 25 years

Repayment type Principal and interest

With an interest rate of 5.6 % p.a.

Your principal and interest repayments would be $1,241 per month

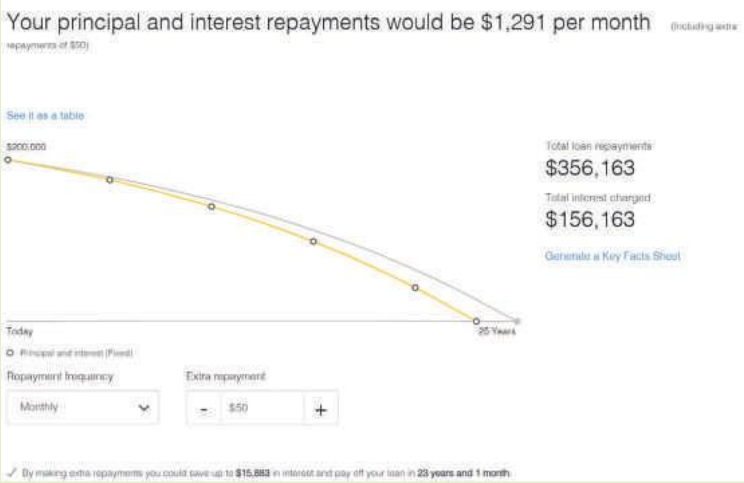
Total loan repayments $372,045

Total interest charged $172,045

Repayment frequency Monthly

We can change the size of the repayment here

  - If we pay an extra $50 per month, we would pay off the loan faster and make savings on the amount of interest we need to pay.



Your principal and interest repayments would be $1,291 per month (including extra repayments of $50)

Total loan repayments $356,163

Total interest charged $156,163

Repayment frequency Monthly

Extra repayment $50

✓ By making extra repayments you could save up to **$15,883** in interest and pay off your loan in **23 years and 1 month**

- A common method used to save time and money on a home loan is to pay half the monthly repayment each fortnight. The Moneysmart Mortgage Calculator has a tab ‘How can I repay my home loan sooner?’ for this calculation.
  - *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator*

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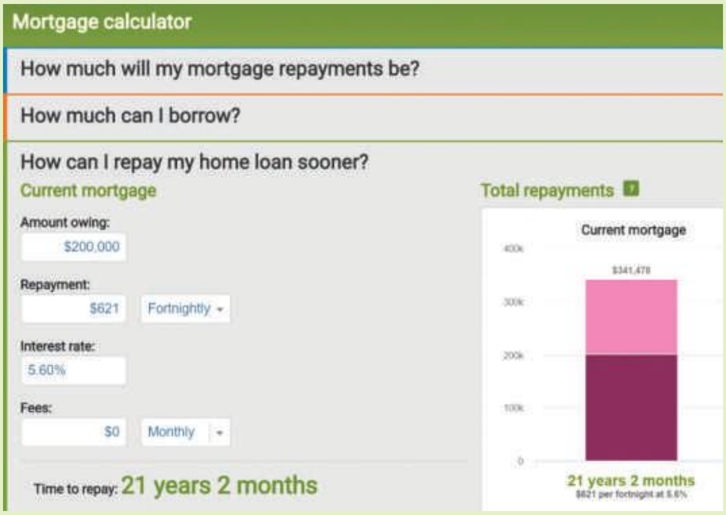
- For example: If half of the $1241 monthly repayment is paid each fortnight (Repayment $= 1241 \div 2 \approx \$621$), we can see that the loan has been paid off in 21 years and 2 months at a total cost of $341 478, or $30 567 less than our original loan cost!



Amount owing: $200,000

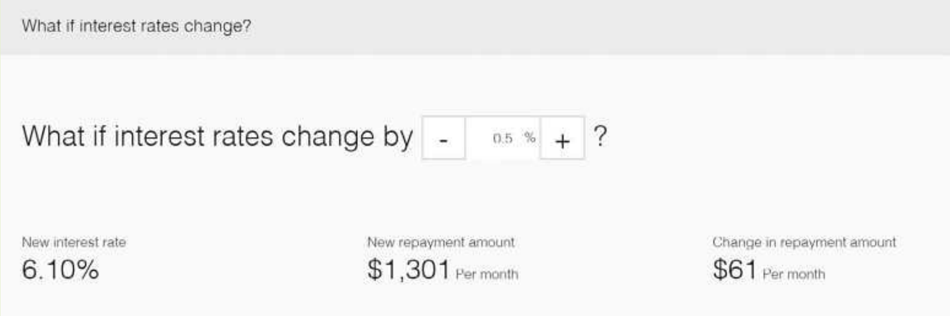
Repayment: $621 Fortnightly

Interest rate: 5.60%

Fees: $0 Monthly

Time to repay: 21 years 2 months

- Interest rates go up and down all the time. Rate changes can make a big difference to the size of the loan repayments and can affect the family budget. The Commonwealth Bank home loan calculator also covers this, so you can plan for worst case scenarios.



What if interest rates change by - 0.5 % + ?

New interest rate 6.10%

New repayment amount $1,301 Per month

Change in repayment amount $61 Per month

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Example 6 Using an online calculator to investigate the effect of the interest rate, payment frequency and repayment amount

Clayton borrows \$200 000 to buy a home. Bank interest rates are 5.95% p.a. reducible interest and he wants to pay his loan off in 25 years.

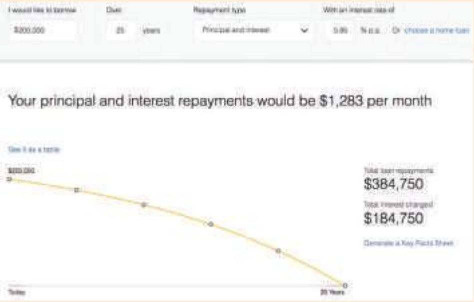
a Use the Commonwealth Bank home loan calculator, *https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments?ei=tools_repayments*, to determine:
  - **i** the monthly repayment
  - **ii** the interest charges if the loan is paid monthly

b Clayton has heard that he can save money by paying fortnightly. Determine:
  - **i** the fortnightly repayment
  - **ii** the savings over 25 years

c Clayton decides to pay an extra \$50 each fortnight. Calculate:
  - **i** the amount he would save in interest costs
  - **ii** the time he would save on his loan

d Clayton is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Clayton would need to pay each fortnight if rates go up 1% p.a.

WORKING	THINKING
a i Monthly repayment = \$1283 **ii** Total interest charges = \$184 750	Input values into the online calculator and read off the answers.



Your principal and interest repayments would be \$1,283 per month

\$384,750

\$184,750

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WORKING | **THINKING**

b **i** Fortnightly repayment = $592

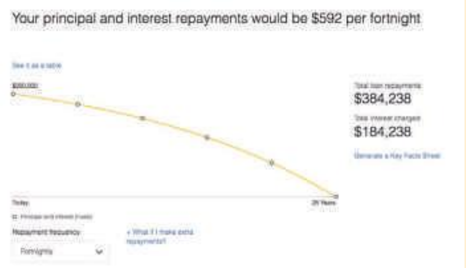
Total interest = $184 238

ii Savings = 184 750 – 184 238

= $512

Note: still takes 25 years to repay the loan.

Change the Repayment frequency to fortnightly (see bottom left of screen).



Your principal and interest repayments would be $592 per fortnight

$384,238

$184,238

c By paying an extra $50 per fortnight:

i Interest = $150 549

Savings = 184 750 – 150 549

= $34 201

ii Time to repay = 21 years 1 month

Saving = 25 years – 21 years 1 month

= 3 years 11 months

Add extra repayment below the graph.

Your principal and interest repayments would be $642 per fortnight

$350,549

$150,549

d If rates increase by 1%, Clayton would need to find another $58 each fortnight to repay his loan.

Scroll down the page and change the rate by 1% p.a.

What if interest rates change?

What if interest rates change by - 1 % + ?

6.95%

$700

$58

Example 7 Using an online calculator to investigate the effect of the interest rate, payment frequency and repayment amount

Hannah has borrowed $275 000 to buy a home. Bank interest rates are 5.49% p.a. reducible interest and she wants to pay her loan off in 25 years.
Use the Moneysmart Mortgage Calculator, *https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-much-will-my-repayments-be*.

a Determine:
- **i** the monthly repayment
- **ii** the interest charges if the loan is paid monthly

b Hannah has heard that she can save money by paying half the monthly repayment each fortnight.
- **i** Calculate her fortnightly repayment to the nearest dollar.
- **ii** Use the 'How can I repay my home loan sooner?' tab to determine the interest and time savings on her loan.

c Hannah is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Hannah would need to pay each fortnight if rates go up 0.5% p.a.

WORKING | **THINKING**

Mortgage calculator

How much will my mortgage repayments be?

Mortgage details

Amount borrowed: $275 000

Interest rate: 5.49%

Repayment frequency: Monthly

Length of loan: 25 years

Fees: $0 Monthly

Your repayments will be: $1,687 per month

Enter values in the Moneysmart Mortgage Calculator. Hover over the pink part of the bar to read off the total interest charges.

a **i** Monthly repayment = $1687
ii Total interest = $231 130

b **i** Fortnightly payment = 1687 ÷ 2
= 843.5 = $844

Round up and enter values in each tab.

ii Interest = $191 054
Savings = 231 130 – 191 054
= $40 076
Time to repay = 21 years 3 months

Saving = 25 years – 21 years 3 month
= 3 years 9 months

Mortgage calculator

How much will my mortgage repayments be?

How much can I borrow?

How can I repay my home loan sooner?

Current mortgage

Amount owing: $275 000

Repayment: $844 Fortnightly

Interest rate: 5.49%

Fees: $0 Monthly

Time to repay: 21 years 3 months

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WORKING	THINKING
c New rate $= 5.49 + 0.5$ $= 5.99\%$ p.a. New monthly repayment $= \$1770$ New fortnightly payment $= 1770 \div 2$ $= \$885$ Increase per fortnight $= 885 - 844$ $= \$41$ Hannah would need to pay an extra \$41 per fortnight to repay her loan.	Go back to ‘How much will my mortgage payments be?’ and change the interest rate before calculating the new fortnightly repayment and any changes.

Mortgage calculator

How much will my mortgage repayments be?

Mortgage details

Amount borrowed: $270,000

Interest rate: 5.99%

Repayment frequency: Monthly

Length of loan: 25 years

Fees: $0 Monthly

Your repayments will be: $1,770 per month

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Exercise 12C

FUNDAMENTALS

1 Determine the missing words in the following sentences.
Changes to the size of repayments, frequency of repayments and interest rate changes all have an _____ on the time taken to _____ a loan and the amount of _____ that will be paid.

APPLICATIONS

SF: 2–8	CF: –	CU: –

Use the Commonwealth Bank home loan calculator to answer the following questions: *https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments?ei=tools_repayments*

Example 6

2 Preston has borrowed \$287 000 to buy a home. Bank interest rates are 5.37% p.a. reducible interest and he wants to pay his loan off in 25 years.
- **a** Use the Commonwealth Bank home loan calculator to determine:
  - **i** the monthly repayment
  - **ii** the interest charges if the loan is paid monthly
- **b** Preston has heard that he can save money by paying fortnightly. Determine:
  - **i** his fortnightly repayment
  - **ii** the savings over 25 years
- **c** Preston decides to pay an extra \$50 each fortnight. Calculate:
  - **i** the amount he would save in interest costs
  - **ii** the time he would save on his loan
- **d** Preston is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Preston would need to pay each fortnight if rates go up 0.5% p.a.

3 Bella has borrowed \$185 000 to buy a townhouse. Bank interest rates are 6.24% p.a. reducible interest and she wants to pay her loan off in 20 years.
- **a** Use the Commonwealth Bank home loan calculator to find the following:
  - **i** the monthly repayment
  - **ii** the interest charges if the loan is monthly
- **b** Bella has heard that she can save money by paying fortnightly. Determine:
  - **i** her fortnightly repayment
  - **ii** the savings over 20 years

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- **c** Bella decides to pay an extra $40 each fortnight. Calculate:
  - **i** the amount she would save in interest costs
  - **ii** the time she would save on her loan
- **d** Bella has heard that interest rates may go down with a new government. Use the online calculator to determine how much less Bella would need to pay each fortnight if rates go down by 0.5% p.a.

Use the Moneysmart Mortgage Calculator to answer the following questions. *https://www.moneysmart.gov.au/tools-and-resources*

Example 7

4 Sianta has borrowed $180 000 to buy a home over 20 years with a reducible interest rate of 6.2% p.a.

- **a** Use the Moneysmart Mortgage Calculator to determine the size of her monthly repayment.
- **b** Calculate the amount of interest will she pay.
- **c** Interest rates drop to 6.05% p.a., calculate her new monthly repayment.
- **d** Calculate the amount of interest she will save because of the rate drop.

5 Michael has borrowed $40 000 to set up a small business over 5 years with a reducible interest rate of 8.2% p.a.

- **a** Use the Moneysmart Mortgage Calculator to determine the size of his monthly repayment.
- **b** Calculate the amount of interest will he pay.
- **c** Interest rates increase to 8.5% p.a., calculate his new monthly repayment.
- **d** Discuss the impact the new interest rate might have on Michael's small business.
- **e** Calculate the amount of extra interest he will need to pay because of the rate increase.

6 Botrus is investigating borrowing $265 000 to buy a home. The bank is offering 6.15% p.a. reducible rates and he thinks he will take 25 years to repay the loan.

- **a** Use the Moneysmart Mortgage Calculator to determine the size of his monthly repayment.
- **b** Determine the total cost of the loan over 25 years.
- **c** When Botrus applies for the loan, he is advised to pay half the monthly repayment each fortnight. Determine his fortnightly repayment.
- **d** Use the 'How can I repay my home loan sooner?' tab to determine how long it will take Botrus to repay his loan by paying half the monthly repayment each fortnight.
- **e** Calculate the total cost of the loan if he pays half the monthly repayment each fortnight.
- **f** Discuss the advice you would give Botrus about his home loan.

7 Maddie borrows $315 000 to buy a home. Bank interest rates are 6.19% p.a. reducible interest and she wants to pay her loan off in 25 years.

- **a** Use the Moneysmart Mortgage Calculator to determine:
  - **i** the monthly repayment
  - **ii** the interest charges if the loan is paid monthly

- **b** Maddie has heard that she can save money by paying half the monthly repayment each fortnight.
  - **i** Calculate her fortnightly repayment to the nearest dollar.
  - **ii** Use the 'How can I repay my home loan sooner?' tab to determine the interest and time savings on her loan.
- **c** Maddie is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Maddie would need to pay each fortnight if rates go up 0.5% p.a.

Hint The fortnightly payment will continue to be half of the monthly repayment.

8 Ben has borrowed $286 000 to buy a home. Bank interest rates are 5.99% p.a. reducible interest and he wants to pay his loan off in 20 years.

- **a** Use the Moneysmart Mortgage Calculator to determine:
  - **i** the monthly repayment
  - **ii** the interest charges if the loan is paid monthly
- **b** Ben is not sure if he can afford to pay the monthly loan repayment over 20 years. If he opts to pay over 25 years, calculate:
  - **i** the monthly repayment
  - **ii** the interest charges
  - **iii** the additional interest charges by taking longer to repay the loan
- **c** Ben wants to take advantage of the savings to be made by paying half the monthly repayment each fortnight.
  - **i** Calculate his fortnightly repayment to the nearest dollar.
  - **ii** Use the 'How can I repay my home loan sooner?' tab to determine the interest and time savings on his loan.
- **d** Ben has heard that interest rates may go down with a new government. Use the online calculator to determine how much less Ben would need to pay each fortnight if rates go down 0.25% p.a.

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12D Investigating the effect of changing interest rates, compounding periods and repayments using spreadsheets COMPLEX

LEARNING GOALS

- Investigate the effect of the interest rate and the number of compounding periods on the future value of a loan by using technology (spreadsheet)
- Investigate the effect of the interest rate and repayment amount on the time taken to repay a loan by using technology (spreadsheet)

Why is using technology to investigate changes to a loan essential?

As we saw in the previous section, when dealing with compound interest, changes to rates, compounding frequency and the amount regularly paid can influence the outcome of a loan. Using technology to investigate how you can make small changes to a loan has the potential to save you a lot of money.

Being well informed about loans has the potential to save you lots in interest charges.

WHAT YOU NEED TO KNOW

- Making changes to the interest rate and frequency of compounds will have an impact on interest charges, the size of repayments required to pay off a loan and the total cost of a loan.
- Making changes to the size of the repayment will change the length of time taken to repay the loan and the amount of interest charged.
- We can use spreadsheets in a similar way to what we have done in previous sections to determine the impact of changes to interest rates, compounding frequency and repayments.

Example 8 Calculating the impact of changes to interest on loans using a spreadsheet

Ned has recently taken out a $320 000 home loan. Interest rates are currently 5.7% p.a. and he is planning to repay the loan in 25 years.

a Use a spreadsheet (like the one used in section 12B) to determine the monthly repayment.

b Calculate the total amount he will repay.

c Calculate the total amount of interest he will pay.

d Interest rates are increased to 5.85% p.a., calculate his new monthly repayment.

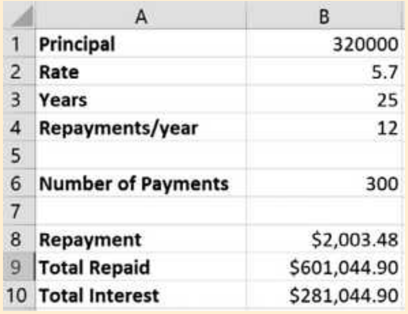
e Determine how much extra will he pay each month.

f Determine how much extra will he now pay in total for the loan.

g Discuss the impact of the increased interest rate.

WORKING | **THINKING**

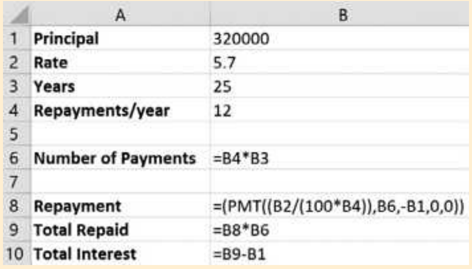
a



	A	B
1	Principal	320000
2	Rate	5.7
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	300
7		
8	Repayment	$2,003.48
9	Total Repaid	$601,044.90
10	Total Interest	$281,044.90

Monthly repayment = $2003.48
($2003 to nearest dollar)

Set up spreadsheet as done in section 12B, example 4.



	A	B
1	Principal	320000
2	Rate	5.7
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=(PMT((B2/(100*B4)),B6,-B1,0,0))
9	Total Repaid	=B8*B6
10	Total Interest	=B9-B1

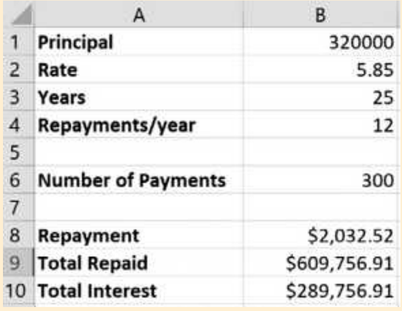
b Amount repaid = $601 044.90

Read from spreadsheet.

c Total interest = $281 044.90

Read from spreadsheet.

d



	A	B
1	Principal	320000
2	Rate	5.85
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	300
7		
8	Repayment	$2,032.52
9	Total Repaid	$609,756.91
10	Total Interest	$289,756.91

Adjust spreadsheet for new interest rate.

Monthly repayment = $2032.52
($2033 to nearest dollar)

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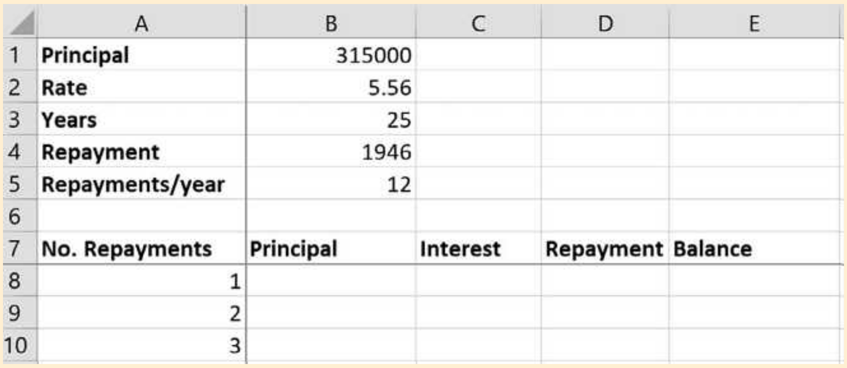
WORKING	THINKING
e Extra payment = 2033 – 2003 = $30 per month	Subtract monthly repayments.
f Extra paid in total = 609 756.91 – 601 044.90 = $8712.01	Subtract total paid.
g Extra interest paid = 289 756.91 – 281 044.90 = $8712.01	Subtract total interest.

Example 9 Investigating the effect of the repayment frequency on the time taken to repay a loan using a spreadsheet

Daniel borrows $315 000 to buy a home. Bank rates are 5.56% p.a. reducible interest and he will make monthly repayments over 25 years.

a Use the spreadsheet from Example 8 to find the size of the monthly repayment to the nearest dollar.

b Set up a new spreadsheet (like the one used in section 12B, example 5) to model his loan with the following headings:



	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	1946			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1				
9	2				
10	3				

c Use the spreadsheet to calculate the amount of interest he will pay over the 25 years.

d Daniel wants to pay half his monthly repayment each fortnight to save interest. Calculate the amount he will need to repay each fortnight.

e Change the repayment amount and the number of repayments per year to determine how long it would take to repay the loan if paid fortnightly.

f Use the spreadsheet to determine the total amount of interest if he pays half the monthly repayment each fortnight. Calculate how the amount he will save in interest and time.

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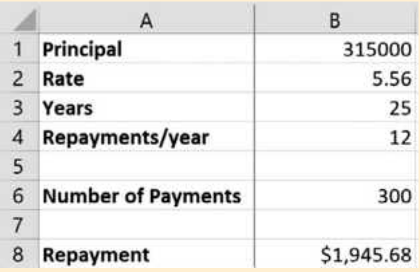
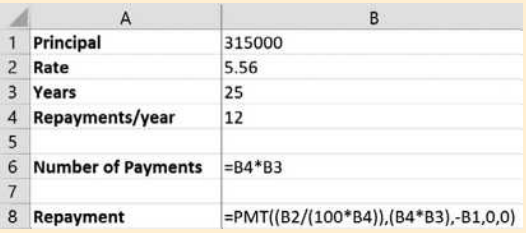
WORKING | **THINKING**

a The monthly repayment is $1946.



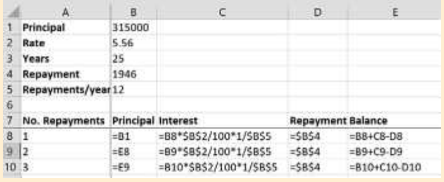
	A	B
1	Principal	315000
2	Rate	5.56
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	300
7		
8	Repayment	$1,945.68

Set up spreadsheet as in section 12B, example 5 to find the repayment.



	A	B
1	Principal	315000
2	Rate	5.56
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=PMT((B2/(100*B4)),(B4*B3),-B1,0,0)

b Set up a new spreadsheet as follows:



	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	1946			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	=B1	=B8*$B$2/100*1/$B$5	=$B$4	=B8+C8-D8
9	2	=E8	=B9*$B$2/100*1/$B$5	=$B$4	=B9+C9-D9
10	3	=E9	=B10*$B$2/100*1/$B$5	=$B$4	=B10+C10-D10

Interest = *Pin*

Using the dollar sign in the cell reference means we always use the interest value in cell B2 even when we fill down.

Cell B5 has the number of payments made per year. We want to use that to adjust time, *n*, in the interest calculation.

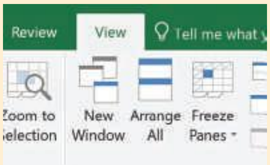
The repayment is in cell B4

Balance = principal + interest – repayment

c 25 years = 25 × 12 = 300 months

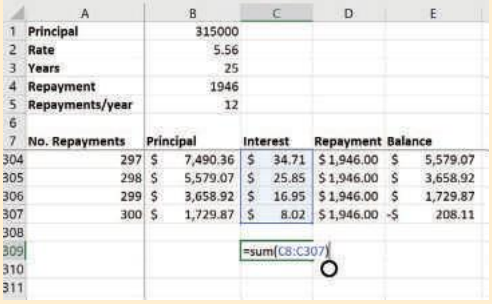
Total interest = $268 591.89

Tip: to continue to show the table headings when you scroll down. Click in cell B8>View>Freeze Panes



Fill the table down for 300 months to model the entire loan.

Type =SUM(C8:C307) into a cell below the interest column.



	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	1946			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
304	297	$ 7,490.36	$ 34.71	$ 1,946.00	$ 5,579.07
305	298	$ 5,579.07	$ 25.85	$ 1,946.00	$ 3,658.92
306	299	$ 3,658.92	$ 16.95	$ 1,946.00	$ 1,729.87
307	300	$ 1,729.87	$ 8.02	$ 1,946.00	-$ 208.11
308					
309			=sum(C8:C307)		
310					
311					

WORKING **THINKING**

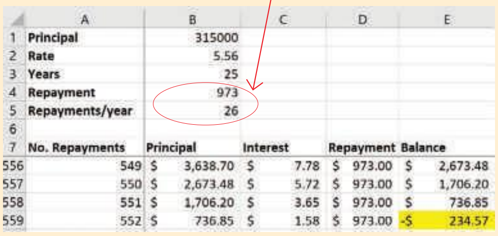
d Fortnightly repayment $= 1946 \div 2$
$= \$973$

Halve the monthly repayment.

e Repayment = \$973
Repayments/year = 26(fortnights)

Time to repay loan = 552 fortnights
$= 552 \div 26$
= 21.23 years
= 21 years 3 months

Change values at the top only and fill down until the balance falls below \$0 for the first time.



	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	973			
5	Repayments/year	26			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
556	549	\$ 3,638.70	\$ 7.78	\$ 973.00	\$ 2,673.48
557	550	\$ 2,673.48	\$ 5.72	\$ 973.00	\$ 1,706.20
558	551	\$ 1,706.20	\$ 3.65	\$ 973.00	\$ 736.85
559	552	\$ 736.85	\$ 1.58	\$ 973.00	-\$ 234.57

To change years from a decimal to months

$21.23 - 21 = 0.23$

$0.23 \times 12 = 2.76$ (or 3 months to nearest month) so

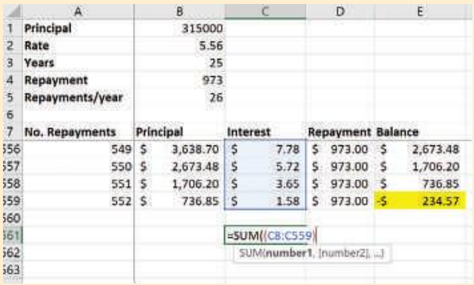
21.23 years = 21 years 3 months

f Interest = \$221861.43
Saving = 268 591.89 − 221861.43
= \$46 730.46

Time saving = 25 years
− 21 years 3 months
= 3 years 9 months

By paying half the monthly payment each fortnight, Daniel repays the loan 3 years and 9 months sooner and saves \$46 730.46 in interest.

Type =SUM(C8:559) into a cell below the interest column to find total interest.



	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	973			
5	Repayments/year	26			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
556	549	\$ 3,638.70	\$ 7.78	\$ 973.00	\$ 2,673.48
557	550	\$ 2,673.48	\$ 5.72	\$ 973.00	\$ 1,706.20
558	551	\$ 1,706.20	\$ 3.65	\$ 973.00	\$ 736.85
559	552	\$ 736.85	\$ 1.58	\$ 973.00	-\$ 234.57
560					
561			=SUM((C8:C559)		
562			SUM(number1, [number2], ...)		
563					

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Exercise 12D

FUNDAMENTALS

1 Determine the missing words in the following sentences.

- **a** Making changes to the ________ rate and ________ of compounds will have an ________ on the interest charges, size of repayments required to ________ off a loan and the total ________ of a loan.
- **b** Making ________ to the size of the repayment will change the ________ of time taken to repay the loan and the amount of ________ charged.

2 Complete the following conversions.

- **a** 1 year = ________ months
- **b** 1 year = ________ weeks
- **c** 1 year = ________ days
- **d** 1 year = ________ fortnights
- **e** 1 year = ________ 6 months
- **f** 1 year = ________ 4 weeks
- **g** 1 year = ________ quarters

APPLICATIONS

SF: –	CF: 3–5	CU: –

Spreadsheet Example 8 applies to question 3.

Example 8

3 Primrose borrows $296 000 to buy a home. She agrees to repay the loan in monthly repayments over 25 years. Reducible interest is calculated at 6.35% p.a.

- **a** Use a spreadsheet to calculate the:
  - **i** size of her monthly repayment
  - **ii** total amount repaid
  - **iii** total interest paid
- **b** Another bank offers her a loan with only 6.25% p.a. interest. Adjust your spreadsheet to calculate:
  - **i** size of her monthly repayment
  - **ii** total amount repaid
  - **iii** total interest paid
- **c** Calculate the savings to be made by taking the loan with the smaller interest rate.

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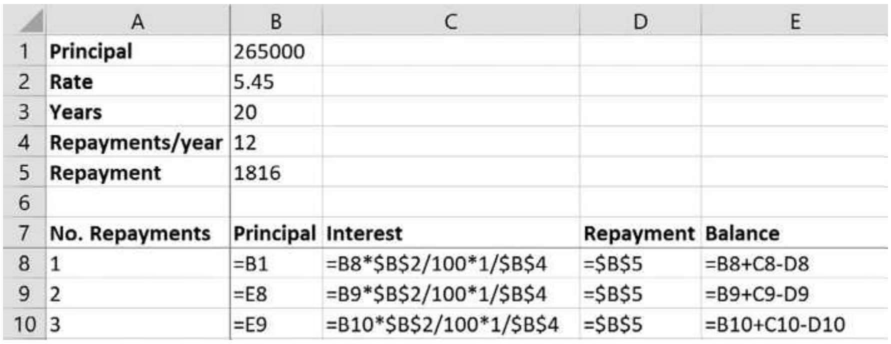
Spreadsheet Example 9 applies to the following questions.

Example 9

4 Jewel borrows $265 000 to buy a home. Bank interest rates are 5.45% p.a. reducible interest and she will repay the loan with monthly repayments over 20 years.

a Use a spreadsheet to determine the size of the monthly repayment. Round UP to the next dollar.

b Set up a new spreadsheet to model her loan with the following headings:



	A	B	C	D	E
1	**Principal**	265000			
2	**Rate**	5.45			
3	**Years**	20			
4	**Repayments/year**	12			
5	**Repayment**	1816			
6					
7	**No. Repayments**	**Principal**	**Interest**	**Repayment**	**Balance**
8	1	=B1	=B8*$B$2/100*1/$B$4	=$B$5	=B8+C8-D8
9	2	=E8	=B9*$B$2/100*1/$B$4	=$B$5	=B9+C9-D9
10	3	=E9	=B10*$B$2/100*1/$B$4	=$B$5	=B10+C10-D10

c Fill down until the loan is repaid. Use the spreadsheet to calculate how much she will pay in interest.

d Jewell wants to pay half her monthly repayment each fortnight to save interest. Calculate how much will she need to repay each fortnight.

e Change the repayment amount and the number of repayments per year to determine how long it would take to repay the loan if paid fortnightly.

f **i** Use the spreadsheet to calculate the total amount of interest if she pays half the monthly repayment each fortnight.

ii Determine how much she will save in interest and time.

5 Lachlan borrows $305 000 to buy a home. Bank interest rates are 5.16% p.a. reducible interest and he will repay the loan with monthly repayments for 20 years.

a Use a spreadsheet to calculate the size of the monthly repayment. Round UP to the next dollar.

b Set up a new spreadsheet to model his loan with the same headings used in question **4b**. Fill down until the loan is repaid.

c Use the spreadsheet to calculate how much he will pay in interest.

d Lachlan wants to pay $40 extra each month. Determine how long it will take him now to pay off the loan.

e **i** Use the spreadsheet to calculate the total amount of interest if he repays an extra $40 each month.

