

Foundation Mathematics

Unit 2

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Links to the Australian Curriculum are sourced from: Australian Curriculum, Assessment and Reporting Authority (ACARA), downloaded from the Australian Curriculum website on (14th May 2015).

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Note to Teachers

About This Book

This book is the second in the Foundation Mathematics series, designed to help older students to learn to become more numerate in their daily lives; in their personal life and in work, education and community aspects of their lives. This book is broken into sections, with each linked to a content area from the Mathematics Foundation Course and the Australian Curriculum.

Planning Across the Semester

This book is designed to be used for one school semester. Some students may need to move more slowly through the content, and some might need to move faster. The schedule below is a guide only, and should be modified to suit the needs of your students.

One semester consisting of 20 weeks of classes, with 5 lessons per week = 100 lessons. Ten of these may be allocated to assessment or other activities. Therefore this schedule is based on 90 lessons (of 45-60 minutes).

| Section | Number of lessons allocated |
|--|-----------------------------|
| 1 Fractions and Decimals | 20 |
| 2 Multiplication and Division of Whole Numbers and Money | 40 |
| 3 Measurement | 20 |
| 4 Probability | 10 |

The content is arranged in order and builds from one section to the next, and from one topic to the next. Therefore it is suggested that students work through the book in the order of presentation. However, some sections could, and should be revisited throughout the semester.

The Mathematics Foundation Course

The Mathematics Foundation Course consists of the following four units. This book supports the second unit.

| Unit One | Unit Two | Unit Three | Unit Four |
|--|---|---|---|
| 1.1: Whole numbers and money | 2.1: Understanding fractions and decimals | 3.2: Percentages linked with fractions and decimals | 4.1: Rates and ratios |
| 1.2: Addition and subtraction with whole numbers and money | 2.2: Multiplication and division with whole numbers and money | 3.1: The four operations: whole numbers and money 3.3: The four operations: fractions and decimals | |
| 1.3: Length, mass and capacity 1.4: Time | 2.3: Metric relationships 2.4: Perimeter, area and volume | 3.4: Location, time and temperature 3.5: Space and design | |
| 1.5: Data, graphs and tables | 2.5: The probability of everyday events | | 4.2: Statistics and probability |
| | | | 4.3: Application of the Mathematical Thinking Process |

The course is designed to support students to learn, not only the mathematics content, but the mathematical thinking and decision making processes they will need as adults. This is integrated into this book and throughout the course.

For more information about the WACE Mathematics Foundation Course and Assessment ideas go to:

<http://wace1516.scsa.wa.edu.au/mathematics/>

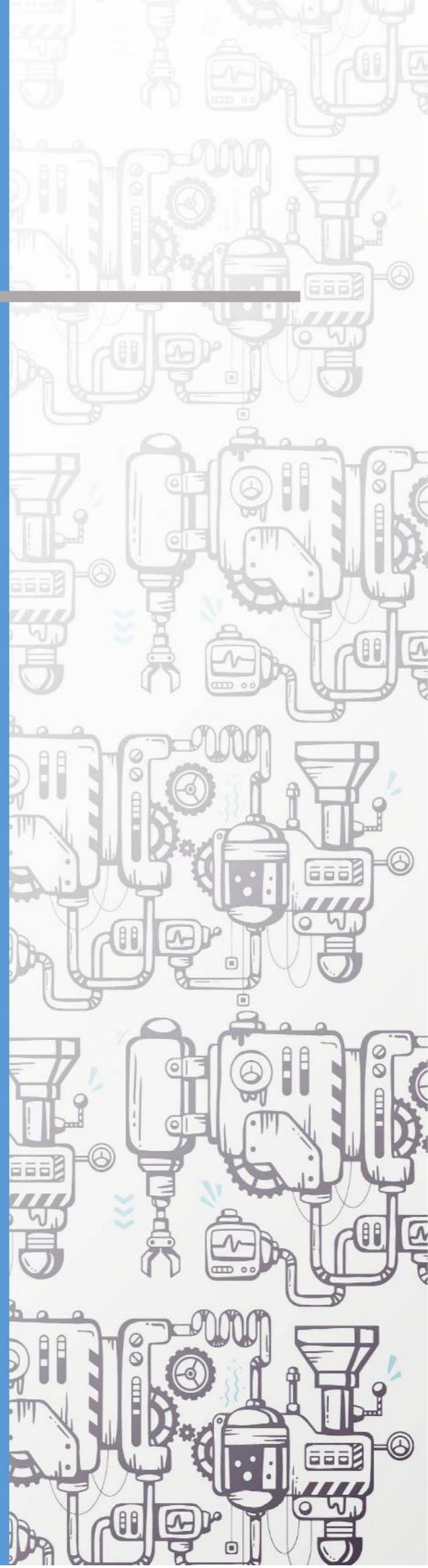


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Section One

Understanding Fractions and Decimals



Content Focus

Mathematics Foundation

- 2.1.1 Identify and describe the purpose of fractions in various texts and media from everyday life and work
- 2.1.3 Read, write and count with fractions, including unit and common non-unit fractions
- 2.1.4 Understand the meaning and magnitude of commonly used fractions, and compare and order them
- 2.1.5 Understand the link between unit fractions and division; for example, finding $\frac{1}{4}$ of a quantity is the same as dividing by 4
- 2.1.6 Use readily visualised equivalent fractions to compare and order the size of fractions
- 2.1.7 Extend whole number place value to the right of the units place to understand decimal numbers as between consecutive whole numbers
- 2.1.8 Use place value to understand the meaning and magnitude of commonly used decimal numbers
- 2.1.9 Use patterns in the number system to read, write, count with and order familiar decimal numbers in everyday contexts involving money and measurements
- 2.1.10 Recognise that fractions and decimals are used to name the same quantity in different ways
- 2.1.11 Make connections between commonly used fractions and decimals to name the same quantity in different ways
- 2.1.12 Determine and explain whether the magnitude of a fraction or decimal is reasonable within everyday contexts

Australian Curriculum Link

- ACMNA058 Model and represent unit fractions including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$ and their multiples to a complete whole
- ACMNA077 Investigate equivalent fractions used in contexts
- ACMNA078 Count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line
- ACMNA079 Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation
- ACMNA102 Compare and order common unit fractions and locate and represent them on a number line
- ACMNA104 Recognise that the place value system can be extended beyond hundredths
- ACMNA105 Compare, order and represent decimals
- ACMNA125 Compare fractions with related denominators and locate and represent them on a number line
- ACMNA131 Make connections between equivalent fractions, decimals and percentages
- ACMNA157 Connect fractions, decimals and percentages and carry out simple conversions

Topic 1

Read, Write and Count with Fractions

Mathematics Discussion

We use fractions in many everyday contexts such as time (a quarter to four) and when measuring various attributes, such as length (two and three quarter metres), mass (one and a half kilograms) and capacity (two and a half cups).

A fraction is a number which is used to describe parts of objects and collections. We write fractions by placing one number on top of another. The number on the top is called the numerator, the number on the bottom is called the denominator. They are separated by a line which is called the vinculum. The denominator shows how many parts an object or collection has been partitioned into, and the numerator shows how many of these we are counting.

$$\begin{array}{ccc} & \mathbf{2} & \text{numerator} \\ \text{vinculum} & \mathbf{\underline{\quad}} & \\ & \mathbf{3} & \text{denominator} \end{array}$$

Typically we read fractions by saying the numerator followed by the denominator. The fraction above is said as *two thirds*, showing that an object or collection has been partitioned into three and we are counting two of these. However some fractions have unique denominator names that do not reflect the number of parts; for example, we read $\frac{1}{2}$ as *a half* or *one half*, and $\frac{1}{4}$ as *a quarter* or *one quarter* (or *one fourth*, which links to the idea of four parts).

We find a fraction of an object or amount by partitioning, folding, cutting, drawing, weighing or pouring into equal sized portions. The portions need not look alike, but they must be the same size.

We find a fraction of a collection by sharing out the items equally into groups. Finding a fraction of a collection is the same as division, just written in a different way. One quarter of 12 is the same as $12 \div 4$. In both of these 12 is broken into four equal portions, with three in each. Indeed, the vinculum can be thought of as the line in the division symbol \div , with the dots being replaced with the numbers. Hence the three in $\frac{3}{4}$ shows that 3 things have been shared between 4 people.

Each person will get $\frac{3}{4}$ of 1 of them.

Whole Class Activity 1

Think: How do I read and write fraction numbers?

1 whole orange



cut into 6 equal parts,
each part is called one sixth



4 of the pieces
 $\frac{4}{6}$ (four sixths)



While most denominators have a 'th' sound at the end there are some exceptions to this.

How do you say this fraction, $\frac{1}{2}$?

How do you say this fraction, $\frac{1}{3}$?

How do you say this fraction, $\frac{1}{4}$?

Sometimes you have to listen carefully when someone says a number to work out whether it is a fraction or not. Say each of the following.

eight

eighteen

eighty

one eighth

six

sixteen

sixty

one sixth

one sixteenth

Use grid paper to draw a model of each of the numbers above – show how each one is different.

Practice Exercise 1

1. Next to each number write how you would say it

a) $\frac{1}{3}$

e) $\frac{3}{5}$

b) $\frac{2}{4}$

f) $\frac{2}{2}$

c) $\frac{2}{6}$

g) $\frac{6}{10}$

d) $\frac{5}{8}$

h) $\frac{5}{4}$

2. Write each of these as a fraction number.

a) one quarter

d) three fourths

b) two fifths

e) two sevenths

c) seven eighths

f) four sixths

3. The teacher picked up the following errors in her students' work.

What errors did they make? Why?

a) A student read $\frac{3}{6}$ as "six threes."

b) A student said there was "one twoth" of the apple left.

c) A student wrote $\frac{4}{3}$ for three quarters.

d) A student said two thirds when they saw this number $\frac{3}{2}$.

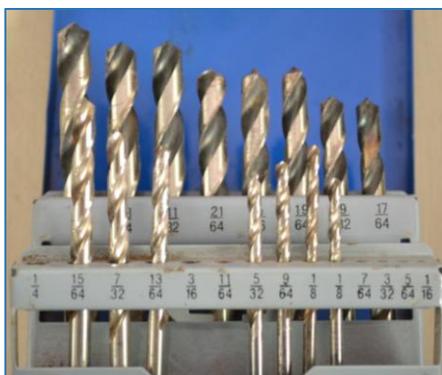
Whole Class Activity 2

Think: Where might I see and hear fraction words or numbers?

Here are 4 examples of fraction words and numbers being used in very different situations.

Ingredients

$1\frac{1}{2}$ cups self-raising flour
2 cups grated tasty cheese
150g salami
1 zucchini grated
 $\frac{1}{4}$ cup thinly sliced chives
 $\frac{3}{4}$ cup milk
1 egg
 $\frac{1}{4}$ cup tomato salsa



Hawthorn kicked five goals in the opening quarter and carried that momentum into the next stanza, booting another six to Sydney's three goals to establish a 42-point lead at the long break.

Source: <http://www.abc.net.au/news/2014-09-27/2014-afl-grand-final-live-blog/5770992>

For each example discuss what the fraction word or number is describing.

What is the 'whole' that each fraction is a part of?



Practice Exercise 2

1. Read each situation and highlight the fraction words and numbers. The first one is done for you.

- a) Mum said to meet her at **quarter to 4**, outside Woollies.
- b) Add $\frac{1}{4}$ cup of milk to the $\frac{1}{2}$ cup of flour.
- c) Nora still had three quarters of the cross country track to go.
- d) Talon sold $\frac{2}{3}$ of the hot dogs before 10am.
- e) Kendall only needed to use four tenths of the length of wood to make the frame.
- f) Use the $\frac{1}{16}$ masonry drill bit.

2. For each of the situations above, describe the whole the fraction is part of. The first one is done for you.

- a) **Quarter to 4** is part of an hour.
- b) _____ is part of _____ and _____ is part of _____
- c) _____ is part of _____
- d) _____ is part of _____
- e) _____ is part of _____
- f) _____ is part of _____

Whole Class Activity 3

Think: What do the numerator and denominator really mean?

You will need four 30cm strips of paper and a pair of scissors.

Take one of the strips and fold in half.

Cut along the fold. Glue one half in the space below.

Take another strip of paper and fold it in half, then half again to make quarters.

Cut along the folds. Glue one of the quarter pieces in the space below.

Take another strip of paper and fold it in half 3 times to make eighths.

Cut along the folds. Glue one of the eighth pieces in the space below.

Take another strip of paper and fold it in half 4 times to make sixteenths.

Cut along the folds. Glue one of the sixteenths in the space below.

Next to each piece write the fraction amount in words and as a number.

What is the same about the numerators? Why?



What is different about the denominators? Why?



What happens to the denominator each time you halve the paper another time?



What happens to the size of each piece, as you cut the strip into more pieces?



Whole Class Activity 4

Think: How do we find a fraction of an object?

Look at each item and jot down how you would find one third of it.

| | | |
|--|---|---|
|  | | |
|  |  |  |

Reflection and Discussion

We use fractions when we tell the time.



A quarter past eleven.



A quarter to two.

What do the fractions above mean?



What is the whole that the fraction 'a quarter' is referring to?



Why don't we say *three quarters past two*?



Whole Class Activity 5

Think: How do we find fractions of a collection?

Here is a collection of lollies.



Divide this collection into fifths? How did you do it?



Circle and label each fifth. How many lollies are in each fifth?



How many lollies would be in $\frac{3}{5}$?



Reflection and Discussion

Find $\frac{1}{4}$ of a packet of smarties. When you find this $\frac{1}{4}$, what are finding a quarter of?



Find $\frac{1}{4}$ of one smartie. When you find this $\frac{1}{4}$, what are finding a quarter of?



How are fractions of a collection different from fractions of an object?



How are fractions of a collection the same as fractions of an object?



Practice Exercise 3

1. Look at each picture and explain how you would make the given fraction.

a) $\frac{2}{3}$



b) $\frac{1}{4}$



c) $\frac{4}{6}$



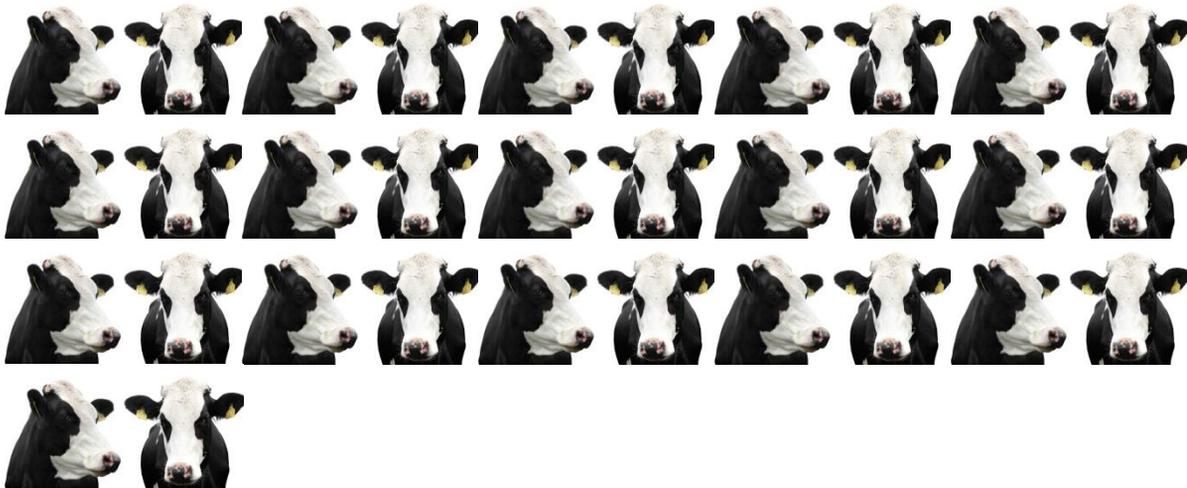
d) $\frac{6}{8}$



e) $\frac{7}{8}$



f) $\frac{5}{16}$



2. Draw a diagram for each of the following fractions.

a) $\frac{1}{3}$ of a plate of 12 cupcakes

d) $\frac{5}{8}$ of a cup of milk

b) $\frac{3}{4}$ of an hour

e) $\frac{4}{6}$ of 18 eggs

c) $\frac{4}{5}$ of a metre length of wire

f) $\frac{1}{10}$ of a length of wood

3. Write the fraction and draw two diagrams for each. One to show the fraction of an object, and one to show the fraction of a collection.

a) two thirds

d) one sixteenth

b) three fifths

e) five tenths

c) six sixths

Practice Exercise 4

1. Gather five things from the classroom, or from home. E.g. a cup of water, a length of wood, a paper plate, play dough.

Find $\frac{3}{4}$ of the length of one thing. How did you do this?

Find $\frac{3}{4}$ of the weight of one thing. How did you do this?

Find $\frac{3}{4}$ of the area of one thing. How did you do this?

Choose how to find $\frac{3}{4}$ of the remaining two items. How did you do this?

2. Make or find three different collections of items. E.g. nails, paper clips, pens.

Find $\frac{3}{4}$ of the first collection. How did you do this?

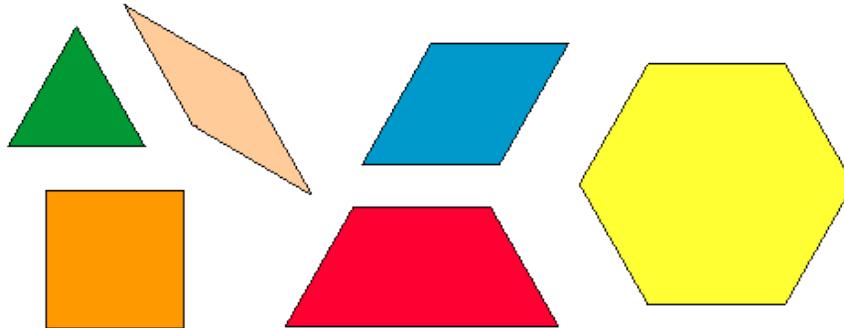
Find $\frac{2}{3}$ of the second collection. How did you do this?

Find $\frac{1}{5}$ of the third collection. How did you do this?

Whole Class Activity 6

Think: how can I find the whole amount from the size of the fractional amount?

Use pattern blocks, or cut out shapes for this activity.



Take one red trapezium. This red trapezium is $\frac{1}{2}$ of a shape.

What does the whole look like? Draw it.

Take the blue rhombus. This rhombus is a $\frac{1}{4}$ of a shape.

What does the whole look like? Draw it.

Take the triangle. This triangle is $\frac{1}{6}$ of a shape.

What does the whole look like? Draw it.

Take four squares. These four squares are $\frac{1}{3}$ of the whole shape.

What does the whole look like? Draw it.

Compare your whole shapes to other student's. Do you have the same shapes? Why or why not?



What strategy did you use in order to work out the whole from the fractional part?



Use this strategy to work out the following:

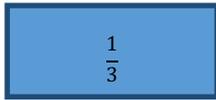
\$14 is $\frac{1}{2}$ of the cost of a T shirt. How much does the shirt cost?

\$2 300 is $\frac{1}{3}$ the cost of a car. How much is does the car cost?

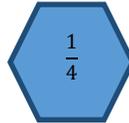
Practice Exercise 5

1. Work out the whole from the fractional part shown.

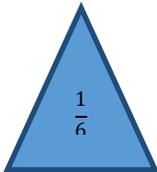
a)



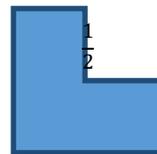
c)



b)



d)



2. Work out the size of the whole amount from the fractional portion.

a) \$100 is $\frac{1}{2}$ so the whole is _____

b) 2.5 litres is $\frac{1}{3}$ so the whole is _____

c) 12 500 bricks is $\frac{1}{4}$ of what is needed to build a wall.

How many bricks for the whole wall? _____

d) \$60 000 is the deposit for a house. This is $\frac{1}{5}$ the price of the whole house.

How much does the whole house cost? _____

e) \$50 is $\frac{2}{5}$ of the cost of small television. How much does the television cost? _____

Whole Class Activity 7

Think: How do I count with fractions?



Annie was filling a catering order for sandwiches cut into quarters. She lost count of the number of rounds of sandwiches she had made, so had to count the quarters to work it out.

This is what the start of Annie's counting sounded like;

"One quarter, two quarters, three quarters, one whole, one and one quarter, one and two quarters, one and three quarters, two wholes, two and one quarter..."

Read the sequence aloud. What patterns do you see and hear?



What happens after three quarters are counted?



Here are the numbers to match the words of Annie's counting.

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1 $1\frac{1}{4}$ $1\frac{2}{4}$ $1\frac{3}{4}$ 2 $2\frac{1}{4}$...

When you combine a whole number and a fraction number, such as $1\frac{1}{4}$, it is called a mixed number.

What is happening to the numerator during this count? Why?



What is happening to the denominator during this count? Why?



Annie kept counting the sandwich quarters.

"...six and three quarters, seven wholes, seven and one quarter, seven and two quarters, seven and three quarters..."

There were 5 more quarters to count. Write the next five numbers in the count?



How do you say this part of the sequence in words? Read to a partner.



How many whole sandwiches had Annie made?



How many quarters were there?



Whole Class Activity 8

Fraser was counting pieces of doughnuts by quarters but kept losing track of when he got to a whole.

This is how he wrote his count of quarters.

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{10}{4}$ $\frac{11}{4}$ $\frac{12}{4}$ $\frac{13}{4}$

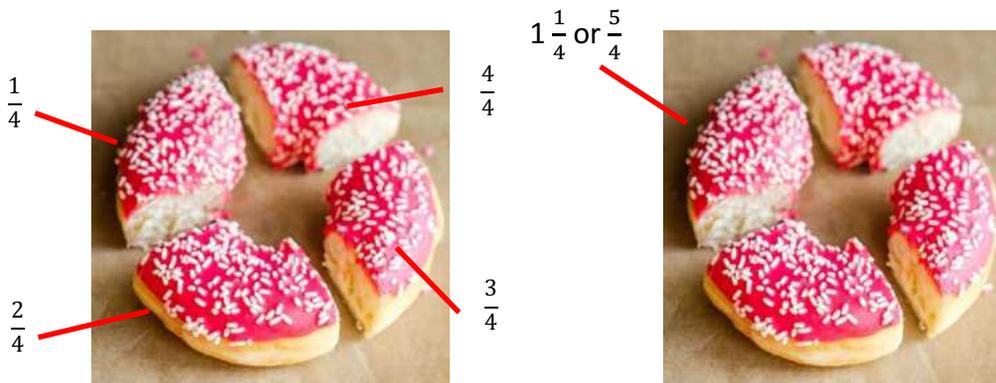
Circle each fraction that could also be written as a whole number.



When the numerator is larger than the denominator, such as $\frac{5}{4}$, we call this an improper fraction. An improper fraction means the number of parts goes beyond the whole.

In the case of $\frac{5}{4}$, there are $\frac{4}{4}$ which make 1 whole. There is also $\frac{1}{4}$ of the next whole.

$\frac{5}{4}$ can be written as $1\frac{1}{4}$.



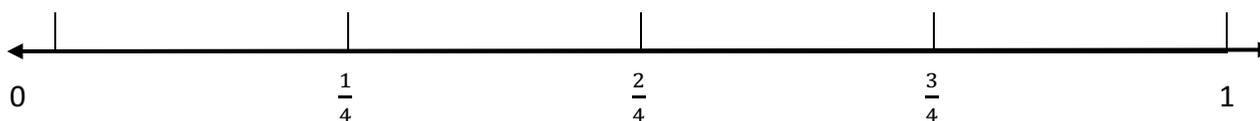
Re-write Fraser's count with numbers, making sure there are no improper fractions.



Fraser thought he could use a number line to keep track of his count better.

Zero is placed at one end. One is placed at the other, to show one whole.

The line is divided into the number of parts the denominator tells you to.



What happened when the count got to $\frac{4}{4}$?



Where does $\frac{5}{4}$ go on the number line?



How could you show fractions more than one whole on a number line?



Use the line below to show how to count by quarters to 2 wholes, using mixed numbers.

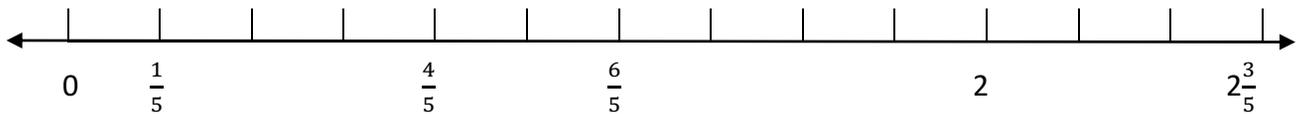


Highlight where the numbers change to a whole.

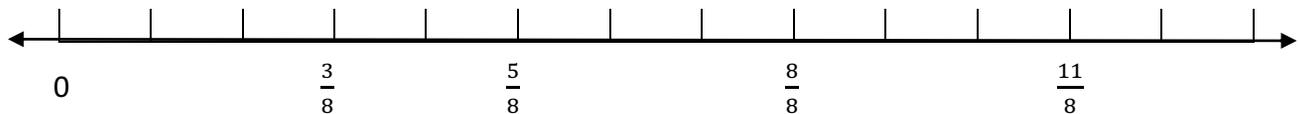
Practice Exercise 6

1. Fill in the blanks.

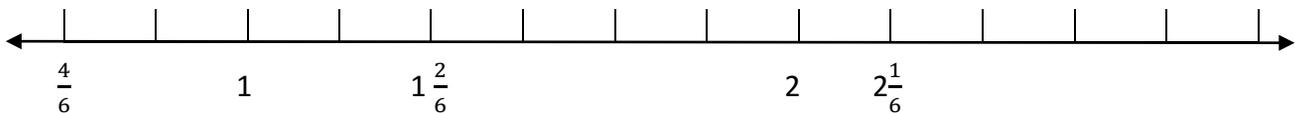
a)



b)



c)



2. Place following counting sequences in the correct order on a number line.

a) $\frac{3}{3}$ $\frac{4}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{6}{3}$ $\frac{5}{3}$

b) $2\frac{1}{2}$ 1 3 $2\frac{1}{2}$ $1\frac{1}{2}$

c) $\frac{3}{10}$ $\frac{5}{10}$ $\frac{2}{10}$ $\frac{10}{10}$ $\frac{1}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{6}{10}$ $\frac{4}{10}$ $\frac{9}{10}$

3. Gary is making a cake and can only find the $\frac{1}{3}$ and $\frac{1}{4}$ measuring cups.

Which measuring cup can Gary use for each ingredient? How many times does he need to fill the cup to make the amount shown?

| Ingredient | Which Measuring Cup | Number of Cups |
|------------------------------|---------------------|----------------|
| $1\frac{2}{3}$ cups of flour | | |
| $\frac{3}{4}$ cup of milk | | |
| $1\frac{1}{4}$ cups of cocoa | | |
| 2 cups of pecans | | |

4. Ally orders one bag of carrots each week to feed her horse. She divides the bag of carrots into sevenths, so the horse gets the same amount each day.

Here are the first two sevenths.



- a) How many more sevenths are needed to make 1 whole?
- b) How many carrots will the horse eat during the week?
- c) How many sevenths in 3 bags of carrots?
- d) How many carrots in $1\frac{3}{7}$ bags?

5. If each of these apples was cut into sixths.



- a) How many sixths would there be?
- b) If $1\frac{2}{6}$ were eaten how much would be left?
- c) If another half an apple was added, how many sixths would there be?

Whole Class Activity 9

Sometimes when sharing out a collection, there are less items to be shared than people. For example; there are 3 pieces of bread left but four people want toast for breakfast.

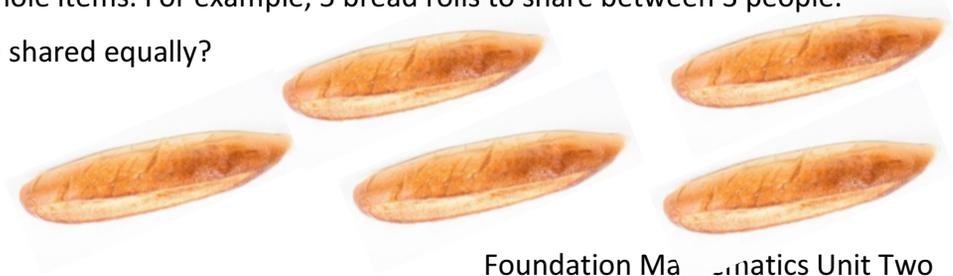
How can the toast be shared equally?



This number sentence represents the sharing situation. $3 \div 4 = \frac{3}{4}$

At other times there may be more items to be shared than there are people, but not enough for an even shares of whole items. For example, 5 bread rolls to share between 3 people.

How can the rolls be shared equally?





This number sentence represents the sharing situation. $5 \div 3 = \frac{5}{3}$

$\frac{5}{3}$ is an improper fraction. Write it as a mixed number.



Practice Exercise 7

1. For each situation draw a diagram showing how to divide and share the items. Write a number sentence to match the situation.

a) 2 metres of wood to make 3 shelves.

Number Sentence:

b) 3 blocks of chocolate shared between 4 children.

Number Sentence:

c) 4 bread rolls shared between 6 people.

Number Sentence:

d) 2 cups of blueberries shared between 3 mixing bowls.

Number Sentence:

e) 3 bags of cement between 4 cement mixers.

Number Sentence:

f) 6 cubic metres of mulch across 10 garden beds.

Number Sentence:

2. For each situation draw a diagram showing how to divide the items. Write a number sentence to match the situation.

a) 3 blocks of chocolate shared between 2 people.

Number Sentence:

b) 4 cups of milk shared between 3 bowls.

Number Sentence:

c) 10 pizzas shared between 6 people.

Number Sentence:

d) 5 lengths of celery shared between 3 rabbits.

Number Sentence:

e) 8 metres of fencing shared between 2 neighbours.

Number Sentence:

f) 12 tonnes of wheat shared between 3 silos.

Number Sentence:

Reflection and Discussion

For each of the previous examples, consider the relationship between the numbers in the number sentence.

$3 \div 4 = \frac{3}{4}$ represents the situation of 3 pieces of toast being shared between 4 people.

Describe the relationship between the numbers on either side of the equals sign.



$5 \div 3 = \frac{5}{3}$ represents the situation of 5 bread rolls shared between 3 people.

Describe the relationship between the numbers on either side of the equals sign.



Practice Exercise 8

Download the following videos and activities to learn more about fractions.

<http://vimeo.com/58508269>

<http://vimeo.com/60215928>

<http://topdrawer.aamt.edu.au/Fractions/Activities>

<https://www.youtube.com/watch?v=D-kLjl1gaQQ>

Reflection on Learning 1

What is the difference between these numbers?

$\frac{1}{2}$

$\frac{2}{1}$

$1\frac{1}{2}$

$\frac{2}{2}$

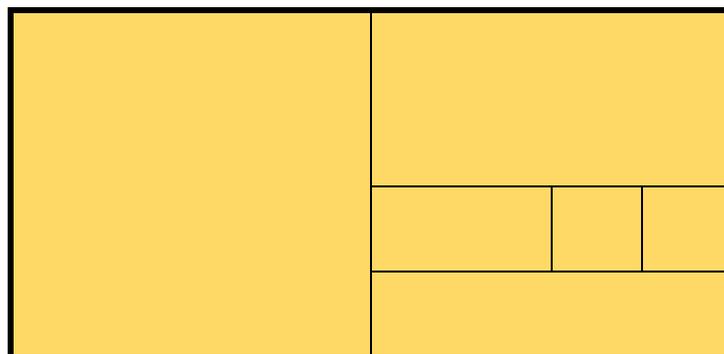
$2\frac{1}{2}$

Discuss and draw a model of each to demonstrate your thinking.



Reflection on Learning 2

How many different fractions can you see in the diagram below? Label as many as you can.



Compare your labels to others. Do you have the same? Why or why not?



Reflection on Learning 3

Discuss the following with a partner and record your thinking.

How many different ways could you find $\frac{3}{8}$ of a bag of flour?



How many different ways could you find $\frac{1}{8}$ of a collection of 100 nails?



OLNA Practice Questions

1. Kim counted seven halves. What number is equal to seven halves?

A. 5

B. 9

C. $\frac{2}{7}$

D. $3\frac{1}{2}$

E. $7\frac{1}{2}$

2. Lena ate $\frac{3}{4}$ of her 20 smarties and gave $\frac{1}{4}$ of them to her little sister. How many smarties did she eat?

3. $\frac{1}{4}$ of a flock of birds is 8. How many birds are in the whole flock?

Topic 2

Comparing and Ordering Fractions

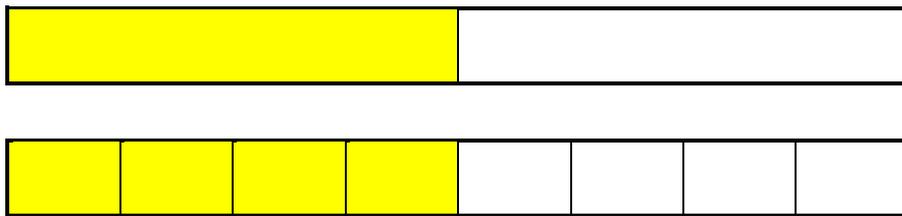
Mathematics Discussion

We can compare the size of fractions when they refer to the same sized whole object or collection. For example, we can say $\frac{1}{2}$ is bigger than $\frac{1}{4}$ when they both refer to a litre of milk. However, we cannot compare fractions if they refer to different things. One half of \$20 is less than one quarter of \$100.

We can compare the size of fractions by drawing diagrams, using a number line, using benchmark numbers and fractions, by visualising the quantities or using equivalent fractions.

We use equivalent fractions in order to compare fractions with different denominators. For example to compare $\frac{3}{8}$ to $\frac{1}{2}$ we can change the half into eighths.

We find out how many eighths are equal to (equivalent to) a half by subdividing the whole.



As can be seen from the diagram, $\frac{1}{2}$ is equivalent to $\frac{4}{8}$ so $\frac{1}{2}$ is bigger than $\frac{3}{8}$. The diagram shows $\frac{1}{2}$ and $\frac{4}{8}$ refer to the same amount, just partitioned and written in different ways.

Whole Class Activity 1

Think: How can we compare fractions that have the same denominator?

Vikrim and Simon were building a cupboard together. Vikrim had to cut a piece of timber $\frac{3}{4}$ of a metre long. Simon had to cut a piece of timber $\frac{1}{4}$ of a metre long.

Who cut the longer piece? How do you know?



Isaia and Ethan were in the cross country race. After 10 minutes Isaia had completed $\frac{3}{16}$ of the race and Ethan had completed $\frac{5}{16}$. At the 10 minute mark, who was winning? How do you know?



Why do you only need to compare the numerator when the denominators are the same?



Reflection and Discussion

To compare mixed numbers and improper fractions with the same denominator make them all the same – all improper fractions or all mixed numbers.

How do you compare $1\frac{3}{4}$ and $\frac{5}{4}$ to work out which is bigger?



Practice Exercise 1

1. Circle the biggest fraction in each set, draw a diagram to prove your answer.

a) $\frac{4}{10}$ $\frac{2}{10}$

d) $\frac{4}{5}$ $\frac{1}{5}$ $\frac{3}{5}$

b) $\frac{3}{8}$ $\frac{7}{8}$

e) $\frac{7}{6}$ $\frac{2}{6}$ $\frac{1}{6}$

c) $\frac{3}{4}$ $\frac{1}{4}$

f) $1\frac{3}{7}$ $\frac{3}{7}$ $\frac{6}{7}$

2. Circle the smallest fraction in each set, draw a diagram to prove your answer.

a) $\frac{6}{4}$ $\frac{1}{4}$

d) $\frac{4}{10}$ $1\frac{2}{10}$ $\frac{7}{10}$

b) $1\frac{1}{3}$ $\frac{5}{3}$

e) $\frac{8}{6}$ $1\frac{1}{6}$ $\frac{3}{6}$

c) $\frac{4}{5}$ $\frac{3}{5}$

f) $\frac{9}{16}$ $\frac{15}{16}$ $\frac{2}{16}$

3. Draw a diagram of a collection of 24 items, eg, nails, balloons, emojis.

Use the diagram to show which is bigger:

a) $\frac{1}{4}$ or $\frac{2}{4}$

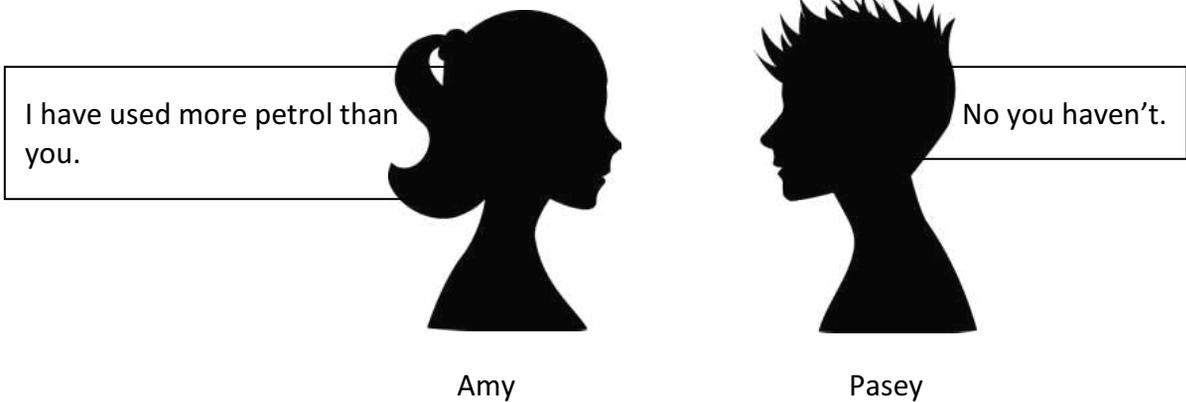
c) $\frac{4}{12}$ or $\frac{6}{12}$

b) $\frac{4}{6}$ or $\frac{3}{6}$

d) $\frac{1}{3}$ or $\frac{3}{3}$

Whole Class Activity 2

Pasey and Amy have both used $\frac{1}{4}$ of a tank of petrol.



Could Pasey have used more petrol than Amy?

Why? Or Why not?



Reflection and Discussion

When you compare the size of different fraction amounts the wholes must be the same.

Discuss this statement.

Write down the main points discussed.



Whole Class Activity 3

Think: How can we compare fractions with unlike denominators?

Nancy was wondering if $\frac{3}{5}$ of the chocolate bar was bigger than $\frac{2}{3}$.

Nancy doesn't want to chop up the chocolate bar to find out which fraction is bigger because this would get messy and confusing!



Nancy decides to draw two rectangles the same size as the chocolate to work out which is bigger $\frac{3}{5}$ or $\frac{2}{3}$.

Use rectangle A to show $\frac{2}{3}$.

Use rectangle B to show $\frac{3}{5}$.

A



B



Which amount is larger? How do you know?



Alternatively, we can compare fractions by placing both onto the same whole.

Follow the instructions to partition the box below to allow $\frac{3}{5}$ and $\frac{2}{3}$ to be compared.

1. Partition the rectangle into fifths using lines that go across the shape.
2. Colour 3 of the fifths blue to show $\frac{3}{5}$.
3. Partition the rectangle into thirds using lines that go up and down.
4. Colour 2 of the thirds yellow to show $\frac{2}{3}$.



Which amount is larger? How do you know?



Reflection and Discussion

How many equal parts is the rectangle divided into after partitioning it into thirds and fifths?



How many fifteenths are equal to $\frac{3}{5}$?



How many fifteenths are equal to $\frac{2}{3}$?



Whole Class Activity 4

Nancy was working with a bag of 15 chocolate bars. She needed to know if $\frac{4}{5}$ of the bag was more than $\frac{2}{3}$ of the bag.



Nancy shared the chocolate bars into 5 equal groups. Draw circles around the bars to show how she did this.

How many chocolate bars is $\frac{1}{5}$ of the bag?



How many chocolate bars are $\frac{4}{5}$ of the bag?



Next Nancy shared the chocolate bars into 3 equal groups.

Use a different colour and draw circles around the bars to show how she did this.

How many chocolate bars is $\frac{1}{3}$ of the bag?



How many chocolate bars are $\frac{2}{3}$ of the bag?



Is $\frac{2}{3}$ of the bag more than $\frac{4}{5}$ of the bag? How do you know?



Practice Exercise 2

1. Circle the biggest fraction in each pair. Use a diagram to prove your answer.

a) $\frac{3}{8}$ and $\frac{2}{4}$

c) $\frac{6}{10}$ and $\frac{2}{5}$

b) $\frac{4}{6}$ and $\frac{10}{12}$

d) $\frac{5}{6}$ and $\frac{2}{3}$

2. Draw a diagram of a collection of 12 items, e.g., nails, balloons, emojis.

Use the diagram to show which is bigger.

a) $\frac{1}{4}$ or $\frac{1}{3}$

b) $\frac{1}{2}$ or $\frac{1}{12}$

c) $\frac{3}{4}$ or $\frac{3}{8}$

3. Draw a diagram of a collection of 24 items, e.g., nails, balloons, emojis.

Use the diagram to show which is smaller.

a) $\frac{5}{12}$ or $\frac{3}{4}$

b) $\frac{4}{6}$ or $\frac{4}{8}$

c) $\frac{3}{6}$ or $\frac{3}{8}$ or $\frac{3}{12}$

4. Visualise drawing a diagram in order to decide which fraction is larger. Place $<$ or $>$ in the space provided.

a) $\frac{2}{3}$ of a dozen eggs _____ $\frac{3}{4}$ of a dozen eggs

b) $\frac{1}{3}$ of half a dozen hot cross buns _____ $\frac{1}{4}$ of half a dozen hot cross buns

c) $\frac{1}{2}$ of 24 cookies _____ $\frac{5}{8}$ of 24 cookies

Whole Class Activity 5

Think: How can you use number lines to compare fractions with different denominators?

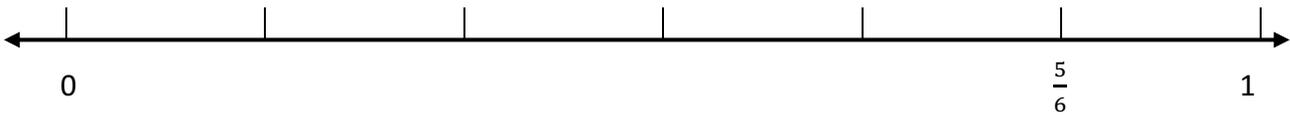


Nancy knew that drawing rectangles and partitioning them worked to compare the size of fractions, but it took a long time. She decided to use a number lines instead.

To compare $\frac{5}{6}$ and $\frac{2}{3}$ Nancy drew two number lines. Nancy divided one into three equal parts to mark the thirds.



She then divided the other line into six equal parts to mark the sixths.



Which fraction is closer to 1? $\frac{2}{3}$ or $\frac{5}{6}$

Nancy can see that $\frac{5}{6}$ is larger than $\frac{2}{3}$ because it is closer to 1.

Alternatively, Nancy can place both onto the same number line.

Use the directions below to compare the fractions on the number line.

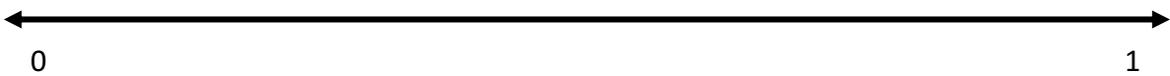
1. Partition the line into sixths, in blue pen, along the top of the line.
2. Label the mark that shows $\frac{5}{6}$.
3. Partition the line into thirds, in red pen, along the bottom of the line.
4. Label the mark that shows $\frac{2}{3}$.



Practice Exercise 3

1. Circle the biggest fraction in each pair, use the number lines to prove your answer.

a) $\frac{1}{3}$ and $\frac{2}{5}$



b) $\frac{4}{6}$ and $\frac{6}{8}$



c) $\frac{3}{5}$ and $\frac{2}{10}$



d) $1\frac{2}{4}$ and $\frac{12}{8}$



e) $\frac{7}{3}$ and $\frac{8}{6}$



2. Visualise a number line in order to decide which of these fractions is the smallest.

a) $\frac{1}{7}$ or $\frac{1}{9}$ of a cake.

c) $\frac{3}{8}$ or $\frac{2}{4}$ of 16 cupcakes.

b) $\frac{4}{12}$ or $\frac{2}{3}$ of a bag of cement.

d) $\frac{12}{5}$ or $\frac{15}{10}$ bottles of water.

Reflection and Discussion

Can number lines be used to compare fractions of collections?

Which is larger $\frac{1}{3}$ of \$30 or $\frac{1}{2}$ of \$30? Explain your thinking with a diagram.



Practice Exercise 4

1. Use a number line to say which fraction of a collection is larger.

a) $\frac{1}{2}$ of \$20 or $\frac{2}{5}$ of \$20

b) $\frac{1}{4}$ of a box of 100 CD's or $\frac{6}{10}$ of a box of 100 CD's

2. Visualise a number line to say which fraction of each collection is smaller.

a) $\frac{2}{3}$ of \$30 or $\frac{5}{10}$ of \$30

b) $\frac{4}{10}$ of a packet of 50 nails or $\frac{3}{5}$ of a packet of 50 nails

c) $\frac{1}{6}$ of a packet of 24 biscuits or $\frac{1}{4}$ of a packet of 24 biscuits

d) $\frac{3}{4}$ of 100 lengths of timber or $\frac{2}{5}$ of 100 lengths of timber

Whole Class Activity 6

Think: How can we use benchmarks to compare fractions?

Kayden is an apprentice plumber and he needs to know which drill bit is smaller, $\frac{1}{16}$ or $\frac{1}{4}$. He thinks about how close each fraction is to a benchmark number or fraction.



$\frac{1}{16}$ must be smaller because $\frac{1}{16}$ is closer to 0 than $\frac{1}{4}$.

Kayden needs to know which is larger, $\frac{7}{8}$ of a piece of pipe or $\frac{7}{16}$. He thinks;



$\frac{7}{8}$ must be larger because $\frac{7}{8}$ is closer to 1 than $\frac{7}{16}$.

Kayden needs to know which spanner is smaller, $\frac{5}{8}$ or $\frac{7}{16}$. He thinks;



$\frac{7}{16}$ must be smaller because it is less than $\frac{1}{2}$ and $\frac{5}{8}$ must be larger because it is more than $\frac{1}{2}$.

Kayden is using benchmark numbers or fractions to decide which is larger. The common benchmarks are 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.

Practice Exercise 5

- Use Kayden's thinking to order these fractions from smallest to largest.
 - $\frac{2}{3}$, $\frac{6}{8}$, and $\frac{5}{6}$ of a length of wire.
 - $\frac{9}{10}$, $\frac{2}{5}$, and $\frac{3}{4}$ of a pallet of tiles.
 - $\frac{1}{8}$, $\frac{5}{16}$, and $\frac{5}{32}$ sized sockets.
 - $\frac{6}{10}$, $\frac{2}{6}$, and $\frac{6}{8}$ of a bag of cement.

Whole Class Activity 7

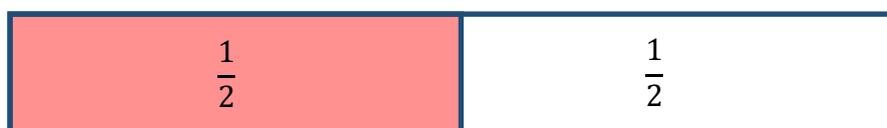
Think: How do you know if fractions are equivalent?

Two different fractions that show the same amount are called equivalent fractions. There are relationships between numerators and denominators that help us to know if fractions are equivalent just by looking at them.

Fraction Wall

Make 10 strips of paper, 21cm long. Take the first strip of paper and glue in at the top of your Fraction Wall. This represents one whole. Label it with '1'.

Fold the second strip in half. Trace over the fold line and label each section.



Glue your strip in your fraction wall in the space below your first strip.

Take the next strip of paper and fold it in half and half again. Unfold it and trace over the fold lines.

How many parts is the strip divided into? Label each part and glue below the second strip in your fraction wall.

Fold the next strip in half, three times. Unfold it and trace over the fold lines.

How many parts is the strip divided into? Label each part and glue below the other strips.

Fold the next strip in half, four times. Unfold it and trace over the fold lines.

How many parts is the strip divided into? Label each part and glue in.

Fold the next strip into three. Unfold it and trace over the fold lines.

How many parts is the strip divided into? Label each part and glue in.

Fold the next strip into three and then fold this in half. Unfold it and trace over the fold lines.

How many parts is the strip divided into? Label each part and glue in.

Fold the next strip into three and then fold this in half twice. Unfold it and trace over the fold lines. How many parts is the strip divided into? Label each part and glue in.

Fold the next strip into five. Unfold it and trace over the fold lines.

How many parts is the strip divided into? Label each part and glue in.

Fold the next strip into five and then fold this in half. Unfold it and trace over the fold lines.
How many parts is the strip divided into? Label each part and glue in.

Use your fraction wall to identify and list all of the fractions that are equivalent to:

a) $\frac{1}{2}$

d) $\frac{2}{5}$

b) $\frac{1}{4}$

e) $\frac{3}{12}$

c) $\frac{1}{3}$

f) $\frac{12}{16}$

Reflection and Discussion

Using the fraction wall we found that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$. They are equivalent.

Look at the numerators.

What pattern can you see in the numerators?



Look at the denominators.

What pattern can you see in the denominators?



What is the relationship between the numerators and denominators?



What would be the next two equivalent fractions in this pattern?

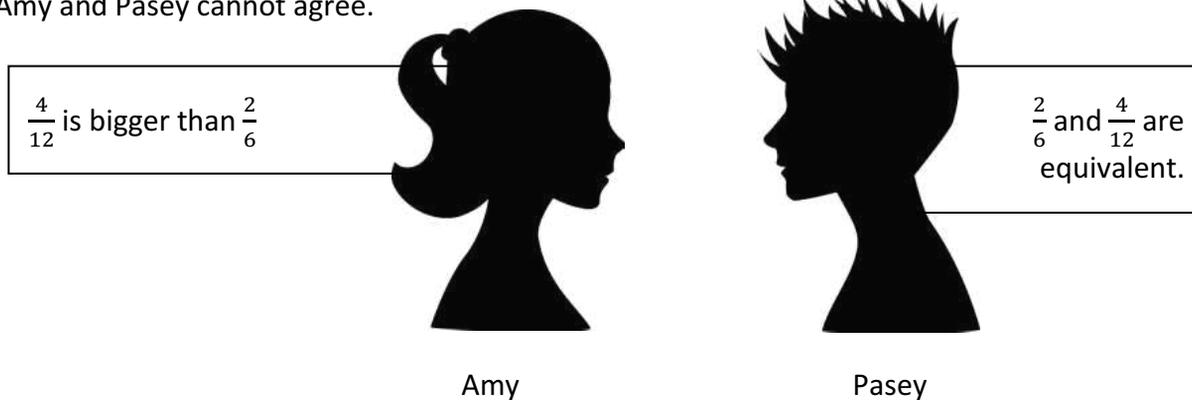


Fraction Wall



Practice Exercise 6

1. Amy and Pasey cannot agree.



Who is right? Explain your choice using a diagram.

2. Place = or \neq in each circle to make the statements correct. Use a diagram, number line, benchmark or patterning to prove your answer.

a) $\frac{1}{6}$ $\frac{3}{12}$

d) $\frac{3}{4}$ $\frac{9}{12}$

b) $\frac{5}{10}$ $\frac{10}{20}$

e) $\frac{3}{8}$ $\frac{12}{32}$

c) $\frac{2}{3}$ $\frac{8}{19}$

3. Write an equivalent fraction next to each fraction below.

a) $\frac{1}{6}$

d) $\frac{2}{8}$

b) $\frac{8}{16}$

e) $\frac{3}{4}$

c) $\frac{2}{5}$

f) $\frac{2}{3}$

Whole Class Activity 8

Think: How do we simplify fractions?

Watch this video <http://topdrawer.aamt.edu.au/Fractions/Good-teaching/Equivalence/Grids-and-arrays/Overlay-grids>

In the video, what are the two ways that they make twelfths?



How many twelfths are equal to one half?

How many twelfths are equal to one third?

Copy the circles from the next page onto a transparency and cut them out. Use a whiteboard marker to colour $\frac{2}{3}$ of the circle divided into thirds.

Lay this over the circle divided into sixths. How many sixths are in each third?



What do you multiply each third by to get sixths?



How many sixths are covered by the shaded section?



Colour this amount on the sixths circle. Record the equivalence below.


$$\frac{2}{3} =$$

Now, lay the sixth circle over the twelfth circle. What happens to the sixths?



How many twelfths are covered by the shaded section?



Colour this amount on the twelfth circle.

Record the equivalence below.


$$\frac{2}{3} =$$

For each equivalent fraction in the list on page 37 compare the numerators and denominators focusing on how they are related to each other through the operations of multiplication and division.

For example $\frac{2}{3} = \frac{4}{6}$

What do you multiply 2 by to get 4?

What do you multiply 3 by to get 6?

What do you divide 4 by to get 2?

What do you divide 6 by to get 3?

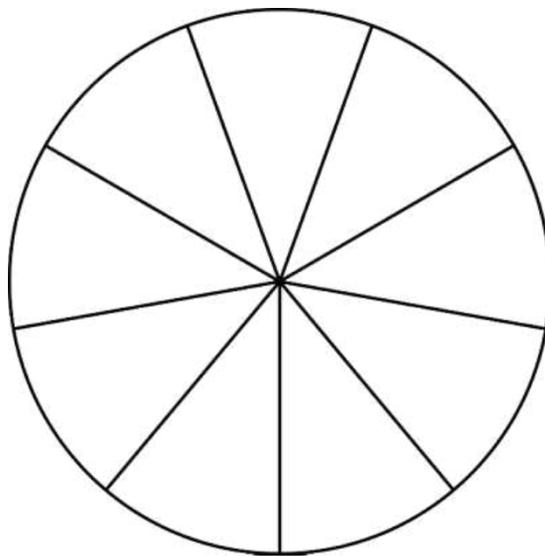
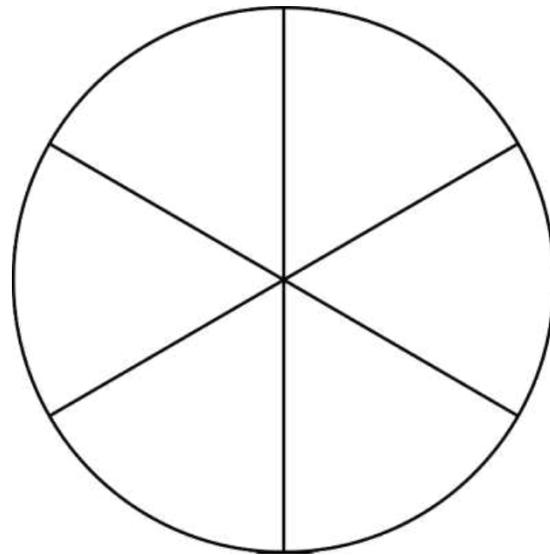
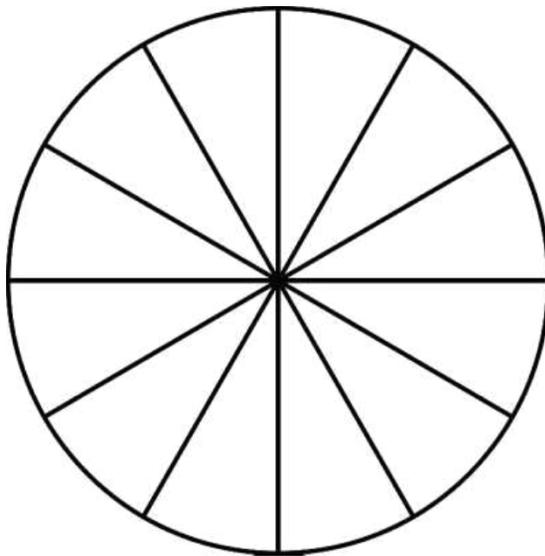
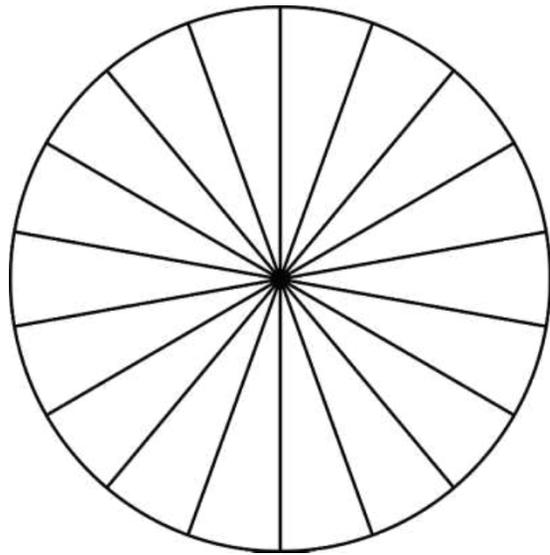
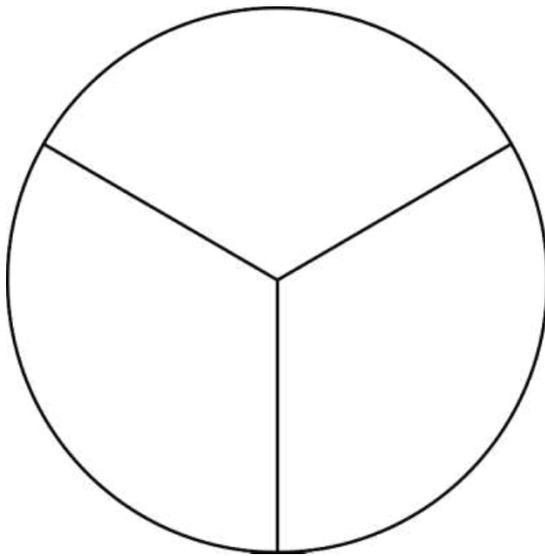
Compare $\frac{2}{3}$ to $\frac{8}{12}$ using multiplication and division.



Compare $\frac{4}{6}$ to $\frac{8}{12}$ using multiplication and division.



Copy the following images onto transparency film and cut them out.



Now use the third, ninth and eighteenth circles to explore equivalences.

Colour $\frac{1}{3}$ and lay this over the other circles to find out what it is equivalent to.

Record the equivalences below.



Explore how the numerators and denominators are related to each other using multiplication and division. Record what you find out below.



When we compare and order fraction numbers we sometimes need to deal with fractions that have large numerators and denominators, such as $\frac{16}{24}$.

It is often useful to reduce a fraction to its simplest form. That is, you find the smallest possible equivalent fraction.

To do this you use the relationships we explored with the transparent circles.

Consider $\frac{16}{24}$. How could you find the smallest possible equivalent fraction?



Practice Exercise 7

1. Create a sequence of equivalent fractions.

a) $\frac{1}{6} = \quad = \quad = \quad =$

d) $\frac{2}{7} = \quad = \quad = \quad =$

b) $\frac{1}{16} = \quad = \quad = \quad =$

e) $\frac{3}{4} = \quad = \quad = \quad =$

c) $\frac{2}{5} = \quad = \quad = \quad =$

f) $\frac{1}{8} = \quad = \quad = \quad =$

2. Reduce these fractions to their simplest form.

a) $\frac{25}{50}$

d) $\frac{6}{8}$

b) $\frac{8}{12}$

e) $\frac{8}{32}$

c) $\frac{4}{10}$

f) $\frac{21}{28}$

Practice Exercise 8

Card Game – Fraction More

Number of Players: 2 – 4

Aim: to win all the cards from the other players.

Equipment: a set of cards each with different fractions

Rules of the Game:

1. Deal out all of the cards. Place them face down.
2. Each person turns over one card at the same time.
3. The person with the fraction that is the largest takes all of the cards and place them at the bottom of their pile (face down).
4. If two fractions are the same or equivalent, the players who own these cards turn over another card. The person with the largest second card wins.
5. Play continues until one player has no cards left. The player with the most cards is the winner.

Variation: Include mixed numbers and improper fractions.

| | | | | | | | | | |
|----------------|----------------|----------------|----------------|-----------------|---------------|----------------|----------------|----------------|----------------|
| $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | $\frac{1}{6}$ | $\frac{1}{8}$ | $\frac{1}{10}$ | $\frac{1}{12}$ | $\frac{1}{9}$ | $\frac{2}{3}$ |
| $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{3}{6}$ | $\frac{2}{6}$ | $\frac{4}{8}$ | $\frac{5}{8}$ | $\frac{2}{10}$ |
| $\frac{3}{10}$ | $\frac{6}{10}$ | $\frac{9}{12}$ | $\frac{9}{10}$ | $\frac{8}{12}$ | $\frac{4}{6}$ | $\frac{5}{6}$ | $\frac{1}{7}$ | $\frac{4}{10}$ | $\frac{4}{12}$ |
| $\frac{2}{8}$ | $\frac{5}{10}$ | $\frac{6}{8}$ | $\frac{3}{8}$ | $\frac{10}{12}$ | $\frac{7}{8}$ | $\frac{5}{25}$ | $\frac{8}{10}$ | 0 | 1 |

Fraction Line-Up.

Number of Players: 2 – 4

Aim: to place all of the cards in order.

Equipment: a set of cards each with different fractions

Rules of the Game:

1. Find the zero card and the one card and place at either end of the desk. Create eight spaces between the two cards.
2. Deal out 5 cards to each player and place the rest of the deck face down in the centre. Players hold their cards face up.
3. Players take turns to place a card in one of the spaces between the zero and one, so they are in the correct order. After they place a card they take a replacement card from the pack.
4. An equivalent fraction can be placed on top of a previous card.
5. If a card cannot be placed in order, the player misses a turn and swaps one of their cards for one in the deck.
6. Winner is the person who places the last card into the line.

Variation: Include mixed numbers and improper fractions and place the numbers between 0 and 3. Include more spaces between the two end numbers.

Fraction Wall Game ¹

Number of Players: 2

Aim: to fill your game board by colouring equivalent fractions.

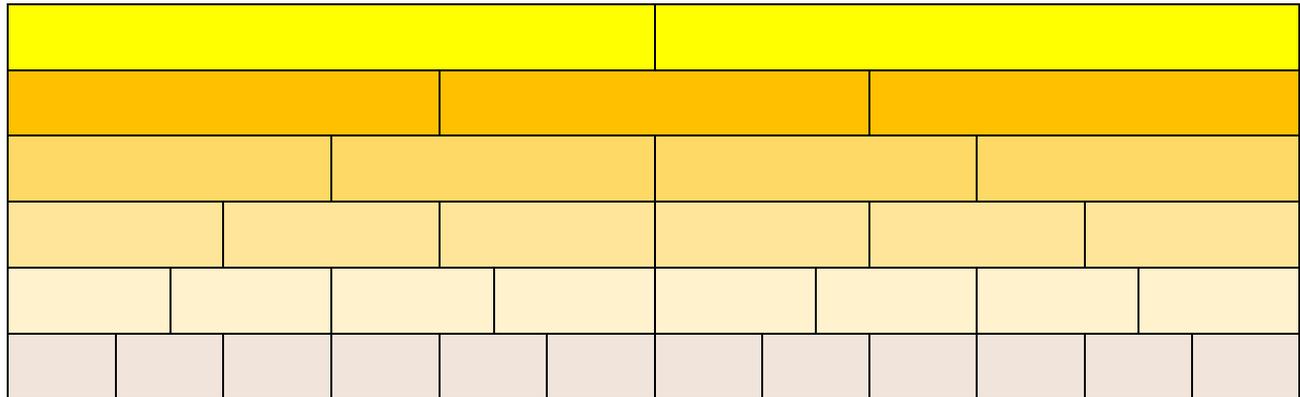
Equipment:

- one die labelled 1, 2, 2, 3, 3, 4, one die labelled $\frac{*}{2}$, $\frac{*}{3}$, $\frac{*}{4}$, $\frac{*}{6}$, $\frac{*}{8}$, $\frac{*}{12}$
- Fraction Wall game board for each player
- Coloured pencils

Rules of the Game:

1. Player A rolls the two dice and makes a fraction using the numbers rolled. The first die is the numerator, the second is the denominator.
2. Colour the fraction or an equivalent to it on your game board.
3. Record the details on the table below the game board.
4. Player B takes their turn rolling the dice, making a fraction and colouring the amount on their game board.
5. If a player cannot colour an amount on their game board, they pass.
6. The first player to colour the whole game board is the winner. To finish the game you must roll the exact amount needed to fill the game board.

