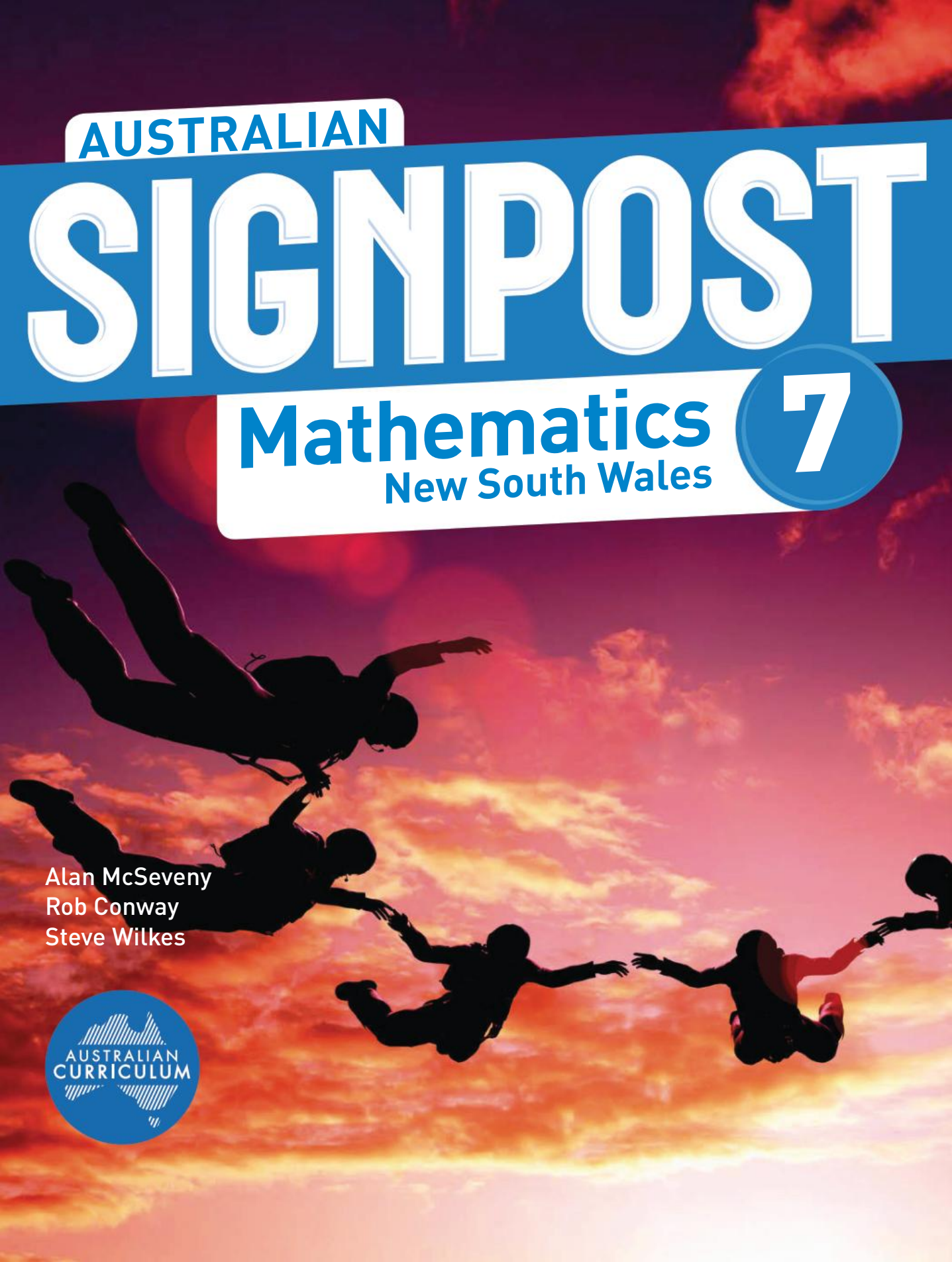
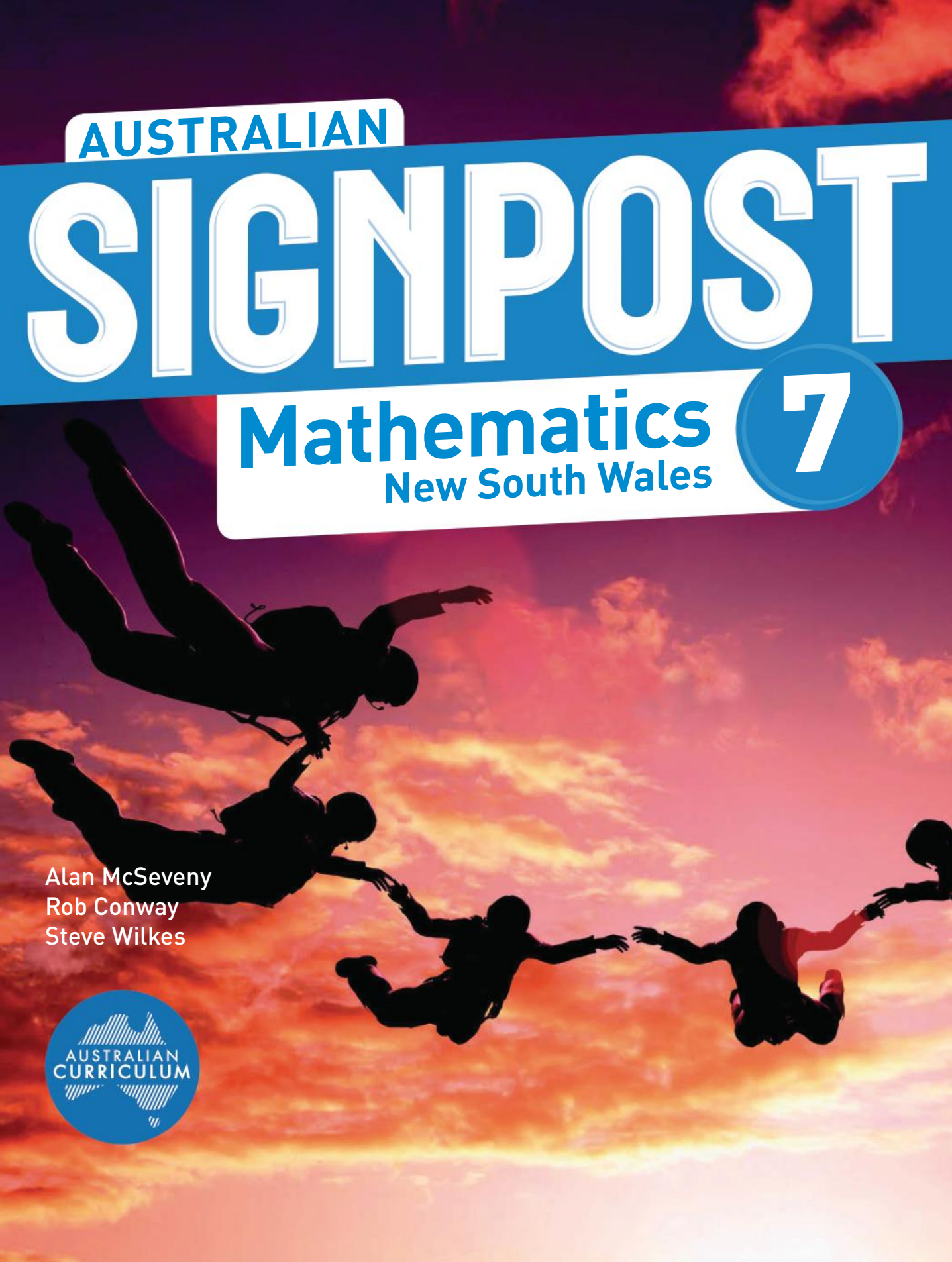
AUSTRALIAN SIGNPOST

Mathematics 7
New South Wales



Alan McSeveny
Rob Conway
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AUSTRALIAN CURRICULUM


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Mathematics
New South Wales
7
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Train a child in the way he should go, and
when he is old he will not turn from it.

Proverbs 22:6

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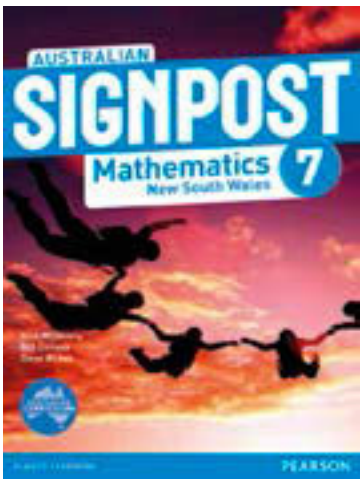
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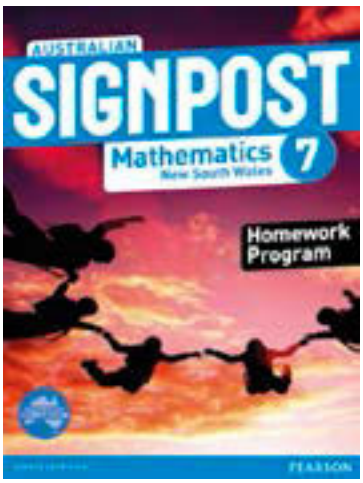
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The **Australian Signpost Mathematics New South Wales** series has been completely updated to comprehensively cover the NSW Syllabus for the Australian Curriculum. Students and teachers are supported with a Student Book, an eBook, a Homework Program and a Teacher Companion for each stage of development. Features within each component, along with additional resources on the eBook, allow the creation of a personalised learning package for the individual. Improvements in technology, teaching resources and design allow students and teachers to approach the NSW syllabus with confidence.



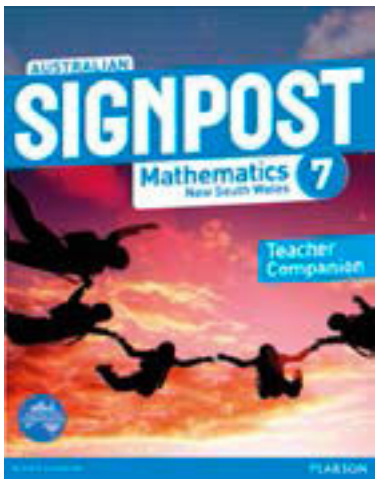
Student Book

The *Australian Signpost Mathematics New South Wales 7* and *Australian Signpost Mathematics New South Wales 8* learning package addresses Stage 4 of the NSW Syllabus for the Australian Curriculum. This new series continues the Signpost tradition of expanding students' ability in a broad range of mathematical skills while emphasising problem-solving and working mathematically.



Homework Program

The Homework Program consists of tear-out worksheets that mirror and supplement the content of the student book, providing opportunity for further practice and application of key skills.

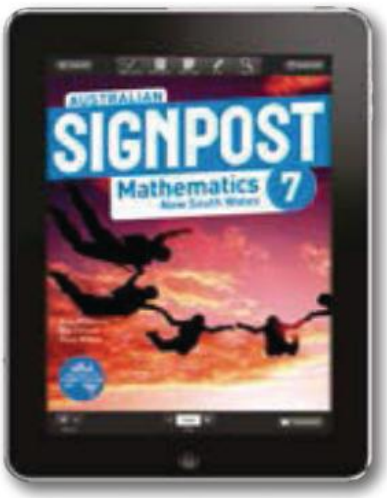


Teacher Companion

The Teacher Companion is an invaluable resource that provides guidance and support to teachers using the **Australian Signpost Mathematics New South Wales** package. It is a practical resource featuring teaching strategies, activities, reference to Stage 4 outcomes and content statements, and links to digital learning materials.

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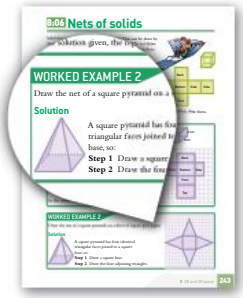
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HOW TO USE THE STUDENT BOOK

The **Australian Signpost Mathematics New South Wales 7** and **8** learning package gives complete coverage of Stage 4 of the NSW Syllabus for the Australian Curriculum.

Features of the Student Book

- a complete year's work with full coverage of the NSW Syllabus for the Australian Curriculum
- a flexible structure with carefully graded exercises, colour-coded to indicate level of difficulty
- technological applications with GeoGebra activities and Pearson eBook 3.0 resources
- integration of a broad range of mathematical skills, expanding students' ability to problem-solve, work mathematically and investigate
- diagnostic tests, chapter reviews and cumulative revision components for every chapter



Worked examples

take students through key processes with step-by-step demonstrations of questions similar to those they will encounter.



Well-graded exercises

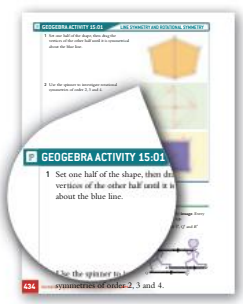
indicate each question's level of difficulty.

- 2 Foundation
- 5 Stage 4 level
- 12 Extension



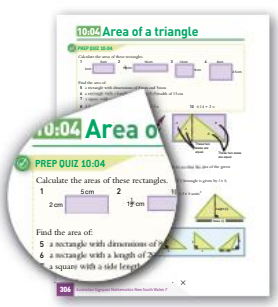
Chapter opener pages

summarise the key content and present the content statements addressed in each chapter. Features within each exercise such as GeoGebra activities, Fun spots and Investigations are listed here.

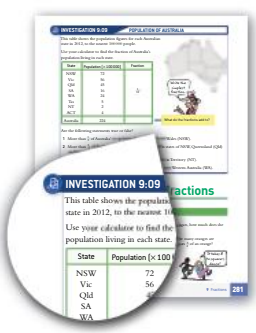


GeoGebra activities

provide a deeper understanding of concepts covered within the exercise while providing the opportunity to integrate ICT into the classroom.

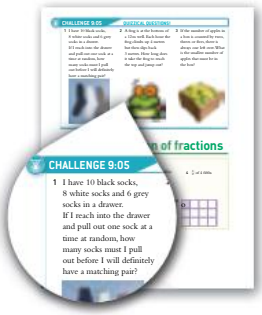


Prep quizzes review important skills and knowledge that students will need to complete an exercise.



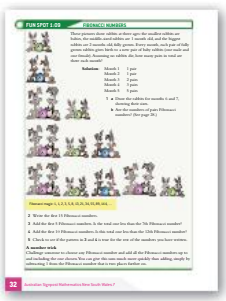
Investigations

encourage students to seek knowledge and develop research skills. The investigative tasks provide opportunities for students to interpret, question, model and communicate mathematically.



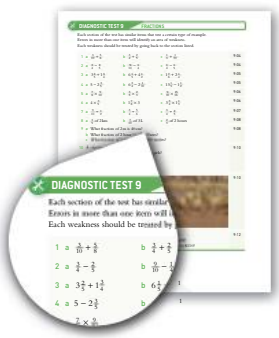
Challenge activities

provide more difficult questions and investigations. They can be used to help students with a stronger understanding extend their knowledge.



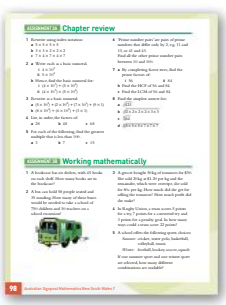
Fun spots

provide amusement and interest while reinforcing concepts learnt within the chapter. They encourage creativity and divergent thinking, and show that mathematics is enjoyable.



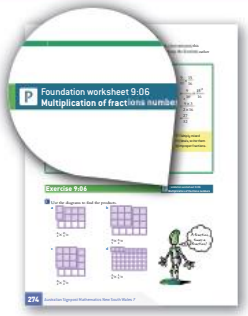
Diagnostic tests

at the end of each chapter test students' achievement of outcomes. Importantly, they also indicate student weaknesses that need to be addressed and provide links to the relevant sections in the text or digital resources.



Assignments

are provided at the end of each chapter. They concentrate on revising the content of previous chapters and developing students' ability to work mathematically.

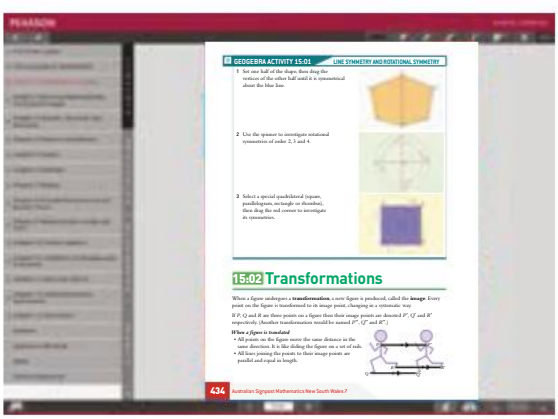


Foundation and Challenge worksheets

are indicated in the student book wherever a worksheet is available on the eBook. Foundation worksheets provide alternative exercises for consolidation of earlier stages. Challenge worksheets help students apply their learning in more advanced activities.

Pearson digital resources

A vast range of technology applications are available with the eBook to complement the content presented in the student book. Digital resources include interactive activities, practice and chapter tests, and Foundation and Challenge worksheets. From the eBook, students can also link directly to the GeoGebra applets referred to in the student book exercises.



The eBook aims to create a classroom environment that embraces mathematical technologies in a way that consolidates and extends student learning.

TREATMENT OF SYLLABUS CONTENT

The NSW Syllabus for the Australian Curriculum outlines the expected outcomes of students in Mathematics. The syllabus outcomes relevant to Stage 4 are listed below.

Working Mathematically (encompassing five components: Communicating, Problem Solving, Reasoning, Understanding and Fluency) is an integral part of the structure of each chapter. Special attention is given to Working Mathematically in Chapter 2 of the Year 7 text, in Chapter 1 of the Year 8 text and in the second Assignment of every chapter of both texts.

Working Mathematically	
A student:	
> communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols	**MA4-1WM**
> applies appropriate mathematical techniques to solve problems	**MA4-2WM**
> recognises and explains mathematical relationships using reasoning	**MA4-3WM**
Number and Algebra	
A student:	
> compares, orders and calculates with integers, applying a range of strategies to aid computation	**MA4-4NA**
> operates with fractions, decimals and percentages	**MA4-5NA**
> solves financial problems involving purchasing goods	**MA4-6NA**
> operates with ratios and rates, and explores their graphical representation	**MA4-7NA**
> generalises number properties to operate with algebraic expressions	**MA4-8NA**
> operates with positive-integer and zero indices of numerical bases	**MA4-9NA**
> uses algebraic techniques to solve simple linear and quadratic equations	**MA4-10NA**
> creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane	**MA4-11NA**
Measurement and Geometry	
A student:	
> calculates the perimeters of plane shapes and the circumference of circles	**MA4-12MG**
> uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area	**MA4-13MG**
> uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume	**MA4-14MG**
> performs calculations of time that involve mixed units, and interprets time zones	**MA4-15MG**
> applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems	**MA4-16MG**
> classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles	**MA4-17MG**
> identifies and uses angle relationships, including those related to transversals on sets of parallel lines	**MA4-18MG**
Statistics and Probability	
A student:	
> collects, represents and interprets single sets of data, using appropriate statistical displays	**MA4-19SP**
> analyses single sets of data using measures of location, and range	**MA4-20SP**
> represents probabilities of simple and compound events	**MA4-21SP**

Stage 4 content statements

The Stage 4 content statements are divided into three strands:

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

Content statements are listed below with their Australian Curriculum code or NSW syllabus reference. The chapters that address each content statement are shown in the right-hand column.

Number and Algebra	Reference	Chapters
Computation with Integers		
• Apply the associative, commutative and distributive laws to aid mental and written computation.	ACMNA151 (Stage 4)	1 (Y7) 1 (Y8)
• Compare, order, add and subtract integers.	ACMNA280 (Stage 4)	7 (Y7) 1, 2 (Y8)
• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies.	ACMNA183 (Stage 4)	7 (Y7) 2 (Y8)
Fractions, Decimals and Percentages		
• Compare fractions using equivalence; locate and represent positive and negative fractions and mixed numerals on a number line.	ACMNA152 (Stage 4)	9 (Y7) 1 (Y8)
• Solve problems involving addition and subtraction of fractions, including those with unrelated denominators.	ACMNA153 (Stage 4)	9 (Y7) 1 (Y8)
• Multiply and divide fractions and decimals using efficient written strategies and digital technologies.	ACMNA154 (Stage 4)	6, 9 (Y7) 1 (Y8)
• Express one quantity as a fraction of another, with and without the use of digital technologies.	ACMNA155 (Stage 4)	9 (Y7)
• Round decimals to a specified number of decimal places.	ACMNA156 (Stage 4)	6 (Y7) 1 (Y8)
• Investigate terminating and recurring decimals.	ACMNA184 (Stage 4)	6 (Y7)
• Connect fractions, decimal and percentages and carry out simple conversions.	ACMNA157 (Stage 4)	6, 11 (Y7) 1 (Y8)
• Investigate the concept of irrational numbers, including π.	ACMNA186 (Stage 4)	3 (Y8)
• Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies.	ACMNA158 (Stage 4)	11 (Y7) 1, 4 (Y8)
• Solve problems involving the use of percentages, including percentage increases and decreases, with and without the use of digital technologies.	ACMNA187 (Stage 4)	4 (Y8)
Financial Mathematics		
• Investigate and calculate the Goods and Services Tax (GST), with and without the use of digital technologies.	NSW (Stage 4)	4 (Y8)
• Investigate and calculate 'best buys', with and without the use of digital technologies.	ACMNA174 (Stage 4)	9 (Y7)
• Solve problems involving profit and loss, with and without the use of digital technologies.	ACMNA189 (Stage 4)	4 (Y8)
Ratios and Rates		
• Recognise and solve problems involving simple ratios.	ACMNA173 (Stage 4)	9 (Y7) 1, 10 (Y8)
• Solve a range of problems involving ratios and rates, with and without the use of digital technologies.	ACMNA188 (Stage 4)	10 (Y8)
• Investigate, interpret and analyse graphs from authentic data.	ACMNA180 (Stage 4)	14 (Y7) 10 (Y8)
Algebraic Techniques 1		
• Introduce the concept of variables as a way of representing numbers using letters.	ACMNA175 (Stage 4)	4 (Y7)
• Extend and apply the laws and properties of arithmetic to algebraic terms and expressions.	ACMNA177 (Stage 4)	4 (Y7) 1, 5 (Y8)
• Simplify algebraic expressions involving the four operations.	ACMNA192 (Stage 4)	4 (Y7) 1, 2, 5 (Y8)

Algebraic Techniques 2		
• Create algebraic expressions and evaluate them by substituting a given value for each variable.	ACMNA176 (Stage 4)	4 (Y7) 1, 2, 5 (Y8)
• Extend and apply the distributive law to the expansion of algebraic expressions.	ACMNA190 (Stage 4)	4 (Y7) 1, 5 (Y8)
• Factorise algebraic expressions by identifying numerical factors.	ACMNA191 (Stage 4)	5 (Y8)
• Factorise algebraic expressions by identifying algebraic factors.	NSW (Stage 4)	5 (Y8)
Indices		
• Investigate index notation and represent whole numbers as products of powers of prime numbers.	ACMNA149 (Stage 4)	3 (Y7) 1 (Y8)
• Investigate and use square roots of perfect square numbers.	ACMNA150 (Stage 4)	3 (Y7) 1 (Y8)
• Use index notation with numbers to establish the index laws with positive-integer indices and the zero index.	ACMNA182 (Stage 4)	2 (Y8)
Equations		
• Solve simple linear equations.	ACMNA179 (Stage 4)	13 (Y7) 1, 9 (Y8)
• Solve linear equations using algebraic techniques and verify solutions by substitution.	ACMNA194 (Stage 4)	13 (Y7) 9 (Y8)
• Solve simple quadratic equations.	NSW (Stage 4)	9 (Y8)
Linear Relationships		
• Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point.	ACMNA178 (Stage 4)	7 (Y7) 1 (Y8)
• Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates.	ACMMG181 (Stage 4)	15 (Y7)
• Plot linear relationships on the Cartesian plane, with and without the use of digital technologies.	ACMNA193 (Stage 4)	11 (Y8)
• Solve linear equations using graphical techniques.	ACMNA194 (Stage 4)	11 (Y8)

Measurement and Geometry	Reference	Chapters
Length		
• Find perimeters of parallelograms, trapeziums, rhombuses and kites.	ACMMG196 (Stage 4)	10 (Y7) 8 (Y8)
• Investigate the concept of irrational numbers, including π.	ACMNA186 (Stage 4)	12 (Y8)
• Investigate the relationship between features of circles, such as the circumference, radius and diameter; use formulas to solve problems involving circumference.	ACMMG197 (Stage 4)	12 (Y8)
Area		
• Choose appropriate units of measurement for area and convert from one unit to another.	ACMMG195 (Stage 4)	8 (Y8)
• Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.	ACMMG159 (Stage 4)	10 (Y7) 8 (Y8)
• Find areas of trapeziums, rhombuses and kites.	ACMMG196 (Stage 4)	8 (Y8)
• Investigate the relationship between features of circles, such as the area and the radius; use formulas to solve problems involving area.	ACMMG197 (Stage 4)	12 (Y8)
Volume		
• Draw different views of prisms and solids formed from combinations of prisms.	ACMMG161 (Stage 4)	8 (Y7)
• Choose appropriate units of measurement for volume and convert from one unit to another.	ACMMG195 (Stage 4)	10 (Y7) 8 (Y8)
• Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume.	ACMMG198 (Stage 4)	10 (Y7) 8 (Y8)
• Calculate the volumes of cylinders and solve related problems.	ACMMG217 (Stage 4)	12 (Y8)
Time		
• Solve problems involving duration, including using 12-hour and 24-hour time within a single time zone.	ACMMG199 (Stage 4)	8 (Y8)
• Solve problems involving international time zones.	NSW (Stage 4)	8 (Y8)
Right-Angled Triangles (Pythagoras)		
• Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles.	ACMMG222 (Stage 4)	3 (Y8)
• Investigate the concept of irrational numbers.	ACMNA186 (Stage 4)	3 (Y8)

Properties of Geometrical Figures 1		
• Classify triangles according to their side and angle properties and describe quadrilaterals.	ACMMG165 (Stage 4)	8 (Y7)
• Identify line and rotational symmetries.	ACMMG181 (Stage 4)	15 (Y7)
• Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral.	ACMMG166 (Stage 4)	8 (Y7) 15 (Y8)
• Use the properties of special triangles and quadrilaterals to solve simple numerical problems with appropriate reasoning.	NSW (Stage 4)	5, 8 (Y7) 15 (Y8)
Properties of Geometrical Figures 2		
• Define congruence of plane shapes using transformations.	ACMMG200 (Stage 4)	13 (Y8)
• Develop the conditions for congruence of triangles.	ACMMG201 (Stage 4)	13 (Y8)
• Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.	ACMMG202 (Stage 4)	13 (Y8)
Angle Relationships		
• Use the language, notation and conventions of geometry.	NSW (Stage 4)	5 (Y7)
• Recognise the geometrical properties of angles at a point.	NSW (Stage 4)	5 (Y7) 15 (Y8)
• Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal.	ACMMG163 (Stage 4)	5 (Y7) 15 (Y8)
• Investigate conditions for two lines to be parallel.	ACMMG164 (Stage 4)	5 (Y7) 15 (Y8)
• Solve simple numerical problems using reasoning.	ACMMG164 (Stage 4)	5 (Y7) 15 (Y8)

Statistics and Probability	**Reference**	**Chapters**
Data Collection and Representation		
• Investigate techniques for collecting data, including census, sampling and observation.	ACMSP284 (Stage 4)	14 (Y7) 14 (Y8)
• Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.	ACMSP206 (Stage 4)	14 (Y8)
• Identify and investigate issues involving numerical data collected from primary and secondary sources.	ACMSP169 (Stage 4)	14 (Y8)
• Construct and compare a range of data displays, including stem-and-leaf plots and dot plots.	ACMSP170 (Stage 4)	14 (Y7) 14 (Y8)
Single Variable Data Analysis		
• Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data.	ACMSP171 (Stage 4)	14 (Y7)
• Investigate the effect of individual data values, including outliers, on the mean and median.	ACMSP207 (Stage 4)	14 (Y8)
• Describe and interpret data displays using mean, median and range.	ACMSP172 (Stage 4)	14 (Y7) 14 (Y8)
• Explore the variation of means and proportions of random samples drawn from the same population.	ACMSP293 (Stage 4)	14 (Y8)
Probability 1		
• Construct sample spaces for single-step experiments with equally likely outcomes.	ACMSP167 (Stage 4)	12 (Y7)
• Assign probabilities to the outcomes of events and determine probabilities for events.	ACMSP168 (Stage 4)	12 (Y7)
• Identify complementary events and use the sum of probabilities to solve problems.	ACMSP204 (Stage 4)	12 (Y7) 6 (Y8)
Probability 2		
• Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'.	ACMSP205 (Stage 4)	6 (Y8)
• Represent events in two-way tables and Venn diagrams and solve related problems.	ACMSP292 (Stage 4)	6 (Y8)

Metric equivalents

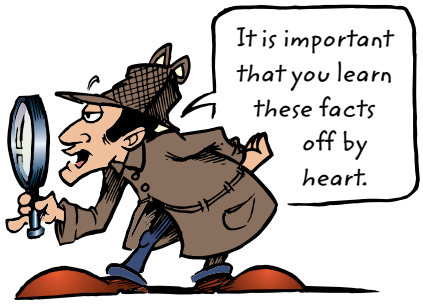
Length
1 m = 1000 mm = 100 cm = 10 dm 1 cm = 10 mm 1 km = 1000 m
Area
$1\ m^2 = 10\,000\ cm^2$ 1 ha = $10\,000\ m^2$ $1\ km^2$ = 100 ha
Mass
1 kg = 1000 g 1 t = 1000 kg 1 g = 1000 mg
Volume
$1\ m^3 = 1\,000\,000\ cm^3$ = $1000\ dm^3$ 1 L = 1000 mL 1 kL = 1000 L $1\ m^3$ = 1 kL $1\ cm^3$ = 1 mL $1000\ cm^3$ = 1 L
Time
1 min = 60 s 1 h = 60 min 1 day = 24 h 1 year = 365 days 1 leap year = 366 days

Months of the year

30 days each has September,
April, June and November.
All the rest have 31, except February alone,
Which has 28 days clear and 29 each leap year.

Seasons

Summer: December, January, February
Autumn: March, April, May
Winter: June, July, August
Spring: September, October, November





The language of mathematics

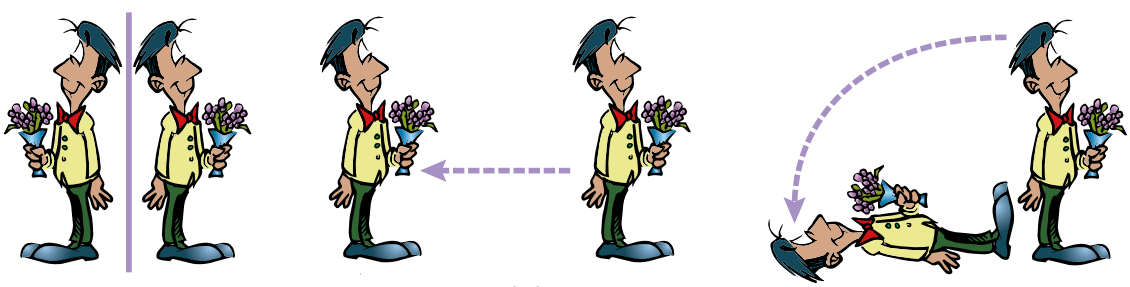
You should regularly test your knowledge by identifying the items on each card.

ID Card 1 (Metric units)			
1 m	2 dm	3 cm	4 mm
5 km	6 m^2	7 cm^2	8 km^2
9 ha	10 m^3	11 cm^3	12 s
13 min	14 h	15 m/s	16 km/h
17 g	18 mg	19 kg	20 t
21 L	22 mL	23 kL	24 °C

For answers see page 466.

ID Card 2 (Symbols)			
1 $+$	2 $-$	3 $\times$	4 $\div$
5 $=$	6 $\div$ or $\approx$	7 $\neq$	8 $<$
9 $\leq$	10 $\not<$	11 $>$	12 $\geq$
13 $\not>$	14 $\not\geq$	15 4^2	16 4^3
17 $\sqrt{25}$	18 $\sqrt[3]{27}$	19 %	20 $\therefore$
21 e.g.	22 i.e.	23 $\frac{3}{4}$ n.....	24 $\frac{3}{4}$ d.....

For answers see page 466.



reflection (or flip)

translation (or slide)

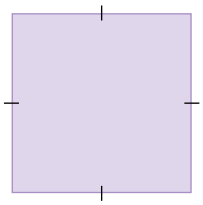
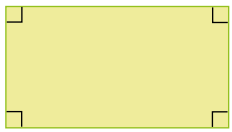
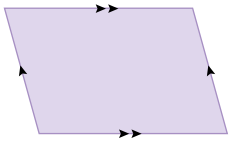
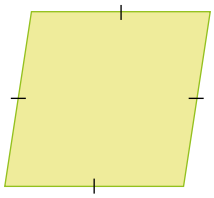
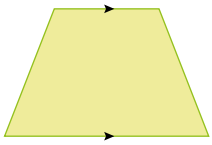
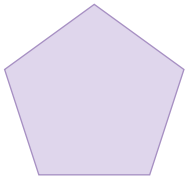
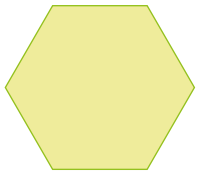
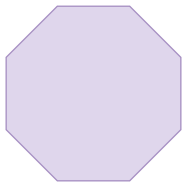
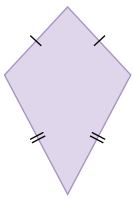
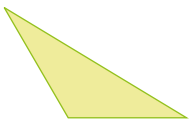
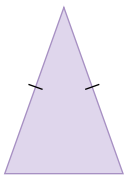
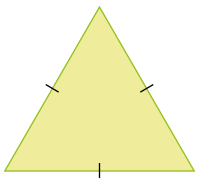
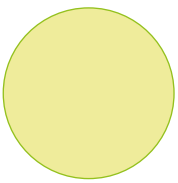
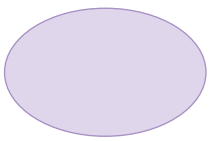
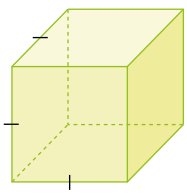
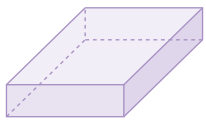
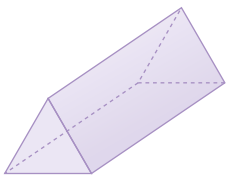
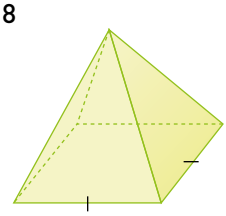
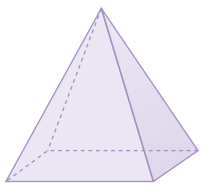
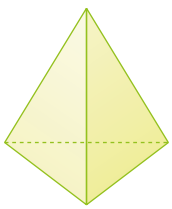
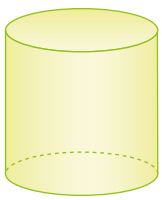
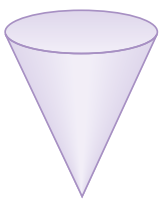
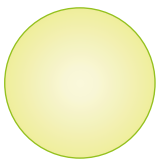
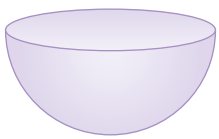
rotation (or turn)

ID Card 3 (Language)			
1 6 minus 2	**2** the sum of 6 and 2	**3** divide 6 by 2	**4** subtract 2 from 6
5 the quotient of 6 and 2	**6** $2\overline{)6}$ the divisor is	**7** $2\overline{)6}$ the dividend is	**8** 6 lots of 2
9 decrease 6 by 2	**10** the product of 6 and 2	**11** 6 more than 2	**12** 2 less than 6
13 6 squared	**14** the square root of 36	**15** 6 take away 2	**16** multiply 6 by 2
17 average of 6 and 2	**18** add 6 and 2	**19** 6 to the power of 2	**20** 6 less 2
21 the difference between 6 and 2	**22** increase 6 by 2	**23** share 6 between 2	**24** the total of 6 and 2

For answers see page 466.



ID Card 4 (Language)

1 	2 	3 	4 
5 	6 	7 	8 
9 	10  All sides different	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 

For answers see page 466.

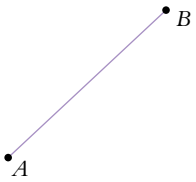
ID Card 5 (Language)

1



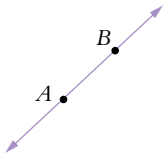
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2



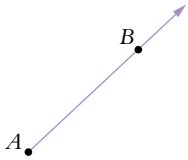
.....

3



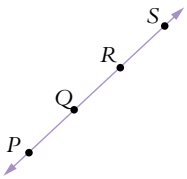
.....

4



.....

5



..... points

6



C is the .....

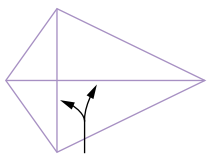
7



.....

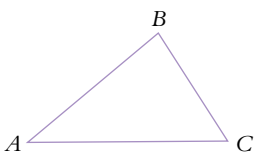
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8



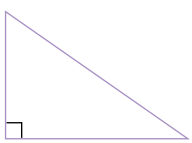
.....

9



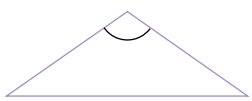
all angles less than 90°

10



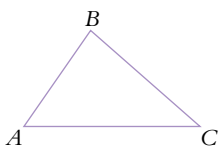
one angle 90°

11



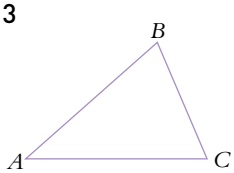
one angle greater than 90°

12



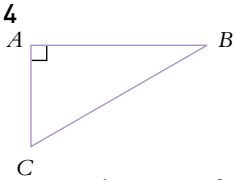
A, *B* and *C* are ..... of the triangle.

13



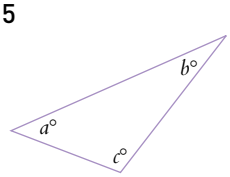
Use the vertices to name the Δ.

14



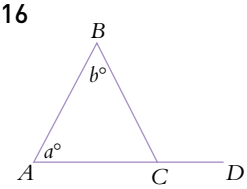
BC is the ..... of the right-angled Δ.

15



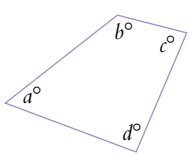
$a° + b° + c° =$ .....

16



$\angle BCD =$ .....

17



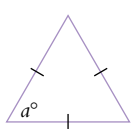
$a° + b° + c° + d° =$ ....

18



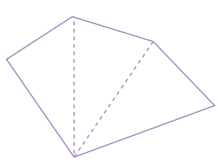
Which is true?
(a) $a° < b°$
(b) $a° = b°$
(c) $a° > b°$

19



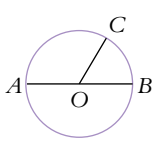
$a° =$ .....

20



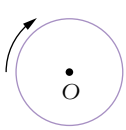
Angle sum = ...

21



AB is a .....
OC is a .....

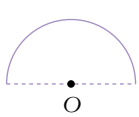
22



Name of distance around the circle.

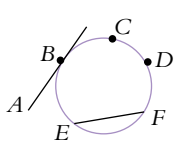
.....

23



s.....

24

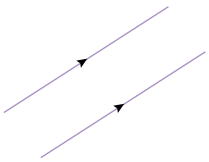


AB is a .....
CD is an .....
EF is a .....

For answers see page 466.

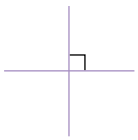
ID Card 6 (Language)

1



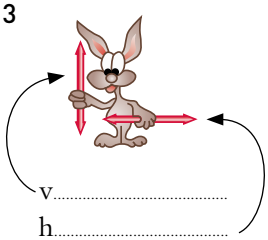
..... lines

2



..... lines

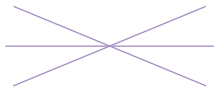
3



v.....

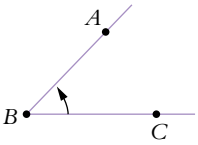
h.....

4



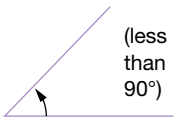
..... lines

5



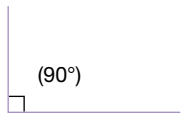
angle .....

6



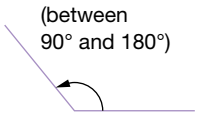
..... angle

7



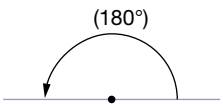
..... angle

8



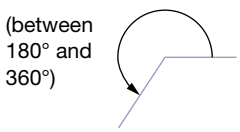
..... angle

9



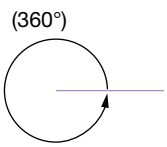
..... angle

10



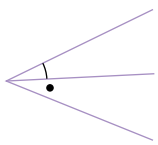
..... angle

11



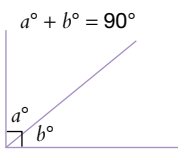
.....

12



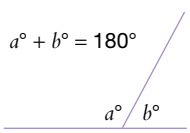
..... angles

13



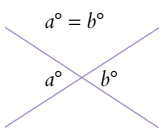
..... angles

14



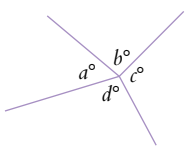
..... angles

15



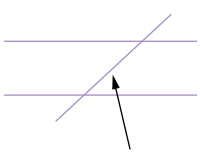
..... angles

16



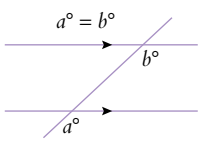
$a° + b° + c° + d° =$ ....

17



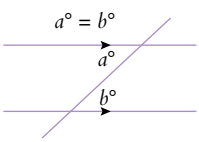
.....

18



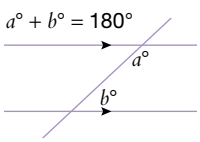
..... angles

19



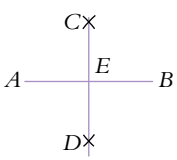
..... angles

20



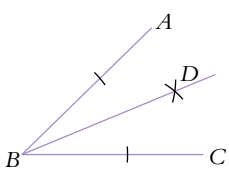
..... angles

21



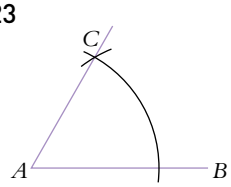
b..... an interval

22



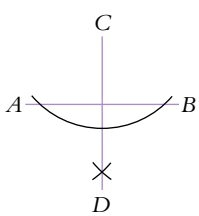
b..... an angle

23



$\angle CAB =$ .....

24



CD is p..... to AB.

For answers see page 467.

ID Card 7 (Language)

1

CE

C..... E.....

AD

A..... D.....

2

BCE

B..... C..... E.....

BC

B..... C.....

3

am

a..... m.....

means

b..... m.....

4

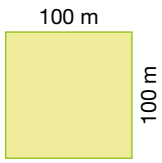
pm

p..... m.....

means

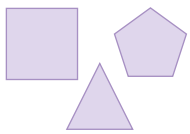
a..... m.....

5



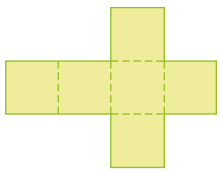
area is 1 .....

6



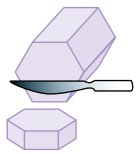
r..... shapes

7



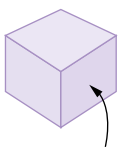
..... of a cube

8



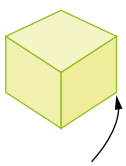
c.....-s.....

9



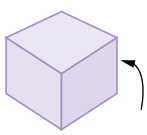
f.....

10



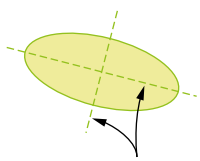
v.....

11



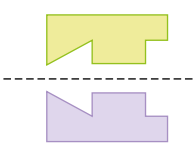
e.....

12



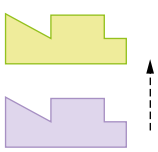
axes of .....

13



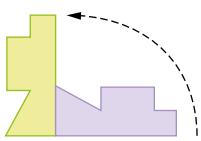
r.....

14



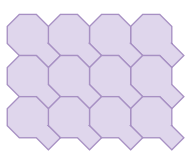
t.....

15



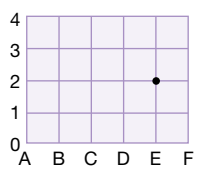
r.....

16



t.....

17



The c.....

of the dot are E2.

18

Cars sold

Mon	𝍸 I
Tues	𝍸 𝍸 𝍸 I
Wed	𝍸 𝍸 II
Thurs	𝍸 𝍸 𝍸
Fri	𝍸 𝍸 II

t.....

19

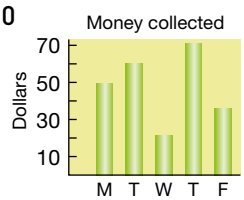
Money collected

Mon	● ● ● ●
Tues	● ● ● ◕
Wed	● ◔
Thurs	● ● ● ●
Fri	● ● ◔

● stands for $10

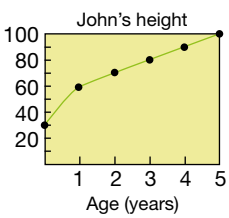
p..... graph

20



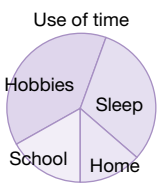
c..... graph

21



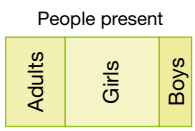
l..... graph

22



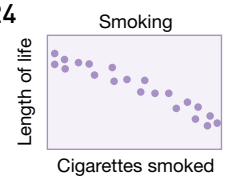
s..... graph

23



b..... graph

24



s..... d.....

For answers see page 467.

Arithmetic and algebra cards

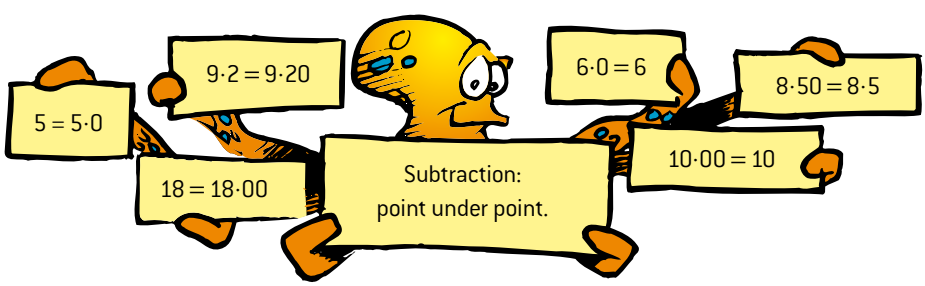
Arithmetic card

	A	B	C	D	E	F	G	H	I	J	K	L
1	2	5	$\frac{1}{2}$	100	0·2	5%	$\frac{2}{3}$	90	0·9	15%	13	56
2	3	8	$\frac{1}{3}$	90	0·1	40%	$\frac{1}{7}$	30	0·5	35%	18	36
3	9	7	$\frac{3}{4}$	80	0·7	20%	$\frac{1}{5}$	20	0·3	1%	15	21
4	5	12	$\frac{1}{5}$	20	0	60%	$\frac{1}{6}$	80	0·1	44%	10	96
5	4	15	$\frac{3}{5}$	80	0·5	25%	$\frac{1}{8}$	100	0·6	95%	19	88
6	1	18	$\frac{2}{3}$	30	0·4	10%	$\frac{1}{4}$	60	0·2	65%	12	24
7	0	4	$\frac{4}{5}$	60	0·8	50%	$\frac{1}{2}$	10	0·4	90%	20	43
8	11	3	$\frac{1}{4}$	40	0·6	80%	$\frac{4}{5}$	60	0·2	100%	14	62
9	10	10	$\frac{3}{10}$	70	0·6	30%	$\frac{2}{5}$	30	0·8	$33\frac{1}{3}$%	18	75
10	7	9	$\frac{2}{5}$	50	0·9	75%	$\frac{1}{3}$	70	0·7	85%	16	45

How to use these cards

As an example, if the instruction given for the Arithmetic card is 'column B + column K', then you would write answers for the following problems:

1 5 + 13 **2** 8 + 18 **3** 7 + 15 **4** 12 + 10 **5** 15 + 19
6 18 + 12 **7** 4 + 20 **8** 3 + 14 **9** 10 + 18 **10** 9 + 16



Algebra card

	A	B	C	D	E	F	G	H	I	J	K	L
1	4	-7	-11	x	$2y$	$-3x$	$-6y$	x^2	$2m$	x^7	$-5y$	-4
2	3	-6	7	$5x$	$7y$	$-x$	$-5y$	x^5	$3d$	x^2	$8y$	-3
3	5	-8	-9	$2x$	$10y$	$4x$	$4y$	x^3	$-2c$	x^8	$-12y$	-2
4	8	-2	-7	$3x$	$4y$	$-9x$	$-9y$	x^6	$-8h$	x^4	$0y$	-1
5	7	-10	-8	$10x$	$5y$	$-5x$	$-2y$	x	$9w$	x^3	$-7y$	0
6	2	-12	6	$7x$	$15y$	$-7x$	$-3y$	x^8	$-4d$	x^5	$0y$	1
7	0	-15	-5	$4x$	$6y$	$5x$	$-8y$	x^4	$-9n$	x	$-17y$	2
8	1	-5	9	$11x$	$3y$	$8x$	$-y$	x^7	$3x$	x^9	$-9y$	3
9	9	-3	8	$6x$	y	$-4x$	$-12y$	x^3	$-2w$	x^6	$5y$	4
10	10	-4	-3	$8x$	$9y$	$-8x$	$-7y$	x^4	$-7m$	x^{10}	$-8y$	5

For convenience, you might stick copies of these pages in your book.



BEGINNINGS IN NUMBER

1



Contents

1:01 The history of number
1:02 Place value
Investigation 1:02 Estimation
Fun spot 1:02 A nest of squares
1:03 The four operations
Challenge 1:03 Long division
1:04 Speed and accuracy
1:05 Using a calculator
1:06 Order of operations
1:06A Grouping symbols
1:06B Rules for the order of operations
1:06C Order of operations on a calculator
Fun spot 1:06 What's the difference between a boxer and a telephone?
1:07 Using number properties
1:07A Properties involving addition or multiplication
1:07B The distributive property
1:08 Language and symbols used in mathematics
Investigation 1:08 Odds and evens
1:09 Special sets of whole numbers
Fun spot 1:09 Fibonacci numbers
1:10 Estimating answers
1:11 Diagnostic check-ups
Check-up 1:11A Operations
Check-up 1:11B Fractions
Check-up 1:11C Decimals and money
Check-up 1:11D Percentages
Fun spot 1:11 Magic squares
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Review of earlier work from ***Whole Numbers 1*** and ***2***, ***Addition and Subtraction 1*** and ***2***, ***Multiplication and Division 1*** and ***2***, ***Fractions and Decimals 1*** and ***2***, and ***Patterns and Algebra 1*** and ***2***. [Stage 3]

Selections from ***Computation with Integers*** [Stage 4]

- Apply the associative, commutative and distributive laws to aid mental and written computation. (ACMNA151)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

1:01 The history of number

Ancient groups of people used number systems according to the needs of their everyday lives. Making notches on a stick or bone, using knots in string, or using pebbles to represent numbers may have completely satisfied the needs of many communities. For example, early Indigenous Australians had little need for a complex number system because most trade was in the form of barter, where groups of people would trade with neighbouring groups by swapping small amounts of goods directly. Many Indigenous Australian languages used concrete methods to indicate numbers of peoples or objects, such as the use of fingers to show the number. Words for 'one' and 'two' were common. In some languages these were combined to make three and four, e.g. 'two-one' would be three and 'two-two' would be four. Larger numbers of objects could be described as 'big mob' or 'little mob'. The life and culture of ancient peoples around the world was often rich in elements needed to ensure survival and harmony, but a complex number system was usually not one of those elements.

As communities expanded over time, their need for larger numbers grew, and so many civilisations invented symbols for large numbers. The Egyptian number system is a good example of this, as is the Roman number system. Roman numerals are still often used today in the prefaces of books, on the faces of clocks, on certain buildings (e.g. to show the date of construction) and in the credits of movies (to show the year of release).



Egyptian numerals

By about 5000 years ago, the Egyptians had developed a tally system based on the number ten. Ten of one symbol can be replaced by one of another.

Number	Symbol	Meaning
1		a vertical staff
10		a heel bone
100		a coiled rope
1000		a lotus flower
10 000		a bent reed or pointing finger
100 000		a burbot fish or tadpole
1 000 000		an amazed man or God of infinity
10 000 000 (or infinity)		a religious symbol

In the ancient Egyptian number system:

- the order of symbols does not affect the value of the numeral
- the value of a numeral can be found by adding the values of all the symbols used.

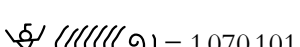
WORKED EXAMPLES

1 364 = 

or



2 11 321 143 = 

3  = 1 070 101

Roman numerals

With the rise to power of the ancient Roman empire, around 2000 years ago, the Roman number system spread to many other places in Europe and elsewhere.

The Roman number symbols originated as combinations of notches on sticks, a system used by the ancient Etruscans before them. The later Roman symbols conveniently look like things that the numbers represent, which makes them easier to remember.

Number	Symbol	How to remember
1	I	one finger
5	V	one hand
10	X	two Vs:
50	L	half a C
100	C	*centum* = Latin word for 'hundred'
500	D	half an
1000	M	*mille* = Latin word for 'thousand'



Roman numbers developed into a system that included the idea of subtraction. When a smaller number symbol appears before a larger one, it is subtracted from the larger one. This means that the position of the symbols is important.

- LX means 50 and 10.
- XL means 50 less 10 (i.e. 40).
- Larger numerals can be formed by drawing a stroke above the symbol:

$\overline{V} = 5000$

$\overline{X} = 10\,000$

$\overline{L} = 50\,000$

$\overline{C} = 100\,000$

$\overline{D} = 500\,000$

$\overline{M} = 1\,000\,000$

WORKED EXAMPLES

1 $392 = 300 + 90 + 2$
$= CCCXCII$

2 $1987 = 1000 + 900 + 80 + 7$
$= MCMLXXXVII$

3 $56\,049 = 50\,000 + 5000 + 1000 + 40 + 9$
$= \overline{L}VMXLIX$

Hindu–Arabic numerals

The modern numerals we use today are also called Hindu–Arabic numerals. They are based on numerals invented by the ancient Hindu civilisation of India around 300–100 BCE (with possible Chinese influence). By the 1300s the Hindu–Arabic system was well known across Western Europe. From the mid-1400s, the development of the printing press in Europe helped to standardise the shape of the numerals as we know them today.

Evolution of Hindu–Arabic symbols				
Now	150 BCE	876 CE	1000 CE	1400 CE
1	—	)	١	۱
2	=	?	২	2
3	≡	३	३	3
4	⊥	४	۷	٤
5	ᔑ	५	५	५
6	६	६	६	६
7	७	७	७	٧
8	ऽ	८	८	٨
9	९	९	९	٩
0		०		٠

- As in Roman numerals, the position of a symbol is very important. The system has *place value*, based on ten.
- The invention of a symbol for zero was a significant step, because an empty space for zero could be easily misunderstood.



WORKED EXAMPLES

1 The 1000 CE numeral for 453 is ۷۹۳

2 The 876 CE numeral for 54 901 is ५४९०)

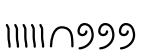
Exercise 1:01

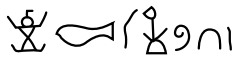


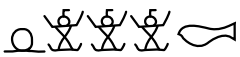
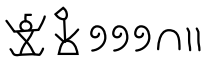
1 Write answers in your own words. Refer to previous pages in this chapter if necessary.

- **a** Why didn't the early Indigenous tribes of Australia have symbols for large numbers?
- **b** Did early Indigenous Australians need a complex number system?
- **c** Which number was the Egyptian number system based on?
- **d** In an Egyptian numeral, does the position of a symbol affect its value?
- **e** In a Roman numeral, what is meant when a symbol for a smaller unit appears before a symbol for a larger unit?
- **f** Where do you think the Roman symbol V came from?
- **g** Why do you think the Roman symbol X was used to represent ten?
- **h** In which country did our modern number system have its beginnings?
- **i** Which of the number systems described in this chapter has a symbol for zero? Why is the zero important to that system?

2 Change these Egyptian numerals into modern Hindu–Arabic numerals.

a  b  c  d 

e  f  g  h 

i  j  k  l 

3 Change these to Egyptian numerals.

a 48	b 91	c 706	d 1989	e 3965
f 31 024	g 93 708	h 102 420	i 150 000	j 371 213
k 4 600 000	l 23 000 000	m 11 111 111	n 3 053 000	o 15 680 000

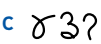
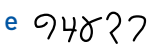
4 Change these Roman numerals into modern numerals.

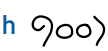
a CCCXV	b XXXIV	c CXXVIII	d DCLXXXII	e DCCIX
f MDCCXXIV	g MMMCD	h DXLVIII	i CMXLVII	j DCCCVIII
k MDV	l $\overline{V}$DLV	m $\overline{L}$MMD	n CDXCIX	o CMXCIX

5 Change these Hindu–Arabic numerals to Roman numerals.

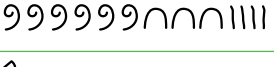
a 37	b 213	c 86	d 637	e 684
f 49	g 290	h 645	i 1452	j 778
k 988	l 1989	m 8489	n 5384	o 543 627

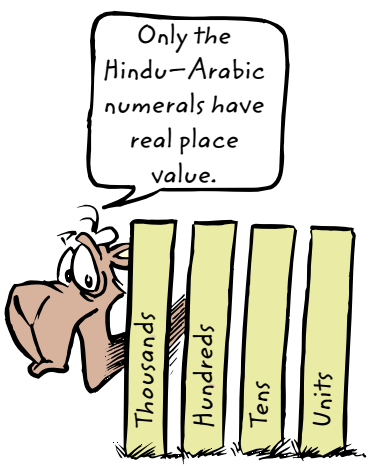
6 Write these 876 CE Hindu–Arabic numerals as modern numerals.

a  b  c  d  e 

f  g  h  i  j 

7 Copy this table into your book and complete it.

Hindu–Arabic numeral (876 CE)	Egyptian numeral	Roman numeral
		
		
		
		MMMDCCLXXVI
		MCMLXXXVIII



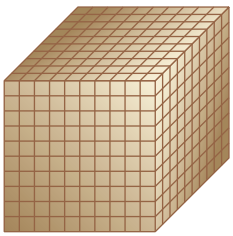
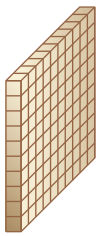
8 What advantages does the modern Hindu–Arabic system have over the Egyptian and Roman systems of numeration?

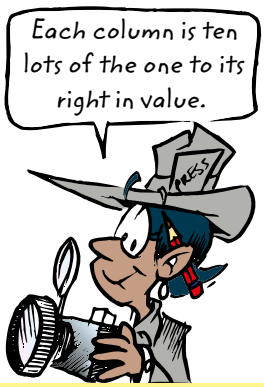
9 What is the longest Egyptian numeral between 500 and 1500?

10 What is the longest Roman numeral between 500 and 1500?

1:02 Place value

We can see that our modern Hindu–Arabic numerals are more useful than the Egyptian or Roman numerals because we use place value.

Thousands	Hundreds	Tens	Units
			
$10 \times (100)$ (1000)	$10 \times (10)$ (100)	$10 \times (1)$ (10)	1 (1)



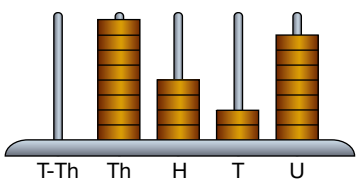
For numbers bigger than 9999, numerals are written with gaps to mark 'thousands', e.g. 10 234.

WORKED EXAMPLE

Write the number 8427 using expanded notation and show it on an abacus.

	8		4		2		7
	8 thousands	+	4 hundreds	+	2 tens	+	7 units
expanded notation:	(8×1000)	+	(4×100)	+	(2×10)	+	(7×1)

8427 on a 'spike' abacus:



WORKED EXAMPLES

1 Write 314 903 213 in words.

2 Write the value of the 5, the 6 and the 7 in 546 703.

Solutions

1 314 million

903 thousand

213 units

3	1	4		9	0	3		2	1	3

We write 'three hundred and fourteen million nine hundred and three thousand two hundred and thirteen'.

2 546 thousand

7 hundred and 3

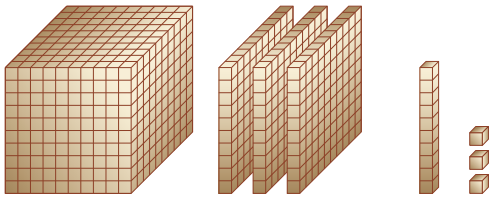
100 000	10 000	1000	100	10	1
5	4	6	7	0	3

The value of the 5 is 500 000.
The value of the 6 is 6000.
The value of the 7 is 700.

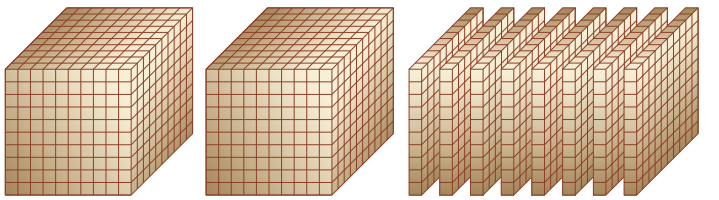
Exercise 1:02

1 What numbers are represented by each of the following?

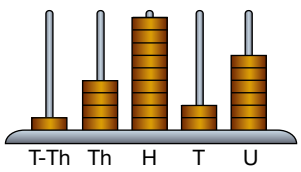
a



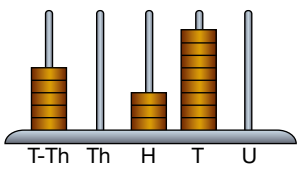
b



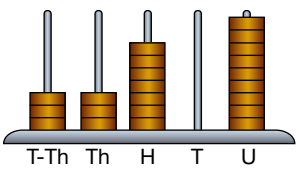
c



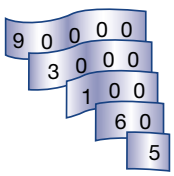
d



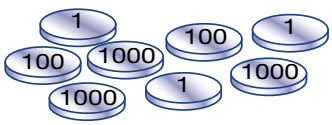
e



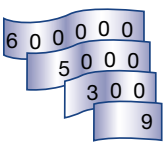
f



g



h



2 In each of the following, write as a numeral the number written in words.

a Five thousand eight hundred and fifteen.
b Three hundred and twenty-four thousand six hundred and seventy-nine.
c Eight hundred and six thousand and fifty.
d One million nine hundred and twenty-seven thousand four hundred and sixty-three.
e Fifty-six million two thousand nine hundred and fourteen.
f Eighty-three million nine hundred and seven thousand two hundred and one.
g Thirty-seven million seventy thousand eight hundred and forty-seven.
h Nine hundred and twenty-seven million one hundred thousand and seventy.
i Two hundred and four million forty-two thousand four hundred and twenty.

3 Write each of the following numerals in words.

a 8405	b 43 627	c 90 046
d 130 215	e 927 004	f 6 360 064
g 17 004 988	h 9 302 850	i 443 200 000

A *number* is the idea of 'how many'. A *numeral* is what we write to stand for the number.

4 Write the value of the 5, the 6 and the 7 in each of the following.

a 567	b 5607	c 53 067
d 570 600	e 63 075	f 635 700
g 6 354 073	h 1 567 214	i 58 673 121

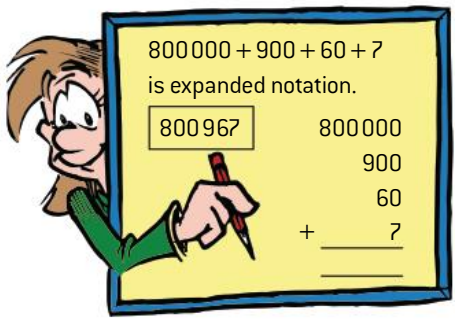
Example: 234 000	
Digit	Value
2	200 000
3	30 000
4	4 000
0	0

5 Write the value of each of the non-zero digits in the following.

a 6421	**b** 80 179	**c** 786
d 91 032	**e** 6094	**f** 1 340 627
g 7 346 912	**h** 27 000 000	**i** 675 000 000

6 Write each of these as a simple (basic) numeral.

a 50 000 + 7000 + 600 + 50 + 7
b 900 000 + 20 000 + 8000 + 600 + 70 + 8
c 3 000 000 + 800 000 + 60 000 + 7000 + 9
d 1 000 000 + 70 000 + 4000 + 600 + 10 + 2
e 800 000 + 900 + 60 + 7
f 5 000 000 + 800 000 + 1000 + 600 + 50 + 7
g 3 000 000 + 800 000 + 60 000 + 70 + 2
h 400 000 + 9000 + 800 + 70 + 1



7 Write these numbers in expanded notation.

a 59 675 **b** 806 307 **c** 9 137 826

8 In each part, write the numbers in order from smallest to largest.

a 47 341, 9841, 63 425, 120 070, 1688	**b** 1903, 24 106, 100 520, 91 000, 65 125
c 635 188, 86 314, 219 414, 9999, 10 112	**d** 132 145, 58 096, 8014, 72 143, 88 000
e 1 090 040, 938 497, 138 096, 365 214	**f** 77 717, 8987, 637 114, 123 000, 97 312
g 47 314, 100 000, 9060, 8914, 621 114	**h** 3 156 214, 1 500 000, 5 937 193, 980 000

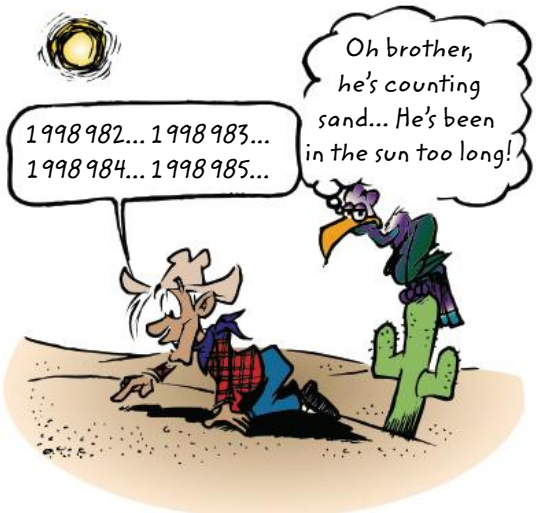
INVESTIGATION 1:02 ESTIMATION

1 Estimating large numbers

In groups of two or four, discuss and then complete these estimations. (You may use a calculator.)

a Given a thimble, a cup and a bucket of sand, it is possible to find an estimate of the number of grains of sand in the bucket without trying to count each grain of sand. Write in full sentences the steps you would use to find your estimate.

b Estimate the number of home-line phone numbers that would be found in a phone directory of your choice. Explain a method that would allow you to find the approximate number of phone numbers.



2 Counting to 1 000 000

Measure how much time it takes you to count from 1000 to 1050; from 121 150 to 121 200; and from 999 950 to 1 000 000. Estimate the time it would take you to count from 1 to 1 000 000.

FUN SPOT 1:02 A NEST OF SQUARES

The diagram shows a nest of squares.

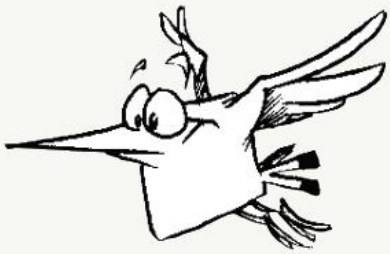
On the four corners (or *vertices*) of the largest square, any four numerals can be written. The positive difference of neighbouring numerals is written half-way between them.

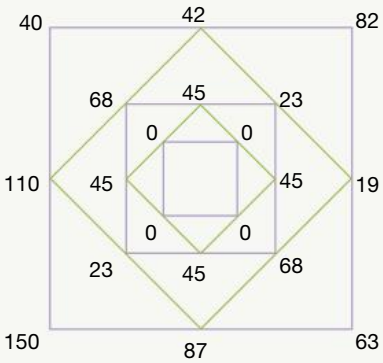
If this process is repeated, eventually you will have a square with a zero on each corner (or *vertex*).

Choose any four starting numbers and show that this happens for your nest of squares.

Will it also happen for a nest of triangles? Pentagons? Octagons?

Draw some nests of other shapes to find out.



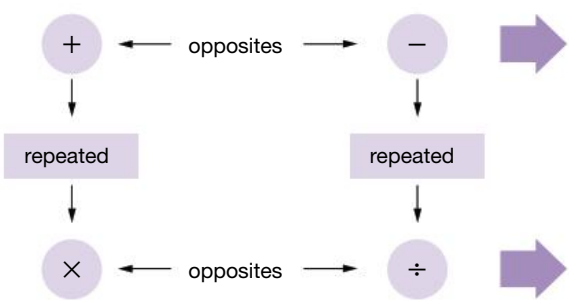


1:03 The four operations

Constant revision is the secret to removing errors and increasing your speed.



The four operations are related:



$(123 + 7) - 7 = 123$
Adding 7 and subtracting 7 are opposite operations.

$(175 \times 5) \div 5 = 175$
Multiplying by 5 and dividing by 5 are opposite operations.

Exercise 1:03

P Foundation worksheet 1:03A, B
Know your tables
Challenge worksheet 1:03
The four operations

1 Complete:

a $6 + 6 + 6 + 6 + 6 = \ldots$
5 lots of $6 = \ldots$
$5 \times 6 = \ldots$

b $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 = \ldots$
8 lots of $9 = \ldots$
$8 \times 9 = \ldots$

Multiplication can be done by repeated addition.

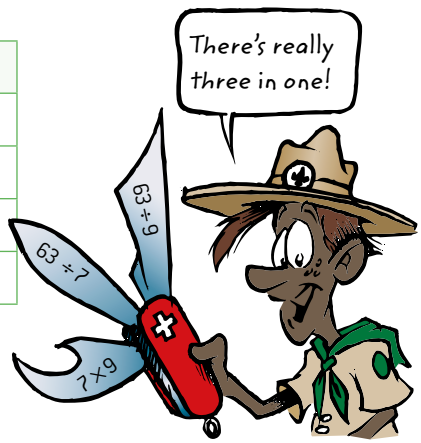
c How many 10s can be subtracted from 80?
How many lots of 10 in 80?
What is $80 \div 10$?

d How many 6s can be subtracted from 24?
How many lots of 6 in 24?
What is $24 \div 6$?

Division can be done by repeated subtraction.

2 Copy and complete this multiplication table.

×	4	1	3	6	0	10	2	5	9	7	11	8
2												
8												
4												
7												



If $7 \times 9 = 63$ then $63 \div 7 = 9$ and $63 \div 9 = 7$.

3 Each set should be completed within 20 seconds.

a

$36 \div 6$	
$56 \div 8$	
$32 \div 4$	
$27 \div 9$	
$42 \div 7$	

b

$36 \div 4$	
$49 \div 7$	
$48 \div 8$	
$45 \div 5$	
$63 \div 9$	

c

$42 \div 6$	
$24 \div 8$	
$36 \div 9$	
$16 \div 4$	
$56 \div 7$	

d

$24 \div 4$	
$63 \div 7$	
$64 \div 8$	
$72 \div 9$	
$35 \div 5$	

e

$48 \div 6$	
$28 \div 4$	
$21 \div 7$	
$72 \div 8$	
$81 \div 9$	

4 Division can be seen as sharing.

a Share 48 lollies between 4 girls. How many lollies does each get?
b $84 is shared by 6 students. How much does each student receive?
c 66 kg of fertiliser is used to treat 6 paddocks. If the same amount is used on each paddock, how much is used on each paddock?
d How many slices of bread would 8 sailors each get if they shared 120 slices?

5 Use the fact that division is the opposite of multiplication to answer the following.

a $(156 \times 9) \div 9$
b $(409 \times 7) \div 7$
c $(1196 \div 6) \times 6$
d $(6784 \times 11) \div 11$
e $(86 \times 13) \div 13$
f $(1160 \div 8) \times 8$
g $(318 \times 97) \div 97$
h $(3156 \div 4) \times 4$

6 To find $\frac{1}{8}$ of 160 we would divide 160 by 8. Use this method to find:

a $\frac{1}{4}$ of 48 **b** $\frac{1}{2}$ of 366 **c** $\frac{1}{3}$ of 300 **d** $\frac{1}{5}$ of 55

e $\frac{1}{8}$ of 160 **f** $\frac{1}{10}$ of 460 **g** $\frac{1}{9}$ of 36 **h** $\frac{1}{7}$ of 63

7 Division can be seen as repeated subtraction.

a How many times can 8 be subtracted from 33? What is the remainder?

b How many times can 14 be subtracted from 140? Is there a remainder?

c How many times can 9 be subtracted from 47? What is the remainder?

d How many 4 cm long pieces of wood can be cut from a length of 47 cm?

8 **a** $33 - 8 - 8 - 8 - 8$

b $138 - (10 \times 6) - (10 \times 6) - (2 \times 6) - (1 \times 6)$

c $2354 - (100 \times 14) - (50 \times 14) - (15 \times 14) - (3 \times 14)$

WORKED EXAMPLE

Four people share 19 slices of toast. How much toast should each receive?



Solution

$4\overline{)19}$ r 3 or $4\overline{)19}$ $4\frac{3}{4}$ or $\frac{19}{4} = 4\frac{3}{4}$

Each person should receive 4 slices. (The remaining 3 slices could also be cut into quarters and shared, so each person would receive 4 whole slices and 3 quarter slices.)

9 Write each answer as a mixed numeral.

a $8 \div 3$ **b** $11 \div 4$ **c** $37 \div 10$ **d** $33 \div 5$

e $7\overline{)11}$ **f** $6\overline{)5}$ **g** $\frac{13}{5}$ **h** $\frac{8}{3}$

i $5\overline{)608}$ **j** $4\overline{)375}$ **k** $10\overline{)108}$ **l** $9\overline{)757}$

10 Find the answer, writing any remainder as a fraction.

a $10\overline{)1089}$ **b** $10\overline{)36\,000}$ **c** $10\overline{)51086}$ **d** $10\overline{)71052}$

e $7\overline{)1086}$ **f** $5\overline{)9135}$ **g** $3\overline{)27111}$ **h** $4\overline{)61041}$

i $6\overline{)36066}$ **j** $7\overline{)71449}$ **k** $5\overline{)15007}$ **l** $8\overline{)245761}$

11 To find the average of a set of scores, add them all and divide your answer by the number of scores. The average of 4, 6 and 14 is $(4 + 6 + 14) \div 3$. Find the average of:

a 7, 4 and 10 **b** 9 and 21 **c** 16, 4 and 9 **d** 18, 8, 6 and 8

e 18 and 36 **f** 2, 11, 9 and 2 **g** 1, 1, 1, 1 and 6 **h** 22, 15 and 80

For the following questions, first decide which of the four operations is required to solve the problem, then write a mathematical statement and calculate the answer.

12 Sage borrowed $4500 from her parents to buy a new car. So far she has repaid $2740.

a How much money does Sage still owe her parents?

b If Sage repays her parents $110 a month, how much time will it take her to pay back the full amount of the loan?

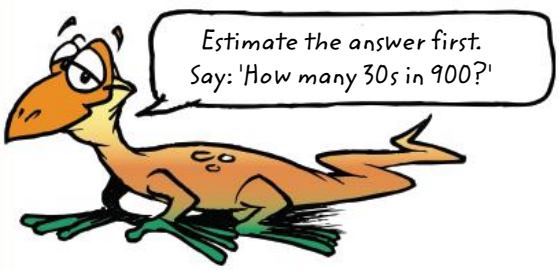
13 The Oxworth family's shower delivers 15 litres of water per minute. Each of the five family members has a 6-minute shower each morning.

a How much water is used by the family in the shower in one week?

b How much water will be saved per week if all family members reduce their shower to 4 minutes?



CHALLENGE 1:03 LONG DIVISION



After we have placed the '2' above the '7', we must also place a number above the '8'.

Method 1: Contracted form

$$\begin{array}{r} 28\text{ r }10 \\ 31\,)\,878 \\ \underline{-\,62} \\ 258 \\ \underline{-\,248} \\ 10 \end{array}$$

Method 2: Preferred multiples

$$\begin{array}{r|l} 28\text{ r }10 & \\ 31\,)\,878 & \\ \underline{-\,620} & \underline{20} \\ 258 & \\ \underline{-\,248} & \underline{8} \\ 10 & (28) \end{array}$$

Answer: 28 remainder 10 or $28\frac{10}{31}$.

1 Do not use a calculator for these. Use Method 1 or Method 2 from above.

a $98 \div 14$	**b** $91 \div 13$	**c** $63 \div 12$	**d** $79 \div 11$
e $214 \div 15$	**f** $196 \div 17$	**g** $238 \div 18$	**h** $256 \div 16$
i $647 \div 23$	**j** $555 \div 24$	**k** $858 \div 13$	**l** $973 \div 16$

2 If $838 \div 12 = 69$ r 10, then $838 = 69 \times 12 + 10$. Complete:

a $88 \div 44 = 2$, $\therefore$ $88 = 2 \times \ldots$

b $708 \div 23 = 30$ r 18, $\therefore$ $708 = 30 \times \ldots + \ldots$

c $182 \div 13 = 14$, $\therefore$ $182 = 14 \times \ldots$

d $654 \div 36 = 18$ r 6, $\therefore$ $654 = 18 \times \ldots + \ldots$

3 We can change a division question into multiplication. $356 \div 52 = \ldots$ becomes $52 \times \ldots = 356$. We then guess and check.

a $96 \div 31$	**b** $128 \div 39$	**c** $99 \div 22$	**d** $107 \div 18$
e $345 \div 99$	**f** $281 \div 47$	**g** $152 \div 29$	**h** $288 \div 62$

Check your answers to these using Method 1 or Method 2 from above.

$365 \div 12 = 30\frac{5}{12}$ means that $365 = 30 \times 12 + 5$

1:04 Speed and accuracy

Exercise 1:04

Time how quickly you can do each of the following sets of questions. Add 10 seconds to your time for each mistake. Record your times in a table like the one at the bottom of the page.

1 **a** $6 + 8$ **b** 9×9 **c** $35 \div 5$
d $15 - 7$ **e** 14×100 **f** $\$2 - \$1.35$
g Find the cost of 3 cups at $2.15 per cup.
h John earns five times as much as $4.20 per hour. How much does he earn per hour?
i How many 60-cent stamps can I buy with $7.40?
j $16.80 was shared by four girls. How much did each get?

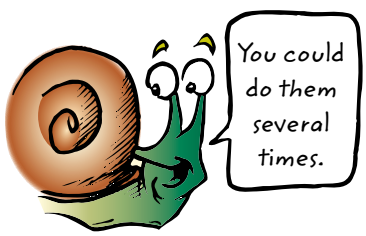
2 **a** $7 + 5$ **b** 8×6 **c** $42 \div 6$
d $14 - 8$ **e** 18×100 **f** $\$2 - \$1.85$
g Find the cost of 4 fans at $3.15 per fan.
h Kai earns six times as much as $5.10 per hour. How much does she earn per hour?
i How many 60-cent stamps can I buy with $6.40?
j $18.60 was shared by three friends. How much did each get?

3 **a** $9 + 4$ **b** 6×9 **c** $21 \div 7$
d $16 - 9$ **e** 17×100 **f** $\$2 - \$1.55$
g Find the cost of 5 pens at $1.15 per pen.
h Spiro earned eight times as much as $2.20. How much did he earn?
i How many 60-cent stamps can I buy with $9.70?
j $15.50 was shared by five friends. How much did each get?

4 **a** $8 + 7$ **b** 7×7 **c** $28 \div 4$
d $13 - 7$ **e** 13×100 **f** $\$2 - \$1.65$
g Find the cost of 3 books at $8.25 per book.
h Shubha earns four times as much as $3.40 per hour. How much does she earn per hour?
i How many 60-cent stamps can I buy with $8.90?
j $20.40 was shared by four friends. How much did each get?

Record of results

Number	1	2	3	4	1	2	3	4	1	2	3	4
Time												



1:05 Using a calculator

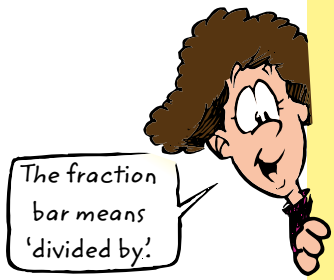
Calculators are great tools to help us solve maths problems, but they are not always right. If you press the wrong key or enter a calculation in the wrong order, the answer will not be correct.

Always estimate the answer before entering the calculation into the calculator. If the answer is very different from your estimate, check that it is correct!



Exercise 1:05

Use a calculator to complete this exercise.



$\frac{25}{4} = 25 \div 4$
$= 6{\cdot}25$

$\frac{3}{8} = 3 \div 8$
$= 0{\cdot}375$

$\frac{2}{7} = 2 \div 7$
$= 0{\cdot}\dot{2}8571\dot{4}$

Use your calculator.
$\frac{1}{8}$ of 36·9 metres:
Enter: 1 ÷ 8 × 36.9 =
Answer: 4·6125 m

1 **a** 3·4 + 29·725 **b** 17 050 − 9347
c 4428 ÷ 36 **d** 25·3 × 541
e Which is the best buy: 2 kg of potatoes for $5 or 7 kg for $12.50?
f How many 60-cent stamps can you buy with $10.35?
g What is the cost of one litre of petrol, if 28 litres cost $38.78?
h Change $\frac{7}{16}$ to a decimal.
i Find the value of $\frac{1272}{3} + 6{\cdot}142$.
j Find $\frac{1}{12}$ of 12·3 metres.

2 **a** 8·6 + 999·56 **b** 90 761 − 4172
c 22 442 ÷ 49 **d** 98 × 52·4
e Which is the best buy: 4 kg of potatoes for $7.92 or 9 kg for $18?
f How many 60-cent stamps can you buy with $12.35?
g What is the cost of one litre of petrol, if 26 litres cost $36.14?
h Change $\frac{7}{8}$ to a decimal.
i Find the value of $\frac{3471}{15} + 81{\cdot}67$.
j Find $\frac{1}{15}$ of 37·2 metres.



3 **a** 14·98 + 7·65 **b** 19 674 − 8475 **c** 20 412 ÷ 28 **d** 81·6 × 493
e Which is the best buy: 500 g of tuna for $3 or 300 g for $2.05?
f How many 60-cent stamps can you buy with $14.15?
g What is the cost of one litre of petrol, if 46 litres cost $54.97?
h Change $\frac{11}{16}$ to a decimal. **i** Find the value of $\frac{4369}{17} + 6{\cdot}142$.
j Find $\frac{1}{14}$ of 32·2 metres.

4 a $374{\cdot}1 + 88{\cdot}8$ b $17\,340 - 8096$ c $15\,456 \div 92$ d $741 \times 8{\cdot}66$
e Which is the best buy: 450 g of tuna for \$2.85 or 250 g for \$1.60?
f How many 89-cent rulers can you buy with \$8.85?
g What is the cost of one litre of petrol, if 16 litres cost \$23.92?
h Change $\frac{5}{8}$ to a decimal. i Find the value of $\frac{7470}{15} + 17{\cdot}86$.
j Find $\frac{1}{18}$ of 97·2 metres.

5 a $419 + 34{\cdot}361$ b $91\,310 - 694$ c $12\,168 \div 156$ d $4{\cdot}886 \times 9{\cdot}1$
e Which is the best buy: 375 g of tuna for \$2.37 or 300 g for \$1.92?
f How many 78-cent rulers can you buy with \$6.56?
g What is the cost of one litre of petrol, if 72 litres cost \$120.24?
h Change $\frac{9}{16}$ to a decimal. i Find the value of $\frac{8412}{12} + 32{\cdot}45$. j Find $\frac{1}{11}$ of 73·7 metres.

1:06 Order of operations

1:06A Grouping symbols

Grouping symbols are often used to tell us which operations to perform first. Three commonly used grouping symbols are parentheses, brackets and braces:

parentheses	brackets	braces
( )	[ ]	{ }

Where grouping symbols occur inside other grouping symbols, deal with the innermost grouping symbols first.

WORKED EXAMPLES

1 $7 \times (11 - 6)$
$= 7 \times 5$
$= 35$

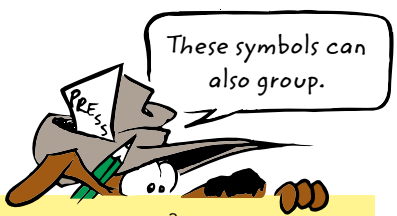
2 $17 - (6 + 10)$
$= 17 - 16$
$= 1$

3 $(6 + 7) \times (10 - 3)$
$= 13 \times 7$
$= 91$

4 $50 - (25 - [3 + 19])$
$= 50 - (25 - 22)$
$= 50 - 3$
$= 47$

5 $6 + 5^2$
$= 6 + (5 \times 5)$
$= 6 + 25$
$= 31$

6 $\dfrac{10 + 30}{10}$
$= (10 + 30) \div 10$
$= 40 \div 10$
$= 4$



- **Power** e.g. $5^2 = (5 \times 5)$
- **Fraction** bar e.g. $\dfrac{5+25}{11-6}$
  $= (5 + 25) \div (11 - 6)$
- **Square root** sign e.g. $\sqrt{9+16}$
  $= \sqrt{(9+16)}$

Exercise 1:06A

P Foundation worksheet 1:06A
Grouping symbols

1 Find the basic numeral for each.

a $(4 + 8) \times 10$	**b** $(19 - 9) \div 5$	**c** $25 - (50 - 30)$	**d** $8 \times (4 + 5)$
e $(4 + 2) \times 7$	**f** $(20 - 14) \times 8$	**g** $14 + (36 \div 6)$	**h** $16 - (12 - 3)$
i $70 \div (35 \div 5)$	**j** $(93 - 12) \div 3$	**k** $40 \div (13 - 8)$	**l** $40 \div (5 \times 4)$
m $15 + (100 \div 20)$	**n** $(16 - 9) + 23$	**o** $15 + (100 \div 5)$	**p** $100 - (7 \times 5)$
q $20 - (28 - 19)$	**r** $45 - (64 \div 8)$	**s** $(13 - 13) \div 6$	**t** $10 \div (28 - 23)$
u $73 \times (35 - 34)$	**v** $115 - (54 + 29)$	**w** $(0 \div 5) \times 15$	**x** $93 \div (14 + 79)$

2 Simplify:

a $(3 + 8) \times (15 - 9)$	**b** $(9 - 4) \times (7 + 5)$
c $(9 + 16) - (4 + 9)$	**d** $(25 + 7) - (9 + 6)$
e $(10 + 10) \div (7 + 3)$	**f** $(23 + 17) \div (5 + 3)$
g $(3 \times 6) + (4 \times 6)$	**h** $(7 \times 3) + (3 \times 3)$
i $(17 - 3) - (35 - 33)$	**j** $(9 - 3) \times (15 - 5)$
k $(7 - 3) - (9 - 7)$	**l** $(28 - 5) - (62 - 59)$
m $(105 - 5) \div (5 \times 10)$	**n** $(9 \times 7) \div (27 \div 3)$
o $(19 + 13) + (93 + 7)$	**p** $(43 + 17) + (60 + 40)$
q $(15 \times 3) \times (25 \times 4)$	**r** $(7 \times 8) \times (5 \times 2)$
s $(88 - 15) - (7 \times 10)$	**t** $(56 - 37) - (56 - 47)$

The simplest answer is called the basic numeral.

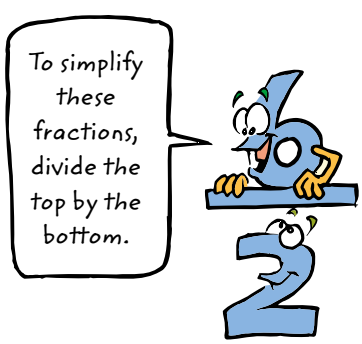
Grouping symbols come first.

3 Give the simplest answer for the following.

a $4 + [6 - (7 - 3)]$	**b** $9 + [5 - (9 - 5)]$	**c** $8 + [60 - (4 + 16)]$
d $20 - \{15 + (40 \div 8)\}$	**e** $64 - \{25 - (14 \div 14)\}$	**f** $\{8 \times [5 \times 2]\} - 60$
g $[\{28 + 14\} \div 7] \times 11$	**h** $[(10 + 8) \div 9] \times 10$	**i** $8 \times (35 \div [4 + 3])$
j $[(8 + 4) \times 3] \div 9$	**k** $16 \div [54 - (28 + 18)]$	**l** $(200 \div [31 + 9]) \div 5$
m $[(60 - 14) - (53 - 19)] \times 10$	**n** $[(96 - 10) - (54 - 27)] \times 6$	
o $83 - [(8 \times 6) + (4 \times 6)]$	**p** $5015 \div [(60 + 15) \div (3 \times 5)]$	

4 Find the basic numeral in each case.

a $3 + 7^2$	**b** $14 - 3^2$
c 5×4^2	**d** $\dfrac{14+6}{2}$
e $\dfrac{31-10}{10-3}$	**f** $\dfrac{100-40}{5\times 2}$
g $\dfrac{8+20}{14\div 2}$	**h** $\dfrac{8\times 4}{2+2}$
i $\dfrac{45+939}{8}$	



5 Find the basic numeral for each expression.

a $0{\cdot}5 + (3 \times 0{\cdot}1)$	**b** $0{\cdot}44 - (0{\cdot}63 - 0{\cdot}22)$	**c** $3{\cdot}9 - (0{\cdot}2 + 0{\cdot}7)$
d $(0{\cdot}2 + 0{\cdot}3) \times 9$	**e** $(0{\cdot}7 - 0{\cdot}3) \times 4$	**f** $(0{\cdot}5 + 0{\cdot}5) \times 12$
g $9 \div (0{\cdot}5 + 0{\cdot}5)$	**h** $0{\cdot}8 \div (16 \div 8)$	**i** $4{\cdot}8 \div (16 - 12)$
j $(6 \times 0{\cdot}1) + (4 \times 0{\cdot}1)$	**k** $(2{\cdot}6 - 0{\cdot}4) + (1{\cdot}3 - 0{\cdot}8)$	**l** $(1{\cdot}4 + 1{\cdot}6) \times (0{\cdot}8 + 1{\cdot}2)$
m $\$6.50 - (\$10 - \$7)$	**n** $\$10 - (3 \times \$2.50)$	**o** $\$20 \times (153 - 144)$
p $(6 \times \$1.20) + (8 \times \$1.20)$	**q** $4 \times (\$8.32 + \$9.63)$	**r** $7 \times (\$100 - \$9.50)$

6 Insert grouping symbols to make each of the following number sentences true.

a $5 \times 3 + 8 = 23$	**b** $5 \times 3 + 8 = 55$	**c** $10 \times 7 + 5 = 120$
d $6 - 3 \times 2 = 0$	**e** $6 - 3 \times 2 = 6$	**f** $10 \times 7 + 5 = 75$
g $40 \div 4 + 1 = 8$	**h** $40 \div 4 + 1 = 11$	**i** $80 \div 8 + 2 = 8$
j $29 - 15 - 6 = 8$	**k** $29 - 15 - 6 = 20$	**l** $18 - 8 + 2 = 8$
m $40 \div 2 \div 2 = 10$	**n** $40 \div 2 \div 2 = 40$	**o** $144 \div 6 \div 3 = 72$
p $6 + 2 \times 8 + 2 = 80$	**q** $6 + 2 \times 8 + 2 = 24$	**r** $10 \times 3 - 16 - 2 = 16$

1:06B Rules for the order of operations

Mathematicians have agreed on an order of operations to avoid confusion.

Order of operations

Step 1 Do operations within grouping symbols.

Step 2 Do multiplications and divisions as they appear (from left to right).

Step 3 Do additions and subtractions as they appear (from left to right).

Order of operations:
1 [ ]
2 × and ÷
3 + and –



First, we do any operations that are inside grouping symbols.
Second, we go from left to right doing any multiplications and divisions.
Finally, we go from left to right doing any additions and subtractions.

WORKED EXAMPLES

1 $20 - 4 + 8 - 2$
(only + and –, so left to right)
$= 16 + 8 - 2$
$= 24 - 2$
$= 22$

2 $40 \div 5 \times 3 \div 2$
(only × and ÷, so left to right)
$= 8 \times 3 \div 2$
$= 24 \div 2$
$= 12$

3 $20 - 6 \times 3 + 15$
(× before + or –)
$= 20 - 18 + 15$
(only + and –, so left to right)
$= 2 + 15$
$= 17$

4 $9 + 15 \div (3 + 2)$
(grouping symbols first)
$= 9 + 15 \div 5$
(÷ before +)
$= 9 + 3$
$= 12$

Exercise 1:06B

P Challenge worksheet 1:06B
The four fours problem

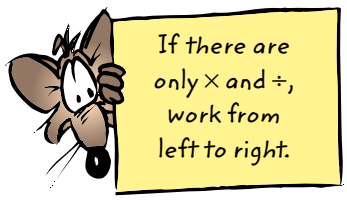
1 Simplify:

a $16 + 4 + 7 + 3$
b $8 + 9 + 3 + 4$
c $20 - 3 - 2 - 1$
d $15 - 4 - 5 - 3$
e $8 + 6 - 3 + 4$
f $9 + 4 - 6 - 3$
g $9 - 8 + 4 - 3$
h $15 - 10 + 6 - 7$
i $16 - 4 - 3 - 5 - 1$
j $18 - 8 - 3 - 5 - 2$
k $14 - 3 - 8 + 5 - 1$
l $12 - 2 - 5 + 4 - 6$



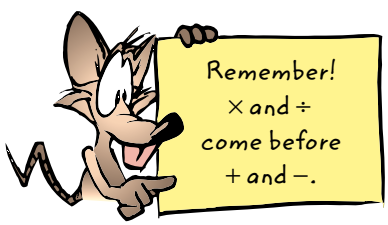
2 Write the basic numeral for each.

a $10 \times 2 \times 2 \times 2$
b $2 \times 3 \times 3 \times 3$
c $120 \div 6 \div 2 \div 2$
d $40 \div 2 \div 2 \div 2$
e $10 \times 5 \div 2 \times 4$
f $6 \times 4 \div 8 \times 11$
g $20 \div 2 \div 5 \times 8$
h $16 \div 8 \div 2 \times 7$
i $8 \times 7 \div 2 \times 4$
j $10 \div 2 \times 8 \times 2$
k $2 \times 12 \div 3 \times 8$
l $42 \div 7 \times 10 \div 5$



3 Simplify the following.

a $46 - 3 \times 8$
b $40 - 10 \times 3$
c $20 \div 5 - 4$
d $9 - 35 \div 5$
e $6 \times 2 + 6 \times 3$
f $10 \times 4 + 10 \times 5$
g $10 \times 8 - 15 \div 3$
h $6 + 5 \times 6 + 3$
i $20 \div 5 - 4 \div 4$
j $10 - 6 \div 3 + 3$
k $81 - 19 \times 2$
l $15 + 30 \div 5 + 4$



4 Use the rules for the order of operations to find the answers to these.

a $3 \times (4 + 6)$
b $8 + 16 \div 4$
c $(25 - 15) \times 9$
d $(8 - 3) \div 5 + 5$
e $(8 - 3) + 5 \div 5$
f $5 \div 5 + (8 - 3)$
g $8 + (3 \times 4) \div 4$
h $(8 + 3) \times 4 \div 4$
i $(8 + 3 \times 4) \div 4$
j $(10 - 4 \times 2) \div 2$
k $10 - (4 \times 2) \div 2$
l $10 - 4 \times 2 \div 2$
m $40 \div 5 \div (7 - 5)$
n $6 \times (2 + 1) \times 4$
o $(6 + 3) \times 4 + 11$
p $6 \times 5 + 8 \times 3$
q $5 \times 8 + 7 \times 3$
r $6 \times 9 + 4 \times 5$
s $(8 + 8) - 6 \times 2$
t $100 - 20 \times (3 + 1)$
u $(15 - 6) \times 4 \times 25$
v $(6 + 4 \times 5) - 3 \times 2$
w $5 \times 8 - (16 - 4 \div 2)$
x $8 - (13 - 8 - 2) \times 2$

5 Find the basic numeral for each.

a $6 + 8 \times 3 - 25 \div 5 \times 2 + 4$
b $100 - 10 \times 4 - 20 \div 2 \div 5 + 20$
c $11 - (8 - 3) \times 2 + 25 \div 5$
d $44 - 11 \times (9 - 7) + 100 \div 5$
e $8 \times \$5.10 + 10 \times \$5.10 + 2 \times \$5.10$
f $\$3.30 \times 3 + \$5.50 \div 5 + \$8$
g $0{\cdot}8 + 5 \times 0{\cdot}1 - 6 \times 0{\cdot}2$
h $8 + 9 \times (0{\cdot}3 + 0{\cdot}7) - 9$
i $80 - [(3 + 2) \div (10 - 5)] \times 2$
j $96 + [(15 \div 3) + (35 \div 7)] \times 10$
k $16 + 4 \times 2 - (7 \times 3 - 10) \div 11 + 6 - (10 \times 5 - 10 \times 4) - [6 \times (18 - 15)]$

1:06C Order of operations on a calculator

Calculators usually have the order of operations built in. Check the operation of your calculator by entering the following and comparing your answer with the one given.

Question	Calculator sentence	Answer
1 162 ÷ (16 + 2)	162 ÷ (16 + 2) =	9
2 6·8 + 14 ÷ 0·5 − 4	If the calculator uses the order of operations: 6.8 + 14 ÷ 0.5 − 4 =	30·8

Note: Different types of calculator may use different keys. Check the instruction booklet for your calculator.

Exercise 1:06C

1 Use a calculator to find the answers.

a 600 ÷ (3 + 3)
b 35 − (10 + 5)
c 10 − (20 − 10)
d 10 + 3 × 10
e 20 − 4 × 5
f 9 − 14 ÷ 2
g 9 + 12 ÷ 3 − 10
h 6 − 10 ÷ 2 + 4
i 10 + 4 × 5 − 20
j 3007 + 51·8 × 95
k 475 − 14 050 ÷ 562
l 1 247 000 − 946 × 384
m 89·14 × 3 − 205·65
n 7044 × 81 − 70 562
o 223 584 ÷ 32 + 3013
p 8·9 × 3·8 + 1·1 × 3·8
q 104 × 7·3 − 4 × 7·3
r 9·8 × 7 − 9·8 × 6

FUN SPOT 1:06 — WHAT'S THE DIFFERENCE BETWEEN A BOXER AND A TELEPHONE?

Find the answer for each letter. Match each letter with its answer below to solve the riddle.

N 6 × 5	**O** 0 × 8	**S** 17 × 1	**N** 4 × 7	**R** 8 × 3
N 2 × 13	**X** 8 × 8	**O** 9 × 9	**B** 9 + 4	**I** 7 + 8
I 6 + 5	**E** 8 + 6	**E** 17 − 8	**E** 13 − 6	**I** 20 − 17
I 100 − 53	**R** 125 ÷ 5	**N** 24 ÷ 3	**X** 42 ÷ 7	**O** 100 ÷ 20
T $\frac{1}{5}$ of 20	**T** $\frac{1}{10}$ of 100	**H** $\frac{1}{2}$ of 58	**H** $\frac{1}{3}$ of 66	**R** 3 × 3 × 3
G 5×10^2	**S** 2 × 2 × 2 × 2	**N** 5 × 5 × 5	**A** \$1.20 × 5	**O** \$2.00 ÷ 4
B \$12 ÷ 3	**E** \$8 × 7			

Change these Roman numerals into our modern numerals.

A CCCLXIV **G** MDXCI

5	30	\$56	13	0	6	14	17	11	125	\$6	27	15	28	500				
4	22	9	81	10	29	7	24	25	3	26	1591	16	47	8	364	\$4	\$0.50	64

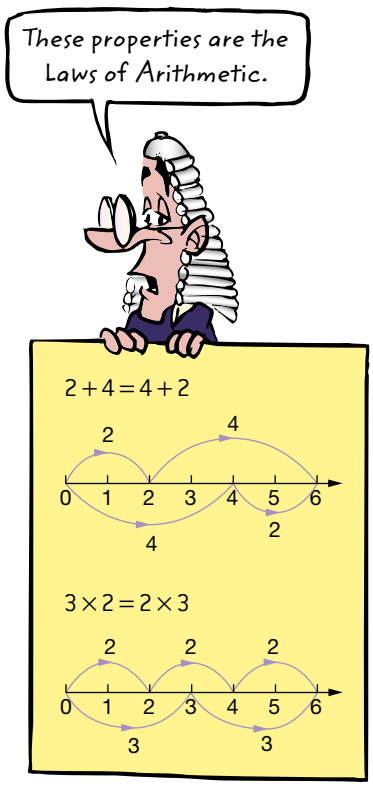


1:07 Using number properties

1:07A Properties involving addition or multiplication

Knowing these properties will give you a greater understanding of our number system.

Property	Example
1 Multiplying any number by one leaves it unchanged.	$6583 \times 1 = 6583$
2 Multiplying any number by zero gives the answer zero.	$749 \times 0 = 0$
3 Adding zero to any number leaves it unchanged.	$96 + 0 = 96$
4 When adding two numbers, the order does not change the answer.	$14 + 9 = 9 + 14$ (commutative law)
5 When multiplying two numbers, the order does not change the answer.	$9 \times 7 = 7 \times 9$ (commutative law)
6 When adding more than two numbers, we can add them in any order.	$41 + 154 + 59 + 6$ $= (41 + 59) + (154 + 6)$ $= 260$ (associative law)
7 When multiplying more than two numbers, we can multiply them in any order.	$4 \times 836 \times 25$ $= (4 \times 25) \times 836$ $= 83\,600$ (associative law)

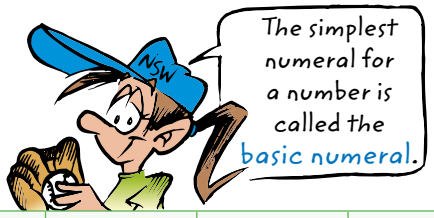


Exercise 1:07A

P Challenge worksheet 1:07A
Using the number properties

1 Draw a table like the one shown.
Write each expression listed below under its basic numeral in the table.

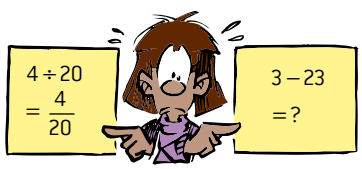
$50 + 6$	$(6 + 5) + 9$	13×0
8×0	$6 + (5 + 9)$	4×9
$(2 \times 6) \times 3$	$6 + 50$	9×4
$2 \times (6 \times 3)$	20×1	$0 \times (8 \times 11)$
$0 + 36$	1×56	$(0 \times 8) \times 11$
56×1	$39 + 16 + 1$	1×20
$20 + 0$	$39 + 1 + 16$	$3 + 17$
$2 \times 9 \times 2$	98×0	$17 + 3$
$2 \times 2 \times 9$		



0	20	36	56
	$(6 + 5) + 9$		$50 + 6$

2 Work out the value of each side to write whether the number sentence is true or false.

a $84 + 46 = 46 + 84$
b $125 \times 8 = 8 \times 125$
c $683 \times 1 = 1 \times 683$
d $836 + 0 = 0 + 836$
e $989 \times 0 = 0 \times 989$
f $384 + 0 = 384 \times 1$
g $4\frac{1}{2} + 2\frac{1}{2} = 2\frac{1}{2} + 4\frac{1}{2}$
h $0{\cdot}9 + 0{\cdot}7 = 0{\cdot}7 + 0{\cdot}9$
i $6\% + 8\% = 8\% + 6\%$
j $8 \times \frac{1}{4} = \frac{1}{4} \times 8$
k $20 \times 0{\cdot}1 = 0{\cdot}1 \times 20$
l $5\% \times 4 = 4 \times 5\%$
m $20 \div 4 = 4 \div 20$
n $26 \div 13 = 13 \div 26$
o $100 \div 5 = 5 \div 100$
p $16 \div 4 = 4 \div 16$
q $23 - 3 = 3 - 23$
r $34 - 9 = 9 - 34$
s $18 - 0 = 0 - 18$
t $15 - 8 = 8 - 15$

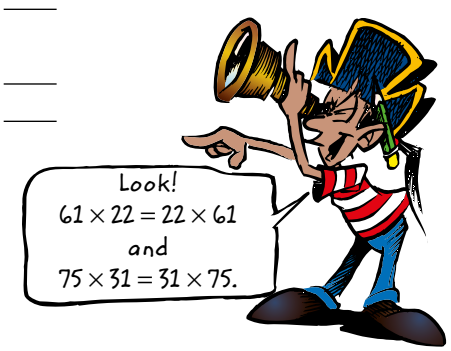


3 Write a numeral that will replace the ☐ to make each number sentence true.

a $6 \times \square = 0$
b $\square \times 1 = 416$
c $5800 + \square = 5800$
d $\square + 0 = 365$
e $67 \times \square = 67$
f $\square \times 0 = 0$
g $9 + 20 = \square + 9$
h $18 + 10 = 10 + \square$
i $6 + \square = 15 + 6$
j $9 \times 8 = \square \times 9$
k $7 \times 4 = 4 \times \square$
l $\square \times 11 = 11 \times 9$
m $0{\cdot}4 \times \square = 0$
n $1{\cdot}6 + \square = 1{\cdot}6$
o $\square \times 1 = 0{\cdot}9$
p $\frac{3}{10} + \frac{4}{10} = \square + \frac{3}{10}$
q $6 \times \frac{3}{100} = \frac{3}{100} \times \square$
r $\frac{3}{4} \times 0 = \square$

4 Find the answer to each of the following.

a 61×22
b 75×31
c 22×61
d 31×75



5 Answer after swapping the top and bottom.

a 31×658
b 21×1040
c 13×6839

6 In each of the following, does one person contribute more than the other? If so, which person?

a Jan poured fifteen 4-litre containers of water onto the garden.
Zac poured four 15-litre containers of water onto the garden.

b Bai scored 12 points in her first basketball game and 16 in her second.
Heather scored 16 points in her first basketball game and 16 in her second.

c André walked for 6 hours at a speed of 3 kilometres per hour.
Rex walked for 3 hours at a speed of 6 kilometres per hour.

d In 5 days Rahul gave his father a total of $10.
In 10 days Maria gave her father a total of $5.

e Phuong paid for the petrol for our car to travel 50 km at 50 km/h.
Alan paid for the petrol for our car to travel 30 km at 30 km/h.

f Jacqui goes fishing once and gives her catch of 15 fish to the nursing home.
Jeff gives everything he catches to the nursing home. He went fishing fifteen times and caught no fish each time.

7 True or false?

a $(18 + 6) + 3 = 18 + (6 + 3)$
b $(100 + 10) + 5 = 100 + (10 + 5)$
c $(18 \times 6) \times 3 = 18 \times (6 \times 3)$
d $(100 \times 10) \times 5 = 100 \times (10 \times 5)$
e $(18 - 6) - 3 = 18 - (6 - 3)$
f $(100 - 10) - 5 = 100 - (10 - 5)$
g $(18 \div 6) \div 3 = 18 \div (6 \div 3)$
h $(100 \div 10) \div 5 = 100 \div (10 \div 5)$
i $25 \times 37 \times 4 = 100 \times 37$
j $99 + 87 + 1 = 100 + 87$

Do what is in grouping symbols first.

8 Write a numeral that will replace the ☐ to make each number sentence true.

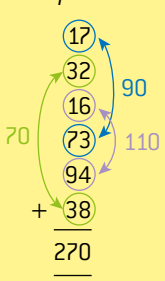
a $5 \times 83 \times 20 = \square \times 83$
b $198 + 37 + 63 = 198 + \square$
c $90 + 73 + 10 + 7 = 100 + \square$
d $16 \times 45 \times 5 = \square \times 45$
e $960 + 127 + 40 = \square + 127$
f $2 \times 368 \times 5 = \square \times 368$
g $14 \times \frac{13}{100} \times 5 = 70 \times \square$
h $1\frac{1}{2} + \frac{7}{10} + \frac{3}{10} = 1\frac{1}{2} + \square$
i $99{\cdot}9 + 13{\cdot}8 + 0{\cdot}1 = \square + 13{\cdot}8$
j $\$3.80 \times 5 \times 5 \times 4 = \$3.80 \times \square$

9 Use the number properties to find the answers quickly.

a $72 \times 3 \times 8 \times 0$
b $\$0.25 \times 17 \times 4$
c $\$4.50 \times 14 \times 2$
d $1453 + 700 + 300$
e $5 + 476 + 795$
f $466 + 58 + 42$
g $5 \times 18 \times 10 \times 2$
h $4 \times 356 \times 5 \times 5$
i $25 \times 9 \times 2 \times 7 \times 2$
j $52 + 96 + 18 + 4$
k $800 + 496 + 4$
l $25 \times 8 \times 4 \times 6$

10 Use the example as a guide to perform these additions by grouping in tens.

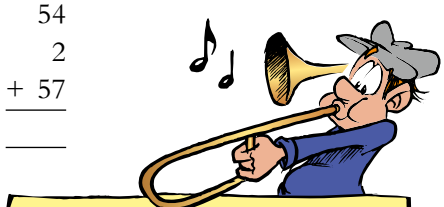
Example:



a
44
27
36
83
25
+ 61

b
29
35
43
85
21
+ 67

c
90
18
63
54
2
+ 57



You can check your answers by adding up or adding down.

11 **a** A cyclist travels for 10 hours at an average speed of 8 kilometres per hour. Would she travel the same distance in 8 hours at a speed of 10 kilometres per hour?

b Each time Alana went to the shop she bought 17 mini packets of chips, which cost 50 cents per packet. If she went to the shop 4 times, how much did she spend?

c Who would work the greater number of hours: 30 people working for 20 hours or 20 people working for 30 hours?

d In each carton of candles there are 85 boxes. In each box there are a dozen (12) candles. How many dozen candles would be in 4 cartons?



12 True or false?

a To multiply a number by 12, you could first multiply by 6 and then double the result.

b To multiply a number by 40, you could first multiply by 4 and then multiply the result by 10.

c To divide by 30, you could first divide by 3 and then divide the result by 10.

d To divide by 27 you could first divide by 3 and then divide the result by 9.

1:07B The distributive property

The **distributive property** involves multiplication as well as addition or subtraction.

PREP QUIZ 1:07B

What numeral will replace the ☐ to make each sentence true?

1 $9 \times 4 = 4 \times \square$ **2** $123 + 7 = \square + 123$ **3** $147 \times \square = 147$

4 $147 \times \square = 0$ **5** $147 + \square = 147$ **6** $25 \times 97 \times 4 = \square$

In each basket of fruit sold we placed 15 oranges and 5 apples. We sold 8 baskets of fruit.

7 How many oranges did we sell?

8 How many apples did we sell?

9 How many pieces of fruit did we sell?

10 Question **9** might have been worked out in two ways:

$8 \times (15 + 5)$ or $(8 \times 15) + (8 \times 5)$.

Is it true that $8 \times (15 + 5) = 8 \times 15 + 8 \times 5$?

In Question **10** above we saw that:

8 lots of (15 + 5) = 8 lots of 15 + 8 lots of 5

i.e. $8 \times (15 + 5) = 8 \times 15 + 8 \times 5$

There are eight 15s and eight 5s.



WORKED EXAMPLES

1 Nine full buses carried spectators to see the NSW netball finals. Each bus held 100 people including the driver. How many passengers were there?

Number of passengers
= 9 lots of (100 − 1) i.e. 9 × (99)
= 9 lots of 100 − 9 lots of 1
= 900 − 9
= 891
Note: 9 × (100 − 1) = 9 × 100 − 9 × 1

2 In last year's ballroom dancing finals, 16 couples competed in each of the twelve events. How many competitors were there altogether?

Number of competitors
= 12 lots of (16 + 16) i.e. 12 × (32)
= 12 lots of 16 + 12 lots of 16
= 192 + 192
= 384
Note: 12 × 16 + 12 × 16 = 12 × (16 + 16)

In these types of examples, multiplication can be *distributed* over addition or subtraction.

3 6 × 108
= 6 × (100 + 8)
= 6 lots of 100 + 6 lots of 8
= 600 + 48
= 648
[6 × 108 = 6 × 100 + 6 × 8]

4 4 × 98
= 4 × (100 − 2)
= 4 lots of 100 − 4 lots of 2
= 400 − 8
= 392
[4 × 98 = 4 × 100 − 4 × 2]



5 5 × 432 = 5 × (400 + 30 + 2)
= 5 × 400 + 5 × 30 + 5 × 2
= 2160

Exercise 1:07B

1 **a** Five shepherds are each caring for 100 sheep. If each shepherd loses one sheep, how many sheep are left altogether?

b To each of five classes of 30 students, two extra students are added. How many students are there now altogether?



2 Which is equal to $7 \times (11 + 4)$ [i.e. 7×15]?

A $7 \times 11 + 4$ **B** $7 \times 11 + 7 \times 4$ **C** $11 + 7 \times 4$

3 True or false? (Try working each side out separately.)

a $6 \times (1 + 7) = 6 \times 1 + 6 \times 7$
b $9 \times (8 + 3) = 9 \times 8 + 9 \times 3$
c $4 \times (5 + 2) = 4 \times 5 + 4 \times 2$
d $6 \times (7 - 1) = 6 \times 7 - 6 \times 1$
e $9 \times (8 - 3) = 9 \times 8 - 9 \times 3$
f $4 \times (5 - 2) = 4 \times 5 - 4 \times 2$

4 What numeral should replace the □?

a $8 \times 6 + 8 \times 3 = 8 \times (6 + \square)$
b $7 \times 3 + 7 \times 5 = 7 \times (\square + 5)$
c $11 \times 9 + 11 \times 6 = \square \times (9 + 6)$
d $5 \times 9 - 5 \times 1 = 5 \times (\square - 1)$
e $4 \times 6 - 4 \times 2 = 4 \times (6 - \square)$
f $6 \times 7 - 6 \times 2 = \square \times (7 - 2)$
g $8 \times 5 + 8 \times 10 = 8 \times \square$
h $5 \times 3 + 10 \times 3 = \square \times 3$
i $17 \times 8 + 3 \times 8 = \square \times 8$
j $7 \times 92 = \square \times 90 + \square \times 2$

5 Use the method given in examples **3** and **4** (page 24) to simplify:

a 5×99
b 7×99
c 3×98
d 9×98
e 6×95
f 8×95
g 12×95
h 18×95
i 71×101
j 24×102
k 8×107
l 5×108
m 35×102
n 42×101
o 15×104
p 8×201
q 195×4
r 98×5
s 703×6
t 980×45



6 We can sometimes use the distributive property to shorten working,

e.g. $97 \times 13 + 3 \times 13 = (97 + 3) \times 13$
$= 1300$

Use the distributive property to answer the following.

a $40 \times 9 + 60 \times 9$
b $7 \times 8 + 3 \times 8$
c $4 \times 9 + 7 \times 9$
d $5 \times 6 + 6 \times 6$
e $9 \times 14 + 1 \times 14$
f $91 \times 8 + 9 \times 8$
g $63 \times 8 - 3 \times 8$
h $104 \times 7 - 4 \times 7$
i $15 \times 12 - 5 \times 12$
j $23 \times 6 - 19 \times 6$
k $67 \times 14 - 65 \times 14$
l $96 \times 73 - 94 \times 73$

7 True or false?

a To multiply a number by 13 you can first multiply by 10, then add 3 times the number.
b To multiply a number by 36 you can first multiply by 30, then add 6 times the number.
c To multiply 72 by 9 you can first multiply by 10 and then subtract 72.
d To multiply 13 by 99 you can first multiply by 100 and then subtract 13.

Note:

$(116) \div 4 = (100 \div 4) + (16 \div 4)$

This is like the distributive law for multiplication.

1:08 Language and symbols used in mathematics

- Use operation symbols to write and answer **1** to **24** of **ID Card 3** on page xvi.

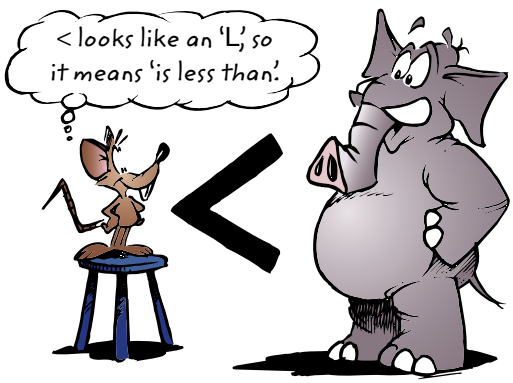
Abbreviations used in mathematics

%	means *per cent*	am	means *before midday* (from *ante meridiem*)
∴	means *therefore*	pm	means *after midday* (from *post meridiem*)
e.g.	means *for example*	CE	means *Common Era*
i.e.	means *that is*	BCE	means *Before Common Era*
K	means *thousand*		

$70K = $70 000
K stands for kilo, which comes from the Greek word *khilioi* meaning 'thousand'.

Other names for CE and BCE are AD (for *Anno Domini*, 'the Year of our Lord') and BC (for *Before Christ*).

Symbols used in mathematics



=	means *is equal to*
≠	means *is not equal to*
≒ or ≈	means *is approximately equal to*
<	means *is less than*
≤	means *is less than or equal to*
≮	means *is not less than*
>	means *is greater than*
≥	means *is greater than or equal to*
≯	means *is not greater than or equal to*
$\sqrt{9}$	means *the square root of 9* or the number that is multiplied by itself to give 9, e.g. $3 \times 3 = 9 \therefore \sqrt{9} = 3$
$\sqrt[3]{8}$	means *the cube root of 8* or the number used in a product three times to give 8, e.g. $2 \times 2 \times 2 = 8 \therefore \sqrt[3]{8} = 2$

WORKED EXAMPLES

1 1·987 ≒ 2 means 1·987 is approximately equal to 2.

2 35 000 ≮ 33 999 means 35 000 is not less than 33 999.

3 $99 > $90.50 means $99 is greater than $90.50.

Exercise 1:08

P Foundation worksheet 1:08 Language and symbols

1 **a** Find the sum of 15 and 27.
b Find the difference between 91 and 23.
c To the quotient of 18 and 3, add 45.
d From the product of 6 and 8, take away 9.
e Decrease 80 by 6 lots of 9.
f Increase 476 by 12.
g Find the average of 13 and 3.
h Find 6 less than the total of 12, 8 and 3.
i What is 5 less than the square of 4?
j By how much does 362 exceed 285?

2 Rewrite each of these using the symbols introduced earlier.
a 15 is less than 105.
b 8·8 is not equal to 8·08.
c 0·7 is equal to 70 per cent.
d 5 squared multiplied by 7 cubed.
e 6 times 9, minus 3.
f 12 divided by 4, plus 6.
g Therefore, the square root of 9 is 3.
h That is, the cube root of 8 is 2.
i π is approximately equal to 3·142.
j $\square$ is less than or equal to 10.
k 0·3 is greater than 0·03.
l For example, 5·1 is not less than 5·09.
m The product of 12 and 4 is greater than or equal to the quotient of 12 and 4.

3 Rewrite each as a sentence by replacing the symbols with words.
a $0{\cdot}499 < \frac{1}{2}$
b $3999 \div 4000$
c $61\,000 \neq 61 \times 100$
d $6 + \square \geq 9$
e $0{\cdot}1 > 0{\cdot}099$
f $6 \times 199 = 6 \times 200 - 6 \times 1$
g $9{\cdot}1 \not< 9\frac{1}{10}$
h $5{\cdot}1 + 4 = 4 + 5{\cdot}1$
i $\square - 3 \leq 6$

4 True or false?
a $16 > 4$
b $399 \div 400$
c $14 \leq 13$
d $3{\cdot}0 = 3$
e $97 \times 5 \times 20 \neq 97 \times 100$
f $800 < 199 \times 4$
g $\frac{12}{2} = 12 \div 2$
h $\frac{1}{2} \not< \frac{2}{4}$
i $\frac{1}{10} < \frac{1}{2}$
j $18 \div 3 \neq 3 \div 18$
k $9 \times 7 \geq 7 \times 9$
l $3821 \div 1283$
m $8000 \not< 8 \times 1000$
n $0{\cdot}3 > 0{\cdot}29$
o $1 + 2 + 3 + 4 < 5$

INVESTIGATION 1:08 ODDS AND EVENS

Investigate what happens when you start with a number (for example, 15) and follow the rules below, continuing to apply the rules over and over again.

If your last answer is …
- odd: multiply by 3 and add 1
- even: halve the number.

Example
15 ⇒ 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1, …

Do all numbers below 20 eventually lead to 1?
In each case, find the number of steps needed.



1:09 Special sets of whole numbers

PREP QUIZ 1:09

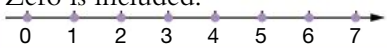
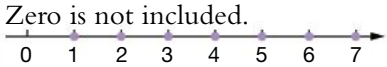
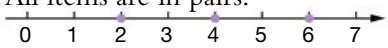
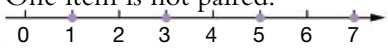
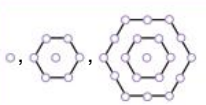
Write the next three terms of each of the number patterns below.

1 2, 4, 6, 8, … , … , …
2 1, 3, 5, 7, … , … , …
3 1, 2, 3, 4, … , … , …
4 1, 4, 9, 16, … , … , …

True or false?

5 The last digit of an even number is always either 0, 2, 4, 6, or 8.
6 The last digit of an odd number is always either 1, 3, 5, 7, or 9.
7 The sum of two even numbers is odd.
8 The sum of two odd numbers is even.
9 The product of an odd and an even number is even.
10 5 squared is equal to 25.

Note:
Zero is a special number and is sometimes included as an even number and as a multiple, but in this chapter we will not include it.

Name of set	Pattern	Diagram or explanation	Comment or rule
Cardinal numbers	0, 1, 2, 3, ...	Zero + counting numbers	Zero is included. 
Counting numbers	1, 2, 3, 4, ...		Zero is not included. 
Even numbers	2, 4, 6, 8, ...		All items are in pairs. 
Odd numbers	1, 3, 5, 7, ...		One item is not paired. 
Square numbers	1, 4, 9, 16, ...		A counting number times itself, e.g. $1 \times 1, 2 \times 2, 3 \times 3$
Triangular numbers	1, 3, 6, 10, 15, ...		1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, ... The sum of counting numbers.
Hexagonal numbers	1, 7, 19, 37, ...		1, 1 + (6 × 1), 1 + (6 × 1) + (6 × 2), 1 + (6 × 1) + (6 × 2) + (6 × 3), ... i.e. 1, $^{+6}$ 7, $^{+12}$ 19, $^{+18}$ 37, $^{+24}$ 61, $^{+30}$ 91, …
Fibonacci numbers	1, 1, 2, 3, 5, 8, ...	Except for the first two numbers in the pattern, each term is the sum of the two terms before it.	
Palindromic numbers	e.g. 929 7337	Forwards or backwards it is the same number.	'Able was I ere I saw Elba' is a *palindromic sentence*.

Exercise 1:09

P Challenge worksheet 1:09
Special sets of whole numbers

1 What is the next odd number:
a after 7006 **b** after 9999 **c** before 975 **d** before 10 340?

2 Write the odd numbers:
a between 85 and 98 **b** between 992 and 1003.

3 What is the next even number:
a after 8146 **b** after 111 **c** before 3195 **d** before 10 000?

4 Write the even numbers:
a between 73 and 83 **b** between 1996 and 2001.

5 **a** All odd numbers must end in one of five digits. What are the digits?
b All even numbers must end in one of five digits. What are the digits?

6 Continue the following patterns.
a 78, 80, 82, … , … , … , … , …
b 193, 195, 197, … , … , … , … , …
c 989, 991, 993, … , … , … , … , …
d 1012, 1014, 1016, … , … , … , … , …

7 Say whether the following are odd or even:
a the sum of two odd numbers
b the product of two odd numbers
c the sum of two even numbers
d the product of two even numbers
e the sum of an odd and an even number
f the product of an odd and an even number
g the square of an odd number
h the square of an even number
i the difference between two odd numbers
j the difference between two even numbers.

8 Copy and complete the following table.

Number group	Term of pattern (e.g. term 3 = T_3)														
	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}
Counting numbers	1	2	3	4	5	6	7	8							
Even numbers	2	4	6	8											
Odd numbers	1	3	5	7											
Square numbers	1	4	9	16											
Triangular numbers	1	3	6	10											
Fibonacci numbers	1	1	2	3											



9 Complete the following:

a $1 + 3 =$
b $1 + 3 + 5 =$
c $1 + 3 + 5 + 7 =$
d $1 + 3 + 5 + 7 + 9 =$
e $1 + 3 + 5 + 7 + 9 + 11 =$
f $1 + 3 + 5 + 7 + 9 + 11 + 13 =$
g $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 =$
h $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =$

10 Use the answers to Question **9** to answer true or false for the following statements.

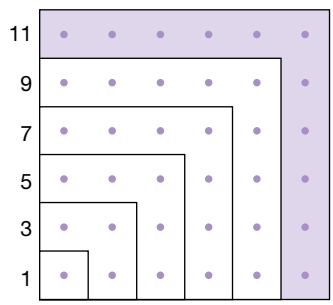
a The sum of the first four odd numbers is equal to 4×4.
b The sum of the first five odd numbers is equal to 5 squared.
c The sum of the first six odd numbers is 6^2.
d The sum of the first eight odd numbers is 8^2.

11 Complete the following table.

Question	Answer as a square
$0^2 + 1$	$0 + 1 = 1 = 1^2$
$1^2 + 3$	$1 + 3 = 4 = 2^2$
$2^2 + 5$	$4 + 5 = 9 = 3^2$
$3^2 + 7$	$9 + 7 = \ldots = \ldots^2$
$4^2 + 9$	
$5^2 + 11$	
$6^2 + 13$	
$7^2 + 15$	
$8^2 + 17$	
$9^2 + 19$	
$10^2 + 21$	
$11^2 + 23$	

The diagram below shows that:

- the sum of the first five odd numbers is 5^2
- the sum of the first six odd numbers is 6^2.



We can also see that this pattern will continue.

The sum of the first 50 odd numbers is 50^2.

$50^2 = 50 \times 50 = 2500$

12 Which of the numbers in the box are:

a even
b odd
c square
d both square and even?

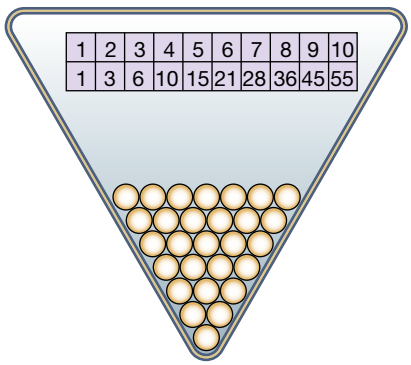
37	15	25	46	100	81
16	33	111	2000	64	144
1	2	49	4	77	84
9	71	66	36	73	122

13 a What triangular number is shown here?
b If the top 5 balls were removed, would the picture still show a triangular number?
c Write the triangular numbers that are less than 10.

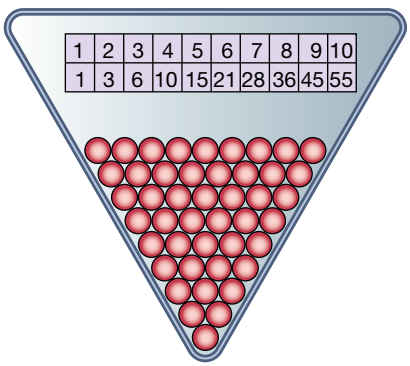


14 Find the number of pills contained in each of the pharmacist's pill trays below.

a



b





15 a How many pins are used in tenpin bowling?
b Is this a triangular number?

16 2002 is a palindromic number. List all the palindromic numbers between 1000 and 3000.

17 Which of the numbers in this box are:
a triangular numbers
b Fibonacci numbers
c palindromic numbers?

55	14	78	66	17
37	21	16	89	91
707	10	610	15	100

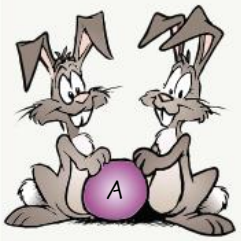
18 Write as much as you can about the different numbers that might be involved in this tower of cards.



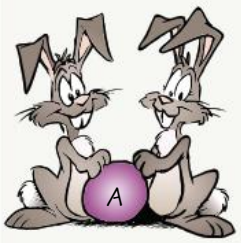
FUN SPOT 1:09 FIBONACCI NUMBERS

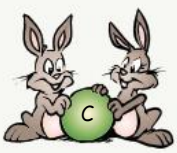


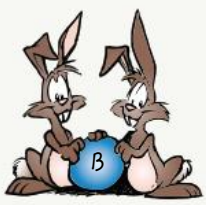


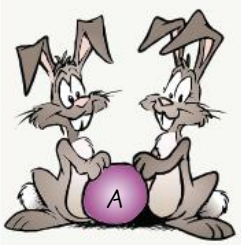






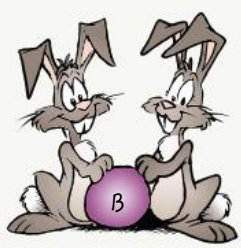














These pictures show rabbits at three ages: the smallest rabbits are babies, the middle-sized rabbits are 1 month old, and the biggest rabbits are 2 months old, fully grown. Every month, each pair of fully grown rabbits gives birth to a new pair of baby rabbits (one male and one female). Assuming no rabbits die, how many pairs in total are there each month?

Solution:

Month 1	1 pair
Month 2	1 pair
Month 3	2 pairs
Month 4	3 pairs
Month 5	5 pairs

1 **a** Draw the rabbits for months 6 and 7, showing their sizes.

b Are the numbers of pairs Fibonacci numbers? (See page 28.)

Fibonacci magic: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

2 Write the first 15 Fibonacci numbers.

3 Add the first 5 Fibonacci numbers. Is the total one less than the 7th Fibonacci number?

4 Add the first 10 Fibonacci numbers. Is this total one less than the 12th Fibonacci number?

5 Check to see if the pattern in **3** and **4** is true for the rest of the numbers you have written.

A number trick

Challenge someone to choose any Fibonacci number and add all the Fibonacci numbers up to and including the one chosen. You can give this sum much more quickly than adding, simply by subtracting 1 from the Fibonacci number that is two places further on.

1:10 Estimating answers

PREP QUIZ 1:10

Round each number to the nearest ten.

1 84 **2** 47 **3** 16 **4** 38·15

Round each to the nearest hundred.

5 435 **6** 851 **7** 118 **8** 274·85

Round each to the nearest thousand.

9 6469 **10** 9710

Leading figure estimation

4947 + 3214 is hard to work out quickly in your head. But by rounding each number to the closest thousand, you can quickly estimate the result.

$$4947 + 3214 \fallingdotseq 5000 + 3000$$
$$\fallingdotseq 8000$$

$\fallingdotseq$ means 'is approximately equal to'.

To use leading figure estimation, we round each number involved at its leading figure and then perform the operation.

WORKED EXAMPLES

Use leading figure estimation to estimate an answer for each of these.

1 618 + 337 + 159 + 409 **2** 38 346 − 16 097

3 3520 × 11·4 **4** 1987 ÷ 4



Note:
It is a convention that no space is left for 'thousands' in a four-digit number.

Remember:
If the digit that follows the leading figure is 5 or more, **round up**.

Solutions

1
$$\begin{array}{r} 600 \\ 300 \\ 200 \\ +\ 400 \\ \hline 1500 \end{array}$$

2
$$\begin{array}{r} 40\,000 \\ -\ 20\,000 \\ \hline 20\,000 \end{array}$$

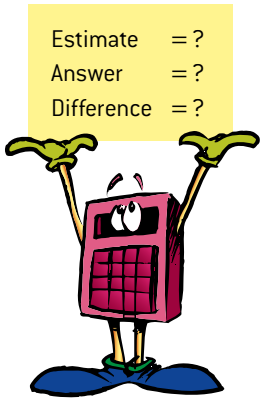
3
$$\begin{array}{r} 4000 \\ \times\ 10 \\ \hline 40\,000 \end{array}$$

4
$$4\overline{)2000} \quad 500$$

When making an estimate, round each number to the place holding the first non-zero digit, then perform the operation, e.g. $8174 \times 3{\cdot}8147 \fallingdotseq 8000 \times 4$ or $32\,000$

Exercise 1:10

Use leading figure estimation to estimate an answer. Then use a calculator to find the answer and the difference between the estimate and the answer.



1
- **a** $659 + 527 + 305$
- **b** $1963 + 3211 + 2416$
- **c** $65\,342 + 31\,450$
- **d** $3\,140\,942 + 6\,095\,811$
- **e** $947 - 382$
- **f** $8342 - 2625$
- **g** $8611 - 8093$
- **h** $340\,321 - 127\,999$
- **i** 3813×3
- **j** 1716×6
- **k** $84\,694 \times 1{\cdot}85$
- **l** $736\,308 \times 2{\cdot}65$
- **m** $8192 \div 4$
- **n** $64\,035 \div 3$
- **o** $3870 \div 1{\cdot}25$
- **p** $479\,352 \div 4{\cdot}8$

2 For each part, estimate the answer before you calculate, then work out the difference between your estimate and answer.



- **a** Crowd attendances at the National Tennis Centre over three days were 8146, 7964 and 9193. What was the total attendance?
- **b** Mindy took 88 410 balloons to Dubbo but sold only 2365. How many balloons were not sold?
- **c** There are 86 400 seconds in one day. How many seconds are there in 7 days?
- **d** Three prospectors found a gold nugget, which they sold for $27 360. They shared the money equally. How much did each receive?
- **e** How many groups of 175 soldiers can be formed from 24 290 soldiers?
- **f** Alan received two cheques in April, one for $13 714.93 and the other for $9854.19. He received two more in October, for $19 142.55 and $12 740.02. How much did he receive altogether?
- **g** Of the money Alan received in part **f**, he paid the government $23 657.40 in taxes. How much did Alan get to keep?
- **h** Kayla sold seven cars to Firstcars.com for $56 315 each. What was the total amount she received for the sale?

3 **Estimation speed test**

- Have a friend measure the time (in seconds) you take to complete the ten estimates below.
- Try to finish all ten estimates in four minutes.
- When you have finished:
  - i work out the answers with a pencil and paper or with a calculator.
  - ii for each one, work out the difference between your estimate and the correct answer.

	Question	Working	Estimate	Answer	Difference
a	11 615 + 58 107				
b	92 094 ÷ 6				
c	48 314 − 27 199				
d	37 118 × 7				
e	2 416 809 + 4 369 170				
f	567 180 ÷ 5				
g	69 847 × 9				
h	486·76 ÷ 1·72				
i	735·15 − 275·35				
j	2315 × 7·8				

4 Where two amounts have different numbers of digits, we usually estimate to a similar place for both numbers when doing addition or subtraction,

e.g. 14 384 + 4256 ≒ 14 000 + 4000
≒ 18 000

≒ means 'is approximately equal to'.

If we had used leading digit estimation, it would have been:

14 384 + 4256 ≒ 10 000 + 4000
≒ 14 000

This last estimate is even smaller than the first of the original numbers!

Which part of Question 2 would be better suited to this method?



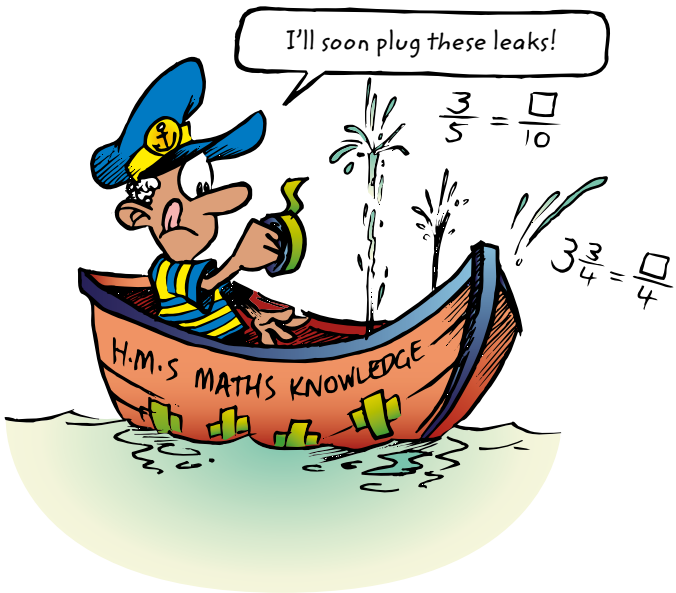
Estimate the number of terracotta figures in the photo. Describe the method of estimation you used.

1:11 Diagnostic check-ups

Check-up and treatment

- Complete each diagnostic check-up test, one row at a time.
- Correct your work using the answers at the back of the book.
- If you make mistakes in a row of questions, go to the eBook and click on the icon next to that row. This will link you to an Appendix page with an explanation and practice exercise.

Note:
You may not have covered all the work included in these check-ups. They are designed to make you and your teachers aware of what is known and what still needs attention.



CHECK-UP 1:11A — OPERATIONS

P Appendix B B:01–B:04

1 **a** 64 + 26 + 37 + 9 **b** 515 + 307 + 96 + 983 **c** 19 208 + 7 537 + 38 690 **d** 76 215 + 4 806 + 193 521 + 60 433

2 **a** 647 − 193 **b** 3862 − 1477 **c** 56 312 − 17 297 **d** 491 625 − 38 163

3 **a** 659 × 6 **b** 2506 × 8 **c** 789 × 10 **d** 1305 × 70

4 **a** 78 × 32 **b** 1907 × 93 **c** 8653 × 45 **d** 473 × 50

5 **a** $6\overline{)660}$ **b** $5\overline{)564}$ **c** $7\overline{)2374}$ **d** $4\overline{)1624}$

6 **a** $20\overline{)8950}$ **b** $90\overline{)9630}$ **c** $400\overline{)6800}$ **d** $40\overline{)73\,400}$

CHECK-UP 1:11B FRACTIONS

P Appendix B B:05–B:10

1 Complete the following to make equivalent fractions.

a $\frac{3}{4} = \frac{\square}{8}$ b $\frac{1}{5} = \frac{\square}{10}$ c $\frac{3}{10} = \frac{\square}{100}$

2 Write each fraction in its simplest form.

a $\frac{8}{10}$ b $\frac{6}{8}$ c $\frac{20}{50}$

3 Which of the two fractions is smaller?

a $\frac{1}{5}, \frac{1}{10}$ b $\frac{3}{4}, \frac{6}{10}$ c $\frac{2}{5}, \frac{1}{2}$

4 Arrange in order from smallest to largest:

a $\{\frac{1}{5}, \frac{1}{2}, \frac{1}{10}\}$ b $\{\frac{3}{10}, \frac{7}{10}, \frac{1}{10}\}$

5 Write these mixed numerals as improper fractions.

a $2\frac{1}{2}$ b $5\frac{3}{10}$ c $1\frac{1}{4}$

6 Write these improper fractions as mixed numerals.

a $\frac{7}{4}$ b $\frac{87}{10}$ c $\frac{9}{2}$

Give the simplest answer for:

7 a $\frac{3}{10} + \frac{4}{10}$ b $\frac{1}{5} + \frac{2}{5}$ c $\frac{8}{10} + \frac{3}{10}$

8 a $\frac{9}{10} - \frac{8}{10}$ b $\frac{3}{4} - \frac{1}{4}$ c $\frac{5}{8} - \frac{1}{8}$

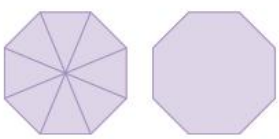
9 a $\frac{1}{2} + \frac{1}{4}$ b $\frac{2}{3} + \frac{2}{9}$ c $\frac{3}{10} + \frac{2}{5}$

10 a $\frac{9}{10} - \frac{1}{5}$ b $\frac{1}{3} - \frac{1}{6}$ c $\frac{7}{16} - \frac{1}{4}$

11 a $4 \times \frac{3}{4}$ b $5 \times \frac{3}{10}$ c $10 \times \frac{2}{5}$

12 a $\frac{1}{24}$ of 24 b $\frac{1}{10}$ of 90 c $\frac{1}{8}$ of 160





$\frac{8}{8} = 1$

8 eighths make 1 whole.

CHECK-UP 1:11C DECIMALS AND MONEY

P Appendix B B:11–B:18

1 Write these fractions as decimals.

a $\frac{7}{10}$ b $2\frac{13}{100}$ c $\frac{9}{1000}$

2 Write these decimals as fractions in simplest form.

a 0·317 b 0·59 c 0·5

3 Which of the two decimals is smaller?

a 0·3, 0·1 b 0·11, 0·2 c 0·5, 0·49

4 Write each set in order from smallest to largest.

a 1, 0·3, 0·8 b 0·11, 0·91, 0·51 c 0·12, 0·8, 0·509

5 a 1·9 + 3·6 b 2·74 + 0·4 c 5·18 + 3

6 a
$$\begin{array}{r} 0{\cdot}65 \\ 4{\cdot}2 \\ +\ 63{\cdot}4 \\ \hline \\ \hline \end{array}$$

b
$$\begin{array}{r} 5{\cdot}85 \\ 0{\cdot}6 \\ +\ 7 \\ \hline \\ \hline \end{array}$$

c $7{\cdot}2 + 16 + 4{\cdot}1$

7 a $9{\cdot}2 - 1{\cdot}25$ b $10{\cdot}63 - 6{\cdot}3$ c $18{\cdot}415 - 9{\cdot}31$

8 a $1{\cdot}6 \times 5$ b $0{\cdot}75 \times 4$ c $0{\cdot}181 \times 3$

9 a $3\overline{)48{\cdot}6}$ b $6\overline{)1{\cdot}8}$ c $3\overline{)0{\cdot}123}$

10 a $3{\cdot}1 \div 10$ b $0{\cdot}05 \times 100$ c $0{\cdot}8 \times 1000$

11 Write each amount as dollars, using a decimal.
a 5 dollars 65 cents b 8 dollars 5 cents c 7 dollars 40 cents

12 a $\$5.80 + \$9$ b $\$63.20 + \$2.50$ c $\$201 + \$1.20$

13 a $\$9.60 - \$3.40$ b $\$50 - \$3.50$ c $\$5 - \$1.85$

14 a $\$4.50 \times 4$ b $\$1.85 \times 6$ c $\$9.05 \times 5$

15 a $\$18 \div 4$ b $\$12.15 \div 3$ c $\$70 \div 8$

CHECK-UP 1:11D PERCENTAGES

P Appendix B B:19–B:23

1 Write these as hundredths.
a 7% b 33% c 15%

2 Write each as a fraction in its simplest form.
a 20% b 30% c 15%

3 Write each as a whole number.
a 100% b 300% c 200%

4 Write each as a mixed numeral in simplest form.
a 250% b 105% c 125%

Write each as a percentage.

5 a $\frac{8}{100}$ b $\frac{37}{100}$ c $\frac{50}{100}$

6 a $\frac{3}{20}$ b $\frac{7}{50}$ c $\frac{7}{10}$

7 a 4 b 1 c 3

8 a $3\frac{1}{2}$ b $1\frac{3}{4}$ c $7\frac{3}{10}$

Write these percentages as decimals.

9 a 93% b 7% c 10%

10 a 150% b 113% c 425%



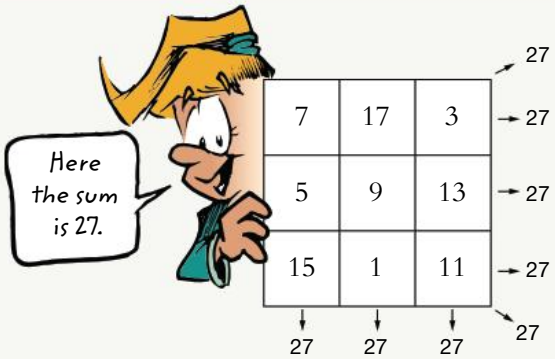
Write these decimals as percentages.

11 a 0·07 b 0·85 c 0·9

12 a 3·15 b 1·08 c 5·9

P Challenge worksheet 1:11
Solving puzzles

The numbers in every row, column and diagonal have the same total.



1 Find the missing numbers in these magic squares.

a

8		6
3	5	
4	9	

b

	18	
14	10	6
		16

c

6		
	7	9
		8

2 **a** Complete each of the magic squares below.

b Show that by putting the nine small magic squares together, you get a large magic square. Make sure that every row, column and diagonal has the same sum.

4	14	12
18		
8	6	16

9	7	17
19	11	3
	15	

3	10	5
	6	
7	2	9

6	7	
1	5	9
	3	4

15	5	7
	9	17
11	13	

10		14
17		9
12	11	16

	8	18
20	12	4
6		14

10		8
5	7	9
6	11	

5	10	
12	8	
7	6	11

MATHS TERMS 1

average (arithmetic mean)

- the result of 'evening out' a set of numbers
- to find the average, divide the sum of the terms by the number of terms,
  e.g. The average of 3, 5, 3, 7, 10 and 2
  $= (3 + 5 + 3 + 7 + 10 + 2) \div 6$
  $= 30 \div 6$
  $= 5$

difference

- the result of subtracting one number from a larger number

digit

- any one of the ten Hindu–Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

distributive property

- the way that multiplication (or division) of a number can be done by breaking the number into an addition (or subtraction) of separate parts,
  e.g. $6 \times 108 = 6 \times (100 + 8)$
  $= (6 \times 100) + (6 \times 8)$
  or $8 \times 99 = 8 \times (100 - 1)$
  $= (8 \times 100) - (8 \times 1)$

estimate

- to calculate roughly
- a good guess or the result of calculating roughly

even numbers

- whole numbers that are divisible by 2
- numbers that end in 0, 2, 4, 6 or 8

expanded notation

- a way of writing a number as the sum of its parts,
  e.g. $612 = (6 \times 100) + (1 \times 10) + (2 \times 1)$

Fibonacci numbers

- the set of numbers 1, 1, 2, 3, 5, 8, …
- the next number in the pattern is the sum of the two numbers before it

fraction

- one or more parts of a unit or whole
- equivalent fractions: are equal in size, e.g. $\frac{1}{2} = \frac{2}{4}$
- improper fraction: the numerator is bigger than the denominator, e.g. $\frac{7}{4}$
- mixed number: has a whole number part and a fraction part, e.g. $3\frac{2}{5}$

grouping symbols

- symbols that are used to tell us which operation to do first
- three main types:

parentheses	brackets	braces
( )	[ ]	{ }

- when one set is within another, do the operation in the innermost grouping symbols first

odd numbers

- whole numbers that are not divisible by 2
- numbers that end in 1, 3, 5, 7 or 9

opposite operations

- one operation undoes what the other does, e.g. $117 \times 3 \div 3 = 117$
- addition is the opposite of subtraction
  multiplication is the opposite of division

palindromic number

- a number that is the same backwards as forwards, e.g. 929, 2002, 18 700 781

power

- a number formed by repeated multiplication, e.g. $5^3 = 5 \times 5 \times 5$
  $= 125$

square number

- the result of multiplying a counting number by itself, e.g. 1, 4, 9, 16, 25

triangular numbers

- numbers that are the sum of consecutive counting numbers beginning from 1,
  e.g. 1, 3 (i.e. 1 + 2),
  6 (i.e. 1 + 2 + 3),
  10 (i.e. 1 + 2 + 3 + 4), 15, …

DIAGNOSTIC TEST 1 BEGINNINGS IN NUMBER

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Change these Egyptian numerals into our modern numerals. 1:01

a  b  c 

2 Change these to Egyptian numerals. 1:01

a 365 b 4029 c 2 320 000

3 Change these Roman numerals into our modern numerals. 1:01

a CCCLXIV b MMII c MCMXCIX

4 Change these to Roman numerals. 1:01

a 372 b 948 c 3409

5 Write the value of the 7 in each case. 1:02

a 374 240 b 6075 c 7 300 004

6 a A standard pack of playing cards contains 52 cards. Four friends share a pack of cards. How many cards does each receive? 1:03

b There are 156 students on a school camp. They are divided into 6 activity groups. How many students are in each group?



7 What is the cost of one movie ticket if the total cost for five tickets is $69.75? 1:05

8 Simplify: 1:06

a $(6 + 8) \times (11 - 9)$ b $33 - (16 - [4 + 10])$ c $\frac{10 + 40}{5}$

9 Simplify: 1:06

a $18 - 3 \times 4 + 5$ b $12 + 18 \div (10 - 4)$ c $120 - 80 \div 8 - 6 \times 10$

10 Copy and replace the $\square$ to make each number sentence true. 1:07B

a $6 \times (900 + 8) = \square \times 900 + \square \times 8$

b $7 \times (600 - 3) = \square \times 600 - \square \times 3$

c $4 \times 398 = 4 \times 400 - 4 \times \square$

11 a List the first three square numbers. 1:09

b List the first three triangular numbers.

c List the first three odd numbers.

12 Use leading figure estimation to estimate the answer to each. 1:10

a 865 109 + 186 422 b 163 241 × 7·6 c 39 487 ÷ 4·3

ASSIGNMENT 1A Chapter review

1 Refer to **ID Card 2** on page xv to identify the figures numbered:

a 6 **b** 7
c 8 **d** 11
e 19 **f** 20
g 21 **h** 22
i 23 **j** 24

2 Write the short form for each part of **ID Card 3** on page xvi.

3 Multiply Column A by Column B on the **Arithmetic card** on page xxi.

4 Write in words:

a 270 307 **b** 2 456 325
c 97 005 000 **d** 294 167 000

5 Write the value of the 7 in each.

a 214 703 **b** 871 000
c 7 415 216 **d** 13 701 233

6 Write each as a numeral in its simplest form.

a $(5 \times 10\,000) + (9 \times 1000) + (3 \times 100) + (7 \times 10) + (6 \times 1)$
b $(8 \times 100\,000) + (6 \times 1000) + (5 \times 100) + (4 \times 10) + (5 \times 1)$
c $(3 \times 1\,000\,000) + (2 \times 100\,000) + (1 \times 10\,000) + (5 \times 1000)$

7 Write in expanded notation:

a 8247 **b** 56 015
c 714 850 **d** 3 420 000

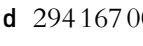
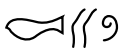
8 Use leading figure estimation to estimate the answer to:

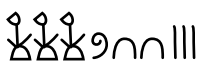
a $37\,145 + 24\,810$
b $417\,816 \div 8$
c $28\,143 \times 7$

9 Simplify:

a $15 - 6 \div 2$ **b** $16 \times 4 - 10 \times 4$
c $14 - 6 + 4 \times 2$ **d** $70 - 7 \times 9 + 7$

10 Change these Egyptian numerals into our modern numerals.

a  **b** 

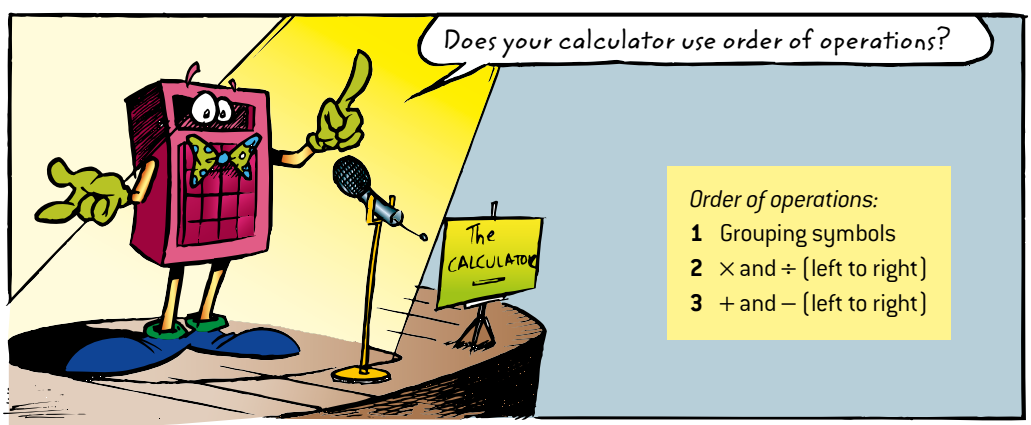
c  **d** 

11 Change these Roman numerals into our modern numerals.

a CMXXVIII **b** DCXLIX
c MMDC **d** $\overline{X}\overline{X}$DCCX

12 Write each as a Roman numeral.

a 327 **b** 419
c 2555 **d** 1991



ASSIGNMENT 1B Working mathematically

1 Read each puzzle and see if you can find the correct answer.

- **a** If it takes 4 minutes to boil an egg, how long will it take to boil 6 eggs?
- **b** If 5 cats can catch 5 mice in 5 minutes, how many cats are needed to catch 100 mice in 100 minutes?
- **c** If 4 people take 4 hours to dig a hole, how long will it take 8 people to dig a similar hole?
- **d** A boy says: 'I have the same number of brothers as I have sisters.' His sister says: 'I have twice as many brothers as sisters.' How many boys and how many girls are in the family?
- **e** A train leaves Katoomba for Sydney at a speed of 150 km/hour. Another train leaves Sydney for Katoomba an hour later, at a speed of 120 km/hour. When the two trains meet, which one is closer to Sydney?

Use a calculator to answer these questions.

2

- **a** Tyndale school bought 4350 exercise books at 89 cents per book. How much did these books cost altogether?
- **b** Five stamp albums contain Alan's stamp collection. There are 3814, 1426, 899, 1796 and 99 stamps in the albums. How many stamps are there altogether?
- **c** Mr Rich planned to start a rabbit farm in Bourke. His intention was to increase the number of rabbits on the farm from 480 000 in 2013 to 645 800 in 2014. What increase was he planning for in that year?
- **d** If $15 000 million worth of goods are to be exported from NSW over 12 months, what value of goods need to be exported in the first month to be on target?
- **e** Inside a beehive, 7560 bees are beating their wings 185 times every second to keep the hive cool. How many beats would occur altogether in one minute?
- **f** Light travels at about 17·9 million kilometres per minute. How many million kilometres would light travel in one year? (Use 1 year = 365 days.) The distance that light travels in one year is called a *light year*.
- **g** Cyclone Blowhard travelled across the countryside at 1·08 km per minute for 128 minutes, and at 0·84 km per minute for a further 117 minutes. How far did the cyclone travel altogether in that 245 minutes?

3 A special discount price is often given when a large number of items is bought. Find the cost (to the nearest cent) for one item, if:

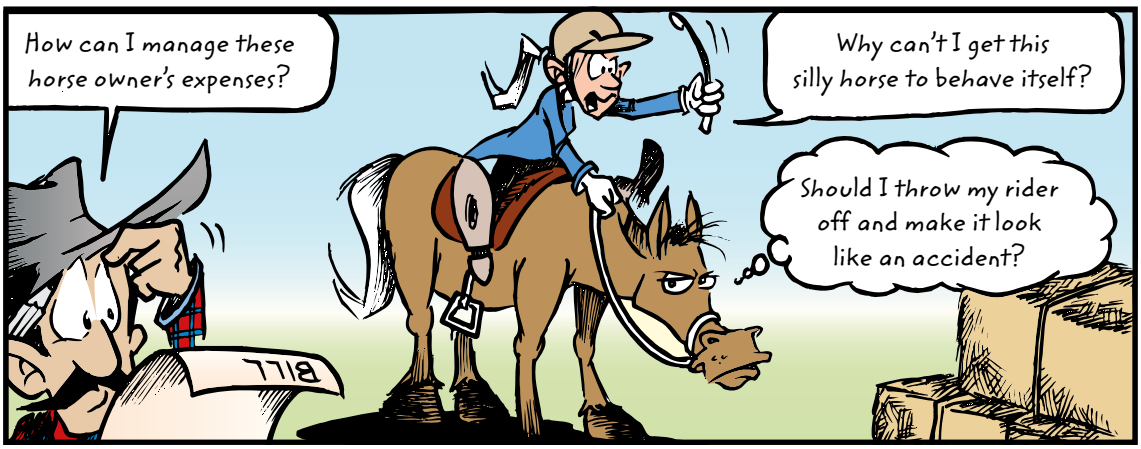
- **a** 1000 nails cost $38
- **b** $45 will buy 400 screws
- **c** $200 gives you 144 cans
- **d** 600 books cost $350
- **e** 3000 pamphlets cost $67
- **f** 40 000 sheets of paper cost $860
- **g** 600 calculators cost $16 300
- **h** 450 tennis balls cost $860
- **i** $1800 buys 700 glue-sticks
- **j** 800 rubber bands cost $3.20

4 Explain how 800 rubber bands can cost $3.20 even though one rubber band costs $0.00 to the nearest cent.



WORKING MATHEMATICALLY

2



Contents

2:01 Direct computation
Fun spot 2:01 Can you play '31'?
2:02 Strategies for problem solving
Investigation 2:02 A standard pack of cards
2:03 Trial and error
2:04 Making a drawing, diagram or model
Investigation 2:04 Catching the crooks
2:05 Making a list, chart, table or tally
2:06 Eliminating possibilities
2:07 Working backwards
Challenge 2:07 Backtracking
2:08 Acting it out
2:09 Looking for patterns
Fun spot 2:09 Where should the numbers go?
2:10 Solving a simpler problem
Investigation 2:10 Let's post a parcel
Assignments

Syllabus references (See pages x–xiii for details.)

The **Working Mathematically** strand is interwoven throughout the content strands **Number and Algebra**, **Measurement and Geometry**, **Statistics and Probability**

Outcomes from ***Working Mathematically*** [Stage 4]

- Communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.
- Applies appropriate mathematical techniques to solve problems.
- Recognises and explains mathematical relationships using reasoning.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

2:01 Direct computation

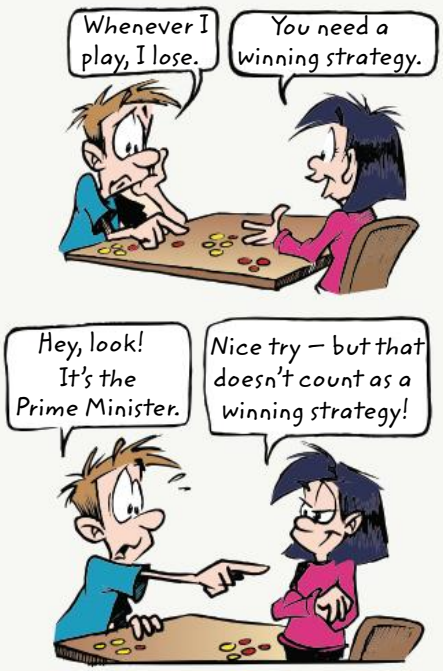
FUN SPOT 2:01 CAN YOU PLAY '31'?

This is a game for two players.

- Start with 31 counters.
- The first player chooses a number from 1 to 6 and removes that number of counters from the pile.
- The second player also chooses a number from 1 to 6 and removes that number of counters.
- The players continue taking turns, removing up to 6 counters each time until all counters are gone.
- The player who is forced to take the last counter is the loser.

A winning strategy

- Play this game with a partner.
- Can you work out a winning strategy? (The person who goes first should always be able to win. See the answers for an explanation of the winning strategy.)



In your lifetime you will meet thousands of real problems waiting to be solved: 'What is the quickest way from Blacktown to Dural?' 'How can I rearrange my working day so I can attend a meeting at Ryde?' 'How can I build a billycart?' 'How should I arrange the dinner party?'

We can see life as an exciting adventure or as a burden. We each have our own ways of solving problems. In this chapter we will examine some exciting and challenging strategies to help us solve unusual problems.

Solving problems

Step 1	Read the question carefully.
Step 2	Decide what you are asked to find.
Step 3	Look for information that might be helpful.
Step 4	Decide on the method you will use.
Step 5	Set out your solution clearly, using solution sentences.
Step 6	Make sure that your answer makes sense.

Always try to show clearly how you have solved a problem, so that others can understand and investigate what you have done.



Setting out	
Find = method = = solution Answer (in words):	Working if needed:

WORKED EXAMPLE 1

Anna was told that she would be given $15 pocket money each week if she was helpful around the home. She decided to bank $2.75 each week and do her best to help around the home. How much would she expect to bank in nine weeks?

Solution

Considering the problem:

- What are we asked to find?
  What Anna would expect to bank in 9 weeks.
- What information is given?
  What is needed?
  Given: Pocket money is $15, banks $2.75 per week.
  Needed: Banks $2.75 per week.
- What method should we use?
  Multiply the amount banked weekly by the number of weeks.
- What solution sentences should we use?
  Amount banked = 9 × $2.75

Setting out (one-step problem)		
Amount banked = 9 × $2.75 = $24.75	$2.75 × 9 $24.75	OR 
In nine weeks, Anna banks $24.75.		



WORKED EXAMPLE 2

Giulio purchased a table for $132 and a vintage radio for $86. After spending $26.50 on repairs, he sold the two items to a customer for $349.95. How much profit did he make altogether on the two items?

Solution

Considering the problem:

- What are we asked to find?
  The profit made altogether on two items.
- What information is given?
  Cost of items and repairs and selling price.
- What method should we use?
  First, find the total cost. Then subtract this from the selling price of the items.
- What solution sentences should we use?
  (1) Total cost = cost 1 + cost 2 + repairs
  (2) Profit on the items = selling price − total cost.

Setting out (two-step problem)

Total cost for the items
= $132 + $86 + $26.50
= $244.50

Profit on the items
= $349.95 − $244.50
= $105.45

Giulio made $105.45 profit on the two items.

```
   $132.00
   $ 86.00
+  $ 26.50
   $244.50
```

OR

```
   $349.95
−  $244.50
   $105.45
```



Note: This problem could have been done as a one-step problem:

Profit on the items = selling price − (total costs)
= $349.95 − ($132 + $86 + $26.50)
= $105.45

∴ Giulio made $105.45 profit on the two items.

Giulio must decide if $105.45 is enough money to earn for his work.

Exercise 2:01

P Foundation worksheet 2:01 Problem solving

1 After comparing prices at three stores, Peter found that the cheapest price for mixed canary seed was $8.15 for 5 kg. He bought 15 kg of seed. How much did he pay for each kilogram of seed and how much did he pay altogether? Solve this problem by completing parts **a**–**e** below.

a How much seed did Peter buy?
b What was the cost of 5 kg of seed?
c To compare prices, how many stores did Peter go to?
d What two things are we asked to find?
e Solve the problem, setting out your solution as shown in the examples above.

2 At a bookshop I bought 3 of the Narnia books at $15.20 each, a Bible for $47.50 and 10 bookmarks at 85 cents each. How much did I spend altogether?

- **a** What am I asked to find?
- **b** How many bookmarks were bought?
- **c** What was the total cost of the bookmarks?
- **d** How many items were bought altogether?
- **e** Solve the problem, setting out your answer as shown earlier.

Solve the following problems, setting out your answers as shown earlier (in the examples on pages 46–47).

3 How many eight-page books can be made from 618 pages?

4 Jennifer bought a television set for $1850.75 and a digital video recorder for $460.60. If she had saved $3000 to buy these two items, how much would she have left?

5 Mr and Mrs Lucre were returning home after spending a day at the beach. After travelling for three hours at 60 kilometres per hour, they were still 11 kilometres from home. How far from home were they when they began the trip home?

6 In May, Alan bought 55 different animal magazines for 25 cents each. In August he bought 47 more, all different from those purchased already. If there are 120 different magazines in the complete set, how many magazines is Alan missing?



7 Luke has 40 toy cars and 100 marbles. If he shares these with his four sisters, what would his oldest sister be given?

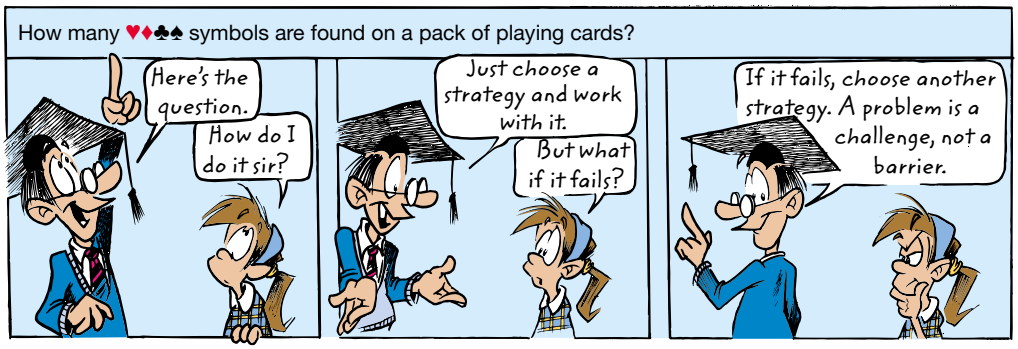
8 Kai cut an apple into 12 equal parts. If she eats $\frac{1}{4}$ of the apple, how many parts will remain?

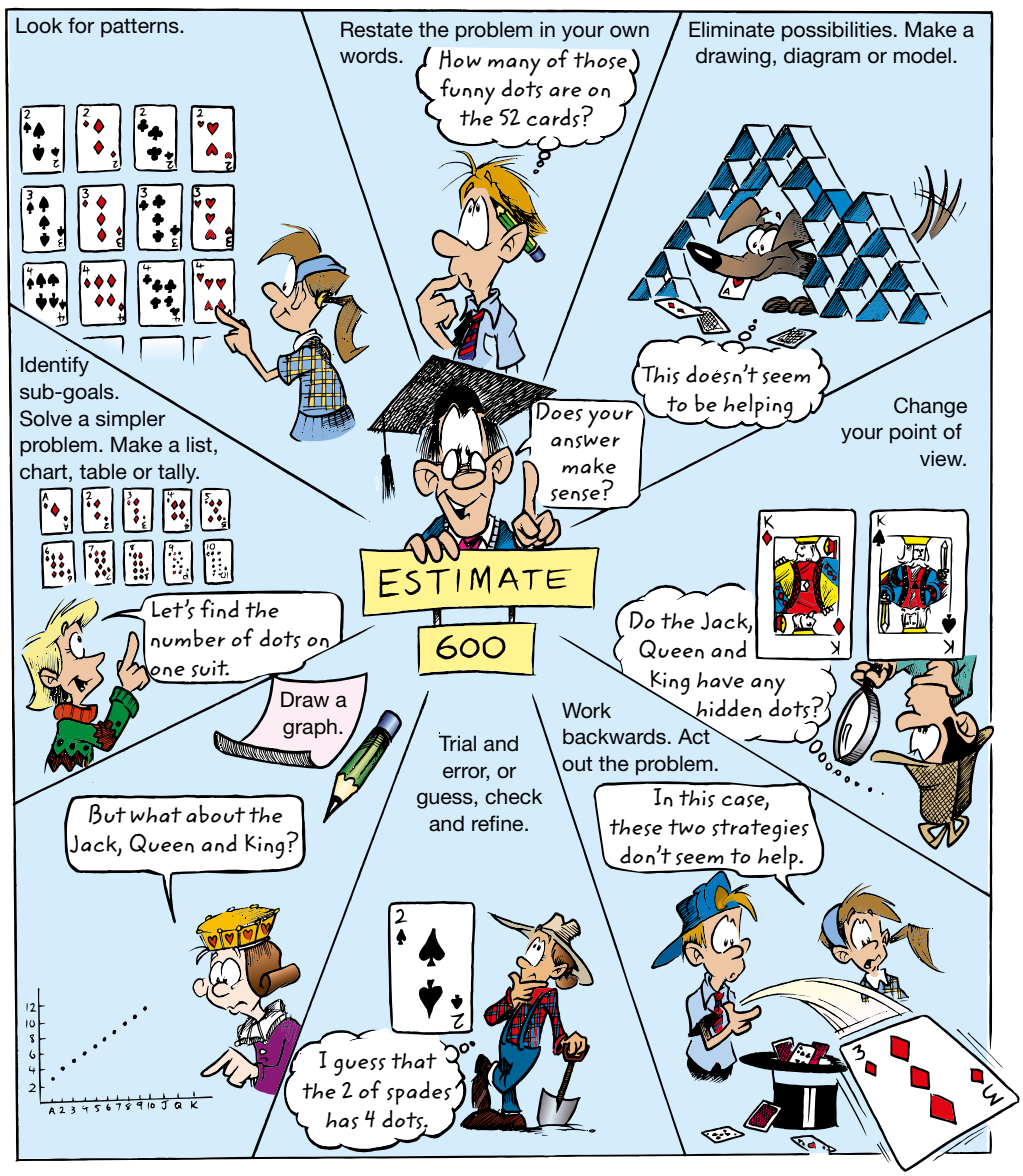
9 The total number of votes cast in three electorates was 654 816. If in one of these electorates 211 302 votes were cast and in another 284 965 votes were cast, how many votes were cast in the third electorate?

10 Marie and Asif went to a Chinese restaurant to celebrate their engagement. The dishes they liked included Sweet and Sour Chicken at $17, Steamed Duck with Sweet Corn at $21.90 and Braised Lobster with Seafood at $41.90. Dessert was $7.50 per serving and tea was $2.50 per person. What would be the total cost if Marie chose the lobster, Asif chose the duck and they each had dessert and tea?



2:02 Strategies for problem solving





Working mathematically

Throughout this course you will apply the strategies treated in this chapter.

As well as applying strategies, you will be developing your skills in questioning, communicating, reasoning and reflecting. These are all ways in which we work mathematically.

You now need to practise using these strategies, so that you can develop skill in choosing which strategies to use when you are solving a problem. Very often you can use more than one strategy to get an answer. But before you begin, you should always **estimate the answer**.

To estimate, ask:

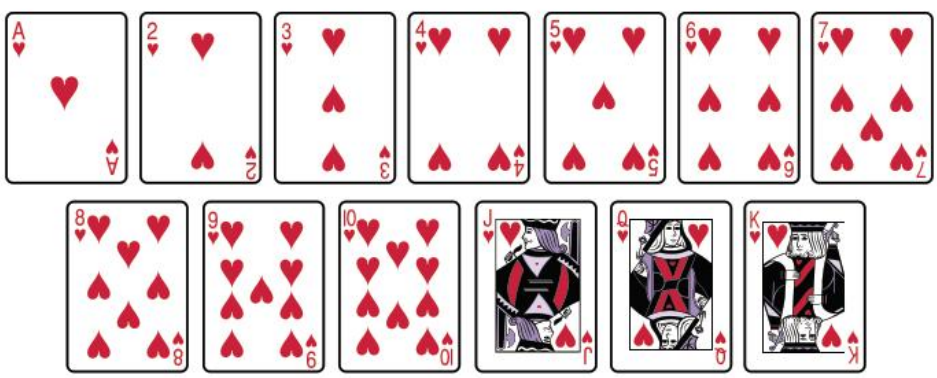
Will the answer be big or small?
How big? How small?
Will the answer be a whole number?
Does my estimate make sense? Is it reasonable?

INVESTIGATION 2:02 A STANDARD PACK OF CARDS

A standard pack of playing cards has 4 'suits': *hearts*, *diamonds*, *clubs* and *spades*. Hearts and diamonds are red. Clubs and spades are black.



In each suit there are 13 cards: Ace (A), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q) and King (K). All the hearts are shown below.



Because there are 4 suits with 13 cards in each suit, the number of cards in a standard pack is 52. (Sometimes there is also one or two Jokers, but these are not standard.)

2:03 Trial and error

Another name for *trial and error* is *guess, check and refine*. Trial and error is a useful problem-solving strategy. Its main drawback is that for some problems it is too slow and time-consuming.

WORKED EXAMPLE 1

Osu is five years older than Erna. Twice Osu's age plus three times Erna's age is equal to 25. What are the ages of the two girls?

Solution

Osu is 5 years older than Erna.

Guess 1: Osu is 10, Erna is 5
Check 1: $(2 \times 10) + (3 \times 5) = 20 + 15$
$= 35$

This is too large.

Guess 2: Osu is 8, Erna is 3
Check 2: $(2 \times 8) + (3 \times 3) = 16 + 9$
$= 25$

This is correct.

$\therefore$ Osu is 8 years old and Erna is 3 years old.

WORKED EXAMPLE 2

Sandra's cycle repair shop fixes normal bicycles and tricycles. Sandra has to assemble a number of bicycles and tricycles. She counts 15 handlebars and 36 wheels among the parts she has to put together. How many of each type of cycle are to be assembled?

Solution



Because there are 15 handlebars, there must be 15 cycles.

Guess 1: 7 bicycles, 8 tricycles
Check 1: Number of wheels $= (7 \times 2) + (8 \times 3)$
$= 14 + 24$
$= 38$

This is too big, so try fewer tricycles.

Guess 2: 9 bicycles, 6 tricycles
Check 2: Number of wheels $= (9 \times 2) + (6 \times 3)$
$= 18 + 18$
$= 36$

This is the correct number of wheels.

$\therefore$ There are 9 bicycles and 6 tricycles to be assembled.

Exercise 2:03

1 Mary is six years older than Jean. The sum of their ages is 20. How old is Mary?

2 Su-Lin is four years older than Mark. Twice Su-Lin's age plus five times Mark's age is equal to 64. What are the ages of Su-Lin and Mark?

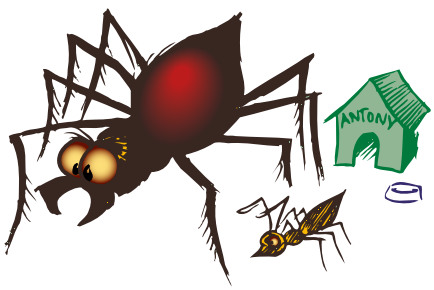
3 Derek was 12 years old when his sister was born. Now the sum of their ages is 58. How old is Derek now?

4 The sum of two numbers is 140 and their difference is 6. Find the two numbers.

5 The product of two numbers is 72 and their sum is 27. What are the numbers?

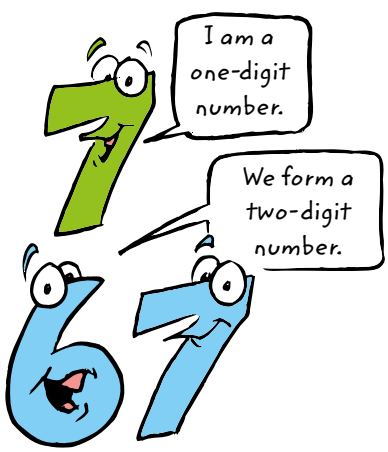
6 The product of two numbers is 741 and their sum is 70. What are the numbers?

Operation	Answer is called …
+	sum
−	difference
×	product
÷	quotient



7 A farmer has chickens and cows. If there are 18 heads and 52 feet on these animals altogether, how many cows are there?

8 Diane keeps ants and spiders as pets. She counted the legs and heads of these pets and found that there were 136 legs and 21 heads. How many spiders were there?



9 From the digits 3, 4 and 5, two numbers are formed: a two-digit number and a one-digit number. What are these two numbers if their product is:

- **a** as big as possible
- **b** as small as possible
- **c** 170?

10 Two 2-digit numbers are made from four of the digits 1, 2, 3, 4 and 5, with no digit being used more than once. Which numbers would produce:

- **a** the smallest product
- **b** the largest product
- **c** a product closest to 700?

11 Oshi sold bananas for 15 cents and apples for 20 cents. He received $3.50 for a mixture of bananas and apples. If twice as many bananas as apples were sold, how many apples were sold?

12 Kai-Lin answered all 25 questions on a test in which she received 4 marks for each correct answer but lost 1 mark for each incorrect answer. If her mark was 65, how many questions did she get right?

13 Find the counting number that would replace the □ to make the sentence true:

$16 \times \square - 72 = 56$

14 Louie, Verity and their three children went to the movies. The children were charged half price. If the total cost of the tickets was $52.50, how much did each adult ticket cost?

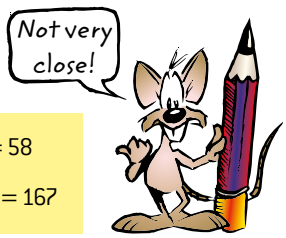
15 Four non-zero digits are placed in these four squares to make two 2-digit numbers across and two 2-digit numbers down. If the sum of these four 2-digit numbers is 77, what are they?

16 Write the numbers below in two groups of four, so that the sum of one group is as close as possible to the sum of the other group:

{9, 10, 18, 21, 25, 42, 49, 51}

$9 + 10 + 18 + 21 = 58$

$25 + 42 + 49 + 51 = 167$



17 Leonie is 35 years older than her daughter Louise but 3 years younger than her husband Max. If the sum of the three ages is 94, how old is Leonie?

18 Kalmna, April and Lindsay scored 196 marks altogether on a test. If April scored 15 less than Kalmna and 7 less than Lindsay, how many marks did April score?

19 I think of a counting number, add one, multiply the answer by itself, then take away twice the number I first thought of. If my answer is 26, what was the number I first thought of?

20 Twice Alana's age plus three times Luke's age is equal to 25. What are the possible totals of their ages?

2:04 Making a drawing, diagram or model

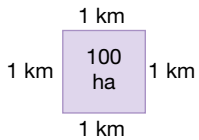
When you draw a picture to represent a problem, you can often see from the picture what must be done. As people say, 'a picture is worth a thousand words'!

WORKED EXAMPLE 1

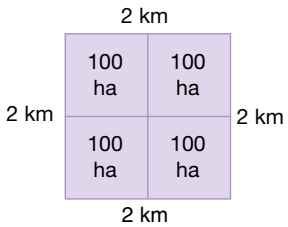
4 km of fencing encloses a square paddock of area 100 hectares. If 8 km of fencing is used to enclose a square paddock, how many hectares are enclosed?

Solution

A diagram or drawing allows us to see the problem more clearly.



Even though the length of fencing has only been doubled, the area has become 4 times as great.

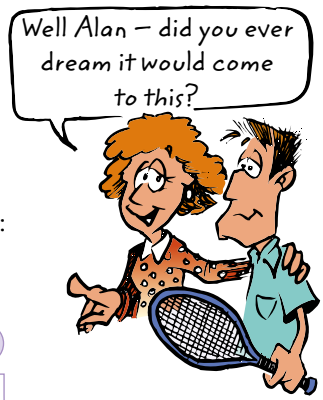


∴ 400 hectares are enclosed by the 8 km of fencing.

WORKED EXAMPLE 2

Rhonda and Alan married and had five children. Each child married and had three children. Assuming no-one has died, how many people are in this extended family altogether?

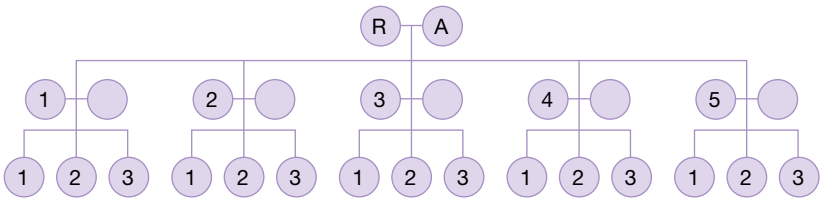
Solution



Unless you draw a diagram, you may miss some of the people.

For example, you might forget to include the people who married the five children.

From the diagram you can see that there are 27 people in the family:

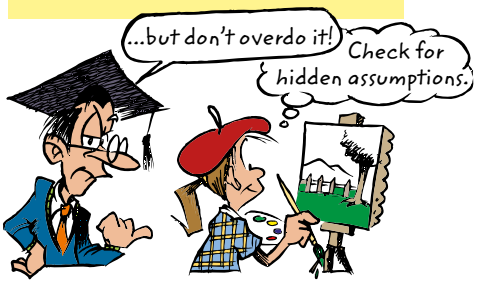


Exercise 2:04

Draw a diagram or picture to help you solve the questions below.

1 4 km of fencing encloses a square paddock of area 100 hectares. If 16 km of fencing is used to enclose a square paddock, how many hectares are enclosed?

You must draw a diagram or picture as part of your answer.



2 Alana and Dave married and had four children. Three of the children married and had four children each. Assuming that no-one died, how many people are in this extended family altogether?

3 Each child in the Paresh family has at least two brothers and one sister. What is the least possible number of children in the family?

4 **a** I want to make a line of posts 8 m long. If I place the posts 1 m apart, how many posts will I need?

b If I use 24 posts and place them 2 m apart, how far does the line go?

c A line of posts 112 m long is made with posts equally spaced. If 8 posts are used, what is the distance between posts?

5 How many counting numbers are less than 3 units from the number 4?

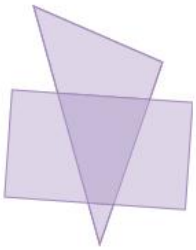
6 **a** Eight equally spaced posts are used as uprights to fence a square yard. How many posts are placed on each side of the yard?

b A square yard is fenced using four posts on each side. How many posts are used?

7 If it takes Dari 6 minutes to cut a pipe into two pieces, how long would it take her to cut it into five pieces?

8 It takes Luke $1\frac{1}{2}$ seconds to join two connecting blocks. How long would it take him to join seven connecting blocks into one row?

9 A triangle and a rectangle are drawn on a page so that they overlap, as shown. Inside these shapes are placed the ten digits 0 to 9. If there are six digits in the triangle and eight digits in the rectangle, how many digits are in the part that belongs to both shapes?



10 **a** Nineteen students were waiting in line to be chosen to play in a game. Mr Archbold chose the first student in line and every third one after that. How many students were chosen?

b 99 Roman soldiers who fled from battle were to be punished. The group was lined up and decimated (meaning that every 10th one was killed). How many were killed?

c How many of the 99 soldiers would be killed if the second soldier was killed, then every tenth soldier after that?

11 A doctor prescribes tablets for June to relieve her pain. They must be taken at least an hour apart and no more than four tablets may be taken in any six-hour period. What is the largest number of tablets June may take in $14\frac{1}{2}$ hours?

12 Two proofreaders are reading the same work. One finds 50 mistakes and the other finds 63 mistakes. If 40 of the mistakes were found by both readers, how many mistakes were found?

13 At the start of each week, May puts $400 into her bank account. During each week she withdraws $300. Assuming there are no bank or government charges, in which week does she have $1000 in the bank for the first time?

14 The 30 students in 7E study both History and Geography. All students like at least one of these two subjects. If 18 students like History and 7 students like both subjects, how many students like Geography?

15 Jock decided to swim 10 km up Bega River. He swam 2 km every hour, then floated on his back for 10 minutes. Each time he floated, the current dragged him back 1 km. How long did it take Jock to complete the swim?



INVESTIGATION 2:04 CATCHING THE CROOKS

Prince Charming was suspicious of his crown-makers. He suspected that they were not using all the gold that he provided to make his crowns. He decided to test the crown-makers, so he ordered three different crown-makers to make identical crowns for him. He made each crown-maker think that only one crown was being made.



How could he discover whether the crown-makers were cheating him?

2:05 Making a list, chart, table or tally

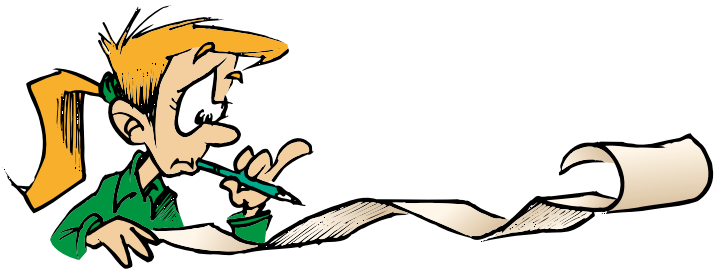
You will often need to make a complete list to solve a problem. It is important that you select the best way to make your list.

WORKED EXAMPLE 1

Alana has to list three universities in the order she prefers. The universities she likes are New South Wales University (NSW), Macquarie University (M) and Sydney University (S). How many different ways are there to list these universities in order?

Solution

The way you make your list is very important. Your method should help you to not leave anything out. We will use the university abbreviations NSW, M and S.



Note: (1) After the first choice is made, there are two left to choose from for choice 2.
(2) After the first two choices are made, only one choice is left for choice 3.

Choice 1	Choice 2	Choice 3		Order		
NSW	M	S	=	NSW	M	S
	S	M	=	NSW	S	M
M	NSW	S	=	M	NSW	S
	S	NSW	=	M	S	NSW
S	NSW	M	=	S	NSW	M
	M	NSW	=	S	M	NSW

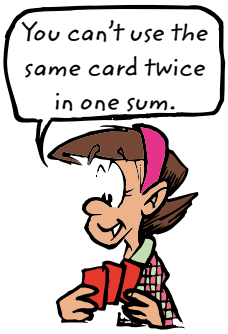
From the list, we can see that there are six different ways to list the universities.

WORKED EXAMPLE 2

The numerals 1 to 10 are written on ten separate cards, one on each card.

a How many pairs of cards have a sum of 8?

b How many groups of three cards have a sum of 20?



Solution

a The numbers must add to give 8.

1 7, 2 6, 3 5

We can see that three pairs of cards have a sum of 8.

b As 20 is fairly big, we will begin with the 10 card and see which pairs will go with it to give a sum of 20:

10 1 9, 10 2 8, 10 3 7, 10 4 6

Now consider the 9 with other pairs:

9 3 8, 9 4 7, 9 5 6

Note: (1) $9 + 1 + 10$ has already been listed.

(2) $9 + 2 + 9$ is impossible because it uses 9 twice.

Now consider the 8 with other pairs: 8 5 7

Note: $8 + 2 + 10$ and $8 + 3 + 9$ have already been listed.

Now consider the 7 with other pairs. All that add to give 20 have already been used, so no other groups of three cards will add to give 20.

$\therefore$ There are eight groups of three cards that have a sum of 20.

Exercise 2:05

1 **a** A 'Carols by Candlelight' program order must be decided. The items to be put in order are Carols (C), Entertainers (E) and Sermon (S). If the sermon must be either the second or the third of the items, in how many ways can the program be arranged?

b It is decided that the two entertainers Julie Anthony and Kamahl should be considered as two separate items, while the sermon must be placed third or fourth on the program. How many ways are there of arranging the items Carols (C), Anthony (A), Kamahl (K) and Sermon (S)?

2 From five girls, two must be chosen to represent our club at tennis. How many different pairs could be chosen? Call the girls A, B, C, D and E.
(*Note:* AB is the same pair as BA.)



3 The numerals 1 to 7 are written on seven separate cards, one on each card.

a How many pairs of cards add up to 8?

b How many groups of three cards have a sum of 10?

4 How many counting numbers less than 100 contain the digit 5?

5 **a** If each digit is used only once, how many two-digit numbers can be made using the digits 1, 2, 3, 4?
(*Note:* 21 is different from 12.)
b If each digit can be used more than once, how many two-digit numbers can be made using the digits 1, 2, 3, 4?
c If each digit can be used only once, how many two-digit numbers can be made using the digits 0, 1, 2, 3?

6 Five people were waiting for a lift. When the lift arrived there was room for only three people. How many different groups of three people could get in the lift? (Call the people A, B, C, D, E.)



7 David and Paul own a sailing boat that carries only two people. Four friends, Luke, Naomi, Rachel and Alana, would like to ride in the boat too. Because only David and Paul know how to sail the boat, at least one of these two must go out in the boat whenever it sails.
How many different pairs could be chosen to go out in the sailing boat?

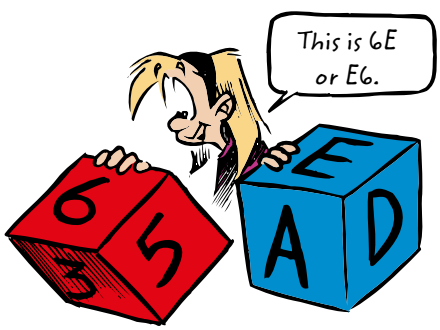
8 **a** How many different ways can the letters A, E, B and T be arranged so that T is the third or fourth letter? (AETB and AEBT are two ways.)
b How many of the letter arrangements in part **a** are words in English?

9 Maurice Mixup saw the number plate of a car involved in a bank robbery. He told the police that the first letter was E and the other two letters were Z and A, but he was unsure of the order of the Z and A. The three digits following were 2, 3 and 8, but he did not know their order. How many number plates would fit this description?



10 A school captain and vice-captain are to be elected from seven candidates. How many different results can occur?

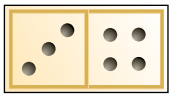
11 Two dice are rolled. One dice has the digits 1 to 6 written on its faces while the other has the letters A, B, C, D, E and F on its faces (one letter on each face).
a How many results could contain an A or an E (a vowel)?
b How many different results are possible?



12 If every letter of the alphabet is followed by a number with a single digit, how many of these pairs can be formed?

13 I own 3 ties, 4 shirts and 2 belts. How many different ways can I wear a tie, a shirt and a belt?
(Hint: Call the ties T1, T2, T3, the shirts S1, S2, S3, S4, and the belts B1, B2.)

14 A set of dominoes is a collection of rectangular pieces that each have two parts. On each part, one of the numbers 0 to 6 is represented by dots. All possible combinations of numbers are available, but no two dominoes are the same.



a How many dominoes are in a set?

b What is the sum of all the numbers shown on a set of dominoes?

15 A number of cards can be shared between 4 people exactly, but when shared between 5 people there are two cards left over. When shared between 6 people there are four cards left over. If there are fewer than 80 cards, how many cards are there?

16 How many different amounts can be made using three of the six Australian coins, 5c, 10c, 20c, 50c, $1, $2?



2:06 Eliminating possibilities

Sometimes you can find the right answer by eliminating answers that are wrong.

WORKED EXAMPLE

Three students have the last names Sze, Kalra and Poon. Their first names are Sundeep, Christopher and Andy, but not necessarily in that order. Sze was born in Australia, Poon has never been to the Philippines, Andy was born in Malaysia and Sundeep was born in the Philippines. If the students were all born in different countries, what is the full name of each student?

Solution

We can use tables to eliminate as many possibilities as we can. We hope that what is left will be the answer.

- Sze was born in Australia, so we can write ✗ ✗ ✓ for Sze.
- Poon has never been to the Philippines, so for Poon we can cross the Philippines box: ✗.
- Andy was born in Malaysia, so we write ✓ ✗ ✗ for Andy.
- Sundeep was born in the Philippines, so we write ✗ ✓ ✗ for Sundeep.

Because each person was born in a different country, the fact that neither Sze nor Poon was born in the Philippines means that Kalra must have been. We can tick the box for Philippines/Kalra: ✓.

In this way the other boxes can be filled in too.

		Malaysia	Philippines	Australia
Last name	Sze	✗	✗	✓
	Kalra	✗	✓	✗
	Poon	✓	✗	✗
First name	Sundeep	✗	✓	✗
	Chris	✗	✗	✓
	Andy	✓	✗	✗

From the table, we can see: Sze and Chris were born in Australia.
∴ Chris Sze was born in Australia.
Kalra and Sundeep were born in the Philippines.
∴ Sundeep Kalra was born in the Philippines.
∴ Andy Poon was born in Malaysia.

Exercise 2:06

1 Find the number between 1 and 10 that is not even and that when divided by 3 gives a remainder of 1.

2	3	4	5
6	7	8	9

2 Find the number less than 50 that has the sum of its digits equal to 6 and the difference of its digits equal to 4.

A **multiple** of 5 is a number that can be exactly divided by 5, e.g. 5, 10, 15, 20, …

3 Find the number between 0 and 120 that is even, is a multiple of 5, and is the result of multiplying a counting number by itself.

4 Find the number between 1 and 50 that is not a multiple of 2 or 3 and whose two digits differ by 4.

5 If ties sell for $8.60, $9.20, $10.80 and $12.40, which of these could be the total cost of 6 ties?

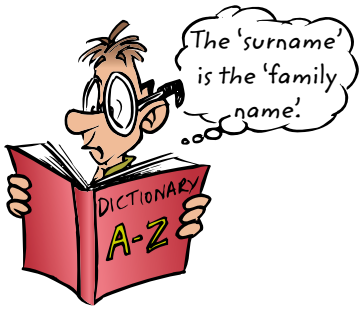
Work out the least and greatest possible cost of six ties.

A $51.40 **B** $74.80 **C** $63.30 **D** $62.60

6 Cecil and his wife Lillian owned a grocery store at Sans Souci. They received ten sample tins, each of which had a net mass of 225 g, 400 g, 425 g or 450 g. Which of the following could be the total net mass of the ten sample tins?

A 4600 g **B** 3830 g **C** 3625 g **D** 1850 g

7 Three children, Jennifer, Kylie and Bronwyn, are friends. Their surnames are Conway, Collison and Wilkes, but not necessarily in that order. Jennifer lives in a red house, Bronwyn lives in a yellow house, Conway lives in a blue house and Wilkes does not live in a red house. What are the full names of each of the girls?



8 Which fruit did I grow? My fruit:

- is not red
- is not narrow
- is not green.

A 

B 

C 

D 

E 

F 

G 

H 

2:07 Working backwards

For some problems, working backwards gives a solution.

WORKED EXAMPLE

Maria is 35 years younger than Tom. Fred is half the age of Maria. Jamila is 17 years older than Fred. If Jamila is 35, how old is Tom?