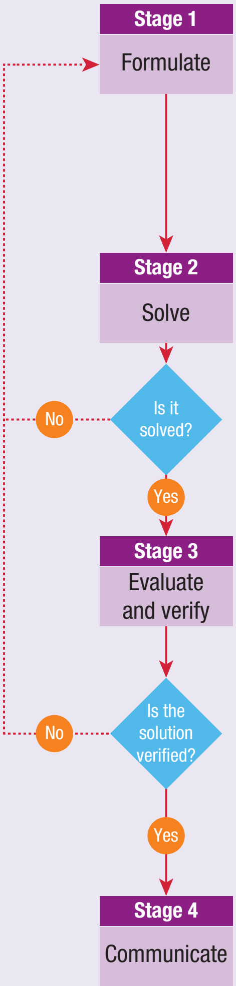
ii Determine how much he will save in interest and time.

Problem-solving and modelling task

Background: The great Australian dream is to own your own home. But is it really the best financial decision? Are you better off renting and saving the extra money you would have spent on rates, insurance and home maintenance?

Task: Compare the costs of buying and maintaining your own home over 10 years against the costs of renting a similar property over the same time and having a regular savings plan. After 10 years, the house will have increased in value but so will the renters' savings. Who is better off?

Approach to problem-solving and modelling task:



Stage 1: Formulate

1 What are you required to do?
2 What information do you have?
3 What other information do you need?
4 How could you gather more information?
5 What assumptions will you make?

Stage 2: Solve

6 Gather more information.
7 Use mathematics from Chapters 11 and 12 to calculate information required to solve the problem
8 Produce graphs/tables required to solve the problem.
9 Have you been able to make a decision?

Stage 3: Evaluate and verify

10 Have you answered the question?
11 Is the answer you have reasonable?

Stage 4: Communicate

12 State your main point.
13 Include the evidence in the form of statements, graphs and tables.
14 Explain the evidence. Use a sentence starter like 'This means …'
15 In the conclusion, link back to your main point.

Chapter checklist

☐ I understand that reducing balance loans are compound interest loans with periodic repayments.

1 Brooke borrowed $16 000 to buy a car. The terms of her loan were 11.56% p.a. reducible interest over two years and her monthly repayments are $750 per month.

a Complete this table to determine how much Brooke will owe on her loan after the first three months.

Month	Principal	Interest	Repay	Balance
1				
2				
3				

b Determine how much interest Brooke paid in the first month.

c Calculate how much Brooke reduced her debt by in the first month.

d Determine how much interest Brooke paid in the third month.

e Calculate how much Brooke reduced her debt by in the third month.

f Interpret your answers to parts **c** and **e**.

☐ I can use technology (spreadsheet) to model a reducing balance loan. **[complex]**

2 Lachlan borrows $23 000 to buy a car. He repays the loan with monthly repayments of $596 over four years with 11.15% p.a. reducible interest.

a Set up a spreadsheet to model the loan schedule over four years.

b Determine how much is left owing after 48 months.

c Calculate the size of the last repayment to fix the underpayment.

d Calculate the total loan repayments.

e Calculate the total interest charges.

f Determine how many months will it take to pay off half the loan.

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☐ I can use technology (online calculator) to model a reducing balance loan.

3 Megan borrows $24 000 to buy a car. Interest is calculated at 10.85% p.a. on the reducing balance (compound interest) and she will repay the loan over four years. Use an online calculator to calculate:

- **a** the monthly repayment required
- **b** the total repaid
- **c** the total interest

☐ I can compare simple and compound interest loans.

4 Joe borrows $24 000 to buy a car. Simple interest is calculated at 10.85% p.a. and he will repay the loan over four years. Calculate:

- **a** the total interest
- **b** the total to repay
- **c** the number of repayments
- **d** how much he will need to repay each month
- **e** the difference in monthly repayments and total cost of Joe's simple interest loan and Megan's compound interest loan in question **3**.

☐ I can use technology (online calculator) to investigate the effect of the interest rate and repayment amount on the time taken to repay a loan.

5 Isabelle borrows $213 000 to buy a townhouse. Bank interest rates are 5.82% p.a. reducible interest and she wants to pay her loan off in 20 years.

- **a** Use the Commonwealth Bank home loan Calculator to determine:
  - **i** her monthly repayment
  - **ii** interest charges if she pays her loan monthly
- **b** Isabelle decides to pay an extra $50 each month:
  - **i** determine how much she would save in interest costs
  - **ii** calculate how much time will she save on her loan
- **c** Isabelle has heard that interest rates may go down with a new government. Use the online calculator to determine how much less she would need to pay each month if rates go down by 0.5% p.a.

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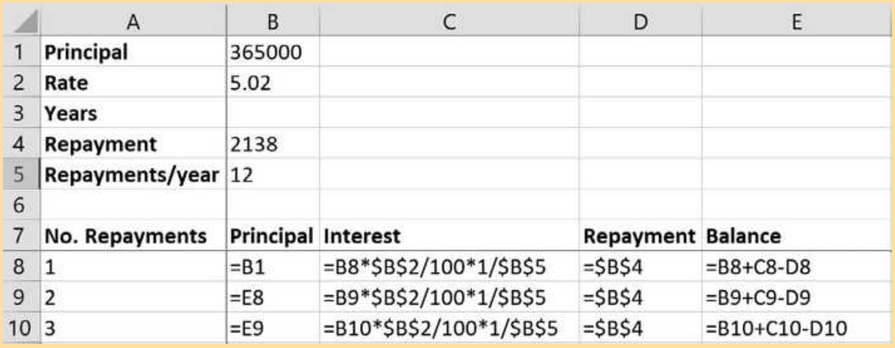
☐ I can use technology (spreadsheet) to investigate the effect of the interest rate and repayment amount on the time taken to repay a loan. **[complex]**

6 Primrose borrows $365 000 to buy a home. Bank interest rates are 5.02% p.a. reducible interest and her monthly repayments will be $2138.

a Set up a spreadsheet to model her loan with the following headings:



	A	B	C	D	E
1	**Principal**	365000			
2	**Rate**	5.02			
3	**Years**				
4	**Repayment**	2138			
5	**Repayments/year**	12			
6					
7	**No. Repayments**	**Principal**	**Interest**	**Repayment**	**Balance**
8	1	=B1	=B8*$B$2/100*1/$B$5	=$B$4	=B8+C8-D8
9	2	=E8	=B9*$B$2/100*1/$B$5	=$B$4	=B9+C9-D9
10	3	=E9	=B10*$B$2/100*1/$B$5	=$B$4	=B10+C10-D10

b Fill the spreadsheet down to determine how long it will take Primrose to repay the loan.

c Use the spreadsheet to calculate how much she will pay in interest.

d Primrose wants to pay half her monthly repayment each fortnight to save interest. Calculate how much she will need to repay each fortnight.

e Change the repayment amount and the number of repayments per year to determine how long it would take to repay the loan if she paid half the monthly amount each fortnight.

f Use the spreadsheet to find the total amount of interest if she pays half the monthly repayment each fortnight. Calculate how much will she save in interest and time.

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Chapter review

All questions in the review are assessment-style.

Simple familiar

Section 12A

1 Betty borrowed $3200 to purchase a new laptop. She agreed to pay off the loan in 12 months with reducible interest calculated at 9.85% p.a. Her monthly repayments are $281.00. Betty is given a table of her loan schedule as follows.

Month	Principal	Interest	Repayment	Balance
1	$3200.00	$26.27	$ 281.00	$2945.27
2	$2945.27	$24.18	$ 281.00	$2688.44
3	$2688.44	$22.07	$ 281.00	$2429.51
4	$2429.51	$19.94	$ 281.00	$2168.45
5	$2168.45	$17.80	$ 281.00	$1905.25
6	$1905.25	$15.64	$ 281.00	$1639.89
7	$1639.89	$13.46	$ 281.00	$1372.35
8	$1372.35	$11.26	$ 281.00	$1102.62
9	$1102.62	$ 9.05	$ 281.00	$ 830.67
10	$ 830.67	$ 6.82	$ 281.00	$ 556.49
11	$ 556.49	$ 4.57	$ 281.00	$ 280.05
12	$ 280.05	$ 2.30	$ 281.00	$ 1.35

a Determine if Betty will completely pay off the loan in 12 months. Explain why this will happen.
b Calculate how much she should pay in the last month to completely pay off the loan.
c Determine how much Betty still owes on her loan after 6 months.
d Determine how much interest Betty paid in the first month.
e Calculate how much Betty reduced her debt by in the first month.
f Determine how much interest Betty paid in the fourth month.
g Calculate how much Betty reduced her debt by in the fourth month.
h Describe the difference in your answers to parts **e** and **g**.
i Explain why the interest reduces each month.
j Calculate how much interest Betty paid on the total loan.
k Calculate 9.85% of $3200. Explain why this is different to your answer in part **j**.

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2 Braydon repays \$512 each month for four years on his \$19 500 loan to buy a car.

- **a** Determine how many repayments he will make.
- **b** Calculate how much he repaid in total.
- **c** Calculate how much he paid in interest.

3 Joseph repays \$1915 each month for twenty-five years on his \$285 000 home loan.

- **a** Determine how many repayments he will make.
- **b** Calculate how much he repays in total.
- **c** Calculate how much he paid in interest.

4 Harry borrows \$2800 for a new computer. Simple interest is charged at 11.8% p.a. and he will repay the loan over 2 years in equal monthly repayments. Calculate how much he will need to repay each month.

5 Jasper borrowed \$18 500 to buy a car. The terms of his loan were 11.12% p.a. reducible interest over four years and his monthly repayments are \$479 per month.

- **a** Complete this table to determine how much Jasper will owe on his loan after the first three months.

Month	Principal	Interest	Repay	Balance
1				
2				
3				

- **b** Calculate the total of Jasper's monthly repayments over the first three months.
- **c** Calculate the total of the interest charges over the first three months.
- **d** Determine the difference between the amount repaid and the interest charges over the first three months.
- **e** Determine how much Jasper reduced his debt by in the first three months.

Note: The following questions require technology beyond a scientific calculator and are not considered exam style questions. They do, however, cover the syllabus content for this topic.

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6 Libby borrows $318 000 to buy a home. She repays the loan monthly over 25 years with 5.58% p.a. reducible interest. Use an online calculator from a bank website to calculate the:

- **a** size of the monthly repayments
- **b** total loan repayments
- **c** total interest charges
- **d** amount left owing after 20 years
- **e** number of years it takes to pay off half the loan

Section 12C

7 Nick borrows $247 000 to buy a home. Bank interest rates are 5.63% p.a. reducible interest and he wants to pay his loan off in 25 years.

- **a** Use the Commonwealth Bank home loan calculator to calculate:
  - **i** his monthly repayment
  - **ii** the interest charges if he pays his loan monthly
- **b** Nick has heard there are savings to be made by paying fortnightly, calculate his fortnightly repayment and savings over 25 years.
- **c** Nick decides to pay an extra $50 each fortnight, calculate:
  - **i** how much he would save in interest costs
  - **ii** how much time he would save on his loan
- **d** Nick is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Nick would need to pay each fortnight if rates go up 1% p.a.

8 Frank borrows $386 000 to buy a home. Bank interest rates are 5.17% p.a. reducible interest and he wants to pay his loan off in 20 years.

- **a** Use the Moneysmart Mortgage Calculator to determine:
  - **i** his monthly repayment
  - **ii** the interest charges if he pays his loan monthly
- **b** Frank is not sure he can afford to pay the monthly loan repayment over 20 years. If he opts to pay over 25 years, calculate:
  - **i** his monthly repayment
  - **ii** the interest charges
  - **iii** his additional interest charges by taking longer to repay the loan
- **c** Frank wants to take advantage of the savings to be made by paying half the monthly repayment each fortnight.
  - **i** Calculate his fortnightly repayment to the nearest dollar.
  - **ii** Use the ‘How can I repay my home loan sooner?’ tab to determine interest and time savings on his loan.

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d Frank has heard that interest rates may go down with a new government. Use the online calculator to determine how much less Frank would need to pay each fortnight if rates go down 0.15% p.a.

Complex familiar

Section 12B

9 Ben borrows $31 000 to buy a car. He repays the loan with monthly repayments of $790 over 4 years with 10.25% p.a. reducible interest.

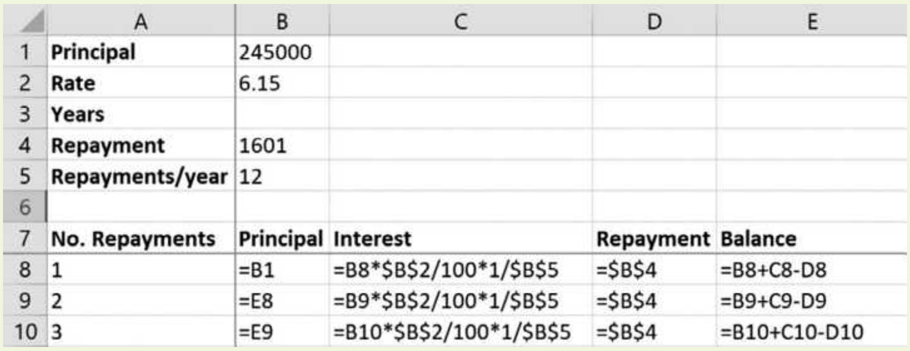
- **a** Set up a spreadsheet to model the loan schedule over four years.
- **b** Determine how much is left owing after 48 months.
- **c** Calculate the size of the last repayment to fix the overpayment.
- **d** Calculate the total loan repayments.
- **e** Calculate the total interest charges.
- **f** Determine how many months it will take to pay off half the loan.

Complex unfamiliar

Section 12D

10 Harry borrows $245 000 to buy a home. Bank interest rates are 6.15% p.a. reducible interest and his monthly repayments will be $1601.

- **a** Set up a spreadsheet to model his loan with the following headings:



	A	B	C	D	E
1	**Principal**	245000			
2	**Rate**	6.15			
3	**Years**				
4	**Repayment**	1601			
5	**Repayments/year**	12			
6					
7	**No. Repayments**	**Principal**	**Interest**	**Repayment**	**Balance**
8	1	=B1	=B8*$B$2/100*1/$B$5	=$B$4	=B8+C8-D8
9	2	=E8	=B9*$B$2/100*1/$B$5	=$B$4	=B9+C9-D9
10	3	=E9	=B10*$B$2/100*1/$B$5	=$B$4	=B10+C10-D10

- **b** Use it to determine how long it will take Harry to repay the loan.
- **c** Use the spreadsheet to calculate how much he will pay in interest.
- **d** Harry wants to pay half his monthly repayment each fortnight to save interest. Calculate how much will he need to repay each fortnight.
- **e** Use the spreadsheet to calculate how long it would take to repay the loan if paid fortnightly.
- **f** Use the spreadsheet to calculate the total amount of interest if he pays half the monthly repayment each fortnight. Determine how much he will save in interest and time.

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11 Tom borrows \$335 000 to buy a home. Bank interest rates are 6.20% p.a. reducible interest and his monthly repayments will be \$2200.

- **a** Set up a spreadsheet to model his loan with the same headings used in question **10**.
- **b** Use the spreadsheet to determine how long it will take Tom to repay the loan.
- **c** Use the spreadsheet to calculate how much he will pay in interest.
- **d** Tom wants to pay \$50 extra each month. Use the spreadsheet to determine how long it will take him now to pay off the loan.
- **e** Use the spreadsheet to calculate the total amount of interest if he repays an extra \$50 each month. Determine how much will he save in interest and time.

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Formula sheet

Please note: this is not an official formula sheet and it should not be used for assessment.

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a + b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$	arc length	$l = \frac{\theta}{180}\pi r$
area of a sector	$A = \frac{\theta}{360}\pi r^2$		

Finance			
simple interest	$I = Pin$	compound interest	$A = P(1 + \mathrm{i})^n$

Trigonometry			
Pythagoras' theorem	$c^2 = a^2 + b^2$		
trigonometric ratios	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

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Location and time					
distance	$d = s \times t$	speed	$s = \frac{d}{t}$	time	$t = \frac{d}{s}$

Statistics	
mean	$\bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$
range	range = highest score – lowest score
interquartile range (IQR)	$\text{IQR} = Q_3 - Q_1$

Conversions

length unit conversion

$$\begin{tikzcd}
\boxed{\text{km}} \arrow[r, bend left, "\times1000"] & \boxed{\text{m}} \arrow[l, bend left, "\div1000"] \arrow[r, bend left, "\times100"] & \boxed{\text{cm}} \arrow[l, bend left, "\div100"] \arrow[r, bend left, "\times10"] & \boxed{\text{mm}} \arrow[l, bend left, "\div10"]
\end{tikzcd}$$

area unit conversion

$$\begin{tikzcd}
\boxed{\text{km}^2} \arrow[r, bend left, "\times1000^2"] & \boxed{\text{m}^2} \arrow[l, bend left, "\div1000^2"] \arrow[r, bend left, "\times100^2"] & \boxed{\text{cm}^2} \arrow[l, bend left, "\div100^2"] \arrow[r, bend left, "\times10^2"] & \boxed{\text{mm}^2} \arrow[l, bend left, "\div10^2"]
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{\text{km}^2} \arrow[r, bend left, "\times100"] & \boxed{\text{ha}} \arrow[l, bend left, "\div100"] \arrow[r, bend left, "\times100^2"] & \boxed{\text{m}^2} \arrow[l, bend left, "\div100^2"]
\end{tikzcd}$$

volume unit conversion

$$\begin{tikzcd}
\boxed{\text{km}^2} \arrow[r, bend left, "\times1000^3"] & \boxed{\text{m}^2} \arrow[l, bend left, "\div1000^3"] \arrow[r, bend left, "\times100^3"] & \boxed{\text{cm}^2} \arrow[l, bend left, "\div100^3"] \arrow[r, bend left, "\times10^3"] & \boxed{\text{mm}^2} \arrow[l, bend left, "\div10^3"]
\end{tikzcd}$$

mass unit conversion

$$\begin{tikzcd}
\boxed{\text{tonne}} \arrow[r, bend left, "\times1000"] & \boxed{\text{kg}} \arrow[l, bend left, "\div1000"] \arrow[r, bend left, "\times1000"] & \boxed{\text{g}} \arrow[l, bend left, "\div1000"] \arrow[r, bend left, "\times1000"] & \boxed{\text{mg}} \arrow[l, bend left, "\div1000"]
\end{tikzcd}$$

capacity unit conversion

$$\begin{tikzcd}
\boxed{\text{ML}} \arrow[r, bend left, "\times1000"] & \boxed{\text{kL}} \arrow[l, bend left, "\div1000"] \arrow[r, bend left, "\times1000"] & \boxed{\text{L}} \arrow[l, bend left, "\div1000"] \arrow[r, bend left, "\times1000"] & \boxed{\text{mL}} \arrow[l, bend left, "\div1000"]
\end{tikzcd}$$

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Answers

Chapter 1

Pre-test

1 **a** 4 sides **b** 4 corners
  c 12 edges **d** 8 corners
2 **a** Degrees **b** 3
3 **a** Angle *B* **b** 90 degrees
4 Lines *A* and *C*
5 **a** **i** 6 faces **ii** 6 rectangles
  b **i** 0 faces **ii** 2 circles, 1 curved rectangle

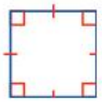
Exercise 1A

1 **a** hexagon **b** square
  c triangle **d** rhombus
  e rectangle **f** trapezium
  g pentagon **h** octagon
  i parallelogram

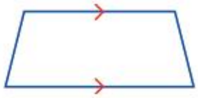
2

Property	Square	Rectangle	Parallelogram	Trapezium	Triangle	Rhombus	Kite
Four sides	✓	✓	✓	✓		✓	✓
Three sides					✓		
All sides are equal length	✓					✓	
All angles measure 90°	✓	✓					
Not all angles measure 90°			✓	✓	✓	✓	✓
Two pairs of parallel sides only	✓	✓	✓			✓	
Two pairs of parallel sides of different lengths		✓	✓				

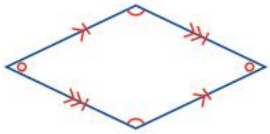
3 **a** **i** Square
  ii



  iii All 4 sides equal, all 4 angles 90°, opposite sides parallel
b **i** Trapezium
  ii



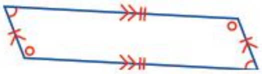
  iii 4 sides, one pair of parallel sides
c **i** Rhombus
  ii



  iii All 4 sides equal, opposite sides parallel, opposite angles equal

d i Parallelogram

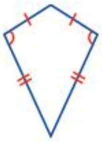
ii



iii 4 sides, opposite sides parallel, opposite sides equal, opposite angles equal

e i Kite

ii



iii 4 sides, adjacent sides equal, one part of opposite angles equal

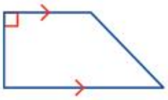
f i Rectangle

ii

iii 4 sides, all 4 angles 90°, opposite sides parallel, opposite sides equal

g i Trapezium

ii



iii 4 sides, one pair of parallel sides
Note: the right angle in this case is not significant.

h i Rectangle

ii

iii 4 sides, all 4 angles 90°, opposite sides parallel, opposite sides equal

i i Square

ii

iii All 4 sides equal, all 4 angles 90°, opposite sides parallel

4 a cylinder

b cone

c cube

d rectangular prism

e triangular pyramid (tetrahedron)

f square-based pyramid

g triangular prism

5 a face

b corners

c edge

6

Name	Faces	Edges	Vertices
Rectangular prism or cube	6	12	8
Triangular prism	5	9	6
Square and rectangular-based pyramid	5	8	5
Triangular-based pyramid (tetrahedron)	4	6	4
Pentagonal prism	7	15	10
Hexagonal prism	8	18	12

7 Shapes in picture: circles, triangles, rectangles, trapeziums, squares and semi-circles

8 Triangles and trapeziums

9 Rectangle – living area, dining room, table, kitchen benches, master bedroom, bed, cinema and garage
Circle – 3 gardens
Trapezium – patio, driveway and linen
Triangle – storage
Other answers are possible.

10 a Squares, rectangles and trapeziums

b Rectangles

c The left side of the roof is a square (7 m × 7 m), while the other is a rectangle (5 m × 7m).

11 Circles, rhombuses, triangles, trapeziums and rectangles

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12 a, b, c

Shape	Rectangle	Parallelogram	Trapezium
House numbers	106–118, 135–137, 143–154	127–133, 138–142, 204–208, 211–247, 306–343	101–105, 119–126, 134, 201–203, 209, 210, 301–305

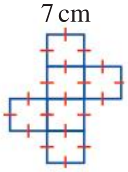
13 a Cone and spheres
  b Tetrahedron
  c Cubes
  d Triangular prism
  e Rectangular prism
  f Cube
  g Rectangular prism
  h Cylinder
  i Triangular prism

14 Square-based pyramid

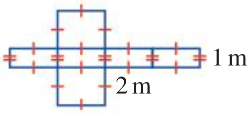
Exercise 1B

1 a B **b** C **c** B
  d A **e** B

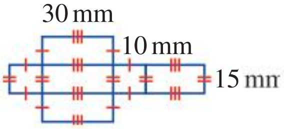
2 a



b

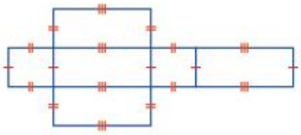


c

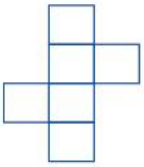


3 a Glass box, toilet roll
  b Phone, tool box
  c Ice cubes, dice
  d Chocolate, calendar
  e Ice cream cone
  f Clock

4

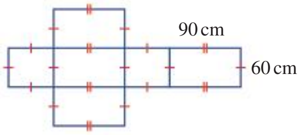


5 a



  b He would need to fold the metal net 5 times.

6



Exercise 1C

1 a 500 cm **b** 70 mm
  c 2 cm **d** 2000 m
  e 370 cm **f** 1700 m
  g 4.9 m **h** 7000 mm
  i 1 200 000 mm **j** 0.03 km

2 a 3 cm **b** 5 m **c** 25 mm
  d 2.5 km **e** 500 cm **f** 1700 m

3 Any sensible response will be acceptable.

4 a m **b** mm **c** mm
  d cm **e** km **f** mm
  g cm **h** m **i** km
  j km

5 5.86 m

6 4500 m

7 4200 mm

8 1.8 m

9 15 m

10 Diamond – 6 mm
Mobile – 12 cm
Gum tree – 32 m
Skyscraper – 0.1 km
Brisbane to Cairns – 1700 km

11 a 5 cm **b** 1.75 cm **c** 10 cm

12 4 m

Exercise 1D

1 a 40 cm **b** 66 cm **c** 200 mm
  d 6 m **e** 17.1 m **f** 630 mm
  g 18 km **h** 36 cm **i** 24 mm

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2 **a** 56 m **b** 320 mm
c 4010 m **d** 64 cm
3 **a** 6.28 m **b** 2513.27 mm
c 125.66 km **d** 188.50 cm
4 **a** 59.34 cm **b** 0.35 m
c 1483.53 mm **d** 3.14 km
5 **a** 139.34 cm **b** 2.35 m
c 2163.53 mm **d** 5.14 km
6 80 cm
7 300 m
8 119.38 cm
9 1.27 m
10 11.24 m
11 **a** 85.6 m **b** 256.8 m
12 5.89 cm

Exercise 1E

1 **a** 40 cm **b** 66 mm **c** 6 m
d 9.42 km **e** 12.85 m **f** 37 km
2 **a** 48 cm **b** 480 mm **c** 20.28 m
d 14.6 km **e** 200 m **f** 85.42 mm
g 37.70 cm **h** 7.14 km **i** 101.56 mm
3 56 m
4 57.4 m
5 384.22 m
6 1.96 m
7 863.79 cm
8 66.6 m
9 11.94 m

Chapter checklist

1 Square: All 4 sides equal, all 4 angles 90°, opposite sides parallel
Rectangle: 4 sides, all 4 angles 90°, opposite sides parallel, opposite sides equal
Triangle: 3 sides, 3 vertices

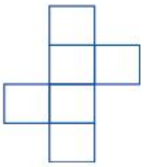
2 Rectangles, squares, trapeziums, triangles, hexagons, parallelograms

3

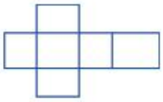
Name	Faces	Edges	Vertices
Cube	6	12	8
Triangular prism	5	9	6
Square and rectangular-based pyramid	5	8	5
Triangular-based pyramid (tetrahedron)	4	6	4
Pentagonal prism	7	15	10
Hexagonal prism	8	18	12

4 Triangular-based pyramid, cube, square prism, triangular prism, cylinder

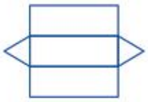
5 **a**



b



c



6 **a** cm **b** m
c mm **d** km
7 Using rulers, estimation
8 **a** iii **b** ii
c i **d** iii
9 **a** 40 mm **b** 3 cm
c 4500 m **d** 730 cm
10 **a** 13.7 cm **b** 7 m **c** 350 mm
11 **a** 23.25 m **b** 238.76 mm
12 **a** **i** 108.38 m **ii** 163.58 m
b **i** 170.87 cm **ii** 348.87 cm
13 **a** 11 m **b** 400 cm **c** 799.91 mm

Chapter review

1 **a** Triangles – 4, rectangle – 3, trapezium – 1
b Triangles – bracing
Properties: 3 sides, 3 vertices
Rectangles – a door, roller door and the basic shape of the entire shed

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Properties: 4 sides, 4 vertices, all angles are 90°, two pairs of parallel sides, opposite sides are equal
Trapezium – patio
Properties: 4 sides, 4 vertices, one pair of parallel sides

2 **a** Triangular prism
b Cube
c Rectangular prism
d Cylinder
e Cone
3 **a** 3800 cm **b** 150 mm
c 4.4 cm **d** 179 cm
4 1.8 m
5 28 m
6 **a** iii **b** ii
c ii **d** i
7 21 m
8 19 m
9 31.14 m

Chapter 2

Pre-test

1 3440 mm, 3.44 m, 0.00344 km
2 120
3 **a** 56 **b** 7
4 **a** 1 **b** 81 **c** 100
d 10 000 **e** 1 000 000 **f** 169
5 **a** 4 units **b** 4.5 units
6 Pi is the symbol and it represents the ratio of the circumference of a circle to its diameter (e.g. for every 1 unit of diameter in a circle there will be 3.14 units of circumference).
7 **a** Right angle
b Angle measure
c Degrees
8 **a** **i** All equal sides
ii Square
b **i** Opposite sides equal
ii Rectangle
c **i** Parallel lines
ii Paralellogram

Exercise 2A

1 **a** multiplication, smaller
b smaller, larger
c 10 000
d 0.0001
2 **a** 50 000 cm^2 **b** 700 mm^2
c 2 cm^2 **d** 2 000 000 m^2
e 500 ha **f** 37 000 cm^2
g 1 700 000 m^2 **h** 0.049 m^2
i 0.0527 ha **j** 0.05 cm^2
k 0.0005 km^2 **l** 375 ha
3 **a** 7 000 000 mm^2
b 4 000 000 000 cm^2
c 0.0028 m^2
d 1 200 000 000 000 mm^2
e 0.0000003 km^2
4 0.025 km^2
5 15 826 000 000 m^2
6 1900 mm^2
7 0.16 m^2
8 3.236 ha
9 0.92 m^2
10 7.14 m^2
11 18 000 mm^2

Exercise 2B

1 **a** 20.25 cm^2 **b** 7 m^2
c 11 040 mm^2 **d** 272 cm^2
e 22.90 m^2 **f** 51 710.76 mm^2
g 209 cm^2 **h** 24.96 m^2
i 16 200 mm^2 **j** 804.25 cm^2
k 38.53 m^2 **l** 900 cm^2
2 **a** 9 cm^2 **b** 40 cm^2
c 6716 mm^2 **d** 20 cm^2
e 1 km^2 **f** 769 mm^2
3 5.25 m^2
4 7.84 m^2
5 35 m^2
6 13.12 m^2