Fraction Wall Game Board:



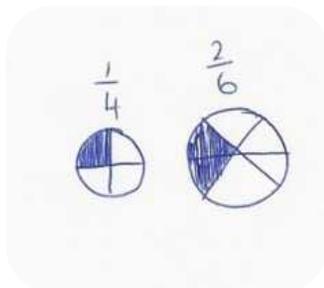
| What I rolled | What I shaded | What I rolled | What I shaded |
|---------------|---------------|---------------|---------------|
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1 Clark, D. & Roche, A., (2010), The Power of a Single Game to Address a Range of Important Ideas in Fraction Learning, Australian Primary Mathematics Classroom, Vol 15, No 3, pp18 – 24

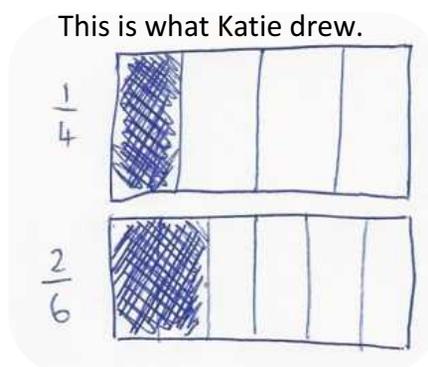
Reflection on Learning 1

Josie and Katie were arguing about the size of some fractions. Josie thought $\frac{1}{4}$ was bigger than $\frac{2}{6}$. Katie disagreed with Josie, so they decided to do some drawings to prove which was bigger.

This is what Josie drew.



This is what Katie drew.



Whose set of drawings is better for comparing the fractions? Why?



How could Katie and Josie improve their drawings?



Which is bigger $\frac{1}{4}$ or $\frac{3}{8}$? Create your own drawings to prove it.



Reflection on Learning 2

Sometimes tools are labelled using fractions. For example, Spanners, Socket Sets and Allen Keys. This Spanner set includes the following sizes:

$\frac{1}{4}$ " $\frac{9}{32}$ " $\frac{5}{16}$ " $\frac{11}{32}$ " $\frac{3}{8}$ " $\frac{7}{16}$ " $\frac{1}{2}$ " $\frac{9}{16}$ " $\frac{19}{32}$ " $\frac{5}{8}$ " $\frac{11}{16}$ " $\frac{3}{4}$ " $\frac{7}{8}$ " $\frac{13}{16}$ "

What do the fractions refer to? The first size is $\frac{1}{4}$ ", what is this one quarter of?



Match the fraction labels to the Allen keys in the photo above. $\frac{3}{8}$ $\frac{1}{16}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{5}{32}$

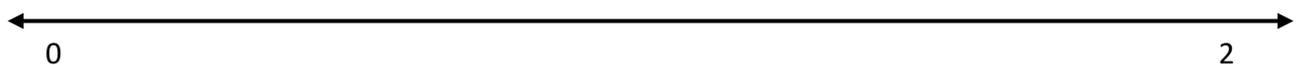
Your boss is working on a car using a socket which is $\frac{5}{16}$ ". He asks you to find the next size up. What might this be? Explain how you know.



Reflection on Learning 3

Place the following fractions on the number line.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{4}{5}$ $\frac{8}{6}$ $1\frac{2}{3}$



Choose 3 more fractions of your own choice to place on the number line.

OLNA Practice Questions

1. Some friends shared a bag of lollies. Brad ate $\frac{1}{3}$ of the bag. Cherry ate half as much as Brad.

What fraction of the bag of lollies did Cherry eat?

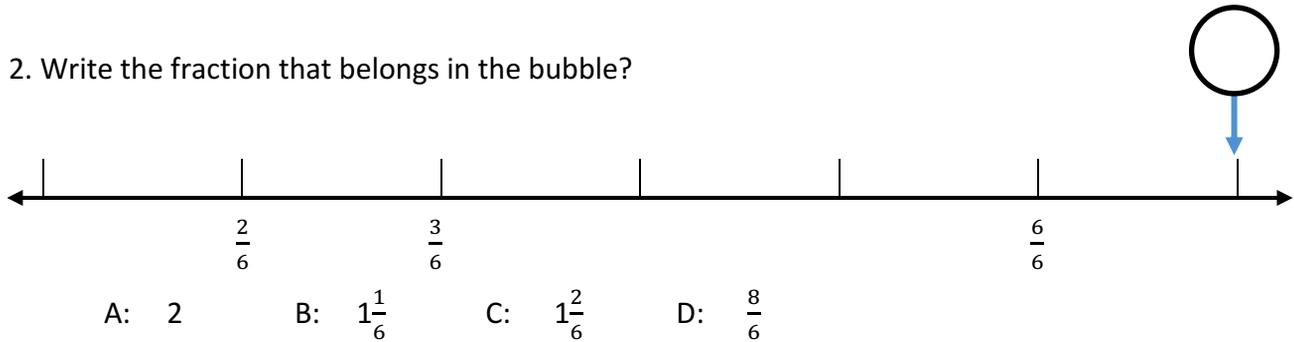
A: $\frac{1}{2}$

B: $\frac{1}{8}$

C: $\frac{1}{3}$

D: $\frac{1}{6}$

2. Write the fraction that belongs in the bubble?



A: 2

B: $1\frac{1}{6}$

C: $1\frac{2}{6}$

D: $\frac{8}{6}$

Topic 3

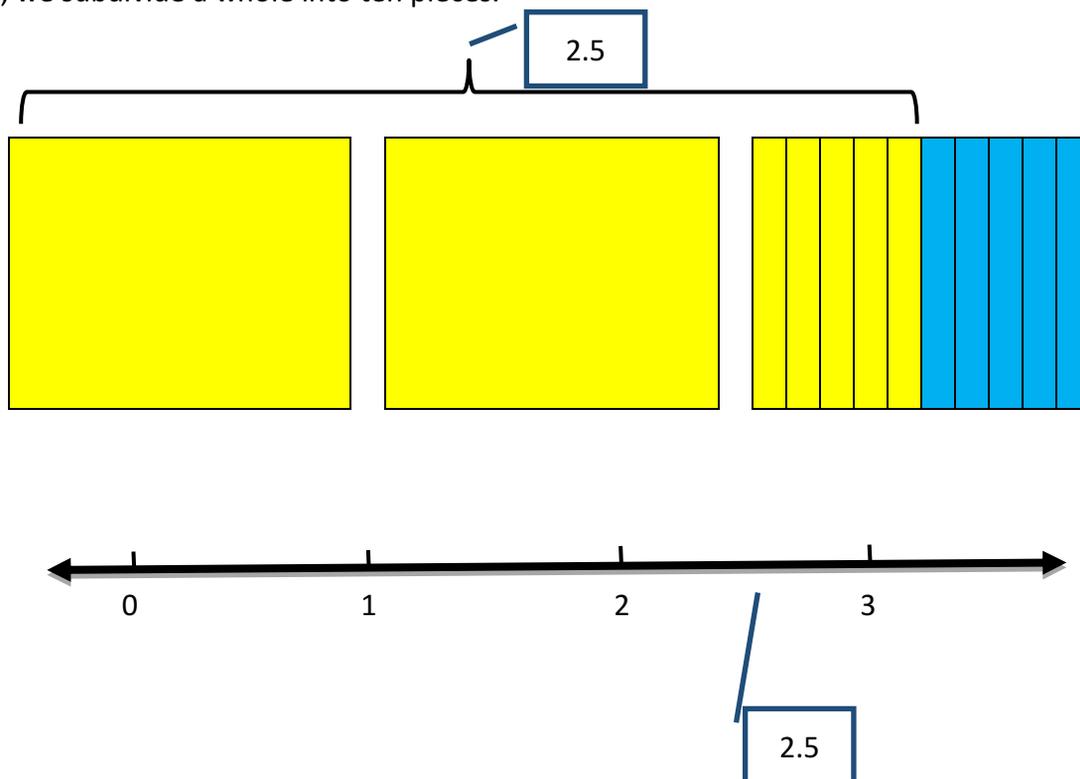
Decimals - Tenths

Mathematics Discussion

Decimals are numbers which are used to show amounts between consecutive whole numbers. For example, 2.5 is a number which is half way between 2 and 3.

There are patterns built into our number system that we can use to read, write, compare, order, count and calculate with decimal numbers.

In Unit 1 we saw that our number system is based on place value, with the places being named in a cyclical way, according to their value. Decimal numbers are written and read using the same place value system as we use for whole numbers. A dot, or decimal point, is used to separate the whole number from the decimal, which shows the part of the next whole. The first place to the right of the decimal point is the tenths place. In the number 2.5, the 5 means five tenths. This number can be said in two different ways; as *two point five* or *two and five tenths*. To make tenths, we subdivide a whole into ten pieces.



Whole Class Activity 1

Think: What does 1.1 look like?

Collect a variety of materials such as string, straws, streamers, A4 paper, paper plates, clay, play dough, rice, sand and water.

Choose one of the materials above. You will need two wholes of the selected material.

Leave one whole intact and divide the other into 10 equal pieces.

What do we call each of the pieces?



How do you write this as a fraction? How do you write this as a decimal?



Use the pieces to show 1.1 then draw a diagram to show your model of 1.1



Which of the materials was the easiest to use to make the model? Why?



Use the material you have divided into tenths to make these numbers. Draw a model of three of these.

1.3

1.6

1.9

0.1

0.3

0.5

Write the numbers onto the place value grid.

| ONES | | | DECIMALS |
|----------|------|------|----------|
| Hundreds | Tens | Ones | Tenths |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

Use your material to make a model of 2.2?



If you did not have any wholes, how many 0.1's would be needed to make 2.2?



Reflection and Discussion

What does the zero mean in - 0.1, 0.3, 0.5?



How much bigger is 1.3 than 0.3?

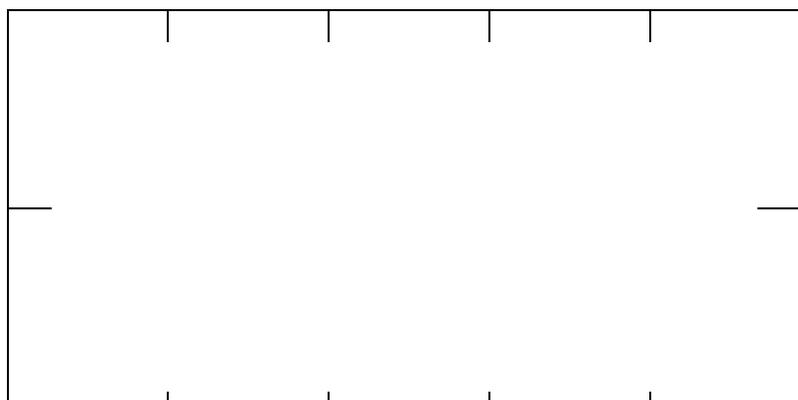


How many 0.1's are needed to make:

- a) 0.5 b) 1 c) 2 d) 10

Practice Exercise 1

1. Use a ruler to complete the partitioning of this box into tenths.



Colour 2 tenths blue.

Colour $\frac{4}{10}$ green.

Colour 0.1 black.

2. a) Use a ruler to divide this line into tenths.



Label each of the marks on the number line with its decimal number. Include 0 at one end and 1 at the other end.

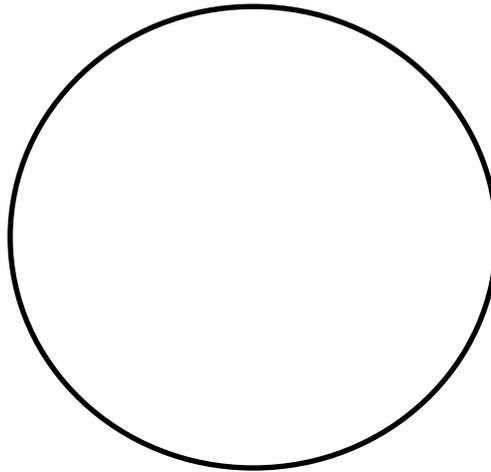


b) There are decimal numbers between each whole number in the number system. On the line below mark and label the tenths between 1 and 2.



3. Divide this circle into tenths by dividing it in half and then dividing each half into fifths.

- a) Colour 4 tenths blue.
- b) Colour 0.2 red.
- c) Colour $\frac{3}{10}$ green.



Practice Exercise 2

1. For each situation below draw a diagram and label the decimal amount.

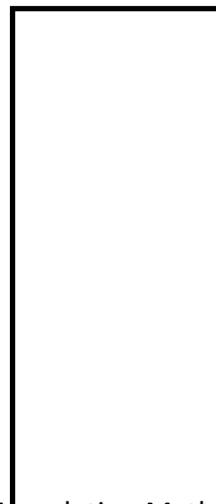
| | |
|----------------------------|-------------------------|
| a) 0.5 of a sandwich | d) 1.2m of pipe |
| b) 0.3 of a sheet of paper | e) 3.5kg flour |
| c) 0.9 of a length of wood | f) 2.5 glasses of water |

2.

a) Colour 0.3 of the square blue.



b) Colour 0.7 of the rectangle red.



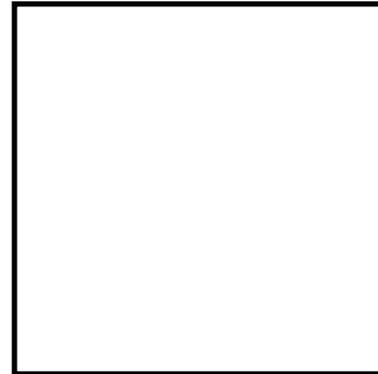
c) Colour 0.1 of the rectangle green and 0.8 black.

How much is not coloured?



d) Colour 0.2 of the square blue, 0.4 red.

How much is not coloured?



Whole Class Activity 2

Take a strip of paper 1m long and divide it into ten equal parts. Draw lines to show each tenth.

Label each line with its decimal number as you count along the strip. Include 0 and 1.

Explain how you know you have made tenths?



Lay the paper strip next to a one metre ruler or tape measure.

Compare where the tenth markings you made on the paper strip are to the markings on the tape measure.

If you need to improve the accuracy of your markings, do so.

How long is one tenth of a metre in centimetres?



How do you write one tenth of a metre as a decimal?



Select some objects and use your paper strip to measure it using tenths.

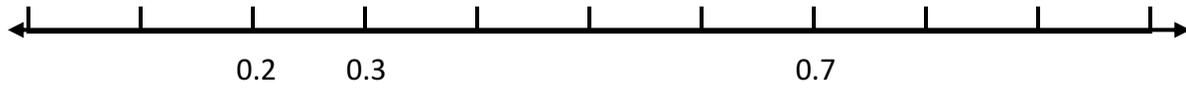
Record the measurements on the table below. If the measurement does not match any of the tenth markings, describe the numbers it is between.

| Object | Measurement |
|--------|-----------------------|
| Desk | Between 0.5m and 0.6m |
| | |
| | |
| | |
| | |

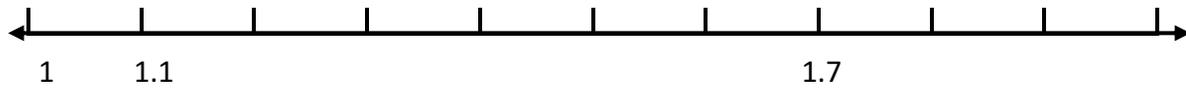
Practice Exercise 3

1. Label the missing numbers on each number line.

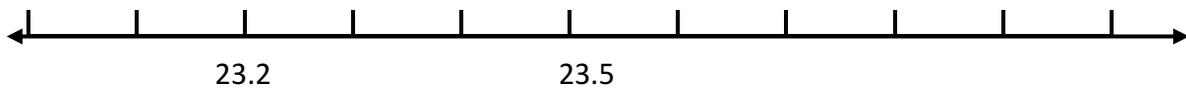
a)



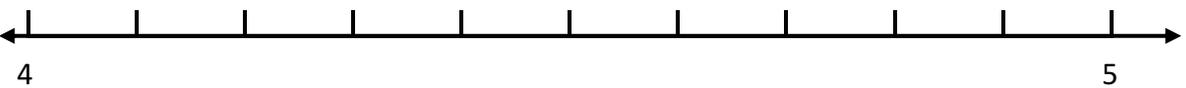
b)



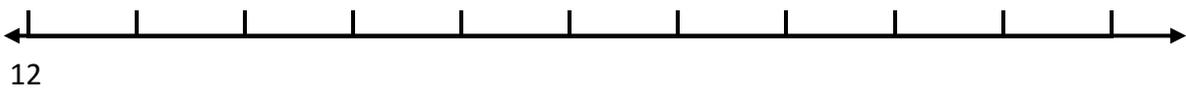
c)



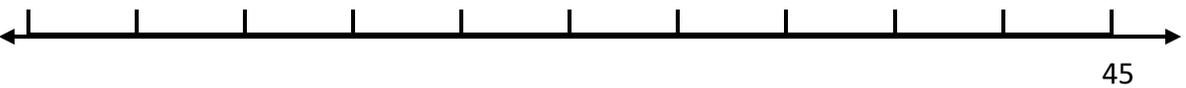
d)



e)



f)



2. Draw a number line and label 0.4, 1.4 and 2.4.

Whole Class Activity 3

Ned, Ben, Kat and Sia used a calculator to work out how much money they could save.

Ned said "That's one point five dollars."

Ben said "So we save one dollar and fifty cents."

Kat said, "No, we are saving one dollar and five cents."

Sia said "We're saving one dollar and five tenths of a dollar."

Who is wrong? Why?



How would you read 1.5 in this situation?



Ned, Ben, Kat and Sia used a calculator to work out how much wood they needed for a shelf.

Ned said this as “That’s one point two metres.”

Ben said “It’s one metre and two centimetres.”

Kat said, “That means one metre and twenty centimetres.”

Sia said “No, that’s one metre and two tenths of a metre.”

Who is right? Why?



How would you read 1.2 if it were referring to kilometres?



Practice Exercise 4

1. Read each number and write two different ways of saying it. What would each mean if it were referring to money? What would it mean if it were referring to metres? The first one has been done for you.

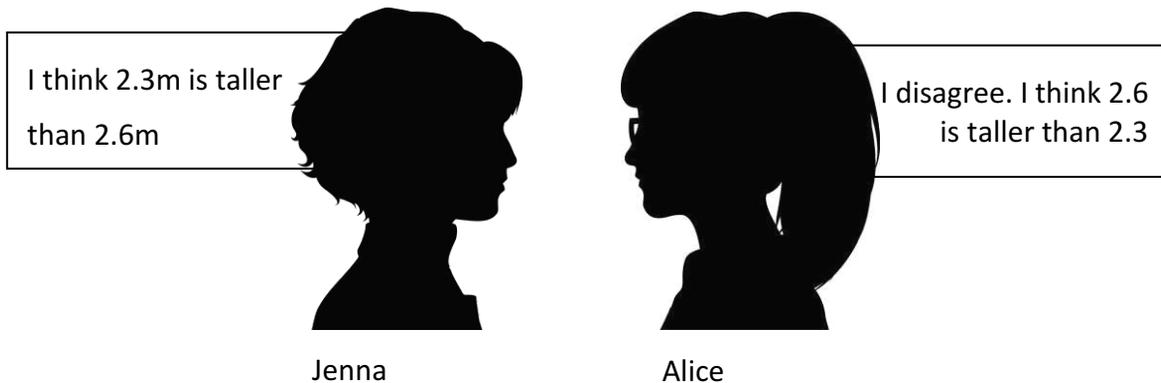
| | | |
|--------|--------------------------------|---------------------------|
| a) 0.4 | Ways of Saying | What does it mean? |
| | 1. Zero point four of a dollar | Money |
| | 2. Forty cents | Four tenths of one dollar |
| | 1. Zero point four of a metre | Metres |
| | 2. Four tenths of a metre | Four tenths of one metre |
| b) 1.2 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |
| c) 3.8 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |
| d) 0.3 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |
| e) 1.6 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2.. | |
| | 1. | Metres |
| | 2. | |

Practice Exercise 5

- Use counting forwards or backwards to complete these number sequences. Use the constant function on the calculator to help if needed.
 - 1.3, 1.2, 1.1, _____, _____, _____
 - 10.4, 10.6, 10.8, _____, _____, _____
 - 8, 8.5, 9, 9.5, _____, _____, _____
 - 13.8, 13.6, 13.4, 13.2, _____, _____, _____
- Use counting by decimals to solve the following problems.
 - A cook had 17 half kilo bags (0.5kg) of flour in his storeroom. How many kilos of flour did he have altogether?
 - The cook had 4 kilograms of butter. He used 0.1 kilograms for his hot cross bun recipe, and he made 12 batches of hot cross buns. How much butter did he have left?
 - He had 2 kilograms of sugar. He used 0.2 kilogram of sugar in his hot cross buns. Did he have enough for his 12 batches? Why or why not?

Whole Class Activity 5

Jenna and Alice are trying to figure out which garden shed is taller.



To work out who was right they each chose a different way of comparing the amounts.

Jenna drew a number line like this.

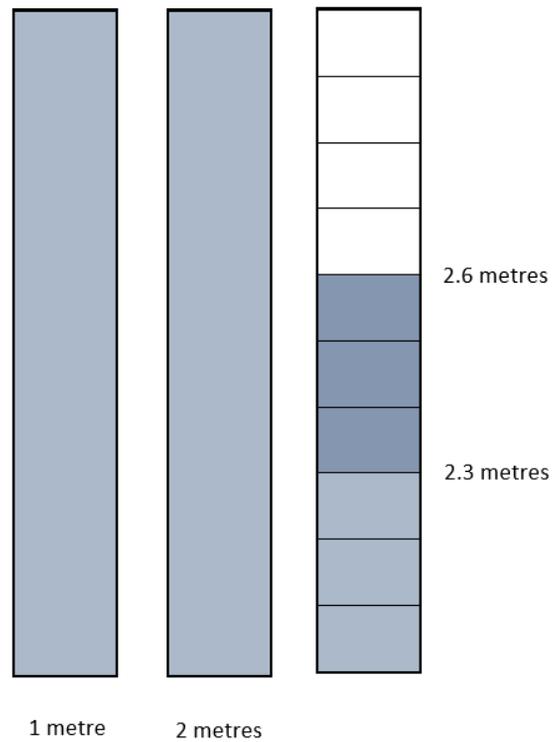


Alice drew rectangles like this.

Which is taller, 2.3 or 2.6?



What else could Jenna and Alice have done to show measurement is taller?



What is the difference between the measurements?
How do you know?



Practice Exercise 5

1. Draw a model to decide which is bigger. Put a $<$ or $>$ in the circle to show which is bigger.

a) 0.8 0.5

c) 2.1m 1.2m

b) \$1.60 \$1.90

d) 3.5kg 3.8kg

2. Order the amounts from smallest to largest.

a) 1.1, 1.0, 10.1, 11.1

d) \$3.60, \$6.30, \$0.30, \$0.60

b) 0.2, 2.1, 1.2, 0.1

e) 0.5L, 1.5L, 5.1L, 0.1L

c) 0.4m, 4.1m, 4.4m, 1.4m

f) 9.1kg, 0.9kg, 1.9kg, 9.9kg

3. Use counting by decimals to work out the difference between the amounts. Use a number line or jottings to help. Record how you worked out the answer.

a) 0.5 and 0.9

d) 3.5m and 2.1m

b) 1.7 and 1.1

e) 4.9kg and 3.3kg

c) \$2.70 and \$2.20

f) 2.4L and 4.2L

Practice Exercise 6

Card Game – Decimal More

Number of Players: 2 – 4

Aim: to win all the cards from the other players.

Equipment: Each person makes a set of cards with a decimals from 0 to 2 written in two ways, e.g. 0.1 and $\frac{1}{10}$, 1.5 and 1 and $\frac{5}{10}$

Rules of the Game:

1. Deal out all of the cards. Place them face down.
2. Each person turns over one card at the same time.
3. The person with the largest number takes all of the cards and places them at the bottom of their pile (face down).
4. If two numbers are the same or equivalent, the players who own these cards turn over another card. The person with the largest second card wins.
5. Play continues until one player has no cards left. The player with the most cards is the winner.

Variation: Use the fraction cards from the previous topic with the decimal cards.

Reflection on Learning

Tape measures, show different amounts

On the ruler below, label the following measurements; 0.8cm, 8.2cm and 0.1m



Explain how you knew where each measurement was.



OLNA Practice Questions

1. Which of these is the longest distance?
A. 13cm B. 1.3m C. 0.3m D. 103cm
2. This is a count by 0.2 pattern. What two numbers comes next in this pattern?
0.2, 0.4, 0.6, 0.8, _____, _____

Topic 4

Decimals - Hundredths

Mathematics Discussion

We use the patterns built into our number system to read, write, compare, order, count and calculate with decimal numbers.

The first place to the right of the decimal point is the tenths place. The second place to the right is the hundredths place. To make tenths, we subdivide a whole into ten pieces, to make hundredths, we subdivide one tenth into ten pieces or we subdivide a whole into a hundred pieces.

| ONES | | | DECIMALS | |
|----------|------|------|----------|------------|
| Hundreds | Tens | Ones | Tenths | Hundredths |
| | | 2 | 7 | 5 |

In the number 2.75, the 5 means five hundredths. When a decimal refers to a measurement, we typically say this number as *two point seven five*. However we can also say this as *two and seventy five hundredths*. When a decimal refers to money we typically say 2.75 as *two dollars and seventy five cents*.

We can rename decimal numbers in the same way that we rename whole numbers. For example, we can think of 2.7 as $2 + \frac{7}{10}$ or as $2 + \frac{70}{100}$.

To compare decimals we need to consider both the tenths place and the hundredths place.

Whole Class Activity 1

Think: What does 1.11 look like?

Collect a variety of materials such as string, straws, streamers, A4 paper, paper plates, clay, play dough, rice, sand and water.

Choose one of the materials above. You will need three wholes of the selected material.

Leave one whole intact and divide the second into 10 equal pieces.

What do we call each piece?



How do you write this as a fraction? How do you write this as a decimal?



Divide the third whole into tenths, then divide each tenth into ten equal pieces.

How many pieces are there? What do we call each piece?



How do you write this as a fraction? How do you write this as a decimal?



Use the material to show 1.11 Draw a diagram to show your model of 1.11



Which of the materials was the easiest to use to make the model? Why?



Use the material you have divided into tenths and hundredths to make the numbers below. Alternatively, you could choose different material for this part of the activity.

Draw a model of two of them.

1.03 1.24 1.99 0.05 0.06 0.55

Write the numbers onto the place value grid.

| ONES | | | DECIMALS | |
|----------|------|------|----------|------------|
| Hundreds | Tens | Ones | Tenths | Hundredths |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Use your material to make a model of 2.22? If you did not have any wholes, how many 0.01's would be needed to make 2.22?



Reflection and Discussion

What do the zeros mean in - 1.03, 0.05, 0.50?



How many 0.01's are needed to make:

- a) 0.5 b) 1 c) 2 d) 10

How much bigger is 1 than 0.01?

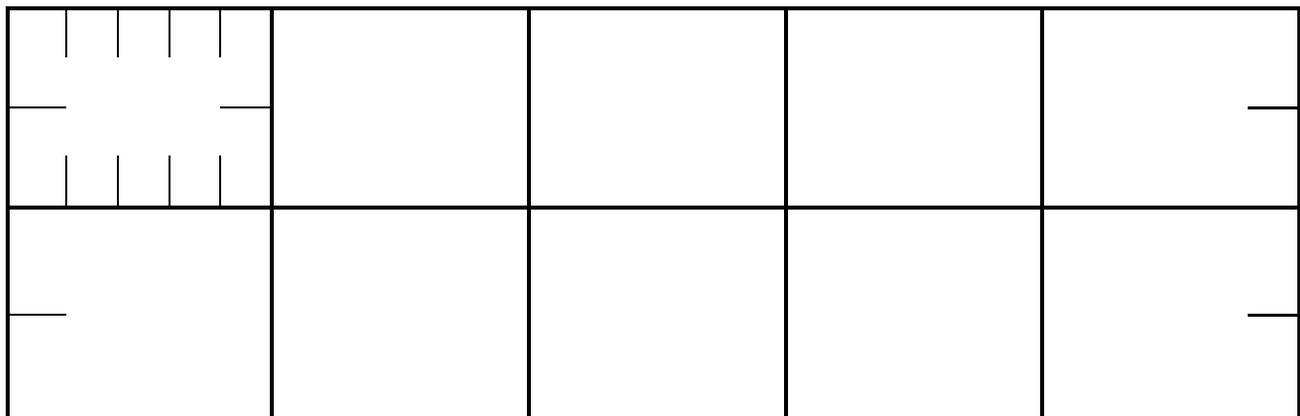


How much bigger is 0.1 than 0.01?



Practice Exercise 1

1. Use a ruler to complete the partitioning of the grid so it is divided into hundredths.

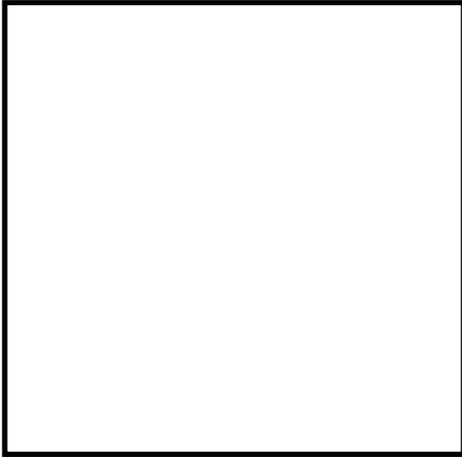


- a) Colour 3 hundredths blue d) Colour 20 hundredths red
 b) Colour 0.04 green e) Colour 0.19 orange
 c) Colour $\frac{1}{100}$ black f) Colour $\frac{36}{100}$ yellow

2. For each situation below draw a diagram on a sheet of paper and label the decimal amount.

- a) 0.52 of a chocolate bar d) 1.25km of pipe
 b) 0.38 of a sheet of paper e) 3.05kg flour
 c) 0.95 length of wood f) 2.75 litres of water

3. a) Colour 0.23 of the square blue.



b) Colour 0.75 of the rectangle red.



c) Colour 0.05 of the rectangle green and 0.80 black.
How much of the rectangle is coloured?
How much is not coloured?



d) Colour 0.25 of the square blue and 0.45 red.
How much of the square is coloured?
How much is not coloured?



Whole Class Activity 2

Take a strip of paper 1m long and divide it into ten equal parts. Draw lines to show each tenth. Label each line with its decimal number as you count along the strip. Include 0 and 1.

Divide each tenth into ten equal parts, so you have hundredths.

Explain how you know you have made hundredths?



Lay the paper strip next to a one metre tape measure.

Compare where the tenth and hundredths markings you made on the paper strip are on the tape measure.

If you need to improve the accuracy of your markings, do so. How long is $\frac{1}{100}$ of a metre?



How do you do you write this as a decimal?



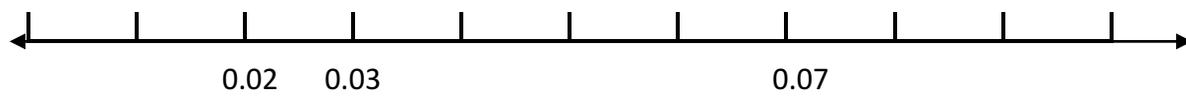
Select some objects and use your paper strip to measure each to the nearest hundredth of a metre. Record the measurement in metres.

| Object | Measurement |
|--------|-------------|
| | |
| | |
| | |
| | |
| | |

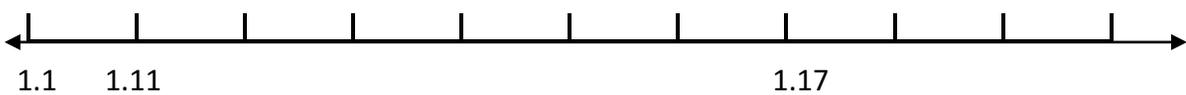
Practice Exercise 3

1. Label the missing numbers on each number line.

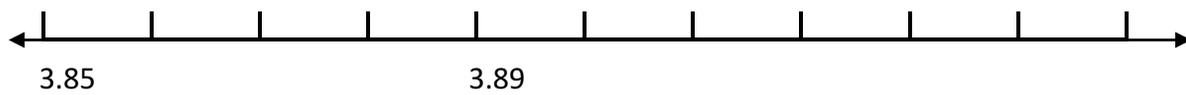
a)



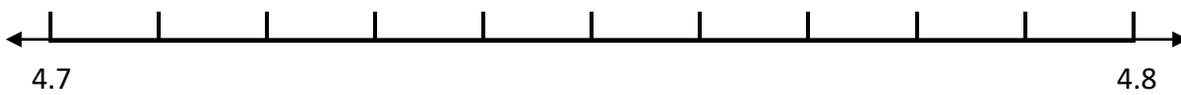
b)



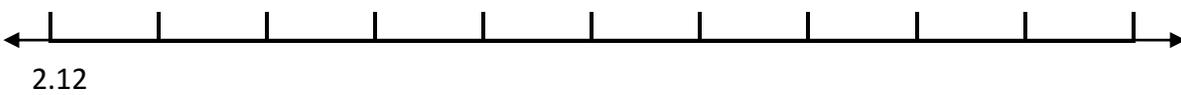
c)



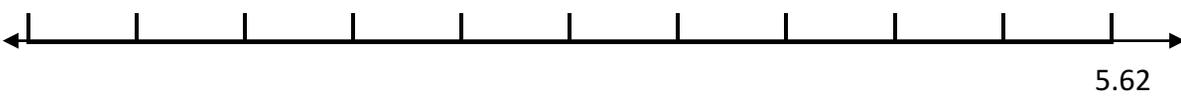
d)



e)



f)



2. Draw a number line and label 0.3, 0.38 and 0.41

Whole Class Activity 3

Ned, Ben, Kat and Sia used a calculator to work out how much money they would save after getting a discount.

Ned said, "That's four point two dollars."

Ben said, "So we save four dollars and twenty cents."

Kat said, "No, we save four dollars and two cents."

Sia said, "We're saving four dollars and two hundredths of a dollar."



Who is wrong? How do you know?



How would you read 4.02 in this situation?



Ned, Ben, Kat and Sia used a calculator to work out the length of a corridor in order to lay new carpet.

Ned said this as "That's four point two metres."

Ben said "It's four metres and two centimetres."

Kat said, "That means four metres and twenty centimetres."

Sia said "No, that's four metres and two hundredths of a metre."

Who is right? Why?



How would you read 4.02 if it was referring to kilometres?



Practice Exercise 4

1. Read each number and write two ways of saying it for money and two ways of saying it for metres.

What would each mean if it were referring to money?

What would it mean if it were referring to metres?

The first one has been done for you.

| | Ways of Saying | What does it mean? |
|---------|---|--|
| a) 0.45 | 1. Zero point four five dollars 2. Forty five cents | Money Forty-five hundredths of one whole dollar |
| | 3. Zero point four five of a metre 4. Forty five hundredths of a metre | Metres Forty-five hundredths of one whole metre |

| | | |
|---------|----------------|--------------------|
| b) 1.26 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |

| | | |
|---------|----------------|--------------------|
| c) 3.08 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |

| | | |
|---------|----------------|--------------------|
| d) 0.39 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |

| | | |
|---------|----------------|--------------------|
| e) 1.09 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |

| | | |
|---------|----------------|--------------------|
| f) 3.88 | Ways of Saying | What does it mean? |
| | 1. | Money |
| | 2. | |
| | 1. | Metres |
| | 2. | |

2. Write the number that matches each.

a) three hundredths

e) one and 30 hundredths of a metre

b) one and nineteen hundredths

f) one and five hundredths of a kilogram

c) two dollars and seventy three cents

g) ten metres and fifteen centimetres

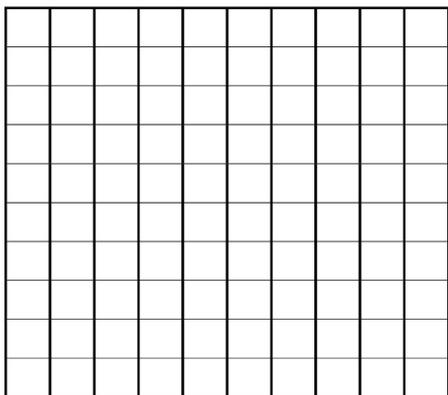
d) five dollars and four cents

h) one metre and ninety eight centimetres

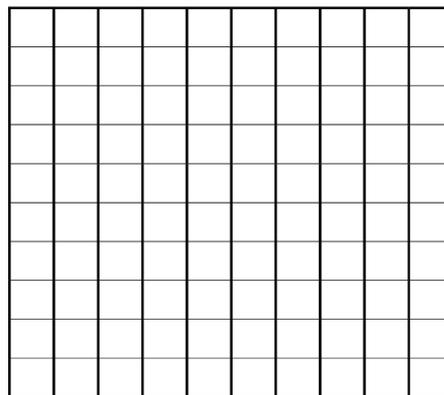
Whole Class Activity 4

Think: how can we partition decimal numbers?

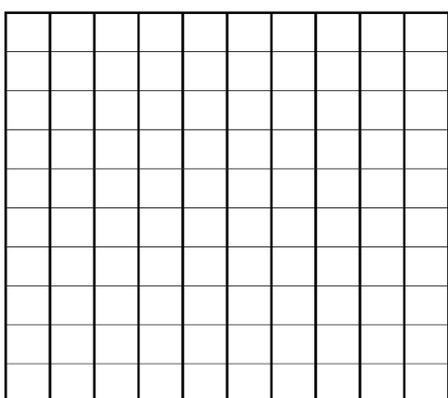
Shade each of the grids to match the numbers given.



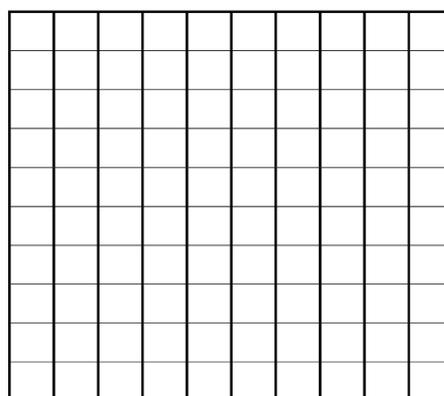
Colour four tenths of the whole grid above.
Write as a decimal.
Write using fractions.



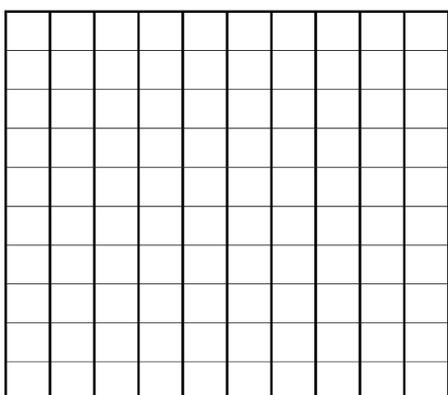
Colour 40 hundredths of the whole grid above.
Write as a decimal.
Write using fractions.



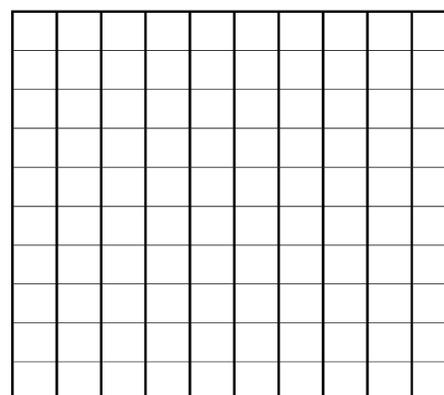
Colour six tenths and eight hundredths of the whole grid.
Write as a decimal.
Write using fractions.



Colour sixty eight hundredths of the whole grid.
Write as a decimal.
Write using fractions.



Colour seven tenths and five hundredths of the whole grid.
Write as a decimal.
Write using fractions.



Colour seventy five hundredths of the whole grid.
Write as a decimal.
Write using fractions.

Reflection and Discussion

Look at each pair of grids you coloured in the activity above.

Explain what is different about the two ways of describing the decimal.



Explain what is the same about the two ways of describing the decimal.



Practice Exercise 5

1. How many ways can you make each number below? Use the partitions listed in the box below to make each number. Record the different ways as you go.

For example the number 23.54 can be made using these numbers in the boxes.

2 tens 3 ones 5 tenths 4 hundredths

2 tens 3 ones 54 hundredths

23 ones 5 tenths and 4 hundredths

23 ones 4 tenths 14 hundredths

| | | | | |
|----------|---------------|---------|-----------|---------|
| 5 tenths | 4 hundredths | 1 ten | 1 hundred | 2 ones |
| 8 tenths | 5 hundredths | 2 tens | | 3 ones |
| 9 tenths | 6 hundredths | 3 tens | | 4 ones |
| | 14 hundredths | 13 tens | | 6 ones |
| | 15 hundredths | | | 12 ones |
| | 50 hundredths | | | 14 ones |
| | 54 hundredths | | | 16 ones |
| | 90 hundredths | | | 23 ones |
| | 95 hundredths | | | 34 ones |

a) 12.56

b) 26.95

c) 134.04

Whole Class Activity 5

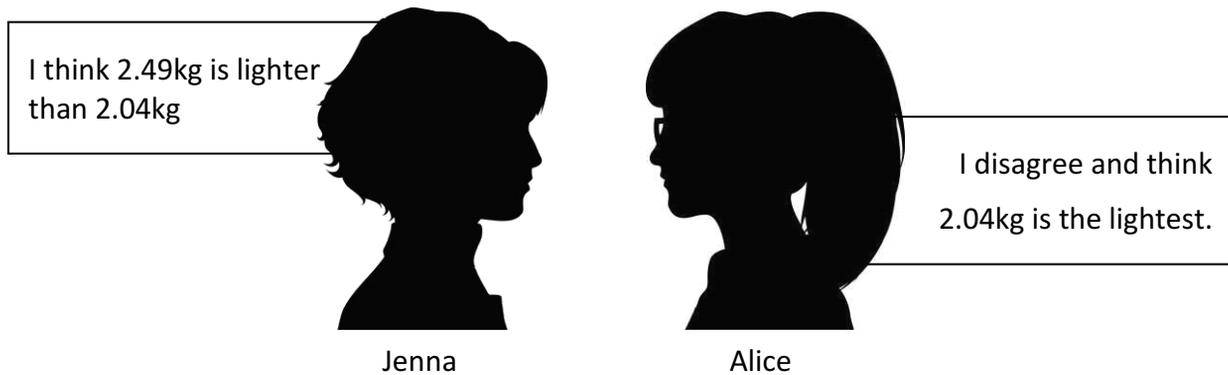
Use the constant function on your calculator to count with decimal numbers.

2. Use counting by decimals to solve the following problems.

- a) A tiler had 18 tubs of grout mix, each weighing 0.75kg. How many kilos of grout mix altogether?
- b) The chef had 6 litres of soup prepared. Each serve is 0.25 litres. 13 people ordered soup for lunch. How much soup was left?
- c) Kerry's closest bus stop is 0.45km from her front door. If she walks to the bus stop 5 times in a week, how far has she walked?
- d) Play Swamp Survival: Hundredth Patterns
<http://www.scootle.edu.au/ec/viewing/L7902/index.html>

Whole Class Activity 6

Jenna and Alice are doing a science experiment and are trying to figure out which rock sample weighs less.



Who is right? How do you know?



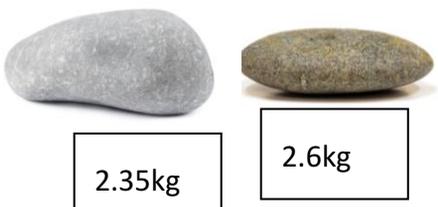
What would you do/say to prove to Jenna and Alice which measurement is lighter?



How much lighter is the lightest rock? How do you know?

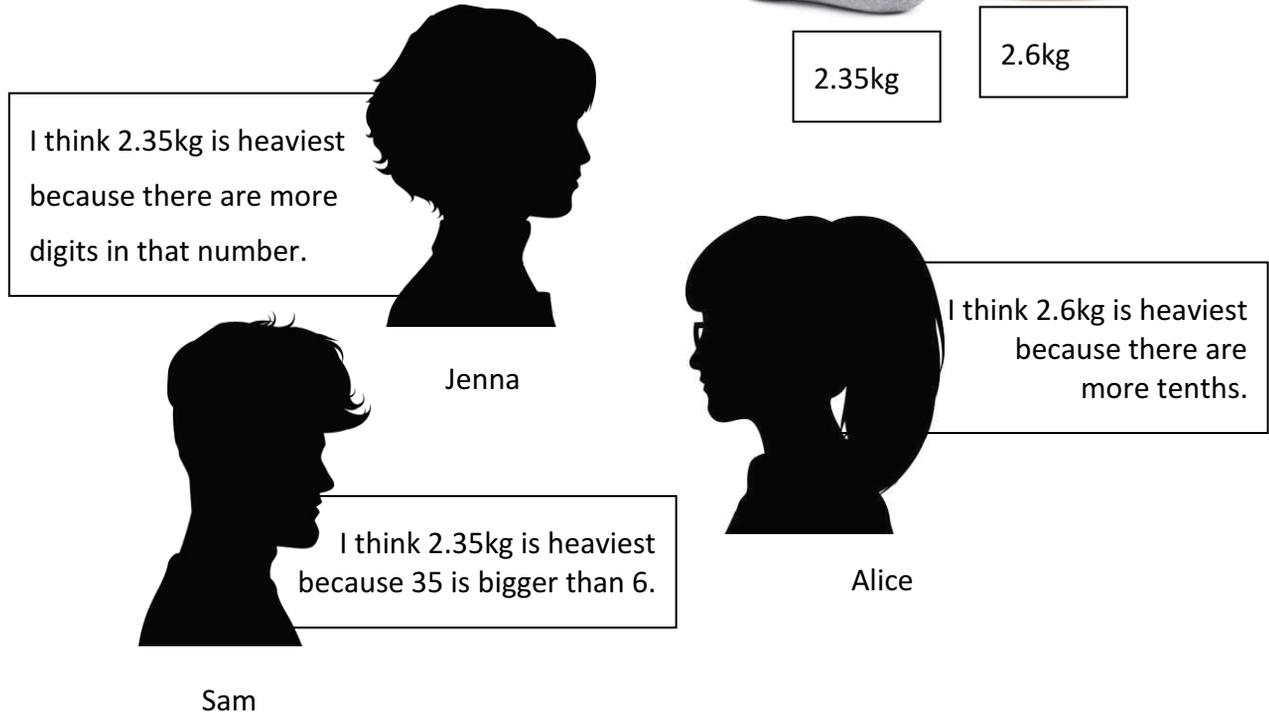


Sam, Jenna and Alice weighed some more samples and got stuck trying to work out which of these was heavier.



2.35kg

2.6kg



I think 2.35kg is heaviest because there are more digits in that number.

Jenna

Sam

I think 2.35kg is heaviest because 35 is bigger than 6.

Alice

I think 2.6kg is heaviest because there are more tenths.

Who is right? How do you know?



What could you do/say to prove to Sam, Jenna and Alice which measurement is heavier?



Reflection and Discussion

Discuss with a partner the strategies you can use when you compare the size of each pair of numbers.

1.06 and 1.6

0.8 and 0.19

4.61 and 4.6

What do you do when you compare different sized decimal numbers?



Practice Exercise 7

1. Draw a model to work out which is bigger. Put a < or > in the circle to show which is bigger.

a) 0.28 ○ 0.51

d) 0.8km ○ 0.31km

b) \$1.65 ○ \$1.56

e) 7.03m ○ 7.2m

c) 2.17m ○ 2.71m

2. Order the amounts from smallest to biggest.

a) 1.11, 1.01, 10.11, 11.10

c) \$8.60, \$8.36, \$0.38, \$0.68

b) 1.48m, 4.81m, 8.4m, 1.84m

d) 0.5L, 1.05L, 5.1L, 0.15L

3. Use counting by decimals (with a number line, calculator or mentally) to work out the difference between the amounts. Record how you worked out the answer.

a) 2.85 and 0.95

c) 4.1kg and 3.31kg

b) \$2.25 and \$2.82

d) 2.48L and 4.8L

4. Play Swamp Survival: Hundredths Challenge

<http://www.scootle.edu.au/ec/viewing/L7903/index.html>

Practice Exercise 8

Card Game – Decimal More

Number of Players: 2 – 4

Aim: to win all the cards from the other players.

Equipment: a set of cards each with a two digit decimals written in three ways, e.g., 0.15 and $\frac{15}{100}$, $\frac{1}{10} + \frac{5}{100}$

Rules of the Game:

1. Deal out all of the cards. Place them face down.
2. Each person turns over one card at the same time.
3. The person with the largest number takes all of the cards and place them at the bottom of their pile (face down).
4. If two numbers are the same or equivalent, the players who own these cards turn over another card. The person with the largest second card wins.
5. Play continues until one player has no cards left. The player with the most cards is the winner.

Variation: Include the one digit decimal cards from the previous topic.

| | | | | | | | | | |
|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 0.15 | 0.24 | 0.38 | 0.41 | 0.53 | 0.62 | 0.79 | 0.86 | 0.97 | 0.03 |
| $\frac{15}{100}$ | $\frac{24}{100}$ | $\frac{38}{100}$ | $\frac{41}{100}$ | $\frac{53}{100}$ | $\frac{62}{100}$ | $\frac{79}{100}$ | $\frac{86}{100}$ | $\frac{97}{100}$ | $\frac{3}{100}$ |
| $\frac{1}{10} + \frac{5}{100}$ | $\frac{2}{10} + \frac{4}{100}$ | $\frac{3}{10} + \frac{8}{100}$ | $\frac{4}{10} + \frac{1}{100}$ | $\frac{5}{10} + \frac{3}{100}$ | $\frac{6}{10} + \frac{2}{100}$ | $\frac{7}{10} + \frac{9}{100}$ | $\frac{8}{10} + \frac{6}{100}$ | $\frac{9}{10} + \frac{7}{100}$ | $\frac{0}{10} + \frac{3}{100}$ |
| 0.05 | 0.29 | 0.9 | $\frac{5}{100}$ | $\frac{29}{100}$ | $\frac{90}{100}$ | $\frac{0}{10} + \frac{5}{100}$ | $\frac{2}{10} + \frac{9}{100}$ | $\frac{9}{10} + \frac{0}{100}$ | 0 |

Decimal Line-Up.

Number of Players: 2 – 4

Aim: to place all of the cards in order.

Equipment: a set of cards each with a different two digit decimal (from previous game)

Rules of the Game:

1. Find the zero card and the one card and place at either end of the desk. Create eight spaces between the two cards.
2. Deal out 5 cards to each player and place the rest of the deck face down in the centre. Players hold their cards face up.
3. Players take turns to place a card in one of the spaces between the zero and one, so that they are in the correct order. After they place a card they take a replacement card from the pack.
4. An equivalent decimal can be placed on top of a previous card.
5. If a card cannot be placed in order, the player misses a turn and swaps one of their cards for one in the deck.
6. Winner is the person who places the last card into the line.

Variation: Include the one digit decimal cards.

Reflection on Learning

1. Usain Bolt is the Olympic record holder for the men's 100 metre sprint. His time is 9.63 seconds. Research this time. What does the .63 refer to?



Watch a YouTube video of the race and count how many steps he takes to complete the race. Use a calculator to divide 100 metres by the number of steps to find the length of one of his strides. What does the decimal part of the number mean? Measure this out along the ground.

2. Renaud Lavillenie is the Olympic record holder for the mens pole vault. He can pole vault 5.97m. What does the .97 refer to?



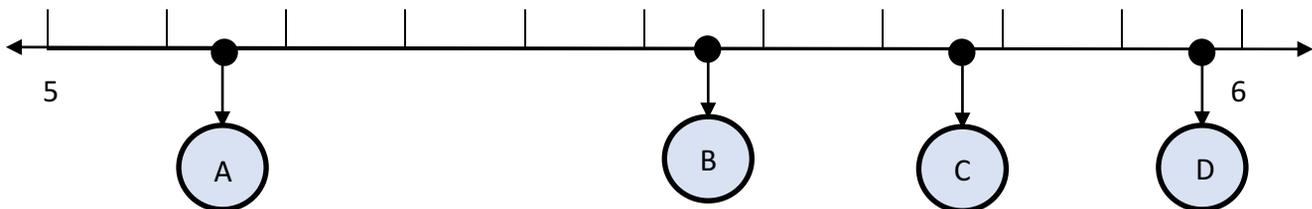
Measure out 5.97m along the ground. Find something in your environment that is about the same height.

3. Why are decimals important in measuring sporting events?

OLNA Practice Questions

1. The dog weighs 5.75kg

Which dot on the number line best represents this amount?



2. Which of these shows the decimals in order from smallest to largest?

- a) 0.05, 0.1, 0.5, 0.15, 0.51
- b) 0.05, 0.1, 0.15, 0.5, 0.51
- c) 0.1, 0.5, 0.05, 0.15, 0.51
- d) 0.51, 0.15, 0.5, 0.1, 0.05
- e) 0.51, 0.15, 0.05, 0.1, 0.5

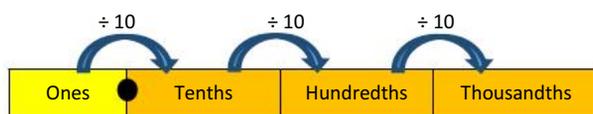
Topic 5

Decimals - Thousandths

Mathematics Discussion

We use the patterns built into our number system to read, write, compare, order, count and calculate with decimal numbers. Decimal numbers are simply an extension of our whole number place value system. Decimal numbers are parts of one, and the names of decimals show how many parts the 'one' has been broken into. Our place value system is built on powers of ten, which means that each place, moving from left to right is ten times smaller.

To make thousandths, we subdivide a whole into ten pieces, subdivide each of these into ten pieces, and then subdivide each of these into ten pieces.



In the number 43.125 below, the 5 means five thousandths. Five thousandths are ten times smaller than five hundredths, which are ten times smaller than five tenths. We tend to use thousandths when we need to be very exact in our measurements.

| Thousands | | | Ones | | | Decimals | | |
|-----------|------|------|----------|------|------|----------|------------|-------------|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths |
| | | | | 4 | 3 | 1 | 2 | 5 |

When a decimal refers to a measurement, we typically say the whole number section of the number differently from the decimal. We say the number above as *forty three point one two five*. However we can also say this as *forty three, and one hundred and twenty five thousandths*.

We can partition decimal numbers in the same way that we partition whole numbers. We can think of 3.125 as $3 + \frac{125}{1000}$, or as $3 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$ or as $3 + \frac{1}{10} + \frac{25}{1000}$.

To compare decimals we need to consider each of the decimal places in turn, starting with the tenths place, as this is larger than the hundredth or the thousandths places. Comparing decimals is more difficult when they have an unequal number of decimal places.

Whole Class Activity 1

Think: What does 1.111 look like?

Collect a variety of materials such as string, streamers, newspaper, paper plates, clay, play dough, rice, sand and water.

Choose one of the materials above. Which is likely to be the easiest to use to make a model? (Hint: a larger 'one' will be easier to subdivide.)

Work with a partner to make a chart showing a model of 1.111. Label the parts of your model so that you can clearly see the size difference between each of the parts.

Draw a diagram below of your model of 1.111



Explain how you divided the material into thousandths.



Use any of the materials listed above to make a model of these numbers.

1.032, 1.154, 1.999, 0.058, 0.762, 0.555

Write the numbers onto the place value grid.

| Ones | | | Decimals | | |
|----------|------|------|----------|------------|-------------|
| Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Use your material to make a model of 1.002?

What does each of the zeros mean in the number above?



Reflection and Discussion

How many 0.001's are needed to make

a) 0.01 b) 0.1 c) 1 d) 2 e) 10

How much bigger is 1 than 0.001?

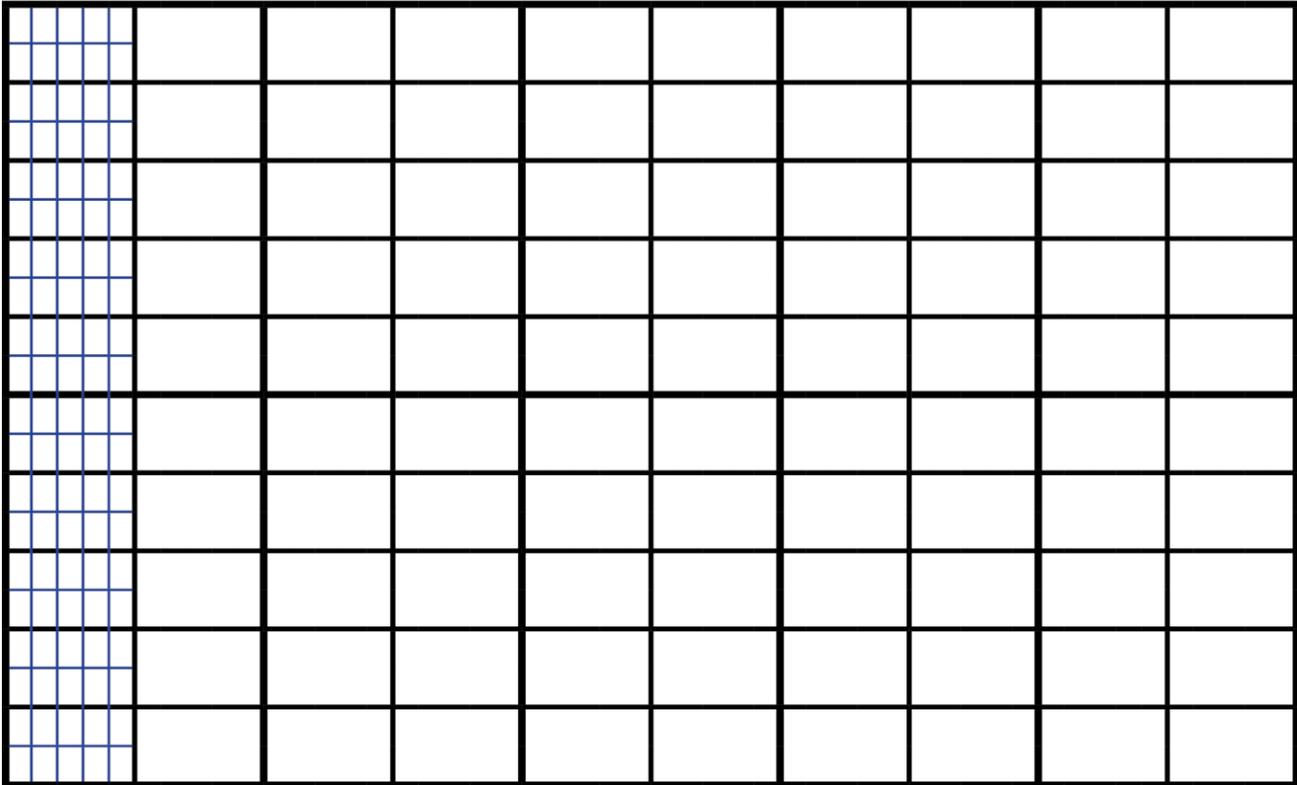
How much bigger is 0.1 than 0.001?

How much bigger is 0.01 than 0.001?

Practice Exercise 1

The bold lines on the grid show tenths and the thinner lines show hundredths.

1. Use a ruler to complete the partitioning of the grid so it is divided into thousandths.

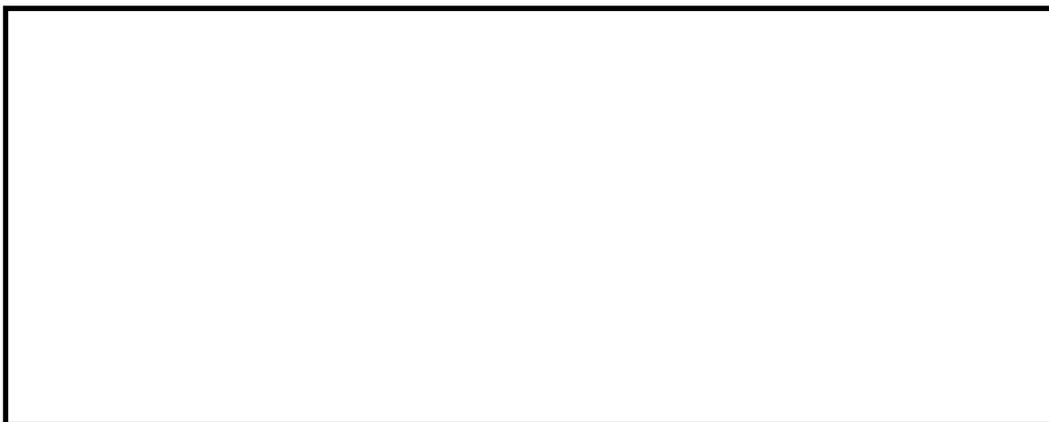


- a) Colour 3 thousandths blue
b) Colour 0.004 green
c) Colour $\frac{1}{1000}$ black
d) Colour 200 thousandths red
e) Colour 0.199 orange
f) Colour $\frac{36}{1000}$ yellow

2. For each situation below draw a diagram on a sheet of paper and label the decimal amount.

- a) 2.359 km of pipe
b) 3.725 litres of water
c) 0.525 of a sheet of paper
d) 5.205kg flour

3. a) Colour 0.259 of the rectangle blue.



b) Colour 0.005 of the rectangle green and 0.008 of the rectangle black.

How much of the rectangle is coloured?

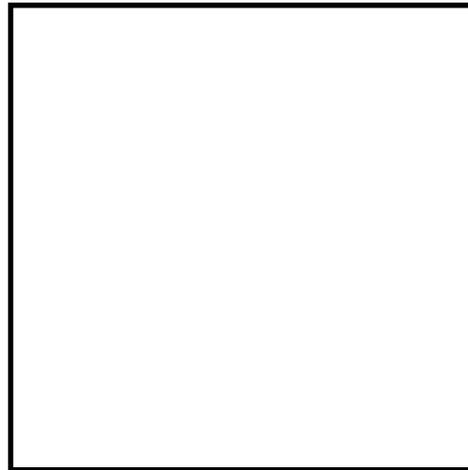
How much is not coloured?



c) Colour 0.025 of the square blue, 0.045 of the square red.

How much of the square is coloured?

How much is not coloured?



Whole Class Activity 1

In the previous topic on hundredths you divided a 1m strip of paper into hundredths.

How long is $\frac{1}{100}$ of a metre?

How do you do you write this as a decimal?

Use the same strip of paper. How would you make thousandths on the strip?



Draw lines to show the thousandths between 0 and 1 on your strip.

Lay the paper strip next to a one metre tape measure.

Compare where the thousandth markings are. How long is $\frac{1}{1000}$ of a metre?



How do you do you write this as a decimal?



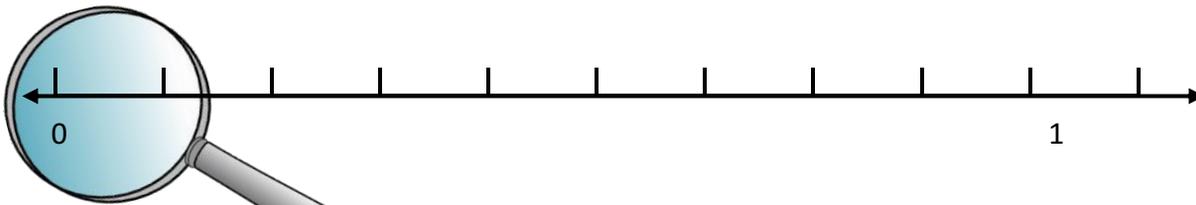
Select some objects and use your paper strip to measure. Record the measurement in metres.

| Object | Measurement |
|--------|-------------|
| | |
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| | |

Whole Class Activity 3

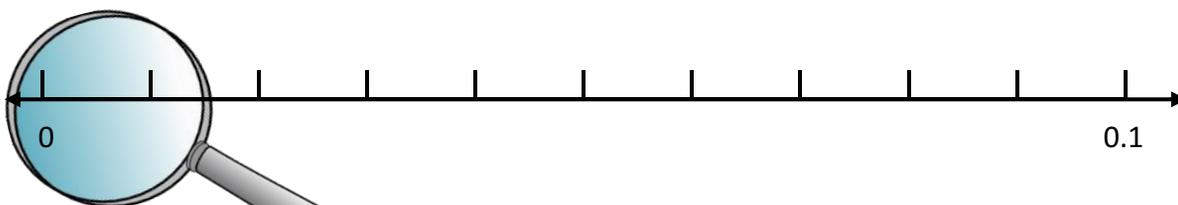
To show tenths on a number line we divide the space between two whole numbers into ten equal parts. This number line shows the tenths between 0 and 1.

Label the tenths on the number line below.