Solution

When working backwards we often use opposite operations.

We know that Jamila is 35.
We know that Jamila is 17 years older than Fred.
So Fred's age is $35 - 17$, which is 18.
We know that Fred is half the age of Maria.
So Maria's age is 18×2, which is 36.
We know that Maria is 35 years younger than Tom.
So Tom's age is $36 + 35$, which is 71.

$\therefore$ Tom is 71 years old.



Exercise 2:07

1 Nick is 14 years younger than Hayley. Lisa is 26 years older than Nick. If Lisa is 37, how old is Hayley?

2 Four people weighed themselves. Young Louise was 17 kg lighter than Adele, Gavin was twice as heavy as Louise, and Andy was 9 kg heavier than Gavin. If Andy's weight was 73 kg, what was Adele's weight?

3 My age is 60. I was married 23 years ago and graduated 5 years after that. How old was I when I graduated?

4 Hungry Harry has $144 now, but during the last week he has bought ten hamburgers at $5.55 each and eight milkshakes at $2.70 each. He also earned $64.50 mowing lawns during the last week. How much money did he have one week ago?

5 I think of a number, multiply it by 4, then subtract 8 from that answer. I am left with 72. What was the number I first thought of?

6 When Vu arrived in Australia he had some money to help him get started. In the first three years this amount grew to 10 times its original size. With this money he started a plant nursery and with a lot of hard work built up his personal wealth by a further $98 000. If his personal wealth was then $848 000, how much did he have when he arrived in Australia?



CHALLENGE 2:07 BACKTRACKING

$\square \xrightarrow{+10} 84$ means '$\square$ is changed into 84 by adding 10'.

Find the numbers represented by the $\square$, $\bigcirc$ and $\square$ in each of the examples below.

1 $\square \xrightarrow{+10} 84$

2 $\square \xrightarrow{-8} 37$

3 $\square \xrightarrow{\times 2} 106$

4 $\square \xrightarrow{\div 3} 17$

5 $\square \xrightarrow{\times 2} \bigcirc \xrightarrow{+10} 36$

6 $\square \xrightarrow{\div 3} \bigcirc \xrightarrow{-8} 1$

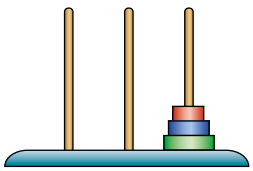
7 $\square \xrightarrow{\times 7} \bigcirc \xrightarrow{-7} \square \xrightarrow{+4} 67$

8 $\square \xrightarrow{\div 5} \bigcirc \xrightarrow{+6} \square \xrightarrow{-9} 11$

2:08 Acting it out

Sometimes it's possible to act out the problem to find the solution.

WORKED EXAMPLE

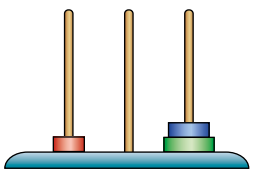


Three circular discs with holes in their centres must be moved from the right peg to the left peg. The discs must be moved one at a time and no disc can be placed on top of a smaller disc. What is the least number of moves needed to move all the discs to the left peg?

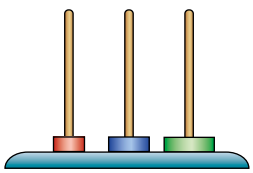
Solution

You could cut out 3 paper circles and act out this problem yourself.

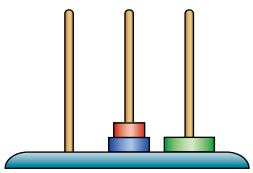
Step 1



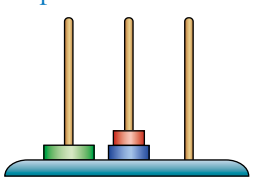
Step 2



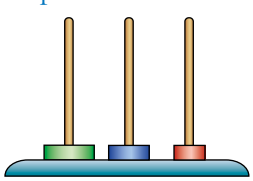
Step 3



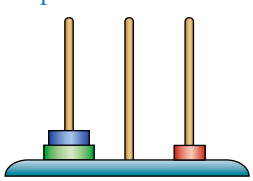
Step 4



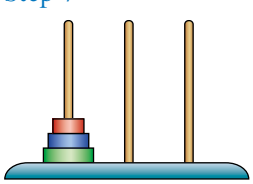
Step 5



Step 6



Step 7

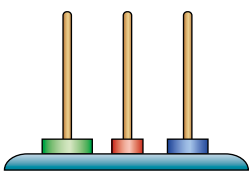


The least number of moves needed to move the discs to the left peg is 7.

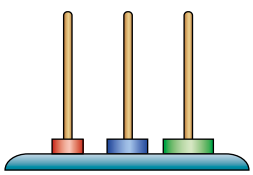
Exercise 2:08

1 Three circular discs with holes in their centres must be moved to the left peg. The discs must be moved one at a time and no disc can be placed on top of a smaller disc. In each of the following cases, what is the least number of moves needed to move all the discs to the left peg?

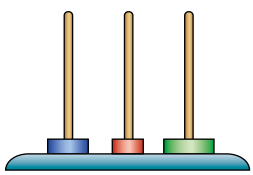
a



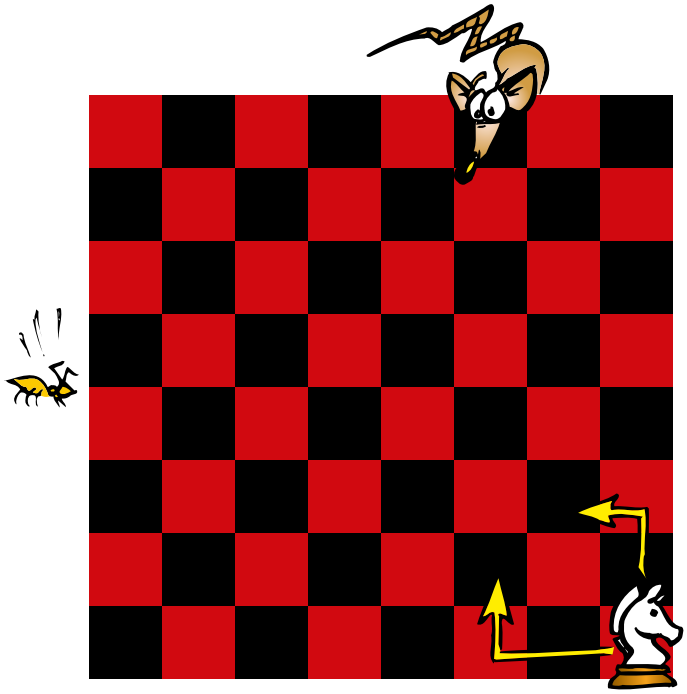
b



c



2 A worker ant wants to cross from one side of a chessboard to the other. She always keeps a black square on her immediate left and always walks along the lines on the board. If the squares on the board have sides of length 3 cm, what is the shortest distance she must travel?

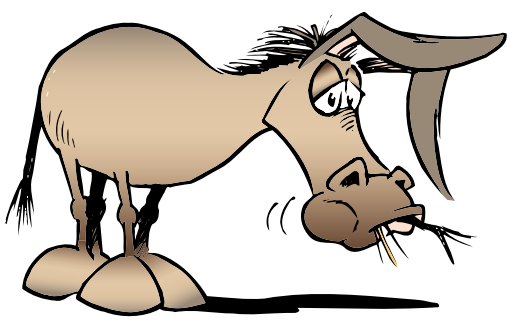


3 A mouse sits on a black square on one edge of a chessboard. It decides to step only on black squares and to move only to squares that are touching the square it is on. How many squares must it step on to reach each of the other three edges and then return to its starting point?

4 The knight (a chess piece that looks like the head of a horse) moves on the board by going two squares in one direction and then one square to the side (or one square in one direction, then two to the side). This is counted as one move. What is the least number of moves necessary to get the knight from one corner of the board to the opposite corner?

5 If the mouse moves as described in Question 3, starting from where it is on the board shown above, what is the greatest number of black squares that it can step on in one trip without stepping on the same square twice?

6 Walking to church dressed in their new clothes, Mr and Ms Ritchie and their two children Andrew and Vanessa came to a very muddy part of the road. Fortunately Mr Ritchie's donkey was grazing nearby. However, it could carry only one adult or two children at one time, and it would not move without at least one person riding it. Explain how the whole family crossed over the mud using the donkey.



2:09 Looking for patterns

If you can find a pattern, it may help you solve the problem.

WORKED EXAMPLES

Look for patterns that will help you find the answer to each of these.

1 $7-6+7-6+7-6+7-6+7-6+7-6+7-6+7-6+7-6+7-6+7$

2 $9+5+2+9+5+2+9+5+2+9+5+2+9+5+2+9+5+2$

3 $(1+2+3+4+5+6+7+8+9)+(9+8+7+6+5+4+3+2+1)$

Solutions

1 By grouping the numbers in pairs, the answer is quickly found:
$(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+(7-6)+7$
= (ten lots of 1) + 7
= 17

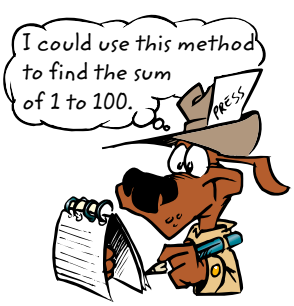
2 Here we can arrange the numbers in groups of three:
$(9+5+2)+(9+5+2)+(9+5+2)+(9+5+2)+(9+5+2)+(9+5+2)$
= 6 lots of 16
= 96

3 If we add the first numbers in each group, we get 10. The second numbers also have a sum of 10, and this pattern continues:

$$\begin{array}{rl} & (1+\ 2+\ 3+\ 4+\ 5+\ 6+\ 7+\ 8+\ 9) \\ + & (9+\ 8+\ 7+\ 6+\ 5+\ 4+\ 3+\ 2+\ 1) \\ \hline \text{Adding:} & 10+10+10+10+10+10+10+10+10 \end{array}$$

Even without showing working, we could have seen that there were 9 lots of 10.
∴ The answer is 90.

We can look for patterns like these to solve other sums.



To add all the counting numbers from 1 to 200, we could add them all twice and then divide that answer by 2, for example:

$$\begin{array}{rl} & (\ 1+\ \ 2+\ \ 3+\ \ 4+\ \ 5+\ \ 6+\ldots+199+200) \\ + & (200+199+198+197+196+195+\ldots+\ \ 2+\ \ 1) \\ \hline & 201+201+201+201+201+201+\ldots+201+201 \end{array}$$

Altogether, twice the sum gives us 200 lots of 201.
∴ The sum of the counting numbers from 1 to 200 = $\frac{1}{2}$ of (200×201)
$= 100 \times 201$
$= 20\,100$

Other problems involving patterns will be met in Chapter 4, 'Patterns and algebra', page 100.

Exercise 2:09

1 Look for patterns that will help you find the answer to each of these.

a $9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9-7+9$

b $1+2+3+4+1+2+3+4+1+2+3+4+1+2+3+4+1+2+3+4$

c $\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9}+\frac{1}{9}-\frac{1}{10}+\frac{1}{10}$

d $29-28+27-26+25-24+23-22+21-20+19-18+17-16+15-14$

e $197-187+177-167+157-147+137-127+117-107+97-87+77-67$

2 Use the method given before this exercise (page 65) to find the sum of the counting numbers from:

a 1 to 400 **b** 1 to 1000 **c** 1 to 100 **d** 1 to 50

3 At a party, each person who arrived shook the hand of every other person present. Upon arrival, how many hands would be shaken by:

a the first person **b** the second person

c the third person **d** the fourth person

e the fifth person **f** the sixth person

g the seventh person **h** the eighth person?



4 If every person at the party in Question **3** shakes hands with every other person there, how many handshakes will there be altogether if:

a 2 people are there **b** 3 people are there **c** 4 people are there

d 5 people are there **e** 6 people are there **f** 7 people are there?

5 Use the answers to Question **4** to complete this table.

Number of people	2	3	4	5	6	7
Number of handshakes						

Can you see a pattern in the number of handshakes? What is it?

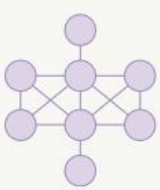
Extend this pattern to find the number of handshakes if 20 people are at the party.

Note: Other methods, such as drawing a picture, making a list or acting it out, can be used to help solve this problem.

FUN SPOT 2:09 — WHERE SHOULD THE NUMBERS GO?

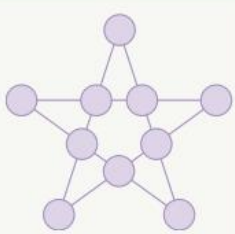
1 Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8 inside these circles so that no two consecutive numbers are in connected circles.

(*Remember:* 'Consecutive' means 'one after the other', e.g. 2 and 3, or 6 and 7.)



2 Place ten of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles on this star so that each line of four numbers adds to give 24.

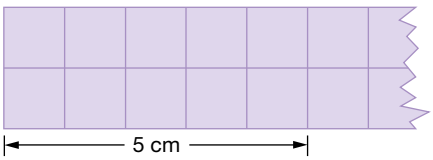
(*Note:* Two numbers cannot be used; but which ones?)



2:10 Solving a simpler problem

If a problem seems complicated, try solving a simpler problem, or see if you can solve the problem using easier numbers.

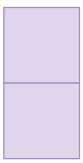
WORKED EXAMPLE



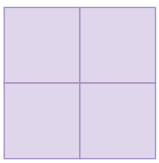
A machine produces a roll of grid paper 2 cm wide. Within the grid there are squares with sides 1 cm in length and squares with sides 2 cm in length. How many squares are there if the roll of grid paper is 100 cm long?

Solution

To solve this problem would take a long time, so we solve simpler problems and look for a pattern. Try the smallest lengths first.



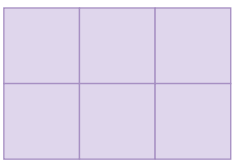
Length	1 cm
Number of squares	2



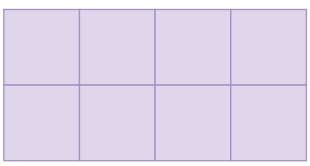
Length	2 cm
Number of squares	4 + 1



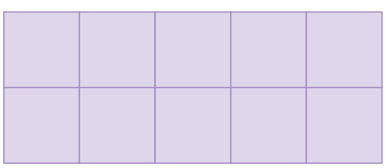
4 small squares
1 large square



Length	3 cm
Number of squares	6 + 2



Length	4 cm
Number of squares	8 + 3



Length	5 cm
Number of squares	10 + 4

In each case: Number of 1 cm squares = 2 × length
Number of 2 cm squares = 1 × length − 1
∴ Total number of squares = 3 × length − 1

This can be seen in the table below.

Length (cm)	1	2	3	4	5	100
Number of squares	2	5	8	11	14	299

∴ 299 squares can be found in 100 cm of grid paper.



Exercise 2:10

1 The hare challenges the tortoise to a race. The hare can travel 10 m for every 1 m travelled by the tortoise. If they continue to travel at this rate, by how much will the hare beat the tortoise in races over the following distances?

a 10 m **b** 20 m **c** 100 m **d** 1000 m **e** 2 km

2 How much of a head start could the hare in Question **1** give the tortoise in a race over 50 m and still expect to win?

3 Blackbeard the pirate was following an unusual treasure map: 'Jump from Dead Man's Rock onto the wet sand and then take 54 steps along the beach.'



a If he obeyed the instructions exactly, how many shoeprints would he have made in the sand?

b If he only had one good leg and a 'pegleg' on the other, how many shoeprints would he have made?

4 How many diagonals can be drawn from one corner of:

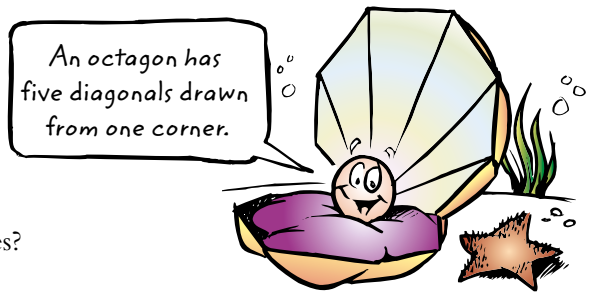
a a square

b a pentagon (5 sides)

c a hexagon (6 sides)

d a heptagon (7 sides)

e a figure on a flat surface that has 100 sides?



5 Ten prospectors are working in a gold mine. Each prospector fills a small bag of gold each month. After the first month one prospector disappears, stealing all the gold that was dug up that month. The same thing happens in each of the following months until only one prospector is left. How many bags of gold were stolen?

6 If there had been five honest prospectors among those in Question **5**, how many bags of gold would have been stolen?

7 Looking again at the worked example before this exercise (on page 67), write how many lines of length 1 cm are needed to draw a grid of length:

a 1 cm **b** 2 cm **c** 3 cm **d** 4 cm **e** 5 cm

8 In Question **7**, how many extra 1 cm lines are needed for each centimetre increase in grid length? Use this information to find the number of 1 cm lines needed to draw a grid of length 100 cm.

INVESTIGATION 2:10 LET'S POST A PARCEL

The cost to post a parcel from Sydney (in parcel zone N1) depends on the parcel's weight and on its destination (where it is going).

Parcel charging zones (Destination)

Postcode	Zone	Postcode	Zone
New South Wales			
0200–0299	N2	2720–2730	N2
1000–2263	N1	2731–2739	V2
2264–2499	N2	2740–2786	N1
2500–2530	N1	2787–2879	N2
2531–2554	N2	2880	S2
2555–2574	N1	2881–2889	N2
2575–2647	N2	2890	N1
2648	V2	2891–2898	N2
2649–2714	N2	2899	NF
2715	V2	2900–2999	N2
2716	N2	3000–3220	V1
2717–2719	V2		

Note: Some destinations in New South Wales are so far away that they use zones designated for Victoria (V1 and V2).

Regular parcel charges

Weight	Sydney N1	Other NSW N2	Melbourne V1	Other Victoria V2
	$	$	$	$
Up to 250 g	4.75	4.75	4.75	4.75
Over 250 g to 500 g	6.05	6.05	6.05	6.05
Over 500 g to 1 kg	8.00	8.65	10.95	11.05
Over 1 kg to 2 kg	8.00	9.30	11.80	12.00
Over 2 kg to 3 kg	8.00	9.95	12.65	12.95
Over 3 kg to 4 kg	8.00	10.60	13.50	13.90
Over 4 kg to 5 kg	8.00	11.25	14.35	14.85
Over 5 kg to 6 kg	8.00	11.90	15.20	15.80
Over 6 kg to 7 kg	8.00	12.55	16.05	16.75
Over 7 kg to 8 kg	8.00	13.20	16.90	17.70
Over 8 kg to 9 kg	8.00	13.85	17.75	18.65
Over 9 kg to 10 kg	8.00	14.50	18.60	19.60
Over 10 kg to 11 kg	8.00	15.15	19.45	20.55
Over 11 kg to 12 kg	8.00	15.80	20.30	21.50
Over 12 kg to 13 kg	8.00	16.45	21.15	22.45
Over 13 kg to 14 kg	8.00	17.10	22.00	23.40
Over 14 kg to 15 kg	8.00	17.75	22.85	24.35
Over 15 kg to 16 kg	8.00	18.40	23.70	25.30

WORKED EXAMPLE

How much would it cost to send a 10·5 kg parcel to area postcode 2650 (Wagga Wagga)?

Solution

We use the first table to find that postcode 2650 is in zone N2.

Then we use the next table to find the charge for 10·5 kg (i.e. 'over 10 kg to 11 kg') in the N2 column.

∴ The cost of sending the parcel is $15.15.



1 How much would it cost to send a parcel from Sydney (zone N1) to area postcode:

a 2041 (Balmain) weight: 15 kg

b 2795 (Gemalla) weight: 8·4 kg

c 2640 (Albury) weight: 260 g

d 2731 (Womboota) weight: 600 g

e 3000 (Melbourne) weight: 13·8 kg

f 2530 (Yallah) weight: 200 g

2 Use the postcode list on the White Pages website or at the back of the White Pages book to make up your own questions, then answer them.

ASSIGNMENT 2A Chapter review

1 Ross purchased a table for $285 and a cupboard for $119. After spending $74.50 on repairs, he sold the two items for $775. How much profit did he make altogether on the two items?
(Direct computation)

2 Rhonda's cycle repair shop fixes normal bicycles and tricycles. Rhonda had to assemble a number of bicycles and tricycles. She counted 16 handlebars and 37 wheels among the parts she had to put together. How many of each type of cycle were to be assembled?
(Trial and error)



3 8 km of fencing encloses a square paddock of area 400 hectares. If 16 km of fencing was used to enclose a square paddock, how many hectares would be enclosed?
(Making a drawing, diagram or model)

4 Rahul has to list his favourite three movies in order. The three movies are *Forrest Gump* (FG), *Harry Potter* (HP) and *The Dish* (TD). How many different ways can these movies be listed in order?
(Making a list, chart, table or tally)



5 Three students who were born in different countries have the last names Sze, Pieris and Chan. Their first names are Rohan, Maja and Dennis, but not necessarily in that order. Pieris was born in Australia, Sze has never been to Malaysia, Dennis was born in Malaysia and Maja was born in Sri Lanka. What is the full name of each student?
(Eliminating possibilities)

6 Tom is 7 years younger than Diane. Lynn is twice Tom's age. Peter is 18 years older than Lynn. If Peter is 42, how old is Diane?
(Working backwards)

7 Look for patterns to help you find the answer.

a $13 - 11 + 13 - 11 + 13 - 11 + 13 - 11 + 13 - 11 + 13 - 11 + 13 - 11 + 13 - 11$

b $11 + 5 + 4 + 3 + 2 + 1 + 5 + 4 + 3 + 2 + 1 + 5 + 4 + 3 + 2 + 1 + 5 + 4 + 3 + 2 + 1$

(Looking for patterns)

ASSIGNMENT 2B Working mathematically

1 Use **ID Card 4** on page xvii to give the mathematical term for:

a 1 **b** 2
c 3 **d** 4
e 5 **f** 6
g 7 **h** 8
i 9 **j** 13

2 Using the **Arithmetic Card** on page xxi:
a multiply column A by column B
b add column K to column L.

3 A student cuts six lengths of cloth from a roll 20 m long. If each cut length is 2·4 m, how much cloth remains?

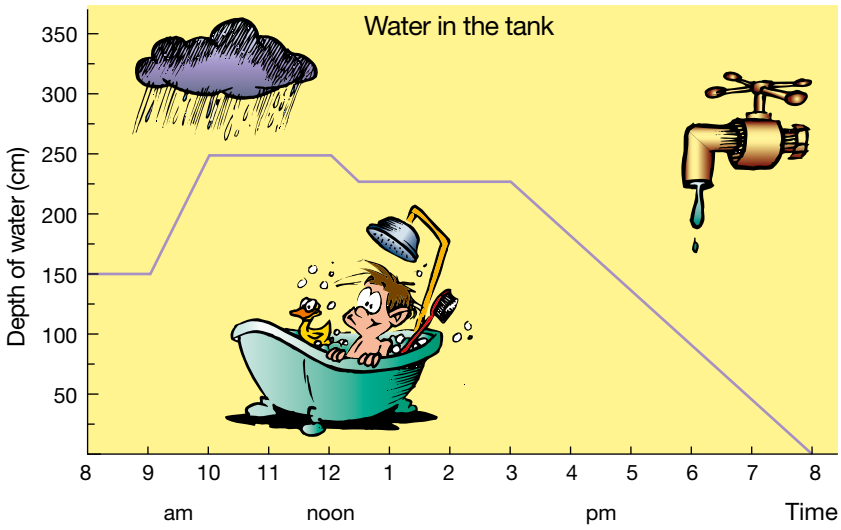
4 A machine caps bottles at a speed of one bottle every 2 seconds. How many bottles does the machine cap in 4 hours?

5 How could you estimate how many households have a phone in your capital city?

6 You are running a basketball competition in which six teams (Teams 1 to 6) are playing. List the matches that should be played if each team has to play each other team only once.

7 A suburban railway line has 10 stations. How many different tickets would be needed to cover any trip from one station to another, if every different trip needs a different ticket?

8 Dave depended on the water stored in his tank to meet his everyday needs. This graph shows the depth of water in the tank from 8 am to 8 pm on one day.



On the day of the graph, it rained, Dave had a bath and Dave's son Brian left the tap running.

a Between what two times did it rain?
b What was the highest level of water in the tank during this 12-hour period?
c When did Brian leave the tap on?
d When was the tank empty?
e How much water was in the tank at 2:30 pm?

ASSIGNMENT 2C Cumulative revision

1 What is the value of the 1 in each of the following numerals? 1:02

a 314 **b** 1608 **c** 21 753 **d** 153 246

2 Simplify: 1:11A

a $317 + 72 + 4825 + 196$ **b** $2176 - 413$

c 15×12 **d** $265 \div 5$

3 Write these decimals as fractions. 1:11C

a 0·7 **b** 0·01 **c** 0·201 **d** 0·033

4 Write these fractions as decimals. 1:11C

a $\frac{3}{10}$ **b** $\frac{17}{100}$ **c** $\frac{45}{1000}$ **d** $1\frac{3}{100}$

5 Simplify the following: 1:11C

a $0{\cdot}5 + 0{\cdot}21$ **b** $4{\cdot}2 + 3{\cdot}17$

c $12 + 1{\cdot}5$ **d** $7{\cdot}4 + 10{\cdot}05 + 3{\cdot}15$

6 Simplify the following: 1:11C

a $12 - 3{\cdot}7$ **b** $4{\cdot}8 - 1{\cdot}6$ **c** $2{\cdot}1 - 0{\cdot}85$ **d** $15{\cdot}17 - 7$

7 Simplify the following: 1:11C

a $6 \times 0{\cdot}1$ **b** $6 \times 0{\cdot}01$ **c** $0{\cdot}6 \times 10$ **d** $0{\cdot}6 \times 0{\cdot}1$

8 Simplify the following: 1:11C

a $12{\cdot}6 \div 2$ **b** $1{\cdot}04 \div 2$ **c** $5{\cdot}5 \div 5$ **d** $17{\cdot}24 \div 4$

9 Simplify: 1:11C

a $\$1.55 + \$3.75$ **b** $\$15 - \$7.90$ **c** $\$42.20 \times 7$ **d** $\$24 \div 5$

10 Simplify: 1:11B

a $\frac{3}{5} + \frac{3}{5}$ **b** $\frac{37}{100} + \frac{42}{100}$ **c** $\frac{15}{20} - \frac{7}{20}$ **d** $\frac{19}{100} - \frac{19}{100}$

11 Find the basic numeral for each. 1:06

a $16 - 4 \times 2$ **b** $24 - 8 \div 2 + 2$ **c** $20 \div (2 + 2)$ **d** $5 \times 8 + 5 \times 20$

12 Insert grouping symbols to make each number sentence true. 1:06

a $6 \times 5 + 3 = 48$ **b** $10 - 3 \times 3 = 21$

c $4 - 3 + 1 = 0$ **d** $9 + 8 - 1 + 1 = 15$

13 Find the value of the missing number. 1:06

a $198 \times 7 = 200 \times 7 - \square \times 7$ **b** $705 \times 4 = 700 \times 4 + 5 \times \square$

14 Write the meaning of each symbol. 1:08

a $\div$ **b** $\leq$ **c** $\neq$

d % **e** $\therefore$ **f** BCE

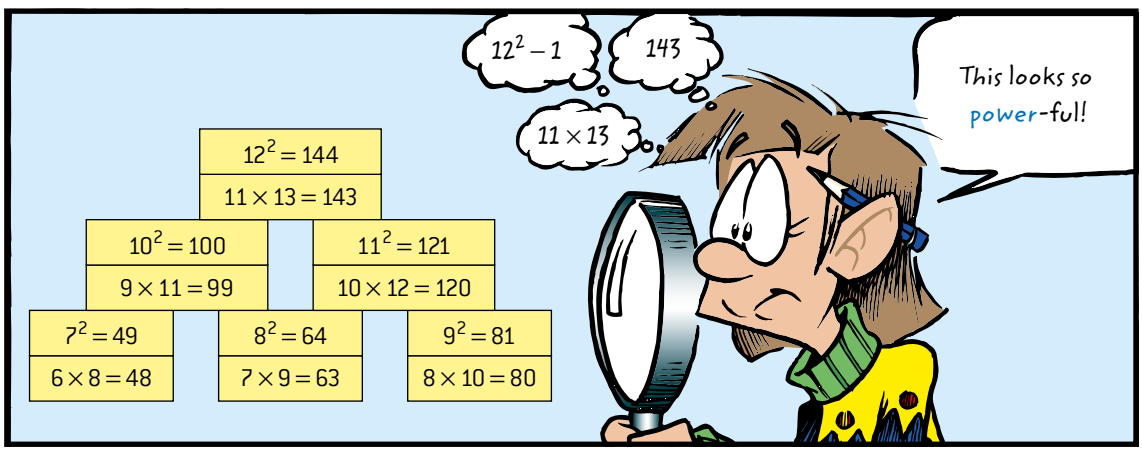
15 List: 1:09

a the odd numbers between 3 and 12 **b** the square numbers less than 50.

16 Give an example of a four-digit palindromic number. 1:09

NUMBER AND INDICES

3



Contents

3:01 Index notation
Challenge 3:01 Now that's a google
3:02 Expanded notation
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3:05 Divisibility tests
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Challenge 3:06 What are the factors?
3:07 Square and cube roots
3:08 The binary system
Fun spot 3:08 Making magic squares
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Indices*** [Stage 4]

- Investigate index notation and represent whole numbers as products of powers of prime numbers. (ACMNA149)
- Investigate and use square roots of perfect square numbers. (ACMNA150)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

3:01 Index notation

PREP QUIZ 3:01

Find the value of:

1 $3 \times 3 \times 3$ **2** 5×5 **3** $2 \times 2 \times 2 \times 2 \times 2$
4 $5 \times 5 \times 5$ **5** 10×10 **6** $10 \times 10 \times 10$
7 $10 \times 10 \times 10 \times 10$ **8** $10 \times 10 \times 10 \times 10 \times 10$ **9** $7 \times 10 \times 10$
10 $4 \times 10 \times 10 \times 10$

We use a **power** when a number is in a product more than once.

$5^2 = 5 \times 5$
This is five squared, or five to the power of two.
$10^3 = 10 \times 10 \times 10$
This is ten cubed, or ten to the power of three.

Note:
Another name for **power** is **index**.

WORKED EXAMPLES

1 Write 5^3 in expanded form and write its basic numeral.

2 Write $10 \times 10 \times 10 \times 10$ as a power of ten.

3 What is the basic numeral of 7×10^3?

5^3 is $5 \times 5 \times 5$ written using **index notation**.

Solutions

1 5^3
$= 5 \times 5 \times 5$
$= 25 \times 5$
$= 125$

2 $10 \times 10 \times 10 \times 10$
$= 10^4$

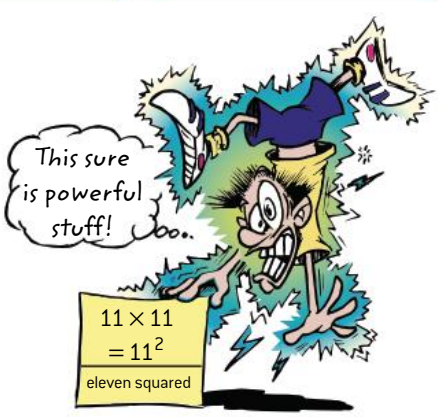
3 7×10^3
$= 7 \times (10 \times 10 \times 10)$
$= 7 \times 1000$
$= 7000$

Exercise 3:01

P Foundation worksheet 3:01 Powers of numbers

1 Write each in expanded form and as a basic numeral.

a 4^2 b 10^2 c 2^2
d 1^2 e 7^2 f 9^2
g 2^3 h 5^4 i 3^3
j 10^3 k 10^4 l 10^5



2 Rewrite using index notation.

a 7×7 b $8 \times 8 \times 8$
c $10 \times 10 \times 10 \times 10$ d $10 \times 10 \times 10$
e $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ f $2 \times 2 \times 2 \times 2$
g $10 \times 10 \times 10 \times 10 \times 10$

3 Write the basic numeral for each.

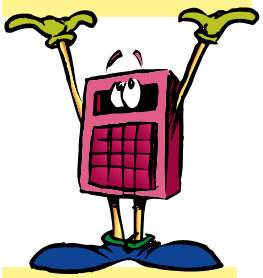
a 6×10^1	b 3×10^2	c 5×10^3
d 2×10^4	e 7×10^3	f 1×10^2
g 4×10^3	h 9×10^2	i 8×10^4
j $3^2 \times 10^3$	k $2^3 \times 10^4$	l $7^2 \times 10^5$

A basic numeral is the simplest answer.

4 Rewrite each expression using index notation.

a $2 \times 2 \times 2 \times 3 \times 3$	b $5 \times 5 \times 2 \times 2 \times 2$
c $4 \times 4 \times 3 \times 3 \times 3 \times 3$	d $6 \times 6 \times 6 \times 7 \times 7$
e $5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3$	f $8 \times 8 \times 8 \times 10 \times 10$
g $2 \times 2 \times 3 \times 2 \times 3 \times 3$	h $5 \times 6 \times 5 \times 5 \times 6$
i $4 \times 4 \times 5 \times 4 \times 4$	j $3 \times 7 \times 3 \times 7 \times 7 \times 3$

$3 \times 3 \times 3 \times 3 \times 5 \times 5$
$= 3^4 \times 5^2$



$2^3 \times 5^2$
$= 2 \times 2 \times 2 \times 5 \times 5$
$= 8 \times 25$
$= 200$

5 Rewrite each in expanded form and then as a basic numeral.

a $2^2 \times 3^2$	b $5^2 \times 3^2$
c $4^2 \times 2^3$	d $3^3 \times 2^2$
e $3^2 \times 2^5$	f $5^3 \times 2^3$
g $4^3 \times 3^2 \times 2^2$	h $2^3 \times 3^2 \times 4^1$

6 Rewrite each in expanded form, then as one term with only one power.

a $2^2 \times 2^3$	b $3^4 \times 3^3$
c $5^3 \times 5^5$	d $4^3 \times 4^2 \times 4^1$

Do you see any pattern or rule?

7 Write each expression as a basic numeral, then express this numeral as a power of a single number.

a $2^2 \times 3^2$	b $2^2 \times 5^2$
c $4^2 \times 2^2$	d $5^3 \times 2^3$

Do you see any pattern or rule?

$20^2 = (2 \times 10)^2$
$= 2^2 \times 10^2$

8 Evaluate each expression, remembering the rules for order of operations.

a $2 \times 2 + 3 \times 3$	b $2^2 + 3^2$	c $3^2 - 2^2$
d $2 \times 2 \times 2 + 3 \times 3 \times 3$	e $2^3 + 3^3$	f $3^3 - 2^3$
g $2 \times 3^2 + 3 \times 2^3$	h $4 \times 2^3 + 5^2$	i $3^3 - 5 \times 2^2$
j $5 \times 2^2 + 2 \times 6^2$	k $4^2 + 2^2 \times 5^2$	l $5 \times 2^4 - 3^2 \times 2^3$
m $5^3 \times 2^2 - 2^4 \times 6$	n $4^3 \div 2^2 \times 5$	o $5^2 \times 2^3 - 10^2 \div 2^2$

9 Use a calculator to find the value of the following. (See the examples at right.)

a 2^5	b 3^4	c 4^4
d 2^9	e 3^7	f 5^6
g 6^4	h 7^5	i 9^4
j 12^3	k 20^4	l 18^5
m $5^3 + 2^6$	n $4^4 - 3^5$	
o $3^6 + 7^3$	p $10^3 - 5^4$	
q $8^3 + 2^5 \times 6$	r $6^5 \div 3^4 + 4$	
s $5^5 - 2^6 \times 4^2$	t $4^7 \div 8^3 \times 5^4$	
u $9^3 \times 2^{10} - 7^4 \times 6^3$	v $6^7 - 5^6 \times 4^8 \div 8^4$	

To find 3^6 using a calculator:
Enter: 3 $x^{\blacksquare}$ 6 =
Answer: 729

For $2^7 + 5^4$: NOTE
Enter: 2 $x^{\blacksquare}$ 7) + 5 $x^{\blacksquare}$ 4 =
Answer: 753

CHALLENGE 3:01 NOW THAT'S A GOOGLE



Almost everyone is familiar with the word 'google' from the internet. However, this brand name originally comes from the word **googol** (with different spelling), which was first used in 1938 to refer to the very large number 10^{100}.

So 1 googol = 10^{100}.
This number is 1 followed by 100 zeros:

10 000 000 000

WORKED EXAMPLES

1 Write 6 millions + 9 thousands + 2 hundreds + 5 tens as a simple numeral.

2 Write $(5 \times 10^4) + (7 \times 10^2) + (2 \times 10^1) + (1 \times 1)$ as a numeral in its simplest form.

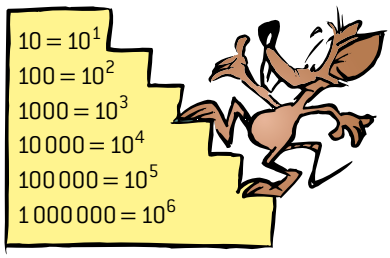
3 Write 932 014 in expanded notation.

Solutions

1 6 millions + 9 thousands + 2 hundreds + 5 tens

Column values						
1 000 000	100 000	10 000	1 000	100	10	1
6	0	0	9	2	5	0

Zeros act as place holders, allowing the other digits to be in their correct columns.



2 $(5 \times 10\,000) + (0 \times 1000) + (7 \times 100) + (2 \times 10) + (1 \times 1)$
$= 50\,721$

3 $932\,014$
$= (9 \times 100\,000) + (3 \times 10\,000) + (2 \times 1000) + (0 \times 100) + (1 \times 10) + (4 \times 1)$
$= (9 \times 10^5) + (3 \times 10^4) + (2 \times 10^3) + (0 \times 10^2) + (1 \times 10^1) + (4 \times 1)$

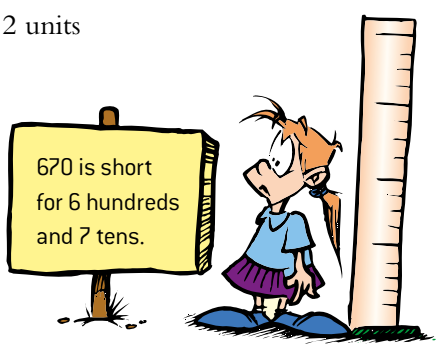
Exercise 3:02

1 Write each as a simple numeral (a basic numeral).

- **a** 6 thousands + 4 hundreds + 5 tens + 9 units
- **b** 2 ten-thousands + 8 thousands + 6 hundreds + 3 tens
- **c** 9 hundred-thousands + 8 hundreds + 7 tens + 4 units
- **d** 4 millions + 3 hundred-thousands + 8 ten-thousands
- **e** 7 ten-thousands + 4 thousands + 5 hundreds + 8 tens + 6 units
- **f** 1 million + 1 hundred-thousand + 1 thousand + 1 ten
- **g** 5 millions + 6 ten-thousands + 8 hundreds + 4 tens + 2 units
- **h** 4 ten-thousands + 8 thousands + 3 hundreds + 9 tens

2 Write these numerals in simplest form.

- **a** $(8 \times 1000) + (5 \times 100) + (3 \times 10) + (9 \times 1)$
- **b** $(7 \times 100) + (3 \times 10) + (8 \times 1)$
- **c** $(7 \times 1000) + (3 \times 100) + (0 \times 10) + (4 \times 1)$
- **d** $(9 \times 1000) + (0 \times 100) + (6 \times 10) + (7 \times 1)$
- **e** $(9 \times 1000) + (0 \times 100) + (0 \times 10) + (3 \times 1)$
- **f** $(8 \times 1000) + (2 \times 100) + (1 \times 10) + (0 \times 1)$



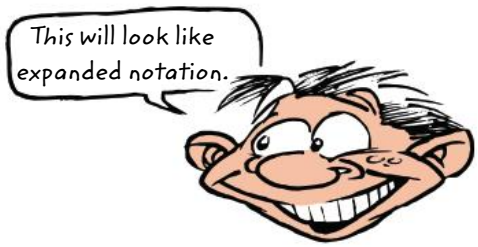
g $(6 \times 1\,000\,000) + (8 \times 100\,000) + (1 \times 10\,000) + (9 \times 1000) + (3 \times 100) + (4 \times 10) + (6 \times 1)$
h $(1 \times 1\,000\,000) + (4 \times 100\,000) + (0 \times 10\,000) + (2 \times 1000) + (0 \times 100) + (7 \times 10) + (5 \times 1)$
i $(3 \times 1\,000\,000) + (0 \times 100\,000) + (9 \times 10\,000) + (4 \times 1000) + (3 \times 100) + (6 \times 10) + (3 \times 1)$
j $(6 \times 1\,000\,000) + (9 \times 100\,000) + (0 \times 10\,000) + (0 \times 1000) + (0 \times 100) + (0 \times 10) + (0 \times 1)$

3 Write each of the following in words, as in Question 1.

a 1360 b 53 200 c 1 600 000 d 9386
e 4 307 200 f 800 305 g 1989 h 7 345 831

4 Write each of the following in expanded form, as in Question 2.

a 6709 b 13 304 c 9084
d 103 243 e 97 670 f 207 116
g 1 344 562 h 9 765 813



5 Write each of the numerals in Question 2 in expanded notation using powers of ten,
e.g. part a: $(8 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (9 \times 1)$

6 Write each of the numerals in Question 3 in expanded form using powers of ten.

3:03 Factors and multiples

PREP QUIZ 3:03

Complete these number sentences.

1 $\square \times 6 = 42$ **2** $42 \times \square = 42$ **3** $7 \times \square = 42$ **4** $1 \times \square = 42$

5 List all the counting numbers that divide 42 leaving no remainder.

What is the product of each pair of factors?

6 4 and 9 **7** 3 and 12 **8** 1 and 36 **9** 2 and 18 **10** 6 and 6

A **factor** of a counting number divides that counting number exactly.

A **multiple** of a counting number is found when you multiply that counting number by another counting number, e.g. $3 \times 8 = 24$.

Here 3 and 8 are factors of 24, and 24 is a multiple of both 3 and 8.

WORKED EXAMPLES

1 List all the factors of 102.
2 List the first nine multiples of 7.
3 Find the highest common factor (called the **HCF**) of 102 and 153.
4 Find the lowest common multiple (called the **LCM**) of 15 and 9.

Solutions



1 1 divides 102 exactly.
$1 \times 102 = 102$
$\therefore$ 1 and 102 are both factors.

2 divides 102 exactly.
$2 \times 51 = 102$
$\therefore$ 2 and 51 are both factors.

3 divides 102 exactly.
$3 \times 34 = 102$
$\therefore$ 3 and 34 are both factors.

If a number is divisible by 24, then the number is also divisible by the factors of 24 (i.e. also by 2, 12, 3, 8, 4, 6 and 1).

4 and 5 do not divide 102 exactly.

6 divides 102 exactly.
$6 \times 17 = 102$
$\therefore$ 6 and 17 are both factors.

7, 8, 9, 10 and 11 do not divide 102 exactly.

Note: $102 \div 11 = 9{\cdot}27\dot{2}\dot{7}$

When your answer ($9{\cdot}27\dot{2}\dot{7}$) is smaller than your divisor (11), you have found all factors.

The factors of 102 are 1, 102, 2, 51, 3, 34, 6, 17.

2 $7 \times 1 = 7$ $7 \times 2 = 14$ $7 \times 3 = 21$ $7 \times 4 = 28$ $7 \times 5 = 35$
$7 \times 6 = 42$ $7 \times 7 = 49$ $7 \times 8 = 56$ $7 \times 9 = 63$

The first nine multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63.

3

Number	Factors
102	1, 102, 2, (51), 3, 34, 6, 17
153	1, 153, 3, (51), 9, 17

The common factors are in blue in the table above. The highest common factor is 51.

4

Number	Multiples
15	15, 30, (45), 60, 75, 90, …
9	9, 18, 27, 36, (45), 54, …

There will be other common multiples, but the lowest common multiple is 45.

Exercise 3:03

1 Complete the table below.

Product	24	24	24	24	20	20	20	12	12	12	18	18	18	25	25
Factor	1	2	3	4	4	2	1	12	6	4	6	9	18	5	1
Factor	24	12													

2 Use the table above to list all factors of:

a 24 **b** 20 **c** 12 **d** 18 **e** 25



3 List all the factors of:

a 10 **b** 8 **c** 15 **d** 9 **e** 7
f 100 **g** 27 **h** 45 **i** 36 **j** 40
k 144 **l** 160 **m** 132 **n** 121 **o** 152

4 True or false?

a 3 is a factor of 313.
b 99 is a multiple of 9.
c 3 is a factor of 300.
d 2 is a factor of 11 012.
e 210 is a factor of 210.
f 210 is a multiple of 210.
g 35 is a multiple of 5.
h 42 is a multiple of 6.
i 1 is a factor of all counting numbers.
j 1 is a multiple of 9.

5 List the first nine multiples of:

a 3 **b** 5 **c** 2 **d** 10 **e** 4
f 11 **g** 6 **h** 9 **i** 7 **j** 8

6 Complete the tables to find the highest common factor (HCF) in each case.

a

Number	Factors
36	
24	

The HCF is …

b

Number	Factors
90	
135	

The HCF is …

c

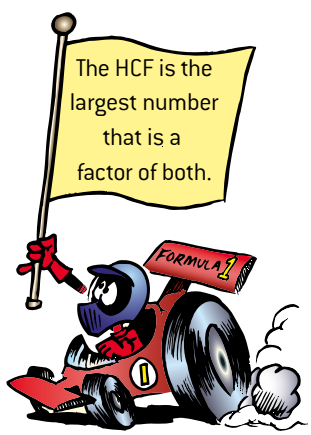
Number	Factors
116	
348	

The HCF is …

d

Number	Factors
105	
165	

The HCF is …



7 Find the highest common factor of each of the following pairs of numbers.

a 16 and 22 **b** 45 and 60 **c** 80 and 100 **d** 28 and 42
e 102 and 132 **f** 700 and 750 **g** 240 and 144 **h** 64 and 27

8 Complete the tables to find the lowest common multiple (LCM) in each case.

a

Number	Multiples
7	
5	

The LCM is …

b

Number	Multiples
10	
12	

The LCM is …

c

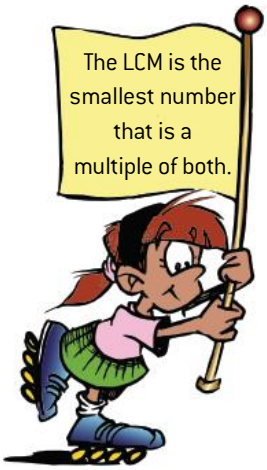
Number	Multiples
6	
15	

The LCM is …

d

Number	Multiples
30	
25	

The LCM is …



9 Find the lowest common multiple of the following:

a 6 and 4 **b** 25 and 40 **c** 12 and 18 **d** 4 and 14

e 8 and 10 **f** 24 and 30 **g** 20 and 8 **h** 22 and 33

10 **a** Find the multiples of 5 between 60 and 90.

b List all common factors of 160 and 200.

c What are the multiples of 2 between 1 and 20?

d Which of the multiples of 5 that are less than 100 are also multiples of 6?

e Find the multiples of 6 between 20 and 50.

3:04 Prime and composite numbers

PREP QUIZ 3:04

1 6×8 **2** 4×12 **3** 48×1 **4** 24×2

5 List all the factors of 48.

6 Complete: $6 \times 1 = \ldots, 6 \times 2 = \ldots, 6 \times 3 = \ldots, 6 \times 4 = \ldots, 6 \times 5 = \ldots$

7 List the first five multiples of 6.

8 List the factors of 30.

9 List the factors of 48.

10 The larger of two numbers is 601 and their difference is 20. What is their sum?

A **prime number** is a counting number that has exactly two factors: itself and 1.
For example, 7 has only two factors (7 and 1), so 7 is a prime number.

A **composite number** has more than two factors.
For example, 25 has three factors (25, 1 and 5), so 25 is a composite number.

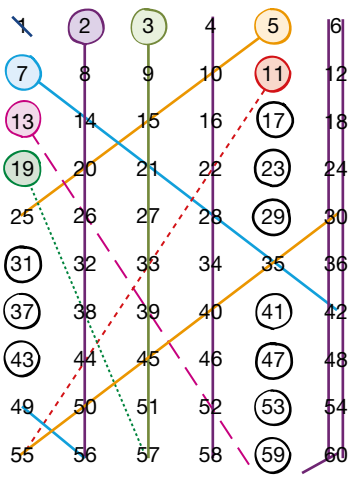
The number 1 has only one factor (1), so 1 is neither prime nor composite.

WORKED EXAMPLE 1

Use the 'Sieve of Eratosthenes' to list all primes less than 60.

Solution

Eratosthenes was a Greek mathematician who is thought to have lived from 276 to 194 BCE.
He invented a way of sorting prime numbers from composite numbers.



Step 1 Cross out 1, because it is not a prime.
Step 2 Circle the prime number 2, then cross out all multiples of 2.
Step 3 Circle the next number, 3. This is a prime. Now cross out any multiples of 3 not already crossed out.
Step 4 Circle the next number, 5. This is a prime. Now cross out any multiples of 5 not already crossed out.
Step 5 Continue using this method until all composite numbers have been crossed out.

The primes less than 60 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59.



Eratosthenes (c. 276–194 BCE)
Greek mathematician, astronomer, poet and geographer. Born at Cyrene in northern Africa, he went to Alexandria to take charge of the great library there.

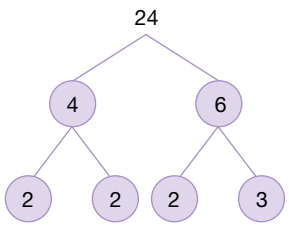
WORKED EXAMPLE 2

Use factor trees to write the following as products of prime numbers:

a 24 **b** 252

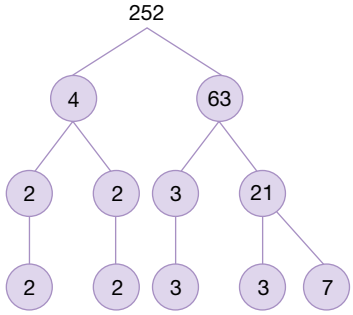
Solution

a



$\therefore 24 = 2 \times 2 \times 2 \times 3$
$= 2^3 \times 3$

b



$\therefore 252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$

Exercise 3:04

1 Copy and complete the tables below, listing all factors of the numbers 1 to 20.

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 4, 2
5	
6	
7	
8	
9	
10	

Number	Factors
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	1, 20, 2, 10, 4, 5

a List all numbers from 1 to 20 that have only two factors. These are prime numbers.

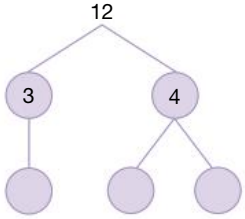
b List all numbers from 1 to 20 that have more than two factors. These are composite numbers.

2 Look back to worked example 1 (page 82) and use the Sieve of Eratosthenes to list all prime numbers that are less than 120. Start by listing all the numbers from 1 to 120 in rows of six numbers.

3 a Write all of the even prime numbers.
b How many odd prime numbers are less than 20?
c List all the odd composite numbers that are less than 20.
d How many composite numbers are between 20 and 40?
e Write each of the even numbers from 6 to 24 as the sum of two prime numbers, e.g. $12 = 5 + 7$.
(The mathematician Christian Goldbach suggested that every even number greater than 4 is the sum of two odd prime numbers. This is thought to be true, but it has never been proved.)

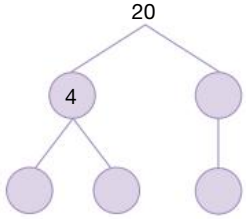
4 Complete these factor trees.

a



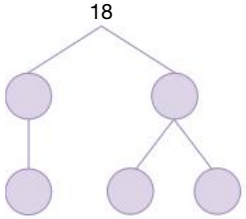
$12 = \ldots \times \ldots \times \ldots$

b



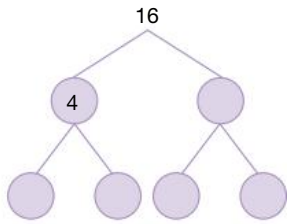
$20 = \ldots \times \ldots \times \ldots$

c



$18 = \ldots \times \ldots \times \ldots$

d



$16 = \ldots \times \ldots \times \ldots \times \ldots$

5 Use factor trees to write these numbers as products of their prime factors.
a 30 b 36 c 100 d 144 e 96 f 225

6 Another way to write a number as a product of its prime factors is to continue to divide by prime factors (starting from the smaller ones) until you get 1 as a result. Use this method (as shown at right) to write these numbers as products of prime factors:
a 216 b 675 c 735 d 504
e 1125 f 1232 g 1323 h 3375

Example: 360

$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$360 = 2^3 \times 3^2 \times 5$

6 is called a *perfect number* because the sum of all of its factors (other than itself) is equal to 6, i.e. $1 + 2 + 3 = 6$.

3:05 Divisibility tests

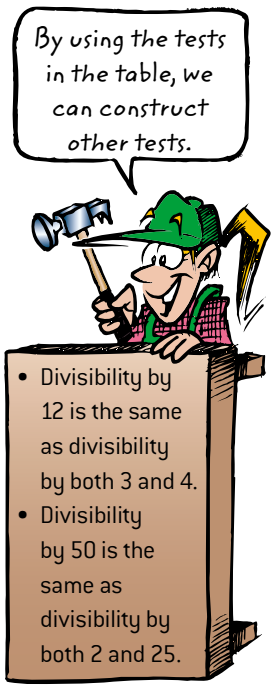
PREP QUIZ 3:05

1 List all the factors of 40.
2 Write the next number after 60 that is divisible by 10.
3 Write the next three multiples of 6 after 12.
4 What is the largest even number that is less than 1000?
5 What is the next odd number after 999?
6 In this number sentence at right, which number is the quotient?
7 Which is the divisor?
8 Which is the dividend?
9 318 813 ÷ 11
10 102 102 102 ÷ 9

$255 \div 5 = 51$

If 4 divides a number and leaves no remainder, then the number is divisible by 4.
Here are some simple divisibility tests you should know.

Divisor	Divisibility test	Example
2	The number must be even, i.e. it must end in 0, 2, 4, 6 or 8.	4136 is divisible by 2 because it is even.
3	The sum of the digits is divisible by 3.	30 012 is divisible by 3 because (3 + 0 + 0 + 1 + 2) is divisible by 3.
4	The number formed by the last two digits must be divisible by 4.	76 112 is divisible by 4 because 12 is divisible by 4.
5	The last digit must be 5 or 0.	11 225 is divisible by 5 because it ends with a 5.
6	The number must be divisible by both 2 and 3.	40 002 is even and its digit sum is divisible by 3.
8	The number formed by the last three digits is divisible by 8.	963 216 is divisible by 8 because 216 is divisible by 8.
9	The sum of the digits is divisible by 9.	142 128 is divisible by 9 because (1 + 4 + 2 + 1 + 2 + 8) is divisible by 9.
10	The last digit must be 0.	814 710 is divisible by 10 because it ends with 0.
11	The sum of the digits in odd-numbered places is equal to the sum of the digits in even-numbered places, or will differ by a multiple of 11.	7 081 426 is divisible by 11 because (7 + 8 + 4 + 6) − (0 + 1 + 2) is 22, which is divisible by 11.
25	The last two digits must be 00, 25, 50 or 75.	80 925 is divisible by 25 because it ends in 25.
100	The last two digits must be 00.	81 700 is divisible by 100 because it ends in 00.



Exercise 3:05

P Foundation worksheet 3:05
Divisibility tests

1 Which of these numbers are divisible by 2?
571 3842 5816 2221 887 9000 374 555 8774 8166 7008

2 Which of these numbers are divisible by 3?
7114 830 3006 7110 21 441 30 031 8145 60 001 211 002 78

3 Which of these numbers are divisible by 4?
1004 67 814 7118 2222 6124 8156 98 61 852 934

4 Which of these numbers are divisible by 9?
9994 31 024 30 024 81 810 41 238 3333 727 411 765 936

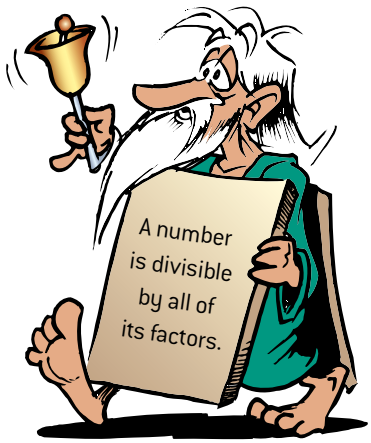
5 Which of these numbers are divisible by 11?
4115 8003 6633 7228 860 74 186 10 406 92 180 999 61 809

6 Find which of the numbers 3, 4, 5, 6, 8, 9 and 11 will exactly divide:
a 440 **b** 3883 **c** 3916 **d** 1485 **e** 3993

7 Which of the numbers 1078, 7600, 13 476 and 33 885 are divisible by the following?
a 2 **b** 3 **c** 4 **d** 6 **e** 9
f 10 **g** 11 **h** 25 **i** 100 **j** 50

8 What divisibility test could we use for 24?

9 The test for divisibility by 6 is that the number must be divisible by both 2 and 3.
a What divisibility test could we use for 15?
b What divisibility test could we use for 18?
c What divisibility test could we use for 12?



10 Use divisibility tests where possible to list all of the factors of:
a 720 **b** 346 **c** 270 **d** 1260
e 567 **f** 1575 **g** 729 **h** 5280

11 Find the smallest number that is greater than 1000 and also:
a divisible by 3 **b** divisible by 8
c divisible by 9 **d** divisible by 11
e a multiple of 25 **f** a multiple of 100
g a multiple of 15 **h** a multiple of 12

12 **a** Find the lowest common multiple of 2, 5, 9, 10 and 11.
b Find the highest common factor of 810, 324 and 1944.
c Find the lowest common multiple of 2, 3, 4, 5, 9 and 100.
d List all factors of $3 \times 5^2 \times 7$. (There are twelve.)
e List all factors of $3^3 \times 11$.

3:06 HCF and LCM by prime factors

PREP QUIZ 3:06

1 List all factors of 30.
2 List all factors of 54.
3 What is the highest common factor of 30 and 54?
4 List all multiples of 4 that are less than 40.
5 List all multiples of 3 that are less than 40.
6 What is the lowest common multiple of 4 and 3?
7 Use a factor tree to write 48 as a product of its prime factors.

Write the following products of prime numbers using index notation.
8 $2 \times 2 \times 2 \times 2 \times 2$
9 $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
10 $3 \times 3 \times 5 \times 5 \times 5 \times 7$

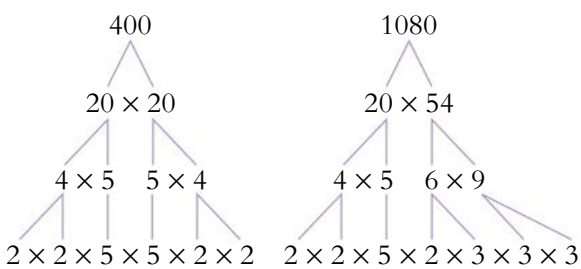
If we express numbers as products of their prime factors, it is sometimes easier to find their highest common factor (HCF) and their lowest common multiple (LCM).

WORKED EXAMPLE 1

Find the highest common factor of 400 and 1080.

Highest Common Factors (HCF) can also be called Greatest Common Divisors (GCD).

Solution



From the factor trees:

Number	Product of prime factors
400	$2 \times 2 \times 2 \times 2 \times 5 \times 5$ or $2^4 \times 5^2$
1080	$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$ or $2^3 \times 3^3 \times 5$

The two numbers have $2 \times 2 \times 2 \times 5$ in common. $2 \times 2 \times 2 \times 5 = 2^3 \times 5 = 40$, so the highest common factor of 400 and 1080 is 40.

WORKED EXAMPLE 2

Find the lowest common multiple of 400 and 1080.

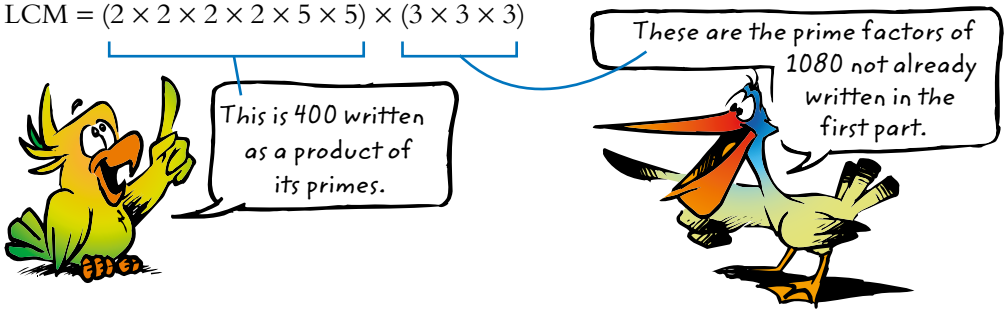
Solution

We write each number as a product of its primes:

400	$2 \times 2 \times 2 \times 2 \times 5 \times 5$ or $2^4 \times 5^2$

1080	$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$ or $2^3 \times 3^3 \times 5$

LCM = $(2 \times 2 \times 2 \times 2 \times 5 \times 5) \times (3 \times 3 \times 3)$



LCM = $2^4 \times 3^3 \times 5^2 = 10\,800$, so the lowest common multiple of 400 and 1080 is 10 800.

WORKED EXAMPLE 3

Find the highest common factor and the lowest common multiple of 12 150 and 39 375, if:

$12\,150 = 2 \times 3^5 \times 5^2$ and $39\,375 = 3^2 \times 5^4 \times 7$.

Solution

$12\,150 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
$39\,375 = 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 7$

$3 \times 3 \times 5 \times 5$ is contained in both,
$\therefore$ the HCF is $3^2 \times 5^2$ or 225.

$2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 7$ contains both numbers,
$\therefore$ the LCM is $2 \times 3^5 \times 5^4 \times 7$ or 2 126 250.

The highest common factor of 12 150 and 39 375 is 225 and the lowest common multiple of the two numbers is 2 126 250.

Exercise 3:06

1 Use this table to answer the questions.

Number	Products of prime factors	
144	$2 \times 2 \times 2 \times 2 \times 3 \times 3$	$2^4 \times 3^2$
324	$2 \times 2 \times 3 \times 3 \times 3 \times 3$	$2^2 \times 3^4$
1890	$2 \times 3 \times 3 \times 3 \times 5 \times 7$	$2 \times 3^3 \times 5 \times 7$
4900	$2 \times 2 \times 5 \times 5 \times 7 \times 7$	$2^2 \times 5^2 \times 7^2$
1960	$2 \times 2 \times 2 \times 5 \times 7 \times 7$	$2^3 \times 5 \times 7^2$
3375	$3 \times 3 \times 3 \times 5 \times 5 \times 5$	$3^3 \times 5^3$
8232	$2 \times 2 \times 2 \times 3 \times 7 \times 7 \times 7$	$2^3 \times 3 \times 7^3$
1568	$2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$	$2^5 \times 7^2$

a Find the highest common factor (HCF) of:
- i 144 and 324
- ii 324 and 1890
- iii 1890 and 4900
- iv 4900 and 1960
- v 1960 and 3375
- vi 3375 and 8232
- vii 8232 and 1568
- viii 1960 and 8232
- ix 1890 and 3375
- x 1568 and 3375

b Find the lowest common multiple of:
- i 144 and 324
- ii 324 and 1890
- iii 1890 and 4900
- iv 4900 and 1960
- v 1960 and 3375
- vi 3375 and 8232
- vii 8232 and 1568
- viii 1960 and 8232
- ix 1890 and 3375
- x 1568 and 3375

2 Copy and complete this table, then answer the questions.

Number	Products of prime factors	
18		
36		
24		
300		
1050		
1250		
2475		
2310		

a Find the highest common factor (HCF) of:
- i 18 and 36
- ii 36 and 24
- iii 24 and 300
- iv 300 and 1050
- v 1250 and 1050
- vi 2475 and 1250
- vii 2475 and 2310
- viii 1050 and 2475
- ix 1250 and 2310
- x 24 and 2475

b Find the lowest common multiple of:
- i 18 and 36
- ii 36 and 24
- iii 24 and 300
- iv 300 and 1050
- v 1250 and 1050
- vi 2475 and 1250
- vii 2475 and 2310
- viii 1050 and 2475
- ix 1250 and 2310
- x 24 and 2475

3 a Find the highest common factor of 1155 and 2079.
b Find the lowest common multiple of 1155 and 2079.
c What is the highest common factor of 264 and 1386?
d What is the lowest common multiple of 264 and 1386?

4 **a** Two people are jogging around an oval. They start together, but one takes 168 seconds to complete exactly one lap while the other takes 189 seconds. After they start, how much time will it take before they again meet at the starting point?

b Two cannons are fired together, then one is fired every 72 minutes while the other is fired every 108 minutes. After the first shot, how much time will it take until they are fired together again?

c Jun is told that she can buy chairs for $44 each and tables for $231 each, but only if she pays exactly the same amount for chairs as for tables. What is the least amount she needs to spend to take advantage of these prices?



CHALLENGE 3:06 WHAT ARE THE FACTORS?

The number 101 is prime, but the numbers 1001, 10 001 and 100 001 are not.

Use a calculator to determine the prime factors of these numbers.

3:07 Square and cube roots

PREP QUIZ 3:07

Give the basic numeral for:

1 5^2 **2** 4^2 **3** 14^2 **4** 2^3 **5** 3^3 **6** 4^3

7 Find the number that has a square of 36.

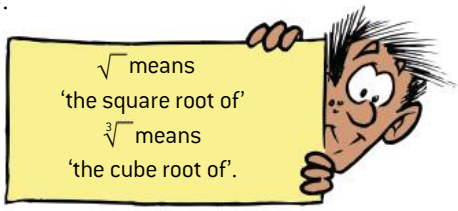
8 Find the number that has a square of 100.

9 If $1764 = (2 \times 3 \times 7) \times (2 \times 3 \times 7)$, what is the number that has a square of 1764?

10 What number has a square of 10 000?

If the square of 15 is 225, then the square root of 225 is 15.

If $15^2 = 225$, then $\sqrt{225} = 15$.



If the cube of 8 is 512, then the cube root of 512 is 8.

If $8^3 = 512$, then $\sqrt[3]{512} = 8$

The number that gives 225 when squared is called the **square root** of 225.
The number that gives 512 when cubed is called the **cube root** of 512.

WORKED EXAMPLES

1 a If $3 \times 3 = 9$,
then $\sqrt{9} = 3$

b If $4 \times 4 \times 4 = 64$,
then $\sqrt[3]{64} = 4$

c If $5^2 = 25$,
then $\sqrt{25} = 5$

d If $6^3 = 216$,
then $\sqrt[3]{216} = 6$

e If $(2 \times 7) \times (2 \times 7) = 196$
then $\sqrt{196} = 2 \times 7$
$= 14$

f If $(2 \times 3 \times 7) \times (2 \times 3 \times 7) = 1764$,
then $\sqrt{1764} = 2 \times 3 \times 7$
$= 42$

g If $(2 \times 2 \times 7)^3 = 21\,952$,
then $\sqrt[3]{21952} = 2 \times 2 \times 7$
$= 28$

2 $\sqrt{2 \times 2 \times 2 \times 2 \times 13 \times 13}$
$= \sqrt{(2 \times 2 \times 13) \times (2 \times 2 \times 13)}$
$= 2 \times 2 \times 13$
$= 52$

3 $\sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$
$= \sqrt[3]{(2 \times 7) \times (2 \times 7) \times (2 \times 7)}$
$= 2 \times 7$
$= 14$

4 a $\sqrt{25} = 5$,
because $5 \times 5 = 25$

b Find $\sqrt{576}$:
$576 = (2 \times 2 \times 2 \times 3)^2$
$\sqrt{576} = 24$

5 a $\sqrt[3]{125} = 5$,
because $5 \times 5 \times 5 = 125$

b Find $\sqrt[3]{8000}$:
By product of primes,
$8000 = (2 \times 2 \times 5)^3$
$\sqrt[3]{8000} = 20$

6 $\sqrt{135}$ is between 11 and 12,
because 135 is between 11^2 and 12^2.

Exercise 3:07

P Foundation worksheet 3:07
Square and cube roots

1 Complete:

a If $7 \times 7 = 49$,
then $\sqrt{49} = \square$

b If $11 \times 11 = 121$,
then $\sqrt{121} = \square$

c If $13 \times 13 = 169$,
then $\sqrt{169} = \square$

d If $5 \times 5 \times 5 = 125$,
then $\sqrt[3]{125} = \square$

e If $9 \times 9 \times 9 = 729$,
then $\sqrt[3]{729} = \square$

f If $12 \times 12 \times 12 = 1728$,
then $\sqrt[3]{1728} = \square$

g If $16^2 = 256$,
then $\sqrt{256} = \square$

h If $21^2 = 441$,
then $\sqrt{441} = \square$

i If $14^2 = 196$,
then $\sqrt{196} = \square$

j If $7^3 = 343$,
then $\sqrt[3]{343} = \square$

k If $4^3 = 64$,
then $\sqrt[3]{64} = \square$

l If $9^3 = 729$,
then $\sqrt[3]{729} = \square$

m If $(2 \times 9) \times (2 \times 9) = 324$,
then $\sqrt{324} = \ldots \times \ldots$
$= \square$

n If $(3 \times 7) \times (3 \times 7) = 441$,
then $\sqrt{441} = \ldots \times \ldots$
$= \square$

o If $(9 \times 11) \times (9 \times 11) = 9801$,
then $\sqrt{9801} = \ldots \times \ldots$
$= \square$

p If $(2 \times 3 \times 5) \times (2 \times 3 \times 5) = 900$,
then $\sqrt{900} = \ldots \times \ldots \times \ldots$
$= \square$

q If $(7 \times 11 \times 13) \times (7 \times 11 \times 13) = 1\,002\,001$,
then $\sqrt{1\,002\,001} = \ldots \times \ldots \times \ldots$
$= \square$

r If $(5 \times 5 \times 11)^3 = 20\,796\,875$,
then $\sqrt[3]{20\,796\,875} = \ldots \times \ldots \times \ldots$
$= \square$

2 Find the square root of each product.

a $2 \times 2 \times 3 \times 3 \times 3 \times 3$
b $3 \times 3 \times 5 \times 5$
c $3 \times 3 \times 3 \times 3 \times 5 \times 5$
d $5 \times 5 \times 7 \times 7 \times 11 \times 11$
e $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7$
f $2 \times 2 \times 3 \times 3 \times 5 \times 5$

3 Find the cube root of each product.

a $3 \times 3 \times 3 \times 5 \times 5 \times 5$
b $2 \times 2 \times 2 \times 11 \times 11 \times 11$
c $2 \times 2 \times 2 \times 3 \times 3 \times 3$
d $2 \times 2 \times 2 \times 2 \times 2 \times 2$
e $3 \times 3 \times 3 \times 3 \times 3 \times 3$
f $2 \times 2 \times 2 \times 5 \times 5 \times 5$

4 Find the basic numeral for:

a $\sqrt{9}$ **b** $\sqrt{1}$ **c** $\sqrt{4}$ **d** $\sqrt{16}$ **e** $\sqrt{64}$ **f** $\sqrt{36}$
g $\sqrt{0}$ **h** $\sqrt{100}$ **i** $\sqrt{81}$ **j** $\sqrt{144}$ **k** $\sqrt{324}$ **l** $\sqrt{400}$
m $\sqrt{1296}$ **n** $\sqrt{1764}$ **o** $\sqrt{225}$

5 Find the basic numeral for:

a $\sqrt[3]{8}$ **b** $\sqrt[3]{27}$ **c** $\sqrt[3]{1}$ **d** $\sqrt[3]{64}$ **e** $\sqrt[3]{1000}$ **f** $\sqrt[3]{0}$
g $\sqrt[3]{216}$ **h** $\sqrt[3]{512}$ **i** $\sqrt[3]{1331}$ **j** $\sqrt[3]{3375}$ **k** $\sqrt[3]{4096}$ **l** $\sqrt[3]{5832}$

6 Evaluate each pair of expressions. What pattern do you see?

a **i** $\sqrt{4} \times \sqrt{9}$ **ii** $\sqrt{4 \times 9}$
b **i** $\sqrt{9} \times \sqrt{16}$ **ii** $\sqrt{9 \times 16}$
c **i** $\sqrt{25} \times \sqrt{4}$ **ii** $\sqrt{25 \times 4}$
d **i** $\sqrt{4} \times \sqrt{9} \times \sqrt{25}$ **ii** $\sqrt{4 \times 9 \times 25}$

$\sqrt{9} \times \sqrt{4} = \sqrt{9 \times 4}$

7 By using the pattern observed in Question **6**, evaluate the following:

a $\sqrt{2} \times \sqrt{8}$ **b** $\sqrt{18} \times \sqrt{2}$ **c** $\sqrt{20} \times \sqrt{5}$ **d** $\sqrt{8} \times \sqrt{50}$

8 Evaluate the following, remembering the rules for order of operations:

a $\sqrt{16} + \sqrt{9}$ **b** $\sqrt{16} + 9$ **c** $\sqrt{16} - \sqrt{9}$
d $16 - \sqrt{9}$ **e** $\sqrt{25} + \sqrt{36} \times \sqrt{4}$ **f** $\sqrt{25} - \sqrt{36} \div \sqrt{4}$
g $\sqrt{25} \times \sqrt{36} \div \sqrt{4}$ **h** $\sqrt{25} + \sqrt{36} - \sqrt{4}$ **i** $\dfrac{\sqrt{16} + \sqrt{9}}{7}$
j $\dfrac{\sqrt{25} + \sqrt{9}}{4}$ **k** $\dfrac{\sqrt{100 - 64}}{6}$ **l** $\sqrt{\dfrac{100 - 64}{9}}$

9 Use the table of squares below to give the two counting numbers that each square root is between.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196

a $\sqrt{13}$ **b** $\sqrt{72}$ **c** $\sqrt{20}$ **d** $\sqrt{40}$ **e** $\sqrt{90}$ **f** $\sqrt{3}$

g $\sqrt{180}$ **h** $\sqrt{30}$ **i** $\sqrt{132}$ **j** $\sqrt{17}$ **k** $\sqrt{147}$ **l** $\sqrt{116}$

10 Write an estimate for the square root, then use a calculator to find the answer.

a $\sqrt{256}$ **b** $\sqrt{324}$ **c** $\sqrt{676}$

d $\sqrt{1521}$ **e** $\sqrt{1089}$ **f** $\sqrt{484}$

11 Write an estimate for the cube root, then use a calculator to find the answer.

a $\sqrt[3]{125}$ **b** $\sqrt[3]{729}$ **c** $\sqrt[3]{1331}$

d $\sqrt[3]{8000}$ **e** $\sqrt[3]{4913}$ **f** $\sqrt[3]{1728}$

Follow these examples.

Find: $\sqrt{576}$

Enter: $\sqrt{\blacksquare}$ 576 = 24

Find: $\sqrt[3]{2744}$

Enter: SHIFT $\sqrt[3]{\blacksquare}$ 2744 = 14

3:08 The binary system

PREP QUIZ 3:08

Write in expanded form:

1 31 627 **2** 15 038

Write as a basic numeral:

3 $(6 \times 10^4) + (3 \times 10^3) + (7 \times 10^2) + (8 \times 10^1) + (9 \times 1)$

4 $(1 \times 10^4) + (0 \times 10^3) + (5 \times 10^2) + (2 \times 10^1) + (0 \times 1)$

Complete:

5 $2^1 = \ldots, 2^2 = \ldots, 2^3 = \ldots, 2^4 = \ldots, 2^5 = \ldots, 2^6 = \ldots$

Simplify:

6 $(1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$

7 $(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 1)$

8 $(1 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$

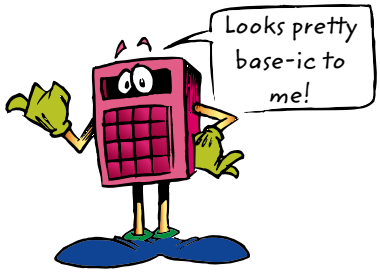
9 $(1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 1)$

10 $(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 1)$

1011_{two} is one-zero-one-one base two, i.e. $(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 1) = 8 + 0 + 2 + 1 = 11$



To use a number system based on ten, we need ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. This is our decimal ('base ten') number system. But machines that work by using electrical currents (e.g. computers) can only use two digits: 'off' and 'on'. These are their digits for 0 and 1. These digital machines use a **base two system that has place value**.



WORKED EXAMPLES

1 Change 111011_{two} into a base ten numeral.
2 Write 54_{ten} as a numeral in the binary system (base two).

Solutions

1

2^5	2^4	2^3	2^2	2^1	1
×32	×16	×8	×4	×2	×1
1	1	1	0	1	1
ON	ON	ON	OFF	ON	ON

The column values are powers of 2:

$(1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 1)$
$= 32 + 16 + 8 + 0 + 2 + 1$
$= 59$
$\therefore 111011_{two} = 59_{ten}$

2 Starting with the larger column values, distribute 54 across the columns.
54 is made up of 32 and 16 and 4 and 2.
$\therefore 54 = (1 \times 32) + (1 \times 16) + (1 \times 4) + (1 \times 2)$
Write zeros in the other columns.

2^6	2^5	2^4	2^3	2^2	2^1	1
×64	×32	×16	×8	×4	×2	×1
	1	1	0	1	1	0

Method
There are no 64s in 54.
There is one 32 in 54, i.e. $54 = (1 \times 32) + 22$. Write the 1 in the 32s column.
There is one 16 in the 22 that remains, i.e. $22 = (1 \times 16) + 6$. Write the 1 in the 16s column.
There is one 4 and one 2 in the 6 that remains, i.e. $6 = (1 \times 4) + (1 \times 2)$. Write a 1 in both the 4s column and the 2s column.
$\therefore 54_{ten} = 110110_{two}$

Exercise 3:08

1 Change these base two numerals to numerals in base ten.

a 10_{two}	**b** 11_{two}	**c** 100_{two}	**d** 101_{two}	**e** 110_{two}
f 111_{two}	**g** 1000_{two}	**h** 1001_{two}	**i** 1010_{two}	**j** 1011_{two}
k 101101_{two}	**l** 11000_{two}	**m** 100011_{two}	**n** 111111_{two}	**o** 1101_{two}

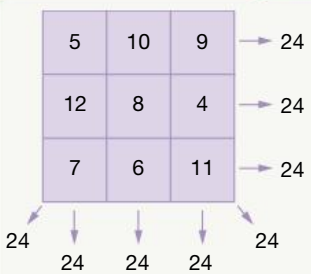
2 Write these base ten numerals as binary numerals.

a 17	**b** 20	**c** 64	**d** 35	**e** 40

3 Complete the table below.

Numeral (base ten)	2^6 ×64	2^5 ×32	2^4 ×16	2^3 ×8	2^2 ×4	2^1 ×2	1 ×1	Numeral (base two)
								1010001_{two}
								1101011_{two}
	1	0	0	0	1	1	1	
80								
100								
97								
125								

FUN SPOT 3:08 MAKING MAGIC SQUARES



The *rows, columns and diagonals* of a 'magic square' *have the same sum*. In this magic square, each row, column and diagonal has a sum of 24.

To make a 'three-by-three' magic square, you need nine numbers. These numbers, when placed in order, must form a pattern with the same difference between each pair of numbers. The numbers used here are: 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Example

2, 4, 6, 8, 10, 12, 14, 16, 18. These all have a difference of 2. We can now make a magic square using these numbers.

Step 1

Write the numbers in rows of three.

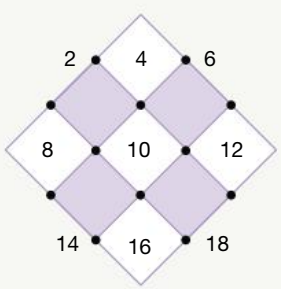
2 4 6

8 10 12

14 16 18

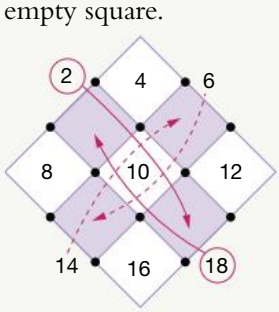
Step 2

Join the half-way points as shown.

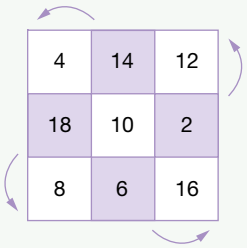


Step 3

Write each number that is outside a square into the opposite empty square.



Step 4
Rotate the square anticlockwise to give it the appearance below.



Activity

1 Use this method to make magic squares with the following numbers:
   a 3, 4, 5, 6, 7, 8, 9, 10 and 11
   b 5, 10, 15, 20, 25, 30, 35, 40 and 45
   c 36, 33, 30, 27, 24, 21, 18, 15 and 12

2 What is the sum of a column in the magic squares of **1** (above)?

3 Use the pattern 1, 2, 4, 8, 16, 32, 64, 128, 256 to make a magic square using this method. What is the product of numbers in each row, column and diagonal?

MATHS TERMS 3

basic numeral
- the simplest way to write a number, e.g. The basic numeral for $(4 + 8) \times 2$ is 24.

composite number
- a number that has more than two factors, e.g. 9 is composite because it has three factors: 1, 3 and 9

cube (number)
- the answer when a whole number is a product of itself 3 times, e.g. $5^3 = 5 \times 5 \times 5 = 125$

cube root ($\sqrt[3]{\ }$)
- to find the cube root of a number, e.g. 8, find the number that needs to be cubed to give 8: $2^3 = 8$ so $\sqrt[3]{8} = 2$

expanded notation
- a way of writing a number as the sum of its parts, e.g. $612 = (6 \times 100) + (1 \times 10) + (2 \times 1)$

factor
- a factor of a counting number divides it exactly, e.g. The factors of 6 are 1, 2, 3 and 6.
- common factor: a number that is a factor of all numbers being considered, e.g. 7 is a common factor of 14, 21 and 70.
- highest common factor (HCF): the largest of the common factors, e.g. 18 and 24 have common factors 2, 3 and 6, but the highest common factor is 6.

index (plural: indices)
- a number indicating how many of a base number need to be multiplied together, e.g. for 5^3, the index is 3

multiple
- a multiple of a counting number is found by multiplying it by another counting number, e.g. The multiples of 5 are 5, 10, 15, 20, …
- common multiple: a number that is a multiple of all numbers being considered, e.g. 50 is a common multiple of 2 and 5.
- lowest common multiple (LCM): the smallest of the common multiples, e.g. 10 is the LCM of 2 and 5. 20 is the LCM of 2, 5 and 10.

power
- another word for *index*

prime number
- a counting number that has exactly two factors, itself and 1, e.g. 17, 31, 2

square number
- the result of multiplying a counting number by itself, e.g. 1, 16, 25

square root ($\sqrt{\ }$)
- the square root of a number, e.g. What is the number that must be squared to give 64? $8^2 = 64$ so $\sqrt{64} = 8$

DIAGNOSTIC TEST 3 NUMBER AND INDICES

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Write each as a power. **3:01**
a $8 \times 8 \times 8$ **b** $3 \times 3 \times 3 \times 3 \times 3$ **c** $2 \times 2 \times 2 \times 2$

2 Write the basic numeral for each. **3:01**
a 8×10^2 **b** 2×10^4 **c** 7×10^6

3 Write each as a basic numeral. **3:02**
a $(7 \times 10^3) + (2 \times 10^2) + (2 \times 10^1) + (8 \times 1)$
b $(5 \times 10^3) + (0 \times 10^2) + (8 \times 10^1) + (0 \times 1)$

4 Write each in expanded form using powers of ten. **3:02**
a 1824 **b** 4307 **c** 2 415 286

5 List all the factors of: **3:03**
a 24 **b** 63 **c** 100

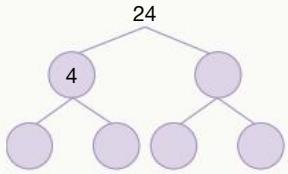
6 List the first five multiples of: **3:03**
a 4 **b** 8 **c** 11

7 Find the highest common factor of: **3:03**
a 36 and 48 **b** 60 and 75 **c** 70 and 98

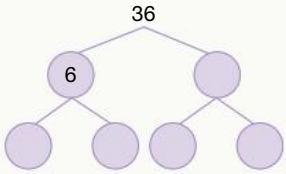
8 Find the lowest common multiple of: **3:03**
a 12 and 9 **b** 6 and 8 **c** 20 and 14

9 **a** Write all prime numbers that are less than 10. **3:04**
b Write all composite numbers that are less than 10.
c Which counting number is neither prime nor composite?

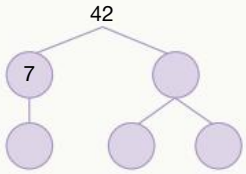
10 Complete these factor trees. **3:04**
a



b



c



11 Find the highest common factor and the lowest common multiple of: **3:06**
a $2 \times 2 \times 2 \times 3$ and $2 \times 2 \times 5$ **b** $3 \times 3 \times 3 \times 5 \times 7$ and $3 \times 5 \times 5 \times 7$
c $2 \times 3 \times 5 \times 7$ and $3 \times 3 \times 5$

Find the simplest answer for:

12 **a** $\sqrt{25}$ **b** $\sqrt{(7 \times 11) \times (7 \times 11)}$ **c** $\sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 3}$ **3:07**

13 **a** $\sqrt[3]{27}$ **b** $\sqrt[3]{11 \times 11 \times 11}$ **c** $\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$ **3:07**

ASSIGNMENT 3A Chapter review

1 Rewrite using index notation:
- **a** $5 \times 5 \times 5 \times 5$
- **b** $3 \times 3 \times 2 \times 2 \times 2$
- **c** $7 \times 4 \times 7 \times 4 \times 7$

2 **a** Write each as a basic numeral.
- **i** 4×10^3
- **ii** 5×10^4

b Hence, find the basic numeral for:
- **i** $(4 \times 10^3) + (5 \times 10^4)$
- **ii** $(4 \times 10^3) \times (5 \times 10^4)$

3 Rewrite as a basic numeral:
- **a** $(5 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (9 \times 1)$
- **b** $(8 \times 10^4) + (6 \times 10^2) + (3 \times 1)$

4 List, in order, the factors of:
a 28 **b** 48 **c** 68

5 For each of the following, find the greatest multiple that is less than 100.
a 3 **b** 7 **c** 15

6 'Prime number pairs' are pairs of prime numbers that differ only by 2, e.g. 11 and 13, or 41 and 43.
Find all the other prime number pairs between 10 and 100.

7 **a** By completing factor trees, find the prime factors of:
i 56 **ii** 84
b Find the HCF of 56 and 84.
c Find the LCM of 56 and 84.

8 Find the simplest answer for:
- **a** $\sqrt{121}$
- **b** $\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$
- **c** $\sqrt[3]{64}$
- **d** $\sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7}$

ASSIGNMENT 3B Working mathematically

1 A bookcase has six shelves, with 45 books on each shelf. How many books are in the bookcase?

2 A bus can hold 58 people seated and 35 standing. How many of these buses would be needed to take a school of 750 children and 30 teachers on a school excursion?



3 A grocer bought 50 kg of tomatoes for $30. She sold 20 kg at $1.20 per kg and the remainder, which were overripe, she sold for 80c per kg. How much did she get for selling the tomatoes? How much profit did she make?

4 In Rugby Union, a team scores 5 points for a try, 7 points for a converted try and 3 points for a penalty goal. In how many ways could a team score 22 points?

5 A school offers the following sports choices:
Summer: cricket, water polo, basketball, volleyball, tennis
Winter: football, hockey, soccer, squash

If one summer sport and one winter sport are selected, how many different combinations are available?

ASSIGNMENT 3C Cumulative revision

1 Write the basic numeral in each case. 1:06

a $12 \div (28 - 24)$ **b** $84 - 10 \times 8$ **c** $100 - [50 - (20 - 7)]$

d $23 + 5 - 2 + 3$ **e** $6 \times 5 + 6 \times 3$ **f** $3 + 7(0{\cdot}7 + 1{\cdot}3)$

2 We sell buckets of blocks. In each bucket there are 157 blocks. We sold 93 buckets of blocks on Monday and 7 on Tuesday. How many blocks did we sell on those two days altogether? 1:03

3 Given that $12 \times 1256 = 15\,072$, what is the value of: 1:07A

a 1256×12 **b** 13×1256 **c** 11×1256?

4 True or false? 1:07

a $2 \times 3186 \times 5 = 31\,860$ **b** $9 \times 888 = 8880 - 888$

c $555 \times 12 = (555 \times 6) \times 2$ **d** $18\,158 \div 20 = (18\,158 \div 2) \div 10$

e $4 \times 186 \times 25 = (4 \times 25) \times 186$ **f** $16 \times 100 - 1 = 16 \times 99$

5 Write as a basic numeral: 1:03

a 1846×1 **b** $0 \times 26\,040$ **c** $8145 + 0$

d $446 - 0$ **e** $0 \div 186$ **f** $4186 \div 1$

g $23 \times 11 - 22 \times 11$ **h** $31 \times 98 + 69 \times 98$ **i** $98 \times 37 + 2 \times 37$

6 True or false? 1:08

a $8 \times 3 \neq 3 \times 8$ **b** $563 < 560$ **c** $213 \times 0 \geq 0$

d $\sqrt{16} = 8$ **e** $16 \times 5 \leq 16 \times 4$ **f** $5 \times 816 > 0$

g $816 \div 1 \not< 816 \times 1$ **h** $999 < 100 - 0$ **i** $\sqrt[3]{1} = 1$

7 Find the answer to each calculation. 1:06

a $5 \times 4 + 3 \times 2$ **b** $5 + 4 \times 3 + 2$ **c** $5 \times (4 + 3) - 2$

d $(5 + 4) \div 3 + 2$ **e** $5 \times 4 \div (3 + 2)$ **f** $(5 - 4 + 3) \div 2$

8 **a** List the factors of 100. 3:03

b List the factors of 125.

c List all the common factors of 100 and 125.

9 **a** List the first 10 multiples of 8. 3:03

b List the first 10 multiples of 12.

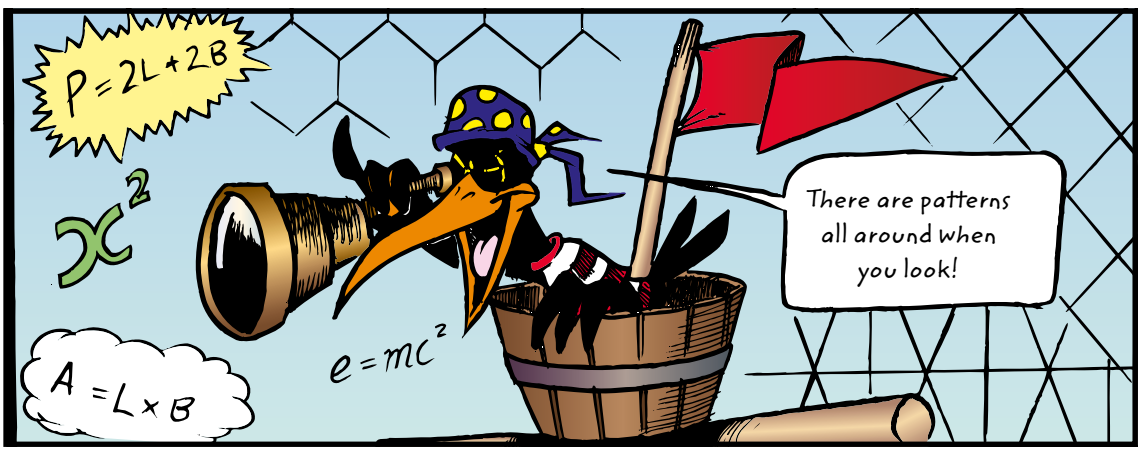
c What is the lowest common multiple of 8 and 12?

10 From the set $\{2, 5, 7, 12, 15, 36, 41\}$, write: 3:04

a the prime numbers **b** the composite numbers.

PATTERNS AND ALGEBRA

4



Contents

4:01 Number patterns
Fun spot 4:01 Mysteries mixing matches
4:02 Pronumerals and variables
4:03 Using variables
4:04 Algebraic abbreviations
4:05 Making sense of algebra
4:06 Substitution
4:07 Simplifying algebraic expressions
Fun spot 4:07 Let's improve our algebra skills
4:08 The laws of arithmetic
4:09 Using grouping symbols
4:10 Simplifying expressions with index notation
Investigation 4:10 Quality of life
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Patterns and Algebra 2*** [Stage 3], ***Algebraic Techniques 1***, ***Algebraic Techniques 2*** [Stage 4]

- Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence. [Stage 3] (ACMNA133)
- Introduce the concept of variables as a way of representing numbers using letters. (ACMNA175)
- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions. (ACMNA177)
- Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)
- Simplify algebraic expressions involving the four operations. (ACMNA192)
- Extend and apply the distributive law to the expansion of algebraic expressions. (ACMNA190)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

4:01 Number patterns

PREP QUIZ 4:01

For each of the following patterns, what is the next number?

1 2, 4, 6, …
2 7, 9, 11, …
3 1, 4, 7, …
4 4, 8, 12, …
5 5, 10, 15, …
6 20, 18, 16, …
7 2, 4, 8, 16, …
8 27, 9, 3, …
9 $0, \frac{1}{2}, 1, 1\frac{1}{2}, \ldots$
10 1·0, 1·3, 1·6, …

In Prep quiz 4:01, the next number can be found because each set of numbers is following a pattern or rule.

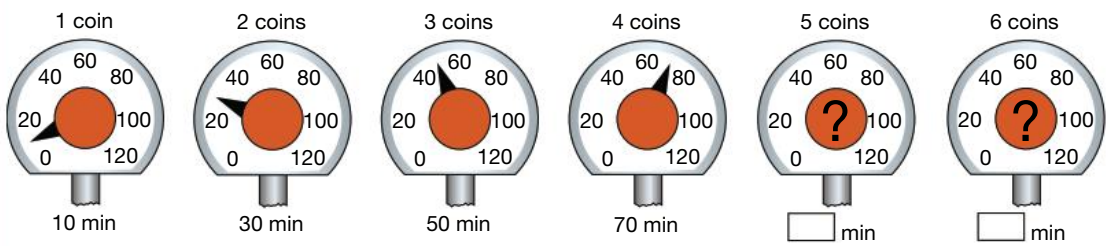


WORKED EXAMPLE 1

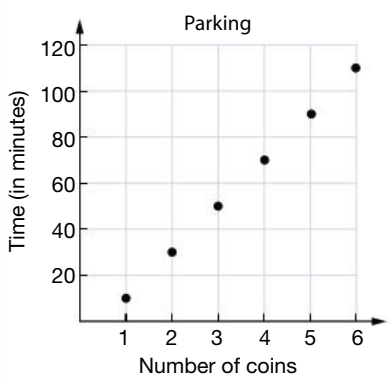
A parking meter takes only $1 coins. The pictures below show what happens as Tran puts in his coins.

Is there a pattern? What is the rule used? What parking time would you expect for:

a 5 coins **b** 6 coins?



Solution



Draw a graph:

There is a pattern. Start at 10 and then add 20 for each extra coin.

a For 5 coins you could park for 90 minutes.
b For 6 coins you could park for 110 minutes.

WORKED EXAMPLE 2

To make display counters for our school display, we joined boxes (cubes) and painted all the square faces we could see.

If one box is used, 5 square faces can be seen, because one face is on the ground. If two boxes are used there are 2 squares on top and 6 around the sides to be painted.

How many squares would need to be painted if the number of boxes joined is:

a 3 **b** 4 **c** 5?



1 box
5 squares

2 boxes
8 squares

3 boxes
..... squares

4 boxes
..... squares

As we add extra boxes, does the number of square sides to be painted form a pattern?
If we know any number in the pattern, how do we find the next number?

Solution

First, make a table:

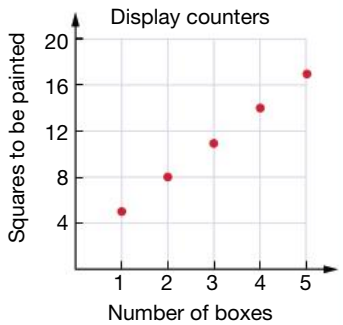
Number of boxes	1	2	3	4
Sides to paint	5	8	11	14

From the table we see that for each extra box, 3 more sides need to be painted.

a For 3 boxes we paint 11 squares.
b For 4 boxes we paint 14 squares.
c For 5 boxes we paint 17 squares.

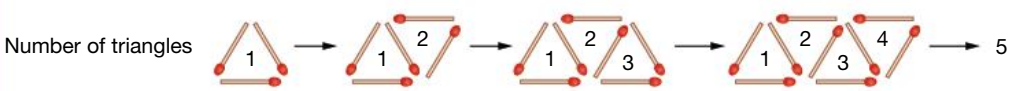
The squares form a pattern. To find the next term of the pattern, 'add 3'.

Tables and graphs can be used to show patterns.



WORKED EXAMPLE 3

Complete the number pattern for the number of matches needed to make these triangles.



What is the rule to get from one number in the pattern to the next?

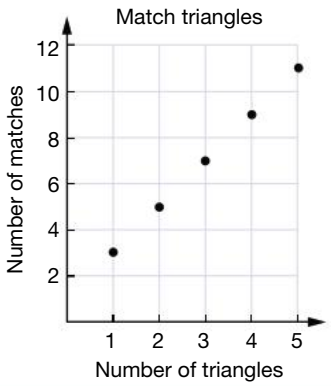
Solution

Make a table or draw a graph.

Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

3, 5, 7, 9, 11, …

The rule to make each new triangle is 'add 2'.



If we can recognise and use patterns, we will better understand the world around us.

Exercise 4:01

1 Using the number and the rule given, write the next *two* numbers in each pattern.

a 5; add 4
b 7; add 11
c 12; add 9
d 26; subtract 9
e 37; subtract 13
f 45; subtract 15
g 3; multiply by 3
h 9; multiply by 4
i 7; multiply by 3
j 100; divide by 2
k 500; divide by 5
l 81; divide by 3
m 3; multiply by 2 and add 1
n 5; multiply by 3 and add 4
o 2; multiply by 4 and subtract 3
p 81; add 1 then divide by 2
q 125; divide by 5 and add 10
r 3; multiply the number by itself

2 For each pattern, write the next number *and the rule* used to find that number.

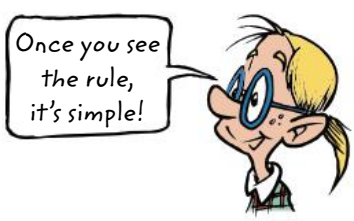
a 3, 7, 11, …
b 5, 11, 17, …
c 1, 7, 13, …
d 20, 15, 10, …
e 37, 30, 23, …
f 17, 15, 13, …
g 2, 4, 8, …
h 1, 3, 9, …
i 1, 10, 100, …
j 80, 40, 20, …
k 81, 27, 9, …
l 64, 16, 4, …

3 For each pattern, write the next three numbers.

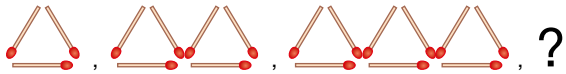
a 10, 25, 40, … , … , …
b 16, 27, 38, … , … , …
c 36, 59, 82, … , … , …
d 76, 62, 48, … , … , …
e 117, 99, 81, … , … , …
f 151, 123, 95, … , … , …
g 8, 16, 32, … , … , …
h 33, 66, 132, … , … , …
i 10, 30, 90, … , … , …
j 1, 3, 6, 10, … , … , …
k 4, 9, 16, 25, … , … , …
l 50, 40, 31, 23, … , … , …

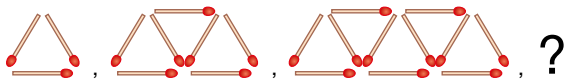
4 Write the missing number in each pattern.

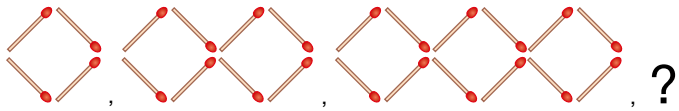
a 4, 9, ☐, 19, 24
b 3, 10, ☐, 24, 31
c 11, 23, ☐, 47, 59
d 24, 31, ☐, 45, 52
e 29, 22, ☐, 8, 1
f 43, 32, ☐, 10
g 2, ☐, 18, 54, 162
h 6, ☐, 24, 48, 96
i 4, 9, 19, ☐, 54, 79
j 5, 8, ☐, 17, 23
k 100, ☐, 25, $12\frac{1}{2}$
l 1000, 200, ☐, 8, 1·6

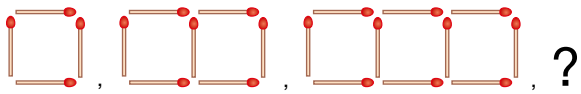


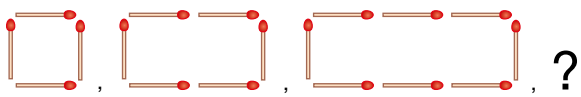
5 These patterns are made with matches. Write the number pattern that each match pattern represents, then write the next number in each pattern.

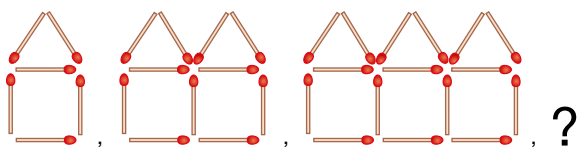
a 

b 

c 

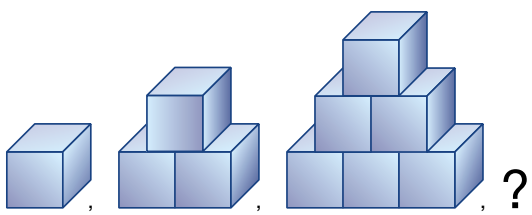
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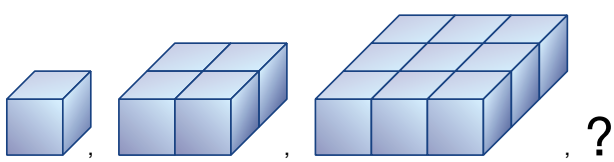
e 

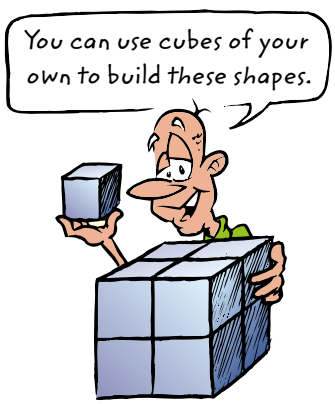
f 



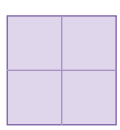
6 Find the number pattern represented by the number of blocks in each set of figures. Also find the next number in each pattern.

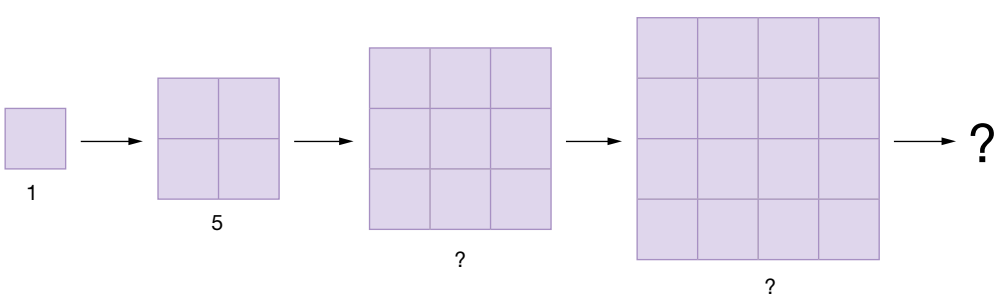
a 

b 



7 Can you see that there are 5 squares in the figure at right? There are four small squares with sides 1 unit long and one bigger square with a side length of 2 units. Find the total number of squares in the third and fourth figures below. What is the next number in the pattern?

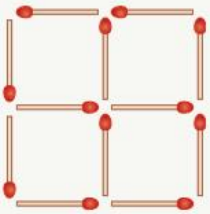




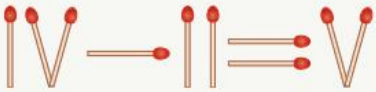
FUN SPOT 4:01 MYSTERIES MIXING MATCHES

Try to solve these tricky match problems.

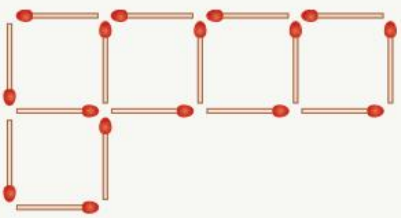
1 Arrange 12 matches to look like this. Then move 3 matches to make only 3 squares.



2 This equation using Roman numerals is not correct. Move just 1 match to make the equation correct.



3 Here are 5 equal squares made from 16 matches. Move 3 matches to make only 4 squares of equal area.



4 Move just 1 match to make this equation correct. (There are two different solutions.)





Patterns can take many forms.

4:02 Pronumerals and variables

PREP QUIZ 4:02

In each case, find the number represented by the box.

1 $\square + 3 = 10$
$\square = \ldots$

2 $3 \times \square = 24$
$\square = \ldots$

3 $\square \div 2 = 11$
$\square = \ldots$

4 $\square - 4 = 10$
$\square = \ldots$

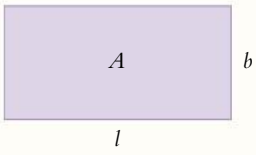
For a rectangle, *A*rea = *l*ength × *b*readth ($A = l \times b$)
Find the area of the rectangle if:

5 $l = 5\,\text{m}$, $b = 3\,\text{m}$

6 $l = 10\,\text{cm}$, $b = 4\,\text{cm}$

7 $l = 8\,\text{cm}$, $b = 2\,\text{cm}$

8 $l = 7\,\text{m}$, $b = 3\,\text{m}$



9 What is the cost of 5 apples, if each apple costs 80c?

10 What is the cost of 10 plants, if each plant costs $6?

Pronumerals are letters or symbols used to represent numbers.



Pronumerals we have used
$\square + \triangle = 7$
$A = l \times b$
Rule: $C = N \times \$1.50$

- Usually, we use a letter as a pronumeral.
- The same letter can stand for different numbers in different questions.

WORKED EXAMPLES

1 **a** $\square - 10 = 4$
$\square = 14$

b $16 - \square = 7$
$\square = 9$

c $6 \times a = 30$
$a = 5$

d $m \div 2 = 10$
$m = 20$

2 If $A = l \times b$, find the value of A when $l = 7$ and $b = 3$.
$A = l \times b$
$= 7 \times 3$
$\therefore A = 21$

3 The cost of N bananas is given by $C = N \times \$1.50$. What is the cost of 5 bananas?
$C = N \times \$1.50$
$= 5 \times \$1.50$
$\therefore$ Cost $= \$7.50$

The symbol $\therefore$ means 'therefore'.

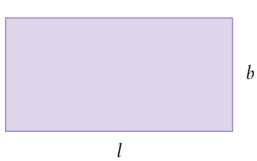
Exercise 4:02

1 Find the number represented by each pronumeral.



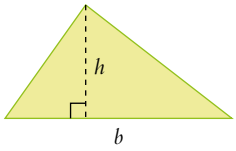
a $\square + 6 = 13$
b $\square - 2 = 16$
c $\square + \square = 8$
d $4 \times \square = 20$
e $\square \div 3 = 7$
f $\square \times 9 = 27$
g $x + 7 = 12$
h $x - 4 = 11$
i $x + x = 22$
j $5 \times y = 40$
k $y \times y = 9$
l $y \times 4 = 8$
m $a + 10 = 90$
n $12 - b = 0$
o $6 \times h = 42$

2 $A = l \times b$ is the formula for the area of a rectangle. Find the area of a rectangle if:



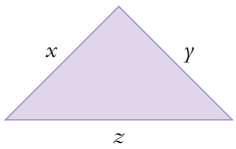
a $l = 10\,\text{m}$ and $b = 6\,\text{m}$
b $l = 5\,\text{m}$ and $b = 3\,\text{m}$
c $l = 11\,\text{cm}$ and $b = 5\,\text{cm}$

3 $\frac{1}{2} \times b \times h$ is the formula for the area of a triangle.
Find the area of a triangle if:



a $b = 10\,\text{m}$ and $h = 4\,\text{m}$
b $b = 8\,\text{m}$ and $h = 9\,\text{m}$
c $b = 12\,\text{cm}$ and $h = 4\,\text{cm}$

4 $P = x + y + z$ gives the perimeter of a triangle.
Find the perimeter if:



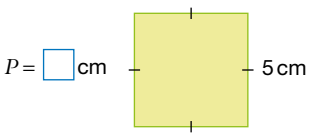
a $x = 4\,\text{m}$, $y = 5\,\text{m}$ and $z = 6\,\text{m}$
b $x = 10\,\text{cm}$, $y = 8\,\text{cm}$ and $z = 7\,\text{cm}$
c $x = 7\,\text{m}$, $y = 10\,\text{m}$ and $z = 8\,\text{m}$
d $x = 4{\cdot}38\,\text{m}$, $y = 6{\cdot}02\,\text{m}$ and $z = 5{\cdot}14\,\text{m}$

5 Find the value of each expression if $a = 8$, $b = 10$, $c = 5$, $d = 4$ and $e = 100$.

a $7 \times b$	b $8 \times d$	c $6 \times e$
d $9 \times a$	e $b \div c$	f $b - d$
g $e - a - b$	h $4 \times e + a$	i $3 \times e + b$
j $e + 3 \times b$	k $a + c \times d$	l $e - (a + b)$
m $7 + 6 \times d$	n $2 \times e + 3 \times b$	o $5 \times e + 4 \times b$

Order of operations
1 ( )
2 × and ÷
3 + and –

6 a For a square, the perimeter is equal to 4 times the length of a side: $P = 4 \times l$. Use this formula to complete the table, to show the value of P as l varies.



l	2	2·5	3	3·5	4	10	12·5	17	26
P									

b The hire charge for a taxi is \$3.30 plus \$2 for each kilometre travelled. The formula is $C = 3{\cdot}3 + 2 \times d$, where C is in dollars and d is in kilometres. Use this formula to complete the table, to show the values of C as d varies.



d	8	10	12	20	33	1·5	4·8	7·4	100
C									

c Apples cost 85 cents each, bananas 180 cents and pears 120 cents. The formula for buying this fruit is $C = (85 \times a) + (180 \times b) + (120 \times p)$, where a, b and p are the numbers of each fruit and C is the cost in cents. Use the formula to complete the table.

a	1	2	10	5	11
b	1	2	3	8	15
p	1	2	2	3	9
C					

Example
What is the cost of 7 apples, 4 bananas and 8 pears?
$C = (85 \times a) + (180 \times b) + (120 \times p)$
$= (85 \times 7) + (180 \times 4) + (120 \times 8)$
$= 2275$
The cost is \$22.75.

4:03 Using variables

PREP QUIZ 4:03

Write the next two numbers in each pattern.

1 5, 11, 17, … , …
2 30, 24, 18, … , …
3 3, 6, 12, … , …
4 80, 40, 20, … , …
5 4, 9, 16, … , …
6 64, 32, 16, … , …

In each table of values, what is the missing number?

7

A	1	2	3	4
B	3	6		12

8

A	2	4	6	8
B	5		9	11

9

A	1	3	5	7
B	0	2		6

10 Multiply 7 by 4 and subtract 8.

We can make the writing of number pattern rules much easier by using *symbols* to represent the numbers in the pattern. These symbols are called **variables** (or 'pronumerals'). The most commonly used variables are letters of the alphabet.

A *variable* (or 'pronumeral') takes the place of a *numeral*.

WORKED EXAMPLE

If apples cost 50 cents each, then to find the cost of a number of apples we follow the rule:

cost of apples = number of apples times 50 cents.

Now we let C represent the cost of apples in cents and n represent the number of apples. C and n are called variables. The rule can be simply written as:

$C = n \times 50$

A table of values will look like this:

n	1	2	3	4	5
C	50	100	150	200	250



$C = n \times 50$ can also be written as $C = 50 \times n$.

Exercise 4:03

P Foundation worksheet 4:03 Using variables
Challenge worksheet 4:03 Describing number patterns

1 Flowers cost \$2 each. The rule for the cost of flowers is $C = \$2 \times F$, where F is the number of flowers. Find the cost of:
a 2 flowers **b** 3 flowers **c** 10 flowers.

2 Each cow has 4 legs. The rule for the number of legs (L) is $L = 4 \times C$, where C is the numbers of cows. Find the number of legs on:

a 6 cows **b** 8 cows **c** 10 cows.

3 This table links the number of bananas to their cost.

Number of bananas (N)	1	2	3	4
Cost in cents (C)	80	160	240	320

$C = N \times \ldots$ cents



a What is the rule that links the number of bananas to their cost?

b What is the cost of 9 bananas?

4 **a** Complete the table of values for this pattern made by joining square tiles.

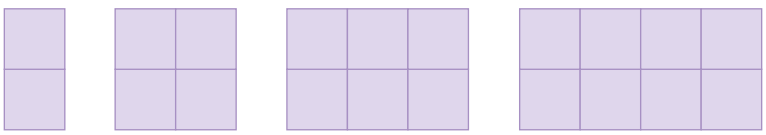


Figure 1 Figure 2 Figure 3 Figure 4

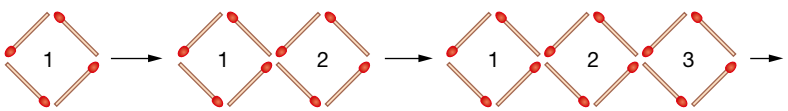
Figure number (F)	1	2	3	4
Number of tiles (N)				

$N = F \times \ldots$

b What rule links the number of square tiles to the number of the figure?

c How many tiles would be in the 15th figure?

5 Matches have been used to make the following figures.



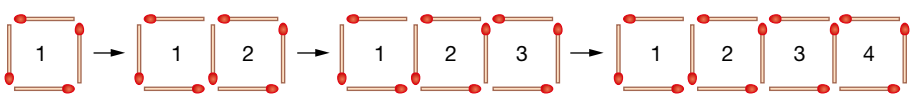
$M = S \times \ldots$

a Find a rule that links the number of matches (M) to the number of squares (S) in each figure.

b Complete this table.

Number of squares (S)	1	2	3	4	5	6	7	8
Number of matches (M)								

6



$M = \ldots + \ldots \times \ldots$

a How many matches are added each time to make the next figure?

b Find a rule that links the number of matches (M) to the number of squares (S) in each figure.

c Complete a table of values showing the number of matches for figures up to 8 squares.

7 Complete each table of values using the rule shown.

a $y = x + 3$

x	0	1	2	3
y				

b $y = 3 \times a$

a	1	2	3	4
y				

c $q = p + 7$

p	1	3	5	7
q				

d

k	2	4	6	8
$k \times 5$				

e

n	8	9	10	11
$n - 5$				

f

t	1	2	3	4
$10 - t$				

8 In the following tables, each second number is obtained from the first number by following a rule. For example:

First number (A)	3	4	5	6
Second number (B)	10	13	16	19

Rule:
'Second' = 3 times 'First' plus 1
or $B = 3 \times A + 1$

Find the rule for each table below from this list of rules.

Rules:

$B = 2 \times A + 1$ $B = A - 2$ $B = 5 \times A$ $B = A + 3$
$B = 3 \times A - 1$ $B = 2 \times A$ $B = A + 6$ $B = 2 \times A + 1$

a

First number (A)	1	2	3	4
Second number (B)	5	10	15	20

b

First number (A)	1	2	3	4
Second number (B)	7	8	9	10

c

First number (A)	2	4	6	8
Second number (B)	5	7	9	11

d

First number (A)	1	3	5	7
Second number (B)	2	6	10	14

e

First number (A)	1	2	3	4
Second number (B)	3	5	7	9

f

First number (A)	1	2	3	4
Second number (B)	2	5	8	11

g

First number (A)	2	3	4	5
Second number (B)	0	1	2	3

h

First number (A)	5	6	7	8
Second number (B)	11	13	15	17

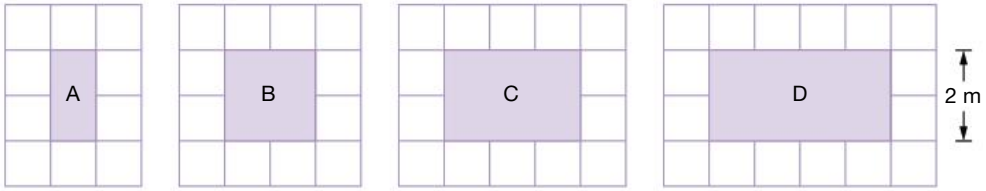
9 Sue-Lin was at the circus. During the parade, she noticed that three unicycles (cycles with only one wheel) were always present, but the number of bicycles varied. She made the following table, showing the total number of wheels present as the number of bicycles changed:



Number of bicycles (B)	1	2	3	4	5	6
Number of wheels (W)	5	7				

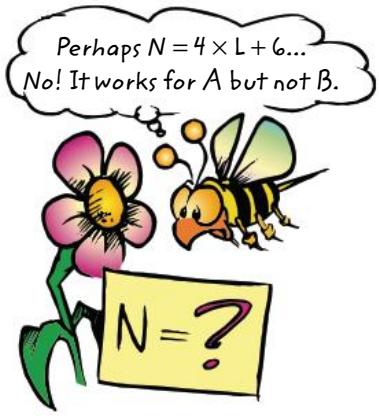
a Complete the table.
b Find a rule that links the total number of wheels to the number of bicycles.
c Can the total number of wheels be 36? Why or why not?

10



These diagrams show gardens that are each 2 m wide. Each garden is surrounded by a path made of one-metre squares of concrete. The table shows the number of concrete squares needed for different garden lengths:

Length of garden (L)	1	2	3	4
Number of squares (N)	10	12	14	16



For each extra metre of garden length, we must 'add two squares' of concrete path.

a Find a rule that links the number of squares to the garden length.

b How many concrete squares are needed for a garden of length:
- **i** 5 metres
- **ii** 30 metres?

c Does the rule $N = 2 \times (L + 1) + 6$ describe this situation?

4:04 Algebraic abbreviations

PREP QUIZ 4:04

If x is replaced by the numeral 4, what is the value of each expression?

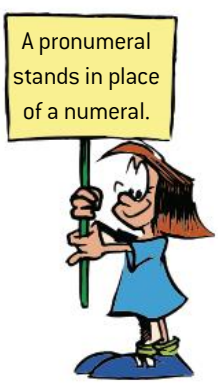
1 $x + 5$ **2** $x - 4$ **3** $3 \times x$

4 $x \times x$ **5** $x \div 2$ **6** $5 \times (x + 3)$

Find the value of C, if $C = 5 \times n + 2$ and n is replaced by the numeral:

7 3 **8** 1 **9** 0 **10** 13

Algebra provides a short way to write ideas in mathematics. Pronumerals take the place of numerals, so they behave like numerals when we perform operations with them.



When writing an algebraic expression, we try to shorten and simplify the expression as much as possible:

- $3 \times a$ is written as $3a$ and $a \times b$ is written as ab
- $x \div 5$ is written as $\frac{x}{5}$ and $a \div b$ is written as $\frac{a}{b}$

- $3 + 3 + 3 + 3$ is the same as 4 lots of 3 or 4 times 3, so $y + y + y + y$ is 4 lots of y or $4y$.
- $2 \times m = 2m$, $y \times 6 = 6 \times y = 6y$, $2a \div 3 = \frac{2a}{3}$, $5 \times a \times b = 5ab$

Note: $n \times 5$ is always written as $5n$, never as $n5$.

When a numeral is multiplied by a pronumeral, the numeral should be written first.

$m^2 = m \times m$ in the same way that $7^2 = 7 \times 7$.
m^2 is read 'm squared'.

The product of a and b is ab.
$5(y + 7) = 5 \times (y + 7)$

WORKED EXAMPLES

1 Rewrite these expressions without multiplication or division signs.

a $5 \times k$ **b** $a \div 7$ **c** $a \times b \times a$ **d** $3 \times x \times y$

e $(a + 3) \div 2$ **f** $5 \times b \div 8$ **g** $6 \times (c + 7)$ **h** $(c \times d) \div (e \times f)$

2 Rewrite these expressions using multiplication and division signs where possible.

a $3a + b$ **b** $5c - \frac{d}{2}$ **c** $3(x + 2)$ **d** $\frac{a+7}{3}$

3 Simplify:

a $3 \times 2m$ **b** $5a \times 3$ **c** $4 \times 2a \times b$ **d** $6 \times 3c \times 10$

Solutions

1 a $5 \times k$
$= 5k$

b $a \div 7$
$= \frac{a}{7}$

c $a \times b \times a$
$= a^2b$

d $3 \times x \times y$
$= 3xy$

e $(a + 3) \div 2$
$= \frac{a+3}{2}$

f $5 \times b \div 8$
$= \frac{5b}{8}$

g $6 \times (c + 7)$
$= 6(c + 7)$

h $(c \times d) \div (e \times f)$
$= \frac{cd}{ef}$

2 a $3a + b$
$= 3 \times a + b$

b $5c - \frac{d}{2}$
$= 5 \times c - d \div 2$

c $3(x + 2)$
$= 3 \times (x + 2)$

d $\frac{a+7}{3}$
$= (a + 7) \div 3$

Here we must use grouping symbols, because the whole numerator is divided by 3.



3 a $3 \times 2m$
$= 3 \times 2 \times m$
$= 6m$

b $5a \times 3$
$= 3 \times 5a$
$= 3 \times 5 \times a$
$= 15a$

c $4 \times 2a \times b$
$= 4 \times 2 \times a \times b$
$= 8ab$

d $6 \times 3c \times 10$
$= 6 \times 10 \times 3 \times c$
$= 180c$

Note: $a \div bc = a \div (b \times c)$ or $\frac{a}{bc}$

$a \div b \times c = \frac{a}{b} \times c$ or $\frac{a \times c}{b}$

Exercise 4:04

1 **a** How can 'three lots of 5' be written in a shorter way?
b How can 'three lots of n' be written in a shorter way?
c How can $3 \times n$ be written in a shorter way?
d Why can't 3×5 be written in the same way?
e What is another way of writing $d \times t$?
f Rewrite $5ab$, showing the multiplication signs.
g Write $q \div 3$ in another way.
h What does $\frac{m}{2}$ mean?
i What is the difference between $34y$ and $3 \times 4 \times y$?



2 Write the following without multiplication signs.

a $7 \times a$	**b** $10 \times x$	**c** $2 \times p$	**d** $8 \times q$
e $9 \times b$	**f** $6 \times m$	**g** $3 \times r$	**h** $12 \times c$
i $m \times 9$	**j** $n \times 5$	**k** $y \times 4$	**l** $t \times 12$
m $1 \times a$	**n** $1 \times x$		
o $n \times 1$	**p** $b \times 1$		
q $1 \times y$	**r** $1 \times m$		
s $5 \times (a + 2)$	**t** $6 \times (x + 7)$		
u $2 \times (3x - 2)$	**v** $5 \times (m - n)$		
w $5 + 2 \times q$	**x** $1 \times n - 10$		
y $4 \times a + 5 \times b$	**z** $q \times 7 - p \times 6$		

If you multiply something by 1, it does not change:

$1 \times n = n \times 1 = n$
$1 \times m = 1m = m$

3 Write these divisions as fractions.

a $x \div 2$	**b** $m \div 4$	**c** $b \div 3$	**d** $n \div 5$
e $a \div 1$	**f** $y \div 10$	**g** $t \div 8$	**h** $k \div 100$
i $2b \div 3$	**j** $3x \div 5$	**k** $5m \div 6$	**l** $3a \div 4$
m $a \div$ b	**n** $6 \div n$	**o** $2a \div x$	**p** $10 \div a$
q $(a + 1) \div 4$	**r** $(8 - x) \div 5$	**s** $7a \div 8b$	**t** $4x \div 3y$

4 Rewrite each expression in its simplest form.

a $5 \times a \times b$	**b** $4 \times m \times n$	**c** $a \times b \times c$	**d** $3 \times a \times a$
e $x \times 3 \times y$	**f** a $\times b \times 10$	**g** $6 \times r \times s$	**h** $m \times 1 \times n$
i $3 \times 8 \times a$	**j** $6 \times 5 \times b$	**k** $4 \times 7 \times x$	**l** $9 \times 9 \times p$
m $5 \times 3a$	**n** $4 \times 2b$	**o** $3 \times 10y$	**p** $2 \times 6m$
q $6 \times m \times 5$	**r** $p \times 3 \times 2$	**s** $4a \times 7$	**t** $8b \times 5$
u $7 \times 3a$	**v** $10 \times 8x$	**w** $10 \times 6b$	**x** $3 \times 3y$
y $4 \times a + 5 \times b$	**z** $q \times 7 - p \times 6$		

5 Rewrite these in a shortened form.

a $2 \times p \times (q - 1)$	**b** $5 \times 3 \times (a - 3)$	**c** a $\times b \times (c - 7)$
d $4 \times (3 \times a + 1)$	**e** $6 \times (1 + 2 \times x)$	**f** $3(5 \times y - 2 \times z)$
g $(x + 2) \times (x + 3)$	**h** $(a - 3) \times (a + 3)$	**i** $(4p + 1) \times (3q - 1)$

6 Show the meaning of each expression by including multiplication signs.

a $5x$	**b** $7a$	**c** $12y$	**d** $14p$
e $2p + 7$	**f** $9x - 5$	**g** $11 - 7m$	**h** $1 + 10a$
i $7x + 3y$	**j** $10a - 9b$	**k** $7a + 3p$	**l** $11m - 9n$
m $ab + 3$	**n** $k - 3l$	**o** abc	**p** $8xy$
q $6(a + 3)$	**r** $4a(6 - a)$	**s** $3(2x + 1)$	**t** $(a + 7)(a - 7)$
u x^2	**v** $a^2 + c^2$	**w** $2y^2$	**x** $m^2 - 3$

7 Rewrite each expression showing all multiplication and division signs. Remember to use grouping symbols if they are needed.

a $\frac{a}{3}$	**b** $\frac{5}{k}$	**c** $\frac{m}{7}$	**d** $\frac{n}{a}$	**e** $\frac{2x}{5}$	**f** $\frac{3y}{2}$
g $\frac{5}{8m}$	**h** $\frac{6}{5p}$	**i** $\frac{5a}{6b}$	**j** $\frac{11m}{10n}$	**k** $\frac{a+7}{3}$	**l** $\frac{m-1}{2}$
m $\frac{6+a}{a-5}$	**n** $\frac{9+x}{7-x}$	**o** $\frac{3(x+1)}{5}$	**p** $\frac{a+6}{3a}$		

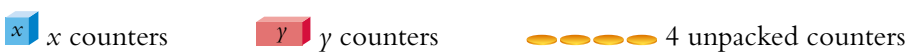
8 If each packet holds n cards, how many cards are there altogether?



4:05 Making sense of algebra

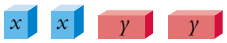
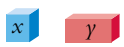
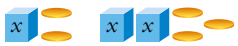
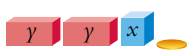
Pronumerals in algebra can stand for numbers that we need to find, or for variables that we use to explain rules.

Consider the following:
There are two different ways of packaging counters. We don't know how many counters are in each type of package, so we let x represent the number of counters in one type of box and y represent the number in the other type of box.



We have many of each box, along with a number of unpacked counters. x might be 12 and y might be 20; we have not yet opened the boxes.

These models will help you understand the algebra used:

Model	Number	Model	Number
	$x + 3$		$x + x + y + y$ $= 2x + 2y$
	$x + x + x$ $= 3x$		$(x + x) + (y + y)$ $= 2x + 2y$
	$x + x + 1$ $= 2x + 1$		$(x + y) + (x + y)$ $= 2(x + y)$ $= 2x + 2y$
	$x + y$		$(x + 2) + (2x + 3)$ $= 3x + 5$
	$(x + 1) + (x + 1)$ $= 2x + 2$		$(x + 2x) + (2 + 3)$ $= 3x + 5$
	$2 + x + x + x$ $= 2 + 3x$		$3x + 5$
	$y + y + x + 1$ $= 2y + x + 1$		$(x + y) \div 2$ $= \frac{x + y}{2}$ $= \frac{x}{2} + \frac{y}{2}$

From the examples above, it can be seen that:

- $x + x + y + y = (x + x) + (y + y)$
  $= (x + y) + (x + y)$
  $= 2(x + y)$
  $= 2x + 2y$
- $(x + 2) + (2x + 3) = (x + 2x) + (2 + 3)$
  $= 3x + 5$

Exercise 4:05

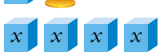
P Foundation worksheet 4:05
Making sense of algebra

1 Use pronumerals to write the number of counters.

a 

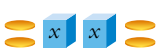
b 

c 

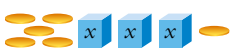
d 

e 

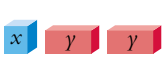
f 

g 

h 

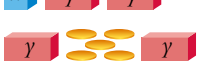
i 

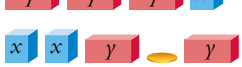
j 

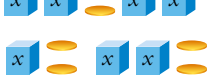
k 

l 

m 

n 

o 

p 

q 

r 

2 Use pronumerals to write the number of counters as simply as you can.

a 

b 

c 

d 

e 

f 

g 

h 

i 

j 

k 

3 Write the number of counters in simplest form.

a 

b 

c 

d 

e 

f 

4 If there are n wrap-ups in each packet, how many wrap-ups are in each picture?

a 

b 

c 

4:06 Substitution

PREP QUIZ 4:06

Rewrite without multiplication or division signs:

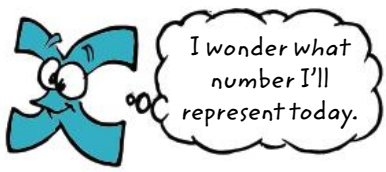
1 $4 \times y$ **2** $m \times 2$ **3** $6 \times a \times b$ **4** $9 \times n + 2$

5 $9 \times (n + 2)$ **6** $p \div 7$ **7** $a \times b \div c$

Show the meaning of the following by using multiplication signs.

8 $3pq$ **9** y^2 **10** $6a + 4b$

An algebraic expression will take on different values as the replacement for the variable changes. Look at the following examples.



If we have 3 boxes of counters (x counters in each), then we can find the total number of counters if we are told that $x = 12$:



$x + x + x = 3x = 3 \times x = 3 \times 12 = 36$

$\therefore$ There are 36 counters if $x = 12$.

WORKED EXAMPLES

1 If x is replaced by the numeral 3, find the value of each expression.

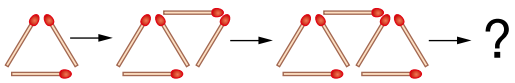
a $6x$ **b** $2x + 5$ **c** $5x^2$ **d** $x(4x - 7)$

2 If $a = 2$ and $b = 5$, evaluate:

a ab **b** $3a + 7b$ **c** $\dfrac{10a}{b}$ **d** $(b + a)(b - a)$

Evaluate means 'find the value of'.

3 The expression $2n + 1$ gives the number of matches needed to make n triangles in the pattern. How many matches are needed to make 25 triangles?



4 Complete each table using the formula given.

a $b = 3t$

t	1	2	3	4
b				

b $m = 5 + 4n$

n	2	4	6	8
m				

c $y = 10 - 2x$

x	0	1	2	3
y				

Solutions

1 If we substitute 3 for x, each expression becomes:

a $6x \to 6 \times 3$
$= 18$

b $2x + 5 \to 2 \times 3 + 5$
$= 11$

c $5x^2 \to 5 \times 3^2$
$= 5 \times 9$
$= 45$

d $x(4x - 7) \to 3 \times (4 \times 3 - 7)$
$= 3 \times 5$
$= 15$

Note: When pronumeral values are substituted, multiplication signs must be written.

2 If $a = 2$ and $b = 5$, then:

a $ab = 2 \times 5$
$= 10$

b $3a + 7b = 3 \times 2 + 7 \times 5$
$= 6 + 35$
$= 41$

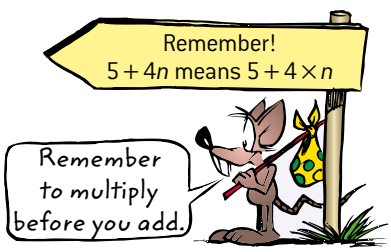
c $\dfrac{10a}{b} = \dfrac{10 \times 2}{5}$
$= 20 \div 5$
$= 4$

d $(b + a)(b - a) = (5 + 2)(5 - 2)$
$= 7 \times 3$
$= 21$

3 In the expression $2n + 1$, n represents the number of triangles. To find the number of matches in 25 triangles, substitute 25 for n:

$2n + 1 = 2 \times 25 + 1$
$= 51$

$\therefore$ number of matches needed = 51



4 To complete a table of values, we find the value of the second variable by substituting the top numbers one at a time into the rule. Check to see that you can get the given answer.

a $b = 3t$

t	1	2	3	4
b	3	6	9	12

b $m = 5 + 4n$

n	2	4	6	8
m	13	21	29	37

c $y = 10 - 2x$

x	0	1	2	3
y	10	8	6	4

Exercise 4:06

P Foundation worksheet 4:06 Substitution

1 Find the value of each expression if n is replaced by 4.

a $2n$ **b** $7n$ **c** $3n + 1$ **d** n^2

e $n^2 + 3$ **f** $n^2 - n$ **g** $5 - n$ **h** $7 + 2n$

i $3(n + 1)$ **j** $\frac{n}{2}$ **k** $\frac{6n}{3}$ **l** $\frac{20}{n}$

2 If $p = 2$ and $q = 5$, find the value of:

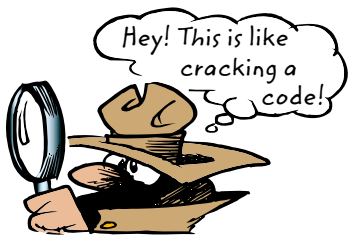
a pq **b** $p + q$ **c** $2p + 3$ **d** $3q - 2$

e $2p + q$ **f** $p + 3q$ **g** $3p + 2q$ **h** $4p - q$

i $3pq$ **j** $\frac{10p}{q}$ **k** $\frac{5+q}{p}$ **l** $\frac{5p}{2q}$

3 Evaluate each expression, replacing the pronumerals with the values shown in this table.

a	b	c	d	e	f	g	h	i	j
0	1	2	3	4	5	6	7	8	9



a ef **b** bj **c** di

d $3a + g$ **e** $4h + 2c$ **f** $9d - 2f$

g beh **h** ace **i** dij **j** d^2f

k $c^2 + d^2$ **l** $5h^2$ **m** $2c + 3d - 5b$ **n** $ac + bd + fh$

o $3(b + g)$ **p** $f(h + 3)$ **q** $j(2b - c)$ **r** $(b + d)(f - c)$

s $\frac{c+e}{g}$ **t** $\frac{i}{b+d}$ **u** $\frac{h+j}{2e}$ **v** $\frac{di}{eg}$

4 Evaluate each expression using the values given.

a $5a$ given that $a = \ldots$

i 3 **ii** 0 **iii** 11 **iv** $\frac{1}{2}$

b $2x + 7$ given that $x = \ldots$

i 5 **ii** 1 **iii** 15 **iv** 0·5

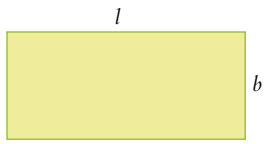
c $20 - 3n$ given that $n = \ldots$

i 0 **ii** 6 **iii** 1·5 **iv** $\frac{1}{3}$

d $5(2y + 1)$ given that $y = \ldots$

i 3 **ii** 10 **iii** 0 **iv** 2·5

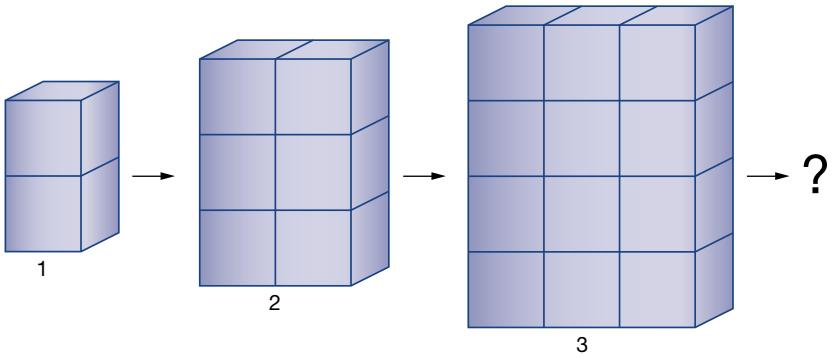
5 The perimeter P of a rectangle is given by the formula $P = 2l + 2b$, where l is the length and b is the breadth of the rectangle. Find the value of P if:



a $l = 5, b = 3$ **b** $l = 7, b = 6$

c $l = 21, b = 15$ **d** $l = 1{\cdot}6, b = 1{\cdot}2$

6 The total number of cubes in each figure is given by the expression $n(n + 1)$, where n is the number of cubes along the bottom of each stack.



How many cubes would there be if:

a $n = 8$

b $n = 12$

c $n = 20$

d $n = 100$?

7 If apples cost 15 cents each and pears cost 20 cents each, then the cost of buying n apples and m pears would be given by the expression $15n + 20m$.

What is the total cost of buying:

a 10 apples and 5 pears **b** 5 apples and 10 pears **c** 25 of each?

8 Complete each table using the formula given.

a $b = 4t$

t	2	3	4	5
b				

b $y = 3x$

x	0	1	2	3
y				

c $n = 7m$

m	1	3	5	7
n				

d $b = 3t + 1$

t	0	1	2	3
b				

e $y = 2x + 3$

x	2	4	6	8
y				

f $q = 5p + 2$

p	1	4	5	10
q				

g $b = 5a - 2$

a	1	2	3	4
b				

h $w = 20 - 2t$

t	0	2	4	6
w				

i $g = 3h - 7$

h	3	4	5	6
g				

j $y = \frac{x}{2}$

x	2	4	6	8
y				

k $b = \frac{a}{5}$

a	0	5	10	20
b				

l $n = m^2$

m	0	1	2	3
n				

9 Write each of the following as an equation. Let the first number be x and the second number y.

a Add 5 to the first number to get the second number.

b If you subtract 23 from the first number, you get the second number.

c Doubling the first number gives the second number.

d Dividing the first number by 7 gives the second number.

10 Match each table of values with the correct rule or formula.

a

x	1	2	3	4
y	6	12	18	24

b

x	0	2	4	6
y	0	6	12	18

c

x	3	4	5	6
y	10	11	12	13

d

x	5	7	9	11
y	1	3	5	7

e

x	1	2	3	4
y	3	5	7	9

f

x	0	2	4	6
y	2	8	14	20

g

x	5	6	7	8
y	6	8	10	12

h

x	9	12	15	18
y	3	4	5	6

A $y = x + 7$
B $y = 2x + 1$
C $y = 6x$
D $y = 2x - 4$
E $y = \frac{x}{3}$
F $y = 3x + 2$
G $y = 3x$
H $y = x - 4$

Write these equations in everyday language, as in Question 9.

4:07 Simplifying algebraic expressions

PREP QUIZ 4:07

Rewrite each expression without multiplication signs.

1 $3 \times a$ **2** $m \times n$ **3** $6 \times p \times 2 \times q$ **4** $3 \times (5 + a)$ **5** $2 \times a + 5 \times b$

Evaluate each expression if $x = 3$ and $y = 5$.

6 $4x$ **7** xy **8** $2y + x$ **9** $2x - y$ **10** $x(y + 3)$

An algebraic expression is made up of one or more **terms** joined together by plus or minus symbols.

Expression	$3x + 5$	$5x - 2t + 7$	$a^2 - 2a + 4x - 5$
Number of terms	2	3	4

Like terms have identical pronumeral parts, e.g. $2x$ and $3x$, $5ax$ and $7ax$, $10n$ and $\frac{n}{2}$.

If the pronumeral parts are different, they are not like terms. Instead they are called **unlike terms**, e.g. $5a$ and $2b$, $2xy$ and $3xz$, $7m^2$ and $7m$.

Discussion

1 Let n represent the number of marbles in a packet. If Jane buys 2 packets and Alan buys 5 packets, then:

The number of marbles Jane has $= 2 \times n$, or $2n$.

The number of marbles Alan has $= 5 \times n$, or $5n$.

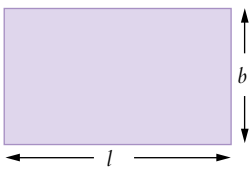
Now the number of marbles altogether is given by the expression $2n + 5n$.

We also know that the number of marbles bought altogether was $7n$, because 7 packets were bought:

$\therefore\ 2n + 5n = 7n$

(That is, 2 lots of n marbles plus 5 lots of n marbles must equal 7 lots of n marbles.)

2



The area of a rectangle is equal to its length multiplied by its breadth.

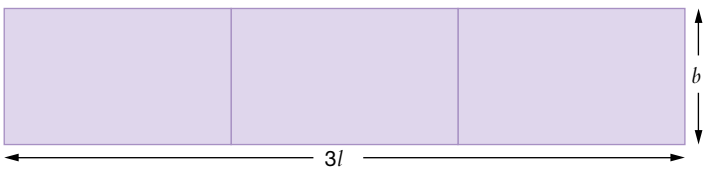
$\therefore$ Area $= l \times b$ or lb

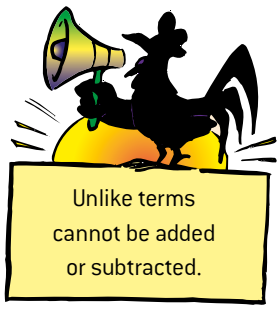
If we join 3 rectangles, the total area is:

$lb + lb + lb$

But the area can also be seen to be $3lb$:

$\therefore\ lb + lb + lb = 3lb$





Only like terms can be added or subtracted.

WORKED EXAMPLES

1 $7a + 5a = 12a$

2 $20pq - 11pq = 9pq$

3 $8x + x = 9x$

4 $5m + 7m - 10m = 2m$

5 $5x + 2y + 7y = 5x + 9y$

6 $7p + 2q + 3p + q = 10p + 3q$

7 $6a + 7b - 2a + 5b$

$= \begin{array}{|c|c|c|c|} \hline 6a & -2a & +7b & +5b \\ \hline \end{array}$

$= 4a + 12b$

Important!
The plus or minus sign always belongs to the term after it!

8 $7a^2 - 4a + 2a^2$

$= 9a^2 - 4a$



Note: a^2 and a are not like terms.

Exercise 4:07

P Foundation worksheet 4:07 Simplifying algebraic expressions

1 How many terms in each expression?

a $7a + 2a$ **b** $5x$ **c** $7p + 9q$

d $7xy$ **e** $9 - m + n$ **f** 2

g $ab + 3a - 4b$ **h** $2x - 5y - x + y$ **i** $9 - 2x + x^2$

j $9x - 4y + 7z - 8w + t$ **k** $6a + 4b - 2c + 11d - e$ **l** $7 + x - x^2 + y - y^2$

m $25a + 9bc - 8def$

2 Write the *like terms* in each group.

a $2x$ $3y$ $4x$

b a $3b$ $7b$

c $4m$ $7n$ $8m$ $6p$

d $2a$ $2b$ $3a$ $3c$

e ab a^2 $3a$ $2a^2$

f xy yz $3yx$

g $9x$ $7y$ x $5b$ $3x$

h $2mn$ $7m$ $8n$ $6mn$ 9

i a $2a$ a^2 $3ab$ $7a$

j ab abc a ba bc ac

k a^2x $2ax$ $3ax^2$ $4a^2x$

l $9m$ $7mn$ $8m^2$ n^2 $5m^2$

Notice:
$ab = ba$
because
$a \times b = b \times a$

3 Simplify each expression.

a $5a + 2a$

b $3x + 4x$

c $3m + 10m$

d $8y + 2y$

e $6n + n$

f $b + 7b$

g $a + a$

h $7x + x$

i $9a - 2a$

j $8m - 5m$

k $10k - 2k$

l $19b - 12b$

m $8q - q$

n $11x - x$

o $a - a$

p $15m - 14m$

q $5x + 2x + 3x$

r $7a + 2a + 4a$

s $9m + 4m + m$

t $6x + x + 5x$

u $8m + 2m - 9m$

v $10y + 4y - 6y$

w $5p - 4p + 2p$

x $8a - a + 2a$

y $4ab + 7ab - 2ab$

z $9xy + 4yx$

Note:
$1x = x$

4 Simplify each expression by adding or subtracting like terms only.

a $7a + 2a + 3b$

b $8p + 2p - 7q$

c $9a^2 + 2a^2 + 5a$

d $5ab + 2ab - 7a$

e $7 + 2a - a$

f $2a + 3b + 5a + 2b$

g $6p + 2q - 4p + 4q$

h $9m^2 + 7m^2 + 5m - 3m$

i $7a^2 + 4a - 2a^2 + 7$

j $10p + 3q + p - 7p$

k $3x^2 + 2x - 2x^2 + 3x + x^2$

l $9x + 2x + 7y$

m $10k + 5l - 2k$

n $4x^2 + 5x + 3x^2$

o $11x^2 + 7xy - 2xy$

p $6x + 9 - 5x$

q $6x + 2y + 4x + y$

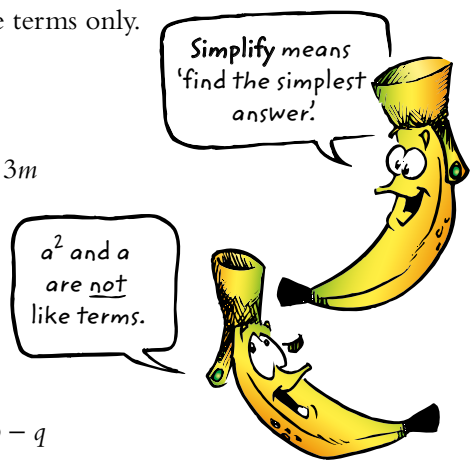
r $8k + 10l - 7l + 2k$

s $6ab + 3bc - 4ab + bc$

t $10k + 5l - 2m + 3l$

u $15a + 3a + 6b + 2a$

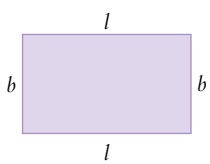
v $10p + 7q - 2q + 3p - q$



5 **a** If there are n chocolates in each box, and Jules buys 4 boxes while Micah buys 7 boxes, how many chocolates are bought altogether?

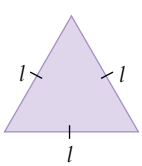
b There are b bananas in a kilogram. Jeff buys 5 kg, but they are so good that he goes back to buy 3 kg more. How many bananas does Jeff buy altogether?

6 **a** The perimeter of this rectangle can be written as $l + b + l + b$. How can this be written in a shorter form?

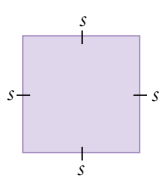


b Write the simplest expression for the perimeter of each figure:

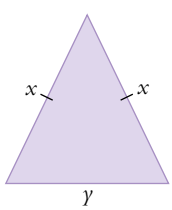
i



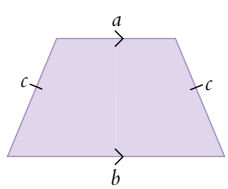
ii



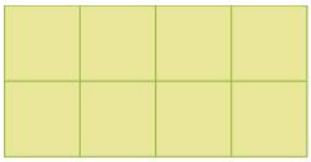
iii



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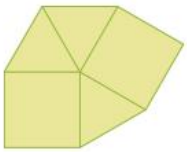


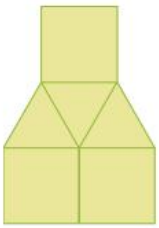
7 If the area of each square tile is s cm^2 and the area of each triangular tile is t cm^2, find the total area of each pattern.

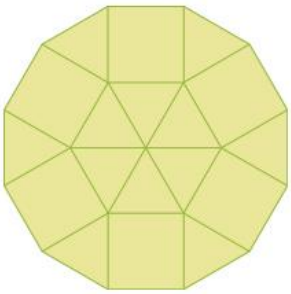
a 

b 

c 

d 

e 

f 

g If a pattern contains 10 squares and 12 triangles, what is its area?

h If a pattern contains n squares and m triangles, what is its area?

FUN SPOT 4:07 LET'S IMPROVE OUR ALGEBRA SKILLS

Mr Breakwell, the principal, visited my Year 7 class and gave us a series of instructions to see how many students could get the right answer.

'Begin with x,' he said. 'Multiply it by 8. Add 4 to your answer. Subtract 2 from your last answer. Divide the result by 2, then find the value of this latest answer if x is equal to 7.' Stuart, who was good at maths, did it this way:

x	$\times 8$	$8x$	$+ 4$	$8x + 4$	$- 2$	$8x + 2$	$\div 2$	$4x + 1$	$x = 7$	$4(7) + 1 = 29$

Activity

See if you can do as well as Stuart.

1 Begin with x, multiply by 6, add 20, subtract 6, divide by 2, let x equal 8.

2 Begin with y, multiply by 10, add 30, multiply by 2, subtract 20, divide by 10, let y equal 5.

3 Begin with m, multiply by 3, add 24, multiply by 2, add 6, divide by 3, let m equal 2.

4 Begin with $3d$, add $5d$, add 8, multiply by 3, subtract 24, let d equal 4.

5 Begin with $10k$, subtract k, multiply by 4, add 18, divide by 2, let k equal 2.



4:08 The laws of arithmetic

PREP QUIZ 4:08

1 134×0
2 67×1
3 $86 + 0$
4 $8 + 13 = 13 + \square$, $\square = \ldots$
5 $9 \times 12 = 12 \times \square$, $\square = \ldots$
6 $(6 + 8) + 11 = 6 + (8 + \square)$, $\square = \ldots$
7 $(8 \times 5) \times 20 = 8 \times (5 \times \square)$, $\square = \ldots$
8 $\square \times 9 = 9 \times 12$, $\square = \ldots$
9 $3 \times 47 = (3 \times 40) + (3 \times \square)$, $\square = \ldots$
10 $9 \times 28 = (9 \times 20) + (9 \times \square)$, $\square = \ldots$

Prep quiz 4:08 should remind you of the number properties we have met so far.

- These number properties also apply to the variables that represent numbers.
- Variables allow us to show the laws of arithmetic more clearly:
  $b + c = c + b$ means that reversing the order of numbers in addition does not change the answer.
  Because b and c can stand for any two numbers, we are stating a rule for all numbers.
- The abbreviations of algebra allow us to express some laws even more simply:
  $1b = b$ $\quad bc = cb$ $\quad a(b + c) = ab + ac$ $\quad 3bc = 3cb$



The number laws

Number law	Using numbers	Using variables
Adding and multiplying with zero	$6 + 0 = 6$ $6 \times 0 = 0$	$b + 0 = b$ $b \times 0 = 0$
Multiplying by 1	$6 \times 1 = 6$	$b \times 1 = b$
Commutative properties for addition and multiplication	$6 + 7 = 7 + 6$ $6 \times 7 = 7 \times 6$	$b + c = c + b$ $b \times c = c \times b$
The commutative properties mean that we may reverse the order when adding or multiplying.		
Associative properties for addition and multiplication	$(6 + 7) + 8 = 6 + (7 + 8)$ $(6 \times 7) \times 8 = 6 \times (7 \times 8)$	$(b + c) + d = b + (c + d)$ $(b \times c) \times d = b \times (c \times d)$
The associative properties mean that we can add numbers in any order. We can also multiply numbers in any order.		
Distributive property of multiplication over addition or subtraction	$6 \times (7 + 8) = (6 \times 7) + (6 \times 8)$ or $6 \times (7 - 8) = (6 \times 7) - (6 \times 8)$	$a \times (b + c) = (a \times b) + (a \times c)$ or $a \times (b - c) = (a \times b) - (a \times c)$
The distributive property can be written as $a(b + c) = ab + ac$ or $a(b - c) = ab - ac$.		

WORKED EXAMPLES

1 We packed 12 baskets. In each basket there were 10 apples and 3 oranges. How many pieces of fruit did we pack?

Number of pieces of fruit
$= 12 \times (10 + 3)$
$= 12 \times 10 + 12 \times 3$
$= 120 + 36$
$= 156$

2 Nine full buses carried spectators to see the NSW tennis finals. Each bus holds 100 people including the driver. How many spectators were on the buses?

Number of spectators
$= 9$ lots of $(100 - 1)$ i.e. $9 \times (99)$
$= 9$ lots of $100 - 9$ lots of 1
$= 900 - 9$
$= 891$

Note: $9 \times (100 - 1) = 9 \times 100 - 9 \times 1$

Exercise 4:08

1 Is each number sentence true or false?

a $37 + 18 = 18 + 37$ **b** $250 \times 4 = 4 \times 250$ **c** $82 \times 0 = 82$
d $987 \times 1 = 987$ **e** $892 + 0 = 892$ **f** $1{\cdot}6 + 3{\cdot}8 = 3{\cdot}8 + 1{\cdot}6$
g $m \div 2 = 2 \div m$ **h** $7 \times n = n \times 7$ **i** $1 \times m = m$
j $5ab \times 0 = 0$ **k** $m - a = a - m$ **l** $7mn \times 1 = 7mn$

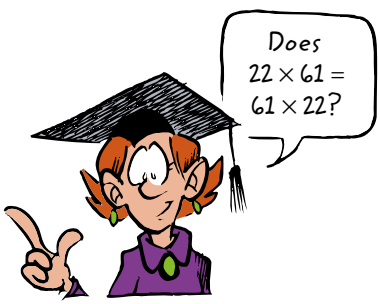
2 Write the numeral or term that will replace the box to make each number sentence true.

a $12 \times \square = 0$ **b** $\square \times 1 = 516$ **c** $1083 + \square = 1083$
d $\square + 0 = 399$ **e** $65 \times \square = 65$ **f** $1548 \times 0 = \square$
g $15 + b = \square + 15$ **h** $25 + m = m + \square$ **i** $1 \times m = \square$
j $12 \times 4 = \square \times 12$ **k** $6 \times y = y \times \square$ **l** $\square \times 5 = 5 \times ab$
m $2bc \times \square = 2bc$ **n** $2bc + \square = 2bc$ **o** $\square \times 1 = 3ab$
p $\frac{3}{4} + c = \square + \frac{3}{4}$ **q** $ab + bc = bc + \square$ **r** $6m \times 0 = \square$
s $3a \times \square = 6 \times 3a$ **t** $\square \times 1 = 12b$ **u** $\square + 0 = 9b$
v $\square + 2ac = 2ac + 1$ **w** $2b \times 4c = 4c \times \square$ **x** $0 \times 5c = \square$
y $7 \times abc = \square \times 7$ **z** $21 + mn = \square + 21$

3 Find the answer to each of the following.

a
$$\begin{array}{r} 61 \\ \times\ 22 \\ \hline \\ \\ \hline \\ \hline \end{array}$$
2×61
20×61

b
$$\begin{array}{r} 22 \\ \times\ 61 \\ \hline \\ \\ \hline \\ \hline \end{array}$$
1×22
60×22



4 In the following examples, does one person contribute more than the other? If so, who?

a Tom carried 10 four-litre containers of milk into the kitchen.
Lachlan carried 4 ten-litre containers of milk into the kitchen.

b Trinh found 6 eggs before lunch and 8 eggs after lunch.
Zac found 8 eggs before lunch and 6 eggs after lunch.

c Abigail stacked 3 shelves each hour for 4 hours.
Leah stacked 4 shelves each hour for 3 hours.

d During the last 3 weeks Madeline has found 6 stray dogs.
During the last 10 weeks Greg has found 3 stray dogs.

5 True or false?

a $(28 + 15) + 35 = 28 + (15 + 35)$
b $(100 + 20) + 9 = 100 + (20 + 9)$
c $(36 \times 4) \times 5 = 36 \times (4 \times 5)$
d $(87 \times 5) \times 20 = 87 \times (5 \times 20)$
e $(27 - 5) - 4 = 27 - (5 - 4)$
f $(100 - 9) - 1 = 100 - (9 - 1)$
g $(24 \div 4) \div 2 = 24 \div (4 \div 2)$
h $(100 \div 20) \div 5 = 100 \div (20 \div 5)$
i $20 \times 86 \times 5 = 100 \times 86$
j $97 + 186 + 3 = 100 + 186$

Do what is in the grouping symbols first.

6 Write the numeral or term that will replace each box to make the sentence true.

a $20 \times 114 \times 5 = \square \times 114$
b $888 + 49 + 51 = 888 + \square$
c $62 + 83 + 17 + 31 = 100 + \square$
d $14 \times 74 \times 5 = \square \times 74$
e $989 + 629 + 11 = \square + 629$
f $5 \times 679 \times 2 = \square \times 679$
g $0{\cdot}5 \times 65{\cdot}47 \times 2 = \square \times 65{\cdot}47$
h $\frac{7}{10} + \frac{5}{8} + \frac{3}{10} = \square + \frac{5}{8}$
i $1{\cdot}99 + 17{\cdot}8 + 0{\cdot}01 = \square + 17{\cdot}8$
j $\$47.55 \times 5 \times 4 \times 5 = \$47.55 \times \square$

7 Use the number laws to find the answers quickly.

a $98 \times 5 \times 6 \times 0$
b $\$0.20 \times 65 \times 5$
c $\$3.50 \times 8 \times 2$
d $618 + 40 + 60$
e $105 + 2547 + 95$
f $25 + 17 + 75 + 2$
g $10 \times 36 \times 5 \times 2$
h $4 \times 138 \times 5 \times 5$
i $25 \times 15 \times 2 \times 3 \times 2$
j $789 + 35 + 4 + 65$
k $960 + 993 + 7$
l $379 + 777 + 622$

8 True or false? (Try working out each side separately.)

a $6 \times (1 + 7) = 6 \times 1 + 6 \times 7$
b $9 \times (8 + 3) = 9 \times 8 + 9$
c $4 \times (5 + 2) = 4 \times 5 + 4 \times 2$
d $6 \times (7 - 1) = 6 \times 7 - 6 \times 1$
e $9 \times (8 - 3) = 9 \times 8 - 9 \times 3$
f $4 \times (5 - 2) = 4 \times 5 - 4 \times 2$

9 What numeral should replace the box?

a $8 \times 6 + 8 \times 3 = 8 \times (6 + \square)$
b $7 \times 3 + 7 \times 5 = 7 \times (\square + 5)$
c $11 \times 9 + 11 \times 6 = \square \times (9 + 6)$
d $5 \times 9 - 5 \times 1 = 5 \times (\square - 1)$
e $4 \times 6 - 4 \times 2 = 4 \times (6 - \square)$
f $6 \times 7 - 6 \times 2 = \square \times (7 - 2)$

10 Use the distributive property to find the answers. *Note:* $5 \times 99 = 5 \times 100 - 5 \times 1$.

a 5×99
b 7×99
c 3×98
d 9×98
e 6×95
f 8×95
g 12×95
h 18×95

4:09 Using grouping symbols

PREP QUIZ 4:09

Evaluate:

1 two groups of $(6 + 7)$
2 two groups of 6 + two groups of 7
3 $3 \times (6 + 10)$
4 $3 \times 6 + 3 \times 10$
5 $10 \times (7 + 3)$
6 $10 \times 7 + 10 \times 3$
7 3 groups of $(100 - 1)$
8 3 groups of 100 − 3 groups of 1
9 $5 \times (100 - 3)$
10 $5 \times 100 - 5 \times 3$

An expression like $3(2 + 5)$ means $3 \times (2 + 5)$ or 3 lots of $(2 + 5)$:

$$\begin{aligned} 3(2 + 5) &= (2 + 5) + (2 + 5) + (2 + 5) \\ &= 2 + 5 + 2 + 5 + 2 + 5 \\ &= 2 + 2 + 2 + 5 + 5 + 5 \\ &= 3 \times 2 + 3 \times 5 \end{aligned}$$

$\therefore 3(2 + 5) = 3 \times 2 + 3 \times 5$

Similarly: $6(5 - 2) = 6 \times 5 - 6 \times 2$

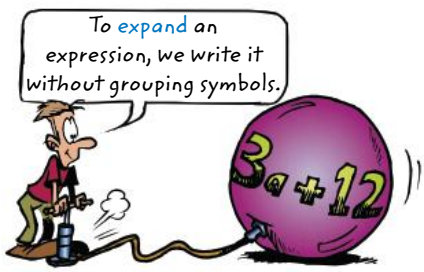
$(10 + 7)8 = 10 \times 8 + 7 \times 8$

This process is the same if pronumerals are involved.

$$\begin{aligned} 3(a + 4) &= (a + 4) + (a + 4) + (a + 4) \\ &= a + 4 + a + 4 + a + 4 \\ &= a + a + a + 4 + 4 + 4 \\ &= 3 \times a + 3 \times 4 \\ &= 3a + 12 \end{aligned}$$

$\therefore 3(a + 4) = 3a + 12$

Note:
The most common grouping symbols are:
- parentheses ( )
- brackets [ ]



To 'remove' the grouping symbols, multiply each term inside the grouping symbols by the term outside:

$$\begin{aligned} 7(2a + 3) &= 7 \times 2a + 7 \times 3 \\ &= 14a + 21 \end{aligned}$$

WORKED EXAMPLES

1 $5(x + 2) = 5 \times x + 5 \times 2$
$= 5x + 10$

2 $10(2a - 3) = 10 \times 2a - 10 \times 3$
$= 20a - 30$

3 $m(m + 2) = m \times m + m \times 2$
$= m^2 + 2m$

Expand and simplify:

4 $5(3x + 8) + 10 = 15x + 40 + 10$
$= 15x + 50$

5 $2(8m - 3) + 4m = 16m - 6 + 4m$
$= 20m - 6$

$a(b + c) = ab + ac$ $\quad a(b - c) = ab - ac$ $\quad (a + b)c = c(a + b)$

Exercise 4:09

P Foundation worksheet 4:09
Using grouping symbols

1 Remove the grouping symbols.

a $2(a + 3)$	**b** $3(x + 5)$	**c** $5(y + 3)$	**d** $3(x - 1)$
e $7(n - 7)$	**f** $4(a - 4)$	**g** $7(2 + x)$	**h** $6(1 + y)$
i $9(5 - y)$	**j** $(5 + x)2$	**k** $(x + 9)3$	**l** $(a - 2)3$

2 Expand:

a $2(3a + 2)$	**b** $3(2x + 1)$	**c** $5(4y + 3)$	**d** $10(3p + 7)$
e $6(2y - 1)$	**f** $5(3q - 4)$	**g** $4(2k - 7)$	**h** $9(4n - 3)$
i $5(4 + 2x)$	**j** $7(3 + 4x)$	**k** $5(1 + 3p)$	**l** $8(3 - 2y)$
m $(2m + 3)4$	**n** $(3y + 6)2$	**o** $(2 + 4x)5$	**p** $(7 + 3q)10$
q $(5n - 3)6$	**r** $(8m - 1)7$	**s** $(2 - 3x)7$	**t** $(5 - 9y)11$

3 Remove the grouping symbols.

a $x(x + 2)$	**b** $y(y + 5)$	**c** $m(m + 7)$
d $q(q - 1)$	**e** $m(m - 10)$	**f** $n(n - 4)$
g $a(3 + a)$	**h** $k(4 + k)$	**i** $p(6 - p)$
j $x(2x + 3)$	**k** $y(3y + 4)$	**l** $n(5n - 1)$
m $2p(p + 7)$	**n** $3q(q - 4)$	**o** $(m + 3)2m$
p $5a(2a + 3)$	**q** $2m(4m - 1)$	**r** $6n(5 - 3n)$



4 Expand:

a $5(x + y)$	**b** $4(a + b)$	**c** $7(p - q)$	**d** $10(m - n)$
e $6(2a + b)$	**f** $7(3x + 2y)$	**g** $9(4p - 2q)$	**h** $5(2k - 7l)$
i $x(x + y)$	**j** $p(p - q)$	**k** $2y(y + z)$	**l** $5g(3g - 2h)$

5 Expand and simplify:

a $2(a + 4) + 3$	**b** $4(m + 6) + 5$	**c** $8(x + 1) + 6$
d $3(a + 6) + 2a$	**e** $5(7 + y) + 3y$	**f** $10(m + 5) + 3m$
g $6(2n + 3) + 2$	**h** $4(2n + 1) + 7$	**i** $3(4a + 2) + 11$
j $3(m + 7) - 1$	**k** $8(a + 6) - 48$	**l** $3(m + 3) - 8$
m $2(5a + 1) - 4a$	**n** $3(2m + 2) - 6a$	**o** $5(10m + 3) - 20m$
p $4(2a - 1) + 3a$	**q** $5(2x - 4) + x$	**r** $7(5y - 2) + 10y$





Each packet holds n cards.
How many cards are there altogether?
$3(2n + 2) = \ldots$

4:10 Simplifying expressions with index notation

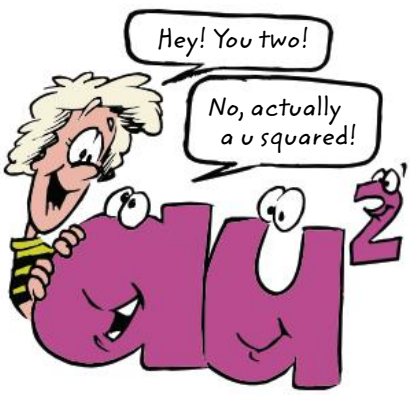
PREP QUIZ 4:10

Simplify these expressions.

1 $5a + 7a$ **2** $10p - 6p$ **3** $11x + x$ **4** $7m - m$
5 $9p + 7q - 2q$ **6** $5ab + 6ba$ **7** $9a + 5 + 2a + 3$
8 $7p + 8q - 2p$ **9** $5x + 7y - 2x + 3y$ **10** $10t^2 + 7t - 4t - 2t^2$

$a \times a$ is best written as a^2.

When a pronumeral is multiplied by itself a number of times, we can simplify the expression using index notation.



WORKED EXAMPLES

1 $a \times a \times a \times a = a^4$

2 $p \times p \times p \times p \times p \times p \times p = p^7$
(seven ps multiplied together)

3 $m \times m \times m \times n \times n \times n \times n \times n = m^3n^5$

4 $5 \times t \times t \times 7 \times y = 35t^2y$

Note that the index belongs to the t only.

The plural of index is *indices* (pronounced 'in-diss-ease').

Exercise 4:10

1 Simplify using index notation.

a $a \times a \times a$ **b** $p \times p \times p \times p \times p$ **c** $t \times t \times t \times t$
d $y \times y$ **e** $k \times k \times k \times k \times k$ **f** $m \times m \times m$
g $n \times n \times n \times n \times n \times n$ **h** $b \times b \times b \times b$ **i** $a \times a \times b \times b \times b$
j $x \times x \times x \times y \times y \times y$ **k** $a \times a \times a \times a \times t$ **l** $m \times n \times n \times n$
m $p \times q \times p \times p \times q$ **n** $a \times a \times b \times a \times b \times b$ **o** $x \times y \times x \times y \times x$

2 Rewrite in expanded form (e.g. $t^4 = t \times t \times t \times t$).

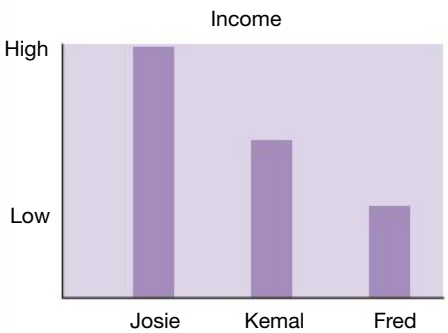
a a^5	**b** q^4	**c** t^7	**d** x^3
e m^2	**f** n^5	**g** p^1	**h** a^6
i m^2n^3	**j** a^4b^3	**k** xy^5	**l** k^{4l}
m a^3y	**n** h^2n^5	**o** tw^3	**p** x^4y^4
q $n^2p^3q^4$	**r** ab^5c^2	**s** $x^3y^2z^4$	**t** m^3np^4

3 Simplify using index notation.

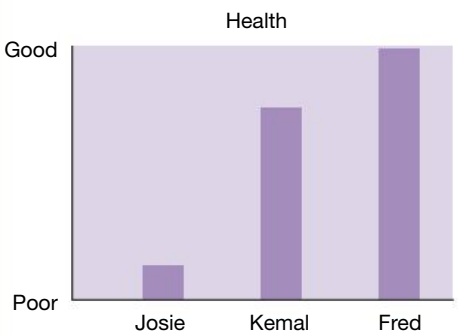
a $5 \times a \times 6 \times a$	**b** $3 \times y \times 4 \times y \times y$	**c** $9 \times m \times m \times 2 \times m$
d $t \times t \times 7 \times t$	**e** $7 \times y \times y \times 4 \times y \times y$	**f** $8 \times n \times 3 \times n \times 2$
g $5 \times m \times m \times m \times 9$	**h** $6 \times a \times 3 \times a \times 4 \times a$	**i** $2 \times x \times 3 \times y \times 4 \times x$
j $10 \times a \times 3 \times b \times 5 \times a \times b$	**k** $k \times l \times 3 \times l \times 5 \times k$	**l** $t \times 5 \times t \times w \times 3 \times t$

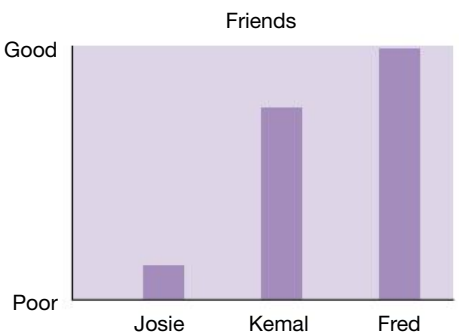
INVESTIGATION 4:10 QUALITY OF LIFE

Carefully study each graph below.







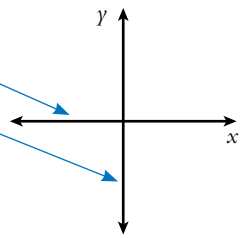


a Write as much as you can about Josie's life.
b Write as much as you can about Kemal's life.
c Write as much as you can about Fred's life.
d Which lifestyle would you prefer? Why?

MATHS TERMS 4

algebra (adjective: **algebraic**)
- the use of pronumerals to represent numbers and solve problems

axis (plural: **axes**)
- horizontal axis
- vertical axis



cubed
- multiplied as a factor three times,
  e.g. $a^3 = a \times a \times a$

evaluate
- to find the value of a term or expression

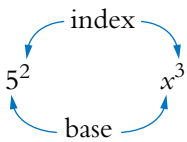
expand
- to remove grouping symbols,
  e.g. $3(2x - 7) = 6x - 21$

expression
- an algebraic expression is made up of one or more terms joined by + or − signs,
  e.g. $2x - 7$, $5x + 2y + 1$, $a^2 - 2a + 1$

formula
- a rule (or equation) written using pronumerals,
  e.g. $C = 50M + 6$, $A = l \times b$

grouping symbols
- symbols that group numbers or terms,
  e.g. parentheses ( ), brackets [ ],
  braces { }, fraction bar $\frac{x+1}{2}$,
  square root sign $\sqrt{4x+1}$

index (plural: **indices**)
- the raised symbol when a number is written as a power.



integer
- whole number,
  e.g. … , −3, −2, −1, 0, 1, 2, 3, …

like terms
- terms that have the same pronumeral parts,
  e.g. $3xy$ and $7xy$, $8x^2$ and $-x^2$
- unlike terms have different pronumeral parts, e.g. x^2 and x, $5xy$ and $2y$

number pattern
- a set of numbers connected by a rule or relationship

power
- when a number is repeated as a factor,
  e.g. 3 to the power of 5 = 3^5
  $= 3 \times 3 \times 3 \times 3 \times 3$

pronumeral
- a symbol (usually a letter) that takes the place of a numeral,
  e.g. x or θ

rule
- an instruction or formula that explains how the numbers in a pattern are related

simplify
- to give the shortest or simplest answer

squared
- multiplied by itself,
  e.g. $5^2 = 5 \times 5$
- multiplied as a factor twice

substitute (noun: **substitution**)
- to 'put in place of'
- usually, when a numeral replaces a pronumeral,
  e.g. 'Substitute $x = 7$ into $2x + 3$'

value
- the number represented by a pronumeral or expression,
  e.g. 'For the rule $y = x + 7$, find the value of y if x is given the value 4.'

variable
- a symbol that can take on different values

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Write the next number in each pattern. 4:01

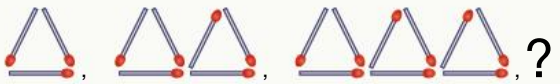
a 2, 6, 10, … b 15, 12, 9, … c 2, 6, 18, … d 40, 20, 10, …

2 Write the rule you used to find each answer in Question 1. 4:01

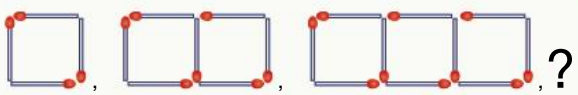
3 Find the number represented by each pronumeral. 4:02

a $\square + 7 = 10$ b $x - 4 = 11$ c $6 \times m = 24$

4 Complete the table of values for each matchstick pattern. 4:03

a 

Number of triangles	1	2	3	4
Number of matches				

b 

Number of squares	1	2	3	4
Number of matches				

5 Complete each table using the rule given. 4:03

a $y = x + 4$

x	1	2	3	4
y				

b $b = 4 \times t$

t	0	2	4	6
b				

c $n = 2 \times m + 3$

m	0	1	2	3
n				

6 Write each expression without multiplication signs. 4:04

a $5 \times y$ b $a \times b$ c $12 \times m \times n$ d $2 \times s + 5 \times t$

7 Write each expression in a shorter form. 4:04

a $a \times a$ b $6 \times p \times p$ c $2 \times m \times 3 \times m$ d $t \times w \times t$

8 Write these divisions in fraction form. 4:04

a $y \div 5$ b $10 \div a$ c $(a + 2) \div 3$ d $x \div y$

9 There are x counters in each blue box and y counters in each red box. 4:05
Use pronumerals to write the total number of counters.

a 

b 

c 

d 

e 

f 

10 Find the value of each expression if $n = 5$. 4:06

a $4n$ **b** n^2 **c** $2n + 1$ **d** $\frac{n+7}{3}$

11 If $a = 3$ and $b = 6$, evaluate: 4:06

a ab **b** $2a + b$ **c** $a^2 + 2b$ **d** $\frac{b}{a}$

12 Complete each table of values. 4:06

a $b = 3t$

t	1	2	3	4
b				

b $y = 2x + 1$

x	0	2	4	6
y				

c $m = n^2$

n	0	1	2	3
m				

13 How many terms are there in each expression? 4:07

a $2x + 1$ **b** ab **c** $3a + 2b + 3c$ **d** $6 - 2x^2$

14 Simplify each expression. 4:07

a $2a + 5a$ **b** $7y - y$ **c** $2ab + 3ba$ **d** $6m - 5m$

15 Simplify: 4:07

a $2a + 3 + a$ **b** $6x + 2y - 4x$

c $2m + 3n + 4m + 5n$ **d** $3p^2 + 5p + 2p^2 - 3p$

16 True or false? 4:08

a $134 \times 8 = 8 \times 134$ **b** $186 + 4 = 4 + 186$

c $5(2 + 9) = 52 + 59$ **d** $6 \times (4 + 8) = 6 \times 4 + 6 \times 8$

17 Expand (remove grouping symbols). 4:09

a $2(x + 4)$ **b** $5(1 - a)$ **c** $(2y + 5)3$ **d** $x(2x + y)$

18 Expand and simplify. 4:09

a $4(x + 3) + 5$ **b** $3(2a + 1) + 8a$ **c** $5(2m - 4) - 8m$

19 Rewrite using index notation. 4:10

a $a \times a \times a \times a$ **b** $y \times y \times y \times y \times y$

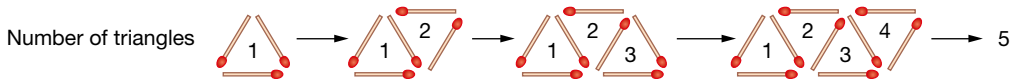
c $3 \times a \times 4 \times a \times a$ **d** $x \times y \times x \times y \times x$

20 Rewrite in expanded form. 4:10

a m^5 **b** a^2b **c** xy^3

ASSIGNMENT 4A Chapter review

1

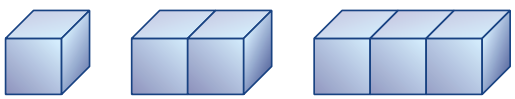


a Complete this table using the diagram above.

Number of triangles	1	2	3	4	5
Number of matches					

b To make an extra triangle, how many matches must be added?

2



Number of blocks (b)	1	2	3	4
Number of faces showing (f)				

a Complete the table.

b Write a rule connecting f (the number of faces showing) and b (the number of blocks), e.g. $f = \square \times b + \triangle$.

3 Rewrite these rules using the pronumerals indicated in colour,
e.g. *C*ost of bananas = *n*umber of bananas times 18 cents. Answer: $C = n \times 18$

a *D*istance travelled = *S*peed multiplied by *T*ime

b *A*rea of a rectangle = *l*ength multiplied by *b*readth

c *P*erimeter of a square = four times its side *l*ength

d *T*otal number of students = number of *b*oys plus the number of *g*irls

e *P*ieces of fruit in a bag = number of *a*pples plus the number of *p*ears

f *M*other's age = *S*on's age plus 27

g *C*ost per item = *T*otal cost divided by the *n*umber of items

h *A*verage of two numbers = first number *a* plus the second number *b*, then divide by 2

4 Write each expression in its simplest form.

a $15 \times y$
b $12x \div 4$
c $5 \times m \times 4$
d $6 \times 3y$
e $6n + 8n$
f $3n - n$
g $3n - 3n$
h $3n - 3n + 3$
i $b + b + b$
j $2a + 4 + 3a$
k $6x - 2x + 3$
l $6p + 6 - p$
m $b \times b \times b$
n $2a^2 + a + a^2$
o $6x \times 0$
p $5ap \times 1$

5 Expand and simplify:

a $7(x + y) - 7y$
b $5(2x + 5) - 11$
c $3(a + 7) + 2(a + 7)$
d $5x + 3(2x - 5)$
e $11 + 2(m - 4)$
f $11(3a + 2b) + 3(7a + b)$

ASSIGNMENT 4B Working mathematically

1 Refer to **ID Card 2** on page xv and identify the symbols **1** to **24**. Learn the symbols that you do not know.

2 Looking at a truck, a student sees that it carries 12 pallets. Each pallet is stacked with 100 cases and each case holds 24 cans of drink. How many cans of drink are on the truck?

3 In a cricket Test match, each team has two turns (called 'innings') to score runs. If the West Indies team scores 310 runs and 337 runs in their two innings, and Australia scores 287 runs in their first innings, how many runs does Australia need to score in their second innings to win?



4 At a restaurant I bought 3 orange juices and a glass of wine. The drinks cost $12.60 altogether. Later I bought 4 orange juices and the cost was $12. What was the cost of the glass of wine?

5 A house is to be built from 30 000 bricks. For every thousand bricks that are laid, 1 tonne of sand and 4 bags of cement are used. How much will it cost to buy and lay all the bricks if the bricks cost $700 per thousand, the bricklayer charges $600 to lay a thousand bricks, sand costs $30 per tonne and a bag of cement costs $20?

6 Soula and her brother Kosta have $20 and $10 pocket money respectively. After going to the shops, Soula has three times as much money left as Kosta. If they spent $20 altogether, how much money does each of them have left?

7 From the graph:
- **a** Which country produces the most steel?
- **b** Which country produces the least steel?
- **c** Which country produces more steel, Poland or Australia?
- **d** Japan's steel production is 117 million tonnes. If this were marked on the graph, how many countries would have a higher production than Japan?
- **e** How much steel does Germany produce?



ASSIGNMENT 4C Cumulative revision

1 Simplify: 1:11A

a
$$\begin{array}{r} 121 \\ 78 \\ +\ 4362 \\ \hline \end{array}$$

b
$$\begin{array}{r} 4296 \\ -\ 709 \\ \hline \end{array}$$

c
$$\begin{array}{r} 107 \\ \times\ 9 \\ \hline \end{array}$$

d $4\overline{)2060}$

2 **a** $\$7.65 + \$3.80 + \$19.05$ **b** $\$100 - \$63.90$ 1:11C
c $\$2.75 \times 22$ **d** $\$16.80 \div 8$

3 Simplify: 1:06
a $3 + 2 \times 2$ **b** $16 \div 2 - 2$
c $3 \times 2 + 4 \times 5$ **d** $12 - 6 \times 2 \div 3$

4 Write as a basic numeral: 3:01
a 4^2 **b** 10^3 **c** $\sqrt{100}$ **d** 8×10^4

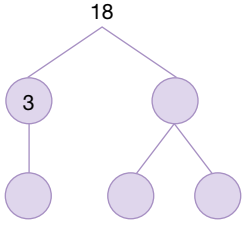
5 Write as a power: 3:01
a $4 \times 4 \times 4$ **b** 10×10 **c** $3 \times 3 \times 3 \times 3$ **d** $7 \times 7 \times 7$

6 List all the factors of each number. 3:03
a 15 **b** 24 **c** 7 **d** 100

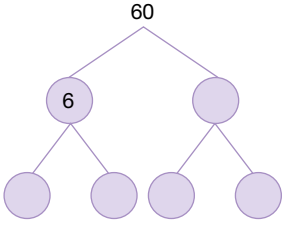
7 **a** Write the first three multiples of 10. 3:03, 3:04, 3:06
b Write the prime numbers between 10 and 20.
c What is the highest common factor of 30 and 40?
d What is the lowest common multiple of 6 and 8?

8 Complete these factor trees. 3:03

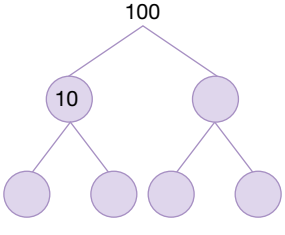
a



b



c



9 Simplify: 1:11B
a $\frac{5}{10}$ **b** $\frac{2}{8}$ **c** $\frac{3}{10} + \frac{6}{10}$ **d** $\frac{3}{4} - \frac{2}{4}$

10 Which fraction is larger? 1:11B
a $\frac{3}{4}$ or $\frac{1}{2}$ **b** $\frac{3}{4}$ or $\frac{1}{10}$ **c** $\frac{9}{10}$ or $\frac{7}{8}$ **d** $\frac{3}{5}$ or $\frac{2}{10}$

ANGLES

5



Contents

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GeoGebra activity 5:01 Naming angles
5:02 Measuring the size of an angle
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Investigation 5:02 Triangulation
5:03 Types of angles
GeoGebra activity 5:03 Acute, obtuse or reflex?
Investigation 5:03 Trails
5:04 Discovering more about angles
GeoGebra activity 5:04 Discovering more about angles
5:05 Angles and parallel lines
GeoGebra activity 5:05 Angles and parallel lines
Challenge 5:05 Getting the angle on things—triangulation
5:06 Identifying parallel lines
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Angles 2*** [Stage 3], ***Angle Relationships*** [Stage 4]

- Use the language, notation and conventions of geometry.
- Investigate, with and without the use of digital technologies, angles on a straight line, angles at a point, and vertically opposite angles; use the results to find unknown angles. [Stage 3] (ACMMG141)
- Recognise the geometrical properties of angles at a point.
- Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal. (ACMMG163)
- Investigate conditions for two lines to be parallel. (ACMMG164)
- Solve simple numerical problems using reasoning. (ACMMG164)

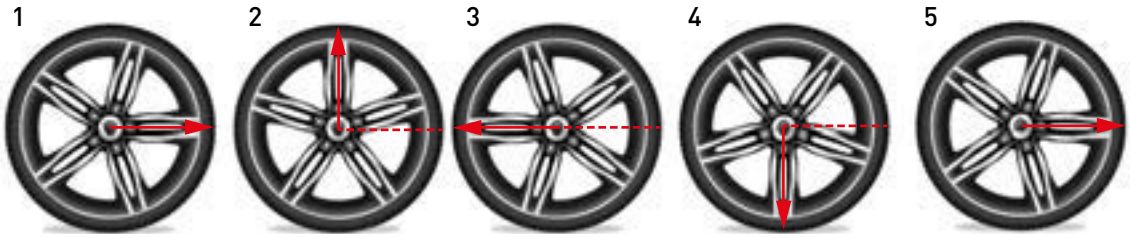
Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

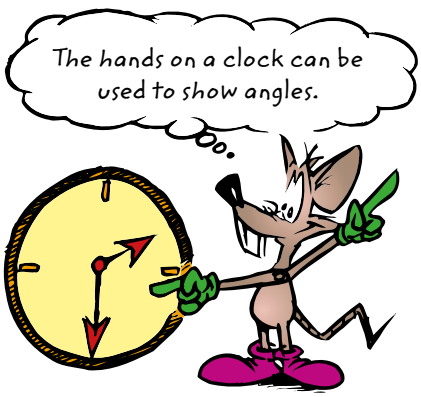
5:01 Introduction to angles

Angles are used almost everywhere, from rowing a boat to building a bridge. Among other things, they are used in manufacturing, surveying, navigation and engineering.

Angles are used to measure how far something has turned. This can be shown with a wheel:

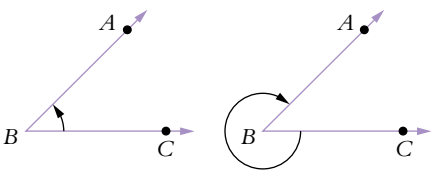


- The wheel can spin about its centre. The diagrams show how the wheel makes one revolution.
- The first diagram shows the starting position, with the red line pointing horizontally to the right.
- As the wheel turns, the red line moves in an anticlockwise direction from its starting position (which is shown by the dashed line) and the angle increases in size.
- The last diagram shows the wheel returned to its original starting position after one revolution.



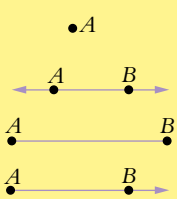
Angles can also be found in two- and three-dimensional shapes such as triangles, quadrilaterals and prisms.

If we look at an angle, we see that it is made of two **arms** or **rays**, which meet at a point called the **vertex**.



- In the angles above, BA and BC are the arms of the angle while B is the vertex.
- Arrows show the direction of rotation, which can be clockwise or anticlockwise.
- When there is no curved line to show the angle, we assume we are looking at the smaller of the two possible angles.

Capital letters are used to name points, lines, intervals and rays.



The point A

The line AB

The interval AB

The ray AB



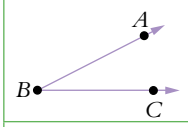
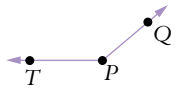
A **point** is a position. It has no size.

A **straight line** continues in both directions. It has no width.

An **interval** is the part of a line that connects two given points.

Naming angles

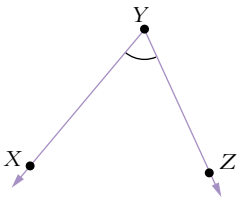
Angles can be named in different ways. Two are shown here:

Angle	Method 1	Method 2
	$\angle ABC$ $\angle CBA$ or $A\hat{B}C$	$\angle B$ or $\hat{B}$
	$\angle TPQ$ $\angle QPT$ or $Q\hat{P}T$	$\angle P$ or $\hat{P}$

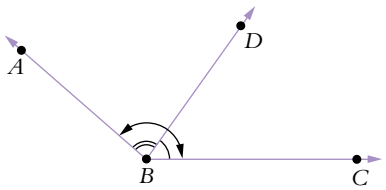
- In method 1, we read from the letter on one arm to the vertex and then to the letter on the other arm. The vertex letter is in the middle.
- Method 2 can only be used when there is no chance of confusion (i.e. when the vertex has only one possible angle).

WORKED EXAMPLES

1 Name the angle pictured below.

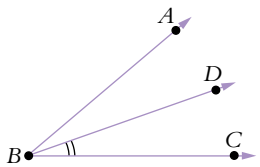


2 This figure has three angles marked, all with vertex B. Name each angle.

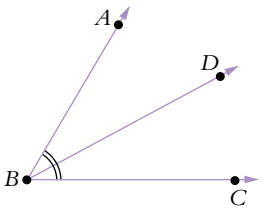


3 Name the angle marked with the double curved lines '))' in each of the following.

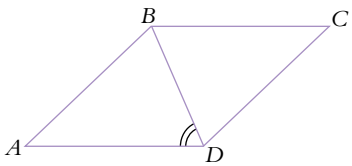
a



b



c



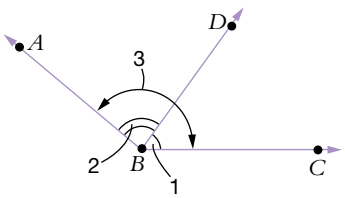
Solutions

1 As Y is the vertex, we can write:

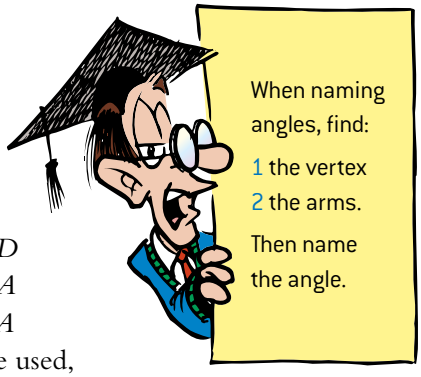
$\angle XYZ$ or $\angle ZYX$ or $X\hat{Y}Z$ (method 1)

$\angle Y$ or $\hat{Y}$ (method 2)

2



1 is $\angle DBC$ or $\angle CBD$
2 is $\angle ABD$ or $\angle DBA$
3 is $\angle ABC$ or $\angle CBA$
(Method 2 cannot be used, because there are different angles at the same vertex.)



3 **a** B is the vertex. BD and BC are the arms. Hence the name of the angle is $\angle DBC$ (or $\angle CBD$).

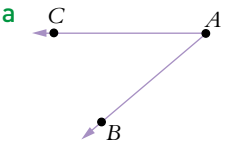
b B is the vertex. AB and BC are the arms. The angle is named $\angle ABC$ (or $\angle CBA$).

c D is the vertex. AD and BD are the arms. The angle is named $\angle ADB$ (or $\angle BDA$).

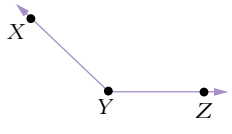
Exercise 5:01

1 Name the vertex of the angle.

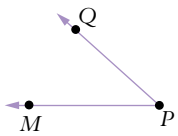
a



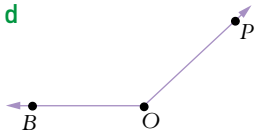
b



c



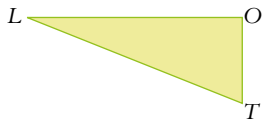
d



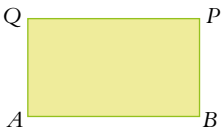
2 Name the angles in Question 1.

3 Use methods 1 and 2 (see page 139) to name the angles in the following shapes.

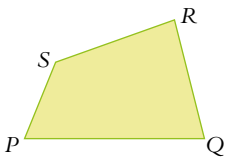
a



b

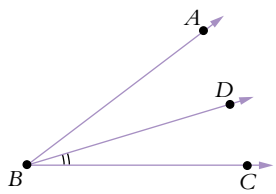


c

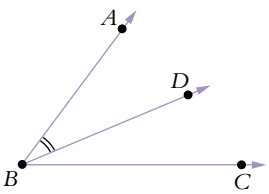


4 Name the angle marked with the double curved lines '))' in each of the following.

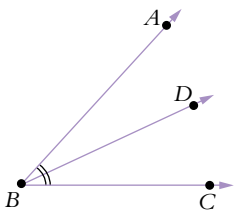
a



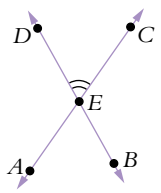
b



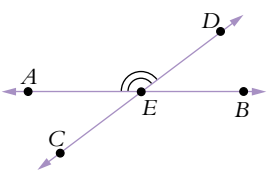
c



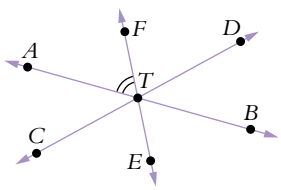
d



e

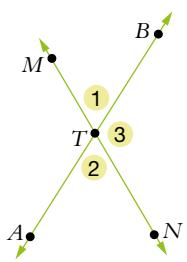


f

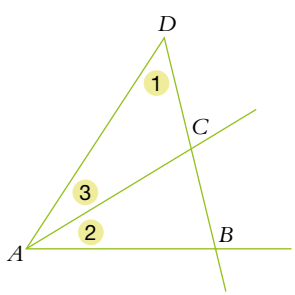


5 Name the angles 1, 2 and 3 in each of the following.

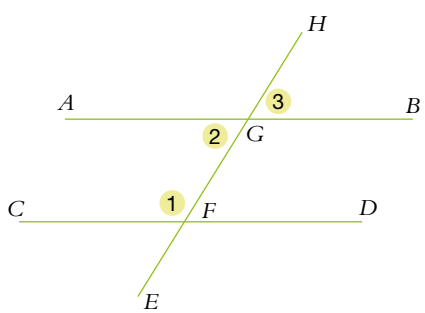
a



b

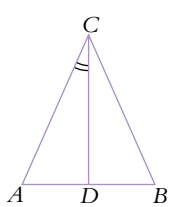


c

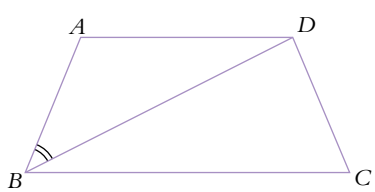


6 Name the angle marked with the double curved lines '))' in each of the following.

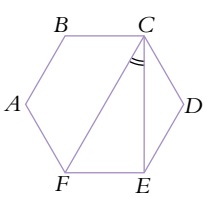
a



b

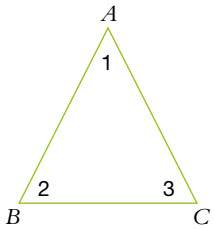


c

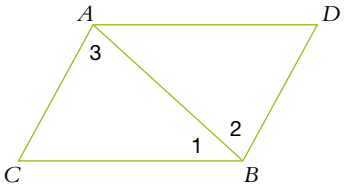


7 Which of the numbers 1, 2, or 3 marks $\angle ABC$?

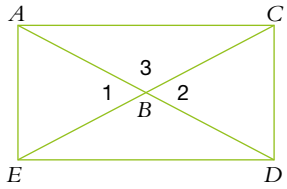
a



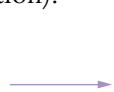
b



c



In the diagrams below, the angles increase in size from Figure 1 (no rotation) to Figure 8 (one revolution).