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7 2 544 690.049 mm^2

8 17.1 m^2

9 **a** 2.2 ha **b** 22 sheep

Exercise 2C

1 **a** 15 m^2 **b** 150 cm^2 **c** 5525 mm^2

2 **a** 10.05 m^2 **b** 12 399.38 mm^2

c 13.68 km^2

3 **a** 3.14 m^2 **b** 30 cm^2

c 800 mm^2 **d** 53.76 m^2

4 **a** 95 cm^2 **b** 5600 mm^2 **c** 26.28 m^2

d 8.6 km^2 **e** 1323 m^2 **f** 92.55 cm^2

g 3.27 cm^2

5 **a** 12.86 m^2 **b** 19.50 cm^2 **c** 16.25 m^2

d 4500 mm^2 **e** 10.37 m^2 **f** 2100 mm^2

6 63 m^2

7 1.195 m^2

8 **a** 226.98 cm^2 **b** 680.94 cm^2

9 131 m^2

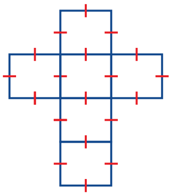
10 2 tins

11 1512.74 cm^2

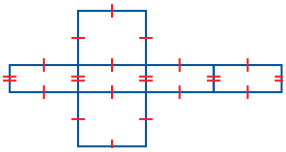
12 362.43 m^2

Exercise 2D

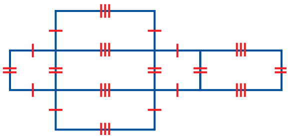
1 **a**



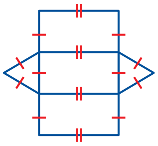
b



c



d



2 **a** 864 cm^2 **b** 121.5 m^2

c 42 m^2 **d** 288 cm^2

3 **a** 21.4 m^2 **b** 99.6 cm^2

c 42 m^2 **d** 236 mm^2

4 **a** 39 mm^2 **b** 224 cm^2 **c** 9.01 m^2

5 **a** 67.9 m^2 **b** 2169 cm^2 **c** 76.86 m^2

6 **a** 20.55 m^2 **b** 328.8 cm^2 **c** 692 mm^2

7 21 600 cm^2

8 5.27 m^2

9 30.84 m^2

10 19 360 cm^2

11 Any sensible drawing will be accepted.

19.36 m^2

12 71 600 m^2

Exercise 2E

1 **a** $A = 4\pi r^2$ **b** $A = 2\pi rh + 2\pi r^2$

c curved surface

d two circles or two ends

2 **a** 2827.43 cm^2 **b** 66.48 m^2

c 20 106.19 mm^2 **d** 530.93 km^2

3 **a** 1055.58 cm^2 **b** 75.65 m^2

c 5.34 km^2 **d** 75.40 m^2

e 82.94 m^2 **f** 9424.78 mm^2

4 854.51 cm^2

5 1.55 m^2

6 136.85 cm^2

7 29.04 m^2

8 510 064 471.9 km^2

9 1306.90 cm^2

Exercise 2F

1 **a** 19.98 m^2 **b** 3518.58 cm^2 **c** 7.47 m^2

2 **a** 84.19 m^2 **b** 66 m^2

c 27.04 m^2 **d** 382 cm^2

e 137.17 m^2 **f** 9896.02 cm^2

g 593.92 m^2 **h** 648.2 m^2

3 573.4 m^2

4 150.80 cm^2

5 2100 cm^2

6 8858.69 m^2

7 406.19 cm^2

8 3656.38 cm^2

Chapter checklist

1 **a** mm^2 **b** km^2
2 **a** square centimetres **b** square metres
c hectares
3 **a** cm^2 **b** m^2
c m^2 **d** ha
4 **a** 60 000 cm^2 **b** 900 mm^2
c 3.5 cm^2 **d** 2 700 000 m^2
e 600 ha **f** 7 000 000 mm^2
5 **a** 33.64 cm^2 **b** 231 mm^2 **c** 52 m^2
d 6.15 m^2 **e** 314.16 cm^2 **f** 310 mm^2
6 **a** 8 m^2 **b** 28 cm^2 **c** 60 mm^2
7 **a** 128 cm^2 **b** 13.65 m^2
8 **a** 212.06 cm^2 **b** 8.73 m^2
9 **a** 18 m^2 **b** 552.73 cm^2
10 **a** 253.5 cm^2 **b** 27 m^2 **c** 966 mm^2
11 $4\pi r^2$
12 $2\pi rh + 2\pi r^2$
13 **a** 9160.88 cm^2 **b** 113.10 m^2
14 **a** 528 cm^2 **b** 56.01 m^2 **c** 19.65 mm^2
15 **a** 10.28 m^2 **b** 722.57 cm^2
16 **a** 108.38 m^2 **b** 7385 cm^2

Chapter review

1 192 000 cm^2
2 2.719 ha
3 1.81 m^2
4 915.6 m^2
5 15.21 m^2
6 3840 cm^2
7 432 841.65 m^2
8 7754.82 m^2
9 29 400 cm^2
10 2.91 m^2
11 4.63 m^2
12 452 389.34 mm^2
13 3.27 m^2
14 No
15 6985.5 cm^2
16 Cooper = 481.06 cm^2, wife = 360.79 cm^2
17 6 656 160 mm^2
18 601.23 m^2

Chapter 3

Pre-test

1 **a** Milli represents one thousandth of a metre
b Kilo represents one thousand metres
2 **a** 1 mm^3 **b** 1 cm^3 **c** 1 m^3
3 1000 mL
4 **a** six, right
b five, triangles
c parallel, circle
5 B
6 grams, kilograms and tonnes

Exercise 3A

1 **a** larger, smaller
b division, cm^3 or mm^3
c hold, solid, liquid
d millilitre
e 1000
f kilolitre
g kilolitres
2 **a** 860 000 000 m^3
b 18 400 mm^3
c 2 700 000 cm^3
d 0.276 cm^3
e 1 380 000 000 m^3
f 68 000 mm^3
g 0.000129 m^3
h 2 700 000 000 mm^3
3 **a** 265 mL **b** 412 m^3
c 6000 cm^3 **d** 2.128 L
e 27 cm^3 **f** 0.001247 kL
g 2700 kL **h** 27.5 kL
i 0.267 ML **j** 800 mL
k 0.628 L **l** 3 200 000 L
4 0.8 m^3
5 1 089 000 000 cm^3
6 12 m^3
7 5000 mL
8 2 500 000 L
9 10 000 L
10 562 000 ML

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Exercise 3B

1 **a** 15 625 cm^3 **b** 6 m^3
c 229 405 mm^3 **d** 49.48 cm^3
e 3.375 m^3 **f** 0.08 km^3
g 437 920 cm^3 **h** 36.95 m^3
i 783.43 cm^3
2 **a** 27 cm^3 = 27 mL
b 4.284 m^3 = 4.284 kL
c 102 240 cm^3 = 102.24 L
d 19.09 m^3 = 19.09 kL
e 21952 m^3 = 21.952 ML
f 48 720 mm^3 = 48.72 mL
3 **a** 2295.61 cm^3 = 2.30 L
b 9.03 m^3 = 9.03 kL
c 39 549.82 mm^3 = 39.55 mL
4 13.57 m^3
5 0.76 m^3
6 91 125 cm^3
7 367.57 cm^3, 367.57 mL
8 413.1 mL
9 5.56 kL
10 4.9 L
11 22.58 kL
12 134.4 kL

Exercise 3C

1 **a** 901.33 cm^3 **b** 1848 mm^3
c 523.60 m^3 **d** 2572.44 cm^3
e 3.648 m^3 **f** 40.5 m^3
2 **a** 8.18 L **b** 23.33 kL
c 0.224 mL
3 **a** 1381.21 mm^3 = 1.38 mL
b 10 305.99 m^3 = 10.31 ML
c 9926.28 cm^3 = 9.9 L
4 288 cm^3
5 0.11 m^3
6 110 976 mm^3
7 37.04 L
8 1.65 L
9 114333 L
10 89 797 L

Exercise 3D

1 **a** 850 g **b** 973.4 g
c 2.3 kg **d** 7500 kg
e 32 000 mg **f** 3.57 t
g 4 700 000 mg **h** 85 000 g
i 0.078 kg **j** 3.56 t
2 3450 g
3 450 000 mg
4 52 310 000 kg
5 1.26 g
6 **a** 2900 kg **b** 3020 kg
7 4.05 kg
8 3700 g
9 13.64 t

Exercise 3E

1 **a** milligram, extremely, (any sensible response)
b Grams
c water, kilogram
d tonnes
2 **a** mg **b** g
c t **d** g
3 Any sensible answers will be accepted
4 **a** Tonnes
b Kilograms
c Grams
d Kilograms or tonnes
e Grams
f Grams
g Milligrams
h Tonnes
i Grams
j Kilograms
k Tonnes
l Milligrams
5 **a** ii **b** iii **c** i
d iii **e** ii **f** i
g i **h** iii **i** ii
j i
6 Any sensible response will be accepted

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7 Weight ranges from 7.5 kg to 15 kg total, hence hire ute.

8 Answers can vary from 200 kg to 350 kg or even more, hence only the ute is needed.

9 Any sensible answer above 1 t and 3 t will be accepted, hence hire the 3-tonne truck.

10 Any sensible answer above 5 tonnes will be accepted, hire the 10-tonne truck.

11 Difficult to tell without more information.

Chapter checklist

1 **a** mm^3 **b** cm^3 **c** m^3

2 **a** 1740 mm^3
b 2 670 000 cm^3
c 2.34 cm^3
d 100 000 000 000 cm^3

3 **a** 75 mL **b** 2500 cm^3
c 6.172 L **d** 3200 kL

4 **a** 636.06 mm^3 **b** 8.4 m^3
c 2250 cm^3

5 **a** 91.13 mL **b** 2.73 ML
c 95.42 mL

6 **a** 22 99.97 cm^3 = 2.30 L
b 16.8 m^3 = 16.8 kL
c 91 751.81 mm^3 = 91.75 mL

7 **a** 3645 cm^3 **b** 8.18 m^3
c 289.5 cm^3

8 **a** 2.4 ML **b** 63.62 L **c** 1.10 kL

9 **a** 43.68 m^3 = 43.68 kL
b 4813.11 mm^3 = 4.81 mL
c 323 830.24 cm^3 = 323.83 L

10 **a** mg **b** g
c kg **d** t

11 **a** 27 g **b** 973.4 g **c** 2.3 kg
d 7500 kg **e** 3.57 t **f** 0.094 kg

12 **a** Kilograms **b** Tonnes
c Grams **d** Milligrams

13 Answers can vary, accept as long as they are sensible
a 700 kg **b** 5400 t
c 20 g **d** 4 mg

Chapter review

1 0.002987 m^3

2 1 400 000 cm^3

3 125 L

4 37.22 L

5 1.14 kL

6 **a** 117 649 cm^3 **b** 1056 cm^3
c 28 652 mm^3

7 0.3817 ML

8 245.7 cm^3

9 **a** 14 L **b** 19.70 L
c 7.24 mL **d** 117.8 mL

10 9.6 kg

11 23.36 kg

12 **a** Milligrams **b** Grams
c Tonnes **d** Kilograms

13 **a** ii **b** iii
c i **d** ii

Chapter 4

Pre-test

1 **a** 1 : 12 **b** 1 : 8 **c** 1 : 15
d 1 : 20 **e** 1 : 30

2 **a** 2.4 m **b** 1.8 m
c 14 m **d** 36 m

3 **a** 2250 mm **b** 1400 mm
c 3840 mm **d** 279 mm

4 **a** 50 m **b** 56 cm
c 70 m **d** 640 mm

5 **a** \$620 **b** \$1488
c \$2604 **d** \$3720

Exercise 4A

1 **a** like **b** Symbols
c abbreviations **d** divide
e multiply

2 **a** 500 cm **b** 280 mm **c** 373 cm
d 27.5 mm **e** 4000 mm **f** 6750 mm
g 2.5 m **h** 1.2 m

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3 **a** 1 : 400 **b** 1 : 200 **c** 1 : 2000
d 1 : 3750 **e** 1 : 60 **f** 1 : 48
g 1 : 600 **h** 1 : 300

4 **a** Laundry
b Sliding robe door
c Refrigerator
d Wardrobe
e Aluminium sliding door
f Aluminium sliding window

5 **a** Laundry sink
b Kitchen sink
c Bathtub
d Dishwasher
e Bathroom vanity
f Kitchen stove
g Doorway
h Toilet

6 **a** Walk-in robe (storage area for clothes)
b Powder room (a toilet area with a sink)
c Walk-in linen (storage area for linen)

7 **a** Stairs (stairs that have a landing half way and continue up the other side)
b Shower (a rectangular shower area)
c Void (an area where the ground floor ceiling doesn't exist giving a sense of great space)
d Lounge chair (marks the area of a living room in a house)
e Coffee table (another indicator of a living room)
f A large window (it would show the window opens from the outside on both sides)

8 AS Australian Standards (designed to meet Australian standards)
U/G Underground (marks underground power and water)
ENS Ensuite (private bathroom linked to a bedroom)

9 FW Floor waste (a drain in the floor)
DP Down pipe (pipes connecting the roof gutters to the storm water or tank pipes)
HWS Hot water system (indicates where the hot water system is situated)

10

Exercise 4B

1 **a** perimeter **b** area
c substitute, formulas **d** length, width
e base, height

2 **a** **i** 1 : 200 **ii** 18 m
b **i** 1 : 80 **ii** 10 m
c **i** 1 : 50 **ii** 9 m
d **i** 1 : 500 **ii** 60 m

3 **a** 25 cm^2 **b** 192 cm^2 **c** 30 cm^2
d 80.6 cm^2 **e** 217.5 m^2 **f** 220 mm^2

4 **a** 40 mm : 6 m = 40 mm : 6000 mm = 1 : 150
b **i** 22 mm by 24 mm
3300 mm by 3600 mm
ii 7 mm by 14 mm
1050 mm by 1350 mm
iii 13 mm by 14 mm
1950 mm by 1350 mm
iv 32 mm by 44 mm
4800 mm by 6600 mm
v 13 mm by 16 mm
1950 mm by 2400 mm
vi 40 mm by 45 mm
6000 mm by 6750 mm
c **i** $P = 3.3 + 3.3 + 3.6 + 3.6 = 13.8$ m
ii $P = 1.05 + 1.05 + 1.35 + 1.35 = 4.8$ m
iii $P = 1.95 + 1.95 + 1.35 + 1.35 = 6.6$ m
iv $P = 4.95 + 4.95 + 6.75 + 6.75$
$= 23.4$ m
v $P = 1.95 + 1.95 + 2.4 + 2.4 = 8.7$ m
vi $P = 6 + 6 + 6.75 + 6.75 = 25.5$ m

5 **a** 50 mm : 5 m = 50 mm : 5000 mm = 1 : 100
b **i** 26 mm by 37 mm
2600 mm by 3700 mm
ii 50 mm by 49 mm
5000 mm by 4900 mm
iii 24 mm by 15 mm
2400 mm by 1500 mm

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iv 15 mm by 24 mm
1500 mm by 2400 mm

c i 9.62 m^2
ii 24.5 m^2
iii 3.6 m^2
iv 3.6 m^2

6 30 mm : 3.6 m = 1 : 120

7 50 mm : 4 m = 1 : 80

8 a 25 mm : 20 m =
25 mm : 20 000 mm = 1 : 800
b Length 58 mm
Actual length $58 \times 800/1000 = 46.4$ m
$A = 20\,\text{m} \times 46.4\,\text{m} = 928\,\text{m}^2$

9 a 40 mm : 20 m = 1 : 500
b Other side 60 mm = 30 m
$P = 100\,\text{m}$

10 a 80 mm : 6000 mm = 1 : 75
b Other side measures 100 mm
Actual length = 7500 mm = 7.5 m
$A = 6\text{ m} \times 7.5\text{ m} = 45.0\text{ m}^2$
c Yes, it meets council requirements.

Exercise 4C

1 a simplify **b** scale factor
c price or cost; units

2 a 20 m **b** $400

3 a 10.5 m^2 **b** $546

4 a 33 m^2 **b** $2145

5 a 80 mm : 6000 mm = 1 : 75
b $100\,\text{mm} \times 75 = 7500\,\text{mm}$ or 7.5 m
c $P = 27\,\text{m}$
d $540

6 a 40 mm by 42 mm
b 3 m by 3.150 m
c 12.3 m
d $58.43

7 a 56 mm by 77 mm
b 4.20 m by 5.775 m
c 24.255 m^2
d $1503.81

8 15 mm : 20 m = 1 : 1333
Sides measure 25 mm, 57 mm, 28 mm, 46 mm
$P = 135$ m
Cost = $6412.50

9 40 mm : 6000 mm = 1 : 150
Laundry 20 mm by 12 mm
$A = 3 \times 1.8 = 5.4\text{ m}^2$
Cost = $561.60

10 a 23 mm by 28 mm
$A = 14.5\,\text{m}^2$
Cost = $779.38
b It would be cheaper to buy carpet for the flat at $600.

11 a Garage 40 mm by 43 mm
$A = 6.0 \times 6.45 = 38.7\text{ m}^2$
5 tins of paint are needed.
b Cost = $270

12 a $11.50 × 38.7 = $445.05
b It is cheaper for Jang to do it herself.

13 90 mm by 126 mm
Area is approximately 255 m^2
Area to turf = 600 − 255 = 345 m^2
Cost is approximately $2700

14 a 32 mm : 6400 mm = 1 : 200
b Perimeter approx 382 mm on plan.
Actual perimeter approx. = 76.4 m
c Cost = $916.80

Exercise 4D

1 a plans, 1
b symbols
c measurements, plan, measurements
d arrows

2 a 4 cm, 7 cm **b** 7 cm, 2 cm
c 3 m, 1 m **d** 7 cm

3 720 mm

4 a 5740 mm by 5740 mm = 5.74 m by 5.74 m
b 6200 mm by 6200 mm = 6.2 m by 6.2 m
c 230 mm
d 2940 mm by 5570 mm or 2.94 m by 5.57 m

5 a 4000 mm
b 4000 mm by 1200 mm
c 800 mm

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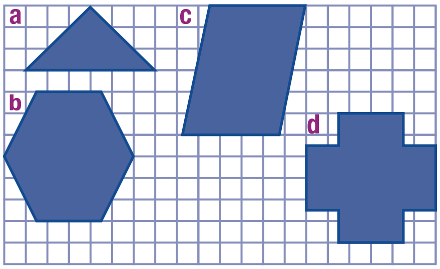
6 **a** plan 1: 100 mm; plan 2: 70 mm
  b **i** 2700 mm by 6100 mm
    ii 2895 mm by 2000 mm
  c **i** width of bath/laundry including external wall
    ii length of the robe in bedroom 2
  d **i** 3500 mm by 3900 mm
    ii 4400 mm by 2895 mm
  e Plan 1: 57.12 m^2 (meets council requirements); plan 2: 62.4 m^2 (does not meet council requirements).

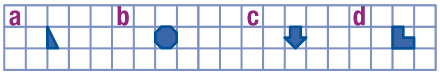
Exercise 4E

1 **a** Construction, share
  b hand, technology
  c architect, drawings, builder

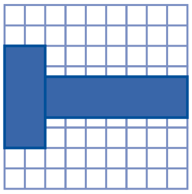
2



3

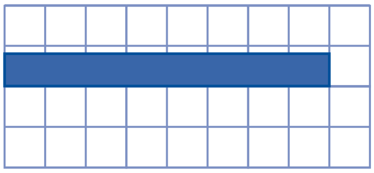


4 squares 10 mm by 10 mm



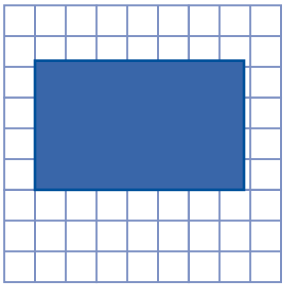
Scale 5 : 1

5 squares 10 mm by 10 mm



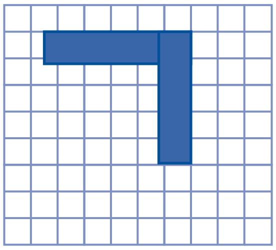
Scale 4 : 1

6 **a** 6.9 m by 4.1 m = 6900 mm by 4100 mm
  b 69 mm by 41 mm
  c squares 10 mm by 10 mm



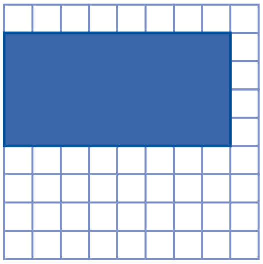
Scale 1 : 100

7 **a** 55.4 mm by 12 mm and 49.4 mm by 12 mm
  b squares 10 mm by 10 mm



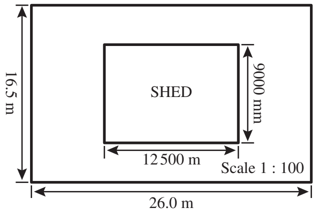
Scale 1 : 50

8 3000 mm by 6000 mm
40 mm by 80 mm
squares 10 mm by 10 mm



Scale 1 : 75

9 The drawing should be 260 mm by 165 mm, with details as shown in this thumbnail view:



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10 Show your teacher what you have produced.

11 Show your teacher what you have produced.

Chapter checklist

1 3540 mm

2 1 : 60

3 Wardrobe where clothes are stored

4 The laundry sink

5 33.92 m^2

6 1 : 100

7 **a** 16.8 m^2 **b** \$806.40

8 1 : 200

22 mm by 15 mm

$A = 2.64 \times 1.8 = 4.75\ \text{m}^2$

Cost = \$294.50

9 6 cm, 2 cm

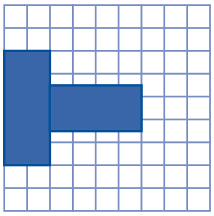
10 **a** 200 mm

b 3600 mm × 9600 mm

c The door width

d $4\ \text{m} \times 10\ \text{m} = 40\ \text{m}^2$

11 Squares 10 mm by 10 mm

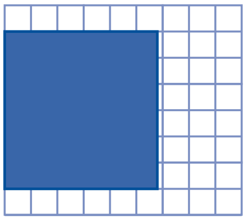


Scale 4 : 1

12 5800 mm by 6000 mm

58 mm by 60 mm

Squares 10 mm by 10 mm



Chapter review

1 W.C. is the water closet or toilet

W.I.R. is the walk-in robe, a closet that you can actually walk into like a room.

2 **a** 1 : 250 **b** 15.0 m^2 **c** 20.0 m^2

3 $A = 6.75\ \text{m}^2$

Cost = \$521.43

4 12 litres required so 3 tins of paint need to be purchased

5 Cost = \$382.50

6 **a** 75 mm

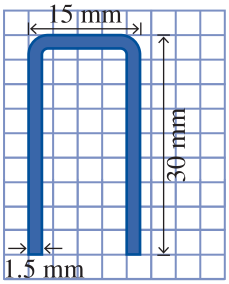
b 3600 mm by 4350 mm

c The length of the bathroom

d 5600 mm by 4350 mm

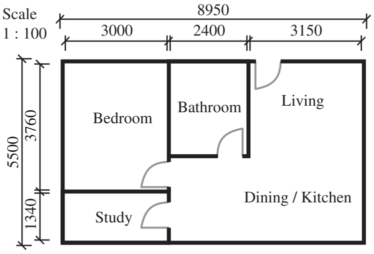
e $A = 60.75\ \text{m}^2$. It does not meet council requirements.

7 Squares 10 mm by 10 mm



Scale 3 : 1

8 In the drawing the external dimensions should be 89.5 mm by 55 mm, with details as shown in this thumbnail view:



Chapter 5

Pre-test

1 **a** 36 **b** 64 **c** 121

d 576 **e** 1089

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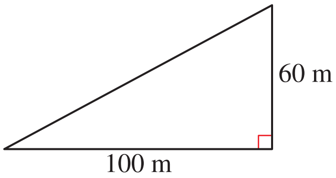
2 **a** 13 **b** 17 **c** 17.89
d 60 **e** 65.85

3 **a** 20 **b** 26 **c** 47.85
d 36 **e** 15.49

4 **a** 14.10 **b** 11.48
c 0.05 **d** 79.74

5



Exercise 5A

1 **a** Pythagoras, hypotenuse, sum
b Pythagoras, right-angled
c c, a, b

2 **a** 5 **b** 13 **c** 6.4

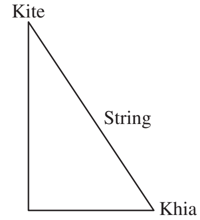
3 **a** 5 **b** 13
c 13.0 **d** 27.3

4 **a** **i** $a = x,\ b = 6,\ c = c$
ii $c = \sqrt{8^2 + 6^2}$
iii $c = 10$
b **i** $a = 10,\ b = 24,\ c = x$
ii $x = \sqrt{10^2 + 24^2}$
iii $x = 26$
c **i** $a = 3,\ b = 5,\ c = y$
ii $y = \sqrt{3^2 + 5^2}$
iii $y = 5.8$
d **i** $a = 8,\ b = 7,\ c = x$
ii $x = \sqrt{8^2 + 7^2}$
iii $x = 10.6$
e **i** $a = 6,\ b = 13,\ c = y$
ii $y = \sqrt{6^2 + 13^2}$
iii $y = 14.3$

5 **a** $8^2 + 15^2 = 17^2$, $289 = 289$ verified
b $7^2 + 24^2 = 25^2$, $625 = 625$ verified
c $33^2 + 56^2 = 65^2$, $4225 = 4225$ verified
d $4.5^2 + 6.3^2 = 7.74^2$, $59.94 = 59.91$ approx. verified
e $12.7^2 + 18.6^2 = 22.52^2$, $507.25 = 507.15$ approx. verified

6 **a** $c = \sqrt{3^2 + 10^2}$ **b** $c = 10.4$ m

7 **a**



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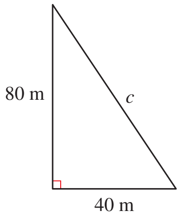
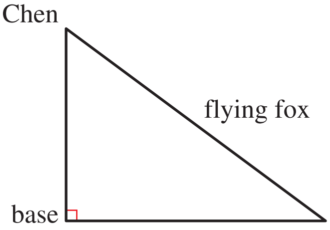


c $c = \sqrt{80^2 + 40^2}$ **d** $c = 89.4$ m

8 **a**



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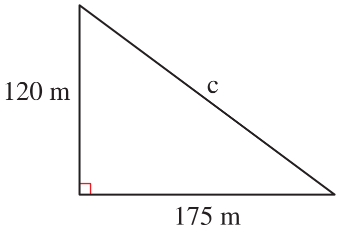

c $c = \sqrt{120^2 + 175^2}$
d $c = 212.2$ m

9 12.8 m

10 44.7 m

11 2.9 m

12 166.4 cm

13 $6^2 + 8^2 = 10^2$
$36 + 64 = 100$
$100 = 100$ verified

14 $3^2 + 3.6^2 = 4.69$
$9 + 12.96 \neq 21.96$
$21.96 = 21.96$ verified

Exercise 5B

1 **a** Pythagoras, unknown, hypotenuse, side
b calculator, solution
c a, c, b

2 **a** $a = \sqrt{u^2 - t^2}$ **b** $a = \sqrt{d^2 - e^2}$
c $a = \sqrt{j^2 - k^2}$

3 **a** 5 **b** 15 **c** 9.6

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4 a i $c = 10,\ a = x,\ b = 8$

ii $x = \sqrt{10^2 - 8^2}$

iii 6

b i $c = 26,\ a = f,\ b = 24$

ii $f = \sqrt{26^2 - 24^2}$

iii 10

c i $c = 9,\ a = x\ ,\ b = 4$

ii $x = \sqrt{9^2 - 4^2}$

iii 8.1

d i $c = 28,\ a = k,\ b = 23$

ii $k = \sqrt{28^2 - 23^2}$

iii 16.0

e i $c = 14,\ a = w, b = 7$

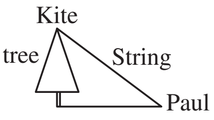
ii $w = \sqrt{14^2 - 7^2}$

iii 12.1

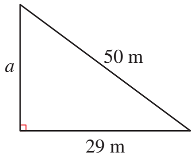
5 a $a = \sqrt{500^2 - 300^2}$

b $a = 400$ m

6 a



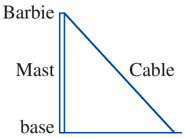
b



c $a = \sqrt{50^2 - 29^2}$

d $a = 40.7$ m

7 a



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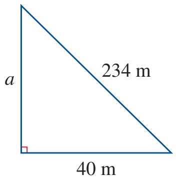


c $a = \sqrt{234^2 - 40^2}$

d $a = 230.6$ m

8 4 m

9 1.8 m

10 129.9 cm

11 30 m

12 3.91 m

Exercise 5C

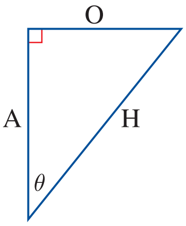
1 a opposite, adjacent

b opposite, hypotenuse

c adjacent, hypotenuse

d diagram, problems

2 a



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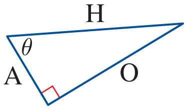
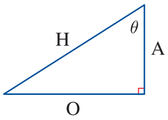


c



3 a 15.4 **b** 19.9

c 6.7 **d** 68.5

4 a 6.64 **b** 58.65 **c** 7.65

d 9.29 **e** 24.39 **f** 251.35

5 a i H = 6 m , O = x

ii SOH

iii $\sin\ 40 = \frac{x}{6}$

iv 3.9 m

b i A = x, O = 9 m

ii TOA

iii $\tan\ 32 = \frac{9}{x}$

iv 14.4 m

c i A = x, H = 16 km

ii CAH

iii $\cos 62 = \frac{x}{16}$

iv 7.5 km

d i A = 7 cm, H = x

ii CAH

iii $\cos 27 = \frac{7}{x}$

iv 7.86 cm

e i A = 12 cm, O = x

ii TOA

iii $\tan 19 = \frac{x}{12}$

iv 4.1 cm

f i H = x, O = 43 km

ii SOH

iii $\sin 53 = \frac{43}{x}$

iv 53.8 km

6 a A = x, O = 45 m **b** TOA

c $\tan 40 = \frac{45}{x}$ **d** $x = 53.6$ m

7 a A = d, O = 31 m **b** TOA

c $\tan 20 = \frac{31}{d}$ **d** $d = 85.2$ m

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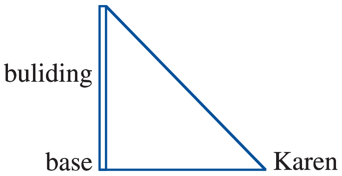
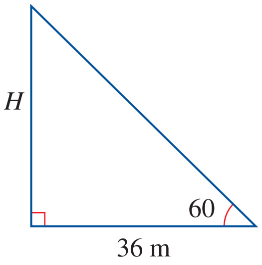
8 **a**



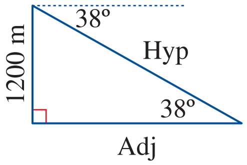


b TOA

c $\tan 60 = \frac{H}{36}$

d $H = 62.35$ m

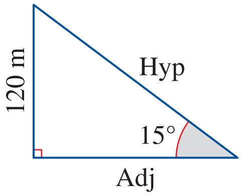
9 **a**



b $\sin\theta = \frac{\text{opp}}{\text{hyp}}$

c $\sin 38 = \frac{1200}{H}$ **d** $c = 1949.1$ m

10 **a**



b $\tan\theta = \frac{\text{opp}}{\text{adj}}$

c $\tan 15 = \frac{120}{\text{a}}$ **d** a = 447.8 m

11 35.3 m

12 39.6 m

13 115.4 m

14 19.5 m

15 3.27 m

16 1677.9 m

Exercise 5D

1 **a** tangent, opposite

b cosine, hypotenuse

c ratio, opposite

d draw, label

2 **a** 22.62 **b** 37.34

c 10.44 **d** 48.37

3 **a** 67.38 **b** 67.38

c 39.47 **d** 20.45

4 **a** **i** O = 26 m, H = 40 m

ii SOH

iii $\sin\theta = \frac{26}{40}$

iv 40.5°

b **i** O = 32 m, A = 14 m

ii TOA

iii $\tan\theta = \frac{32}{14}$

iv 66.4°

c **i** A = 20 km , H = 29 km

ii CAH

iii $\cos\theta = \frac{20}{29}$

iv 46.4°

d **i** O = 17 cm, H = 27 cm

ii SOH

iii $\sin\theta = \frac{17}{27}$

iv 39.0°

e **i** O = 19 m , A = 12 m

ii TOA

iii $\tan\theta = \frac{19}{12}$

iv 57.7°

f **i** A = 43 km, H = 71 km

ii CAH

iii $\cos\theta = \frac{43}{71}$

iv 52.7°

5 **a** O = 300, A = 400 **b** TOA

c $\tan\theta = \frac{300}{400}$ **d** 36.9°

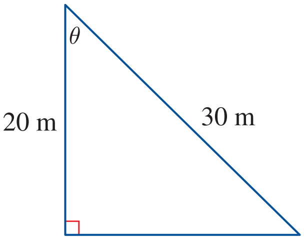
6 **a** O = 50 m, H = 70 m

b SOH

c $\sin\theta = \frac{50}{70}$

d 45.6°

7 **a**



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b O = 20 m , H = 30 m

c $\cos\theta = \frac{20}{30}$

d 48.2°

8 10.7°

9 40.9°

10 26.6°

11 44.4°

12 30°

13 48°

Chapter checklist

1 11.7 cm

2 19.7 m

3 3162 mm

4 13.3 m

5 23.6 m

6 3.4 km

7 12.6

8 **a** 14.4 m **b** 28.9 m

9 0.9 m

10 32.6°

11 19.7°

Chapter review

1 32.0 m

2 5 m

3 5 km

4 2.4 m

5 1.5 m

6 51.2 m

7 2130 m

8 23.8°

9 24.1°

10 296.1 m

Chapter 6

Pre-test

1 **a** 20.71 **b** 0.5 **c** 0.33

2 1, 4, 5, 6, 7, 9, 10, 11, 13, 13, 17, 19, 21
Median = 10

3 **a** 26.5 **b** 44 **c** 63.5

4 **a** 25% **b** 43.75%

5 **a** $x = 1$ **b** $x = 9$

6 **a** 4 **b** 49 **c** 0.25

7 24 minutes

8 smallest: 41
largest: 95
most common (mode): 78

Exercise 6A

1 **a** centre
b mode, mean, median
c Mode
d two, bimodal
e average, sum, divided
f $\bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$
g middle, ascending
h even, two

2 **a** **i** 5.2 **ii** 5 **iii** 5 and 6
b **i** 77.2 **ii** 85 **iii** 85
c **i** 8.0 **ii** 7.95 **iii** no mode
d **i** 20.1 **ii** 19 **iii** 18
e **i** 2.1 **ii** 2 **iii** 3

3 **a** Mode = 1 **b** Mode = 12
c Mode = 75%

4 **a** Mean = 121 **b** Median = 115
c Mode = 115

5 **a** Mean = $886 **b** Median = $850
c Mode = Bimodal ($700 and $900)

6 **a** Mean = 1.9 **b** Median = 2
c Mode = Bimodal (1 and 2)

7 **a** **i** Mean = 16.5 **ii** Mode = 12
iii Median = 14
b **i** Mean = 222.5 **ii** Mode = 225
iii Median = 225
c **i** Mean = 57.4 **ii** Mode = 52
iii Median = 55
d **i** Mean = 115.9 **ii** Mode = 100
iii Median = 115

8 **a** **i** Mean = 2.1 **ii** Mode = 2
iii Median = 2
b **i** Mean = 5.2 **ii** Mode = 6
iii Median = 5.5

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c **i** Mean = 1.7 **ii** Mode = 1
iii Median = 1.5

d **i** Mean = 1.8
ii Mode = no mode
iii Median = 2

Exercise 6B

1

Measure of central tendency	Advantages	Disadvantages
Mean	All data is taken into account. Easy to understand and calculate.	Extreme values distort data. Percentages and ratios could be challenging.
Median	Simple to understand and calculate. Not affected by extreme values.	May not be able to sort. All items may not be taken into account. An even number of data points requires an extra calculation step.
Mode	Simple to find. Not affected by extreme values.	Not based on all the data Can have more than one mode or no modes at all.