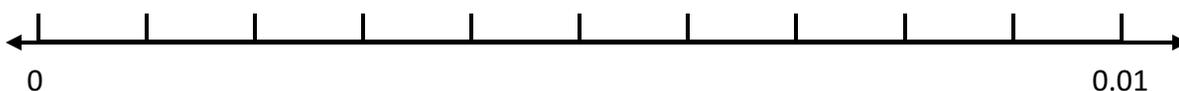
The number line below shows the 0 to 0.1 section from the line above magnified. To show hundredths on a number line we divide each tenth into ten equal parts.

Label the hundredths on the number line below.



The number line below shows the 0 to 0.01 section from the line magnified. To show thousandths on a number line we divide each hundredth into ten equal parts.

Label the thousandths on the number line below.



Reflection and Discussion

What happens when you move from 0.9 to the next tenth?

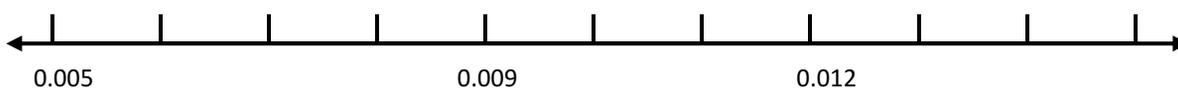
What happens when you move from 0.09 to the next hundredth?

What happens when you move from 0.009 to the next thousandth?

Practice Exercise 2

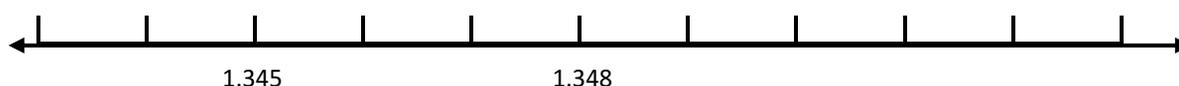
1. Label the missing numbers on each number line.

a)



What does the 1 mean in 0.013?

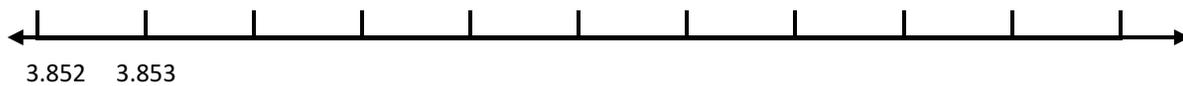
b)



What does the 5 mean in 1.350?

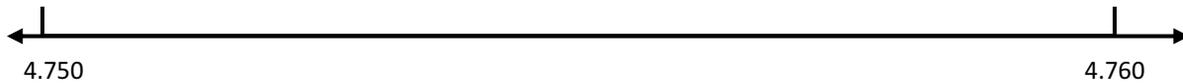
What does the 0 mean in 1.350?

c)



What does the 8 mean in each of these numbers?

d) Draw and label the lines to show each 0.001 between 4.750 and 4.760



e) Draw and label the lines to show each 0.001 from 2.555 onwards.



f) Draw and label the lines to show each 0.001 backwards from 3.99.



2. Draw a number line then label 0.81, 0.82 and 0.812

3. Draw a number line and subdivide it to work out the following:

- a) What is a number that is half way between 1 and 2?
- b) What is a number that is half way between 1.1 and 1.2?
- c) What is a number that is half way between 1.05 and 1.06?

4. Draw a number line and subdivide it to show the difference in size of each of the following decimals?

0.5 0.05 0.005

Whole Class Activity 4

Ned, Ben, Kat and Sia used a calculator to work out the width of a window so they could buy glass to fit.

Ned said, "That's one point two five five metres."

Ben said, "No it's one point two hundred and fifty five metres."

Kat said, "That means 1 metre and two hundred and fifty five thousandths of a metre."

Sia said, "I think its 1 metre and two hundred and fifty five centimetres."



Who is right? How do you know?



How should you read 1.255 in this situation?



Practice Exercise 3

1. Read each number and write two ways of saying it each. What would it mean if it were referring to kilometres? The first one has been completed for you.

| | |
|----------|--|
| a) 0.459 | Ways of Saying |
| | 1. zero point four five nine |
| | 2. four hundred and fifty nine thousandths |

| | |
|----------|----------------|
| b) 1.768 | Ways of Saying |
| | 1. |
| | 2. |

| | |
|----------|----------------|
| c) 3.089 | Ways of Saying |
| | 1. |
| | 2. |

| | |
|-----------|----------------|
| d) 0.83 m | Ways of Saying |
| | 1. |
| | 2. |

| | |
|--------------|----------------|
| e) 23.572 km | Ways of Saying |
| | 1. |
| | 2. |

| | |
|-------------|----------------|
| f) 5.704 kg | Ways of Saying |
| | 1. |
| | 2. |

2. Write the number that matches each.

- a) five thousandths
- b) two and one hundred and twenty two thousandths
- c) one and two thousandths of a kilometre
- d) ten and three thousandths of a metre
- e) twenty and fifty five thousandths of a kilogram
- f) five metres and nine hundred and eighty millimetres
- g) eight metres and fifteen millimetres

Whole Class Activity 5

Use the constant function on your calculator to count with decimal numbers.

Use the table below to record the numbers as you count.

| Count by 0.001, start from 0 | Count by 0.002, start from 0 | Count by 0.005, start from 0 |
|------------------------------|------------------------------|------------------------------|
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What patterns can you see in the numbers?

Counting by 0.001

Counting by 0.002

Counting by 0.005

How many times did you add each decimal amount to make a whole.

How many 0.001's make a whole?

How many 0.002's make a whole?

How many 0.005's make a whole?

Practice Exercise 4

1. Use counting forwards or backwards to complete these number sequences. Use the constant function on the calculator to help if needed.

a) 1.004, 1.005, 1.006, _____, _____, _____

b) 10.456, 10.455, 10.454, _____, _____, _____

c) 5, 5.025, 5.05, 5.075, _____, _____, _____

d) 14.648, 14.643, 14.638, 14.633, _____, _____, _____

2. Use counting by decimals to solve the following problems.

- a) A carpenter was marking off sections on a piece of dowelling. Each section was 0.025 m. After marking off 12 sections, what length of dowelling had he used?
- b) A chef was weighing up meat to package and place in the freezer. He made 15 packages, and each package weighed 0.125 kg. He counted forward to work out how many kilograms of meat he had altogether. How many kilograms of meat did he place in the freezer?
- c) A goldsmith at the Perth Mint made gold ingots that weighed 0.005 kilograms. He had 2 kilograms of ingots and sold 25 ingots. What was the weight of the gold that he had left?

3. Practise counting by thousands by playing the following games on Scootle:

Swamp Survival: Thousandths Counting

<https://www.scootle.edu.au/ec/resolve/view/L7904>

Swamp Survival: Thousandths Patterns

<http://www.scootle.edu.au/ec/viewing/L7905/index.html>

Swamp Survival: Thousandths Challenge

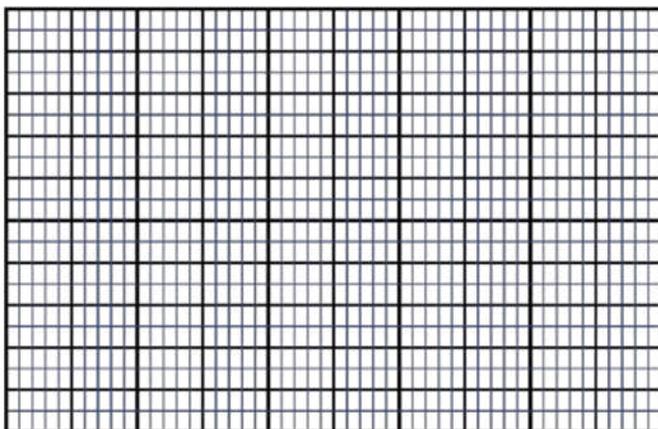
<https://www.scootle.edu.au/ec/resolve/view/L7906>

Wishball: Thousandths

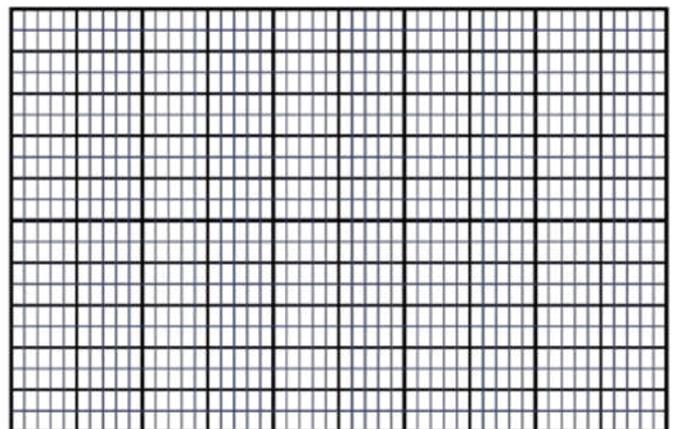
<http://www.scootle.edu.au/ec/viewing/L495/index.html>

Whole Class Activity 6

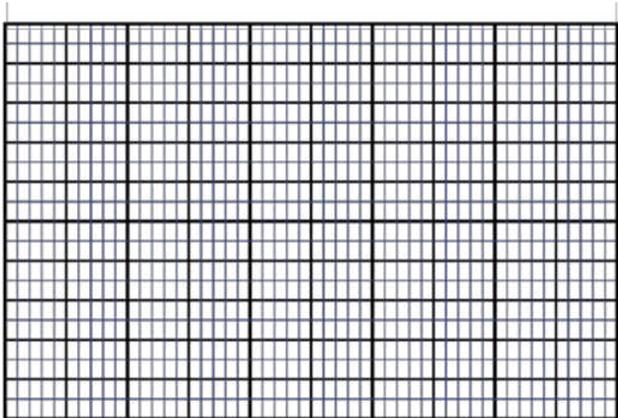
Think: How do we partition decimal numbers?



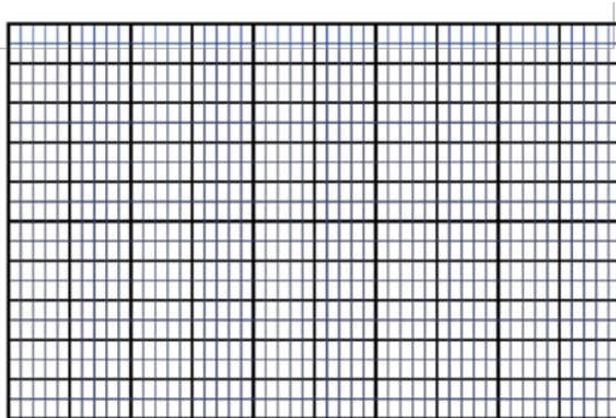
Colour 40 hundredths of the whole grid above.
Write as a decimal.
Write using fractions.



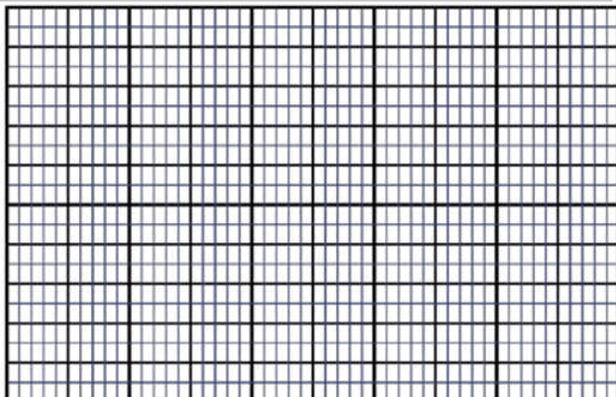
Colour 400 thousandths of the whole grid above.
Write as a decimal.
Write using fractions.



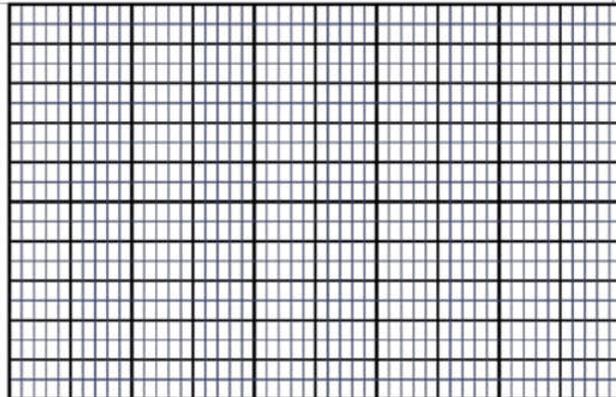
Colour 70 hundredths and 5 thousandths of the whole grid above.
Write as a decimal.
Write using fractions.



Colour 705 thousandths of the whole grid.
Write as a decimal.
Write using fractions.



Colour 38 hundredths and 7 thousandths of the whole grid above.
Write as a decimal.
Write using fractions.



Colour 387 thousandths of the whole grid.
Write as a decimal.
Write using fractions.

Reflection and Discussion

Look at each pair of grids you coloured in the activity above.

Explain what is different about the two ways of describing the decimal.



Explain what is same about the two ways of describing the decimal.



Practice Exercise 5

Decimat Game ¹

Number of Players: 2

Aim: to fill your game board by colouring equivalent fractions.

Equipment:

- 1 die with numbers 1 to 6
- 1 die with sides labelled $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{1000}$ and $\frac{1}{1000}$
- Decimat colouring grid and recording table for each player
- Coloured pencils

Rules of the Game:

1. Player A rolls the two dice and then shades the product of the two numbers displayed using a coloured pencil or texta.
2. Player A records what has been rolled as a fraction and as a decimal on the table below the game board, using the same colour. In the last column, Player A records “how much is shaded altogether?” (as a decimal) at the completion of their turn.
3. Player B then rolls the two dice and continues the game using her/his own Decimat gameboard.
4. Players take turns, each player recording how much of their Decimat is shaded after their turn. A new colour is chosen at each roll of the dice. The place value chart is provided as a reference to help you when you need to interpret the decimal numbers.
5. It is permitted for tenths/hundredths to be further divided into hundredths/thousandths, if more hundredths/thousandths are required.
6. The first player to shade the whole Decimat (which equals 1) or the player closest to one.

Whole Class Activity 7

Jenna and Toby are trying to figure out which distance is the longest.

Route A: 23.638km

Route B: 23.83km

"I think 23.638 km is further than 23.83 km because 638 is larger than 83."



Jenna

"I disagree and think 23.83 km is further than 23.638 because 8 tenths is bigger than 6 tenths."



Toby

Who is right?



Draw a diagram or a number line to prove to Jenna and Toby which measurement is larger?



How much longer is the longest distance? How do you know?



Jenna, Alice and Toby are planning another part of their trip. They are trying to decide which route gives the shortest distance.

I think 285.95km is shorter because there are less digits in that number.

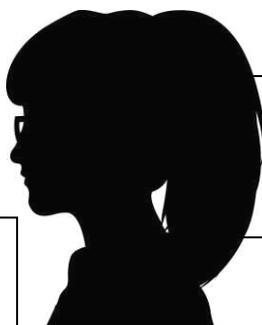


Jenna

Route A: 285.95km

Route B: 285.905km

I think 285.905km is longer because 905 is bigger than 95.



Alice

I think 285.905km is the shortest because 9 tenths and 5 hundredths is the same as 950 thousandths, which is more than 905 thousandths.



Toby

Who is right? How do you know?



What could you do/say to prove to Toby, Jenna and Alice which measurement is shorter?



Reflection and Discussion

Discuss with a partner the strategies you can use to compare the size of each pair of numbers.

1.687 and 1.86

0.98 and 0.892

3.614 and 3.64

What do you do when you compare different sized decimal numbers?



Practice Exercise 6

1. Draw a model or use a number line to work out which is bigger.

Put a $<$ or $>$ in the circle to show which is bigger.

a) 0.851 0.518

c) 5.983kg 5.389kg

b) 7.123m 7.231m

d) 0.273 0.23

2. Order the amounts from smallest to biggest.

a) 3.04, 4.3, 4.343, 4.043

b) 1.04m, 4.81m, 18.4m, 1.840m

c) 0.005L, 1.55L, 5.1L, 0.105L

d) 9.198kg, 1.98kg, 18.8kg, 8.908kg

3. Use counting by decimals to work out the difference between the amounts. Use a number line or jottings to help. Record how you worked out the answer.

a) 0.558 and 0.552

b) 1.255 and 1.552

c) 9.07km and 10.073km

d) 8.48L and 8.842L

Reflection on Learning

1. Search for decimal numbers in any texts available to you. Find three examples: one with one decimal place, one with two decimal places, and one with three decimal places.

Record decimal numbers in the table below. Record the purpose of the decimal and how you would say the number.

| Decimal Number | How to say it | Purpose |
|----------------|---------------|---------|
| | | |
| | | |
| | | |

Why does the number with three decimal places have three decimal places?



Why would you choose to use a decimal to thousandths?



OLNA Practice Questions

1. Which two numbers are larger than 1.452?

A: 1.387 and 1.456

B: 1.459 and 1.52

C: 1.005 and 1.458

D: 1.49 and 1.451

2. How many 0.001 are needed to make 0.1? _____

3. Which number is closest to 1?

A. 0.009

B. 1.019

C. 0.999

D. 1.1

Connecting Fractions and Decimals

Mathematics Discussion

Decimals and fractions are two different types of numbers that we can use to show the same amount of an object or collection.

Decimal numbers are an extension of our place value system, which uses powers of ten (tenths, hundredths, thousandths - see Thousandths Topic). Fractions can be converted into decimals so that we can compare them to other numbers using place value.

Decimals can also be converted into fractions in order to compare them with other fractional amounts. We can use the names of the decimal places to convert decimals to fractions. 1.5 can be said as *one point five*, or as *one and five tenths*. This shows that 1.5 can also be thought of as the fraction $1\frac{5}{10}$. We can also convert the *point five* into the common fraction $\frac{1}{2}$ as 0.5 is half way between 1 and 2. So, 1.5 can also be thought of as $1\frac{1}{2}$.

$$1.5 = 1\frac{5}{10} = 1\frac{1}{2}$$

Decimals are made by dividing a whole into ten, a hundred or a thousand pieces. Fractions are also made by dividing. For example, if we divide 1 bag of oats between 4 horses, each horse will get $\frac{1}{4}$ of a bag of oats. We can use division to convert fractions into decimals using a calculator.

$$\frac{1}{4} = 1 \div 4 = 0.25$$

Whole Class Activity 1

Think: How do you work out the fraction equivalent of a decimal number?

| Ones | | | Decimals | | |
|----------|------|------|--------------------------|-------------------------------|---------------------------------|
| Hundreds | Tens | Ones | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Write 0.4 in the place value table above. What does the digit 4 mean in this number?



0.4 written as a fraction of one whole is $\frac{4}{10}$. Therefore $0.4 = \frac{4}{10}$

Write 0.48 in the place value table above. What do the digits 4 and 8 mean in this number?



Write this amount as a fraction of one whole.



Write 0.485 in the table above. What do the digits 4, 8 and 5 mean in this number?



Write this amount as a fraction of one whole.



Write 2.8 on the place value table above. What do each of the digits mean in this number?



Write this amount as a mixed fraction.



Practice Exercise 1

1. Write these decimals as a fraction with a base ten denominator.

a) 0.8

e) 2.03

b) 1.3

f) 0.829

c) 0.25

g) 0.203

d) 3.59

h) 1.062

2. Write each of these decimal measurements as a fraction with a base ten denominator.

a) 0.4m

e) 4.09cm

b) 3.2kg

f) 0.125kg

c) 0.55km

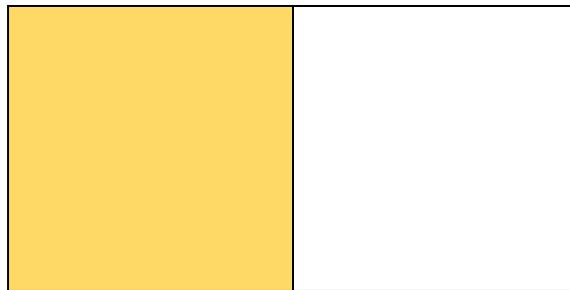
g) 2.115t

d) 10.25t

h) 5.075km

Whole Class Activity 2

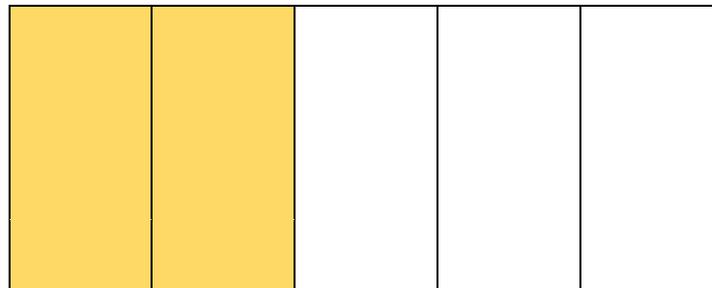
Think: How do you work out the decimal equivalent of a fraction number?



$\frac{1}{2}$ of this rectangle is shaded. Partition the whole rectangle into tenths. How many tenths are shaded?



Partition the whole rectangle above into hundredths. How many hundredths are shaded?



$\frac{2}{5}$ of this rectangle is shaded. Partition the whole rectangle into tenths. How many tenths are shaded?



Partition the whole rectangle above into hundredths. How many hundredths are shaded?



The vinculum in a fraction can be thought of as a division symbol and this helps to find a decimal equivalent for a fraction number.

Use your calculator to divide $1 \div 2 =$

Use your calculator to divide $2 \div 5 =$

Why does the calculator show the result with one decimal place, not two decimal places?



Consider the fraction $\frac{1}{5}$. Think of it as $1 \div 5$.

Use a calculator to find the decimal equivalent of $\frac{1}{5}$.

Write the decimal number onto the place value table below.

| Ones | | | Decimals | | |
|----------|------|------|--------------------------|-------------------------------|---------------------------------|
| Hundreds | Tens | Ones | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Consider the fraction $\frac{1}{4}$. Think of it as $1 \div 4$.

Use a calculator to find the decimal equivalent of $\frac{1}{4}$.

Write the decimal number onto the place value table above.

Consider the fraction $\frac{1}{8}$. Think of it as $1 \div 8$.

Use a calculator to find the decimal equivalent of $\frac{1}{8}$.

Write the decimal number onto the place value table above.

Consider the fraction $4\frac{3}{8}$. Think of it as $3 \div 8$ plus 4 wholes.

Use a calculator to find the decimal equivalent of $4\frac{3}{8}$.

Write $4\frac{3}{8}$ as a decimal in the table above.

Practice Exercise 2

1. Write these base ten fractions as decimals.

a) $\frac{1}{10}$

e) $\frac{7}{100}$

b) $\frac{5}{10}$

f) $\frac{205}{1000}$

c) $\frac{10}{100}$

g) $\frac{95}{1000}$

d) $\frac{25}{100}$

h) $\frac{6}{1000}$

2. Write these fractions as decimals.

a) $\frac{1}{4}$

b) $\frac{3}{15}$

c) $\frac{5}{8}$

d) $\frac{9}{12}$

e) $\frac{3}{5}$

f) $2\frac{4}{8}$

g) $1\frac{3}{12}$

h) $6\frac{9}{36}$

3. Write each of these measurements as decimals.

a) $\frac{1}{2}$ cm

b) $\frac{1}{5}$ kg

c) $\frac{5}{10}$ m

d) $\frac{3}{4}$ km

e) $4\frac{3}{100}$ t

f) $2\frac{3}{5}$ g

g) $10\frac{57}{1000}$ km

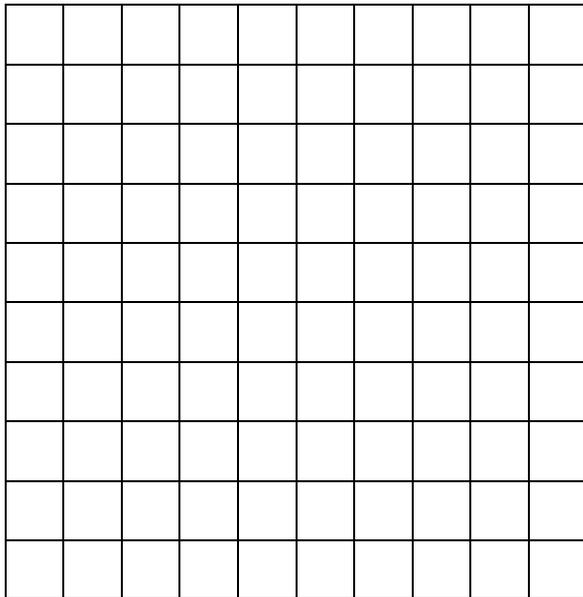
h) $5\frac{5}{1000}$ m

Whole Class Activity 3

Think: Which decimal and fraction benchmarks are useful to know?

Some fraction and decimal numbers are used commonly in everyday situations.

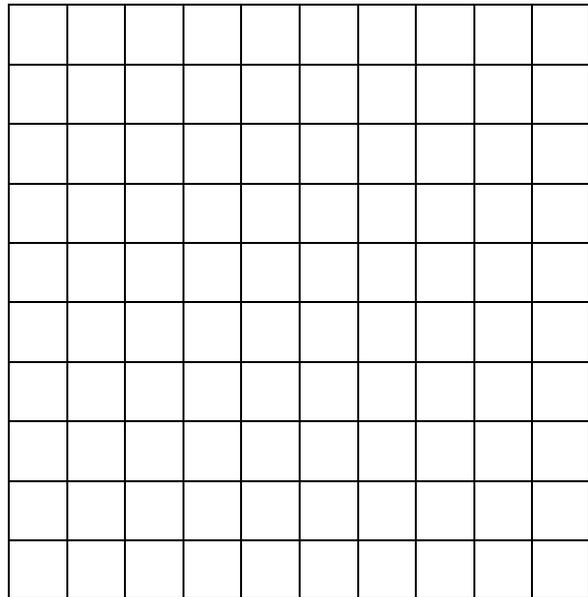
On the grid below show the fraction 0.1



What fraction of the grid is shaded?

Write other fractions that are equivalent to 0.1

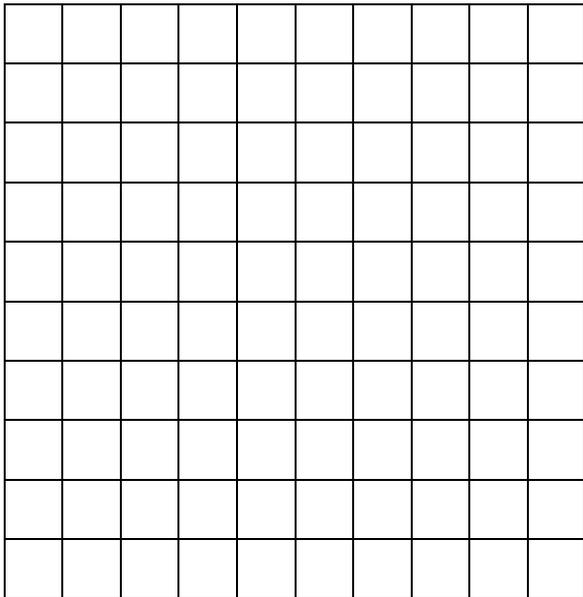
On the grid below show the fraction 0.5



What fraction of the grid is shaded?

Write other fractions that are equivalent to 0.5

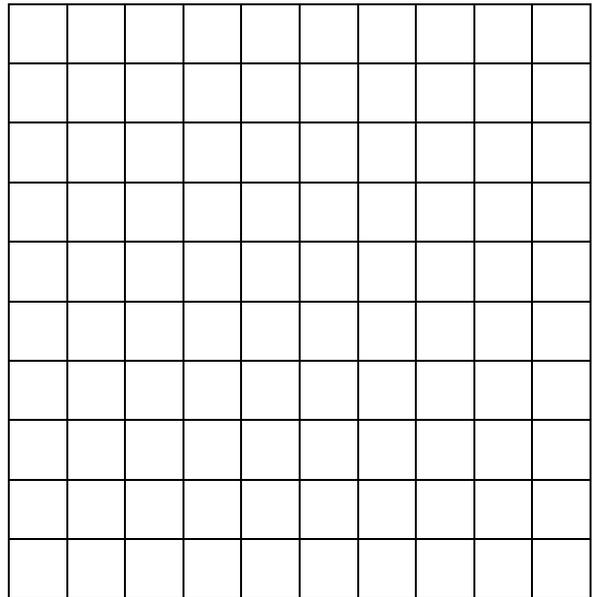
On the grid below show the fraction 0.25



What fraction of the grid is shaded?

Write other fractions that are equivalent to 0.25

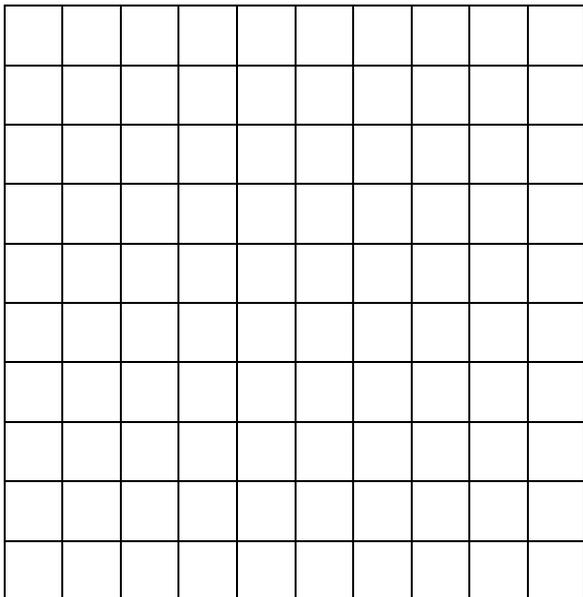
On the grid below show the fraction 0.01



What fraction of the grid is shaded?

Write other fractions that are equivalent to 0.01

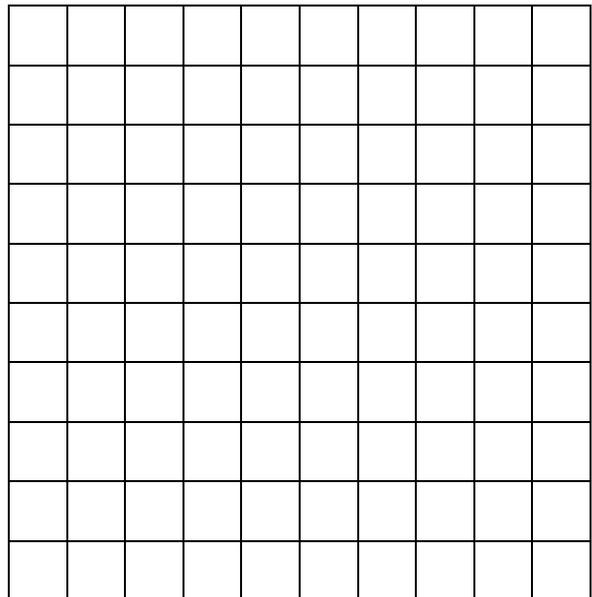
On the grid below show the fraction 0.75



What fraction of the grid is shaded?

Write other fractions that are equivalent to 0.75

On the grid below show the fraction 0.001



What fraction of the grid is shaded?

Write other fractions that are equivalent to 0.001

Practice Exercise 3

1. Use the given equivalent fraction and decimal to work out the others.

a) $\frac{1}{4} = 0.25$ so, $\frac{2}{4} = 0.5$ and $\frac{3}{4} =$

b) $\frac{1}{5} = 0.2$ so $\frac{2}{5} =$ $\frac{3}{5} =$ $\frac{4}{5} =$

c) $\frac{1}{8} = 0.125$ so $\frac{2}{8} =$ $\frac{3}{8} =$ $\frac{5}{8} =$

2. Complete the table.

| Decimal | Base Ten Fraction | Equivalent Fractions |
|---------|-------------------|----------------------|
| | $\frac{1}{10}$ | |
| 0.01 | | |
| | $\frac{20}{100}$ | |
| | $\frac{25}{100}$ | |
| 0.5 | | |
| 0.75 | | |
| | $\frac{1}{1000}$ | |
| 0.15 | | |

3. Fraction and Decimal Memory Game

Aim

To collect the most cards by matching fraction and decimal equivalents.

Equipment

2 players, 1 set of cards copied below.

Rules of the Game

1. Shuffle the cards and arrange in an array with the numbers facing down.
2. Player A turns over three cards, leaving them in their place. If all three cards are equivalent then player A takes them. If they are not equivalent turn the cards back over.
3. Player B takes their turn.
4. The game ends when all the cards have been matched and taken off the playing area.
5. The winner is the player with the most cards.

| | | |
|-----------------|-------|--------------------|
| $\frac{1}{4}$ | 0.25 | $\frac{25}{100}$ |
| $\frac{1}{5}$ | 0.2 | $\frac{2}{10}$ |
| $\frac{1}{2}$ | 0.5 | $\frac{5}{10}$ |
| $\frac{3}{4}$ | 0.75 | $\frac{75}{100}$ |
| $\frac{1}{8}$ | 0.125 | $\frac{125}{1000}$ |
| $\frac{1}{10}$ | 0.1 | $\frac{10}{100}$ |
| $\frac{1}{100}$ | 0.01 | $\frac{10}{1000}$ |
| $\frac{1}{20}$ | 0.05 | $\frac{5}{100}$ |

Practice Exercise 4

Download the following activity to learn more about fractions.

<http://www.scottle.edu.au/ec/viewing/L586/index.html>

Whole Class Activity 4

Think: How do you compare fractions and decimals?

Hang a piece of string across the classroom and peg up two cards; the first labelled 0 and the other 2, at either end of the string.

Write 1, $\frac{1}{5}$, 1.3 and 0.2 onto cards. Discuss where they belong on the line and peg them up. Equivalent numbers can be pegged under each other.

Each pair of students selects a number from the list below, writes it onto a piece of card and decides where on the string it should go and the reason for its position.

| | | | | |
|-----------------|-----------------|------------------|------|------|
| $\frac{9}{12}$ | $\frac{1}{100}$ | $\frac{10}{100}$ | 0.85 | 0.75 |
| $\frac{12}{10}$ | $\frac{3}{8}$ | 1.5 | 0.9 | 1.95 |
| $\frac{3}{5}$ | $1\frac{1}{4}$ | 0.125 | 0.8 | 0.1 |

Where does your number belong on the line? Justify.



Reflection and Discussion

What strategies were your classmates using to decide where the numbers belonged?



Which numbers were the easiest to visualise where they belong on the string? Why?



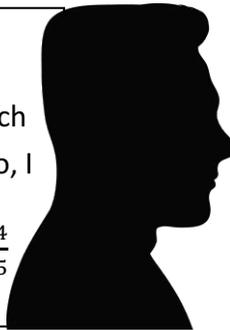
Which numbers were the most difficult to visualise where they belong on the string? Why?



Whole Class Activity 5

Tully and River are trying to work out which distance is longer; $\frac{4}{5}$ of a metre or 0.45m

I am pretty sure $\frac{4}{5}$ is equivalent to $\frac{8}{10}$, which is 0.8 as a decimal. So, I know 0.8m is bigger than 0.45m, making $\frac{4}{5}$ the longest distance.



River

I think they are equal because they have the same numbers in them.



Tully

Who is wrong? How do you know?



Reflection and Discussion

To decide which number is larger $\frac{3}{8}$ or 0.4 should you convert $\frac{3}{8}$ to a decimal, or convert 0.4 to a fraction? Why?



Why is it useful to convert one number so both are fractions or both are decimals?



How can you use your knowledge of equivalent fractions to help you convert a fraction to a decimal?



Write an equivalent base ten fraction for each of these numbers?

$$\frac{1}{5}$$

$$\frac{5}{2}$$

$$2\frac{3}{4}$$

Whole Class Activity 6

Allocate each student in class with letter A, B, C or D.

Students with the same letter must compare the numbers in the box below to work out which is larger. Jot down how you worked it out.

| | |
|--------------------------------|----------------------------|
| A: 0.75 and $\frac{5}{8}$ | B: $\frac{3}{4}$ and 0.575 |
| C: $1\frac{28}{100}$ and 1.028 | D: 0.382 and $\frac{2}{5}$ |



Talk with another student who had the same letter as you and compare your reasoning.

What was the same/different about what the other person did?



Talk with a student who had a different letter to you. Listen to their methods for comparing the numbers and share what you did with your numbers.

Practice Exercise 5

1. Compare each pair of numbers and place a $<$, $>$ or $=$ in the circle.

a) 0.85 $\frac{2}{8}$

c) $\frac{3}{4}$ kg 0.705 kg

b) 2.55 m $\frac{255}{1000}$ m

d) 0.25L $\frac{2}{5}$ L

2. Place each set of numbers in order from the smallest to largest.

a) 0.04, $\frac{4}{10}$, 4.1

b) 1.25m, 1.505m, $\frac{12}{10}$ m

c) $\frac{4}{20}$ L, $\frac{2}{5}$ L, 0.120L

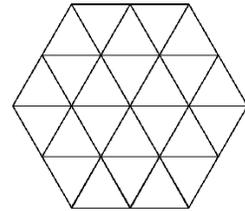
d) 2.12kg, $\frac{200}{1000}$ kg, $2\frac{1}{4}$ kg

Reflection on Learning

Copy the puzzle pieces below.

Cut along the black lines to form 24 triangles.

To complete the puzzle you must match the equivalent fractions and decimals. When complete the puzzle will make a hexagon that looks like this.



When you complete the puzzle, glue it onto a sheet of paper.

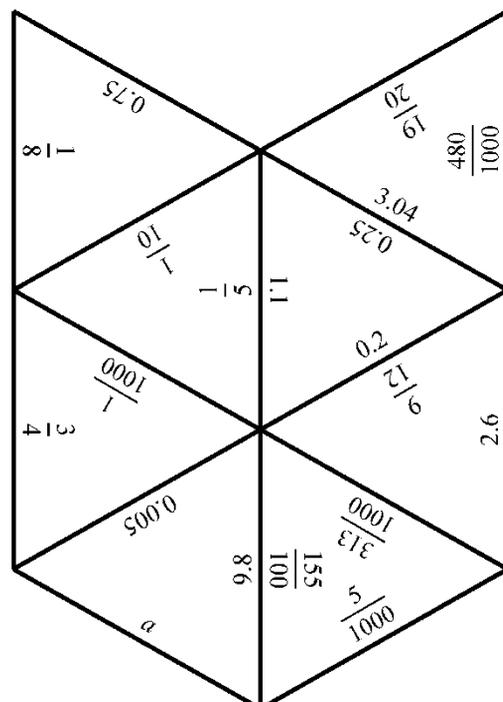
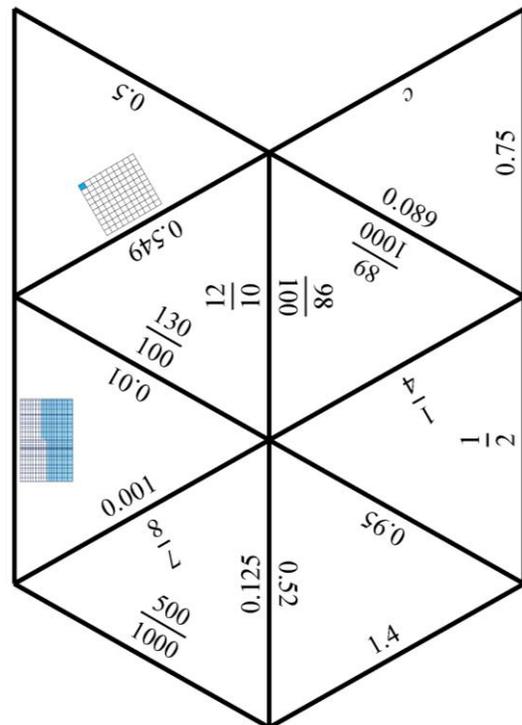
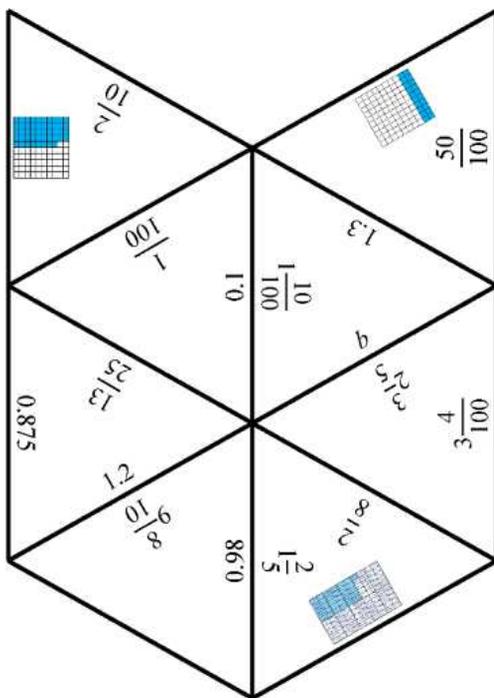
Three of the numbers are missing, they are labelled with the letters a, b and c. Work these out and record below.

a)

b)

c)

Which matching numbers were the most difficult to work out? Explain why they were difficult.



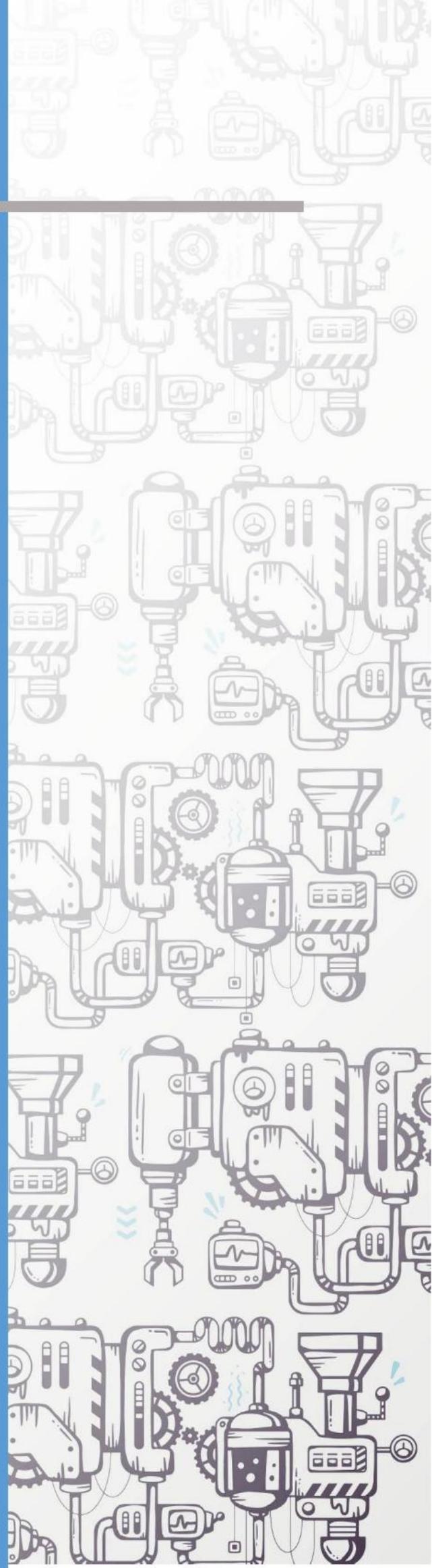
OLNA Practice Question

1. Which amount is bigger than 0.75?

- A: $\frac{1}{2}$ B: $\frac{7}{10}$ C: $\frac{7}{8}$ D: $\frac{2}{3}$

Section Two

Multiplication and Division with Whole Numbers and Money



Content Focus

Mathematics Foundation

- 2.2.1 Determine whether an estimation or an exact answer is needed in everyday situations
- 2.2.2 Choose when it is appropriate to use multiplication or division to solve a range of everyday problems
- 2.2.3 Understand and use the inverse relationship between multiplication and division to assist in mental calculation
- 2.2.4 Understand, recall, use and extend basic multiplication facts to facilitate mental calculation
- 2.2.5 Recall, use connections between and apply basic multiplication facts to facilitate mental calculation
- 2.2.6 Use mental calculation strategies, with informal jottings to keep track if required, to solve everyday problems involving multiplication with simple whole numbers
- 2.2.7 Use a calculator efficiently and appropriately when more complex numbers or tasks are involved
- 2.2.8 Interpret remainders resulting from division in relation to the context
- 2.2.9 Determine whether an answer is reasonable by using estimation and the context of the problem
- 2.2.10 Communicate solutions using language and symbols consistent with the context

Australian Curriculum Link

- ACMNA031 Recognise and represent multiplication as repeated addition, groups and arrays
- ACMNA032 Recognise and represent division as grouping into equal sets and solve simple problems using these representations
- ACMNA056 Recall multiplication facts of two, three, five and ten and related division facts
- ACMNA057 Represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies
- ACMNA074 Investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9
- ACMNA075 Recall multiplication facts up to 10×10 and related division facts
- ACMNA076 Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder
- ACMNA082 Solve word problems by using number sentences involving multiplication or division where there is no remainder
- ACMNA099 Use estimation and rounding to check the reasonableness of answers to calculations
- ACMNA100 Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies
- ACMNA101 Solve problems involving division by a one digit number, including those that result in a remainder
- ACMNA121 Use equivalent number sentences involving multiplication and division to find unknown quantities
- ACMNA123 Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers
- ACMNA291 Use efficient mental and written strategies and apply appropriate digital technologies to solve problems

Topic 1

Understanding and Recalling Basic Multiplication Facts

Mathematics Discussion

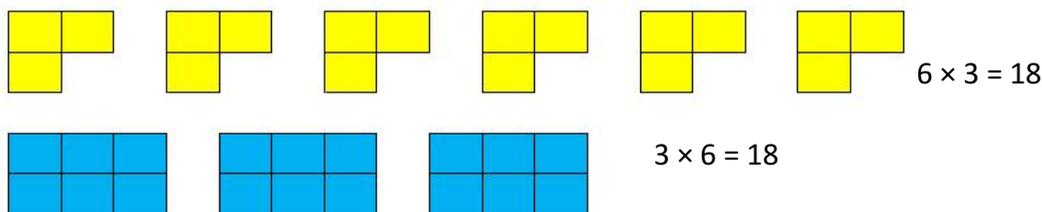
Maths is easier if you can remember and recall your *times tables*. The times tables are sometimes called the basic multiplication facts. It is helpful to know all of the basic multiplication facts up to 10×10 .

We use basic multiplication facts to mentally solve both multiplication and division problems.

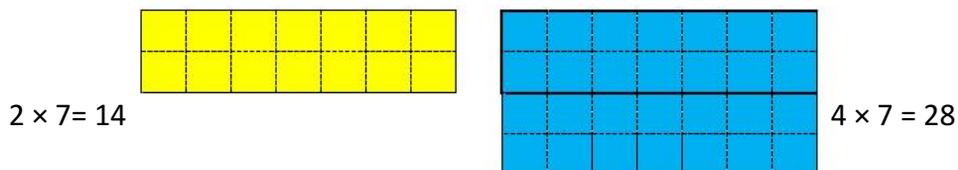
For example, we can use $6 \times 3 = 18$ to solve 600×3 , $18 \div 3$, $18 \div 6$ and $1800 \div 30$.

There are 5 main strategies for learning and remembering the basic multiplication facts:

1. Commutativity. Six groups of three and three groups of six give the same result, even though they may look different when we see the groups. If we know one multiplication fact, we can use it to find another one. $6 \times 3 = 3 \times 6$.



2. Counting patterns. The patterns in the way we say skip counting sequences can help us with some facts. For example, the count by five's sequence; 5, 10, 15, ... can be used to work out 4×5 .
3. Doubling. If we know a times table, then we can double it to find another. For example, we can double the two times table to work out the four times table. Four times seven is double two times seven, because four is double two.



4. Halving. Like doubling above, we can halve some facts to find others. For example, 5×6 is half of 10×6 .
5. Using facts we know. For example, 7×3 is the same as 6×3 plus one more group of 3.

Learning basic multiplication facts takes time, and after they are learnt, you need to practise them so that they are not forgotten.

Whole Class Activity 1

Commutativity

Think: What does $3 \times 4 = 12$ really mean?

What are all of the different ways you can read and say $3 \times 4 = 12$? Write them in the space below.



Do they all mean the same thing? Explain why.

A class at Lansdon Lake Grammar were asked to produce a poster explaining 'What is the same

3×4 **4×3**

• There are 3 groups with 4 in each group and 12 squares in total
 $4 + 4 + 4 = 12$

• There are 4 groups with 3 in each group and 12 squares in total
 $3 + 3 + 3 + 3 = 12$

This means the number of groups is different and the number of squares in each group is different but the **TOTAL** squares is the same.
That is why $4 \times 3 = 3 \times 4$

and what is different about 3×4 and 4×3 ? Tom, one of the students, produced the following poster.

Katie, another student at Lansdon Lake Grammar thought that $3 \times 4 = 12$ and $4 \times 3 = 12$ could also be shown on an array model.





groups of 4
on the array below.

Show Katie how to circle 4 groups of 3
on the array below.



Show Katie
how to
circle 3

Write the addition number sentence formed by circling the groups, beneath each array
An array model can simply be 'turned' to show that $3 \times 4 = 4 \times 3$.
It also shows that $4 + 4 + 4 = 3 + 3 + 3 + 3$.

Answer the following question after viewing “The Array,” an online activity at:
<http://www.scootle.edu.au/ec/viewing/L106/index.html>

‘How do we use arrays to explain multiplication?’

Record your response to the focus question in the space below:



Work with a partner to make a poster about a different multiplication fact and its commutative fact. (e.g. $7 \times 2 = 14$ and $2 \times 7 = 14$). Use the array model shown by Katie and the website. Show what is the same and what is different about the two number sentences. In small groups share and discuss your posters.

Whole Class Activity 2

Using doubling for $\times 2$

Think: How can we show that doubling is the same as ‘Multiplying by 2’?

From a sheet of 1cm grid paper, cut out 1 row of 7 squares. Cut out another row of 7 squares. Place the two rows underneath each other to form **an array** and paste in the space below.

Circle two sevens in red.

Circle seven twos in blue.

In the table below, write two multiplication number sentences and their corresponding addition number sentences shown by this array.

| Multiplication Number Sentence | Addition Number Sentence |
|--------------------------------|--------------------------|
| | |
| | |

Circle the pair of number sentences that help show that multiplying a number by 2 is the same as doubling the number

Repeat the activity by cutting out 2 separate columns of 5 squares and pasting in the space in the space below.

Circle two fives in red.

Circle five twos in blue.

In the table below write the two multiplication number sentences, next to their two corresponding addition number sentences.

| Multiplication Number Sentence | Addition Number Sentence |
|--------------------------------|--------------------------|
| | |
| | |

Circle the pair of number sentences that shows that multiplying a number by 2 is the same as doubling the number.

Fill in the gaps in the following table. The first has been completed for you

| Doubles (Addition) | Two Groups \times Number | Answer |
|--------------------|----------------------------|--------|
| 6 + 6 | 2×6 | 12 |
| 7 + 7 | | |
| 9 + 9 | | |
| | 2×3 | |
| | | 10 |
| | 2×2 | |
| | 2×8 | |
| | | 2 |
| | 0×0 | |
| 10 + 10 | | |

What do you notice about the answers to each of the doubling problems? What pattern can you see?



Reflection and Discussion – $\times 1$, $\times 0$

Draw arrays to show the answers to:

1×5

8×1

3×0

0×2

Discuss your answers to the $\times 1$ and $\times 0$ problems above.

Complete the following statements:

When we multiply a number by 1 or find 1 group of the number, the answer is:

When we multiply a number by 0 or find 0 groups of the number, the answer is:

Whole Class Activity 3

Using counting patterns and halving for $\times 10$ and $\times 5$

Using counting patterns

We can generate skip counting patterns using the constant function on a basic calculator. To generate a counting by 10's pattern we press the following buttons:



Write the counting by 10's sequence up to 100, in the space below.



How is this sequence related to the ten times table? Why is this so?



Use the constant function on the calculator to generate the counting by 5's sequence. Write the sequence up to 100, in the space below.



How is this sequence related to the five times table? Why is this so?



Fill in the answers to 'Multiplying by 10' and 'Multiplying by 5'. Start with the $\times 0$, $\times 1$ and $\times 2$ facts you know. Complete the rest using the skip counting patterns and the constant function on your calculator.

Multiplying by 10

$0 \times 10 =$

$1 \times 10 =$

$2 \times 10 =$

$3 \times 10 =$

$4 \times 10 =$

$5 \times 10 =$

$6 \times 10 =$

$7 \times 10 =$

$8 \times 10 =$

$9 \times 10 =$

$10 \times 10 =$

Multiplying by 5

$0 \times 5 =$

$1 \times 5 =$

$2 \times 5 =$

$3 \times 5 =$

$4 \times 5 =$

$5 \times 5 =$

$6 \times 5 =$

$7 \times 5 =$

$8 \times 5 =$

$9 \times 5 =$

$10 \times 5 =$

What rule can be used to determine answers to multiplying by 10?



What rule can be used to determine answers to multiplying by 5?



How is every multiplying by 5 fact related to the corresponding multiplying by 10 fact?



Using halving

Think: How can we show that the $\times 5$ tables are half the associated $\times 10$ tables?

From a sheet of 1cm grid paper, cut out a 3×10 array. Glue it below. Cut out another 3×10 array, then cut it in half to form two 3×5 arrays. Glue the two cut arrays in the space below and write a sentence to connect the 3×10 and 3×5 arrays.

Repeat the above activity by cutting out two 7×10 arrays. Cut one in half. Glue the arrays in the space below. Write a sentence to connect the 7×10 and 7×5 arrays.

In the space below describe the connection between multiplying by 10 and by 5. How could you use 9×10 to find the answer to 9×5 ?



How could you use 90×10 to find the answer to 90×5 ?



The table below is a way of representing all the multiplication facts.

Place the 'Multiplying by 10 and 5' facts into the table together with the 'Multiplying by 0, 1 and 2' facts.

These facts represent 85 of the 121 Times Tables

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|---|---|---|---|---|---|---|---|---|---|----|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

Practice Exercise 1

1. Write the following situations as multiplication number sentences. The first has been done for you.

a) 5, 10, 15, 20, 25, 30

$6 \times 5 = 30$

b) 2, 4, 6, 8, 10, 12, 14

c) ○○○○○ ○○○○○ ○○○○○

d) ○○ ○○ ○○ ○○ ○○ ○○ ○○ ○○ ○○

e) ○○○○
 ○○○○
 ○○○○
 ○○○○
 ○○○○

f) ○○○○○○○○○○○
 ○○○○○○○○○○○
 ○○○○○○○○○○○

2. Complete the following table

| | $\times 0$ | $\times 1$ | $\times 2$ | $\times 10$ | $\times 5$ |
|----|------------|------------|------------|-------------|------------|
| 3 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 2 | | | | | |
| 9 | | | | | |
| 10 | | | | | |
| 4 | | | | | |
| 1 | | | | | |
| 7 | | | | | |

3. *Winning arrays*

A Game to Practise Basic $\times 0$, $\times 1$, $\times 2$, $\times 5$, $\times 10$ Multiplication Facts

Aim

To win the largest area of grid paper using multiplication arrays.

Equipment

- A pack of cards with Jokers, Queens and Kings removed. The cards should be separated into the following two piles:
Pile 1 - All aces, twos, fives, tens and Jacks (use as 0)
Pile 2 - The remainder of the cards
- 2 coloured pens. Player A has one colour pen and Player B another
- One piece of 1 cm grid paper
- Calculator

Rules of the Game

1. Place the cards face down in the two piles between the two players.
2. Player 1 takes one card from each pile (eg a 2 and 8) and draws an array anywhere on the grid paper. The player write the multiplication ($2 \times 8 = 16$) in the middle of the rectangle. If the player gets a Jack they cannot draw an array (eg 0×7), so they miss a turn.
3. Player 2 then does the same. The new array must NOT overlap an existing array drawn by Player 1.
4. The cards can be shuffled within their piles and reused if necessary.
5. The game continues until it is impossible to fit a new array on the paper. At this point, each player uses a calculator to find his or her total number of squares.
6. The winner is the player with the highest score.

Variations

Play on 1mm graph paper with the cards ten times their value eg a 5 becomes 50.
Once all the basic facts are learned, play with two complete sets of cards in each pile.

Reflection and Discussion - Extending the Basic Multiplication Facts; $\times 0$, $\times 1$, $\times 2$, $\times 5$ and $\times 10$

A maths class at Onslow District High School was asked:

'How can you extend a fact that you know such as $3 \times 5 = 15$ to help with other mental calculations.'

The class responded as follows;

$$\begin{array}{c} \mathbf{3 \times 5 = 15} \\ 3 \times 50 = 150 \quad 30 \times 5 = 150 \\ 3 \times 500 = 1\,500 \quad 3 \times 5\,000 = 15\,000 \\ 3 \times 5 \text{ million} = 15 \text{ million} \\ 3 \times 6 = 18 \quad 4 \times 5 = 20 \quad 6 \times 5 = 30 \\ 15 \div 5 = 3 \quad 15 \div 3 = 5 \\ 3 \times 50 \text{ cents} = \$1.50 \\ 30 \text{ cents} \times 5 = 150 \text{ cents which is } \$1.50 \end{array}$$

How is each fact connected to $3 \times 5 = 15$? Discuss.

Complete a similar activity with the following facts on a blank sheet of paper. Share your responses.

$8 \times 10 = 80$

$6 \times 1 = 6$

$9 \times 2 = 18$

$9 \times 5 = 45$

4. Use your knowledge of multiplying by 0, 1, 2, 10 and 5 to solve harder problems:

- | | | | | |
|---------------------|------------------------------|-------------------|-------------------------------|-----------------|
| a) 4×0 | b) 6×1 | c) 7×2 | d) 3×10 | e) 6×5 |
| 40×0 | 60×1 | 70×2 | 300×10 | 60×5 |
| $0 \times 4\,000$ | 1×600 | 2×700 | $10 \times 3\,000$ | 5×12 |
| $400\,000 \times 0$ | $1 \times 6 \text{ million}$ | $2 \times 7\,000$ | $10 \times 3 \text{ million}$ | 5×24 |

Whole Class Activity 4

Using doubling for $\times 4$ and $\times 8$

Think: How can we use a fact that we know such as $7 \times 2 = 14$ to solve problems such as 7×4 and 7×8 ?

Using $7 \times 2 = 14$ to calculate 7×4

From a sheet of 1cm grid paper, cut out two, 7×2 arrays. Connect the two arrays to form a 7×4 array and paste in the space below.

Write a statement beneath the arrays describing the connection between 7×2 and 7×4 .

Using $7 \times 4 = 28$ to calculate 7×8

Use 1cm graph paper to demonstrate the connection between 7×4 and 7×8 . Glue your arrays in the space below.

Complete the following statements:

We can use the $\times 2$ facts to develop the $\times 4$ facts by _____

We can use the $\times 4$ facts to develop the $\times 8$ facts by _____

Therefore,

We can use the $\times 2$ facts to develop the $\times 8$ facts by _____

Use your conclusions to complete the following table.

| $\times 2$ Facts (Doubled) | $\times 4$ Facts (Doubled $\times 2$) (Doubled and Doubled Again) | $\times 8$ Facts (Doubled $\times 4$) (Doubled, Doubled and Doubled Again) |
|--|--|---|
| $0 \times 2 = 0$ | $0 \times 4 = 0$ | $0 \times 8 = 0$ |
| $1 \times 2 = 2$ | $1 \times 4 = 4$ | $1 \times 8 = 8$ |
| $2 \times 2 = 4$ | $2 \times 4 = 8$ | $2 \times 8 = 16$ |
| $3 \times 2 = 6$ | | |
| $4 \times 2 = 8$ | | |
| $5 \times 2 = 10$ | | |
| $6 \times 2 = 12$ | | |
| $7 \times 2 = 14$ | | |
| $8 \times 2 = 16$ | | |
| $9 \times 2 = 18$ | | |
| $10 \times 2 = 20$ | | |

Place the 'Multiplying by 4 and 8' facts into the table below. Add the multiplying by 0, 1, 2, 5 and 10 facts.

These facts represent 103 of the 121 Times Tables.

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|---|---|---|---|---|---|---|---|---|---|----|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

Practice Exercise 2

1. Write the following as multiplication number sentences. The first has been done for you.

a) 4, 8, 12, 16, 20, 24, 28, 32, 36

$$9 \times 4 = 36$$

b) 8, 16, 24, 32, 40, 48

c) ○○○○○○○○○ ○○○○○○○○○ ○○○○○○○○○

d) ○○ ○○ ○○ ○○ ○○
 ○○ ○○ ○○ ○○ ○○

e) ○○○○○○○○○
 ○○○○○○○○○
 ○○○○○○○○○
 ○○○○○○○○○
 ○○○○○○○○○

f) ○○○○
 ○○○○
 ○○○○
 ○○○○
 ○○○○

2. Complete the following table

| × | 5 | 9 | 3 | 0 | 7 | 8 | 10 | 4 | 1 | 6 |
|---|---|---|---|---|---|---|----|---|---|---|
| 2 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 8 | | | | | | | | | | |

3. **Winning Piles**

A Game to Practice Basic $\times 4$, $\times 8$ Multiplication Facts

Aim

To have the most cards in your pile

Equipment

- A pack of playing cards with picture cards removed
- A coin for each player (Heads = $\times 4$, Tails = $\times 8$).

Rules of the Game

1. Deal the cards equally between 3 or 4 players. Place cards face down in a pile.
2. Players take a card from their pile and toss their coin. Calculate the result (eg a 3 and a tail means $3 \times 8 = 24$)
3. Players compare their results and the player with the highest total gets the cards from the other players and places them at the bottom of their pile.
4. The game continues for 10 minutes. The winner is the player with the most cards.

Variation

The game can be played with Heads = $\times 40$ and Tails = $\times 80$.

Reflection and Discussion - Extending the Basic Multiplication Facts; $\times 2$, $\times 4$ and $\times 8$

A maths class at Derby District High School was asked:

'If you know $9 \times 4 = 36$, how can you extend this to help with other mental calculations?'

The class responded as follows:

| $9 \times 4 = 36$ | |
|---|---------------------|
| $9 \times 40 = 360$ | $90 \times 4 = 360$ |
| 9 thousand $\times 4 = 36$ thousand | |
| 9×40 cents = 360 cents or \$3.60 | |
| $9 \times 2 = 18$ | $9 \times 8 = 72$ |
| $10 \times 4 = 40$ | $8 \times 4 = 32$ |
| $9 \times 5 = 45$ | |
| $36 \div 4 = 9$ | $36 \div 9 = 4$ |

How is each fact related to $9 \times 4 = 36$? Discuss.

Complete a similar activity with the following facts on a blank sheet of paper. Share your responses.

$$6 \times 4 = 24$$

$$5 \times 8 = 40$$

$$8 \times 4 = 32$$

$$7 \times 8 = 56$$

4. Use your knowledge of multiplying by 2, 4 and 8 to solve harder problems:

a) 3×2

b) 3×4

c) 3×8

$$30 \times 2$$

$$30 \times 4$$

$$30 \times 8$$

$$2 \times 3\,000$$

$$4 \times 3\,000$$

$$8 \times 3\,000$$

$$2 \times 3\,003$$

$$4 \times 3\,003$$

$$8 \times 3\,003$$

Whole Class Activity 5

Using known facts for $\times 3$ and doubling $\times 3$ for $\times 6$

Think: How can we use a fact that we know such as $6 \times 2 = 12$ to work out one that we do not know, such as 6×3 ?

Using known facts for $\times 3$

From a sheet of 1cm grid paper, cut out a 6×2 array and paste in the space below.

Cut out a 6×1 array and paste it below the 6×2 array.

Write a number sentence beneath the arrays describing how 6 groups of 3 are equal to 6 groups of 2 plus 6 groups of 1.

How can we use a fact that we know such as:

$$9 \times 2 = 18 \text{ to find } 9 \times 3?$$

$$40 \times 2 = 80 \text{ to find } 40 \times 3?$$

$$700 \times 2 = 1\,400 \text{ to find } 700 \times 3?$$

Using doubling $\times 3$ for $\times 6$

From a sheet of 1cm grid paper, cut out two, 6×3 arrays. Place the two arrays above and below each other to form a 6×6 array and paste in the space below.

Write a number sentence beneath the arrays describing how 6 groups of 6 are equal to double 6 groups of 3.