Figure 1

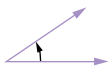
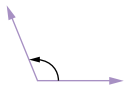
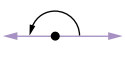
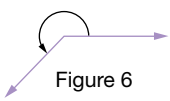
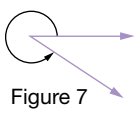

Figure 2

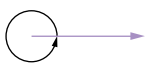

Figure 3


Figure 4


Figure 5

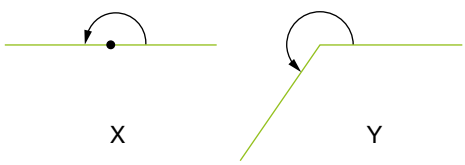

Figure 6


Figure 7


Figure 8

8 Which angle is the larger in each of the following pairs?

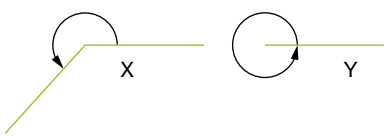
a



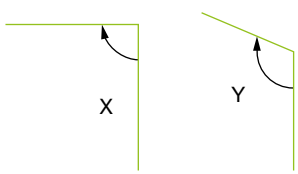
b



c

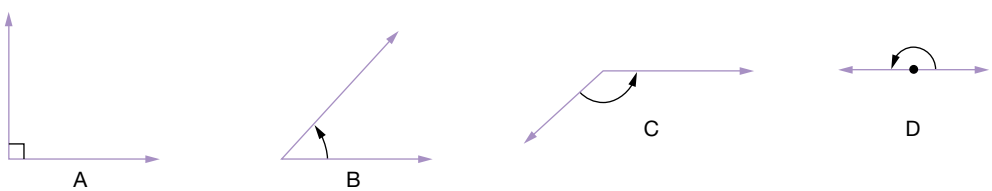


d

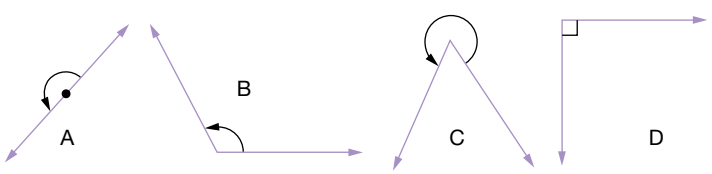


9 In each part, arrange the angles A, B, C and D in order of increasing size.

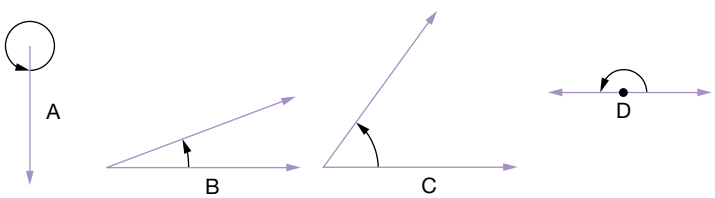
a

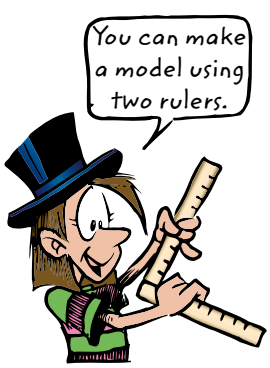


b



c



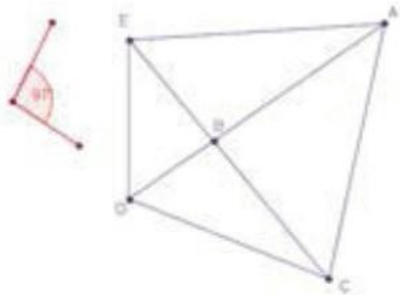


P GEOGEBRA ACTIVITY 5:01 NAMING ANGLES

1 Identify each angle in the diagram from its three-letter name, then tick its box to see if you are correct.



2 Click and drag the red angle to make it match each of the angles given.

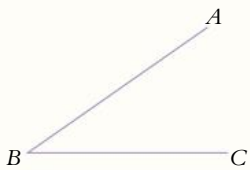


5:02 Measuring the size of an angle

PREP QUIZ 5:02

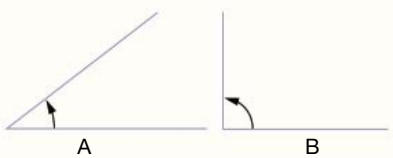
For the angle shown:

1 name the vertex
2 name the arms
3 name the angle.

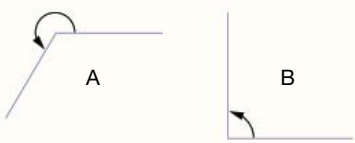


Which angle is the larger in each of the following pairs?

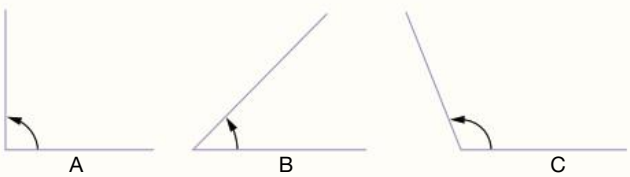
4



5

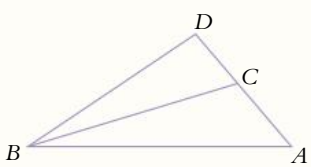


6 Arrange the angles A, B and C in order of size, from smallest to largest.

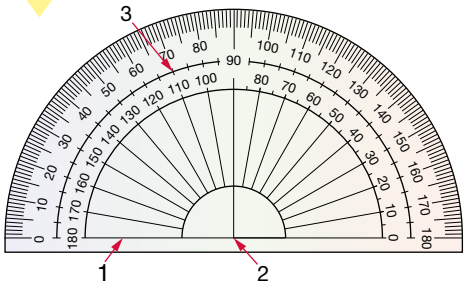


For the diagram shown:

7 Which angle is larger, $\angle ABC$ or $\angle ABD$?
8 Which is larger, $\angle D$ or $\angle A$?
9 Which is larger, $D\hat{C}B$ or $B\hat{C}A$?
10 Which is larger, $\angle BDC$ or $\angle BCA$?



Study this carefully and you will know all the angles!



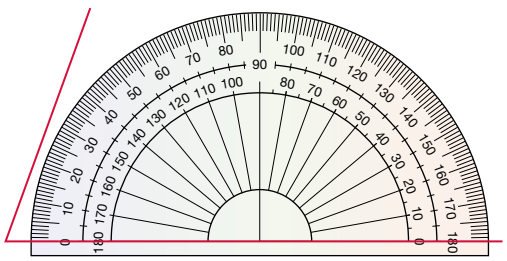
Earlier in this chapter we learned that angles are used to measure turning. The instrument used to measure angles is called a **protractor** and the most common unit of measurement is the **degree**. The degree symbol is a raised circle, °.

As the diagram shows, a protractor usually has:

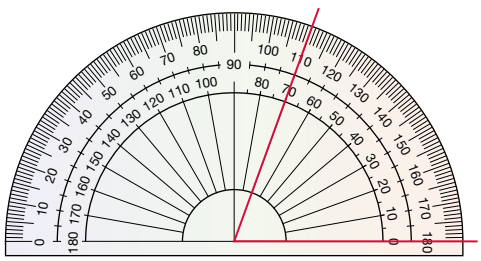
1 a base line
2 a centre point where the 90° line meets the base line
3 two scales ('inside' and 'outside'), each ranging from 0° to 180°.

To measure an angle, follow these steps.

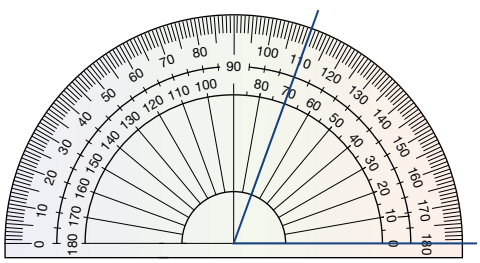
Step 1 Set the protractor down with its base along the lower arm of the angle.



Step 2 Slide the protractor along until the centre of the protractor meets the vertex of the angle.



Step 3 Find 0° on the lower arm and use the scale to measure the angle.

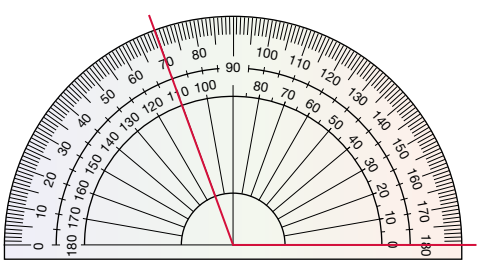




Measure the angle.

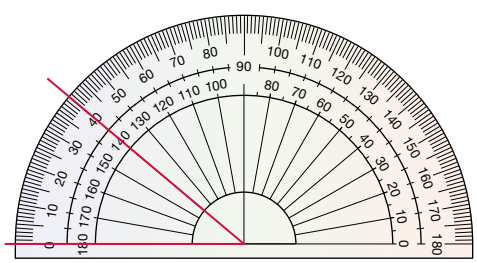
WORKED EXAMPLES

1



This angle measures 110°

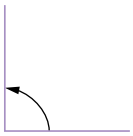
2



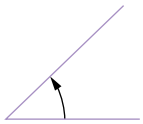
This angle measures 40°.

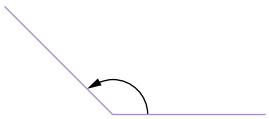
Exercise 5:02

1 Without using a protractor, estimate the size of each angle. Match each angle with a measurement from the list.

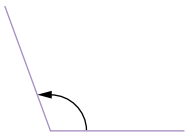
a 

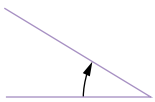
b 

c 

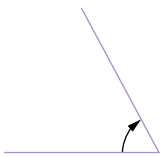
d 

e 

f 

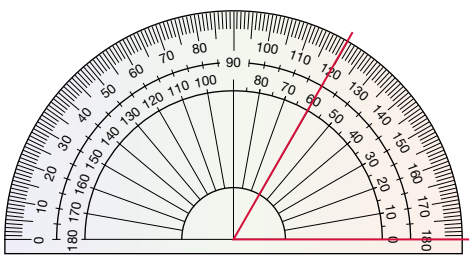
g 

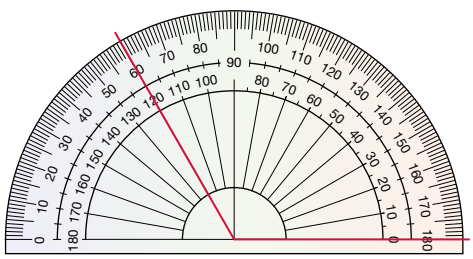
h 

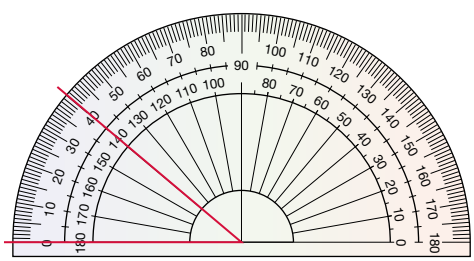
i 

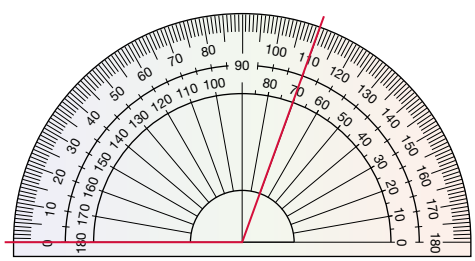
Size
10°
30°
45°
60°
90°
110°
135°
160°
180°

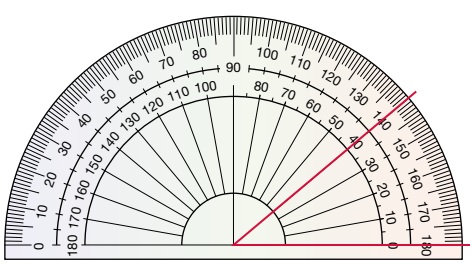
2 Write the size of the angle in each of the following.

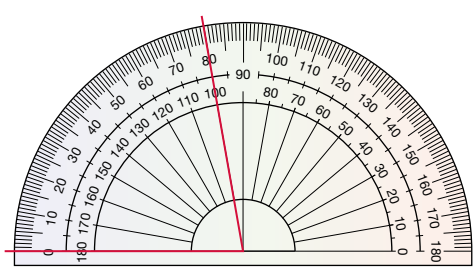
a 

b 

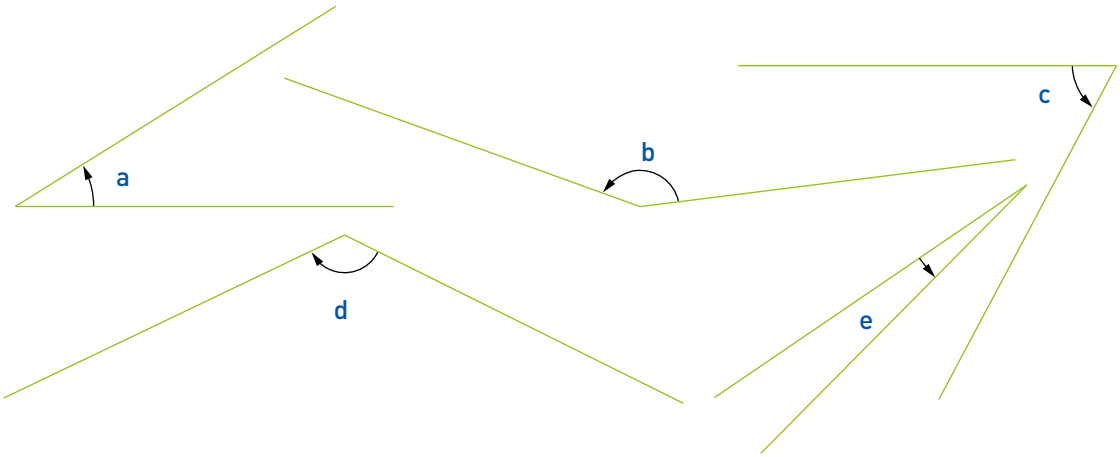
c 

d 

e 

f 

3 Estimate the size of each angle. Then use a protractor to measure its size, giving the answer to the nearest ten degrees.



4 Use a protractor to draw angles of the following sizes:

a 40° **b** 80° **c** 120° **d** 160° **e** 90°

5 Using a ruler (but NOT a protractor), draw angles as close as you can to the following:

a 90° **b** 45° **c** 30° **d** 60° **e** 135°

Check your answers by measuring each one with a protractor.

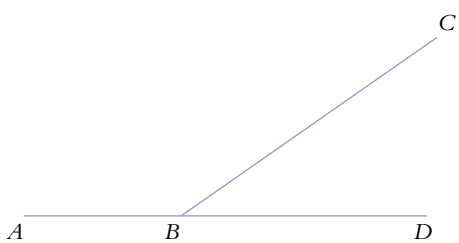
6 Measure the size of each angle in Question **3** to the nearest degree.

7 Use a protractor to draw these angles.

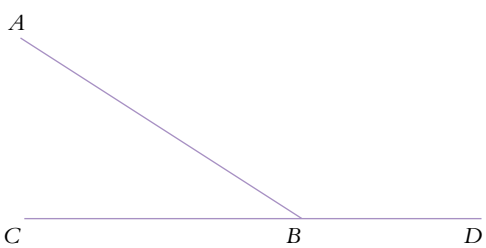
a 35° **b** 75° **c** 87° **d** 104° **e** 138°

8 Measure the size of $\angle ABC$ in each of the following.

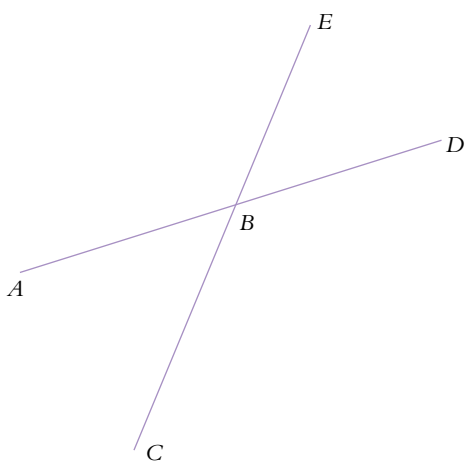
a



b



c

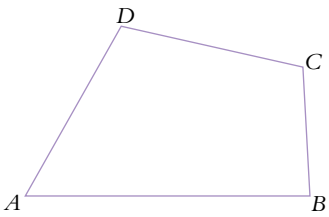




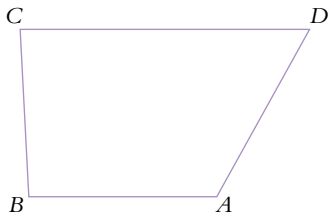
Woodworkers use mitre saws marked with protractors to make angled cuts in wood.

9 Measure the size of $\angle A$ and $\angle C$ in each of the following.

a

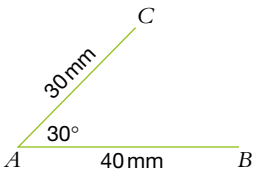


b

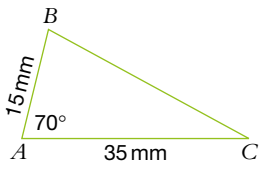


10 Make accurate full-size drawings of the following using a ruler, compass and protractor. (You may not need every instrument for each drawing.)

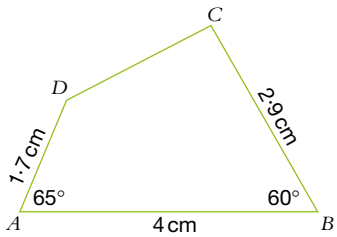
a



b

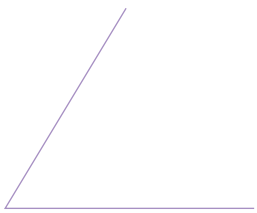


c

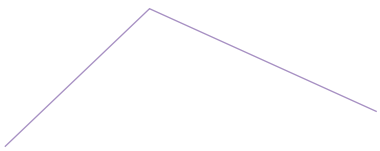


11 Use a protractor to bisect the following angles.

a



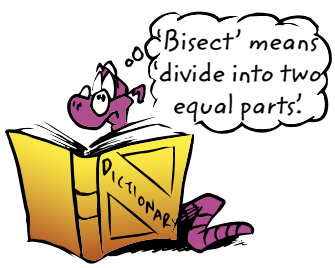
b



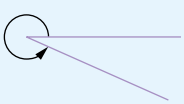
c



12 Use a ruler to draw three different angles. For each angle, estimate where the bisector (the line that bisects the angle) would be, then draw it. Check your estimate with a protractor.



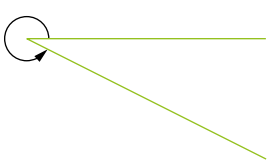
To measure an angle larger than 180°, measure the smaller angle and then subtract this from 360°.



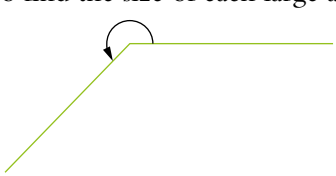
- The smaller angle is 23°.
- The larger angle is 360° – 23°,
  ∴ the larger angle is 337°.

13 Use the method described above to find the size of each large angle.

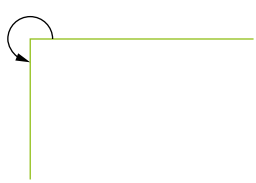
a



b

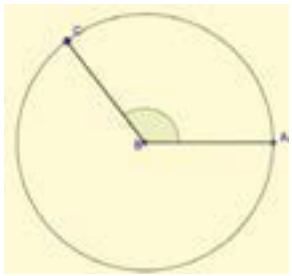


c

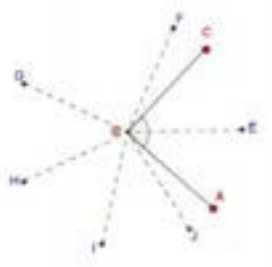


P GEOGEBRA ACTIVITY 5:02 ESTIMATING AND MEASURING ANGLES

1 Drag point C to change $\angle ABC$. Estimate the size of $\angle ABC$, then tick its box to check your estimate.

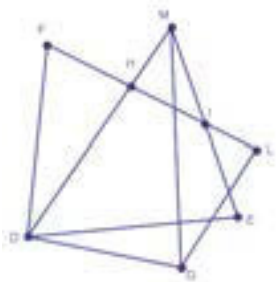


2 Estimate the size of each angle in the diagram. Check your estimates by dragging $\angle ABC$ to match each angle.



3 Use the **Angle** tool  to measure each angle in the diagram.

Make sure you click on the points of each angle in a clockwise order. If you make a mistake, click **Undo**  .



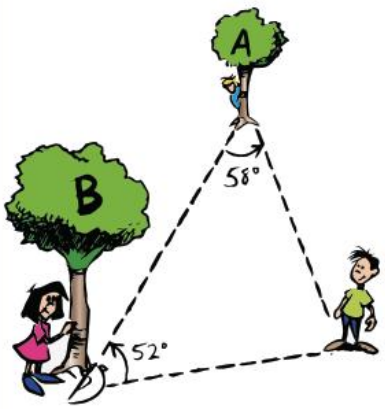


The direction on a compass is measured as an angle from magnetic north.

INVESTIGATION 5:02 TRIANGULATION

Triangulation is a way to fix a position so that it can be found later by someone else. It is also useful when we want to make a scale drawing.

Bronwyn and Barney wanted to plant a time capsule in the school grounds. They would leave directions in the school records so that the capsule could be found again in 20 years.



Step 1 They selected two strong young trees, A and B, and joined them with string.

Step 2 A friend stood at the spot where the time capsule would be buried.

Step 3 Bronwyn and Barney used a blackboard protractor to measure the angles between the string and the lines from each tree to their friend.

Step 4 They recorded the results: from the string joining A and B the angles were 58° at A, 52° at B.

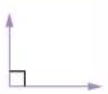
1 In a group of three, choose a spot where you would place a time capsule. Record the position of the spot using the steps listed above.

2 Swap information with another group and locate their spot.

3 Investigate the use of a **theodolite** in surveying.

5:03 Types of angles

Angles can be classified into different types depending on their size, as follows:

Type	acute	right	obtuse	straight	reflex	revolution
Size	less than 90°	90°	between 90° and 180°	180°	between 180° and 360°	360°
Diagram						

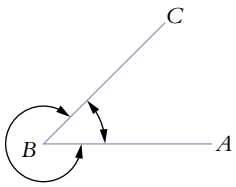


- A revolution is one complete turn.
- A straight angle is a half turn.
- A right angle is a quarter turn.
- The arms of a straight angle form a straight line.
- The special symbol for a right angle is  .
- The right angle, straight angle and angle of revolution each have only one size: 90°, 180°, 360°.



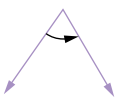
Why are right angles important for buildings?

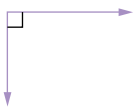
In the diagram below, $\angle ABC$ can be measured in either a clockwise or anticlockwise direction. When there is no marking, we assume it is the smaller angle.

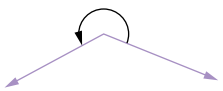


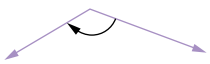
WORKED EXAMPLES

1 Classify the following angles.

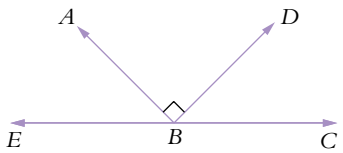
a 

b 

c 

d 

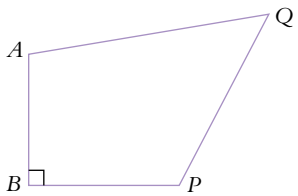
2 From the diagram, classify the following angles.



a $\angle ABC$ **b** $\angle EBC$

c $\angle ABD$ **d** $\angle DBC$

3 In this diagram, what type of angle are the following?



a $\angle ABP$

b $\angle BAQ$

c $\angle BPQ$

Solutions

1 **a** The angle is smaller than a right angle, so it is acute.

b A right angle.

c This angle is bigger than a straight angle, so it is a reflex angle.

d This angle is bigger than a right angle but smaller than a straight angle, so it is an obtuse angle.

2 **a** $\angle ABC$ is obtuse.

b $\angle EBC$ is a straight angle.

c $\angle ABD$ is a right angle.

d $\angle DBC$ is acute.

3 **a** A right angle.

b An obtuse angle.

c An obtuse angle.

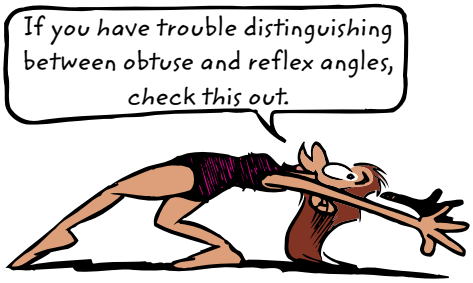


Figure 1

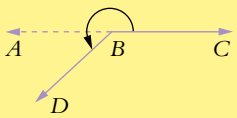
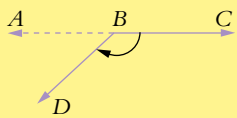


Figure 2

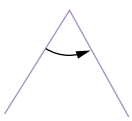


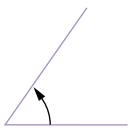
In both figures $\angle ABC$ is a straight angle.

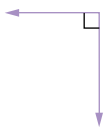
- In Figure 1 $\angle DBC$ is bigger than a straight angle, so $\angle DBC$ is reflex.
- In Figure 2 $\angle DBC$ is smaller than a straight angle, so $\angle DBC$ is obtuse.

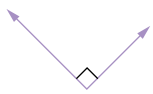
Exercise 5:03

1 Classify the following angles.

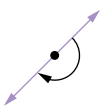
a 

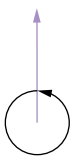
b 

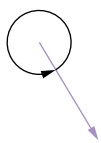
c 

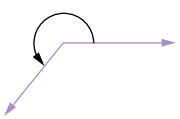
d 

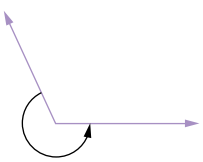
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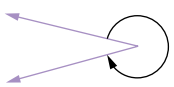
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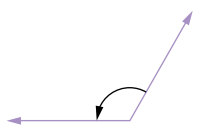
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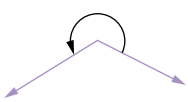
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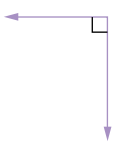
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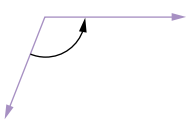
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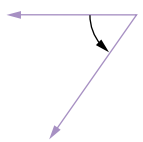
k 

l 

m 

n 

o 

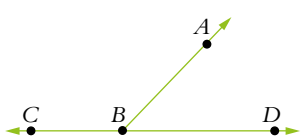
p 

2 Classify angles of the following sizes.

a 90° b 180° c 360° d 100° e 89° f 98°
g 210° h 270° i 27° j 135° k 300° l 170°

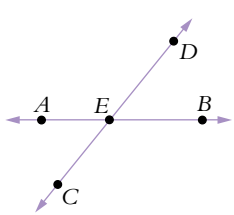
3 In this diagram, what type of angle is:

a $\angle ABD$ b $\angle ABC$ c $\angle CBD$?



4 In this diagram, what type of angle is:

a $\angle AEB$ b $\angle DEC$ c $\angle AEC$
d $\angle BED$ e $\angle AED$ f $\angle BEC$?

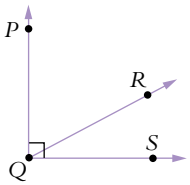


5 In the diagram, what type of angle is:

a $\angle PQS$

b $\angle PQR$

c $\angle RQS$?

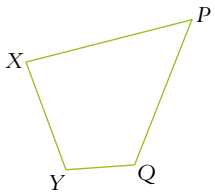


6 In the diagram, what type of angle is:

a $\angle XYQ$

b $\angle PXY$

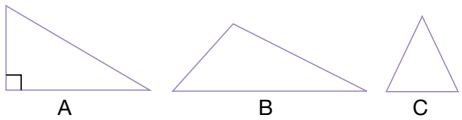
c $\angle XPQ$?



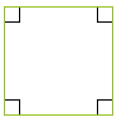
7 Study the triangles and then answer the questions.

a Which triangle has 3 acute angles?

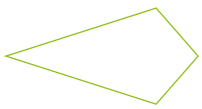
b Which triangle has an obtuse angle?

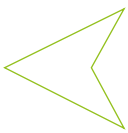


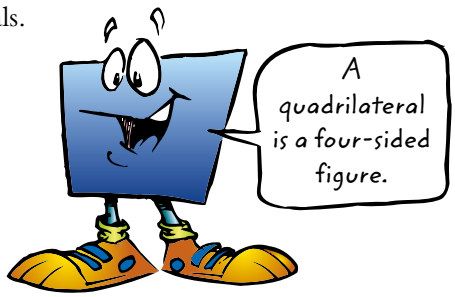
8 Classify the angles in each of the following quadrilaterals.

a 

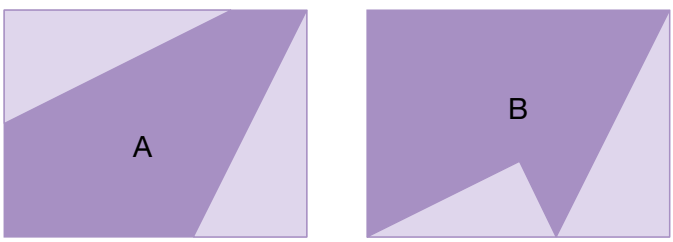
b 

c 

d 



9



Two triangular pieces are removed from each rectangle to form the two dark purple pentagons. What are the types of angles inside:

a pentagon A

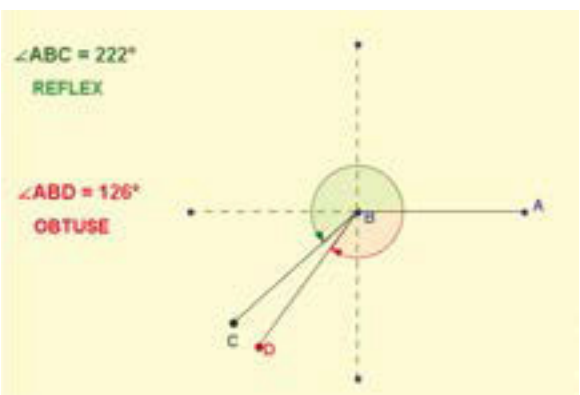
b pentagon B?

c Show how removing two right-angled triangles from a rectangle can make a hexagon with a right angle and a reflex angle.

P GEOGEBRA ACTIVITY 5:03 ACUTE, OBTUSE OR REFLEX?

Create different types of angles by dragging points C and D around the point B.

Choose a blue angle and measure it by dragging it across so that it sits on top of $\angle ABC$ or $\angle ABD$.



INVESTIGATION 5:03 TRAILS

These trails have been formed by counting squares on grid paper to represent number patterns, making a left-hand turn (i.e. an anticlockwise 90° turn) for each new number. For example, Figure A has been formed by counting from 1 to 2 repeatedly until the trail arrives back at its starting point. Let's call it T2 (where '2' means a trail of size 2).

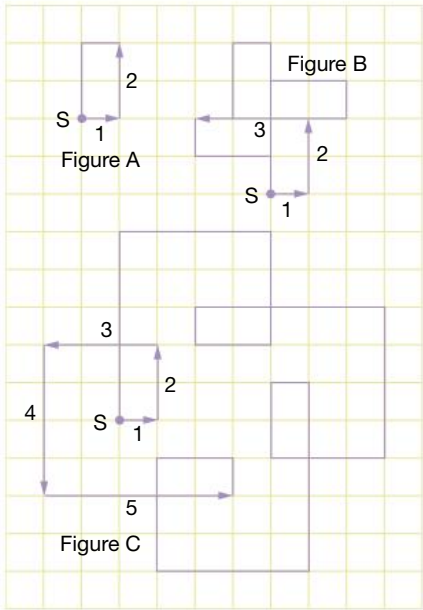


Figure B has been formed by counting to 3 repeatedly until the trail arrives back at its starting point. Call it T3.

Similarly, for Figure C a count to 5 has been repeated (T5).

- Try these patterns yourself on grid paper, then try counting to other numbers.

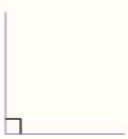
The following questions suggest some investigations of these trails:

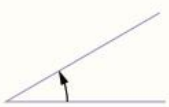
- Do you always return to the starting point?
- What is the length of the trail?
- How many times do you need to repeat the count to return to the starting point?
- What happens to a trail, e.g. T3, if instead of counting up from 1 to 3 you count down from 3 to 1?
- Can you predict what will happen to a trail of a given size?

5:04 Discovering more about angles

PREP QUIZ 5:04

What type of angle is each of the following?

1 

2 

3 

4 

What is the size (in degrees) of:

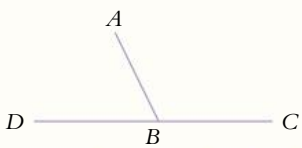
5 a right angle **6** a straight angle **7** an angle of revolution?

Copy this diagram.

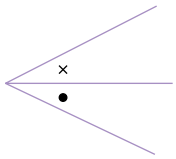
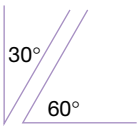
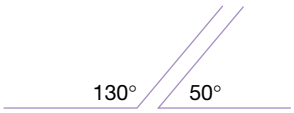
8 Mark $\angle ABD$ with a '★'.

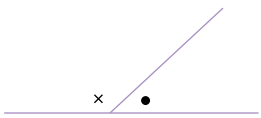
9 Mark $\angle CBA$ with a '•'.

10 What is the size of $\angle DBC$?

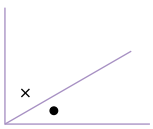


Describing angles

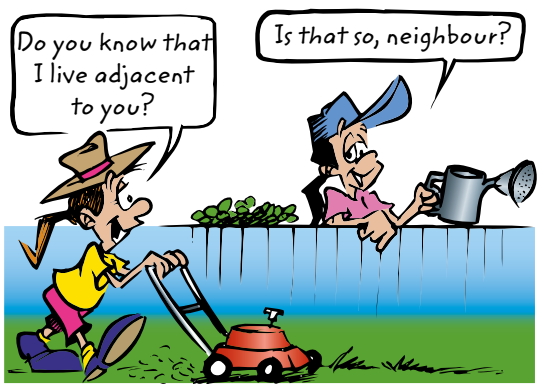
Adjacent angles	Complementary angles	Supplementary angles
• are next to each other  • have the same vertex and are alongside each other.	• add up to 90°.  • 30° is the complement of 60°.	• add up to 180°.  • 130° is the supplement of 50°.



These angles are adjacent *and* supplementary.



These angles are adjacent *and* complementary.



Adjacent angles
- have a common vertex
- have a common arm
- are on opposite sides of the common arm.

Exercise 5:04

P Challenge worksheet 5:04 More about angles

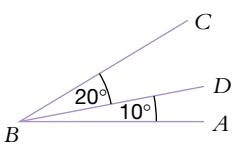
1 Write the complement of each angle.

a 20° **b** 50° **c** 70° **d** 63°

2 Write the supplement of each angle.

a 50° **b** 120° **c** 75° **d** 105°

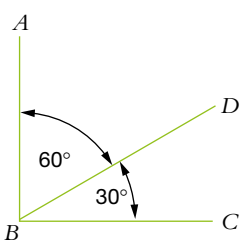
3



a From the diagram, name a pair of adjacent angles.

b Find the size of $\angle ABC$.

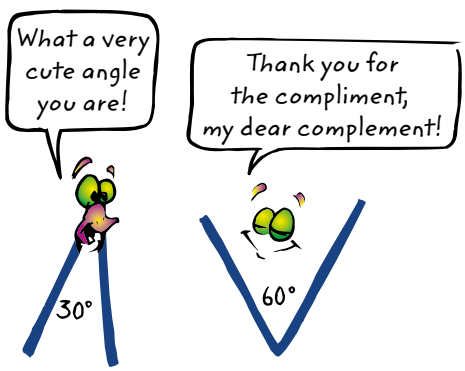
4



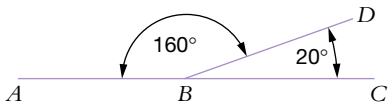
a From the diagram, name a pair of adjacent angles.

b Name a pair of complementary angles.

c Find the size of $\angle ABC$.

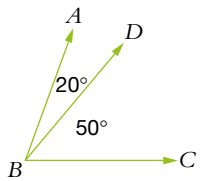


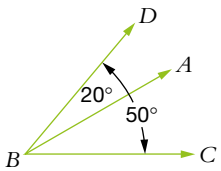
5

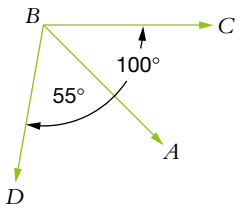


a From the diagram, name a pair of adjacent angles.
b Name a pair of supplementary angles.
c Is $\angle ABC$ a straight angle?

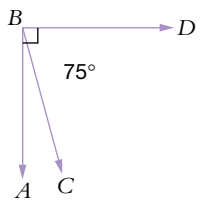
6 Find the size of $\angle ABC$ in each of the following.

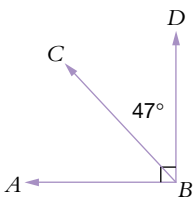
a 

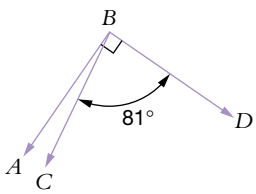
b 

c 

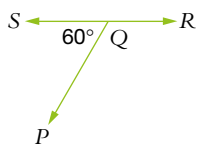
7 Each diagram shows a pair of adjacent complementary angles. In each case, find the size of $\angle ABC$.

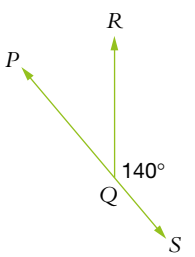
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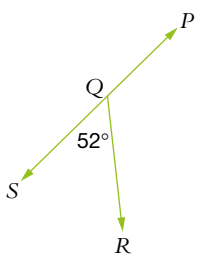
b 

c 

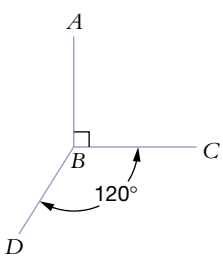
8 Each diagram shows adjacent supplementary angles or angles on a straight line. In each case, find the size of $\angle PQR$.

a 

b 

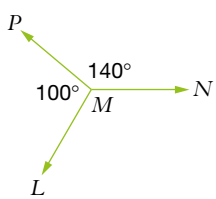
c 

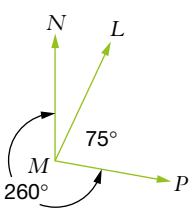
9 Construct the figure shown in the diagram. Can you predict the size of $\angle ABD$ from what you know about an angle of revolution? Check your prediction by measurement.

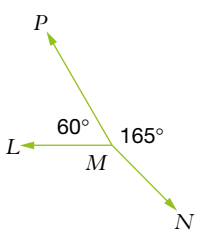


The angles around a point add up to 360°.

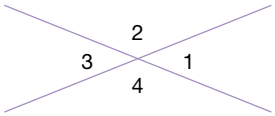
10 These diagrams show angles around a point. What is the size of $\angle LMN$ in each case?

a 

b 

c 

11 When two straight lines cross, two pairs of vertically opposite angles are formed. (In the diagram shown, the pairs are 1, 3 and 2, 4.)



a Which angles look to be equal?

b If the size of angle 1 is 60°, what is the size of:

i angle 2 ii angle 4?

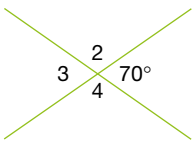
c If you know the size of angles 1, 2 and 4, how can you find the size of angle 3?

d What can you say about vertically opposite angles?

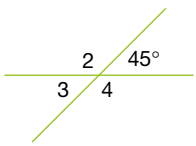
12 In each of the following, what is the value of:

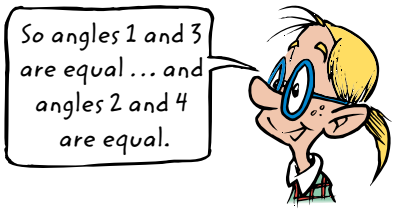
i angle 3 ii angles 2 and 4?

a



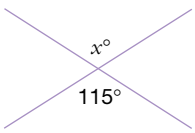
b



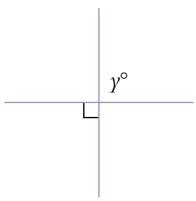


13 Find the value of each pronumeral.

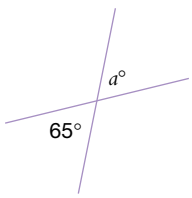
a



b



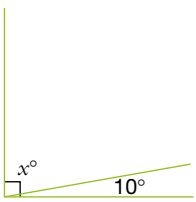
c



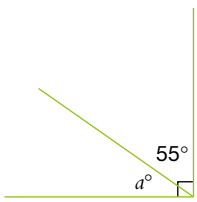
Vertically opposite angles are equal.

14 Find the value of each pronumeral. In each case, give a reason for your answer.

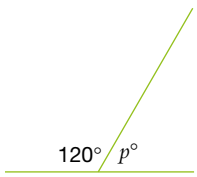
a



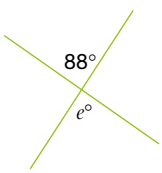
b



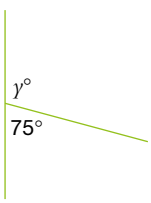
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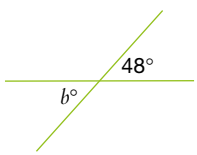
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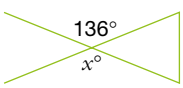
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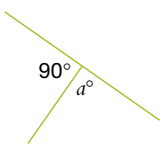
f



g



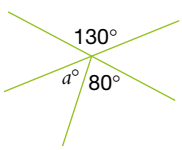
h



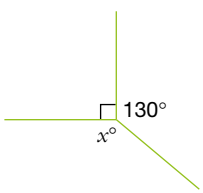
i



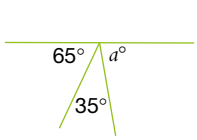
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k

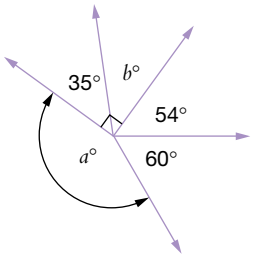


l

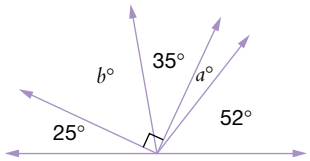


15 Find the value of each pronumeral.

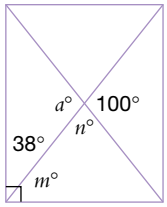
a



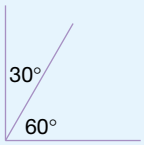
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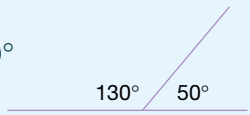
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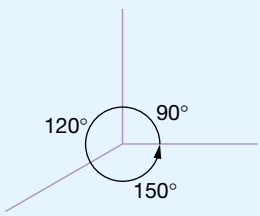
- *Adjacent complementary* angles form a right angle, e.g. 30° + 60° = 90°



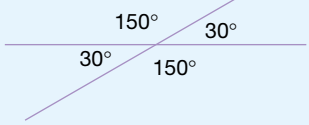
- *Adjacent supplementary* angles form a straight angle, e.g. 130° + 50° = 180°



- *Angles around a point* form an angle of revolution, e.g. 120° + 90° + 150° = 360°



- *Vertically opposite angles*, formed when straight lines intersect, are equal, e.g. 150° = 150°, 30° = 30°



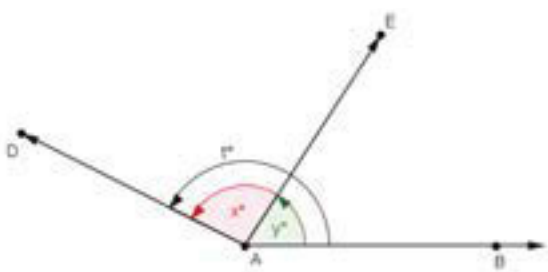
P GEOGEBRA ACTIVITY 5:04 DISCOVERING MORE ABOUT ANGLES

1 Click and drag points *B* and *D* to vary the size of the angles. Use the tick boxes to show and hide each angle and try to find what its value must be.

2 Make ∠*DAB* a right angle or a straight angle. What do we call angles that add to 90° or 180°?

3 Drag point *D* all the way around so that it sits on point *B*. What is the sum of the angles in a full revolution?





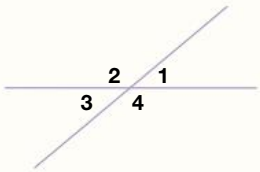
Go to the eBook to find three more GeoGebra activities investigating the work covered in this exercise.

5:05 Angles and parallel lines

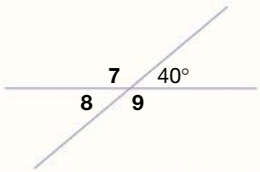
PREP QUIZ 5:05

From the diagram:

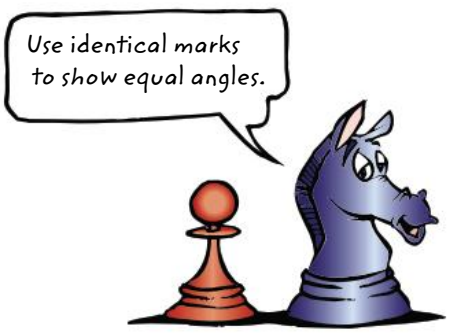
1 Which angle is equal to angle 1?
2 Which angle is equal to angle 2?
3 What can you say about the sizes of angles 2 and 3?
4 What can you say about the sizes of angles 3 and 4?
5 What can you say about the sizes of angles 1 and 4?
6 $\angle 1 + \angle 2 + \angle 3 + \angle 4 = \ldots$

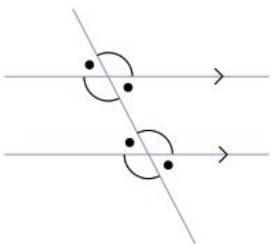


From the diagram at right, find the size of angles **7**, **8** and **9**.

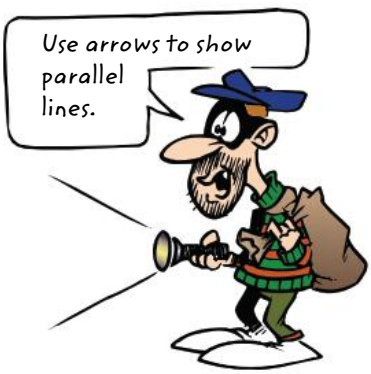


10 Are vertically opposite angles equal?

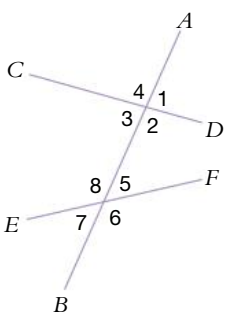


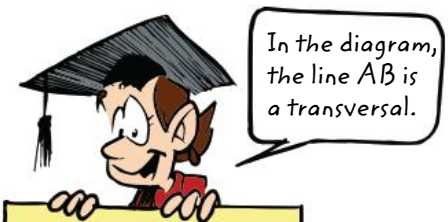


Angles marked in the same way are equal.



If two lines are cut by another line (which is called a **transversal**), then eight angles are formed, as shown:





A **transversal** is a straight line that cuts two or more other lines.

When the two lines are parallel, there are special relationships between the eight angles that are formed. These relationships are investigated in the following exercise.

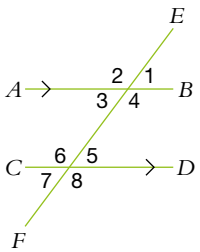
Parallel lines are lines in the same plane that do not meet. They are always the same distance apart.

Exercise 5:05

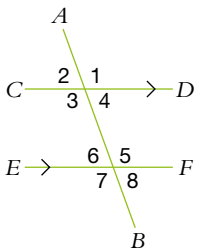
P Challenge worksheet 5:05
Angles and parallel lines

1 In each diagram, name a transversal and a pair of parallel lines.

a



b



2 For each diagram in Question 1, state the angles that are equal.

3 From your answers to Question 2, what did you find about the sizes of the following pairs of angles?

a 1 and 5 **b** 3 and 7 **c** 2 and 6 **d** 4 and 8

The pairs of angles in Question 3 are called **corresponding angles** because they are in corresponding positions relative to the transversal and a parallel line. For instance:

- 1 and 5 are both above a parallel line and to the right of the transversal.
- 2 and 6 are both above a parallel line and to the left of the transversal.

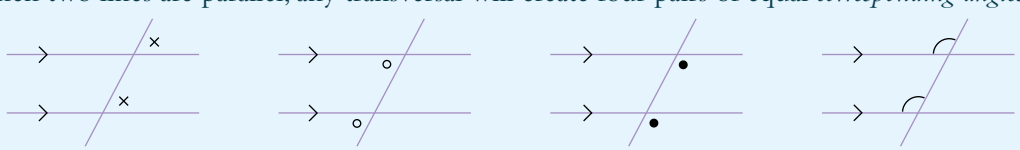
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4 How would you describe the positions of the following angles from Question 1?

a 3 and 7 **b** 4 and 8

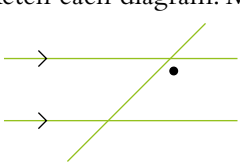


When two lines are parallel, any transversal will create four pairs of equal *corresponding angles*.

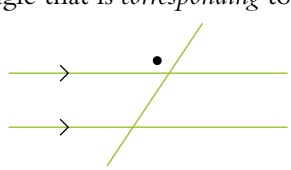


5 Sketch each diagram. Mark the angle that is *corresponding* to the angle marked with the dot.

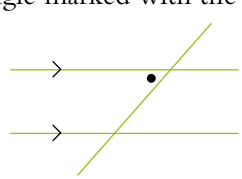
a



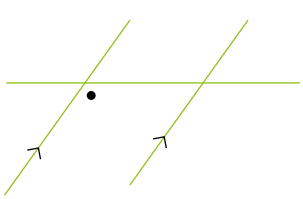
b



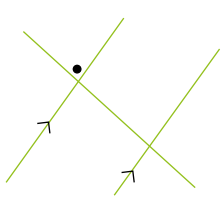
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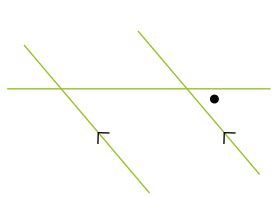
d



e

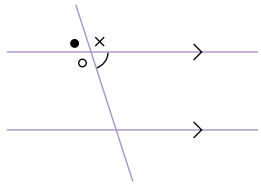


f

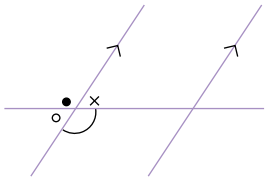


6 Sketch each diagram and mark the four pairs of corresponding angles, using the symbols ×, •, °, and).

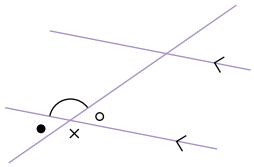
a



b



c

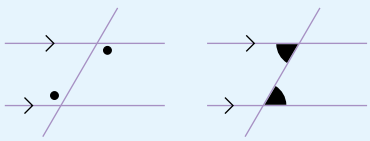


7 Using the diagrams from the start of this exercise, what do you find about the sizes of the following pairs of angles?

a 3 and 5 **b** 4 and 6

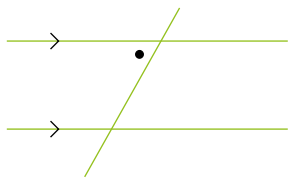
These pairs of angles are called **alternate angles**.

When two lines are parallel, any transversal will create two pairs of equal *alternate angles*.

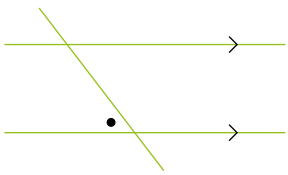


8 Sketch each diagram. Mark the angle that is *alternate* to the angle marked with the dot.

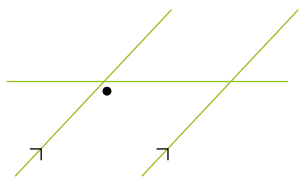
a



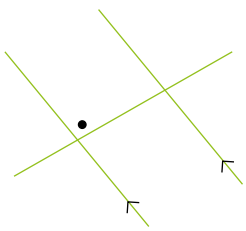
b



c

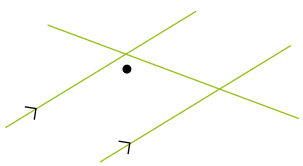


d



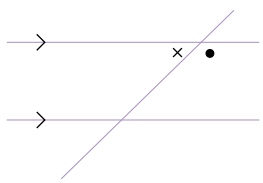


e

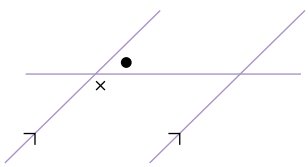


9 Sketch each diagram. Mark the pairs of alternate angles using the symbols × and •.

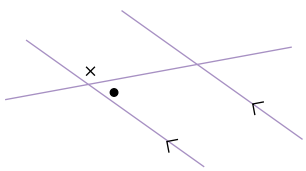
a



b



c



10 Using the diagrams from the start of this exercise, what do you find about the sizes of the following pairs of angles?

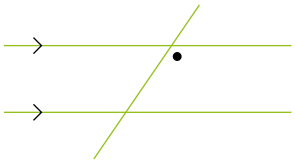
a 4 and 5 **b** 3 and 6

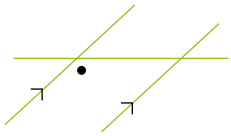
These pairs of angles are called **co-interior angles**. Co-interior angles are between the parallel lines and on the same side of the transversal.

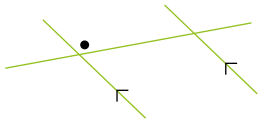
When two lines are parallel, any transversal will create two pairs of supplementary *co-interior angles*.



11 Sketch each diagram. Mark the angle that is *co-interior* to the angle marked with the dot.

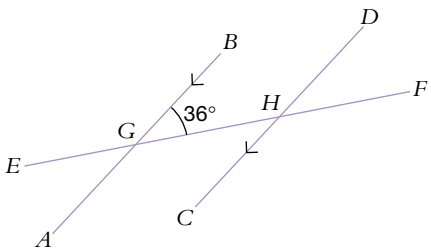
a 

b 

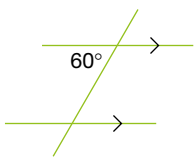
c 

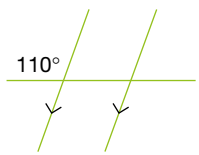
12 From the diagram:

a Which angle is co-interior to $\angle BGH$?
b Which angle is co-interior to $\angle CHG$?
c Which angle is alternate to $\angle AGH$?
d Which angle is alternate to $\angle BGH$?
e Which angle is corresponding to $\angle EGB$?
f Which angle is corresponding to $\angle CHF$?
g Name the three other angles that measure 36°.
h What is the size of each of the other four angles?

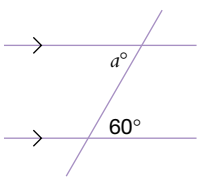


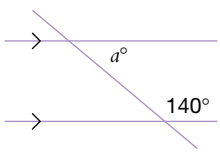
13 Copy the following diagrams and state the sizes of all the missing angles.

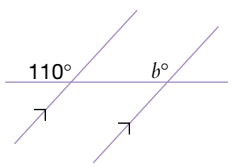
a 

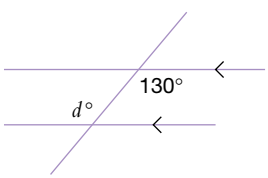
b 

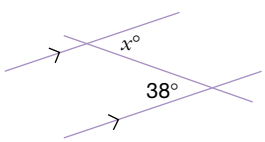
14 Find the value of each pronumeral by first identifying corresponding, alternate or co-interior angles.

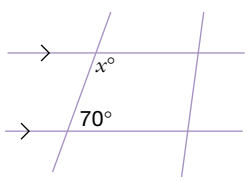
a 

b 

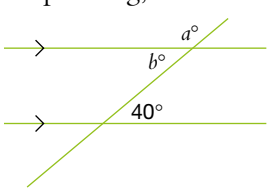
c 

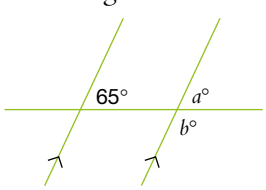
d 

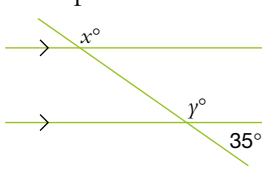
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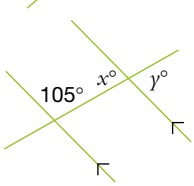
f 

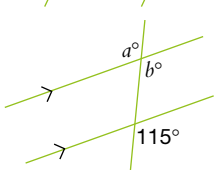
15 Use the properties of vertically opposite angles, adjacent angles making straight angles, and corresponding, alternate and co-interior angles to find the value of each pronumeral.

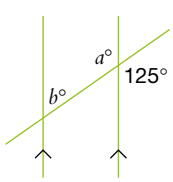
a 

b 

c 

d 

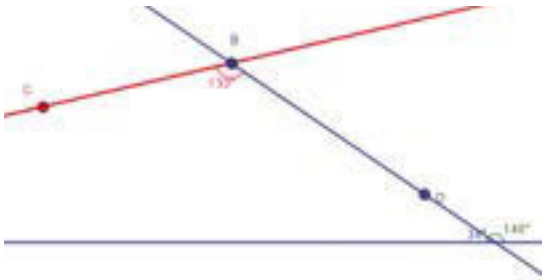
e 

f 

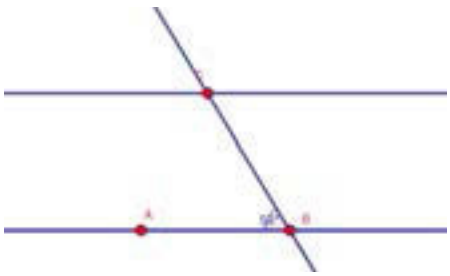
Refer to **ID Card 6** on page xix and identify figures **1** to **20**. Learn the terms that you do not know.

P GEOGEBRA ACTIVITY 5:05 ANGLES AND PARALLEL LINES

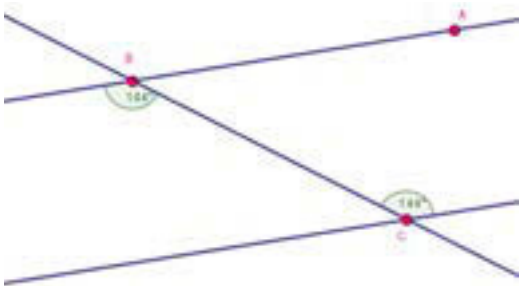
1 Rotate the red line about B until it is parallel to the blue line. What you notice about the two angles that appear?



2 Given the value of $\angle ABC$, can you work out the values of the other seven angles? Tick the box to check your answers.



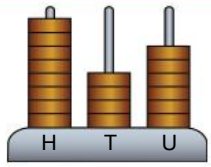
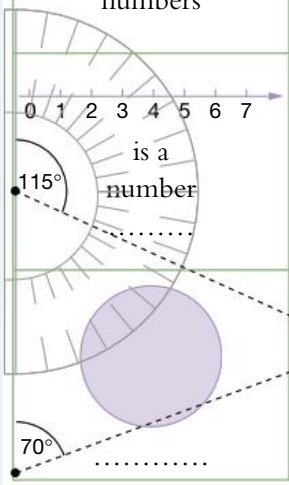
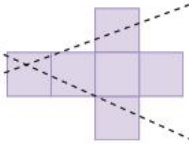
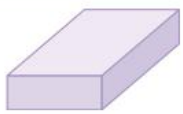
3 Choose a pair of angles to investigate (alternative, co-interior or corresponding). Click and drag points A, B and C. What do you notice about the value of the angle in each pair?

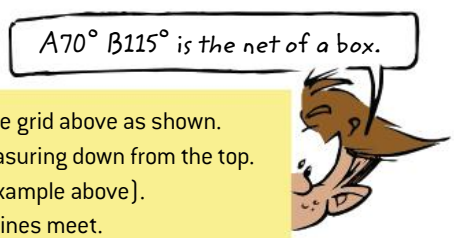




You can see a set of parallel lines and transversals in the timber frame of this house.

CHALLENGE 5:05 GETTING THE ANGLE ON THINGS—TRIANGULATION

2, 4, 6, … are ..... numbers	1, 3, 5, … are ..... numbers	814 has three	 .....
	 line of		 .....
.....	 ..... of a box	 ..... prism	 .....

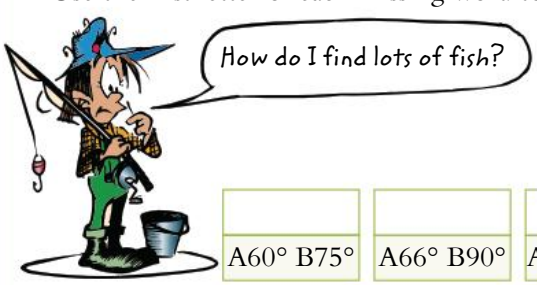


- To solve this challenge, place your protractor on the side of the grid above as shown.
- Find the angles from A and from B to the reference given, measuring down from the top. Imagine dashed lines extending for these angles (as in the example above).
- Write the missing word from the square in which the dashed lines meet.

1 Find the missing word for each reference.

a A55° B85°	**b** A10° B25°	**c** A70° B115°	**d** A40° B60°
e A60° B75°	**f** A52° B70°	**g** A70° B87°	**h** A25° B70°
i A66° B90°	**j** A80° B105°	**k** A50° B140°	**l** A85° B102°

2 Use the first letter of each missing word to decode the answer to the question below.



	A60° B75°	A66° B90°	A85° B102°	A10° B25°	A70° B115°	A52° B70°
A55° B85°	A50° B140°	A70° B87°	A40° B60°	A40° B60°	A25° B70°	A55° B85°

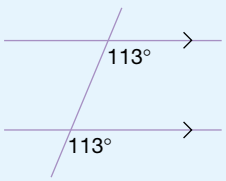
5:06 Identifying parallel lines

To determine whether a pair of lines are parallel, we can extend them in both directions to see if they meet. However, even if it seems they do not meet, we can't be sure that they will never meet (i.e. are parallel)—we may not have extended the lines far enough.

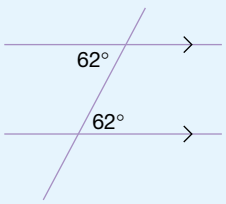
In real life, there are also problems caused by inaccurate drawing and measurement. In diagrams, however, knowing the size of angles around a transversal will tell us for sure whether two lines are parallel.

Two lines are parallel if:

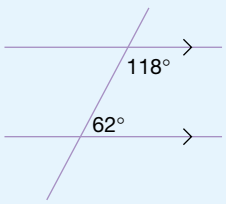
- corresponding angles are equal



- alternate angles are equal



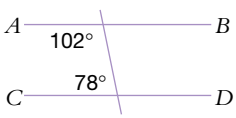
- co-interior angles are supplementary.



In each of these cases, the lines must be parallel.

WORKED EXAMPLE

Is AB parallel to CD?

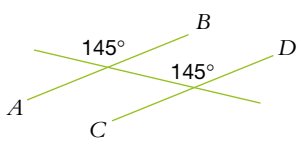


Yes. $AB \parallel CD$ because co-interior angles are supplementary.

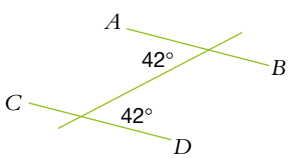
Exercise 5:06

1 In each case, is AB parallel to CD? Give reasons.

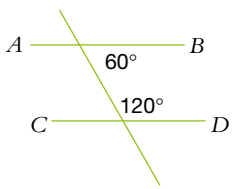
a



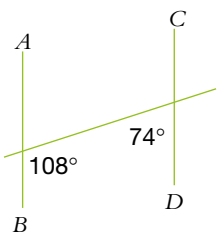
b



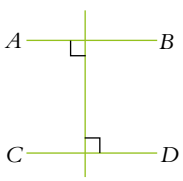
c



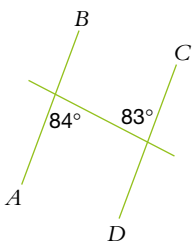
d



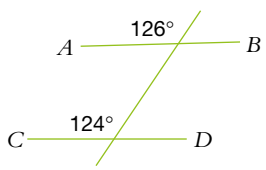
e



f



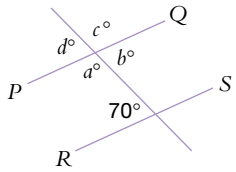
g



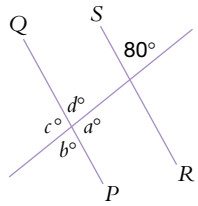
$\parallel$ means 'is parallel to'
$\perp$ means 'is perpendicular to'

2 In each of the following diagrams, PQ is parallel to RS. What are the pronumeral values needed for this to be true?

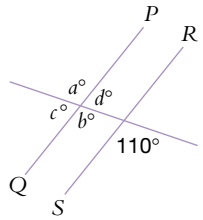
a



b



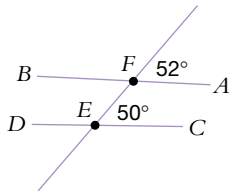
c



3 The lines AB and CD are not parallel. AB and CD can rotate about the points F and E respectively.

a If DC was fixed, would A have to move clockwise or anticlockwise for the lines to become parallel?

b If AB was fixed, would C have to move clockwise or anticlockwise for the lines to become parallel?



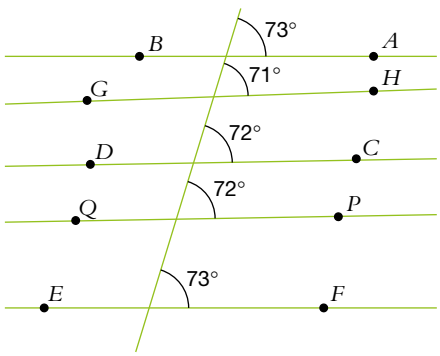
4 This diagram shows five lines that are parallel or almost parallel to each other. For each of the following pairs of lines, state if they are parallel or not. If they are not parallel, state on which side of the transversal they cross.

a AB and EF

b GH and PQ

c CD and PQ

d AB and CD

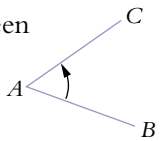




Parallel lines appear in many things and places. It is usually easy to tell if a line is not parallel to the others.

MATHS TERMS 5

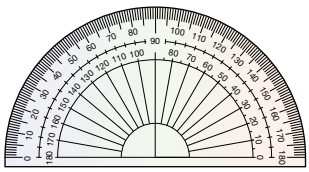
angle
- the amount of turning between two rays (half-lines) that start from the same point



arm
- a ray of an angle

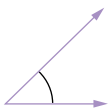
vertex
- the point where the two arms of an angle meet

protractor
- an instrument used for measuring the size of an angle

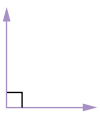


degree
- a unit used to measure angles, indicated by the symbol '°', e.g. 60°

acute angle
- an angle less than 90°



right angle
- a 90° angle



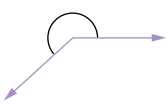
obtuse angle
- an angle between 90° and 180°



straight angle
- a 180° angle



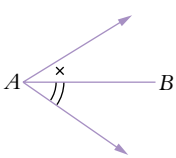
reflex angle
- an angle between 180° and 360°



revolution
- a 360° angle
- one complete turn or rotation



adjacent angles
- angles that have the same vertex and are on opposite sides of a common arm



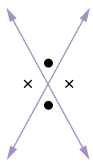
complementary angles
- two angles that add to 90°

complement
- the angle that needs to be added to a given angle to give 90°, e.g. The complement of 60° is 30°.

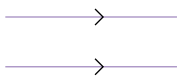
supplementary angles
- two angles that add to 180°

supplement
- the angle that needs to be added to a given angle to give 180°, e.g. The supplement of 120° is 60°.

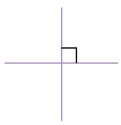
vertically opposite angles
- the equal angles formed when two straight lines intersect



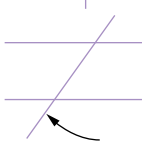
parallel lines
- straight lines, in the same plane, that do not meet



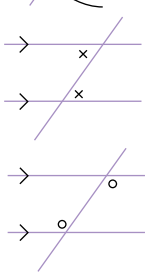
perpendicular lines
- straight lines that meet at right angles



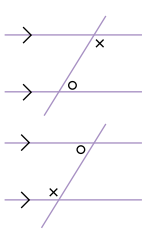
transversal
- a line that crosses two or more other lines



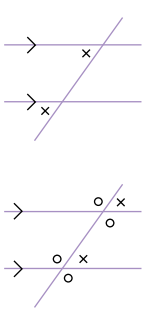
alternate angles
- angles that are 'inside' straight lines and on opposite sides of a transversal
- alternate angles are equal if the lines are parallel



co-interior angles
- angles that are 'inside' straight lines and on the same side of a transversal
- co-interior angles are supplementary if the lines are parallel



corresponding angles
- angles that are in corresponding (matching) positions relative to straight lines and a transversal
- corresponding angles are equal if the lines are parallel
- There are 4 pairs of corresponding angles marked in these two diagrams.

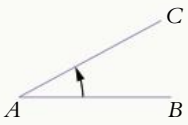


DIAGNOSTIC TEST 5 ANGLES

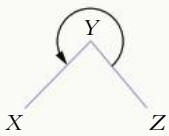
Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Name the following angles and their angle type. 5:01, 5:03

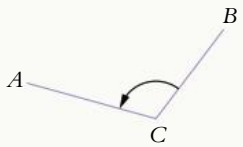
a



b

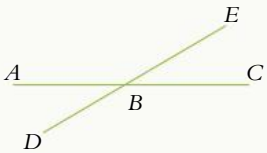


c

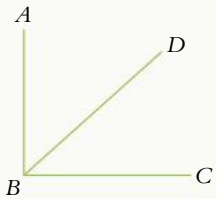


2 Copy each figure and mark $\angle ABC$ with double curved lines '))'. 5:01

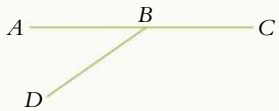
a



b

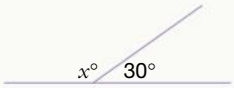


c

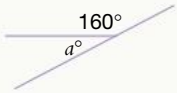


3 Find the value of each pronumeral. 5:04

a



b

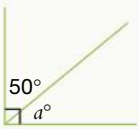


c

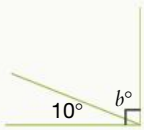


4 Find the value of each pronumeral. 5:04

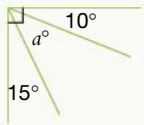
a



b

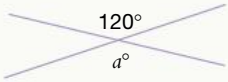


c

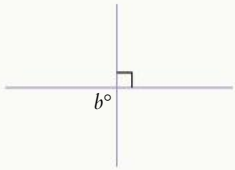


5 Find the value of each pronumeral. 5:05

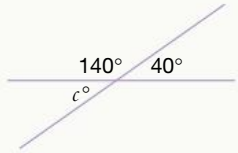
a



b

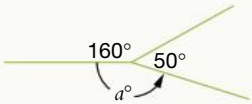


c

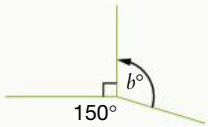


6 Find the value of each pronumeral. 5:05

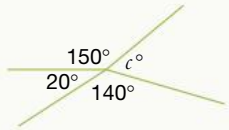
a



b

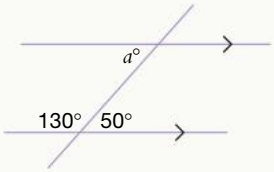


c

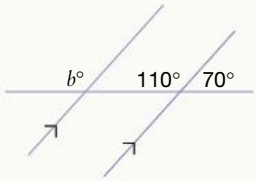


7 Find the value of each pronumeral. 5:05

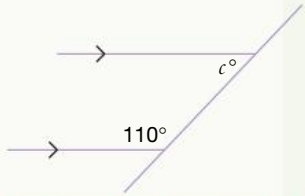
a



b



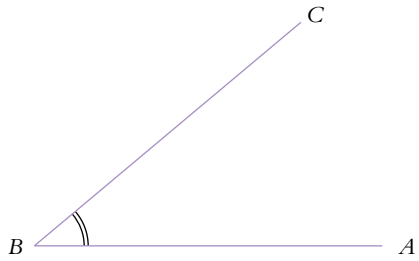
c



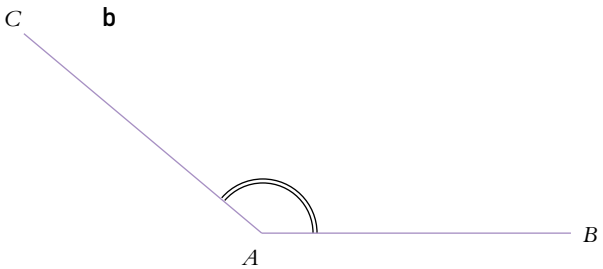
ASSIGNMENT 5A Chapter review

1 In each of the following, name the angle, identify what type of angle it is and measure its size to the nearest degree.

a

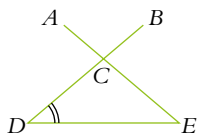


b

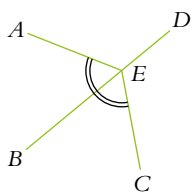


2 In each of the following, name the angle marked with double curved lines '))'.

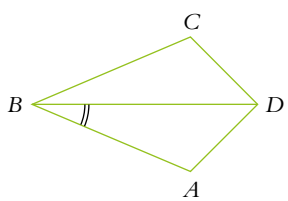
a



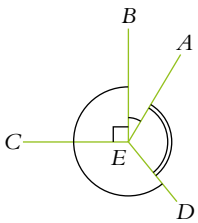
b



c



3 From the diagram, what type of angle is each of the following?



a $\angle AED$

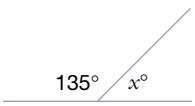
b $\angle BEC$

c $\angle AEB$

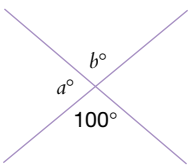
d $\angle BED$

4 Find the value of each pronumeral, giving a reason for your answer.

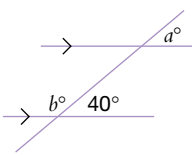
a



b

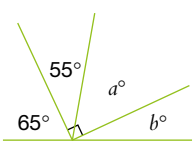


c

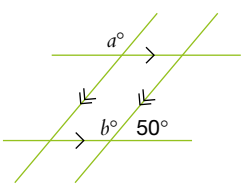


5 Find the value of each pronumeral.

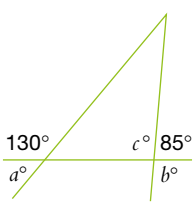
a



b



c



ASSIGNMENT 5B Working mathematically

1 A salesperson drove from Town A to Town B and back again. At the start of the journey, her car's odometer read 21 857 km, while at the end of her journey it read 22 173 km. How far was Town A from Town B?

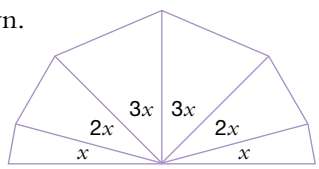
2 Jan works 4 hours each day from Monday to Friday and 6 hours on Saturday. He is paid $14.50 an hour from Monday to Friday and $21.75 an hour on Saturday. What are his weekly earnings?



Global Positioning System (GPS) technology is based on a thorough knowledge of angles.

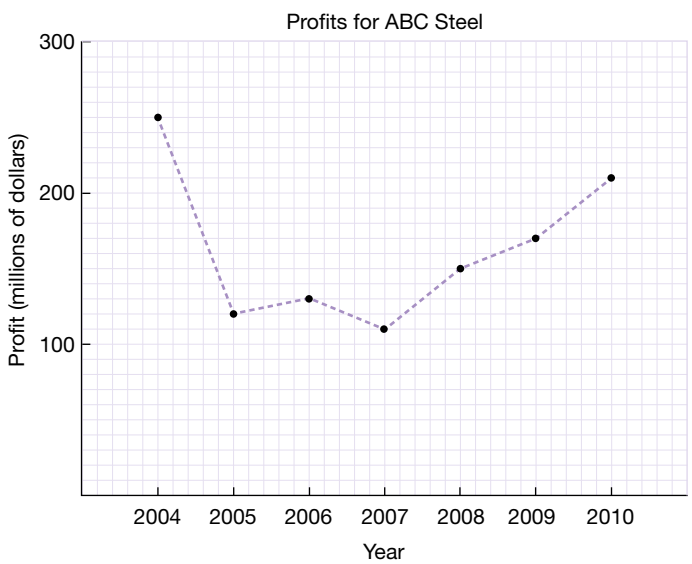
3 At the hardware store I buy twelve 150 mm bolts and eight 100 mm bolts. I notice that the longer bolt costs 80c more than the smaller bolt. If the total cost is $35.60, what is the price of the 100 mm bolt?

4 An architect designs a symmetrical window with angles as shown. What is the value of x (in degrees)?



5 The time on a clock is correct at midday, but it loses 3 minutes every hour. What time does the clock show at 9 am the next day?

6



a What was the profit in:
- **i** 2004
- **ii** 2008?

b When was the largest profit made?

c When was the smallest profit made?

d By how much did the profit fall from 2004 to 2005?

e In which years was the profit greater than $150 million?

f Do the points that are between the dots on the line have meaning, other than to give an impression of the trend?

ASSIGNMENT 5C Cumulative revision

1 **a** Write the numeral for one million three hundred and twenty-six thousand seven hundred and sixty-three. 1:02

b Write 16 020 in words.

c What is the value of the 1 in 716 250?

d Write 42 375 in expanded notation.

2 Simplify: 1:06

a $(4 + 3) \times (8 - 2)$ **b** $17 - (12 - [2 + 10])$

c $\dfrac{25 + 35}{5}$ **d** $\dfrac{12}{1 + 3}$

3 Simplify: 1:06

a $16 + 2 \times 4 - 5$ **b** $18 + 24 \div (10 - 4)$

c $200 - 120 \div 4 - 3 \times 5$ **d** $18 \div [54 - (24 \times 2)]$

4 Are the following true or false? 1:08

a $6 \times 96 > 6 \times 100$ **b** $0{\cdot}3 < 0{\cdot}0333$ **c** $\frac{1}{10} < \frac{1}{2}$ **d** $\frac{12}{2} = \frac{2}{12}$

5 Complete each number sentence to make it true. 1:07

a $6 \times (300 + 6) = \square \times 300 + \square \times 6$ **b** $73 \times 10 = 10 \times \square$

c $6 \times 199 = 6 \times 200 - 6 \times \square$ **d** $98 + 47 + 2 = 98 + \square + 47$

6 Change these Roman numerals into our modern numerals. 1:01

a LXXXVII **b** CCXCIII **c** MDCLXIV **d** MCMXC

7 Write each in expanded form and as a basic numeral. 3:01

a 4^3 **b** 3^3 **c** 10^6 **d** 2×10^4 **e** $3^2 \times 2^4$

8 Convert to decimals: 1:11C

a $\frac{7}{10}$ **b** $\frac{42}{100}$ **c** $\frac{175}{1000}$ **d** $\frac{5}{100}$ **e** $\frac{27}{1000}$

9 **a** What is the lowest common multiple of 6 and 4? 3:03

b What is the highest common factor of 15 and 30?

c What is the greatest common divisor of 48 and 60?

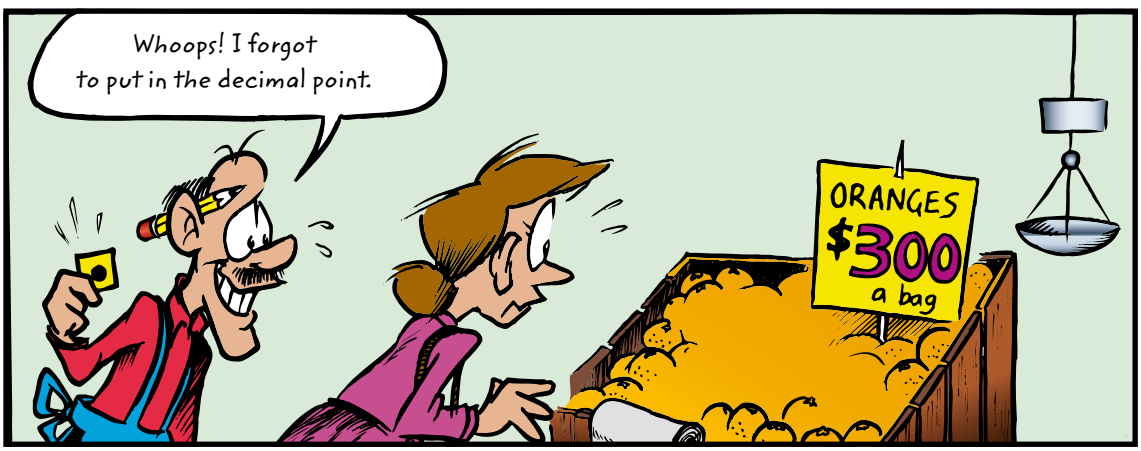
d What numbers are factors of 200 and also multiples of 50?

10 If $x = 20$ and $y = 30$, what is the value of: 4:06

a $x + y$ **b** xy **c** $2x - 10$ **d** $3x - 2y$ **e** $3 \times (x - 5)$

DECIMALS

6



Contents

6:01 Review of decimals
6:02 Addition and subtraction of decimals
6:03 Multiplying a decimal by a whole number
6:04 Dividing a decimal by a whole number
6:05 Multiplying a decimal by a decimal
6:06 Dividing by a decimal
Fun spot 6:06 Why did the orange stop in the middle of the road?
6:07 Changing fractions to decimals
6:08 Rounding
6:09 Applications of decimals
6:10 Using a calculator
Fun spot 6:10 Olympic diving
Investigation 6:10 Applying decimals
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Fractions and Decimals 2*** [Stage 3], ***Fractions, Decimals and Percentages*** [Stage 4]

- Add and subtract decimals, with and without the use of digital technologies, and use estimation and rounding to check the reasonableness of answers. [Stage 3] (ACMNA128)
- Multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without the use of digital technologies. [Stage 3] (ACMNA129)
- Multiply and divide decimals by powers of 10. [Stage 3] (ACMNA130)
- Make connections between equivalent fractions, decimals and percentages. [Stage 3] (ACMNA131)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies. (ACMNA154)
- Round decimals to a specified number of decimal places. (ACMNA156)
- Connect fractions, decimals and percentages and carry out simple conversions. (ACMNA157)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

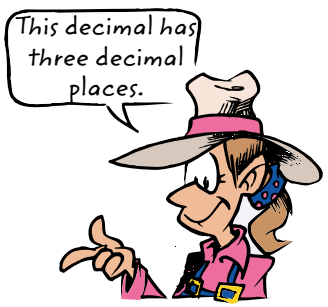
6:01 Review of decimals

With our world's increasing dependence on computers and other digital devices, decimals are extremely important. Digital technologies must use decimals in their internal calculations and in their output. Where parts of a whole number are to be shown, a decimal point is used.

WORKED EXAMPLE

Write 734·625 in expanded notation.

Hundreds	Tens	Units		Tenths	Hundredths	Thousandths
7	3	4	•	6	2	5



Solution

In expanded form, this is:

$(7 \times 100) + (3 \times 10) + (4 \times 1) + (6 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

Exercise 6:01

P Foundation worksheet 6:01 Review of decimals

1 True or false?

a 0·3 means 3 of 10 equal parts.
b 0·4 is the same 0·40.
c 0·8 is equal to $\frac{8}{10}$.
d $2 \times \frac{1}{100} = \frac{2}{100}$
e 0·14 has two decimal places.
f The value of the 7 in 6·174 is $\frac{7}{100}$.
g 0·7 is the same as 0·007.
h 0·35 is the same as $\frac{35}{100}$.
i 7 = 7·00
j Ten lots of 0·1 gives 1.

WORKED EXAMPLE

$\frac{7}{100} + \frac{5}{1000}$

= 0·075

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	•	0	7	5



2 Write each sum as a decimal.

a $\frac{6}{10} + \frac{4}{100}$
b $\frac{1}{10} + \frac{7}{100}$
c $\frac{2}{10} + \frac{3}{100} + \frac{8}{1000}$
d $\frac{5}{100} + \frac{1}{1000}$
e $\frac{3}{10} + \frac{9}{1000}$
f $\frac{6}{100} + \frac{6}{1000}$
g $2 + \frac{3}{10} + \frac{5}{100}$
h $9 + \frac{4}{10}$
i $15 + \frac{3}{10} + \frac{6}{100}$
j $11 + \frac{2}{100}$
k $6 + \frac{8}{100} + \frac{1}{1000}$
l $2 + \frac{5}{10} + \frac{3}{1000}$

3 Write these decimals in expanded form.

a 7·342	**b** 3·483	**c** 4·215
d 45·03	**e** 30·75	**f** 40·07
g 256·04	**h** 24·125	**i** 69·345
j 246·379	**k** 482·375	**l** 238·667

4 Which is smaller?

a 0·7 or 1	**b** 0·05 or 2	**c** 3 or 1·2
d 0·4 or 0·05	**e** 0·7 or 0·69	**f** 0·03 or 0·22
g 0·2 or 0·1	**h** 2·8 or 4·1	**i** 0·99 or 0·612
j 0·5 or 0·77	**k** 0·05 or 0·049	**l** 0·333 or 0·29

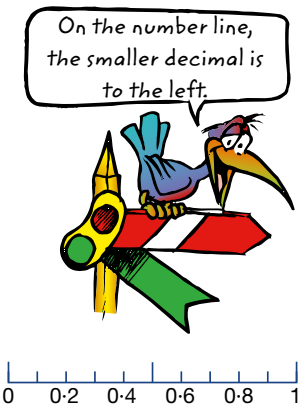
To compare decimals, give them the same number of decimal places, e.g.

0·3	0·300
0·33	0·330
0·303	0·303

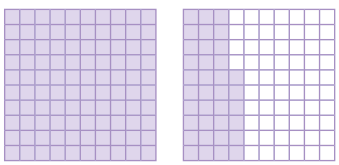
'Ascending order' means 'smallest to largest'.

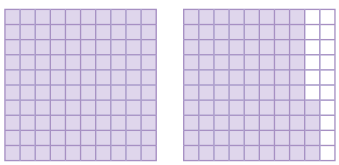
5 Arrange in ascending order.

a 0·3, 0·8, 0·6	**b** 3·85, 1·2, 7·5
c 0·8, 1, 0·55	**d** 3, 0·3, 0·03
e 0·3, 0·33, 0·03	**f** 0·39, 0·41, 0·4
g 0·85, 0·9, 0·792	**h** 0·1, 0·09, 0·8

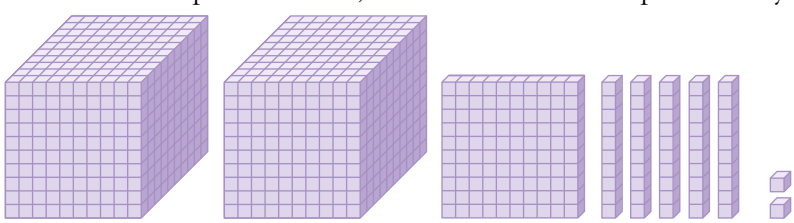


6 Each hundred square represents one whole. In each example, what decimal is shaded?

a 

b 

7 If the large cube represents one whole (1·0), the 'flat' represents 0·1, the 'long' represents 0·01 and the 'mini' represents 0·001, then what number is represented by the following?



8 How many decimal places in each number?

a 8·14	**b** 23·123	**c** 963·5
d 0·6	**e** 0·06	**f** 0·066
g 5·068	**h** 15·03	**i** 900·1
j 463·05	**k** 6·004	**l** 0·7

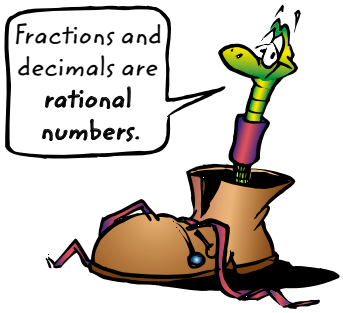


9 Write as a decimal:

a $\frac{23}{100}$	**b** $\frac{12}{100}$	**c** $\frac{7}{1000}$	**d** $\frac{125}{1000}$
e $\frac{15}{1000}$	**f** $\frac{604}{1000}$	**g** $2\frac{93}{100}$	**h** $15\frac{103}{1000}$
i $10\frac{9}{100}$	**j** $7\frac{999}{1000}$	**k** $11\frac{33}{1000}$	**l** $8\frac{5}{1000}$

10 Write as a fraction:

a 0·7	**b** 0·9	**c** 0·3
d 0·21	**e** 0·77	**f** 0·91
g 0·447	**h** 0·601	**i** 0·853
j 2·99	**k** 8·3	**l** 9·007



11 Write as a decimal:

a $\frac{14}{10}$	**b** $\frac{86}{10}$	**c** $\frac{73}{10}$	**d** $\frac{724}{100}$
e $\frac{806}{100}$	**f** $\frac{903}{100}$	**g** $\frac{1186}{1000}$	**h** $\frac{7406}{1000}$
i $\frac{8014}{1000}$	**j** $\frac{4504}{100}$	**k** $\frac{6504}{10}$	**l** $\frac{842}{1000}$

12 Write in simplest fraction form:

a 0·30	**b** 0·3	**c** 0·300
d 0·7	**e** 0·70	**f** 0·700
g 0·01	**h** 0·010	**i** 0·0100
j 0·090	**k** 0·030	**l** 0·0010

0·3 = 0·30 = 0·300
Zeros at the end of a decimal do not change its value.

13 What is the value of the 5 in each number?

a 24 560·7	**b** 28·53	**c** 0·125	**d** 6·25
e 0·005	**f** 7·152	**g** 4·6125	**h** 0·054

14 True or false?

a $14 + \frac{6}{100} = 14{\cdot}6$	**b** $0{\cdot}97 > 1$	**c** $5 + 0{\cdot}007 < 5 + 0{\cdot}07$
d $0{\cdot}606 < 0{\cdot}66$	**e** $0{\cdot}125 = 0{\cdot}1 + 0{\cdot}02 + 0{\cdot}005$	**f** $0{\cdot}083 = 0{\cdot}08 + 0{\cdot}003$
g 10 lots of 0·01 equals 0·1.	**h** 10 lots of 0·001 equals 0·01.	

6:02 Addition and subtraction of decimals

The column in which a digit is placed determines its size. The decimal point separates the whole number from the parts of a whole, so the decimal points must be lined up under one another.

To add or subtract decimals, use the PUP rule: place Points Under Points.
Any empty space can be filled with a zero.

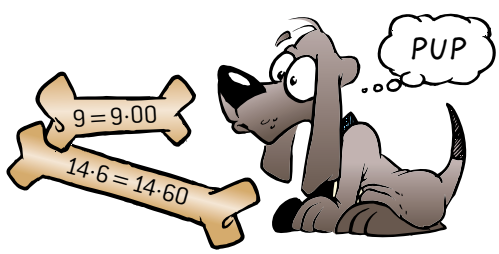
WORKED EXAMPLES

1 $14{\cdot}6 + 3{\cdot}15 + 9$

$$\begin{array}{r} 14{\cdot}60 \\ 3{\cdot}15 \\ +\ 9{\cdot}00 \\ \hline 26{\cdot}75 \end{array}$$

2 $97{\cdot}3 - 8{\cdot}95$

$$\begin{array}{r} 97{\cdot}30 \\ -\ 8{\cdot}95 \\ \hline 88{\cdot}35 \end{array}$$



Exercise 6:02

P Foundation worksheet 6:02
+ and − of decimals

1 **a**
$$\begin{array}{r} 3{\cdot}85 \\ 1{\cdot}6 \\ +\ 0{\cdot}41 \\ \hline \end{array}$$

b
$$\begin{array}{r} 6 \\ 5{\cdot}8 \\ +\ 4{\cdot}25 \\ \hline \end{array}$$

c
$$\begin{array}{r} 9{\cdot}175 \\ 4{\cdot}7 \\ +\ 9{\cdot}36 \\ \hline \end{array}$$

d
$$\begin{array}{r} 26\,459{\cdot}6 \\ +\ 9006{\cdot}8 \\ \hline \end{array}$$

e
$$\begin{array}{r} 54\,000 \\ +\ 12\,673{\cdot}5 \\ \hline \end{array}$$

f
$$\begin{array}{r} 3{\cdot}1486 \\ +\ 9{\cdot}852 \\ \hline \end{array}$$

g 9·08 + 12·16
h 4·018 + 0·582
i 6·9 + 3·1
j 12 + 0·12
k 6·3 + 8·05
l 150·86 + 9·1
m 0·9 + 15 + 3·125
n 8·6 + 9·3 + 0·75
o 8·25 + 6·85 + 9·125
p 0·185 + 0·25 + 0·03

We write tenths under tenths, hundredths under hundredths, and thousandths under thousandths.

2 **a**
$$\begin{array}{r} 12{\cdot}30 \\ -\ 9{\cdot}85 \\ \hline \end{array}$$

b
$$\begin{array}{r} 15{\cdot}4 \\ -\ 6{\cdot}75 \\ \hline \end{array}$$

c
$$\begin{array}{r} 8{\cdot}35 \\ -\ 1{\cdot}148 \\ \hline \end{array}$$

d
$$\begin{array}{r} 18{\cdot}000 \\ -\ 6{\cdot}375 \\ \hline \end{array}$$

e
$$\begin{array}{r} 25 \\ -\ 19{\cdot}625 \\ \hline \end{array}$$

f
$$\begin{array}{r} 9 \\ -\ 0{\cdot}025 \\ \hline \end{array}$$

g
$$\begin{array}{r} 9{\cdot}68 \\ -\ 2{\cdot}90 \\ \hline \end{array}$$

h
$$\begin{array}{r} 40{\cdot}125 \\ -\ 9{\cdot}04 \\ \hline \end{array}$$

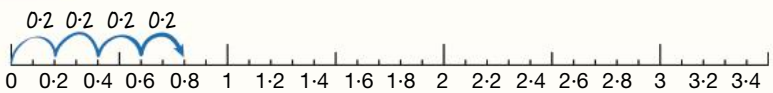
i
$$\begin{array}{r} 37{\cdot}889 \\ -\ 7{\cdot}8 \\ \hline \end{array}$$

j 9·3 − 1·6
k 6·75 − 0·45
l 8·875 − 6·125
m 9 − 0·366
n 8·4 − 0·11
o 23·8 − 0·08
p 8·799 − 3·89
q 1·086 − 0·7
r 3·467 − 0·35
s 689·5 − 18·25
t 634·8 − 115·85
u 4725 − 6·845



6:03 Multiplying a decimal by a whole number

PREP QUIZ 6:03



Use the number line above to find:

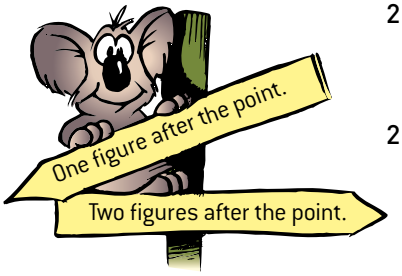
1 4 lots of 0·2
2 5 lots of 0·4
3 6 lots of 0·4
4 3 lots of 0·6
5 4 lots of 0·8
6 5 lots of 0·5
7 4 × 0·3
8 5 × 0·6
9 3 × 0·7
10 (7 lots of 0·2) + (3 lots of 0·6)

When multiplying a decimal by a whole number, the number of digits after the decimal point is the same in the question as in the answer.

WORKED EXAMPLES

1 $2{\cdot}8 \times 5$

2 $\$0.85 \times 7$





Solutions

1
$$\begin{array}{r} 2{\cdot}8 \\ \times\ 5 \\ \hline 14{\cdot}0 \end{array}$$

2
$$\begin{array}{r} \$0.85 \\ \times\ 7 \\ \hline \$5.85 \end{array}$$

Multiplying by 10, 100 and 1000:

3 $0{\cdot}81 \times 10$
$= 8{\cdot}10$
$= 8{\cdot}1$

4 $0{\cdot}031 \times 100$
$= 3{\cdot}100$
$= 3{\cdot}1$

5 $1{\cdot}015 \times 1000$
$= 1015{\cdot}000$
$= 1015$

6 $0{\cdot}6 \times 1000$
$= 600{\cdot}0$
$= 600$

From the worked examples above, we can see an easier way to get the answer:

> When we multiply by 10, 100 or 1000, we move the decimal point 1, 2, or 3 places to the right.

From the worked examples:

3 $0{\cdot}81 \times 10$
$= 8{\cdot}1$

4 $0{\cdot}031 \times 100$
$= 3{\cdot}1$

5 $1{\cdot}015 \times 1000$
$= 3{\cdot}1$

6 $0{\cdot}600 \times 1000$
$= 600$

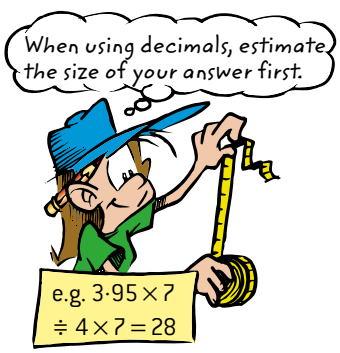
Exercise 6:03

P Foundation worksheet 6:03 Multiplying a decimal

1 **a** $3 \times 0{\cdot}2$ **b** $2 \times 1{\cdot}2$ **c** $4 \times 1{\cdot}1$ **d** $5 \times 0{\cdot}08$ **e** $7 \times 0{\cdot}03$
f $8 \times 0{\cdot}06$ **g** $0{\cdot}11 \times 6$ **h** $0{\cdot}7 \times 3$ **i** $0{\cdot}04 \times 9$

2 **a** $\begin{array}{r} 2{\cdot}4 \\ \times\ 6 \\ \hline \end{array}$ **b** $\begin{array}{r} 1{\cdot}9 \\ \times\ 4 \\ \hline \end{array}$ **c** $\begin{array}{r} 0{\cdot}25 \\ \times\ 4 \\ \hline \end{array}$ **d** $\begin{array}{r} 3{\cdot}8 \\ \times\ 5 \\ \hline \end{array}$ **e** $\begin{array}{r} 4{\cdot}62 \\ \times\ 5 \\ \hline \end{array}$

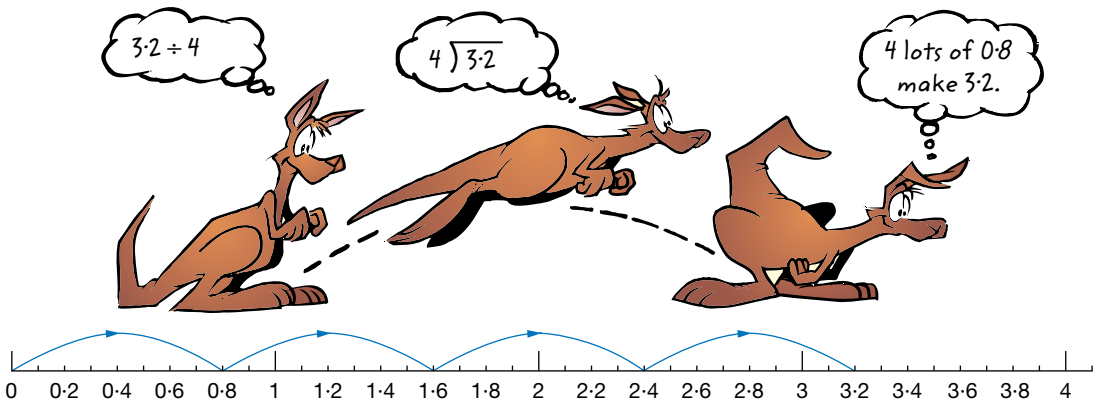
f $\begin{array}{r} 2{\cdot}4 \\ \times\ 5 \\ \hline \end{array}$ **g** $\begin{array}{r} 0{\cdot}63 \\ \times\ 3 \\ \hline \end{array}$ **h** $\begin{array}{r} 2{\cdot}318 \\ \times\ 4 \\ \hline \end{array}$ **i** $\begin{array}{r} 17{\cdot}2 \\ \times\ 8 \\ \hline \end{array}$



3 **a** $3{\cdot}8 \times 2$ **b** $4{\cdot}11 \times 3$ **c** $3{\cdot}08 \times 3$
d $0{\cdot}375 \times 8$ **e** $5{\cdot}11 \times 9$ **f** $6{\cdot}07 \times 5$
g $16{\cdot}43 \times 7$ **h** $26{\cdot}08 \times 6$ **i** $78{\cdot}46 \times 4$

4 **a** $1{\cdot}25 \times 10$ **b** $9{\cdot}125 \times 10$ **c** $84{\cdot}35 \times 10$
d $6{\cdot}5 \times 10$ **e** $13{\cdot}8 \times 10$ **f** $3{\cdot}7 \times 10$
g $2{\cdot}315 \times 100$ **h** $6{\cdot}85 \times 100$ **i** $0{\cdot}12 \times 100$
j $0{\cdot}3 \times 100$ **k** $8{\cdot}1 \times 100$ **l** $0{\cdot}125 \times 100$
m $0{\cdot}1111 \times 1000$ **n** $0{\cdot}015 \times 1000$ **o** $8{\cdot}1 \times 1000$
p $0{\cdot}75 \times 1000$ **q** $30{\cdot}05 \times 1000$ **r** $6{\cdot}75 \times 1000$

6:04 Dividing a decimal by a whole number



When dividing a decimal by a whole number, place the decimal point in the answer above the decimal point in the question.

WORKED EXAMPLES

1 $1{\cdot}624 \div 8$ **2** $62{\cdot}1 \div 6$ **3** $0{\cdot}0136 \div 4$

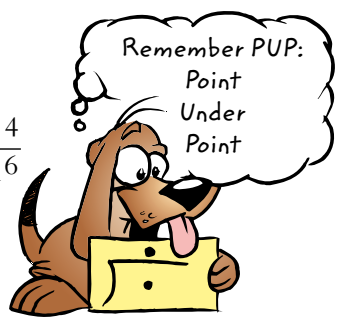
4 $81{\cdot}3 \div 10$ **5** $75{\cdot}84 \div 100$

Solutions

1 $8\overline{)1{\cdot}_{1}6\ 2_{2}4}$ → $0{\cdot}203$

2 $6\overline{)6\ 2{\cdot}_{2}1_{3}0}$ → $10{\cdot}35$

3 $4\overline{)0{\cdot}0\ 1_{1}3_{1}6}$ → $0{\cdot}0034$



Dividing by 10, 100 and 1000:

4 $10\overline{)8\ 1{\cdot}_{1}3_{3}0}$ → $8{\cdot}13$

5 $100\overline{)7\ 5{\cdot}_{75}8_{58}4_{84}0_{40}0}$ → $0{\cdot}7584$

From worked examples **4** and **5** above, we can see an easier way to get the answer:

When we divide by 10, 100 or 1000, we move the decimal 1, 2 or 3 places to the left.

From the worked examples:

4 $81{\cdot}3 \div 10$
$= 8{\cdot}13$

5 $75{\cdot}84 \div 100$
$= 0{\cdot}7584$

And, for example: $000{\cdot}6 \div 1000$
$= 0{\cdot}0006$

Exercise 6:04

P Foundation worksheet 6:04 Dividing a decimal

1 **a** $4\overline{)0\cdot84}$ **b** $3\overline{)6\cdot3}$ **c** $2\overline{)0\cdot008}$ **d** $5\overline{)30\cdot15}$ **e** $4\overline{)12\cdot12}$

f $3\overline{)9\cdot27}$ **g** $9\overline{)82\cdot8}$ **h** $7\overline{)0\cdot014}$ **i** $6\overline{)7\cdot308}$

2 **a** $0{\cdot}35 \div 5$ **b** $9{\cdot}8 \div 2$ **c** $5{\cdot}4 \div 6$

d $0{\cdot}11 \div 11$ **e** $0{\cdot}125 \div 5$ **f** $0{\cdot}027 \div 9$

g $8{\cdot}82 \div 7$ **h** $243{\cdot}2 \div 8$ **i** $1{\cdot}008 \div 8$

3 **a** $2\overline{)0\cdot5}$ **b** $2\overline{)0\cdot7}$ **c** $4\overline{)0\cdot6}$ **d** $5\overline{)1\cdot3}$

e $8\overline{)2\cdot8}$ **f** $4\overline{)1\cdot34}$ **g** $6\overline{)2\cdot19}$ **h** $8\overline{)1\cdot1}$

i $6\overline{)14\cdot1}$ **j** $4\overline{)3\cdot}$ **k** $6\overline{)9\cdot}$ **l** $8\overline{)5\cdot}$

4 **a** $34{\cdot}1 \div 10$ **b** $75{\cdot}5 \div 10$ **c** $64{\cdot}8 \div 10$

d $0{\cdot}6 \div 10$ **e** $0{\cdot}15 \div 10$ **f** $0{\cdot}05 \div 10$

g $751{\cdot}6 \div 100$ **h** $0{\cdot}38 \div 100$ **i** $6784{\cdot}5 \div 100$

j $0{\cdot}04 \div 100$ **k** $0{\cdot}35 \div 100$ **l** $35 \div 100$

m $876{\cdot}4 \div 1000$ **n** $3156{\cdot}4 \div 1000$ **o** $3437 \div 1000$

p $34{\cdot}6 \div 1000$ **q** $8{\cdot}75 \div 1000$ **r** $0{\cdot}8 \div 1000$

5 **a** $1846 \div 1000$ **b** $6{\cdot}75 \div 10$

c $87{\cdot}3 \div 100$ **d** $976{\cdot}5 \div 10$

e $346{\cdot}8 \div 100$ **f** $0{\cdot}8 \div 10$

g $34{\cdot}6 \div 1000$ **h** $4 \div 100$

i $75{\cdot}8 \div 10$

- When dividing a decimal, you can write extra zeros to the decimal.
- You can write extra zeros before the whole number part, too.

WORKED EXAMPLE

$0{\cdot}31 \div 100$

$= 00{\cdot}31 \div 100$

$= 0{\cdot}0031$

6:05 Multiplying a decimal by a decimal

PREP QUIZ 6:05

1 $8 \times 0{\cdot}3$ **2** $0{\cdot}5 \times 6$ **3** $2{\cdot}35 \times 3$

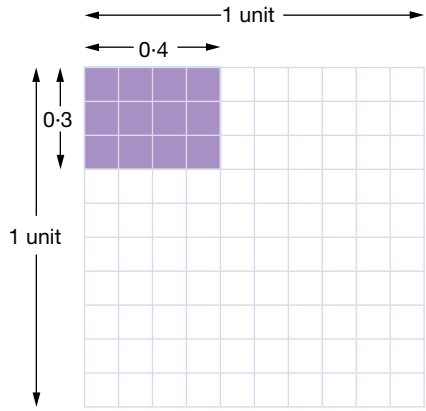
4 $2 \times 0{\cdot}076$ **5** $0{\cdot}35 \times 10$ **6** $10 \times 7{\cdot}1$

7 $0{\cdot}01 \times 8$ **8** $15 \times 0{\cdot}1$ **9** $3{\cdot}105 \times 4$

10 $0{\cdot}3 \times 70$

We need to find a rule for multiplying a decimal by a decimal.

Let the large square here be 1 square unit of area.



$\therefore$ 1 small square $= \frac{1}{100}$ of the large square
$= 0{\cdot}01$ of the large square
$= 0{\cdot}01$ square units

Shade a rectangle 0·4 units long and 0·3 units wide.

Area of shaded rectangle = length × breadth
$= 0{\cdot}4 \times 0{\cdot}3$ square units

But the shaded rectangle has 12 small squares:

Area of shaded rectangle $= \frac{12}{100}$ of large square
$= 0{\cdot}12$ square units

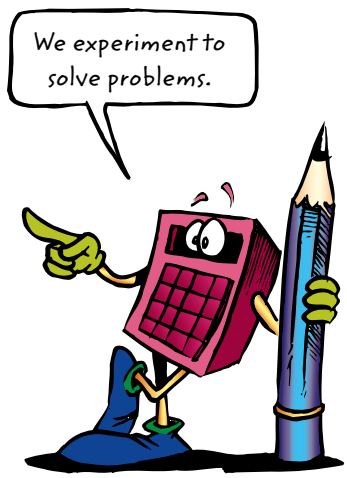
$\therefore$ $0{\cdot}4 \times 0{\cdot}3 = 0{\cdot}12$

It seems the answer must have the same number of digits after its decimal point as the question has after its decimal points altogether.

Let's use a calculator to test this idea.

Copy this table into your workbook and use a calculator to complete it.

Question	Number of decimal places in question	Calculator answer	Number of decimal places in answer
$0{\cdot}7 \times 6$	1		
$0{\cdot}3 \times 0{\cdot}2$	2		
$0{\cdot}15 \times 0{\cdot}3$	3		
$0{\cdot}92 \times 0{\cdot}11$	4		
$0{\cdot}123 \times 0{\cdot}1$	4		



Your solution should show that what we suspected was true.

When multiplying decimals, the number of digits after the decimal point in the answer must be the same as the total number of digits after the decimal points in the question.

WORKED EXAMPLES

1 $0{\cdot}008 \times 0{\cdot}5$
$= 0{\cdot}0040$
$= 0{\cdot}004$

You can delete 'trailing' zeros like this, but only after you have made sure your decimal multiplication is correct.

2 $5{\cdot}5 \times 1{\cdot}7$
$= 9{\cdot}35$

3 $(0{\cdot}03)^2$
$= 0{\cdot}03 \times 0{\cdot}03$
$= 0{\cdot}0009$

Exercise 6:05

P Foundation worksheet 6:05 Multiplying decimals

1 How many decimal places does each numeral have?

a 9·35 b 0·7 c 0·014 d 0·0004 e 0·15
f 721·5 g 156·84 h 927·1 i 3·861

2 a $0{\cdot}4 \times 0{\cdot}6$ b $0{\cdot}11 \times 0{\cdot}8$ c $0{\cdot}7 \times 0{\cdot}1$ d $3{\cdot}1 \times 0{\cdot}2$
e $6 \times 0{\cdot}7$ f $0{\cdot}3 \times 0{\cdot}3$ g $0{\cdot}04 \times 0{\cdot}7$ h $0{\cdot}8 \times 0{\cdot}03$
i $0{\cdot}07 \times 0{\cdot}07$ j $0{\cdot}8 \times 0{\cdot}5$ k $0{\cdot}05 \times 0{\cdot}6$ l $0{\cdot}15 \times 4$
m $0{\cdot}09 \times 6$ n $0{\cdot}08 \times 0{\cdot}7$ o $0{\cdot}4 \times 0{\cdot}4$

3 a $0{\cdot}125 \times 0{\cdot}3$ b $2{\cdot}14 \times 0{\cdot}6$ c $6{\cdot}7 \times 0{\cdot}4$
d $3{\cdot}825 \times 700$ e $0{\cdot}426 \times 80$ f $7{\cdot}6 \times 900$
g $8{\cdot}125 \times 0{\cdot}8$ h $0{\cdot}048 \times 0{\cdot}5$ i $620 \times 0{\cdot}07$
j $6800 \times 0{\cdot}002$ k $312\,000 \times 0{\cdot}09$ l $540 \times 0{\cdot}5$
m $0{\cdot}01 \times 346{\cdot}7$ n $0{\cdot}6 \times 87{\cdot}2$ o $0{\cdot}11 \times 73{\cdot}1$

Multiply as if there are no decimal points, then use the rule to write the decimal point in your answer.

4 a
$$\begin{array}{r} 9{\cdot}6 \\ \times\ 3{\cdot}1 \\ \hline \end{array}$$
b
$$\begin{array}{r} 0{\cdot}412 \\ \times\ 2{\cdot}4 \\ \hline \end{array}$$
c
$$\begin{array}{r} 1{\cdot}33 \\ \times\ 8{\cdot}3 \\ \hline \end{array}$$
d
$$\begin{array}{r} 36{\cdot}51 \\ \times\ 0{\cdot}14 \\ \hline \end{array}$$

e $15{\cdot}6 \times 1{\cdot}7$ f $23{\cdot}8 \times 0{\cdot}12$ g $654 \times 0{\cdot}18$
h $0{\cdot}16 \times 0{\cdot}16$ i $3{\cdot}5 \times 3{\cdot}5$ j $0{\cdot}25 \times 0{\cdot}25$
k $365{\cdot}1 \times 1{\cdot}4$ l $365{\cdot}1 \times 0{\cdot}14$

$$\begin{array}{rl} 15{\cdot}6 & \leftarrow \text{1 decimal place} \\ \times\ 1{\cdot}7 & \leftarrow \text{1 decimal place} \\ \hline 109\,2 & \\ 156\,0 & \\ \hline 26{\cdot}52 & \leftarrow \text{2 decimal places} \end{array}$$

5 a $(0{\cdot}2)^2$ b $(0{\cdot}03)^2$ c $(0{\cdot}6)^2$ d $(0{\cdot}8)^2$ e $(1{\cdot}1)^2$
f $(0{\cdot}05)^2$ g $(0{\cdot}021)^2$ h $(1{\cdot}5)^2$ i $(0{\cdot}17)^2$

$0{\cdot}03^2 = 0{\cdot}03 \times 0{\cdot}03$

6:06 Dividing by a decimal

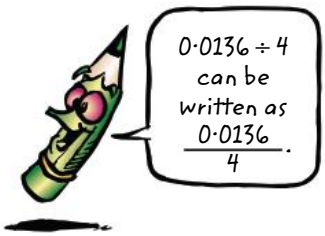
PREP QUIZ 6:06

1 $0{\cdot}3 + 0{\cdot}14$ **2** $6 - 0{\cdot}05$ **3** $6 + 0{\cdot}04 + 0{\cdot}3$
4 $2{\cdot}5 \times 6$ **5** $(0{\cdot}2)^2$ **6** $0{\cdot}3 + 0{\cdot}6 \times 4$
7 $2\overline{)2{\cdot}08}$ **8** $2\overline{)2{\cdot}09}$ **9** $7\overline{)38{\cdot}5}$ **10** $4\overline{)0{\cdot}0136}$

WORKED EXAMPLE 1

$$\begin{array}{r} 0\cdot0034 \\ 4\overline{)0\cdot0\,1\,{}_{1}3\,{}_{1}6} \end{array}$$

When dividing by a whole number, write the decimal point in the answer above the point in the question.



To divide by a decimal, change the question so that you are dividing by a whole number.

WORKED EXAMPLE 2

$0{\cdot}0136 \div 0{\cdot}04$

$= \dfrac{0{\cdot}0136}{0{\cdot}04}$

$= \dfrac{0{\cdot}0136 \times 100}{0{\cdot}04 \times 100}$

$= \dfrac{1{\cdot}36}{4}$

$= 0{\cdot}34$

To multiply by 100, move the decimal point two places right.

$\dfrac{1{\cdot}36}{4} = 4\overline{)\,1{\cdot}36}$ $\quad 0{\cdot}34$

From these worked examples we can see that if the decimal point is moved the same number of places in both numbers, we will get the same right answer.

$0{\cdot}0136 \div 0{\cdot}04$

$= 1{\cdot}36 \div 4$

MORE WORKED EXAMPLES

1 $7{\cdot}816 \div 0{\cdot}4$

$= 78{\cdot}16 \div 4$

$= 19{\cdot}54$

2 $8{\cdot}600 \div 0{\cdot}002$

$= 8600 \div 2$

$= 4300$

3 $\dfrac{875}{0{\cdot}05} = \dfrac{875{\cdot}00}{0{\cdot}05} = \dfrac{87\,500}{5} = 17\,500$

Exercise 6:06

1 Change each division into a division by a whole number.

a $8{\cdot}35 \div 0{\cdot}5$	**b** $0{\cdot}344 \div 0{\cdot}08$	**c** $0{\cdot}651 \div 0{\cdot}003$
d $0{\cdot}049 \div 0{\cdot}7$	**e** $0{\cdot}8 \div 0{\cdot}2$	**f** $7{\cdot}125 \div 0{\cdot}05$
g $38 \div 0{\cdot}04$	**h** $18 \div 0{\cdot}9$	**i** $8 \div 0{\cdot}008$
j $2{\cdot}7 \div 0{\cdot}03$	**k** $4{\cdot}72 \div 0{\cdot}8$	**l** $2{\cdot}2 \div 0{\cdot}11$

When moving the decimal point to the right, you may need to add zeros.

$8{\cdot}600$

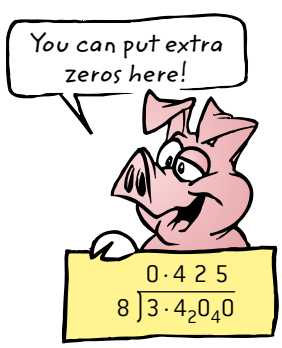
2 Find the answer to each division in Question 1.

3

a $0{\cdot}9 \div 0{\cdot}3$	**b** $3{\cdot}6 \div 0{\cdot}9$	**c** $0{\cdot}24 \div 0{\cdot}6$
d $0{\cdot}08 \div 0{\cdot}008$	**e** $0{\cdot}8 \div 0{\cdot}04$	**f** $42 \div 0{\cdot}007$
g $365 \div 0{\cdot}04$	**h** $14 \div 0{\cdot}07$	**i** $15 \div 0{\cdot}3$
j $49 \div 0{\cdot}1$	**k** $16 \div 0{\cdot}01$	**l** $2 \div 0{\cdot}001$

4

a $5{\cdot}75 \div 2{\cdot}5$	**b** $11{\cdot}11 \div 1{\cdot}1$	**c** $4{\cdot}5 \div 0{\cdot}15$
d $7 \div 3{\cdot}5$	**e** $48 \div 2{\cdot}4$	**f** $8{\cdot}76 \div 0{\cdot}12$
g $0{\cdot}322 \div 0{\cdot}04$	**h** $0{\cdot}081 \div 0{\cdot}05$	**i** $34{\cdot}96 \div 2{\cdot}3$
j $7{\cdot}23 \div 0{\cdot}6$	**k** $5{\cdot}58 \div 0{\cdot}18$	**l** $0{\cdot}47 \div 0{\cdot}04$
m $\dfrac{25{\cdot}5}{0{\cdot}5}$	**n** $\dfrac{31{\cdot}71}{0{\cdot}04}$	**o** $\dfrac{2{\cdot}718}{0{\cdot}03}$



FUN SPOT 6:06 — WHY DID THE ORANGE STOP IN THE MIDDLE OF THE ROAD?

Find the answer for each letter. Match each letter with its answer below to solve the riddle.

A $1{\cdot}58 + 2{\cdot}3$
Y $8{\cdot}6 - 6$
T $5{\cdot}2 \times 0{\cdot}2$
N $12{\cdot}1 \div 5$
T $0{\cdot}3 + 0{\cdot}008$
S $11{\cdot}11 \div 11$

W $3 + 8{\cdot}6$
P $0{\cdot}7 \times 6$
I $0{\cdot}02 \times 15$
U $0{\cdot}3 \div 4$
E $0{\cdot}25 \times 4$
Q $0{\cdot}7 + 0{\cdot}2 \times 3$

T $3{\cdot}91 - 0{\cdot}01$
A $(0{\cdot}3)^2$
O $4{\cdot}06 \div 2$
D $0{\cdot}24 \div 8$
S $0{\cdot}08 \times 1000$
L $0{\cdot}5 \div 100$
H Write $\frac{3}{10} + \frac{7}{100} + \frac{5}{1000}$ as a decimal.



A Which is the smallest: 0·38, 0·378, 0·4?

0·3	1·04	11·6	3·88	2·42	3·9	1	0·03	0·308	2·03	4·2	0·005	0·09	2·6	1·01	1·3	0·075	0·378	80	0·375

6:07 Changing fractions to decimals

PREP QUIZ 6:07

Write as a decimal:

1 $\frac{3}{10}$ **2** $\frac{7}{100}$ **3** $\frac{196}{100}$ **4** $\frac{5}{1000}$ **5** $\frac{37}{1000}$

Simplify:

6 $\frac{16}{2}$ **7** $\frac{6}{3}$ **8** $\frac{120}{10}$ **9** $\frac{148}{4}$ **10** $\frac{8315}{5}$

$\frac{3}{10}$ ← numerator
← denominator

The Prep quiz shows two methods that we can use to change fractions to decimals.

Method 1 If a fraction is written as a 10th, 100th, 1000th etc., change it to a decimal by writing the last digit of the numerator in the matching place-value column.

WORKED EXAMPLES

Change these fractions to decimals.

1 $\frac{37}{1000}$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	•	0	3	7

$\therefore \frac{37}{1000} = 0{\cdot}037$

2 $\frac{451}{100}$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
4	•	5	1	

$\therefore \frac{451}{100} = 4{\cdot}51$

3 $\frac{16}{5} = \frac{32}{10}$

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	•	2		

$\therefore \frac{16}{5} = 3{\cdot}2$

Method 2 To change any fraction to a decimal, divide the numerator by the denominator (divide the top by the bottom), e.g. $\frac{3}{4} = 3 \div 4$ or $4\overline{)3 \cdot 00}$.

WORKED EXAMPLES

Change these fractions to decimals.

1 $\frac{3}{4} = 3 \div 4$

$$4\overline{)3 \cdot {}_{3}0\ {}_{2}0} \quad \begin{matrix} 0 \cdot 7\ 5 \end{matrix}$$

$\therefore \frac{3}{4} = 0{\cdot}75$

2 $1\frac{5}{8} = 1 + (5 \div 8)$

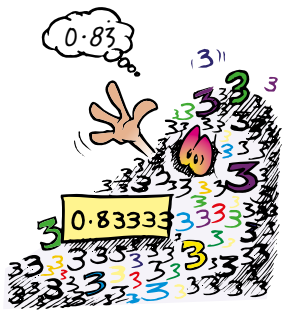
$$8\overline{)5 \cdot {}_{5}0\ {}_{2}0\ {}_{4}0} \quad \begin{matrix} 0 \cdot 6\ 2\ 5 \end{matrix}$$

$\therefore 1\frac{5}{8} = 1{\cdot}625$

3 $\frac{5}{6} = 5 \div 6$

$$6\overline{)5 \cdot {}_{5}0\ {}_{2}0\ {}_{2}0\ {}_{2}0\ {}_{2}0\ {}_{2}\ldots} \quad \begin{matrix} 0 \cdot 8\ 3\ 3\ 3\ 3\ \ldots \end{matrix}$$

$\therefore \frac{5}{6} = 0 \cdot 8\dot{3}$



Fractions can be written as either **terminating** or **repeating** decimals.

1 A **terminating** decimal comes to a definite end. It has a definite number of decimal places, e.g. 3·6, 0·000 125, 0·75.

2 A **repeating** or **recurring** decimal does not come to an end, but forms a pattern that is repeated indefinitely (i.e. forever). To show that a set of digits is repeated, we write a dot over the first and last of the repeating digits,
e.g. $0{\cdot}666\ldots = 0 \cdot \dot{6}$ $1{\cdot}272\,727\ldots = 1 \cdot \dot{2}\dot{7}$ $32{\cdot}460\,860\,8\ldots = 32 \cdot 4\dot{6}0\dot{8}$

Exercise 6:07

1 Is each decimal terminating or repeating?

a 0·65	**b** 0·333…	**c** 21·9	**d** $8 \cdot \dot{7}$
e 0·777	**f** 0·777…	**g** $0 \cdot \dot{7}$	**h** 7·7
i $14 \cdot 3\dot{5}\dot{5}$	**j** 4·0408	**k** 9·105 105…	**l** $2 \cdot 1\dot{6}\dot{5}$

2 Write these repeating decimals in the shorter way.

a 0·999…	**b** 3·1444…	**c** 0·181 818…	**d** 0·1666…
e 0·909 090…	**f** 6·745 745…	**g** 66·666…	**h** 0·142 857 142 857…

3 Write as a decimal.

a $\frac{3}{10}$	**b** $\frac{6}{100}$	**c** $\frac{9}{1000}$	**d** $\frac{19}{1000}$
e $\frac{55}{100}$	**f** $1\frac{1}{10}$	**g** $3\frac{77}{100}$	**h** $\frac{709}{1000}$
i $6\frac{2}{100}$	**j** $\frac{1}{2}$	**k** $\frac{44}{50}$	**l** $\frac{175}{500}$
m $\frac{7}{20}$	**n** $\frac{1}{4}$	**o** $\frac{3}{250}$	

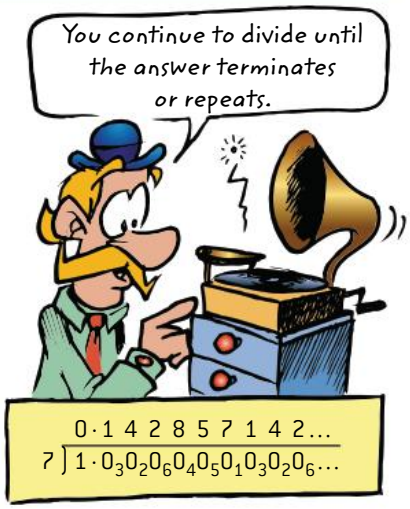
If you can change the denominator to 10, 100 or 1000, you can still use method 1 (from page 181),

e.g. $\frac{11}{20} \begin{matrix} \times 5 \\ \times 5 \end{matrix} = \frac{55}{100} = 0{\cdot}55$

4 Use method 2 (from page 182) to change these to decimals.

a $\frac{4}{5}$	b $\frac{1}{8}$	c $\frac{1}{4}$	d $\frac{3}{8}$
e $\frac{5}{8}$	f $\frac{7}{8}$	g $\frac{7}{20}$	h $\frac{33}{50}$
i $\frac{3}{4}$	j $\frac{1}{3}$	k $\frac{2}{3}$	l $\frac{1}{9}$
m $\frac{2}{9}$	n $\frac{4}{9}$	o $\frac{7}{9}$	p $\frac{5}{6}$
q $\frac{1}{6}$	r $\frac{5}{9}$		

If using a calculator, you would use method 2.



5 Change these to decimals.

a $3\frac{8}{9}$	b $2\frac{3}{5}$	c $6\frac{2}{3}$	d $\frac{3}{11}$
e $\frac{5}{11}$	f $\frac{10}{11}$	g $2\frac{5}{12}$	h $\frac{11}{15}$
i $1\frac{3}{16}$	j $\frac{4}{7}$	k $\frac{3}{7}$	l $\frac{1}{14}$

6 a Convert $6\frac{7}{8}$ to a decimal. b Express $\frac{9}{11}$ as a decimal.

c Change $12\frac{1}{4}$ to a decimal. d How would you write $\frac{5}{16}$ as a decimal?

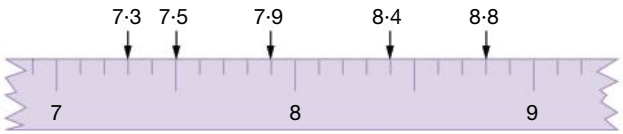
e Write $8\frac{5}{6}$ as a decimal. f Convert $2\frac{124}{200}$ to a decimal.

6:08 Rounding

PREP QUIZ 6:08

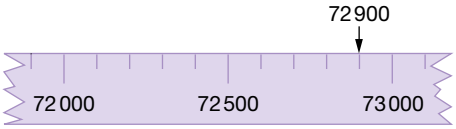
1 Is 16 closer to 10 or 20?
2 Is 153 closer to 150 or 160?
3 Is 153 closer to 100 or 200?
4 Is 4·1 closer to 4 or 5?
5 Is 4·7 closer to 4 or 5?
6 Is \$7.66 closer to \$7.60 or \$7.70?
7 Is 7·66 closer to 7·6 or 7·7?
8 Is 3·111 closer to 3·11 or 3·12?
9 Which two numbers is 14·63 between: 14·5 and 14·6 *or* 14·6 and 14·7?
10 What decimal number is half-way between 6·7 and 6·8?

To **round** a decimal to the nearest whole number, we write the whole number closest to it.



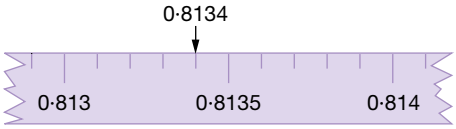
7·3 is closer to 7	7·9 is closer to 8	8·4 is closer to 8	8·8 is closer to 9
7·5 is exactly half-way between 7 and 8. In cases like this it is common to round up. We say 7·5 = 8, correct to the nearest whole number.			

To round 72 900 to the nearest thousand, we write the thousand closest to it.



72 900 is closer to 73 000 than to 72 000.
∴ 72 900 = 73 000, correct to the nearest thousand.

To round 0·8134 to the nearest thousandth, we write the thousandth closest to it.



0·8134 is closer to 0·813 than to 0·814.
∴ 0·8134 = 0·813, correct to the nearest thousandth.

To round (or approximate) a number correct to a given place:

- if the next digit is 5 or more, round up
- if the next digit is less than 5, round down.

WORKED EXAMPLES

Round:

1 56 700 000 to the nearest million

2 0·0851 to the nearest hundredth

3 86·149 to one decimal place

4 0·666 66 to four decimal places

Solutions

1 56 700 000 has a 6 in the millions column. The digit after the 6 is 5 or more (i.e. 7).
∴ 56 700 000 = 57 000 000, correct to the nearest million.

2 0·0851 has an 8 in the hundredths column. The digit after the 8 is 5 or more (i.e. 5).
∴ 0·0851 = 0·09, correct to the nearest hundredth.

3 86·149 has a 1 in the first decimal place. The digit after the 1 is less than 5 (i.e. 4).
∴ 86·149 = 86·1, correct to one decimal place.

4 0·666 66 has a 6 in the fourth decimal place. The digit after the 6 is 5 or more (i.e. 6).
∴ 0·666 66 = 0·6667, correct to four decimal places.

Exercise 6:08

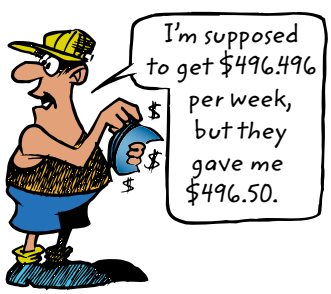
P Foundation worksheet 6:08 Rounding

1 Round each number to the nearest hundred.

a 7923	**b** 1099	**c** 67 314	**d** 853·461
e 609·99	**f** 350	**g** 74 932	**h** 7850

2 Round each number to the nearest whole number.

a 9·3	**b** 79·5	**c** 45·1	**d** 2·7
e 2·314	**f** 17·81	**g** 236·502	**h** 99·5



3 Round each number to the nearest hundredth.

a 243·128	**b** 79·664	**c** 91·351	**d** 9·807
e 0·3046	**f** 0·0852	**g** 0·097	**h** 1·991

4 Round each number to one decimal place.

a 6·70	**b** 8·45	**c** 2·119	**d** 6·092
e 0·05	**f** 246·739	**g** 29·88	**h** 9·99

5 The cost of petrol is 169·3 cents per litre. Angelika bought 12 litres of petrol. How much did this cost, correct to the nearest cent?

6 Leonie bought 12 doors at $59.30 each. What is the cost of the doors, rounded to the nearest dollar?



7 To change $\frac{4}{9}$ to a decimal, Min divided 9 into 4·000…
Her answer was $0\cdot\dot{4}$. Give this answer correct to:

a 1 dec. pl.	**b** 2 dec. pl.
c 3 dec. pl.	**d** 4 dec. pl.

'dec. pl.' is short for 'decimal place'.

8 Peter needed to divide a 3-metre length into 11 equal parts. After dividing he got the answer $0\cdot\dot{2}\dot{7}$. Give this answer correct to:

a 1 dec. pl. **b** 2 dec. pl. **c** 3 dec. pl. **d** 4 dec. pl.

9 These calculator displays represent an answer in cents.
Round the answers to the nearest cent.

a 1362.836	**b** 1941.341853
c 6118.91	**d** 509.5008
e 549.9342	**f** 1768049.5

10 Round the values shown in Question **9** correct to the nearest dollar.



6:09 Applications of decimals

PREP QUIZ 6:09

1 10 lots of 10c **2** 10 × $0.10 **3** 10 × 0·1

4 I have eleven coins in my pocket. There are three dollar coins, seven 10-cent coins and one 5-cent coin. How much money do I have?

5 0·97 + 0·033 **6** $20 − $1.67 **7** $10 − $0.32
8 $3.45 × 6 **9** 0·3 × 0·2 **10** 0·84 ÷ 0·4

Exercise 6:09

P Foundation worksheet 6:09
Application of decimals

1 Round each amount to the nearest 5 cents.

a $34.27	**b** $8.52	**c** $0.99	**d** $17.11
e $85.44	**f** $9.55	**g** $103.33	**h** $11.68
i $27.16	**j** $277.37	**k** $108.08	**l** $654.72

> Australian 1-cent and 2-cent coins were withdrawn from circulation in 1992.
> *If you pay cash, the final bill is rounded to the nearest five cents.*

2

a $3.45 + $15.52	**b** $18 + $9.65	**c** $0.95 + $3.15
d $8.95 − $3.58	**e** $20 − $6.72	**f** $50.00 − $0.35
g $7.45 × 6	**h** $0.94 × 10	**i** $1.30 × 9
j $8.45 ÷ 5	**k** $7.60 ÷ 10	**l** $15.40 ÷ 7

3 Petrol is sold for 135.9 cents per litre. For the following amounts, find the price to be paid to the nearest 5 cents:

a 8 L	**b** 3 L	**c** 6 L	**d** 7 L	**e** 10 L	**f** 30 L
g 20 L	**h** 100 L	**i** 27 L	**j** 34 L	**k** 52 L	**l** 41 L
m 25 L	**n** 19 L	**o** 8·5 L	**p** 0·5 L		

4 Find the value of each expression if $a = 0.2$, $b = 0.3$ and $c = 0.5$

a $a + b$	**b** $c - b$	**c** $b - a$	**d** $c + a$
e $a + b + c$	**f** $b + c - a$	**g** $c + a - b$	**h** $c - b - a$

5 Find the value of each expression if $x = 0.5$, $y = 0.8$ and $z = 1.2$.

a $4x$	**b** $5z$	**c** $y \div 2$	**d** $4 \div x$	**e** xy	**f** yz
g xyz	**h** $xz - xy$	**i** $\frac{y}{x}$	**j** $\frac{x}{y}$	**k** $\frac{xz}{y}$	**l** $\frac{y+z}{4x}$

6 Dermot kept a record of the distance he travelled to football grounds during one weekend. The trips were 68·4 km, 8·9 km, 42·4 km and 9 km.

a What was the total distance Dermot travelled?

b What was the difference between the longest and shortest trips?

c Two of the trips were to the same ground. Which two do you think they were? Why would the numbers be different?

7 A photocopier takes 0·8 seconds to produce each copy. How long would it take to produce 100 copies?

8 Coola Soft Drinks sends out their bottles in containers that hold 1000 bottles, 100 bottles or 10 bottles. 8360 bottles are to be sent out.

a What is the least number of full containers that could be used?

b What is the greatest number of full containers that could be used?

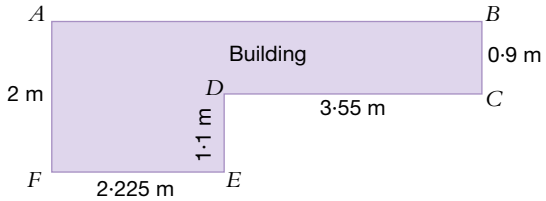


9 Katherine Collison owns a fabric shop. She can buy coloured ribbon on rolls that contain 0·1 km, 0·01 km or 0·001 km of ribbon. She needs 0·836 km of coloured ribbon.

a If she wanted to buy as few rolls as possible, how many of each type of roll would she buy? How many rolls is this altogether?

b What is the greatest number of rolls that she could buy to get 0·836 km of ribbon?

10 $BC = 0{\cdot}9$ m, $DC = 3{\cdot}55$ m, $DE = 1{\cdot}1$ m, $FE = 2{\cdot}225$ m, $AF = 2$ m.



a What two lengths have a sum equal to AB?

b What is the length of AB?

c What is the perimeter of the building?

d What is the difference in length between FE and DC?

11 These are the winning times for the Olympic Games women's 100 m sprint, from 1964 to 1980:

1964: 11·4 s 1968: 11·0 s 1972: 11·07 s
1976: 11·08 s 1980: 11·06 s

Arrange these times in order from least time taken to most time taken.



12 In the World Championship pairs skating finals, Jasmine and Chris were given the scores 9·9, 9·8, 10, 10, 10, 10, 10, 10, 9·9, 9·1, 10 and 9·4 by the twelve judges. The lowest score and one of the equal highest scores are not counted.

a What is the total of the remaining ten scores?

b To find the average score we divide the total (found in part **a**) by 10. Write the average score as a decimal.

13 When building her house, Sian hammered 5 nails into a wall. The nails were each 0·75 metres apart in a straight line. What was the distance from the first nail to the last? (Hint: Draw a diagram.)

14 A box is filled with 100 pencils. Each pencil has a mass of 9·6 grams. If the empty box has a mass of 54·5 grams, what is the mass of the box full of pencils?

15 Helena bought 6·6 litres of petrol at 161·3 cents per litre. How much did she pay?

16 A carton contains 0·6 litres of milk. How many 0·25 litre glasses can be filled from one carton?

17 In a 400-metre race, the runner who came second ran the distance in 44·37 seconds. The race was won in a time of 43·79 seconds.

a What was the time difference between 1st and 2nd?

b If the winner broke the previous record by 0·07 seconds, what was the previous record?

18 A container holds less than 0·8 litres of water. Which of the following could be the amount in four of these containers?

A 5 litres **B** 4·2 litres **C** 3·2 litres **D** 2·4 litres

19 The paint in one tin will cover an area of 100 square metres. Trinh wants to paint an area of 460 square metres.

- **a** How many tins of paint will she need?
- **b** What is the total cost of the paint, if each tin costs $27.55?

20 Joel has a salary of $32 640 per year. He divided this amount by 365·25 to find his salary per day, then multiplied by 7 to find his weekly salary. He calculated his weekly salary to be $625.544, but he is paid only $625.54 each week.

- **a** How much less than his 'correct' weekly salary is he paid?
- **b** If there are 52 pays this year, how much will he have lost in payments?
- **c** If 1000 employees lost an average of $0.005 per week, how much would be lost by them altogether in 52 weeks?
- **d** Who could profit by the loss (which would probably go unnoticed by the employees)?

6:10 Using a calculator

When calculations are difficult or time-consuming, it is sensible to use a calculator. However, it is easy to press the wrong key without realising. Knowing how to calculate without a calculator can help you notice when a calculator has given the wrong answer.

This exercise below contains questions similar to those in the rest of this chapter. The questions are not 'harder' to understand, but the numbers involved make using a calculator the sensible option.

WORKED EXAMPLES

1 Use a calculator to answer the following:

- **a** 29·6 + 19·57 − 8·683
- **b** 23·61 × 5·83
- **c** 73·4 ÷ 19·7 (correct to 1 decimal place)

Always check that the decimal point is in the correct position.

2 Write $\frac{5}{7}$ as a decimal, correct to 3 decimal places.

3 Find the cost of 15 ice-creams at $2.70 each plus 12 drinks at $3.25 each.

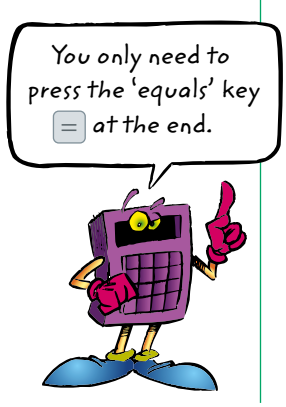
Solutions

1 a Enter: 29.6 [+] 19.57 [−] 8.683 [=]
Answer: = 40·487

b Enter: 23.61 [×] 5.83 [=]
Answer: = 137·6463

c Enter: 73.4 [÷] 19.7 [=]
Answer: = 3·725 888…
Rounding to 1 dec. pl.: = 3·7

Note: There are 4 digits after the decimal point in the question and in the answer.



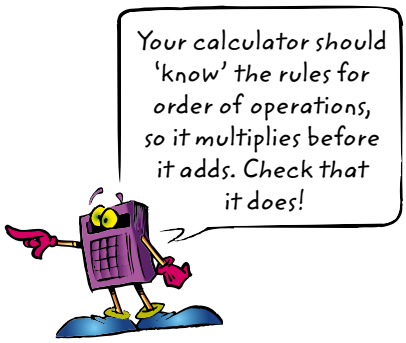
2 We need to divide 5 by 7.
Enter: 5 [÷] 7 [=]
Answer: = 0·714 285…
Rounding to 3 dec. pl.: = 0·714

3 We need to calculate \$2.70 × 15 plus \$3.25 × 12.

Enter: 2.7 × 15 + 3.25 × 12 =

Answer: = 79·5

Because this is money, we should write the answer as \$79.50.



Exercise 6:10

1 **a** 6·98 + 5·37 **b** 19·6 + 7·94 **c** 241·73 + 67·094 **d** 6·53 − 4·37
e 27·3 − 9·68 **f** 172·9 − 83·06 **g** 9·73 + 6·48 + 8·15
h 8·6 + 3·72 − 5·017 **i** 21·46 − 17·9 + 6·432 **j** 107·6 − 73·94 − 19·004
k 8·61 + 73·5 + 23·073 + 2·46 + 37·9 **l** 172·3 + 347 − 246·43 + 82·63 − 97·5

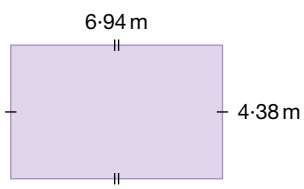
2 **a** 6·4 × 7·9 **b** 17·6 × 5·74 **c** 9·61 × 11·7 **d** 29·73 × 6·51
e 37·5 × 6·474 **f** 276·9 × 37·67 **g** 9·6 × 4·7 × 3·5
h 12·8 × 4·63 × 25·3 **i** 27·41 × 9·67 × 5·4 **j** 5·96 × 7·47 × 13·14

3 Answer each division correct to the number of decimal places indicated.
a 19·6 ÷ 5·3 (1 dec. pl.) **b** 27·61 ÷ 3·9 (1 dec. pl.)
c 37·81 ÷ 12·27 (2 dec. pl.) **d** 87·3 ÷ 9·57 (2 dec. pl.)
e 6·917 ÷ 9.61 (3 dec. pl.) **f** 27·31 ÷ 39·7 (3 dec. pl.)
g 251·47 ÷ 76·8 (4 dec. pl.) **h** 75·38 ÷ 561·9 (4 dec. pl.)

4 Convert each fraction to a decimal numeral, correct to the number of decimal places indicated.
a $\frac{3}{7}$ (1 dec. pl.) **b** $\frac{7}{8}$ (2 dec. pl.) **c** $\frac{7}{9}$ (2 dec. pl.)
d $\frac{8}{15}$ (2 dec. pl.) **e** $\frac{7}{12}$ (3 dec. pl.) **f** $\frac{13}{17}$ (3 dec. pl.)
g $\frac{10}{13}$ (3 dec. pl.) **h** $\frac{15}{19}$ (4 dec. pl.) **i** $\frac{115}{130}$ (4 dec. pl.)

5 **a** Find the cost of 16 cakes at \$4.20 each plus 20 cups of coffee at \$3.40.
b What is the cost of 45 litres of petrol at 138·6 cents per litre?
c A carton of drinks costs \$12.60. How many complete cartons can be purchased for \$250?

6 **a** What is the perimeter of a rectangle with a length of 6·94 m and a width of 4·38 m?
b To make the perimeter 25 m, how much must be added to the length?



7 Shura is paid a salary of \$78 350 per year.
a Divide this by 365·25 to find her daily rate of pay to the nearest cent.
b Multiply by 14 to find her fortnightly pay.
c There are 26 complete fortnights in a year. Multiply your answer to part **b** by 26. Is this the same as the stated annual salary? Explain.

FUN SPOT 6:10 OLYMPIC DIVING

At the Olympic diving competition there were seven judges.

6	5·5	7	7	7·5	8	7·5
Degree of difficulty 2·2						



Scoring:

- Eliminate the two highest and two lowest scores awarded and find the total of the rest.
- Multiply this total by the degree of difficulty to find the final score.

More difficult dives have a higher degree of difficulty, e.g. *DD* = 2·8.

WORKED EXAMPLE

Points (from 7 judges)	Total × *DD* (*DD* = 'degree of difficulty')	Final score
~~5·5~~, ~~6~~, 7, 7, 7·5, ~~7·5~~, ~~8~~	21·5 × 2·2	47·3
~~7·5~~, ~~8~~, 8, 8·5, 8·5, ~~9~~, ~~9~~	25 × 2·8	70

Calculate the final score for each set of points.

1	8, 7, 7, 7, 7, 7, 6·5	*DD* = 2·0	**2**	6, 6, 6, 5·5, 5, 5, 4	*DD* = 2·4
3	4, 3·5, 3, 3, 3, 2, 2·5	*DD* = 2·8	**4**	9, 9, 8, 8, 8, 7·5, 7	*DD* = 1·8
5	6, 6, 6, 6, 6, 6, 6	*DD* = 2·2	**6**	7, 6·5, 6·5, 6, 6, 5, 4·5	*DD* = 2·6
7	8, 8, 8, 7, 6·5, 6·5, 6	*DD* = 2·4	**8**	2, 2, 1·5, 1·5, 1, 1, 0	*DD* = 2·8
9	9·5, 9·5, 9, 9, 9, 9, 9	*DD* = 2·6	**10**	8, 7, 6·5, 6·5, 6·5, 6, 6	*DD* = 2·0

INVESTIGATION 6:10 APPLYING DECIMALS

Calculate the cost of running a family car for one year.

You will need to estimate or discover all the costs involved, such as:

- distance travelled
- petrol consumption
- petrol costs
- service and oil-change costs
- tyres
- repairs
- insurance
- registration.



MATHS TERMS 6

ascending order
- from smallest to largest

descending order
- from largest to smallest

decimal (or decimal fraction)
- a way of writing a common fraction using place value, e.g. $\frac{3}{8} = 0{\cdot}375$

decimal places
- the number of digits used in a decimal that are to the right of the decimal point

decimal point
- the point in a decimal that separates the whole number part from the fraction part

estimate
- to make a close guess or approximation

round (or approximate)
- to write in a simpler (less precise) way, e.g. $0{\cdot}9499 \doteqdot 0{\cdot}95$

recurring (or repeating) decimal
- a decimal whose digits form a repeating pattern,
  e.g. $3{\cdot}7777\ldots = 3{\cdot}\dot{7}$
  $0{\cdot}5454\ldots = 0{\cdot}\dot{5}\dot{4}$

terminating decimal
- a decimal that has a definite number of decimal places, e.g. 0·375

DIAGNOSTIC TEST 6 DECIMALS

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1	Write each decimal in expanded notation.			6:01
	a $36{\cdot}152$	b $0{\cdot}3333$	c $7{\cdot}039$	
2	Arrange in order from smallest to largest.			6:01
	a $0{\cdot}3, 0{\cdot}9, 0{\cdot}6$	b $0{\cdot}4, 0{\cdot}11, 0{\cdot}6$	c $0{\cdot}7, 0{\cdot}77, 0{\cdot}077$	
3	a $0{\cdot}45 + 12{\cdot}3 + 4$	b $0{\cdot}008 + 0{\cdot}15 + 0{\cdot}9$	c $0{\cdot}95 + 0{\cdot}005 + 0{\cdot}5$	6:02
4	a $23{\cdot}587 - 4{\cdot}35$	b $880{\cdot}6 - 77{\cdot}03$	c $50 - 6{\cdot}9$	6:02
5	a $0{\cdot}015 \times 8$	b $3{\cdot}9 \times 6$	c $4{\cdot}72 \times 4$	6:03
6	a $5{\cdot}35 \div 5$	b $6{\cdot}2 \div 5$	c $11{\cdot}031 \div 6$	6:04
7	a $0{\cdot}4 \times 100$	b $3{\cdot}375 \times 10$	c $1{\cdot}56 \times 1000$	6:03
8	a $134{\cdot}65 \div 100$	b $9{\cdot}135 \div 1000$	c $0{\cdot}045 \div 10$	6:04
9	a $0{\cdot}008 \times 0{\cdot}5$	b $11{\cdot}2 \times 0{\cdot}11$	c $(0{\cdot}03)^2$	6:05
10	a $0{\cdot}0136 \div 0{\cdot}04$	b $8{\cdot}6 \div 0{\cdot}002$	c $\dfrac{875}{0{\cdot}05}$	6:06
11	Is each decimal terminating or repeating?			6:07
	a $0{\cdot}75$	b $1{\cdot}272727\ldots$	c $1{\cdot}\dot{2}\dot{7}$	
12	Write as a decimal.	a $3\frac{77}{100}$	b $3 + \frac{7}{10} + \frac{7}{100}$	c $6\frac{7}{1000}$	6:07
13	Write as a decimal.	a $\frac{5}{8}$	b $\frac{1}{6}$	c $1\frac{10}{11}$	6:07
14	Round each decimal correct to two decimal places.			6:08
	a $78{\cdot}484$	b $0{\cdot}6666\ldots$	c $9{\cdot}295$	

ASSIGNMENT 6A Chapter review

1 Write as a simple fraction:
a 0.6 **b** 0.07 **c** 0.008

2 Write in ascending order:
0.5 0.51 0.501 0.051

3 Answer the following (without using a calculator):
a $6.73 + 67.3 + 0.673$
b $12.69 - 1.269$
c 2.51×1000 **d** $13.6 \div 1000$
e 6.4×0.5 **f** $3.72 \div 0.3$

4 Convert each fraction to a decimal.
a $\frac{4}{5}$ **b** $\frac{5}{8}$ **c** $\frac{7}{20}$ **d** $\frac{3}{25}$

5 Rewrite each fraction as a recurring decimal.
a $\frac{5}{6}$ **b** $\frac{2}{7}$

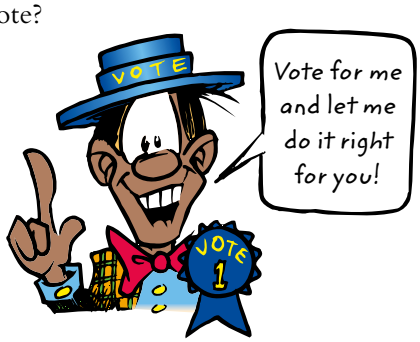
6 Rewrite the number 9.817 263 correct to:
a 1 dec. pl. **b** 2 dec. pl.
c 3 dec. pl. **d** 4 dec. pl.

7 $\frac{4}{15}$ is equivalent to the recurring decimal number $0 \cdot 2\dot{6}$. Write this correct to:
a 1 dec. pl. **b** 2 dec. pl. **c** 3 dec. pl.

8 **a** Kevin ran 100 m five times, for the following results: 12.1 s, 12.7 s, 12.4 s, 13.0 s, 12.2 s. What was his average time?
b A Mazda car has a petrol tank that holds 55 litres.
i How much would it cost to fill the tank, if petrol costs 141.7 cents per litre?
ii If it takes 7.9 litres to travel 100 km, how far can the car travel on one full tank of petrol? (You may need a calculator for this question.)

ASSIGNMENT 6B Working mathematically

1 In an election one candidate received 9873 votes, another received 7828 votes, and the third 2314 votes. If 215 voting papers were informal (i.e. not filled out properly, hence not counted) and there were 20 609 voters in the electorate, how many voters did not vote?



2 A school hall has 24 rows of chairs with 20 chairs in each row. If the chairs are rearranged in rows of 16, how many rows will there be?

3 A publishing company wants to send some advertising material to 650 schools. If an envelope costs 6 cents, the advertising pamphlet costs 38 cents to produce and the stamp for the letter costs 60 cents, how much does it cost to send the advertising material to the schools?

4 A tennis tournament has 64 players. If only the winner of each match stays in the tournament, how many matches must be played to find the final winner of the tournament?

5 A group of units is made up of four different buildings. Each building has five floors, with six apartments on the first four floors and four apartments on the fifth floor. How many apartments are there in the group of units?

ASSIGNMENT 6C Cumulative revision

1 Simplify: 1:06

a $22 - 4 \times 3$ **b** $2 + 4 \times 2 \times 3$ **c** $\frac{24+6}{4}$ **d** $\frac{6+4}{6-4}$

2 What is the value of the 5 in each? 1:02

a 537 **b** 15 384 **c** 2·54 **d** 20·05

3 Write each expanded notation in its simplest form. 3:02

a $(7 \times 1000) + (8 \times 100) + (2 \times 10) + (9 \times 1)$

b $(9 \times 10) + (7 \times 1) + (3 \times \frac{1}{10}) + (8 \times \frac{1}{100})$

c $(4 \times 10^5) + (3 \times 10^4) + (0 \times 10^3) + (7 \times 10^2) + (5 \times 10) + (1 \times 1)$

4 Write in expanded notation. 3:02

a 817 **b** 2143 **c** 6·35 **d** 60 081

5 Find the value of the $\square$ in each. 1:11B

a $\frac{1}{2} = \frac{\square}{8}$ **b** $\frac{3}{4} = \frac{\square}{100}$ **c** $\frac{50}{100} = \frac{\square}{2}$ **d** $\frac{4}{16} = \frac{\square}{4}$

6 Simplify: 3:01

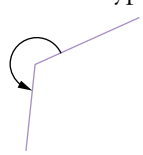
a 5^2 **b** 6^2 **c** 7^2 **d** 8^2

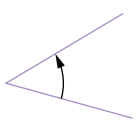
7 Find: 1:11C

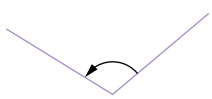
a $5 \times \$1.50$ **b** half of \$3.70

c $\$10.50 \div 2$ **d** $\$10 - \$1.75$

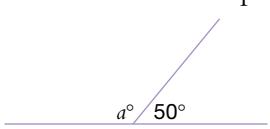
8 Name the type of angle in each case. 5:03

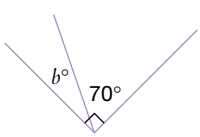
a 

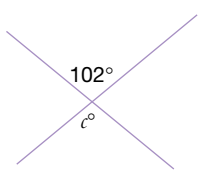
b 

c 

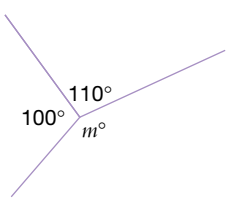
9 Find the value of each pronumeral. 5:04

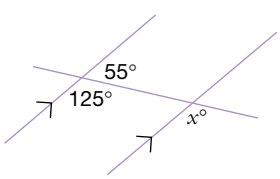
a 

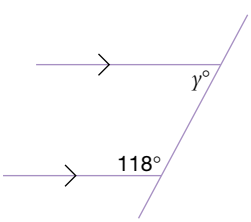
b 

c 

10 Find the value of each pronumeral. 5:05

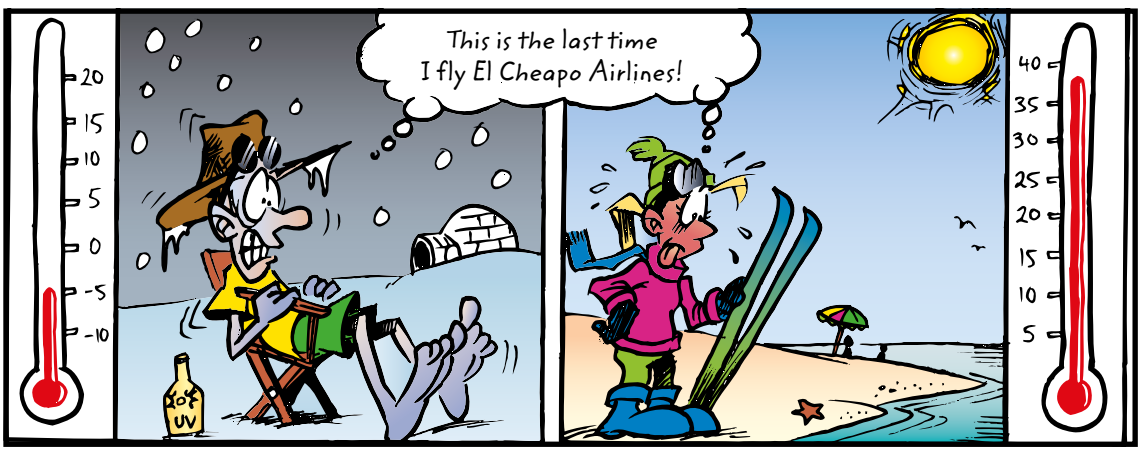
a 

b 

c 

DIRECTED NUMBERS AND THE NUMBER PLANE

7



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7:02 The number plane
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7:03 The number plane extended
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Investigation 7:04 Directed numbers
7:05 Addition and subtraction of directed numbers
7:06 Subtracting a negative number
7:07 Multiplication of directed numbers
7:08 Division of directed numbers
7:09 Using directed numbers
Fun spot 7: 09 Try this quick quiz!
7:10 Directed numbers and algebra
Fun spot 7:10A What happened to the thief who was caught in the rubber band factory?
Fun spot 7:10B Why do sumo wrestlers like to eat peanuts?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Computation with Integers***, ***Linear Relationships*** [Stage 4]

- Compare, order, add and subtract integers. (ACMNA280)
- Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies. (ACMNA183)
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point. (ACMNA178)

Working Mathematically

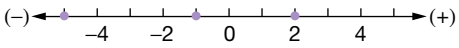
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

7:01 Directed numbers

Many of the numbers we use represent situations that have direction as well as size. We call these numbers **directed numbers**.

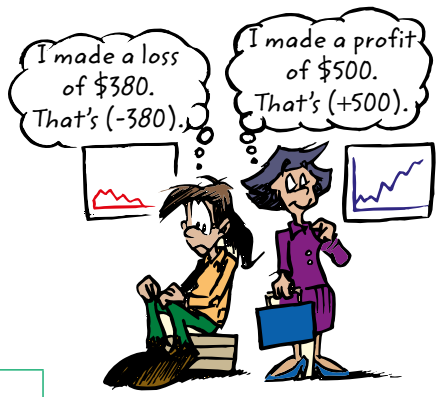
Once a direction is chosen to be positive (+), the opposite direction is taken to be negative (–).

The set of numbers $\{-5, -1, 2\}$ has been graphed on this number line.



Choosing directions for numbers:

- If north is positive, then south is negative.
- If east is positive, then west is negative.
- If above zero degrees is positive, then below zero degrees is negative.
- If to the right is positive, then to the left is negative.



WORKED EXAMPLES

1 Mount Everest is 8000 metres above sea level, so its height could be written as +8000 m or just 8000 m.

2 The Dead Sea is 100 metres below sea level, which could be written as –100 m.

3 In cold countries the temperature is often below the freezing point of water (0°C). The temperature on a cold day could be –5°C.

–20 is read 'negative 20' or 'minus 20'.
+20 is read 'positive 20' or just '20'.

Note: +20 can be written more simply as 20.

Exercise 7:01

1 **a** Use a directed number to represent each of the following.

A profit of $50	**B** 50 km south	**C** fall of 50 points
D gain of 50 kg	**E** $50 deposit	**F** 50 m above sea level
G 50° below zero	**H** 3 floors above ground	**I** increase of 50
J 50 cm to the right	**K** 50 s after take-off	**L** 50 km east
M 50 s before take-off	**N** 50° above zero	**O** 50 cm to the left
P rise of 50 points	**Q** loss of $50	**R** loss of 50 kg
S 50 km west	**T** 50 m below sea level	**U** decrease of 50
V $50 withdrawal	**W** 3 floors below ground	**X** 50 km north

b Match up each of A to L in part **a** with its opposite chosen from M to X.

Directed numbers have both size and direction.

2 Write a directed number to show the size and direction of the number in each sentence.

a Jan's Mathematics mark was 8 marks higher this time.

b The temperature today is 4°C below yesterday's temperature.

c John took fourteen steps backwards.

d After playing marbles, Melissa had lost 15 marbles.

e The price of Telstra shares rose by twelve cents last week.

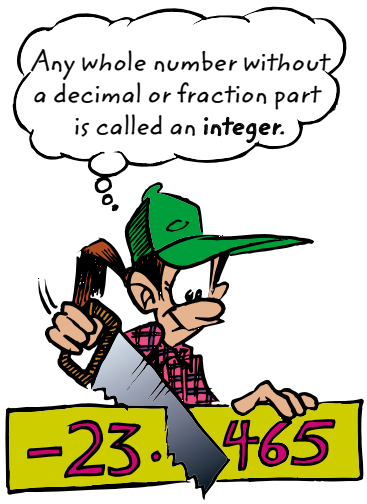
f The level of water in the reservoir has dropped two metres.

g The price of fish has not changed this week.

h My profits have increased by $1800 this year.

i In the last hour we have travelled 60 km south.

j I have just discovered that my bank account is overdrawn by $135.



3 Write two different sentences that might be represented by the directed number −3.

4 Write two different sentences that might be represented by the directed number +7.

5 Write a directed number to show the change described in each sentence.

a The temperature fell from 37°C to 26°C.

b I had ten golf balls when I arrived. Now I have only three.

c I was 4 km east of home. Now I am 2 km west of home.

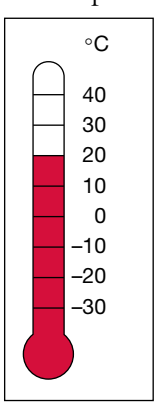
d I bought shares for $23 per share and sold them for $19 per share.

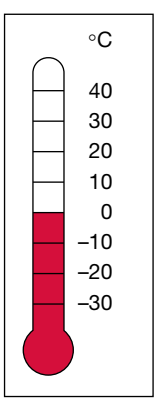
e The diver was 2·4 m below the surface. She is now 4·4 m below the surface.

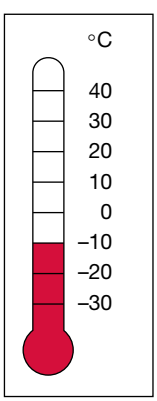
f The frog was 2 cm behind the line. After the jump it was 5 cm in front of the line.

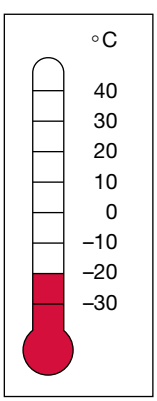
g We entered the lift on the tenth floor. We left the lift one floor below ground level.

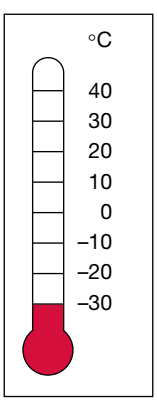
6 What temperature is shown on each thermometer below?

a


b


c


d


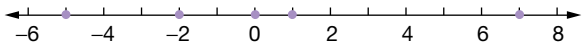
e


7 True or false?

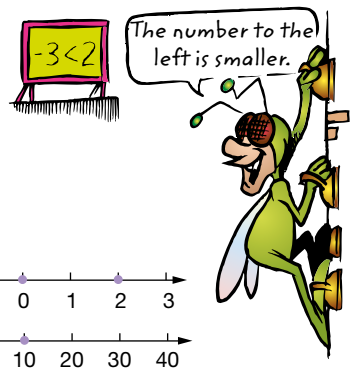
a $20 < 0$ **b** $0 > -10$ **c** $-20 < -30$ **d** $-10 > -30$ **e** $-30 < 0$

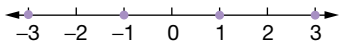
To graph a number on a number line, place a large dot at the position of that number on the line.

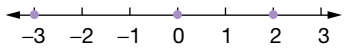
8 We can extend our number line to the left of zero to include negative numbers. Use braces (curly brackets) to show the set of numbers graphed on each number line; e.g. the set shown on

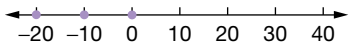


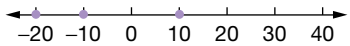
is $\{-5, -2, 0, 1, 7\}$.

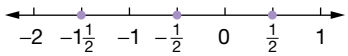


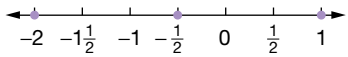
a


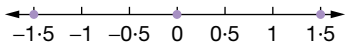
b


c


d


e


f


g


h


9 Graph each set of numbers below on a separate number line.

a $\{-2, -1, 0\}$ **b** $\{-4, -2, 0, 2\}$ **c** $\{-1, 0, 1\}$

d $\{-1{\cdot}5, -0{\cdot}5, 0{\cdot}5\}$ **e** $\{-2, -0{\cdot}5, 1\}$ **f** $\{-0{\cdot}8, -0{\cdot}6, -0{\cdot}2\}$

10 Which is the lower temperature?

a −2°C or 2°C **b** −5°C or 0°C **c** −5°C or 2°C

d −5°C or −2°C **e** −5°C or −10°C **f** 7°C or −7°C

An integer is a directed whole number. … −3, −2, −1, 0, 1, 2, 3, …

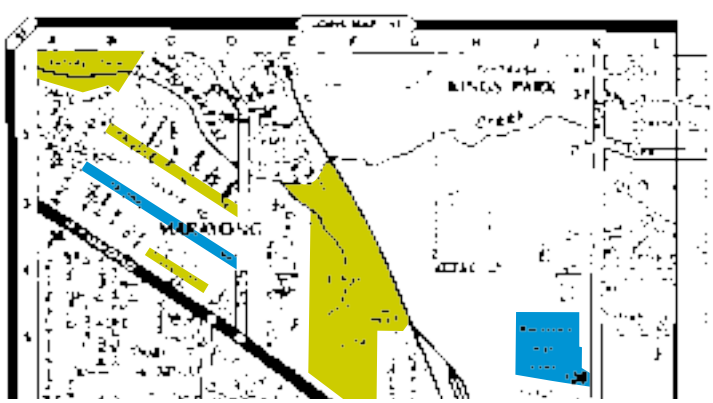
11 Arrange these integers in order from smallest to largest.

a 1, 5, −2, 4 **b** 0, −4, 1, −1 **c** −1, −2, −3, −4

d 2, −2, 3, −3 **e** −2, 4, −3, 0 **f** 7, −8, 9, −10

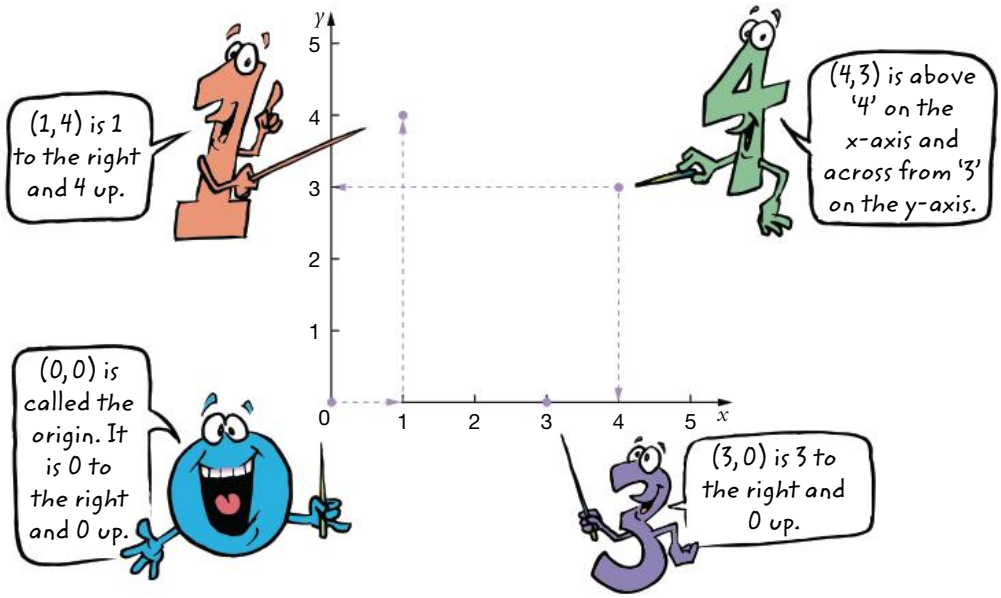
7:02 The number plane

This street directory uses a grid of letters and numbers. You can look down from the letter and across from the number to find a location. For example, 'J5' gives Blacktown High School.



On most street directories, a letter and a number are used to name a position, but on the **number plane** two numbers are used. These are called **coordinates** and are written in parentheses and separated by a comma. The two axes of the number plane are called the ***x*-axis** (horizontal) and the ***y*-axis** (vertical). The ***x*-coordinate** is always written before the ***y*-coordinate**.

(2,5) is called a *number pair* or the *coordinates of a point*.
- The first number is the reading on the x-axis (how far right).
- The second number is the reading on the y-axis (how far up).
- The point (0,0) is called the **origin**. ('Origin' means 'starting point' or 'beginning'.)



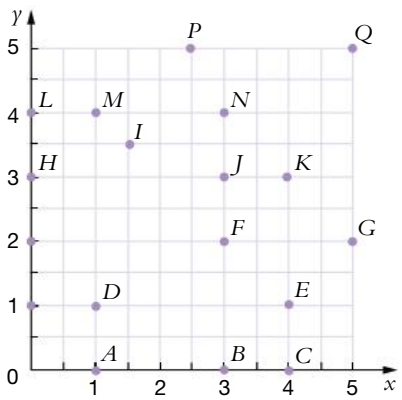
WORKED EXAMPLES

1 From the diagram, write the letter naming each point:

a (3,4) **b** $(1\frac{1}{2},3\frac{1}{2})$ **c** (1,0)

2 State the coordinates of:

a G **b** L **c** P



Solutions

1 To find each point, start at the origin.

a (3,4) is 3 to the right and 4 up.
$\therefore$ (3,4) is N.

b $(1\frac{1}{2},3\frac{1}{2})$ is $1\frac{1}{2}$ to the right and $3\frac{1}{2}$ up.
$\therefore$ $(1\frac{1}{2},3\frac{1}{2})$ is I.

c (1,0) is 1 to the right and 0 up.
$\therefore$ (1,0) is A.

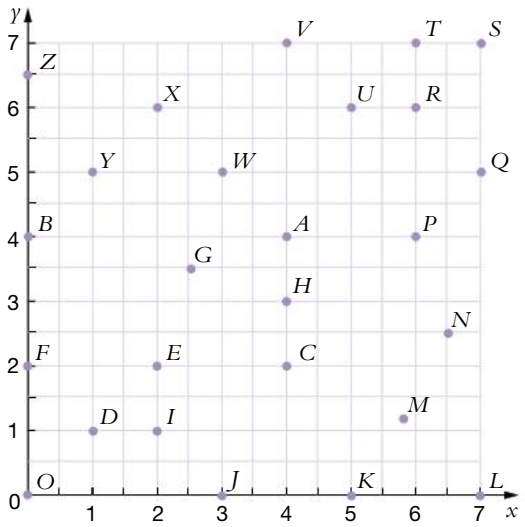
2 **a** G is above 5 and to the right of 2.
$\therefore$ G is (5,2).

b L is above 0 and on 4.
$\therefore$ L is (0,4).

c P is above $2\frac{1}{2}$ and to the right of 5.
$\therefore$ P is $(2\frac{1}{2},5)$.

Exercise 7:02

1 From the diagram, write the letter naming each point.



a $(2\frac{1}{2}, 3\frac{1}{2})$ **b** (2,2)
c (5,6) **d** (4,7)
e (0,0) **f** (7,5)
g (3,5) **h** (6,6)
i $(0, 6\frac{1}{2})$ **j** (0,2)
k (3,0) **l** (5,0)
m (4,3) **n** (6,4)
o $(6\frac{1}{2}, 2\frac{1}{2})$ **p** (1,5)
q (2,1) **r** (4,4)
s (4,2) **t** (7,7)
u (0,4) **v** (5·8,1·2)
w (7,0) **x** (6,7)
y (1,1) **z** (2,6)



2 From the number plane in Question 1, write the coordinates of each point.

a *A* **b** *B* **c** *C* **d** *D* **e** *E* **f** *F* **g** *G*
h *H* **i** *I* **j** *J* **k** *K* **l** *L* **m** *Z* **n** *N*
o *O* **p** *P* **q** *Q* **r** R **s** *S* **t** *T* **u** *U*

3 Make a number plane grid with 10 equal units on the *x*-axis and 7 equal units on the *y*-axis. (Draw your own grid or use graph paper.) Plot the points below on your number plane. What shapes are formed when each set of points is joined in the given order?

a (7,1) (7,3) (9,3) (9,1) (7,1)
b (4,1) (4,4) (5,4) (5,1) (4,1)
c (6,0) (6,3) (7,0) (6,0)
d (0,4) (2,6) (5,6) (3,4) (0,4)
e (1,0) (0,1) (0,2) (1,3) (2,3) (3,2) (3,1) (2,0) (1,0)

4 Now draw a grid with 10 units on the *x*-axis and 16 units on the *y*-axis. All units must be the same size. Draw shapes on this number plane as you did in Question 3, using the following points:

a (2,5) (2,3) (1,1) (1,0) (2,0) (3,1) (4,4)
b (8,14) (7,14) (7,12) (8,11) (8,9) (9,9)
c (2,11) (3,11) (4,12) (5,12) (5,10) (4,10) (1,8) (0,9) (0,10) (1,12) (5,16) (6,16) (9,13) (10,10) (9,9) (9,7) (7,4) (7,3) (8,1) (8,0) (7,0) (6,1) (5,4) (4,4) (2,5) (1,7) (1,8)
d Draw a big nose on your picture by starting from (5,14) and ending at (6,14).
e At (5,15) and (6,15), mark the pupils of the eyes as large dots. Draw the eyes around these dots.
f Draw a small mouth and fingers. Colour your picture.

5 Use 5-millimetre grid paper with 30 units on the *x*-axis and 24 units on the *y*-axis. (All units must be the same size.) Draw shapes as indicated by the following sets of coordinates, but join the points with curved lines.

a (6,0) (8,4) (10,6) (12,8) (10,10) (8,11) (2,10) (4,12) (8,13) (12,14) (6,14) (2,12) (2,10) (1,12) (2,16) (4,18) (6,18) (8,20) (12,22) (16,23) (20,23) (24,22) (28,20) (26,20) (29,16) (28,16) (29,12) (28,12) (29,8) (28,8) (30,4) (29,4) (30,0)

b (8,13) (6,14)

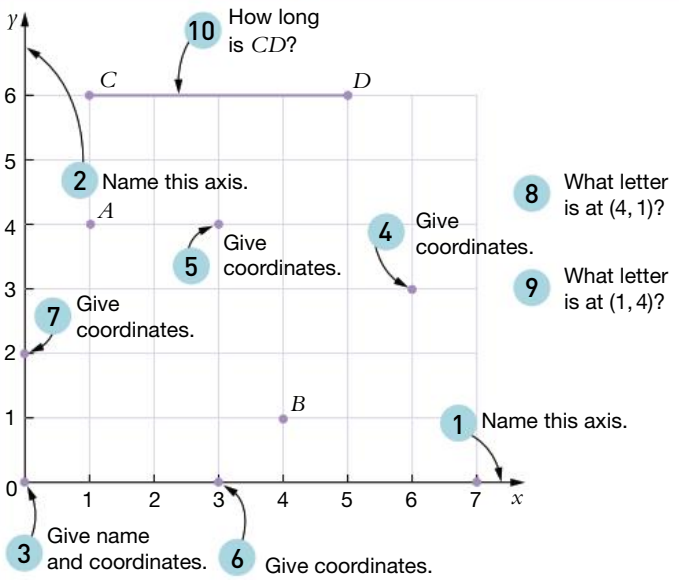
c (11,17) (11,19) (12, $19\frac{1}{2}$) (13,19) (13,17) (12, $16\frac{1}{2}$) (11,17)

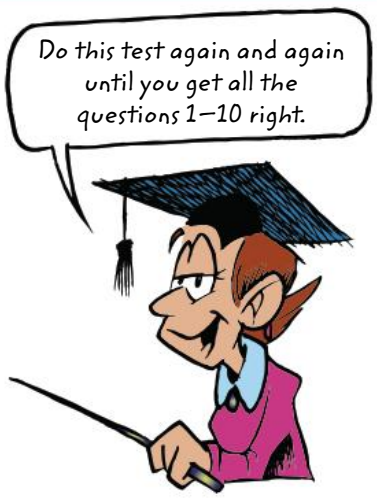
d Colour the inside of the mouth, above and to the right of (8,13).

e Draw a large black dot in the eye.

f Add feathers and colour the eagle.

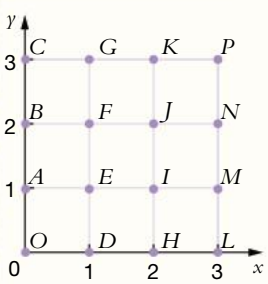
CHALLENGE 7:02 THE NUMBER PLANE





7:03 The number plane extended

PREP QUIZ 7:03



1 What are the coordinates of the origin?

Write the letter used to name each point:

2 (1,2)	**3** (3,3)	**4** (2,3)
5 (0,3)	**6** (1,0)	**7** (3,1)
8 (0,0)	**9** (2,2)	**10** (0,2)



The number plane is also called the 'Cartesian plane' after René Descartes (1596–1650), who largely invented it. This number plane contains four quadrants, as used in the examples and questions below. Just as we extended the number line, the four quadrants now extend the number plane.

WORKED EXAMPLES

1 (−3,2) is 3 to the left and 2 up.

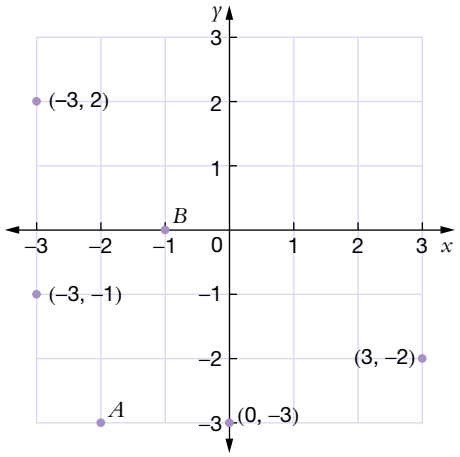
2 (−3,−1) is 3 to the left and 1 down.

3 (0,−3) is 0 to the right and 3 down.

4 (3,−2) is 3 to the right and 2 down.

5 A is below −2 and across from −3,
$\therefore$ A is (−2,−3).

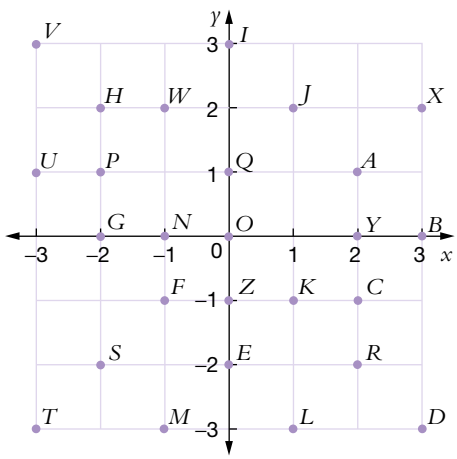
6 B is on −1 and across from 0,
$\therefore$ B is (−1,0).



Exercise 7:03

1 Write the letter used to name each point.

a (3,2)	**b** (1,−1)	**c** (0,0)
d (−2,−2)	**e** (−1,2)	**f** (0,3)
g (−2,0)	**h** (1,−3)	**i** (−3,1)
j (−1,−3)	**k** (1,2)	**l** (−2,1)
m (0,1)	**n** (0,−1)	**o** (2,−1)
p (−3,−3)	**q** (2,1)	**r** (3,0)
s (−1,−1)	**t** (3,−3)	**u** (−2,2)
v (2,−2)	**w** (2,0)	**x** (−3,3)
y (−1,0)	**z** (0,−2)	



2 Use the number plane in Question 1 to write the coordinates of each of these points.

a A	**b** B	**c** C	**d** D	**e** E	**f** F	**g** G
h H	**i** I	**j** J	**k** K	**l** L	**m** M	**n** N
o O	**p** P	**q** Q	**r** R	**s** S	**t** T	**u** U
v V	**w** W	**x** X	**y** Y	**z** Z		

3 Use the letters on the number plane in Question 1 to decode these messages.

a (−2,−2) (0,−2) (0,−2) (1,−1) … (−1,2) (0,3) (−2,−2) (3,−3) (0,0) (−1,−3) … (−2,−2) (0,−2) (0,−2) (1,−1) … (1,−1) (−1,0) (0,0) (−1,2) (1,−3) (0,−2) (3,−3) (−2,0) (0,−2)

b (3,−3) (0,3) (−2,−2) (2,−1) (0,3) (−2,1) (1,−3) (0,3) (−1,0) (0,−2) … (2,0) (0,0) (−3,1) (2,−2) … (2,−1) (−2,2) (0,3) (1,−3) (3,−3) (2,−2) (0,−2) (−1,0) … (−1,2) (−2,2) (0,3) (1,−3) (0,−2) … (−3,−3) (−2,2) (0,−2) (2,0) … (2,1) (2,−2) (0,−2) … (2,0) (0,0) (−3,1) (−1,0) (−2,0) … (0,−2) (−1,0) (0,0) (−3,1) (−2,0) (−2,2) … (−3,−3) (0,0) … (1,−3) (0,−2) (2,1) (2,−2) (−1,0)

4 The grid size of this number plane allows the coordinates of the points to be given to one decimal place.

a Write the coordinates of:

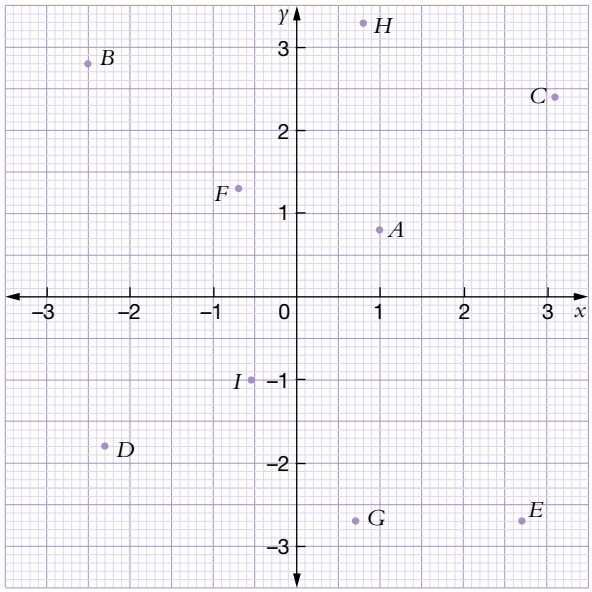
i A **ii** B

iii C **iv** D

b Write the letter used to name each point:

i $(-0{\cdot}7, 1{\cdot}3)$ **ii** $(-0{\cdot}5, -1{\cdot}0)$

iii $(2{\cdot}7, -2{\cdot}7)$ **iv** $(0{\cdot}8, 3{\cdot}3)$



5 Use a grid with x-axis values from -16 to 20 and y-axis values from -20 to 16 (5 mm graph paper is ideal). On this number plane, graph the points listed below and join them in the order given. Colour the picture that you make.

a (−1,−6) (−1,−4) (1,−1) (7,1) (10,8) (13,11) (19,8) (17,7) (17,6) (18,6) (16,4)

b (15,10) (13,8) (13,7) (15,6) (16,4) (15,2) (15,−2) (13,−7) (6,−11) (7,−16) (8,−18) (6,−17) (4,−18) (6,−16) (5,−11)

c (5,−11) (4,−12) (3,−12) (4,−9) (3,−12) (2,−16) (3,−18) (1,−17) (−1,−18) (1,−16) (2,−12) (3,−9) (2,−12) (1,−11) (−1,−6)

d (−1,−6) (−3,−2) (−1,−10) (−6,−1) (−6,−3) (−7,−1) (−8,−7) (−9,2) (−12,−7) (−11,4) (−14,−2) (−11,9) (−13,9) (−9,11) (−4,11) (2,7) (1,−1)

e (12,10) (9,13) (10,14) (11,13) (12,15) (13,14) (14,15) (15,15) (16,14) (18,13) (19,11) (17,9)

f (15,8) (15,9) (16,9) (16,8) (15,8)



6 A is the point (3,4). Give the coordinates of the point that is:

a 3 units to the right of A (call this point B)

b 3 units to the left of A (call this point C)

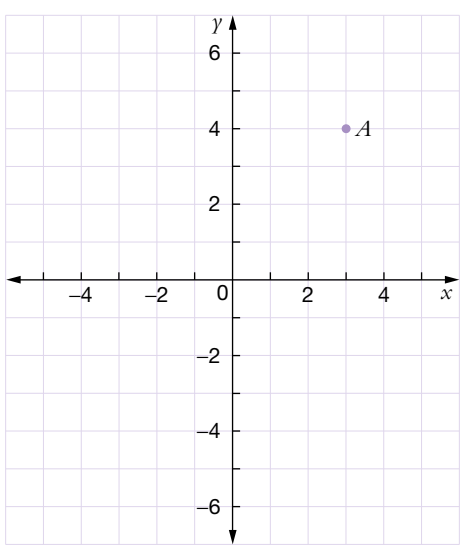
c 3 units above A (call this point D)

d 3 units below A (call this point E)

e the reflection of A in the y-axis

f the reflection of A in the x-axis.

g Join the points in order from B to C to D to E and then to B again. Describe the shape you have made.



7 What would the coordinates of the point $A(3, 4)$ become if A were:

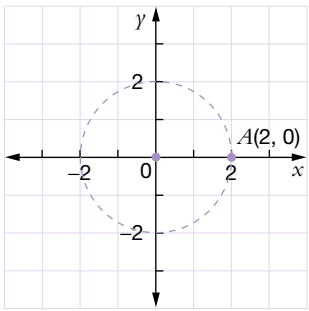
a translated 4 units to the left **b** translated 5 units down

c reflected in the x-axis **d** reflected in the y-axis?

(Hint: Use the diagram in Question **6**.)

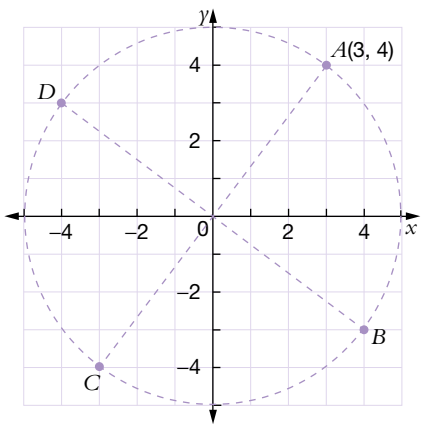
8 Find the coordinates of the new position of the point $A(2, 0)$ when it is rotated in a clockwise direction about the origin through an angle of:

a 90° **b** 180° **c** 270° **d** 360°



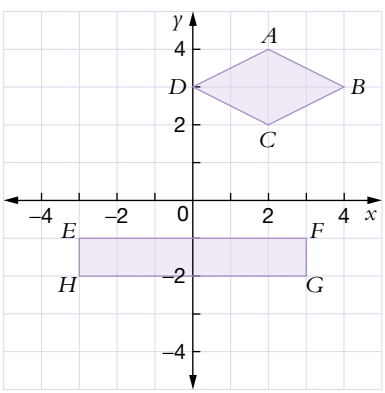
9 B, C and D are the images of the point A when it is rotated in a clockwise direction about the origin through 90°, 180° and 270°. What are the coordinates of:

a B **b** C **c** D?



10 **a** From the diagram, write the coordinates of A, B, C, D, E, F, G and H.

b Sketch a rectangle that has vertices at (3,0) and (–1,–4), at opposite ends of a diagonal.



7:04 Wilderness adventures



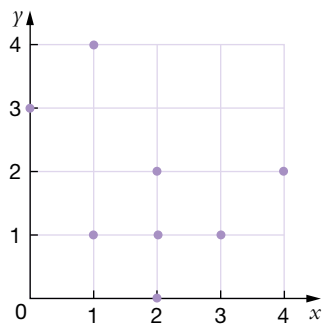
Ms Travers decided to play a game with her Year 7 class.

She drew a number plane on the board and secretly chose coordinates for each part of her favourite adventure. She then gave a positive or negative score to each part of the adventure. Each part of the adventure was represented by a point on the number plane and by a score.

Keeping the adventure coordinates secret, she asked students in two teams to take turns to choose points on the number plane, giving them a positive or negative score when they selected the coordinates for a part of the adventure. Two students wrote the total score for each team on the board.

The game continued until each team had chosen ten points on the plane. The team with the highest score won the game.

	Part of the adventure	Score (points)	Secret coordinates
1	Photograph zebra	1	(0,2)
2	Photograph gnu	2	(1,3)
3	Photograph lion	3	(2,4)
4	Photograph tiger	4	(1,1)
5	Photograph rhinoceros	5	(4,0)
6	Meet friendly locals	6	(3,2)
7	Helped by Tarzan	7	(2,0)
8	Discover a lost city	8	(0,0)
9	Bridge destroyed	−3	(1,0)
10	Lose your compass	−4	(0,4)
11	Lost in the jungle	−5	(3,1)
12	Shoot yourself in foot	−6	(2,2)
13	Bitten by snake	−7	(3,4)
14	Fall into quicksand	−8	(4,4)
	Selecting coordinates already chosen	−2	The secret coordinates change for each new game.
	All other coordinates (miss)	−1	



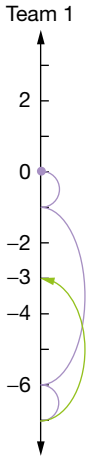
As each point is chosen by the players, it is graphed on the number plane and the scores are updated.

Team 1	Team 2
−1	7
−6	6
−7	5
−3	

WORKED EXAMPLE

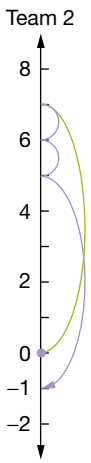
Each team hopes to guess the secret coordinates that have positive values.