2 **a** Mean = 8
b Median = 8
c Mode = 7
d Mean and median shoe sizes do not help determine which sizes the shop should stock. But the size which is the mode (7) should be stocked the most, and sizes which occur the least should have the least stock. So mode is the better measure.

3 **a** Mean = 16
b Median = 15
c Mode = no mode
d The better measure would be the mean, as it takes all results into account and gives Noah the best idea of his overall performance. There is no mode and the median does not take all results into account.

4 **a** Jamie:
i Mean = 82.4 **ii** Mode = 85
iii Median = 83.5
Scarlett:
i Mean = 86.7 **ii** Mode = 95
iii Median = 89

b Scarlett's higher scores were shown in the mean and the median, as she had several high results. The mode scores were irrelevant to their actual results.

5 Median = 1

6 **a** Mean = $257
b Median = $75
c Mode = $50
d The son would suggest the mean, as it takes into account the extreme $2350, which brings the mean up to $257. The father is trying to be fair and puts his son in the middle of his friends' range. The younger sister may just be choosing the mode as it happens to be at the lower end of the range and is closer to what she gets.

In this situation the median would be the best measure of tendency, as it is not affected by the outlier. Since it is the middle of the ordered values, there are as many friends who get less as there are friends who get more.

7 Student A Mean = 37.3; Student B Mean = 38.3; Student C Mean = 37.3
Student A Median = 45; Student B Median = 43; Student C Median = 42
Student A Mode = 45; Student B Mode = Bimodal (30 and 50);
Student C Mode = 48
The fairest measure of central tendency for this scenario would be the mean, as it takes into account all results of the students,

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whether high or low. The median only takes into account the middle result and the mode is only found by results being the same rather than the overall score of the student.

The math award should go to Student B.

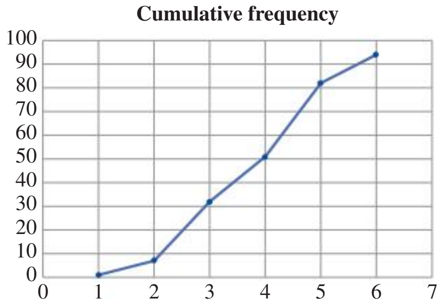
Exercise 6C

1 **a** quantiles **b** running total
c three, four **d** value
e nine, ten **f** 99, 100

2

Score	Frequency	Cumulative frequency	Cumulative frequency %
1	1	1	1.1%
2	6	7	7.4%
3	25	32	34.0%
4	19	51	54.3%
5	31	82	87.2%
6	12	94	100.0%
Total	94		

3



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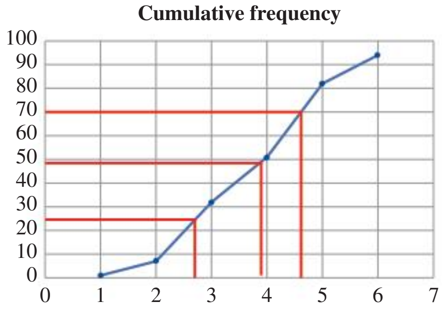

Q_1: 2.7
Q_2: 3.9
Q_3: 4.7

5 **a** 7
b 5
c 9
d A quarter of the matches won lie below Q_1 and three-quarters lie above it.
Half the matches won lie below Q_2 and half lie above it.
Three-quarters of the matches won lie below Q_3 and a quarter lie above it.

6 **a** Age 16.5 years **b** Age 15 years
c Age 17.7 years

7 **a** Andrew’s score lies between the 7th and 8th deciles.
Brad’s score lies between the 9th and 10 deciles.
b 7 tenths of the students did worse than Andrew and two tenths (one fifth) did better.
9 tenths of the students did worse than Brad and less than one tenth did better.

8 **a** Brolga A’s mass is between the 5th and 6th percentiles.
Brolga B’s mass is between the 81st and 82nd percentiles.
b Brolga A is heavier than 5% of the brolgas and lighter than 94%.
Brolga B is heavier than 81% of the brolgas and lighter than 18%.

9 **a** Virat is 17.7 years old.
b Zara is 15 years old.
c 50% or the visitors to the park are between the ages of Zara and Virat.

10 It would be better to receive a high percentile, as the higher percentile means a higher grade.

11 The 85th percentile would mean that Troy has been waiting longer than others. It means that 85% of the other people waited 32 minutes or less and 15% waited 32 minutes or longer, which is a bad outcome.

12 Katrina and Elliot can only afford 25% of the houses in their area, as the other 75% cost $350 000 or more.

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Exercise 6D

1 **a** displayed
b difference(s)
c histogram, column
d mean
e cluster
f gap

2 **a** Not accurate. The distribution does not have an outlier.
b Not accurate. The distribution does not have a gap.
c Accurate: The data is tightly packed around the mean.

3 The data is spread out and loosely packed around the mean.
The dispersion is widely scattered from 8 to 19.
There is one cluster between 8 and 19.
The data has one gap between 2 and 7.
0–1 could be considered as an outlier.

4 January's data shows a spread that is tightly packed around the mean with a wide value. The data has a gap between 149 and 179, with what could be considered an outlier at 179.
February's data is spread out around the mean with a wide value. There are two main clusters, with a small gap between 143 and 161. February does not contain an outlier.
February's data is loosely packed and more spread out than January. Only January's data contains an outlier. Both contain wide values. February has 3 less observations making it slightly less dense.

5 Graph A's data is spread out around the mean. The data has a gap between 2.25 and 6.75. Graph A has what could be considered an outlier at 6.75.
Graph B's data is tightly packed around the mean with a small value. This graph does not contain an outlier.
Graph A's data is loosely packed and more spread out than Graph B. Only Graph A's data contains an outlier. Both contain small values and only one cluster.

6 Semester 1's data is spread out around the mean. The data has a gap between 30 and 51. There are two clusters, one from 0–30, and the other from 51–90.
Semester 2's data is spread out around the mean. This graph could contain an outlier at 0–10. Semester 2 has two gaps, one at 10–21, and the other at 50–81. Semester 2 also has two clusters, the first at 21–50, the second at 81–100.
Both graphs have the same size spread, and both have two distinct clusters.
Semester 2 is the only graph that could contain an outlier. Semester 1 has one gap, whereas Semester 2 has two gaps.

Exercise 6E

1 **a** range
b difference, highest, lowest
c interquartile range
d measure, average
e outside
f mean, median

2 Range: 15, IQR 9, SD 5.5

3 **a** $Q_1 = 14$, $Q_3 = 20$, $\text{IQR} = 6$
b 5. One value (0) is lower than this.
c 29. No values are greater than this.
d There is an outlier in the data, the value 0.

4 **a** 3 and 25
b $25 - 3 = 22$, range is 22.

5 **a** 20, 37, 40, 75, 80, 85, 100
b $Q_2 = 75$
$Q_1 = 37$
$Q_3 = 85$
c $\text{IQR} = 48$

6 **a** 5 **b** 2.4

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7 **a** Mean 89

b The standard deviation for Carrie's friends losing weight is 16.66 from the mean (89).

c 72, 75, 85, 105, 120

d Range = 48

e IQR = 30

8 **a** The mean = 135.85

b The standard deviation = 18.19.

c 102, 120, 140, 148.5, 163

d Range = 61

e IQR = 28.5

9 Mean 29.38, standard deviation 2.29

10 Mean 3.28%, standard deviation 0.62%

11 **a** 14.9

b 2 is an outlier.

c Mean without outlier = 17. The outlier 2 does affect the mean, lowering it from 17 to 14.9.

d The outlier is either a valid score because Cooper had an off day, in which case it should be kept, or it is an error in writing down the score, in which case it could be removed. Cooper would know if he had an off day, if he didn't then he must have made an error recording it.

12 **a** The median is $40 000.

b $120 000 is an outlier.

c The median without the outlier is $37 000. The outlier does not affect the median much (with the outlier it is $40 000).

d The outlier can be removed as it does not have much effect on Faith's research into the prices of cars.

Chapter checklist

1 Mode = 7

2 Mean = 6.5

Median = 7

3 Median is the better measure of central tendency, as it is not so affected by the outlier ($1 700 000), which would affect the mean. There is no mode.

4 **a** Mode and median

b The mean, because when you calculate it you add up all the values in the dataset before dividing by the number of values.

5 If the dataset has outliers and is not very big, the mean is misleading as a measure of central tendency.

If the dataset is skewed towards one particular section, the median is misleading as a measure of central tendency.

If the dataset does not have a value that occurs many times, or is bimodal, the mode is misleading as a measure of central tendency.

6 Q_1 Lower Quartile = 5

Q_2 Median = 7

Q_3 Upper Quartile = 8

7 5

8 3

9 98th percentile, 2% of the class did better.

10 The data is spread out with its dispersion being widely scattered from 0 to 49 500. There is one main cluster between 0 and approximately 11 250. The data has five gaps; one between 11 250 and 13 500; one between 13 500 and 22 500; one between about 23 000 and 27 000; one between about 27 500 and 31 500; and one between 31 500 and 49 500. 49 500 could be considered as an outlier.

11 Range = 45, IQR = 24,

Standard deviation = 14.19

12 The range is quite large, so the data is spread out. The interquartile range is a lot less and may be more representative. The standard deviation is quite high also indicating that the data is quite scattered.

13 107 is an outlier. The mean is 38.9 with the outlier and 32.7 without it. The median is 38.5 with the outlier and 37 without it.

14 Features of a dataset that could make the range misleading:

Datasets with outliers

Features of dataset that could make the

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standard deviation misleading:
Datasets with outliers
Widely scattered with dense and less dense regions

Chapter review

1 **a** Mode = 65 **b** Mean = 62.15
c Median = 65

2 **a** The median or mode would be better than the mean because calculation of the mean adds up the values of all data points and extreme values will give a mean that is quite different from most of the data points. The median and the mode are not affected much by the inclusion of extreme values.
b The mean is best as its calculation adds up the values of all data points.
c The median is best because it is the middle value when the data is ordered so it always has an equal number of data points above it as below it.
d The mode is the best as it is the only measure based on the value that occurs most often in the dataset.

3 The data is spread out with its dispersion being widely scattered from 0 to 21. There is one main cluster between 0 and 13. The data has three gaps; one around 13; one around 16; one between 18 and 20. 21 could be considered as an outlier.

4 **a** Mean = 7.25
b Median = 6.5
c Mode = There is no mode
d Mean is the better measure of central tendency, as it is the average score of all grades. The median score does not take into account all grades recorded and there is no mode in this dataset.

5 **a** 40 **b** 26
c 60 **d** 26

6 **a** 64 g **b** 83%

7 Range = 45 – 0 = 45
IQR = 7 – 1 = 6
Standard deviation = 13.36

8 Range = 27 – 16 = 11
IQR = 25 – 19 = 6
Mean = 22.33
Five-number summary = 16, 19, 22.5, 25, 27
Standard deviation = 3.5

9 Range = 4
Mean = 6.17
Standard deviation = 1.40

10 **a** **i** Median = 35.5
ii Upper quartile = 46
iii Lower quartile = 15
iv IQR = 31
b 107 is an outlier.
c There may have been a lot of fish around that day.
d The outlier is more than double any of the other observations, therefore it may be an error. It should only be removed if there is evidence that it is an error (i.e. a mistake in adding up or recording the number that day).

11 Mitchell needs to make at least $80 on Saturday to meet his budget.

12 Jackie earns more than 78% of her colleagues' wages and 22% of them have a higher wage. Jackie needs to consider whether they do more hours or more skilled work than her. If she believes she contributes considerably more to the success of the workplace than 78% of her colleagues she is probably justified in asking for a raise.

13 **a** The standard deviation of each group is:
A = 2.28
B = 0
C = 8.28
b Dataset C has the largest standard deviation.

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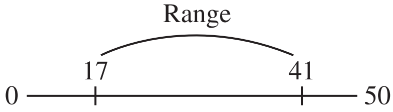
c Yes, since by inspection a mean can be estimated for each dataset, and dataset C has data values that are further away from the mean compared to the other two datasets.

Chapter 7

Pre-test

1 12, 23 and 46

2 Range = 24



3

Stem and Leaf Plot

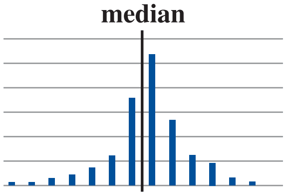
4	1
5	2 7 8
6	5 6
7	0 5 8 8 8
8	0 0
9	5

4 Answer is C

5 Most students: class E

Least students: class A

6



Exercise 7A

1 **a** smallest, largest
  b minimum
  c $Q_{1,}$ Q_3
  d median, middle
  e maximum

2 **a** 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 5, 7, 9
  b 7.7, 7.9, 8.5, 8.6, 8.7, 8.7, 8.9, 9, 9.2, 9.6
  c 41, 42, 42, 43, 45, 45, 46, 46, 47, 48, 49, 65
  d \$34.70, \$35.80, \$36.00, \$36.30, \$36.50, \$45.90

3 **a** Min = 3; Max = 9
  b Min = 8.0; Max = 8.9
  c Min = 15; Max = 28
  d Min = 0.5; Max = 0.88

4 **a** 5 **b** 4 **c** 3.5
  d 15.5 **e** 14 **f** 60.5

5 4, 6, 7, 7, 12, 13, 13, 16, 16, 17, 27, 29, 45, 46, 48, 50, 54, 67, 72, 83, 84, 86

6 **a** 0, 0, 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 7
  b Min. = 0 and Max. = 7
  c Median = 2
  d $Q_1 = 0$
  e $Q_3 = 3$
  f Min. = 0, $Q_1 = 0$, Median = 2, $Q_3 = 3$, Max. = 7

7 **a** 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 7, 9
  b Min. = 2 and Max. = 9
  c Median = 5
  d $Q_1 = 4$
  e $Q_3 = 6$
  f Min = 2, $Q_1 = 4$, Median = 5, $Q_3 = 6$, Max. = 9

8 **a** 8, 9, 10, 10, 11, 12, 12, 12, 12, 12, 15, 15, 16, 17, 18, 19, 21, 28, 29, 34
  b Min. = 8 and Max. = 34
  c Median = 13.5
  d $Q_1 = 11.5$
  e $Q_3 = 18.5$
  f Min. = 8, $Q_1 = 11.5$, Median = 13.5, $Q_3 = 18.5$, Max. = 34

9 **a** 98, 101, 103, 105, 107, 109, 112, 114, 116, 118, 119, 127, 127, 127, 128, 134, 135, 135, 137, 138, 139, 145, 145, 146, 147, 148, 149, 150, 150, 150, 155, 155, 155, 157
  b Min. = 98 and Max. = 157
  c Median = 135
  d $Q_1 = 116$
  e $Q_3 = 148$
  f Min. = 98, $Q_1 = 116$, Median = 135, $Q_3 = 148$, Max. = 157

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10 **a** Min. = 24, $Q_1 = 32$, Median = 53.5, $Q_3 = 67$, Max. = 87

b Min. = 27, $Q_1 = 43$, Median = 65, $Q_3 = 88$, Max. = 105

c Min. = 12, $Q_1 = 35$, Median = 73, $Q_3 = 97.5$, Max. = 117

d Min. = 1.05, $Q_1 = 2.84$, Median = 3.975, $Q_3 = 6.5$, Max. = 9.56

11 Min. = 106, $Q_1 = 107.5$, Median = 115, $Q_3 = 121$, Max. = 130

12 Min. = 200, $Q_1 = 400$, Median = 775, $Q_3 = 900$, Max. = 1 200

13 Min. = 0, $Q_1 = 1$, Median = 2, $Q_3 = 3$, Max. = 5

14 **a** Interest rates

3	2 5 6 7 7
4	0 5 6
5	3 4 4 6 8
6	0 0 4 4 5
7	0 0 0 0 3
8	4

key 3|2 = 3.2

b Min. = 3.2, $Q_1 = 4.25$, Median = 5.7, $Q_3 = 6.75$, Max. = 8.4

15 D

16 B

17 **a** Number of cars per 20 min

1	9
2	0 0 5 6 7 8 8
3	0 4 6
4	0 4 5 5 5 5 6

b Min. = 19, $Q_1 = 26$, Median = 32, $Q_3 = 45$, Max. = 46

Exercise 7B

1 **a** visual, dataset

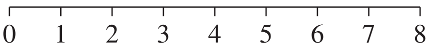
b scale

c whisker

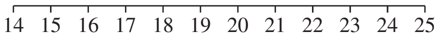
d Q_3

e inconsistent, surprising

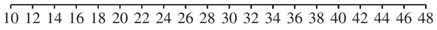
2 **a**



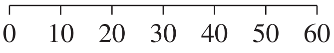
b



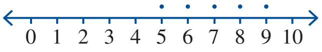
c



d Answers may vary (could also use increments of 5)

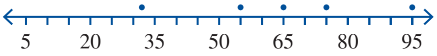


3 **a**



b

c



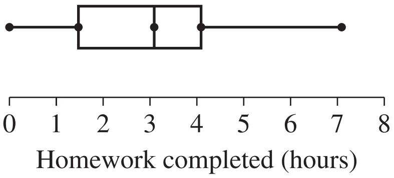
4 **a** 3, 9, 12, 18, 28

b 5, 30, 35, 45, 80

c 20, 120, 180, 320, 360

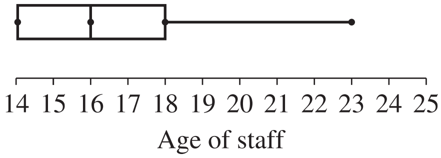
5 **a** Min. = 0, $Q_1 = 1.5$, Median = 3, $Q_3 = 4$, Max. = 7

b



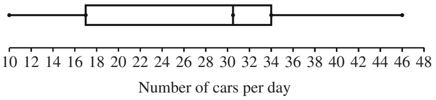
6 **a** Min. = 14, $Q_1 = 14$, Median = 16, $Q_3 = 18$, Max. = 23

b



7 **a** Min. = 10, $Q_1 = 16.5$, Median = 30.5, $Q_3 = 34$, Max. = 46

b

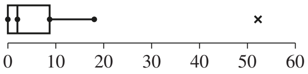


8 **a**

0 11.5

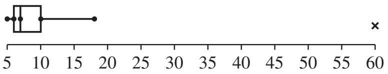
⓪0 0 0 0 0|0 0 1 1 1 2 $\boxed{2}$ 4 4 5 7 8 11|12 13 13 14 ⑱(52)

b



9 **a** ⑤ 5 5 6 6 $\boxed{6}$ 6 6 6 7 7 $\boxed{7\ 8}$ 8 8 9 10 10 $\boxed{11}$ 14 14 15 ⑱ (60)

b



10 Min. = 30, $Q_1 = 20$, Median = 51, $Q_3 = 70$, Max. = 78

11 Min. = 9, $Q_1 = 20$, Median = 51, $Q_3 = 68$, Max. = 76

12 **a** At least 75% of the children are over 8 years of age.

b No, as there are many datasets that could fit this information.

c 25% of the children

d 5 years of age

e 75% of the children

13 **a** 75% of the trees are older than 32 years of age.

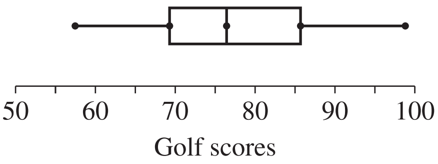
b 72 years

c 25% of the trees

d 87 years old

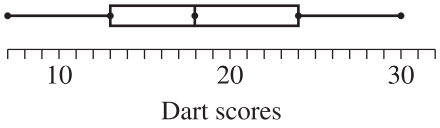
14 **a** Min. = 58, $Q_1 = 69$, Median = 76, $Q_3 = 86$, Max. = 99

b



15 **a** Min. = 7, $Q_1 = 13$, Median = 18, $Q_3 = 24$, Max. = 30

b



16 C

17 C

Exercise 7C

1 **a** two

b same

c measure, sensitive, outliers

d outlier

e single, two

f separated, compared

2 **a** minimum

b lower quartile whisker

c Q_1

d median, Q_2

e IQR (Interquartile Range)

f Q_3

g upper quartile whisker

h maximum

i range

3 **a** Title

b Stem column

c Girls' leaf column

d Boys' leaf column

e Girls' key

f Boys' key

4 **a** The median for males is 65, which is lower than the females' median of 75 beats per minute.

b The range of the male's pulse rates is 40, compared to the range of the females being larger at 55.

c The IQR for males is 15, being less than the females' IQR of 25.

d Modelled response: In general the resting pulse rate for females is higher, and so is the variability. Both genders had the same minimum pulse rates.

5 **a** The median for both datasets is 15 spelling errors.

b The range of the Year 11 errors is 45, compared to the range of the Year 12 being smaller at 35.

c The IQR for Year 11 is 10, being less than the Year 12 IQR of 15.

d Modelled response: Judged by the median number of errors being the

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same, the general number of errors is similar in Year 11 and 12. The variability is also similar: while the range is greater in Year 11, the IQR is greater in Year 12, so it evens out. The maximum number of errors was highest for Year 11 at 50, but this could be an outlier. The minimum number was lower in Year 12, but again this could be an outlier.

NOTE: *this is an example where it would have been very helpful to know the means of the datasets, in order to decide if there really is a difference between them.*

6 **a** The median for Year 7 is 9 hours, which is lower than the Year 12 median of 14 hours per week.

b The range of the Year 7 students homework time is 12 hours, compared to the range of the Year 12 being 4 hours longer at 16.

c The IQR for Year 7 is 9, being less than the Year 12 IQR of 11 hours of homework per week.

d Modelled response: The homework time for Year 12 students is longer, as shown by the median, but also has greater variability.

7 **a** The median for performing burpees before a game of netball is 32, which is lower than the end of season median of 37 burpees.

b The range of burpees before the season is 27, compared to the range after the season being slightly larger at 30.

c The IQR for before is 15, being less than the end of season IQR of 22.

d Modelled response: The ability to do burpees at the end of the season of netball is higher, though the range of ability is greater, meaning that a few players did not improve very much.

8 **a** The mean values of each distribution cannot be calculated since we don't have the data values; however, the median of the 2016 Olympics distribution is represented as being higher than the median of the Australian qualifiers distribution. These datasets show that the Australian qualifiers' long jumps were generally shorter than the Olympic long jumps, though there is one outlier in the Australian qualifiers of 8.15 m which is the same as the longest jump at the 2016 Olympics.

b Ignoring the outlier in the Australian qualifiers' distribution, the distribution of the Olympic long jumps is more spread out than the Australian qualifiers' distribution. The datasets show that the IQR and range of the Olympic jumps is larger. Therefore, the Olympic long jumps vary more (are more spread out) than the Australian qualifiers' jumps.

c The shortest jump at the Olympics (the minimum value, 7.7 m) was the same as the third quartile of the Australian qualifiers' jumps. This means that all the Olympic jumps were longer than 75% of the Australian qualifiers' jumps, therefore it can be concluded that most of the Olympic jumps were longer.

9 Answers may vary. Modelled response. Range: Popular 8–30 min, new 10–30 min. Mean: Popular 17.5 min, new 17.3 min. Median: Popular 16 min, new 16 min. Mode: popular 13 and 23 min, new 10 and 16 min. The datasets are very similar, and they do not justify the new café in claiming to have the fastest service. Although the popular café had the two shortest delivery times, overall there was very little difference between them.

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10 The median for the highest score for the girls was 130, while the median for the highest score for the boys was 143. The mode for the highest score for the girls was both 125 and 150, while the mode for the highest score for the boys was 162. The range for the highest score for the girls was 100, while the range for the highest score for the boys was 90.
The highest score for the girls was 203, while the highest score for the boys was 196. We are told that the mean highest score was slightly higher for the boys than the girls, and the median for the boys was higher. The mode does not tell us anything useful. While the highest score of 203 was by a girl, overall the girls' highest scores were slightly lower than the boys' highest scores.

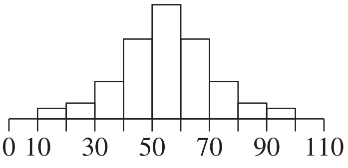
11 Answers may vary. Modelled response. The delay distributions from the 20th of December to the 25th of December are different from the delay distributions for the month before and after those dates. Both the median delay and the variability (spread) of the delays are greater from December 20th to 25th. By the 26th the delays are almost the same as before the 20th, but still slightly higher.

Exercise 7D

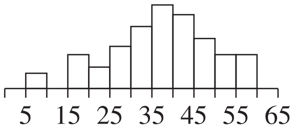
1 a mirror
b approximately
c left, right
d Negatively, left
e Bimodal, two

2 a Approximately symmetric
b Negatively skewed
c Bimodal
d Symmetric

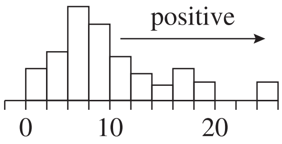
3 a



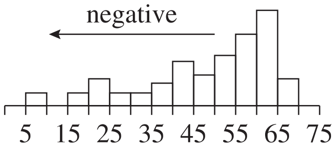
b



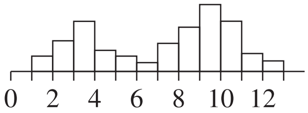
c



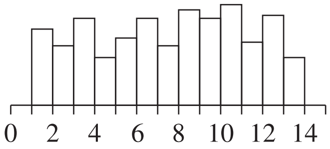
d



e



f



4 a The First 11 throws histogram is very nearly a symmetric shape, whereas the Second 11 throws histogram is only very roughly symmetric.

b Both histograms have a centre of 3.5. The spread for both histograms is exactly the same, being five.

5 a The Monday histogram is positively skewed, as the distribution's peak is higher on the left, with the tail stretching to the right.
The Tuesday histogram is negatively skewed, as the distribution's peak is higher on the right, with the tail stretching to the left.

b The Monday histogram has a centre of 10.5, with the Tuesday histogram also having a centre of 10.5. The spread of both histograms is exactly the same at 19 m/s.

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6 **a** The Term one histogram's shape is bimodal as it is double-peaked at 30% and 90%, whereas the Term two histogram is approximately symmetric.

b The Term one histogram has a centre of 55, whereas the Term two histogram has a centre of 60. Therefore, the difference for the centres is 5. The spread for Term one is 90 and the spread for Term two is 80, which is also a difference of 10.

7 **a** Course A is positively skewed, as the results are higher on the left. Course B is approximately symmetric as the results are approximately the same on both sides of the peak.

b Course A's results lean towards scores that are failing the course. More students on Course B achieve high marks and the centre of spread is higher too. So the student may feel that she has a higher chance of passing and getting a high score on Course B.

8 The Before program histogram is positively skewed, as the peak is higher on the left of the distribution. The After program histogram displays an approximately symmetric shape, as the distributions are approximately the same on both sides of the peak.

The locations are different for the histograms, with the Before program's location covering 40–55% and the After program's location covering 20–40%.

The spread for the After program (25%) is larger than the spread for the Before program (20%).

9 **a** The Zambia histograms are both positively skewed, with the shorter bars being in the positive section of the distribution.

The Zambia histograms have the same location and same range of 80. Females outnumber males slightly for ages up to about 24, and above that the sexes have approximately the same numbers in each age group.

b The Swedish histograms are both bimodal, with peaks at 50–54 and 35–39. There are slightly more males than females in each age group up to about 60, and above that, there are more females and males. While there are a few thousand females older than 100, the number of males is too small to appear on the graph.

The Swedish histograms have similar locations and ranges.

c In Zambia, the youngest in the population are the most numerous, indicating a rapidly-growing population. Each older age group is smaller than the preceding one, indicating high birth or death rates or a combination of the two. In Sweden a greater proportion of the population is old compared to Zambia, and people live longer. In Sweden the total population is probably not growing and, as death rates for children are unlikely to have increased, we can say the birth rate has declined over the past 60 years. In Zambia there are more females than males under 24 and then the sexes have about equal numbers in each age group, but in Sweden males exceed females up to about 60, and above that there are more females than males.

Chapter checklist

1 minimum, lower quartile, median, upper quartile, maximum

2 Minimum – the smallest number in the dataset

Lower quartile – the median of the lower half of the dataset

Median – the number that falls exactly in the middle of the dataset

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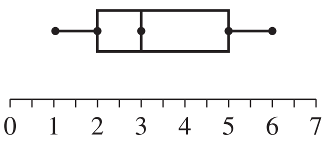
Upper quartile – the median of the upper half of the dataset

Maximum – the largest number in the dataset

3 Min. = 4, $Q_1 = 5.5$, Median = 8, $Q_3 = 9$, Max. = 13

4 Min. = 4, $Q_1 = 5.5$, Median = 8, $Q_3 = 9$, Max. = 13

5 $(\boxed{1}, 2, \overset{2}{|}\, 2, 3)\,\boxed{3}, (4, 5, \overset{5}{|}\, 5, \boxed{6})$



6 The barramundi in location B are longer, the median for location A is 80 cm, whereas for location B it is 40 cm longer at 120 cm. 25% of the barramundi at B are longer than 170 cm, while the longest barramundi at A was only just over 170 cm. The variability (spread) of length at the two locations is similar, location B has a slightly smaller IQR (70 cm compared to 75 cm) but a larger range (140 cm compared to 130 cm).

7 Batch B tomatoes took a little bit longer to ripen than batch A. The median was 11 days for A and 13 days for batch B. The variability was similar, with B being slightly less (A had a range of 10 days and IQR of 8 days, B had a range of 9 days and an IQR of 6 days).

8 **a** Cow A's histogram is bimodal, as it has a double peak. Cow B's histogram is uniform or multimodal, having a fairly even distribution.

b Both Ty and James' histograms are approximately symmetric, as the shape of both graphs are approximately the same on both sides.

c Team A's histogram is negatively skewed, whereas Team B's histogram is positively skewed.