How can we use a fact that we know such as: $7 \times 3 = 21$ to find 7×6 ?

$$30 \times 3 = 90 \text{ to find } 30 \times 6?$$

$$800 \times 3 = 2\,400 \text{ to find } 800 \times 6?$$

Complete the following statements:

We can work out the $\times 3$ facts by _____

We can work out the $\times 6$ facts by _____

Use your conclusions to complete the following table.

| $\times 2$ Facts | $\times 1$ Facts | $\times 3$ Facts | $\times 6$ Facts |
|--------------------|--------------------|------------------|-------------------|
| $0 \times 2 = 0$ | $0 \times 1 = 0$ | $0 \times 3 = 0$ | $0 \times 6 = 0$ |
| $1 \times 2 = 2$ | $1 \times 1 = 1$ | $1 \times 3 = 3$ | $1 \times 6 = 6$ |
| $2 \times 2 = 4$ | $2 \times 1 = 2$ | $2 \times 3 = 6$ | $2 \times 6 = 12$ |
| $3 \times 2 = 6$ | $3 \times 1 = 3$ | | |
| $4 \times 2 = 8$ | $4 \times 1 = 4$ | | |
| $5 \times 2 = 10$ | $5 \times 1 = 5$ | | |
| $6 \times 2 = 12$ | $6 \times 1 = 6$ | | |
| $7 \times 2 = 14$ | $7 \times 1 = 7$ | | |
| $8 \times 2 = 16$ | $8 \times 1 = 8$ | | |
| $9 \times 2 = 18$ | $9 \times 1 = 9$ | | |
| $10 \times 2 = 20$ | $10 \times 1 = 10$ | | |

Place the 'Multiplying by 0, 1, 2, 10, 5, 4, 8, 3 and 6 facts, into the table.

These facts represent 117 of the 121 Times Tables.

| \times | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|---|---|---|---|---|---|---|---|---|---|----|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

List the remaining facts to be learnt?



Describe strategies to help learn these remaining facts?



Practice Exercise 3

1. Write the following situations as multiplication number sentences. The first has been done for you.

a) 6, 12, 18, 24, 30, 36, 42

$$7 \times 6 = 42$$

b) 3, 6, 9, 12, 15, 18, 21, 24, 27

c) ○○○ ○○○ ○○○ ○○○ ○○○ ○○○ ○○○

d) ○○○○○○ ○○○○○○ ○○○○○○

e) ○○○○○○
 ○○○○○○
 ○○○○○○
 ○○○○○○
 ○○○○○○
 ○○○○○○
 ○○○○○○
 ○○○○○○

f) ○○○
 ○○○
 ○○○

2. Complete the following table

| × | 2 | 6 | 9 | 3 | 0 | 5 | 10 |
|---|---|---|---|---|---|---|----|
| 2 | | | | | | | |
| 3 | | | | | | | |
| 6 | | | | | | | |

3. Dominoes

A Game to Practice Basic $\times 3$, $\times 6$ Multiplication Facts

Aim: To have the most dominoes

Equipment: A set of dominoes

Rules of the Game

1. Place all dominoes face down between 3 – 4 players.
2. Each player chooses a domino. They multiply the smaller number of dots by 6 and the larger number of dots by 3 and add the result. If the number of dots is the same on each side of the domino, multiply both numbers by 4 and add the result.
3. The player with the highest result collects the dominoes from the other players.
4. Players choose another domino and repeat the calculation.

5. Play continues until there are no more dominoes face down.
6. The winner is the player with the most dominoes.

Variation

The side of the domino with the least number of dots could be multiplied by 60 and the side with the larger number of dots could be multiplied by 30.

Reflection and Discussion - Extending the Basic Multiplication Facts; $\times 3$ and $\times 6$.

A maths class at Harvey Agricultural School was asked:

‘If you know $7 \times 3 = 21$, then what else do you know.’

The class responded as follows:

| | | |
|--|---------------------|-------------------|
| $7 \times 3 = 21$ | | |
| $7 \times 30 = 210$ | $70 \times 3 = 210$ | |
| $700 \times 3 = 21\ 000$ | | |
| $70\ \text{thousand} \times 3 = 210\ \text{thousand}$ | | |
| $7 \times 4 = 28$ | $7 \times 6 = 42$ | $8 \times 3 = 24$ |
| $30\ \text{cents} \times 7 = 210\ \text{cents or } \2.10 | | |
| $21 \div 7 = 3$ | $21 \div 3 = 7$ | |

How is each fact connected to $7 \times 3 = 21$?

Complete a similar activity with the following facts on a blank sheet of paper. Share your responses.

| | | | |
|-------------------|-------------------|-------------------|-------------------|
| $5 \times 3 = 15$ | $7 \times 6 = 42$ | $9 \times 3 = 27$ | $8 \times 6 = 48$ |
|-------------------|-------------------|-------------------|-------------------|

4. Use your knowledge of multiplying by 2, 3 and 6 to solve harder problems:

- | | | | |
|--------------------|--------------------|-------------------|--------------------|
| a) 7×2 | b) 6×3 | c) 5×6 | d) 9×6 |
| 70×2 | 60×3 | 50×6 | 90×6 |
| $2 \times 7\ 000$ | $3 \times 6\ 000$ | 6×500 | $6 \times 9\ 000$ |
| $2 \times 70\ 000$ | $3 \times 60\ 000$ | $6 \times 5\ 000$ | $6 \times 90\ 000$ |

Whole Class Activity 6

Using known facts for $\times 9$

Think: How can we use a fact that we know such as $7 \times 10 = 70$ to solve a problem such as 7×9 ?

Using a sheet of 1cm grid paper, cut out a 7×10 array. From the array, cut off a 1×7 column. Glue the 2 pieces (a 7×9 and a 7×1 array) next to each other in the space below.

Complete the statements below using the whole array 7×10 and its parts; 7×9 and 7×1

$$7 \times 10 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$7 \times 9 = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$$

How can we use a fact such as $5 \times 10 = 50$ to find 5×9 ? $8 \times 10 = 80$ to find 8×9 ?



Use the ten times table to develop the nine times table by multiplying the number by 10 and taking away one of the numbers (e.g. $6 \times 9 = 6 \times 10 - 6$). Complete the table below:

| $\times 10$ Facts | $\times 1$ Facts | $\times 9$ Facts ($\times 10$ Facts Subtract $\times 1$ Facts) | |
|--------------------|------------------|--|-----------------|
| $0 \times 10 = 0$ | $0 \times 1 = 0$ | $0 \times 9 = 0$ | $(0 - 0 = 0)$ |
| $1 \times 10 = 10$ | $1 \times 1 = 1$ | $1 \times 9 = 9$ | $(10 - 1 = 9)$ |
| $2 \times 10 = 20$ | $2 \times 1 = 2$ | $2 \times 9 = 18$ | $(20 - 2 = 18)$ |
| $3 \times 10 =$ | $3 \times 1 =$ | | |
| $4 \times 10 =$ | $4 \times 1 =$ | | |
| $5 \times 10 =$ | $5 \times 1 =$ | | |
| $6 \times 10 =$ | $6 \times 1 =$ | | |
| $7 \times 10 =$ | $7 \times 1 =$ | | |
| $8 \times 10 =$ | $8 \times 1 =$ | | |
| $9 \times 10 =$ | $9 \times 1 =$ | | |
| $10 \times 10 =$ | $10 \times 1 =$ | | |

What patterns can you see in the answers to the nine times table? Discuss and record your findings below.



Whole Class Activity 7

Using commutativity to develop $\times 7$

We can learn the $\times 7$ tables using commutativity and the strategies we have developed for other facts. Complete the following table.

| $\times 7$ Tables | Strategy | Answer |
|-------------------|---|--------|
| 0×7 | Any number multiplied by 0 is 0 | 0 |
| 1×7 | Any number multiplied by 1 is the number itself | 7 |
| 2×7 | Same as 7×2 | 14 |
| 3×7 | | |

| | | |
|---------------|--|--|
| 4×7 | | |
| 5×7 | | |
| 6×7 | | |
| 7×7 | | |
| 8×7 | | |
| 9×7 | | |
| 10×7 | | |

From this we can see that there is one fact (7×7) that we have not seen in this section. Whilst we can demonstrate a strategy such as $5 \times 7 + 2 \times 7 = 49$, it is just easier to remember this fact, e.g. ***Seven, sevens are 49 – oh so fine!!***

Practice Exercise 4

1. Use your knowledge of multiplying by 7 and 9 to solve harder problems:

a) 4×7

40×7

$7 \times 4\,000$

$7 \times 400\,000$

b) 8×9

80×9

9×800

$9 \times 80\,000$

c) 7×7

7×70

700×7

$700\,000 \times 7$

An Approach to Learning and Retaining Basic Multiplication Facts

Strategies have now been developed for all basic multiplication facts (Times Tables). Complete the two-way table below to record all of the facts.

From this table, we can see that there are 121 different facts. We can halve the number we have to remember, by using the commutative property of multiplication (that is, if we can remember 4×3 , we can also remember 3×4). Use a highlighter on the table to show which facts can be thought of as a 'turn around' of another fact.

What other facts are easy to remember? Discuss.

On the two-way table, place a line through all the facts you can recall instantly (within 3 seconds).

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|---|---|---|---|---|---|---|---|---|---|----|
| 0 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

Circle the facts on the table that you cannot recall instantly.

Write them in the space below:



Circle each of the above facts into groups of 3.

Use a strategy shown in this topic to learn each of the facts in groups of 3.

Ask your partner to 'test' you on these 3 facts on a regular basis until you are ready to work on a new group of 3 facts.

The testing should always include some of the facts from the previous group of unlearned facts.

When you are confident that you can recall a learned fact, cross it off on the two-way table.

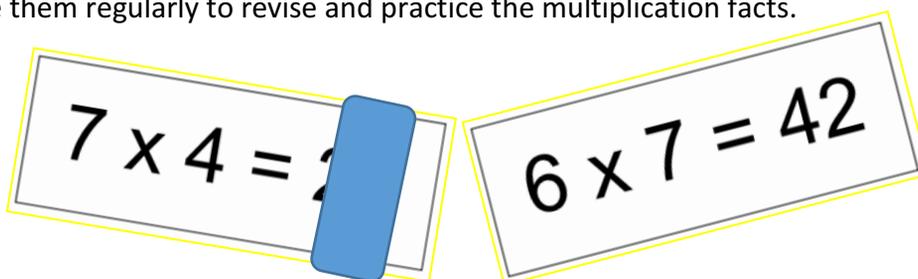
To help practise your Basic Multiplication Facts:

Make a set of 'flash cards' and use them to practise the multiplication facts by yourself.

A 'sleeve' can be placed over a card to hide each of the numbers in turn.

Use your flash cards to play games such as 'Memory' and 'Fish' with a partner.

Use them regularly to revise and practice the multiplication facts.



Alternatively, use online flash cards such as:

<http://www.multiplication.com/games/play/quick-flash-ii>

<http://www.memory-improvement-tips.com/support-files/multiplication0-12.pdf>

Practice Exercise 5

1. Puzzle

Did you hear about the cannibal who was late for dinner?'

Complete these questions to decode the answer to this joke.

T: 5×8 R: 4×8 D: 3×7 E: 9×3 H: 2×7
D: 10×3 O: 5×7 H: 8×9 T: 6×6 O: 7×9
C: 9×6 H: 8×3 G: 7×7 L: 9×2 L: 6×10
E: 7×8 U: 6×7 E: 5×4 O: 4×4 S: 4×7

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 24 | 56 | 49 | 35 | 36 | 40 | 14 | 20 | 54 | 63 |
| 60 | 30 | 28 | 72 | 16 | 42 | 18 | 21 | 27 | 32 |

2. Complete the following table.

| \times | 3 | 5 | 2 | 10 | 80 | 600 | 4000 | 20 | 90 | 10000 | 300 |
|----------|---|---|---|----|----|-----|------|----|----|-------|-----|
| 4 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |

3. Phone/Tablet Apps or Websites

a) Download one of the following free Apps onto your phone or tablet to practise your basic multiplication facts.

Basic Math with Smarty; Otto App Studio

Simple Sums V1.0; Sygem Software

Basic Math V3.30; ExplorerTechnologies

Multiply Time; Rudie Ekkelenkamp

Multiply Wiz; The Rocket Studio

b) Play these computer games from the following websites

<http://www.mathsisfun.com/timestable.html>

<http://www.multiplication.com/games/all-games>

Reflection on Learning

Complete the following two-way table as quickly as possible, using commutativity, counting patterns, doubling, halving and using known facts.

| × | 7 | 2 | 9 | 1 | 3 | 5 | 4 | 6 | 8 | 10 | 0 |
|----|---|---|---|---|---|---|---|---|---|----|---|
| 4 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 1 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |
| 0 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |

Reflect on your learning in this topic.

Have your basic multiplication skills improved? Discuss.



REMEMBER: Learning basic facts takes time. After they are learnt, you need to practice them so that they are not forgotten. The brain is like any other muscle in the body, it gets better at something with practice.

Learn then practise.

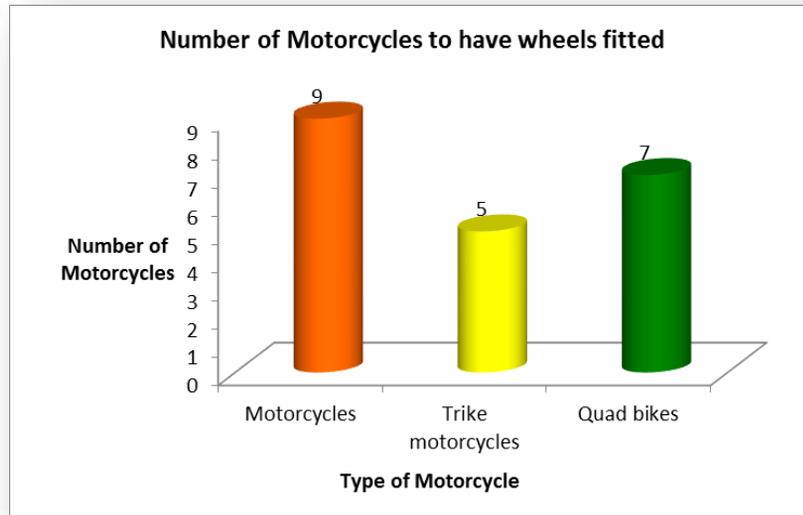
Then after that, practise, practise, practise....

OLNA Practice Questions

1. Which one of the following gives the same answer as 9×4 ?

- A. $10 \times 4 - 10$ B. $10 \times 4 - 4$ C. $18 \times 1 \times 1$ D. $8 \times 4 + 8$

2. Jak assembles motorcycles for a living. He fits wheels on Motorcycles (2 wheels), Trike Motorcycles (3 wheels) and Quad bikes (4 wheels). The graph below shows how many wheels he fitted in one week.



How many wheels does Jak fit over the week in total?

- A. 18 B. 42 C. 63 D. 61

3. A number is multiplied by itself and then 9 is added. The answer is 45. What is the number?

- A. 7 B. 18 C. 8 D. 6

Topic 2

Multiplication of Whole Numbers and Money

Mathematics Discussion

We can solve multiplication problems mentally by:

1. Extending the strategies we use for basic multiplication facts. For example,

To solve 25×2 we can use doubles.

To solve 25×4 we can double 25×2

To solve 25×8 we can double 25×4

To solve 25×10 we can use place value patterns

To solve 25×5 we can find half of 25×10

To solve 25×9 we can take one lot of 25 from 25×10

2. Using place value partitioning to break up the numbers and multiplying the parts. For example,

To solve 25×3 we can split 25 into $20 + 5$ and then multiply each of the parts by 3
i.e. (20×3) add (5×3)

This method can also be used for all multiplication problems.

3. Doubling one number and halving the other. This strategy works more easily with some numbers than with others. For example,

To solve 25×6 , we can double 25 and halve 6 i.e. $25 \times 6 = 50 \times 3$

Most people tend to use written jottings to help keep track of calculations that cannot be completely stored in their heads. For problems involving 'difficult' numbers a calculator should be used.

Whole Class Activity 1

Tom's Strategies for Solving Harder Multiplication Problems

Think: How can doubling be used to solve harder problems?

Tom knows he can extend the strategies developed in Multiplication of Basic Facts to solve harder problems.

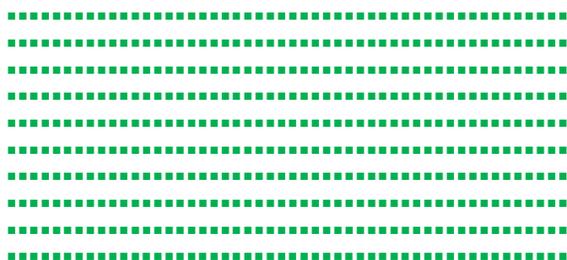


*With harder problems I can use the strategies I know:
Doubling for $\times 2$, $\times 4$, $\times 8$
Patterning for $\times 10$.
I could use $\times 10$ and halving for $\times 5$*

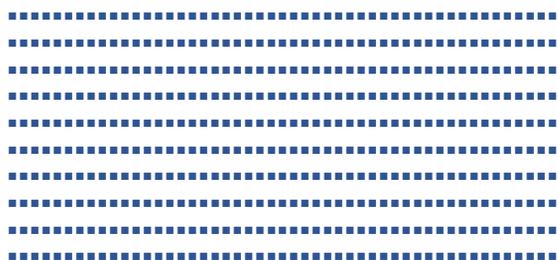
Tom purchased 2 punnets of strawberries each weighing 500 grams. What is the total weight of both punnets?

Tom knew this was a multiplication problem because he knew the NUMBER OF GROUPS (2) and the SIZE OF EACH GROUP (500 grams) and he needed to calculate the TOTAL weight. He thought of the problem as $2 \times 500 = ?$

From his understanding of multiplication basic facts, Tom knew he could use doubling.



500



another 500



2×500 is the same as $500 + 500$.
This gives a total weight of 1 000 grams of strawberries (which is the same as 1 kilogram).

Practice Exercise 1

1. Use Tom's thinking to mentally solve the following problems by doubling. Use jottings to help keep track.

a) 2×32

d) $2\,300 \times 2$

b) 2×410

e) $15\,250 \times 2$

c) 2×63

f) $2 \times 6\,550\,000$

2. Write the two hardest multiplying by 2 problems you can mentally calculate, in the space below. Test your partner with your problems.

Whole Class Activity 2

Using doubling for $\times 4$ and $\times 8$

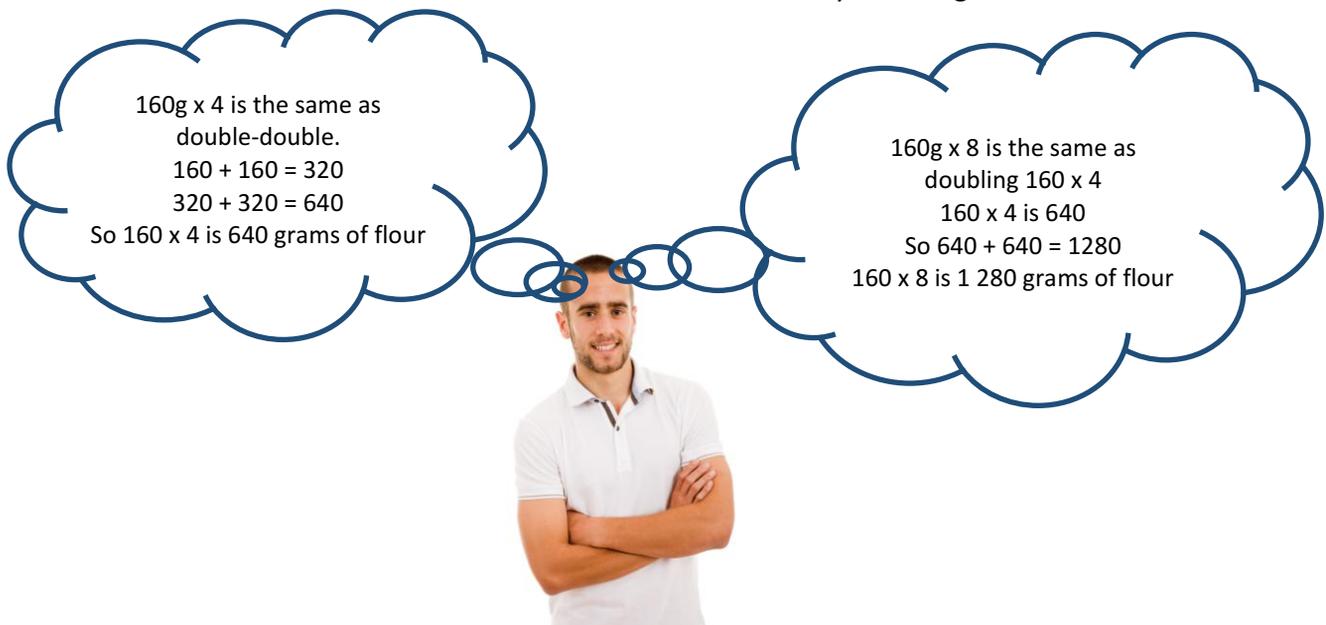
Think: How can doubling be used to solve harder multiplication problems involving $\times 4$ and $\times 8$?

Tom is making two large chocolate cakes from a recipe that uses 160g of flour. The first cake uses 4 times the amount of flour. The second cake is larger and requires 8 times the amount of flour. How much flour does Tom use in each cake?

In multiplication of basic facts Tom learnt that:

- $\times 4$ is the same as doubling the number and doubling again
- $\times 8$ is the same as doubling, doubling and doubling the number

Tom worked out how much flour he needed for the two cakes by doubling:



Practice Exercise 2

1. Use doubling to complete the $\times 2$, $\times 4$ and $\times 8$ problems in the table below.

| $\times 2$ Facts (Double) | $\times 4$ Facts (Doubled $\times 2$) (Double-Double) | $\times 8$ Facts (Doubled $\times 4$) (Doubled, Doubled and Doubled Again) |
|------------------------------|---|--|
| $1 \times 2 = 2$ | $1 \times 4 = 4$ | $1 \times 8 = 8$ |
| $2 \times 2 = 4$ | $2 \times 4 = 8$ | $2 \times 8 = 16$ |
| $3 \times 2 = 6$ | $3 \times 4 = 12$ | $3 \times 8 = 24$ |
| $12 \times 2 =$ | $12 \times 4 =$ | $12 \times 8 =$ |
| $15 \times 2 =$ | $15 \times 4 =$ | $15 \times 8 =$ |
| $25 \times 2 =$ | $25 \times 4 =$ | $25 \times 8 =$ |
| $150 \times 2 =$ | $150 \times 4 =$ | $150 \times 8 =$ |
| $18 \times 2 =$ | $18 \times 4 =$ | $18 \times 8 =$ |
| $45 \times 2 =$ | $45 \times 4 =$ | $45 \times 8 =$ |
| $135 \times 2 =$ | $135 \times 4 =$ | $135 \times 8 =$ |

2. Use Tom's method to mentally solve the following problems. Use jottings to help keep track.

a) 13×2

d) 4×65

b) 14×4

e) 250×8

c) 8×400

f) 4×225

3. Write the two hardest multiplying by 4 and by 8 problems you can mentally calculate, in the space below. Test your partner with your problems.

Whole Class Activity 3

Strategies for multiplying money

Tom was confident that his doubling methods for multiplying by 2, 4 and 8 would work with money.

Calculate the cost of purchasing two concert tickets, four concert tickets and eight concert tickets at \$37 each using this method.



Check your solutions with a calculator.

Does this method work when the money has both dollars and cents? Calculate the cost of buying 4 cans of soft drink at \$3.25 each. Discuss with a partner and check your solutions with a calculator.



Practice Exercise 3

1. Use a ruler to draw a line to connect the question to the correct answer

- | | |
|-----------------------|-----------|
| a) $\$27 \times 4$ | $\$98$ |
| b) $8 \times \$13$ | $\$94$ |
| c) $\$50.50 \times 2$ | $\$94.50$ |
| d) $4 \times \$23.50$ | $\$108$ |
| e) $\$47.25 \times 2$ | $\$104$ |
| f) $\$12.25 \times 8$ | $\$101$ |

2. Mentally calculate the solutions to the following problems. Use jottings if necessary.

- | | |
|------------------------|-----------------------|
| a) $2 \times \$54$ | d) $\$12.50 \times 8$ |
| b) $4 \times \$350$ | e) $2 \times \$3.27$ |
| c) $\$5\ 600 \times 4$ | f) $\$16.75 \times 4$ |

3. For the following everyday problems:

- Write the appropriate number sentence for each situation.(e.g $23 \times 4 = ?$)
- Find the answer using Tom's method of doubling
- Think about your answer. Does it make sense?

a) How much would 2 pairs of sunglasses at $\$62.50$ each cost?

| | | | |
|-----------------|----------|--------|---------------------|
| Number Sentence | Doubling | Answer | Does it make sense? |
|-----------------|----------|--------|---------------------|

b) A strawberry smoothie requires 145g of strawberries. How many grams of strawberries would be needed for 8 smoothies?

| | | | |
|-----------------|----------|--------|---------------------|
| Number Sentence | Doubling | Answer | Does it make sense? |
|-----------------|----------|--------|---------------------|

c) The perimeter of a square is found by multiplying the side length by 4. If one side of a square is 28 cm, what is the perimeter?

| | | | |
|-----------------|----------|--------|---------------------|
| Number Sentence | Doubling | Answer | Does it make sense? |
|-----------------|----------|--------|---------------------|

d) Find the wage earned by Kerry who works 8 hours for $\$22.50$ per hour?

| | | | |
|-----------------|----------|--------|---------------------|
| Number Sentence | Doubling | Answer | Does it make sense? |
|-----------------|----------|--------|---------------------|

e) The cost of cherries is expected to double in the week leading up to Christmas. If cherries are currently \$18.25 per kilogram, how much will they cost in the week before Christmas?

| | | | |
|-----------------|----------|--------|---------------------|
| Number Sentence | Doubling | Answer | Does it make sense? |
|-----------------|----------|--------|---------------------|

f) Jill's cycling speed is 36 kilometres per hour. Jill cycled for 4 hours. How far did she ride?

| | | | |
|-----------------|----------|--------|---------------------|
| Number Sentence | Doubling | Answer | Does it make sense? |
|-----------------|----------|--------|---------------------|

4. At the start of the year, Rory's parents decided to give him pocket money. They asked him to choose one of the following options:

OPTION A: 5 cents for the first day, doubled to 10 cents on the second day, doubled to 20 cents the day after and so on for a fortnight and no other money for the rest of the year.

OPTION B: \$8 a week for every week for the year.

OPTION C: \$4 per week for the 41 school weeks in the year and \$20 per week for each of the 11 weeks of holidays.

a) Which option do you think Rory should choose? Why?



b) Work out how much Rory would receive for options A, B and C in the space below each option.

c) Did you choose the correct option for Rory? Why/why not?



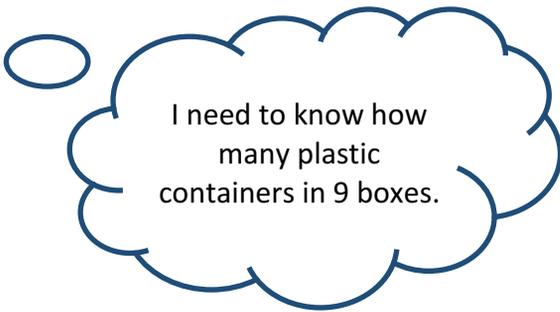
Whole Class Activity 4

Using Patterning and Place Value for $\times 10$

Think: How can we use place value patterns and halving to solve multiplication problems?



Tom was doing work experience at a warehouse.



He knows this is a multiplication problem because there are 9 boxes with 10 000 plastic containers in each. He needs to know the TOTAL number of containers. $9 \times 10\,000 = ?$

Tom knows that $9 \times 1 = 9$ and $9 \times 10 = 90$. On 1mm grid paper, draw arrays to show what these facts would look like. Then draw a 9×100 array. What is the answer? What is happening to the value of the 9 as it is multiplied by 1, then 10, then 100 etc?



He placed these facts in the Place Value Chart and used patterning to multiply by 100, 1 000 and 10 000 (i.e. increasing powers of 10).

| | THOUSANDS | | | ONES | | |
|----------------------|-----------|------|------|----------|------|------|
| | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| $9 \times 1 =$ | | | | | | 9 |
| $9 \times 10 =$ | | | | | 9 | 0 |
| $9 \times 100 =$ | | | | | | |
| $9 \times 1\,000 =$ | | | | | | |
| $9 \times 10\,000 =$ | | | | | | |

Complete the above table

What is happening to the place value of the 9, as it is multiplied by 1, then 10, then 100 etc?



How many plastic containers are in the warehouse?



What is the answer to $9 \times 100\,000$?



What does the 9 mean in 900 000?



Predict the answer for:

a) $9 \times 1\,000\,000$

b) $9 \times 10\,000\,000$

c) $9 \times 100\,000\,000$

What does the 9 mean in each of these numbers?

Practice Exercise 4

1. Solve the following problems using the Place Value Chart below.

| | THOUSANDS | | | ONES | | |
|-----------------------|-----------|------|------|----------|------|------|
| | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| $43 \times 1 =$ | | | | | | |
| $43 \times 1000 =$ | | | | | | |
| $710 \times 1 =$ | | | | | | |
| $710 \times 10 =$ | | | | | | |
| $9\ 400 \times 1 =$ | | | | | | |
| $9\ 400 \times 100 =$ | | | | | | |

2. Visualize the Place Value Chart to solve the following problems.

- | | |
|------------------------|---------------------------|
| a) 11×10 | d) $10\ 000 \times 80$ |
| b) 25×100 | e) $9\ 000 \times 100$ |
| c) $1\ 000 \times 351$ | f) $4\ 350 \times 1\ 000$ |

3. Write the two hardest multiplying by 10 problems you can mentally calculate, in the space below. Test your partner with your problems.

Whole Class Activity 5

Tom decided he could multiply money amounts in the same way.

In January 2015, the price of gold was approximately \$46.50 per gram whilst the price of silver was \$0.63 per gram.

Place these prices in the Place Value Chart below. Use the chart to calculate the price of:

- 10 grams of the metal (i.e. price \times 10)
- 100 grams of the metal
- 1000 grams of the metal

| | THOUSANDS of dollars | | | ONES of dollars | | | Cents (parts of next dollar) | |
|-------------------------|----------------------|------|------|-----------------|------|------|------------------------------|------------|
| | Hundreds | Tens | Ones | Hundreds | Tens | Ones | Tenths | Hundredths |
| Price of gold = | | | | | | | | |
| 10 grams of gold = | | | | | | | | |
| 100 grams of gold = | | | | | | | | |
| 1000 grams of gold = | | | | | | | | |
| Price of silver = | | | | | | | | |
| 10 grams of silver = | | | | | | | | |
| 100 grams of silver = | | | | | | | | |
| 1 000 grams of silver = | | | | | | | | |

Predict the price of:

(i) 10 000 grams of gold

(ii) 100 000 grams of silver

What is happening to the value of the 6, as \$0.63 is multiplied by 1, then 10, then 100 etc.?

Discuss with a partner.



Practice Exercise 5

1. Calculate the solutions to the following problems. Draw yourself a Place Value Chart to help if needed.

a) $10 \times \$61$

d) $\$29.50 \times 10$

b) $10 \times \$415$

e) $100 \times \$4.28$

c) $\$33\ 000 \times 10$

f) $\$376.10 \times 1\ 000$

Whole Class Activity 6

Using $\times 10$ and halving for $\times 5$

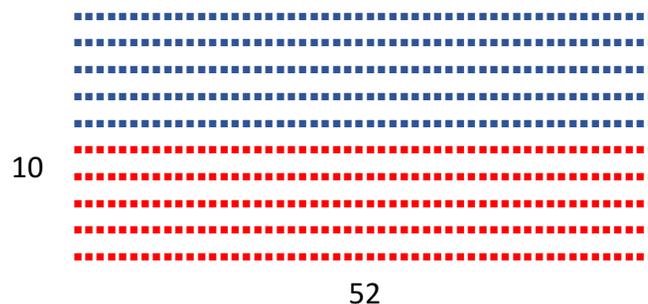
Think: How can we use $\times 10$ to work out $\times 5$ multiplication problems?

How many weekdays (Monday – Friday) are in a year (52 weeks)?

Tom knew this was a multiplication problem because he knew the NUMBER OF GROUPS (52) and the NUMBER IN EACH GROUP (5 days) and he needed to find the TOTAL NUMBER OF DAYS. He thought of this problem as $52 \times 5 = ?$

Tom remembered that multiplying by 5 is the same as multiplying the number by 10 and halving it (see page 106). He realised he could use the same strategy for larger numbers.

To help with his thinking, Tom drew and cut out a 52×10 array. He cut the 52×10 in half to form two 52×5 arrays as shown in the following diagram:





52 x 5 is the same as half of 52 x 10.
52 x 10 is 520.
Half of 520 is 260.
There are 260 weekdays in a year.

Practice Exercise 6

1. Use Tom's thinking to solve the following. The first one has been done for you.

- | | |
|-------------------------|--------------------|
| a) $14 \times 10 = 140$ | $14 \times 5 = 70$ |
| b) $24 \times 10 =$ | $24 \times 5 =$ |
| c) $46 \times 10 =$ | $46 \times 5 =$ |
| d) $17 \times 10 =$ | $17 \times 5 =$ |
| e) $53 \times 10 =$ | $53 \times 5 =$ |
| f) $136 \times 10 =$ | $136 \times 5 =$ |

2. Use the method to mentally solve the following problems. Use jottings to help keep track.

- | | |
|-------------------|----------------------|
| a) 12×5 | d) 5×500 |
| b) 64×5 | e) $4\,800 \times 5$ |
| c) 5×410 | f) 5×843 |

3. Write the two hardest multiplying by 5 problems you can mentally solve in the space below. Test your partner with your problems.

Whole Class Activity 7

Tom thought the $\times 5$ strategy would work with money to solve problems such as:

- Buying 5 litres of petrol at \$1.40/L
- Buying 5 litres of diesel at \$1.48/L
- Buying 5 litres of LPG at \$0.69/L

Calculate the answers using the $\times 5$ strategy. Check with your calculator. Does the strategy work?



Practice Exercise 7

1. Use Tom's method to mentally calculate the solutions to the following problems. Use jottings to help keep track.

- | | |
|---------------------|-----------------------|
| a) $5 \times \$26$ | d) $\$8.60 \times 5$ |
| b) $5 \times \$47$ | e) $5 \times \$14.40$ |
| c) $\$620 \times 5$ | f) $\$62.80 \times 5$ |

2. Puzzle

'What do you call a shoe made from a banana?'

Complete these questions to decode the answer to this joke.

- | | |
|------------------------|------------------------|
| P: $5 \times \$27$ | S: $5 \times \$26.60$ |
| L: $10 \times \$13.55$ | P: $\$14.10 \times 10$ |
| I: $\$28.40 \times 5$ | R: $10 \times \$13$ |
| E: $\$13.25 \times 10$ | A: $5 \times \$25.80$ |

| | | | | | | | |
|-------|-------|----------|-------|-------|-------|----------|-------|
| \$129 | \$133 | \$135.50 | \$142 | \$135 | \$141 | \$132.50 | \$130 |
|-------|-------|----------|-------|-------|-------|----------|-------|

3. For the following everyday problems:

- Write the appropriate multiplication number sentence for each situation (e.g $23 \times 4 = ?$).
- Find the answer using Tom's method of patterning for $\times 10$ and $\times 100$, and halving for $\times 5$.
- Think about your answer. Does it make sense?

a) 84 new cars were fitted with 5 tyres each (4 and a spare). How many tyres were fitted?

| | | | |
|-----------------|--------|--------|---------------------|
| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|

b) The length of a desk was 62 centimetres. If one centimetre is equal to 10 millimetres, how many millimetres is the desk?

| | | | |
|-----------------|--------|--------|---------------------|
| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|

c) The height of a bird on a scale drawing was 21mm. The bird in real life is 100 times taller. How tall is the bird?

| | | | |
|-----------------|--------|--------|---------------------|
| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|

d) What is the total cost of 5 lollies costing \$1.20 each?

| | | | |
|-----------------|--------|--------|---------------------|
| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|

e) Calculate the area of a rectangle measuring 5cm by 34cm.

| | | | |
|-----------------|--------|--------|---------------------|
| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|

f) How much would 10kg of apples cost at \$3.45 per kilogram?

| | | | |
|-----------------|--------|--------|---------------------|
| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|

Whole Class Activity 8

Using $\times 10$ and subtracting one group, to solve $\times 9$ problems

Think: How can we use $\times 10$ to help solve multiplication problems involving $\times 9$?

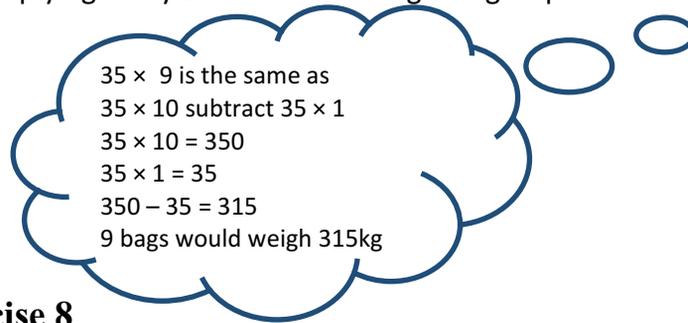
If bags of wheat weigh 35kg, how much would 9 bags weigh?

We know this is a multiplication problem because we know the AMOUNT IN EACH GROUP (35kg) and the NUMBER OF GROUPS (9) and we need to find the TOTAL WEIGHT. We can think of this problem as $35 \times 9 = ?$

Use 5mm grid paper to make an array, 35×10 . How many squares in this array?
Cut off 35×1 . How many squares are left?

Glue the arrays below and label them.

Tom looked at the arrays above and saw that 35×9 could be thought of as multiplying 35 by 10 and subtracting one group of 35.



Practice Exercise 8

1. Use Tom's method to solve the following $\times 9$ problems by multiplying by 10 and subtracting one group. Use jottings to help keep track.

a) 12×9

d) 9×45

b) 22×9

e) 250×9

c) 34×9

f) 9×360

2. Write the two hardest multiplying by 9 problems you can calculate, in the space below. Test your partner with your problems.

Reflection and Discussion

Tom wondered whether he could use this method to multiply by 99, 299, or 4999.

Calculate 7×99 by multiplying 7 by 100 (99 rounded UP) and then subtracting 7. Use jottings to keep track. (Or an array on 1mm grid paper)



Check your solution with a calculator

Calculate 7×299 by multiplying 7 by 300 (299 rounded UP) and subtracting 7. Use jottings to keep track.



Check your solution with a calculator.

Calculate $7 \times 4\,999$ by multiplying 7 by 5 000 (4 999 rounded UP) and then subtracting 7. Use jottings to keep track.



Check your solution with a calculator. Does this method work with larger numbers?

3. Solve the following problems mentally. Use jottings to keep track of your thinking.

a) 8×99

d) 199×5

b) 7×999

e) 6×399

c) $9\,999 \times 3$

f) 999×40

Whole Class Activity 9

Tom noticed that prices of items were often close to the next power of ten. For example:



sandwiches - \$3.99 each



computer - \$1999 each

Why are prices in shops often \$99.99, \$1 999, \$5.99 etc.? Discuss with your class.



Tom thought he could use the strategy for multiplying by 9 to mentally solve shopping problems.

To work out the cost of 5 sandwiches, round the price of the sandwich UP to the nearest dollar (\$4.00). Calculate the cost by multiplying by 5 and subtracting 5 cents. Discuss your jottings with a partner.



To work out the cost of the 6 computers, round the price UP to the nearest thousand dollars (\$2 000). Calculate the cost of the computers. Discuss your thinking with a partner.



If the computers were actually \$1 995 each, how could you use the same method to calculate the total cost of 6 computers?



Practice Exercise 9

1. Solve the following problems by subtracting one or more groups. Use jotting to keep track.

a) $4 \times \$99$

d) $15 \times \$99$

b) $\$9.99 \times 8$

e) $\$3\,995 \times 6$

c) $\$69.99 \times 6$

f) $\$2\,999.95 \times 8$

2. Reuben mentally checked his shopping receipts. He was sure some of the totals were wrong.

Mentally calculate each purchase. Circle any errors.

Write the correct calculation in the space below.

| RECEIPT 12/3/15 | |
|----------------------|---------|
| MILK; 4 @ \$1.99 | \$7.96 |
| BUTTER; 3 @ \$4.99 | \$14.93 |
| DOGFOOD; 2 @ \$19.99 | \$39.98 |
| RICEPOPS; 3 @ \$5.95 | \$17.70 |

| RECEIPT 13/3/15 | |
|---------------------|---------|
| CORN; 3 @ \$0.99 | \$1.97 |
| PEACHES; 8 @ \$3.99 | \$32.92 |
| SOAP; 7 @ \$0.95 | \$6.65 |
| YOGHURT; 3 @ \$2.99 | \$5.97 |

Whole Class Activity 10

Using Place Value Partitioning

Think: How can we break up numbers to make multiplication easier?

Using a sheet of 5mm grid paper, cut out the array formed by 5×18

Use place value to cut the array into two pieces that would help make the multiplication easier.

Glue the cut pieces in the space below.

Write the multiplication number sentence formed by each of the cut pieces.

What is the answer to 5×18 ?

Label your diagram so that the answer can be easily seen in the two parts of your array.

Share your thinking with your partner. Did you cut up the array in the same way? What other strategies were used in the class?



Decide as a class, what is the best strategy for this particular problem?





6×52 is the same as
 6×50 added to 6×2 .
That is, $300 + 12 = 312$

Katie decided she could solve problems like these mentally, by visualizing the partitioned rectangles.

However, she found she needed to write down a few numbers to keep track of where she was up to.

To multiply 6 and 26 she wrote down 120 and 36



6×26 is the same as
 6×20 add 6×6
 6×20 is 120
 6×6 is 36
120 add 36 is 156

Practice Exercise 10

1. Use Katie's thinking and 5 mm grid paper to solve the following problems.

a) 7×21

c) 35×3

b) 6×13

d) 6×34

2. Draw rectangles to *represent* arrays and solve the following:

a) 3×51

d) 7×42

b) 2×63

e) 6×125

c) 84×2

f) 5×264

3. Use Katie's method to mentally solve the following problems by visualizing rectangles. Use jottings to help.

a) 6×31

d) 6×64

b) 3×43

e) 2×346

c) 35×7

f) 235×3

Reflection and Discussion

Katie used the same method for solving money problems. For example, she needed to buy 7 T-Shirts for work; each costing \$18.

Katie thought of this as 7×18 . She broke 18 up into tens and ones and calculated:

$$7 \times 18 = 7 \times 10 \text{ add } 7 \times 8$$

$$= 70 + 56$$

$$= 126 \quad \text{Katie thought a total of \$126 seemed reasonable.}$$

However, Katie wondered whether this method would work when multiplying money which had both dollars and cents. For example, $3 \times \$2.25$

To solve the problem Katie broke the \$2.25 into \$2, \$0.20 and \$0.05 and calculated:

$$3 \times \$2.25 = 3 \times \$2, \quad \text{add } 3 \times \$0.20, \quad \text{add } 3 \times \$0.05$$

$$= \$6 + \$0.60 + \$0.15$$

$$= \$6.75$$

Use grid paper to show why this partitioning works.

How could you partition \$2.25 in a different way to do this calculation?



4. Use Katie's thinking and jottings, to solve the following:

a) $3 \times \$39$

d) $6 \times \$2.50$

b) $4 \times \$315$

e) $\$6.50 \times 7$

c) $2 \times \$15.35$

f) $\$4.25 \times 6$

Practice Exercise 11

1. For the following everyday problems:

- Write the appropriate number sentence for each situation (e.g $23 \times 4 = ?$).
- Find the answer using Katie's method.
- Think about your answer. Does it make sense?

a) A baker has 7 muffin trays, each with 24 holes. How many muffins can be baked at one time?

| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|
| | | | |

b) The owner of a small business expects to make a profit of \$620 per week. How much profit does she expect in 6 weeks?

| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|
| | | | |

c) The average weight of a bag of cement is 22kg. The rear section of a four-wheel drive can carry 9 bags. How much weight can it carry?

| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|
| | | | |

d) At a community environment meeting, there are 8 rows of 35 chairs. How many people can be seated at the meeting?

| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|
| | | | |

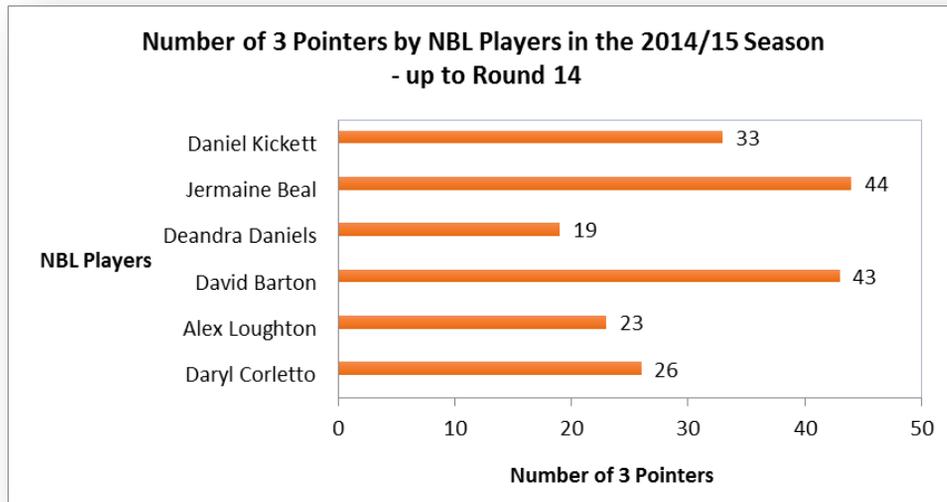
e) A car travels at 85km/h. How far will it travel in 3 hours?

| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|
| | | | |

f) A cupcake recipe uses 175grams of flour. Brodey wants to make 4 times the number of cupcakes as shown in the recipe. How much flour will he need?

| Number Sentence | Method | Answer | Does it make sense? |
|-----------------|--------|--------|---------------------|
| | | | |

2. The graph below shows the number of '3 pointers' shot by six players in the National Basketball League (NBL) in the 2014/15 season up until Round 14.



a) Calculate the number of points each player had contributed to his team through scoring 3 pointers

| <u>Player</u> | <u>Total number of points</u> | <u>Player</u> | <u>Total number of points</u> |
|---------------|-------------------------------|---------------|-------------------------------|
| Kickett | | Barton | |
| Beal | | Loughton | |
| Daniels | | Corletto | |

b) Kickett and Barton both play for Melbourne United. How many 3 pointers had they both scored in total up until Round 14? How many points is this?

3. Use Katie's method to check all calculations in the following table. Circle any errors in the table and write the problem and the correct solution in the space below.

| | | | |
|----------------------|-----------------------|-----------------------|-------------------------|
| $4 \times 23 = 92$ | $14 \times 8 = 112$ | $5 \times 27 = 135$ | $93 \times 4 = 382$ |
| $215 \times 4 = 860$ | $6 \times 141 = 946$ | $12 \times 9 = 108$ | $5 \times \$33 = \165 |
| $3 \times 34 = 102$ | $19 \times 3 = 57$ | $2 \times 755 = 1510$ | $2 \times \$3.50 = \7 |
| $5 \times 17 = 95$ | $3 \times 416 = 1248$ | $3 \times 37 = 121$ | $132 \times 5 = 660$ |

Whole Class Activity 12

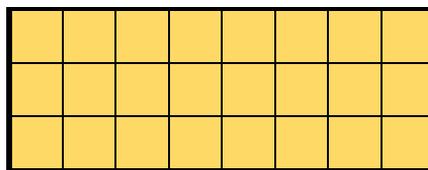
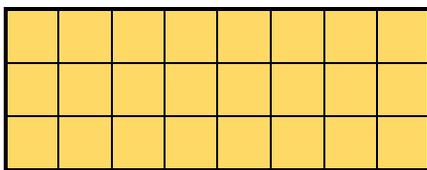
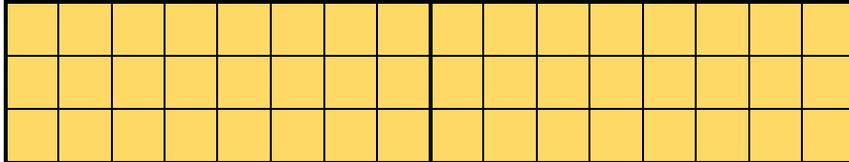
Penny's Strategy for Solving Multiplication Problems

Think: How can we use doubling and halving to solve other multiplication problems?

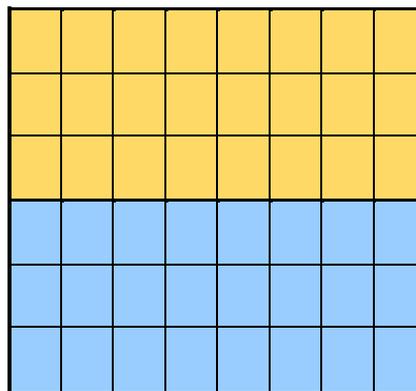
Penny used doubling and halving to calculate the area of a rectangle with a length of 3cm and a width of 16cm?

Firstly she cut out a 3×16 array on grid paper.

She then cut the array in half forming two arrays of 3×8



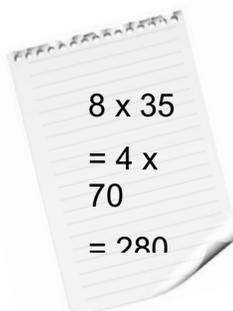
Penny then placed one of the 3×8 arrays below the other to form a 6×8 array



Penny could see that $3 \times 16 = 6 \times 8$. She realised that if she doubled one of the numbers and halved the other one, the total number of squares in the array would be the same.

Hence the total area of a rectangle measuring 3cm by 16cm is 48 square centimetres.

Penny decided she could solve problems like these mentally, by visualizing cutting the arrays in half. She found she needed to write down a few numbers to keep track. To multiply 8 and 35 she wrote down 4 and 70



8×35 is the same as
 4×70
 4×70 is 280

Practice Exercise 12

1. Cut out the following arrays on 5mm grid paper. Then cut them in half and rearrange them to show Penny's thinking

a) 5×18

c) 12×15

b) 3×16

d) 24×5

2. Use Penny's method to mentally solve by visualizing cutting the arrays in half.

Use jottings to help keep track.

a) 3×18

d) 16×35

b) 25×8

e) 12×25

c) 15×18

f) 55×14

3. Penny knew that her method would also work with money. She could see that buying 12 items at \$2.50 was the same as buying 6 items at \$5.00 and would cost \$30.

Use Penny's method to solve the following. Use jottings to help keep track.

a) $18 \times \$150$

d) $16 \times \$4.50$

b) $16 \times \$2\ 500$

e) $3 \times \$1.60$

c) $\$12\ 500 \times 12$

f) $\$450 \times 14$

Reflection and Discussion

Penny needed to solve the following multiplication problems.

8×35

16×36

$\$15 \times 35$

$26 \times \$33$

Circle the problems that can be easily solved using Penny's doubling and halving method?



What type of problems cannot be solved using Penny's method?



Write 4 problems best suited to being solved by Penny's method in the space below. Write your problems on cards and swap with a partner to find the answers to each other's problems.



Practice Exercise 13

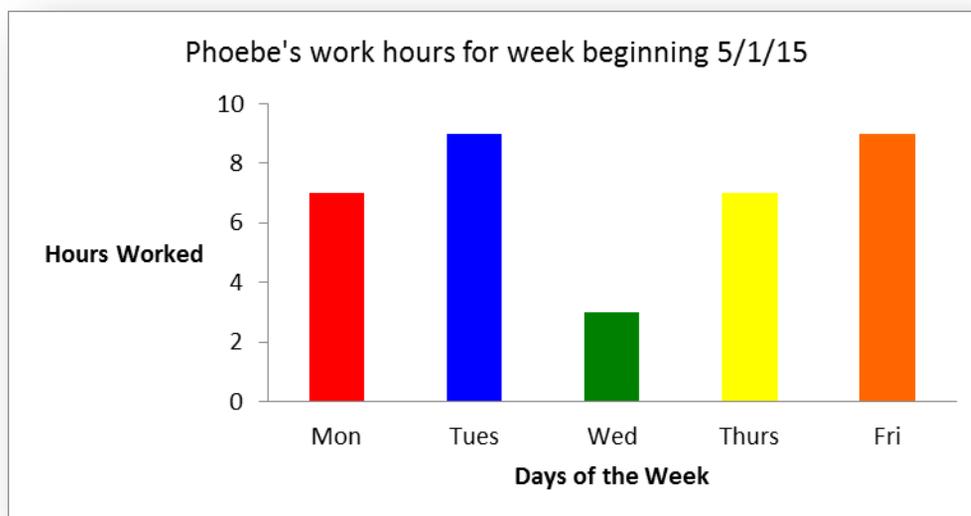
1. Complete the following table

| × | 14 | 25 | \$35 | 18 | 60 | \$2.50 | 100 | \$1.25 |
|---|----|----|------|----|----|--------|-----|--------|
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 9 | | | | | | | | |
| 7 | | | | | | | | |
| 6 | | | | | | | | |
| 5 | | | | | | | | |
| 8 | | | | | | | | |

2. Play these computer games from the following websites to practise the multiplication strategies shown in this topic

- <http://www.mathsisfun.com/timestable.html> (Using strategies to solve $\times 11$, $\times 12$, $\times 13$, $\times 14$, and $\times 15$)
- <http://www.mathplayground.com/multiplication03.html> (Multiplying 2 digits by 1 digit)

3. Phoebe is paid \$14 per hour in her part-time job during the school holidays. The graph shows how many hours she worked each day in one week.



a) How much did Phoebe earn on Monday?

b) How much did Phoebe earn altogether during the week beginning 5/1/15?

4. Travis, Jake and Michael's Grandmother passed her family recipe for one 24cm Chocolate Caramel Tart onto her grandchildren.

| Recipe: CHOCOLATE CARAMEL TART (24cm Tart) | |
|--|-------------------------|
| PASTRY | FILLING |
| 150g Butter | 180mL Cream |
| 110g Icing sugar | 90g Butter |
| 35g Cocoa Powder | 330g Castor Sugar |
| 225g Plain Flour | 125mL Water |
| 2 Egg Yolks | Icing Sugar for dusting |

a) Travis wants to make 3 tarts. How much of each ingredient will he need for the pastry?

b) Jake loves the filling so he decides to make 5 times the amount. What quantities of each of the filling ingredients will he need?

c) Michael wants to make 8 tarts for the school fete. Rewrite the recipe with the new quantities.

Reflection on Learning

Problem: Isobel babysat every week for 8 weeks and was paid \$45 per week. How much was she paid altogether?

a) Why is this a multiplication problem?



b) Represent this problem as a number sentence



c) Calculate the solution to the problem by the 3 different methods outlined in this unit of work. Check that your answers make sense.

Show jottings to support your thinking in the table below.

| Method | Jottings |
|--|----------|
| Tom (Extending Basic Fact Strategies) | |

| | |
|-------------------------------------|--|
| Katie (Place Value Partitioning) | |
| Penny (Doubling and Halving) | |

c) Circle the method that was easiest to use in this example. Explain why.



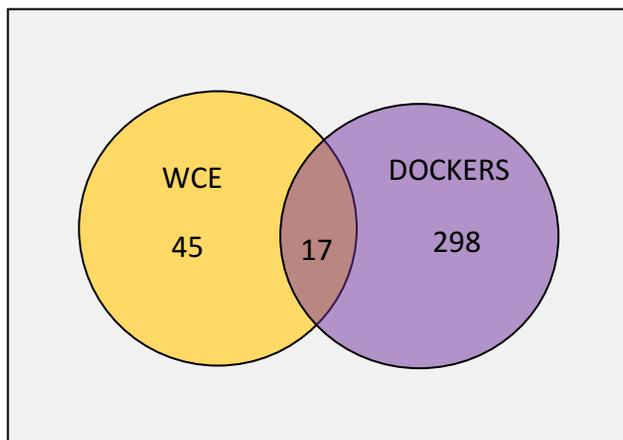
OLNA Practice Questions

1. Avijoy bought a TV from a major appliance store. He paid \$132 deposit and then 18 monthly payments of \$45 each.

The total amount that Avijoy would pay the store is

- A. \$2 475 B. \$1 350 C. \$810 D. \$942

2. A survey by the West Coast Eagles (WCE) marketing team was taken at a Fremantle shopping centre asking which West Australian AFL team people support.



The marketing team, hope to increase the people who support WCE only, to five times that shown.

How many people do they expect to say support the WCE when the next survey is taken?

- A. 85 B. 255 C. 225 D. 310

Understanding and Recalling Basic Division Facts

Mathematics Discussion

There are two types of division: sharing and grouping. In a problem such as $12 \div 3$, sometimes we say 12 shared between 3 and sometimes we say how many 3's in 12. We use sharing or grouping strategies to solve division problems.

Sharing division: In $12 \div 3$, the 12 items are given out one at a time into three groups until they are all used. The answer is the number of items in a group.

Total \div the number of groups = number in a group

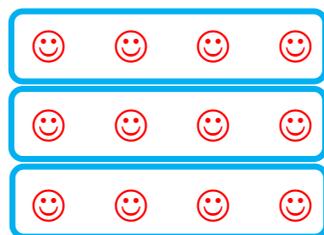
Grouping division: In $12 \div 3$, the 12 items are placed into groups of three until they are all used. The answer is the number of groups. This tends to be quicker as the items are given out in groups rather than one at a time.

Total \div the number in a group = number of groups

We can use arrays to show both types of division:

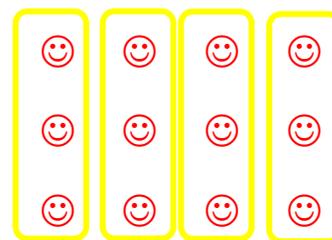
SHARING

12 items shared between 3



GROUPING

12 items in groups of 3



We can use arrays to show the relationship between multiplication and division.

$$3 \times 4 = 12$$

$$12 \div 4 = 3$$

$$4 \times 3 = 12$$

$$12 \div 3 = 4$$

Typically, we use basic multiplication facts to work out the basic division facts because multiplication and division are related. We think of $12 \div 4$ as, *how many fours in twelve* ($? \times 4 = 12$), or *four times something equals twelve* ($4 \times ? = 12$).

Using the array we can also see how fractions are related to division. $\frac{1}{4}$ of 12 is 3, which is the same as $12 \div 4$ and $\frac{1}{3}$ of 12 is 4 which is the same as $12 \div 3$.

From an array diagram such as the one above we can generate many different, related, number sentences. These are sometimes called 'Fact Families'.

$$4 \times 3 = 12$$

$$12 \div 4 = 3$$

$$\frac{1}{4} \text{ of } 12 = 3$$

$$3 \times 4 = 12$$

$$12 \div 3 = 4$$

$$\frac{1}{3} \text{ of } 12 = 4$$

$$12 = 4 \times 3$$

$$3 = 12 \div 4$$

$$3 = \frac{1}{4} \text{ of } 12$$

$$12 = 3 \times 4$$

$$4 = 12 \div 3$$

$$4 = \frac{1}{3} \text{ of } 12$$

We can solve division problems using arrays, counting how many groups by skip counting or by changing the division problem into a multiplication problem and using our knowledge of basic multiplication facts. We can also use halving to work out some division problems. For example, $16 \div 2$ is the same as half of 16, which is 8. To divide by four we can halve and then halve again.

Whole Class Activity 1

Think: What does $12 \div 3$ mean? What does $3 \overline{)12}$ mean?

How do you read and say them?

What are all of the different ways you can read and say $12 \div 3 = 4$? Write them in the space below



What are all of the different ways you can read and say $3 \overline{)12}^4$? Write them in the space below



Do they all mean the same thing? Explain why.



Two types of Division

Collect 24 objects. Place them into two groups of 12 to solve the following problems:

1. Take one set of 12. Share them out between 3 people. How many does each person get? Draw the result.
2. Take the other set of 12 and give them out 3 at a time. How many people can get a group of 3? Draw the result.

How are the two different? How are they the same?



Get two 30cm pieces of string, wool or similar to solve the following problems:

1. Share the first 30 cm length between 3 people? How long is each piece? Draw the result.
2. Cut the second piece of string into 3cm pieces and give each person one piece. How many people get a piece of string? Draw the result.

How are the two different? How are they the same?



Practice Exercise 1

1. Use sharing to solve the following. Draw a diagram for each.

| | |
|----------------|----------------|
| a) $16 \div 2$ | c) $35 \div 7$ |
| b) $24 \div 6$ | d) $56 \div 8$ |

2. Use grouping to solve the following. Draw a diagram for each.

| | |
|----------------|----------------|
| a) $16 \div 2$ | c) $35 \div 7$ |
| b) $24 \div 6$ | d) $56 \div 8$ |

Whole Class Activity 2

Understanding Arrays

We can use arrays to solve division problems. They can be used to solve **both** grouping and sharing division problems.

Work with a partner to draw arrays to show the meaning of the following number sentences.

| | | | |
|-------------------|---|---|-------------------------|
| $6 \times 2 = 12$ | $12 \div 6 = 2$ (12 shared between 6 is 2) | $12 \div 6 = 2$ (12 how many groups of 6 is 2) | $\frac{1}{2}$ of 12 = 6 |
| $3 \times 5 = 15$ | $15 \div 3 = 5$ (15 shared between 3 is 5) | $15 \div 3 = 5$ (15 how many groups of 3 is 5) | $\frac{1}{5}$ of 15 = 3 |
| $5 \times 4 = 20$ | $20 \div 5 = 4$ (20 shared between 5 is 4) | $20 \div 5 = 4$ (20 how many groups of 5 is 4) | $\frac{1}{4}$ of 20 = 5 |

What do you notice about the answers to $15 \div 3$ when it is thought of as:

(i) grouping?

(ii) sharing?

Discuss with your class.



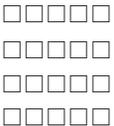
Practice Exercise 2

1. Write two multiplication, two division and two fraction number sentences for the following diagrams. The first has been completed for you

a) $\square\square\square\square$
 $\square\square\square\square$
 $\square\square\square\square$

$3 \times 4 = 12$; $4 \times 3 = 12$; $12 \div 3 = 4$; $12 \div 4 = 3$; $\frac{1}{4}$ of 12 = 3; $\frac{1}{3}$ of 12 = 4

b) $\square\square$
 $\square\square$
 $\square\square$
 $\square\square$

c) 

d) 

2. Draw arrays to show:

a) 14 divided into groups of 2 makes 7 in each

d) 40 divided into groups of 5 is 8.

b) 18 shared between 6 is 3

e) $30 \div 5 = 6$

c) 24 shared between 3 is 8

f) $80 \div 10 = 8$

3. a) Draw as many different arrays as you can that contain 12 squares.

Circle the array that best shows how to calculate $12 \div 6$.

What is the answer to $12 \div 6$? Show this on the array.

b) Draw as many different arrays as you can that contain 16 squares

Circle the array that best shows how to calculate $\frac{1}{4}$ of 16

What is the answer to $\frac{1}{4}$ of 16? Show this on the array.

Reflection and Discussion

An array can show two multiplications and two divisions – a Fact Family.

$$\text{NUMBER OF GROUPS} \times \text{NUMBER IN EACH GROUP} = \text{TOTAL}$$

$$\text{NUMBER IN EACH GROUP} \times \text{NUMBER OF GROUPS} = \text{TOTAL}$$

$$\text{TOTAL} \div \text{NUMBER OF GROUPS} = \text{NUMBER IN EACH GROUP}$$

$$\text{TOTAL} \div \text{NUMBER IN EACH GROUP} = \text{NUMBER OF GROUPS}$$

Use your understanding of Fact Families and an array to explain why multiplication is commutative (e.g. $10 \times 1 = 1 \times 10$) and division is NOT (e.g. $10 \div 1 \neq 1 \div 10$)



Whole Class Activity 3

Strategies for $\div 2$, $\div 3$, $\div 5$, $\div 10$

Think: What methods can be used to solve division facts?

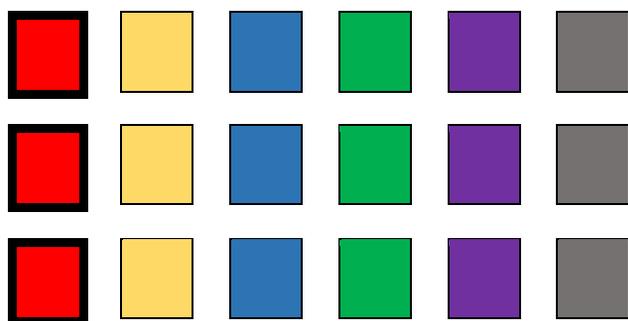
Three students Tegan, Tristan and Sann were solving the problem $18 \div 3$.

They each solved the problem in different ways:

Tegan's Method

Tegan thought of $18 \div 3$ as a grouping division problem. That is, 'how many groups of three in 18?' ($? \times 3 = 18$)

She drew one group of 3 squares, then another next to it to form an array. She continued doing this until she had 18 squares in total.