Team 1			
Coordinates chosen	Result	Score	Total so far
(1,4)	(miss)	−1	−1
(3,1)	Lost in the jungle	−5	−6
(4,2)	(miss)	−1	−7
(1,1)	Photograph tiger	4	−3





Team 2			
Coordinates chosen	Result	Score	Total so far
(2,0)	Helped by Tarzan	7	7
(2,1)	(miss)	−1	6
(0,3)	(miss)	−1	5
(2,2)	Shoot yourself in foot	−6	−1



Exercise 7:04

1 Copy and complete the following tables using the secret coordinates from page 204.

a

Coordinates chosen	Result	Score	Total so far
(0,0)			
(0,4)			
(2,1)			
(1,0)			
(1,3)			
(3,4)			

b

Coordinates chosen	Result	Score	Total so far
(3,1)			
(3,2)			
(0,2)			
(2,2)			
(4,1)			
(1,1)			

2 Copy and complete the following:

a

Coordinates chosen	Score	Total so far
(0,4)		
(1,4)		
(2,4)		
(3,4)		
(4,4)		

b

Coordinates chosen	Score	Total so far
(4,4)		
(3,3)		
(2,2)		
(1,1)		
(0,0)		

c

Coordinates chosen	Score	Total so far
(3,0)		
(3,1)		
(3,2)		
(3,3)		
(3,4)		

3 Copy and complete the following:

a

Score	Total so far
−1	
4	
−2	
1	
−5	

b

Score	Total so far
1	
−6	
−4	
−3	
5	

c

Score	Total so far
−1	
−1	
−1	
−1	
−1	

d

Score	Total so far
−8	
1	
2	
3	
4	

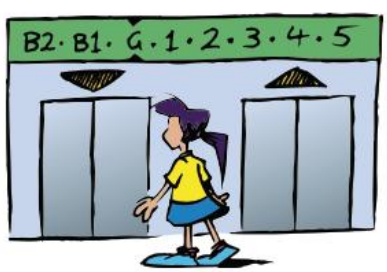
4 Combine the following scores to find the total so far.

a a loss of 6, a gain of 3, a loss of 1
b a gain of 5, a loss of 3, a gain of 6
c a gain of 2, a loss of 1, a loss of 2
d a loss of 1, a gain of 3, a loss of 7
e a loss of 5, a loss of 4, a loss of 8
f a loss of 7, a gain of 2, a gain of 4
g −3, −4
h −2, 5
i 1, −4
j 6, −6
k −8, −7
l −6, 4
m −3, −2
n 8, 6
o −1, 8
p −7, −6
q −1, −2
r −8, −1
s −3, 4, −1
t 2, −5, 1
u 7, −5, −3
v 4, −7, 3

INVESTIGATION 7:04 DIRECTED NUMBERS

Find newspaper, magazine or internet articles (including graphs and diagrams) that use directed number quantities. In each case, explain how the directed numbers are used.





7:05 Addition and subtraction of directed numbers

PREP QUIZ 7:05

In the game of indoor cricket, 1 point is gained for every run made and 5 points are lost every time a player gets out. 1 run and 1 player out would give a total score of (1) + (−5) = −4. Find the total score for each situation.



1 6 runs and 1 player out
2 2 runs and 1 player out
3 5 runs and 1 player out
4 4 runs and 1 player out
5 1 player out and 10 runs
6 1 player out and 3 runs
7 1 player out and 5 runs
8 1 player out and no runs
9 2 players out and 0 runs
10 8 runs and 2 players out

WORKED EXAMPLES

1 *Adding directed numbers*

(We have been adding directed numbers in Exercise 7:04 and Prep quiz 7:05.)

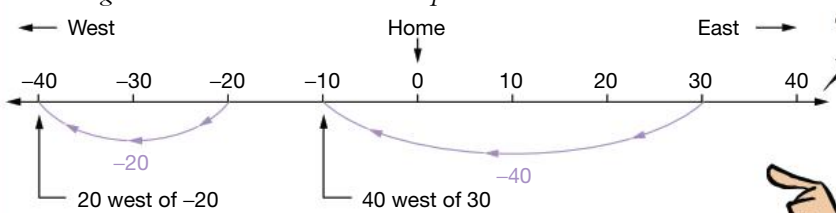
a −5 + 3 = −2 or (−5) + (3) = −2
A loss of 5 and a gain of 3 gives a loss of 2.

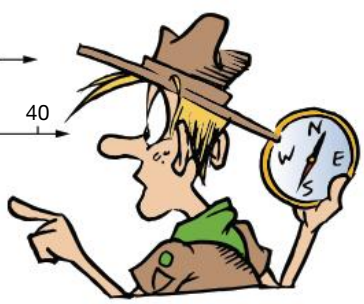
b −3 + −5 = −8 or (−3) + (−5) = −8
A loss of 3 and a loss of 5 gives a loss of 8.

c 1 + −10 = −9 or (1) + (−10) = −9
A gain of 1 and a loss of 10 gives a loss of 9.

A negative number can represent a loss. A positive number can represent a gain.

2 *Using the east–west road to subtract positive numbers*

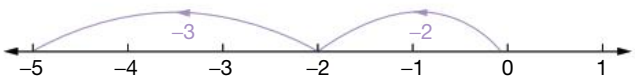




a −20 − 20 = −40
−20 take away 20 gives −40.

b 30 − 40 = −10
30 take away 40 gives −10.

3 *Using the number line for addition and subtraction*



The number line could be used to show that
(−2) + (−3) = −5 or −2 −3 = −5.
A loss of 2 and a loss of 3 gives a loss of 5.

Adding the negative of a number is the same as subtracting the number,
e.g. −2 + −3 = −2 − 3

Exercise 7:05

1 **a** $-8 + 4$ **b** $-3 + 5$ **c** $-6 + 6$ **d** $-10 + 3$
e $-1 + 12$ **f** $-6 + 8$ **g** $-11 + 8$ **h** $-3 + 7$
i $(-2) + (-3)$ **j** $(-6) + (-1)$ **k** $(-7) + (-10)$ **l** $(-4) + (-4)$
m $-10 + -3$ **n** $-8 + -1$ **o** $-3 + -5$ **p** $-8 + -2$
q $(4) + (-3)$ **r** $(9) + (-15)$ **s** $(2) + (-1)$ **t** $(100) + (-7)$
u $9 + -1$ **v** $8 + -8$ **w** $6 + -11$ **x** $1 + -7$

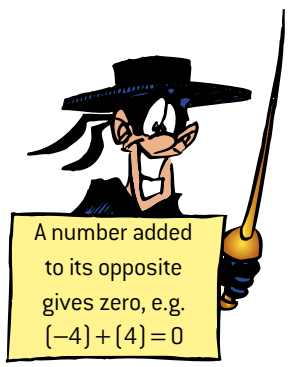
2 Use the east–west road on page 207 to write a number sentence for each, giving the answer as a directed number.



a 30 west of 10 **b** 20 east of −10
c 30 east of 10 **d** 20 west of −10
e 50 west of 5 **f** 40 east of −20
g 20 east of −53 **h** 100 west of 33
i 33 east of −4 **j** 33 east of −6

3 **a** $2 - 0$ **b** $2 - 1$ **c** $2 - 2$ **d** $2 - 3$
e $2 - 4$ **f** $2 - 5$ **g** $0 - 2$ **h** $-1 - 2$
i $-6 - 1$ **j** $1 - 4$ **k** $3 - 5$ **l** $5 - 15$
m $-1 - 1$ **n** $-3 - 4$ **o** $-4 - 2$ **p** $-5 - 3$
q $2 - 12$ **r** $5 - 9$ **s** $3 - 13$ **t** $1 - 19$
u $30 - 90$ **v** $-20 - 30$ **w** $-15 - 45$ **x** $9 - 30$

4 **a** $-3 + 3$ **b** $-6 + 6$ **c** $-5 + 5$
d $(4) + (-4)$ **e** $(9) + (-9)$ **f** $(1) + (-1)$
g $6 + -6$ **h** $2 + -2$ **i** $7 + -7$
j $-8 + 8$ **k** $-11 + 11$ **l** $-9 + 9$
m $0{\cdot}5 + -0{\cdot}5$ **n** $(-1{\cdot}4) + (1{\cdot}4)$ **o** $(-0) + (0)$
p $(-\frac{3}{4}) + (\frac{3}{4})$ **q** $(\frac{3}{10}) + (-\frac{3}{10})$ **r** $(-1\frac{1}{2}) + (1\frac{1}{2})$



5 **a** $(6) + (-3)$ **b** $(1) + (-3)$ **c** $(12) + (-5)$
d $6 - 3$ **e** $1 - 3$ **f** $12 - 5$
g $4 + -2$ **h** $1 + -5$ **i** $7 - +2$
j $4 - 2$ **k** $1 - 5$ **l** $7 - 2$

The answers to Question **5** suggest that when an addition and a subtraction sign are side by side (with no numeral between them), the two *unlike* signs make a minus, e.g. $7 + -3 = 7 - 3$ $(2) + (-8) = 2 - 8$ $(6) - (+4) = 6 - 4$

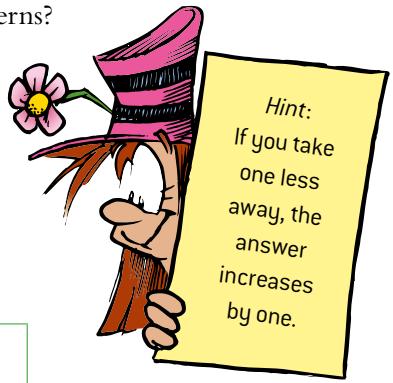
6 **a** $-5 + -7$ **b** $6 + -3$ **c** $8 + -10$ **d** $-2 + -3$
e $6 - (-3 + 5)$ **f** $-2 - (-8 + 9)$ **g** $-5 - (-3 + 6)$ **h** $4 - (7 - 2)$
i $(3 - 11) + 2$ **j** $(-1 - 5) - 1$ **k** $(2 + 8) - 12$ **l** $9 + (-3 - 4)$
m $-8 + (4 \times 2)$ **n** $-5 + (3 \times 2)$ **o** $-16 + (7 \times 2)$ **p** $(8 \times 2) - 20$

7:06 Subtracting a negative number

Discussion

What does it look like when we complete these subtraction patterns?

$5 - 4 = 1$	$8 - 2 = 6$
$5 - 3 = 2$	$8 - 1 =$
$5 - 2 =$	$8 - 0 =$
$5 - 1 =$	$8 - -1 =$
$5 - 0 =$	$8 - -2 =$
$5 - (-1) =$	$8 - -3 =$
$5 - (-2) =$	$8 - -4 =$



Investigation	$5 - 3 = 2 = 5 - 3$ $5 - 2 = 3 = 5 - 2$ $5 - 1 = 4 = 5 - 1$ $5 - 0 = 5 = 5 - 0$ $5 - (-1) = 6 = 5 + 1$ $5 - (-2) = 7 = 5 + 2$	$8 - 1 = 7 = 8 - 1$ $8 - 0 = 8 = 8 - 0$ $8 - -1 = 9 = 8 + 1$ $8 - -2 = 10 = 8 + 2$ $8 - -3 = 11 = 8 + 3$ $8 - -4 = 12 = 8 + 4$
Pattern and discovery	The answers are increasing by 1 each time. To take away a negative, add its opposite: e.g. $5 - (-3) = 5 + 3$ $\quad 8 - -5 = 8 + 5$ $5 - (-4) = 5 + 4$ $\quad 8 - -6 = 8 + 6$	
Rule	When two minus signs are side by side (with no numeral between them), the two *like* signs make a plus.	



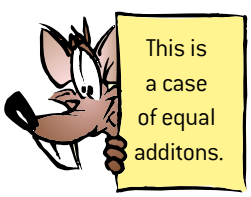
Using opposites

$$6 - -3 = (6) - (-3)$$

If we add 3 to both numbers, the answer won't change:

$$= (6 + 3) - (-3 + 3)$$
$$= (6 + 3) - (0)$$
$$= 6 + 3$$

$\therefore 6 - -3 = 6 + 3$



The two minus signs become a plus.

Using reasoning

In the 'Wilderness adventures' game (see 7:04), if our total so far is 9 and we lose 3, what is the new total?

$$9 + (-3) = 6$$

If the loss of 3 was a mistake, then we have to take that score away in order to get back to our previous score.

$\therefore 6 - (-3) = 9$ or $6 + 3 = 9$

To take away −3, add its opposite.

Exercise 7:06

1 **a** $3 - -4$ **b** $8 - -1$ **c** $0 - -6$ **d** $6 - -10$
e $5 - (-3)$ **f** $4 - (-8)$ **g** $9 - (-2)$ **h** $16 - (-7)$
i $-9 - -8$ **j** $-6 - -10$ **k** $-5 - -12$ **l** $-10 - -9$
m $-3 - (-6)$ **n** $-9 - (-4)$ **o** $-2 - (-7)$ **p** $-8 - (-5)$
q $(-10) - (-20)$ **r** $(-7) - (-1)$ **s** $(-2) - (-8)$ **t** $(-60) - (-4)$
u $8 - -7$ **v** $-8 - -7$ **w** $-9 - -9$ **x** $-11 - -11$

2 We have seen that $- -3$ or $-(-3)$ can mean 'the opposite of -3'. Simplify:
a $-(-5)$ **b** $-(-15)$ **c** $-(-2)$ **d** $-(-8)$
e $- -34$ **f** $- -7$ **g** $- -11$ **h** $- -9$

3 Simplify these mixed expressions.
a $8 - -9$ **b** $6 - 8$ **c** $-3 + 4$ **d** $16 - 9$
e $-4 - 13$ **f** $17 - -1$ **g** $2 - 23$ **h** $-4 - 6$
i $-5 - (-10)$ **j** $-2 - -12$ **k** $14 + (-4)$ **l** $-9 + 5$
m $6 - (-7 + 3)$ **n** $3 - (6 - 10)$ **o** $5 - (-1 - 8)$ **p** $-10 - (6 - 7)$
q $- -10$ **r** $0 - (-10)$ **s** $0 - -13$ **t** $-(-13)$
u $6 - (-3 + 8)$ **v** $12 - (6 - 2)$ **w** $-5 - (-1 + 10)$ **x** $-3 - (-8 + 9)$

4 Simplify:
a $0{\cdot}2 - -0{\cdot}3$ **b** $2{\cdot}2 - -0{\cdot}7$ **c** $-1{\cdot}4 - -0{\cdot}4$
d $-0{\cdot}11 - 0{\cdot}43$ **e** $0{\cdot}8 - 0{\cdot}9$ **f** $0{\cdot}1 - 0{\cdot}8$
g $\frac{3}{10} - \frac{9}{10}$ **h** $-\frac{3}{4} - -\frac{1}{4}$ **i** $-\frac{2}{10} - -\frac{5}{10}$
j $-(-\frac{3}{8})$ **k** $0 - -\frac{2}{3}$ **l** $(-\frac{3}{5}) + (-\frac{2}{5})$

The same methods can be used with decimals and with fractions.

5 We produced 843 coconuts from our farm in Hawaii and 1104 from our farm in Queensland. We calculated that the number left after filling an order for 2000 coconuts will be -53. What does this answer mean?





Write the question and answer for **1** to **24** on **ID Card 3**, page xvi.

7:07 Multiplication of directed numbers

PREP QUIZ 7:07

Complete:

1 $6 + 6 + 6 + 6$ = four lots of 6
$= \ldots \times \ldots$

2 $(-7) + (-7) + (-7)$ = three lots of (-7)
$= \ldots \times \ldots$

3 $8 + 8 + 8$ = three lots of 8
$= \ldots \times \ldots$

4 $(-2) + (-2) + (-2)$ = three lots of (-2)
$= \ldots \times \ldots$

Evaluate:

5 3 lots of -10

6 6 lots of -3

7 10 lots of -7

Continue each pattern.

8 $6, 4, 2, 0, \ldots, \ldots$

9 $9, 6, 3, 0, \ldots, \ldots$

10 $-9, -6, -3, 0, \ldots, \ldots$

We can discover the rules for multiplication by extending number patterns.

Investigation	$3 \times 3 = 9$ $3 \times 2 = 6$ $3 \times 1 = 3$ $3 \times 0 = 0$ $3 \times -1 = -3$ $3 \times -2 = -6$	$3 \times 3 = 9$ $2 \times 3 = 6$ $1 \times 3 = 3$ $0 \times 3 = 0$ $-1 \times 3 = -3$ $-2 \times 3 = -6$	3 lots of $-3 = -9$ $2 \times -3 = -6$ $1 \times -3 = -3$ $0 \times -3 = 0$ $-1 \times -3 = 3$ $-2 \times -3 = 6$
Pattern and discovery	The answers are decreasing by 3 each time.		The answers are increasing by 3 each time.
	plus × minus = minus minus × plus = minus		minus × minus = plus
Rule	Multiplying two *unlike* signs gives a minus. $-8 \times 7 = -56$		Multiplying two *like* signs gives a plus. $-8 \times -7 = 56$



WORKED EXAMPLES

1 $-3 \times -8 \times -5$
$= (-3 \times -8) \times -5$
$= 24 \times -5$
$= -120$

2 $(-6)^2$
$= -6 \times -6$
$= 36$

Exercise 7:07

1 **a** $1 \times (-1)$ **b** 4×-2 **c** $6 \times (-2)$ **d** 5×-5
e -8×1 **f** -3×4 **g** -11×4 **h** -2×8
i $(-2) \times (-5)$ **j** -3×-3 **k** -2×-4 **l** $(-1) \times (-1)$
m -8×0 **n** 0×-7 **o** -12×0 **p** 0×-9
q -3×7 **r** 3×-2 **s** -2×-6 **t** -4×3
u 4×-5 **v** -1×-7 **w** -8×2 **x** -7×-2

2 a 7×-5 b 4×-7 c -3×9 d 9×-5
e -4×-4 f -8×-4 g -7×-6 h -8×6
i $(-6) \times (-4)$ j 9×-9 k 5×-8 l 4×-9
m -8×7 n $7 \times (-7)$ o -7×-4 p $(-8) \times 9$
q -6×-6 r 9×-8 s -9×-7 t -4×6
u 11×11 v -9×-6 w -8×8 x -7×-9

3 a $(-1)^2$ b $(-2)^2$ c $(-3)^2$ d $(-4)^2$
e $(-5)^2$ f $(-6)^2$ g $(-7)^2$ h $(-8)^2$
i $(-2)^3$ j $(-2)^4$ k $(-2)^5$ l $(-2)^6$

4 a -4×80 b -5×-30 c -9×10 d -80×3
e $6 \times -0{\cdot}3$ f $8 \times (-0{\cdot}5)$ g $-0{\cdot}2 \times 0{\cdot}2$ h $-0{\cdot}3 \times -0{\cdot}6$
i $0{\cdot}7 \times -0{\cdot}1$ j $-0{\cdot}11 \times 0{\cdot}7$ k $0{\cdot}06 \times -8$ l $-0{\cdot}7 \times -0{\cdot}8$

5 a $-3 \times -3 \times -3$ b $-2 \times -2 \times -2$ c $-5 \times -5 \times -5$ d $-5 \times -2 \times 5$
e $-1 \times -3 \times 6$ f $-4 \times -8 \times 2$ g $-19 \times 3 \times 5$ h $4 \times -2 \times 5$
i $3 \times 4 \times -3$ j $2 \times 6 \times -1$ k $6 \times -3 \times -1$ l $-3 \times -8 \times -1$

6 What is the sign of the answer when a negative number is raised to:
a an odd power b an even power?

7:08 Division of directed numbers

PREP QUIZ 7:08

Find the value of each ☐ to make the number sentence true.

1 $6 \times 5 = \square \quad \therefore \square \div 5 = 6$ **2** $6 \times -5 = \square \quad \therefore \square \div -5 = 6$
3 $-6 \times -5 = \square \quad \therefore \square \div -5 = -6$ **4** $-6 \times 5 = \square \quad \therefore \square \div 5 = -6$

Copy and complete:

5 $6 \times \ldots = 30$ **6** $6 \times \ldots = -30$ **7** $-6 \times \ldots = 30$ **8** $-6 \times \ldots = -30$
9 How many losses of 2 are needed to make a loss of 10? (i.e. $-10 \div -2 = \ldots$)
10 What would you multiply 5 by to get -10? (i.e. $-10 \div 5 = \ldots$)

From the questions **1** to **4** above, we can see:

1 $30 \div 5 = 6$
2 $-30 \div -5 = 6$
→ Division with two like signs gives a plus.

3 $30 \div -5 = -6$
4 $-30 \div 5 = -6$
→ Division with two unlike signs gives a minus.

If $-6 \times -5 = 30$
then $30 \div -6 = -5$
and $30 \div -5 = -6$

For multiplication and division:
- two *like* signs give a *plus*
- two *unlike* signs give a *minus*.

Exercise 7:08

P Foundation worksheet 7:08 Division of directed numbers

1 **a** $18 \div 3$ **b** $-18 \div 3$ **c** $18 \div -3$ **d** $-8 \div -2$
e $16 \div -4$ **f** $-20 \div 2$ **g** $9 \div (-1)$ **h** $(-6) \div (-1)$
i $(-10) \div 2$ **j** $-28 \div 4$ **k** $-24 \div -3$ **l** $60 \div -6$
m $-35 \div -7$ **n** $20 \div -5$ **o** $-30 \div 10$ **p** $-16 \div 2$
q $-27 \div -3$ **r** $20 \div -4$ **s** $6 \div (-6)$ **t** $(-9) \div 9$
u $(-7) \div (-7)$

2 **a** $\frac{-30}{6}$ **b** $\frac{6}{-2}$ **c** $\frac{-21}{-3}$
d $\frac{42}{-6}$ **e** $\frac{-21}{-7}$ **f** $\frac{30}{-5}$
g $\frac{-32}{8}$ **h** $\frac{-56}{-7}$ **i** $\frac{45}{-9}$
j $\frac{49}{-7}$ **k** $\frac{-48}{-8}$ **l** $\frac{36}{6}$
m $\frac{-630}{-9}$ **n** $\frac{810}{-9}$ **o** $\frac{-300}{-10}$



7:09 Using directed numbers

PREP QUIZ 7:09

Write the basic (simplest) numeral for the following.

1 $8 - 15$ **2** $-9 - 4$ **3** $-8 + 6$ **4** $6 - -3$
5 $5 + (-7)$ **6** $- -6$ **7** -8×5 **8** $(-400) \div (-5)$
9 $\frac{32}{-4}$ **10** $(-8)^2$

The order of operations introduced in 1:06 also applies to directed numbers (of course).

Remember, the directed numbers … $-4, -3, -2, -1, 0, 1, 2, 3, 4$, … are also called **integers**.

Order of operations
1 [ ]
2 × and ÷
3 + and −

WORKED EXAMPLES

1 $8 - 5 \times 3 = 8 - 15$
$= -7$

2 $-6 + (-3)^2 - (-1 - 3) = -6 + 9 - (-4)$
$= -6 + 9 + 4$
$= 7$

Exercise 7:09

P Foundation worksheet 7:09
Using directed numbers

1 **a** $8 - 5 \times 2$ **b** $15 - 4 \times 4$ **c** $6 + 5 - 25$
d $11 - 15 + 8$ **e** $6 \times -4 \div 2$ **f** $-50 \div 10 \times 2$
g $(-3 - 7) \times -9$ **h** $3 \div (-3 + 6)$ **i** $9 \div (3 \times -3)$
j $(-3 - 3) \div (1 - 7)$ **k** $6 - 3 \times 4 - 3$ **l** $10 - 12 \div 4 - 7$
m $-1 \times [-8 \div (-4 + 6)]$ **n** $6 - [15 - (-3 - 2)]$

$(-3)^2 = -3 \times -3$
$= 9$

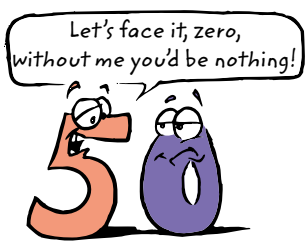
2 **a** $-3{\cdot}8 + 9$ **b** $-4{\cdot}7 - 5{\cdot}4$ **c** $8{\cdot}6 - 10{\cdot}4$
d $4{\cdot}11 - -0{\cdot}11$ **e** $-100 + 9{\cdot}56$ **f** $14{\cdot}65 - 20$
g $6{\cdot}35 \times -6$ **h** $-0{\cdot}5 \times -0{\cdot}5$ **i** $(-0{\cdot}3)^2$
j $-103 \div 10$ **k** $-6{\cdot}24 \div -6$ **l** $0{\cdot}05 \div (-5)$
m $-100 \times (0{\cdot}1)^2$ **n** $-1{\cdot}3 + 0{\cdot}3$ **o** $(-2)^3 \times 0{\cdot}2$

3 Continue each number pattern.
a 9, 7, 5, 3, … , … , … **b** −10, −8, −6, −4, … , … , …
c −2, −4, −8, −16, … , … , … **d** 2, −4, 8, −16, … , … , …
e −6, 6, −6, 6, … , … , … **f** 64, −32, 16, −8, … , … , …

4 **a** $\dfrac{-6+10}{-2}$ **b** $\dfrac{-32-16}{4}$ **c** $\dfrac{-15+65}{-10}$
d $\dfrac{-10\times-4}{8}$ **e** $\dfrac{-6-4}{-2+7}$ **f** $\dfrac{3-48}{1-10}$

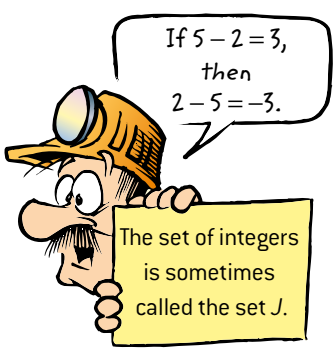
The fraction bar groups the top and groups the bottom.

5 Travelling in a lift 3 floors up and then 4 floors down could be shown as $3 - 4 = -1$. Make up stories of your own for the following.
a $-4 - 7 = \square$ **b** $-2 + 10 = \square$ **c** $6 - 10 = \square$
d $6 \times -3 = \square$ **e** $-60 \div 4 = \square$ **f** $-8 + 2 = \square$



6 True or false?
a $-6 \times 0 = -6$ **b** $-8 \times 1 = -8$
c $3 - 5 = 5 - 3$ **d** $3 - 5 = -5 + 3$
e $-3 - 5 = -5 - 3$ **f** $-6 \times -3 = -3 \times -6$
g $5 \times (-6 + 3) = 5 \times -6 + 5 \times 3$
h $(-6 - 7) - 8 = -6 + (-7 - 8)$
i $(-3 \times -4) \times -5 = -3 \times (-4 \times -5)$

7 Which of the integers −10, 7, 3 and −2 is closest to zero on the number line?



8 From the integers −7, −5, −3, 1, 4, 7, find the three integers whose sum is:
a −2 **b** 2 **c** 0

9 Find the value of □ to make each number sentence true.
a $-8 \times \square = 304$ **b** $6 \times \square = -6{\cdot}12$ **c** $6 + \square = -17$

10 One more than −4 is added to the product of 2 and four less than 2. What is the result?

11 Start with the integer 2. Subtract 7. From your answer subtract 1, then multiply the result by −3. Now subtract 20. What is your final answer?

12 On a very cold day, the temperature at 6 am was −4°C. By 12 noon the temperature had risen by 6°C, but over the following 8 hours it fell by 11°C. What was the temperature at 8 pm?

13 The yachts *Kooka* and *Aussie* raced over a course that had six sections. *Aussie* started with a 10-second advantage. Over the six sections *Aussie* gained 20 seconds, then lost 1 minute, then gained 15 seconds, then lost 27 seconds, then gained 41 seconds and then lost 6 seconds. Which yacht won the race and by how much?

14 Local time in Sydney is always 3 hours ahead of Singapore time, and Singapore is always 5 hours behind Wellington, which is 12 hours ahead of London. What is the time difference between Sydney and London?

15 **a** Two numbers have a product of 12 but their sum is −7. What are the numbers?
b The sum of two numbers is 1 but their product is −30. What are the numbers?

16 At a gym for body builders, the most weight gained by one member in a week was 0·7 kg; the most weight lost by a member was 2·4 kg. Five members worked out the total of their changes since the previous week. Which of the following could be their answer?
A a gain of 4 kg **B** a loss of 0·1 kg **C** a loss of 15 kg **D** a loss of 144 kg

When playing Snakes & Ladders:
- going up a ladder is a move in a positive direction
- going down a snake is a move in a negative direction.

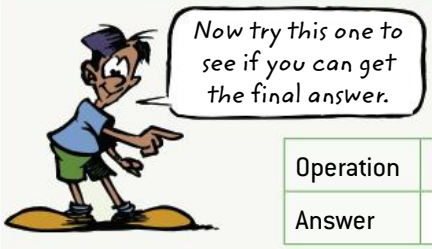


FUN SPOT 7:09 TRY THIS QUICK QUIZ!

You can practise working with integers by completing these tables, starting with the number given and then performing the operations in order. Look at the first table and check to see that you can get each answer.

Operation	Start	+ 3	−10	× 2	÷ (−4)	−1
Answer	5	8	−2	−4	1	0

← Final answer



Operation	Start	+ 6	× (−4)	+ 8	÷ (−4)
Answer	−3				1

← Final answer

Complete each table.

1

Operation	Start	× (−2)	+ 10	÷ 5	−3	+ 6
Answer	10					

2

Operation	Start	−10	× 3	+ 8	÷ 2	+ 2
Answer	6					

3

Operation	Start	+ 3	× (−5)	−5	÷ 2	+ 10
Answer	−2					

4

Operation	Start	−3	÷ 4	+ 4	× (−3)	+ 7
Answer	−5					

5

Operation	Start	−10	× 5	+ 8	÷ 2	−4
Answer	8					



To enter directed numbers on your calculator, use the (−) (or (+/−)) key.

For more practice, you can make more tables of your own. You could make much longer tables and check your answers with a calculator. (Press (=) after each step.)

7:10 Directed numbers and algebra

PREP QUIZ 7:10

1 $-4 - 10$ **2** $-2 + 5$ **3** -3×2 **4** $5 - 6$

5 $-3 - 3 - 3$ **6** $12 - 5 - 8$ **7** $-3 + 4 \times 6$ **8** $\frac{10}{4} - \frac{10}{2}$

9 Find the value of $3a + 2$ if $a = 4$.

10 If $x = 7$ and $y = 2$, find the value of $2x - 10y$.

The rules for directed numbers are the same in algebra.

When evaluating an algebraic expression, substitute the directed numbers and then calculate.

WORKED EXAMPLES

1 $4x - 10x$
$= -6x$
because
$4 - 10 = -6$

2 $-2a + 5a$
$= 3a$
because
$-2 + 5 = 3$

3 $-3 \times 2c$
$= -6c$
because
$-3 \times 2 = -6$

4 $2f - (-5f)$
$= 7f$
because
$2 - (-5) = 7$

5 If $x = -8$ and $y = 2$, find:

a $x + 3$
$= -8 + 3$
$= -5$

b $\frac{x}{y}$
$= \frac{-8}{2}$
$= -4$

c $x - y$
$= -8 - 2$
$= -10$

d $2y - x$
$= 2 \times 2 - -8$
$= 4 + 8$
$= 12$

Exercise 7:10

1 Simplify:

a $-3 - 2$ **b** $2 - 6$ **c** $4 - (-7)$ **d** $-8 + 3$ **e** $-6 + 11$

f $7 - 13$ **g** $-8 - 6$ **h** $-2 + 7$ **i** $-5 - (-4)$ **j** $-7 - 7$

2 Simplify:

a $-2x - x$ **b** $2y - 7y$ **c** $5a - 6a$ **d** $-2m + 5m$ **e** $-3h - 7h$

f $7c - (-3c)$ **g** $-5y + y$ **h** $10p - (-p)$ **i** $x - 7x$ **j** $4x - 10x$

k $-4t + 9t$ **l** $d - (-d)$ **m** $-8w + 8w$ **n** $-4x - (-7x)$ **o** $3y - 5y$

3 Simplify:

a -4×5 **b** $16 \div (-8)$ **c** $-7 \times (-5)$ **d** $6 \times (-3)$ **e** $-45 \div 5$

f $-40 \div (-10)$

4 Simplify:

a $6 \times (-2a)$ b $15y \div (-3)$ c $-10 \times 2k$

d $-20m \div (-2)$ e $-4 \times 9b$ f $6 \times (-5n)$

5 Evaluate the following using $x = 7$ and $y = 3$.

a $2x$ b $x + 3$ c $y - x$

d $x + y$ e $5y$ f $x + 3y$

g $y - 10$ h $3x + 4$ i $2y - 8$

6 For $x = -2$ and $y = 3$, evaluate the following.

a $x + 1$ b $x + y$ c $x - y$ d $y - x$ e $5x$

f $y - 7$ g $x - 5$ h $-2y$ i $3x - 4$ j $x \div 2$

k x^2 l $4x + y$ m xy n $x + 4y$ o $2x + 3y$

7 For $a = 8$ and $b = -2$, find the following.

a $a + b$ b $a - b$ c $b - a$ d $3b$ e $b + 5$

f $2a + 3b$ g ab h $b - 6$ i $3b + 1$ j $a + 4b$

k $a \div b$ l $10 \div b$ m $a - 2b$ n $10b$ o $3ab$

Multiplying or dividing two unlike signs gives a negative.
$-8 \times 7 = -56$
$16 \div (-2) = -8$
Multiplying or dividing two like signs gives a positive.
$-8 \times -7 = 56$
$-20 \div (-5) = 4$

FUN SPOT 7:10A — WHAT HAPPENED TO THE THIEF WHO WAS CAUGHT IN THE RUBBER BAND FACTORY?

Simplify each expression. Match each letter with its answer below to solve the riddle.

A $4a - 6a$ **C** $-3 \times 2a$ **E** $-3a + 5a$ **G** $-10a \div 2$ **H** $2a - (-a)$

I $-5a - 2a$ **L** $-7 \times 2a$ **N** $3a - 6a$ **O** $-20a \div (-5)$ **R** $9a \div (-9)$

S $6a - (-2a)$ **T** $a - 9a$ **V** $5 \times (-3a)$ **W** $-4a \times 4$

$3a$	$2a$	$-16a$	$-2a$	$8a$	$-5a$	$-7a$	$-15a$	$2a$	$-3a$	$-2a$				
$-14a$	$4a$	$-3a$	$-5a$	$8a$	$-8a$	$-a$	$2a$	$-8a$	$-6a$	$3a$				

FUN SPOT 7:10B — WHY DO SUMO WRESTLERS LIKE TO EAT PEANUTS?

Evaluate using $m = -10$ and $n = 4$. Match each letter with its answer below.

A $m + 5$ **E** $m + n$ **H** $2m$ **I** mn **K** $3n$

L $m - n$ **N** $m - 2$ **O** $n - 4$ **P** $n - m$ **R** $3m + 5$

S $-m$ **T** $-n$ **U** $2n - 3$ **V** $4m + 10$ **Y** $5n - 4$

-20	-5	-30	-6	16	0	5	-6	-30	-6	-25	10	-6	-6	-12	
															?
-5	10	12	-40	-12	-12	16	-6	-14	-6	14	-20	-5	-12	-4	

MATHS TERMS 7

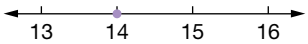
number line

- a line with a scale showing the relative positions of numbers,

e.g. 

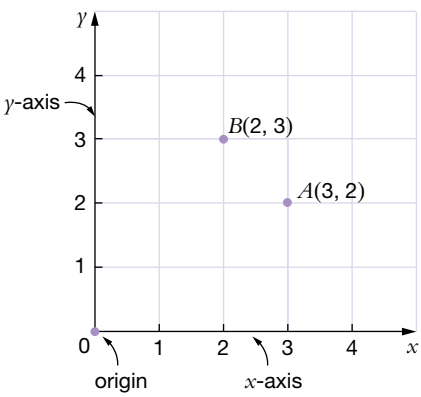
graph (to graph a number)

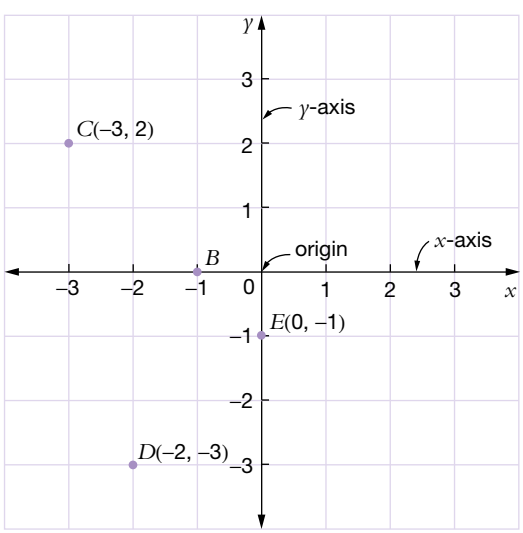
- to use a dot to mark the position of a number on a number line,
  e.g. The number 14 has been graphed on the number line.



number plane (or **number lattice**)

- a rectangular grid that allows the position of any point in the plane to be identified by an ordered pair of numbers





x-axis

- the horizontal number line in a number plane

y-axis

- the vertical number line in a number plane

origin

- the point on a number plane where the x-axis and y-axis intersect, i.e. (0,0)

coordinates

- an ordered pair of numbers that gives the position of a point in a number plane relative to the origin,
  e.g. (3,2), (2,3), (–3,2), (–2,–3) and (0,–1) are coordinates for the points A–E on the graphs below

x-coordinate

- the first of a coordinate pair, a number that tells how far right (or left) the point is from the origin

y-coordinate

- the second of a coordinate pair, a number that tells how far above (or below) the point is from the origin

directed number

- a number that has both size and direction

positive

- greater than zero

negative

- less than zero

integer

- a positive or negative whole number,
  e.g. … –3, –2, –1, 0, 1, 2, 3, …

sign (of a number)

- whether a number is positive or negative

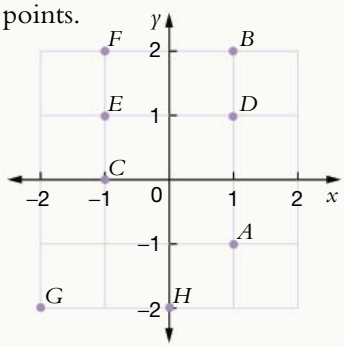
opposite number

- a number of the same size but opposite sign,
  e.g. The opposite of 3 is –3.

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Graph each set on a separate number line. 7:01
a $\{0, 3, 5\}$ **b** $\{-2, 1, 3\}$ **c** $\{-5, -4, -3, 0\}$

2 From this number plane, write the coordinates of the points. 7:03
a A **b** B **c** C



3 Which points have the following coordinates? 7:03
a $(-2,-2)$ **b** $(1,1)$
c $(-1,2)$ **d** $(0,-2)$

4 **a** $-5 + 3$ **b** $(-8) + 12$ **c** $-17 + 2$ 7:05

5 **a** $(-6) + (-3)$ **b** $-8 + -2$ **c** $9 + (-10)$ 7:05

6 **a** $-5 - 3$ **b** $4 - 9$ **c** $0 - 7$ 7:05

7 **a** $10 - -3$ **b** $-6 - -4$ **c** $(-4) - (-3)$ 7:06

8 **a** -8×7 **b** -5×-8 **c** 6×-4 7:07

9 **a** $(-6)^2$ **b** $(-1)^2$ **c** $(-10)^2$ 7:07

10 **a** $-30 \div 3$ **b** $(-42) \div (-6)$ **c** $77 \div -11$ 7:08

11 **a** $\frac{-15}{-3}$ **b** $\frac{36}{-6}$ **c** $\frac{-16}{2}$ 7:08

12 **a** $10 - 7 \times 2$ **b** $-4 + 8 \div 2$ **c** $(3 - 17) \div (-10 + 8)$ 7:09

13 Write the next three terms of each pattern. 7:09
a $12, 9, 6, 3, \ldots, \ldots, \ldots$ **b** $-40, -30, -20, -10, \ldots, \ldots, \ldots$
c $1, -3, 9, -27, \ldots, \ldots, \ldots$ **d** $-64, -32, -16, -8, \ldots, \ldots, \ldots$

14 **a** $\frac{-3+8}{5}$ **b** $\frac{4-12}{6-4}$ **c** $\frac{-9+12}{2-5}$ 7:09

15 **a** One more than -4. **b** One less than -10. 7:09

ASSIGNMENT 7A Chapter review

1 Use a directed number to represent each of the following.

a increase of 512 kg

b decrease of 17 kg

c 230 m below sea level

2 Write a directed number to show the change described in each case.

a The temperature fell from 5 degrees Celsius to −3 degrees Celsius.

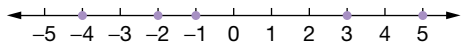
b I took an elevator from two floors below ground level to seven floors above ground level.

c My bank account balance has changed from $487 to $415.

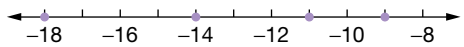
d My bank account balance has changed from −$487 to −$415.

3 Write the set of numbers that have been graphed on each number line.

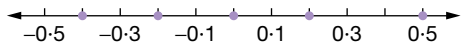
a



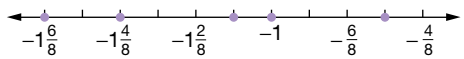
b

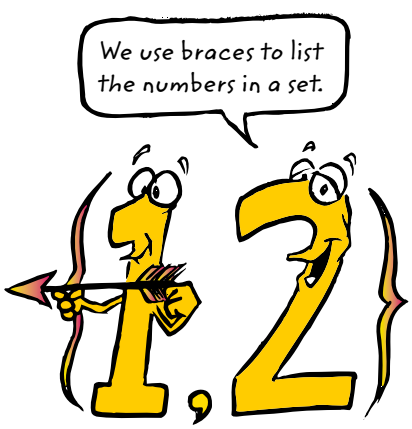


c



d





4 Arrange the directed numbers in order from smallest to largest.

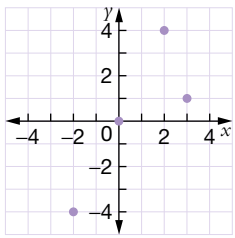
a −23, 15, −33, −25

b 1·8, −0·9, −1·1, 0·22

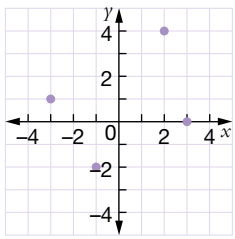
c $-\frac{1}{2}, -\frac{1}{4}, \frac{1}{10}, -\frac{3}{4}$

5 List the set of points graphed on each number plane.

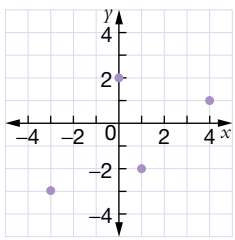
a



b



c



6 Are the points in each set on a straight line? Graph each set of points to find out.

a {(−1,1), (1,2), (3,5)}

b {(−2,3), (0,1), (2,−1)}

c {(−2,−3), (1,0), (4,3)}

7 Write the basic numeral for each.

a 2 − 7 + 3 + 2

b 8 − 5 − 7 + 1

c −6 + 3 + 9

d 15 − 25 + 8 + 11

e 72 − 3 − 89 + 10

f −18 + 6 + 24

g 5 + (−2 − 2)

h − 8 − (4 + 6)

i −2 + 3 × 9

j 5 − (12 + 32)

k − 8 − (4 + 6 + 10)

l −2 − 3 × 9

m 5 − (−2 − 2)

n − 8 − (4 − 6)

o −6 − (−17 −13)

8 Evaluate:

a (16 − 4) ÷ (−6 + 2)

b (8 − 5) × (−7 + 1)

c (−6 + 3) × (−9 − 9)

ASSIGNMENT 7B Working mathematically

1 On **ID Card 7** on page xx, identify **6** to **16**.

2 Write the different combinations that will produce a total of 7 when a green dice and a red dice are rolled.

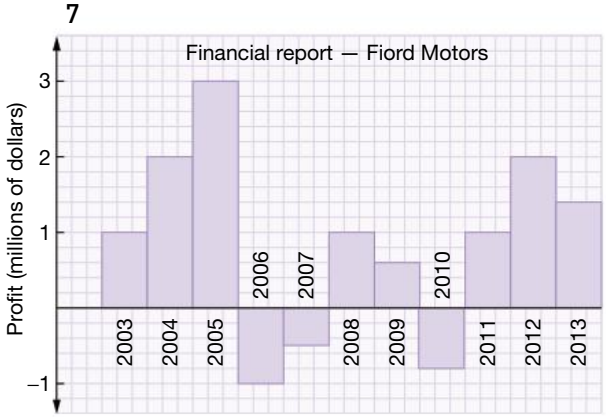
3 A person buys 300 mining shares at $4.50 each and 1000 oil shares at $1.80 each. After one year, she sells the oil shares for $1.95 each and the mining shares for $4.10 each. How much money has she made or lost overall?

4 A builder hires three wheelbarrows. Each wheelbarrow costs $6 a day or $24 a week to hire. If the wheelbarrows are hired from 27 October to 16 November (inclusive), how much will the builder pay?

5 In a chess tournament each player scores 1 point for a win, $\frac{1}{2}$ a point for a draw and no points for a loss. If the score is currently $3\frac{1}{2}$ to $1\frac{1}{2}$, how many games have been played? How could this score have come about?

6 A ferry service operates a 'scenic cruise' to and from the same wharf. The cruise takes 1 hour, but it also takes 20 minutes to load and 10 minutes to unload the passengers. The first cruise leaves the wharf at 9 am and the last cruise must be back by 5 pm. Create a timetable for the ferry, allowing at least half an hour for lunch.

7



From the graph:

a In what year was the greatest profit made?

b In what year was the first loss made?

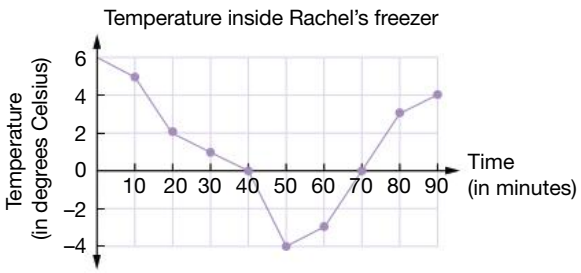
c In how many years was a loss made?

d In how many years was the profit over $1 000 000?

e What was the profit in 2009?

f What was the loss in 2010?

8 Rachel recorded the temperature inside her freezer every 10 minutes, then drew the graph below.



a What was the temperature at:

i 20 min **ii** 40 min

iii 50 min **iv** 60 min

v 90 min?

b At what time was the temperature lowest?

c At what times was the temperature 0°C?

d What do you think the temperature would have been at 45 minutes?

e Between which two times was the change in temperature greatest? What was this change?

9 I am playing the card game called '500'. My score is 170 and I have bid '8 hearts'. If I succeed in making my bid, then my score will increase by 300. If I fail, it will decrease by 300. After the hand is played, what two possible scores might I have?



ASSIGNMENT 7C Cumulative revision

1 Simplify:

a $3 - 4 \times 5 + 17$ **b** $20 \div (4 \div 4) - 12$ **c** $14 - (6 - 4) \times 8$ 1:06

2 Write each equivalent fraction. 1:11B

a $\frac{3}{4} = \frac{\square}{20}$ **b** $\frac{9}{27} = \frac{\square}{3}$ **c** $\frac{6}{8} = \frac{3}{\square}$ **d** $\frac{16}{30} = \frac{\square}{15}$

3 **a** $\frac{1}{8}$ of 32 **b** $\frac{1}{4}$ of 40 **c** $\frac{1}{3}$ of 330 **d** $\frac{1}{8}$ of 640 1:11B

4 **a** $8{\cdot}6 + 1{\cdot}26$ **b** $8{\cdot}5 \times 4$ **c** $0{\cdot}4 \div 8$ **d** $1 - 0{\cdot}016$ 1:11C

5 Write as a percentage. 1:11D

a $\frac{13}{100}$ **b** $\frac{3}{10}$ **c** $1\frac{1}{2}$ **d** $0{\cdot}25$

6 Rewrite each expression using index notation. 3:01

a $5 \times 5 \times 5 \times 5$ **b** $2 \times 2 \times 2 \times 7 \times 7$ **c** $8 \times 10 \times 10 \times 10$

7 Find the value of each expression if $m = 8$, $n = 4$ and $p = 12$. 4:02, 4:04, 4:06

a $mn \div p$ **b** $m^2 - 2n$ **c** $\frac{m+n}{p}$ **d** $\frac{n}{m} \times 2p$

8 Simplify each expression. 4:07, 4:09

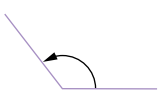
a $7(y + 2) - y$ **b** $4x \times 5 - 16x$ **c** $106 + a - a$ **d** $10x \div 10$

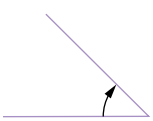
9 Simplify using index notation. 4:10

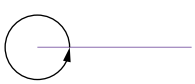
a $m \times m \times m \times m$ **b** $x \times x \times x \times 6 \times x$ **c** $7 \times 3 \times y \times y \times y$

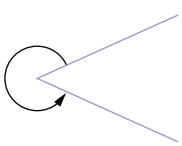
10 Write the angle type of each. 5:03

a 

b 

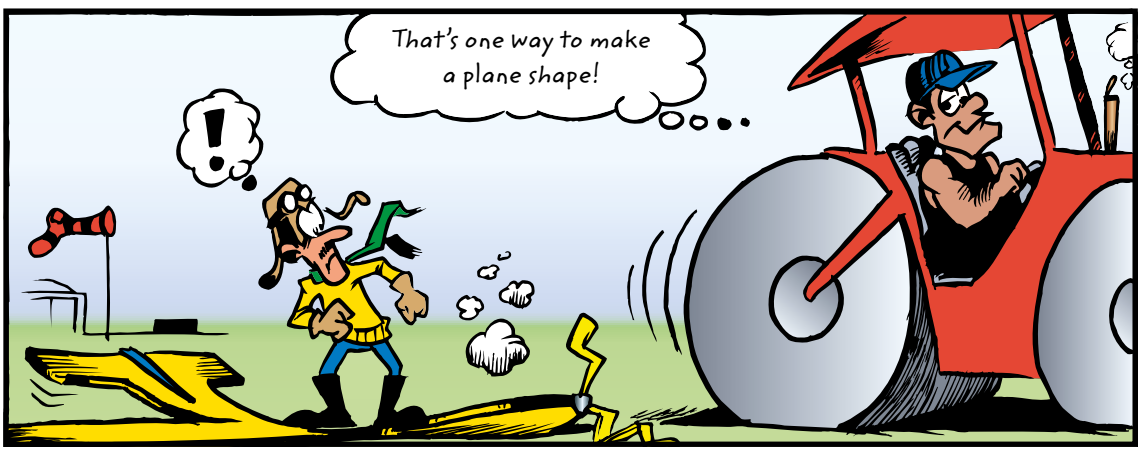
c 

d 

e 

2D AND 3D SPACE

8



Contents

8:01 Plane shapes
8:02 Types of triangles and their properties
GeoGebra activity 8:02 Investigating properties of triangles
8:03 Describing quadrilaterals
GeoGebra activity 8:03 Investigating properties of quadrilaterals
8:04 Finding the size of an angle
8:05 Solids
8:06 Nets of solids
GeoGebra activity 8:06 Nets
8:07 Drawing pictures of solids
8:08 Looking at solids from different views
Investigation 8:08 The Platonic solids
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Volume***, ***Properties of Geometrical Figures 1*** [Stage 4]

- Draw different views of prisms and solids formed from combinations of prisms. (ACMMG161)
- Classify triangles according to their side and angle properties and describe quadrilaterals. (ACMMG165)
- Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral. (ACMMG166)
- Use the properties of special triangles and quadrilaterals to solve simple numerical problems with appropriate reasoning.

Working Mathematically

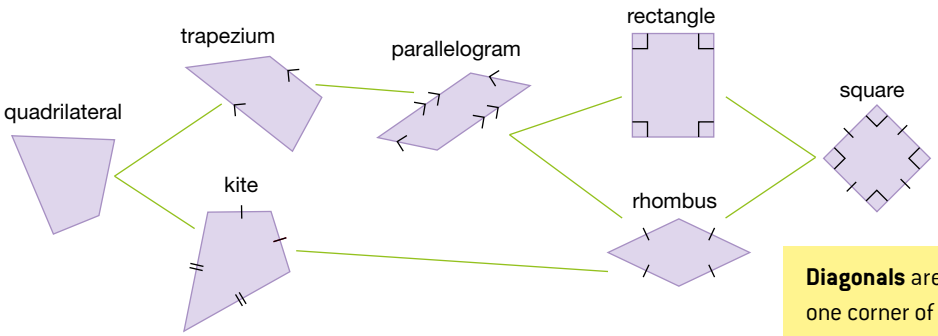
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

8:01 Plane shapes

Plane shapes are shapes that lie in one plane, or flat surface. They have area but not volume. We say that they are **two-dimensional** (2D).

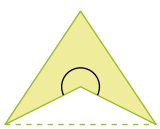
Some of the common plane shapes are shown below. Together they are the family of **quadrilaterals**, or four-sided shapes.

Each type of quadrilateral is a special kind of the type to its left. (For example, a rectangle is a special kind of parallelogram.)

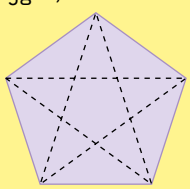


The quadrilaterals above are **convex**. All the diagonals of each shape are inside the shape.

Non-convex quadrilaterals have at least one diagonal outside the shape. They also have at least one reflex angle.

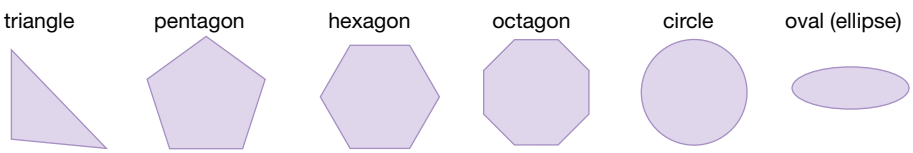


Diagonals are lines drawn from one corner of a polygon, across the polygon, to another corner.



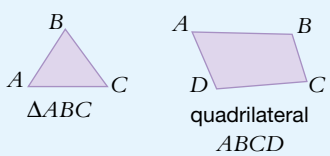
This shape has 5 diagonals.

Other plane shapes include:



Shapes that have only straight sides are called **polygons**.

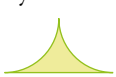
We name shapes using the letters at the vertices.



Exercise 8:01

The following questions refer to the plane shapes shown above.

1 Why are the figures below not triangles?

a 

b 

c 

2 A plane shape with four straight sides is called a quadrilateral. Name six special types of quadrilaterals.

3 A parallelogram has two pairs of parallel sides.

- **a** What does *parallel* mean?
- **b** Name all of the special types of quadrilateral that are also parallelograms.
- **c** Which type of quadrilateral has only one pair of parallel sides?
- **d** Give a real-life example of a trapezium.

4 Name two plane shapes that do not have straight sides.



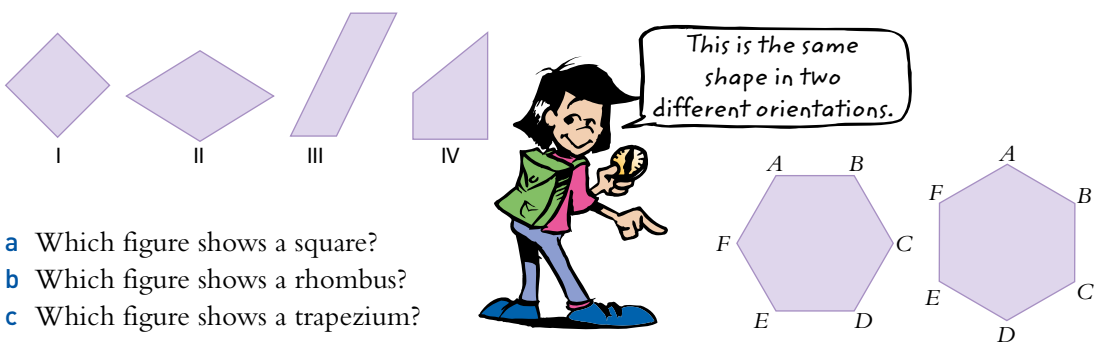
5 What is another name for an ellipse?

6 Copy and complete this table.

Shape	Number of sides	Number of angles	Number of diagonals
triangle			
quadrilateral			
pentagon			
hexagon			
octagon			

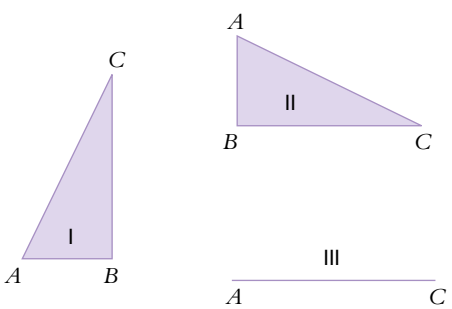
7 Some people think of a rhombus as a squashed square. What other name is often given to the rhombus?

8 When a shape is drawn in a different position or *orientation*, it can sometimes look like a different shape. Figures I to IV are four plane shapes drawn in unusual orientations.



- **a** Which figure shows a square?
- **b** Which figure shows a rhombus?
- **c** Which figure shows a trapezium?

9 The figures below show a triangle in three different orientations, but Figure III shows only one side of the triangle. Copy and complete the sketch.



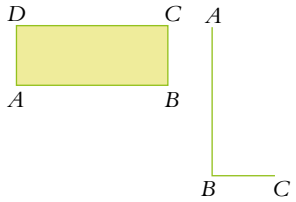
Did you know?

Two constellations of stars are named after polygons and both are triangles. One constellation is called Triangulum and is in the northern sky. The other is called Triangulum Australe and is in the southern sky.

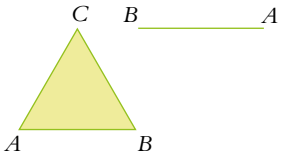
Find out when these constellations are visible from where you live and see if you can find them in the night sky.

10 Each of the following shows the same plane shape in two different orientations, but only a part of the second shape is shown. Complete the sketch of the shape in its second orientation.

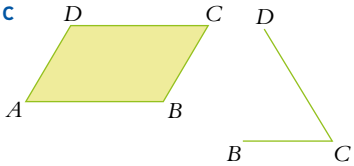
a



b

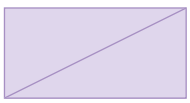


c

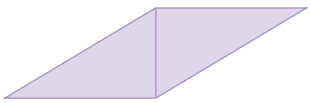


11 Name the quadrilateral formed by joining the two identical triangles in each of the following.

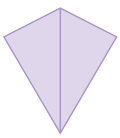
a



b

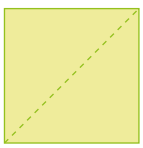


c

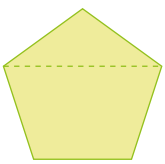


12 If each of these plane shapes were cut along the dotted line, what new shapes would be formed?

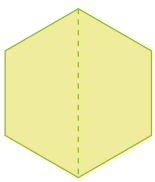
a



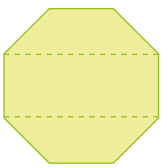
b



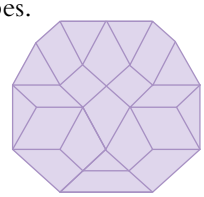
c



d



13 This decagon is called a *composite figure* because it is composed of smaller shapes.



a Name the different types of quadrilaterals that you can see in the decagon.

b What is the only quadrilateral that is always seen in the same orientation?

c How many different-shaped triangles are there?

d Sketch the three trapeziums that are in the decagon.

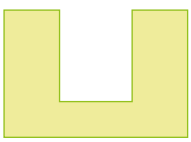
14 In later work it will be important to be able to divide a complicated shape into simpler shapes. Divide each of the following into the shapes indicated.

a



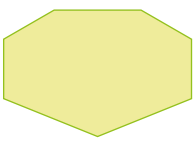
2 rectangles

b



3 rectangles

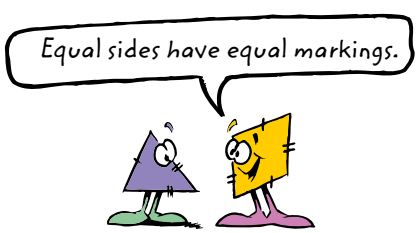
c



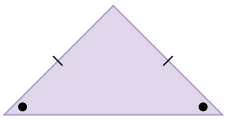
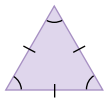
triangle, rectangle, trapezium

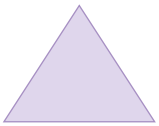
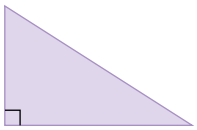
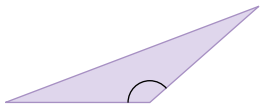
8:02 Types of triangles and their properties

Triangles have three sides and three angles. They can be sorted into different types according to the lengths of their sides or the sizes of their angles.

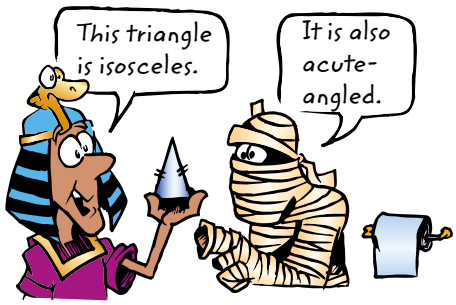


These are 6 terms used to sort triangles into different types:

Scalene	Isosceles	Equilateral
		
• No sides equal. • No angles equal.	• Two sides equal. • Angles opposite equal sides are equal.	• All sides equal. • Three 60° angles.

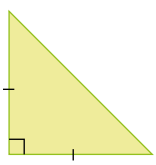
Acute-angled	Right-angled	Obtuse-angled
		
• All angles are acute.	• One angle is a right angle.	• One angle is obtuse.

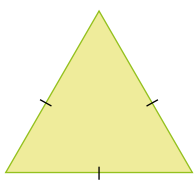
Triangles can be of more than one type. For example, a triangle can be both isosceles and acute-angled: we would call it an *acute-angled isosceles triangle*.

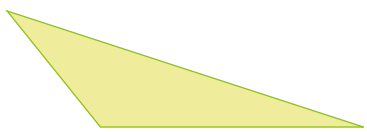


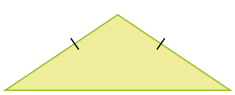
Exercise 8:02

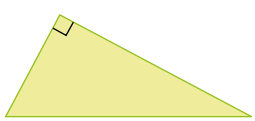
1 Classify each triangle according to the lengths of its sides and the sizes of its angles.

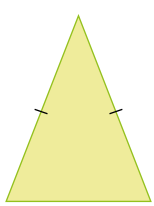
a


b


c


d


e


f


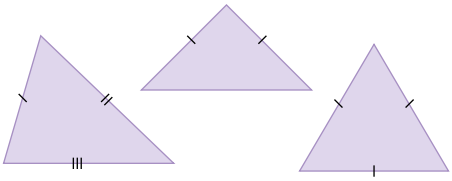
2 Use a ruler to draw isosceles triangles as follows:

a acute-angled, two sides 5 cm

b right-angled, two sides 5 cm

c obtuse-angled, two sides 5 cm

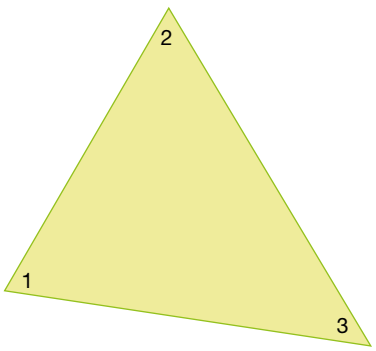
3 Use the following diagrams to answer the questions below.

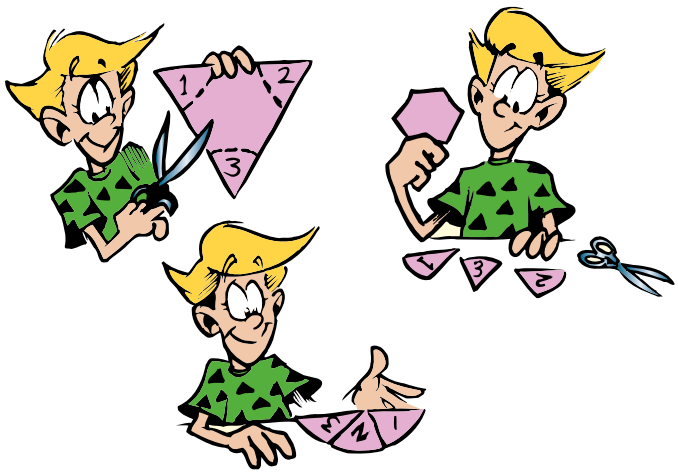


- **a** In a scalene triangle, are any angles equal in size?
- **b** In an isosceles triangle, are any angles equal in size?
- **c** In an equilateral triangle, are any angles equal in size?
- **d** Is the largest angle of a triangle opposite the largest side?
- **e** Is the smallest angle of a triangle opposite the smallest side?

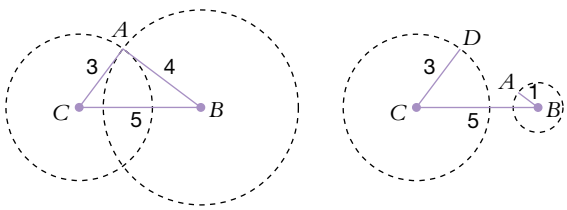
4 Draw any triangle and measure its angles. What is the sum of the angles?

5
- **a** Draw any triangle. Mark the angles 1, 2 and 3.
- **b** Cut off the corners of the triangle and arrange them so that they form a straight angle.
- **c** What does this suggest about the angles of a triangle?





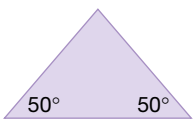
6 Students were asked to draw two triangles: the first with sides of length 5 cm, 4 cm and 3 cm, and the second with sides 5 cm, 3 cm and 1 cm. The diagrams show that although the first triangle could be drawn, the second was impossible.



- **a** Why was it impossible to draw the second triangle?
- **b** How big would the smallest side need to be to ensure that the second triangle could be drawn?
- **c** Is it possible for a triangle to have the following side lengths?
  - **i** 1, 2, 4
  - **ii** 6, 8, 12
  - **iii** 9, 4, 7
  - **iv** 11, 8, 20

7 Draw an isosceles triangle (two sides equal) and measure its angles. Are any of the angles equal? If so, how could you describe their position in the triangle?

8 The diagram shows a triangle with two 50° angles.



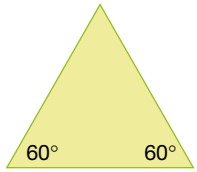
- **a** What is the size of the third angle?
- **b** Are any of the sides equal in length? If so, how would you describe their position in the triangle?

9 A triangle with two 60° angles is shown.

a What is the size of the third angle?

b Are all the sides the same length?

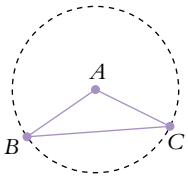
c Is the triangle equilateral?



Triangle discoveries:

1 The angles of a triangle add to 180°.
2 A scalene triangle has no equal angles.
3 An isosceles triangle has two equal angles opposite its equal sides.
4 An equilateral triangle has three 60° angles.
5 The largest angle of a triangle is opposite the largest side and the smallest angle is opposite the smallest side.

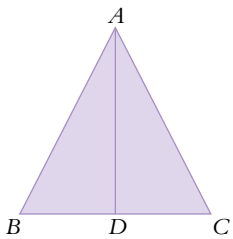
10 Take any two points on a circle and join them to the centre of the circle to form a triangle. What type of triangle is it?



11 The diagram shows an isosceles triangle *ABC*. *AD* is an axis of symmetry of the triangle.

a Write the lengths in the diagram that are equal.

b Write the angles in the diagram that are equal.



12 a A triangle has three sides of length 5 cm. What type of triangle is it? What are the sizes of its angles?

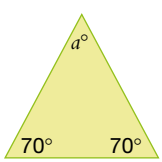
b A triangle has all its angles 60° in size. What can you say about the length of its sides?

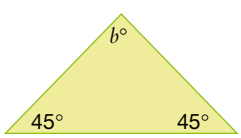
c A triangle has sides of length 3 cm, 4 cm and 5 cm. What type of triangle is it?

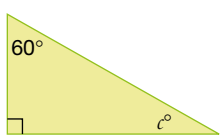
d A triangle has angles of 40°, 60° and 80°. Can it have any sides equal in length?

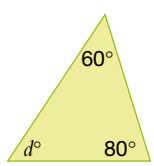
e Draw a triangle that has a 90° angle and two sides of length 2 cm. What are the sizes of the other angles?

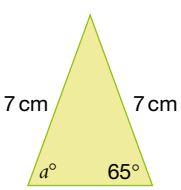
13 Find the value of each pronumeral. Give a reason for each answer.

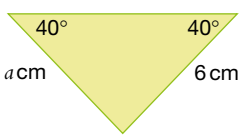
a 

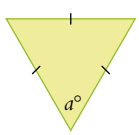
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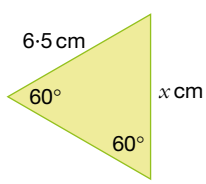
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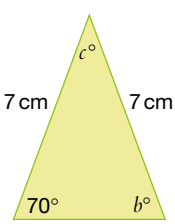
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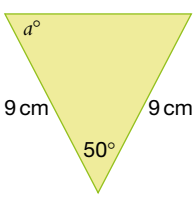
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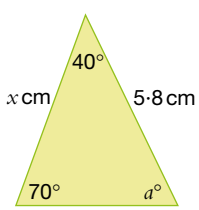
f 

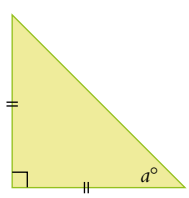
g 

h 

i 

j 

k 

l 

14 In each of the following, a triangle is described. Use the information to sketch and label the triangle.

- **a** Triangle ABC is a right-angled isosceles triangle with the right angle at A.
- **b** Triangle ABC is an obtuse-angled triangle with the obtuse angle at A and with a longest side of 7 cm.
- **c** In triangle ABC, $\angle A$ is the largest angle and $\angle C$ is the smallest. The triangle has sides of length 5 cm, 6 m and 7 cm.
- **d** Triangle ABC is right-angled at C. The smallest angle B is 35° and the smallest side is 10 cm.

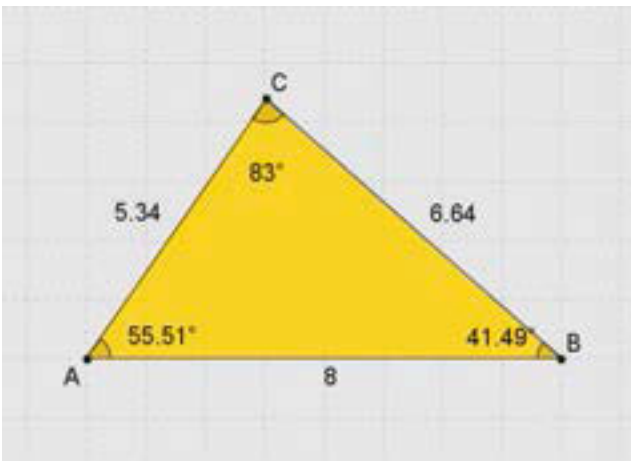
15 For each of the following, make a sketch showing the information given in the question as well as anything else that you can deduce about the angles and sides.

- **a** Triangle ABC is equilateral with a side of 10 cm.
- **b** Triangle ABC is right-angled and isosceles with a longest side of 10 cm.
- **c** Triangle ABC has a 50° angle, a 60° angle and a longest side of 7 cm. Angle A is the largest angle.
- **d** Triangle ABC is an obtuse-angled isosceles triangle, with $AB = AC$. Angle A measures 100° and the equal sides are 7 cm in length.

P GEOGEBRA ACTIVITY 8:02 INVESTIGATING PROPERTIES OF TRIANGLES

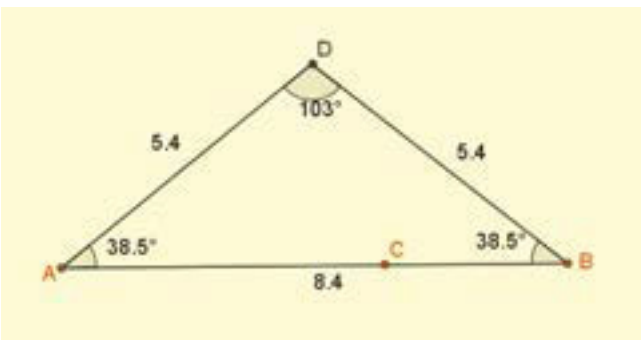
1 *Investigating the side and angle properties of triangles*

Click and drag points B and C to change the angle sizes and side lengths of the triangle. What is the sum of the three angles?
Can you position B and C so that two of the sides are equal?
Can you position B and C so that all three of the sides are equal?
What happens to the angles when you do this?

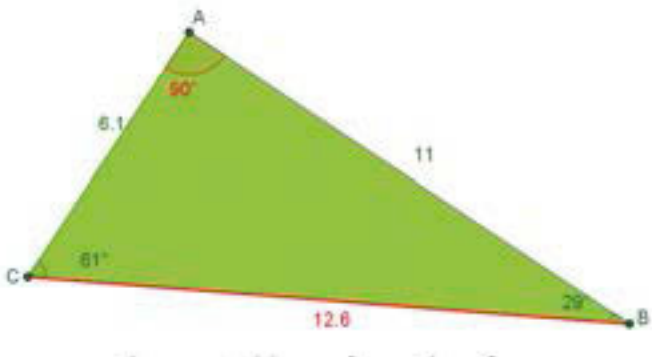


2 *Isosceles and equilateral triangles*

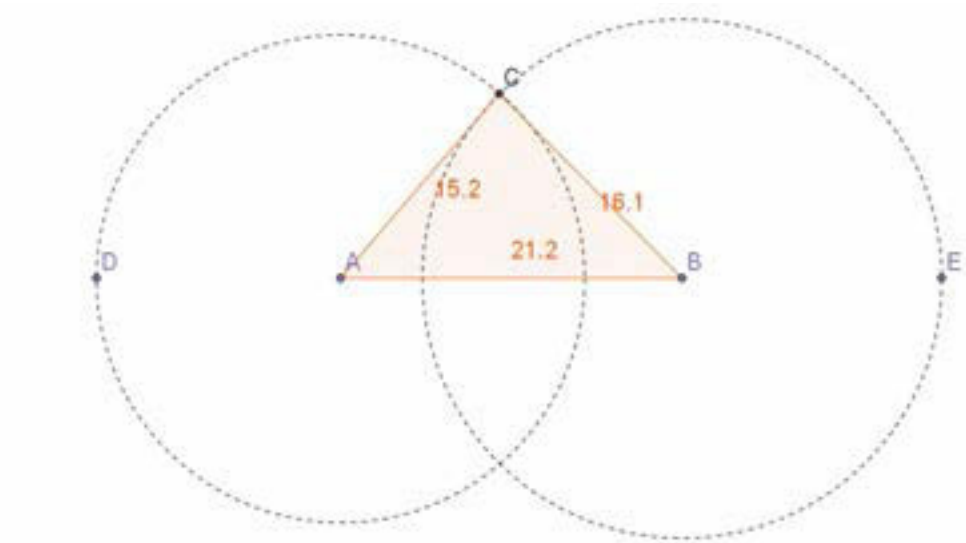
An isosceles triangle is shown.
Click and drag points A and B.
What do you notice about the sizes of the angles? What do you notice about the lengths of the sides?
Click and drag point C towards point B. What happens to the angles and sides when C reaches B?
What type of triangle is this?



3 Drag points A, B and C of the triangle to investigate the possible angles and side lengths. Is the largest side always opposite the largest angle? Is the smallest side always opposite the smallest angle?



4 Drag points A, B, D and E to change the lengths of the different sides. Can any three intervals be used to form the sides of a triangle?



8:03 Describing quadrilaterals

The square, rectangle, parallelogram, rhombus, trapezium and kite are all special types of quadrilaterals. The shapes have many special properties that are related to their sides, angles and diagonals. These properties are used to identify and describe the shapes.

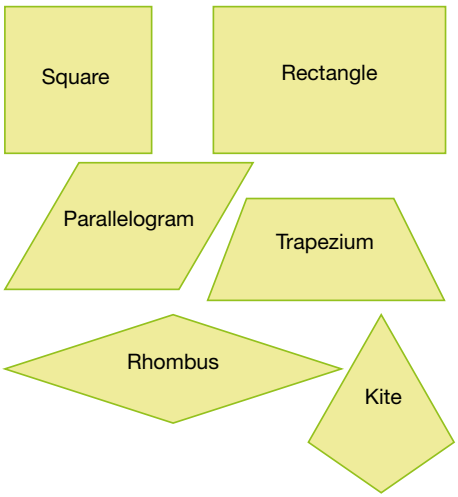
When describing the relationships between sides and diagonals, we often use the words **parallel** and **perpendicular**.

- 'AB is parallel to CD' is written $AB \parallel CD$.
- 'AB is perpendicular to CD' is written $AB \perp CD$.

Perpendicular lines intersect at right angles.

Exercise 8:03

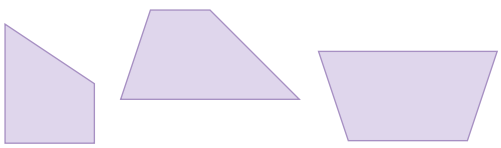
You can use your ruler, protractor or compass where necessary.



1 Use the figures at left to answer the following questions.

- **a** Which shapes have opposite sides parallel?
- **b** Which shapes have opposite sides equal in length?
- **c** Which shapes have all sides equal in length?
- **d** In which shapes are all the angles right angles?
- **e** Measure the angles of the parallelogram and rhombus. What can you say about them?

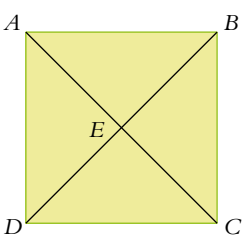
2 The following shapes are all trapeziums. What do they have in common?



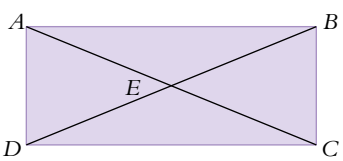
3
- **a** What is the only special quadrilateral that has no pairs of parallel sides?
- **b** How would you describe the lengths of the sides of a kite?
- **c** What can you say about the angles of a kite?
- **d** Do the diagonals of a kite cross at right angles? Do they bisect each other?

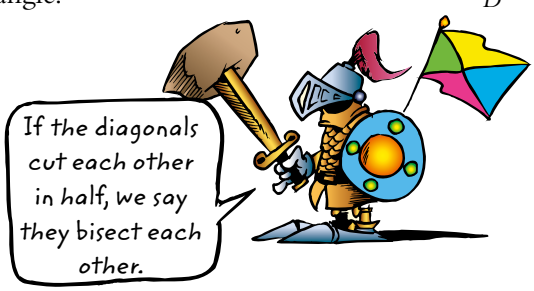
4 The diagram shows a square $ABCD$ with its diagonals coloured. By measurement, find the following:

- **a** Are the diagonals AC and BD equal in length?
- **b** Do the diagonals meet at right angles (i.e. $AC \perp BD$)?
- **c** Do the diagonals cut each other in half?

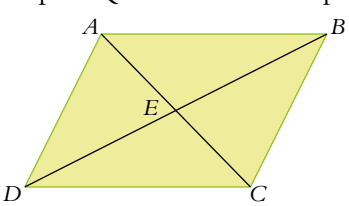


5 Repeat Question **4** for this rectangle.

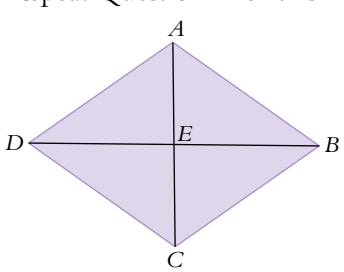




6 Repeat Question **4** for this parallelogram.



7 Repeat Question **4** for this rhombus.

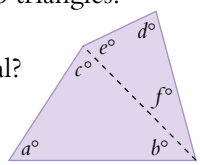


8 **a** In which of the four shapes drawn in questions **4**, **5**, **6** and **7** are the diagonals axes of symmetry?
b In which of the shapes do the diagonals bisect the angles of the shape?

9 Draw any quadrilateral. Measure its angles. What is the sum of the angles?

10 Draw any quadrilateral and cut it out. Mark the angles 1, 2, 3 and 4 and cut them off. Rearrange these angles to form an angle of revolution. What does this tell you about the angles of a quadrilateral?

11 The diagram shows a quadrilateral divided by a diagonal into two triangles.
a Draw a non-convex quadrilateral and show how it can be divided into two triangles.
b What do the angles of a triangle add to?
c Do the angles of two triangles together make the angles of the quadrilateral?
d What is the sum of the angles of the quadrilateral?

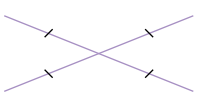


12 Sketch and label the quadrilateral described in each of the following.
a $ABCD$ is a rectangle with $AB = 10$ cm and $AD = 4$ cm.
b The quadrilateral $ABCD$ is a parallelogram with $AB = 5$ cm, $AD = 3$ cm and $\angle BAD = 65°$.
c $ABCD$ is a trapezium with $AB \parallel DC$. $AB = AD = 5$ cm, $\angle BAD = 120°$ and $\angle ABC = 90°$.
d The kite $ABCD$ has $AB = 5$ cm, $BC = 7$ cm and $\angle ABC = 110°$.
e $ABCD$ is a trapezium with $AB \parallel DC$. The sides AB and AD are equal to 5 cm, $\angle DAB = 120°$ and $\angle ABC = 100°$.
f $ABCD$ is a quadrilateral with $AB = 10$ cm, $AD = 9$ cm and $BC = 8$ cm. $AD \perp AB$ and $\angle ABC = 120°$.

13 What special quadrilaterals could be described by each of the following statements?
a All the angles are equal. **b** All the sides are equal. **c** Opposite sides are equal.
d All the sides are equal and the diagonals are unequal.
e All the angles are equal and the diagonals meet at right angles.
f Opposite sides are equal and diagonals are equal.

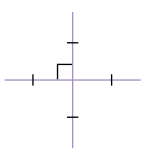
14 The following diagrams show the diagonals of a quadrilateral. If the ends of the diagonals were joined, what quadrilateral would be formed?

a



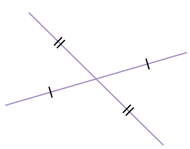
- diagonals are equal
- diagonals bisect each other

b



- diagonals are equal
- diagonals bisect each other at right angles

c



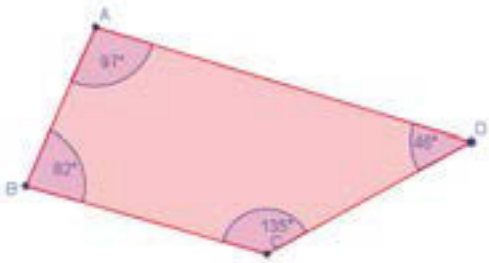
- diagonals are not equal
- diagonals bisect each other

15 The following descriptions of shapes were given by some students. What shape is each describing?
a a parallelogram with four right angles
b a parallelogram with all its sides equal
c a rectangle with all its sides equal
d a quadrilateral with all its angles equal
e a quadrilateral with all its sides equal
f a quadrilateral with all its sides and angles equal
g a rhombus with all its angles equal

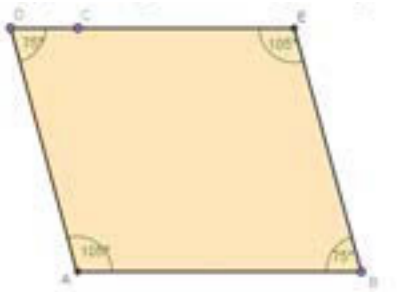
Refer to **ID Card 4** on page xvii and identify the figures **1** to **14**. Learn the terms that you do not know.

P GEOGEBRA ACTIVITY 8:03 INVESTIGATING PROPERTIES OF QUADRILATERALS

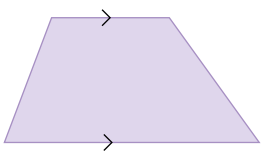
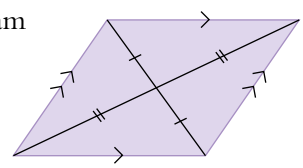
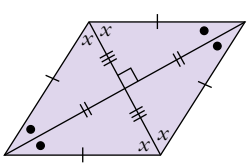
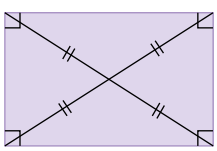
1 Change the shape of the quadrilateral by clicking and dragging points A, B, C and D. What happens to the individual angles? What happens to the sum of the angles?

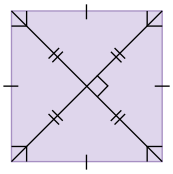
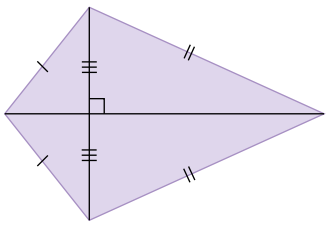


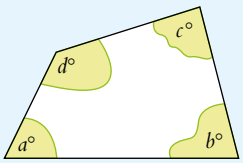
2 Click and drag points B and C to change the parallelogram into a rhombus, rectangle or square. Click the boxes to investigate the angles, side lengths and diagonals for each shape.



Summary of quadrilateral properties

Quadrilateral	Properties
1 Trapezium 	• One pair of opposite sides are parallel.
2 Parallelogram 	• Two pairs of parallel sides. • Opposite sides are equal. • Opposite angles are equal. • Diagonals bisect each other.
3 Rhombus 	All the properties of a parallelogram. Also: • All sides are equal. • Diagonals bisect each other at right angles. • Diagonals bisect the angles that they pass through.
4 Rectangle 	All the properties of a parallelogram. Also: • All angles are right angles. • Diagonals are equal.

Quadrilateral	Properties
5 Square 	All the properties of a rhombus and a rectangle. Also: • Four sides equal. • Four right angles.
6 Kite 	• Two pairs of adjacent sides are equal. • Diagonals are perpendicular. • One diagonal is an axis of symmetry.





The angle sum of any quadrilateral is 360°.

$a° + b° + c° + d°$ makes a revolution.

$$a° + b° + c° + d° = 360°$$

8:04 Finding the size of an angle

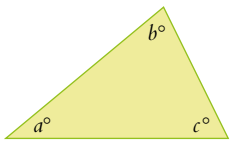
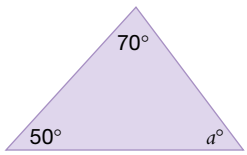
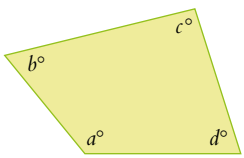
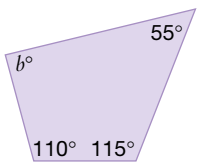
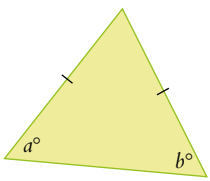
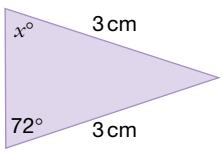
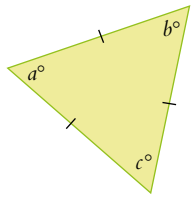
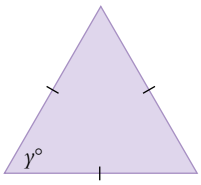
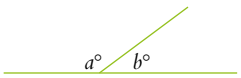
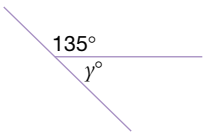
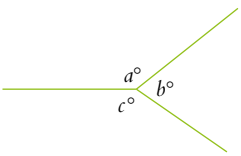
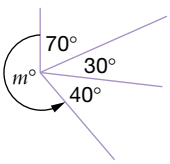
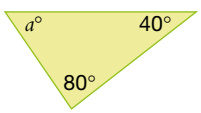
Figure	Rule	Example
	triangle $a° + b° + c° = 180°$	 Find the value of a: $a + 50 + 70 = 180$ $a + 120 = 180$ $\therefore a = 60$
	quadrilateral $a° + b° + c° + d° = 360°$	 Find the value of b: $b + 55 + 115 + 110 = 360$ $b + 280 = 360$ $\therefore b = 80$
	isosceles triangle $a° = b°$	 Find the value of x: Two sides are equal, so this is an isosceles triangle. $\therefore x = 72$

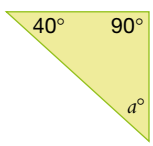
Figure	Rule	Example
	equilateral triangle $a° = b° = c°$	 Find the value of y: All sides are equal, so this is an equilateral triangle. $\therefore$ All angles are equal. $3y = 180$ $y = 60$
	straight angle $a° + b° = 180°$	 Find the value of y: $135 + y = 180$ $\therefore\ y = 45$
	angles at a point $a° + b° + c° = 360°$	 Find the value of m: $m + 70 + 30 + 40 = 360$ $m + 140 = 360$ $\therefore\ m = 220$

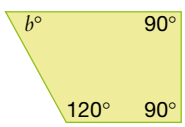
Exercise 8:04

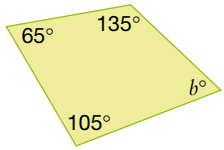
P Foundation worksheet 8:04
Finding the size of an angle

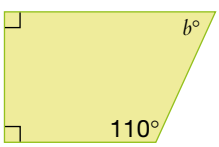
1 Find the value of each pronumeral.

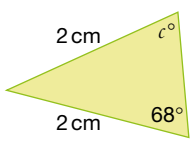
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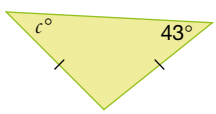
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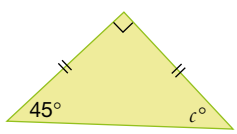
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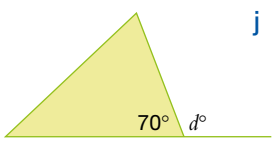
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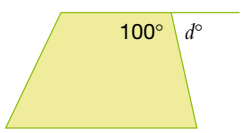
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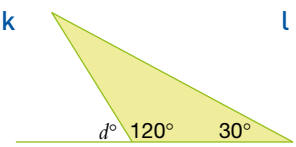
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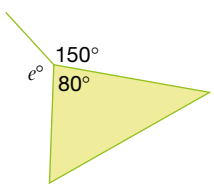
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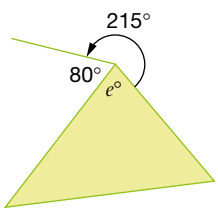
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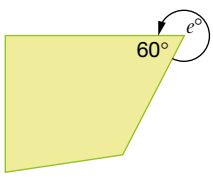
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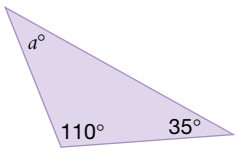
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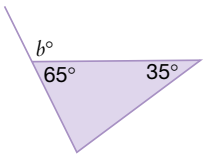
n


2 Find the value of each pronumeral. Give a reason for your answer.

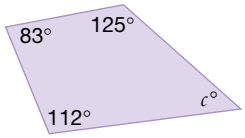
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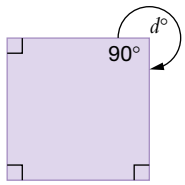
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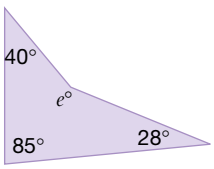
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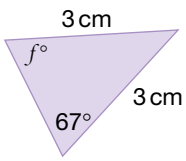
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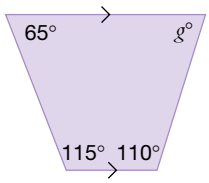
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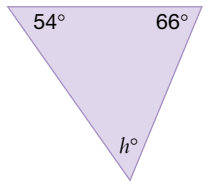
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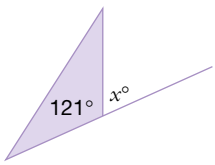
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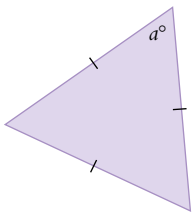
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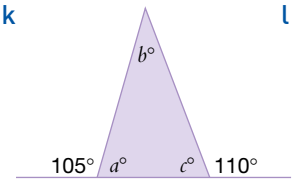
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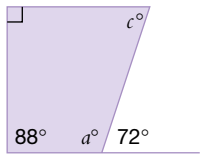
j



k

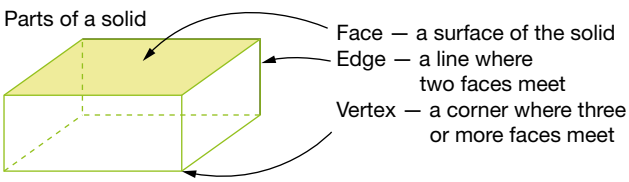


l



8:05 Solids

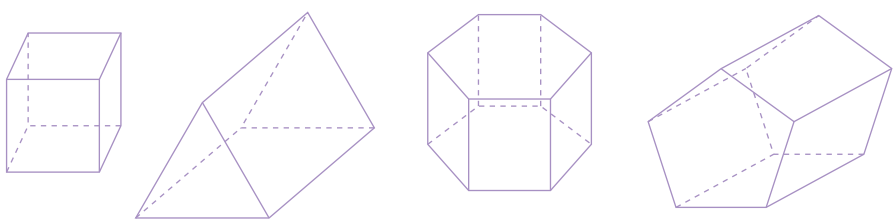
Solid shapes have thickness as well as length and breadth.
We say that they are **three-dimensional** (3D). Our world is filled with solid shapes.



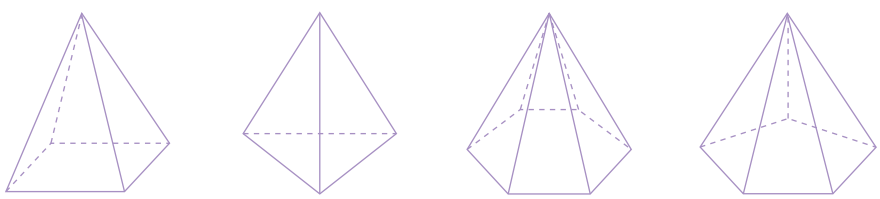
A solid shape may be full or empty inside.

There are two main families of solids: the **prisms** and the **pyramids**. Look carefully at the examples below and see what each group has in common. Not all members of each are shown.

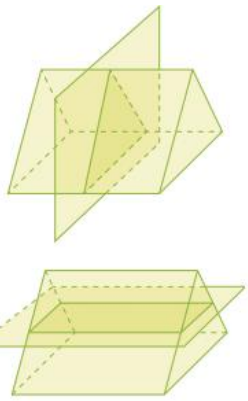
Prisms



Pyramids



Cross-sections



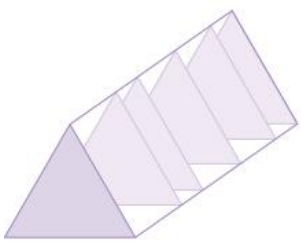
When a solid is cut by a plane, the shape on the cut face is called a **cross-section**. A shape can have different cross-sections depending on the angle of the cut.

In the first diagram at right, the cut is vertical and the cross-section is triangular in shape. In the second diagram, the cut is horizontal and the cross-section is rectangular.

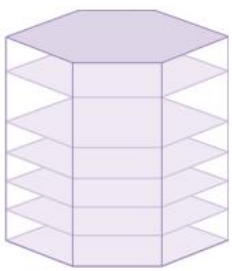
Prisms

All prisms have a special pair of parallel faces. These two faces are the only prism faces that do not need to be rectangular.

If a prism is 'sliced' at any point parallel to these faces, the cross-section is the same. Because of this, the prism is said to have a *uniform* cross-section.







A *prism* is named according to the shape of its uniform cross-section. The *base* of a prism is the same as the uniform cross-section.



This steel girder is called an I- beam because its cross-section is shaped like the letter I.



The rectangular prism shape of a shipping container makes it easy to stack and to move from ship to truck.

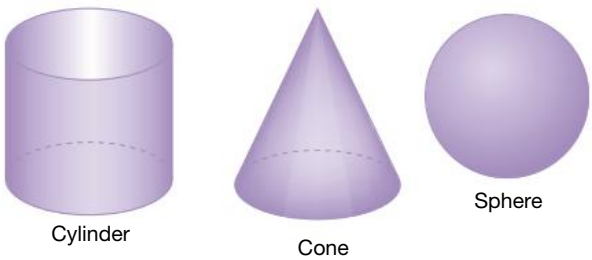
Pyramids

All the faces of a pyramid need to be triangular, except for the base. The pyramid is named according to the shape of its base. Unlike prisms, pyramids cannot be sliced to always make an identical shape, so pyramids are said to have a *non-uniform* cross section.

> A *pyramid* is named according to the shape of its non-triangular face. If all its faces are triangles, then it is a triangular pyramid.

Other solids

Some solids are neither prisms nor pyramids. The most common of these are the cylinder, cone and sphere.





The cylinder is one of the more common solid shapes.

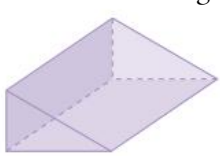


The famous glass pyramid at the Louvre Museum in Paris.

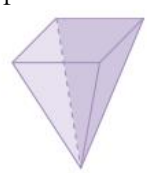
WORKED EXAMPLE

Name the following solid shapes.

a



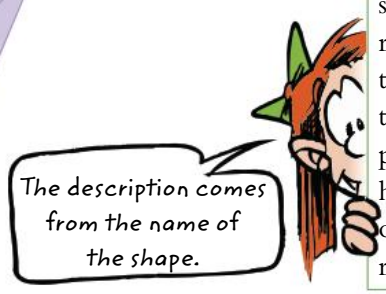
b



Solution

a triangular prism

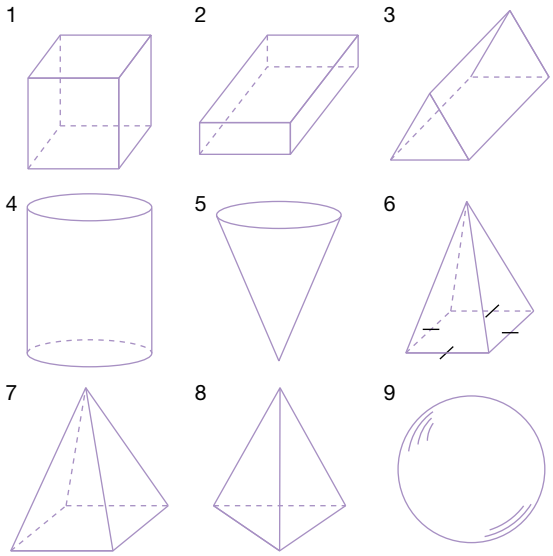
b square pyramid



Shape	Description
square	square
rectangle	rectangular
triangle	triangular
trapezium	trapezoidal
pentagon	pentagonal
hexagon	hexagonal
octagon	octagonal
rhombus	rhombic

Exercise 8:05

Questions 1 to 11 refer to the nine numbered shapes below.



1 What is the number of the following?
a cube **b** cone
c sphere **d** cylinder

2 What is the name of each numbered solid?
a 4 **b** 2
c 9 **d** 5

3 Which of the solids have a surface that is curved?

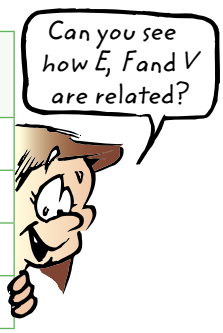
4 Which of the solids are prisms?

5 Which of the solids are pyramids?

6 Which of the solids are neither prisms nor pyramids?

7 Copy and complete the following table.

Solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)	$F + V$
cube				
triangular prism				
rectangular pyramid				
triangular pyramid				



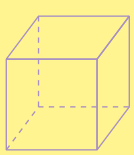
Questions 8 to 11 refer to the nine numbered shapes at the start of the exercise.

8 Which solid shapes have no vertices?

9 Which solid shapes can roll?

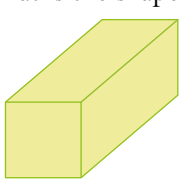
10 What are the shapes of the faces on the solids numbered:
a 3 **b** 6?

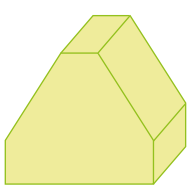
Dotted lines or dashed lines represent edges that are hidden from view. In diagrams, hidden edges are not always shown.

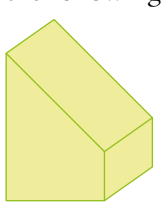


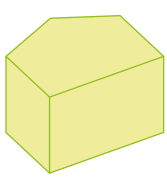
11 What is the name of the each numbered solid?
a 2 **b** 3 **c** 4 **d** 8

12 What is the shape of the uniform cross-section for the following prisms?

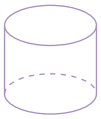
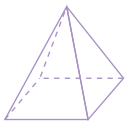
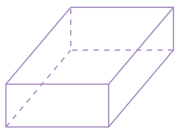
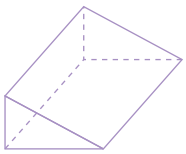
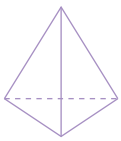
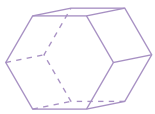
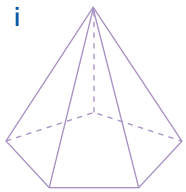
a


b


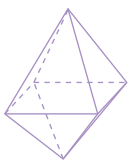
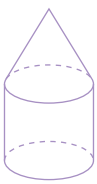
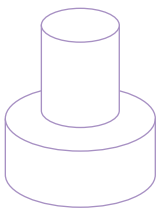
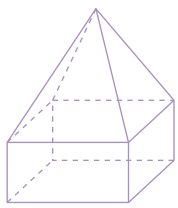
c


d


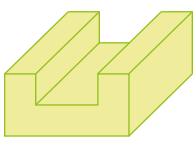
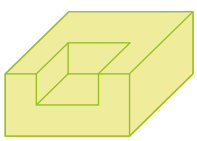
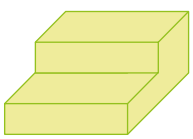
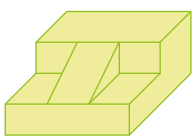
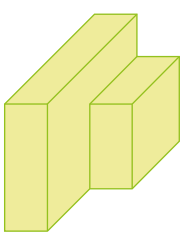
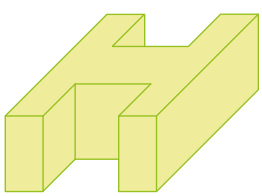
13 From the list of names below, choose the correct name for each solid shape.
Names: hexagonal prism, square pyramid, sphere, pentagonal pyramid, triangular prism, cone, triangular pyramid, rectangular prism, cylinder

a 
b 
c 
d 
e 
f 
g 
h 
i 

14 What solids have been joined to form each composite solid pictured?

a 
b 
c 
d 

15 Does each of the following solids have a uniform cross-section?

a 
b 
c 
d 
e 
f 

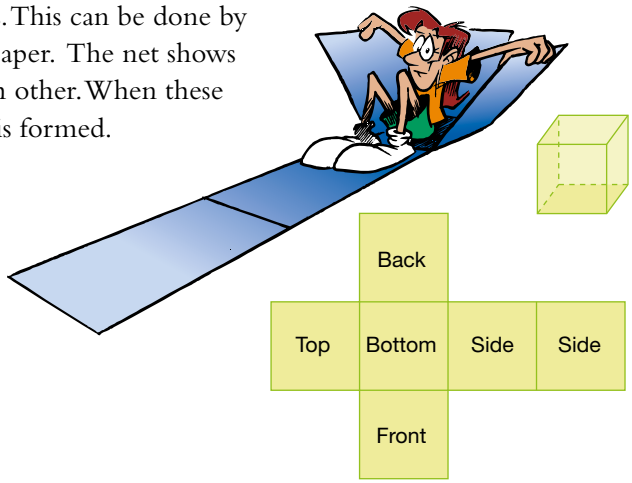

A pyramid-inspired building in San Francisco.

16 For each solid, show that Euler's formula holds.
- **a** triangular prism
- **b** square prism
- **c** triangular pyramid
- **d** square pyramid
- **e** double pyramid (see Question **14** part **a**)

Euler's formula:
For any convex polyhedron, $F + V = E + 2$ where F = number of faces, V = number of vertices, E = number of edges.

8:06 Nets of solids

Solid shapes can be made from plane shapes. This can be done by drawing the **net** of the solid on a piece of paper. The net shows how the faces of the solid are joined to each other. When these faces are folded along their edges, the solid is formed.



Some nets are quite easy to make, while others are quite difficult. The net of a cube is shown on the right.

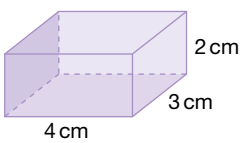
When trying to draw a net, you need to answer the following questions:

- What types of faces does the solid have?
- How many of each type are there?
- Which faces are joined to each other?

Select a face of the solid and draw it. Look at the faces that join this face and then draw them. Continue until all the faces have been drawn.

WORKED EXAMPLE 1

Draw the net of this rectangular prism on square grid paper.

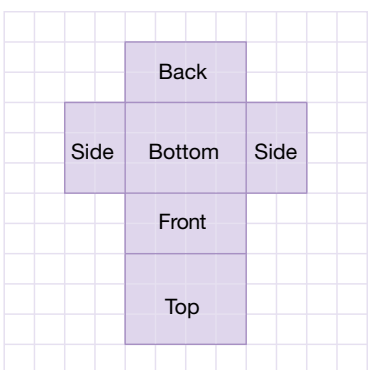


Solution

The solid has six rectangular faces. There are three pairs of different-sized rectangles.

Step 1 Draw the bottom. This is 4 cm long and 3 cm wide.
Step 2 Draw the two sides that join to the bottom.
Step 3 Draw the front and back that join to the bottom.
Step 4 Finally draw the top, which can be joined to either the back, front or side.

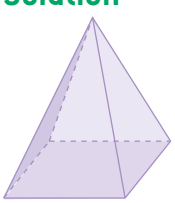
In the solution given, the top has been joined to the front.



WORKED EXAMPLE 2

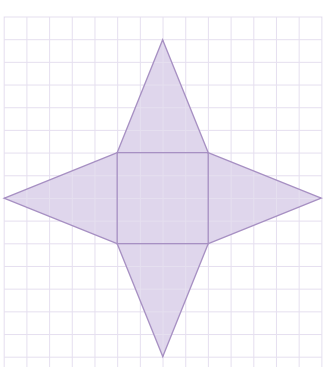
Draw the net of a square pyramid on a sheet of square grid paper.

Solution



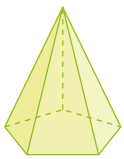
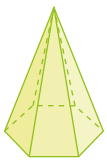
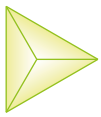
A square pyramid has four identical triangular faces joined to a square base, so:

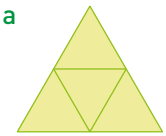
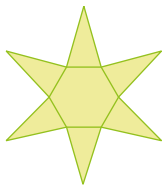
Step 1 Draw a square base.
Step 2 Draw the four adjoining triangles.



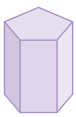
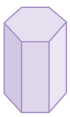
Exercise 8:06

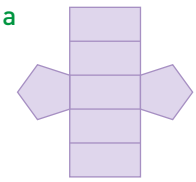
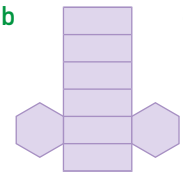
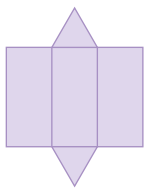
1 Match the nets with the solids A, B and C.

A  B  C 

a  b  c 

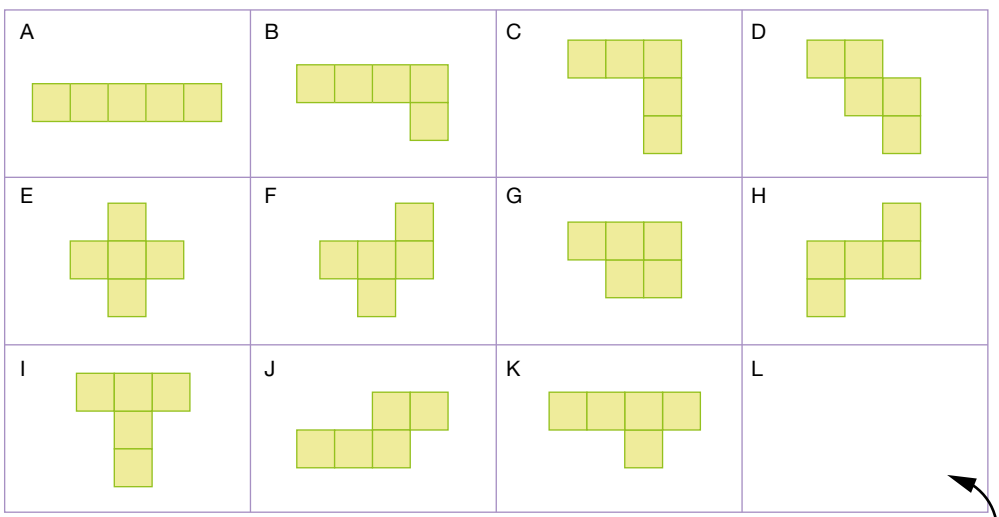
2 Match the nets with the solids D, E and F.

D  E  F 

a  b  c 

3 Five connected squares form a *pentomino*.

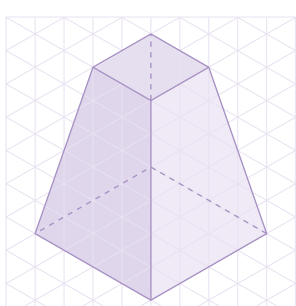
a Which of the pentominoes below form the net of an open box?



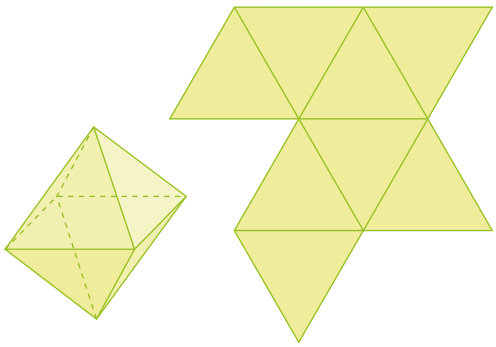
b If two pentominoes match exactly when rotated or flipped over, they are considered to be the same pentomino. Draw the only pentomino that is not shown above.

4 Make an accurate model of a rectangular prism that is 5 cm long, 3 cm wide and 2 cm high by drawing its net on square grid paper.

5 This diagram shows a square pyramid with the top part removed. Construct the net of this solid on square grid paper.



6 The diagram below is an attempt to draw the net of the solid shown, which is an octahedron (double pyramid). By copying and assembling the attempted net, find the error in the net and draw the correct version.



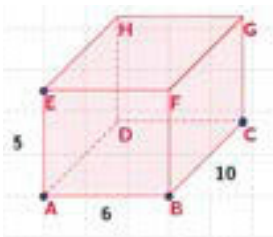


How would the folding flaps of this cubic box change the shape of its net?

P GEOGEBRA ACTIVITY 8:06 NETS

Choose a prism and its net. Change the shape of the prism by clicking and dragging points and watch the changes to the shape of the net.

Next, un-tick the 'Net' box and change the shape of the prism. Draw the net of this new prism in your book, then tick the 'Net' box again to check your drawing.

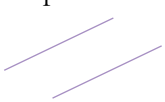
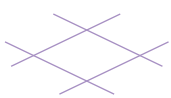
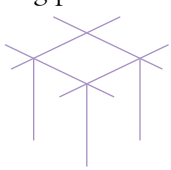
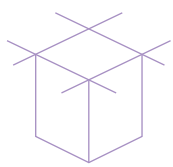
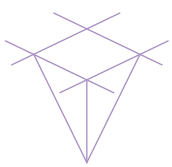


8:07 Drawing pictures of solids

Drawing pictures of solids can be made easier by using one of the methods below.

Method 1: Parallel lines

Some prisms and pyramids can be drawn using parallel lines.

A 
B 
C 
D 
E 

- Draw two pairs of parallel lines that cross.
- Draw three lines of equal length straight down from the corners. Join the ends to make a prism.

Or:

- Choose a point in the middle, below (or above) the crossed parallel lines. Draw lines from three corners to this point to make a pyramid.

Method 2: Projections

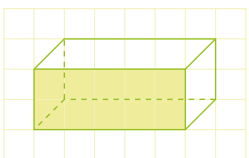


To draw three-dimensional objects on paper, artists use different techniques called **projections**. On the grids below, four different prisms have been drawn using two different projections.

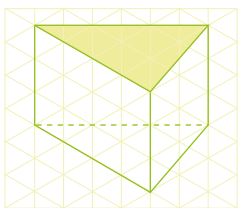
When drawing a prism, start with the base. This is the same shape as the uniform cross-section. Note that the base is not always the face that is 'closest to the ground'.

The base of each prism below is shaded. (Also note that for a cube or rectangular prism, any face could be the base.)

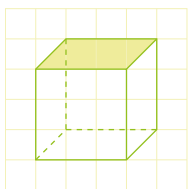
Rectangular prism



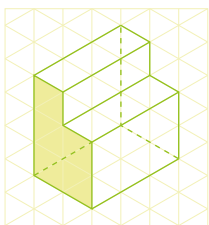
Triangular prism



Cube



Other prism



Notice that in both projections, things have been deliberately distorted to make them 'look right'. For instance, the square and rectangular faces have not all been drawn as squares and rectangles.

Also, in the drawings made on the square grid, the face that appears to be going backwards into the paper is drawn shorter than its correct length. This is to compensate for the optical illusion of perspective.

Notice also that in both projections, edges that would be hidden from view behind faces are shown with dashed lines.

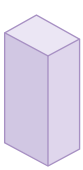
It is important that:
- you are able to draw pictures of different solids
- you can understand a picture of a solid.

Exercise 8:07

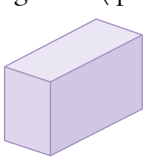
Equipment needed: ruler, square and isometric grid paper

1 Use Method 1 from page 245 ('parallel lines') to draw these solids.

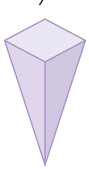
a



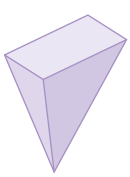
b



c



d



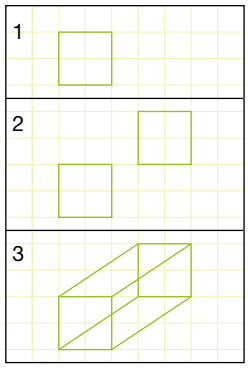
e



2 Follow these steps to draw a prism. It will have a uniform cross-section that is a square.

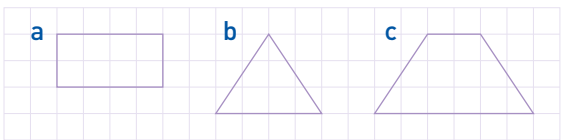
1 Draw a square on square grid paper.
2 Translate the square sideways and upwards and draw it again.
3 Join the corresponding corners of the squares.

What type of solid is this?





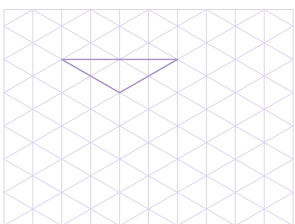
3 Repeat the steps from Question 2 to draw prisms with the following uniform cross-sections. In each case, name the prism you have drawn.



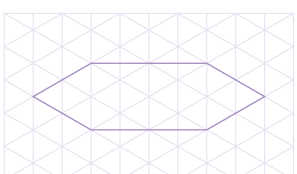
4 Follow these steps to draw a prism with a uniform cross-section that is a triangle.

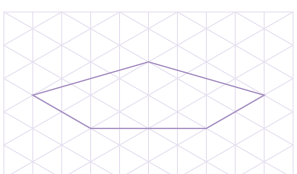
1 On a piece of isometric grid paper, draw the triangle given.
2 Translate the triangle downwards and draw it in its new position.
3 Join the corresponding corners of the triangles.

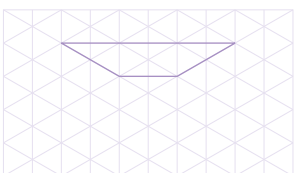
What type of solid is this?



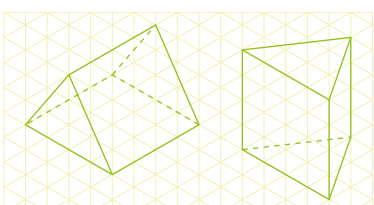
5 Repeat the steps from Question 4 to draw solids with the following uniform cross-sections. In each case, name the prism you have drawn.

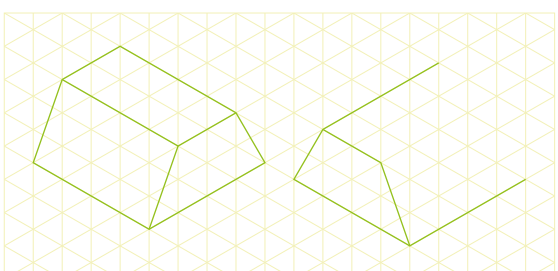
a 

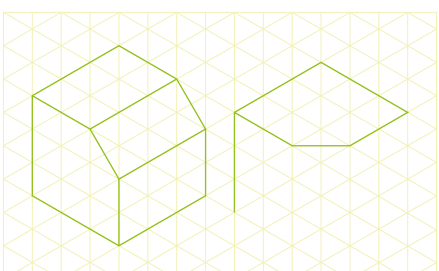
b 

c 

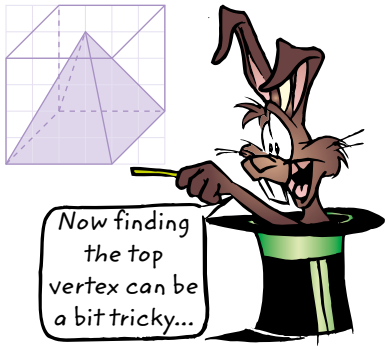
6 These diagrams show two drawings of the same solid in different orientations. Complete the drawing of each solid given below in its new orientation.



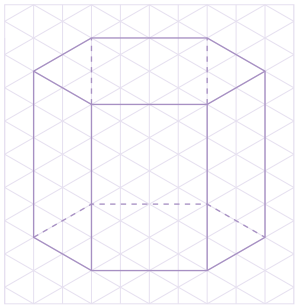
a 

b 

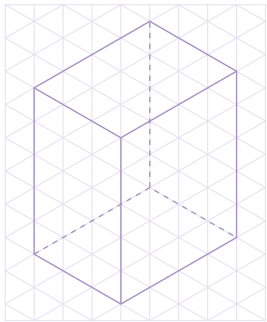
7 You can sketch a pyramid by first making a prism, as shown at right.
On isometric grid paper, copy the prisms below in pencil and then use each prism to draw a pyramid. After drawing the pyramid, erase the prism.



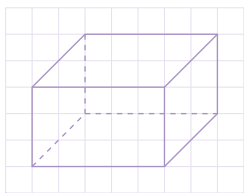
a



b

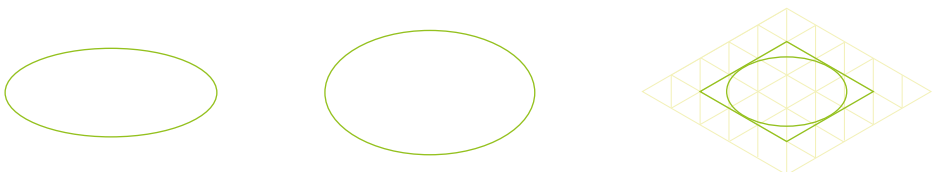


c



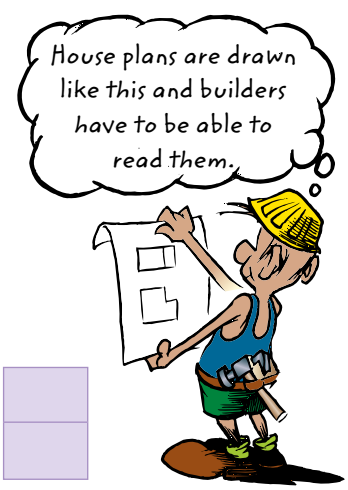
8 Find how many different solids can be built from three identical cubes. Sketch each solid on isometric grid paper.

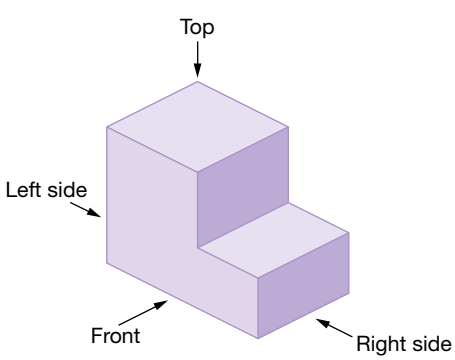
9 When drawing cylinders and cones in three dimensions, the circles are represented by ellipses. Some ellipses are shown below, including one on an isometric grid. Practise drawing ellipses and use them to draw pictures of cones and cylinders.

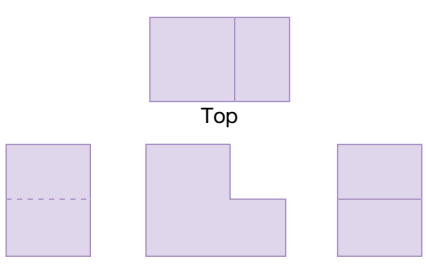


8:08 Looking at solids from different views

In many technical situations, a solid is represented by being drawn from different views. Usually a solid will be drawn from the front, the top and the sides. Together these drawings can be used to completely 'describe' the solid. This is illustrated below.







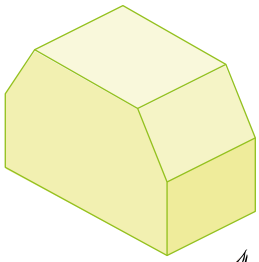
From each view, you can obtain two of the three dimensions of the solid:

- From the front view you can get the length and height of the solid.
- From the top view you can get the length and width of the solid.
- From the side views you can get the width and height of the solid.

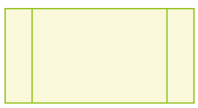
Dashed or dotted lines are used to show any hidden edges, as in the left side view of the solid above.

WORKED EXAMPLE 1

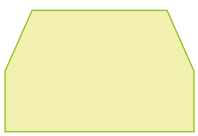
Draw the front view, top view and right side view of the solid pictured.



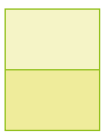
Solution



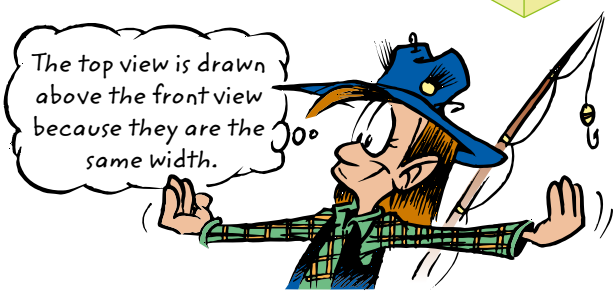
Top



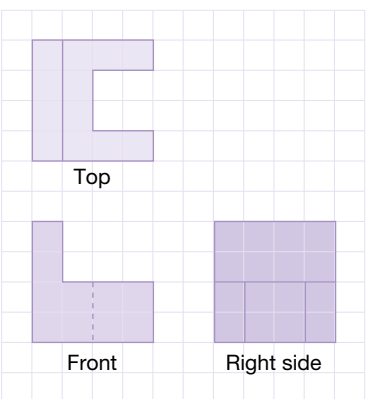
Front



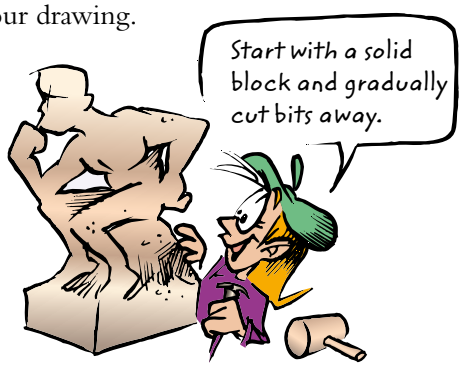
Right side



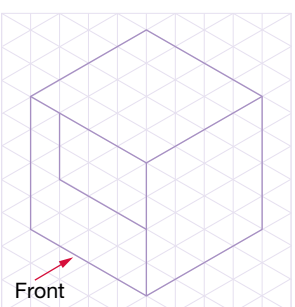
WORKED EXAMPLE 2



Using the views given in the diagram, draw a picture of the solid on isometric grid paper. Use the grid to work out the size of your drawing.

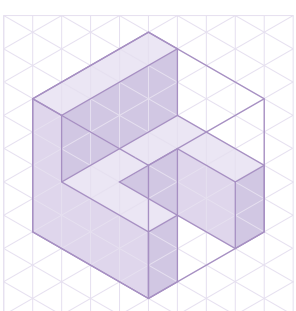


Solution



Step 1 Determine the length, width and height from the views. Construct a rectangular prism with these dimensions to enclose the solid.

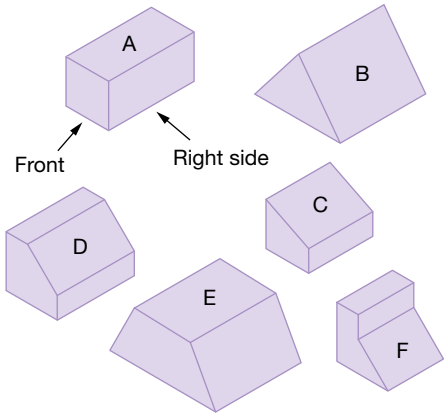
Step 2 Draw the front face. Use the other two views to complete the drawing.



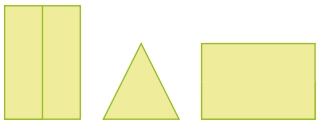
Exercise 8:08

Challenge worksheet 8:08
Looking at solids from different views

1 Match the prisms A to F with the following sets of views.



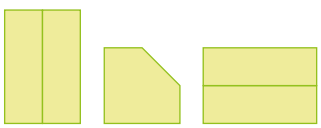
a



b



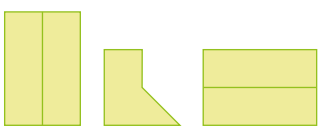
c



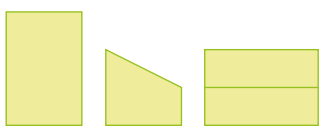
d



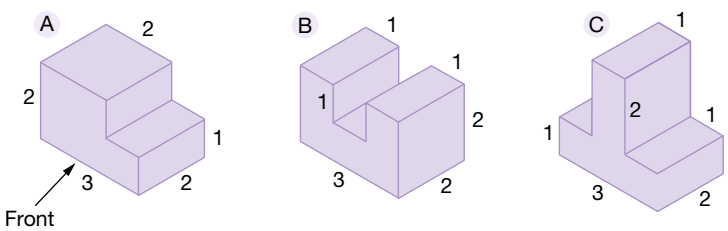
e



f



2

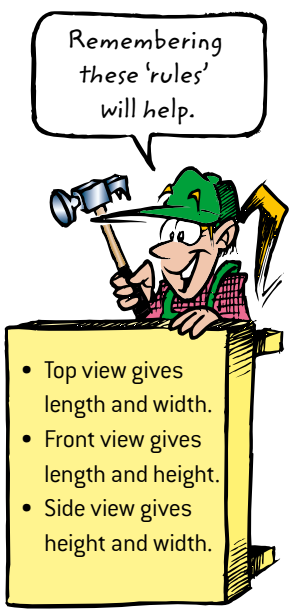


a Which of the solids above has a different top view to the other two?

b Which solid's left side view and right side view are different?

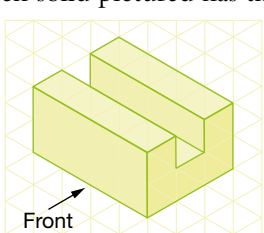
c Which views give the height of a solid?

d What dimensions of the solid are obtained from the top view?

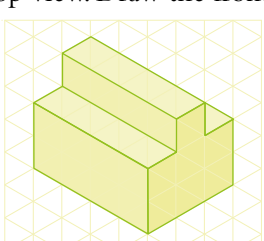


3 Each solid pictured has the same top view. Draw the front and side views of each.

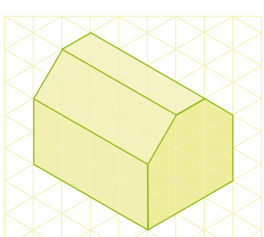
a



b

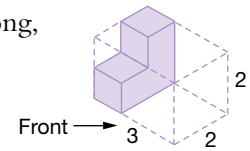


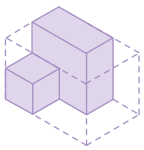
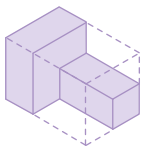
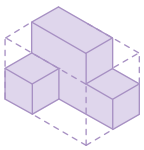
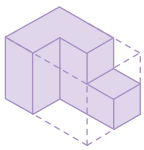
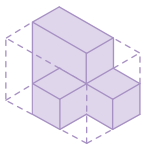
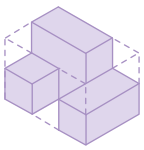
c



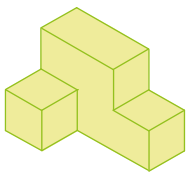
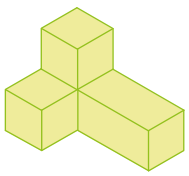
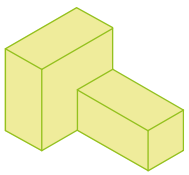
4 Draw the front view, right side view and top view of each solid from Question 2.

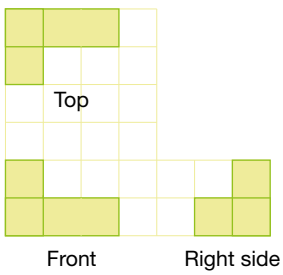
5 Each solid below has been cut from a square prism 3 units long, 2 units wide and 2 units high. An example is shown at right. Draw the front view and top view of each solid.

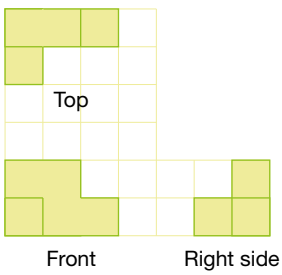


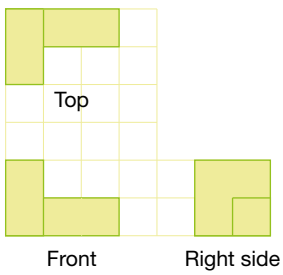
a 
b 
c 
d 
e 
f 

6 Which of the solids shown is represented by each set of views?

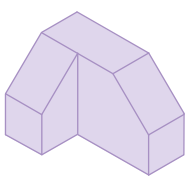
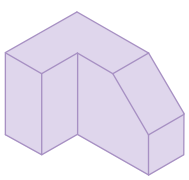
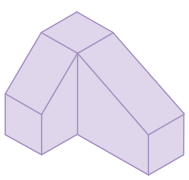
A 
B 
C 

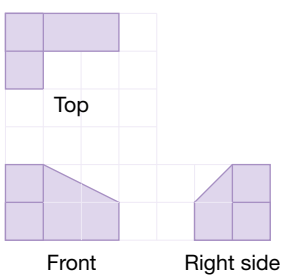
a 

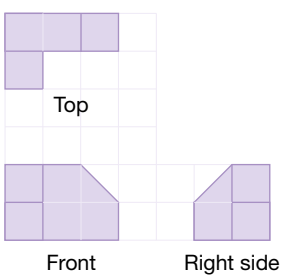
b 

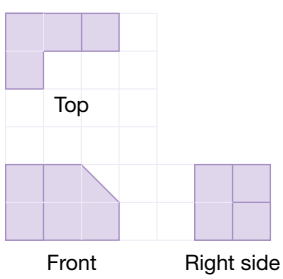
c 

7 Which of the solids shown is represented by each set of views?

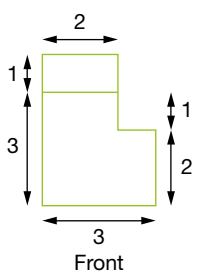
A 
B 
C 

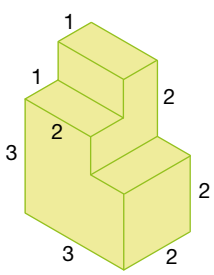
a 

b 

c 

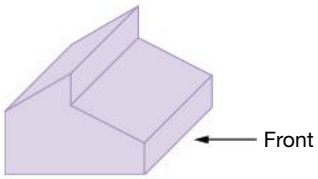
8 A solid and its front view are shown. Use the dimensions given to draw the top view and the two side views of the solid.



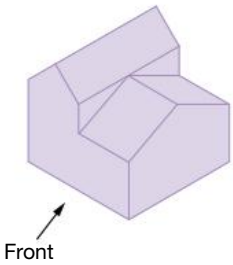


9 Draw the front view and top view of these 'prism-based' buildings.

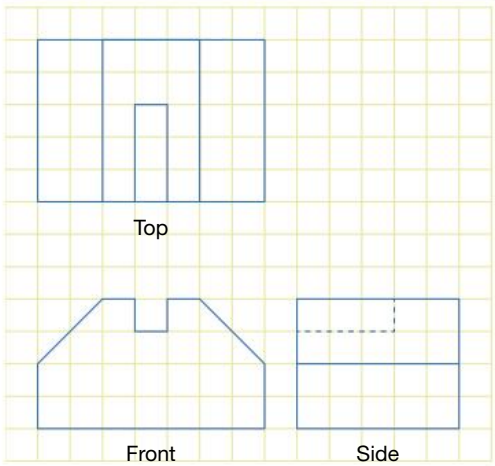
a

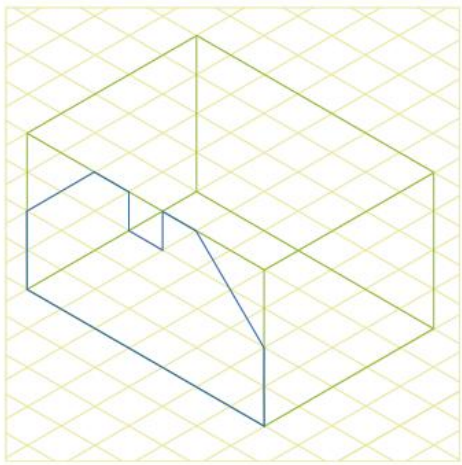


b



10 Use the views given to complete the three-dimensional isometric drawing of the solid. Each grid square represents 1 unit of distance.





INVESTIGATION 8:08 THE PLATONIC SOLIDS

Polyhedrons are three-dimensional figures whose faces are plane shapes with straight edges. The solids we have been studying that have only flat surfaces are polyhedrons.

Polyhedra is another word for ***polyhedrons***.

Ancient Greek mathematicians realised that there are only five polyhedrons that can be made using regular polygons such as the equilateral triangle, the square and the regular pentagon. These **Platonic solids** are the tetrahedron, octahedron, icosahedron, cube and dodecahedron.



Tetrahedron



Octahedron



Icosahedron



Hexahedron (cube)



Dodecahedron

This table shows the properties of the Platonic solids.

Platonic solid	Shape used	Number of shapes at each vertex	Number of:		
			faces	vertices	edges
tetrahedron	 triangle	3	4	4	6
octahedron	 triangle	4	8	6	12
icosahedron	 triangle	5	20	12	30
hexahedron (cube)	 square	3	6	8	12
dodecahedron	 pentagon	3	12	20	30

1 How many faces do these solids have?
a hexahedron **b** octahedron **c** tetrahedron **d** dodecahedron

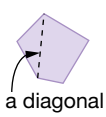
2 Show that Euler's formula (see page 242) is true for each of the Platonic solids.

3 Make models of at least two of the Platonic solids.

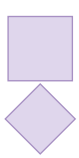
MATHS TERMS 8

2D SHAPES

diagonal
- a line drawn from one corner of a polygon across the polygon to another corner



orientation
- the way in which a figure is drawn
- two different orientations of a square are shown here:



plane shape
- a shape that lies on a flat surface
- a two-dimensional (2D) shape

polygon
- a plane figure that has only straight sides
- many polygons are named according to their number of sides

regular polygon
- a polygon with all of its sides and all angles equal
- a polygon that is not regular is called *irregular*

quadrilateral
- a plane figure that has four straight sides
- quadrilaterals with special properties are given special names, e.g. square, rectangle, rhombus, parallelogram, trapezium, kite

convex quadrilateral
- a quadrilateral that has diagonals that are all inside the quadrilateral

TRIANGLE

- a plane figure that has three straight sides

acute-angled triangle

- has three acute angles



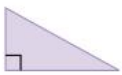
obtuse-angled triangle

- has an obtuse angle



right-angled triangle

- has one right angle



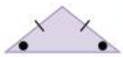
equilateral triangle

- has three equal sides; each angle is 60°



isosceles triangle

- has two equal sides; angles opposite the equal sides are equal



scalene triangle

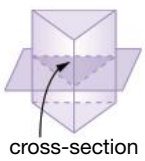
- has no equal sides or angles



3D SHAPES

cross-section

- the shape on a face where a solid has been 'cut through'



cube

- a prism with six square faces

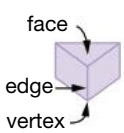
cylinder

- a prism-like solid that has a circle as its uniform cross-section



Euler's formula

- $F + V = E + 2$
  This relates the number of faces (F), vertices (V) and edges (E) of all solids whose faces are flat.



edge

- a line where two faces meet

face

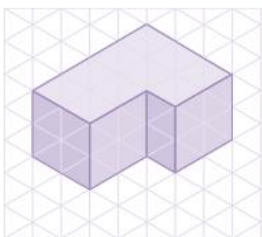
- a flat surface

vertex

- a corner

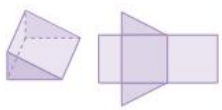
isometric projection

- a way of drawing 3D pictures of solids on an isometric grid
- shapes are drawn as if observed at an angle from above, with sides receding to right and left



net (of a solid)

- a 2D drawing of the surfaces of a solid that can be folded to make the solid



Platonic solids

- the five regular polyhedrons: tetrahedron, cube (hexahedron), octahedron, dodecahedron, icosahedron

polyhedron (plural: **polyhedrons** or **polyhedra**)

- a three-dimensional figure whose faces are plane shapes with straight edges

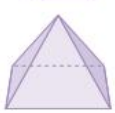
prism

- a solid that has two identical ends joined by rectangular faces



pyramid

- a solid that has a base from which triangular faces rise to meet at a point



DIAGNOSTIC TEST 8 2D AND 3D SPACE

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

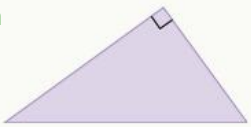
1 Show how each shape can be divided into the pieces written below it. 8:01

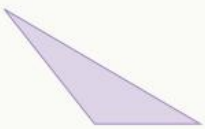
a 
rectangle and triangle

b 
two triangles

c 
trapezium and triangle

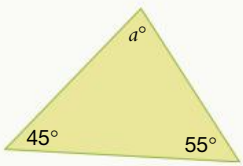
2 Use one of the terms 'acute-angled', 'obtuse-angled' or 'right-angled' to describe the following triangles. 8:02

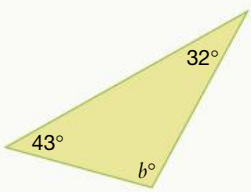
a 

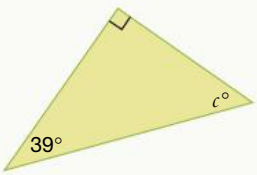
b 

c 

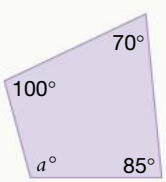
3 Find the value of each pronumeral. 8:02

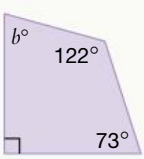
a 

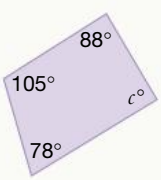
b 

c 

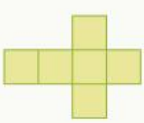
4 Find the value of the each pronumeral. 8:03, 8:04

a 

b 

c 

5 What solids are formed from the following nets? 8:06

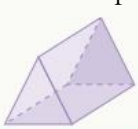
a 

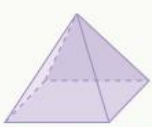
b 

c 

6 Using isometric grid paper, draw: 8:07
a a rectangular prism **b** a triangular prism **c** a hexagonal prism.

7 Draw the top view, front view and side view of each shape. 8:08

a 

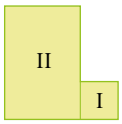
b 

c 

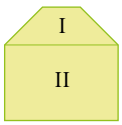
ASSIGNMENT 8A Chapter review

1 Name the two shapes used to form each of the following hexagons.

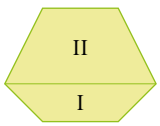
a



b

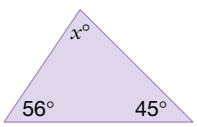


c

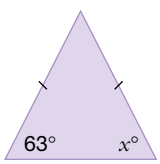


2 Find the value of the each pronumeral.

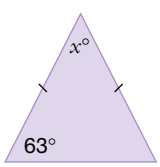
a



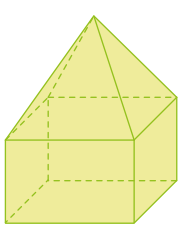
b



c



3 This solid has a square base.



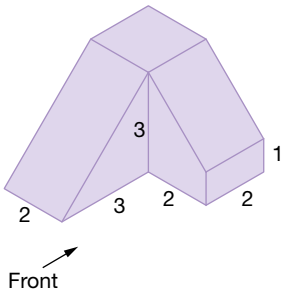
a Name the two solids that have been joined to make this solid.

b For the solid, find the number of:

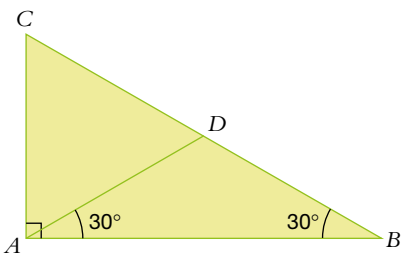
- **i** faces
- **ii** edges
- **iii** vertices.

c Sketch the net of the solid.

4 Draw the front view, top view and both side views of the solid shown.



5 The diagram shows three triangles.



a Write the size of each angle:

- **i** $\angle ACB$
- **ii** $\angle CAD$
- **iii** $\angle ADC$

b What type of triangle is:

- **i** ΔABC
- **ii** ΔABD
- **iii** ΔADC?

c If $AC = 6$ cm, what is the length of:

- **i** AD
- **ii** BD?

ASSIGNMENT 8B Working mathematically

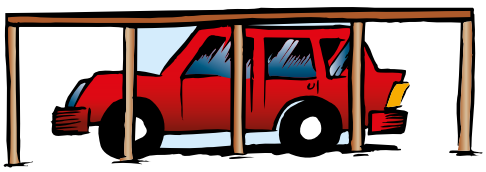
1 The digits 5, 4, 3, 2 and 1 are used to make numbers with five digits.
   a What are the largest and smallest numbers that can be made?
   b What are the fifth-largest and fifth-smallest numbers that can be made?

2 It takes me 25 minutes to drive from home to work. At what time must I leave home to reach work 10 minutes early, if I start work at 7:25 am?

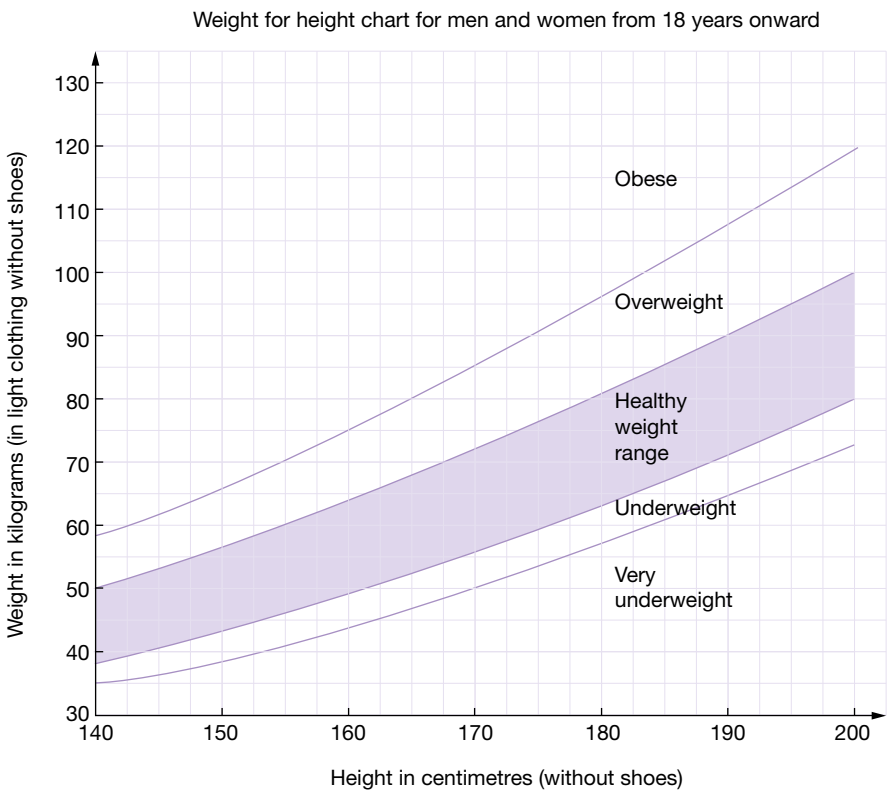
3 Beads of four different colours are threaded onto a string in the order red, blue, yellow, green. What is the colour of the 182nd bead?

4 You go to the post office and buy 10 stamps, which come in one long strip. You fold along all the perforations to form a single stack. What is the fewest number of folds needed to do this?

5 The length of a carport is 10 000 mm. On one side the carport is supported by five posts, which are each 100 mm wide. If the posts are evenly spaced, how far apart are they?



6 Refer to the graph to answer the following:



   a Bill is 170 cm tall and weighs 70 kg. Does this put him in the healthy weight range?
   b Jill is 150 cm tall. What is the lightest weight she can be and still be in the healthy weight range?
   c If a man weighs 70 kg and is in the healthy weight range, what heights must he be between?
   d If a woman is 180 cm tall and is in the healthy weight range, what weights must she be between?
   e I am 175 cm tall and weigh 80 kg. How much weight must I lose to be in the healthy weight range?

ASSIGNMENT 8C Cumulative revision

1 **a** $0{\cdot}6 + 0{\cdot}7$ **b** $2{\cdot}15 + 3{\cdot}6$ **c** $7{\cdot}8 + 1{\cdot}95$ **d** $12{\cdot}16 + 3{\cdot}178$ 6:02

2 **a** $0{\cdot}9 - 0{\cdot}3$ **b** $1{\cdot}6 - 0{\cdot}8$ **c** $12{\cdot}1 - 6{\cdot}72$ **d** $5{\cdot}68 - 1{\cdot}7$ 6:02

3 **a** $0{\cdot}6 \times 0{\cdot}5$ **b** $(0{\cdot}2)^2$ **c** $11{\cdot}5 \times 2$ **d** $3{\cdot}1 \times 0{\cdot}4$ 6:05

4 **a** $12{\cdot}6 \div 0{\cdot}2$ **b** $0{\cdot}3 \div 0{\cdot}1$ **c** $2{\cdot}5 \div 0{\cdot}02$ **d** $0{\cdot}7 \div 2{\cdot}5$ 6:06

5 **a** $1{\cdot}1 \times 5$ **b** $0{\cdot}11 \times 5$ **c** $0{\cdot}011 \times 5$ **d** $1{\cdot}1 \times 0{\cdot}5$ 6:05

6 **a** $6{\cdot}5 \div 5$ **b** $0{\cdot}65 \div 5$ **c** $6{\cdot}5 \div 0{\cdot}5$ **d** $0{\cdot}65 \div 0{\cdot}5$ 6:06

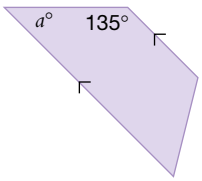
7 True or false? 1:08

a $16 > 12$ **b** $398 \div 400$

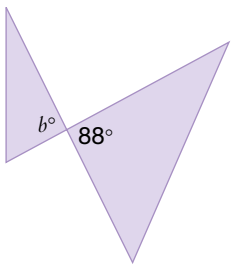
c $0{\cdot}3 > 0{\cdot}033$ **d** $5{\cdot}1 + 4 = 4 + 5{\cdot}1$

8 Find the value of each pronumeral. 5:04, 5:05

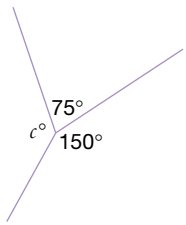
a



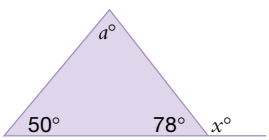
b



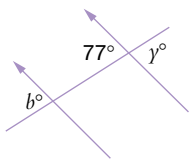
c



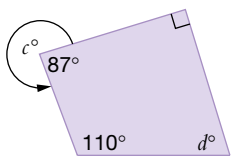
d



e



f



9 Use the given rules to complete each table of values. 4:06

a $L = M + 4$

M	1	2	5	10
L				

b $P = 2 \times B + 4$

B	0	1	2	4
P				

c $n = (m + 3) \times (m - 2)$

m	2	3	4	10
n				

d $y = x + 2$

x	1	2	3	4
y				

e $m + n = 6$

m	0	1	2	3
n				

f $P = 2 \times M + 1$

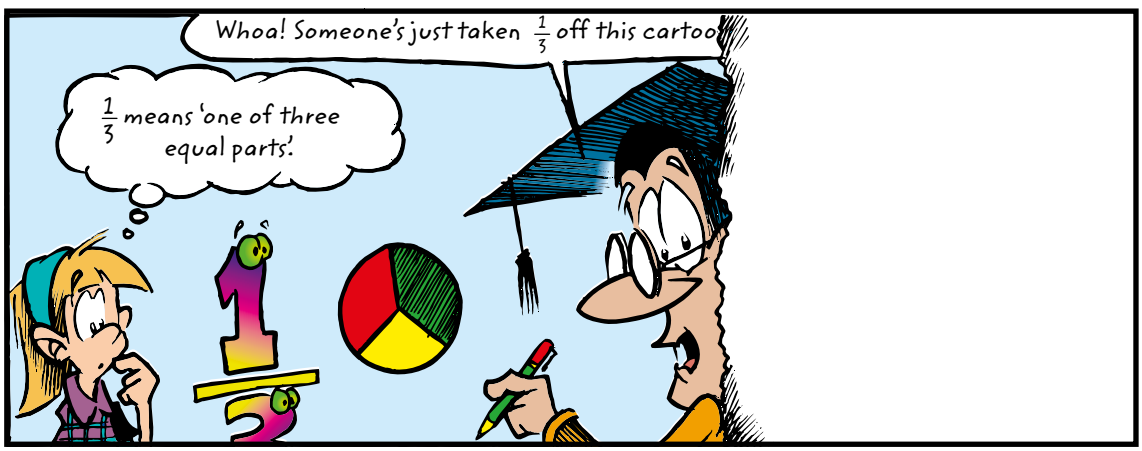
M	2	3	4	5
P				

10 If $x = 3$, find the value of y for each of the following rules. 4:06

a $y = 5 \times x$ **b** $y = 7 - x$ **c** $y = 2 \times x + 3$ **d** $y = 24 \div x$

FRACTIONS

9



Contents

9:01 Exploring fractions
9:02 Comparing fractions
9:03 Review of fractions
Investigation 9:03 Fractions and shapes
9:04 Addition and subtraction of fractions
9:05 Addition and subtraction of mixed numbers
Challenge 9:05 Quizzical questions!
9:06 Multiplication of fractions
9:07 Division involving fractions
9:08 Fractions of quantities
9:09 Using a calculator
Investigation 9:09 Population of Australia
9:10 Applications of fractions
Fun spot 9:10 Where did the king keep his armies?
9:11 Ratios
9:12 Best buys
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Fractions and Decimals 2*** [Stage 3], ***Fractions, Decimals and Percentages***, ***Financial Mathematics***, ***Ratios and Rates*** [Stage 4]

- Compare fractions using equivalence; locate and represent positive and negative fractions and mixed numerals on a number line. (ACMNA152)
- Find a simple fraction of a quantity where the result is a whole number, with and without the use of digital technologies. [Stage 3] (ACMNA127)
- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators. (ACMNA153)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies. (ACMNA154)
- Express one quantity as a fraction of another, with and without the use of digital technologies. (ACMNA155)
- Recognise and solve problems involving simple ratios. (ACMNA173)
- Investigate and calculate 'best buys', with and without the use of digital technologies. (ACMNA174)

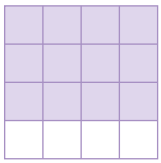
Working Mathematically

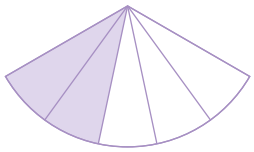
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

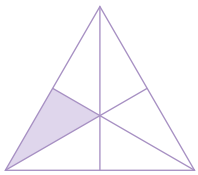
9:01 Exploring fractions

Exercise 9:01

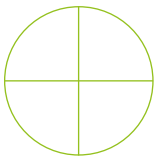
1 What fraction of each figure is shaded?

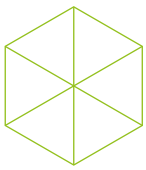
a 

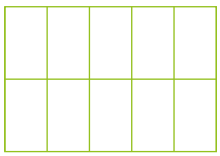
b 

c 

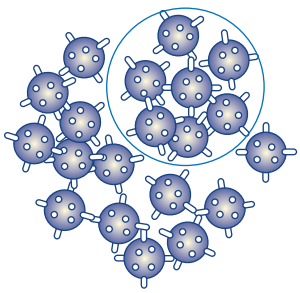
2 Copy the figure and shade the fraction indicated.

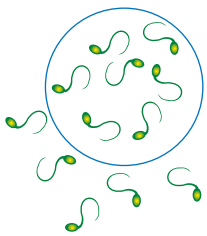
a  $\frac{3}{4}$

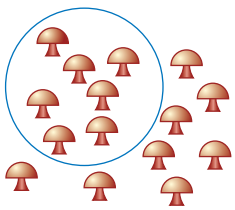
b  $\frac{6}{6}$

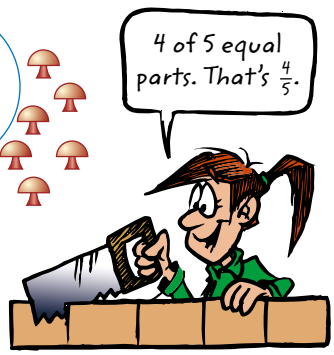
c  $\frac{7}{10}$

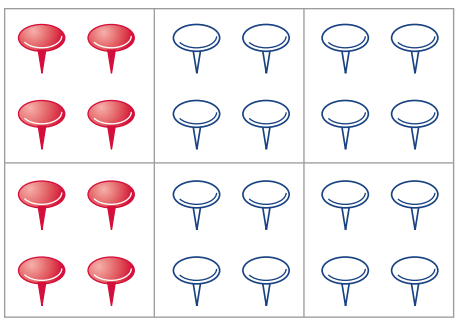
3 What fraction of each group is circled?

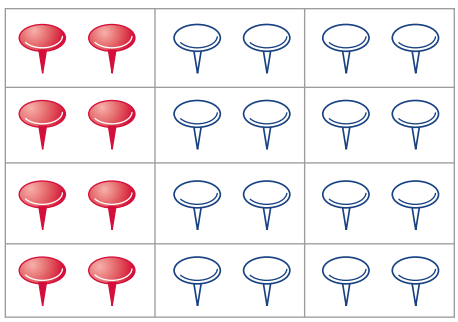
a 

b 

c 



4 A 

B 

The same fraction is coloured in both A and B. Use the diagrams to complete:

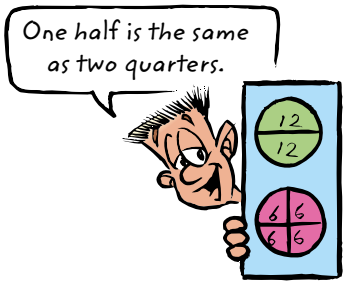
$\frac{8}{24} = \frac{\quad}{12} = \frac{\quad}{6} = \frac{\quad}{3}$

5 Use the diagrams in Question **4** to find the following:

a $\frac{1}{2}$ of 24 **b** $\frac{1}{3}$ of 24 **c** $\frac{1}{4}$ of 24

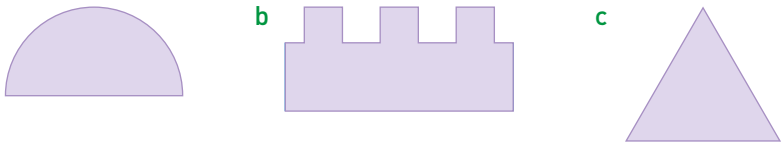
d $\frac{1}{6}$ of 24 **e** $\frac{1}{8}$ of 24 **f** $\frac{2}{3}$ of 24

g $\frac{3}{4}$ of 24 **h** $\frac{5}{6}$ of 24 **i** $\frac{3}{8}$ of 24

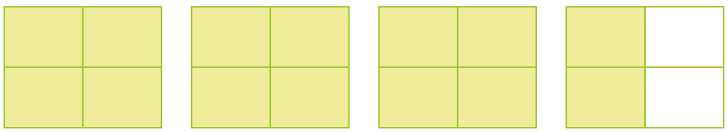


6 Copy and shade $\frac{2}{3}$ of each figure.

a **b** **c**



7



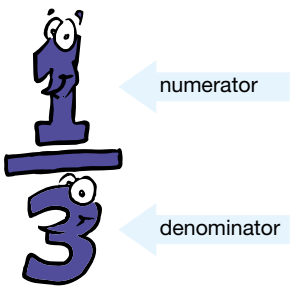
a How many quarters are shaded?

b How many quarters are in one whole?

c How many quarters are in $3\frac{1}{2}$?

d What is 8 lots of $\frac{1}{4}$? **e** What is 10 lots of $\frac{1}{4}$?

f $12 \times \frac{1}{4}$ **g** $6 \times \frac{1}{4}$ **h** $16 \times \frac{1}{4}$ **i** $14 \times \frac{1}{4}$



8 **a** If $\frac{1}{4}$ of my money is \$3.15, how much do I have?

b If $\frac{1}{10}$ of my swap cards is 23 cards, how many do I have?

c If $\frac{1}{3}$ of Australia's population is 7 500 000 people, what is Australia's population?

9 When a water tank is $\frac{3}{4}$ full, it holds 3390 litres of water.

a How much does the tank hold when it is $\frac{1}{4}$ full?

b How much does the tank hold when it is full?

10 **a** How many people could share a cake, if each received $\frac{1}{10}$?

b How many people could share a cake, if each received $\frac{2}{100}$?

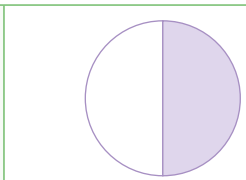
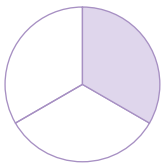
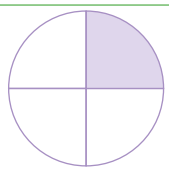
11 What fraction of the top of the yellow block is covered?



9:02 Comparing fractions

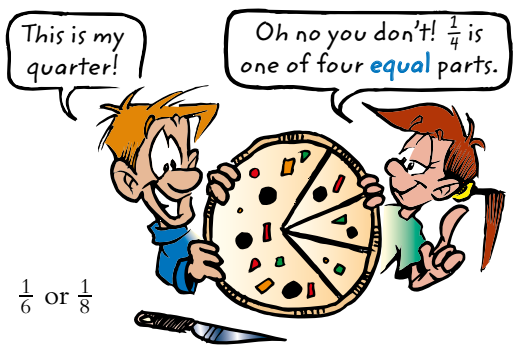
Exercise 9:02

1

Figure			
Fraction shaded	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

From the table above:

a Which fraction has the smallest denominator?
b Which fraction has the largest shaded part?
c Which fraction has the largest denominator?
d Which fraction has the smallest shaded part?
e Complete this sentence: 'As the denominator increases, the value of the fraction .....'



2 Which fraction is bigger?

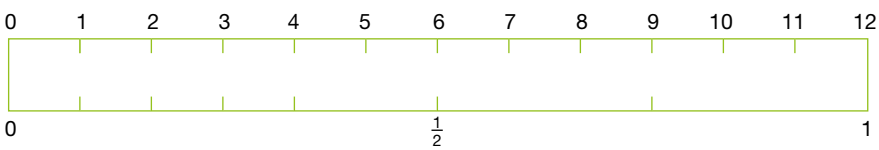
a $\frac{1}{4}$ or $\frac{1}{2}$ **b** $\frac{1}{3}$ or $\frac{1}{10}$ **c** $\frac{1}{6}$ or $\frac{1}{8}$

3 Write as a whole number:

a $\frac{1}{2}$ of 12 **b** $\frac{1}{4}$ of 12 **c** $\frac{1}{3}$ of 12

d $\frac{1}{6}$ of 12 **e** $\frac{1}{12}$ of 12 **f** $\frac{3}{4}$ of 12

4



Copy the diagram above into your book. Use the answers to Question **3** to write the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$ and $\frac{3}{4}$ in their correct places along the lower edge.

5 List the fractions graphed on the number line below.



6 Graph each set of fractions on a separate number line.

a $\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$ **b** $\{\frac{2}{10}, \frac{7}{10}, 1\frac{1}{10}\}$ **c** $\{\frac{5}{12}, \frac{1}{3}, \frac{3}{4}\}$

7 By referring to the number line in Question **5**, write either < or > between each pair of fractions.

a $\frac{1}{2}$ $\frac{3}{10}$ **b** $\frac{9}{10}$ $1\frac{1}{10}$ **c** $1\frac{1}{2}$ $\frac{13}{10}$



Any number that can be written as a fraction is called a **rational number**.

< means 'is less than'.

8 Write each set of fractions in ascending order.

a $\{\frac{3}{10}, \frac{1}{2}, \frac{1}{4}\}$ **b** $\{\frac{1}{3}, \frac{3}{4}, \frac{1}{5}\}$ **c** $\{\frac{7}{10}, \frac{1}{2}, \frac{3}{4}\}$

Note: Any fraction over zero, such as $\frac{2}{0}$, is not a real fraction. You cannot have any number of zero equal parts.

9 Lachlan used fraction blocks to demonstrate equivalent fractions. Complete his findings.

1									← 1 whole
$\frac{1}{2}$					$\frac{1}{2}$				← 2 halves
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	← 5 fifths
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$ ← 10 tenths

a $\frac{1}{1} = \frac{\quad}{2} = \frac{\quad}{5} = \frac{\quad}{10}$

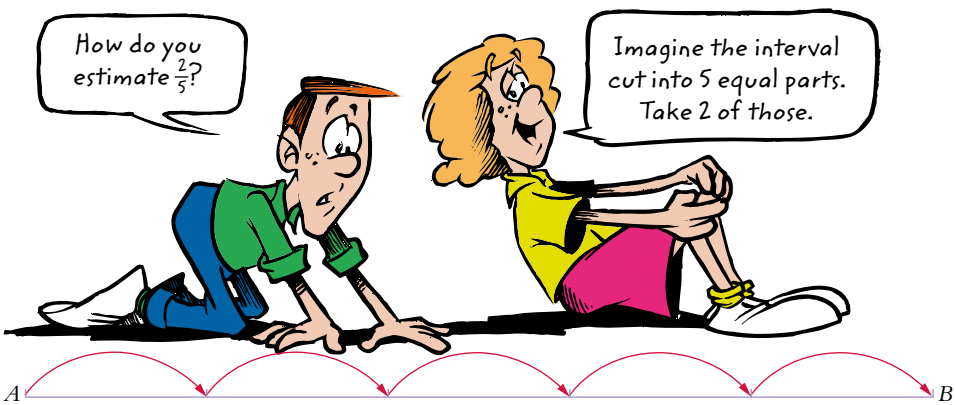
b $\frac{1}{2} = \frac{\quad}{10}$ **c** $\frac{1}{5} = \frac{\quad}{10}$

d $\frac{2}{5} = \frac{\quad}{10}$ **e** $\frac{3}{5} = \frac{\quad}{10}$

f $\frac{4}{5} = \frac{\quad}{10}$

10 Draw an interval 10 centimetres long. Let this represent one unit. Using only a ruler and pencil, mark and label the positions on the interval that would represent $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$ and $\frac{5}{8}$.

11



Draw four intervals like the interval AB above, each 120 mm long.

a On your first interval, estimate the position that is $\frac{1}{2}$ of the length from A to B. What is $\frac{1}{2}$ of 120 mm? Measure the distance from A to your estimate and calculate your error.

b What is $\frac{1}{3}$ of 120 mm? Estimate this position on your second interval, then calculate your error.

c What is $\frac{2}{3}$ of 120 mm? Estimate this position on your third interval, then calculate your error.

d What is $\frac{3}{5}$ of 120 mm? Estimate this position on your fourth interval, then calculate your error.

12



Consider the interval XY above. Which points show the following fractions?

a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$ **d** $\frac{1}{3}$ **e** $\frac{3}{4}$ **f** $\frac{2}{3}$ **g** $\frac{9}{10}$

9:03 Review of fractions

If you have difficulty with this review, try the diagnostic check-up test 1:11B on page 37.

Summary of important ideas

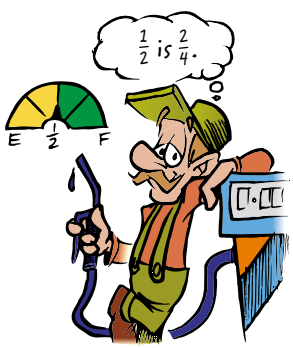
- If the numerator and denominator are both multiplied or divided by the same number, then the size of the fraction is not changed.
  e.g. $\frac{3}{8} = \frac{3\times5}{8\times5} = \frac{15}{40}$, so $\frac{3}{8}$ and $\frac{15}{40}$ are equivalent fractions.
- To compare fractions, make their denominators the same, then compare the numerators.
  e.g. $\frac{3}{8} < \frac{4}{8}$
- An *improper fraction* has a numerator that is greater than its denominator.
  e.g. $\frac{5}{2}, \frac{13}{10}$
- A *mixed number* has a whole number part and a fraction part.
  e.g. $2\frac{1}{2}, 1\frac{3}{10}$
- When fractions have the *same denominator*, we can add them by adding the numerators.
  e.g. $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$
  We can also subtract them by subtracting the numerators.
  e.g. $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$
- $5 \times \frac{3}{4}$ means '5 lots of $\frac{3}{4}$', which is also:
  $\frac{5\times3}{4} = \frac{15}{4} = 3\frac{3}{4}$
- To find $\frac{7}{8}$ of a number, find $\frac{1}{8}$ of the number and then multiply by 7.
  (To find $\frac{1}{8}$ of a number, divide it by 8.)
- When giving answers, reduce fractions to their simplest form. Write $\frac{6}{8}$ as $\frac{3}{4}$ and $2\frac{4}{16}$ as $2\frac{1}{4}$.

Exercise 9:03

1 Complete each set of equivalent fractions.

a $\frac{1}{2} = \frac{_}{4} = \frac{_}{10}$ **b** $\frac{1}{4} = \frac{_}{8} = \frac{_}{100}$ **c** $\frac{1}{5} = \frac{_}{10} = \frac{_}{100}$

d $\frac{3}{2} = \frac{_}{4} = \frac{_}{10}$ **e** $\frac{3}{4} = \frac{_}{8} = \frac{_}{100}$ **f** $\frac{2}{5} = \frac{_}{10} = \frac{_}{100}$



2 In each case, find the value of n that makes the sentence true.

a $\frac{4}{5} = \frac{n}{10}$ **b** $\frac{3}{10} = \frac{n}{100}$ **c** $\frac{3}{5} = \frac{n}{20}$

d $\frac{n}{20} = \frac{3}{2}$ **e** $\frac{n}{18} = \frac{2}{3}$ **f** $\frac{n}{100} = \frac{7}{20}$

3 Find the value of x to make each pair of fractions equivalent.

a $\frac{3}{10}, \frac{x}{40}$ b $\frac{3}{8}, \frac{x}{24}$ c $\frac{4}{5}, \frac{x}{20}$

d $\frac{x}{100}, \frac{8}{25}$ e $\frac{x}{3}, \frac{12}{18}$ f $\frac{x}{48}, \frac{3}{4}$

4 Simplify:

a $\frac{6}{10}$ b $\frac{15}{20}$ c $\frac{12}{36}$ d $\frac{50}{80}$ e $\frac{35}{100}$

f $\frac{14}{42}$ g $\frac{24}{72}$ h $\frac{35}{49}$ i $\frac{24}{40}$

5 What fraction is each of the following?

a 60° of 180° b 10° of 180° c 90° of 180°

d 120° of 180° e 70° of 180° f 135° of 180°

g 5° of 360° h 40° of 360° i 220° of 360°

6 By writing each fraction with a common denominator, arrange these fractions in ascending order.

a $\frac{7}{10}, \frac{1}{2}, \frac{1}{5}$ b $\frac{5}{8}, \frac{1}{2}, \frac{3}{4}$ c $\frac{11}{24}, \frac{1}{2}, \frac{5}{8}$

d $\frac{2}{3}, \frac{5}{12}, \frac{3}{4}$ e $\frac{3}{4}, \frac{4}{5}, \frac{73}{100}$ f $\frac{3}{10}, \frac{11}{40}, \frac{1}{8}$

7 Which of the symbols $<, =$ or $>$ should be placed in each ◯?

a $\frac{7}{10}$ ◯ $\frac{3}{4}$ b $\frac{5}{8}$ ◯ $\frac{15}{24}$ c $\frac{18}{24}$ ◯ $\frac{14}{20}$

d $\frac{32}{100}$ ◯ $\frac{1}{3}$ e $\frac{4}{5}$ ◯ $\frac{5}{7}$ f $\frac{8}{10}$ ◯ $\frac{6}{7}$

8 Write each improper fraction as a mixed number.

a $\frac{7}{4}$ b $\frac{11}{8}$ c $\frac{3}{2}$

d $\frac{8}{5}$ e $\frac{13}{10}$ f $\frac{723}{100}$

9 Write each mixed number as an improper fraction.

a $3\frac{1}{2}$ b $2\frac{3}{4}$ c $1\frac{1}{3}$ d $10\frac{2}{3}$ e $9\frac{3}{5}$ f $76\frac{7}{10}$

10 a $\frac{7}{10} + \frac{1}{10}$ b $\frac{5}{10} + \frac{3}{10}$ c $\frac{37}{100} + \frac{18}{100}$

d $\frac{3}{4} + \frac{3}{4}$ e $\frac{7}{10} + \frac{3}{10}$ f $\frac{13}{20} + \frac{8}{20}$

g $\frac{5}{8} + \frac{1}{8}$ h $\frac{4}{5} + \frac{3}{5}$ i $\frac{2}{3} + \frac{2}{3}$

11 a $\frac{9}{10} - \frac{3}{10}$ b $\frac{7}{10} - \frac{1}{10}$ c $\frac{8}{10} - \frac{3}{10}$

d $\frac{7}{4} - \frac{3}{4}$ e $\frac{12}{5} - \frac{7}{5}$ f $\frac{13}{20} - \frac{4}{20}$

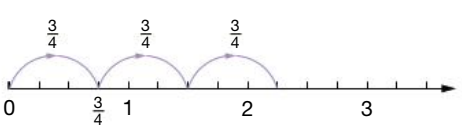
g $\frac{78}{100} - \frac{31}{100}$ h $\frac{11}{24} - \frac{3}{24}$ i $\frac{16}{16} - \frac{3}{16}$

12 a $1 + \frac{3}{4}$ b $4 + \frac{3}{10}$ c $6 + \frac{2}{3}$

d $1 - \frac{3}{4}$ e $1 - \frac{3}{10}$ f $1 - \frac{2}{3}$

13 Use this number line to find:

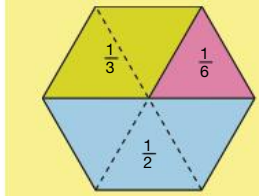
a two lots of $\frac{3}{4}$ b three lots of $\frac{3}{4}$ c four lots of $\frac{3}{4}$



d $3 \times \frac{3}{4}$

e $2 \times \frac{3}{4}$

f $4 \times \frac{3}{4}$

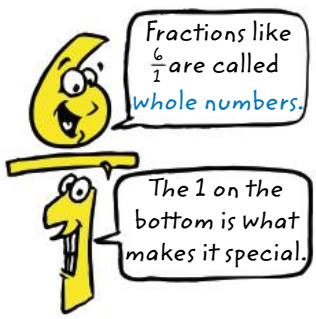


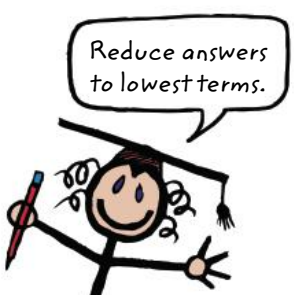
The pattern blocks show:

$\frac{1}{3} = \frac{2}{6}$

$\frac{1}{2} = \frac{3}{6}$

'Common' here means 'belonging to both'.





$\frac{55}{100}$ can be simplified by dividing the top and the bottom by 5.

14 a $2 \times \frac{2}{5}$ b $5 \times \frac{2}{10}$ c $7 \times \frac{2}{3}$

d $3 \times \frac{3}{10}$ e $6 \times \frac{9}{10}$ f $8 \times \frac{2}{5}$

g $9 \times \frac{13}{100}$ h $4 \times \frac{3}{8}$ i $19 \times \frac{1}{2}$

15 a $\frac{1}{4}$ of 20 b $\frac{1}{5}$ of 100 c $\frac{1}{8}$ of 24

d $\frac{3}{4}$ of 20 e $\frac{4}{5}$ of 100 f $\frac{7}{8}$ of 24

g $\frac{7}{10}$ of 150 h $\frac{3}{8}$ of 800 i $\frac{11}{100}$ of 9000

WORKED EXAMPLE

$9 \times \frac{11}{20} = \frac{9 \times 11}{20}$

$= \frac{99}{20}$

$= 4\frac{19}{20}$

16 a $\frac{3}{4} + 0$ b $\frac{5}{8} \times 1$ c $\frac{7}{10} - 0$ d $\frac{2}{3} \times 0$

e $\frac{17}{20}$ of 0 f $0 \times \frac{91}{100}$ g $0 + \frac{9}{10}$ h $1 \times \frac{7}{8}$

i $\frac{4}{5} + \frac{3}{5} = \frac{3}{5} + \frac{4}{5}$: true or false? j $5 \times \frac{2}{5} = \frac{2}{5} \times 5$: true or false?

k Are $\frac{3}{5}$ and $\frac{21}{35}$ equivalent? l Are $\frac{75}{85}$ and $\frac{7}{8}$ equivalent?

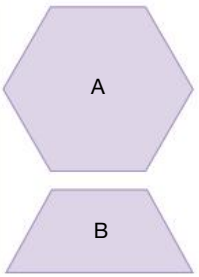
m $6 \times \frac{3}{5} + 6 \times \frac{2}{5}$ n $6 \times (\frac{3}{5} + \frac{2}{5})$

o Is $6 \times \frac{3}{5} + 6 \times \frac{2}{5}$ equal to $6 \times (\frac{3}{5} + \frac{2}{5})$?

p Is $10 \times 1\frac{1}{2} + 10 \times \frac{1}{2}$ equal to $10 \times (1\frac{1}{2} + \frac{1}{2})$?



INVESTIGATION 9:03 FRACTIONS AND SHAPES



Blocks like these are called pattern blocks.

- How many blocks like B are needed to cover A?
- How many blocks like C are needed to cover A?
- What fraction of A is block C?



Complete the table below: in each white square, write the fraction that the left figure is of the top figure.

What fraction?				
				
				

Extension

What numbers should go in the blue squares?

Refer to **ID Card 4** on page xvii and identify the figures **1** to **24**. Learn the terms that you do not know.

9:04 Addition and subtraction of fractions

PREP QUIZ 9:04

Complete or solve:

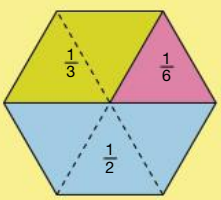
1 $\frac{3}{4} = \frac{\quad}{12}$ 2 $\frac{1}{3} = \frac{\quad}{12}$ 3 $\frac{9}{12} + \frac{4}{12}$ 4 $\frac{9}{12} - \frac{4}{12}$

5 $\frac{4}{5} = \frac{\quad}{15}$ 6 $\frac{2}{3} = \frac{\quad}{15}$ 7 $\frac{12}{15} + \frac{10}{15}$ 8 $\frac{12}{15} - \frac{10}{15}$

9 $\frac{1}{2} = \frac{\quad}{4} = \frac{\quad}{6} = \frac{\quad}{8} = \frac{\quad}{10} = \frac{\quad}{12}$

10 $\frac{1}{5} = \frac{\quad}{10} = \frac{\quad}{15} = \frac{\quad}{20} = \frac{\quad}{25} = \frac{\quad}{30}$

The Prep quiz questions should remind you that fractions must have the same denominator if we want to add or subtract the numerators.



The pattern blocks show:

$\frac{1}{2} + \frac{1}{3}$

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

Before adding or subtracting fractions, we must express each fraction with the same denominator, called a common denominator. We always try to use the **lowest common denominator** (**LCD**).

e.g. $\frac{4}{5} = \frac{12}{15}$ $\frac{2}{3} = \frac{10}{15}$ $\therefore \frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15}$

Any rational number can be represented by an infinite number of equivalent fractions. For example, consider $\frac{3}{10}$ and $\frac{5}{8}$:

$\frac{3}{10} = \frac{6}{20} = \frac{9}{30} = \frac{12}{40} = \frac{15}{50} = \ldots$ $\frac{5}{8} = \frac{10}{16} = \frac{15}{24} = \frac{20}{32} = \frac{25}{40} = \ldots$

To add $\frac{3}{10}$ and $\frac{5}{8}$, we look for a common denominator in each row.

WORKED EXAMPLES

1 $\frac{3}{10} + \frac{5}{8}$

10 and 8 could each be changed to 40 or to 80. The LCD is 40:

$\frac{3}{10} + \frac{5}{8} = \frac{3}{10} \times 4 + \frac{5}{8} \times 5$

$= \frac{12}{40} + \frac{25}{40}$

$= \frac{37}{40}$

2 $\frac{3}{4} - \frac{2}{5}$

4 and 5 both go into 20 exactly. 20 is the LCD.

$\frac{3}{4} - \frac{2}{5} = \frac{3}{4} \times 5 - \frac{2}{5} \times 4$

$= \frac{15}{20} - \frac{8}{20}$

$= \frac{7}{20}$

Exercise 9:04

P Foundation worksheet 9:04
Addition and subtraction of fractions

1 a $\frac{7}{10}+\frac{2}{10}$ b $\frac{1}{6}+\frac{4}{6}$ c $\frac{5}{8}+\frac{2}{8}$

d $\frac{3}{20}+\frac{8}{20}$ e $\frac{2}{10}+\frac{5}{10}$ f $\frac{1}{12}+\frac{4}{12}$

g $\frac{5}{15}+\frac{2}{15}$ h $\frac{17}{30}+\frac{2}{30}$ i $\frac{40}{100}+\frac{17}{100}$

2 a $\frac{9}{10}-\frac{2}{10}$ b $\frac{11}{15}-\frac{4}{15}$ c $\frac{7}{8}-\frac{6}{8}$

d $\frac{29}{30}-\frac{6}{30}$ e $\frac{3}{4}-\frac{2}{4}$ f $\frac{14}{20}-\frac{7}{20}$

g $\frac{20}{20}-\frac{11}{20}$ h $\frac{100}{100}-\frac{47}{100}$ i $\frac{10}{10}-\frac{7}{10}$

3 Write the simplest answer for each.

a $\frac{3}{10}+\frac{1}{10}$ b $\frac{9}{15}-\frac{4}{15}$ c $\frac{3}{8}+\frac{1}{8}$

d $\frac{20}{20}-\frac{14}{20}$ e $\frac{7}{8}+\frac{1}{8}$ f $\frac{11}{12}-\frac{8}{12}$

g $\frac{3}{5}+\frac{3}{5}$ h $\frac{7}{10}+\frac{3}{10}$ i $\frac{3}{8}-\frac{3}{8}$

j $\frac{11}{100}-\frac{6}{100}$ k $\frac{9}{15}-\frac{9}{15}$ l $\frac{14}{15}+\frac{16}{15}$

4 Simplify:

a $\frac{7}{10}+\frac{1}{5}$ b $\frac{7}{10}+\frac{3}{5}$ c $\frac{1}{10}+\frac{4}{5}$

d $\frac{1}{2}+\frac{1}{10}$ e $\frac{1}{2}+\frac{3}{10}$ f $\frac{1}{2}+\frac{7}{10}$

g $\frac{3}{4}+\frac{1}{3}$ h $\frac{1}{4}+\frac{2}{3}$ i $\frac{3}{4}+\frac{2}{3}$

j $\frac{1}{3}+\frac{1}{2}$ k $\frac{2}{3}+\frac{1}{2}$ l $\frac{4}{3}+\frac{1}{2}$

m $\frac{3}{5}+\frac{1}{4}$ n $\frac{2}{5}+\frac{3}{4}$ o $\frac{4}{5}+\frac{3}{4}$

p $\frac{3}{4}+\frac{7}{10}$ q $\frac{2}{3}+\frac{3}{5}$ r $\frac{3}{4}+\frac{1}{2}$

s $\frac{3}{5}+\frac{1}{2}$ t $\frac{1}{3}+\frac{3}{10}$ u $\frac{3}{10}+\frac{1}{2}$

5 Simplify:

a $\frac{9}{10}-\frac{1}{2}$ b $\frac{7}{10}-\frac{1}{2}$

c $\frac{8}{10}-\frac{1}{2}$ d $\frac{3}{8}-\frac{1}{5}$

e $\frac{5}{8}-\frac{2}{5}$ f $\frac{7}{8}-\frac{4}{5}$

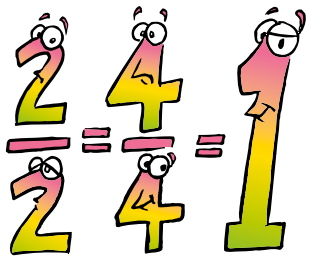
g $\frac{3}{5}-\frac{3}{10}$ h $\frac{3}{4}-\frac{2}{3}$ i $\frac{1}{2}-\frac{1}{3}$

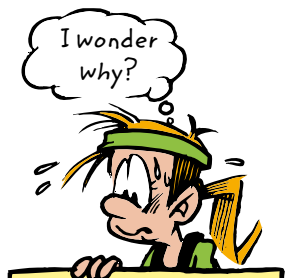
j $\frac{3}{4}-\frac{1}{5}$ k $\frac{3}{4}-\frac{7}{10}$ l $\frac{1}{4}-\frac{1}{5}$

m $\frac{1}{4}-\frac{1}{10}$ n $\frac{2}{3}-\frac{2}{5}$ o $\frac{3}{5}-\frac{1}{2}$

p $\frac{2}{3}-\frac{3}{10}$ q $\frac{9}{10}-\frac{1}{2}$ r $\frac{97}{100}-\frac{3}{4}$

s $\frac{7}{8}-\frac{2}{3}$ t $\frac{9}{20}-\frac{7}{10}$ u $\frac{79}{100}-\frac{3}{5}$



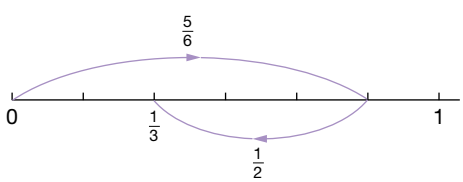


Third, fifth, sixth etc. in a race are spelt the same way as the fractions one third, one fifth, one sixth etc.

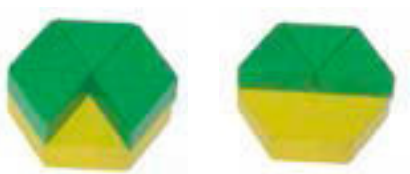
Discuss: Why are these wrong?

A $\frac{1}{2}+\frac{1}{2}=\frac{2}{4}$

B $\frac{3}{5}-\frac{2}{4}=\frac{3-2}{5-4}=\frac{1}{1}$



$\frac{5}{6}-\frac{1}{2}=\frac{1}{3}$



6 Simplify:

a $\frac{3}{4}+\frac{1}{6}$ **b** $\frac{7}{18}-\frac{1}{6}$ **c** $\frac{5}{12}+\frac{3}{8}$ **d** $\frac{1}{2}-\frac{7}{25}$ **e** $\frac{3}{25}+\frac{1}{4}$ **f** $\frac{5}{6}-\frac{3}{8}$

g $\frac{2}{3}-\frac{3}{8}$ **h** $\frac{3}{50}+\frac{11}{20}$ **i** $\frac{3}{4}-\frac{1}{12}$ **j** $\frac{3}{40}+\frac{2}{3}$ **k** $\frac{3}{4}-\frac{2}{7}$ **l** $\frac{4}{9}-\frac{1}{4}$



How many quarters in $3\frac{1}{4}$?

9:05 Addition and subtraction of mixed numbers

PREP QUIZ 9:05

Heather wrote the answers to multiplication tables along some ice-block sticks. She spaced the answers 1 cm apart. When she placed one stick under another she was surprised to see a whole list of equivalent fractions.

1-stick	1	2	3	4	5	6	7	8	9	10	
2-stick	2	4	6	8	10	12	14	16	18	20	
3-stick	3	6	9	12	15	18	21	24	27	30	
5-stick	5	10	15	20	25	30	35	40	45	50	

Use the sticks to complete these equivalent fractions:

1 $\frac{1}{3}=\frac{\quad}{24}$ **2** $\frac{1}{3}=\frac{\quad}{21}$ **3** $\frac{9}{27}=\frac{\quad}{30}$ **4** $\frac{4}{12}=\frac{\quad}{18}$

5 $\frac{2}{5}=\frac{\quad}{35}$ **6** $\frac{2}{5}=\frac{\quad}{50}$ **7** $\frac{8}{20}=\frac{\quad}{30}$ **8** $\frac{18}{45}=\frac{\quad}{25}$

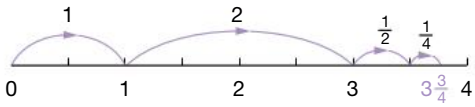
What is the LCD (lowest common denominator) of the following?

9 $\frac{1}{3}$ and $\frac{2}{5}$ **10** $\frac{3}{5}$ and $\frac{1}{2}$

Addition of mixed numbers

You can use a number line to model addition of mixed numbers:

$1\frac{1}{2} + 2\frac{1}{4} = (1+2) + \left(\frac{1}{2} + \frac{1}{4}\right)$



$\therefore 1\frac{1}{2} + 2\frac{1}{4} = 3\frac{3}{4}$



The average family has $1\frac{1}{2}$ girls and $1\frac{1}{4}$ boys.

WORKED EXAMPLE

$3\frac{2}{5} + 1\frac{3}{4} = (3+1) + \left(\frac{2}{5} + \frac{3}{4}\right)$

$= 4 + \left(\frac{2}{5} \times \frac{4}{4} + \frac{3}{4} \times \frac{5}{5}\right)$

$= 4 + \left(\frac{8}{20} + \frac{15}{20}\right)$

$= 4\frac{23}{20}$ (i.e. $4 + 1\frac{3}{20}$)

$= 5\frac{3}{20}$

$\frac{20}{20} = 1$

Subtraction of mixed numbers

WORKED EXAMPLES

1 $5 - 2\frac{3}{5} = (5-2) - \frac{3}{5}$

$= 3 - \frac{3}{5}$

$= 2 + \left(1 - \frac{3}{5}\right)$

$= 2\frac{2}{5}$

2 $6\frac{3}{4} - 2\frac{1}{10} = (6-2) + \left(\frac{3}{4} - \frac{1}{10}\right)$

$= 4 + \left(\frac{3}{4} \times \frac{5}{5} - \frac{1}{10} \times \frac{2}{2}\right)$

$= 4 + \left(\frac{15}{20} - \frac{2}{20}\right)$

$= 4\frac{13}{20}$

3 *Harder type:* Method 1

$13\frac{1}{5} - 1\frac{1}{2} = (13-1) + \left(\frac{1}{5} - \frac{1}{2}\right)$

$= 12 + \left(\frac{1}{5} - \frac{1}{2}\right)$

($\frac{1}{2}$ is bigger than $\frac{1}{5}$, bring back 1):

$= 11 + \left(1\frac{1}{5} - \frac{1}{2}\right)$

$= 11 + \left(\frac{6}{5} - \frac{1}{2}\right)$

$= 11 + \left(\frac{12}{10} - \frac{5}{10}\right)$

$= 11 + \frac{7}{10}$

$= 11\frac{7}{10}$

4 *Harder type:* Method 2

$13\frac{1}{5} - 1\frac{1}{2} = (13-1) + \left(\frac{1}{5} - \frac{1}{2}\right)$

$= 12 + \left(\frac{1}{5} - \frac{1}{2}\right)$

($\frac{1}{2}$ is bigger than $\frac{1}{5}$, use directed numbers):

$= 12 + \left(\frac{2}{10} - \frac{5}{10}\right)$

$= 12 + \left(-\frac{3}{10}\right)$

$= 12 - \frac{3}{10}$

$= 11\frac{7}{10}$

$1 - \frac{3}{10} = \frac{7}{10}$

Exercise 9:05

P Foundation worksheet 9:05
Addition and subtraction of mixed numbers

1 a $1+\frac{3}{4}$ b $3+\frac{1}{5}$ c $4\frac{1}{10}+3$ d $7\frac{1}{2}+6$

e $2+3\frac{1}{3}$ f $8+1\frac{7}{8}$ g $6+4\frac{1}{4}$

2 a $3\frac{3}{10}+2\frac{4}{10}$ b $6\frac{3}{8}+\frac{4}{8}$ c $1\frac{2}{4}+1\frac{1}{4}$

d $1\frac{3}{5}+4\frac{2}{5}$ e $6\frac{1}{4}+3\frac{3}{4}$ f $2\frac{7}{8}+1\frac{1}{8}$

g $4\frac{1}{4}+4\frac{1}{4}$ h $9\frac{3}{10}+4\frac{5}{10}$ i $1\frac{1}{8}+6\frac{3}{8}$

3 a $1\frac{3}{4}+\frac{1}{2}$ b $3\frac{3}{10}+\frac{8}{10}$ c $2\frac{7}{8}+\frac{5}{8}$

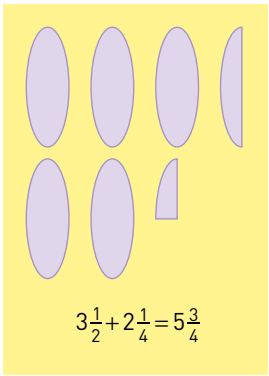
d $2\frac{3}{10}+1\frac{1}{2}$ e $6\frac{7}{10}+7\frac{1}{5}$ f $8\frac{3}{4}+2\frac{1}{2}$

g $4\frac{3}{8}+2\frac{1}{3}$ h $6\frac{1}{5}+1\frac{1}{4}$ i $2\frac{5}{8}+1\frac{1}{5}$

j $1\frac{7}{8}+2\frac{3}{5}$ k $6\frac{1}{2}+5\frac{2}{3}$ l $4\frac{7}{10}+1\frac{2}{3}$

m $6\frac{2}{3}+2\frac{4}{5}$ n $3\frac{3}{20}+2\frac{3}{4}$ o $10\frac{7}{50}+2\frac{19}{100}$

p $4\frac{1}{6}+3\frac{2}{5}$ q $6\frac{3}{4}+1\frac{5}{6}$ r $4\frac{8}{15}+2\frac{5}{8}$



4 a $1-\frac{1}{10}$ b $1-\frac{7}{10}$ c $1-\frac{3}{4}$

d $1-\frac{5}{8}$ e $1-\frac{9}{30}$ f $1-\frac{17}{20}$

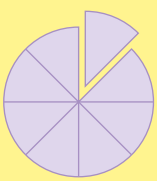
g $1-\frac{93}{100}$ h $1-\frac{3}{12}$ i $1-\frac{4}{15}$

$1-\frac{17}{20}=\frac{1}{1}-\frac{17}{20}$
$=\frac{20}{20}-\frac{17}{20}$
$=\frac{3}{20}$

5 a $2-\frac{1}{10}$ b $5-\frac{3}{10}$ c $7-\frac{3}{8}$

d $4-\frac{7}{12}$ e $9-\frac{1}{30}$ f $3-\frac{2}{15}$

g $10-\frac{21}{30}$ h $1-\frac{5}{24}$ i $1-\frac{31}{100}$



6 a $8\frac{3}{4}-6$ b $14\frac{1}{2}-9$ c $8\frac{2}{5}-3$

d $6\frac{7}{10}-2\frac{6}{10}$ e $7\frac{9}{12}-1\frac{4}{12}$ f $4\frac{80}{100}-3\frac{9}{100}$

g $5\frac{7}{8}-5\frac{4}{8}$ h $9\frac{3}{10}-5\frac{3}{10}$ i $2\frac{17}{20}-2\frac{7}{20}$

7 a $6\frac{8}{10}-2\frac{1}{2}$ b $4\frac{7}{8}-1\frac{1}{4}$ c $5\frac{9}{10}-2\frac{3}{5}$

d $5\frac{2}{3}-3\frac{1}{4}$ e $10\frac{7}{10}-1\frac{2}{3}$ f $7\frac{3}{8}-5\frac{1}{10}$

g $2\frac{11}{20}-2\frac{2}{5}$ h $12\frac{11}{16}-12\frac{3}{8}$ i $9\frac{2}{3}-9\frac{1}{2}$

8 a $4\frac{1}{2}-\frac{7}{8}$ b $6\frac{3}{5}-1\frac{9}{10}$ c $7\frac{1}{4}-2\frac{7}{12}$

d $7\frac{1}{2}-2\frac{2}{3}$ e $12\frac{3}{10}-4\frac{2}{3}$ f $9\frac{3}{8}-2\frac{4}{5}$

g $6\frac{1}{100}-3\frac{1}{8}$ h $2\frac{7}{100}-1\frac{3}{40}$ i $15\frac{7}{15}-8\frac{11}{12}$



CHALLENGE 9:05 QUIZZICAL QUESTIONS!