Chapter review

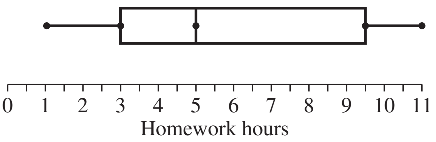
1 Min. = 3, $Q_1 = 4$, Median = 5, $Q_3 = 7.5$, Max. = 14

2 Min. = 0, $Q_1 = 1$, Median = 2, $Q_3 = 2.5$, Max. = 4

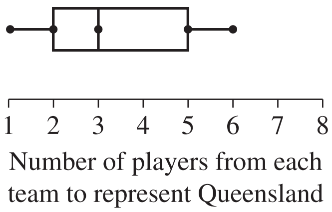
3 Min. = 4, $Q_1 = 5.5$, Median = 8, $Q_3 = 9$, Max. = 13

4 **a** Min. = 1, $Q_1 = 3$, Median = 5, $Q_3 = 9.5$, Max. = 11

b $(1, 2, 3, \overset{3}{\uparrow} 3, 4, 4)\overset{\boxed{5}}{|}(6, 8, 9, \overset{9.5}{\uparrow} 10, 10, 11)$



5 $(1, 2, \overset{\boxed{2}}{\uparrow} 2, 3)\,\boxed{3}\,(4, 5, \overset{\boxed{5}}{\uparrow} 5, 6)$



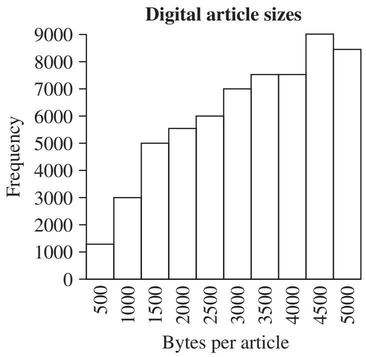
6 Answers may vary. Modelled response. Varieties A and B are shorter than C and D. The median heights are similar for A (55 cm) and B (50 cm). The median height for C is 65 cm and is greatest for D (85 mm). The variability indicated by the range is similar for all four varieties, but the IQR indicates variety B has the lowest variability.

7 **a** The median life expectancy of males in 1988 (69) was 6 years less than in 2018 (75). The range is 36 for 1988 and 35 for 2018. The IQR in 1998 was 17 and was less than in 2018 (IQR = 25).

b In conclusion, the median life expectancy for these particular countries has increased over the last 30 years, and the variability in life expectancy between countries has increased as shown by the increase in the IQR.

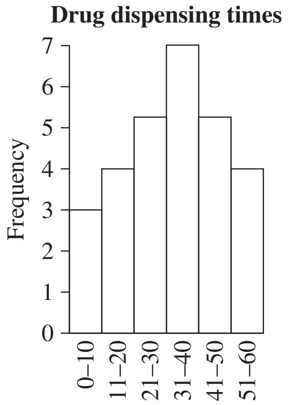
8 The 2017 histogram's shape is bimodal as it is double-peaked at April and December. The 2018 histogram's shape is uniform, as it has a fairly constant distribution. Both histograms have their centre between June and July. The spread for both histograms is 12.

9 a i



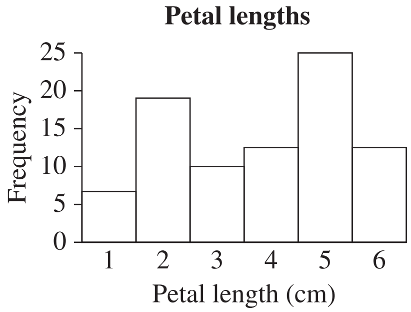
ii The Digital article sizes histogram is negatively skewed, and the peak of the distribution is higher on the right.

b i



ii The drug dispensing histogram is approximately symmetric with a peak almost in the centre.

c i



ii The Petal length histogram is bimodal, as the frequency numbers have two peaks, which would produce a bimodal histogram.

10 Answers may vary. Modelled response. The median heights for GA/GD and GK/GS (both about 166 cm) are different from the height distributions of WA/WD and C (about 158 and 156 cm). The variability is similar for GA/GD and GK/GS, and is less than for WA/WD and C. GK/GS has the lowest variability and C has the highest. Positions involving goal have higher median heights and lower variability than those that do not.

11 Both male and female histograms are bimodal as they display double-peaks. The male histogram has peaks at ages 25–29 and ages 40–44, whereas, the female histogram has peaks at ages 25–29 and ages 50–54. The location for both histograms is similar, as well as the spread.

Chapter 8

Pre-test

1 a 3 **b** 12
c −5 **d** 0
2 a 2 **b** 13
c −6 **d** 0
3 a \$66 **b** \$176
c \$55 **d** \$143
e \$104.50
4 a \$90 **b** \$170
c \$150 **d** \$230
e \$120
5 a 8 **b** 12
c 13 **d** 23

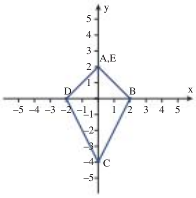
Exercise 8A

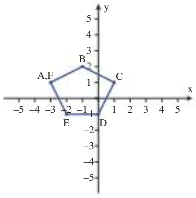
1 **a** coordinate, horizontal
b distance, vertical
c Cartesian, positive, negative

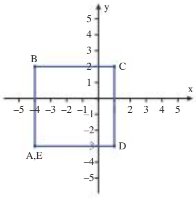
2 **a** 3 **b** −1 **c** −2
d 0 **e** −2 **f** 0
g −3 **h** 0

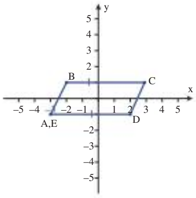
3 **a** (1, 1) **b** (5, 0) **c** (3, 4)
d (0, 4) **e** (−1, 2) **f** (−3, 3)
g (−5, 1) **h** (−3, 0) **i** (−4, −2)
j (−2, −5) **k** (0, −3) **l** (2, −3)
m (5, −5)

4 **a** It is a kite.
b It is a pentagon.
c It is a rectangle.
d It is a parallelogram.



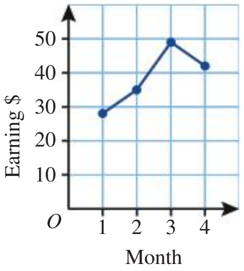






5 **a**

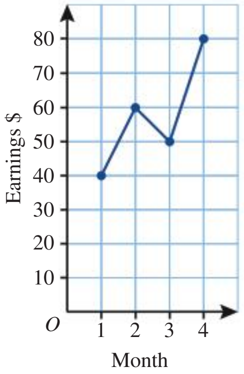
Month	1	2	3	4
Earning $	28	35	49	42



b See graph **c** See graph
d Month 3 **e** $154

6 **a**

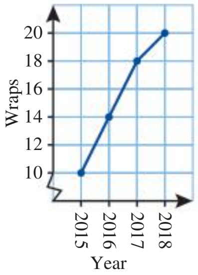
Month	1	2	3	4
Earning $	40	60	50	80



b See graph **c** See graph
d Month 4 **e** $230

7 **a**

Year	2015	2016	2017	2018
Wraps	10	14	18	20



b See graph **c** 2018
d 62

8

Year	2016	2017	2018	2019
Stock price $	26.50	27.50	27.00	26.50

He should have sold his stock in 2017 to make $27 500.

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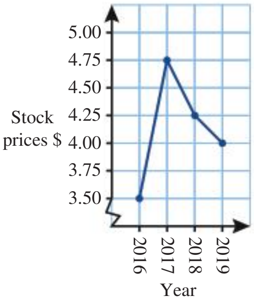
9

Year	2016	2017	2018	2019
Stock price $	3.50	4.75	4.25	4.00

She should have sold her stock in 2017 to make $9500.



Exercise 8B

1 **a** table, substitute **b** negative, number

c positive, equals

2 **a** 6 **b** –10 **c** 5

d –2 **e** 0 **f** –4

3 **a** –12 **b** 6 **c** 1

d –6 **e** –3 **f** 15

4 **a**

x	–2	0	2
y	–6	0	6

b

x	–3	0	3
y	6	0	–6

c

x	–2	0	2
y	1	3	5

d

x	–3	0	3
y	–5	–2	1

e

x	–2	0	2
y	–7	1	9

f

x	–3	0	3
y	4	–2	–8

5 **a**

x	1	2	3
y	110	170	230

b $200

6 **a**

x	2	4	6
y	150	240	330

b $262.50

7

x	0	2	4
y	15	11	7

8

x	1	3	5
y	850	550	250

9 **a** Approx. 8 weeks **b** Approx. 7 weeks

10 $y = 50x + 120$

x	1	2	3	4	5
y	170	220	270	320	370

Exercise 8C

1 **a** graph, predictions

b create, range

c Computer, easily, accurately

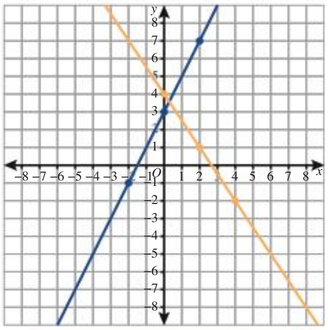
2 **a** x from 1 to 3, y from –3 to 7

b x from –2 to 2, y from –5 to 3

c x from –3 to 3, y from –12 to 16

d x from 0 to 6, y from 0 to 26

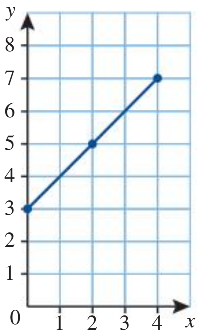
3 **a** and **b**



4 **a** **i**

x	0	2	4
y	3	5	7

ii x from 0 to 4, y from 3 to 7



iii See graph

iv See graph

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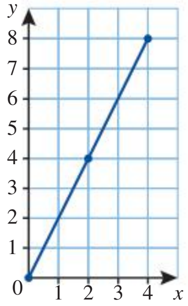
b **i**

x	0	2	4
y	0	4	8

ii *x* from 0 to 4, *y* from 0 to 8



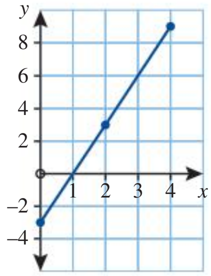
iii See graph

iv See graph

c **i**

x	0	2	4
y	–3	3	9

ii *x* from 0 to 4, *y* from –3 to 9



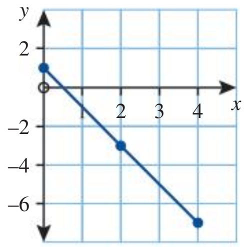
iii See graph

iv See graph

d **i**

x	0	2	4
y	1	–3	–7

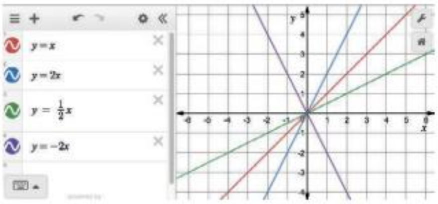
ii *x* from 0 to 4, *y* from –7 to 1



iii See graph

iv See graph

5 **a**



$y=x$

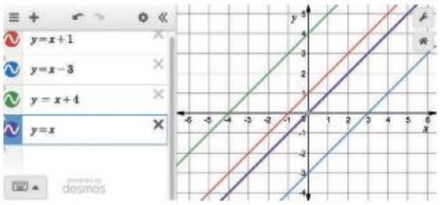
$y=2x$

$y=\frac{1}{2}x$

$y=-2x$

b The bigger number makes the slope steeper and the negative number makes the line go downhill.

c



$y=x+1$

$y=x-3$

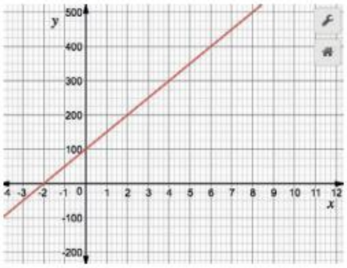
$y=x+4$

$y=x$

d The number at the end is where the line crosses the *y*-axis.

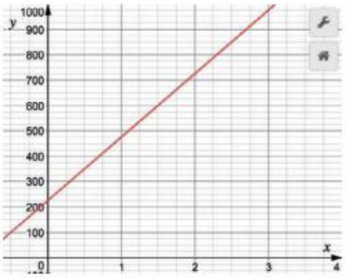
6 **a** See graph

b $450

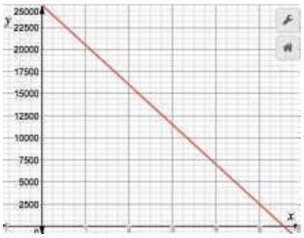


7 **a** See graph

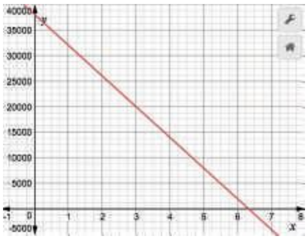
b $975



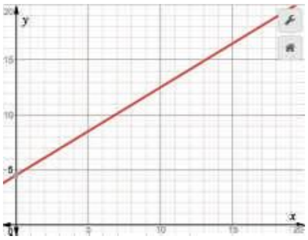
8 **a** See graph
b 5 years



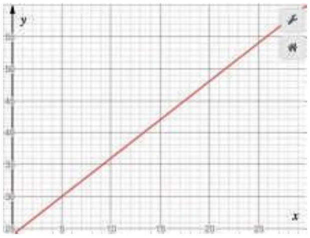
9 **a** See graph
b 4 years



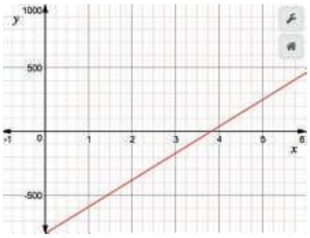
10 \$18.10



11 \$55.20



12 $y = 210x - 800$. At least 4 surfboards need to be sold each day to make a profit.



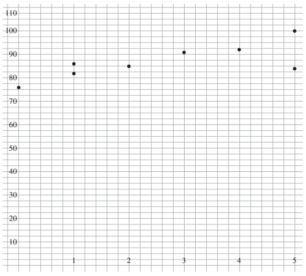
Exercise 8D

1 **a** positive, left, right
b scatterplot, straight
c pattern
d weak

2 **a** Yes **b** No
c Yes **d** No
e Yes **f** Yes

3 **a** Positive, linear, strong
b Negative, linear, weak
c Positive, linear, strong
d Positive, linear, moderate
e Positive, non-linear, strong
f None, non-linear, none

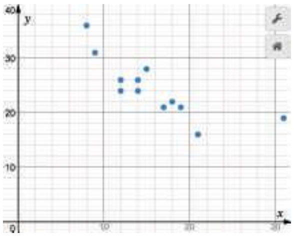
4 **a** Positive **b** Linear
c Moderate



5 Positive, linear and moderate correlation exists, which indicates that students achieve higher if they get more sleep.

6 Negative, linear and weak correlation exists, which indicates that age definitely does affect the eyesight of drivers but at varying rates of decline.

7 There is a negative, linear, strong correlation between the number of police and the incidence of crime with only one outlier. The police union are correct with their assertions.



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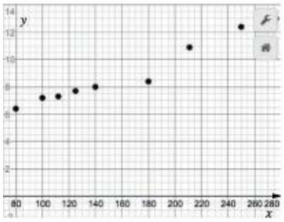
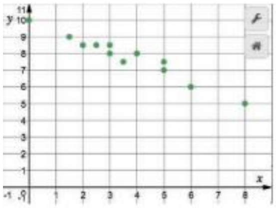
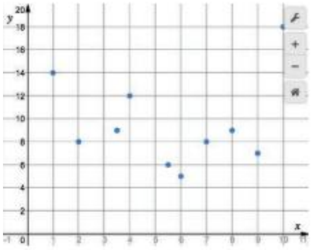
8 There is a positive, linear, strong correlation between the amount of fertiliser and the crop yield. The CSIRO can demonstrate the positive effect of fertiliser on crop yields.



9 There is a negative, linear, strong correlation that the more hours of video games being played on average results in less average sleep.

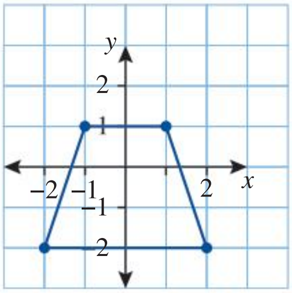


10 There is a negative, linear, weak correlation with an outlier. This would indicate that, although practice does generally improve the points scored by the players, some players improve at a greater rate than others.



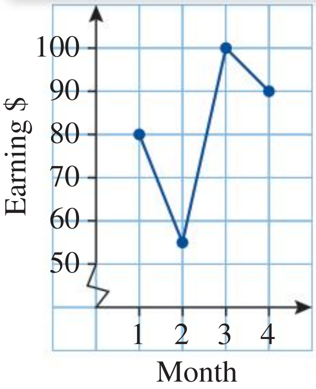
Chapter checklist

1 trapezium



2 a

Month	1	2	3	4
Earning $	80	55	100	90



b See graph

c See graph

d 3rd month

e $325

3

x	−2	0	2
y	−8	−2	4

4 a

x	1	2	3
y	110	180	250

b $215

5 a

x	−2	0	2
y	−1	3	7

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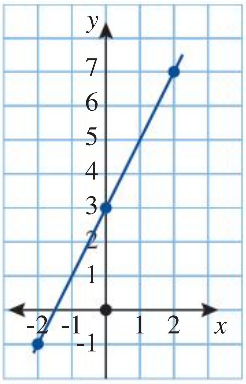
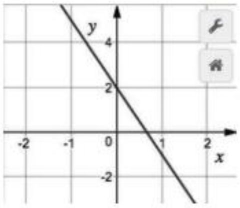
b x from −2 to 2, y from −1 to 7



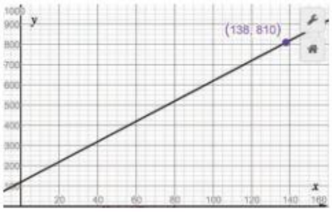
c See graph

d See graph

6

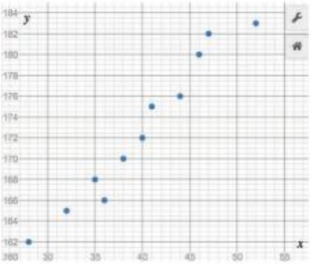


7 The total fare will be $810.



8 positive, linear, strong correlation

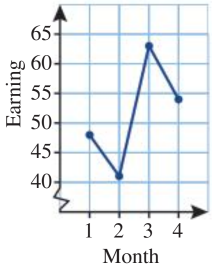
9 There is a positive, linear, strong correlation between the length of the students forearms and their heights as shown on the scatterplot.



Chapter review

1 a

Month	1	2	3	4
Earning	$48	$41	$63	$54



b See graph

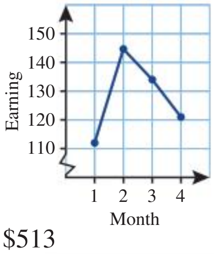
c See graph

d 3rd month

e $206

2

Week	1	2	3	4
Earning	$112	$146	$134	$121



$513

3 a

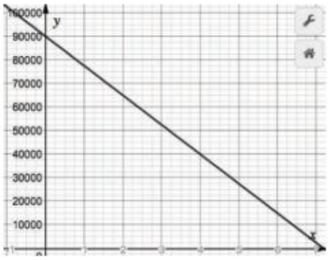
x	0	1	2	3
y	$20	$58	$96	$134

b At 3 hours, he has earned more than $100.

4

x	1	3	5
y	$1700	$1100	$500

5 a See graph



b After 6 years

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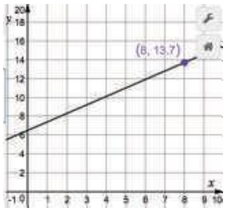
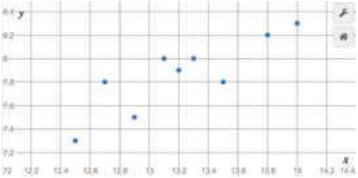
6 The total fare for 8 km is \$13.70.



7 There is a positive, linear, moderate correlation with an outlier, which would indicate that it is correct to believe that the warmer the temperature, the more visitors come to the beach.

8 The graph indicates that there is a positive, linear, weak correlation between the temperature and the area of ice at Antarctica.

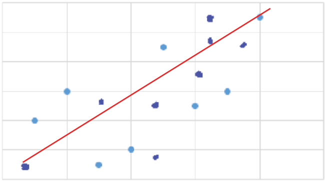


This would say that temperature has a definite effect on the area of ice but there are probably other factors affecting it as well.

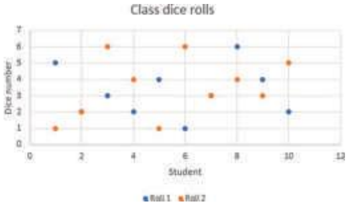
Chapter 9

Pre-test

1



2



3 **a** 15 **b** 3

Exercise 9A

1 **a** change
  b horizontal (x-axis), vertical (y-axis)
  c dependent, variable
  d independent, variable
  e change, controlled, response

2 Answers may vary
  a A variable is something that can be measured. It can be basically anything, such as things, time, feelings, events, or age.
  b Independent variable can be changed by the researcher.
  c Dependent variable changes because of the independent variable.

3 **a** Independent variable: amount of medication, because it is the thing that Alix changes to see what effect it has.
  b Dependent variable: breathing rate, because it is the thing that Alix measures to see if it is changed by the amount of medication.

4 **a** Independent variable: amount of fertiliser. It is what Michael is changing.
  b Dependent variable: amount (mass) of beans produced per plant. It is what we measure in the expectation that it will be changed by the independent variable.

5 **a** Independent variable: ocean temperature. Although it has to be measured, Sonia ‘changes’ it by selecting different locations which have different temperatures.
  b Dependent variable: amount of coral. It is what is being measured.

6 **a** Independent variable: amount of time listening to classical music. It is what Noah sets.
  b Dependent variable: reading test results each student received. It is what is being measured in the expectation that it may be changed by the time spent listening to classical music.

7 **a** Independent variable: how long each student studies. It is what each student decides. It is also on the *x*-axis.

b Dependent variable: score each student received on their exam. It is what is being measured because we think it is affected by hours of study. It is also on the *y*-axis.

8 **a** Independent variable: how many pamphlets were delivered. It is the cause. ‘Number of pamphlets delivered’

b Dependent variable is the amount of pay received. It is what is being affected, the effect. ‘Pay ($)’

Exercise 9B

1 **a** relationship

b straight, scatter

c trend, regression

d represent, trend

e independent, dependent

f relationship, variables, scatterplot

2 **a** To draw a scatter plot:

- Always use grid paper.
- Determine which observations are dependent and which are independent.
- Place the dependent variable on the *y*-axis.
- Place the independent variable on the *x*-axis.
- Use each paired value to plot the graph.

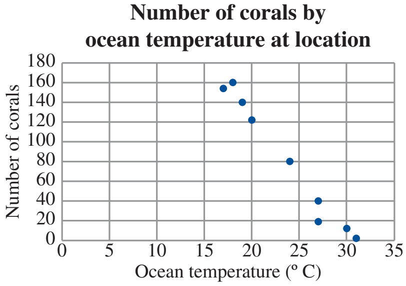
b To find the line of best fit by eye:

- Find the mean of the independent variables on the *x*-axis.
- Find the mean of the dependent variable on the *y*-axis.
- Plot the mean, using a different colour pen.
- Put your ruler on its edge so you can see both sides of the plot easily.
- Position the ruler on the mean to have approximately the same numbers of plots either side of it, keeping the line going through the mean.
- Use your ruler to draw the line of best fit.

3 **a** Independent variable: the ocean temperature. It is the temperature at the locations Felicity has chosen.

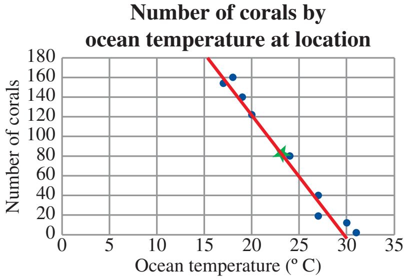
Dependent variable: the amount of coral. It is what is being measured to see if it is affected by the ocean temperature.

b



c 23.67, 81, shown on the graph below (★)

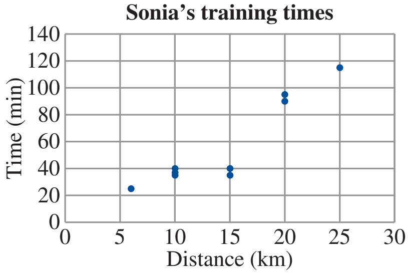
d



4 **a** Independent variable is the distance Sonia ran.

Dependent variable is the time each run took. It is what is being affected by the distance.

b



c 56.9, 14.6 shown on the graph in part **d** (★)

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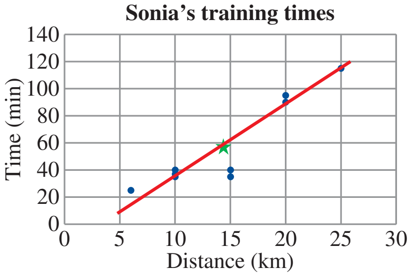
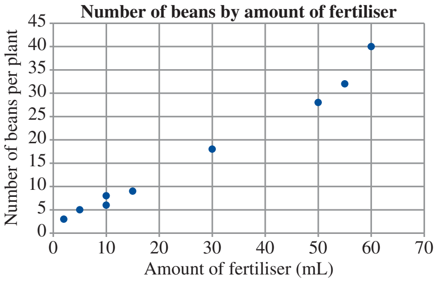
d



5 **a** Independent variable: the amount of fertiliser. It is what David chooses. Dependent variable: amount of beans produced by each plant. It is what is being affected.

b



c 26.33, 16.56 (★)

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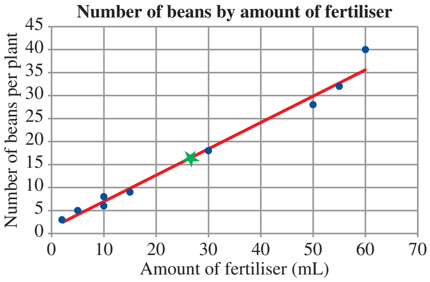
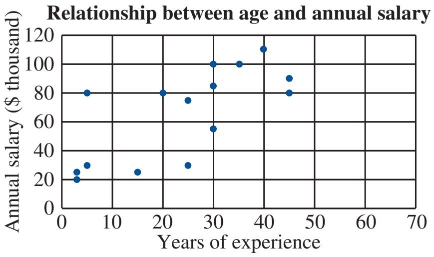


6 **a**



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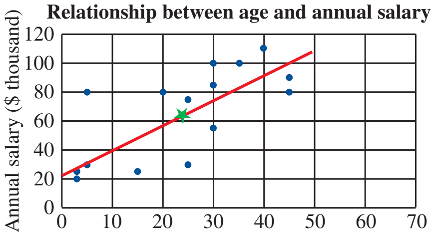
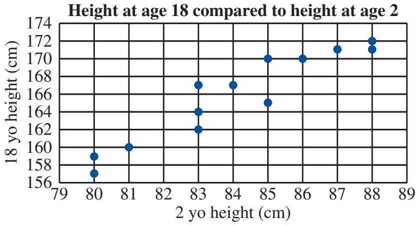


7 **a**



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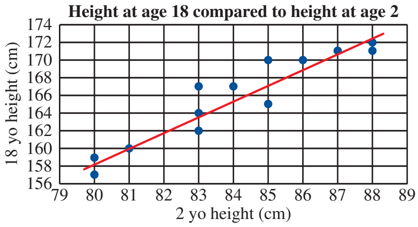
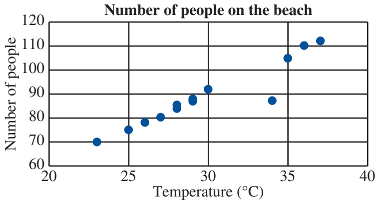


8 **a**



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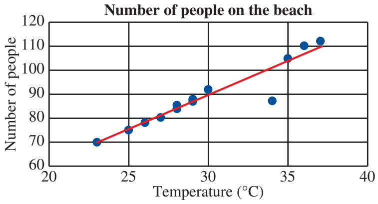
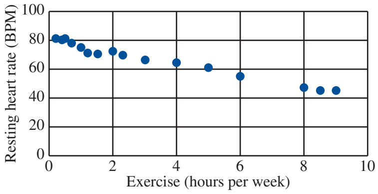


9 **a**



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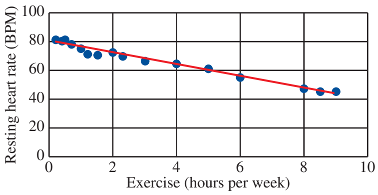
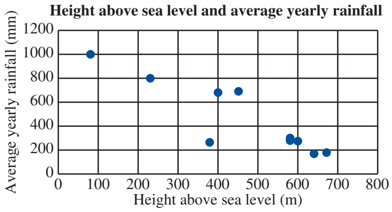


10 **a**



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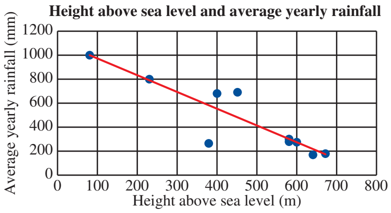


Exercise 9C

1 **a** Regression, relationships

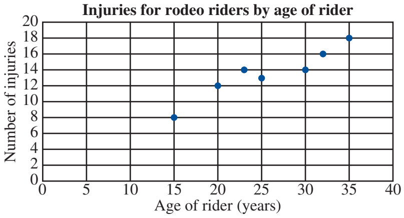
b strength

c positively, negatively, together

d increases, decreases

2 **a** Dependent variable: number of injuries; independent variable: age of the riders.

b



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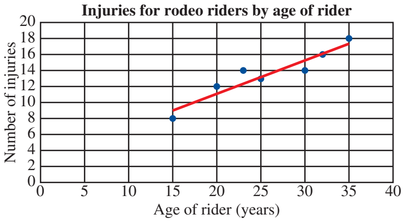

d The correlation is positive, linear and strong.

e There is a strong relationship between the age of the rodeo riders and the number of injuries. The correlation is positive, as both variables are increasing in tandem with each other. As the age of the rider increases, the number of their injuries also increases.

f Answers may vary: As the rodeo rider ages they need to be more aware of the risks involved, to avoid injuries.

3 **a** Dependent variable: amount of sleep; independent variable: age of the person.

b Overall, correlation is negative, linear and moderate. However, the correlation looks to be different for children compared to adults. A line of best fit for ages 0 to 10 is negative, linear and strong. For older children and teenagers, the relationship may be non-linear.

c Over a lifetime, there is a moderate relationship between the amount of sleep someone requires and their age. The correlation is negative, as when one variable increases the other decreases. As the age of the person increases, the amount of sleep they have decreases. The correlation is stronger for young children and may be non-linear for older children and teenagers.

d Answers may vary: The data confirms that children sleep more than adults. People may need to adjust to having less sleep as they get older.

4 **a** Independent variable: amount of time playing video games; dependent variable: their grades.

b The correlation is negative, linear and moderate.

c There is a moderate relationship between the amount of time spent playing video games and their grades. The correlation is negative, as the time spent playing increases, the grade score decreases.

d Answers may vary: If you want to receive higher grades, you need to lessen the amount of time playing video games.

5 **a** Dependent variable: weight (kg); independent variable: amount of exercise done each day (min)

b The correlation is negative, linear and strong.

c There is a strong relationship between the amount of time spent exercising and people's weight. The correlation is negative, as when one variable increases the other decreases. It is strong, as most of the points are directly on the line of

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best fit. As the time spent exercising increases their weight decreases.

d Answers may vary: If you want to reduce your weight, you need to increase the amount of time you spend exercising.

6 a The correlation is positive, linear and weak. There is a very weak relationship between the overall marks and the maths marks. The correlation is positive, i.e. an increase in the overall mark correlates with an increase in the maths mark. It is weak as most of the points are scattered from the line of best fit. The line of best fit is nearly horizontal; therefore, the relationship is very weak.

b There is a slight relationship between the two variables; however, the teacher would need to obtain more results each year to be able to determine if there is an actual relationship between his students' maths results and overall results.

7 a There is no correlation, as the line of best fit is horizontal.There is no relationship between their BMI and hand grip. The BMI can vary and the hand grip can vary, but there is no correlation between them.

b According to this data there is no relationship between the two variables. BMI does not seem to change due to a person's strength in their hand grip.