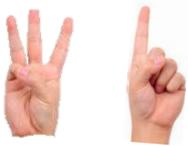
Tegan could see that there were 6 groups of 3 in 18.

Draw an array using Tegan's method to solve $15 \div 5$?



Tristan's Method

Tristan thought of the problem as a sequence. He started at 3 and counted by groups of 3 until he reached 18. He used his fingers to keep track of how many 3's he counted.



| | | | | | |
|----------|----------|----------|----------|----------|----------|
| 3 | 6 | 9 | 12 | 15 | 18 |
| that's 1 | that's 2 | that's 3 | that's 4 | that's 5 | that's 6 |

How could you use Tristan's method to solve $50 \div 5$?



Sann's Method

Sann knew that a division problem such as $18 \div 3 = ?$ was related to a multiplication fact family.

He wrote down:

$$18 \div 3 = ?$$
$$3 \times ? = 18$$
$$? \times 3 = 18$$

He thought of $18 \div 3 = ?$ as how many 3's make 18, $? \times 3 = 18$.

He knew $6 \times 3 = 18$, so the answer to $18 \div 3$ must be 6.

How could you use Sann's method to solve $30 \div 5$? Write the related multiplication first.



Practice Exercise 3

1. Draw arrays and use Tegan's method to solve the following problems by grouping.

a) $18 \div 2$

d) $25 \div 5$

b) $15 \div 5$

e) $40 \div 10$

c) $21 \div 3$

f) $27 \div 3$

Complete the following statements:

When we divide a number by 1 the answer is _____

When we divide a number by the same number the answer is _____

Whole Class Activity 4

Using halving for $\div 2$, $\div 4$, $\div 8$

Dividing by 2

Using an array we can see how dividing by two is the same as finding a half.

For example: $10 \div 2$.



Circle two groups – how many in each?

$$10 \div 2 = 5$$

$$\frac{1}{2} \text{ of } 10 = 5$$

We also know this from our knowledge of fact families.

Practice Exercise 4

1. Solve the following problems by halving. Draw an array to help if needed.

a) $6 \div 2$

d) $16 \div 2$

b) $14 \div 2$

e) half of 12

c) $18 \div 2$

f) split 20 into 2

2. Extensions to Basic Facts: Use your knowledge of dividing by 2 to solve harder problems:

a) $8 \div 2$

b) $12 \div 2$

c) $20 \div 2$

$80 \div 2$

$120 \div 2$

$200 \div 2$

$8\,000 \div 2$

12 million $\div 2$

$20\,000 \div 2$

80 cents $\div 2$

$\$1200 \div 2$

$40 \div 2$

half of 800

$\frac{1}{2}$ of 120

$\frac{1}{2}$ of 2 000

Whole Class Activity 5

Dividing by 4

Dividing by 4 is the same as finding half of a half. Use an array to answer $20 \div 4$. Circle the solution on the array.

Using this diagram we can see how dividing a number by 4 is the same as finding a quarter of the number. That is $20 \div 4 = \frac{1}{4}$ of 20. Finding a quarter means halving the total and then halving again. Check that this makes sense using the diagram above.

Practice Exercise 5

1. Solve the following problems by halving the total and then halving again. Draw an array to help if needed.

a) $8 \div 4$

d) $16 \div 4$

b) $24 \div 4$

e) quarter of 20

c) $32 \div 4$

f) split 28 into 4

2. Extensions to Basic Facts: Use your knowledge of dividing by 4 to solve harder problems:

a) $12 \div 4$

b) $28 \div 4$

c) $36 \div 4$

$120 \div 4$

$280 \div 4$

$3\ 600 \div 4$

$12\ 000 \div 4$

$28\ 000 \div 4$

$360\ 000 \div 4$

$\$1.20 \div 4$

$\$2800 \div 4$

$\$0.36 \div 4$

quarter of 12

$\frac{1}{4}$ of 280

$\frac{1}{4}$ of 360

Whole Class Activity 6

Dividing by 8

We can use halving to divide by 8. Use an array to show the answer to $48 \div 8$.

Find one half of 48, find one half of this, and then find one half again. Circle the solution on the array.

Using this diagram we can see how dividing a number by 8 is the same as finding one eighth of the number. That is $48 \div 8 = \frac{1}{8}$ of 48. Finding an eighth means halving the total, halving again and then halving again. Check that this makes sense using the diagram above.

Practice Exercise 6

1. Solve the following problems by halving the total, halving and then halving again. Draw an array to help if needed.

a) $24 \div 8$

d) $72 \div 8$

b) $64 \div 8$

e) One eighth of 40

c) $32 \div 8$

f) split 56 into 8

2. Extensions to Basic Facts: Use your knowledge of dividing by 8 to solve harder problems:

a) $16 \div 8$

b) $56 \div 8$

c) $80 \div 8$

$160 \div 8$

$560 \div 8$

$8\ 000 \div 8$

$16\ 000 \div 8$

$56\ 000 \div 8$

8 million $\div 8$

$\$1.60 \div 8$

$\$5.60 \div 8$

$\$0.80 \div 8$

One eighth of 160

$\frac{1}{8}$ of 5 600

$\frac{1}{8}$ of 800

Practice Exercise 7

1) Complete the following division 'Operating Tables'

a)

| IN | OUT |
|--------------------------|-----|
| 16 | |
| 20 | |
| 8 | |
| 12 | |
| 6 | |
| 14 | |
| 40 | |
| 100 | |
| RULE: DIVIDE BY 2 | |

b)

| IN | OUT |
|--------------------------|-----|
| 16 | |
| 32 | |
| 20 | |
| 64 | |
| 12 | |
| 24 | |
| 40 | |
| 160 | |
| RULE: DIVIDE BY 4 | |

c)

| IN | OUT |
|--------------------------|-----|
| 16 | |
| 32 | |
| 80 | |
| 64 | |
| 72 | |
| 24 | |
| 40 | |
| 480 | |
| RULE: DIVIDE BY 8 | |

d)

| IN | RULE | OUT |
|-------|------------|-----|
| | $\times 2$ | 12 |
| 32 | $\div 4$ | |
| | $\times 8$ | 56 |
| 20 | $\div 4$ | |
| | $\times 4$ | 24 |
| 48 | $\div 8$ | |
| | $\times 4$ | 40 |
| 1 400 | $\div 2$ | |
| | $\times 2$ | 600 |

Whole Class Activity 7

Which strategy for $\div 6$, $\div 7$, $\div 9$?

When dividing by 6, 7 or 9 is it best to use Tegan, Tristan or Sann's strategies for division? Instant recall of multiplication facts makes Sann's method the quickest. However, Tegan and Tristan's methods can be used if a fact cannot be remembered.

Work with a partner to decide which strategy you would use to solve each problem.

| Problem | Circle Your Chosen Strategy | Strategy and Answer |
|-------------|-----------------------------|---------------------|
| $42 \div 6$ | Tegan Tristan Sann | |
| $81 \div 9$ | Tegan Tristan Sann | |
| $35 \div 7$ | Tegan Tristan Sann | |
| $54 \div 6$ | Tegan Tristan Sann | |
| $63 \div 9$ | Tegan Tristan Sann | |
| $28 \div 7$ | Tegan Tristan Sann | |

Reflection and Discussion

Tegan, Tristan and Sann were asked to solve $0 \div 7$

Tegan thought about how many groups of 7 were in zero.

Tristan tried to count from 7 up by 7 and soon realized he would never reach 0

Sann thought if $0 \div 7 = ?$, then $? \times 7 = 0$

What is the answer to $0 \div 7$?



What would the students be thinking if the problem was $0 \div 9$ or $0 \div 6$?



The three students were asked to solve $7 \div 0$

Tegan thought about drawing groups of zero until she reached 7

Tristan tried to count from 0 up by 0 and soon realized he would never reach 7

Sann thought if $7 \div 0 = ?$, then $? \times 0 = 7$

What is the answer to $7 \div 0$?



What would the students be thinking if the problem was $5 \div 0$ or $9 \div 0$?



Complete the following:

When zero is divided by any number the answer is _____

When any number is divided by zero the answer is _____

Practice Exercise 8

1. Solve the following

a) $12 \div 6$

e) $45 \div 9$

i) $54 \div 9$

b) $27 \div 9$

f) $63 \div 7$

j) $70 \div 7$

c) $9 \div 0$

g) $0 \div 6$

k) $48 \div 6$

d) $60 \div 6$

h) $42 \div 7$

l) $36 \div 9$

2. Phone/Tablet Apps and Websites

a) Download one of the following free Apps onto your phone or tablet to practice your basic division facts.

- Basic Math with Smarty; Otto App Studio
- Simple Sums V1.0; Sygem Software
- Basic Math V3.30; ExplorerTechnologies
- Division Wiz; The Rocket Studio
- The Math Game – Division Facts; Aeralas Apps

b) Play these computer games from the following websites

- www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/bingo/index.html
- http://www.sheppardsoftware.com/mathgames/fruitshoot/fruitshoot_division.htm

3. *Divide and Conquer*

A game to assist in practising basic division facts.

Number of Players – 3; two players and a caller.

Aim To have the largest number of cards at the end of the game.

Equipment: A set of playing cards with all picture cards and jokers removed. Ace = 1.

Rules of the game

1. The two players A and B and the 'caller' sit in a triangle.
2. Each player draws a card from the deck and without looking at it, places it on their forehead with the number pointing outward. The players can see each others' cards but not their own.
3. The 'caller' mentally multiplies the two numbers on the player's heads and calls out the answer.
4. Player A calculates what the number on his/her forehead must be, based on knowing the product and one factor. Player A then collects the 3 cards and places them in a pile.
5. If Player A cannot calculate the number on his/her forehead then Player B gets the chance to calculate what the number on his/her own forehead must be and collects the 3 cards to place in their own pile.
6. New cards are then drawn with Player B calculating the number on his/her forehead first.
7. Play continues until the deck runs out. The winner is the player with the most cards.

VARIATION: Use Jacks = 100, Queens = 1000 and Kings = 10 000, to extend the basic division facts into powers of 10 (e.g. $8 \times 1000 = 8\ 000$).

Reflection and Discussion - Extending the Basic Division Facts

A maths class at Kellerberin District High School was asked:

'How can you extend a fact that you know such as $72 \div 8 = 9$ to help with other mental calculations?'

The class responded as follows;

| | |
|-----------------------------------|----------------------|
| $72 \div 8 = 9$ | |
| $72 \div 9 = 8$ | $9 \times 8 = 72$ |
| $8 \times 9 = 72$ | $90 \times 8 = 720$ |
| $720 \div 8 = 90$ | $720 \div 90 = 8$ |
| $7200 \div 8 = 900$ | |
| $\frac{1}{8}$ of $72 = 9$ | |
| $720 \div 9 = 80$ | $7\ 200 \div 80 = 9$ |

How is each fact connected to $72 \div 8 = 9$? Discuss

Can you add some more to this chart?

Complete a similar activity with the following facts. Share your responses.

$$45 \div 5 = 9$$

$$90 \div 10 = 9$$

$$21 \div 3 = 7$$

$$14 \div 7 = 2$$

4. Extensions to Basic Facts: Use your knowledge of division to solve harder problems

a) $28 \div 7$

b) $36 \div 6$

c) $50 \div 5$

$280 \div 7$

$360 \div 6$

$500 \div 5$

$280 \div 70$

$360 \div 60$

$500 \div 50$

$2\ 800 \div 7$

$3\ 600 \div 600$

$5\ 000 \div 5$

$28\ 000 \div 700$

$36\ 000 \div 60$

$5\ 000 \div 50$

one seventh of 280

$\frac{1}{6}$ of 360

$\frac{1}{5}$ of 50 000

Reflection on Learning

1. Complete the following table.

| Problem | Diagram or Array | Problem Written as a Multiplication of Whole Numbers | Answer |
|-----------------------------|------------------|--|--------|
| Share 27 between 3 | | | |
| How many groups of 6 in 60? | | | |
| $8 \div 2$ | | | |
| $42 \div 7$ | | | |
| $\frac{1}{8}$ of 64 | | | |

2. Complete the following table by summarizing the division strategies shown in this topic.

| | Advantages of this strategy | Disadvantages of this strategy | Examples in which you use this strategy |
|--|-----------------------------|--------------------------------|---|
| Tegan's Method Description of this strategy | | | |
| Tristan's Method Description of this strategy | | | |
| Sann's Method Description of this strategy | | | |
| Halving Method for $\div 2, \div 4, \div 8$ | | | |

3. Record all of the Division basic fact combinations that give an answer to the number indicated in the table below.

| Answer | Combinations | Number of Combinations |
|--------|--|------------------------|
| 1 | 10÷10, 9÷9, 8÷8, 7÷7, 6÷6, 5÷5, 4÷4, 3÷3, 2÷2, 1÷1 | 10 |
| 2 | 20÷10, 18÷9 | |
| 3 | 30÷10 | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |

What patterns did you use when you were working out the combinations?



OLNA Practice Questions

1. Tahlia divided 3 600 by a number. She got 40 as the answer.

What number did Tahlia divide by?

A 90

B 900

C 3 200

D 120

2. These are 4 number cards



Insert each of the number cards once into the following number sentence to make it true.

$$\square \div \square \square = \square$$

Topic 4

Division of Whole Numbers and Money

Mathematics Discussion

We can solve division problems mentally by:

1. Extending the strategies we use for basic division facts. For example,

To solve $720 \div 2$ we can halve 720.

To solve $720 \div 4$ we can halve 720 and halve it again.

To solve $720 \div 8$ we can halve 720, halve it again and halve it again.

To solve $720 \div 10$ we can use place value patterns.

2. Using one of the two different types of partitioning to break up the numbers to make the mental division easier.

- Place value partitioning: for example to solve $215 \div 5$ we can break 215 into $200 + 15$ and divide the parts.

$$200 \div 5 = 40$$

$$15 \div 5 = 3$$

Therefore, $215 \div 5 = 40 + 3$ (i.e.43).

- Non-standard partitioning: for example, to solve $720 \div 6$ we can split 720 into numbers that are easily divided by 6 like $600 + 120$. We can then divide the parts.

$$600 \div 6 = 100$$

$$120 \div 6 = 20$$

Therefore, $720 \div 6 = 100 + 20$ (i.e.120).

Most people tend to use written jottings to help keep track of division calculations that cannot be completely stored in their heads. For problems involving 'difficult' numbers a calculator should be used.

Whole Class Activity 1

Tom's Strategies for Solving Harder Division Problems

Think: How can we use basic division fact for harder division problems?

Tom knows he can extend the strategies developed in Division of Basic Facts to solve harder problems.

With harder problems I can use the strategies I know:
Halving for $\div 2$, $\div 4$, $\div 8$
Patterning for $\div 10$.



Using halving for $\div 2$

Think: How can halving be used to solve division problems?

Tom works at a factory that packs Easter Eggs into cartons. The Easter Eggs arrive in batches of 48 and 88.

Tom packs 48 eggs into 2 cartons for the JUMBO size, or 4 cartons for the REGULAR size.

He packs 88 eggs into 8 cartons for the MINI size.

How many Easter Eggs will be in each carton if Tom splits the batches into:

2 JUMBO cartons?

4 REGULAR cartons?

8 MINI cartons?

We know these are division problems because we know the TOTAL number of eggs (48 or 88) and the NUMBER OF GROUPS (2,4,or 8). We need to find the NUMBER of eggs in EACH GROUP; i.e. TOTAL \div NUMBER OF GROUPS = NUMBER IN EACH GROUP.

We are solving:

$$48 \div 2 = ?$$

$$48 \div 4 = ?$$

$$88 \div 8 = ?$$

We can use halving to divide by 2.

In the space below, draw an array of 48 dots.

Draw a circle on the array to show half. How many eggs would be in the JUMBO carton? Write a division number sentence next to the array (i.e. complete $48 \div 2 = ?$)

Compare your array and your number sentences with your classmates. Did you all use the same array? What number sentences did your classmates make?



Whose array best shows $48 \div 2$?



Complete this sentence: 'Each JUMBO carton will have _____ eggs.'

The array shows that dividing by 2 is the same as halving the total. Therefore, we can think of Tom's problem as $48 \div 2 = \frac{1}{2}$ of 48.

We know that a half of 48 is 24 because half of 40 is 20 and half of 8 is 4 and $20 + 4 = 24$.

Therefore, each JUMBO bag will have 24 eggs.

Practice Exercise 1

1. Solve the following by drawing arrays and circling the answer on the array. Next to each diagram write the matching division number sentence.

a) $26 \div 2$

b) $30 \div 2$

2. Solve the following problems by visualizing the array and halving the total.

a) $50 \div 2$

d) $110 \div 2$

b) $34 \div 2$

e) $154 \div 2$

c) $66 \div 2$

f) $1\,450 \div 2$

3. Write the two hardest dividing by 2 problems you can mentally calculate, in the space below. Test your partner with your problems.

Whole Class Activity 2

Using halving and halving again for $\div 4$

Draw an array to solve $48 \div 4 = ?$ Use halving and halving again to divide by 4.

Draw circles on the array to show how many eggs would be in the REGULAR carton. Write a division number sentence next to the array (i.e. complete $48 \div 4 = ?$)

Compare your array and your number sentences with your classmates. Did you all use the same array? What number sentences did your classmates make?



Whose array best shows $48 \div 4$?



Complete this sentence: 'Each REGULAR carton will have _____ eggs.'

This array shows that dividing by 4 is the same as finding a quarter of the total. Therefore in Tom's problem: $48 \div 4 = \frac{1}{4}$ of 48. Finding a quarter means halving the total and then halving again.

$$\begin{aligned} \text{For example: } 48 \div 4 &= 48 \div 2 \div 2 \\ &= 24 \div 2 \\ &= 12 \end{aligned}$$

Therefore, each REGULAR bag will have 12 eggs.

Practice Exercise 2

1. Solve the following by drawing arrays and circling the answer on the array. Next to each diagram write the matching division number sentence.

a) $52 \div 4$

b) $60 \div 4$

2. Solve the following problems by visualizing the array and finding a quarter of the total by halving and halving again.

a) $80 \div 4$

d) $2\,200 \div 4$

b) $92 \div 4$

e) $3\,000 \div 4$

c) $140 \div 4$

f) $3\,280 \div 4$

3. Write the two hardest dividing by 4 problems you can mentally calculate, in the space below. Test your partner with your problems.

Whole Class Activity 3

Using halving, halving and halving again for $\div 8$

Draw an array to solve $88 \div 8 = ?$ Remember – dividing by 8 is the same as finding half, halving again, and then again.

Draw circles on the array to show how many eggs would be in the MINI carton. Write a division number sentence next to the array (i.e. complete $88 \div 8 = ?$)

Compare your array and your number sentences with your classmates. Did you all use the same array? What number sentences did your classmates make?



Whose array best shows $88 \div 8$?



Complete this sentence: 'Each MINI carton will have _____ eggs.'

This array shows that dividing by 8 is the same as finding one eighth of the total. Therefore in Tom's problem: $88 \div 8 = \frac{1}{8}$ of 88. Finding an eighth means halving, halving and halving the total again.

$$\begin{aligned} \text{For example: } 88 \div 8 &= 88 \div 2 \div 2 \div 2 \\ &= 44 \div 2 \div 2 \\ &= 22 \div 2 \\ &= 11 \end{aligned}$$

Therefore, each MINI bag will have 11 eggs.

Practice Exercise 3

1. Solve the following by drawing arrays and circling the answer on the array. Next to each diagram write the matching division number sentence.

a) $96 \div 8$

b) $104 \div 8$

2. Solve the following problems by visualizing the array and finding an eighth of the total by halving, halving and halving again.

a) $120 \div 8$

d) $3\,200 \div 8$

b) $280 \div 8$

e) $2\,000 \div 8$

c) $144 \div 8$

f) $2\,080 \div 8$

3. Write the two hardest dividing by 8 problems you can mentally calculate, in the space below. Test your partner with your problems.

4. Some basic division facts have been completed in the table below. Use your understanding of $\div 2$, $\div 4$ and $\div 8$ to complete the harder problems in the table

| $\div 2$ Facts (Halve) | $\div 4$ Facts (Halve and halved again) | $\div 8$ Facts (Halve, halved and halved again) |
|---------------------------|--|--|
| $8 \div 2 = 4$ | $8 \div 4 = 2$ | $8 \div 8 = 1$ |
| $16 \div 2 = 8$ | $16 \div 4 = 4$ | $16 \div 8 = 2$ |
| $40 \div 2 =$ | $40 \div 4 =$ | $40 \div 8 =$ |
| $600 \div 2 =$ | $600 \div 4 =$ | $600 \div 8 =$ |
| $96 \div 2 =$ | $96 \div 4 =$ | $96 \div 8 =$ |
| $440 \div 2 =$ | $440 \div 4 =$ | $440 \div 8 =$ |
| $248 \div 2 =$ | $248 \div 4 =$ | $248 \div 8 =$ |
| $1\ 000 \div 2 =$ | $1\ 000 \div 4 =$ | $1\ 000 \div 8 =$ |
| $10\ 400 \div 2 =$ | $10\ 400 \div 4 =$ | $10\ 400 \div 8 =$ |
| $50\ 000 \div 2 =$ | $50\ 000 \div 4 =$ | $50\ 000 \div 8 =$ |

Whole Class Activity 4

Solving division problems involving money

Tom wondered whether the halving method would work when solving problems with dollars and cents. Solve the following shopping problems using halving.

1. The total cost of 8 apples is \$10.00. How much does one cost?



- Write the problem as a division number sentence
- Find the answer using halving
- Check that your solution makes sense

2. Two punnets of strawberries cost \$6.14. How much does one punnet cost?



- Write the problem as a division number sentence
- Find the answer using halving
- Check that your solution makes sense

3. The total price for four mandarins is \$3.40. How much for one?



- Write the problem as a division number sentence
- Find the answer using halving
- Check that your solution makes sense

Check your answers with your calculator. Does the halving method work when solving problems involving dollars and cents. Discuss with a partner.



Practice Exercise 4

1. Use a ruler to draw a line to connect the question to the correct answer. Solve the problems mentally using halving. Use jottings to help keep track if needed.

- | | |
|--------------------|-----|
| a) $300 \div 2$ | 180 |
| b) $880 \div 4$ | 225 |
| c) $426 \div 2$ | 150 |
| d) $860 \div 4$ | 220 |
| e) $1\ 800 \div 8$ | 215 |
| f) $1\ 440 \div 8$ | 213 |

2. Solve the following problems by mentally halving. Use jottings to help keep track if needed.

- | | |
|--------------------|-----------------------|
| a) $\$54 \div 2$ | d) $\$14 \div 8$ |
| b) $\$128 \div 4$ | e) $\$16.80 \div 4$ |
| c) $\$8.50 \div 2$ | f) $\$80\ 800 \div 8$ |

3. For the following everyday problems:

- Write the appropriate division number sentence for each situation (e.g. $104 \div 4 = ?$).
- Find the answer using Tom's method of halving.
- Think about your answer. Does it make sense? Tick the box when you have checked.

a) How many \$2 coins in \$186?

| Number Sentence | Halving Strategy and Answer | Does it make sense? |
|-----------------|-----------------------------|---------------------|
| | | |

b) Dinner for 4 friends comes to \$142. The friends decide to split the bill. How much does each person pay?

| | | |
|-----------------|-----------------------------|---------------------|
| Number Sentence | Halving Strategy and Answer | Does it make sense? |
|-----------------|-----------------------------|---------------------|

c) Calculate the side length of a square with a perimeter of 30cm.

| | | |
|-----------------|-----------------------------|---------------------|
| Number Sentence | Halving Strategy and Answer | Does it make sense? |
|-----------------|-----------------------------|---------------------|

d) Abbi buys 2kg of peaches for \$14.70. What is the price per kg?

| | | |
|-----------------|-----------------------------|---------------------|
| Number Sentence | Halving Strategy and Answer | Does it make sense? |
|-----------------|-----------------------------|---------------------|

e) A baker bakes 56 bread rolls in columns of 4. How many rows of bread rolls will there be?

| | | |
|-----------------|-----------------------------|---------------------|
| Number Sentence | Halving Strategy and Answer | Does it make sense? |
|-----------------|-----------------------------|---------------------|

f) Sophie swims 400 metres. If she has swum 8 laps, what is the length of one lap?

| | | |
|-----------------|-----------------------------|---------------------|
| Number Sentence | Halving Strategy and Answer | Does it make sense? |
|-----------------|-----------------------------|---------------------|

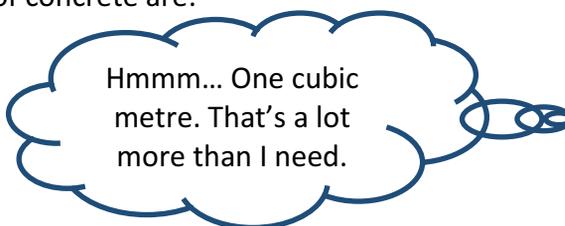
4. Tristan is mixing up concrete to fix a small section of his driveway. The components for producing one cubic metre of concrete are:

CONCRETE MIX

320 kg cement

640 kg sand

960 kg aggregate



a) Sally, Tristan's neighbour, needs half a cubic metre of concrete. How much of each part of the concrete mix does she need?

b) Tristan thinks he needs $\frac{1}{8}$ of the above formula. How much of each part of the concrete mix does he need?

Whole Class Activity 5

Using Patterning and Place Value for $\div 10$

Think: How can we use place value patterns to solve division problems?

Tom was driving a semi trailer truck that could carry 20t (20 000kg) of cement. Cement is packed in either bags of 100kg or crates of 1 000kg.

- If the truck was packed with all bags, how many bags could it carry?
- If the truck was packed with all crates, how many crates could it carry?

Tom knows these are division problems because he knows the TOTAL weight (20 000) and the NUMBER IN EACH GROUP (100 or 1000). He needs to find the NUMBER OF GROUPS.

He is solving:

- $20\ 000 \div 100 = ?$
- $20\ 000 \div 1\ 000 = ?$

Tom decided to use arrays and patterning to help with his thinking:

He firstly used arrays to think through the problems.

Using 1mm grid paper draw arrays to show:

1. $20 \div 10$ (circle the number of groups of 10 in 20)
2. $200 \div 10$ (circle the number of groups of 10 in 200)
3. $2\ 000 \div 10$ (circle the number of groups of 10 in 2000)

Next to each array, write the equivalent multiplication number sentence and the answer to the problem.

Tom used the answers from the arrays and patterning to think through the problems:

1. $20 \div 10 = 2$; because $2 \times 10 = 20$
2. $200 \div 10 = 20$; because $20 \times 10 = 200$
3. $2\ 000 \div 10 = 200$; because $200 \times 10 = 2\ 000$

Therefore: 4. $20\ 000 \div 10 = 2\ 000$; because $2\ 000 \times 10 = 20\ 000$.

Tom realized he could use the same patterning for $20\ 000 \div 100 = ?$

1. $200 \div 100 = 2$; because $2 \times 100 = 200$
2. $2\ 000 \div 100 = 20$; because $20 \times 100 = 2\ 000$
3. $20\ 000 \div 100 = 200$; because $200 \times 100 = 20\ 000$

How could Tom solve $20\ 000 \div 1\ 000$ using similar patterning? Discuss with your classmates and write the patterning in the space below:



Complete the Place Value Chart below using your answers from the arrays and patterns. Check your answers with a calculator.

| EXAMPLE | THOUSANDS | | | ONES | | |
|------------------------|-----------|------|------|----------|------|------|
| | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| $20\ 000 \div 1$ | | 2 | 0 | 0 | 0 | 0 |
| $20\ 000 \div 10$ | | | 2 | 0 | 0 | 0 |
| $20\ 000 \div 100$ | | | | | | |
| $20\ 000 \div 1\ 000$ | | | | | | |
| $20\ 000 \div 10\ 000$ | | | | | | |

What is happening to the place value, and hence the size of the 2 in 20 000, as it is divided by 1, then 10, then 100 etc.?



How many bags could Tom's truck carry? Write your solution as a sentence using the words in the question.



How many crates could Tom's truck carry? Write your solution as a sentence using the words in the question.



Predict a solution for $20\ 000 \div 10\ 000$.



Practice Exercise 5

1. Solve the following problems using the Place Value Chart below.

| EXAMPLE | THOUSANDS | | | ONES | | |
|-----------------------|-----------|------|------|----------|------|------|
| | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| $620 \div 10$ | | | | | | |
| $91\ 000 \div 100$ | | | | | | |
| $780\ 000 \div 1000$ | | | | | | |
| $456\ 000 \div 10$ | | | | | | |
| $9\ 820\ 000 \div 10$ | | | | | | |
| $871\ 000 \div 100$ | | | | | | |

2. Visualize the Place Value Chart to solve the following problems.

- | | |
|---------------------------|----------------------------|
| a) $80\ 000 \div 10$ | d) $530\ 000 \div 10\ 000$ |
| b) $6\ 500 \div 100$ | e) $7\ 000\ 000 \div 100$ |
| c) $700\ 000 \div 1\ 000$ | f) $890\ 000 \div 1\ 000$ |

3. Write the two hardest dividing by powers of 10 problems you can mentally calculate, in the space below. Test your partner with your problems.

Whole Class Activity 6

Tom decided he could divide money amounts in the same way.

The prize pool for Wednesday Lotto was \$635 891.27 for Division 1 and \$4953 for Division 2.

- 100 workers from a mine in Collie shared the Division 1 prize. How much would each miner receive?
- 10 teachers from St Mary’s school shared the Division 2 prize. How much would each teacher receive?

Write each of the problems as a number sentence in the space below

-
-

Place the Division 1 and 2 Prizes in the Place Value Chart. Use the chart, and your knowledge of dividing by 10 from above to calculate the prize winnings for:

- (i) 1 person (i.e. PRIZE ÷ 1)
- (ii) 10 people (i.e. PRIZE ÷ 10)
- (iii) 100 people (i.e. PRIZE ÷ 100)

| | THOUSANDS of dollars | | | ONES of dollars | | | Cents | |
|-------------------------|----------------------|------|------|-----------------|------|------|--------|------------|
| | Hundreds | Tens | Ones | Hundreds | Tens | Ones | Tenths | Hundredths |
| Division 1 Prize | | | | | | | | |
| Shared between 1 | | | | | | | | |
| Shared between 10 | | | | | | | | |
| Shared between 100 | | | | | | | | |
| Division 2 Prize | | | | | | | | |
| Shared between 1 | | | | | | | | |
| Shared between 10 | | | | | | | | |
| Shared between 100 | | | | | | | | |

For the Division 1 amount:

What is happening to the place value of the 8 in \$635 891.27 as it is divided by 1, 10 and then 100?



What is happening to the place value of the 1 in \$635 891.27 as it is divided by 1, 10 and then 100?



What is happening to the place value of the 2 in \$635 891.27 as it is divided by 1, 10 and then 100?



How much should each Collie miner receive? Write your solution as a sentence using the words in the question. Discuss with a partner.



For the Division 2 amount:

What is happening to the place value of the 9 in \$4 953 as it is divided by 1, 10 and then 100?



What is happening to the place value of the 5 in \$4 953 as it is divided by 1, 10 and then 100?



What is happening to the place value of the 3 in \$4 953 as it is divided by 10 and then 100?



How much should each St Mary's teacher receive? Write your solution as a sentence using the words in the question. Discuss with a partner.



Predict the prize winnings if:

- (i) 1 000 people shared in Division 1
- (ii) 1 000 people shared in Division 2

Reflection and Discussion

Sometimes it is easiest to solve a division problem by turning it into a multiplication problem and extending our basic fact families.

For example, when solving a problem like $\$160 \div \$16 = ?$, we can think of the problem as how many 16's in 160 ($? \times 16 = 160$). It is then easy to see that the answer is 10.

Solve the following by thinking of the problem as multiplication

- | | |
|--------------------|-------------------|
| a) $250 \div 25$ | d) $\$28 \div 14$ |
| b) $1\ 500 \div 3$ | e) $48 \div 4$ |
| c) $\$55 \div \5 | f) $63 \div 3$ |

Write the two hardest division problems you can mentally calculate, that are best solved by changing the division into a multiplication, in the space below. Test your partner with your problems.



b) Maureen measured a bookcase as being 930mm high. She drew a scale drawing of the bookcase 10 times smaller. How tall was the scale drawing?

| | | |
|--------------------------|--------|---------------------|
| Division Number Sentence | Answer | Does it make sense? |
|--------------------------|--------|---------------------|

c) Lara treks 290km, averaging 10km/day. How many days is her journey?

| | | |
|--------------------------|--------|---------------------|
| Division Number Sentence | Answer | Does it make sense? |
|--------------------------|--------|---------------------|

d) A rectangle that is 1 000 mm wide has an area of 750 000 square millimetres. What is its length?

| | | |
|--------------------------|--------|---------------------|
| Division Number Sentence | Answer | Does it make sense? |
|--------------------------|--------|---------------------|

e) A truck can carry 35 000kg of soil. It delivers 100kg to different building sites until there is no more sand on the truck. How many deliveries does it make?

| | | |
|--------------------------|--------|---------------------|
| Division Number Sentence | Answer | Does it make sense? |
|--------------------------|--------|---------------------|

f) A beautician paints 190 fingernails in a day. How many clients does she manicure in a day?

| | | |
|--------------------------|--------|---------------------|
| Division Number Sentence | Answer | Does it make sense? |
|--------------------------|--------|---------------------|

Whole Class Activity 7

Katie's Strategy for Solving Harder Division Problems

Think: How can we partition numbers to help solve harder division problems?

Katie knows from her work in Multiplication Strategies that partitioning numbers into parts using addition can help make calculations easier.

View the "The Divider" online using the following URL, to answer the 'Think' question above.

- <http://www.scottle.edu.au/ec/viewing/L2008/index.html>

Record your response to the 'Think' question in the space below:



Use the ideas from the above website and 1cm grid paper to solve $42 \div 3$

Draw an array with 42 squares. Cut the array into two pieces that would help make the division easier.

Glue the cut pieces in the space below.

Write the division number sentence formed by each of the cut pieces.

What is the answer to $42 \div 3$?

Label your diagram so that the answer can be easily seen in the two parts of your array.

Share your thinking with your partner. Did you cut up the array in the same way? What other strategies were used in the class?



Decide as a class, what is the best strategy for this particular problem?



Using Standard Place Value Partitioning for Division

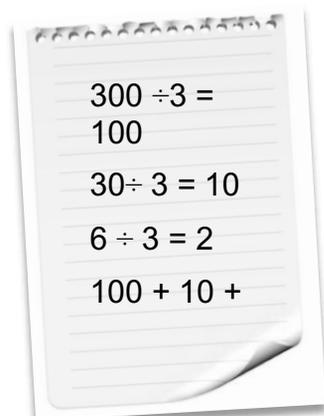
Katie works in a hospital cafeteria. She uses 3 slices of ham in every sandwich she makes. How many sandwiches can Katie make if she has 336 slices of ham?

We know this is a division problem because we know the TOTAL slices of ham (336) and the NUMBER IN EACH GROUP (3). We need to find the NUMBER OF GROUPS.

i.e. TOTAL \div NUMBER IN EACH GROUP = NUMBER OF GROUPS

We can solve this $336 \div 3 = ?$ problem by standard partitioning the 336 into $300 + 30 + 6$.

We can then divide the parts separately and use jottings to keep track of the parts.



Using 336 slices of ham with 3 slices per sandwich, Katie can make 112 sandwiches.

Using Non Standard Partitioning for Division

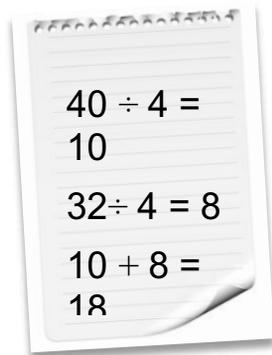
In Katie's job at the hospital cafeteria, she has to place 4 bundles of serviettes on each of the tables. If she has 72 bundles, how many tables can Katie put serviettes on?

We know this is a division problem because we know the TOTAL number of serviettes (72) and the NUMBER IN EACH GROUP (4). We need to find the NUMBER OF GROUPS.

i.e. $\text{TOTAL} \div \text{NUMBER IN EACH GROUP} = \text{NUMBER OF GROUPS}$

We can solve this $72 \div 4 = ?$ problem by non-standard partitioning the 72 into $40 + 32$.

We can then divide the parts separately and use jottings to keep track of the parts.



With 4 bundles of serviettes on each table, Katie can distribute the 72 bundles onto 18 tables.

Work with a partner to find three different partitions of 124 that would make it easier to divide by 4. Calculate the answer to the problem $124 \div 4$ using each of the different partitions.



Work with a partner to find three different partitions of 378 that would make it easier to divide by 3. Calculate the answer to the problem $378 \div 3$ using each of the different partitions.



Practice Exercise 7

1. Work with a partner using partitioning to solve the following problems. Use jottings to support your thinking, such as those shown when solving $72 \div 4$.

| | | |
|--------------|--------------|--------------|
| $135 \div 5$ | $78 \div 6$ | $45 \div 3$ |
| $147 \div 7$ | $918 \div 9$ | $215 \div 5$ |
| $132 \div 6$ | $348 \div 3$ | $161 \div 7$ |

2. Write the two hardest division problems you can calculate using partitioning, in the space below. Test your partner with your problems.

Whole Class Activity 8

Katie also used partitioning for money problems involving whole dollar amounts. For example, sharing \$780 between 6 people:

Break \$780 into \$600 + \$180; $600 \div 6 = 100$, $180 \div 6 = 30$; therefore, $\$780 \div 6 = 130$.

Katie wondered how partitioning would work when dividing dollars and cents?

Work with a partner to calculate the cost of one item in the table below, using the partitioning method of division. Discuss your solutions and methods with the class.

| Item | Jottings | Price Per Item |
|--|----------|----------------|
|  3 for \$9.45 | | |
|  5 for \$9.50 | | |
|  6 for \$25.20 | | |
|  3 for \$7.32 | | |

Practice Exercise 8

1. Puzzle

'What goes "Ha Ha Thump"?

Complete these questions to decode the answer to this joke. The letter of the question goes above the answer found in each box below.

L: $48 \div 3$

G: $459 \div 9$

A: $56 \div 4$

S: $639 \div 3$

F: $128 \div 4$

D: $336 \div 6$

I: $84 \div 7$

M: $1\,250 \div 5$

N: $147 \div 7$

E: $170 \div 5$

U: $117 \div 9$

H: $330 \div 15$

N: $424 \div 8$

I: $132 \div 4$

H: $672 \div 6$

O: $264 \div 24$

| | | | | | | | | | | |
|----|-----|----|-----|----|----|----|----|-----|----|----|
| 14 | 250 | 14 | 53 | 16 | 14 | 13 | 51 | 112 | 33 | 21 |
| 51 | 112 | 12 | 213 | 22 | 34 | 14 | 56 | 11 | 32 | 32 |

2. For the following everyday problems:

- Write the situation as a division number sentence (e.g. $426 \div 6 = ?$).
- Find the answer by using partitioning.
- Check your solution to see if it is reasonable. Tick the box when you have checked.

a) How many weeks is 154 days?

| Number Sentence | Partitioning Method | Answer | Does it make sense? |
|-----------------|---------------------|--------|---------------------|
| | | | |

b) 216 tins of tomatoes are stacked in rows of 9. How many columns of tins will there be?

| Number Sentence | Partitioning Method | Answer | Does it make sense? |
|-----------------|---------------------|--------|---------------------|
| | | | |

c) How many hours in 6 120 minutes?

| Number Sentence | Partitioning Method | Answer | Does it make sense? |
|-----------------|---------------------|--------|---------------------|
| | | | |

d) Calculate the side length of a regular hexagon if its perimeter is 900cm.

| | | | |
|-----------------|---------------------|--------|---------------------|
| Number Sentence | Partitioning Method | Answer | Does it make sense? |
|-----------------|---------------------|--------|---------------------|

e) Claire buys 7kg of flour for \$10.50. How much is the flour per kilogram?

| | | | |
|-----------------|---------------------|--------|---------------------|
| Number Sentence | Partitioning Method | Answer | Does it make sense? |
|-----------------|---------------------|--------|---------------------|

f) Charlie's grandfather shares out \$1610 between 7 grandchildren. How much does each grandchild receive?

| | | | |
|-----------------|---------------------|--------|---------------------|
| Number Sentence | Partitioning Method | Answer | Does it make sense? |
|-----------------|---------------------|--------|---------------------|

3. Jasmine sells cleaning products via telemarketing for 5 hours each weekday. The following chart shows her sales for each day in the week beginning January 19th 2015.



Calculate her sales per hour on each of the 5 days:

Monday

Tuesday

Wednesday

Thursday

Friday

Revision: The best strategy?

In this unit on 'Division of Whole Numbers' we have looked at 2 main strategies:

- Tom's strategy of using and extending the basic division fact strategies
- Katie's strategy of using partitioning

Circle the most suitable strategy. Mentally solve the following problems using jottings to help keep track.

| | Whose strategy? | | Whose strategy? |
|---------------------|-----------------|------------------------|-----------------|
| a) $600 \div 200$ | Tom Katie | b) $78 \div 6$ | Tom Katie |
| c) $48 \div 2$ | Tom Katie | d) $5\,600 \div 10$ | Tom Katie |
| e) $39 \div 13$ | Tom Katie | f) $\$20 \div 8$ | Tom Katie |
| g) $\$245 \div \5 | Tom Katie | h) $759\,000 \div 100$ | Tom Katie |
| i) $\$5.80 \div 10$ | Tom Katie | j) $\$12.80 \div 4$ | Tom Katie |

OLNA Practice Questions

1. Consider the following number sentences:

$$1\,350 \div 10 = 135 \times 100$$

$$220 \div 4 = 275 \div 5$$

$$126 \div 3 = 294 \div 7$$

If these problems were to be marked True (T) or False (F) in order, the answer would be:

- A. F, T, F B. T, T, F C. F, T, T D. F, F, T

2. The area of this  rectangle is 102 square centimetres.

6cm

(Not to scale)

What is the length of the rectangle? _____ cm

Topic 5

Solving Division Problems with Remainders

Mathematics Discussion

In the previous topics we solved division problems with numbers that could be shared equally. Sometimes, however, when dividing an amount into equal parts or groups, there is something left over. This is called the *remainder*. When this happens we need to think about the answer in relation to the problem and decide whether to:

- Round up to the nearest whole number
- Round down to the nearest whole number
- Leave the answer as is, with a remainder.
- Continue to break up the remainder and distribute fractional amounts.

The context of the problem will help indicate which of these is the most sensible. For example; if 11 people were going to the movies in cars that can seat 5 people, how many cars would be needed? $11 \div 5$ is 2 with a remainder of 1. In this case it is better to round up. Three cars would be needed, not 2 cars, unless 1 person decided they didn't want to go to the movies!

Whole Class Activity 1

Round up or down or leave it the same?

Think: What do we do when the answer to a division problem does not come out evenly?

How can 17 lollies be shared between 3 friends equally?

We can think of this as how many 3's in 17 and use an array to calculate the answer.



We can write the answer as $17 \div 3 = \underline{\quad} \text{ r } \underline{\quad}$

In this problem, it is impossible to share the 2 leftover lollies with the 3 friends.

Therefore, our answer stays the same:

Each friend would get 5 lollies and there would be 2 left over.

Choosing a Strategy

In the previous sections we used a range of strategies to solve division problems. Some of these strategies are difficult to use when there is a remainder.

Cross out the strategies, listed below, that are difficult to use when solving a problem such as 'Share a crate of 53 apples between 4 families.'

changing the division into a multiplication

arrays

skip counting

halving

recall of basic facts

extensions to basic facts

place value patterns

place value partitioning

Use a highlighter to highlight the strategies that work best when solving a division problem where there is a remainder? Solve $53 \div 4$ using each of these strategies.



List the best strategies below and use them in the rest of this section.



Choose a strategy to solve the following problems and decide what to do with the remainder.

| Problem | Problem as a Number Sentence | Strategy. (Use this space to work it out) | Answer in Context of the Question |
|---|------------------------------|--|---|
| How many cars (5 seaters) are needed to drive 22 people to a Concert? | | | __ cars are needed for 22 people to drive to the concert. |
| Share 14 large cakes between 3 people. | | | |
| How many separate squares can be made from 30 matches? | | | |
| 47 cows need to be divided between 3 paddocks. How many in each? | | | |
| Split \$17 between 2 people | | | |

How was the remainder treated differently in each problem? Discuss.



Circle the problems in blue where the answer involves rounding up to the nearest whole number.

Circle the problems in red where the answer involves rounding down to the nearest whole number.

Circle the word problems in black where the answer is given exactly.

Practice Exercise 1

1. Solve the following division problems. Write the remainders as whole numbers (e.g. 7r3)

a) $11 \div 2 = ?$

d) $38 \div 5 = ?$

b) $23 \div 4 = ?$

e) $17 \div 7 = ?$

c) $14 \div 3 = ?$

f) $42 \div 8 = ?$

2. Solve the following word problems using mental calculations supported by written jottings. Decide what to do with the remainder: round up, round down or leave it as it is.

a) Dale baked 39 cupcakes for the school fete. How many boxes will Dale need if he packs 6 cupcakes in each box?

| | | |
|-----------------|----------|------------------------------------|
| Number Sentence | Solution | What do you do with the remainder? |
|-----------------|----------|------------------------------------|

b) Four friends split their restaurant bill of \$37. How much would each person pay?

| | | |
|-----------------|----------|------------------------------------|
| Number Sentence | Solution | What do you do with the remainder? |
|-----------------|----------|------------------------------------|

c) Each school mini bus carries 8 students. How many buses are needed for 67 students?

| | | |
|-----------------|----------|------------------------------------|
| Number Sentence | Solution | What do you do with the remainder? |
|-----------------|----------|------------------------------------|

d) What is the maximum number of \$5 notes in \$47?

| | | |
|-----------------|----------|------------------------------------|
| Number Sentence | Solution | What do you do with the remainder? |
|-----------------|----------|------------------------------------|

e) Share 8 balls of play dough between 3 children. How much will each child receive?

| | | |
|-----------------|----------|------------------------------------|
| Number Sentence | Solution | What do you do with the remainder? |
|-----------------|----------|------------------------------------|

f) How many separate triangles can be formed from 20 popsticks?

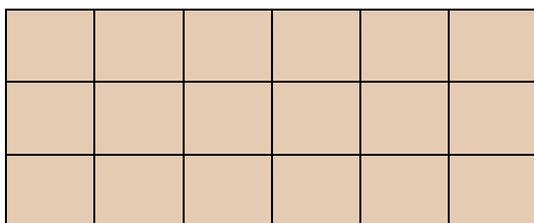
| | | |
|-----------------|----------|------------------------------------|
| Number Sentence | Solution | What do you do with the remainder? |
|-----------------|----------|------------------------------------|

Whole Class Activity 2

Remainders as fractions.

Think: How do we write the answers when we distribute all of the remainder and there is nothing left over?

How can we share a chocolate bar that has 18 squares between 4 friends equally?



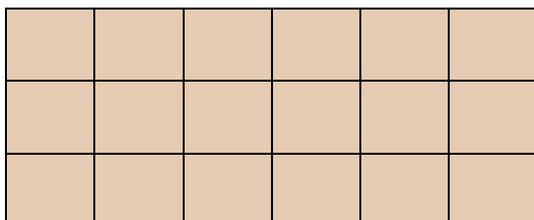
On the array above, shade each friend's share using a different colour for each friend.

Write the answer. $18 \div 4 = \underline{\quad} \text{ r } \underline{\quad}$

In this case, the remainder can be cut up and shared between the four friends. Show each friend's share of the remainder using their colour.

How much chocolate does each person get? Write the answer with the remainders as a fraction.
 $18 \div 4 =$

Another friend arrives so they decide to share the 18 squares between 5 people. How much do they each get?



On the array above, shade each friend's share of chocolate using a different colour for each friend.

Write the answer. $18 \div 5 = \underline{\quad} \text{ r } \underline{\quad}$

Divide *each* remaining piece into 5. Shade each of the fifths a different colour to show each person's share.

How much chocolate does each person get?

Write the answer with the remainders as a fraction. $18 \div 5 = \underline{\hspace{2cm}}$

How could you write the answers to the following with the remainders written as fractions?
Discuss with a partner.

$$23 \div 5$$

$$16 \div 7$$

$$14 \div 3$$



Practice Exercise 2

1. Solve the following division problems using mental calculations supported by written jottings. Write the answer with a remainder then change this into a fraction (e.g. $7\frac{3}{4}$).

a) $41 \div 4 = ?$

d) $19 \div 2 = ?$

b) $35 \div 10 = ?$

e) $47 \div 5 = ?$

c) $10 \div 3 = ?$

f) $31 \div 4 = ?$

2. Solve the following word problems using mental calculations supported by written jottings. Write the answer with a remainder, then change this into a fraction.

a) Farmer Bob plants 28 basil seedlings in rows of 8. How many rows did he plant?

| Number Sentence | Solution with remainder | Solution with fraction |
|-----------------|-------------------------|------------------------|
| | | |

b) A rectangle of 32 square centimetres has a width of 5 centimetres. What is its length?

| Number Sentence | Solution with remainder | Solution with fraction |
|-----------------|-------------------------|------------------------|
| | | |

c) A 38cm string is cut into 8 pieces. What is the length of each piece?

| Number Sentence | Solution with remainder | Solution with fraction |
|-----------------|-------------------------|------------------------|
| | | |

d) 26 lamingtons are shared between 4 people. How many does each person receive?

| Number Sentence | Solution with remainder | Solution with fraction |
|-----------------|-------------------------|------------------------|
| | | |

e) Judy draws a picture of a plant ten times smaller than the original, which is 65cm tall. How tall is the drawing?

| Number Sentence | Solution with remainder | Solution with fraction |
|-----------------|-------------------------|------------------------|
| | | |

f) Peter travelled 22km in 8 minutes on his motorcycle. How many kilometres did he travel in one minute?

| | | |
|-----------------|-------------------------|------------------------|
| Number Sentence | Solution with remainder | Solution with fraction |
|-----------------|-------------------------|------------------------|

Whole Class Activity 3

Using the calculator to solve division problems with remainders

Think: How can a restaurant bill of \$187.88 be split between 4 friends?

We can think of this division problem as $\$187.88 \div 4 = ?$

The numbers in this problem are 'difficult' and the problem is hard to solve using a mental strategy. In this case, it is best to use a calculator.

We can enter:



Calculate the answer to $187.88 \div 4 = \underline{\hspace{2cm}}$

The numbers after the decimal point can be considered as 'remainders'. Interpret the remainder in context of the problem. How much should each friend contribute to the restaurant bill?



Solve the following division word problems using your calculator. Look at the remainder and decide whether to round up, round down, leave it as a remainder or continue to distribute it so that there is a fractional amount.

| | | | |
|------------------------------------|---|--|-----------------------------------|
| | a) A playdough recipe uses 3 125 grams of salt. If you wish to make a recipe 4 times smaller than this, how much salt would you need? | b) At a school camp, dormitories could sleep 12. If there were 278 students, how many dormitories would they need? | c) Share \$423 between 18 people. |
| What do you do with the remainder? | | | |

How was the remainder treated differently in each of the word problems? Why? Discuss.



Practice Exercise 3

1. Solve the following word problems using division on the calculator and the context of the problem. Decide what you should do with the remainder: round up, round down, leave it as it is or continue to distribute it so that there is a fractional amount.

a) How many committees of 24 can be made from 17 789 delegates at an AIDS World conference?

| | | |
|-----------------|---------------------|------------------------------------|
| Number Sentence | Calculator solution | What do you do with the remainder? |
|-----------------|---------------------|------------------------------------|

b) A lotto syndicate of 5 players won \$19 921.11. How much should each player receive?

| | | |
|-----------------|---------------------|------------------------------------|
| Number Sentence | Calculator solution | What do you do with the remainder? |
|-----------------|---------------------|------------------------------------|

c) 7 645 eggs were packed into dozen cartons. How many dozen eggs were packed?

| | | |
|-----------------|---------------------|------------------------------------|
| Number Sentence | Calculator solution | What do you do with the remainder? |
|-----------------|---------------------|------------------------------------|

d) Gina was raising money for *The 40 Hour Famine*. She hoped to collect \$2 500 from 42 relatives and friends. How much was she hoping each friend or relative would donate?

| | | |
|-----------------|---------------------|------------------------------------|
| Number Sentence | Calculator solution | What do you do with the remainder? |
|-----------------|---------------------|------------------------------------|

e) 4 572 bricks are to be piled in groups of 8. How many piles of bricks will there be? How many bricks will be in the last pile?

| | | |
|-----------------|---------------------|------------------------------------|
| Number Sentence | Calculator solution | What do you do with the remainder? |
|-----------------|---------------------|------------------------------------|

f) The school is planning an excursion. There are 1 232 students and each school bus seats 36 students. How many buses will be needed for the excursion?

| | | |
|-----------------|---------------------|------------------------------------|
| Number Sentence | Calculator solution | What do you do with the remainder? |
|-----------------|---------------------|------------------------------------|

Revision Questions

1. Complete the following table. Solve the problem mentally, using written jottings. Write any remainders as fractions. The first one has been done for you.

| | $\div 2$ | $\div 6$ | $\div 9$ | $\div 3$ | $\div 10$ | $\div 5$ | $\div 4$ |
|----|-----------------|----------|----------|----------|-----------|----------|----------|
| 23 | $11\frac{1}{2}$ | | | | | | |
| 35 | | | | | | | |
| 41 | | | | | | | |
| 68 | | | | | | | |

2. A Game to Assist in Practising Division Problems with Remainders

'Four in a Row'

Aim

To cross out 4 'remainders' in a horizontal, vertical or diagonal row on the grid shown below after calculating a series of division problems.

Equipment

- One normal dice
- Scrap paper for jottings
- Counters for each player to place over the remainders on the grid.

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 0 | 1 | 0 |
| 2 | 2 | 0 | 3 | 5 |
| 1 | 2 | 3 | 4 | 0 |
| 3 | 4 | 4 | 0 | 5 |
| 5 | 4 | 0 | 5 | 3 |

Rules of the game

1. Player 1 states a number between 30 and 80 to the other players.
2. Player 1 rolls the dice and divides their stated number by the number on the dice and calculates the remainder either mentally or using jottings on scrap paper. The player then places one of their counters over the remainder on their own grid in their own book.
3. Step (b) is repeated with the other players.
4. If a player cannot cover a number, they miss a turn.
5. The winner is the first player to have 4 counters in a row, across, down or diagonal.

Variations

Players state a number between 200 and 500 and use a 10-sided dice for division with a grid produced on scrap paper showing 25 numbers between 0 and 9 (some repeated)

3. Complete the table below. Choose between mental or calculator. Decide what to do with the remainder. The first example has been completed for you.

| Problem | | Number Sentence, Strategy and Solution | Interpret Answer in Context of Problem |
|--|--|---|--|
| A zookeeper divides 19 Sumatran Tigers between 3 enclosures. How many tigers would be in each enclosure? |   | $19 \div 3 = ?$ is the same as $3 \times ? = 19$ $3 \times 6 = 18$ $19 \div 3 = 6r1$ | <i>There would be 2 enclosures with 6 tigers in and one with 7 tigers.</i> |
| 11 bowls of rice were shared between 4 restaurant customers at 'Moon Restaurant' in Hong Kong. How many bowls did each customer receive? |   | | |
| James purchases 4 tickets to the Singapore Zoo, costing \$131. What is the price per ticket? |   | | |
| A Vietnamese rice grower plants rice plants in rows of 200. If there are 17 253 rice plants, how many rows will he plant? |   | | |
| Singapore Orchids were wrapped in bunches of 5. If there were 87 orchids, how many bunches would there be? |   | | |
| The height of an Indian Elephant is 321cm. If a sculptor wishes to produce a brass model 14 times smaller, how tall will the model be? |   | | |

Reflection on Learning

1. Complete the following activities online using the following URL to answer the question:

'How do YOU solve division problems that have remainders?'

- <http://www.scootle.edu.au/ec/viewing/L2809/index.html>
- <http://www.scootle.edu.au/ec/viewing/L2008/index.html>
- <http://www.scootle.edu.au/ec/viewing/L2055/index.html>

Record your response to the focus question in the space below:



2. Write division word problems using $67 \div 5$ where the answer is best given:

- With the remainder as a whole number
- With the remainder as a fraction
- With the remainder being rounded up so the answer is a whole number
- With the remainder being rounded down so the answer is a whole number

Swap your questions with a partner. Solve their problems. Discuss their solutions in context of their word problems.

OLNA Practice Questions

1. Maddy is packing 75 cupcakes into tins that hold 6 cupcakes for a School Fete.

What is the smallest number of tins Maddy needs to pack the cakes?

2. What is the division number sentence that matches this array?



A. $21 \div 5 = 4\frac{1}{4}$

B. $21 \div 4 = 5\frac{1}{4}$

C. $21 \div 4 = 6r1$

D. $21 \div 5 = 5\frac{1}{5}$

Topic 6

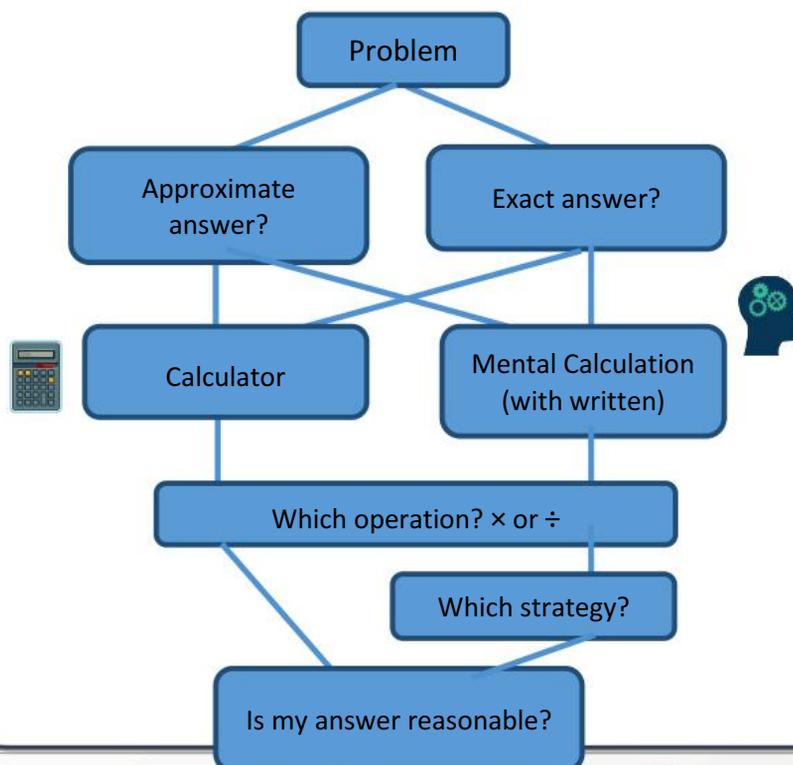
Choosing between mental or a calculator to solve multiplication and division problems

Mathematics Discussion

When solving multiplication and division problems, we make many decisions. Firstly, we decide whether an exact answer is needed or whether an approximate answer will be good enough. Next we decide whether to solve the problem mentally, (using written jottings to keep track) or to use a calculator.

As described in 'Solving Division Problems with Remainders', we tend to solve a problem mentally or with written jottings if the numbers are not too difficult, and use a calculator for all other problems. After solving the problem, we need to decide whether our solution makes sense.

In this section we are dealing with problems that require exact answers. We will be focussing on making the choice between a calculator and a mental calculation.



Whole Class Activity 1

Working in pairs:

STUDENT A is to complete Set A problems mentally.

STUDENT B is to complete Set A problems by using a calculator.

Both start working at exactly the same time.

The student who finishes first is to call out “Finished!”

Repeat with Set B, but this time STUDENT A uses a calculator and STUDENT B uses mental.

| Set A | Solution |
|-------------------|----------|
| 14×100 | |
| 15×2 | |
| $1600 \div 2$ | |
| $8\ 000 \div 8$ | |
| 7×21 | |
| $37\ 000 \div 10$ | |
| $5 \times \$11$ | |
| $11 \div 5$ | |

| Set B | Solution |
|--------------------|----------|
| $180\ 000 \div 10$ | |
| 14×2 | |
| $8\ 000 \div 4$ | |
| 23×100 | |
| 3×23 | |
| $50\ 000 \div 2$ | |
| $\$1.20 \times 3$ | |
| $7 \div 2$ | |

Who finished first in each Set? What did you notice?



Whole Class Activity 2

Complete the following, decide whether to solve the problem using a calculator or mental methods (with written jottings if needed). The first has been completed for you.

| Problem | | | Explanation of Choice and Solution |
|---------------------|--|---|--|
| $\$13 \times 5$ | | ✓ | Thinking of place value partitioning can solve the problem. The basic facts involved are easy to use. 5×10 add 5×3 $= 50 + 15$ $= 65$ |
| | | | |
| $\$56.78 \times 24$ | | | |
| | | | |

| | | | |
|-------------|---|--|--|
| 35 ÷ 3 |  | | |
| |  | | |
| 3 × 415 |  | | |
| |  | | |
| \$18.74 ÷ 5 |  | | |
| |  | | |
| 132 ÷ 6 |  | | |
| |  | | |

What types of problems are best solved using a calculator?

What types of problems are best solved using mental strategies with jottings?

Discuss.



Choosing Which Operation

After deciding that a problem is best solved using either mental strategies or a calculator, the next choice is 'Which Operation, × or ÷' to use.

Our work on multiplication and division showed us:

NUMBER OF GROUPS × NUMBER IN EACH GROUP = TOTAL

NUMBER IN EACH GROUP × NUMBER OF GROUPS = TOTAL

TOTAL ÷ NUMBER OF GROUPS = NUMBER IN EACH GROUP

TOTAL ÷ NUMBER IN EACH GROUP = NUMBER OF GROUPS

Therefore:

- If the TOTAL is missing, we use MULTIPLICATION
- If the TOTAL is present, we use DIVISION

It is obvious which operation to use in problems such as:

a) $\$137.24 \times 54 = ?$ b) $\$18\,700.74 \div 14 = ?$

And more complex when solving problems such as:

c) $? \div 5 = 4\,500$ d) $? \times 315 = 53\,500$ e) $368 \div ? = 4$ f) $12 \times ? = 132$

When solving these more complex problems, it helps to remember our multiplication and division statements from above.

g) $? \div 5 = 4\,500$

The TOTAL is missing, so we use MULTIPLICATION.

$$? = 5 \times 4\,500$$

h) $? \times 315 = 53\,500$

The TOTAL, 53 500, is present, so we use DIVISION.

$$53\,500 \div 315 = ?$$

i) $368 \div ? = 4$

The TOTAL, 368, is present, so we use DIVISION.

$$368 \div 4 = ?$$

j) $12 \times ? = 132$

The TOTAL, 132, is present, so we use DIVISION.

$$132 \div 12 = ?$$

Whole Class Activity 3

Choose whether to use multiplication or division to solve the following problems on a calculator, discussing your choices as you go.

- Write a number sentence to enter into the calculator.
- Use the calculator to find the answer.

a) $1\,436 \div ? = 4$

What would you enter into a calculator?

Answer

b) $? \div 7 = 21\,350$

What would you enter into a calculator?

Answer

c) $4 \times ? = 806$

What would you enter into a calculator?

Answer

d) $? \times 5 = 3\,986$

What would you enter into a calculator?

Answer

Whole Class Activity 4

The following table guides the decision making process as outlined by the flow diagram on page 197.

After completing each calculation check to see if your answer is reasonable. This is particularly important when using a calculator, as it is easy to push the wrong button.

Complete the table as a class. The first example has been completed for you.

| PROBLEM | | WHICH OPERATION ? | STRATEGY AND SOLUTION | Is your answer reasonable? Yes or No |
|---------------------|---|-------------------|--|--------------------------------------|
| $? \times 5 = 460$ |   | Division | $460 \div 5 = ?$ $460 = 450 + 10$ $450 \div 5 = 90$ $10 \div 5 = 2$ <i>Therefore, $460 \div 5 = 92$</i> | Yes |
| $89756 \div ? = 11$ |   | | | |
| $6 \times 531 = ?$ |   | | | |
| $? \div 4 = 55$ |   | | | |

| | | | | |
|-----------------------|--|--|--|--|
| $? \div 13 = 67981$ |   | | | |
| $\$78.29 \times 10 =$ |   | | | |
| $3 \times ? = 726$ |   | | | |

Reflection and Discussion

When using a calculator it is easy to make a mistake. We might input a number more than once, press the wrong number or symbol, or input numbers and symbols in the wrong order. For example:

$$72 \times 43 = 3\,096 \quad \text{could become} \quad 724 \times 3 = 2\,172$$

Thinking about the size of the number in the answer can help.