1 I have 10 black socks, 8 white socks and 6 grey socks in a drawer. If I reach into the drawer and pull out one sock at a time at random, how many socks must I pull out before I will definitely have a matching pair?



2 A frog is at the bottom of a 12 m well. Each hour the frog climbs up 4 metres but then slips back 3 metres. How long does it take the frog to reach the top and jump out?



3 If the number of apples in a box is counted by twos, threes or fives, there is always one left over. What is the smallest number of apples that must be in the box?



9:06 Multiplication of fractions

PREP QUIZ 9:06

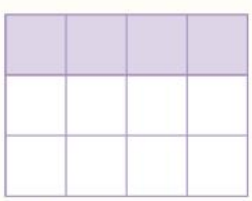
1 7 lots of $\frac{3}{8}$ **2** $4 \times \frac{7}{10}$ **3** $\frac{1}{3}$ of 6 fifths **4** $\frac{1}{2}$ of 4 fifths

Change these mixed numbers to improper fractions.

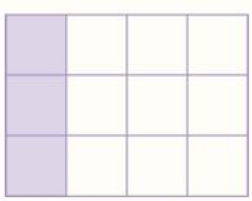
5 $2\frac{1}{4}$ **6** $3\frac{2}{3}$ **7** $1\frac{4}{5}$

What fraction of each rectangle has been shaded?

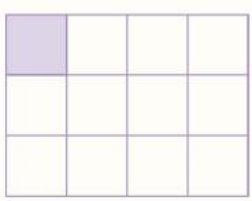
8



9



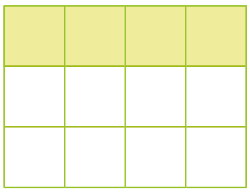
10



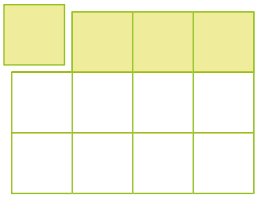
Discussion: Multiplying a fraction by a fraction



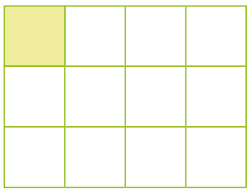
1



$\frac{1}{3}$ has been shaded.



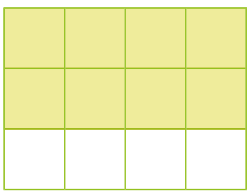
$\frac{1}{4}$ of the shaded part has been cut out.



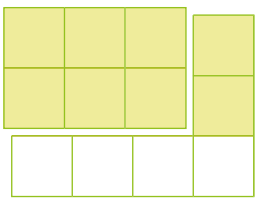
($\frac{1}{4}$ of $\frac{1}{3}$) or ($\frac{1}{4} \times \frac{1}{3}$) is $\frac{1}{12}$.

Conclusion: $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

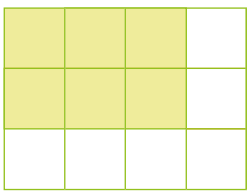
2



$\frac{2}{3}$ has been shaded.



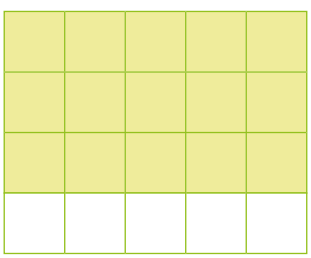
$\frac{3}{4}$ of the shaded part has been cut out.



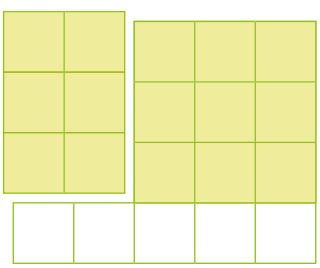
($\frac{3}{4}$ of $\frac{2}{3}$) or ($\frac{3}{4} \times \frac{2}{3}$) is $\frac{6}{12}$.

Conclusion: $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

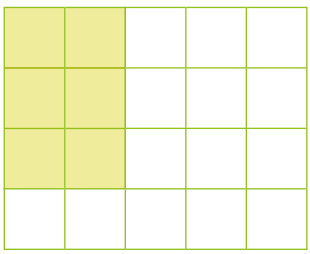
3



$\frac{3}{4}$ has been shaded



$\frac{2}{5}$ of the shaded part has been cut out



($\frac{2}{5}$ of $\frac{3}{4}$) or ($\frac{2}{5} \times \frac{3}{4}$) is $\frac{6}{20}$.



Conclusion: $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$

Note: $\frac{\cancel{6}^{3}}{\cancel{20}_{10}} = \frac{3}{10}$

In the discussion above we have discovered how to multiply a fraction by a fraction.

To multiply two fractions, multiply the numerators and multiply the denominators.

$$Answer = \frac{\text{numerator 1} \times \text{numerator 2}}{\text{denominator 1} \times \text{denominator 2}}$$

WORKED EXAMPLES

1 $\frac{1}{4} \times \frac{1}{3} = \frac{1 \times 1}{4 \times 3} = \frac{1}{12}$

2 $\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12}$ or $\frac{1}{2}$

3 $\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20}$ or $\frac{3}{10}$

Simplifying the working when multiplying fractions

If there is a number that will divide exactly into both a numerator and a denominator, this division should be done first. It is usually easier to do this before multiplying the fractions, rather than at the end when reducing the answer.

WORKED EXAMPLES

1 $\frac{7}{8} \times \frac{9}{10}$

$= \frac{7 \times 9}{8 \times 10}$

$= \frac{63}{80}$

2 $\frac{3}{10} \times \frac{7}{10}$

$= \frac{3 \times 7}{10 \times 10}$

$= \frac{21}{100}$

(In these two examples there is no number that will exactly divide a numerator and a denominator.)

3 $\frac{3}{5} \times \frac{2}{3}$

$= \frac{\cancel{3}^{1}}{5} \times \frac{2}{\cancel{3}_{1}}$

$= \frac{1 \times 2}{5 \times 1}$

$= \frac{2}{5}$

4 $\frac{9}{10} \times \frac{15}{16}$

$= \frac{9}{\cancel{10}_{2}} \times \frac{\cancel{15}^{3}}{16}$

$= \frac{9 \times 3}{2 \times 16}$

$= \frac{27}{32}$

5 $4 \times \frac{3}{5}$

$= \frac{4}{1} \times \frac{3}{5}$

$= \frac{12}{5}$

$= 2\frac{2}{5}$

6 $1\frac{7}{8} \times 3$

$= \frac{15}{8} \times \frac{3}{1}$

$= \frac{45}{8}$

$= 5\frac{5}{8}$

7 $3\frac{3}{4} \times 1\frac{1}{3}$

$= \frac{\cancel{15}^{5}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}}$

$= \frac{5}{1}$

$= 5$

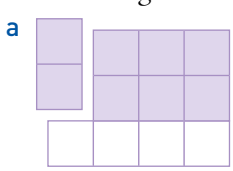
To multiply mixed numerals, write them as improper fractions.

Exercise 9:06

P Foundation worksheet 9:06 Multiplication of fractions numbers

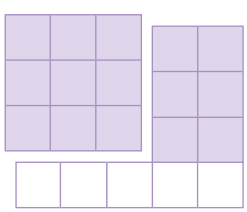
1 Use the diagrams to find the products.

a



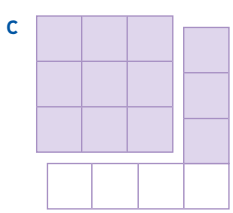
$\frac{1}{4} \times \frac{2}{3} =$

b



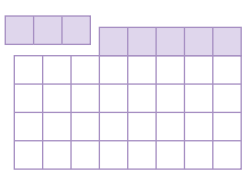
$\frac{3}{5} \times \frac{3}{4} =$

c

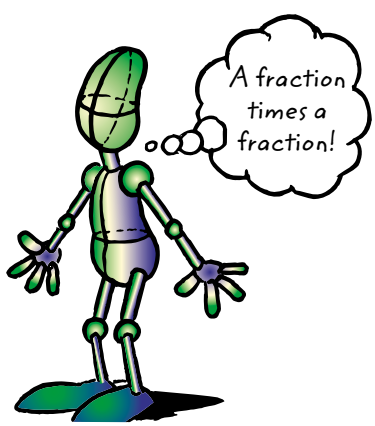


$\frac{3}{4} \times \frac{3}{4} =$

d



$\frac{3}{8} \times \frac{1}{5} =$



2 a $\frac{1}{10} \times \frac{1}{10}$ b $\frac{3}{10} \times \frac{7}{10}$ c $\frac{7}{10} \times \frac{1}{10}$

d $\frac{7}{10} \times \frac{9}{10}$ e $\frac{1}{10} \times \frac{3}{10}$ f $\frac{9}{10} \times \frac{3}{10}$

g $\frac{11}{100} \times \frac{7}{10}$ h $\frac{3}{100} \times \frac{1}{10}$ i $\frac{9}{10} \times \frac{7}{100}$

3 a $\frac{1}{2} \times \frac{1}{2}$ b $\frac{1}{4} \times \frac{1}{2}$ c $\frac{1}{2} \times \frac{1}{3}$

d $\frac{1}{2} \times \frac{3}{10}$ e $\frac{1}{4} \times \frac{9}{10}$ f $\frac{1}{4} \times \frac{3}{4}$

g $\frac{2}{5} \times \frac{3}{10}$ h $\frac{7}{8} \times \frac{5}{6}$ i $\frac{9}{10} \times \frac{7}{8}$

4 Give answers as basic fractions.

a $\frac{1}{2}$ of $\frac{4}{5}$ b $\frac{2}{3}$ of $\frac{3}{10}$ c $\frac{3}{5}$ of $\frac{5}{8}$

d $\frac{1}{2} \times \frac{2}{5}$ e $\frac{2}{3} \times \frac{3}{8}$ f $\frac{3}{5} \times \frac{1}{6}$

g $\frac{5}{6} \times \frac{3}{4}$ h $\frac{1}{2} \times \frac{2}{3}$ i $\frac{6}{15} \times \frac{7}{8}$

j $\frac{3}{4} \times \frac{4}{5}$ k $\frac{3}{4} \times \frac{9}{10}$ l $\frac{2}{3} \times \frac{1}{4}$

m $\frac{15}{20} \times \frac{3}{4}$ n $\frac{27}{100} \times \frac{5}{6}$ o $\frac{18}{25} \times \frac{3}{4}$

p $\frac{3}{10} \times \frac{15}{16}$ q $\frac{2}{3} \times \frac{3}{4}$ r $\frac{2}{3} \times \frac{9}{16}$

5 a $\frac{3}{4} \times \frac{4}{3}$ b $\frac{2}{3} \times \frac{3}{2}$ c $\frac{5}{8} \times \frac{8}{5}$

d $\frac{3}{10} \times \frac{10}{3}$ e $\frac{1}{3} \times \frac{3}{1}$ f $\frac{6}{5} \times \frac{5}{6}$

g $\frac{8}{3} \times \frac{3}{8}$ h $\frac{4}{5} \times \frac{5}{4}$ i $\frac{1}{10} \times \frac{10}{1}$

6 Give the simplest answer for each.

a $1\frac{1}{2} \times \frac{3}{4}$ b $2\frac{1}{2} \times \frac{2}{3}$ c $1\frac{2}{3} \times \frac{3}{4}$

d $\frac{5}{8} \times 2\frac{3}{5}$ e $\frac{7}{10} \times 1\frac{1}{3}$ f $\frac{3}{4} \times 2\frac{4}{5}$

g $\frac{7}{8} \times 5$ h $\frac{3}{5} \times 7$ i $\frac{7}{10} \times 4$

j $1\frac{3}{4} \times 1\frac{1}{2}$ k $2\frac{1}{4} \times 1\frac{4}{5}$ l $1\frac{2}{3} \times 2\frac{1}{2}$

m $3\frac{1}{5} \times 5\frac{3}{4}$ n $2\frac{2}{5} \times 1\frac{3}{4}$ o $1\frac{9}{10} \times 1\frac{3}{5}$

p $3 \times 2\frac{2}{3}$ q $8 \times 1\frac{4}{5}$ r $6 \times 2\frac{3}{8}$

s $\frac{5}{8} \times 1\frac{3}{5}$ t $1\frac{1}{3} \times \frac{3}{4}$ u $\frac{3}{10} \times 3\frac{1}{3}$

$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$ OR $\frac{3}{10} \times \frac{3}{10} = 0{\cdot}3 \times 0{\cdot}3$

$= 0{\cdot}09$

$\therefore 0{\cdot}3 \times 0{\cdot}3 = 0{\cdot}09$

The rules for fractions and decimals agree.

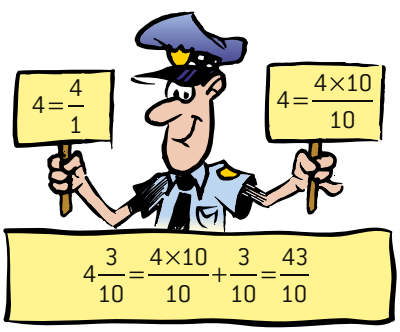


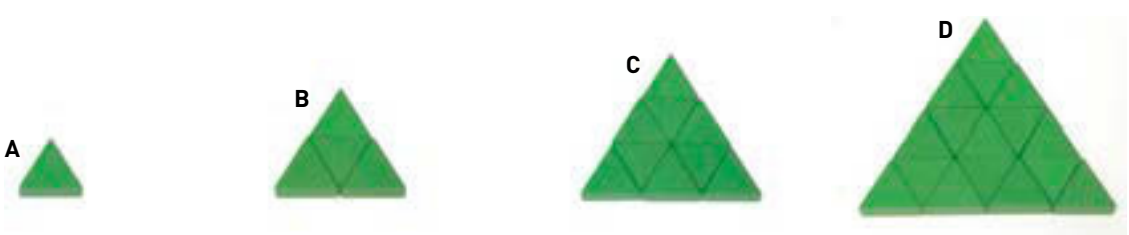
$\frac{5}{6} \times \frac{3}{4} = \frac{\cancel{15}^{5}}{\cancel{24}_{8}} = \frac{5}{8}$ OR $\frac{5}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{4} = \frac{5}{8}$

You can reduce fractions before or after you multiply.

If two numbers multiply to give 1, they are called **reciprocals** of each other.

Discuss: What happens when you multiply by a number less than 1?





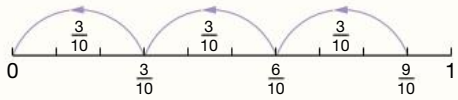
What fraction is:

- A of B
- A of C
- A of D
- B of C
- B of D
- C of D?

9:07 Division involving fractions

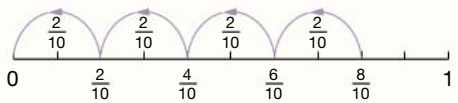
PREP QUIZ 9:07

1



How many lots of $\frac{3}{10}$ in $\frac{9}{10}$? (i.e. $\frac{9}{10} \div \frac{3}{10} = ?$)

2



How many lots of $\frac{2}{10}$ in $\frac{8}{10}$? (i.e. $\frac{8}{10} \div \frac{2}{10} = ?$)



How many $\frac{1}{5}$ are in $\frac{4}{5}$?

3 If $11 \times 4 = 44$ then $44 \div 4 = \ldots$

4 If $5 \times \frac{3}{4} = \frac{15}{4}$ then $\frac{15}{4} \div \frac{3}{4} = \ldots$

5 If $3 \times \frac{4}{5} = \frac{12}{5}$ then $\frac{12}{5} \div \frac{4}{5} = \ldots$

6 Is $\frac{40}{2}$ equal to 20?

7 $\frac{40}{2} \div \frac{10}{2}$ (or $20 \div 5$)

8 $\frac{60}{3} \div \frac{6}{3}$ (or $20 \div 2$)

9 $\frac{27}{100} \div \frac{3}{100}$

10 $\frac{99}{100} \div \frac{9}{100}$

WORKED EXAMPLES

Method 1

1 $\frac{9}{10} \div \frac{1}{2} = \frac{9}{10} \div \frac{1}{2}$ $\times 5$ $\times 5$

Make each denominator the same.

How many groups of $\frac{5}{10}$ can we get from $\frac{9}{10}$?

$\frac{9}{10} \div \frac{5}{10} = 9 \div 5$

$= \frac{9}{5}$

$= 1\frac{4}{5}$

Method 2

2 $\frac{9}{10} \div \frac{1}{2} = \frac{\frac{9}{10} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}}$

Make the denominator equal to 1.

$\frac{9}{10} \div \frac{1}{2} = \frac{\frac{9}{10} \times \frac{2}{1}}{1}$

$= \frac{9}{\cancel{10}_5} \times \frac{\cancel{2}^1}{1}$

$= \frac{9}{5}$

$= 1\frac{4}{5}$

$25 \div 5 = \frac{25}{5}$

so $\frac{9}{10} \div \frac{1}{2} = \frac{\frac{9}{10}}{\frac{1}{2}}$

When fractions have the same denominator, you can divide the numerators to get the answer.

e.g. $\frac{3}{4} \div \frac{5}{8} = \frac{6}{8} \div \frac{5}{8} = 6 \div 5$

To divide by a fraction, invert the fraction and then multiply (i.e. multiply by the reciprocal).

e.g. $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2}$

Invert means 'turn upside down'.

Exercise 9:07

P Foundation worksheet 9:07 Division involving fractions

1 a $\frac{4}{5} \div \frac{2}{5}$ b $\frac{8}{10} \div \frac{4}{10}$ c $\frac{3}{100} \div \frac{11}{100}$

d $\frac{9}{10} \div \frac{5}{10}$ e $\frac{7}{8} \div \frac{3}{8}$ f $\frac{4}{5} \div \frac{1}{5}$

g $\frac{1}{5} \div \frac{4}{5}$ h $\frac{3}{8} \div \frac{5}{8}$ i $\frac{7}{10} \div \frac{8}{10}$

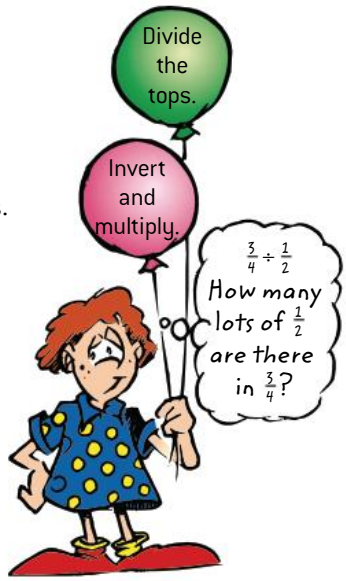
2 Use Method 2 from page 276 to complete these number sentences.

a $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \square$ b $\frac{5}{8} \div \frac{1}{3} = \frac{5}{8} \times \square$

c $\frac{7}{10} \div \frac{1}{4} = \frac{7}{10} \times \square$ d $\frac{2}{5} \div \frac{5}{8} = \frac{2}{5} \times \square$

e $\frac{3}{10} \div \frac{7}{8} = \frac{3}{10} \times \square$ f $\frac{1}{3} \div \frac{1}{10} = \frac{1}{3} \times \square$

g $\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \times \square$ h $\frac{1}{5} \div \frac{4}{5} = \frac{1}{5} \times \square$



3 Use either method from page 276 to find the basic fraction.

a $\frac{3}{4} \div \frac{1}{2}$ b $\frac{5}{8} \div \frac{1}{3}$ c $\frac{7}{10} \div \frac{1}{4}$ d $\frac{2}{5} \div \frac{5}{8}$

e $\frac{3}{10} \div \frac{7}{8}$ f $\frac{1}{3} \div \frac{1}{10}$ g $\frac{3}{8} \div \frac{2}{3}$ h $\frac{1}{5} \div \frac{4}{5}$

i $\frac{7}{100} \div \frac{49}{100}$ j $\frac{5}{8} \div \frac{1}{2}$ k $\frac{3}{4} \div \frac{2}{3}$ l $\frac{7}{10} \div \frac{2}{3}$ m $\frac{1}{2} \div \frac{3}{5}$

n $\frac{1}{2} \div \frac{1}{10}$ o $\frac{1}{2} \div \frac{3}{100}$ p $\frac{49}{100} \div \frac{7}{8}$ q $\frac{15}{16} \div \frac{3}{10}$ r $\frac{18}{25} \div \frac{3}{5}$

4 Simplify by first changing mixed numbers to improper fractions.

a $5 \div \frac{1}{4}$ b $2 \div \frac{5}{8}$ c $1 \div \frac{2}{3}$

d $1 \div \frac{4}{10}$ e $3 \div \frac{4}{5}$ f $2 \div \frac{3}{8}$

g $6 \div \frac{2}{3}$ h $5 \div \frac{5}{8}$ i $7 \div \frac{3}{4}$

j $1\frac{3}{8} \div \frac{3}{4}$ k $2\frac{1}{2} \div \frac{1}{3}$ l $4\frac{4}{5} \div \frac{3}{4}$

m $3\frac{1}{2} \div 6$ n $5\frac{3}{8} \div 4$ o $6\frac{3}{4} \div 5$

p $1\frac{3}{4} \div 1\frac{1}{2}$ q $3\frac{1}{3} \div 1\frac{1}{4}$ r $6\frac{3}{5} \div 4\frac{1}{10}$

s $3\frac{3}{8} \div 1\frac{1}{4}$ t $10\frac{1}{2} \div 1\frac{3}{8}$ u $7\frac{1}{4} \div 3\frac{4}{5}$

WORKED EXAMPLES

1 $4 \div \frac{3}{5}$

$= \frac{4}{1} \div \frac{3}{5}$

$= \frac{4}{1} \times \frac{5}{3}$

$= \frac{20}{3}$

$= 6\frac{2}{3}$

2 $1\frac{1}{2} \div 3\frac{3}{4}$

$= \frac{3}{2} \div \frac{15}{4}$

$= \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{\cancel{15}_{5}}$

$= \frac{2}{5}$

5 a When you divide by a number less than 1, is the original number increased or decreased?

b When you divide by a number greater than 1, is the original number increased or decreased?

9:08 Fractions of quantities

PREP QUIZ 9:08

Write as mixed numerals: **1** $\frac{11}{4}$ **2** $\frac{22}{7}$

Write as improper fractions: **3** $3\frac{1}{2}$ **4** $7\frac{3}{10}$

Complete: **5** $\frac{3}{4} = \frac{}{24}$ **6** $\frac{9}{10} = \frac{}{100}$

Simplify: **7** $\frac{30}{80}$ **8** $\frac{72}{108}$ **9** $\frac{9}{10} + \frac{3}{10}$ **10** $1 - \frac{7}{10}$

WORKED EXAMPLES

1 Find $\frac{3}{5}$ of 2 km.

2 What fraction of 2 m is 40 cm?

Solutions

1 $\frac{1}{5}$ of 2 km $= \frac{1}{5}$ of 2000 m
$= 2000 \div 5$ m
$= 400$ m

$\therefore \frac{3}{5}$ of 2 km $= 3 \times 400$ m
$= 1200$ m or 1·2 km

2 Fraction $= \frac{40\text{ cm}}{200\text{ cm}}$
$= \frac{4}{20}$
$= \frac{1}{5}$

Exercise 9:08

P Foundation worksheet 9:08
Fractions of quantities

1 Find:

a $\frac{1}{5}$ of 1 m b $\frac{1}{4}$ of 1 h (1 hour) c $\frac{1}{8}$ of 1 kg

d $\frac{1}{10}$ of 3 m e $\frac{1}{3}$ of 90° f $\frac{1}{6}$ of 2 h

g $\frac{1}{100}$ of 200 g h $\frac{1}{100}$ of 1 kg i $\frac{1}{100}$ of 2 L



2 Use your answers to Question 1 to find:

a $\frac{4}{5}$ of 1 m b $\frac{3}{4}$ of 1 h (1 hour) c $\frac{5}{8}$ of 1 kg

d $\frac{9}{10}$ of 3 m e $\frac{2}{3}$ of 90° f $\frac{5}{6}$ of 2 h

g $\frac{13}{100}$ of 200 g h $\frac{21}{100}$ of 1 kg i $\frac{99}{100}$ of 2 L

3 What fraction is the first quantity of the second quantity?

a 50c of $1 b 20 g of 40 g c 30 m of 100 m

d 400 of 500 e 3 g of 12 g f 4 m of 100 m

g $25 of $100 h $30 of $70 i $4 of $200

4 Find:

a $\frac{3}{4}$ of 6000 L b $\frac{5}{8}$ of $48 000 c $\frac{3}{10}$ of $1 600 000

d $\frac{3}{5}$ of 3 kg e $\frac{9}{10}$ of 7 kg f $\frac{7}{8}$ of 9 km

g $\frac{2}{3}$ of $6.30 h $\frac{3}{4}$ of 7·56 kg i $\frac{3}{10}$ of 8·6 m

5 What fraction is the first quantity of the second quantity?

a 60 cm of 2 m b 200 mL of 1 L c 50 g of 3 kg

d 1c of $10 e 8 min of 3 h f 90° of 1 revolution

g 6·6 m of 13·2 m h 13·6 cm of 4 m i 14·7 cm of 4·9 m

9:09 Using a calculator

Fraction calculations in this chapter can also be done using a calculator. Most calculators have a 'fraction' key to enter fractions. Fraction keys are labelled differently on different calculators, but the most common labels are the following:



The fraction key allows you to enter a fraction or a mixed numeral as part of a calculation on your calculator. Work through the exercise below to see how this can be done.

WORKED EXAMPLES

Note: Different types of calculator may use different keys. Check the instruction booklet for your calculator. The workings below are for a Casio fx-82AU PLUS calculator.

Some calculators can be preset to give answers as mixed numerals or improper fractions.

1 Calculate $\frac{5}{6} + \frac{3}{4}$

Enter: 5 $\square\!\frac{\square}{\square}$ 6 ▶ + 3 $\square\!\frac{\square}{\square}$ 4 =

Answer: $1\frac{7}{12}$

2 Calculate $2\frac{2}{3} \div 3\frac{1}{5}$

Enter: SHIFT $\square\!\frac{\square}{\square}$ 2 ▶ 2 ▼ 3 ▶ ÷ SHIFT $\square\!\frac{\square}{\square}$ 3 ▶ 1 ▼ 5 =

Answer: $\frac{5}{6}$

3 Convert $3\frac{2}{5}$ to an improper fraction.

Enter: SHIFT $\square\!\frac{\square}{\square}$ 3 ▶ 2 ▼ 5 = then SHIFT $a\frac{b}{c} \Leftrightarrow \frac{d}{c}$

Answer: $\frac{17}{5}$

Note: Pressing SHIFT $a\frac{b}{c} \Leftrightarrow \frac{d}{c}$ again will convert the improper fraction back to a mixed numeral.

Exercise 9:09

1 Enter each fraction into your calculator and press = to find its simplest form.

a $\frac{2}{4}$ b $\frac{10}{12}$ c $\frac{24}{30}$ d $\frac{14}{16}$ e $\frac{15}{25}$ f $\frac{60}{100}$ g $\frac{32}{128}$ h $\frac{150}{225}$

2 Change these improper fractions to mixed numerals. (Enter the fraction and press =.)

a $\frac{5}{4}$ b $\frac{7}{2}$ c $\frac{10}{3}$ d $\frac{12}{5}$ e $\frac{15}{4}$

f $\frac{9}{2}$ g $\frac{20}{8}$ h $\frac{24}{10}$ i $\frac{20}{4}$

Press SHIFT $a\frac{b}{c} \Leftrightarrow \frac{d}{c}$ if you need to.

3 Change these mixed numerals to improper fractions.

a $1\frac{1}{4}$ b $2\frac{3}{5}$ c $3\frac{2}{3}$ d $5\frac{5}{6}$ e $4\frac{7}{8}$ f $7\frac{1}{3}$ g $10\frac{3}{4}$ h $15\frac{2}{7}$

4 Complete these additions and subtractions, giving your answer as a proper fraction or mixed numeral in its simplest form.

a $\frac{1}{2}+\frac{1}{3}$ b $\frac{2}{5}+\frac{1}{4}$ c $\frac{3}{8}+\frac{2}{5}$ d $\frac{7}{8}-\frac{3}{8}$

e $\frac{5}{6}-\frac{2}{3}$ f $\frac{9}{10}-\frac{3}{5}$ g $\frac{3}{4}+\frac{1}{3}$ h $\frac{5}{8}+\frac{3}{4}$

i $\frac{4}{5}+\frac{2}{3}$ j $5\frac{1}{2}+\frac{2}{3}$ k $3\frac{1}{3}+2\frac{1}{5}$ l $7\frac{3}{4}+3\frac{7}{10}$

m $4\frac{2}{5}-2\frac{1}{10}$ n $6\frac{1}{3}-4\frac{1}{2}$ o $10\frac{3}{7}-8\frac{4}{5}$

5 Complete these subtractions with negative answers.

a $\frac{2}{5}-\frac{2}{3}$ b $\frac{1}{2}-\frac{7}{8}$ c $\frac{3}{7}-\frac{4}{5}$

d $\frac{7}{8}-1\frac{1}{4}$ e $2\frac{1}{3}-3\frac{1}{2}$ f $5\frac{3}{4}-7\frac{1}{5}$

6 Calculate these products.

a $\frac{1}{2}\times\frac{1}{5}$ b $\frac{1}{3}\times\frac{3}{5}$ c $\frac{2}{3}\times\frac{3}{4}$ d $\frac{9}{10}\times\frac{2}{3}$

e $\frac{5}{8}\times\frac{3}{10}$ f $\frac{5}{6}\times\frac{9}{10}$ g $\frac{3}{4}\times1\frac{1}{2}$ h $2\frac{2}{3}\times\frac{3}{4}$

i $1\frac{3}{5}\times2\frac{1}{4}$ j $3\frac{2}{5}\times2\frac{1}{4}$ k $5\frac{3}{5}\times3\frac{3}{10}$ l $7\frac{2}{9}\times3\frac{9}{10}$

7 Use your calculator to find the answer to these divisions.

a $\frac{1}{2}\div\frac{3}{4}$ b $\frac{5}{6}\div\frac{1}{3}$ c $\frac{9}{10}\div\frac{3}{5}$ d $\frac{7}{8}\div\frac{2}{5}$

e $\frac{1}{4}\div\frac{5}{8}$ f $\frac{3}{4}\div\frac{6}{7}$ g $2\frac{1}{4}\div\frac{1}{2}$ h $\frac{9}{10}\div1\frac{1}{5}$

i $3\frac{1}{2}\div1\frac{1}{4}$ j $5\frac{7}{10}\div3\frac{1}{3}$ k $4\frac{1}{4}\div6\frac{1}{3}$ l $8\frac{5}{6}\div5\frac{3}{10}$

8 Use your calculator to find each fraction.

a $\frac{3}{4}$ of 60 L b $\frac{2}{5}$ of 40 kg c $\frac{1}{6}$ of 2 h d $\frac{2}{3}$ of 120°

e $\frac{7}{8}$ of 2·4 m f $\frac{5}{6}$ of \$1500 g $\frac{7}{10}$ of 450 g h $\frac{17}{20}$ of \$24 000

INVESTIGATION 9:09 POPULATION OF AUSTRALIA

This table shows the population figures for each Australian state in 2012, to the nearest 100 000 people.

Use your calculator to find the fraction of Australia's population living in each state.

State	Population (× 100 000)	Fraction
NSW	72	
Vic	56	
Qld	45	
SA	16	$\frac{1}{14}$
WA	24	
Tas	5	
NT	2	
ACT	4	
Australia	224	



What do the fractions add to?

Are the following statements true or false?

1 More than $\frac{1}{3}$ of Australia's population lives in New South Wales (NSW).

2 More than $\frac{3}{4}$ of Australia's population lives in the eastern states of NSW, Queensland (Qld) and Victoria (Vic).

3 Less than $\frac{1}{100}$ of Australia's population lives in the Northern Territory (NT).

4 Less than $\frac{1}{10}$ of Australia's population lives in the largest state, Western Australia (WA).

9:10 Applications of fractions

Exercise 9:10

1 If Naomi shares 350 g of birdseed equally among her seven budgerigars, how much does she put in the cage containing two budgerigars?

2 If 18 football players are each given $\frac{1}{4}$ of an orange at half-time, how many oranges are needed? How many oranges are needed if each player instead receives $\frac{3}{4}$ of an orange?

3 On Tuesday night 312 students attended the square dance. This is exactly $\frac{1}{3}$ of my school.

a How many students belong to my school?

b How many students did not attend the dance?

c If each 'square' consists of 8 students, how many squares could be formed for dancing?



4 Our school committee has 60 members. Two-thirds of the members must be present before any decisions can be made. Yesterday only $\frac{3}{5}$ of the members were present.

- **a** How many members attended the meeting yesterday?
- **b** How many members are needed before decisions can be made?
- **c** How many more members were needed yesterday for a decision to be made?

5 It takes me $\frac{3}{4}$ of an hour to write one page of a letter. How long would it take me to write a letter seven pages long?

6 For 20 hours of one day Tom stayed awake to build a canoe. He was actually at work on the canoe for $\frac{3}{4}$ of this time.

- **a** For how many hours did Tom work on the canoe?
- **b** For how many hours did Tom sleep during this day?

7 In a terrible battle, the Roman soldiers lost three of every eight soldiers in their army. If the army had 10 000 soldiers before the battle, how many soldiers were lost?

8 Ravi enrolled in a speed-reading course. He was given a two-minute speed test at the beginning and end of the course. In the first test he read $2\frac{3}{4}$ pages of the test book. In the second test he read $5\frac{1}{2}$ pages of the same book.

- **a** How much of the book did he read altogether?
- **b** What was his average speed in the first test, in pages per minute?
- **c** What was his average speed in the second test, in pages per minute?

9 In Africa, 3 out of every 10 people attend church. If 2000 people in Africa were chosen at random, how many from this group would you expect to be church attenders?

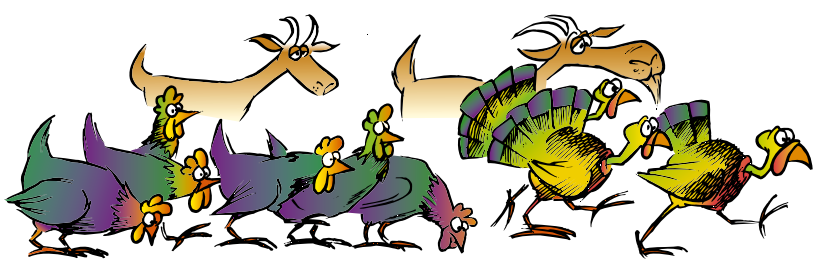
10 Miriam's petrol tank was half full. She then used $\frac{1}{3}$ of a tank of petrol travelling to Dubbo and back. What fraction of the petrol remained?

11 Ten full drums of oil were stored in a warehouse. Three-eighths of one drum was lost. If the value of the oil that was lost was $90, what is the value of the remaining oil?

12 Miriam's petrol tank was half full when she started out, but she used $\frac{1}{4}$ of this petrol travelling to Tamworth and back. What fraction of the whole tank did she use, and what fraction remained after the trip?

13 Taxation takes $\frac{1}{3}$ of Joel's salary. He banks $\frac{1}{10}$ of what is left. If he banks $40 every week, how much does he earn each week before tax is paid?

14 It takes $\frac{3}{8}$ of a bottle of milk to fill a large glass. How many of these glasses can be filled with 40 bottles of milk?



15 James left the farm gate open, so $\frac{3}{4}$ of the turkeys, $\frac{3}{4}$ of the chickens and $\frac{1}{2}$ of the goats got out. What fraction of all the turkeys and chickens together got out?

16 Tom practises his violin for exactly the same length of time each day. After five days he had practised for $8\frac{3}{4}$ hours altogether. How much did he practise each day?

17 The Hudsons buy three pizzas to share between eight people. If each person is given the same amount, how much does each receive?

18 Ms Daley had $\frac{1}{3}$ of a pie. She gave each of her two sons half of this. What fraction of the pie was each son given?

FUN SPOT 9:10 — WHERE DID THE KING KEEP HIS ARMIES?

Find the answer for each letter. Match each letter with its answer below to solve the riddle.

L $\frac{1}{4}+\frac{1}{2}$	**V** $\frac{5}{8}+\frac{1}{4}$	**H** $\frac{5}{8}-\frac{1}{4}$	**S** $\frac{5}{6}-\frac{2}{3}$	**I** $1\frac{2}{3}-\frac{5}{6}$
U $\frac{1}{4}\times\frac{1}{2}$	**S** $\frac{3}{5}\times\frac{1}{6}$	**E** $1\frac{1}{2}\times\frac{2}{5}$	**E** $\frac{3}{8}\div\frac{3}{4}$	
P $\frac{5}{6}\div\frac{2}{3}$	**I** $1\frac{1}{4}\div3\frac{1}{2}$	**E** $\left(\frac{3}{4}\right)^2$	**S** $\left(2\frac{1}{3}\right)^2$	



$\frac{1}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{14}$	$\frac{1}{6}$			
$5\frac{4}{9}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{9}{16}$	$\frac{1}{10}$

9:11 Ratios

PREP QUIZ 9:11

Simplify these fractions.

1 $\frac{2}{4}$ **2** $\frac{6}{8}$ **3** $\frac{4}{20}$ **4** $\frac{12}{36}$ **5** $\frac{70}{100}$

I have 8 footy cards. How many does my sister have, if she has:

6 twice as many **7** five times as many?

In a pet shop there are 3 dogs for every 2 cats.

8 If there are 9 dogs, how many cats are there?

9 If there are 10 cats, how many dogs are there?

10 What fraction of these pets are dogs?

A **ratio** is a comparison of like quantities.

If there are 18 girls and 12 boys in a class of 30 students, then the ratio of boys to girls is 12 to 18. Dividing by the highest common factor of 6 gives the ratio of boys to girls as 2 to 3. This means that for every 2 boys there are 3 girls in the class.

Ratios like this are written as 2:3, or sometimes in fraction form as $\frac{2}{3}$.

Adding the two numbers in a ratio represents the 'whole' amount.



WORKED EXAMPLES

1 If there are 5 blue marbles and 15 red marbles in a jar, what is the:
- **a** ratio of blue marbles to red marbles
- **b** ratio of red marbles to blue marbles
- **c** fraction of blue marbles in the jar?

A ratio is written as $a:b$ or $\frac{a}{b}$
It is read as 'a to b'.

2 **a** If $\frac{2}{5}$ of the girls in a school are blonde, what fraction are not blonde?
- **b** What is the ratio of blonde to not blonde?
- **c** If there are 250 girls in the school, how many are blonde?

3 **a** If 1 cm on a map represents 10 m, what is the scale for the map?
- **b** What real length is represented on the map by:
  - **i** 3 cm
  - **ii** 5·6 cm?

A **scale** is a ratio.
1:10 000 means that a scaled length of 1 cm represents 10 000 cm (or 100 m) in real life.

Solutions

1 **a** Ratio of blue to red is 5:15.
Dividing by the HCF of 5 gives the simplest ratio as 1:3.

Entering 5 : 15 as $\frac{5}{15}$ in your calculator will simplify to $\frac{1}{3}$, i.e. 5 : 15 = 1 : 3

- **b** The order of numbers in a ratio is important. Reversing the terms means the ratio of red to blue = 3:1.
- **c** The number of marbles in the jar is 20, so the fraction that are blue is $\frac{5}{20}$, which simplifies to $\frac{1}{4}$.
Alternatively:
The ratio of blue to red = 1:3. From this ratio we can see that for every 4 (= 1 + 3) marbles, 1 is blue, so we can see from the ratio that the fraction of blue marbles is $\frac{1}{4}$ (and the fraction of red marbles is $\frac{3}{4}$).



2 **a** The fraction that are not blonde is $1 - \frac{2}{5}$, i.e. $\frac{3}{5}$.

b If $\frac{2}{5}$ are blonde and $\frac{3}{5}$ are not blonde then the ratio of blonde to not blonde is $\frac{2}{5} : \frac{3}{5}$. Multiplying by the common denominator of 5 gives the simplest ratio as 2:3.

c Number of blondes $= \frac{2}{5}$ of 250
$= 100$

3 **a** For a ratio or scale, the units must be the same for both terms.
For the scale 1 cm:10 m, we change the metres to centimetres,
i.e. 1 cm:10 m = 1 cm:(100 × 10) cm
= 1 cm:1000 cm
Now the units are the same, we can say the scale is 1:1000.

b **i** The scale is 1:1000, so 3 cm represents 3 × 1000 cm on the map:
i.e. 3 cm represents 3000 cm, or 30 m.

ii Similarly, 5·6 cm represents 5·6 × 1000 = 5600 cm, or 56 m.

Exercise 9:11



1 There are 3 white marbles and 12 black marbles in a bag. Find:

a the ratio of white marbles to black marbles (in simplest form)

b the simplest ratio of black marbles to white marbles

c the total number of marbles in the bag

d the fraction of marbles that are white

e the fraction of marbles that are black.



2 The families in Pound Road own 8 dogs and 12 cats.

a Divide by the HCF to find the simplest ratio of dogs to cats.

b What is the ratio of cats to dogs?

c What fraction of these pets are dogs?

d What fraction are cats?

3 Allyson has $16 and Paul has $24.

a What is the ratio of Allyson's amount to Paul's amount?

b What is the ratio of Paul's amount to Allyson's amount?

c What is the total amount of money they have?

d What fraction of the whole amount is Allyson's?

e What fraction of the whole amount is Paul's?

Always give your ratio in its simplest form.



4 At a picnic there are 10 adults and 20 children.

a What fraction of the people at the picnic are:

i adults ii children?

b What is the ratio of:

i adults to children ii children to adults?



5 In a bowl of fruit, $\frac{1}{4}$ are oranges and the rest are apples.

a What fraction are apples?

b What is the ratio of oranges to apples?

c If there are 16 pieces of fruit in the bowl, how many are:

i oranges ii apples?

6 The ratio of a son's weight to his father's weight is 3 : 7. Their combined weight is 120 kg.

a What fraction of the combined weight is the weight of the:

i son ii father?

b What is the weight of the:

i son ii father?

Hint: Total = 3 + 7



7 The ratio of teachers to students in a school is 1 : 14. Everyone at the school is either a teacher or a student.

a What fraction of the school's population are:

i teachers ii students?

b If there are 10 teachers, how many students are there?

8 1 mm on a map represents 1 metre.

a What is the scale (ratio) for this map?

b What real length is represented on the map by:

i 8 mm ii 25 mm iii 4 cm?

Remember: 1 m = 1000 mm

9 The scale on a map is 1 : 100 000.

a What real distance is represented on the map by a length of:

i 1 cm ii 5 cm iii 7·3 cm?

b What length on the map would represent a real distance of:

i 300 000 cm ii 4 km iii 10·5 km?



10 In a bag of 72 jelly beans, 16 are black.

a What fraction of jelly beans are:

i black ii not black?

b What is the ratio of black to not black jelly beans?

c How many not black jelly beans need to be added for the ratio to be 1 : 5?

d If 12 black jelly beans are added to the bag, what is the new ratio?



9:12 Best buys

PREP QUIZ 9:12

Find the cost of one apple if:

1 5 apples cost $2 **2** 8 apples cost $2.40 **3** 10 apples cost $5.

4 Which example above has the cheapest apples?

If 3 DVDs cost $24, what is the cost of:

5 1 DVD **6** 12 DVDs **7** 20 DVDs?

What is the cost per pair of socks if I buy:

8 3 pairs for $12 **9** 5 pairs for $15?

10 Which example above is the cheapest price per pair of socks?

When we know the cost of a certain amount of a product, we usually find the cost of 1 unit so we can compare prices. This is called the **unitary method**.



If I know that a bag of 10 oranges costs $4 and another bag of 12 oranges costs $6, then I can calculate that the cost of each orange in the bag of 10 is 40 cents, while each orange from the bag of 12 costs 50 cents.

This means that the bag of 10 oranges is the better buy. (We assume that all the oranges are the same.)

WORKED EXAMPLES



1 8 lollies cost $5.60. Find the cost of 10 lollies.

2 A packet of 12 biscuits costs $3.60 and a packet of 20 costs $5.00.
What is the price per biscuit for each packet?
Which is the better buy?



3 A 130 mL can of baked beans costs 55 cents.
A 420 mL can costs $1.60. Which is the better buy?

Solutions

1 Using the unitary method, we find the cost of 1 lolly,
i.e. $5.60 ÷ 8 = 70 cents
∴ cost of 10 lollies is 70c × 10 = $7.00.

2 To find the 'unit' price of one biscuit in each packet, we divide by the number of biscuits in each packet.
For the packet of 12, unit cost = $3.60 ÷ 12
= 0.30c
For the packet of 20, unit cost = $5.00 ÷ 20
= 0.25c
∴ The packet of 20 is the better buy.

3 For this example we need to use a calculator.
We can divide the cost of the 130 mL can by 130 to find the cost of 1 mL (1 unit), then multiply by 420 to find the cost of 420 mL at this price.

Enter: 0.55 ÷ 130 × 420 =

The answer is $1.78 (to the nearest cent).
This is more expensive than the cost of the 420 mL can at $1.60, so the cost of the 420 mL can is the better buy.



Exercise 9:12

1 **a** If 6 cans of soft drink costs $4.20, how much would 10 cost?
b If the cost of 10 sweets is $3.50, how much for 3?
c If 4 ice-creams cost $10, what is the cost of 7?
d 20 packs of footy cards cost $30. What is the cost of 8 packs?
e 10 kg of potatoes costs $4.50. What is the cost of 25 kg?

2 Supplier *A* will sell me 5 tonnes of soil for $60.
Supplier *B* will sell me 8 tonnes for $100.
a Find the cost per tonne for:
i supplier *A* **ii** supplier *B*.
b Which supplier sells soil at the cheaper rate?



3 I can buy 5 metres of timber for $18 or 8 metres for $28.
a Find the cost of 1 metre for each price.
b Which is the better price per metre?

4 A 300 g can of a product costs $6.60 while a 500 g can costs $10.50.
a Find the cost of 100 g for each price.
b Which size can is sold at the cheaper rate?

5 **a** 500 g of an item costs $12. Use the unitary method to find the equivalent cost of 320 g.
b 100 m costs $180. Find the cost of 225 m.
c 5 kg costs $8.40. Find the cost of 24 kg.

Remember:
Divide by 500, then multiply by 320.

6 For each example, which is the better buy?
a 500 g for $12 or 750 g for $17
b 100 g for $3.50 or 250 g for $10
c 250 g for $8 or 400 g for $12
d 50 g for $1.60 or 150 g for $4.50
e 5 m for $12 or 8 m for $20
f 100 m for $170 or 160 m for $280
g 30 cm for $6 or 50 cm for $9
h 200 g for $6.40 or 500 g for $15.50
i 5 t for $360 or 7 t for $400
j 150 g for $9 or 500 g for $31
k 4.2 kg for $8 or 6 kg for $11.50
l 400 g for $18 or 50 g for $2.20

MATHS TERMS 9

fraction (or common fraction)



- part of a whole,
  e.g. $\frac{1}{4}$ of a pie
- a number between 0 and 1 formed by dividing a number by a bigger number,
  e.g. $\frac{7}{8}$ or $7 \div 8$

denominator

- the bottom number of a fraction,
  e.g. 8 in $\frac{7}{8}$

numerator

- the top number of a fraction,
  e.g. 7 in $\frac{7}{8}$

equivalent fractions

- fractions of equal size,
  e.g. $\frac{1}{2} = \frac{5}{10} = \frac{7}{14} = \ldots$

improper fraction

- a fraction with a numerator that is bigger than the denominator,
  e.g. $\frac{9}{8}$

common denominator

- a denominator that allows two or more fractions to be written with the same denominator,
  e.g. some common denominators for $\frac{3}{4}$ and $\frac{5}{6}$ are 12, 24, 36, …

lowest common denominator (LCD)

- the smallest possible common denominator for some fractions

mixed number

- a number that has a whole number part and a fraction part,
  e.g. $1\frac{2}{3}$

rational number

- a number that can be written as a common or improper fraction

ratio

- a comparison of like quantities or numbers written in a definite order

reciprocal

- the number formed by turning a fraction upside down,
  e.g. the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

  the reciprocal of 4 is $\frac{1}{4}$
- reciprocals multiply to give 1

scale

- the ratio of a drawing's length to the original length
- if a map has a scale of 1 : 10 000, then 1 cm on the map represents 10 000 cm (i.e. 100 m) in real terms

DIAGNOSTIC TEST 9 FRACTIONS

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1	a $\frac{3}{10} + \frac{5}{8}$	b $\frac{3}{4} + \frac{2}{5}$	c $\frac{1}{3} + \frac{1}{10}$	9:04
2	a $\frac{3}{4} - \frac{2}{5}$	b $\frac{9}{10} - \frac{1}{4}$	c $\frac{1}{4} - \frac{1}{5}$	9:04
3	a $3\frac{2}{5} + 1\frac{3}{4}$	b $6\frac{1}{5} + 4\frac{1}{2}$	c $1\frac{7}{8} + 2\frac{1}{4}$	9:05
4	a $5 - 2\frac{3}{5}$	b $6\frac{3}{4} - 2\frac{1}{10}$	c $13\frac{1}{5} - 1\frac{1}{2}$	9:05
5	a $\frac{7}{8} \times \frac{9}{10}$	b $\frac{3}{5} \times \frac{2}{3}$	c $\frac{9}{10} \times \frac{15}{16}$	9:06
6	a $4 \times \frac{3}{5}$	b $1\frac{7}{8} \times 3$	c $3\frac{3}{4} \times 1\frac{1}{3}$	9:06
7	a $\frac{9}{10} \div \frac{1}{2}$	b $\frac{3}{4} \div \frac{5}{8}$	c $\frac{3}{4} \div \frac{2}{5}$	9:07
8	a $\frac{3}{5}$ of 2 km	b $\frac{7}{10}$ of 3 L	c $\frac{2}{3}$ of 2 hours	9:08

9 a What fraction of 2 m is 40 cm? 9:08
b What fraction of 2 hours is 12 minutes?
c What fraction of 5 kilometres is 300 metres?

10 A class has 8 boys and 16 girls. 9:10
a What is the simplest ratio of boys to girls?
b What is the ratio of girls to boys?
c What fraction of the class are boys?
d What fraction of the class are girls?

11 The ratio of ewes to rams in a paddock is 9:1 and there are 60 sheep altogether. 9:10
a What fraction of the sheep are rams?
b What fraction are ewes?
c How many rams are in the paddock?



12 a If 5 chocolate bars cost $4, how much for 8? 9:12
b Which is the better buy: 10 kg for $18 or 12 kg for $24?
c Which is the better buy: 25 cans for $75 or 40 cans for $110?

ASSIGNMENT 9A Chapter review

1 Write the answer in simplest form.

a $\frac{3}{4} + \frac{5}{8}$ **b** $\frac{7}{10} - \frac{1}{2}$

c $1\frac{2}{3} + 2\frac{1}{4}$ **d** $5\frac{1}{4} - 3\frac{3}{8}$

2 Write the answer in simplest form.

a $\frac{3}{4} \times \frac{2}{5}$ **b** $\frac{3}{8} \times \frac{4}{9}$

c $2\frac{1}{2} \times 1\frac{1}{3}$ **d** $4\frac{1}{4} \times 3\frac{3}{5}$

3 **a** Knowing that $a^2 = a \times a$, find the value of:

i $\left(\frac{3}{4}\right)^2$ **ii** $\left(2\frac{1}{2}\right)^2$

b Hence, find the value of:

i $\sqrt{\frac{9}{16}}$ **ii** $\sqrt{6\frac{1}{4}}$

4 Find the answer to these divisions.

a $\frac{1}{2} \div \frac{1}{3}$ **b** $\frac{7}{10} \div \frac{4}{5}$

c $4\frac{1}{2} \div 1\frac{1}{6}$ **d** $5 \div 1\frac{1}{4}$

5 Find the value of:

a $\frac{3}{5}$ of 1 metre **b** $\frac{3}{8}$ of \$3.20

c $\frac{5}{6}$ of 1 day

6 Complete the following, remembering the 'order of operations' rules:

a $\frac{1}{2} + \frac{1}{3} \times \frac{1}{4}$ **b** $\frac{3}{8} \div \frac{3}{4} + \frac{2}{3}$

c $(2\frac{1}{3} - \frac{1}{4}) \div (\frac{3}{4} + \frac{2}{3})$

7 **a** A length of timber measures $3\frac{1}{2}$ metres. If $\frac{2}{7}$ of the length is cut off, what length remains?

b A 50-metre length of rope has pieces $3\frac{1}{2}$ metres long cut from it.

i How many pieces can be cut off?

ii What length is left over?

8 A firm employs 24 men and 16 women.

a What fraction of the employees are:

i men **ii** women?

b What is the ratio of men to women?

9 A car dealer sells Holden and Ford cars in the ratio 9:11.

a What fraction of the cars sold are:

i Holdens **ii** Fords?

b If 600 cars are sold altogether, how many are Holdens?

10 Determine which option is the best buy, if a cleaning product comes in these three sizes:

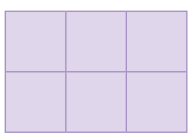
A 500 mL for \$12.40

B 1250 mL for \$33.50

C 2 L for \$48

ASSIGNMENT 9B Working mathematically

1 How many squares can you find in this rectangle? (Be careful!)



2 When staying at a motel, a person notices that the menu has a choice of 3 entrees, 4 main meals and 2 desserts. How many different three-course meals are possible?

3 A train 1 km long is travelling at 60 km/h. The train enters a tunnel 1 km long. How much time does it take before the train emerges completely from the tunnel?

4 A business makes plastic figures representing the digits 0 to 9. It receives an order from a school for a set of room numbers from 1 to 50. How many plastic figures of each digit are needed for this order?

5 A plumber, a carpenter and an electrician all work for a large building company. Their names are Bagnell, Jones and Farrant, but not necessarily in that order. It is known that the plumber who was an only child earns the least, while Bagnell, who married Jones' sister, earns more than the electrician. Give the names of the plumber, the carpenter and the electrician in that order.

ASSIGNMENT 9C Cumulative revision

1 Write as a numeral the number written in words. 1:01

a six thousand and thirty-five
b seven hundred thousand
c twelve thousand six hundred and twenty-five
d thirteen million four hundred thousand and eighty-nine

2 Simplify these algebraic expressions. 4:07

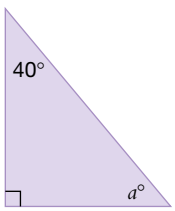
a $5a + 7a$ **b** $10x - x$ **c** $9m + 6m - 7m$
d $4a + 5b - 3a + b$ **e** $2(a + 3b) - 5b$ **f** $6m + 4 + 3(m - 2)$

3 Simplify: 6:03, 6:04

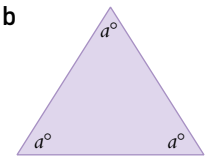
a $1{\cdot}5 \times 10$ **b** $1{\cdot}5 \times 100$ **c** $15{\cdot}6 \div 10$ **d** $15{\cdot}6 \div 100$

4 Find the value of a in each figure. 8:02, 8:04

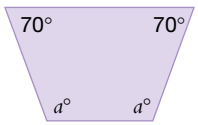
a



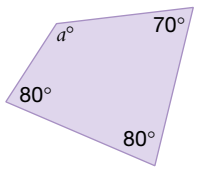
b



c



d



5 Find: 7:05–7:08

a $-4 + 8$ **b** $5 - -8$ **c** $-30 \div 6$ **d** $(-4)^2$

6 Copy and complete: Metric table, page xiv

a 4 cm = … mm **b** 3·5 km = … m
c 25 000 mm = … m **d** 420 m = … km

7 **a** Find all the factors of 18 and 24. 3:03
b What is the highest common factor (HCF) of 18 and 24?
c What is the lowest common multiple (LCM) of 18 and 24?

8 Find the value of 52·3647 correct to: 6:08

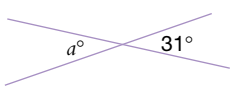
a the nearest whole number **b** one decimal place
c two decimal places **d** three decimal places.

9 Expand each expression by removing the grouping symbols. 4:09

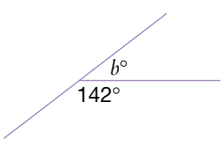
a $4(x + 3)$ **b** $7(a - 5)$ **c** $n(n + 8)$
d $x(x - 1)$ **e** $4(3y + 5)$ **f** $4k(2k - 3)$

10 Find the value of each pronumeral. 5:04

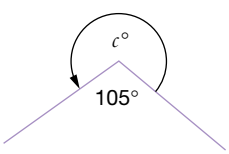
a



b

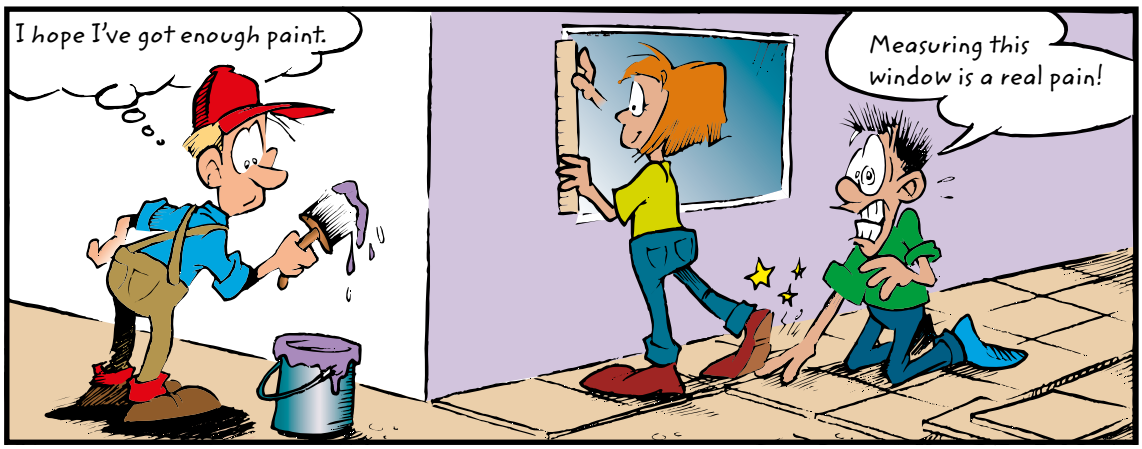


c



PERIMETER, AREA AND VOLUME

10



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10:02 The definition of area
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10:03 Area of a rectangle
GeoGebra activity 10:03 Area of a rectangle
Investigation 10:03 Area of a rectangle
10:04 Area of a triangle
GeoGebra activity 10:04 Area of a triangle
10:05 Area of a parallelogram
GeoGebra activity 10:05 Area of a parallelogram
10:06 Area problems
10:07 Volume of a rectangular prism
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Investigation 10:07 Stacking cubes
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Investigation 10:08 Estimating capacity and volume
Challenge 10:08 The fantastic Soma cube
Fun spot 10:08 What makes money after 8 pm?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Length***, ***Area***, ***Volume*** [Stage 4]

- Find perimeters of parallelograms, trapeziums, rhombuses and kites. (ACMMG196)
- Choose appropriate units of measurement for area and convert from one unit to another. (ACMMG195)
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving. (ACMMG159)
- Choose appropriate units of measurement for volume and convert from one unit to another. (ACMMG195)
- Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume. (ACMMG198)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

10:01 Perimeter

PREP QUIZ 10:01

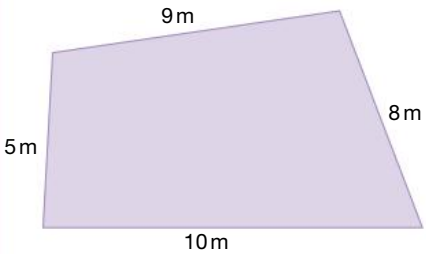
Add together the following measurements.

1 5·1 m + 6·9 m
2 1·5 m + 2·3 m + 4·6 m
3 1·2 m + 0·9 m + 5 m
4 1·6 m + 73 cm
5 80 cm + 60 mm
6 2·5 km + 1750 m

Find in metres:

7 4 × 2·5 m
8 3 × 35 cm
9 2 × 1·2 m + 2 × 0·8 m
10 2 × (1·2 m + 0·8 m)

WORKED EXAMPLES





The perimeter P of this figure is given by:

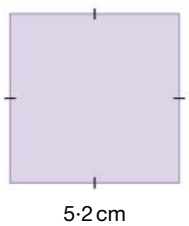
$P = (5 + 10 + 8 + 9)$ m
$= 32$ m

The *perimeter* of a figure is the sum of the lengths of the sides of the figure. It is the distance around the figure.

Sometimes the lengths of some sides are the same. This can be indicated by putting the *same mark* on equal sides.

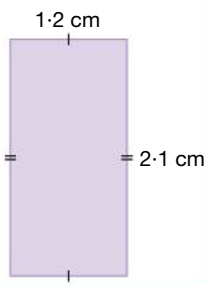
For a square, all 4 sides are equal in length, so the perimeter P is given by:

$P = 4 \times 5{\cdot}2$ cm
$= 20{\cdot}8$ cm



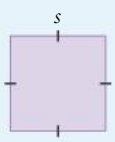
For a rectangle, two pairs of sides are equal in length, so the perimeter P is given by:

$P = (2 \times 2{\cdot}1 + 2 \times 1{\cdot}2)$ cm
$= (4{\cdot}2 + 2{\cdot}4)$ cm
$= 6{\cdot}6$ cm



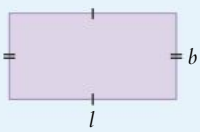
For a square, if s stands for the length of one side, then a formula for perimeter would be:

$P = 4 \times s$
(or simply $P = 4s$)



For a rectangle with a length of l and a breadth of b, the perimeter would be:

$P = 2 \times l + 2 \times b$
(or simply $P = 2l + 2b$)

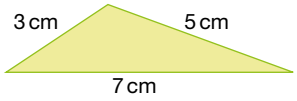


Exercise 10:01

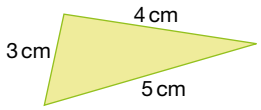
P Foundation worksheet 10:01 Perimeter

1 Calculate the perimeter of each figure.

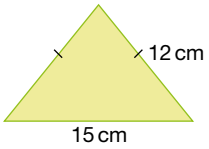
a



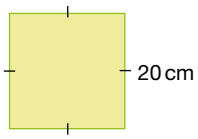
b



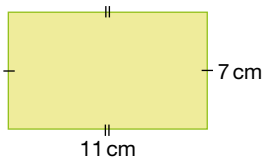
c



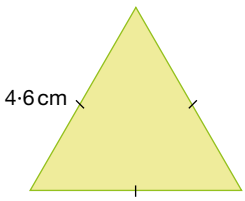
d



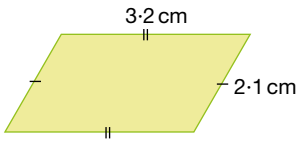
e



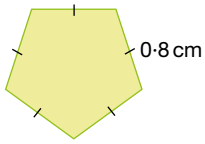
f



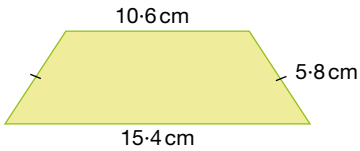
g



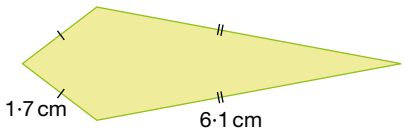
h



i



j



2 Find the perimeter of:

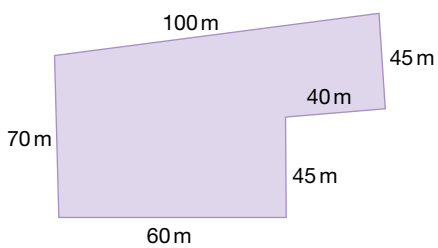
- **a** a square with sides of length 3·2 cm
- **b** a square with sides of length 56 mm
- **c** a rectangle with a length of 2·4 cm and a breadth of 1·6 cm
- **d** a rectangle with sides of length 5·2 cm and 3·6 cm
- **e** a parallelogram with sides of length 8·2 m and 4·8 m
- **f** a rhombus with sides of length 6·2 cm
- **g** a kite with sides of length 3 cm and 5 cm



Remember:
Squares and rhombuses have all sides equal. Rectangles, parallelograms and kites have 2 pairs of equal sides.

3 A field has the dimensions shown in the diagram.

- **a** What is the perimeter of this field?
- **b** How much would it cost to fence this field if the price of fencing is $75 for 10 metres?



4 A farmer has a rectangular paddock that is 350 m long and 190 m wide.

- **a** Find the perimeter of this paddock.
- **b** How much would it cost to fence the paddock at 85 cents per metre?

5 A jogger runs around a square park that has sides of length 145 m. How far will she jog if she runs around the park 10 times?

6

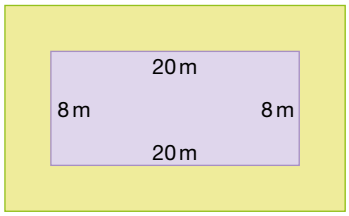
- **a** The perimeter of a square is 60 cm. What is its side length?
- **b** The perimeter of an equilateral triangle is 30 cm. What is the triangle's side length?

7

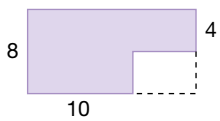
- **a** A rectangle has a perimeter of 80 cm and a length of 30 cm. What is its breadth (or width)?
- **b** The perimeter of a rectangle with a length of 20 m is 70 m. What is the breadth of the rectangle?

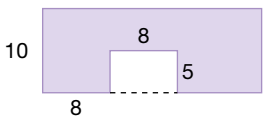
8 a The perimeter of a square is 14·8 cm. What is its side length?
b The perimeter of a rectangle with a length of 5·6 cm is 14·4 cm. What is the width (or breadth) of the rectangle?

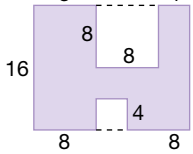
9 a A concrete path is constructed around a lawn as shown in the diagram. If the path is 1 m wide, what is the perimeter around the outside of the path?
b A similar 1 m wide path is placed around another rectangular lawn, which has a perimeter of 42 m. What would be the perimeter around the outside of this path?



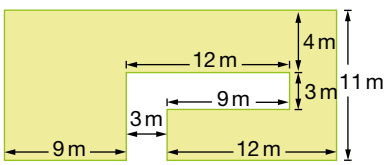
10 By determining missing lengths (or otherwise), find the perimeter of each figure. (All angles are right angles and the measurements are in centimetres.)

a


b


c


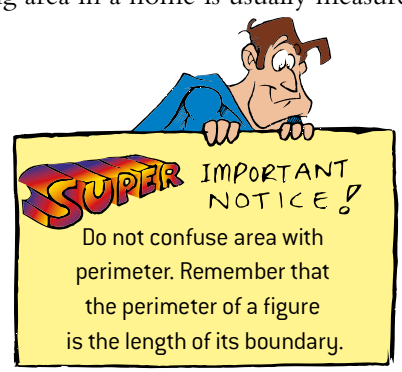
11 Find the perimeter of the building whose plan is shown at right.



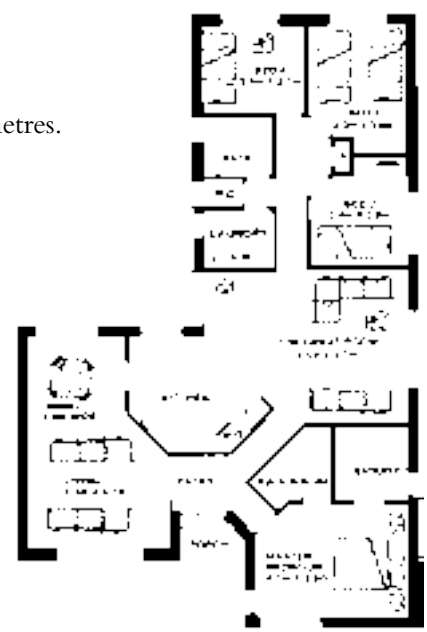
10:02 The definition of area

When we measure the *area* of a shape, we are measuring the amount of space inside that shape.

The living area in a home is usually measured in square metres.

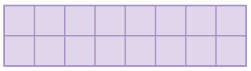


We measure area by dividing it into *square units* and counting how many square units there are inside the figure.



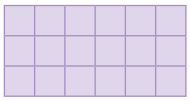
Floor plan courtesy of Pioneer Homes

A

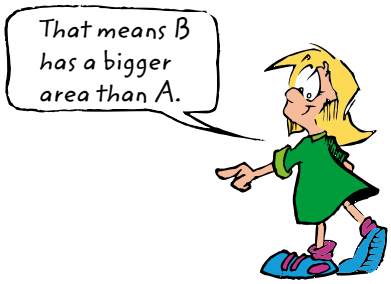


Number of squares = 16

B



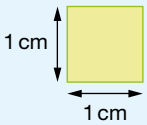
Number of squares = 18



We say that the area of figure A is 16 square units, and the area of figure B is 18 square units.

If we are going to be consistent, we obviously need a standard unit of area. A convenient unit is the square centimetre.

The area of a square with sides of 1 cm is 1 square centimetre.

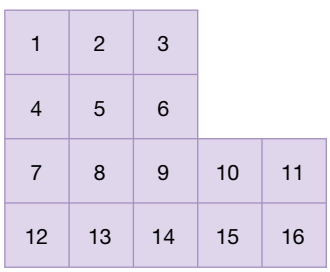


This can be written as:

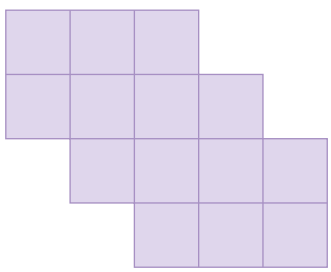
$1\ \text{cm}^2$

Smaller areas might be measured in square millimetres (mm^2). Larger areas could be measured in square metres (m^2), hectares (ha) or square kilometres (km^2).

The areas of the figures below would be:



Area = 16 units^2



Area = 14 units^2



Exercise 10:02

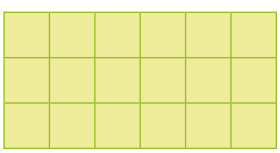
P Foundation worksheet 10:02 The definition of area

1 Find the area, in square units, of each figure.
(You can write 'square units' as 'units^2'.)

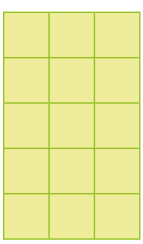
a



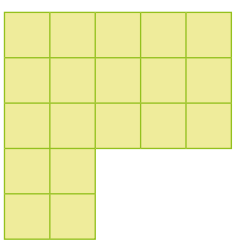
b



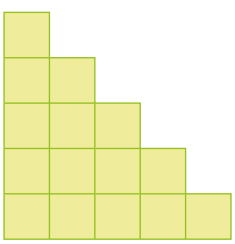
c



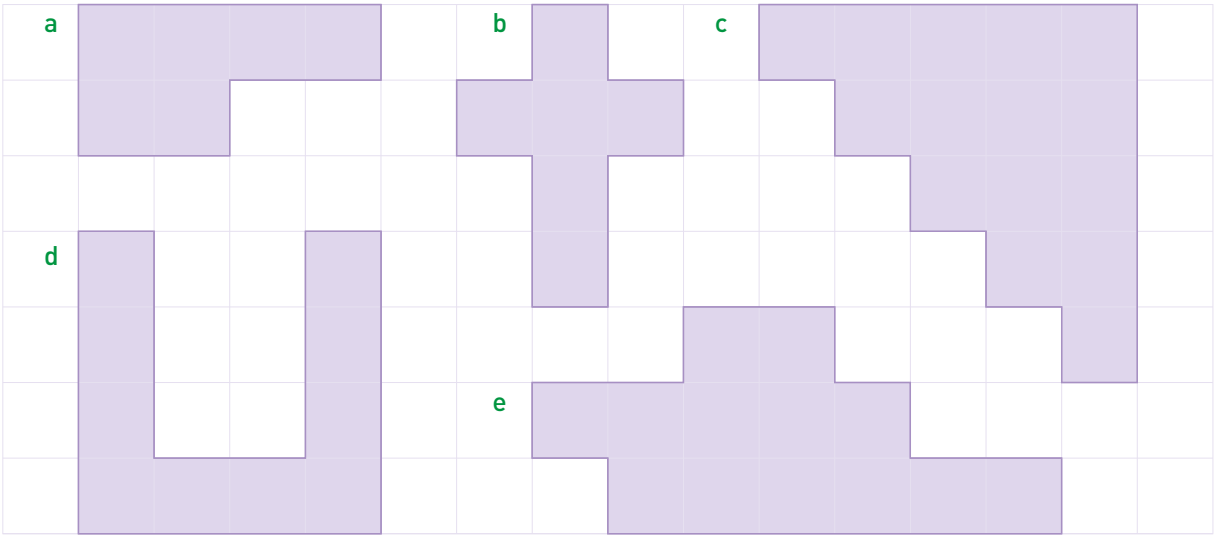
d



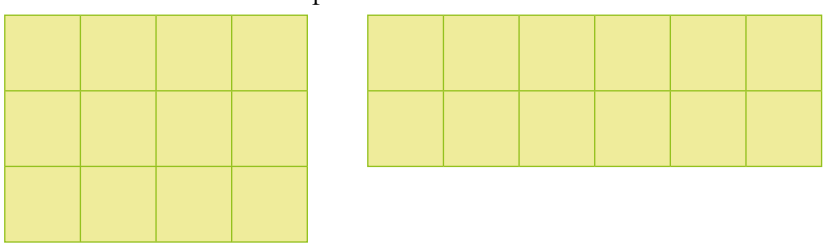
e



2 Each of these shapes is drawn on centimetre grid paper. Find the area of each shape in cm^2.



3 a What is the area of each shape?



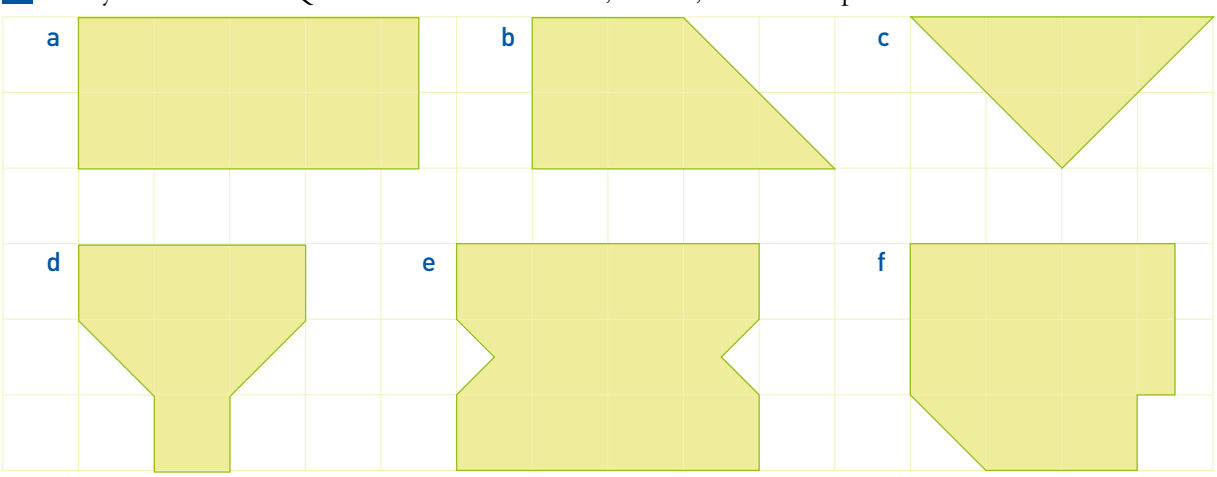
b What is the perimeter of each shape?

c Do you think shapes with the same area will always have the same perimeter?

4 Not all shapes are made up of whole unit squares. What fraction of a square centimetre would these shaded areas be?

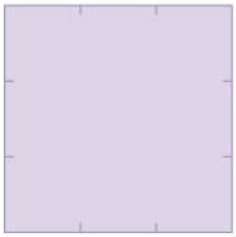


5 Use your answers to Question 4 to find the area, in cm^2, of these shapes.

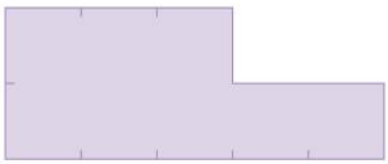


6 Each of the following shapes is made from 1 cm squares. The markings on each side are 1 cm apart. Find the area of each shape in square centimetres.

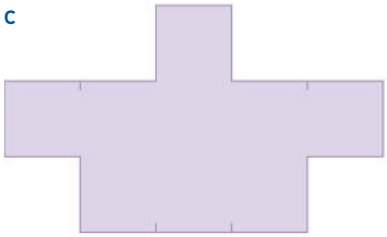
a



b



c

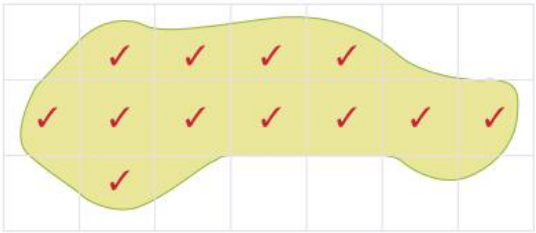


INVESTIGATION 10:02 FINDING AREA

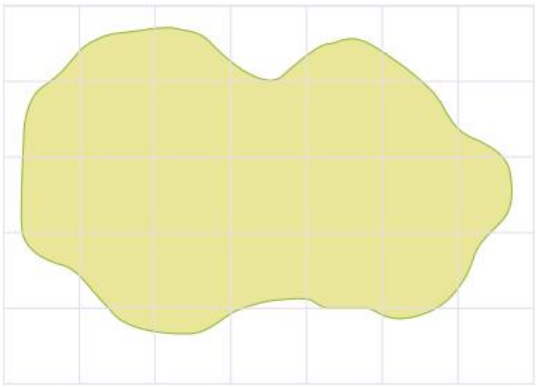
Discuss how you could find the area of a figure with curved sides.

1 An approximation for the area inside a curved boundary can be found by placing a grid over it and counting squares. If more than $\frac{1}{2}$ of a square is included in the figure, count it. If less than $\frac{1}{2}$ of a square is included in the figure, don't count it. Find the approximate areas of these shapes.

a



b





2 Try to estimate the areas of various shapes and then trace the shapes on grid sheets. Calculate the areas by counting squares.

3 Use the method above to find the area of your hand or your shoe.

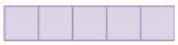
4 Tape or glue sheets of paper together to make a square with sides of 1 metre. The area of this square will be 1 m^2. Use this square metre to find the area of some large shapes.

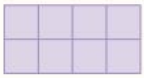
5 Draw several different rectangles on grid paper, each with a perimeter of 24 cm. Find the area of each. Do you notice any pattern?

10:03 Area of a rectangle

PREP QUIZ 10:03

Write the area and perimeter of each rectangle.







1 $A =$ … units^2
2 $P =$ … units

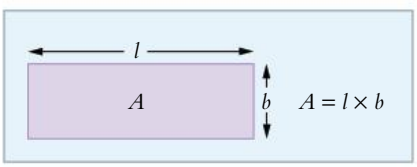
3 $A =$ … units^2
4 $P =$ … units

5 $A =$ … units^2
6 $P =$ … units

7 2·2 cm = … mm **8** 5·3 m = … cm **9** 400 mm = … m **10** 70 cm = … m

In the questions above you may have seen a quicker way to find a rectangle's area than counting the squares, if you didn't know it already.

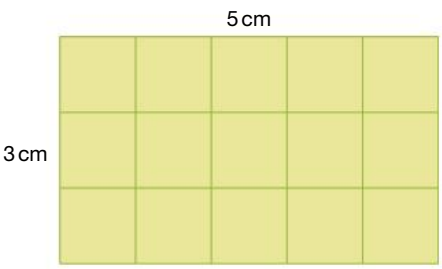
If we know the number of units of both the length and the breadth (or width) of the rectangle, we can simply use the following rule or formula:



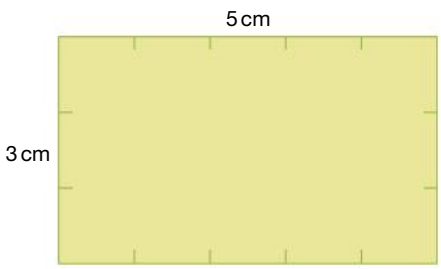


The *breadth* is also known as the *width*.

This can be seen from the following diagrams.



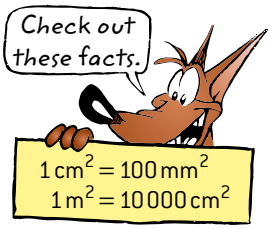
Number of squares = 15
∴ Area = 15 cm^2



$A = l \times b$
$= 5 \times 3$
$= 15$
∴ Area = 15 cm^2

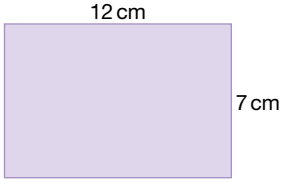
Important note!
To use this rule we must be careful. ***The length and breadth must be measured in the same units.*** If not, then the dimensions must be *converted* to the same units.

WORKED EXAMPLES



1 Calculate the areas of these rectangles.

a



Here the units are both cm, so:

$A = 12 \times 7$
$= 84$
$\therefore$ Area $= 84\,\text{cm}^2$

b



The units are different.

2·2 cm could be written as 22 mm.

Then $A = 22 \times 6$
$= 132$
$\therefore$ Area $= 132\,\text{mm}^2$

OR

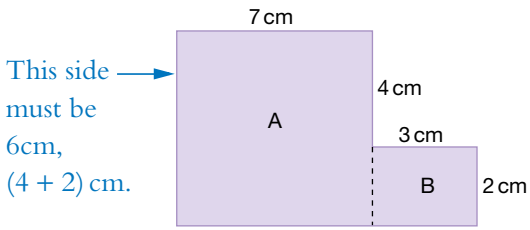
6 mm could be written as 0·6 cm.

Then $A = 2{\cdot}2 \times 0{\cdot}6$
$= 1{\cdot}32$
$\therefore$ Area $= 1{\cdot}32\,\text{cm}^2$

(*Note:* $132\,\text{mm}^2 = 1{\cdot}32\,\text{cm}^2$)

2 The areas of some shapes can often be found by dividing them into smaller rectangles. Before finding the area of each smaller rectangle, you may need to find some unknown lengths.

a



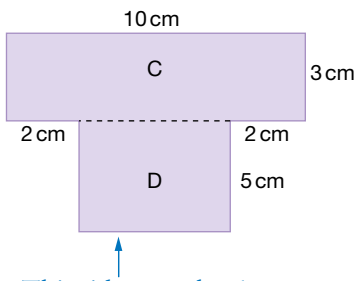
This side must be 6cm, (4 + 2) cm.

Area of A
$= (7 \times 6)\,\text{cm}^2$
$= 42\,\text{cm}^2$

Area of B
$= (3 \times 2)\,\text{cm}^2$
$= 6\,\text{cm}^2$

$\therefore$ Total area of figure
$= 42\,\text{cm}^2 + 6\,\text{cm}^2$
$= 48\,\text{cm}^2$

b



This side must be 6cm, [10 − (2 + 2)] cm.

Area of C
$= (10 \times 3)\,\text{cm}^2$
$= 30\,\text{cm}^2$

Area of D
$= (5 \times 6)\,\text{cm}^2$
$= 30\,\text{cm}^2$

$\therefore$ Total area of figure $= 30\,\text{cm}^2 + 30\,\text{cm}^2$
$= 60\,\text{cm}^2$

(*Note:* The figures are not drawn to scale.)

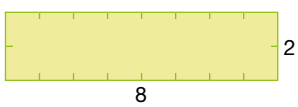
Exercise 10:03

P Foundation worksheet 10:03
Area of a rectangle

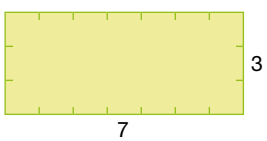
Challenge worksheet 10:03
Area of simple composite shapes involving rectangles

1 Find the area of each rectangle in square units.

a

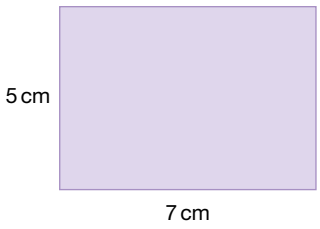


b

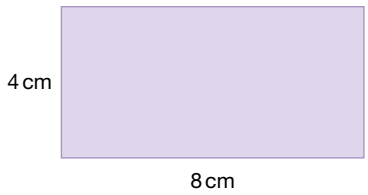


2 The following rectangles all have a perimeter of 24 cm. What is the area of each rectangle?

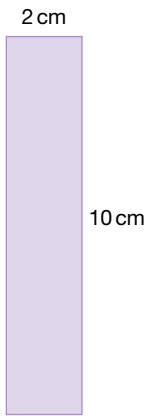
a



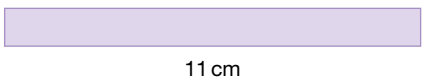
b



c



d



3 Find the areas of rectangles with the following lengths and breadths.

a $l = 10\,\text{cm}, b = 5\,\text{cm}$ **b** $l = 6\,\text{cm}, b = 4\,\text{cm}$ **c** $l = 5\,\text{cm}, b = 8\,\text{cm}$

d $l = 14\,\text{cm}, b = 2\,\text{cm}$ **e** $l = 9\,\text{cm}, b = 5\,\text{cm}$ **f** $l = 50\,\text{cm}, b = 20\,\text{cm}$

4 **a** The area of a rectangle is $40\,\text{m}^2$. Find the length if the width is:

i 5 cm

ii 4 cm

iii 2 cm

b The area of a rectangle is $48\,\text{m}^2$. Find the width if the length is:

i 8 m **ii** 12 m **iii** 16 m

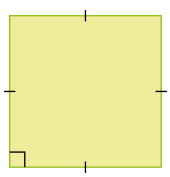
c Find the length and breadth of two different rectangles that each have an area of $50\,\text{cm}^2$. Do these rectangles have the same perimeter?

If the sides of the rectangles are integers, then the length and breadth are factors of the area.

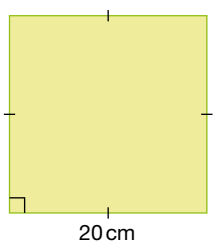


5 A square is, of course, a rectangle whose length is equal to its breadth. Hence, if a square has sides of s cm then its area $= s \times s\ \text{cm}^2$. Find the areas of these squares.

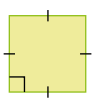
a



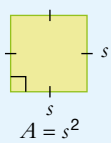
b



c



Area of a square:



6 Find the area of a square whose side length is:

a 3 cm **b** 10 mm **c** 7 km **d** 18 m

7 Find the side length of a square that has an area of:

a $4\,\text{cm}^2$ **b** $25\,\text{mm}^2$ **c** $16\,\text{cm}^2$ **d** $144\,\text{km}^2$

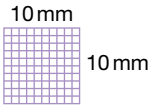
To find the area of a square, you square the side length.

To find the side length, you find the square root of the area.

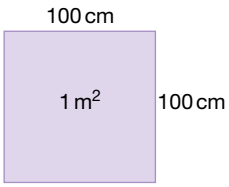
8 Conversions between different units of area can be found by calculating the area of the same square in two different ways. This is shown in **a** and **b** below



a Each side of this 1 square centimetre is, of course, 10 mm. How many square mm are in this 1 square cm?

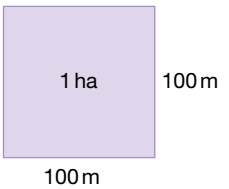


b A square with sides of 1 m would measure 100 cm on each side. How many square cm would fit into 1 square m?



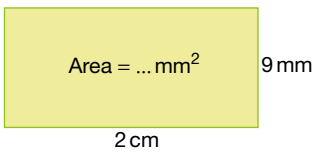
c A square with sides 100 m in length has an area of 1 hectare (ha). How many square metres would fit into 1 hectare?

d A square with sides of 1 km has an area of 1 km^2. What is the area, in square metres, of a square with sides of 1000 m? Complete the following: 1 km^2 = … m^2

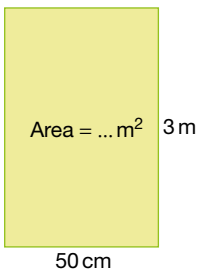


9 Calculate the area of each rectangle in the units indicated.

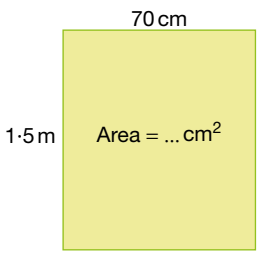
a



b

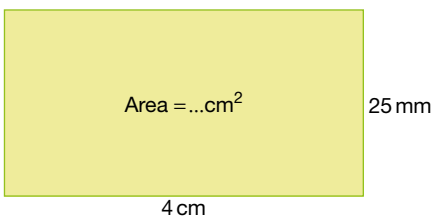


c



To calculate the area, both sides must be given the same units.

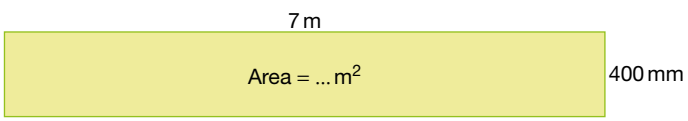
d





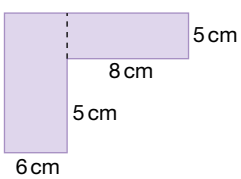
10 mm = 1 cm
100 cm = 1 m
1000 mm = 1 m

e

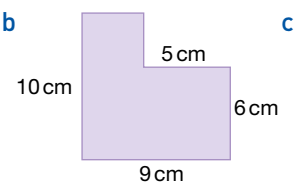


10 Calculate the areas of these figures by dividing them into rectangles.

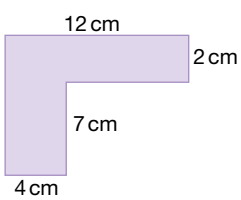
a



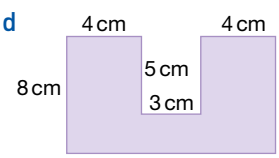
b



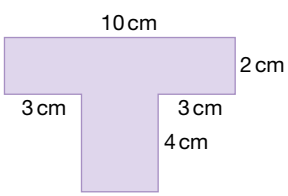
c



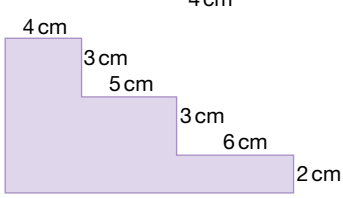
d

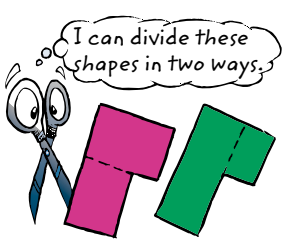


e



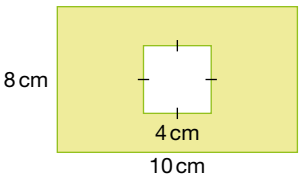
f



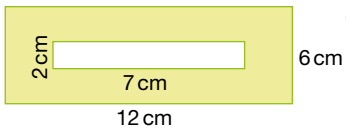


11 Determine the coloured area in each figure.

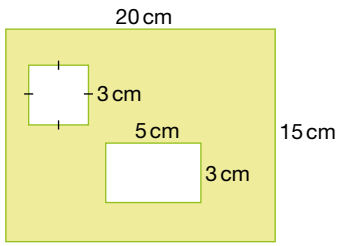
a



b



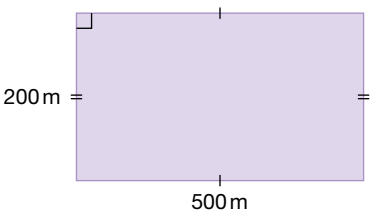
c



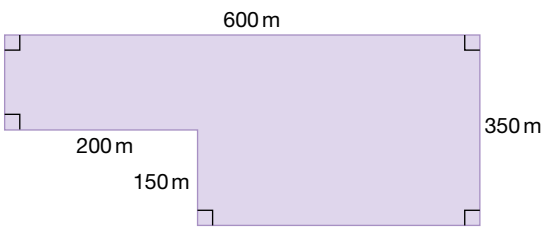
Note: We can take the smaller white areas away from the large area to find the coloured area.

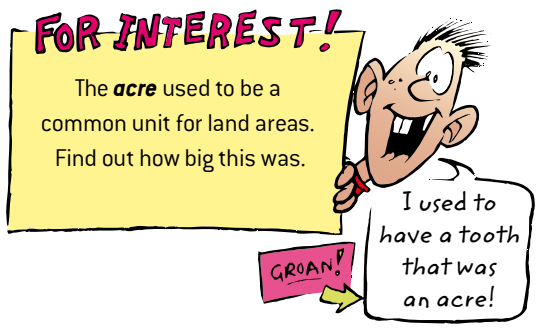
12 Find the area, in hectares, of these fields. (*Remember:* 1 ha = 10 000 m^2)

a



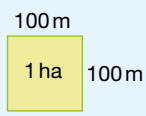
b





Land areas

A square with sides 100 m long has an area of 1 hectare (ha).

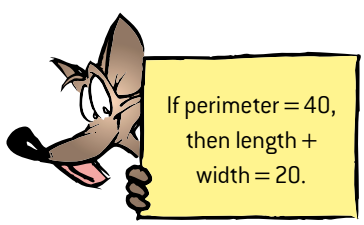


1 ha = 10 000 m^2

13 The perimeter of a rectangle is 40 cm. Calculate the width and area of the rectangle if the length of the rectangle is:

a 19 cm **b** 18 cm **c** 15 cm **d** 12 cm

e What dimensions for this rectangle would make the largest area?



14 This holiday house has the dimensions shown. Find the floor area, in m^2, of the:

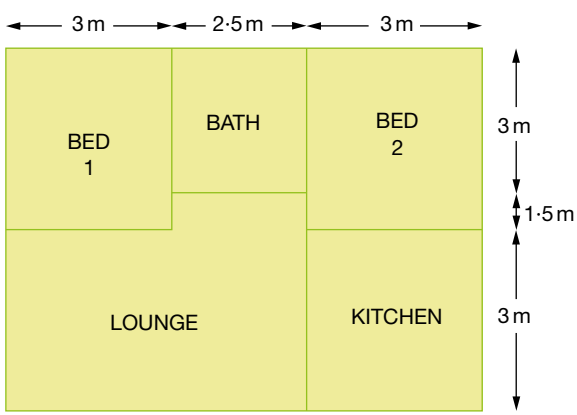
a kitchen

b bathroom

c lounge

d two bedrooms

e whole house.

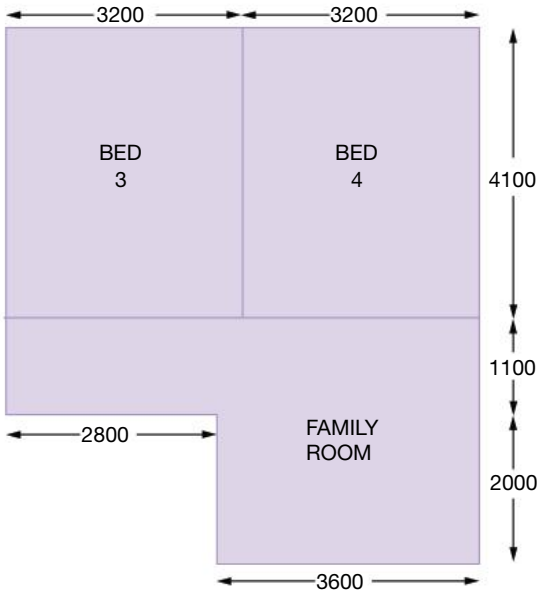


15 Calculate the area of a rectangle with the following lengths and breadths:

a length = 27 cm, breadth = 18 cm
b length = 88 m, breadth = 68 m
c length = 16·8 m, breadth = 12·5 m
d length = 1·8 m, breadth = 0·15 m

16 The plans shown are for an extension to a house. Measurements on plans are normally given in millimetres. Find the area, in square metres, of:

a bedroom 4
b the family room
c the whole extension



GEOGEBRA ACTIVITY 10:03 AREA OF A RECTANGLE

Click and drag points B and D to change the length and breadth. Note how the area changes.



INVESTIGATION 10:03 AREA OF A RECTANGLE

1 Advertisements in newspapers cost more if they take up more area on the page. Take a page from a newspaper and find the areas, in cm^2, taken up by advertisements, articles and photographs.

2 Calculate the area of the face of a brick. Use this to determine the number of bricks needed to build a wall 10 m long and 2 m high.

3 Measure the area of a paving brick and calculate the number of bricks needed to pave a rectangular driveway 3 m by 6 m.

4 Mark out an area 100 m by 100 m. This area is 1 hectare.

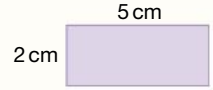
5 Determine the area of your school grounds in hectares. Estimate how many housing blocks would fit into it.

10:04 Area of a triangle

PREP QUIZ 10:04

Calculate the areas of these rectangles.

1



2



3



4



Find the area of:

5 a rectangle with dimensions of 8 mm and 5 mm

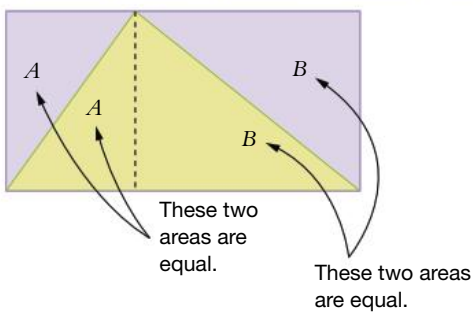
6 a rectangle with a length of 20 cm and a breadth of 15 cm

7 a square with a side length of 7 cm

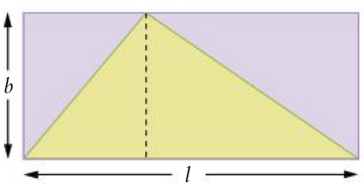
8 $3{\cdot}8 \times 10 =$ **9** $10{\cdot}4 \times 3 =$ **10** $6{\cdot}14 \div 2 =$







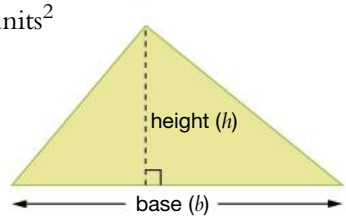
Looking at either of these two figures, you should be able to see that the area of the green triangle is always half the area of the rectangle around it.



Knowing that the area of a rectangle is given by $l \times b$, we could say:

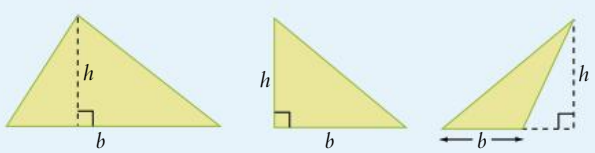
area of a triangle $= \frac{1}{2} \times l \times b$ units^2

But we usually call the dimensions of a triangle its ***base*** and ***height***. (*Note:* These two measurements must be perpendicular to one another.)



Area of a triangle is given by:

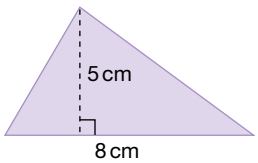
$$A = \frac{b \times h}{2} \quad \text{or} \quad \frac{1}{2}bh$$



WORKED EXAMPLES

1 Find the area of each triangle.

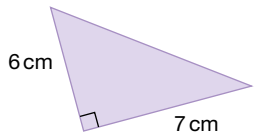
a



$$A = \frac{b \times h}{2}$$
$$= \frac{8 \times 5}{2}$$
$$= 20$$

$\therefore$ Area $= 20\,\text{cm}^2$

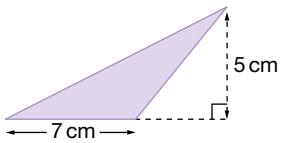
b



$$A = \frac{b \times h}{2}$$
$$= \frac{7 \times 6}{2}$$
$$= 21$$

$\therefore$ Area $= 21\,\text{cm}^2$

c

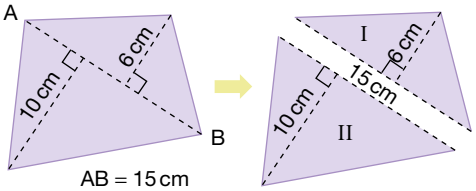


$$A = \frac{b \times h}{2}$$
$$= \frac{7 \times 5}{2}$$
$$= 17\tfrac{1}{2} \text{ or } 17{\cdot}5$$

$\therefore$ Area $= 17{\cdot}5\,\text{cm}^2$

2 These shapes may be divided into triangles and rectangles to find their areas.

a



Area of triangle I

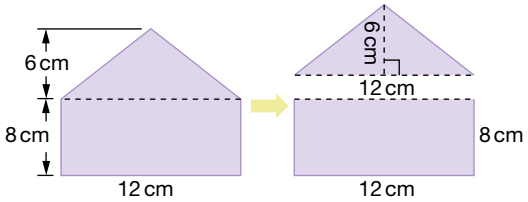
$$A = \frac{15 \times 6}{2}$$
$$= 45$$

Area of triangle II

$$A = \frac{15 \times 10}{2}$$
$$= 75$$

$\therefore$ Total area of figure $= 45\,\text{cm}^2 + 75\,\text{cm}^2$
$= 120\,\text{cm}^2$

b



Area of triangle

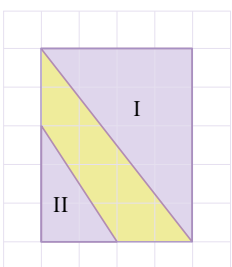
$$A = \frac{12 \times 6}{2}$$
$$= 36$$

Area of rectangle

$$A = 12 \times 8$$
$$= 96$$

$\therefore$ Total area of figure $= 36\,\text{cm}^2 + 96\,\text{cm}^2$
$= 132\,\text{cm}^2$

3 Areas like the green figure shown can be calculated by making a rectangle around the figure, then subtracting the area of the resulting triangles from the area of the rectangle. The remainder must be the area of the figure.



Area of triangle I $= \dfrac{4 \times 5}{2}\,\text{cm}^2$
$= 10\,\text{cm}^2$

Area of triangle II $= \dfrac{2 \times 3}{2}\,\text{cm}^2$
$= 3\,\text{cm}^2$

Area of rectangle $= 4 \times 5\,\text{cm}^2$
$= 20\,\text{cm}^2$

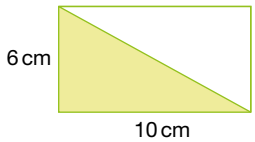
$\therefore$ Area of green figure $= 20 - (10 + 3)\,\text{cm}^2$
$= 7\,\text{cm}^2$

Exercise 10:04

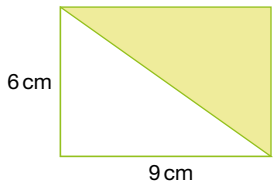
Foundation worksheet 10:04
Area of a triangle

1 Calculate the area of the green triangle inside each rectangle.

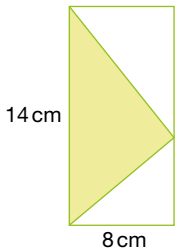
a



b

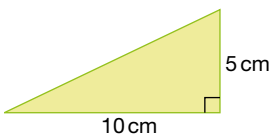


c

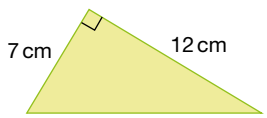


2 Calculate the area of each triangle.

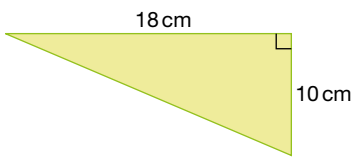
a



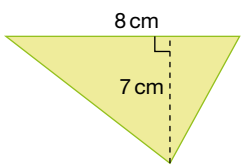
b



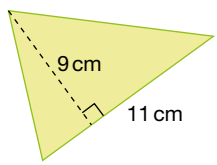
c



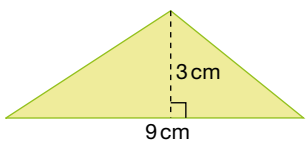
d



e

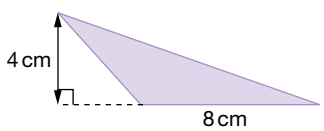


f

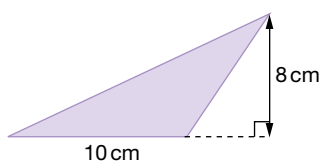


3 Calculate the areas of these triangles.

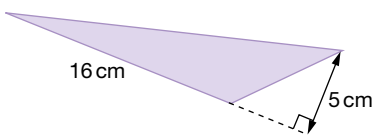
a



b

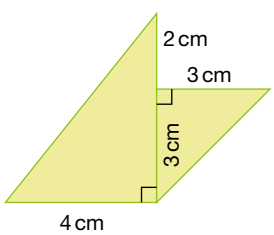


c

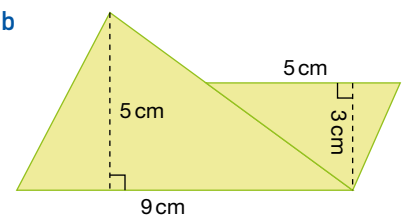


4 Each figure has been made by joining two triangles. Find the area of each triangle and add them together to find the area of the figure.

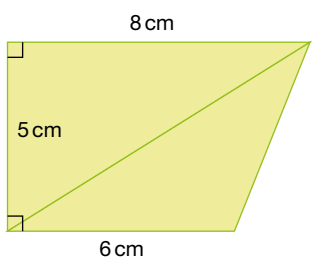
a



b



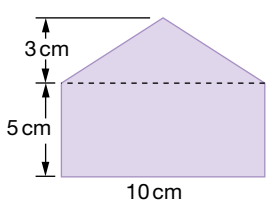
c



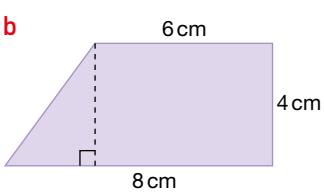
5 a A triangle has an area of 20 cm^2 and a base length of 5 cm. What is its height?
b A triangle has an area of 48 mm^2 and a height of 12 mm. What is the length of its base?
c A triangle has an area of 72 cm^2 and a base length of 6 cm. What is its height?

6 Determine the areas of these figures.

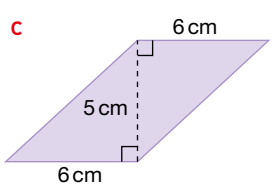
a



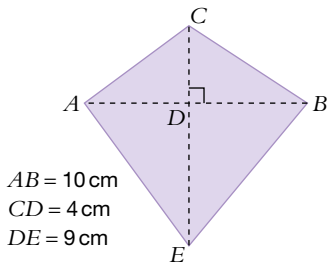
b



c

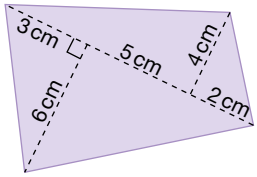


d

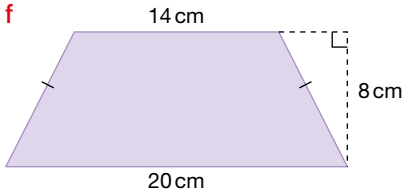


$AB = 10\,\text{cm}$
$CD = 4\,\text{cm}$
$DE = 9\,\text{cm}$

e

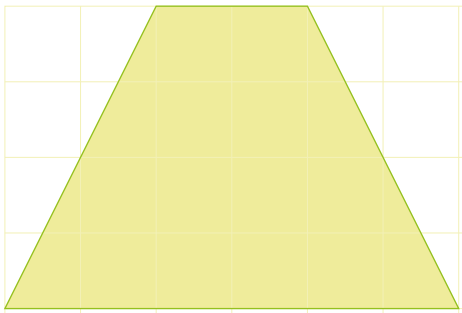


f

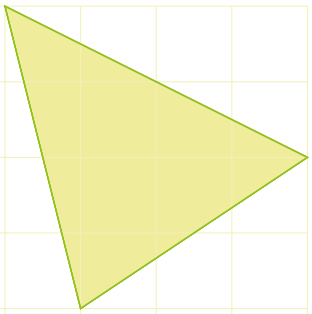


7 Determine the area of each shaded figure using the centimetre grid it is drawn on.

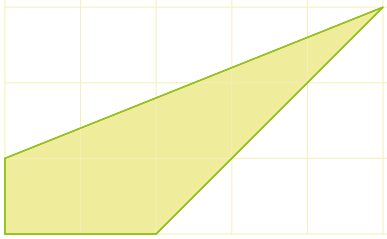
a



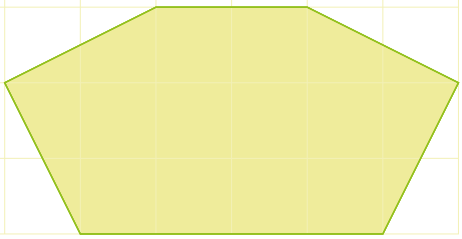
b



c

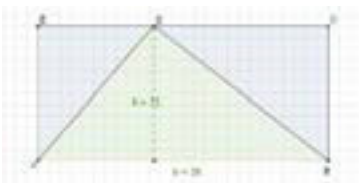


d

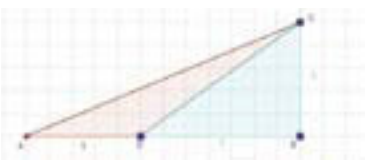


P GEOGEBRA ACTIVITY 10:04 AREA OF A TRIANGLE

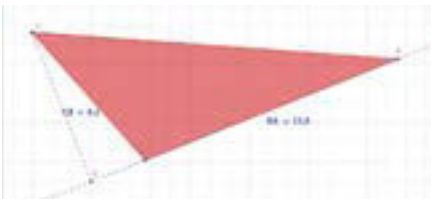
1 Drag points A, B and C to change the area of the rectangle and the triangle. How are these two areas related?



2 Drag points B, C and D to change the area of the obtuse-angled triangle. Check the area by applying a formula.



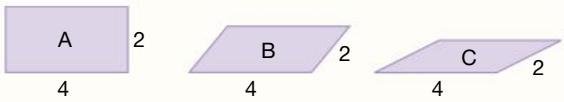
3 Drag points A, B and C to vary the base and height of the triangle. How are the base and height related to the area?



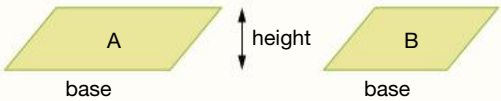
10:05 Area of a parallelogram

PREP QUIZ 10:05

A is a rectangle. B and C are parallelograms.



1 What is the area of A?
2 Do the figures A, B and C have the same perimeter?
3 Which of the figures has the biggest area?
4 Which of the figures has the smallest area?
5 These parallelograms have the same height but different base lengths. Which has the larger area?

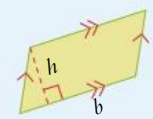


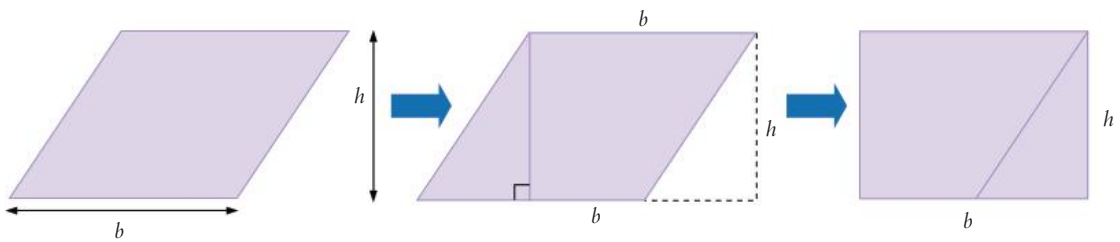
6 $5{\cdot}8 \times 10 = \ldots$
7 $1{\cdot}2 \times 3{\cdot}5 = \ldots$
8 $140 \div 100 = \ldots$
9 $1{\cdot}3\,\text{m} = \ldots\ \text{cm}$
10 $210\,\text{cm} = \ldots\ \text{m}$

By cutting off one end of a parallelogram and adding it to the other end, a rectangle is formed. The area of the figure has not changed.

Area of parallelogram $= b \times h$
$= bh$

The area of a parallelogram
= base × perpendicular height
$A = bh$

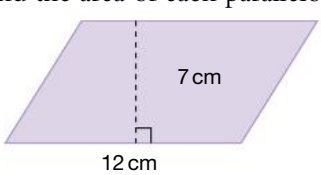




WORKED EXAMPLES

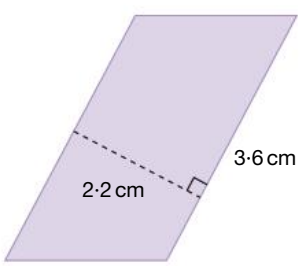
1 Find the area of each parallelogram.

a

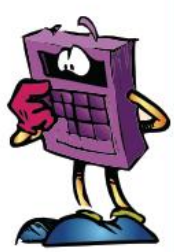


$A = bh$
$= 12 \times 7$
$= 84\,\text{cm}^2$

b



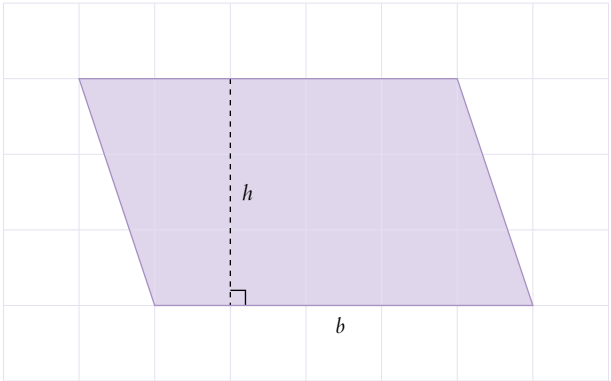
$A = bh$
$= 3{\cdot}6 \times 2{\cdot}2$
$= 7{\cdot}92\,\text{m}^2$



2 Find the area of the parallelogram using the centimetre grid.

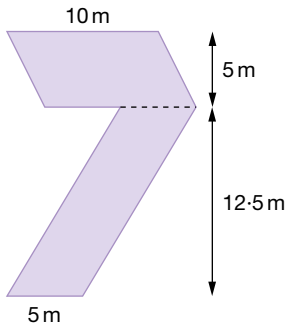
The base = 5 cm
The height = 3 cm

$A = bh$
$= 5 \times 3$
$= 15\,\text{cm}^2$

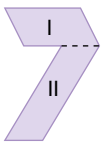


3 These figures have been formed by joining different shapes. Calculate the area of each figure.

a



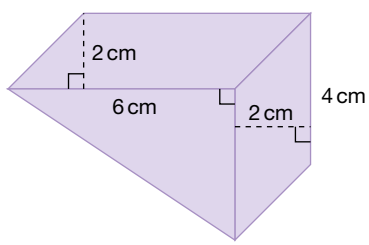
Area of parallelogram I $= b \times h$
$= 10 \times 5$
$= 50\,\text{m}^2$



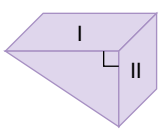
Area of parallelogram II $= b \times h$
$= 5 \times 12{\cdot}5$
$= 62{\cdot}5\,\text{m}^2$

Area of figure $= 50 + 62{\cdot}5$
$= 112{\cdot}5\,\text{m}^2$

b



Area of parallelogram I $= b \times h$
$= 6 \times 2$
$= 12\,\text{cm}^2$



Area of parallelogram II $= b \times h$
$= 4 \times 2$
$= 8\,\text{cm}^2$

Area of triangle $= \dfrac{bh}{2}$

$= \dfrac{6 \times 4}{2}$

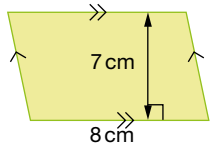
$= 12\,\text{cm}^2$

Area of figure $= 12 + 8 + 12$
$= 32\,\text{cm}^2$

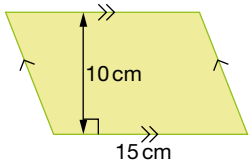
Exercise 10:05

1 Use the formula $A = bh$ to find the area of each parallelogram.

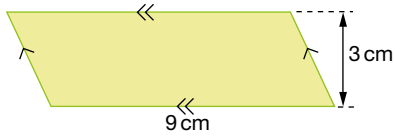
a



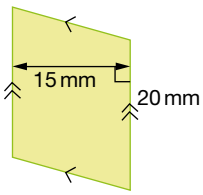
b



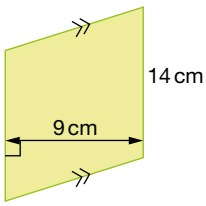
c



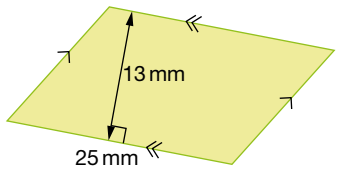
d



e

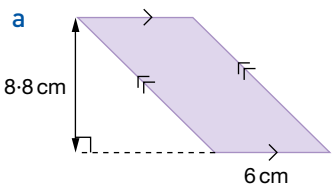


f

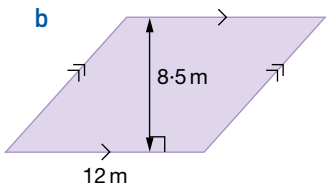


2 Calculate the areas of the following parallelograms. Make sure all side lengths are in the same units.

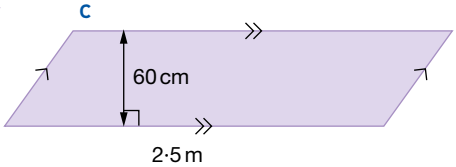
a



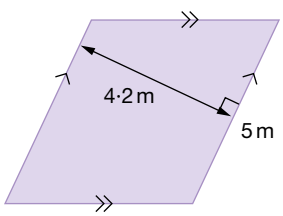
b



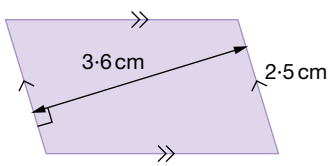
c



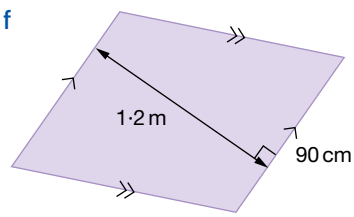
d



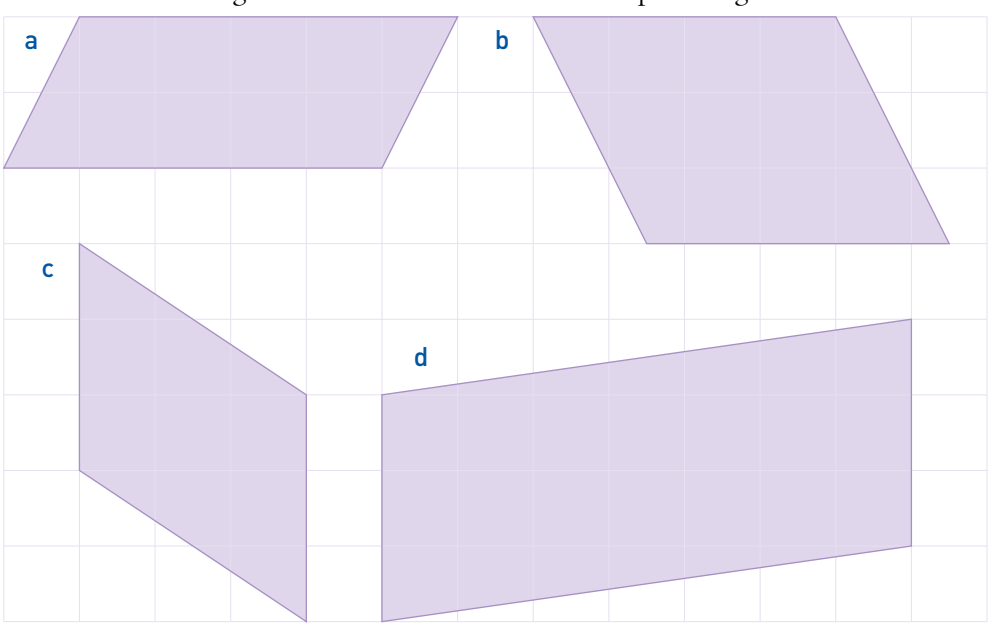
e



f

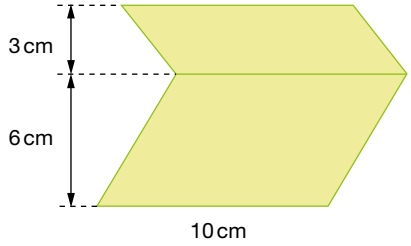


3 Use the centimetre grid to determine the area of each parallelogram.

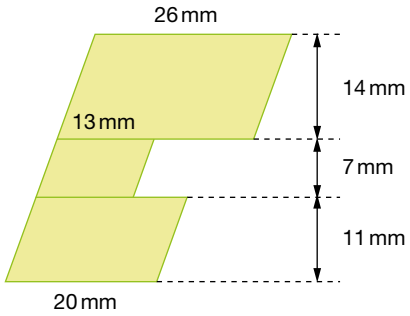


4 Each figure has been made by joining parallelograms. Find the area of each figure.

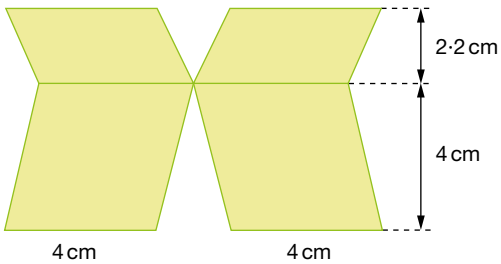
a



b

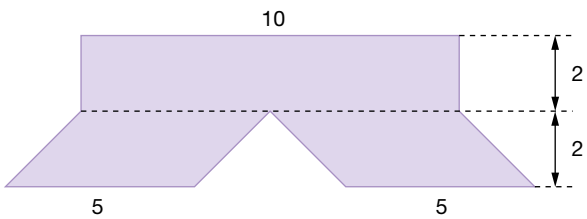


c

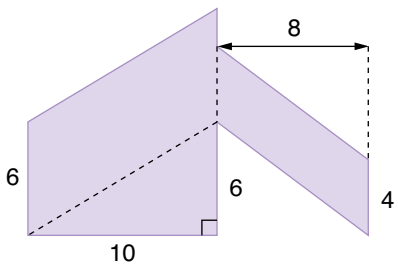


5 Determine the areas of these figures. All measurements are in centimetres.

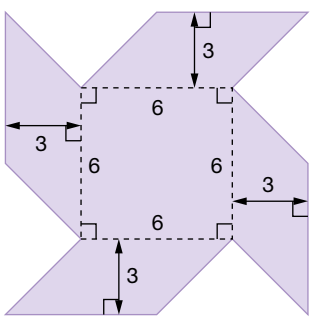
a



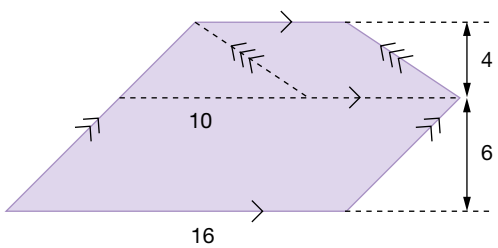
b



c

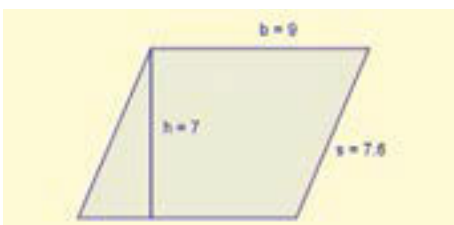


d

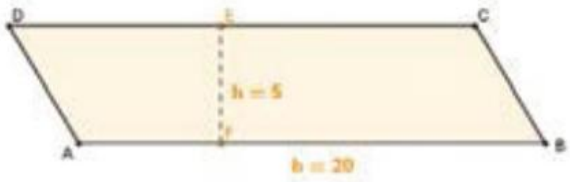


P GEOGEBRA ACTIVITY 10:05 AREA OF A PARALLELOGRAM

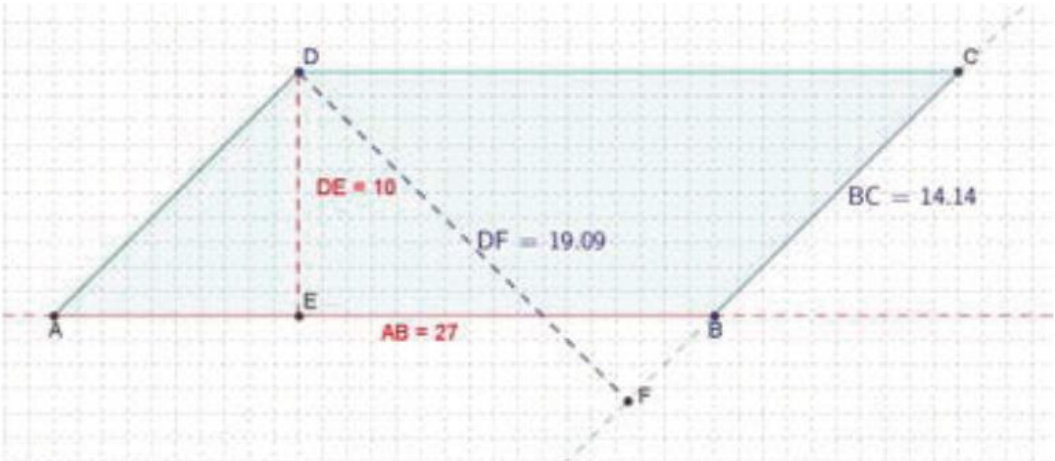
1 Click and drag to vary the base and height.
How are the base and height related to the area?



2 Use the formula to calculate the area of the parallelogram and then check your answer. Drag points B and D to change the parallelogram and repeat the process.



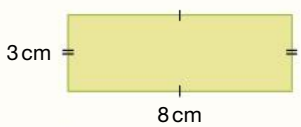
3 Drag points B and D to vary the base and height. Does it matter which side is the base and which is the height?

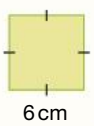


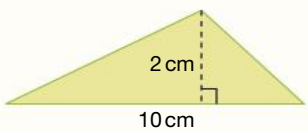
10:06 Area problems

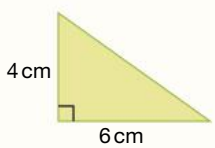
PREP QUIZ 10:06

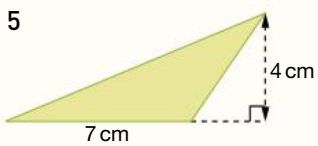
Calculate the area of each figure.

1


2


3


4


5


How many square tiles 10 cm by 10 cm will it take to cover the following rectangular areas?

6 10 cm × 80 cm **7** 10 cm × 2 m **8** 50 cm × 50 cm **9** 50 cm × 5 m

10 What is the area of a rectangular wall that is 3·6 m long and 2·4 m high?

Measurement of area occurs in a variety of everyday situations, such as tiling, wallpapering, making curtains, laying turf or painting.

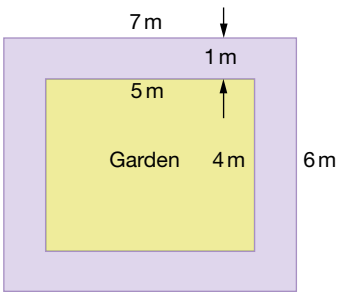
The problems in this section ask you to apply your ability to find the areas of rectangles and triangles in various situations. It is also possible to find the areas of different common shapes from the area of a rectangle.

WORKED EXAMPLE 1

A rectangular garden bed is 5 m by 4 m. A path 1 m wide is laid around the garden. What is the area of the path?

Solution

The path and garden would look like the diagram below.



Important!
Always draw a diagram of the problem if there is not a diagram already.

Because the path is 1 m wide, the outer dimensions would be 7 m by 6 m.

Area of path = area of outer rectangle – area of garden
$= (7 \times 6)\ m^2 - (5 \times 4)\ m^2$
$= 42\,m^2 - 20\,m^2$
$= 22\,m^2$

$\therefore$ Area of path will be $22\,m^2$.

WORKED EXAMPLE 2

A room and corridor are made up of two rectangular shapes as shown. Find the cost of tiling this area if the tiles and labour costs \$75 per m^2.

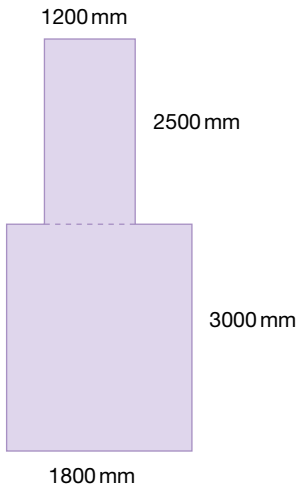
Solution

To find the area in square metres, change the length measurements to metres by dividing by 1000.

e.g. 1200 mm ÷ 1000 = 1.2 m

Area = area of rectangle I + area of rectangle II
$= 1{\cdot}2 \times 2{\cdot}5 + 1{\cdot}8 \times 3$
$= 8{\cdot}4\,m^2$

$\therefore$ Cost = $8{\cdot}4 \times \$75$
= \$630



Exercise 10:06



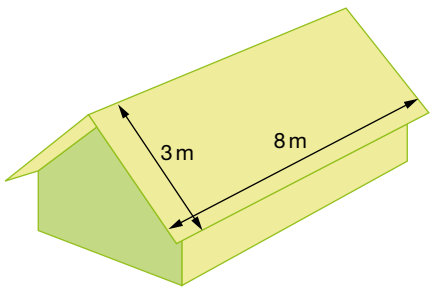
1 Tanith's parents are going to carpet her bedroom. Her bedroom is rectangular, measuring 4 m × 3 m. If the carpet costs $67 per square metre, what is the cost of carpeting the room?

2 Jason's fence needs painting. Each litre of paint will cover 16 square metres. If the fence is rectangular and is 80 m long by 2 m high, how many litres of paint will be needed?

3 A hallway is 8 m long and 2 m wide. It is being tiled at a cost of $80 per square metre. What is the total cost of the tiling?

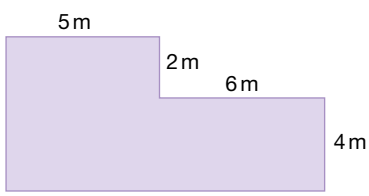
4 A gymnasium floor is rectangular. It is 40 m wide and 60 m long. What is the cost of resurfacing the floor if the contractor charges $30 per square metre?

5 The roof shown consists of two rectangles.



- **a** Calculate the area of the roof.
- **b** Calculate the number of concrete roof tiles required for the roof if it takes 11 tiles to cover 1 square metre.
- **c** Roof tiles cost $280 for 100 tiles and must be bought in lots of 100. How much will it cost to buy the tiles for the roof?

6 The diagram shows the shape of a lawn.



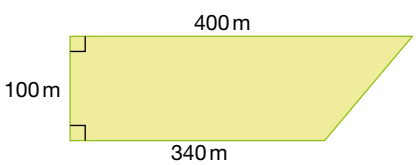
- **a** Calculate the area of the lawn in square metres.
- **b** If 1 kg of fertiliser will cover 10 m^2, how much fertiliser will be needed to cover the lawn?

7 In a large shopping mall, a rectangular garden has a paved section 5 m wide around its border. If the garden is 10 m by 5 m, find:

- **a** the area of the garden
- **b** the total area of the garden and the path
- **c** the area of the paved section.

8 **a** Fred the farmer has a rectangular field measuring 80 m by 400 m. Calculate the area of the field in square metres and also in hectares. (*Remember:* 1 hectare = 10 000 m^2)

- **b** Fred's neighbour Bella has a field like the one shown in the diagram. By dividing the field into a triangle and a rectangle, calculate the area of the field in hectares.

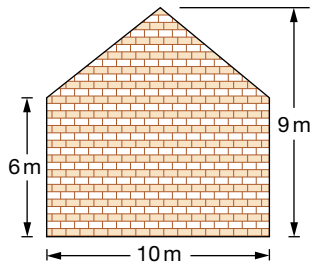


9 Brent wants to cover his maths textbook because he needs to take care of it. The book measures 40 cm by 26 cm when opened flat. What area of plastic will Brent need to cover the book, if he wants an extra 2 cm around each edge to tuck the plastic over?

10 A wall is shaped as a triangle on top of a rectangle with the measurements shown.

a Find the area of the wall in m^2.

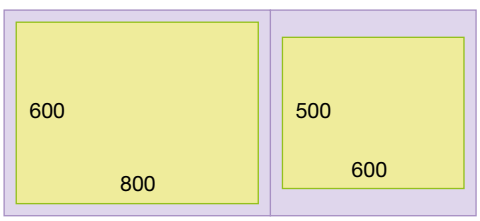
b Using the fact that approximately 40 bricks are needed to cover 1 square metre, calculate the number of bricks required to build the wall.



11 The photo shows part of the triangular section of a roof that needs to be painted. Calculate the area of the triangle if the base is 8·8 m long and the height is 1·6 m.



12 The diagram shows a window that contains two pieces of glass. What is the total area of the glass? What would it cost to replace it with new glass that costs $90 per square metre? (Measurements are in millimetres.)



13 Wallpaper comes in rolls that are 50 cm wide. What length of wallpaper would be needed to cover two walls 4 m by 3 m high and one wall 2·5 m by 3 m high?

14 Floor tiles measure 300 mm by 300 mm. How many would be needed to cover a kitchen floor measuring 2·4 m by 3·6 m?

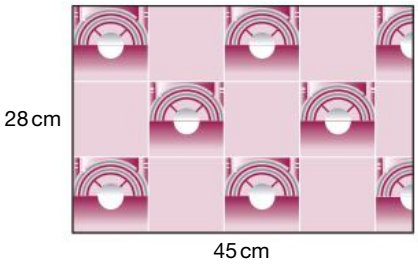
15 a A company supplies turf, which comes in rolls 40 cm wide and 2 m long. How many rolls must be ordered to cover an area 28 m by 24 m?

b If each roll costs $4.20, what is the cost of the turf?

c The same area could have been seeded instead. If a box of seed covers 16 m^2 and costs $18.50, how much money would have been saved on the cost of turf?

16 When laying tiles, an exact number may not cover an area, or a whole number may not lie along each edge. Look at the diagram at right:

If each tile is 10 cm × 10 cm, we can see that 15 tiles are needed, presuming that the pieces of tiles cut off are not good enough to be used elsewhere.



This is true even though the area is $28\text{ cm} \times 45\text{ cm} = 1260\text{ cm}^2$.
Dividing this by 100 cm^2 (the area of a tile) would suggest that only 12·6 or 13 whole tiles are needed.

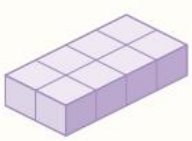


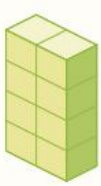
a How many 10 cm × 10 cm tiles would be needed to cover an area 3·25 m by 2·17 m?
b How many 300 mm by 300 mm tiles would be needed to cover an area 2·5 m by 3·8 m?

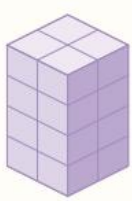
10:07 Volume of a rectangular prism

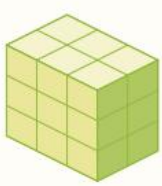
PREP QUIZ 10:07

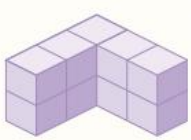
What is the volume, in cubic units, of each solid? (The solids have uniform cross-sections.)

1 

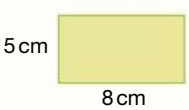
2 

3 

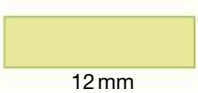
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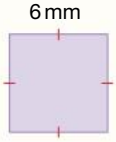
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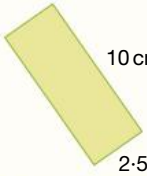
What is the area of each rectangle?

6 

7 

8 

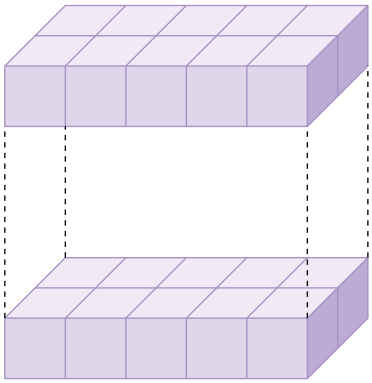
9 

10 



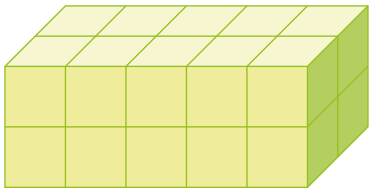
Do you count the boxes one at a time, or is there a better way?

To calculate the volume of a rectangular prism, we can work out how many cubes are in one layer and then multiply by the number of layers.





For this prism we can see there are 2 layers of 5×2 cubes.

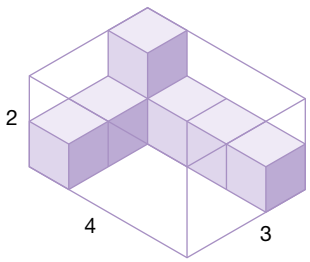


WORKED EXAMPLES

1 To determine the volume of this prism, we can see that there would be 4×3 cubes in the bottom layer, i.e. 12 cubic units.

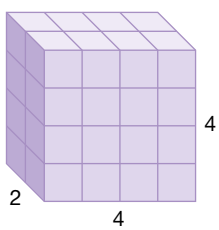
Since there are 2 layers, the volume must be:

$V = (4 \times 3) \times 2$ cubic units
$= 24$ cubic units



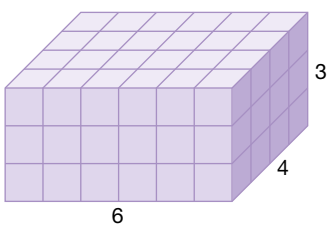
Note: The number of cubes in each layer is the same as the area of the base rectangle in square units.

2 a



The volume of this solid would be given by:
$V = (4 \times 2) \times 4$
$= 32$ cubic units

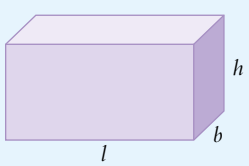
b



The volume of this rectangular prism would be given by:
$V = (6 \times 4) \times 3$
$= 72$ cubic units

To find the volume of a rectangular prism, use the rule:

$V = l \times b \times h$ or simply $V = lbh$

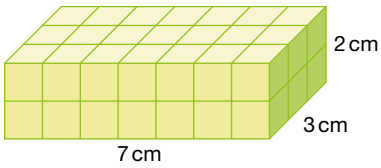


Exercise 10:07

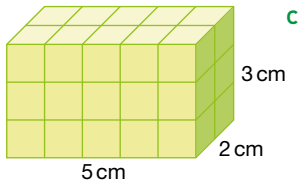
P Foundation worksheet 10:07
Volume of a rectangular prism
Challenge worksheet 10:07
Volume

1 Find the volume of each rectangular prism in cubic centimetres.

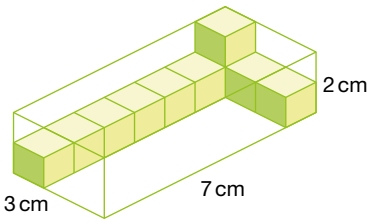
a



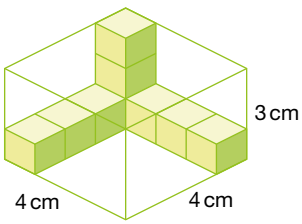
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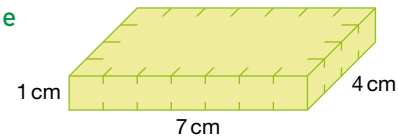
c



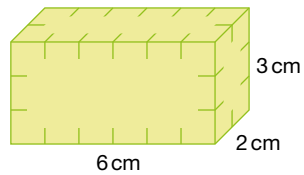
d



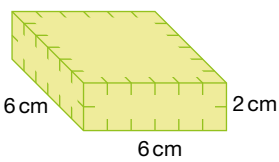
e



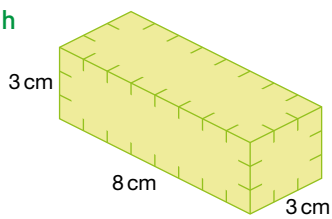
f



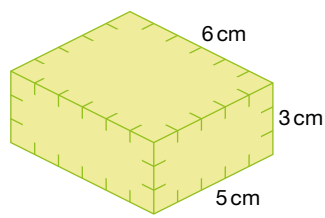
g



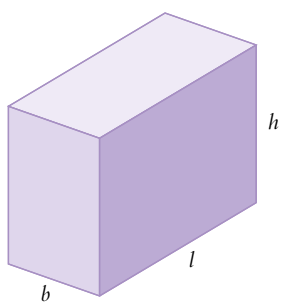
h



i



2 By using the formula $V = l \times b \times h$, calculate the volume of each rectangular prism described below.



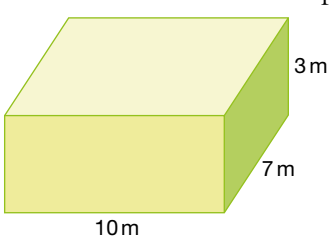
a length = 3 cm, breadth = 2 cm, height = 1 cm
b length = 5 cm, breadth = 3 cm, height = 4 cm
c length = 2 cm, breadth = 1 cm, height = 6 cm
d $l = 9$ cm $b = 3$ cm $h = 2$ cm
e $l = 5$ cm $b = 5$ cm $h = 5$ cm
f $l = 10$ cm $b = 8$ cm $h = 4$ cm

3 Calculate the volume of a cube that has a side length of:

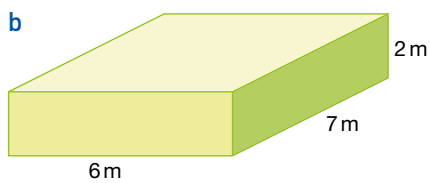
a 3 cm **b** 5 cm **c** 9 cm **d** 4 mm

4 Calculate the volume of each prism in m^3.

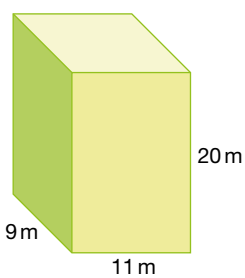
a



b

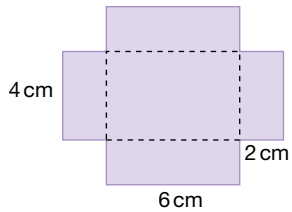


c

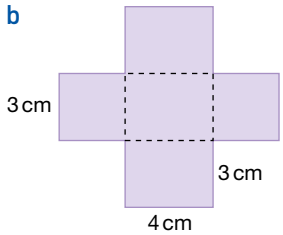


5 If open boxes are made by folding along the dotted lines, what is the volume of each box?

a



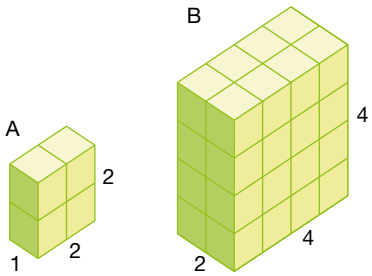
b



6 Look at the dimensions of these prisms. The dimensions of prism A have been doubled to give prism B.

a What is the volume of each prism?

b How many of prism A would be needed to make prism B?



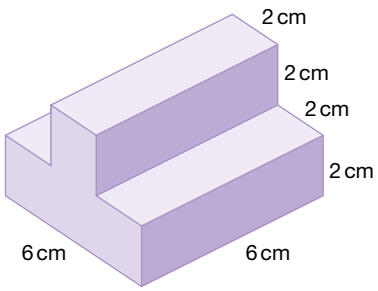
7 The dimensions of a rectangular prism are $5\text{ cm} \times 4\text{ cm} \times 2\text{ cm}$.

a If these dimensions are doubled, what does the volume become?

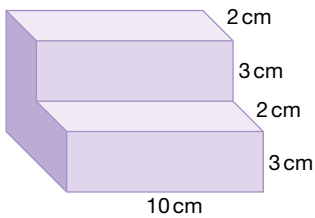
b How many of the original prism would fit into the larger prism?

8 Find the volumes of these solids formed by joining together rectangular prisms.

a

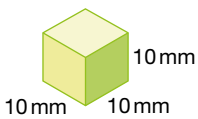


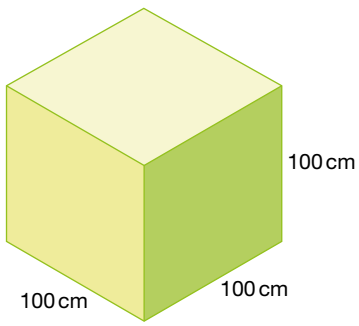
b



9 **a** A cube with a side length of 1 metre would have a volume of 1 cubic metre (1 m^3). What would be the volume of this cube in cubic centimetres (cm^3)?

b A centimetre cube would have a side length of 10 mm. What would be the volume of this cube in cubic millimetres (mm^3)?



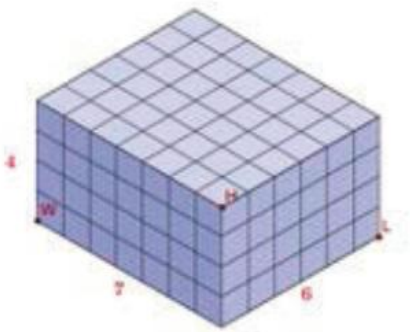


10 Use a calculator and the formula for the volume of a rectangular prism to find the volumes of prisms with the following dimensions.

	Length (l)	Breadth (b)	Height (h)		Length (l)	Breadth (b)	Height (h)
a	3·6 m	2·5 m	1·8 m	**b**	15 cm	28 cm	17 cm
c	5·2 m	5·5 m	40 cm	**d**	1·2 m	80 cm	12 cm

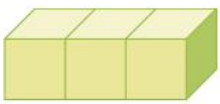
GEOGEBRA ACTIVITY 10:07 VOLUME OF A RECTANGULAR PRISM

Drag points to change the dimensions of the prism.
Find the volume and check the answer.



INVESTIGATION 10:07 STACKING CUBES

1 There are only 2 differently shaped solids that can be formed by joining three cubes together at their faces, as shown by these diagrams.





There are 8 different solids that can be formed using four cubes. After stacking cubes together, draw them all.

2 By stacking cubes together, find out how many *rectangular prisms* can be made that have different dimensions and a volume of:

a 8 cubic units **b** 20 cubic units **c** 24 cubic units

10:08 Capacity and volume

When we measure the amount of a liquid that a container can hold, we are measuring its ***capacity***.

Capacity is the volume of *liquids*.



We might say that an amount of a liquid is so many 'cupfuls' or 'spoonfuls', but as with the volume of solids, we need some standard units.

The basic unit that we use for capacity is the *litre* (L).
1000 millilitres (mL) = 1 litre (L)
1000 litres (L) = 1 kilolitre (kL)
1000 kilolitres (kL) = 1 megalitre (ML) = 1 000 000 litres (L)







Exercise 10:08

P Foundation worksheet 10:09 Capacity

1 How many millilitres would be in these amounts?

a 2 L **b** 5 L **c** $\frac{1}{2}$ L **d** 0·4 L **e** 3·5 L **f** 100 L **g** 0·05 L **h** 1 kL

2 How many litres would be in these amounts?

a 3000 mL **b** 9000 mL **c** 7500 mL **d** 8300 mL
e 500 mL **f** 300 mL **g** 20 000 mL **h** 50 mL

How many kilolitres (kL) would be in these amounts?

i 2 ML **j** 11 ML **k** 1·5 ML **l** 6·25 ML

How many megalitres (ML) would be in these amounts?

m 3000 kL **n** 70 000 kL **o** 4500 kL **p** 23 000 kL
q 3 000 000 L **r** 68 000 000 L **s** 3 500 000 L **t** 3 250 000 L

3 Which measure of capacity, 50 mL, 500 mL or 50 L, would be most likely for:

a a bottle of drink **b** the petrol tank of a car **c** a medicine glass?

4 Would the capacity of a cup be closest to 25 mL, 250 mL or 2500 mL?

5 How many 300 mL mugs could be filled completely from a 2 L kettle?

6 How many 375 mL cans of drink do I need to buy to have at least 5 L of drink?

7 Elizabeth's backyard pool contains 20 000 L of water. How many kL is this?

8 How many times would a 1·5 L jug need to be filled to pour out 50 drinks at a party, if each party glass holds 300 mL?

9 A 600 mL bottle of drink costs $1.20 and a 2 L bottle costs $4.20. Which is the better buy, and by how much money per litre?

10 A leaking tap loses 1 mL of water every 10 seconds.
How much water will be lost in:

a 10 minutes
b 1 hour
c 1 day
d 1 week
e 1 year?



There is a link between the units used for the capacity of liquids and for the volume of solids.

A container that holds a volume of 1 cubic centimetre holds 1 mL of liquid.

$$1\text{ mL} = 1\text{ cm}^3$$

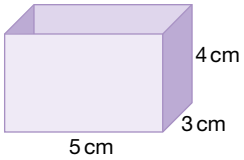
This means that $1\text{ L} = 1000\text{ cm}^3$ and $1\text{ m}^3 = 1\text{ kL}$

11 Complete:

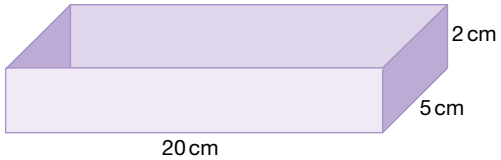
a 15 mL = … cm^3 **b** 8 cm^3 = … mL **c** 183 mL = … cm^3
d 5000 cm^3 = … L **e** 35 L = … cm^3 **f** 2·5 L = … cm^3
g 15 m^3 = … kL **h** 4·5 kL = … m^3 **i** 8·2 m^3 = … kL

12 How many mL of liquid would these prisms hold?

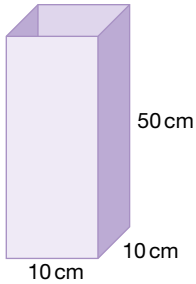
a



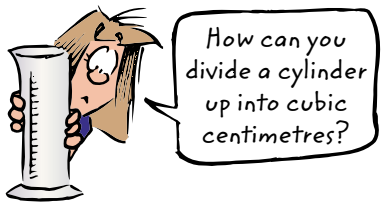
b



c

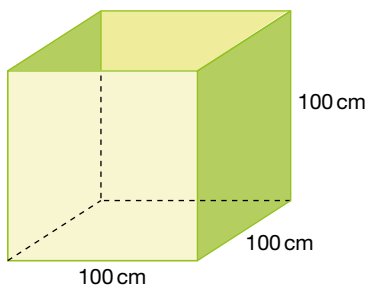


13 If this measuring cylinder has a capacity of 500 mL, what volume can it hold in cubic centimetres?



14 **a** What is the volume, in cm^3, of this cubic container?

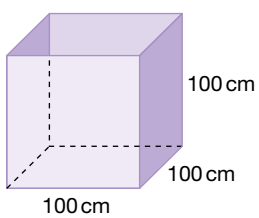
b How many litres of water would fill this container?



15 **a** What is the volume, in cm^3, of a cubic metre?

b What is the capacity of a 1 m^3 container in:

- **i** millilitres
- **ii** litres
- **iii** kilolitres?



1 cubic metre = 1 kilolitre

16 A petrol can is the shape of a rectangular prism with dimensions of 15 cm × 20 cm × 25 cm. How much petrol will it hold?

17 The capacities of car engines are sometimes quoted in litres and sometimes in cubic centimetres (often using the abbreviation cc).

'cc' stands for 'cubic centimetre'.

a A car's engine is said to be 2·4 L. How many cubic centimetres is this?

b Another engine has a capacity of 1600 cc. What is this in litres?

18 **a** A large water tank is in the shape of a square prism, with dimensions of 5 m, 5 m and 20 m. What is the capacity of the tank in:

- **i** cubic metres (m^3)
- **ii** kilolitres (kL)
- **iii** megalitres (ML)?

1 megalitre (ML) = 1 000 000 L
= 1000 kL
= 1000 m^3

b The average dairy cow produces 14·4 L of milk per day. If Sydney consumes 1·3 ML of milk per day, how many cows are needed to produce the milk? Answer to the nearest thousand.

c A large city building used an average of 230 kL of water per day. How many megalitres of water would be used in 365 days? What would be the cost of this water if each litre of water costs 4 cents?

INVESTIGATION 10:08 ESTIMATING CAPACITY AND VOLUME

1 Work with a friend. You will need a large measuring cylinder (big enough to put your clenched fist into) and a tray in which to stand the cylinder.

Begin by estimating the volume of your fist in cubic centimetres. Then, fill the measuring cylinder to the brim with water and gently put your clenched fist into the water. Measure how many millilitres of water have overflowed into the tray. Decide who has the bigger fist.

2 On average, each person in a town uses 30 litres of water each day. If there are 2500 people in the town and the town needs a water tank that can hold 14 days' supply, find how many litres of water the tank should hold. Design a possible tank in the shape of a rectangular prism.

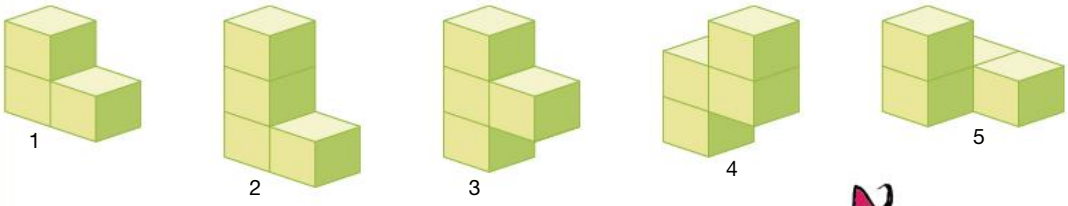
3 Collect various drink containers and estimate their capacities. Check your accuracy by reading the labels.



If a container is not labelled, the capacity can be measured by filling it with water and then pouring the water into a container whose capacity is known.

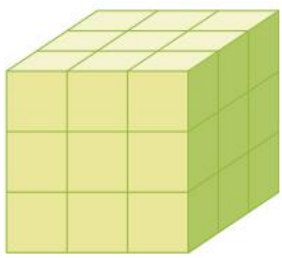
CHALLENGE 10:08 THE FANTASTIC SOMA CUBE

A Soma cube is formed by joining these 7 pieces together.



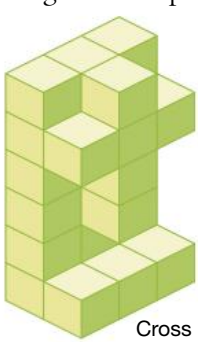


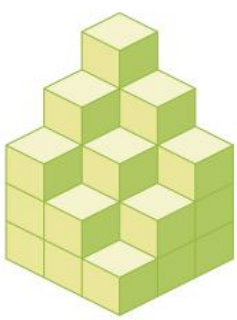
The first piece is formed by joining 3 cubes together. The other six pieces are formed from 4 cubes. The seven pieces represent all of the ways to join 3 or 4 cubes together that include at least one corner.

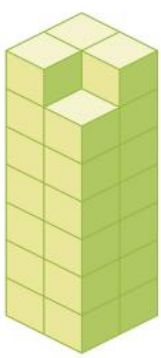


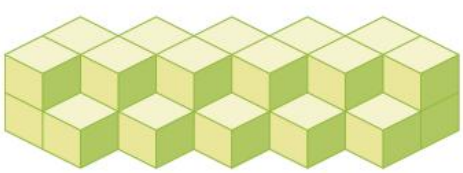
There are many, many different ways of putting the pieces together to make the Soma cube. See how many you can find.

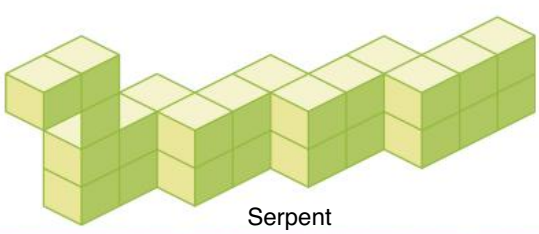
The pieces can also be put together to form other interesting shapes. Some are shown below. See if you can form them. Also try making some shapes of your own.


Cross


Crystal


Tower


Wall


Serpent

FUN SPOT 10:08 WHAT MAKES MONEY AFTER 8 PM?

Find the answer for each letter. Match each letter with its answer below to solve the riddle.

A 1 m = … cm	**A** 1 cm = … mm	**D** 5 kg = … g	**E** 1 L = … mL
E 3 h = … min	**F** 5 min = … s	**I** 3 km = … m	**I** 4 t = … kg
M 2 kL = … L	**N** 1 day = … h	**N** 6 kg = … g	**N** 5 cm^2 = … mm^2
N 4 ha = … m^2	**R** 4 km^2 = … ha	**R** 6 m^2 = … cm^2	**T** 1·8 m = … mm
T 4 000 000 cm^3 = … m^3			

10	40 000	100	300	4	180	400	5000	4000	24	6000	1000	60 000	2000	3000	500	1800



An Olympic-size swimming pool holds a volume of 2500 m^3 or 2 500 000 L (2·5 ML) of water.

MATHS TERMS 10

area
- the amount of space inside a two-dimensional shape
- *common units of area:* square millimetre (mm^2), square centimetre (cm^2), square metre (m^2), hectare (ha), square kilometre (km^2)

hectare
- an area of 10 000 m^2
- 100 m × 100 m

perimeter
- the distance around a plane figure (or the length of its boundary)

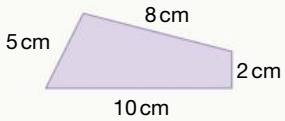
volume
- the amount of space inside a three-dimensional shape
- *common units of volume:* cubic millimetre (mm^3), cubic centimetre (cm^3), cubic metre (m^3)

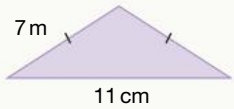
capacity
- the amount of fluid that can be held by a container
- *common units of capacity:* litre (L), millilitre (mL), kilolitre (kL)

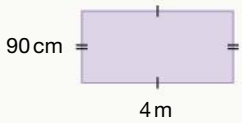
Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

10.01

1 What is the perimeter of each figure?

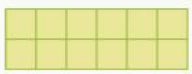
a 

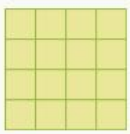
b 

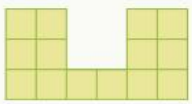
c 

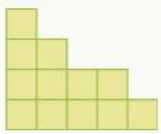
10.02

2 What is the area of each figure, in square units?

a 

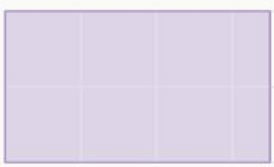
b 

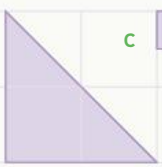
c 

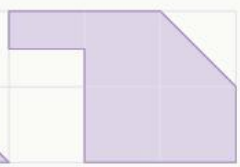
d 

10.02

3 What is the area of each figure, in square centimetres?

a 

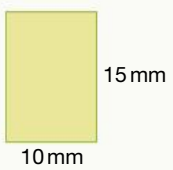
b 

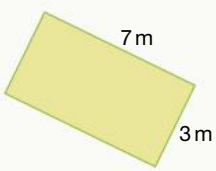
c 

10.03

4 Find the areas of these rectangles.

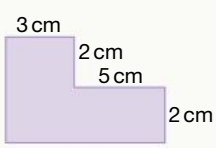
a 

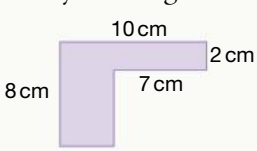
b 

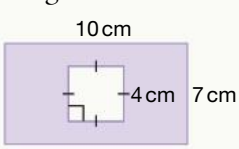
c 

10.03

5 Calculate the areas of these figures by dividing them into rectangles.

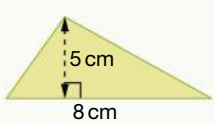
a 

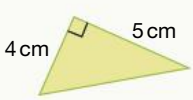
b 

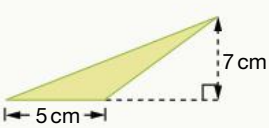
c 

10.04

6 Find the areas of these triangles.

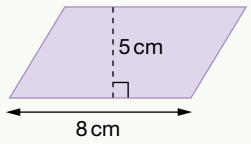
a 

b 

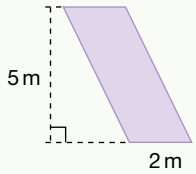
c 

7 Find the area of each parallelogram. 10.05

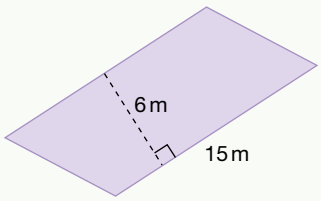
a



b

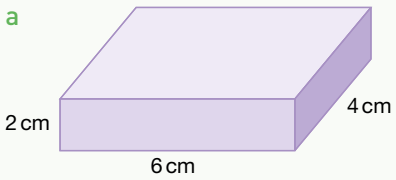


c

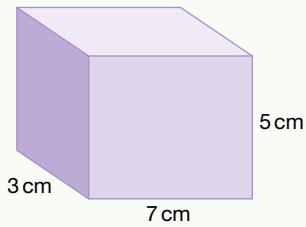


8 What are the volumes of these rectangular prisms, in cm^3? 10.07

a



b



9 What is the volume of the following? 10.07

a a cube with an edge length of 4 cm

b a rectangular prism that has length 7 cm, breadth 3 cm and height 4 cm

c a rectangular prism with dimensions of 2 m × 3 m × 5 m

10 Complete the following. 10.08

a 1 L = … mL

b 2000 L = … kL

c 3500 mL = … L

d 1·5 kL = … L

11 Complete the following. 10.08

a 3 mL = … cm^3

b 50 cm^3 = … mL

c 1 L = … cm^3

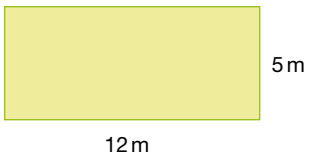
d 1 m^3 = … kL

12 What would be the capacity of each prism in Question **8**, in millilitres? 10.08

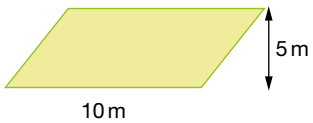
ASSIGNMENT 10A Chapter review

1 What are the areas of the following shapes?

a



b



2 **a** How many 200 mL cups could be filled from a 2 L bottle?

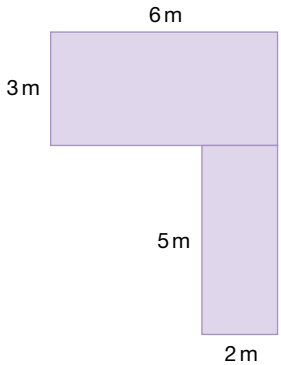
b A box holds twenty-four 600 mL bottles of water. How much water is this in litres?



3 A stack of boxes is in the shape of a rectangular prism. The stack is 5 boxes long, 4 boxes wide and 10 boxes high. How many boxes are in the stack?

4 A bag of fertiliser will cover an area of 25 m^2. How many bags are needed to cover a rectangular lawn that is 35 m long and 20 m wide?

5 An area to be paved is in the shape of two adjoining rectangles, as shown. Find the total cost of paving the area at a cost of $80 per square metre.



6 **a** A square paver has a side length of 500 mm. What is its area in square metres?

b How many square pavers are needed to cover 1 m^2?

c How many pavers would be needed to cover a rectangular area 6 m by 3 m?

7 An aquarium is in the shape of a rectangular prism. It is 1 m long, 30 cm wide and 50 cm deep. What is the capacity of the tank in litres?

8 A concrete path is the shape of a rectangular prism. It is 15 m long, 1·2 m wide and 10 cm thick. Calculate the volume of concrete in the path.



ASSIGNMENT 10B Working mathematically

1 **a** A book has 100 pages. How many digits are used to number the pages?

b In a book it is found that 495 digits were used to number the pages. How many pages were there?

2 **a** How much will it cost to post 500 letters if the postage is $1.20 per letter?

b Stamps come in rectangular sheets. Each sheet has 5 rows of 10 stamps. If each stamp sells for $1.20, how much will 8 sheets of stamps cost?

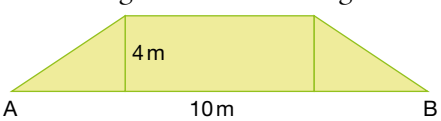


3 Timber is bought by the linear metre. If the timber is 150 mm wide, how many linear metres would I need to buy to cover a rectangular area 3 m long and 1 m high?

4 Approximately 55 bricks are needed to build 1 square metre of wall. How many bricks would be needed to build a wall 14 m long and 2·8 m high if the wall has two windows, each 2·4 m long and 1·2 m high?



5 Two identical triangles are placed either side of a rectangle, as shown. The total area of the triangles is equal to the area of the rectangle. What is the length of AB?



6 Twenty-seven 1 cm cubes are put together to build a 3 cm cube. The cube is then painted red. How many of the 1 cm cubes will have:

a only 1 face painted

b only 2 faces painted

c 3 faces painted?

ASSIGNMENT 10C Cumulative revision

1 Write each expression without multiplication signs. 4:04

a $6 \times m$ **b** $3 \times m \times n$ **c** $4 \times y \times 3$ **d** $2 \times a + 3 \times b$

2 Find the value of each expression if $a = 3$ and $b = 6$. 4:06

a ab **b** $(a + b)^2$ **c** $a^2 + b^2$ **d** $\frac{b+6}{a}$

3 Evaluate the following for $a = 0{\cdot}6$ and $b = 0{\cdot}8$. 1:06, 6:02–6:06

a $a + b$ **b** $a - b$ **c** ab

d $4a - 3b$ **e** $4a \div b$

4 **a** What is the highest common factor of 24 and 12? 3:03, 3:04

b What is the lowest common multiple of 24 and 12?

c What is the prime factorisation of 240?

d Which of the numbers from 2 to 10 are factors of 840?

5 Simplify: 7:06

a $2 - 5$ **b** $-2 - 5$ **c** $-2 - -5$ **d** $2 - -5$

6 Write in index form: 3:01, 4:10

a $2 \times 2 \times 2$ **b** $3 \times 3 \times 3 \times 3$

c $a \times a \times b \times b \times b$ **d** $2 \times a \times a \times b \times a$

7 How many sides do these shapes have? 8:01

a pentagon **b** hexagon **c** octagon **d** trapezium

8 **a** Which fraction is larger, $\frac{2}{3}$ or $\frac{3}{4}$? 9:02, 9:03

b How many fifths in $3\frac{3}{5}$?

c Write $\frac{7}{2}$ as a mixed numeral.

d Which is larger, $\frac{5}{3}$ or $\frac{5}{4}$?

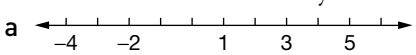
9 If I have these coins, how much do I have in dollars and cents? 1:11C

a 30 twenty-cent coins **b** 100 five-cent coins

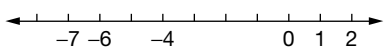
c 200 ten-cent coins **d** 500 twenty-cent coins

10 Place a number under every mark to complete the number lines below. 7:01

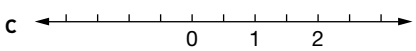
a



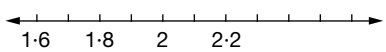
b



c

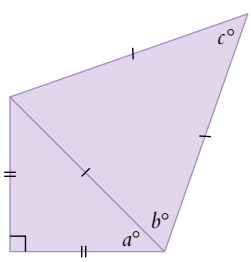


d

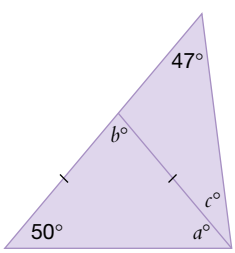


11 Find the value of each pronumeral. 8:02

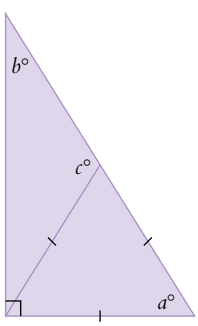
a



b

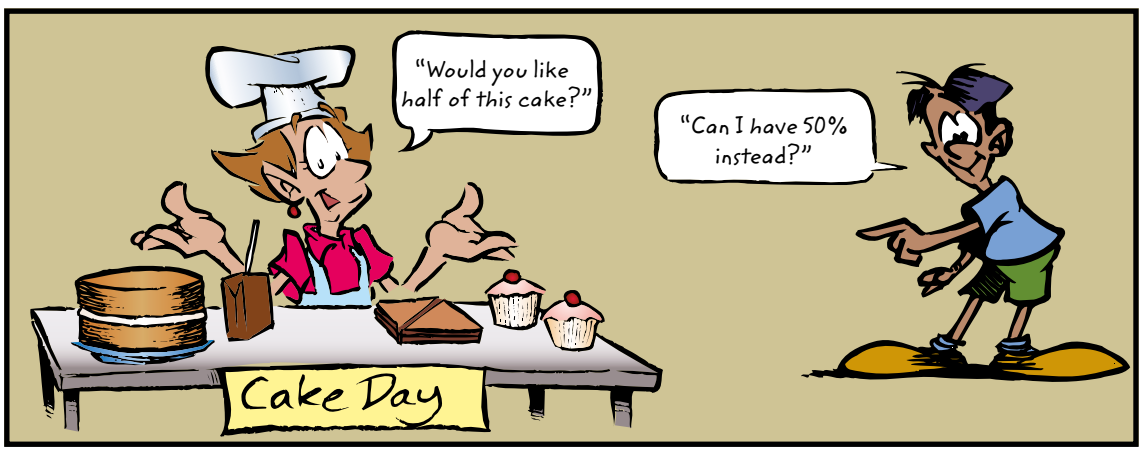


c



PERCENTAGES

11



Contents

11:01 Review of percentages
11:02 Changing fractions and decimals to percentages
11:03 Changing percentages to fractions and decimals
11:04 Finding a percentage of a quantity
11:05 One quantity as a percentage of another
11:06 Using a calculator
11:07 Applications of percentages
Investigation 11:07 What's in food?
Fun spot 11:07 Why was the butcher worried?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Fractions and Decimals 2*** [Stage 3], ***Fractions, Decimals and Percentages*** [Stage 4]

- Make connections between equivalent fractions, decimals and percentages. [Stage 3] (ACMNA131)
- Connect fractions, decimals and percentages and carry out simple conversions. (ACMNA157)
- Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies. (ACMNA158)

Working Mathematically

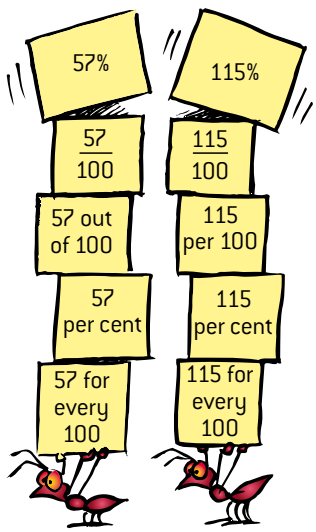
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

11:01 Review of percentages

If you have difficulty with this review, try the diagnostic check-up test 1:11D on page 38.

Summary of important ideas

- A percentage is a convenient way to write fractions that have a denominator of 100. 'Per cent', written %, means 'per 100' or 'for every 100'.
  e.g. $7\% = \frac{7}{100}$
- To write a percentage as a fraction or mixed number, first write it as a fraction with denominator 100, then simplify.
  e.g. $125\% = \frac{125}{100} = 1\frac{1}{4}$
- To change fractions to percentages, first change the denominator of the fraction to 100.
  e.g. $\frac{3}{20} = \frac{15}{100} = 15\%$
- To change a percentage to a decimal, we can write it first as hundredths and then as a decimal.
  e.g. $93\% = \frac{93}{100} = 0{\cdot}93$
- To change a decimal to a percentage, we can first write it as a fraction and then change to a decimal, OR we can multiply the decimal by 100%:
  e.g. $0{\cdot}93 = 0{\cdot}\overset{\curvearrowright}{93} \times 100\% = 93\%$



Exercise 11:01

P Foundation worksheet 11:01 Review of percentages

1 Change these percentages to fractions.

a 9%	b 7%	c 1%	d 37%	e 99%
f 97%	g 11%	h 13%	i 33%	

2 Change each percentage to its equivalent basic fraction.

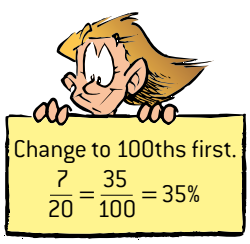
a 50%	b 25%	c 80%	d 10%	e 5%
f 26%	g 44%	h 35%	i 2%	

3 Change each percentage to a whole or mixed number.

a 100%	b 150%	c 225%	d 310%	e 175%
f 400%	g 203%	h 112%	i 109%	

4 Change each to a percentage.

a $\frac{3}{100}$	b $\frac{7}{100}$	c $\frac{9}{100}$
d $\frac{18}{100}$	e $\frac{27}{100}$	f $\frac{69}{100}$
g $\frac{50}{100}$	h $\frac{44}{100}$	i $\frac{90}{100}$



5 Express each as a percentage.

a $\frac{1}{20}$ b $\frac{3}{4}$ c $\frac{7}{10}$ d $\frac{4}{5}$ e $\frac{3}{10}$

f $\frac{13}{50}$ g 2 h $1\frac{1}{2}$ i $3\frac{1}{10}$

6 Write each percentage as a decimal.

a 14% b 37% c 81% d 7%

e 2% f 1% g 60% h 30%

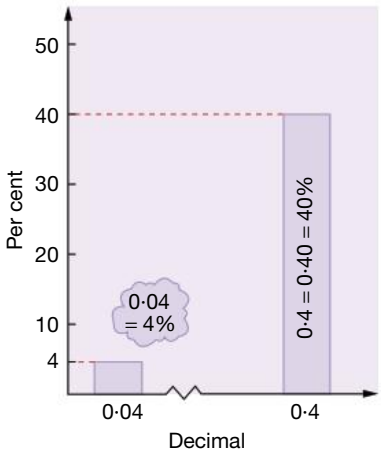
i 90% j 100% k 126% l 215%

7 Write each decimal as a percentage.

a 0·35 b 0·76 c 0·88 d 0·04

e 0·06 f 0·05 g 0·4 h 0·6

i 0·5 j 1·87 k 2·6 l 1·02



8 a $7 \times 10\%$ b $8 \times 9\%$ c $5 \times 35\%$

d $12\% \div 3$ e $95\% \div 5$ f $100\% \div 20$

g $31\% + 9\%$ h $8\% + 11\%$ i $51\% + 49\%$

j $9\% - 5\%$ k $17\% - 7\%$ l $100\% - 37\%$

m If I spend 20% of my money, what percentage of it is left?

n I have lost 3% of the worms from my worm farm. What percentage remains?

o Three friends are each given 25% of a pie. What percentage are they given altogether?

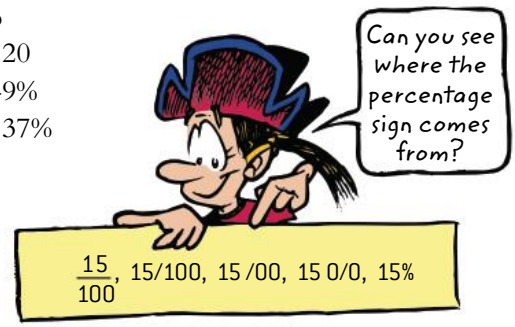
p 50% of an inheritance is shared among four sisters. What percentage does each receive?

q 20% of my money is \$12. What is 10% of my money?

r 30% of my mass is 21 kg. What is 10% of my mass?

s 100% of my money is \$400. What amount is:

i 50% ii 10% iii 25% iv 20%?



11:02 Changing fractions and decimals to percentages

PREP QUIZ 11:02

1 Write 39% as a fraction.

2 Change $\frac{3}{100}$ to a percentage.

3 Write $\frac{7}{20}$ as a percentage.

4 Write 16% as a decimal.

5 Write 0·13 as a percentage.

6 Write 100% as a whole number.

7 Write the number 1 as a percentage.

8 $\frac{3}{20} \times \frac{100}{1}$

9 $0{\cdot}07 \times 100$

10 $0{\cdot}325 \times 100$

You have already learned: $\frac{62}{100} = 0{\cdot}62 = 62\%$

However, $\frac{62}{100} \times 100 = 62$ and $0{\cdot}62 \times 100 = 62$.

Multiplying a fraction or decimal by 100 will give the number per cent.

WORKED EXAMPLES

1 Change $\frac{5}{8}$ to a percentage.

2 Change 0·625 to a percentage.

Solutions

1 $\frac{5}{8} = \frac{5}{8} \times 1$

$= \frac{5}{8} \times 100\%$

$= \frac{5}{2\,8} \times \frac{\cancel{100}^{25}}{1}\%$

$= \frac{125}{2}\%$

$= 62{\cdot}5\%$

2 $0{\cdot}625 = 0{\cdot}625 \times 1$

$= 0{\cdot}625 \times 100\%$

$= 62{\cdot}5\%$

- To express a fraction as a percentage, multiply by $\frac{100}{1}\%$.
- To express a decimal as a percentage, multiply by 100%.

Exercise 11:02

P Foundation worksheet 11:02
Changing fractions and decimals to percentages

1 Write these fractions as percentages by multiplying by $\frac{100}{1}\%$.

a $\frac{3}{5}$	b $\frac{9}{10}$	c $\frac{7}{50}$	d $\frac{17}{20}$	e $\frac{1}{4}$	f $\frac{73}{100}$
g $\frac{1}{2}$	h $\frac{8}{25}$	i $\frac{3}{20}$	j $\frac{3}{4}$	k $\frac{4}{5}$	l $\frac{9}{100}$
m $\frac{2}{3}$	n $\frac{1}{6}$	o $\frac{3}{8}$	p $\frac{5}{8}$	q $2\frac{1}{4}$	r $\frac{9}{500}$
s $\frac{27}{400}$	t $\frac{61}{300}$	u $1\frac{1}{2}$	v $2\frac{1}{4}$	w $3\frac{1}{3}$	x $3\frac{1}{7}$

$3\frac{1}{7} = \frac{22}{7}$

$\frac{22}{7} = \frac{22}{7} \times 100\%$

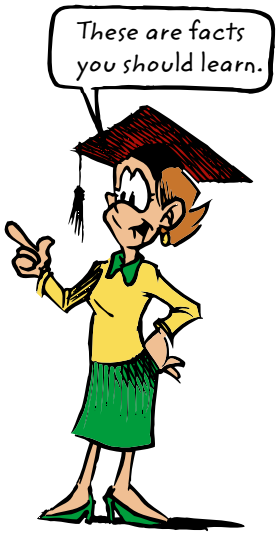
2 Write these decimals as percentages by multiplying by 100%.

a 0·37	b 0·71	c 0·95	d 0·04	e 0·7	f 0·2
g 0·1	h 0·6	i 0·345	j 0·125	k 0·375	l 0·825
m 0·3	n 0·045	o 0·85	p 0·02	q 1·8	r 1·1
s 2·5	t 3·6	u 1	v 3	w 2	x 5

3 Use the table on the right to find:

a 0·01 as a percentage
b 5% as a fraction
c $\frac{1}{3}$ as a percentage
d $\frac{1}{20}$ as a decimal
e 10% as a decimal
f 0·125 as a fraction
g 20% as a decimal
h $\frac{1}{8}$ as a percentage
i $0.\dot{3}$ as a fraction
j $12\frac{1}{2}\%$ as a decimal.

Percentage	Decimal	Fraction
1%	0·01	$\frac{1}{100}$
5%	0·05	$\frac{1}{20}$
10%	0·1	$\frac{1}{10}$
$12\frac{1}{2}\%$	0·125	$\frac{1}{8}$
20%	0·2	$\frac{1}{5}$
25%	0·25	$\frac{1}{4}$
$33\frac{1}{3}\%$	$0.\dot{3}$	$\frac{1}{3}$
50%	0·5	$\frac{1}{2}$
100%	1	1



4 To complete the table below, use the table above and the methods in the worked examples at right.

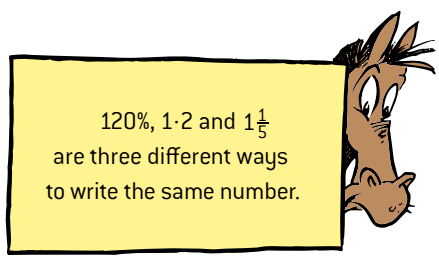
	Percentage	Decimal	Fraction
a	3%		
b			$\frac{3}{20}$
c		0·9	
d		1·5	
e	60%		
f			$\frac{2}{3}$
g			$\frac{9}{10}$
h		0·09	
i	75%		
j			$\frac{3}{8}$

WORKED EXAMPLES

1 $10\% = 0{\cdot}1 = \frac{1}{10}$
Multiply each by 6:
$60\% = 0{\cdot}6 = \frac{6}{10}$

2 $5\% = 0{\cdot}05 = \frac{1}{20}$
Multiply each by 7:
$35\% = 0{\cdot}35 = \frac{7}{20}$

3 $20\% = 0{\cdot}2 = \frac{1}{5}$
Add 1 or 100% to each:
$120\% = 1{\cdot}2 = 1\frac{1}{5}$



11:03 Changing percentages to fractions and decimals

These are harder examples. For easier examples, see 1:11D (page 38).

WORKED EXAMPLES

1 Write $3\frac{1}{4}\%$ as a fraction and as a decimal.

2 Convert 18·6% to a fraction and to a decimal.

Solutions

1 $3\frac{1}{4}\% = \dfrac{3\frac{1}{4}}{100}$

$= \dfrac{\frac{13}{4}}{100} \times 4 \times 4$

$= \dfrac{13}{400}$

$3\frac{1}{4}\% = 3 \cdot 25\%$

$= \dfrac{3 \cdot 25}{100}$

$= 3{\cdot}25 \div 100$

$= 0{\cdot}0325$

2 $18{\cdot}6\% = \dfrac{18 \cdot 6}{100}$

$= \dfrac{18 \cdot 6 \times 10}{100 \times 10}$

$= \dfrac{186}{1000}$

$= \dfrac{93}{500}$

$18{\cdot}6\% = \dfrac{18 \cdot 6}{100}$

$= 18{\cdot}6 \div 100$

$= 0{\cdot}186$

Exercise 11:03

1 Change these percentages to fractions.

a $2\frac{1}{4}\%$ **b** $1\frac{3}{4}\%$ **c** $4\frac{1}{2}\%$ **d** $6\frac{3}{4}\%$ **e** $12\frac{3}{4}\%$ **f** $16\frac{1}{2}\%$

g $14\frac{3}{4}\%$ **h** $19\frac{1}{2}\%$ **i** 19·6% **j** 14·3% **k** $106\frac{1}{2}\%$ **l** $108\frac{3}{4}\%$

2 Write these percentages as decimals.

a 4·3% **b** 18·6% **c** 7·7% **d** 11·2% **e** $21\frac{1}{2}\%$ **f** $17\frac{1}{4}\%$

g $13\frac{3}{4}\%$ **h** $4\frac{1}{2}\%$ **i** 97·3% **j** $37\frac{1}{2}\%$ **k** $112\frac{1}{2}\%$ **l** 253·8%

3 **a** $87\frac{1}{2}\%$ of children in Clareville were vaccinated against infectious diseases by the time they were 6 years old. What fraction were vaccinated?

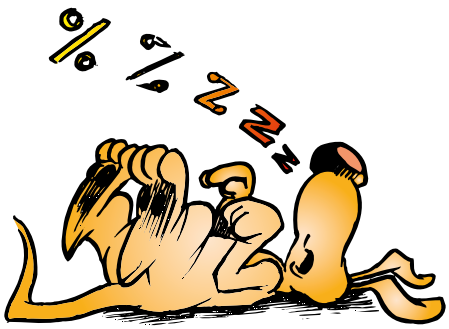
b June invested $87 000 in a savings account with an interest rate of $6\frac{1}{4}\%$ per annum (which means 'each year'). What fraction of the invested amount does she earn as interest in the first year?

c Three authors, Alan, Rob and Steve, each receive $3\frac{1}{3}\%$ of the recommended retail price of their textbook as royalties for writing the book. What fraction of the recommended retail price does each receive?

11:04 Finding a percentage of a quantity

To find a percentage of a quantity, write the percentage as a decimal (or fraction) and multiply by the quantity: e.g. 11% of 800 = 0·11 × 800 = 88.

Using decimals



WORKED EXAMPLES

1 8% of 450 g
= 0·08 × 450
= 36·00
= 36 g

2 90% of 870
= 0·90 × 870
= 783·00
= 783

3 11% of $800
= 0·11 × 800
= 88.00
= $88

Using fractions

WORKED EXAMPLES

4 8% of 450 g

$$= \frac{8}{100} \times 450$$

$$= \frac{\cancel{8}^{4}}{\cancel{100}_{2}} \times \frac{\cancel{450}^{9}}{1}$$

= 36 g

5 90% of 870

$$= \frac{9}{\cancel{10}_{1}} \times \frac{\cancel{870}^{87}}{1}$$

= 9 × 87
= 783

6 11% of $800

$$= \frac{11}{\cancel{100}_{1}} \times \frac{\cancel{800}^{8}}{1}$$

= 11 × 8
= $88

Exercise 11:04

P Foundation worksheet 11:04
Finding a percentage of a quantity

1 Find the simplest answer for each of the following.

a 10% of 880	**b** 33% of 300	**c** 6% of 1000 h	**d** 3% of 800 mg
e 16% of 50 km	**f** 24% of 25 g	**g** 25% of 440 ha	**h** 75% of 24 min
i 40% of 760 m	**j** 80% of 700 L	**k** 80% of 140	**l** 5% of 660 m

2 Give answers correct to the nearest cent.

a 15% of $400	**b** 8% of $300	**c** 2% of $900	**d** 10% of $470
e 6% of $850	**f** 4% of $11 000	**g** 40% of $3.00	**h** 7% of $5.00
i 12% of $8.50	**j** 9% of $348	**k** 11% of $333	**l** 5% of $438.60

3 **a** 30% of our football team is injured. If there are 20 in the team, how many people are injured?
b Ben decided to give 5% of his money to charity. If he has $1600, how much does he give?
c Heather has 30 birds. Of these, 60% are doves. How many doves does Heather have?
d There are 6000 library books in our library. 85% of these are in the fiction section. How many books are in this section? How many of the books are not in this section?
e Of 4000 worms in our worm farm, 3% escaped. How many do we have left?
f A housing development is built on 600 ha of bushland. Of this land, 8% is made into parks. What is the area of the parks?

11:05 One quantity as a percentage of another

PREP QUIZ 11:05

Write the first quantity as a fraction of the second.

1 90°, 180°	**2** 2 m, 7 m	**3** 5 s, 20 s	**4** 50c, $1
5 15 min, 1 h	**6** $13, $50	**7** 20 cm, 1 m	**8** 10 g, 50 g
9 $1.50, $4.50	**10** 750 cm, 800 cm		

The fractions obtained above could be changed to percentages by multiplying by $\frac{100}{1}$:

e.g. $\frac{5\text{ s}}{20\text{ s}} = \frac{1}{4}$

$= \frac{1}{\cancel{4}_1} \times \frac{\cancel{100}^{25}}{1}\%$

$= 25\%$



Estimate what percentage is white.

To write one quantity as a percentage of another:
- make sure the quantities are in the same units
- write the first as a fraction of the second
- change the fraction to a percentage by multiplying by $\frac{100}{1}$.

WORKED EXAMPLES

1 I have 800 Australian stamps. My whole collection has 6000 stamps. What percentage of my stamps are Australian?

2 For every kilogram of sugar bought, 200 g will be wasted. What percentage is wasted?

Solutions

1 $\text{Fraction} = \frac{800 \text{ stamps}}{6000 \text{ stamps}} = \frac{800}{6000} = \frac{8}{60} = \frac{2}{15}$

$\text{Percentage} = \frac{2}{\cancel{15}_{3}} \times \frac{\cancel{100}^{20}}{1}\% = \frac{40}{3}\%$

$= 13\frac{1}{3}\%$

$\therefore 13\frac{1}{3}\%$ of my stamps are Australian.

2 $\text{Fraction} = \frac{200\,\text{g}}{1\,\text{kg}} = \frac{200\,\text{g}}{1000\,\text{g}} = \frac{200}{1000} = \frac{1}{5}$

$\text{Percentage} = \frac{1}{\cancel{5}_{1}} \times \frac{\cancel{100}^{20}}{1}\%$

$= 20\%$

$\therefore$ 20% of the sugar is wasted.



Exercise 11:05

P Foundation worksheet 11:05
One quantity as a percentage of another

1 **a** Express 10 cents as a percentage of \$1.
b What percentage of 400 g is 20 g?
c What percentage of 200 m is 160 m?
d Express 70 points as a percentage of 100 points.
e Express 36 g as a percentage of 300 g.

2 Write the following as percentages:
a 500 g of 1 kg
b 15 min of 1 h
c 180° of 1 revolution
d 7500 m^2 of 1 ha
e 60 mm of 8 cm
f 40 years of 1 century
g 13 of 200
h 420 of 8000

3 **a** I have 308 sheets of blue paper. Altogether I have 1400 sheets of paper. What percentage of the paper is blue?
b 12 of my 30 birds are Gouldian finches. What percentage are not Gouldian finches?
c I sold my computer for \$500. What percentage is this of the cost price, if I bought the computer for \$4000?
d From \$200 000 I received in one year, I paid \$85 000 in wages. What percentage of the money received did I pay as wages?
e Of the 40 books Alan wrote, 5 are now out of print. What percentage of Alan's books are out of print?



Discuss: 50 g is 125% of 40 g.

4 At the school concert there were 8 teachers, 30 boys, 40 girls and 82 parents. What percentage of those at the concert were:

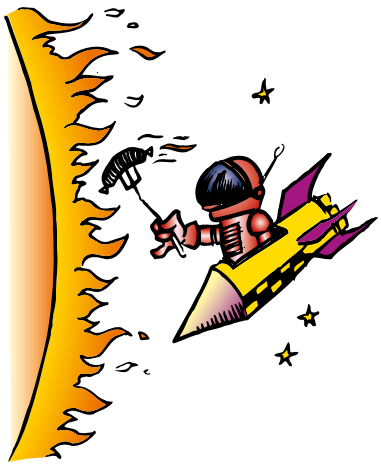
a girls **b** teachers **c** boys
d parents **e** not girls **f** boys or girls?

5 **a** The time I took to finish the ski course was 48 s. My previous best time was 50 s. By what percentage was the time better than my previous best time?

b To get to school, 13 students travel by bus, 10 walk and 7 come by car. What percentage of these students walk to school?

c The Davis Cup is an international tennis competition held each year. From 1947 to 1986, Australia won 19 times. What was the percentage of times that Australia won during the period?

d A spaceship travelling to the sun from Earth has travelled 120 000 000 km. If the sun is 150 000 000 km from Earth, what percentage of the journey has the spaceship still to complete?



e The lead in a propelling pencil is 6 cm long before use. What percentage of the lead has been used when its length is 57 mm?

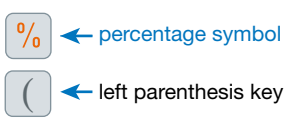
f A container holds 10 L of juice. What percentage of the juice remains after seven 230 mL cups have been filled?

11:06 Using a calculator

Calculations in this chapter can also be done using a calculator. Most calculators have a 'percentage' (%) key, either as a key itself or as a secondary function.



On a Casio fx-82AU PLUS calculator, the percentage key is a secondary function above the left parenthesis key.



The percentage key allows you to enter a percentage directly instead of as a fraction of one hundred. Work through the examples below to see how this can be done.

WORKED EXAMPLES

Note: Different types of calculator may use different keys. Check the instruction booklet for your calculator. The workings below are for a Casio fx-82AU PLUS calculator.

1 Find 12% of $500.

Enter: 12 SHIFT % × 500 =

Answer: $60

Don't forget to add correct units to your answer if necessary, e.g. $, m, cm, %

2 Calculate $17\frac{1}{2}$% of 68 m.

Enter: SHIFT ▬ 17 ▶ 1 ▼ 2 ▶ SHIFT % × 68 =

Answer: 11.9 m

You may or may not need to enter parentheses around a fraction, depending upon the fraction mode used.