8 a The correlation is positive, linear and strong. There is a strong relationship between the number of weeks and the income. The relationship is positive, as the data shows when the number of weeks that have passed increases the income also increases. The relationship is strong as the data is very close to the line of best fit.

b Will's business is doing well, and he should expect to increase his income as each week increases.

9 a Looking at the data there does seem to be a strong relationship between exporting local apples and the fatality rate on the highway. The points show a strong, linear negative correlation, as the points are very close to the line of best fit. This also shows that as the exporting of local apples increases, the fatality rate on the main highway decreases.

b Answers may vary: Although there appears to be a relationship there is no common-sense explanation for it and we can't say that exporting more apples causes the road fatalities to reduce. The most likely explanation is that it is a coincidence that apple exports have gone up while fatalities have decreased. Another possibility is that improvements to the highway out of the area has made the road safer and also made the cost of road transport cheaper so increasing the apples exported.

Exercise 9D

1 a strength, measured
b strength, coefficient
c (r), strength, direction
d $-1, +1$
  - **i** perfect positive relationship
  - **ii** 0.75 – 1
  - **iii** 0.50 – 0.75
  - **iv** weak positive relationship
  - **v** 0
  - **vi** −0.25 – −0.50
  - **vii** moderate negative relationship
  - **viii** −0.75 – −1
  - **ix** −1

2 a Correlation is a measure of the strength of the linear relationship between two variables.

b The correlation coefficient (r) measures the strength and direction of a linear relationship between variables.

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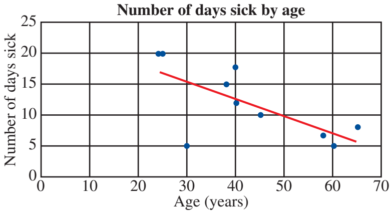
3 **a** Correlation coefficient is –0.997.

b The correlation coefficient is very close to being –1; therefore, the relationship is a very strong negative relationship. This means that as the study time increases the errors decrease.

4 **a** Correlation coefficient is 0.05.

b The correlation coefficient is very close to being 0; therefore, there is no relationship. This means that according to this data there is no relationship between the men's heights and their weights.

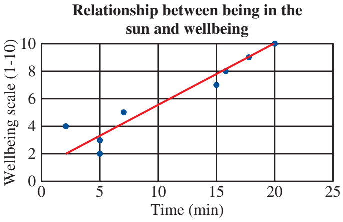
5 **a**



b Correlation coefficient is –0.68.

c The correlation coefficient is between –0.5 and –0.75. This means that the relationship between the age of the patient and their sick days is a moderate negative correlation. As age increases the number of sick days decrease. This suggests that the condition becomes less severe with age.

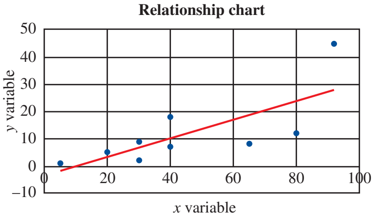
6 **a**



b Correlation coefficient is 0.96.

c The correlation coefficient is nearly +1, which means that there is a very strong correlation between the amount of time in the sun and a sense of wellbeing. As the time in the sun increases, the sense of wellbeing also increases.

7 **a** Answers may vary



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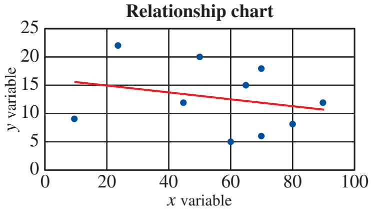
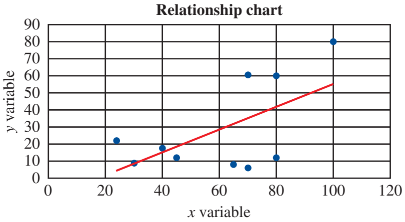


Answers may vary

c



Answers may vary

Exercise 9E

1 **a** predictions, in, out

b $y = mx + c$

c gradient, slope

d y-intercept, y-axis

e Interpolation

f Extrapolation

2 **a** Interpolation is predicting an outcome based on estimating a value on the line of best fit.

b Extrapolation is predicting an outcome based on extending the line of best fit, not within the range of the particular dataset.

3 A: $r = -0.94$, is the most reliable as it is the closest to either 1 or –1.

4 A: weak linear negative is the least reliable as it has the weakest relationship between its variables.

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5 **a** Mindy's weight would have been about 75 kg when she had exercised for 50 days.
b According to the line of best fit Mindy had been exercising for 70 days when her weight reached 70 kg.
c The equation of the line of best fit is $y = -0.4x + 94$
d 46 kg
e The prediction in **d** is unreliable because 46 kg is an unhealthily low body weight for an adult female and she is likely to either have reached her target weight or she would have been advised to stop losing weight. This example illustrates the dangers of extrapolating beyond the data: the line of best fit predicts Mindy would eventually reach the point of weighing nothing, which is impossible.

6 **a** The average amount of sleep for someone who is 25 would be about 9 hours.
b 52 years old
c $y = -0.14x + 13.6$
d 2.4 hours
e The prediction is unreliable because extrapolation also predicts that very old people have no sleep to a negative amount of sleep, which is impossible. Common sense tells us that the elderly still sleep on average more than a couple of hours per night.

7 **a** 0.7 kg per week.
b 19 chocolate bars per week.
c $y = 0.05x + 0.08$
d 98.4 chocolates per day
e This prediction is not reliable and also unrealistic, as eating that number of bars per week would be difficult and likely to cause illness that would slow or stop consumption. In addition, weight gain has a lot more variables to consider than eating chocolates.

8 **a** Approximately 12 errors.
b About 55 minutes of study.
c $y = -0.4x + 26$
d −10 hours.
e This prediction is not just unreliable but impossible. It is not possible to study a negative number of hours, which demonstrates the dangers of extrapolation.

9 Extrapolation in business is fraught with danger. The correlation coefficient of 0.98 is very strong, but one must not let it lead to assuming that it is possible to increase sales beyond 80 000 per week. There are many possible barriers to this: there may simply not be enough customers or demand to sell more than this, or the business may not have the capacity to go beyond this, or the cost of selling and distributing greater numbers may be prohibitive. Extrapolation is only valid in situations that are realistically attainable.

Exercise 9F

1 **a** relationship, causes
b impact
c strength, variables
d imply

2 **a** Correlation is a measure of the strength of the relationship between two variables.
b Causation states that any change in one variable will cause a change in another variable.

3 **a** There is a strong positive correlation between the energy and fat. As the amount of fat increases, the amount of energy increases.
b There is causality, as fat in the diet can be used by the body for energy. The amount of fat affects the amount of energy.

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4 a There is a strong negative correlation between reading books and weight loss. As the time spent reading books increases, the weight decreases.

b There is no causality, as logically a change in book-reading habits should not affect weight. There may be other factors at play, for example, she may also be eating less or exercising more on days she reads more.

5 a The line of best fit is nearly horizontal, which means there is no relationship between people's BMI and the strength of their hand grip.

b There is no causality. The strength of a person's hand grip does not cause a change in their BMI. There are many other variables that determine the change in people's BMI (weight, exercise, and eating habits).

6 a The data does show a strong positive correlation ($r = 0.9971$), as the correlation coefficient is nearly one.

b There is no obvious causality between the variables. For causality there would need to be something prevalent in organic food known to cause autism. This graph does not show a cause or effect between organic food sales and autism, and if different scales were used for the graph the curves would not match so well.

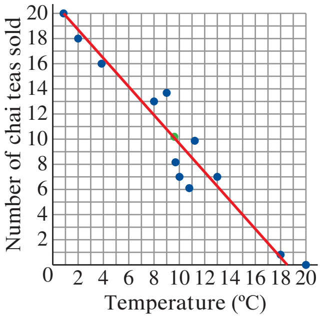
7 a The data does show a positive correlation, as lines of best fit for both datasets would be closely matched.

b There is no causality between the variables, as there is no mechanism known for consuming mozzarella cheese leading to someone studying engineering.

Chapter checklist

1 The independent variable is the temperature, as it is independent of the number of chai teas sold. The dependent variable is the number of chai teas sold, as it is dependent on the temperature; the colder the day, the more people feel like having chai.

2



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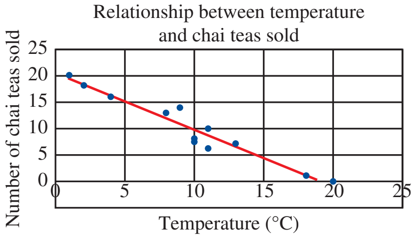

4 The correlation is strong negative. As the temperature decreases, the number of chai teas sold increases.

5 The correlation coefficient is –0.962647417.

6 Answers may vary.

When the temperature is 5°C, 15 cups of chai tea will be sold.

When the temperature is 30°C, –12 cups of chai tea will be sold.

7 It is impossible to sell a negative number of cups of chai; therefore, the prediction by extrapolation in the previous question is not valid. The danger of extrapolation is extending it to impossible situations.

8 With this data there is a strong correlation of –0.96, which being very close to 1, shows a very strong negative relationship. This data has some causality, as some

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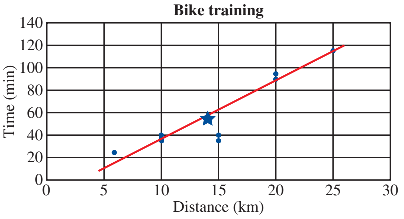
people may be influenced by the temperature to cause them to choose to have chai tea. However, there are many other influences, for example tea/coffee preference, taste preferences, socioeconomic influences.

Chapter review

1 **a** The dependent variable is the growth of his crops, as it is affected by high summer temperatures, not the other way around.

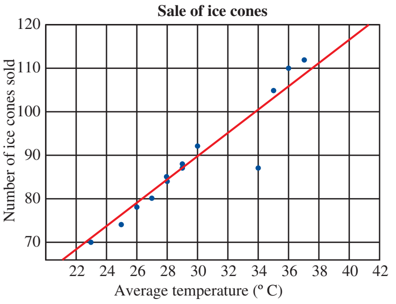
b The independent variable is the temperature, as it is not affected by what Jon does, but happens naturally.

2



3 Dependent variable is growth of snowpeas; independent variables include sunlight, water, fertiliser, seed type, temperature

4



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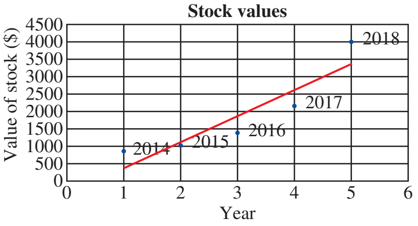


6 **a** There is a moderate relationship between energy output and fat. The correlation is positive, as fat (g) in the diet increases, the energy output (kilojoules) increases.

b There is a weak relationship between energy output and salt in the diet. The correlation is negative, as when one variable increases the other decreases. As salt (mg) in the diet decreases, the energy output (kilojoules) increases.

c There is no relationship in this graph between energy output and fibre in the diet, as the line of best fit is nearly horizontal.

7 The correlation coefficient for the data is 0.939290.

8 **a** Rachel was able to row 40 times in week 4.

b 160 rows

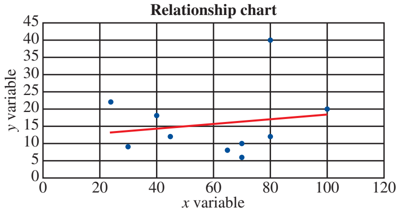
c 200 rows

d There is a risk, as Rachel's muscles may not be able to cope with the continuous increase. She may not be physically able to increase every week, showing the dangers of extrapolation. There is a limit to the maximum number of rows she can achieve, i.e. the graph can't keep going upwards.

9 **a** There is a strong negative correlation between study time and number of errors. As the amount of study time increases, the number of errors decreases.

b There is causality, as there is a reasonable mechanism to explain how more study time leads to fewer errors. The amount of study affects the number of errors.

10 **a** Weak positive (answers will vary)



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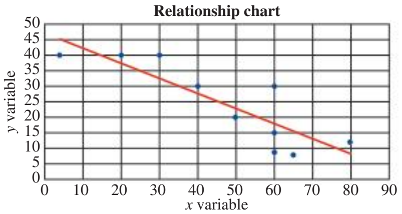
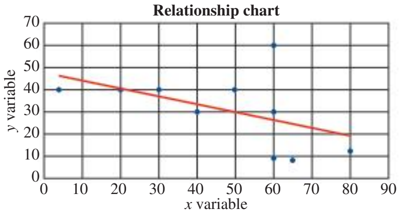
b Strong negative (answers will vary)



c Moderate negative (answers will vary)



11 Answers will vary.
Interpolation: After approximately 62 000 sales, the profit will be \$8000.
Extrapolation: After approximately 100 000 sales, the profit will be \$12 335.
The dangers of extrapolation in this graph is that sales and profit are very unpredictable and cannot be guaranteed to hold their trend. It may not be physically possible to sell more than 80 000.

12 **a** Lines of best fit for these data points would show a weak positive relationship between the consumption of sour cream and motorcycle accidents.

b There is no causality between the consumption of sour cream and motorcycle accidents, as there is no mechanism to explain how one variable could change the other.

Chapter 10

Pre-test

1 **a** 0.45 **b** 0.44 **c** 0.60

2 **a** 0.17 **b** 0.31 **c** 0.05

3 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{6}{7}$

4 **a** 19.75% **b** 13.89% **c** 22.22%

5 **a** 48.75 **b** 42.86 **c** 22.17

6 **a** 4.5 **b** 82.8 **c** 1.8

7 **a** $\{1,2,3,4,5,6\}$

b **i** $\frac{1}{6}$ **ii** $\frac{3}{6} = \frac{1}{2}$ **iii** $\frac{4}{6} = \frac{2}{3}$

iv $\frac{4}{6} = \frac{2}{3}$

8 **a**

Outcome	**Tally**	**Frequency**
Odd/Boy	|||| |||	8
Even/Girl	||||	4
Total	12	12

b Relative frequency of boys $= \frac{8}{12}$

c Expected boys $= 12 \div 2 = 6$

d The simulation resulted in more boys than would be expected. You would expect half the babies to be boys, which would be 6 in this size sample.

Exercise 10A

1 **a** simulation, coin, die, spinner

b technology, random, calculator, decimal, Excel, between

2 **a** Answers will vary but should be around 5/10/5.

b Answers will vary but should be around 50%.

c The three rows of the table may have been deceptive. You could think that the three outcomes are equally likely, but BG is the same as GB so is more common than either of the single sex combinations.

3 **a** Answers will vary but should be around 3/7/7/3.

b Answers will vary but should be around 75%.

c The four rows of the table may have been deceptive. You could think that the four outcomes are equally likely but as an example, BBG is the same as GBB and BGB so is more common than either of the single sex combinations.

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4 a

Toy number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency	1	3	2	4	4	3	6	2	8	2	4	2	4	3	5	1	3	5	5	2

b 69 simulations

c $69 \times \$30 = \$2070$

d Toy number 9 was collected 8 times. Toy number 1 was the last one to complete the collection.

e Average $= \frac{69}{20} = 3.45$

5 a

Tie number	1	2	3	4	5	6	7
Frequency	4	1	3	1	5	0	6

b Tie number 7 is worn 6 times in 4 weeks. Tie number 6 is not worn at all.

c

Week	Monday	Tuesday	Wednesday	Thursday	Friday
One	5	7	1	1	4
Two	1	3	7	3	7
Three	1	5	7	5	7
Four	7	5	5	3	2

6 times the same tie is worn more than once in the same week.

d

Week	Monday	Tuesday	Wednesday	Thursday	Friday
One	5	7	1	1	4
Two	1	3	7	3	7
Three	1	5	7	5	7
Four	7	5	5	3	2

On two occasions, the simulation shows Ian wears the same tie on two consecutive days. Tie number one in week 1 and Tie number 5 in week 4.

e Expect each tie to be worn $\frac{20}{7} \approx 3$ times each. The results for ties 2, 4, 5, 6 and 7 are different to what you would expect.

6 a i Not suitable as the chance of selecting a particular card is lowered once it has been removed from the pack.

ii Suitable method. The chance of selecting a card remains the same for each trial.

iii Not suitable. The lowest sum is $1+1=2$ so impossible to get a January outcome in the simulation.

iv Suitable. All outcomes are equally likely.

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b

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally	|		|		||	|		|	|	|	|	|
Frequency	1		1		2	1		1	1	1	1	1

c It took 10 trials to repeat a result.

7 **a**

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally	𝍸	|||	𝍸	||	𝍸	||	||||	𝍸 |	|	|||	||	||
Frequency	5	3	5	2	5	2	4	6	1	3	2	2

b The months were not equally represented. Month 8, August, appeared 6 times in the simulations while month 9, September, only appeared once.

8 **a** Results will vary but will be at most 13 trials.

b Results will vary.

9 **a** Results will vary.

b Results will vary but will be around 2.5–3 children.

10 **a** $\% = \frac{283}{315} \times 100 = 89.8\% \approx 90\%$

b Results will vary but should be around 27 successes and 3 misses.

c Results will vary but should be approximately 90%.

d Expected goals $= \frac{90}{100} \times 30 = 27$

e Results will vary but should be similar if simulation is successful.

Exercise 10B

1 **a** previous, chance

b equal, different, same

c unusual, trials

d reliability

2 **a** 2

b Approximately $20 \div 2 = 10$

c

H	T	H	T	H	H	T	H	T	T
T	T	H	H	T	T	T	T	H	H

9 times. This is very close to what was expected.

d $20 \div 2 = 10$

e Heads = 9, Tails = 11. This is close to what was expected.

3 **a** 6

b Approximately $30 \div 6 = 5$ times

c

4	3	4	5	2	1	5	6	5	6
1	3	1	2	3	5	5	1	4	4
2	4	1	1	4	3	4	4	4	4

6 times. This is close to what we expected

d $30 \div 6 = 5$ times

e There are more 4s and less 2s and 6s than expected.

Outcome	1	2	3	4	5	6
Frequency	6	3	4	10	5	2

4 **a** $20 \div 6 = 3.33$ So around 3 times each

b **i** More 4s and 5s than expected, less 1s

ii More 1s and 2s than expected, no 5s

c

	Dice Outcome Frequency						
	1	2	3	4	5	6	Total
Total	37	32	30	33	36	32	200

d $200 \div 6 = 33.33$ So around 33 times each

e There are more 1s and 5s than expected and less 3s.

5 **a** No. Each number has the same chance of being drawn each time.

b **i** 1 **ii** 3

c More people are choosing the ‘Hot Numbers’ so when they come up, they are not as profitable.

d **i** 0 **ii** 3

iii 0 **iv** 1

e Yes, the Cold Numbers would have won a division 6 prize.

6 Technology activity

As the number of trials increases, the two outcomes are evenly distributed.

7 **a** As the number of trials increases, the six outcomes are evenly distributed.

b As the number of trials increases, the distribution becomes symmetrical with outcomes around 7 having the highest frequency.

Exercise 10C

1 **a** chance

b favourable, outcomes

c ranges, not, will

d estimate, survey

e frequency, observed

2 **a** 0.56 **b** 0.47

c 0.62 **d** 0.02

3 **a** 159 **b** 539

c 216 **d** 85

4 **a** 26

b $\frac{10}{26} \approx 0.38$

c $\frac{4+2+0+1}{26} = \frac{7}{26} \approx 0.27$

d $\frac{10+4+2+0+1}{26} = \frac{17}{26} \approx 0.65$

e $\frac{3+6}{26} = \frac{9}{26} \approx 0.35$

5 **a** $\text{RF}(1) = \frac{2}{5} = 0.40$, $\text{RF}(2) = \frac{1}{4} = 0.25$, $\text{RF}(3) = \frac{2}{4} = 0.50$, $\text{RF}(4) = \frac{1}{4} = 0.25$, $\text{RF}(5) = \frac{1}{6} = 0.17$, $\text{RF}(6) = \frac{1}{3} = 0.33$, $\text{RF}(7) = \frac{1}{3} = 0.33$

b $\text{RF}(1) = \frac{3}{5} = 0.60$, $\text{RF}(2) = \frac{1}{4} = 0.25$, $\text{RF}(3) = \frac{3}{4} = 0.75$, $\text{RF}(4) = \frac{2}{4} = 0.50$, $\text{RF}(5) = \frac{3}{6} = 0.50$, $\text{RF}(6) = \frac{2}{3} = 0.67$, $\text{RF}(7) = \frac{1}{3} = 0.33$

c **i** Horse 3 has the best chance of winning based on past performances.

ii Horse 3 has the best chance of placing based on past performances.

6 **a** 5361

b $\frac{137}{5361} = 0.03$

c Numbers 1–31 are favourable, $\frac{4147}{5361} = 0.77$

d All numbers are equally likely. Past performance does not influence future outcomes.

7 **a** 192 582 **b** 0.16

c 0.23 **d** 0.04

8 **a** $105 \div 5 = 21$ pieces of candy per 20 grams

b $20\text{ g} \times 9 = 180\text{ g}$, $21 \times 9 = 189$ pieces of candy

c $\text{RF} = \frac{7}{38} \approx 0.18$

d $\frac{7}{38} \times 189 = 34.8$, so approximately 35 green candies per 180 g bag

e $50 \div 35 = 1.4$, so she will need $2 \times 180\text{ g}$ bags

Exercise 10D

1 **a** techniques, simulation

b selecting, return, next

c selection, same

2 **a** Answers will vary.

b ii, iii, iv all have problems with maintaining consistently random results as they are either not returning selected cards or shuffling between selections.

3 **a** Answers will vary.

b i: issue is always starting in the same place

ii: starting with previous result may cause issues if not tossed properly

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iv: low toss may not get enough spin in to get a random result

4 a Answers will vary.

b ii: unlikely to get enough spin to obtain a random result

5 a Answers will vary but should be fairly evenly distributed.

b Answers will vary but should be around $\frac{0 \times 13 + 0.5 \times 13 + 1.00 \times 13 + 1.5 \times 13}{52} = \frac{39}{52} = \$0.75$

c i \$0 discount is no longer an option

ii Answers will vary but should be around 13.

iii Answers will vary but should be around.

Discount	0	0.50	1.00	1.50
Relative frequency	$\frac{4}{52} \approx 0.08$	$\frac{16}{52} \approx 0.31$	$\frac{16}{52} \approx 0.31$	$\frac{16}{52} \approx 0.31$

iv Answers should be around

$$\frac{0 \times 4 + 0.5 \times 16 + 1.00 \times 16 + 1.5 \times 16}{52} = \frac{48}{52}$$
$$= \$0.92$$

d Only have three options available.

Exercise 10E

1 a two, combinations

b space, product

c systematically, second

d cannot

2 a i Size = 6 **ii** Size = 2

iii Size = 2 **iv** Size = 4

b i Sample Space = 1, 2, 3, 4, 5, 6

ii Sample Space = Heads, Tails

iii Sample Space = Red, Black

iv Sample Space = Hearts, Diamonds, Clubs, Spades

3 a Size = 3 × 4 = 12

b

Body type

Colour

	Hatch	**Sedan**	**SUV**
White	White Hatch	White Sedan	White SUV
Red	Red Hatch	Red Sedan	Red SUV
Grey	Grey Hatch	Grey Sedan	Grey SUV
Blue	Blue Hatch	Blue Sedan	Blue SUV

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4 **a** Size $= 2 \times 2 = 4$

b

		Baby 1	
		Boy	**Girl**
Baby 2	**Boy**	Boy, Boy	Girl, Boy
	Girl	Boy, Girl	Girl, Girl

5 **a** Size $= 3 \times 3 = 9$

b

Chicken, Steamed Rice	Chicken, Fried Rice	Chicken, Noodles
Beef, Steamed Rice	Beef, Fried Rice	Beef, Noodles
Prawns, Steamed Rice	Prawns, Fried Rice	Prawns, Noodles

6 **a** Size $= 3 \times 5 = 15$

b

S & P Calamari, Steak	Garlic Prawns, Steak	Satay, Steak
S & P Calamari, Schnitzel	Garlic Prawns, Schnitzel	Satay, Schnitzel
S & P Calamari, Barra	Garlic Prawns, Barra	Satay, Barra
S & P Calamari, Parmi	Garlic Prawns, Parmi	Satay, Parmi
S & P Calamari, Pasta	Garlic Prawns, Pasta	Satay, Pasta

7 **a** Size $= 6 \times 5 = 30$

b

		Sock 1					
		Stripe 1	**Stripe 2**	**Dotty 1**	**Dotty 2**	**Orange**	**Black**
Sock 2	**Stripe 1**	X	S2, S1	D1, S1	D2, S1	O, S1	B, S1
	Stripe 2	S1, S2	X	D1, S2	D2, S2	O, S2	B, S2
	Dotty 1	S1, D1	S2, D1	X	D2, D1	O, D1	B, D1
	Dotty 2	S1, D2	S2, D2	D1, D2	X	O, D2	B, D2
	Orange	S1, O	S2, O	D1, O	D2, O	X	B, O
	Black	S1, B	S2, B	D1, B	D2, B	O, B	X

8 **a** Size $= 5 \times 4 = 20$

b

		1st SRC				
		Mike	**Daniel**	**Brooke**	**Abby**	**Holly**
2nd SRC	**Mike**	X	D, M	B, M	A, M	H, M
	Daniel	M, D	X	B, D	A, D	H, D
	Brooke	M, B	D, B	X	A, B	H, B
	Abby	M, A	D, A	B, A	X	H, A
	Holly	M, H	D, H	B, H	A, H	X

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Exercise 10F

1 **a** indicator, event
b number, outcomes
c total, size
d Favourable, needs

2 **a** **i** Size = 2
ii Red, Black
iii $P(\text{Red}) = \frac{1}{2}$
b **i** Size = 4
ii Hearts, Diamonds, Clubs, Spades
iii $P(\text{Heart}) = \frac{1}{4}$
c **i** Size = 13
ii A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
iii $P(\text{Ace}) = \frac{1}{13}$

3 **a** Size $= 2 \times 2 = 4$
b

		Coin 1	
		Head	**Tail**
Coin 2	**Head**	H, H	T, H
	Tail	H, T	T, T

c Number of favourable outcomes = 1, P(no tails) $= \frac{1}{4}$
d Number of favourable outcomes = 2, P(one tail) $= \frac{2}{4}$ or $\frac{1}{2}$
e Number of favourable outcomes = 1, P(two tails) $= \frac{1}{4}$
f Number of favourable outcomes = 3, P(at least one tail) $= \frac{3}{4}$

4 **a** Size $= 2 \times 4 = 8$
b

		Flavour			
		Chocolate	**Vanilla**	**Strawberry**	**Salted Caramel**
Cone	**Waffle**	Choc, Waffle	Van, Waffle	Straw, Waffle	Cara, Waffle
	Sugar	Choc, Sugar	Van, Sugar	Straw, Sugar	Cara, Sugar

c Number of favourable outcomes = 1, P(Salted Caramel in a Waffle cone) $= \frac{1}{8}$
d Number of favourable outcomes = 2, P(Salted Caramel) $= \frac{2}{8} = \frac{1}{4}$
e Number of favourable outcomes = 4, P(Waffle cone) $= \frac{4}{8} = \frac{1}{2}$
f Number of favourable outcomes = 5, P(Waffle cone or chocolate) $= \frac{5}{8}$

5 **a** Size $= 4 \times 13 = 52$
b

		Face												
		A	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**
Suit	**H**	A, H	2, H	3, H	4, H	5, H	6, H	7, H	8, H	9, H	10, H	J, H	Q, H	K, H
	D	A, D	2, D	3, D	4, D	5, D	6, D	7, D	8, D	9, D	10, D	J, D	Q, D	K, D
	C	A, C	2, C	3, C	4, C	5, C	6, C	7, C	8, C	9, C	10, C	J, C	Q, C	K, C
	S	A, S	2, S	3, S	4, S	5, S	6, S	7, S	8, S	9, S	10, S	J, S	Q, S	K, S

c Number of favourable outcomes = 1, P(Ace Hearts) $= \frac{1}{52}$
d Number of favourable outcomes = 12, P(Picture Card) $= \frac{12}{52} = \frac{3}{13}$
e Number of favourable outcomes = 8, P(Ace or Queen) $= \frac{8}{52} = \frac{2}{13}$
f Number of favourable outcomes = 16, P(Spade or a Queen) $= \frac{16}{52} = \frac{4}{13}$
g Number of favourable outcomes = 22, P(Picture Card or a club) $= \frac{22}{52} = \frac{11}{26}$

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6 **a** Size $= 4 \times 4 = 16$

b

Dice 1

	+	1	2	3	4
Dice 2	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

c Number of favourable outcomes = 4, P(total of 5) $= \frac{4}{16} = \frac{1}{4}$

d Number of favourable outcomes = 10, P(total of at least 5) $= \frac{10}{16} = \frac{5}{8}$

e Number of favourable outcomes = 6, P(total more than 5) $= \frac{6}{16} = \frac{3}{8}$

f Number of favourable outcomes = 6, P(total less than 5) $= \frac{6}{16} = \frac{3}{8}$

7 **a** Size $= 4 \times 4 = 16$

b

Question 1

		Correct	Wrong	Wrong	Wrong
Question 2	**Correct**	✓✓	✗✓	✗✓	✗✓
	Wrong	✓✗	✗✗	✗✗	✗✗
	Wrong	✓✗	✗✗	✗✗	✗✗
	Wrong	✓✗	✗✗	✗✗	✗✗

c Number of favourable outcomes = 1, P(both correct) $= \frac{1}{16}$

d Number of favourable outcomes = 6, P(one correct) $= \frac{6}{16} = \frac{3}{8}$

e Number of favourable outcomes = 7, P(at least one correct) $= \frac{7}{16}$

f Number of favourable outcomes = 9, P(none correct) $= \frac{9}{16}$

8 **a** Size $= 3 \times 3 = 9$

b

Hayley

		Rock	Paper	Scissors
Ben	**Rock**	R, R	P, R	S, R
	Paper	R, P	P, P	S, P
	Scissors	R, S	P, S	S, S

c Number of favourable outcomes = 3, P(Ben plays Scissors) $= \frac{3}{9} = \frac{1}{3}$

d Number of favourable outcomes = 1, P(both play rock) $= \frac{1}{9}$

e Number of favourable outcomes = 3, P(both play same move) $= \frac{3}{9} = \frac{1}{3}$

f Number of favourable outcomes = 1, P(Hayley wins with paper) $= \frac{1}{9}$

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Exercise 10G

1 **a** determining, two
b probabilities, more
c columns, outcomes

2 **a** Red, Petrol, Manual
Red, Petrol, Auto
Red, Hybrid, Manual
Red, Hybrid, Auto
Blue, Petrol, Manual
Blue, Petrol, Auto
Blue, Hybrid, Manual
Blue, Hybrid, Auto

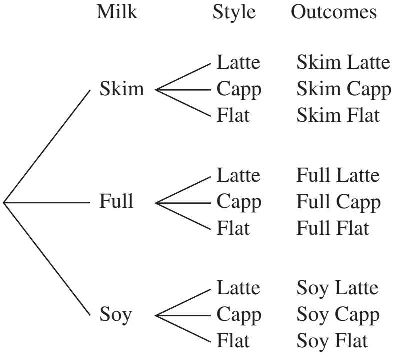
b 8 combinations

c Number of favourable outcomes = 1,
P(Red, Hybrid, Auto) = $\frac{1}{8}$

d Number of favourable outcomes = 2,
P(Red, Auto) = $\frac{2}{8} = \frac{1}{4}$

e Number of favourable outcomes = 4,
P(Auto) = $\frac{4}{8} = \frac{1}{2}$

3 **a**



b 9 combinations

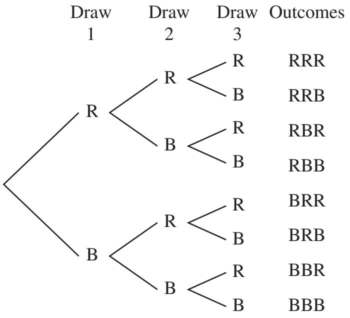
c Skim Milk, Latte
Skim Milk, Cappuccino
Skim Milk, Flat White
Full Cream, Latte
Full Cream, Cappuccino
Full Cream, Flat White
Soy Milk, Latte
Soy Milk, Cappuccino
Soy Milk, Flat White

d Number of favourable outcomes = 1,
P(Skim Milk, Latte) = $\frac{1}{9}$

e Number of favourable outcomes = 3,
P(Latte) = $\frac{3}{9} = \frac{1}{3}$

f Number of favourable outcomes = 6,
P(no Soy) = $\frac{6}{9} = \frac{2}{3}$

4 **a**



b 8 outcomes

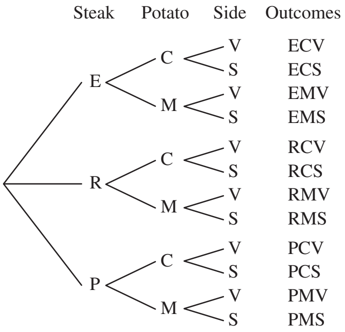
c RRR, RRB, RBR, RBB, BRR, BRB,
BBR, BBB

d Number of favourable outcomes = 1,
P(RRR) = $\frac{1}{8}$

e Number of favourable outcomes = 3,
P(two Red cards) = $\frac{3}{8}$

f Number of favourable outcomes = 4,
P(less than two Red cards) = $\frac{4}{8} = \frac{1}{2}$

5 **a**



Key:
Steak: Eye (E), Rib (R), Rump (P)
Potato: Chips (C), Mash (M)
Side: Vegetables (V), Salad (S)

b 12 combinations

c ECV, ECS, EMV, EMS, RCV, RCS,
RMV, RMS, PCV, PCS, PMV, PMS

d Number of favourable outcomes = 1,
P(Rump, Chips, Salad) = $\frac{1}{12}$

e Number of favourable outcomes = 2,
P(Rib, Chips) = $\frac{2}{12} = \frac{1}{6}$

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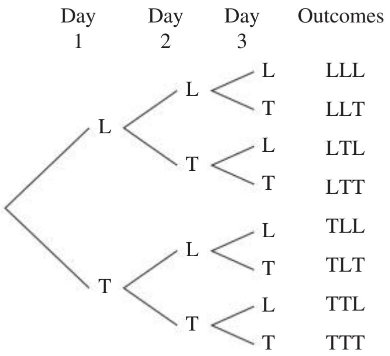
6 **a**



b There are 8 outcomes.