In the example above, we could think, 'Seventy times forty is two thousand eight hundred. So two thousand one hundred cannot possibly be right.'

Which of these could be the correct answer? Circle your choice.

52×24

$1\,248 \text{ or } 2\,088?$

25×42

$508 \text{ or } 1\,050?$

How do you know which is more likely to be correct?



Practice Exercise 1

1. Circle the problems that are best solved by calculator.

$153 \times ? = \$45\,786$

$? \div 4 = 55$

$80 \div ? = 40$

$? \div 3 = 21$

$? \times 10 = 2\,600$

$? \div 37 = 89\,754$

$\$100 \div ? = 5$

$12 \times ? = \$568.45$

$\$367 \div ? = \5.24

$? \times 2 = 184$

$3 \times ? = 60$

$? \div \$1.23 = \65.45

2. Use a calculator and choose between multiplication and division to solve the following problems by:

- Writing a number sentence to enter into the calculator.
- Using the calculator to find the answer.
- Checking to see if your answer is sensible.

a) $? \times 24\,578 = 187\,900$

What would you enter in the calculator?

Answer

Is this a sensible answer?

b) $? \div 48 = \$679.50$

What would you enter in the calculator?

Answer

Is this a sensible answer?

c) $\$21.20 \times ? = \347

What would you enter in the calculator?

Answer

Is this a sensible answer?

d) $\$8\,490 \div ? = \13.25

What would you enter in the calculator?

Answer

Is this a sensible answer?

e) $? = \$9\,803\,000 \div 77$

What would you enter in the calculator?

Answer

Is this a sensible answer?

3. Make decisions using the flow diagram on page 197 to complete the following table.

Decide:

- Calculator or mental?
- Multiplication or division?
- Which strategy?
- Is your answer reasonable?

| Problem | Decisions and Solution | Is your answer reasonable? Justify. |
|-------------------------|------------------------|-------------------------------------|
| $\$84 \div ? = \2 | | |
| $? = 6\ 803 \div 551$ | | |
| $? \times 8 = \$2\ 000$ | | |
| $? \div 5 = \$1.99$ | | |
| $456 \times ? = \$3479$ | | |
| $25 = ? \div 3$ | | |

4. Use the decision making process outlined in the flow diagram on page 197 to calculate the solutions to the following.

- | | |
|------------------------------|----------------------------------|
| a) $? \div \$3 = \42 | g) $1000 \times ? = \$89\ 000$ |
| b) $? \times 18 = \$25\ 678$ | h) $? = \$985\ 600 \div \67.50 |
| c) $\$800 \div \$5 = ?$ | i) $? \times 4 = \$18\ 000$ |
| d) $10 \times ? = \$47\ 800$ | j) $\$560 \div ? = \2 |
| e) $? \div 6 = \$3.99$ | k) $5 \times ? = \$12$ |
| f) $57\ 489 \div ? = 356$ | l) $? \div 35\ 689 = 23$ |

Reflection on Learning

Choose two problems from the list below; one for each of the following flow charts.

The two problems must follow the decision making process shown in yellow. They both require exact answers.

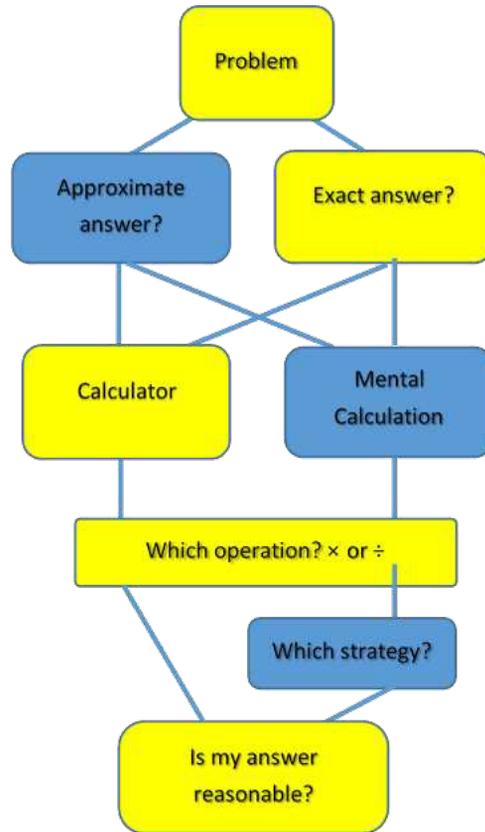
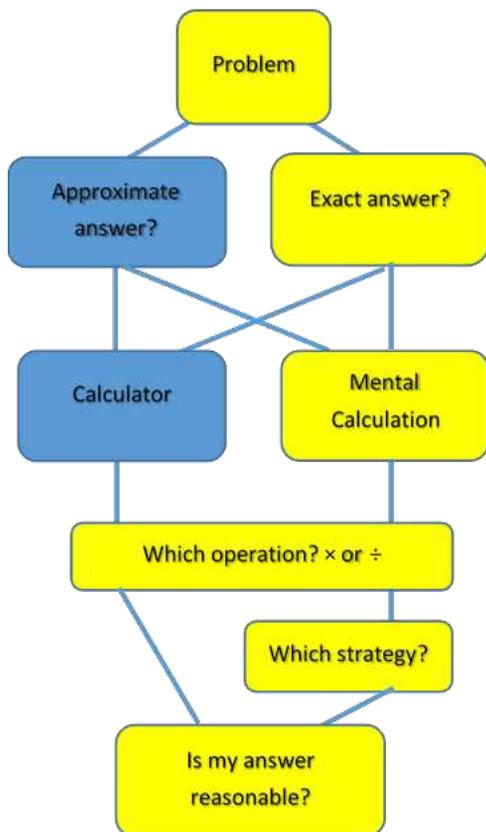
The first flow chart is to show a problem that is best solved mentally and must use multiplication to solve.

The second flow chart is to show a problem that is best solved using a calculator and must use division to solve.

- | | |
|----------------------------|-----------------------------|
| $? = \$325\ 600 \times 73$ | $? \div 43 = \$965$ |
| $4 \times ? = \$300$ | $456 \div ? = 3$ |
| $\$6.50 \div ? = 2$ | $? \div 7 = \$1.99$ |
| $? \times 26 = \$45\ 752$ | $100 \times ? = \$310\ 000$ |

Compare your flow charts with a partner. What do you notice?





OLNA Practice Question

1. What is the answer to $1540 \div ? = 5$?

A. 4700

B. 1535

C. 308

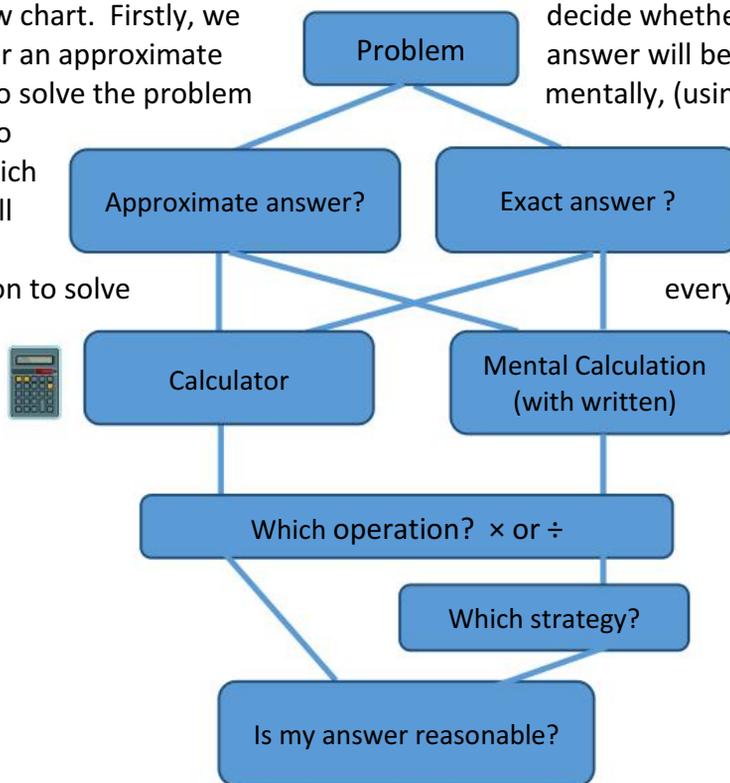
D. 1545

Topic 7

Choosing Multiplication or Division to Solve Everyday Problems

Mathematics Discussion

When solving problems, we make many decisions. The list of decisions is shown below in the flow chart. Firstly, we decide whether an approximate answer or whether an exact answer is needed. Next we decide whether to solve the problem mentally, (using written jottings to keep track) or to use a calculator. Next we decide which operation to use. This focus on choosing between operations of multiplication and division to solve everyday problems.



We can use our understanding of multiplication and division to help decide which of the following most closely matches the 'story' in the situation.

NUMBER OF GROUPS \times NUMBER IN EACH GROUP = **TOTAL**

NUMBER IN EACH GROUP \times NUMBER OF GROUPS = **TOTAL**

TOTAL \div NUMBER OF GROUPS = NUMBER IN EACH GROUP

TOTAL \div NUMBER IN EACH GROUP = NUMBER OF GROUPS

Sometimes an open number sentence will be required. For example, *how many \$2 coins make \$10* could be represented with $? \times 2 = 10$. This could then be solved mentally using either multiplication or division.

If the question was more difficult, for example: *how many weeks in 438 days*, then the open number sentence which matches this situation, $? \times 7 = 438$, needs to be changed into a division in order to solve it on a calculator. $438 \div 7 = ?$ In this case it helps to think:

- If the **TOTAL** is missing, we use **MULTIPLICATION**
- If the **TOTAL** is present, we use **DIVISION**

After solving a problem it is important to consider the answer in relation to the problem to decide what the number is referring to. For example:

What is your speed if you travel 100 kilometres in one hour? $100 \text{ kilometres} \times 1 \text{ hour} = 100$. The answer of 100 refers to kilometres per hour, not 100 kilometres nor 100 hours.

Whole Class Activity 1

Think: When solving a word problem, how do I choose between multiplication and division?

Use the following tables to guide the decision making process as outlined by the flow diagram on page 206.

A. 'The Subsandwich Store' has only \$5 notes in its cash register. If there is \$185 in the till, how many \$5 notes should there be?

| |
|--|
| 1. Circle: APPROXIMATE ANSWER or EXACT ANSWER? |
| 2. Circle: MENTAL/WRITTEN or CALCULATOR |
| 3. Write the problem as a number sentence that matches the 'Story'. |
| 4. Circle: MULTIPLICATION (total missing) or DIVISION (total present)? Rewrite the number sentence to help with the calculation if needed |
| 5. Decide: If MENTAL, which STRATEGY? If CALCULATOR, enter the numbers. |
| 6. Calculate the SOLUTION |

7. IS THE ANSWER REASONABLE? What is the number referring to? Write the answer in a sentence using the words in the question.

B. Tins of tomatoes are packed in boxes of 42. If there are 3 boxes, how many tins of tomatoes are there?

1. Circle: APPROXIMATE ANSWER or EXACT ANSWER?

2. Circle: MENTAL/WRITTEN or CALCULATOR

3. Write the problem as a number sentence that matches the 'Story'

4. Circle: MULTIPLICATION (total missing) or DIVISION (total present)?

Rewrite the number sentence to help with the calculation if needed

5. Decide: If MENTAL, which STRATEGY? If CALCULATOR, enter the numbers.

6. Calculate the SOLUTION

7. IS THE ANSWER REASONABLE? What is the number referring to? Write the answer in a sentence using the words in the question.

C. The area of a rectangle is 1120 square centimetres. If the width is 32cm, what is the length?

1. Circle: APPROXIMATE ANSWER or EXACT ANSWER?

2. Circle: MENTAL/WRITTEN or CALCULATOR

3. Write the problem as a number sentence that matches the 'Story'

4. Circle: MULTIPLICATION (total missing) or DIVISION (total present)?

Rewrite the number sentence to help with the calculation if needed

5. Decide: If MENTAL, which STRATEGY? If CALCULATOR, enter the numbers.

6. Calculate the SOLUTION

7. IS THE ANSWER REASONABLE? What is the number referring to? Write the answer in a sentence using the words in the question.



D. Denise has 5 DVD's. How many times as many DVD's does James have if he has 70?

| |
|--|
| 1. Circle: APPROXIMATE ANSWER or EXACT ANSWER? |
| 2. Circle: MENTAL/WRITTEN or CALCULATOR |
| 3. Write the problem as a number sentence that matches the 'Story' |
| 4. Circle: MULTIPLICATION (total missing) or DIVISION (total present)? Rewrite the number sentence to help with the calculation if needed |
| 5. Decide: If MENTAL, which STRATEGY? If CALCULATOR, enter the numbers. |
| 6. Calculate the SOLUTION |
| 7. IS THE ANSWER REASONABLE? What is the number referring to? Write the answer in a sentence using the words in the question. |

Which of the above questions did you need to multiply to solve? Which did you need to divide?
How did you know?



Which problems did you have to change the initial number sentence into another number sentence in order to solve with a calculator?



Practice Exercise 1

1. For the word problems below:

- Circle those where the total is missing (ie require MULTIPLICATION to solve) in blue.
- Circle those where the total is present (ie require DIVISION to solve) in red.
- Write a number sentence below each problem.
- What does each number in your number sentence refer to?
- Calculate the answer. What does the number in the answer refer to?

a) Sophie has 14 books. Charlie has 3 times more. How many does Charlie have?

b) Barbara packs eggs. If she has packed 420 eggs, how many dozen has she packed?

c) A bag of oranges costs \$10.50 for 3 kilograms. How much for one kilogram?

d) Calculate the area of a rectangle with length 73cm and width 6cm.

e) Jackie gives 18 lollies to each of 6 friends. How many lollies did she have to start with?

f) George plants 84 corn plants in rows of 7. How many rows does he plant?

g) Mitch cycles 240km in 8 hours. What is his average speed per hour?

h) Sophie has 43 socks. How many pairs of socks does she have?

2. The following word problems all require exact answers and are best solved using a calculator. For each problem:

- Write the problem as a multiplication or division number sentence.
- Write a number sentence to enter into the calculator.
- Use the calculator to find the answer.

a) The Division 1 Lotto winnings were shared between 3 winners. If each winner received \$724 563, what was the total prize pool?

| |
|--|
| Write the problem as a multiplication or division number sentence: |
| Write a number sentence to enter into the calculator: |
| Sentence Answer: |

b) Jalen calculated that he completed 495 minutes of homework on a weekend. How many hours was this?

| |
|---|
| Write the problem as a multiplication or division number sentence |
| Write a number sentence to enter into the calculator. |
| Sentence Answer: |

c) Paddo packs 210 loaves of bread per tray. If the bread is packed in columns of 15, how many rows of bread are packed?

Write the problem as a multiplication or division number sentence

Write a number sentence to enter into the calculator.

Sentence Answer:

d) Lauren drove for 1924km between Noosa and Melbourne. How many kilometres per day did she travel if it took her $4\frac{1}{2}$ days to complete the journey?

Write the problem as a multiplication or division number sentence

Write a number sentence to enter into the calculator.

Sentence Answer:

e) James works in a department store and earns \$7.37 per hour. Grace works at the local Recreation centre and earns three times as much per hour. How much does Grace earn per hour?

Write the problem as a multiplication or division number sentence

Write a number sentence to enter into the calculator.

Sentence Answer:

f) The picture of a cupcake in a recipe book is 21mm. The cupcake in real life is 94.5mm. How many times bigger is the cupcake in real life than in the book?

Write the problem as a multiplication or division number sentence

Write a number sentence to enter into the calculator.

Sentence Answer:

3. The following table guides the decision making process as outlined by the flow diagram on page 206. All problems require exact answers.

Complete the table. The first example has been done for you.

| Problem | Choose | Circle Operation Number Sentence(s) | Strategy and Solution | Write the answer in a sentence |
|--|--|--|--|---|
| George bought 4 tickets for himself and 3 friends to see hip-hop artist 'Drake' at Perth Arena. The tickets cost him \$633.68. How much was each ticket? |    | MULTIPLICATION DIVISION $4 \times ? = \$633.68$ $\$633.68 \div 4 = ?$ | Enter: $\$633.68 \div 4 = ?$ into calculator. $? = \$158.42$ | Each ticket to the Drake concert costs \$158.42 |
| George travelled for 5 hours at an average of 82km/h to get to the 'Drake' concert. How far did George travel? |   | MULTIPLICATION DIVISION | | |
| George sat in the front row of a section of Perth Arena that had 50 seats in the row. If there were 1100 seats in the section, how many rows were there? |   | MULTIPLICATION DIVISION | | |
| The 'Drake' concert was sold out. If the capacity of the Perth Arena for the concert was 13423 and the average ticket price was \$148, what was the gross income of the Perth Arena for the concert? |   | MULTIPLICATION DIVISION | | |
| At interval, George bought snacks costing \$50 to be shared with the 4 friends. What was the price of each friend's share? |   | MULTIPLICATION DIVISION | | |
| Drake performed 30 songs in two hours. What was the average length of each song? |   | MULTIPLICATION DIVISION | | |

4. The answer to ALL of the following problems is 35. What does the 35 refer to in each situation?
For example, \$35 or 35 pieces etc.

- a) How much would each person receive if \$175 was shared between 5 people?
- b) How many \$5 notes in \$175?
- c) What is the area of a rectangle of length 5cm and width 7cm?
- d) How many pieces would you have if you cut a 175cm ribbon into 5cm pieces?
- e) How many days in 5 weeks?
- f) How many 5 minute time intervals in 175 minutes?

5. The answer to ALL of the following problems is 8. What does the 8 refer to in each situation?
For example, 8 kilograms or 8km/h etc.

- a) A person cycles 16km in 2 hours. What is the average speed of the cyclist?
- b) A 4kg box of apricots costs \$32? What is the cost per kilogram?
- c) A moped can travel 56km on 7 Litres of petrol. What is its petrol consumption?
- d) It takes Jill 6 hours to swim 48km in the Rottneest swim. What is her average speed?
- e) Cashews costs \$0.80 for 100grams. What is the cost per kilogram?
- f) Half a kilogram of mince costs \$4. What is the price per kilogram of mince?

6. Use the decision making process and choose between multiplication and division to solve the following. What does the answer refer to? Use this when writing your answer in a sentence.

- a) Danielle bought 8 DVD's for \$29.99 each. What was the total cost?
- b) In one year in Bunbury, it rained for 91 days. How many weeks was this?
- c) Michael bought 2 punnets of strawberries. One punnet weighed 450g. How much would 2 punnets weigh?
- d) Sarah's father inherited some money and shared it equally between his 3 daughters. If each daughter received \$25 783.55, how much did he inherit?
- e) The area of a rectangular farm was 385 square kilometres. If the length was 27.5 kilometres, what was the width?
- f) Tia drew a triangular based pyramid and then constructed a model of the pyramid 3 times larger than the drawing. If the drawing was 328mm high, what was the height of the model?

Practice Exercise 2

1. The following payments occur for workers at a Margaret River winery

Hours worked:
 Monday – Friday: 38 hours at a normal rate, time and a half thereafter.
 Saturday: All hours at time and a half
 Sunday: All hours at double time

Choose between multiplication and division to fill the gaps in the following wage slips for each of the employees.

a)

| Ryan Smith: Normal Rate \$22/hr | | | |
|---------------------------------|-------------|-------|-------|
| | Hourly Rate | Hours | Wage |
| Mon – Fri Normal | | 30 | |
| Mon – Fri Overtime | | 0 | |
| Saturday | | | \$132 |
| Sunday | | 7 | |
| TOTAL WAGE | | | |

c)

| Jake Morgan: Normal Rate \$32.28/hr | | | |
|-------------------------------------|-------------|-------|----------|
| | Hourly Rate | Hours | Wage |
| Mon – Fri Normal | | 20 | |
| Mon – Fri Overtime | | 0 | |
| Saturday | | | \$145.26 |
| Sunday | | 4 | |
| TOTAL WAGE | | | |

b)

| Travis Jones: Normal Rate \$26/hr | | | |
|-----------------------------------|-------------|-------|------|
| | Hourly Rate | Hours | Wage |
| Mon – Fri Normal | | 38 | |
| Mon – Fri Overtime | | | \$78 |
| Saturday | | 3 | |
| Sunday | | 4 | |
| TOTAL WAGE | | | |

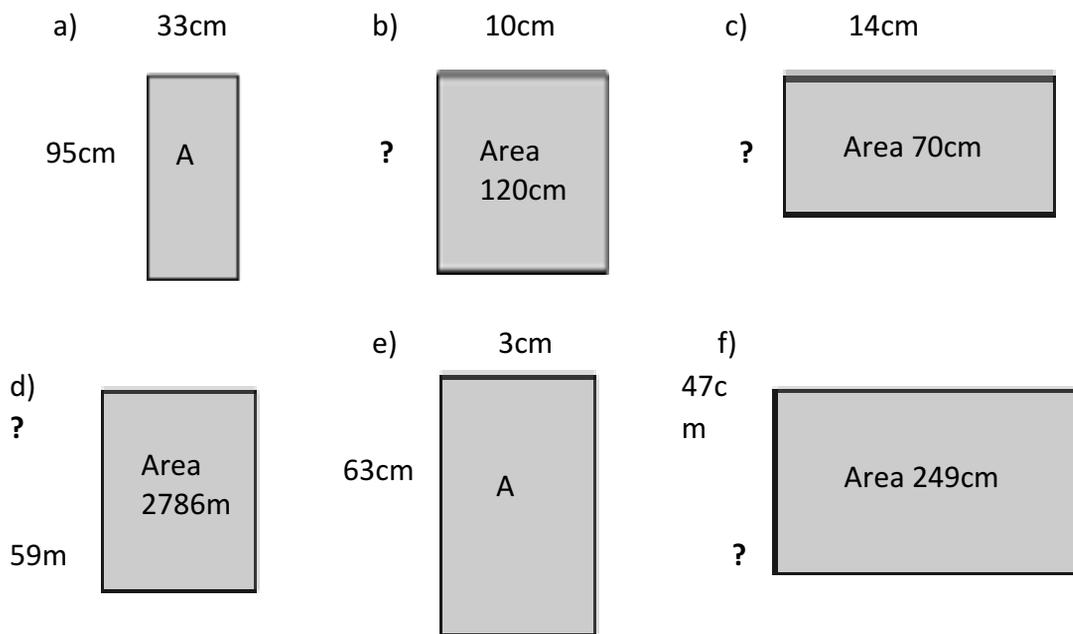
d)

| Sam Harris: Normal Rate \$?/hr | | | |
|--------------------------------|-------------|-------|-------|
| | Hourly Rate | Hours | Wage |
| Mon – Fri Normal | | 38 | \$608 |
| Mon – Fri Overtime | | | \$24 |
| Saturday | | 6 | |
| Sunday | | 5 | |
| TOTAL WAGE | | | |

2. The following pictures of animals were being used as illustrations in a children’s book that also showed the height of the animals in real life. Choose between multiplication and division to fill the gaps in the table. The first has been done for you.

| Animal | Height of Picture | Scale of Picture to Real Life | Height in Real Life |
|---|-------------------|-------------------------------|---------------------|
|  | 5cm | 4.5 × larger | 22.5cm |
|  | | 6 × larger | 157.46cm |
|  | 24mm | 3 × Larger | |
|  | 37mm | | 370mm |
|  | | 1.5 × Larger | 27mm |
|  | 44mm | 9 × Larger | |

3. The Area of a rectangle is calculated by multiplying the length by the width; i.e. $A = L \times W$. Choose between multiplication and division to calculate the missing side length (?) or area (A) in each of the following. (Diagrams are not to scale)



4. People attending an Asian Travel Fair were asked which of two countries in South-East Asia they had visited. The results were as follows:

| | Visited Indonesia | Not Visited Indonesia | Total |
|----------------------|-------------------|-----------------------|-------|
| Visited Malaysia | 18 | 12 | 30 |
| Not Visited Malaysia | 65 | 5 | 70 |
| Total | 83 | 17 | 100 |

As a result of promotions at the Fair, the South-East Asian tourist bureau were hoping that if they conducted the survey at next year's Fair the visitors:

- To both countries would increase four-fold
- Who haven't visited either country would stay the same
- Who go to Indonesia but not Malaysia would triple
- Who go to Malaysia but not Indonesia would double.

Choose between multiplication and division to complete the predicted survey results for next year's Fair in the table below:

| | Visited Indonesia | Not Visited Indonesia | Total |
|----------------------|-------------------|-----------------------|-------|
| Visited Malaysia | | | |
| Not Visited Malaysia | | | |
| Total | | | |

How many people are they hoping to survey next year?

Reflection on Learning

- Read the following problems.
- Choose a problem that matches each of the four boxes in the table below.
- Write the number of the problem in the matching box.
- Write a number sentence that matches your chosen problems.
- Solve the problems and write each answer in a sentence.

1. How many eggs in 25 dozen?
2. Sarah had \$127.36. Tia had four times more. How much did Tia have?
3. Sophie travelled 48 km in 3 hours. What was her average speed?
4. 500 grams of chicken costs \$4.24. What is the price per kilogram?
5. Playing cards were shared between 4 people. Each person got 13 cards. How many cards were there?
6. 456 789 cents is how many dollars?
7. The area of a rectangle is 5678 square centimetres. If one of its sides is 23cm, what is the length of the other side?

| | |
|---|--|
| SOLVE BY MULTIPLICATION (total missing) AND MENTAL STRATEGY | SOLVE BY MULTIPLICATION (total missing) AND CALCULATOR |
| SOLVE BY DIVISION AND MENTAL STRATEGY | SOLVE BY DIVISION AND CALCULATOR |

Reflect on your learning in this topic. Has your ability to choose whether to multiply or divide in problems improved? Discuss with a partner.

OLNA Practice Questions

1. Towns A and B are 13 cm apart on a map. On the map 1 cm represents 5 km. What is the actual distance between the two towns?

- A. 2.6 km B. 26km C. 18km D. 65 km

2. Points are scored for each medal won at a sports carnival. Governor Stirling SHS won a total of 40 medals. It won 17 Gold, 18 Silver and the rest Bronze.

| Medal | Points per Medal |
|--------|------------------|
| Gold | 3 |
| Silver | 2 |
| Bronze | 1 |

How many points did Governor Stirling SHS score?

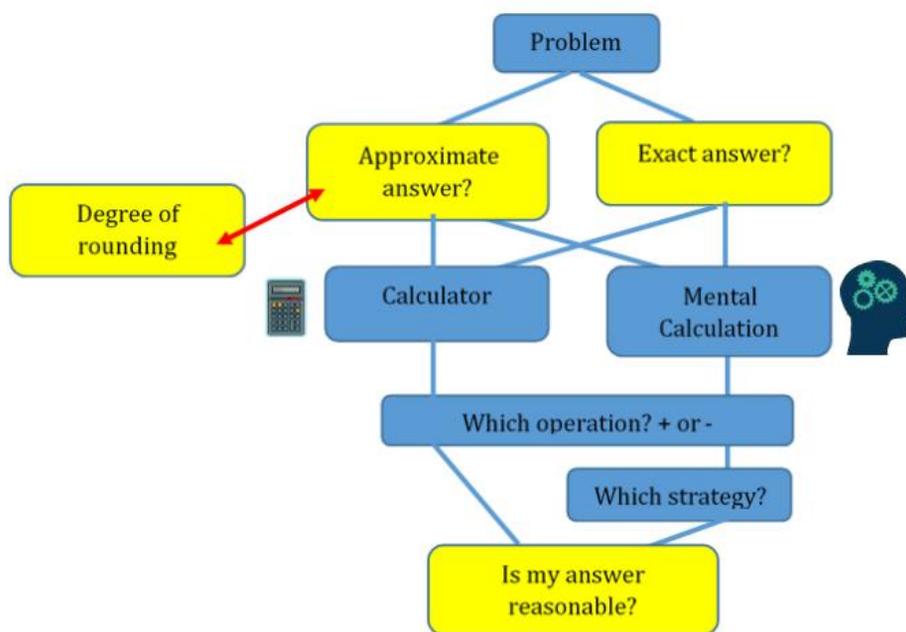
- A. 92 B. 240 C. 87 D. 1 600

Topic 8

Using Estimation for Solving Problems and Checking Reasonableness of Answers

Mathematics Discussion

In this topic we will be focussing on the first section of the flow chart; 'Do we need an exact answer or an approximate answer?' and the last section; 'Is my answer reasonable'.



Many real life situations require an exact answer, but many require an approximate answer.

If we decide that the situation requires an exact answer, we solve the problem using mental strategies, jottings or a calculator.

If we decide the situation requires an approximate answer, we use estimation strategies to simplify the problem. We then use mental strategies or jottings to approximate a solution.

The estimation strategies that help us approximate an answer are:

- Rounding.
- Upper and Lower Limits.

When we round, we must decide how accurate our rounding needs to be and whether we need to round up or down. For example, \$285.64 could be rounded to \$285.60, \$285.70, \$286 or \$300, depending on the situation. Mostly we tend to round up to ensure that we have enough, e.g., food, money, building supplies etc.

The last decision in the flow chart is 'Is my answer reasonable?' We make judgements as to whether an answer is reasonable based on:

- Properties of numbers
- Estimation strategies
- The context of the problem

In this section we will be focussing on whether an exact or an approximate answer is needed, using estimation strategies and deciding whether answers are reasonable, for problems involving multiplication and division.

Whole Class Activity 1

Approximate or Exact Answer?

Think: *When should I choose to use an approximate answer instead of an exact answer?*

1. Discuss each of these scenarios and decide whether an approximation or an exact answer is needed in the following everyday situations. Justify your decision.

| Scenario | Circle One | Justification |
|---|-------------------------------|---------------|
| Total wage if you are paid \$11.25 per hour for 8 hours | Approximation Exact answer | |
| Apples cost \$7.95/kg. Lou bought 2.5kg. Would \$20 be enough to pay for it? | Approximation Exact answer | |
| Painting the 5.8m by 2.9m ceiling of your living room and knowing that one tin of paint covers 18 square metres | Approximation Exact answer | |
| Check that \$903 collected from 86 students paying \$10.50 each to see a class movie is correct. | Approximation Exact answer | |
| Fit skirting board on a square room of length 3.8m | Approximation Exact answer | |

2. Create a class poster that lists common situations where class members and teachers have used estimation instead of a precise calculation. How close did the class members and teachers estimations have to be to the exact answer?

Add to the poster as you work through this topic.

Practice Exercise 1

1. For the following problems:

Circle the problems in blue where an approximate answer would be best.

Circle the problems in red where an exact answer would be best.

a) A barrel holds 25 000mL of water. It is used to fill bottles holding 675mL. How many bottles will be needed?

b) Share 17 cakes between 11 people. How much does each person receive?

c) The change received from \$20 when purchasing 2 cans of soft drink at \$3.75 per can.

d) Jamie cycles at 33km/h. How long will it take him to cycle 209km?

e) Janice wondered whether \$10 was enough to purchase $2\frac{1}{2}$ kg of apricots costing \$3.95 per kilogram

f) Jack paints a ceiling measuring 3.65m by 4.02m and wonders whether a tin of paint covering 16 square metres will be enough.

Reflection and Discussion – Underestimate or Overestimate?

An underestimation means our approximation is less than the accurate answer whilst an overestimation means our approximation is more than the accurate answer. In most cases it is best to get an overestimation of the answer to ensure we have enough food, money, time etc.

For each of the problems in Practice Exercise 1 where an approximate answer was selected, write next to each question whether it should be an overestimate or an underestimate.

Discuss with the class.

When should you use an overestimate?



When should you use an underestimate?



Whole Class Activity 2

Underestimating and Overestimating with Multiplication and Division Problems.

Think: How does rounding up or down affect the result of multiplication and division calculations?

A plumber charges \$896.25 per day to work on a building site for 15.75 days.

Estimate how much the plumber earned.

This is a multiplication problem because the TOTAL is missing.

We are estimating $\$896.25 \times 15.75$.

We could round both numbers down to the nearest whole number to estimate.

That is $\$896 \times 15$. However, this is difficult to calculate mentally.

It is easier to round both numbers down to the nearest ten to find an approximate solution.

Calculate $\$890 \times 10 =$

Is this answer an underestimation or an overestimation?



We could round both numbers up to the nearest whole number to estimate.

That is $\$897 \times 16$. However, this is also difficult to calculate mentally.

It is easier to round both numbers up to the nearest ten to find an approximate solution.

Calculate $\$900 \times 20 =$

Is this answer an underestimation or an overestimation?



Using this information, it is safe to say that the plumber earned between _____ and _____ for his work on the building site.

Why is there such a big difference between the two answers?



What is approximately half way between these two answers?



We could round one number UP to the nearest whole 'friendly' number and one number DOWN to the nearest whole 'friendly' number to estimate an answer.

Hence, calculate $\$900 \times 15 =$



Which approximation is closest to the exact answer? Use a calculator to check.



If you were the plumber which is the best approximation to use? Why?



If you were the building contractor, which is the best approximation to use? Why?



Discuss the difference between the different rounding and estimating strategies. How would you decide which strategy to use?



A Carpenter receives \$6 641.37 for a 16.25 day job. What was her rate per day?

This is a division problem because the TOTAL is present.

We are estimating $\$6\,641.37 \div 16.25$.

We could round both numbers down to the nearest 'friendly number' to find an approximate solution. Calculate $\$6\,000 \div 10 =$

Is this answer an underestimation or an overestimation?



We could round both numbers up to the nearest 'friendly number' to find an approximate solution. Calculate $\$7\,000 \div 20 =$

Is this answer an underestimation or an overestimation?



Using this information, it is safe to say that the carpenter earned between _____ and _____ per day for her work on the building site.

Why is there such a big difference between the two answers?



What is approximately half way between these two answers?



We could round one number UP to the nearest whole 'friendly' number and one number DOWN to the nearest whole 'friendly' number to estimate an answer.

Hence, calculate $\$6\,000 \div 20 =$

Which approximation is closest to the exact answer? Use a calculator to check.



If you were the carpenter which is the best approximation to use? Why?



If you were the building contractor, which is the best approximation to use? Why?



Discuss the difference between the different rounding and estimating strategies. How would you decide which strategy to use?

Reflection and Discussion

Rounding up or down affects overestimation and underestimation differently for multiplication and division.

Complete the following table:

| Problem | Round each number down to nearest whole number and solve | Round each number up to nearest whole number and solve | Exact Answer Using a Calculator |
|---------------------|--|--|---------------------------------|
| $\$8.23 \times 2.4$ | | | |
| $\$8.23 \div 2.4$ | | | |

Complete the following sentences using the words OVERESTIMATION or UNDERESTIMATION

- When we round both numbers DOWN in a multiplication problem we get an _____ of the result.
- When we round both numbers UP in a multiplication problem we get an _____ of the result.
- When we round both numbers DOWN in a division problem we get an _____ of the result.
- When we round both numbers UP in a division problem we get an _____ of the result

Practice Exercise 2

1. Calculate an approximate solution using underestimation, overestimation, or rounding one number up and one number down.

a) $\$59.75 \div 4.8 = ?$

d) $\$0.89 \times 19 = ?$

b) $\$7.81 \times 11 = ?$

e) $235\ 869\text{kg} \div 11.25\text{kg} = ?$

c) $\$8\ 999 \div \$2.75 = ?$

f) $\$1\ 567\ 000 \times 3.21 = ?$

2. From supermarket catalogues, choose 5 items (e.g. butter, milk, cereal). Cut out the items and glue in the table below.

Round the cost of each of the items **up** to the nearest (a) fifty cents, (b) dollar, (c) five dollars, (d) ten dollars. Write the rounded amounts in the column labelled 'ROUNDED COST'. Choose the most appropriate rounded cost to perform each of the remaining multiplication calculations in the table.

| Item | Rounded Cost | Approximate Cost of 4 Items or 4kg of Item | Approximate Cost of 10 Items or 10kg of Item | Approximate Cost of 12 Items or 12kg of Item |
|------|--------------------------|--|--|--|
| | (a) (b) (c) (d) | | | |

a) Why do you think you were asked to round the cost of the items up? Is this an overestimation or an underestimation?

b) Did you need to round the cost of the item when buying 10 of that item or 10kg of that item? Explain.

c) What is the maximum number of each item you can purchase for \$20?

d) What is the maximum number of each item you can purchase for \$50?

e) Could you purchase 3 of one item and 2 of a different item for less than \$20? List the combinations in the space below.

3. When measuring it is often best to overestimate in order to have enough materials or time to complete the job. Overestimate to calculate approximate solutions to the following.

Remember in multiplication problems, we round both numbers UP to overestimate; in division problems we round both numbers DOWN to overestimate.

a) Find the approximate area of a rectangular room measuring 3.75m by 4.2m.

b) Find the approximate length of string required to tie 84 parcels, each requiring 97cm of string.

c) A barrel holding 2230L is to be poured into bottles each holding 1.25L. Approximately how many bottles will be needed?

d) How long will it take to pack 9 freezers into a delivery truck if each freezer takes approximately 3.7 minutes to pack?

e) How many test questions could Abbi complete if the test is 42 minutes and each question takes approximately 2.25 minutes to complete?

f) What will the size of a model of 1.59m kangaroo be, if it is to be made 5.5 times smaller?

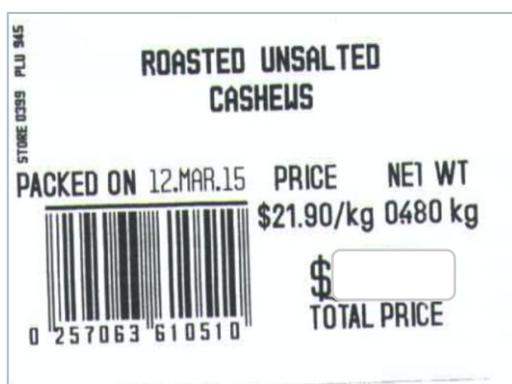
4. When solving sharing or grouping division problems, sometimes it is best to underestimate (i.e. round up) in order for there to be enough of the amount to distribute equally. Underestimate to calculate approximate solutions to the following.

a) Share 117 Easter Eggs between 18 people at an Easter egg hunt.

b) Cut a 17.95m piece of ribbon into 2.74m pieces. How many pieces would there be?

c) How many 7.5 minute sprint intervals in 79 minutes of athletics training?

- d) Share a \$177 564 inheritance between 6 people. How much would each person receive?
- e) Ken needs to plant 13 435 zucchini plants in rows of 18. Approximately how many rows will Ken need to plant?
- f) How many classes of at least 27 students could be made from a group of 234 students?
5. The following items were purchased at a local supermarket.
- a) Estimate the missing price for the Roasted Unsalted Cashews
- b) Estimate the missing weight for the Soya Crisps



Whole Class Activity 3

How accurate should my estimation be?

Think: When I estimate, how accurate should I be?

The accuracy of our estimation depends upon the types of numbers, the difficulty of the mental calculation and the context of the problem.

How close does your estimation need to be in the following situations:

1. A school woodwork project involves a class of 28 students constructing a square photo frame of side length 28.45cm from wood.

How much wood is needed for each student to make the photo frame? Should the answer be within 10m, 1m, 50cm, 10cm, 5 cm, 1cm or 1mm? Justify your answer.



How much wood should the teacher order for the class to make the photo frames? Should the answer be within 10m, 1m, 50cm, 10cm, 5cm, 1cm or 1mm? Justify your answer



Round the numbers to calculate the estimation mentally or with jottings. Write number sentences and solve.



How do the numbers and the context of this problem affect the choice of rounding?



Describe the level of accuracy needed in this problem. Use words such as overestimation, underestimation, rounding, cost, waste and error.



2. Pip is tying packets of biscuits with a ribbon for a market stall. Each packet requires 77cm of ribbon. She plans to tie 32 packs. Ribbon costs \$2.37 per metre.

How much does ribbon for one packet cost? Should the answer be within \$10, \$5, \$1, 50c, 10c or 5c?



How much does ribbon for the total project cost? Should the answer be within \$10, \$5, \$1, 50c, 10c or 5c?



Round the numbers in the problems to calculate the estimation mentally or with jottings. Write number sentences and solve.



How do the numbers and the context of this problem affect the choice of rounding?



Describe the level of accuracy needed in this problem. Use words such as overestimation, underestimation, rounding, cost, waste and error.



3. A building project cost \$15 378 942.89 and takes 287 days to complete.

A politician wants a 'ball park' figure on the daily cost of the project. Should the answer be within \$5 million, \$1 million, \$500 000, \$100 000, \$50 000, \$10 000, \$1000, \$500 or \$100?



Round the numbers to calculate the estimation mentally or with jottings. Write as a number sentence and solve.



How do the numbers and the context of this problem affect the choice of rounding?



Describe the level of accuracy needed in this problem. Use words such as overestimation, underestimation, rounding, cost, and error.



Practice Exercise 3

1. Circle the most appropriate degree of accuracy to use for each problem.

a) A living room has ceiling dimensions of 6.84m by 4.35m. What is the area of the ceiling so as to buy enough paint to paint the ceiling?

Round the length and width to the nearest:

1 metre

50 centimetres

10 centimetres

1 centimetre

b) Becky purchases 4 apartment buildings for \$357 899 each. How much does she tell her sister she paid in total for the properties?

Round the price of one apartment building to the nearest:

\$500 000

\$100 000

\$50 000

\$10 000

\$5000

c) It takes 2 minutes and 22 seconds to assemble a cardboard pizza box. How long would it take Josie to assemble 16 boxes?

Round the time taken to assemble one box to the nearest:

10 minutes

5 minutes

1 minute

30 seconds

10 seconds

5 seconds

d) Nick purchases 0.47 hectares of land for \$367 954. What is the estimated cost per hectare for insurance purposes?

Round the hectares to the nearest:

10 ha

1 ha

0.5 ha

Round the cost per hectare to the nearest:

Million

\$500 000

\$100 000

\$50 000

\$10 000

2. Estimate the solutions to the following problems.

| Problem | Write the most appropriate rounded cost (nearest 50c, \$1, \$5, \$10) | Written as a rounded \times or \div number sentence | Approximate Solution |
|---|--|---|----------------------|
| 3 blocks of Cheese at \$5.47 each | | | |
| 4 Movie Tickets at \$13.65 each | | | |
| 3kg of Apples cost \$20.18. What is the cost of 1kg? | | | |
| Dog food is 6 tins for \$11.34. What is the price per tin? | | | |
| Jack won 2 Division prize pools each paying \$67 589.43. How much did Jack win? | | | |
| Mary bought 2ha of land for \$987 450. What was the price per hectare? | | | |

Whole Class Activity 4

Is my Answer Reasonable?

Think: How do I know if my answer to a multiplication or division problem is reasonable?

Kevin estimated the cost of 2.29kg of trout at \$24.68 per kg as being about 17kg.

Jill checked Kevin's answer. She immediately said 'I can tell your estimate is wrong for three reasons.'

1. 'When you multiply by a number **bigger than one**, the answer gets bigger. The answer to $2.29 \times \$24.68$ must be a lot bigger than 17.'

Explore this idea with a partner. Use examples such as 25×1 ; 25×1.5 ; 25×2 etc. Write your answers to these problems in the space below:



Is Jill right? Discuss.

What happens to answers when dividing by a number **bigger than one**. Discuss with a partner using examples such as $25 \div 1$; $25 \div 2$; $25 \div 3$ etc.



2. 'We can use underestimation and overestimation to work out that the answer cannot be 17.'

Calculate the underestimation and overestimation of Kevin's trout problem.



Is Jill right? Discuss.



3. 'How can the cost be 17kg? The answer does not make sense! It has to be dollars.'

Explore this idea with a partner. Use examples such as $\$5 \times 1$, $5 \times \$1$, $\$5 \times \1 , $5 \times 1\text{kg}$, $5\text{kg} \times \$1$, $5\text{kg} \times 1\text{kg}$ and decide whether the answer should be dollars or kilograms. Use similar problems with division. Which of these problems do not make sense? When is an answer given in money? weight? a quantity? How can you tell how to write the answer to a problem?



Is Jill right? Discuss.



Reflection and Discussion

What happens to a number when you multiply or divide it by a number between 0 and 1?

MULTIPLYING BY NUMBERS BETWEEN 0 AND 1

Use your calculator to investigate, using examples such as 25×0.75 , 25×0.5 and 25×0.25 . Write your answers to the problems and your conclusions in the space below.



DIVIDING BY NUMBERS BETWEEN 0 AND 1

Use your calculator to investigate, using examples such as $25 \div 0.75$, $25 \div 0.5$ and $25 \div 0.25$. Write your answers to the problems and your conclusions in the space below.



Formalize your thinking by completing the statements below:

When you multiply by a number between 0 and 1, the answer is _____ than the original number.

When you divide by a number between 0 and 1 the answer is _____ than the original number

Practice Exercise 4

1. Without doing the calculation, decide whether the following number sentences are reasonable. Justify your answer using your knowledge of estimation.

a) $\$412.37 \times 2 = \84.74

d) $\$36 \times 1.88\text{kg} = 67.68\text{kg}$

b) $\$33 \times 5 = \165

e) $250 \div 0.44 = 110$

c) $1\ 200 \div 100 = 12\ 000$

f) $\$53 \div \$24 = \$2.21$

2. Use your understanding of reasonableness of result to circle the most likely answer.

a) 22×0.9 Choose from: 198, 24, 19.8, 13.8

b) $\$42.87 \times 3$ Choose from: $\$128.61$, $\$41.33$, 128.61, $\$1286.10$

c) $456\text{mins} \div 3\text{mins}$ Choose from: 152, 1368, 156, 152mins

d) $315\text{cm} \div 1.5$ Choose from: 480cm, 165cm, 210cm, 103cm

e) $2.2\text{kg} \times \$54.95$ Choose from: $\$121$, 115kg, $\$24.98$, $\$168.55$

f) $\$3.21 \times 31.45\text{m}$ Choose from: 313m, 28.45, $\$100$, $\$92$

3. Use properties of numbers, estimation strategies and the context of the problem to determine if the conclusions to the following situations are reasonable. Justify your answer.

a) Shannon had $\$248.97$ in her wallet for the Royal Show. She wanted to share it between her 4 children. Each child received $\$60$.

b) At the Royal Show, Show Bags were packed in boxes of 27. There were 39 boxes of 'Gertie Gums' Show Bags. Millie, who works in the Show Bag stall, said there were 605 'Gertie Gums' Show Bags.

c) Grace bought 7 Show Bags for a total of $\$46.50$. She thought the average price of each Show Bag was 6.

d) Dale had $\$30$. He purchased 5 buckets of chips each costing $\$5.99$. Did Dale have enough money?

e) At the Royal Show rides were sold in voucher books of 6 for $\$32$. Reuben said that was $\$5.33333$ per ride.

f) Lauren saw the dimensions of the hen cages at the Royal Show were 82cm by 77cm. She calculated the area to be roughly 64 square centimetres.

Reflection on Learning

Solve the following problems using the decision making table below. The first decision is whether an exact answer is needed or an approximation.

- The Student Council at Melville Senior High School were responsible for the School Ball organisation. 147 tickets were sold at a cost of \$79.95 per ticket. The school Principal asked 'What is an estimate of the gross income from the sale of tickets?'
- The three different music bands who played at the Ball all charged the same amount. The total cost of band hire was \$2 610. What was the price of each band?
- Harry, a member of the Student Council, purchased 27 packets of balloons costing \$2.87 per packet. He paid for the balloons himself and then asked the Student Council to reimburse him the cost. How much was he reimbursed?
- The food costs at the Ball totalled \$4 410. The best estimate for the price of food per head for each of the 147 guests would be A. \$40 per head; B. \$50 per head; C. \$30 per head.
- Harry bought 11.74 metres of ribbon at \$3.95 per metre to decorate the tables. Harry took \$35 from the Student Council petty cash fund to pay for it. Did Harry take enough?

| EXACT OR APPROXIMATE ANSWER? | MENTAL OR CALCULATOR? | WHICH OPERATION? Number Sentence(s) | STRATEGY AND SOLUTION | IS YOUR ANSWER REASONABLE? JUSTIFY. |
|------------------------------|---|--|-----------------------|-------------------------------------|
| 1. |   | | | |
| 2. |   | | | |
| 3. |   | | | |
| 4. |   | | | |
| 5. |   | | | |

OLNA Practice Question

- 3.88 kg of pears costs \$17.95. About how much is the cost of pears per kilogram?
A. 4.5 B. \$72 C. \$7 D. \$4.50

Section Three

Measurement



Content Focus

Mathematics Foundation

2.3.1 Identify and describe the purpose of commonly used metric units within various everyday contexts

2.3.2 Use prefix names of measurement units to work out the relationship with the base units: metre, gram and litre. For example milli means a thousandth, kilo means times by a thousand

2.3.3 Link decimal place value and the prefix names of metric units to read, write, compare and order length, mass and capacity measurement

2.3.4 Use place value understanding to convert from one unit to another; that is, 250 mm = 25 cm

2.3.5 Use relationships between decimals and fractions to convert from one unit to another; for example, 1500 mm is 1.5 m because 1000 = 1 metre and 500 is half a metre. (Note: perimeter, area and volume all use length measures)

2.4.1 Identify and describe the purpose of perimeter, area and volume measures in everyday life and work

2.4.2 Determine whether an estimate or an accurate perimeter, area and volume measurement is needed in everyday situations

2.4.3 Calculate perimeter by adding length measurements and connect this to the formulae

2.4.4 Measure area (including irregular shapes) by counting squares, and volume by counting cubes

2.4.5 Connect the idea of counting squares for area with the formula of $A = l \times w$

2.4.6 Connect the idea of counting cubes for volume with the formula of $V = h \times l \times w$

2.4.7 Use perimeter and area formula for shapes based on rectangles, and volume formula for shapes based on rectangular prisms

2.4.8 Develop and use a sense of size of commonly used standard area and volume units; for example:

- 1 square centimetre (1 cm^2), 1 square metre (1 m^2),
- 1 cubic centimetre (1 cm^3), 1 cubic metre (1 m^3) to estimate in familiar situations

2.4.9 Estimate perimeter, area and volume in practical situations using familiar standard units and rounding.

Australian Curriculum Link

ACMMG061 Measure, order and compare objects using familiar metric units of length, mass and capacity

ACMMG084 Use scaled instruments to measure and compare lengths, masses, capacities and temperatures

ACMMG109 Calculate the perimeter and area of rectangles using familiar metric units

ACMMG135 Connect decimal representations to the metric system

ACMMG136 Convert between common metric units of length, mass and capacity

ACMMG137 Solve problems involving the comparison of lengths and areas using appropriate units

ACMMG138 Connect volume and capacity and their units of measurement

ACMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving

ACMMG160 Calculate volumes of rectangular prisms

ACMMG290 Compare objects using familiar metric units of area and volume

Topic 1

Metric Relationships

Mathematics Discussion

The international system of units for metric measurement is based on a set of prefixes added to a base unit, to show the relative size of each unit.

The table below shows the most common prefixes and how they relate to the base unit.

| Kilo-unit | Hecto-unit | Deca-unit | Base unit | Deci-unit | Centi-unit | Milli-unit |
|------------|------------|-----------|-----------|---------------------|----------------------|-----------------------|
| k | h | D | | d | c | m |
| 1000 units | 100 units | 10 units | 1 unit | $\frac{1}{10}$ unit | $\frac{1}{100}$ unit | $\frac{1}{1000}$ unit |

Each place to the left of the base unit is ten times larger as you move from one prefix name to the next. Each place to the right of the base unit is ten times smaller as you move from one prefix name to the next. For example, 2 millimetres is ten times smaller than 2 centimetres.

This connection to the place value system is helpful when converting from one unit to another. For example, to convert 1 metre into centimetres, we break the one metre into ten equal sized pieces to make decimetres, and then break each of these into ten equal sized pieces to make centimetres. This means that to convert 1 metre to centimetres we multiply $10 \times 10 = 100$. One metre is equal to 100 centimetres. Moving in the other direction across the prefix table, one centimetre is $\frac{1}{100}$ of a metre.

In fact, we can move in either direction from one unit to the next. Moving from right to left, each place is ten times larger, or $\times 10$. Moving from left to right, each place is ten times smaller, or $\div 10$.

| Kilo-unit | Hecto-unit | Deca-unit | Base unit | Deci-unit | Centi-unit | Milli-unit |
|-------------|-------------|-------------|-----------|-------------|-------------|-------------|
| $\times 10$ | $\times 10$ | $\times 10$ | | $\times 10$ | $\times 10$ | $\times 10$ |
| $\div 10$ | $\div 10$ | $\div 10$ | | $\div 10$ | $\div 10$ | $\div 10$ |

The prefix names in the metric system are consistent across the different attributes. For example, the prefix 'milli' means one thousandth, whether it is a millimetre, a millilitre or a milligram. This makes it easier to remember the names and the sizes of the different units.

Whole Class Activity 1

Think: What do the measurement prefixes tell about the size of a unit?

Read the unit names below. Sort them according to the attribute they measure and write them into the appropriate box below.

| | | |
|------------|------------|------------|
| gram | hectogram | millimetre |
| litre | kilometre | kilogram |
| decametre | decalitre | decigram |
| milligram | metre | hectolitre |
| decilitre | kilolitre | centimetre |
| hectometre | centilitre | decagram |
| centigram | decimetre | millilitre |

| Length | Mass | Capacity |
|--------|------|----------|
| | | |

Highlight the units you are most familiar with.

What is the same about each list of units?



What is different about each list of units?



The table below shows the most common prefixes and how they relate to the base unit.

| Kilo-unit | Hecto-unit | Deca-unit | Base unit | Deci-unit | Centi-unit | Milli-unit |
|------------|------------|-----------|-----------|---------------------|----------------------|-----------------------|
| k | h | D | | d | c | m |
| 1000 units | 100 units | 10 units | 1 unit | $\frac{1}{10}$ unit | $\frac{1}{100}$ unit | $\frac{1}{1000}$ unit |

Use the prefix table to order the words above, from smallest to biggest.

| Length | Mass | Capacity |
|--------|------|----------|
| | | |

Circle the base unit in each list. What is the abbreviation for each base unit? Write them below.



Write the abbreviation for each unit next to it in the list above.

Use the prefix table to describe the following unit relationships.

How are base units related to kilo units?



How are deci units related to base units?



How are milli units related to base units?



Reflection and Discussion

Why do we use the international system of units for metric measurement?

Watch one of the following youtube clips to answer the question:

<https://www.youtube.com/watch?v=NValmBwli1Q>

<https://www.youtube.com/watch?v=dvVCNhWJvvo>



Whole Class Activity 2

Think: What are the relationships between the length units?

What is the base unit for length?



Cut a piece of paper tape 1m long.

Fold it into 10 parts and mark each fold with a line. What fraction of a metre is each part?



What is the name of this unit? (Use the prefix table to help.)



Divide the first segment into 10 parts. What is the name of this unit?



Divide the first segment into 10 parts. What is the name of this unit?



Now take 10 of the 1m tapes and lay them end to end. What is the name of this unit?



Visualise 100 of the tapes laying end to end. What is the name of this unit?



Visualise 1000 of the tapes laying end to end. What is the name of this unit?



Fill in the names of the units in the prefix table below. Add their abbreviations in the last row.

| Kilo-unit | Hecto-unit | Deca-unit | Base unit | Deci-unit | Centi-unit | Milli-unit |
|-------------|------------|-----------|-----------|---------------------|----------------------|-----------------------|
| 1000 units | 100 units | 10 units | 1 unit | $\frac{1}{10}$ unit | $\frac{1}{100}$ unit | $\frac{1}{1000}$ unit |
| 1 kilometre | | | 1 metre | | | |
| | | | m | | | |

Use the table to work out the following relationships.

How many decimetres = 1 metre?

How many metres = 1 decametre?

How many centimetres = 1 metre?

How many metres = 1 hectometre?

How many millimetres = 1 metre?

How many metres = 1 kilometre?

Use the tape and the prefix table to describe as many other relationships between the units as you can. E.g 100 millimetres = 1 decimetre



List two objects that match each measurement.

| Measurement | Object |
|----------------|--------|
| 2 decimetres | |
| 5 centimetres | |
| 15 millimetres | |

Whole Class Activity 3

Using estimation, cut a piece of string less than 1m.

Use measuring equipment to work out how long it is.

Use the prefix table to describe this measurement in three different ways.

For example, A piece of string 93cm long could also be described as 0.93m, 930mm and 9.3dm.

| Length of string | Measurement 1 | Measurement 2 | Measurement 3 |
|------------------|---------------|---------------|---------------|
| | | | |
| | | | |
| | | | |

Discuss your measurements with two other people and record their measurements on your table.

Practice Exercise 1

1. Complete the table.

| Symbols | Words | Equivalent |
|----------|-------------------------|------------|
| a) 5cm | | 0.05m |
| b) | Fifteen kilometres | |
| c) 890mm | | |
| d) 57m | | |
| e) | Five hundred kilometres | |
| f) 70cm | | |

2. Make and use a prefix table to order these measurements from smallest to largest.

a) 5m, 5dm, 15cm, 555mm

c) 25mm, 2cm, 0.2m, 22cm

b) 12cm, 21m, 0.012m, 2km

d) 104km, 1.4m, 4001mm, 40m

3. Use your prefix table to convert the following measurements.

a) $4\text{m} = \quad \text{cm}$

d) $5\text{km} = \quad \text{m}$

b) $5\text{cm} = \quad \text{m}$

e) $68\text{m} = \quad \text{km}$

c) $25\text{mm} = \quad \text{cm}$

f) $110\text{cm} = \quad \text{km}$

4. Use your prefix table to convert and add the amounts together.

a) $5\text{km} + 2\,500\text{m} = \quad \text{m}$

c) $45\text{cm} + 1\text{km} = \quad \text{km}$

b) $3\text{m} + 56\text{cm} = \quad \text{cm}$

d) $58\text{cm} + 22\text{mm} = \quad \text{mm}$

5. Complete the following part whole diagrams, showing the whole measurement being made up of 2 parts.

a)

| | |
|-------|------|
| 1m | 60cm |
| 160cm | |

e)

| | |
|-------|----|
| 1m | mm |
| 160cm | |

b)

| | |
|-------|------|
| m | 80cm |
| 180cm | |

f)

| | |
|-------|----|
| 1m | mm |
| 180cm | |

c)

| | |
|-------|----|
| m | cm |
| 105cm | |

g)

| | |
|-------|----|
| m | mm |
| 105cm | |

d)

| | |
|-------|----|
| m | cm |
| 107cm | |

h)

| | |
|-------|----|
| m | mm |
| 107cm | |

6. Convert the measurements to solve the problems.

a) Jack needs to order 'rebar' (reinforced bar) from the supplier. The construction drawings show he needs to lay 600 000mm of rebar within the formwork. How many metres of rebar does he need to order?

b) An open fireplace needs to be built 450mm inset from the perimeter wall of a house. How many cm is this?

c) Maddy measured the length of the driveway as 8.24m and the width as 3.5m. The bricklayer needs to know the length in mm. What measurements does Maddy need to give the bricklayer?

Whole Class Activity 4

Think: What are the relationships between the mass units?

You will need: 1 bucket, 30 plastic cups, 10L of water (rice or sand), measuring cups that show millilitres and litres.

What is the base unit for capacity?



Measure 1 litre of water into a jug. Divide the litre of water evenly between 10 cups.

What fraction of a litre is in one of these cups?



What is the name of this unit?



Divide one of the cups of water into 10 equal parts. What is the name of this unit?



Take one of the amounts of water from the previous division and divide into 10 parts.

What is the name of this unit?



Put the parts of the litre back together in a container and make sure it is 1 litre.

Now put 10, 1L amounts together in a bucket. What is the name of this unit?



Visualise 100 litres in a container. What is the name of this unit?



Visualise 1000 litres in a container. What is the name of this unit?



Fill in the names of the units in the prefix table below. Add their abbreviations in the last row.

| Kilo-unit | Hecto-unit | Deca-unit | Base unit | Deci-unit | Centi-unit | Milli-unit |
|-------------|------------|-----------|-----------|---------------------|----------------------|-----------------------|
| 1000 units | 100 units | 10 units | 1 unit | $\frac{1}{10}$ unit | $\frac{1}{100}$ unit | $\frac{1}{1000}$ unit |
| 1 kilolitre | | | 1 litre | | | |
| | | | L | | | |

Use the table to work out the following relationships.

How many decilitres = 1 litre?

How many litres = 1 decalitre?

How many centilitres = 1 litre?

How many litres = 1 hectolitre?

How many millilitres = 1 litre?

How many litres = 1 kilolitre?

Use the prefix table to describe as many other relationships between the units as you can.
e.g 10 decilitres = 1 litre



Whole Class Activity 5

Research or measure to find two objects that match each measurement.

| Measurement | Object |
|---------------|--------|
| 5 decalitres | |
| 3 centilitres | |
| 15 kilolitres | |

Without measuring, place an amount of water that is less than one litre into a cup.

Use measuring equipment to work out how much water you have.

Use the prefix table to describe this measurement in three different ways.

For example, 250mL could also be described as 0.25L, 2.5dL or 0.025DL

| Amount of Water | Measurement 1 | Measurement 2 | Measurement 3 |
|-----------------|---------------|---------------|---------------|
| | | | |
| | | | |
| | | | |

Discuss your measurements with two other people and record their measurements on your table.

Practice Exercise 2

1. Complete the table.

| Symbols | Words | Equivalent |
|----------|-----------------------------------|------------|
| a) 12mL | | |
| b) | Five litres | |
| c) 0.3L | | |
| d) 205kL | | |
| e) | Four hundred and fifty decilitres | |
| f) 10dL | | |

2. Make and use a prefix table to order the measurements from smallest to largest.

a) 3mL, 0.3L, 30kL, 30mL

c) 18dL, 8kL, 80L, 0.8L

b) 20DL, 2L, 200mL, 12mL

d) 7cL, 700mL, 7.7L, 70DL

3. Use your prefix table to convert the following measurements.

a) $6\text{L} = \quad \text{dL}$

d) $15\text{kL} = \quad \text{L}$

b) $9\text{cL} = \quad \text{L}$

e) $64\text{L} = \quad \text{mL}$

c) $36\text{mL} = \quad \text{cL}$

f) $119\text{L} = \quad \text{kL}$

4. Use your prefix table to convert and add the amounts together.

a) $5\text{L} + 500\text{mL} = \quad \text{mL}$

c) $140\text{L} + 3\text{kL} = \quad \text{kL}$

b) $4\text{L} + 34\text{cL} = \quad \text{cL}$

d) $5\text{dL} + 22\text{mL} = \quad \text{mL}$

5. Complete the following part whole diagrams, showing the whole measurement being made up of 2 parts.

a)

| | |
|---------|-------|
| 2L | 800mL |
| 2 800mL | |

d)

| | |
|-----|------|
| 5kL | 340L |
| L | |

b)

| | |
|---------|-------|
| L | 170mL |
| 2 170mL | |

e)

| | |
|------|------|
| 10kL | 250L |
| L | |

c)

| | |
|---------|----|
| L | mL |
| 2 450mL | |

f)

| | |
|-----|-----|
| 1kL | 50L |
| L | |

Whole Class Activity 6

Think: What relationships are there between the mass units?

You will need: 1kg lumps of play dough (or clay), scales that show grams and kilograms.

Break your 1kg lump of play dough into ten pieces that all weigh the same.

What fraction of a kilogram is each piece?



What is the name of this unit? (Use the prefix table to help).



Take one of these pieces and divide it into ten pieces that all weigh the same.

What fraction of a kilogram is each piece?



What is the name of this unit?



Take one of these pieces and divide it into ten pieces that all weigh the same.

What fraction of a kilogram is each piece?



What is the name of this unit?



A gram is the base unit for weight. Visualise breaking up one gram into ten pieces that all weigh the same.

What is the name of this unit?



How would you make a milligram from one of the gram pieces?



Milligrams are very, very small. What everyday things would you measure using milligrams?



Fill in the names of the units in the prefix table below.

Add their abbreviations in the last row.

| Kilo-unit | Hecto-unit | Deca-unit | Base unit | Deci-unit | Centi-unit | Milli-unit |
|------------|------------|-----------|-----------|---------------------|----------------------|-----------------------|
| 1000 units | 100 units | 10 units | 1 unit | $\frac{1}{10}$ unit | $\frac{1}{100}$ unit | $\frac{1}{1000}$ unit |
| 1 kilogram | | | 1 gram | | | |
| | | | g | | | |

Use the table to work out the following relationships.