3 What percentage is $16 of $80?

Enter: 16 ÷ 80 SHIFT % =

Answer: 20%

4 I paid $18 230 tax on my salary of $81 430. What percentage is this, correct to one decimal place?

Enter: 18230 ÷ 81430 SHIFT % =

Answer: 22·4%

Exercise 11:06

1 Find:

a 10% of $200	**b** 20% of $350	**c** 40% of $480	**d** 25% of $240
e 30% of $550	**f** 75% of $960	**g** 15% of 240 m	**h** 24% of 700 L
i 45% of 40 km	**j** 7% of 250 mL	**k** 120% of $35	**l** 250% of 48 t

2 Determine the percentage:

a $5 of $20	**b** $12 of $60	**c** $15 of $120
d $7 of $35	**e** $18 of $360	**f** $3 of $150
g 5 m of 25 m	**h** 12 cm of 150 cm	**i** 30 mL of 200 mL
j 14 km of 70 km	**k** $100 of $50	**l** 240 g of 150 g

3 Find each value correct to the nearest cent:

a 10% of $24	**b** 20% of $36	**c** 45% of $48
d 17% of $34	**e** 55% of $555	**f** 65% of $9.60
g 15·5% of $20	**h** 2·4% of $750	**i** 37·5% of $94
j $7\frac{1}{2}$% of $250	**k** $12\frac{1}{4}$% of $75	**l** $16\frac{2}{3}$% of $98

4 Find the following values correct to one decimal place:

a 14% of 20 m b 35% of 35 L c 45% of 730 g
d 2·5% of 25 km e $33\frac{1}{3}$% of 530 cm f $62\frac{1}{2}$% of 92 t

5 Determine each percentage, correct to two decimal places where necessary:

a \$5 of \$15 b \$12 of \$45 c \$18 of \$48
d \$5.50 of \$9 e 12 cm of 18 cm f 7·5 m of 30 m
g \$12.20 of \$20 h 9·6 kg of 56 kg i $7\frac{1}{2}$ pies of 24 pies

6 You can avoid using the percentage key by entering each percentage into the calculator as a decimal. This is quicker. For example, 24% can be entered as 0·24. Calculate these percentages:

a 12% of \$250 b 25% of \$420
c 40% of \$75 d 7% of \$45
e 36% of 360 m f 27% of 90 L
g 150% of 400 kg h 2.8% of \$500

To find 24% of \$75:
Enter: .24 × 75 =
Answer: \$18
Note: 5% = 0·05 and 125% = 1·25

7 As in Question 6, the percentage key can be ignored when finding what percentage one quantity is of another, simply by moving the decimal point two places to the right after dividing. This is quicker. Find these percentages to the nearest whole per cent:

a \$15 of \$25 b \$16 of \$48
c \$7 of \$20 d \$12 of \$240
e 35 m of 50 m f 144 g of 600 g
g \$3.80 of \$20 h 12 km of 56 km

To find what percentage \$24 is of \$75:
Enter: 24 ÷ 75 =
Answer: 0·32 (i.e. 32%)
Note: 0·5 = 50% and 0·05 = 5%

11:07 Applications of percentages

PREP QUIZ 11:07

Write as a percentage:

1 0·37 **2** 0·7 **3** $\frac{3}{4}$ **4** $1\frac{1}{5}$

Calculate:

5 20% of \$15 **6** 75% of 48 m **7** 150% of 8 L

What percentage is:

8 8 cm of 20 cm **9** \$12 of \$60 **10** 2·5 kg of 10 kg?

The following exercise requires all the skills met in this chapter.

Exercise 11:07

1 Rui has $\frac{3}{10}$ of her money invested in shares, $\frac{6}{10}$ in a credit union account and the rest kept in a safe at home. What fraction of her money is kept in the safe? What percentage of her money is that?

2 Charlie has 11 guinea pigs and one rabbit. What fraction of his pets are guinea pigs? What percentage is this?



3 During the football season, I. Kickemover was successful with exactly $\frac{3}{5}$ of his attempts to kick a goal.

- **a** What was his percentage success?
- **b** What fraction of his attempts were unsuccessful?

4 76% of the students in my class have brown hair. What percentage of the students in my class do not have brown hair? If my class has less than 50 students, how many people are in my class?

5 Alan and Rhonda have five children: 4 girls and 1 boy.

- **a** What percentage of the children are girls?
- **b** What percentage of the children are boys?
- **c** What fraction of the family are male?
- **d** What percentage of the family are female? (Give your answer to the nearest whole per cent.)



6 There are 1000 students in our school. Males make up 0·52 of the school.

- **a** How many students are male?
- **b** What percentage of students are male?
- **c** If all students are female or male, what fraction of students are female?

7 Interest of $150 was charged on a loan of $750 for one year.

- **a** What fraction of the loan is the interest?
- **b** What percentage of the loan is the interest?

8 When Marita was 14 years old she had a fine collection of stamps. After a further 16 years she had increased her collection by 300%.

- **a** If she had 2500 stamps when she was 14, how many did she have when she was 30?
- **b** How many times bigger was her collection when she was 30 than when she was 14?

9 If I scored $\frac{18}{20}$ in my spelling test, what was my score as a percentage?

10 Zac spent 20% of his money to buy a car for $8600. How much did he have before buying the car?

INVESTIGATION 11:07 WHAT'S IN FOOD?

Do you ever wonder what is in the food you are eating?

Food	% protein	% fat	% carbohydrate	kJ/100 g
Bacon, fried streaky	24·0	46·0	0	2200
Bananas	1·1	[trace]	19·2	322
Beef, roast	24·2	23·8	0	1343
Butter	0·4	85·1	[trace]	3318
Carrots	0·7	[trace]	5·4	96
Eggs, poached	12·4	11·7	[trace]	669
Chocolate bar	5·0	20·0	70·0	1920
Milk	3·4	3·7	4·8	276
Peas	5·0	[trace]	7·7	205

'[trace]' means that there is only a trace amount present (i.e. too little to measure accurately), but more than zero.



1 What percentage of each food is protein?
   a beef **b** carrots **c** chocolate bar

2 What percentage of each food is fat?
   a bacon **b** poached eggs **c** milk

3 How many kilojoules (kJ) per 100 g are there in:
   a bananas **b** butter **c** peas?

FUN SPOT 11:07 WHY WAS THE BUTCHER WORRIED?

Answer the question for each letter. Match each letter with its answer below to solve the riddle.

Find:

T 10% of 120	**A** 20% of 80	**I** 30% of 50
O 40% of 90	**S** 5% of 60	**T** 25% of 160
A 75% of 36	**H** 15% of 40	

What percentage is the first number of the second?

E 30, 60	**W** 12, 48	**A** 7, 35
B 9, 30	**J** 36, 200	**S** 49, 70
S 4, 80	**K** 45, 75	

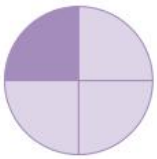


☐☐☐	☐☐☐	☐☐☐	☐☐	☐☐☐☐☐
6 15 70	18 36 30	25 16 3	27 12	5 40 50 20 60

MATHS TERMS 11

fraction (or common fraction)

- part of a whole, e.g. $\frac{1}{4}$ of a pie
- a number between 0 and 1 formed by dividing a number by a bigger number, e.g. $\frac{7}{8}$ or $7 \div 8$



percentage (or percentage fraction)

- 'per cent' means 'out of 100'
- a percentage fraction has a denominator of 100 and uses the symbol %, e.g. $15\% = \frac{15}{100}$

DIAGNOSTIC TEST 11 PERCENTAGES

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1	Change these percentages to simple fractions. a 7% b 30% c 25% d 15%	11:01
2	Change these percentages to decimals. a 17% b 30% c 25% d 150%	11:01
3	Convert these fractions to percentages. a $\frac{23}{100}$ b $\frac{3}{10}$ c $\frac{1}{4}$ d $\frac{4}{5}$	11:01
4	Convert these decimals to percentages. a 0·15 b 0·7 c 0·09 d 0·35	11:01
5	Convert these fractions to percentages. a $\frac{1}{8}$ b $\frac{2}{3}$ c $\frac{1}{6}$ d $\frac{9}{200}$	11:02
6	Convert these decimals to percentages. a 0·155 b 1·7 c 0·093 d 2·35	11:02
7	Change these percentages to fractions. a $7\frac{1}{2}\%$ b $33\frac{1}{3}\%$ c $37\frac{1}{2}\%$ d $16\frac{2}{3}\%$	11:03
8	Find: a 10% of \$70 b 25% of 48 m c 5% of 360 kg	11:04
9	Write as a percentage: a \$20 of \$80 b 16 cm of 20 cm c 21 g of 56 g	11:05
10	Find: a 38% of \$260 b $33\frac{1}{3}\%$ of 87 m c 17·5% of 840 kg	11:06
11	Write as a percentage: a \$23 of \$184 b 16 min of 1 h c 7·5 kg of 24 g	11:06

ASSIGNMENT 11A Chapter review

1 Change each of these to a percentage.

a $\frac{3}{4}$ **b** $\frac{7}{20}$ **c** $\frac{2}{3}$

d 0·08 **e** 0·8 **f** 1·35

2 Rewrite as a fraction in its simplest form.

a 70% **b** 15% **c** 40%

d 6% **e** 120% **f** 17%

3 Write each percentage as a simple fraction.

a $33\frac{1}{3}\%$ **b** $12\frac{1}{2}\%$ **c** $62\frac{1}{2}\%$

4 Convert these percentages to decimal numerals.

a 4% **b** 40% **c** 140%

d 12·5% **e** 32·2% **f** 7·5%

5 Find:

a 30% of \$150 **b** 25% of \$240

c 50% of 3·2 m **d** 60% of 20 L

e 150% of 16 g

6 Write as a percentage:

a \$10 of \$40

b 20 marbles of 50 marbles

c 15 points of 25 points

d 30 L of 40 L

e 90 m of 60 m

7 Express the first quantity as a percentage of the second.

a 25 cm of 1 m

b 600 mL of 2 L

c 80 cents of \$4

d 13 years of 1 century

e 36 degrees of a right angle

8 There are 40 pets in Critter Crescent.

a 30% of the pets are cats. How many cats are there?

b 24 of the pets are dogs. What percentage of the pets is this?

c What percentage of the pets are neither cats nor dogs?

9 **a** 20% is added on to a price of \$50. What is the increased price?

b The increased price in part **a** is now discounted by 20%. What is this new reduced price?

10 **a** What percentage of 40 is 16?

b What percentage is 16 of 40?

c What percentage of 16 is 40?

ASSIGNMENT 11B Working mathematically

1 Two fruit growers send their fruit to market. One grower sends 70 cases and the other sends 96 cases. They receive \$1660 altogether. How much should each grower get?

2 My car averages 9 km per litre of petrol. How much will it cost for petrol when I drive from Bega to Bombala, a distance of 126 km, if petrol costs 159 cents per litre?

3 A block of land is rectangular in shape. Its perimeter is 360 m and it is known to be 8 times longer than it is wide. Find its length and width.

4 While waiting at the airport I decide to buy some magazines to read. In the airport bookstore I see five magazines priced at \$2.20, \$3.40, \$1.20, \$1.60 and \$2.95. What possible combinations of magazines could I buy for exactly \$5?

5 When a truck is loaded with 500 boxes of mass 5·8 kg each, the total mass of the loaded truck is 8·1 tonnes. What is the mass of the truck when it is unloaded?



ASSIGNMENT 11C Cumulative revision

1 Simplify:

a $(6 + 3) \times 2$ **b** $(7 - 2) \times (7 + 3)$ **c** $20 - 3 \times 2 + 5$ **d** $4 - 3 + 2 \times 4$

1:06

2 Simplify:

a $\frac{1}{2}$ of 42 **b** $\frac{1}{4}$ of 72 **c** $\frac{1}{5}$ of 60 **d** $\frac{2}{3}$ of 48

9:06

3 If $x = 7$, find the value of:

a $5 \times x$ **b** $5 \times x + 3$ **c** $5 \times x - 4$ **d** $5 \times x - x$

4:06

4 Answer the following:

a $7 - 4$ **b** $4 - 7$ **c** $-4 + 7$

d $-4 - 7$ **e** $4 - (-7)$ **f** $-4 - (-7)$

7:06

5 What is the value of the 5 in each of the following?

a 0·15 **b** 1·52 **c** 5·16 **d** 12·715

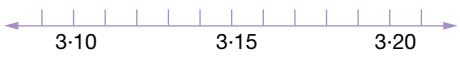
6:01

6 Copy these number lines and write a number under each mark.

a



b



6:01

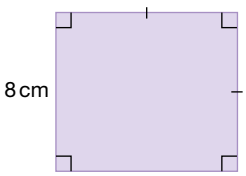
7 Write as decimals:

a $\frac{72}{100}$ **b** $\frac{105}{1000}$ **c** $\frac{7}{100}$ **d** $\frac{45}{1000}$

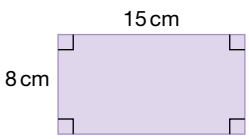
6:07

8 Find the area of each figure.

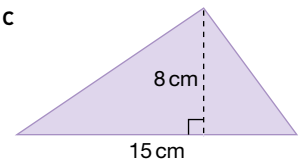
a



b



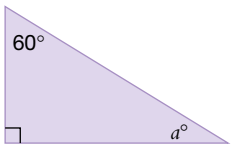
c



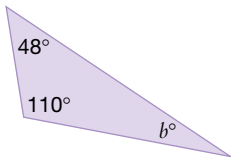
10:03, 10:04

9 Find the value of each pronumeral.

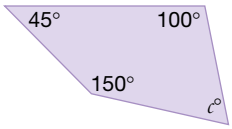
a



b



c



8:04

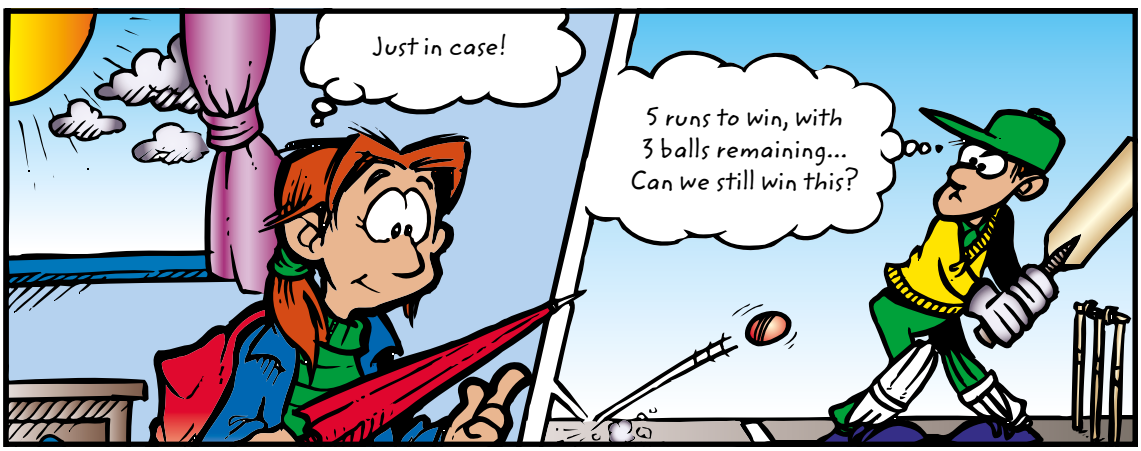
10 **a** Find the volume of a rectangular prism that has a length of 50 cm, a breadth of 40 cm and a height of 30 cm.

b How many litres of water would fill this rectangular prism?

10:07, 10:08

PROBABILITY

12



Contents

12:01 The language of probability
12:02 Sample spaces and experiments
12:03 The probability of simple events
12:04 What chance of survival?
Investigation 12:04 Describing probability
12:05 Using probability
Investigation 12:05 Choosing cards
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Statistics and Probability

Selections from ***Probability 1*** [Stage 4]

- Construct sample spaces for single-step experiments with equally likely outcomes. (ACMSP167)
- Assign probabilities to the outcomes of events and determine probabilities for events. (ACMSP168)
- Identify complementary events and use the sum of probabilities to solve problems. (ACMSP204)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

12:01 The language of probability

We can describe the *chance of something happening* using either words or numbers.

Words:	impossible	unlikely	(fifty-fifty) even chance	likely	certain
Numbers:	0% or 0	between 0 and $\frac{1}{2}$	50%, $\frac{1}{2}$ or 0·5	between $\frac{1}{2}$ and 1	100% or 1

PREP QUIZ 12:01

For each of the events listed below, estimate its chance to be:

A an even chance
B less than an even chance
C more than an even chance.

Give a reason for each answer.



1 If you toss a coin it will come down tails.
2 The next time you catch a bus it will break down.
3 On your next birthday you will be given a present.
4 You will score 100 runs when you next play cricket.
5 The next child born in the nearest hospital will be a girl.

6 Match each term in the middle column below to its meaning and its percentage chance of occurring, e.g. 'impossible': 0%, 'The event will never happen.'

	impossible	
0%	certain	The event is more likely to happen than not.
50%	unlikely	The event is more likely not to happen.
between 50% and 100%	likely	The event will always happen.
100%	even chance	The event will never happen.
between 0% and 50%	fifty-fifty chance	The event is as likely to happen as not.
	no chance	

WORKED EXAMPLE

Order the following events from *least likely* to *most likely*.

A A normal dice is rolled. The result is either a 1, a 2 or a 3.

B The next person to walk into the classroom has a birthday in December.

C If I pick a counter at random from this group, it will be red.



Solution

The least likely option is B, because December is 1 of 12 possible months in which a person could be born. For a person to be born in December is unlikely.

The next most likely is A, because half of the possible dice rolls will be successful. To roll a 1, 2 or 3 is a *fifty-fifty* (or *even*) chance.

The most likely is C, because 4 out of 6 counters are red, which is more than half. It is more likely than not that a red counter would be chosen.

Exercise 12:01

1 **a** What does it mean for something to be *impossible*?
b What does it mean for something to be *certain*?
c What does it mean for an event to have an *even chance* of happening?
d What does it mean for an event to be *unlikely* to happen?

2 Choose a label that describes the chance of each event happening.

impossible | unlikely | even chance (fifty-fifty) | very likely | certain

a You will travel on a train next week.
b Your next teacher will be more than 50 years old.
c You will get more than 95% in your next Maths test.
d You will see a living *Tyrannosaurus rex* today.
e If you roll a dice you will get a number less than 7.
f If you toss a coin it will come up tails.



3 **a** Arrange Question 2 parts **a** to **c** in order from least likely to most likely.
b Arrange Question 2 parts **d** to **f** in order from least likely to most likely.

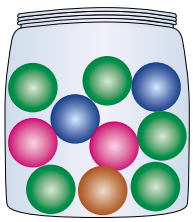
4 For each event, choose one of these categories:
impossible, unlikely, even chance, likely, certain.

a A dice is rolled.



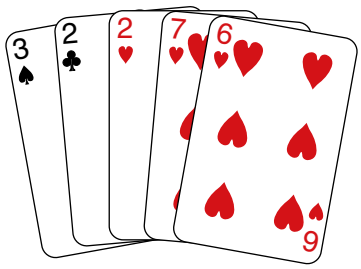
Event 1: A 6 is rolled.
Event 2: A number less than 4 is rolled.
Event 3: A number greater than 3 is rolled.
Event 4: A zero is rolled.
Event 5: A number less than 7 is rolled.

b A ball is drawn at random from the jar shown.



Event 1: The ball is green.
Event 2: The ball is brown.
Event 3: The ball is red.
Event 4: The ball is coloured.
Event 5: The ball is either green or blue.

c One card is chosen at random from these five cards.



Event 1: The card is a heart ♥.
Event 2: The card is less than 8.
Event 3: The card is a diamond ♦.
Event 4: The card is a 2.
Event 5: The card is not a 3.

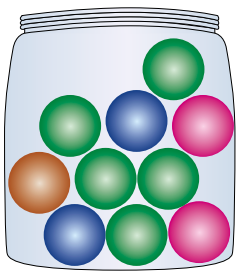
5 Order the following events from *least likely* to *most likely*.

a A dice is rolled.



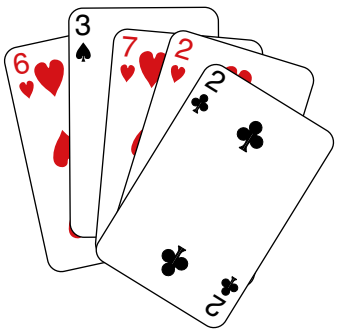
A The number showing is even.
B The number showing is larger than 4.
C The number showing is less than 6.

b A ball is drawn at random from the jar shown at right.



A The ball is brown.
B The ball is green.
C The ball is blue or red.

c One card is chosen at random from these five cards.



A The card is a 2.
B The card is a 7.
C The card is greater than 3.

6 **a** Research the meaning of **complementary events**.
b Give two examples of complementary events.

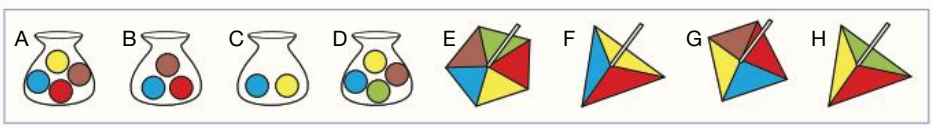
12:02 Sample spaces and experiments

When we carry out a chance (random) experiment, it is important to know what outcomes are possible. The set of all possible outcomes is called the **sample space**.

PREP QUIZ 12:02

List all possible outcomes when we:

1 toss a coin
2 roll a normal dice
3 choose a ball from bag A
4 choose a ball from bag B
5 choose a ball from bag C
6 choose a ball from bag D
7 spin spinner E
8 spin spinner F
9 spin spinner G
10 spin spinner H



Probability terms you should know

- **sample space** — The set of all possible outcomes of a random process.
  e.g. If you roll a normal dice once, the sample space is 1, 2, 3, 4, 5, 6.
- **chance experiment** — An experiment where the outcomes are *random*, in that they occur by chance, e.g. choosing a coloured ball from a bag without looking.
- **trial** — One case of carrying out an experiment, e.g. spinning a spinner once.
- **chance event** — A category that relates to a chance experiment, e.g. rolling a 3 or 4 on a normal dice.
- **favourable outcome** — An outcome from a trial that fits the category or the event being considered.
  e.g. A coin toss coming up heads is a 'favourable' outcome if we are counting the number of heads tossed in 10 tosses (or trials).
- **equally likely outcomes** — A situation where the outcomes have the same chance of occuring in any trial, e.g. picking a card from a standard pack.
- **probability** — The chance of an event happening, given in words, as a fraction, as a decimal or as a percentage.
  e.g. The probability of a coin toss coming up heads is 50%.
- **complementary events** — The complement of an event happening is that the event does not happen.
  e.g. 'Our team wins' and 'our team does not win' are complementary events.

Exercise 12:02

1 Write the sample space for each of these experiments.

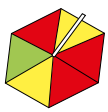
- **a** I will toss a coin to see which side shows.
- **b** I will drop a drawing pin onto the table to see how it lands.
- **c** I will drop a cone (3D solid) onto the floor to find how it lands.
- **d** I will ask a person I see, 'Have you met the Prime Minister?'
- **e** I will roll a normal dice to see what number is showing on the top face.
- **f** I will choose a card from a standard pack to see which colour I select.

2 Read this passage and answer the following questions.

> Trinh wanted to find the experimental probability of rolling 'a number greater than 4' on a normal dice. To do this, she rolled a normal dice 50 times.

- **a** When Trinh rolled a dice, was this a chance experiment? Why is this so?
- **b** What did Trinh roll on her first trial?
- **c** What chance event was being considered by Trinh?
- **d** What is the sample space of a roll of the dice?
- **e** Was the result of Trinh's first trial a favourable result?
- **f** For each roll of the dice, are the outcomes equally likely?
- **g** Is the probability of rolling 'a number greater than 4' greater or less than 50%?

3 The spinner shown is spun once.



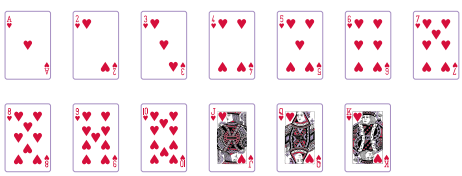
- **a** What is the sample space?
- **b** Is the chance of spinning each colour equally likely? Explain your answer.
- **c** Which is the most likely outcome?
- **d** Which is the least likely outcome?

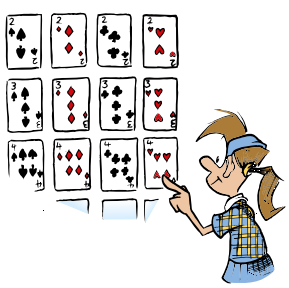
4



- **a** If three coins are tossed, what are the possible outcomes?
- **b** Are all of the possible outcomes equally likely?
- **c** Carry out this experiment at home to check your answer to part **b**.

5 A standard pack of playing cards consists of the cards Ace (A), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q) and King (K), in each of the four suits: hearts ♥, diamonds ♦, clubs ♣ and spades ♠. All of the hearts are shown below. The Jack, Queen and King are called 'court cards' (or 'picture cards').





One card is chosen at random from a standard pack.

- **a** List the sample space if we are considering the type of card but not its suit.
- **b** List the sample space if we are considering only the suit.
- **c** List the sample space if each card is considered to be different.

6 In your own words, explain the meaning of each term:

- **a** sample space
- **b** chance event
- **c** complementary events
- **d** probability
- **e** favourable outcome
- **f** equally likely outcomes

12:03 The probability of simple events

PREP QUIZ 12:03

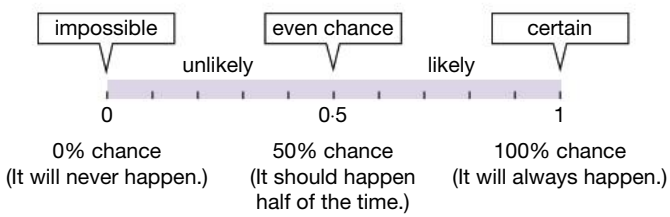
True or false? Give a reason for each answer.

1. On my last three attempts to jump 1·5 metres, I failed. I am likely to succeed the next time I try.
2. If I roll two dice, their total must be 12 or less.
3. If I choose at random a number between 1 and 10, it will have 1 digit.
4. Even if I have tossed 4 heads in a row, the chance of getting a head with the next toss of a coin is an even chance.
5. If I choose at random a counter from a jar that contains 999 red counters and 1 yellow counter, it is sure to be red.

The *probability* of something happening is its *chance* of happening.

We often use probability in everyday life: 'Where is the most likely place to find my wallet?' 'How can I best avoid an accident?' 'Which is the most likely answer?' 'Is it possible that a pedestrian will step out from behind that car?'

- Probability can be expressed as words, e.g. 'There is an even chance that we will win'.
- Probability can be expressed as a fraction, e.g. 'Our chance of winning is $\frac{1}{2}$'.
- Probability can be expressed as a decimal, e.g. 'Our chance of winning is 0·5'.
- Probability can be expressed as a percentage, e.g. 'We have a 50% chance of winning'.



An **unlikely** event has a probability of between 0% and 50%.

A **likely** event has a probability of between 50% and 100%.



If all outcomes are *equally likely*, then the probability of an event A is given by:

$$P(A) = \frac{\text{number of favourable outcomes}}{n}$$

where n is the total number of outcomes in the sample space.

When all outcomes are equally likely, the event is called a **simple event**.

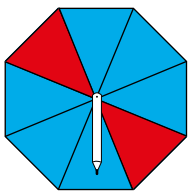
WORKED EXAMPLES

1 One of these pictures will be chosen at random. What is the probability that the picture chosen is of the following?

a a woman
b a man
c a man or a woman
d someone wearing a hat



2 What is the probability that this spinner will stop on red?



Solutions

1 **a** $P(\text{a woman}) = \frac{3}{4}$ or 75%
b $P(\text{a man}) = \frac{1}{4}$ or 25%
c $P(\text{a man or a woman}) = \frac{4}{4}$ or 100%
d $P(\text{wearing a hat}) = \frac{3}{4}$ or 75%

2 There are 8 sections on the spinner, so the number of outcomes is 8.
There are 2 favourable (red) outcomes.

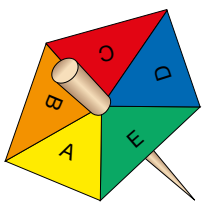
$$P(\text{red}) = \frac{\text{number of favourable outcomes}}{n}$$
$$= \frac{2}{8}$$
$$= \frac{1}{4} \text{ or } 0 \cdot 25 \text{ or } 25\%$$

Exercise 12:03

P Foundation worksheet 12:03 Probability

1 List all possible outcomes of each chance experiment (i.e. list the sample space).

a A dice is rolled.
b A coin is tossed.
c This spinner is spun.
d A coloured ball is chosen from the jar at right.





Note: If there were two green balls, we would name them 'green 1' and 'green 2'.

2 How many outcomes are in each sample space?

a A number between 4 and 7 is chosen at random.
b A ticket is chosen from tickets numbered 1 to 99.
c A ticket is chosen from tickets numbered 0 to 100.
d Ten different cards are placed in a hat and one is chosen at random.

3 Liam rolls a dice. Write, as a fraction, the probability that he will roll:

a a 2 **b** a zero

c an even number **d** an odd number

e either a 1 or a 6 **f** a number less than 4

g a number less than 7 **h** a number other than 2.



4 A coin is tossed.

a Write, as a fraction, the probability of tossing:

i a head **ii** a tail.

b Write, as a percentage, the probability of tossing:

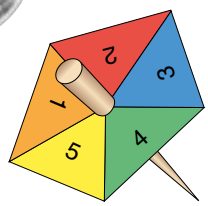
i a head **ii** a tail.



5 The spinner at right (marked 1 to 5) is spun. Write, as a fraction, the probability of spinning:

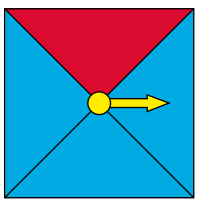
a a 3 **b** a number that is not 3

c an even number **d** a zero.

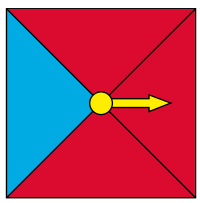


6 What is the probability, as a fraction, that each spinner below will stop on red?

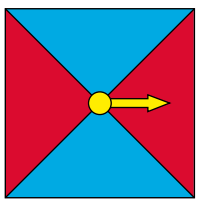
a



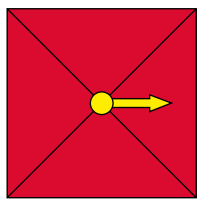
b



c



d



7 Write each of the probabilities in Question **6** as a percentage.

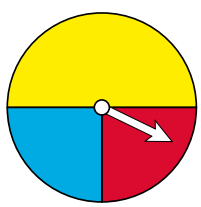
8 Use the diagrams in Question **6** to find the probability that each spinner will stop on blue, as a fraction and as a percentage.

9 **a** What is the probability, as a fraction, that this spinner will stop on:

i yellow **ii** blue **iii** red?

b Are the outcomes in part **a** equally likely?

c What do you get when you add all three possible spinner probabilities together?



Event: tossing a coin

$P(\text{head}) = \frac{1}{2}$

$P(\text{tail}) = \frac{1}{2}$

$P(\text{head}) + P(\text{tail}) = 1$



Event: rolling a dice

$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$

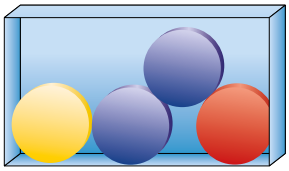
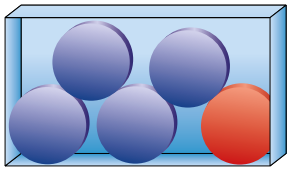
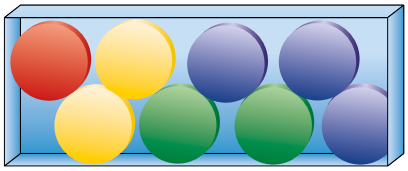
$P(4) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$



$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

The sum of the probabilities of the possible outcomes is 1 or 100%.

'Rolling a 6' and 'not rolling a 6' are *complementary events*, so their probabilities add to 1.

10 A  B  C 

Counters have been slid into these narrow containers. One counter will be chosen at random from each container. For each container, find the probability of choosing:

a a red counter **b** a blue counter **c** a yellow counter

d a counter that is either red or blue **e** a counter that is not red.

11 The 5 on this dice is covered by a sticker showing a red 1.



When I roll this dice, what is the probability that I will roll:

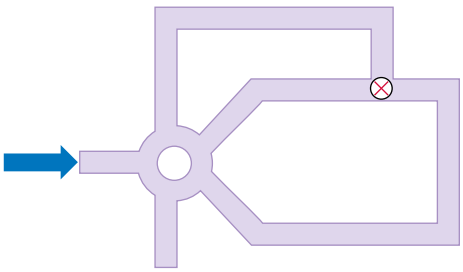
a a 1 **b** a 3 **c** 3 or less

d a 5 **e** a number greater than 1?

12 Naomi approaches a roundabout from the west. To leave the roundabout she chooses one of the other exit roads at random.

a What is the probability that she chooses the dead end?

b What is the probability that she does not choose the dead end (i.e. the complement of part **a**)?

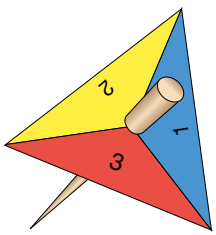


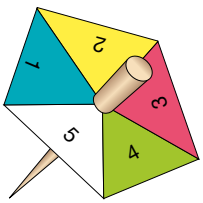
13 Of 100 tickets sold in a raffle, Luke bought 10, Heather bought 5 and Alan bought 1. A ticket was chosen at random to determine who won the prize. What is the probability that the prize was won by:

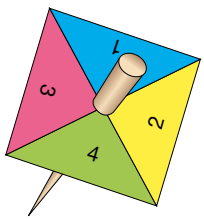
a Alan **b** Luke **c** Heather **d** none of these three people?

How many of the 100 tickets would I need to buy to make my chance of winning be the following? Explain why.

e 0 **f** 1

14 A 

B 

C 

For which of these spinners is the probability of spinning a 3 equal to the following?

a 20% **b** 25% **c** $\frac{1}{3}$

15 From the 13 cards shown below, one card is chosen at random. What is the probability that the card chosen is:

a the 7 **b** the Ace **c** a picture card **d** a heart ♥

e a spade ♠ **f** either a 9 or a 10 **g** a card that is not a picture card?



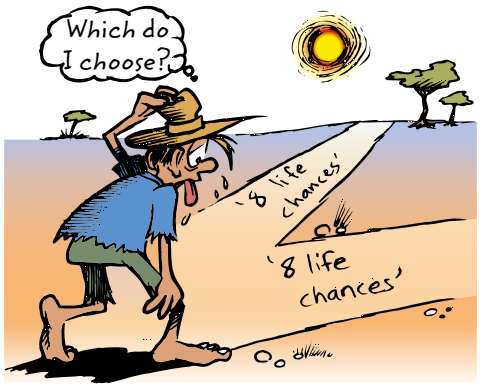
16 From a standard pack of 52 cards, a card is chosen at random. What is the probability that the card will be:

a a diamond ♦ **b** a red card **c** not a red card **d** a 3

e a picture card **f** either an Ace or a King **g** a card that is not the Ace of hearts?

12:04 What chance of survival?

Lost in the outback, Mac the prospector decides to follow whatever tracks he finds, because some of the tracks are sure to lead to water. When he comes to a point where there is more than one path to choose from, he is just as likely to choose any of them.



In each of the cases below, Mac begins with a number of 'life chances' and we will work out how many of these 'chances' lead to water. After we have found what fraction of his chances lead to water, we can calculate his percentage chance of survival.

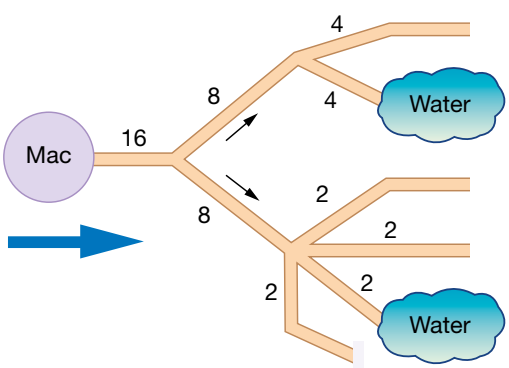
Note: Mac never retraces his steps.

WORKED EXAMPLE

Starting with 16 life chances, work out Mac's percentage chance of survival for the following tracks. At each fork in the track, the life chances must be shared equally among the new tracks.

Solution

From the diagram, we can see that 6 out of 16 life chances reach water.



$\therefore$ Mac's chance of survival

$$= \frac{\cancel{6}^{3}}{\cancel{16}_{2\,4}} \times \frac{\cancel{100}^{25}}{1}\,\%$$

$$= \frac{75}{2}\%$$

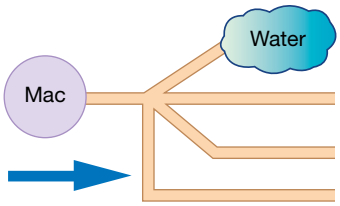
$$= 37\tfrac{1}{2}\%$$

Mac has a $37\frac{1}{2}\%$ chance of survival.

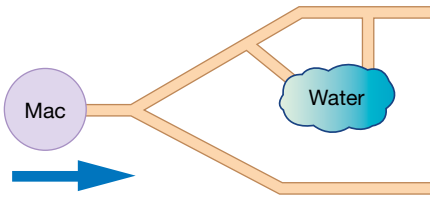
Exercise 12:04

Find Mac's percentage chance of survival for each of the following cases.

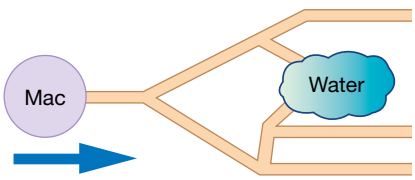
1 Start with 4 life chances.



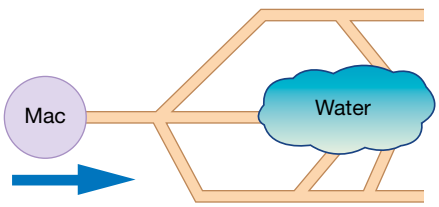
2 Start with 8 life chances.



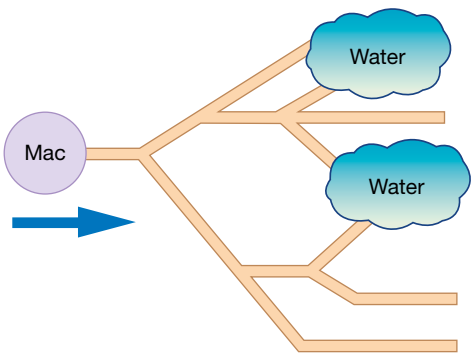
3 Start with 8 life chances.



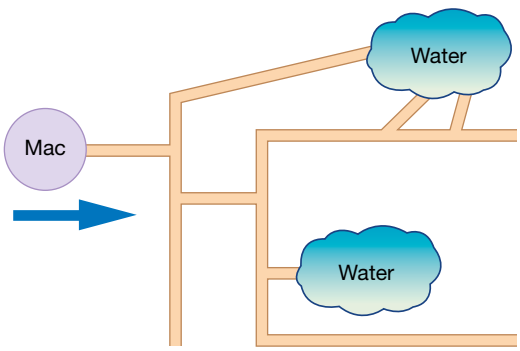
4 Start with 12 life chances.



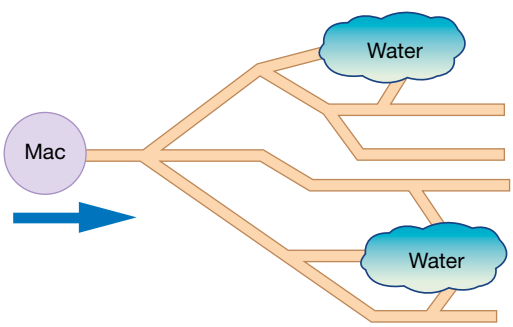
5 Start with 24 life chances.



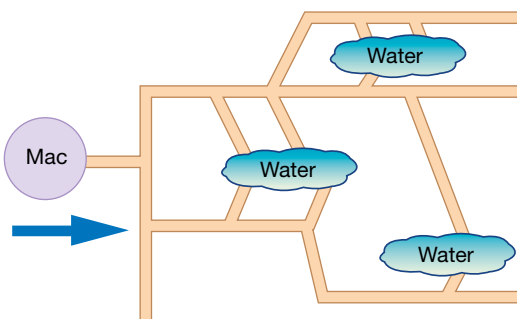
6 Start with 32 life chances.



7 Start with 24 life chances.



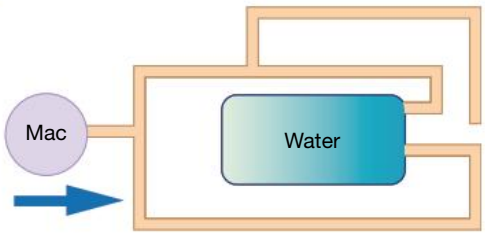
8 Start with 96 life chances.



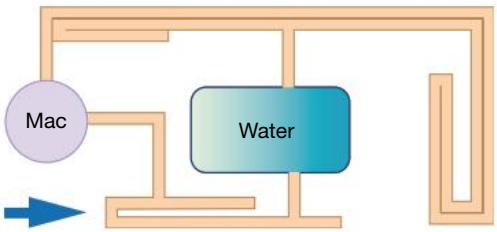
In the following questions find the chance (as a percentage) that Mac will find water without having to turn back.

Note: If we start with 100 life chances, the answer will be a percentage.

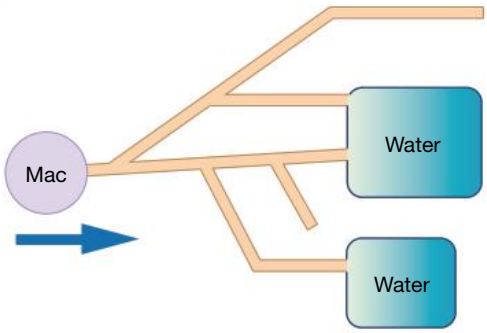
9 Start with 100 chances.



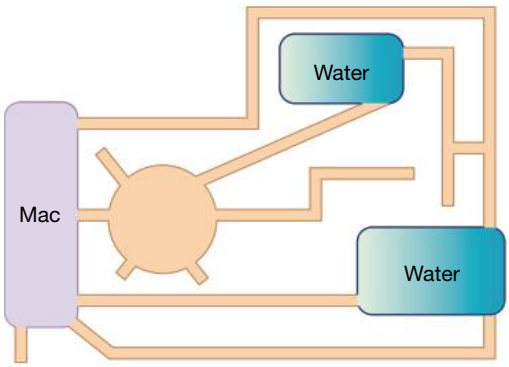
10 Start with 100 chances.



11 Start with 100 chances.



12 Start with 100 chances.



INVESTIGATION 12:04 DESCRIBING PROBABILITY

1 Describe a chance experiment in which the outcomes are equally likely.

2 Describe a chance experiment in which the outcomes are not equally likely.

3 Design a six-sided dice so that the outcome '2' is two times as likely as the outcome '3', while the outcome '4' is three times as likely as the outcome '3'.

4 Design a spinner with the colours red, white and blue so that spinning 'white' is three times as likely as spinning 'red', while spinning 'blue' is twice as likely as spinning 'white'.



12:05 Using probability

PREP QUIZ 12:05

1 A fair coin is tossed. Write as a percentage the probability of tossing a head.
2 The same fair coin is tossed again. Write as a fraction the probability of tossing a tail.
3 If the fair coin is tossed and shows heads four times in a row, is the chance of tossing a head the next time still 50%?

True or false?

4 If the probability of an event is 50%, the event should happen about half the time.
5 An *even chance* is the same as a 50% chance or a fifty-fifty chance.
6 If the probability of an event is 0·0001, it will never happen.
7 If an event is impossible, it will hardly ever happen.

Write the sample space for each of the following:

8 a coin is tossed 9 a normal dice is rolled 10 a suit is chosen from a standard pack of cards.

The theoretical probability of an event A is found using the formula:

$$P(A) = \frac{\text{number of favourable outcomes}}{n}$$



For this formula to work, each outcome needs to be equally likely. We also need to know how many outcomes (n) there are.

But what can we do when these things are not known?

Answer:

- We can carry out an experiment many times, to estimate the probability from the results.
  This would give us the **experimental probability**.
- We can collect statistics that would help us make an informed judgement.
  This would allow us to make a **subjective estimate**.

Experimental probability

If we don't know the probability of an event, we can carry out an experiment many times to see how often the event occurs. The more trials we use, the more reliable our estimate of the probability will be.

Subjective estimate

We can use information gathered through investigation to estimate probability. The accuracy of this estimate is increased if we increase the amount of information we gather.

For example: if an event has occurred 31% of the time in the past, then we could estimate that its probability is 31%. (This could also be written as $\frac{31}{100}$ or 0·31.)

- If the probability of drawing a red counter from a bag is 4%, then in 100 trials we would expect to draw out a red counter about 4 times. We call this the **expected result**.

- If there is a 4% chance of an event happening, then there is a 96% chance that it will not happen, i.e. 100% − 4% = 96%. The chance of an event happening and the chance of it not happening are called **complementary events**.

WORKED EXAMPLES

1 I want to know the chance of choosing a red counter at random from a bag of counters. I don't know the colours of the counters in the bag or the number of counters.

Method

I drew a coloured counter from the bag, noted its colour and then returned it to the bag. I repeated this experiment 100 times and recorded the results in the table below.

Colour of counters chosen in 100 trials		
Red	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40
Blue	𝍸 𝍸 𝍸 𝍸 𝍸 ||	27
Yellow	𝍸 𝍸 𝍸	15
White	𝍸 𝍸 𝍸 |||	18

Conclusions

The experimental probabilities of choosing each counter are as follows:

- red counter 40%
- blue counter 27%
- yellow counter 15%
- white counter 18%

We cannot be sure that other colours are not also in the bag.

The experimental probability would be more accurate if we had done 500 trials. The more trials, the closer we should get to the theoretical probability.

From these results, we would expect that a red counter would be chosen about 40% of the time.

2 After researching the weather for August over the last 10 years, Rachel found that it had rained only 10% of the time. The table shows the information she gathered.

Weather in August	Number of days (over 10 years)
Cloudy	132
Raining	31
Sunny	147

Limitations

- Only 310 days are considered (10 years × 31 days).
- Weather patterns may be changing over time.
- The categories are not clear. Would scattered showers be called 'raining' or 'cloudy'?

Conclusions

$$P(\text{cloudy}) = \frac{\text{number of cloudy days}}{\text{number of days}}$$
$$= \frac{132}{310} \times \frac{100}{1}\%$$
$$= 42 \cdot 6\%$$
$$P(\text{raining}) = \frac{\text{number of raining days}}{\text{number of days}}$$
$$= \frac{31}{310} \times \frac{100}{1}\%$$
$$= 10\%$$
$$P(\text{sunny}) = \frac{\text{number of sunny days}}{\text{number of days}}$$
$$= \frac{147}{310} \times \frac{100}{1}\%$$
$$= 47 \cdot 4\%$$

Exercise 12:05

P Foundation worksheet 12:05
Using probability

1 Use one of the terms *impossible*, *very unlikely*, *unlikely*, *even chance*, *likely*, *very likely* or *certain* to rewrite each statement.

a Parramatta is *expected* to continue its winning way this Saturday.
b We *predict* early showers today.
c The drought has been severe and we are *hoping* it might rain tomorrow.
d We *believe* that our company is *on the rocks*.
e This election is a *cliffhanger* between the Labor Party and the Liberal–National Coalition.
f Everybody overboard! The ship is *doomed*!

2 7 women and 4 men draw straws to decide who will buy morning tea. (To *draw straws* is a way of making a random choice.) What is the probability that the person chosen is:

a a man **b** a woman **c** not a woman
d not a man **e** either a man or a woman?

3 I have 3 pairs of shoes in a box. I have chosen one shoe and must now choose a second shoe at random. What is the probability that the second shoe:

a will match the first
b will not match the first?



4 To find the probabilities of choosing coloured counters at random from a bag of counters, I drew a counter from the bag, noted its colour and then returned it to the bag. I repeated this experiment 100 times and tallied the results.

Colour of counters chosen in 100 trials		
Blue	𝍸 𝍸 𝍸 𝍸 ||||	24
Red	𝍸 𝍸 𝍸 𝍸 𝍸 |||	28
White	𝍸 𝍸 𝍸 𝍸 𝍸 |	26
Yellow	𝍸 𝍸 𝍸 𝍸 ||	22



What is the experimental probability of choosing:

a a red counter **b** a blue counter
c a yellow counter **d** a white counter?
e Could there be the same number of each colour in the bag?
f Could there be five different colours in the bag?
g Use the experimental probability to estimate the number of each colour in the bag if there are 50 counters in total.

5 After researching the weather for May over the last 2 years, Rhonda found that it had rained about 20% of the time. She decided to choose the 4th of May as her wedding date. The table at right shows the information she gathered.
Use the table to find the experimental probability (to the nearest per cent) that on a day in May the weather will be:

Weather in May	Number of days (over 2 years)
Cloudy	22
Raining	12
Sunny	28

a raining **b** cloudy
c sunny **d** not sunny.
e Is it possible that every day in May will be sunny?

6 In a bag I have 3 red counters and 7 blue counters. I draw one counter from this bag, record its colour and then return it to the bag. What would be the expected result for each colour if I carry out this experiment:
a 100 times **b** 50 times?

7 What would be the expected result (for each possible outcome) if:
a a coin is tossed 40 times
b a normal dice is rolled 60 times
c 80 babies are born in a hospital?

8 My last ten cricket batting scores were 0, 33, 14, 87, 15, 8, 2, 10, 49 and 50. If one of these scores is selected at random, what is the probability that it will be:
a 0 **b** greater than 40 **c** greater than 100
d less than 50 **e** between 5 and 30 **f** less than 100?

9 If I toss a coin three times and record the results in order, there are eight possible outcomes: (head, head, head), (head, head, tail), (head, tail, head), (head, tail, tail), (tail, head, head), (tail, head, tail), (tail, tail, head) and (tail, tail, tail). If I toss a coin three times, what is the probability of getting:
a three heads **b** two heads **c** one head
d no heads **e** at least one head?

10 300 tickets are to be sold in a raffle.
How many tickets must I buy to have:
a a 10% chance of winning
b a 30% chance of winning
c a probability of $\frac{1}{4}$ of winning
d a probability of 1 of winning?



INVESTIGATION 12:05 CHOOSING CARDS

- Have someone else choose 10 cards from a standard pack, then shuffle them.
- Choose one of these cards at random, record its suit (hearts ♥, diamonds ♦, clubs ♣ or spades ♠) in a tally, then return the card and shuffle the cards again. Do this 50 times.
- After 50 trials, write your experimental probability of choosing each suit from this set of 10 cards.

	Tally	Total
♥		
♦		
♣		
♠		

Experimental probability (%)	
♥	
♦	
♣	
♠	

- Compare your results with the actual numbers of each suit that are in the set of 10 cards.
- Write a conclusion to your investigation.



MATHS TERMS 12

at random
- a way of choosing that does not affect the likelihood of any outcome

certain
- sure, inevitable
- a probability of 1 (i.e. 100%)

complementary events
- the complement of an event is all possible outcomes not in that event,
  e.g. the complementary event to *rolling a six* is *not rolling a six*
- the probabilities of an event and its complement add to 1

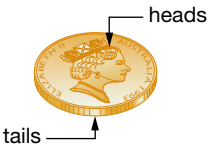
equally likely outcomes
- a situation where each outcome has the same chance of occurring,
  e.g. picking any particular card from a standard pack

even chance (or fifty-fifty chance)
- there is an equal chance of the event happening or not happening,
  e.g. a coin toss coming up heads
- 50% chance of happening and 50% chance of not happening

event
- a category that relates to a chance experiment,
  e.g. rolling a 3 or 4 on a fair dice

favourable outcome
- an outcome from a trial that fits the category or the event under consideration,
  e.g. a coin toss coming up heads is a 'favourable' outcome if we are counting the number of heads tossed in 100 tosses

heads and tails
- each coin has two sides, a 'head' and a 'tail'



impossible
- cannot happen, not possible
- having a probability of zero (i.e. 0%)

probability
- the chance of an event happening
- probability must be in the range 0 to 1 (i.e. 0% to 100%)
- Probability = $\dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
- $P(E) = \dfrac{n(E)}{n}$ where

  $n(E)$ = number of ways the event E can occur

  n = number of possible outcomes (i.e. the number in the sample space)

sample space
- all of the possible outcomes,
  e.g. The sample space for tossing a coin is *head*, *tail*.

simple event
- an event where each outcome is equally likely

standard pack of cards
- 4 suits are used:

  ♥ hearts ♦ diamonds ♣ clubs ♠ spades
- 52 cards are used, 13 in each of the four suits: Ace (A), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q) and King (K) in each suit
- 'picture cards' (or 'court cards') are Jacks, Queens and Kings (12 in total)



trial
- one case of carrying out an experiment,
  e.g. spinning a spinner once

DIAGNOSTIC TEST 12 PROBABILITY

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Order the following events A, B and C from least likely to most likely. 12:01

a **A** A dice is rolled. The result will be either a 1, a 2 or a 3.
B The next person to walk into our classroom will have their birthday in December.
C A red counter picked at random from a group containing one green, one yellow and four red counters will be red.

b When I roll a dice:
A I will roll a zero
B I will roll a number less than 3
C I will roll a number greater than 3.

2 List all possible outcomes for the following: 12:02

a A coin is tossed. **b** A dice is rolled.

3 How many outcomes are in each sample space below? 12:02

a A dice is rolled.
b A card is selected from a standard pack of cards.

4 One of these pictures is chosen at random. What is the probability that the picture chosen is of: 12:03



a a woman
b a man
c someone wearing a hat
d someone who is not a man?

5 A letter is chosen at random from the vowels *a*, *e*, *i*, *o* and *u*. What is the probability that it is: 12:03

a either the *a* or the *e* **b** a vowel
c the letter *t* **d** a letter that is not *u*?

6 **a** Luke's chance of clearing the high jump is $\frac{7}{10}$. What is Luke's chance of not clearing the high jump? 12:05

b There is a probability of 25% of choosing a club ♣ from a pack of cards. If we drew a card from the pack 200 times, returning the card each time, what would be the expected result for 'choosing a club'?

c What would be the expected result for 'heads' if I tossed a coin 300 times?

ASSIGNMENT 12A Chapter review

1 Choose one of these labels for the probability of each event described below.

impossible | unlikely | even chance | likely | certain

If I throw a fair dice, the result will be:

a even **b** a 2

c a zero **d** less than 7.

2 Of 100 tickets in a raffle, Hiro bought 70, Mia bought 25, Marika bought 3 and Vu bought none. One ticket is chosen at random. What is the chance that it was bought by:

a Hiro

b Mia

c Marika

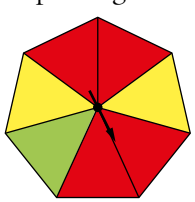
d someone other than Marika?

3 **a** If a coin is tossed, is a head more likely than a tail? Explain your answer.

b If two coins are tossed, what could be the result?

c When a dice is rolled, Suri wins if an even number is rolled, Arlo wins if a 5 is rolled and Luke wins if a 1 or 3 is rolled. Write the names in order of their chance of winning.

4 This spinner is spun once. Write the chance (as a fraction) of spinning:



a red

b green

c yellow

d brown

e red or yellow

f red, green or yellow.

5 **a** What does it mean for an event to have an 'even chance' of happening?

b What does it mean for four events to have an 'equal chance' of occuring?

6 One of these shapes is chosen at random.



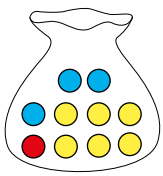
Write the probability (as a percentage) of choosing:

a an oval

b a square

c a quadrilateral.

7 This bag contains 10 coloured counters. One counter is taken at random from the bag. Write the probability (as a decimal) that the counter will be:



a red

b yellow

c blue

d not blue

e red or blue

f not red or blue.

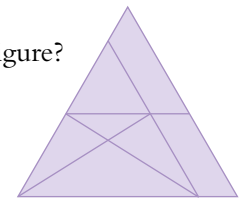
8 What is the sample space for choosing one shape from those in Question **6**?

9 What is the probability of choosing a King from a standard pack of cards?

10 What is the complementary event to 'choosing a King from a standard pack of cards'?

ASSIGNMENT 12B Working mathematically

1 How many triangles can you find in this figure?



2 The Eiffel Tower in Paris is 312 m high. 500 sheets of paper make a pile 55 mm high. How many sheets of paper would make a pile as high as the Eiffel Tower? (Answer correct to the nearest 1000.)

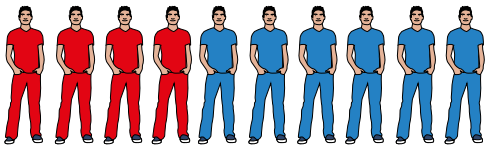
3 If *24 H in a D* stands for *24 Hours in a Day*, what might the following cryptic statements mean?

a 365 D in a Y
b 9 L of a C
c 52 C in a S P
d 26 L in the A
e 12 M in a Y
f 1000 Y in a M
g 64 S on a C B



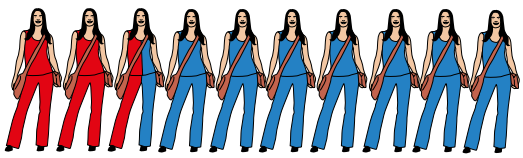
4 Causes of disability:

Young men



Accident Sickness

Young women

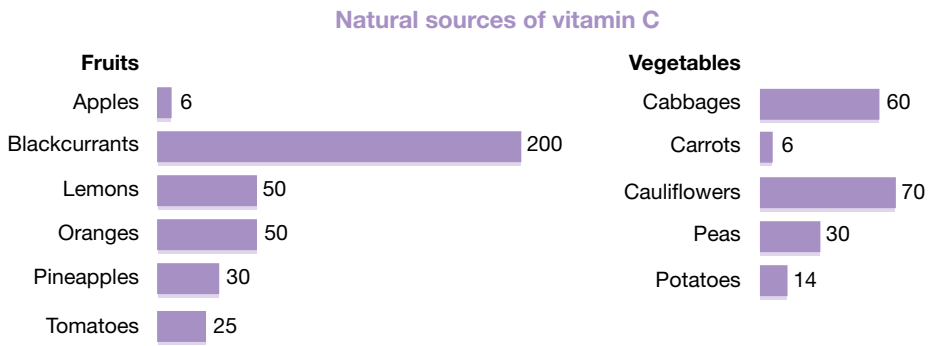


Accident Sickness

Compare the results for young men and young women shown above.

a What information is communicated by this graph?
b What could account for the differences in the results for men and women?

5



The figures quoted show the average number of milligrams of vitamin C per 100 g.

a Which fruit is the richest source of vitamin C?
b Which vegetable is the richest source of vitamin C?
c How many grams of tomatoes must be eaten to provide the same amount of vitamin C as 100 g of oranges?
d The recommended daily intake of vitamin C is 32 mg. If I eat 50 g of an apple and 50 g of an orange, how many milligrams short of the recommended daily intake am I?
e In one day, I eat the following:

100 g of tomato	20 g of peas	50 g of pineapple
50 g of cauliflower	50 g of carrot	100 g of potato

How many milligrams of vitamin C have I consumed?

ASSIGNMENT 12C Cumulative revision

1 Change these Roman numerals into modern (Hindu–Arabic) numerals. 1:01

a XIX **b** CCLX **c** MCMIII **d** MMXIV

2 Simplify the following. 1:06B

a $15 - (4 + 8)$ **b** $120 - 80 \div 4$ **c** $5 \times 9 - (20 - 18 \div 2)$

3 Rewrite as a basic numeral. 3:02, 3:07

a $(7 \times 10^3) + (2 \times 10^2) + (9 \times 10^1) + 4 \times 1$ **b** $(5 \times 10^4) + (7 \times 10^3)$

c $\sqrt{64}$ **d** $\sqrt[3]{27}$ **e** $\sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$

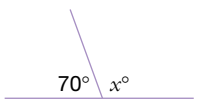
4 Simplify: 4:07, 4:09

a $5a + 3b - a$ **b** $6 \times 4m - 4$ **c** $20x \div 5 + 3x \times 7$

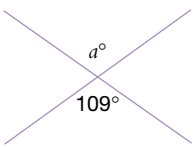
d $7(2x + 2) - 4$ **e** $5(x + 7) - 5x$ **f** $3(2y + 9) + 6(5y - 4)$

5 Find the value of each pronumeral. 5:04, 5:05, 8:02, 8:04

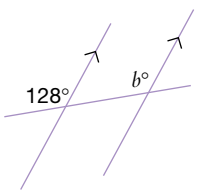
a



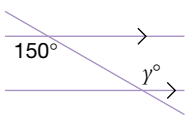
b



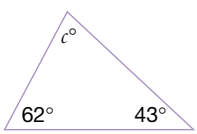
c



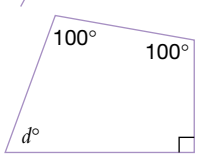
d



e



f



6 **a** $-36 + 6$ **b** $-17 - 8$ 7:05, 7:06

c $8 - (3 - 18)$ **d** $3 - 1 - 1 - 1 - 1$

7 **a** $\frac{3}{4} - \frac{2}{3}$ **b** $3\frac{2}{5} + 5\frac{1}{2}$ 9:04–9:07

c $1\frac{7}{8} \times 4$ **d** $\frac{3}{8} \div \frac{1}{4}$

8 The ratio of boys to girls in the playground was 3:2 and there were 60 children altogether. 9:10

a What fraction of the children were girls?

b How many girls were in the playground?

9 **a** What is 10% of \$850? 11:04, 11:05

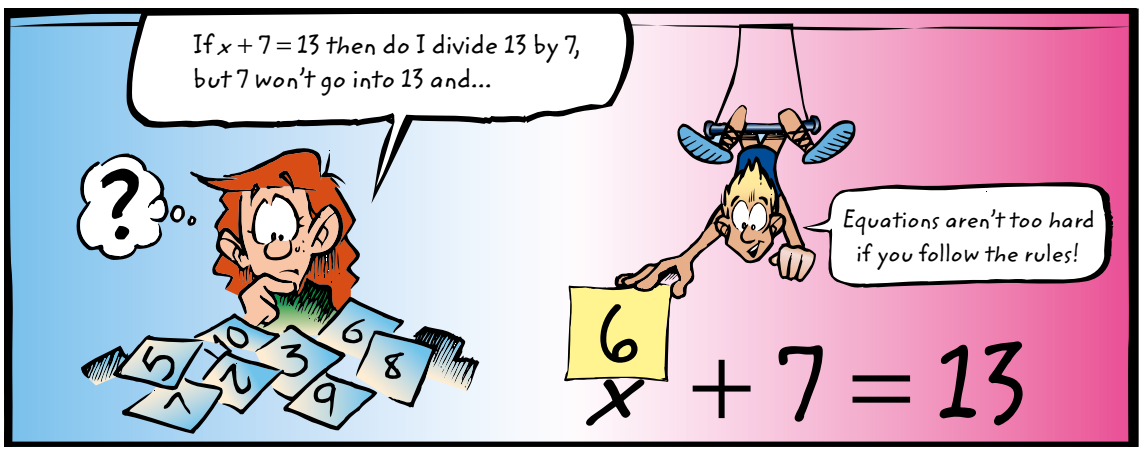
b What is 3% of 6·8 m?

c What percentage is 24 g of 600 g?

10 What is the probability of choosing a red King at random from a standard pack of cards? 12:03

EQUATIONS

13



Contents

13:01 Simple equations
Fun spot 13:01 How do vampires cross the sea?
13:02 Inverse operations—backtracking
13:03 Solving equations
13:04 Two-step equations
13:05 Solving problems using equations
13:06 Investigation of real problems
Fun spot 13:06 Solving pyramids
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Equations*** [Stage 4]

- Solve simple linear equations. (ACMNA179)
- Solve linear equations using algebraic techniques and verify solutions by substitution. (ACMNA194)
- Solve simple quadratic equations.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

13:01 Simple equations

An **equation** is a number sentence in which one or more of the numbers is missing or unknown and is represented by a pronumeral.

> Equations are sometimes called algebraic sentences.

PREP QUIZ 13:01

Make each number sentence true by finding the missing number.

1 $\square + 3 = 7$ **2** $\triangle - 5 = 2$ **3** $\star \times 5 = 20$ **4** $12 - \square = 3$ **5** $24 \div \triangle = 6$

True or false?

6 If $6 + \square = 15$, then $\square = 11$.

7 If $5 \times \triangle = 45$, then $\triangle = 9$.

8 If $\theta - 6 = 10$, then $\theta = 4$.

9 If $2 \times \square + 1 = 11$, then $\square = 5$.

10 If $\frac{\Delta + 1}{3} = 4$, then $\Delta = 11$.

- To **solve** an algebraic sentence or equation, we must find the numerical value of the pronumeral that makes the sentence a true mathematical statement. Sometimes we can just 'see' the answer to the equation, especially if we say in words what the sentence means.
- To **check** a solution, we substitute the numerical value into the original equation to see if it makes a number sentence that is true.

WORKED EXAMPLES

1 $x + 5 = 9$
We have to find the value of x, so we say:
'What number plus 5 is equal to 9?'
Answer: $x = 4$

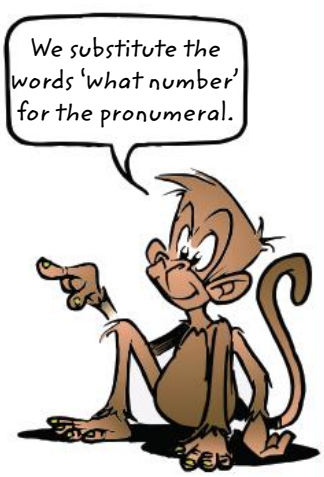
2 $12 - a = 5$
For this equation we would say:
'12 minus what number is equal to 5?'
Answer: $a = 7$

3 $6m = 42$
'6 times what number is equal to 42?'
Answer: $m = 7$

4 $\frac{x}{3} = 21$
'What number divided by 3 is equal to 21?'
Answer: $x = 63$

5 $y - 11 = 3$
'What number minus 11 is equal to 3?'
Answer: $y = 14$

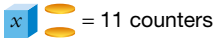
6 $18 \div n = 6$
'18 divided by what number is equal to 6?'
Answer $n = 3$

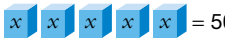


Exercise 13:01

P Foundation worksheet 13:01
Solving equations 1

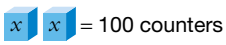
1 There are x counters in each blue box and y counters in each red box. In each question the x and y can stand for a different number. In each question find the value of x or y.

a  = 11 counters

b  = 50 counters

c  = 24 counters

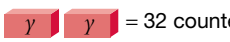
d  = 33 counters

e  = 100 counters

f  = 20 counters

g  = 15 counters

h  = 21 counters

i  = 32 counters

j  = 27 counters

2 Write in words what each equation means, then solve it.

a $x + 5 = 7$ **b** $a + 9 = 21$ **c** $10 - p = 7$ **d** $m - 5 = 2$

e $4n = 20$ **f** $7m = 56$ **g** $q \div 2 = 7$ **h** $\frac{n}{4} = 4$

3 Solve these equations by saying in words what each equation means.

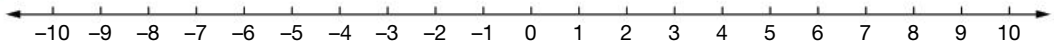
	i	ii	iii	iv
a	$a + 3 = 6$	$x + 9 = 10$	$m + 3 = 11$	$k + 7 = 15$
b	$n + 4 = 12$	$q + 10 = 17$	$p + 8 = 13$	$a + 11 = 22$
c	$6 + x = 9$	$7 + b = 14$	$10 + q = 23$	$14 + m = 29$
d	$12 + l = 30$	$15 + x = 19$	$21 + a = 40$	$25 + n = 48$
e	$10 - a = 9$	$12 - x = 8$	$9 - q = 0$	$8 - m = 8$
f	$15 - p = 9$	$19 - m = 8$	$21 - x = 12$	$30 - a = 18$
g	$x - 9 = 1$	$p - 7 = 3$	$q - 10 = 4$	$t - 7 = 10$
h	$y - 6 = 6$	$x - 5 = 0$	$k - 12 = 13$	$l - 21 = 19$
i	$6x = 12$	$3p = 18$	$9m = 27$	$5m = 25$
j	$20 = 4y$	$24 = 6m$	$15 = 5a$	$16 = 2k$
k	$8m = 0$	$9n = 9$	$100 = 25x$	$144 = 12n$
l	$x \div 2 = 3$	$p \div 5 = 2$	$a \div 3 = 4$	$n \div 8 = 2$
m	$10 \div a = 5$	$12 \div x = 4$	$24 \div m = 4$	$20 \div m = 5$
n	$\frac{x}{3} = 6$	$\frac{a}{2} = 6$	$\frac{m}{5} = 3$	$\frac{n}{7} = 4$
o	$\frac{16}{x} = 2$	$\frac{12}{a} = 1$	$\frac{32}{m} = 4$	$\frac{48}{p} = 4$

4 Check the given solution for each equation by substituting it back into the equation. Is it correct or incorrect?

a $x + 12 = 26$, $x = 14$ **b** $14 + a = 29$, $a = 16$ **c** $15 - m = 9$, $m = 4$ **d** $n - 19 = 22$, $n = 3$

e $7a = 63$, $a = 9$ **f** $13x = 65$, $x = 6$ **g** $p \div 7 = 3$, $p = 21$ **h** $m \div 3 = 6$, $m = 2$

5 To solve these equations you need to remember how to operate with integers.



a $3 - x = -1$	**b** $5 - m = -5$	**c** $4 - a = -2$
d $a - 5 = -1$	**e** $x - 3 = -6$	**f** $p - 2 = -5$
g $x + 3 = 1$	**h** $a + 1 = -4$	**i** $m + 7 = 1$
j $a + (-2) = 1$	**k** $x + (-3) = -1$	**l** $n + (-9) = -10$
m $6m = -12$	**n** $3x = -18$	**o** $-5m = -15$
p $a \div 4 = -6$	**q** $x \div (-2) = 3$	**r** $20 \div n = -4$
s $x - (-2) = 5$	**t** $a - (-1) = -3$	**u** $-6 - x = 1$



FUN SPOT 13:01 HOW DO VAMPIRES CROSS THE SEA?

Find the answer for each letter. Match each letter with its answer below to solve the riddle.



Solve:

L $a + 3 = 7$	**D** $m + 10 = 10$	**I** $x - 5 = 9$
E $n - 4 = 4$	**E** $10 - k = 1$	**N** $6 + n = 11$
B $3a = 18$	**V** $7x = 7$	**S** $6x = 42$
S $4y = 44$	**O** $12k = 24$	
O $\frac{x}{3} = 9$	**L** $\frac{n}{7} = 3$	**S** $\frac{a}{5} = 2$

| | | |

We build algebraic expressions by starting with the pronumeral.

Algebraic expression	Meaning	Arrow diagram
$x + 5$	x has 5 added to it.	$x \xrightarrow{+5} x + 5$
$x - 5$	x has 5 subtracted from it.	$x \xrightarrow{-5} x - 5$
$5x$	x is multiplied by 5.	$x \xrightarrow{\times 5} 5x$
$\frac{x}{5}$	x is divided by 5.	$x \xrightarrow{\div 5} \frac{x}{5}$

Arrow diagrams can be used to show how the expression has been made.
The operation that has caused the change is written above the arrow.

When solving equations, we need to reverse the steps to get back to the pronumeral. In other words, we need to perform the **inverse** operations. For instance:

$x \xrightarrow{+5} x + 5$ then $x + 5 \xrightarrow{-5} x$

$x \xrightarrow{-5} x - 5$ then $x - 5 \xrightarrow{+5} x$

$x \xrightarrow{\times 5} 5x$ then $5x \xrightarrow{\div 5} x$

$x \xrightarrow{\div 5} \frac{x}{5}$ then $\frac{x}{5} \xrightarrow{\times 5} x$



Inverse means *opposite*.

$2 \xrightarrow{+5} 7$

$7 \xrightarrow{-5} 2$

Adding 5 and subtracting 5 are inverse operations.

This process is called **backtracking**.

WORKED EXAMPLES

1 See how each expression is built from the pronumeral.

a $y \xrightarrow{-8} y - 8$

b $a \xrightarrow{\times 3} 3a \xrightarrow{+5} 3a + 5$

c $q \xrightarrow{+6} q + 6 \xrightarrow{\times 2} 2(q + 6)$

d $m \xrightarrow{\div 2} \frac{m}{2} \xrightarrow{-4} \frac{m}{2} - 4$

e $n \xrightarrow{-11} n - 11 \xrightarrow{\div 7} \frac{n - 11}{7}$

2 Note the inverse operations used to get back to the pronumeral.

a $9m \xrightarrow{\div 9} m$

b $8x + 15 \xrightarrow{-15} 8x \xrightarrow{\div 8} x$

c $4(y + 3) \xrightarrow{\div 4} y + 3 \xrightarrow{-3} y$

d $\frac{a-1}{5} \xrightarrow{\times 5} a - 1 \xrightarrow{+1} a$

Backtrack to the pronumeral.



Exercise 13:02

P Foundation worksheet 13:02 Equations algebra review

1 Complete the following arrow diagrams by writing the correct operation above each arrow.

a $x \rightarrow 9x$

b $a \rightarrow a + 2$

c $q \rightarrow q - 7$

d $y \rightarrow 5y \rightarrow 5y - 1$

e $m \rightarrow \frac{m}{10}$

f $n \rightarrow n + 6 \rightarrow \frac{n+6}{5}$

g $a \rightarrow \frac{a}{3} \rightarrow \frac{a}{3} + 9$

h $a \rightarrow a - 8 \rightarrow \frac{a-8}{7}$

i $n \rightarrow \frac{n}{-5} \rightarrow \frac{n}{-5} + 3$

j $p \rightarrow p + 3 \rightarrow 2(p + 3)$

k $x \rightarrow -3x \rightarrow -3x + 5$

2 Complete these arrow diagrams, showing how to get back to the pronumeral.

a $3m \rightarrow m$

b $\frac{x}{2} \rightarrow x$

c $n + 5 \rightarrow n$

d $p - 7 \rightarrow p$

e $10x \rightarrow x$

f $\frac{a}{3} \rightarrow a$

g $8 + t \rightarrow t$

h $q - 5 \rightarrow q$

i $2m + 1 \rightarrow 2m \rightarrow m$

j $5p - 2 \rightarrow 5p \rightarrow p$

k $7x + 3 \rightarrow 7x \rightarrow x$

l $9q - 4 \rightarrow 9q \rightarrow q$

m $6 + 3x \rightarrow 3x \rightarrow x$

n $7 + \frac{m}{2} \rightarrow \frac{m}{2} \rightarrow m$

o $\frac{x}{5} - 2 \rightarrow \frac{x}{5} \rightarrow x$

p $\frac{a+7}{3} \rightarrow a + 7 \rightarrow a$

q $\frac{b-2}{5} \rightarrow b - 2 \rightarrow b$

r $5 - 3x \rightarrow -3x \rightarrow x$

s $8 - 2p \rightarrow -2p \rightarrow p$

t $5 - \frac{x}{3} \rightarrow \frac{-x}{3} \rightarrow x$

3 Draw arrow diagrams to show how the following expressions are made from the pronumeral.

a $8m$ **b** $\frac{p}{7}$ **c** $5 + x$ **d** $q - 10$

e $3x + 2$ **f** $5n - 3$ **g** $6p + 7$ **h** $9a - 1$

i $\frac{x}{3} + 5$ **j** $7 + \frac{a}{4}$ **k** $\frac{y+3}{4}$ **l** $\frac{n-1}{5}$

m $7(m + 2)$ **n** $9(x - 3)$ **o** $5(2 + x)$ **p** $3(q - 4)$

q $3 - 2x$ **r** $-5a + 7$ **s** $6 - \frac{x}{3}$ **t** $10 - 9p$

4 Complete the following arrow diagrams.

a $a \xrightarrow{-2} \square$

b $x \xrightarrow{\times 4} \square$

c $m \xrightarrow{\div 3} \square$

d $t \xrightarrow{+9} \square$

e $p \xrightarrow{\times 5} \square \xrightarrow{+2} \square$

f $x \xrightarrow{\times 6} \square \xrightarrow{-4} \square$

g $y \xrightarrow{+1} \square \xrightarrow{\times 7} \square$

h $b \xrightarrow{\div 4} \square \xrightarrow{+2} \square$

i $q \xrightarrow{-3} \square \xrightarrow{\div 3} \square$

j $n \xrightarrow{\times(-2)} \square \xrightarrow{+6} \square$

k $b + 10 \xrightarrow{-10} \square$

l $a - 7 \xrightarrow{+7} \square$

m $5m \xrightarrow{\div 5} \square$

n $\frac{n}{4} \xrightarrow{\times 4} \square$

o $2x + 3 \xrightarrow{-3} \square \xrightarrow{\div 2} \square$

p $5p - 4 \xrightarrow{+4} \square \xrightarrow{\div 5} \square$

q $6 + 7t \xrightarrow{-6} \square \xrightarrow{\div 7} \square$

r $\frac{m+5}{4} \xrightarrow{\times 4} \square \xrightarrow{-5} \square$

s $\frac{a}{3} - 5 \xrightarrow{+5} \square \xrightarrow{\times 3} \square$

t $5 - 2x \xrightarrow{-5} \square \xrightarrow{\div(-2)} \square$

5 What operations must be performed on x to get the following expressions?

a $x - 6$ **b** $\frac{x}{3}$ **c** $x + 5$ **d** $7x$

e $2x + 3$ **f** $5x - 2$ **g** $6 + 3x$ **h** $7x - 1$

i $\frac{x+7}{5}$ **j** $\frac{x-4}{3}$ **k** $\frac{x}{8} + 5$ **l** $\frac{x}{3} - 4$

m $2(x + 3)$ **n** $5(x - 9)$ **o** $-6x + 8$ **p** $10 - 3x$



6 What must be done to these expressions to make them equal to a?

a $a - 7$ **b** $5a$ **c** $6 + a$ **d** $\frac{a}{3}$

e $-4a$ **f** $2a + 5$ **g** $3a - 1$ **h** $5 + 7a$ **i** $5a - 6$ **j** $8 - 3a$

k $\frac{a+3}{7}$ **l** $\frac{a}{7} + 3$ **m** $5(a + 2)$ **n** $3(a - 1)$ **o** $\frac{a-9}{5}$

7 What must be done to these expressions to make them equal to n?

a $-3n$ **b** $-8n$ **c** $\frac{n}{-4}$ **d** $\frac{-n}{2}$

e $5 - 2n$ **f** $3 - n$ **g** $\frac{4-2n}{3}$ **h** $\frac{9-n}{5}$

8 What order of inverse operations must be performed to get back to the pronumeral?

a $\frac{2a+3}{5}$ **b** $6(3x + 7)$ **c** $\frac{7-3p}{6}$ **d** $9 + \frac{2m}{3}$

e $5(2 - 3m)$ **f** $\frac{5-n}{7}$ **g** $6 - \frac{5q}{3}$ **h** $\frac{7(3y+4)}{5}$

13:03 Solving equations

PREP QUIZ 13:03

Simplify the following:

1 $7x \div 7$ **2** $x + 4 - 4$ **3** $a - 7 + 7$ **4** $\frac{m}{5} \times 5$

What is the inverse of the following?

5 multiplying by 3 **6** adding 7 **7** subtracting 1 **8** dividing by 5

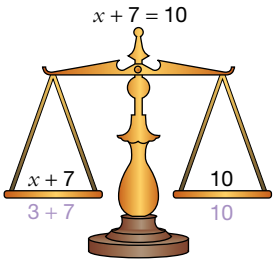
Complete these arrow diagrams, writing the correct operations above each arrow.

9 $\boxed{m + 3} \rightarrow \boxed{m}$ **10** $\boxed{8a} \rightarrow \boxed{a}$

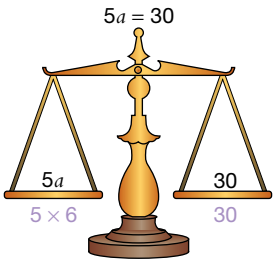
Solving equations is like balancing scales.



With equations, we know that one side is equal to the other. The solution of the equation is the value of the pronumeral that 'balances' the equation.



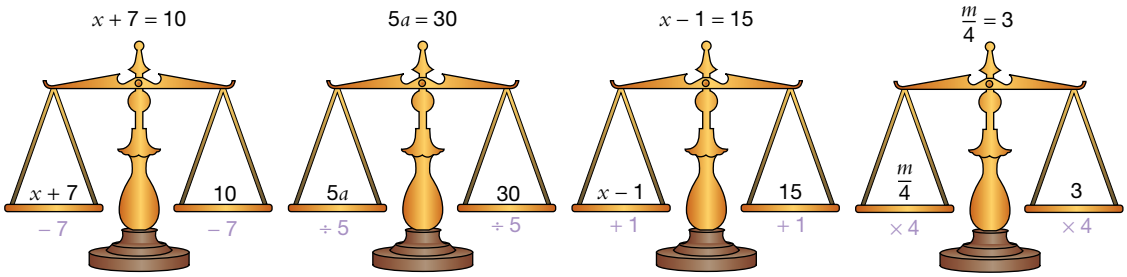
$x = 3$ balances the scale, so $x = 3$ is the solution.

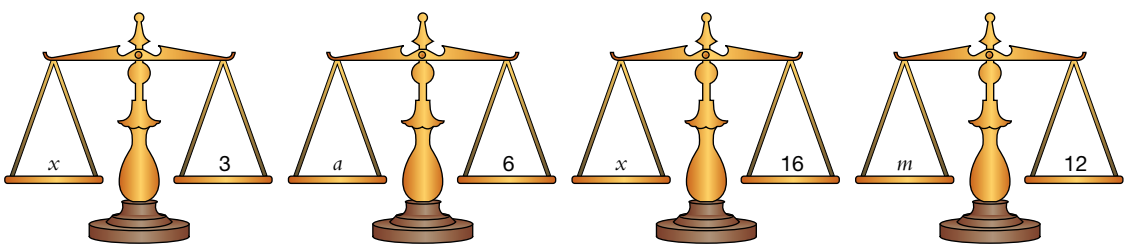


$a = 6$ balances the scale, so $a = 6$ is the solution.

We can also solve equations by adding (+), subtracting (–), multiplying (×) or dividing (÷) both sides of the equation by the same number.

Look at the solutions of these equations. Note that both sides remain balanced as the same operation is done to both sides.





$x = 3$ is the solution | $a = 6$ is the solution | $x = 16$ is the solution | $m = 12$ is the solution



We perform operations to leave *only the pronumeral* on one side of the equation.

Examine the solutions to the equations in the following worked examples, noting the operations that have been done to both sides.

WORKED EXAMPLES

These solutions involve only one step.

1 $m + 27 = 59$
$-27 \quad -27$
$m = 59 - 27$
$\therefore m = 32$

2 $a - 16 = 9$
$+16 \quad +16$
$a = 9 + 16$
$\therefore a = 25$

3 $3p = 23$
$\div 3 \quad \div 3$
$p = \frac{23}{3}$
$\therefore p = 7\frac{2}{3}$

4 $\frac{x}{7} = 12$
$\times 7 \quad \times 7$
$x = 12 \times 7$
$\therefore x = 84$

If a number is multiplied by itself to give 9, then the number could be either 3 or −3.
This is because $3 \times 3 = 9$ and $(-3) \times (-3) = 9$.
So if $c^2 = 16$, ask: what multiplies itself to give 16?
$c = 4$ or -4 because $4 \times 4 = 16$ and $(-4) \times (-4) = 16$.

WORKED EXAMPLES

Give two solutions for each equation.

1 $m^2 = 25$
$m = 5$ or -5

2 $a^2 = 100$
$a = 10$ or -10

3 $x^2 = 49$
$x = 7$ or -7

Exercise 13:03

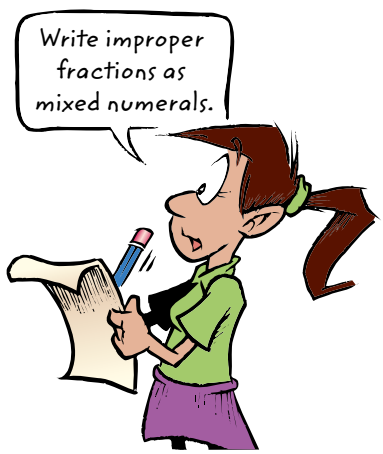
P Foundation worksheet 13:03 Algebraic sentences

1 Solve each of these one-step equations.

a $x + 7 = 15$ **b** $p + 9 = 11$ **c** $y + 7 = 21$
d $15 + n = 18$ **e** $7 + q = 15$ **f** $20 + k = 29$
g $y - 5 = 3$ **h** $m - 7 = 2$ **i** $x - 10 = 3$
j $5x = 15$ **k** $7x = 42$ **l** $3p = 36$
m $12 = 3y$ **n** $18 = 9y$ **o** $24 = 8x$
p $\frac{n}{3} = 2$ **q** $\frac{x}{5} = 3$ **r** $\frac{m}{4} = 4$

2 Solve these equations where the answers are not whole numbers.

a $x + 1{\cdot}5 = 3$ **b** $a + 6{\cdot}2 = 10$ **c** $m + 2{\cdot}9 = 3{\cdot}6$
d $n + 2\frac{1}{2} = 4$ **e** $p + 3\frac{1}{4} = 5$ **f** $5\frac{1}{3} + k = 7\frac{2}{3}$
g $x - 2{\cdot}4 = 1{\cdot}2$ **h** $n + 5{\cdot}7 = 17.4$ **i** $y - 3 = 8{\cdot}4$
j $5n = 8$ **k** $3m = 10$ **l** $6p = 15$
m $9 = 2x$ **n** $17 = 8k$ **o** $22 = 4y$
p $\frac{n}{3} = 0{\cdot}4$ **q** $\frac{a}{5} = 0{\cdot}5$ **r** $\frac{m}{7} = 1{\cdot}2$



3 Solve these equations, giving two possible solutions in each case.

a $m^2 = 4$ **b** $x^2 = 36$ **c** $y^2 = 64$
d $x^2 = 81$ **e** $c^2 = 9$ **f** $y^2 = 1$
g $m^2 = 16$ **h** $x^2 = 100$ **i** $y^2 = 49$

$x^2 = -25$ has no solution, because no real number can multiply itself to give a negative.

4 The solution to an equation can be checked by substituting the value back into the equation. Check to see if the solution to each equation below is correct.

a $x + 16 = 43$
$x = 27$

b $9 + y = 13$
$y = 5$

c $m - 10 = 3$
$m = 13$

d $n - 12 = 15$
$n = 27$

e $6 - q = 4$
$q = 2$

f $8 - y = 10$
$y = 2$

g $7m = 10$
$m = \frac{10}{7}$

h $3a = 2$
$a = \frac{3}{2}$

i $-5y = -15$
$y = -3$

j $\frac{x}{3} = 6$
$x = 18$

k $\frac{m}{5} = 10$
$m = 2$

l $\frac{n}{6} = 3$
$n = \frac{1}{2}$

5 These one-step equations involve negative integers.

a $x + 3 = 2$ **b** $m + 5 = 1$ **c** $n + 6 = 4$
d $7 + p = -1$ **e** $6 + q = 0$ **f** $10 + y = -2$
g $a - 7 = -2$ **h** $x - 10 = -3$ **i** $p - 9 = -4$
j $q - 2 = -1$ **k** $y - 4 = -4$ **l** $t - 6 = -2$
m $3m = -6$ **n** $8p = -16$ **o** $6w = -6$
p $-2n = 12$ **q** $-3q = 27$ **r** $-4m = -8$
s $\frac{x}{3} = -2$ **t** $\frac{m}{4} = -20$ **u** $\frac{n}{7} = -28$
v $x - (-2) = 5$ **w** $-5 - y = 3$ **x** $6 - (-x) = 3$

Note:
If $-a = 3$, then $a = -3$ (multiply both sides by -1).

13:04 Two-step equations

PREP QUIZ 13:04

What is the inverse of:

1 adding 8 **2** subtracting 5 **3** multiplying by 3 **4** dividing by 2?

Complete these arrow diagrams, writing the operations above each arrow.

5 $\boxed{5m + 3} \rightarrow \boxed{5m} \rightarrow \boxed{m}$

6 $\boxed{3x + 1} \rightarrow \boxed{3x} \rightarrow \boxed{x}$

7 $\boxed{7 + 2x} \rightarrow \boxed{2x} \rightarrow \boxed{x}$

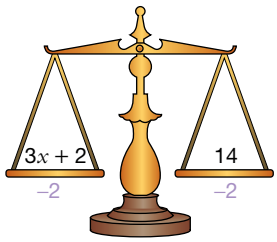
8 $\boxed{8a - 2} \rightarrow \boxed{8a} \rightarrow \boxed{a}$

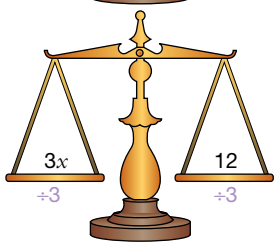
9 $\boxed{\frac{n+2}{3}} \rightarrow \boxed{n + 2} \rightarrow \boxed{n}$

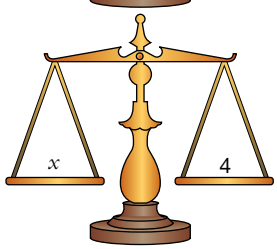
10 $\boxed{\frac{a}{5} - 4} \rightarrow \boxed{\frac{a}{5}} \rightarrow \boxed{a}$

Look at the solutions of these two equations. Both require two steps (or two *operations*) to be solved. Both sides of the equation remain balanced as the same operations are done to both sides.

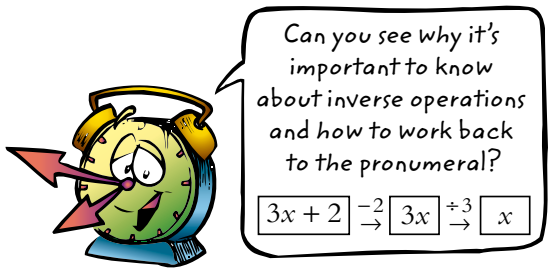
$3x + 2 = 14$







$x = 4$ is the solution



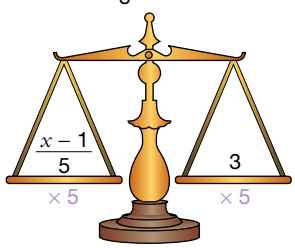


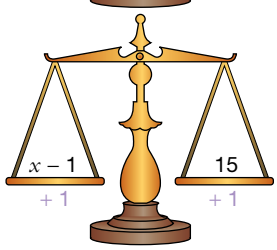
Note here that the inverse operations are:

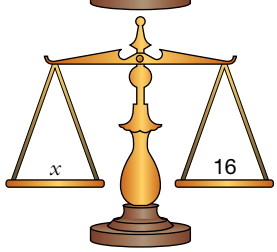
$\boxed{\frac{x-1}{5}} \xrightarrow{\times 5} \boxed{x-1} \xrightarrow{+1} \boxed{x}$



$\frac{x-1}{5} = 3$







$x = 16$ is the solution

Examine the solutions to the equations in the following worked examples. Note the operations that have been done to both sides.

WORKED EXAMPLES

The solutions of these equations involve two steps.

1 $2a + 5 = 7$

$-5 \quad -5$

$2a = 2$

$\div 2 \quad \div 2$

$\therefore a = 1$

2 $\frac{x+3}{5} = 2$

$\times 5 \quad \times 5$

$x + 3 = 10$

$-3 \quad -3$

$\therefore x = 7$

3 $\frac{p}{3} - 6 = 15$

$+6 \quad +6$

$\frac{p}{3} = 21$

$\times 3 \quad \times 3$

$\therefore p = 63$

4 $10 - 3m = 25$

$-10 \quad -10$

$-3m = 15$

$\div (-3) \quad \div (-3)$

$\therefore m = -5$

Exercise 13:04

1 Solving these equations involves two steps. Clearly show each step in your working. (All the answers are integers.)

a $2x + 1 = 5$ **b** $3a + 2 = 8$ **c** $2m + 7 = 13$ **d** $3n - 2 = 7$

e $5k - 1 = 24$ **f** $6t - 4 = 8$ **g** $6 + 5a = 26$ **h** $10 + 3w = 13$

i $12 + 4q = 16$ **j** $15 = 2x - 3$ **k** $7 = 5y - 3$ **l** $10 = 2 + 4a$

2 Solving these equations also involves two steps. (The solutions are all positive integers.)

a $\frac{3x}{2} = 3$ **b** $\frac{2x}{5} = 4$ **c** $\frac{3m}{4} = 6$

d $\frac{x+1}{4} = 3$ **e** $\frac{a+2}{3} = 1$ **f** $\frac{n+7}{2} = 10$

g $\frac{m-4}{2} = 5$ **h** $\frac{n-1}{3} = 6$ **i** $\frac{a-2}{4} = 10$

j $\frac{y}{3} + 1 = 3$ **k** $\frac{x}{4} + 2 = 1$ **l** $\frac{m}{5} - 3 = 2$

m $5 + \frac{a}{2} = 10$ **n** $2 + \frac{x}{3} = 8$ **o** $5 + \frac{m}{2} = 7$

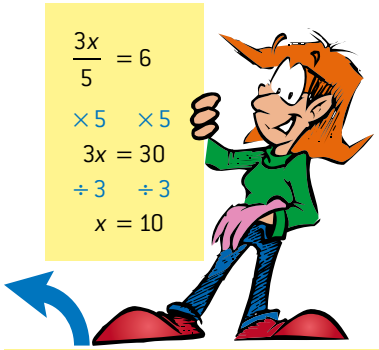
$\frac{3x}{5} = 6$

$\times 5 \quad \times 5$

$3x = 30$

$\div 3 \quad \div 3$

$x = 10$



Multiply each term on *both sides* by the denominator.

3 The solutions to these equations involve fractions.

a $2x + 1 = 4$ **b** $2a + 5 = 10$

c $3m + 4 = 5$ **d** $5n + 2 = 8$

e $2p - 3 = 2$ **f** $4q - 1 = 2$

g $5n - 5 = 4$ **h** $6y - 3 = 1$

i $5 + 2k = 12$ **j** $7 + 3x = 9$

k $8 + 3a = 10$ **l** $1 + 2a = 4$

m $2m + 6 = 3$ **n** $5p + 7 = 1$

$3a + 2 = 6$

$-2 \quad -2$

$3a = 4$

$\div 3 \quad \div 3$

$a = \frac{4}{3}$ or $1\frac{1}{3}$



4 To solve these equations you need to divide by a negative number.

a $10 - 3x = 1$ **b** $15 - 2m = 11$ **c** $20 - 5q = 0$

d $4 - x = -7$ **e** $-3 - x = -2$ **f** $2 = 4 - x$

g $3 - 2a = 6$ **h** $5 - 3a = 1$ **i** $10 - 4n = 12$

5 Now try these equations, which are either one-step or two-step types.

a $5x = 85$ **b** $m + 23 = 47$ **c** $2x + 1 = 8$ **d** $35 - n = 0$

e $3a - 1 = 10$ **f** $a - 2 = -3$ **g** $y + 3 = 1$ **h** $3m - 1 = 5$

i $-4x = 16$ **j** $5 + 3n = 10$ **k** $3x = 2$ **l** $6 - q = 10$

m $\frac{x}{5} = 10$ **n** $\frac{3p}{2} = 6$ **o** $\frac{x+2}{5} = 1$ **p** $\frac{a}{3} - 4 = 2$

q $6 + \frac{a}{3} = 8$ **r** $\frac{5+a}{2} = 1$ **s** $\frac{m-4}{7} = 6$ **t** $\frac{5m}{2} = 3$

u $6 - 2a = 8$ **v** $3p + 5 = 2$ **w** $\frac{q+6}{2} = 1$ **x** $\frac{3-x}{2} = 5$

13:05 Solving problems using equations

PREP QUIZ 13:05

What mathematical symbol (+ − × ÷) is correct for each word?

1 sum **2** product **3** minus **4** difference
5 total **6** times **7** quotient

Solve these equations.

8 $x + 12 = 23$ **9** $a - 7 = 10$ **10** $6m = 42$

The main use of algebra, especially with equations, is to help us solve problems.

> The use of algebra is a problem-solving strategy.



Take note!
Many of the problems in this section are very easy. You may be able to see the answer immediately. But you must practise writing out the solutions correctly now to be able to solve harder equations and problems later!

We are going to *translate* (or rewrite) problems written in words into algebraic sentences, which we call **equations**. It is therefore important to know the key words and their meanings, like those in Prep quiz 13:05 above.

We will let a pronumeral represent what we are trying to find.

Look at the following examples and note the way the solutions are set out.

WORKED EXAMPLES

∴ means 'therefore'.

1 The sum of a certain number and 7 is 12. What is the number?		**2** The product of a number and 9 is 72. What is the number?
Let the number be n.	**Step 1** Introduce pronumeral	Let the number be x.
$n + 7 = 12$	**Step 2** Write down equation	$9x = 72$
$n = 5$	**Step 3** Solve equation	$x = 8$
∴ The number is 5.	**Step 4** Answer the problem	∴ The number is 8.

3 After buying a drink I have 35 cents left from $3.
How much was the drink?
Let the cost of the drink be c cents.
$c + 35 = 300$ [or $300 - c = 35$]
$c = 265$
∴ The cost of the drink
was 265 cents, or $2.65.

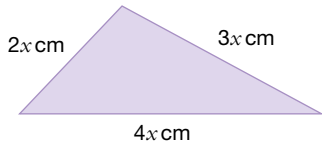


4 If an integer is multiplied by 7 and then 3 is added to the product, the result is 38.
What is the integer?
Let the integer be x.
$7x + 3 = 38$ ← This is a two-step equation.
$7x = 35$
$x = 5$
∴ The integer is 5.

In the next two examples you need to collect like terms before solving the equation.

5 One number is 5 more than a second number. Their sum is 21.
What are the numbers?
Let the first number be n.
The second number must be $n - 5$.
So: $n + n - 5 = 21$
i.e. $2n - 5 = 21$ ← Collect like terms.
$2n = 26$
$n = 13$
$n - 5 = 8$
∴ The numbers are 8 and 13.

6 The perimeter of this triangle is 36 cm.
Find the length of each side.



In this question the pronumerals are given.
$2x + 3x + 4x = 36$
$9x = 36$
$x = 4$
As the sides are $2x$, $3x$ and $4x$, the side lengths are 8 cm, 12 cm and 16 cm.

Exercise 13:05

P Foundation worksheet 13:05
Solving problems using algebra

1 Solve each problem by first writing an equation.
- **a** The sum of a certain number and 4 is 13. What is the number?
- **b** The sum of 9 and a certain number is 17. What is the number?
- **c** A certain number minus 7 is equal to 9. What is the number?
- **d** The product of 8 and a number is 56. What is the number?
- **e** A certain number is bigger than 7 and its difference from 7 is 3. What is the number?
- **f** The total of 6, 14 and a number is 35. What is the number?
- **g** If a number is divided by 7, the result is 12. What is the number?
- **h** The product of a number and 9 is 117. What is the number?
- **i** If 96 is divided by a number, the result is 16. What is the number?
- **j** If a number is subtracted from 73, the result is 31. What is the number?

2 Write an equation and then solve each problem.

a An exercise book and pen cost me $2.20. If the pen cost 75 cents, how much did the book cost? (Hint: Let the cost of the book be b cents.)

b It takes me 30 minutes to get dressed and have breakfast each morning. If it takes me 17 minutes to have breakfast, how much time do I need to get dressed? (Hint: Let the time to get dressed be t minutes.)

c A set of books cost me $63. If each book cost $5.25, how many were in the set? (Hint: Let the number of books be n.)

d Tara, Matilda and Harry's ages add to give 52. If Tara's age is 18 and Matilda is 15, how old is Harry?

e 156 cubes are divided among a group of children so that each child gets 12 cubes. How many children are there?

3 To solve each of these problems you will need a two-step equation.

a If a number is multiplied by 3 and then 5 is added to that product, the answer is 17. What is the number?

b If 10 is subtracted from the product of 2 and a certain number, the result is 4. What is the number?

c A number is divided by 5 and then 3 is added to the result, giving 7. What is the number?

d The product of a number and 8 is subtracted from 100. The result is 28. What is the number?

4 For each of these problems, collect like terms before solving the equation.

a One number is 7 more than another number. Their sum is 13. What are the numbers?

b The sum of two numbers is 20. Their difference is 6. What are the numbers?

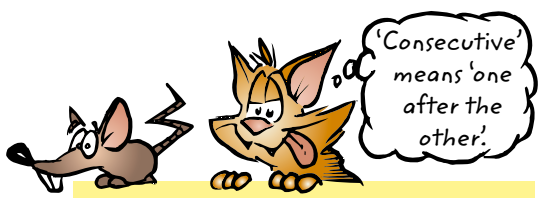
c The difference of two numbers is 19 and their sum is 29. Find the smaller number.

d The sum of two consecutive integers is 17. What are these integers?

e The sum of three consecutive integers is 18. What is the smallest integer?

f Ahmed is 5 years older than Meg. Their ages add to give 37. How old is Ahmed? (Hint: Let Meg's age be x.)

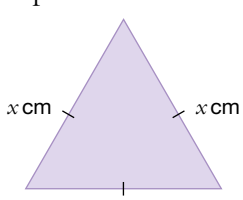
g Kylie has twice as many toys as Michelle. Together they have 36. How many toys does Michelle have?



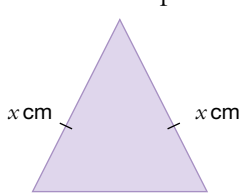
Note: If one integer is x, then the next **consecutive** integers are $x + 1$, $x + 2$, etc.

5 The perimeter of each figure is given. For each problem, find the value of x.

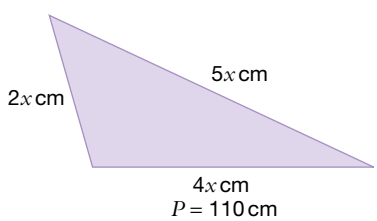
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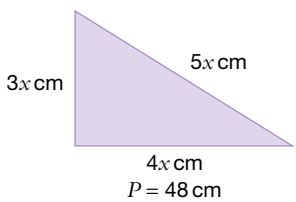
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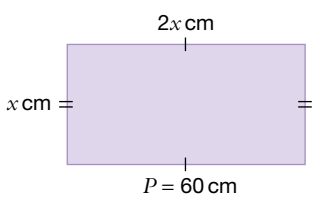
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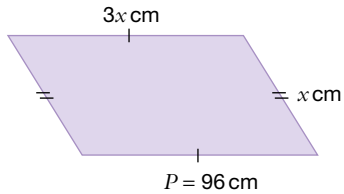
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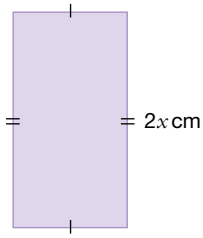
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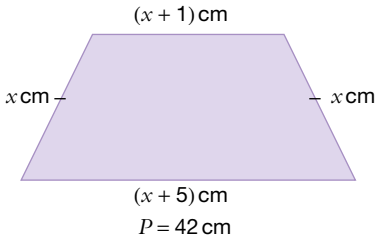
f



g



h



6 These problems may involve negative integers and/or decimal fractions. (Remember to write the equations first.)

- **a** The sum of a number and 3 is −4. Find the number.
- **b** The product of a number and 1·3 is 1·17. What is the number?
- **c** If 7 is subtracted from the product of 3 and a number, the result is −10. What is the number?
- **d** If the sum of a number and 5 is divided by 4, the result is 3. What is the number?
- **e** If twice a number is subtracted from 10, the result is 12. What is the number?
- **f** Three drinks and five ice-creams cost a total of $17.45. If a drink costs 75 cents more than an ice-cream, find the cost of an ice-cream.
- **g** The sum of three consecutive even numbers is 36. What are the numbers? (*Hint:* If the first even number is n, what is the next even number?)

13:06 Investigation of real problems

A variety of real-life problem-solving tasks, investigations and discussions are considered here. You will find that algebra can be a valuable tool.

Exercise 13:06

1 A car-hire company uses this formula to find the rental per day on a van that they have for hire:

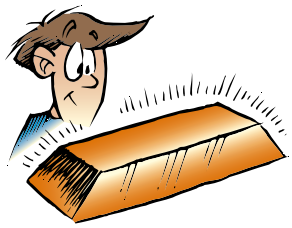
$$\text{Rental per day} = \$\left(25 + \frac{\text{kilometres travelled}}{2}\right)$$



Lisa wants to hire the van for one day. Why does she not know exactly how much it will cost? If she hires the van and travels 120 km, what will be the rental for the day? If Yasar hired the van and paid a total of $111, how many kilometres did he travel? (Set this up as an equation to solve.)

What would it cost for you to hire the van and travel to your favourite place?

2 Asif wants to make a picture frame from a single strip of wood 300 cm long. He wants the frame to be 12 cm longer than the width. Write an equation to represent this situation and use the equation to solve the problem.



3 A 400 g ingot of gold is made into a 40 g brooch and some 15 g rings.
Using r to represent the number of rings, complete the equation:
$\ldots = 40 + \ldots \times r$
Solve this equation to find the value of r, the number of rings made.

4 An industrious student decided to investigate the relationship between shoe size and foot length (in inches) among her classmates. The table shows the results she obtained.

Foot length	Shoe size
7	1
8	4
9	7
10	10



Can you find a rule connecting shoe size (S) and foot length (L)? Is this rule useful for people of all ages? Predict the foot length (in inches) of someone with a size 13 shoe.
Investigate for yourself among your classmates to see if a rule for shoe size and foot length can be found, using centimetres for foot length.

5 A building contractor uses wooden boards to 'box' concrete while it sets. Making a concrete path, the contractor records the amount of concrete used and the number of boards laid down along the path. The results appear below.

Concrete (m^3)	Number of boards
2	7
3	12
4	17
5	22



The contractor wants to find a rule connecting the amount of concrete with the number of boards used. Can you find a rule?
For a large project, about 30 m^3 of concrete is needed. How many boards are required?

6 A company executive found a formula that gave a good approximation to the relationship between sales and profit for her company in any one year:
Profit = (2 × Sales − 300) thousand dollars.
Over a five-year period, the sales figures have been 180, 210, 300, 170, 140.

a Use the formula to find the profit in each year.
b What can you say about the profit in the last year?
c If the company wants to make a profit of $50 000, how many sales would they need to make?
d How many sales need to be made for the company to break even (exactly zero profit)?

7 A student decides to study the relationship between the area of a circle and the radius. She is only able to measure the circles roughly, but she comes up with the following results:

Radius (cm)	Area (cm^2)
2	12
3	27
4	48
5	75

a Can you find a rule to fit these results?
(Hint: Try comparing the radius with one-third of the area.)

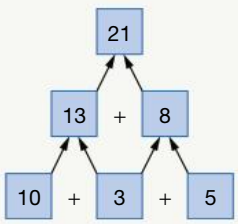
b If the radius of the centre circle on a football oval is about 400 cm, use your result to find an approximate area for the circle.

c Find out the actual formula connecting radius and area.

d Is the rule you have found a good approximation?

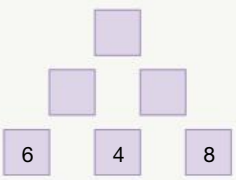
FUN SPOT 13:06 SOLVING PYRAMIDS

In each pyramid, the numbers in two squares are added to give the number in the square above. By doing this we can fill in any blank squares.

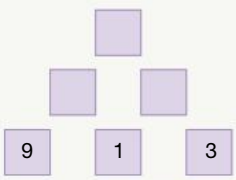


Complete these pyramids.

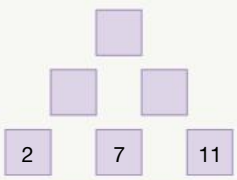
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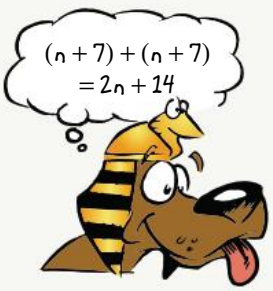
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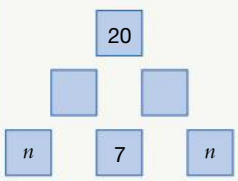


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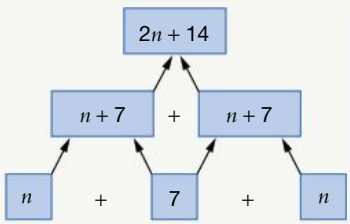


If one of the bottom numbers is missing, we can call it n. If enough information is given, we may be able to find the value of n.





Question

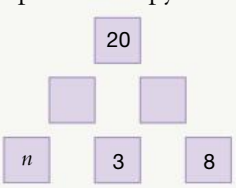


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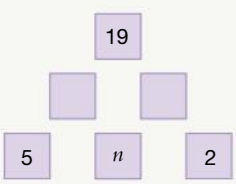
We can see that $2n + 14 = 20$
$2n = 6$
$\therefore\ n = 3$

Complete these pyramids and find the value of n.

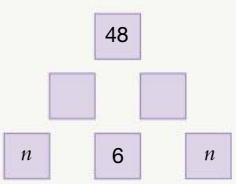
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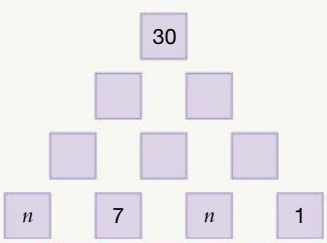
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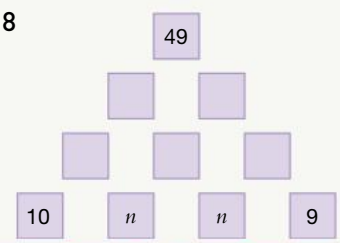
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7



8



MATHS TERMS 13

equation

- a number sentence where one or more of the numbers is missing or unknown
- the unknown number is represented by a pronumeral,
  e.g. $x + 5 = 8$, $\frac{3x+1}{7} = \frac{x-5}{2}$

evaluate

- to find the value of an expression,
  e.g. Find the value of $3a + 7$ if $a = 3$.
- to find the answer (usually after substitution)

integer

- whole number (directed number),
  e.g. … , −3, −2, −1, 0, 1, 2, 3, …

inverse operation

- the operation that will reverse or 'undo' a previous operation,
  e.g. addition is the inverse operation of subtraction; division is the inverse operation of multiplication

pronumeral

- a symbol (usually a letter) that takes the place of a numeral,
  e.g. x or $\square$

solution

- the method of finding the answer to a problem
  OR
  the answer to a problem
- the solution to an equation is a number that makes it a true sentence,
  e.g. $x = 3$ is the solution to $x + 2 = 5$

solve

- find the solution or answer to a problem or equation

substitute (noun: **substitution**)

- to 'put in place of'
- usually, when a numeral replaces a pronumeral,
  e.g. 'Substitute $x = 7$ into $2x + 3$'

DIAGNOSTIC TEST 13 EQUATIONS

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Solve these equations by writing in words what each equation means. 13:01

a $x + 7 = 11$ b $n + 3 = 3$ c $y - 6 = 3$ d $9 - m = 7$

2 Solve these equations by writing in words what each equation means. 13:01

a $5y = 20$ b $4m = 24$ c $\frac{x}{3} = 2$ d $\frac{h}{6} = 1$

3 Complete each arrow diagram by writing the correct operation above each arrow. 13:02

a $\boxed{x} \xrightarrow{\quad} \boxed{5x}$ b $\boxed{m} \xrightarrow{\quad} \boxed{7m} \xrightarrow{\quad} \boxed{7m + 1}$ c $\boxed{n} \xrightarrow{\quad} \boxed{n - 3} \xrightarrow{\quad} \boxed{\frac{n-3}{2}}$

4 Complete each arrow diagram by writing the correct operation above each arrow. 13:02

a $\boxed{a + 3} \xrightarrow{\quad} \boxed{a}$ b $\boxed{6x + 2} \xrightarrow{\quad} \boxed{6x} \xrightarrow{\quad} \boxed{x}$ c $\boxed{\frac{m}{2} - 7} \xrightarrow{\quad} \boxed{\frac{m}{2}} \xrightarrow{\quad} \boxed{m}$

5 Complete each arrow diagram. 13:02

a $\boxed{x - 5} \xrightarrow{+5} \boxed{}$ b $\boxed{3a + 4} \xrightarrow{-4} \boxed{} \xrightarrow{\div 3} \boxed{}$ c $\boxed{\frac{m-3}{5}} \xrightarrow{\times 5} \boxed{} \xrightarrow{+3} \boxed{}$

6 What operations must be performed on a to make each expression? 13:02

a $a + 4$ b $4a + 5$ c $\frac{a+7}{2}$ d $\frac{a}{4} - 5$

7 Solve each one-step equation by performing an inverse operation. 13:03

a $k + 8 = 11$ b $p - 7 = 3$ c $3a = 33$ d $\frac{x}{4} = 8$

8 Solve these equations that involve negative numbers. 13:03

a $n + 7 = 3$ b $m - 8 = -5$ c $-4x = 12$ d $\frac{a}{6} = -3$

9 Solve these two-step equations. The answers are integers. 13:04

a $2x + 5 = 9$ b $5m - 3 = 2$ c $\frac{n}{4} + 2 = 5$ d $\frac{m-2}{3} = 1$

10 Solve these equations. The answers involve fractions. 13:04

a $3a + 2 = 3$ b $5x - 4 = 4$ c $8 + 7y = 5$ d $3n - 7 = 6$

11 Write an equation to represent each problem. Let the unknown number be n. Solve each problem. 13:05

a The sum of a number and 7 is 19. What is the number?
b The product of 3 and a certain number is 36. Find the number.
c An unknown number minus 13 is equal to 23. What is the number?
d If 72 is divided by a certain number, the result is 12. Find the number.

ASSIGNMENT 13A Chapter review

1 Solve each of these one-step equations.

a $x + 7 = 20$ **b** $a + 15 = 23$

c $a - 7 = 20$ **d** $m - 15 = 23$

e $6 + n = 15$ **f** $16 - n = 5$

g $6n = 30$ **h** $4n = 36$

i $\frac{x}{3} = 5$ **j** $\frac{n}{4} = 8$

2 Solve each of these one-step equations. The answers are negative integers.

a $x + 7 = 2$ **b** $a + 15 = 10$

c $a - 3 = -10$ **d** $m - 1 = -4$

e $6 + n = 5$ **f** $6 - n = 10$

g $6n = -12$ **h** $-4n = 16$

i $\frac{x}{3} = -5$ **j** $\frac{n}{-3} = 8$

3 Solve each of these two-step equations.

a $3x + 7 = 19$ **b** $8a - 5 = 19$

c $9 + 2y = 19$ **d** $5 - 3n = 2$

e $30 = 2m - 6$ **f** $10 = 20 - 5h$

g $\frac{x+2}{3} = 4$ **h** $\frac{a}{3} - 5 = 4$

4 Check the given solution for each equation to see if it is correct.

a $3x + 2 = 17$ $x = 5$

b $7a - 5 = 19$ $a = 2$

c $8 + 3y = 19$ $y = 9$

d $15 - 2n = 5$ $n = 5$

e $38 = 7m - 4$ $m = 6$

f $14 = 30 - 4h$ $h = 4$

g $\frac{x-5}{3} = 2$ $x = 1$

h $\frac{a}{6} + 5 = 4$ $a = -6$

5 Solve each of these two-step equations.

a $3x + 5 = 9$ **b** $8a - 1 = 12$

c $9 + 4y = 19$ **d** $5 - 3n = 10$

6 Solve each problem by first writing an equation.

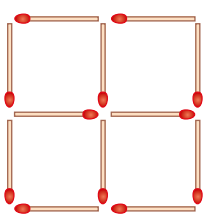
a The product of a certain number and 12 is 96. What is the number?

b If a number is subtracted from 23, the result is 14. What is the number?

c If the sum of a number and 10 is divided by 3, the result is 9. What is the number?

ASSIGNMENT 13B Working mathematically

1 Five squares are made from 12 matches, as shown in the diagram. By removing two of the matches, form only two squares.



2 You have five coins, one of which is known to be counterfeit (fake). The counterfeit coin is either lighter or heavier than the real coins. What is the smallest number of weighings on the balance needed to be sure of identifying the counterfeit coin?



3 A car park charges $11.80 per hour for parking. How much will it cost to park a car that is left at 6:45 pm and collected at 9:10 pm? (*Note:* A fraction of an hour costs the same as 1 full hour.)

4 Thirty-five children are to go on an excursion. They each bring $6 to cover the cost of the bus. At the last moment, an extra fifteen children decide to go on the excursion too. How much do the extra children pay and how much is returned to each of the children who have already paid?

5 A party is being organised for 30 children. If each child has to have 2 glasses of lemonade and each glass holds 125 mL, how many 1250 mL bottles of lemonade must be bought?

ASSIGNMENT 13C Cumulative revision

1 Complete:

a 70 cm = … mm **b** 8500 m = … km **c** 960 mm = … cm

Metric table, page xiv

2 Simplify:

a 0·6 + 0·12 **b** 12 − 2·5 **c** 6 × 0·4 **d** 12·36 ÷ 6

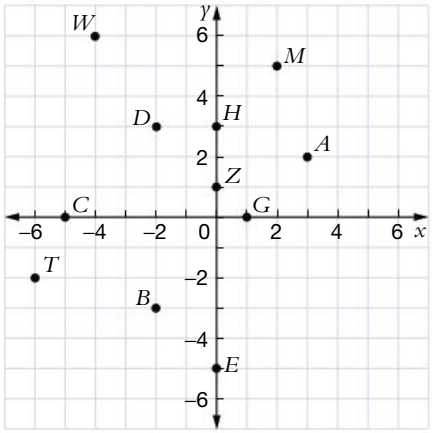
6:02–6:04

3 Write the coordinates of these points on the graph.

a A **b** B

c C **d** D

e E



7:03

4 On the graph at right, which point is at:

a (0,3) **b** (−6,−2)

c (2,5) **d** (−4,6)

e (0,1)?

7:03

5 Simplify:

a $\frac{7}{5} - \frac{3}{5}$ **b** $\frac{2}{10} + \frac{1}{10}$ **c** $5 \times \frac{2}{3}$ **d** $\frac{8}{10} \div 2$

9:04–9:06

6 **a** What fraction of 1 m is 40 cm? **b** Change $\frac{3}{10}$ to a percentage.

c Change 0·8 to a percentage. **d** Write 56% as a decimal.

11:02, 11:03

7 Simplify these expressions.

a $3x + 2y - x + 3y$ **b** $3a - 4 + 5a + 7$ **c** $3x \times 4y$

d $6n \times 5m$ **e** $8x \div 4$ **f** $8x \div 4x$

4:07

8 Find the value of each expression if $a = 3$, $b = 5$ and $c = -4$.

a $a + b + c$ **b** $a - b - c$ **c** $ab + c$

d abc **e** $ab - ac$ **f** $a^2 + b^2 + c^2$

4:06

9 Write each numeral in its simplest form.

a $(7 \times 1000) + (3 \times 100) + (2 \times 10)$ **b** $(6 \times 1000) + (4 \times 10) + (8 \times 1)$

c $(6 \times 10\,000) + (7 \times 1000) + (4 \times 100)$ **d** $(3 \times 1\,000\,000) + (5 \times 10\,000)$

3:02

10 Complete the tables, carrying out each operation on the previous answer found.

7:09

a

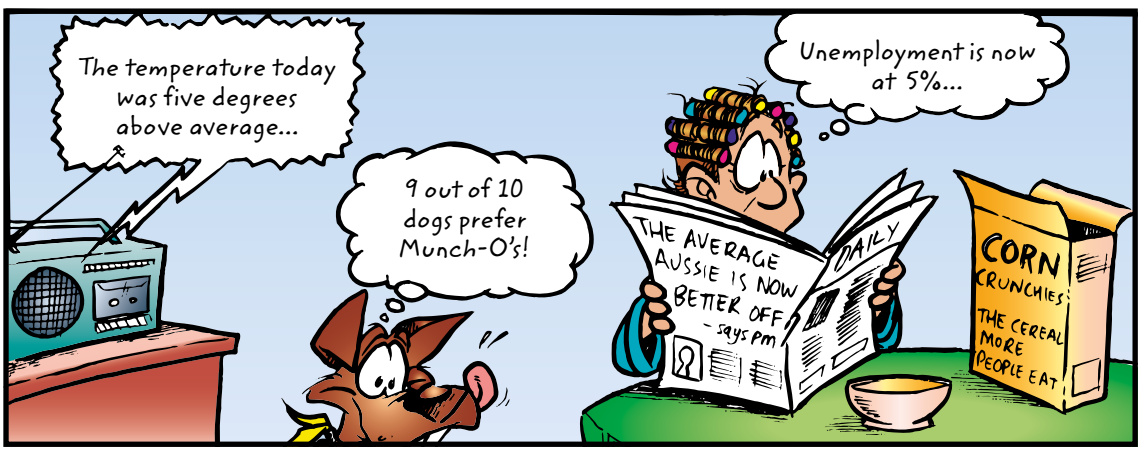
Operation	Start	−7	×3	+11	−35	÷6
Answer	5					

b

Operation	Start	×3	−45	÷9	+5	×96
Answer	−3					

STATISTICS

14



Contents

14:01 Types of data
14:02 Collecting data
Investigation 14:02 Nothing but the facts
14:03 Sorting data
Investigation 14:03 Sorting it out
14:04 Analysing data
Investigation 14:04 Spreadsheets
14:05 Dot plots
Investigation 14:05 Mean, mode and median
14:06 Stem-and-leaf plots
Fun spot 14:06 Anyone for tennis?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Statistics and Probability

Selections from ***Data Collection and Representation***, ***Single Variable Data Analysis*** [Stage 4]

- Investigate techniques for collecting data, including census, sampling and observation. (ACMSP284)
- Construct and compare a range of data displays, including stem-and-leaf plots and dot plots. (ACMSP170)
- Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data. (ACMSP171)
- Describe and interpret data displays using mean, median and range. (ACMSP172)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

We are bombarded with statistics every day, particularly in advertising and in the news. **Statistics** is the branch of mathematics that is concerned with the gathering and organisation of numerical material called **data**. In this chapter we will look at three basic steps:

- collecting data
- sorting data
- analysing data.

But before we do this, we need to recognise what types of data are being collected.

14:01 Types of data

The data we collect consists of **variables**. These are pieces of information, such as quantities or characteristics, that can be observed or measured. They may change over time or between individual observations.

There are two main types of variables, which can be identified by the following types of questions:

What is your favourite colour?	The answer may be: red, blue, orange etc.
How many brothers and sisters do you have?	The answer may be: 0, 1, 2, 3 etc.

The first question needs a **categorical** answer. (The answer must be a category.)
The second question needs a **numerical** answer. (The answer must be a number.)

Categorical data

These variables can be put into various categories. The categories may indicate some order or they may be distinct (i.e. with no order to them). For example:

Categorical variable	Ordered categories
Exam grade	A, B, C, D, E, F
Movie rating	★, ★★, ★★★, ★★★★, ★★★★★
Quality rating	Poor, Satisfactory, Good, Excellent
Garment size	XS, S, M, L, XL, XXL

Categorical variable	Distinct categories
Car brand	Ford, Holden, Mazda, Nissan, Toyota
Hair colour	Black, Blonde, Brunette, Red
Eye colour	Blue, Brown, Green, Hazel
Favourite music	Blues, Classical, Country, Jazz, Metal, Pop, Rock



Numerical data

These variables are represented by numbers. Numerical variables can be divided into two more types: **discrete** or **continuous**.

Discrete data can only be certain fixed values, usually whole numbers. This usually results from a counting process, in response to a question such as 'how many?'

Continuous data can be a range of continuous values. This usually result from a measuring process. Continuous values are not distinct units and their accuracy may depend on the precision of a measuring device.

Discrete numerical variables
Number of children in a family
Number of sixes when rolling a dice
Runs scored by each batter
Number of cars per household

Continuous numerical variables
Height of a person
Time to run a race
Volume of rainfall
Distance for a javelin throw



Continuous numerical data is usually measured to a given degree of accuracy. For example, weight may be measured to the nearest kilogram. It may be possible to obtain measurements in smaller units, but real-life measurements usually cannot achieve perfect accuracy no matter how small the units.

WORKED EXAMPLES

1 For a class of students, what type of data is represented by the following?

a their shoe size
b the time they woke up this morning
c the cereal they ate for breakfast
d the grade for effort on their last report

2 Consider data that might be collected about cars. Think of an example for each data type.

a categorical, ordered
b categorical, distinct
c numerical, discrete
d numerical, continuous

Solutions

1 **a** Shoe sizes are numerical. Even with 'half' sizes such as $6\frac{1}{2}$, the sizes are not continuous, so they are discrete.

b The time a student woke would be recorded as numerical data, and time is continuous. A judgement must be made to determine the degree of accuracy, e.g. to the nearest minute.

c The type of cereal is clearly not numerical, it is categorical. There is no rating or judgement of one type over another, so the data is distinct, not ordered.

d Grades for effort are usually recorded as a letter (A, B, C etc.) or as a comment such as 'excellent', 'good', 'satisfactory' etc. The type of data is categorical and a ranking is involved, so the data is ordered.

2 There are many possible answers.
For example:

a categorical, ordered: a safety rating by a motoring magazine (e.g. 0 to 5 stars)

b categorical, distinct: brand of tyre used

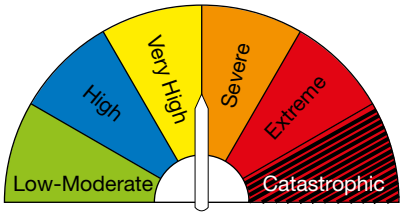
c numerical, discrete: number of cylinders in a motor

d numerical, continuous: petrol consumption in km/L



Exercise 14:01

1 State whether each variable is numerical or categorical.



- **a** school enrolments
- **b** maximum daily temperatures for May
- **c** eye colour of students
- **d** censorship classification for movies
- **e** religion of students in a class
- **f** number of heads in 100 coin tosses
- **g** weight of babies born in June
- **h** fire hazard ratings
- **i** height of trees in a park
- **j** survey on the quality of service

2 Determine whether these numerical variables are discrete or continuous.

- **a** length of a person's foot
- **b** shoe size
- **c** phone calls in a day
- **d** duration of a phone call
- **e** number of laps swum
- **f** time to swim one lap of a pool
- **g** time taken to finish a quiz
- **h** number of questions answered in a quiz
- **i** number of posts in a fence
- **j** length of a fence

3 Determine whether the categorical data in the following examples is distinct or ordered.



- **a** Patrons of Lily's Pad Cafe are asked to rate the meals as excellent, good, OK or poor.
- **b** Viewers are asked to nominate their favourite TV show.
- **c** Students are asked to count the number of trucks, cars and motorcycles that pass the school in one hour.
- **d** A survey asks readers to rank their response to an opinion as 'strongly agree', 'agree', 'disagree' or 'strongly disagree'.
- **e** In a particular election, voters cast their vote for the political candidate of their choice: Labor, Liberal, Greens, National or Independent.
- **f** Movies for cinemas in Australia are rated G, PG, M, MA or R.

4 Do these tables display categorical or numerical data? If categorical, state whether it is distinct or ordered. If numerical, state whether it is discrete or continuous.

a

Employee	Salary
Wahab	$52 000
Wells	$39 900
Wilson	$62 300
Wolfe	$59 150
Wu	$48 370

b

Employee	Efficiency
Wahab	B
Wells	C
Wilson	A
Wolfe	A
Wu	B

c

Employee	Age
Wahab	39 y, 127 d
Wells	42 y, 246 d
Wilson	45 y, 037 d
Wolfe	36 y, 314 d
Wu	40 y, 193 d

d

Employee	Car
Wahab	Holden
Wells	Toyota
Wilson	Mazda
Wolfe	Ford
Wu	Mazda

e

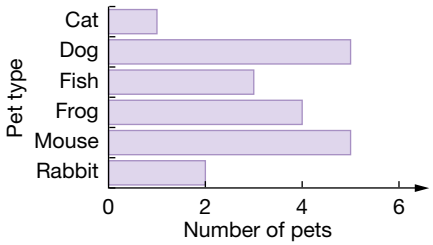
Employee	Dependants
Wahab	4
Wells	3
Wilson	5
Wolfe	0
Wu	2

f

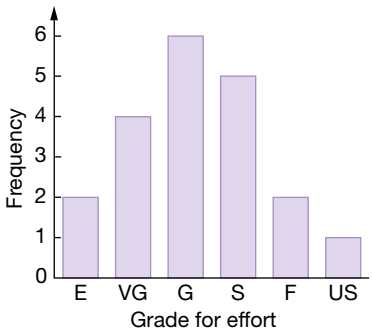
Employee	Movie type
Wahab	Western
Wells	Drama
Wilson	Comedy
Wolfe	Western
Wu	Action

5 Do these graphs display categorical or numerical data?

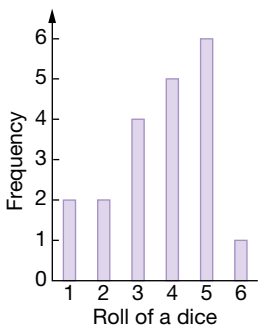
a



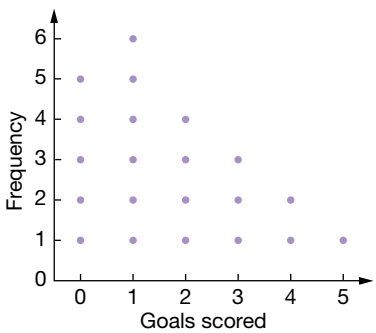
b



c



d



14:02 Collecting data

Information is collected in different ways, depending on what it is used for and how accurate it needs to be. Three main ways of collecting data are by **census**, by **sample** or by **observation**.

Census

A census collects information from the entire population studied. For example, if every student in a school is questioned to determine how many students catch a bus to school, this is a census of the school students.

In Australia a national census is conducted every five years, collecting information from every household including details of income, employment, education, housing and many other things. This is a huge task, of course, and it takes a lot of time and effort to sort and analyse the data. An Australian census was taken on 8 August 2006 and on 9 August 2011.



Sample



A sample collects data from only a portion of a population. (Samples are common because a census is often impractical, particularly if a population is large.) The size of the sample must be large enough for the sample to give a fair indication of the population. For the same reason, the members of the sample should be chosen at random.

For example, if a sample of five students is randomly selected and asked if they catch a bus to school, and four say yes, this sample is too small to reasonably conclude that 4 out of 5 (or 80%) of *all* students catch a bus to school. If the sample was 100 students, then the data collected would have a greater chance of reflecting the total student population.

Observation



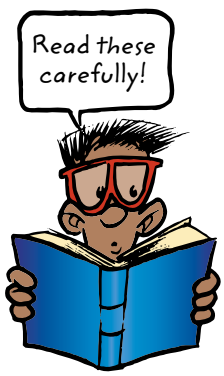
A lot of statistics does not involve either a census or a sample, but instead merely observation, collection and tabulation of numerical facts. For example:

- Finance—prices to buy and sell, exchange rates
- Sports—cricket: bowling and batting averages, run rates; tennis: first serves in, unforced errors, double faults; football: tackle counts, goals kicked, points for and against
- Weather—temperatures, barometer readings, rainfall, tides

Work through the following short exercise and then try some of the practical suggestions given.

Exercise 14:02

1 Mr Hines suspected that Maths was the favourite subject of students in Year 8, so he asked students in his class to conduct a survey to establish if this was true.



- Matthew decided to ask five of his friends. As three of them liked Maths the best, he reported that 3 out of 5 (or 60%) of students liked Maths best.
- Tai decided to survey his class. As 20 out of the class of 30 liked Maths best, he reported that $\frac{2}{3}$ of the year were Maths lovers.
- Casey decided to ask 10 students from each of the four classes their opinion. From this she concluded that 70% of students liked Maths best.
- Alexis decided that the only way to be really sure was to ask every student in the year.

a Which student actually carried out a census?

b Three students surveyed a sample of the year. Which sample is likely to have been the most accurate?

c Why do you think the other two samples might have been less accurate?

d What is one problem with conducting a census, rather than surveying a sample?

2 Melody wanted to see which type of music was most popular in the community, so she handed out this survey form to fellow students at school. She discovered that Rock was by far the most popular.

a What is wrong with Melody's sample?

b How could she get a better sample of the community's opinion?

Tick your favourite music
- Classical ☐
- Country & Western ☐
- Folk ☐
- Jazz ☐
- Rock ☐

3 Ben wanted to discover the sport that was most popular in the community, so he handed out this survey form to fellow students at school. He discovered that Rugby League was by far the most popular.

a What is wrong with Ben's sample?

b What is wrong with his questionnaire?

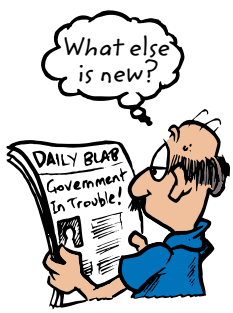
c How could he get a better sample of the community's opinions?

d Design a questionnaire that would be more useful than this one.

Tick your favourite sport
- Rugby League ☐
- Australian Rules ☐
- Soccer ☐
- Cricket ☐
- Basketball ☐
- Tennis ☐
- Softball ☐

4 Before elections, polls are carried out by different organisations to try to predict who voters will vote for. Often the polls come up with different results. How can this happen?





5 Roland rolled a dice and got these results:

1 4 6 5 6 3 2 6

Because three times as many 6s were rolled as any other number, Roland concluded that the dice must be 'weighted' to give more 6s.

a Is this conclusion necessarily correct?

b What should Roland do to make sure that his conclusion is correct?

c How does this question relate to taking a sample?

6 The *TV Now* website runs an opinion poll to see what shows and personalities are the most popular. Why might the results not be accurate?

7 What is wrong with these claims?

a 9 out of 10 dogs prefer MUNCH-O'S!

b 4 out of 5 dentists use AJAX toothpaste!

c 20% more people drink FIZ soft drink!

d ELECTRO batteries last 30% longer!



Stratified random sampling

If a population contains subgroups that need to be acknowledged, random sampling should be done on a sectional basis to ensure that all subgroups are fairly represented:

1 Determine the fraction that the group is of the whole.

2 Make sure that the sample contains the same fraction of this group.

For example, if our sample is 60 people and we know that $\frac{1}{10}$ of the whole population are Year 7 students, then we should survey $\frac{1}{10}$ of 60 (i.e. 6) Year 7 students.

8 A survey is to be held at Black Rock High School. It is decided to use stratified random sampling to sample students from Years 8 to 12. If 80 students are to be chosen, how many from each year should be chosen? The number of students in each year is shown in this table.

Year 8	Year 9	Year 10	Year 11	Year 12	Total
160	203	245	200	202	1010

9 A survey of Victorians aged 15 years and older is to be carried out to discover what percentage have seen the Sydney Opera House. A stratified random sample of 400 people is to be surveyed, with groups considered according to age. How many of each age group should be surveyed, if of the 3 510 000 people aged 15 years and older:

- there are 339 000 in the 15–19 years category
- there are 2 467 000 in the 20–59 years category
- there are 188 000 in the 60–64 years category
- there are 516 000 in the 65 years and over category?



10 If a stratified random sample of 100 students is selected from your school, how many students should be selected from each year?

INVESTIGATION 14:02 NOTHING BUT THE FACTS

Practical activities

(Data collected in these activities may be used for Investigation 14:03, page 409.)



1 Conduct a survey to find out how many children are in the family of each class member. (You can decide which definition of 'family' is used for the survey.)

2 Go to a car park or a point beside a fairly busy road and collect various data on the vehicles you see, for example:
Type of vehicle: car, truck, bus, motorcycle
Make of vehicle: Ford, Holden, Toyota etc.
Colour of vehicle: red, white etc.
Number plate: first letter or number on the plate.

3 Roll two dice a large number of times (at least 100 times!) and record the totals.

14:03 Sorting data

Frequency distribution tables



When a large amount of data is collected, it is usually fairly useless until it has been *sorted* in some way and presented in a form that is easily read. A common method of sorting data is to use a **frequency distribution** table. Look at the example below to see how it is done.

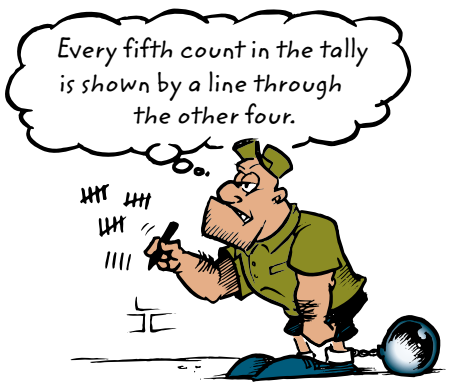
Example

Elizabeth surveyed the 30 students in her class to determine the number of children in each family. She collected the following data:

2	3	1	2	5	4	1	2	3	3	2	4	6	4	4
2	3	1	1	3	2	1	5	3	3	2	1	2	4	2

When the data is organised in a frequency distribution table, it looks like this:

Outcome	Tally	Frequency
1	𝍸 |	6
2	𝍸 ||||	9
3	𝍸 ||	7
4	𝍸	5
5	||	2
6	|	1
	Total:	30



Note the following features of the frequency distribution table:

- The **Outcome** column shows all the possible results for the data.
- The **Tally** column is used for recording the data, one entry at a time.
- The **Frequency** is the total tally for each outcome (i.e. the number of times each outcome appears in the data).
- The **Total** of the frequencies should be the same as the total number of data.

From the table it can easily be seen that the most common number of children per family is 2; that only one family has 6 children; that 6 students are the only children in their families; and so on.

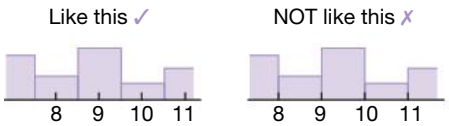
Using graphs

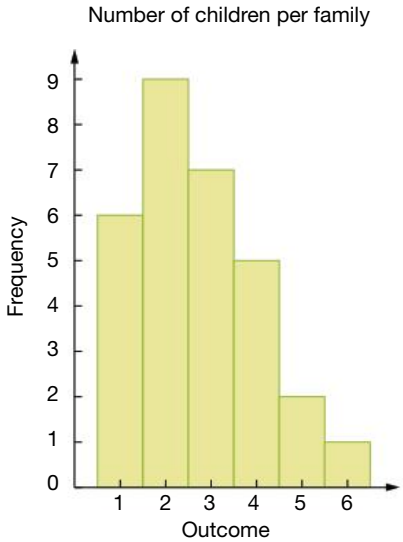
Another way to show data in an easily read form is by drawing a graph. Two commonly used types of graphs are the frequency histogram and the frequency polygon. The data in the table on page 404 can be graphed in these ways.

Frequency histogram

This is a type of column graph where the columns are drawn next to each other to show the range of outcomes. You should note the following features:

- The graph has a title.
- Each axis is labelled.
- The columns are centred on the outcome values.
- The first column begins one-half of a column width in from the vertical axis.



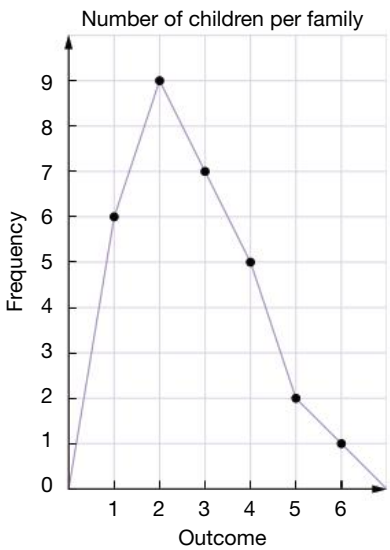


Frequency polygon

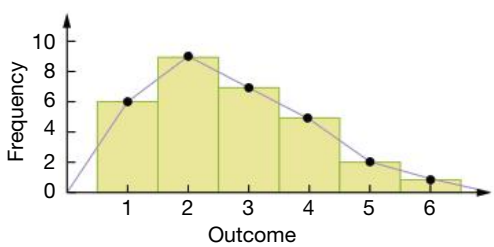
This is a type of line graph. Note that the axes are the same as for the histogram. You should note the following features:

- The graph has a title.
- Each axis is labelled.
- The dots showing the data are joined by straight lines.
- The first and last dots are connected to the horizontal axis as shown.
- The first outcome is one unit in from the vertical axis.





It is interesting to see that if a frequency histogram and polygon are drawn on the same axes, the polygon joins the midpoints of the top of each column in the histogram. This is shown in the diagram at right.



Exercise 14:03

1 **a** Complete the frequency column of this table.

b How many items of data (or 'scores') are there altogether?

c Which outcome occurred the most?

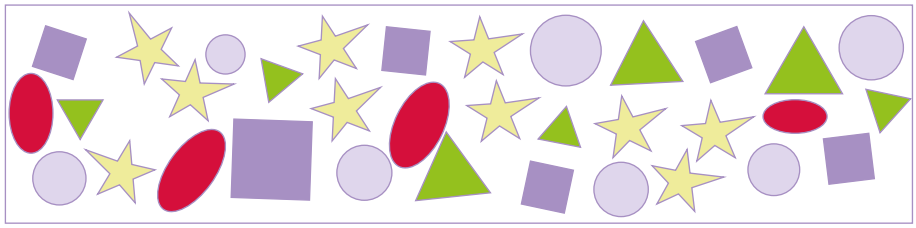
d What is the frequency of 4?

e How many scores are 4 or more?

Outcome	Tally	Frequency
2	|||	
3	𝍸 ||||	
4	𝍸 |	
5	||||	
6	||	

Total: …

2



a Complete the frequency distribution table for the shapes shown above.

b How many shapes are there altogether?

c Which shape occurs most frequently?

d Are there more triangles or squares?

Outcome	Tally	Frequency
circle		
oval		
square		
star		
triangle		

Total: …

3 A dice was rolled 36 times and the following results were recorded:

4	2	3	1	3	4	6	1	5	3	4	6
6	3	4	2	1	1	4	5	6	3	4	5
1	3	5	4	2	6	1	2	5	3	6	4



a Sort the data into a frequency distribution table.

b Which outcome has the highest frequency?

c Which outcome has the lowest frequency?

d If you rolled this dice another 36 times, would you expect to get the same results?

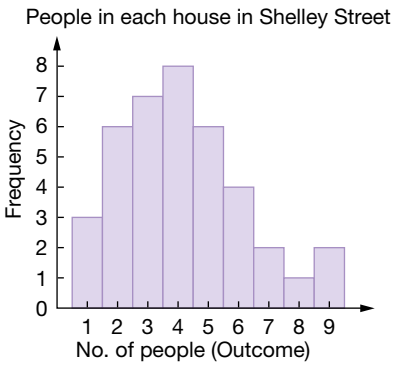
4 Ms Morris gave her class a spelling test of 10 Maths words. The number of words that each of the 30 students spelt correctly was:

9	7	8	6	4	7	10	9	8	7
6	7	5	5	10	9	8	8	7	9
10	5	7	6	8	8	7	9	6	9

a What was the highest score?
b What was the lowest score?
c Organise the data into a frequency distribution table.
d What was the most common score?
e How many students spelt fewer than 7 words correctly?
f How many students made only 2 mistakes or fewer?

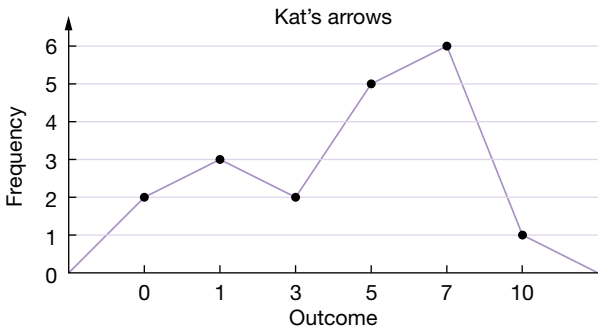
Outcome	Frequency
4	
5	
6	
7	
8	
9	
10	
Total:	…

5 This histogram shows the number of people living in each house in Shelley Street.



a What is the most common number of people per house?
b How many people live by themselves?
c What is the largest number of people in any one house?
d How many houses have 5 people in them?
e Work out the total number of houses in Shelley Street.
f Calculate how many people live in Shelley Street altogether.

6 This frequency polygon shows the scores for the arrows that Kat fired at a target.





a How many arrows hit the bullseye for a score of 10?
b How many arrows missed the target, scoring zero points?
c How many arrows scored 7 or more?
d How many arrows were fired altogether?

7 Draw a frequency histogram for each set of data.

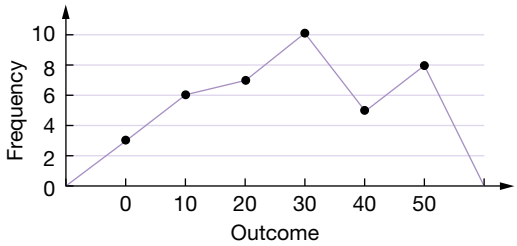
a

Outcome	Frequency
1	2
2	4
3	6
4	5
5	1

b

Outcome	Frequency
7	1
8	3
9	2
10	6
11	3
12	5

8 **a** Read the frequency polygon below to fill in this frequency table.
b How many scores are there altogether?
c How many scores are 20 or less?



Outcome	Frequency

Total: ...

9 A bag of Goody Gumdrops contains 30 lollies—at least, that's what it says on the packet. However, when 50 bags were surveyed for their contents, the following results were obtained:

31	30	29	30	30	31	32	29	28	30
30	29	30	31	33	32	31	30	28	29
27	31	30	32	30	30	31	30	32	31
30	29	28	30	31	28	30	30	27	30
33	29	32	30	30	31	30	31	32	30



a Determine the highest and lowest scores in the data.
b Organise the data into a frequency distribution table.
c How many packets did not have at least 30 lollies in them?
d How many packets had more than 30 lollies?
e Graph these results by drawing a combined frequency histogram and polygon.

10 Luke was concerned about gun laws. He asked 100 people to respond to the following statement:

It should be made harder for people to own a gun in New South Wales.
A strongly agree **B** agree
C disagree **D** strongly disagree

These were his results:

A	B	A	A	C	A	A	D	A	B	C	B	D	A	B	B	C	B	A	B
B	C	B	B	A	B	C	C	B	B	D	B	A	A	B	C	C	B	B	B
B	B	A	B	C	B	B	C	A	B	B	A	A	B	B	C	A	A	A	A
B	A	C	D	A	B	B	A	C	B	B	B	B	C	A	B	B	C	B	D
B	A	B	B	B	C	C	C	A	B	B	C	C	B	B	B	A	A	C	A

a Organise these results into a frequency distribution table.
b How many people strongly agree with the statement?
c What percentage of the people asked strongly disagree with the statement?
d What percentage of the people asked think that it should be made harder for people to own a gun in New South Wales?
e What percentage think that it should not be made harder?
f Draw a frequency histogram of these results.

INVESTIGATION 14:03 SORTING IT OUT

Practical activities

(Some of these activities refer to data collected in the practical activities of Investigation 14:02, page 403.)

1 Sort the data collected on the number of children in the family of each class member into a frequency distribution table. Also draw a graph showing this information. How would this information reflect the whole population of NSW? What differences might there be and why?

2 Sort and graph the data collected on motor vehicles.

3 When two dice are rolled a large number of times, the results are very interesting. What total would you expect to occur most often? Sort the results into a table to find out.



14:04 Analysing data

After data has been sorted, certain key numbers can be determined.

Range—a measure of spread



The **range** gives a rough idea of how the data values are spread.

It is defined as the difference between the highest and lowest values.

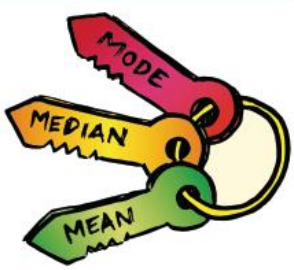
WORKED EXAMPLE

For the scores 5, 9, 3, 4, 7, 3, 2, the highest score is 9 and the lowest score is 2.

Range = highest score − lowest score

Range = 9 − 2
= 7

Measures of location



The next three key numbers tell us how the data tends to cluster. These numbers are called **mode**, **median** and **mean**, and are together called the 'measures of location' or 'measures of centre'.

Mode

The mode is the outcome that occurs the most often, i.e. it has the highest frequency.

WORKED EXAMPLE

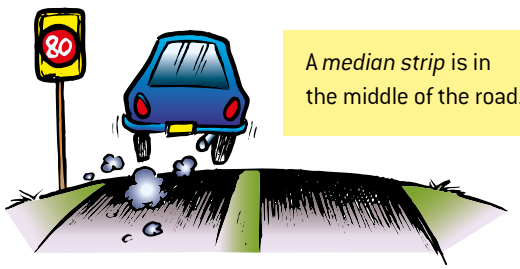
For the scores 2, 5, 3, 2, 5, 2, 4, the outcome 2 occurs the most often, so the mode = 2.

The *mode* is the outcome that occurs most often.

Median

When a set of data is arranged in order, the median is the 'middle score'.

This is only strictly true if there is an odd number of scores. For an even number of scores, the median is the average of the middle two scores.



A *median strip* is in the middle of the road.

WORKED EXAMPLES

1 If the scores 5, 9, 3, 7, 4 are arranged in order, we get 3, 4, 5, 7, 9.
The middle score is 5, so the median = 5.

2 If the scores 5, 9, 3, 7, 4, 6 are arranged in order, we get 3, 4, 5, 6, 7, 9.
The middle two scores are 5 and 6, so the median $= \frac{5+6}{2} = 5\frac{1}{2}$ or 5·5.

For an odd number of scores, the *median* is the middle score.
For an even number of scores, the *median* is the average of the middle two scores.

Mean

The term 'average' usually refers to the mean.

To calculate the mean, the sum of the scores is divided by the number of scores.

Note:
The word 'average' may not always refer to the mean, especially in everyday language. In a statement like 'the average family car is a Holden', the term 'average' is really referring to the *mode*, i.e. saying that more families own Holdens than any other make of car.

WORKED EXAMPLE

For the scores 6, 7, 8, 4, 6, 8, the sum of the scores is 39 and the number of scores is 6.

$\therefore$ Mean = 39 ÷ 6
= 6·5 or $6\frac{1}{2}$

The *mean* is the arithmetic average.

Finding the mean from a frequency distribution table

To more easily calculate the mean from a frequency distribution table, another column is added: the frequency × outcome column. This can be abbreviated to fx, where f stands for frequency and x stands for outcome.

Each number in the fx column is obtained by multiplying the outcome and frequency together.

The numbers in the fx column give the sums of all 0s, 1s, 2s etc.

Now the sum of the fx column gives the sum of all the scores and the sum of the frequency column gives the number of scores, so:

$$\text{Mean} = 49 \div 20$$
$$= 2{\cdot}45 \text{ or } 2\tfrac{9}{20}$$

Outcome (x)	Frequency (f)	fx
0	2	0
1	3	3
2	4	8
3	7	21
4	3	12
5	1	5
Total:	20	49



$$\text{The } \textit{mean} = \frac{\text{sum of all the scores}}{\text{total number of scores}} = \frac{\text{sum of } fx \text{ column}}{\text{sum of frequency column}}$$

Finding the mean using a calculator

Many calculators can be used for statistics, but different calculators may need to be operated differently.

- For sets of individual scores, it is easy to simply add the scores in the calculator and then divide this sum by the number of scores.
- Most calculators have a statistics mode that enables the use of various statistical functions. In Year 7 the most useful will be the 'mean' function, which is usually labelled as $\overline{x}$.

Note: Different types of calculator may use different keys. Check the instruction booklet for your calculator. The workings below are for a Casio fx-82AU PLUS calculator.

To find the mean for the scores in this table:

Outcome (x)	Frequency (f)
0	2
1	3
2	4
3	7
4	3
5	1

1 Press SHIFT, then SET UP and toggle to the second screen of functions. Select *3:STAT*, then select *1:ON* to choose a frequency column.
2 Press MODE, then select *2:STAT* to enter *statistics* mode.
3 Select *1:1-VAR*. Two columns should appear on the screen.
4 Enter the 'Outcome' scores into the column labelled x, using the = key.
5 Toggle across to the top of the second column (labelled *FREQ*). Enter the frequency for each score.

The next step is **VERY IMPORTANT**:

6 Press AC ('All Clear') to remove the table from the screen.
7 Press SHIFT then STAT (above *1*). Select *4:VAR*. You will be presented with four choices.
8 Select *2* $\overline{x}$ and then press =. This should give you the mean of 2·45.

Note: If you don't enter any numbers in the frequency column, then most calculators will assume there is only 1 of each score.

WORKED EXAMPLES

1 For this set of scores determine the range, mode, median and mean:

6 9 7 3 9 4 7 6 5 7

Range = highest score − lowest score
= 9 − 3
= 6
∴ Range is 6.

Mode = outcome that occurs the most.
7 occurs the most (frequency = 3)
∴ Mode is 7.

Median—first, the scores must be rearranged in order:

Don't forget!

3 4 5 6 6 | 7 7 7 9 9 [10 scores]
As there is an even number of scores,
Median = average of two middle scores

$$= \frac{6+7}{2}$$

$$= 6\tfrac{1}{2} \text{ or } 6{\cdot}5$$

∴ Median is 6·5.

Mean $= \dfrac{\text{sum of all the scores}}{\text{total number of scores}}$

$$= \frac{63}{10}$$

$$= 6{\cdot}3$$

∴ Mean is 6·3.

2 From this table of data determine the range, mode, median and mean.

Outcome	Frequency	fx
5	1	5
6	5	30
7	5	35
8	4	32
9	3	27
10	2	20
Total:	20	149

Range = highest score − lowest score
= 10 − 5
= 5
∴ Range is 5.

Mode: these two scores (6 and 7) both have the highest frequency (5).
∴ Modes are 6 and 7.

Note:
There can be more than one mode.

Median: we can use the frequency column to find the median. There are 20 scores (an even number), so we want the average of the 10th and 11th scores.



Outcome	Frequency	
5	1	1
6	5	6
7	5	11
8	4	15
9	3	
10	2	

We add down the frequency column (i.e. counting the number of scores) until we reach the 10th and 11th scores. This shows that there were 6 scores of 6 or less and 11 scores of 7 or less. This means that both the 10th and 11th scores must have been 7.
Of course the average of two 7s is 7:
∴ Median is 7.

$$\text{Mean} = \frac{\text{sum of } fx \text{ column}}{\text{sum of frequency column}}$$

$$= \frac{149}{20}$$

$$= 7{\cdot}45$$

$\therefore$ Mean is 7·45.

Exercise 14:04

P Foundation worksheet 14:04 Working with data

1 a Determine the range of each set of data.

i 2, 5, 3, 4, 5

ii 7, 9, 6, 9, 8, 7, 7

iii 10, 20, 10, 50, 40, 30

iv 17, 20, 19, 22, 21, 17, 20

v 49, 50, 48, 51, 50

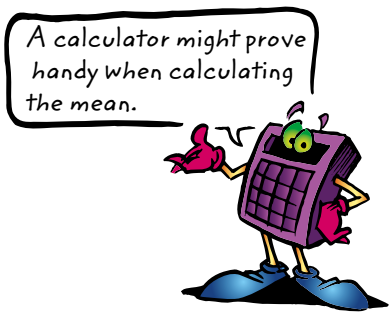
vi 8, 10, 12, 7, 8, 10, 9, 8, 10, 8, 11

vii 5, 4, 6, 3, 2, 4, 6, 9, 4, 7, 3, 2

viii 1·1, 1·3, 1·0, 1·2, 1·1, 1·3, 1·4

ix 16, 17, 19, 15, 17, 19, 14, 16, 17, 20

x 147, 151, 148, 150, 148, 152, 151



b Determine the mode for each set of data above.

c Find the median of each set of data.

d Calculate the mean for each set of data (correct to 1 decimal place).

2 a A cricketer has scores of 63, 27, 19, 5, 47 and 21. Find the mean (average) score.

b Samantha had percentage test results of 56, 67, 81, 74 and 93. What was her mean test result?

c The mean of 5 scores is 8.

i What must be the total of these scores?

ii If four of the scores are 7, 10, 8 and 4, what must the fifth score be?

d The mean of six scores is 3·5. If five of the scores are 2, 4, 4, 3 and 6, what must the sixth score be?

3 Amelia's and Ehsan's scores for 6 spelling tests of ten words were:

Amelia: 6, 9, 10, 9, 9, 10

Ehsan: 6, 7, 6, 7, 7, 10

The range of scores for both students was $10 - 6 = 4$.

a Does this mean that both students had equally good results?

b What would be a better measure of their abilities?

4 Complete these tables and calculate the mean for each set of data.

a

Outcome	Frequency	fx
1	2	
2	4	
3	5	
4	6	
5	3	
Total:	…	…

b

Outcome	Frequency	fx
5	1	
6	5	
7	3	
8	8	
9	5	
10	3	
Total:	…	…

c

Outcome	Frequency	fx
18	1	
19	3	
20	1	
21	4	
22	5	
23	3	
24	1	
25	2	
Total:	…	…

5 Look at the tables in Question **4**. What is the mode for each set of data?

6 Calculate the median for each set of data in Question **4** by looking at the frequency column.

7 The following are the numbers of goals that Mr Blackwell's soccer team scored in its matches last season:

1 0 2 3 0 1 4 2 3 0 1 1 5 4 3 2 1 2

a Sort the data into a frequency distribution table.

b What is the: **i** range **ii** mode **iii** median **iv** mean?

c Graph the data on a frequency histogram.

d How can you tell the mode from the graph?

8 Six coins were tossed 64 times. The number of heads was recorded each time, with the following results:

3 5 3 4 2 2 3 3 5 1 0 4 3 2 3 3
2 1 3 2 3 4 2 1 4 6 3 4 2 5 4 4
3 2 4 1 2 4 3 5 2 3 4 3 1 3 6 4
4 4 3 4 2 3 2 2 3 5 1 2 3 4 1 3



a Sort the data into a frequency distribution table.

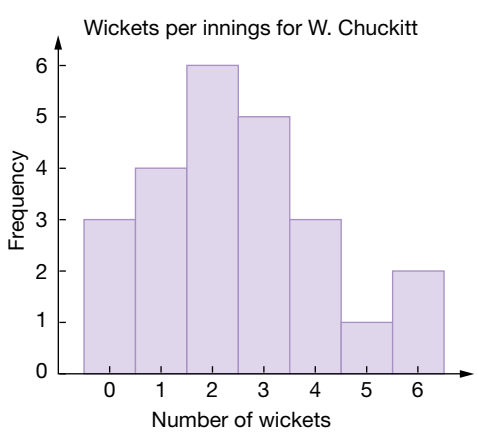
b Calculate the mean number of heads.

c How many tosses displayed 2 heads?

d How many tosses must have shown 4 tails?

e Draw a frequency polygon for these results.

9 This histogram shows the number of wickets taken by Willie Chuckitt in each innings for the last cricket season.



a Complete a frequency distribution table for this set of scores. (*Note:* You don't need a tally column.)

b What is the mode for this data?

c Determine Willie's median score.

d Calculate the mean number of wickets that Willie took per innings.

e How many more wickets would Willie have had to take to have a mean of 3 wickets per innings?

10 Ally and Rob kept a record of the number of pins knocked down with their first balls in tenpin bowling.

Ally's results									
8	7	3	10	9	8	8	7	10	9
8	8	7	9	10	6	10	5	8	7
9	3	5	9	10	8	9	7	6	8
7	9	8	10	8	7	5	8	7	9
10	6	8	9	7	6	10	5	8	9

Rob's results									
9	5	3	8	10	10	10	10	4	6
8	10	7	6	10	4	5	7	9	10
9	7	6	4	10	7	6	8	5	9
10	4	8	7	6	10	9	9	8	10
7	10	6	10	9	8	10	6	10	9

a Record this information in two frequency distribution tables, one showing Ally's results and the other showing Rob's.

b Graph the results on two frequency histograms.

c For each set of results, find the:

i range **ii** mode

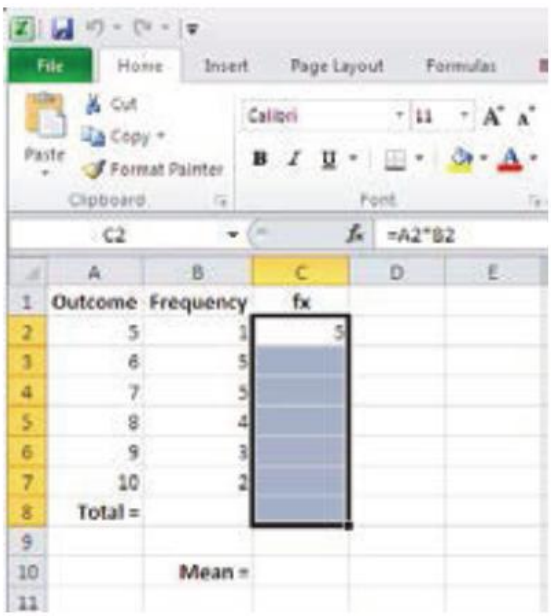
iii median **iv** mean.

d Whose results were more impressive?



INVESTIGATION 14:04 SPREADSHEETS

Data can be entered into a spreadsheet (using software such as Microsoft® Excel®), as shown in this diagram.



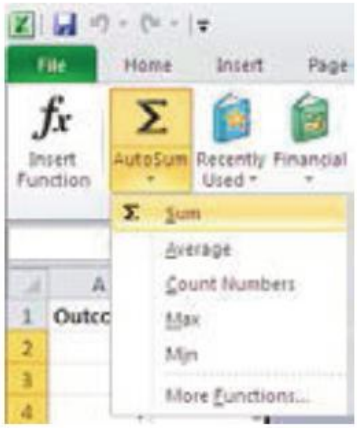
Note: Different spreadsheet software and different versions of Excel may require slightly different procedures and commands. Check the instructions for your software. The workings below are for Microsoft Excel 2010 for Windows.

- First, enter the headings 'Outcome', 'Frequency' and 'fx' in the first three cells of row 1. (You can enter 'Total =' and 'Mean =' into cells A8 and B10 now, or you can do that at the end.) Then:
- Enter the outcomes of 5 to 10 into column A.
- Enter the corresponding frequencies into column B.
- In the cell C2, enter the formula '=A2*B2'. (On the spreadsheet, this is shown in the 'formula bar' labelled f_x.)
- Copy this formula into cells C3 to C7 using 'Fill Down'. The product of each outcome and its frequency is then calculated automatically.

This data is the same as for Worked example **2** on page 412.

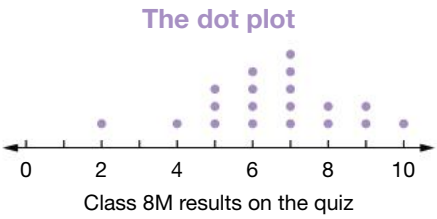
On the toolbar at the top of the spreadsheet, select the 'Formulas' tab and then select the option labelled 'AutoSum'. This reveals a menu of basic statistical functions, the first being 'Sum'.

- Highlight cells B2 to B7, then select 'Sum' from the 'AutoSum' menu. The total of these cells will appear in cell B8.
- Repeat this step for cells C2 to C7. Their total will appear in cell C8.
- In cell C10, enter the formula '=C8/B8'. The total of the 'fx' column will be divided by the total of the frequency column to give the mean in cell C10.



Experiment with some of the other statistical functions (under 'AutoSum') to see what they do.

14:05 Dot plots



The scores on the dot plot above may have been given as a list,

e.g. 4 6 8 7 6 5 2
7 7 8 9 7 5 9
5 6 7 6 10

Features

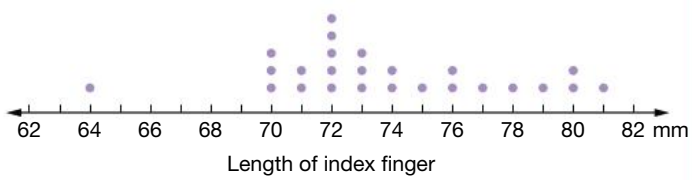
- The dot plot uses a number line.
- By placing one dot plot above another, comparisons can be made easily.
- It is very similar to a column graph or simple picture graph.

Details

- Only one axis is used.
- The axis should be labelled.
- When drawing the dot plot, decide what part of the number line will be used (e.g. −5 to 5 or 0 to 10).

WORKED EXAMPLE

The length of the index fingers of 25 teachers was measured. The results are shown on this dot plot.



- The mode can be seen at a glance. The mode is 72.
- The shape of the distribution is easily observed. A gap occurs between 64 and 70.
- Outliers are easily identified. 64 is an outlier.

Outliers are outcomes that are distant or separate from the main body of data.

- The median, the range and the frequency of data can be calculated.
  Median = middle score = 73
  Range = 81 − 64 = 17

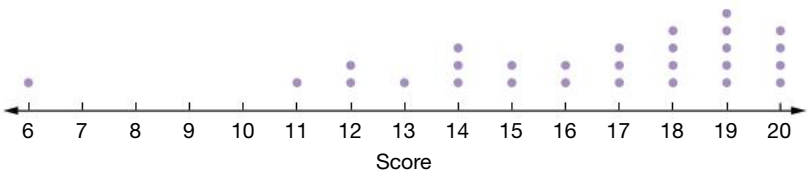
Score	64	70	71	72	73	74	75	76	77	78	79	80	81
Frequency	1	3	2	5	3	2	1	2	1	1	1	2	1

- The skew, which is the tendency of scores to group together on one side of the distribution, can be seen on the dot plot.
  The tendency of scores to form clusters can also be observed.
  The distribution is skewed to the left, as the bulk of the scores are from 70 to 73 while the others trail off to the right. (When determining the skew, outliers are ignored.)

Exercise 14:05

P Foundation worksheet 14:05 Dot plots and column graph

1 This dot plot shows the scores of students on a driving test.
For the scores shown on the dot plot, find:



a the mode **b** the range **c** the median **d** any outliers
e the frequency of the score 17. **f** Explain the meaning of the term *outlier*.

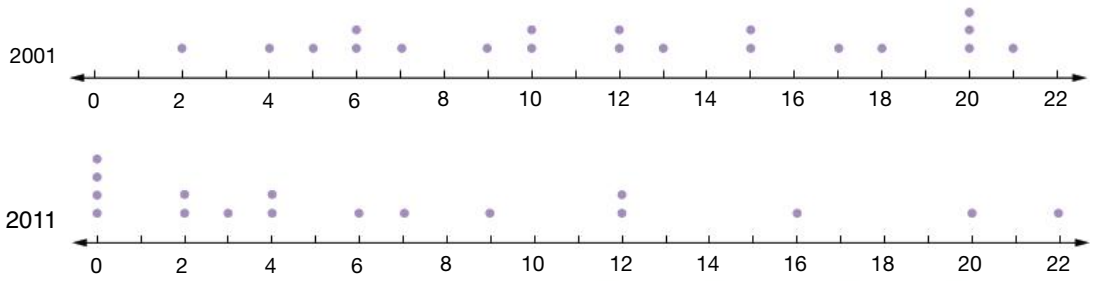
2 Draw a dot plot for this data.

8	11	15	10	12	9	14	10	13	11
4	7	11	19	13	10	12	11	11	9
3	8	12	14	16	12	11	6	5	7

For this set of scores, find:
a the mode
b the range
c the median
d any outliers
e the frequency of the score 11.



3 Twenty adult smokers were chosen at random in 2001. The number of cigarettes each smoked in a day was recorded. Ten years later, in 2011, the process was repeated with the same people. The data is shown on the dot plots below.



a Complete this table using the dot plots above.

	Mode	Median	Mean	Range
2001				
2011				

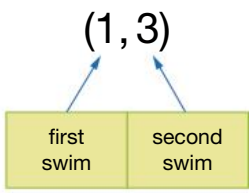
b How did the smoking pattern change for these people over 10 years? Explain three factors that may have contributed to the change in pattern.

4 Represent the following data on a dot plot:

1·6 1·4 2·0 1·8 2·0 1·7 2·1 1·8 2·1 1·7 1·1 1·9
2·1 2·0 1·6 2·0 1·4 1·7 1·6 2·0 1·9 2·1 1·7 1·8

5 Thirty students entered a swimming program, hoping to improve their swimming. At the beginning and end of the program, each was asked to swim as many laps as they could in 10 minutes. The number pairs below show each student's results.

(1,3)	(2,6)	(5,5)	(4,12)	(2,4)	(2,11)
(10,12)	(1,5)	(6,10)	(10,10)	(3,11)	(3,10)
(3,8)	(5,8)	(7,9)	(8,10)	(8,11)	(5,8)
(1,7)	(10,12)	(0,1)	(8,12)	(8,11)	(4,10)
(2,10)	(2,7)	(5,8)	(2,9)	(5,12)	(4,9)



a Draw two dot plots, one above the other, to compare the ability of the group before and after the course.

b What information is lost when the data is shown on the dot plots?

c Use the dot plots to complete the table below.

	Mode	Median	Mean	Range
Beginning				
End				



INVESTIGATION 14:05 MEAN, MODE AND MEDIAN

Practical activities

(Some of these activities refer to data collected in the practical activities of Investigation 14:02, page 403.)

1. If you have completed a table showing the number of children in the families of your class members, determine the mode, median and mean for this data.
2. You might also determine the mean and median heights of the students in your class.
3. Look for advertisements that use the word 'average'. Determine whether the average they refer to is the mean, the mode or perhaps even the median.
4. Choose an item of interest and survey people to discover the mean, mode or median for that item, as appropriate. For example:
   - How many phone calls do you make per week?
   - How far from school do you live?
5. When an 'average' is quoted for the cost of houses, the median is usually used. Explain why the median is the most appropriate measure.



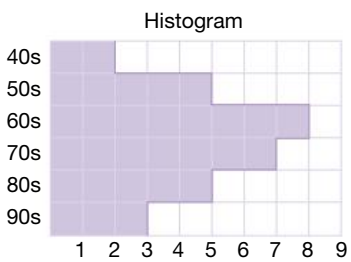
The height of the man in the suit (on the left) is 178 cm. Estimate:
1. the mean height of the group
2. the median height of the group.

14:06 Stem-and-leaf plots

A stem-and-leaf plot resembles a histogram (on its side) where the data has been grouped. (Compare the histogram on the left with the stem-and-leaf plots on the next page.)

The great advantage is that the identity of individual scores is not lost.

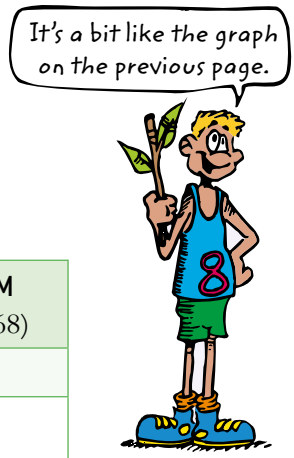
A back-to-back stem-and-leaf plot lets us compare two groups of data together at a glance.



WORKED EXAMPLE

Prepare an ordered stem-and-leaf plot for the following test marks of class 8M.

68	59	85	53	57	93	84	73	41	65
77	73	66	50	97	67	62	54	64	88
48	80	68	66	71	91	79	73	84	75



Solution

Stem-and-leaf plot

- The stem-and-leaf plot needs a title.
- An example of the stem/leaf data could be supplied above the table.
- In this case the stem is the tens digit and the leaf is the units digit.
- The stem is written with the smallest stem at the top.
- The leaf is built up by first working through the data in the order given.

Test results, 8M
(6/8 represents 68)

Stem	Leaf
4	1 8
5	9 3 7 0 4
6	8 5 6 7 2 4 8 6
7	3 7 3 1 9 3 5
8	5 4 8 0 4
9	3 7 1

Ordered stem-and-leaf plot

- To make an ordered plot, rewrite the table with the leaves in order.
- From the ordered plot, we can find that the range is $97 - 41 = 56$, the mode is 73 and the median is the average of 68 and 71 (i.e. 69·5).

A limitation of this sort of plot is that each leaf needs to be a single digit, so decimals may need to be rounded. Alternatively, the stem could be made whole numbers and the leaf could be decimals.

Stem	Leaf
4	1 8
5	0 3 4 7 9
6	2 4 5 6 6 7 8 8
7	1 3 3 3 5 7 9
8	0 4 4 5 8
9	1 3 7

Exercise 14:06

1 Complete a stem-and-leaf plot for each set of test marks.

a

26	15	62
51	47	19
33	40	58
26	38	35
47	41	36
29	17	16
62	28	18
47	30	26

Test marks
(2/6 represents 26)

Stem	Leaf
1	
2	
3	
4	
5	
6	

b

53	81	74
72	91	45
80	57	44
82	70	67
59	80	81
60	49	77
65	42	73
63	92	88

Test marks
(5/3 represents 53)

Stem	Leaf
4	
5	
6	
7	
8	
9	

2 Rewrite each stem-and-leaf plot in Question 1 as an ordered stem-and-leaf plot. Find the median for each.

3 The number of runs scored by 22 players in a cricket match are recorded below as two-digit numbers. 8 runs is shown as 08.

08	15	46	62	20	18	06	04	19	36	27
33	45	26	11	29	32	40	01	09	02	43

a Complete this table for the data above.

Runs scored (0/8 represents 8 runs)		
Stem	Leaf	Frequency (f)
0	8 6 4 1 9 2	
1	5 8 9 1	
2	0 7 6 9	
3	6 3 2	
4	6 5 0 3	
5		
6	2	
Total:		…



b Rewrite this data as an ordered stem-and-leaf plot.

c Find the range, median and mean for this set of scores. List any outliers.

d What percentage of players scored less than 20 runs? (Answer correct to 1 decimal place.)

4 Every student in Year 8 was asked to throw three darts at a dartboard. The total score for each student is listed below.

08	14	45	54	30	52	20	12	55	33
22	27	43	39	11	05	42	03	09	47
51	25	60	18	31	01	46	53	02	01
30	14	11	42	14	09	24	45	15	06
05	10	48	16	09	19	44	04	54	35
53	13	08	52	26	07	52	12	37	17
42	01	46	08	17	44	13	44	55	46
28	39	14	49	17	55	41	50	14	51



a Draw an ordered stem-and-leaf plot for this data.

b Find the range, mode and median of this data.

c Describe any distinctive features in the data, including outliers, clusters of scores and distribution shape.

d How does a score of 41 compare with the other scores?

FUN SPOT 14:06 ANYONE FOR TENNIS?

Six married couples took part in a tennis tournament.



The husbands' names were:

Harry Keith Mike Rashid Lachlan Shu

The wives' names were:

Maria Sarah Lin Diane Georgina Vanessa

Each of the women came from a different town in New South Wales. They were:

Orange Dubbo Bathurst Parkes Lithgow Tamworth

Also, each woman had a different hair colour, namely:

black blonde brown grey red white



Use the information given above and below to work out each woman's:

- husband
- home town
- hair colour.

The following doubles games took place:

- Harry and Keith played Georgina and Sarah.
- Mike and Rashid played Lin and Sarah.
- The women with black and brown hair played Harry and Mike, then Rashid and Keith.

Then, these singles games took place:

- Georgina played Mike, Rashid and Lachlan.
- The woman with grey hair played Maria, Diane and Vanessa.
- The woman from Tamworth played Maria, Lin and Diane.
- Maria played the women with white and blonde hair.
- The woman from Dubbo played Harry and Mike.
- Keith played Lin and Vanessa.
- Mike played Diane and Vanessa.
- The woman from Bathurst played women with red and black hair.
- The woman from Parkes played the woman with grey hair.
- Shu played the woman with black hair.
- Diane played the woman with white hair.
- Lachlan played the woman from Tamworth.
- Lin played the woman from Bathurst.
- The woman from Tamworth played the woman with white hair.
- The woman from Dubbo played Vanessa.
- Rashid played the woman from Bathurst.
- The woman from Orange played the woman with red hair.
- Rashid played the women from Parkes and Lithgow.



Note: No married couple ever played in the same game!

MATHS TERMS 14

average (arithmetic mean)

- the result of 'evening out' a set of numbers
- to find the average, divide the sum of the terms by the number of terms,
  e.g. The average of 3, 5, 3, 7, 10 and 2
  $= (3 + 5 + 3 + 7 + 10 + 2) \div 6$
  $= 30 \div 6$
  $= 5$

census

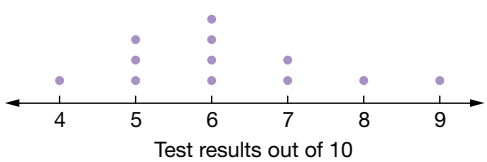
- a survey of a whole population

data

- the pieces of information (or 'scores') to be examined
- categorical: data that uses non-numerical categories
  —*ordered* data involves a ranking,
  e.g. exam grades, garment sizes
  —*distinct* data has no order,
  e.g. colours, types of cars
- numerical: data that uses numbers to show 'how much'
  —*continuous* data can have any reasonable numerical value, e.g. height
  —*discrete* data is restricted to certain numerical values, e.g. age in years

dot plot

- a type of graph that uses one axis and a number of dots above the axis



frequency

- the number of times an outcome occurs in the data,
  e.g. For the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6, the outcome 5 has a frequency of 3.

frequency distribution table

- a table that shows all the possible outcomes and their frequencies (often extended by adding other columns, such as a frequency times score column),

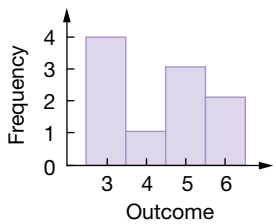
e.g.

Outcome	Frequency
3	4
4	1
5	3
6	2

frequency histogram

- a type of column graph showing outcomes and their frequencies,

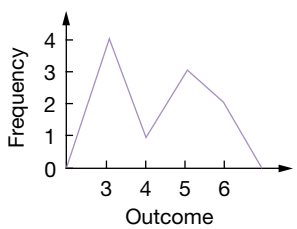
e.g.



frequency polygon

- a type of line graph showing outcomes and their frequencies
- to complete the polygon, the outcomes immediately above and below the actual outcomes are used (the height of these columns is zero),

e.g.



mean

- the mean is the arithmetic average:

$$\text{Mean} = \frac{\text{sum of all the scores}}{\text{total number of scores}}$$

- the number obtained by 'evening out' all the data until they are equal,
  e.g. if the scores 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 are 'evened out', the number obtained is 4·3

median

- when data is put in order, the median is the middle score (for an odd number of scores) or the mean of the middle two scores (for an even number of scores)

mode

- the outcome that occurs most often

outcome

- a possible value of the data

outlier

- a data value that is separated from the main body of data

range

- the difference between the highest and lowest data values

sample

- a part (usually a small part) of a large population
- random sample: a sample taken so that each member of the population has the same chance of being included

statistics

- the collection, organisation and interpretation of data

stem-and-leaf plot

- a type of graph that shows the spread of data without losing the identity of each value,

e.g.

Test results, 8M
(6/8 represents 68)

Stem	Leaf
4	1 8
5	9 3 7 0 4
6	8 5 6 7 2 4 8 6
7	3 7 3 1 9 3 5
8	5 4 8 0 4
9	3 7 1

- ordered stem-and-leaf plot: a stem-and-leaf plot where the leaves have been placed in order,

e.g.

Stem	Leaf
4	1 8
5	0 3 4 7 9
6	2 4 5 6 6 7 8 8
7	1 3 3 3 5 7 9
8	0 4 4 5 8
9	1 3 7

variable

- something that can be observed, measured or counted to provide data

DIAGNOSTIC TEST 14 STATISTICS

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

14:01

1 State whether each variable is categorical or numerical. If categorical, is it ordered or distinct? If numerical, is it continuous or discrete?

a runs scored in cricket
b favourite TV show
c time taken to swim 100 metres
d effort grades on a school report

2 Sort each set of data into a frequency distribution table. 14:03

a 1 2 5 4 3 3 2 5 1 4 4 3 2 1 2 2 3 2 4 2

b 0 2 4 4 2 4 6 8 6 8 4 6 4 2 0 4 6 0 2 4 4 6 2 4 8

c 6 7 9 10 5 3 5 7 8 9 7 8 6 7 9 10 7 8 7 6 4 3 5 9 6

3 a Draw a frequency histogram for the data in Question 2a. 14:03

b Draw a frequency polygon for the data in Question 2b.

c Draw a combined frequency histogram and polygon for the data in Question 2c.

4 State the range of each set of data. 14:04

a 5, 6, 9, 4, 3 b 0, 7, 3, 5, 6, 3 c 1·1, 1·3, 1·7, 1·4, 1·5, 1·0

5 State the mode for each set of data. 14:04

a 5, 6, 7, 6, 4, 9 b 3, 5, 6, 3, 5, 3, 7 c 2, 4, 6, 5, 4, 6, 5, 2, 6

6 State the mode for each set of data in Question 2. 14:04

7 State the median for each set of data. 14:04

a 3, 9, 4, 7, 8 b 5, 9, 4, 6, 7, 8, 6 c 5, 3, 2, 6, 4, 8

8 Determine the median for each set of data in Question 2. 14:04

9 Calculate the mean for each set of data. 14:04

a 6, 7, 4, 8, 5 b 8, 9, 4, 3, 6, 8, 4, 2 c 8, 9, 3, 7, 7, 6, 3, 5, 9, 3

10 Calculate the mean for each set of data in Question 2. 14:04

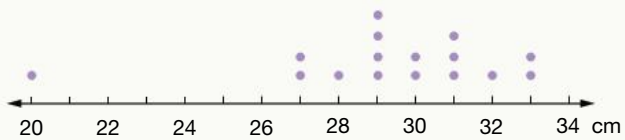
11 The lengths of 16 fish were measured. The results are shown on this dot plot. 14:05

a What is the:

i mode

ii median

iii range?



b Which measurement is an outlier?

12 Year 4 students were tested on their knowledge of multiplication tables. The results are shown on this stem-and-leaf display. 14:06

a List any outliers in these results.

b Find the mode, median and range for Year 4.

Test results, Year 4
(5/1 represents 51)

Stem	Leaf
3	5
4	
5	1 4 6 6 9
6	0 2 2 5 8
7	3 6 8 8 9 9 9
8	0 0

ASSIGNMENT 14A Chapter review

1 30 Year 7 students were surveyed and it was found that 18 owned a bicycle.

a If this sample is representative of the whole of Year 7, which has 150 students, then how many in the whole of Year 7 own a bicycle?

b There are 90 students in Year 12, so 54 should own a bicycle. Is this a fair statement? Why or why not?

2 A survey was taken of Year 7 students, asking them to state:

a the time they leave home to go to school

b what they eat for breakfast

c the distance they travel to school

d the type of transport they use to get to school

e how many brothers and sisters they have who also go to the school

f whether they think the amount of homework they get is too much, just right or not enough.

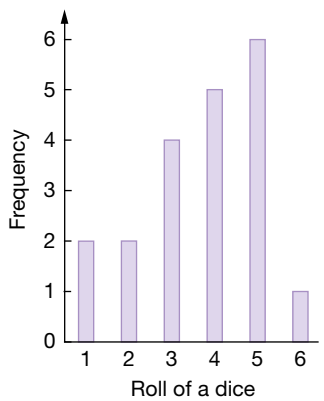
State which type of data is collected by each of these questions: categorical (ordered or distinct) or numerical (discrete or continuous).

3 Determine the range, mode, mean and median for these sets of data.

a 5, 7, 4, 3, 7, 5, 9, 7, 9

b 12, 14, 15, 11, 14, 14, 16, 20, 19, 18

4 This graph shows the number of times each number was rolled on a six-sided dice.



a How many times was the dice rolled?

b What is the range for this data?

c What is the mode for this data?

d What is the average (mean) result?

e Determine the median result.

f Write the mode, mean and median in order from smallest to largest.

5 **a** The following are the goals scored by a football team during their season of 18 games. Organise this data into a frequency distribution table.

3	2	1	2	0	4
3	1	1	2	5	4
0	2	0	3	3	2

b For this data, calculate the:

i range **ii** mode

iii mean **iv** median.

c The team has two more games to play in the finals. How many goals do they need to score in these games so that their goal average for the season will be at least 2 goals per game?

6 Construct a dot plot for the data in Question **5**.

7 For this stem-and-leaf plot:

Stem	Leaf
3	8
4	2 6 9
5	3 3 6 7
6	0 1 3 3 4 7 8
7	4 6 6 8 9
8	3 4 8 9

a Find the range.

b How many modes are there? What are they?

c Determine the median score.

d Calculate the sum of the scores.

e Calculate the mean, correct to 1 dec. pl.

f If one more score of 83 is added to this data, what is the new mean and median?

ASSIGNMENT 14B Working mathematically

1 Fred inherited a large sum of money. He gave $5000 to charity, $1000 to each of his three brothers and $1500 each to his school and community centre. He then paid $5328 towards his house loan. After this he had $3572 left. How much did he inherit?

2 The petrol tank of my car holds 55 litres. I drive into a petrol station and fill the tank, which costs me $52.15.

a If the petrol costs 146·9 cents a litre, how many litres did I buy?

b How much was in the tank before it was filled?

(Give answers correct to 1 decimal place.)



3 In a bag there are six black cards and four red cards.

a If cards are taken from the bag one at a time (without looking), what is the smallest number of cards you must take to be sure of getting a pair of cards the same colour?

b What is the smallest number of cards you must take to be sure of getting a pair of red cards and a pair of black cards?

4 If 25 people enter a knockout tennis competition, how many byes should be given in the first round so that there are no byes in the following rounds? (A 'bye' is when a player does not play in a round.)



5 Is it possible to have a year without a Friday the 13th?

January

Mon	Tue	Wed	Thu	Fri	Sat	Sun
31	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1	2	3
4	5	6	7	8	9	10

February

Mon	Tue	Wed	Thu	Fri	Sat	Sun
28	29	30	31	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	1	2	3
4	5	6	7	8	9	10

March

Mon	Tue	Wed	Thu	Fri	Sat	Sun
25	26	27	28	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
4	5	6	7	8	9	10

April

Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	1	2	3	4	5
6	7	8	9	10	11	12

May

Mon	Tue	Wed	Thu	Fri	Sat	Sun
29	30	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31	1	2
3	4	5	6	7	8	9

June

Mon	Tue	Wed	Thu	Fri	Sat	Sun
27	28	29	30	31	1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
1	2	3	4	5	6	7

July

Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	1	2	3	4
5	6	7	8	9	10	11

August

Mon	Tue	Wed	Thu	Fri	Sat	Sun
29	30	31	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	1
2	3	4	5	6	7	8

September

Mon	Tue	Wed	Thu	Fri	Sat	Sun
26	27	28	29	30	31	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	1	2	3	4	5	6

October

Mon	Tue	Wed	Thu	Fri	Sat	Sun
30	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1	2	3
4	5	6	7	8	9	10

November

Mon	Tue	Wed	Thu	Fri	Sat	Sun
28	29	30	31	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	1
2	3	4	5	6	7	8

December

Mon	Tue	Wed	Thu	Fri	Sat	Sun
25	26	27	28	29	30	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31	1	2	3	4	5

ASSIGNMENT 14C Cumulative revision

1 Simplify: 1:06

a $7 \times 2 + 8 \times 10$ **b** $14 - (15 - 1)$

c $7 - 36 \div 4$ **d** $100 \div (80 \div [20 - 16])$

e $(7 \times 10\,000) + (2 \times 1000) + (1 \times 100) + (8 \times 10) + (5 \times 1)$

2 **a** $\frac{7}{10} + \frac{9}{10}$ **b** $\frac{3}{8} + \frac{2}{5}$ **c** $\frac{9}{10} - \frac{1}{4}$ **d** $\frac{3}{5} \times \frac{3}{4}$ 9:04, 9:06, 9:07

e $\frac{17}{100} \times \frac{1}{2}$ **f** $4 \times \frac{3}{8}$ **g** $\frac{9}{10} \div \frac{3}{4}$ **h** $\frac{3}{4} \div 5$

3 **a** $18{\cdot}65 + 4{\cdot}4$ **b** $10 - 3{\cdot}15$ **c** $41{\cdot}5 \div 100$ **d** $1{\cdot}3 \times 0{\cdot}5$ 6:02, 6:05, 6:06, 6:08

e $4{\cdot}5 \div 3$ **f** $8{\cdot}26 \div 8$ **g** $4{\cdot}6 \div 0{\cdot}02$ **h** $(0{\cdot}05)^2$

i Round $0{\cdot}\dot{6}$ correct to 2 decimal places.

4 Write as a percentage: 11:02

a $\frac{3}{4}$ **b** $1\frac{2}{5}$ **c** $0{\cdot}145$ **d** $2{\cdot}05$

5 **a** Find 8% of \$43 000. **b** Find 13% of 800 km. 11:07

c Increase \$1600 by 5%. **d** Decrease \$1680 by 5%.

e 20% of my wages is \$130. How much are my wages?

6 Simplify: 4:07

a $8x + 3x$ **b** $10a - 9a$ **c** $3x^2 + 5x^2$ **d** $6m - m$

e $10 \times 4y$ **f** $20b \div 2$ **g** $\frac{1}{4} \times 16a$ **h** $2a \times 7b$

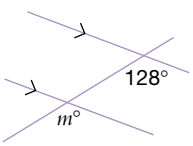
7 **a** Of 200 tickets sold in a raffle, Rachel bought 10, Naomi bought 8 and Ben bought 2. A ticket is chosen at random to determine who wins the prize. What is the probability that the prize is won by: 12:03

i Rachel **ii** Naomi **iii** Ben?

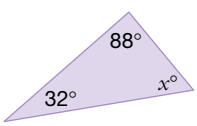
b The chance of our boat sinking as we cross the river is $\frac{3}{20}$. What is the chance that our boat will not sink?

8 Find the value of each pronumeral, giving a reason for your answer. 5:05, 8:04

a



b



9 Solve these equations. 13:04

a $x + 7 = 15$ **b** $a - 7 = 5$ **c** $2k + 5 = 25$

d $4x - 7 = 13$ **e** $5y + 8 = 12$ **f** $3p - 9 = 2$

g $\frac{n}{2} + 7 = 15$ **h** $\frac{m+3}{2} = 3$

SYMMETRY AND TRANSFORMATIONS

15



Contents

15:01 Symmetry
GeoGebra activity 15:01 Line symmetry and rotational symmetry
15:02 Transformations
15:03 Translations
GeoGebra activity 15:03 Translations
15:04 Reflections
GeoGebra activity 15:04 Reflections
15:05 Rotations
GeoGebra activity 15:05 Rotations
15:06 Combining transformations
Investigation 15:06 Using technology to create patterns
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra and **Measurement and Geometry**

Selections from ***Linear Relationships***, ***Properties of Geometrical Figures 1*** [Stage 4]

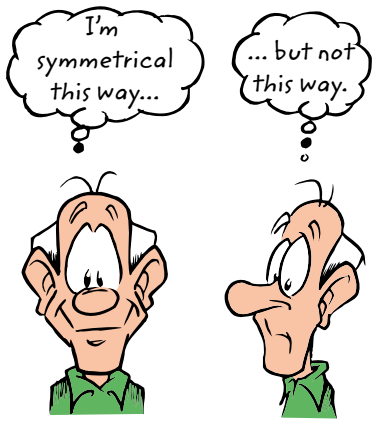
- Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. (ACMMG181)
- Identify line and rotational symmetries. (ACMMG181)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

15:01 Symmetry

Many shapes are *symmetrical*. This means that they look 'balanced' and in the right proportions. In maths, there are two special types of symmetry: **line symmetry** and **rotational symmetry**.

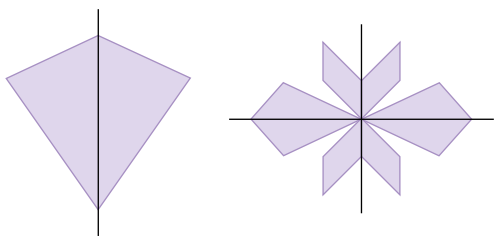


A shape has *line symmetry* if it can be divided by a line into two identical parts that are mirror images of one another. The dividing line is called an **axis of symmetry**.

(*Note:* The plural of axis is *axes*, pronounced 'axe-ease'.)

The black lines in the figures below right are axes of symmetry. If a mirror was placed on these lines, you could see that the two halves of the figure are mirror images of each other.

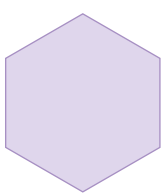
- When a figure is folded along an axis of symmetry, the two halves of the figure will overlap.
- Symmetrical figures are also produced by the *reflection* transformation.
  We will see more about transformations later in the chapter.