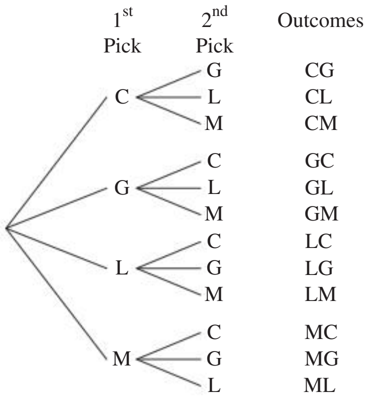
c LLL, LLT, LTL. LTT, TLL, TLT, TTL, TTT

d Number of favourable outcomes = 1, P(TTT) = $\frac{1}{8}$

e Number of favourable outcomes = 3, P(Late once) = $\frac{3}{8}$

f Number of favourable outcomes = 7, P(Late at least once) = $\frac{7}{8}$

7 **a**



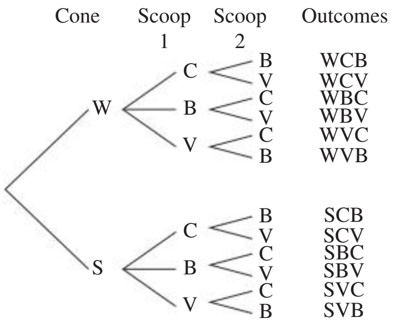
b There are 12 outcomes.

c CG, CL, CM, GC, GL, GM, LC, LG, LM, MC, MG, ML

d Number of favourable outcomes = 2, P(Libby and Gabby) = $\frac{2}{12} = \frac{1}{6}$

e Number of favourable outcomes = 6, P(Chloe is in the team) = $\frac{6}{12} = \frac{1}{2}$

8 **a**



b There are 12 combinations.

c WCB, WCV, WBC, WBV, WVC, WVB, SCB, SCV, SBC, SBV, SVC, SVB

d Number of favourable outcomes = 2, P(Waffle, Berry and Choc) = $\frac{2}{12} = \frac{1}{6}$

e Number of favourable outcomes = 4, P(no Choc) = $\frac{4}{12} = \frac{1}{3}$

Chapter checklist

1 Answers will vary.

2 Expected = 300 ÷ 6 = 50 times each

3 Total outcomes = 42 + 57 + 24 + 2 = 125

a P(plays one sport) = $\frac{57}{125} = 0.46$

b P(plays fewer than two sports)

$= \frac{42 + 57}{125} = 0.79$

4 Adjacent cards may be of the same suit or in numerical order, so they may not be drawn radnomly.

5 The coin should not always be held the same way up when tossing, because if the tossing action is very similar each time, the outcomes may not be random.

6 **a**

		Coin 1	
		Head	**Tail**
Coin 2	**Head**	H, H	T, H
	Tail	H, T	T, T

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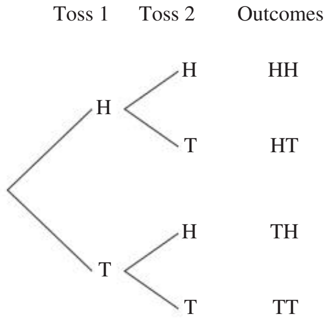
b HH, HT, TH, TT

c

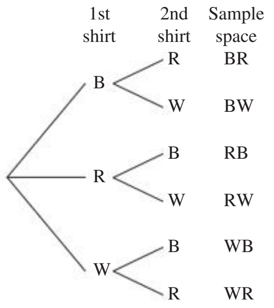


7 a $P(TT) = \frac{1}{4} = 0.25$

b $P(\text{one Tail}) = \frac{2}{4} = 0.50$

c $P(\text{at least one Tail}) = \frac{3}{4} = 0.75$

8 a



b $P(B + R) = \frac{2}{6} = \frac{1}{3}$

c $P(W) = \frac{4}{6} = \frac{2}{3}$

Chapter review

1 a

Result	Frequency
heads/girls	11
tails/boys	19

b $\% \text{ boys} = \frac{19}{30} \times 100 = 63\%$

c Expected = 30 ÷ 2 = 15

d Yes, you would expect half the sample to be boys, but it was $\frac{19}{30}$ or 63%.

2 a

2-child combination	Tally	Frequency
boy + boy	𝍸	5
boy + girl	𝍸 𝍸 |	11
girl + girl	||||	4

b $RF(\text{boy/girl}) = \frac{11}{20} = 0.55$

3 a 4 outcomes: H, D, C, S

b Expected repeats = 20 ÷ 4 = 5 times

c 2 times. This is less than expected but not too unusual.

d Expected outcomes = 20 ÷ 4 = 5 times

4 a, b

	Suit	Hearts	Diamonds	Clubs	Spades
a	**Frequency**	5	7	4	4
b	**Relative freq**	$\frac{5}{20} = 0.25$	$\frac{7}{20} = 0.35$	$\frac{4}{20} = 0.20$	$\frac{4}{20} = 0.20$

c Diamonds appear more than expected.

d Increasing the size of the sample to a large number would result in the outcomes being more evenly distributed.

5 a Total shoppers = 122, P(2 Bags) $= \frac{18}{122} = 0.15$

b Number with less than 4 bags = 15 + 24 + 18 + 35 = 92

P(less than 4 Bags) $= \frac{92}{122} = 0.75$

6 a Not returning the card between selections changes the chance of that suit/colour etc. being selected in the next draw.

b If shuffling technique is poor, this could impact results.

c Depending on technique, starting on heads could result in the same outcome occurring more frequently.

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d No issue.

e If the dice does not get an opportunity to spin, the outcome could be the same as last time.

f No issue.

7 a Size = 6 × 6 = 36

b

Dice 1 (across), Dice 2 (down)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

8 a Sample size = 2 × 2 = 4

b HH, HT, TH, TT

9 a Number of favourable outcomes = 5, P(total 8) = $\frac{5}{36}$

b Number of favourable outcomes = 15, P(total at least 8) = $\frac{15}{36} = \frac{5}{12}$

c Number of favourable outcomes = 10, P(total is more than 8) = $\frac{10}{36} = \frac{5}{18}$

d Number of favourable outcomes = 21, P(total is less than 8) = $\frac{21}{36} = \frac{7}{12}$

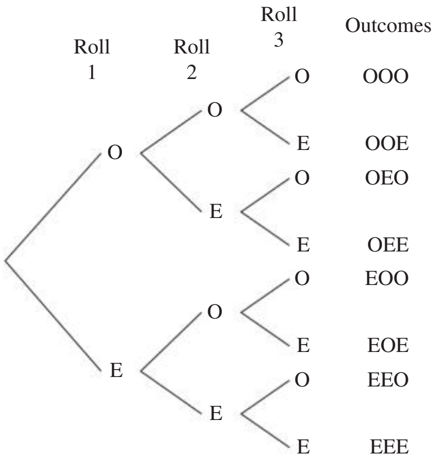
10 a Number of favourable outcomes = 1, P(HH) = $\frac{1}{4}$

b Number of favourable outcomes = 2, P(one Head) = $\frac{2}{4} = \frac{1}{2}$

c Number of favourable outcomes = 1, P(no Heads) = $\frac{1}{4}$

d Number of favourable outcomes = 3, P(at least one head) = $\frac{3}{4}$

11 a



b 8 outcomes

c OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE

d Number of favourable outcomes = 3, P(two even) = $\frac{3}{8}$

e Number of favourable outcomes = 1, P(three even) = $\frac{1}{8}$

f Number of favourable outcomes = 4, P(less than two even) = $\frac{4}{8} = \frac{1}{2}$

12 a P(bus is late) = $\frac{1}{5}$ = 0.20

b i Answers will vary. Example response:

0.448	0.534	0.727	0.973	0.471
0.069	0.858	0.606	0.230	0.193
0.728	0.495	0.010	0.176	0.563
0.935	0.702	0.184	0.228	0.746

ii The bus is late on 5 occasions in the simulation shown.

c i $\frac{20}{100} \times 20 = 4$ times

ii Answers will vary. A simulation result of 3, 4, 5 would not be considered too unusual. If less than 3 or more than 5, the simulation result could be considered unusual.

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13 **a** Sample size = $4 \times 3 = 12$

b

		Dish 1			
		C	**L**	**P**	**B**
Dish 2	**C**	X	L, C	P, C	B, C
	L	C, L	X	P, L	B, L
	P	C, P	L, P	X	B, P
	B	C, B	L, B	P, B	X

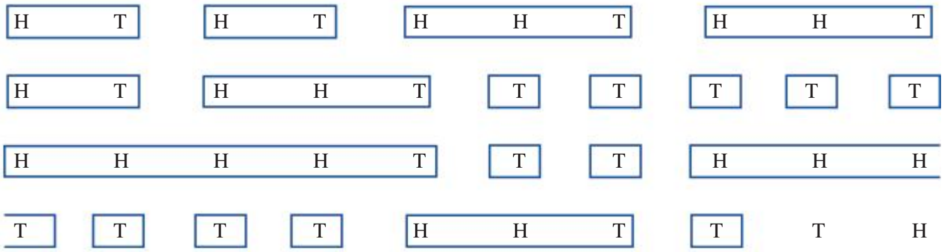
c Number of favourable outcomes = 6, P(orders Pork) = $\frac{6}{12} = \frac{1}{2}$

d Number of favourable outcomes = 2, P(Chicken and Beef) = $\frac{2}{12} = \frac{1}{6}$

e Number of favourable outcomes = 4, P(Chicken and Beef or Lamb) = $\frac{4}{12} = \frac{1}{3}$

f Number of favourable outcomes = 10, P(Chicken or Beef) = $\frac{10}{12} = \frac{5}{6}$

14 **a**



b

Number of children	Tally	Frequency
1	𝍸 𝍸 |	11
2	|||	3
3	||||	4
4	|	1
5	|	1
Total	𝍸 𝍸 𝍸 𝍸	20

c Average = $\frac{1 \times 11 + 2 \times 3 + 3 \times 4 + 4 \times 1 + 5 \times 1}{20} = \frac{38}{20} = 1.9$

d **i** RF(2 Children) = $\frac{3}{20} = 0.15$

ii RF (1 child) = $\frac{11}{20} = 0.55$

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Chapter 11

Pre-test

1 **a** $\frac{15}{100}$ **b** $\frac{6}{100}$

c $\frac{2.4}{100}$ **d** $\frac{11.25}{100}$

2 **a** $\frac{12}{100} \times 45 = \$5.40$

b $\frac{8}{100} \times 1590 = \$127.20$

c $\frac{1.8}{100} \times 24\,900 = \$448.20$

d $\frac{3.1}{100} \times 800 = \$24.80$

3 **a** $\frac{12}{100} \times 5750 = \$690;\ 5750 + 690 = \$6440$

b $\frac{3}{100} \times 600 = \$18;\ 600 + 18 = \$618$

c $\frac{4}{100} \times 2120 = \$84.80;$
$2\,120 - 84.80 = \$2035.20$

d $\frac{15}{100} \times 30\,000 = \$4500;$
$30\,000 - 4500 = \$25\,500$

4 **a** $5 \times 12 = 60$ months

b $3 \times 52 = 156$ weeks

c 365 days

d $7 \times 26 = 182$ fortnights

5 **a** 32.81 **b** 1.08

c 0.32 **d** 0.17

Exercise 11A

1 **a** borrowing, investing, percentages

b interest, same

c borrowed, interest

2 **a** $\frac{7}{12}$ **b** $\frac{31}{52}$

c $\frac{25}{365}$ **d** $\frac{18}{12}$

e $\frac{270}{365}$ **f** $\frac{9}{52}$

3 **a** $\frac{30}{100} \times 23.70 = \$7.11$

b $\frac{10}{100} \times 80.90 = \$8.09$

c $\frac{5}{100} \times 126.45 = \$6.32$

d $\frac{3.45}{100} \times 456.21 = \$15.74$

e $\frac{1.05}{100} \times 364.85 = \$3.83$

f $\frac{0.65}{100} \times 63.90 = \$0.42$

4 **a** $I = 780 \times \frac{4.57}{100} \times 4 = \$142.58$

b $I = 4260 \times \frac{6.32}{100} \times 3 = \$807.70$

c $I = 2030 \times \frac{9.05}{100} \times 5 = \$918.58$

d $I = 625 \times \frac{5.9}{100} \times 2 = \$73.75$

e $I = 1650 \times \frac{4.2}{100} \times 5 = \$346.50$

f $I = 5050 \times \frac{3.9}{100} \times 7 = \$1378.65$

g $I = 8880 \times \frac{5.79}{100} \times 6 = \$3084.91$

h $I = 7280 \times \frac{5.36}{100} \times 2 = \$780.42$

5 **a** $I = 10\,000 \times \frac{3.65}{100} \times 4 = \$1460.00$

b $\text{Value} = 10\,000 + 1\,460.00 = \$11\,460.00$

6 **a** $I = 6200 \times \frac{6.8}{100} \times \frac{4}{12} = \$140.53$

b $I = 590 \times \frac{5.11}{100} \times \frac{20}{52} = \$11.60$

c $I = 8290 \times \frac{4.48}{100} \times \frac{15}{365} = \$15.26$

d $I = 6700 \times \frac{2.26}{100} \times \frac{2}{52} = \$5.82$

e $I = 7740 \times \frac{5.35}{100} \times \frac{14}{365} = \$15.88$

f $I = 858 \times \frac{7.07}{100} \times \frac{7}{12} = \$35.39$

g $I = 3700 \times \frac{6.1}{100} \times 9\frac{6}{12} = \$2144.15$

h $I = 6160 \times \frac{5.3}{100} \times 8\frac{3}{12} = \$2693.46$

7 **a** $I = 2\,000\,000 \times \frac{8}{100} \times \frac{30}{365} = \$13\,150.68$

b $\text{Value} = 2\,000\,000 + 13\,150.68$
$= \$2\,013\,150.68$

8 **a** $I = 3000 \times \frac{9.7}{100} \times \frac{18}{12} = \$436.50$

b $\text{Repay} = 3000 + 436.50 = \$3436.50$

c $\text{Monthly repayment} = \$3436.50 \div 18$
$= \$190.92$

9 **a** $I = 5000 \times \frac{11.2}{100} \times 4 = \$2240.00$

b $\text{Repay} = 5000 + 2240 = \$7240.00$

c $\text{Monthly repayment} = \$7240.00 \div 48$
$= \$150.83$

10 **a** **i** 2.60% p.a.

ii $I = 15\,000 \times \frac{2.6}{100} \times 2 = \$780$

iii $15\,780

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b **i** 2.30% p.a.

ii $I = 65\,000 \times \frac{2.3}{100} \times 1 = \$1495$

iii $66 495

c **i** 2.05% p.a.

ii $I = 11\,400 \times \frac{2.05}{100} \times \frac{6}{12} = \$116.85$

iii $11 516.85

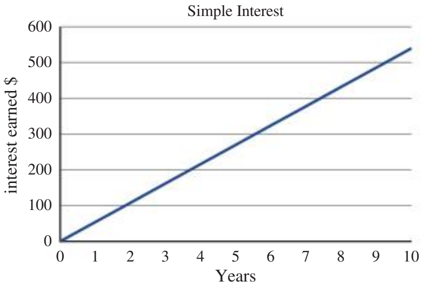
11 **a**

Year	Simple interest	Balance
1	54	1554
2	108	1608
3	162	1662
4	216	1716
5	270	1770
6	324	1824
7	378	1878
8	432	1932
9	486	1986
10	540	2040

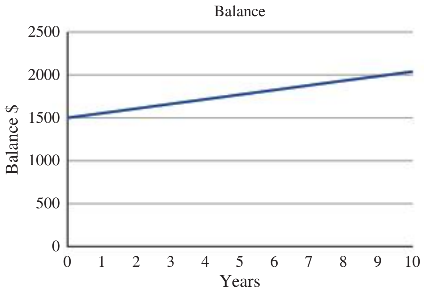
b **i** $270 **ii** $540

c **i** $1770 **ii** $2040

d



e



Exercise 11B

1 **a** interest, interest

b **i** accumulated **ii** principal

iii interest **iv** number

c must, frequency

2 **a** $\frac{15}{100}$ **b** $\frac{6}{100}$

c $\frac{3.4}{100}$ **d** $\frac{2.03}{100}$

3 **a** 1 year = 12 months

b 1 year = 52 weeks

c 1 year = 2 half-years

d 1 year = 365 days

e 1 year = 4 quarters

f 1 year = 26 fortnights

4 **a** $\frac{2.4}{2}\% = 1.2\%$ per half year

b $\frac{2.4}{4}\% = 0.6\%$ per quarter

c $\frac{2.4}{12}\% = 0.2\%$ per month

d $\frac{2.4}{26}\%$ per fortnight

e $\frac{2.4}{52}\%$ per week

f $\frac{2.4}{365}\%$ per day

5 **a**

Year	Principal	Interest	Balance
1	$2480	$2480 \times \frac{3.2}{100} = \$79.36$	$2480 + 79.36 = \$2559.36$
2	$2559.36	$\$2559.36 \times \frac{3.2}{100} = \$81.90$	$2559.36 + 81.90 = \$2641.26$
3	$2641.26	$\$2641.26 \times \frac{3.2}{100} = \$84.52$	$2641.26 + 84.52 = \$2725.78$

b $\$2725.78 - \$2480 = 245.78$

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6 a

Year	Principal	Interest	Balance
1	\$3850	$3850 \times \frac{2.1}{100} = \$80.85$	$3850 + 80.85 = \$3930.85$
2	\$3930.85	$\$3930.85 \times \frac{2.1}{100} = \$82.55$	$3930.85 + 82.55 = \$4013.40$
3	\$4013.40	$\$4013.40 \times \frac{2.1}{100} = \$84.28$	$4013.40 + 84.28 = \$4097.68$
4	\$4097.68	$\$4097.68 \times \frac{2.1}{100} = \$86.05$	$\$4097.68 + 86.05 = \$4183.73$

b $\$4183.73 - \$3850 = \$333.73$

7 a $P = \$9600,\ i = \frac{3.2}{100},\ n = 2$

$A = P(1+i)^n$

$A = 9600\left(1+\frac{3.2}{100}\right)^2$

$A = \$10\,224.23$

b $\$10\,224.23 - 9600 = \$624.23$

8 a $P = \$4500,\ i = \frac{2.2}{100}\ n = 4$

$A = P(1+i)^n$

$A = 4500\left(1+\frac{2.2}{100}\right)^4$

$A = \$4909.26$

b $4909.26 - 4500 = \$409.26$

9 a $P = \$680,\ i = \frac{5}{100},\ n = 3,$

$A = 680\left(1+\frac{5}{100}\right)^3 = \$787.19$

b $P = \$275,\ i = \frac{3}{100},\ n = 6,$

$A = 275\left(1+\frac{3}{100}\right)^6 = \$328.36$

c $P = \$682,\ i = \frac{4}{100},\ n = 5,$

$A = 682\left(1+\frac{4}{100}\right)^5 = \$829.76$

d $P = \$1240,\ i = \frac{2.2}{100},\ n = 4,$

$A = 1240\left(1+\frac{2.2}{100}\right)^4 = \$1352.77$

e $P = \$5760,\ i = \frac{1.8}{100},\ n = 7,$

$A = 5760\left(1+\frac{1.8}{100}\right)^7 = \$6526.15$

f $P = \$4030,\ i = \frac{2.15}{100},\ n = 5,$

$A = 4030\left(1+\frac{2.15}{100}\right)^5 = \$4482.26$

10 $P = \$5000,\ i = \frac{3.6}{1200},\ n = 12 \times 5 = 60$

$A = P(1+i)^n$

$A = 5000\left(1+\frac{3.6}{1200}\right)^{60}$

$A = \$5984.47$

11 $P = \$3800,\ i = \frac{2.4}{5200},\ n = 2 \times 52 = 104$

$A = P(1+i)^n$

$A = 3800\left(1+\frac{2.4}{5200}\right)^{104}$

$A = \$3986.80$

12 a $P = \$1370,\ i = \frac{8.85}{1200},\ n = 3 \times 12 = 36,$

$A = 1370\left(1+\frac{8.85}{1200}\right)^{36} = \$1784.85$

b $P = \$54\,600,\ i = \frac{1.2}{200},\ n = 8 \times 2 = 16,$

$A = 54\,600\left(1+\frac{1.2}{200}\right)^{16} = \$60\,084.21$

c $P = \$4300,\ i = \frac{5.07}{5200},\ n = 52 \times 2 = 104,$

$A = 4300\left(1+\frac{5.07}{5200}\right)^{104} = \$4758.66$

d $P = \$3350,\ i = \frac{6.1}{1200},\ n = 12 \times 3 = 36,$

$A = 3350\left(1+\frac{6.1}{1200}\right)^{36} = \$4020.86$

e $P = \$7960,\ i = \frac{4.92}{2600},\ n = 26 \times 2 = 52,$

$A = 7960\left(1+\frac{4.92}{2600}\right)^{52} = \$8782.28$

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f $P = \$4220,\ i = \frac{4.66}{400},\ n = 4 \times 4 = 16,$

$A = 4220\left(1 + \frac{4.66}{400}\right)^{16} = \$5079.22$

13 a i B2

ii = B2*4 / 100

iii = B2 + C2

iv = D2

b ii Balance = \$4258.29

Exercise 11C

1 a percentage, compound

b increasing

c measured

d value

e decrease, interest

f reducing

2 a increasing **b** decreasing

c decreasing **d** increasing

e increasing

3 a $P = \$10.65,\ i = \frac{2.1}{100},\ n = 4,$

$A = 10.65\left(1 + \frac{2.1}{100}\right)^{4} = \$11.57$

b $P = \$20\,490,\ i = \frac{2.1}{100},\ n = 20,$

$A = 20\,490\left(1 + \frac{2.1}{100}\right)^{20} = 31\,049.66$

4 a $P = 93\,000,\ i = \frac{1.5}{100},\ n = 1,$

$A = 93\,000\left(1 + \frac{1.5}{100}\right)^{1} = \$94\,395$

b $P = 93\,000,\ i = \frac{1.9}{100},\ n = 1,$

$A = 93\,000\left(1 + \frac{1.9}{100}\right)^{1} = \$94\,767$

5 a $P = 145.9,\ i = \frac{1.9}{100},\ n = 20,$

$A = 145.9\left(1 + \frac{1.9}{100}\right)^{20}$

= 212.6 cents per litre

b $P = 145.9,\ i = \frac{2.1}{100},\ n = 10,$

$A = 145.9\left(1 + \frac{2.1}{100}\right)^{10}$

= 179.6 cents per litre

6 a $P = 24.8,\ i = \frac{1.32}{100},\ n = 10,$

$A = 24.8\left(1 + \frac{1.32}{100}\right)^{10} = 28.28$ million

b $P = 24.8,\ i = \frac{1.32}{100},$

$n = (2100 - 2018) = 82,$

$A = 24.8\left(1 + \frac{1.32}{100}\right)^{82} = 72.69$ million

7 a $P = \$447\,500,\ i = \frac{3.1}{100},\ n = 15,$

$A = 447\,500\left(1 + \frac{3.1}{100}\right)^{15} = \$707\,412.97$

b $P = \$447\,500,\ i = \frac{3.1}{100},\ n = 20,$

$A = 447\,500\left(1 + \frac{3.1}{100}\right)^{20} = \$824\,074.25$

8 a $P = \$320\,000,\ i = -\frac{15}{100},\ n = 5,$

$A = 320\,000\left(1 - \frac{15}{100}\right)^{5} = \$141\,985.70$

b $P = \$320\,000,\ i = -\frac{15}{100},\ n = 10,$

$A = 320\,000\left(1 - \frac{15}{100}\right)^{10} = \$62\,999.81$

c $P = \$320\,000,\ i = -\frac{15}{100},\ n = 15,$

$A = 320\,000\left(1 - \frac{15}{100}\right)^{15} = \$27\,953.35$

9 a i $P = 6500,\ i = -\frac{20}{100},\ n = 3,$

$A = 6500\left(1 - \frac{20}{100}\right)^{3} = 3328$

ii $P = 6500,\ i = -\frac{20}{100},\ n = 5,$

$A = 6500\left(1 - \frac{20}{100}\right)^{5} = 2130$

b The disease is not fully eradicated because it is reducing by 20% of the current amount each year, not 20% of the original amount each year.

10 a $P = \$1549,\ i = -\frac{45}{100},\ n = 2,$

$A = 1549\left(1 - \frac{45}{100}\right)^{2} = \$468.57$

b $P = \$2099,\ i = -\frac{45}{100},\ n = 3,$

$A = 2099\left(1 - \frac{45}{100}\right)^{3} = \$349.22$

Exercise 11D

1 a plan, successful

b calculate

c value, interest

d same, every

e added, increases

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2 a i \$0 **ii** \$250
iii fortnight **iv** monthly
v 20 **vi** 6%
b \$250 272
c \$120 272

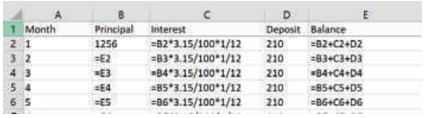
3 a i \$0 **ii** fortnight
iii annually **iv** $55 - 18 = 37$
v 5.6%
b \$331 (by guess and check)
c \$681 826

4 a i \$12 000 **ii** \$500
iii monthly **iv** monthly
v 2 **vi** 3.2%
b \$25 167
c \$1167
d 3 years
e \$696 (by guess and check)

5 a \$21 276 **b** \$46 616

6 a \$300 **b** \$42 553
c No he was not able to catch up to Nicole. She has \$46 616 which is \$4063 more than he has. This is because she has been earning interest for a longer time.

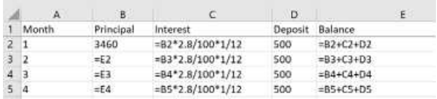
7 b



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2	1	1256	=B2*3.15/100*1/12	210	=B2+C2+D2
3	2	=E2	=B3*3.15/100*1/12	210	=B3+C3+D3
4	3	=E3	=B4*3.15/100*1/12	210	=B4+C4+D4
5	4	=E4	=B5*3.15/100*1/12	210	=B5+C5+D5
6	5	=E5	=B6*3.15/100*1/12	210	=B6+C6+D6

c \$3852.84 **d** \$76.84
e 18 months

8 b



	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2	1	3460	=B2*2.8/100*1/12	500	=B2+C2+D2
3	2	=E2	=B3*2.8/100*1/12	500	=B3+C3+D3
4	3	=E3	=B4*2.8/100*1/12	500	=B4+C4+D4
5	4	=E4	=B5*2.8/100*1/12	500	=B5+C5+D5

c \$15 986.63 **d** \$526.63
e 32 months

Exercise 11E

1 changes, impact, interest

2 a 12 **b** 52
c 365 **d** 26
e 2 **f** 13
g 4

3 a $12 \times 2 = 24$ months
b i 2.6% p.a.
ii 2.5% p.a.
iii 2.4% p.a
c i once
ii $12 \div 6 = 2$ times
iii $52 \div 4 = 13$ times
d i $P = \$20\,000,\ i = \frac{2.6}{100},\ n = 2$

$$A = 20\,000\left(1 + \frac{2.6}{100}\right)^2 = \$21053.52$$

ii $P = \$20\,000,\ i = \frac{2.5}{200},\ n = 2 \times 2 = 4,$

$$A = 20\,000\left(1 + \frac{2.5}{200}\right)^4 = \$21\,018.91$$

iii $P = \$20\,000,\ i = \frac{2.4}{1300},$

$n = 2 \times 13 = 26,$

$$A = 20\,000\left(1 + \frac{2.4}{1300}\right)^{26} = \$20\,982.48$$

e Best option is to invest the money with annual compounds at 2.6% p.a.

4 a i \$17 670.89 **ii** \$35 506.61
b Difference = \$35 506.61 – \$17 670.89
= \$17 835.72
c Interest earned at 5% p.a. = \$9170.89
Interest earned at 10% p.a. = \$27 006.61
Doubling the interest rate led to approximately three times more interest being earned.
d The curve increases a lot faster and is much steeper when the interest rate increases.

5 a i \$56 044.11 **ii** \$96 462.93
b Difference = \$96 462.93 – \$56 044.11
= \$40 418.82

c Increasing the interest rate by 3% (or one third) led to an increase interest of approximately \$40 000, almost double, over the same period.
d The curve increases a lot faster and is much steeper when the interest rate increases.

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6 a i Annual compounds, interest = \$900

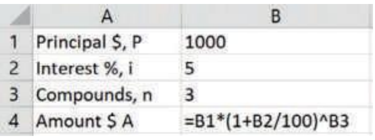
ii Monthly compounds, interest = \$911

b She earns more interest.

7 a \$5369 **b** \$5341

c The first option, compounding annually at 2.4% p.a. gave the best return on her investment.

8 a



	A	B
1	Principal \$, P	1000
2	Interest %, i	5
3	Compounds, n	3
4	Amount \$ A	=B1*(1+B2/100)^B3

b i B1 **ii** B2

iii B3

c = B1 * (1 + B2/100)^B3

d i \$ 1157.63 **ii** \$1331.00

iii \$1061.21

e i \$78 750.57 **ii** \$92 989.38

iii \$73 243.63

f Higher rate = more money

9 b i B1 **ii** B2

c i the number of times the interest compounds per year

ii The power in the equation, n, is the number of compounds over the duration of the investment. It will be the product of years (B3) and how many compounds per year (B4)

iii The interest rate needs to match the compounding periods (so divide by B4) and written as a fraction (so divide by 100)

d i \$1197.09 **ii** \$1196.68

iii \$1195.62

e i \$78 990.40 **ii** \$78 974.39

iii \$78 933.00

f More interest is earned when interest compounds more frequently.