How many decigrams = 1 gram?

How many grams = 1 decagram?

How many centigrams = 1 gram?

How many grams = 1 hectogram?

How many milligrams = 1 gram?

How many grams = 1 kilogram?

Use the playdough and the prefix table to describe as many other relationships between the units as you can. E.g 100 milligrams = 1 decigram



List two objects that match each measurement.

| Measurement | Object |
|----------------|---|
| 4 grams | 1. 2. |
| 100 milligrams | 1. Amount of caffeine in a cup of coffee. 2. |
| 2 kilograms | 1. 2. |

Whole Class Activity 7

Using estimation, take a chunk of play dough weighing less than 500 grams.

Use measuring equipment to work out how heavy it is.

Use the prefix table to describe this measurement in two different ways.

For example, A piece of play dough weighing 200 grams could also be described as 0.2kg.

| Weight of Playdough | Measurement 1 | Measurement 2 |
|---------------------|---------------|---------------|
| | | |
| | | |
| | | |

Discuss your measurements with two other people and record their measurements on your table.

Practice Exercise 3

1. Complete the table.

| Symbols | Words | Equivalent to |
|------------|-------------------------|---------------|
| a) 25g | | |
| b) | Three kilograms | |
| c) 0.9kg | | |
| d) 2 300mg | | |
| e) | Ninety eight centigrams | |
| f) 28kg | | |

2. Make and use a prefix table to order these measurements from smallest to largest.

a) 1kg, 11g, 11.1Dg, 110mg

c) 8dg, 800g, 80kg, 8.8g

b) 50g, 5.5kg, 0.55kg, 500mg

d) 14kg, 41g, 4.1kg, 4000mg

3. Use your prefix table to convert the following measurements.

a) 50g = kg

d) 780g = mg

b) 6kg = g

e) 800g = kg

c) 2 100mg = g

f) 90 000mg = kg

4. Use your prefix table to convert and add the amounts together.

- a) $5\text{kg} + 250\text{g} =$ g c) $900\text{mg} + 30\text{g} =$ g
 b) $70\text{g} + 2\,000\text{mg} =$ mg d) $23\text{kg} + 75\text{g} =$ kg

5. Complete the following part whole diagrams, showing the whole measurement being made up of 2 parts.

a)

| | |
|-----|------|
| 1kg | 500g |
| g | |

b)

| | |
|-------|---|
| 6kg | g |
| 6075g | |

c)

| | |
|---------|---|
| 10kg | g |
| 10 025g | |

d)

| | |
|----|------|
| 4g | 10mg |
| mg | |

e)

| | |
|--------|----|
| g | mg |
| 8100mg | |

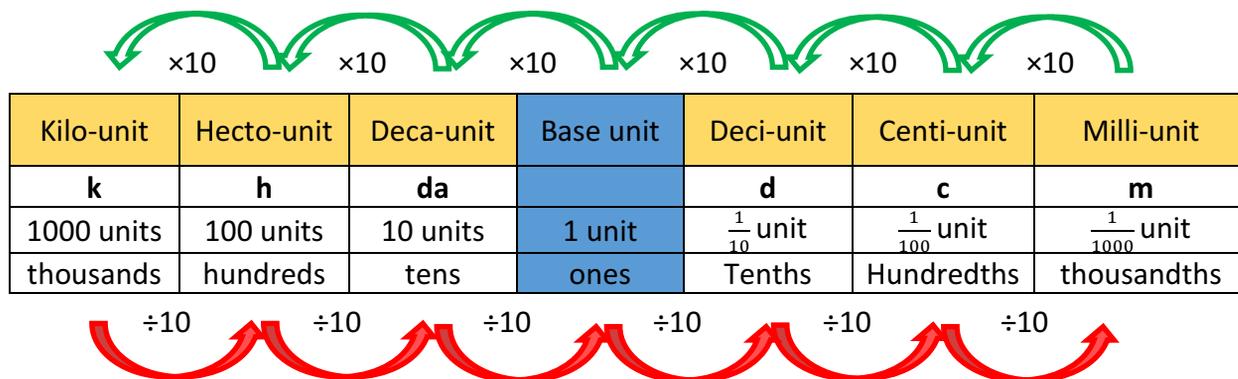
f)

| | |
|---------|----|
| g | mg |
| 9 099mg | |

Whole Class Activity 8

Think: How does the measurement prefix system relate to the place value system?

The metric system of measurement is directly linked to the place value system. We can multiply or divide to convert from one unit to another.



Use a calculator to multiply and divide the measurement by 10, to find how much of each unit the measurement is equal to.

| Kilo-unit | Hecto-unit | Deca-unit | Measurement | Deci-unit | Centi-unit | Milli-unit |
|-----------|------------|-----------|-------------|-----------|------------|------------|
| km | hm | dam | 1m | dm | cm | mm |
| | | | 5m | | | |
| | | | 100m | | | |
| | | | 1 000 | | | |
| | | | 1L | | | |
| | | | 10L | | | |

| | | | | | | |
|--|--|--|--------|--|--|--|
| | | | 150L | | | |
| | | | 1g | | | |
| | | | 10g | | | |
| | | | 250g | | | |
| | | | 1 000g | | | |

Reflection and Discussion

Some metric conversions are used more often than others.

In the building industry metres and millimetres are used more often than centimetres.

What are the reasons for this?



Which building measurements use metres?



Which building measurements use millimetres?



How do you convert metres to millimetres?



How do you convert millimetres to metres?



Practice Exercise 4

1. Complete the sentences describing how many times bigger or smaller the first unit is compared to the second unit.

- A decimetre is 10 times smaller than a metre.
- A kilogram is _____ times _____ than a gram.
- A millilitre is _____ times _____ than a litre.
- A metre is _____ times _____ than a hectometre.
- A milligram is _____ times _____ than a gram.
- A kilolitre is _____ times _____ than a litre.

2. There are more prefixes in the metric system than those we have discussed. Research the prefixes mega, giga, tera, micro and nano. Where are they used in everyday life? Create a table below to show how each of these units relate to a base unit.

Reflection on Learning

England uses a mixture of imperial and metric measurements. Imperial measurements include yards, feet, inches, pounds and ounces.

Research on the internet to find out the difference between the two measurement systems.



Standard Imperial measures on the gate at the Royal Observatory in Greenwich.

Write a persuasive text to an English person explaining the benefits of using the metric system.

OLNA Practice Questions

1. 7.4 kilometres is the same as:

- A: 74 metres B: 7400 metres C: 740 metres D: 0.74 metres

2. Which of these shows the units in order from smallest to largest.

- A: centimetre, kilometre, metre, hectometre
B: litre, kilolitre, millilitre, decilitre
C: decagram, centigram, milligram, decigram
D: millimetre, centimetre, metre, hectometre, kilometre

3. A water tank has a capacity of 8.75 kilolitres. How many litres does the water tank hold when it is full?

- A: 875 L B: 8075 L C: 8750 L D: 87 500 L

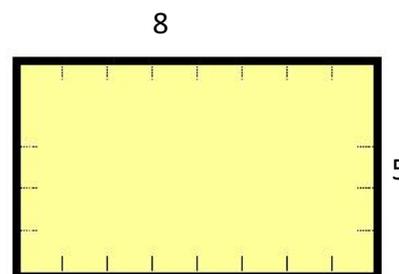
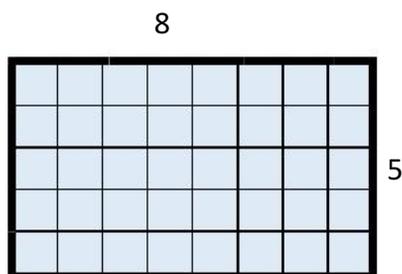
Topic 2

Area, Perimeter and Volume

Mathematics Discussion

We use perimeter and area to measure two dimensional shapes. Perimeter is a measure of how far it is around the outside of a shape. Area is a measure of how much space is within a shape. Volume is a measure of the amount of space within, or taken up by, a three dimensional object.

We use length measures to calculate perimeter, area and volume measures. They can all be calculated using a formula, however it is helpful to know where the formula comes from. In the shapes below, the perimeter can be found by counting the squares along the length and width of the shape $8 + 5 + 8 + 5 = 26$ or using the formula $P = 2(l + w)$. The area can be found by counting how many squares are inside the shape, $8 + 8 + 8 + 8 + 8 = 40$, which is the same as $8 \times 5 = 40$. The formula for area is $A = l \times w$



Volume can be found by counting the cubes needed to fill an object or to take up as much space as an object. In the object to the right, the base is made up of 4 rows of 3 cubes, $3 + 3 + 3 + 3 = 12$ ($4 \times 3 = 12$) and there are two layers, $2 \times 12 = 24$. The formula for Volume is $V = l \times w \times h$.



We typically measure perimeter using linear units such as, millimetres, centimetres, metres and kilometres. We measure area using square units such as square centimetres, square metres and square kilometres. We typically measure volume using cubic units such as cubic centimetres and cubic metres.

Whole Class Activity 1

Think: What is perimeter?

Read through the situations listed below. Colour the situations that are about perimeter.

| | | |
|--|---|--|
| You need to buy tiles to cover the bathroom floor. | Frank needs to know if the load of mulch will fit into the tray of his ute? | The greenkeeper at the golf course is working out how much sand is needed for the sand trap on hole 3. |
| The fence around the chicken coop needs to be replaced on all sides. | Pip has built a new house and she needs to know how much turf to order to cover the front yard. | The local primary school needs a security fence installed around its boundary. |

Explain what you are measuring when you measure the perimeter of something?



On the following page there are some coloured shapes.

Discuss with a classmate how you would find the perimeter of each shape.



Work out the perimeter for the shapes A to E and record the amounts below.

A B C D E

What did you do to work out the perimeter?



Is there a different way to calculate the perimeter?



How would you write this as a rule?



Try out this method with shapes D and E. Show your working below.



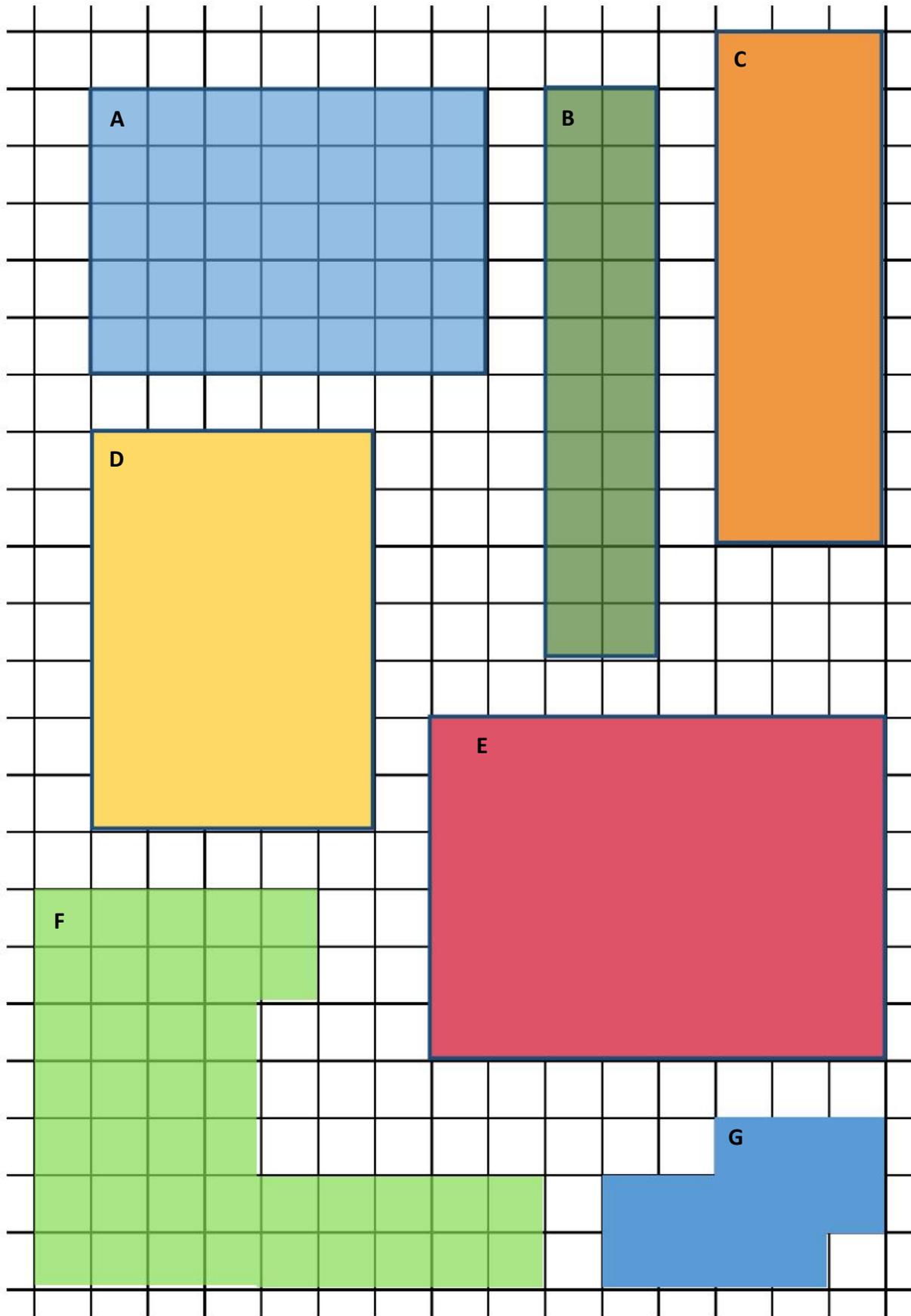
Look at shapes F and G.

How could you work out the perimeter of these shapes, without counting each square?



Calculate the perimeter of shapes F and G, showing your working below.





1 square is 1cm × 1cm

Reflection and Discussion

“Perimeter is a measurement of length.”

Discuss the meaning of this statement. Do you agree? Why/Why not?



Practice Exercise 1

1. Use the plan of the vegetable garden on page 252 to answer the following questions. Show your workings.

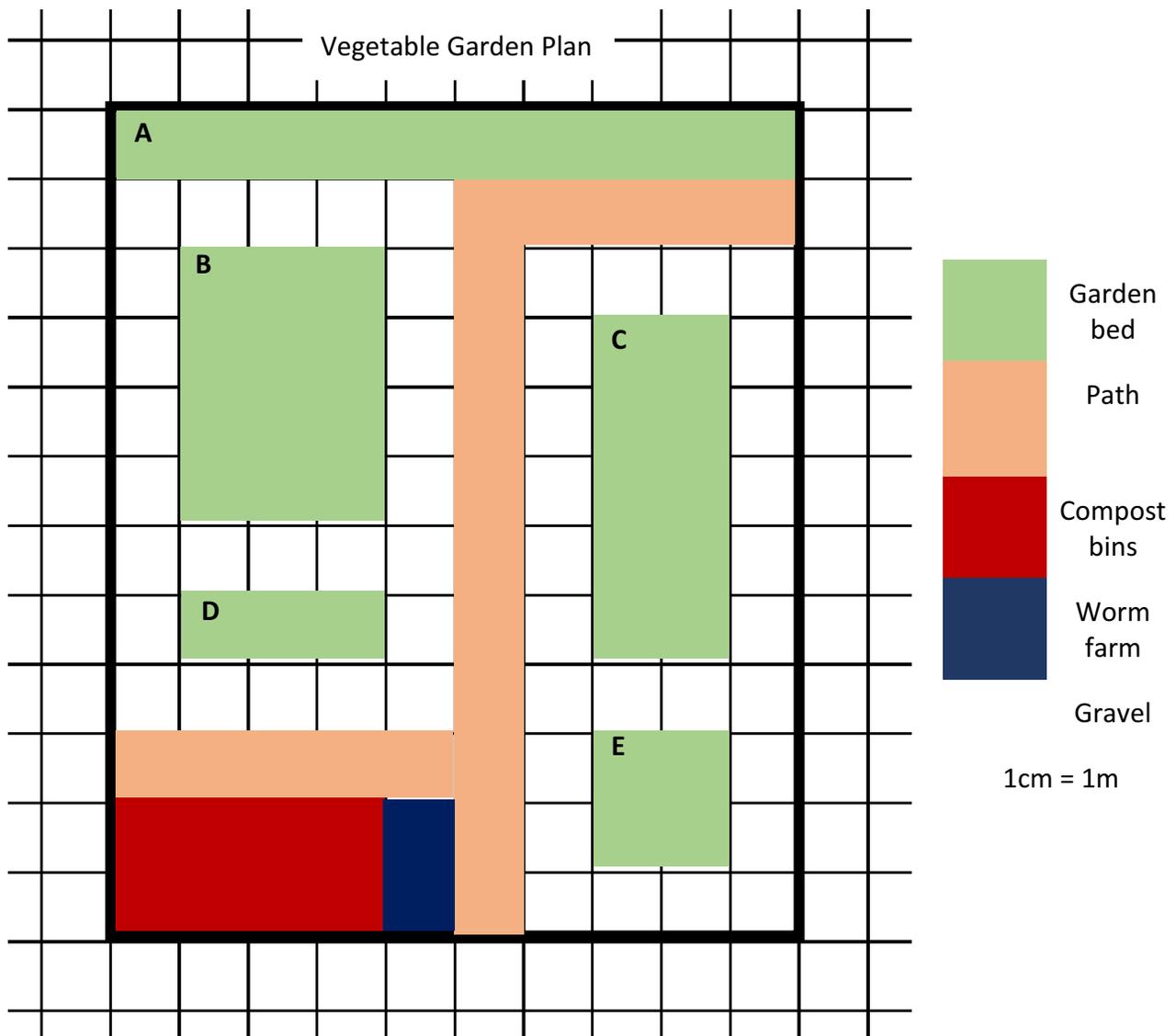
a) Fencing needs to be purchased to go around the boundary of the whole garden. What length of fencing needs to be ordered?

b) Garden bed B is a raised bed, so the edges will be made out of corrugated steel. What length of steel is needed to make the bed?

c) The path needs to be edged with timber. What length of timber is needed to complete this task?

d) Garden beds C and E need reticulation pipe around their edges. What length of piping is needed to reticulate both beds?

e) Garden bed D is for growing climbing plants and needs wire along all four sides, at three different heights. What is the total length of wire needed?



2. Read each student's estimate. Use the vegetable garden plan to decide if their estimate is reasonable or not. Justify your choice. If the estimate is unreasonable provide a reasonable estimate.



a) Tara estimates the perimeter of the compost bins and worm farm is 7000cm.

This estimate is reasonable unreasonable.

Justify:

What would be a more reasonable estimate?



b) Sann estimates the perimeter of garden bed D is half the size of garden bed B.

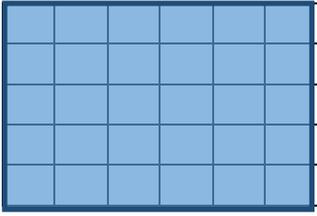
This estimate is reasonable unreasonable.

Justify

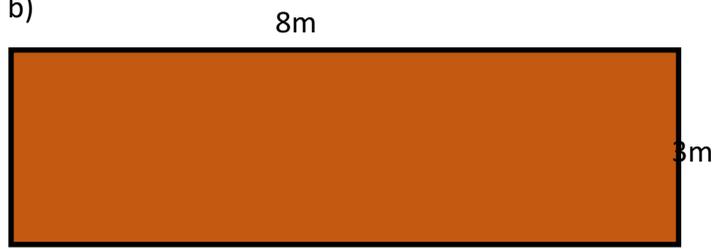
What would be a more reasonable estimate?

3. Calculate the perimeter of the following shapes.

a)

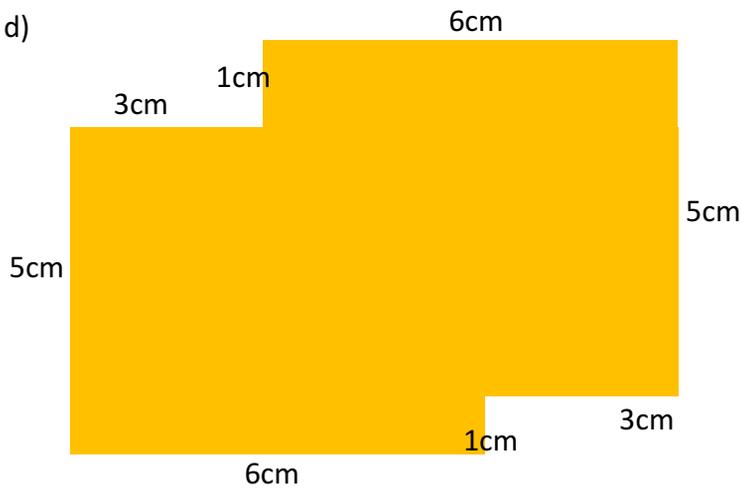


b)

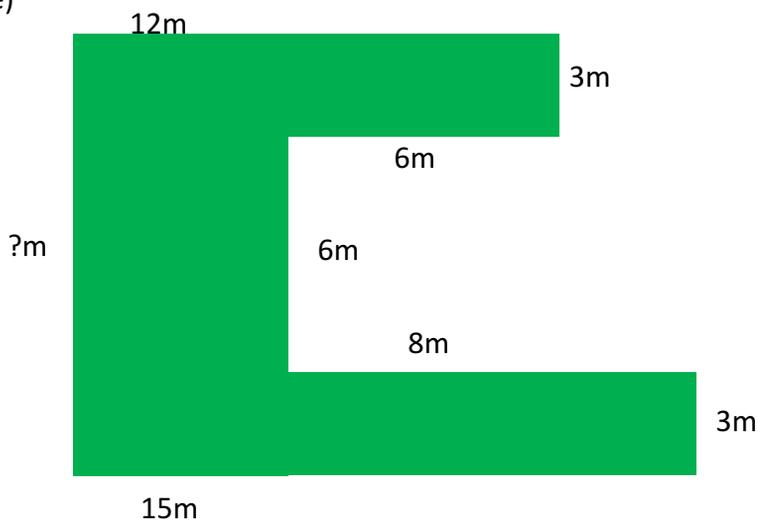


c) A rectangle with long sides 23m and short sides 15m.

d)



e)



4. What unit should be used for each of the answers in exercise 1 to 3 above? If you have not written the units, then go back and write them now.

Whole Class Activity 2

Think: What is area?

Read through the situations listed below. Colour the situations that are about area.

| | | |
|--|---|--|
| You need to buy tiles to cover the bathroom floor. | Frank needs to know if the load of mulch will fit into the tray of his ute? | The greenkeeper at the golf course is working out how much sand is needed for the sand trap on hole 3. |
| The fence around the chicken coop needs to be replaced on all sides. | Pip has built a new house and she needs to know how much turf to order to cover the front yard. | The local primary school needs a security fence installed around its boundary. |

Explain what you are measuring when you measure the area of something?



On page 250 there are some coloured shapes.

Discuss with a classmate how you would find the area of each one.



Work out the area for the shapes A to E. Record the areas below.

A B C D E

What did you do to work out the area?



Is there a different way to calculate the area?



How would you write this as a rule?



Try out this method with shapes C and E.



Look at shapes F and G.

How could you work out the area of these shapes, without counting each square?



Calculate the area of shapes F and G, showing your working out below.



Reflection and Discussion

Area measurements are expressed in square units. In the previous activity you found out how many square centimetres of area each coloured box covered.

What is the symbol for squared centimetres?



What other units have you seen or heard area measured with?



Whole Class Activity 3

Think: What does one square metre look like?

You will need: Newspaper, sticky tape, scissors, measuring tapes.

Use the newspaper to make a 1 square metre.

What is the length of each side?



Use your 1 square metre to measure the areas of some parts of the school. Record your measurements on the table.

| Location | Estimated number of square metres | Actual number of square metres |
|----------|-----------------------------------|--------------------------------|
| | | |
| | | |
| | | |

A square metre does not always have to be a 1m by 1m square, it just needs to take up the same amount of space.

Using the newspaper and sticky tape, make a model of 1 square metre, where the lengths of the sides are **not** both 1 metre long.

Draw a picture of your model and label the length of its sides. Explain how you know it is still 1 square metre of area.



Reflection and Discussion

We abbreviate *centimetres* to *cm* and *metres* to *m*.

How should we abbreviate *one square centimetre*?

How should we abbreviate *one square metre*?

Practice Exercise 2

1. Use the vegetable garden plan on page 252 to answer the following questions. Show your working out.

a) How much area are the compost bins taking up?

b) What is the total area of the vegetable garden?

c) Which garden bed takes up the largest area?

2. Read each student's estimate. Use your knowledge of units and the vegetable garden plan to decide if their estimate is reasonable or unreasonable. Justify your choice. If the estimate is unreasonable provide a more reasonable estimate.



a) Tegan estimates there will be 50m^2 of gravel needed for the garden.

This estimate is reasonable unreasonable.

Justify:

What would be a more reasonable estimate?



b) Tristan estimates that the total area of the garden beds is 600m^2 .

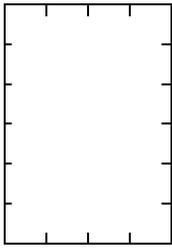
This estimate is reasonable unreasonable.

Justify:

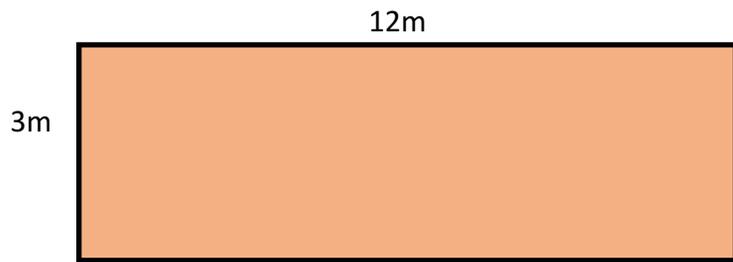
What would be a more reasonable estimate?

3. Work out the area of each of these shapes

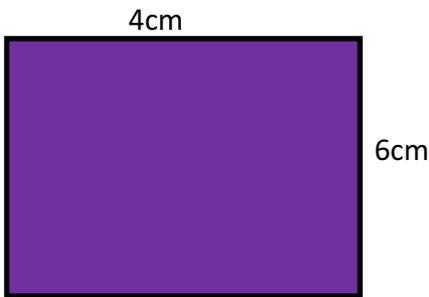
a)



c)



b)



d) A square with a perimeter of 12 metres.

4. What unit should be used for each of the answers in exercise 1 to 3 above? If you have not written the units, then go back and write them now.

Whole Class Activity 4

Think: How do you find the area of shapes made up of rectangles or shapes that are irregular?

Here is the path that goes through the vegetable garden.

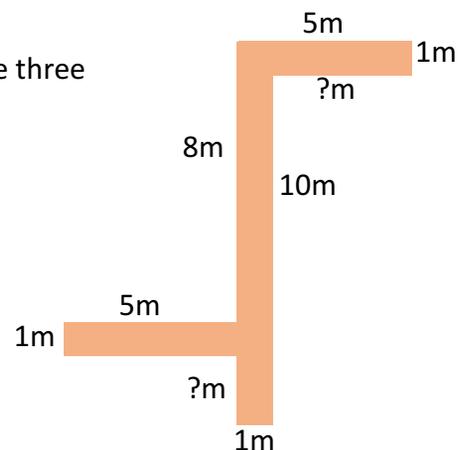
It is made up of 3 rectangles. Draw lines so you can see the three rectangles clearly.



Compare with a partner, where they put their lines.



Work out the measurements shown by the question marks.



The path will be made using roll on turf, so you need to know the area of the path.

How could you work out the area of the path?



Share your idea with the class.

Work out the area of the path. What measurement will you use to describe the area of the path?



What shortcuts did you use to calculate the area of the path?



Reflection and Discussion

What should you do whenever you have to find the area of a shape that is made up of more than one rectangle?

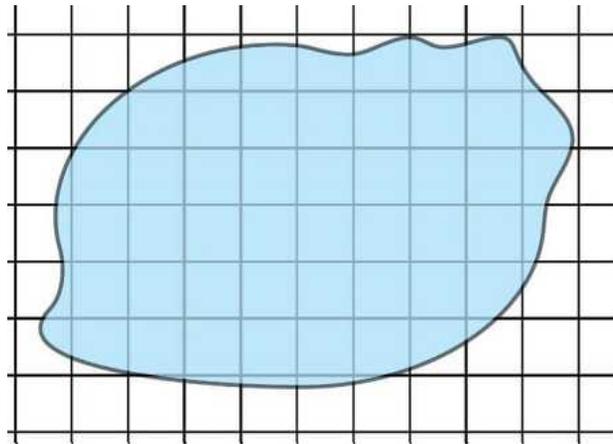
Discuss with a partner and write down your ideas.



Here is a plan for the frog pond that will go next to the vegetable garden. Each square is 1 metre by 1 metre.

The builders need to know the area of the pond.

How could you work out the area, without counting each square?



Share your idea with the class.

Work out the area of the frog pond.



What shortcuts did you use to calculate the area of the path?



What unit of measurement will you use to describe the area of the frog pond?



Reflection and Discussion

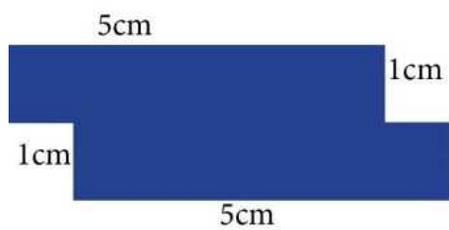
What should you do whenever you have to find the area of an irregular shape?

Discuss with a partner and write down your ideas.

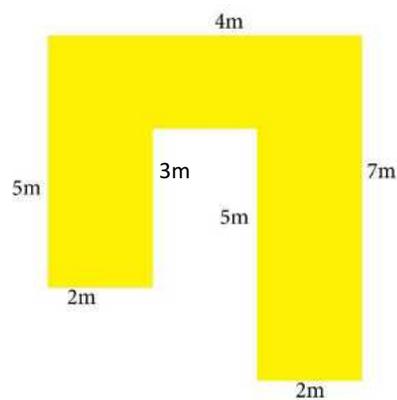


Practice Exercise 3

1. Calculate the area of these shapes.



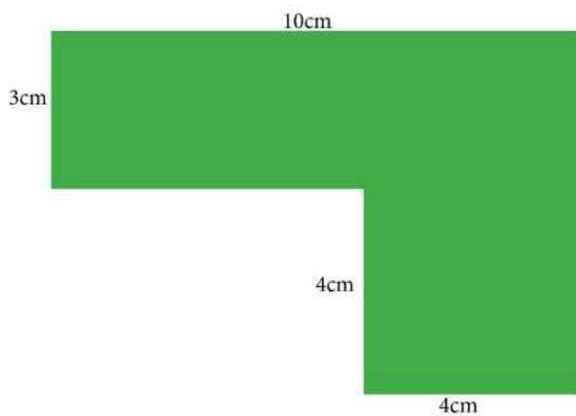
a)



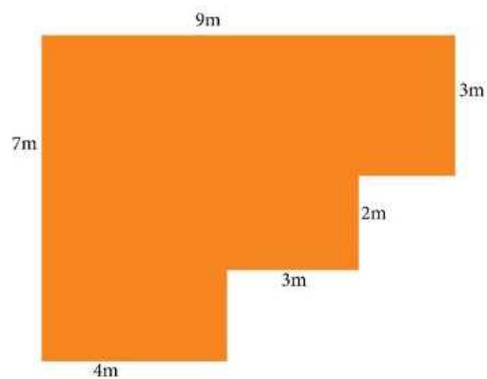
c)

6m

b)

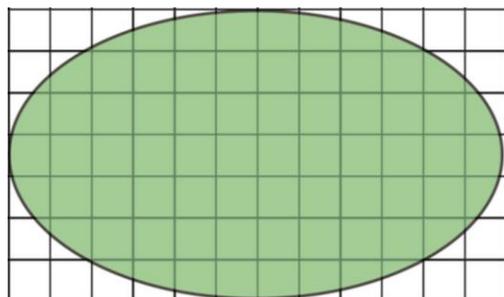


d)

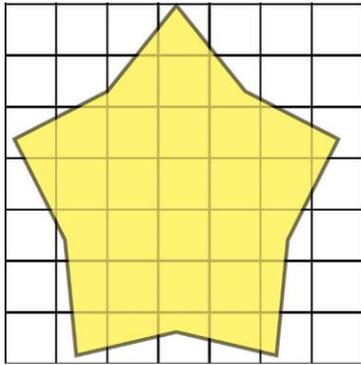


2. Calculate the area of the irregular shapes.

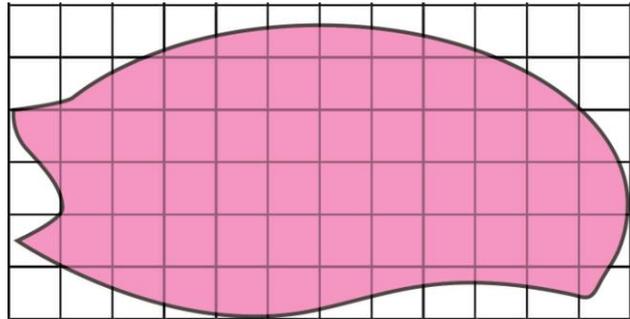
a)



b)



c)



Whole Class Activity 5

Think: What is volume?

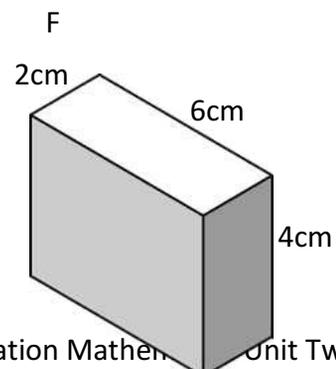
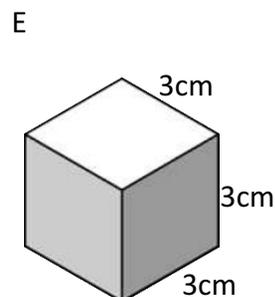
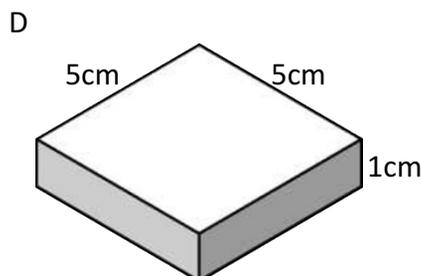
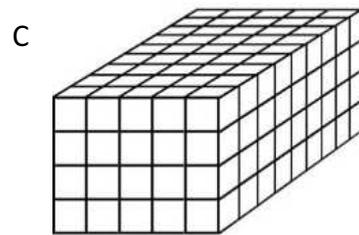
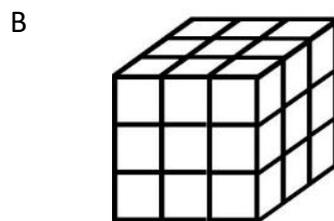
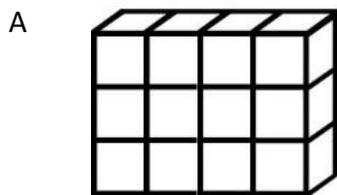
Read through the situations listed below. Colour the situations that are talking about volume.

| | | |
|--|---|--|
| You need to buy tiles to cover the bathroom floor. | Frank needs to know if the load of mulch will fit into the tray of his ute? | The greenkeeper at the golf course is working out how much sand is needed for the sand trap on hole 3. |
| The fence around the chicken coop needs to be replaced on all sides. | Pip has built a new house and she needs to know how much turf to order to cover the front yard. | The local primary school needs a security fence installed around its boundary. |

Explain what you are measuring when you measure the volume of something?



Here are some rectangular prisms. Discuss with a classmate how you would find the volume of each one.



Work out the volume for the shapes.

A

B

What did you do to work out the volume?



Is there a different way to calculate the volume?



How would you write this as a rule?



Try out this method with shapes C and D.



Look at shapes E and F.

How could you work out the volume of these shapes, without counting each square?



Calculate the volume of shapes E and F, showing your working below.



Reflection and Discussion

Volume measurements are expressed in cubic units. In the previous activity you found out how many cubic centimetres of area each coloured box was.

What is the symbol for cubic centimetres?



What other units have you seen or heard volume measured with?



Reflection and Discussion

Mulch is used to prevent weeds from growing and help conserve water. Mulch is sold in bulk by the cubic metre.

How many cubic metres of mulch is needed to cover all the garden beds with 10cm of mulch?



Whole Class Activity 6

Think: What does one cubic metre look like?

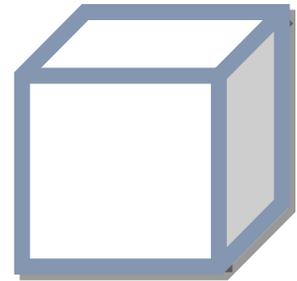
You will need: Newspaper, sticky tape, scissors, measuring tapes.

Tightly roll the pages of newspaper, from corner to corner, into cylinders. Use these to make a skeletal model of one cubic metre.

What is the length of each side?



Use your 1 cubic metre to measure the volume of some parts of the school. Record your measurements on the table.



| Location | Estimated number of cubic metres | Actual number of cubic metres |
|----------|----------------------------------|-------------------------------|
| | | |
| | | |
| | | |

A cubic metre does not always have to be a 1m by 1m by 1m cube, it just needs to take up the same amount of space.

Using the newspaper cylinders and sticky tape, make a model of 1 cubic metre, where the lengths of the sides are **not** 1 metre long.

Draw a picture of your model and label the length of its sides. Explain how you know it is still 1 cubic metre of volume.



Reflection and Discussion

We abbreviate *centimetres* to *cm* and *metres* to *m*.

How should we abbreviate *one cubic centimetre*?

How should we abbreviate *one cubic metre*?

Practice Exercise 4

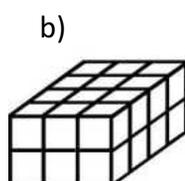
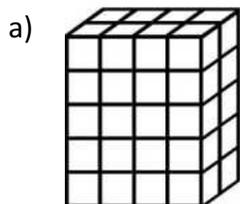
1. The models below are components of the vegetable garden on page 252.

Garden Bed B

Worm Farm

Compost Bins

Use the vegetable garden plan to help you match the label to the model and work out the answers to the problems.

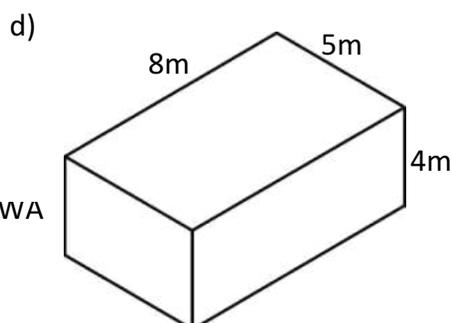
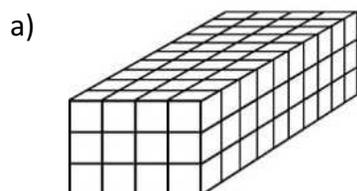


a) How many cubic metres of compost will fit in the compost bins?

b) How many cubic metres of soil is needed to fill garden bed B?

c) What is the volume of the worm farm?

2. Work out the volume of each of these rectangular prisms.



b)

c)

e)

4. What unit should be used for each of the answers in exercise 1 & 2 above? If you have not written the units, then go back and write them now.

Whole Class Activity 7

Think: How do you find the volume of shapes that are made up of rectangular prisms?

Here is a mould for pouring concrete to make steps. It is made up of two rectangular prisms.

Draw lines so you can see the two rectangular prisms clearly.



Compare with a partner, where they put their lines.



How could you work out the volume of cement needed to fill the mould?

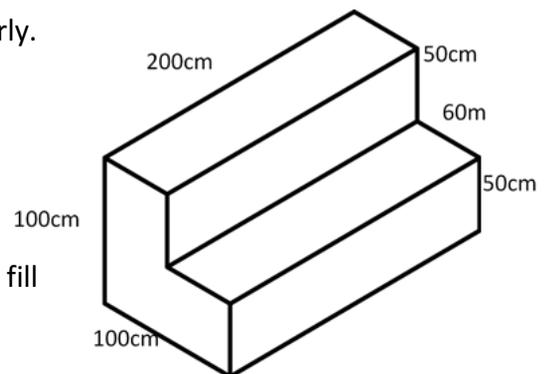


Share your idea with the class.

Work out the volume of the mould. What measurement will you use to describe the volume of the mould?



What shortcuts did you use to calculate the volume of the mould?



Reflection and Discussion

What should you do whenever you have to find the volume of a shape that is made up of more than one rectangular prism?

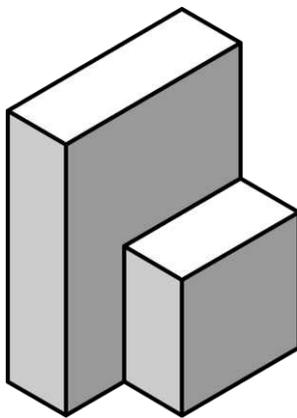
Discuss with a partner and write down your ideas.



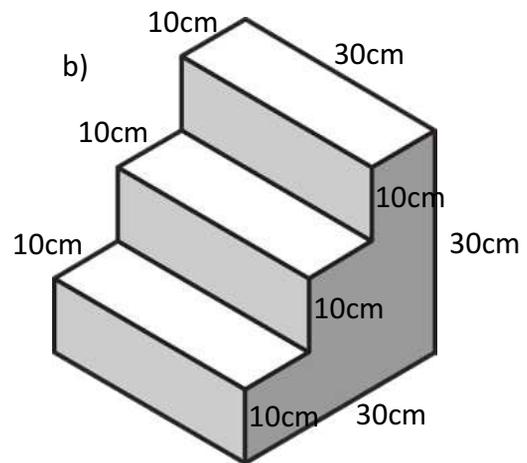
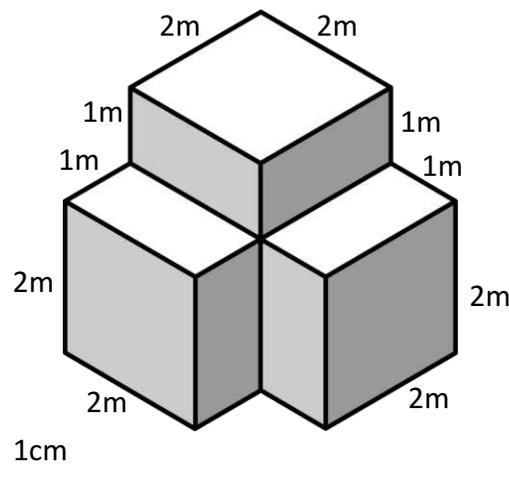
Practice Exercise 5

1. Calculate the volume of these solids.

a)



c)



Reflection on Learning

Dave wants to paint his bedroom walls. What information will help him decide how much paint to buy?

- 4cm
- volume of room 2cm
- capacity of room 2cm
- perimeter of all walls
- area of all walls

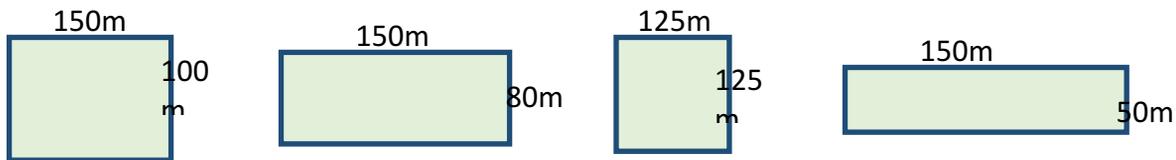
Explain the reasons for your choice and giving reasons why the other information choices are not helpful.



OLNA Practice Questions

1. A rectangular paddock has a perimeter of 70 metres. Each long side has a length of 25 metres. What is the length of each short side?

2. A farm has 4 paddocks. Which paddock has the largest area?



A

B

C

D

3. A factory makes rectangular metal boxes. The height of each box is 0.8 metres. Which box has a volume of 0.16 cubic metres? (base $-b$, sides $-s$)

A. $b = 0.4$ m, $s = 0.4$ m

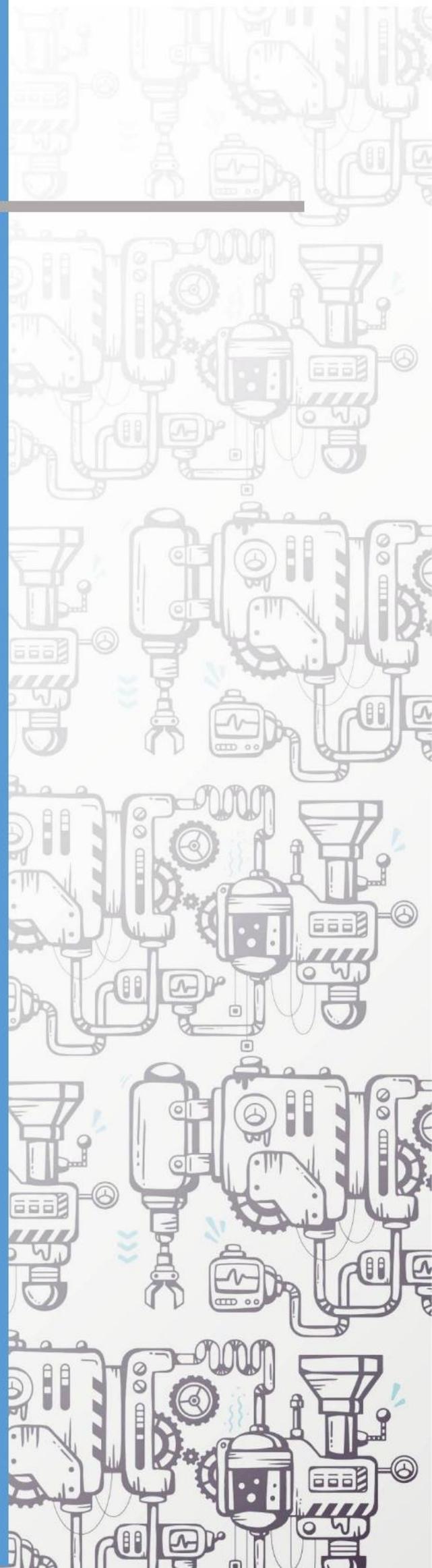
B. $b = 0.5$ m, $s = 0.3$ m

C. $b = 0.5$ m, $s = 0.5$ m

D. $b = 0.4$ m, $s = 0.5$ m

Section Four

Probability



Content Focus

Mathematics Foundation

- 2.5.1 Identify and describe situations which involve the element of chance in everyday life and work
- 2.5.2 Describe the likelihood of everyday chance events using terms such as certain, likely, equally likely, fifty/fifty, impossible
- 2.5.3 Compare and order chance events from least likely to most likely, providing reasoning from personal experience or based on data
- 2.5.4 Recognise and use elements of chance to make decisions in everyday life and work
- 2.5.5 Explain decisions based on likelihood of events

Australian Curriculum Link

- ACMSP047 Identify practical activities and everyday events that involve chance. Describe outcomes as 'likely' or 'unlikely' and identify some events as 'certain' or 'impossible'
- ACMSP092 Describe possible everyday events and order their chances of occurring
- ACMSP093 Identify everyday events where one cannot happen if the other happen
- ACMSP094 Identify events where the chance of one will not be affected by the occurrence of the other
- ACMSP116 List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions
- ACMSP117 Recognise that probabilities range from 0 to 1

Topic 1

The Probability of Everyday Events

Mathematics Discussion

We tend to use the everyday language of chance without really noticing. We say that it is 'likely' to rain, or that we 'might' go to the shops after school. We are predicting how likely an event is to happen (or not to happen), based on our past experiences or on data.

We can compare and order everyday chance events from least likely to most likely to happen. Some events are impossible – that is, they have no chance of happening and some events are certain – that is, they will definitely happen. All other events have a degree of uncertainty, which is why we like to predict what might happen. Predicting the likelihood of events allows us to have some control over the events in our daily lives. For example, predicting how long it takes to get to the bus stop will ensure that we don't miss the bus.

We use a range of words to describe the likelihood of events happening, such as certain, likely, equally likely, fifty/fifty, fair, probable, improbable, unlikely and impossible.

Sometimes we can collect data or use existing data on how often an event has happened to help us predict how likely it is to happen in the future. For example, we may look at data on previous wins and losses of AFL football teams when participating in a Footy Tipping Competition.

Whole Class Activity 1

The Language of Chance

Think: *Where do we see situations involving chance in our everyday work and life?*

1. Brainstorm with your classmates examples of chance that we see in our everyday lives (e.g. weather, Lotto). List the ideas in the space below.

2. Brainstorm with your classmates words and phrases used in everyday life that describe chance (e.g. highly likely, impossible). List the ideas in the space below

3. Circle 8 words or phrases from your list above. Rank them in order from least likely to most likely in the space below.

4. Write the following phrases on paper and then take turns to place them in order from least likely to most likely.

- You will drink a glass of water tomorrow
- You will see a cat wearing pyjamas singing the national anthem
- You will roll an even number on a dice
- The capital of Bali will still be called Denpasar tomorrow
- Your Maths teacher will arrive at school tomorrow with purple hair
- When you toss two coins you will get two tails
- You will win Division 1 Lotto
- The temperature tomorrow will be between 25 and 35 degrees Celsius

Did you all agree with the order these events were placed in? Why or why not?



Which of these events are impossible? What does 'impossible' mean?



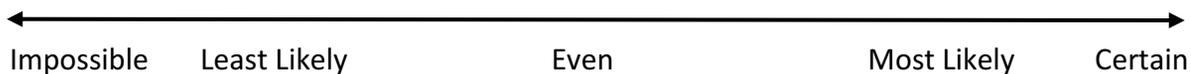
Which of these events are certain? What does 'certain' mean?



Which of these events have an even chance of happening? What other words or phrases are used for 'even chance'?

Practice Exercise 1

1. Write down 4 chance events that you feel have no chance of happening (i.e. impossible)
2. Write down 4 chance events that you feel certainly WILL happen.
3. Write down 4 chance events that you feel have a fifty-fifty chance of happening?
4. Place the following events on the number line below using the letters A, B, C e.t.c.
 - A. The next baby being born at Port Hedland Hospital being a girl
 - B. A 5 being rolled from a normal 6 sided dice
 - C. A diamond being drawn from a pack of cards
 - D. The siren at the end of the day being late
 - E. A red or black card being drawn from a standard 52 card pack.
 - F. Raining in July in Perth



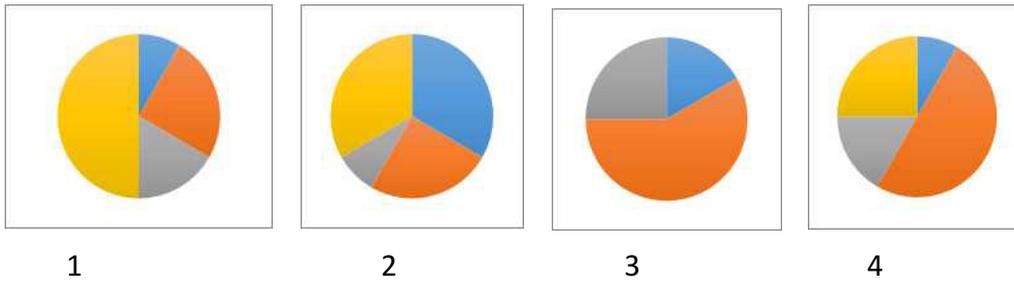
5. Play the activity called 'Foul Food!' on the following website:

<http://www.scootle.edu.au/ec/viewing/L213/index.html>

What words and phrases did they use to describe chance? Write the words below, and describe the meaning of each in relation to the number of choices available for each food.

6. Six marbles, 3 red, 2 green and 1 blue are placed in a bag. One marble is randomly drawn out and the colour noted.
 - a) What colour is least likely to be drawn out?
 - b) What colour has no chance of being drawn?
 - c) What colour has an even chance of being drawn?
 - d) What colour is more likely to be drawn out than blue but less likely than red?
 - e) A red marble is drawn out and then put back in the bag. What is the chance of a red marble being drawn out again on the next draw? Justify your answer
 - f) A blue marble is drawn out and then NOT put back in the bag. What is the chance of it being drawn out again on the next draw? Justify your answer

7. The following are spinners in a game of chance:



- Which spinner has no chance of landing on yellow?
- Order the spinners from least likely to most likely to land on yellow.
- Order the spinners from most likely to least likely to land on blue.
- Which spinner has a fifty-fifty chance of landing on orange?
- Jayden notices there are 4 colours on spinner 2. He tells his friend Julie that there is a quarter chance of the spinner landing on blue. Is Jayden correct? Why or why not?

8. Watch the following you tube clip showing a cat playing a game of chance:

<https://www.youtube.com/watch?v=GoXHVs65NFO>

Describe the chances of the cat:

- Picking the cup with the bell in it when there were 3 cups.
- Not picking the cup with the bell in it when there were 3 cups.
- Picking the cup with the bell in it when there were 4 cups.
- Not picking the cup with the bell in it when there were 4 cups.
- Do you think the cat was randomly choosing the correct cup each time? What did the cat do to improve its chances of picking the cup with the bell in it?

Whole Class Activity 2

The Likelihood of Events based on data

Play 3 games of 'Hangman' as a class. Record the letters used and the number of guesses until the word is found.



Do all letters have an equal chance of occurring? Discuss with your class



Work out the best letters to choose:

Read a paragraph of text and count how many times each letter occurs. Allocate a letter or letters to each class member. Record the number of times each letter occurs in the table using a tally. An example is shown for 'X' below.

| | | | | | | | | | | | | |
|-------|---|---|---|---|---|---|---|---|---|-----------|---|---|
| A | B | C | D | E | F | G | H | I | J | K | L | M |
| | | | | | | | | | | | | |
| Total | | | | | | | | | | | | |
| N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| | | | | | | | | | | e.g. IIII | | |
| Total | | | | | | | | | | | | |

Which letter(s) occurs most often? Which letter(s) occur least often? Which letters should you choose when playing 'Hangman'? What order should you call them?



Play the game again using EXACTLY the same words as before. This time, use the letters that occur most often as shown by the tally. Record the number of guesses needed.

Did knowing which letters occur most often help reduce the number of 'guesses' needed to 'solve' the word? Discuss.



How can we use chance and data to help us predict outcomes? Write a paragraph describing the purpose of this activity. Include the words 'more likely', 'chance', 'predict' and 'data' in your paragraph.



Practice Exercise 2

1a) Play three games of noughts and crosses with a partner in the space below:

b) Play three more games in the space below but after placing each nought or cross, predict how likely you are to win.

c) Were your predictions accurate? Why or why not?

d) How could you improve the likelihood of winning in a game of noughts and crosses?

2. The game of 'Heads and Tails' is played by people standing up and placing two hands on their head for two 'Heads', two hands on their buttocks for two 'Tails' and one hand on their head and one on their buttock for a 'Head and a Tail'.

A spinner tosses two coins in the air and calls out how they land. Players remain standing if they have their hands in the position called by the spinner. All others sit down.

Standing players may choose to keep their hands in their current position or switch to a new head/tail combination. The spinner then tosses the coins for the next round of the game.

The last player standing is the winner.

a) List the possible head/tail combinations for the game. Which head/tail combination is most likely to happen? Why?

b) Julie decided to place both hands on her head. She thought she had a one in three chance of winning. Is Julie correct? Discuss.

c) A head and a tail comes up 4 times in a row. Julie is sure the next toss will result in two heads or two tails. Is Julie correct? Discuss.

d) If the two coins were tossed 64 times, how many times would you expect:

- Two 'Heads' to happen?
- Two 'Tails' to happen?
- A 'Head and a Tail' to happen?

e) Toss two coins 32 times and then double the results. Are your results the same as you predicted? Why or why not?

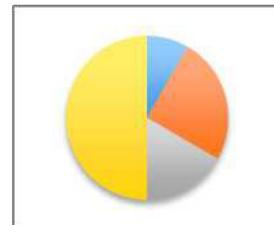
3. Jayden was playing a board game using spinner 1 from Practice Exercise 1.

If the spinner was spun 120 times, predict the number of times you would expect it to land on:

a) yellow?

b) orange?

c) black?



4. The following website shows weather in towns across Australia:

www.weatherzone.com.au/

a) Use the website to record the maximum temperature in your town or suburb for the last 5 days in the space below.

b) Use the information to predict the maximum temperature for the following day.

c) What was the website's prediction for the maximum temperature the following day? What factors would they base their prediction on?

d) Check your prediction on the following day. Was your prediction accurate? Was 'weatherzone's' prediction accurate? Why or why not?

e) Repeat steps a – d using the rainfall in your town over the last 5 days.

5. The Cancer Council of WA reports that *'At least 2 in 3 Australians will be diagnosed with skin cancer before the age of 70. The risk is higher in men (2 in 3) than in women (3 in 5)'*. (<https://www.cancerwa.asn.au/prevention/sunsmart/>)

a) The population of Australia in 2014 was approximately 24 000 000. Use your calculator to find the approximate number of these Australians who will have skin cancer by the time they are 70.

b) The population of women in Western Australia in 2011 was approximately 1 200 000. Use your calculator to find the approximate number of these women, who will have skin cancer by the time they are 70.

The Cancer Council of WA list the following factors that increase a person's chance of getting skin cancer.

Risk factors

Most people living in Australia are at risk of developing skin cancer. You are at higher risk of developing skin cancer if you have:

- Fair skin that burns easily and does not tan
- Blue or green eyes and/or fair or red hair
- Suffered sunburn as a child
- Spent your childhood in Australia
- A large number of freckles or moles
- A family or personal history of skin cancer
- Used a solarium
- Worked or spent a lot of leisure time in the sun

c) Circle the risk factors above that relate to you.

d) List some ways of reducing the chance of getting skin cancer by referring to the risk factors above. You may wish to view the Cancer Council website: ([https://www.cancerwa.asn.au/prevention/sunsmart/.](https://www.cancerwa.asn.au/prevention/sunsmart/))

6. Tom and Katie were planning a trip to Beijing in China in 2017. They used data showing average maximum temperatures and average rainfall to work out the best time to visit the city.

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|--------------------------------|-----|-----|------|------|------|------|-------|-------|------|------|------|-----|
| Average Maximum Temperature °C | 1.6 | 4 | 11.3 | 19.9 | 26.4 | 30.3 | 30.8 | 29.5 | 25.8 | 19 | 10.1 | 3.3 |
| Average Rainfall (mm) | 2.6 | 5.9 | 9 | 26.4 | 28.7 | 70.7 | 175.6 | 182.2 | 48.7 | 18.8 | 6 | 2.3 |

Use the data above to predict:

- The month(s) where it is most likely to rain.
- The month(s) where it is least likely to rain.
- The month(s) where it is likely to be very hot.
- The month(s) where it is likely to be very cold.
- The weather in Beijing in November.
- The weather in Beijing in July.
- The best month(s) for travelling to Beijing. Justify your answer using the temperature and rainfall data.

Reflection on Learning

Think: how do you predict the events of daily life?

a) Complete the following in each column;

COLUMN A: List 8 things that you think will happen in your life over the next 24 hours.

COLUMN B: Use a word or phrase such as certain, highly likely, unlikely etc. to describe each of the events.

COLUMN C: Describe how you could improve the likelihood of this event happening.

COLUMN D: Describe how you could decrease the likelihood of the event happening.

| A | B | C | D |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

b) 24 hours later, refer to your list. Place a tick next to the events that happened. Place a cross next to the events that did not happen.

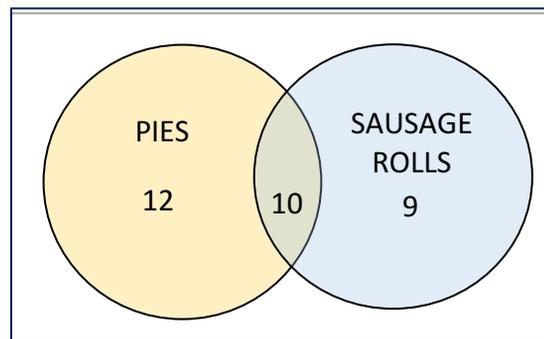
- c) What did you base your predictions of the likelihood of the events happening on (refer to COLUMN B)? Were your predictions accurate?
- d) In hindsight, was there anything else that you could have done to ensure an event did happen?
- e) How could you use the knowledge you have gained from this reflection activity to improve the chances of events happening in your life?

OLNA Practice Questions

1. Janice throws a standard 6-sided die. Janice's chance of throwing a number bigger than 1 could best be described as:

- A. Fifty-fifty B. Highly Likely C. Impossible D. A fair chance

2. The following Venn Diagram shows the results from a survey of 50 people who were asked 'Do you like pies or sausage rolls?'



A person is chosen at random from the survey and asked about their pie or sausage roll preference.

Who is this person most likely to be? A person who:

- A. Likes pies but not sausage rolls C. Likes sausage rolls only
- B. Likes both pies and sausage rolls D. Likes neither pies nor sausage rolls

Answers

Section 1 Fractions and Decimals

Topic 1 Read, Write and Count Fractions

PRACTICE EXERCISE 1

1 a) one third b) two quarters c) two sixths d) five eighths e) three fifths f) two halves g) six tenths h) five quarters

2 a) $\frac{1}{4}$ b) $\frac{2}{5}$ c) $\frac{7}{8}$ d) $\frac{3}{4}$ e) $\frac{2}{7}$ f) $\frac{4}{6}$

3 a) three sixths b) one half c) $\frac{3}{4}$ d) three halves

PRACTICE EXERCISE 2

1 b) Add $\frac{1}{4}$ cup of milk to the $\frac{1}{2}$ cup of flour.

c) Nora still had **three quarters** of the cross country track to go.

d) Talon sold $\frac{2}{3}$ the hot dogs before 10am.

e) Kendall only needed to use **four tenths** of the length of wood to make the frame.

f) Use the $\frac{1}{16}$ masonry drill bit.

2 b) $\frac{1}{4}$ is part of one cup of milk and $\frac{1}{2}$ is part of one cup of flour.

c) three quarters is part of the whole cross country track.

d) $\frac{2}{3}$ is part of the whole amount of hot dogs.

e) four tenths is part of the whole length of wood.

f) $\frac{1}{16}$ is part of an inch.

PRACTICE EXERCISE 3

1 Answers will vary, discuss with teacher and classmates.

2 Answers will vary, discuss with teacher and classmates.

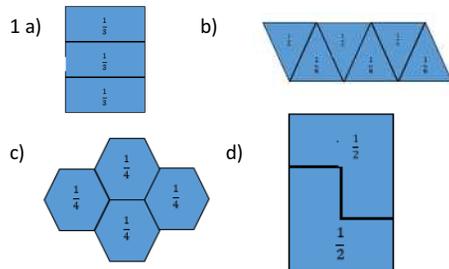
3 Answers will vary, discuss with teacher and classmates.

PRACTICE EXERCISE 4

1 Answers will vary, discuss with teacher and classmates.

2 Answers will vary, discuss with teacher and classmates.

PRACTICE EXERCISE 5

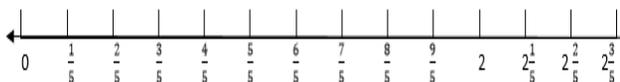


2 a) \$200 b) 7.5L c) 50 000 d) \$300 000 e) \$125

PRACTICE EXERCISE 6

1. Fill in the blanks.

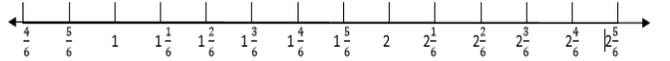
a)



b)



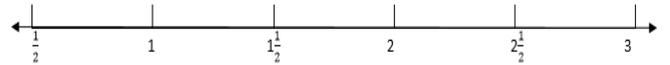
c)



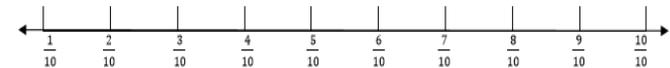
2 a)



b)



c)



3

| Ingredient | Which Measuring Cup | Number of Cups |
|-------------------------------|--------------------------------|----------------|
| $1\frac{2}{3}$ cups of flour. | $\frac{1}{3}$ | 5 |
| $\frac{3}{4}$ cup of milk. | $\frac{1}{4}$ | 3 |
| $1\frac{1}{4}$ cups of cocoa. | $\frac{1}{4}$ | 5 |
| 2 cups of pecans. | $\frac{1}{4}$ or $\frac{1}{3}$ | 8 or 6 |

4 a) 5 b) 42 c) 21 d) 60

5 a) 18 b) $1\frac{4}{6}$ c) 21

PRACTICE EXERCISE 7

1 a) $2 \div 3 = \frac{2}{3}$ b) $3 \div 4 = \frac{3}{4}$ c) $4 \div 6 = \frac{4}{6}$ d) $2 \div 3 = \frac{2}{3}$ e) $3 \div 4 = \frac{3}{4}$

f) $6 \div 10 = \frac{6}{10}$

2 a) $3 \div 2 = \frac{3}{2}$ b) $4 \div 3 = \frac{4}{3}$ c) $10 \div 6 = \frac{10}{6}$ d) $5 \div 3 = \frac{5}{3}$ e) $8 \div 2 = \frac{8}{2}$

f) $12 \div 3 = \frac{12}{3}$

REFLECTION ON LEARNING 1

Answers will vary, check with teacher and classmates.