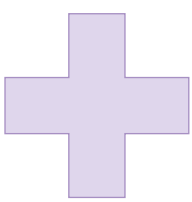
WORKED EXAMPLES

1 Mark in the axes of symmetry of the shapes below.

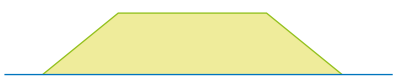
a



b

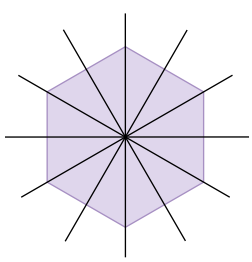


2 Complete this drawing, making the blue line an axis of symmetry of the shape.

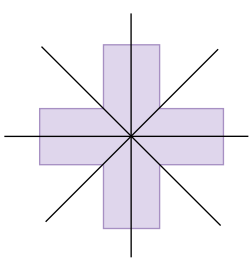


Solutions

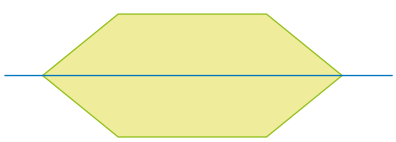
1 a



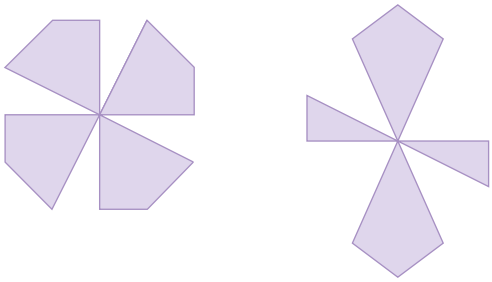
b



2



Some shapes do not possess line symmetry, but still appear to have some symmetrical properties.





A shape has **rotational symmetry** if it can be spun about a point to repeat its shape more than once in a complete rotation (revolution).

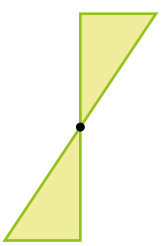
The number of times that its shape repeats per revolution is called the **order of rotational symmetry**.

The point about which the shape turns is called the **centre of symmetry**.

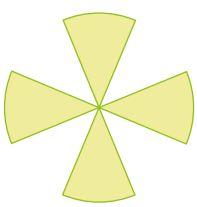
When investigating rotational symmetry, it is often useful to make a tracing of the figure. You can then place the tracing over the original figure and rotate it.

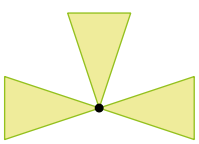
WORKED EXAMPLES

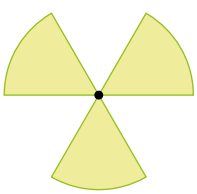
Do these shapes have rotational symmetry? If so, give the order of rotational symmetry.

1 

2 

3 

4 

5 

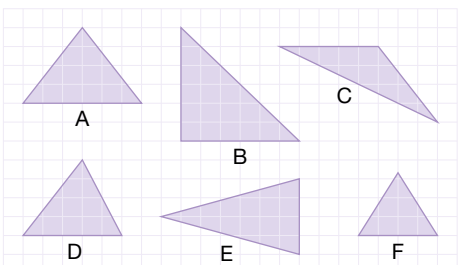
Solutions

1 yes, order 2 **2** no **3** yes, order 4 **4** no **5** yes, order 3

Exercise 15:01

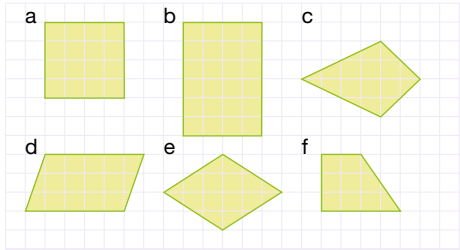
1 **a** Copy or trace the following triangles and mark the axes of symmetry.

b Can a scalene triangle ever be symmetrical?



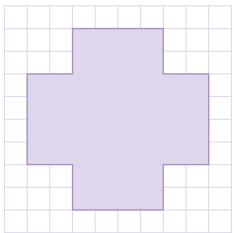
2 How many axes of symmetry does each quadrilateral have?

a square
b rectangle
c kite
d parallelogram
e rhombus
f trapezium

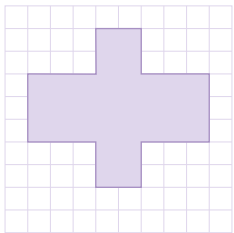


3 Copy or trace the following shapes and mark all the axes of symmetry.

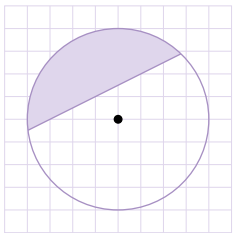
a



b

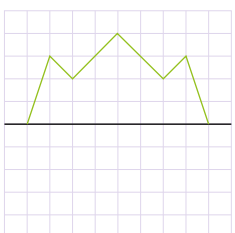


c

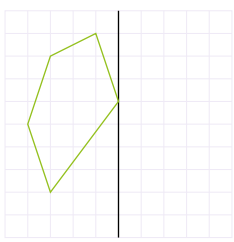


4 Copy and complete the figures, making each black line an axis of symmetry.

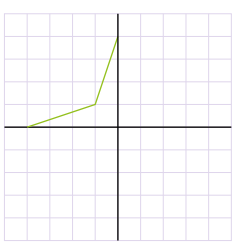
a



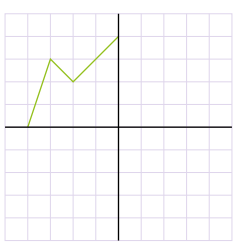
b

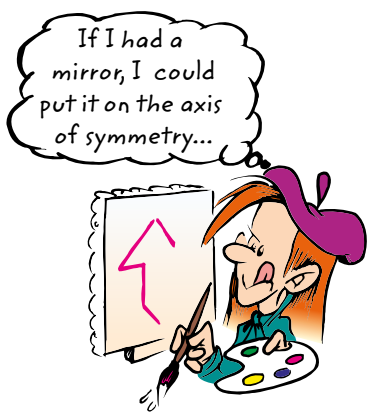


c



d

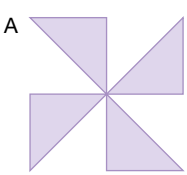




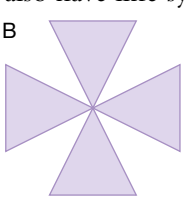
5 State whether the quadrilaterals in Question 2 have rotational symmetry and give the order of rotational symmetry for those that do.
(Hint: Imagine that they are rotated about the point of intersection of their diagonals.)

6 **a** State the order of rotational symmetry for each shape below.
b Which of the shapes also have line symmetry?

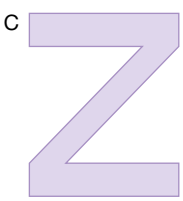
A



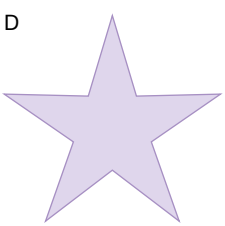
B



C



D





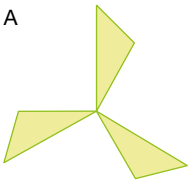
What rotational symmetries can you see in this paving pattern?

7 Which of the figures show:

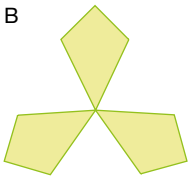
a rotational symmetry **b** line symmetry **c** line and rotational symmetry

d rotational symmetry only **e** line symmetry only?

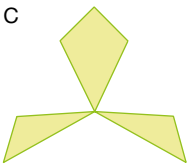
A



B

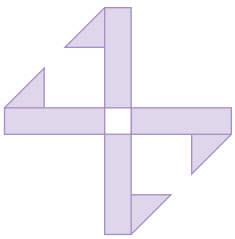


C

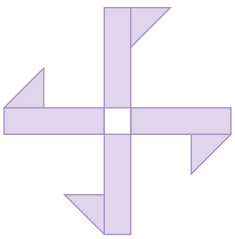


8 State whether the following designs show both line symmetry and rotational symmetry, either one or the other, or neither.

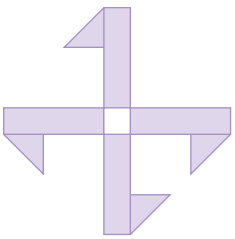
a



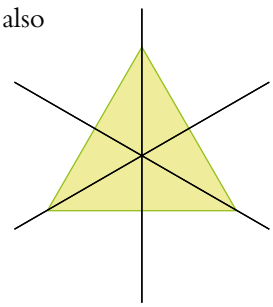
b



c



9 The equilateral triangle shown at right has three axes of symmetry and also rotational symmetry of order 3. This means that it repeats its shape 3 times in one revolution. The diagram also shows that it does not have **point symmetry** (i.e. symmetry so that the shape repeats itself after half a turn).

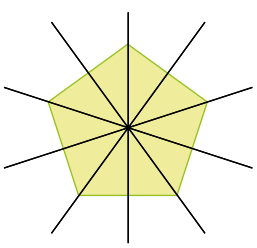


For the regular pentagon and octagon below, find:

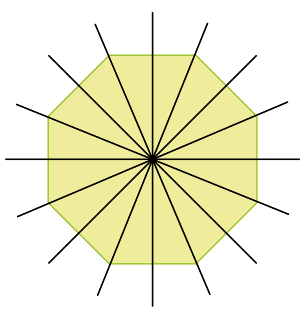
i the number of axes of symmetry

ii the order of rotational symmetry.

a



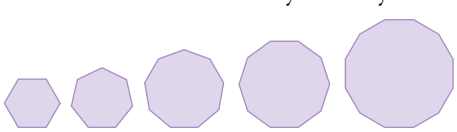
b



10 For each regular polygons below, find:

i the number of axes of symmetry

ii the order of rotational symmetry.



a hexagon **b** heptagon **c** nonagon

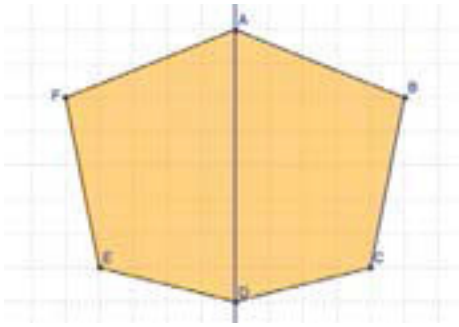
d decagon **e** dodecagon



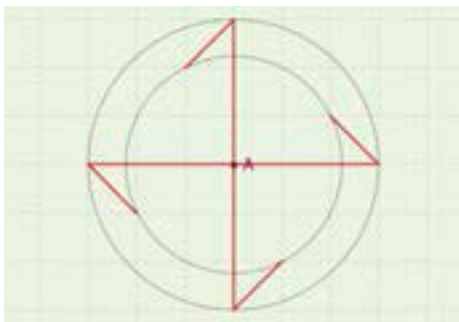
What polygon is the shape of this wheel based on?

P GEOGEBRA ACTIVITY 15:01 LINE SYMMETRY AND ROTATIONAL SYMMETRY

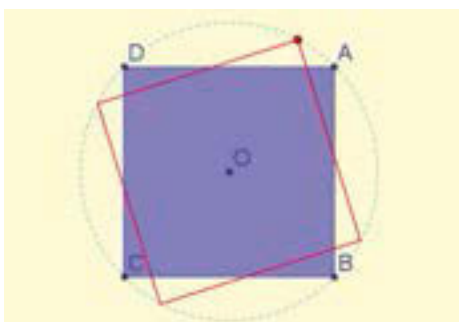
1 Set one half of the shape, then drag the vertices of the other half until it is symmetrical about the blue line.



2 Use the spinner to investigate rotational symmetries of order 2, 3 and 4.



3 Select a special quadrilateral (square, parallelogram, rectangle or rhombus), then drag the red corner to investigate its symmetries.



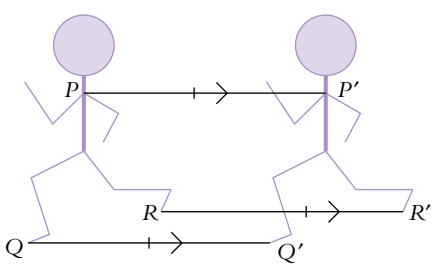
15:02 Transformations

When a figure undergoes a **transformation**, a new figure is produced, called the **image**. Every point on the figure is transformed to its image point, changing in a systematic way.

If P, Q and R are three points on a figure then their image points are denoted P', Q' and R' respectively. (Another transformation would be named P'', Q'' and R''.)

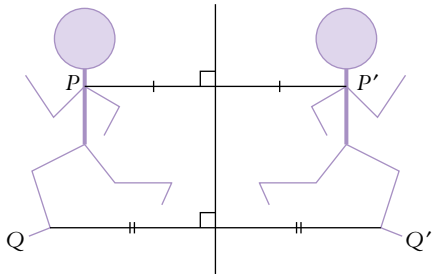
When a figure is translated

- All points on the figure move the same distance in the same direction. It is like sliding the figure on a set of rails.
- All lines joining the points to their image points are parallel and equal in length.



When a figure is reflected in a line

- Every point on the figure moves to a position on the opposite side of the line and the same distance away from the line.



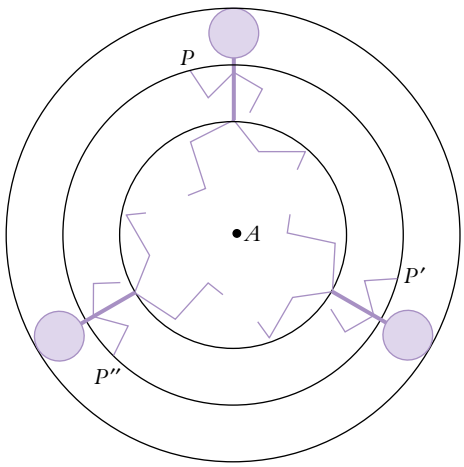
- This makes a *mirror image*, i.e. the way the figure's reflection in a mirror would look.



This 2D X-ray is a beautiful example of the symmetry of the human body.

When a figure is rotated

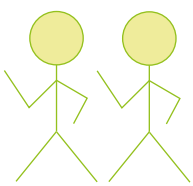
- The points on the object move in circles whose centre is called the centre of rotation. In this diagram, A is the centre of rotation.
  It is like fixing the image to a wheel.
- The further a point is from the centre of rotation, the further it moves.



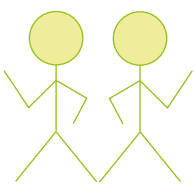
Exercise 15:02

1 Identify each of the following transformations as a translation (slide), reflection (flip) or rotation (turn).

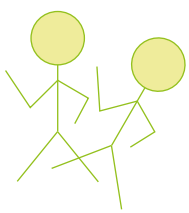
a



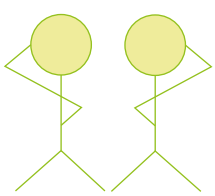
b



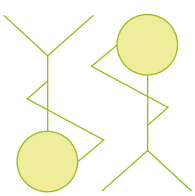
c



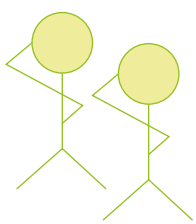
d



e



f



g



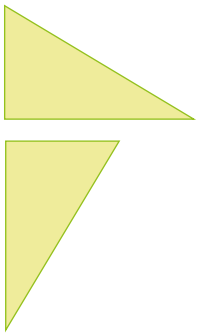
h



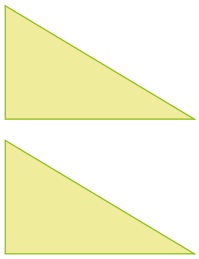
i



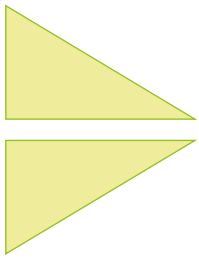
j



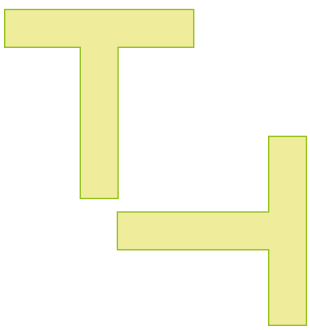
k



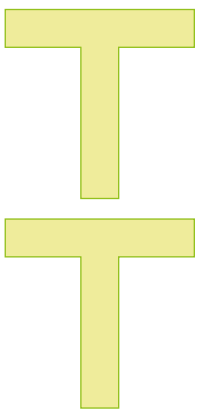
l



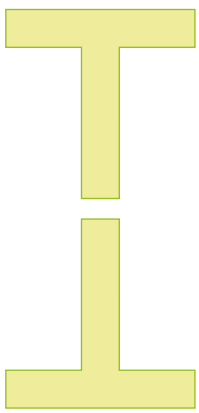
m



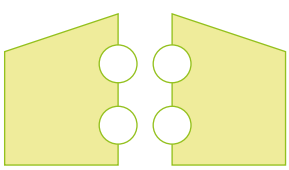
n



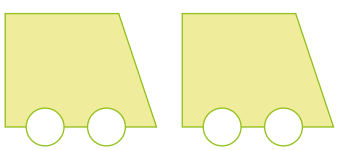
o



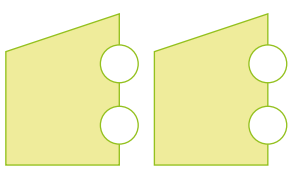
p



q

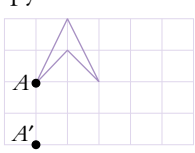


r

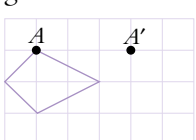


2 Copy or trace the following. Sketch the shape made when A is translated to A'.

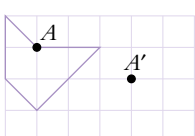
a



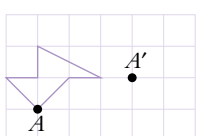
b



c

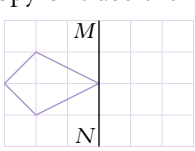


d

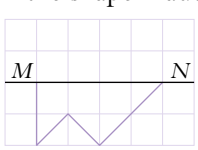


3 Copy or trace the following. Sketch the shape made when it is reflected in the line MN.

a

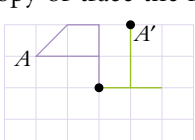


b

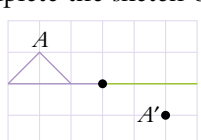


4 Copy or trace the following. Complete the sketch of the shape made when A is rotated to A'.

a



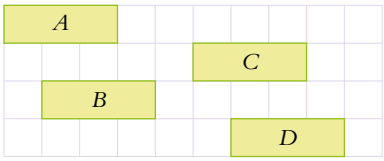
b



5 Describe how you would make the following translations (e.g. A to B is '1 right, 2 down').

a A to C **b** A to D

c D to C **d** C to B

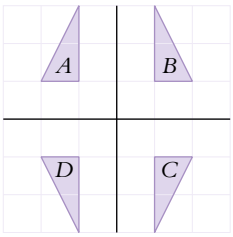


6 The triangles have been reflected in the horizontal and vertical lines. Are the following reflections horizontal or vertical?

a A to B **b** B to A

c C to B **d** C to D

e D to A **f** A to D

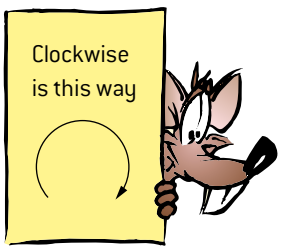


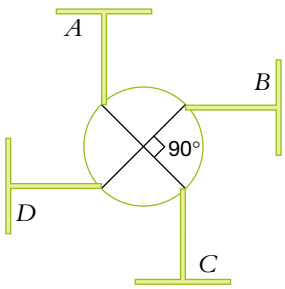
7 The shapes have been rotated through 90°, 180° , or 270°. Describe the clockwise rotations that transform:

a A to B **b** A to C

c A to D **d** B to C

e B to A **f** C to A

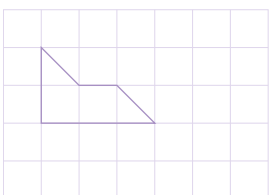




8 Copy or trace this figure, then draw the figure when it is translated:

a 1 unit to the left and 2 units down

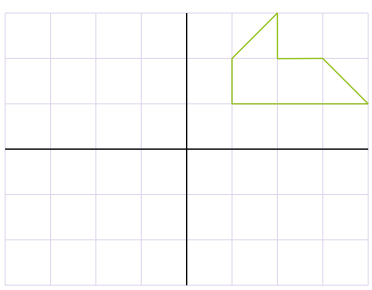
b 3 units to the right and 1 unit up.



9 Copy or trace the figure, then reflect the figure:

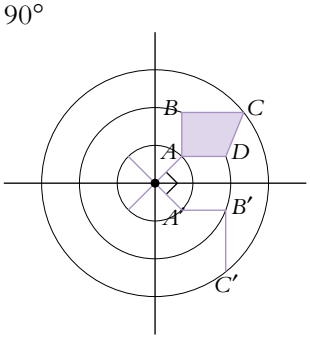
a in the horizontal line

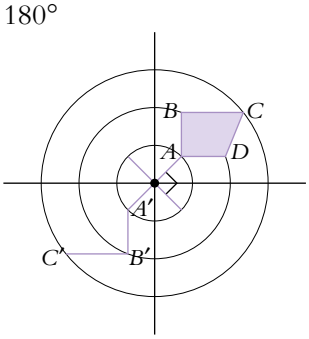
b in the vertical line.

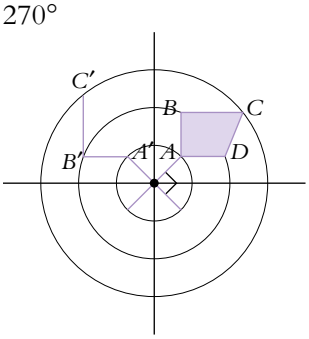


10 The purple trapezium has been rotated clockwise through 90°, 180° and 270°. In each of the following, trace and complete the purple trapezium and its image.

a 90° **b** 180° **c** 270°





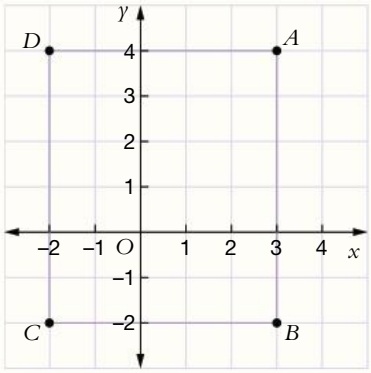


15:03 Translations

PREP QUIZ 15:03

From the graph, write the coordinates of:

1 A **2** B **3** C **4** D



How many units of distance is it from:

5 A to B **6** A to D?

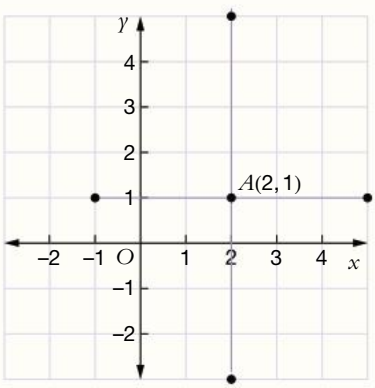
What are the coordinates of A, when it is moved:

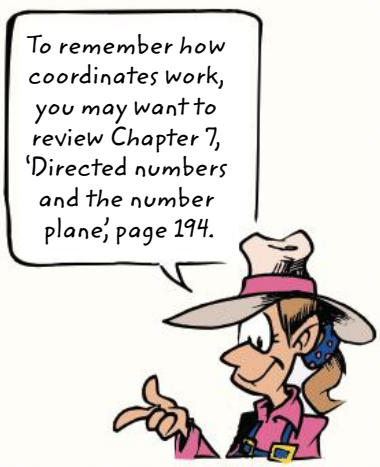
7 3 units to the right

8 3 units to the left

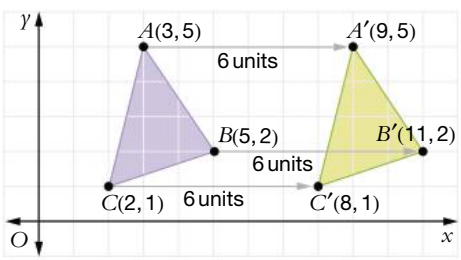
9 4 units up

10 4 units down?





In this diagram, the purple triangle is translated horizontally to the right to give the green triangle.

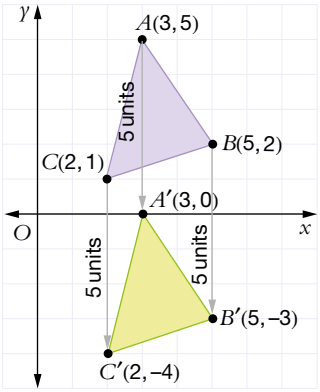


The points A, B and C all move 6 units to the right, i.e. the x-coordinates increase by 6:

$A(3,5)$ becomes $A'(9,5)$
$B(5,2)$ becomes $B'(11,2)$
$C(2,1)$ becomes $C'(8,1)$.

If the purple triangle is translated 6 units to the left, the x-coordinates will decrease by 6.

In this diagram, the purple triangle is translated vertically to give the green triangle.



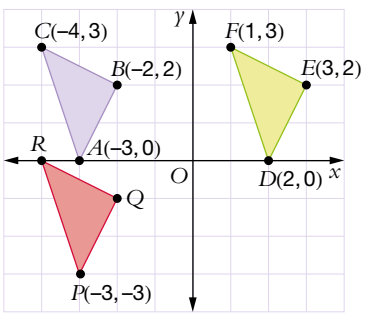
The points A, B and C all move 5 units down, i.e. the y-coordinates decrease by 5:

A (3,5) becomes A'(3,0)
B (5,2) becomes B'(5,–3)
C (2,1) becomes C'(2,–4).

If the purple triangle were translated 7 units up, the y-coordinates would increase by 7.

Translations on a number plane can be described by looking at how far a point moves in the horizontal and vertical directions.

WORKED EXAMPLES



1 From the graph, describe the following translations.
- **a** ΔABC is translated to ΔDEF.
- **b** ΔDEF is translated to ΔABC.

2
- **a** What are the coordinates of Q and R?
- **b** If the points A, B and C were moved down 3 units, would they become P, Q and R?
- **c** Describe the translation of ΔABC to ΔPQR.
- **d** Describe the translation that would move ΔPQR to ΔABC.

Solutions

1
- **a** ΔABC has been moved 5 units to the right.
- **b** ΔDEF has been moved 5 units to the left.

2
- **a** Q is (–2,–1), R is (–4,0).
- **b** yes
- **c** translation 3 units down
- **d** translation 3 units up



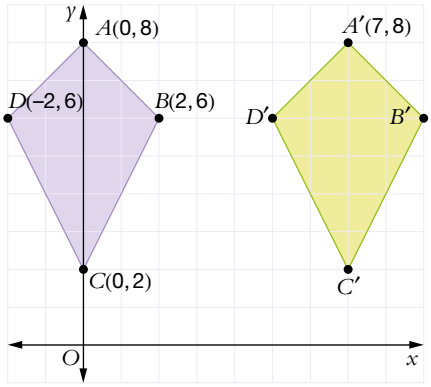
Translating a car, the hard way ...

WORKED EXAMPLES

1 Describe the translation that transforms the kite $ABCD$ into the kite $A'B'C'D'$.

2 What are the coordinates of B', C' and D'?

3 If the kite $ABCD$ is translated 5 units down, what are the new coordinates of its vertices?



Solutions

1 $ABCD$ is translated 7 units to the right.

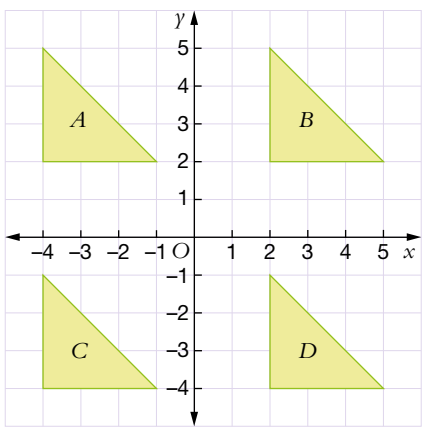
2 B' is (9,6), C' is (7,2), D' is (5,6).

3 The y-coordinate decreases by 5:
A becomes (0,3), B becomes (2,1),
C becomes (0,−3) and D becomes (−2,1).

Exercise 15:03

1 Which triangle is produced, if:

- **a** A is moved 6 units to the right
- **b** B is moved 6 units to the left
- **c** D is moved up 6 units
- **d** A is moved down 6 units
- **e** C is moved 6 units to the right
- **f** C is moved up 6 units
- **g** D is moved 6 units to the left
- **h** B is moved down 6 units?



2 Describe the following translations:

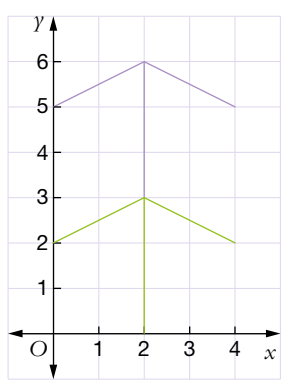
- **a** A to B
- **b** D to B
- **c** D to C
- **d** A to C

3 In each of the following, the purple arrow is translated to the position of the green arrow. Describe each translation using a description like those in Question 1.

a



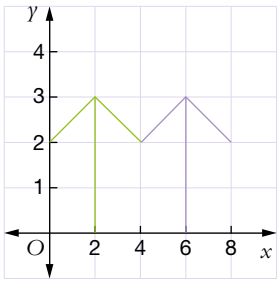
b



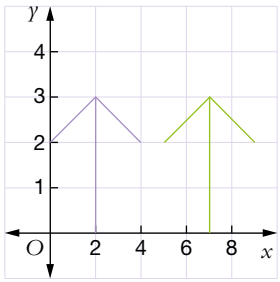
c



d



e



4 Triangle ABC is translated 6 units to the right.

a Write the coordinates of:

i A' ii B' iii C'

b Look at the coordinates of A and A'.

i Are their y-coordinates the same?

ii Do their x-coordinates differ by 6?

c Look at the coordinates of B and B'

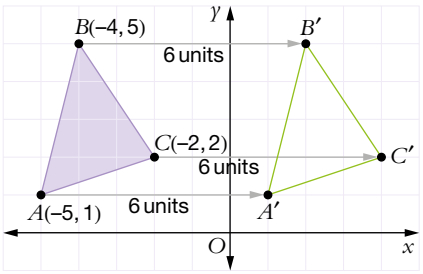
i Are their y-coordinates the same?

ii Do their x-coordinates differ by 6?

d Look at the coordinates of C and C'.

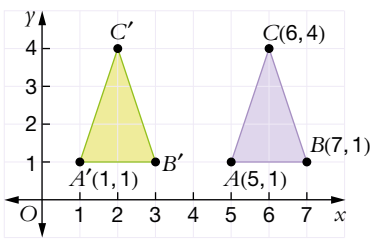
i Are their y-coordinates the same?

ii Do their x-coordinates differ by 6?

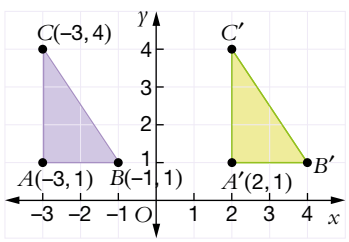


5 Triangle ABC is translated to triangle $A'B'C'$. In each case, describe the translation and find the coordinates of B' and C'.

a



b



> Translating a shape *horizontally* increases or decreases the *x-coordinates* of the points by the length of the translation.

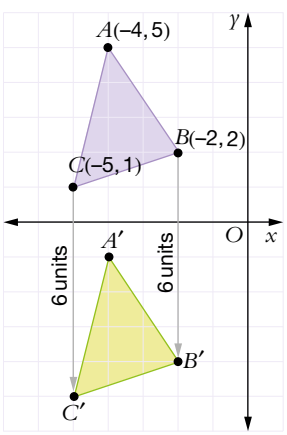
6 The purple triangle is translated 6 units downwards.
Write the coordinates of:

a i A' ii B' iii C'

b Are the x-coordinates of each point and its image the same?

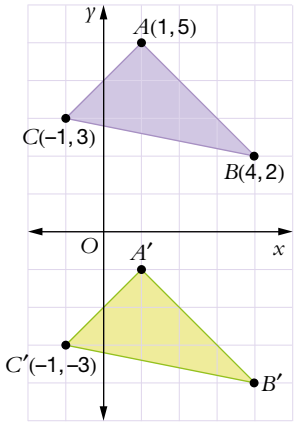
c Subtract 6 from the y-coordinates of each of the points A, B and C. Do you get the y-coordinates of A', B' and C'?

> Translating a shape *vertically* increases or decreases the *y-coordinates* of the points by the length of the translation.

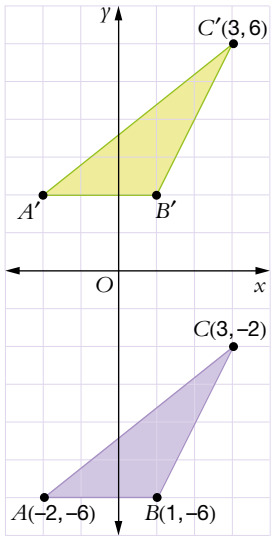


7 Triangle ABC is translated to triangle $A'B'C'$. In each case, describe the translation and find the coordinates of A' and B'.

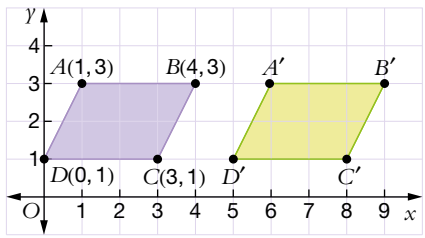
a



b

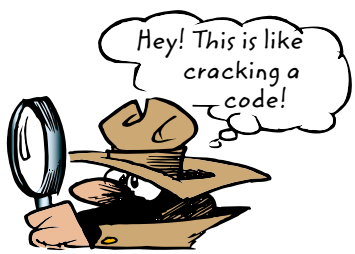


8 **a** Describe the translation that moves $ABCD$ to $A'B'C'D'$ in the graph at right.



b Copy and complete this table by writing the coordinates of each point's image.

Point	Image
$A(1,3)$	
$B(4,3)$	
$C(3,1)$	
$D(0,1)$	

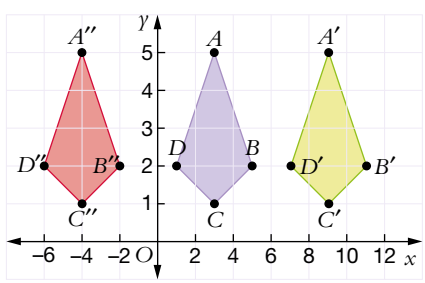


c Look at the coordinates of each point and its image.

i What do you notice about the y-coordinates?

ii What can you say about the x-coordinates?

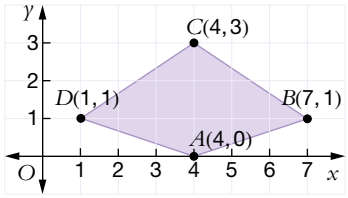
9 The kite $ABCD$ is translated 6 units to the right to give the kite $A'B'C'D'$, and also translated 7 units to the left to give the kite $A''B''C''D''$.



a Add 6 to the x-coordinates of the points A, B, C and D. Do you get the x-coordinates of the points A', B', C' and D'?

b Subtract 7 from the x-coordinates of the points A, B, C and D. Do you get the x-coordinates of the points A'', B'', C'' and D''?

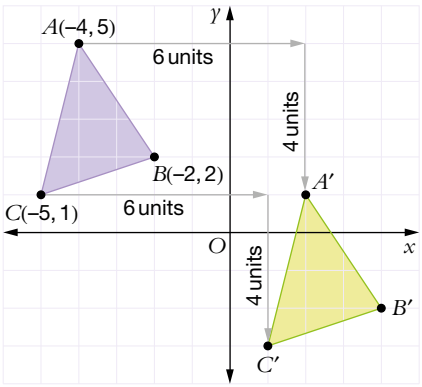
10 The quadrilateral $ABCD$ is translated up 4 units to produce the quadrilateral $A'B'C'D'$.



a On your own graph, plot and name the coordinates of the points:
i A' ii B' iii C' iv D'

b If $ABCD$ is translated down 2 units to produce $A''B''C''D''$, find the coordinates of:
i A'' ii B'' iii C'' iv D''

11 Triangle ABC is translated 6 units to the right and 4 units down to produce triangle $A'B'C'$.



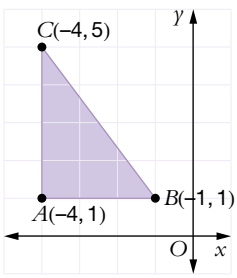
a Write the coordinates of:
i A' ii B' iii C'

b Add 6 to the x-coordinates of A, B and C. Do you get the x-coordinates of A', B' and C'?

c Subtract 4 from the y-coordinates of A, B and C. Do you get the y-coordinates of A', B' and C'?

d What do the coordinates of triangle ABC become if it is translated 4 units to the left and 3 units up?

12 a Triangle ABC is translated 5 units to the right and 2 units down to give triangle $A'B'C'$. By plotting the triangles, find the coordinates of A', B' and C'.



b Triangle $A''B''C''$ is the image of triangle ABC after it is translated 5 units to the left and 4 units up. What are the coordinates of A'', B'' and C''?

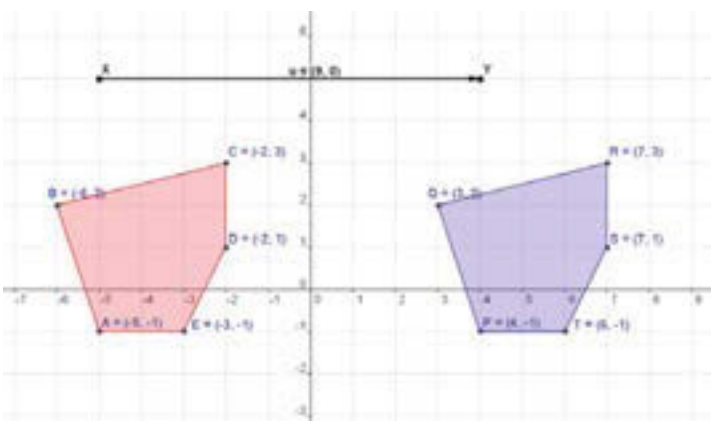
13 A triangle has coordinates $A(-1,2)$, $B(5,-3)$ and $C(7,4)$. What do the coordinates of the triangle become if it is translated:

a 10 units to the right b 10 units to the left

c 10 units up d 10 units down?

P GEOGEBRA ACTIVITY 15:03 TRANSLATIONS

Change the shape of the figure, then change the length and direction of the translation by clicking and dragging the points X and Y.



15:04 Reflections

PREP QUIZ 15:04

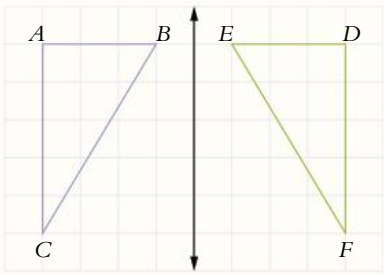
EDF is the reflection of *BAC*.

Which point is the image of:

1 *A* **2** *B* **3** *C*?

Which point is the same distance away from the line of reflection as:

4 *A* **5** *B* **6** *C*?



MN is reflected in the *y*-axis to give *CD*.

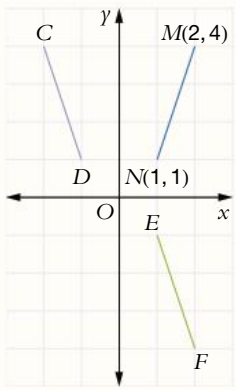
What are the coordinates of:

7 *C* **8** *D*?

MN is reflected in the *x*-axis to give *FE*.

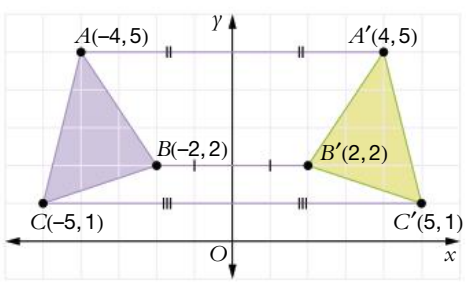
What are the coordinates of:

9 *E* **10** *F*?



Reflection in the *y*-axis

In the diagram, ΔABC is reflected in the *y*-axis. Its image is $\Delta A'B'C'$.



The points *A* and *A*′, *B* and *B*′, and *C* and *C*′ are images of each other. They are the same distance away from the *y*-axis but on opposite sides of it.

Looking at the coordinates of the points and their images shows that the *y*-coordinates are equal but the *x*-coordinates are opposites (i.e. the negative *x* become positive, the positive *x* become negative):

$A(-4,5) \rightarrow A'(4,5)$
$B(-2,2) \rightarrow B'(2,2)$
$C(-5,1) \rightarrow C'(5,1)$

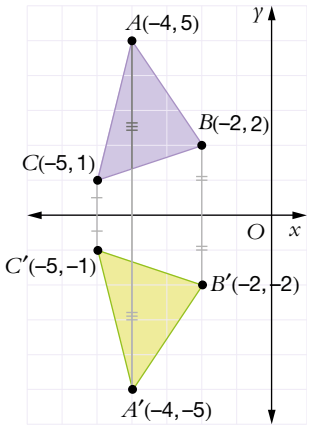
To find the image of a point reflected in the *y*-axis, replace the *x*-coordinate with its opposite but leave the *y*-coordinate unchanged.

Reflection in the *x*-axis

When ΔABC is reflected in the x-axis, each point and its image are the same distance away from the x-axis but on opposite sides of it.

The coordinates of the points and their images show that the x-coordinates are the same but the y-coordinates are opposites:

$A(-4,5) \rightarrow A'(-4,-5)$
$B(-2,2) \rightarrow B'(-2,-2)$
$C(-5,1) \rightarrow C'(-5,-1)$



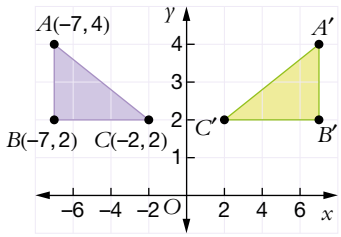
To find the image of a point reflected in the x-axis, leave the x-coordinate unchanged but replace the y-coordinate with its opposite.

WORKED EXAMPLE 1

Triangle ABC is reflected in the y-axis to produce triangle $A'B'C'$.

Write the coordinates of:

a A' **b** B' **c** C'



Solution

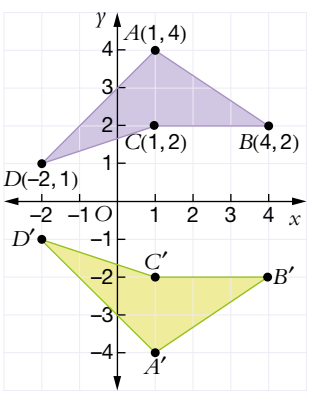
Reading the coordinates from the graph gives:

a (7,4) **b** (7,2) **c** (2,2)

WORKED EXAMPLE 2

The quadrilateral $ABCD$ is reflected in the x-axis.

a Write the coordinates of:
  i A' **ii** B' **iii** C' **iv** D'
b What can you say about the y-coordinates of each point and its image?
c What can you say about the x-coordinates of each point and its image?



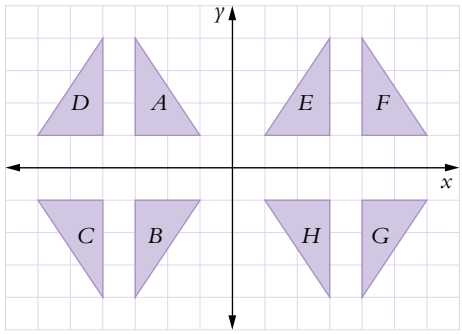
Solution

a **i** (1,–4) **ii** (4,–2) **iii** (1,–2) **iv** (–2,–1)
b The y-coordinates of each point and its image are opposites.
c The x-coordinates of each point and its image are equal.

Exercise 15:04

1 What triangle is produced, if each of the following triangles is reflected in the y-axis?

a A **b** B **c** C
d D **e** E **f** F
g G **h** H



2 What triangle is produced, if each of the following triangles is reflected in the x-axis?

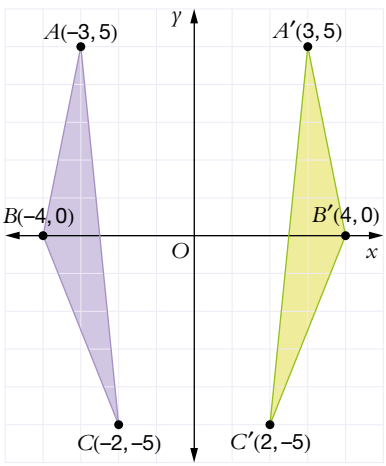
a A **b** B **c** C
d D **e** E **f** F
g G **h** H

3 Triangles ABC and $A'B'C'$ are reflections of each other in the y-axis.

a Write the coordinates of:
- **i** A and A'
- **ii** B and B'
- **iii** C and C'

b What do you notice about the y-coordinates of each pair of points in part **a**?

c What do you notice about the x-coordinates of each pair of points in part **a**?

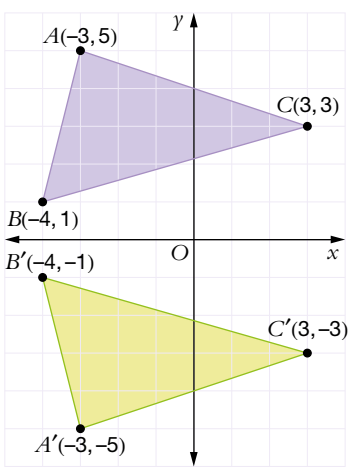


4 Triangles ABC and $A'B'C'$ are reflections of each other in the x-axis.

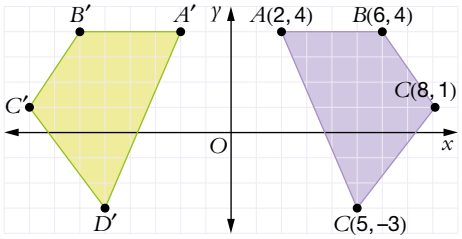
a Write the coordinates of:
i A and A' **ii** B and B' **iii** C and C'

b What do you notice about the x-coordinates of each pair of points in part **a**?

c What do you notice about the y-coordinates of each pair of points in part **a**?

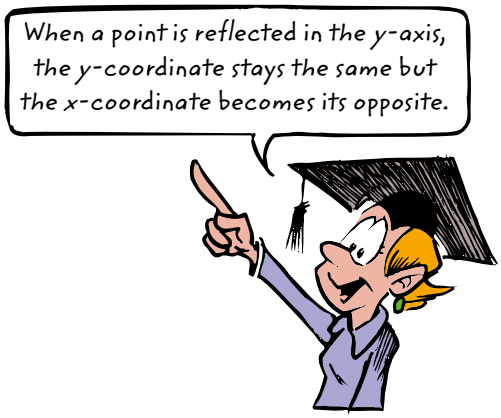


5 The quadrilateral $ABCD$ is reflected in the y-axis.

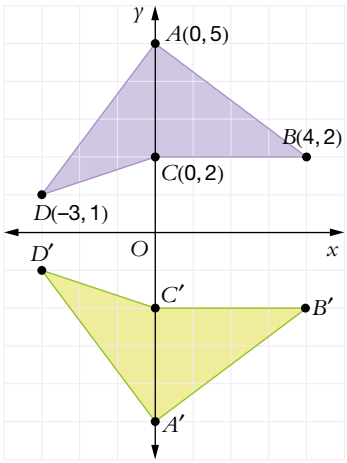


Write the coordinates of:

a A' **b** B' **c** C' **d** D'



6 The quadrilateral $ABCD$ is reflected in the x-axis.



When a point is reflected in the x-axis, the x-coordinate stays the same but the y-coordinate becomes its opposite.



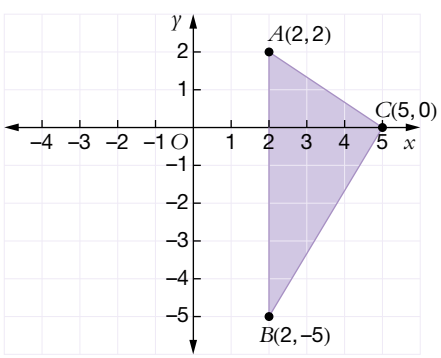
Write the coordinates of:

a A' **b** B' **c** C' **d** D'

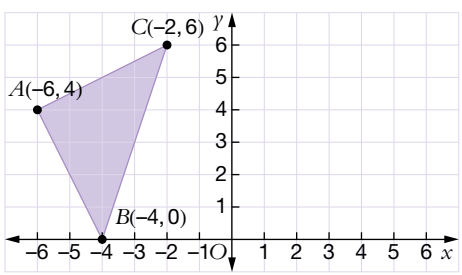
7 In each of the following, reflect triangle ABC in the y-axis and write the coordinates of the image of:

i A **ii** B **iii** C

a



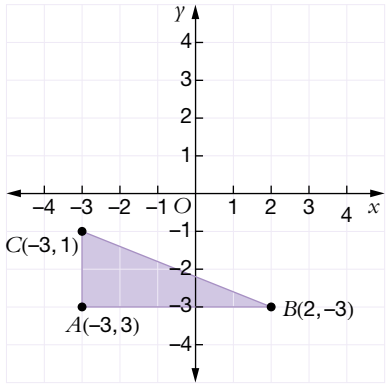
b



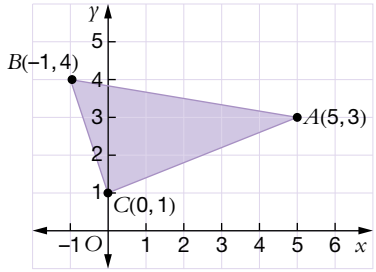
8 In each of the following, reflect triangle ABC in the x-axis and write the coordinates of the image of:

i A ii B iii C

a

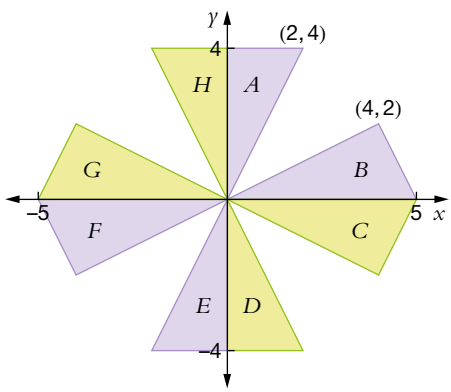


b



9 **a** Which triangle is the reflection of:

i ΔA in the x-axis
ii ΔB in the x-axis
iii ΔA in the y-axis
iv ΔB in the y-axis
v ΔG in the x-axis
vi ΔH in the x-axis
vii ΔG in the y-axis
viii ΔH in the y-axis



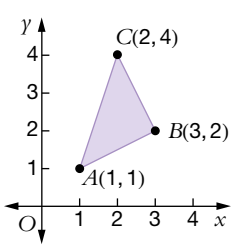
b For each of the following triangles, find the coordinates of the vertex that is not on either the x-axis or the y-axis.

i triangle H
ii triangle D
iii triangle C
iv triangle G

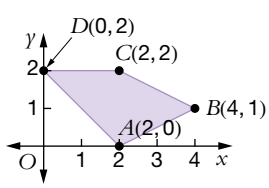
10 Find the coordinates of the vertices of the following shapes, when they are reflected:

i in the x-axis ii in the y-axis.

a



b



11 Triangle ABC is formed by joining the points $A(3,2)$, $B(-2,-3)$ and $C(0,6)$.

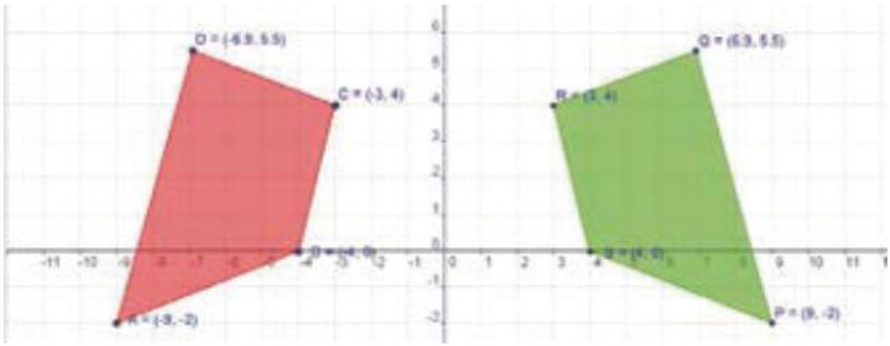
a Triangle $A'B'C'$ is its reflection in the x-axis. Find the coordinates of A', B' and C'.

b Triangle $A''B''C''$ is its reflection in the y-axis. Find the coordinates of A'', B'' and C''.

P GEOGEBRA ACTIVITY 15:04 REFLECTIONS

You can change the shape of the figure by moving its vertices. Select the options to reflect the figure in the x-axis, the y-axis or another line.

The coordinates of all points are shown. What is the relationship between a point and its image?

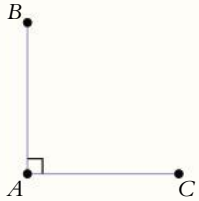


15:05 Rotations

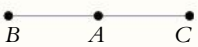
PREP QUIZ 15:05

In each of the following, write the clockwise angle that AB must turn through to become AC.

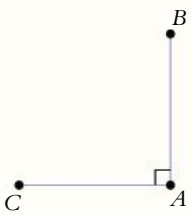
1



2



3



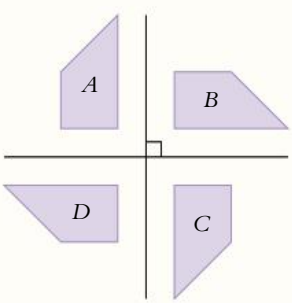
Which figure does A become, when it is rotated:

4 90° clockwise

5 90° anticlockwise

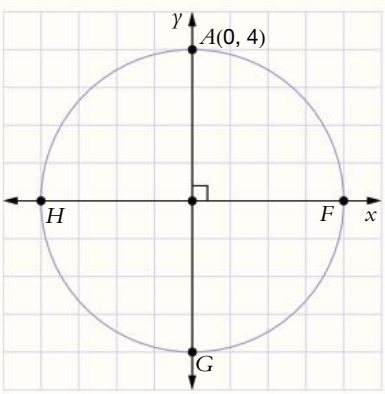
6 180°

7 270° clockwise?



What are the coordinates of:

8 F **9** G **10** H?



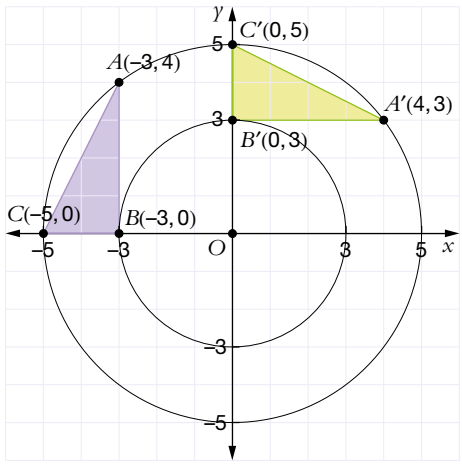
When an object is rotated, all the points move on circles around the centre of rotation.

In the following diagrams, triangle ABC is rotated about the point O through angles of 90°, 180° and 270° in a clockwise direction to produce triangle $A'B'C'$.

The points C, A, C' and A' are all on a circle of radius 5 units. The points B and B' are on a circle of radius 3 units.

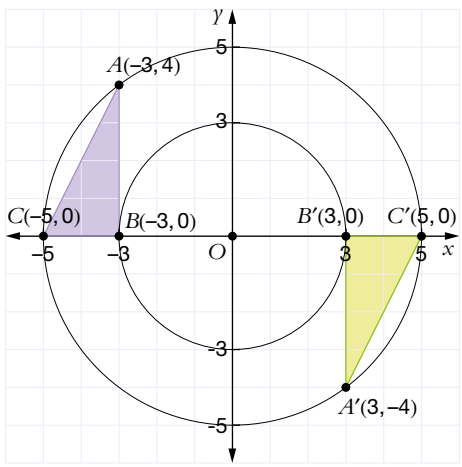
In the first diagram, triangle ABC is rotated through 90° in a clockwise direction (or 270° in an anticlockwise direction):

$A(-3,4) \rightarrow A'(4,3)$
$B(-3,0) \rightarrow B'(0,3)$
$C(-5,0) \rightarrow C'(0,5)$.



In the next diagram, triangle ABC is rotated 180° (this is exactly the same in either direction):

$A(-3,4) \rightarrow A'(3,-4)$
$B(-3,0) \rightarrow B'(3,0)$
$C(-5,0) \rightarrow C'(5,0)$.

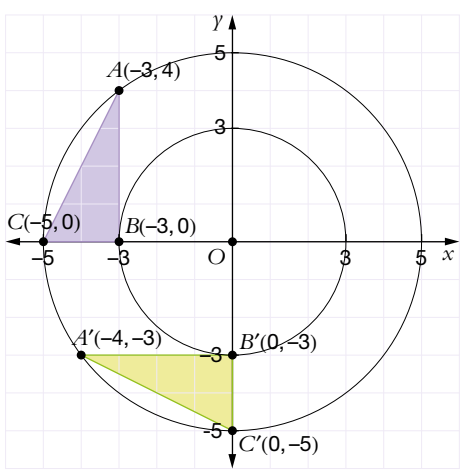


To find the coordinates of the image, use the following rule:

> In a 180° rotation, the coordinates of the image are found by replacing the x- and y-coordinates of each point with their opposites,
> e.g. $A(-3,4) \rightarrow A'(3,-4)$

In the last diagram, triangle ABC is rotated 270° in a clockwise direction (or 90° in an anticlockwise direction):

$A(-3,4) \rightarrow A'(-4,-3)$
$B(-3,0) \rightarrow B'(0,-3)$
$C(-5,0) \rightarrow C'(0,-5)$



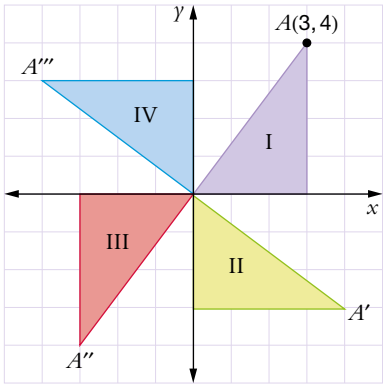
WORKED EXAMPLE 1

a Name the triangle produced when triangle I is rotated in a clockwise direction through:

i 90° ii 180° iii 270°

b Give the name and coordinates of the image of A when it is rotated in an anticlockwise direction through:

i 90° ii 180° iii 270°



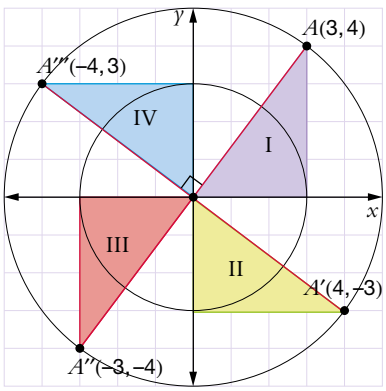
Solution

a In the diagram, one set of four matching sides is highlighted to show that they form a set of four right angles. Hence, the answers are:

i II ii III iii IV

b Starting at A and rotating anticlockwise gives:

i $A'''(-4,3)$ ii $A''(-3,-4)$ iii $A'(4,-3)$



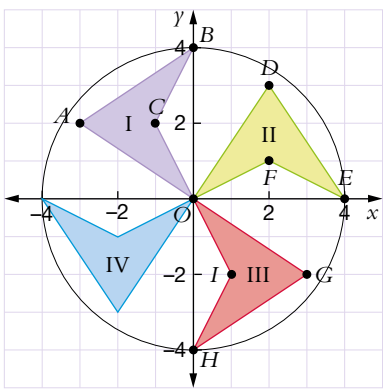
WORKED EXAMPLE 2

a In this diagram, figure I is rotated through 90°, 180° and 270°.

i What are the coordinates of the points A, B and C?

ii Which points do A, B and C become when figure I is rotated clockwise through 90° and 180°?

b What are the coordinates of the vertices of figure IV?



Solution

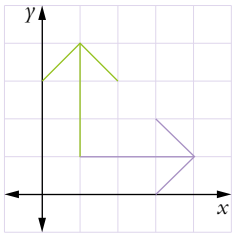
a i Reading from the number plane gives $A(-3,2)$, $B(0,4)$ and $C(-1,2)$.

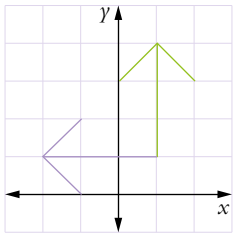
ii A becomes D and G, B becomes E and H, C becomes F and I.

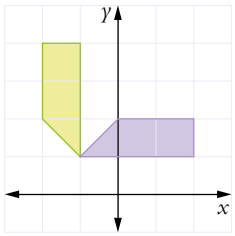
b Reading from the graph, the vertices of figure IV are $(0,0)$, $(-2,-3)$, $(-4,0)$ and $(-2,-1)$.

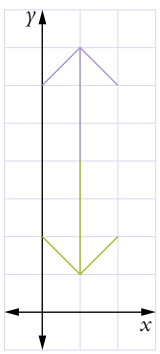
Exercise 15:05

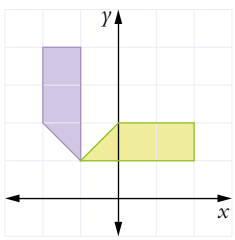
1 The green figures are rotated in a clockwise direction through 90°, 180° or 270° to produce the purple figures. What angle describes each rotation?

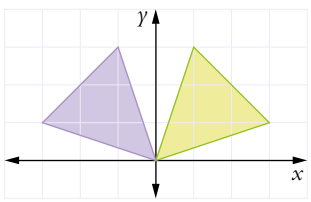
a 

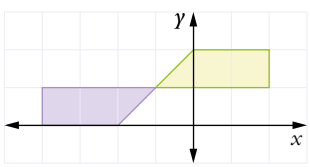
b 

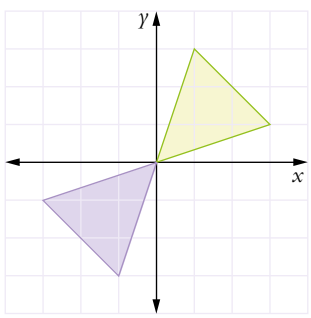
c 

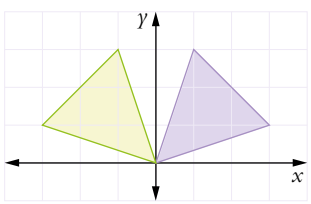
d 

e 

f 

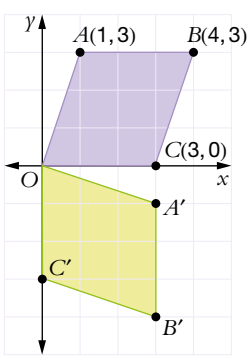
g 

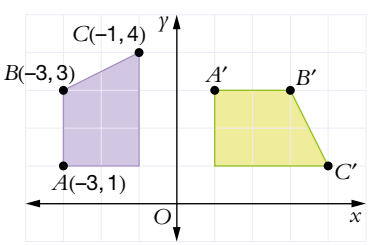
h 

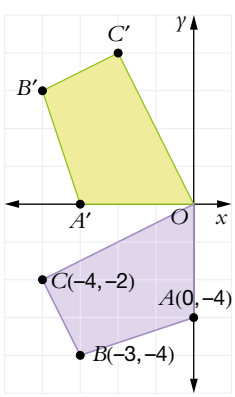
i 

2 What would the answers to Question 1 be if the rotation were anticlockwise?

3 In each of the following, the purple shape is rotated clockwise through 90° to produce the green shape. What are the coordinates of A', B' and C'?

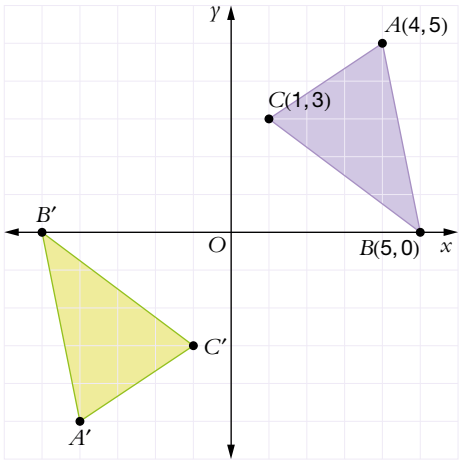
a 

b 

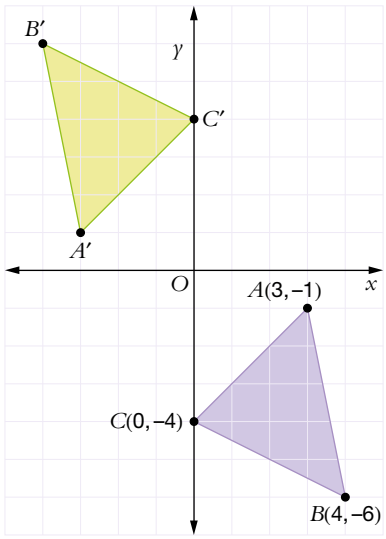
c 

4 In each of the following, the purple triangle is rotated through 180° to produce the green triangle. What are the coordinates of A', B' and C'?

a



b

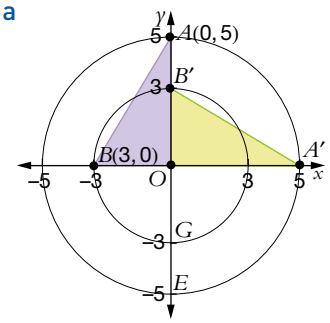


5 Write the coordinates of each point and its image in Question **4**. What do you notice about the coordinates of each pair of points?

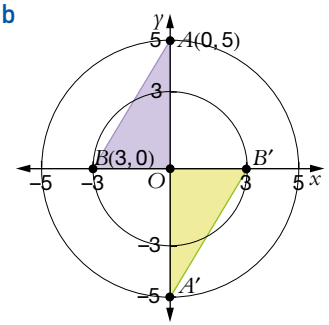
6 In each of the following, ΔABO has been rotated 90°, 180° or 270° in a clockwise direction to produce $\Delta A'B'O'$. For each diagram:

i What is the angle of rotation? ii What are the coordinates of A' and B'?

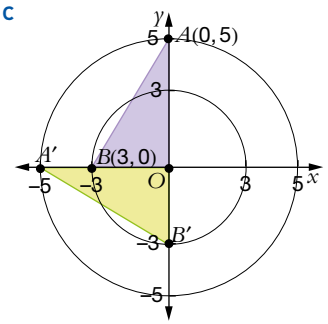
a



b



c



7 Rectangle I has been rotated to produce rectangle II.

a Which point is the image of:

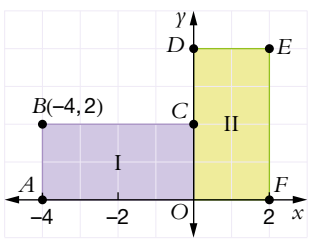
i A ii B

iii C iv O?

b Write the coordinates of the image of:

i A ii B

iii C iv O



8 Rectangle I has been rotated to produce rectangle II.

a Which point is the image of:

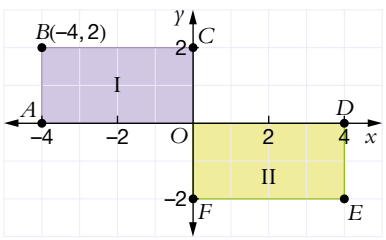
i A ii B

iii C iv O?

b Write the coordinates of the image of :

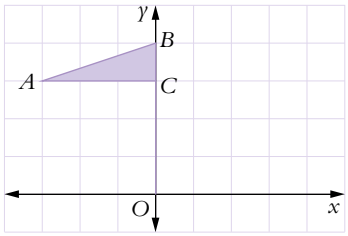
i A ii B

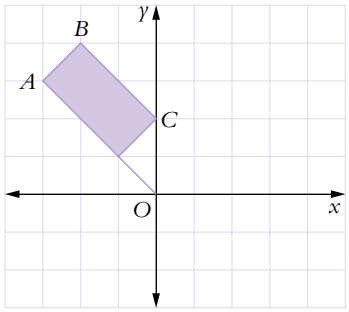
iii C iv O

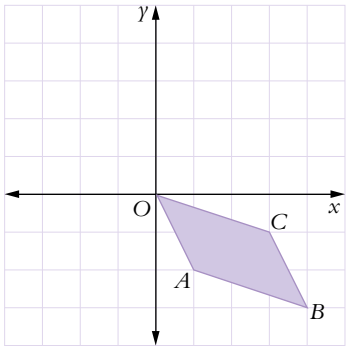


9 Rotate each purple figure by the given angle in a clockwise direction about the origin to create figure $A'B'C'$. What are the coordinates of A', B' and C'?

a 90° **b** 180° **c** 270°







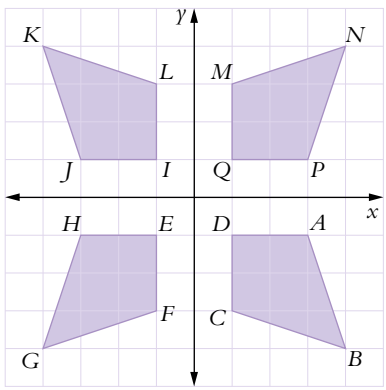
10 The kite $ABCD$ is rotated through angles of 90°, 180° and 270° about O.

a Which points are the images of B?

b Which points are the images of D?

c Which points are the images of A?

d Which points are the images of C?



11 The kite $ABCO$ is rotated through angles of 90°, 180° and 270° about O to produce this figure.

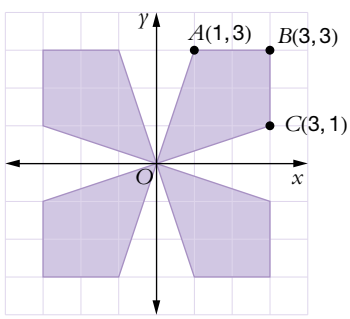
Use the grid and the given coordinates to find the coordinates of the image of:

a A when it is rotated clockwise through 90°

b B when it is rotated through 180°

c C when it is rotated clockwise through 270°

d A when it is rotated through 180°.



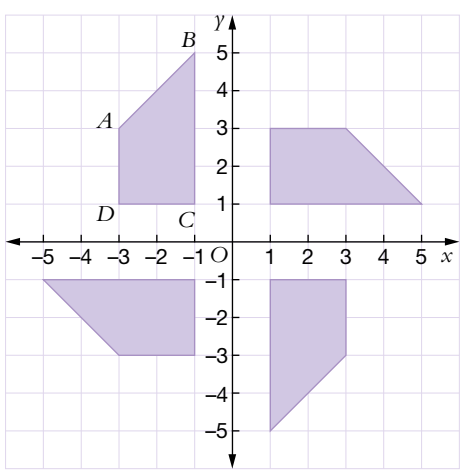
12 The trapezium $ABCD$ is rotated through 90°, 180° and 270°.

a What are the coordinates of A and its image points?

b What are the coordinates of B and its image points?

c What are the coordinates of C and its image points?

d What are the coordinates of D and its image points?



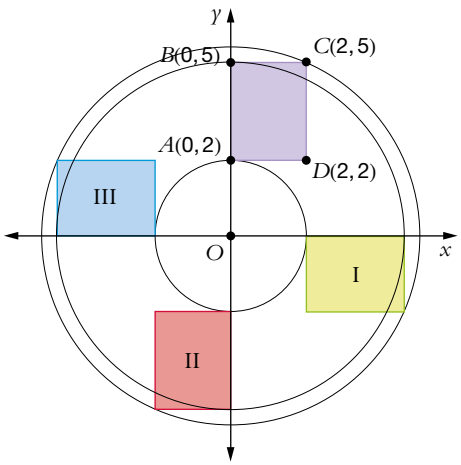
13 Rectangle $ABCD$ is rotated through angles of 90°, 180° and 270° to produce rectangles I, II and III respectively.
Find the coordinates of the images of A, B, C and D when it is rotated:

a 90° clockwise

b 180°

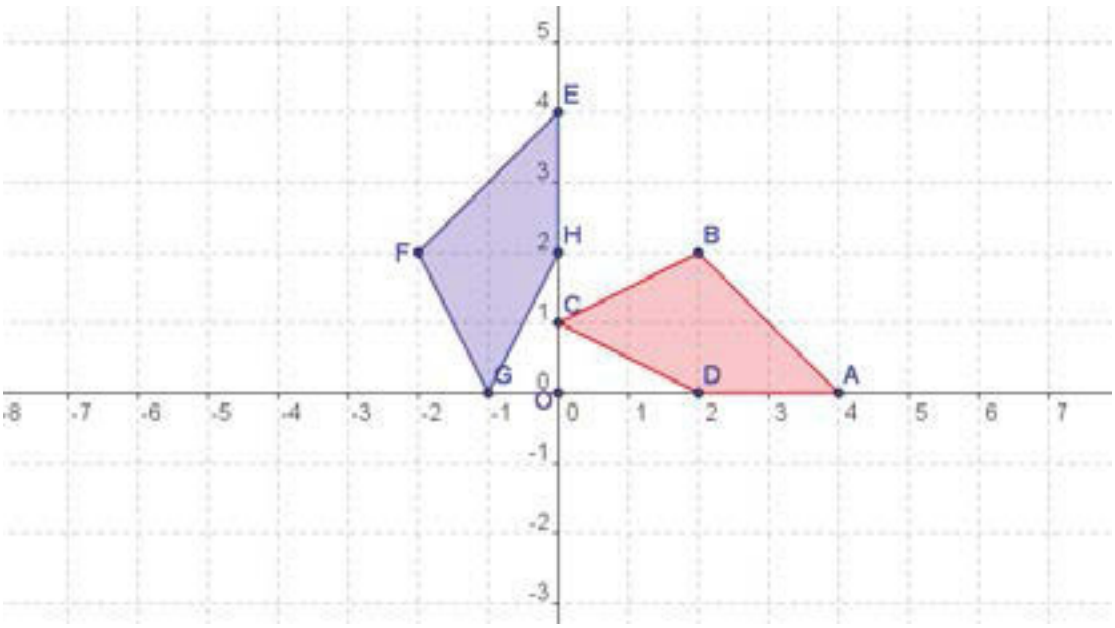
c 270° clockwise

d 90° anticlockwise.



P GEOGEBRA ACTIVITY 15:05 ROTATIONS

Change the shape of the figure by moving its vertices. Tick the boxes to see the image of the figure rotated through angles of 90°, 180° or 270°. You can also see the circles on which the points rotate.



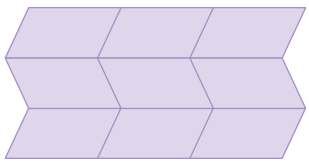
15:06 Combining transformations

Transformations can be used to produce patterns and designs. You can often clearly see how a pattern could be made using different combinations of translations, rotations and reflections. For example:



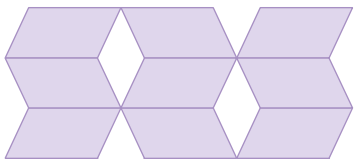
Pattern 1

- In Pattern 1, the obvious way to form the pattern is by a series of translations. However, it could also be formed using a combination of rotations and translations.



Pattern 2

- In Pattern 2, the pattern could be produced using translations and reflections, or using translations and rotations.

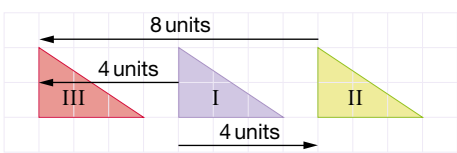


Pattern 3

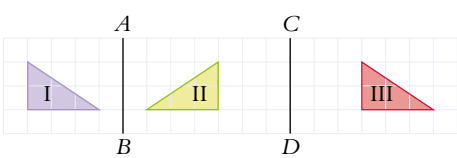
- In Pattern 3, both horizontal and vertical reflections can be used to produce the pattern. However, this pattern could also have been produced by a combination of reflections and translations and also rotations and translations.

A number of successive transformations can often be replaced by a single transformation. For example:

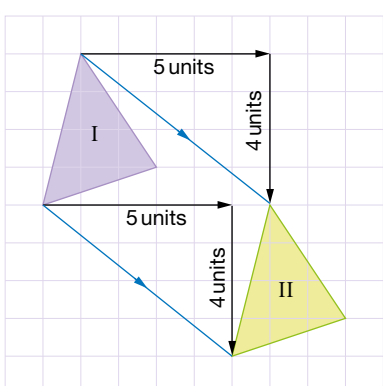
- A translation of 4 units to the right followed by a translation of 8 units to the left is the same as a single translation of 4 units to the left.



- Figure I is reflected in the line AB to produce figure II. Figure II is then reflected in the line CD to produce figure III. This double reflection could also be achieved by a single translation of 14 units to the right.

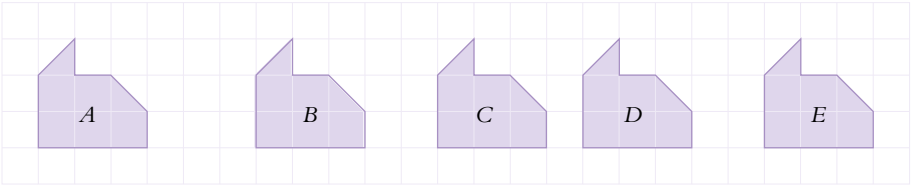


Sometimes it is better to think of one transformation being replaced by two simpler ones. In the diagram at right, figure I can be translated diagonally along the blue lines to produce figure II. The same result can be achieved by translating it 5 units to the right and then 4 units down. The horizontal and vertical translations are mathematically simpler than a diagonal translation.



Exercise 15:06

1 Which figures would be produced by the following successive translations?

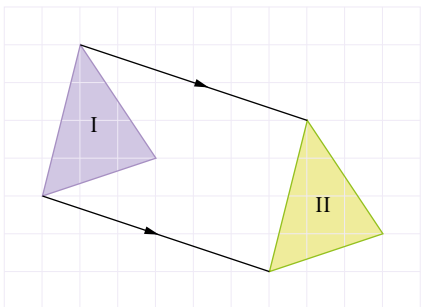


a B moves 5 units to the right, then 4 units to the right
b D moves 4 units to the left, then 11 units to the left
c A moves 15 units to the right, then 4 units to the left
d C moves 5 units to the left, then 14 units to the right

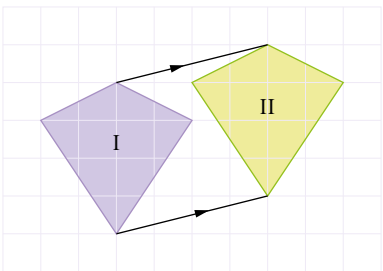
2 Replace each series of successive translations in Question 1 with one single translation that produces the same change.

3 In each of the following, figure I has been translated to produce figure II. Write the two simpler translations that produce the same result as the diagonal translation.

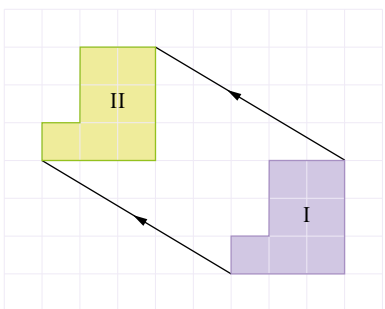
a



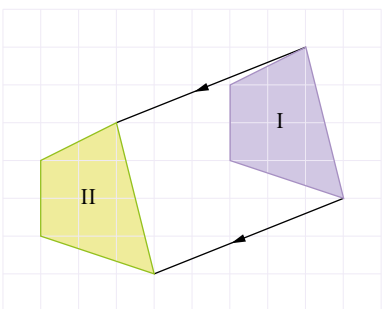
b



c

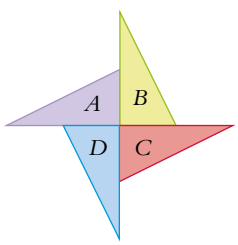


d



4 Which triangle is produced by the following successive rotations of triangle A?

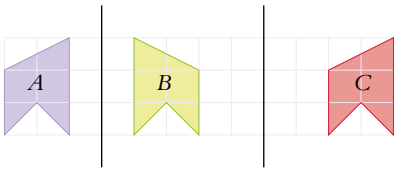
a rotation of 90° clockwise, then rotation of 180° clockwise
b rotation of 90° anticlockwise, then rotation of 90° anticlockwise
c rotation of 180° anticlockwise, then rotation of 90° clockwise



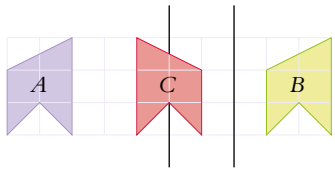
5 In Question 4, what single rotations would make the same change as each set of successive rotations?

6 In the following diagrams, A is reflected to produce B and then B is reflected to produce C. What single translation would produce the same change as each double reflection?

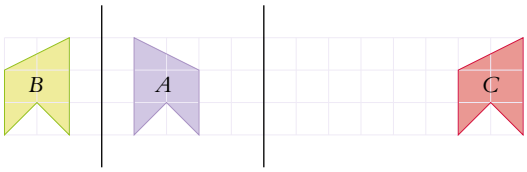
a



b



c



7 **a** In diagram 1:

- **i** Is B a reflection of A in the vertical line?
- **ii** Is C a reflection of B in the horizontal line?
- **iii** Is C a possible result of successive reflections of A?

b In diagram 2, what rotation has changed A into C?

c What transformation produces the same result as successive reflections in the vertical and horizontal lines?

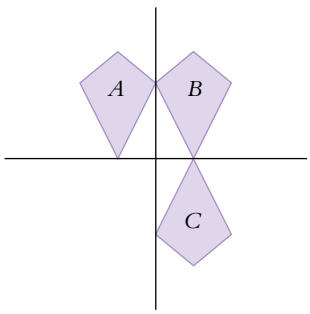


Diagram 1

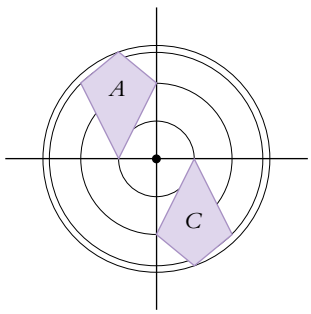
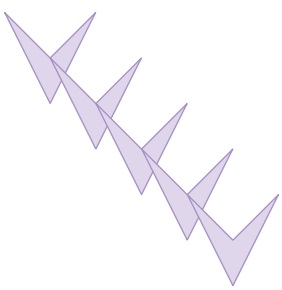


Diagram 2

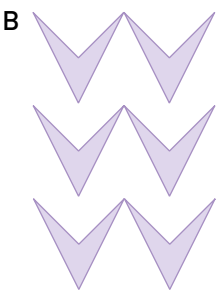
8 Which of the figures below can be produced from a single shape, using:

a translations **b** reflections **c** rotations **d** translations and reflections **e** translations and rotations?

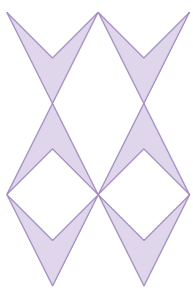
A



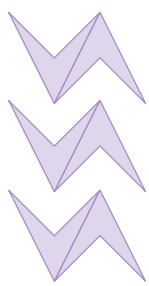
B



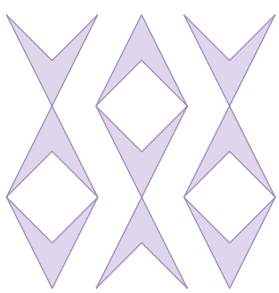
C



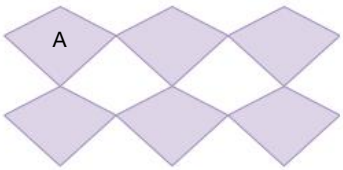
D



9 Describe a combination of transformations that could be used to make this pattern from a single shape.

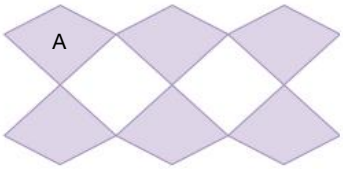


10 Could this pattern be produced from the shape A using:



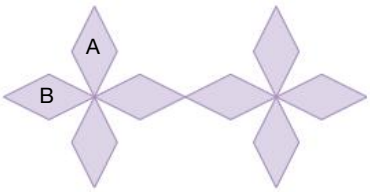
- **a** only translations
- **b** only reflections
- **c** only rotations?

11 Could this pattern be produced from the shape A using:



- **a** only translations
- **b** only reflections
- **c** only rotations
- **d** translations and reflections
- **e** reflections and rotations?

12 **a** Can rhombus A be moved to the position of rhombus B using:



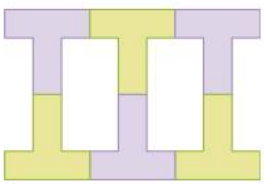
- **i** rotation
- **ii** translation
- **iii** horizontal or vertical reflection?

b Can this design be made from a single rhombus using:

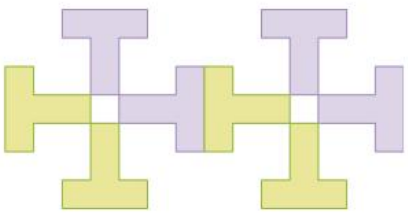
- **i** different translations
- **ii** horizontal and vertical reflections
- **iii** rotations and translations?

13 Ignoring the colours of the 'T' shapes, describe two different ways in which the following designs could be generated from a single 'T' using translations, rotations and/or reflections.

a

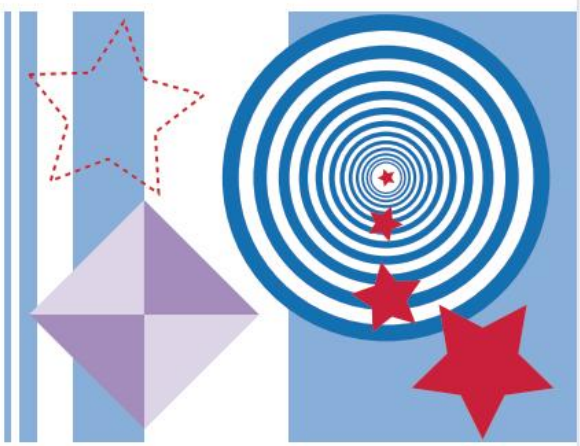


b



INVESTIGATION 15:06 USING TECHNOLOGY TO CREATE PATTERNS

Use a computer drawing program to experiment with, create and re-create patterns using combinations of flips, slides, turns, enlargements and reductions.



MATHS TERMS 15

image
- the figure that results after a transformation

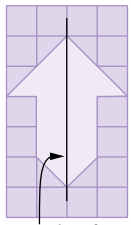
transformation
- a change to a figure using actions such as reflections (flips), translations (slides) or rotations (turns)

reflection (flip)
- the process of 'flipping' a figure over one side of a line to the other side
- the line is called the *line of reflection*

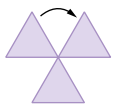
rotation (turn)
- the process of turning a figure about a point through an angle
- the point is called the *centre of rotation*

translation (slide)
- the process of 'sliding' a shape so that every point on the figure moves the same distance in the same direction

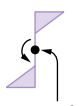
symmetry
- a balanced arrangement
- axis of symmetry: a line that divides a figure into two parts that are mirror images of each other



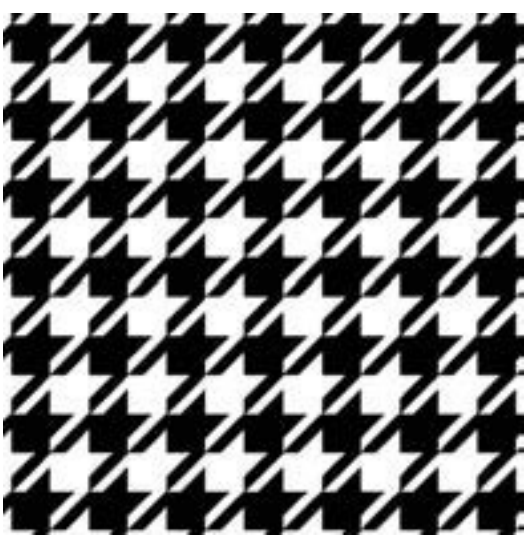
- line symmetry: a property of a figure where one half is the mirror image of the other
- rotational symmetry: a property of a figure where it can be turned around a point so that it repeats its shape more than once in a complete rotation (revolution)



- order (of rotational symmetry): the number of times that a figure repeats its shape in one complete rotation
- centre of symmetry: the point about which a figure is symmetrical when rotated



- point symmetry: a property where a figure repeats itself after half a turn, i.e. the order of rotational symmetry is an even number



How many different combinations of translations, reflections and rotations could have been used to make this pattern?

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 How many axes of symmetry does each of the following shapes have? 15:01

a  b  c 

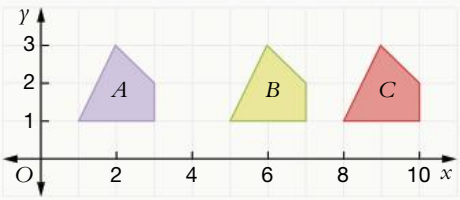
2 What is the order of rotational symmetry of each of the shapes in Question 1? 15:01

3 Identify each of the following as a translation, reflection or rotation. 15:02

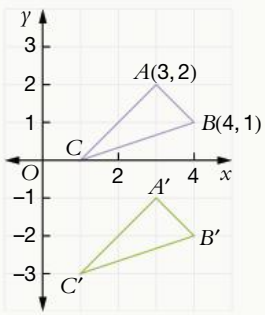
a  b  c 

4 Describe the translation that moves: 15:03

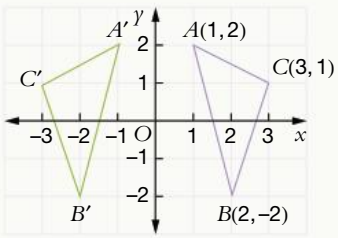
a A to B
b A to C
c C to B



5 a Is $A'B'C'$ a translation, rotation or reflection of ABC? 15:03
Write the coordinates of:
b A' c B' d C'



6 a Is $A'B'C'$ a translation, rotation or reflection of ABC? 15:04
Write the coordinates of:
b A' c B' d C'

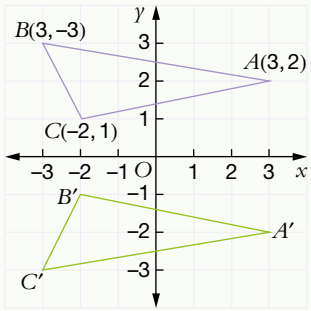


15:04

7 a Is $A'B'C'$ a translation, rotation or reflection of ABC?

Write the coordinates of:

b A' c B' d C'

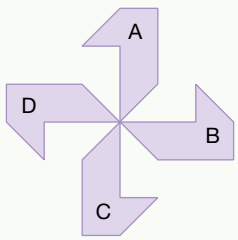


15:05

8 This shape is rotated through 90°, 180° and 270°. Which of these angles describes the following clockwise rotations?

a A to B b B to D

c A to D d D to A



15:05

9 If triangle I is rotated 90° clockwise, which point is the image of:

a A b B c C?

15:05

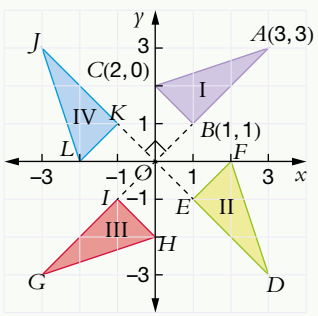
10 If triangle I is rotated 180°, which point is the image of:

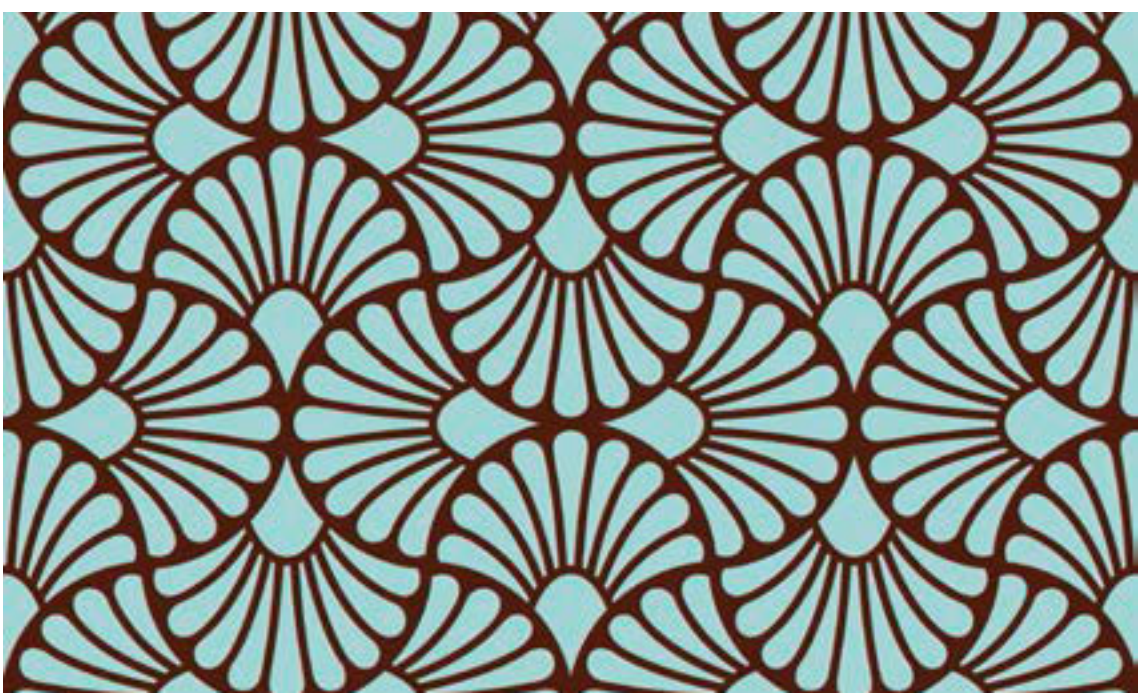
a A b B c C?

15:05

11 If triangle I is rotated 270° clockwise, which point is the image of:

a A b B c C?

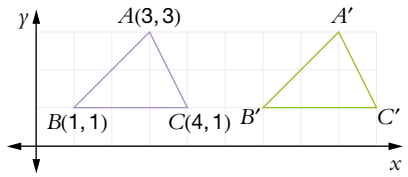




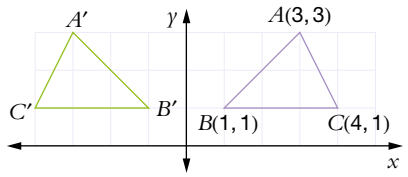
ASSIGNMENT 15A Chapter review

1 In each of the following, state what type of transformation is shown and give the coordinates of A', B' and C'.

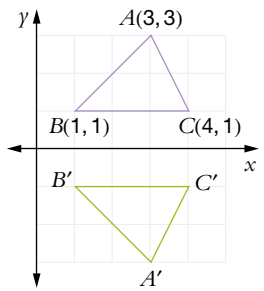
a



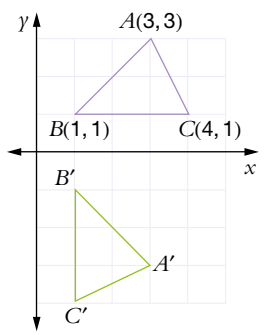
b



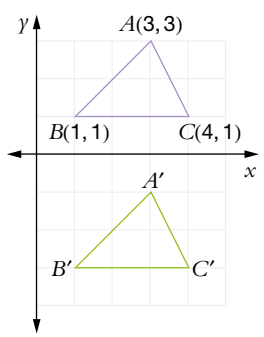
c



d

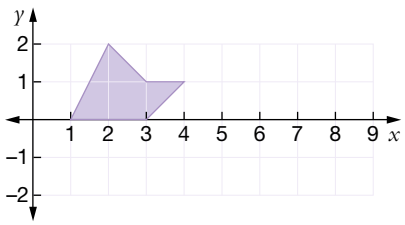


e



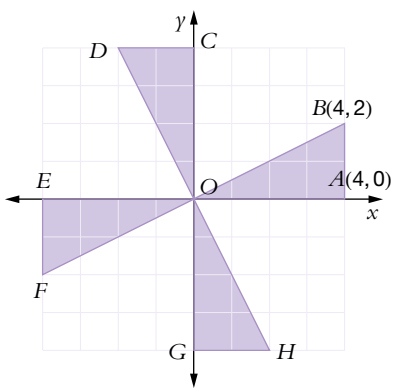
2 The triangle formed by joining $A(1,2)$, $B(5,-4)$ and $C(-3,-8)$ is reflected in the x-axis to produce triangle $A'B'C'$. What are the coordinates of A', B' and C'?

3 **a** If this figure is translated 5 units to the right, what are the coordinates of the vertices of the new figure?



b If the figure is translated 2 units down, what are the coordinates of the vertices of the new figure?

4 The triangle OAB is rotated through 90°, 180° and 270°.



a Which points are the images of A? What are the coordinates of these points?

b Which points are the images of B? What are the coordinates of these points?

5 A quadrilateral $ABCD$ is reflected in the y-axis to produce the quadrilateral $A'B'C'D'$. Find the coordinates of A', B', C' and D' if A is $(10,8)$, B is $(-8,-6)$, C is $(-4,5)$ and D is $(6,16)$.

ASSIGNMENT 15B Working mathematically

1 On her birthday, Bronwyn noticed that in 30 years she would be three times older than she is now. How old is she?

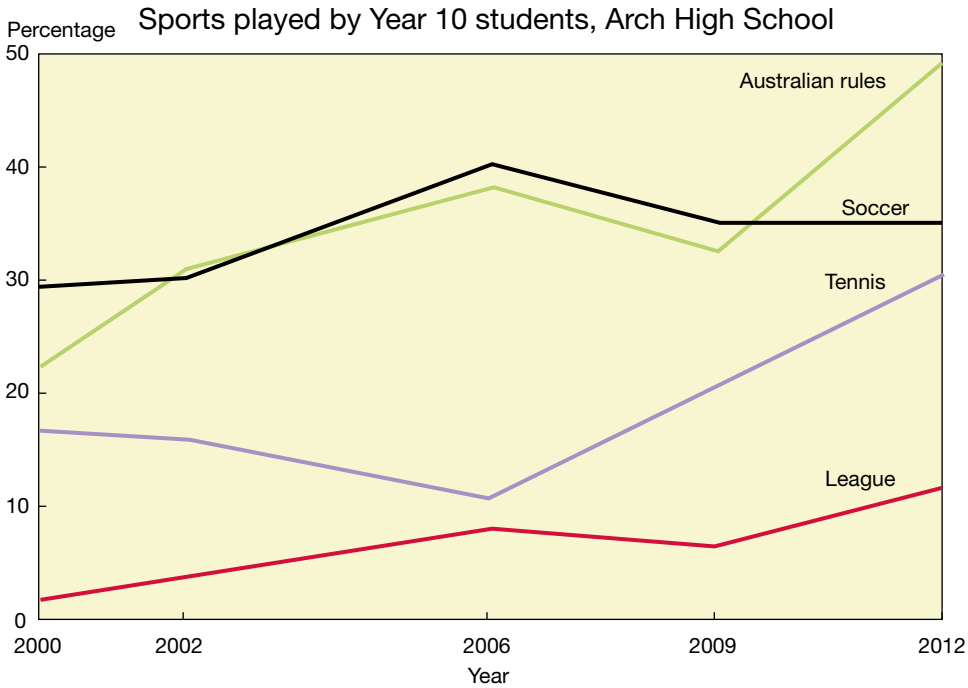
2 A spool of wire is 100 m long. How many pieces of wire 30 cm long could be cut from this spool?

3 A girl has 4 tops that can be worn with each of 5 skirts. How many different outfits can she wear?

4 A shopkeeper reduced the price of an item by $\frac{1}{3}$ and sold it for $18.60. How much was the item before the reduction?

5 From Monday to Thursday, a jogger runs 5 km every morning and 8 km every evening. On Friday she runs 10 km after work. She rests on Saturday, then runs 15 km on Sunday afternoon. How far does she run in one week? How many weeks will it take her to run 1000 km?

6



From the graph:

a What was the most commonly played sport in 2002?

b What was the most commonly played sport in 2006?

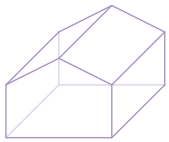
c Approximately what percentage of students played Australian rules in 2012?

d By what percentage did the playing of soccer increase from 2000 to 2012?

e If you add together the percentages of students who played Australian rules, soccer and tennis in 2012, you get more than 100%. How do you explain this?

ASSIGNMENT 15C Cumulative revision

1 **a** How many faces does the solid have?
b How many vertices does the solid have?
c How many edges does the solid have ?
d Name the solid.



8:05

2 **a** Write 15% as a decimal.
b Write 45% as a fraction in simplified form.
c Find 10% of 600.
d Find 30% of 45.

11:03, 11:04

3 **a** What percentage is 16 of 20?
b What percentage is 20 of 16?
c What percentage of 40 is 50?

11:05

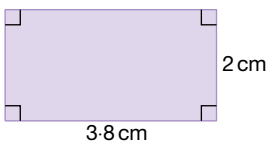
4 Solve each equation.

a $5x - 14 = 31$ **b** $18 - 2a = 4$ **c** $\frac{x}{3} - 4 = 7$

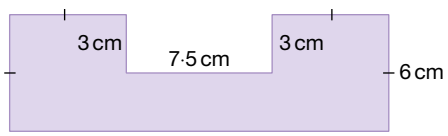
13:04

5 Find the area and perimeter of the following shapes.

a



b



10:01, 10:03

6 Evaluate the following.

a $3 + 4 \times 6$ **b** $8 \times 6 \div 4 \times 3$
c $25 \times (6 + 5) \times 2$ **d** $3 \times 5 + 3 \times 95$

1:06B, 1:07B

7 **a** Two angles of a triangle are 35° and 55°. What is the size of the third angle?
b A parallelogram has two 65° angles. What are the sizes of the other two angles?
c The largest angle of an isosceles triangle is 100°. What are the sizes of its other angles?

8:02, 8:03

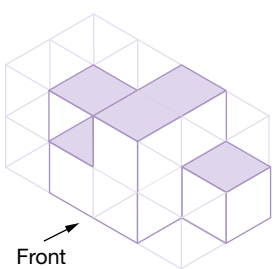
8 A is the point (3,2). Write the coordinates of a point that is:
a 5 units to its right **b** 5 units to its left
c 5 units above it **d** 5 units below it.

7:03

9 If $m = 10$ and $n = 7$, write the value of:
a $m - n$ **b** $n - m$ **c** $2mn$ **d** $m^2 + n^2$

4:06

10 Draw the front view, top view and side views of the solid shown.



8:08

ANSWERS

ID Cards

ID Card 1 (Metric units)

1 metres	**2** decimetres	**3** centimetres	**4** millimetres
5 kilometres	**6** square metres	**7** square centimetres	**8** square kilometres
9 hectares	**10** cubic metres	**11** cubic centimetres	**12** seconds
13 minutes	**14** hours	**15** metres per second	**16** kilometres per hour
17 grams	**18** milligrams	**19** kilograms	**20** tonnes
21 litres	**22** millilitres	**23** kilolitres	**24** degrees Celsius

ID Card 2 (Symbols)

1 add (or plus)	**2** subtract (or minus)	**3** multiplied by (or times)
4 divided by	**5** is equal to	**6** is approximately equal to
7 is not equal to	**8** is less than	**9** is less than or equal to
10 is not less than	**11** is greater than	**12** is greater than or equal to
13 is not greater than	**14** is not greater than or equal to	**15** 4 squared
16 4 cubed	**17** the square root of 25	**18** the cube root of 27
19 per cent	**20** therefore	**21** for example
22 that is	**23** numerator	**24** denominator

ID Card 3 (Language)

1 $6 - 2 = 4$	**2** $6 + 2 = 8$	**3** $6 \div 2 = 3$	**4** $6 - 2 = 4$
5 $6 \div 2 = 3$	**6** 2	**7** 6	**8** $6 \times 2 = 12$
9 $6 - 2 = 4$	**10** $6 \times 2 = 12$	**11** $2 + 6 = 8$	**12** $6 - 2 = 4$
13 $6^2 = 36$	**14** $\sqrt{36} = 6$	**15** $6 - 2 = 4$	**16** $6 \times 2 = 12$
17 $(6 + 2) \div 2 = 4$	**18** $6 + 2 = 8$	**19** $6^2 = 36$	**20** $6 - 2 = 4$
21 $6 - 2 = 4$	**22** $6 + 2 = 8$	**23** $6 \div 2 = 3$	**24** $6 + 2 = 8$

ID Card 4 (Language)

1 square	**2** rectangle	**3** parallelogram	**4** rhombus (or diamond)
5 trapezium	**6** regular pentagon	**7** regular hexagon	**8** regular octagon
9 kite	**10** scalene triangle	**11** isosceles triangle	**12** equilateral triangle
13 circle	**14** oval (or ellipse)	**15** cube	**16** rectangular prism
17 triangular prism	**18** square pyramid	**19** rectangular pyramid	**20** triangular pyramid
21 cylinder	**22** cone	**23** sphere	**24** hemisphere

ID Card 5 (Language)

1 point A	**2** interval AB	**3** line AB
4 ray AB	**5** collinear points	**6** midpoint
7 number line	**8** diagonals	**9** acute-angled triangle
10 right-angled triangle	**11** obtuse-angled triangle	**12** vertices
13 ΔABC	**14** hypotenuse	**15** $180°$
16 $(a+b)°$	**17** $360°$	**18** $a° = b°$
19 $a° = 60°$	**20** $3 \times 180° = 540°$	**21** AB is a diameter, OC is a radius
22 circumference	**23** semicircle	**24** AB is a tangent, CD is an arc, EF is a chord

ID Card 6 (Language)

1 parallel lines
2 perpendicular lines
3 vertical, horizontal
4 concurrent lines
5 angle *ABC* or *CBA*
6 acute angle
7 right angle
8 obtuse angle
9 straight angle
10 reflex angle
11 revolution
12 adjacent angles
13 complementary angles
14 supplementary angles
15 vertically opposite angles
16 360°
17 transversal
18 corresponding angles
19 alternate angles
20 co-interior angles
21 bisecting an interval
22 bisecting an angle
23 $\angle CAB = 60°$
24 CD is perpendicular to AB

ID Card 7 (Language)

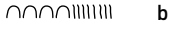
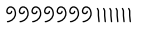
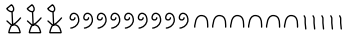
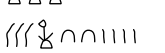
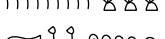
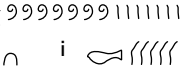
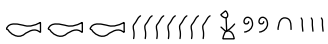
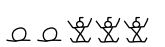
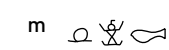
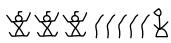
1 Common Era, Anno Domini
2 Before Common Era, Before Christ
3 ante meridiem (before midday)
4 post meridiem (after midday)
5 hectare
6 regular shapes
7 net of a cube
8 cross-section
9 face
10 vertex
11 edge
12 axes of symmetry
13 reflection (or flip)
14 translation (or slide)
15 rotation (or turn)
16 tessellation
17 coordinates
18 tally
19 picture graph
20 column graph
21 line graph
22 sector (or pie) graph
23 bar graph
24 scatter diagram

Chapter 1

Exercise 1:01

1 **a** didn't need them **b** no **c** 10 **d** no
e subtract smaller unit from larger one
f Answers will vary. (It came from crossed counting notches on sticks, but can also represent the V formed by a thumb and the fingers of a hand.)
g Answers will vary. (It came from crossed counting notches on sticks, but can also represent two Vs.)
h India
i Hindu–Arabic; to indicate an 'empty space'

2 **a** 35 **b** 71 **c** 1235 **d** 315
e 1 111 111 **f** 1032 **g** 131 200
h 12 007 **i** 11 000 000 **j** 13 100 000
k 2 230 000 **l** 1 001 312

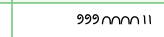
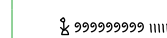
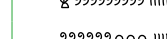
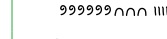
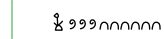
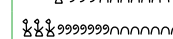
3 **a** 
b 
c 
d 
e 
f 
g
h 
i 
j 
k
l 
m 
n 
o 

4 **a** 315 **b** 34 **c** 128 **d** 682 **e** 709
f 1724 **g** 3400 **h** 548 **i** 947
j 808 **k** 1505 **l** 5555 **m** 52 500
n 499 **o** 999

5 **a** XXXVII **b** CCXIII **c** LXXXVI
d DCXXXVII **e** DCLXXXIV **f** XLIX
g CCXC **h** DCXLV **i** MCDLII
j DCCLXXVIII **k** CMLXXXVIII
l MCMLXXXIX **m** $\overline{V}$MMMCDLXXXIX
n $\overline{V}$CCCLXXXIV **o** $\overline{D}\overline{X}\overline{L}$MMMDCXXVII

6 **a** 586 **b** 2107 **c** 432 **d** 8006
e 95 427 **f** 4330 **g** 1004 **h** 90 012
i 85 430 **j** 127

7

Hindu–Arabic numeral (876 CE)	Egyptian numeral	Roman numeral
٣٥٢		CCCXLII
١٩٠٤		MCMV
٤٣٥		DCXXXIV
١٣٤٢		MCCCLXII
٣٧٧٤		MMMDCCLXXVI
١٩٨٨		MCMLXXXVIII

8 It takes fewer symbols to write large numbers.
9 999 **10** 888 or 1388

Exercise 1:02

1 **a** 1313 **b** 2801 **c** 14 926 **d** 50 380
e 33 709 **f** 93 165 **g** 3203 **h** 605 309

2 **a** 5815 **b** 324 679 **c** 806 050
d 1 927 463 **e** 56 002 914 **f** 83 907 201
g 37 070 847 **h** 927 100 070 **i** 204 042 420

3 **a** eight thousand four hundred and five
b forty-three thousand six hundred and twenty-seven
c ninety thousand and forty-six
d one hundred and thirty thousand two hundred and fifteen
e nine hundred and twenty-seven thousand and four
f six million three hundred and sixty thousand and sixty-four
g seventeen million four thousand nine hundred and eighty-eight
h nine million three hundred and two thousand eight hundred and fifty
i four hundred and forty-three million two hundred thousand

4 **a** 500, 60, 7 **b** 5000, 600, 7 **c** 50 000, 60, 7
d 500 000, 70 000, 600 **e** 60 000, 70, 5
f 600 000, 5000, 700 **g** 6 000 000, 50 000, 70
h 500 000, 60 000, 7000
i 50 000 000, 600 000, 70 000

5 **a** 6000 + 400 + 20 + 1
b 80 000 + 100 + 70 + 9
c 700 + 80 + 6
d 90 000 + 1 000 + 30 + 2
e 6000 + 90 + 4
f 1 000 000 + 300 000 + 40 000 + 600 + 20 + 7
g 7 000 000 + 300 000 + 40 000 + 6 000 + 900 + 10 + 2
h 20 000 000 + 7 000 000
i 600 000 000 + 70 000 000 + 5 000 000

6 **a** 57 657 **b** 928 678 **c** 3 867 009
d 1 074 612 **e** 800 967 **f** 5 801 657
g 3 860 072 **h** 409 871

7 **a** 50 000 + 9000 + 600 + 70 + 5
b 800 000 + 6000 + 300 + 7
c 9 000 000 + 100 000 + 30 000 + 7000 + 800 + 20 + 6

8 **a** 1688, 9841, 47 341, 63 425, 120 070
b 1903, 24 106, 65 125, 91 000, 100 520
c 9999, 10 112, 86 314, 219 414, 635 188
d 8014, 58 096, 72 143, 88 000, 132 145
e 138 096, 365 214, 938 497, 1 090 040
f 8987, 77 717, 97 312, 123 000, 637 114
g 8914, 9060, 47 314, 100 000, 612 114
h 980 000, 1 500 000, 3 156 214, 5 937 193

Investigation 1:02

1 **a** Count the grains in the thimble. See how many thimbles of sand it takes to fill a cup and how many cupfuls it takes to fill a bucket, then multiply.
b Count the number of numbers in one column, then the number of columns to a page and the number of pages in the book, then multiply.

2 Estimates will vary.

Exercise 1:03

1 **a** 30, 30, 30 **b** 72, 72, 72 **c** 8, 8, 8
d 4, 4, 4

2

×	4	1	3	6	0	10	2	5	9	7	11	8
2	8	2	6	12	0	20	4	10	18	14	22	16
8	32	8	24	48	0	80	16	40	72	56	88	64
4	16	4	12	24	0	40	8	20	36	28	44	32
7	28	7	21	42	0	70	14	35	63	49	77	56

3 **a**

6
7
8
3
6

b

9
7
6
9
7

c

7
3
4
4
8

d

6
9
8
8
7

e

8
7
3
9
9

4 **a** 12 **b** $14 **c** 11 kg **d** 15
5 **a** 156 **b** 409 **c** 1196 **d** 6784
e 86 **f** 1160 **g** 318 **h** 3156
6 **a** 12 **b** 183 **c** 100 **d** 11 **e** 20
f 46 **g** 4 **h** 9
7 **a** 4, 1 **b** 10, no **c** 5, 2 **d** 11
8 **a** 1 **b** 0 **c** 2
9 **a** $2\frac{2}{3}$ **b** $2\frac{3}{4}$ **c** $3\frac{7}{10}$ **d** $6\frac{3}{5}$
e $1\frac{4}{7}$ **f** $\frac{5}{6}$ **g** $2\frac{3}{5}$ **h** $2\frac{2}{3}$
i $121\frac{3}{5}$ **j** $93\frac{3}{4}$ **k** $10\frac{8}{10}$ or $10\frac{4}{5}$
l $84\frac{1}{9}$

10 **a** $108\frac{9}{10}$ **b** 3600 **c** $5108\frac{6}{10}$ or $5108\frac{3}{5}$
d $7105\frac{2}{10}$ or $7105\frac{1}{5}$ **e** $155\frac{1}{7}$ **f** 1827
g 9037 **h** $15\,260\frac{1}{4}$ **i** 6011 **j** 10 207
k $3001\frac{2}{5}$ **l** $30\,720\frac{1}{8}$

11 **a** 7 **b** 15 **c** $9\frac{2}{3}$ **d** 10 **e** 27
f 6 **g** 2 **h** 39

12 **a** $4500 − $2740 = $1760
Sage owes her parents $1760.
b 1760 ÷ 110 = 176 ÷ 11 = 16
It will take Sage another 16 months to repay her parents if she pays $110 per month.

13 **a** 15 × 5 × 6 × 7 = 3150
The family uses 3150 litres in the shower each week.
b 15 × 5 × 4 × 7 = 2100, 3150 − 2100 = 1050
They would save 1050 litres by reducing their shower time to 4 minutes each.

Challenge 1:03

1 **a** 7 **b** 7 **c** 5 r 3 or $5\frac{3}{12}$ or $5\frac{1}{4}$
d 7 r 2 or $7\frac{2}{11}$ **e** 14 r 4 or $14\frac{4}{15}$
f 11 r 9 or $11\frac{9}{17}$ **g** 13 r 4 or $13\frac{4}{18}$ or $13\frac{2}{9}$
h 16 **i** 28 r 3 or $28\frac{3}{23}$
j 23 r 3 or $23\frac{3}{24}$ or $23\frac{1}{8}$ **k** 66
l 60 r 13 or $60\frac{13}{16}$

2 **a** $88 = 2 \times 44$ **b** $708 = 30 \times 23 + 18$
c $182 = 14 \times 13$ **d** $654 = 18 \times 36 + 6$

3 **a** 3 r 3 or $3\frac{3}{31}$ **b** 3 r 11 or $3\frac{11}{39}$
c 4 r 11 or $4\frac{11}{22}$ or $4\frac{1}{2}$ **d** 5 r 17 or $5\frac{17}{18}$
e 3 r 48 or $3\frac{48}{99}$ or $3\frac{16}{33}$ **f** 5 r 46 or $5\frac{46}{47}$
g 5 r 7 or $5\frac{7}{29}$ **h** 4 r 40 or $4\frac{40}{62}$ or $4\frac{20}{31}$

Exercise 1:04

1 **a** 14 **b** 81 **c** 7 **d** 8 **e** 1400
f \$0.65 **g** \$6.45 **h** \$21 **i** 12
j \$4.20

2 **a** 12 **b** 48 **c** 7 **d** 6 **e** 1800
f \$0.15 **g** \$12.60 **h** \$30.60 **i** 10
j \$6.20

3 **a** 13 **b** 54 **c** 3 **d** 7 **e** 1700
f \$0.45 **g** \$5.75 **h** \$17.60 **i** 16
j \$3.10

4 **a** 15 **b** 49 **c** 7 **d** 6 **e** 1300
f \$0.35 **g** \$24.75 **h** \$13.60 **i** 14
j \$5.10

Exercise 1:05

1 **a** 33·125 **b** 7703 **c** 123 **d** 13 687·3
e 7 kg for \$12.50 **f** 17 **g** 138·5 c/L
h 0·4375 **i** 430·142 **j** 1·025

2 **a** 1008·16 **b** 86 589 **c** 458 **d** 5135·2
e 4 kg for \$7.92 **f** 20 **g** 139·0 c/L
h 0·875 **i** 313·07 **j** 2·48

3 **a** 22·63 **b** 11 199 **c** 729 **d** 40 228·8
e 500 g for \$3 **f** 23 **g** 119·5 c/L
h 0·6875 **i** 263·142 **j** 2·3

4 **a** 462·9 **b** 9244 **c** 168 **d** 6417·06
e 450 g for \$2.85 **f** 9 **g** 149·5 c/L
h 0·625 **i** 515·86 **j** 5·4

5 **a** 453·361 **b** 90 616 **c** 78 **d** 44·4626
e 375 g for \$2.37 **f** 8 **g** 167·0 c/L
h 0·5625 **i** 733·45 **j** 6·7

Exercise 1:06A

1 **a** 120 **b** 2 **c** 5 **d** 72 **e** 42
f 48 **g** 20 **h** 7 **i** 10 **j** 27
k 8 **l** 2 **m** 20 **n** 30 **o** 35
p 65 **q** 11 **r** 37 **s** 0 **t** 2
u 73 **v** 32 **w** 0 **x** 1

2 **a** 66 **b** 60 **c** 12 **d** 17 **e** 2
f 5 **g** 42 **h** 30 **i** 12 **j** 60
k 2 **l** 20 **m** 2 **n** 7 **o** 132
p 160 **q** 4500 **r** 560 **s** 3 **t** 10

3 **a** 6 **b** 10 **c** 48 **d** 0 **e** 40
f 20 **g** 66 **h** 20 **i** 40 **j** 4
k 2 **l** 1 **m** 120 **n** 354 **o** 11
p 1003

4 **a** 52 **b** 5 **c** 80 **d** 10 **e** 3
f 6 **g** 4 **h** 8 **i** 123

5 **a** 0·8 **b** 0·03 **c** 3·0 **d** 4·5 **e** 1·6
f 12 **g** 9 **h** 0·4 **i** 1·2 **j** 1
k 2·7 **l** 6 **m** \$3.50 **n** \$2.50
o \$180 **p** \$16.80 **q** \$71.80 **r** \$633.50

6 **a** $(5 \times 3) + 8 = 23$ **b** $5 \times (3 + 8) = 55$
c $10 \times (7 + 5) = 120$ **d** $6 - (3 \times 2) = 0$
e $(6 - 3) \times 2 = 6$ **f** $(10 \times 7) + 5 = 75$
g $40 \div (4 + 1) = 8$ **h** $(40 \div 4) + 1 = 11$
i $80 \div (8 + 2) = 8$ **j** $(29 - 15) - 6 = 8$
k $29 - (15 - 6) = 20$ **l** $18 - (8 + 2) = 8$
m $(40 \div 2) \div 2 = 10$ **n** $40 \div (2 \div 2) = 40$
o $144 \div (6 \div 3) = 72$ **p** $(6 + 2) \times (8 + 2) = 80$
q $6 + (2 \times 8) + 2 = 24$ **r** $(10 \times 3) - (16 - 2) = 16$

Exercise 1:06B

1 **a** 30 **b** 24 **c** 14 **d** 3 **e** 15
f 4 **g** 2 **h** 4 **i** 3 **j** 0 **k** 7
l 3

2 **a** 80 **b** 54 **c** 5 **d** 5 **e** 100
f 33 **g** 16 **h** 7 **i** 112 **j** 80
k 64 **l** 12

3 **a** 22 **b** 10 **c** 0 **d** 2 **e** 30
f 90 **g** 75 **h** 39 **i** 3 **j** 11
k 43 **l** 25

4 **a** 30 **b** 12 **c** 90 **d** 6 **e** 6 **f** 6
g 11 **h** 11 **i** 5 **j** 1 **k** 6 **l** 6
m 4 **n** 72 **o** 47 **p** 54 **q** 61
r 74 **s** 4 **t** 20 **u** 900 **v** 20
w 26 **x** 2

5 **a** 24 **b** 78 **c** 6 **d** 42 **e** \$102
f \$19 **g** 0·1 **h** 8 **i** 78 **j** 196
k 1

Exercise 1:06C

1 **a** 100 **b** 20 **c** 0 **d** 40 **e** 0
f 2 **g** 3 **h** 5 **i** 10 **j** 7928
k 450 **l** 883 736 **m** 61·77 **n** 500 002
o 10 000 **p** 38 **q** 730 **r** 9·8

Exercise 1:07A

1

0	20	36	56
13×0 8×0 $0 \times (8 \times 11)$ $(0 \times 8) \times 11$ 98×0	$(6 + 5) + 9$ $6 + (5 + 9)$ 20×1 1×20 $20 + 0$ $3 + 17$ $17 + 3$	4×9 $(2 \times 6) \times 3$ 9×4 $2 \times (6 \times 3)$ $0 + 36$ $2 \times 9 \times 2$ $2 \times 2 \times 9$	$50 + 6$ $6 + 50$ 1×56 56×1 $39 + 16 + 1$ $39 + 1 + 16$

2 **a** true **b** true **c** true **d** true
e true **f** true **g** true **h** true
i true **j** true **k** true **l** true
m false **n** false **o** false **p** false
q false **r** false **s** false **t** false

3 **a** 0 **b** 416 **c** 0 **d** 365 **e** 1
f any **g** 20 **h** 18 **i** 15 **j** 8
k 7 **l** 9 **m** 0 **n** 0 **o** 0·9
p $\frac{4}{10}$ **q** 6 **r** 0

4 **a** 1342 **b** 2325 **c** 1342 **d** 2325

5 **a** 20 398 **b** 21 840 **c** 88 907

6 **a** no **b** Heather **c** no **d** Rahul
e Phuong **f** Jacqui

7 **a** true **b** true **c** true **d** true
e false **f** false **g** false **h** false
i true **j** true

8 **a** 100 **b** 100 **c** 80 **d** 80 **e** 1000
f 10 **g** $\frac{13}{100}$ **h** 1 **i** 100 **j** 100

9 **a** 0 **b** \$17 **c** \$126 **d** 2453
e 1276 **f** 566 **g** 1800 **h** 35 600
i 6300 **j** 170 **k** 1300 **l** 4800

10 **a** 276 **b** 280 **c** 284

11 **a** yes **b** \$34 **c** same **d** 340

12 **a** true **b** true **c** true **d** true

Prep quiz 1:07B

1 9 **2** 7 **3** 1 **4** 0 **5** 0 **6** 9700
7 120 **8** 40 **9** 160 **10** yes

Exercise 1:07B

1 **a** 495 **b** 160 **2** B

3 **a** true **b** true **c** true **d** true
e true **f** true

4 **a** 3 **b** 3 **c** 11 **d** 9 **e** 2 **f** 6
g 15 **h** 15 **i** 20 **j** 7

5 **a** 495 **b** 693 **c** 194 **d** 882 **e** 570
f 760 **g** 1140 **h** 1710 **i** 7171
j 2448 **k** 856 **l** 540 **m** 3570
n 4242 **o** 1560 **p** 1608 **q** 780
r 490 **s** 4218 **t** 44 100

6 **a** 900 **b** 80 **c** 99 **d** 66 **e** 140
f 800 **g** 480 **h** 700 **i** 120 **j** 24
k 28 **l** 146

7 **a** true **b** true **c** true **d** true

Exercise 1:08

1 **a** 42 **b** 68 **c** 51 **d** 39 **e** 26
f 488 **g** 8 **h** 17 **i** 11 **j** 77

2 **a** $15 < 105$ **b** $8{\cdot}8 \neq 8{\cdot}08$ **c** $0{\cdot}7 = 70\%$
d $5^2 \times 7^3$ **e** $6 \times 9 - 3$ **f** $12 \div 4 + 6$
g $\therefore \sqrt{9} = 3$ **h** i.e. $\sqrt[3]{8} = 2$ **i** $\pi \div 3{\cdot}142$
j $\square \leq 10$ **k** $0{\cdot}3 > 0{\cdot}03$ **l** e.g. $5{\cdot}1 \not< 5{\cdot}09$
m $12 \times 4 \geq 12 \div 4$

3 **a** 0·499 is less than $\frac{1}{2}$
b 3999 is approximately equal to 4000
c 61 000 is not equal to 61 times 100
d 6 plus $\square$ is greater than or equal to 9
e 0·1 is greater than 0·099
f 6 times 199 is equal to 6 times 200 minus 6 times 1
g 9·1 is not less than $9\frac{1}{10}$
h 5·1 plus 4 is equal to 4 plus 5·1
i $\square$ minus 3 is less than or equal to 6

4 **a** true **b** true **c** false **d** true
e false **f** false **g** true **h** true
i true **j** true **k** true **l** false
m true **n** true **o** false

Prep quiz 1:09

1 10, 12, 14 **2** 9, 11, 13 **3** 5, 6, 7
4 25, 36, 49 **5** true **6** true **7** false
8 true **9** true **10** true

Exercise 1:09

1 **a** 7007 **b** 10 001 **c** 973 **d** 10 339

2 **a** 87, 89, 91, 93, 95, 97
b 993, 995, 997, 999, 1001

3 **a** 8148 **b** 112 **c** 3194 **d** 9998

4 **a** 74, 76, 78, 80, 82 **b** 1998, 2000

5 **a** 1, 3, 5, 7, 9 **b** 0, 2, 4, 6, 8

6 **a** 84, 86, 88, 90, 92 **b** 199, 201, 203, 205, 207
c 995, 997, 999, 1001, 1003
d 1018, 1020, 1022, 1024, 1026

7 **a** even **b** odd **c** even **d** even
e odd **f** even **g** odd **h** even
i even **j** even

8

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}
Counting	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Even	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
Odd	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Triangular	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120
Fibonacci	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610

9 **a** 4 **b** 9 **c** 16 **d** 25 **e** 36
f 49 **g** 64 **h** 81

10 **a** true **b** true **c** true **d** true

11

$3^2 + 7$	$9 + 7 = 16 = 4^2$
$4^2 + 9$	$16 + 9 = 25 = 5^2$
$5^2 + 11$	$25 + 11 = 36 = 6^2$
$6^2 + 13$	$36 + 13 = 49 = 7^2$
$7^2 + 15$	$49 + 15 = 64 = 8^2$
$8^2 + 17$	$64 + 17 = 81 = 9^2$
$9^2 + 19$	$81 + 19 = 100 = 10^2$
$10^2 + 21$	$100 + 21 = 121 = 11^2$
$11^2 + 23$	$121 + 23 = 144 = 12^2$

12 **a** 46, 100, 16, 2000, 64, 144, 2, 4, 84, 66, 36, 122
b 37, 15, 25, 81, 33, 111, 1, 49, 77, 9, 71, 73
c 25, 100, 81, 16, 64, 144, 1, 49, 4, 9, 36
d 100, 16, 64, 144, 4, 36

13 **a** 15 **b** yes **c** 1, 3, 6

14 **a** 28 **b** 45

15 **a** 10 **b** yes

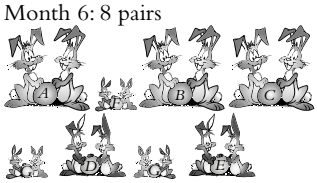
16 1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991, 2002, 2112, 2222, 2332, 2442, 2552, 2662, 2772, 2882, 2992

17 **a** 55, 78, 66, 21, 91, 10, 15 **b** 55, 21, 89, 610
c 55, 66, 707

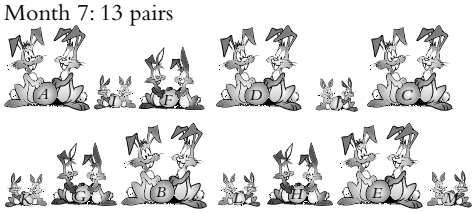
18 Answers will vary.

Fun spot 1:09

1 **a** Month 6: 8 pairs



Month 7: 13 pairs



b yes

2 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610

3 yes **4** yes **5** It is true.

Prep quiz 1:10

1 80 **2** 50 **3** 20 **4** 40
5 400 **6** 900 **7** 100 **8** 300
9 6000 **10** 10 000

Exercise 1:10

E = Estimate, A = Answer, D = Difference

1 **a** E = 1500
A = 1491
D = 9

b E = 7000
A = 7590
D = 590

c E = 100 000
A = 96 792
D = 3208

d E = 9 000 000
A = 9 236 753
D = 236 753

e E = 500
A = 565
D = 65

f E = 5000
A = 5717
D = 717

g E = 1000
A = 518
D = 482

h E = 200 000
A = 212 322
D = 12 322

i E = 12 000
A = 11 439
D = 561

j E = 12 000
A = 10 296
D = 1704

k E = 160 000
A = 156 683·9
D = 3 316·1

l E = 2 100 000
A = 1 951 216·2
D = 148 783·8

m E = 2000
A = 2048
D = 48

n E = 20 000
A = 21 345
D = 1345

o E = 4000
A = 3096
D = 904

p E = 100 000
A = 99 865
D = 135

2 **a** E = 25 000
A = 25 303
D = 303

b E = 88 000
A = 86 045
D = 1955

c E = 630 000 s
A = 604 800 s
D = 25 200 s

d E = $10 000
A = $9120
D = $880

e E = 100
A = 138
D = 38

f E = $50 000
A = $55 451.69
D = $5451.69

g E = $40 000
A = $31 794.29
D = $8205.71

h E = $420 000
A = $394 205
D = $25 795

	Question	Working	Estimate	Answer	Difference
3 a	11 615 + 58 107	10 000 + 60 000	70 000	69 722	278
b	92 094 ÷ 6	90 000 ÷ 6	15 000	15 349	349
c	48 314 − 27 199	50 000 − 30 000	20 000	21 115	1115
d	37 118 × 7	40 000 × 7	280 000	259 826	20 174
e	2 416 809 + 4 369 170	2 000 000 + 4 000 000	6 000 000	6 785 979	785 979
f	567 180 ÷ 5	600 000 ÷ 5	120 000	113 436	6564
g	69 847 × 9	70 000 × 9	630 000	628 623	1377
h	486·76 ÷ 1·72	500 ÷ 2	250	283	33
i	735·15 − 275·35	700 − 300	400	459·8	59·8
j	2315 × 7·8	2000 × 8	16 000	18 057	2057

4 Question **2b** would be better suited to this method.

Check-up 1:11A

1 a 136 **b** 1901 **c** 65 435 **d** 334 975
2 a 454 **b** 2385 **c** 39 015 **d** 453 462
3 a 3954 **b** 20 048 **c** 7890 **d** 91 350
4 a 2496 **b** 177 351 **c** 389 385 **d** 23 650
5 a 110 **b** $112\frac{4}{5}$ **c** $339\frac{1}{7}$ **d** 406
6 a $447\frac{1}{2}$ or 447·5 **b** 107 **c** 17 **d** 1835

Check-up 1:11B

1 a 6 **b** 2 **c** 30
2 a $\frac{4}{5}$ **b** $\frac{3}{4}$ **c** $\frac{2}{5}$
3 a $\frac{1}{10}$ **b** $\frac{6}{10}$ **c** $\frac{2}{5}$
4 a $\frac{1}{10}, \frac{1}{5}, \frac{1}{2}$ **b** $\frac{1}{10}, \frac{3}{10}, \frac{7}{10}$
5 a $\frac{5}{2}$ **b** $\frac{53}{10}$ **c** $\frac{5}{4}$
6 a $1\frac{3}{4}$ **b** $8\frac{7}{10}$ **c** $4\frac{1}{2}$
7 a $\frac{7}{10}$ **b** $\frac{3}{5}$ **c** $1\frac{1}{10}$
8 a $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$
9 a $\frac{3}{4}$ **b** $\frac{8}{9}$ **c** $\frac{7}{10}$
10 a $\frac{7}{10}$ **b** $\frac{1}{6}$ **c** $\frac{3}{16}$
11 a 3 **b** $1\frac{1}{2}$ **c** 4
12 a 1 **b** 9 **c** 20

Check-up 1:11C

1 a 0·7 **b** 2·13 **c** 0·009
2 a $\frac{317}{1000}$ **b** $\frac{59}{100}$ **c** $\frac{1}{2}$
3 a 0·1 **b** 0·11 **c** 0·49
4 a 0·3, 0·8, 1 **b** 0·11, 0·51, 0·91
c 0·12, 0·509, 0·8
5 a 5·5 **b** 3·14 **c** 8·18
6 a 68·25 **b** 13·45 **c** 27·3
7 a 7·95 **b** 4·33 **c** 9·105
8 a 8 **b** 3 **c** 0·543
9 a 16·2 **b** 0·3 **c** 0·041
10 a 0·31 **b** 5 **c** 800
11 a \$5.65 **b** \$8.05 **c** \$7.40
12 a \$14.80 **b** \$65.70 **c** \$202.20
13 a \$6.20 **b** \$46.50 **c** \$3.15
14 a \$18 **b** \$11.10 **c** \$45.25
15 a \$4.50 **b** \$4.05 **c** \$8.75

Check-up 1:11D

1 a $\frac{7}{100}$ **b** $\frac{33}{100}$ **c** $\frac{15}{100}$
2 a $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{3}{20}$
3 a 1 **b** 3 **c** 2
4 a $2\frac{1}{2}$ **b** $1\frac{1}{20}$ **c** $1\frac{1}{4}$
5 a 8% **b** 37% **c** 50%
6 a 15% **b** 14% **c** 70%
7 a 400% **b** 100% **c** 300%
8 a 350% **b** 175% **c** 730%
9 a 0·93 **b** 0·07 **c** 0·1
10 a 1·5 **b** 1·13 **c** 4·25
11 a 7% **b** 85% **c** 90%
12 a 315% **b** 108% **c** 590%

Fun spot 1:11

1 a

8	1	6
3	5	7
4	9	2

b

4	18	8
14	10	6
12	2	16

c

6	11	4
5	7	9
10	3	8

2 a

4	14	12	9	7	17	3	10	5
18	10	2	19	11	3	8	6	4
8	6	16	5	15	13	7	2	9
6	7	2	15	5	7	10	15	14
1	5	9	1	9	17	17	13	9
8	3	4	11	13	3	12	11	16
10	8	18	10	3	8	5	10	9
20	12	4	5	7	9	12	8	4
6	16	14	6	11	4	7	6	11

b Every row, column and diagonal has a sum of 81.

Diagnostic test 1

1 **a** 1023 **b** 1 130 200 **c** 200 312

2 **a**  **b** 

c 

3 **a** 364 **b** 2002 **c** 1999

4 **a** CCCLXXII **b** CMXLVIII

c MMMCDIX

5 **a** 70 000 **b** 70 **c** 7 000 000

6 **a** 13 **b** 26

7 $13.95

8 **a** 28 **b** 31 **c** 10

9 **a** 11 **b** 15 **c** 50

10 **a** $6 \times 900 + 6 \times 8$ **b** $7 \times 600 - 7 \times 3$

c $4 \times 400 - 4 \times 2$

11 **a** 1, 4, 9 **b** 1, 3, 6 **c** 1, 3, 5

12 **a** 1 100 000 **b** 1 600 000 **c** 10 000

ASSIGNMENT 1A

1 **a** is approximately equal to **b** is not equal to

c is less than **d** is greater than **e** per cent

f therefore **g** for example **h** that is

i numerator **j** denominator

2 **1**: $6 - 2$ **2**: $6 + 2$ **3**: $6 \div 2$

4: $6 - 2$ **5**: $6 \div 2$ **6**: 2

7: 6 **8**: 6×2 **9**: $6 - 2$

10: 6×2 **11**: $2 + 6$ **12**: $6 - 2$

13: 6^2 **14**: $\sqrt{36}$ **15**: $6 - 2$

16: 6×2 or 2×6 **17**: $(6 + 2) \div 2$ **18**: $6 + 2$

19: 6^2 **20**: $6 - 2$ **21**: $6 - 2$

22: $6 + 2$ **23**: $6 \div 2$ **24**: $6 + 2$

3 **1**: 10 **2**: 24 **3**: 63 **4**: 60 **5**: 60

6: 18 **7**: 0 **8**: 33 **9**: 100 **10**: 63

4 **a** two hundred and seventy thousand three hundred and seven

b two million four hundred and fifty-six thousand three hundred and twenty-five

c ninety-seven million five thousand

d two hundred and ninety-four million one hundred and sixty-seven thousand

5 **a** 700 **b** 70 000 **c** 7 000 000 **d** 700 000

6 **a** 59 376 **b** 806 545 **c** 3 215 000

7 **a** $(8 \times 1000) + (2 \times 100) + (4 \times 10) + (7 \times 1)$

b $(5 \times 10\,000) + (6 \times 1000) + (1 \times 10) + (5 \times 1)$

c $(7 \times 100\,000) + (1 \times 10\,000) + (4 \times 1000) + (8 \times 100) + (5 \times 10)$

d $(3 \times 1\,000\,000) + (4 \times 100\,000) + (2 \times 10\,000)$

8 **a** 60 000 **b** 50 000 **c** 210 000

9 **a** 12 **b** 24 **c** 16 **d** 14

10 **a** 321 **b** 120 100 **c** 3123 **d** 1 040 101

11 **a** 928 **b** 649 **c** 2600 **d** 20 710

12 **a** CCCXXVII **b** CDXIX **c** MMDLV

d MCMXCI

ASSIGNMENT 1B

1 **a** 4 minutes **b** 5 cats **c** 2 h

d 4 boys and 3 girls are in the family.

e They are both the same distance from Sydney.

2 **a** $3871.50 **b** 8034 **c** 165 800

d $1250 million **e** 83 916 000

f 9 408 240 million km **g** 236·52 km

3 **a** $0.04 **b** $0.11 **c** $1.39 **d** $0.58

e $0.02 **f** $0.02 **g** $27.17 **h** $1.91

i $2.57 **j** $0.00

4 Each rubber band costs less than $\frac{1}{2}$ of one cent.

Chapter 2

Fun spot 2:01

If you want to win, try to make the number of counters remaining after your turn either 29, 22, 15, 8 or 1. From each of these values it is impossible for your opponent to then leave one of these values after his or her choice. After their move you can then move down to the next number until you leave only one counter for your opponent.

Exercise 2:01

1 **a** 15 kg **b** $8.15 **c** 3

d price per kg and total cost **e** $1.63, $24.45

2 **a** total spent **b** 10 **c** $8.50 **d** 14

e $101.60

3 77 **4** $688.65 **5** 191 km **6** 18

7 8 cars, 20 marbles **8** 9 **9** 158 549

10 $83.80

Exercise 2:03

1 13 **2** 12 and 8 **3** 35 **4** 67 and 73

5 3 and 24 **6** 13 and 57 **7** 8 **8** 5

9 **a** 43 and 5 **b** 3 and 45 **c** 5 and 34

10 **a** 13 and 24 **b** 43 and 52 **c** 13 and 54

11 7 **12** 18 **13** 8 **14** $15

15 12, 31, 13, 21 or 21, 22, 22, 12 or 11, 41, 14, 11

16 {9, 10, 42, 51}, {18, 21, 25, 49} **17** 42

18 58 **19** 5 **20** 9, 10, 11 or 12

Exercise 2:04

1 1600 **2** 21 **3** 5

4 (The thickness of the posts has been ignored.)

a 9 **b** 46 m **c** 16 m

5 5 **6** **a** 3 **b** 12 **7** 24 min

8 9 s **9** 4 **10** **a** 7 **b** 9 **c** 10

11 10 **12** 73 **13** 7th **14** 19

15 10 h 20 min

Investigation 2:04

Many answers are acceptable. He could compare the mass and volume of the three crowns with a mass of gold equal to that given to each crown-maker.

Exercise 2:05

1 **a** 4 **b** 12 **2** 10 **3** **a** 3 **b** 4
4 19 **5** **a** 12 **b** 16 **c** 9
6 10 **7** 9 **8** **a** 12 **b** 4 **9** 12
10 42 **11** **a** 12 **b** 36 **12** 260 **13** 24
14 **a** 28 **b** 168 **15** 52
16 20 if coins can be used only once in each group of three (or 49 if coins can be used more than once)

Exercise 2:06

1 7 **2** 15 **3** 100 **4** 37 **5** D **6** C
7 Kylie Conway, Bronwyn Wilkes, Jennifer Collison
8 G

Exercise 2:07

1 25 **2** 49 kg **3** 42 **4** $156.60 **5** 20
6 $75 000

Challenge 2:07

1 74 **2** 45 **3** 53 **4** 51 **5** 13
6 27 **7** 10 **8** 70

Exercise 2:08

1 **a** 2 **b** 5 **c** 7 **2** 45 cm **3** 14
4 6 **5** 28
6 two children across, one child back, one adult across, other child back, two children across, one child back, second adult across, other child back, two children across

Exercise 2:09

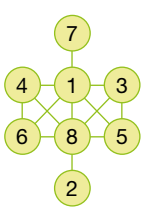
1 **a** 29 **b** 50 **c** $\frac{1}{2}$ **d** 8 **e** 70
2 **a** 80 200 **b** 500 500 **c** 5050 **d** 1275
3 **a** 0 **b** 1 **c** 2 **d** 3
e 4 **f** 5 **g** 6 **h** 7
4 **a** 1 **b** 3 **c** 6 **d** 10 **e** 15 **f** 21
5

Number of people	2	3	4	5	6	7
Number of handshakes	1	3	6	10	15	21

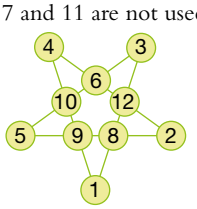
The difference between the number of handshakes is increasing by 1 each time, 190

Fun spot 2:09

1



2 7 and 11 are not used



Exercise 2:10

1 **a** 9 m **b** 18 m **c** 90 m **d** 900 m
e 1800 m
2 less than 45 m
3 **a** 56 **b** 28
4 **a** 1 **b** 2 **c** 3 **d** 4 **e** 97
5 45 bags of gold were stolen if prospectors own the gold they dig, 54 were stolen if they are working for someone else
6 35 bags of gold were stolen (40 if they were working for someone else)
7 **a** 7 **b** 12 **c** 17 **d** 22 **e** 27
8 5, 502

Investigation 2:10

1 **a** $8.00 **b** $13.85 **c** $6.05 **d** $11.05
e $22.00 **f** $4.75
2 Answers will vary.

ASSIGNMENT 2A

1 $296.50 **2** 11 bicycles and 5 tricycles
3 1600 ha **4** 6
5 Maja Sze, Rohan Pieris, Dennis Chan
6 Diane is 19 **7** **a** 16 **b** 71

ASSIGNMENT 2B

1 **a** square **b** rectangle **c** parallelogram
d rhombus (or diamond) **e** trapezium
f regular pentagon **g** regular hexagon
h regular octagon **i** kite **j** circle
2 **a** 10, 24, 63, 60, 60, 18, 0, 33, 100, 63
b 69, 54, 36, 106, 107, 36, 63, 76, 93, 61
3 5·6 m **4** 7200
5 You could count the number of households represented on one page of the phone directory (or one page of search results online) and then multiply by the number of pages in the directory (or pages of search results offerred).
6 1 v 2, 3 v 4, 5 v 6
1 v 3, 2 v 5, 4 v 6
1 v 4, 2 v 6, 3 v 5
1 v 5, 2 v 4, 3 v 6
1 v 6, 2 v 3, 4 v 5
7 90 tickets
8 **a** 9 am and 10 am **b** 250 cm **c** 3 pm
d 8 pm **e** water to a depth of 225 cm

ASSIGNMENT 2C

1 **a** 10 **b** 1000 **c** 1000 **d** 100 000
2 **a** 5410 **b** 1763 **c** 180 **d** 53
3 **a** $\frac{7}{10}$ **b** $\frac{1}{100}$ **c** $\frac{201}{1000}$ **d** $\frac{33}{1000}$
4 **a** 0·3 **b** 0·17 **c** 0·045 **d** 1·03
5 **a** 0·71 **b** 7·37 **c** 13·5 **d** 20·6
6 **a** 8·3 **b** 3·2 **c** 1·25 **d** 8·17
7 **a** 0·6 **b** 0·06 **c** 6·0 **d** 0·06

8 **a** 6·3 **b** 0·52 **c** 1·1 **d** 4·31
9 **a** $5.30 **b** $7.10 **c** $295.40 **d** $4.80
10 **a** $1\frac{1}{5}$ **b** $\frac{79}{100}$ **c** $\frac{2}{5}$ **d** 0
11 **a** 8 **b** 22 **c** 5 **d** 140
12 **a** $6 \times (5 + 3) = 48$ **b** $(10 - 3) \times 3 = 21$
c $4 - (3 + 1) = 0$ **d** $9 + 8 - (1 + 1) = 15$
13 **a** 2 **b** 4
14 **a** is approximately equal to
b is less than or equal to
c is not equal to **d** per cent
e therefore **e** Before Common Era
15 **a** 5, 7, 9, 11 **b** 1, 4, 9, 16, 25, 36, 49
16 Answers will vary. The 4-digit number should be the same if read forwards or backwards, e.g. 1991.

Chapter 3

Prep quiz 3:01

1 27 **2** 25 **3** 32 **4** 125 **5** 100
6 1000 **7** 10 000 **8** 100 000 **9** 700
10 4000

Exercise 3:01

1 **a** $4 \times 4 = 16$ **b** $10 \times 10 = 100$
c $2 \times 2 = 4$ **d** $1 \times 1 = 1$ **e** $7 \times 7 = 49$
f $9 \times 9 = 81$ **g** $2 \times 2 \times 2 = 8$
h $5 \times 5 \times 5 \times 5 = 625$ **i** $3 \times 3 \times 3 = 27$
j $10 \times 10 \times 10 = 1000$
k $10 \times 10 \times 10 \times 10 = 10\,000$
l $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$
2 **a** 7^2 **b** 8^3 **c** 10^4 **d** 10^3 **e** 4^6
f 2^4 **g** 10^5
3 **a** 60 **b** 300 **c** 5000 **d** 20 000
e 7000 **f** 100 **g** 4000 **h** 900
i 80 000 **j** 9000 **k** 80 000 **l** 4 900 000
4 **a** $2^3 \times 3^2$ **b** $5^2 \times 2^3$ **c** $4^2 \times 3^4$
d $6^3 \times 7^2$ **e** $5^3 \times 3^4$ **f** $8^3 \times 10^2$
g $2^3 \times 3^3$ **h** $5^3 \times 6^2$ **i** $4^4 \times 5^1$ **j** $3^3 \times 7^3$
5 **a** $2 \times 2 \times 3 \times 3$; 36
b $5 \times 5 \times 3 \times 3$; 225
c $4 \times 4 \times 2 \times 2 \times 2$; 128
d $3 \times 3 \times 3 \times 2 \times 2$; 108
e $3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2$; 288
f $5 \times 5 \times 5 \times 2 \times 2 \times 2$; 1000
g $4 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2$; 2304
h $2 \times 2 \times 2 \times 3 \times 3 \times 4$; 288
6 **a** $2 \times 2 \times 2 \times 2 \times 2$; 2^5
b $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; 3^7
c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$; 5^8
d $4 \times 4 \times 4 \times 4 \times 4 \times 4$; 4^6
(When multiplying, add the powers.)
7 **a** 36, 6^2 **b** 100, 10^2 **c** 64, 8^2 or 4^3 or 2^6
d 1000, 10^3
(If powers are the same, then base numbers can be multiplied together.)
8 **a** 13 **b** 13 **c** 5 **d** 35 **e** 35
f 19 **g** 42 **h** 57 **i** 7 **j** 92
k 116 **l** 8 **m** 404 **n** 80 **o** 175
9 **a** 32 **b** 81 **c** 256 **d** 512 **e** 2187
f 15 625 **g** 1296 **h** 16 807 **i** 6561
j 1728 **k** 160 000 **l** 1 889 568 **m** 189
n 13 **o** 1072 **p** 375 **q** 704 **r** 100
s 2101 **t** 20 000 **u** 227 880 **v** 29 936

Prep quiz 3:02

1 100 **2** 500 **3** 800 **4** 1000 **5** 2000
6 9000 **7** 63 **8** 251 **9** 7935 **10** 8132

Exercise 3:02

1 **a** 6459 **b** 28 630 **c** 900 874
d 4 380 000 **e** 74 586 **f** 1 101 010
g 5 060 842 **h** 48 390
2 **a** 8539 **b** 738 **c** 7304 **d** 9067
e 9003 **f** 8210 **g** 6 819 346
h 1 402 075 **i** 3 094 363 **j** 6 900 000
3 **a** 1 thousand + 3 hundred + 6 tens
b 5 ten-thousand + 3 thousand + 2 hundred
c 1 million + 6 hundred-thousand
d 9 thousand + 3 hundred + 8 tens + 6 units
e 4 million + 3 hundred-thousand + 7 thousand + 2 hundred
f 8 hundred-thousand + 3 hundred + 5 units
g 1 thousand + 9 hundreds + 8 tens + 9 units
h 7 million + 3 hundred-thousand + 4 ten-thousand + 5 thousand + 8 hundred + 3 tens + 1 unit
4 **a** $(6 \times 1000) + (7 \times 100) + (0 \times 10) + (9 \times 1)$
b $(1 \times 10\,000) + (3 \times 1000) + (3 \times 100) + (0 \times 10) + (4 \times 1)$
c $(9 \times 1000) + (0 \times 100) + (8 \times 10) + (4 \times 1)$
d $(1 \times 100\,000) + (0 \times 10\,000) + (3 \times 1000) + (2 \times 100) + (4 \times 10) + (3 \times 1)$
e $(9 \times 10\,000) + (7 \times 1000) + (6 \times 100) + (7 \times 10) + (0 \times 1)$
f $(2 \times 100\,000) + (0 \times 10\,000) + (7 \times 1000) + (1 \times 100) + (1 \times 10) + (6 \times 1)$
g $(1 \times 1\,000\,000) + (3 \times 100\,000) + (4 \times 10\,000) + (4 \times 1000) + (5 \times 100) + (6 \times 10) + (2 \times 1)$
h $(9 \times 1\,000\,000) + (7 \times 100\,000) + (6 \times 10\,000) + (5 \times 1000) + (8 \times 100) + (1 \times 10) + (3 \times 1)$
5 **a** $(8 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (9 \times 1)$
b $(7 \times 10^2) + (3 \times 10^1) + (8 \times 1)$
c $(7 \times 10^3) + (3 \times 10^2) + (0 \times 10^1) + (4 \times 1)$
d $(9 \times 10^3) + (0 \times 10^2) + (6 \times 10^1) + (7 \times 1)$
e $(9 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (3 \times 1)$
f $(8 \times 10^3) + (2 \times 10^2) + (1 \times 10^1) + (0 \times 1)$
g $(6 \times 10^6) + (8 \times 10^5) + (1 \times 10^4) + (9 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + (6 \times 1)$
h $(1 \times 10^6) + (4 \times 10^5) + (0 \times 10^4) + (2 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) + (5 \times 1)$
i $(3 \times 10^6) + (0 \times 10^5) + (9 \times 10^4) + (4 \times 10^3) + (3 \times 10^2) + (6 \times 10^1) + (3 \times 1)$

j $(6 \times 10^6) + (9 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (0 \times 1)$

6 a $(1 \times 10^3) + (3 \times 10^2) + (6 \times 10^1) + (0 \times 1)$

b $(5 \times 10^4) + (3 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (0 \times 1)$

c $(1 \times 10^6) + (6 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (0 \times 1)$

d $(9 \times 10^3) + (3 \times 10^2) + (8 \times 10^1) + (6 \times 1)$

e $(4 \times 10^6) + (3 \times 10^5) + (0 \times 10^4) + (7 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (0 \times 1)$

f $(8 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (3 \times 10^2) + (0 \times 10^1) + (5 \times 1)$

g $(1 \times 10^3) + (9 \times 10^2) + (8 \times 10^1) + (9 \times 1)$

h $(7 \times 10^6) + (3 \times 10^5) + (4 \times 10^4) + (5 \times 10^3) + (8 \times 10^2) + (3 \times 10^1) + (1 \times 1)$

Prep quiz 3:03

1 7 **2** 1 **3** 6 **4** 42

5 1, 2, 3, 6, 7, 14, 21, 42

6 36 **7** 36 **8** 36 **9** 36 **10** 36

Exercise 3:03

1

Factor	24	12	8	6	5	10	20	1	2	3	3	2	1	5	25

2 a 1, 2, 3, 4, 6, 8, 12, 24 **b** 1, 2, 4, 5, 10, 20

c 1, 2, 3, 4, 6, 12 **d** 1, 2, 3, 6, 9, 18 **e** 1, 5, 25

3 a 1, 2, 5, 10 **b** 1, 2, 4, 8 **c** 1, 3, 5, 15

d 1, 3, 9 **e** 1, 7

f 1, 2, 4, 5, 10, 20, 25, 50, 100

g 1, 3, 9, 27 **h** 1, 3, 5, 9, 15, 45

i 1, 2, 3, 4, 6, 9, 12, 18, 36

j 1, 2, 4, 5, 8, 10, 20, 40

k 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

l 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160

m 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132

n 1, 11, 121 **o** 1, 2, 4, 8, 19, 38, 76, 152

4 a false **b** true **c** true **d** true

e true **f** true **g** true **h** true

i true **j** false

5 a 3, 6, 9, 12, 15, 18, 21, 24, 27

b 5, 10, 15, 20, 25, 30, 35, 40, 45

c 2, 4, 6, 8, 10, 12, 14, 16, 18

d 10, 20, 30, 40, 50, 60, 70, 80, 90

e 4, 8, 12, 16, 20, 24, 28, 32, 36

f 11, 22, 33, 44, 55, 66, 77, 88, 99

g 6, 12, 18, 24, 30, 36, 42, 48, 54

h 9, 18, 27, 36, 45, 54, 63, 72, 81

i 7, 14, 21, 28, 35, 42, 49, 56, 63

j 8, 16, 24, 32, 40, 48, 56, 64, 72

6 a 12 **b** 45 **c** 116 **d** 15

7 a 2 **b** 15 **c** 20 **d** 14 **e** 6

f 50 **g** 48 **h** 1

8 a 35 **b** 60 **c** 30 **d** 150

9 a 12 **b** 200 **c** 36 **d** 28 **e** 40

f 120 **g** 40 **h** 66

10 a 65, 70, 75, 80, 85 **b** 1, 2, 4, 5, 8, 10, 20, 40

c 2, 4, 6, 8, 10, 12, 14, 16, 18 **d** 30, 60, 90

e 24, 30, 36, 42, 48

Prep quiz 3:04

1 48 **2** 48 **3** 48 **4** 48

5 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 **6** 6, 12, 18, 24, 30

7 6, 12, 18, 24, 30 **8** 1, 30, 2, 15, 3, 10, 5, 6

9 1, 48, 2, 24, 3, 16, 4, 12, 6, 8 **10** 1182

Exercise 3:04

1

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 4, 2
5	1, 5
6	1, 6, 2, 3
7	1, 7
8	1, 8, 2, 4
9	1, 9, 3
10	1, 10, 2, 5

Number	Factors
11	1, 11
12	1, 12, 2, 6, 3, 4
13	1, 13
14	1, 14, 2, 7
15	1, 15, 3, 5
16	1, 16, 2, 8, 4
17	1, 17
18	1, 18, 2, 9, 3, 6
19	1, 19
20	1, 20, 2, 10, 4, 5

a 2, 3, 5, 7, 11, 13, 17, 19

b 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113

3 a 2 **b** 7 **c** 9, 15 **d** 15

e $6 = 3 + 3$, $8 = 5 + 3$, $10 = 5 + 5$, $12 = 5 + 7$, $14 = 7 + 7$, $16 = 5 + 11$, $18 = 7 + 11$

4 a $3 \times 2 \times 2$ **b** $2 \times 2 \times 5$ **c** $2 \times 3 \times 3$

d $2 \times 2 \times 2 \times 2$

5 a $2 \times 3 \times 5$ **b** $2 \times 2 \times 3 \times 3$

c $2 \times 2 \times 5 \times 5$ **d** $2 \times 2 \times 2 \times 2 \times 3 \times 3$

e $2 \times 2 \times 2 \times 2 \times 2 \times 3$ **f** $3 \times 3 \times 5 \times 5$

6 a $2^3 \times 3^3$ **b** $3^3 \times 5^2$ **c** $3 \times 5 \times 7^2$

d $2^3 \times 3^2 \times 7$ **e** $3^2 \times 5^3$ **f** $2^4 \times 7 \times 11$

g $3^3 \times 7^2$ **h** $3^3 \times 5^3$

Prep quiz 3:05

1 1, 2, 4, 5, 8, 10, 20, 40 **2** 70 **3** 18, 24, 30

4 998 **5** 1001 **6** 51 **7** 5 **8** 255

9 28 983 **10** 11 344 678

Exercise 3:05

1 3842, 5816, 9000, 8774, 8166, 7008

2 3006, 7110, 21 441, 8145, 211 002, 78

3 1004, 6124, 8156, 61 852

4 30 024, 81 810, 41 238, 765, 936

5 6633, 10 406, 92 180, 61 809

6 a 4, 5, 8, 11 **b** 11 **c** 4, 11

d 3, 5, 9, 11 **e** 3, 11

7 **a** 1078, 7600, 13 476 **b** 13 476, 33 885
c 7600, 13 476 **d** 13 476 **e** 33 885
f 7600 **g** 1078 **h** 7600 **i** 7600
j 7600

8 Test for divisibility of both 4 and 6.

9 **a** 3 and 5 **b** 2 and 9 **c** 3 and 4

10 **a** 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720
b 1, 2, 173, 346
c 1, 2, 3, 5, 6, 9, 10, 15, 18, 27, 30, 45, 54, 90, 135, 270
d 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 28, 30, 35, 36, 42, 45, 60, 63, 70, 84, 90, 105, 126, 140, 180, 210, 252, 315, 420, 630, 1260
e 1, 3, 7, 9, 21, 27, 63, 81, 189, 567
f 1, 3, 5, 7, 9, 15, 21, 25, 35, 45, 63, 75, 105, 175, 225, 315, 525, 1575
g 1, 3, 9, 27, 81, 243, 729
h 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 15, 16, 20, 22, 24, 30, 32, 33, 40, 44, 48, 55, 60, 66, 80, 88, 96, 110, 120, 132, 160, 165, 176, 220, 240, 264, 330, 352, 440, 480, 528, 660, 880, 1056, 1320, 1760, 2640, 5280

11 **a** 1002 **b** 1008 **c** 1008 **d** 1001
e 1025 **f** 1100 **g** 1005 **h** 1008

12 **a** 990 **b** 162 **c** 900
d 1, 3, 5, 7, 15, 21, 25, 35, 75, 105, 175, 525
e 1, 3, 9, 11, 27, 33, 99, 297

Prep quiz 3:06

1 1, 2, 3, 5, 6, 10, 15, 30 **2** 1, 2, 3, 6, 9, 18, 27, 54
3 6 **4** 4, 8, 12, 16, 20, 24, 28, 32, 36
5 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39
6 12 **7** $2 \times 2 \times 2 \times 2 \times 3$ **8** 2^5 **9** $2^3 \times 3^4$
10 $3^2 \times 5^3 \times 7$

Exercise 3:06

1 **a** **i** 36 **ii** 54 **iii** 70 **iv** 980 **v** 5
vi 3 **vii** 392 **viii** 392 **ix** 135
x 1
b **i** 1296 **ii** 11 340 **iii** 132 300
iv 9800 **v** 1 323 000 **vi** 9 261 000
vii 32 928 **viii** 41 160 **ix** 47 250
x 5 292 000

2

Number	Products of prime factors	
18	$2 \times 3 \times 3$	2×3^2
36	$2 \times 2 \times 3 \times 3$	$2^2 \times 3^2$
24	$2 \times 2 \times 2 \times 3$	$2^3 \times 3$
300	$2 \times 2 \times 3 \times 5 \times 5$	$2^2 \times 3 \times 5^2$
1050	$2 \times 3 \times 5 \times 5 \times 7$	$2 \times 3 \times 5^2 \times 7$
1250	$2 \times 5 \times 5 \times 5 \times 5$	2×5^4
2475	$3 \times 3 \times 5 \times 5 \times 11$	$3^2 \times 5^2 \times 11$
2310	$2 \times 3 \times 5 \times 7 \times 11$	$2 \times 3 \times 5 \times 7 \times 11$

a **i** 18 **ii** 12 **iii** 12 **iv** 150 **v** 50
vi 25 **vii** 165 **viii** 75 **ix** 10 **x** 3
b **i** 36 **ii** 72 **iii** 600 **iv** 2100
v 26 250 **vi** 123 750 **vii** 34 650
viii 34 650 **ix** 288 750 **x** 19 800

3 **a** 231 **b** 10 395 **c** 66 **d** 5544

4 **a** 1512 s, or 25 min 12 s **b** 216 min, or 3 h 36 min
c $1848 ($924 for chairs and $924 for tables)

Challenge 3:06

$1001 = 7 \times 11 \times 13$, $10\,001 = 73 \times 137$,
$100\,001 = 11 \times 9091$

Prep quiz 3:07

1 25 **2** 16 **3** 196 **4** 8 **5** 27
6 64 **7** 6 **8** 10 **9** 42 **10** 100

Exercise 3:07

1 **a** 7 **b** 11 **c** 13 **d** 5 **e** 9 **f** 12
g 16 **h** 21 **i** 14 **j** 7 **k** 4 **l** 9
m 18 **n** 21 **o** 99 **p** 30 **q** 1001 **r** 275

2 **a** 18 **b** 15 **c** 45 **d** 385 **e** 140 **f** 30

3 **a** 15 **b** 22 **c** 6 **d** 4 **e** 9 **f** 10

4 **a** 3 **b** 1 **c** 2 **d** 4 **e** 8 **f** 6
g 0 **h** 10 **i** 9 **j** 12 **k** 18 **l** 20
m 36 **n** 42 **o** 15

5 **a** 2 **b** 3 **c** 1 **d** 4 **e** 10 **f** 0
g 6 **h** 8 **i** 11 **j** 15 **k** 16 **l** 18

6 **a** **i** 6 **ii** 6 **b** **i** 12 **ii** 12
c **i** 10 **ii** 10 **d** **i** 30 **ii** 30
The pattern is the rule shown, $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

7 **a** 4 **b** 6 **c** 10 **d** 20

8 **a** 7 **b** 13 **c** 1 **d** 13 **e** 17 **f** 2
g 15 **h** 9 **i** 1 **j** 2 **k** 1 **l** 2

9 **a** 3, 4 **b** 8, 9 **c** 4, 5 **d** 6, 7
e 9, 10 **f** 1, 2 **g** 13, 14 **h** 5, 6
i 11, 12 **j** 4, 5 **k** 12, 13 **l** 10, 11

10 **a** 16 **b** 18 **c** 26 **d** 39 **e** 33 **f** 22

11 **a** 5 **b** 9 **c** 11 **d** 20 **e** 17 **f** 12

Prep quiz 3:08

1 $(3 \times 10^4) + (1 \times 10^3) + (6 \times 10^2) + (2 \times 10^1) + (7 \times 1)$
2 $(1 \times 10^4) + (5 \times 10^3) + (0 \times 10^2) + (3 \times 10^1) + (8 \times 1)$
3 63 789 **4** 10 520 **5** 2, 4, 8, 16, 32, 64
6 13 **7** 13 **8** 30 **9** 30 **10** 20

Exercise 3:08

1 **a** 2 **b** 3 **c** 4 **d** 5 **e** 6 **f** 7
g 8 **h** 9 **i** 10 **j** 11 **k** 45 **l** 24
m 35 **n** 63 **o** 13

2 **a** 10001 **b** 10100 **c** 1000000
d 100011 **e** 101000

3

Numeral (base ten)	2^6 ×64	2^5 ×32	2^4 ×16	2^3 ×8	2^2 ×4	2^1 ×2	1 ×1	Numeral (base two)
81	1	0	1	0	0	0	1	1010001_{two}
107	1	1	0	1	0	1	1	1101011_{two}
71	1	0	0	0	1	1	1	1000111_{two}
80	1	0	1	0	0	0	0	1010000_{two}
100	1	1	0	0	1	0	0	1100100_{two}
97	1	1	0	0	0	0	1	1100001_{two}
125	1	1	1	1	1	0	1	1111101_{two}

Fun spot 3:08

1 a

4	9	8
11	7	3
6	5	10

b

10	35	30
45	25	5
20	15	40

c

33	18	21
12	24	36
27	30	15

2 a 21 **b** 75 **c** 72 **3** 4096

2	64	32
256	16	1
8	4	128

Diagnostic test 3

1 a 8^3 **b** 3^5 **c** 2^4
2 a 800 **b** 20 000 **c** 7 000 000
3 a 7228 **b** 5080
4 a $(1 \times 10^3) + (8 \times 10^2) + (2 \times 10^1) + (4 \times 1)$
b $(4 \times 10^3) + (3 \times 10^2) + (0 \times 10^1) + (7 \times 1)$
c $(2 \times 10^6) + (4 \times 10^5) + (1 \times 10^4) + (5 \times 10^3) + (2 \times 10^2) + (8 \times 10^1) + (6 \times 1)$
5 a 1, 2, 3, 4, 6, 8, 12, 24 **b** 1, 3, 7, 9, 21, 63
c 1, 2, 4, 5, 10, 20, 25, 50, 100
6 a 4, 8, 12, 16, 20 **b** 8, 16, 24, 32, 40
c 11, 22, 33, 44, 55
7 a 12 **b** 15 **c** 14
8 a 36 **b** 24 **c** 140
9 a 2, 3, 5, 7 **b** 4, 6, 8, 9 **c** 1
10 a $2 \times 2 \times 2 \times 3$ **b** $2 \times 3 \times 2 \times 3$
c $7 \times 2 \times 3$
11 a HCF = 4, LCM = 120
b HCF = 105, LCM = 4725
c HCF = 15, LCM = 630
12 a 5 **b** 77 **c** 12
13 a 3 **b** 11 **c** 6

ASSIGNMENT 3A

1 a 5^4 **b** $3^2 \times 2^3$ **c** $4^2 \times 7^3$
2 a i 4000 **ii** 50 000
b i 54 000 **ii** 200 000 000
3 a 5279 **b** 80 603
4 a 1, 2, 4, 7, 14, 28
b 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
c 1, 2, 4, 17, 34, 68
5 a 99 **b** 98 **c** 90
6 17 and 19, 29 and 31, 59 and 61, 71 and 73
7 a i $2 \times 2 \times 2 \times 7$ **ii** $2 \times 2 \times 3 \times 7$
b 28 **c** 168
8 a 11 **b** 12 **c** 4 **d** 35

ASSIGNMENT 3B

1 270 **2** 9 **3** \$48; \$18 **4** 4 **5** 20

ASSIGNMENT 3C

1 a 3 **b** 4 **c** 63
d 29 **e** 48 **f** 17
2 15 700
3 a 15 072 **b** 16 328 **c** 13 816
4 a true **b** true **c** true **d** true
e true **f** false
5 a 1846 **b** 0 **c** 8145 **d** 446 **e** 0
f 4186 **g** 11 **h** 9800 **i** 3700
6 a false **b** false **c** true **d** false
e true **f** true **g** true **h** false **i** true
7 a 26 **b** 19 **c** 33 **d** 5 **e** 4 **f** 2
8 a 1, 100, 2, 50, 4, 25, 5, 20, 10 **b** 1, 125, 5, 25
c 1, 5, 25
9 a 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
b 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 **c** 24
10 a 2, 5, 7, 41 **b** 12, 15 36

Chapter 4

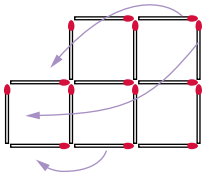
Prep quiz 4:01

1 8 **2** 13 **3** 10 **4** 16 **5** 20
6 14 **7** 32 **8** 1 **9** 2 **10** 1·9

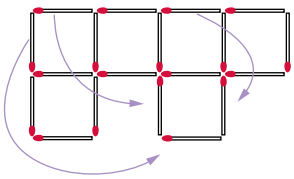
Exercise 4:01

1 a 9, 13 **b** 18, 29 **c** 21, 30 **d** 17, 8
e 24, 11 **f** 30, 15 **g** 9, 27 **h** 36, 144
i 21, 63 **j** 50, 25 **k** 100, 20 **l** 27, 9
m 7, 15 **n** 19, 61 **o** 5, 17 **p** 41, 21
q 35, 17 **r** 9, 81
2 a 15, +4 **b** 23, +6 **c** 19, +6 **d** 5, − 5
e 16, − 7 **f** 11, − 2 **g** 16, ×2 **h** 27, ×3
i 1000, ×10 **j** 10, ÷2 **k** 3, ÷3 **l** 1, ÷4
3 a 55, 70, 85 **b** 49, 60, 71 **c** 105, 128, 151
d 34, 20, 6 **e** 63, 45, 27 **f** 67, 39, 11
g 64, 128, 256 **h** 264, 528, 1056
i 270, 810, 2430 **j** 15, 21, 28 **k** 36, 49, 64
l 16, 10, 5
4 a 14 **b** 17 **c** 35 **d** 38 **e** 15
f 21 **g** 6 **h** 12 **i** 34 **j** 12
k 50 **l** 40
5 a 3, 6, 9, 12 **b** 3, 7, 11, 15 **c** 4, 8, 12, 16
d 4, 7, 10, 13 **e** 4, 6, 8, 10 **f** 6, 11, 16, 21
6 a 1, 3, 6, 10 **b** 1, 4, 9, 16 **7** 14, 30, 55

Fun spot 4:01

1 

2 IV + I = V

3 

4 II + II = IV or III + I = IV

Prep quiz 4:02

1 7 2 8 3 22 4 14 5 $15\,m^2$
6 $40\,cm^2$ 7 $16\,cm^2$ 8 $21\,m^2$ 9 $4
10 $60

Exercise 4:02

1 a $\square = 7$ b $\square = 18$ c $\square = 4$ d $\square = 5$
e $\square = 21$ f $\square = 3$ g $x = 5$ h $x = 15$
i $x = 11$ j $y = 8$ k $y = 3$ l $y = 2$
m $a = 80$ n $b = 12$ o $h = 7$

2 a area = $60\,m^2$ b area = $15\,m^2$
c area = $55\,cm^2$

3 a area = $20\,m^2$ b area = $36\,m^2$
c area = $24\,cm^2$

4 a perimeter = 15 m b perimeter = 25 cm
c perimeter = 25 m d perimeter = 15·54 m

5 a 70 b 32 c 600 d 72 e 2
f 6 g 82 h 408 i 310 j 130
k 28 l 82 m 31 n 230 o 540

6 a

l	2	2·5	3	3·5	4	10	12·5	17	26
P	8	10	12	14	16	40	50	68	104

b

d	8	10	12	20	33	1·5	4·8	7·4	100
C	19·3	23·3	27·3	43·3	69·3	6·3	12·9	18·1	203·3

(*Note:* Answers could have a zero at the end as they represent dollars.)

c

a	1	2	10	5	11
b	1	2	3	8	15
p	1	2	2	3	9
C	385	770	1630	2225	4715

(*Note:* The cost C is shown in cents.)

Prep quiz 4:03

1 23, 29 2 12, 6 3 15, 18 4 10, 5
5 25, 36 6 8, 4 7 9 8 7 9 4
10 20

Exercise 4:03

1 a $4 b $6 c $20

2 a 24 (legs) b 32 (legs) c 40 (legs)

3 a cost = number × 80 cents or $C = N \times 80c$
b $7.20

4 a

Figure number (F)	1	2	3	4
Number of tiles (T)	2	4	6	8

b number of tiles = (figure number) × 2 or $N = F \times 2$ or $N = 2 \times F$
c 30

5 a number of matches = (number of squares) × 4 or $M = S \times 4$ or $M = 4 \times S$

b

Number of squares (S)	1	2	3	4	5	6	7	8
Number of matches (M)	4	8	12	16	20	24	28	32

6 a 3
b number of matches = 1 + (number of squares) × 3 or $M = 1 + S \times 3$ or $M = 1 + 3 \times S$

c

Number of squares (S)	1	2	3	4	5	6	7	8
Number of matches (M)	4	7	10	13	16	19	22	25

7 a

x	0	1	2	3
y	3	4	5	6

b

a	1	2	3	4
y	3	6	9	12

c

p	1	3	5	7
q	8	10	12	14

d

k	2	4	6	8
$k \times 5$	10	20	30	40

e

n	8	9	10	11
$n - 5$	3	4	5	6

f

t	1	2	3	4
$10 - t$	9	8	7	6

8 a $B = 5 \times A$ b $B = A + 6$ c $B = A + 3$
d $B = 2 \times A$ e $B = 2 \times A + 1$
f $B = 3 \times A - 1$ g $B = A - 2$
h $B = 2 \times A + 1$

9 a

Number of bicycles (B)	1	2	3	4	5	6
Number of wheels (W)	5	7	9	11	13	15

b $W = 2 \times B + 3$,
(number of wheels = 2 × number of bicycles + 3)
c No. The bicycles have an even number of wheels. When we add 3, for the unicycles, the answer must be an odd number.

10 a $N = 2 \times L + 8$ would be the simplest rule. Others are possible.
b i 18 squares ii 68 squares c yes

Prep quiz 4:04

1 9 2 0 3 12 4 16 5 2 6 35
7 17 8 7 9 2 10 67

Exercise 4:04

1 a 3×5 or 15 b $3 \times n$ or $3n$ c $3n$
d because 3×5 is not equal to 35 e dt
f $5 \times a \times b$ g $\frac{q}{3}$ h $m \div 2$ or half of m
i $34y = 34 \times y$, $3 \times 4 \times y = 12y$

2 **a** $7a$ **b** $10x$ **c** $2p$ **d** $8q$ **e** $9b$
f $6m$ **g** $3r$ **h** $12c$ **i** $9m$ **j** $5n$
k $4y$ **l** $12t$ **m** a **n** x **o** n
p b **q** y **r** m **s** $5(a+2)$
t $6(x+7)$ **u** $2(3x-2)$ **v** $5(m-n)$
w $5+2q$ **x** $n-10$ **y** $4a+5b$ **z** $7q-6p$

3 **a** $\frac{x}{2}$ **b** $\frac{m}{4}$ **c** $\frac{b}{3}$ **d** $\frac{n}{5}$ **e** $\frac{a}{1}$
f $\frac{y}{10}$ **g** $\frac{t}{8}$ **h** $\frac{k}{100}$ **i** $\frac{2b}{3}$ **j** $\frac{3x}{5}$
k $\frac{5m}{6}$ **l** $\frac{3a}{4}$ **m** $\frac{a}{b}$ **n** $\frac{6}{n}$ **o** $\frac{2a}{x}$
p $\frac{10}{a}$ **q** $\frac{a+1}{4}$ **r** $\frac{8-x}{5}$ **s** $\frac{7a}{8b}$ **t** $\frac{4x}{3y}$

4 **a** $5ab$ **b** $4mn$ **c** abc **d** $3a^2$ **e** $3xy$
f $10ab$ **g** $6rs$ **h** mn **i** $24a$ **j** $30b$
k $28x$ **l** $81p$ **m** $15a$ **n** $8b$ **o** $30y$
p $12m$ **q** $30m$ **r** $6p$ **s** $28a$ **t** $40b$
u $21a$ **v** $80x$ **w** $60b$ **x** $9y$
y $4a+5b$ **z** $7q-6p$

5 **a** $2p(q-1)$ **b** $15(a-3)$ **c** $ab(c-7)$
d $4(3a+1)$ **e** $6(1+2x)$ **f** $3(5y-2z)$
g $(x+2)(x+3)$ **h** $(a-3)(a+3)$
i $(4p+1)(3q-1)$

6 **a** $5\times x$ **b** $7\times a$ **c** $12\times y$ **d** $14\times p$
e $2\times p+7$ **f** $9\times x-5$ **g** $11-7\times m$
h $1+10\times a$ **i** $7\times x+3\times y$
j $10\times a-9\times b$ **k** $7\times a+3\times p$
l $11\times m-9\times n$ **m** $a\times b+3$
n $k-3\times l$ **o** $a\times b\times c$ **p** $8\times x\times y$
q $6\times(a+3)$ **r** $4\times a\times(6-a)$
s $3\times(2\times x+1)$ **t** $(a+7)\times(a-7)$
u $x\times x$ **v** $a\times a+c\times c$ **w** $2\times y\times y$
x $m\times m-3$

7 **a** $a\times 3$ **b** $5\times k$ **c** $m\times 7$ **d** $n\times a$
e $(2\times x)\times 5$ **f** $(3\times y)\times 2$ **g** $5\times(8\times m)$
h $6\times(5\times p)$ **i** $(5\times a)\times(6\times b)$
j $(11\times m)\times(10\times n)$ **k** $(a+7)\times 3$
l $(m-1)\times 2$ **m** $(6+a)\times(a-5)$
n $(9+x)\times(7-x)$ **o** $3\times(x+1)\times 5$
p $(a+6)\times(3\times a)$

8 $2n+2$

Exercise 4:05

1 **a** $x+1$ **b** $y+4$ **c** $2y$ **d** $4x$
e $2x+1$ **f** $2y+2$ **g** $2x+4$ **h** $y+4$
i $3x+6$ **j** $3x+y$ **k** $x+2y$
l $3y+x$ (or $x+3y$) **m** $4x+2$ **n** $2y+5$
o $2x+2y+$ **p** $3x+4$ **q** $2y+4$
r $3x+7$

2 **a** a **b** $a+1$ **c** $a+2$ **d** $2a+1$
e $2a+2$ **f** $2a+2$ **g** $2a+3x$
h $2a+3x$ **j** $2a+3x$ **j** $3a+5$ **k** $3a+5$

3 **a** $2x+3a+3$ (or $3a+2x+3$)
b $4y+2x+9$ (or $2x+4y+9$) **c** $2a+17$
d $3y+x+15$ (or $x+3y+15$)
e $3x+11$ **f** $5a+10$

4 **a** $n+2$ **b** $2n+1$ **c** $2n+4$

Prep quiz 4:06

1 $4y$ **2** $2m$ **3** $6ab$ **4** $9n+2$
5 $9(n+2)$ **6** $\frac{p}{7}$ **7** $\frac{ab}{c}$ **8** $3\times p\times q$
9 $y\times y$ **10** $6\times a+4\times b$

Exercise 4:06

1 **a** 8 **b** 28 **c** 13 **d** 16 **e** 19
f 12 **g** 1 **h** 15 **i** 15 **j** 2
k 8 **l** 5

2 **a** 10 **b** 7 **c** 7 **d** 13 **e** 9
f 17 **g** 16 **h** 3 **i** 30 **j** 4
k 5 **l** 1

3 **a** 20 **b** 9 **c** 24 **d** 6 **e** 32
f 17 **g** 28 **h** 0 **i** 216 **j** 45
k 13 **l** 245 **m** 8 **n** 38 **o** 21
p 50 **q** 0 **r** 12 **s** 1 **t** 2
u 2 **v** 1

4 **a** **i** 15 **ii** 0 **iii** 55 **iv** $2\frac{1}{2}$
b **i** 17 **ii** 9 **iii** 37 **iv** 8
c **i** 20 **ii** 2 **iii** 15·5 **iv** 19
d **i** 35 **ii** 105 **iii** 5 **iv** 30

5 **a** 16 **b** 26 **c** 72 **d** 5·6

6 **a** 72 **b** 156 **c** 420 **d** 10 100

7 **a** \$2.50 **b** \$2.75 **c** \$8.75

8 **a** $b=8, 12, 16, 20$ **b** $y=0, 3, 6, 9$
c $n=7, 21, 35, 49$ **d** $b=1, 4, 7, 10$
e $y=7, 11, 15, 19$ **f** $q=7, 22, 27, 52$
g $b=3, 8, 13, 18$ **h** $w=20, 16, 12, 8$
i $g=2, 5, 8, 11$ **j** $y=1, 2, 3, 4$
k $b=0, 1, 2, 4$ **l** $n=0, 1, 4, 9$

9 **a** $y=x+5$ **b** $y=x-23$
c $y=2\times x$ or $y=2x$ **d** $y=x\div 7$ or $y=\frac{x}{7}$

10 **a** C **b** G **c** A **d** H **e** B
f F **g** D **h** E

A You add 7 to the first number to get the second number.
B You double the first number and add 1 to get the second number.
C Multiply the first number by 6 to get the second number.
D Double the first number then subtract 4 to get the second number.
E Dividing the first number by 3 produces the second number.
F Multiplying the first number by 3 then adding 2 gives the second number.
G Multiplying the first number by 3 gives the second number.
H If you subtract 4 from the first number, you get the second number.