10 a i 2.5% p.a. **ii** 2.6% p.a.

iii 2.7% p.a.

b i \$126 146.47 (13 compounds per year)

ii \$126 362.74

iii \$ 126 567.48

c Normally you would expect to earn more interest from more frequent compounding periods, but the lower rate means that the yearly compounds result in the most amount of interest earned.

Chapter checklist

1 a $I = Pin = 2682 \times \frac{4.2}{100} \times 3 = \$337.93$

b Balance = 2682 + 337.93 = \$3019.93

2 a $A = P(1 + i)^n$

$A = 3540\left(1 + \frac{2.7}{100}\right)^4 = \$3938.08$

b Interest = 3938.08 – 3540 = \$398.08

3 a $A = P(1 + i)^n$

$A = 5.04\left(1 + \frac{1.6}{100}\right)^{15} = 6.39 \text{ million}$

b Time = 2050 – 2018 = 32 years,

$A = P(1 + i)^n$

$A = 5.04\left(1 + \frac{1.6}{100}\right)^{32} = 8.38 \text{ million}$

4 He has \$19 098 after 2 years.

5 a 500

b = B2 * 4 / 100 * 1 / 12 (simple interest formula)

c = B2 + C2 (principal plus interest)

d = D2 (the previous months balance becomes the principal in the next month)

6 a i $A = P(1 + i)^n$

$A = 7500\left(1 + \frac{3.75}{100}\right)^5 = \$9015.75$

ii $A = P(1 + i)^n$

$A = 7500\left(1 + \frac{5.15}{100}\right)^5 = \$9640.68$

b The higher rate generated more interest and the final balance is larger as a result.

7 a i $A = P(1 + i)^n$

$A = 7500\left(1 + \frac{5}{100}\right)^5 = \$9572.11$

ii $A = P(1 + i)^n$

$A = 7500\left(1 + \frac{5}{2600}\right)^{130} = \$9627.88$

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b The more frequent compounding periods generated more interest and the final balance is larger as a result.

Chapter review

1 a $I = Pin = 60\,000 \times \frac{2.65}{100} \times 3 = \$4770$

b Total value $= 60\,000 + 4770 = \$64\,770$

2 a

$I = Pin = 5\,000\,000\frac{6.7}{100} \times \frac{90}{365} = \$82\,602.74$

b Total value $= \$5\,082\,602.74$

3 a

Year	Principal	Interest	Balance
1	$1400.00	$53.20	$1453.20
2	$1453.20	$55.22	$1508.42
3	$1508.42	$57.32	$1565.74

b Interest $= \$165.74$

4 $A = P(1+i)^n$

$A = 2000\left(1+\frac{4.6}{1200}\right)^{48} = \$2403.19$

5 a $A = P(1+i)^n$

$A = 5200\left(1+\frac{3.2}{100}\right)^{2} = \$5538.12$

b Interest $= 5538.12 - 5200 = \$338.12$

6 a i Initial deposit $= \$0$

ii Regular deposit $= \$450$

iii Deposit frequency = fortnightly

iv Compound frequency = monthly

v Number of years $= 5$

vi Interest rate $= 3.6$

b i Balance $= \$63\,991$

ii $5491

7 a i $A = P(1+i)^n$

$A = 3500\left(1+\frac{4.75}{100}\right)^{3} = \$4022.82$

ii $A = P(1+i)^n$

$A = 3500\left(1+\frac{5.25}{100}\right)^{3} = \$4080.70$

b The larger interest rate earns more interest and the final balance is greater.

8 a i $i = \frac{4.5 \div 2}{100} = \frac{4.5}{200},\ n = 5 \times 2 = 10$

$A = P(1+i)^n$

$A = 3500\left(1+\frac{4.5}{200}\right)^{10} = \$4372.21$

ii $i = \frac{4.5 \div 52}{100} = \frac{4.5}{5200},$

$n = 5 \times 52 = 260\ A = P(1+i)^n$

$A = 3500\left(1+\frac{4.5}{5200}\right)^{260} = \$4382.70$

b The more frequent compounding period earns more interest and the final balance is greater.

9 a $A = P(1+i)^n$

$A = 2.4\left(1+\frac{3.42}{100}\right)^{5} = 2.8\text{ million}$

b Time $= 2100 - 2018 = 82$ years

$A = P(1+i)^n$

$A = 2.4\left(1+\frac{3.42}{100}\right)^{82} = 37.8\text{ million}$

10 a $A = P(1+i)^n$

$A = 3.30\left(1+\frac{2.1}{100}\right)^{15} = \$4.51$

b Time $= 2050 - 2018 = 32$ years

$A = P(1+i)^n$

$A = 3.30\left(1+\frac{2.1}{100}\right)^{32} = \$6.42$

11 a i $1250.00 **ii** $1381.41

b They are both calculated on a principal of $5000.

c Simple interest is always calculated on the original principal. Compound interest is calculated on the principal and any interest that has been added to the account.

12 $A = P(1+i)^n$

$A = 1899\left(1-\frac{45}{100}\right)^{2} = \$574.45$

13 a A2: 1 **b** B2: 3400

c C2: =B2 * 4.25 / 100 * 1 / 12

d D2: 360

e E2: =B2 + C2 + D2

f B3: =E2

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Chapter 12

Pre-test

1 a $2.25 b $0.94
c $0.60 d $16.81
e $10.50

2 a $I = Pin = 4000 \times \frac{6}{100} \times 2 = \$480$
b 4000 + 480 = $4480
c 4480 ÷ 24 = $186.67

3 a 48 ÷ 12 = 4 years
b 260 ÷ 52 = 5 years
c 78 ÷ 26 = 3 years
d 42 ÷ 12 = 3.5 years = 3 years 6 months
e 338 ÷ 52 = 6.5 years = 6 years 6 months
f 91 ÷ 26 = 3.5 years = 3 years 6 months
g 200 ÷ 12 = 16.67 years = 16 years + 0.67 × 12 months = 16 years 8 months
h 200 ÷ 52 = 3.846 years = 3 years + 0.846 × 12 months = 3 years 10 months
i 200 ÷ 26 = 7.692 years = 7 years + 0.692 × 12 months = 7 years 8 months

Exercise 12A

1 a borrow, interest
b remaining, interest
c formula
d adding, subtracting
e principal
f reduces
g end, balance
h calculators, total

2 a No. The repayments were likely rounded to the nearest dollar and have not quite covered the debt.
b 205 + 0.66 = $205.66
c $1186.49
d The interest gets less each month.
e Total interest = 24.25 + … + 2.15 = $160.68
f $\frac{12.65}{100} \times 2300 = \$290.95$. Reducing balance loans have the interest calculated on the remaining debt. Simple interest loans have the same amount of interest every payment period.

3 a Number of repayments = 12 × 4 = 48
b Total repaid = 373 × 48 = $17 904
c Interest = 17 904 – 15 000 = $2904

4 a

Month	Principal	Interest	Repayment	Balance
1	$ 14 700.00	$ 140.26	$ 383.00	$ 14 457.26
2	$ 14 457.26	$ 137.95	$ 383.00	$ 14 212.21
3	$ 14 212.21	$ 135.61	$ 383.00	$ 13 964.82

b 383 × 3 = $1149
c 140.26 + 137.95 + 135.91 = $413.82
d 1149 – 413.82 = $735.18
e 14 700 – 13 964.82 = $735.18

5 a

Month	Principal	Interest	Repayment	Balance
1	$ 5600.00	$ 56.47	$ 264.00	$ 5392.47
2	$ 5392.47	$ 54.37	$ 264.00	$ 5182.84
3	$ 5182.84	$ 52.26	$ 264.00	$ 4971.10
4	$ 4971.10	$ 50.13	$ 264.00	$ 4757.23

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b \$56.47

c 5600 – 5392.47 = \$ 207.53

d \$50.13

e 4971.1 – 4757.23 = \$213.87

f As the loan progresses, interest reduces each month and Stephanie is able to reduce her loan by more each payment.

6 **a** 2 × 4 = 8 repayments

b

Quarter	Principal	Interest	Repayment	Balance
1	\$ 28 000.00	\$ 304.50	\$3673.00	\$ 24 631.50
2	\$ 24 631.50	\$ 267.87	\$3673.00	\$ 21 226.37
3	\$ 21 226.37	\$ 230.84	\$3673.00	\$ 17 784.20
4	\$ 17 784.20	\$ 193.40	\$3673.00	\$ 14 304.61
5	\$ 14 304.61	\$ 155.56	\$3673.00	\$ 10 787.17
6	\$ 10 787.17	\$ 117.31	\$3673.00	\$ 7231.48
7	\$ 7231.48	\$ 78.64	\$3673.00	\$ 3637.12
8	\$ 3637.12	\$ 39.55	\$3673.00	\$ 3.68

c **i** \$3.68 is left owing after 8 payments.

ii The repayment amount is likely rounded to the nearest dollar.

iii He will need to pay \$3.68 as well as the \$3673 to clear the debt with his last payment.

d $I = Pin = 28\,000 \times \frac{4.35}{100} \times 2 = \$2436$

e Interest = 304.50 + … 39.55 = \$1387.68

7 **a** \$455

b Total repaid = \$21 840

c Interest = 21 840 – 18 000 = \$3840

8 **a** \$392

b Total repaid = 392 × 36 = \$14 112

c Interest = 14 112 – 12 000 = \$2112

9 **a** **i** \$1588

ii Total repaid = 1588 × 12 × 25
= \$476 400

iii Interest = 476 400 – 250 000
= \$226 400

b **i** It would now take 21 years, 8 months to pay off the loan.

ii Payments = 21 × 12 + 8 = 260
Total repaid = 260 × 1700 = \$442 000

iii Interest = 442 000 – 250 000
= \$192 000

iv Saving = 226 400 – 192 000
= \$34 400

v Extra payment/month = 1700 – 1588
= \$112

vi If you can afford to pay a little more each month, it generates a large saving in interest costs and pays the loan off more quickly.

10 **a** Monthly repayment = \$1459

b \$437 544

c \$177 544

d Years remaining = 25 – 20 = 5 years.
Balance remaining = \$76 900

e Half loan = 260 000 ÷ 2 = 130 000,
9 years remaining is first time below \$130 000.

10	\$139,217
9	\$127,869
8	\$115,986

Time taken = 25 – 9 = 16 years

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11 **a** Monthly repayment = \$468

b \$28 047

c \$6047

d After 3 years, leaves 2 years remaining

3	\$14,139
2	\$9,746
1	\$4,893

Balance remaining = \$9746

e Half the loan = 22 000 ÷ 2 = \$11 000

It takes 3 years for the balance to fall below \$11 000.

12 **a** Monthly repayment = \$94 105

b \$5 646 264

c \$646 264

d After 4 years, leaves 1 year remaining

2	\$2,062,062
1	\$1,010,279
0	\$0

Balance remaining = \$1 010 279

e Half the loan = 5 million ÷ 2 = \$2.5 million. It takes 3 years (2 years remaining) for the debt to fall below \$2.5 million.

Exercise 12B

1 **a** online, spreadsheets

b costs

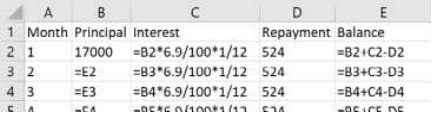
c calculations

2 **a** Monthly repayment = \$524

b Total repayments = \$18 868.81

c Total interest = \$1868.81

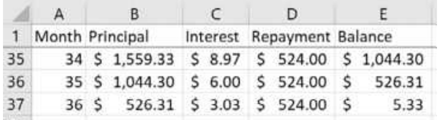
3 **a**



	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
2	1	17000	=B2*6.9/100*1/12	524	=B2+C2-D2
3	2	=E2	=B3*6.9/100*1/12	524	=B3+C3-D3
4	3	=E3	=B4*6.9/100*1/12	524	=B4+C4-D4

	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
2	1	\$17,000.00	\$97.75	\$ 524.00	\$16,573.75
3	2	\$16,573.75	\$95.30	\$ 524.00	\$16,145.05
4	3	\$16,145.05	\$92.83	\$ 524.00	\$15,713.88

b \$5.33 still left owing



	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
35	34	\$ 1,559.33	\$ 8.97	\$ 524.00	\$ 1,044.30
36	35	\$ 1,044.30	\$ 6.00	\$ 524.00	\$ 526.31
37	36	\$ 526.31	\$ 3.03	\$ 524.00	\$ 5.33

c Last payment = 524 + 5.33 = \$529.33

d Half loan = 17 000 ÷ 2 = \$8500

It takes 19 months to pay the first half of the loan.

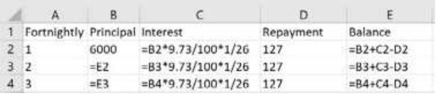
4 **a** Fortnightly payment = \$127

b 2 × 26 = 52

c Total loan repayments = \$6613.91

d Total interest = \$613.91

5 **a**



	A	B	C	D	E
1	Fortnightly	Principal	Interest	Repayment	Balance
2	1	6000	=B2*9.73/100*1/26	127	=B2+C2-D2
3	2	=E2	=B3*9.73/100*1/26	127	=B3+C3-D3
4	3	=E3	=B4*9.73/100*1/26	127	=B4+C4-D4

b $I = Pin = 6000 \times \frac{9.73}{100} \times \frac{1}{26} = \$22.45$

c \$10.91

d 127 + 10.91 = \$137.91

e Half loan = 6000 ÷ 2 = \$3000. It takes 28 fortnights to repay half the loan.

6 **a** **i** Interest $= Pin = 20\,000 \times \frac{11.5}{100} \times 5$

$= \$11500$

ii Total to repay = 20 000 + 11 500 = \$31 500

iii Number of repayments = 12 × 5 = 60

iv Monthly repayment = 31 500 ÷ 60 = \$525

b **i** Simple interest for one month = $Pin = 20\,000 \times \frac{11.5}{100} \times \frac{1}{12}$

ii The balance owed at the end of the month is the previous amount owed plus interest and take away the repayment made.

iii Simple interest is the same every time period as it is always calculated on the original loan amount.

c Yes, the amount corresponds. Both are \$11 500.

d Yes, the amount corresponds. Both are \$31 500.

e They are always the same amount.

7 **a** **i** Monthly repayment = \$440 to nearest dollar

ii Total to repay = \$26 391.13

iii Total interest = \$6391.13

b **i** Interest for one month = $Pin = \text{Principal} \times \frac{11.5}{100} \times \frac{1}{12}$

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ii The balance owed at the end of the month is the previous amount owed plus interest and take away the repayment made.

iii The principal in the second month is the same as the closing balance of the previous month.

c The values are reducing.

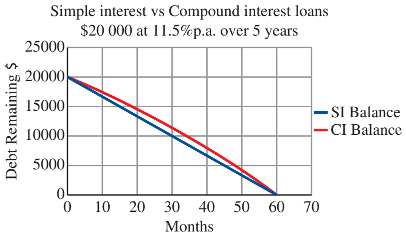
d i It is not quite the same $6388.09 vs $6391.13. This is because of rounding.

ii Compound loan interest = $6388.09, Simple loan interest = $11 500, Difference in interest = 11 500 – 6388.09 = $5111.91

e i It is not quite the same $26 400 vs $26 391.13. This is because of rounding.

ii Compound loan total = $26 400, Simple loan total = $31 500, Difference in total paid = 31 500 – 26 400 = $5100.00

8 a, b



c The simple interest loan produces a straight-line graph. The compound interest loan produces a curved-line graph.

Exercise 12C

1 impact, repay, interest

2 a i $1741 per month

ii Total interest = $235 066

b i $804 per fortnight, Interest = $234 440

ii Savings = 235 066 – 234 440 = $626

c i Total interest = $203 714, Savings = 235 066 – 203 714 = $31 352

ii Repaid in 22 years 2 months. Time saved = 2 years 10 months

d An extra $40 per fortnight

3 a i $1352 per month

ii Total interest = $139 274

b i $624 per fortnight.

ii Interest = $138 822, Savings = 139 274 – 138 822 = $452

c i Interest = $121 140, Savings = 139 274 – 121 140 = $18 134

ii Repaid in 17 years 10 months, Time saved = 2 years 2 months

d Her new repayment would be $24 less per fortnight.

4 a $1310 per month

b Interest = (1310 × 12 × 20) – 180 000 = $134 400

c $1295 per month

d Interest = (1295 × 12 × 20) – 180 000 = $130 800, Saving = $3600

5 a $815 per month

b Interest = (815 × 12 × 5) – 40 000 = $8900

c $821 per month

d He needs to find an extra $6 per month for his loan repayment. If he had a bigger loan, finding extra money could be difficult.

e Interest = (821 × 12 × 5) – 40 000 = $9260, Extra interest = $360

6 a $1732 per month

b Total cost = 1732 × 12 × 25 = $519 600

c 1732 ÷ 2 = $866 per fortnight

d 21 years

e Total cost = 866 × 26 × 21 = $472 836

f He should pay his loan each fortnight. He pays it off 4 years faster and saves himself (519 600 – 472 836) $46 764 in interest charges.

7 a i $2066 per month

ii Total interest = $304 887 (from website)

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b **i** Fortnightly payment = 2066 ÷ 2
= $1033

ii Total interest = $247 374 (from website)
Savings = 304 887 – 247 374 = $57 513
Time to repay = 21 years, Time saved = 4 years

c New rate = 6.19 + 0.5 = 6.69% p.a.
New monthly payment = $2164
New fortnightly payment = $1082
Additional payment = 1082 – 1033 = $49 per fortnight

8 **a** **i** $2047 per month
ii Total interest = $205 362 (from website)

b **i** $1841 per month
ii Total interest = $266 286 (from website)
iii Extra interest = 266 286 – 205 362 = $60 924

c **i** Fortnightly payment = 1841 ÷ 2
= $921
ii Total interest = $216 978 (from website)
Savings = 266 286 – 216 978 = $49 308
Time = 21 years 1 month, Time saving = 3 years 11 months

d New rate = 5.99 – 0.25 = 5.74% p.a.
New monthly payment = $1798
New fortnightly payment = $899
Reduction in payment = 921 – 899 = $22 per fortnight

Exercise 12D

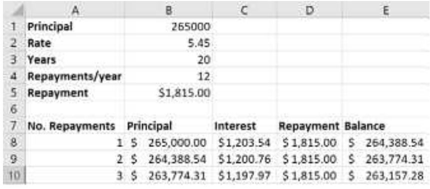
1 **a** interest, frequency, impact, pay, cost
b changes, length, interest

2 **a** 12 **b** 52
c 365 **d** 26
e 2 **f** 13
g 4

3 **a** **i** $1970.96 **ii** $591 287.49
iii $295 287.49
b **i** $1952.62 **ii** $585 786.41
iii $289 786.41
c $295 287.49 – $289 786.41 = $5501.08

4 **a** Repayment = $1816

b

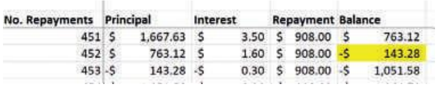


	A	B	C	D	E
1	Principal	265000			
2	Rate	5.45			
3	Years	20			
4	Repayments/year	12			
5	Repayment	$1,815.00			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	$ 265,000.00	$1,203.54	$ 1,815.00	$ 264,388.54
9	2	$ 264,388.54	$1,200.76	$ 1,815.00	$ 263,774.31
10	3	$ 263,774.31	$1,197.97	$ 1,815.00	$ 263,157.28

c Total interest = $170 591.37

d Repay = 1816 ÷ 2 = $908 per fortnight

e First balance less than $0 at 452 fortnightly payments
Time = 452 ÷ 26 = 17.38 years
= 17 years 5 months

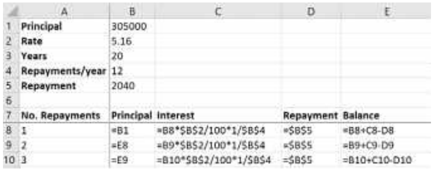


No. Repayments	Principal	Interest	Repayment	Balance
451	$ 1,667.63	$ 3.50	$ 908.00	$ 763.12
452	$ 763.12	$ 1.60	$ 908.00	-$ 143.28
453	-$ 143.28	-$ 0.30	$ 908.00	-$ 1,051.58

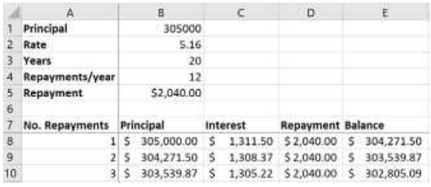
f **i** Total interest = $145 272.72,
ii Interest saved = 170 591.37 – 145 272.72 = $25 318.65
Time saved = 20 years – 17 years 5 months = 2 years 7 months

5 **a** Monthly repayment = $2040

b



	A	B	C	D	E
1	Principal	305000			
2	Rate	5.16			
3	Years	20			
4	Repayments/year	12			
5	Repayment	2040			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	=B1	=B8*$B$2/100*1/$B$4	=$B$5	=B8+C8-D8
9	2	=E8	=B9*$B$2/100*1/$B$4	=$B$5	=B9+C9-D9
10	3	=E9	=B10*$B$2/100*1/$B$4	=$B$5	=B10+C10-D10



	A	B	C	D	E
1	Principal	305000			
2	Rate	5.16			
3	Years	20			
4	Repayments/year	12			
5	Repayment	$2,040.00			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	$ 305,000.00	$ 1,311.50	$ 2,040.00	$ 304,271.50
9	2	$ 304,271.50	$ 1,308.37	$ 2,040.00	$ 303,539.87
10	3	$ 303,539.87	$ 1,305.22	$ 2,040.00	$ 302,805.09

c Total interest = $184 566.75

d New payment = $2040 + 40 = $2080
Time taken to repay loan is now 233 months = 19 years 5 months

e **i** Total interest = $177 667.51,
ii Interest saved = 184 566.75 – 177 667.51 = $6 899.24
Time saved = 240 – 233 = 7 months

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Chapter checklist

1 a

Month	Principal	Interest	Repayment	Balance
1	$ 16 000.00	$ 154.13	$ 750.00	$ 15 404.13
2	$ 15 404.13	$ 148.39	$ 750.00	$ 14 802.52
3	$ 14 802.52	$ 142.60	$ 750.00	$ 14 195.12

b $154.13

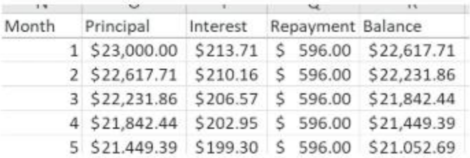
c 16 000 – 15 404.13 = $595.87

d $142.60

e 14 802.52 – 14 195.12 = $607.40

f Brooke is increasing the amount she pays off her debt each month as the interest reduces.

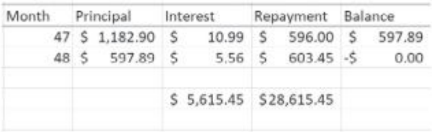
2 a



Month	Principal	Interest	Repayment	Balance
1	$23,000.00	$213.71	$ 596.00	$22,617.71
2	$22,617.71	$210.16	$ 596.00	$22,231.86
3	$22,231.86	$206.57	$ 596.00	$21,842.44
4	$21,842.44	$202.95	$ 596.00	$21,449.39
5	$21.449.39	$199.30	$ 596.00	$21.052.69

b $7.45 is left owing

c $596 + 7.45 = $603.45



Month	Principal	Interest	Repayment	Balance
47	$ 1,182.90	$ 10.99	$ 596.00	$ 597.89
48	$ 597.89	$ 5.56	$ 603.45	-$ 0.00
		$ 5,615.45	$28,615.45	

d Total repaid = $28 615.45

e Total Interest = $5615.45

f Half the loan = 23 000 ÷ 2 = $11 500
Balance fall below $11 500 after 27 months

3 a The monthly repayment will be $619

b Total repaid = 619 × 48 = $29 712

c Interest = 29 712 – 24 000 = $5712

4 a $I = Pin = 24\,000 \times \frac{10.85}{100} \times 4 = \$10\,416$

b Total to repay = 24 000 + 10 416
= $34 416

c No. = 4 × 12 = 48

d Repayment = $34 416 ÷ 48 = $717

e Monthly repayments: Megan = $619, Joe = $717. Joe pays $98 more per month. Overall cost: Megan = $29 712, Joe = $34 416. Joe pays $4704 more for his loan.

5 a i Repayment = $1504 per month

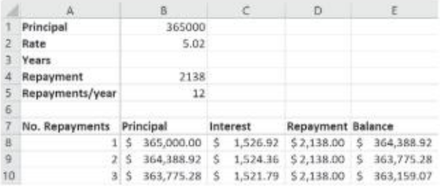
ii Total interest $147 951

b i Interest = $138 046, Interest saving
= 147 951 – 138 046 = $9905

ii Time to repay = 18 years 10 months,
Time saved = 1 year 2 months

c She would save $60 per month on her loan repayment.

6 a



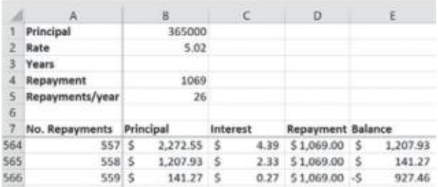
	A	B	C	D	E
1	Principal	365000			
2	Rate	5.02			
3	Years				
4	Repayment	2138			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	$ 365,000.00	$ 1,526.92	$ 2,138.00	$ 364,388.92
9	2	$ 364,388.92	$ 1,524.36	$ 2,138.00	$ 363,775.28
10	3	$ 363,775.28	$ 1,521.79	$ 2,138.00	$ 363,159.07

b Loan repaid in 301 months = 25 years 1 month. (Only $5.36 left owing after 300 months)

c Total interest = $276 405.39

d Fortnightly payment = 2138 ÷ 2
= $1069

e Loan repaid in 559 fortnights = 21.5 years



	A	B	C	D	E
1	Principal	365000			
2	Rate	5.02			
3	Years				
4	Repayment	1069			
5	Repayments/year	26			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
564	557	$ 2,272.55	$ 4.39	$ 1,069.00	$ 1,207.93
565	558	$ 1,207.93	$ 2.33	$ 1,069.00	$ 141.27
566	559	$ 141.27	$ 0.27	$ 1,069.00	-$ 927.46

f Total interest = \$231 643.54
Saving in interest = 276 405.39 – 231 643.54 = \$44 761.85
Time saving = 25 years, 1 month – 21 years 6 months = 3 years 7 months

Chapter review

1 **a** No. There is still \$1.35 left owing. This is because the monthly repayments must have been rounded down to the nearest dollar.

b 281 + 1.35 = \$282.35

c She still owes \$1639.89 after 6 months.

d \$26.27 interest in the first month

e 3200 – 2945.27 = \$254.73 reduced loan balance in first month

f \$19.94 interest in the fourth month

g 2429.51 – 2168.45 = \$261.06 reduced loan balance in fourth month

h She is reducing the debt by a greater amount each month as the interest reduces.

i The interest gets less each month as it is calculated on the reducing balance.

j \$173.36 total interest

k $\frac{9.85}{100} \times 3200 = \$315.20$ This is simple interest and it is calculated on the original balance every month, unlike reducing balance interest which is calculated on a smaller amount each month.

2 **a** Repayments = 4 × 12 = 48

b Repaid = 512 × 48 = \$24 576

c Interest = 24 576 – 19 500 = \$5076

3 **a** Repayments = 25 × 12 = 300

b Repaid = 1915 × 300 = \$574 500

c Interest = 574 500 – 285 000 = \$289 500

4 $I = Pin = 2800 \times \frac{11.8}{100} \times 2 = \$660.80$
Repay = 2800 + 660.8 = \$3460.80
No. Repayments = 2 × 12 = 24
Repayment = 3460.80 ÷ 24 = \$144.20

5 **a**

Month	Principal	Interest	Repayment	Balance
1	\$18 500.00	\$171.43	\$ 479.00	\$ 18 192.43
2	\$18 192.43	\$168.58	\$ 479.00	\$ 17 882.02
3	\$17 882.02	\$165.71	\$ 479.00	\$ 17 568.72

b Total repayments = \$1437.00

c Total interest = \$505.72

d Difference = 1437.00 – 505.72
= \$931.28

e Debt reduced by = 18 500 – 17 568.72
= \$931.28

6 **a** Monthly repayment = \$1969

b Total repaid = \$590 406

c Total interest = \$272 406

d \$101 342

e Half the loan = \$159 000; With 8 years remaining she still owes \$150 852. Takes 17 years to pay the first half of the loan.

7 **a** **i** Repayment = \$1537 per month

ii Interest charges = \$213 810

b Repayment = \$709 per fortnight
Interest = \$213 230 Savings = 213 810 – 213 230 = \$580

c **i** Interest = \$181 015, Savings = 213 810 – 181 015 = \$32 795

ii Time to repay = 21 years 9 months, Saving = 3 years 3 months

d Repayments would go up by \$71 per fortnight

8 **a** **i** \$2584 per month

ii Interest = \$234 116

b **i** Repayment = \$2295 per month

ii Interest \$302 474

iii Additional interest = 302 474 – 234 116 = \$68 358

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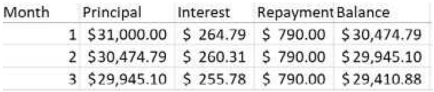
c **i** Fortnightly payment = 2295 ÷ 2
= $1148

ii Interest = $252 245, Saving = 302 474 – 252 245 = $50 229
Time to repay = 21 years, 5 months,
Time saving = 3 years 7 months

d New interest rate = 5.17 – 0.15
= 5.02% p.a.
New monthly repayment = $2261
New fortnightly repayment
= $1131(nearest dollar)
Savings per fortnight = 1148 – 1131
= $17

9 **a**



Month	Principal	Interest	Repayment	Balance
1	$31,000.00	$ 264.79	$ 790.00	$30,474.79
2	$30,474.79	$ 260.31	$ 790.00	$29,945.10
3	$29,945.10	$ 255.78	$ 790.00	$29,410.88

b Overpaid loan by $1.94

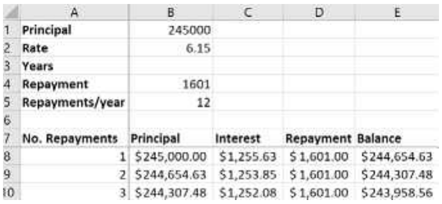
c 790.00 – 1.94 = $788.06

d Total loan repayments = $37 918.06

e Total interest = $6918.06

f Half the loan = 31 000 ÷ 2 = $15 500. The 27th month balance falls below $15 500.

10 **a**



	A	B	C	D	E
1	Principal	245000			
2	Rate	6.15			
3	Years				
4	Repayment	1601			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	$245,000.00	$1,255.63	$1,601.00	$244,654.63
9	2	$244,654.63	$1,253.85	$1,601.00	$244,307.48
10	3	$244,307.48	$1,252.08	$1,601.00	$243,958.56

b Loan repaid after 301 months, or 25 years, 1 month (only $56.03 owing after 300 months)

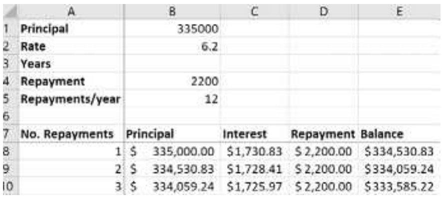
c Interest = $235 356.31

d 1601 ÷ 2 = $801 (to nearest dollar)

e Loan repaid after 545 fortnights, or 21 years

f Interest = $190 837.47, Saving = 235 356.31 – 190 837.47 = $44 518.84
Time saving = 25 years 1 month – 21 years = 4 years 1 month

11 **a**



	A	B	C	D	E
1	Principal	335000			
2	Rate	6.2			
3	Years				
4	Repayment	2200			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	$ 335,000.00	$1,730.83	$2,200.00	$334,530.83
9	2	$ 334,530.83	$1,728.41	$2,200.00	$334,059.24
10	3	$ 334,059.24	$1,725.97	$2,200.00	$333,585.22

b Loan repaid after 300 months, or 25 years

c Total interest = $324 678.39

d New payment = $2250, Time to repay = 285 months or 23 years, 9 months

e Interest = $305 269.69, Saving = 324 678.39 – 305 269.69 = $19 408.70
Time saving = 25 years = 23 years 9 months = 1 year 3 months

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