REFLECTION ON LEARNING 2

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$

REFLECTION ON LEARNING 3

Answers will vary, check with teacher and classmates.

OLNA PRACTICE QUESTIONS

1. D 2. 15 3. 32

Topic 2 Compare and Order Fractions

PRACTICE EXERCISE 1

1 a) $\frac{4}{10}$ b) $\frac{7}{8}$ c) $\frac{3}{4}$ d) $\frac{4}{5}$ e) $\frac{7}{6}$ f) $1\frac{3}{7}$

2 a) $\frac{1}{4}$ b) $1\frac{1}{3}$ c) $\frac{3}{5}$ d) $1\frac{4}{10}$ e) $\frac{3}{6}$ f) $\frac{2}{16}$

3 a) $\frac{2}{4}$ b) $\frac{4}{6}$ c) $\frac{6}{12}$ d) $\frac{3}{3}$

PRACTICE EXERCISE 2

1 a) $\frac{2}{4}$ b) $\frac{10}{12}$ c) $\frac{6}{10}$ d) $\frac{5}{6}$

2 a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$

3 a) $\frac{5}{12}$ b) $\frac{4}{8}$ c) $\frac{3}{12}$

4 a) $<$ b) $>$ c) $<$

PRACTICE EXERCISE 3

1 a) $\frac{1}{3}$ b) $\frac{6}{8}$ c) $\frac{3}{5}$ d) equal e) $\frac{7}{3}$

2 a) $\frac{1}{9}$ b) $\frac{4}{12}$ c) $2\frac{3}{8}$ d) $\frac{15}{10}$

PRACTICE EXERCISE 4

1 a) $\frac{1}{2}$ b) $\frac{6}{10}$

2 a) $\frac{5}{10}$ b) $\frac{4}{10}$ c) $\frac{1}{6}$ d) $\frac{2}{5}$

PRACTICE EXERCISE 5

a) $\frac{2}{3}$, $\frac{6}{8}$, $\frac{5}{6}$ b) $\frac{2}{5}$, $\frac{3}{4}$, $\frac{9}{10}$ c) $\frac{1}{8}$, $\frac{5}{32}$, $\frac{5}{16}$ d) $\frac{2}{6}$, $\frac{6}{10}$, $\frac{6}{8}$

PRACTICE EXERCISE 6

1. Pasey is correct

2. a) \neq b) $=$ c) \neq d) $=$ e) $=$

3. a) $\frac{2}{12}$, $\frac{3}{18}$, $\frac{4}{24}$ b) $\frac{2}{4}$, $\frac{4}{8}$, $\frac{16}{32}$ c) $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$ d) $\frac{1}{4}$, $\frac{6}{24}$, $\frac{4}{16}$ e) $\frac{6}{8}$, $\frac{9}{12}$, $\frac{12}{16}$ f) $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$

PRACTICE EXERCISE 7

1. a) $\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30}$ b) $\frac{1}{16} = \frac{2}{32} = \frac{3}{48} = \frac{4}{64} = \frac{5}{80}$

c) $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25}$

d) $\frac{2}{7} = \frac{4}{14} = \frac{6}{21} = \frac{8}{28} = \frac{10}{35}$ e) $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20}$ f) $\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40}$

2. a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{2}{5}$ d) $\frac{3}{4}$ e) $\frac{1}{4}$ f) $\frac{3}{4}$

REFLECTION ON LEARNING 1

Katie's pictures are better because the rectangles are roughly the same size and lined up under each other, so it is easy to see which amount takes up more space.

Josie needs to draw circles the same size. Katie could measure and draw the lines with a ruler to ensure they are the same dimensions.

$\frac{3}{8}$ is larger. Diagrams will vary, check with teacher and classmates.

REFLECTION ON LEARNING 2

The fractions are a measurement of the diameter of the opening on the tool. $\frac{1}{4}$ refers to one quarter of 1 inch.

Allen key sizes left to right: $\frac{5}{32}$ $\frac{3}{8}$ $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{16}$

Reflection on Learning 3



OLNA PRACTICE QUESTIONS

1 D 2 B

Topic 3 Tenths

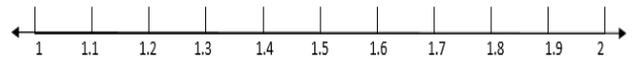
PRACTICE EXERCISE 1



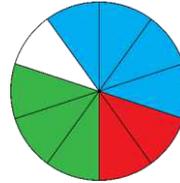
2 a)



b)



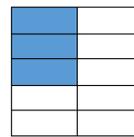
3



PRACTICE EXERCISE 2

1 Answers will vary, check with teacher and classmates.

2 a)



b)



c) 0.1 is not shaded

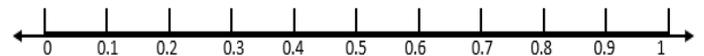


d) 0.4 is not shaded

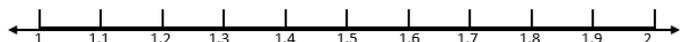


Practice Exercise 3

1 a)



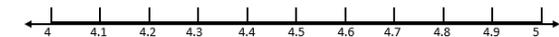
b)



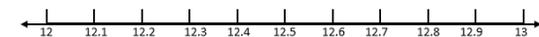
c)



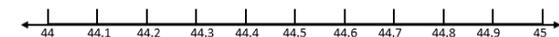
d)



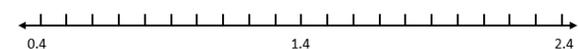
e)



f)



2



PRACTICE EXERCISE 4

1. b) one point two dollars, one dollar and twenty cents, One whole dollar and two tenths of a dollar, One whole metre and two tenths of a metre

c) three point eight dollars; three dollars and eighty cents; three point eight metres; three metres and eighty centimetres; Three whole dollars and eight tenths of a dollar; Three whole metres and eight tenths of a metre

d) Zero point three of a dollar; thirty cents; Zero point three of a metre; Thirty centimetres; Three tenths of a dollar; Three tenths of a metre

e) One point six dollars; One dollar and sixty cents; One point six metres; One metre and sixty centimetres; One whole dollar and six tenths of a dollar; One whole metre and six tenths of a metre

f) Three point seven dollars; Three dollars and seventy cents; Three point seven metres; Three metres and seventy centimetres; Three whole dollars and seven tenths of a dollar; Three whole metres and seven tenths of a metre.

2. a) 0.2 b) 1.9 c) \$2.70 d) \$5.40 e) 1.6m f) 1.5km g) 4.1m h) 1.4m

PRACTICE EXERCISE 5

1. a) 1, 0.9, 0.8 b) 11, 11.2, 11.4 c) 10, 10.5, 11 d) 13, 12.8, 12.6

2. a) 8.5kg b) 2.8kg c) No he didn't because $0.2\text{kg} \times 12 = 2.4\text{kg}$

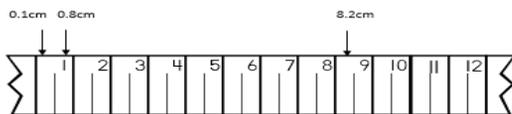
PRACTICE EXERCISE 6

1a) > b) < c) > d) <

2. a) 1.0, 1.1, 10.1, 11.1b) 0.1, 0.2, 1.2, 2.1 c) 0.4m, 1.4m, 4.1m, 4.4m d) \$0.30, \$0.60, \$3.60, \$6.30 e) 0.1L, 0.5L, 1.5L, 5.1L f) 0.9kg, 1.9kg, 9.1kg, 9.9kg

3. a) 0.4 b) 0.6 c) \$0.50 d) 1.4m e) 1.6kg f) 1.8L

REFLECTION ON LEARNING

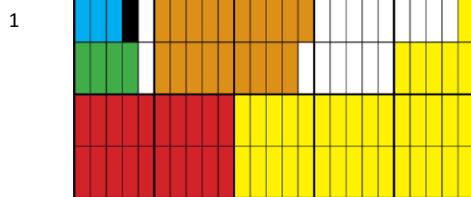


OLNA PRACTICE QUESTIONS

1. B 2. 1, 1.2

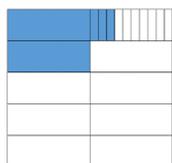
Topic 4 Hundredths

PRACTICE EXERCISE 1



2 Answers will vary, discuss with teacher and classmates.

3a)



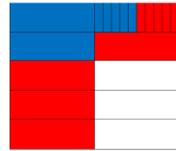
b)



c) 0.85 is shaded, 0.15 is not shaded



d) 0.7 is shaded, 0.3 is not shaded

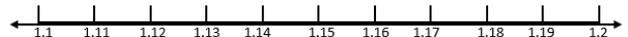


PRACTICE EXERCISE 3

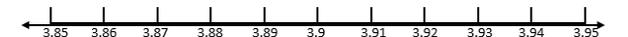
1 a)



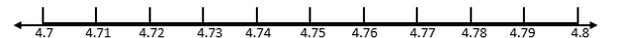
b)



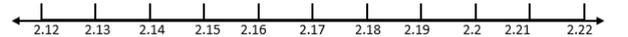
c)



d)



e)



f)



2



PRACTICE EXERCISE 4

1 b) three point zero eight dollars, three dollars and eight cents, three point zero eight metres, three metres and eight centimetres; Three whole dollars and eight hundredths of a dollar, Three whole metres and eight hundredths of a metre.

c) zero point three nine dollars, thirty nine cents, three point zero eight metres, three metres and eight centimetres, Three whole dollars and eight hundredths of a dollar, One whole metre and nine hundredths of a metre

d) one point zero nine dollars, one dollar and nine cents, one point zero nine metres, one metre and nine centimetres, One whole dollar and nine hundredths of a dollar, One whole metre and nine hundredths of a metre

e) three point eight eight dollars, three dollars and eighty eight cents, three point eight eight metres, three metres and eighty eight centimetres, Three whole dollars and eighty eight hundredths of a dollar, Three whole metres and eighty eight metres

2. a) 0.03 b) 1.19 c) \$2.73 d) \$5.04 e) 1.30m f) 1.05kg g) 10.15m h) 1.98m

PRACTICE EXERCISE 5

a) 1 ten, 2 ones, 5 tenths, 6 hundredths; 12 ones, 5 tenths, 6 hundredths

b) 2 tens, 6 ones, 9 tenths, 5 hundredths; 2 tens, 6 ones, 95 hundredths; 2 tens, 6 ones, 90 hundredths, 5 hundredths; 1 ten, 16 ones, 9 tenths, 5 hundredths

c) 1 hundred, 3 tens, 4 ones, 4 hundredths; 1 hundred, 34 ones, 4 hundredths; 1 hundred, 34 ones, 4 hundredths; 1 hundred, 2 tens, 14 ones, 4 hundredths; 13 tens, 4 ones, 4 hundredths

PRACTICE EXERCISE 6

1 a) 1, 0.99, 0.98 b) 10.6, 10.62, 10.64 c) 9, 9.25, 9.5

d) 13.02, 12.82, 12.62

2 a) 13.5kg b) 2.75L c) 2.25km

PRACTICE EXERCISE 7

1 a) < b) > c) < d) > e) <

2. a) 1.01, 1.11, 10.11, 11.10 b) 1.48m, 1.84m, 4.81, 8.4m

d) \$0.38, \$0.68, \$8.36, \$8.60

e) 0.15L, 0.5L, 1.05L, 5.1L

3. a) 1.9 b) \$0.57 c) 0.79kg d) 2.32L

REFLECTION ON LEARNING

1. The 0.63 refers to hundredths of 1 second. Usain Bolt took 41 steps over the 100m, so $100m \div 41 = 2.439m$. The decimal part of the number refers to thousandths of 1 metre, so each stride was 2 metres and 43.9cm.

2. The 0.97 refers to hundredths of 1 metre. Answers will vary, please discuss with teacher and classmates.

3. Decimals allow us to make measurements that are highly accurate, in sport this is important to determine the winner of an event where the competitors achieve results so close to each other.

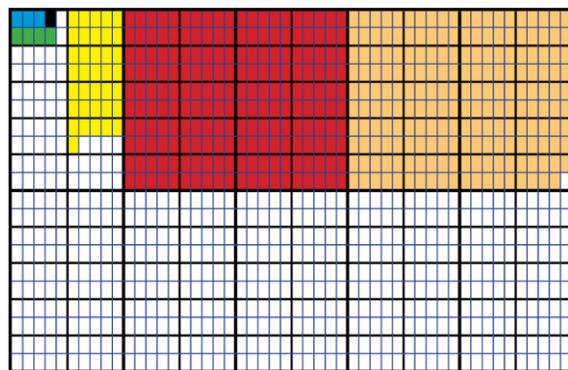
OLNA PRACTICE QUESTIONS

1. C 2. B

Topic 5 Thousandths

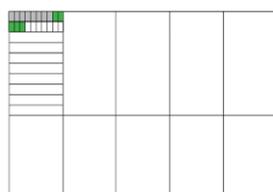
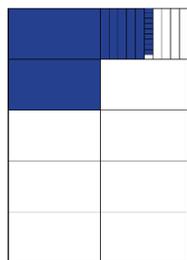
PRACTICE EXERCISE 1

1

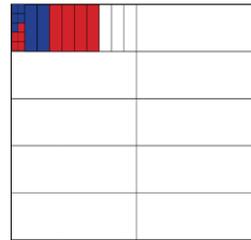


2. Answers will vary discuss with teacher and classmates.

3. a) b) 0.013 is shaded. 0.987 is not shaded.

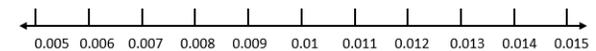


c) 0.07 is shaded. 0.93 is not shaded.



PRACTICE EXERCISE 2

1.a)



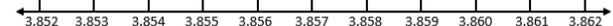
The 1 in 0.013 means 1 hundredth or 10 thousandths

b)



The 5 in 1.350 means 5 hundredths or 50 thousandths. The 0 in 1.350 means there are no thousandths

c)

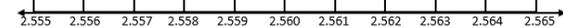


The 8 in each of these numbers means 8 tenths.

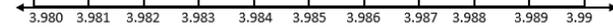
d)



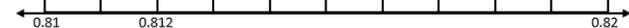
e)



f)

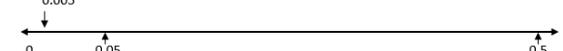


2.



3. a) 1.5 b) 1.15 c) 1.055

4.



PRACTICE EXERCISE 3

1 b) one point seven six eight, one and seven hundred and sixty eight thousandths c) three point zero eight nine, three and eighty nine thousandths d) zero point eight three, eighty three hundredths of a metre e) twenty three point five seven two, twenty three and five hundred and seventy two thousandths of a kilometre f) five point seven zero four, five and seven hundred and four thousandths

2. a) 0.005 b) 2.122 c) 1.002km d) 10.003m e) 20.055kg f) 5.980m g) 8.015m

PRACTICE EXERCISE 4

1. a) 1.007, 1.008, 1.009 b) 10.453, 10.452, 10.451 c) 5.1, 5.125, 5.15 d) 14.628, 14.623, 14.618

2. a) 0.3m b) 1.875kg c) 1.875kg

PRACTICE EXERCISE 6

1. a) > b) < c) > d) >

2. a) 3.04, 4.043, 4.3, 4.343 b) 1.04m, 1.840m, 4.81m, 18.4m c) 0.005L, 0.105L, 1.55L, 5.1L

d) 1.98kg, 8.908kg, 9.198kg, 18.8kg

3. a) 0.006 b) 0.297 c) 1.003km d) 0.362L

REFLECTION ON LEARNING

Answers will vary, discuss with teacher and classmates.

OLNA PRACTICE QUESTIONS

1. B 2. 100

Topic 6 Connecting Fractions and Decimals

PRACTICE EXERCISE 1

1. a) $\frac{8}{10}$ b) $1\frac{3}{10}$ c) $\frac{25}{100}$ d) $3\frac{59}{100}$ e) $2\frac{3}{100}$ f) $\frac{829}{1000}$ g) $\frac{203}{1000}$ h) $1\frac{62}{1000}$
 2. a) $\frac{4}{10}$ m b) $3\frac{2}{10}$ kg c) $\frac{55}{100}$ km d) $10\frac{25}{100}$ t e) $4\frac{9}{100}$ cm f) $\frac{125}{1000}$ kg g) $2\frac{115}{100}$ t h) $5\frac{75}{1000}$ km

PRACTICE EXERCISE 2

1. a) 0.1 b) 0.5 c) 0.1 d) 0.25 e) 0.07 f) 0.205 g) 0.095 h) 0.006
 2. a) 0.25 b) 0.2 c) 0.625 d) 0.75 e) 0.6 f) 2.5 g) 1.25 h) 6.25
 3. a) 0.5 cm b) 0.2 kg c) 0.5m d) 0.75 km e) 4.03t f) 2.6g g) 10.057km h) 5.005m

PRACTICE EXERCISE 3

1. a) 0.75 b) 0.4, 0.6, 0.8 c) 0.25, 0.375, 0.625
 2.

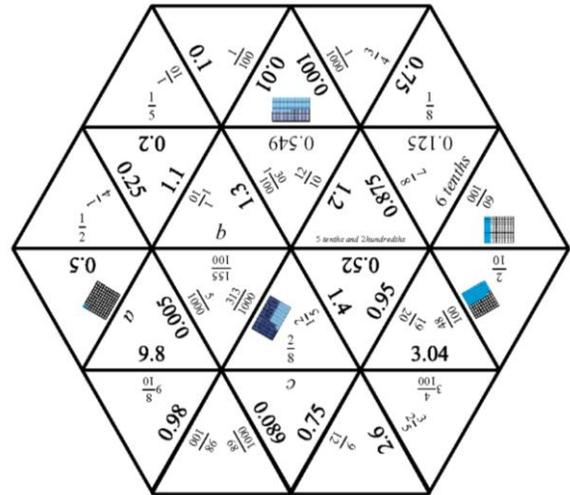
| Decimal | Base Ten Fraction | Equivalent Fractions (Answers will vary) |
|---------|-------------------|--|
| 0.1 | $\frac{1}{10}$ | $\frac{2}{20}, \frac{3}{30}, \frac{10}{100}$ |
| 0.01 | $\frac{1}{100}$ | $\frac{2}{200}, \frac{3}{300}, \frac{10}{1000}$ |
| 0.2 | $\frac{2}{10}$ | $\frac{1}{5}, \frac{3}{15}, \frac{20}{100}$ |
| 0.25 | $\frac{25}{100}$ | $\frac{1}{4}, \frac{2}{8}, \frac{250}{1000}$ |
| 0.5 | $\frac{5}{10}$ | $\frac{1}{2}, \frac{10}{20}, \frac{50}{100}$ |
| 0.75 | $\frac{75}{100}$ | $\frac{3}{4}, \frac{6}{8}, \frac{750}{1000}$ |
| 0.001 | $\frac{1}{1000}$ | $\frac{2}{2000}, \frac{3}{3000}, \frac{4}{4000}$ |
| 0.15 | $\frac{15}{100}$ | $\frac{3}{20}, \frac{30}{200}, \frac{150}{1000}$ |

PRACTICE EXERCISE 5

1. a) > b) > c) > d) <
 2. a) 0.04, $\frac{4}{10}$, 4.1 b) $\frac{12}{10}$ m, 1.25m, 1.505m
 c) 0.120L, $\frac{4}{20}$ L, $\frac{2}{5}$ L d) $\frac{200}{1000}$ kg 2.12kg, $2\frac{1}{4}$ kg

REFLECTION ON LEARNING

a) 0.01 b) 1.55 c) 0.25



OLNA PRACTICE QUESTION

1: C

Section 2 Whole Numbers and Money

Topic 1: Multiplication of Basic Facts

PRACTICE EXERCISE 1

1. a) $6 \times 5 = 30$ b) $7 \times 2 = 14$ c) $3 \times 5 = 15$ d) $9 \times 2 = 18$ e) $5 \times 4 = 20$ f) $3 \times 10 = 30$

2.

| | | | | |
|---|----|----|-----|----|
| 0 | 3 | 6 | 30 | 15 |
| 0 | 5 | 10 | 50 | 25 |
| 0 | 6 | 12 | 60 | 30 |
| 0 | 2 | 4 | 20 | 10 |
| 0 | 9 | 18 | 90 | 45 |
| 0 | 10 | 20 | 100 | 50 |
| 0 | 4 | 8 | 40 | 20 |
| 0 | 1 | 2 | 10 | 5 |
| 0 | 7 | 14 | 70 | 35 |

4. a) 0,0,0 b) 6,60,600,6 million c) 14,140,1 400,14 000
 d) 30,3 000, 30 000, 30 million e) 30,300,60,120

PRACTICE EXERCISE 2

1. a) $9 \times 4 = 36$ b) $6 \times 8 = 48$ c) $3 \times 8 = 24$ d) $5 \times 4 = 20$ e) $5 \times 8 = 40$
 f) $7 \times 4 = 28$

2.

| | | | | | | | | | |
|----|----|----|---|----|----|----|----|---|----|
| 10 | 18 | 6 | 0 | 14 | 16 | 20 | 8 | 2 | 12 |
| 20 | 36 | 12 | 0 | 28 | 32 | 40 | 16 | 4 | 24 |
| 40 | 72 | 24 | 0 | 56 | 64 | 80 | 32 | 8 | 48 |

4. a) 6,60,6 000,6 006 b) 12,120,12 000, 12 012
 c) 24, 240 24 000, 24 024

PRACTICE EXERCISE 3

1. a) $7 \times 6 = 42$ b) $9 \times 3 = 27$ c) $7 \times 3 = 21$ d) $3 \times 6 = 18$ e) $8 \times 6 = 48$ f) $4 \times 3 = 12$

2.

| | | | | | | |
|----|----|----|----|---|----|----|
| 4 | 12 | 18 | 6 | 0 | 10 | 20 |
| 6 | 18 | 27 | 9 | 0 | 15 | 30 |
| 12 | 36 | 54 | 18 | 0 | 30 | 60 |

4a) 14,140,14 000,140 000 b) 18,180,18 000,180 000
c) 30,300,3 000,30 000 d) 54,540,54 000,540 000

PRACTICE EXERCISE 4

1a) 28,280,28 000, 2 800 000 b) 72,720,7 200, 720 000
c) 49,490,4 900,4 900 000

PRACTICE EXERCISE 5

1. HE GOT THE COLD SHOULDER

2.

| | | | | | | | | | | |
|----|----|----|----|-----|------|-------|-----|-----|-------|------|
| 12 | 20 | 8 | 40 | 320 | 2400 | 16000 | 80 | 360 | 40000 | 1200 |
| 18 | 30 | 12 | 60 | 480 | 3600 | 24000 | 120 | 540 | 60000 | 1800 |
| 21 | 35 | 14 | 70 | 560 | 4200 | 28000 | 140 | 630 | 70000 | 2100 |
| 27 | 45 | 18 | 90 | 720 | 5400 | 36000 | 180 | 810 | 90000 | 2700 |
| 24 | 40 | 16 | 80 | 640 | 4800 | 32000 | 160 | 720 | 80000 | 2400 |

REFLECTION ON LEARNING

| | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|---|
| 28 | 8 | 36 | 4 | 12 | 20 | 16 | 24 | 32 | 40 | 0 |
| 49 | 14 | 63 | 7 | 21 | 35 | 28 | 42 | 56 | 70 | 0 |
| 42 | 12 | 54 | 6 | 18 | 30 | 24 | 36 | 48 | 60 | 0 |
| 7 | 2 | 9 | 1 | 3 | 5 | 4 | 6 | 8 | 10 | 0 |
| 21 | 6 | 27 | 3 | 9 | 15 | 12 | 18 | 24 | 30 | 0 |
| 63 | 18 | 81 | 9 | 27 | 45 | 36 | 54 | 72 | 90 | 0 |
| 70 | 20 | 90 | 10 | 30 | 50 | 40 | 60 | 80 | 100 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 56 | 16 | 72 | 8 | 24 | 40 | 32 | 48 | 64 | 80 | 0 |
| 35 | 10 | 45 | 5 | 15 | 25 | 20 | 30 | 40 | 50 | 0 |
| 14 | 4 | 18 | 2 | 6 | 10 | 8 | 12 | 16 | 20 | 0 |

Topic 2: Multiplication of Whole Numbers

PRACTICE EXERCISE 1

1a) 64 b) 820 c) 126 d) 4 600 e) 30 500 f) 13 100 000

2. Answers will vary. Check with a partner.

PRACTICE EXERCISE 2

1.

| | | |
|----------------------|----------------------|-----------------------|
| $12 \times 2 = 24$ | $12 \times 4 = 48$ | $12 \times 8 = 96$ |
| $15 \times 2 = 30$ | $15 \times 4 = 60$ | $15 \times 8 = 120$ |
| $25 \times 2 = 50$ | $25 \times 4 = 100$ | $25 \times 8 = 200$ |
| $150 \times 2 = 300$ | $150 \times 4 = 600$ | $150 \times 8 = 1200$ |
| $18 \times 2 = 36$ | $18 \times 4 = 72$ | $18 \times 8 = 144$ |
| $45 \times 2 = 90$ | $45 \times 4 = 180$ | $45 \times 8 = 360$ |
| $135 \times 2 = 270$ | $135 \times 4 = 540$ | $135 \times 8 = 1080$ |

2a) 26 b) 56 c) 3 200 d) 260 e) 2 000 f) 900

3. Answers will vary. Check with a partner.

PRACTICE EXERCISE 3

1a) \$27 \times 4 matches \$108 b) 8 \times \$13 matches \$104
c) \$50.50 \times 2 matches \$101 d) 4 \times \$23.50 matches \$94
e) \$47.25 \times 2 matches \$94.50 f) \$12.25 \times 8 matches \$98

2a) \$108 b) \$1400 c) \$22 400 d) \$100 e) \$6.54 f) \$67

3a) \$125 b) 1160g c) 112cm d) \$180 e) \$36.50 f) 144km

4a) B. It looks the most per week b) Option A - \$819.15;

OPTION B - \$416; OPTION C - \$384

c) Option A increased rapidly due to the doubling and far exceeded the other OPTIONS

PRACTICE EXERCISE 4

1.

| THOUSANDS | | | ONES | | |
|-----------|------|------|----------|------|------|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | 4 | 3 |
| | 4 | 3 | 0 | 0 | 0 |
| | | | 7 | 1 | 0 |
| | | 7 | 1 | 0 | 0 |
| | | 9 | 4 | 0 | 0 |
| 9 | 4 | 0 | 0 | 0 | 0 |

2a) 110 b) 2 500 c) 351 000 d) 800 000 e) 900 000 f) 4 350 000

3. Answers will vary. Check with a partner.

PRACTICE EXERCISE 5

1a) \$610 b) \$4 150 c) \$330 000 d) \$295.00 e) \$428 f) \$376 100

PRACTICE EXERCISE 6

1a) $14 \times 10 = 140$ $14 \times 5 = 70$

b) $24 \times 10 = 240$ $24 \times 5 = 120$

c) $46 \times 10 = 460$ $46 \times 5 = 230$

d) $17 \times 10 = 170$ $17 \times 5 = 85$

e) $53 \times 10 = 530$ $53 \times 5 = 265$

f) $136 \times 10 = 1360$ $136 \times 5 = 680$

2a) 60 b) 320 c) 2 050 d) 2 500 e) 24 000 f) 4 215

3. Answers will vary. Check with a partner.

PRACTICE EXERCISE 7

1a) \$130 b) \$235 c) \$3 100 d) \$43 e) \$72 f) \$314

2. PUZZLE: A SLIPPER

3a) 420 b) 620mm c) 2 100mm d) \$6 e) 170 square centimetres
f) \$34.50

PRACTICE EXERCISE 8

1a) 108 b) 198 c) 306 d) 405 e) 2 250 f) 3 240

2. Answers will vary. Check with a partner.

3a) 792 b) 6 993 c) 29 997 d) 995 e) 2 394 f) 39 960

PRACTICE EXERCISE 9

1a) \$396 b) \$79.92 c) \$419.94 d) \$1 485 e) \$23 970 f) \$23 999.60

2. RECEIPT 12/3/15 Errors

Butter; 3 @ \$4.99 \quad \$14.97

Ricepops; 3 @ \$5.95 \quad \$17.85

RECEIPT 13/3/15 Errors

Peaches; 8 @ \$3.99 \quad \$31.92

Yoghurt; 3 @ \$2.99 \quad \$8.97

Corn; 3 @ \$0.99 \quad \$2.97

PRACTICE EXERCISE 10

1a) 147 b) 78 c) 105 d) 204

- 2a) 153 b) 126 c) 168 d) 294 e) 750 f) 1 320
 3a) 186 b) 129 c) 245 d) 384 e) 692 f) 705
 4a) \$117 b) \$1 260 c) \$30.70 d) \$15 e) \$45.50 f) \$25.50

PRACTICE EXERCISE 11

- 1a) 168 b) \$3 720 c) 198kg d) 280 e) 255km f) 700g
 2a) Kickett 99 Beal 132 Daniels 57

Barton 129 Loughton 69 Corletto 78

b) 76; 228

3. ERRORS

$5 \times 17 = 85$, $6 \times 141 = 846$, $3 \times 37 = 111$, $93 \times 4 = 372$

PRACTICE EXERCISE 12

1a) 90 b) 48 c) 180 d) 120

2a) 54 b) 200 c) 270 d) 560 e) 300 f) 770

3a) \$2 700 b) \$40 000 c) \$150 000 d) \$72 e) \$4.80 f) \$6 300

PRACTICE EXERCISE 13

1.

| | | | | | | | |
|-----|-----|-------|-----|-----|---------|-----|---------|
| 28 | 50 | \$70 | 36 | 120 | \$5 | 200 | \$2.50 |
| 42 | 75 | \$105 | 54 | 180 | \$7.50 | 300 | \$3.75 |
| 56 | 100 | \$140 | 72 | 240 | \$10 | 400 | \$5 |
| 126 | 225 | \$315 | 162 | 540 | \$22.50 | 900 | \$11.25 |
| 98 | 175 | \$245 | 126 | 420 | \$17.50 | 700 | \$8.75 |
| 84 | 150 | \$210 | 108 | 360 | \$15 | 600 | \$7.50 |
| 70 | 125 | \$175 | 90 | 300 | \$12.50 | 500 | \$6.25 |
| 112 | 200 | \$280 | 144 | 480 | \$20 | 800 | \$10 |

3a) \$98 b) \$490

4a) PASTRY: 450g Butter, 330g Icing sugar, 6 Egg Yolks, 105g Cocoa Powder and 675g Plain Flour.

b) FILLING: 900mL Cream, 450g Butter, Icing Sugar for dusting, 1650g Caster Sugar and 625mL Water.

c) PASTRY: 1200g Butter, 1440mL Cream, 880g Icing sugar, 720g Butter, 280g Cocoa Powder, 2640g Castor Sugar, 1800g plain Flour, 1000mL Water, 16 Egg Yolks and Icing Sugar for dusting.

REFLECTION ON LEARNING

a) We know Number of groups and number in each group; we need to find the TOTAL

b) $8 \times 45 = ?$ c) Tom; Double, double, double (i.e. 90, 180, 360)

Katie; $(8 \times 40) + (8 \times 5) = 360$ Penny; $8 \times 45 = 4 \times 90$

d) Answers will vary

OLNA PRACTICE QUESTIONS

1. D 2. B

Topic 3: Division of Basic Facts

PRACTICE EXERCISE 1

1a) 8 b) 4 c) 5 d) 7 2a) 8 b) 4 c) 5 d) 7

PRACTICE EXERCISE 2

1 a) $3 \times 4 = 12$; $4 \times 3 = 12$; $12 \div 3 = 4$; $12 \div 4 = 3$; $\frac{1}{4}$ of 12 = 3;
 $\frac{1}{2}$ of 12 = 6

b) $4 \times 2 = 8$; $2 \times 4 = 8$; $8 \div 2 = 4$; $8 \div 4 = 2$; $\frac{1}{4}$ of 8 = 2; $\frac{1}{2}$ of 8 = 4

c) $4 \times 5 = 20$; $5 \times 4 = 20$; $20 \div 5 = 4$; $20 \div 4 = 5$; $\frac{1}{5}$ of 20 = 4;

$\frac{1}{4}$ of 20 = 5 d) $3 \times 10 = 30$; $10 \times 3 = 30$; $30 \div 3 = 10$; $30 \div 10 = 3$;

$\frac{1}{3}$ of 30 = 10; One tenth of 30 = 3

2a) 2 rows of 7 b) 6 rows of 3 c) 3 rows of 8 d) 5 rows of 8

e) 5 rows of 6 f) 10 rows of 8

3a) 2 rows of 6, 6 rows of 2, 3 rows of 4, 4 rows of 3, 1 row of 12, 12 rows of 1 b) 2 rows of 8, 4 rows of 4, 7 rows of 2, 1 row of 16, 16 rows of 1

PRACTICE EXERCISE 3

1a) 9 b) 3 c) 7 d) 5 e) 4 f) 0

2a) 2,4,6,8,10,12 b) 3,6,9,12,15 c) 10,20,30,40,50 d) 3,6,9,12

e) 5,10,15,20,25,30,35,40,45 f) 2,4,6,8,10,12,14,16,18,20

3.

| | | |
|---------------|---------------------|----|
| $24 \div 3$ | $3 \times ? = 24$ | 8 |
| $6 \div 2$ | $2 \times ? = 6$ | 3 |
| $70 \div 10$ | $10 \times ? = 70$ | 7 |
| $5 \div 5$ | $5 \times ? = 5$ | 1 |
| $9 \div 3$ | $3 \times ? = 9$ | 3 |
| $30 \div 10$ | $10 \times ? = 30$ | 3 |
| $60 \div 6$ | $? \times 6 = 60$ | 10 |
| $100 \div 10$ | $10 \times ? = 100$ | 10 |
| $8 \div 1$ | $1 \times ? = 8$ | 8 |

4a) 10 b) 7 c) 3 d) 7 e) 8 f) 9

PRACTICE EXERCISE 4

1a) 3 b) 7 c) 9 d) 8 e) 6 f) 10

2a) 4,40,4 000,40 cents,400 b) 6,60,6 million, \$600, 60

c) 10,100,10 000,20,1 000

PRACTICE EXERCISE 5

1a) 2 b) 6 c) 8 d) 4 e) 5 f) 7

2a) 3,30,3 000,\$0.30,3 b) 7,70,7 000,\$700,70

c) 9,900,90 000,\$0.09,90

PRACTICE EXERCISE 6

1a) 3 b) 8 c) 4 d) 9 e) 5 f) 7

2a) 2,20,2 000,\$0.20,20 b) 7,70,7 000,\$0.70,700

c) 10,1 000,1 million,\$0.10,100

PRACTICE EXERCISE 7

1a) OUT 8, 10, 4, 6, 3, 7, 20, 50

b) OUT 4, 8, 5, 16, 3, 6, 10, 40

c) OUT 2, 4, 10, 8, 9, 3, 5, 60

d) MISSING NUMBERS 6, 8, 7, 5, 6, 6, 10, 700, 300

PRACTICE EXERCISE 8

1a) 2 b) 3 c) DOESN'T EXIST d) 10 e) 5 f) 9 g) 0 h) 6 i) 6
 j) 10 k) 8 l) 4

4.a) 4,40,4,400,40,40 b) 6,60,6,600,60 c) 10,100,10,1000,100,
 10 000

REFLECTION ON LEARNING

Check arrays with a partner

1. $? \times 3 = 27$; $9 \times ? = 60$; $10 \times ? = 8$; $4 \times ? = 42$; $68 \times ? = 64$; 8

2. Discuss advantages, disadvantages and examples with your classmates and teacher

3.

| | | |
|---|--|----|
| 1 | 10÷10, 9÷9, 8÷8, 7÷7, 6÷6, 5÷5, 4÷4, 3÷3, 2÷2, 1÷1. | 10 |
| 2 | 20÷10, 18÷9, 16÷8, 14÷7, 12÷6, 10÷5, 8÷4, 6÷3, 4÷2, 2÷1. | 10 |
| 3 | 30÷10, 27÷9, 24÷8, 21÷7, 18÷6, 15÷5, 12÷4, 9÷3, 6÷2, 3÷1 | 10 |
| 4 | 40÷10, 36÷9, 32÷8, 28÷7, 24÷6, 20÷5, 16÷4, 12÷3, 8÷2, 4÷1 | 10 |
| 5 | 50÷10, 45÷9, 40÷8, 35÷7, 30÷6, 25÷5, 20÷4, 15÷3, 10÷2, 5÷1 | 10 |
| 6 | 60÷10, 54÷9, 48÷8, 42÷7, 36÷6, 30÷5, 24÷4, 18÷3, 12÷2, 6÷1 | 10 |
| 7 | 70÷10, 63÷9, 56÷8, 49÷7, 42÷6, 35÷5, 28÷4, 21÷3, 14÷2, 7÷1 | 10 |
| 8 | 80÷10, 72÷9, 64÷8, 56÷7, 48÷6, 40÷5, 32÷4, 24÷3, 16÷2, 8÷1 | 10 |
| 9 | 90÷10, 81÷9, 72÷8, 63÷7, 54÷6, 45÷5, 36÷4, 27÷3, 18÷2, 9÷1 | 10 |

OLNA PRACTICE QUESTIONS

1. A. $2. 0 \div 17 = 0$ OR $0 \div 71 = 0$

Topic 4: Division of Whole Numbers

PRACTICE EXERCISE 1

1a) $26 \div 2 = 13$ b) $30 \div 2 = 15$

2a) 25 b) 17 c) 33 d) 55 e) 77 f) 725

3. Answers will vary. Check with a partner.

PRACTICE EXERCISE 2

1a) $52 \div 4 = 13$ b) $60 \div 4 = 15$

2a) 20 b) 23 c) 35 d) 550 e) 750 f) 820

3. Answers will vary. Check with a partner.

PRACTICE EXERCISE 3

1a) $96 \div 8 = 12$ b) $104 \div 8 = 13$

2a) 15 b) 35 c) 18 d) 400 e) 250 f) 260

3. Answers will vary. Check with a partner.

4.

| THOUSANDS | | | ONES | | |
|-----------|------|------|----------|------|------|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | 6 | 2 |
| | | | 9 | 1 | 0 |
| | | | 7 | 8 | 0 |
| | 4 | 5 | 6 | 0 | 0 |
| 9 | 8 | 2 | 0 | 0 | 0 |
| | | 8 | 7 | 1 | 0 |

PRACTICE EXERCISE 4

1a) $300 \div 2 = 150$ b) $880 \div 4 = 220$ c) $426 \div 2 = 213$ d) $860 \div 4 = 215$
e) $1\ 800 \div 8 = 225$ f) $1\ 440 \div 8 = 180$

2a) \$27 b) \$32 c) \$4.25 d) \$1.75 e) \$4.20 f) \$10 100

3a) $\$186 \div \$2 = ?$; $\$93$ b) $\$142 \div \$4 = ?$; $\$35.50$ c) $30 \div 4 = ?$; 7.5cm
d) $\$14.70 \div \$2 = ?$; $\$7.35$ e) $56 \div 4 = ?$; 14 rows f) $400 \div 8 = ?$; 50m

4a) 160 kg cement, 320 kg sand, 480 kg aggregate

b) 40 kg cement, 80 kg sand, 120 kg aggregate

PRACTICE EXERCISE 5.

1.

| | | |
|----------------------------|----------------------------|---------------------------|
| $8 \div 2 = 4$ | $8 \div 4 = 2$ | $8 \div 8 = 1$ |
| $16 \div 2 = 8$ | $16 \div 4 = 4$ | $16 \div 8 = 2$ |
| $40 \div 2 = 20$ | $40 \div 4 = 10$ | $40 \div 8 = 5$ |
| $600 \div 2 = 300$ | $600 \div 4 = 150$ | $600 \div 8 = 75$ |
| $96 \div 2 = 48$ | $96 \div 4 = 24$ | $96 \div 8 = 12$ |
| $440 \div 2 = 220$ | $440 \div 4 = 110$ | $440 \div 8 = 55$ |
| $248 \div 2 = 124$ | $248 \div 4 = 62$ | $248 \div 8 = 31$ |
| $1\ 000 \div 2 = 500$ | $1\ 000 \div 4 = 250$ | $1\ 000 \div 8 = 125$ |
| $10\ 400 \div 2 = 5\ 200$ | $10\ 400 \div 4 = 2\ 600$ | $10\ 400 \div 8 = 1\ 300$ |
| $50\ 000 \div 2 = 25\ 000$ | $50\ 000 \div 4 = 12\ 500$ | $50\ 000 \div 8 = 6\ 250$ |

2a) 8000 b) 65 c) 700 d) 53 e) 70 000 f) 890

3. Answers will vary. Check with a partner.

PRACTICE EXERCISE 6

1a) 5cm b) 60cm c) 710cm d) 825.6cm

2a) 3m b) 70m c) 85.5m d) 126.24m

3a) 4L b) 80L c) 120.5L d) 5.765L

4a) \$15 b) \$1.39 c) \$1.42 d) \$870

5a) 151 b) 3000 c) 62 700 d) 900 000

6a) 4 b) 5400 c) 10 000 d) 340 000

7a) $560 \div 10 = ?$; 56 decades: $7000 \div 100 = ?$; 70 centuries.

b) $930 \div 10 = ?$; 93mm c) $290 \div 10 = ?$; 29 days

d) $750\ 000 \div 1\ 000 = ?$; 750mm e) $35\ 000 \div 100 = ?$; 350 deliveries

f) $190 \div 10 = ?$; 19 clients

PRACTICE EXERCISE 7

1.

| | | |
|-------------------|--------------------|-------------------|
| $135 \div 5 = 27$ | $78 \div 6 = 13$ | $45 \div 3 = 15$ |
| $147 \div 7 = 21$ | $918 \div 9 = 102$ | $215 \div 5 = 43$ |
| $132 \div 6 = 22$ | $348 \div 3 = 116$ | $161 \div 7 = 23$ |

2. Answers will vary. Check with a partner.

PRACTICE EXERCISE 8

1. PUZZLE: A man laughing his head off

2.a) $154 \div 7 = ?$; 22 weeks. b) $216 \div 9 = ?$; 24 columns

c) $6\ 120 \div 60 = ?$; 102 hours d) $900 \div 6 = ?$; 150cm

e) $\$10.50 \div 7 = ?$; \$1.50 f) $\$1610 \div 7 = ?$; \$230

3. Monday \$18; Tuesday \$28; Wednesday \$16; Thursday \$44; Friday \$53

Revision: THE BEST STRATEGY?

| | |
|------------------------------------|---------------------------------------|
| a) $600 \div 200 = 3$; Tom | b) $78 \div 6 = 13$; Katie |
| c) $48 \div 2 = 24$; Tom | d) $5\ 600 \div 10 = 560$; Tom |
| e) $39 \div 13 = 3$; Katie | f) $\$20 \div 8 = \2.50 ; Tom |
| g) $\$245 \div \$5 = 49$; Katie | h) $759\ 000 \div 100 = 7\ 590$; Tom |
| i) $\$5.80 \div 10 = \0.58 ; Tom | j) $\$12.80 \div 4 = \3.20 ; Tom |

OLNA PRACTICE QUESTIONS

1. C. 2. 17cm

Topic 5: Division with Remainders

PRACTICE EXERCISE 1

- 1a) 5r2 b) 5r3 c) 4r2 d) 7r3 e) 2r3 f) 5r2

- 2a) $39 \div 6 = ?$; 6r3; round down to 6. b) $37 \div 4 = ?$; 9r1; keep as is; \$1 can be divided by 4 into \$0.25, so bill is \$9.25 each.
 c) $67 \div 8 = ?$; 8r3; round up to 9 d) $47 \div 5 = ?$; 9r2; round down to 9
 e) $8 \div 3 = ?$; 2r2; keep as is; split the remaining 2 balls between the 3.
 f) $20 \div 3 = ?$; 6r2; round down to 6

PRACTICE EXERCISE 2

- 1a) $10\frac{1}{4}$ b) $3\frac{1}{2}$ c) $3\frac{1}{3}$ d) $9\frac{1}{2}$ e) $2\frac{2}{5}$ f) $7\frac{3}{4}$

- 2a) $28 \div 8 = ?$; 3r4; $3\frac{1}{2}$ b) $32 \div 5 = ?$; 6r2; $6\frac{2}{5}$ c) $38 \div 8 = ?$; 4r6; $4\frac{3}{4}$
 d) $26 \div 4 = ?$; 6r2; $6\frac{1}{2}$ e) $65 \div 10 = ?$; 6r5; $6\frac{1}{2}$ f) $22 \div 8 = ?$; 2r6; $2\frac{3}{4}$

PRACTICE EXERCISE 3

- 1a) 741.208. Round down to 741 committees. The remaining delegates could be distributed to other committees so that some committees would have 25. b) \$3 984.222. Round down to \$3 984.22. Lotto could keep the remaining \$0.01 c) 637.08 dozen. Round down to 637 whole dozen d) \$59.52. Round up to \$60
 e) 571.5. Round down to 571 piles with 4 bricks remaining
 f) 34.22. Round up. 35 buses needed

REVISION QUESTIONS

1

| | | | | | | |
|-----------------|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|
| $11\frac{1}{2}$ | $5\frac{5}{6}$ | $5\frac{2}{9}$ | $2\frac{2}{3}$ | $3\frac{3}{10}$ | $4\frac{3}{5}$ | $3\frac{3}{4}$ |
| $17\frac{1}{2}$ | $5\frac{5}{6}$ | $8\frac{8}{9}$ | $11\frac{2}{3}$ | $3\frac{1}{2}$ | 7 | $3\frac{3}{4}$ |
| $20\frac{1}{2}$ | $5\frac{5}{6}$ | $4\frac{5}{9}$ | $13\frac{2}{3}$ | $4\frac{1}{10}$ | $8\frac{1}{5}$ | $10\frac{1}{4}$ |
| 34 | $11\frac{1}{3}$ | $5\frac{5}{9}$ | $22\frac{2}{3}$ | $4\frac{4}{5}$ | $13\frac{3}{5}$ | 17 |

- 3a) MENTAL; $13 \div 4 = ?$, Halve 13 and halve again, $13 \div 2 \div 2 = 3\frac{1}{4}$

Each person would get $3\frac{1}{4}$ bowls of rice b) MENTAL; $300 \div 8 = ?$,

Halve 300, halve again and halve again, $300 \div 2 \div 2 \div 2 = 37\frac{1}{2}$,

Each ticket would cost \$37.50 c) CALCULATOR; $17\ 253 \div 200 = ?$,
 $? = 86.265$; There would be 86 complete rows and one incomplete row
 d) MENTAL; $87 \div 5 = ?$, Use partitioning, $50 \div 5 = 10$,
 $37 \div 5 = 7r2$, $87 \div 5 = 17r2$; There would be 17 bunches.

CALCULATOR; $321 \div 14 = ?$, $? = 22.928$; The model will be 23 times smaller.

REFLECTION ON LEARNING

1. Answers will vary. Check with your teacher and classmates

2. Answers will vary. Check with your teacher and classmates

OLNA PRACTICE QUESTIONS

1. 12 2. B

Topic 6 Choosing Between Mental Calculation or Calculator to Solve Multiplication and Division Problems Solutions

PRACTICE EXERCISE 1

1. The following problems should be circled

$153 \times ? = \$45\ 786$; $? \div 37 = 89754$; $12 \times ? = \$568.45$;
 $\$367 \div ? = \5.24 ; $? \div \$1.23 = \65.45

- 2a) $187\ 900 \div 24\ 578 = ?$; 7.645 b) $\$679.50 \times 48 = ?$; \$32 616
 c) $\$347 \div \$21.20 = ?$; 16.37 d) $\$8\ 490 \div \$13.25 = ?$; 640.75
 e) $\$9\ 803\ 000 \div 77 = ?$; \$127 311.68

3. $\$84 \div ? = \2 ; Mental; Division; Halving; 41; $? = 6\ 803 \div 551$;
 Calculator; Division; 12.35; $? \times 8 = \$2000$; Mental; Division; Halving,
 halving and halving again; \$250; $? \div 5 = \$1.99$; Mental;
 Multiplication; $5 \times \$2 - \$0.05 = \$9.95$; $456 \times ? = \$3479$; Calculator;
 Division; $\$3\ 479 \div 456 = \7.63 ; $25 = ? \div 3$; Mental; Division; $25 \div 3$.
 Partitioning into known facts ($25 = 24 + 1$). As $24 \div 3 = 8$;
 $25 \div 3 = 8r1$

- 4a) \$126 b) \$1 426.56 c) 160 d) \$47 80 e) \$23.94 f) 161.49 g) \$89 h)
 14 601.48 i) \$4 500 j) \$280 k) \$2.40 l) 820 847

REFLECTION ON LEARNING

First flow chart - $? \div 7 = \$1.99$; Second flow chart - $? \times 26 = \$45\ 752$

OLNA PRACTICE QUESTION

1. C

Topic 7 Choosing between Multiplication and Division to Solve Everyday Problems Solutions

PRACTICE EXERCISE 1

MULTIPLICATION PROBLEMS

1. a) Sophie has 14 books. Charlie has 3 times more. How many does Charlie have?; $14 \times 3 = ?$; 14 is number of books. 3 is how many times larger; $? = 42$; 42 is TOTAL number of books

- d) Calculate the area of a rectangle with length 73cm and width 6cm.; $73 \times 6 = ?$; 73 is length; 6 is width; $? = 438$; 438 is TOTAL number of square centimetres in a rectangle

- e) Jackie gives 18 lollies to each of 6 friends. How many lollies did she have to start with?; $6 \times 18 = ?$; 6 is number of groups, 18 is number in each group; $? = 108$; 108 is TOTAL number of lollies

DIVISION PROBLEMS

- b) Barbara packs eggs. If she has packed 420 eggs, how many dozen has she packed?; $420 \div 12 = ?$; 420 is TOTAL eggs; 12 is number in each group; $? = 35$; 35 is number of groups i.e. number of dozen

- c) A bag of oranges costs \$10.50 for 3 kilograms. How much for one kilogram?; $\$10.50 \div 3 = ?$; \$10.50 is TOTAL cost of oranges; 3 is number of groups; $? = \$3.50$; \$3.50 is cost for 1 kg

- f) George plants 84 corn plants in rows of 7. How many rows does he plant?; $84 \div 7 = ?$; 84 is TOTAL number of plants; 7 is number in each group; $? = 12$; 12 is number of groups i.e. number of rows

- g) Mitch cycles 240km in 8 hours. What is his average speed per hour?; $240 \div 8 = ?$; 240 is TOTAL km; 8 is number of groups; $? = 30$; 30 is number of km in each group

- h) Sophie has 43 socks. How many pairs of socks does she have?; $43 \div 2 = ?$; 43 is TOTAL pairs of socks; 2 is number in each group; $? = 21r1$; 21 is number of groups i.e. number of pairs of socks

2. a) $3 \times \$724\,563 = ?$; The total prize pool was \$2 173 689
 b) $? \times 60 = 495$; $495 \div 60 = ?$; 8.25. Jalen spent 8.25 hours on his homework
 c) $? \times 15 = 210$; $210 \div 15 = ?$; 14. Paddo packed 14 rows and 15 columns of bread
 d) $4.5 \times ? = 1924$; $1924 \div 4.5 = ?$; 427.55km. Lauren travelled 427.55km per day
 e) $\$7.37 \times 3 = ?$; \$22.11. Grace earns \$22.11 per hour
 f) $21 \times ? = 94.5$; $94.5 \div 21 = ?$; 4.5. The cupcake is 4.5 times bigger in real life.

3. MENTAL; MULTIPLICATION; $5 \times 82 = ?$; $5 \times 80 = 400$, $5 \times 2 = 10$; Therefore: $5 \times 82 = 410$. George drove 410km to get to the Drake concert

MENTAL; DIVISION; $50 \times ? = 1100$, $1100 \div 50 = ?$; $1000 \div 50 = 20$, $100 \div 50 = 2$. Therefore: $1100 \div 50 = 22$.

There were 22 rows of 50 in the section of the Perth Arena

CALCULATOR; MULTIPLICATION, $13423 \times 148 = ?$; $? = \$1\,986\,604$. The gross income of the Drake concert was \$1 986 604

MENTAL; DIVISION, $4 \times ? = 50$, $50 \div 4 = ?$; Dividing by 4 means halving and halving again.

$50 \div 2 = 25$, $25 \div 2 = \$12.50$. The cost of each friend's share of snacks was \$12.50

MENTAL; DIVISION, $30 \times ? = 120$, $120 \div 30 = ?$ Use basic facts $12 \div 3 = 4$, Therefore: $120 \div 30 = 4$. The average length of each song was 4 minutes.

4a) \$35 b) 35 notes c) 35 square centimetres d) 35 pieces e) 35 days f) 35 intervals

5a) 8km/h b) \$8/kg c) 8L/km d) 8km/h e) \$8/kg f) \$8/kg

6a) Multiplication. Cost. The total COST of the DVD's was \$239.92

b) Division. Number of weeks. The number of weeks that it rained in Bunbury in one year was 13. c) Multiplication. Weight. The total weight of the punnets of strawberries was 900g. d) Multiplication. Money. Sarah's father inherited \$77 350.65 e) Division. Width. The width of the rectangular farm was 14km. f) Multiplication. Height. The height of the model was 984mm.

PRACTICE EXERCISE 2

1a)

| Ryan Smith: Normal Rate \$22/hr | | | |
|---------------------------------|-------------|-------|--------|
| | HOURLY RATE | HOURS | WAGE |
| Mon – Fri Normal | \$22 | 30 | \$660 |
| Mon – Fri Overtime | \$33 | 0 | \$0 |
| Saturday | \$33 | 4 | \$132 |
| Sunday | \$44 | 7 | \$308 |
| TOTAL WAGE | | | \$1100 |

b)

| Travis Jones: Normal Rate \$26/hr | | | |
|-----------------------------------|-------------|-------|--------|
| | HOURLY RATE | HOURS | WAGE |
| Mon – Fri Normal | \$26 | 38 | \$988 |
| Mon – Fri Overtime | \$39 | 2 | \$78 |
| Saturday | \$39 | 3 | \$117 |
| Sunday | \$52 | 4 | \$208 |
| TOTAL WAGE | | | \$1391 |

c)

| Jake Morgan: Normal Rate \$32.28/hr | | | |
|-------------------------------------|-------------|-------|-----------|
| | HOURLY RATE | HOURS | WAGE |
| Mon – Fri Normal | \$32.28 | 20 | \$645.60 |
| Mon – Fri Overtime | \$48.42 | 0 | \$0 |
| Saturday | \$48.42 | 3 | \$145.26 |
| Sunday | \$64.56 | 4 | \$258.24 |
| TOTAL WAGE | | | \$1049.10 |

d)

| Sam Harris: Normal Rate \$7/hr | | | |
|--------------------------------|-------------|-------|-------|
| | HOURLY RATE | HOURS | WAGE |
| Mon – Fri Normal | \$16 | 38 | \$608 |
| Mon – Fri Overtime | \$24 | 1 | \$24 |
| Saturday | \$24 | 6 | \$144 |
| Sunday | \$32 | 5 | \$160 |
| TOTAL WAGE | | | \$936 |

2. Height of Picture of Cow; 26.24cm; Height in Real life of Frog; 72mm; Pig is 10 x larger

Height of Picture of Spider; 18mm; Height in Real life of Duck; 396mm

3a) A = 3135 square centimetres b) $? = 12\text{cm}$ c) $? = 5\text{cm}$

d) $? = 47.22\text{cm}$ e) A = 189 square centimetres

f) $? = 5.298\text{cm}$ or 5.30cm

4.

| | Visited Indonesia | Not Visited Indonesia | Total |
|----------------------|-------------------|-----------------------|-------|
| Visited Malaysia | 72 | 24 | 96 |
| Not Visited Malaysia | 195 | 5 | 200 |
| Total | 267 | 29 | 296 |

They are hoping to survey 296 people

REFLECTION ON LEARNING

SOLVE BY MULTIPLICATION (total missing) AND MENTAL STRATEGY

1. How many eggs in 25 dozen?; $25 \times 12 = ?$ by doubling halving is the same as $50 \times 6 = ?$ Therefore: $25 \times 12 = 300$; There are 300 eggs in 25 dozen.

4. 500 grams of chicken costs \$4.24. What is the price per kilogram?; 2 lots of 500g is 1kg. Therefore: $2 \times \$4.24 = ?$; Therefore: $2 \times \$4.24 = \8.48 ; One kilogram of chicken cost \$8.48/kg

5. Playing cards were shared between 4 people. Each person got 13 cards. How many playing cards were there?; $13 \times 4 = ?$ is the same as $13 \times 2 \times 2 = 52$; There are 52 cards in a pack.

SOLVE BY MULTIPLICATION (total missing) AND CALCULATOR

2. Sarah had \$127.36. Tia had four times more. How much did Tia have?; $\$127.36 \times 4 = \509.44 ; Tia has \$509.44.

SOLVE BY DIVISION AND MENTAL STRATEGY

3. Sophie travelled 48 km in 3 hours. What was her average speed?

$48 \div 3 = ?$; $30 \div 3 = 10$; $18 \div 3 = 6$; Therefore, $48 \div 3 = 16$; Sophie's average speed was 16km/h

6. 456 789 cents is how many dollars?; $456\,789 \div 100 = ?$; $? = \$4567.89$; 456 789 cents is \$4 567.89

SOLVE BY DIVISION AND CALCULATOR

7. The area of a rectangle is 5 678 square centimetres. If one of its sides is 23cm, what is the length of the other side?

$5678 \div 23 = ?$; $? = 246.87$; The length of the other side of the rectangle is 246.87cm

OLNA PRACTICE EXAMPLES

1. D. 2. A

Topic 8: Using Estimation for Solving Problems and Checking Reasonableness of Answers

PRACTICE EXERCISE 1

1. APPROXIMATE SOLUTION – (a), (b), (d) EXACT SOLUTION – (c), (e), (f)

PRACTICE EXERCISE 2

There are many solutions when approximating. Check with your teacher if your answers are different to those below

1a) 12 b) \$88 c) \$3 000 d) \$19 e) 20 000 f) \$4 800 000

2. Catalogue activity. Check with your classmates and teacher

a) We tend to round money amounts up to ensure that we have enough money to pay for the goods. In these multiplication problems, this is an overestimation **b) No rounding is necessary when multiplying by 10 as we can use the place value chart and patterning to find the exact answer.**

c) Answers will vary. Check with your teacher and classmates
d) Answers will vary. Check with your teacher and classmates
e) Answers will vary. Check with your teacher and classmates

3a) Rounded to $4 \times 4.5 = 18$ b) Rounded to $84 \times 100 = 8400$ cm
c) Rounded to $2\ 200 \div 1 = 2\ 200$ d) Rounded to $9 \times 4 = 36$ e)
Rounded to $40 \div 2 = 20$ f) Rounded to $1.5 \div 5 = 0.3$ or 30cm

4. a) Rounded to $120 \div 20 = 6$ b) Rounded to $18 \div 3 = 6$ pieces
c) Rounded to $80 \div 8 = 10$ intervals d) Rounded to $180\ 000 \div 6 = \$30\ 000$ e) Rounded to $14\ 000 \div 20 = 700$ rows f) Rounded to $240 \div 30 = 8$ classes

5a) Rounded to $0.5 \times \$22 = \11 b) Rounded to $14 \div 16 = 0.8$ or 0.9kg

PRACTICE EXERCISE 3

There are many solutions when approximating. Check with your teacher if your answers are different to those below

1a) 50 centimetres b) \$10 000 c) 30 seconds d) 0.5 ha; \$10 000

2. \$5.50, $3 \times \$3.50 = ?$, \$10.50.

\$14, $4 \times \$14 = ?$, \$56.

\$21, $\$21 \div 3 = ?$, \$7.

\$12, $\$12 \div 6 = ?$, \$2.

\$68 000, $2 \times \$68\ 000 = ?$, \$136 000.

\$990 000, $\$990\ 000 \div 2 = ?$, \$495 000.

PRACTICE EXERCISE 4

1a) Not reasonable. Doubling \$412.37 will result in \$824.74; \$84.74 is too small. b) Reasonable c) Not reasonable. Dividing by 100 decreases 1 200, not increases it. d) Not reasonable. Number appears correct but answer should be in \$ not kg e) Not reasonable. Dividing by a number less than one increases the size of the number f) Not reasonable. The number is right but not the dollars. The answer should just be 2.21

2a) 19.8 b) \$128.61 c) 152 d) 210cm e) \$121 f) \$100

3a) This is reasonable. Rounding down to $\$240 \div 4 = \60 .

b) This is not reasonable. Rounding one up and one down gives $40 \times 25 = 1\ 000$. This is much more than 605. c) This is not reasonable. The answer should be in dollars. d) This is reasonable. Dale did have enough as $5 \times \$6 = \30 e) This is not reasonable as our money system is in dollars and cents. Answer should be \$5.33 per ride. f) This is not reasonable. 82×77 is much bigger than 64.

REFLECTION ON LEARNING

APPROXIMATE; MENTAL; $150 \times 80 = ?$, Double, double, double $\times 10$. \$12 000. Yes. \$12 000 seems reasonable

EXACT; MENTAL; $2610 \div 3 = ?$, $2400 \div 3 = 800$, $210 \div 3 = 70$, THEREFORE $2610 \div 3 = \$870$. Yes. \$870 seems reasonable

EXACT; CALCULATOR; $2.87 \times 27 = ?$, $? = \$77.49$. Yes but he would probably be reimbursed \$77.50

APPROXIMATE; MENTAL; $4500 \div 150 = ?$, $4500 \div 150 = 30$. Yes \$30 seems reasonable

APPROXIMATE; MENTAL; $12 \times 4 = ?$, $? = 48$. Yes but Harry did not take enough money

OLNA PRACTICE QUESTION

1. D

Section 3 Measurement

Topic 1 Metric Relationships

PRACTICE EXERCISE 1

1. a) Five centimetres b) 15km
c) Eight hundred and ninety millimetres d) Fifty seven metres
e) 500km f) Seventy centimetres

2. a) 15cm, 5dm, 555mm, 5m b) 0.012m, 12cm, 21m, 2km
c) 2cm, 25mm, 0.2m, 22cm d) 1.4m, 4001mm, 40m, 104km

3. a) 400cm b) 0.05m c) 2.5cm d) 5000m e) 0.068km f) 0.0011km

4. a) 7500m b) 356cm c) 1.045km d) 602mm

5. b) 1m c) 1m 5cm d) 1m 7cm e) 600mm f) 800mm g) 1m 50mm
h) 1m 70mm

6. a) 600m b) 45cm c) 8 240mm

PRACTICE EXERCISE 2

1. a) Twelve millilitres b) 5L c) Zero point three litres
d) Two hundred and five kilolitres e) 450dL f) Ten decilitres

2. a) 3mL, 30mL, 0.3L, 30kL b) 12mL, 200mL, 2L, 20DL
c) 0.8L, 18dL, 80L, 8kL d) 7cL, 700mL, 7.7L, 70DL

3. a) 60dL b) 0.09L c) 3.6cL d) 15 000L e) 64 000mL f) 0.119kL

4. a) 5 500mL b) 434cL c) 3.14kL d) 522mL

5. b) 2L c) 2L 450mL d) 5 340kL e) 10 250L f) 1 050L

PRACTICE EXERCISE 3

1. a) Twenty five grams b) 3kg c) Zero point nine kilograms
d) Two thousand three hundred milligrams e) 98cL f) Twenty eight kilograms

2. a) 110mg, 11g, 11.1Dg 1kg, b) 500mg, 50g, 0.55kg, 5.5kg
c) 8dg, 8.8g, 800g, 80kg d) 4000mg, 41g, 4.1kg, 14kg

3. a) 0.05kg b) 6 000g c) 2.1g d) 780 000mg e) 0.8kg f) 0.09kg

4. a) 5250g b) 72 000mg c) 30.9g d) 23.075kg

5. a) 1500g b) 75g c) 25g d) 4 010mg e) 8 g 100mg f) 9g 99mg

PRACTICE EXERCISE 4

1. b) 1000, bigger c) 1000, smaller d) 100, smaller e) 1000, smaller
f) 1000, bigger