Prep quiz 4:07

1 $3a$ **2** mn **3** $12pq$ **4** $3(5+a)$
5 $2a+5b$ **6** 12 **7** 15 **8** 13 **9** 1
10 24

Exercise 4:07

1 **a** 2 **b** 1 **c** 2 **d** 1 **e** 3 **f** 1
g 3 **h** 4 **i** 3 **j** 5 **k** 5 **l** 5
m 3

2 **a** $2x, 4x$ **b** $3b, 7b$ **c** $4m, 8m$ **d** $2a, 3a$
e $a^2, 3a^2$ **f** $xy, 3yx$ **g** $9x, x, 3x$
h $2mn, 6mn$ **i** $a, 2a, 7a$ **j** ab, ba
k $a^2x, 4a^2x$ **l** $8m^2, 5m^2$

3 **a** $7a$ **b** $7x$ **c** $13m$ **d** $10y$ **e** $7n$
f $8b$ **g** $2a$ **h** $8x$ **i** $7a$ **j** $3m$
k $8k$ **l** $7b$ **m** $7q$ **n** $10x$ **o** 0
p m **q** $10x$ **r** $13a$ **s** $14m$ **t** $12x$
u m **v** $8y$ **w** $3p$ **x** $9a$
y $9ab$ **z** $13xy$

4 **a** $9a + 3b$ **b** $10p - 7q$ **c** $11a^2 + 5a$
d $7ab - 7a$ **e** $7 + a$ **f** $7a + 5b$ **g** $2p + 6q$
h $16m^2 + 2m$ **i** $5a^2 + 4a + 7$ **j** $4p+3q$
k $2x^2 + 5x$ **l** $11x + 7y$ **m** $8k + 5l$
n $7x^2 + 5x$ **o** $11x^2 + 5xy$ **p** $x + 9$
q $10x + 3y$ **r** $10k + 3l$ **s** $2ab + 4bc$
t $10k + 8l - 2m$ **u** $20a + 6b$ **v** $13p + 4q$

5 **a** $11n$ chocolates **b** $8b$ bananas

6 **a** $2l + 2b$
b **i** $3l$ **ii** $4s$ **iii** $2x + y$ **iv** $a + b + 2c$

7 **a** $8s$ cm^2 **b** $4t$ cm^2 **c** $6t$ cm^2
d $2s + 3t$ cm^2 **e** $3s + 3t$ cm^2
f $6s + 12t$ cm^2 **g** $10s + 12t$ cm^2
h $ns + mt$ cm^2

Fun spot 4:07

1 $x, 6x, 6x + 20, 6x+14, 3x+7, 31$
2 $y, 10y, 10y + 30, 20y + 60, 20y+40, 2y+4, 14$
3 $m, 3m, 3m + 24, 6m + 48, 6m + 54, 2m + 18, 22$
4 $3d, 8d, 8d + 8, 24d + 24, 24d, 96$
5 $10k, 9k, 36k, 36k + 18, 18k + 9, 45$

Prep quiz 4:08

1 0 **2** 67 **3** 86 **4** $\square = 8$ **5** $\square = 9$
6 $\square = 11$ **7** $\square = 20$ **8** $\square = 12$ **9** $\square = 7$
10 $\square = 8$

Exercise 4:08

1 **a** true **b** true **c** false **d** true
e true **f** true **g** false **h** true
i true **j** true **k** false **l** true

2 **a** 0 **b** 516 **c** 0 **d** 399 **e** 1
f 0 **g** b **h** 25 **i** m **j** 4 **k** 6
l ab **m** 1 **n** 0 **o** $3ab$ **p** c
q ab **r** 0 **s** 6 **t** $12b$ **u** $9b$
v 1 **w** $2b$ **x** 0 **y** abc **z** mn

3 **a** $122 + 1220 = 1342$ **b** $22 + 1320 = 1342$

4 **a** no **b** no **c** no
d yes, Madeline contributes more

5 **a** true **b** true **c** true **d** true
e false **f** false **g** false **h** false
i true **j** true

6 **a** 100 **b** 100 **c** 93 **d** 70 **e** 1000
f 10 **g** 1 **h** 1 **i** 2 **j** 100

7 **a** 0 **b** \$65 **c** \$56 **d** 718 **e** 2747
f 119 **g** 3600 **h** 13 800 **i** 4500
j 893 **k** 1960 **l** 1778

8 **a** true **b** false **c** true **d** true
e true **f** true

9 **a** 3 **b** 3 **c** 11 **d** 9 **e** 2 **f** 6

10 **a** 495 **b** 693 **c** 294 **d** 882 **e** 570
f 760 **g** 1140 **h** 1710

Prep quiz 4:09

1 26 **2** 26 **3** 48 **4** 48 **5** 100
6 100 **7** 297 **8** 297 **9** 485 **10** 485

Exercise 4:09

1 **a** $2a + 6$ **b** $3x + 15$ **c** $5y + 15$
d $3x - 3$ **e** $7n - 49$ **f** $4a - 16$
g $14 + 7x$ **h** $6 + 6y$ **i** $45 - 9y$
j $10 + 2x$ **k** $3x + 27$ **l** $3a - 6$

2 **a** $6a + 4$ **b** $6x + 3$ **c** $20y + 15$
d $30p + 70$ **e** $12y - 6$ **f** $15q - 20$
g $8k - 28$ **h** $36n - 27$ **i** $20 + 10x$
j $21 + 28x$ **k** $5 + 15p$ **l** $24 - 16y$
m $8m + 12$ **n** $6y + 12$ **o** $10 + 20x$
p $70 + 30q$ **q** $30n - 18$ **r** $56m - 7$
s $14 - 21x$ **t** $55 - 99y$

3 **a** $x^2 + 2x$ **b** $y^2 + 5y$ **c** $m^2 + 7m$
d $q^2 - q$ **e** $m^2 - 10m$ **f** $n^2 - 4n$
g $3a + a^2$ **h** $4k + k^2$ **i** $6p - p^2$
j $2x^2 + 3x$ **k** $3y^2 + 4y$ **l** $5n^2 - n$
m $2p^2 + 14p$ **n** $3q^2 - 12q$ **o** $2m^2 + 6m$
p $10a^2 + 15a$ **q** $8m^2 - 2m$ **r** $30n - 18n^2$

4 **a** $5x + 5y$ **b** $4a + 4b$ **c** $7p - 7q$
d $10m - 10n$ **e** $12a + 6b$ **f** $21x + 14y$
g $36p - 18q$ **h** $10k - 351$ **i** $x^2 + xy$
j $p^2 - pq$ **k** $2y^2 + 2yz$ **l** $15g^2 - 10gh$

5 **a** $2a + 11$ **b** $4m + 29$ **c** $8x + 14$
d $5a + 18$ **e** $35 + 8y$ **f** $13m + 50$
g $12n + 20$ **h** $8n + 11$ **i** $12a + 17$
j $3m + 20$ **k** $8a$ **l** $3m + 1$
m $6a + 2$ **n** $6m + 6 - 6a$ **o** $30m + 15$
p $11a - 4$ **q** $11x - 20$ **r** $45y - 14$

Prep quiz 4:10

1 $12a$ **2** $4p$ **3** $12x$ **4** $6m$ **5** $9p + 5q$
6 $11ab$ **7** $11a + 8$ **8** $5p + 8q$ **9** $3x + 10y$
10 $8t^2 + 3t$

Exercise 4:10

1 **a** a^3 **b** p^5 **c** t^4 **d** y^2 **e** k^5
f m^3 **g** n^6 **h** b^4 **i** a^2b^3 **j** x^3y^3
k a^4t **l** mn^3 **m** p^3q^2 **n** a^3b^3 **o** x^3y^2

2 **a** $a \times a \times a \times a \times a$ **b** $q \times q \times q \times q$
c $t \times t \times t \times t \times t \times t \times t$ **d** $x \times x \times x$
e $m \times m$ **f** $n \times n \times n \times n \times n$
g p **h** $a \times a \times a \times a \times a \times a$
i $m \times m \times n \times n \times n$ **j** $a \times a \times a \times a \times b \times b \times b$
k $x \times y \times y \times y \times y \times y$ **l** $k \times k \times k \times k \times l$
m $a \times a \times a \times y$ **n** $h \times h \times n \times n \times n \times n \times n$
o $t \times w \times w \times w$ **p** $x \times x \times x \times x \times x \times y \times y \times y \times y$
q $n \times n \times p \times p \times p \times q \times q \times q \times q$
r $a \times b \times b \times b \times b \times b \times c \times c$
s $x \times x \times x \times y \times y \times z \times z \times z \times z$
t $m \times m \times m \times n \times p \times p \times p \times p$

3 **a** $30a^2$ **b** $12y^3$ **c** $18m^3$ **d** $7t^3$
e $28y^4$ **f** $48n^2$ **g** $45m^3$ **h** $72a^3$
i $24x^2y$ **j** $150a^2b^2$ **k** $15k^2l^2$ **l** $15t^3w$

Diagnostic test 4

1 **a** 14 **b** 6 **c** 54 **d** 5

2 **a** add 4 **b** subtract 3 **c** multiply by 3
d divide by 2

3 **a** 3 **b** 15 **c** 4

4 **a**

Number of triangles	1	2	3	4
Number of matches	3	6	9	12

b

Number of squares	1	2	3	4
Number of matches	4	7	10	13

5 **a**

x	1	2	3	4
y	5	6	7	8

b

x	0	2	4	6
y	0	8	16	24

c

x	0	1	2	3
y	3	5	7	9

6 **a** $5y$ **b** ab **c** $12mn$ **d** $2s + 5t$

7 **a** a^2 **b** $6p^2$ **c** $6m^2$ **d** wt^2

8 **a** $\frac{y}{5}$ **b** $\frac{10}{a}$ **c** $\frac{a+2}{3}$ **d** $\frac{x}{y}$

9 **a** $2y$ **b** $x + 5$ **c** $2x + y$
d $x + 2y$ (or $2y + x$) **e** $2x + 8$ **f** $3x + 4$

10 **a** 20 **b** 25 **c** 11 **d** 4

11 **a** 18 **b** 12 **c** 21 **d** 2

12 **a**

t	1	2	3	4
b	3	6	9	12

b

x	0	2	4	6
y	1	5	9	13

c

n	0	1	2	3
m	0	1	4	9

13 **a** 2 **b** 1 **c** 3 **d** 2

14 **a** $7a$ **b** $6y$ **c** $5ab$ **d** m

15 **a** $3a + 3$ **b** $2x + 2y$ **c** $6m + 8n$
d $5p^2 + 2p$

16 **a** true **b** true **c** false **d** true

17 **a** $2x + 8$ **b** $5 - 5a$ **c** $6y + 15$
d $2x^2 + xy$

18 **a** $4x + 17$ **b** $14a + 3$ **c** $2m - 20$

19 **a** a^4 **b** y^5 **c** $12a^3$ **d** x^3y^2

20 **a** $m \times m \times m \times m \times m$ **b** $a \times a \times b$
c $x \times x \times y \times y \times y$

ASSIGNMENT 4A

1 **a**

Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

b 2

2 **a**

Number of blocks (b)	1	2	3	4
Number of faces showing (f)	5	8	11	14

b $f = 3 \times b + 2$

3 **a** $D = s \times T$ **b** $A = L \times B$ **c** $P = 4 \times L$
d $T = b + g$ **e** $P = a + p$ **f** $M = S + 27$
g $C = T \div n$ **h** $A = (a + b) \div 2$

4 **a** $15y$ **b** $3x$ **c** $20m$ **d** $18y$ **e** $14n$
f $2n$ **g** 0 **h** 3 **i** $3b$ **j** $5a + 4$
k $4x + 3$ **l** $5p + 6$ **m** b^3 **n** $3a^2 + a$
o 0 **p** $5ap$

5 **a** $7x$ **b** $10x + 14$ **c** $5a + 35$
d $11x - 15$ **e** $3 + 2m$ **f** $54a + 25b$

ASSIGNMENT 4B

1 (See ID Card 2 Answers, page 466.)
2 28 800 **3** 361 **4** \$3.60 **5** \$42 300
6 Soula has \$7.50, Kosta has \$2.50
7 **a** USA **b** Bulgaria **c** Poland **d** two
e about 96 million tonnes

ASSIGNMENT 4C

1 **a** 4561 **b** 3587 **c** 963 **d** 515

2 **a** \$30.50 **b** \$36.10 **c** \$60.50 **d** \$2.10

3 **a** 7 **b** 6 **c** 26 **d** 8

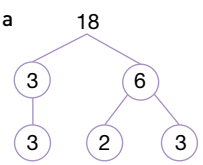
4 **a** 16 **b** 1000 **c** 10 **d** 80 000

5 **a** 4^3 **b** 10^2 **c** 3^4 **d** 7^3

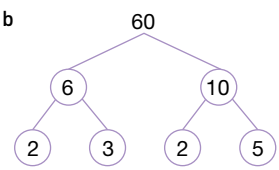
6 **a** 1, 3, 5, 15 **b** 1, 2, 3, 4, 6, 8, 12, 24
c 1, 7 **d** 1, 2, 4, 5, 10, 20, 25, 50, 100

7 **a** 10, 20, 30 **b** 11, 13, 17, 19 **c** 10 **d** 24

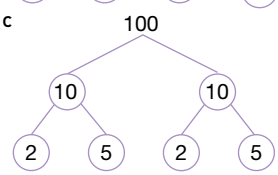
8 **a**



b



c



9 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{9}{10}$ **d** $\frac{1}{4}$

10 **a** $\frac{3}{4}$ **b** $\frac{3}{4}$ **c** $\frac{9}{10}$ **d** $\frac{3}{5}$

Chapter 5

Exercise 5:01

1 **a** A **b** Y **c** P **d** O

2 **a** $\angle CAB$ **b** $\angle XYZ$ **c** $\angle MPQ$
d $\angle BOP$

3 **a** $\angle LOT$ or $\angle O$, $\angle OTL$ or $\angle T$, $\angle TLO$ or $\angle L$
b $\angle AQP$ or $\angle Q$, $\angle QPB$ or $\angle P$, $\angle PBA$ or $\angle B$, $\angle BAQ$ or $\angle A$
c $\angle PSR$ or $\angle S$, $\angle SRQ$ or $\angle R$, $\angle RQP$ or $\angle Q$, $\angle QPS$ or $\angle P$

4 **a** $\angle DBC$ **b** $\angle ABD$ **c** $\angle ABC$
d $\angle DEC$ **e** $\angle AED$ **f** $\angle ATF$

5 **a** **1** $\angle MTB$ **2** $\angle ATN$ **3** $\angle BTN$
b **1** $\angle ADB$ **2** $\angle CAB$ **3** $\angle DAC$
c **1** $\angle CFG$ **2** $\angle AGF$ **3** $\angle HGB$

6 **a** $\angle ACD$ **b** $\angle ABD$ **c** $\angle FCE$

7 **a** 2 **b** 1 **c** 3

8 **a** Y **b** X **c** Y **d** Y

9 **a** B, A, C, D **b** D, B, A, C **c** B, C, D, A

Prep quiz 5:02

1 B **2** BA and BC **3** $\angle ABC$ **4** B
5 A **6** B, A, C **7** $\angle ABD$ **8** $\angle D$
9 $B\hat{C}A$ **10** $\angle BCA$

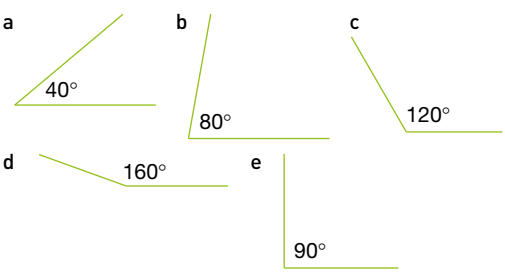
Exercise 5:02

1 **a** 90° **b** 180° **c** 45° **d** 135°
e 10° **f** 110° **g** 30° **h** 160° **i** 60°

2 **a** 60° **b** 120° **c** 40° **d** 110°
e 40° **f** 80°

3 **a** 30° **b** 150° **c** 60° **d** 130° **e** 10°

4 **a** **b** **c**

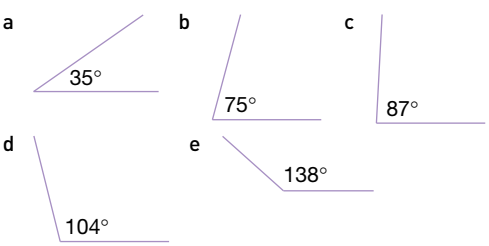


d **e**

5 Check your efforts with a protractor.

6 **a** 32° **b** 153° **c** 62° **d** 128° **e** 11°

7 **a** **b** **c**



d **e**

8 **a** 144° **b** 33° **c** 51°

9 **a** $\angle A = 62°, \angle C = 107°$
b $\angle A = 119°, \angle C = 87°$

10 Accurate drawings will be drawn.

11 **a** **b** **c**



12 Answers will vary.

13 **a** 334° **b** 224° **c** 270°

Exercise 5:03

1 **a** acute **b** acute **c** right **d** right
e straight **f** straight **g** revolution
h revolution **i** reflex **j** reflex **k** reflex
l obtuse **m** reflex **n** right **o** obtuse
p acute

2 **a** right **b** straight **c** revolution
d obtuse **e** acute **f** obtuse **g** reflex
h reflex **i** acute **j** obtuse **k** reflex
l obtuse

3 **a** acute **b** obtuse **c** straight

4 **a** straight **b** straight **c** acute **d** acute
e obtuse **f** obtuse

5 **a** right **b** acute **c** acute

6 **a** obtuse **b** acute **c** acute

7 **a** C **b** B

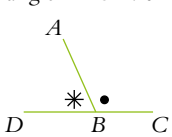
8 **a** all right angles **b** 2 acute and 2 obtuse angles
c 1 acute and 3 obtuse angles
d 1 reflex and 3 acute angles

9 **a** 3 obtuse, 1 right and 1 acute
b 3 acute, 1 right and 1 reflex
c



Prep quiz 5:04

1 right angle **2** acute angle **3** obtuse angle
4 straight angle **5** 90° **6** 180° **7** 360°
8 and **9**



10 about 180°

Exercise 5:04

1 **a** 70° **b** 40° **c** 20° **d** 27°

2 **a** 130° **b** 60° **c** 105° **d** 75°

3 **a** $\angle CBD$ and $\angle DBA$ **b** 30°

4 **a** $\angle ABD$ and $\angle DBC$ **b** $\angle ABD$ and $\angle DBC$
c 90°

5 **a** $\angle ABD$ and $\angle DBC$ **b** $\angle ABD$ and $\angle DBC$
c yes

6 **a** 70° **b** 30° **c** 45°

7 **a** 15° **b** 43° **c** 9°

8 **a** 120° **b** 40° **c** 128°

9 $\angle ABD = 150°$

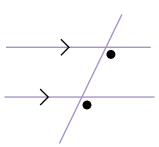
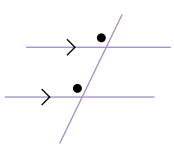
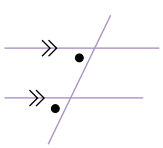
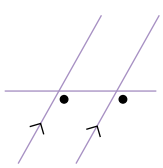
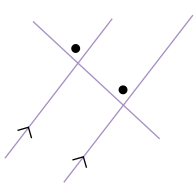
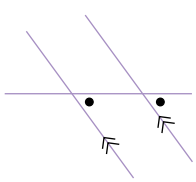
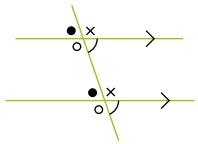
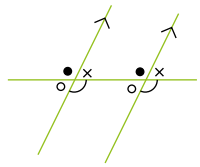
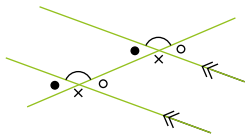
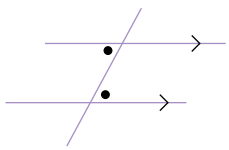
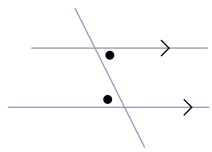
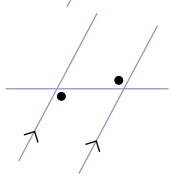
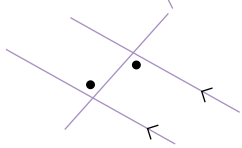
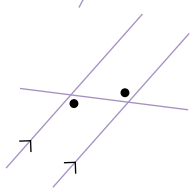
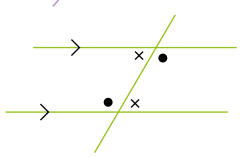
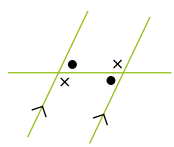
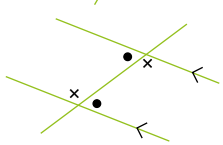
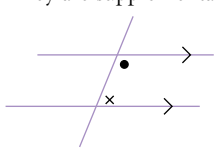
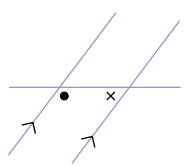
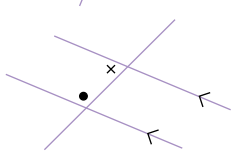
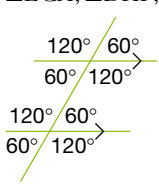
10 **a** 120° (angles at a point add to 360°)
b 25° (angles at a point add to 360°)
c 135°(angles at a point add to 360°)

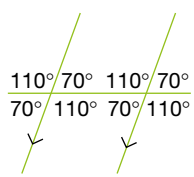
11 **a** 1 and 3, 2 and 4 **b** **i** 120° **ii** 120°
c Add up the sizes of 1,2 and 4 and subtact the answer from 360°.
d Vertically opposite angles are equal.
12 **a** **i** 70° **ii** 110° **b** **i** 45° **ii** 135°
13 **a** 115 **b** 90 **c** 65
14 (*Note:* Wording of reasons may vary.)
a $x = 80$ (adjacent complementary angles)
b $a = 35$ (adjacent complementary angles)
c $p = 60$ (adjacent supplementary angles)
d $e = 88$ (vertically opposite angles)
e $y = 105$ (adjacent supplementary angles)
f $b = 48$ (vertically opposite angles)
g $x = 136$ (vertically opposite angles)
h $a = 90$ (adjacent complementary angles)
i $m = 110$ (vertically opposite angles)
j $a = 50$ (angles at a point)
k $x = 140$ (angles at a point)
l $a = 80$ (adjacent supplementary angles)
15 **a** $a = 156, b = 55$ **b** $a = 13, b = 55$
c $a = 100, m = 52, n = 80$

Prep quiz 5:05

1 angle 3 **2** angle 4 **3** They add to 180°.
4 They add to 180°. **5** They add to 180°.
6 360° **7** 140° **8** 40° **9** 140° **10** yes

Exercise 5:05

1 **a** *AB* and *CD* are parallel lines. *EF* is a transversal.
b *CD* and *EF* are parallel lines and *AB* is a transversal.
2 **a** Angles 1, 3, 5 and 7 are equal and angles 2, 4, 6 and 8 are equal.
b Angles 1, 3, 5 and 7 are equal and angles 2, 4, 6 and 8 are equal.
3 **a** equal **b** equal **c** equal **d** equal
4 **a** below a parallel line and to the left of the transversal
b below a parallel line and to the right of the transversal
5 **a**

b

c

d

e

f

6 **a**

b

c

7 **a** equal **b** equal
8 **a**

b

c

d

e

9 **a**

b

c

10 **a** They are supplementary.
b They are supplementary.
11 **a**

b

c

12 **a** $\angle DHG$ **b** $\angle AGH$ **c** $\angle GHD$
d $\angle GHC$ **e** $\angle GHD$ **f** $\angle AGH$
g $\angle EGA, \angle DHF, \angle GHC$ **h** 144°
13 **a**


b


14 **a** $a = 60$ (alternate angles)
b $a = 40$ (co-interior angles)
c $b = 110$ (corresponding angles)
d $d = 130$ (alternate angles)
e $x = 38$ (alternate angles)
f $x = 110$ (co-interior angles)

15 **a** $a = 140, b = 40$ **b** $a = 65, b = 115$
c $x = 35, y = 145$ **d** $x = 75, y = 75$
e $a = 115, b = 115$ **f** $a = 125, b = 55$

Challenge 5:05

1 **a** symmetry **b** even **c** net **d** odd
e abacus **f** digits **g** hexagon **h** line
i triangle **j** rectangular **k** circle
l thermometer

Exercise 5:06

1 **a** Yes. The corresponding angles are equal.
b Yes. The alternate angles are equal.
c Yes. The cointerior angles are supplementary.
d No. The cointerior angles are not supplementary.
e Yes. The alternate angles are equal.
f No. The alternate angles are not equal.
g No. The corresponding angles are not equal.

2 **a** $a = 110, b = 70, c = 110, d = 70$
b $a = 100, b = 80, c = 100, d = 80$
c $a = 110, b = 110, c = 70, d = 70$

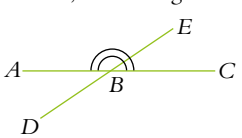
3 **a** anticlockwise **b** clockwise

4 **a** parallel
b No, they would meet on the left of the transversal.
c parallel
d No, they would meet on the right of the transversal.

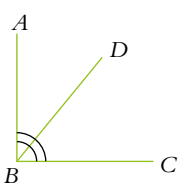
Diagnostic test 5

1 **a** $\angle CAB$, acute angle
b reflex $\angle XYZ$, reflex angle
c $\angle ACB$, obtuse angle

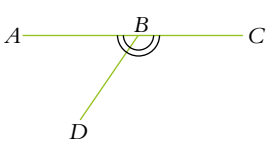
2 **a**



b



c



3 **a** $x = 150$ **b** $a = 20$ **c** $b = 105$

4 **a** $a = 40$ **b** $b = 80$ **c** $a = 65$

5 **a** $a = 120$ **b** $b = 90$ **c** $c = 40$

6 **a** $a = 150$ **b** $b = 120$ **c** $c = 50$

7 **a** $a = 50$ **b** $b = 110$ **c** $c = 70$

ASSIGNMENT 5A

1 **a** $\angle ABC$ (or $\angle CBA$), acute, 37°
b $\angle BAC$ (or $\angle CAB$), obtuse, 137°

2 **a** $\angle CDE$ or $\angle EDC$ **b** $\angle AEC$ or $\angle CEA$
c $\angle ABD$ or $\angle DBA$

3 **a** obtuse **b** right **c** acute **d** reflex

4 **a** $x = 45$ (supplementary angle)
b $a = 80, b = 100$ (supplementary angles)
c $a = 40$ (corresponding angle),
$b = 140$ (supplementary angle)

5 **a** $a = 35, b = 25$ **b** $a = 130, b = 130$
c $a = 50, b = 95, c = 95$ **f** no

ASSIGNMENT 5B

1 158 km **2** $420.50 **3** $1.30
4 15° **5** 7:57am
6 **a** **i** $250 million **ii** $150 million
b 2004 **c** 2007 **d** $130 million
e 2004, 2009 and 2010 **f** no

ASSIGNMENT 5C

1 **a** 1 326 763 **b** sixteen thousand and twenty
c ten thousand
d $(4 \times 10\,000) + (2 \times 1000) + (3 \times 100) + (7 \times 10) + (5 \times 1)$

2 **a** 42 **b** 17 **c** 12 **d** 3

3 **a** 19 **b** 22 **c** 155 **d** 3

4 **a** false **b** false **c** true **d** false

5 **a** $\square = 6$ **b** $\square = 73$ **c** $\square = 1$ **d** $\square = 2$

6 **a** 87 **b** 293 **c** 1664 **d** 1990

7 **a** $4 \times 4 \times 4$, 64 **b** $3 \times 3 \times 3$, 27
c $10 \times 10 \times 10 \times 10 \times 10 \times 10$, 1 000 000
d $2 \times 10 \times 10 \times 10 \times 10$, 20 000
e $3 \times 3 \times 2 \times 2 \times 2 \times 2$, 144

8 **a** 0·7 **b** 0·42 **c** 0·175 **d** 0·05
e 0·027

9 **a** 12 **b** 15 **c** 12 **d** 50, 100, 200

10 **a** 50 **b** 600 **c** 30 **d** 0 **e** 45

Chapter 6

Exercise 6:01

1 **a** true **b** true **c** true **d** true
e true **f** true **g** false **h** true
i true **j** true

2 **a** 0·64 **b** 0·17 **c** 0·238 **d** 0·051
e 0·309 **f** 0·066 **g** 2·35 **h** 9·4
i 15·36 **j** 11·02 **k** 6·081 **l** 2·503

3 **a** $(7 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$
b $(3 \times 1) + (4 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (3 \times \frac{1}{1000})$
c $(4 \times 1) + (2 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$
d $(4 \times 10) + (5 \times 1) + (3 \times \frac{1}{100})$

e $(3 \times 10) + (7 \times \frac{1}{10}) + (5 \times \frac{1}{100})$

f $(4 \times 10) + (7 \times \frac{1}{100})$

g $(2 \times 100) + (5 \times 10) + (6 \times 1) + (4 \times \frac{1}{100})$

h $(2 \times 10) + (4 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

i $(6 \times 10) + (9 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

j $(2 \times 100) + (4 \times 10) + (6 \times 1) + (3 \times \frac{1}{10}) + (7 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$

k $(4 \times 100) + (8 \times 10) + (2 \times 1) + (3 \times \frac{1}{10}) + (7 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$

l $(2 \times 100) + (3 \times 10) + (8 \times 1) + (6 \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$

4 **a** 0·7 **b** 0·05 **c** 1·2 **d** 0·05
e 0·69 **f** 0·03 **g** 0·1 **h** 2·8
i 0·612 **j** 0·5 **k** 0·049 **l** 0·29

5 **a** 0·3, 0·6, 0·8 **b** 1·2, 3·85, 7·5
c 0·55, 0·8, 1 **d** 0·03, 0·3, 3
e 0·03, 0·3, 0·33 **f** 0·39, 0·4, 0·41
g 0·792, 0·85, 0·9 **h** 0·09, 0·1, 0·8

6 **a** 1·36 **b** 1·84 **7** 2·152

8 **a** 2 **b** 3 **c** 1 **d** 1 **e** 2 **f** 3
g 3 **h** 2 **i** 1 **j** 2 **k** 3 **l** 1

9 **a** 0·23 **b** 0·12 **c** 0·007 **d** 0·125
e 0·015 **f** 0·604 **g** 2·93 **h** 15·103
i 10·09 **j** 7·999 **k** 11·033 **l** 8·005

10 **a** $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{3}{10}$ **d** $\frac{21}{100}$ **e** $\frac{77}{100}$
f $\frac{91}{100}$ **g** $\frac{447}{1000}$ **h** $\frac{601}{1000}$ **i** $\frac{853}{1000}$
j $2\frac{99}{100}$ **k** $8\frac{3}{10}$ **l** $9\frac{7}{1000}$

11 **a** 1·4 **b** 8·6 **c** 7·3 **d** 7·24 **e** 8·06
f 9·03 **g** 1·186 **h** 7·406 **i** 8·014
j 45·04 **k** 650·4 **l** 0·842

12 **a** $\frac{3}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{10}$ **d** $\frac{7}{10}$ **e** $\frac{7}{10}$
f $\frac{7}{10}$ **g** $\frac{1}{100}$ **h** $\frac{1}{100}$ **i** $\frac{1}{100}$ **j** $\frac{9}{100}$
k $\frac{3}{100}$ **l** $\frac{1}{1000}$

13 **a** 500 **b** $\frac{5}{10}$ **c** $\frac{5}{1000}$ **d** $\frac{5}{100}$
e $\frac{5}{1000}$ **f** $\frac{5}{100}$ **g** $\frac{5}{10\,000}$ **h** $\frac{5}{100}$

14 **a** false **b** false **c** true **d** true
e true **f** true **g** true **h** true

Exercise 6:02

1 **a** 5·86 **b** 16·05 **c** 23·235 **d** 35 466·4
e 66 673·5 **f** 13·0006 **g** 21·24 **h** 4·6
i 10 **j** 12·12 **k** 14·35 **l** 159·96
m 19·025 **n** 18·65 **o** 24·225 **p** 0·465

2 **a** 2·45 **b** 8·65 **c** 7·202 **d** 11·625
e 5·375 **f** 8·975 **g** 6·78 **h** 31·085
i 30·089 **j** 7·7 **k** 6·3 **l** 2·75
m 8·634 **n** 8·29 **o** 23·72 **p** 4·909
q 0·386 **r** 3·117 **s** 671·25 **t** 518·95
u 4718·155

Prep quiz 6:03

1 0·8 **2** 2 **3** 2·4 **4** 1·8 **5** 3·2
6 2·5 **7** 1·2 **8** 3 **9** 2·1 **10** 3·2

Exercise 6:03

1 **a** 0·6 **b** 2·4 **c** 4·4 **d** 0·4
e 0·21 **f** 0·48 **g** 0·66 **h** 2·1
i 0·36

2 **a** 14·4 **b** 7·6 **c** 1 **d** 19
e 23·1 **f** 12 **g** 1·89 **h** 9·272
i 137·6

3 **a** 7·6 **b** 12·33 **c** 9·24 **d** 3
e 45·99 **f** 30·35 **g** 115·01 **h** 156·48
i 313·84

4 **a** 12·5 **b** 91·25 **c** 843·5 **d** 65
e 138 **f** 37 **g** 231·5 **h** 685
i 12 **j** 30 **k** 810 **l** 12·5
m 111·1 **n** 15 **o** 8100 **p** 750
q 30 050 **r** 6750

Exercise 6:04

1 **a** 0·21 **b** 2·1 **c** 0·004 **d** 6·03
e 3·03 **f** 3·09 **g** 9·2 **h** 0·002
i 1·218

2 **a** 0·07 **b** 4·9 **c** 0·9 **d** 0·01
e 0·025 **f** 0·003 **g** 1·26 **h** 30·4
i 0·126

3 **a** 0·25 **b** 0·35 **c** 0·15 **d** 0·26
e 0·35 **f** 0·335 **g** 0·365 **h** 0·1375
i 2·35 **j** 0·75 **k** 1·5 **l** 0·625

4 **a** 3·41 **b** 7·55 **c** 6·48 **d** 0·06
e 0·015 **f** 0·005 **g** 7·516 **h** 0·0038
i 67·845 **j** 0·0004 **k** 0·0035 **l** 0·35
m 0·8764 **n** 3·1564 **o** 3·437 **p** 0·0346
q 0·00875 **r** 0·0008

5 **a** 1·846 **b** 0·675 **c** 0·873 **d** 97·65
e 3·468 **f** 0·08 **g** 0·0346 **h** 0·04
i 7·58

Prep quiz 6:05

1 2·4 **2** 3 **3** 7·05 **4** 0·152 **5** 3·5
6 71 **7** 0·08 **8** 1·5 **9** 12·42 **10** 21

Exercise 6:05

1 **a** 2 **b** 1 **c** 3 **d** 4 **e** 2
f 1 **g** 2 **h** 1 **i** 3

2 **a** 0·24 **b** 0·088 **c** 0·07 **d** 0·62
e 4·2 **f** 0·09 **g** 0·028 **h** 0·024
i 0·0049 **j** 0·4 **k** 0·03 **l** 0·6
m 0·54 **n** 0·056 **o** 0·16

3 **a** 0·0375 **b** 1·284 **c** 2·68 **d** 2677·5

e 34·08 f 6840 g 6·5 h 0·024
i 43·4 j 13·6 k 28 080 l 270
m 3·467 n 52·32 o 8·041

4 a 29·76 b 0·9888 c 11·039 d 5·1114
e 26·52 f 2·856 g 117·72 h 0·0256
i 12·25 j 0·0625 k 511·14 l 51·114

5 a 0·04 b 0·0009 c 0·36 d 0·64
e 1·21 f 0·0025 g 0·000441 h 2·25
i 0·0289

Prep quiz 6:06

1 0·44 2 5·95 3 6·34 4 15 5 0·04
6 2·7 7 1·04 8 1·045 9 5·5 10 0·0034

Exercise 6:06

1 a 83·5 ÷ 5 b 34·4 ÷ 8 c 651 ÷ 3
d 0·49 ÷ 7 e 8 ÷ 2 f 712·5 ÷ 5
g 3800 ÷ 4 h 180 ÷ 9 i 8000 ÷ 8
j 270 ÷ 3 k 47·2 ÷ 8 l 220 ÷ 11

2 a 16·7 b 4·3 c 217 d 0·07
e 4 f 142·5 g 950 h 20
i 1000 j 90 k 5·9 l 20

3 a 3 b 4 c 0·4 d 10
e 20 f 6000 g 9125 h 200
i 50 j 490 k 1600 l 2000

4 a 2·3 b 10·1 c 30 d 2
e 20 f 73 g 8·05 h 1·62
i 15·2 j 12·05 k 31 l 11·75
m 51 n 792·75 o 90·6

Prep quiz 6:07

1 0·3 2 0·07 3 1·96 4 0·005 5 0·037
6 8 7 2 8 12 9 37 10 1663

Exercise 6:07

1 a terminating b repeating c terminating
d repeating e terminating f repeating
g repeating h terminating i repeating
j terminating k repeating l repeating

2 a $0.\dot{9}$ b $3.1\dot{4}$ c $0.1\dot{8}$ d $0.1\dot{6}$
e $0.\dot{9}\dot{0}$ f $6.\dot{7}4\dot{5}$ g $66.\dot{6}$ h $0.\dot{1}42857\dot{}$

3 a 0·3 b 0·06 c 0·009 d 0·019
e 0·55 f 1·1 g 3·77 h 0·709
i 6·02 j 0·5 k 0·88 l 0·35
m 0·35 n 0·25 o 0·012

4 a 0·8 b 0·125 c 0·25 d 0·375
e 0·625 f 0·875 g 0·35 h 0·66
i 0·75 j $0.\dot{3}$ k $0.\dot{6}$ l $0.\dot{1}$
m $0.\dot{2}$ n $0.\dot{4}$ o $0.\dot{7}$ p $0.8\dot{3}$
q $0.1\dot{6}$ r $0.\dot{5}$

5 a $3.\dot{8}$ b 2·6 c $6.\dot{6}$ d $0.2\dot{7}$
e $0.4\dot{5}$ f $0.\dot{9}\dot{0}$ g $2.41\dot{6}$ h $0.7\dot{3}$
i 1·1875 j $0.\dot{5}71428\dot{5}$ k $0.\dot{4}28571\dot{}$
l $0.07\dot{1}42857\dot{}$

6 a 6·875 b $0.8\dot{1}\dot{}$ c 12·25 d 0·3125
e $8.8\dot{3}$ f 2·62

Prep quiz 6:08

1 20 2 150 3 200 4 4
5 5 6 $7.70 7 7·7 8 3·11
9 14·6 and 14·7 10 6·75

Exercise 6:08

1 a 7900 b 1100 c 67 300 d 900
e 600 f 400 g 74 900 h 7900

2 a 9 b 80 c 45 d 3
e 2 f 18 g 237 h 100

3 a 243·13 b 79·66 c 91·35 d 9·81
e 0·30 f 0·09 g 0·10 h 1·99

4 a 6·7 b 8·5 c 2·1 d 6·1
e 0·1 f 246·7 g 29·9 h 10·0

5 $20.32 6 $712

7 a 0·4 b 0·44 c 0·444 d 0·4444

8 a 0·3 b 0·27 c 0·273 d 0·2727

9 a 1363c b 1941c c 6119c d 510c
e 550c f 1768 050c

10 a $14 b $19 c $61 d $5
e $5 f $17 680

Prep quiz 6:09

1 $1 2 $1 3 1 4 $3.75 5 1·003
6 $18.83 7 $9.68 8 $20.70 9 0·06 10 2·1

Exercise 6:09

1 a $34.25 b $8.50 c $1.00 d $17.10
e $85.45 f $9.55 g $103.35 h $11.70
i $27.15 j $277.35 k $108.10 l $654.70

2 a $18.97 b $27.65 c $4.10 d $5.37
e $13.28 f $49.65 g $44.70 h $9.40
i $11.70 j $1.69 k $0.76 l $2.20

3 a $10.85 b $4.10 c $8.15 d $9.50
e $13.60 f $40.75 g $27.20 h $135.90
i $36.70 j $46.20 k $70.65 l $55.70
m $34.00 n $25.80 o $11.55 p $0.70

4 a 0·5 b 0·2 c 0·1 d 0·7
e 1·0 f 0·6 g 0·4 h 0

5 a 2·0 b 6·0 c 0·4 d 8·0
e 0·4 f 0·96 g 0·48 h 0·2
i 1·6 j 0·625 k 0·75 l 1

6 a 128·7 km b 59·5 km
c 8·9 km and 9 km. He may have entered the ground by different gates, parked in a different spot or travelled a different route.

7 80 seconds

8 a 17 b 836

9 a 8 (of the 0·1 km), 3 (of the 0·01 km), 6 (of the 0·001 km); 17 altogether
b 836 rolls

10 a *FE* and *DC* (2·225 m and 3·55 m)

b 5·775 m **c** 15·55 m **d** 1·325 m

11 11·0 s, 11·06 s, 11·07 s, 11·08 s, 11·4 s

12 **a** 99 **b** 9·9

13 3 m (ignoring the width of the nails)

14 1014·5 g

15 $10.65

16 2 **17** **a** 0·58 s **b** 43·86 s

18 D **19** **a** 5 **b** $137.75

20 **a** $0.004 or 0·4 cents **b** 20·8 cents **c** $260

d The employer or anyone who instructed the firm's computer to put these parts of one cent into their own account. This could be a form of 'computer crime'.

Exercise 6:10

1 **a** 12·35 **b** 27·54 **c** 308·824 **d** 2·16
e 17·62 **f** 89·84 **g** 24·36 **h** 7·303
i 9·992 **j** 14·656 **k** 145·543 **l** 258

2 **a** 50·56 **b** 101·024 **c** 112·437
d 193·5423 **e** 242·775 **f** 10 430·823
g 157·92 **h** 1499·3792 **i** 1431·29538
j 585·008568

3 **a** 3·7 **b** 7·1 **c** 3·08 **d** 9·12
e 0·720 **f** 0·688 **g** 3·2743 **h** 0·1342

4 **a** 0·4 **b** 0·88 **c** 0·78 **d** 0·53
e 0·583 **f** 0·765 **g** 0·769 **h** 0·7895
i 0·8846

5 **a** $135.20 **b** $62.37 **c** 19

6 **a** 22·64 m **b** 2·36 m

7 **a** $214.51 **b** $3003.14
c $78 081.64; no, because 26 fortnights is equal to only 364 days.

Fun spot 6:10

1 42 **2** 39·6 **3** 25·2 **4** 43·2 **5** 39·6
6 48·1 **7** 51·6 **8** 11·2 **9** 70·2 **10** 39

Diagnostic test 6

1 **a** $(3 \times 10) + (6 \times 1) + (1 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$

b $(3 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (3 \times \frac{1}{1000}) + (3 \times \frac{1}{10000})$

c $(7 \times 1) + (3 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$

2 **a** 0·3, 0·6, 0·9 **b** 0·11, 0·4, 0·6
c 0·077, 0·7, 0·77

3 **a** 16·75 **b** 1·058 **c** 1·455

4 **a** 19·237 **b** 803·57 **c** 43·1

5 **a** 0·120 **b** 23·4 **c** 18·88

6 **a** 1·07 **b** 1·24 **c** 1·8385

7 **a** 40 **b** 33·75 **c** 1560

8 **a** 1·3465 **b** 0·009135 **c** 0·0045

9 **a** 0·004 **b** 1·232 **c** 0·0009

10 **a** 0·34 **b** 4300 **c** 17 500

11 **a** terminating **b** repeating **c** repeating

12 **a** 3·77 **b** 3·77 **c** 6·007

13 **a** 0·625 **b** $0·1\dot{6}$ **c** $1·\dot{9}\dot{0}$

14 **a** 78·48 **b** 0·67 **c** 9·30

ASSIGNMENT 6A

1 **a** $\frac{3}{5}$ **b** $\frac{7}{100}$ **c** $\frac{1}{125}$

2 0·051, 0·5, 0·501, 0·51

3 **a** 74·703 **b** 13·959 **c** 2510 **d** 0·0136
e 3·2 **f** 12·4

4 **a** 0·8 **b** 0·625 **c** 0·35 **d** 0·12

5 **a** $0·8\dot{3}$ **b** $0·\dot{2}85714\dot{}$

6 **a** 9·8 **b** 9·82 **c** 9·817 **d** 9·8173

7 **a** 0·3 **b** 0·27 **c** 0·267

8 **a** 12·48 s **b** **i** $77.95 **ii** 696·2 km

ASSIGNMENT 6B

1 379 **2** 30 rows **3** $676
4 63 (singles matches) **5** 112

ASSIGNMENT 6C

1 **a** 10 **b** 26 **c** $7\frac{1}{2}$ or 7·5 **d** 5

2 **a** 500 **b** 5000 **c** $\frac{5}{10}$ or 0·5
d $\frac{5}{100}$ or 0·05

3 **a** 7829 **b** 97·38 **c** 430 751

4 **a** $(8 \times 100) + (1 \times 10) + (7 \times 1)$
b $(2 \times 1000) + (1 \times 100) + (4 \times 10) + (3 \times 1)$
c $(6 \times 1) + (3 \times \frac{1}{10}) + (5 \times \frac{1}{100})$
d $(6 \times 10\,000) + (0 \times 1000) + (0 \times 100) + (8 \times 10) + (1 \times 1)$

5 **a** 4 **b** 75 **c** 1 **d** 1

6 **a** 25 **b** 36 **c** 49 **d** 64

7 **a** $7.50 **b** $1.85 **c** $5.25 **d** $8.25

8 **a** reflex angle **b** acute angle **c** obtuse angle

9 **a** $a = 130$ **b** $b = 20$ **c** $c = 102$

10 **a** $m = 150$ **b** $x = 125$ **c** $y = 62$

Chapter 7

Exercise 7:01

1 **a** **A** 50 **B** −50 **C** −50 **D** 50
E 50 **F** 50 **G** −50 **H** 3
I 50 **J** 50 **K** 50 **L** 50
M −50 **N** 50 **O** −50 **P** 50
Q −50 **R** −50 **S** −50 **T** −50
U −50 **V** −50 **W** −3 **X** 50

b A and Q, B and X, C and P, D and R, E and V, F and T, G and N, H and W, I and U, J and O, K and M, L and S

2 **a** 8 **b** −4 **c** −14 **d** −15 **e** 12
f −2 **g** 0 **h** 1800 **i** −60 **j** −135

3–4 There are many possible answers.

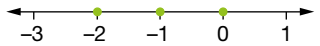
5 **a** −11 **b** −7 **c** −6 **d** −4 **e** −2 **f** 7 **g** −11

6 **a** 20°C **b** 0°C **c** −10°C **d** −20°C **e** −30°C

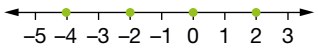
7 **a** false **b** true **c** false **d** true **e** true

8 **a** {−3, −1, 1, 3} **b** {−3, 0, 2} **c** {−20, −10, 0} **d** {−20, −10, 0} **e** $\{-1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\}$ **f** $\{-2, -\frac{1}{2}, 1\}$ **g** {−1·5, 0, 1·5} **h** {−0·5, 0·5, 1·5}

9 **a**



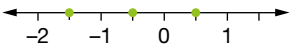
b



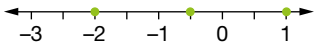
c



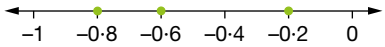
d



e



f



10 **a** −2°C **b** −5°C **c** −5°C **d** −5°C **e** −10°C **f** −7°C

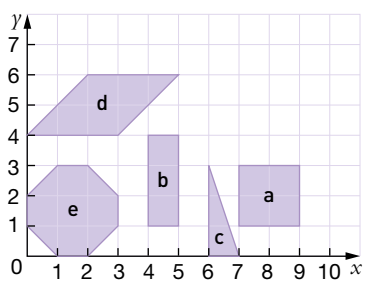
11 **a** −2, 1, 4, 5 **b** −4, −1, 0, 1 **c** −4, −3, −2, −1 **d** −3, −2, 2, 3 **e** −3, −2, 0, 4 **f** −10, −8, 7, 9

Exercise 7:02

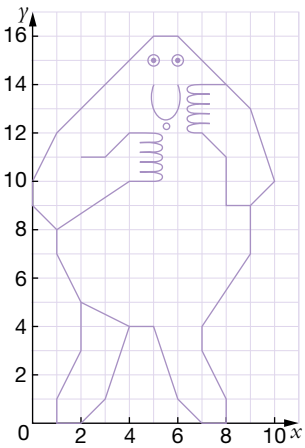
1 **a** *G* **b** *E* **c** *U* **d** *V* **e** *O* **f** *Q* **g** *W* **h** *R* **i** *Z* **j** *F* **k** *J* **l** *K* **m** *H* **n** *P* **o** *N* **p** *Y* **q** *I* **r** *A* **s** *C* **t** *S* **u** *B* **v** *M* **w** *L* **x** *T* **y** *D* **z** *X*

2 **a** (4, 4) **b** (0, 4) **c** (4, 2) **d** (1, 1) **e** (2, 2) **f** (0,2) **g** $(2\frac{1}{2}, 3\frac{1}{2})$ **h** (4, 3) **i** (2, 1) **j** (3, 0) **k** (5, 0) **l** (7, 0) **m** $(0, 6\frac{1}{2})$ **n** $(6\frac{1}{2}, 2\frac{1}{2})$ **o** (0, 0) **p** (6, 4) **q** (7, 5) **r** (6, 6) **s** (7, 7) **t** (6, 7) **u** (5, 6)

3 **a** square **b** rectangle **c** triangle **d** parallelogram **e** octagon



4



5



Challenge 7:02

1 *x*-axis **2** *y*-axis **3** origin (0, 0) **4** (6, 3) **5** (3, 4) **6** (3, 0) **7** (0, 2) **8** *B* **9** *A* **10** 4 units

Prep quiz 7:03

1 (0, 0) **2** *F* **3** *P* **4** *K* **5** *C* **6** *D* **7** *M* **8** *O* **9** *J* **10** *B*

Exercise 7:03

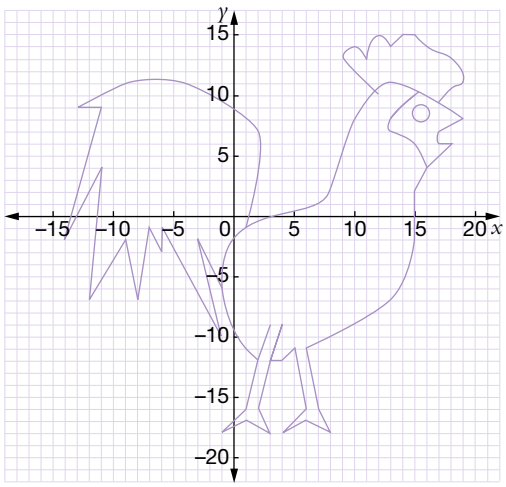
1 **a** *X* **b** *K* **c** *O* **d** *S* **e** *W* **f** *I* **g** *G* **h** *L* **i** *U* **j** *M* **k** *J* **l** *P* **m** *Q* **n** *Z* **o** *C* **p** *T* **q** *A* **r** *B* **s** *F* **t** *D* **u** *H* **v** *R* **w** *Y* **x** *V* **y** *N* **z** *E*

2 **a** (2, 1) **b** (3, 0) **c** (2, −1) **d** (3, −3) **e** (0, −2) **f** (−1, −1) **g** (−2, 0) **h** (−2, 2) **i** (0, 3) **j** (1, 2) **k** (1, −1) **l** (1, −3) **m** (−1, −3) **n** (−1, 0) **o** (0, 0) **p** (−2, 1) **q** (0, 1) **r** (2, −2) **s** (−2, −2) **t** (−3, −3) **u** (−3, 1) **v** (−3, 3) **w** (−1, 2) **x** (3, 2) **y** (2, 0) **z** (0, −1)

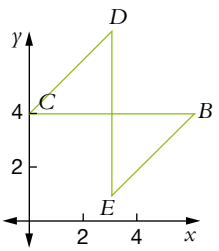
3 **a** Seek wisdom seek knowledge.
b Discipline your children while they are young enough to learn.

4 **a** **i** (1·0, 0·8) **ii** (−2·5, 2·8)
iii (3·1, 2·4) **iv** (−2·3, −1·8)
b **i** *F* **ii** *I* **iii** *E* **iv** *H*

5

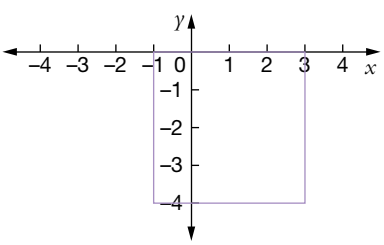


6 **a** (6, 4) **b** (0, 4) **c** (3, 7)
d (3, 1) **e** (−3,4) **f** (3,−4)
g Two triangles have been drawn.



7 **a** (−1, 4) **b** (3, −1) **c** (3, −4) **d** (−3, 4)
8 **a** (0, −2) **b** (−2, 0) **c** (0, 2) **d** (2, 0)
9 **a** (4, −3) **b** (−3, −4) **c** (−4, 3)
10 **a** *A*(2, 4) *B*(4, 3) *C*(2, 2) *D*(0, 3)
E(−3, −1) *F*(3, −1) *G*(3, −2) *H*(−3, −2)

b



Exercise 7:04

1 **a**

Coordinates chosen	Result	Score	Total so far
(0,0)	Discover a lost city	8	8
(0,4)	Lose your compass	−4	4
(2,1)	(miss)	−1	3
(1,0)	Bridge destroyed	−3	0
(1,3)	Photograph gnu	2	2
(3,4)	Bitten by snake	−7	−5

b

Coordinates chosen	Result	Score	Total so far
(3,1)	Lost in the jungle	−5	−5
(3,2)	Meet friendly locals	6	1
(0,2)	Photograph zebra	1	2
(2,2)	Shoot yourself in foot	−6	−4
(4,1)	(miss)	−1	−5
(1,1)	Photograph tiger	4	−1

2 **a**

Coordinates chosen	Score	Total so far
(0, 4)	−4	−4
(1, 4)	−1	−5
(2, 4)	3	−2
(3, 4)	−7	−9
(4, 4)	−8	−17

b

Coordinates chosen	Score	Total so far
(4, 4)	−8	−8
(3, 3)	−1	−9
(2, 2)	−6	−15
(1, 1)	4	−11
(0, 0)	8	−3

c

Coordinates chosen	Score	Total so far
(3, 0)	−1	−1
(3, 1)	−5	−6
(3, 2)	6	0
(3, 3)	−1	−1
(3, 4)	−7	−8

3 **a**

Score	Total so far
−1	−1
4	3
−2	1
1	2
−5	−3

b

Score	Total so far
1	1
−6	−5
−4	−9
−3	−12
5	−7

c

Score	Total so far
−1	−1
−1	−2
−1	−3
−1	−4
−1	−5

d

Score	Total so far
−8	−8
1	−7
2	−5
3	–
4	2

4 **a** a loss of 4 **b** a gain of 8 **c** a loss of 1
d a loss of 5 **e** a loss of 17 **f** a loss of 1
g −7 **h** 3 **i** −3 **j** 0 **k** −15
l −2 **m** −5 **n** 14 **o** 7 **p** −13
q −3 **r** −9 **s** 0 **t** −2 **u** −1
v 0

Prep quiz 7:05

1 1 **2** −3 **3** 0 **4** −1 **5** 5 **6** −2
7 0 **8** −5 **9** −10 **10** −2

Exercise 7:05

1 **a** −4 **b** 2 **c** 0 **d** −7 **e** 11 **f** 2
g −3 **h** 4 **i** −5 **j** −7 **k** −17
l −8 **m** −13 **n** −9 **o** −8 **p** −10
q 1 **r** −6 **s** 1 **t** 93 **u** 8 **v** 0
w −5 **x** −6

2 **a** 10 − 30 = −20 **b** −10 + 20 = 10
c 10 + 30 = 40 **d** −10 − 20 = −30
e 5 − 50 = −45 **f** −20 + 40 = 20
g −53 + 20 = −33 **h** 33 − 100 = −67
i −4 + 33 = 29 **j** −6 + 33 = 27

3 **a** 2 **b** 1 **c** 0 **d** −1 **e** −2
f −3 **g** −2 **h** −3 **i** −7 **j** −3
k −2 **l** −10 **m** −2 **n** −7 **o** −6
p −8 **q** −10 **r** −4 **s** −10 **t** −18
u −60 **v** −50 **w** −60 **x** −21

4 **a** 0 **b** 0 **c** 0 **d** 0 **e** 0 **f** 0
g 0 **h** 0 **i** 0 **j** 0 **k** 0 **l** 0
m 0 **n** 0 **p** 0 **q** 0 **r** 0

5 **a** 3 **b** −2 **c** 7 **d** 3 **e** −2 **f** 7
g 2 **h** −4 **i** 5 **j** 2 **k** −4 **l** 5

6 **a** −12 **b** 3 **c** −2 **d** −5 **e** 4
f −3 **g** −8 **h** −1 **i** −6 **j** −7
k −2 **l** 2 **m** 0 **n** 1 **o** −2 **p** −4

Exercise 7:06

1 **a** 7 **b** 9 **c** 6 **d** 16 **e** 8 **f** 12
g 11 **h** 23 **i** −1 **j** 4 **k** 7
l −1 **m** 3 **n** −5 **o** 5 **p** −3
q 10 **r** −6 **s** 6 **t** −56 **u** 15
v −1 **w** 0 **x** 0

2 **a** 5 **b** 15 **c** 2 **d** 8 **e** 34 **f** 7
g 11 **h** 9

3 **a** 17 **b** −2 **c** 1 **d** 7 **e** −17
f 18 **g** −21 **h** −10 **i** 5 **j** 10
k 10 **l** −4 **m** 10 **n** 7 **o** 14
p −9 **q** 10 **r** 10 **s** 13 **t** 13
u 1 **v** 8 **w** −14 **x** −4

4 **a** 0·5 **b** 2·9 **c** −1 **d** −0·54 **e** −0·1
f −0·7 **g** $-\frac{6}{10} = -\frac{3}{5}$ **h** $-\frac{1}{2}$
i $\frac{3}{10}$ **j** $\frac{3}{8}$ **k** $\frac{2}{3}$ **l** −1

5 There are not enough coconuts to fill the order.
53 more coconuts are needed.

Prep quiz 7:07

1 4 × 6 **2** 3 × −7 **3** 3 × 8 **4** 3 × −2
5 3 × − 10 = −30 **6** 6 × −3 = −18
7 10 × −7 = −70 **8** −2, −4 **9** −3, −6
10 3, 6

Exercise 7:07

1 **a** −1 **b** −8 **c** −12 **d** −25 **e** −8
f −12 **g** −44 **h** −16 **i** 10 **j** 9
k 8 **l** 1 **m** 0 **n** 0 **o** 0 **p** 0
q −21 **r** −6 **s** 12 **t** −12 **u** −20
v 7 **w** −16 **x** 14

2 **a** −35 **b** −28 **c** −27 **d** −45 **e** 16
f 32 **g** 42 **h** −48 **i** 24 **j** −81
k −40 **l** −36 **m** −56 **n** −49 **o** 28
p −72 **q** 36 **r** −72 **s** 63 **t** −24
u 121 **v** 54 **w** −64 **x** 63

3 **a** 1 **b** 4 **c** 9 **d** 16 **e** 25
f 36 **g** 49 **h** 64 **i** −8 **j** 16
k −32 **l** 64

4 **a** −320 **b** 150 **c** −90 **d** −240
e −1·8 **f** −4 **g** −0·04 **h** 0·18
i −0·07 **j** −0·077 **k** −0·48 **l** 0·56

5 **a** −27 **b** −8 **c** −125 **d** 50 **e** 18
f 64 **g** −285 **h** −40 **i** −36 **j** −12
k 18 **l** −24

6 **a** negative **b** positive

Prep quiz 7:08

1 30 **2** −30 **3** 30 **4** −30
5 6 × 5 = 30 **6** 6 × −5 = −30 **7** −6 × −5 = 30
8 −6 × 5 = −30 **9** 5 **10** −2

Exercise 7:08

1 **a** 6 **b** −6 **c** −6 **d** 4 **e** −4
f −10 **g** −9 **h** 6 **i** −5 **j** −7
k 8 **l** −10 **m** 5 **n** −4 **o** −3
p −8 **q** 9 **r** −5 **s** −1 **t** −1
u 1

2 **a** −5 **b** −3 **c** 7 **d** −7 **e** 3
f −6 **g** −4 **h** 8 **i** −5 **j** −7
k 6 **l** 6 **m** 70 **n** −90 **o** 30

Prep quiz 7:09

1 −7 **2** −13 **3** −2 **4** 9 **5** −2
6 6 **7** −40 **8** 80 **9** −8 **10** 64

Exercise 7:09

1 **a** −2 **b** −1 **c** −14 **d** 4 **e** −12
f −10 **g** 90 **h** 1 **i** −1 **j** 1
k −9 **l** 0 **m** 4 **n** −14

2 **a** 5·2 **b** −10·1 **c** −1·8 **d** 4·22
e −90·44 **f** −5·35 **g** −38·1 **h** 0·25
i 0·09 **j** −10·3 **k** 1·04 **l** −0·01
m −1 **n** −1 **o** −1·6

3 **a** 1, −1, −3 **b** −2, 0, 2 **c** −32, −64, −128
d 32, −64, 128 **e** −6, 6, −6 **f** 4, −2, 1

4 **a** −2 **b** −12 **c** −5 **d** 5 **e** −2
f 5

5 **a** −11 **b** 8 **c** −4 **d** −18 **e** −15
f −6

6 **a** false **b** true **c** false **d** true
e true **f** true **g** true **h** true **i** true
7 −2 **8** **a** −7, 1, 4 **b** −3, 1, 4 **c** −5, 1, 4
9 **a** −38 **b** −1·02 **c** −23
10 **a** −7 **11** −2 **12** −9°C
13 Kooka won by 7 seconds
14 Sydney is 10 hours ahead
15 **a** −4 and −3 **b** 6 and −5
16 B: a loss of 0·1 kg

Fun spot 7:09

1

Operation	Start	× (-2)	+ 10	÷ 5	−3	+ 6
Answer	10	−20	−10	−2	−5	1

2

Operation	Start	−10	× 3	+ 8	÷ 2	+ 2
Answer	6	−4	−12	−4	−2	0

3

Operation	Start	+ 3	× (−5)	−5	÷ 2	+ 10
Answer	−2	1	−5	−10	−5	5

4

Operation	Start	−3	÷ 4	+ 4	× (−3)	+ 7
Answer	−5	−8	−2	2	−6	1

5

Operation	Start	−10	× 5	+ 8	÷ 2	−4
Answer	8	−2	−10	−2	−1	−5

Prep quiz 7:10

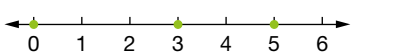
1 −14 **2** 3 **3** −6 **4** −1 **5** −9
6 −1 **7** 21 **8** $-\frac{10}{4}$ **9** 14 **10** −6

Exercise 7:10

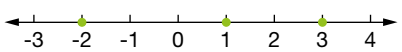
1 **a** −5 **b** −4 **c** 11 **d** −5 **e** 5
f −6 **g** −14 **h** 5 **i** −1 **j** −14
2 **a** $-3x$ **b** $-5y$ **c** $-a$ **d** $3m$ **e** $-10h$
f $10c$ **g** $-4y$ **h** $11p$ **i** $-6x$ **j** $-6x$
k $5t$ **l** $2d$ **m** 0 **n** $3x$ **o** $-2y$
3 **a** −20 **b** −2 **c** 35 **d** −18 **e** −9
f 4
4 **a** $-12a$ **b** $-5y$ **c** $-20k$ **d** $10m$ **e** $-36b$
f $-30n$
5 **a** 14 **b** 10 **c** −4 **d** 10 **e** 15
f 16 **g** −7 **h** 25 **i** −2
6 **a** −1 **b** 1 **c** −5 **d** 5 **e** −10
f −4 **g** −7 **h** −6 **i** −10 **j** −1
k 4 **l** −5 **m** −6 **n** 10 **o** 5
7 **a** 6 **b** 10 **c** −10 **d** −6 **e** 3
f 10 **g** −16 **h** −8 **i** −5 **j** 0
k −4 **l** −5 **m** 12 **n** −20 **o** −48

Diagnostic test 7

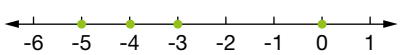
1 **a**



b



c



2 **a** (1, −1) **b** (1, 2) **c** (−1, 0)
3 **a** *G* **b** *D* **c** *F* **d** *H*
4 **a** −2 **b** 4 **c** −15
5 **a** −9 **b** −10 **c** −1
6 **a** −8 **b** −5 **c** −7
7 **a** 13 **b** −2 **c** −1
8 **a** −56 **b** 40 **c** −24
9 **a** 36 **b** 1 **c** 100
10 **a** −10 **b** 7 **c** −7
11 **a** 5 **b** −6 **c** −8
12 **a** −4 **b** 0 **c** 7
13 **a** …, 0, −3, −6 **b** … 0, 10, 20
c …, 81, −243, 729 **d** …, −4, −2, −1
14 **a** 1 **b** −4 **c** −1
15 **a** −3 **b** −11

ASSIGNMENT 7A

1 **a** 512 or +512 **b** −17 **c** −230
2 **a** −8 **b** +9 **c** −72 **d** +72
3 **a** {−4, −2, −1, 3, 5} **b** {−18, −14, −11, −9}
c {−0·4, −0·2, 0, 0·2, 0·5}
d $\{-1\frac{6}{8}, -1\frac{4}{8}, -1\frac{1}{8}, -1, -\frac{5}{8}\}$
4 **a** −33, −22, −23, 15 **b** −1·1, −0·9, 0·22, 1·8
c $-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{10}$
5 **a** {(−2, −4), (0,0), (2, 4), (3, 1)}
b {(−3, 1), (−1, −2), (2, 4), (3, 0)}
c {(−3, −3), (0, 2), (1, −2), (4, 1)}
6 **a** no **b** yes **c** yes
7 **a** 0 **b** −3 **c** 6 **d** 9 **e** −10
f 12 **g** 1 **h** −18 **i** 25 **j** −39
k −28 **l** −29 **m** 9 **n** −6 **o** 24
8 **a** −3 **b** −18 **c** 54

ASSIGNMENT 7B

1 (See ID Card 7 Answers, page 467.)
2 1 (green) and 6 (red), 6 (green) and 1 (red),
2 (green) and 5 (red), 5 (green) and 2 (red),
3 (green) and 4 (red), 4 (green) and 3 (red)
3 She made a profit of $30. **4** $216
5 5 games have been played. There have been either 3 draws and 2 wins or 3 wins, 1 draw and a loss in 5 games.
6 The ferry will leave at 9 am, 10:30 am, 12 pm, 2 pm and 3:30 pm. Lunch will be from 1:20 pm to 1:50 pm.
7 **a** 2005 **b** 2006 **c** 3 **d** 4
e $600 000 **f** $800 000
8 **a** **i** 2°C **ii** 0°C **iii** −4°C **iv** −3°C
v 4°C
b after 50 min **c** after 40 min and 70 min
d approximately −2°C
e between 40 min and 50 min after the start of timing; the change was −4°C
9 470 or −130

ASSIGNMENT 7C

1 **a** 0 **b** 8 **c** −2
2 **a** $\frac{15}{20}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$ **d** $\frac{8}{15}$
3 **a** 4 **b** 10 **c** 110 **d** 80
4 **a** 9·86 **b** 34·0 (or 34) **c** 0·05
d 0·984
5 **a** 13% **b** 30% **c** 150% **d** 25%
6 **a** 5^4 **b** $2^3 \times 7^2$ **c** 8×10^3
7 **a** $\frac{8}{3}$ or $2\frac{2}{3}$ **b** 56 **c** 1 **d** 12
8 **a** $6y + 14$ **b** $4x$ **c** 106 **d** x
9 **a** m^4 **b** $6x^4$ **c** $21y^3$
10 **a** straight angle **b** obtuse angle
c acute angle **d** revolution **e** reflex angle

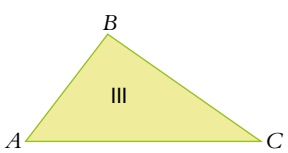
Chapter 8

Exercise 8:01

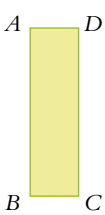
1 **a** It does not have straight sides.
b It has four sides.
c It does not have three straight sides.
2 square, rectangle, rhombus, parallelogram, trapezium, kite
3 **a** Two lines are parallel if they lie in the same plane and do not meet.
b square, rhombus, rectangle **c** trapezium
d trapeze at the circus
4 circle, oval (or ellipse) **5** oval
6

Shape	Number of sides	Number of angles	Number of diagonals
triangle	3	3	0
quadrilateral	4	4	2
pentagon	5	5	5
hexagon	6	6	9
octagon	8	8	20

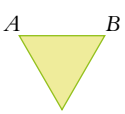
7 diamond **8** **a** I **b** II **c** IV
9



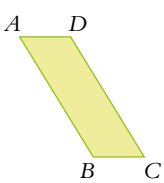
10 **a**



b



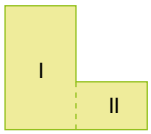
c



11 **a** rectangle **b** parallelogram **c** kite
12 **a** two triangles **b** triangle and trapezium
c two trapeziums
d rectangle and two trapeziums
13 **a** square, parallelogram, trapezium, kite
b square **c** 3 **d**



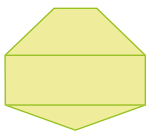
14 **a**



b

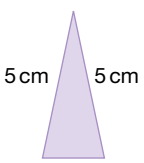


c

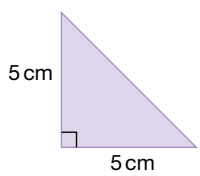


Exercise 8:02

1 **a** isosceles, right-angled
b equilateral, acute-angled
c scalene, obtuse-angled
d isosceles, obtuse-angled
e scalene, right-angled
f isosceles, acute-angled
2 **a**



b

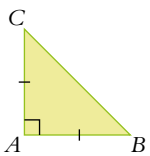


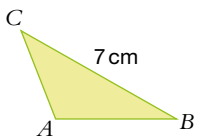
c

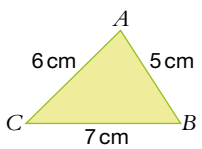


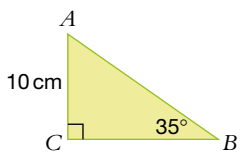
These answers have been scaled down to save space.
3 **a** no **b** yes, 2 angles are equal
c yes, 3 angles are equal **d** yes **e** yes
4 In all triangles the angles sum to 180°.
5 **c** The angle sum is 180°.
6 **a** The two shorter sides are not long enough to meet. The sum of their lengths needs to be longer than the length of the longest side.
b It would have to be longer than 2 units.
c **i** no **ii** yes **iii** yes **iv** no
7 Yes, the equal angles are opposite the equal sides.
8 **a** 80° **b** Yes, the equal sides are opposite the equal angles.
9 **a** 60° **b** yes **c** yes
10 isosceles
11 **a** AB, AC and BD, DC
b $\angle ABC$, $\angle ACB$; $\angle BAD$, $\angle CAD$; $\angle ADB$, $\angle ADC$
12 **a** equilateral, 60° **b** They are all equal in length.
c The largest angle is 90°; the triangle is right-angled and scalene.
d no **e** 45°
13 **a** $a = 40$ (angles of a triangle add to 180°)
b $b = 90$ (angles of a triangle add to 180°)
c $c = 30$ (angles of a triangle add to 180°)
d $d = 40$ (angles of a triangle add to 180°)
e $a = 65$ (equal angles of isosceles triangle)

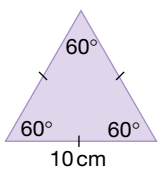
f $a = 6$ (equal sides of an isosceles triangle)
g $a = 60$ (angle of an equilateral triangle)
h $x = 6{\cdot}5$ (equal side of isosceles triangle)
i $b = 70$ (equal angles of isosceles triangle), $c = 40$ (angles of a triangle add to 180°)
j $a = 65$ (The triangle is isosceles and equal angles are opposite equal sides. So the three angles must be $a°$, $a°$ and 50°. As these must add up to 180°, $a = 65$)
k $a = 70$ (angles of a triangle add to 180°)
$x = 5{\cdot}8$ (triangle has two equal angles and the sides opposite equal angles are equal)
l $a = 45$ (angle of a right-angled isosceles triangle)

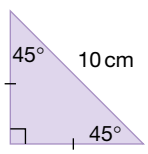
14 a


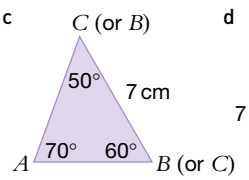
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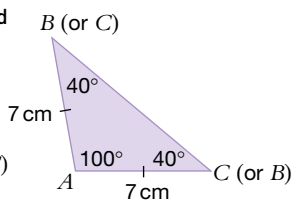
c


d


15 a


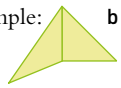
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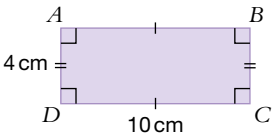
c


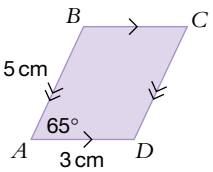
d


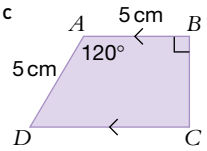
Exercise 8:03

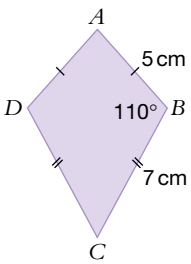
1 a square, rectangle, parallelogram, rhombus
b square, rectangle, parallelogram, rhombus
c square, rhombus
d square, rectangle
e Opposite angles are equal.
2 They have one pair of parallel sides.
3 a kite **b** It has two pairs of equal adjacent sides.
c One pair of opposite angles is equal.
d Yes. No, only one diagonal is bisected.
4 a yes **b** yes **c** yes
5 a yes **b** no **c** yes
6 a no **b** no **c** yes
7 a no **b** yes **c** yes
8 a square and rhombus **b** square and rhombus
9 The angles add up to 360°.
10 They add up to 360°.

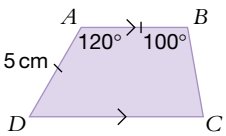
11 a Example:

b 180° **c** yes **d** 360°

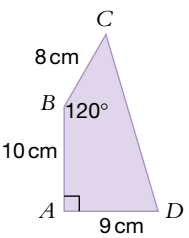
12 a


b


c


d


e


f


13 a rectangle or square **b** rhombus or square
c parallelogram, rhombus, rectangle, square
d rhombus **e** square **f** square or rectangle
14 a rectangle **b** square **c** parallelogram
15 a rectangle or square **b** rhombus or square
c square **d** rectangle or square
e square or rhombus **f** square **g** square

Exercise 8:04

1 a $a = 60$ **b** $a = 50$ **c** $b = 60$
d $b = 55$ **e** $b = 70$ **f** $c = 68$
g $c = 43$ **h** $c = 45$ **i** $d = 110$
j $d = 80$ **k** $d = 60$ **l** $e = 130$
m $e = 65$ **n** $e = 300$
2 (*Note*: Wording of reasons may vary. Not all possible answers are given.)
a $a = 35$ (the angles of a triangle add to 180°)
b $b = 115$ (adjacent angles on a straight line)
c $c = 40$ (angles of a quadrilateral add to 360°)
d $d = 270$ (angles at a point add to 360°)
e $e = 207$(angles of a quadrilateral add to 360°)
f $f = 67$ (equal angles of isosceles triangle)
g $g = 70$ (angles of a quadrilateral add to 360°)
h $h = 60$ (the angles of a triangle add to 180°)
i $x = 59$ (adjacent angles on a straight line)
j $a = 60$ (angles of equilateral triangle equal 60°)

k $a = 75$ (adjacent angles on a straight line)
$c = 70$ (adjacent angles on a straight line)
$b = 35$ (the angles of a triangle add to 180°)

l $a = 108$ (adjacent angles on a straight line)
$c = 74$ (the angles of a quadrilateral add to 360°)

Exercise 8:05

1 **a** 1 **b** 5 **c** 9 **d** 4

2 **a** cylinder **b** rectangular prism
c ball or sphere **d** cone

3 4, 5, 9 **4** 1, 2, 3 **5** 6, 7, 8 **6** 4, 5, 9

7

Solid	F	V	E	$F+V$
cube	6	8	12	14
triangular prism	5	6	9	11
rectangular pyramid	5	5	8	10
triangular pyramid	4	4	6	8

8 cylinder, sphere

9 cylinder, sphere, cone

10 **a** triangles and rectangles **b** triangles and square

11 **a** rectangular prism **b** triangular prism
c cylinder **d** triangular pyramid

12 **a** square **b** hexagon **c** trapezium
d pentagon

13 **a** cone **b** cylinder **c** square pyramid
d rectangular prism **e** sphere
f triangular prism **g** triangular pyramid
h hexagonal prism **i** pentagonal pyramid

14 **a** two square pyramids **b** cone and cylinder
c two cylinders
d rectangular prism and rectangular pyramid

15 **a** yes **b** no **c** yes **d** no **e** no
f yes

16

	F	V	E	$F+V$	$E+2$
a	5	6	9	11	11
b	6	8	12	14	14
c	4	4	6	8	8
d	5	5	8	10	10
e	8	6	12	14	14

Euler's formula holds in every case.

Exercise 8:06

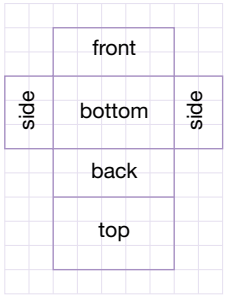
1 **a** C **b** A **c** B

2 **a** E **b** F **c** D

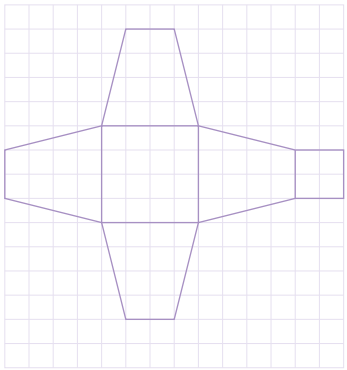
3 **a** B, D, E, F, H, I, J, K **b**



4



5



(*Note:* This is only one possible answer. Different nets would have different-sized trapeziums depending on the height of the pyramid.)

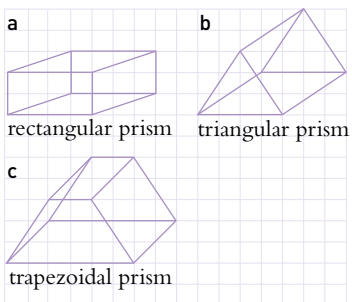
6



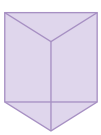
Exercise 8:07

2 square prism

3

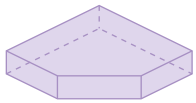


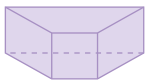
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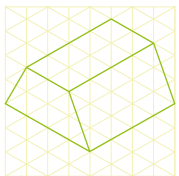


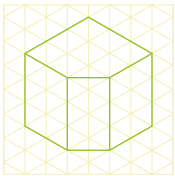
triangular prism

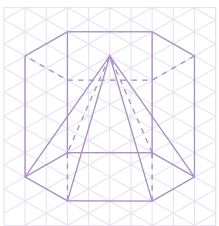
5 a 
hexagonal prism

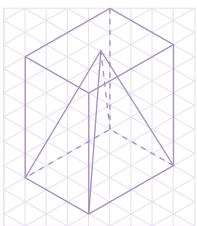
b 
pentagonal prism

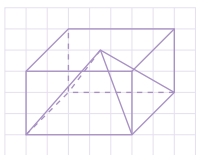
c 
trapezoidal prism

6 a 

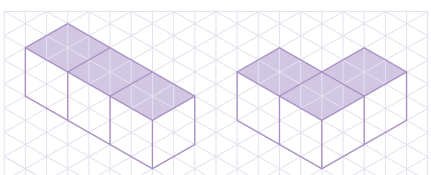
b 

7 a 

b 

c 

8 The question assumes that the cubes must be joined face to face. If this condition is applied there are only two possible arrangements.



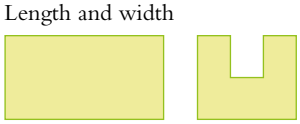
9 Cones and cylinders should be drawn.

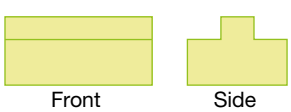
Exercise 8:08

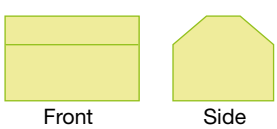
1 a B b E c D d A e F f C

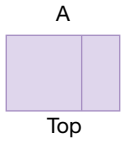
2 a A b A c front view, side view

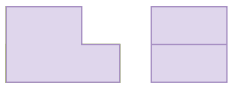
d Length and width

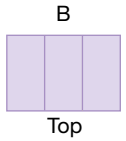
3 a 

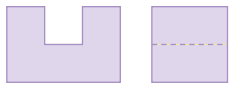
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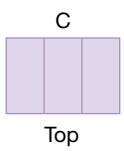
c 

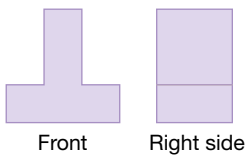
4 a 



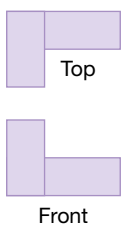
b 

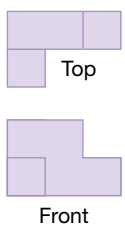


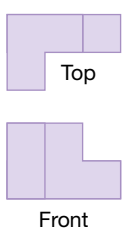
c 

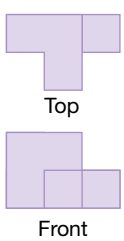


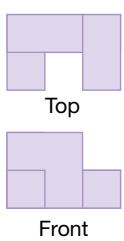
5 a 

b 

c 

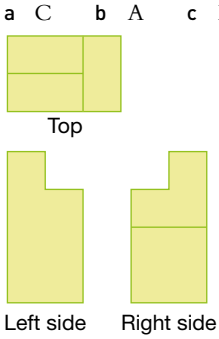
d 

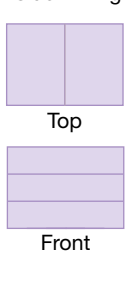
e 

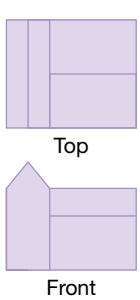
f 

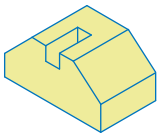
6 a B b A c C

7 a C b A c B

8 

9 a 

b 

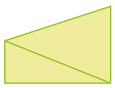
10 

Investigation 8:08

1 **a** 6 **b** 8 **c** 4 **d** 12

2 Euler's formula holds for each of the Platonic solids.

Diagnostic test 8

1 **a**  **b**  **c** 

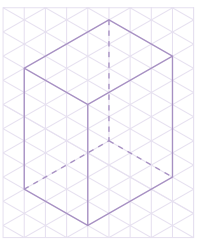
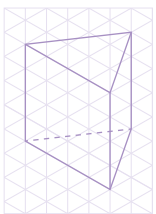
2 **a** right-angled **b** obtuse-angled
c acute-angled

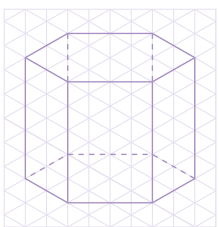
3 **a** $a = 80$ **b** $b = 105$ **c** $c = 51$

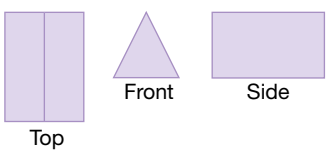
4 **a** $a = 105$ **b** $b = 75$ **c** $c = 89$

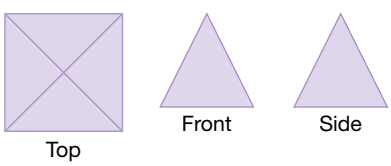
5 **a** cube **b** triangular prism
c triangular pyramid

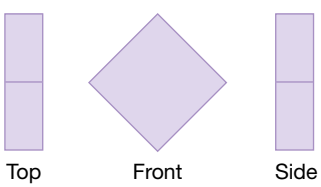
6 Examples:

a  **b** 

c 

7 **a** 

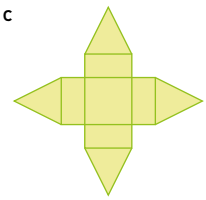
b 

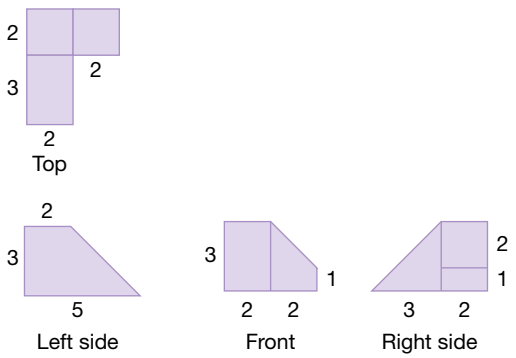
c 

ASSIGNMENT 8A

1 **a** **i** square **ii** rectangle
b **i** trapezium **ii** rectangle
c **i** trapezium **ii** trapezium

2 **a** $x = 79$ **b** $x = 63$ **c** $x = 54$

3 **a** square prism, square pyramid
b **i** 9 **ii** 16 **iii** 9 **c** 

4 

5 **a** **i** 60° **ii** 60° **iii** 60°
b **i** right-angled **ii** isosceles **iii** equilateral
c **i** 6 cm **ii** 6 cm

ASSIGNMENT 8B

1 **a** Largest number is 54 321, smallest number is 12 345
b Fifth largest number is 54 132, fifth smallest number is 12 543

2 6:50 am **3** blue **4** 4 **5** 2375 mm

6 **a** yes **b** about 43 kg
c about 168 cm and 189 cm
d about 63 kg and 81 kg **e** about 2·5 kg

ASSIGNMENT 8C

1 **a** 1·3 **b** 5·75 **c** 9·75 **d** 15·338

2 **a** 0·6 **b** 0·8 **c** 5·38 **d** 3·98

3 **a** 0·3 **b** 0·04 **c** 23 **d** 1·24

4 **a** 63 **b** 3 **c** 125 **d** 0·28

5 **a** 5·5 **b** 0·55 **c** 0·055 **d** 0·55

6 **a** 1·3 **b** 0·13 **c** 13 **d** 1·3

7 **a** true **b** true **c** true **d** true

8 **a** $a = 45$ **b** $b = 88$ **c** $c = 135$
d $a = 52, x = 102$ **e** $b = 77, y = 77$
f $c = 273, d = 73$

9 **a**

M	1	2	5	10
L	5	6	9	14

b

B	0	1	2	4
P	4	6	8	12

c

m	2	3	4	10
n	0	6	14	104

d

x	1	2	3	4
y	3	4	5	6

e

m	0	1	2	3
n	6	5	4	3

f

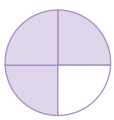
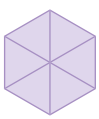
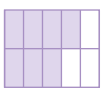
M	2	3	4	5
P	5	7	9	11

10 **a** $y = 15$ **b** $y = 4$ **c** $y = 9$ **d** $y = 8$

Chapter 9

Exercise 9:01

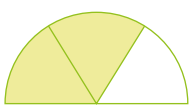
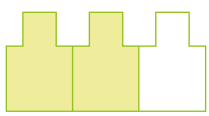
1 **a** $\frac{3}{4}$ **b** $\frac{2}{5}$ **c** $\frac{1}{6}$

2 **a**  **b**  **c** 

3 **a** $\frac{7}{20}$ **b** $\frac{7}{12}$ **c** $\frac{7}{15}$

4 $\frac{8}{24} = \frac{4}{12} = \frac{2}{6} = \frac{1}{3}$

5 **a** 12 **b** 8 **c** 6 **d** 4 **e** 3 **f** 16
g 18 **h** 20 **i** 9

6 **a**  **b** 
c 

7 **a** 14 **b** 4 **c** 14 **d** 2 **e** $2\frac{1}{2}$
f 3 **g** $1\frac{1}{2}$ **h** 4 **i** $3\frac{1}{2}$

8 **a** \$12.60 **b** 230 **c** 22 500 000
9 **a** 1130 L **b** 4520 L **10** **a** 10 **b** 50
11 $\frac{1}{3}$

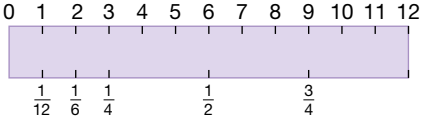
Exercise 9:02

1 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{4}$ **e** decreases

2 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$

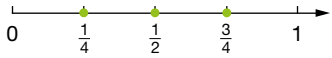
3 **a** 6 **b** 3 **c** 4 **d** 2 **e** 1 **f** 9

4 0 1 2 3 4 5 6 7 8 9 10 11 12

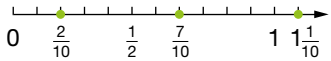


5 $\{\frac{3}{10}, \frac{7}{10}, \frac{9}{10}, 1\frac{1}{10}, 1\frac{3}{10}\}$

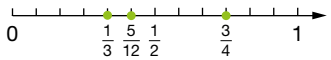
6 **a**



b



c

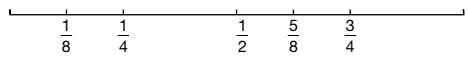


7 **a** $\frac{1}{2} > \frac{3}{10}$ **b** $\frac{9}{10} < 1\frac{1}{10}$ **c** $1\frac{1}{2} > \frac{13}{10}$

8 **a** $\frac{1}{4}, \frac{3}{10}, \frac{1}{2}$ **b** $\frac{1}{5}, \frac{1}{3}, \frac{3}{4}$ **c** $\frac{1}{2}, \frac{7}{10}, \frac{3}{4}$

9 **a** $\frac{2}{2} = \frac{5}{5} = \frac{10}{10}$ **b** $\frac{5}{10}$ **c** $\frac{2}{10}$ **d** $\frac{4}{10}$
e $\frac{6}{10}$ **f** $\frac{8}{10}$

10



11 **a** your estimate, 60 mm **b** your estimate, 40 mm
c your estimate, 80 mm **d** your estimate, 72 mm
12 **a** D **b** A **c** B **d** C **e** F
f E **g** G

Exercise 9:03

1 **a** $\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$ **b** $\frac{1}{4} = \frac{2}{8} = \frac{25}{100}$
c $\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$ **d** $\frac{3}{2} = \frac{6}{4} = \frac{15}{10}$
e $\frac{3}{4} = \frac{6}{8} = \frac{75}{100}$ **f** $\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$

2 **a** 8 **b** 30 **c** 12 **d** 30 **e** 12 **f** 35
3 **a** 12 **b** 9 **c** 16 **d** 32 **e** 2 **f** 36
4 **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{1}{3}$ **d** $\frac{5}{8}$ **e** $\frac{7}{20}$ **f** $\frac{1}{3}$
g $\frac{1}{3}$ **h** $\frac{5}{7}$ **i** $\frac{3}{5}$

5 **a** $\frac{1}{3}$ **b** $\frac{1}{18}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $\frac{7}{18}$
f $\frac{3}{4}$ **g** $\frac{1}{72}$ **h** $\frac{1}{9}$ **i** $\frac{11}{18}$

6 **a** $\frac{1}{5}, \frac{1}{2}, \frac{7}{10}$ **b** $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$ **c** $\frac{1}{2}, \frac{11}{24}, \frac{5}{8}$
d $\frac{5}{12}, \frac{2}{3}, \frac{3}{4}$ **e** $\frac{73}{100}, \frac{3}{4}, \frac{4}{5}$ **f** $\frac{1}{8}, \frac{11}{40}, \frac{3}{10}$

7 **a** < **b** = **c** > **d** < **e** >
f <

8 **a** $1\frac{3}{4}$ **b** $1\frac{3}{8}$ **c** $1\frac{1}{2}$ **d** $1\frac{3}{5}$
e $1\frac{3}{10}$ **f** $7\frac{23}{100}$

9 **a** $\frac{7}{2}$ **b** $\frac{11}{4}$ **c** $\frac{4}{3}$ **d** $\frac{32}{3}$ **e** $\frac{48}{5}$
f $\frac{767}{10}$

10 **a** $\frac{4}{5}$ **b** $\frac{4}{5}$ **c** $\frac{11}{20}$ **d** $1\frac{1}{2}$ **e** 1
f $1\frac{1}{20}$ **g** $\frac{3}{4}$ **h** $1\frac{2}{5}$ **i** $1\frac{1}{3}$

11 **a** $\frac{3}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{2}$ **d** 1 **e** 1
f $\frac{9}{20}$ **g** $\frac{47}{100}$ **h** $\frac{1}{3}$ **i** $\frac{13}{16}$

12 a $1\frac{3}{4}$ b $4\frac{3}{10}$ c $6\frac{2}{3}$ d $\frac{1}{4}$ e $\frac{7}{10}$
f $\frac{1}{3}$

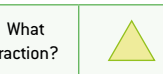
13 a $1\frac{1}{2}$ b $2\frac{1}{4}$ c 3 d $2\frac{1}{4}$ e $1\frac{1}{2}$
f 3

14 a $\frac{4}{5}$ b 1 c $4\frac{2}{3}$ d $\frac{9}{10}$ e $5\frac{2}{5}$
f $3\frac{1}{5}$ g $1\frac{17}{100}$ h $1\frac{1}{2}$ i $9\frac{1}{2}$

15 a 5 b 20 c 3 d 15 e 80
f 21 g 105 h 300 i 990

16 a $\frac{3}{4}$ b $\frac{5}{8}$ c $\frac{7}{10}$ d 0 e 0
f 0 g $\frac{9}{10}$ h $\frac{7}{8}$ i true
j true k yes l no m 6 n 6
o yes p yes

Investigation 9:03

What fraction?				
	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$
	3	1	$\frac{1}{2}$	$\frac{3}{4}$

Prep quiz 9:04

1 9 **2** 4 **3** $1\frac{1}{12}$ **4** $\frac{5}{12}$ **5** $\frac{12}{15}$
6 $\frac{10}{15}$ **7** $1\frac{7}{15}$ **8** $\frac{2}{15}$
9 $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$
10 $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25} = \frac{6}{30}$

Exercise 9:04

1 a $\frac{9}{10}$ b $\frac{5}{6}$ c $\frac{7}{8}$ d $\frac{11}{20}$ e $\frac{7}{10}$
f $\frac{5}{12}$ g $\frac{7}{15}$ h $\frac{19}{30}$ i $\frac{57}{100}$

2 a $\frac{7}{10}$ b $\frac{7}{15}$ c $\frac{1}{8}$ d $\frac{23}{30}$ e $\frac{1}{4}$
f $\frac{7}{20}$ g $\frac{9}{20}$ h $\frac{53}{100}$ i $\frac{3}{10}$

3 a $\frac{2}{5}$ b $\frac{1}{3}$ c $\frac{1}{2}$ d $\frac{3}{10}$ e 1
f $\frac{1}{4}$ g $1\frac{1}{5}$ h 1 i 0 j $\frac{1}{20}$
k 0 l 2

4 a $\frac{9}{10}$ b $1\frac{3}{10}$ c $\frac{9}{10}$ d $\frac{3}{5}$ e $\frac{4}{5}$
f $1\frac{1}{5}$ g $1\frac{1}{12}$ h $\frac{11}{12}$ i $1\frac{5}{12}$ j $\frac{5}{6}$
k $1\frac{1}{6}$ l $1\frac{5}{6}$ m $\frac{17}{20}$ n $1\frac{3}{20}$
o $1\frac{11}{20}$ p $1\frac{9}{20}$ q $1\frac{4}{15}$ r $1\frac{1}{4}$
s $1\frac{1}{10}$ t $\frac{19}{30}$ u $\frac{4}{5}$

5 a $\frac{2}{5}$ b $\frac{1}{5}$ c $\frac{3}{10}$ d $\frac{7}{40}$ e $\frac{9}{40}$
f $\frac{3}{40}$ g $\frac{3}{10}$ h $\frac{1}{12}$ i $\frac{1}{6}$
j $\frac{11}{20}$ k $\frac{1}{20}$ l $\frac{1}{20}$ m $\frac{3}{20}$ n $\frac{4}{15}$
o $\frac{1}{10}$ p $\frac{11}{30}$ q $\frac{2}{5}$ r $\frac{11}{50}$ s $\frac{5}{24}$
t $-\frac{1}{4}$ u $\frac{19}{100}$

6 a $\frac{11}{12}$ b $\frac{2}{9}$ c $\frac{19}{24}$ d $\frac{11}{50}$ e $\frac{37}{100}$
f $\frac{11}{24}$ g $\frac{7}{24}$ h $\frac{61}{100}$ i $\frac{2}{3}$
j $\frac{89}{120}$ k $\frac{13}{28}$ l $\frac{7}{36}$

Prep quiz 9:05

1 8 **2** 7 **3** 10 **4** 6 **5** 14 **6** 20
7 12 **8** 10 **9** 15 **10** 10

Exercise 9:05

1 a $1\frac{3}{4}$ b $3\frac{1}{5}$ c $7\frac{1}{10}$ d $13\frac{1}{2}$
e $5\frac{1}{3}$ f $9\frac{7}{8}$ g $10\frac{1}{4}$

2 a $5\frac{7}{10}$ b $6\frac{7}{8}$ c $2\frac{3}{4}$ d 6 e 10
f 4 g $8\frac{1}{2}$ h $13\frac{4}{5}$ i $7\frac{1}{2}$

3 a $2\frac{1}{4}$ b $4\frac{1}{10}$ c $3\frac{1}{2}$ d $3\frac{4}{5}$
e $13\frac{9}{10}$ f $11\frac{1}{4}$ g $6\frac{17}{24}$ h $7\frac{9}{20}$
i $3\frac{33}{40}$ j $4\frac{19}{40}$ k $12\frac{1}{6}$ l $6\frac{11}{30}$
m $9\frac{7}{15}$ n $5\frac{9}{10}$ o $12\frac{33}{100}$ p $7\frac{17}{30}$
q $8\frac{3}{4}$ r $7\frac{19}{120}$

4 a $\frac{9}{10}$ b $\frac{3}{10}$ c $\frac{1}{4}$ d $\frac{3}{8}$ e $\frac{7}{10}$
f $\frac{3}{20}$ g $\frac{7}{100}$ h $\frac{3}{4}$ i $\frac{11}{15}$

5 a $1\frac{9}{10}$ b $4\frac{7}{10}$ c $6\frac{5}{8}$ d $3\frac{5}{12}$
e $8\frac{29}{30}$ f $2\frac{13}{15}$ g $9\frac{3}{10}$ h $\frac{19}{24}$
i $\frac{69}{100}$

6 a $2\frac{3}{4}$ b $5\frac{1}{2}$ c $5\frac{2}{5}$ d $4\frac{1}{10}$
e $6\frac{5}{12}$ f $1\frac{71}{100}$ g $\frac{3}{8}$ h 4 i $\frac{1}{2}$

7 a $4\frac{3}{10}$ b $3\frac{5}{8}$ c $3\frac{3}{10}$ d $2\frac{5}{12}$
e $9\frac{1}{30}$ f $6\frac{3}{8}$ g $\frac{3}{20}$ h $\frac{5}{16}$ i $\frac{1}{6}$

8 a $3\frac{5}{8}$ b $4\frac{7}{10}$ c $4\frac{2}{3}$ d $4\frac{5}{6}$
e $7\frac{19}{30}$ f $6\frac{23}{40}$ g $2\frac{177}{200}$ h $\frac{199}{200}$
i $6\frac{11}{20}$

Challenge 9:05

1 4 **2** 9 hours **3** 31

Prep quiz 9:06

1 $2\frac{5}{8}$ **2** $2\frac{4}{5}$ **3** $\frac{2}{5}$ **4** $\frac{2}{5}$ **5** $\frac{9}{4}$
6 $\frac{11}{3}$ **7** $\frac{9}{5}$ **8** $\frac{1}{3}$ **9** $\frac{1}{4}$ **10** $\frac{1}{12}$

Exercise 9:06

1 a $\frac{1}{6}$ b $\frac{9}{20}$ c $\frac{9}{16}$ d $\frac{3}{40}$

2 a $\frac{1}{100}$ b $\frac{21}{100}$ c $\frac{7}{100}$ d $\frac{63}{100}$
e $\frac{3}{100}$ f $\frac{27}{100}$ g $\frac{77}{1000}$ h $\frac{3}{1000}$
i $\frac{63}{1000}$

3 a $\frac{1}{4}$ b $\frac{1}{8}$ c $\frac{1}{6}$ d $\frac{3}{20}$ e $\frac{9}{40}$
f $\frac{3}{16}$ g $\frac{3}{25}$ h $\frac{35}{48}$ i $\frac{63}{80}$

4 a $\frac{2}{5}$ b $\frac{1}{5}$ c $\frac{3}{8}$ d $\frac{1}{5}$ e $\frac{1}{4}$
f $\frac{1}{10}$ g $\frac{5}{8}$ h $\frac{1}{3}$ i $\frac{7}{20}$ j $\frac{3}{5}$
k $\frac{27}{40}$ l $\frac{1}{6}$ m $\frac{9}{16}$ n $\frac{9}{40}$ o $\frac{27}{50}$
p $\frac{9}{32}$ q $\frac{1}{2}$ r $\frac{3}{8}$

5 a 1 b 1 c 1 d 1 e 1 f 1
g 1 h 1 i 1

6 a $1\frac{1}{8}$ b $1\frac{2}{3}$ c $1\frac{1}{4}$ d $1\frac{5}{8}$ e $\frac{14}{15}$
f $2\frac{1}{10}$ g $4\frac{3}{8}$ h $4\frac{1}{5}$ i $2\frac{4}{5}$
j $2\frac{5}{8}$ k $4\frac{1}{20}$ l $4\frac{1}{6}$ m $18\frac{2}{5}$
n $4\frac{1}{5}$ o $3\frac{1}{25}$ p 8 q $14\frac{2}{5}$
r $14\frac{1}{4}$ s 1 t 1 u 1

Prep quiz 9:07

1 3 **2** 4 **3** 11 **4** 5 **5** 3 **6** yes
7 4 **8** 10 **9** 9 **10** 11

Exercise 9:07

1 a 2 b 2 c $\frac{3}{11}$ d $1\frac{4}{5}$ e $2\frac{1}{3}$
f 4 g $\frac{1}{4}$ h $\frac{3}{5}$ i $\frac{7}{8}$

2 a $\frac{2}{1}$ b $\frac{3}{1}$ c $\frac{4}{1}$ d $\frac{8}{5}$ e $\frac{8}{7}$
f $\frac{10}{1}$ g $\frac{3}{2}$ h $\frac{5}{4}$

3 a $1\frac{1}{2}$ b $1\frac{7}{8}$ c $2\frac{4}{5}$ d $\frac{16}{25}$ e $\frac{12}{35}$
f $3\frac{1}{3}$ g $\frac{9}{16}$ h $\frac{1}{4}$ i $\frac{1}{7}$ j $1\frac{1}{4}$
k $1\frac{1}{8}$ l $1\frac{1}{20}$ m $\frac{5}{6}$ n 5 o $16\frac{2}{3}$
p $\frac{14}{25}$ q $3\frac{1}{8}$ r $1\frac{1}{5}$

4 a 20 b $3\frac{1}{5}$ c $1\frac{1}{2}$ d $2\frac{1}{2}$ e $3\frac{3}{4}$
f $5\frac{1}{3}$ g 9 h 8 i $9\frac{1}{3}$ j $1\frac{5}{6}$
k $7\frac{1}{2}$ l $6\frac{2}{5}$ m $\frac{7}{12}$ n $1\frac{11}{32}$
o $1\frac{7}{20}$ p $1\frac{1}{6}$ q $2\frac{2}{3}$ r $1\frac{25}{41}$
s $2\frac{7}{10}$ t $7\frac{7}{11}$ u $1\frac{69}{76}$

5 a increased b decreased

Prep quiz 9:08

1 $2\frac{3}{4}$ **2** $3\frac{1}{7}$ **3** $\frac{7}{12}$ **4** $\frac{73}{10}$
5 $\frac{3}{4} = \frac{18}{24}, \square = 18$ **6** $\frac{9}{10} = \frac{90}{100}, \square = 90$ **7** $\frac{3}{8}$
8 $\frac{2}{3}$ **9** $1\frac{1}{5}$ **10** $\frac{3}{10}$

Exercise 9:08

1 a 20 cm b 15 min c 125 g d 30 cm
e 30° f 20 min g 2 g h 10 g
i 20 mL

2 a 80 cm b 45 min c 625 g d 270 cm
e 60° f 1 h 40 min g 26 g h 210 g
i 1980 mL

3 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{3}{10}$ d $\frac{4}{5}$ e $\frac{1}{4}$
f $\frac{1}{25}$ g $\frac{1}{4}$ h $\frac{3}{7}$ i $\frac{1}{50}$

4 a 4500 L b \$30 000 c \$480 000
d 1·8 kg e 6·3 kg f 7·875 km
g \$4.20 h 5670 g i 258 cm

5 a $\frac{3}{10}$ b $\frac{1}{5}$ c $\frac{1}{60}$ d $\frac{1}{1000}$ e $\frac{2}{45}$
f $\frac{1}{4}$ g $\frac{1}{2}$ h $\frac{17}{500}$ i $\frac{3}{100}$

Exercise 9:09

1 a $\frac{1}{2}$ b $\frac{5}{6}$ c $\frac{4}{5}$ d $\frac{7}{8}$ e $\frac{3}{5}$
f $\frac{3}{5}$ g $\frac{1}{4}$ h $\frac{2}{3}$

2 a $1\frac{1}{4}$ b $3\frac{1}{2}$ c $3\frac{1}{3}$ d $2\frac{2}{5}$
e $3\frac{3}{4}$ f $4\frac{1}{2}$ g $2\frac{1}{2}$ h $2\frac{2}{5}$ i 5

3 a $\frac{5}{4}$ b $\frac{13}{5}$ c $\frac{11}{3}$ d $\frac{35}{6}$ e $\frac{39}{8}$
f $\frac{22}{3}$ g $\frac{43}{4}$ h $\frac{107}{7}$

4 a $\frac{5}{6}$ b $\frac{13}{20}$ c $\frac{31}{40}$ d $\frac{1}{2}$ e $\frac{1}{6}$
f $\frac{3}{10}$ g $1\frac{1}{12}$ h $1\frac{3}{8}$ i $1\frac{7}{15}$
j $6\frac{1}{6}$ k $5\frac{8}{15}$ l $11\frac{9}{20}$ m $2\frac{3}{10}$
n $1\frac{5}{6}$ o $1\frac{22}{35}$

5 a $-\frac{4}{15}$ b $-\frac{3}{8}$ c $-\frac{13}{35}$ d $-\frac{3}{8}$
e $-1\frac{1}{6}$ f $-1\frac{9}{20}$

6 a $\frac{1}{10}$ b $\frac{1}{5}$ c $\frac{1}{2}$ d $\frac{3}{5}$ e $\frac{3}{16}$
f $\frac{3}{4}$ g $1\frac{1}{8}$ h 2 i $3\frac{3}{5}$ j $7\frac{13}{20}$
k $18\frac{12}{25}$ l $28\frac{1}{6}$

7 a $\frac{2}{3}$ b $2\frac{1}{2}$ c $1\frac{1}{2}$ d $2\frac{3}{16}$ e $\frac{2}{5}$
f $\frac{7}{8}$ g $4\frac{1}{2}$ h $\frac{3}{4}$ i $2\frac{4}{5}$ j $1\frac{71}{100}$
k $\frac{51}{76}$ l $1\frac{2}{3}$

8 a 45 L b 16 kg c 20 min d 80°
e 2·1 m f \$1250 g 315 g h \$20 400

Investigation 9:09

1 false **2** true **3** true **4** false

Exercise 9:10

1 100 g **2** 5, 14
3 a 936 b 624 c 39
4 a 36 b 40 c 4
5 $5\frac{1}{4}$ h **6** a 15 h b 4 h **7** 3750
8 a $8\frac{1}{4}$ pages b $1\frac{3}{8}$ pages per min
c $2\frac{3}{4}$ pages per min
9 600 **10** $\frac{1}{6}$ **11** \$2310 **12** $\frac{1}{8}$; $\frac{3}{8}$
13 \$600 **14** 106 **15** $\frac{3}{4}$ **16** $1\frac{3}{4}$ h
17 $\frac{3}{8}$ **18** $\frac{1}{6}$

Prep quiz 9:11

1 $\frac{1}{2}$ **2** $\frac{3}{4}$ **3** $\frac{1}{5}$ **4** $\frac{1}{3}$ **5** $\frac{7}{10}$

6 16 **7** 40 **8** 6 **9** 15 **10** $\frac{3}{5}$

Exercise 9:11

1 a 1:4 **b** 4:1 **c** 15 **d** $\frac{1}{5}$ **e** $\frac{4}{5}$

2 a 2:3 **b** 3:2 **c** $\frac{2}{5}$ **d** $\frac{3}{5}$

3 a 2:3 **b** 3:2 **c** \$40 **d** $\frac{2}{5}$ **e** $\frac{3}{5}$

4 a i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **b i** 1:2 **ii** 2:1

5 a $\frac{3}{4}$ **b** 1:3 **c i** 4 **ii** 12

6 a i $\frac{3}{10}$ **ii** $\frac{7}{10}$ **b i** 36 kg **ii** 84 kg

7 a i $\frac{1}{15}$ **ii** $\frac{14}{15}$ **b** 140

8 a 1:1000 **b i** 8 m **ii** 25 m **iii** 40 m

9 a i 1 km **ii** 5 km **iii** 7·3 km

b i 3 cm **ii** 4 cm **iii** 10·5 cm

10 a i $\frac{2}{9}$ **ii** $\frac{7}{11}$ **b** 2:9 **c** 24 **d** 1:3

Prep quiz 9:12

1 40 cents **2** 30 cents **3** 50 cents

4 8 apples for \$2.40 **5** \$8 **6** \$96 **7** \$160

8 \$4 **9** \$3 **10** 5 pairs for \$15

Exercise 9:12

1 a \$7 **b** \$1.05 **c** \$17.50 **d** \$12

e \$11.25

2 a i \$12 **ii** \$12.50 **b** Supplier A

3 a \$3.60, \$3.50 **b** \$3.50 (8 m for \$28)

4 a \$2.20, \$2.10 **b** 500 g

5 a \$7.68 **b** \$405 **c** \$40.32

6 a 750 g for \$17 **b** 100 g for \$3.50

c 400 g for \$12 **d** 150 g for \$4.50

e 5 m for \$12 **f** 100 m for \$170

g 50 m for \$9 **h** 500 g for \$15.50

i 7 t for \$400 **j** 500 g for \$31

k 4·2 kg for \$8 **l** 50 g for \$2.20

Diagnostic test 9

1 a $\frac{37}{40}$ **b** $1\frac{3}{20}$ **c** $\frac{13}{30}$

2 a $\frac{7}{20}$ **b** $\frac{13}{20}$ **c** $\frac{1}{20}$

3 a $5\frac{3}{20}$ **b** $10\frac{7}{10}$ **c** $4\frac{1}{8}$

4 a $2\frac{2}{5}$ **b** $4\frac{13}{20}$ **c** $11\frac{7}{10}$

5 a $\frac{63}{80}$ **b** $\frac{2}{5}$ **c** $\frac{27}{32}$

6 a $2\frac{2}{5}$ **b** $5\frac{5}{8}$ **c** 5

7 a $1\frac{4}{5}$ **b** $1\frac{1}{5}$ **c** $1\frac{7}{8}$

8 a 1200 km **b** 2100 mL **c** 1 h 20 min

9 a $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** $\frac{3}{50}$

10 a 1:2 **b** 2:1 **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

11 a $\frac{1}{10}$ **b** $\frac{9}{10}$ **c** 6

12 a \$6.40 **b** 10 kg for \$18 **c** 40 cans for \$110

ASSIGNMENT 9A

1 a $1\frac{3}{8}$ **b** $\frac{1}{5}$ **c** $3\frac{11}{12}$ **d** $1\frac{7}{8}$

2 a $\frac{3}{10}$ **b** $\frac{1}{6}$ **c** $3\frac{1}{3}$ **d** $15\frac{3}{10}$

3 a i $\frac{9}{16}$ **ii** $6\frac{1}{4}$ **b i** $\frac{3}{4}$ **ii** $2\frac{1}{2}$

4 a $1\frac{1}{2}$ **b** $\frac{7}{8}$ **c** $3\frac{6}{7}$ **d** 4

5 a 60 cm **b** \$1.20 **c** 20 hours

6 a $\frac{7}{12}$ **b** $1\frac{1}{6}$ **c** $1\frac{8}{17}$

7 a $3\frac{3}{14}$ m **b i** 14 **ii** 2 m

8 a i $\frac{3}{5}$ **ii** $\frac{2}{5}$ **b** 3:2

9 a i $\frac{9}{20}$ **ii** $\frac{11}{20}$ **b** 270

10 a 2 L for \$48

ASSIGNMENT 9B

1 8 **2** 24 **3** 2 min

4 fifteen each of 1 to 4; six of 5, five each of numbers 6, 7, 8, 9, 0

5 plumber is Farrant, electrician is Jones, carpenter is Bagnell

ASSIGNMENT 9C

1 a 6035 **b** 700 000 **c** 12 625

d 13 400 089

2 a $12a$ **b** $9x$ **c** $8m$ **d** $a + 6b$

e $2a + b$ **f** $9m - 2$

3 a 15 **b** 150 **c** 1·56 **d** 0·156

4 a 50 **b** 60 **c** 110 **d** 130

5 a 4 **b** 13 **c** −180 **d** 16

6 a 40 **b** 3500 **c** 25 **d** 0·42

7 a {1, 2, 3, 6, 9, 18} {1, 2, 3, 4, 6, 8, 12, 24}

b 6 **c** 72

8 a 52 **b** 52·4 **c** 52·36 **d** 52·365

9 a $4x + 12$ **b** $7a - 35$ **c** $n^2 + 8n$

d $x^2 - x$ **e** $12y + 20$ **f** $8k^2 - 12k$

10 a $a = 31$ **b** $b = 38$ **c** $c = 255$

Chapter 10

Prep quiz 10:01

1 12 m **2** 8·4 m **3** 7·1 m **4** 2·33 m

5 86 cm **6** 4·25 km **7** 10 m **8** 1·05 m

9 4 m **10** 4 m

Exercise 10:01

1 a 15 cm **b** 12 cm **c** 39 cm **d** 80 cm

e 36 cm **f** 13·8 cm **g** 10·6 cm **h** 4 cm

i 37·6 cm **j** 15·6 cm

2 **a** 12·8 cm **b** 224 mm **c** 8 cm **d** 17·6 cm
e 26 m **f** 24·8 cm **g** 16 cm
3 **a** 360 m **b** $2700
4 **a** 1080m **b** $918
5 5·8 km
6 **a** 15 cm **b** 10cm
7 **a** 10 cm **b** 15 m
8 **a** 3·7 cm **b** 1·6 cm
9 **a** 64 m **b** 50 m
10 **a** 48 cm **b** 82 cm **c** 96 cm
11 102 m

Exercise 10:02

1 **a** 5 units^2 **b** 18 units^2 **c** 15 units^2
d 19 units^2 **e** 15 units^2
2 **a** 6 cm^2 **b** 6 cm^2 **c** 15 cm^2 **d** 10 cm^2
e 13 cm^2
3 **a** 12 cm^2, 12 cm^2 **b** 14 cm, 16 cm **c** no
4 **a** $\frac{1}{2}$ cm^2 **b** $\frac{1}{2}$ cm^2 **c** $\frac{1}{4}$ cm^2 **d** $\frac{3}{4}$ cm^2
5 **a** 9 cm^2 **b** 6 cm^2 **c** 4 cm^2 **d** 6 cm^2
e $11\frac{1}{2}$ cm^2 **f** $9\frac{1}{2}$ cm^2
6 **a** 9 cm^2 **b** 8 cm^2 **c** 9 cm^2

Investigation 10:02

1 **a** 12 cm^2 **b** 20 cm^2
2–5 Answers will vary.

Prep quiz 10:03

1 5 **2** 12 **3** 8 **4** 12 **5** 24 **6** 20
7 22 **8** 530 **9** 0·4 **10** 0·7

Exercise 10:03

1 **a** 16 units^2 **b** 21 units^2
2 **a** 35 cm^2 **b** 32 cm^2 **c** 20 cm^2 **d** 11 cm^2
3 **a** 50 cm^2 **b** 24 cm^2 **c** 40 cm^2
d 28 cm^2 **e** 45 cm^2 f 1000 cm^2
4 **a** **i** 8 cm **ii** 10 cm **iii** 20 cm
b **i** 6 m **ii** 4 m **iii** 3 m
c Rectangles with integer side lengths:
$l = 50$ cm, $b = 1$; $l = 25$, $b = 2$; $l = 10$, $b = 5$.
They do not have the same perimeter.
5 **a** 64 cm^2 **b** 400 cm^2 **c** 49 mm^2
6 **a** 9 cm^2 **b** 100 mm^2 **c** 49 km^2 **d** 324 m^2
7 **a** 2 cm **b** 5 mm **c** 4 cm **d** 12 km
8 **a** 100 **b** 10 000 **c** 10 000
d 1 000 000 m^2; 1 km^2 = 1 000 000 m^2
9 **a** 180 mm^2 **b** 1·5 m^2 **c** 10 500 cm^2
d 10 cm^2 **e** 2·8 m^2
10 **a** 100 cm^2 **b** 70 cm^2 **c** 52 cm^2 **d** 73 cm^2
e 36 cm^2 **f** 69 cm^2
11 **a** 64 cm^2 **b** 58 cm^2 **c** 276 cm^2
12 **a** 10 ha **b** 18 ha
13 **a** width = 1 cm, area = 19 cm^2
b width = 2 cm, area = 36 cm^2
c width = 5 cm, area = 75 cm^2
d width = 8 cm, area = 96 cm^2
e length = 10 cm, width = 10 cm
14 **a** 9 m^2 **b** 7·5 m^2 **c** 20·25 m^2 **d** 27 m^2
e 63·75 m^2
15 **a** 486 cm^2 **b** 5984 m^2 **c** 210 m^2
d 0·27 m^2
16 **a** 13·12 m^2 **b** 14·24 m^2 **c** 40·48 m^2

Prep quiz 10:04

1 10 cm^2 **2** 15 cm^2 **3** 108 cm^2 **4** 15 cm^2
5 40 mm^2 **6** 300 cm^2 **7** 49 cm^2 **8** 38
9 31·2 **10** 3·07

Exercise 10:04

1 **a** 30 cm^2 **b** 27 cm^2 **c** 56 cm^2
2 **a** 25 cm^2 **b** 42 cm^2 **c** 90 cm^2
d 28 cm^2 **e** $49\frac{1}{2}$ cm^2 **f** 13·5 cm^2
3 **a** 16 cm^2 **b** 40 cm^2 **c** 40 cm^2
4 **a** 14·5 cm^2 **b** 30 cm^2 **c** 35 cm^2
5 **a** 8 cm **b** 8 mm **c** 24 cm
6 **a** 65 cm^2 **b** 28 cm^2 **c** 30 cm^2
d 65 cm^2 **e** 50 cm^2 **f** 136 cm^2
7 **a** 16 cm^2 **b** 7 cm^2 **c** $5\frac{1}{2}$ cm^2 **d** 14 cm^2

Prep quiz 10:05

1 8 cm^2 **2** yes **3** A **4** C **5** A
6 58 **7** 4·2 **8** 1·4 **9** 130 **10** 2·1

Exercise 10:05

1 **a** 56 cm^2 **b** 150 cm^2 **c** 27 cm^2
d 300 mm^2 **e** 126 cm^2 **f** 325 mm^2
2 **a** 52·8 cm^2 **b** 102 m^2 **c** 1·5 cm^2
d 21 m^2 **e** 9 cm^2 **f** 1·08 m^2
3 **a** 10 cm^2 **b** 12 cm^2 **c** 9 cm^2 **d** 21 m^2
4 **a** 90 cm^2 **b** 675 mm^2 **c** 49·6 cm^2
5 **a** 40 cm^2 **b** 122 cm^2 **c** 108 cm^2
d 140 cm^2

Prep quiz 10:06

1 24 cm^2 **2** 36 cm^2 **3** 10 cm^2 **4** 12 cm^2
5 14 cm^2 **6** 8 **7** 20 **8** 25 **9** 250
10 8·64 m^2

Exercise 10:06

1 $804 **2** 10 L **3** $1280 **4** $72 000
5 **a** 48 m^2 **b** 4·36 tiles
6 **a** 54 m^2 **b** 5·4 kg
7 **a** 50 m^2 **b** 300 m^2 **c** 250 m^2
8 **a** 32 000 m^2, 3·2 hectares **b** 3·7 hectares
9 1320 cm^2 **10** **a** 75 m^2 **b** 3000
11 7·04 m^2 **12** 0·78 m^2, $70.20
13 63 m **14** 96
15 **a** 840 **b** $3528
c $2751 could have been saved
16 **a** 706 **b** 106

Prep quiz 10:07

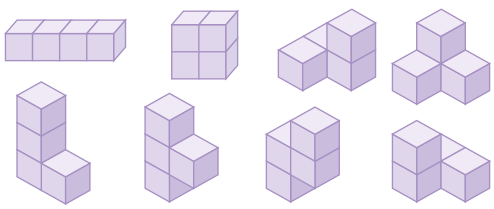
1 8 units^3 **2** 8 units^3 **3** 16 units^3
4 18 units^3 **5** 10 units^3 **6** 40 cm^2
7 36 cm^2 **8** 36 mm^2 **9** 36 mm^2 **10** 25 cm^2

Exercise 10:07

1 **a** 42 cm^3 **b** 30 cm^3 **c** 42 cm^3
d 48 cm^3 **e** 28 cm^3 **f** 36 cm^3 **g** 72 cm^3
h 72 cm^3 **i** 90 cm^3
2 **a** 6 cm^3 **b** 60 cm^3 **c** 12 cm^3 **d** 54 cm^3
e 125 cm^3 **f** 320 cm^3
3 **a** 27 cm^3 **b** 125 cm^3 **c** 729 cm^3
d 64 mm^3
4 **a** 210 m^3 **b** 84 m^3 **c** 1980 m^3
5 **a** 48 m^3 **b** 36 m^3
6 $A = 4$ unit^3, $B = 16$ unit^3
7 **a** 320 cm^3 **b** 8
8 **a** 96 cm^3 **b** 180 cm^3
9 **a** 1000 000 cm^3 **b** 1000 mm^3
10 **a** 16·2 m^3 **b** 7140 cm^3 **c** 11·44 m^3
d 0·1152 m^3

Investigation 10:07

1



2 **a** 3 **b** 4 **c** 6

Exercise 10:08

1 **a** 2000 **b** 5000 **c** 500 **d** 400
e 3500 **f** 100 000 **g** 50 **h** 1 000 000
2 **a** 3 **b** 9 **c** 7·5 **d** 8·3 **e** 0·5
f 0·3 **g** 20 **h** 0·05 **i** 2000 kL
j 11 000 kL **k** 1500 kL **l** 6250 kL
m 3 ML **n** 70 ML **o** 4·5 ML
p 23 ML **q** 3 ML **r** 68 ML
s 3·5 ML **t** 3·25 ML
3 **a** 500 mL **b** 50 L **c** 50 mL **4** 250 mL
5 6 **6** 14 **7** 20
8 10 **9** 600 mL bottle, by 10c per L
10 **a** 60 mL **b** 360 mL **c** 8·64 L
d 60·48 L **e** 3144·96 L
11 **a** 15 cm^3 **b** 8 mL **c** 183 cm^3 **d** 5 L
e 35 000 cm^3 **f** 2500 cm^3
g 15 kL **h** 4·5 m^3 **i** 8·2 kL
12 **a** 60 mL **b** 200 mL **c** 5000 mL
13 500 cm^3
14 **a** 1 000 000 cm^3 **b** 1000 L
15 **a** 1 000 000 cm^3
b **i** 1 000 000 mL **ii** 1000 L **iii** 1 kL
16 7·5 L **17** **a** 2400 cm^3 **b** 1·6 L
18 **a** **i** 500 m^3 **ii** 500 kL **iii** 0·5 ML
b 90 000 **c** 83·95 ML, $3 358 000

Diagnostic test 10

1 **a** 25 cm **b** 25 cm **c** 9·8 m or 980 cm
2 **a** 12 units^2 **b** 16 units^2 **c** 14 units^2
d 12 units^2
3 **a** 7 cm^2 **b** 2 cm^2 **c** 4 cm^2
4 **a** 36 cm^2 **b** 150 mm^2 **c** 21 m^2
5 **a** 22 cm^2 **b** 38 cm^2 **c** 54 cm^2
6 **a** 20 cm^2 **b** 10 cm^2 **c** 17·5 cm^2
7 **a** 40 cm^2 **b** 10 m^2 **c** 90 m^2
8 **a** 48 cm^3 **b** 105 cm^3
9 **a** 64 cm^3 **b** 84 cm^3 **c** 30 m^3
10 **a** 1000 mL **b** 2 kL **c** 3·5 L **d** 1500 L
11 **a** 3 cm^3 **b** 50 mL **c** 1000 cm^3 **d** 1 kL
12 **a** 48 mL **b** 105 mL

ASSIGNMENT 10A

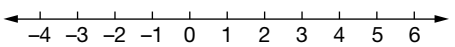
1 **a** 60 m^2 **b** 50 m^2 **2** **a** 10 **b** 14·4 L
3 200 boxes **4** 28 **5** $2240
6 **a** 0·25 m^2 **b** 4 **c** 72
7 150 L **8** 1·8 m^3

ASSIGNMENT 10B

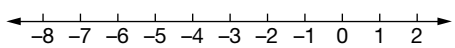
1 **a** 192 **b** 201 **2** **a** $ 600 **b** $480
3 20 m **4** 1840 **5** 30m
6 **a** 6 **b** 12 **c** 8

ASSIGNMENT 10C

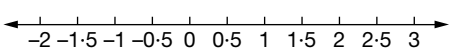
1 **a** $6m$ **b** $3mn$ **c** $12y$ **d** $2a + 3b$
2 **a** 18 **b** 81 **c** 45 **d** 4
3 **a** 1·4 **b** 0·2 **c** 0·48 **d** 0 **e** 3
4 **a** 12 **b** 24 **c** $2^4 \times 3 \times 5$
d 2, 3, 4, 5, 6, 8, 10
5 **a** -3 **b** -7 **c** 3 **d** 7
6 **a** 2^3 **b** 3^4 **c** a^2b^3 **d** $2a^3b$
7 **a** 5 **b** 6 **c** 8 **d** 4
8 **a** $\frac{3}{4}$ **b** 18 **c** $3\frac{1}{2}$ **d** $\frac{5}{3}$
9 **a** $6.00 **b** $5.00 **c** $20.00 **d** $100.00
10 **a**



b



c



d

11 **a** $a = 45, b = 60, c = 60$
b $a = 50, b = 80, c = 33$
c $a = 60, b = 30, c = 120$

Chapter 11

Exercise 11:01

1 **a** $\frac{9}{100}$ **b** $\frac{7}{100}$ **c** $\frac{1}{100}$ **d** $\frac{37}{100}$ **e** $\frac{99}{100}$
f $\frac{97}{100}$ **g** $\frac{11}{100}$ **h** $\frac{13}{100}$ **i** $\frac{33}{100}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{1}{10}$ **e** $\frac{1}{20}$
f $\frac{13}{50}$ **g** $\frac{11}{25}$ **h** $\frac{7}{20}$ **i** $\frac{1}{50}$

3 **a** 1 **b** $1\frac{1}{2}$ **c** $2\frac{1}{4}$ **d** $3\frac{1}{10}$ **e** $1\frac{3}{4}$
f 4 **g** $2\frac{3}{100}$ **h** $1\frac{3}{25}$ **i** $1\frac{9}{100}$

4 **a** 3% **b** 7% **c** 9% **d** 18% **e** 27%
f 69% **g** 50% **h** 44% **i** 90%

5 **a** 5% **b** 75% **c** 70% **d** 80% **e** 30%
f 26% **g** 200% **h** 150% **i** 310%

6 **a** 0·14 **b** 0·37 **c** 0·81 **d** 0·07 **e** 0·02
f 0·01 **g** 0·6 **h** 0·3 **i** 0·9 **j** 1
k 1·26 **l** 2·15

7 **a** 35% **b** 76% **c** 88% **d** 4% **e** 6%
f 5% **g** 40% **h** 60% **i** 50% **j** 187%
k 260% **l** 102%

8 **a** 70% **b** 72% **c** 175% **d** 4% **e** 19%
f 5% **g** 40% **h** 19% **i** 100% **j** 4%
k 10% **l** 63% **m** 80% **n** 97% **o** 75%
p 12·5% **q** \$6 **r** 7 kg
s **i** \$200 **ii** \$40 **iii** \$100 **iv** \$80

Prep quiz 11:02

1 $\frac{39}{100}$ **2** 3% **3** 35% **4** 0·16 **5** 13%
6 1 **7** 100% **8** 15 **9** 7 **10** 32·5

Exercise 11:02

1 **a** 60% **b** 90% **c** 14% **d** 85%
e 25% **f** 73% **g** 50% **h** 32%
i 15% **j** 75% **k** 80% **l** 9%
m $66\frac{2}{3}$% **n** $16\frac{2}{3}$% **o** $37\frac{1}{2}$% **p** $62\frac{1}{2}$%
q $66\frac{1}{2}$% **r** $1\frac{4}{5}$% **s** $6\frac{3}{4}$% **t** $20\frac{1}{3}$%
u 150% **v** 225% **w** $333\frac{1}{3}$% **x** $314\frac{2}{7}$%

2 **a** 37% **b** 71% **c** 95% **d** 4%
e 70% **f** 20% **g** 10% **h** 60%
i 34·5% **j** 12·5% **k** 37·5% **l** 82·5%
m 30% **n** 4·5% **o** 85% **p** 2%
q 180% **r** 110% **s** 250% **t** 360%
u 100% **v** 300% **w** 200% **x** 500%

3 **a** 1% **b** $\frac{1}{20}$ **c** $33\frac{1}{3}$% **d** 0·05
e 0·1 **f** $\frac{1}{8}$ **g** 0·2 **h** $12\frac{1}{2}$%
i $\frac{1}{3}$ **j** 0·125

4 **a** 3%, 0·03, $\frac{3}{100}$ **b** 15%, 0·15, $\frac{3}{20}$
c 90%, 0·9, $\frac{9}{10}$ **d** 150%, 1·5, $1\frac{1}{2}$
e 60%, 0·6, $\frac{3}{5}$ **f** $66\frac{2}{3}$%, $0·\dot{6}$, $\frac{2}{3}$
g 90%, 0·9, $\frac{9}{10}$ **h** 9%, 0·09, $\frac{9}{100}$
i 75%, 0·75, $\frac{3}{4}$ **j** $37\frac{1}{2}$%, 0·375, $\frac{3}{8}$

Exercise 11:03

1 **a** $\frac{9}{400}$ **b** $\frac{7}{400}$ **c** $\frac{9}{200}$ **d** $\frac{27}{400}$
e $\frac{51}{400}$ **f** $\frac{33}{200}$ **g** $\frac{59}{400}$ **h** $\frac{39}{200}$
i $\frac{49}{250}$ **j** $\frac{143}{1000}$ **k** $\frac{213}{200}$ **l** $\frac{435}{400}$

2 **a** 0·043 **b** 0·186 **c** 0·077 **d** 0·112
e 0·215 **f** 0·1725 **g** 0·1375 **h** 0·045
i 0·973 **j** 0·375 **k** 1·125 **l** 2·538

3 **a** $\frac{7}{8}$ **b** $\frac{1}{16}$ **c** $\frac{1}{30}$

Exercise 11:04

1 **a** 88 **b** 99 **c** 60 h **d** 24 mg
e 8 km **f** 6 g **g** 110 ha **h** 18 min
i 304 m **j** 560 L **k** 112 **l** 33 m

2 **a** \$60 **b** \$24 **c** \$18 **d** \$47 **e** \$51
f \$440 **g** \$1.20 **h** \$0.35 **i** \$1.02
j \$31.32 **k** \$36.63 **l** \$21.93

3 **a** 6 people are injured.
b Ben gave \$80.
c Heather has 18 doves.
d 5100 books are in the fiction section. 900 books are not in this section.
e We had 3880 worms left.
f 48 ha of parks were made.

Prep quiz 11:05

1 $\frac{1}{2}$ **2** $\frac{2}{7}$ **3** $\frac{1}{4}$ **4** $\frac{1}{2}$ **5** $\frac{1}{4}$
6 $\frac{13}{50}$ **7** $\frac{1}{5}$ **8** $\frac{1}{5}$ **9** $\frac{1}{3}$ **10** $\frac{15}{16}$

Exercise 11:05

1 **a** 10% **b** 5% **c** 80% **d** 70%
e 12%

2 **a** 50% **b** 25% **c** 50% **d** 75%
e 75% **f** 40% **g** $6\frac{1}{2}$% **h** $5\frac{1}{4}$%

3 **a** 22% **b** 60% are not Gouldian finches.
c $12\frac{1}{2}$% **d** $42\frac{1}{2}$% **e** $12\frac{1}{2}$%

4 **a** 25% **b** 5% **c** $18\frac{3}{4}$% **d** $51\frac{1}{4}$%
e 75% **f** $56\frac{1}{4}$%

5 **a** 4% **b** $33\frac{1}{3}$%
c $47\frac{1}{2}$% (*Note:* There are 40 years from 1947 to 1986 inclusive.)
d 20% **e** 5% **f** 100% − 16% = 84%

Exercise 11:06

1 **a** \$20 **b** \$70 **c** \$192 **d** \$60
e \$165 **f** \$720 **g** 36 m **h** 168 L
i 18 km **j** 17·5 mL **k** \$42 **l** 120 t

2 **a** 25% **b** 20% **c** 12·5% **d** 20%
e 5% **f** 2% **g** 20% **h** 8% **i** 15%
j 20% **k** 200% **l** 160%
3 **a** \$2.40 **b** \$7.20 **c** \$21.60 **d** \$5.78
e \$305.25 **f** \$6.24 **g** \$3.10 **h** \$18.00
i \$35.25 **j** \$18.75 **k** \$9.19 **l** \$16.33
4 **a** 2·8 m **b** 12·3 L **c** 328·5 g
d 0·6 km **e** 176·7 cm **f** 57·5 t
5 **a** 33·33% **b** 26·67% **c** 37·5%
d 61·11% **e** 66·67% **f** 25% **g** 61%
h 17·14% **i** 31·25%
6 **a** \$30 **b** \$105 **c** \$30 **d** \$3.15
e 129·6 m **f** 24·3 L **g** 600 kg **h** \$14
7 **a** 60% **b** 33·3% **c** 35% **d** 5%
e 70% **f** 24% **g** 19%
h 21·43% (to 2 dec. pl.)

Prep quiz 11:07

1 37% **2** 70% **3** 75% **4** 120%
5 \$3 **6** 36 m **7** 12 L **8** 40%
9 20% **10** 25%

Exercise 11:07

1 $\frac{1}{10}$; 10% **2** $\frac{11}{12}$; $91\frac{2}{3}$%
3 **a** 60% **b** $\frac{2}{5}$ **4** 24%; 25
5 **a** 80% **b** 20% **c** $\frac{2}{7}$ **d** 71%
6 **a** 520 **b** 52% **c** $\frac{12}{25}$
7 **a** $\frac{1}{5}$ **b** 20%
8 **a** 10 000 **b** 4 **9** 90% **10** \$43 000

Investigation 11:07

1 **a** 24·2% **b** 0·7% **c** 5·0%
2 **a** 46% **b** 11·7% **c** 3·7%
3 **a** 322 kJ/100 g **b** 3318 kJ/100 g
c 205 kJ/100 g

Diagnostic test 11

1 **a** $\frac{7}{100}$ **b** $\frac{3}{10}$ **c** $\frac{1}{4}$ **d** $\frac{3}{20}$
2 **a** 0·17 **b** 0·3 **c** 0·25 **d** 1·5
3 **a** 23% **b** 30% **c** 25% **d** 80%
4 **a** 15% **b** 70% **c** 9% **d** 35%
5 **a** $12\frac{1}{2}$% **b** $66\frac{2}{3}$% **c** $16\frac{2}{3}$%
d $4\frac{1}{2}$%
6 **a** 15·5% **b** 170% **c** 9·3%
d 235%
7 **a** $\frac{3}{40}$ **b** $\frac{1}{3}$ **c** $\frac{3}{8}$ **d** $\frac{1}{6}$
8 **a** \$7 **b** 12 m **c** 18 kg
9 **a** 25% **b** 80% **c** 37·5%
10 **a** \$98.80 **b** 29 m **c** 147 kg
11 **a** 12·5% **b** 26·6̇% **c** 31·25%

ASSIGNMENT 11A

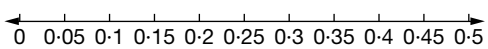
1 **a** 75% **b** 35% **c** $66\frac{2}{3}$% **d** 8%
e 80% **f** 135%
2 **a** $\frac{7}{10}$ **b** $\frac{3}{20}$ **c** $\frac{2}{5}$
d $\frac{3}{50}$ **e** $1\frac{1}{5}$ **f** $\frac{17}{100}$
3 **a** $\frac{1}{3}$ **b** $\frac{1}{8}$ **c** $\frac{5}{8}$
4 **a** 0·04 **b** 0·4 **c** 1·4
d 0·125 **e** 0·322 **f** 0·075
5 **a** \$45 **b** \$60 **c** 1·6 m
d 12 L **e** 24 g
6 **a** 25% **b** 40% **c** 60% **d** 75%
e 150%
7 **a** 25% **b** 30% **c** 20%
d 13% **e** 40%
8 **a** 12 **b** 60% **c** 10%
9 **a** \$60 **b** \$48
10 **a** 40% **b** 40% **c** 250%

ASSIGNMENT 11B

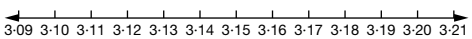
1 \$700 and \$960 **2** \$22.26 **3** 160 m and 20 m
4 \$3.40 + \$1.60; \$2.20 + \$1.20 + \$1.60 **5** $5{\cdot}2t$

ASSIGNMENT 11C

1 **a** 18 **b** 50 **c** 19 **d** 9
2 **a** 21 **b** 18 **c** 12 **d** 32
3 **a** 35 **b** 38 **c** **31** **d** 28
4 **a** 3 **b** −3 **c** 3 **d** −11
e 11 **f** 3
5 **a** $\frac{5}{100}$ **b** $\frac{5}{10}$ **c** 5 **d** $\frac{5}{1000}$
6 **a**



b



7 **a** 0·72 **b** 0·105 **c** 0·07 **d** 0·045
8 **a** 64 cm^2 **b** 120 cm^2 **c** 60 cm^2
9 **a** 30 **b** 22 **c** 65
10 **a** 60 000 cm^3 **b** 60 L

Chapter 12

Prep quiz 12:01

1 A: The result 'tails' is just as likely as the result 'heads'.
2 B: Buses don't often break down.
3 C: Most people on their birthdays are given a present.
4 B: Not many people score 100 runs when they play cricket.
5 A: A girl being born is just as likely as a boy being born.
6 impossible: 0%, The event will never happen.
certain: 100%, The event will always happen.
unlikely: between 0% and 50%, The event is more likely not to happen.

likely: between 50% and 100%, The event is more likely to happen than not.
even chance: 50%, The event is as likely to happen as not.
fifty-fifty chance: 50%, The event is as likely to happen as not.
no chance: 0%, The event will never happen.

Exercise 12:01

1 a For something to be impossible means that it will never happen.
b For something to be certain means that it will always happen.
c The chance that it will happen is 50%.
d It has between 0% and 50% chance of happening.
2 a Answers will vary. b Answers will vary.
c Answers will vary. d impossible
e certain f even chance
3 a Answers will vary. b d, f, e
4 a Event 1: unlikely; Event 2: even chance; Event 3: even chance; Event 4: impossible; Event 5: certain
b Event 1: even chance; Event 2: unlikely; Event 3: impossible; Event 4: certain; Event 5: likely
c Event 1: likely; Event 2: certain; Event 3: impossible; Event 4: unlikely; Event 5: likely
5 a B, A, C b A, C, B
c B is the least likely. A and C have the same probability, which is greater than that of B.
6 a The term should be researched. Answers will vary. The complementary event to something happening is for that event not to happen.
b Examples: 'we will win the game', 'we will not win the game'; or 'rolling a 6', 'not rolling a 6'

Prep quiz 12:02

1 head, tail
2 1, 2, 3, 4, 5, 6
3 yellow, blue, red, brown
4 blue, red, brown
5 blue, yellow
6 yellow, blue, green, brown
7 blue, brown, green, red, yellow
8 blue, yellow, red
9 yellow, brown, red, blue
10 yellow, green red

Exercise 12:02

1 a head, tail b point up, point down
c on its side, on its end d yes, no
e 1, 2, 3, 4, 5, 6 f red, black
2 a yes, the outcomes were random events
b a 6 c 'a number greater than 4'
d 1, 2, 3, 4, 5, 6 e yes f yes
g less than 50%
3 a red, yellow, green
b no, There are more red sections than yellow and green.
c red d green
4 a (three heads), (two heads and a tail), (one head and two tails) and (three tails)
b no, There is less chance of getting three of the same kind than two of one and one of the other.
c Results will vary.
5 a A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
b hearts ♥, diamonds ♦, clubs ♣, spades ♠
c A♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥, J♥, Q♥, K♥, A♦, 2♦, 3♦, 4♦, 5♦, 6♦, 7♦, 8♦, 9♦, 10♦, J♦, Q♦, K♦, A♣, 2♣, 3♣, 4♣, 5♣, 6♣, 7♣, 8♣, 9♣, 10♣, J♣, Q♣, K♣, A♠, 2♠, 3♠, 4♠, 5♠, 6♠, 7♠, 8♠, 9♠, 10♠, J♠, Q♠, K♠
6 Answers will vary. (See the definitions at the start of this section.)

Prep quiz 12:03

1 False. Having failed on the last three attempts, I am not likely to be successful on the fourth attempt. Also, getting tired or despondent has a detrimental effect.
2 True. The total shown on two dice can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.
3 True. All numbers between 1 and 10 have 1 digit.
4 True. For each toss, the chance of a head will always be $\frac{1}{2}$, regardless of the number of heads previously tossed.
5 False. There is 1 chance in 1000 that the counter chosen will be yellow.

Exercise 12:03

1 a 1, 2, 3, 4, 5, 6 b head, tail
c A(yellow), B(orange), C(red), D(blue), E(green)
d red, blue, green, pink, brown, yellow, orange
2 a 2 b 99 c 101 d 10
3 a $\frac{1}{6}$ b 0 c $\frac{1}{2}$ d $\frac{1}{2}$ e $\frac{1}{3}$
f $\frac{1}{2}$ g 1 h $\frac{5}{6}$
4 a i $\frac{1}{2}$ ii $\frac{1}{2}$ b i 50% ii 50%
5 a $\frac{1}{5}$ b $\frac{4}{5}$ c $\frac{2}{5}$ d 0
6 a $\frac{1}{4}$ b $\frac{3}{4}$ c $\frac{2}{4}$ (or $\frac{1}{2}$) d $\frac{4}{4}$ (or 1)
7 a 25% b 75% c 50% d 100%
8 a $\frac{3}{4}$, 75% b $\frac{1}{4}$, 25% c $\frac{2}{4}$ or $\frac{1}{2}$, 50%
d $\frac{0}{4}$ (or 0), 0%
9 a i $\frac{1}{2}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$
b no c $\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=1$
10 a A $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{8}$ b A $\frac{1}{2}$, B $\frac{4}{5}$, C $\frac{3}{8}$
c A $\frac{1}{4}$, B 0, C $\frac{1}{4}$ d A $\frac{3}{4}$, B 1, C $\frac{1}{2}$
e A $\frac{3}{4}$, B $\frac{4}{5}$, C $\frac{7}{8}$
11 a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{2}{3}$ d 0 e $\frac{2}{3}$

12 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$

13 **a** $\frac{1}{100}$ **b** $\frac{1}{10}$ **c** $\frac{1}{20}$ **d** $\frac{21}{25}$ (ie $\frac{84}{100}$)

e **i** 0. If I don't buy any tickets, it is impossible for me to win.

ii 100. Only if I buy all the tickets can I be certain to win.

14 **a** B **b** C **c** A

15 **a** $\frac{1}{13}$ **b** $\frac{1}{13}$ **c** $\frac{3}{13}$ **d** 1 **e** 0

f $\frac{2}{13}$ **g** $\frac{10}{13}$

16 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{13}$ **e** $\frac{3}{13}$

f $\frac{2}{13}$ **g** $\frac{51}{52}$

Exercise 12:04

1 25% **2** $37\frac{1}{2}\%$ **3** $37\frac{1}{2}\%$ **4** 75%

5 $54\frac{1}{6}\%$ **6** $65\frac{5}{8}\%$ **7** $62\frac{1}{2}\%$

8 $67\frac{68}{96}\%$ (or $67\frac{17}{24}\%$ or $\frac{65}{96}$ or $67{\cdot}708\dot{3}\%$) **9** 75%

10 $37\frac{1}{2}\%$ **11** $62\frac{1}{2}\%$ **12** 59%

Prep quiz 12:05

1 50% **2** $\frac{1}{2}$ **3** yes **4** true

5 true **6** false **7** false **8** head, tail

9 1, 2, 3, 4, 5, 6

10 hearts ♥, diamonds ♦, clubs ♠, spades ♣

Exercise 12:05

1 Wording of answers may vary, but the correct probability terms must be used. Examples:

a Parramatta football team is very likely to win this Saturday.

b Early showers are likely today.

c The drought has been severe and it is very unlikely it will rain tomorrow.

d We believe that our company is very likely to fail.

e This election has both the Labor party and the Liberal–National Coalition with an even chance of victory.

f Everybody overboard. The ship is certain to sink!

2 **a** $\frac{4}{11}$ **b** $\frac{7}{11}$ **c** $\frac{4}{11}$ **d** $\frac{7}{11}$ **e** 1

3 **a** $\frac{1}{5}$ **b** $\frac{4}{5}$ **4** 0·97

4 **a** $\frac{28}{100} = \frac{7}{25}$ **b** $\frac{24}{100} = \frac{6}{25}$ **c** $\frac{22}{100} = \frac{11}{50}$

d $\frac{26}{100} = \frac{13}{50}$ **e** yes **f** yes, but unlikely

g 12 blue, 14 red, 13 white, 11 yellow

5 **a** 19% **b** 35% **c** 45% **d** 55%

e yes

6 **a** 30 red, 70 blue **b** 15 red, 35 blue

7 **a** 20 heads, 20 tails

b Each number 1–6 to appear 10 times each

c 40 girls, 40 boys

8 **a** $\frac{1}{10}$ **b** $\frac{3}{10}$ **c** 0 **d** $\frac{4}{5}$ **e** $\frac{2}{5}$

f 1

9 **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{3}{8}$ **d** $\frac{1}{8}$ **e** $\frac{7}{8}$

10 **a** 30 **b** 90 **c** 75 **d** 300

Diagnostic test 12

1 **a** B, A, C **b** A, B, C

2 **a** head, tail **b** 1, 2, 3, 4, 5, 6

3 **a** 6 **b** 52

4 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{3}{4}$

5 **a** $\frac{2}{5}$ **b** 1 **c** 0 **d** $\frac{4}{5}$

6 **a** $\frac{3}{10}$ **b** 50 clubs **c** 150 heads

ASSIGNMENT 12A

1 **a** even chance **b** unlikely **c** impossible

d certain

2 **a** $\frac{70}{100}$ (or $\frac{7}{10}$ or 70%) **b** $\frac{25}{100}$ (or $\frac{1}{4}$ or 25%)

c $\frac{3}{100}$ (or 3%) **d** $\frac{97}{100}$ (or 97%)

3 **a** no, heads and tails are equally likely

b two heads, a head and a tail, two tails

c Suri, Luke, Arlo

4 **a** $\frac{4}{7}$ **b** $\frac{1}{7}$ **c** $\frac{2}{7}$ **d** $\frac{0}{7}$ or 0

e $\frac{6}{7}$ **f** $\frac{7}{7}$ or 1

5 **a** The event has a 50% chance of occurring. It is as likely to occur as not.

b Each event is equally likely to occur. Each would have a 25% chance of occurring.

6 **a** 40% **b** 20% **c** 50%

7 **a** 0·1 **b** 0·6 **c** 0·3 **d** 0·7

e 0·4 **f** 0·6

8 oval, square, rhombus (or diamond), triangle

9 $\frac{1}{13}$ **10** not choosing a King

ASSIGNMENT 12B

1 14 **2** 2 836 000 sheets

3 **a** 365 Days in a Year **b** 9 Lives of a Cat

c 52 Cards in a Standard Pack

d 26 Letters in the Alphabet

e 12 Months in a Year

f 1000 Years in a Millennium

g 64 Squares on a Chess Board

4 **a** A greater percentage of young men (40%) are disabled through accident than are young women (25%). 60% of young men and 75% of young women are disabled through sickness.

b Answers will vary. More men might be more likely to be involved in dangerous occupations or behaviours.

5 **a** blackcurrants **b** cauliflowers **c** 200 g

d 4 mg **e** 98 mg

ASSIGNMENT 12C

1 **a** 19 **b** 260 **c** 1903 **d** 2014
2 **a** 3 **b** 100 **c** 34
3 **a** 7294 **b** 57 000 **c** 8 **d** 3
e 45
4 **a** $4a + 3b$ **b** $24m - 4$ **c** $25x$
d $14x + 10$ **e** 35 **f** $36y + 3$
5 **a** 110 **b** 109 **c** 128 **d** 150
e 75 **f** 70
6 **a** −30 **b** −25 **c** 23 **d** −1
7 **a** $\frac{1}{12}$ **b** $8\frac{9}{10}$ **c** $7\frac{1}{2}$ **d** $1\frac{1}{2}$
8 **a** $\frac{2}{5}$ **b** 24
9 **a** \$85 **b** 0·204 m or 20·4 cm **c** 4%
10 $\frac{1}{26}$

Chapter 13

Prep quiz 13:01

1 4 **2** 7 **3** 4 **4** 9 **5** 4
6 false **7** true **8** false **9** true **10** true

Exercise 13:01

1 **a** $x = 9$ **b** $x = 10$ **c** $y = 20$ **d** $y = 11$
e $x = 50$ **f** $x = 8$ **g** $y = 12$ **h** $y = 10$
i $y = 16$ **j** $x = 7$
2 **a** What number plus 5 is equal to 7? $x = 2$
b What number plus 9 is equal to 21? $a = 12$
c 10 minus what number is equal to 7? $p = 3$
d What number minus 5 is equal to 2? $m = 7$
e 4 times what number is equal to 20? $n = 5$
f 7 times what number is equal to 56? $m = 8$
g What number divided by 2 is equal to 7? $q = 14$
h What number divided by 4 is equal to 4? $n = 16$
3 **a** **i** 3 **ii** 1 **iii** 8 **iv** 8
b **i** 8 **ii** 7 **iii** 5 **iv** 11
c **i** 3 **ii** 7 **iii** 13 **iv** 15
d **i** 18 **ii** 4 **iii** 19 **iv** 23
e **i** 1 **ii** 4 **iii** 9 **iv** 0
f **i** 6 **ii** 11 **iii** 9 **iv** 12
g **i** 10 **ii** 10 **iii** 14 **iv** 17
h **i** 12 **ii** 5 **iii** 25 **iv** 40
i **i** 2 **ii** 6 **iii** 3 **iv** 5
j **i** 5 **ii** 4 **iii** 3 **iv** 8
k **i** 0 **ii** 1 **iii** 4 **iv** 12
l **i** 6 **ii** 10 **iii** 12 **iv** 16
m **i** 2 **ii** 3 **iii** 6 **iv** 4
n **i** 18 **ii** 18 **iii** 15 **iv** 28
o **i** 8 **ii** 12 **iii** 8 **iv** 12
4 **a** correct **b** incorrect **c** incorrect
d incorrect **e** correct **f** incorrect
g correct **h** incorrect
5 **a** 4 **b** 10 **c** 6 **d** 4 **e** −3
f −3 **g** −2 **h** -5 **i** −6 **j** 3
k 2 **l** −1 **m** −2 **n** −6 **o** 3
p −24 **q** −6 **r** −5 **s** 3 **t** −4
u −7

Prep quiz 13:02

1 $x + 5$ **2** $3a$ **3** $\frac{y}{8}$ **4** $p - 2$ **5** $3x + 2$
6 $7(y - 2)$ **7** subtracting 8 **8** adding 5
9 dividing by 4 **10** multiplying by 3

Exercise 13:02

1 **a** × 9 **b** + 2 **c** − 7
d × 5, − 1 **e** ÷ 10 **f** + 6, ÷ 5
g ÷ 3, + 9 **h** − 8, ÷ 7 **i** ÷ (−5), + 3
j + 3, × 2 **k** × (−3), + 5
2 **a** ÷ 3 **b** × 2 **c** − 5
d + 7 **e** ÷ 10 **f** × 3
g − 8 **h** + 5 **i** − 1, ÷ 2
j + 2, ÷ 5 **k** − 3, ÷ 7 **l** + 4, ÷ 9
m − 6, ÷ 3 **n** − 7, × 2 **o** + 2, × 5
p × 3, − 7 **q** × 5, + 2 **r** − 5, ÷ (−3)
s − 8, ÷ (−2) **t** − 5, × (−3)

3 **a** $m \xrightarrow{\times 8} 8m$ **b** $p \xrightarrow{\div 7} \frac{p}{7}$
c $x \xrightarrow{+5} 5 + x$ **d** $q \xrightarrow{-10} q - 10$
e $x \xrightarrow{\times 3} 3x \xrightarrow{+2} 3x + 2$
f $n \xrightarrow{\times 5} 5n \xrightarrow{-3} 5n - 3$
g $p \xrightarrow{\times 6} 6p \xrightarrow{+7} 6p + 7$
h $a \xrightarrow{\times 9} 9a \xrightarrow{-1} 9a - 1$
i $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+5} \frac{x}{3} + 5$
j $a \xrightarrow{\div 4} \frac{a}{4} \xrightarrow{+7} 7 + \frac{a}{4}$
k $y \xrightarrow{+3} y + 3 \xrightarrow{\div 4} \frac{y+3}{4}$
l $n \xrightarrow{-1} n - 1 \xrightarrow{\div 5} \frac{n-1}{5}$
m $m \xrightarrow{+2} m + 2 \xrightarrow{\times 7} 7(m + 2)$
n $x \xrightarrow{-3} x - 3 \xrightarrow{\times 9} 9(x - 3)$
o $x \xrightarrow{+2} 2 + x \xrightarrow{\times 5} 5(2 + x)$
p $q \xrightarrow{-4} q - 4 \xrightarrow{\times 3} 3(q - 4)$

q $x \xrightarrow{\times(-2)} -2x \xrightarrow{+3} 3-2x$

r $a \xrightarrow{\times(-5)} -5a \xrightarrow{+7} -5a+7$

s $x \xrightarrow{\div(-3)} -\frac{x}{3} \xrightarrow{+6} 6-\frac{x}{3}$

t $p \xrightarrow{\times(-9)} -9p \xrightarrow{+10} 10-9p$

4 **a** $a-2$ **b** $4x$ **c** $\frac{m}{3}$

d $t+9$ **e** $5p, 5p+2$ **f** $6x, 6x-4$

g $y+1, 7(y+1)$ **h** $\frac{b}{4}, \frac{b}{4}+2$

i $q-3, \frac{q-3}{3}$ **j** $-2n, -2n+6$

k b **l** a **m** m

n n **o** $2x, x$ **p** $5p, p$

q $7t, t$ **r** $m+5, m$ **s** $\frac{a}{3}, a$

t $-2x, x$

5 **a** -6 **b** $\div 3$ **c** $+5$ **d** $\times 7$

e $\times 2, +3$ **f** $\times 5, -2$ **g** $\times 3, +6$ **h** $\times 7, -1$

i $+7, \div 5$ **j** $-4, \div 3$ **k** $\div 8, +5$ **l** $\div 3, -4$

m $+3, \times 2$ **n** $-9, \times 5$ **o** $\times(-6), +8$

p $\times(-3), +10$

6 **a** $+7$ **b** $\div 5$ **c** -6 **d** $\times 3$

e $\div(-4)$ **f** $-5, \div 2$ **g** $+1, \div 3$ **h** $-5, \div 7$

i $+6, \div 5$ **j** $-8, \div(-3)$ **k** $\times 7, -3$

l $-3, \times 7$ **m** $\div 5, -2$ **n** $\div 3, +1$ **o** $\times 5, +9$

7 **a** $\div(-3)$ **b** $\div(-8)$ **c** $\times(-4)$ **d** $\times(-2)$

e $-5, \div(-2)$ **f** $-3, \div(-1)$

g $\times 3, -4, \div(-2)$ **h** $\times 5, -9, \div(-1)$

8 **a** $\times 5, -3, \div 2$ **b** $\div 6, -7, \div 3$

c $\times 6, -7, \div(-3)$ **d** $-9, \times 3, \div 2$

e $\div 5, -2, \div(-3)$ **f** $\times 7, -5, \div(-1)$

g $-6, \times 3, \div(-5)$ **h** $\times 5, \div 7, -4, \div 3$

Prep quiz 13:03

1 x **2** x **3** a **4** m **5** dividing by 3

6 subtracting 7 **7** adding 1

8 multiplying by 5 **9** -3 **10** $\div 8$

Exercise 13:03

1 **a** $x=8$ **b** $p=2$ **c** $y=14$ **d** $n=3$

e $q=8$ **f** $k=9$ **g** $y=8$ **h** $m=9$

i $x=13$ **j** $x=3$ **k** $x=6$ **l** $p=12$

m $y=4$ **n** $y=2$ **o** $x=3$ **p** $n=6$

q $x=15$ **r** $m=16$

2 **a** $x=1{\cdot}5$ **b** $a=3{\cdot}8$ **c** $m=0{\cdot}7$

d $n=1\frac{1}{2}$ **e** $p=1\frac{3}{4}$ **f** $k=2\frac{1}{3}$

g $x=3{\cdot}6$ **h** $n=11{\cdot}7$ **i** $y=11{\cdot}4$

j $n=1\frac{3}{5}$ **k** $m=3\frac{1}{3}$ **l** $p=2\frac{1}{2}$

m $x=4\frac{1}{2}$ **n** $k=2\frac{1}{8}$ **o** $y=5\frac{1}{2}$

p $n=1{\cdot}2$ **q** $a=2{\cdot}5$ **r** $m=8{\cdot}4$

3 **a** $m=2$ or -2 **b** $x=6$ or -6

c $y=8$ or -8 **d** $x=9$ or -9

e $c=3$ or -3 **f** $y=1$ or -1

g $m=4$ or -4 **h** $x=10$ or -10

i $y=7$ or -7

4 **a** correct **b** incorrect **c** correct

d correct **e** correct **f** incorrect

g correct **h** incorrect **i** incorrect

j correct **k** incorrect **l** incorrect

5 **a** $x=-1$ **b** $m=-4$ **c** $n=-2$

d $p=-8$ **e** $q=-6$ **f** $y=-12$

g $a=5$ **h** $x=7$ **i** $p=5$ **j** $q=1$

k $y=0$ **l** $t=4$ **m** $m=-2$ **n** $p=-2$

o $w=-1$ **p** $n=-6$ **q** $q=-9$ **r** $m=2$

s $x=-6$ **t** $m=-80$ **u** $n=-196$

v $x=3$ **w** $y=-8$ **x** $x=-3$

Prep quiz 13:04

1 subtracting 8 **2** adding 5 **3** dividing by 3

4 multiplying by 2 **5** $-3, \div 5$ **6** $-1, \div 3$

7 $-7, \div 2$ **8** $+2, \div 8$ **9** $\times 3, -2$

10 $+4, \times 5$

Exercise 13:04

1 **a** $x=2$ **b** $a=2$ **c** $m=3$ **d** $n=3$

e $k=5$ **f** $t=2$ **g** $a=4$ **h** $w=1$

i $q=1$ **j** $x=9$ **k** $y=2$ **l** $a=2$

2 **a** $x=2$ **b** $x=10$ **c** $m=8$ **d** $x=11$

e $a=1$ **f** $n=13$ **g** $m=14$ **h** $n=19$

i $a=42$ **j** $y=6$ **k** $x=-4$ **l** $m=25$

m $a=10$ **n** $x=18$ **o** $m=4$

3 **a** $x=\frac{3}{2}$ **b** $a=\frac{5}{2}$ **c** $m=\frac{1}{3}$ **d** $n=\frac{6}{5}$

e $p=\frac{5}{2}$ **f** $q=\frac{3}{4}$ **g** $n=\frac{9}{5}$ **h** $y=\frac{2}{3}$

i $k=\frac{7}{2}$ **j** $x=\frac{2}{3}$ **k** $a=\frac{2}{3}$ **l** $a=\frac{3}{2}$

m $m=-\frac{3}{2}$ **n** $p=-\frac{6}{5}$

4 **a** $x=3$ **b** $m=2$ **c** $q=4$ **d** $x=11$

e $x=-1$ **f** $x=2$ **g** $a=-1\frac{1}{2}$

h $a=1\frac{1}{3}$ **i** $n=-\frac{1}{2}$

5 **a** $x=17$ **b** $m=24$ **c** $x=3\frac{1}{2}$ **d** $n=35$

e $a=3\frac{2}{3}$ **f** $a=-1$ **g** $y=-2$ **h** $m=2$

i $x=-4$ **j** $n=1\frac{2}{3}$ **k** $x=\frac{2}{3}$ **l** $q=-4$

m $x=50$ **n** $p=4$ **o** $x=3$ **p** $a=18$

q $a=6$ **r** $a=-3$ **s** $m=46$ **t** $m=1\frac{1}{5}$

u $a=-1$ **v** $p=-1$ **w** $q=-4$ **x** $x=-7$

Prep quiz 13:05

1 $+$ **2** $\times$ **3** $-$ **4** $-$ **5** $+$ **6** $\times$

7 $\div$ **8** $x=11$ **9** $a=17$ **10** $m=7$

Exercise 13:05

1 **a** $x + 4 = 13; x = 9$ **b** $9 + x = 17; x = 8$
c $x - 7 = 9; x = 16$ **d** $8x = 56; x = 7$
e $x - 7 = 3; x = 10$ **f** $x + 6 + 14 = 35; x = 15$
g $\frac{x}{7} = 12; x = 84$ **h** $9x = 117; x = 13$
i $\frac{96}{x} = 16; x = 6$ **j** $73 - x = 31; x = 42$

2 **a** $b + 75 = 220$; book = \$1.45
b $t + 17 = 30$; time to get dressed = 13 min
c $5{\cdot}25n = 63$; number = 12
d $H + 18 + 15 = 52$; Harry is 19
e $\frac{156}{n} = 12$; 13 children

3 **a** $3x + 5 = 17; x = 4$ **b** $2x - 10 = 4; x = 7$
c $\frac{x}{5} + 3 = 7; x = 20$ **d** $100 - 8x = 28; x = 9$

4 **a** $x + (x + 7) = 13$; numbers are 3 and 10.
b $x - (20 - x) = 6$; numbers are 7 and 13.
c $x + (x + 19) = 29$; smallest number is 5.
d $x + (x + 1) = 17$; numbers are 8 and 9.
e $x + (x + 1) + (x + 2) = 18$; smallest number is 5.
f $x + (x + 5) = 37$; Ahmed is 21.
g $x + 2x = 36$; Michelle has 12 toys.

5 **a** 9 **b** 14 **c** 10 **d** 4 **e** 10
f 12 **g** 8 **h** 9

6 **a** −7 **b** 0·9 **c** −1 **d** 7 **e** −1
f \$1.90 **g** 10, 12, 14

Exercise 13:06

1 The distance travelled needs to be known before the cost can be calculated. Lisa pays \$85 rent for the day. Yasar travelled 172 km. $(111 = 25 + \frac{K}{2})$

2 $2W + 2(W + 12) = 300$. The picture frame has width 69 cm and length 81 cm. (Complications will arise if we consider the width and join of the frame.)

3 $400 = 40 + 15r, r = 24$

4 $S = 3L - 20$. If $S = 13, L = 11$.

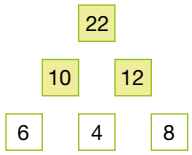
5 $N = 5C - 3$ (or $C = \frac{N + 3}{5}$). He will need 147 boards.

6 **a** 180 sales → \$60 000 profit,
210 sales → \$120 000 profit,
300 sales → \$300 000 profit,
170 sales → \$40 000 profit,
140 sales → − \$20 000 profit.
b In the last year there was a loss of \$20 000.
c For a profit of \$50 000, 175 sales are needed.
d The company breaks even with 150 sales.

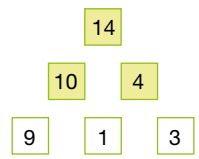
7 **a** $A = 3r^2$
b Area of circle ÷ 480 000 cm^2 = 48 m^2
c The actual formula is $A = \pi r^2$ (i.e. $A \div 3{\cdot}142r^2$).
d Answers will vary.

Fun spot 13:06

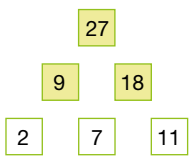
1



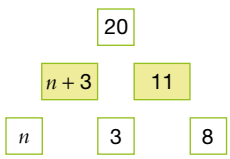
2



3

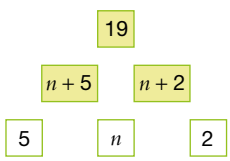


4



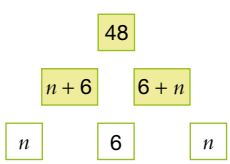
$(n + 3) + 11 = 20$
$\therefore\ n = 6$

5



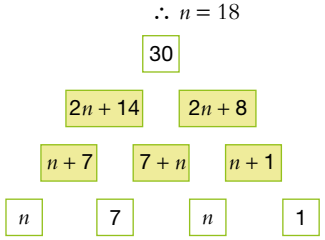
$(n + 5) + (n + 2) = 19$
$2n + 7 = 19$
$\therefore\ n = 6$

6



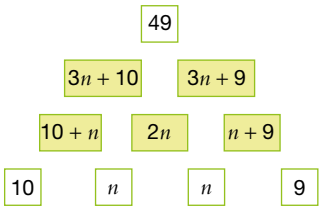
$(n + 6) + (6 + n) = 48$
$2n + 12 = 48$
$\therefore\ n = 18$

7



$(2n + 14) + (2n + 8) = 30$
$4n + 22 = 30$
$\therefore\ n = 2$

8



$(3n + 10) + (3n + 9) = 49$

$6n + 19 = 49$

$\therefore\ n = 5$

Diagnostic test 13

1 **a** x plus 7 equals 11; $x = 4$
b n plus 3 equals 3; $n = 0$
c y minus 6 equals 3; $y = 9$
d 9 subtract m equals 7; $m = 2$

2 **a** 5 times y equals 20; $y = 4$
b 4 times m equals 24; $m = 6$
c x divided by 3 equals 2; $x = 6$
d h divided by 6 equals 1; $h = 6$

3 **a** $\times 5$ **b** $\times 7, + 1$ **c** $- 3, \div 2$

4 **a** $- 3$ **b** $- 2, \div 6$ **c** $+ 7, \times 2$

5 **a** x **b** $3a, a$ **c** $m - 3, m$

6 **a** $+ 4$ **b** $\times 4, + 5$ **c** $+ 7, \div 2$
d $\div 4, - 5$

7 **a** $k = 3$ **b** $p = 10$ **c** $a = 11$
d $x = 32$

8 **a** $n = -4$ **b** $m = 3$ **c** $x = -3$
d $a = -18$

9 **a** $x = 2$ **b** $m = 1$ **c** $n = 12$ **d** $m = 5$

10 **a** $a = \frac{1}{3}$ **b** $x = 1\frac{3}{5}$ **c** $y = -\frac{3}{7}$
d $n = 4\frac{1}{3}$

11 **a** $n + 7 = 19; n = 12$ **b** $3n = 36; n = 12$
c $n - 13 = 23; n = 36$ **d** $\frac{72}{n} = 12; n = 6$

ASSIGNMENT 13A

1 **a** $x = 13$ **b** $a = 8$ **c** $a = 27$ **d** $m = 38$
e $n = 9$ **f** $n = 11$ **g** $n = 5$ **h** $n = 9$
i $x = 15$ **j** $n = 32$

2 **a** $x = -5$ **b** $a = -5$ **c** $a = -7$
d $m = -3$ **e** $n = -1$ **f** $n = -4$
g $n = -2$ **h** $n = -4$ **i** $x = -15$
j $n = -24$

3 **a** $x = 4$ **b** $a = 3$ **c** $y = 5$ **d** $n = 1$
e $m = 18$ **f** $h = 2$ **g** $x = 10$ **h** $a = 27$

4 **a** correct **b** incorrect **c** incorrect
d correct **e** correct **f** correct
g incorrect **h** correct

5 **a** $x = 1\frac{1}{3}$ **b** $a = 1\frac{5}{8}$ **c** $y = 2\frac{1}{2}$
d $n = -1\frac{2}{3}$

6 **a** $12n = 96, n = 8$ **b** $\frac{n + 10}{3} = 9, n = 17$

ASSIGNMENT 13B

1



2 3 **3** \$35.40 **4** \$4.20, \$1.80 **5** 6

ASSIGNMENT 13C

1 **a** 700 mm **b** 8·5 km **c** 96 cm

2 **a** 0·72 **b** 9·5 **c** 2·4 **c** 2·06

3 **a** (3,2) **b** (−2,−3) **c** (−5,0)
d (−2,3) **e** (0,−5)

4 **a** H **b** T **c** M **d** W **e** Z

5 **a** $\frac{4}{5}$ **b** $\frac{3}{10}$ **c** $\frac{10}{3} = 3\frac{1}{3}$ **d** $\frac{2}{5}$

6 **a** $\frac{2}{5}$ **b** 30% **c** 80% **d** 0·56

7 **a** $2x + 5y$ **b** $8a + 3$ **c** $12xy$ **d** $30mn$
e $2x$ **f** 2

8 **a** 4 **b** 2 **c** 11 **d** −60 **e** 27
f 50

9 **a** 7320 **b** 6048 **c** 67 400
d 3 050 000

10 **a**

Operation	Start	− 7	× 3	+ 11	− 35	÷ 6
Answer	5	−2	−6	5	−30	−5

b

Operation	Start	× 3	−45	÷ 9	+ 5	× 96
Answer	−3	−9	−54	−6	−1	−96

Chapter 14

Exercise 14:01

1 **a** numerical **b** numerical **c** categorical
d categorical **e** categorical **f** numerical
g numerical **h** categorical **i** numerical
j categorical

2 **a** continuous **b** discrete **c** discrete
d continuous **e** discrete **f** continuous
g continuous **h** discrete **i** discrete
j continuous

3 **a** ordered **b** distinct **c** distinct
d ordered **e** distinct **f** ordered

4 **a** numerical, continuous **b** categorical, ordered
c numerical, continuous **d** categorical, distinct
e numerical, distinct **f** categorical, distinct

5 **a** categorical **b** categorical **c** numerical
d numerical

Exercise 14:02

1 **a** Alexis **b** Casey's
c not a representative sample
d more time-consuming

2 **a** Fellow students are all in a particular age group.
b Survey people in the street.

3 **a** His sample is limited to students of his own school, who would all have a similar range of ages and experiences.
b He has not listed all sports. The order of sports could also influence the result.
c He could use stratified random sampling.
d Answers will vary. (Each person could be asked to write their favourite sport.)

4 They survey different samples of people.

5 **a** no
b Roll the dice a large number of times.
c The number of times a person rolls a dice is limited by the time available. We would get closer to the real probability if we rolled the dice millions of times. In this sense, it is like a sample.

6 The accuracy of the survey would depend on the size of the sample and the sample is limited to the readers of the magazine or website.

7 **a** What is MUNCH-O's preferred to?
b Which 5 dentists? **c** 20% more than what?
d Longer than what?

8 Year 8, 13; Year 9, 16; Year 10, 19; Year 11, 16; Year 12, 16

9 15–19 years, 39; 20–59 years, 281; 60–64 years, 21; 65 years and over, 59

10 Answers will vary.

Exercise 14:03

1 **a**

Outcome	Frequency
2	3
3	9
4	6
5	4
6	2
Total:	24

b 24 **c** 3 **d** 6 **e** 12

2 **a**

Shape	Tally	Frequency
circle	𝍸 ||	7
oval	𝍸	4
square	𝍸 |	6
star	𝍸 𝍸	10
triangle	𝍸 ||	7
Total:		34

b 34 **c** star **d** triangles

3 **a**

Outcome	Tally	Frequency
1	𝍸 |	6
2	||||	4
3	𝍸 ||	7
4	𝍸 |||	8
5	𝍸	5
6	𝍸 |	6
Total:		36

b 4 **c** 2 **d** no

4 **a** 10 **b** 4 **c**

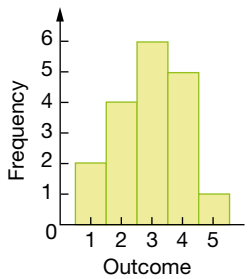
Outcome	Tally	Frequency
4	|	1
5	|||	3
6	||||	4
7	𝍸 ||	7
8	𝍸 |	6
9	𝍸 |	6
10	|||	3
Total:		30

d 7 **e** 8 **f** 15

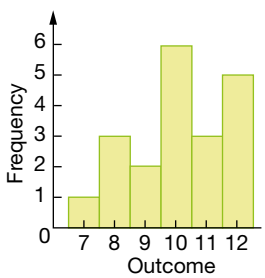
5 **a** 4 **b** 3 **c** 9 **d** 6 **e** 39 **f** 162

6 **a** 1 **b** 2 **c** 7 **d** 19

7 **a**



b



8 **a**

Outcome	Frequency
0	3
10	6
20	7
30	10
40	5
50	8
Total:	39

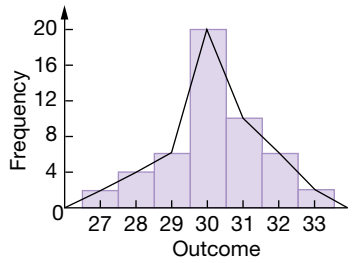
b 39 **c** 16

9 **a** 33, 27
b

Shape	Tally	Frequency
27	||	2
28	||||	4
29	𝍸 |	6
30	𝍸 𝍸 𝍸 𝍸	20
31	𝍸 𝍸	10
32	𝍸 |	6
33	||	2
Total:		50

c 12 **d** 18

e

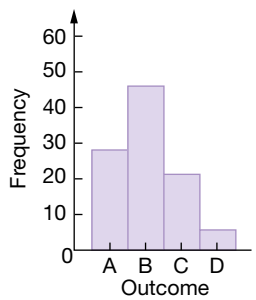


10 a

Responses	Tally	Frequency
strongly agree(A)	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	28
agree (B)	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	46
disagree (C)	𝍸 𝍸 𝍸 𝍸 |	21
strongly disagree (D)	𝍸	5
	Total:	100

b 28 **c** 5% **d** 74% **e** 26%

f



Exercise 14:04

1 a i 3 **ii** 3 **iii** 40 **iv** 5 **v** 3
vi 5 **vii** 7 **viii** 0·4 **ix** 6 **x** 5
b i 5 **ii** 7 **iii** 10 **iv** 17, 20 **v** 50
vi 8 **vii** 4 **viii** 1·1, 1·3 **ix** 17
x 148, 151
c i 4 **ii** 7 **iii** 25 **iv** 20 **v** 50
vi 9 **vii** 4 **viii** 1·2 **ix** 17 **x** 150
d i 3·8 **ii** 7·6 **iii** 26·7 **iv** 19·4 **v** 49·6
vi 9·2 **vii** 4·6 **viii** 1·2 **ix** 17 **x** 149·6

2 a 30·3 **b** 74·2 **c i** 40 **ii** 11 **d** 2

3 a no **b** the mean

4 a

Outcome	Frequency	*fx*
1	2	2
2	4	8
3	5	15
4	6	24
5	3	15
Total:	20	64

Mean = 3·2

b

Outcome	Frequency	*fx*
5	1	5
6	5	30
7	3	21
8	8	64
9	5	45
10	3	30
Total:	25	195

Mean = 7·8

c

Outcome	Frequency	*fx*
18	1	18
19	3	57
20	1	20
21	4	84
22	5	110
23	3	69
24	1	24
25	2	50
Total:	20	432

Mean = 21·6

5 a 4 **b** 8 **c** 22

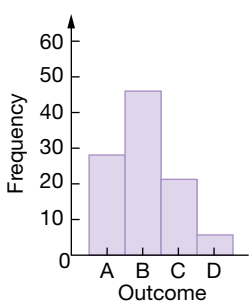
6 a 3 **b** 8 **c** 22

7 a

Outcome	Tally	Frequency	*fx*
0	|||	3	0
1	𝍸	5	5
2	||||	4	8
3	|||	3	9
4	||	2	8
5	|	1	5
	Total:	18	35

b i 5 **ii** 1 **iii** 2 **iv** $1{\cdot}9\dot{4}$ or $1\frac{17}{18}$

c



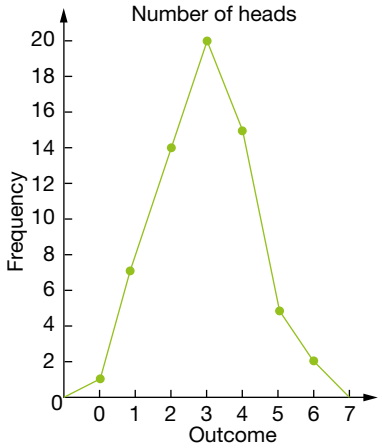
d highest column

8 a

Outcome	Tally	Frequency	*fx*
0	|	1	0
1	𝍸 ||	7	7
2	𝍸 𝍸 𝍸	14	28
3	𝍸 𝍸 𝍸 𝍸	20	60
4	𝍸 𝍸 𝍸	15	60
5	𝍸	5	25
6	||	2	12
	Total:	64	192

b 3 **c** 14 **d** 14

e



9 a

Outcome	Frequency	fx
0	3	0
1	4	4
2	6	12
3	5	15
4	3	12
5	1	5
6	2	12
Total:	24	60

b 2 **c** 2 **d** 2·5 **e** 12

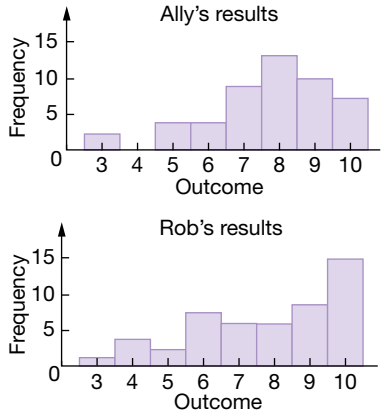
10 a Ally's results

Outcome	Tally	Frequency
3	||	2
4		0
5	||||	4
6	||||	4
7	𝍸 ||||	9
8	𝍸 𝍸 |||	13
9	𝍸 𝍸	10
10	𝍸 |||	8
	Total:	50

Rob's results

Outcome	Tally	Frequency
3	|	1
4	||||	4
5	|||	3
6	𝍸 ||	7
7	𝍸 |	6
8	𝍸 |	6
9	𝍸 |||	8
10	𝍸 𝍸 𝍸	15
	Total:	50

b



c Ally **i** 7 **ii** 8 **iii** 8 **iv** 7·74
Rob **i** 7 **ii** 10 **iii** 8 **iv** 7·76

d Rob's results were more impressive.

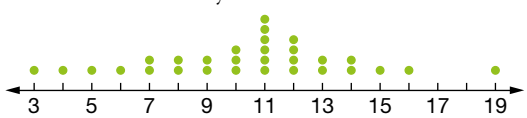
Exercise 14:05

1 a 19 **b** 20 − 6 = 14 **c** 17

d 6 is the only outlier. **e** 3

f An outlier is any score that is distant or separate from the main body of scores.

2



a 11 **b** 16 **c** 11 **d** 19 is an outlier.

e 6

3 a

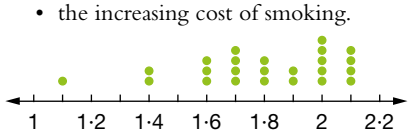
	Mode	Median	Mean	Range
2001	20	12	12·1	19
2011	0	4	7	22

b Overall they are smoking much less and, apart from the three who have died, four have given up smoking.

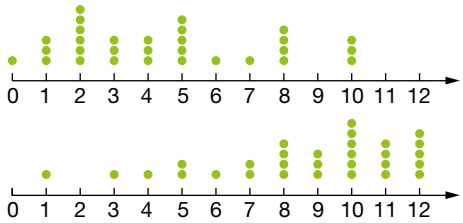
Factors could include:

- an increased awareness of the dangers of smoking
- decreased health that forces a reduction in smoking
- an awareness of reduced fitness because of smoking
- consideration for others
- restrictions on where people can smoke
- the increasing cost of smoking.

4



5 a



b The improvement made by particular individuals can no longer be seen.

c

	Mode	Median	Mean	Range
Beginning	2	4	4·5	10
End	10	9·5	8·7	11

Exercise 14:06

1 a

Stem	Leaf
1	5 9 7 6 8
2	6 6 9 8 6
3	3 8 5 6 0
4	7 0 7 1 7
5	1 8
6	2 2

b

Stem	Leaf
4	5 4 9 2
5	3 7 9
6	7 0 5 3
7	4 2 0 7 3
8	1 0 2 0 1 8
9	1 2

2 a

Stem	Leaf
1	5 6 7 8 9
2	6 6 6 8 9
3	0 3 | 5 6 8
4	0 1 7 7 7
5	1 8
6	2 2

median $= \frac{33+35}{2}$

$= 34$

b

Stem	Leaf
4	2 4 5 9
5	3 7 9
6	0 3 5 7
7	0 | 2 3 4 7
8	0 0 1 1 2 8
9	1 2

median $= \frac{70+72}{2}$

$= 71$

3 a

Stem	Leaf	Frequency (f)
0	8 6 4 1 9 2	6
1	5 8 9 1	4
2	0 7 6 9	4
3	6 3 2	3
4	6 5 0 3	4
5		0
6	2	1
	Total:	22

b

Stem	Leaf	Frequency (f)
0	1 2 4 6 8 9	6
1	1 5 8 9	4
2	0 6 7 9	4
3	2 3 6	3
4	0 3 5 6	4
5		0
6	2	1
	Total:	22

c range = 61, median $= \frac{20+26}{2} = 23$,

mean = 24·18 (to 2 dec. pl.)

The only outlier is 62.

d 45·5%

4 a

Stem	Leaf
0	1 1 1 2 3 4 5 5 6 7 8 8 8 9 9 9
1	0 1 1 2 2 3 3 4 4 4 4 4 5 6 7 7 7 8 9
2	0 2 4 5 6 7 8
3	0 0 1 3 5 7 9 9
4	1 2 2 2 3 4 4 4 5 5 6 6 6 7 8 9
5	0 1 1 2 2 2 3 3 4 4 5 5 5
6	0

b range = 59, mode = 14, median = 26·5

c 60 is the only outlier. There are no obvious clusters. The scores tend to be either good or poor. This means the distribution has a bulge at each end of the scale.

d 41 is 12·7 above the mean, is 19 below the highest score and is the 30th highest of the 80 scores.

Fun spot 14:06

Husband	Wife	Town	Hair
Mike	Maria	Parkes	Red
Lachlan	Sarah	Dubbo	Black
Harry	Lin	Lithgow	Grey
Keith	Diane	Bathurst	Blonde
Shu	Georgina	Tamworth	Brown
Rashid	Vanessa	Orange	White

Diagnostic test 14

1 a numerical, discrete b categorical, distinct

c numerical, continuous d categorical, ordered

2 a

Outcome	Tally	Frequency
1	|||	3
2	𝍸 ||	7
3	||||	4
4	||||	4
5	||	2
	Total:	20

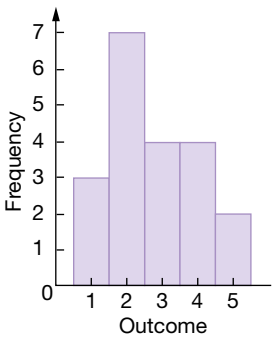
b

Outcome	Tally	Frequency
0	|||	3
2	𝍸	5
4	𝍸 ||||	9
6	𝍸	5
8	|||	3
	Total:	25

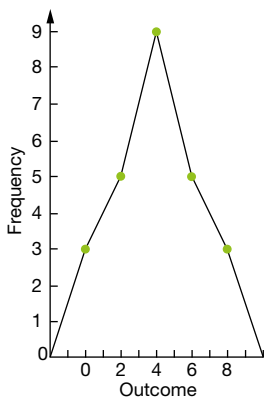
c

Outcome	Tally	Frequency
3	||	2
4	|	1
5	|||	3
6	||||	4
7	$\cancel{\|\|\|\|}$ |	6
8	|||	3
9	||||	4
10	||	2
	Total:	25

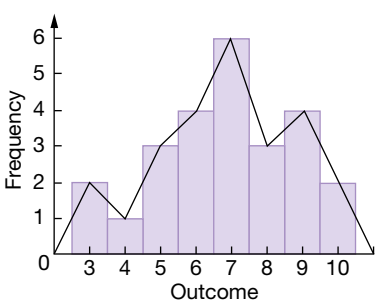
3 a



b



c



4 a 6 **b** 7 **c** 0·7

5 a 6 **b** 3 **c** 6

6 a 2 **b** 4 **c** 7

7 a 7 **b** 6 **c** 4·5

8 a 2·5 **b** 4 **c** 7

9 a 6 **b** 5·625 **c** 6

10 a 2·75 **b** 4 **c** 6·84

11 a i 29 cm **ii** 29·5 cm **iii** 13 cm

b 20 cm

12 a 35

b mode = 79, median = (65 + 68) ÷ 2 = 66·5, range = 45

ASSIGNMENT 14A

1 a 90

b No, Year 7 students are not representative of Year 12 students.

2 a numerical, continuous **b** categorical, distinct

c numerical, continuous **d** categorical, distinct

e numerical, discrete **f** categorical, ordered

3 a range = 6, mode = 7, mean = 6·2 (to 1 d.p.), median = 7

b range = 9, mode = 14, mean = 15·3, median = 14·5

4 a 20 **b** 5 **c** 5 **d** 3·7 **e** 4

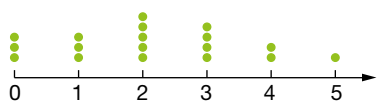
f mean, median, mode

5 a

Score	Tally	Frequency
0	|||	3
1	|||	3
2	$\cancel{\|\|\|\|}$	5
3	||||	4
4	||	2
5	|	1
	Total:	18

b i 5 **ii** 2 **iii** $2\frac{1}{9}$ **iv** 2 **c** 2

6



7 a 51 **b** 3, (53, 63, 76) **c** 63·5 **d** 1567

e 65·3 **f** mean = 66, median = 64

ASSIGNMENT 14B

1 $19 900

2 a 35·5 L **b** 19·5 L

3 a 3 cards **b** 8 cards

4 7 byes **5** no

ASSIGNMENT 14C

1 a 94 **b** 0 **c** −2 **d** 5 **e** 72 185

2 a $1\frac{6}{10}$ or $1\frac{3}{5}$ **b** $\frac{31}{40}$ **c** $\frac{13}{20}$ **d** $\frac{9}{20}$

e $\frac{17}{200}$ **f** $1\frac{1}{2}$ **g** $1\frac{1}{5}$ **h** $\frac{3}{20}$

3 a 23·05 **b** 6·85 **c** 0·415 **d** 0·65

e 1·5 **f** 1·0325 **g** 230 **h** 0·0025

i 0·67

4 a 75% **b** 140% **c** 14·5% **d** 205%

5 a $3440 **b** 104 km **c** $1680 **d** $1596

e $650

6 a $11x$ **b** a **c** $8x^2$ **d** 5 m **e** $40y$

f $10b$ **g** $4a$ **h** $14ab$

7 **a** i $\frac{1}{20}$ ii $\frac{1}{25}$ iii $\frac{1}{100}$ **b** $\frac{17}{20}$

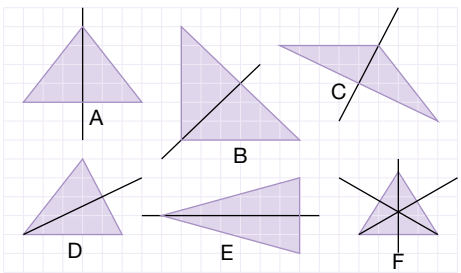
8 **a** $m = 128$, corresponding angles and parallel lines
b $x = 60$, angle sum of a triangle

9 **a** $x = 8$ **b** $a = 12$ **c** $k = 10$ **d** $x = 5$
e $y = \frac{4}{5}$ **f** $p = 3\frac{2}{3}$ **g** $n = 16$ **h** $m = 3$

Chapter 15

Exercise 15:01

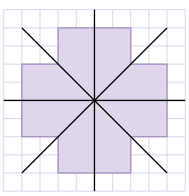
1 **a**



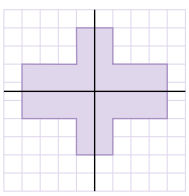
b no

2 **a** 4 **b** 2 **c** 1 **d** 0 **e** 2 **f** 0

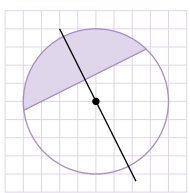
3 **a**



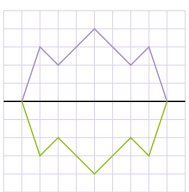
b



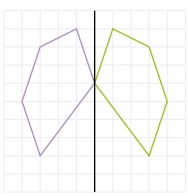
c



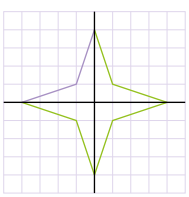
4 **a**



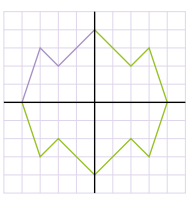
b



c



d



5 **a** yes, order 4 **b** yes, order 2 **c** no
d no **e** yes, order 2 **f** no

6 **a** A: order 4, B: order 4, C: order 2, D: order 5
b B and D

7 **a** A and B **b** B and C **c** B **d** A
e C

8 **a** both **b** rotational symmetry only
c neither

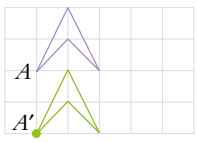
9 **a** i 5 ii 5 **b** i 8 ii 8

10 **a** i 6 ii 6 **b** i 7 ii 7
c i 9 ii 9 **d** i 10 ii 10
e i 12 ii 12

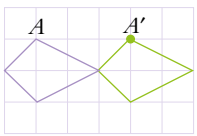
Exercise 15:02

1 **a** translation **b** reflection **c** rotation
d reflection **e** rotation **f** translation
g rotation **h** translation **i** rotation
j rotation **k** translation **l** reflection
m rotation **n** translation **o** reflection
p reflection **q** translation **r** translation

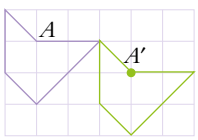
2 **a**



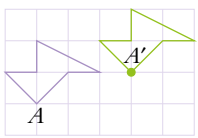
b



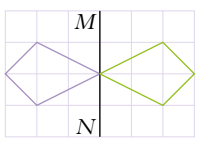
c



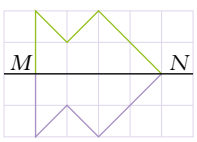
d



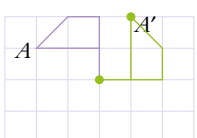
3 **a**



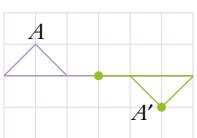
b



4 **a**



b

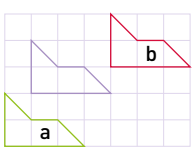


5 **a** 5 right 2 down **b** 6 right 3 down
c 1 left 2 up **d** 4 left 2 down

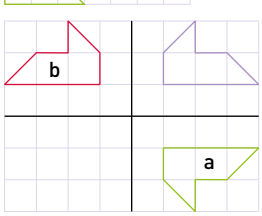
6 **a** vertical **b** vertical **c** horizontal
d vertical **e** horizontal **f** horizontal

7 **a** 90° **b** 180° **c** 270° **d** 90°
e 270° **f** 180°

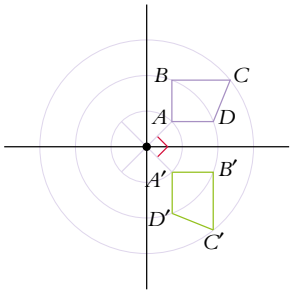
8



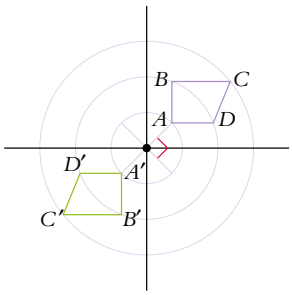
9



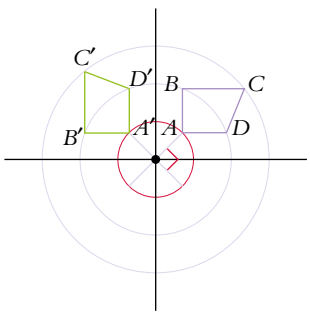
10 **a** 90°



b 180°



c 270°



Prep quiz 15:03

1 (3,4) **2** (3,−2) **3** (−2,−2) **4** (−2,4)
5 6 **6** 5
7 (5,1) **8** (−1,1) **9** (2,5)
10 (2,−3)

Exercise 15:03

1 **a** *B* **b** *A* **c** *B* **d** *C* **e** *D* **f** *A*
g *C* **h** *D*
2 **a** 6 right **b** up 6 **c** 6 to the left
d down 6
3 **a** up 3 **b** down 3 **c** down 4
d 4 to the left **e** 5 to the right
4 **a** **i** (1,1) **ii** (2,5) **iii** (4,2)
b **i** yes **ii** yes
c **i** yes **ii** yes **d** **i** yes **ii** yes
5 **a** 4 to the left; $B'(3,1)$, $C'(2,4)$
b 5 to the right; $B'(4,1)$, $C'(2,4)$
6 **a** **i** (−4,−1) **ii** (−2,−4) **iii** (−5,−5)
b yes **c** yes
7 **a** down 6; $A'(1,-1)$, $B'(4,-4)$
b up 8, $A'(-2,2)$, $B'(1,2)$
8 **a** 5 to the right
b $A'(6,3)$, $B'(9,3)$, $C'(8,1)$, $D'(5,1)$
c **i** They are the same. **ii** They differ by 5.
9 **a** yes **b** yes
10 **a** **i** (4,4) **ii** (7,5) **iii** (4,7) **iv** (1,5)
b **i** (4,−2) **ii** (7,−1) **iii** (4,1)
iv (1,−1)
11 **a** **i** (2,1) **ii** (4,−2) **iii** (1,−3)
b yes **c** yes **d** (−8,8), (−6,5), (−9,4)
12 **a** $A'(1,-1)$, $B'(4,-1)$, $C'(1,3)$
b $A''(-9,5)$, $B''(-6,5)$, $C''(-9,9)$
13 **a** (9,2), (15,−3), (17,4)
b (−11,2), (−5,−3), (−3,4)
c (−1,12), (5,7), (7,14)
d (−1,−8), (5,−13), (7,−6)

Prep quiz 15:04

1 *D* **2** *E* **3** *F* **4** *D* **5** *E* **6** *F*
7 (−2,4) **8** (−1,1) **9** (1,−1) **10** (2,−4)

Exercise 15:04

1 **a** *E* **b** *H* **c** *G* **d** *F* **e** *A* **f** *D*
g *C* **h** *B*
2 **a** *B* **b** *A* **c** *D* **d** *C* **e** *H*
f *G* **g** *F* **h** *E*
3 **a** **i** $A(-3,5)$, $A'(3,5)$ **ii** $B(-4,0)$, $B'(4,0)$
iii $C(-2,-5)$, $C'(2,-5)$
b They are equal.
c They are opposites of each other.
4 **a** **i** $A(-3,5)$, $A'(-3,-5)$ **ii** $B(-4,1)$, $B'(-4,-1)$
iii $C(3,3)$, $C'(3,-3)$
b They are equal.
c They are opposites of each other.
5 **a** (−2,4) **b** (−6,4) **c** (−8,1) **d** (−5,−3)
6 **a** (0,−5) **b** (4,−2) **c** (0,−2) **d** (−3,−1)
7 **a** **i** (−2,2) **ii** (−2,−5) **iii** (−5,0)
b **i** (6,4) **ii** (4,0) **iii** (2,6)
8 **a** **i** (−3,3) **ii** (2,3) **iii** (−3,1)
b **i** (5,−3) **ii** (−1,−4)**iii** (0,−1)
9 **a** **i** *D* **ii** *C* **iii** *H* **iv** *G*
v *F* **vi** *E* **vii** *B* **viii** *A*
b **i** (−2,4) **ii** (2,−4) **iii** (4,−2) **iv** (−4,2)
10 **a** **i** (1,−1), (3,−2), (2,−4)
ii (−1,1), (−3,2), (−2,4)
b **i** (0,−2), (2,0), (4,−1), (2,−2)
ii (0,2), (−2,0), (−4,1), (−2,2)
11 **a** (3,−2), (−2,3), (0,−6)
b (−3,2), (2,−3), (0,6)

Prep quiz 15:05

1 90° **2** 180° **3** 270° **4** B **5** D
6 C **7** D **8** (4,0) **9** (0,−4) **10** (−4,0)

Exercise 15:05

1 **a** 90° **b** 270° **c** 90° **d** 180°
e 270° **f** 270° **g** 180° **h** 180°
i 90°

2 **a** 270° **b** 90° **c** 270° **d** 180°
e 90° **f** 90° **g** 180° **h** 180°
i 270°

3 **a** $A'(3,-1)$, $B'(3,-4)$, $C'(0,-3)$
b $A'(1,3)$, $B'(3,3)$, $C'(4,1)$
c $A'(3,0)$, $B'(-4,3)$, $C'(-2,4)$

4 **a** $A'(-4,-5)$, $B'(-5,0)$, $C'(-1,-3)$
b $A'(-3,1)$, $B'(-4,6)$, $C'(0,4)$

5 **a** $A(4,5)$, $A'(-4,-5)$; $B(5,0)$, $B'(-5,0)$; $C(1,3)$, $C'(-1,-3)$
b $A(3,-1)$, $A'(-3,1)$; $B(4,-6)$, $B'(-4,6)$; $C(0,-4)$, $C'(0,4)$
The x and y values of the image are opposite to the x and y values of the original point.

6 **a** **i** 90° **ii** (5,0), (0,3)
b **i** 180° **ii** (0,−5), (3,0)
c **i** 270° **ii** (−5, 0), (0, −3)

7 **a** **i** D **ii** E **iii** F **iv** O
b **i** (0,4) **ii** (2,4) **iii** (2,0) **iv** (0,0)

8 **a** **i** D **ii** E **iii** F **iv** O
b **i** (4,0) **ii** (4,−2) **iii** (0,−2) **iv** (0,0)

9 **a** $A'(3,3)$, $B'(4,0)$, $C'(0,0)$
b $A'(3,-3)$, $B'(2,-4)$, $C'(0,-2)$
c $A'(2,1)$, $B'(3,4)$, $C'(1,3)$

10 **a** G, K, N **b** E, I, Q **c** F, J, M
d H, L, P

11 **a** (3,−1) **b** (−3,−3) **c** (−1,3)
d (−1,−3)

12 **a** $A(-3,3)$, image points of A are :(3,3), (3,−3), (−3,−3)
b $B(-1,5)$; image points of B are: (5,1), (1,−5), (−5,−1)
c $C(-1,1)$; image points of C are: (1,1), (1,−1), (−1,−1)
d $D(-3,1)$; image points of D are: (1,3), (3,−1), (−1,−3)

13 **a** (2,0), (5,0), (5,−2), (2,−2)
b (0,−2), (0,−5), (−2,−5), (−2,−2)
c (−2,0), (−5,0), (−5,2), (−2,2)
d (−2,0), (−5,0), (−5,2), (−2,2)

Exercise 15:06

1 **a** D **b** A **c** C **d** E

2 **a** Move figure B 9 units to the right.
b Move figure D 15 units to the left.
c Move figure A 11 to the right.
d Move figure C 9 to the right.

3 **a** 6 right then down 2 **b** 4 right then up 1
c 5 left then up 3 **d** 5 left then down 2

4 **a** D **b** C **c** D

5 **a** 270°clockwise or 90° anticlockwise
b 180° **c** 90° anticlockwise or 270° clockwise

6 **a** Translate 10 units right.
b Translate 4 units to the right.
c Translate 10 units to the right.

7 **a** **i** yes **ii** yes **iii** yes
b 180° rotation **c** a 180° rotation.

8 **a** A, B **b** C **c** C
d A, B, C, D **e** C, D

9 translation and rotation *or* translation and reflection

10 **a** yes **b** no **c** no

11 **a** no **b** yes **c** yes **d** yes **e** yes

12 **a** **i** yes **ii** no **iii** no
b **i** no **ii** no **iii** yes

13 **a** translations and horizontal reflections *or* horizontal and vertical reflections *or* rotations and translations *or* reflections and rotations
b rotations *or* rotations and translations *or* reflections and rotations

Diagnostic test 15

1 **a** 2 **b** 0 **c** 3

2 **a** 2 **b** 4 **c** 3

3 **a** translation **b** reflection **c** rotation

4 **a** 4 right **b** 7 right **c** 3 left

5 **a** translation **b** (3,−1) **c** (4,−2)
d (1,−3)

6 **a** reflection **b** (−1,2) **c** (−2,−2)
d (−3,1)

7 **a** reflection **b** (3,−2) **c** (−3,−3)
d (−2,−1)

8 **a** 90° **b** 180° **c** 270° **d** 90°

9 **a** D **b** E **c** F

10 **a** G **b** I **c** H

11 **a** J **b** K **c** L

ASSIGNMENT 15A

1 **a** translation, $A'(8,3)$, $B'(6,1)$, $C'(9,1)$
b reflection, $A'(-3,3)$, $B'(-1,1)$, $C'(-4,1)$
c reflection, $A'(3,-3)$, $B'(1,-1)$, $C'(4,-1)$
d rotation, $A'(3,-3)$, $B'(1,-1)$, $C'(1,-4)$
e translation, $A'(3,-1)$, $B'(1,-3)$, $C'(4,-3)$

2 $A'(1,-2)$, $B'(5,4)$, $C'(-3,8)$

3 **a** (6,0), (7,2), (8,1), (9,1), (8,0)
b (1,−2), (2,0), (3,−1), (4,−1), (3,-2)

4 **a** C, E and G; C is (0,4), E is (−4,0), G is (0,−4)
b D, F, H; D is (−2,4), F is (−4,−2), H is (2,−4)

5 $A'(-10,8)$, $B'(8,-6)$, $C'(4,5)$, $D'(-6,16)$

ASSIGNMENT 15B

1 15 **2** 333 **3** 20 **4** $27.90
5 77 km, 13 weeks
6 **a** Australian rules **b** soccer
c 49% **d** 5%
e Some students played more than one sport.

ASSIGNMENT 15C

1 **a** 7 **b** 10 **c** 15 **d** pentagonal prism
2 **a** 0·15 **b** 0·45 **c** 60 **d** 13·5
3 **a** 80% **b** 125% **c** 125%
4 **a** 9 **b** 8 **c** 33
5 **a** area = 7·6 cm^2, perimeter = 11·6 cm
b area = 94·5 cm^2
6 **a** 27 **b** 36 **c** 60 **d** 300
7 **a** 90° **b** They are both 115°.
c They are both 40°.
8 **a** (8,2) **b** (−2,2) **c** (3,7) **d** (3,−3)
9 **a** 3 **b** −3 **c** 140 **d** 149
10

